RADIO ENGINEERING HANDBOOK

HENNEY

McGRAW-HILL BOOK COMPANY

THE RADIO ENGINEERING HANDBOOK

PREPARED BY A STAFF OF
TWENTY-TWO SPECIALISTS

KEITH HENNEY, Editor-in-Chief
Member, The Institute of Radio Engineers; Author, "Principles of Radio"; Associate Editor, "Electronics"

First Edition

McGRAW-HILL BOOK COMPANY, Inc.
NEW YORK AND LONDON
1933

PRINTED IN THE UNITED STATES OF AMERICA

THE MAPLE PRESS COMPANY, YORK, PA.

PREFACE

For several years the need for a handbook for radio engineers has been apparent. Although many of the fundamental principles of electrical engineering apply as well to radio, the whole task of designing, manufacturing, and operating equipment for radio communication is vastly different from that for electrical-power apparatus.

Radio engineering moves forward rapidly. New circuits, new tubes, new portions of the frequency spectrum, new applications of existing apparatus are explored annually. In fact, the developments are so extensive that a textbook can scarcely cover both theory and practice adequately without becoming hopelessly large. A handbook dealing more with practice than with theory is therefore essential.

In addition to the practical material, much of which appears in tables representing many man-hours of effort, there is an essential amount of fundamental discussion. The circuits described quantitatively are those in use today, or soon to be widely used, while description of the past art has been limited.

The twenty-odd engineers and physicists who contributed to this handbook were chosen because of their expert knowledge of a particular phase of the subject matter. In many cases the authors are daily engaged in the design, manufacture, or operation of the apparatus they describe here.

The editor's contribution is largely that of coordination and of the necessary, though laborious, work incidental to publishing. He wishes to express his gratitude to the authors for keeping the subject matter an up-to-date record of the rapidly changing art. Although his name does not appear among the list of authors, Mr. Howard E. Rhodes has been of assistance in originally laying out the contents and in much later consultation.

KEITH HENNEY.

NEW YORK, N. Y.,
October, 1932.

CONTENTS

PAGE

PREFACE v

SECTION 1

MATHEMATICAL AND ELECTRICAL TABLES. 1
Greek alphabet—Trigonometric functions—Table of wave lengths and *LC* products—Copper-wire tables—Decibel chart—Graphic symbols—International communication bands.

SECTION 2

ELECTRIC AND MAGNETIC CIRCUITS, *by* E. A. UEHLING 20
Fundamentals of electric circuits—Ohm's law—Continuous and alternating currents—Direct-current circuits—Alternating-current circuits—Magnetic circuits—Radiation.

SECTION 3

RESISTANCE, *by* JESSE MARSTEN. 40
General concepts—Properties of materials as conductors—Resistance as a function of frequency—Types of resistors—Composition of resistors.

SECTION 4

INDUCTANCE, *by* GOMER L. DAVIES. 51
Magnetic flux—Measurement of inductance—Inductors at radio frequencies—Types of inductors—Design of inductance coils—Calculation of inductance—Inductance standards—Mutual inductance.

SECTION 5

CAPACITY, *by* E. L. HALL 79
Units of capacity—Dielectric materials—Power factor of insulating materials—Calculation of capacity—Types of condensers—Design of variable air condensers—Methods of measuring capacity.

SECTION 6

COMBINED CIRCUITS OF *L*, *C*, AND *R*, *by* W. F. LANTERMAN 104
General impedances—Equivalent impedance—Resonance—Coupled circuits—Recurrent networks—Artificial lines—Wave filters.

SECTION 7

MEASURING INSTRUMENTS, *by* R. F. FIELD 145
Current measuring instruments—Moving diaphragm meters—Hot-wire meters—Thermocouple meters—Rectifier meters—

PAGE

Voltage measuring instruments—Cathode-ray meters—Measurement of frequency, capacity, resistance—Standards—Comparison measurement—Direct-current bridge measurements—Alternating-current bridge measurements.

SECTION 8

VACUUM TUBES, *by* J. M. STINCHFIELD 182
Electrons—Space charge—Thermionic emission—Filament calculations—Two-electrode tubes—Three-electrode tubes—Calculation of space current—Calculation of amplification factor—Screen-grid tetrode—Power-output pentode—Radio-frequency pentode—Interelectrode capacity—Multi-element, varied-purpose tubes.

SECTION 9

OSCILLATING CIRCUITS, *by* D. C. PRINCE 221
Hartley circuit—Oscillator circuit design—Oscillator calculations—Parasitic oscillations—Short-wave circuits—Use of tubes in parallel and push pull—Magnetron oscillator—Dynatron oscillator.

SECTION 10

DETECTION AND MODULATION, *by* K. W. JARVIS 244
Detection characteristic—Triode as detector—Plate detection—Grid-current detection—High-amplitude detectors—Heterodyne detector—Demodulation of one signal by another—Translation ratio—Modulation—Analysis of modulation—Heising modulation—Frequency inversion; scrambling—Measuring modulation—Band-width necessary—Frequency modulation.

SECTION 11

AUDIO-FREQUENCY AMPLIFIERS, *by* JULIUS G. ACEVES. 281
Methods of coupling—Coupling circuits—Compensating Circuits—Galvanic-coupled stages—Push-pull amplifiers—Class B amplifiers.

SECTION 12

RADIO-FREQUENCY AMPLIFIERS, *by* R. S. GLASGOW 300
Class A amplification—Impedance-coupled amplifier—Transformer-coupled amplifiers—Cascade amplifiers—Band-pass filters—Neutralizing circuits—Class B amplifiers—Class C amplifiers—Frequency multipliers.

SECTION 13

RECEIVING SYSTEMS, *by* G. L. BEERS 322
Types of receiving circuits—Methods of testing and rating—Design of receiving systems—Neutralization—Tuned radio-frequency amplifier—Intermediate-frequency amplifiers—Frequency converters—Power detectors—Audio-frequency amplifiers—Tone control—Volume control—Automatic volume control—Power-supply systems—Superheterodyne receivers—Superregenerator receivers—Direction finders—Television receivers.

PAGE

SECTION 14

BROADCASTING, by C. W. HORN 354
Volume range—Microphones—Speech-input amplifiers—Wire lines—Harmonics in transmitters—Power supply.

SECTION 15

RECTIFIERS AND POWER-SUPPLY SYSTEMS, by R. C. HITCHCOCK . . . 374
Power-supply design—Voltage divider—Voltage regulation—Filters—Filter choke design—Rectifiers—Small transformer design.

SECTION 16

LOUD-SPEAKERS AND ACOUSTICS, by IRVING WOLFF. 403
Sound waves in air—Acoustical impedance—Acoustics of rooms—Absorption coefficients—Characteristics of the ear—Loud-speakers—Calculation of loud-speaker efficiency—Loud-speaker motor—Loud-speaker radiator—Horns—Acoustic measurements.

SECTION 17

HIGH-FREQUENCY TRANSMISSION AND RECEPTION, by A. HOYT TAYLOR. 433
Peculiarities of high frequencies—Kennelly-Heaviside layer—Fading—Theory of high-frequency wave propagation—Ultra-high frequencies—Skip-distance and range tables—Band-width required—Allocation of frequencies—Magnetron oscillator—Barkhausen oscillator.

SECTION 18

CODE TRANSMISSION AND RECEPTION, by JOHN B. MOORE. 453
Codes—Frequency range required—Speeds attainable—Radio-telegraphic services—Transmitting systems and equipment—Arc transmitters—Antenna systems—Short-wave technique—Receiving systems and equipment—Control methods and equipment—Transcribing methods and equipment.

SECTION 19

TELEVISION, by C. H. W. NASON. 477
The television problem—Scanning systems—Transmission methods—Propagation phenomena—Light valves—Synchronism.

SECTION 20

FACSIMILE TRANSMISSION, by R. H. RANGER. 489
General requirements—Scanning—Photocell amplifiers—Dot-dash system—Carrier systems—Light valves—Recording methods—Synchronizing methods—Systems in use.

SECTION 21

AIRCRAFT RADIO, by HARRY DIAMOND 511
Government weather service—Ground-station equipment—Airplane equipment—Power requirements—Aircraft power

PAGE
equipment—Airplane receiving and transmitting antennas—Radio shielding and bonding—Course navigation—Position determination—Beacon systems—Radio aids to blind landing—Landing beam—Absolute altimeters.

SECTION 22

PHOTOCELLS, *by* H. C. RENTSCHLER 543
Photoelectricity—Color sensitivity—Vacuum and gas phototubes—Photo-conductivity—Selenium cells—Photovoltaic effect.

SECTION 23

SOUND MOTION PICTURES, *by* FRANKLIN S. IRBY 554
Methods of recording—Microphones—Amplifiers—RCA Photophone system—Western Electric system—Glowlamp recorder—Portable sound equipment—Motor systems—Sound-film processing—Disk recording—Sound-film standards—Theater reproducing equipment.

INDEX 575

THE RADIO ENGINEERING HANDBOOK

SECTION 1

MATHEMATICAL AND ELECTRICAL TABLES

1. Greek Alphabet.

Name	Letters		Commonly used to designate
	Cap.	Small	
Alpha	A	α	Angles. Coefficients. Area
Beta	B	β	Angles. Coefficients
Gamma	Γ	γ	Specific gravity. Conductivity
Delta	Δ	δ	Decrements. Variation. Density
Epsilon	E	ε	E.m.f. Base of hyperbolic logarithms
Zeta	Z	ζ	Impedance. Coordinates
Eta	H	η	Hysteresis coefficient. Efficiency
Theta	Θ	θϑ	Angular phase displacement. Time constant
Iota	I	ι	Current in amperes
Kappa	K	κ	Dielectric constant. Susceptibility. Kilo. Visibility
Lambda	Λ	λ	(Small) Wave length
Mu	M	μ	Permeability. Amplification factor. Prefix micro-
Nu	N	ν	Reluctivity
Xi	Ξ	ξ	
Omicron	O	ο	
Pi	Π	π	Circumference divided by diameter 3.1416
Rho	P	ρ	Resistivity
Sigma	Σ	σς	(Cap) Sign of summation
Tau	T	τ	Time constant. Time-phase displacement
Upsilon	Υ	υ	
Phi	Φ	φϕ	Flux. Angle of lag or lead
Chi	X	χ	Reactance
Psi	Ψ	ψ	Angular velocity in time. Phase difference. Dielectric flux
Omega	Ω	ω	Resistance in ohms. Resistance in megohms. $2\pi F$. Angular velocity

2. Decimal Equivalents of Parts of One Inch.

1/64	0.015625	17/64	0.265625	33/64	0.515625	49/64	0.765625
1/32	0.031250	9/32	0.281250	17/32	0.531250	25/32	0.781250
3/64	0.046875	19/64	0.296875	35/64	0.546875	51/64	0.796875
1/16	0.062500	5/16	0.312500	9/16	0.562500	13/16	0.812500
5/64	0.078125	21/64	0.328125	37/64	0.578125	53/64	0.828125
3/32	0.093750	11/32	0.343750	19/32	0.593750	27/32	0.843750
7/64	0.109375	23/64	0.359375	39/64	0.609375	55/64	0.859375
1/8	0.125000	3/8	0.375000	5/8	0.625000	7/8	0.875000
9/64	0.140625	25/64	0.390625	41/64	0.640625	57/64	0.890625
5/32	0.156250	13/32	0.406250	21/32	0.656250	29/32	0.906250
11/64	0.171875	27/64	0.421875	43/64	0.671875	59/64	0.921875
3/16	0.187500	7/16	0.437500	11/16	0.687500	15/16	0.937500
13/64	0.203125	29/64	0.453125	45/64	0.703125	61/64	0.953125
7/32	0.218750	15/32	0.468750	23/32	0.718750	31/32	0.968750
15/64	0.234375	31/64	0.484375	47/64	0.734375	63/64	0.984375
1/4	0.250000	1/2	0.500000	3/4	0.750000	1	1

3. Trigonometric Functions.

° ′	sin	tan	cot	cos		° ′	sin	tan	cot	cos	
0 0	0.0000	0.0000	infinit.	1.0000	0 90	8 0	0.1392	0.1405	7.1154	0.9903	0 82
10	0.0029	0.0029	343.7737	1.0000	50	10	0.1421	0.1435	6.9682	0.9899	50
20	0.0058	0.0058	171.8854	1.0000	40	20	0.1449	0.1465	6.8269	0.9894	40
30	0.0087	0.0087	114.5887	1.0000	30	30	0.1478	0.1495	6.6912	0.9890	30
40	0.0116	0.0116	85.9398	0.9999	20	40	0.1507	0.1524	6.5606	0.9886	20
50	0.0145	0.0145	68.7501	0.9999	10	50	0.1536	0.1554	6.4348	0.9881	10
1 0	0.0175	0.0175	57.2900	0.9998	0 89	9 0	0.1564	0.1584	6.3138	0.9877	0 81
10	0.0204	0.0204	49.1039	0.9998	50	10	0.1593	0.1614	6.1970	0.9872	50
20	0.0233	0.0233	42.9641	0.9997	40	20	0.1622	0.1644	6.0844	0.9868	40
30	0.0262	0.0262	38.1885	0.9997	30	30	0.1650	0.1673	5.9758	0.9863	30
40	0.0291	0.0291	34.3678	0.9996	20	40	0.1679	0.1703	5.8708	0.9858	20
50	0.0320	0.0320	31.2416	0.9995	10	50	0.1708	0.1733	5.7694	0.9853	10
2 0	0.0349	0.0349	28.6363	0.9994	0 88	10 0	0.1736	0.1763	5.6713	0.9848	0 80
10	0.0378	0.0378	26.4316	0.9993	50	10	0.1765	0.1793	5.5764	0.9843	50
20	0.0407	0.0407	24.5418	0.9992	40	20	0.1794	0.1823	5.4845	0.9838	40
30	0.0436	0.0437	22.9038	0.9990	30	30	0.1822	0.1853	5.3955	0.9833	30
40	0.0465	0.0466	21.4704	0.9989	20	40	0.1851	0.1883	5.3093	0.9827	20
50	0.0494	0.0495	20.2056	0.9988	10	50	0.1880	0.1914	5.2257	0.9822	10
3 0	0.0523	0.0524	19.0811	0.9986	0 87	11 0	0.1908	0.1944	5.1446	0.9816	0 79
10	0.0552	0.0553	18.0750	0.9985	50	10	0.1937	0.1974	5.0658	0.9811	50
20	0.0581	0.0582	17.1693	0.9983	40	20	0.1965	0.2004	4.9894	0.9805	40
30	0.0610	0.0612	16.3499	0.9981	30	30	0.1994	0.2035	4.9152	0.9799	30
40	0.0640	0.0641	15.6048	0.9980	20	40	0.2022	0.2065	4.8430	0.9793	20
50	0.0669	0.0670	14.9244	0.9978	10	50	0.2051	0.2095	4.7729	0.9787	10
4 0	0.0698	0.0699	14.3007	0.9976	0 86	12 0	0.2079	0.2126	4.7046	0.9781	0 78
10	0.0727	0.0729	13.7267	0.9974	50	10	0.2108	0.2156	4.6382	0.9775	50
20	0.0756	0.0758	13.1969	0.9971	40	20	0.2136	0.2186	4.5736	0.9769	40
30	0.0785	0.0787	12.7062	0.9969	30	30	0.2164	0.2217	4.5107	0.9763	30
40	0.0814	0.0816	12.2505	0.9967	20	40	0.2193	0.2247	4.4494	0.9757	20
50	0.0843	0.0846	11.8262	0.9964	10	50	0.2221	0.2278	4.3897	0.9750	10
5 0	0.0872	0.0875	11.4301	0.9962	0 85	13 0	0.2250	0.2309	4.3315	0.9744	0 77
10	0.0901	0.0904	11.0594	0.9959	50	10	0.2278	0.2339	4.2747	0.9737	50
20	0.0929	0.0934	10.7119	0.9957	40	20	0.2306	0.2370	4.2193	0.9730	40
30	0.0958	0.0963	10.3854	0.9954	30	30	0.2334	0.2401	4.1653	0.9724	30
40	0.0987	0.0992	10.0780	0.9951	20	40	0.2363	0.2432	4.1126	0.9717	20
50	0.1016	0.1022	9.7882	0.9948	10	50	0.2391	0.2462	4.0611	0.9710	10
6 0	0.1045	0.1051	9.5144	0.9945	0 84	14 0	0.2419	0.2493	4.0108	0.9703	0 76
10	0.1074	0.1080	9.2553	0.9942	50	10	0.2447	0.2524	3.9617	0.9696	50
20	0.1103	0.1110	9.0098	0.9939	40	20	0.2476	0.2555	3.9136	0.9689	40
30	0.1132	0.1139	8.7769	0.9936	30	30	0.2504	0.2586	3.8667	0.9681	30
40	0.1161	0.1169	8.5555	0.9932	20	40	0.2532	0.2617	3.8208	0.9674	20
50	0.1190	0.1198	8.3450	0.9929	10	50	0.2560	0.2648	3.7760	0.9667	10
7 0	0.1219	0.1228	8.1443	0.9925	0 83	15 0	0.2588	0.2679	3.7321	0.9659	0 75
10	0.1248	0.1257	7.9530	0.9922	50	10	0.2616	0.2711	3.6891	0.9652	50
20	0.1276	0.1287	7.7704	0.9918	40	20	0.2644	0.2742	3.6470	0.9644	40
30	0.1305	0.1317	7.5958	0.9914	30	30	0.2672	0.2773	3.6059	0.9636	30
40	0.1334	0.1346	7.4287	0.9911	20	40	0.2700	0.2805	3.5656	0.9628	20
50	0.1383	0.1376	7.2687	0.9907	10	50	0.2728	0.2836	3.5261	0.9621	10
8 0	0.1392	0.1405	7.1154	0.9903	0 82	16 0	0.2756	0.2867	3.4874	0.9613	0 74
	cos	cot	tan	sin	′ °		cos	cot	tan	sin	′ °

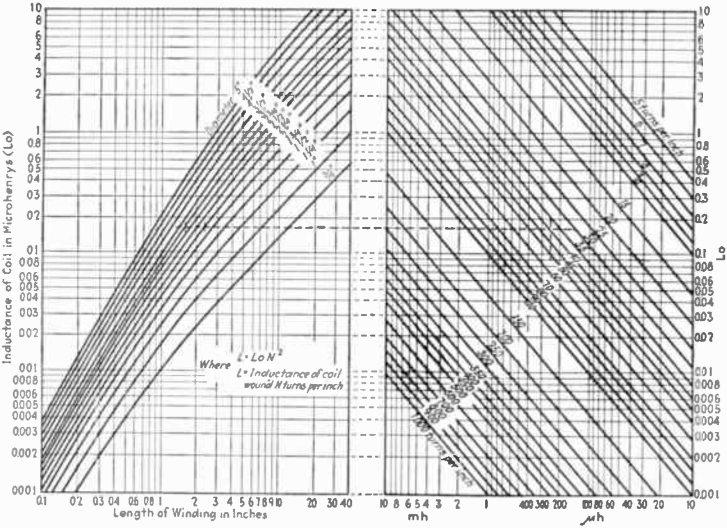
° ′	sin	tan	cot	cos	
16 0	0.2756	0.2867	3.4874	0.9613	0 74
10	0.2784	0.2899	3.4495	0.9605	50
20	0.2812	0.2931	3.4124	0.9596	40
30	0.2840	0.2962	3.3759	0.9588	30
40	0.2868	0.2994	3.3402	0.9580	20
50	0.2896	0.3026	3.3052	0.9572	10
17 0	0.2924	0.3057	3.2709	0.9563	0 73
10	0.2952	0.3089	3.2371	0.9555	50
20	0.2979	0.3121	3.2041	0.9546	40
30	0.3007	0.3153	3.1716	0.9537	30
40	0.3035	0.3185	3.1397	0.9528	20
50	0.3062	0.3217	3.1084	0.9520	10
18 0	0.3090	0.3249	3.0777	0.9511	0 72
10	0.3118	0.3281	3.0475	0.9502	50
20	0.3145	0.3314	3.0178	0.9492	40
30	0.3173	0.3346	2.9887	0.9483	30
40	0.3201	0.3378	2.9600	0.9474	20
50	0.3228	0.3411	2.9319	0.9465	10
19 0	0.3256	0.3443	2.9042	0.9455	0 71
10	0.3283	0.3476	2.8770	0.9446	50
20	0.3311	0.3508	2.8502	0.9436	40
30	0.3338	0.3541	2.8239	0.9426	30
40	0.3365	0.3574	2.7980	0.9417	20
50	0.3393	0.3607	2.7725	0.9407	10
20 0	0.3420	0.3640	2.7475	0.9397	0 70
10	0.3448	0.3673	2.7228	0.9387	50
20	0.3475	0.3706	2.6985	0.9377	40
30	0.3502	0.3739	2.6746	0.9367	30
40	0.3529	0.3772	2.6511	0.9356	20
50	0.3557	0.3805	2.6279	0.9346	10
21 0	0.3584	0.3839	2.6051	0.9336	0 69
10	0.3611	0.3872	2.5826	0.9325	50
20	0.3638	0.3906	2.5605	0.9315	40
30	0.3665	0.3939	2.5386	0.9304	30
40	0.3692	0.3973	2.5172	0.9293	20
50	0.3719	0.4006	2.4960	0.9283	10
22 0	0.3746	0.4040	2.4751	0.9272	0 68
10	0.3773	0.4074	2.4545	0.9261	50
20	0.3800	0.4108	2.4342	0.9250	40
30	0.3827	0.4142	2.4142	0.9239	30
40	0.3854	0.4176	2.3945	0.9228	20
50	0.3881	0.4210	2.3750	0.9216	10
23 0	0.3907	0.4245	2.3559	0.9205	0 67
10	0.3934	0.4279	2.3369	0.9194	50
20	0.3961	0.4314	2.3183	0.9182	40
30	0.3987	0.4348	2.2998	0.9171	30
40	0.4014	0.4383	2.2817	0.9159	20
50	0.4041	0.4417	2.2637	0.9147	10
24 0	0.4067	0.4452	2.2460	0.9135	0 66
	cos	cot	tan	sin	′ °

° ′	sin	tan	cot	cos	
24 0	0.4067	0.4452	2.2460	0.9135	0 66
10	0.4094	0.4487	2.2286	0.9124	50
20	0.4120	0.4522	2.2113	0.9112	40
30	0.4147	0.4557	2.1943	0.9100	30
40	0.4173	0.4592	2.1775	0.9088	20
50	0.4200	0.4628	2.1609	0.9075	10
25 0	0.4226	0.4663	2.1445	0.9063	0 65
10	0.4253	0.4699	2.1283	0.9051	50
20	0.4279	0.4734	2.1123	0.9038	40
30	0.4305	0.4770	2.0965	0.9026	30
40	0.4331	0.4806	2.0809	0.9013	20
50	0.4358	0.4841	2.0655	0.9001	10
26 0	0.4384	0.4877	2.0503	0.8988	0 64
10	0.4410	0.4913	2.0353	0.8975	50
20	0.4436	0.4950	2.0204	0.8962	40
30	0.4462	0.4986	2.0057	0.8949	30
40	0.4488	0.5022	1.9912	0.8936	20
50	0.4514	0.5059	1.9768	0.8923	10
27 0	0.4540	0.5095	1.9626	0.8910	0 63
10	0.4566	0.5132	1.9486	0.8897	50
20	0.4592	0.5169	1.9347	0.8884	40
30	0.4617	0.5206	1.9210	0.8870	30
40	0.4643	0.5243	1.9074	0.8857	20
50	0.4669	0.5280	1.8940	0.8843	10
28 0	0.4695	0.5317	1.8807	0.8829	0 62
10	0.4720	0.5354	1.8676	0.8816	50
20	0.4746	0.5392	1.8546	0.8802	40
30	0.4772	0.5430	1.8418	0.8788	30
40	0.4797	0.5467	1.8291	0.8774	20
50	0.4823	0.5505	1.8165	0.8760	10
29 0	0.4848	0.5543	1.8040	0.8746	0 61
10	0.4874	0.5581	1.7917	0.8732	50
20	0.4899	0.5619	1.7796	0.8718	40
30	0.4924	0.5658	1.7675	0.8704	30
40	0.4950	0.5696	1.7556	0.8689	20
50	0.4975	0.5735	1.7437	0.8675	10
30 0	0.5000	0.5774	1.7321	0.8660	0 60
10	0.5025	0.5812	1.7205	0.8646	50
20	0.5050	0.5851	1.7090	0.8631	40
30	0.5075	0.5890	1.6977	0.8616	30
40	0.5100	0.5930	1.6864	0.8601	20
50	0.5125	0.5969	1.6753	0.8587	10
31 0	0.5150	0.6009	1.6643	0.8572	0 59
10	0.5175	0.6048	1.6534	0.8557	50
20	0.5200	0.6088	1.6426	0.8542	40
30	0.5225	0.6128	1.6319	0.8526	30
40	0.5250	0.6168	1.6212	0.8511	20
50	0.5275	0.6208	1.6107	0.8496	10
32 0	0.5299	0.6249	1.6003	0.8480	0 58
	cos	cot	tan	sin	′ °

° ′	sin	tan	cot	cos	
32 0	0.5299	0.6249	1.6003	0.8480	0 58
10	0.5324	0.6289	1.5900	0.8465	50
20	0.5348	0.6330	1.5798	0.8450	40
30	0.5373	0.6371	1.5697	0.8434	30
40	0.5398	0.6412	1.5597	0.8418	20
50	0.5422	0.6453	1.5497	0.8403	10
33 0	0.5446	0.6494	1.5399	0.8387	0 57
10	0.5471	0.6536	1.5301	0.8371	50
20	0.5495	0.6577	1.5204	0.8355	40
30	0.5519	0.6619	1.5108	0.8339	30
40	0.5544	0.6661	1.5013	0.8323	20
50	0.5568	0.6703	1.4919	0.8307	10
34 0	0.5592	0.6745	1.4826	0.8290	0 56
10	0.5616	0.6787	1.4733	0.8274	50
20	0.5640	0.6830	1.4641	0.8258	40
30	0.5664	0.6873	1.4550	0.8241	30
40	0.5688	0.6916	1.4460	0.8225	20
50	0.5712	0.6959	1.4370	0.8208	10
35 0	0.5736	0.7002	1.4281	0.8192	0 55
10	0.5760	0.7046	1.4193	0.8175	50
20	0.5783	0.7089	1.4106	0.8158	40
30	0.5807	0.7133	1.4019	0.8141	30
40	0.5831	0.7177	1.3934	0.8124	20
50	0.5854	0.7221	1.3848	0.8107	10
36 0	0.5878	0.7265	1.3764	0.8090	0 54
10	0.5901	0.7310	1.3680	0.8073	50
20	0.5925	0.7355	1.3597	0.8056	40
30	0.5948	0.7400	1.3514	0.8039	30
40	0.5972	0.7445	1.3432	0.8021	20
50	0.5995	0.7490	1.3351	0.8004	10
37 0	0.6018	0.7536	1.3270	0.7986	0 53
10	0.6041	0.7581	1.3190	0.7969	50
20	0.6065	0.7627	1.3111	0.7951	40
30	0.6088	0.7673	1.3032	0.7934	30
40	0.6111	0.7720	1.2954	0.7916	20
50	0.6134	0.7766	1.2876	0.7898	10
38 0	0.6157	0.7813	1.2799	0.7880	0 52
10	0.6180	0.7860	1.2723	0.7862	50
20	0.6202	0.7907	1.2647	0.7844	40
30	0.6225	0.7954	1.2572	0.7826	30
40	0.6248	0.8002	1.2497	0.7808	20
50	0.6271	0.8050	1.2423	0.7790	10
39 0	0.6293	0.8098	1.2349	0.7771	0 51
	cos	cot	tan	sin	′ °

° ′	sin	tan	cot	cos	
39 0	0.6293	0.8098	1.2349	0.7771	0 51
10	0.6316	0.8146	1.2276	0.7753	50
20	0.6338	0.8195	1.2203	0.7735	40
30	0.6361	0.8243	1.2131	0.7716	30
40	0.6383	0.8292	1.2059	0.7698	20
50	0.6406	0.8342	1.1988	0.7679	10
40 0	0.6428	0.8391	1.1918	0.7660	0 50
10	0.6450	0.8441	1.1847	0.7642	50
20	0.6472	0.8491	1.1778	0.7623	40
30	0.6494	0.8541	1.1708	0.7604	30
40	0.6517	0.8591	1.1640	0.7585	20
50	0.6539	0.8642	1.1571	0.7566	10
41 0	0.6561	0.8693	1.1504	0.7547	0 49
10	0.6583	0.8744	1.1436	0.7528	50
20	0.6604	0.8796	1.1369	0.7509	40
30	0.6626	0.8847	1.1303	0.7490	30
40	0.6648	0.8899	1.1237	0.7470	20
50	0.6670	0.8952	1.1171	0.7451	10
42 0	0.6691	0.9004	1.1106	0.7431	0 48
10	0.6713	0.9057	1.1041	0.7412	50
20	0.6734	0.9110	1.0977	0.7392	40
30	0.6756	0.9163	1.0913	0.7373	30
40	0.6777	0.9217	1.0850	0.7353	20
50	0.6799	0.9271	1.0786	0.7333	10
43 0	0.6820	0.9325	1.0724	0.7314	0 47
10	0.6841	0.9380	1.0661	0.7294	50
20	0.6862	0.9435	1.0599	0.7274	40
30	0.6884	0.9490	1.0538	0.7254	30
40	0.6905	0.9545	1.0477	0.7234	20
50	0.6926	0.9601	1.0416	0.7214	10
44 0	0.6947	0.9657	1.0355	0.7193	0 46
10	0.6967	0.9713	1.0295	0.7173	50
20	0.6988	0.9770	1.0235	0.7153	40
30	0.7009	0.9827	1.0176	0.7133	30
40	0.7030	0.9884	1.0117	0.7112	20
50	0.7050	0.9942	1.0058	0.7092	10
45 0	0.7071	1.0000	1.0000	0.7071	0 45
	cos	cot	tan	sin	′ °

4. Inductance of Various Windings.



5. Table of Circuit Constants.

Values of ω, $1/\omega$, inductive and capacitive reactance, wave length, and LC products for frequencies from 10 cycles to 100 mc.

The following table, in conjunction with the multiplying factors given below, gives the values of frequently used circuit constants, for any frequency between 10 cycles and 100 mc:

Multiplying Factors

For frequencies between	Mult. ω by	Mult. $1/\omega$ by	Mult. λ (wave length) by	Mult. LC by
10.5 cycles and 100 cycles	1.0	10^{-4}	10^{5}	10^{-6}
105 cycles and 1,000 cycles	10.0	10^{-5}	10^{4}	10^{-8}
1,050 cycles and 10,000 cycles	10^{2}	10^{-6}	10^{3}	10^{-10}
10.5 kc and 100 kc	10^{3}	10^{-7}	10^{2}	10^{-12}
105 kc and 1,000 kc	10^{4}	10^{-8}	10^{1}	10^{-14}
1,050 kc and 10,000 kc	10^{5}	10^{-9}	1.0	10^{-16}
10.5 mc and 100 mc	10^{6}	10^{-10}	0.1	10^{-18}

Inductive Reactance. To obtain the inductive reactance of an inductance of L henrys at any frequency:

a. Apply the proper multiplying factor to column 2.
b. Multiply by L, the number of henrys.

Capacitive Reactance. To obtain the capacitive reactance of a condenser of C μf at any frequency:

a. Apply the proper multiplying factor to column 3.
b. Divide the result by C, the number of microfarads.
c. Multiply by 10^6.

If C is in micromicrofarads instead of microfarads, multiply by 10^{12} instead of 10^6.

Frequency	$\omega = 2\pi f$ or $X_L = \omega L$	$1/\omega = 1/2\pi f$ or $X_c = 1/\omega c$	λ Wave length	LC
105	65.974	151.57	285.71	229.75
110	69.115	144.79	272.73	209.34
115	72.257	138.49	260.87	191.52
120	75.398	132.63	250.00	175.90
125	78.540	127.33	240.00	162.18
130	81.682	122.43	230.77	149.88
135	84.823	117.89	222.22	138.99
140	87.965	113.68	214.28	129.23
145	91.106	109.76	206.90	120.48
150	94.248	106.10	200.00	112.58
155	97.389	102.60	193.55	105.44
160	100.53	99.472	187.50	98.945
165	103.67	96.459	181.82	93.040
170	106.81	93.624	176.47	87.646
175	109.96	90.983	171.43	82.708
180	113.10	88.418	166.67	78.179
185	116.24	86.030	162.16	74.011
190	119.38	83.766	157.90	70.167
195	122.52	81.618	153.85	66.615
200	125.66	79.562	150.00	63.325
205	128.81	77.633	146.35	60.274
210	131.95	75.785	142.85	57.637
215	135.09	74.024	139.54	54.796
220	138.23	72.395	136.36	52.335
225	141.37	70.736	133.33	50.035
230	144.51	69.245	130.43	47.880
235	147.65	67.727	127.66	45.866
240	150.80	66.315	125.00	43.975
245	153.94	64.959	122.45	42.198
250	157.08	63.665	120.00	40.545
255	160.22	62.415	117.65	38.954
260	163.36	61.215	115.38	37.470
265	166.50	60.060	113.20	36.068
270	169.65	58.995	111.11	34.747
275	172.89	57.841	109.09	33.494
280	175.93	56.840	107.14	32.307
285	179.07	55.844	105.26	31.185
290	182.21	54.880	103.45	30.120
295	185.35	53.952	101.70	29.107
300	188.47	53.050	100.00	28.145
305	191.64	52.181	98.36	27.229
310	194.78	51.300	96.77	26.360
315	197.92	50.525	95.238	25.528
320	201.06	49.736	93.700	24.736
325	204.20	48.977	92.308	23.981
330	207.35	48.229	90.910	23.260
335	210.49	47.508	89.559	22.571
340	213.63	46.812	88.245	21.911
345	216.77	46.132	86.956	21.281
350	219.91	45.491	85.715	20.677
355	223.05	44.833	84.390	20.099
360	225.20	44.209	83.335	19.565
365	229.34	43.602	82.192	19.013
370	232.48	43.015	81.080	18.503
375	235.62	42.440	80.000	18.013

Frequency	$\omega = 2\pi f$ or $X_L = \omega L$	$1/\omega = 1/2\pi f$ or $X_c = 1/\omega c$	λ Wave length	LC
380	238.76	41.883	78.950	17.542
385	241.90	41.339	77.922	17.089
390	245.04	40.809	76.975	16.654
395	248.19	40.293	75.948	16.234
400	251.33	39.781	75.000	15.831
405	254.47	39.298	74.073	15.442
410	257.61	38.816	73.175	15.068
415	260.75	38.355	72.288	14.707
420	263.89	37.892	71.425	14.409
425	267.04	37.448	70.588	14.023
430	270.18	37.012	69.770	13.699
435	273.32	36.587	68.965	13.386
440	276.46	36.197	68.180	13.084
445	279.60	35.764	67.416	12.788
450	282.74	35.368	66.666	12.509
455	285.89	34.980	65.934	12.238
460	288.03	34.622	65.215	11.970
465	292.17	34.227	64.516	11.715
470	295.31	33.863	63.830	11.466
475	298.45	33.505	63.161	11.227
480	301.59	33.157	62.500	10.994
485	304.74	32.815	61.856	10.768
490	307.88	32.479	61.225	10.549
495	311.02	32.152	60.604	10.337
500	314.16	31.832	60.000	10.136
505	317.30	31.516	59.406	9.9322
510	320.44	31.207	58.825	9.7380
515	323.59	30.903	58.251	9.5524
520	326.73	30.607	57.690	9.3675
525	329.87	30.317	57.142	9.1898
530	333.01	30.030	56.600	9.0170
535	336.15	29.748	56.075	8.8498
540	339.29	29.497	55.555	8.6867
545	342.43	29.203	55.045	8.5276
550	345.58	28.920	54.545	8.3735
555	348.72	28.676	54.054	8.2234
560	350.86	28.420	53.570	8.0767
565	355.00	28.169	53.097	7.9348
570	358.14	27.922	52.630	7.7962
575	361.28	27.679	52.174	7.6610
580	364.43	27.440	51.725	7.5296
585	367.57	27.207	51.280	7.4013
590	370.71	26.976	50.850	7.2767
595	373.85	26.749	50.420	7.1547
600	376.99	26.525	50.000	7.0362
605	380.13	26.308	49.586	6.9200
610	383.28	26.090	49.180	6.8072
615	386.42	25.878	48.780	6.6968
620	389.56	25.650	48.385	6.5900
625	392.70	25.468	48.000	6.4844
630	395.84	25.262	47.619	6.3820
635	398.98	25.063	47.244	6.2819
640	402.12	24.868	46.850	6.1840
645	405.27	24.674	46.511	6.0885
650	408.41	24.488	46.154	5.9952

Frequency	$\omega = 2\pi f$ or $X_L = \omega L$	$1/\omega = 1/2\pi f$ or $X_c = 1/\omega c$	λ Wave length	LC
655	411.55	24.298	45.801	5.9040
660	413.69	24.114	45.455	5.8150
665	417.83	23.933	45.113	5.7279
670	420.97	23.754	44.779	5.6425
675	424.12	23.578	44.445	5.5466
680	427.26	23.406	44.122	5.4777
685	430.39	23.238	43.796	5.3982
690	433.54	23.066	43.478	5.3202
695	436.68	22.900	43.166	5.2441
700	439.82	22.745	42.857	5.1492
705	442.97	22.575	42.553	5.0962
710	446.11	22.416	42.195	5.0247
715	449.25	22.259	41.957	4.9546
720	452.39	22.104	41.667	4.8912
725	455.53	21.953	41.379	4.8189
730	458.67	21.801	41.096	4.7532
735	461.82	21.655	40.817	4.6887
740	464.96	21.507	40.540	4.6257
745	468.10	21.363	40.268	4.5636
750	471.24	21.220	40.000	4.5032
755	474.38	21.080	39.735	4.4436
760	476.52	20.941	39.475	4.3855
765	480.67	20.804	39.215	4.3282
770	483.81	20.669	38.961	4.2722
775	486.95	20.536	38.710	4.2173
780	490.09	20.404	38.487	4.1635
785	493.23	20.275	38.216	4.1105
790	496.37	20.146	37.974	4.0585
795	499.51	20.019	37.735	4.0076
800	502.66	19.891	37.500	3.9577
805	505.80	19.770	37.267	3.9087
810	508.94	19.649	37.036	3.8605
815	512.08	19.528	36.810	3.8134
820	515.22	19.408	36.587	3.7670
825	518.36	19.292	36.364	3.7216
830	521.51	19.177	36.144	3.6767
835	524.65	19.060	35.927	3.6337
840	527.79	18.946	35.712	3.6022
845	530.93	18.835	35.502	3.5474
850	534.07	18.724	35.294	3.5062
855	537.21	18.614	35.087	3.4657
860	539.36	18.506	34.885	3.4242
865	543.50	18.399	34.682	3.3852
870	546.64	18.293	34.487	3.3465
875	549.78	18.189	34.285	3.3082
880	552.92	18.098	34.090	3.2710
885	556.06	17.988	33.898	3.2341
890	558.92	17.882	33.708	3.1970
895	562.35	17.783	33.520	3.1622
900	565.49	17.689	33.333	3.1272
905	568.63	17.586	33.150	3.0926
910	571.77	17.490	32.967	3.0595
915	574.91	17.378	32.787	3.0254
920	578.05	17.311	32.607	2.9925
925	581.20	17.206	32.432	2.9604

Frequency	$\omega = 2\pi f$ or $X_L = \omega L$	$1/\omega = 1/2\pi f$ or $X_c = 1/\omega c$	λ Wave length	LC
930	584.34	17.113	32.258	2.9287
935	587.48	17.022	32.086	2.8974
940	590.62	16.931	31.915	2.8665
945	593.76	16.842	31.746	2.8364
950	596.90	16.752	31.580	2.8067
955	600.05	16.665	31.414	2.7774
960	602.19	16.578	31.250	2.7485
965	606.33	16.492	31.088	2.7200
970	609.47	16.407	30.928	2.6920
975	612.61	16.324	30.770	2.6646
980	615.75	16.239	30.617	2.6372
985	618.90	16.158	30.456	2.6106
990	622.04	16.071	30.302	2.5842
995	625.18	15.995	30.150	2.5586
1000	628.32	15.916	30.000	2.5330

6. Dimensions, Weights, and Resistances of Pure, Solid, Bare Copper Wire.

Copper-wire Tables, *Circular* 31, Bur. Standards.

B. & S. or American wire gage	Diam. in mils at 20°C. (68°F.)	Cross-sectional area at 20°C. (68°F.)		Carrying capacities			Weight	
		Circular mils (d^2), 1m = 0.001 in.	Square inches	Rubber insul. amps. A	Varn. cloth insul. amps. B	Other insul. amps. C	Pounds per 1,000 ft.	Pounds per mile
0000	460.0	211,600.0	0.166,2	225	270	325	640.5	3,381.840
000	409.6	167,800.0	0.131,8	175	210	275	507.9	2,681.712
00	364.8	133,100.0	0.104,5	150	180	225	402.8	2,126.784
0	324.9	105,500.0	0.082,89	125	150	200	319.5	1,686.960
1	289.3	83,690.0	0.065,73	100	120	150	253.3	1,337.424
2	257.6	66,370.0	0.052,13	90	110	125	200.9	1,060.752
3	229.4	52,640.0	0.041,34	80	95	100	159.3	841.104
4	204.3	41,740.0	0.032,78	70	85	90	126.4	667.392
5	181.9	33,100.0	0.026,00	55	65	80	100.2	529.056
6	162.0	26,250.0	0.020,62	50	60	70	79.46	419.548,8
7	144.3	20,820.0	0.016,35	38	...	54	63.02	332.745,6
8	128.5	16,510.0	0.012,97	35	40	50	49.98	263.894,4
9	114.4	13,090.0	0.010,28	28	...	38	39.63	209.246,1
10	101.9	10,380.0	0.008,155	25	30	30	31.43	165.950,4
11	90.74	8,234.0	0.006,467	20	...	27	24.02	131.577,6
12	80.81	6,530.0	0.005,129	20	25	25	19.77	104.385,6
13	71.96	5,178.0	0.004,067	17	...	...	15.08	82.790,4
14	64.08	4,107.0	0.003,225	15	18	20	12.43	65.630,4
15	57.07	3,257.0	0.002,558	...	...	...	9.858	52.050,24
16	50.82	2,583.0	0.002,028	6	...	10	7.818	41.279,04
17	45.26	2,048.0	0.001,609	...	...	...	6.200	32.736,00
18	40.30	1,624.0	0.001,276	3	...	6	4.917	25.961,76
19	35.89	1,288.0	0.001,012	...	...	...	3.899	20.586,72
20	31.96	1,022.0	0.000,802,3	The above values are those specified in the 1931 National Electrical Code. In lighting work, no wire smaller than No. 14 is used, except in fixtures			3.092	16.325,70
21	28.46	810.1	0.000,636,3				2.452	12.946,56
22	25.35	642.4	0.000,504,6				1.945	10.269,60
23	22.57	509.5	0.000,400,2				1.542	8.141,76
24	20.10	404.0	0.000,317,3				1.223	6.457,44
25	17.90	320.4	0.000,251,7				0.969,9	5.121,072
26	15.94	254.1	0.000,199,6				0.769,2	4.061,376
27	14.20	201.5	0.000,158,3				0.610,0	3.220,800
28	12.64	159.8	0.000,125,5				0.483,7	2.553,936
29	11.26	126.7	0.000,099,53				0.383,6	2.025,408
30	10.03	100.5	0.000,078,94				0.304,2	1.608,176
31	8.928	79.70	0.000,062,60				0.241,3	1.274,060
32	7.950	63.20	0.000,049,64				0.191,3	1.010,064
33	7.080	50.13	0.000,039,37				0.151,7	0.800,976
34	6.305	39.75	0.000,031,22				0.120,3	0.635,184
35	5.615	31.52	0.000,024,76				0.095,42	0.513,717,6
36	5.000	25.00	0.000,019,64				0.075,68	0.399,590,4
37	4.453	19.83	0.000,015,57				0.060,01	0.316,852,8
38	3.965	15.72	0.000,012,35				0.047,59	0.251,275,2
39	3.531	12.47	0.000,009,793				0.037,74	0.199,267,2
40	3.145	9.888	0.000,007,766				0.029,93	0.158,030,4

Length, 25°C. (77°F.)		Resistance at 25°C. (77°F.)			B. & S. or American wire gage
Feet per pound	Feet per ohm	R ohms per 1,000 ft.	Ohms per mile	Ohms per pound	
1.561	20,010.0	0.049,98	0.263,894.4	0.000,078,03	0000
1.968	15,870.0	0.063,02	0.332,745.6	0.000,124,1	000
2.482	12,580.0	0.079,47	0.419,501.6	0.000,197,3	00
3.130	9,980.0	0.100,2	0.529,056	0.000,313,7	0
3.947	7,914.0	0.126,4	0.667,392	0.000,498,8	1
4.977	6,270.0	0.159,3	0.841,104	0.000,793,1	2
6.276	4,977.0	0.200,9	1.060,752	0.001,261	3
7.914	3,947.0	0.253,3	1.337,424	0.002,005	4
9.980	3,130.0	0.319,5	1.686,960	0.003,188	5
12.58	2,482.0	0.402,8	2.126,784	0.005,069	6
15.87	1,969.0	0.508,0	2.682,240	0.008,061	7
20.01	1,561.0	0.640,5	3.381,840	0.012,82	8
25.23	1,238.0	0.807,7	4.264,656	0.020,38	9
31.82	981.8	1.018	5.375,04	0.032,41	10
40.12	778.7	1.284	6.779,52	0.051,53	11
50.59	617.5	1.619	8.548,32	0.081,93	12
63.80	489.7	2.042	10.781,76	0.130,3	13
80.44	388.3	2.575	13.596,00	0.207,1	14
101.4	308.0	3.247	17.144,16	0.329,4	15
127.9	244.2	4.094	21.616,32	0.523,7	16
161.3	193.7	5.163	27.260,64	0.832,8	17
203.4	153.6	6.510	34.372,80	1.324	18
256.5	121.8	8.210	43.348,80	2.105	19
323.4	96.60	10.35	54.648,0	3.348	20
407.8	76.61	13.05	68.904,0	5.323	21
514.2	60.75	16.46	86.908,8	8.464	22
648.4	48.18	20.76	109.612,8	13.46	23
817.7	38.21	26.17	138.177,6	21.40	24
1,031.0	30.30	33.00	174.240,0	34.03	25
1,300.0	24.03	41.62	219.753,6	54.11	26
1,639.0	19.06	52.48	277.094,4	86.03	27
2,067.0	15.11	66.17	349.377,6	136.8	28
2,607.0	11.98	83.44	440.563,2	217.5	29
3,287.0	9.504	105.2	555.456	345.9	30
4,145.0	7.537	132.7	700.656	549.9	31
5,227.0	5.977	167.3	883.344	874.4	32
6,591.0	4.740	211.0	1,114.080	1,390.0	33
8,310.0	3.759	266.0	1,404.480	2,211.0	34
10,480.0	2.981	335.5	1,771.440	3,515.0	35
13,210.0	2.364	423.0	2,233.440	5,590.0	36
16,600.0	1.875	533.4	2,816.352	8,888.0	37
21,010.0	1.487	672.6	3,551.328	14,130.0	38
26,500.0	1.179	848.1	4,477.968	22,470.0	39
33,410.0	0.935	1,069.0	5,644.32	35,730.0	40

7. Tensile Strength of Pure Copper Wire in Pounds.

Size, B. & S. gage	Hard drawn		Annealed		Size, B. & S. gage	Hard drawn		Annealed	
	Actual	Average per square inch	Actual	Average per square inch		Actual	Average per square inch	Actual	Average per square inch
0000	8,260	49,700	5,320	32,000	7	1050.0	64,200	556.0	34,000
000	6,550	49,700	4,220	32,000	8	843.0	65,000	441.0	34,000
00	5,440	52,000	3,340	32,000	9	678.0	66,000	350.0	34,000
0	4,530	54,600	2,650	32,000	10	546.0	67,000	277.0	34,000
1	3,680	56,000	2,100	32,000	12	343.0	67,000	174.0	34,000
2	2,970	57,000	1,670	32,000	14	219.0	68,000	110.0	34,000
3	2,380	57,600	1,323	32,000	16	138.0	68,000	68.9	34,000
4	1,900	58,000	1,050	32,000	18	86.7	68,000	43.4	34,000
5	1,580	60,800	884	34,000	19	68.8	68,000	34.4	34,000
6	1,300	63,000	700	34,000	20	54.7	68,000	27.3	34,000

8. Insulated Copper Wire.

Size, B. & S. gage	Enamel wire			Single-silk covered			Double-silk covered		
	Outside diameter, mils	Turns per linear inch	Pounds per 1,000 ft.	Outside diameter, mils	Turns per linear inch	Pounds per 1,000 ft.	Outside diameter, mils	Turns per linear inch	Pounds per 1,000 ft.
8	130.6	7.7	50.6						
9	116.5	8.6	40.2						
10	104.0	9.6	31.8						
11	92.7	10.8	25.3						
12	82.8	12.1	20.1						
13	74.0	13.5	15.90						
14	66.1	15.1	12.60						
15	59.1	16.9	10.00						
16	52.8	18.9	7.930	52.8	18.9	7.89	54.6	18.3	8.00
17	47.0	21.3	6.275	47.3	21.1	6.26	49.1	20.4	6.32
18	42.1	23.8	4.980	42.4	23.6	4.97	44.1	22.7	5.02
19	37.7	26.5	3.955	37.9	26.4	3.94	39.7	25.2	3.99
20	33.7	29.7	3.135	34.0	29.4	3.13	35.8	28.0	3.17
22	26.9	37.2	1.970	27.3	36.6	1.98	29.1	34.4	2.01
24	21.5	46.5	1.245	22.1	45.3	1.25	23.9	41.8	1.27
26	17.1	58.5	0.785	17.9	55.9	0.791	19.7	50.8	0.810
28	13.6	73.5	0.494	14.6	68.5	0.498	16.4	61.0	0.514
30	10.9	91.7	0.311	12.0	83.3	0.316	13.8	72.5	0.333
32	8.7	115	0.196	9.9	101	0.210	11.8	84.8	0.217
34	6.9	145	0.123	8.3	121	0.129	10.1	99.0	0.141
36	5.5	180	0.078	7.0	143	0.082	8.8	114	0.092
38	4.4	227	0.049	6.0	167	0.053	7.8	128	0.062
40	3.5	286	0.031	5.1	196	0.035	6.9	145	0.043

9. Insulated Copper Wire.

Size, B. & S. gage	Ohms per 1,000 ft.	Single-cotton covered			Double-cotton covered		
		Outside diameter, mils	Turns per linear inch	Pounds per 1,000 ft.	Outside diameter, mils	Turns per linear inch	Pounds per 1,000 ft.
0000	0.0500	467	2.14	......	477	2.10	
000	0.0630	418	2.39	......	428	2.34	
00	0.0795	373	2.68	......	382	2.62	
0	0.100	334	3.00	......	343	3.00	
1	0.126	300	3.33	......	308	3.25	
2	0.159	267	3.75	......	275	3.64	
3	0.201	239	4.18	......	248	4.03	
4	0.253	214	4.67	......	222	4.51	
5	0.319	192	5.21	......	200	5.00	
6	0.403	170	5.88	......	175	5.62	
7	0.508	153	6.54	......	160	6.25	
8	0.641	136	7.35	50.6	142	7.05	51.2
9	0.808	121	8.26	40.2	127	7.87	40.6
10	1.02	108	9.25	31.9	113	8.85	32.2
11	1.28	97	10.3	25.3	102	9.80	25.6
12	1.62	87	11.5	20.1	92	10.9	20.4
13	2.04	78	12.8	16.0	82	12.2	16.2
14	2.58	70	14.3	12.7	74	13.5	12.9
16	4.1	56	17.9	8.03	60	16.7	8.21
18	6.5	45	22.2	5.08	49	20.4	5.24
20	10.4	37	27	3.22	41	24.4	3.37
22	16.6	29.5	33.9	2.05	33.3	30.0	2.17
24	26.2	24.1	41.5	1.3	28.1	35.6	1.4
26	41.6	19.9	50.2	0.834	23.9	41.8	0.914
28	66.2	16.6	60.2	0.533	20.6	48.6	0.608
30	105	14	71.4	0.340	18.0	55.6	0.400
32	167	12	83.4	0.223	16.0	62.9	0.270
34	266	10.3	97.1	0.148	14.3	70.0	0.193
36	423	9.0	111	0.099	13.0	77.0	0.136
38	673	8.0	125	0.070	12.0	83.3	0.105
40	1.070	7.1	141	0.052	11.1	90.9	0.084

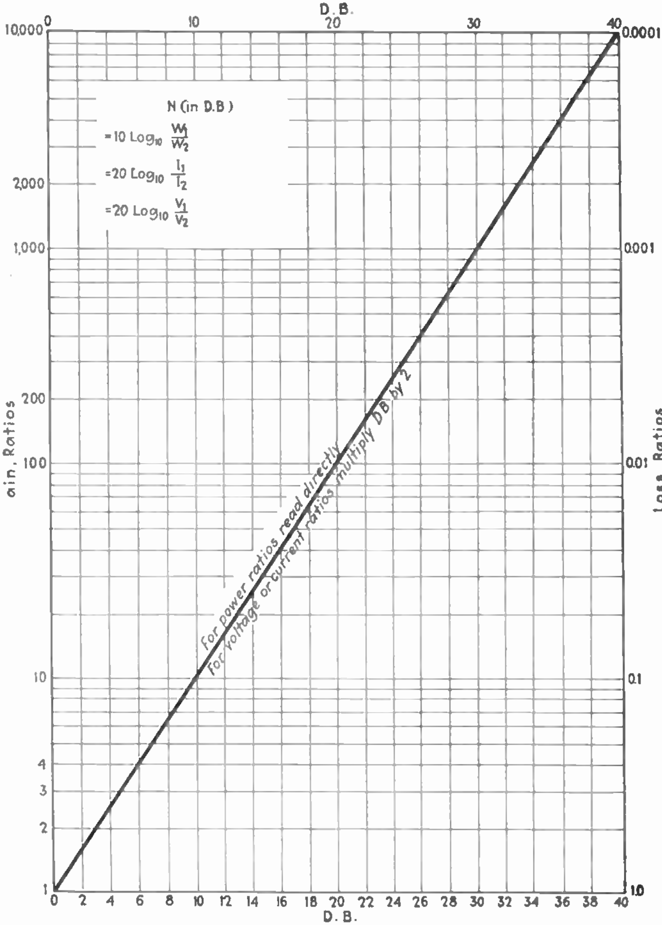
10. Approximate Wave Lengths of 4-ft. Coil Antennae with Various Values of Condenser Capacity Across the Coil Terminals.

Number of turns	Condenser capacity, microfarads						Distribution in slots ½ in. apart, turns per slot
	0.00005	0.0001	0.0005	0.001	0.002	0.003	
1	.....	65	128	178	250	310	1
3	130	155	290	400	550	675	1
6	230	280	500	710	1,000	1,200	1
12	430	490	920	1,250	1,700	2,050	1
24	760	880	1,600	2,100	3,000	3,600	1
48	1,550	1,775	3,150	4,300	6,000	7,000	2
72	2,200	2,650	4,800	6,400	8,800	11,000	3
120	3,930	4,500	7,900	10,000	14,700	17,700	5
240	7,600	9,000	15,650	20,500	27,200	32,900	10

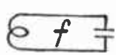
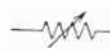
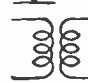
11. Number of Volts Required to Produce a Spark between Balls in Air.

Length of spark gap in		Diameter of balls, volts		
Centimeters	Inches	1 cm = 0.3937 in.	2 cm = 0.787 in.	6 cm = 2.36 in.
0.02	0.0079	1,560	1,530	
0.04	0.0157	2,460	2,430	
0.06	0.0236	3,300	3,240	
0.08	0.0315	4,050	3,990	
0.10	0.0394	4,800	4,800	4,500
0.20	0.0787	8,400	8,400	7,800
0.30	0.1181	11,400	11,400	10,800
0.40	0.1575	14,400	14,400	13,500
0.50	0.1969	17,100	17,100	16,500
0.60	0.2362	19,500	19,800	19,500
0.70	0.2756	21,600	22,500	22,500
0.80	0.3150	23,400	24,900	26,100
0.90	0.3543	24,600	27,300	29,000
1.00	0.3937	25,500	29,100	32,700

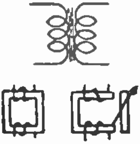
12. Chart for Converting Loss or Gain into Decibels.



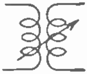
13. Standard Graphic Symbols Used in Radio Communication.

1. Aerial (antenna)	
2. Ammeter	
3. Arc	
4. Battery (the positive electrode is indicated by the long line)	
5. Coil antenna	
6. Condenser fixed	
7. Condenser, fixed, shielded	
8. Condenser, variable	
9. Condenser, variable (with moving plate indicated)	
10. Condenser, variable shielded	
11. Counterpoise	
12. Crystal detector	
13. Frequency meter (wave meter)	
14. Galvanometer	
15. Glow lamp	
16. Ground	
17. Inductor	
18. Inductor, adjustable	
19. Inductor, iron core	
20. Inductor, variable	
21. Jack	
22. Key	
23. Lightning arrester	
24. Loud-speaker	
25. Microphone (telephone transmitter)	
26. Photoelectric cell	
27. Piezoelectric plate	
28. Resistor	
29. Resistor, adjustable	
30. Resistor, variable	
31. Spark gap, rotary	
32. Spark gap, plain	
33. Spark gap, quenched	
34. Telephone receiver	
35. Thermoelement	
36. Transformer, air core	

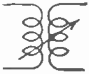
37. Transformer, iron core



38. Transformer with variable coupling



39. Transformer, with variable coupling (with moving coil indicated



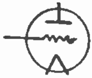
40. Voltmeter

41. Wires, joined

42. Wires, crossed, not joined

43. Diode (or half-wave rectifier)

44. Triode (with directly heated cathode)

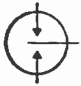


45. Triode (with indirectly heated cathode)

46. Screen-grid tube (with directly heated cathode)

47. Screen-grid tube (with indirectly heated cathode)

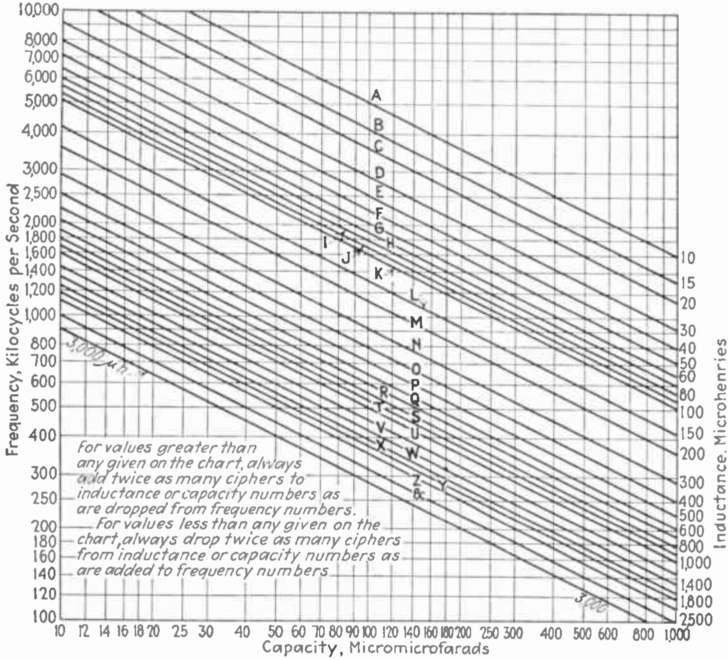
48. Rectifier tube, full wave (filamentless)



49. Rectifier tube, full wave (with directly heated cathode)

50. Rectifier tube, half wave (filamentless)

14. L, C, λ Chart.



15. Width of Authorized Communication Band.
Federal Radio Commission, General Order 119, Sept. 3, 1931.

Type of emission	Frequency range, kilocycles	Normal width of communication band, kilocycle[s]
A1: C. W. Morse telegraphy; printer and slow-speed facsimile	10 to 100	0.100
	100 to 550	0.250
	1,500 to 6,000	0.500
	6,000 to 12,000	1.000
	12,000 to 28,000	2.000
A2: I. C. W.	10 to 100	(To be specified in instru- ment of authorization)
	100 to 550	1.500
	1,500 to 6,000	2.000
	6,000 to 12,000	3.000
	12,000 to 28,000	4.000
A3: Commercial telephony:		
Single side band	....	3.000
Double side band	....	6.000
A3: Broadcasting:		
Double side band	550 to 1,500	10.000
Special:		
High-speed facsimile; picture transmission; high-quality telephony; television, etc.	10 to 550 1,500 to 28,000	The authorized width of th[e] communication band fo[r] special types of transmis[-] sion shall be specified i[n] the instrument of authori[-] zation

16. Tolerance Table.

Every station shall be required to maintain frequency within the tolerance as provided by the following table:

Frequency range, kilocycles	Frequency tolerance, per cent	
	A Applicable to stations licensed and authorized by construction permits prior to effective date of this order	B Applicable to all equipment authorized subsequent to effective date of this order
A. 10 to 550:	Plus or minus	Plus or minus
a. Fixed stations	0.1	0.1
b. Land stations	0.1	0.1
c. Mobile stations except those using damped waves or simple oscillator transmitters	0.5	0.5
d. Mobile stations using damped wave or simple oscillator transmitters	1.0	0.5
B. 550 to 1,500:		
a. Broadcasting stations	See General Order 116	
C. 1,500 to 6,000:		
a. Fixed stations	0.05	0.03
b. Land stations	0.05	0.04
c. Mobile stations	0.1	0.1
D. 6,000 to 23,000:		
a. Fixed stations	0.05	0.02
b. Land stations	0.05	0.04
c. Mobile stations using frequencies assigned to land stations or those in bands shared between mobile and fixed services	0.05	0.04
d. Mobile stations using frequencies other than those specified under *c*	0.1	0.1
e. Broadcasting stations	0.03	0.01

17. Separation between Assigned Frequencies.

Frequency range, kilocycles	Frequency separation, kilocycles
10 to 15	0.15
15 to 20	0.2
20 to 25	0.25
25 to 30	0.3
30 to 40	0.4
40 to 50	0.5
50 to 60	0.6
60 to 100	0.8
100 to 390	1
390 to 550	2
550 to 1,500	10
1,500 to 3,000	4
3,000 to 6,000	5
6,000 to 11,000	10
11,000 to 16,400	15
16,400 to 21,550	20
21,550 to 28,000	25

NOTE. The separation between assignments may be greater than those indicated where this is required by the type of emission authorized.

SECTION 2

ELECTRIC AND MAGNETIC CIRCUITS

By E. A. Uehling[1]

FUNDAMENTALS OF ELECTRIC CIRCUITS

1. Nature of Electric Charge. According to modern views all natura[l] phenomena may be explained on the basis of fundamental postulate[s] regarding the nature of electric charge. In the neighborhood of a[n] electric charge is postulated the existence of an electric field to explai[n] such phenomena as repulsion and attraction. The force which act[s] between electric charges by virtue of the electric fields surrounding the[m] is expressed by Coulomb's law which states that

$$F = \frac{q_1 q_2}{r^2}$$

The value of the unit charge in the electrostatic system is based on thi[s] law and is defined, therefore, as that value of electric charge which whe[n] placed at 1 cm distance from an equal charge repels it with a force o[f] 1 dyne.

2. Electrons and Protons. There are two types of electricity: positiv[e] and negative. The electron is representative of the latter and th[e] proton of the former. All matter is made up simply of electrons an[d] protons. Exhaustive experiment has proved that all electrons, n[o] matter how derived, are identical in nature. They are easily isolate[d] and as a consequence have been thoroughly studied. Among the mos[t] important results of this study are the following facts:[2]

Charge of the electron	4.770×10^{-10} e.s.u.
Mass	9.04×10^{-28} g
Radius	2×10^{-13} cm, approx.

The proton has not been so thoroughly studied. It is not so easil[y] isolated, and the effects of electric and magnetic fields on its motion ar[e] considerably smaller than similar effects obtained when electrons ar[e] studied. The proton apparently has a mass of about 1,838 times tha[t] of the electron and a considerably smaller radius.

The mass of electrons and protons is purely inertial in character. I[n] other words these fundamental units of electric charge consist simpl[y] of pure electricity. For the sake of completeness it should be adde[d] that this mass is not independent of velocity and that the values give[n] for both the electron and proton assume velocities which are smal[l] in comparison with that of light.

[1] Department of Physics, University of Michigan.
[2] MILLIKAN, R. A., "The Electron."

3. Atomic Structure. The atoms of matter consist of a central positive nucleus surrounded by such a number of electrons as will neutralize the nuclear charge. The central positive nucleus consists of both electrons and protons with an excess of the latter. This excess determines the chemical characteristics of the atom by determining the number of electrons outside the nucleus, while the total number of protons determines the atomic weight of the element. According to one view the electrons outside the nucleus move in planetary elliptic orbits about it. The radius of the different orbits varies within a single atom, and as a consequence the strength of the bond existing between the nucleus and the different electrons varies.

4. Ionization. The outer electrons are in general loosely bound to the nucleus and under favorable conditions may be completely dissociated from the remainder of the atom. This process of the removal of an electron is known as *ionization*. It is the process by which electrons are removed from a heated filament in a vacuum tube, from an alkali metal surface in the photoelectric cell, and from the plate and grid of vacuum tubes when bombarded by the filament electrons giving rise to the secondary emission so commonly experienced.

5. The Nature of Current. The modern view of electricity regards a current as a flow of negative charge in one direction plus a flow of positive charge in the opposite direction. In electrolytic conduction the unit of negative charge is an atom with one or more additional electrons called a *negative ion*, and the unit of positive charge is an atom with one or more electrons less than its normal number known as the *positive ion*.

In conduction through gases, as, for example, through the electric arc, the negative ion is usually a single electron, whereas the positive ion is as before an atom with one or more electrons removed.

In conduction through solids, however, the current is strictly electronic and is not made up of two parts as in the previous cases. The electrons constituting the current are the outer orbital electrons of the atoms. Since these electrons are less tightly bound to the atom than the other electrons they are comparatively free and are often spoken of as *free electrons*. These electrons move through the solid under the influence of an electric field colliding with the atoms as they move and continuously losing energy gained from the field. As a consequence the motion of the electrons in the direction of the field is of a comparatively small velocity[1] (of the order of 1 cm per second), whereas the velocity of thermal agitation of the free electrons is high (about 10^7 cm per second). According to this view of the electric current in solids, conductors and insulators differ only in the relative number of free electrons possessed by the substance.

Since current consists of a motion of electric charges, it may be defined as a given amount of charge passing a point in a conductor per unit time. In the electrostatic system the unit of current is defined to be a current such that an electrostatic unit of electricity crosses any selected cross section of a conductor in unit time. In the practical system the unit of current is the ampere which is approximately equal to 3×10^9 electrostatic units of current and is defined on the basis of material constants as that current which will deposit 0.00111800 g of silver from a solution of silver nitrate in 1 sec.

[1] Jeans, J. H., "Electricity and Magnetism," p. 306.

6. The Nature of Potential. An electric charge that is resident in an electric field experiences a force of repulsion or attraction depending on the nature of the charge. Its position in the field may be considered as representing a certain quantity of potential energy which may be taken as the amount of work which is capable of being done when the electric charge moves from the point in question to an infinite distance. If the convention of considering a unit positive charge as the test charge is adopted, the potential energy at a point may be taken as characteristic of the field and consequently will be regarded simply as the potential.

In a similar manner the difference of potential of two points may be described as the amount of work required to move a unit positive test charge from one point to another. More specifically a difference of potential in a conductor may be spoken of as equal to the energy dissipated when an electron moves through the conductor from the point of low potential to the point of high potential. This energy is dissipated in the form of heat caused by the bombardment of the molecules of the conductor by the electrons as they proceed from one point to another.

7. Concept of E.M.F. The idea of potential leads directly to a conception of an electromotive force. If a difference of potential between two points of a conductor is maintained by some means or other, electrons will continue to flow, giving rise to a continuous current. A difference in potential maintained in this way while the current is flowing is known as an *electromotive force.* Only two important methods of maintaining a constant e.m.f. exist: the battery and the generator. Other methods, as, for example, the thermocouple, are not primarily intended for the purpose of maintaining a current.

The unit of e.m.f. in the practical system is the *volt.* It is defined as 10^8 e.s.u. of potential or as 1.0000/1.0183 of the voltage generated by a standard Weston cell.

8. Ohm's Law and Resistance. The free electrons which contribute to the electric current have a low drift velocity in the negative direction of the field within the conductor. In moving through the metal in a common general direction they enter into frequent collisions with the molecules of the metal, and as a consequence they are continually retarded in their forward motion and are not able to attain a velocity greater than a certain terminal velocity u, which depends on the value of the field and the nature of the substance. The collisions which tend to reduce the drift velocity of the electrons act as a retarding force. When a current is flowing, this retarding force must be exactly equal to the accelerating force of the field. The retarding force is proportional to N, the number of free electrons per unit length of conductor, and to u, their drift velocity. It may be designated as kNu. The accelerating force is proportional to the field E per unit length of conductor, to the number N of electrons per unit length, and to the electronic charge e and may be represented as NEe. Then $NEe = kNu$. Since the current i has been given as

$$i = Neu$$

$$NEe = k\frac{i}{e}$$

$$E = \frac{k}{Ne^2}i = Ri$$

where

$$R = \frac{k}{Nc^2}$$

The statement $E = Ri$ is known as Ohm's law. R is here defined as the *resistance* per unit length. The unit of resistance is the *ohm.* It may be obtained from Ohm's law when the e.m.f. is expressed in volts and the current in amperes.

9. Inductance. Circuits possess inductance by virtue of the electromagnetic field which surrounds a conductor carrying a current. The coefficient of self-inductance is defined as the total number of lines of force passing through a circuit and due entirely to one c.g.s. unit of current traversing the circuit. If N is the number of lines of force linked with any circuit of inductance L and conveying C c.g.s. units of current, $N = LC$.

The practical unit of inductance is the *henry.* It is equal to 10^9 c.g.s. units of inductance. If the number of lines of force N through a circuit is changed, an e.m.f. due to this change of flux is induced in the circuit. This e.m.f. is given by the equation

$$e = -\frac{dN}{dt} = -L\frac{dC}{dt}$$

The inductance of a circuit is equal to 1 henry if an opposing e.m.f. of 1 volt is set up when the current in the circuit varies at the rate of 1 amp. per second.

10. Mutual Inductance. The coefficient of mutual inductance is defined in the same way as that of self-inductance and is given in c.g.s. units as the total magnetic flux which passes through one circuit when the other is traversed by one c.g.s. unit of current, or

$$N = MC$$

$$e = -\frac{dN}{dt} = -M\frac{dC}{dt}$$

The practical unit is the henry as in self-inductance.

11. Energy in Magnetic Field. Energy is stored in the electromagnetic field surrounding a circuit representing the energy accumulated during the time when the free electrons were initially set in motion and the current established. This energy is given by the equation, $W = \frac{1}{2}LI^2$, where, if L is in henrys and I in amperes, the energy is in *joules.*

12. Capacitance. The ratio of the *quantity* of charge on a conductor to the *potential* of the conductor represents its *capacity.* If one conductor is at zero potential and another at the potential V, the capacity is given as the ratio of the charge stored to the potential difference of the conductors

$$C = \frac{Q}{V}$$

If Q is in coulombs (the quantity of charge carried by 1 amp. flowing for 1 sec.) and V is in volts, C is known as the *farad.*

The energy stored in a condenser is given by the equation, $W = \frac{1}{2}CV^2$, where, if V is in volts and C is in farads, W is in joules.

The force acting per unit area on the conductors of the condenser tending to draw them together is

$$F = \frac{E^2}{8\pi} = \frac{V^2}{8\pi d^3}$$

where d is the distance separating the condenser plates, and V is the potential difference.

Other expressions relating charge or current to capacity and potential difference are

$$V = \frac{\int i dt}{C}$$

and

$$i = C\frac{dV}{dt}$$

13. Units. The practical units that have been described are related to the electrostatic units as shown by the following table. A third set of units, known as the electromagnetic, is also related to the practical units, the ratios of which are given in this table.

Quantity	Name of unit	Measure in electromagnetic units	Measure in electrostatic units
Charge of electricity	Coulomb	10^{-1}	3×10^9
Potential	Volt	10^8	$1/300$
Capacity	Farad	10^{-9}	9×10^{11}
Current	Ampere	10^{-1}	3×10^9
Resistance	Ohm	10^9	$1/9 \times 10^{-11}$
Inductance	Henry	10^9	

14. Continuous and Alternating Currents. If the free electrons of a conductor move with a constant drift velocity under the impelling force of an invariant electric field, the electric current in the conductor is spoken of as being *continuous*, or *direct*. If, however, the impressed electric field is varying in both direction and magnitude, the drift velocity of the electrons will vary in both direction and magnitude, since electrons always flow in a direction opposite to that of the electric field. A current of this kind which varies periodically with the time is known as an *alternating* current.

15. Wave Form. The current or the e.m.f. may be represented graphically as a function of the time by assigning to successive values of the latter variable the value of the former. There is an infinite variety of functional relationships between current and time, but of all the laws by which these two variables may be connected there is one that can be differentiated from all others. This law is that of the *sine* or *cosine* function. All other relationships can be resolved into a linear combination of functions of this simple type.

The form of the sine function is shown in Fig. 1*a*. It is represented analytically by the following type of equations

$$i = I_0 \sin \omega t$$
$$e = E_0 \sin \omega t$$

where i and e are the instantaneous values of the current and voltage, I_0 and E_0 are the maximum values, and ω is 2π times the frequency with which the current or voltage alternates. The sine wave is the ideal toward which practical types approach more or less closely. Since it cannot be resolved into other types, it is the pure wave form.

16. Harmonics. Current and voltage waves, in practice, are not pure and may therefore be resolved into a series of sine or cosine functions. One of the functions into which the original wave is resolved will have a frequency term equal to that of the original wave. All of the other functions will have frequency terms of higher value, which will in general be designated as *harmonics* of the lowest or fundamental frequency. A few types of complex waves which may be resolved into two or more pure sine waves are shown in Fig. 1*b* and *c*. The resolution of a complex wave into its component parts may be accomplished physically as well as mathematically. This may be demonstrated by means of high- and low-pass filters in the output circuit of an ordinary vacuum-tube oscillator.

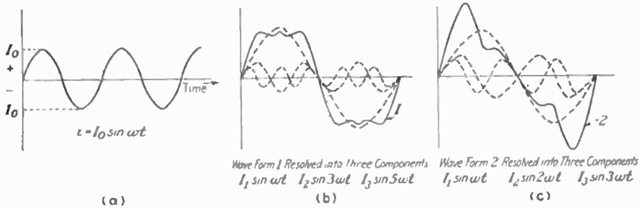


FIG. 1.—Sine wave and complex waves.

17. Effective and Average Values. The *effective value* of an a-c wave is the value of continuous current which gives the same power dissipation as the a. c. in a resistance. For a sine wave this value of continuous current is equal to the maximum value divided by $\sqrt{2}$. The average value of an alternating current is equal to the integral of the current over the time for one-half period divided by the elapsed time. For a sine wave the average value is equal to the maximum value of the current divided by $\pi/2$. The ratio of the effective value of the current to the average value is often taken as the *form factor* of the wave. Thus all types of waves may be simply characterized by means of this ratio.

Direct-current meters read average values of currents over a complete period. Such meters therefore read zero in an a-c circuit. Thermocouple and hot-wire-type meters read effective values. Such meters are therefore used for making a-c measurements at radio- as well as at audio-frequencies.

18. Phase. The current in a circuit may have its maximum and zero values at the same time as those of the e.m.f. wave, or these values may occur earlier or later than those of the latter. These three cases are illustrated in Fig. 2. When the corresponding values of the current and e.m.f. occur at the same time they are said to be in phase. If the current values occur before the corresponding values of the voltage wave, the current is said to be in *leading* phase, and if these values occur after the corresponding values of the voltage wave, it is said to be in *lagging* phase.

19. Power. The power consumed in a continuous-current circuit is $W = EI = I^2R$, where R is the *effective resistance* of the circuit. The power consumed in an a-c circuit having negligible inductance and capacitance is given by the same equation with the necessary restrictions on I so that it represents the effective value of the current and not the average value. The power consumed in an inductive or capacitative circuit is $W = EI \cos \varphi$, where φ is the *phase angle*, that is, the angle of lag or lead of current. The term "$\cos \varphi$" is commonly referred to as the *power factor* of the circuit.

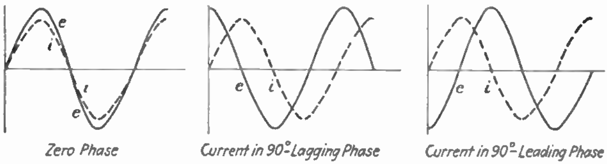


FIG. 2.—Phase in a-c circuits.

DIRECT-CURRENT CIRCUITS

20. Direction of Current Flow. An electric current is a flow of electric charges. Electric charges will move through a medium of finite resistance if a difference of electric potential exists between two points of that medium. In metallic conductors there is but one type of charge which is free to move, the negative charge or the free electrons of the conductor. The current in a metallic conductor then consists solely of an electron current. The convention arose historically of speaking of an electric current as flowing from the high potential (positive) to the low potential (negative) point, while, as a matter of fact, the electrons of the conductor actually move in the opposite direction. It is necessary to distinguish, therefore, between the direction of current flow in the historical sense and the direction of flow of electrons.

21. Constant Positive Resistance, Negative Resistance, and Infinite Resistance. In a d-c circuit the relationship between voltage and current is governed solely by the resistance of the circuit and all equivalent resistances such as counter e.m.fs. Some knowledge regarding the nature of this resistance is needed. Three cases present themselves. In the first case are those circuits in which

$$\frac{de}{di} = R$$

where R is positive and is constant in value over a rather large range. Conduction in solids and electrolytes is of this type. In the second class are those circuits in which de/di has a value which is negative and is usually not constant. Conduction in arcs and glow discharges is generally of this type. In the third class are those circuits in which

$$\frac{de}{di} = \infty$$

Conduction in the plate circuit of a vacuum tube under saturation conditions is of this type.

Circuits of the first class, in which the differential coefficient de/di has a positive value, may be subdivided into two other classes. If the alue of de/di is constant over the entire range of voltage and current rom zero to the maximum value, and if this value is designated by the uantity R, then Ohm's law may be used and $e = iR$. In this case, R s both the d-c and a-c resistance. If, however, R is not constant over his range of values, the value of R given at a particular value of e and given by the equation

$$R = \frac{de}{di}$$

s only the a-c resistance of the circuit at the particular value of e and i hosen. The a-c resistance given by this equation may be quite different rom the d-c value as given by the equation

$$R = \frac{e}{i}$$

In a vacuum-tube plate circuit the d-c value of the resistance is frequently about twice as high as the a-c value.

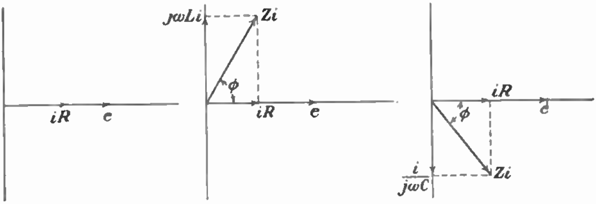


FIG. 3.—Vector representation of a-c circuits.

ALTERNATING-CURRENT CIRCUITS

22. Impedance. The resistance to the flow of an electric current having the value $i = I_0 \sin \omega t$ depends on the circuit element through which the current is passing. In a pure resistance the potential fall would be $E_1 = I_0 R \sin \omega t$, which is seen to be in phase with the current passing through it. In an inductance the potential fall would be

$$E_2 = L\frac{di}{dt} = \omega L I_0 \cos \omega t = j\omega L I_0 \sin \omega t = j\omega L i$$

and therefore leads the current by a phase angle of 90 deg. In a capacitance the potential fall would be

$$E_3 = \frac{1}{C}\int i\,dt = -\frac{I_0}{\omega C}\cos \omega t = -\frac{jI_0}{\omega C}\sin \omega t$$
$$= -\frac{ji}{\omega C}$$
$$= \frac{i}{j\omega C}$$

and is therefore led by the current by a phase angle of 90 deg. Th potential fall through all three elements taken together is equal to

$$E = \left(R + j\omega L + \frac{1}{j\omega C}\right)i$$

The coefficient of i is termed the *impedance* of the circuit. It is written in general, as

$$z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

where R is the total series resistance of the circuit, L is the total serie inductance, and C is the effective series capacitance. The term involv ing j is of special importance, for it is this term which gives to the curren its leading or lagging characteristics depending on whether ωL is smalle or larger than $1/\omega C$. This quantity is known as the circuit *reactanc*

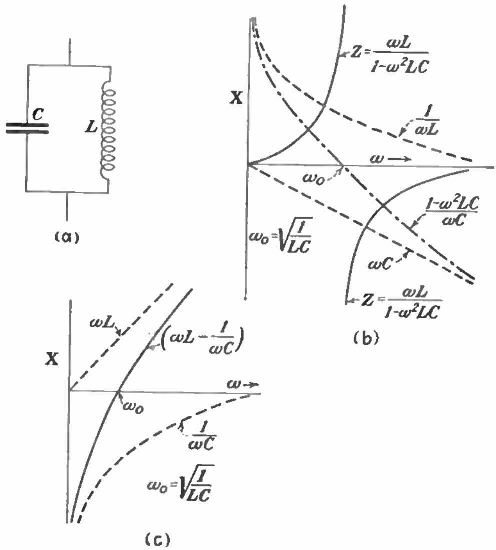


FIG. 4.—Reactance and impedance of parallel circuit.

and is designated by the letter X. The impedance may be written, therefore,

$$z = R + jX$$

Occasionally the absolute value of the circuit impedance is required. It is then written in the following form

$$z = Ze^{j\phi}$$

where

$$Z = \sqrt{R^2 + X^2}$$

$$\phi = \text{arc tan}\,\frac{X}{R}$$

n this expression Z represents the absolute value of the impedance, z he complex value, and ϕ the phase angle.

The impedance of a single circuit will be given to illustrate the method ·f obtaining this quantity for any circuit. For a parallel combination f circuit elements, such as illustrated in Fig. 4*a*, it would be obtained ;s follows:

$$z = \frac{1}{\dfrac{1}{1/j\omega C} + \dfrac{1}{j\omega L}} = \frac{j\omega L}{1 - \omega^2 LC}$$

'his equation shows that when $\omega^2 = 1/LC$ the impedance is infinite. t may be represented graphically as a function of ω as shown in Fig. *b*. The figure and the equation illustrate the case of parallel resonance. 'he case of series resonance is illustrated in Fig. 4*c*, and the equation is $= j\left(\omega L - \dfrac{1}{\omega C}\right)$, which holds for a circuit having only an inductance L nd capacitance C in series with the e.m.f. In the series case, the npedance is zero at resonance; that is, when $\omega^2 = 1/LC$ and in the arallel case the impedance is infinite at resonance.

23. Circuit Parameters. Every electric circuit, no matter how comlicated, is made up of a particular combination of inductances, capaciınces, and resistances. These parameters and the manner in which ıey are combined with one another completely govern the performance f a circuit and determine the value of the current at any point of the ırcuit at any time for any given value of the impressed e.m.f. or combinaon of e.m.fs.

Inductances, capacitances, and resistances may be lumped or distribted in nature. They are regarded as of the former type if their values re more or less concentrated at one or a finite number of points in a rcuit. For example, the inductance of a circuit would be considered s lumped if a definite number of places in the circuit is found where ıductance exists, and at all other points a comparative non-existence of ıductance. On the other hand the inductance of a uniform telephone ne is considered as distributed since it exists along the entire line and ıay, at no point in the line, be neglected.

24. Circuit Equations. Every circuit may be completely expressed y a system of simultaneous equations. Having expressed a particular rcuit in this manner, a solution may be obtained frequently without ifficulty. Since the equations are of primary importance, methods of ɔtaining them will be given.

There are two distinct cases. When a sinusoidal voltage or combinaon of sinusoidal voltages is impressed on a circuit, a.c. flows in every ranch of the circuit as a consequence of the impressed e.m.f. This ırrent may be divided into two parts. One part is known as the *ansient* current, and the other as the current of the *steady state*. The 'ansient current disappears very shortly after the voltage has been ıpressed. The steady state continues as long as the e.m.f. continues its initial state of voltage, frequency, and wave form. Often only ıe steady state is of interest. Examples of this are to be found in udies of r-f transformer performance and in studies of electric filters ' the low-pass, high-pass, or band-pass types and in the studies of the ırious characteristics of different antenna-coupling methods. At other times the transient condition may be of primary interest; as, for example in the study of the fidelity of reproduction with regard to wave form o an electromagnetic or electrodynamic loud-speaker motor.

If interest centers only in the steady state the following method i to be used: Apply Kirchhoff's second law which states that the sum c all the e.m.fs. around any circuit is zero, writing one equation for eac branch of the circuit, and using as the potential falls the values $j\omega LI$ fo each inductance, $I/j\omega L$ for each capacitance, and IR for each resistance If inductances, capacitances, and resistances occur that are commo to two or more branches, they will be used once for each of the commo branches paying due regard to the sign of the term.

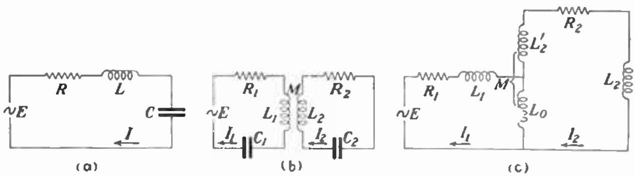


FIG. 5.—Circuits illustrating use of Kirchhoff's laws.

This method may be illustrated by the examples of Fig. 5 and the followin equations:
For circuit a:

$$E = IR + j\omega LI + \frac{I}{j\omega C} = I\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right]$$
$$= I(R + jX)$$
$$I = \frac{E}{R + jX}$$

For circuit b:

$$E = I_1R_1 + j\omega L_1I_1 + \frac{I_1}{j\omega C_1} - j\omega MI_2 = I_1z_1 - j\omega MI_2$$
$$0 = I_2R_2 + j\omega L_2I_2 + \frac{I_2}{j\omega L_2} - j\omega MI_1 = I_2z_2 - j\omega MI_1$$

where z_1 is the total complex impedance of circuit 1, and z_2 is the total con plex impedance of circuit 2.

For circuit c:

$$E = I_1R_1 + j\omega L_1I_1 + j\omega L_0I_1 - j\omega MI_2 - j\omega L_0I_2$$
$$= I_1z_1 - j\omega I_2(M + L_0)$$
$$0 = I_2R_2 + j\omega L'_2I_2 + j\omega L_0I_2 + j\omega L_2I_2 - j\omega MI_1 - j\omega L_0I_1$$
$$= I_2z_2 - j\omega I_1(M + L_0)$$

In these equations I is the maximum value of the sinusoidal current, an E is the maximum value of the sinusoidal e.m.f. These equations may l solved for any of the currents by the method of simultaneous equation

In the transient values of the various currents, Kirchhoff's second law ma be used as before, but instead of using the values of potential fall as give in the preceding equations, use the instantaneous values. The equation fo circuit a of Fig. 5 is then written

$$e = iR + L\frac{di}{dt} + \frac{1}{C}\int idt$$

$$\frac{de}{dt} = L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C}$$

here e and i are the instantaneous values of the impressed e.m.f. and cur-
ent respectively. For circuit b,

$$e = i_1R_1 + L_1\frac{di_1}{dt} + \frac{1}{C_1}\int i_1dt - M\frac{di_2}{dt}$$

$$0 = i_2R_2 + L_2\frac{di_2}{dt} + \frac{1}{C_2}\int i_2dt - M\frac{di_1}{dt}$$

o obtain the transient solution, e and de/dt are replaced by zero and the
quation solved by the methods used for linear, homogeneous equations of
e first degree.

25. General Characteristics of A-c Circuits. The general equations
pplied to a number of the more important radio circuits yield the
llowing results.

Current Flow in an Inductive Circuit:

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right)$$

here E is the constant impressed e.m.f.

Time Constant of an Inductive Circuit: The time required for a current to
se to $\left(1 - \frac{1}{\epsilon}\right)$ or to about 63 per cent of its final value. This time is equal
L/R.

Current Flow in a Capacitative Circuit:

$$i = \frac{E}{R}\epsilon^{-\frac{t}{RC}}$$

here E is the constant impressed e.m.f.

Time Constant of a Capacitative Circuit. The time required for the current
fall from its initial value to $1/\epsilon$ or about 0.37 of this value. This time is
ual to RC.

Current Flow in an Inductive-capacitative Circuit:

$$i = \frac{E}{\omega L}\epsilon^{-\frac{Rt}{2L}} \sin \omega t, \text{ if } R^2 < \frac{4L}{C}$$

$$i = \frac{E}{\omega L}\epsilon^{-\frac{Rt}{2L}} \qquad, \text{ if } R^2 = \frac{4L}{C}$$

here ω is 2π times the natural frequency of the circuit which is given by the
uation

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

Logarithmic Decrement. Ratio of successive maxima of the current in an
cillatory discharge is equal to

$$\epsilon^{\frac{RT}{2L}} = \epsilon^{\frac{R}{2Lf}}$$

here $R/2Lf$ is called the log. dec. of the circuit, T is the natural period, and
he natural frequency of the circuit.

Currents in Two Circuits Coupled by a Mutual Impedance, M, when Sinusoidal E.M.F., E, Exists in Circuit 1:

$$I_1 = \frac{E}{z_1 + \frac{\omega^2 M^2}{z_2}}$$

$$I_2 = \frac{j\omega M I_1}{z_2} = \frac{j\omega M E}{z_1 z_2 + \omega^2 M^2}$$

where z_1 and z_2 are the complex impedances of circuits 1 and 2 respective

Effective Reactance of One Circuit Coupled to a Second Circuit:

$$X' = X_1 - \frac{\omega^2 M^2}{Z_2^2} X_2$$

where X_1 and X_2 are the actual reactances of circuits 1 and 2 respective and Z_2 is the absolute value of the complex impedance of circuit 2.

Effective Resistance of One Circuit Coupled to a Second Circuit:

$$R' = R_1 + \frac{\omega^2 M^2}{Z_2^2} R_2$$

where R_1 and R_2 are the actual resistances of circuits 1 and 2 respective

Effective Total Impedance of One Circuit Coupled to a Second Circuit:

$$z' = z_1 + \frac{\omega^2 M^2}{z_2} = R_1 + jX_1 + \frac{\omega^2 M^2}{R_2 + jX_2}$$

$$= R_1 + \frac{\omega^2 M^2}{Z_2^2} R_2 + j\left\{X_1 - \frac{\omega^2 M^2}{Z_2^2} X_2\right\}$$

Partial Resonance Relation Obtained When Only the Reactance of Circu Is Variable:[1]

$$X_1 = \frac{\omega^2 M^2}{Z_2^2} X_2$$

Partial Resonance Relation Obtained when only the Reactance of Circu Is Variable:[1]

$$X_2 = \frac{\omega^2 M^2}{Z_1^2} X_1$$

Total Optimum Resonance Relation when the Reactance of Both Circ 1 and 2 Are Variable:[1]

Case I: If $\omega^2 M^2 < R_1 R_2$

Resonance relation $X_1 = 0$ and $X_2 = 0$

Case II: If $\omega^2 M^2 > R_1 R_2$

Resonance relation $\frac{R_2}{R_1} = \frac{\omega^2 M^2}{Z_1^2} = \frac{X_2}{X_1}$

Case III: If $\omega^2 M^2 = R_1 R_2$

Resonance relation $X_1 = 0,\ X_2 = 0$

$$\frac{R_2}{R_1} = \frac{\omega^2 M^2}{Z_1^2}$$

Total Secondary Current at Total Optimum Resonance Relation, the E.M E, Being Impressed in Circuit 1.

Case I: If $\omega^2 M^2 < R_1 R_2$

$$I_2 = \frac{\omega M E}{R_1 R_2 + \omega^2 M^2}$$

[1] PIERCE, G. W., *Electric Oscillations and Electric Waves*, Chap. XI.

Cases II and III: If $\omega^2M^2 \geq R_1R_2$

$$I_2 = \frac{E}{2\sqrt{R_1R_2}}$$

for cases *II* and *III* is seen to be greater than for case *I* and is independent ωM.

MAGNETIC CIRCUITS

26. The Fundamental Quantities of Magnetic Circuits. The first ndamental quantity is the *magnetic flux* or *induction*. The unit of ıx is known as the *maxwell* and is defined by the statement that from a ıit magnetic pole, 4π maxwells, or lines of force, radiate.

The second fundamental quantity is the *reluctance*. It is analogous the resistance of electric circuits, as the flux is analogous to the current. ıe unit of reluctance is the *oersted* and is defined as the reluctance fered by 1 cm cube of air.

The third fundamental quantity is the *magnetomotive force* (m.m.f.). is analogous to the e.m.f. of electrical circuits. The unit of m.m.f. is e *gilbert* and is defined as the m.m.f. required to force a flux of 1 maxwell rough a reluctance of 1 oersted. Thus the fundamental equation in ıich these three quantities are related to one another is:

$$M = \phi R$$

Other important quantities of magnetic currents may be defined as llows: the *magnetic field strength* is represented by the quantity H and is ual to the number of maxwells per unit of area when the medium rough which the flux is passing is air. This unit is known as the *gauss* the unit of area is the square centimeter.

In any medium other than air the lines of force are known as *lines of duction* and the symbol B is used instead of H to represent them. In r the induction B and the field strength H are equal to one another, but other mediums this is not true.

The *permeability* μ is the ratio between the magnetic induction B and ıe field strength H. In air this ratio is unity. In *paramagnetic* mateıls the permeability is greater than unity, in *ferromagnetic* materials may have a value of several thousand, and in *diamagnetic* materials it ıs a value of less than unity.

The intensity of magnetization I is the *magnetic moment* per unit ılume or the *pole strength* per unit area. The unit of magnetic pole rength is a magnetic pole of such a value that when placed 1 cm from a ke pole, a force of repulsion of 1 dyne will exist between them. The agnetic pole strength per unit area of any pole is measured in terms of ıis unit. The magnetic moment of a magnet is the product of the pole rength and the distance between the poles.

The *susceptibility* K of a material is equal to the ratio of the magnetizaon I produced in the material to the field strength H producing it. All these quantities are connected by the following equations

$$\begin{aligned} B &= \mu H \\ I &= KH \\ B &= 4\pi I + H \\ \mu &= 4\pi K + 1 \end{aligned}$$

Magnetization curves are of great importance in the design of magnetic ructures and should be immediately available for all materials with hich one intends to work. These curves may give either the values of

B as a function of H for the material, or the values of I as a function H. A typical B-H curve is shown in Fig. 6. The ratio of the coordinat of a B-H curve gives the va ue of μ for the material at t particular value of H chose The ratio of the coordinat in an I-H curve similar gives the value of the su ceptibility K.

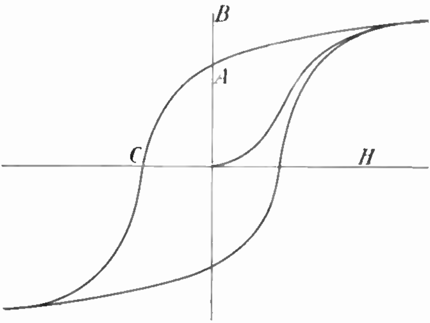


FIG. 6.—Typical B-H curve.

Magnetic *saturation* is phenomenon occurring large values of H when th induction B increases at much lower rate with i crease of H than is the ca for small values of H.

The *retentivity* of a su stance is the value of B in th material when the field H reduced to zero after havi first been raised to above its saturation value. It is given by the point of the B-H curve of Fig. 6.

The *coercivity* of a material is the minimum negative value of required to just reduce the induction to zero after the field strength has first been raised to a positive value sufficiently large to saturate th material. It is given by the point C of the B-H curve of Fig. 6.

27. Magnetic Properties of Iron and Steel.

Material	Coercivity	Retentivity	Maximum permeability	$4\pi I$ at saturation
Electrolytic iron	2.83	11,400	1,850	21,620
Annealed	0.36	10,800	14,400	21,630
Annealed electrical iron in sheets	1.30	9,400	3,270	20,500
Cast steel	1.51	10,600	3,550	21,420
Annealed	0.37	11,000	14,800	21,420
Steel hardened	52.4	7,500	110	18,000
Cast iron	11.4	5,100	240	16,400
Annealed	4.6	5,350	600	16,800
Tungsten magnet steel	64.0	9,600	105	13,600
Chrome magnet steel	64.0	9,000	94	12,600
Cobalt steel (15 per cent)	192.0	8,000		

28. Electromagnetic Structures. In this type of structure the ma netic material is usually very soft; its coercivity is very low; and as consequence the m.m.f. must be supplied by a continuous electric curren The m.m.f., M, due to an electric current, is given by the equatio $M = 0.4\pi NI$, where I is the current in amperes, and N is the number turns on the electromagnet.

By our most fundamental relation for magnetic circuits

$$M = \phi R$$
$$0.4\pi NI = R\phi$$
$$NI = \frac{R\phi}{0.4\pi}$$

The design of a magnetic structure is usually begun by a consideration the flux requirements in a particular air gap. The size and shape of e air gap are generally given, and the flux density desired in the air p is known. From these data one can compute R and ϕ. For the ıantity ϕ, $\phi = BA$, where A is the area of the air gap and B is the flux nsity desired. This equation assumes no leakage flux, and since this a condition never realized in practice and from which there may be a r from negligible departure, one must add to the value of ϕ given by is equation a correction the value of which is dictated by experience. ır the quantity R, $R = L/A$, where L is the length of the air gap and is the area. This equation neglects the reluctance of the magnet elf and of all other iron parts of the magnetic circuit. Since all uctances but that residing in the air gap are very small in comparison, is procedure is usually justified, although there are cases in which ditional reluctance must be taken into account. In such cases the uctance of the other parts of the circuit is computed in the same manner that of the air gap, except that an estimate of the permeability of e part in the circuit in question must be made and its equivalent air-gap uctance computed by dividing by this permeability. Finally,

$$NI = \frac{R\phi}{0.4\pi} = \frac{LBA}{0.4\pi A} = \frac{LB}{0.4\pi}$$

ıis equation then completely determines the value of the ampere-rns NI from the original data. This is the important quantity in the sign of the electromagnet. The separate values of N and I are unde-·mined by this equation, other considerations such as the nature of e current supply, the size of the coil, the heat dissipation that can be rmitted and the cost being of paramount importance.

29. Permanent-magnet Type of Magnetic Structure.[1] One begins e consideration of this type of structure with the dimensions of the · gap and the flux density in the air gap as fixed quantities. As a st choice select the material of the magnet and its desired length and ›ss section. These choices are based principally on considerations of st, on the amount of available space, and, to a large extent, on previous perience. Calculate the quantity tan θ in the equation

$$\tan\,\theta = \frac{A_mL_a}{A_aL_m}$$

.ere A_m and A_a are the cross-sectional areas of the magnet and the air ɔ, and L_m and L_a the lengths of the magnet and air gap, respectively. .is value of tan θ defines the quantity θ shown above and therefore gives e magnetic flux density B in the magnet. In this way various values tan θ and θ should be obtained by selecting various values of A_m d L_m keeping, however, the product A_mL_m and thus the quantity of e magnet material constant. A certain value of A_m and L_m will be ınd to yield a maximum value of BA_m, the total flux delivered by the .gnet. This particular value of A_m and L_m gives the most economical .gnet yielding the total flux BA_m to the air gap. The flux density the air gap is then BA_m/A_s.

[1] *Radio Broadcast*, March, 1930, p. 265.

If this flux density is less than desired, a larger magnet or a magnet different material is required. If it is too large, a smaller magnet ma be used. Again there is neglected leakage flux, a by no means negligil quantity. However, these equations constitute only a guide, and will be necessary to depend upon experience with magnetic circui when taking into account factors neglected in the use of these equatio

RADIATION

30. Nature of Radiation. Electromagnetic energy may arise fro continuously varying electronic currents in a conductor, displaceme currents, or oscillating dipoles. In order that this energy may appreciable it is necessary that the system of conductors be of such form that the electromagnetic field will not be confined in any way a that the frequency of oscillation of the current or charges be high. T various forms of antennas and the employment of radio frequenci satisfy these requirements.

The nature of radiation may be understood only after a comple examination of Maxwell's equations and the various transformatio of the wave equation. Any attempt to give a simple yet accurate pictu of the phenomenon of radiation must be fruitless, though such pictu may aid in an understanding of the subject. Such descriptions m be found in any text on radio. An exact analysis of Maxwell's equatio shows that whenever an electric wave moves through space an associat magnetic wave having its vectors at right angles to that of the electi wave must accompany it. Both vectors, furthermore, are at rig angles to the direction of propagation. This analysis also shows th an electromagnetic field due to an oscillating dipole or to an oscillati current in a conductor has two components. One of these varies inverse as the first power of the distance from the source and is, furthermo directly proportional to the frequency, and the other varies inverse as the second power of the distance. The former is known as t *radiation field* and the latter as the *induction field*. Though ind tinguishable physically, the induction and radiation fields have a separa mathematical existence accounting completely for the phenomenon energy radiation. The energy of the induction field returns to the co ductor with the completion of each cycle. Its existence is confined, one might expect, to the neighborhood of the conductor, whereas t radiation field may be thought of as a detached field traveling outwa into space with the velocity of light and varying much more slowly intensity with distance from the conductor than the other.

31. Vertical Antenna. The most simple form of antenna is the verti wire. The electromagnetic radiation field depends on the strength the current in the wire, and as a consequence its intensity is increas if the current throughout the vertical wire is uniform. It is for th reason that a counterpoise is usually attached to the lower end of t antenna and a horizontal aerial to the upper end. The capacity the counterpoise and aerial may be made so high that the current throug out the vertical portion of the wire is practically uniform.

Under these conditions the magnetic field at any distant point is given the equation

$$H = -\frac{\omega h I_0}{10cl} \cos \omega\left(t - \frac{l}{c}\right) \text{ gauss}$$

here $\omega = 2\pi f$
f = frequency of oscillation
I_0 = maximum value of the current in the antenna
c = velocity of light in centimeters per second in vacuum
l = distance from the source in centimeters
h = height of antenna or length of vertical wire in centimeters

nd

$$E = -\frac{300\omega h I_0}{10cl}\cos\omega\left(t - \frac{l}{c}\right)\text{ volts}$$

hese equations[1] are derived by considering the antenna as an oscillating ertzian doublet of separation h. The effective values of the magnetic and ectric fields are

$$H_e = -\frac{\omega h I_e}{10cl} = -\frac{2\pi h I_e}{10\lambda l}$$

$$E_e = -\frac{300\omega h I_e}{10cl} = -\frac{600\pi h I_e}{10\lambda l}$$

here I_e is the effective value of the antenna current, and λ is the wave length the electromagnetic wave.

32. Loop Antenna. The field due to a loop antenna is given by the uations

$$H_e = \frac{4\pi h I_e}{10\lambda l}\sin\frac{\pi s}{\lambda}$$

$$E_e = \frac{1{,}200\pi h I_e}{10\lambda l}\sin\frac{\pi s}{\lambda}$$

here s is the distance of separation of the vertical portions of the loop in ntimeters.

33. Coil Antenna. For a coil of N turns having negligible capacity tween turns at the frequency considered so that the current in all turns substantially the same, the field is given by the equations

$$H_e = \frac{4\pi N h I_e}{10\lambda l}\sin\frac{\pi s}{\lambda}$$

$$E_e = \frac{1{,}200\pi N h I_e}{10\lambda l}\sin\frac{\pi s}{\lambda}$$

34. The fundamental and harmonic frequencies of oscillation in an itenna may be calculated in many cases. If the inductance and pacity of the vertical wire of the antenna are neglected, the low frequency pacity and inductance are given by the equations[2]

$$C = lC_i$$

$$L = \frac{l}{3}L_i$$

here C_i and L_i are the capacity and inductance per unit length of conictor, and l is the length of conductor. These equations may be lculated by means of accurate formulas which are available.[3]

Then the low-frequency reactance of the antenna is

$$X_l = \frac{\omega l L_i}{3} - \frac{1}{\omega l C_i}$$

[1] Berg, "Electrical Engineering," Advanced Course, pp. 278 *ff.*; Morecroft, "Princies of Radio Communication," p. 706.
[2] *Bur. Standards Circ.* 74, pp. 72 *ff.*
[3] *Bur. Standards Circ.* 74, pp. 237–243.

The high-frequency reactance of the antenna is given by the equati

$$X_h = -\sqrt{\frac{L_i}{C_i}} \cot \omega l\sqrt{L_iC_i}$$

The reactance of the antenna becomes zero when

$$\omega l\sqrt{C_iL_i} = n\frac{\pi}{2}(n = 1, 3, 5 \cdots)$$

that is, when

$$f = \frac{\omega}{2\pi} = \frac{n}{4l\sqrt{C_iL_i}}$$

The reactance becomes infinite when

$$\omega l\sqrt{C_iL_i} = m\frac{\pi}{2}(m = 0, 2, 4 \cdots)$$

that is, when

$$f = \frac{\omega}{2\pi} = \frac{m}{4l\sqrt{C_iL_i}}$$

If the inductance of the vertical wire is to be considered, or if a series ind tance is used with the antenna

$$X = \omega L_s - \sqrt{\frac{L_i}{C_i}} \cot \omega l\sqrt{C_iL_i}$$

where L_s is the total inductance of the vertical wire and any coils in se with the antenna.

The harmonic frequencies of the antenna at which the reactance is z do not differ by multiples of π as before. The natural frequency of osci tion is given, however, quite generally by the equation

$$\omega L_s - \sqrt{\frac{L_i}{C_i}} \cot \omega l\sqrt{C_iL_i} = 0$$

$$\frac{\cot \omega l\sqrt{C_iL_i}}{\omega\sqrt{C_iL_i}} = \frac{L_s}{L_i}$$

35. Antenna Resistance. The resistance of an antenna may be divic into three parts in which the power dissipation is of the following kin

1. Radiation.
2. Joule heat.
3. Dielectric absorption.

The power radiated depends on the form of the antenna. It is prop tional to the square of the frequency of oscillation and to the square the current flowing in the antenna. Due to the latter consideration may write $P = AI^2$, where A is a constant factor depending on the fo of the antenna and the frequency. It may be called the radiat resistance. For a given antenna the radiation resistance varies invers as the square of the wave length. The ohmic resistance to which joule heat is due is approximately constant, the skin effect and ot factors being comparatively small. The resistance due to dielec absorption is directly proportional to the wave length. When these th components of resistance are added to obtain the total resistance, finds that for every antenna there is a wave length for which the to resistance is a minimum.

36. Energy in the Field. The energy of an electromagnetic field at ny point is given by the equation[1]

$$U = \frac{1}{8\pi}(\epsilon E^2 + \mu H^2)$$

here E is in electrostatic units instead of volts as in the previous equaions, ϵ is the dielectric constant, and μ the permeability of the medium. n free space

$$U = \frac{1}{8\pi}(E^2 + H^2)$$

But, in general,

$$H = \sqrt{\frac{\epsilon}{\mu}}E$$

$$U = \frac{\epsilon}{4\pi}E^2 = \frac{\mu}{4\pi}H^2$$

$$= \frac{E^2}{4\pi} = \frac{H^2}{4\pi} \text{ in free space.}$$

The energy flux through 1 sq cm of surface, perpendicular to the irection of propagation, is given by the equation

$$S = vU = \frac{c}{\sqrt{\epsilon\mu}}U = \frac{c}{4\pi}\sqrt{\frac{\epsilon}{\mu}}E_e^2 = \frac{c}{4\pi}\sqrt{\frac{\mu}{\epsilon}}H_e^2$$

$$\left.\begin{aligned} &= \frac{c}{4\pi}E_e^2 = \frac{c}{4\pi}H_e^2 \\ &= \frac{c}{8\pi}E_m^2 = \frac{c}{8\pi}H_m^2 \end{aligned}\right\} \text{in free space.}$$

here E_e and H_e represent effective values, and E_m and H_m the maximum alues of the electric and magnetic fields respectively. Therefore, for he effective values of the electric and magnetic fields due to a vertical ire antenna,

$$E_e = -\frac{2\pi h I_e}{10\lambda l} \text{ e.s.u.}$$

$$H_e = -\frac{2\pi h I_e}{10\lambda l}$$

$$S = \frac{c}{4\pi}\left(\frac{2\pi h I_e}{10\lambda l}\right)^2 = \frac{c\pi h^2 I_e^2}{10^2\lambda^2 l^2}$$

hen the total radiation from a vertical antenna, assuming that H has s maximum value in the equatorial plane of the antenna and that its ariation in a vertical plane at a distance l from the antenna follows a ne law, is given by the expression

$$2\pi l^2\left(\frac{c\pi h^2 I_e^2}{10^2\lambda^2 l^2}\right) \text{ ergs per second}$$

r

$$\frac{60\pi^2 h^2 I_e^2}{\lambda^2} \text{ watts}$$

[1] JEANS, J. H., "Mathematical Theory of Electricity and Magnetism," p. 518.

SECTION 3

RESISTANCE

By Jesse Marsten, B.S.[1]

1. General Concepts. In any electrical conductor or system in whic there is a flow of current there is a certain amount of energy continuall being lost or converted into forms not readily available for use. As fa as is known at present this dissipation of energy may take one of tw forms: there may be an evolution of heat, and there may be radiatic of energy into space. Such energy dissipation is attributed to a propert of electric conductors or systems termed *resistance*.

When dealing with continuous currents, the resistance of a conducto or network, R, is adequately defined by *Ohm's law*,

$$E = iR \qquad (\!$$

where E is the voltage drop across the conductor or network and i the current through it. This assumes no back e.m.f. due to polarizatic or other causes. In this case the dissipation of energy takes place entire in the form of heat generation, and the rate at which electrical energ is thus converted into heat is given by *Joule's law*,

$$P = i^2R \qquad (\!$$

where P is the power or rate at which electrical energy is being dissipate in the form of heat, i is the continuous current in the circuit, and R th resistance of the circuit.

Ohm's law is insufficient to define resistance in a-c circuits. It found experimentally that the rate at which heat is evolved in a circu exceeds that which would be necessitated by the resistance of the circu as determined by Ohm's law. This is due to the fact that the electr magnetic and electrostatic fields around the circuit vary with time ar introduce effects which increase the losses in the circuit. Among the effects may be enumerated the following major ones:

1. *Eddy-current* losses in conductors and other masses of metals in and ne the circuit.
2. *Hysteresis* losses in magnetic materials.
3. *Dielectric* losses in the insulating mediums.
4. *Absorption* of energy by neighboring conductors or circuits by inductio
5. *Radiation* of electromagnetic energy into space.
6. *Skin Effect.* Increase of conductor resistance due to non-unifor current density.

[1] Member, Institute of Radio Engineers; associate member, American Institute Electrical Engineers, chief engineer, International Resistance Company.

All these effects result in an increase in energy loss in the circuit over and above that given by Ohm's law. It therefore becomes necessary to introduce the concept of *a-c resistance* or *effective resistance*, which is defined by the more general joulean relationship,

$$P = i^2R \text{ effective} \tag{3}$$

where P is the power loss in the circuit due to all causes and i is the effective current in the circuit. Ohm's law for continuous currents follows directly from this more general definition.

2. Units of Resistance. The practical unit of resistance is the *ohm* and is defined by Ohm's law when the voltage and current are unity in the practical system. It has, however, been arbitrarily defined as the resistance at 0°C. of a column of mercury having a uniform cross section, a height of 106.3 cm, and weighing 14.4521 g. Owing to the increasing use of resistors having resistances of the order of millions of ohms, the *megohm* unit is employed. The megohm is equal to 10^6 ohms.

3. Specific Resistance. It is found experimentally that the resistance of an electric conductor is directly proportional to its length and inversely to its cross section:

$$R = \rho \frac{l}{A} \tag{4}$$

The proportionality factor ρ is called the *specific resistance* of the conductor and is a function of the material of the conductor.

From this definition of specific resistance it is apparent that any number of units may be derived for specific resistance, depending upon the units chosen for l and A. The unit generally employed in practical engineering is the *ohms per circular mil foot*, and is the resistance of a 1 ft. length of the conductor having a section of 1 cir. mil (diameter 1 mil for a circular conductor).

4. Volume Resistivity. If, in the above definition, l and A are both unity, in the same system of units, then ρ is the resistance of a unit cube of the material and may be defined as the *volume resistivity* of the material. It should be noted that volume resistivity is not the resistance of any unit volume of the material but is specifically the resistance of unit volume measured across faces whose areas are each unity.

With a knowledge of the dimensions of a conductor and its specific resistance the resistance of the conductor to d.c. may be computed from Eq. (4). Consistent units must be employed. The resistance thus computed will be correct at the temperature for which the specific resistance applies. To obtain the resistance of the conductor at any other temperature a correction will have to be applied.

5. Temperature Coefficient. The resistance of a conductor is a function not only of the material and dimensions of the conductor but also of its temperature. Within the temperature limits generally encountered in practice the change in resistance due to temperature variation is directly proportional to the change in temperature:

$$R_{t_2} = R_{t_1}[1 + \alpha(t_2 - t_1)] \tag{5}$$

R_{t_1} and R_{t_2} are the conductor resistances at temperature t_1 and t_2 respectively.

The proportionality factor α is defined as the *temperature coefficient c resistance* of the material and is the change in resistance of any materia per ohm per degree rise in temperature.

All conductors do not react alike to changes in temperature. Metal for example, have a positive temperature coefficient. Some alloy such as manganin and constantan, have practically zero temperatur coefficient and are therefore used primarily for resistance standards.

A knowledge of the temperature coefficient of conductor material enables one at times to make more accurate determinations of tem perature change than is possible by thermometer measurements, especiall in cases where parts to be measured are not readily accessible. Resist ance determinations of the conductor are made at both temperature and the temperature change computed from Eq. (5).

6. Properties of Materials as Conductors.

Material	Specific resistance at 0°C., ohms per cir. mil ft.	Temperature coefficient per °C. between 20 to 100°C., ohm per °C.
Silver	9.75	0.004
Copper	10.55	0.004
Aluminum	17.3	0.0039
Nickel (pure)	58.0	0.0041
Iron (pure)	61.1	0.0062
Phosphor bronze	70.0	0.004
Lead	114.7	0.0041
Nickel silver, 18 per cent (German silver)	180 to 190	0.00027
Manganin (copper, 82 per cent; manganese, 14 per cent; nickel, 4 per cent)	290	0.00002
Constantan (Advance, Cupron, Ideal, Ia-Ia) (copper, 55 per cent; nickel, 45 per cent)	294	0.00002
Nichrome (nickel, 60 per cent; chromium, 15 per cent; iron, balance)	650 to 675	0.0001 to 0.0001

7. Resistors in Series and Parallel. Simple and complex networks c resistors may be represented by an equivalent resistor which may b expressed in terms of the individual resistances making up the network

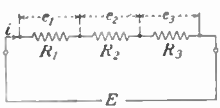


FIG. 1.—Simple series circuit.

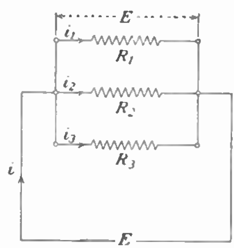


FIG. 2.—Parallel circuit.

The equivalent resistance of a number of resistors connected in serie is equal to the sum of the individual resistances. Referring to Fig. 1:

$$= iR_{\text{equiv.}} = e_1 + e_2 + \cdots + e_n = R_1 i + R_2 i + \cdots + R_n i = i(R_1 + R_2 + \cdots + R_n)$$

$$= R_{\text{equiv.}} = (R_1 + R_2 + \cdots + R_n)$$

$$R_{\text{equiv.}} = \sum_1^n R$$

The reciprocal of the equivalent resistance of a number of resistors onnected in parallel is equal to the sum of the reciprocals of the indi- dual resistances. Referring to Fig. 2:

$$i = i_1 + i_2 + \cdots + i_n = \frac{E}{R_1} + \frac{E}{R_2} + \cdots + \frac{E}{R_n}$$

$$\frac{i}{E} = \frac{1}{R_{\text{equiv.}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

$$\frac{1}{R_{\text{equiv.}}} = \sum_1^n \frac{1}{R}$$

RESISTANCE AS FUNCTION OF FREQUENCY

8. Skin Effect. It may be shown that the resistance of a conductor a minimum when the current density is uniformly distributed over the oss section of the conductor. This condition obtains for d.c. The sistance increases for non-uniform distribution of current density over e cross section of the conductor. This latter condition obtains in onductors carrying a.c. This is a result of the distribution of magnetic- ux lines, outside and inside the conductor. If the conductor is assumed be made up of a number of conducting elements in parallel, then the terior elements, being surrounded by more flux lines than the exterior, ill have greater reactance and, therefore, the current in the interior ements will be less than that in the exterior elements. As a result the urrent crowds toward the surface of the conductor, giving a non- niform current density. This imperfect penetration of current in a onductor, resulting in an increase in resistance, is termed *skin effect.* Skin effect in a conductor is a function of the following factors:

$$t\sqrt{\frac{\mu f}{\rho}} \qquad (6)$$

here t = thickness of the conductor
f = frequency of current
μ = permeability of the conductor
ρ = specific resistance of the conductor.

It is possible to compute accurately the h-f resistance of simple round ylindrical conductors from involved functions of the above factor. To cilitate these computations tables have been prepared from which e ratio of h-f resistance R_f to d-c resistance R_0 may be quickly deter- ined. From this factor and the easily measured d-c resistance the h-f sistance may be computed.

The table below gives the values of R_f/R_0 for different values of th factor

$$x = \pi d \sqrt{\frac{2\mu f}{\rho}}$$

where d is the diameter of the wire in centimeters, x may be computed fo any particular case, and R_0 may be measured at d.c. or compute Knowing x and R_0, R_f may be determined.

9. Ratio of H-f Resistance to the D-c Resistance for Different Value of $x = \pi d \sqrt{2\mu f/\rho}$.

x	R_f/R_0	x	R_f/R_0	x	R_f/R_0
0	1.0000	5.2	2.114	14.0	5.209
0.5	1.0003	5.4	2.184	14.5	5.386
0.6	1.0007	5.6	2.254	15.0	5.562
0.7	1.0012	5.8	2.324		
0.8	1.0021	6.0	2.394	16.0	5.915
0.9	1.0034	6.2	2.463	17.0	6.268
				18.0	6.621
1.0	1.005	6.4	2.533	19.0	6.974
1.1	1.008	6.6	2.603	20.0	7.328
1.2	1.011	6.8	2.673		
1.3	1.015	7.0	2.743	21.0	7.681
1.4	1.020	7.2	2.813	22.0	8.034
1.5	1.026	7.4	2.884	23.0	8.387
				24.0	8.741
1.6	1.033	7.6	2.954	25.0	9.094
1.7	1.042	7.8	3.024		
1.8	1.052	8.0	3.094	26.0	9.447
1.9	1.064	8.2	3.165	28.0	10.15
2.0	1.078	8.4	3.235	30.0	10.86
				32.0	11.57
2.2	1.111	8.6	3.306	34.0	12.27
2.4	1.152	8.8	3.376		
2.6	1.201	9.0	3.446	36.0	12.98
2.8	1.256	9.2	3.517	38.0	13.69
3.0	1.318	9.4	3.587	40.0	14.40
				42.0	15.10
3.2	1.385	9.6	3.658	44.0	15.81
3.4	1.456	9.8	3.728		
3.6	1.529	10.0	3.799	46.0	16.52
3.8	1.603	10.5	3.975	48.0	17.22
4.0	1.678	11.0	4.151	50.0	17.93
				60.0	21.47
4.2	1.752	11.5	4.327	70.0	25.00
4.4	1.826	12.0	4.504		
4.6	1.899	12.5	4.680	80.0	28.54
4.8	1.971	13.0	4.856	90.0	32.07
5.0	2.043	13.5	5.033	100.0	35.61

It is frequently useful to know the largest diameter of wire of differen materials which will give a ratio of R_f/R_0 of 1.01 for different frequencie For a ratio of R_f/R_0 equal to 1.001, the diameters given below shoul be multiplied by 0.55; and for R_f/R_0 equal to 1.1, the diameters should b multiplied by 1.78.

10. Maximum Diameter of Wires for R-f Resistance Ratio of 1.01.

Frequency, kilocycles	100	200	400	600	800	1,000	1,200	1,400	1,600	1,800	2,000	3,000
Wave length, meters	3,000	1,500	750	500	375	300	250	214.3	187.5	166.7	150	100
Material	Diameter, centimeters											
Copper	0.356	0.0251	0.0177	0.0145	0.0125	0.0112	0.0102	0.0095	0.0089	0.0084	0.0079	0.0065
Silver	0.0345	0.0244	0.0172	0.0141	0.0122	0.0109	0.0099	0.0092	0.0086	0.0082	0.0077	0.0063
Gold	0.0420	0.0297	0.0210	0.0172	0.0149	0.0133	0.0121	0.0112	0.0105	0.0099	0.0094	0.0077
Platinum	0.1120	0.0793	0.0560	0.0457	0.0396	0.0354	0.0323	0.0300	0.0280	0.0264	0.0250	0.0205
Mercury	0.264	0.187	0.132	0.1080	0.0936	0.0836	0.0763	0.0706	0.0661	0.0623	0.0591	0.0483
Manganin	0.1784	0.1261	0.0892	0.0729	0.0631	0.0564	0.0515	0.0477	0.0446	0.0420	0.0399	0.0325
Constantan	0.1892	0.1337	0.0946	0.0772	0.0664	0.0598	0.0545	0.0506	0.0473	0.0446	0.0423	0.0345
German silver	0.1942	0.1372	0.0970	0.0792	0.0692	0.0614	0.0560	0.0518	0.0485	0.0458	0.0434	0.0354
Graphite	0.765	0.541	0.383	0.312	0.271	0.242	0.221	0.204	0.191	0.180	0.171	0.140
Carbon	1.60	1.13	0.801	0.654	0.566	0.506	0.462	0.428	0.400	0.377	0.358	0.292
Iron												
μ = 1,000	0.00263	0.00186	0.00131	0.00108	0.00094	0.00083	0.00076	0.00070	0.00066	0.00062	0.00059	0.00048
μ = 500	0.00373	0.00264	0.00187	0.00152	0.00132	0.00118	0.00108	0.00100	0.00093	0.00088	0.0084	0.00068
μ = 100	0.00838	0.00590	0.00418	0.00340	0.00295	0.00264	0.00241	0.00223	0.00209	0.00197	0.00186	0.00152

11. Reduction of Skin Effect. In view of the tendency of the current to crowd to the surface of the conductor at high frequencies, the remedies which have been found practical in effecting an improvement in the resistance ratio R_f/R_0 have been those in which the conductor has been designed so that it presents a skin to the current flow. These are:

1. *Use of Flat Copper Strip.* While skin effect is present, for the same cross-sectional area a flat strip gives a lower resistance ratio than do round conductors.

2. *Use of Tubular Conductors.* Here the external magnetic field is much greater than the internal field, and therefore all parts of the conductor are affected alike by the field, thus reducing the skin effect.

3. *Use of Litzendraht.* According to Eq. (6) the smaller the diameter of the wire the less the skin effect. Litzendraht is a braided cable made up of a large number of fine strands of wire. When certain precautions are taken this braid shows a very much lower resistance ratio than does a solid copper wire of equal section. These precautions are:

a. Each strand must be thoroughly insulated from every other strand to avoid contact resistance.

b. Braiding must be such that each strand passes from the center to the outside of the conductor at regular intervals—a sort of transposition. This insures that all strands are affected alike by the magnetic flux.

c. Each strand must be continuous.

12. Types of Resistors. Resistors generally used in radio and allied applications may be broadly classified as:

1. Fixed resistors.
2. Variable resistors.

Each of these groups may be further classified on the basis of the natur of the conducting material of the resistor, as:

1. Wire wound.
2. Composition (employing carbon).

13. Fixed Wire-wound Resistors. As commonly employed, thes are wound on strips of fiber or bakelite, and on ceramic forms. Th former are used where the power-dissipation requirements are generall negligible, for example, as center-tapped resistors across vacuum-tub filaments for hum balancing. Resistors wound on ceramic forms ar generally used where the power requirements exceed 2 or 3 watts. Sucl resistors are made with a protective coat of enamel or cement baked ove the winding, thus affording a measure of protection against mechanica injury and penetration of moisture. The characteristics of the wire wound resistor are those of the particular wire employed and generall show a negligible or slight temperature coefficient and no voltage coeffi cient, that is, the resistance is independent of the applied voltage.

14. Variable Wire-wound Resistors. These are usually of the con tinuously variable type, made by winding resistance wire on a fla strip of fiber, bakelite, or other insulating material. This strip may b formed into an arc and placed in a protecting container. A metalli sliding arm is arranged to travel over the winding, thus making contac with each turn as it is rotated. The choice of wire and size is determine by the range and space requirements.

In general, wire-wound continuously variable resistors are wound s that the resistance changes uniformly with the motion of the slidin contact. For certain uses, as, for example, antenna-type volume con trols, it is desirable that the resistance change be non-uniform. I this case the form on which the wire is wound is sometimes tapered s that the resistance per degree rotation is not constant. Other methods c tapering employed are winding with variable pitch, winding sections c the control with different sizes of wire, and copper plating start an finish of the winding.

Some of the factors to be considered in design are:
1. Contact between slider and resistor element should be positive.
2. Winding should not become loose on the form.
3. Sliding contact should not wear away resistance wire.
4. Resistance change per turn should be as small as possible.
5. Slider material should be such that it will not oxidize.

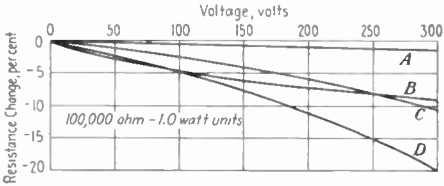
15. Composition-type (Radio) Resistors. The term *composition-typ* resistor is employed to cover that group of resistors in which a conducto is mixed with binder in definite proportions and suitably treated t produce a resistor material. This type of resistor has attained a wid popularity because of the following advantages: (1) Flexibility in range– it may be made in any value up to several megohms; (2) compactness– its physical dimensions are small for any range; they may be made i sizes down to $\frac{3}{16}$ by $\frac{1}{2}$ in. or smaller.

Numerous types of these resistors have been produced, but they tak two general forms:

1. *Solid-body Resistor.* In this type the resistor material is extrude pressed or molded into its final physical form, which generally is a solid ro after which it may be subjected to some form of heat treatment. Th so-called *carbon* resistors are examples of this type.

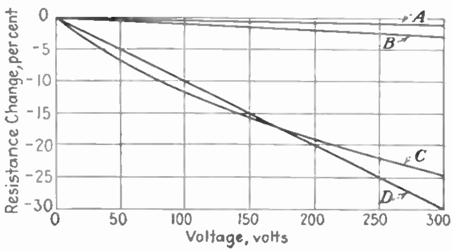
2. *Filament-coated Resistors.* In this type a conducting coat is baked n the surface of a continuous glass filament or other form. In the case of he glass filament this is completely enclosed in an insulating tube. The o-called *metallized-filament* resistors are examples of this type.

16. Characteristics of Composition-type Resistors. Composition-ype (commercially known as *radio*) resistors possess properties differing



'IG. 3*a*.—Voltage characteristic of various resistors. Curve *A* is metallized-filament type; others are carbon type.

'ery markedly from those of metallic resistors. The most important nes are as follows, and are possessed by all these types in varying degree:



'IG. 3*b*.—Voltage coefficient of 1.0 megohm units. Curve *A* is metallized-filament type; others are carbon type.

a. Voltage Characteristics. The resistance is not independent of the applied 'oltage and generally falls with increasing voltage. Typical curves showing

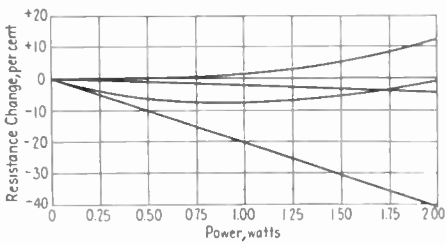


FIG. 4.—Load characteristic of 100,000 ohm-1.0 watt resistors.

he manner in which the resistance varies with voltage (heating effect due to oad not present or corrected for) are shown in Figs. 3*a* and *b*.

b. Load Characteristics. The characteristics shown in Fig. 4 show how the esistance varies with load, the load being applied for about fifteen minutes to permit steady state to be reached. No specific trend is present, so[me] types showing a negative change, some a positive one, and others chan[ge] from a negative to a positive coefficient.

c. Humidity Characteristics. The effect of humidity in general is to ca[use] a rise of resistance. This effect may sometimes be reduced by suitab[le] treatment.

d. Noise. These types of resistors all show, in varying degree, the presen[ce] of microphonic noise. The degree of noise is a function of the load.

17. Variable Carbon-type Resistors. In numerous radio applicatio[ns] high variable resistors are required, for example, for controlling t[he] sensitivity of a receiver by varying the C-bias on the r-f tubes a variab[le] resistor up to 50,000 ohms maximum is commonly employed. F[or] adjusting the audio-signal level in automatic volume-control sets [a] variable resistor up to 0.5 megohm is not uncommon. From the poi[nt] of view of cost, wire-wound resistors of this order of magnitude a[re] prohibitive. Furthermore it is desirable to have a non-uniform rate [of] change of resistance with respect to angular rotation, which is ve[ry] difficult to secure with wire-wound resistors. Carbon or graphitic typ[es] of variable resistors which are capable of being made to meet the[se] requirements at reasonable cost are therefore widely used. Su[ch] resistors generally consist of a resistive solution applied to some fl[at] form, such as paper or ceramic, and baked on. A rotating slider or so[me] other form of contact travels over this resistive element producing [a] continuous variation of resistance. Since the resistor is essential[ly] painted on the form, its geometrical form may be varied by desig[n]. Also different concentrations of the resistor ink or paint may be employ[ed] at different positions of the resistor element. By the use of these tw[o] expedients the resistor may be designed to give any variation of resistan[ce] desired.

18. Rating Wire-wound Resistors. In view of the low temperatu[re] coefficient of the resistance wires generally employed in radio wire-wou[nd] resistors, the resistance change with loads normally encountered is sma[ll]. The rating is, therefore, primarily determined by the power the resist[or] can dissipate continuously for an unlimited time without excessive tem[-] perature rise or deterioration of the resistor. Some manufacturers ra[te] resistors on the basis of the power that will produce a temperature rise [of] 250°C. in an ambient temperature of 40°C., when the resistor is mount[ed] in free air. Such perfect ventilation conditions are seldom encountere[d]. As a result, it is generally recommended that such resistors be used [at] one-fourth to one-half the nominal rating, which results in a temperatu[re] rise of 100°C. to 150°C. In practice even these temperature rises ma[y] be excessive owing to such factors as poor ventilation, proximity [of] resistors to parts which may not be subjected to elevated temperature[s] and Fire Underwriter's approval. The specific application therefo[re] limits the practical use of a resistor rather than any nominal rating.

19. Rating Composition-type Resistors. The rating of compositio[n-] type resistors is a more complicated matter. The temperature coefficie[nt] of this type of resistor being larger, it is possible for a resistance change t[o] become quite appreciable, before a temperature limitation is exceede[d]. Furthermore, with the higher ranges, such as 0.25 megohm and over, i[n] which the power dissipation may be very low, the voltage characteristi[c] may be a determining factor instead of the load-carrying characteristic. It is therefore customary to rate this type of unit on the basis of the maxi[-] mum load it can carry, or the maximum voltage which can be applied t[o] it, without exceeding prescribed resistance changes. No definite stan[d-] ls have been set as yet, but designs are such that power dissipations of out 0.5 to 1 watt per square inch of radiating surface are employed.

20. Composition of Resistors. Radio resistors of the carbon and ıment types generally employ a conducting material of high specific sistance mixed with a filler and binder. The most widely used conduct- g material is some form of carbon or graphite. The fillers or binders ıployed vary with the type of resistor. Examples of these are clay, bber, and bakelite. The filler and conductor are mixed in various pro- rtions to obtain resistors with different ranges. The method of making e resistor varies also with its type. The solid-body types are generally her molded or extruded. The filament resistor is made by baking the sistance material on a glass rod which is sealed in a ceramic container.

21. R.M.A. Color Code. The use of resistors has increased to such an tent and so many are employed in adio set that it has become desirable identify each resistor for range in a ick and simple manner. Such entification simplifies assembly of ese units in radio sets and helps in rvicing. A color code has therefore en adopted by the Radio Manufacturers' Association as follows:

FIG. 5.—Standard resistor of R.M.A.

Ten colors shall be assigned to the figures as shown in the table:

igure	Color	Color to be equivalent to
0	Black	
1	Brown	Cable 60113
2	Red	Cable 60149
3	Orange	Cable 60041
4	Yellow	Cable 60187
5	Green	Cable 60105
6	Blue	Cable 60102
7	Violet	Cable 60010
8	Gray	Cable 60034
9	White	

Cable designations indicate the color shades as shown on the Standard Color Card of America, 8th ed., 1928, issued by the Textile Color Card Association of the United States.

The body *A* of the resistor shall be colored to represent the first figure of the resistance value. One end *B* of the resistor shall be colored to represent the second figure. A band, or dot, *C*, of color, representing the number of ciphers following the first two gures, shall be located within the body color. Two diagrams below illustrate zo interpretations of this standard, both of which are deemed to be in accordıce with the standard.

Examples illustrating the standard are as follows:

Ohms	*A*	*B*	*C*
10	Brown 1	Black 0	Black, no ciphers
200	Red 2	Black 0	Brown, one cipher
3,000	Orange 3	Black 0	Red, two ciphers
3,400	Orange 3	Yellow 4	Red, two ciphers
40,000	Yellow 4	Black 0	Orange, three ciphers
44,000	Yellow 4	Yellow 4	Orange, three ciphers
43,000	Yellow 4	Orange 3	Orange, three ciphers

22. Test Specifications. Up to the present time there has been agreement on a standard specification acceptable to all radio mar facturers. An illustration of specifications sometimes called for is giv here.

1. *Material.* The resistance material shall be carbon, graphite, metal, any other substance which will give resistors which meet the requireme of the specifications.

2. *Rating Wattage.* Unless otherwise specified the ratings of radio res tors shall be assumed to correspond with those in the following table:

Assumed rating, watts	Over-all length (±1/16 in.), inches	Diameter (±1/32 in.) inches	Leads: Length, ±1/8 in.; diameter, ±0.005 ir spacing, ±1/8 in.		
			Tinned, inches	Copper, inches	Wire, inches
0.25	1 1/16	1/4	1 1/2	0.032	1/2
0.50	1	1/4	1 1/2	0.040	1 1/16
1.00	1 3/4	1/4	1 1/2	0.040	1 1/2
2.00	2	7/16	1 1/2	0.040	1 5/8

3. *Resistance.* The resistance of the resistor shall, when measured by approved method, fall within the limits specified. There shall be no grea variation than one per cent between the resistance of a resistor obtained measuring with the current flowing through it in one direction and th obtained with the direction of current reversed.

4. *Life Test.* The permanence of the resistors shall be such that wh tested as here specified, the permanent change shall not be greater thar per cent when measured by the bridge method at 77°F. (25°C.). The t shall consist of intermittent operation at the rated wattage or voltage of t resistor, for 1,000 hr., each cycle consisting of a load period of 2 hr. anc no-load period of 30 min.

5. *Humidity.* Resistors shall be capable of passing Tests 3 and 4 af conditioning for 100 hr. in a humidifier operating at 38°C. and a relati humidity of 90 per cent. Free moisture shall be removed from the surfa of the resistor.

6. *Finish.* The finish shall consist of a smooth uniform coating of enar or lacquer in colors in accordance with the color code specified by the R.M. standards. The surface shall be reasonably free from stains, blisters, crac dirt, or other blemishes.

23. Representative Values of Resistors Employed in Radio Se The range of resistors usually employed in radio sets extends from 1 ol up to 10 megohms. These resistors are used for various purposes, su as providing grid bias to radio, audio, and detector tubes; plate couplir voltage dividers, and filters. Typical values employed for these vario applications are enumerated below:

1. Detector bias resistors	5,000 to 50,000 ohms
2. Power bias resistors	200 to 3,000 ohms
3. Voltage dividers	1,000 to 100,000 ohms
4. Plate-coupling resistors	50,000 to 250,000 ohms
5. Grid leaks	100,000 to 10 megohms
6. Filter resistors	100 to 100,000 ohms

References

Bur. Standards Circ. 74.
DRIVER-HARRIS COMPANY: *Bull.* R-28.
GILBY WIRE COMPANY: "Resistance Handbook."
KENNELLY, LAWS, and PIERCE: Experimental Research in Skin Effect in Conducto *Trans. A.I.E.E.*, 1915.
MORECROFT: "Principles of Radio Communication."
PENDER: "Handbook for Electrical Engineers."

SECTION 4

INDUCTANCE

By Gomer L. Davies, B.S.[1]

1. Magnetic Flux. The property of electrical circuits called *inductance* epends upon the magnetic effects associated with a flow of electric curent. In a magnetic system the magnitude of the force of magnetic traction or repulsion is proportional to the product of the strengths the poles and inversely proportional to the square of the distance etween them. A *unit magnetic pole* is defined as that pole which repels similar pole at a distance of 1 cm with a force of 1 dyne. The force etween two poles acts along the line joining the poles. Consequently unit north pole in the vicinity of a magnet is acted upon by two forces: ne of repulsion, due to the north pole of the magnet; and one of attracon, due to the south pole. The resultant is the total force exerted by ne magnet upon the unit pole. Thus the magnet is surrounded by a *eld of force* or *magnetic field* whose direction and magnitude at any point re defined as the direction and magnitude of the force acting upon a unit orth pole at that point.

If a unit north pole is allowed to move freely in a magnetic field, it will iove in the direction of the field at each point and will trace out a path hich is called a *line of force*. The total field is considered to be made up a large number of such lines. In any region of space the total of all ne lines of force in that region is called the *magnetic flux* in that region, nd the number of lines of force passing through a unit area of a surface erpendicular to the direction of the field is the *flux density* and is deterined by the strength of the field.

2. Magnetic Effects of Current-carrying Conductors. Magnetic fects are exhibited not only by magnets but also by wires carrying ectric currents. The magnetic field near a straight current-carrying onductor consists of circular lines of force surrounding the conductor; ne flux density at any point outside the wire is proportional to the curent and inversely proportional to the distance of the point from the xis of the conductor. If the wire carrying the current is wound in one r more layers on a cylindrical form, the field inside of this coil is parallel the axis of the cylinder and is proportional to the product of the curent and the number of turns on the coil. This product of current (in mperes) and number of turns is called the *ampere-turns* of the coil. The ux density along the axis of the coil may be expressed as the product of ne ampere-turns by a constant. If the winding is of infinite length, this onstant is 4π.

[1] Junior physicist, Radio Section, Bureau of Standards.

3. Inductance—Definition and Units.[1] When the current in a circu varies Ohm's law in the form in which it is stated for constant-curre circuits, no longer serves to define the current.

The magnetic flux associated with the circuit varies with the current a induces a voltage in the circuit which is given by the equation

$$e = -\frac{d\phi}{dt} \qquad ($$

where e is the induced voltage, ϕ the flux, and t the time. As the flux proportional to the current, it may be written

$$\phi = Li \qquad ($$

where L is a constant and i the current. Then

$$e = -\frac{d}{dt}(Li) = -L\frac{di}{dt} \qquad ($$

If the current is increasing, the induced e.m.f. opposes the current, an work must be done to overcome this e.m.f. If the work is W,

$$\frac{dW}{dt} = ei = -Li\frac{di}{dt} \qquad ($$

and

$$W = -\int_0^{i_0} Li\,di = -\frac{Li_0^2}{2} \qquad ($$

i_0 being the final value of the current, the initial value being taken as zero.

The quantity L in these equations is the *coefficient of self-inductio self-inductance*, or simply *inductance* of the circuit. It may be defined i three ways: from Eq. (2), as *the flux associated with the circuit when un current is flowing in it;* from Eq. (3), as *the back e.m.f. in the circuit caus by unit rate of change of current;* and from Eq. (5), as *twice the work do in establishing the magnetic flux associated with unit current in the circu* These three definitions give identical and constant values of L provide there is no material of variable permeability near the circuit, and pr vided the current does not change so rapidly that its distribution i the conductors differs materially from that of a constant current. these conditions do not hold, L is not constant and the values obtaine from the three definitions will in general be different.

The units used for inductance must conform to the units used for t other quantities used in the defining equations. The practical unit the *henry*, which is the inductance of a circuit when a back e.m.f. of volt is induced in the circuit by a current changing at the rate of 1 am per second. The relations between units are as follows:

$$\begin{aligned} 1 \text{ henry} &= 10^9 \text{ e.m.u.} \\ &= 1.1124 \times 10^{-12} \text{ e.s.u.} \end{aligned}$$

The henry is subdivided into two smaller units, the millihenry and th microhenry. The millihenry is one-thousandth of a henry, and th microhenry is one-millionth of a henry. The millihenry and microhenr are abbreviated mh and μh respectively. Thus

$$1 \text{ henry} = 1{,}000 \text{ mh} = 1{,}000{,}000\ \mu\text{h}$$

[1] STARLING, S. G., "Electricity and Magnetism," Chap. XI, 1926.

The term "inductance" refers to a property of an electrical circuit or ece of apparatus but not to any material object. A piece of apparatus sed to introduce inductance into a circuit is properly called an *inductor* · coil.

4. Current in Circuits Containing Inductance. If a circuit containing source of constant e.m.f. and pure resistance only is closed, the current ses instantly to its full value as determined by Ohm's law. If the circuit ntains inductance, a back e.m.f. of the value $L\frac{di}{dt}$ acts during the time ie current is changing, so that, if the e.m.f. of the source is E, the actual m.f. available to force current through the resistance is $E - L\frac{di}{dt}$.

The equation for the current in the circuit is

$$E - L\frac{di}{dt} = Ri \tag{6}$$

$$L\frac{di}{dt} + Ri = E \tag{7}$$

The solution of this equation is

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right) \tag{8}$$

he time t is reckoned from the instant at which the switch is closed, and ϵ the base of natural logarithms.

At a time $t = L/R$ after the circuit is closed, the current has a value equal · $I_0\left(1 - \frac{1}{\epsilon}\right)$, or about 63 per cent of its final value. The quantity L/R called the *time constant* of the circuit. The time constant, or the time quired for the current to rise to a value of $1 - \frac{1}{\epsilon}$ times its final value, does ıt depend upon the actual values of inductance and resistance but only ›on their ratio.

The current in such a circuit is shown in Fig. 1 for several values of $/R$. Theoretically the current does not reach its maximum value I_0

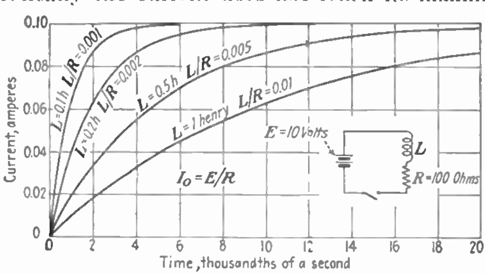


FIG. 1.—Rise of current in inductive circuit.

xcept at an infinite time after the circuit is closed, but practically the ifference between the actual current and the value I_0 becomes negligible :ter a relatively short time.

If, after the steady current I_0 has been established in the circuit, th source of the e.m.f. is short-circuited, the current does not fall to zer instantly but decreases according to the equation

$$i = \frac{E}{R}\epsilon^{-\frac{Rt}{L}} \tag{9}$$

This equation is plotted in Fig. 2 for the same values of the circu constants as were used in Fig. 1. In this case the time constant $L/$

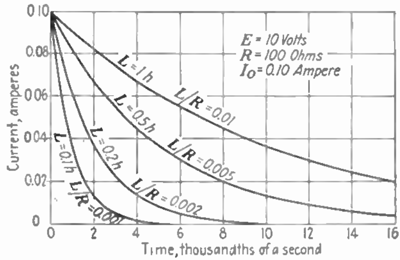


FIG. 2.—Fall of current in inductive circuit.

represents the time required for the current to fall to $1/\epsilon$ or about 37 pe cent of its initial value.

If, instead of the source of e.m.f. being short-circuited, the circuit i opened, the resistance becomes extremely large and the current falls t zero almost instantly. As a result of this rapid change of current, a larg e.m.f. is induced in the circuit, causing a spark or arc at the point a which the circuit is opened.

FIG. 3.—Series circuit containing resistance and inductance.

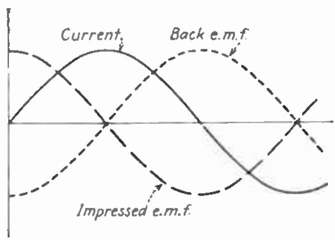


FIG. 4.—Phase relations in inductive circuit.

When the current in an inductive circuit is changing, a back e.m.f other than that due to resistance acts in the circuit. This back e.m.f is proportional to the current and to the quantity ωL, which is calle the *inductive reactance* and usually written X_L. Also, the phase of th back e.m.f. is 90 deg. behind that of the current. To force a curren through a pure inductance, therefore, requires an impressed e.m.f 180 deg. out of phase with the back e.m.f., or one leading the curren by 90 deg. (Fig. 4).

Now, if a sinusoidal e.m.f. is impressed on a circuit containing resistance ıd inductance in series (Fig. 3), the current in the circuit will also be ıusoidal, provided the resistance and inductance are independent of e current. The portion of the impressed e.m.f. required to force .rrent through the resistance will be in phase with the current, while the ›rtion required to force current through the inductance will lead the .rrent by 90 deg. The resultant phase of the impressed e.m.f. with spect to the current will have some value between zero and 90 deg., :pending upon the values of resistance and inductance in the circuit.

To determine mathematically the behavior of the circuit described ›ove, it is necessary to set up and solve the differential equation for e circuit. This equation will have the same form as Eq. (7) with E placed by $E_M \sin \omega t$; that is,

$$L\frac{di}{dt} + Ri = E_M \sin \omega t \tag{10}$$

The solution is

$$i = \frac{E_M}{\sqrt{R^2 + \omega^2 L^2}} \sin (\omega t - \phi) + c\epsilon^{-\frac{Rt}{L}} \tag{11}$$

ıere $\tan \phi = \omega L/R$, and c is a constant to be determined. The first rm is the only one of importance after the current has been flowing r a short time. Thus the current has a peak or maximum amplitude $E_M/\sqrt{R^2 + \omega^2 L^2}$ and lags the impressed e.m.f. by the phase angle ϕ ıose tangent is $\omega L/R$. The quantity $\sqrt{R^2+\omega^2 L^2}$ is called the *impedance* the circuit and is denoted by Z. In terms of the effective values of rrent and e.m.f. I and E, the equation for the current may be written

$$I = \frac{E}{Z} \text{ or } I_M = \frac{E_M}{Z} \tag{12}$$

In complex notation this form is

$$i = \frac{E_M \sin \omega t}{R + j\omega L} \tag{13}$$

, in terms of the instantaneous e.m.f.,

$$i = \frac{e}{R + j\omega L} = \frac{e}{z} \tag{14}$$

The quantity z is called the *complex* or *vector impedance*. It is a vector th a magnitude $\sqrt{R^2 + \omega^2 L^2}$ or Z, and angle ϕ whose tangent is $\omega L/R$. A ctor diagram showing these relations given in Fig. 5. Thus the relation tween current and e.m.f. in an a-c cirit containing resistance and inductce in series may be expressed in the me form as Ohm's law for d-c circuits, ›vided instantaneous values of current d voltage and vector impedance are ›d [Eq. (14)]. A similar relation may written using effective values of curıt and voltage and the magnitude of › vector impedance. Both the vector impedance z and its magnitude Z

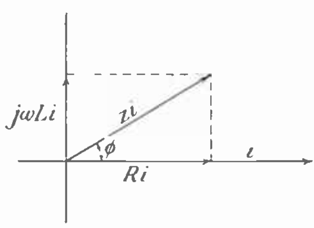


FIG. 5.—Vector relations of inductive circuit.

are generally referred to simply as impedance, the context usually indicat ing which quantity is meant.

The impedance Z increases as the frequency is increased. Cor sequently, for constant values of E, R, and L, the current I will decreas

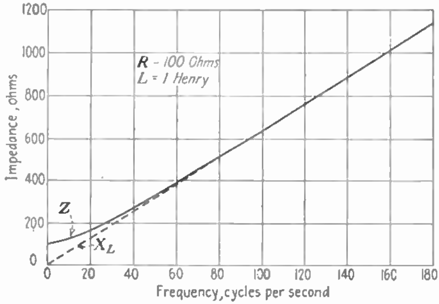


FIG. 6.—Impedance of inductive circuit with frequency.

as the frequency increases. Figure 6 shows values of Z plotted agains frequency, and Fig. 7 shows how the current in the circuit of Fig. varies with the frequency of the impressed voltage.

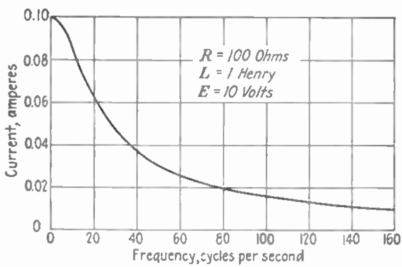
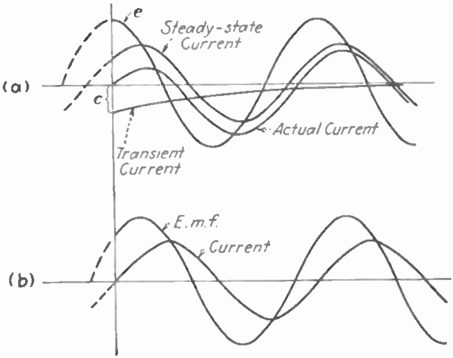


FIG. 7.—Current *vs.* frequency in inductive circuit.

Consider Eq. (11). After tl switch has been closed for som time, the values of current an voltage bear a definite relatic to each other at each instar during a cycle, and this series relations is repeated durir every cycle. The circuit is no said to be in the steady-sta condition, and the first term the right-hand side of Eq. (1 completely defines the curre in terms of the voltage and in pedance. However, for a sho interval of time after the swit is closed, the second or transie term generally has an appreciable value and must be considered. F comparison with Eq. (9) it is seen that this transient current has tl form shown in Fig. 2. It is evident that the duration of tl transient current will depend upon the time constant L/R. The initi value of the current, which is equal to the constant c, must, however, l determined. Now the current must be zero at the instant the switch closed (since it cannot rise to some finite value instantaneously becau of the inductance in the circuit) and, therefore, if t is taken as zero the instant of closing the switch, the value of c may be found math matically to be defined by the equation

$$c = \frac{E_M}{Z} \sin \phi = I_M \sin \phi \qquad (1$$

The physical significance of this equation is most readily seen by erence to Fig. 8.[1] In *a* of this figure, the curve *e* represents the voltage pressed upon the circuit and the curve marked "Steady state current" licates the value the current would have if the switch had been closed a time much earlier than the time represented in the figure. Accord-



G. 8.—Effect on transient current of closing circuit at different times in the cycle.

gly, at the instant of closing the switch, the current should have e value given by the intersection of the steady-state current curve th the vertical axis in the figure. But the actual current must be ro at this instant; therefore, the transient current must have the lue *c*, just neutralizing the fictitious steady-state current. This ansient current then decreases cording to the curve labeled ransient current," and the actual rrent is the sum of the steady-state rrent and the transient current. If e switch should be closed at an stant at which the steady-state curnt would be zero, as in Fig. 8*b*, the nstant *c* would be equal to zero and ere would be no transient term. onsequently the quantity ϕ in Eq. 5) represents the phase angle of the stant of closing the switch with ference to the nearest time at which

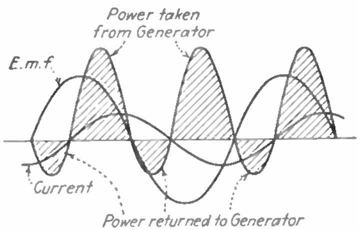


FIG. 9.—Power in inductive circuit.

e steady-state current crosses the zero axis in passing from negative to sitive values. In Fig. 8*a*, the switch was assumed to be closed shortly ter the steady-state current passed through such a zero value; therefore, this case, the so-called "phase angle" is a lag angle, and sin ϕ is negative, aking *c* negative as shown.

5. Power in Inductive Circuit. The instantaneous power used in the cuit of Fig. 3 is the product of the instantaneous values of current and ltage. Figure 9[2] shows this power at times to be negative because

[1] MORECROFT, J. H., "Principles of Radio Communication," 2d ed., 1927.
[2] *Ibid.*

the current and voltage have opposite signs. Such negative pow represents a restoration to the source of some of the energy stored in th magnetic field. In a circuit containing inductance only, the curren and voltage are 90 deg. out of phase and the negative loops of the instan taneous-power curve are exactly equal to the positive loops, so that th average power taken by the inductance is zero.

In general, the instantaneous power is given by[1]

$$
\begin{aligned}
p &= E_M \sin \omega t \times I_M \sin (\omega t - \phi) \\
&= E_M I_M (\sin^2 \omega t \cos \phi - \sin \omega t \cos \omega t \sin \phi) \\
&= \frac{E_M I_M}{2} (\cos \phi - \cos 2\omega t \cos \phi - \sin 2\omega t \sin \phi) \qquad (1
\end{aligned}
$$

The average value of the second and third terms in the last parenthesis zero, so that the average power taken by the circuit is that expressed by th first term, or

$$
P = \frac{E_M I_M}{2} \cos \phi = EI \cos \phi \qquad (1
$$

where, as before, E_M and I_M are maximum values, and E and I are effecti values of the voltage and current. Since

$$
E = IZ
$$

and

$$
\cos \phi = \frac{R}{Z}
$$

$$
P = IZ \times I \times \frac{R}{Z} = I^2 R \qquad (1
$$

This last equation is often used to define the effective resistance of an a circuit.

As a consequence of Eq. (17), the power in an a-c circuit containin inductance and resistance cannot be determined by measuring the curren and voltage unless the value of the phase angle ϕ can also be measure As this is usually difficult, the power must generally be measured wit a wattmeter.

The quantity $\cos \phi$ is called the *power factor* of the circuit. In a circu containing only resistance, the power factor is unity; in a circuit con taining only inductance, the power factor would be zero. As applied t a coil used as an inductor, the power factor at a given frequency gives th ratio of the resistance of the coil to its impedance and may be used as figure of merit for the coil. As the ideal inductor would have zero pow factor, a good coil should have a very small power factor.

6. Measurements of Inductance at Low Frequencies. The measure ment of the inductance of air-core coils at low frequencies is relativel simple, as the inductance is sensibly constant with change in frequenc and current. Iron-core inductors, for reasons which will be examined i detail later, do not have a fixed inductance under all conditions, an measurements on them must be made under conditions which duplica as nearly as possible the conditions under which the inductor is use

[1] *Ibid.*

A simple method of approximate measurement uses the circuit of Fig.). An a-c voltage of known frequency is applied at E, and the current nd voltage read on the meters. The voltmeter reading divided by the mmeter reading gives the impedance and, if the resistance is measured y a d-c-bridge or voltmeter-ammeter method, the inductance may be ulculated from the equation

$$L = \frac{Z^2 - R^2}{4\pi^2 f^2} = 0.025\frac{Z^2 - R^2}{f^2} \tag{19}$$

he method is usable for iron-core coils that carry a.c. only, provided ne measuring current is adjusted to the value that the coil carries in use. measurements are made at a number of current values, the curve of ductance against current may be plotted. The results obtained by nis method are generally slightly larger than the true values of inducnce because the a-c resistance, particularly in iron-core coils, is eater than the d-c resistance.

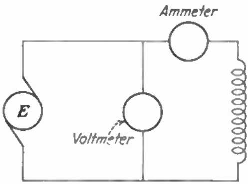


FIG. 10.—Circuit for measurement of inductance.

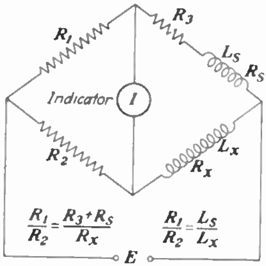


FIG. 11.—Bridge for comparing inductances.

There are several bridge methods for the measurement of inductance id a-c resistance at low frequencies; they give very accurate results, but e generally satisfactory only for air-core coils. For the comparison of o inductances, or the determination of the value of an unknown inducnce in terms of a standard, the circuit shown in Fig. 11 is used. L_s id R_s are the inductance and resistance of the standard and L_x and R_x present the unknown coil values. A galvanometer may be used as an dicator and a battery connected at E; or a telephone receiver or vibran galvanometer may be used with an a-c source connected at E. If e galvanometer and battery are used, the bridge is first balanced for c. and then for intermittent current (produced by a switch or comutator in the battery circuit).

For the steady-current balance,

$$\frac{R_1}{R_2} = \frac{R_s + R_3}{R_x} \tag{20}$$

is assumed here that the resistance of the standard is less than that of e coil to be measured; if the reverse is true, R_3 must be connected in the arm containing the coil being measured. For the intermitten current balance,

$$\frac{R_1}{R_2} = \frac{L_s}{L_x} \tag{21}$$

R_x and L_x are determined from these equations. If the telephone or vibration galvanometer and a.c. are used, adjustment of the bridge for minimum signal in the indicator satisfies Eqs. (20) and (21) simultaneously. The wave form of the voltage impressed upon the bridge is immaterial unless, with a telephone receiver as indicator, the inductance being measured is a function of the frequency of the current through it; in such a case, balance for the fundamental would not occur under the same conditions as for the harmonics. The use of the vibration galvanometer as indicator removes this limitation, as the response of the galvanometer to the harmonics of the impressed voltage is negligible in comparison with the response to the fundamental. The sensitivity of the vibration galvanometer, however, is inversely proportional to the frequency, so that it is useful only at comparatively low frequencies. This method of inductance measurement has one serious disadvantage: the steady-current and intermittent-current balances are not independent, so that final balance must be approached by a series of approximations.

A number of bridge methods for comparing an inductance with a capacity have been developed. Several are described in the Section on measurements.

7. Measurement of Inductance of Iron-core Coils. When an iron-core coil must carry relatively large d.c. upon which is superimposed a

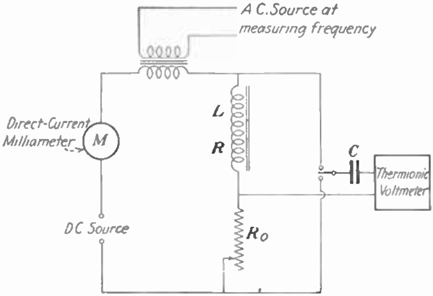


FIG. 12.—Measurement of iron core carrying a.c. and d.c.

small value of a.c., its inductance is dependent upon the magnitudes of the two currents flowing through it, and the methods already given are not directly applicable to measurements under such conditions.

The impedance of an iron-core coil carrying d.c. and a.c. may be measured by the circuit of Fig. 12. The d.c. through the circuit is adjusted to the value carried by the coil during operation, and the a-c source adjusted to impress a voltage across the coil (measured by the thermionic voltmeter) equal to the a-c voltage across it under operating conditions. The resistance R_0 is then varied until the alternating voltage ross it is equal to that across the coil, as measured by the thermionic ›ltmeter. Then the impedance of the coil at the measuring frequency equal to R_0. Readjustments of the impressed direct and alternating ›ltages may be necessary as R_0 is changed. The condenser C prevents e direct voltages across the coil and resistor from affecting the thermi-ic voltmeter. From the impedance and the resistance of the coil, e inductance under the conditions of measurement may be calculated · Eq. (19).

8. Turner Constant-impedance Method.—For measurements involv-g a.c. only, the constant-impedance method (of Turner[1]), shown in g. 13, is used. The method is based upon the ct that, when $1 - \omega^2 LC = 0$, the impedance of e parallel circuit is equal to ωC and is independ-t of the resistance in the inductive branch. ›nsequently the line current will have the same agnitude with the switch open or closed. To easure any value of inductance, then, it is only cessary to adjust the capacity so that the read-g of the ammeter A is the same for both posi-›ns of the switch, when the inductance may be lculated by the equation

$$L = \frac{1}{2\omega^2 C} \qquad (22)$$

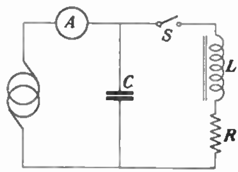


FIG. 13.—Turner constant-impedance method of measuring inductance.

When the coil must carry d.c. as well as a.c., the circuit of Fig. 14 may used for the inductance measurement. Two similar inductors are ed, the d.c. through them being adjusted to the proper value by means the resistor R_1 and measured by means of the d-c ammeter M. The itch S' is then thrown to the right and the resistor R_2 adjusted to make

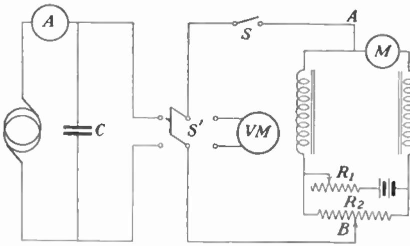


FIG. 14.—Measuring circuit for coils carrying a.c. and d.c.

› constant-potential difference between the points A and B zero. ıen, with S' thrown to the left, the inductance measurement may be rried out in the manner already described. The result is the inductance the two coils in parallel, which is one-half the inductance of one coil.

9. Measurements of Inductance at High Frequencies. Very often the v-frequency inductance of a coil, determined by one of the methods ·eady given, may also be used as the high-frequency inductance. In

[1] TURNER, H. M., Constant Impedance Method for Measuring Inductance of Choke ils, *Proc. I.R.E.*, **16**, 1559, 1928.

some instances it is desirable to determine the inductance at the operat frequency. Bridge methods are not suitable for measurements at h frequencies. Two other methods are commonly used: comparison the coil with a standard, and measurement of the capacity required tune the coil to resonance with a known frequency, from which inductance may be calculated. Both methods give the appar inductance.

In the comparison method, a standard inductor, having an appar inductance L_s at the measuring frequency, is connected in parallel wit calibrated variable condenser, coupled to an oscillator and the c condenser circuit tuned to resonance, the capacity C_s of the conden being noted at the resonance setting. The coil to be measured, wh inductance is denoted by L_x, is then substituted for L_s, the circuit retun and the condenser capacity C_x again observed. Since the frequency the same in both cases,

$$L_xC_x = L_sC_s \quad (\text{:}$$

If the low-frequency inductance L_0 and internal capacity C_0 of standard coil are known,

$$L_xC_x = L_0(C_s + C_0) \quad (\text{'}$$

In the second method, it is necessary to determine accurately frequency of the source. The coil to be measured is connected t calibrated variable condenser, coupled loosely to the generator a tuned to resonance. If f is the frequency of the source, L_x the appar inductance of the coil, and C_x the condenser capacity at resonar

$$L_x = \frac{1}{39.48f^2C_x} = \frac{0.02533}{f^2C_x} \quad ($$

In this equation, L_x is expressed in henrys and C_x in farads. For in μh and C_x in $\mu\mu$f, the equation becomes

$$L_x = \frac{25.33 \times 10^{15}}{f^2C_x} \quad ($$

If the capacity necessary to tune the coil to resonance at a numbe different frequencies is determined, a graph of the squares of the wa lengths corresponding to several measuring frequenc against the measured values capacity will be a straight l whose slope is the pure ind tance and whose intercept w the negative-capacity axis is internal capacity of the c This is illustrated in Fig. 15.

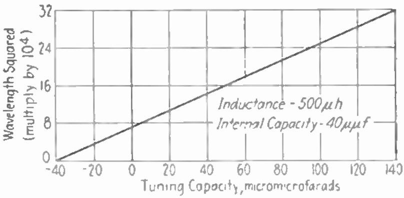


FIG. 15.—Method of determining inductance and distributed capacity of a coil.

10. Inductance of Iron-c Coils. Iron-core coils mainly useful at relatively l frequencies, and their use is g erally confined to circuits car ing currents within the a-f range.

The inductance of a circuit is not constant if any material of varia permeability is within the magnetic field of the circuit. Consequent hen a coil is wound on an iron core, its inductance is dependent upon e circumstances under which it is used. Accordingly, to use iron-core ils most advantageously, it is necessary to study their characteristics ıder varying conditions. Three important cases must be distinguished: e current through the coil is a.c. of single frequency; the current consists a d-c component upon which is superimposed a single-frequency a-c mponent; the current is comprised of two a-c mponents of different frequencies.

The average inductance of an iron-core coil rrying a.c. of single frequency is dependent ɔon the magnitude of the current. Also, the c resistance of such a coil is higher than that an air core coil with an identical winding. ıerefore all inductance measurements of ironre coils should be made with the measuring ırrent equal to the current which will flow rough the coil in operation, or the inductance ay be measured for a number of different ırrents and a curve of inductance against ırrent plotted.

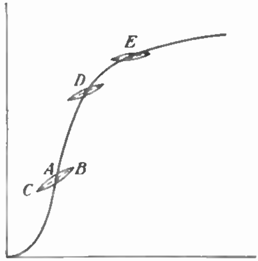
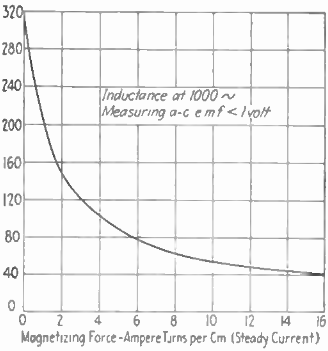


FIG. 16.—Characteristic of coil carrying large value of d.c. and small value of a.c.

In many radio applications a coil carries a latively large d.c. with a small a-c component perimposed. The inductance of an ironre coil under such conditions is a function of e magnitudes of the d-c and a-c components of the current. This is ustrated by Fig. 16. The constant magnetizing force (due to the d.c.) ay be such as to cause the core to be magnetized to the point A. The alternating component of the magnetizing force (due to the a.c.) will then carry the iron through the small hysteresis loop CB whose slope is not the same as the slope of the magnetization curve. The permeability represented by the slope of this small hysteresis loop is called the *incremental permeability*. As the constant component of the magnetizing force or current is increased, the point A moves farther up the magnetization curve and the incremental permeability decreases, as indicated by the small loops at D and E. As saturation of the core is approached, the incremental permeability, and hence the inductance, becomes very small. As the magnitude of the a-c component is increased, the slope of the 'steresis loop, and accordingly the incremental permeability, increases, us increasing the inductance. Consequently the inductance of an ironre coil under these conditions decreases with increase of the d-c comɔnent of the current, and increases with increase of the a-c component. gure 17 shows the decrease in inductance with increase in constant agnetizing force.



G. 17.—Effect of d.c. on inductance of coil.

If an *air gap* is introduced in the magnetic circuit of an iron-core co the inductance of the coil is generally diminished. If, however, the co is carrying both d.c. and a.c., the air gap may so decrease the constan flux that the incremental permeability is actually increased, so that th effective inductance for the a-c component is increased. The effecti resistance of the inductor is also decreased by the introduction of an a gap. These effects are illustrated in Fig. 18.[1] As a consequence these characteristics, iron-core inductors that are intended for use circuits where they must carry d.c. as well as a.c. are usually made wi an air gap in the magnetic circuit of the core.

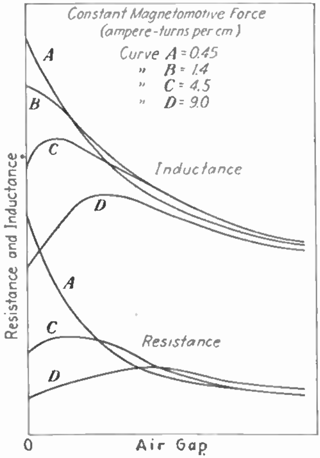


FIG. 18.—Effect of air gap on coil characteristics.

When the inductor carries two alterna ing currents of different frequencies, t effects of the variable permeability of t iron are somewhat more complicated a of relatively less practical importan than in the cases already treated.[2]

11. Inductors at Radio Frequencie When inductors are used at radio fr quencies, many factors affecting th performance come into prominenc The h-f resistance of a coil is mu larger than its d-c resistance because a number of losses which come in existence with the operation of the c in h-f circuits. The factors causing th increase are skin effect, eddy curren dielectric losses, and internal capacity

When the wire is wound into a co the effect of the magnetic field of t coil is such as to concentrate t current on the inner surfaces of t turns. Figure 19 illustrates this effe the depth of shading indicating the cu rent density. This concentration current causes a further increase in t effective resistance of the coil, and also causes a decrease in the indu tance as the frequency increases. However, the variation of inductan with frequency is generally small in comparison with the variation caus by internal capacity.

Eddy currents in the conductors composing the coil constitute serious source of loss at frequencies over 3,000 kc. These losses are minimized by the use of wire as small as possible without unduly increasing the conductor resistance, or by the use of tubing instead of wire. Because of these losses at frequencies higher than 3,000 kc there is an optimum wire size giving a minimum resistance in inductance coils.

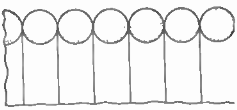


FIG. 19.—Concentrati of current at surface high frequencies.

Any dielectric in the field of the coil also introduces losses which become important at

[1] MORECROFT, J. H., "Principles of Radio Communication," 2d ed., 1927.

[2] TURNER, H. M., Inductance as Affected by Initial Magnetic State, Air Gap, a Superposed Currents, *Proc. I.R.E.*, 17, 1822, 1929.

ese frequencies, so that the type and amount of dielectric within the field the coil must be carefully regulated. The dielectric should be of the st quality and its volume must be kept at a minimum. The conductors the coil should, in general, come in contact with the dielectric as little possible. Coils are often wound upon skeleton or ribbed winding forms that each turn touches the supporting insulating material at only a few ints and is surrounded for the greater part of its length solely by air.

12. Effect of Coil Capacity. Every inductor behaves not as a pure ductance and resistance in series but as an inductance and resistance unted by a small capacity. This behavior is caused by the self- or ternal capacity of the coil. The resistance and inductance of the uivalent parallel circuit at any frequency are called the *apparent sistance* and *apparent inductance* of the coil at that frequency. The parent resistance is given approximately[1] by the equation

$$R_A = \frac{R}{(1 - \omega^2 LC_0)^2} \tag{27}$$

d the apparent inductance by

$$L_A = \frac{L}{1 - \omega^2 LC_0} \tag{28}$$

iere R and L are the resistance and inductance the coil would have the frequency $\omega/2\pi$ if the internal capacity C_0 were absent. These uations do not hold for frequencies near the natural frequency of the il; that is, the frequency for which $1 - \omega^2 LC_0 = 0$. These equations e derived on the assumption that the e.m.f. in the circuit is introduced some manner other than by induction in the coil itself. If the e.m.f. induced in the coil, the internal capacity is merely added to any other pacity which may be connected in parallel with the coil. Since a il is practically always used at frequencies for which $1 - \omega^2 LC_0$ is sitive, the apparent resistance and inductance of the coil will increase the frequency increases, the apparent resistance becoming very large $1 - \omega^2 LC_0$ approaches zero. The percentage change in resistance r a given change in frequency is about twice as great as the change inductance. At frequencies for which $1 - \omega^2 LC_0$ is negative, the coil haves as a capacity rather than an inductance.

It has been found[2] that the internal capacity of a single-layer coil is ughly proportional to the radius and practically independent of the umber of turns and the length. For a closely wound solenoid, the ternal capacity in $\mu\mu$f is very approximately equal to six-tenths of e radius in centimeters.

13. Types of Inductors. A straight wire has a certain amount of ductance, but to make inductors small enough to be convenient it is cessary to wind the wire in the form of a coil thus utilizing a great ngth of wire in a small space and also increasing the interlinkages of ix and wire.

The simplest inductor consists of a single square turn of wire. The ductance of this arrangement may be calculated accurately, but it has

[1] Radio Instruments and Measurements, *Bur. Standards Circ.* 74.
[2] Howe, G. W. O., *Jour. I.E.E.* (London), **60**, 63, 1922; also Moullin, E. B., "Radio equency Measurements," p. 340, 1931.

few other advantages. This type is sometimes used as a fundament standard.

The *single-layer solenoid* consists of one layer of wire on a cylindrical for: the turns either adjacent to one another or spaced. Sometimes the coil made self-supporting by means of a binder, such as collodion, and the foı removed after winding.

Multilayer coils must be used when a single-layer coil of the required indı tance would be inconveniently large. The multilayer coil may take one three forms: *layer wound, bank wound,* and *honeycomb or duolateral.*

The *layer-wound coil* is useful only at low frequencies because of its hi internal capacity caused by the proximity of turns of greatly differing pote tials. The wire is wound on the coil in laye each layer being completed before another begun. Iron-core coils are usually wound in tl manner. If a very large number of turns mı be used, it is better for the whole coil to be ma up of a number of "pies," each pie being a sh layer-wound coil. The pies are assembled side side to form the complete coil. Insulation greatly facilitated by this type of construction, and the internal capacity somewhat reduced.

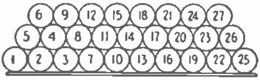


FIG. 20.—Bank winding.

Bank winding is one result of the attempt to devise a multilayer coil wi relatively low internal capacity. The turns are wound in the order shov by the cross-sectional view in Fig. 20.

Honeycomb and *duolateral* windings are further results of the same effo: The wire zigzags back and forth from one side of the winding space to t other, adjacent turns of the same layer being spaced from each other several times the wire diameter. The effect of this type of winding is cause turns of adjacent layers to cross each other at an angle and to separa parallel turns by at least the diameter of the wire. A coil of this type is se supporting and quite compact.

Basket-weave and *spider-web* windings were developed also to minimize t internal capacity. In the basket-weave coil the wire is wound in and out a number of pegs set in a circle. Adjacent turns cross at an angle. T pegs are usually removed after the winding is completed and the coil is se supporting. This is essentially a single-layer coil. The spider web, the other hand, is primarily a multilayer coil of one turn per layer. T wire is wound back and forth between a series of pegs fastened radially a circular form. This coil may also be self-supporting.

The *toroidal* coil is wound around a doughnut-shaped form. Its field almost entirely internal, so that it may be placed close to other coils a: apparatus.

The *flat spiral* type of coil is self-explanatory—the wire being wound in the form of a spiral, each turn having a greater radius than the preceding one.

14. Variable Inductors. Any of the previous types of coils may be tapped and the number of turns in circuit varied with a tap switch or clip. This method gives only a step-by-step variation, and considerable loss may be introduced by the unused portions of the coil.

A continuously variable inductor may be made by connecting in series or parallel two coils having a variable mutual inductance. The coils may be single-layer or multilayer solenoids and their mutual inductance may be varied by changing the distance between the coils or by rotating one with respect to t other. The most common form of variable inductor, however, is t arrangement commonly called a *variometer,* a cross section of which

FIG. 2
Variab
inductoı

ɪwn in Fig. 21. The inner coil is rotatable about the axis A, which is ·pendicular to the plane of the figure. The two coils may be connected either series or parallel, thus increasing the range of the instrument ɪsiderably. The mutual inductance between the coils may be increased winding the outer coil upon the interior of a spherical surface, instead ɪsing the cylindrical form shown.

f a slight increase of resistance of a coil is not objectionable, and the ired range of inductance variation is small, a copper disk slightly aller than the inside of the coil form may be mounted on a shaft pendicular to the axis of the coil. The inductance of the coil will be ɔreciably decreased when the plane of the disk is perpendicular the coil axis, the decrease of inductance becoming less as the disk is ated away from this position.

.5. Design of Inductance Coils. It is desirable that the inductance uld be as large as possible, while the resistance is kept at a minimum. ɜre are some cases in which a relatively high resistance is permissible ɜven desirable. Choke coils for use at high frequencies must have a h impedance with a minimum internal capacity.

'o determine a basis for comparison between coils of different char-eristics, a factor of merit for an inductor must be defined. Coils use at frequencies above 300 or 400 kc are usually small in size, so t volume is relatively unimportant and the desirable characteristics high inductance (and, therefore, high reactance) and low resistance. ɜ ratio of inductance (or reactance) to resistance may then be taken ı factor of merit, the ideal coil having a large ratio. Sometimes the ʋer factor of the coil, which is equal to the ratio of resistance to ɪedance, is taken as a factor of merit, an ideal coil having zero power ɔor. The ratio of reactance to resistance ($L\omega/R$) is sometimes called Q of the coil. (See Table I, Sec. 6.)

. coil to be used at frequencies below 300 kc is likely to be somewhat ;e if wound in a manner that would be entirely appropriate at higher ɪuencies. Consequently the factor of merit for coils designed for use he lower radio frequencies should include the volume of the inductor may be defined as the inductance-resistance ratio divided by the ɪme of the coil.

'or a given length of wire, maximum inductance is obtained when the ɜ is wound as compactly as possible; that is, in a bank-wound coil ı a winding cross section as nearly square as possible. The bank-ɪnd type is mentioned because the simple multilayer coil is practically ess at radio frequencies because of its high internal capacity. A ely wound single-layer coil made up of the same length of wire has a siderably lower inductance than the bank-wound coil. However, at o frequencies, the resistance of the single-layer coil is so much lower ı that of the multilayer coil that the L/R ratio of the former is much er than that of the latter. In view of its simplicity of construction, single-layer solenoid wound with solid wire would appear to be the t desirable coil type at medium and high radio frequencies, even ɪgh within certain ranges of frequency some other types have certain antages. At high frequencies (above 3,000 kc), the single-layer noid, either closely wound or spaced, is used almost exclusively.

or a given wire length, this type of coil has a maximum inductance n the ratio of diameter to length of coil is 2.46,[1] although this value

[1] Radio Instruments and Measurements, *Bur. Standards Circ.* 74.

is not critical. The inductance decreases somewhat rapidly as this rat becomes much smaller than 2.46, while the decrease is only slight f larger values of the ratio. Since the internal capacity of the coil approximately proportional to the diameter, it is advantageous to u a ratio of diameter to length somewhat smaller than 2.46, provid that the coil is to be used under such conditions that the decrease internal capacity effected in this way more than compensates for t slightly lower inductance-resistance ratio.

A multilayer coil has a maximum inductance when the cross secti of the winding is a square. It has also been shown[1] that, with a squa cross section given, the inductance of this type of coil is maximum wh the mean diameter is 3.02 times the depth of the winding.

Below 300 kc the volume of the coil must be included in the fact of merit. In these circumstances, the honeycomb and bank-wound co outstrip all others, the honeycomb type being somewhat superior to t bank wound. Table I gives the characteristics of honeycomb coils.

TABLE I.—HONEYCOMB-COIL DATA

Turns on coil	Size of wire, B. & S. gage	Inductance, mh	Distributed capacity, $\mu\mu f$	Natural wave length, meters	Wave lengths with the following shunt-condense capacities, μf			
					0.001	0.0005	0.00025	0.0
25	24	0.038	26.8	60	372	267	193	
35	24	0.076	30.8	91	528	378	277	
50	24	0.150	36.4	139	743	534	391	
75	24	0.315	28.6	179	1,007	770	560	
100	24	0.585	36.1	274	1,470	1,055	771	
150	24	1.29	21.3	313	2,160	1,546	1,110	
200	25	2.27	18.9	391	2,870	2,050	1,470	
250	25	4.20	22.9	585	3,910	2,800	2,020	1,
300	25	6.60	19.0	669	4,900	3,490	2,510	1,
400	25	10.5	17.4	806	6,160	4,400	3,160	2,
500	25	18.0	17.3	1,052	8,070	5,750	4,140	2,
600	28	37.5	19.2	1,600	11,600	8,300	5,980	3,
750	28	49.0	18.3	1,785	13,300	9,500	6,830	4,
1,000	28	85.3	16.8	2,260	17,600	12,500	9,000	5,
1,250	28	112.0	15.5	2,490	20,100	14,300	10,250	6,
1,500	28	161.5	15.8	3,000	24,200	17,200	12,350	8,

16. Coils for Various Frequency Ranges. A study of the characte ties of various types of inductors in the frequency range of 300 to 1,500 has been made by Hund and De Groot.[2] Their results show that in t frequency band the single-layer solenoid and the loose basket-wea coils have the highest inductance-resistance ratios of the coils wou with solid wire, with the radial basket weave or spider web a close th Coils wound with 32–38 Litz wire were found to be somewhat better in respects than solid-wire coils. Contrary to a somewhat generally accep belief, a few broken strands in the Litz wire made only a slight differe in the r-f resistance of a coil.

[1] Radio Instruments and Measurements, *Bur. Standards Circ.* 74.

[2] HUND, AUGUST, and H. B. DE GROOT, Radio Frequency Resistance and Inducta of Coils Used in Broadcast Reception, *Bur. Standards Tech. Paper* 298, Vol. 19, p. 1925.

In solid-wire coils, little is gained by using a wire size larger than No. AWG, although No. 16 gives a slightly lower resistance between 300 l 1,200 kc. Spacing the turns does not decrease the resistance apprebly—not enough to compensate for the extra length necessary. A mber of binders were tried on single-layer coils, all of them causing a ght increase in the r-f resistance of the coil. Collodion appeared to the best of these binders.

At frequencies above 3,000 kc, dielectric losses, eddy currents, and ernal capacity are important. The first two cause relatively large reases in the coil resistance. The third increases both the resistance l inductance of the coil if the voltage in the circuit is not induced in coil itself. If the circuit e.m.f. is introduced by induction in the l, the internal capacity, acting as a parallel condenser, determines the hest frequency to which the coil can be tuned. As the upper limit of rallel tuning capacity is not very large (in order that the L/C ratio not too small), a large internal capacity seriously restricts the range er which the coil may be tuned efficiently. It is for these reasons that single-layer solenoid is used almost exclusively at such frequencies. Dielectric losses are so important that it is generally essential to use her a self-supported coil or a skeleton or ribbed winding form. It is hly desirable that the wire touch the solid dielectric in as few points possible. The loss in the insulation between turns is readily elimited by the use of bare wire, the turns being spaced to prevent short cuits.

Spacing of the turns is also efficacious in reducing the internal capacity. coil whose turns are spaced by the diameter of the wire has approxitely half the internal capacity of a closely wound coil.

17. Calculation of Inductance of Air-core Coils. The inductance of ny types of air-core coils may be calculated by means of formulas volving the dimensions of the coil and the number of turns.[1] Several mulas from *Circular* 74 of the Bureau of Standards are given here. w of the available corrections to inductance formulas are included, ce they apply only to the calculation of the l-f inductance. The h-f luctance of a coil cannot be calculated with a high degree of accuracy cause of the skin effect and coil capacity.

In the following formulas all dimensions are expressed in centimeters and inductance is in microhenrys.

18. Straight Round Wire. If l is the length of the wire, d is the diameter the cross section, and μ is the permeability of the material of the wire,

$$L_0 = 0.002l\left[\log_\epsilon \frac{4l}{d} - 1 + \frac{\mu}{4}\right] \tag{29}$$

$$= 0.002l\left[2.303 \log_{10} \frac{4l}{d} - 1 + \frac{\mu}{4}\right] \tag{30}$$

$\mu = 1$ (for all materials except iron),

$$L_0 = 0.002l\left[2.303 \log_{10} \frac{4l}{d} - 0.75\right] \tag{31}$$

e return conductor is assumed to be remote. These formulas give the inductance.

[1] Rosa, E. B., and F. W. Grover, *Bur. Standards Sci. Paper* 169; Grover, F. W., *r. Standards Sci. Papers* 320, 1917; 455, 1922; 468, 1923.

As the frequency increases, the inductance decreases, its value at infin frequency being

$$L_\infty = 0.002l\left[2.303 \log_{10}\frac{4l}{d} - 1\right]$$ (3

A general expression for the inductance at any frequency is

$$L = 0.002l\left[2.303 \log_{10}\frac{4l}{d} - 1 + \mu\delta\right]$$ (3

The quantity δ is obtained from the table below, as a function of t argument x, where

$$x = 0.1405d\sqrt{\frac{\mu f}{\rho}}$$ (3

and f is the frequency and ρ is the volume resistivity of the wire in microh centimeters. For copper at 20°C.,

$$x_c = 0.1071d\sqrt{f}$$

This quantity δ will be used in several of the following formulas witho further definition.

VALUE OF δ IN INDUCTANCE FORMULAS

x	δ	x	δ	x	δ	x	δ	x	δ	x	δ
0	0.250	2.5	0.228	6.0	0.116	12.0	0.059	25.0	0.028	70.0	0.[illegible]
0.5	0.250	3.0	0.211	7.0	0.100	14.0	0.050	30.0	0.024	80.0	0.[illegible]
1.0	0.249	3.5	0.191	8.0	0.088	16.0	0.044	40.0	0.0175	90.0	0.[illegible]
1.5	0.247	4.0	0.1715	9.0	0.078	18.0	0.039	50.0	0.014	100.0	0.[illegible]
2.0	0.240	5.0	0.139	10.0	0.070	20.0	0.035	60.0	0.012	∞	0.[illegible]

19. Two Parallel Round Wires—Return Circuit. The current is assum to flow in opposite directions in two parallel wires of length l and diameter the distance between centers of wires being D. Then

$$L = 0.004l\left[2.303 \log_{10}\frac{2D}{d} - \frac{D}{l} + \mu\delta\right]$$ (3

This neglects the inductance of the wires connecting the two main wir If these wires are long, their inductance may be calculated by Eq. (33) a added to the result from Eq. (35), or the whole system may be treated as rectangle and the inductance calculated by Eq. (37).

20. Square of Round Wire. The length of one side of the square is denot by a; other letters have already been defined.

$$L = 0.008a\left[2.303 \log_{10}\frac{2a}{d} + \frac{d}{2a} - 0.774 + \mu\delta\right]$$ (3

21. Rectangle of Round Wire. The sides of the rectangle are a and and the diagonal $g = \sqrt{a^2 + a_1{}^2}$. Then

$$L = 0.00921\left[(a + a_1)\log_{10}\frac{4aa_1}{d} - a\log_{10}(a + g) - a_1\log_{10}(a_1 + g)\right]$$

$$+0.004\left[\mu\delta(a + a_1) + 2\left(g + \frac{d}{2}\right) - 2(a + a_1)\right]$$ (3

22. Grounded Horizontal Wire. The wire is assumed to be parallel to ie earth which acts as the return circuit. In addition to symbols already ied, h denotes the height of the wire above ground. Then

$$L = 0.004605l\left[\log_{10}\frac{4h}{d} + \log_{10}\left\{\frac{l + \sqrt{l^2 + \frac{d^2}{4}}}{l + \sqrt{l^2 + 4h^2}}\right\}\right]$$

$$+0.002\left[\sqrt{l^2 + 4h^2} - \sqrt{l^2 + \frac{d^2}{4}} + \mu l\delta - 2h + \frac{d}{2}\right] \tag{38}$$

23. Circular Ring of Circular Section. If a is the mean radius of the ring,

$$L = 0.01257a\left[2.303 \log_{10}\frac{16a}{d} - 2 + \mu\delta\right] \tag{39}$$

rovided that $d/2a \leqq 0.2$.

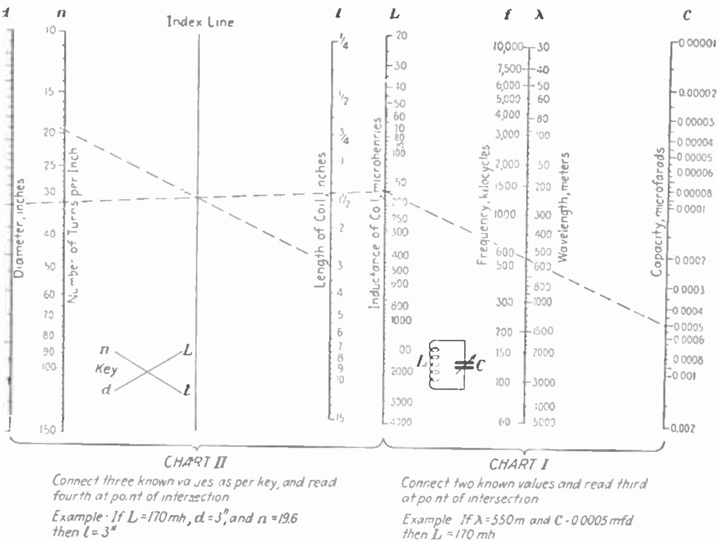


Fig. 22.—Inductance design chart.

24. Single-layer Coil or Solenoid.

$$L = \frac{0.0395a^2n^2}{b}K \tag{40}$$

iere n is the number of turns, a is the radius of the coil measured from the is to the center of the wire, b is the length of the coil, and K is a function of $/b$, the value of which may be determined by means of the table below. ie chart given in Fig. 22 may also be used for the calculation of the inducnce of a given coil, or for the design of a coil to have a given inductance. iis figure also includes a chart for the determination of the resonant freiency of any combination of inductance and capacity in the range of the art.

VALUE OF K IN FORMULA 40

Diameter to length	K	Difference	Diameter to length	K	Difference	Diameter to length	K	Differ ence
0.00	1.0000	−0.0209	2.00	0.5255	−0.0118	7.00	0.2584	−0.00
.05	.9791	203	2.10	.5137	112	7.20	.2537	
.10	.9588	197	2.20	.5025	107	7.40	.2491	
.15	.9391	190	2.30	.4918	102	7.60	.2448	
.20	.9201	185	2.40	.4816	97	7.80	.2406	
0.25	0.9016	−0.0178	2.50	0.4719	−0.0093	8.00	0.2366	−0.00
.30	.8838	173	2.60	.4626	89	8.50	.2272	
.35	.8665	167	2.70	.4537	85	9.00	.2185	
.40	.8499	162	2.80	.4452	82	9.50	.2106	
.45	.8337	156	2.90	.4370	78	10.00	.2033	
0.50	0.8181	−0.0150	3.00	0.4292	−0.0075	10.0	0.2033	−0.01
.55	.8031	146	3.10	.4217	72	11.0	.1903	1
.60	.7885	140	3.20	.4145	70	12.0	.1790	
.65	.7745	136	3.30	.4075	67	13.0	.1692	
.70	.7609	131	3.40	.4008	64	14.0	.1605	
0.75	0.7478	−0.0127	3.50	0.3944	−0.0062	15.0	0.1527	−0.00
.80	.7351	123	3.60	.3882	60	16.0	.1457	
.85	.7228	118	3.70	.3822	58	17.0	.1394	
.90	.7110	115	3.80	.3764	56	18.0	.1336	
.95	.6995	111	3.90	.3708	54	19.0	.1284	
1.00	0.6884	−0.0107	4.00	0.3654	−0.0052	20.0	0.1236	−0.00
1.05	.6777	104	4.10	.3602	51	22.0	.1151	
1.10	.6673	100	4.20	.3551	49	24.0	.1078	
1.15	.6573	98	4.30	.3502	47	26.0	.1015	
1.20	.6475	94	4.40	.3455	46	28.0	.0959	
1.25	0.6381	−0.0091	4.50	0.3409	−0.0045	30.0	0.0910	−0.01
1.30	.6290	89	4.60	.3364	43	35.0	.0808	
1.35	.6201	86	4.70	.3321	42	40.0	.0728	
1.40	.6115	84	4.80	.3279	41	45.0	.0664	
1.45	.6031	81	4.90	.3238	40	50.0	.0611	
1.50	0.5950	−0.0079	5.00	0.3198	−0.0076	60.0	0.0528	−0.00
1.55	.5871	76	5.20	.3122	72	70.0	.0467	
1.60	.5795	74	5.40	.3050	69	80.0	.0419	
1.65	.5721	72	5.60	.2981	65	90.0	.0381	
1.70	.5649	70	5.80	.2916	62	100.0	.0350	
1.75	0.5579	−0.0068	6.00	0.2854	−0.0059			
1.80	.5511	67	6.20	.2795	56			
1.85	.5444	65	6.40	.2739	54			
1.90	.5379	63	6.60	.2685	52			
1.95	.5316	61	6.80	.2633	49			

25. Multilayer Coils: Circular Coils of Rectangular Cross Section. F long coils of a few layers, the following formula may be used:

$$L = L_s - \frac{0.0126n^2ac}{b}(0.693 + B_s) \qquad (4$$

where L_s is the inductance calculated by Eq. (40), n and b are the same as Eq. (40), a is the radius of coil measured from axis to center of winding cro section, c is the radial depth of winding, and B_s is the correction given on p. 7

VALUE OF B_s IN FORMULA 43

c	B_s	b/c	B_s	b/c	B_s	b/c	B_s	b/c	B_s	b/c	B_s
	0.0000	6	0.2446	11	0.2844	16	0.3017	21	0.3116	26	0.3180
	0.1202	7	0.2563	12	0.2888	17	0.3041	22	0.3131	27	0.3190
	0.1753	8	0.2656	13	0.2927	18	0.3062	23	0.3145	28	0.3200
	0.2076	9	0.2730	14	0.2961	19	0.3082	24	0.3157	29	0.3209
	0.2292	10	0.2792	15	0.2991	20	0.3099	25	0.3169	30	0.3218

For short multilayer coils, the dimensions shown in Fig. 23 are used. Two rmulas are required, one for use when $b > c$, and the other for use when $b <$ In the first case:

$$L = 0.01257an^2\left[\left(1 + \frac{b^2}{32a^2} + \frac{c^2}{96a^2}\right)\log_\epsilon \frac{8a}{d} - y_1 + \frac{b^2}{16a^2}y_2\right]$$

$$= 0.01257an^2\left[2.303\left(1 + \frac{b^2}{32a^2} + \frac{c^2}{96a^2}\right)\log_{10} \frac{8a}{d} - y_1 + \frac{b^2}{16a^2}y_2\right] \quad (42)$$

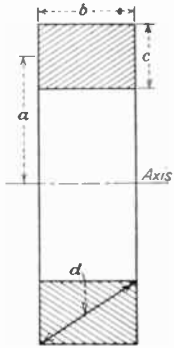


FIG. 23.—Multilayer coil.

hen $b < c$:

$$L = 0.01257an^2\left[\left(1 + \frac{b^2}{32a^2} + \frac{c^2}{96a^2}\right)\log_\epsilon \frac{8a}{d} - y_1 + \frac{c^2}{16a^2}y_3\right]$$

$$= 0.01257an^2\left[2.303\left(1 + \frac{b^2}{32a^2} + \frac{c^2}{96a^2}\right)\log_{10} \frac{8a}{d} - y_1 + \frac{c^2}{16a^2}y_3\right] \quad (43)$$

. y_2, and y_3 may be obtained from the table shown on page 78. These forulas are quite accurate as long as the diagonal of the cross section (d Fig. 23) es not exceed the mean radius. The accuracy decreases considerably as b comes large in comparison with a.

For very accurate results, a correction must be added if the insulation of e wire occupies a considerable percentage of the winding space. This rrection is given by

$$\Delta L = 0.01257an\left[2.303 \log_{10} \frac{D}{d} + 0.155\right] \quad (44)$$

here D is the distance between the centers of adjacent wires, and d is the ameter of the bare wire.

26. Multilayer Square Coil. If n is the number of turns and a is the side the square measured to the center of the rectangular cross section which s length b and depth c, then

$$L = 0.008an^2\left[2.303 \log_{10} \frac{a}{b + c} + 0.2235\frac{b + c}{a} + 0.726\right] \quad (45)$$

the cross section is square ($b = c$), this becomes

$$L = 0.008an^2\left[2.303 \log_{10} \frac{a}{b} + 0.447\frac{b}{a} + 0.033\right] \quad (46)$$

VALUE OF CONSTANTS IN FORMULAS 42 AND 43

b/c or c/b	y_1	c/b	y_2	b/c	y_3
0	0.5000	0	0.125	0	0.597
0.025	0.5253				
0.05	0.5490	0.05	0.127	0.05	0.599
0.10	0.5924	0.10	0.132	0.10	0.602
0.15	0.6310	0.15	0.142	0.15	0.608
0.20	0.6652	0.20	0.155	0.20	0.615
0.25	0.6953	0.25	0.171	0.25	0.624
0.30	0.7217	0.30	0.192	0.30	0.633
0.35	0.7447	0.35	0.215	0.35	0.643
0.40	0.7645	0.40	0.242	0.40	0.654
0.45	0.7816	0.45	0.273	0.45	0.665
0.50	0.7960	0.50	0.307	0.50	0.677
0.55	0.8081	0.55	0.344	0.55	0.690
0.60	0.8182	0.60	0.384	0.60	0.702
0.65	0.8265	0.65	0.427	0.65	0.715
0.70	0.8331	0.70	0.474	0.70	0.729
0.75	0.8383	0.75	0.523	0.75	0.742
0.80	0.8422	0.80	0.576	0.80	0.756
0.85	0.8451	0.85	0.632	0.85	0.771
0.90	0.8470	0.90	0.690	0.90	0.786
0.95	0.8480	0.95	0.752	0.95	0.801
1.00	0.8483	1.00	0.816	1.00	0.816

Formula (43) may be used to correct for insulation by replacing the fact 0.01257 by 0.008.

For a single-layer square coil,

$$L = 0.008an^2\left[2.303 \log_{10}\frac{a}{b} + 0.2231\frac{b}{a} + 0.726\right] - 0.008an(A + B) \quad (4$$

A and B are given below, where d is the diameter of the bare wire and D the distance between turns, measured to the centers of the wires.

VALUE OF A IN FORMULA 47

d/D	A	d/D	A	d/D	A
1.00	0.557	0.40	−0.359	0.15	−1.340
0.95	0.506	0.38	−0.411	0.14	−1.409
0.90	0.452	0.36	−0.465	0.13	−1.483
0.85	0.394	0.34	−0.522	0.12	−1.563
0.80	0.334	0.32	−0.583	0.11	−1.650
0.75	0.269	0.30	−0.647	0.10	−1.746
0.70	0.200	0.28	−0.716	0.09	−1.851
0.65	0.126	0.26	−0.790	0.08	−1.969
0.60	0.046	0.24	−0.870	0.07	−2.102
0.55	−0.041	0.22	−0.957	0.06	−2.256
0.50	−0.136	0.20	−1.053	0.05	−2.439
0.48	−0.177	0.19	−1.104	0.04	−2.662
0.46	−0.220	0.18	−1.158	0.03	−2.950
0.44	−0.264	0.17	−1.215	0.02	−3.355
0.42	−0.311	0.16	−1.276	0.01	−4.048

VALUE OF B IN FORMULA 47

Number of turns, n	B	Number of turns, n	B
1	0.000	40	0.315
2	0.114	45	0.317
3	0.166	50	0.319
4	0.197	60	0.322
5	0.218	70	0.324
6	0.233	80	0.326
7	0.244	90	0.327
8	0.253	100	0.328
9	0.260	150	0.331
10	0.266	200	0.333
15	0.286	300	0.334
20	0.297	400	0.335
25	0.304	500	0.336
30	0.308	700	0.336
35	0.312	1,000	0.336

27. Inductance Standards. Like all other standards, inductance andards must be rugged, permanent, and constant. The simplest ndamental standard is a single square turn of round wire. The inducnce of such a standard can be calculated with great accuracy.

When a standard having a large value of inductance is desired, the ngle square turn becomes too large for use, and it is necessary to design me more compact form. The resistance and internal capacity must kept to a minimum. Furthermore the turns must be held rigidly in ace so they cannot change their relative positions. The dielectric the field of the coil must have a minimum volume and be of such aterial that the losses in it are as small as possible.

These requirements are best met by a single-layer solenoid with a aced winding. For a minimum conductor resistance, the ratio of ameter to length should be 2.46, but a somewhat smaller value of this tio is desirable to reduce the internal capacity, this being proportional the radius.

One excellent form of standard inductor is made by winding silkvered Litz wire in slots in the edges of strips of hard rubber, the ends which are supported by hard-rubber rings. With this skeleton type winding form, the cross section of the coil is polygonal rather than rcular. In order that the proper ratios of diameter to length may be aintained, the coils must be of large size, their diameters ranging from to 40 cm. for inductance values that are necessary in the frequency nge from 15 to 1,500 kc. Such a coil must be given relatively careful ndling, however, since jolts might cause some of the wires to change eir positions. A more rugged coil consists of bare wire wound upon a readed cylindrical form, the turns being cemented in place with a very tle cement, preferably collodion. The form should be as thin as is nsistent with adequate strength. Glass forms may also be used, hough it is then necessary to cement the turns more thoroughly than the case of a threaded form.

With recent advances in the precision of frequency determination and provement in standard condensers, the temperature coefficient of a standard inductance may become an important factor. It is possibl in this case, to reduce the temperature coefficient by a special desig of the winding form.

28. Mutual Inductance. As the changing magnetic field due to varying current in a circuit induces an e.m.f. in the circuit itself, so ma it induce an e.m.f. in any neighboring circuit. The e.m.f. induced in t first circuit depends upon the self-inductance of that circuit, and, in t same way, the e.m.f. induced in the second circuit depends upo the *mutual inductance* between the two circuits. Mutual inductan is defined in three ways exactly analogous to the three ways of defini self-inductance: (1) as the magnetic flux linking the second circuit wh unit current flows in the first circuit; (2) as the e.m.f. induced in circu 2 when the current in circuit 1 changes at the rate of one unit per secon (3) as twice the work done in establishing the magnetic flux, linki circuit 2, associated with unit current in circuit 1. These three definitio give constant and equal values for the mutual inductance if there is material of variable permeability near the circuits and if the curre does not vary so rapidly that its distribution in the cross section of t conductors differs greatly from a uniform one. The change in curre distribution at high frequencies, however, has a very slight effect upon t mutual inductance.

The units of mutual inductance are the same as those of self-inductan in the practical system they are the henry and its subdivisions, the mil henry (mh) and microhenry (μh).

29. Measurement of Mutual Inductance. When two inducto having a mutual inductance, are connected in series so that their ma netic fields aid each other, the total inductance of the combination is

$$L' = L_1 + L_2 + 2M \qquad (4$$

where L' is the inductance of the combination, L_1 and L_2 are the indu tances of the coils, and M is their mutual inductance. If the connectio to one of the coils are reversed, the total inductance becomes

$$L'' = L_1 + L_2 - 2M \qquad (4$$

Then, from these two equations,

$$M = \frac{L' - L''}{4} \qquad (:$$

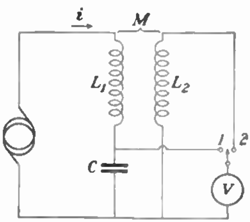


Fig. 24.—Circuit for measuring mutual inductance.

These relations furnish a convenient meth for the measurement of mutual inductance. T inductance of the two coils connected in series measured by any suitable method, the connectio to one coil reversed, and the inductance aga measured. The larger of the two measured valu is then denoted by L' and the smaller by L'', a M is calculated by means of Eq. (50). T method is applicable at any frequency, provid the inductance-measurement method is appropr ate at that frequency. It is not very accura when M is small in comparison with the inductan of the larger of the two coils.

A method applicable for all values of M is illustrated in Fig. 24.[1] represents a voltage-measuring device of high impedance, preferably thermionic voltmeter. A voltage source of frequency $\omega/2\pi$ is connected

[1] Moullin, E. B., "Radio Frequency Measurements," p. 383, 1932.

Values of F for Formula 56

r_2/r_1	F	Difference
0	∞	
0.010	0.05016	−0.00120
.011	4897	109
.012	4787	100
0.013	4687	−0.00093
.014	4594	87
.015	4507	81
.016	4426	148
.018	4278	132
0.020	0.04146	−0.00119
.022	4027	109
.024	3918	100
.026	3818	93
.028	3725	86
0.030	3639	−0.00081
.032	3558	76
.034	3482	71
.036	3411	68
.038	3343	64
0.040	0.03279	−0.00061
.042	3218	58
.044	3160	55
.046	3105	53
.048	3052	51
0.050	0.03001	−0.00226
.060	2775	191
.070	2584	164
.080	2420	144
.090	2276	128
0.100	0.02148	−0.00116
.11	2032	104
.12	1928	96
.13	1832	89
.14	1743	82
0.15	0.01661	−0.00075
.16	1586	71
.17	1515	66
.18	1449	62
.19	1387	59
0.20	0.01328	−0.00055
.21	1273	52
.22	1221	50
.23	1171	47
.24	1124	45
0.25	0.010792	−0.000425
.26	10366	408
.27	0.009958	388
.28	9570	371
.29	9199	355

r_2/r_1	F	Difference
0.30	0.008844	−0.000341
.31	8503	328
.32	8175	314
.33	7861	302
.34	7559	290
0.35	0.007269	−0.000280
.36	6989	270
.37	6720	260
.38	6460	249
.39	6211	241
0.40	0.005970	−0.000232
.41	5738	225
.42	5514	217
.43	5297	210
.44	5087	202
0.45	0.004885	−0.000195
.46	4690	189
.47	4501	183
.48	4318	178
.49	4140	171
0.50	0.003969	−0.000166
.51	3803	160
.52	3643	156
.53	3487	150
.54	3337	146
0.55	0.003191	−0.000141
.56	3050	137
.57	2913	133
.58	2780	128
.59	2652	125
0.60	0.002527	−0.000120
.61	2407	117
.62	2290	113
.63	2177	109
.64	2068	106
0.65	0.001962	−0.000103
.66	1859	99
.67	1760	96
.68	1664	93
.69	1571	90
0.70	0.001481	−0.000087
.71	1394	84
.72	1310	81
.73	1228	78
.74	1150	76
0.75	0.0010741	−0.0000731
.76	10010	704
.77	9306	680
.78	8626	653
.79	7973	628

r_2/r_1	F	Difference
0.80	0.0007345	−0.0000604
.81	6741	579
.82	6162	555
.83	5607	531
.84	5076	507
0.85	0.0004569	−0.0000484
.86	4085	460
.87	3625	437
.88	3188	413
.89	2775	389
0.90	0.0002386	−0.0000365
.91	2021	341
.92	1680	316
.93	1364	290
.94	1074	263
0.95	0.00008107	−0.00002351
.96	5756	2046
.97	3710	1706
.98	2004	1301
.99	703	703
1.00	0	
0.950	0.00008170	−0.00000494
.952	7613	482
.954	7131	470
.956	6661	458
.958	6202	446
0.960	0.00005756	−0.00000436
.962	5320	421
.964	4899	409
.966	4490	397
.968	4093	383
0.970	0.00003710	−0.00000370
.972	3340	356
.974	2984	341
.976	2643	327
.978	2316	312
0.980	0.00002004	−0.00000296
.982	1708	278
.984	1430	262
.986	1168	242
.988	926	223
0.990	0.00000703	−0.00000201
.992	502	177
.994	326	148
.996	177	115
.998	062	62

the terminals A and B, the current being denoted by i. When the switc is connected to point 1, the voltage measured is

$$e_1 = \frac{i}{\omega C}$$ (51

With the switch on point 2, the measured voltage

$$e_2 = \omega M i = \omega^2 M C e_1$$ (52

Then

$$M = \frac{e_2}{e_1} \cdot \frac{1}{\omega^2 C}$$ (53

The capacity C may be replaced by a resistance R. Then

$$M = \frac{e_2 R}{e_1 \omega}$$ (54

If a variable standard of mutual inductance is available, any other mutua inductance whose value falls within the range of the standard may be readil measured. The primaries are connected in series to a voltage source, th secondaries in opposition to a telephone receiver or other indicating devic and the standard is varied until a null indication is obtained. The unknow mutual inductance then has the value indicated by the standard.

30. Calculation of Mutual Inductance.[1] The mutual inductance of tw parallel coaxial circles may be calculated by the following method: first calculate

$$\frac{r_2}{r_1} = \sqrt{\frac{\left(1 - \frac{a}{A}\right)^2 + \frac{D^2}{A^2}}{\left(1 + \frac{a}{A}\right)^2 + \frac{D^2}{A^2}}}$$ (55

where a is the radius of the smaller circle, A the radius of the larger circl and D the distance between the planes of the two circles. From the tabl shown on page 77 the value of F corresponding to the calculated value of r_2/r is obtained. Then

$$M = F\sqrt{Aa}$$ (56

The units are the same as in the formulas for self-inductance already giver

For two parallel coaxial multilayer coils of square or nearly square cros section, a good approximation is given by

$$M = n_1 n_2 M_0$$ (58

where n_1 and n_2 are the numbers of turns on the two coils, and M_0 is th mutual inductance of two circles located at the centers of the cross section of the two coils.

The same formula may be used as a rough approximation for the mutua inductance of two coaxial single-layer solenoids.

References

BUTTERWORTH, S.: The High-frequency Copper Losses in Inductance Coils, *Exp Wireless*, **2**, 613, 1925.

COURSEY, P. R.: Calculation and Design of Inductances, *Electrician*, **75**, 841, 1915.

GRIFFITHS, W. H. F.: Notes on Standard Inductances for Wave Meters and Othe Radio-frequency Purposes, *Exp. Wireless and Wireless Eng.*, **6**, 543, 1929.

MAXWELL: "Electricity and Magnetism," Vol. II.

PERNOT, F. E.: Formulae and Tables for Design of Air-Core Inductance Coils, *Uni Calif. Pub., Eng.*, **1**, 117, 1916.

WHITTEMORE, L. E., and G. BREIT: Inductance, Capacity, and Resistance of Coils a Radio Frequency, *Phys. Rev.*, **14**, 170, 1919.

WILMOTTE, R. M.: Parasitic Losses in Inductance Coils at Radio Frequency, *Exp Wireless*, **2**, 451 and 477, 1925.

[1] ROSA, E. B., and F. W. GROVER, *Bur. Standards Sci. Paper* 169; GROVER, F. W *Bur. Standards Sci. Papers* 320 and 498.

SECTION 5

CAPACITY

By E. L. Hall,[1] E.E.

1. Capacity. Capacity is one of the three electrical quantities present all radio circuits. The radio engineer endeavors to concentrate .pacity in definite well-known forms at definite points in the circuits, it capacity exists between different conductors in the circuits and etween the various conductors and the ground. Such capacities, sually small, are ordinarily of no importance in the case of low or idio-frequency currents but may be of great consequence in radio-equency circuits.

A *condenser* is an electrical device in which capacity plays the main le. While some inductance and some resistance may be present, these antities are usually of such minor importance that they are negligible.

A condenser has three essential parts, two of which are usually metal ates separated or insulated by the third part called the *dielectric*.

The amount of electricity which the condenser will hold depends on ie voltage applied to the condenser. This may be expressed as $Q = \times V$. The *capacity* of the condenser is the ratio of the quantity of ectricity and the potential difference or voltage, or $C = Q/V$ where is given in coulombs, C in farads, and V in volts. The capacity of a ondenser is dependent on the size and spacing of the plates and the ind of dielectric between the plates.

2. Units of Capacity. The unit of capacity is the *farad*. A condenser as a capacity of one farad when one coulomb of electricity can be dded to it by an applied voltage of one volt. This unit is too large or practical use so that a smaller unit, the microfarad, abbreviated f, or one-millionth of a farad, is used. A condenser having a capacity one microfarad is much larger than is used in radio circuits. Condensers for such circuits usually have capacities between a few thouandths and a few millionths of a microfarad. Another unit, the iicromicrofarad, is often used. It is abbreviated $\mu\mu$f.

Another unit of capacity sometimes used is the *centimeter*. The centiieter is equal to 1.1124 micromicrofarads.

3. Electrical Energy of Charged Condenser. Work is done in charging condenser because the dielectric opposes the setting up of the electric train or displacement of the electric field in the dielectric. The energy f the charging source is stored up as electrostatic energy in the dielectric.

The work done in placing a charge in the condenser is

$$W = \frac{1}{2}Q \times V = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

[1] Radio Section, Bureau of Standards.

where W is expressed in joules
Q is expressed in coulombs
V is expressed in volts.

The work done in charging the condenser is independent of the time taken charge it.

4. Power Required to Charge Condenser. The average power requir to charge a condenser is given by the equation

$$P = \frac{1}{2}\frac{CV^2}{t}$$

where P is expressed in watts
C is expressed in microfarads
V is expressed in volts
t is expressed in seconds.

If the condenser is charged and discharged N times per second the abo equation becomes

$$P = \tfrac{1}{2}CV^2N$$

If an alternating e.m.f. of frequency f is used in charging the condenser, equation may be written

$$P = CE_0^2f$$

where P = power in watts
C = capacity in farads
E_0 = maximum value of voltage
f = frequency in cycles per second.

5. Dielectric Materials. The dielectric of a condenser is one of three essential parts. It may be found in solid, liquid, or gaseous fo or in combinations of these forms in a given condenser.

The simplest form of condenser consists of two electrodes or pla separated by air. This represents a condenser having a gaseous diel tric. If this imaginary condenser has the air between the plates repla by a non-conducting liquid, such as transformer oil, and if the dista between the plates is the same as in the first case, it would be fo that the capacity was increased several times because the oil has a hig value of dielectric constant than air which is usually taken as 1.

If the space between the plates is occupied by a solid insulato condenser would result, which would be practical, as far as the possibil of constructing it is concerned. It would be found, in this case a that the capacity of the condenser was several times larger than wl air was the dielectric.

The mechanical construction of either air or liquid dielectric conden requires the use of a certain amount of solid dielectric for holding two sets of plates.

There are a great many dielectric or insulating materials available the engineer to choose from. It often is found that a material whic very good from the electrical standpoint is poor mechanically, or *versa.*

Air is the only gas generally used as a dielectric. Compressed air been used in some high-voltage condensers.

Several kinds of oil have been used in condensers, such as castor cottonseed oil, and transformer oil. More recently electrolytic c densers have come into use in radio equipment for use as filters and by ndensers where a large capacity is required and either a direct current pulsating direct current is applied.

Among the solids used as the condenser dielectric are mica, glass, and per. Solid insulators used as mechanical supports in condensers lude quartz, glass, Isolantite, porcelain, Bakelite, mica, amber, hard bber, Victron, etc.

6. Dielectric Properties of Insulating Materials. Such properties surface and volume resistivity, dielectric strength or puncture voltage, electric constant, and absorption, are often considered in direct-rrent and commercial-frequency applications. Such data are of little lue if the insulating material is to be used at radio frequencies. For e latter application r-f measurements of various properties of the aterial are essential. A material which may be a satisfactory insulator low frequencies may be worthless as an insulator at radio frequencies.

One of the most important properties of an insulator for radio frequencies is its power loss. This includes several factors which are ficult to separate, but together indicate its suitability for radio purses. The general idea of the imperfection of a condenser is brought t in several names such as "power loss," "power factor," and "phase ference," but they are not identical terms.

Dielectric constant is another important property of a material which s a definite bearing upon its use at radio frequencies.

Neither power loss nor dielectric constant alone can be used in selecting e best insulator for a particular application at radio frequencies. Some vestigators have published results in which a product of the power ss and dielectric constant appears. This factor has no recognized name yet but has certain merits in use for indicating more completely the itability of an insulating material for radio uses.

7. Dielectric Constant. The *dielectric constant* K of an insulating aterial is the ratio of the capacity C_x of a condenser using the material the dielectric, to the capacity C_a of the condenser using air as the electric, or $K = C_x/C_a$. This property of the material is sometimes lled *inductivity* or *specific inductive capacity*.

The dielectric constant of a material is not a constant in the true sense the word, but varies with the frequency, moisture content, temperare, voltage applied, and manner of applying it.

8. Values of Dielectric Constant for Electrical Insulating Materials Radio Frequencies.

Material	Frequency, kilocycles	Dielectric constant,	Source
lluloid, photographic film	A	6.7	1
llulose nitrate, laboratory product	A	3.8	1
ment, de Khotinski, medium hard	A	3.9	1
er, black	A	7.6	1
ed	A	4.8	1
oil impregnated	A	5.8	1
ass	30	5.1 to 7.9[a]	3
rown	150	6.4	4
rown	800	6.2	4

Material	Frequency, kilocycles	Dielectric constant,	Sour
cobalt	500	7.3	
flint	500	7.0	2
	890	7.0	
heat resisting	[A]	5.7	
photographic, with gelatin coating	[A]	7.5	1
without gelatin coating	[A]	7.5	
plate, American	[A]	7.6	
plate	500	6.8	2
pyrex	30	4.8	3
	500	4.9	2
window	[A]	8.0	1
Hard rubber	210	3.0	2
	1,126	3.0	
Isolantite	[B]	6.1	5
Marble	44	8.4	2
	1,400	7.3	
white	[A]	9.3	
gray	[A]	11.6	1
blue	[A]	9.4	
Mica, clear, India	[A]	6.4	
built-up, shellac binder	[A]	5.6	
Phenolic insulation, laminated (Bakelite)	190	5.4 to 5.8	2
	1,000	5.1 to 5.6	
Slate, electrical	[A]	30.0	1
	44	20.5	4
	2,650	8.95	
Varnish, spar	[A]	5.5	
insulating	[A]	4.8	
Wax, beeswax	[A]	3.2	
ceresin	[A]	2.5	
paraffin	[A]	2.6	1
Wood, basswood, quite dry		2.0	
baywood, quite dry		2.4	
cypress, quite dry	[A]	2.0	
fir, quite dry		3.1	
maple, quite dry		2.6	
oak, quite dry		3.1	
birch	500	5.2	
maple	500	4.4	
	300	3.2	2
oak	425	3.3	
	635	3.3	
	1,060	3.3	

[a] range of nine samples of various chemical compositions reported.
[A] measurements made between 80 and 1,875 kc.
[B] average of a number of values between 1 kc and 3,130 kc.
[1] PRESTON, J. L., and E. L. HALL, Radio-frequency Properties of Insulating Materi *QST*, **9**, 26–28, February, 1925.
[2] HOCH, E. T., Power Losses in Insulating Materials, *Bell System Tech. Jour.* November, 1922.
[3] DECKER, WILLIAM C., Power Losses in Commercial Glasses, *Elec. World*, **89**, 601 6 Mar. 19, 1927.
[4] BAIRSTO, G. E., Conductivity and Dielectric Constant of Dielectrics for Hi frequency Oscillations, *Proc. Roy. Soc. (London) A*, **96**, 363–382, January, 1920.
[5] Isolantite *circ.*

9. Power Loss, Phase Difference, and Power Factor. Electrical sulating materials are not perfect in their insulating qualities and there a certain amount of power absorbed in them when used in an a-c rcuit. A measurement of the power loss is the best single property iat gives an indication of the suitability of an insulating material for se in radio circuits. Power loss can be expressed by a number of quanties, the most commonly used being resistance, power factor, phase fference, and phase angle.

When a.c. flows in a condenser, the voltage across ie condenser lags somewhat less than 90 deg. behind ie current as shown by the angle θ (Fig. 1), called ie *phase angle*. The complement ψ of the phase igle, is called the *phase difference*. The cosine of ie phase angle is called the *power factor*. The ower loss in the insulating material is

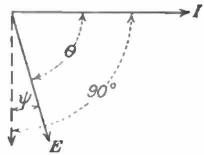


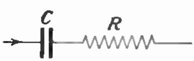
FIG. 1.—Phase in a capacitive circuit.

$$P = EI \cos \theta$$

.

$$P = EI \sin \psi$$

here E = voltage across the condenser
I = current in amperes through the condenser
plus ψ = 90 deg. as shown in Fig. 1. From the above, $\sin \psi = \cos \theta$, the sine of the phase difference is equal to the power factor.



IG. 2.—Condenser ith dielectric losses.

When considering a condenser having dielectric losses, such as current leakage, brush discharge or corona, dielectric absorption or resistance in the plates, joints, contacts, leads, etc., it is customary to think of it as a perfect condenser C with a resistance R in series as shown in Fig. 2.

The voltage vectors may be shown as in Fig. 3, where the resultant voltage flowing in the circuit is obtained by completing the vector diagram. The igle ψ is quite small for materials suitable for radio-frequency insulators. or small angles the angle $\psi = \tan \psi$. In Fig. 3

$$\tan \psi = \frac{RI}{I/\omega C} = R\omega C = 2\pi fRC.$$

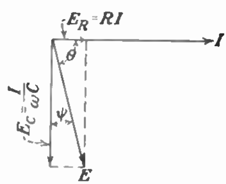


FIG. 3.—Vector relations in a condenser with dielectric losses.

the resistance, capacity, and frequency can be easured, the phase difference can be calculated from

$$\psi = 2\pi fRC,$$

here ψ = phase difference in radians
f = frequency in cycles per second
R = resistance in ohms
C = capacity in farads.

The following equation is sometimes convenient hen wave length in meters is given

$$\psi = 0.1079\frac{RC}{\lambda},$$

here ψ = phase difference in degrees
R = resistance in ohms
C = capacity in micromicrofarads
λ = wave length in meters.

or small angles, phase difference in radians is equal to power factor (nearly).

Power factor in per cent is 1.745 times phase difference in degrees. Pow factor in per cent is given by the following equation:

$$\cos\theta = 2\pi f R C \times 10^{-7}$$

where $\cos\theta$ = power factor in per cent
f = frequency in kilocycles
R = resistance in ohms
C = capacity in micromicrofarads.

The leakage of electricity by conduction through the dielectric or along i surface contributes to the phase difference but is generally negligible at hi frequencies. A condenser having leakage may be represented by a perfe condenser with a resistance in parallel as shown in Fig. 4. The curre

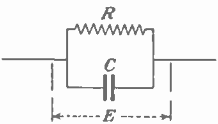


FIG. 4.—Equivalent of condenser with leakage.

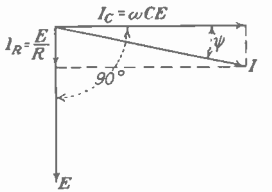


FIG. 5.—Vectors in condenser with leakage.

divides between the capacity and the resistance, I_R through the resistan being in phase with the applied voltage E, and I_C through the capacity leadi E by 90 deg. as shown in Fig. 5. The resultant current I leads E by (deg. $-\psi$), where ψ is the phase difference. In Fig. 5

$$\tan\psi = \frac{E/R}{\omega CE} = \frac{1}{\omega RC}$$

or

$$\psi = \frac{1}{\omega RC}$$

Power factor is a term that involves all the power losses in a condens If the total power loss in a condenser is W watts, the voltage applied to it V volts (r-m-s), and the current flowing through it is I amperes (r-m- the power factor, of the condenser is W/VI. The relation between I (ampere and V (volts) for a condenser of capacity C (microfarads) operating at frequency f is

$$I = \frac{2\pi f C V}{10^6} = \frac{\omega C V}{10^6}$$

The power factor of a condenser in per cent may be written

$$\cos\theta = \frac{W \times 10^6}{2\pi f C V^2} = \frac{W \times 10^6}{\omega C V^2}$$

Referring again to Fig. 2 showing the perfect condenser C and resistance replacing the actual condenser, the value of R can be calculated from t equation $W = I^2R$. The quantity R is known as the *equivalent resistance* the condenser at the given frequency.

The expression $W \times 10^6/\omega C V^2$ for power factor can be changed into t expression involving resistance, capacity and ω by substituting I^2R for and then substituting $\omega C V/10^6$ for I, giving power factor equal to $RC\omega$ 10^{-6}.

10. Values of Power Factor for Electrical Insulating Materials at adio Frequencies.

Material	Frequency, kilocycle	Power factor, per cent	Source
	187.5	0.459	
	300	0.476	
nber	429	0.478	1
	600	0.495	
	1,000	0.513	
lluloid, photographic film	A	4.2	
llulose nitrate, laboratory product	A	2.8	
ment, de Khotinski, medium hard	A	3.68	5
ber, black	A	4.55	
red	A	4.89	
oil impregnated	A	3.68	
ass	30	0.35 to 2.98[a]	4
	600	0.04 to 0.65[b]	1
cobalt	500	0.70	
	500	0.42	2
flint	720	0.42	
	890	0.40	
heat resisting	A	0.61	
photographic, with gelatin coating	A	1.00	5
without gelatin coating	A	0.86	
	14	0.97	
	100	0.77	3
plate	500	0.66	
	1,000	0.62	
	500	0.70	2
American plate	A	0.93	5
	14	0.88	
	100	0.74	
	500	0.67	3
Pyrex	750	0.68	
	30	0.56	4
		0.26	
	500	0.42	2
Window	A	0.87	5
	210	0.88	
	440	0.88	2
ard rubber	710	0.88	
	1,126	1.05	
	600	0.62	
	1,000	0.68	1
olantite	B	0.18	6
arble, white	A	0.52	
gray	A	4.2	5
blue	A	1.22	
ica, clear, India	A	0.07	5
	600	0.017	1
Built-up, shellac binder	A	1.75	5

Material	Frequency, kilocycles	Power factor, per cent	Source
Phenolic insulation, laminated (Bakelite)	190	3.85 to 7.35	2
	1,000	4.20 to 6.65	
Slate, electrical	A	63.	5
Varnish, spar	A	3.15	
insulating	A	5.25	
Wax, beeswax	A	1.63	
ceresin	A	0.04	
paraffin	A	0.49	
	14	0.042	3
	100	0.031	
	500	0.026	
Wood, basswood, quite dry	A	1.92	5
baywood, quite dry	A	2.45	
cypress, quite dry	A	2.1	
fir, quite dry	A	3.5	
maple, quite dry	A	2.45	
oak, quite dry	A	2.97	
birch	500	6.48	2
maple	500	3.33	
oak	300	3.68	
	425	3.50	
	635	3.85	
	1,060	4.20	

[a] Range of nine samples of various chemical compositions reported.
[b] Range of 27 samples of various chemical compositions reported.
[A] Measurements made between 80 and 1,875 kc.
[B] Between 250 kc and 1,500 kc.
[1] SCHOTT, ERICH, "Hochfrequenzverluste von Gläsern und einigen anderen Dielectricis," *Jahr. Drahtlosen Tele. Tele.*, **18**, 82–122, August, 1921.
[2] HOCH, E. T., Power Losses in Insulating Materials, *Bell System Tech. Jour.* November, 1922.
[3] MACLEOD, H. J., Power Losses in Dielectrics, *Phys. Rev.*, **21**, 53–73, 1923.
[4] DECKER, WILLIAM C., Power Losses in Commercial Glasses, *Elec. World*, **89**, 601–60 Mar. 19, 1927.
[5] PRESTON, J. L., and E. L. HALL, "Radio-frequency Properties of Insulating Materials, *QST*, **9**, 26–28, February, 1925.
[6] Isolantite *circ.*

wer Factor of Various Insulating Materials at 1,000,000 Cycles
(General Electric Company)

Material	Power factor	Dielectric constant
ar linseed-oil varnish film	0.012	2.2
ck asphaltic varnish film	0.008	2.0
llac film	0.025	4.1
ar gum varnish film	0.011	3.2
eral oil	0.0008	2.7
in	0.0026 to 0.0037	3.3 to 4.7
affin wax	0.0097	2.5
esin wax	0.0012 to 0.0021	2.5 to 2.6
swax	0.025	2.9
tland cement	0.018 to 0.029	6.8 to 8.0
celain (wet process)	0.006 to 0.008	6.5 to 7.0
e glass	0.01	8.4
ed quartz	0.0002	4.1
calex	0.002	8.0
d rubber	0.015 to 0.02	3.0 to 5.0
nolic resin laminated compound (highest grade, aper base)	0.035	5.0
nolic resin laminated compound (highest grade, oth base)	0.045	5.0
nolic resin molded compound (wood-flour filler)	0.035	5.5
nolic resin molded compound (mica filler)	0.01	6.0
d fiber (dry)	0.05	5.0
a (clear muscovite)	0.0001 to 0.0006	6.5 to 8.0
a (amber)	0.0004 to 0.071	5.4 to 5.8
nished cloth, (yellow)	0.03	2.5
lack)	0.02	2.0
e	0.45 to 0.63	12.4 to 19.0

1. **Dielectric Strength.** The *dielectric strength* of an insulating erial is the minimum value of electric field intensity required to ture it. Dielectric strength is usually expressed in kilovolts per timeter of dielectric thickness. The fall in insulation resistance with in temperature is a factor of great importance in connection with breakdown of a dielectric under the applied voltage. Insulating erials are not strictly homogeneous. The current leak through an ilating material may perhaps be concentrated in a few small paths ough the material, and the energy loss due to the leakage, while small, be large compared with the area through which it is flowing. The ns of the current flowing through the dielectric become heated with a lting lowering of the resistance of the path and an increase in the ent leakage. The heating of the dielectric may lead to rapid deteri- ion, particularly if moisture is present, and ultimate breakdown. length of time of the application of the voltage has a definite bearing n the breakdown voltage. Most dielectrics will withstand for a

very brief period a much higher voltage than they can when the vol is applied for a longer period.

These effects have dictated two tests for condensers, a high flash-voltage of very brief duration, and the application of a much lc voltage for a longer period.

The dielectric strength of a material is usually found to be lowe r-f voltages than for a-f or d-c voltages. The rupturing voltage at r frequencies depends on the rapidity with which the voltage is raised is not nearly so definite a phenomenon as low-frequency puncture volt Dielectric strength of solid insulators is difficult to measure because of complexity of the experimental effects. As the r-f currents flow in material, heating, corona, flash-over, and possible deterioration, bli ing, or charring may result with consequent changing of voltage current as the time of application elapses.

If high r-f voltages are applied to an air condenser, a corona discharge be set up which appears as a visible glow around high potential metal p points and sharp edges, and is usually distinctly audible. These co effects represent a power loss in the condenser. Hence, the constructi air condensers for high voltages requires the rounding of all edges and co and the avoiding of sharp points which encourage the formation of co and flash-over.

12. Dielectric Absorption. When a condenser is connected to a source of e.m.f. the instantaneous charge is followed by the flow of a s and steadily decreasing current into the condenser. The additional ch is absorbed by the dielectric. Similarly, the instantaneous disch of a condenser is followed by a continuously decreasing current. condenser does not become fully charged immediately, nor does it pletely discharge immediately when its terminals are shorted, but se discharges may be secured when the condenser possesses dielectric abs tion. The maximum charge in a condenser cyclically charged discharged varies with the frequency of charge.

If a condenser evidencing dielectric absorption is used at radio quencies, a power loss occurs which appears as heat in the conde The existence of power loss indicates a component of e.m.f. in phase the current as though a resistance were in series with the condens shown in Fig. 2. The effect of dielectric absorption can be meas along with other losses in the condenser, although dielectric absor represents the chief power loss in solid dielectrics.

13. Calculation of Capacity. Formulas are available for us calculating the capacity for a large number of geometrical shap conducting surfaces such as spheres, and cylinders, either separat concentric, and flat surfaces of various shapes. The usual type of denser calculations are concerned with two or more flat conductors.

When two conducting plates are parallel, close together and of large the capacity of the condenser is given by

$$C = 0.0885 \times \frac{KS}{t}$$

where C = capacity in micromicrofarads
K = dielectric constant (which is 1 for air)
S = area of one plate in square centimeters
t = distance between plates in centimeters.

ıen more than two plates are used in the condenser, the formula becomes

$$C = 0.0885 \times \frac{KS(N - 1)}{t}$$

ere N = number of plates

Γhe actual capacity of a parallel plate condenser is slightly larger than the ue as calculated from the above formula, because of the fringing of the ctric lines of force beyond the space between the plates. A correction ı be made for this fringing by slightly increasing the dimensions of the .tes. A narrow strip of width w can be added to the actual plate dimenns. In the case of circular plates $w = 0.4413t$ and for plates with straight ges $w = 0.110t$, where t is the distance between the plates in centimeters.

l4. Combinations of Condensers. Combinations of two or more ıdensers in a circuit are often arranged in either series or parallel. ndensers connected in parallel give a total capacity equal to the sum the capacities of the individual condensers. Condensers connected series give a resulting capacity which may be calculated from the lowing:

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots}$$

is formula gives the following expression in the case of two condensers series

$$C = \frac{C_1 \times C_2}{C_1 + C_2}.$$

Γhe various elements such as tubes, sockets, mountings, wiring, etc., radio apparatus contain many small capacities by virtue of the differ:e of potential existing between the numerous conductors insulated m one another. These small capacities are known as stray capacities. ıile they are unimportant in some kinds of work, in other types of rk, such as in amplifier design they must be taken into account. the case of resistance-coupled amplifiers, for example, these capacities uce the amplification at the higher audio frequencies and make a flat ıracteristic with high overall gain impossible.

Γhe effect of stray capacities is eliminated in the case of condensers d as capacity standards by shielding the insulated plates and groundthe shield. In this manner a definite capacity is always assured a given scale setting.

.5. Effect of Frequency on Condenser Capacity. One of the most ɔortant considerations is the effect of frequency upon the capacity ue of a condenser. In the best condensers this effect is nil. In fact, · of the criterions of a suitable condenser for a capacity standard is t its capacity shall be the same for two different sets of charging and :harging conditions. A variable air condenser, such as the Bureau Standards type described on page 120 of the Bureau's *Circ.* 74, gives same capacity at 100 and at 1,000 charges and discharges per second. condenser having considerable solid dielectric in its make-up will w a difference in capacity with frequency. The quantity of electricity ich flows into a condenser during a finite charging period is greater n would flow in during an infinitely short charging period. Consequently, the measured or apparent capacity with a.c. of any finite f: quency is greater than the capacity on infinite frequency, the latt being called the geometric capacity. The capacity of a conden: decreases as the frequency increases.

The length of the internal leads of a condenser should be kept as sho and direct as possible to minimize the inductance of the leads whi acts to give an apparent change of capacity with frequency. The amou of this change can be calculated from $C_a = C[1 + \omega^2 CL \times 10^{-12}]$ whe C_a is the apparent or measured capacity, C is in μf, and L in μh.

16. Types of Condensers. There are many ways in which condens might be classified, having to do with their construction, size, volta rating, use, dielectric, or whether the capacity is fixed or variab The condensers used in various radio applications are found in innum able sizes, shapes, and uses. The two simplest divisions into whi condensers may be classified have to do with their capacity: *i.e.*, wheth it is fixed or variable.

17. Types of Fixed Condensers.—Fixed condensers are available all capacity ranges from a few $\mu\mu$f to several μf, for any voltage rati up to 45,000 volts or higher, and in innumerable shapes and sizes, depending upon the use for which the condenser is intended.

Paper formerly was used as the dielectric for condensers for use lower voltages, while mica was used in condensers for higher voltag More recently as the art of condenser manufacture has progressed, oil-impregnated paper dielectric is used in condensers for the high voltages, the whole condenser being mounted within an oil-filled contain

For paper dielectric, 100 per cent pure linen paper is used, which mu meet severe requirements as to thickness, porosity, uniformity, wid freedom from conducting particles, alkalinity, and acidity. Two more layers of paper are used between the metal foil plates, dependi upon the voltage for which the condenser is designed. Paper condens are impregnated with special high melting point waxes and sealed with metal containers, thus being protected from moisture.

Paper condensers are formed by winding two metal foil electrodes ribbons in conjunction with the paper ribbons. There are two types winding, *inductive* and *non-inductive*. The latter type is recommended r-f and for the higher a-f work. The inductive type is satisfactory for work.

In winding the inductive type of condenser, the foil used is narrower th the paper and the contact is made with the foils by tinned copper str inserted in the winding. The non-inductive type of winding is made w the foils about the same width as the paper. The foil is staggered so that condenser plates project over the ends of the paper. The terminals soldered to the extending foil at the opposite ends and thus make conta with every turn of the foil. The latter type of construction makes minimum plate resistance and minimum power loss.

Mica has been used very extensively for condensers for use at radio f quencies. India mica has been used almost exclusively as it has been gen ally considered as of superior quality for radio use.

Selected mica is split into sheets of definite thickness, gauged and test for punctures or other defects. A condenser is built up of alternating m and metal foil sheets, the sets of plates of opposite polarity being broug out at opposite ends where they are soldered together, forming the t terminals. The whole stack of plates is rigidly clamped together in such way as to firmly grip the plates in the center and expel all dielectric other th mica. The condenser may be mounted in a suitable container.

If a condenser is to be used with higher voltages, the practice is to construct ıe condenser with two or more condenser sections in series, rather than to .crease the thickness of the mica. The former method is more flexible than ıe latter, permitting the construction of condensers for 45,000 volts or gher.

It is customary to mount the large high voltage condensers in steel nks which are filled with a high flash-point insulating oil which serves to 'event access of dirt and moisture, prevents flash-over along the con- ɔnser sections, insulates the condenser from the tank and conducts ·at away from the condenser elements.

18. Electrolytic Condensers. Another type of fixed condenser of gh capacity for use on voltages not exceeding about 500 volts has cently come into use known as the *electrolytic condenser*. The chief vantage of the electrolytic condenser is its low cost and its small size r its large capacity as compared to other older types of fixed condensers. ɔr example, an 8-μf 500-volt condenser is about 1⅜ in. in diameter d 4½ in. long.

These condensers, however, are not always interchangeable with ndensers using paper or mica dielectric, because they can be used only direct or pulsating direct current circuits, and must be correctly nnected with respect to polarity.

Electrolytic condensers can be obtained for operation in low-voltage ıment circuits, for use as filter condensers in "B" power supply units d "A" eliminators. The capacities available run as high as 4,000 μf · the low-voltage types. Other electrolytic condensers are available voltages of 100 and 180 volts with capacities from 10 to 100 μf, while : condensers for 350 to 400 volts have capacities of from 1 to 32 μf.

Electrolytic condensers have small leakage currents which increase h the operating temperature of the condenser and with the voltage ɔlied. This leakage is less than 0.2 milliamp per microfarad at 400 500 volts.

.9. Electrolytic Condenser Characteristics. The electrolytic con- ıser has found general use in the filter circuits of radio receivers and : made possible the design of compact but effective filter systems. addition to the advantages mentioned above, the electrolytic is also -healing, momentary overloads of voltage simply causing a temporary ·ture of the dielectric, the rupture healing itself as soon as the voltage educed to normal.

Electrolytic condensers in general use today are frequently divided › three classes although the divisions are not clearly marked.

. *Liquid* electrolytic condensers in which the electrolyte is a liquid gener- containing a fairly large percentage of water.

. *Semidry* electrolytic condensers in which the electrolyte is a liquid ı a viscosity usually between about 3 to 4.5.

Dry electrolytic condensers in which the electrolyte is in the form of a e.

Vhile the liquid condenser contains a considerable quantity of water e of the condensers is entirely without moisture.

he electrolytic condenser consists of four essential parts: the anode, cathode, the electrolyte, and the dielectric film formed electrochemi- ⁄ usually on the surface of the anode (Fig. 6). The anode is almost invariably made of aluminum, the cathode of either copper or aluminum and the electrolyte composition depends upon the type of condenser and the service to which it is to be put; in one type of semidry condenser the electrolyte is composed of boric acid, glycerin, and ammonia either gaseous or as ammonia water. The proportions are 1,000 g of glycerin, 620 g of boric acid, and about 5 cc of 26 per cent ammonia water or the equivalent amount of ammonia gas.

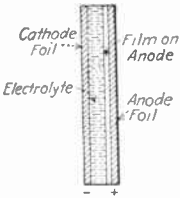


FIG. 6.—Electrolytic condenser construction.

20. Characteristics of Dielectric Film. The properties of the electrolytic condenser are due to the film formed on the anode, the composition of which is not accurately known. The extreme thinness of the film makes it possible to obtain high capacities per unit area and its dielectric strength enables it to withstand high voltages. The unit functions as a condenser only so long as a positive potential is applied to the anode. Ordinary electrolytic condensers can therefore be used only on d.c. or on pulsating d.c. This characteristic does not limit the application of the condenser to radio or audio circuits since most of the currents in such systems are pulsating d.c.

Commercial electrolytic condensers for radio applications have been made in ratings up to 600 volts peak; by a series arrangement of two or more condensers the voltage rating may be increased in direct proportion to the number of units connected in series. Experiments with several 500-volt condensers have indicated that when using a series arrangement of electrolytic condensers, shunting resistors to equalize the voltages are not required as is the case when several paper condensers are connected in series.

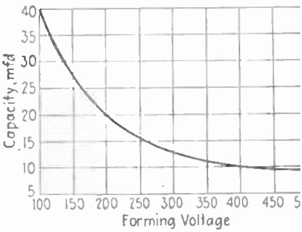


FIG. 7.—Electrolytic condenser characteristic.

The capacity per unit area depends upon the thickness of the film on the anode which in turn is an inverse function of the voltage to which the film is formed in manufacture. For a constant anode area the capacity is therefore inversely proportional to the forming voltage. If the anode area is such as to give 8 μf if the forming voltage is 500 volts d.c. then the same anode area formed to any lower voltage will give a capacity as indicated by the curve of Fig. 7.

21. Leakage Current. If an unformed electrolytic unit is connected across a d-c circuit the initial current is limited only by the resistance of the electrolyte. An anode film rapidly forms however and the current drops, finally reaching values in the order of 0.2 per μf in the case of condensers such as are generally used in the filter circuits of radio receivers. If the condensers are left on a d-c voltage for a long period (several hundred hours) the d-c current through the unit will drop to but a few microamperes per microfarad.

Condensers which have not been in use for some time will give a high leakage current; when voltage is again applied, this current rapidly decreases.

:2. Effect of Temperature. Figure 8 shows how the capacity of a ical electrolytic condenser varies with temperature; all electrolytic densers show a similar dependence of capacity upon temperature. ›jecting such condensers to temperatures below 0°F. causes a tem- ary change in characteristics, but the condensers regain the normal racteristics after the return to room temperature.

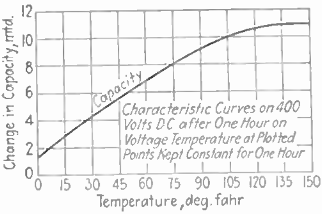


FIG. 8.—Temperature coefficient, electrolytic condenser.

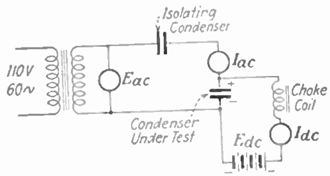
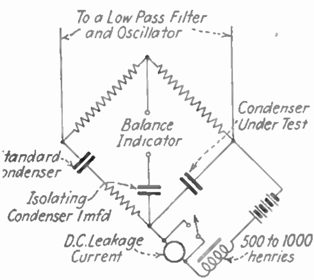


FIG. 9.—Production testing circuit for electrolytic condenser.

:3. Testing. The circuit of Fig. 9 is generally used to test electrolytics in duction. E_{dc} supplies a polarizing voltage so that the voltage across the denser will be pulsating d. c. The isolating condenser prevents short- uiting the polarizing voltage. If E_{ac} is maintained at a constant value the milliammeter may be calibrated in terms of the capacity of the condenser ler test. I_{dc} reads the d-c leakage current through the condenser. For the accurate measurement of capacity and power factor bridge systems h as those shown in Fig. 10 or 11 should be used. They are essentially



IG. 10.—Circuit for measuring electrolytic condenser capacity.

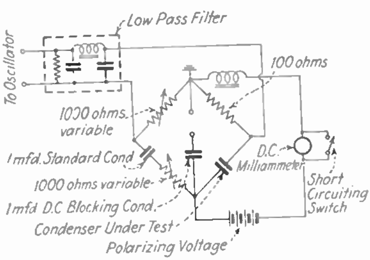


FIG. 11.—Capacity and power factor measurement.

ındard bridge systems rearranged to permit the application of a polarizing ltage.

24. Types of Variable Condensers. The most common type of riable condenser consists of a series of parallel metal plates fastened a shaft capable of rotation so that the moving plates intermesh with a t of fixed plates. Air is the main dielectric in such condensers, although me solid insulating material is required to insure that the two sets of ates are correctly located with respect to each other. Many ways of sulating the plates from each other have been devised, using one or ore pieces of the insulating material in sheet, rod, or bar form. Bakelite, hard rubber, Pyrex, porcelain, fused quartz, and Isolantite some of the materials used for such insulators.

The most common use of a variable condenser is in association wit coil, the combination forming a circuit resonant to a band of ra frequencies depending upon the coil constants and the capacity ra of the condenser. For a number of applications it is more conveni to have the capacity change in a different way than proportional to angle of rotation of the plates. This first resulted in the "decremet plate and the straight-line wave-length plate. As the use of freque rather than wave length became common, the straight-line freque plate came into use and later the "mid-line" plate. There are ot possibilities such as straight-line percentage wave length and straig line percentage frequency, the latter being of advantage in freque measurements. In any of the above shapes or classifications, movable plates formerly were so shaped as to give the desired freque or wave-length curve. This resulted in an ill-shaped plate difficult balance or to hold to a desired setting. In some cases semicirc rotating plates were used with the fixed plates cut away so as to obt the desired curve. In any of the special forms of plates, the pl shape may vary. The minimum and maximum capacities of the c denser play a large part in determining the outline of the plate.

Brass or aluminum plates and steel shafts are ordinarily used. the condenser is intended for use on high voltages, the spacing betw opposite plates must be sufficient to avoid a flash-over or arcing betw plates. It is customary to round off all sharp edges and corners in su condensers to avoid flash-over.

Condensers of the air type are often filled with oil, which increa the voltage that they can stand and increases the capacity from two five times depending on the dielectric constant of the oil used.

Compressed-air condensers were formerly used in some radio tra mitting stations. The voltage which such a condenser will stand increased without changing the capacity.

25. Gang Condensers. The single-dial control radio receiver broug problems to the designer in how to tune two to five circuits accurat using a corresponding number of similar coils and variable conden operating on the same shaft. As it is practically impossible convenien to manufacture two condensers exactly alike, to say nothing of three four alike, so that their capacities shall be exactly the same througho the complete rotation of the condenser plates and accurately tune t condensers with the same number of similar coils which differ slightly value, it has been customary to balance or equalize these tuned circu by the addition of small paralleling condensers sometimes called trimmi condensers. Such condensers can be obtained matched to one-half 1 per cent. It is possible to obtain two to four condensers called ga condensers for radio receivers arranged with their shafts in line a operated by one dial, matched to one-half of 1 per cent. The individu condensers may be separated from one another by metal shields if desir

26. Design Equations for Variable Air Condensers. The capaci of a condenser made up of three plates as indicated in Fig. 12 can obtained by determining the area of the overlapping plates, the distan between the adjacent plates, and substitution of these values in t general equation given above. The area of the shaded portion of Fi 12 is $\frac{1}{2}\pi(r_1^2 - r_2^2)$. The distance between the plates is $\frac{1}{2}(s -$

ıbstituting these values in the general equation, the capacity of the ›ndenser is given by

$$C = \frac{0.0885\frac{1}{2}\pi(r_1^2 - r_2^2) \times (3 - 1)}{\frac{1}{2}(s - t)}$$

The maximum capacity of a condenser with N plates can be obtained ; using a similar equation which may be written

$$C = \frac{0.278(r_1^2 - r_2^2)(N - 1)}{(s - t)}$$

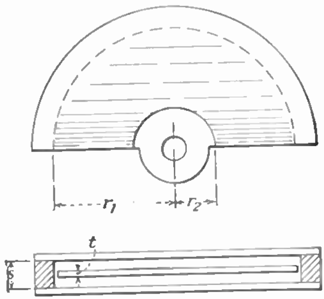


Fig. 12.—Dimensions useful in determining condenser capacity.

ı the above equations C is in micromi- ›ofarads and the dimensions r_1, r_2, s, ıd t in centimeters. These equations ›glect the capacity through the solid sulation which is used in the con- ›nser and the fringing effect, the correc- on for which is on page 89. Many ›ndensers are made to have as small a ıinimum capacity as possible, giving a ırge ratio of maximum to minimum ıpacity, but this is of doubtful advan- ıge, as slight changes of capacity due › warping of plates or wear in bearings ›ill cause a relatively large error at the ›wer end of the scale but practically no noticeable effect at the maximum ıpacity end of the scale.

A semicircular plate condenser gives a capacity calibration curve milar to C shown in Fig. 13. With the exception of the portions near the ends of the curve, it is practically a straight line. In practice, the lower ten and upper five or ten degrees of a 180-deg. scale are not used, so as to avoid the curvature in the calibration curve in these regions. Zero setting does not give zero capacity.

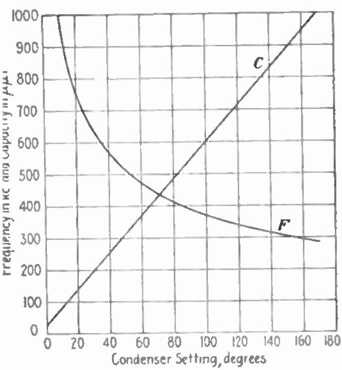
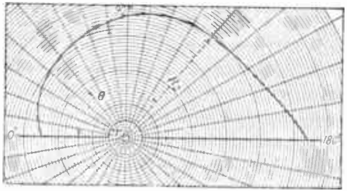
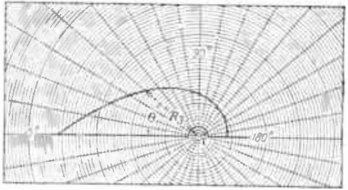
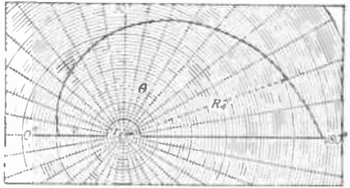


Fig. 13.—Semicircular plate condenser characteristic.

The frequency curve for such a condenser is shown at F in Fig. 13. The frequency changes very rapidly on the lower part of the scale. A slight capacity change would make a large frequency change. Therefore, when using frequency meters having semicircular plate condensers which constitute the main capacity of the circuit, the coils should be so designed as to give overlaps without resort to the low-capacity end of the scale.

As the wave length λ of a wave-meter circuit is proportional to $\sqrt{LC}$, f L is assumed to be constant, $\lambda \propto \sqrt{C}$ and $\sqrt{C}$ is proportional to the quare root of the setting θ. For a uniform wave-length condenser it is ıecessary to have C vary as the square of the setting θ, or $C \propto \theta^2$.

Straight-line wave length	Straight-line frequency	Straight-line percentage wave length or frequency
		
$C_2 = (a_2\theta + b_2)^2$ $A_2 = k\{(a_2\theta + b_2)^2 - \text{resid. cap.}\} + K\theta$ $R_2 = \left[114.6\{2ka_2(a_2\theta + b_2) + K\}\right]^{1/2}$ Constants: $a_2 = \dfrac{\sqrt{\text{max. cap.}} - \sqrt{\text{resid. cap.}}}{180}$ $b_2 = \sqrt{\text{resid. cap.}}$	$C_3 = \dfrac{1}{(a_3\theta + b_3)^2}$ $A_3 = k\left\{\dfrac{1}{(a_3\theta + b_3)^2} - \text{resid. cap.}\right\} + K(180 - \theta)$ $R_3 = \left[114.6\left\{\dfrac{2ka_3}{(a_3\theta + b_3)^3} + K\right\}\right]^{1/2}$ Constants: $a_3 = \dfrac{1}{180}\left\{\dfrac{1}{\sqrt{\text{resid. cap.}}} - b_3\right\}$ $b_3 = \dfrac{1}{\sqrt{\text{max. cap.}}}$	$C_4 = a_4\epsilon^{b_4\theta}$ $A_4 = k\{a_4\epsilon^{b_4\theta} - \text{resid. cap.}\} + K\theta$ $R_4 = \left[114.6\{ka_4b_4\epsilon^{b_4\theta} + K\}\right]^{1/2}$ Constants: $a_4 = \text{resid. cap.}$ $b_4 = \dfrac{\log(\text{max. cap.}) - \log(\text{resid. cap.})}{78.174}$

Common constants $k = \dfrac{\text{total plate area} - 180\,K}{\text{max. cap.} - \text{resid. cap.}}$

$$K = \frac{r^2}{114.6}$$

Again, it may be desirable that the percentage change in capacity r a given angle of rotation of the plates be the same for all parts of the ale as in the Kolster decremeter.[1] The polar equation for the boundary rve is

$$r = \sqrt{2C_0 a \epsilon^{a\theta} + r_2^2}$$

here C_0 = capacity when angle $\theta = 0$
a = constant = percentage change of capacity per scale division
ϵ = 2.71828
r_2 = radius of cut-out portion to clear washers separating variable plates.

The foregoing equations and tables have been compiled by Griffiths.[2] ne four types of plates given are for equivalent condensers having a pacity at zero setting of 36 $\mu\mu$f and a maximum of 500 $\mu\mu$f, with a ate area of 20 sq. cm.

The paper mentioned above gives the following data for the radii at fferent angles for the condensers mentioned in the table of equations ı page 96.

θ, degrees	Radius, centimeters		
	R_2	R_3	R_4
0	2.49	8.25	1.93
5	2.56		
10	2.60	6.70	2.02
20	2.76	5.62	2.13
30	2.89	4.80	2.24
40	....	4.17	2.36
60	3.18	3.32	2.64
80	....	2.75	2.98
90	3.56		
100	....	2.37	3.38
120	3.86	2.10	3.85
140	....	1.90	4.40
150	4.12	....	4.71
160	....	1.76	5.04
170	....	....	5.40
180	4.38	1.65	5.80

27. Effect of Putting Odd-shaped Plate Condensers in Series or arallel. If any of the above condensers are placed in parallel or in ries with another condenser, the straight-line calibration will be altered. paralleling condensers are used, the plate shape would require recalcution, after which the plate would become more nearly semicircular. a condenser is added in series, the calculation of the plate shape is more fficult. Griffiths[3] gives complete equations for a number of series

[1] *Bur. Standards Circ.* 74, p. 117. *Bur. Standards Sci. Paper* 235.
[2] GRIFFITHS, W. H. F., Notes on the Laws of Variable Air Condensers, *Exp. Wireless d Wireless Eng.*, **3**, 3–14, January, 1926.
[3] GRIFFITHS, W. H. F., Further Notes on the Laws of Variable Air Condensers, *Exp. ireless and Wireless Eng.*, **3**, 743–755, December, 1926.

combinations, the following table applying to the cases indicated whe maximum capacity of variable condenser = 500 $\mu\mu$f, minimum capaci of variable condenser = 36$\mu\mu$f, series fixed capacity = 500$\mu\mu$f, tot plate area = 20 sq. cm., r = radius of inactive semicircular area moving plate = 1.2 cm.

θ, degrees	Radius, centimeters			
	R_5	R_6	R_7	R_8
0	2.74	2.16	9.25	1.82
10	2.80	....	6.95	
20	....	2.35	5.57	1.96
30	2.92	....	4.65	
40	....	2.56	....	2.15
50	3.06			
60	....	2.78	3.32	2.38
70	3.22			
90	3.40	....	2.42	2.85
100	....	3.37		
110	3.66			
120	....	....	2.02	3.57
130	3.88			
140	....	4.25		
150	4.18	....	1.78	4.74
160	....	4.85		
170	4.52			
180	4.73	5.66	1.62	7.16

R_5, straight-line capacity with series fixed capacity.
R_6, corrected square law of capacity with series fixed capacity.
R_7, inverse square law of capacity with series fixed capacity.
R_8, exponential law of capacity with series fixed capacity.

28. Important Considerations in Design. It is not difficult to find large number of condensers on the market which will answer the need of any condenser application in radio receivers. The manufacture condensers for such use has been brought to a high stage of developmen both electrically and mechanically. The design problems here ar simpler in that low power and low voltage are to be handled.

When condensers for radio transmitters are considered, high powe and high voltage are to be provided for. More recently the use of ver high radio frequencies was added to the problem by requiring bette insulating materials. Insulators which were satisfactory at low radi frequencies have been found to heat up and be unsuited for frequencie such as 30,000 kc.

The following classification shows how condensers for transmitting set could be divided with respect to the voltages to which they are subjected:

Those subjected to steady d-c voltages only.
Those subjected to low-frequency voltages only.
Those subjected to damped r-f voltages only (obsolete).
Those subjected to steady CW r-f voltages only.
Those subjected to modulated CW r-f voltages only.
Those subjected to d-c voltages with superimposed r-f voltage.
Those subjected to low-frequency voltage and superimposed r-f voltage.

The last four of the above divisions could be further subdivided into thos for use on frequencies up to about 3,000 kc, those for use on frequencies fror

0 to about 25,000 kc, and those for use on frequencies of 30,000 kc and ve. The two latter classes require special construction.

ı specifying the rating of condensers for use in radio transmitters, following data should be given: capacity, current, frequency, nature oltage to be applied. A knowledge of the maximum radio-frequency age and maximum current permissible is important. A condenser ıld never be operated at more than half the breakdown voltage. the case of radio-frequency voltages, this fraction should be much ller.

9. Standards of Capacity. Fixed condensers using the best grade of a or fixed air condensers are used as capacity standards for radio uencies. For some work a variable air condenser is essential as a ıdard.

n important requirement of a standard condenser is that the capacity ain constant, the prerequisite of which is rigidity of construction, which nore difficult to secure in a variable than in a fixed condenser. re should be no relative motion possible between the movable plates and pointer. There should be no stops against which the pointer or movable es may strike and thus destroy the calibration. The manner of insu-ıg the two sets of plates is of great importance not only in fulfilling the lity requirement but in minimizing the power loss. An insulating mate-having a low temperature coefficient of expansion should be used, so that capacity will not change perceptibly with temperature. As small an ount of solid insulating material as possible should be employed, keeping ell out of the electric field. This field is quite intense near the high-ntial post. All insulation should be avoided in the vicinity of that ninal if power factor is to be kept low.

he condenser should be provided with a metal shield, which may be ınded during measurements, if the capacity is to remain constant. The s inside the condenser should be as short and direct as possible. The stance of leads, plates and contacts should be kept to the minimum. cible connection to the moving plates should not be used in a standard.

Iica condensers can be employed as standards after calibration as to acity and power factor over the range of frequencies at which they are to ısed.

0. Methods of Measuring Capacity. There are two general methods apacity measurement: (1) absolute measurements in terms of other trical or physical units; (2) comparison methods, where a condenser ınknown capacity is compared with a known calibrated condenser. absolute methods are not carried out at radio frequencies. Approxi-te calibrations of condensers for r-f use can be obtained using some n of bridge operating at 1,000 cycles. A very convenient instrument rapid checking work is found in the direct-reading microfarad meter ch operates on 60-cycle current.

Condenser calibrations at radio frequencies are conveniently made by ıbstitution method in a resonance circuit. The standard used must one which is constructed for use as a standard at radio frequencies. should give the same calibration at two widely different charge and charge rates, such as 100 and 1,000 charges and discharges per second. t fills this requirement, it may be assumed to give the same calibration adio frequencies.

simple tuned circuit consisting of a coil and the condenser under is arranged with a double-throw switch so that the standard con-ser may be readily substituted. Resonance may be indicated by a sensitive meter coupled to the main coil by a few turns of wire. crystal detector and 1-ma d-c meter makes a very convenient indicat device. Power is supplied electromagnetically by a small vacuum t oscillator. The measurement circuit is shown in Fig. 14. The shiel side of the condenser should be grounded. It is essential that the le connecting the switch points to each condenser be of the same lengtl each case as otherwise the cuits will not have the sa amount of inductance w one condenser is substitu for the other, which will re in an error in the calibrat The coupling between the circuit and the oscillator sho be kept quite loose, which be necessary if a sensitive re nance indicating instrumen used.

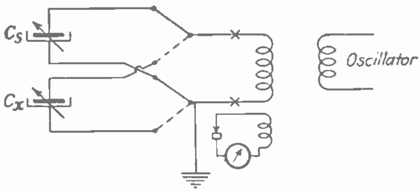


FIG. 14.—Measurement of condenser capacity.

If in the circuit shown in Fig. 14 a fixed inductor is used, the calibrat will be made at various frequencies depending upon the capacity the different condenser settings. A variable air condenser of suita size could be connected across the coil at XX and used to keep resonance frequency the same for any setting of C_x. If such a cir is carefully set up, no errors will result if the two circuits connected to and C_s are similar. The frequency at which the measurements are m can be measured with a frequency meter. The frequency or freque range over which a calibration is made should always be stated.

For rougher calibration work, the circuit shown in Fig. 15 may be used where C_s is tuned both with and without C_x in the circuit. It should be noted that the leads and switch connecting C_x to the circuit will introduce errors in the calibration.

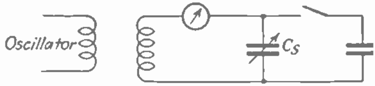


FIG. 15.—Simple scheme for measu capacity.

31. Precautions in Measurement of Very Small Capacities. I difficult to get agreement between different laboratories in the measu ment of capacities of the order of 15 or 20 $\mu\mu f$ or less. The reasons this are several and include differences in methods of measurem different lengths of leads used, different sizes and spacing of lea stray capacities to neighboring objects, and differences of a few mi microfarads in the capacity standards of the various laboratories. Hen it is not unusual to find a disagreement as much as 30 per cent or m in the measurement of a capacity of the order of 10 $\mu\mu f$.

For measurements of small capacities it is essential to keep all connec leads of minimum length, and have them occupy definite positions, so t corrections for their inductance and capacity can be applied if desi Apparatus not actually needed should be kept away from the measu circuit. A standard having a finely graduated scale is essential for s measurements. It should be capable of repeating its capacity value for given setting. Its capacity curve should preferably be a straight line with y crooks in it, so that interpolations can be accurately made from calibrated .nts.

32. Methods of Measuring Condenser Resistance and Power Factor d Dielectric Constant of Insulating Materials at Radio Frequencies. ›asurements of condenser resistance and power factor of insulating .terials are made in practically the same manner, as the sample of ulating material is prepared so as to form a condenser. Methods of asuring condenser resistance[1] and power factor of insulating materials[2] ve been given in publications of the Bureau of Standards. The ierican Society for Testing Materials has one or more standard methods testing electrical insulating materials for power factor and dielectric ıstant.[3]

The circuit shown in Fig. 16 may be used for measurements of resist- ;e, power factor and dielectric constant. Assuming that the power tor of a sample of insulating material is to be measured, the sample sheet form is made into a condenser :apacity between 100 and 1,000 μμf, as ›resented by C_x (Fig. 16). The re- inder of the circuit consists of the coil thermoelement T, and double-pole ible-throw switch S, in which radio- quency resistors R may be inserted. e galvanometer G gives deflections ich are proportional to the square of · current flowing in the circuit LTC_xR, electromagnetically induced from the lio-frequency oscillator O.

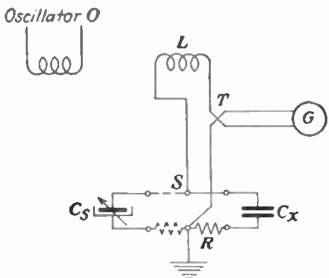


Fig. 16.—Circuit for measuring properties of insulators.

The deflections of galvanometer G are .ed for several values of inserted resist- :e R and for the case when R is a link practically zero resistance. Using the :ro resistance" deflection and the deflec- ı for a known value r of resistance inserted in switch S, the resistance R_T of total circuit LTC_xR is given by

$$R_T = \frac{r}{\sqrt{\frac{d_0}{d_1}} - 1}$$

ə average of the values of R_T calculated for various values of r should taken as the resistance of the complete circuit. The resistance R_S of circuit when C_S is substituted for C_X should be obtained in the same nner. The resistance R_X of the condenser C_x is then given by $R_x =$ $- R_S$. It is essential for this measurement that the two parts of the :uit which are interchanged should be as nearly identical as possible.

After the resistance R_x of the insulating material condenser is obtained, power factor or phase difference can be calculated from the equations en above. The dielectric constant K can be calculated from the

[1] Radio Instruments and Measurements, *Bur. Standards Circ.* 74, pp. 190–193.

[2] Methods of Measurement of Properties of Electrical Insulating Materials, *Bur. ıdards Sci. Paper* 471.

[3] "Proposed Revised Tentative Methods of Testing Electrical Insulating Materials Power Factor and Dielectric Constant at Frequencies of 100 to 1,500 kc," serial gnation D150–27T, American Society for Testing Materials, 1315 Spruce Street, ladelphia, Pa.

equation $K = Ct/0.0885S$, where C = capacity of sample in micromic farads, t = thickness of sample in centimeters, and S = area of smaller pla in square centimeters. The capacity is known, as given by Cs, and the ar of one plate and the thickness of the sample can easily be measured.

The method described above operates satisfactorily at frequencies from 1 to 1,500 kc.

A bridge method is sometimes used for these measurements although t apparatus is considerably more complicated than that described above.

A comparative method for testing insulating materials at very hi radio frequencies has been used by certain laboratories. In this meth the insulating material sample is placed in an intense electric field p duced by a 30-megacycle transmitter, and the temperature rise in t sample measured for a definite time interval. While such results ha not as yet been definitely tied up with power factor, dielectric consta etc., yet they represent in a very practical manner a means for determ ing the suitability of different types of materials for use at very hi radio frequencies. An insulator which is entirely satisfactory at low radio frequencies such as 1,000 or 2,000 kc may prove to be unusable 20 or 30 megacycles. Hence data on power factor and dielectric co stant are meaningless without a statement of the frequency at whi the data were obtained.

33. Life Tests on Paper Condensers (Dubilier). Accelerated life te of paper condensers can be made with d-c voltages only. Excessi alternating voltages produce heating, which in turn so alters the chara teristics of the dielectric of the condensers that no definite relationsh between these voltages and life has yet been obtained. So that wh tests with alternating voltages of higher than rated value have so produced results that cannot be coordinated, high d-c voltages ha given fairly consistent results—so much so, that it has been possible express the life of condensers in terms of impressed voltage.

Engineers of the Dubilier laboratories have taken samples of kinds of paper condensers and subjected them to voltages ranging fro that rated to four times rated voltage, keeping them on until the co densers broke down. A record of the kind of condenser, voltage, a life at the particular voltage was kept. When enough data were acc mulated—which represented the test results of thousands of condense with dielectric thicknesses of from 0.8 to 6.0 mils, and voltages of 200 2,000—it was found that the life could be expressed conservatively terms of the fifth power of voltage. In other words, the life of pap condensers on d.c. was found to vary inversely as the fifth power of t impressed voltage.

Expressed mathematically:

$$L = K \left\{ \frac{V_1}{V_2} \right\}^5$$

where L = life in hours
K = a constant depending upon the design of the conden (usually 10,000)
V_1 = rated voltage
V_2 = applied voltage

It is therefore clear that if a proper sample is taken from a lot of co densers and is subjected to a higher voltage to hasten its breakdown, i y short time the sample will reveal the quality of the entire lot. an example, twice the rated voltage will reduce the life to only about er cent, and hence, instead of waiting about 10,000 hr. to find the life ı condenser, only about 300 hr. are required at the accelerated life of twice rated voltage.

n this as in any other test, a sufficiently large sample must be taken e really representative of the entire lot. This is governed by the l known probability laws of sampling.

'he fifth power relationship is a conservative one, and in well con-icted condensers, as high as a seventh power relation between life voltage holds. At no time, even with the poorest of condensers has wer than fifth power been obtained.

SECTION 6

COMBINED CIRCUITS OF L, C, AND R

By W. F. Lanterman[1]

GENERAL IMPEDANCES

1. Impedances in A-c Circuits. The *impedance* of a circuit carry alternating current is the ratio of the voltage impressed across its minals to the current flowing through it. If the circuit consists of res ance only, the current is in phase with the impressed voltage and impedance is resistive. If the circuit consists solely of inductance, current lags one-fourth cycle phase behind the voltage, or if circuit is made up of pure cap tance, the current leads the volt by one-fourth cycle. In the la two cases, the impedance is sai be reactive.

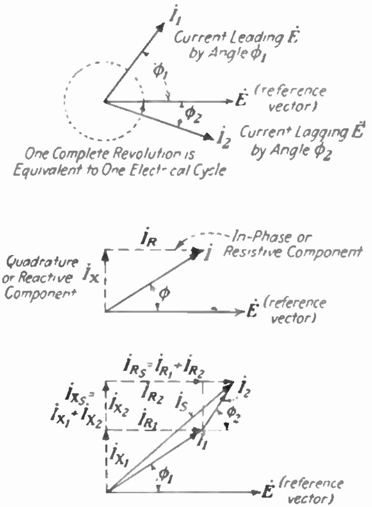


Fig. 1.—Vector diagrams of typical circuits.

2. Vector Diagrams. Vec diagrams are graphical represe tions showing both magnitudes phase angles of currents and v ages with respect to some kn voltage or current called the *refer vector* (Fig. 1). Leading vectors displaced from the reference vec by counterclockwise angles equ the time phase angle; lagging vec are similarly displaced in the cl wise direction.

The two projections of a ve upon lines parallel to and per dicular to the reference vector respectively, the in-phase (resist and the quadrature (reactive) c ponents of the projected vec The algebraic sum of any two vec is the resultant of the algeb sum of the in-phase components the sum of the quadrature components, added vectorially, as show Fig. 1.

3. Complex Notation. Algebraic vector notation requires the use vector operator j as a factor in each expression for a quadrature vec

[1] National Broadcasting Co.

distinguish such quantities from in-phase vectors. Thus, a vector is itten

$$\dot{I} = I_R + jI_X$$

ere I_R and I_X are the magnitudes of the in-phase and quadrature nponents, respectively. The operator j signifies that I_X, the reactive nponent, is leading the reference vector (and the in-phase component, by 90 electrical degrees, or one-fourth cycle. If the reactive comnent is *lagging*, the expression is written with a negative operator j):

$$\dot{I} = I_R - jI_X$$

A vector operated on twice by the operator j—a double operation tten $j \times j$, or j^2—is rotated twice through 90 deg., or 180 deg. This ounts merely to reversing the original direction of the vector, which is oted by $j^2\dot{I} = -\dot{I}$: hence $j \times j$ or $j^2 = -1$ and j is sometimes conered equivalent to $\sqrt{-1}$. In vector notation, however, the radical -1 is real and signifies an operation, which, if performed twice, reverses direction of a vector; this usage is in contradistinction to the purely ebraic conception of the radical $\sqrt{-1}$, wherein it is an imaginary neric.

. **Impedance in Form of a Vector.** Instead of writing the magnitude current due to each resistance and reactance, it is often convenient to te the resistances and reactances themselves in the form of a vector ch expressions are not true vectors; they are a form of vector operator). is a resistance R in series with a reactance X may be expressed as $= R + jX$. The current when a voltage $\dot{E}$ is impressed is

$$\dot{I} = \frac{\dot{E}}{Z} = \frac{\dot{E}}{R + jX} \tag{1}$$

m this it follows that $\dot{I}R + j\dot{I}X = \dot{E}$. $\dot{I}R$ is the voltage across the stor and is in phase with $\dot{E}$ while $j\dot{I}X$ is the voltage across the reacte and is leading $\dot{E}$ by 90 deg.

To convert (1) into an expression of the form $\dot{I} = I_R + jI_X$, both nerator and denominator are multiplied by $(R - jX)$. Since j^2 is ivalent to -1,

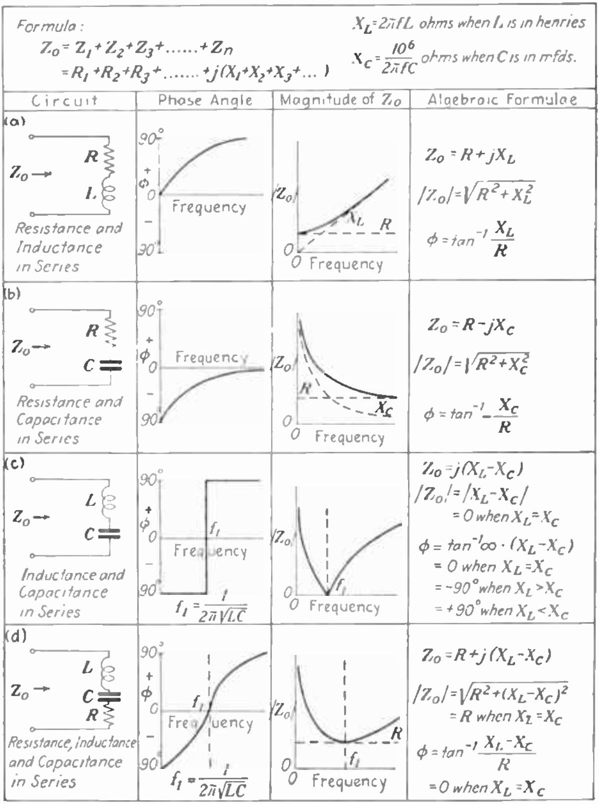
$$\dot{I} = \frac{\dot{E}(R - jX)}{(R + jX)(R - jX)} = \frac{\dot{E}R}{R^2 + X^2} - j\frac{\dot{E}X}{R^2 + X^2} \tag{2}$$

. **Values of the Reactance X of Coils and Condensers.** In the above ressions, reactances have been symbolized by X. If the reactance is oil having an inductance of L henrys, $X = \omega L$ ohms, where $\omega = 2\pi f$; is a condenser having a capacitance of C farads, $X = -\dfrac{1}{\omega C}$ ohms; is composed of both L and C, $X = \left(\omega L - \dfrac{1}{\omega C}\right)$ ohms. Capacitance ys has negative reactance, and inductance always has positive tance.

6. Equivalent Impedance. The equivalent impedance of a network impedances is the ratio of voltage to current at the terminals of network.

7. Equivalent Impedance of Impedances in Series. If two impeda Z_1 and Z_2 are in series, the resistance component of their equivalent imp ance is the sum of their resistances, and the reactance component is the s of their reactances:

$$Z_0 = Z_1 + Z_2 = (R_1 + jX_1) + (R_2 + jX_2) = (R_1 + R_2) + j(X_1 + X_2)$$



Equivalent impedances of series combinations of L, C, and R.

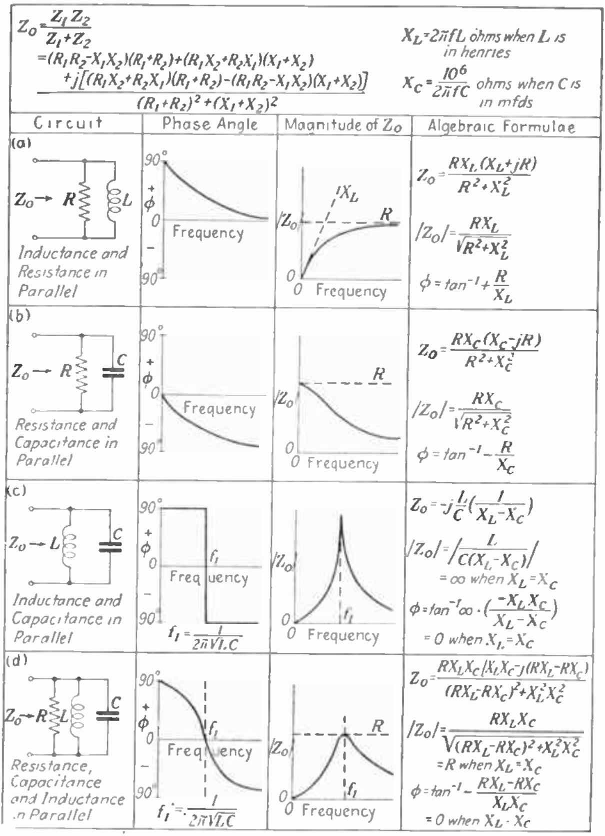
8. Equivalent Impedance of Impedances in Parallel. If two impedan Z_1 and Z_2, are in parallel, their equivalent impedance is

$$Z_0 = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(R_1 + jX_1)(R_2 + jX_2)}{(R_1 + jX_1) + (R_2 + jX_2)}$$

$$= \frac{[(R_1 + R_2)(R_1R_2 - X_1X_2) - (X_1 + X_2)(X_1R_2 - R_1X_2)]}{(R_1 + R_2)^2 + (X_1 + X_2)^2} + j\frac{[(R_1 + R_2)(X_1R_2 + X_2R_1) + (X_1 + X_2)(R_1R_2 - X_1X_2)]}{(R_1 + R_2)^2 + (X_1 + X_2)^2} \qquad (4)$$

his expression, while somewhat involved, is seen still to be of the form

$$Z_0 = R_0 + jX_0$$



Equivalent impedance of parallel combinations of L, C, and R.

. Equivalent Impedance of Networks Having More than Two Impedances. By applying the foregoing principles as many times as necessary in a step-by-step process, it is possible to reduce any network to a single equi alent impedance. Thus in Fig. 2,

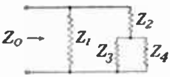


FIG. 2.—Network with branch impedances.

$$Z_{34} = \frac{Z_3Z_4}{Z_3 + Z_4}$$

$$Z_{234} = Z_2 + Z_{34}$$

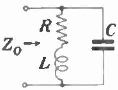
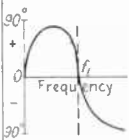
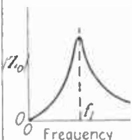
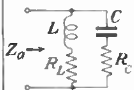
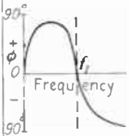
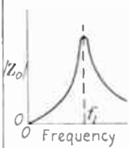
and finally,

$$Z_0 = \frac{Z_1(Z_2 + Z_{34})}{Z_1 + Z_2 + Z_{34}}.$$

$$Z_0 = \frac{Z_1Z_2}{Z_1+Z_2} = \frac{(R_1R_2-X_1X_2)(R_1+R_2)+(R_1X_2+R_2X_1)(X_1+X_2) + j[(R_1X_2+R_2X_1)(R_1+R_2)-(R_1R_2-X_1X_2)(X_1+X_2)]}{(R_1+R_2)^2+(X_1+X_2)^2}$$

$X_L = 2\pi fL$ ohms when L is in henries

$X_C = \frac{10^6}{2\pi fC}$ ohms when C is in mfds

Circuit	Phase Angle	Magnitude of Z_0	Algebraic Formulae
(e)  Resistance and Inductance in Series and in Parallel with Capacitance	 $f_1 = \frac{1}{2\pi\sqrt{LC}}$ Max. + ϕ angle occurs when $f = 0.577 f_1$ $Q > 1$ Not valid for $Q \leqq 1$		$Z_0 = \frac{RX_C - j[X_L(X_L-X_C)+R^2]}{R^2+(X_L-X_C)^2}$ $\lvert Z_0 \rvert = X_C\sqrt{\frac{R^2+X_L^2}{R^2+(X_L-X_C)^2}}$ $\doteq \frac{L}{RC}$ when $X_L = X_C$ $\phi = \tan^{-1} - \frac{X_L(X_L-X_C)+R^2}{RX_C}$ $\doteq 0$ when $X_L = X_C$ and R is small
(f)  Resistance and Inductance in Series and in Parallel with Capacitance and Resistance in Series	 $f_1 = \frac{1}{2\pi\sqrt{LC}}$ $Q > 1$ Not valid for $Q \leqq 1$		$Z_0 = \frac{M+N}{(R_L^2+R_C^2)+(X_L-X_C)^2}$ $\lvert Z_0 \rvert = \frac{\sqrt{M^2+N^2}}{\sqrt{(R_L+R_C)^2+(X_L-X_C)^2}}$ $\doteq \frac{L}{(R_L+R_C)C}$ when $X_L = X_C$ and R_L and R_C are small $\phi = \tan^{-1}\frac{N}{M}$ $\doteq 0$ when $X_L = X_C$ and R_L and R_C are small $M = (R_LR_C + X_LX_C)(R_L+R_C) + (R_CX_L - R_LX_C)(X_L-X_C)$ $N = (R_CX_L - RX_C)(R_L+R_C) - (R_LR_C+X_LX_C)(X_L-X_C)$

Equivalent impedance of parallel combinations of L, C, and R.

10. Absolute Values of an Impedance. In many cases the magnitude an impedance is all that it is required to know; this is given by

$$|Z_0| = \sqrt{R_0^2 + X_0^2} \tag{5}$$

11. Dissipation Factor Q. The ratio Q of reactance to resistance has nificance as a figure of merit of a coil or condenser and is called the *sipation constant.*

For a coil,

$$Q = \frac{\omega L}{R} \tag{6}$$

For a condenser,

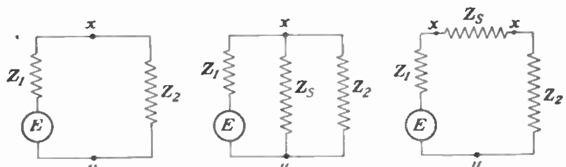
$$Q = \frac{1}{\omega RC} \tag{7}$$

Since both reactance and resistance vary with frequency, it is usually ssible by proper design and choice of materials to arrive at some timum value of Q for a coil or condenser at any particular frequency. Values of Q representative of those which are attainable in practice shown in Table I.

BLE I.—REPRESENTATIVE VALUES OF Q FOR VARIOUS COILS AND CONDENSERS

Frequency, cycles	Coils with powdered iron cores	Air-cored coils	Condensers with paper dielectric	Condensers with mica dielectric
100	25 to 50	3 to 10	1,000	
1,000	50 to 75	25 to 50	500	3,000
10,000	100 to 150	200 to 350	100 to 200	500
100,000	150 to 200	1,000 to 2,000	50 to 100	200 to 300
1,000,000	....	50,000	....	100 to 200

12. Loss Due to Inserting Series or Shunt Impedance in Audio rcuits. In audio circuits, attenuation-frequency characteristics are



(a)-Circuit with Applied Voltage, E, Impedance of Source, Z_1, and Load Impedance, Z_2

(b)-Shunt Impedance Inserted

(c)-Series Impedance Inserted

G. 3.—Shunt and series impedances inserted in audio-frequency circuits.

ten purposely modified by the insertion of corrective impedances. xamples of this practice are the use of equalizers, "tone controls," d scratch filters. It is desirable to be able to predict the effect on the frequency response of the insertion of such devices either in series or shunt with the original circuit. The following formulae give the amou of loss in decibels for each case:

a. Shunt Impedance. The loss due to inserting a shunt impedance (Fig. 3a and b) is

$$L = 20 \log_{10} \left(1 + \frac{Z_1 Z_2}{Z_s(Z_1 + Z_2)} \right) \text{ db}$$

The shunting impedance can usually be located at a point in the circ where the impedances Z_1 and Z_2 are matched, and where each is substantia a pure resistance through the range of frequencies involved. Then, letti $Z_1 = Z_2 = R_0$, the loss is

$$L = 20 \log_{10} \left| \frac{2Z_s + R_0}{2Z_s} \right| \text{ db}$$

$$= 20 \log_{10} \sqrt{1 + \frac{\cos \phi}{K} + \frac{1}{4K^2}} \text{ db}$$

where $K = |Z_s|/R_0$ and ϕ is the phase angle of Z_s. For various values K and ϕ, the loss can be read from the curve (Fig. 4).

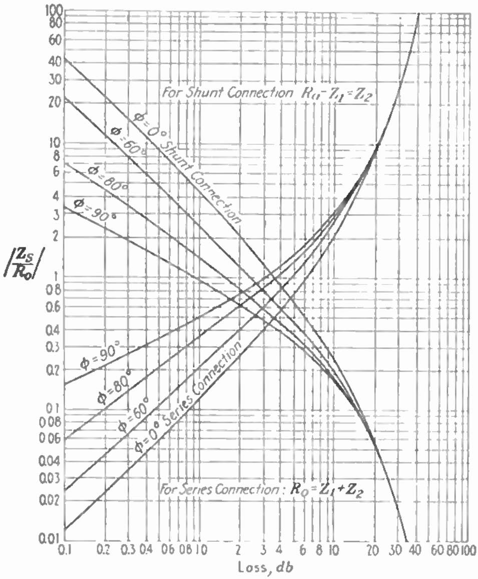


FIG. 4.—Transmission loss due to insertion of shunt or series impedanc

b. Series Impedance. The loss in decibels due to inserting a series impe ance Z_s (Fig. 4a and c) is

$$L = 20 \log_{10} \left(\frac{Z_1 + Z_2 + Z_s}{Z_1 + Z_2} \right) \text{ db} \quad (1$$

he series impedance can usually be inserted at a point in the circuit where e impedances Z_1 and Z_2 are matched, and where each is substantially a pure sistance through the range of frequencies involved. Then, letting $Z_1 + {}_2 = R_0$, the loss is

$$L = 20 \log_{10} \left| \frac{R_0 + Z_s}{R_0} \right| \text{db}$$

$$= 20 \log_{10} \sqrt{1 + 2K \cos\phi + K^2}\,\text{db} \tag{11}$$

here $K = |Z_s|/R_0$ and ϕ is the phase angle of Z_s. The loss can be read om Fig. 4 for various values of K and ϕ.

RESONANCE

13. Definition. In circuits containing both inductive and capacitive eactances, the current drawn from the source may be in phase with the .m.f., under which condition the reactance component of the equivalent npedance becomes equal to zero. At such frequencies the circuit is in *hase resonance*, or merely in *resonance*.

Physically, resonance depends upon the periodic shift of stored energy om the magnetic field of the coil to the electrostatic field of the conenser, in the form of a circulating current. If the coil and condenser re in series the phenomenon is called *series resonance* and the circulating urrent I_0 flows through the source and line. The impedance of this rrangement is low at resonance and the line current is large. If the coil nd condenser are in shunt the phenomenon is called *parallel resonance* or *anti-resonance*). The impedance at resonance is large in this case. he value of the circulating current in the case of parallel resonance epends inversely on the L/C ratio and the resistance and is usually rge compared to the current flowing in the external supply circuit. ecause of its ability to store energy a parallel resonant circuit is often eferred to as a *tank* circuit.

14. Series Resonance. The impedance of a coil is

$$Z_L = R_L + jX_L$$

here R_L is the resistance and X_L the reactance of the coil. If the coil carrying a current $\dot{I}$, the countervoltage due to the impedance of the oil is $\dot{I}Z_L$ or

$$\dot{E}_L = \dot{I}Z_L = \dot{I}R_L + j\dot{I}X_L$$

his is shown graphically by the vector diagram (Fig. 5*a*).

In many condensers the resistance is negligible compared to X_C, so hat the impedance of a condenser is very nearly equal to the reactance, or

$$Z_C \doteqdot -jX_C \doteqdot -j\frac{1}{2\pi fC}$$

f the value of the current is I amperes, the countervoltage across the ondenser is

$$\dot{E}_C = \dot{I}Z_C \doteqdot -j\dot{I}X_C \doteqdot -j\frac{\dot{I}}{2\pi fC}$$

This is shown graphically by the vector diagram (Fig. 5*b*).

If the coil and condenser are connected in series, the same curren flows through both. The voltage across the system is $\dot{E}_0 = \dot{E}_L + \dot{E}_C =$ $\dot{I}[R + j(X_L - X_C)]$ and the countervoltages developed by the reactance are in phase opposition to each other (Fig. 5c). If $X_L = X_C$, the react ance term becomes zero and $\dot{E}_0 = \dot{I}R$. This occurs when $2\pi fL =$ $1/2\pi fC$ or when the frequency is

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{12}$$

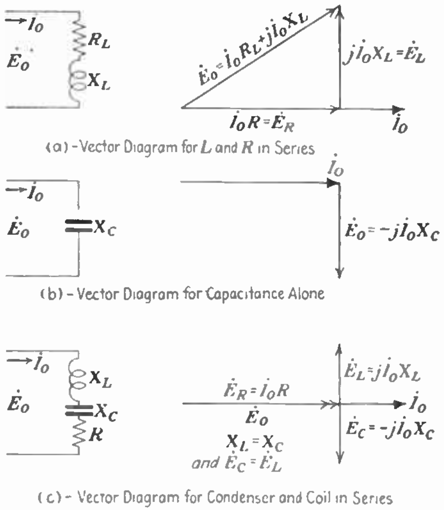


FIG. 5.—Vectors in series circuits.

Under these conditions, the circuit is in *series resonance*. The resonanc frequency f_r depends only upon the product of L and C. The vecto diagram for this condition is shown by Fig. 5c.

15. Impedance of Series Resonant Circuit at Frequencies Other tha Resonant Frequency. At frequencies other than f_r the impedance c the circuit is greater than the resistance, due to the reactive component which at any frequency f_1 is

$$X_1 = 2\pi L\left(\frac{f_1^2 - f_r^2}{f_1}\right) \tag{13}$$

With X_1 and R known, the absolute value of impedance is

$$|Z_1| = \sqrt{R^2 + X_1^2} \tag{14}$$

16. Design of Series Resonant Circuits. The magnitude of Z_0 (looking to a series circuit) is $|Z_0| = \sqrt{R^2 + (X_L - X_C)^2}$ which can be written

$$\frac{|Z_0|}{\omega_r L} = \sqrt{\frac{1}{Q^2} + n^2 + \frac{1}{n^2} - 2} \tag{15}$$

here $\omega_r = 2\pi \times$ resonance frequency
$\omega_1 = 2\pi \times$ any frequency f_1

$$n = \frac{\omega_1}{\omega_r}$$

$$Q = \frac{\omega_r L}{R}$$

The phase angle of Z_0 is given by $\phi = \tan^{-1} \dfrac{X_0}{R_0}$

$$= \tan^{-1} Q\left(n - \frac{1}{n}\right) \tag{16}$$

Values of $|Z_0|/\omega_r L$ and ϕ for various values of Q and n may be read from he curves (Figs. 6 and 7). Since (15) is symmetrical with regard to n and

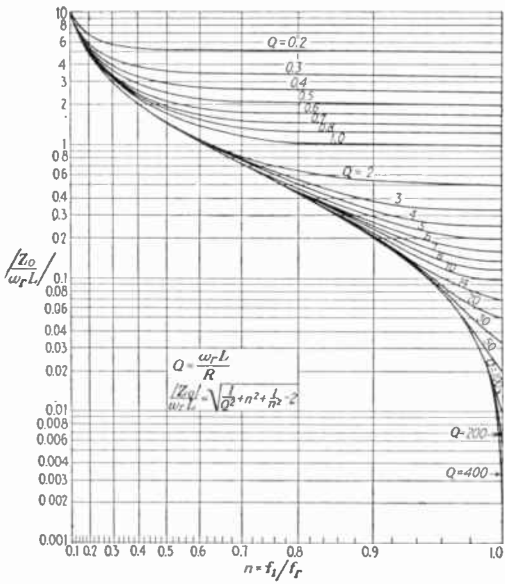


FIG. 6. $\dfrac{|Z_0|}{\omega_r L}$ vs. n for series circuits.

$/n$, and (16) is also symmetrical except for a reversal of sign, the same curves Figs. 6 and 7) may be used when $n = f_1/f_r$ or when $n = f_r/f_1$. Thus the esonance curve may be plotted for frequencies above and below resonance.

Example of Design of Series Resonant Circuit. Assume that a seri resonant circuit is to be designed to have an impedance, Z_0, of 100 ohms at resonant frequency of 1,000 cycles, and of 500 ohms at 0.9 resonance fre quency. At resonance, the impedance is the resistance, so $R = 100$ ohms.

At $n = 0.9$, the impedance is to be five times the impedance at $n = 1.$ From Fig. 6 we find that to secure the desired 5/1 ratio between $|Z_0|/\omega_r$ at $n = 1.0$ and at $n = 0.9$, Q must be 23, giving $|Z_0|/\omega_r L = 0.043$ an $|Z_0|/\omega_r L = 0.215$.

Then

$$\omega_r L = QR = 2{,}300$$

For

$$f_r = 1{,}000$$
$$\omega_r = 6{,}280$$

and

$$L = \frac{\omega_r L}{\omega_r} = 0.366 \text{ henry}$$

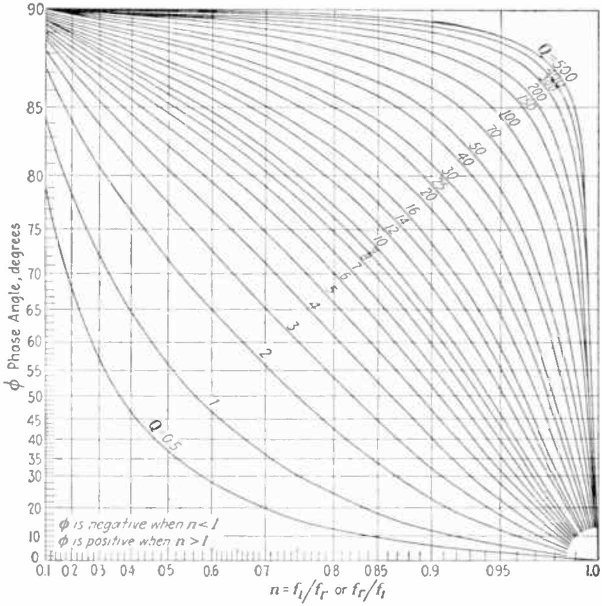


FIG. 7.— Phase angles in terms of n and Q, series circuits.

$$\phi = \tan^{-1}\left[Q\left(n - \frac{1}{n}\right)\right].$$

From the table in Section 1,

$$LC = 25.33 \times 10^{-8}$$

l

$$C = \frac{LC}{L} = 0.692 \times 10^{-6} \text{ farad}$$

en we have $R = 100$ ohms, $L = 0.366$ henry, and $C = 0.692 \times 10^{-6}$ ıd as the constants of the circuit.

Гhe impedance of the circuit at other frequencies can be found from the io $|Z_0|/\omega_r L$ read from Fig. 6 along the curve $Q = 23$, and the phase angles . be read from the corresponding curve of Fig. 7.

17. Table of Circuit Constants and Impedances at Various Frequencies. e table in Section 1 gives frequently d constants in impedance and resoıce calculations, for frequencies from 10 ·les to 100 megacycles.

18. Properties of Series Resonant Circuits. A series resonant circuit has the owing properties at resonance: (1) The rent flowing is in phase with the im-ssed voltage; (2) the current is limited y by the resistance of the coil; (3) the intervoltage across the coil is always ater than the impressed voltage, if the istance of the coil is the only resistance he circuit; (4) the countervoltage across : condenser may or may not be greater than the impressed voltage, pending upon the ratio between X_C and R; (5) the reactance and impedce of the circuit vary in magnitude and sense with the frequency shown in Fig. 8.

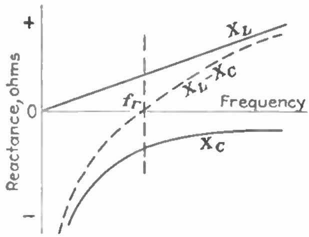


FIG. 8.—Series circuit reactance.

Items 3 and 4 are of importance in cases where $\dot{I}Z_L$ and $\dot{I}X_C$ are several ıes higher than the impressed voltages, under which conditions such :h voltages may occur across L and C as to endanger their insulation.

19. Amplifier Using Series Resonant Circuits. The fact that the countervoltages may be made to exceed the impressed voltage is useful as a voltage amplification scheme in vacuum tube circuits, such as that shown in Fig. 9. If C and R are small and L is large, the voltage E_g applied to the grid of the tube at resonance may be several times the impressed voltage E_o. If C and L are calibrated and one or both are made variable, the plate current is proportional to E_L, the circuit can be used as a frequency meter. The phase relation of E_L and · is determined by the values of L, R, and C, so that the circuit is o useful as a phase changing device. At resonance the input impedance the circuit viewed from the source is equal to R, and since a small value R must be used to secure voltage amplification and sharpness of resonce, the circuit is essentially "current operated" and works efficiently ly out of low impedance sources.

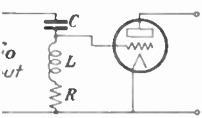


FIG. 9.—Use of series onance circuit as tage amplifier.

20. Use of Series Resonant Circuit for Frequency Regulation. Anıer application of a series resonant circuit is shown in Fig. 10. At sonance, the excitation voltages applied to the grids are the reactance ɔps IX_C and IX_L. The tubes are biased to the cut-off point so that :tification takes place. As long as the frequency of the applied voltage is $f = 1/2\pi\sqrt{LC}$, the excitation voltages and therefore the plate currents of the two tubes will be equal, but if the frequency varies, the vo age drop across one reactance will increase and that across the ot will decrease, causing the plate current of one tube to exceed the oth This difference in plate currents may be read on a meter to indicate frequency of applied voltage, or may be utilized through a differen relay to operate an automatic frequency contr ling device.

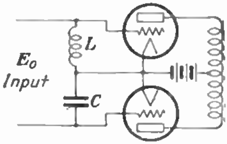


FIG. 10.—Use of series resonance circuit for frequency regulation.

21. Series Resonant Circuits as Equalize Series resonant circuits are often used as equaliz where it is required to eliminate or attenuat certain frequency or a small band of frequenc The resonant circuit with a variable resistance series is connected in shunt across the line or t minals of the circuit to be equalized, and more less readily by-passes currents of the resonant a adjacent frequencies, depending upon the adju ment of R.

22. Scratch Filters. Resonant circuits are also used as filters reducing needle scratch in electrical phonograph reproducers and reducing carbon hiss in microphone circuits. In this case the resona frequency is usually about 4,500 cycles; L may be 140 mh (1,500-t honeycomb coil) and $C = 0.0075$ µf. The value of R may be adjusted to give the desired attenuation of the high frequency. The loss-frequency characteristic of such a filter is shown in Fig. 12.

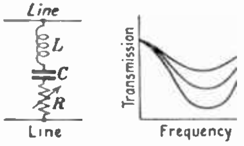


FIG. 11.—Series reson equalizer.

23. Tone Control. A series-resonant circuit tuned to 5,000 or 6,000 cycles is also applied as a "tone control" in audio systems, where it is desired to accentuate the low-frequency reproduction at the expense of the higher frequencies. For this purpose, a smaller L/C ratio than that used in scratch filters is desirable; L may about 20 mh, $C = 0.5$ µf and R variable to obtain the desired effect.

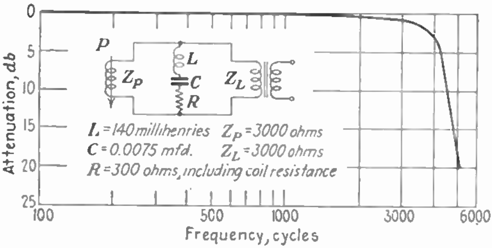


FIG. 12.—Transmission characteristic of scratch filter used with magnetic phonograph pick-up.

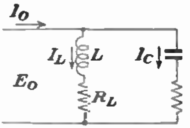


FIG. 13.—Parallel onance.

24. General Parallel Circuits. The parallel circuit shown in Fig. is widely used in audio and radio circuits. The resistance R_L is pr cipally that of the coil; the resistance R_C of the condenser is usua small.

equivalent impedance is

$$= \frac{Z_L Z_C}{Z_L + Z_C} = (R_L + R_C)(R_L R_C + X_L X_C) + (R_C X_L - R_L X_C)(X_L - X_C)$$

$$\frac{+j[(R_L + R_C)(R_C X_L - R_L X_C) - (X_L - X_C)(R_L R_C + X_L X_C)]}{(R_L + R_C)^2 + (X_L - X_C)^2} \qquad (17)$$

5. Resonance Relations in Parallel Circuits. The reactive com-
ent of the equivalent impedance is

$$= \frac{(R_L + R_C)(R_C X_L - R_L X_C) - (X_L - X_C)(R_L R_C + X_L X_C)}{(R_L + R_C)^2 + (X_L - X_C)^2} \qquad (18)$$

: X_0 is equal to zero, Z_0 becomes pure resistance, I_0 is in phase with
and the circuit is in resonance.

s condition exists if

$$\omega_r = \frac{1}{\sqrt{LC}}\sqrt{\frac{L - R_L^2 C}{L - R_C^2 C}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{L - R_L^2 C}{L - R_C^2 C}} \qquad (19)$$

en, also, $R_L = R_C$,

$$f_r = \frac{1}{2\pi\sqrt{LC}}. \qquad (19a)$$

reasing the ratio of R_L/R_C in (19) tends to decrease the frequency of
nance.

quation (19a) gives the condition under which the frequency of
allel resonance exactly equals that of a series circuit of the same
nd C—that is, when the resistances of the branches are equal.

6. Special Case Where $R_L = R_C$ and $X_L = X_C$.

$$R_L = R_C = R \text{ and } X_L = X_C = X,$$

$$Z_0 = \frac{R^2 + X^2}{2R} \qquad (20)$$

if $X_L = X_C \left(\omega L = \frac{1}{\omega C}\right)$, $\omega_r = \frac{1}{\sqrt{LC}}$ or $f_r = \frac{1}{2\pi\sqrt{LC}}$.

n $X^2 = (\omega L)^2 = L/C$, and (20) becomes

$$Z_0 = \frac{R^2 + \frac{L}{C}}{2R} \qquad (21)$$

en the resistances of the two parallel branches are equal, the equivalent
edance is a pure resistance at the frequency $f = 1/2\pi\sqrt{LC}$ and has the
e shown in (20). If also $R^2 = L/C$, (20) reduces to

$$Z_0 = R \qquad (22)$$

Thus if $R_L = R_C = R$ and $R = \sqrt{L/C}$, the circuit is resonant at all quencies (*i.e.*, the current and voltage are in phase), and the impedan equal to R.

27. Approximate Value of Resonance Frequency When R_C and R_L Small. In many actual circuits the resistance R_C is negligible, R_L consists principally of the resistance of the coil, in which case becomes

$$f_r \doteq \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_L^2C}{L}}$$

If, also the quantity R_L^2C/L is small compared with 1, then

$$f_r \doteq \frac{1}{2\pi\sqrt{LC}}$$

The latter relation is identical with the Eq. (12) for series reson and is sufficiently accurate for most circuit calculations.

28. Properties of Parallel Resonant Circuits. At its resonant quency, a parallel circuit has the following properties: (1) The cur in the external circuit is in phase with the impressed voltage; (2) current circulating in the parallel circuit itself is generally much la than the current flowing in the external circuit; (3) as far as the exte circuit is concerned, the parallel circuit behaves as a resistance appr mately equal to L/RC, which is usually large.

29. Absolute Value of Impedance at Resonance in Parallel Reso Circuit. Letting $\omega = 1/\sqrt{LC}$ in (17) gives for the impedance of a par circuit at resonance

$$Z_0 \doteq \frac{(R_LR_C + X_LX_C) + j(R_CX_L - R_LX_C)}{R_L + R_C}$$

The absolute value of this impedance is

$$|Z_0| \doteq \sqrt{\frac{(R_LR_C + \omega^2L^2)^2 + \omega^2L^2(R_CR_L)^2}{(R_L + R_C)^2}}$$

30. Absolute Value of Impedance in General Parallel Circuit, with N gible Resistance in Capacity Branch. In this case $R_C \doteq 0$, and from (1

$$Z_0 \doteq X_C\left[\frac{R_LX_C - j[R_L^2 + X_L^2 - X_LX_C]}{R_L^2 + (X_L - X_C)^2}\right]$$

The absolute magnitude of Z_0 is

$$|Z_0| \doteq \frac{X_C\sqrt{R_L^2 + X_L^2}}{\sqrt{R_L^2 + (X_L - X_C)^2}}$$

If R_L is small compared with X_L,

$$|Z_0| \doteq \frac{X_LX_C}{\sqrt{R_L^2 + (X_L - X_C)^2}} \doteq \frac{L}{C}\frac{1}{\sqrt{R_L^2 + (X_L - X_C)^2}}$$

At resonance, $X_L = X_C$ (R_L and R_C being assumed negligible), and

$$|Z_0| \doteq \frac{L}{RC}$$

he equivalent impedance of a low-resistance parallel circuit is therefore ›ry nearly a pure resistance at the resonant frequency and has the value $/RC$.

31. Design of Parallel Resonant Circuits. The magnitude of Z_0 is

$$|Z_0| \doteq X_C \frac{\sqrt{R_L^2 + X_L^2}}{\sqrt{R_L^2 + (X_L - X_C)^2}} \tag{27}$$

hich can be written

$$\frac{|Z_0|}{\omega_r L} = \left[\frac{1 + Q^2}{nQ^2}\right] \frac{\sqrt{\frac{1}{Q^2} + n^2}}{\sqrt{\frac{1}{Q^2} + \left(n - \frac{1 + Q^2}{nQ^2}\right)^2}} \tag{30}$$

here $Q = \frac{\omega_r L}{R}$ and $n = \frac{f_1}{f_r}$.

›r values of $Q = 10$ or larger, this reduces to

$$\frac{|Z_0|}{\omega_r L_0} \doteq \frac{\sqrt{\frac{1}{Q^2} + n^2}}{n\sqrt{\frac{1}{Q^2} + n^2 + \frac{1}{n^2} - 2}} \tag{30a}$$

›om the latter expression, it can be shown that $|Z_0| = 1.414\sqrt{L/C}$ at $= 0.707f_r$. Hence the L/C ratio of a parallel resonant circuit is expressible a function of its impedance at 70.7 per cent of resonance frequency, or *ce versa*. The ratio of $|Z_0|$ at 70.7 per cent resonance frequency to $|Z_0|$ at sonance is

$$\frac{|Z_0| \text{ at } 70.7\% f_r}{|Z_0| \text{ at } f_r} = \frac{1.414}{Q} \tag{31}$$

The phase angle of Z_0 is given by

$$\phi = \tan^{-1}\left[-\left(\frac{R_L^2 \times X_L^2 - X_L X_C}{R X_C}\right)\right]$$

$$= \tan^{-1}\left[-nQ\left(\frac{1}{Q^2} + n^2 - 1\right)\right]$$

$$\doteq \tan^{-1}[-nQ(n^2 - 1)] \tag{32}$$

ıen $Q \geqq 10$, say.

Values of $|Z_0|/\omega_r L$ and ϕ for various values of n and Q can be read from the rves (Figs. 14 and 15).

As an example, assume that a parallel circuit similar to that in Fig. 13 is be designed to be resonant at 5,000 cycles, with an impedance of 4,000 ms at resonance ($n = 1$) and an impedance of 100 ohms at 3,000 cycles $= 0.6$). From Fig. 14, $|Z_0|/\omega_r L = 0.9$ for all values of Q when $n = 0.6$. resonance $|Z_0|/\omega_r L$ is to be $\frac{4{,}000}{100} \times 0.9 = 36$. From the curves it found that $Q = 36$ gives $|Z_0|/\omega_r L = 36$ at $n = 1$ where $\omega_r = 31{,}416$. ıen for $n = 1$,

$$Z_0 = 36\omega_r L = 4{,}000, \text{ or } L = \frac{4{,}000}{36 \times 31{,}416} = 0.00354 \text{ henry}$$

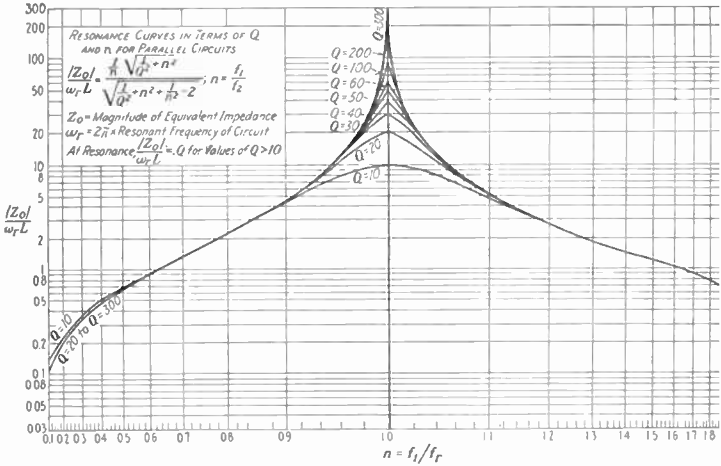


FIG. 14.—Parallel resonance curves.

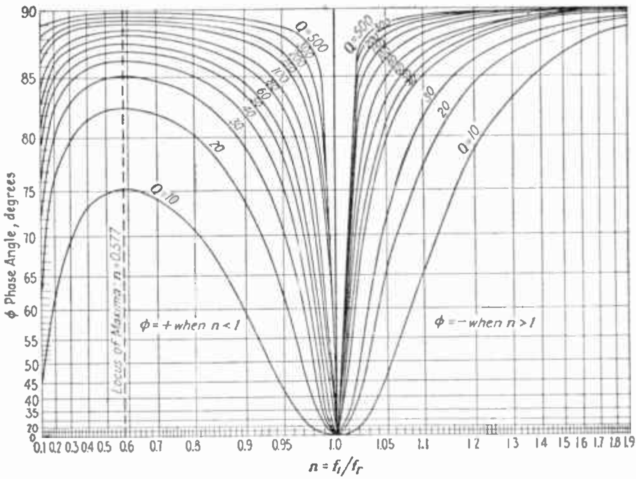


FIG. 15.—Phase angle of parallel LC circuit in terms of n and Q.

C for 5,000 cycles $= 10.136 \times 10^{-10}$. Then $C = LC/L = 0.286 \times 10^{-6}$ rad, and $R = \omega_r L/Q = 3.08$ ohms.

As a second example, suppose that a parallel circuit resonant at 1,000 kc to have an impedance of 10,000 ohms at that frequency, and 100 ohms 707 kc (70.7 per cent resonance frequency). By (32),

$$\frac{|Z_0| \text{ at } 70.7\% f_r}{|Z_0| \text{ at } f_r} = \frac{1.414}{Q} = 0.01$$

$$Q = 141.4$$

From Fig. 14, $|Z_0|/\omega_r L = 141.4$ when $Q = 141.4$ and $n = 1$; and

$$\omega_r L = \frac{|Z_0|}{141.4}$$
$$= 70.7 \text{ ohms}$$

hen

$$L = \frac{\omega_r L}{\omega_r} = 0.0112 \times 10^{-3} \text{ henry}$$

$$C = \frac{LC}{L} = 2{,}260 \times 10^{-12} \text{ farad}$$

ıd

$$R = \frac{\omega_r L}{Q} = 0.5 \text{ ohm}$$

The impedances at other frequencies can be computed from $|Z_0|/\omega_r L$ ılues read from Fig. 14, and the phase angles can be read from Fig. 15.

32. Split-tank Circuits. Parallel resonant circuits are high-impedance ɾcuits; this property makes them peculiarly suitable for use with ıcuum tubes, where the latively high impedance grid ıd plate circuits necessary to gh amplification are easily alized. In other instances, ɔwever, the reverse is true ıd the high impedance of a sonant circuit is a disadvange, as for instance, at the end a transmission line, where e termination circuit must fer a low impedance equal to that of the line (about 500 ohms). In ch cases an impedance adjustment may be made by the insertion of a ansformer, or by using a "split" form of the resonant circuit itself. ıe latter is equivalent to "tapping" off at a midpoint of the tank circuit ıd may be done in either the inductance or capacitance branch, as ustrated in Fig. 16. The result is a coupled circuit, that part of the actance between points B and C in each case being the mutual impedance.

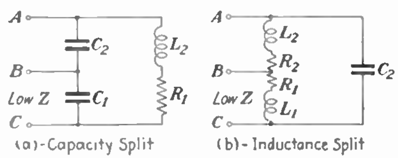


FIG. 16.—Split-tank circuits for matching low-impedance to high-impedance circuits.

a. Capacity Split. In Fig. 16*a*, the impedance at B–C is

$$|Z_{BC}| = \frac{\sqrt{L_2^2 C_2^2 \left(\frac{1}{C_1(C_1 + C_2)}\right)^2 + \frac{R_2^2 L_2 C_2}{C_1(C_1 + C_2)}}}{R_2} \tag{33}$$

If R_2 is small,

$$|Z_{BC}| \doteqdot \frac{L_2C_2}{R_2C_1(C_1 + C_2)} \qquad \text{(3}$$

and its ratio to the impedance Z_{AC} is

$$\frac{|Z_{BC}|}{|Z_{AC}|} = \frac{C_2^2}{(C_1 + C_2)^2} \qquad \text{(3}$$

The resonant frequency is

$$f_r = \frac{1}{2\pi\sqrt{L\dfrac{C_1C_2}{C_1 + C_2}}} \qquad \text{(3}$$

and the impedances Z_{AC} and Z_{BC} are both purely resistive at resonance.
The ratio of C_1 to C_2 for a given ratio between Z_{AC} and Z_{BC} is

$$\frac{C_1}{C_2} = \left[\sqrt{\frac{Z_{AC}}{Z_{BC}}} - 1\right] \qquad \text{(3}$$

In terms of the resonant frequency, inductance, and the impedance rat

$$C_1 = \frac{1}{4\pi^2 f_r^2 L}\sqrt{\frac{Z_{AC}}{Z_{BC}}} \qquad \text{(3}$$

$$C_2 = \frac{1}{4\pi^2 f_r^2 L\left(1 - \sqrt{\dfrac{Z_{BC}}{Z_{AC}}}\right)} \qquad \text{(3}$$

b. Inductance Split. In Fig. 16*b* the inductance is split, and the impedar at *B-C* is (assuming no mutual inductance between L_1 and L_2)

$$|Z_{BC}| = \frac{\sqrt{\left(R_1R_2 - \dfrac{L_1L_2}{(L_1 - L_2)C_2} + \dfrac{L_2}{C_2}\right)^2 + \left(\dfrac{R_2L_1}{\sqrt{(L_1 + L_2)C_2}} + \dfrac{R_1L_2}{\sqrt{(L_1 + L_2)C_2}} - \dfrac{R_1\sqrt{(L_1 + L_2)C_2}}{C_2}\right)^2}}{R_1 + R_2} \qquad \text{(}$$

If R_1 and R_2 are small,

$$|Z_{BC}| \doteqdot \frac{L_2}{C_2(R_1 + R_2)} \cdot \frac{L_2}{(L_1 + L_2)} \qquad \text{(}$$

and its ratio to the total impedance Z_{AC} is

$$\frac{|Z_{BC}|}{|Z_{AC}|} = \frac{L_2^2}{(L_1 + L_2)^2} \qquad \text{(}$$

The resonant frequency is

$$f_r = \frac{1}{2\pi\sqrt{(L_1 + L_2)C_2}} \qquad \text{(}$$

and the impedances Z_{AC} and Z_{BC} are both resistive at resonance.
The ratio of L_1 to L_2 for a given ratio between Z_{AC} and Z_{BC} is

$$\frac{L_1}{L_2} = \sqrt{\frac{Z_{AC}}{Z_{BC}}} - 1 \qquad \text{(}$$

:n terms of the frequency, capacity, and the impedance ratio,

$$L_1 = \frac{1}{4\pi^2 f_r^2 C_2}\left(1 - \sqrt{\frac{Z_{BC}}{Z_{AC}}}\right) \tag{45}$$

$$L_2 = \frac{1}{4\pi^2 f_r^2 C_2}\sqrt{\frac{Z_{BC}}{Z_{AC}}} \tag{46}$$

13. Measurement of Parallel Resonance Impedance. A convenient thod of experimentally determining the resonance impedance of a ·allel circuit is shown in Fig. 17. LC is the circuit to be measured. is method is based on the fact that the circuit just commences to illate when the "negative resistance" of the tube characteristic is nerically equal to the impedance of the LC plate circuit. In practice, ·ype −22 or −24 tube is satisfactory, in which case B should be about ' volts and C about 25 volts. The potentiometers G and P control grid bias and plate voltages, respectively. The latter should be ween 60 and 80 volts for the B voltage ıtioned. A receiver is loosely coupled to to detect the point where oscillation :ts. To make a measurement, G and P adjusted until the circuit is on the verge of llation. The LC is short-circuited by ing the key S, and P is varied a few volts ve and below the setting at which oscilla- ı occurred and the values of plate current ed. The values of G and B are of course hanged during this latter adjustment. : slope of the $e_p - i_p$ curve through the ıe of e_p where oscillation occurred is the ıtive resistance and is numerically equal he impedance $|Z_0|$. If L and C are known, ın be computed from Eq. (29):

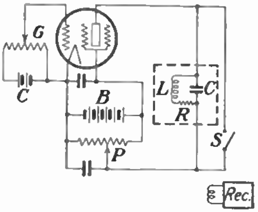


FIG. 17.—Circuit for measuring resonant impedance of parallel circuit.

$$|Z_0| = \frac{L}{RC} \tag{29}$$

$$R = \frac{L}{|Z_0|C}$$

; also suggests the use of the above circuit for measuring r-f resistance, nserting an unknown resistance in series in the LC circuit and meas- g its impedance before and after the insertion is made. By a similar ess, capacity or inductance may also be measured. The method as ined is limited by tube characteristics to impedances of about 10,000 s and over.

COUPLED CIRCUITS

:. **Coupling.** If two circuits have one or more common impedances, are said to be electrically **coupled.** A common impedance is any ·dance so situated that it causes the current in one circuit to influence :urrent in the other. The impedance may be resistive, reactive, or .

35. Coefficient of Coupling. The coefficient of coupling is

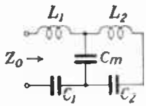
$$K = \frac{X_m}{\sqrt{X_1 X_2}} \qquad ($$

where X_m is any one component of the mutual impedance (resistar capacitive reactance or inductive reactance) and X_1 and X_2 are the t impedance components of the same kind in the respective circu K varies in value between zero and 1; if it is nearly 1, the couplin close or *tight;* if near zero, the coupling is *loose.*

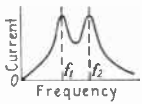
Coupled Circuits: Direct Capacitive

Impedance:

$$Z_0 = j\frac{\frac{1}{\omega C_m}\left(\omega L_1 - \frac{1}{\omega C_1}\right) - \left(\omega L_1 - \frac{1}{\omega C_1}\right)\left(\omega L_2 - \frac{1}{\omega C_2}\right) + \frac{1}{\omega C_m}\left(\omega L_2 - \frac{1}{\omega C_2}\right)}{\frac{1}{\omega C_m} - \omega L_2 + \frac{1}{\omega C_2}}$$



Circuit



Resonance Curve

General case: L_1, L_2, C_1, C_2 and C_m unrestri

$$f_1 = \sqrt{\frac{f_a^2 + f_b^2 - \sqrt{(f_a^2 - f_b^2)^2 + 4k^2 f_a}}{2}}$$

$$f_2 = \sqrt{\frac{f_a^2 + f_b^2 + \sqrt{(f_a^2 - f_b^2)^2 + 4k^2 f_a}}{2}}$$

where $f_a = \dfrac{1}{2\pi\sqrt{L_1 \dfrac{C_1 C_m}{C_1 + C_m}}}$ $\qquad f_b = \dfrac{1}{2\pi\sqrt{L_2 \dfrac{C_2 C_m}{C_2 + C_m}}}$

Coefficient of coupling:

$$k = \sqrt{\frac{C_1 C_2}{(C_1 + C_m)(C_2 + C_m)}}$$

Special cases:

a Both circuits tuned to same frequency ($f_a = f_b$)

$f_1 = f_a\sqrt{1 - k}$ $\qquad f_2 = f_b\sqrt{1 + k}$

b Loose coupling ($f_a = f_b$ and $C_m >> C_1$ and C_2; $k \neq 0$).

$$f_1 \neq f_2 \neq f_a \neq \frac{1}{2\pi\sqrt{L_1 C_1}} \neq \frac{1}{2\pi\sqrt{L_2 C_2}}$$

c Close coupling ($f_a = f_b$ and $C_m << C_1$ and C_2; $k \neq 1$).

$$f_1 \neq 0 \text{ and } f_2 \neq \sqrt{2} f_a \neq \frac{\sqrt{2}}{2\pi\sqrt{L_1 C_m}} \neq \frac{\sqrt{2}}{2\pi\sqrt{L_2 C_m}}$$

d Both circuits identical

$$\begin{cases} f_a = f_b \\ L_1 = L_2 \\ C_1 = C_2 \end{cases}$$

$f_1 = \dfrac{1}{2\pi\sqrt{L_1 C_1}}$ $\qquad f_2 = \dfrac{1}{2\pi\sqrt{L_1 \dfrac{C_1 C_m}{2C_1 + C_m}}}$

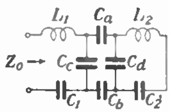
Coupled Circuits: Indirect Capacitive

uivalent impedance

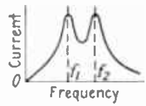
$$Z_0 = j\left[\omega L_1 - \frac{1}{\omega C_a}\,\frac{\left(\omega L_2 - \frac{1}{\omega C_d}\right)\frac{1}{\omega C''} + \frac{L_2}{C_d}}{\left(\omega L_2 - \frac{1}{\omega C_d}\right)\frac{1}{\omega C'''} + \frac{L_2}{C_d}}\right]$$

ere $$C' = \frac{C_aC_b}{C_a + C_b}$$

$$C'' = \frac{C_aC_bC_c}{C_aC_b + C_aC_c + C_bC_c}$$



Circuit



Resonance Curve

neral case: L_1, L_2, C_a, C_b, C_c and C_d unrestricted.

$$f_1 = \sqrt{\frac{f_a^2 + f_b^2 - \sqrt{(f_a^2 - f_b^2)^2 + 4k^2f_a^2f_b^2}}{2}}$$

$$f_2 = \sqrt{\frac{f_a^2 + f_b^2 + \sqrt{(f_a^2 - f_b^2)^2 + 4k^2f_a^2f_b^2}}{2}}$$

ere $$f_a = \frac{1}{2\pi\sqrt{L_1\left(C_c + \frac{C_dC''}{C_d + C''}\right)}} \qquad f_b = \frac{1}{2\pi\sqrt{L_2\left(C_d + \frac{C_cC'}{C_c + C'}\right)}}$$

efficient of coupling:

$$k = \frac{C''}{\sqrt{(C_c + C')(C_d + C')}}$$

ecial cases:

a Both circuits tuned to same frequency ($f_a = f_b$).

$$f_1 = f_a\sqrt{1 - k} \qquad f_2 = f_a\sqrt{1 + k}$$

b Loose coupling ($(C_a + C_b) << C_c$ and C_d) $k \neq 0$ ($f_a = f_b$).

$$f_1 \neq f_2 \neq f_a \neq \frac{1}{2\pi\sqrt{L_1C_c}} \neq \frac{1}{2\pi\sqrt{L_2C_d}}$$

c Close coupling ($f_a = f_b$)($(C_a + C_b) >> C_c$ and C_d; $k = 1$.
$f_1 \neq 0$

$$f_2 \neq \sqrt{2}\, f_a \neq \frac{1}{\pi\sqrt{2L_1(C_c + C_d)}} \neq \frac{1}{\pi\sqrt{2L_2(C_c + C_d)}}$$

d Both circuits identical.

$$\begin{cases} f_a = f_b \\ L_1 = L_2 \\ C_1 = C_2 \end{cases}$$

$$f_1 \neq \frac{1}{2\pi\sqrt{L_1(C_c + 2C')}}$$

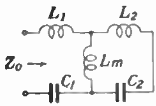
$$f_2 \neq \frac{1}{2\pi\sqrt{L_1C_c}}$$

$$k = \frac{C'}{C_c + C'}$$

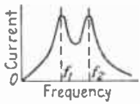
Coupled Circuits: Direct Inductive

Equivalent impedance:

$$Z_0 = j\frac{\omega L_m\left(\omega L_1 - \frac{1}{\omega C_1}\right) + \left(\omega L_1 - \frac{1}{\omega C_1}\right)\left(\omega L_2 - \frac{1}{\omega C_2}\right) + \omega L_m\left(\omega L_2 - \frac{1}{\omega C_2}\right)}{\omega L_m + \omega L_2 - \frac{1}{\omega C_2}}$$



Circuit



Resonance Curve

General case: L_1, L_2, L_m, C_1 and C_2 unrestricte

$$f_1 = \sqrt{\frac{f_a^2 + f_b^2 - \sqrt{(f_a^2 - f_b^2)^2 + 4k^2f^2f}}{2(1 - k^2)}}$$

$$f_2 = \sqrt{\frac{f_a^2 + f_b^2 + \sqrt{(f_a^2 - f_b^2)^2 + 4k^2f_a^2f}}{2(1 - k^2)}}$$

where $f_a = \frac{1}{2\pi\sqrt{(L_1 + L_m)C_1}}$ $\qquad f_b = \frac{1}{2\pi\sqrt{(L_2 + L_m)C_2}}$

Coefficient of coupling $\qquad k = \frac{L_m}{\sqrt{(L_1 + L_m)(L_2 + L_m)}}$

Special cases:

a Both circuits tuned to same frequency ($f_a = f_b$).

$$_1 = \frac{f_a}{\sqrt{1 + k}} \qquad f_2 = \frac{f_a}{\sqrt{1 - k}}$$

b Loose coupling ($f_a = f_b$ and $L_m << L_1$ and L_2) $k \neq 0$.

$$f_1 \neq f_2 \neq f_a \neq \frac{1}{2\pi\sqrt{L_1C_1}} \neq \frac{1}{2\pi\sqrt{L_2C_2}}$$

c Close coupling ($f_a = f_b$ and $L_m >> L_1$ and L_2; $k \neq 1$).

$$f_1 \neq \frac{f_a}{\sqrt{2}} \neq \frac{1}{2\pi\sqrt{2L_mC_1}}$$

$$f_2 \neq \infty$$

d Both circuits identical.

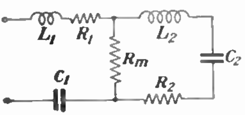
$$\begin{cases} f_a = f_b \\ L_1 = L_2 \\ C_1 = C_2 \end{cases}$$

$$f_1 = \frac{1}{2\pi\sqrt{(L_1 + 2L_m)C_1}}$$

$$f_2 = \frac{1}{2\pi\sqrt{L_1C_1}}$$

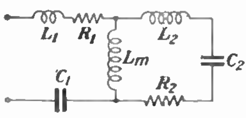
$$k = \frac{L_m}{L_1 + L_m}$$

36. Direct and Indirect Coupling. If the common impedance is resistance, inductance or capacitance connected directly between tl



$$K = \frac{R_m}{\sqrt{R_1'R_2'}}$$
$$R_1' = R_1 + R_m$$
$$R_2' = R_2 + R_m$$

FIG. 18.—Direct resistive coupling.



$$K = \frac{L_m}{\sqrt{L_1'L}}$$
$$L_1' = L_1 +$$
$$L_2' = L_2 +$$

FIG. 19.—Direct inductive coupli

two circuits, the coupling is *direct*. Such circuits are shown in Fig 18, 19, and 20. If the common impedance is a transformer, the couplir

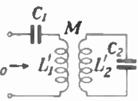
Coupled Circuits: Indirect Inductive

ivalent impedance:

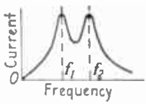
$$Z_0 = j\left[\left(\omega L_1' - \frac{1}{\omega C_1}\right) - \frac{(\omega M)^2}{\left(\omega L_2' - \frac{1}{\omega C_2}\right)}\right]$$

ivalent direct-coupled circuit: Indirect induc- coupling is equivalent to direct inductive)ling if

$$L_1 = L_1' - M$$
$$L_2 = L_2' - M$$
$$L_m = M$$



Circuit



Resonance Curve

·e L_1' and L_2' are the self-inductances of the

$$f_1 = \sqrt{\frac{f_a^2 + f_b^2 - \sqrt{(f_a^2 - f_b^2)^2 + 4k^2f_a^2f_b^2}}{2(1 - k^2)}} \qquad f_a = \frac{1}{2\pi\sqrt{L_1'C_1}}$$

$$f_2 = \sqrt{\frac{f_a^2 + f_b^2 + \sqrt{(f_a^2 - f_b^2)^2 + 4k^2f_a^2f_b^2}}{2(1 - k^2)}} \qquad f_b = \frac{1}{2\pi\sqrt{L_2'C_2}}$$

$$k = \frac{M}{\sqrt{L_1'L_2'}}$$

ial cases:

Both circuits tuned to the same frequency ($f_a = f_b$)

$$f_1 = \frac{f_a}{\sqrt{1 + k}} \qquad f_2 = \frac{f_a}{\sqrt{1 - k}}$$

Loose coupling ($f_a = f_b$ and $M << L_1'$ and L_2'; $k \neq 0$).

$$f_1 \neq f_2 \neq f_a \neq \frac{1}{2\pi\sqrt{L_1'C_1}} \neq \frac{1}{2\pi\sqrt{L_2'C_2}}$$

Close coupling ($f_a = f_b$ and $M >> L_1'$ and L_2'; $k \rightarrow 1$).

$$f_1 \neq \frac{f_a}{\sqrt{2}} \neq \frac{1}{2\pi\sqrt{2MC_1}}$$

$$f_2 \neq \infty$$

Both circuits identical

$$\begin{cases} f_a = f_b \\ L_1' = L_2' \\ C_1 = C_2 \end{cases}$$

$$f_1 = \frac{1}{2\pi\sqrt{(L_1' + M)C_1}}$$

$$f_2 = \frac{1}{2\pi\sqrt{(L_1' - M)C_1}}$$

$$k = \frac{M}{L_1'}$$

ndirect, and is usually called merely *inductive coupling.* This type coupling is illustrated in Fig. 21. Indirect capacitive coupling is strated in Fig. 22.

'rom Figs. 18 to 20 it is apparent that direct-coupled circuits may be sidered as networks of impedances in series and parallel, as in Fig. 23. notion of "equivalent impedance" (paragraph 6) is a useful concept he treatment of such circuits. In the present treatment of coupled circuits the equivalent impedance is determined by combining the vari impedance elements of the circuits according to the laws of para and series combination as discussed in articles 7 and 8.

The equivalent impedance of the network of Fig. 23 is

$$Z_0 = Z_1 + \frac{Z_m Z_2}{Z_m + Z_2}$$
$$= \frac{Z_1 Z_m + Z_1 Z_2 + Z_m Z_2}{Z_m + Z_2}$$

Coupled Circuits: Inductive or Transformer with Resistance

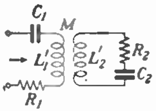
Equivalent Impedance:

$$Z_0 = R_0 + jX_0 = \left(R_1 + \frac{\omega^2 M^2 R_2}{|Z_2{}^2|}\right) + j\left(X_1 - \frac{\omega^2 M^2 X_2}{|Z_2{}^2|}\right)$$

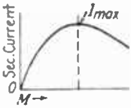
where $|Z_2{}^2| = R_2{}^2 + X_2{}^2$

$$X_1 = \omega L_1' - \frac{1}{\omega C_1}$$

$$X_2 = \omega L_2' - \frac{1}{\omega C_2}$$



Circuit



Current Cur

Special case:

If M is variable, and both circuits tuned to the same frequency, the current in secondary varies with M as shown in the figure.
The maximum secondary current occurs at

$$\omega M = \sqrt{R_1 R_2}$$

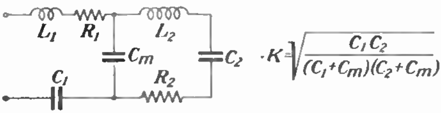


FIG. 20.—Direct capacitive coupling.

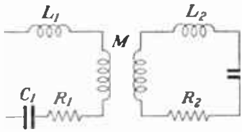


FIG. 21.—Indirect or ductive coupling.

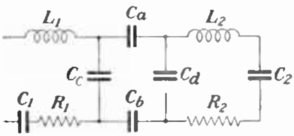


FIG. 22.—Indirect capacitive coupling.

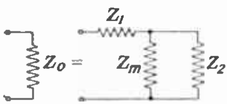


FIG. 23.—Equivalent impedance of direct-coupled circuits.

37. Use of Resistanceless Circuits in Calculations. Each impeda in (48) is in general of the form $R_0 + jX_0$, so that the expression becon somewhat involved if an exact solution is made. In many act applications, however, coupled circuits are also sharply tuned, whicl tantamount to saying that their resistances are small compared to tl tances. For such cases, computations are much simplified without ue sacrifice of accuracy if the circuits are assumed to be resistanceless.

3. Combined Inductive and Capacitive Coupling in Radio-frequency ector Circuits. A combination of inductive and capacitive coupling been utilized in a radio-frequency "preselector" circuit designed by A. Uehling.[1] The circuit functions as a band-pass filter and has, as name implies, especial application as the coupling link between antenna and first tube of a broadcast receiver. For this purpose it equired to transmit a band of frequencies about 10 kc wide and to w this band to be shifted over the broadcast range (500 to 1,500 by tuning, without substantial change in its width.

he band-pass characteristic is obtained by use of the double resonance nomenon in coupled circuits, the difference between the two fre- ncies determining the width of the transmission band. If the two oled circuits are identical these resonant frequencies are functions of $X_m^2 - R^2$ where X_m is the mutual impedance and R the resistance ach circuit. The band width is approximately

$$f_s = f_1 - f_2 \doteq \frac{\sqrt{X_m^2 - R^2}}{2\pi L} \tag{49}$$

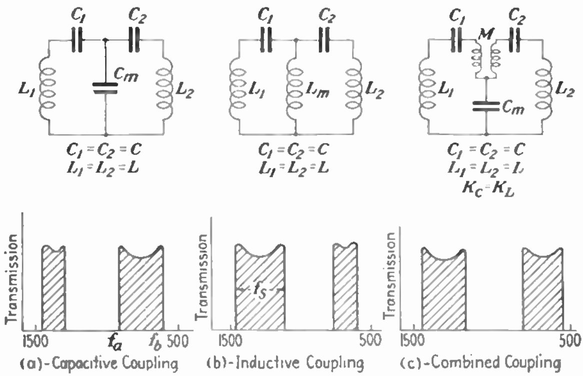


FIG. 24.—Coupled circuits as band-pass filters.

e both X_m and R vary with frequency, the band width will in general vary with frequency, as shown in Fig. 24. However, the variation inductive coupling is opposite in effect to that with capacitive oling, as shown by the figure, so that a combination of both can obtained which will give a practically constant band width.

ehling has shown that this condition obtains when

$$X_{m_n} = \pm\sqrt{R_n^2 + 4\pi^2 L^2 f_s^2} \tag{50}$$

ere R_n is the resistance and L the total inductance of each branch and f_s e band width. With X_m computed for the two boundary frequencies d f_b of the tuning range, the values of M and C_m required are given by

[1] Electronics, p. 279, September, 1930.

$$M = \frac{X_{m_b} f_b - X_{m_a} f_a}{2\pi(f_a^2 - f_b^2)}$$

$$C_m = \frac{f_a^2 - f_b^2}{2\pi f_a f_b (X_{m_a} f_b - X_{m_b} f_a)}$$

Representative values of M and C_m for $f_a = 1{,}500$ kc, $f_b = 550$ kc, R_a = ohms, $R_b = 10$ ohms, $L = 200 \times 10^{-6}$ henrys, and $f_s = 10$ kc, whicl typical constants of broadcast circuits, are

$$M = 3.2 \times 10^{-6} \text{ henry}$$

and

$$C_m = 0.06 \text{ mfd}$$

The inductive coupling M must be negative so that its effect will be a tive to that of C_m. This may be contained by winding the coils M (24) of two wires side by side, and connecting the "start" ends of the coi C_1 and C_2 and the "finish" ends to C_m.

39. Stray Coupling. Because of the apparent increase in resist of a circuit when another circuit is coupled to it, spurious and unin tional coupling due to stray fields and the proximity of other appar may appreciably affect the resistance of r-f circuits and introduce un essary losses unless precautions are taken to avoid it. Stray effects due principally to capacity coupling and stray inductive coupling. former varies with the areas of conductors and a-c voltages invol and inversely with the distances between the conductors, while the la varies with ampere turns, the diameter of the heavy current path in circuit and inversely with the distance between the circuit and o conductors in which induced currents flow.

RECURRENT NETWORKS

40. General Types. *Recurrent networks* are iterative combinat of L, C, and R, such as those shown in Fig. 25.

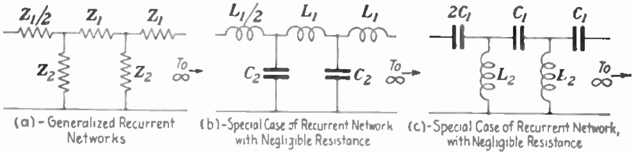


FIG. 25.—Types of infinite recurrent network structures.

The transmission characteristics of such structures vary with quency in a singular manner and introduce both useful and detrime effects in radio- and audio-frequency circuits. Examples of recur networks are transmission lines (actual and artificial) and wave filt

41. Terminating Conditions for No Reflection and Maximum Po Transfer. If a recurrent network is terminated at the nth section ir impedance equal to its image impedance, there is no reflection at termination, and the network behaves as though it had an infinite nun of sections, in so far as its input terminals are concerned.

A long line so isolates its terminating impedances (the source and l impedances) that the apparent value of each as measured from the op site end of the line is very nearly equal to the line impedance and p ally independent of the terminations. Consequently, to obtain a ximum transfer of power from source to line and from line to load, the irce and load impedances must equal the characteristic impedance of line, or be matched to the line by transformers whose turns ratios equal to the square root of the ratio of termination and line imped- ces. A line terminated in its characteristic impedance at both ends has a minimum reflection from its terminals, and in general a line is operated has the lowest total transmission loss.

n a structure having lumped constants, and terminated at one of its ies elements the series impedance in each end section is one-half the ue of the series impedance in the internal sections (Fig. 25). If the mination is at a shunt element, the shunt impedance at each end is de twice the shunt impedance in the internal sections.

2. Transmission Lines. Transmission lines are recurrent structures ing continuously distributed impedances. Two wires in space e, besides their ohmic resistance, nt capacity and series inductance l are thus equivalent to the recur- t structure of Fig. 26, where L, C, R are the constants of a very short gth (Δl) of the line and G is the con- tance due to leakage between the es in the same length.

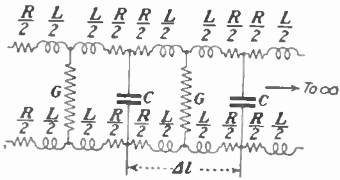


FIG. 26.

3. General Properties of a Transmission Line. The *characteristic edance* is

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \text{ ohms} \tag{53}$$

$$|Z_0| = \sqrt[4]{\frac{(R^2 + \omega^2 L^2)}{(G^2 + \omega^2 C^2)}} \text{ ohms} \tag{54}$$

$$Z_0 = \sqrt{Z_{oc} Z_{sc}} \text{ ohms} \tag{55}$$

re Z_{oc} and Z_{sc} are the input impedances with the far end open- and short- uited, respectively.

he *propagation constant* is

$$P = \sqrt{(R + j\omega L)(G + j\omega C)} = A + jB \tag{56}$$

, G, and C being the resistance, inductance, leakance, and capacitance unit length of the line.

ttenuation Constant. The real part (A) of P is the attenuation constant is

$$6.141\sqrt{\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} + RG - \omega^2 LC} \text{ db per unit length} \tag{57}$$

ave-length Constant. The quadrature part (B) of P is the wave-length tant and is

$$0.707\sqrt{\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} - RG + \omega^2 LC} \text{ radians per unit length} \tag{58}$$

The *velocity of propagation* is

$$V = \frac{\omega}{B} = \frac{2\pi f}{B} \text{ unit lengths per second}$$

The *wave length* is

$$\lambda = \frac{2\pi}{B} \text{ unit lengths}$$

The *retardation time* is

$$t = \frac{B}{\omega} = \frac{B}{2\pi f} \text{ sec. per unit length}$$

Input Impedance of a Line Terminated at Its Far End by an Impedance

Let Z_i = input impedance of the line
Z_0 = characteristic impedance of the line
Z_a = terminating impedance at the far end
θ = propagation factor.

The input impedance of a line so terminated is

$$Z_i = Z_0\left[\frac{Z_a \cosh\theta + Z_0 \sinh\theta}{Z_0 \cosh\theta + Z_a \sinh\theta}\right]$$

The *propagation factor* is

$$\theta = lP$$

where l = length
P = propagation constant per unit length

In the communication field, transmission lines may be classified accord to the frequencies they are used to transmit, as *audio-* or *radio-frequency* li Simplified forms of the general transmission line formulae result from introduction of approximations appropriate to each case.

44. Audio-frequency Lines. In open-wire lines and large-gage cab G is negligible, so that

$$A \doteq 6.14\sqrt{\omega C\sqrt{R^2 + \omega^2L^2} - \omega^2LC} \text{ db per unit length}$$

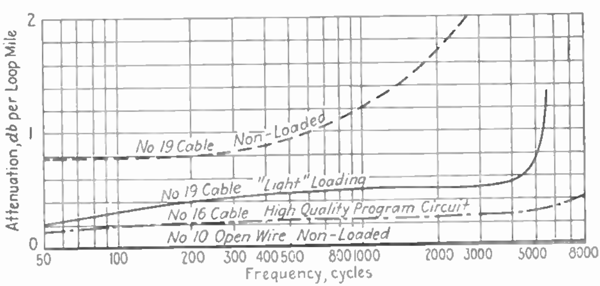


FIG. 27.—Attenuation-frequency characteristics of various audio-freque circuits.

and

$$B \doteq 0.707\sqrt{\omega C\sqrt{R^2 + \omega^2L^2} + \omega^2LC} \text{ radians per unit length}$$

small-gage cables, both L and G me negligibly small, and

$$\doteqdot 15.39\sqrt{fRC} \text{ db per unit length} \qquad (66)$$

$$\doteqdot 1.772\sqrt{fRC} \text{ radians per unit length} \qquad (67)$$

both cases, the attenuation is seen ry with frequency. The attenuation-iency characteristics of various kinds f circuits are shown in Fig. 27, and r characteristics of typical audio lines hown in Table II.

i. Equalization of Attenuation-iency Characteristic. From the es in Fig. 27 it is evident that if a l of frequencies is transmitted over a the higher frequencies will suffer more uation than the low frequencies, ting in distortion. The prevention iis condition necessitates the use of uation equalizers in high quality iits. A typical 5,000-cycle equalizer his purpose and its attenuation curves llustrated in Fig. 28 and the curves he bare line, equalizer alone, and the lized line are shown in Fig. 29. The lizer is usually connected in shunt ss the receiving end of the line, eding other apparatus.

. Artificial Lines. An artificial line compact network of lumped imped-s to simulate the electrical charac-ties of an actual line. Such a network ng approximately the characteristics ı unloaded cable may be constructed hown in Fig. 30 and is useful laboratory measurements and stigations.

ie constants R_1 and C_2 are the loop ance and capacity of the full length e line to be represented. For stand-able, R_1 = 88 ohms and C_2 = 0.054 r loop mile; values for various other are given in Table II. As the arity between the artificial and the il line increases with the number of ons in the former, it is preferable to it least ten sections, and not more

TABLE II.—CHARACTERISTICS OF NON-LOADED AUDIO-FREQUENCY CIRCUITS
(Per loop mile at 1,000 cycles)

Type of circuit	R Ohms	L Henrys	G μ mhos	C Micro-farads	Z Ohms	X Miles	V Miles per second	A db per mile	B Radians per second
No. 10 open-wire NL	10.4	0.00394	0.8	0.0078	739	177	176,600	0.65	0.0356
No. 16 cable NL	42.2	0.001	0.87	0.062	331	64.5	64,500	0.73	0.0975
No. 19 cable NL	83.2	0.001	0.87	0.062	462	47.5	47,500	1.065	0.1322
No. 22 cable NL	171	0.001	1.75	0.073	610	31.7	31,700	1.72	0.198

than one mile of cable or ten miles of open wire should be represe

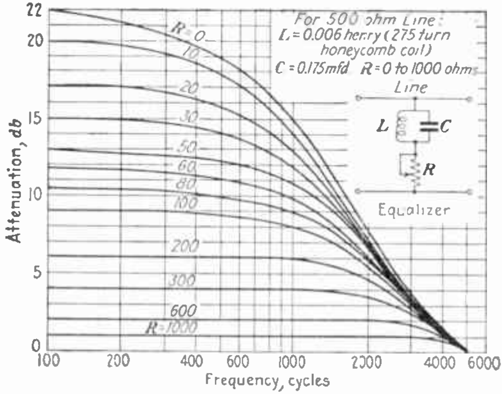


FIG. 28.—Attenuation-frequency characteristic of equalizer shunted a a 500-ohm circuit.

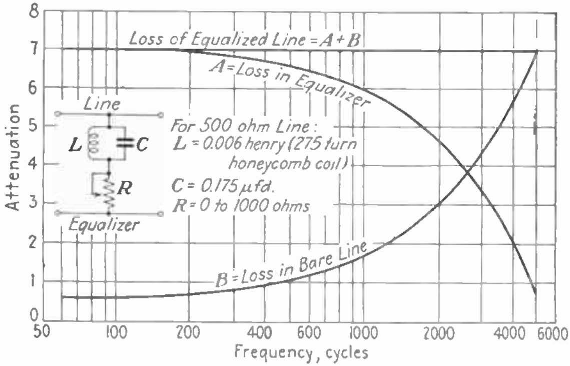


FIG. 29.—Attenuation equalizer for short cable circuits.

by one section. The end sections should be "mid-series" terminated—their series impedances should be one-half of the internal sections.

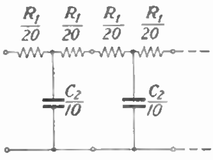


FIG. 30.—Artificial non-loaded cable.

47. Characteristic Impedance of R-f Line. high frequencies R and G usually become negli as compared with ωL and ωC respectively. characteristic impedance of a line at radio quencies is then

$$Z_0 = \sqrt{\frac{L}{C}} \text{ ohms}$$

where L and C are in henrys and farads per unit length.

ı. *Special Case: Line of Two Parallel Wires.* In terms of the dimensions the line

$$Z_0 = 277 \log_{10} \frac{2s}{d} \text{ ohms} \tag{69}$$

parallel wire where s is the spacing from center to center of the wires, l d the diameter, both being measured in the same units. Equation) is based on the assumption that s is at least ten times d and that the ght of the line above the ground is at least ten times s. The charac-istic impedances of radio-frequency lines of commonly used dimensions shown in Fig. 31.

). *Special Case: Line of Two Coaxial Conductors.* Radio-frequency ·s are often constructed with one conductor in the form of a metal)e, and the other a coaxially placed wire or tube of smaller diameter. e advantage of such construction lies principally in the effective shield-which can be obtained by grounding the outer tube.

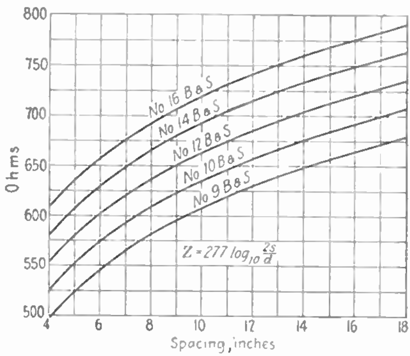


FIG. 31.—Characteristic impedance of r-f transmission line.

The characteristic impedance of a line having such coaxial conductors is

$$Z_0 = 138.5 \log_{10} \frac{r_0}{r_i} \text{ ohms} \tag{70}$$

ɘre r_0 is the inside radius of the outer tube, and r_i is the outside radius the inner conductor. For a line whose outer and inner conductors are pectively $\frac{3}{4}$ and $\frac{1}{4}$ in. in diameter, $Z = 65$ ohms.

8. Other Properties of R-f Lines.

elocity of propagation is

$$V \doteq \frac{1}{\sqrt{L_1 C_2}} \doteq 186{,}000 \text{ miles per second} \tag{71}$$

ɘ speed of light)

Vave-length constant is

$$B = \omega\sqrt{L_1 C_2} \text{ radians per unit length} \tag{72}$$

$$= \frac{\omega}{186{,}000} \text{ radians per mile} \tag{73}$$

Wave length is

$$\lambda = \frac{2\pi}{\omega\sqrt{L_1C_2}} = \frac{1}{f\sqrt{L_1C_2}} \text{ unit lengths}$$

$$= \frac{186,000}{f} \text{ miles}$$

$$= \frac{3,000,000,000}{f} \text{ m}$$

Retardation time is

$$t = \sqrt{L_1C_2} \text{ sec. per unit length}$$
$$= 5.39 \times 10^{-6} \text{ sec. per mile}$$

Attenuation constant is

$$A = 4.346R\sqrt{\frac{C}{L}} \text{ db per unit length}$$

For *parallel wires* this becomes

$$A = \frac{0.0157R}{\log_{10}\frac{2s}{d}} \text{ db per unit length}$$

where R = loop resistance per unit length
s = spacing of wires, center to center
d = diameter of each wire, s and d being measured in the same uni

For *coaxial conductors*, the attenuation is

$$A = \frac{0.0314R}{\log_{10}\frac{r_o}{r_i}} \text{ db per unit length}$$

where R = loop resistance (sum of the resistance of the two conductors)
r_o = radius of outer tube
r_i = radius of inner conductor, r_o and r_i being measured in the sa units.

49. Input Impedance of Line Terminated in Impedance Z_a at Far End. *Special Cases for Radio Frequencies.* At high frequencies, attenuation constant A of a line approaches zero and the propagat constant is nearly equal to the wave-length constant B:

$$P \doteq jB \doteq j\omega\sqrt{LC}$$

and from (63)

$$\theta = lP = jlB = j\omega l\sqrt{LC}$$

Then Eq. (62) becomes:

$$Z_i = Z_0\left[\frac{Z_a \cos lB + jZ_0 \sin lB}{Z_0 \cos lB + jZ_a \sin lB}\right] \text{ ohms}$$

This input impedance has certain interesting and useful values when length of the line is a multiple of a quarter or half wave length.

a. Lines Quarter Wave Length Long. In this case,

$$l = \frac{\lambda}{4}, B = \frac{2\pi}{\lambda}, \text{ and } lB = \frac{\pi}{2}.$$

Then (84) reduces to

$$Z_i = \frac{Z_0^2}{Z_a} \text{ ohms}$$

ıe to this property, quarter wave lines are made use of as impedance-ıtching transformers. If, for instance, a line whose characteristic pedance is Z_1 is to be connected to an antenna system whose input pedance is Z_2, a quarter wave line having characteristic impedance $= \sqrt{Z_1 Z_2}$ is inserted. Since $Z_2 = Z_a$ the impedance facing the line is $= Z_1Z_2/Z_2 = Z_1$ ohms, and the impedance facing the antenna is $= Z_1Z_2/Z_1 = Z_2$ ohms, which results in a perfect impedance match at ch junction.

Quarter-wave Line Short-circuited at Far End. In this case $Z_a = 0$ d $Z_i = \infty$. Such a line is thus anti-resonant at the radio frequency rresponding to four times its length and is often used in antenna stems to by-pass low-frequency current around large radio-frequency pedances, for melting sleet. Such a use is illustrated in Fig. 32.

Quarter-wave Line Open-circuited at the Far d. In this case, $Z_a = \infty$ and $Z_i = 0$. ch a line thus has practically no impedance the radio frequency which corresponds to ır times its length.

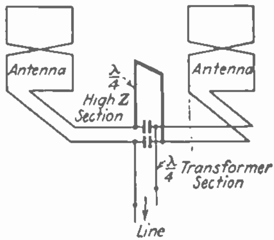


FIG. 32.—Use of quarter-wave short-circuited line to by-pass low-frequency currents for sleet melting without disturbing the r-f impedance of the system.

Half-wave line Terminated in Impedance Z Far End. Here, $l = \lambda/2$ and $lB = \pi$. nsequently, (84) becomes

$$Z_i = Z_a \tag{86}$$

ıus the input impedance of a half-wave line equal to the termination impedance at its end and is independent of the characteristic pedance of the line.

Lines Whose Lengths Are Integral Multiples Quarter- or Half-wave Lines. Such lines can shown to have the same properties as arter of half-wave lines, due to the riodicity of the sine and cosine functions (84).

50. Termination Impedances at Radio Frequencies. At radio freencies, proper termination of lines is even more important than at dio frequencies, since reflection resulting from mismatched impedances the junctions produces standing waves which in turn cause radiation ong the line and a decrease in efficiency. Impedance irregularities a line also tend to set up reflections, and bends in the line should there-e be gradual, with a minimum radius of about one-fourth wave length. r the same reason, the line should be kept free (at least one-fourth ve length) from large masses of conducting or dielectric materials.

51. Efficiency of Lines at Radio Frequencies. In a properly conucted and terminated line the power losses are practically all due to inherent ohmic resistance of the line, and the efficiency may be rly high. For ordinary designs, the efficiency is approximately

$$(100 - 2l) \text{ per cent} \tag{87}$$

ıere l is the length of the line in wave lengths.

52. Tapered Lines as Impedance Transformers. A gradual smooth ange with length in the inductance and capacity of a line causes the aracteristic impedance to vary along the line, and can be shown to introduce no reflections. Consequently, a section of line with varial spacing or diameter of the wires is, like the quarter wave-length line useful impedance matching transformer, the dimensions being so chos that the end impedances of the line equal their respective terminati impedances.

WAVE FILTERS

53. Wave filters are forms of recurrent networks, such as those Fig. 25*b* and *c*, purposely designed to transmit efficiently currents in desired band of frequencies and more or less completely to suppress other frequencies.

A wave-filter design may be based on the assumption that its elemen are pure reactances. The impedances out of which and into which t filter works should be pure resistances of such value that the series a shunt impedance elements of the filter are of approximately the sa order of magnitude and the $Q[=\omega L/R]$ ratio of the coils lies with limits of the following order:

25 to 50 at 110 cycles
100 to 150 at 10,000 cycles
150 to 200 above 50,000 cycles

At low frequencies, the Q ratio $[=1/\omega RC]$ of the condenser is usua so large that the condenser resistance may be neglected. At ra frequencies, however, the ratio is often 100 to 300 and approaches th of the coils.

When the Q ratios of the elements of any filter have reasonable valu such as those suggested above, the effect of dissipation on the charact istic of a filter may ordinarily be neglected.

54. Wave-filter Design. In filter designs, non-dissipative structu terminated in pure resistances equal to their image impedances assumed. Let R represent the terminating resistance.

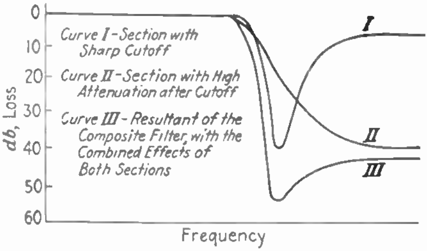


FIG. 33.—Transmission curves for composite low-pass filter.

The basis of filter design is the section, which represents an elemen length or unit in a recurrent structure.

A *uniform filter* consists of identical sections and approximates transmission characteristics of a uniform smooth line. Both the sha ness of cut-off and the attenuation of frequencies beyond cut-off va directly with the number of sections. However, a composite filter usually give the same results with fewer sections.

A *composite filter* consists of sections whose sections are dissimilar a are chosen to combine desirable effects of several types of sections. I nple, in Fig. 33, I represents the curve of a section having a steep cut- which is desired, but also having objectionably low attenuation for

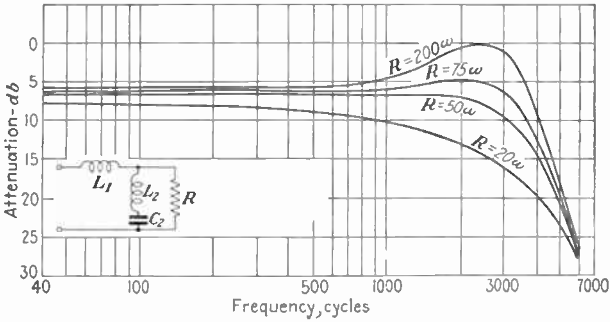


FIG. 34.—Effect of varying resistance at termination of filter.

uencies to the right of the cut-off; II, a second section having a lual cut-off but with the desired high attenuation for the higher

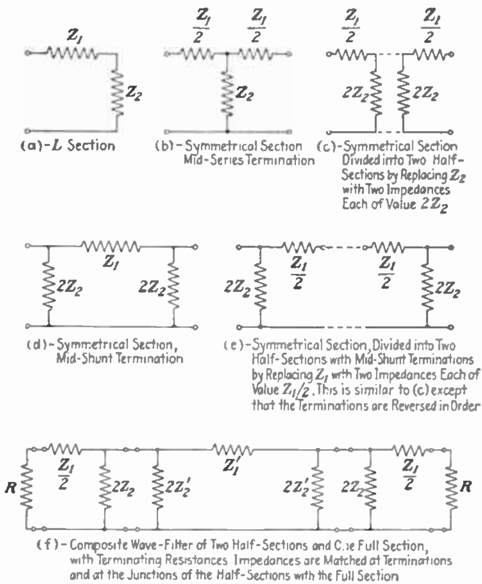


FIG. 35.—Wave-filter structures.

uencies; III, the characteristic of the composite filter made up of combined sections, which retains both the sharp cut-off and the attenuation beyond cut-off. In general, a sharp cut-off as in I in an m-derived type filter is obtained by choosing a small value of m (s as $m = 0.4$), while a gradual cut-off as in II results when m approa 1 (see Art. 55, Fig. 36).

The *end sections* of a multi-section filter should, in general, be sections, to present the correct image impedance for matching the f to the termination. A *half section* is an "L" structure whose series impedance is one-half that of the L-section series arm and whose sh arm impedance is twice that of the L-section shunt arm. Such sections are shown in Fig. 35c and e and the method of connecting internal sections at f.

A single L-type or full-series section is a simple and easily constru filter for many purposes w sharp cut-off and extr selectivity are not requi The transmission character approximates that of a s metrical section, but the off is flattened somewhat.

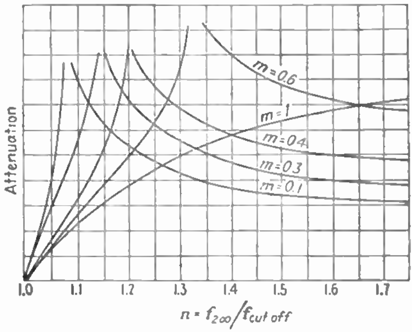


FIG. 36.—Effect of m upon sharpness of cut-off in a low-pass filter structure.

The transmission prope of a given filter can be alt considerably by adjusting value of the terminating re ance. Quantitative result this are demonstrated in 34. In general, values c larger than normal tend increase the transmission frequencies near the cut resulting in sharper cut-of

55. Filter-design Formu

The basis of filter-design formulae is the L-type or full-series section (35). From the relations shown in Fig. 35, symmetrical sections, half tions, and composite groups are readily derived from the basic L- constants.

The prototype filter section is the so-called constant-K struc which derives its name from the fact that the geometric mean of its se and shunt impedances is a constant at all frequencies which, for fi design purposes, is equal to the resistance of the termination. ' structure corresponds to a natural line and has a transmission charac istic which falls off gradually beyond cut-off and has infinite attentua at no finite value of frequency.

A form of filter called the m-type has been derived from the constar structure by Otto J. Zobel.[1] The method consists of introducing ope factors in the series and shunt impedances to give infinite attenua at some finite frequency, which produces a sharper cut-off. The e of m upon the transmission characteristic of a filter is illustrated in 36. It is seen that the constant-K structure is a special case w $m = 1$. Figure 36 is useful in selecting diverse values of m for sect of a composite filter such as that referred to in Art. 54 (Fig. 33). general, unless otherwise restricted, m is usually chosen between 0.4 0.6.

Formulae for L-type sections of the most commonly used filters given on page 141.

[1] *Bell Tech. Jour.*, January, 1923.

Filter-design Formulae

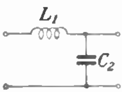
Formulae for full L sections

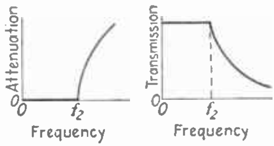
R = termination impedance
m = 0.6 usually

For simple filter, use one-half series impedance and twice shunt impedance shown, in L-type half-section.

I. LOW PASS FILTERS

(a)-Constant K Type

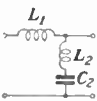




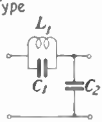
$$L_1 = \frac{R}{\pi f_2}$$

$$C_2 = \frac{1}{\pi f_2 R}$$

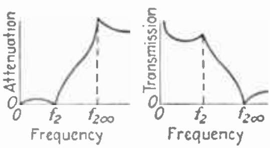
(b) m-Derived Type



Series



Shunt



Series:

$$L_1 = \frac{mR}{\pi f_2}$$

$$L_2 = \frac{(1-m^2)R}{4m\pi f_2}$$

$$C_2 = \frac{m}{\pi f_2 R}$$

Shunt:

$$L_1 = \frac{mR}{\pi f_2}$$

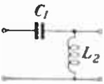
$$C_1 = \frac{(1-m^2)}{4m\pi f_2 R}$$

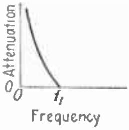
$$C_2 = \frac{m}{\pi f_2 R}$$

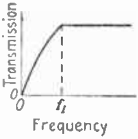
$$m = \sqrt{1 - \frac{f_2^2}{f_{2\infty}^2}}$$

II - HIGH PASS FILTERS

(a) - Constant K Type



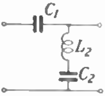




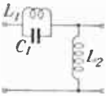
$$C_1 = \frac{1}{4\pi f_1 R}$$

$$L_2 = \frac{R}{4\pi f_1}$$

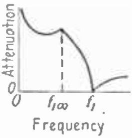
(b) - m - Derived Types

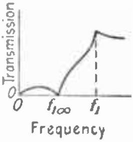


Series



Shunt



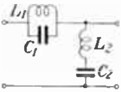


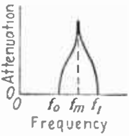
Series	Shunt
$C_1 = \frac{1}{4\pi f_1 m R}$	$L_1 = \frac{mR}{(1-m^2)\pi f_1}$
$L_2 = \frac{R}{4\pi f_1 m}$	$C_1 = \frac{1}{4\pi f_1 m R}$
$C_2 = \frac{m}{(1-m^2)\pi f_1 R}$	$L_2 = \frac{R}{4\pi f_1 m}$

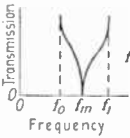
$$m = \sqrt{1 - \frac{f_{1\infty}^2}{f_1^2}}$$

III - BAND ELIMINATION FILTERS

(a) - Constant K Type







$$f_m = \sqrt{f_0 f_1}$$

$$L_1 = \frac{(f_1 - f_0)R}{\pi f_0 f_1}$$

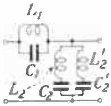
$$C_1 = \frac{1}{4\pi(f_1 - f_0)R}$$

$$L_2 = \frac{R}{4\pi(f_0 - f_1)}$$

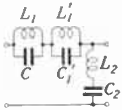
$$C_2 = \frac{f_1 - f_0}{\pi R f_0 f_1}$$

III - BAND ELIMINATION FILTERS (continued)

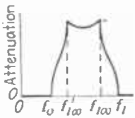
(b) - *m*-Derived Types



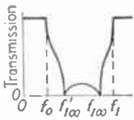
Series



Shunt



Frequency



Frequency

Series

$$L_1 = \frac{mR(f_1-f_0)}{\pi f_0 f_1}$$

$$C_1 = \frac{1}{4\pi(f_1-f_0)mR}$$

$$L_2 = \frac{aR}{4\pi(f_1-f_0)}$$

$$C_2 = \frac{(f_1-f_0)}{\pi f_0 f_1 bR}$$

$$L_2' = \frac{bR}{4\pi(f_1-f_0)}$$

$$C_2' = \frac{(f_1-f_0)}{\pi f_0 f_1 aR}$$

Shunt

$$L_1 = \frac{(f_1-f_0)R}{\pi f_0 f_1 b}$$

$$C_1 = \frac{a}{4\pi(f_1-f_0)R}$$

$$L_1' = \frac{(f_1-f_0)R}{\pi f_0 f_1 a}$$

$$C_1' = \frac{b}{4\pi(f_1-f_0)R}$$

$$L_2 = \frac{R}{4\pi(f_1-f_0)m}$$

$$C_2 = \frac{m(f_1-f_0)}{\pi f_0 f_1 R}$$

$$m = \sqrt{\frac{\left(1-\frac{f_0^2}{f_{1\infty}^2}\right)\left(1-\frac{f_{1\infty}^2}{f_1^2}\right)}{1-\frac{f_0}{f_1}}}$$

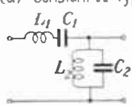
$$a = \frac{1}{m}\left(1+\frac{f_0 f_1}{f_{1\infty}^2}\right)$$

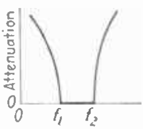
$$b = \frac{1}{m}\left(1+\frac{f_{1\infty}^2}{f_0 f_1}\right)$$

$$f_{1\infty}' = \frac{f_0 f_1}{f_{1\infty}}$$

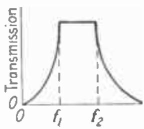
IV. BAND PASS FILTERS

(a) - Constant *K* Type





Frequency



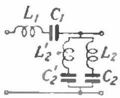
Frequency

$$L_1 = \frac{R}{\pi(f_2-f_1)} \qquad L_2 = \frac{(f_2-f_1)R}{4\pi f_2 f_1}$$

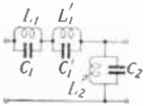
$$C_1 = \frac{(f_2-f_1)}{4\pi f_2 f_1 R} \qquad C_2 = \frac{1}{\pi(f_2-f_1)R}$$

IV. BAND PASS FILTERS (continued)

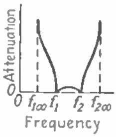
(b) - *m* - Derived Types

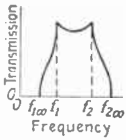


Series



Shunt





Series	Shunt
$L_1 = \dfrac{mR}{\pi(f_2-f_1)}$	$L_1 = \dfrac{(f_2-f_1)R}{4\pi f_1 f_2 b}$
$C_1 = \dfrac{(f_2-f_1)}{4\pi f_1 f_2 mR}$	$C_1 = \dfrac{a}{\pi(f_2-f_1)R}$
$L_2 = \dfrac{aR}{\pi(f_2-f_1)}$	$L_1' = \dfrac{(f_2-f_1)R}{4\pi f_1 f_2 a}$
$C_2 = \dfrac{(f_2-f_1)}{4\pi f_1 f_2 bR}$	$C_1' = \dfrac{b}{\pi(f_2-f_1)R}$
$L_2' = \dfrac{bR}{\pi(f_2-f_1)}$	$L_2 = \dfrac{(f_2-f_1)R}{4\pi f_1 f_2 m}$
$C_2' = \dfrac{(f_2-f_1)}{4\pi f_1 f_2 aR}$	$C_2 = \dfrac{m}{\pi(f_2-f_1)R}$

$$m = \frac{h}{1 - \dfrac{f_1 f_2}{f_{2\infty}^2}}$$

$$h = \sqrt{\left(1 - \frac{f_1^2}{f_{2\infty}^2}\right)\left(1 - \frac{f_2^2}{f_{2\infty}^2}\right)}$$

$$a = \frac{(1-m^2) f_1 f_2}{4h f_{1\infty}^2}\left(1 - \frac{f_{1\infty}^2}{f_{2\infty}^2}\right)$$

$$b = \frac{(1-m^2)}{4h}\left(1 - \frac{f_{1\infty}^2}{f_{2\infty}^2}\right)$$

$$f_{1\infty} = \frac{f_1 f_2}{f_{2\infty}}$$

SECTION 7

MEASURING INSTRUMENTS

BY R. F. FIELD[1]

nstruments for the measurement of electrical quantities, such as rent, voltage, and power, are usually indicating instruments, in which torque developed by the current acting in a magnetic field or by the tage acting in an electrostatic field moves a coil or vane against the ntertorque of a spring or other mechanical device. The electrical ntity is compared with a non-electrical quantity, usually mechanical sometimes physical or chemical. By the application of Ohm's law, uit elements such as resistance and reactance may be measured by icating instruments. The accuracy of indicating instruments is ited by their scale length. It rarely exceeds 0.1 per cent of full-le reading, with an average of about 1 per cent of full-scale reading. ater accuracy is obtained by the use of comparison instruments, in ch the electrical quantity or circuit element is compared with a ndard which has been calibrated to a sufficient accuracy. This ndard may be of the same kind as the unknown quantity or of a erent kind.

CURRENT MEASURING INSTRUMENTS

. **Moving-coil or D'Arsonval Galvanometers** consist of a coil, usually ınd on a metal frame, which can rotate between the poles of a per-nent magnet, as shown in Fig. 1.

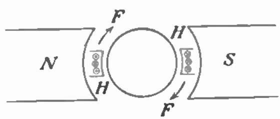


FIG. 1.—Moving-coil galvanometer.

'he current I flowing through the turns of the coil reacts with the magnetic field n the air gap to produce a force F acting each conductor proportional to the product of the current, magnetic field, and length onductor in the field. If the coil is pivoted ts center, a torque will be exerted, tend-to rotate the coil about an axis parallel to sides of the coil and perpendicular to the gnetic field. Some kind of restoring torque is provided which is propor-ıal to the angle θ through which the coil rotates. Expressing the sensitivity f the instrument as the angular deflection per unit current, it is given by

$$S = \frac{\theta}{I} = \frac{HNlb}{\tau} \tag{1}$$

ere b is the diameter of the coil and τ is the restoring torque per unit ;ular displacement. For maximum sensitivity the permanent magnet uld be very strong and the restoring force very weak. The magnetic l obtained from the permanent magnet must be constant so that the

[1] Engineer, General Radio Company, Inc.

electrical characteristics of the instrument may remain unchanged. T usable residual magnetism is between 10 and 30 per cent of the maxim obtainable. Tungsten steel is commonly used. Cobalt steel is availa for instruments requiring the greatest sensitivity. The flux density in air gap is between 500 and 2,500 gauss. A core of soft iron is usually pla inside of the coil to decrease the length of the air gap and to make the magne flux uniform and radial.

The deflection of any sensitive galvanometer is indicated by the angu rotation of a beam of light, the so-called optical lever, which is reflec from a mirror, either plane or convex, mounted above the moving c The older form of telescope and scale is now being replaced by a spot light containing cross hairs which moves along a scale. The use o spot of light is much less fatiguing than observation through a telesc and a wider range of view is obtained. The usual scale length is 50 c with zero in the center. The standard distance from mirror to scal 1 m. The maximum angular deflection is about 14 deg. Practica all pivot instruments use pointers. Full-scale deflection corresponds approximately 90 deg. This is increased to 120 deg. in some cent station meters by careful shaping of the pole pieces. It may be increa to 270 deg. by a radical change in design.

The moving element of every deflection instrument provided with a res ing torque proportional to the angular deflection is in effect a torsic pendulum. As such it has a moment of inertia P, a period T, and a damp factor. The relation between these quantities is given by

$$T = 2\pi\sqrt{\frac{P}{\tau}} = \pi b\sqrt{\frac{ml}{\tau}} \text{ (approx.)}$$

where τ is the restoring torque per unit angular displacement and m is mass of the coil per unit length. The period of a galvanometer is import because the time necessary for any deflection instrument to attain a r position when its deflecting force is altered cannot be less than its peri For sensitive galvanometers it is between 6 and 12 seconds.

Combining Eqs. (1) and (2) by the elimination of the restoring torqu the sensitivity becomes

$$S = \frac{HNlbT^2}{4\pi^2P} = \frac{HNT^2}{\pi^2mb}$$

which shows that the coil should be as light per unit length and as narrow possible. Its length does not affect the sensitivity; the longer the coil greater may be the restoring force for a given period. But since the hea coil produces greater friction at the pivots, freedom from sticking and stabi of zero reading are not much increased by lengthening the coil.

The friction of the suspension and the surrounding air is not suffici to prevent the moving coil oscillating back and forth about its equilibri position when a deflecting force is applied. The amount of damping measured by the rate at which the amplitude of the oscillations decrea The ratio of any two successive swings is constant. The Napierian hyperbolic logarithm of this ratio is called the *logarithmic decrement* of instrument. The smallest amount of damping which will cause the coi come to rest with no oscillation whatever is called the *critical dampi* and the coil is said to be critically damped. Increasing the damp beyond this point increases the time necessary for the coil to come to r and produces overdamping. The shortest time in which the coil ɔme within a given small distance of its position of rest occurs when the ɔil is slightly underdamped. It has a value of about 1.5 times the ɛriod of the coil. The extra damping necessary to critically damp a ɔil is usually obtained magnetically from the motion of the coil in the ɛld of the permanent magnet, which sets up counter electromotive ɔrces. The amount of damping produced by the current in the coil ɛpends upon the total resistance of the coil and connected circuit. hat resistance which produces critical damping is called the *critical amping resistance.* A galvanometer is usually so designed that its itical damping resistance is at least five times its coil resistance so that may be shunted for critical damping without losing much sensitivity. ll but the most sensitive pivot instruments are critically damped on pen circuit by the current set up in the metal winding form and resist- ice of the connected circuit has little effect on the damping.

The *current sensitivity* of any galvanometer varies directly as the num- er of turns on its moving coil and as the square of its period (Eq. 3). or a given winding space on the coil, its resistance varies as the square f the number of turns, assuming that the portion of the winding space ccupied by insulation remains constant. The deflection is proportional ɔ the current and to the square root of the resistance, *i.e.*, to the square ɔot of the power dissipated in the coil.

TABLE I.—CHARACTERISTICS OF D-C GALVANOMETERS

Make	Type	E, μv	I, μa	T, sec.	R coil, Ω	$R_{C.D.}$, Ω	W, $\mu\mu$w
		Suspended coil type with mirror					
	2285a	0.032	0.0027	7.5	12	37	0.00009
	2285b	0.046	0.0038	5	12	52	0.00017
	2285f	0.032	0.00004	20	800	71,000	0.0000013
	2290	0.008	0.00001	40	800	101,000	0.00000008
. & N.	2500b	0.25	0.0005	6	500	10,500	0.00012
	2500e	1.5	0.003	3	500	2,500	0.0045
	2500f	0.05	0.0001	14	500	14,500	0.000005
	2239a	1.7	0.014	8	115	10,000	0.022
	2239b	1.0	0.001	14	1,000	10,000	0.001
	2239f	1.6	0.0002	18	8,000	54,000	0.00032
		Suspended iron type with mirror					
. & N.	2270	0.008	0.0002	5	40	. . . .	0.0000016
		Suspended coil type with self-contained scale					
	2400c	10	0.01	3	1,000	16,000	0.1
. & N.	2420c	25	0.025	3	1,000	16,000	0.62
	2310d	125	0.125	3.5	1,000	11,000	15.6
		Double-pivot type with pointer and scale					
'eston.	440	37	0.25	2.7	150	1,150	9.4
	322	38	0.20	. . . .	190	1,890	7.6

Values of voltage E, current I, and power W are for a scale deflection of 1 mm at a ale distance of 1 m for the galvanometers having mirrors: for those having self- ɔntained scales the values given are for a deflection of the smallest division, usually mm. The voltage drop in the external critical damping resistance is not included in the ɔltage given.

Electrical characteristics of representative commercial galvanometers re shown in Table I. Galvanometers with a single suspension have the reatest sensitivity, those with a taut suspension less, and those with ouble pivots least. For the most sensitive type of galvanometer, icreasing the period from 5 to 40 sec. allows the power to be decreased from 11 to 0.005 $\mu\mu$w. The minimum current sensitivity is 10^{-11} amp per millimeter. The smallest current sensitivity for a taut suspensio is 10^{-8} amp. per millimeter, and for a double-pivot, pointer instrumen 2×10^{-7} amp. per scale division.

Galvanometers of the suspended type are used mainly as null ind cators for d-c bridges and potentiometers and as deflection instrument in comparison methods. In the latter case a *differential galvanometer* sometimes used. This is a galvanometer having two separate insulate windings on the suspended coil. The have equal numbers of turns and are s connected that, when equal currents flo through the two coils, no deflection produced. The sensitivity of a galvanon eter is most easily reduced by shunting When it is desirable to keep the galvanon eter critically damped, the Ayrtor Mather universal shunt shown in Fig. 2 most convenient. The total resistance of the shunt is made approx mately equal to the critical damping resistance of the galvanomete

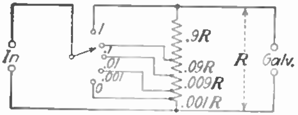


FIG. 2.—Ayrton-Mather universal shunt.

Meters of the pivot type are used as ammeters and voltmeters of a ranges and as the indicating meters of thermocouple and rectifier meter Electrical characteristics of representative commercial ammeters ar shown in Table II. The full-scale range of the ammeters extends fro 25 μa to 20 ka. Above 15 to 30 ma the meters are shunted. The ful scale range of the voltmeters extends from 1 mv to 25 kv. The resistanc in series with the moving coil is self-contained up to 150 to 750 volt Above these values external multipliers are used. The usual resistanc used is 100 ohms per volt, *i.e.*, a current of 10 ma. Some voltmeters ar now built taking a current of 1 ma, and having a resistance of 1000 ohm per volt. *Voltammeters* are combinations of a voltmeter and ammete using the same moving element with suitable multipliers and shunts fc the ranges desired.

TABLE II.—CHARACTERISTICS OF D-C AMMETERS

Make	Type	E, mv	I, μa	R, Ω	W, μw
Weston	1	170	300	570	51
	1	220	1,500	145	330
	45	540	1,500	360	810
	267	42	1,500	28	63
	269	100	1,500	67	150
	280	40	1,500	27	61
	301	27	1,000	27	27
	301	11	200	55	2
	322	4.7	25	190	0.12
	375	30	1,320	23	40
	440	2.2	15	150	0.034
	506	27	1,000	27	27

Values of voltage E, current I, and power W are for full-scale deflection.

2. Moving-coil Vibration Galvanometers. When an alternatin voltage is applied to the coil of a permanent magnet galvanometer, th coil will follow the alternations of the current if the frequency is of th same order as that defined by its period. Maximum amplitude of vibra

›n will occur at the natural frequency of the coil. The relation between ›plitude and frequency is similar to e resonance curve of an electrical ·cuit. The ratio of the maximum ›plitude at its natural frequency to e amplitude for an equal d-c voltage between 25 and 150. The period of e ordinary d-c galvanometer is never s than 1 sec., while the frequencies at ›ich measurements are made are rely less than 30 cycles. The upper ›it for a taut single suspension is ›und 300 cycles. This limit may be sed to 1,000 by the use of a taut ilar suspension. Electrical charac- ·istics of commercial vibration gal- nometers are given in Table III. At cycles their sensitivity is equal to at of a good d-c galvanometer. resonance curve when tuned to frequency of 100 cycles is shown in Fig. 3.

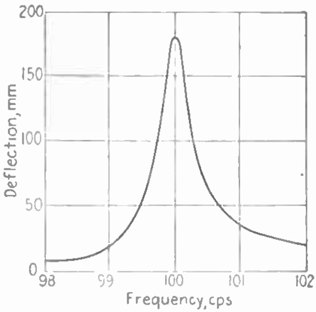


FIG. 3.—Resonance curve of vibration galvanometer.

TABLE III.—CHARACTERISTICS OF A-C GALVANOMETERS

Make	Type	f, cycles	E, μv	I, μa	R, Ω	W, μμw
		Vibrating-coil type				
›nbridge......	Campbell bifilar	50	8.5	0.017	500	0.14
		100	17.5	0.05	350	0.88
		350	53	0.33	160	17
		750	104	2.0	52	200
		1,000	175	5.0	35	800
	Campbell unifilar	30	1.5	0.05	30	0.075
		50	1.2	0.04	50	0.080
		100	3.0	0.025	120	0.075
		200	7.0	0.10	70	0.70
& N.........	2350a	60	17.5	0.025	700	0.44
›nbridge......	Duddell oscillograph	100	5.0	0.02	250	0.10
		1,000	50	0.2	250	10
		2,000	100	0.4	250	40
		Vibrating-string type				
›nbridge......	Einthoven	100	100	0.025	4,000	2.5
		300	800	0.2	4,000	160
R. Co.......	338-L	60	110	2.5	45	280
		250	590	13	45	7,600
		500	1,350	30	45	40,000
		1,000	5,000	110	45	550,000
		Suspended coil type with electromagnet				
& N.........	2570	60	0.06	0.005	12	0.0003
		Electromagnet type				
& N.........	2440	60	16	0.05	325	800,000
		Vibrating-diaphragm type (telephone)				
E. Co.......	....	800	400	0.02	6,000	2.4

›alues of voltage E, current I, and power W are for a scale deflection of 1 mm at a e distance of 1 m for all galvanometers except the telephone, for which the threshold ›udibility is used. The moving system is tuned to the frequencies given for all ›ruments except the suspended coil galvanometer with electromagnet.

The natural frequency may be raised still further by eliminating t coil entirely and using the single-turn loop formed by the bifilar su pension. The mirror is then placed at the center of the taut wires. T general method of construction is shown in Fig. 4. By this means natural frequency of 12 kc may be obtained. The sensitivity decreas inversely as the first power of the frequency. On this account it is sensitive at 10 kc as the bifilar-coil galvanometer was at 1 kc. comparison with other null detectors at these frequencies, its sensitivi is so low that it is not much used in this form.

3. The Einthoven string galvanometer uses the simplest possil moving system for a galvanometer. A single conducting string moves the narrow air gap of the magnetic system, which may be a permane magnet or an electromagnet depending on the sensitivity desired. I motion is observed through a microscope or by its shadow thrown on screen from a point light source. Electrical characteristics of t Einthoven string galvanometer built by the Cambrid Instrument Company are given in Table III, usi a silvered glass string and a magnification of s hundred times. The string galvanometer may al be used as an oscillograph. The shadow of the string observed on a translucent screen as reflected from revolving mirror. The motion of the string may also photographed on film or bromide paper. The usu paper speed is 10 in. per second, but this may increased to a maximum of 100 in. per second. At tl latter speed, phenomena lasting a millisecond appear (in. long. Electrical characteristics of a string oscil graph built by the General Radio Company, using a 0.0004- tungsten string, are also shown in Table III. It may be equipped wi a motor-driven camera and a synchronous shutter for producing 0.01-s timing lines.

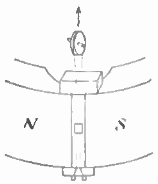


FIG. 4.—Bifilar suspension.

4. Moving-coil A-c Galvanometers. If a steady deflection is desi with a.c., the magnetic field must change in direction with the curr in the coil and must have the same phase. The field of laminat iron is excited at the same frequency, usually 60 cycles, as the movi coil. When used as a null indicator in a bridge network, the field connected across the same supply as the bridge while the moving c is connected to the detector terminals. Since the current through t field and the flux produced will be nearly 90 deg. out of phase with t voltage applied to the bridge, the galvanometer will be most sensit to the reactance balance and will be little affected by the resista balance. These conditions may be equalized or reversed by the int duction of resistance in series with the field or reactance in series with t bridge to make the field current and bridge current differ in phase 45 deg. or be in phase. The phase selectivity of the a-c galvanome may be of advantage in certain special cases, but in general it is a co siderable disadvantage. The electrostatic field of the main field wind exerts a considerable force on the moving coil so that it must be carefu shielded. Its sensitivity is very high, as shown in Table III. It co pares favorably with the best d-c galvanometers.

5. Electrodynamometer. When the iron core is omitted from field winding, the moving coil and field coil may be connected in ser The deflection is then proportional to the square of the current flow the windings, and the instrument is called an *electrodynamometer*. truments of this type read the same on both a.c. and d.c. and are table as transfer instruments, provided certain precautions are taken. otection from external magnetic fields is most important. This is ually accomplished in pivot-type instruments by shielding with soft n. It may also be effected by making the instrument *astatic*. When . is used, an error is introduced if the distribution of current in coils is affected by eddy currents in the conductors themselves— so-called *skin effect*—or by capacitance between the windings. The mer effect is minimized by the use of conductors with insulated ands—so-called *litzendraht*—the latter by careful spacing and by trostatic shielding.

TABLE IV.—CHARACTERISTICS OF A-C AMMETERS

Make	Type	E, v	I, amp.	R, Ω	W, w
			Electrodynamometer type		
ston	326	2.6	1.0	2.6	2.6
	341	1.0	0.5	2.0	0.5
	370	21	0.015	1,400	0.31
			Moving-iron type		
ston	155	31	0.02	1,540	0.62
	433	14	0.03	460	0.41
	476	30	0.015	2,000	0.45
	517	30	0.015	2,000	0.45
	528	30	0.015	2,000	0.45
			Hot-wire type		
tmann & Braun	....	0.6	0.1	6	0.06
R. Co	127	2.3	0.1	23	0.23
		0.9	1	0.9	0.90
		0.52	10	0.052	5.2
			Thermocouple type		
R. Co	493	0.8	0.008	100	0.0064
		0.2	0.10	2	0.020
nbridge	....	0.24	0.008	30	0.0019
		0.12	0.12	1	0.014
		0.08	0.70	0.12	0.059
ston	412	0.25	0.01	25	0.0025
	425	0.13	0.10	1.35	0.0135
		0.62	0.12	5.2	0.075
		0.59	0.50	1.18	0.295
nbridge	Duddell	1.5	0.01	150	0.015
		1.5	0.10	1.5	0.015
			Rectifier type		
ston	301	1	0.001	1,000	0.001
R. Co	488	3	0.00075	4,000	0.00225
		2	0.0005	4,000	0.001
		2	0.00025	8,000	0.0005
		2	0.0001	20,000	0.0002

alues of voltage E, current I, and power W are for full-scale deflection.

Electrodynamometers may be used as galvanometers, ammeters, tmeters, and wattmeters. Their sensitivity as galvanometers is so compared with vibration galvanometers and other meters that they are now rarely used. As ammeters, voltmeters, and wattmeters, th are the standard instruments for use at commercial frequencies. El trical characteristics of electrodynamometer-type ammeters are giv in Table IV. Their sensitivities are one-hundred times less than t least sensitive d-c meters. Their current range is from 15 ma to 50 an The upper limit is set by the difficulty of leading large currents into t moving coil through the torsion springs. Currents up to 5,000 amp. a measured by the use of current transformers. Frequencies up to 1,0 cycles may be used, the normal limit being 133. The voltmeters requ a sufficient series resistance to give them both a negligible temperatu coefficient and a negligible frequency coefficient due to the inductance the windings. Their resistance varies from 2 to 33 ohms per volt. Th voltage range is from 1 to 850 volts. Voltages up to 14 kv are measu by the use of potential transformers. Frequencies up to 600 cycles m be used, the normal limit being 133. The power scale is linear. Th power range is from 25 watts at 50 volts to 75 kw. at 750 vol Frequencies up to 1,200 cycles may be used, the normal limit being 1

6. Moving Iron Meters. Galvanometers are also constructed with stationary coil and a moving magnet. The moving system consists small permanent magnets placed at the center of the coil at right ang to the axis of suspension. To avoid the effect of outside magnetic fiel the system is duplicated with the magnets pointing in the oppos direction to make it astatic and the whole galvanometer is surrounded multiple soft iron shields. Its sensitivity (see Table I) is nearly equal by the best moving-coil galvanometers so that it is very little used.

7. Soft-iron Meters. Soft iron may also be used in the moving e ment, either alone or in conjunction with a fixed piece of soft iron, bo of which are magnetized by the fixed coil.

Soft-iron meters are much used as a-c ammeters and voltmeters in wide variety of ranges and sizes. They may also be used on d.c. El trical characteristics are given in Table IV. The range of the ammet is from 20 ma to 500 amp. The upper limit is ten times that of dy mometer-type meters, because the current coil is fixed. Currents up 5,000 amp. are measured by the use of current transformers. Frequenc to 500 cycles may be used. The range of the voltmeters is from 1 to volts. Their resistances are such as to give from 3 to 167 ohms volt, the values increasing with the voltage. Higher voltages measured by the use of either multipli or potential transformers. Frequenc up to 500 cycles may be used, the norm limit being 133.

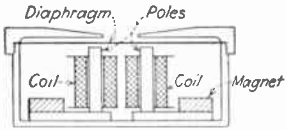


FIG. 5.—Construction of telephone.

MOVING DIAPHRAGM METERS

8. The telephone is a very sensit galvanometer, in which the indication motion is acoustic. It is essentially moving iron vibration galvanome polarized with a permanent magnet. Its construction is sho in Fig. 5. The amplitude of vibration is proportional to product of the steady flux in the air gap produced by the perman magnet and the alternating flux produced by the coils carrying alternating current. The latter flux is much increased by placing coils on laminated soft-iron pole pieces. The reluctance of the harde eel magnet to the alternating flux is so great that most of the a-c flux sses across the gap at the base of the pole pieces. This gap is made the oper length to make the product of the two fluxes at the diaphragm air p a maximum. The diaphragm is a thin steel disk clamped at its outer ge. Its natural frequency of vibration is determined by its mass and ffness. For silicon steel 0.01 in. in diameter, this frequency is about 0 cycles. By plugging the orifice in the earpiece, the natural frequency ay be increased by as much as 50 per cent. The damping of the aphragm is very small, being mainly due to the eddy-current losses the iron. The variation of amplitude th frequency is a sharp resonance rve. Figure 6 shows such a curve a Western Electric telephone. The mping is little affected by changes in ffness and natural frequency. The pedance of a telephone winding reases with frequency in a regular way, cept around the resonance frequencies. e resistance and reactance are generally the same order of magnitude, so that its angle is about 45 deg. At a frequency 1,000 cycles they are about 10 times d-c resistance of the winding. Near onance the motion of the diaphragm roduces a counter e.m.f. into the circuit ich is usually interpreted as additional istance and reactance. These terms referred to as *motional values*. In telephones of low damping, ey may be as much as 70 per cent of the normal values. The actual merical value of the resistance and reactance depends on the number of ns with which the magnets are wound. The d-c resistance varies from to 1,000 ohms. The sensitivity of telephones is somewhat indefinite ause it depends on the acuteness of hearing of the observer. It is ial to express it as the current necessary to produce a just audible ponse. Because of the existence of a threshold of hearing, this nimum current is reasonably definite and reproducible, at least for any person. Values of this minimum current, together with the corre- nding voltage, resistance, and power are given in Table III for a stern Electric receiver. It is much more sensitive than any vibration vanometer and at its resonant frequency is not far behind a good d-c vanometer.

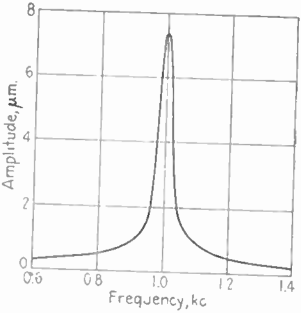


Fig. 6.—Resonance curve of Western Electric telephone.

. Mica-diaphragm Telephone. It is possible to use non-magnetic terials for the diaphragm by providing a separate steel armature so ped and clamped that its natural frequency is higher than that of the phragm, to which it is attached by a stiff rod. The *Baldwin telephone* s a mica diaphragm very similar to that of a phonograph pickup. h its sensitivity and selectivity are very high. Other modifications the use of corrugated diaphragms to broaden the resonance curve and use of a balanced armature in which the polarity of the permanent gnet is so arranged that the armature is not under tension due to them is attracted only by the alternating flux.

0. Dynamic Telephone. The present type of dynamic speaker is a ving coil galvanometer, in which a light paper cone attached to the moving coil acts as a diaphragm. There is no single natural frequenc so that over a wide frequency range the sensitivity is essentially constan A head telephone has been developed by the Bell Telephone Laborato with a moving coil and very light conical diaphragm. Its sensitivity reasonably constant over a wide range of frequencies and holds remarkably at frequencies as low as 100 cycles.

11. Thermophones. When a fine wire is heated by the passage of a. sound waves are produced in the surrounding air if the heat capacity the wire is so small that the temperature of the surface of the wi follows the cyclic variations of the current. Instruments of this sc have been constructed, using gold foil as the heater. They are call *thermophones.* Their sensitivity in terms of sound energy is low. B they can be made small enough to be placed in the ear, so that their ove all sensitivity is quite satisfactory. Their response decreases slowly the frequency is increased. The theory of this instrument has be studied in considerable detail, because of its use as a standard in t production of sound.

HOT-WIRE METERS

12. The expansion of a fine wire when heated by the passage of a cu rent is utilized in hot-wire meters to operate a pointer.

The details of construction of a hot-wire ammeter as built by Hartma and Braun are shown in Fig. 7. The sag the platinum wire AB, which carries the heat current, is taken up by the wire CD, whose is in turn taken up by the cord EG held taut the spring S. The cord passes around a pivo drum F which carries a pointer. The me must be compensated for temperature so th the base on which the various parts are moun has about the same effective coefficient expansion as the hot wire. It is slow in tak up its equilibrium position due to the relativ large heat capacity of the hot wire and as ciated parts. Although the wire itself attain approximate equilibrium in a few secon the final reading takes many minutes as the whole meter heats.

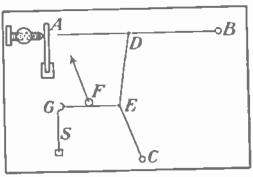


FIG. 7.—Hartmann and Braun hot-wire meter.

Its readings are proportional to the power dissipated in the wire, that is the square of the current, so that a d-c calibration will hold for an alternat current of any frequency up to that at which skin effect becomes apprecial Since the wire is fine and of fairly high specific resistance, this limit usu occurs between 100 and 1,000 kc. It may therefore be used as a tran instrument.

The electrical characteristics of hot-wire meters are given in Table The range of the ammeters is from 100 ma to 10 amp. Larger curre may be measured by shunting or by subdividing the hot wire. Neit of these methods is available at radio frequencies. Their sensitivit somewhat higher than that of either dynamometer or moving i meters. But their low accuracy has been sufficient to prevent their at low frequencies. They have been used extensively at radio frequen in spite of their defects, because there was nothing better. They now completely displaced by thermocouple meters.

THERMOCOUPLE METERS

13. A thermocouple meter consists of a thermocouple and a galvanometer or millivoltmeter. The thermocouple usually consists

·ating element and a thermojunction composed of two dissimilar metals ·ving a high thermoelectric power, as shown in g. 8.

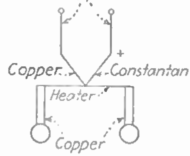
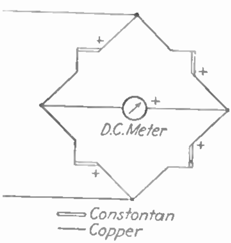


FIG. 8.—Thermo-couple meter.

The metals most frequently used are copper and nstantan (advance). Their thermoelectric power is gh—about 45μv per degree centigrade—and is linear er the temperature range used. The heater is usually high-resistance alloy, either constantan or chromel, the proper diameter and length to give the desired sistance. Carbon or graphitized wire is used for the ghest resistance heaters. The maximum temperature which the heater may be operated is determined by e constancy of generated e.m.f. over a long period time. Temperatures much over 240°C. noticeably shorten the useful of a copper-constantan couple. Metals having higher melting points, ch as platinum and rhodium, may of course be operated at higher nperatures.

All sensitive thermocouples are evacuated. The electrical charac-ristics of various types of thermocouples are shown in Table IV, lculated on the basis of a temperature difference of 215°C.

Contact-type vacuum couples, having high-resistance heaters, are ost sensitive. Vacuum couples are rarely built for currents larger an 1 amp. Air couples range from 100 ma to 1,000 amp., the sizes above amp. taking 0.15 volt per ampere. In the sizes between 100 and 500 ι, the Weston Electrical Instrument Company use a combination of four air couples, arranged in the form of a bridge as shown in Fig. 9. They can be used with direct current if direct and reversed readings are taken. Single self-heater couples, if used, require a suitable choke coil in series with the meter to block the alternating current, and a condenser in the supply circuit to block the direct current. A combination of platinum and tellurium crystal fused together electrically is sometimes used for this type of couple. The large thermoelectric power of these metals makes the couple as sensitive as vacuum couples, but it is not so stable. For a temperature difference of 215°C., a copper-constantan couple produces an open-circuit tage of about 10 mv. Its resistance is about 10 ohms. It can erefore maintain 5 mv across a 10-ohm load or 2.5 μw. This is ficient power to produce full-scale deflection on a good pivot meter. e most sensitive pivot meters (see Table II) require only 2.5 mv for l-scale deflection. Greater sensitivity may, of course, be obtained by ng suspended coil galvanometers.



ı. 9.—Air-couple bridge. (Weston.)

The ratio of the power available to operate the indicating meter hat put into the heater is about 1 to 2,000 for the most efficient couples. e sensitivity of a thermocouple meter must therefore be less than ιt of its d-c indicating meter by at least that amount. The charac-istics of various thermocouple meters are given in Table IV. In the ddell meters, built by the Cambridge Instrument Company, the rmocouple is carried at the lower end of the moving coil, immediately above the separate heater. They are not so sensitive as similar d meters using vacuum thermocouples.

Thermocouple voltmeters are constructed by using one of the mo sensitive couples with sufficient series resistance to give the desir voltage range. Their range is from 0.3 to 150 volts with resistances 125 ohms per volt above 1 volt, and 500 ohms per volt above 10 vol if desired. Their frequency range is determined by that of the seri resistance. The small resistance spools which must be used in mete with self-contained resistors change their resistance rapidly with fr quency so that their frequency limit is 3 kc. Frequencies of 1 megacyc may be attained with an error of 1 per cent with special high-frequen resistors.

Since the e.m.f. produced by the thermocouple is proportional to t power input and hence to the square of the current, this meter will re correctly on both d.c. and a.c. and may therefore be used as a transf instrument. It is necessary, however, to take the average of the readin for both directions when using direct current.

RECTIFIER METERS

14. An alternating current may be changed to a pulsating curre having a steady component by the process of rectification. If t current-voltage characteristic is as shown in Fig. 10*a* the effect is call half-wave rectification. The negative half cycles are eliminated a

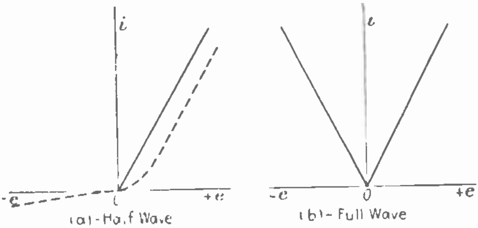


FIG. 10.—Rectifier characteristics.

the positive half cycles reproduced undistorted. The value of the stea component is half the average value of a half sine wave. The ratio of t d.c. to the effective value of an a-c current having a sine wavefor which would flow if the rectifier were replaced by a pure resistan of the same value as that of the rectifier is $\sqrt{2}/\pi$, or 0.450. By combination of rectifiers, it is possible to obtain the characteristic show in Fig. 10*b*, which gives full-wave rectification. The d.c. is then 0.9 of the a.c. Actual rectifiers have a curved characteristic as shown the dotted line in Fig. 10*a*. For negative voltages the resistance is n infinite. The ratio of the positive and negative half-cycle resistances sometimes as low as 8. Because of the curvature of the characteristic, t ratio of d.c. to a.c. is a function both of the magnitude of the current a of waveform.

The crystal rectifiers used with early radio receivers may be used with sensitive d-c meter for rectifying an alternating current. Carborundu galena, silicon, and many other crystals may be used. The crystal is cast

›w melting-point alloy and the top contact made with a fine copper wire. :tification occurs at the points of contact of copper and crystal.

.5. **Copper-oxide Rectifier Meters.** Alternate disks of copper or other t metal and copper oxide, held together under considerable pressure, ʼe proved reasonably satisfactory. Their minimum positive half-cycle stance is about 20 ohms per square inch apparent contact area, l their corresponding negative half-cycle resistance fifty times as ge. Their breakdown voltage, the point where rectification rapidly iinishes, is low, so that a number of contact surfaces are usually nected in series.

'he rectifiers used with small d-c meters have plates about $\frac{3}{16}$ in. square. ger rectifiers having disks 1 in. in diameter are used for battery charging. əse may be used with low-resistance meters and relays. These rectifiers sist of four separate rectifiers connected in a closed loop as shown in Fig. This combination puts two rectifiers in series l gives full-wave rectification. When used h a d-c meter as an a-c micro- or milliam- :er, its current sensitivity is determined ıost entirely by the d-c meter used. Its ctive resistance, although much larger than corresponding d-c meter, is much smaller than · other a-c meter. When used as a voltmeter, icient resistance must be put in series with rectifier, as indicated by *R* in Fig. 11, so that over-all temperature coefficient of resistance y not be too large. For this reason the lower tage limit is 1 volt. On a multiple-range tmeter, the minimum series resistance must larger in order that a single scale may suffice all ranges. Six volts is the lower limit for an accuracy of 2 per cent. econd scale must be added for the lowest range.

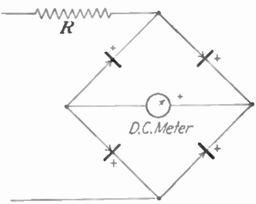


FIG. 11.—Copper-oxide rectifier loop.

Γhe electrical characteristics of these meters are given in Table IV. e range of the ammeters is from 100 μa to 5 ma, of the voltmeters from ɔ 300 volts. The resistances of the latter range from 1,000 to 10,000 ns per volt. The latter figure is higher than any attained by commer- l d-c voltmeters. Their frequency range is limited by the capacitance the rectifier to 10 kc. The decrease in reading is about 0.5 per cent kilocycle up to 35 kc. These meters may also be used on d.c. in ich case they read about 11 per cent high.

\ two-electrode vacuum tube may be used as a rectifier. Its negative f-cycle resistance is infinite and it has no frequency error. Its positive f-cycle resistance is large, being at minimum between 5 and 20 kilohms ›ending on the type of tube. This resistance is too high for its use as ammeter, and as a voltmeter the three-electrode vacuum tube is used.

VOLTAGE-MEASURING INSTRUMENTS

.6. **Moving-vane Meters.** *Electrostatic voltmeters* depend on the ractive force which exists between two conducting plates between ich a difference of potential exists. In their simplest form, the force attraction between a stationary and a movable disk is balanced by alibrated spring. The *Kelvin absolute electrometer* is constructed in s manner. The force of attraction is proportional to the square of difference of potential between the plates. Such meters give the same indication on steady and alternating voltages and have neit waveform nor frequency error.

One type of construction, used in suspended vane meters, is shown Fig. 12. The stationary plates are sections of two concentric cylind into which the cylindrical rotor turns. With the opposite poles c magnet placed outside the stator plates, satisf tory damping is obtained from the curr induced in the loop. This type of constructio that used in the *Ayrton-Mather electrost voltmeter* built by the Cambridge Instrum Company.

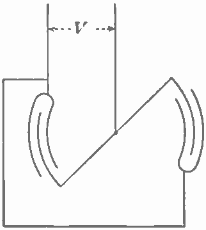


FIG. 12.—Suspended-vane meter.

Electrostatic voltmeters are very useful beca of their high resistance and low power consumpt at low frequencies. They cannot be used on l voltage at frequencies much above a megacy because of the rapid increase of the power loss the necessary insulation. This loss increases dire as the first power of the frequency and the square the voltage. A hard-rubber insulator with a po factor of 0.004 and capacitance of 10 $\mu\mu f$ will ha at a frequency of 10 megacycles and voltage of 2.5 kv. a charging curren 1.5 amp. and a power loss of 15 watts, both of which values are excessiv

17. Spark-gap Meters. The voltage at which air ionizes and allo a spark to pass between two electrodes is a function of its temperat and pressure, the shape of the electrodes and the length of time dur which the voltage is applied. The spark is initiated at the point wh the potential gradient is highest, *i.e.*, where the radius of curvature of electrode is least. It has been found that large spheres give more reliable results than needle points and need not be renewed after each measurement. The distance between the spheres at their closest approach should not be greater than their radius. The voltage range for spheres 50 cm in diameter is from 10 to 400 kv, with the spacing varying from 0.4 to 25 cm. For a-c voltages the spark is determined by the peak voltage. A given calibration is independent of frequency for the lower audio frequencies. The method is also used at high radio frequencies with enclosed electrodes and neon or other inert gas at low pressure.

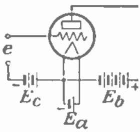


FIG. 13. V uum-tube vo meter.

18. Vacuum-tube Voltmeters. The three-electrode vacuum tu is used as the basis of a number of different types of meters. It is u as a rectifier in the manner discussed above. Its great advantage o the two-electrode tube lies in the fact that its input resistance is pr tically infinite so that it is essentially a potential-operated device. T simplest type of connections is shown in Fig. 13. The grid bias is so chosen that maximum plate rectification occurs, the relation betw plate current and grid voltage being as shown in Fig. 14. When alternating voltage e is applied between grid and filament, the aver plate current increases from I_P to I_P'. This change in plate curr is the quantity in terms of which the instrument is calibrated. T upper limit of applied voltage e is that for which the peak voltage equ the grid bias.

'he zero of the plate-current meter may be suppressed mechanically so t the zero of the voltage scale may coincide with its electrical zero. This pression may also be attained electrically as shown in Fig. 15. Part of filament voltage taken from the potentiometer P_p sends a current through resistance R and the ammeter equal and opposite to the zero plate current. ts success depends upon the fact that the rectifying property of a three-trode tube is nearly independent of plate voltage, provided that the grid ;age is simultaneously adjusted so as to keep the plate current constant. .h the suppressor switch K open, the grid bias is adjusted by the grid

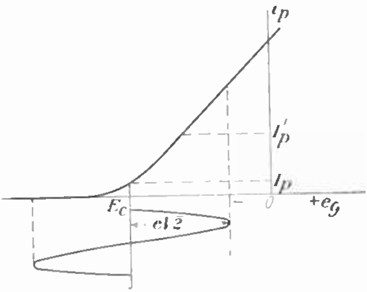


FIG. 14.—Vacuum-tube voltmeter characteristic.

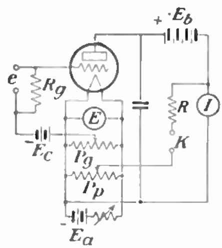


FIG. 15.—Circuit for bucking-out plate current.

entiometer P_g to give the value of plate current for which the calibration made, the filament voltage having been previously adjusted. This ermines the correct grid bias for the plate voltage then existing. Switch s then closed and the zero suppressed electrically. With mechanical pression this procedure reduces to setting the meter to zero by the poten-neter P_g.

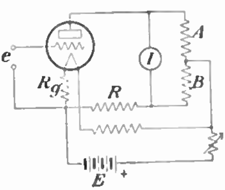


FIG. 16.—Single-battery type of voltmeter.

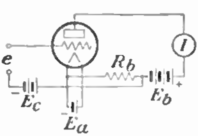


FIG. 17.—Grid bias from plate circuit.

'he use of three separate batteries is a great disadvantage. A method ereby a single 22.5-volt battery supplies all three voltages was sug-ted by Hoare[1] and is shown in Fig. 16. The zero of the meter is pressed electrically by the balance of the bridge formed by the three stances A, B, and R and the plate resistance of the tube. The grid s is obtained from the potential drop in the resistance R_g due to the ment current.

HOARE, *Jour. A. I. E. E.*, **46**, 541–545, 1927.

The grid bias for the voltmeters shown in Figs. 13 and 15 may alsc obtained by connecting the grid return to a resistance R_b in the p circuit as shown in Fig. 17. This method of obtaining the grid bias ca the bias to increase with the applied voltage. The relation result between meter deflection and signal voltage, while nearly a square- relation for small voltages, becomes nearly linear for large voltage from 20 to 100 volts. For a large grid bias plate current flows only du the positive peak so that the error due to waveform may become seri Waveform error is not serious for low voltages and vanishes if the followed by the meter is strictly the square.

The *sensitivity* obtainable with a vacuum-tube voltmeter depe mainly upon that of the indicating meter. The detection coefficie of the various tubes available are not widely different and are not m affected by the value of plate voltage. A full-scale reading of 3 volt usual with a d-c meter showing full-scale deflection on 200 μa. A 20 meter would show a full-scale deflection on 1 volt. Wall galvanomet may be used to obtain increased sensitivity but the difficulty in maint ing the zero setting increases greatly.

The *input resistance* of a vacuum-tube voltmeter is high, being eit the insulation resistance of the input terminals or the resistance R_g Fig. 15 shunted between grid and filament to maintain the grid b This may be as high as 10 megohms. The plate load of the tub sufficiently low so that it does not affect the input resistance. The in capacitance is essentially that of the terminals, socket, and grid-filam capacitance. By careful design this may be made as low as 5 μ

The *calibration* of a vacuum-tube voltmeter is usually independ of frequency over a wide range. At low frequencies an error appe when the reactance of the plate by-pass condenser, connected betw plate and filament to provide a low-impedance path for the alternat component of the plate current, becomes comparable with the plate lo If this condenser is omitted, in order that the meter may be calibra and used at commercial frequencies, errors may appear at frequen below 100 kc due to natural frequencies in the meter and resista of the plate circuit. Finally natural frequencies in the grid circ either in the resistance R_g of Fig. 15 or in the combination of resista R_g of Fig. 16 and the grid-filament capacitance of the tube, set a d nite upper limit below 10 megacycles.

The sensitivity of the vacuum-tube voltmeter may be increased by method suggested by Turner[1] in which two voltages are impressed on t balanced tubes connected as shown in Fig. Equal voltages e_2 are applied to the two grids opposite phase across resistances R and a separ voltage e_1 of the same frequency and the sa phase as either is introduced into the comm grid lead across the resistance R_c. With the g bias adjusted for plate rectification, the differ tial current through the meter connec between the two plates is proportional to product e_1e_2 of the two voltages. The voltage applied to each grid is usually the small volt to be measured and voltage e_1 is a high volt which gives increased sensitivity. A special phase shifting network is genera necessary for the adjustment of voltage e_1. An effective amplification of may be obtained.

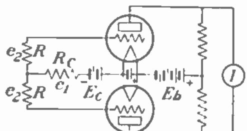


FIG. 18.—Balanced vacuum-tube voltmeter.

[1] TURNER and McNAMARA, *Proc. I. R. E.*, **18**, No. 10, 1743–1747, October, 1930.

f the two voltages are not in phase, the current through the ammeter is portional to $e_1e_2 \cos \theta$, where θ is the phase angle between e_1 and e_2. This he form for the expression for power in an a-c circuit. Hence if e_1 is pro- tional to the voltage across any load, and e_2 is proportional to the current ough that load, obtained as the fall of potential due to the flow of this cur- t through resistances R, the ammeter deflection is proportional to the power ipated in the load. Full-scale deflection may be obtained with powers as ll as 20 μw. The frequency limits are those of the regular vacuum-tube meter.

9. Amplifier-detector Voltmeter. The sensitivity of any a-c volt- ter may be increased by the use of a calibrated amplifier. This should resistance coupled so as to give a constant voltage amplification over ide frequency range. The electrical connections for such an amplifier nanufactured by the General Radio Company are shown in Fig. 19.

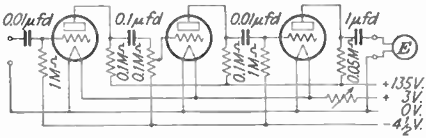


FIG. 19.—Amplifier-detector circuit. (*General Radio.*)

oltage amplification of 100 may be obtained with a voltmeter having esistance of 5 kΩ, 200 for one of 20 kΩ, over a frequency range from cycles to 50 kc. By suitably changing coupling condensers and grid stances, increasing them for lower frequencies and decreasing them higher resistances, this range may be extended to 1 cps and to 200 With a voltmeter giving a full-scale deflection on 2 volts, an input tage of 20 mv will produce a full-scale deflection and 2 mv may be ected.

'he amplifier may be calibrated by means of an attenuator or potenti- eter, adjusted to decrease the voltage applied to it in the ratio of 100 to 1. attenuator is connected directly to the voltmeter and its deflection set to e convenient value. It is then connected to the input terminals of the lifier, and the volume control adjusted to give the same voltmeter deflec- . The effect of the attenuator on the voltage of the source may be alized for the two observations by connecting across the input terminals sistance equal to that of the voltmeter.

0. Electron-stream Meters. A stream of moving electrons is used in *cathode-ray tube* to indicate and measure an electric or magnetic field.

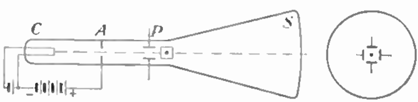


FIG. 20.—Electron-stream meter.

ctrons emitted from a hot cathode are accelerated by a positive ential applied to the anode as shown in Fig. 20. Most of the electrons ke the anode and form the anode or plate current. The remainder s through a small hole in the center of the anode and continue at con- it velocity to a fluorescent screen which is usually the enlarged end of the glass tube in which the various parts are mounted. The coat of the screen is willemite or zinc sulphide. Four deflecting pla are symmetrically disposed around the electron stream near the ano When a difference of potential is applied to either pair of opposite para plates, the electric field deflects the stream toward the positive pl through an angle proportional to the strength of the electric field. T bright spot on the fluorescent screen, which marks where the electr strike the screen, then moves proportionally. A voltage applied betw the other pair of plates produces a deflection of the spot in a direction right angles to the first deflection.

When an alternating voltage is applied to a pair of plates, the electric f set up between the plates is continually varying in magnitude and directi The stream of electrons is deflected back and forth between the plates, and spot of light is drawn out into a line symmetrically disposed about the un flected spot, provided the pair of plates is grounded at a point midway potential between them. An alternating voltage applied to the other pai plates will produce a line at right angles to the first. If the two voltages applied to the two pairs of plates simultaneously the electron stream foll the instantaneous resultant force exerted by both fields and traces on screen a pattern which is closed, and therefore appears stationary, when frequencies used bear a simple relation to one another. These patterns called *Lissajou*[1] *figures*. For two equal frequencies the pattern is an elli of varying eccentricity which at the extremes becomes a straight line circle. The exact figure is determined by the phase difference of the voltages. For other ratios of the two frequencies the patterns become re trant. For the general case the ratio of the number of loops formed adjacent sides of the pattern is that of the two frequencies.

21. Timing Axis. Since the electron stream can follow accurately variations in applied voltage, it is only necessary to spread out the line light which it produces on the screen into a two-dimensional pict to make visible its exact wave form. The second voltage of the sa frequency giving the elliptical pattern just described does this but in s a manner that the whole pattern must be redrawn to be easily interpret The time axis, which the second voltage must provide, should be line not sinusoidal and its return to zero value should be instantaneous.

A very convenient circuit for this purpose employs a neon tube as show Fig. 21. The potential across the condenser C builds up according to exponential law determined by the time constant of the circuit, which over the first part of its rang nearly linear. At some potential between 100 and volts, dependent on the shape of the electrodes the pressure of the gas, the neon tube breaks do and the condenser discharges very rapidly. At so lower voltage the neon tube goes out and the cha ing process is resumed. If the resistance R replaced by a two-electrode vacuum tube, the cur ture of the exponential law of charging may be parti compensated for by the changing resistance of the vacuum tube as the volt across it is varied. The frequency at which the condenser charges and charges depends on the time constant CR of the charging circuit, and is contro by varying these quantities. Frequencies covering the range from 1 to 20, cycles are attainable. The waveform thus spread out on the screen will d along the time axis unless the two frequencies are exactly equal or are sin

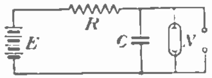


FIG. 21.—Timing circuit for cathode-ray tube.

[1] BARTON, "Textbook on Sound," pp. 555–557.

tiples. It is very convenient to have the pattern stationary. The two quencies may be synchronized by using a thyratron or three-electrode gas-d tube in place of the two-electrode neon tube. Some voltage from the rce of the waveform under observation is applied to the grid of the thyra-1. When the control circuit is adjusted to produce approximately the ect frequency, this added voltage is sufficient to trigger off the discharge maintain exact synchronism.

time axis may also be obtained by viewing the screen on a *revolving ror.* The pattern will be stationary when the speed of revolution of the ror is an exact multiple of the frequency of the given wave.

2. Transient phenomena may be studied by photographing the single e of the electron stream as spread out by any of the above methods btaining a time axis. The time axis may also be obtained by moving photographic film itself.

he cathode is usually of the oxide-coated type which, aside from its 1 efficiency in producing electrons, operates at a temperature suffi-tly low so that light from it does not illuminate the screen.

3. The voltmeter-ammeter method of measuring resistance consists neasuring the voltage and current with deflection instruments and ing their ratio, whence, from n's law,

$$R = \frac{E}{I} \qquad (4)$$

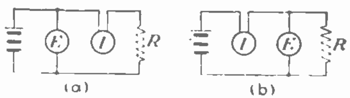
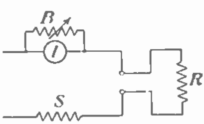


FIG. 22.—Voltmeter-ammeter connections for resistance measurement.

two methods of connecting the ers are shown in Fig. 22*a* and *b*. e all deflection instruments orb power, there is a small voltage drop across the ammeter and nall current flow in the voltmeter. The connection of Fig. 22*a* is l for high resistances and that of Fig. 22*b* for low resistances.

he simplest *direct-reading ohmmeter* consists of an ammeter and battery as shown in Fig. 23. Two readings are made, one with the terminals shorted, the other with the unknown resistance R connected. The fixed resistance S limits the current to about full-scale reading of the ammeter. The deflection is made exactly full scale by adjustment of the ammeter shunt B. The range of this type of meter is usually taken as that resistance which gives a deflection which is 5 per cent of scale. On this basis the usual ranges are 1,000, 10,000, and ,000 ohms.



23.—Direct-reading ohmmeter circuit.

he readings of an ohmmeter may be made independent of the applied age by dispensing with the controlling springs and obtaining the control-torque from a separate coil connected across the supply voltage. Figure hows the circuit used by Evershed and Vignole in their ohmmeters of type.

his construction was first used by Evershed for an ohmmeter designed to sure high resistances up to 100 megohms. The source of voltage was a contained high-voltage magneto generator, giving voltages up to 500 s. It was called a *megger*. The same principle has now been applied to meters of lower range using battery voltages. The resistance range nds from 1 ohm to 5,000 megohms.

24. Measurement of Impedance. When the voltmeter-amm method is used with a source of alternating voltage, the ratio of voltag current gives the impedance of the load

$$Z = \frac{E}{I}$$

With the usual a-c instruments the corrections for the instruments larger and more difficult to make because of their reactance.

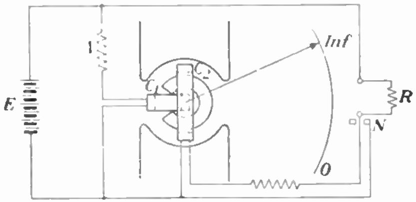


FIG. 24.—Ohmmeter of Evershed and Vignole.

high-resistance rectifier voltmeter and vacuum-tube voltmeter elimi this difficulty.

The separation of impedance into its components requires the use wattmeter. The connections of Fig. 25*a* are usually used when no correc for instrument errors is to be made, while those of Fig. 25*b* allow the correc

FIG. 25.—Measurement of impedance.

to be made quite easily. For this distinction the current coil of the wattm is grouped with the ammeter and its potential coil with the voltmeter. before, the impedance of the load is given by Eq. (5). Its power fact the ratio of the wattmeter readings to the product of voltage and curren

$$\text{P.f.} = \cos\theta = \frac{W}{EI}$$

where θ is the phase angle between voltage and current. The resistance of load is

$$R = \frac{W}{I^2}$$

and the reactance

$$X = \sqrt{W^2 - R^2}$$

With the knowledge as to whether the load is inductive or capacitive inductance or capacitance may be calculated from

$$X = \omega L = -\frac{1}{\omega C}$$

where $\omega = 2\pi f$.

5. Measurement of Capacitance. Since the power factor of the usual lenser is small, its reactance is approximately equal to its impedance. 3 may be measured directly by the voltmeter-ammeter method and capacitance calculated from (9). At a given voltage and uency, a single ammeter read- is sufficient and the ammeter be calibrated to read capaci- e directly.

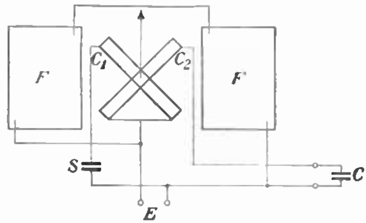
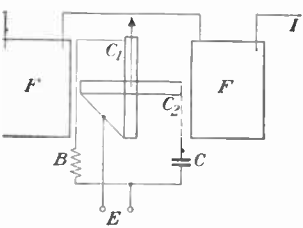


FIG. 26.—High-frequency microfarad meter. (*Weston.*)

apacitance may also be meas- on a single indicating meter se readings are independent of applied voltage. The moving ent consists of two coils set right angles to each other. re are no controlling springs. connections used in the high- uency Weston microfarad meter are shown in Fig. 26.

1e C_1 and C_2 are connected across the supply voltage, one in series with ed capacitance S, the other in series with the unknown C. The station- field coils F are directly connected across the line voltage. With no enser connected in circuit with coil C_2, the coil C_1 sets itself in the plane e field coils F and determines the zero of the scale. The introduction of ows current to flow in the coil C_1 and provides an opposing torque which oportional to the capacitance added. The resulting deflection is of se just as dependent on frequency as on capacitance, so that any par- ar instrument must be used on the exact frequency for which it was calibrated. The low-frequency Weston microfarad meter has the moving coils connected in series instead of in parallel with the field coils.



27.—Power-factor meter. (*Weston.*)

The capacitance range of the Weston Microfarad Meters extends from 0.05 to 10 μf at 60 cycles ,0.001 to 0.05 μf at 500, and 0.0005 μf at 1,000 cycles. The applied voltage must be large enough to provide sufficient torque to give a definite reading.

26. Measurement of Power Factor. Instruments for measuring er factor are very similar to the moving coil capacitance meters ribed above. The connections used in the Weston power-factor er are shown in Fig. 27.

. Measurement of Frequency. Frequency may be measured with ndicating instrument similar to the capacitance meter shown in Fig. a which the capacitance C is fixed and the capacitance S is replaced resistance. The scale is, of course, calibrated in terms of frequency.

e functions of the moving and fixed coils may be transposed, the sta- ry part now consisting of two coils set at right angles to each other. The ng part is simply a vane of soft iron, since its sole function is to indicate irection of the resultant magnetic field set up by the two stationary coils. connections of such a frequency meter are shown in Fig. 28*a*. The tendency of the vane toward rotation is overcome in the Weston freque meter by decreasing the phase difference between the currents in the two as shown in Fig. 28*b*. The rotation of the magnetic field is no longer unif The vane, being long and narrow, takes up a definite position, its in preventing it from following the irregular rotation of the magnetic field. frequency range of the instrument is about 30 per cent of the mid-s reading. These meters are usually built for the commercial frequencie and 60 cycles. The General Ele Company has built them for hi frequencies, up to 2,000 cycles.

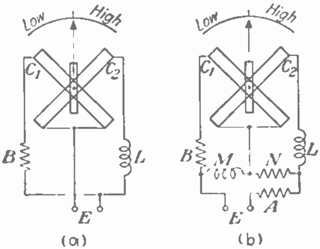


FIG. 28.—Frequency meter. (*Weston.*)

Frequency meters are also structed, which make use of *vibra reeds*. A series of reeds, whose nat frequencies of vibration differ by reg intervals, are arranged in a line or circular arc in the order of ascen frequency. They are mounted c suitably shaped electromagnet, w winding is connected across the su voltage of unknown frequency. reed, having a natural frequency ne to the supply frequency, will vi with an easily visible amplitude, and the frequency intervals bet adjacent reeds are sufficiently small, compared to their damping, so at least one will always vibrate.

STANDARDS

28. Standards of Current. Current is measured absolutely in tern the force of attraction or repulsion between two coils connected in s and carrying that current, and the various dimensions of the This current is then used to deposit silver in the *silver voltammeter* to de mine the electrochemical equivalent of silver. The silver voltamn is thus the standard of current. Its use is tedious and time-consun There is no simple and convenient secondary standard of curr

29. Standard of Resistance. Resistance is measured absol by a number of methods in terms of a speed of revolution of a disk or and its various dimensions. The resistance is then compared wi mercury column of uniform cross section by a suitable bridge met Such a column of mercury of stated length and mass and kept at a t perature of 0°C. is the standard of resistance. Practical secon standards are coils of manganin wire immersed in oil and sealed in m containers. The sealed standards built by Leeds and Northrup Comp to the specifications of the Bureau of Standards are adjusted to accuracy of 0.01 per cent. They may be relied upon to hold their cali tion to 1 part in 100,000 for considerable periods of time.

30. Standards of Voltage. Voltage cannot be measured absolu with an accuracy sufficient to make the measurement desirable on acc of the smallness of the electrostatic forces involved. The secon standard of voltage is the saturated cadmium or Weston cell.

These cells, as built by Weston and by the Eppley Laboratory, are co to 0.01 per cent. They may be depended upon to hold their voltage to 1 in 100,000 when proper correction for temperature is made. The unsatur cadmium cell must be compared with the saturated type for its initial cali tion. Its temperature coefficient is negligible. Its voltage is constant to 1 part in 10,000.

31. Standards of Reactance. The self- and mutual inductance of single-layer air-core coils and the capacitance of two-plate air condensers having guard rings may be calculated from their dimensions, with an accuracy of better than 1 part in 10,000.

32. Standards of Frequency. The absolute standard of frequency is the mean solar day, as measured by astronomical observations. Piezoelectric quartz crystals provide standards of frequency, when permanently connected into suitable vacuum-tube circuits and allowed to oscillate continuously at constant temperature. Over long periods of time their frequency is constant to better than one part in 1,000,000. The frequency of the crystal with which such accuracy may be attained is restricted to a fairly narrow band in the neighborhood of 100 kc. By means of suitable frequency multipliers and dividers all other frequencies from 1 cycle to 100 megacycles may be obtained with the same accuracy.

Quartz crystals whose frequencies remain constant to 1 part in 100,000 may be made for the frequency range 20 kc to 10 megacycles. Metals, such as nickel and certain iron alloys, having the property of *magnetostriction*, may be used as oscillators in suitable vacuum-tube circuits. Their frequency range extends from 5 to 100 kc. Their stability is about 1 part in 10,000. For the lower frequencies tuning forks and metal bars are used. Their frequency range is 25 to 1,000 cycles, and their stability 5 parts in 10,000.

COMPARISON MEASUREMENTS

33. Comparison of Voltages. A steady voltage may be compared with the difference of potential across a resistance carrying current by the use of the simple potentiometer shown in Fig. 29*a*.

A battery E_1 causes a current I to flow in a resistance R. The unknown voltage E is connected to this resistance through a galvanometer G, and the resistance is adjusted to give no deflection of the galvanometer. The voltage E is then equal to the potential drop IR. A second voltage E' may then be made equal to a different potential drop IR'. The two currents in the two cases are the same because at balance no current flows in the galvanometer circuit. The two voltages are thus proportional to the two resistances. The potentiometer may be made direct-reading in voltage by using a standard cell for one of the comparison voltages and connecting it across such a portion of the resistance that the current must be adjusted to a predetermined decimal value in order to obtain balance. The unknown voltage is then connected through the galvanometer and balance is restored by adjustment of resistance R, which may now be calibrated directly in volts. Connections for this type of measurement are shown in Fig. 29*b*.

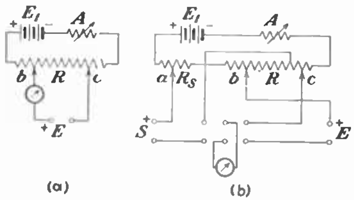


Fig. 29.—Potentiometer types. (*a*) simple; (*b*) with standard cell resistance.

Two alternating voltages may be compared by the potentiometer principle only when they have the same frequency and the same phase. They must at every instant be equal and opposite in order that the galvanometer in series with them shall show no deflection. Hence the potentiometer current must be taken from the same source as the voltage to be measured and some form of phase-shifting device must be provided for which the output current is independent of its phase.

Drysdale used a two-phase induction regulator, feeding one phase through a resistance and the other through a capacitance in order to obtain the two currents in quadrature. Such a device P is shown in Fig. 29c connected to a

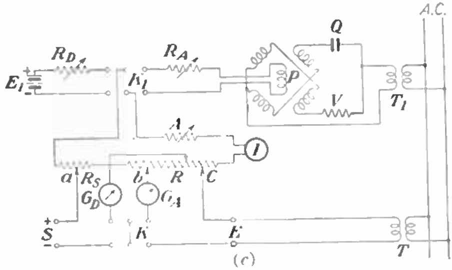


FIG. 29(c).—Drysdale potentiometer.

d-c potentiometer. The galvanometer G_A is an a-c galvanometer having a sensitivity comparable to that of the d-c galvanometer G_D. Since there is no standard of a-c voltage, a standard cell is used to adjust the potentiometer current to its proper value. This value is read on a transfer ammeter I, which may be either of the electrodynamometer or insulated heater thermocouple type. Its zero may be suppressed mechanically to give the effect of a longer scale and hence a greater accuracy of reading. Switches K and K_1 are then thrown to connect the potentiometer to the a-c voltages and the a-c current adjusted to produce the same deflection in ammeter I. Vacuum-tube voltmeters and rectifier voltmeters whose resistances are large compared with the resistance of the potentiometer may be calibrated directly without using the phase shifter, by connecting them directly to the terminals E. The voltage applied to them may be calculated from the settings of the contacts b and c.

34. Comparison of Impedances. An unknown resistance may be compared with a known resistance in a number of different ways. When the known resistance is variable, a *substitution method* may be employed.

The unknown resistance X is connected in series with a battery and shunted galvanometer g, the shunt resistance M having been adjusted to allow a full-scale deflection. The known variable resistance S is then substituted for X and the same current allowed to flow. Its value as thus determined is that of the resistance R. When the known resistance is not continuously variable, the value of the unknown resistance may be interpolated from the two readings of the meter. This method is frequently used for the measurement of very high resistances, such as insulation resistances from a megohm up. The known resistance is rarely larger than 1 megohm so that under these conditions different values of the shunt M are used for the two measurements. The method is not applicable to measurements with alternating current because the phase angles of the source and load are indeterminate.

Two resistances may be compared by connecting them in series and measuring the voltage drops across them by means of a high-resistance voltmeter.

Since the same current flows in both resistances, the value of the unknown resistance is

$$R = S\frac{E_R}{E_S} \tag{10}$$

where E_R and E_S are the voltages across the unknown and known resistances respectively. Except for the case of equal resistances, the resistance of the galvanometer must be either very large compared with the resistances being measured or a correction must be made for the current taken by the galvanometer. This method may be used with alternating current to compare all kinds of impedances. Either a vacuum-tube voltmeter or a high-resistance rectifier voltmeter must be used, since correction for the current taken by the voltmeter is difficult. The polarity of the voltmeter should be maintained as in d-c measurements in order to eliminate the errors of these voltmeters due to even harmonics. The upper limit for frequency is that imposed by the frequency characteristics of the known standard and by the capacitances to ground of the voltmeter in its two positions.

The power factor of an unknown impedance may be determined by the three-voltmeter method, in which the voltages across the unknown and known impedances and that applied to the two in series are read. The same precautions concerning polarity and capacitances to ground apply as in the two-voltmeter method. The vectorial relations between the three voltmeter readings together with the voltage components of the unknown impedance are shown in Fig. 30.

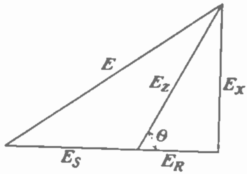


FIG. 30.—Vectorial relations in three-voltmeter circuit.

The expressions giving the unknown impedance Z, its resistance R, reactance X, and power factor $\cos\theta$ are

$$\begin{aligned} Z &= S\frac{E_Z}{E_S} \\ R &= S\frac{E^2 - E_Z{}^2 - E_S{}^2}{2E_S{}^2} \\ X &= \sqrt{Z^2 - R^2} \\ \cos\theta &= \frac{R}{Z} = \frac{E^2 - E_Z{}^2 - E_S{}^2}{2E_ZE_S} \end{aligned} \tag{11}$$

The total resistance of a circuit may be measured by the added resistance method. Since with a constant applied voltage, the current flowing in the circuit is inversely proportional to the total resistance, the circuit resistance is given by

$$R = S\frac{I'}{I - I'} \tag{12}$$

where I is the initial current and I' the current which flows when the resistance S is added. A plot of the reciprocal of the current flowing for different values of the added resistance against that resistance gives a straight line whose negative intercept on the resistance axis is the circuit resistance. The added resistance necessary to halve the current is also the circuit resistance. This method is sometimes used to measure the resistance of a sensitive galvanometer.

The added-resistance method may be used with alternating current provided the circuit is tuned to resonance. The necessary connections are shown in Fig. 31. By reducing the reactance of the circuit to zero the same equations and procedure may be used as for direct current. The ammeter used is usually of the thermocouple type. Halving the current on such a meter quarters the deflection, so that this type of measurement is sometimes called the quarter-deflection method. The ammeter may be replaced by a vacuum-tube voltmeter connected across the condenser. This arrangement is much more sensitive than the thermocouple ammeter, so that the source of alternating current may be of lower power. The upper limit for frequency is set by the frequency characteristic of the known resistance and the capacitances to ground of the different parts of the circuit. This method is the one usually adopted for the measurement of the resistance of inductors at high frequencies.

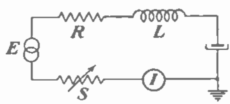


FIG. 31.—Added-resistance method.

The total resistance of the tuned circuit may also be measured by detuning the circuit. The added reactance necessary to halve the squared current (deflection of a thermocouple meter) is equal to the resistance of the circuit. This method is sometimes called the added-reactance method.

Two reactances may be compared in a tuned circuit by a substitution method. The circuit is tuned to resonance both when the unknown reactance is connected in circuit and when it is disconnected. The change in reactance of the variable standard, with which the circuit is tuned, is equal to the unknown reactance. When the unknown and known reactances are both inductive or both capacitive, the value of the unknown inductance or capacitance is obtained directly, independent of frequency, the two reactances being connected in series if inductive, and in parallel if capacitive. For these pairs of measurements it is unnecessary that the currents be kept of the same value.

The resistance of the unknown reactance may be determined by noting the current at resonance when it is connected in circuit and then by adjusting the current to this same value by adding sufficient resistance when it is disconnected. This added resistance, corrected for the change in resistance of the standard reactance with setting, is the resistance of the unknown reactance. The resistance of variable reactors must in general be measured by the added resistance method described above or by one of the bridge methods. The resistance of a variable air condenser follows a definite law and this fact may be used in this type of resistance measurements. The formula is given by Eq. 29 of Art. 37.

35. Comparison of Frequencies. Two nearly equal frequencies may be compared by measuring in a suitable manner their difference in frequency. When the two frequencies are in the audible range, this difference will appear as an audible beat—a waxing and waning in intensity—which may be counted if it is less than 10 beats per second. If the beats are faster than this or if the beating frequencies are above audibility, the beat must be rectified and a beat frequency produced. This beat frequency may then be measured by a suitable frequency meter. The accuracy of the comparison depends both on the accuracy of measurement of the beat frequency and on the ratio of this frequency to the original frequencies. The beat frequency is usually kept in the audible range.

If the two frequencies to be compared are not nearly equal, so that their frequency difference is large and above audibility, audible beats may usually be obtained between some of their harmonics. For a beat frequency b between the mth harmonic of a known frequency f and the nth harmonic of an unknown frequency f', the expression giving f' is

$$f' = \frac{mf \pm b}{n} \tag{13}$$

the sign of b being determined by considering which harmonic, mf or nf' is the larger. Sufficient harmonics are usually present in most frequency sources for the purpose of this comparison, especially when emphasized and isolated by the use of tuned circuits. They can always be produced by the use of a rectifier tube.

In the most precise measurements the known frequency is a multiple or submultiple of a standard crystal frequency, obtained from the various multivibrators driven by the standard. For less precise work a variable standard may be used. The beat frequency is then made zero. Such a variable frequency oscillator, called a heterodyne oscillator, will have a limited frequency range, even though provided with multiple coils. Properly chosen for range, it may be used to measure a super-audio beat frequency, such as might be obtained when comparing two very high frequencies.

Frequency is measured in terms of inductance and capacitance by means of a tuned-circuit frequency meter consisting of a variable capacitance and a set of fixed inductances. The frequency range allotted to each coil determines the accuracy of setting, which ranges from 0.1 per cent to 0.001 per cent. Resonance is indicated in a variety of ways; thermocouple ammeter, heterodyne zero beat, or reaction on an oscillator, these being arranged in the order of their accuracy. In the third method the frequency meter is coupled closely enough to the oscillator whose frequency is being measured so that either the amplitude of its oscillations is affected or its frequency is altered. The frequency alteration is the more precise method, but demands for greatest accuracy a second oscillator set at zero beat with the first. When the frequency meter is in exact resonance, the zero beat note of the two oscillators will be unaffected. In the second method a vacuum-tube oscillator is connected to the wavemeter so that it really becomes a heterodyne oscillator. A screen-grid tube, operating as a dynatron oscillator, may be connected to a frequency meter without the addition of extra coils or taps and converts it into a heterodyne-frequency meter.

DIRECT-CURRENT BRIDGE MEASUREMENTS

36. Whenever two resistances or impedances are compared by matching or comparing the deflections of any deflecting instrument, the accuracy of the measurement is determined by the accuracy of reading of the deflections themselves. This accuracy may be greatly increased by adopting a null method, in which a certain relation of the resistances being compared is indicated by a zero deflection. As this condition is approached, the sensitivity of the indicating instrument may be increased.

37. Four-resistance Network. The simple four-resistance network invented by Christie in 1833 and exploited by Wheatstone ten years later is shown in Fig. 32.

Two paths are provided for the current, one through the ratio arms A and B, the other through the unknown and known resistances U and S. The galvanometer G is connected between the junctions of these pairs of resistances. The condition for a null deflection of the galvanometer is that these two junctions are at the same potential. Equating the voltage drops

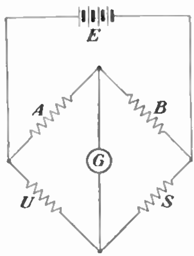


FIG. 32.—Wheatstone bridge.

$$AI_A = UI_U \text{ and } BI_B = SI_S \tag{14}$$

or, since no current flows in the galvanometer,

$$\frac{A}{B} = \frac{U}{S} \text{ or } U = \frac{A}{B}S \tag{15}$$

The ratio arms are usually only variable in steps of ten so that the bridge is balanced by varying the known resistance S.

In commercial bridges the accuracy ranges from 0.1 to 0.02 per cent. In the complete bridges of highest accuracy all switching is by taper plugs and the ratio arms are reversible. There are five decades in the known resistance, tenths to thousands, and nine ratios, 0.0001 to 10,000. Comparisons of resistances on the best bridges may be made to a part in a million, which is beyond the accuracy with which the primary standard of resistance is known.

When the known resistance is fixed, the bridge must be balanced by varying one or both of the ratio arms. In the slide-wire bridge shown in Fig. 33a the ratio arms A and B are parts of a single uniform resistance along which the contact of the lead from the galvanometer may slide.

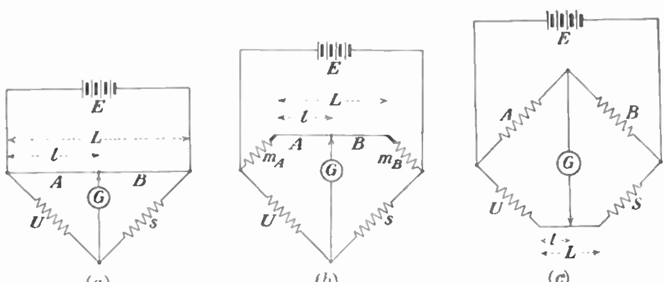


FIG. 33.—(a) Slide-wire bridge; (b) bridge with extension arms; (c) Carey Foster bridge.

The position of the contact is read as a distance measured from one end, the whole length of the scale being L divisions. The value of the unknown resistance in terms of these distances is

$$U = \frac{l}{L - l}S \tag{15a}$$

When the known and unknown resistances are nearly equal the accuracy of measurement may be increased by placing extension coils in series with the slide wire as shown in Fig. 33b.

Two nearly equal resistances may also be compared by means of the Carey Foster bridge shown in Fig. 33*c*. This is a slide-wire bridge in which the slide wire is placed between the two resistances being compared. Two settings of the slide wire l and l' are made with the resistances U and S as shown in Fig. 34 and transposed.

The value of the unknown resistance is

$$U = S - (l - l')\rho \tag{16}$$

where ρ is the resistance per unit length of the slide wire.

In the measurement of a low resistance, a tenth ohm or less, the variation in contact resistance at its terminals and the consequent variation in the lines of current flow near the terminals may produce appreciable errors. To overcome this difficulty, low-resistance standards are always built as four-terminal resistances. All ammeter shunts are so constructed. The two potential terminals are placed between the current terminals and the resistance proper. The value of the resistance is that between the potential terminals.

Such four-terminal resistances cannot be compared on the ordinary Wheatstone bridge. They may be measured on the Kelvin double bridge shown in Fig. 34. The two four-terminal conductors U and S are connected in series, leaving an unknown resistance M between their adjacent potential terminals. The bridge is balanced by adjustment of the standard resistance S. The value of the unknown resistance U is given by

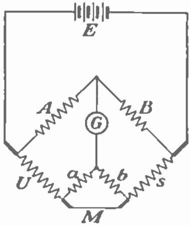


FIG. 34.—Kelvin double bridge.

$$U = \frac{A}{B}S \tag{17}$$

when the double ratio arms are proportional, satisfying the condition $A/B = a/b$.

A-C BRIDGE MEASUREMENTS

38. Four-terminal Network. When an alternating voltage is applied to the simple Wheatstone bridge of Fig. 32, the conditions for balance of the bridge involve the impedances of the four arms.

For a null deflection of the a-c galvanometer or telephones the two junctions, across which it is connected, must be at the same potential at all instants of the a-c cycle. Equating the voltage drops along the two parallel paths offered to the flow of the alternating current

$$Z_A I_A = Z_U I_U \text{ and } Z_B I_B = Z_S I_B \tag{18}$$

where Z_A, Z_B, etc., replace A, B, etc., in Fig. 33. The four impedances are vectors of the form

$$Z = R + jX \tag{19}$$

Hence, since no current flows in the galvanometer,

$$\frac{Z_A}{Z_B} = \frac{Z_U}{Z_S} \tag{20}$$

Expanding these vectors into their rectangular components the two conditions of balance are

$$\frac{A}{B} = \frac{U}{S} + \frac{X_A X_S - X_B X_U}{BS} = \frac{X_U}{X_S} + \frac{UX_B - SX_A}{BX_S} \tag{21}$$

where the resistance components of the four arms are represented by the four letters A, B, U, S without subscripts. If the ratio arms have no reactance, so that $X_A = X_B = 0$, these conditions reduce to

$$\frac{A}{B} = \frac{U}{S} = \frac{X_U}{X_S} \tag{22}$$

The two reactances must have the same ratio as their resistances and as the ratio arms. Considering the reactances as both inductive or both capacitive, Eq. (22) becomes

$$\frac{A}{B} = \frac{U}{S} = \frac{L_U}{L_S} \text{ and } \frac{A}{B} = \frac{U}{S} = \frac{C_S}{C_U} \tag{23}$$

respectively. These equations cover all the types of bridge measurements in which similar impedances are compared.

All parts of an a-c bridge network, power source, arms of the bridge, and null detector have capacitances to ground, which in various combinations are thus in parallel with the bridge arms. At whatever point the bridge is grounded, the capacitance of that point to ground is shorted and all the other capacitances are connected to that point. The effect of a small capacitance in parallel with a low resistance is negligible except at high frequencies, while its effect when placed across a high reactance will introduce serious error. Direct grounding of the bridge is permissible only when the grounded point is the junction of low impedances or when extra bridge balances are made to eliminate the effect of these capacitances. This effect may be minimized by the use of a *Wagner ground*, as shown in Fig. 35. All of the ground capacitances are equivalent to two capacitances C_1 and C_2 between the input leads to the bridge and ground. Their power losses may be represented by parallel resistances R_1 and R_2. The junction of the ratio arms A and B may be brought to ground potential without actual grounding by balancing the bridge formed by these capacitances and the two ratio arms by means of the added resistance W and capacitance C_W, which together comprise a complete Wagner ground.

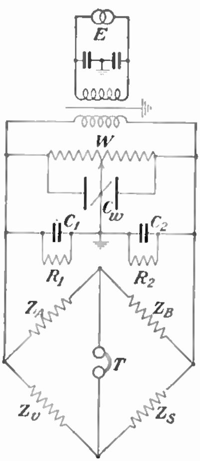


FIG. 35.—Wagner ground.

The greatest capacitances to ground generally occur in the power supply to the bridge and the connecting wiring. Frequently one side of the supply is grounded, either directly or through a large capacitance. The effect of these unequal capacitances may be greatly reduced by the use of a shielded input transformer. The shield is placed between primary and secondary in such a manner as to reduce the direct capacitance between the two windings to a negligible amount. A reduction from 100 to 5 $\mu\mu f$ may be easily obtained. There remains, however, the capacitance between the secondary of this transformer and the grounded shield which may amount to 200 $\mu\mu$f in an iron-core audio-frequency transformer. This capacitance is not in general divided between the terminals of the bridge and ground in the same ratio as the ratio arms so that the equalizing effect of the Wagner ground is still needed.

In the wiring diagrams of the various a-c bridges given below the arrangement of different impedences and the position of the shielded side of any condensers have been so chosen as to minimize the effect of capacitances to ground when that terminal of the telephones is grounded which is nearest to the condenser shields.

The power source at audio and radio frequencies is usually a vacuum-tube oscillator, capable of supplying several hundred milliwatts of power at varying potentials up to 100 volts. At the low audio frequencies, a-c generators with rotating parts may be used, as well as the commercial power supplies at 60 and 25 cycles. The null detector used throughout the audio-frequency range is almost always the head telephone. For the lower frequencies, vibration galvanometers and a-c moving-coil galvanometers are frequently used. Rectifier voltmeters are used for frequencies up to 20 kc, vacuum-tube voltmeters at all frequencies.

Vacuum-tube amplifiers are used with all types of null detectors to give increased sensitivity. The amount of amplification necessary to give any desired accuracy of balance may be determined from the following considerations. For a four-arm bridge, all of whose arms are equal pure resistances A, the ratio of the output voltage e to the input voltage E is given by

$$\frac{e}{E} = \frac{1}{4} \frac{D/A}{1 + \dfrac{D}{A}} d \tag{24}$$

where D is the resistance of the null detector and d is the fractional accuracy of balance demanded. This ratio lies between $\frac{1}{8}d$ and $\frac{1}{4}d$ for ratios of detector and bridge-arm resistance between one and infinity. The minimum voltage detectable on a rectifier voltmeter is 0.2 volt, on a high-grade head telephone at its resonant frequency 0.001 volt. In Table V are given the values of the input voltage E needed to obtain

Table V.—Input Voltage on an Equal-arm Resistance Bridge

Detector	f, kc	E, volts, for d = 1 per cent	E, volts, for d = 0.1 per cent
Telephone	0.1	80	800
	0.2	8	80
	0.5	1.0	10
	1.0	0.4	4
	2.0	0.8	8
	5.0	12	120
	10.0	80	800
Voltmeter	Any	80	800

an accuracy of balance d of 1 and 0.1 per cent with the telephone at various chosen frequencies and with the voltmeter at any frequency, on the assumption that both are connected to the bridge through an amplifier so that the resistance presented to the bridge is infinite. The ratio of the input voltage given to the voltage available is the amplification needed.

At radio frequencies, a heterodyne oscillator and detector may be used to produce an audio-frequency beat note which can then be amplified to any desired degree. In another method the oscillator supplying the bridge may be modulated at an audio frequency. A detector connected to the output rectifies this modulated carrier frequency and reproduces the audio modulation which may then be amplified.

When two reactances are compared on a four-arm bridge, the conditions of balance [Eq. (23)] demands that an added non-reactive resistance be connected in series with one of the reactances, to attain the resistance balance. It would be most unusual for two reactors to have proportional power factors. It must also be possible to connect this resistance in series with either reactance. Figure 36 shows the connections needed for the added resistance, combined with a switch which allows the bridge telephones to be used for the balance of the Wagner ground. The resistance of the standard reactor must be known. When the known reactance is variable, the ratio arms may be fixed or variable in steps, as is the practice in d-c bridges. When the known reactance is fixed, one of the ratio arms at least must be continuously variable.

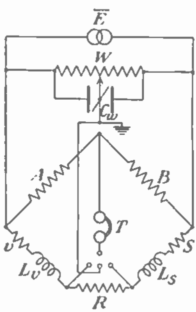


FIG. 36.—Bridge for comparing reactances.

39. Bridge-measurement Errors. Errors introduced into bridge measurements by the presence of reactance in the ratio arms, occurring either in the resistances themselves or in the bridge wiring, and whose magnitudes are indicated by Eq. (21) may be minimized by the use of a substitution method. The effect of capacitances to ground, when a Wagner ground is not used, and the effect of the reactance of the leads to the known and unknown reactances may also be thus greatly reduced. Both reactances are connected in the same arm of the bridge, a similar reactance being placed in the other arm. Two bridge balances are obtained, one with the unknown reactance in circuit, the second with it disconnected and its impedance replaced by the known variable reactance and the added resistance. Inductances are connected in series, placing them far enough apart to reduce their mutual inductance to a negligible amount, and the unknown is removed by shorting. Capacitances are connected in parallel and the unknown is removed by disconnecting its high-potential terminal. Both condensers must be completely shielded and their grounded terminals connected together.

Distinguishing the values for the second balance, when the unknown reactance has been removed, by primes, the values of the unknown reactances are given by the change in reactance of the variable standards.

$$L_U = L_S' - L_S = \Delta L_S \qquad C_U = C_S' - C_S = \Delta C_S \tag{25}$$

The corresponding expressions for the resistances are

$$U = S' - S + R' - R \qquad U = (R' - R)\left(\frac{C_S'}{C_U'}\right)^2 \qquad (26)$$
$$= \Delta S + \Delta R \qquad = \Delta R\left(\frac{C_S'}{C_U'}\right)^2$$

The squared terms appearing in the expression for the condenser resistance result from the law by which the series resistance of condensers connected in parallel is found.

$$R = \frac{R_1C_1^2 + R_2C_2^2 + \cdots}{(C_1 + C_2 + \cdots)^2} = \frac{\sum_1^n R_mC_m^2}{\left(\sum_1^n C_m\right)^2} \qquad (27)$$

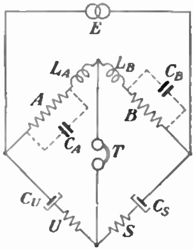


FIG. 37.—Grover method.

The terms containing the resistance of the standard condenser have disappeared because the quantity RC^2 for an air condenser is a constant, independent of the setting of the condenser. This follows from the more general law that, for an air condenser, in which the losses occurring in the solid dielectric are independent of the setting of the plate and for which the power factor of the solid dielectric is independent of frequency, the quantity $R\omega C^2$ is constant. This law holds with increasing frequency until the losses due to skin effect in the plates and supports and to ionization of the air between the plates become appreciable.

40. The resistance balance in a four-impedance bridge may be obtained by introducing suitable inductances in series with the ratio arms, provided that the power factors of the two reactances being compared are small. This method was first suggested by Grover and is shown in Fig. 37.

The balance equations are

$$C_U = \frac{B}{A}C_S \text{ (approx.) and } U = \frac{A}{B}S + \frac{1}{B}\left(\frac{L_A}{C_S} - \frac{L_B}{C_U}\right) \qquad (28)$$

The resistance balance may equally well be obtained by paralleling the ratio arms with small condensers shown in Fig. 37 in dotted lines. The balance equations for this case are

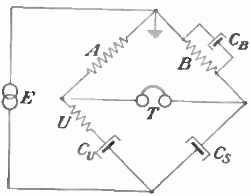


FIG. 38.—Schering bridge.

$$C_U = \frac{B}{A}C_S \text{ (approx.) and}$$

$$U = \frac{A}{B}S + A\left(\frac{C_B}{C_S} - \frac{C_A}{C_U}\right) \qquad (29)$$

In any particular case the reactance needs to be placed in only one arm in order to obtain the resistance balance. Schering's bridge is such a modification of Fig. 37 in which the parallel condenser is placed only across one ratio arm. This bridge is used for measuring the power factor of cables and other small condensers at high voltages. The connections are shown in Fig 38. The input and output terminals are transposed so that the high voltage may be placed across the two condensers in parallel, while keeping the two resistors and the null detector at practically ground potential. The standard condenser C_S has a small capacitance and is so mounted that its losses are reduced to a minimum.

The balance equations are

$$C_U = \frac{B}{A} C_S \text{ and } U = \frac{C_B}{C_S} A \tag{30}$$

whence

$$(pf)_U = U\omega C_U = B\omega C_B$$

41. Comparison of Inductances and Capacitances. An inductance and a capacitance may be compared directly by suitably placing them in the four-impedance network. The connections for Maxwell's bridge are shown in Fig. 39.

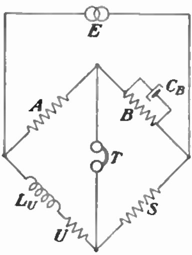


FIG. 39.—Maxwell bridge.

The balance equations are

$$L_U = ASC_B \text{ and } U = \frac{A}{B} S \tag{31}$$

whence

$$Q_U = \frac{\omega L_U}{U} = B\omega C_B$$

Losses in the condenser C_B enter only into the resistance balance and may be made negligible by suitable choice of resistance A. The resistance and reactance balances are not independent unless condenser C_B is continuously variable.

In Owen's bridge an inductance is compared with a capacitance in the manner shown in Fig. 40.

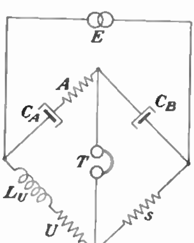


FIG. 40.—Owen bridge.

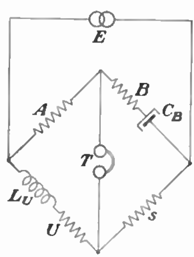


FIG. 41.—Hay bridge.

The balance equations are

$$L_U = ASC_B \text{ and } U = \frac{C_B}{C_A} S \tag{32}$$

whence

$$Q_U = \frac{\omega L_U}{U} = A\omega C_A$$

The resistance balance is made either by having condenser C_A continuously variable or by adding resistance in series with the unknown inductor.

Hay's bridge is similar to Owen's bridge with one of the condensers omitted. On this account, however, it is not independent of frequency. The connections are shown in Fig. 41. The conditions of balance are

$$L_U = \frac{ASC_B}{1 + B^2\omega^2 C_B^2} \text{ and } U = \frac{ABS\omega^2 C_B^2}{1 + B^2\omega^2 C_B^2} \qquad (33)$$

whence

$$(\text{p.f.})_U = \frac{U}{\omega L_U} = B\omega C_B$$

The two bridge balances are not independent.

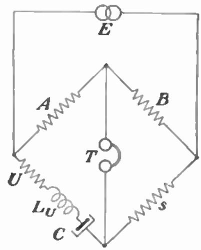


FIG. 42.—Resonance bridge.

42. The resonance bridge shown in Fig. 42 is the simplest bridge in which inductance, capacitance, and frequency enter. At balance the arm containing the reactances is resonated to the applied frequency and becomes a pure resistance. The bridge is then an all-resistance equal-arm bridge, to which the Wagner ground may be applied. For this reason it may be used at high frequencies to measure the resistance and inductance of a reactor.

The balance equations are

$$\omega^2 = \frac{1}{L_U C_U} \text{ and } U = \frac{A}{B}S \qquad (34)$$

This bridge is frequently used to measure frequency, usually in the audio-frequency range. A variable inductor is used and the condenser may be varied in steps. A range from 200 cycles to 4 kc may be covered in three ranges. The frequency scale is irregular, due to the characteristics of variable inductors and the various ranges cannot be made multiples of one another. Due to the large stray field of the variable inductor, its magnetic pickup is considerable. A resistance balance must be provided to allow for the variation of the resistance of the tuned arm with frequency.

43. Wien's Bridge. Capacitances may be measured in terms of resistance and frequency with Wien's bridge, shown in Fig 43. The balance equations expressed in their simplest form are

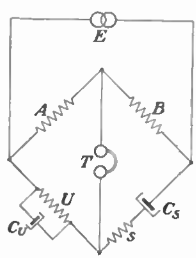


FIG. 43.—Wien bridge.

$$\omega^2 = \frac{1}{USC_U C_S} \text{ and } \frac{C_U}{C_S} = \frac{B}{A} - \frac{S}{U} \qquad (35)$$

Solving for the two capacitances,

$$C_U{}^2 = \frac{BU - AC}{AU^2S\omega^2} \text{ and } C_S{}^2 = \frac{A}{(BU - AS)S\omega^2} \qquad (36)$$

The bridge is valuable because the standards of frequency and resistance are known to a greater accuracy than the standard of capacitance. Ferguson and Bartlett[1] have developed this method to its greatest pre-

[1] FERGUSON and BARTLETT, *Bell System Tech. Jour.* **7**, No. 3, 420–437.

cision. Their estimated accuracy for the determination of capacitance by this method, is 0.003 per cent.

The Wien bridge also furnishes a very convenient means for measuring frequency in the audio-frequency range. The two capacitances are made equal, while the two ratio arms are made such that B is twice A. The two resistances U and S are made variable over a suitable range but are also kept equal. Thus the resistance balance is always satisfied and the reactance balance reduces to

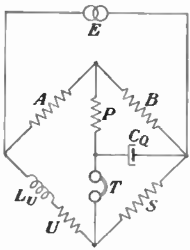


FIG. 44.—Anderson bridge.

$$f = \frac{1}{2\pi U C_U} \tag{37}$$

In the frequency meter built by the General Radio Company the resistances U and S are wound on tapered cards so shaped that the frequency scale is logarithmic. This gives a constant fractional accuracy of reading. There are three frequency ranges, obtained from three different pairs of condensers, each covering a range of 10 to 1 in frequency. The same calibration serves for all ranges. The frequency limits attained are 20 cycles and 20 kc.

44. Six-impedance Network. The six-impedance network was developed by Anderson to provide a modification of Maxwell's bridge which would render the two balance conditions independent even with a fixed capacitance. The connections are shown in Fig. 44.

The general balance condition for the six-impedance network is

$$Z_Q(Z_B Z_U - Z_A Z_S) = Z_P[Z_P(Z_A + Z_B) + Z_A Z_B] \tag{38}$$

For Anderson's bridge this reduces to

$$L_U = SC_Q\left[P\left(1 + \frac{A}{B}\right) + A\right] \text{ and } U = \frac{A}{B}S \tag{39}$$

The effect of losses in the condenser C_Q is usually small.

45. Mutual-inductance Balances. Two mutual inductances may be compared by means of *Felici's mutual-inductance balance* shown in Fig. 45. The known mutual inductance must be variable. For the usual condition of balance, zero voltage across the null detector, the two mutual inductances are equal.

$$M_U = M_S \tag{40}$$

They must be so connected that their induced secondary voltages are in opposition. Mutual inductance between them should be avoided.

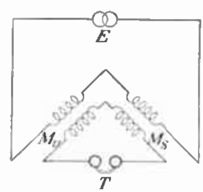


FIG. 45.—Felici mutual-inductance balance.

46. Four-impedance Network with Mutual Inductances. A mutual inductance may be compared with a self-inductance on a four-impedance bridge by placing it between one arm and either an input or output lead of the bridge, as shown in Fig. 46.

The general balance equation for this network is

$$Z_A Z_S - Z_B Z_U - j\omega M(Z_A + Z_B) = 0 \tag{41}$$

For Campbell's arrangement of this bridge the two conditions of balance become

$$L_U = \frac{A}{B}L_S - \left(1 + \frac{A}{B}\right)M$$

and

$$U = \frac{A}{B}S \tag{42}$$

A substitution method is usually adopted so that the inductance and resistance of that portion of the mutual inductance connected in the S arm need not be known. When the ratio arms are equal the extra balancing inductance represented by L_U of Fig. 46 may be eliminated by providing a center tap in

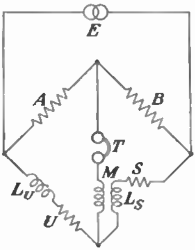


FIG. 46.—Comparison of mutual with self-inductance.

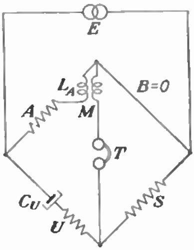


FIG. 47.—Carey Foster mutual-inductance bridge.

one branch of the mutual inductance. This connection is usually referred to as Heaviside's equal arm bridge.

A mutual inductance may be compared with a capacitance by means of *Carey Foster's bridge*, shown in Fig. 47. The conditions of balance are

$$C_U = \frac{M}{AS} \text{ and } U = S\left(\frac{L_A}{M} - 1\right) \tag{43}$$

The impedance of the B arm is made zero in order to make the balance independent of frequency. The method suffers because the resistance and self-inductance of the mutual inductance enter into the expressions for the unknown capacitance and its resistance respectively. Capacitance between the two windings of the mutual inductance causes the voltage induced in its secondary to have a phase angle with reference to the primary current different from 90 deg. This reduces the calculated resistance of the condenser and frequently yields negative values, especially for large mica condensers. The method is perhaps better suited for the measurement of a mutual inductance in terms of a known condenser.

References

BOLTON, E. P.: "Electrical Measuring Instruments," Dutton & Co., New York, 1923.
Cambridge Instrument Company, Catalogue.
General Radio Company, Catalogue.
HAGUE: "Alternating-current Bridges."
JANSKY: "Electrical Meters."
KENNELLY: "Vibration Instruments."
Leeds & Northrup Company, Catalogue.
LAWS: "Electrical Measurements."
STERLING: "Electricity and Magnetism."
Weston Electrical Instrument Corporation, Catalogue.

SECTION 8

VACUUM TUBES

By J. M. Stinchfield, B.S.[1]

1. Electrons. The **electron** is a negatively charged particle of electricity. In 1897 J. J. Thomson discovered that the cathode rays passing from the cathode to the anode in a gaseous discharge, were moving, negatively charged, particles. He measured the ratio of the charge e to the mass m of these particles and termed them corpuscles. Thomson's corpuscles are now commonly known as electrons. The cathode rays or streams of electrons are deflected by either magnetic or electrostatic fields. They exert mechanical force sufficient to turn a vane in a vacuum or to heat the object they strike.

2. Electrons in an Electrostatic Field. An electrostatic field exerts a force upon an electron. If the field intensity is X and the charge on the electron e, the force f acting on the electron is

$$f = Xe \tag{1}$$

If the mass of the electron is m, the acceleration a will be

$$a = \frac{Xe}{m} \tag{2}$$

The force and acceleration on the electron will change if the field intensity changes. The force is in the direction of the field at the point considered, the electron tending to move toward the positive.

In a uniform field the work W done on an electron in moving between two points distance s apart will be

$$\begin{aligned} W &= fs \\ &= Xes \end{aligned} \tag{3}$$

Since Xs is also the potential difference between the two points, calling this potential difference V, the work done on the electron is

$$W = Ve$$

If the field is not uniform the line integral of the force and distance regardless of the path between the two points will give the work done. The work done on a unit charge moved between two points defines the potential difference between the two points. The work done on an electron moved between two points of potential difference V will be

$$W = Ve \tag{4}$$

[1] Engineering Department, E. T. Cunningham, Inc.

If the velocity of an electron is changed by an amount v in passing between two points, the change in kinetic energy will be

$$\frac{mv^2}{2} \tag{5}$$

The change in potential energy or work done in passing between the two points will be

$$Ve$$

The change in kinetic energy is equal to the change in potential energy, and

$$Ve = \frac{mv^2}{2} \tag{6}$$

The velocity acquired by an electron in passing between two points of potential difference V is

$$v = \sqrt{\frac{2Ve}{m}} \tag{7}$$

The potential V is in absolute e.s.u. in the relations above. The potential difference in volts divided by 300 is the potential difference in absolute e.s.u.

The ratio of the charge e to the mass m of the electron is

$$\frac{e}{m} = \frac{4.774 \times 10^{-10}}{8.999 \times 10^{-28}} = 5.305 \times 10^{17} \text{ e.s.u. per gm}$$

The electrons' velocity corresponding to various potential differences are shown in the table. When the velocity becomes greater than about one-tenth the velocity of light, the apparent mass of the electron increases enough to cause a small error. The error in using Eq. (7) is less than one-half of 1 per cent for potential differences less than 300 volts.

Volts	Velocity, Centimeters per Second
1	0.00595×10^{10}
5	0.0133
10	0.0188
20	0.0266
30	0.0326
40	0.0376
50	0.0421
60	0.0461
70	0.0498
80	0.0532
90	0.0564
100	0.0595
200	0.0841
300	0.103
400	0.119
500	0.133
1,000	0.188

Volts	Velocity, Centimeters per Second
10,000	0.586×10^{10}
100,000	1.64
1,000,000	2.82

3. Electrons in an Electromagnetic Field. An electron moving with a velocity v in an electromagnetic field of intensity H is acted on by a force

$$f = Hev \tag{8}$$

The direction of the force is at right angles to both the direction of the field H and the direction of motion of the electron.

The force f is effective in producing an acceleration:

$$a = \frac{Hev}{m} \tag{9}$$

The acceleration is at right angles to the direction of motion. If the electron moves unimpeded and the field H is uniform, the path will be circular and of radius

$$r = \frac{v^2}{a} = \frac{mv}{eH} \tag{10}$$

4. Current Due to a Stream of Electrons. A current i is defined by the quantity of electricity q flowing per unit of time. If there are n electrons per unit of volume in a certain space, the quantity of electricity q in this space is ne per unit of volume. If these electrons are moved with a velocity v, the quantity flowing per unit of time is the current

$$i = nev \tag{11}$$

This is the current per unit of area at right angles to the direction of flow.

5. Space Charge Due to a Cloud of Electrons. If in a given space there are n electrons per unit of volume, the volume density of electrification is

$$\rho = ne \tag{12}$$

The potential distribution in the given space due to the electrons is given by

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi\rho \tag{13}$$

For the case of large parallel plates, only the distance x between plates need be considered. Equation (13) simplifies to

$$\frac{\partial^2 V}{\partial x^2} = -4\pi\rho \tag{14}$$

If a current i is flowing and the electrons move with uniform velocity v the space charge or volume density of electrification is

$$\rho = \left(\frac{i}{v}\right) \tag{15}$$

6. Emission of Electrons. Certain internal forces existing at the surfaces of substances prevent the escape of the free electrons unless a certain amount of energy is supplied to the surface. In the usual type of radio tube, the electron-emitting filament material is supplied with the heat energy of an electrical current sufficient to cause the desired electron emission. Emission excited by heat energy is known as *thermionic emission.*

Electron emission may be produced by electrons impinging upon substances with sufficient velocity. For example the electrons emitted by the hot filament of a radio tube may be accelerated toward the plate by a positive voltage. If a great enough velocity is reached each electron will have sufficient energy to release one or more electrons from the plate. This is known as *secondary emission.*

The energy supplied by light is sufficient to cause emission from some substances. This is the type of emission employed in photoelectric cells and is known as *photoelectric* emission.

Strong electric fields acting on gases or vapors may cause the gas particles to collide with sufficient energy to release electrons from the gas. This process is known as *ionization.* In this case both the electron and the remaining positively charged gas ion are mobile, so that the electron moves toward the positive and the gas ion toward the negative electrodes from which the field originates.

7. Thermionic Emission. The emission of electrons from metals heated to a certain temperature is a characteristic property of the metal. From consideration of thermodynamics and the kinetic theory of gases Richardson obtained an equation for thermionic emission.

$$I_s = A_1 T^{1/2} \epsilon^{-\frac{b_1}{T}} \tag{16}$$

where I_s = emission current in amperes per square centimeter
A_1 = a constant for the emitting substance
T = absolute temperature in degrees Kelvin
ϵ = base of Napierian logarithms
b_1 = a constant depending upon the nature of the emitting surface

A similar equation giving equivalent results was derived by Dushman:

$$I_s = A_2 T^2 \epsilon^{-\frac{b_2}{T}} \tag{17}$$

where I_s = electron emission in amperes per square centimeter
T = absolute temperature of the emitter in degrees Kelvin (C + 273)
ϵ = base of Napierian logarithms (2.718)
b_2 = a constant for the material

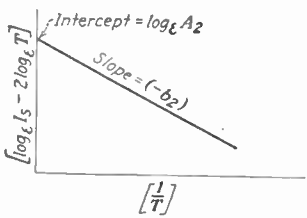


Fig. 1.—Determination of constants in emission equation.

The constants A_2 and b_2 of Eq. (17) can be determined for a given material in the following manner:

$$\log_\epsilon [I_s] = \log_\epsilon \left[A_2 T^2 \epsilon^{-\frac{b_2}{T}} \right]$$

$$[\log_\epsilon I_s - 2 \log_\epsilon T] = \left[\log_\epsilon A_2 - \frac{b_2}{T} \right]$$

Readings of the emission current from the substance at different temperatures are obtained. Values of $[\log I_s - 2 \log_\epsilon T]$ are plotted against $[1/T]$. The result should be a straight line. The intercept of this line with the vertical axis gives the value of $\log_\epsilon A_2$, the slope gives the value of $(-b_2)$.

Equations (16) and (17) are experimentally indistinguishable within the usual range of temperatures. When the constants are known for Eq. (16) the constants for Eq. (17) may be calculated from the following approximate relations.

$$b_2 = \left[b_1 - 1.5 \frac{T_1 + T_2}{2} \right] \tag{18}$$

$$A_2 = [0.223 A_1 T^{-1.5}] \tag{19}$$

CONSTANTS FOR EQS. (16) AND (17)

Substance	A_2	b_2	ϕ_2	Temperature, degrees Kelvin	Milliampere per square centimeter
Carbon	60.2	46,500	4.00	2000	20.
Calcium	60.2	26,000	2.24	1100	4.
Caesium	162.	21,000	1.81	500	2.5×10^{-8}
Caesium on Oxygen on	0.003	8,300	0.72	1000	350.
Tungsten (monatomic layer)					
Molybdenum	60.2	51,500	4.44	2000	1.59
Nickel	26.8	32,100	2.77		
Platinum	60.2	59,000	5.08	1600	1.6×10^{-5}
Tantalum	60.2	47,200	4.51	2000	13.8
Thorium	60.2	38,700	3.35	1600	4.3
Thorium on tungsten (monatomic layer)	3.0	30,500	2.63	1600	40.
Tungsten	60.2	52,400	4.52	2000	1.00
Coating of oxides on platinum					
Al_2O_3	16.2	46,300	3.90	2200	59.
Ba_2O_3	13.2	52,300	4.51	2200	3.1
BaO	2.88	19,500	1.68	1200	355.
CaO	129.	29,200	2.52	1400	219.
CuO	1.55×10^{-8}	20,500	1.76	1673	2.2×10^{-4}
Fe_2O_3	0.0116	44,400	3.82	1723	2.3×10^{-4}
MgO	1,020.	38,400	3.31	1600	102.
M'O	0.091	48,700	4.19	1723	1.5×10^{-4}
SrO	4.07	21,600	1.86	1200	85.
ThO_2	0.57	36,900	3.18	2000	23.

A filament coated with a mixture of the oxides of barium and strontium on a core of 95 per cent platinum and 5 per cent nickel has the following characteristics:

Electrical resistivity of the core

$$= 0.000022(1 + 0.00208t - 0.000,000,46t^2) \text{ ohm cm}$$
$$t = \text{temperature in degrees centigrade}$$

Thermal emissivity (ratio to black body)

$$= [0.4 + 0.000257T]$$

where T = degrees Kelvin lies between 800° and 1200°K.

The electron emission in zero field is given by the equation

$$I_s = 0.01T^2\epsilon^{-\frac{11600}{T}}$$

T = degrees Kelvin
I_s = emission current in amperes per square centimeter

For an anode potential of 150 volts and a current limited by space charge to 0.010 amp. per square centimeter the average life is

$$= 0.000015\epsilon^{\frac{22000}{T}} \text{ hr.}$$

The following values are those most probable when the anode potential equals 150 volts and the electric field is zero:

T	I_s	p_r	p_e	Life
900	20	2.3	0.02	730,000
950	45	3.0	0.045	170,000
1,000	90	3.7	0.09	55,000
1,050	170	4.6	0.17	20,000
1,100	310	5.6	0.31	7,400

T = temperature in degrees Kelvin
I_s = emission current in milliamperes per square centimeter
p_r = power thermally radiated in watts per square centimeter
p_e = power absorbed by electron emission in watts per square centimeter
Life = most probable average life in hours

8. Contact Potential. The rate of emission of electrons from different substances and the contact differences of potential are closely related. The contact potential depends only upon the materials of the electrodes and their temperature, but not upon size, shape, or position of the electrodes.

For example, an electron in escaping from the inner to the outer surface of substance A will do work equal to W_A so that its potential is changed to V_A. Similarly the work for an electron to escape from the surface B is W_B and the potential change V_B. Hence in moving an electron from substance A across a space to substance B the work done will be

$$[W_A + (V_A - V_B)e - W_B] \tag{20}$$

This is the algebraic summation of the work done and would be equal to zero, except for the work done at the junction of the two substances in the return connection. This later potential difference is known as the *Peltier effect* and is negligible in comparison with the other effects

$$W_A = \phi_A e \tag{21}$$
$$W_B = \phi_B e \tag{22}$$
$$(V_A - V_B)e = W_B - W_A = (\phi_B - \phi_A)e \tag{23}$$
$$(V_A - V_B) = (\phi_B - \phi_A) \tag{24}$$

$(V_A - V_B)$ is called the *contact potential difference* between the two substances, and by Eq. (24) it is equal to the difference in the *work function*, or electron affinity ϕ of the two substances.

9. Work Function. When a quantity of electricity q is moved through a potential difference V the work done equals qV. Work must be done when an electron is removed from a surface. If the work done per electron is W_1, the electron charge e, and the potential difference ϕ is required to supply an amount of energy equal to W_1, then,

$$W_1 = \phi e \tag{25}$$

$$\phi = \frac{W_1}{e} = \frac{k_0 b}{e} = (8.62 \times 10^{-5} b) \text{ volts} \tag{26}$$

ϕ is called the *electron affinity* of the substance and is equal to the work function (W_1/e). The smaller the quantity ϕ the easier it will be for an electron to escape from the cathode. A low value of ϕ indicates a large electron emission for a given temperature.

The following table gives the electron affinity or work function of several substances expressed in volts:

Substance	ϕ
Tungsten	4.52
Platinum	4.4
Tantalum	4.3
Molybdenum	4.3
Carbon	4.1
Silver	4.1
Copper	4.0
Bismuth	3.7
Tin	3.8
Iron	3.7
Zinc	3.4
Thorium	3.4
Aluminum	3.0
Magnesium	2.7
Nickel	2.8
Titanium	2.4
Lithium	2.35
Sodium	1.82
Mercury	4.4
Calcium	3.4

10. Filament Calculations. The dimensions of filaments designed to operate at a given voltage and temperature, and to furnish a certain total emission current are related to the physical properties of the material.

Suppose that the required total emission current is I_B ma. From the power-emission chart for the type of filament material being used, find I_s the emission current in milliamperes per square centimeter for a given power input p watts per square centimeter corresponding to good life performance, or to temperature T.

The total surface area of the required filament: $A = (I_B/I_s)$.
The total power input to the filament: $A_p = E_f I_f = P_f$ watts.
At a voltage E_f the filament current $I_f = (A_p/E_f)$.

Filament resistance at the operating temperature: $R_f = (E_f/I_f)$.

The resistance of a circular filament: $R = \left[\rho \dfrac{A}{2\pi^2 r^3}\right]$

where A = area of the filament surface
r = radius of the filament
ρ = specific resistance of the filament material. ρ must be known as a function of the temperature.

The resistance of a rectangular filament is given by

$$R = \left[\rho \frac{A}{2S_1S_2(S_1 + S_2)} \right]$$

where A = area of the filament surface
S_1 = thickness of the filament
S_2 = width of the filament
ρ = specific resistance of the filament material at temperature T

11. Filament-current Filament-radius Relation. For a given type of filament material operating at a specified temperature and filament voltage, the radius or filament cross section is uniquely related to the filament current.

For a circular filament: $I_f = [(2p/\rho)^{1/2}\pi r^{3/2}]$

For a rectangular filament: $I_f = (2p/\rho)^{1/2} \cdot [S_1S_2(S_1 + S_2)]^{1/2}$.

For a square filament: $I_f = (2p/\rho)^{1/2} \cdot 2^{1/2} \cdot S_1^{3/2}$.

12. Filament-voltage Filament-dimensions for a Constant Temperature. For a given filament material to be operated at a given temperature, the filament voltage is related to the filament length and sectional dimensions as follows:

Circular filament: $E_f = (2p\rho)^{1/2} \dfrac{l}{r^{1/2}}$

Rectangular filament: $E_f = (2p\rho)^{1/2} \left(\dfrac{1}{S_1} + \dfrac{1}{S_2} \right)^{1/2} \cdot l$

13. Lead-loss Correction. The cooling effect of the leads connected to a filament decreases the emission from the parts near the junction. The voltage drop in these parts of the filament is also less.

Langmuir and Dushman give the following correction formulas for a V-shaped filament cooled by large leads. The decrease in voltage due to the cooling effect of the two end leads is

$$\Delta V = 0.00026(T' - 400) \text{ volts}$$

T' = degree Kelvin of the central portion of the filament

The correction for the effect on the electron emission is given in terms of the voltage of a length of uncooled filament which would give the same effect as the decrease caused by the cooling of the leads. The correction for the two leads is $\Delta V_H = 2(0.00017T'\phi - 0.05)$ volts. ϕ is a number which depends upon the temperature coefficient of the quantity H, which may represent any property of the metal, such as candlepower, electron emissivity, etc. For the case of electron emission the exponent of the temperature coefficient is $N = \left(2 + \dfrac{b_0}{T}\right)$

Dushman's coefficient for the material b_0 and the temperature T in degrees Kelvin being known, N is calculated.

N..................	0.5	1.0	2.0	2.5	5.0	10.	20.	30.	50.
ϕ..................	0.48	0.85	1.23	1.44	1.72	2.10	2.47	2.69	2.95

N is related to ϕ as shown by the data above which may be plotted as a curve knowing ϕ the correction ΔV_H is determined.

The electron emission per unit area after taking into account the lead-loss correction is

$$I = \left(\frac{i}{S} f\right)$$

where i = observed total emission from any given filament
S = total filament area

The correction factor f is given by

$$f = \left[\frac{V + \Delta V}{V + \Delta V - \Delta V_H}\right]$$

Dushman gives curves of ΔV and ΔV_H plotted against temperature for different values of b_0.

$V + \Delta V$ corresponds to the corrected voltage drop along the filament.

14. Effect of Space Charge. The equations of Richardson and Dushman for thermionic emission give the total electron current, with

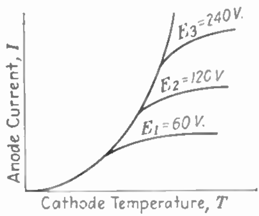


FIG. 2.—Space-charge effect in limiting emission.

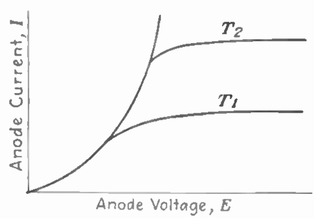


FIG. 3.—Saturation at constant temperatures.

zero field strength at the surface of the cathode. If the electrons are allowed to accumulate just outside the surface they form a negative cloud. If the electrons are drawn to a positive electrode both the negative cloud and to a less degree the cathode surface fields are changed.

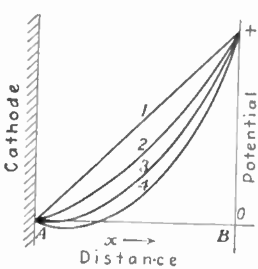


FIG. 4.—Distribution of potential in cathode-plate space.

Langmuir found that if the voltage applied to the anode was not sufficiently high a temperature increase of the cathode did not increase the current indefinitely. This effect is shown in Fig. 2. It is due to the repelling effect of the negative cloud of electrons surrounding the cathode and is known as the *space-charge effect*, or volume density of electrification. Figure 3 shows this effect with constant-cathode temperatures and variable-anode voltage.

The theory of these effects is as follows: The distribution of the potential between two large parallel plates is directly proportional to the distance starting from the low and increasing to the high potential plate. If plate A emits low-velocity electrons (assumed zero) spontaneously, and if plate B is positive with respect to A, electrons will be drawn over to B. Starting with a low temperature T, the distribution of potential between A and B will be uniform as shown by the straight line 1 in Fig. 4. Increasing the temperature of A will cause an electron current of I amp. per square centimeter to flow to B. Laplace's equation connecting the potential distribution with the volume density of electrification ρ is

$$\Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi\rho \tag{27}$$

For large parallel planes Eq. (27) may be simplified to

$$\frac{d^2 V}{dx^2} = -4\pi\rho \tag{28}$$

If ρ is constant and negative, the potential distribution will be a parabolic curve as shown by curve 2 in Fig. 4. A further increase in the temperature of A will cause the parabola to take the form of curve 3 having a horizontal tangent at A. In this case the potential gradient at the cathode is zero ($dV/dx = 0$), and a further increase of temperature will not increase the electron current to B. This accounts for the effect shown in Fig. 2.

In the above discussion the electrons were assumed to be emitted with no initial velocity. Usually small initial velocities exist, so that a slightly negative gradient is necessary at A in order to prevent an increase in current. Curve 4 of Fig. 4 shows the effect of the initial velocities of emission on the potential distribution at the temperature for which a further increase in temperature will not increase the anode current.

15. Schottky Effect. Richardson's and Dushman's equations for the thermionic emission from a substance at a given temperature assumes that the electric field strength is zero at the cathode. In actual practice a definite potential is used. This effect of the potential gradient at the cathode on the observed emission current is called the *Schottky effect.*

Dushman gives the correction for the Schottky effect as follows:

I_0 = electron emission in zero field
I_v = observed emission at an anode voltage V

Then

$$I_v = I_0 \epsilon^{\frac{4.39\sqrt{kV}}{T}}$$

where k = a constant whose value depends upon the relative geometrical arrangement of anode and cathode
T = temperature in degrees Kelvin
ϵ = base of Napierian logarithms

16. Electron Current between Parallel Plates. When the cathode is a large flat surface A and the plate, or anode, B is a parallel surface, the plate current per square centimeter of surface not too near the edges of the plates is given by the equation

$$i = 2.34 \times 10^{-6} \frac{V^{3/2}}{x^2} \tag{29}$$

where i = maximum current density in amperes per square centimeter
x = distance between plates in centimeters
V = potential difference between A and B in volts

This equation assumes that the initial velocities of the electrons leaving A are zero. If the potential of B is large relative to one or two volts, the initial velocities of the electrons can be neglected.

Equation (29) assumes that the anode potential is positive with respect to A so that some current is flowing but that the anode potential is below the value necessary to give the full current emitted at A. When the anode potential is great enough to draw over all of the electrons emitted at A, the current (saturation current) I_s is given by the Richardson-Dushman equation.

17. Electron Current between Concentric Cylinders. Given two concentric cylinders A and B (Fig. 6) having radii of a and r cm and of infinite length. Langmuir's equation for the electron current to the plate B is given by the relation

$$i = 14.7 \times 10^{-6}\frac{V^{3/2}}{r\beta^2}$$

where i = current in amperes per centimeter length
V = potential between A and B in volts
r = radius of the anode in centimeter
a = radius of the cathode in centimeter
β = a factor which varies with the ratio of (r/a)

r/a	β^2	r/a	β^2
1.00	0.000	20	1.072
2.00	0.279	50	1.094
3.00	0.517	100	1.078
4.00	0.667	200	1.056
5.00	0.767	500	1.031
10.00	0.978	1,000	1.017
		∞	1.000

When the inner cylinder is a small wire of less than one-tenth the diameter of the plate, the error is small if β is neglected, and the approximate equation is

$$i = \left[14.7 \times 10^{-6}\frac{V^{3/2}}{r}\right]$$

18. Electron Current with Any Shape Electrodes. Langmuir has demonstrated that under the assumption on which the above equations were derived the current will vary as the three-halves power of the potential difference V regardless of the shape of the electrodes. The derivation of the equations neglects the initial velocities of the electrons and the potential gradient at the cathode.

19. Two-electrode Vacuum Tubes. The three-halves power equation for the plate current of a two-electrode tube is quite accurate when the voltage between cathode and plate is large with respect to the effects of (1) initial velocities of emission; (2) voltage drop in the filament or cathode; (3) contact potential between cathode and plate and the emission of electrons from the cathode is large and the plate voltage well below the value for saturation current. The electrodes are assumed to be in good vacuum, so that the effects of gas are negligible.

In the case of thoriated-tungsten or oxide-coated filaments only a fraction of the total cathode surface is active so that the saturation current may be reached at a plate voltage below the theoretical.

The current is calculated from the formula

$$i = kV^{3/2}$$

where k is the space-charge constant of the tube for a given type of tube structure and depends only upon the geometrical configuration without regard to the dimensions of the tube. The value of k for infinite parallel plates is $\left(2.34 \times 10^{-6}\frac{A}{x^2}\right)$

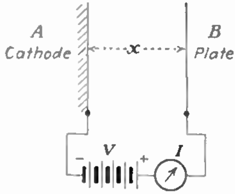


FIG. 5.—Electron current between parallel plates.

where A = the area of the plate in square centimeters
x = the distance from the cathode plate to the anode plate in centimeters.

For concentric cylinders, $k = \left(14.7 \times 10^{-6}\frac{l}{r\beta^2}\right)$

l = length of the cylinders
r = radius of the outer cylinder or anode
β = a function of (r/a) (see table on page 192)

20. Effect of Initial Velocities—Parallel Plates. If the effect of the initial velocity of the electrons is included and they have a Maxwellian distribution,

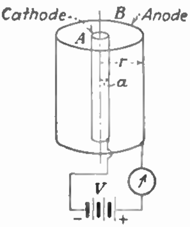


FIG. 6.—Cylindrical structure.

$$i = 2.34\frac{A}{(x_a - x_m)^2}(V_a - V_m)^{1.5}\left(1 + 0.0247\sqrt{\frac{T}{V_a}}\right)$$

where i = total plate current in amperes
A = area of one surface of the anode in square centimeters
T = temperature of the cathode in degrees Kelvin
V_a = potential of the anode above that of the cathode volts
V_m = minimum potential of the space between cathode and anode with respect to the cathode
x_a = distance from cathode to anode in centimeters
x_m = distance from cathode to V_m in centimeters

21. Effect of Magnetic Field. Initial velocities = 0 For coaxial cylinders,

$$i = kV_a^{1.5}, \text{ if } V_a > V^1$$
$$i = 0, \quad \text{ if } V_a < V^1$$
k = same as above

$$V^1 = \left[0.0221H^2r_o^2 + 0.0188I^2\left(\log_{10}\frac{r_o}{r_i}\right)^2\right]$$

H = strength of magnetic field externally applied parallel to axis of cylindrical electrodes
I = current flowing through the inner cylindrical electrode parallel to its axis
r_o = radius of the outer cylinder
r_i = radius of the inner cylinder

22. Characteristics of Typical Commercial Diodes.

Type	i_f	E_f	E_m	i_m	P_n	k
ThW	1.25	7.5	550	0.065	0.0075	1.2
ThW	3.25	10	1,500	0.20	0.050	1.7
ThW	3.85	11	2,500	0.25	0.250	1.1
PW	14.7	11	16,000	0.166	1.00	0.5
PW	24.5	22	17,500	0.833	5.00	1.0
PW	52	22	18,000	3.0	20.00	1.1
PW	10	10	20,000	0.10	....	0.10
PW	10	10	85,000	0.10	....	0.11
PW	32	9	75,000	0.25	....	0.25
PW	10	10	150,000	0.100	....	0.11
PW	32	12.5	150,000	0.25	....	0.11

i_f, E_f = filament current, voltage (amperes and volts)
E_m = maximum effective a-c input voltage (volts)
i_m = maximum rectified tube current (amperes)
P_n = nominal power rating (kilowatts)
ThW, PW = thoriated tungsten, and pure tungsten, filament
k = 0.0001 amp. per volt$^{1.5}$

THREE-ELECTRODE TUBES

23. Effect of the Grid. When a wire mesh or similar electrode having openings through which electrons may pass is placed between the cathode and the plate of a two-electrode tube it exerts a large controlling effect on the flow of electrons to the plate. The meshlike electrode between cathode and plate is termed a *grid.* The tube is then known as a *triode,* or three-electrode tube.

When the grid is connected to a battery or other source of voltage the electrons are attracted if the grid is positive with respect to the cathode and repelled if it is negative. The close proximity of the grid to the space charge surrounding the cathode increases its effectiveness in controlling the electron flow.

In most useful applications the tubes are operated with sufficient electron emission and with plate and grid voltages low enough so that the space charge surrounding the cathode is ample to permit large momentary increases in the electron flow to the plate.

The effect of a large positive plate voltage in drawing the electrons to the plate can be reduced by a relatively small negative voltage applied to the grid. The electrons being negative will avoid the negative grid so that no current will flow in the grid circuit. If the negative grid voltage is not too large with respect to the plate voltage, electrons will be drawn through the openings in the grid mesh to the positive plate.

The resulting plate current is controlled by the grid although no current flows in the grid circuit. The power is equal to the product of the current times the voltage. Zero power in the grid circuit can thus control a considerable amount of power in the plate circuit. Voltage variations of the grid produce corresponding variations of the plate current. The extent to which the plate-current variations are faithful reproductions of the grid-voltage variations depends upon the steady polarizing voltages (A, B, and C voltages) applied to the tube and the range of the voltage variations.

CHARACTERISTICS OF THE THREE-ELECTRODE TUBE

24. Static Characteristics. The effects of constant d-c voltages applied to the electrodes of a tube are shown by curves called the *static characteristics* of the tube. The vertical scale of these curves usually shows the plate current, grid current, total emission current, or filament current of the tube. The horizontal scale shows a range of voltages

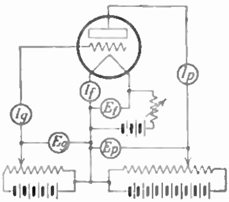


FIG. 7.—Circuit for measuring static characteristics.

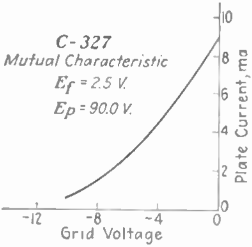


FIG. 8.—Typical grid-voltage plate-current characteristic.

effective on one electrode. The voltage on the other electrodes are held constant at some value specified on the curve.

The circuit for obtaining the static characteristic curve data is shown in Fig. 7. The meters marked I_p, I_g, and I_f read the plate current, grid current, and filament current respectively. Potentiometers connected across batteries permit adjusting the plate and grid voltages.

25. Mutual or Transfer Characteristic. The mutual characteristic, or transfer characteristic of the tube, shows the effect of the grid voltage upon the plate current. The term mutual or transfer indicates that the

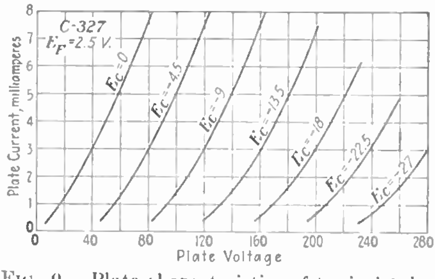


FIG. 9.—Plate characteristics of typical tube.

voltage in one circuit controls the current in another circuit. Figure 8 shows this characteristic for a type C-327 tube. The plate voltage is held constant at 90 volts and the filament voltage at the rated value of 2.5 volts. When the grid is connected to the cathode without any applied grid voltage, we read at zero grid voltage on the curve, a plate current of 9.0 ma. As the grid voltage is made negative the plate current decreases until it is zero at −12.0 volts. The plate current is zero for all grid voltages more than 12.0 volts negative unless the plate voltage is increased.

26. Plate Characteristic. The plate characteristic represents the relation between plate current and plate voltage. These curves for a type C-327 tube are shown in Fig. 9. Each curve is for a specified voltage. The plate current increases as the plate voltage is increased.

27. Grid Characteristic. The grid characteristic shows the grid current-grid voltage relation. Electron flow to the grid starts in the region of zero grid voltage. The exact point at which grid current starts is determined by the initial velocities of emission, and the contact potential of the grid to cathode. The net effect is equivalent to a small positive or negative bias usually not greater than one volt.

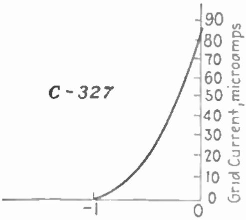


FIG. 10.—Grid-current grid-voltage characteristic.

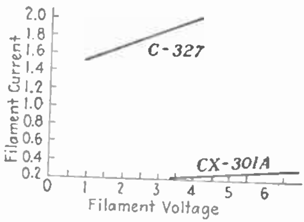


FIG. 11.—Typical filament characteristics.

The grid characteristic of a C-327 tube is shown in Fig. 10. The inherent bias in the tube is nearly 0.9 volt positive so that the grid must be biased negative by 0.9 volt to secure an effective zero grid voltage. In filament-type tubes the point of connection to the filament alters the effective bias voltage. The negative filament terminal is usually considered the zero of potential. This connection gives the greatest negative bias on the grid.

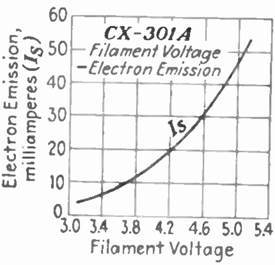


FIG. 12.

28. Filament, or Heater, Characteristic. The filament-voltage filament-current curve obtained with plate and grid terminals disconnected is termed the filament characteristic. The characteristic refers to the heater filament when the tube is of the indirectly heated cathode type. Figure 11 shows the filament characteristic of a filament-type CX-301A tube and an indirectly heated cathode-type C-327 tube.

29. Emission Characteristic. The emission characteristic shows the total electron emission from the cathode for a range of filament voltages or filament power. The emission-current filament-voltage characteristic of a CX-301A tube is shown in Fig. 12. The circuit for obtaining the data for this curve is shown in Fig. 13. The grid and plate terminals are connected together through a suitable meter to +50 volts B supply. The filament voltage is gradually increased. The corresponding total current to grid and plate is read on meter I.

In taking the emission-current readings it is assumed that the B voltage is large enough to eliminate all space-charge effect so that the current is limited only by the temperature of the filament. If the B voltage is too high, the power dissipation at the grid and plate will increase the temperature of the filament. Traces of gas also affect the results more at higher voltages. To reduce these effects the B voltage should only be connected long enough to obtain the emission reading. The emission reading imposes a severe load on the tube. The emission current may be great enough to damage the tube. In tubes employing oxide-coated filaments with 0.25-amp. filament current or less the

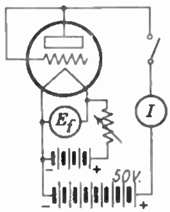


FIG. 13.—Measurement of emission characteristic.

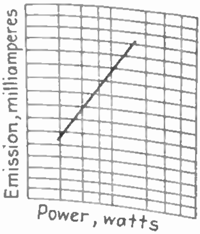


FIG. 14.

emission current is a considerable portion of the filament current. Accurate results can be obtained by means of a rotating contact which closes the circuit for only an instant. The current must be read with an oscillograph or peak-current voltmeter arrangement.

The emission current per watt of filament heating power is fundamentally related to the filament material and the design of the filament. The filament power should be determined from the filament characteristic obtained with grid and plate terminals open. The data can be plotted on a special coordinate paper devised by C. J. Davisson. If the emission follows Richardson's temperature equation and the power is radiated according to the Stefan Boltzman law of radiation, the filament-power emission-current curve will be a straight line. Figure 14 shows such a curve for a type CX-301A tube.

Departure from a straight line may indicate:
1. Cooling by conduction of gas or support leads.
2. Space-charge effect due to too low B voltage.

CALCULATION OF THE SPACE CURRENT OF THE THREE-ELECTRODE TUBE

30. The space current I of a three-electrode tube is equal to the sum of the plate current I_p and the grid current I_g; $I = (I_p + I_g)$. The three-electrode tube is calculated as an equivalent diode $I = k(E_p +$

$\mu E_g)^{3/2}$. The grid voltage E_g is equivalent to a plate voltage μE_g. μ is the amplification factor of the tube.

31. Plane-parallel Elements. For a structure with plane-parallel elements with the filament symmetrically placed between grids and plates:

$$k = 2.34 \times 10^{-6} \times \frac{A}{(\alpha + \beta)^{1/2}[\alpha + \beta(\mu + 1)]^{3/2}}$$

$$\mu = \frac{2\pi \alpha n}{\log_\epsilon \dfrac{1}{2\pi r n}}$$

$$I = 2.34 \times 10^{-6} \frac{A}{(\alpha + \beta)^{1/2}}\left[\frac{E_p + \mu E_g}{\alpha + \beta(\mu + 1)}\right]^{3/2}$$

where I = total space current in amperes
α = distance from plate to grid in centimeters
β = distance from grid to filament in centimeters
n = number of grid wires per centimeter length of the structure
r = radius of the grid wires
A = effective plate area

32. Concentric Elements. For a structure with a cylindrical anode and grid and a coaxial strand of filament,

$$k = 14.7 \times 10^{-6} \frac{L R_p^{1/2}}{[(R_p - R_g) + R_g(\mu + 1)]^{3/2}}$$

$$\mu = \frac{2\pi n R_g^2\left(\dfrac{1}{R_g} - \dfrac{1}{R_p}\right)}{\log_\epsilon \dfrac{1}{2\pi r n}}$$

$$I = 14.7 \times 10^{-6} \frac{L}{R_p}\left[\frac{(R_p - R_f)(E_p + \mu E_g)}{(R_p - R_f) + (R_g - R_f)(\mu + 1)}\right]^{3/2}$$

If R_f is very much smaller than R_p and R_g the equation can be written approximately

$$I = 14.7 \times 10^{-6} L R_p^{1/2}\left[\frac{E_p + \mu E_g}{(R_p - R_g) + R_g(\mu + 1)}\right]^{3/2}$$

where L = length of the structure in centimeters
R_f = radius of the filament in centimeters
R_p = radius of the plate in centimeters
R_g = radius of the grid in centimeters

The above relations are useful in the design of the structures. The k should be determined for the type of tube structure. The μ and the current-voltage characteristics remain the same if all dimensions are changed proportionately. The plate current equals the space current when the grid current is zero.

33. Amplification Factor. The amplification factor is a measure of the effectiveness of the grid voltage relative to that of the plate voltage upon the plate current. It is the ratio of the change in plate voltage to a change in grid voltage in the opposite polarity, under the condition that the plate current remains unchanged. As most precisely used, the term refers to infinitesimal changes as indicated by the defining equation:

$$\mu = -\frac{\partial e_p}{\partial e_g};\ i_p = \text{constant}$$

The amplification factor is indicated by the horizontal spacing of the plate characteristic or mutual characteristic curves of the tube. Since horizontal lines represent constant plate current the plate voltage spacing divided by the grid voltage spacing of the curve is the amplification factor. The amplification factor of three-electrode tubes is nearly constant for a constant plate current. In the region near zero plate current or near the full emission current of the filament, the amplification factor changes greatly with voltage. It is necessary to use smaller increments in regions where the amplification factor changes rapidly.

34. Measurement of Amplification Factor. The amplification factor is measured conveniently with an alternating-current bridge circuit shown schematically in Fig. 15. The resistance R_1 is adjusted for zero sound in the phones. The amplification factor is given by

$$= \frac{R_2}{R_1}$$

Due to tube capacities or other reactances in the circuit it is usually necessary to provide a means for adjusting the phase of the grid and plate a-c voltages for complete balancing out of the sound in the phones. This phase balance is secured with condenser C in Fig. 15. The d-c voltage drop in R_2 should be allowed for when setting the plate voltage. The adjustable ground connection is convenient in eliminating the unbalancing effects of capacity to ground. The a-c tone voltage should be as small as practical. The phones can be preceded by a suitable amplifier.

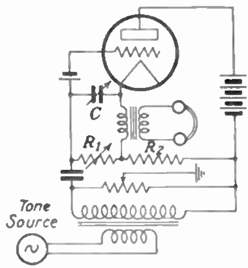


Fig. 15.—Measurement of amplification factor.

CALCULATION OF THE AMPLIFICATION FACTOR

35. Plane-parallel Electrodes. When the diameter of the grid wires is large compared to their spacing the formula derived by Vodges and Elder is most accurate. Figure 16 shows a cross section of the electrodes. The amplification factor is

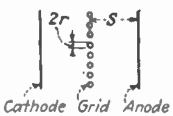


Fig. 16.—Tube with plane-parallel electrodes.

$$\mu = \left[\frac{2\pi ns - \log_\epsilon \frac{1}{2}(\epsilon^{2\pi nr} + \epsilon^{-2\pi nr})}{\log_\epsilon (\epsilon^{2\pi nr} + \epsilon^{-2\pi nr}) - \log_\epsilon (\epsilon^{2\pi nr} - \epsilon^{-2\pi nr})}\right]$$

where r = radius of the grid wire in centimeters
n = number of grid wires per centimeter length of structure
s = distance from plate to grid in centimeters

When the diameter of the grid wires is small compared to their spacing the equation above simplifies to

$$\mu = \frac{2\pi ns}{\log_\epsilon (1/2\pi nr)}$$

36. Concentric Cylindrical Electrodes. The amplification factor of the cylindrical structure shown in Fig. 17 is given by

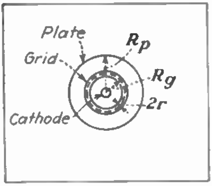


FIG. 17.

$$\mu = \frac{2\pi n R_g \log_\epsilon (R_p/R_g) - \log_\epsilon \frac{1}{2}(\epsilon^{2\pi nr} + \epsilon^{-2\pi nr})}{\log_\epsilon (\epsilon^{2\pi nr} + \epsilon^{-2\pi nr}) - \log_\epsilon (\epsilon^{2\pi nr} - \epsilon^{-2\pi nr})}$$

where R_p = radius of the anode in centimeters
R_g = radius of the grid in centimeters
r = radius of the grid wires in centimeters
n = number of grid wires (turns) per centimeter length of structure.

When the diameter of the grid wires is small compared with their spacing the equation simplifies to

$$\mu = \frac{2\ nRg \log_\epsilon (R_p/R_g)}{\log_\epsilon \left(\frac{1}{2\pi nr}\right)}$$

37. Plate Resistance and Plate Conductance. The plate resistance r_p is defined by the equation,

$$r_p = \frac{1}{S_p} = \frac{\partial e_p}{\partial i_p}$$

It is the reciprocal of the plate conductance S_p.

The plate conductance is the ratio of the change in plate current to the change in plate voltage producing it, all other electrode voltages being maintained constant. As most precisely used, the term refers to infinitesimal changes as indicated by the defining equation

$$S_p = \frac{\partial i_p}{\partial e_p}$$

The plate conductance is given by the slope of the plate-characteristic curves of the tube. When readings are taken on the characteristic curves the current and voltage increments should be made as small as convenient. The plate resistance is the reciprocal slope of the plate-characteristic curve. For example, at the point on the plate characteristics corresponding to the d-c operating voltages, the plate current rises 1.0 ma when the plate voltage increases 10 volts. The conductance is then

$$S_p = \frac{0.001}{10} = 0.0001 \text{ mho} = 100 \text{ micromhos}$$

The plate resistance is

$$r_p = \frac{10.}{0.001} = 10{,}000, \text{ ohms}$$

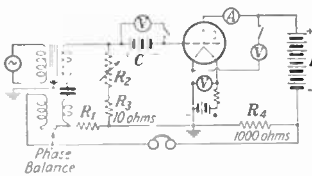


FIG. 18.—Measurement of plate resistance.

The numerical value of the plate resistance changes with the applied d-c operating voltages.

38. Measurement of the Plate Resistance. The plate resistance or plate conductance can be measured directly with the aid of a bridge type of circuit. Figure 18 illustrates a circuit suitable for this purpose. When the bridge is balanced for minimum sound in the phones the plate resistance of the tube is

$$r_p = \frac{R_2 R_3}{R_1}$$

The alternating voltage (tone) applied to the bridge should be as small as practical. The use of an amplifier preceding the phones increases the sensitivity and accuracy of these measurements. The effects of small capacities are sometimes troublesome in circuits of this type. The electrode capacity of the tube causes some phase shift resulting in a poor balance. The phase balance variometer balances the small out-of-phase component permitting a closer adjustment to the null point. The capacity to ground can be balanced by suitable shielding or by means of a Wagner earth connection.

39. Calculation of the Plate Resistance. The plate resistance of a tube depends upon the operating voltages as well as the structural parameters. It is within certain limits inversely proportional to the area of the anode and also to the area of the cathode. Decreasing the distance between filament and plate decreases the plate resistance. Since it is desirable to make (μ/r_p) large, the grid to plate distance controlling μ should not be decreased too much. This requires that the grid be placed near the filament to lower the plate resistance. When the grid is too near to the filament it will be heated. Small amounts of grid emission current resulting from too high grid temperature have an objectionable effect on the operation of the tube.

The plate resistance of a tube may be calculated from the plate-current plate-voltage relation. For a structure with plane-parallel elements in which the filament is symmetrically placed between grids and plates the plate resistance is,

$$r_p = \frac{(\alpha + \beta)^{\frac{1}{2}}[\alpha + \beta(\mu + 1)]^{\frac{3}{2}}}{A(E_p + \mu E_g)^{\frac{1}{2}}} \times 10^6$$

where r_p = plate resistance in ohms
α = distance from plate to grid in centimeters
β = distance from grid to filament in centimeters
μ = amplification factor
E_p = plate voltage
E_g = grid voltage
A = a constant depending on the cathode area, or anode area, and type of structure. For typical filament-type tubes $A = 1.8L$, where L is the length of the filament in centimeters.

The grid voltage E_g is conveniently made zero and the plate voltage taken equal to the value giving normal plate current.

40. Mutual Conductance. The mutual conductance (or grid-plate transconductance) of a tube is defined by the relation

$$S_m = S_{gp} = \frac{\partial i_p}{\partial e_g}$$

It is the ratio of the change in plate current to the change in grid voltage, under the condition that all other voltages remain constant. It is also equal to the ratio of the amplification factor μ to the plate resistance r_p of the tube:

$$S_m = \frac{\mu}{r_p}$$

The mutual conductance determines the plate current change per volt applied to the grid. It is evident that this is the most important characteristic of a tube. It is a figure of merit of the tube and enters into the calculations of the performance of the tube. It is a direct measure of the amplifying properties of the tube operating into a load impedance which is small with respect to the plate resistance.

With high impedance loads the amplification factor and plate resistance are considered separately in determining the tube performance. The mutual conductance may be determined graphically from the slope of the mutual characteristic curve of the tube. Direct measurements are usually most convenient when many readings are required. The bridge circuit described in the following section balances readily with either three-electrode or screen-grid tubes.

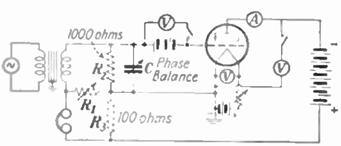


FIG. 19.—Measurement of mutual conductance.

41. Measurement of Mutual Conductance. The mutual conductance can be measured directly in the circuit shown in Fig. 19. The resistance R_1 and the phase balance C are adjusted until the sound in the phones is balanced out. The mutual conductance is given by

$$S_m = \frac{R_1}{R_2R_3}\left(1 + \frac{R_3}{r_p}\right) = \frac{R_1}{R_2R_3} \text{ (approx.)}$$

42. Calculation of the Mutual Conductance. The mutual conductance S_m is equal to the ratio of the amplification factor μ to the plate resistance r_p. Each of these factors can be calculated with a fair degree of accuracy for certain types of structures. The amplification factor depends almost entirely upon the structure of the grid and the grid-plate distance. The plate resistance depends upon the amplification factor, the surface areas of the cathode and anode, the grid-filament distance, and the applied d-c operating voltages. The mutual conductance depends upon all of these factors.

43. Grid-current Coefficients. When the grid of a tube is not biased with sufficient negative voltage and the tube operation extends into the positive range of grid voltage, an electron current will flow to the grid. Under these conditions the current in the grid circuit may change the effective grid voltage. When it is desirable to include these effects in determining the performance of the tube the coefficients relative to the grid current are useful.

The *grid conductance* S_g, or its reciprocal the grid resistance r_g, is defined by the equation

$$S_{gg} = S_g = \frac{\partial i_g}{\partial e_g}$$

$$r_g = \frac{1}{S_g} = \frac{\partial e_g}{\partial i_g}$$

The grid conductance S_g is the ratio of the change in the grid current to the change in grid voltage producing it, other electrode potentials being maintained constant. As most precisely used the term refers to infinitesimal changes, as indicated by the defining equation.

The coefficient showing the relative effectiveness of grid and plate voltages on the grid current has been variously termed reflex factor, inverse amplification factor, and inverse factor. Recent I.R.E. standards term this coefficient the *plate-grid mu factor.* It is the ratio of the change in grid voltage to the change in plate voltage required to maintain a constant value of grid current. As most precisely used the term refers to infinitesimal changes as indicated by the defining equation

$$\mu_{ggp} \equiv \mu_n = -\frac{\partial e_g}{\partial e_p};\ i_g = \text{constant}$$

The coefficient showing the effect of plate voltage on the grid current has been termed *inverse mutual conductance*, or the *plate-grid transconductance* (note that this is not the grid-plate transconductance which is the mutual conductance. The difference in these terms can be easily remembered, since the words grid and plate appear in the same order as the direction of action in the tube). It is the ratio of the change in grid current to the change in plate voltage producing it, all other electrode voltages being maintained constant. As most precisely used the term refers to infinitesimal changes, as indicated by the defining equation

$$S_{gp} \equiv S_n = \frac{\partial i_g}{\partial e_p}$$

The grid-current coefficients of the tube may be determined graphically from the static characteristic curves or measured directly in bridge circuits similar to those employed for the plate current coefficients.

44. Higher-order Coefficients. The tube coefficients in most common use are the amplification factor, plate resistance or conductance, and mutual conductance. These are the first-order plate-current coefficients of a triode. They determine the amplifying properties of the tube and enter into nearly all applications of the tube.

When the tube is operated so that detection, modulation, distortion, cross modulation, frequency conversion, and such effects are of importance, it is necessary to use second-order, third-order, and higher-order coefficients in addition to the first-order coefficients to determine the performance of the tube. For example, in the case of plate circuit detection the tube coefficient determining this effect is the second derivative of the plate current with respect to the grid voltage. The first derivative, or first-order coefficient, is the mutual conductance which is

$$\frac{\partial i_p}{\partial e_g} = S_m$$

The second derivative, or second-order coefficient, is

$$\frac{\partial^2 i_p}{\partial e_g^2} = \frac{\partial S_m}{\partial e_g}$$

The d-c plate-current change with signal voltage and second-harmonic distortion are also determined by the second-order coefficient.

Cross-modulation and modulation distortion in the r-f stages of a receiver are determined by the third-order coefficient

$$\frac{\partial^3 i_p}{\partial e_g{}^3} = \frac{\partial^2 S_m}{\partial e_g{}^2}$$

The third-harmonic distortion in a tube is also determined by the third-order coefficient. The fifth-harmonic distortion would be determined by the fifth-order coefficient,

$$\left(\frac{\partial^5 i_p}{\partial e_g{}^5}\right)$$

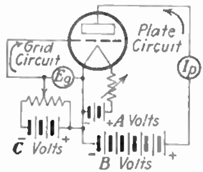


FIG. 20.—Triode circuit.

Higher-order coefficients are usually obtained graphically from the current-voltage characteristics of the tube. When the analytical expression for the current is known the coefficients may be obtained by differentiation. The measurement of an effect depending principally on one coefficient may be used as a measure of the coefficient.

45. Mechanism of the Three-electrode Amplifier. Figure 20 represents a triode connected to a suitable source of A, B, and C voltage. A meter I_p is connected in the plate circuit of the tube for reading the plate current. A potentiometer is connected across the C voltage. The grid voltage E_g will be changed as the slider is changed on the potentiometer. If the slide moves toward the positive, the plate current increases; if toward the negative, the plate current decreases. The plate currents corresponding to different grid voltages are plotted as in curve 1 in Fig. 21. This is a mutual characteristic curve of the tube.

Suppose that the slide is varied in some definite manner. For example, start to count time from zero on curve 2 in Fig. 21. With the slider initially at 5 volts the plate current is 3 ma. Move the slider steadily in the negative direction, until say, in three seconds the grid voltage is 9 volts. The plate current will be 0.5 ma. Now start the slider in the positive direction, moving at the same steady rate. At the end of 6 sec. the slider has returned to its original position. Continuing the motion of the slider in the positive direction at the end of 9 sec. the grid voltage is -1.0 volt and the plate current is 6.5 ma. If the slider is started in the negative direction at the same rate, the grid voltage will be -5 volts at the end of 12 sec., thus completing the cycle.

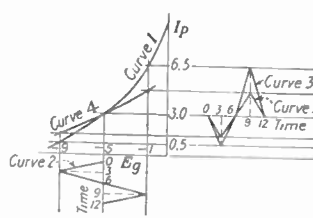


FIG. 21.—Mechanism of amplification.

Curve 3 shows the plate-current change corresponding to the grid-voltage change with time. If the slider is connected to a mechanism arranged to continue this motion, the plate current would contain an alternating current of 1 cycle in 12 seconds or 5 cycles per minute. The waveform of the a.c. will be as shown in curve 3. It is superimposed upon the d-c plate current.

The positive and negative peaks of the plate current as measured from the initial 3-ma point are not equal although the grid-voltage peaks are equal. In this case the plate current is not a faithful reproduction of the input voltage.

If a resistance is connected in the plate circuit, the effective plate voltage is reduced as the plate current increases. The plate current at E_g equals -5 volts can be brought to the initial 3-ma point by a suitable increase in the B voltage to compensate for the voltage lost in the resistance. Starting with the same initial 3-ma point, the resulting characteristic with a resistance load is shown by the curve 4 in Fig. 21. The same alternating grid-voltage curve 2 produces the plate current curve 5. The positive and negative plate current peaks of curve 5 as measured from the initial point are almost identical. The distortion has been eliminated and the voltage developed across the resistance can be used to operate a succeeding stage of amplification or other device.

The potentiometer and slider of Fig. 20 can be replaced with a fixed grid-bias voltage and an a-c voltage. The tube will operate as described above except that a-c cycles usually occur so rapidly that the plate current (d.c.) meter cannot follow them. A meter showing the effective value (r-m-s) of the a.c. can be used to measure the current. The alternating current can be heard when connected to a loud-speaker, if it is within the audible range of frequencies. The waveform of the a.c. can be seen when connected to an oscillograph.

46. Four-electrode Tubes. The four-electrode tube is known as a *tetrode*. A tube having a cathode, two grids, and a plate such as the screen-grid tetrode is of this type. Tubes having two grids may be classified according to their use as screen grid, space-charge grid, or double-function tubes.

47. Screen-grid Tetrode. The screen-grid tetrode is designed especially for use as an r-f amplifier. The grid-plate capacitance is extremely small in this tube so that feedback through the tube is reduced to a low value eliminating the necessity for neutralizing. The unusual characteristics of this tube give higher voltage amplification than can be obtained with three-electrode tubes.

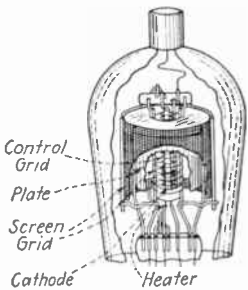


FIG. 22.—Structure of screen-grid tube.

The construction of this tube is shown in Fig. 22. When connected in a screen-grid circuit the inner grid (No. 1 grid) is the control electrode. The screen grid (No. 2 grid) is between the inner grid and plate. The screen grid is also extended to shield the inner grid from the outer surface of the plate.

When the screen grid is effectively grounded the resultant shielding reduces the capacitance between control grid and plate to a small fraction of the value in a three-electrode tube. For example the grid-plate capacitance in a type C-327 triode is 3.3 $\mu\mu$f. The corresponding control grid-plate capacitance in a type C-324 tetrode is less than 0.01 $\mu\mu$f. The screen-grid tube should be surrounded with a grounded metal shield to avoid stray coupling around the outside of the electrodes. The control-grid circuit should be shielded from the plate circuit to eliminate both electrostatic and electromagnetic coupling between coils, condensers, connecting leads, etc.

The fundamental circuit arrangement for a screen-grid tube is shown in Fig. 23.

The screen-grid electrode is connected directly to a positive voltage which may be a tap on the plate-voltage supply. The plate voltage should at all times be greater than the screen-grid voltage. The input circuit is connected to the inner or control grid, and the output circuit is connected to the plate in the usual way. The screen grid is effectively at ground potential for the alternating signal currents.

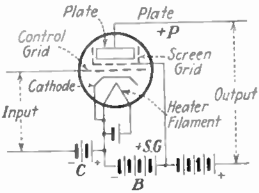


FIG. 23.—Circuit for screen-grid tube.

The internal action in a screen-grid tube may be illustrated by comparison with a triode. In the triode the plate current depends upon the grid and plate voltages, the grid voltage having a greater effect than the plate voltage.

In the screen-grid tube the plate current depends principally upon the voltages of the two grids. The influence of the plate voltage on the plate current is extremely small due to the presence of the two intervening grids. The screen grid has nearly the same effect on the current as the plate of the three-electrode tube. The electrons are attracted by the screen grid. About 75 per cent of electrons pass through the screen grid and are drawn by the higher plate potential to the plate. The plate acts as a collector of the electrons which have passed the screen.

Since the plate voltage has an extremely small effect on the plate current, the plate resistance of the tube is large. In triodes large plate resistance is accompanied by low mutual conductance. In the screen-grid tube the mutual conductance is determined mainly by the voltages of the two grids. The mutual conductance remains nearly the same as a triode having the same filament and control grid.

Since the plate resistance of the screen-grid tube can be made extremely high without reducing the mutual conductance the amplification factor which is equal to the product of mutual conductance and plate resistance is also high.

The equivalent circuit of the screen-grid tube is similar to that for a triode. The screen grid applies a fixed polarizing voltage to the tube, but does not take part in the a-c operation.

The voltage amplification is higher with a screen-grid tube than with a triode having the same mutual conductance. The triode has a low plate resistance. Connecting a load in plate circuit reduces the current flow. The screen-grid tube has a high plate resistance. The same load connected to the plate circuit will have little effect on the current. The output voltage is equal to the product of current and load impedance and is thus higher for the screen-grid tube. For example, consider a triode with mutual conductance 1,000 micromhos, amplification factor 10, and plate resistance 10,000 ohms. The voltage amplification obtained with a 50,000-ohm load would be

$$Av = \frac{\mu R_p}{r_p + R_p} = \left(\frac{10 \times 50,000}{10,000 + 50,000}\right) = 8.3$$

A corresponding screen-grid tube having a mutual conductance of 1,000 micromhos has an amplification factor of 400 and a plate resistance of

400,000 ohms. The voltage amplification obtained with the same load would be

$$Av = \left(\frac{400 \times 50{,}000}{400{,}000 + 50{,}000}\right) = 44.4$$

48. Characteristics of the Screen-grid Tube. Figure 24 shows the mutual characteristics of a C-324 tube. The plate current changes with control-grid voltage in a manner similar to the three-electrode tube. The plate voltage effect on these curves is, however, small compared to this effect in a triode. Figure 25 shows the plate characteristics of a C-324 tube. Operation is normally in the region where the curves are nearly parallel to the voltage axis. If the curves were perfectly parallel to the voltage axis the plate resistance would be infinite. As the plate voltage is decreased below the screen-grid voltage the curves dip due to secondary emission from the plate. This part of the curves is not used in the normal operation.

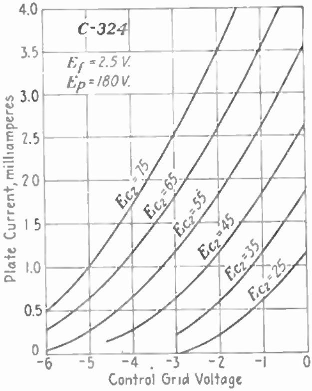


FIG. 24.—Mutual characteristic of screen-grid tube.

The dynamic characteristics of the screen-grid tube of most importance when the screen grid is effectively grounded are the amplification factor μ, the plate resistance r_p, and the mutual conductance s_m.

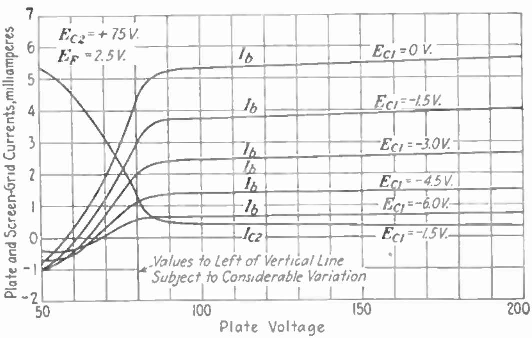


FIG. 25.—Screen-grid tube plate characteristic.

Figure 26 shows the amplification factor, plate resistance, and mutual conductance of a C-324 tube plotted against control-grid negative-bias voltage and in Fig. 27 plotted against plate voltage.

49. Low-distortion r-f Tetrode. A type of screen-grid r-f amplifier tube especially designed to permit control of the amplification by varying the voltage on the control grid is commonly known as the "variable-mu" or "super-control" type. Tubes of this type, such as the types 551, C-335, and RCA-235, are used in r-f, i-f amplifier stages and in the first detector or modulator stage of superheterodynes.

Tubes of this type differ from the type 24 screen-grid tube in the way in which the plate current is decreased as the control-grid negative-bias voltage is increased. The plate current of a type 24 screen-grid tube approaches cut-off when approximately −9.0 volts bias is applied.

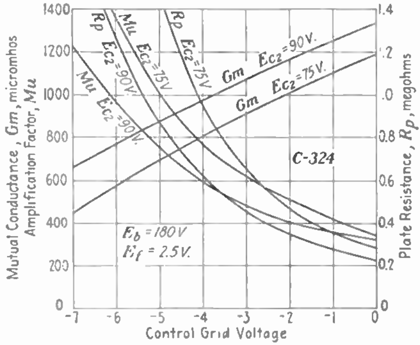


FIG. 26.—Characteristics of a-c screen-grid tube.

With the same plate voltage (250) and screen-grid voltage (90) the type 35 requires −40 volts bias to approximate plate current cut-off. At full gain both types operate with −3.0 volts bias and have a mutual conductance of 1,050 micromhos. The performance of the two types is similar at these voltages. To control volume by means of changing

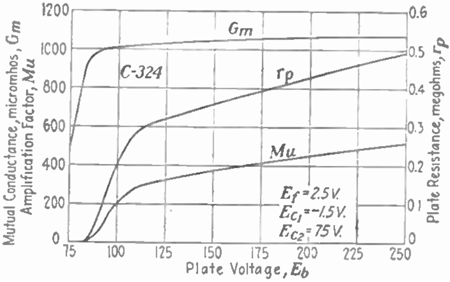


FIG. 27.—Characteristics vs. plate voltage.

the r-f amplification a tube of the type 35 is required if a large range of control is desired without distortion of the signal.

If the peaks of the signal swing beyond cut-off or the curvature of the mutual conductance characteristic is great the signal modulation will be distorted. Interfering signals effective on the grid of the first tube may also produce cross-modulation on the carrier of the desired signal when the curvature of the mutual characteristic is great. To eliminate these

Operating Voltages and Approximate Characteristics of Resistance-coupled Amplifier Circuits
Type C-324

Connection	Filament, volts	Plate supply, volts	Series plate-load resistor	Inner grid, volts	Outer grid, volts	Plate current, milliamperes	G_m	Plate resistance, megohms	Mu	Voltage amplification
Screen grid	2.5	180	100,000	− 1.0	+25	0.5	500	2.0	1,000	47
Screen grid	2.5	250	200,000	− 1.0	+25	0.5	500	2.0	1,000	91
Screen-grid	2.5	300	100,000	− 1.5	+45	1.3	750	1.5	1,100	68
Space-charge grid	2.5	180	100,000	+15	− 0.75	0.3	800	0.3	230	57
Space-charge grid	2.5	180	250,000	+15	− 0.75	0.2	700	0.3	230	104
Space-charge grid	2.5	250	250,000	+15	− 0.75	0.4	1,000	0.2	230	128

The grid-circuit d-c resistance should be kept as small as practicable to avoid loss of bias when a small gas current flows in the grid circuit. Such an effect is usually cumulative when the bias voltage is fixed but will be partially compensated if the tube is biased by the plate current through a resistor.

The use of a 0.25-megohm grid resistor and a 0.1-μf coupling condenser should be satisfactory.

The space-charge grid current is 12.0 ma.

effects which are most troublesome at low values of mutual conductance or near plate current cut-off the characteristics of the 35-type tube have no abrupt curvature and the approach to cut-off is gradual. The mutual conductance varies from 1,050 to 15 micromhos as the control grid bias is varied between −3.0 and −40 volts. A signal of 10 volts peak will not swing beyond cut-off at the 15-micromho condition. The range of control is 70 to 1 per stage. In two stages the range is 4,900 to 1. The use of the type 35 tube in one r-f, one i-f and a small control in the first detector of a superheterodyne receiver permits a range of control greater than 10,000 to 1.

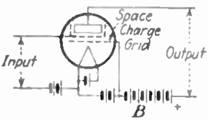


FIG. 28.—Space-charge grid connection.

The type 24 tube has a useful range of control of about 5 to 1. A greater range causes distortion. This tube is superior to the type 35 as a detector. The sharply rising characteristic of the 24 tube is essential to good detector sensitivity and fidelity.

50. Space-charge Grid Tetrode. A tube with two grids connected as a space-charge-grid tube, has the inner grid connected to a small positive voltage to reduce the space charge surrounding the cathode. The outer grid is used as the control electrode. The tube performs exactly as a triode except that the mutual conductance is higher due to the reduction in space charge. The space-charge grid is effective in increasing the mutual conductance from three to four times the value

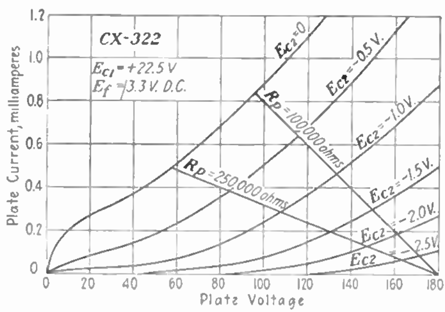


FIG. 29.—Characteristics of space-charge-grid tube.

obtained in the same tube without the space-charge grid. Figure 28 shows the schematic circuit of a space-charge grid tetrode. When a screen-grid tube is used for this purpose the screen-grid electrode becomes the control electrode. Figure 29 shows the plate current-plate voltage curves of the CX-322 tube connected as a space-charge-grid tube. Figure 30 shows the plate current and space-charge-grid current versus space-charge-grid voltage for the CX-322 tube.

51. Five-electrode Tubes. The five-electrode tube is known as a *pentode*. Tubes of this type have a filament or cathode, an inner or first grid, a second grid, and an outer or third grid. The plate or anode surrounds the outer grid.

The five-electrode tube like the tetrode is usually connected either as a screen-grid amplifier or as a space-charge-grid amplifier. The space-charge-grid amplifier is a screen-grid circuit with a space-charge grid.

Power output pentodes such as C-347 and C-333 are designed to operate as screen-grid tubes. In these tubes the control grid (No. 1 grid)

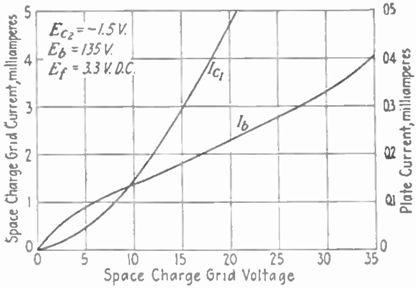


Fig. 30.—Characteristics of space-charge-grid tube as function of grid voltage.

surrounds the filament. A negative bias voltage and the signal input voltage are connected to the control grid. The screen grid (No. 2 grid) surrounds the control grid. A positive voltage, which for a pentode may equal the plate voltage, is connected to the screen grid. The screen grid is effectively grounded to the cathode for a.c. Grid 3 which is known as the *suppressor grid* is located between the screen grid and the plate. It is connected to the filament inside of the tube. The purpose of this grid is to suppress secondary emission effects.

Secondary emission electrons occur at the plate of a tube due to the impact of the primary electrons from the cathode. The secondary electrons are accelerated toward the point of highest positive potential. In a triode they are returned to the plate. In a tetrode the secondary electrons are drawn to the screen grid when the plate voltage is below the screen-grid voltage. This is the reason for the "dynatron kink" in the plate current-plate voltage curves of a tetrode screen-grid tube.

To operate a screen-grid type tube as a power amplifier requires a moderately high screen-grid voltage. The use of a third grid between the plate and screen grid is effective in preventing the secondary emission of the plate from being attracted to the screen grid. The secondary electrons are returned to the plate at all plate voltages. For this reason a relatively low plate voltage, usually equal to the screen-grid voltage, can be used. Almost the entire range of the plate voltage-plate current characteristics of the tube can be utilized to produce power output.

The space-charge grid pentode has the inner grid (No. 1 grid) connected to a small positive voltage to reduce the negative space charge surrounding the cathode. The No. 2 grid is the control grid. It is connected to a negative bias voltage and to the signal input voltage. The third grid is the screen grid located between control grid and plate. The screen grid is connected to a positive voltage and is effectively grounded to the cathode for a.c. The plate is next to the screen grid. When the tube is used as an r-f amplifier, the screen grid is extended over the outside of the plate to reduce the capacity between control grid and plate.

52. Power-output Pentode. The output pentode is a type of screen-grid tube designed for relatively large power output. The schematic circuit for this tube is shown in Fig. 31. The control grid is nearest the filament, the screen grid next, and the suppressor grid is between screen grid and plate. For maximum output the screen grid and plate are operated at the maximum rated voltage, the same voltage being applied to screen grid and plate. The control grid is biased with a negative voltage and operated similar to other types of amplifier tubes. The suppressor grid does not enter into the external circuit. It is connected to the center of the filament inside the tube. The socket connections are shown in Fig. 32.

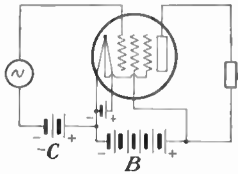


FIG. 31.—Connections of pentode for power-output tube.

The plate-characteristic curves of a type C-347 pentode power-output tube are shown in Fig. 33. The curves are similar to those obtained with other types of screen-grid tubes, except for the elimination of the kink at low plate voltages. This tube is rated at 250 volts maximum on the plate and screen grid, 16.5 volts negative bias on the control grid, and 2.5 volts a.c. on the filament. When d.c. is used on the filament and the bias voltage is connected to the negative filament terminal the rated bias voltage should be decreased by one-half, the filament voltage making it 15.25 volts. The curves of Fig. 33 were obtained with d-c filament voltage. The intersection of the curve for 15.25 volts bias with 250 plate volts represents the operating point. A signal on the control grid

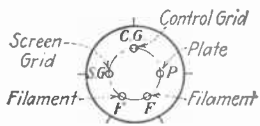


FIG. 32.—Socket connections of pentode.

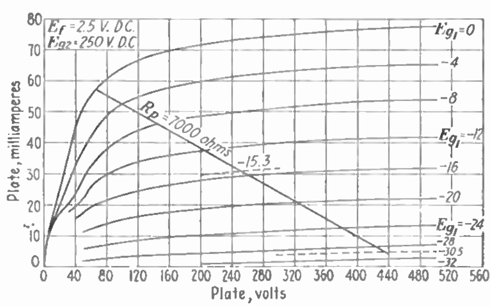


FIG. 33.—Load characteristic of C-347 pentode.

with a 15.25-volts peak would swing the control from zero to −30.5 volts. The plate current would increase to 75. ma and decrease to 3.0 ma, the effective voltage on the plate remaining constant if the load impedance in the plate circuit is zero. Minimum distortion occurs with a load impedance of 7,000 ohms. Drawing a line through the operating point representing a voltage-to-current ratio of 7,000 ohms shows that the plate current will swing from 58.5 ma to 5.2 ma and the effective plate voltage will swing from 65 volts to 434 volts.

Calculation of Power Output and Distortion. To calculate the power output and distortion draw a line on the $I_p - E_p$ characteristic curves representing the load resistance. The line is drawn through the operating point with the reciprocal slope (voltage to current ratio) equal to the resistance of the load.

A pure sine wave (or cosine wave) signal voltage is assumed to be effective on the grid. At certain values of bias voltage E_c corresponding to selected points on the signal voltage wave, the plate current is noted. With these values of plate current the power output and distortion are calculated as shown by the following example for the type C-347 tube.

$$\begin{aligned} E_c &= 0 &&= 0 \qquad & I_{\text{max.}} &= .0585 \\ E_c &= .293E &&= -4.47 & I_x &= .0527 \\ E_c &= E &&= -15.25 & I_{po} &= .0320 \\ E_c &= 1.707E &&= -26.03 & I_y &= .0107 \\ E_c &= 2E &&= -30.50 & I_{\text{min.}} &= .0052 \end{aligned}$$

Static operating point is $E_B = E_{c2} = 250$ volts, $E_{c1} = -15.25$ volts, $E_f = 2.5$ volts d.c., $I_{po} = 32.0$ ma. Load resistance = 7,000 ohms. The plate current corresponding to values of bias voltage not shown on the $I_p - E_p$ curves can be obtained by plotting a curve of the known values of I_p *vs.* E_c from which intermediate points may be read.

$$\begin{aligned} e_g &= E \cos \omega t \\ i_p &= I_0 + I_1 \cos \omega t + I_2 \cos 2\,\omega t + I_3 \cos 3\,\omega t \\ I_0 &= +\tfrac{1}{8}[I_{\text{max.}} + I_{\text{min.}} + 2(I_x + I_{po} + I_y)] \\ I_1 &= +\tfrac{1}{4}[I_{\text{max.}} - I_{\text{min.}} + \sqrt{2}(I_x - I_y)] \\ &= \tfrac{1}{4}[.0585 - .0052 + 1.414(.0527 - .0107)] = .0282 \\ I_2 &= +\tfrac{1}{4}[I_{\text{max.}} + I_{\text{min.}} - 2I_{po}] \\ &= \tfrac{1}{4}[.0585 + .0052 - 2 \times .0320] = -.00007 \\ I_3 &= +\tfrac{1}{4}[I_{\text{max.}} - I_{\text{min.}} - \sqrt{2}(I_x - I_y)] \\ &= \tfrac{1}{4}[.0585 - .0052 - 1.414(.0527 - .0107)] = -.0015 \end{aligned}$$

Power output $= \tfrac{1}{2}I_1^2R = \tfrac{1}{2}(.0282)^2 \times 7{,}000 = 2.77$ watts

Percentage second harmonic $= \dfrac{I_2}{I_1} \times 100$ per cent $= \dfrac{.00007}{.0282} \times 100$ per cent $= 0.25$ per cent

Percentage third harmonic $= \dfrac{I_3}{I_1} \times 100$ per cent $= \dfrac{.0015}{.0282} \times 100$ per cent $= 5.3$ per cent

The power output and distortion with various load impedances are shown in Fig. 34. The second harmonic distortion is a minimum near the rated 7,000-ohm. load. The harmonic distortion increases with the load. The total distortion is the vector sum of the second and third harmonics, since the magnitude of the higher-frequency components is small. The power output for minimum distortion is near the maximum obtainable.

53. Pentode Compared to Triode as Power-output Tube. Some of the advantages and disadvantages of the pentode power-output tube compared to the triode power-output tube are:

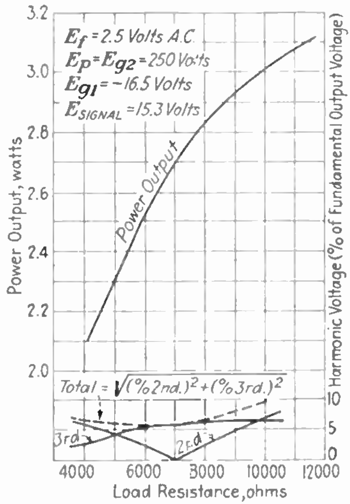


Fig. 34.

1. Higher efficiency is obtained with a pentode. A larger percentage of the *B* power can be converted into undistorted output.
2. Power sensitivity. The amplification is much higher than it is with equivalent triodes.
3. The pentode requires less *B* voltage for a given power output.
4. The load impedance must be maintained constant with the pentode for good quality. Any load impedance above twice the tube resistance gives good quality with the triode.
5. The high-frequency power output is greater with the pentode tube.
6. Better filtering is required with the pentode due to the higher amplification.

54. Suppressor R-f Pentode. Advantages of the pentode type of tube, in which secondary emission is prevented by a suppressor grid connected inside the tube to the cathode, have been applied to the screen-grid type of tube. The tube is designed for amplifiers working at radio frequencies, intermediate frequencies, or first detectors in superheterodynes.

The tube is effective in reducing cross modulation and modulation distortion over the range of received signals. The suppressor grid, situated between plate and screen grid, eliminates secondary emission which otherwise limits the voltage swing permissible in screen-grid tubes particularly if operated with low plate voltages, *i.e.*, at a plate voltage nearly equal to the screen grid.

Characteristics of RCA-239

Heater voltage, d.c.			6.3
Heater current, amperes			0.3
Plate voltage, maximum	90	135	180
Screen voltage, maximum	90	90	90
Grid voltage, variable, minimum	−3	−3	−3
Plate current, milliamperes	4.4	4.4	4.5
Screen current, milliamperes	1.3	1.2	1.2
Plate resistance, ohms	375,000	540,000	750,000
Amplification factor	360	530	750
Mutual conductance, micromhos	960	980	1,000
Mutual conductance, micromhos, at −30 volts bias	10	10	10
Mutual conductance, micromhos, at −40 volts bias	Very small but not zero		
Interelectrode capacitances, micrcfarads			
Effective grid-plate capacitance, maximum		0.007	
Input capacitance, maximum		4	
Output capacitance, maximum		10	

INTERELECTRODE CAPACITANCE

55. Tube-equivalent Network. The capacitances between the grid, plate, and filament of a triode are illustrated in Fig. 35 and also the equivalent mesh network. These are the direct interelectrode capacitances of the tube. In general, an n-electrode tube has N direct interelectrode capacitances, where

$$N = \frac{n}{2}(n - 1)$$

The direct interelectrode capacitance is the standard method of specifying the tube capacitances. It is preferred to the older methods of measurement with one electrode floating or, between one electrode and the other electrodes connected together. Either of these methods leads to results which are not independent of the particular arrangement of apparatus. The direct interelectrode capacitance is the same regardless of the type of measuring circuit. The capacitance of the socket and socket connections is not included. The tube is usually measured with the cathode cold. When the cathode is heated and voltages applied the capacitance may change a small amount.

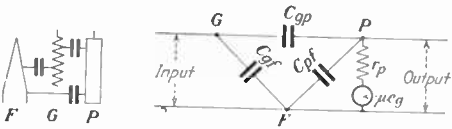


FIG. 35.—Interelectrode capacity network.

The three direct capacitances of a triode are grid-plate capacitance (C_{gp}), grid-cathode capacitance (C_{gf}), and plate-cathode capacitance (C_{pf}). The grid-plate capacitance allows energy feedback from the plate to the grid circuit having an important effect on the stability and input impedance. The grid-cathode capacitance and the plate-cathode capacitance shunt the input and output load impedances having some effect on the tuning or frequency characteristics.

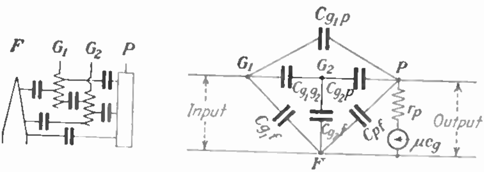


FIG. 36.—Tetrode network

The direct interelectrode capacitances of a tetrode are represented in Fig. 36. The six direct capacitances form a three-mesh network. When the tetrode is connected as a screen-grid tube the screen grid G_2 is effectively grounded. The three-mesh network is reduced to an equivalent single-mesh triode network. The screen-grid cathode capacitance (C_{g2f}) is effectively short-circuited by a large by-pass condenser. The control-grid to screen-grid capacitance (C_{g1g2}) is in parallel with the control-grid to cathode capacitance (C_{g1f}). The screen-grid to plate capacitance (C_{g2p}) is in parallel with the plate-to-cathode capacitance (C_{pf}). The equivalent network is shown in Fig. 37.

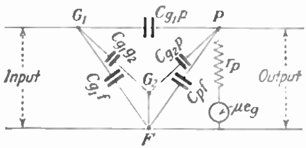


Fig. 37.—Equivalent network of screen-grid tube.

The capacitances of a screen-grid tube are usually stated as the maximum grid-plate capacitance (C_{g1p}), the average input capacitance ($C_{g1f} + C_{g1g2}$), and the average output capacitance ($C_{pf} + C_{g2p}$).

56. Measurement of Interelectrode Capacitance. The direct interelectrode capacitance can be measured with the bridge circuit of Fig. 38. The electrodes to be measured are connected to terminals AB. The remaining electrodes and any shields are connected to the ground terminal G.

When the bridge is balanced the capacitance is

$$C_{AB} = C_{gp} = \frac{R_1C}{R_2}$$

The resistance R corrects the phase and balances the effect of the capacitance across R_2.

Any leakage resistance R_{AB} across C_{AB} will cause an error. If the leakage resistance R_{AB} is known, the capacitance C_{AB} is given by the relation

$$C_{AB} = \frac{R_1C}{R_2} \cdot \sqrt{1 - \frac{1}{\omega^2\left(\frac{R_1C}{R_2}\right)^2 R^2_{AB}}}$$

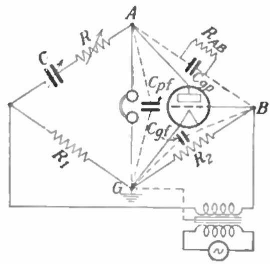


Fig. 38.—Measurement of tube capacities.

For example if (R_1C/R_2) = 5.0 $\mu\mu$f, the frequency is 1,000 cycles, and R_{AB} is 100 megohms, the correction factor is approximately 0.95 and C_{AB} = 4.75 $\mu\mu$f.

57. Radio-frequency Method. An r-f method of measuring the direct interelectrode capacitances is shown schematically in Fig. 39. The r-f oscillator supplies sufficient voltage to cause a current through C_2 which can be measured with the thermocouple TC. The capacitance C_1 does not affect the measured current if the voltage E is held constant. The reactance of capacitance C_3 is high with respect to the low-resistance thermocouple. The indicating microammeter I has one side grounded. An r-f choke L and bypass condenser C keep r-f currents out of the meter I. When the voltage E and current I are known the capacitance C_2 is given by

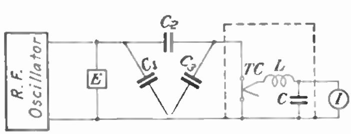


Fig. 39.—Method of measuring tube capacities.

$$C_2 = \frac{I}{\omega E}$$

If a standard variable capacitance of slightly greater range than C_2 is available, a substitution method can be used. The standard capacitance is connected across C_2. It should be enclosed in a grounded shield. The small capacitance to the shield is in parallel with C_1 and C_3.

In use the meter reading I is noted with the tube in place. The tube is then removed and the standard capacitance is increased until the same meter reading I is obtained. The difference in the two readings of the standard capacitance is the value of the tube capacitance C_2. The r-f voltage E should be constant. The absolute value of the voltage and current need not be known. A thermocouple with a filter and meter connected in series with a small capacitance across the oscillator terminals can be used as the voltage indicator.

58. Direct Interelectrode Capacitances.

Tube type		Average value, micromicrofarads		
		Grid plate	Grid cathode	Plate cathode
C-11	WD-11	3.3	2.5	2.5
CX-12	WX-12	3.3	2.5	2.5
C-299	UV-199	3.3	2.5	2.5
CX-299	UX-199	3.3	2.2	2.5
CX-322	UX-222	0.025 (max.)	3.2 (input)	12.0 (output)
CX-220	UX-120	4.1	2.0	2.3
CX-301A	UX-201A	8.1	3.1	2.2
CX-300A	UX-200A	8.5	3.2	2.0
CX-340	UX-240	8.8	3.4	1.5
CX-112A	UX-112A	8.1	4.2	2.1
CX-371A	UX-171A	7.4	3.7	2.1
CX-326	UX-226	8.1	3.5	2.2
C-327	UY-227	3.3	3.5	3.0
C-324	UY-224	0.010 (max.)	5.0 (input)	10.0 (output)
C-335	RCA-235	0.010 (max.)	5.0 (input)	10.0 (output)
CX-345	UX-245	8.0	5.0	3.0
CX-310	UX-210	8.0	5.0	4.0
CX-350	UX-250	9.0	5.0	3.0
C-336	RCA-236	0.010 (max.)	4.0 (input)	9.0 (output)
C-337	RCA-237	2.0	3.3	2.3
C-338	RCA-238	0.25	4.0 (input)	8.1 (output)
CX-330	RCA-230	6.4	3.7	2.1
CX-331	RCA-231	5.6	3.5	2.1
CX-332	RCA-232	0.020 (max.)	5.8 (input)	11.4 (output)

59. Grid-plate Capacitance of Screen-grid Tubes. The direct grid-plate capacitance of screen-grid tubes is a small fraction of a micromicrofarad. Bridge measurements are not generally satisfactory. The radio-frequency substitution method is convenient for this purpose. Figure 40 is the schematic circuit. C is a standard capacitance having a range equal to the range of capacitances to be measured. Coaxial cylinder capacitors can be constructed accurately covering an extremely small capacitance range. The thermocouple current indicator should be replaced with a sensitive indicator such as a tube rectifier or carborundum crystal. The plate of the tube should be shielded from the grid. A balancing tube T_2 of the same type as the tube T_1 being measured serves to maintain the tube input capacitance load on the oscillator. The low-capacity switch S is first thrown to the tube T_1 under test, and the reading of the meter noted. The switch is then thrown to the balance tube T_2 and the standard condenser C adjusted to give the same reading on the meter. The grid-plate capacitance is equal to the change in the standard capacitance.

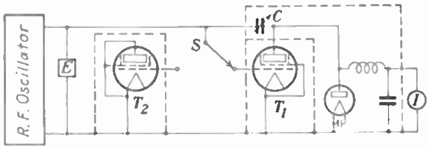


FIG. 40.—Measurement of screen-grid plate-grid capacitance.

MULTI-ELEMENT, VARIED-PURPOSE TUBES

60. Detector Automatic-control Tubes. Tubes especially designed to perform one or more of the functions of detection, automatic volume control, automatic audio-frequency cut-off control, or automatic channel-width control belong to this general class. Type 55 is a 2.5 volt a-c heater-cathode tube of this class. Type 85 is similar with a 6.3 volt d-c heater.

The type 55 and 85 tubes consist of a triode and two diodes in a single bulb. They serve as a combined detector, amplifier, and a.v.c. tube.

The two diodes and the triode are independent of one another except for the common cathode sleeve, which has one emitting surface for the diodes and another for the triode. The diodes can perform the functions of detection and automatic volume control; sensitivity control and time-delay action being confined to the a.v.c. circuit. At the same time the triode operates as an amplifier under its own optimum conditions.

61. Triple-grid Tubes. Triple-grid tubes have, in addition to the cathode and plate electrode, three grid electrodes with separate external connections. Types 57 and 58 are intended for use in amplifier and detector stages of a-c operated receivers. Type 89 is a triple-grid power-amplifier tube for use in motor-car receivers. Type 59 is a similar tube for a-c service.

Type 57 is especially recommended for use as a biased detector or as an a.v.c. tube of the biased-detector type. It is also suitable for use as a screen-grid amplifier for r-f or a-f signals of small amplitude.

The physical characteristics of particular interest are its small over-all size, the dome-top bulb, the internal shield in the dome, the rigidity of electrode assembly, and the suppressor grid with its own base terminal.

Its significant electrical characteristics are the relatively low power required by the heater—only 2.5 watts at 2.5 volts—the high value of mutual conductance and plate resistance, the sharp cut-off of the plate current with respect to the grid voltage, the satisfactory operation at 5-meters wave length, and the adaptability of electrode combinations to various circuit applications.

The pentode type of I_p-E_p curves are obtained with the type 57 tube when grid 1 (inner) is used as a control grid. Grid 2 is used as a screen grid, and grid 3 (next to plate) is used as a suppressor (connected to cathode) grid. With this type of connection, secondary emission from the plate is eliminated and the plate may be operated at any voltage within its rated maximum regardless of the screen-grid voltage.

When the suppressor grid is not connected directly to the cathode, it may be utilized in a number of ways, for obtaining modified tube characteristics or for applying the tube to special circuits. The internal shield in this tube is placed in the bulb dome above the electrode assembly and is connected within the tube directly to the cathode.

The dome-top bulb makes possible close proximity of the external and internal shields. The close spacing of the two shields produces a low effective grid-plate capacitance.

Type 58 is a triple-grid super-control amplifier recommended especially for use in the r-f and i-f stages of a-c receivers. As an amplifier it is capable of amplifying and controlling relatively large input signals with a minimum modulation distortion and cross modulation. It may be used as a frequency converter or superheterodyne first detector. Under the proper conditions of grid and local oscillator it is capable of voltage-producing gain in the first detector stage of about one-third that which can be obtained in an i-f amplifier stage. In addition, this gain can be controlled as in the case of the r-f or i-f amplifier by varying the d-c grid bias.

The physical features of types 57 and 58 are similar. The electrical features are similar except for the remote plate-current cut-off characteristic, which allows an extended mutual-conductance operating range.

The suppressor grid in this tube may be used as a quality control, since a negative bias voltage on this grid reduces the plate resistance of the tube. The change in plate resistance is effective in changing the frequency band width passed by a selective i-f or r-f amplifier.

The types 59 and 89 are triple-grid power-amplifier tubes of the heater-cathode type. The triple-grid construction of these tubes, with external connections for each grid, makes possible their application as (1) a class A power-amplifier triode; (2) a class A power-output pentode; and (3) a class B power-output triode.

A single type 89 connected as a class A triode is capable of driving two type 89 tubes connected as class B power-output tubes. A pair of type 89 tubes so connected in a class B output stage is capable of supplying a large amount of power with relatively low plate voltage and with unusual over-all economy of class B power consumption. Connected as a class A power-output pentode it is capable of giving a large power output with relatively small signal voltage input.

References

CARSON, J. R.: A Theoretical Study of the Three-element Vacuum Tube, *Proc. I. R. E.*, April, 1919.

DUSHMAN, S.: *Gen. Elec. Rev.*, **18**, 156, 1915.

———: Thermionic Emission, *Rev. Modern Phys.*, **2**, October, 1930.

———, and J. W. EWALD: Graphs for Calculation of Electron Emission from Tungsten, Thoriated Tungsten, Molybdenum, and Tantalum, *Gen. Elec. Rev.*, **26**, No. 3, March, 1923.

———, H. N. ROWE, J. EWALD, and C. A. KIDNER: *Phys. Rev.*, **25**, 343.

FRY T. C.: Potential Distribution between Parallel Plane Electrodes, *Phys. Rev.*, **17**, 441, 1921; **22**, 445, 1923.

GROSZKOWSKI, J.: "Les lampes á plusieurs electrodes," Étienne Chiron, Editeur, Paris, 1927.

"International Critical Tables," McGraw-Hill Book Company, Inc.

KING, R. W.: Calculation of the Constants of the Three-electrode Vacuum Tube, *Phys. Rev.*, **15**, No. 4, p. 256, 1920.

KUSUNOSE, Y.: Calculation of Characteristics and the Design of Triodes, *Proc. I. R. E.*, October, 1929.
LANGMUIR, I.: Emission from Thoriated Tungsten Filaments, *Phys. Rev.*, **22**, 357, 1923.
———: *Phys. Rev.*, **7**, 154, 302, 1916.
———: *Phys. Rev.*, **34**, 401, 1912.
———: The Effect of Space Charge and Residual Gases on Thermionic Currents in High Vacuum, *Phys. Rev.*, **2**, No. 6, 1913.
———, and KARL T. COMPTON, Electrical Discharges in Gases, *Rev. Modern Phys.*, **3**, April, 1931.
LLEWELLYN, F. B.: Operation of Thermionic Vacuum Tube Circuits, *Bell System Tech. Jour.*, **5**, July, 1926.
PETERSON, E., and H. P. EVANS: Modulation in Vacuum Tubes Used as Amplifiers, *Bell System Tech. Jour.*, **6**, July, 1927.
RICHARDSON, O. W.: "The Emission of Electricity from Hot Bodies," Longmans, Green & Co., 1921.
"Smithsonian Physical Tables," 1927, p. 403.
THOMSON, J. J.: "Conduction of Electricity through Gases," 3d Ed., Cambridge University Press, 1928.
VAN DER BIJL, H. J.: "The Thermionic Vacuum Tube," McGraw-Hill Book Company, Inc., 1920.
VODGES, F. B., and F. R. ELDER: Formulas for the Amplification Constant for Three-electrode Tubes in Which the Diameter of Grid Wires Is Large Compared to the Spacing, *Phys. Rev.*, **24**, December, 1924.
WARNER, J. C.: Some Characteristics and Applications of Four-electrode Tubes, *Proc. I. R. E.*, April, 1928.

SECTION 9

OSCILLATING CIRCUITS

By D. C. Prince[1]

1. Hartley Circuit. The simplest triode oscillating circuit to explain is the Hartley circuit shown in Fig. 1. A direct current is fed from the generator G through the line choke or inductance L_c to the anode of the triode through which it passes back through ground to the generator negative terminal. The line choke L_c keeps all oscillations out of this circuit. The blocking condenser C_b keeps all direct current out of the oscillating circuit so that only alternating currents need be considered in the latter, which is shown with full lines. The oscillating circuit consists of the plate and grid inductances L_p and L_g tuned by condenser C and the load resistance r. At the resonant frequency the oscillating circuit is equivalent to a resistance R in series with the plate. The value of R may readily be determined from L_pL_gC and r by the well-known circuit laws. The triode characteristic is usually available in the form of Fig. 2. In the circuit the grid voltage is related to the plate voltage by the ratio of L_p to L_g so that $e_g = e_p\frac{L_g}{L_p}$. For any given value of this ratio a combined tube and circuit characteristic can be drawn from Fig. 2. Such a characteristic is shown in Fig. 3. The values e_g and e_p are instantaneous voltages, e_g being measured from zero at the start of oscillations and from the average bias potential for steady-state oscillations and e_p measured from the average impressed direct voltage.

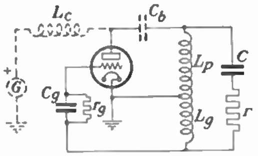


Fig. 1.—Hartley oscillator circuit.

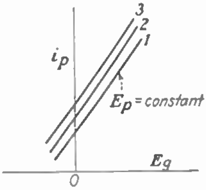


Fig. 2.—Triode characteristic.

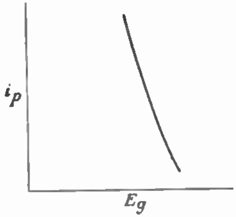


Fig. 3.—Characteristic of tube and circuit combined.

[1] Engineer, Switchgear Department, General Electric Company; fellow A.I.E.E.; fellow American Physical Society.

The instantaneous current value i_p is measured from the average plate current. That is, since the d.c. is prevented from entering the oscillating circuit it is disregarded also in the tube. It appears that positive values of i_p are accompanied by negative values of e_p and *vice versa*, whereas for loss in a resistance, positive or negative values of voltage and current always appear together. Dividing, $-e_p/i_p = -R_1$. That is, the tube is equivalent to a negative resistance of R_1 ohms, at the axis. If the equivalent load resistance is more than this, the circuit will oscillate. With any alternating voltage across the circuit higher load resistance corresponds to lower loss, that is, energy consumed is $W = E^2/R$. If, with a given alternating voltage across the circuit the energy from the tube is greater than that consumed, the circuit will oscillate. The negative resistance curve of the tube may be reversed in sign and superimposed on the output resistance characteristic. The points of intersection then determine the amplitude of oscillation.

2. Efficiency. By the foregoing process it is possible to tell whether oscillations will take place at all or not. In the limiting case where oscillations begin the efficiency of the circuit approaches zero because the energy in the oscillations is small, whereas the average direct current through the tube produces a large loss. As the amplitude of oscillations increases but with current still passing at all times through the tube the efficiency approaches 50 per cent, as shown in Fig. 4. The minimum space-charge loss is here assumed zero, the limiting case, so that for maximum current there is no drop in the tube. For simplicity the current is assumed sinusoidal.

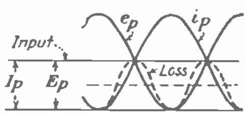


FIG. 4.—Conditions when circuit oscillates.

$$
\begin{aligned}
E_p &= E(1 + \cos\theta) \\
i_p &= I(1 - \cos\theta) \\
\text{Input} &= I \times E \\
\text{Loss} &= e_p i_p = EI[1 - \cos^2\theta]
\end{aligned}
$$

which is shown dotted in Fig. 4. This curve is symmetrical about a line halfway between EI and zero, so that the loss appears from inspection to be half the input. The output is the other half and the efficiency is 50 per cent.

3. Oscillator Circuit Design. In practical oscillating circuits designed for efficient output the current does not flow all the time, the grid being so biased that current only flows when the anode voltage is relatively low. The curves of instantaneous current and voltage then appear as in Fig. 5. The currents through a triode do not follow any simple mathematical laws even when current flows all of the time in the anode circuit. The reason for this is that the three-halves power space-charge law assumes no emission limit, the division of current between anode and grid is uncertain, depending on secondary emission and other irregular factors. For discontinuous currents such as shown in Fig. 5 a mathematical treatment giving efficiency as a function of circuit factors would be even more difficult and has not been attempted. For any given tube it is possible by the process of step-by-step integration to predict the tube and circuit behavior under varying conditions and then select the most suitable values to be incorporated in circuit design.

Practical problems usually arise in such a way that definite voltages, currents, and power are required; but it will be found impossible to tell directly what should be done with a triode to produce these results. The answer to the practical questions is obtained by first calculating all the desirable operating conditions for a tube, and then picking from curves plotted from the results of the calculations the conditions required for any particular application. This appears a rather formidable task, but several factors operate to make it easy. Comparatively few types of tubes are used in practice, and a good set of calculations on one tube will cover a multitude of applications. Moreover, for reasons of economy, a tube is always operated somewhere near the maximum voltage and current of which it is capable. Many sets of conditions are seen upon inspection to be undesirable. Of the desirable ones, it is easy to pick out those which will give substantially the best possible operation, for small changes in the operating conditions cause surprisingly small loss in efficiency. Many workers get excellent tube performance by merely arranging a circuit of material at hand and making adjustments by a little experimenting. This works very well for the smaller circuits; but it is an expensive procedure with high-powered sets, because extra condenser capacity costs heavily, and taps and end turns on inductance coils are apt to be quite a detriment to the apparatus.

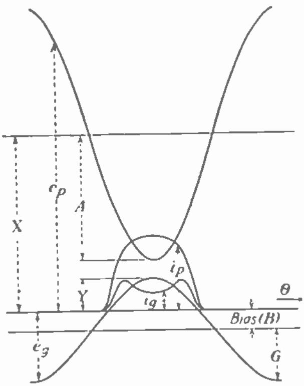


FIG. 5.—Current and voltage in highly biased oscillator.

4. Assumption of Operating Conditions. In picking sets of assumed operating conditions, the nature of the assumptions to be made is determined by the character of oscillating circuits. As long as such circuits have large amounts of energy stored in them, the voltages and currents are sinusoidal; and, even for the minimum amount of energy which they can contain and still give satisfactory operation, it will be found that the wave forms suffer comparatively little distortion. The source of energy is of constant potential, and other steady voltages can be obtained, if desired, either by batteries or by so arranging a condenser that it contains a constant charge. Current waves of any shape may be drawn from condensers. In general, inductances will be found useful only as chokes to pass steady currents while holding back the high frequency or as parts of the main oscillating circuit.

In applying these ideas more specifically, it is seen that the function of the plate-filament circuit through the tube is to connect a sinusoidal and a steady voltage source together at a time when the voltages are substantially equal. Absolute equality, however, will not answer the purpose, for there would then be nothing left to overcome the space-charge drop in the tube. At a time in the cycle of events 180 deg. removed from the passage of current, the voltage across the tube will consist of the direct voltage plus the peak value of the alternating voltage. This is shown in Fig. 5. Here the voltages are expressed with respect to the filament. The potential of the plate e_p, therefore, consists of a sine wave of amplitude A about the line representing the potential X of the supply source as an axis. During the period when the alternating voltage is not quite equal to the direct, *i.e.*, at a time when the total plate potential is lowest, current flows from the direct-voltage source into the oscillating circuit, thus supplying energy with which to overcome its losses. Figure 6 illustrates the type of circuit which would be used for this purpose.

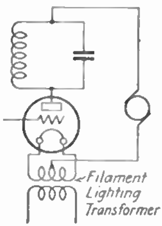


FIG. 6.—Oscillator with tuned plate circuit.

The period during which the tube is conducting is dependent upon the grid excitation. Circuits can be made in which special wave forms are applied to the grid, but this is not done in the ordinary circuit. Here, the voltage at hand is used. This normally consists of a sinusoidal voltage wave obtained either directly from the oscillating circuit, or by coupling with this circuit, and a steady voltage usually obtained from a grid-leak resistance and condenser. This is accomplished as shown in Fig. 7. The sinusoidal voltage is induced in the coil by coupling with the main oscillating circuit and is applied to the grid through a large condenser. A small current will flow to the grid, and it is assumed that the coupling to the main circuit is such that this current will meet no impedance there. The high-frequency components of the current will pass through the condenser without causing any appreciable voltage drop. However, as the current is not sinusoidal, but pulsating in one direction, it will have an average value which may be considered as d.c., and this will be forced to pass through the resistance bridging the condenser, thus creating a steady voltage across its terminals. Of course, the condenser must first charge itself up to this voltage, but this is a phenomenon which is very rarely of any interest. Figure 5 indicates the manner in which this grid voltage e_g will be related to the plate potential. The alternating voltage of peak value G is sinusoidal about an axis representing the drop B in the bias resistance. It will be observed that the alternating voltages applied to the plate and grid are 180 deg. out of phase. In order that large currents may flow in the plate circuit, it is necessary that the grid become positive during that part of the cycle in which current flows in the plate. The bias voltage allows control over the interval during which current may flow. If it were not for this voltage, current would have to flow in the plate circuit for a time corresponding to an angle of over 180 deg., or more than an entire half cycle, a condition which will lead to low efficiency.

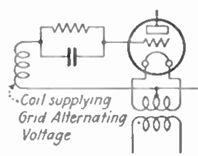


FIG. 7.—Method of exciting and biasing oscillator tube.

Knowing the plate and grid voltages, the corresponding currents through the tube are obtained from the characteristic curves. In general, these curves will have somewhat the shapes shown in Fig. 5 (i_p and i_g). The plate current will flow for a slightly longer period than that during which the grid is positive, for the positive plate potential can cause a current to flow, even though the grid be slightly negative, until the two potentials are in the ratio of the amplification constant. The depression in the center of the grid-current pulse is due to secondary emission from the grid and is not always present in the degree shown but usually is present at least as a slight flattening of a curve which would otherwise be more like the plate-current pulse.

5. Output and Efficiency of Operation. In calculating the performance of a tube, the energy delivered to the oscillating circuit is its output. In general, this circuit will have two effective resistances, one representing useful energy, and the other representing losses. The first resistance is used in calculating the output of the entire apparatus, but the second should also be included when calculating the tube performance; for it is not right to charge any device with losses dependent on another part of the apparatus.

The energy delivered to the oscillating circuit by the tube may be calculated directly by integrating the instantaneous product of plate current and oscillating circuit voltage for a complete cycle. As in many other devices of good efficiency, however, it is found desirable to calculate the input to the tube and losses connected with it and take the difference between the two for the output.

The input is obtained by multiplying the average plate current by the potential of the d-c source of energy. The losses are of two kinds: those inside the tube and those dependent upon the tube's grid requirements. The losses in the tube are obtained by integration of the products of instantaneous values of the plate and grid voltages and the corresponding currents. The only other loss to be charged against the tube is that occurring in the grid-leak resistance. This is obtained by multiplying the average grid current by the bias voltage.

Figure 8 shows the plate and grid voltages with the nomenclature to be used in calculating tube performance: X is the direct potential of the supply source (often termed direct plate voltage) and Z is the minimum instantaneous plate voltage. This gives for the instantaneous plate voltage

$$e_p = Z + (X - Z)(1 - \cos\theta)$$

in which angular displacement θ is measured from the point of minimum voltage.

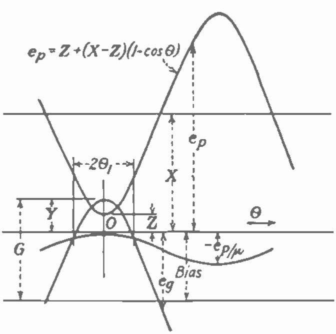


FIG. 8.—Curves for calculating tube performance.

The maximum amplitude of the alternating component of the grid potential is G and it is superimposed on the bias voltage B. The maximum positive value of the grid voltage is Y and its instantaneous value is

$$e_g = Y - G(1 - \cos\theta)$$

In making calculations of tube performance it is convenient to assume a given angle for plate-current flow and, likewise, given values for minimum plate and maximum grid voltages and then to calculate from these assumed values the required grid excitation. Figure 8 shows how the grid excitation, operating angle, and plate and grid voltages are related. When the grid voltage is equal to $-e_p/\mu$ the tube is at the point of cut-off. Letting θ_1 be the corresponding phase angle gives

$$G(1 - \cos\theta_1) = Y + \frac{e_p}{\mu}$$

or

$$G = \frac{Y + \frac{1}{\mu}\{Z + (X - Z)(1 - \cos \theta_1)\}}{1 - \cos \theta_1}$$

and, of course,

$$B = G - Y$$

6. Relation of Grid and Plate Voltages. The connection between the minimum plate and maximum grid voltages, which occur simultaneously in the middle of the current pulses, is obtained by studying the tube characteristics. The plate-voltage drop fixes the instantaneous efficiency, and at this point in the cycle the input should not be allowed to suffer any decrease for this should be the time of maximum efficiency and, therefore, of the greatest importance. For a given instantaneous plate voltage, the maximum plate current is usually obtained with a grid voltage about 90 per cent as large. For this reason, Y is usually taken about 80 per cent of Z. This relationship is due to the secondary emission phenomena. If the plate is at a higher potential than the grid, it will be the gainer by virtue of secondary emission from the grid, while if the grid has a potential high enough to collect the secondary electrons emitted by the plate, the current to the latter will suffer heavily. In different tubes the relation between maximum grid and minimum plate voltages will vary slightly, making somewhat indefinite the point at which best advantage may be taken of this characteristic; but the ratio is usually in the neighborhood of 80 per cent.

The means by which this ratio can be determined exactly may be inquired. This could be accomplished by assuming different ratios and using that which trial calculations showed to be the best. It will be found, however, that a quite appreciable change in the ratio of voltages produces very little difference in performance; in other words, since the objective is the peak of a very flat-topped curve, a good guess at the probable location of the peak instead of the actual calculation of it will give the desired results and save an amount of work which would make calculations of performance laborious almost beyond reason.

Having removed the ratio of minimum plate and maximum grid voltages from the list of variables, there remain to be assumed a series of absolute values for these voltages and, for each pair of values, a series of operating angles. This involves the calculation of about twenty-five or thirty sets of conditions. As the results of these calculations are to be assembled and displayed graphically, it is essential that they be more or less regular; for this reason, the calculations should be made according to some scheme as illustrated in Fig. 9.

7. Oscillator Calculations. In starting upon the calculations there will be found available the tube designation (catalogue number, etc.), its amplification constant, the direct plate potential, and characteristic curves giving the plate and grid currents for a wide range of impressed voltages. After studying the characteristics, it should be possible to pick a series of pairs of values for Y and Z, the maximum grid and minimum plate voltages, and for each of these a series of values of operating angles will be assumed, usually running up to $\theta_1 = 90$ deg. For each of these cases, the grid excitations will be calculated and the plate and grid currents obtained from the characteristic curves at intervals of about 10 deg. Multiplying the corresponding instantaneous currents and voltages together will give the instantaneous plate and grid losses and these will be averaged for the whole cycle. At the same time the average values of the currents should be obtained. The average value of the plate current gives the input when multiplied by the direct plate voltages and the average value of the grid current in connection with the bias voltage gives the grid-leak resistance loss and the ohms resistance required. This

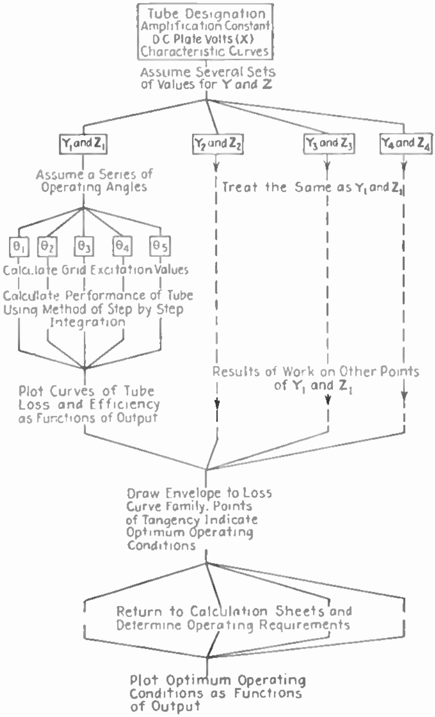


FIG. 9.—Chart for calculating oscillator circuit.

completes the data necessary to compute output, losses and efficiency. The filament heating current is not included in these figures because it is supplied from a separate source and at a much cheaper rate than the high direct-voltage power. The results of these calculations are arranged in a systematic manner so as to indicate the best possible operating conditions for any given output and these values are plotted as the correct operating conditions to be used in circuit design. An example illustrating the method employed will indicate some of the points more clearly.

8. Calculation on Basis of One-kilowatt Tube. Figure 10 shows the characteristics of a one-kilowatt tube. This tube is to be operated at

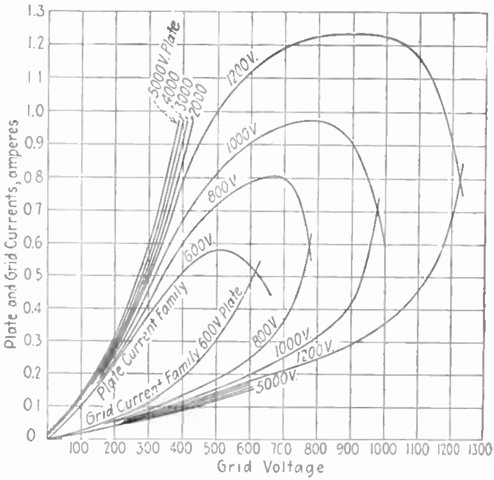


FIG. 10.—Characteristics of 1-kw tube.

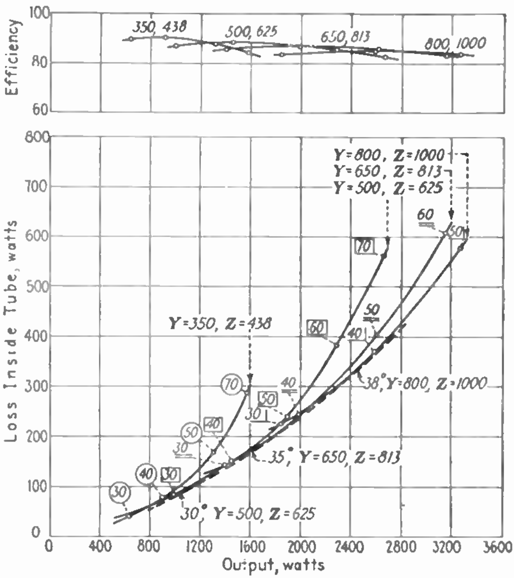


FIG. 11.—Curves of loss and efficiency.

15,000 volts. The allowable heat dissipation due to the plate and grid losses is 350 watts. The amplification constant is 250. The assumed maximum grid and minimum plate voltages with the corresponding operating angles are:

Y, volts	Z, volts	θ_1, degrees
800	1,000	30, 40, 50, 60, 70, 80
650	813	30, 40, 50, 60, 70, 80
500	625	30, 40, 50, 60, 70,
350	438	30, 40, 50, . . ., 70,

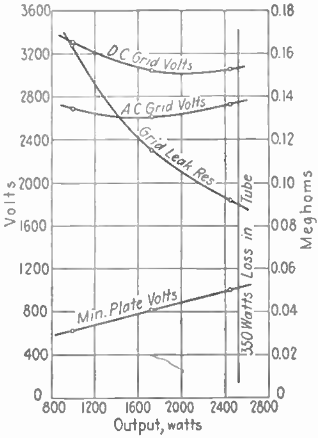


FIG. 12.—Results of oscillator calculations.

The calculations based on these assumptions have been indicated on page 230, etc. Table I shows a form arranged for quickly and systematically attacking the work, and Table II shows some sample calculations.

The corresponding curves of tube loss and efficiency are shown in Fig. 11. The efficiency curves do not necessarily intersect at the same output values as the loss curves because the former include grid-leak loss, while the latter do not. If an envelope to the loss curves were to be drawn in, the points where any curve touched this envelope would indicate the output for which the corresponding values of maximum grid and minimum plate volts give better operation than any other value. It is quite easy to estimate the corresponding operating angle by noting the location on the curve of the points calculated for the assumed angles. Knowing this angle the remainder of the operating conditions may be computed or estimated by inspection of the original calculated data. By this method there result the data included in Table III, which are shown plotted as curves in Fig. 12.

It will be seen that the results might have been varied quite appreciably by the least irregularity in the figures. As the figures contain the results of step-by-step integrations, it is certain that these irregularities are present. It might appear, then, that the curves of Fig. 12 in particular are meaningless. This is far from the case, however, for those factors which are subject to the greatest error are those allowing wide variation without affecting performance and the factors requiring close adjustment are given quite accurately.

9. Value of Grid-leak Resistance. The grid-leak resistance is an example of a factor in which large variations are permissible. All that is desired here is the approximate value. If the resistance be too low, the tube will adjust itself by drawing more grid current, which it can do without any appreciable effect on the other operating conditions. Grid-leak resistances which are too large are often trouble makers, but a slight error in this direction will usually cause no difficulty. In fact, different tubes are likely to vary widely from the grid-leak resistance required in theory because its value depends upon the grid current which is quite apt

TABLE I.—KEY FOR OSCILLATOR CALCULATIONS

Tube......
Amplification constant μ......
Characteristics......

D-c plate volts X......
Maximum grid volts Y......
Minimum plate volts Z......

		a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
		Plate voltage									
	θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
A	$1 - \cos\theta$	0	0.0152	0.0603	0.1340	0.2340	0.3572	0.5000	0.6580	0.8264	1.0000
B	$(X - Z)(1 - \cos\theta)$	$[(X - Z) \times$ line $A]$									
C	e_p	(Z + line B)									
D	e_p/μ	(Line $C \div \mu$)									
		Grid voltage									
	Operating angle θ_1	(Insert only the values required)									
F	$Y + e_p/\mu$	(Y + line D)									
G	$1 - \cos\theta$	0	0.0152	0.0603	0.1340	0.2340	0.3572	0.5000	0.6580	0.8264	1.0000
H	Max. swing of $e_g(G)$	(Line $F \div$ line G)									
J	A-c component of E_g	(Line $H \div \sqrt{2}$)									
K	Bias	(Line $H - Y$)									
		Calculation of an operating point									
	$\theta_1 =$	(Pick one of the values required)									
	$\theta =$	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
L	$1 - \cos\theta$	0	0.0152	0.0603	0.1340	0.2340	0.3572	0.5000	0.6580	0.8264	1.0000
M	$G(1 - \cos\theta)$	(Take value of G corresponding to θ_1)									
N	e_g	(Y − line M)									
P	i_p	(From characteristic curves)									
Q	i_g	(From characteristic curves)									
R	Plate loss	(Line $P \times$ line C)									
S	Grid loss	(Line $Q \times$ line N)									

Eq. (1): av. plate current $= \dfrac{P_a + 2P_b + 2P_c + 2P_d, \text{ etc.}}{36}$

Eq. (2): av. grid current $= \dfrac{Q_a + 2Q_b + 2Q_c + 2Q_d, \text{ etc.}}{36}$

Eq. (3): av. plate loss $= \dfrac{R_a + 2R_b + 2R_c + 2R_d, \text{ etc.}}{36}$

Eq. (4): av. grid loss $= \dfrac{S_a + 2S_b + 2S_c + 2S_d, \text{ etc.}}{36}$

Electron loss = sum of last two losses
Grid-leak loss = bias × av. grid current
Total loss (not including filament) = sum of last two
Input (not including filament) = X × av. plate current
Output = input − total loss
Efficiency (not including filament) $= \dfrac{\text{output}}{\text{input}}$

Table II.—Sample Oscillator Calculations

Tube, 1 kw.
$\mu = 250$
Characteristics (Fig. 10)

D-c plate volts, 15,000
Max. grid volts, 500
Min. plate volts, 625

Plate Voltage

θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$(1 - \cos\theta)$	0	0.0152	0.0603	0.1340	0.2340	0.3572	0.5000	0.6580	0.8264	1.000
$(X - Z)(1 - \cos\theta)$	0	219	867	1,926	3,364	5,135	7,188	9,459	11,880	14,375
e_p	625	844	1,492	2,551	3,989	5,760	7,813	10,084	12,505	15,000
e_p/μ	3	3	6	10	16	23	31	40	50	60

Grid Voltage

θ_1	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$Y + e_p/\mu$	..	......	......	510	516	523	531	540		
$1 - \cos\theta_1$	0	0.0152	0.0603	0.1340	0.2340	0.3572	0.5000	0.6580	0.8264	1.000
G	..	......	......	3,806	2,205	1,464	1,062	821		
A-c component of E_g	..	......	......	2,691	1,559	1,035	751	581		
Bias	..	......	......	3,306	1,705	964	562	321		

1st operating point: $\theta_1 = 30°$

θ	0°	10°	20°	30°	Sum of values from −30° to +30°	Average for entire cycle or 360°	Determined from Table I
$1 - \cos\theta$	0	0.0152	0.0603	0.1340			
$G(1 - \cos\theta)$	0	58	230	510			
e_g	500	442	270	−10			
i_p	0.600	0.672	0.400	0	2.774	0.0762	Eq. (1)
i_g	0.283	0.152	0.070	0	0.727	0.0202	Eq. (2)
Plate loss	375	567	597	0	2,703	75.1	Eq. (3)
Grid loss	142	67	28	0	332	9.2	Eq. (4)

Input (not including filament) = 15,000 × 0.0762 = 1,143 watts
Output = 991.9 watts (not including filament)
Efficiency = 86.8 per cent
Electron loss = 84.3 watts
Grid-leak loss = 66.8 watts
Total loss (not including filament) = 151.1 watts

to vary from tube to tube, as it usually involves secondary emission phenomena.

TABLE III.—CALCULATED OPTIMUM OPERATING CONDITIONS

Output, watts	Tube loss, watts	Y, volts	Z, volts	θ_1, degrees	A-c grid volts, r.m.s.	D-c grid volts	i_g, av.	Grid-leak resistance, ohms	A-c plate volts, r.m.s.	Ratio $\frac{\text{A.C.G.V.}}{\text{A.C.P.V.}}$
992	84.3	500	625	30	2,691	3,306	0.0202	164,000	10,170	0.265
1,720	192.	650	813	35	2,610	3,040	0.0265	115,000	10,040	0.260
2,440	333.	800	1,000	38	2,720	3,050	0.0332	92,000	9,910	0.275

On the other hand, the ratio of the alternating components of the grid and plate voltages will not permit of much variation without a sacrifice in performance. Fortunately, the irregularities in the calculations have very little effect on this factor.

10. Design of the Simpler Vacuum-tube Circuits. It has been shown that a vacuum-tube oscillating circuit must contain a certain amount of stored energy to maintain oscillations. The energy leaves the circuit as output at a different instantaneous rate than it enters as input and, consequently, some flywheel effect is necessary.

Consider a circuit having an inductance L, a capacity C, and oscillating at a frequency f and voltage V with W watts loss. At a point in the cycle when the current is zero, the instantaneous voltage will be $\sqrt{2}V$; all the energy is stored in the condenser; and its amount is CV^2. The r-m-s current in this circuit is $2\pi fCV$, and the joules lost per cycle are W/f. Dividing energy stored by joules lost per cycle gives

$$\frac{\text{Energy stored}}{\text{Energy lost per cycle}} = \frac{CV^2}{W/f} = \frac{fCV^2}{W} = \frac{2\pi fCV^2}{2\pi W} = \frac{VI}{2\pi W}$$

which shows that the ratio of energy stored to energy dissipated per cycle is $1/2\pi$ times the ratio of reactive power to watts. Circuits having less than twice as much energy stored in them as they dissipate per cycle have been found by actual test to have a tendency to erratic operation. Circuits having a greater relative amount of stored energy give excellent operation but require more condenser capacity and more expensive inductance coils, and the useless power losses in the oscillating circuit are higher. Unless, therefore, there is some other determining factor, circulating volt-amperes should approximate 4π times the total power output in watts as a desirable minimum value. Another way of saying this is that the power factor should not be over 8 per cent or, in the older radio nomenclature, that the decrement should not exceed 0.25. This circulating energy has the same effect as the flywheel of a single-acting engine and must be coupled closely to the plate; *i.e.*, the circuit must be so arranged that the sinusoidal voltage wave of the oscillating circuit is available at the tube terminals without distortion due to the current passing between the two. As a mechanical analogue, consider the value of a flywheel driven through a spring—an oscillating circuit connected to a vacuum tube through a loose coupling of any sort would be equally ineffective.

After assurance is obtained that the circulating energy is adequate, the design of an oscillating circuit becomes a matter of obtaining the proper voltages and phase relations for the leads to the tube. Two general types of circuit are in common use and in either of these the production of the desired values is quite simple.

11. Design of a Hartley Circuit. Figure 13 shows a form of the Hartley circuit. The oscillating current is made to flow through two inductances or one inductance with a tap. The voltage drops across the two windings are used as the alternating components of the plate and grid voltages. Energy is supplied at constant potential from the blocking condenser C_2. This condenser supplies the pulses of current drawn through the tube at the frequency of the oscillating circuit. The charge is replaced at a more uniform rate from the direct-current source G through the choke L_c. The oscillating circuit consists of the plate inductance L_p, the grid inductance L_g, the condenser C_1, and the load resistance r_L. It will be noticed that the triode, the oscillating circuit and the immediate source of energy C_2 are connected in series in a manner similar to that already described. The impedance of the load resistance will be quite small compared with the other impedances in the oscillating circuit, and its effect on the voltage distribution may be neglected in an approximate calculation. The function of the grid-leak resistance r_g and bias condenser C_3 is to maintain a bias for the grid voltage. The condenser has sufficient capacity to pass the current pulses through to the grid without distorting the voltage wave, but the average current thus passed is discharged at a uniform rate through the resistance, thereby providing a steady bias voltage.

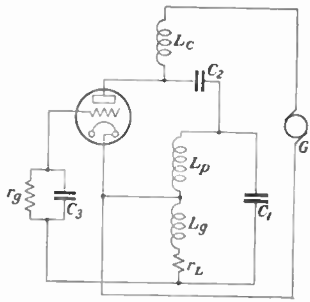


FIG. 13.—Form of Hartley circuit.

The first step in design is to determine the volt-amperes in the oscillating circuit, which value should be at least 4π times the watts output. The voltages across L_p and L_g are obtained from the curves giving optimum operating conditions for the tube and this fixes the current and impedance of the various parts of the oscillating circuit. The value of the grid-leak resistance is also obtained from the optimum operating conditions curves. This leaves only the plate choke L_c, blocking condenser C_2 and grid-leak condenser C_3 to be determined. In general, if the choke has enough inductance so that it passes only a very small current and the condensers have sufficient capacity to present no appreciable impedance to the currents which they carry, the circuit will operate satisfactorily. There are certain refinements to be considered if the best design practice is to be followed, but consideration of these will be deferred.

12. Design of Colpitts Oscillator. Figure 14 shows a form of the Colpitts circuit which differs from the Hartley circuit in that the plate and grid alternating voltages are obtained by taking the voltage drop across two condensers instead of across two inductances. The oscillating circuit, therefore, consists of the plate condenser C_p, the grid condenser

C_g, the inductance L_1, and effective resistance r_L. A plate choke L_c and blocking condenser C_B are used as in the Hartley circuit. The capacity C_g serves also as the grid-leak condenser. In order to prevent a loss in the grid-leak resistance r_g, due to the alternating voltage across the condenser, the former has in series with it a substantial choke coil L_2. The Colpitts circuit offers practically the same design problems as the Hartley circuit and will therefore not be considered separately.

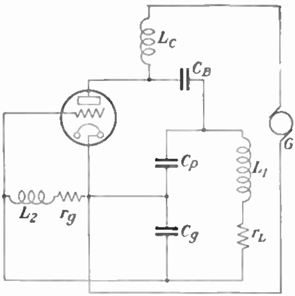


FIG. 14.—Colpitts oscillator.

13. Design Calculations. An example will serve to illustrate the methods of calculation just described. Assume a Hartley circuit, as in Fig. 13, driven by a one-kilowatt triode operating at 15,000 volts direct potential. The curves of optimum operating conditions are given in Fig. 12. The output is to be one kilowatt at 10^6 cycles.

For an output of one kilowatt, the circulating volt-amperes immediately available in the oscillating circuit will be 4π, or about 12.5 kva.

From Fig. 12 the minimum instantaneous plate volts should be 630. Deducting this from 15,000 volts leaves 14,370 for the peak value of the plate alternating voltage, which corresponds to 10,160 volts r-m-s. Figure 12 gives also the grid alternating voltage, which is 2,690 volts in this case. Adding the plate and grid voltages together gives 12,850 volts as the total a-c drop across the oscillating circuit.

TABLE IV

Element of circuit	Volts	Amperes	Impedance	Value at 10^6 cycles
Plate coil L_p	10,160	1.23	8,270	1.32 mh
Grid coil L_g	2,690	1.23	2,185	0.348 mh
Tuning condenser C_1	12,850	1.23	10,450	0.000,152 μf

To be conservative, it will be assumed that only that part of the energy stored in the oscillating circuit which corresponds to the volt-amperes in the plate coil is coupled closely enough with the vacuum tube to be useful as flywheel effect. This means that the plate coil must carry a current corresponding to the 12,500 volt-amp. at 10,160 volts, or 1.23 amp., and this is the circulating current of the oscillating circuit. Hence, there results the accumulation of the data in Table IV.

The grid-leak resistance should be about 165,000 ohms, from Fig. 12, and a condenser of a few thousandths of a microfarad capacity representing an impedance of less than a hundred ohms should operate nicely as a grid-leak condenser C_3. A condenser of similar capacity, but insulated for a higher voltage, should be satisfactory as a plate-blocking condenser C_2. It is very desirable to keep the high-frequency current out of the supply source because of the damage it can do there. As far as the r. f. is concerned, the choke coil L_c is in parallel with the plate coil L_p, for there will be enough capacity in the leads and windings of the generator G to make sure that it will present very little impedance to high frequency. Hence, if the choke coil has an inductance of several hundred or a thousand times that of the plate coil the operation will probably be altogether satisfactory. As a practical precaution, however, it is advisable to shunt G with a condenser to by-pass any high-frequency current reaching its terminals.

14. Effect of Load Resistance. A point of the greatest importance, which is sometimes overlooked, is the fact that the effective resistance of the oscillating circuit is absolutely fixed. Since the voltages in the circuit permit only slight variation without greatly disturbing the efficiency, it is necessary that the load absorb the correct power at the current fixed by this condition. In the case in hand, the current is 1.23 amp. and the output 1 kw, which means the effective resistance of the oscillating circuit must be 660 ohms. If the load itself does not have

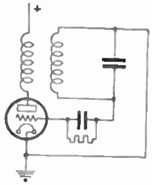


FIG. 15.—Grid-circuit oscillator.

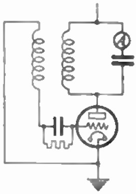


FIG. 16.—Plate-circuit oscillator.

this actual resistance, it must be coupled into the oscillating circuit in such a way that it presents that effective resistance. Considerations which follow, dealing with more complicated circuits, will illustrate some of the ways in which this can be accomplished.

The two simple circuits of Figs. 15 and 16 are calculated as in the foregoing cases but require the calculation of mutual inductance in determining the anode voltage in Fig. 15 and the grid voltage in Fig. 16.

15. Grid Phase-angle Corrections. In dealing with the design of oscillating circuits the effects of the load resistance, the plate choke, and the blocking condenser were described as though only their primary functions were fulfilled and, this accomplished, they exerted no other influence on the circuit. All three parts of the circuit can, however, affect its operation; and, though the changes produced are usually small, it is satisfying to know just what may be expected. In general, the effects consist of the introduction of small voltages or currents into the simple scheme of things first described. This results in a small change in magnitude of practically all of the various quantities, combined with a slight change in the phase relations.

To simplify the discussion, only the fundamental frequency component of the current supplied to the oscillating circuit through the blocking condenser will be considered. The harmonics are relatively much less important, and their omission will entail no serious error. The value of the fundamental component can be obtained by dividing the watts input to the oscillating circuit by the voltage between the leads to the plate and filament of the tube.

Figure 17 illustrates the effect of the load resistance in a Hartley circuit. The oscillating circuit itself $AOBC$ is drawn in such a way that the angles between various parts of the circuit correspond to the electrical phase differences of the voltages across them. Thus the grid coil OB and the condenser AC are in geometrically parallel sections of circuit, for they carry the same current, and both are pure reactances. The load resistance is drawn at right angles to them for similar reasons. Part of the load resistance might be located in the plate coil section AO, but this

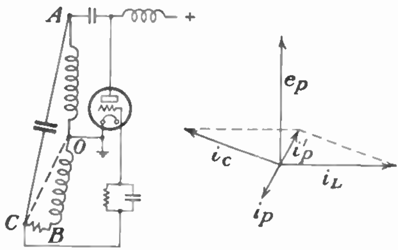


FIG. 17.—Effect of load resistance in Hartley circuit.

would not affect the diagram, so far as the object in hand is concerned, which is to show the relation between plate and grid voltages. It will be observed that the grid current has been neglected. This is not the case with the plate current, which may be represented by a sinusoidal current flowing between the leads to the circuit at A and O. For this reason the plate coil in Fig. 17 is not drawn parallel with the grid coil, but instead there is a phase difference between their voltages. For the

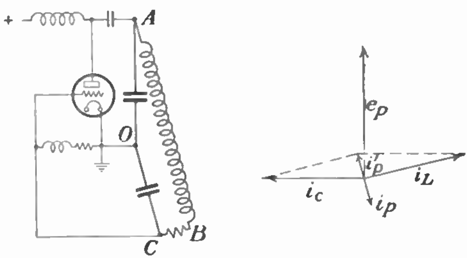


FIG. 18.—Colpitts circuit.

present it will be assumed that there is no drop across the blocking condenser and no current passed by the plate choke. Hence, OA will be the alternating component of the plate voltage.

The grid voltage will be represented by the dotted line OC in Fig. 17 and the plate current will be in phase with it. Thus, in the vector diagram the plate current i_p is drawn parallel with OC. The currents through the condenser and plate coil are 90 deg. out of phase with the corresponding voltages and are represented by i_c and i_L perpendicular to AC and OA respectively. Adding the two vectorially results in i_p'

which must be equal and opposite to the alternating component of the plate current i_p. It will be seen that i_p' must always lag behind e_p, or, in other words, the oscillating circuit absorbs power not as a perfect resistance but as a resistance in connection with an inductance. The circuit itself produces this effect by running slightly below the resonant frequency, thus drawing the lagging component of current by increasing the current through the inductance and decreasing the current through the capacity. The deviation of the frequency from that corresponding to the natural frequency of oscillation can be calculated by applying numerical values to the vectors.

The corresponding phenomena in a Colpitts circuit are shown in Fig. 18. In this case i_p' leads e_p by an angle which requires that the oscillating circuit operate slightly above its natural frequency in order to produce

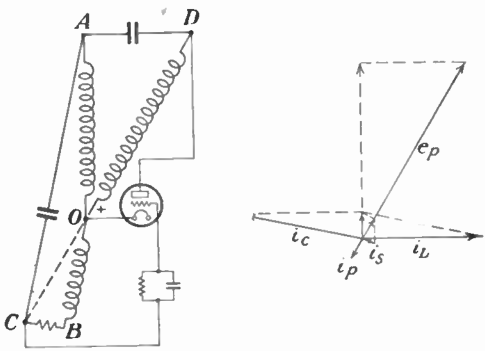


Fig. 19.—Effect of plate choke and blocking condenser on Hartley circuit.

this power factor. This self-adjustment of the simpler types of circuit is a very valuable property under many conditions, as it automatically insures against serious losses due to the grid excitation being out of phase.

16. Effect of Imperfect Chokes and Condensers. Figure 19 indicates the effect of the plate choke and the blocking condenser upon the operation of the circuit. This is the same Hartley circuit as shown in Fig. 17, but the blocking condenser and plate choke are no longer considered to be perfect in operation. The alternating component of the plate voltage is, therefore, no longer represented by OA but by a vector OD displaced from OA by the addition of AD, the drop in the blocking condenser. The plate choke may be considered to be grounded on the side next to the high voltage generator as far as the high frequency is concerned, so this choke appears as an inductance connected between O and D.

By choosing a blocking condenser of the correct impedance, it is possible to bring the plate voltage OD exactly 180 deg. out of phase with the grid voltage, the conditions to be desired for efficient operation. This will result in the oscillating circuit plus the blocking condenser drawing a load with a leading current component. This component may then be neutralized by arranging the choke so that it will draw a lagging current of equal magnitude. In the vector diagram of Fig. 19 the current through AC is represented by i_c and that through AO by i_L. These combine to form a short vertical current vector to which is added the choke current i_s to form a total current equal and opposite to i_p supplied by the tube. The voltage across OA has been taken to be vertical and, if the sum of i_c and i_L is vertical, the drop which this current causes in the blocking condenser must be represented by a horizontal line. This, when added to that representing the voltage across OA, results in the alternating component of the plate voltage e_p. This voltage can, by this means, be made opposite to i_p and e_g, *i.e.*, by control of the drop in the blocking condenser.

To make a complete calculation of a circuit including the points just discussed, the circuit $OBCA$ is treated as though the choke and blocking condenser were not present. This circuit may be equivalent either to a pure resistance between A and O (as shown in the vector diagram) or the equivalent load may contain some reactance. Let it be so proportioned that it will be equivalent to a resistance. The angle COA and its supplement AOD can then be calculated. The equivalent resistance between A and O is known since it represents a given load at a given voltage. The capacity reactance AD can, therefore, be selected so that $AD/OA = \tan (180° - AOC)$.

The alternating voltage is impressed at D and the circuit DAO will draw some leading current, the amount being easily ascertainable. It is then only necessary to choose the choke DO of such a value that it will draw the same amount of lagging current.

17. Absolute Values of Choke and Condensers. The procedure in arriving at the proper values of blocking condenser and line choke will be clearer if the solution of a numerical example is carried through. The first item to be determined is the ratio of the inductances on which the alternating components of the plate and grid voltage depend. In Fig. 19 the plate voltage is represented by OD and the drop across the inductance by OA, and it will be noticed that the two may be considered equal in magnitude unless the circuit is far from normal in design. The effect of the resistance in the grid inductance on the magnitude of the grid voltage can also be neglected. The two inductances will then be in the same ratio as the alternating components of plate and grid potentials E_p and E_g respectively, as determined from the data on optimum operating conditions.

Assume that this gives

$$\frac{E_p}{E_g} = \frac{OA}{OC} = 4$$

Let OA = 100 ohms inductance with 5 ohms resistance
OB = 25 ohms inductive reactance
BC = 2.5 ohms resistance
CA = 125 ohms capacity reactance

Then angle $OAC = \sin^{-1} \dfrac{2.5}{100}$ = 1 deg. 27 min.

If 100 volts be impressed across OA,

$$\begin{aligned} i_L &= 1, \text{ watts in } OA = (i_L)^2 \times 5 = 5.0 \\ i_c &= 1, \text{ watts in } BC = (i_c)^2 \times 2.5 = 2.5 \\ &\text{Total watts} = 7.5 \end{aligned}$$

Equivalent resistance $= \frac{E^2}{W} = \frac{100^2}{7.5} = 1{,}333$ ohms. It will be arranged to have this circuit operate at the resonant points so that its reactance between O and A is zero.

In the triangle AOC

$$\frac{OA}{\sin ACO} = \frac{OC}{\sin OAC} = \frac{AC}{\sin AOC}$$

$$\frac{100}{\sin ACO} = \frac{25}{0.025}, \quad \sin ACO = 0.1$$

Angle $ACO = 5°$ 45 min.
Angle AOD = angle OAC + angle ACO = 7 deg. 12 min.
AD = impedance between A and $O \times \tan AOD$
$= 1{,}333 \times 0.126 = 168$ ohms capacitance.

The total impedance between O and D through A is 1,343 ohms, so that the ratio between the plate and grid voltages suffers no appreciable change due to the presence of the plate-blocking condenser. If 100 volts are impressed between D and O, the current is

$$\frac{100}{1{,}343} = 0.0745 \text{ amp.}$$

and the wattless volt-ampere component is, then,

$$0.0745^2 \times 168 = 0.933 \text{ volt amp.}$$

In order to correct for these leading volt amperes, the choke should draw the same amount lagging, thus giving

$$DO = \frac{100^2}{0.933} = 10{,}700 \text{ ohms}$$

If the impedance of the plate-blocking condenser had been high, the desired ratio between grid and plate voltages would not have been obtained. It would then have been necessary to assume an initial value somewhat larger than desired for the final result, proceeding by a series of approximations. However, for other reasons, it is not likely that a high-impedance blocking condenser would be desirable. A high impedance blocking condenser corresponds to a low impedance choke which would allow radio frequency currents to flow in circuits with high effective resistance and thus, possibly damage power generating apparatus.

18. To Secure Proper Phase Relations. Proportioning the plate-blocking condenser and line choke is not the only method of bringing plate voltage and current into the 180-deg. phase relation. The angle OAC may be compensated for by de-phasing the grid in such a way as to cause the oscillations to occur at the natural resonant frequency and the plate current and voltage to be properly related.

In Fig. 20 diagrams of two methods are shown by which a Hartley circuit may be restored to operation at the natural resonant frequency of the circuit with proper phase relations, while at the right are two equivalent methods for the Colpitts circuit. In the first diagram the phasing is accomplished by connecting resistance CG and inductance GO in series. The grid is attached at C. The same result is accomplished in the second drawing by a resistance from O to G and a condenser from G to C. The elements are reversed in the third and fourth to produce lag instead of lead.

Although grid phasing may correct the various angles, it is probable that adjustments of the choke and the blocking condenser are to be preferred since these devices are normally present and so do not constitute added complication.

19. Grid-bias Condenser. It will be noted that no criteria have yet been developed governing the choice of a grid-blocking condenser. Since the function of this condenser is to pass the alternating current component of grid excitation without the occasion of serious voltage drop while forcing the direct component to flow through the grid leak or biasing resistance, its value is not critical. Its value should be large enough to make the grid-plate capacity small by comparison. The

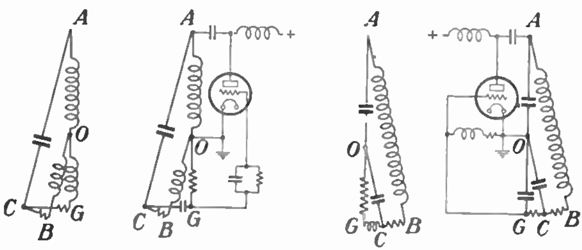


FIG. 20.—Hartley (left) and Colpitts (right) circuits.

values of tube capacity are normally so small that this requirement causes no concern. The individual pulse of direct current should cause no considerable change in the bias, but this requirement also causes small concern. Too large a value of condenser will produce intermittant oscillations because of the time required to charge and discharge.

20. Parasitic Oscillations. Where oscillating circuits are intended for moderate or low frequency it often happens that oscillations may occur at some higher frequency corresponding to the natural period of some inductance in the circuit. Such parasitic oscillations are usually inefficient and cause excessive heating of the plate at high voltages. The circuit must then be rearranged as by putting loading capacities in the grid circuit or inserting resistance at some critical point.

21. Short-wave Circuits. As frequency is increased indefinitely, a point is reached for which the capacity between grid and plate becomes large compared with the capacity, say in a Hartley circuit, and all external capacity may then be omitted. At such frequencies the various leads may possess sufficient inductance for that part of the circuit. Further increases in frequency can then only be made with special tubes. The operating principles remain unchanged but the quantities become very hard to identify.

22. Use of Tubes in Parallel and Push Pull. In case more power is desired than one tube can furnish, it is possible to parallel two tubes. However, it is often more desirable to connect the two tubes at opposite ends of the oscillating circuit in what is called the push-pull arrangement.

The oscillator efficiencies determined in the foregoing paragraphs are all on the assumption that substantially sine waves of voltage are employed. By modifying the voltage wave in such a way as to maintain the potential drop across the tube at a minimum for a considerable time, it is possible to secure considerably increased outputs from a given pair of tubes while still maintaining high efficiency.

In Fig. 21 is shown a circuit of the push-pull type in which, by proper design, it is possible to have approximately square current waves drawn through each tube for a half cycle. While the calculations are somewhat more involved, they are not too difficult and here, as before, comparison of practice with theory has shown that performance can be estimated with a high degree of accuracy.

23. Stability. A self-excited oscillator with a battery bias is not stable for the reason that any decrease in amplitude of oscillations produces a decrease in grid excitation and therefore a decrease in power output from the oscillator tube. For this reason, if the oscillations start to

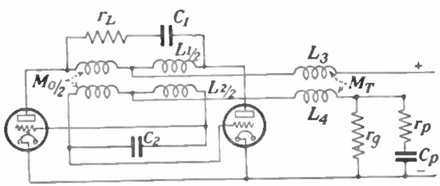


FIG. 21.—Push-pull oscillator with square current waves through each tube per half cycle.

decrease, they will continue to decrease until they stop altogether. Where the usual grid leak and condenser bias is employed, oscillations tend to be stable because in the event of a decrease in amplitude, the grid will draw less current until the condenser has discharged, thus reducing the bias and allowing more power to be drawn from the tube. If the condenser is made too large, the influences toward stability will operate too slowly and a condition of intermittent oscillations will be obtained. The circuit will oscillate violently until the condenser is charged; the oscillations will then be cut off until there has been an opportunity for discharge. Such an arrangement will give the equivalent of modulated output.

24. Magnetron Oscillating Circuits. The characteristic equation of the magnetron is

$$e' = Ki^2$$

e' = voltage required to cause electrons to reach the plate
i = a current flowing to make magnetic field
K = proportionality constant including number of turns and geometry of the oscillating coil

If the potential difference between the electrodes is larger than given by the formula, current will flow in accordance with the three-halves power space-charge law with very little modification due to the presence of the magnetic field. In order to permit current to flow once per cycle, a steady field is set up by means of polarizing coils. This field is opposed by current in the oscillating coil L which is adjusted so that at its maximum, the total field will be reduced to substantially zero during that part of the cycle that it is desired to have current flow. Let the current be

$$I \cos \theta$$

The magnetic effect $KI^2(1 - \cos \theta)^2$
That is,

$$e' = KI^2(1\tfrac{1}{2} - 2 \cos \theta + \tfrac{1}{2} \cos 2\theta)$$

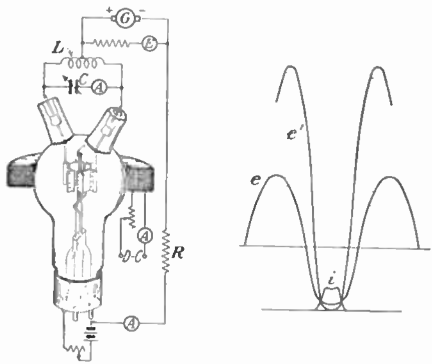


FIG. 22.—Magnetron oscillator.

The voltage required to just make current flow therefore contains a direct-current component, (the impressed anode voltage), a fundamental component, and a double-frequency component.

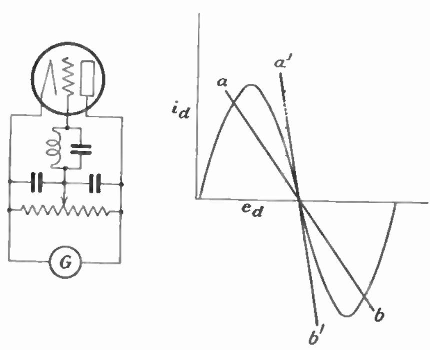


FIG. 23.—Dynatron oscillator.

The shape of the anode voltage wave e and the shape of the voltage which would be required to make current flow at each point in the cycle e' are given in Fig. 22. Where e' is greater than e, no current will flow; where e is greater than e', the current which flows is substantially that determined by the space-charge relations. Since the desired current is in phase with the voltage, the simple circuit is series tuned to give the equivalent of a non-inductive load with current in phase with the anode voltage.

25. Dynatrons. There is no accepted quantitative law for the behavior of a dynatron. Its characteristic will be of the type shown in Fig. 23 and will contain a region where increasing voltage is accompanied by decreasing current. This condition corresponds to negative resistance and should therefore be a possible source of power for oscillations. In the circuit shown, the dynode is connected by means of a potentiometer to a point substantially the middle of the negative resistance portion of the characteristic through a parallel tuned circuit. Oscillations will result at the natural frequency of the tuned circuit provided the equivalent resistance of this circuit is equal to or greater than the negative resistance represented by the slope of the characteristic curve at the point of bias potential. If the equivalent resistance represented by the oscillating circuit is greater than the negative resistance represented by the maximum slope of the characteristic curve, the oscillations will increase in amplitude until the losses are equal to the power obtained from the tube, that is, until the product of current squared times the negative resistance is equal to the product of current squared times the resistance represented by the circuit.

Referring to the characteristic curve, the line *ab* may represent the current-voltage relations through the load which have been reversed in order to find the points at which tube output will no longer be greater than circuit losses. If the resistance is lower than the negative resistance represented by the characteristic curve, the load resistance characteristic will be represented by the line *a′b′* which crosses the tube characteristic at but one point. The crossing point is therefore stable, and no oscillations will result.

SECTION 10

DETECTION AND MODULATION

BY KENNETH W. JARVIS[1]

DETECTION

1. Demodulation, usually called *detection,* is the process necessary to obtain from the incoming signal the initial modulation frequency. The instantaneous signal voltage for the detection processes discussed in this chapter, unless otherwise noted, is as follows:

Instantaneous signal voltage =

$$e = E_A(1 + m \cos Bt) \cos At \tag{1}$$

This may also be written as

$$e = E_A \cos At + mE_A \cos At \cos Bt \tag{2}$$

or as

$$e = E_A \cos At + \frac{mE_A}{2} \cos (A + B)t + \frac{mE_A}{2} \cos (A - B)t \tag{3}$$

where E_A = peak value of the carrier-frequency voltage
m = modulation factor
B = $2\pi f_B$, where f_B is the low-modulation frequency
A = $2\pi f_A$, where f_A is the carrier frequency

(1) is the expression derived as the standard type of modulation signal by customary modulation processes.
(2) represents the normal carrier voltage plus a voltage of the carrier frequency which varies at the modulation frequency.
(3) represents the sum of three individual frequencies termed the carrier and the side bands respectively, which adds to give (at least mathematically) the standard signal.

In the analysis of detection (or demodulation) all three types of equations are occasionally used. For simplicity of expression and ease and accuracy of calculation (3) is customarily employed. The problem of detection is to obtain, from the standard-type signal, currents and voltages of frequency f_B.

2. Detection Characteristic. An *asymmetrical* current voltage characteristic is necessary for detection. Figure 1 shows a standard-type signal applied to such a characteristic and the resulting current wave form. For illustration some of the more important frequency components of the current are shown in approximately correct amplitude. The subscripts refer to corresponding frequencies. Such asymmetrical characteristics may be obtained in certain mineral crystals, both natural

[1] Director of Engineering, Zenith Radio Corporation. The author's indebtedness to Marvin Hobbs for considerable mathematical work is gratefully acknowledged.

and artificial. Electronic devices, such as vacuum tubes, are more commonly used due to greater stability and sensitivity. Figure 2 shows

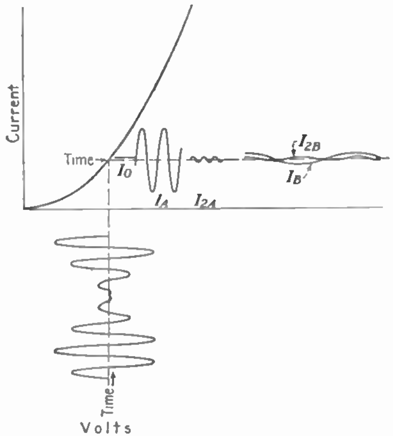


FIG. 1.—Standard detection characteristic.

such a characteristic of a crystal detector, and Figure 3 that of a vacuum tube. In the use of crystals much depends on the pressure of the metallic contact. Galena (PbS) requires a fine contact with light pressure and a sensitive operating point. Under these conditions it is a very sensitive detector. Silicon crystals require

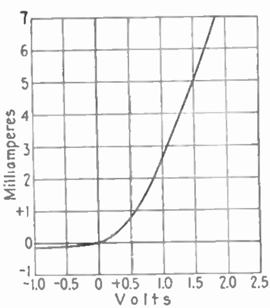


FIG. 2.—"Perikon" Crystal detector characteristic.

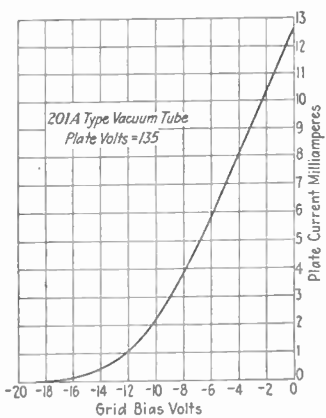


FIG. 3.—Characteristic suitable for detection.

greater contact pressure but are somewhat less sensitive. Carborundum, requiring a high-pressure contact, is less sensitive than silicon but is very stable in operation. The maximum sensitivity is obtained at the point of greatest change in curvature. With some crystals, such as carborundum, a biasing potential is necessary to bring the operating point to this maximum sensitivity.

Copper oxide rectifiers exhibit such a curve as Fig. 1 but as yet have not been used as detectors owing to the action of the rectifier capacity at high carrier frequencies.

3. Detector Equations. The equations relating to the asymmetrical characteristic of Fig. 1 are developed as follows:

$$I = f(E) \tag{4}$$

$$I = I_0 + i = P_0 + P_1 e + \frac{P_2 e^2}{2!} + \frac{P_3 e^3}{3!} + \tag{5}$$

where P_1, P_2, etc., are the first, second, etc., derivatives of I with respect to E evaluated under the circuit operating conditions and at the chosen operating point. Substituting Eq. (1) into (5) and reducing to first-order terms to determine the amplitude of the various frequency components resulting gives a series of terms representing d-c, fundamental, second, third, etc., harmonics of both carrier and modulation frequencies as well as combinations of carrier and side band of the form of

$$\begin{aligned} I = P_0 &+ \left(\frac{P_2 E_A{}^2}{4} + \frac{P_4 E_A{}^4}{64} + \cdots\right) \\ &+ \left(\frac{P_2 m^2 E_A{}^2}{8} + \frac{3P_4 m^2 E_A{}^4}{64} + \frac{3P_4 m^4 E_A{}^4}{512} \cdots\right) \\ &+ \left(P_1 E_A + \frac{P_3 E_A{}^3}{8} + \frac{3P_3 m^2 E_A{}^3}{16} + \cdots\right) \cos At \\ &+ \left(\frac{P_2 m E_A{}^2}{2} + \frac{P_4 m E_A{}^4}{16} + \frac{3P_4 m^3 E_A{}^4}{64} + \cdots\right) \cos Bt \\ &+ (\text{terms}) \cos 2At + (\text{terms}) \cos 2Bt \\ &+ (\text{terms}) \cos 3At + (\text{terms}) \cos 3Bt + \cdots \\ &+ (\text{terms}) [\cos (A + B)t + \cos (A - B)t] \end{aligned} \tag{6}$$

The coefficient of any term in (6) may be found from the double summation indicated below:

$$i_{(pA \pm qB)} = \sum_{s=0}^{s=\infty} \sum_{r=0}^{r=\frac{p-q+2s}{2}} \frac{(p + 2s)!\, P_{p+2s}\, m^{q+2r}\, E_A{}^{p+2s}}{2^{(p+2s+q+2r-n)}(p + 2s - q - 2r)!(p + s)!(q + r)!s!r!} \cos (pA \pm qB)t \tag{7}$$

When $p = q = 0$, $n = 0$. For all other values of p or q, $n = 1$. The expression (7) holds for all values of p and q including zero. The only restriction is in the term $\frac{p - q + 2s}{2}$. In case the values of p, q, and the chosen s make this term a fraction, the next integer below the fraction is used to terminate the r summation.

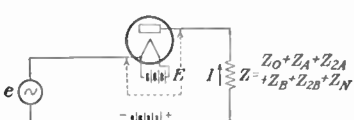


FIG. 4.—Detection circuit.

4. Detector Derivative. The series in Art. 3 is given in terms of P_n, the derivative of I with respect to E, for the entire circuit. This is often difficult to evaluate, especially when a complex wave such as the standard radio signal is applied. The derivative of the detector alone can usually be determined satisfactorily. Determining P and using the circuit constants enable the detector operation to be calculated. Figure 4 shows the circuit where e is the voltage of Eq. (3) and Z is the circuit impedance to any frequency as indicated by the subscript.

Only two terms of the power series

$$i = a_1 e_g + a_2 e_g^2 + \tag{8}$$

will be retained. For simplicity the total current i is given as the sum of the individual frequency components.

$$\begin{aligned}
i = i_0 = {} & \tfrac{1}{2}\left\{\frac{P_2}{1 + P_1 Z_0}\left[1 - \frac{Z_A P_2}{1 + P_1 Z_A} - \frac{\bar{Z}_A P_2}{(1 + P_1 \bar{Z}_A} + \frac{Z_A \bar{Z}_A P_2^2}{(1 + P_1 Z_A)(1 + P_1 \bar{Z}_A)}\right]\right\} E_A^2 \\
& + \tfrac{1}{2}\left\{\frac{P_2}{1 + P_1 Z_0}\left[1 - \frac{Z_{A+B} P_2}{1 + P_1 Z_{A+B}} - \frac{\bar{Z}_{A+B} P_2}{1 + P_1 \bar{Z}_{A+B}} + \frac{Z_{A+B} \bar{Z}_{A+B} P_2^2}{(1 + P_1 Z_{A+B})(1 + P_1 \bar{Z}_{A+B})}\right]\right\} E^2_{A+B} \\
& + \tfrac{1}{2}\left\{\frac{P_2}{1 + P_1 Z_0}\left[1 - \frac{Z_{A-B} P_2}{1 + P_1 Z_{A-B}} - \frac{\bar{Z}_{A-B} P_2}{1 + P_1 \bar{Z}_{A-B}} + \frac{Z_{A-B} \bar{Z}_{A-B} P_2^2}{(1 + P_1 Z_{A-B})(1 + P_1 \bar{Z}_{A-B})}\right]\right\} E^2_{A-B} \\
+ i_A = {} & \left(\frac{P_2}{1 + P_1 Z_A}\right) E_A \cos At \\
+ i_{2A} = {} & \tfrac{1}{2}\left\{\frac{P_2}{1 + P_1 Z_{2A}}\left[1 - \frac{2 Z_{2A} P_2}{1 + P_1 Z_{2A}} + \frac{Z^2_{2A} P_2^2}{(1 + P_1 Z_{2A})^2}\right]\right\} E_A^2 \cos 2At \\
+ i_B = {} & \left\{\frac{P_2}{1 + P_1 Z_B}\left[1 - \frac{Z_A P_2}{1 + P_1 Z_A} - \frac{\bar{Z}_{A-B} P_2}{1 + P_1 \bar{Z}_{A-B}} + \frac{Z_A \bar{Z}_{A-B} P_2^2}{(1 + P_1 Z_A)(1 + P_1 \bar{Z}_{A-B})}\right]\right\} E_A E_{A-B} \cos Bt \\
& + \left\{\frac{P_2}{1 + P_1 Z_B}\left[1 - \frac{Z_{A+B} P_2}{1 + P_1 Z_{A+B}} - \frac{\bar{Z}_A P_2}{1 + P_1 \bar{Z}_A} + \frac{Z_{A+B} \bar{Z}_A P_2^2}{(1 + P_1 Z_{A+B})(1 + P_1 \bar{Z}_A)}\right]\right\} E_A E_{A+B} \cos Bt \\
+ i_{2B} = {} & \left\{\frac{P_2}{1 + P_1 Z_{2B}}\left[1 - \frac{Z_{A+B} P_2}{1 + P_1 Z_{A+B}} - \frac{\bar{Z}_{A-B} P_2}{1 + P_1 \bar{Z}_{A-B}} + \frac{Z_{A+B} \bar{Z}_{A-B} P_2^2}{(1 + P_1 Z_{A+B})(1 + P_1 \bar{Z}_{A-B})}\right]\right\} E_{A+B} E_{A-B} \cos 2\,Bt
\end{aligned} \tag{9}$$

If the circuit conditions ahead of the detector input do not change the initial side-band ratio (due to asymmetrical amplification or side-band cutting),

$$E_{A+B} = E_{A-B} = \frac{m}{2}E_A \tag{10}$$

and if the radio frequency is by-passed so that

$$Z_A = Z_{A+b} = Z_{A-B} = 0$$

the audio currents and resulting voltages IZ are

$$\left.\begin{aligned} i_B &= \frac{P_2}{1 + P_1 Z_B} m E_A^2; \quad e_B = \frac{P_2 Z_B}{1 + P_1 Z_B} m E_A^2 \\ i_{2B} &= \frac{P_2}{1 + P_1 Z_{2B}} \frac{m^2}{4} E_A^2; \quad e_{2B} = \frac{P_2 Z_{2B}}{1 + P_1 Z_{2B}} \frac{m^2}{4} E_A^2 \end{aligned}\right\} \tag{11}$$

If good fidelity is required so that $Z_B = Z_{2B}$, the ratio between second harmonic and fundamental is

$$\text{Percentage distortion} = \frac{m}{4} \tag{12}$$

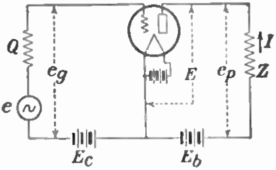


FIG. 5.—Triode as detector.

where

$$Z_A = R + jX_A$$
$$\overline{Z}_A = R - jX_A$$
$$P_1 = \frac{dI}{dE} = \frac{1}{R_p}$$
$$P_2 = \frac{d^2I}{dE^2} = \frac{dR_p}{dE} = R_p'$$

R_p = detector resistance while detecting

5. Triode as Detector. The three-element tube also may serve as a detector, and its performance in terms of the tube parameters and circuit constants developed exactly as in the previous paragraph. More complicated expressions result, for grid current and grid impedances affect the plate current as well as plate impedances. The plate voltage affects the grid current; the grid voltage affects the plate current. Figure 5 shows the circuit conditions. The plate current at any frequency may be determined by substituting the a and b coefficients given by (14) and (15) into (13).

$$i_p = \begin{cases} i_0 = \tfrac{1}{2}[(1 - b_{1(A)}Q_A)(1 - \overline{b}_{1(A)}\overline{Q}_A)a_{2(0A)} - a_{1(0A)}Q_0 b_{2(0A)}]E_A^2 \\ \quad + \tfrac{1}{2}[(1 - b_{1(A+B)}Q_{A+B})(1 - \overline{b}_{1(A+B)}\overline{Q}_{A+B})a_{2(0(A+B))} - a_{1(0(A+B))}Q_0 \\ \qquad\qquad b_{2(0(A+B))}]E^2_{A+B} \\ \quad + \tfrac{1}{2}[(1 - b_{1(A-B)}Q_{A-B})(1 - \overline{b}_{1(A-B)}\overline{Q}_{A-B})a_{2(0(A-B))} - a_{1(0(A-B))}Q_0 \\ \qquad\qquad b_{2(0(A-B))}]E^2_{A-B} \\ i_A = [a_{1(A)}(1 - b_{1(A)}Q_A)]E_A \cos At \\ i_{2A} = \tfrac{1}{2}[(1 - b_{1(A)}Q_A)^2 a_{2(2A)} - a_{1(2A)}Q_{2A}b_{2(2A)}]E_A^2 \cos 2At \\ i_B = [(1 - b_{1(A+B)}Q_{A+B})(1 - \overline{b}_{1(A)}\overline{Q}_A)a_{2(+B)} - a_{1(A+B)}Q_{(A+B)}b_{2(A+B)}] \\ \qquad\qquad E_A E_{A+B} \cos Bt \\ \quad + [(1 - b_{1(A-B)}Q_{A-B})(1 - \overline{b}_{1(A-B)}\overline{Q}_{A-B})a_{2(-B)} - a_{1(A-B)}Q_{(A-B)}b_{2(A-B)}] \\ \qquad\qquad E_A E_{A-B} \cos Bt \\ i_{2B} = [(1 - b_{1(A+B)}Q_{A+B})(1 - \overline{b}_{1(A-B)}\overline{Q}_{A-B})a_{2(2B)} - a_{1(2B)}Q_{2B}b_{2(2B)}] \\ \qquad\qquad E_{A+B}E_{A-B} \cos 2Bt \end{cases} \tag{13}$$

$$a_{1(A)} = \frac{\mu}{R_p + Z_A}; \quad a_{1(A-B)} = \frac{\mu}{R_p + Z_{A-B}}; \quad a_{1(A+B)} = \frac{\mu}{R_p + Z_{A+B}}$$

$$a_{2(B)} = \frac{\frac{1}{2}\left[-\mu^2 R_p R_p' + \mu\frac{\partial\mu}{\partial E_p}(R_p^2 - Z_A\bar{Z}_{A-B}) + \frac{\partial\mu}{\partial E_g}(R_p + Z_A)(R_p + \bar{Z}_{A-B})\right]}{(R_p + Z_A)(R_p + \bar{Z}_{A-B})(R_p + Z_B)}$$

$$a_{2(2B)} = \frac{\frac{1}{2}\left[-\mu^2 R_p R_p' + \mu\frac{\partial\mu}{\partial E_p}(R_p^2 - Z_{A+B}\bar{Z}_{A-B}) + \frac{\partial\mu}{\partial E_g}(R_p + Z_{A+B})(R_p + \bar{Z}_{A-B})\right]}{(R_p + Z_{A+B})(R_p + \bar{Z}_{A-B})(R_p + Z_{2B})} \tag{14}$$

$$b_{1(A)} = \frac{1 - \frac{\mu}{\nu}\frac{Z_A}{R_p + Z_A}}{R_g + \left(1 - \frac{\mu}{\nu}\frac{Z_A}{R_p + Z_A}\right)Q_A}$$

$$b_{2(B)} = \left\{ \frac{\begin{array}{c}\frac{1}{2}\left[-R_gR_g'\left(1 - \frac{\mu}{\nu}\frac{Z_A}{R_p + Z_A}\right)\left(1 - \frac{\mu}{\nu}\frac{\bar{Z}_{A-B}}{R_p + \bar{Z}_{A-B}}\right) - 2a_{2(B)}\frac{R_g^2}{\nu}Z_B \right.\\ - \frac{\partial}{\partial E_g}\left(\frac{1}{\nu}\right)\left(\frac{\mu Z_A R_g^2}{R_p + Z_A} + \frac{\mu\bar{Z}_{A-B}R_g^2}{R_p + \bar{Z}_{A-B}} - \frac{\mu^2 Z_A\bar{Z}_{A-B}R_g^2}{(R_p + Z_A)(R_p + \bar{Z}_{A-B})}\right)\\ \left. + \frac{\partial}{\partial E_p}\left(\frac{1}{\nu}\right)\left(\frac{\mu^2 Z_A\bar{Z}_{A-B}R_g^2}{(R_p + Z_A)(R_p + \bar{Z}_{A-B})}\right)\right]\end{array}}{\begin{array}{c}\left[R_g + \left(1 - \frac{\mu}{\nu}\frac{Z_A}{R_p + Z_A}\right)Q_A\right]\left[R_g + \left(1 - \frac{\mu}{\nu}\frac{\bar{Z}_{A-B}}{R_p + \bar{Z}_{A-B}}\right)\bar{Q}_{A-B}\right]\\ \left[R_g + \left(1 - \frac{\mu}{\nu}\frac{Z_B}{R_p + Z_B}\right)Q_B\right]\end{array}} \right.$$

$$b_{2(2B)} = \left\{ \frac{\begin{array}{c}\frac{1}{2}\left[-R_gR_g'\left(1 - \frac{\mu}{\nu}\frac{Z_{A+B}}{R_p + Z_{A+B}}\right)\left(1 - \frac{\mu}{\nu}\frac{Z_{A-B}}{R_p + \bar{Z}_{A-B}}\right) - 2a_{2(2B)}\frac{R_g^2}{\nu}Z_{2B} \right.\\ - \frac{\partial}{\partial E_g}\left(\frac{1}{\nu}\right)\left(\frac{\mu Z_{A+B}R_g^2}{R_p + Z_{A+B}} + \frac{\mu\bar{Z}_{A-B}R_g^2}{R_p + \bar{Z}_{A-B}} - \frac{\mu^2 Z_{A+B}\bar{Z}_{A-B}R_g^2}{(R_p + Z_{A+B})(R_p + \bar{Z}_{A-B})}\right)\\ \left. + \frac{\partial}{\partial E_p}\left(\frac{1}{\nu}\right)\left(\frac{\mu^2 Z_{A+B}\bar{Z}_{A-B}R_g^2}{(R_p + Z_{A+B})(R_p + \bar{Z}_{A-B})}\right)\right]\end{array}}{\begin{array}{c}\left[R_g + \left(1 - \frac{\mu}{\nu}\frac{Z_{A+B}}{R_p + Z_{A+B}}\right)Q_{A+B}\right]\left[R_g + \left(1 - \frac{\mu}{\nu}\frac{\bar{Z}_{A-B}}{R_p + \bar{Z}_{A-B}}\right)\bar{Q}_{A-B}\right]\\ \left[R_g + \left(1 - \frac{\mu}{\nu}\frac{Z_{2B}}{R_p + Z_{2B}}\right)Q_{2B}\right]\end{array}} \right.$$

$$b_{2(2A)} = \left\{ \frac{\begin{array}{c}\frac{1}{2}\left[-R_gR_g'\left(1 - \frac{\mu}{\nu}\frac{Z_A}{R_p + Z_A}\right)^2 - 2a_{2(2A)}\frac{R_g^2}{\nu}Z_{2A} \right.\\ \left. - \frac{\partial}{\partial E_g}\left(\frac{1}{\nu}\right)\left(\frac{2\mu Z_A R_g^2}{R_p + Z_A} - \frac{\mu^2 Z_A^2 R_g^2}{(R_p + Z_A)^2}\right) + \frac{\partial}{\partial E_p}\left(\frac{1}{\nu}\right)\frac{\mu^2 Z_A^2 R_g^2}{(R_p + Z_A)^2}\right]\end{array}}{\left[R_g + \left(1 - \frac{\mu}{\nu}\frac{Z_A}{R_p + Z_A}\right)Q_A\right]^2\left[R_g + \left(1 - \frac{\mu}{\nu}\frac{Z_{2A}}{R_p + Z_{2A}}\right)Q_{2A}\right]} \right. \tag{15}$$

The barred symbol is the conjugate of the unbarred symbol.

$\mu = \dfrac{\Delta E_p}{\Delta E_g}$ for equal increments of plate current.

$\nu = \dfrac{\Delta E_p}{\Delta E_g}$ for equal increments of grid current.

μ is inherently positive; ν is inherently negative.

6. Plate Detection. The modulation frequency qB components of (13) are derived from both grid and plate-current curvature. To simplify the case for plate detection assume μ constant and that the grid is maintained negative with respect to the cathode.

Then

$$i_B = \frac{-\frac{1}{2}\mu^2 R_p R_p'}{(R_p + Z_A)(R_p + Z_{A-B})(R_p + Z_B)} E_A E_{A+B} - \frac{\frac{1}{2}\mu^2 R_p R_p'}{(R_p + Z_{A+B})(R_p + \bar{Z}_A)(R_p + Z_B)} E_A E_{A-B} \quad (16)$$

$$\left.\begin{array}{l}\text{When the plate is by-passed, the usual case, } Z_{A+B} = Z_A = Z_{A-B} = 0 \\ \text{With a standard signal, } E_{A+B} = E_{A-B} = \dfrac{m}{2}E_A\end{array}\right\} \quad (17)$$

Substituting (17) in (16) gives

$$i_B = -\frac{\frac{1}{2}\mu^2 R_p'}{R_p(R_p + Z_B)} m E_A{}^2 \quad (18)$$

Similarly,

$$i_{2B} = -\frac{\frac{1}{2}\mu^2 R_p'}{R_p(R_p + Z_{2B})} \frac{m^2}{4} E_A{}^2 \quad (19)$$

The similarity of (18) and (19) is apparent. As in (11),

$$\text{Per cent distortion} = \frac{m}{4} \quad (11)$$

In the foregoing analysis R_p and R_p' must be measured under the operating conditions of bias voltage, plate voltage, E_A, etc. When the signal E_A is introduced a change in plate current i_0 takes place, and if an impedance Z_0 is present, the plate voltage at the tube terminals will decrease, increasing R_p, and probably decreasing R_p'. This decreases the audio output as based on measurements of R_p and R_p' with $E_A = 0$. For calculation purposes it is necessary to have a series of curves of R_p and R_p' with E_p and E_A as variables. E_p and the expression for i_0 being known,

$$i_0 = \frac{1}{2}a_{2(0A)} E_A{}^2 + \underbrace{\frac{1}{2}a_2[_{0(A+B)}]\frac{m^2}{4}E_A{}^2 + \frac{1}{2}a_2[_{0(A-B)}]\frac{m^2}{4}E_A{}^2}_{\text{usually neglected}} \quad (20)$$

the plate voltage drop i_0Z_0 may be calculated, assuming R_p and R_p'. The determined plate voltage in this way will give wrong values of R_p and R_p', but observation of the R_p and R_p' curves will show the probable correction. This process of trial and error may be repeated, two checks usually giving the current within 5 or 10 per cent of the correct value. R_p may be conveniently determined by direct measurement of the plate resistance with a Wheatstone bridge using a 1,000-cycle tone source, while the carrier voltage E_A is applied to the grid of the tube. Correct bias and plate potentials must be applied. R_p' can be most accurately determined graphically by drawing tangents to the curve of R_p vs. E_p.

The fidelity curve can be calculated when the characteristics of Z_B are known. With the most sensitive operating point, R_p usually decreases with increase in E_A, giving better fidelity for high input signal voltages. Figure 6 shows a fidelity curve of a radio receiver with three values of input, using a reactive load.

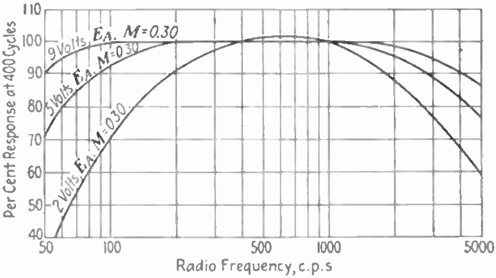


FIG. 6.—Fidelity as function of input voltage.

7. Grid-current detection is more complex. Assume a plate impedance of Z_p which is resistance only, and μ and ν as constants. The grid circuit contains a resistance Q_g shunted by a condenser of negligible impedance to r.f. and infinite impedance to a.f. The plate current then is

$$i_B = \frac{\frac{1}{2}\mu Q_g\left[R_gR_g' - \frac{\mu^2 R_pR_p'}{(R_p + Z_p)^3}\left(\frac{R_g{}^2Z_p}{\nu}\right)\right]}{(R_p + Z_p)R_g{}^2(R_g + Q_g)} mE_A{}^2 \tag{21}$$

As in the case of plate detection, R_g and R_g' must be evaluated under the operating conditions with all voltages, including signal, applied. The second term of the numerator shows that the change in plate current curvature, as expressed by R_p' affects the sensitivity. As ν is inherently negative, this second term aids the detection. As the plate battery is varied the value of the second term changes, reaching a maximum with a rather low plate voltage. This detection action is in addition to that due to plate curvature. As the sign of i_B in (18) is negative, the currents of (18) and (21) are in opposition and the modulation frequency current is less as a result.

Since ν is usually numerically large, terms with ν in the denominator may be dropped. Considering the actual impedance of the grid circuit network, with this simplification the plate current is

$$i_B = \frac{\mu Q_B}{R_p + Z_B}\left\{\frac{\frac{1}{2}R_gR_g'}{(R_g + Q_A)(R_g + Q_{A-B})(R_g + Q_B)} + \frac{\frac{1}{2}R_gR_g'}{(R_g + Q_{A+B})(R_g + Q_A)(R_g + Q_B)}\right\} mE_A{}^2 \tag{22}$$

Substituting the P terms of (12) in (10) gives an i_B of the form in the brackets of (22), which is the a-f current of a two-element detector. Multiplying this current by the a-f grid impedance Q_B gives the audio voltage impressed on the grid of the tube, which as an amplifier produces a plate current $\mu/R_p + Z_B$ times the grid voltage. Grid-leak and condenser detectors are more sensitive than the plate curvature detectors because d^2I_g/dE_g^2 is greater than d^2I_p/dE_g^2, and because of the additional audio amplification. Several disadvantages are evident. The finite value of R_g forms an undesired load on the circuit ahead. The additional audio amplification adds to microphonic and filter problems. To gain sensitivity a low plate voltage is used, with a resulting high value of R_p and poor fidelity if using reactive loads.

Substituting the impedance of the leak-condenser combination Q_B, Q_A, Q_{A+B}, Q_{A-B} in (22) enables the a-f current to be calculated for any value of A and B. If R_q is the leak resistance and C_q the shunt capacity, the maximum current i_B is approximately obtained when

$$C_q^2 = \frac{\sqrt{2}(R_q + R_g)}{ABR_qR_g^2} \qquad (23)$$

If $R_q = 10^6$, $R_g = 5 \times 10^4$, $A = 2\pi \times 10^6$, $B = 2\pi \times 10^3$, then $C_q = 125$ $\mu\mu$f. Decrease of R_g (as due to greater signal or lower plate voltage) means a larger condenser is desirable. Increasing the a. f. means decreasing C_q for maximum i_B. Occasionally the equivalent grid impedance due to tube capacities and circuit elements is more important than R_g. In this case other tube input impedance equations will give an approximate value to use for R_g.

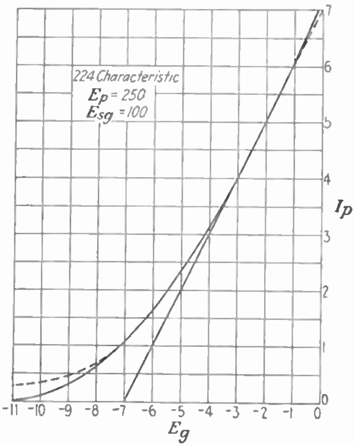


FIG. 7.—Hyperbolic detector characteristic.

8. High-amplitude Detectors. The conclusions reached in Arts. 6 and 7 above are valid only for small signals. For higher amplitude signals more than two terms of Eq. (6) must be used. The complete series may be reduced to terms containing the tube and circuit parameters, but the computation is laborious and without value for inspection purposes. In many cases the equation suiting the E—I curve may be obtained under operating conditions. Consider Fig. 7. The solid curve is the I_p, E_g of a 224-type tube under the conditions stated. The dashed line is the equation of the hyperbola

$$I_p = \frac{b}{a}\sqrt{a^2 + (E_g + c)^2} + \frac{b}{a}(E_g + c) \qquad (24)$$

where $a = 2.69$, $b = 1.41$, $c = 6.35$.

The two curves coincide within reasonable limits. Using Eq. (24) and successively differentiating to determine P_2, P_4, P_6, etc., gives values to be substituted into Eq. (6) to determine i_B and i_{2B}. Determining the maximum value of i_B as a function of E_g gives

$$I_{B(\max.)},\quad E_g = -c \tag{25}$$

To determine the minimum second-harmonic distortion, the coefficient of i_{2B} is divided by the coefficient of i_B, and the minimum value determined.

$$\frac{i_{2B}}{i_B}(\text{min.}),\quad E_g = -c \tag{26}$$

The maximum sensitivity and minimum distortion are obtained with the same bias condition, a fortunate circumstance. In case the cut-off of the tube does not match the assumed hyperbola as in Fig. 7 (hyperbola above tube characteristic) the best operating point is with a slightly greater negative bias. If the straight-line portion of the curve be extended, as shown by the light line of Fig. 7, the intercept on the E_g axis provides approximately the correct operating point. In this case $E_g = -c = 6.35$ volts, while the intercept gives $E_g = -7.0$ volts. Experimentally this tube gave a maximum sensitivity with $E_g = -6.5$ volts, the values not being critical to ± 1 volt.

The straight-line extension method has been used in various comparisons and gives a uniformly satisfactory means of determining the best bias voltage. It should be noted that Fig. 7 is determined under the operating conditions. The value of E_A will affect the shape of the characteristic curve, giving an intercept indicating a required higher bias for large values of E_A. It can be shown that the distortion decreases for increasing values of E_A. As this case is merely intermediate between the restricted input square-law detector and the linear detector, no discussion is necessary.

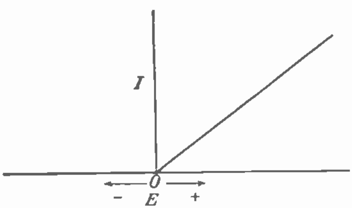


Fig. 8.—Linear detection characteristic.

9. The linear detector characteristic shown in Fig. 8 is ideal for detector operation. This exact characteristic has never been produced, yet a study of its detection operation leads to helpful conclusions. The curve itself may be expressed by a Fourier series.

$$i_p = K\left[\frac{c}{4} + \frac{\theta}{2} - \frac{2c}{\pi^2}\cos\frac{\pi\theta}{c} - \frac{2c}{3^2\pi^2}\cos\frac{3\pi\theta}{c} \cdots \frac{2c}{(2n-1)^2\pi^2}\cos(2n-1)\frac{\pi\theta}{c}\right] \tag{27}$$

where

$$\theta = e_g = E_A(1 + m\cos Bt)\cos At \tag{1}$$

Substituting (1) in (27) gives a series involving Bessel functions. For detection purposes the i_o, i_B, and i_{2B} components of i only need be considered. Tabulating the Bessel functions for these frequencies shows that for all values of E_A and m

$$i_o = \frac{KE_A}{\pi}\sqrt{1 + m^2} \tag{28}$$

$$i_B = \frac{KmE_A}{\pi}\cos Bt \tag{29}$$

$$i_{2B} = 0 \tag{30}$$

Equation (29) shows the linear detector to be truly linear in output, both with respect to m and with respect to E_A. Equation (30) shows the entire lack of distortion. (Calculations for i_{3B}, i_{4B}, etc., show all harmonic terms to be zero.) In the current i_B of (29) is the short-circuit audio-frequency plate current of the tube. The equivalent plate-circuit audio-frequency generator voltage is $i_B R_P$, and therefore the voltage across an external impedance Z_B, assuming $Z_A = Z_{A \pm B} = 0$, is

$$e_B = \frac{KmE_A}{\pi} \frac{R_p Z_B}{R_p + Z_B} \cos Bt \tag{31}$$

Assuming μ holds constant over the entire operating range,

$$R_p = \frac{2\mu}{K} \tag{32}$$

and therefore

$$e_B = \frac{2\mu KmE_A}{\pi} \frac{Z_B}{2\mu + KZ_B} \cos Bt \tag{33}$$

In case the operating point is not at the cut-off point but is biased below cut-off, these equations may be written

$$i_B = \left[\frac{KmE_A}{\pi} \sin \alpha\right] \cos Bt \tag{34}$$

$$e_B = \left[\frac{KmE_A Z_B \mu}{\mu\pi + KZ_B\alpha} \sin \alpha\right] \cos Bt \tag{35}$$

where α is one-half the angle during which current flows. If Eq. 31 is used, R_p must be measured, as previously, with E_A, or the equivalent, on the grid and with other operating conditions normal.

The theoretical advantages of linear detection have led to many attempts to make such a device. The simplest expedient is to operate at the point indicated in Fig. 7 and apply such a large value of E_A that the device is operating on the straight-line portions the major part of the cycle. Linear detection methods need not be confined to the plate circuit, as the input to a grid current curvature detector may be sufficient to approximate linear grid rectification.

10. The heterodyne detector is more properly a modulation device, although in this, as in other similar units, the distinction is a matter of viewpoint. A signal of the type of Eq. (1) is impressed simultaneously with a heterodyne voltage E_H of a frequency $H/2\pi$ upon an asymmetrical device. New frequencies are produced, each of which may be considered as a new carrier frequency. These new carrier frequencies are all possible sum and difference combinations of integer multiples of A and H. With each of these carriers are associated other frequencies differing therefrom by the modulation frequency B (and for higher-order curvature, $\pm 2B$, $\pm 3B$, etc.). Thus the heterodyne voltage e_H is of the form

$$e_H = P_2 \frac{E_A E_H}{4} (1 + m \cos Bt) \cos (A + H)t \tag{36}$$

The voltage $E_{(A-H)}$ is the one commonly used. In general the external impedance of the heterodyne detector is zero to both frequencies A and H and finite at the frequency $A - H$. As in (10) the heterodyne voltage may directly be written. For a square-law characteristic, $i = KE^2$, and operation above cut-off,

$$e_H = \left[\frac{\mu KZ_{(A-H)}}{\mu + 2KZ_{(A-H)}E_0}\right] E_A E_H [1 + m \cos Bt] \cos (A - H)t \tag{37}$$

where E_0 is the initial bias voltage measured above the current cut-off. In case the magnitude of E_H swings the instantaneous voltage off the square-law curve and below cut-off

$$e_H = \frac{\mu K Z_{(A-H)} \frac{1}{\pi}\left(\alpha - \frac{1}{2}\sin 2\alpha\right)}{\mu + 2KZ_{(A-H)} E_H \frac{1}{\pi}(\sin\alpha - \alpha\cos\alpha)} E_A E_H [1 + m\cos Bt]\cos(A - H)t \quad (38)$$

where α is as defined for (35) and is one-half the angle during which current flows.

For a linear detector characteristic $i = KE$ and operation entirely above cut-off, no detection or heterodyne voltage results. The action is as if several frequencies be simultaneously applied to a linear amplifier. Amplification of each frequency results, but no modulation is produced, hence no new frequencies.

When, in the linear detector, the voltage E_H swings the operation below cut-off, a heterodyne voltage results:

$$e_H = \frac{\mu K Z_{(A-H)} \frac{1}{\pi}\sin\alpha}{\mu + KZ_{(A-H)} \frac{1}{\pi}\alpha} E_A [1 + m\cos Bt]\cos(A - H)t \qquad (39)$$

Notice that E_H does not appear in (39). Its only effect is to determine α. Equations (37), (38) and (39) assume that E_A is very small compared with E_H, quite generally the case.

11. Two modulated signals of the type given by Eq. (1) are often impressed simultaneously upon a detector. The response ratio of the desired and undesired stations and the magnitude of the spurious new frequencies depend upon the type of detector used, relative carrier

Frequency	Amplitude-square law	Amplitude linear characteristic, first terms
B	$E_A^2 M$	$E_A M$
$2B$	$E_A^2 M^2/4$	0
b	$E_a^2 m$	$mE_a - E_a k\left(a_0 m - \frac{a_1 m}{2} - \frac{mE_a k}{E}\right) - \frac{3E_a^2 k^3 b_0 m}{2E}$
$2b$	$\frac{E_A^2 m^2}{4}$	$\frac{m^2 E_a^2 k^2}{2E} - \frac{b_0 E_a^2 k^3 m^2}{4E}$
D	$E_A E_a$	$E_a k\left(a_0 - \frac{a_1 M}{2} - \frac{m^2 E_a k}{2E}\right) + \frac{b_0 E_a^2 k^3}{2E}(2 + m^2)$
$2D$	0	$b_0 E_a^2 k^3/4E$
$B \pm D$	$\frac{E_A E_a M}{2}$	$E_a k\left(\frac{a_0 M}{2} - \frac{a_1}{2} + \frac{a_2 M}{4} - \frac{m^2 M E_a k}{4E}\right)$
$b \pm D$	$\frac{E_A E_a m}{2}$	$E_a k\left(\frac{a_0 m}{2} - \frac{a_1 M m}{4} - \frac{m E_a k}{2E}\right) + \frac{b_0 E_a^2 k^3 m}{E}$
$B \pm b \pm D$	$E_A E_a M m$	0

amplitudes and degrees of modulation. The mathematical computation may be made by the use of a sum of infinite series, or with the somewhat more rapidly convergent Bessel series. The relative amplitude terms indicated in the preceding table were obtained from the infinite-series solution.

The voltage e impressed on the detector is of the form

$$e = E_A(1 + M \cos Bt) \cos At + Ea(1 + m \cos bt) \cos at \qquad (40)$$

where A, B, M refer to the desired station signal and a, b, m refer to the interfering station signal. For simplicity of representation let $A - a = D$. The various important frequency components and their amplitudes are noted in the table on page 255. The first terms of the infinite series given may be considered as representing almost completely the amplitude of the frequency components under the following restrictions:

$$E_a \gtreqless 0.1E_A,\ 0.1 < m < 0.5,\ 0.1 < M < 0.5$$

The constants in the table are

$$a_0 = 1 + \frac{M^2K^2}{2} + \frac{3M^4K^4}{8} \qquad b_0 = 1 + 3M^2K^2$$

$$a_1 = MK + \frac{3M^3K^3}{4} + \frac{5M^5K^5}{8}$$

$$a_2 = \frac{M^2K^2}{2} + \frac{M^4K^4}{2} \qquad K = \frac{E_A}{E_A + E_a}$$

12. Demodulation of One Signal by Another. The audio component b of the undesired signal is reduced due to the presence of E_A. This is an important and interesting phenomenon. The linear detector in the presence of more than one signal discriminates against the weaker signal. This ratio has been investigated by means of Bessel functions. Assuming a low and equal percentage of modulation, for convenience in calculation, this gives the following ratio of the audio components of b and B.

Carrier ratio	Acoustic ratio	Carrier ratio	Acoustic ratio
1.0	1.0	0.5	0.137
0.9	0.630	0.4	0.0956
0.8	0.430	0.3	0.0470
0.7	0.308	0.2	0.0202
0.6	0.209	0.1	0.0052

13. Rectification diagrams are experimentally determined curves very useful in deriving detector characteristics. For a two-element detector (including such devices as grid-leak and condenser detector in a screen-grid tube or in a neutralized triode) the only variables considered are direct current, direct voltage, and alternating voltage, and the resulting series of curves is called a *rectification diagram.* For triodes, etc., where the signal is applied to the grid circuit of a plate-current curvature detector, the series of curves is known as a *transrectification* diagram. Figure 9 is a transrectification diagram for a 201-A tube. The plate current is shown as a function of plate voltage for various r-f voltages (r-m-s values) on the grid. Two load-resistance lines for 100,000 and 200,000 ohms are shown. The d-c voltage change across the load resistance for values of r-f voltage is shown in Fig. 10. These curves are derived from Fig. 9.

Considering the standard signal of Eq. (1) to be a voltage E_A varied in magnitude $\pm mE_A$, the output voltage change across the load impedance may be calculated. This voltage change, divided by two, gives the peak value of audio output. Thus in Fig. 10, an r-f voltage of 5

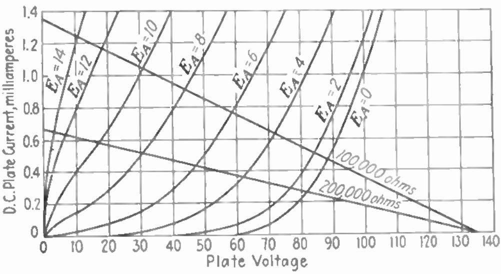


FIG. 9.—Transrectification diagram of 201-A. A tube for various applied r-f voltages.

volts r-m-s, modulated 30 per cent gives 9.3 and 10.4 volts peak, across the 100,000- and 200,000-ohm loads, respectively.

Rectification diagrams need not be taken at radio frequencies. A voltage source such as the 60-cycle supply is satisfactory. The plate impedance to the frequencies E_A, E_{A-B}, E_{A+B} must be approximately zero.

The rectification diagrams may also be used with reactive loads. The slope of the load line through the operating point must correspond to the impedance of the reactance at the audio frequency under consideration. The actual current-voltage relations form an ellipse about this load line, the extremities of which approximate the peak-voltage swings. Complete fidelity curves may be plotted in this manner. In this case, as with the resistance load, the external impedance to E_A, E_{A-B}, and E_{A+B} must be approximately zero. An r-f voltage of 10, r-m-s, modulated 30 per cent to a 227 tube properly biased gives 29 audio peak volts across 200,000 ohms.

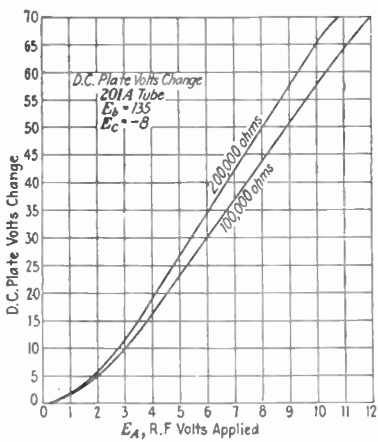


FIG. 10.—Curves derived from characteristic of Fig. 9.

14. Other Detection Characteristics. For simplicity in reference, all voltages given on the curves as E_A are given in r-m-s values. Figure 11 shows detection characteristics of a 227 used as a two-element detector. Figure 12 shows the equivalent detection resistance for the same tube.

In using various tubes for grid-leak and condenser detectors several factors are evident. The grid-leak resistance should be high to increase the tube initial bias, reducing the tube grid conductance and so the loss on the r-f circuit. A high leak resistance also increases the sensitivity. It has a major disadvantage in that in combination with the correct capacity for detection, considerable loss in high modulation frequencies results. Figure 13 shows a series of fidelity curves using different grid-

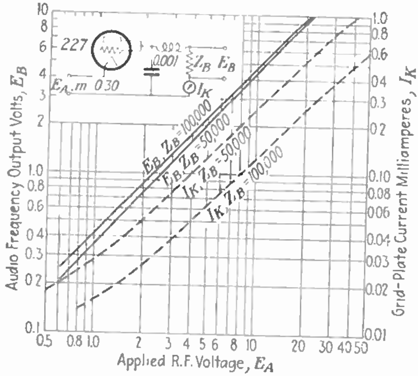


FIG. 11.—Diagram for 227-type tube as a two-element detector.

leak resistances. It has become almost standard practice to use a grid condenser of 250 $\mu\mu$f and a leak resistance of one megohm for all tubes used as grid-leak detectors. Better individual compromises for specific uses are often helpful. Figure 14 shows the r-f input a-f output of a 227 tube used as a grid-leak and condenser detector.

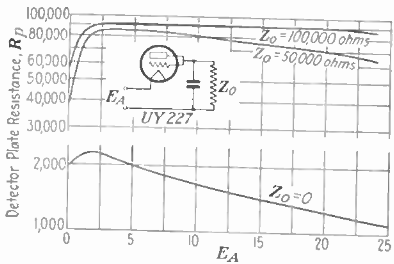


FIG. 12.—Equivalent resistance of 227 as two-element detector.

The plate resistance R_p while detecting, and the change in plate resistance R_p', have been repeatedly used in detection equations. Figure 15 gives values of R_p and Fig. 16 gives values of R_p' for various conditions of a 227 tube.

15. Screen-grid Tube as Detector. The 224-type tube is of great importance as a detector. Figure 17 gives a series of curves, plotting a-f output against r-f input for various bias conditions. The total harmonic distortion introduced by this detector is also shown. With a

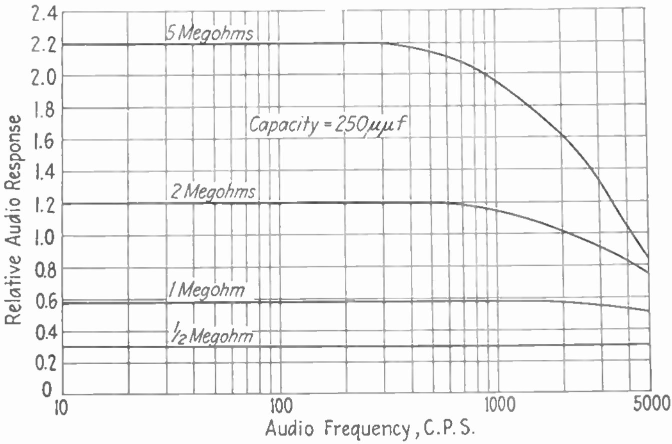


FIG. 13.—Fidelity of grid-leak detector as function of leak resistance.

low battery bias (7 volts) the distortion is initially $m/4$ and decreases with increase in voltage, increasing when grid current begins to flow. With successively increasing bias voltages b, c, d the distortion per-

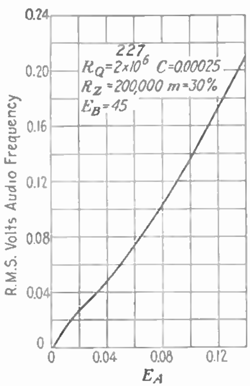


FIG. 14.—Grid-leak and condenser detector.

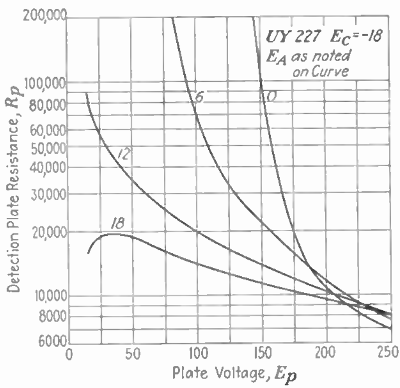


FIG. 15.—Value of R_p, plate resistance while detecting, for various input voltages.

centage increases rapidly from the $m/4$ condition, and the sensitivity decreases. Higher outputs are possible, however. With sufficient r-f input, the second harmonic for these extreme bias voltages will decrease as shown by the descending curves of *b*, *c*, and *d*.

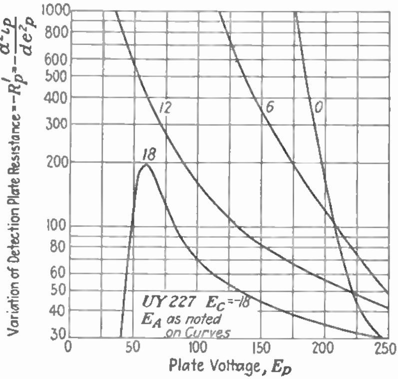


FIG. 16.—Values of change in plate resistance.

Two conditions of self-bias resistance for detection are shown at *e* and *f*. At low signal inputs the plate current is low, the bias is of the order of 6 volts, the distortion is low, and the sensitivity high. Increasing

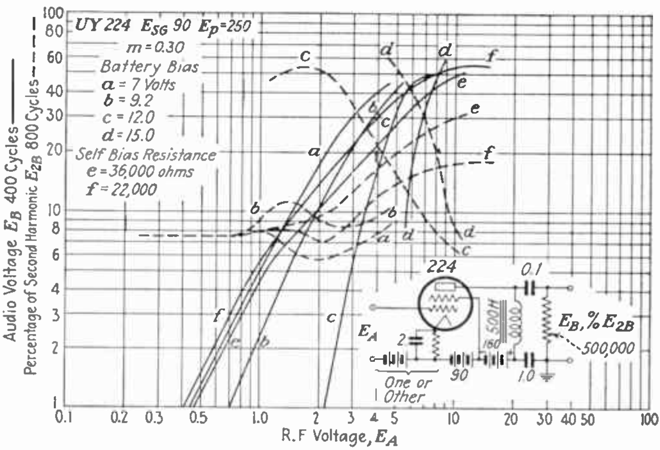


FIG. 17.—Screen-grid detector characteristics.

the signal input increases the current, bias voltage and distortion. Instead of decreasing, as would be normal in a tube having a characteristic as in Fig. 7, the distortion increases with input.

The slope of the input-output curve is often considered as a means of estimating if the detector action is square law, linear or intermediate, and guessing the corresponding harmonic distortion. In such a self-bias arrangement the increasing bias voltage with increasing r-f input may modify the output-input curve to indicate a linear detector characteristic without the distortion conforming. In a sense the output-input curves as shown are static; distortion is due to a dynamic characteristic. It is significant to note that the *e*, *f* output curves cross the *c*, *d* output curves at the same r-f input as the corresponding distortion curves, indicating that a given r-f input and bias voltage produce a definite output and distortion regardless of how obtained.

Figure 18 shows another series of curves giving output and distortion as a function of bias conditions. These bias resistances range from values

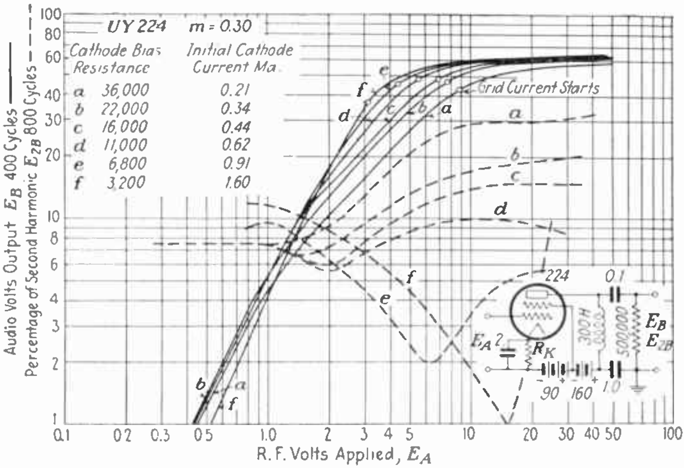


FIG. 18.—Output and distortion as function of bias resistance.

too low to those too high. *a* with 36,000 ohms gives an input-output curve of high initial sensitivity and almost linear characteristic. In spite of this apparent linear characteristic, the distortion, except for the initial $m/4$ value, is high. Decreasing the bias resistance below the optimum value results in an initial increase in distortion, a later decrease in distortion but only after grid current starts, making this decrease of doubtful value. Third harmonic distortion becomes serious only after grid current starts and when feeding from a high-impedance circuit so it is not shown in Figs. 17 and 18.

Other factors greatly influence the sensitivity and distortion of the 224. The output circuit for the curves of Figs. 17 and 18 is an extremely high reactance shunted by a 500,000-ohm resistance. For resistance coupling, higher supply voltages must be used to make up for the IR drop, or other compromises, such as lower plate resistance, be utilized. With signal voltages applied, the plate current increases and the net voltage on the tube plate electrode decreases. When the instantaneous plate potential reaches a value as low as the screen potential serious distortion results. Actual design must insure high plate voltage in addition to correct bias conditions as portrayed in Figs. 17 and 18.

16. Superheterodyne Translation Ratio. The sensitivity of the first detector, or heterodyne detector, of a superheterodyne system is expressed

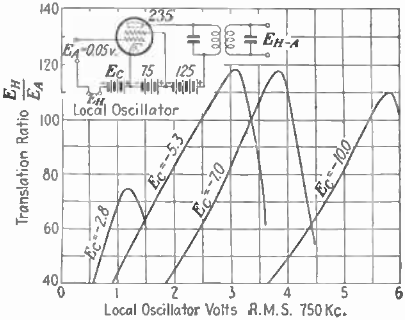


FIG. 19.—Superheterodyne oscillator characteristic.

as the translation ratio. This is the ratio of the intermediate frequency voltage across the i-f impedance in the plate circuit of the detector, and the r-f voltage applied to the grid of the detector. Figure 19 gives typical translation ratio characteristics for a 235 tube under various oscillator

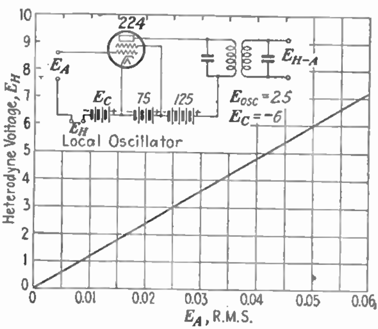


FIG. 20.—I-f output *versus* r-f input.

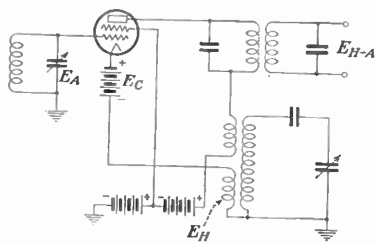


FIG. 21.—Combined detector-oscillator circuit.

voltages. The sudden drops are due to grid current load on the r-f tuned circuit.

The 235-type tubes were not designed as detectors, or modulators; but as large voltages from a local oscillator can be applied to the grid, this type may be used as first detectors.

With a constant oscillator voltage, the i-f output is linear with respect to the r-f input voltage within operating limits. This condition for a 224 is shown in Fig. 20.

17. Combination Detector and Oscillator. The functions of 1st detector and oscillator of a superheterodyne may be simultaneously carried on in a single tube. The grid circuit is tuned to the applied signal E_A, and a plate circuit is made resonant to the resultant intermediate frequency. In addition, the plate circuit is coupled back to the grid circuit by means of a third circuit resonant at the frequency of the applied signal plus the intermediate frequency. The result is that an oscillator voltage E_H (at the frequency $H/2\pi$) is simultaneously applied on the grid with the signal E_A, and a heterodyne voltage E_{H-A} will be developed across the resonant plate circuit. A typical circuit is shown in Fig. 21. Several limitations are necessary in practice. The heterodyne voltage E_H applied to the grid should not be high enough to draw grid current, or distortion will result. This calls for low oscillation voltages. Most satisfactory operation to date has resulted from use of a 224-type tube. The actual voltage conditions vary with design. Bias voltages may range from 5 to 10 volts with the r-m-s heterodyne voltages on the grid about one half of this bias voltage. Translation ratios from 15 to 50 may be obtained.

18. Values of C Bias for Various Tubes as Detectors.

C-327

E_b	E_{c_1}
45	−5.0
90	−10.0
135	−15.0
180	−20.0
250	−28

C-324

E_b	E_{c_2}	E_{c_1}
180	25	−2.5
180	35	−3.5
180	45	−4.5
180	55	−5.5
180	65	−6.5
180	75	−7.5
250	90	−9.0

CX-322

E_b	E_{c_2}	E_{c_1}
135	22½	−4.5
135	45	−9.0
135	55	−11.0
135	67½	−13.5
135	75	−15

CX-330

E_b	E_{c_1}
45	−3.5
90	−9.5
180	−20

CX-332

E_b	E_{c_2}	E_{c_1}
135	45	−3.0
	67½	−5.0
	90	−8.0

E_{c_1}, control-grid voltage; E_{c_2}, screen-grid voltage.

These bias values are all slightly higher than those required for optimum sensitivity. Greater output can be obtained using these values, however.

MODULATION

19. Modulation is the process or result of modifying an energy carrier, the changes conforming to the modulating signal. The simplest form of modulation consists in the intermittent transmission of energy, producing an instantaneous change in energy from zero to maximum amplitude. Such a modulation process is customarily produced by keying and breaks the power supply, disconnects the carrier medium, or diverts the energy carrier into a power-dissipating unit when transmission is undesired. The energy carrier might be direct current in the case of a telegraph wire line, medium or high frequency currents in the cases of carrier current or radio telegraphy transmission. Suitable devices responsive to the modulation are assumed to be at the receiving end of the system.

20. Absorption modulation is typical of modulation processes. With a given impressed voltage in a circuit the current is determined by the resistance of the circuit. In an absorption modulation circuit the resistance comprises the load impedance R_L and a resistance R_0 which varies in amplitude at the modulation frequency and to a degree represented by m and determined by circuit conditions.

The frequency of the impressed voltage is $A/2\pi$ and that of the modulation frequency $B/2\pi$. The voltage across the load impedance R_L is

$$e_L = \frac{E_0 R_L \sin At}{R_L + R_0(1 + m \sin Bt)} \tag{41}$$

Expanded, this gives

$$e_L = E_0 \left\{ \sin At - \frac{R_0}{R_L}(1 + m \sin Bt) \sin At + \cdots \right\} \tag{42}$$

The successive terms dropped from this expansion represent harmonic or distortion terms and may be neglected for this analysis. Combining,

$$e_L = E_0 \left\{ \left(1 - \frac{R_0}{R_L}\right) \sin At - \frac{R_0}{R_L} m \sin At \sin Bt \right\} \tag{43}$$

Equation (43) represents a voltage $E_0\left(1 - \frac{R_0}{R_L}\right) \sin At$ at the impressed frequency, and a second voltage $E_0 \frac{R_0}{R_L} m \sin At \sin Bt$ at the frequency $\frac{A}{2\pi}$ but modulated by the frequency $R/2\pi$. The amplitude of the modulated component is proportional to the variation in resistance R_0 as expressed by m and the ratio R_0/R_L.

21. Circuits for Absorption Modulation. The absorption-modulation circuits of Fig. 22 *a*, *b*, *c* are illustrative of common methods. *a* is a telephone transmitter modulating a current of zero (d-c) frequency; *b* and *c* are methods applied to low-power r-f transmitters for voice-frequency modulation. The coupling ratios used are such as to provide the maximum modulation within the power capacity of the microphone.

22. The power rating of the device used to produce the resistance variation largely determines the modulated output. The greater R_0 with respect to R_L the greater will be the modulated output component, but a smaller percentage of total power supplied will be transmitted. The current of carrier frequency flowing through the modulating unit causes power loss and resultant heat. In the case of a carbon-grain transmitter as shown in Fig. 22, sticking and poor operation will result. The correct relationship between R_0 and R_L is rather a compromise between opposing factors of decreased modulation and increased transmission. Maximum modulated output (maximum signal response) will generally be obtained between the values R_0 equals R_L and R_0 equals $\frac{1}{2}R_L$ depending somewhat on m. The power rating of well designed carbon-grain type microphones is approximately 5 watts, giving reasonable modulation control of from 5 to 20 watts. Special water-cooled micro-

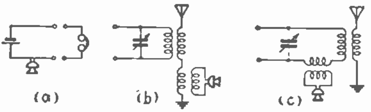


FIG. 22.—Absorption modulation circuits.

phones have been used multiplying the above power values by about five times.

23. Analysis of Modulation. The mathematical expression for an energy carrier of a frequency $A/2\pi$ modulated by a signal of frequency $B/2\pi$ is shown to be a product, as $(1 + m \sin Bt) \sin At$. This means that the per cycle amplitude of the carrier voltage varies as the amplitude of the modulation signal, being greatest when $\sin Bt$ is $+1$ and least when $\sin Bt$ is -1. Such a variation can be obtained by impressing simultaneously voltages of frequencies $A/2\pi$ and $B/2\pi$ upon a device

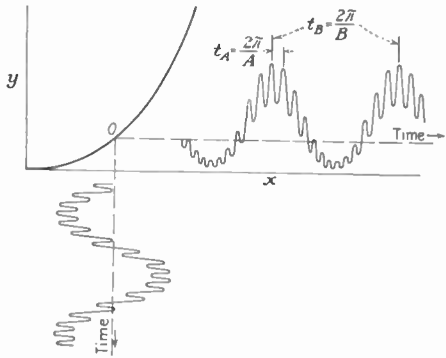


FIG. 23.—Modulation process.

whose response is not proportional to the voltage applied. Figure 23 shows how this result is obtained; x is the applied voltage and y the corresponding response. An initial operating point o is indicated. Time is measured along the dashed lines through o and parallel to the x and y axes. On the vertical time axis is shown the instantaneous sum of the applied voltages. On the horizontal time axis is shown the instantaneous response. Due to the changing slope of the asymmetrical x, y characteristic the per cycle amplitude of the response at the frequency $A/2\pi$ varies as the value of $\sin Bt$ varies. This is the desired relation for a modulated signal, and so indicates that such an asymmetrical characteristic does produce a modulated response.

The response y to an input x can be expressed as a general function

$$y = f(x) \tag{44}$$

Expanding by Maclaurin's theorem

$$y = ax + \frac{bx^2}{2!} + \frac{cx^3}{3!} + \frac{dx^4}{4!} \cdots \tag{45}$$

Let

$$x = E_A \sin At + E_B \sin Bt \tag{46}$$

where E_A is the peak voltage of the carrier frequency input ($f = A/2\pi$) and E_B is the peak voltage of the modulation signal input ($f = B/2\pi$). Substituting (46) in (45) and tabulating vertically, with equivalent first-order trigonometric functions,

$$
\begin{aligned}
&a(E_A \sin At + E_B \sin Bt) + \frac{b}{2}(E_A \sin At + E_B \sin Bt)^2 + \frac{c}{6}(E_A \sin At + E_B \sin Bt)^3 + \frac{d}{24}(E_A \sin At + E_B \sin Bt)^4 + \cdots \\
&= aE_A \sin At + aE_B \sin Bt + \\
&\frac{bE_A{}^2}{4} + \frac{bE_B{}^2}{4} - \frac{bE_A{}^2}{2}\cos 2At - \\
&\frac{bE_B{}^2}{4}\cos 2Bt + bE_AE_B \sin At \sin Bt \\
&\frac{cE_A}{4}\left(\frac{E_A{}^2}{2} + \frac{E_B{}^2}{3}\right)\sin At + \\
&\frac{cE_B}{4}\left(\frac{E_B{}^2}{2} + \frac{E_A{}^2}{3}\right)\sin Bt - \\
&\frac{cE_A{}^3}{24}\sin 3At - \frac{cE_B{}^3}{24}\sin 3Bt - \\
&\frac{cE_A{}^2E_B}{12}(1 - \cos 2At)\sin Bt - \\
&\frac{cE_AE_B{}^2}{12}(1 - \cos 2Bt)\sin At \\
&\frac{dE_A{}^4}{192}\cos 4At - \frac{dE_A{}^2}{48}(E_A{}^2 + 3E_B{}^2) \\
&\cos 2At + \frac{d}{64}(E_A{}^4 + E_B{}^4) + \\
&\frac{dE_AE_B}{8}(E_A{}^2 + E_B{}^2)\sin At \sin Bt - \\
&\frac{dE_A{}^3E_B}{24}\sin 3AT \sin Bt - \\
&\frac{dE_AE_B{}^3}{24}\sin 3Bt \sin At - \\
&\frac{dE_B{}^2}{48}(E_B{}^2 + 3E_A{}^2)\cos 2Bt + \\
&\frac{d}{16}E_A{}^2E_B{}^2 \cos 2AT \cos 2Bt + \\
&\frac{dE_B{}^4}{192}\cos 4Bt \\
&+ \cdots
\end{aligned} \tag{47}
$$

The values of a, b, c, d, etc., represent successive differentials of the function y with respect to x, evaluated at the operating point x. It is to be noted that the functional relationship between x and y is based on the *complete* operating circuit and not merely the asymmetrical characteristic of the modulating device. Due to load impedances straightening out the curve between x and y of the modulating device itself, the differentials indicated will in general be smaller in magnitude than those based upon the x, y characteristic of the modulating device. If the actual value of this function is known, the values of these differentials may be obtained. Two typical curves of $y = f(x)$ are expressed below and the differentials evaluated:

	$y = C_1 + K_1x_0{}^2$	$y = C_2 + K_2X_0{}^{3/2}$
$a = \dfrac{dy}{dx}$	$= 2K_1X_0$	$= \dfrac{3}{2}K_2X_0{}^{1/2}$
$b = \dfrac{d^2y}{dx^2}$	$= 2K_1$	$= \dfrac{3}{4}K_2X_0{}^{-1/2}$

$$c = \frac{d^3y}{dx^3} \qquad = 0 \qquad = -\frac{3}{8}K_2X_0^{-3/2}$$

$$d = \frac{d^4y}{dx^4} \qquad = 0 \qquad = \frac{9}{16}K_2X_0^{-5/2}$$

In the equations the product of two trigonometric values such as $\sin rAt \sin sBt$ represents a carrier of frequency $rA/2\pi$ modulated by a signal of $sB/2\pi$. If s is not unity, higher frequencies than the desired modulation signal frequency are transmitted and in general distortion results. If r is not unity, the modulation signal is also transmitted as a modulated carrier of higher frequency than that desired. The magnitudes of such undesired components may be reduced by keeping E_A and E_B small (in comparison with the operating characteristic of the x, y function) or by using such a function as indicated above where all differentials beyond the second have a value of zero.

For analytical purposes, the instantaneous amplitude of y may be expressed as an equation and written as follows:

$$y = aE_A \sin At + aE_B \sin Bt + \frac{bE_A^2}{4} + \frac{bE_B^2}{4} - \frac{bE_A^2}{4}\cos 2At - \frac{bE_B^2}{4}\cos 2Bt + bE_AE_B \sin At \sin Bt \qquad (48)$$

The desired transmission is at a frequency $A/2\pi$. Dropping other terms,

$$y = aE_A \sin At + bE_AE_B \sin At \sin Bt \qquad (49)$$

The second component of Eq. (49) represents a variable amplitude carrier at the frequency $A/2\pi$ and varying in amplitude at the frequency $B/2\pi$. This variable amplitude adds to and subtracts from the first term of Eq. (49), giving an average peak value of aE_A. The percentage of modulation may be expressed as a percentage change in value of this average peak, due to the second term of Eq. (49).

$$m = \frac{bE_B}{a} \qquad (50)$$

where m is the modulation factor. The percentage of modulation is usually the value discussed, which is $100m$. The factor b/a represents the ability of the device to produce modulation and is termed its modulation efficiency. The percentage of modulation varies directly as the modulation voltage E_B. Substituting (50) in (49) gives

$$y = aE_A\left[\sin At + \frac{m}{2}\cos (A - B)t - \frac{m}{2}\cos (A + B)t\right] \qquad (51)$$

Equation (51) indicates that the desired result of modulation is equivalent to three components, having frequency characteristics of $\frac{A}{2\pi}$, $\frac{A-B}{2\pi}$, $\frac{A+B}{2\pi}$ and having corresponding amplitudes of 1, $m/2$, $m/2$ and initial corresponding phases of 0, +90, −90 deg. respectively. The frequency $A/2\pi$ is referred to as the carrier frequency and the frequencies $\frac{A-B}{2\pi}$ and $\frac{A+B}{2\pi}$ as the lower and upper side bands.

24. Carrier and Side Band Physical Picture. Three viewpoints may be taken of the relations between the carrier and the side bands. The first is illustrated in Fig. 24, where *a*, *b*, and *c* respectively represent instantaneous amplitudes of the carrier, lower and upper side-band components plotted against time; *d* gives the instantaneous sum of *a*, *b*, and *c* and represents exactly the form of the modulated output. The modulation factor is indicated.

A second viewpoint is shown in Fig. 25, which represents the peak (or effective) voltage amplitude of the carrier and side bands plotted

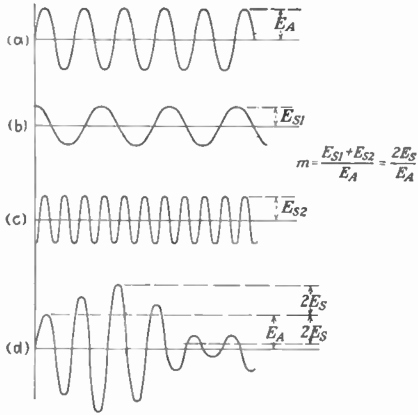


FIG. 24.—Relations between carrier and side bands.

against their corresponding frequencies. The modulation factor is indicated. Its value is obtained by noting that the maximum and minimum amplitudes occur when the sum of the side-band amplitudes adds to and subtracts from the carrier.

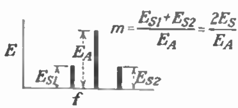


FIG. 25.—Carrier and side bands on a frequency scale.

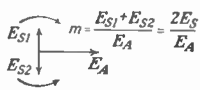


FIG. 26.—Vector diagram of carrier and side bands.

A third viewpoint is shown in Fig. 26, where the vectors indicate peak (or effective) voltage amplitudes of the carrier and side bands and their respective initial phase conditions. With respect to the carrier vector the lower side band rotates clockwise while the upper side band rotates counterclockwise, the relative angular velocities being $+B$ and $-B$. The maximum and minimum amplitudes occur when the side-band vectors are in phase or exactly out of phase with the carrier. The modulation factor is indicated.

25. Modulation Due to Iron Saturation. An asymmetrical characteristic occurs near the saturation point of an iron core reactor. If such a reactor, so operated, be used as a transmission element in a carrier frequency system and if a modulating signal be impressed through an auxiliary winding, modulation will result. Such a system was used for modulating arc-type radio transmitters or those of the Alexanderson or Goldschmidt type. Due to nonpenetration of the flux into the iron such schemes are effective only at frequencies up to 100,000 cycles.

26. Plate-circuit Modulation. The plate-current characteristic of a three-element vacuum tube with respect to its grid and plate voltages may be approximately expressed as

$$I_p = \left(E_g + \frac{E_p}{\mu} + e\right)^x \tag{52}$$

x, in the range in which the tube is effective as a modulation device, is approximately $\frac{3}{2}$. The voltages at the carrier frequency and at the modulation frequency may be independently impressed in the grid or plate circuits of the vacuum tube with resultant modulation. If both voltages are impressed in the grid circuit, smaller amplitudes are necessary to produce an equivalent plate circuit output than if one or both are impressed in the plate circuit. Here μ represents the amplification factor of the tube. It is not constant but varies as I_p cut-off is approached at high negative grid voltages. The result is that in obtaining high percentages of modulation, a greater amount of distortion will be produced when the carrier and modulation voltages are impressed in the grid circuit than when impressed in the plate circuit. Best results may be obtained when the carrier voltage is impressed in the grid circuit and the modulation voltage in the plate circuit. Tubes operating in the manner outlined above are called *modulated amplifiers.*

27. Modulation Quantities. Certain quantitative values are often needed in the solution of modulation problems. These are indicated as follows:

In terms of the grid and plate incremental voltages the power series for the three-element vacuum tube is

$$i_p = F_1e_g + F_2e_p + \tfrac{1}{2}F_3e_g^2 + F_4e_ge_p + \tfrac{1}{2}F_5e_p^2 \cdots \tag{53}$$

$$\left.\begin{aligned} F_1 &= \frac{\mu}{R_p};\quad F_2 = \frac{1}{R_p};\quad F_3 = \frac{1}{R_p}\frac{d\mu}{dE_g} + \frac{\mu}{R_p}\frac{d\mu}{dE_p} - \mu^2\frac{R_p'}{R_p^2};\quad F_4 = \\ &\qquad \frac{1}{R_p}\frac{d\mu}{dE_p} - \mu\frac{R_p'}{R_p^2};\ F_5 = -\frac{R_p'}{R_p^2} \end{aligned}\right\} \tag{54}$$

where
$$\mu = -\frac{dE_p}{dE_g};\quad \frac{1}{R_p} = \frac{dI_p}{dE_p};\quad R_p' = \frac{dR_p}{dE_p} \tag{55}$$

With a resistance R_L in the plate circuit the series of (53) may be expressed as

$$i_p = \frac{\mu e_g}{R_p + R_L} - \frac{1}{2}e_g^2\left[\frac{\mu^2R_pR_p'}{(R_p + R_L)^3} - \frac{2R_p\frac{d\mu}{dE_g}}{(R_p + R_L)^2}\right] + \tag{56}$$

$$e_p = -i_pR_L \tag{57}$$

Maximum modulated voltage across R_L is obtained when

$$R_L(\text{approx.}) = \frac{\mu}{2}\frac{1}{d\mu/dE_g}\left[R_p' + \sqrt{(\mu R_p')^2 - 2R_pR_p'\frac{d\mu}{dE_g}}\right] \tag{58}$$

$$R_L = \frac{R_p}{2}\left(\text{when } \frac{d\mu}{dE_g} = 0\right) \tag{59}$$

Maximum modulated power in R_L is obtained when

$$R_L = \frac{R_p}{5}\left(\text{when } \frac{d\mu}{dE_g} = 0\right) \tag{60}$$

Effective voltage E_f of modulated carrier is

$$E_f = \tfrac{1}{2}E_A\sqrt{m^2 + 2} \tag{61}$$

where E_A is the peak voltage of the unmodulated carrier and m is the modulation factor.

Maximum peak voltage of modulated carrier is

$$E_{p\ \max.} = E_A(1 + m) \tag{62}$$

Minimum peak voltage of modulated carrier is

$$E_{p\ \min.} = E_A(1 - m) \tag{63}$$

Maximum peak voltage of modulated carrier is

$$E_{p\ \max.} = E_f\frac{2(m + 1)}{\sqrt{m + 2}} \tag{64}$$

28. Heising modulation, named after its inventor, is based on the relationship between the amplitude of the carrier frequency voltage from an oscillator and the voltage of the plate power supply to that oscillator. Figure 27 shows the relationship between the voltage across the tuned circuit in a low-power oscillator with respect to the supply voltage applied to the plate. The output is practically linear with respect to the supply voltage. In operation the modulating voltage is added to and subtracted from the plate supply voltage. If the peak of the modulating voltage is equal to the plate supply voltage the oscillator carrier voltage varies between zero and twice the average value, which is equivalent to 100 per cent modulation. The transmitted power from the oscillator varies as the square of the voltage and, therefore, during this maximum voltage period is four times its normal value. At the minimum voltage period no power is radiated. The increased power supply to the oscillator tube must come from the modulation signal source. The modulation supply is generally from a vacuum tube used as a modulation-frequency amplifier. For 100 per cent modulation the undistorted power output of this modulation amplifier must be twice the normal power rating of the oscillator.

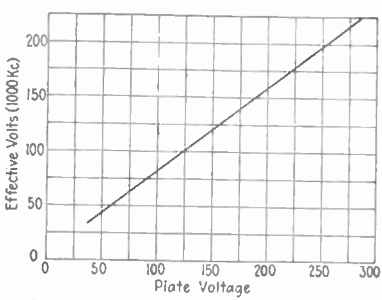


FIG. 27.—Voltage across tuned circuit as function of oscillator plate voltage.

29. Oscillator-modulator System. Figure 28 shows a complete oscillator-modulator system comprised of a Hartley oscillator and a Heising modulator. The plate power supply for the oscillator and modulator tubes is from a common source, the plates being fed through the choke L. The modulation-frequency voltage developed across this choke is added to and subtracted from the supply voltage impressed on the plate of the oscillator tube, giving the resultant modulation as previously indicated. The radio-frequency choke K prevents the effective grounding of the plate of the oscillator tube and prevents radio-frequency energy from being fed back into the modulator tube. To maintain fidelity at low frequencies, the choke L must be of high inductance. About 95 per cent of the 400-cycle voltage amplitude across this choke will be maintained at 60 cycles, if the choke inductance in henrys is 0.008 R_p, where R_p is the plate resistance of the modulator tube. To maintain fidelity

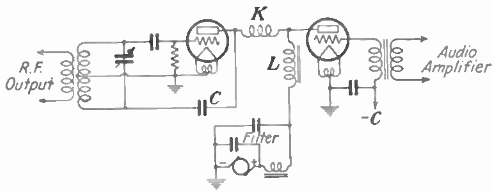


Fig. 28.—Hartley oscillator—Heising modulator.

at high frequencies the feed-back condenser C (plus any equivalent capacity due to wiring) must be kept small. At 5,000 cycles 95 per cent transmission will be maintained if this capacity in microfarads is not greater than $10/R_p$.

30. Oscillation Control. The amplitude of oscillation in a carrier-frequency oscillator may be controlled by varying the grid voltage in a manner similar to the Heising modulator. One effective way of accomplishing this is to control the value of the grid leak used. With proper adjustment the amplitude of oscillation is proportional to the impedance of the grid leak, the current through which is used to provide bias for the oscillator tube. If the plate-filament circuit of a vacuum tube be

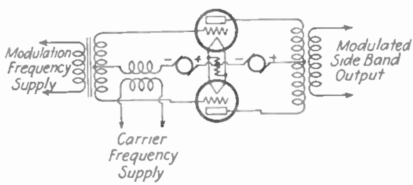


Fig. 29.—Modulated balanced amplifier.

used in place of the customary grid leak, this modulation tube will draw current whenever its plate is positive, thereby giving an equivalent grid-leak resistance. The magnitude of this resistance is determined by the type of tube used, etc., and by the potential impressed upon its grid. If this applied grid potential is in the nature of a modulation signal, the leak resistance and consequently the carrier-frequency output will vary in accordance with this signal.

31. Power Tubes as Modulators.

General information

Model	Main use	Socket mounting	Type of cooling	Max. over-all length, inches	Max. over-all dia., inches	Filament			Average voltage amplification factor
						Types	Volts	Amperes	
UX-171A	Power amplifier	UX	Air	4⅝	1 13/16	Thoriated	5.0	0.25	3
UX-210	General purpose	UX	Air	5⅝	2 3/16	Thoriated	7.5	1.25	8
UV-211	General purpose	UT-541	Air	7⅞	2 5/16	Thoriated	10.0	3.25	12
UX-245	Power amplifier	UX	Air	5⅜	2 3/16	Thoriated	2.5	1.75	3.5
UX-250	A-f power amplifier or modulator only	UX	Air	6¼	2 11/16	Coated	7.5	1.25	3.8
UX-841	Voltage amplifier only	UX	Air	5⅝	2 3/16	Thoriated	7.5	1.25	30
UX-842	A-f power amplifier or modulator only	UX	Air	5⅝	2 3/16	Thoriated	7.5	1.25	3
UV-845	A-f power amplifier or modulator only	UT-541	Air	7⅞	2 5/16	Thoriated	10.0	3.25	5
UV-848	General purpose	Water jacket	Water	20¼	4¼	Tungsten	22.0	52.0	8
UV-849	General purpose	UT-501 & UT-502	Air	14⅜	4 1/16	Thoriated	11.0	5.00	19
UV-851	General purpose	UT-501 & UT-502	Air	17⅝	6⅛	Thoriated	11.0	15.50	20
DE-520M	Modulator	Water jacket	Water	16	4¼	Tungsten	22.0	30.0	10
WE-205D	General purpose	115B	Air	4½	2⅜	Coated	4.5	1.60	7.3
WE-211D	General purpose	112A	Air	7 15/16	2 1/16	Coated	10.0	3.00	12.0
WE-212D	Oscillator or modulator	118A 113A	Air	13⅝	3⅝	Coated	14.0	6.00	16.0

32. Modulated Balanced Amplifier. Figure 29 shows the circuit commonly known as a modulated balanced amplifier. Both the carrier voltage and the modulating signal are impressed on the grids of the amplifier tubes. The voltage on the upper tube is $[E_A \sin At + E_B \sin Bt]$, and the voltage on the lower tube is $[E_A \sin At - E_B \sin Bt]$. The currents in the plate circuit are transferred to a third circuit, the sign of the mutual inductance being positive for one plate circuit and negative for the other. The net result is to cause a cancellation of the carrier frequency in the third circuit, while the side-band components add, giving two components $E_Am \cos (A - B)t$ and $E_Am \cos (A + B)t$.

The radiated energy is concentrated in the side bands and for this reason is often called the *carrier-suppression* method. When E_B is zero; that is, no modulation signal impressed; there is no radiation. This method of operation presupposes the addition of an equivalent carrier frequency voltage at the receiving end. Its principal advantages

Oscillator or r-f power amplifier					A-f power amplifier or modulator							
Max. operating d-c volts modulated	Max. operating d-c volts non-modulated	Max. operating a-c (r-m-s) volts	Max. plate dissipation, watts	Max. r-f grid amperes	Max. operating plate volts	Max. plate dissipation watts	Normal plate, volts*	Grid bias, volts†	Plate, amperes	Normal output, watts	Average a-c plate resistance, ohms	Average mutual conductance, micromhos
135	180	180	5	....	180	3.5	180	− 40.5	0.020	0.7	2,000	1,500
350	450	450	15	5	425	12	425	− 39	0.018	1.7	5,000	1,600
1,000	1,250	1,500	100	7.5	1,250	75	1,000	− 55	0.072	10.0	3,400	3,530
200	250	250	10	....	250	7.5	250	− 50	0.030	1.7	2,000	1,750
......	......	......	......	....	450	30	450	− 84	0.055	4.5	1,800	2,100
......	......	......	......	....	425	12	425	− 8	0.0075	....	21,500	1,400
......	......	......	......	....	425	12	425	− 100	0.028	3.0	2,500	1,200
......	......	......	......	....	1,250	75	1,000	− 147	0.075	20.0	1,800	3,000
12,000	15,000	15,000	10,000	30	12,000	7,500	10,000	−1,000	0.750	....	2,400	3,300
2,000	2,500	2,500	400	10	3,000	300	{ 3,000	− 132	0.100	100	3,200	6,000
							{ 2,000‡	− 73	0.150	65	2,800	6,800
2,000	2,500	2,500	750	10	2,500	600	2,000	− 65	0.300	100	1,400	15,000
8,000	10,000	10,000	5,000	20.0	10,000	5,000	8,000	− 500	0.65			
275	350	350	20	....	350	15	350	− 22.5	0.033	1.0	3,500	2,000
750	1,000	1,000	65	....	1,000	65	750	− 30	0.065	....	3,200	3,900
1,300	......	......	520	....	2,000	250	1,500	− 60	0.015	....	2,150	7,500

* These voltages are maximum values for usual broadcast transmitting uses.
† Measured from mid-point of filament.
‡ At 2,000 volts modulator output is greater than that from UV-204A at 2,000 volts.

lie in the efficiency of operation and in the fact that the service area of the transmitter and its interference area practically coincide. For perfect balance the effective m is infinite.

33. Single Side-band Transmission. If suitable filters be arranged to prevent the transmission of all frequencies below the carrier frequency, the upper side band only will be transmitted. Its principal advantage over the carrier-suppression method is that at the receiving end the supplied carrier frequency does not have to be so nearly identical in frequency with the original carrier frequency for the same degree of distortion. With single side-band transmission the signal may be completely lost due to selective frequency fading, while in the carrier-suppression method at least one of the two side bands may get through, providing half-amplitude response.

34. Frequency Inversion; Scrambling. To insure secrecy of transmission, frequency inversion or frequency scrambling methods are used.

The modulation signal modulates an oscillator of approximately 50,000 cycles. If the modulation frequencies range between 100 and 3,000 cycles, filters are arranged in the output of the modulated oscillator to pass only those frequencies between 47,000 and 49,900 cycles. The frequencies in this selected range are made to heterodyne with an oscillator at a frequency of 42,000 cycles. A second filter system is arranged to pass only those frequencies between 5,000 and 7,900 cycles, which are the resulting beat frequencies. It may be observed that the original 3,000-cycle modulation signal frequency now has a corresponding amplitude at 5,000 cycles. The original 100-cycle modulation frequency signal now has a corresponding amplitude at a frequency of 7,900 cycles. The frequency scale of the original modulation signal has been inverted by the process. These frequencies may, if desired, be further lowered by a second heterodyning step against a 4,900-cycle oscillator which would produce the inverted scale of frequencies ranging from 100 to 3,000 cycles. This inverted modulation frequency is then impressed as the normal modulation upon a carrier-frequency oscillator. To obtain an intelligible signal at the receiving end, it is necessary to again invert the modulation frequencies by the reverse process. To anyone not having at hand the proper equipment and not knowing the correct frequency at which the inverting was done the transmission would be indecipherable.

METHODS OF MEASURING MODULATION

35. Inferential Methods. Percentage of modulation is a most important factor in modulation problems. Its value may be determined in two general ways, the first based on the process of producing the modulation, and the second on measurements of the modulated carrier itself.

The factor of modulation is given in Eq. (50). Determining the values of a and b under the operating conditions and measuring E_B enables m to be calculated.

Another inferential method is based on the oscillator amplitude characteristic as shown in Fig. 27. If the plate supply voltage increased 10 per cent, the output voltage will also increase 10 per cent. This simple relation indicates that

$$m = \frac{E_B}{E_p} \tag{65}$$

where E_B represents the peak value of the modulation signal voltage and E_p represents the applied plate voltage. Occasionally the function shown in Fig. 27 will be a straight line, but its intercept on the E_p axis will not be at the point 0, 0. In this case the value of E_p in Eq. (65), will not be the actual plate voltage but the indicated voltage along the E_p axis, between the straight-line intercept and the applied plate voltage.

Another inferential method is to produce an equivalent modulated signal, not by the actual process of modulation, but by the simultaneous transmission of two or more frequencies differing by the desired modulation signal frequency. The factor of modulation is equal to the amplitude of the transmitted frequency component corresponding to the side band divided by the amplitude of the frequency chosen as the carrier. Independent measurement of the voltage amplitudes of these components enables m to be calculated.

36. Methods Based on Shape of Modulated Output. The first method based on the shape of the modulated output as indicated in *d* (Fig. 24). Before modulation, the peak voltage of the carrier-frequency component is measured with a peak voltmeter. After modulation, the positive peak voltage is again measured. The increase in peak voltage divided by the carrier peak voltage gives the percentage of modulation. The principal disadvantage in this method is that, due to improper adjustment in the oscillator-modulator system, the effective carrier may change in

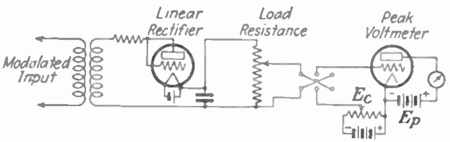


FIG. 30.—Circuit for measuring modulation.

amplitude from its unmodulated value when modulated. This error may be avoided by measuring the amplitude of the maximum and minimum peaks. The factor of modulation is then given by

$$m = \frac{E_{\text{max.}} - E_{\text{min.}}}{E_{\text{max.}} + E_{\text{min.}}} \tag{66}$$

A circuit diagram for such a measuring arrangement is shown in Fig. 30. The rectifier is linear, giving a voltage (d.c. and at the modulation frequency) across the load corresponding (in proportion) to the carrier plus and minus the modulation component. The positive peak is first measured with the peak voltmeter, and then by reversing, the negative peak is measured, both with reference to zero load voltage. Substituting in (66) gives m.

37. Use of Detector to Measure Modulation. A simple way of measuring the percentage of modulation is to apply the modulated signal to the grid of a detector tube, and observing the d-c change in plate current and the alternating component at the modulation frequency. The change in d.c. may be read with a d-c meter while the alternating component may be calculated by measuring the modulation-frequency voltage across a known resistance in the plate circuit. Figure 31 shows a curve between percentage of modulation and the ratio between the effective value of the a.c. and change in the d-c plate circuit.

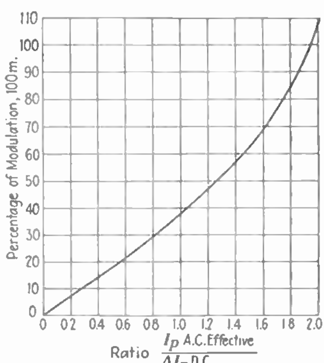


FIG. 31.—Use of change in detector plate current as measure of modulation.

38. Use of Cathode-ray Oscilloscope. The modulated signal is applied to one pair of control plates while the modulation frequency obtained by rectifying a portion of the modulated carrier frequency is applied to the other pair of control plates. The resulting figure, with proper adjustment of phase, will be an isosceles trapezoid. The greater of the parallel lines is the peak amplitude, while the lesser of the two corresponds to the minimum amplitude of the applied voltage.

$$m = \frac{l_1 - l_2}{l_1 + l_2} \tag{67}$$

where l_1, l_2 are the lengths of the parallel sides referred to above. Another method of using an oscilloscope or oscillograph to obtain the value of m is as follows: As in the positive and negative peak measurement method, a linear rectifier is necessary to supply the oscillograph. Instead of measuring peak values of the rectified signal, the curves are actually observed in the image glass or measured from a photograph and calculated as in (66). Customarily a second line image, initially coinciding with the zero input line of the vibrating element, is used to indicate the zero condition. If the negative peaks reach this zero line, 100 per cent modulation is being obtained.

39. Change of Modulation Due to Resonance Characteristic. When a modulated carrier is impressed on a circuit having resonance characteristics the percentage of modulation is changed. The action may be considered from either the frequency amplitude response characteristic of the resonant circuit or from the energy storing and decrement properties of the circuit. If the circuit resonance frequency is made to coincide with the carrier frequency the new factor of modulation is

$$m' = m\left(\frac{R}{\sqrt{R^2 + \frac{4L}{C}\left(\frac{f_m}{f_c}\right)^2}}\right) \tag{68}$$

where L, R, and C are the inductance, series resistance, and capacity of the circuit at resonance to the carrier frequency f_c, and f_m is the modulation frequency. In Eq. (68) the effect of R is to decrease the change in percentage of modulation. In transmitters resistance is often added to the resonance circuits to prevent change in modulation percentage at high modulation frequencies.

40. Frequency Multipliers. It is often desired to produce frequency multiplication by an asymmetrical characteristic such as illustrated in Fig. 23. A further expansion of (47) shows various components having frequencies $2A/2\pi$, $3A/2\pi$, $4A/2\pi$, etc. If double carrier frequency is desired, there are four components having the frequency $2A/2\pi$ in the expansion indicated. Two of these may be considered as the new carrier. The second is a term involving $\cos 2At \sin Bt$, indicating side bands with respect to the new carrier at a frequency difference corresponding to the desired modulation frequency. The fourth component involving $\cos 2At \cos 2Bt$ indicates double-frequency modulation and is undesired. The desired modulation frequency is due to third-order curvature, while the undesired double frequency is due to fourth-order curvature. If two identical tubes are used, the third-order components may be increased by push-pull connection in the grid circuits and by connecting the plate circuits in parallel. This method of connection also tends to cancel the fourth-order effect and so produces a substantially undistorted modulated carrier of twice the initial carrier frequency.

41. Change in Percentage Modulation. If a modulated signal such as given in Eq. (51) be applied as x in the function of Fig. 23 and the components at a frequency corresponding to the carrier frequency only be considered the response will be

$$y = ax + \frac{1}{8}cx^3 + \frac{1}{192}ex^5 + \; + \tag{69}$$

where a, c, e, etc., are the first, third, fifth, etc., differentials of y with respect to x. In case of a triode,

$$\left.\begin{array}{ll} a = \dfrac{di_p}{de_g} & \\ b = \dfrac{d^3i_p}{de_g{}^3} & y = i_p \\ c = \dfrac{d^5i_p}{de_g{}^5} & x = E_A(1 + m \sin Bt) \sin At \end{array}\right\} \tag{70}$$

Equation (70) shows that the percentage of modulation has been increased so that

$$m' = m\left(1 + \frac{1}{8}x^2\frac{c}{a} + \; + \right) \tag{71}$$

This change in modulation is undesired, as it destroys a linear input-output relation and introduces 2nd and higher harmonics of the modulation frequency and consequent distortion. It may be reduced by decreasing the value of c. Vacuum tubes having a value of c small under any voltage conditions are used for variable gain control amplifier tubes. The 235 type is such a tube.

42. Cross modulation results when two modulated signals are impressed simultaneously upon an asymmetrical amplifier. This means that the modulation of both signals may be obtained when tuned on either carrier. The effective percentage of modulation for the interfering signal with respect to the desired carrier is

$$m' = m_i\frac{E_i{}^2}{2}\frac{c}{a} \tag{72}$$

where m' is the effective modulation of the desired carrier E_c, by the undesired modulation frequency, m_i is the modulation factor of the interfering carrier E_i, c and a are the third and first differentials of the x, y function (see Fig. 23) respectively, evaluated under operating conditions. In the case both signals are impressed on the grid of a triode,

$$a = \frac{di_p}{de_g}, \qquad c = \frac{d^3i_p}{de^3_g}$$

43. Band Width Necessary. The transmission of intelligence by modulated carrier frequencies requires a definite portion of the frequency spectrum and implies a limit to the number of such carrier channels available. In the case of wire lines the total number of channels is given by the available frequency spectrum divided by the band width, times the number of pairs of wires, times the multiplexing factor. In the case of a radiated wave the number of channels available is the available frequency spectrum (at present, frequencies between 10^4 and 10^8 cycles per second) divided by the band width.

The total band widths necessary for various types of modulation intelligence symbols are indicated on page 278.

Modulation intelligence symbol	Character or rate	Total band width cycles	
		Necessary by best known methods	Desirable for high quality
Telegraph	English, Continental code, 200 words per minute	40	100
Telephony	Speech	2,500	6,500
Telephony	Music	4,500	15,000
Picture	1 sq. in. 60 lines per inch, per second	1,800	6,000
Television	1 sq. in. 50 lines per inch, 16 pictures per second	15,000	40,000

These band widths are single side-band widths except for picture and television modulation, where both side bands give a more acceptable response.

44. Frequency modulation is the term applied to that type of modulation where the carrier is varied in frequency rather than in amplitude by the modulation signal. If the carrier is wobbled plus and minus $B/2\pi$ times a second, and a device proportionally responsive to the frequency be used at the receiver, an effective response at the frequency $B/2\pi$ will be obtained as a demodulated signal. The response amplitude is supposed to be directly proportional to the carrier-frequency variation, a wobble of 500 cycles giving ten times the response of a 50-cycle wobble. Mathematical considerations for sine-wave frequency modulation give an equation for the various frequency components having Bessel function coefficients as follows:

$$\left.\begin{aligned} q &= J_0 n \cos At \\ &\quad - J_1 n[\cos (A - B)t - \cos (A + B)t] \\ &\quad + J_2 n[\cos (A - 2B)t - \cos (A + 2B)t] \\ &\quad + J_3 n[\cos (A - 3B)t - \cos (A + 3B)t] \\ &\quad + \cdots\cdots \\ \text{where} \qquad n &= \frac{\Delta A}{B}. \end{aligned}\right\} \tag{73}$$

With n having the following values, the respective side-band amplitudes are:

n	Unmodulated carrier amplitude	Modulated carrier amplitude	$\frac{B}{2\pi}$	$\frac{2B}{2\pi}$	$\frac{3B}{2\pi}$	$\frac{4B}{2\pi}$
0.1	1	0.998	0.058			
1.0	1	0.763	0.440	0.115	0.020	0.003

This analysis shows that frequency modulation occupies not less but more of the frequency spectrum.

45. Square-top Waves. For intermittent frequency modulation by keying, the coefficients of the Fourier series are those resulting from the square-top wave. For this case the equation similar to (73) is

$$
\begin{aligned}
q = \frac{2}{\pi}\Bigg[& \frac{n}{n^2} \sin\left(\frac{\pi}{2}n\right) \cos At \\
& + \frac{n}{n^2 - 1^2} \cos\left(\frac{\pi}{2}n\right) \{\cos (A - B)t - \cos (A + B)t\} \\
& - \frac{n}{n^2 - 2^2} \sin\left(\frac{\pi}{2}n\right) \{\cos (A - 2B)t - \cos (A + 2B)t\} \\
& - \frac{n}{n^2 - 3^3} \cos\left(\frac{\pi}{2}n\right) \{\cos (A - 3B)t - \cos (A + 3B)t\} \\
& + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \Bigg]
\end{aligned}
\tag{74}
$$

Bibliography on Detection

AIKEN, C. B.: Detection of Two Modulated Waves Which Differ Slightly in Carrier Frequency, *Proc. I.R.E.*, January, 1931.

BAILEY, AUSTIN, and T. A. MCCANA: Application of Printing Telegraph to Long Wave Radio Circuits, *Bell System Tech. Jour.*, October, 1931.

BALLANTINE, STUART: Detection at High Signal Voltages—Plate Rectification with the High-vacuum Triode, *Proc. I.R.E.*, July, 1929.

———: Detection by Grid Rectification with the High-vacuum Triode, *Proc. I.R.E.*, May, 1928.

BARCLAY, W. A.: The Numerical Estimation of Grid Rectification for Small Signal Amplitudes, *Exper. Wireless and Wireless Eng.*, November, 1929.

BROWN, H. A., and C. T. KNIPP: Behavior of Alkali Vapor Detector Tubes, *Proc. I.R.E.*, January, 1927.

BUTTERWORTH, S.: Note on the Apparent Demodulation of a Weak Station by a Stronger One, *Exper. Wireless and Wireless Eng.*, November, 1929.

CHAFFEE, E. L.: Voltage Detection Coefficient, *Proc. I.R.E.*, November, 1927.

———, and G. H. BROWNING: A Theoretical and Experimental Investigation of Detection for Small Signals, *Proc. I. R. E.*, February, 1927.

COLEBROOK, F. M.: The Theory of the Straight-line Amplifier, *Exper. Wireless and Wireless Eng.*, November, 1930.

DONLE, H. P.: New Applications of the Sodion Detector, *Proc. I.R.E.*, April, 1924.

FREEMAN, H. M.: Detecting Characteristics of Electron Tubes, *Proc. I.R.E.*, October, 1925.

HARRIS, SYLVAN: Notes on the Detection of Large Signals, *Proc. I.R.E.*, October, 1929.

NELSON, J. R.: Detection with the Four-electrode Tube, *Proc. I.R.E.*, June, 1928.

———: Grid Circuit Power Rectification, *Proc. I.R.E.*, March, 1931.

———: Notes on Grid Circuit Detection, *Proc. I.R.E.*, March, 1929.

ROBERTS, W. VAN B.: Note on Detection by Grid Condenser and Leak, *Proc. I.R.E.*, September, 1927.

ROBINSON, G. D.: Test Procedure for Detectors with Resistance Coupled Output, *Proc. I.R.E.*, May, 1931.

RODER, HANS: Amplitude, Phase, and Frequency Modulation, *Proc. I.R.E.*, December, 1931.

SMITH, L. P.: Theory of Detection in a High-vacuum Thermionic Tube, *Proc. I.R.E.*, October, 1926.

TERMAN, F. E.: Some Principles of Grid-leak Grid-condenser Detection, *Proc. I.R.E.*, October, 1928.

———, and BIRNEY DYSART: Detection Characteristics of Screen-grid and Space-charge-grid Tubes, *Proc. I.R.E.*, May, 1929.

———, and THOMAS M. GOOGIN: Detection Characteristics of Three-element Vacuum Tubes, *Proc. I.R.E.*, January, 1929.

WHEELER, H. A.: Discussion on W. A. MacDonald Paper, *Proc. I.R.E.*, April, 1927.

Bibliography on Modulation

AFFEL, H. A., C. S. DEMAREST, and C. W. GREEN: Carrier Systems on Long-distance Telephone Lines, *Bell System Tech. Jour.*, July, 1928.

ARNOLD, H. D., and LLOYD ESPENSCHIED: Transatlantic Radio Telephony, *Bell System Tech. Jour.*, October, 1923.

BALLANTINE and SNOW: Variable Mu Tetrodes, Contribution 22 from the Radio-frequency Laboratories.

BOWN, RALPH, DE LOSS K. MARTIN, and R. K. POTTER: Some Studies in Radio Broadcast Transmission, *Proc. I.R.E.*, February, 1926.

Byrnes, I. F.: Recent Developments in Low Power and Broadcasting Transmitters. *Proc. I.R.E.*, May, 1928.
Carson, J. R.: *Proc. I.R.E.*, February, 1922.
Clokey, A. A.: Automatic Printing Equipment for Long Loaded Submarine Telegraph Cables, *Bell System Tech. Jour.*, July, 1927.
Culver, C. A.: An Improved System of Modulation in Radio Telephony, *Proc. I.R.E.*, October, 1923.
Curtis: The Application of Vacuum-tube Amplifiers to Submarine Cable, *Bell System Tech. Jour.*, July, 1927.
Farnham, P. O., and A. W. Barber: Problems Involved in the Design and Use of Apparatus for Testing Radio Receivers, *Proc. I.R.E.*, August, 1930.
Gray, Frank, J. W. Horton, and R. C. Mathes: The Production and Utilization of Television Signals, *Bell System Tech. Jour.*, October, 1927.
Harris, Sylvan: Cross Modulation in R-f Amplifiers, *Proc. I.R.E.*, February, 1930.
Hartley, R. V. L.: Relations of Carrier and Side Bands in Radio Transmission, *Proc. I.R.E.*, February, 1923.
———: Transmission of Information, *Bell System Tech. Jour.*, July, 1928.
Heising, R. A.: Production of Single Side Band for Transatlantic Radio Telephony, *Proc. I.R.E.*, June, 1925.
Henney, Keith: "Principles of Radio," John Wiley & Sons, Inc.
Hombald, N. E.: Frequency Modulation, *Exper. Wireless and Wireless Eng.*, May and August, 1929.
Horton, J. W.: Vacuum-tube Oscillator, A Graphical Method of Analysis, *Bell System Tech. Jour.*, July, 1924.
Ives, Horton, Parker, and Clark: Transmission of Pictures over Telephone Lines, *Bell System Tech. Jour.*, April, 1925.
Jarvis, K. W.: Radio Receiver Testing Equipment, *Proc. I.R.E.*. April, 1929.
Jolliffe, C. B.: The Uses of the Electron-tube Peak Voltmeter for the Measurement of Modulation, *Proc. I.R.E.*, April, 1929.
Jones, W. C.: Condenser and Carbon Microphones, Their Construction and Use, *Bell System Tech. Jour.*, January, 1931.
King, R. W.: Thermionic Vacuum Tubes and Their Applications, *Bell System Tech. Jour.*, October, 1923.
Lauer, H.: *Exper. Wireless and Wireless Eng.*, August, 1929.
Llewellyn, F. B.: Operation of Thermionic Vacuum-tube Circuits, *Bell System Tech. Jour.*, July, 1926.
Makey: *Exper. Wireless and Wireless Eng.*, July, 1929.
Martin, W. H., and W. F. Davidson: The Trend in Design of Telephone Transmitters and Receivers, *Bell System Tech. Jour.*, October, 1930.
Measurement of Percentage Modulation, *R.C.A. Broadcast Bull.* 3; G. R. Experimentor, March, 1931.
Milnor: Submarine Cable Telegraphy, *Trans. A.I.E.E.*, p. 20, 1922.
Morecroft, J. H.: "Principles of Radio Communication," John Wiley & Sons, Inc.
Moullin: "High-frequency Measurements," J. B. Lippincott Company.
Nelson, E. L.: Radio Broadcasting Transmitters and Related Transmission Phenomena, *Bell System Tech. Jour.*, January, 1930.
———: Radio Transmission System for Television, *Bell System Tech. Jour.*, October, 1927.
Peterson, Eugene, and Clyde R. Keith: Grid-current Modulation, *Bell System Tech. Jour.*, January, 1928.
——— and Herbert P. Evans: Modulation in Vacuum Tubes used as Amplifiers, *Bell System Tech. Jour.*, July, 1927.
——— and F. B. Llewellyn: The Operation of Modulators from a Physical Standpoint, *Proc. I.R.E.*, January, 1930.
Raven-Hart, R.: Discussion of F. K. Vreeland's "On the Distortionless Reception of a Modulated Wave and Its Relation to Selectivity," *Proc. I.R.E.*, October, 1928.
Shackelton, W. J., and J. G. Ferguson: Electrical Measurement of Communication Apparatus, *Bell System Tech. Jour.*, January, 1928.
Terman, F. E.: Discussion on Some Possibilities of Intelligence Transmission when Using a Limited Band of Frequencies, *Proc. I.R.E.*, July, 1930.
———: Some Possibilities of Intelligence Transmission When Using a Limited Band of Frequencies, *Proc. I.R.E.*, January, 1930.
Van der Bijl, H. J.: "The Thermionic Vacuum Tube and Its Applications," McGraw-Hill Book Company, Inc.
Van der Pol, B.: Frequency Modulation, *Proc. I.R.E.*, July, 1930.
Wright, S. B., H. C. Silent: The New York-London Telephone Circuit, *Bell System Tech. Jour.*, October, 1927.

SECTION 11

AUDIO-FREQUENCY AMPLIFIERS

By Julius G. Aceves, E.E.[1]

1. Methods of Coupling Amplifier Tubes. The a-f amplifier consists of several vacuum tubes connected together in concatenation in such manner that frequencies ranging from about fifty to ten thousand cycles are amplified successively from one stage to another following a predetermined frequency-gain law.

In the majority of cases, this law is a straight line, but in some cases it is desirable to depart from it.

There are three general methods of coupling the vacuum tubes with each other in concatenation: (*a*) *inductively;* (*b*) *capacitatively;* and (*c*) *galvanically.*

An example of (*a*) is the transformer coupling; of (*b*), the resistance-capacity coupling; and of (*c*), a direct-coupled system. Combinations of these three methods will constitute a fourth class, by far the most extensive. There are ways in which tube circuits are coupled undesirably and they may include any of the types mentioned above. These effects will either reinforce certain frequencies even to the point of oscillations, or will reduce the gain at some others. This subject will be treated under the heading of distortion.

2. General Requirements of Audio-frequency Amplifiers. The conditions imposed upon the design of an a-f amplifier are as follows:

1. The gain must conform to a certain given pattern of frequency-amplification characteristic.
2. The output e.m.f. across the load must be a sine wave when at the input the impressed e.m.f. is a sine wave.
3. Ability to fulfill the first two requirements within a certain given maximum amplitude of applied e.m.f. and delivered power level.
4. Should not generate any e.m.f. of itself, such as "hum" from the power supply mains, or from tube noises.
5. The total gain should remain substantially constant regardless of operating conditions such as line voltage or temperature of the filaments, etc.

The choice of coupling depends, as a rule, upon the way in which the different parts of the frequency spectrum are to be amplified.

3. Elements of an Audio-frequency Amplifier. Each stage of an a-f amplifier consists of a vacuum tube, an input coupling device, an output coupling device, and a suitable source of power to actuate the vacuum tube.

The input and output coupling devices may be common to two consecutive stages, as well as the sources of power. By the latter may be

[1] Engineer in Charge of Research Laboratory, Amy, Aceves, and King, Inc.

understood more specifically the resistors, condensers, and inductors which bring in the desired A, B, and C voltages and keep out the amplified currents from other parts of the amplifier and the interfering currents proceeding from the power source, which is usually a commercial 110–220 volt a-c or d-c supply except when batteries are used.

4. Stage Gain. In the greater majority of cases a triode is used in each stage of amplification. Pentodes are becoming fashionable for the last, or "power" stage, and tubes of from four to eight electrodes have been introduced in specially designed amplifiers.

The essential properties of the triodes are the amplification factor μ; the mutual conductance G_m; the internal resistance of the anode circuit r_p; the capacity and conductance of the input or grid circuit c and g; the power-handling ability of the tube with a given limit of distortion $P_{\max.}$; and the power sensitivity W. The vacuum tube acts as a source of alternating e.m.f. when connected according to the schematic diagram of Fig. 1, having an internal resistance r_p equal to μ/G_m. If an e.m.f. $e_1 = E_1 \sin \omega t$ is impressed upon the grid input circuit of the tube (Fig. 1) an e.m.f. $e_2 = E_2 \sin \omega t$ will be developed across the output circuit impedance $\dot{Z} = R + jX$ whenever we operate in the linear portion of the tube characteristic. The ratio between output and input e.m.fs is called the *gain* and is given by the expression

$$\gamma = \frac{e_2}{e_1} = \mu\frac{\dot{Z}}{\dot{Z} + r_p} \tag{1}$$

and in terms of G_m:

$$\gamma = G_m\frac{r_p\dot{Z}}{\dot{Z} + r_p} \tag{2}$$

It is convenient to express the gain logarithmically, in which case the unit is the decibel which is equal to twenty times the common logarithm of γ. Decibels are usually logarithmic power ratios, because they do not

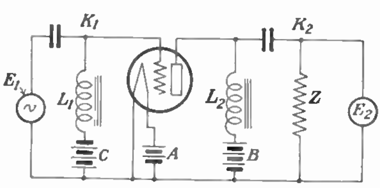


FIG. 1.—Simple amplifier circuit.

involve impedance changes. To express voltage ratios in decibels, it is assumed that these quantities are taken across the same impedances. With this proviso in mind, it is very easy to calculate the total gain of an amplifier by adding the gains of the various stages when expressed in decibels.

5. Input Impedance. In Fig. 1 the d-c potential supply sources A, B, and C are shown separated from the a-c components by means of chokes L_1 and L_2 of infinite impedance and condensers K_1 and K_2 of vanishingly small reactance. Now the question will arise: what is the input impedance of the tube under these conditions? It is easier to consider the reciprocal of the impedance, namely, the admittance $\dot{Y} = G + jB$. These quantities vary considerably depending upon the plate impedance $\dot{Z} = R + jX$ and whether X is positive or negative. The capacity c between grid and filament is equal to $(\gamma + 1)$ times the geometrical capacity of the tube elements (such as would be obtained by measurement with the filament not lighted). The sign of the conductance G may be either positive or negative depending mostly upon the sign of X and its magnitude. When the plate circuit contains inductive reactance with relatively low resistance, G may become negative, and the tube may break into oscillations, particularly at very high frequencies. This may explain certain cases where a very bad distortion takes place in an a-f amplifier even when other conditions seem to be right. In some other cases, the sign of G may be positive and will show as a low resistance input circuit, thereby loading down the source of e.m.f. applied to the grid and reducing the gain. It is of considerable importance to determine the input impedance of a tube under actual conditions, because this factor alone may completely upset the results of carefully designed coupling circuits between stages, and in practice it resolves itself in a limitation of gain per stage. The design of the following examples of inductive, capacitative, and galvanic couplings will show how the input impedance of a triode affects the maximum possible gain per stage without departure from the frequency-amplitude law to which the amplifier must conform.

6. Properties of the Power Tube. The power-handling ability and the power sensitivity relate to the final stage of amplification. The first refers to the rating of the tube for the amplification of a sinusoidal wave without distortion. Obviously this is never possible, as there is no triode with a perfectly linear characteristic. Modern tube design, however, has made great progress in this direction and in order to determine the rating of a tube from the standpoint of distortion, the Institute of Radio Engineers has recommended 5 per cent as the maximum voltage distortion permissible. Consequently, if a tube is placed as an a-f amplifier supplying a non-inductive load of the proper value, and a sinusoidal e.m.f. is applied between grid and filament (or cathode), the harmonic components of the total alternating e.m.f. across the load must not exceed 5 per cent of the fundamental. The *power sensitivity* is the watts output per volt input impressed upon the grid and filament. The output should be measured using a non-reactive load of optimum value, which, it has been shown, for distortionless amplification should be equal to twice the value of r_p or anode a-c resistance. The power sensitivity is ordinarily considered only in connection with the final stage and the modern pentodes have an abnormally high power sensitivity, hence their popularity.

INTERSTAGE COUPLING CIRCUITS

7. Inductively Coupled Stages. The simplest form of this kind of coupling is the transformer. Any transformer may be regarded as a simple circuit having an effective resistance and an effective reactance, given by the expressions

$$R' = R_1 + \frac{\omega^2 M^2 R_2}{\omega^2 L_2^2 + R_2^2} \tag{3}$$

$$X' = \omega\left(L_1 - \frac{\omega^2 M^2 L_2}{\omega^2 L_2^2 + R_2^2}\right) \tag{4}$$

where R_1 is the effective primary resistance, R_2 that of the secondary, M the mutual inductance, L_1 and L_2 the effective inductances of the primary and secondary windings respectively.

In transformers of modern design, $M^2 = L_1L_2$ may be assumed without serious error, and if the ratio of transformation is n, then the circuit of Fig. 2 can be simplified as per Fig. 3. Further simplification may be obtained if we ignore the resistance of the primary winding R_1, and we consider R_2 as made of both the winding resistance and the load. The effective inductance L_2 accounts for the capacities across the secondary, such as exists between grid and anode of the following tube, between terminals of the tube socket, leads, etc., as well as the distributed capacity of the transformer windings. Another capacity is that between the primary and secondary windings; there is no simple circuit that can accurately represent it; it acts more in the fashion of a so-called "wave conductor" such as those in use at radio frequencies for the purpose of shifting the phases of received signals from two antennas in

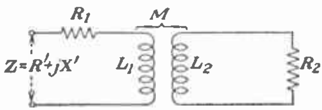


FIG. 2.—Transformer circuit.

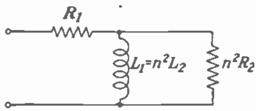


FIG. 3.—Equivalent of transformer.

static-balancing schemes. The effects at the lower audio frequencies of such distributed capacity are negligible, but toward the upper end of the audio spectrum there is a tendency to introduce a rise of potential due to the combination of this capacity and other distributed capacities in the transformer as well as in the attached apparatus with the leakage inductance of the transformer forming a resonant circuit of comparatively high damping. Many frequency-gain curves taken with commercial a-f transformers have a "peak" toward five or six kc, sometimes a little higher. The latter case is far from being objectionable, since in the case of radio reception the extreme ends of the side bands are poorly amplified and the combined result of the two effects is to approach a better fidelity curve all around.

8. Eddy Current and Hysteresis. The eddy-current losses in the laminations of the transformer have been taken into account as part of the resistance R_2; they may be considered as an extra resistance in parallel with the load. The effect of the eddy-current resistance alone upon the primary effective resistance is proportional to the square of the frequency. The hysteresis has an effect of appearing as an added primary resistance proportional to the frequency and to the 1.6 power of the flux density (in transformer-iron laminations); but as a-f transformers are not operated beyond the first knee of the magnetization cycle, hysteresis losses are negligible in all cases except perhaps in the power-stage output transformer. Even then, the secondary circuit being closed on a comparatively low resistance load, the power lost in hysteresis and eddy currents is small compared with the power delivered to the load.

9. Effects of Transformer Impedance at Various Frequencies. It has been seen that the general expression for the gain per stage is

$$\gamma = \mu \frac{\dot{Z}}{\dot{Z} + r_p} \tag{5}$$

By substituting the value of $\dot{Z} = R' + jX'$ in the above equation, the variation of amplification with frequency for a given combination of tube and transformer can be predetermined. Equations 3 and 4 give us the values of R' and X' in general.

If the secondary circuit of a transformer has a very close coupling ($M^2 = L_1L_2$), and assuming for simplicity that the ratio of transformation is unity, it will be seen that the transformer will act as a resistance of constant value and equal to the secondary load, with an inductance in parallel with it, as shown

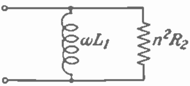


FIG. 4.—Transformer with unity ratio and close coupling.

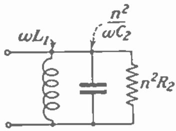


FIG. 5.—Transformer with external and internal capacities.

in Fig. 4. If the stray capacities either internal or external to the transformer are taken into account, and the total capacity as it would appear across the secondary called C_2, then the circuit would behave as in the schematic diagram of Fig. 5. To transform the parallel circuits of Figs. 4 and 5 into equivalent series circuits so that the values of R' and X' may be obtained to be substituted in the above formula for the gain per stage (Eq. 1), the parallel circuits can be expressed in terms of a conductance and susceptance in parallel. The conductance G' is $1/R_1$ (or $1/n^2R_2$) and the susceptance B' is

$$\frac{1}{\omega L_1 - \dfrac{n^2}{\omega C_2}} \tag{6}$$

Then,

$$R' = \frac{G'}{G'^2 + B'^2} \tag{7}$$

and

$$X' = \frac{B'}{G'^2 + B'^2} \tag{8}$$

If both ωL_1 and $n^2/\omega C_2$ are large in comparison with $n^2R_2(= R_1)$ for any value of ω within the a-f range, then the transformer will act as a pure resistance, since B'^2 would be very small in comparison with G'^2 and Eq. (7) will become $R' = 1/G' = n^2R_2 = R_1$ and Eq. (8) will make X' vanishingly small. Hence, to make a transformer-coupled amplifier of uniform gain and without phase distortion, all that is necessary is to have transformers with very little leakage with a high open-circuit primary inductance and very low distributed capacity and to connect a resistance across either the primary or the secondary of such magnitude that the parallel equivalent reactances ωL_1 and $n^2/\omega C_2$ of Fig. 5 will be large compared with the resistance at any frequency which is desired to be transmitted. Some very good transformers used in telephone work have a third winding short-circuited in itself of a comparatively high resistance. This has the same effect as connecting an external resistance across the secondary or primary windings. The eddy-current losses in many commercial types of audio transformers have an equivalent effect.

It should be noted that if in Eq. 1 we make $\dot{Z}$ large in comparison with r_p for any frequency to be amplified uniformly, the gain will be practically equal to μ and chiefly to this fact is due the very flat characteristic that most modern audio-frequency transformer amplifiers show, even when no resistance is connected across either winding of the transformer. To illustrate this, let an actual case be considered of a choke-coil amplifier (which is nothing more than a transformer in which the ratio is unity and the coupling between windings perfect). Let the inductance of the coil be 100 henrys and the total distributed capacity of 0.001013 μf.

At 50 cycles, $\omega L = 314{,}200$ ohms and $1/\omega C = 3{,}142{,}000$ ohms.

At 5,000 cycles, $\omega L = 3{,}142{,}000$ and $1/\omega C = 31{,}420$ ohms.

At 500 cycles both reactances will be of 314,200 ohms, and the impedance of the circuit practically infinite. At this frequency, which is the geometric mean of the audio broadcast range, the amplification will be the mu of the tube, which in the illustration will be assumed to be 8, and the plate resistance 6,000 ohms.

At any of the two extreme frequencies the reactances of the circuit will be $\pm \dfrac{(3{,}142{,}000 \times 31{,}420)}{3{,}142{,}000 - 31{,}420} = \pm 31{,}700$ ohms. Rationalizing Eq. (1),

$$\gamma = \mu \frac{31{,}700}{\sqrt{(31{,}700)^2 + (6{,}000)^2}} = 0.982 \times 8$$

which means that the amplification will go down by 2 per cent at the extreme ends of the frequency band with respect to the value that assumes at the geometric mean frequency.

10. Compensating Circuits with Transformer-coupling Stages. The ideal performance of reproduced speech and music is approached when the original air-pressure variations at the transmitting end are reproduced at the receiving end with the same instantaneous value, or, at least, with proportionally varying amplitudes. As there are many causes of distortion, to obtain as faithful a reproduction as is possible in practice, a distortion equal and opposite should be superimposed, so that the net result shall be corrected from the excess or defect of certain frequencies or frequency bands.

In general, there is a tendency to underamplify the extreme ends of the musical scale and to overamplify certain particular frequencies below the middle, such as originate from acoustic resonance. For this purpose, there are a multitude of circuits which have been proposed or put in operation in the audio-frequency amplifiers.

11. Parallel-feed, Series-resonance Circuit. Figure 6 shows diagrammatically the simplest form of parallel plate-feed, series-resonant circuit coupling. The inductance L_0 may have any value such that its reactance is large in comparison with that of the primary of the transformer L_1 with the condenser C in series, at any frequency.

It will be noted that the impedance of the primary of the transformer at very low audio frequencies is practically ωL_1. The voltage across the secondary will be $n^2\omega L$ times the current.

By substituting the values of the impedance in Eq. (1) the gain per stage may be obtained. To see more clearly how the gain varies with frequency, let ω_0 be the frequency (in radians) for which $\omega L_1 - \dfrac{1}{\omega C} = 0$ and the ratio of any frequency ω to ω_0 will be designated by N; so that $\omega = N\omega_0$. Let Q be the ratio $\omega_0 L_1/r_p$ and assume that the resistance of the primary is small in comparison with r_p; then Eq. (4) in terms of these quantities will become:

$$\gamma = \mu \frac{nNQ}{\sqrt{1 + Q^2\left(N - \frac{1}{N}\right)^2}} \qquad (9)$$

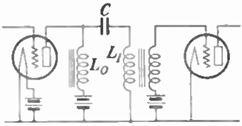


FIG. 6.—Series-resonant coupling circuit.

Analyzing this equation, it will be apparent that if Q is large, say over 4, and N is more than 3 or 4, the amplification or gain γ is practically equal to μn. For frequencies near ω_0 the gain will be greater, and at ω_0 the gain will be $\gamma = nQ$. Hence, the maximum amplification takes place at ω_0 and is Q times as large as it is at all other frequencies far removed from ω_0, say two or more octaves. Figure 7 shows the variation of γ with frequency for different values of Q and ω_0. Graph A shows the gain at various frequencies for $Q = 10$ and $\omega_0 = 314$ radians (50 cycles). It is altogether too sharp for good low-frequency compensation. With $Q = 5$ curves B and C were calculated for $\omega_0 = 314$ and 377 radians respectively (50 and 60 cycles). If one of the stages is tuned to 50 cycles and the next to 60, the combined amplification will be represented by curve D. By shunting the primary of the transformer with a variable resistance, the rise will be controllable even to a point of actually depressing the amplification curve at the frequencies around or below ω_0, and the amplifier will offer the characteristics of a resistance-coupled circuit.

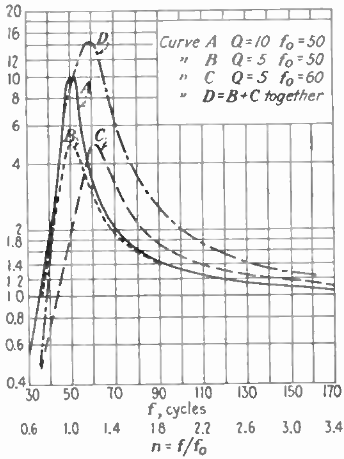


FIG. 7.—Variation of gain (γ) with frequency.

12. Increasing Amplification at High Frequencies. There are cases when it is desirable to increase the amplification at the higher frequencies—for example, in ultraselective superheterodynes, in phonograph reproduction, or in sound moving pictures with film recording where the audio amplifier is far away from the light cell. Among the various schemes used in this connection, two of the simplest expedients will be mentioned. One method consists of utilizing the capacity coupling between windings of the audio-frequency transformers which, combined with the leakage reactance and the capacity of the tube input circuit, form a network similar to the schematic diagram of Fig. 8. It is rather difficult to calculate the gain at various frequencies even if the values of C_0, C_2, L_2, and L_0 were known. It is better to connect the audio-frequency transformer windings with such "polarity" that the capacity coupling C_0 will have an additive effect with the mutual induction between windings, and to choose a suitable type of audio transformer with the required leakage reactance. There are many such transformers on the market, particularly of old design.

A second method lends itself to predetermination of the gain and consists of a combination of choke and resistance in the feed circuit of the anode as shown in Fig. 9. The resistance R_2 may be replaced by a choke or by the primary of an audio-frequency transformer.

If, for simplicity, this resistance or impedance is assumed to be large in comparison with the impedance $Z_1 = R_1 + j\omega L_1$ it will be noted that the gain per stage will be given by substituting the value of Z_1 in the fundamental Eq. (1):

$$\gamma = \mu \frac{R_1 + j\omega L_1}{R_1 + r_p + j\omega L_1}$$

or rationalizing,

$$\gamma = \mu \sqrt{\frac{R_1^2 + \omega^2 L_1^2}{(R + r_p)^2 + \omega^2 L_1^2}} \tag{10}$$

It will be noted that as ω increases, $(\omega L_1)^2$ will be large in comparison with R^2 and $(R + r_p)^2$ and the fraction inside the radical will approach unity.

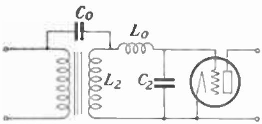


FIG. 8.—Utilizing capacity between windings to raise gain at high frequencies.

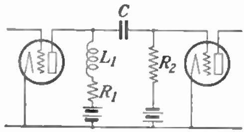


FIG. 9.—Choke and resistance frequency control circuit

For low frequencies, however, $(\omega L_1)^2$ will be small compared to R^2 and to $(R + r_p)^2$ and the gain will tend to the value of

$$\gamma = \mu \frac{R_1}{R_1 + r_p}$$

which would be the gain had there been no choke in the circuit. If we make $R_1 = r_p$, the gain at low frequencies will be nearly one-half the corresponding value at high frequencies, and the effect of duplicating this circuit in two of the stages will make the net ratio between low- and high-frequency gain nearly four, or 12 db. The value of the inductance L_1 should be such that ωL_1 will be sensibly equal to $R_1 + r_p$ at the frequency at which it is desired to begin the boosting, for example, 700 cycles.

There is nothing to prevent the combination of both low- and high-frequency boosting in each stage; all that is necessary is to substitute the resistance R_2 of Fig. 9 for an inductance of the proper value according to previous discussion and Eq. (9).

13. Frequency-band Suppression. There are cases where it is desirable to amplify the low frequencies, the high frequencies, or some intermediate frequency band to a smaller extent than the rest or even to eliminate them altogether. Low-, high-, and band-elimination filters can be employed, but in the majority of cases a simple device is quite suitable, and it may be secured at hand from standard parts. A combination of a variable resistance of about 100 to 100,000 ohms, a fixed condenser, and a multi-tap inductance, such as commercial variable-ratio transformers, will be quite effective in reducing a frequency band.

The resistance with the condenser only will reduce high frequencies; and the resistance with the inductance alone, the low. The place to connect these elements may be either the grid or the anode circuit of any of the stages, according to the values of the resistance, inductance and capacity of these parts and the impedance of the circuit to which they are to be attached. They may be connected in series or in shunt

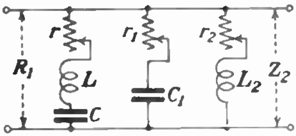


FIG. 10.—Series equalizing circuits.

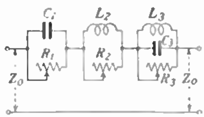


FIG. 11.—Shunt-connected equalizing circuits.

as per Figs. 10 and 11. If the output impedance Z_2 is infinite, as when connecting the circuit to the grid of a tube, the calculation for the voltage reduction at various frequencies is very simple. Representing by Q, as in previous sections, the ratio of the reactance to the resistance of the source (vacuum tube or phonograph pick-up, etc.) and q the ratio of the reactance of the coil to the total resistance of the shunt circuit r, l, c (Fig. 10):

$$Q = \frac{\omega_0 L}{R} \text{ and } q = \frac{\omega_0 L}{r}$$

where

$$\omega_0 = \frac{1}{\sqrt{Lc}} \text{ and } \frac{\omega}{\omega_0} = N$$

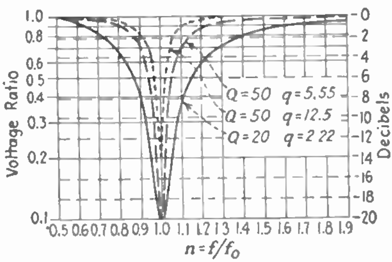


FIG. 12.—Filter cut-off characteristics.

The gain will be

$$\gamma = \mu \sqrt{\frac{Q^{-2} + (N - N^{-1})^2}{(Q^{-1} + q^{-1})^2 + (N - N^{-1})^2}} \tag{11}$$

Figure 12 gives some graphs calculated for various values of Q and q, for $\mu = 1$. It will be noted that the sharpness of the peak is mostly controlled by the selection of Q, while the reduction is governed by the choice of q.

If the impedance Z_2 is not large in comparison with the impedance of the shunt, the results will be only approximate but sufficiently so to enable the designer to make a good choice of parts for an experimental trial from which the final circuit constants can easily be selected. The similarity of the circuits containing inductance or capacity only with resistance (Fig. 10) to the circuit of Fig. 9 will be apparent, and the calculations will be almost identical. In Fig. 11 the circuits in series with the line are quite well known, and only the fact need be pointed out that they are not suitable for operation in connection with a high-impedance load such as the grid circuit of a tube except perhaps in one instance—when the capacity reactance of the grid to filament is low. An inductance with a resistance in shunt with it, such as L_2R_2 (Fig. 11), may work very well in surface-noise elimination in phonograph-record reproduction. The inductance together with the electrode capacity of the tube will act as a single stage of low-pass filter that should be designed to cut off just below 3,500 cycles, as the "scratch" predominent frequency seems to be around 3,700 cycles. The shunt resistance will permit the control of the sharpness and the extent of the cutting off of the upper frequency band so as not to interfere unnecessarily with the reproduction of the overtones and at the same time sufficiently to reduce the obnoxious needle scratch.

14. Capacity-coupled Audio Stages. By this is usually designated the resistance-capacity type of coupling, although we have already discussed some types of coupling involving a condenser in series between the anode of one tube and the grid of the following. There, however, the condenser has been put on mostly for the purpose of obtaining a certain frequency-gain characteristic, while in the resistance-capacity coupling it serves merely to block the steady component of the plate voltage, transmitting only the variable part.

In connection with this type of coupling there are two considerations to examine. First, the value of the coupling condenser for a given value of the grid-leak resistance so as to pass with constant gain all the frequencies above a certain lower limit. Second, the curious instability characteristic of this type of coupling when the B voltage is not of the constant-potential type, commonly called "motor boating" on account of the noise that the loud-speaker makes when this disturbance occurs. This effect has a tendency to limit the capacity of the coupling condenser for a given grid leak. Resistance-capacity amplifiers are coming into use again in connection with television where frequencies of from 20 to 100,000 cycles have to be amplified with equal gain and without phase distortion. Figure 13 shows diagrammatically the actual and effective capacities and resistances in a stage of capacity-coupled amplifier.

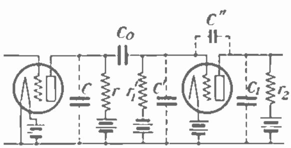


FIG. 13.—Capacity-coupled amplifier.

At very low frequencies, C, C', C'', and C_1 may be ignored in the calculations and the gain will be given approximately by the expression

$$\gamma = \mu\frac{r_p}{r + r_p}\sqrt{\frac{r_1'^2}{r_1'^2 + \frac{1}{\omega^2 C_0^2}}} \tag{12}$$

This formula holds good when r_1' is large in comparison with the anode resistance load r, but it will give the point at which the gain begins to diminish rapidly with decreasing frequency. When $1/\omega C_0$ equals r_1', the gain is about 70 per cent of what it will be at much higher frequencies. Hence, if 20 cycles is the lowest frequency to be amplified, and $1/\omega C_0 = r_1'$ at $\omega = 62.8$ radians (10 cycles), at 20 cycles the reduction will be of $\sqrt{\frac{1}{1 + \frac{1}{2}^2}} = \frac{1}{\sqrt{1.25}}$ $= 89.4$ per cent of the gain at, say, 1,000 cycles or more.

The upper end may suffer more than the lower end of the frequency band on account of the parasitic capacities C'', C''', and C_1. The effect of C'' is easy to compute, but the effects of the two others are quite complicated. As a first approximation, we may put for the three capacities a single condenser of a value equal to the grid-to-ground capacity of the tube and connections multiplied by $(\mu + 1)\frac{r_p}{r_p + r_2}$ and calculate the parallel circuit $r_1'C$ (where C is the resultant capacity calculated as shown).

Inspection of the equations given for the corrections at the low end as well as the high end of the frequency band shows that the best results, so far as uniform gain with frequency is concerned, are obtained by using a large number of stages with comparatively small gain per stage. When very low frequencies such as are met with in submarine cable work, are to be amplified, the coupling capacities will have to be quite large [see Eq. (12)]. In such cases, unless batteries are used to supply the B voltage, motor boating will occur. This form of instability may give rise to audible frequencies of very low period, or just to violent oscillations which may be seen in a d-c instrument in the plate circuit of any one of the tubes of the system.

15. Stabilizing Network. To stop these disturbances, a voltage-stabilizing arrangement will have to be connected to the source, such as a glow tube (UX-874) across the B voltage condensers of the filter or a comparatively low resistance across the same place. In many amplifiers each stage is fed from a different point of the filter condenser units in such manner that the first tube will get more filtering effect than the second, and so on to the last, which may be fed to a line with one or even without a filter section. Motor boating is very likely to occur in these cases because the time elements of both the amplifier circuits and those of the filter sections may result in a feed-back action which, for some frequency, may result in positive regeneration. Even some transformer coupled units without "bass boosting" have been found to suffer from this ailment. It is better to do all the filtering before the voltage is delivered to the amplifier and then to return all the plate circuits to a common point, or to a voltage divider of comparatively low resistance if different plate voltages are required for the various tubes. The "power stage" may be exempt from this rule in many cases.

16. Galvanic-coupled Stages. Under this heading may be classed all forms of d-c amplifiers and direct-coupled a-c and d-c amplifiers.

Figure 14 represents a direct-coupled amplifier with two tubes. The gain, if no stray capacities and mutual paths for the various currents existed, should be strictly independent of the frequency, and this is its most important characteristic. In practice, however, the capacities between elements of the tubes, which in most cases resolve themselves into an input capacity between grid and ground, limit the upper range of the frequency spectrum. This limitation is quite severe, because tubes of very high mu should be used in this type of coupling to obtain a high gain compatible with great simplicity in the circuit. Tubes with a high mu have input effective capacities of large magnitude, as we have seen in previous sections, since there is always a high plate resistance accompanying high-mu tubes, the capacity reactance coming parallel with the plate resistance will make it difficult to extend the frequency range of uniform amplification very much above one kilocycle except at the expense of over-all gain. However, when great simplicity of the circuit is required, sufficiently good results may be obtained with this type of coupling to justify its adoption. In connection with the resistance network shown in Fig. 14 for the proper supply of d-c potentials to the tubes, it should be noted that unless the paths of the a-c components of the anode current of

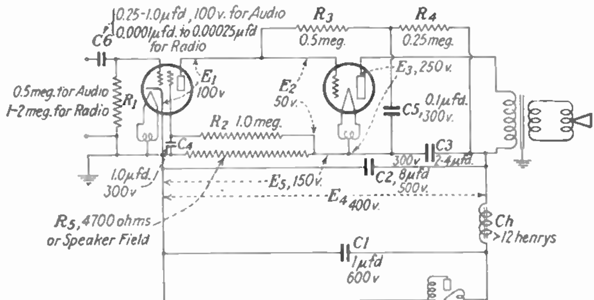


FIG. 14.—Two-tube direct-coupled amplifiers.

the various tubes are returned directly to their proper tube elements by means of large capacities, or by filter sections containing condensers and resistances, positive or negative regeneration will take place, and of a varying intensity at various frequencies, which will defeat the very purpose of the direct-coupled system, namely, to amplify uniformly all the frequencies in the audio band.

Figure 14 shows how a UY-224 with a UX-245 or UX-250 may operate from a phonograph pick-up. The proper voltages, and resistances to obtain them by *IR* drop from a filtered high voltage source, are given together with a very simple power pack. The filter choke may be substituted in many instances by the high-resistance magnetizing winding of a standard electrodynamic reproducer.

17. Power Stage. The last stage in the chain of an amplifier has to deliver not only an e.m.f. but also power to the device which is to convert electrical into sound energy, and hence the name power stage.

In general, to obtain the maximum power output from a given source, the external resistance should be equal to the internal resistance of the source itself. This is also true when it is a question of obtaining power from a vacuum tube regardless of wave form, but when the wave shape is to be faithfully preserved, then the so-called "matching of impedances" is not so simple as it looks.

With ordinary triodes, it has been shown that if the load resistance is twice the value of the internal plate resistance of the tube, maximum power with minimum distortion may be obtained. However, this power does not vary rapidly with the load resistance so that even if the value of this resistance which we shall call r divided by the plate resistance $r_p = (\delta i_p/\delta e_p)$ varies between 0.5 and 4, the maximum power for a given amount of distortion varies only by a few decibels. It is very fortunate that it happens to be so because the effective impedance of loud-speakers is not independent of the frequency.

18. Determination and Measure of Distortion. When a sinusoidal e.m.f. is impressed on any device, the output of which does not vary linearly with the input, the wave shape is altered and it can be expressed in the form of a Fourier series with the impressed frequency as a fundamental.

Such is the case with vacuum tubes. If the impressed grid voltage is $e = E \sin \omega t$ or, more completely, $e = E_c + E \sin \omega t$ where E_c is a steady potential, the e.m.f. across the plate impedance cannot be of the form $e_2 = E' \sin \omega t$ unless the tube characteristic is linear, which is never the case, although it may be approached sufficiently for practical purposes. Actually the output voltage across the load will be of the form

$$e_2 = E' \sin \omega t + E'' \sin (2\omega t + \phi'')E''' \sin (3\omega t + \phi''') + E_4 \sin (\omega t + \phi_4) +$$

Usually all harmonics above the third are weak in comparison with the first two.

If we measure the fundamental component, and then take the r-m-s value of all the harmonics, we can express distortion in more precise terms. Hence, the distortion is defined as the percentage of harmonic currents (or voltages) contained in the total current (or voltage) supplied to the load, when a sinusoidal e.m.f. is impressed between grid and cathode. Obviously the steady plate current is excluded.

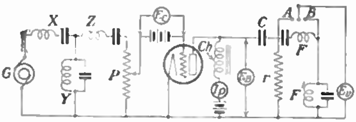


Fig. 15.—Circuit for distortion measurement.

19. Experimental Determination. Figure 15 shows a simple layout that will permit the measurement of distortion of a vacuum tube. An e.m.f. from a source G which has a good sinusoidal wave shape is impressed upon a potentiometer P through a band-pass filter XYZ completely to purify it of harmonics. By sliding the contact arm in the potentiometer, the magnitude of the impressed e.m.f. may be adjusted. In the plate circuit, the steady current is separated from the variable components by means of a choke Ch and a condenser C, to which the load resistance voltmeter E_v may be successively connected to the points A and B. When at A, it will measure the r-m-s voltage across the load r, including fundamental plus harmonics. When connected to B, it will indicate r-m-s voltages of the fundamental alone by means of the filter FF', which should have very large effective reactance for all the harmonics and also a very

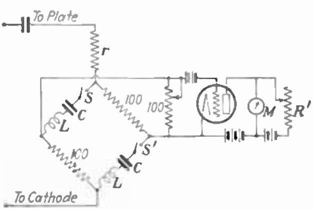


Fig. 16.—Measurement of harmonic content.

large effective resistance for the fundamental. When the vacuum-tube voltmeter is connected to B, the error introduced by the filter is negligible if the resistances of the coils that form it are very small compared to their reactances at the fundamental frequency.

To minimize this error and in order to measure the harmonic r-m-s voltage rather than the fundamental, a circuit shown in Fig. 16 will furnish more accurate results and it is very simple to construct. A low-resistance Wheatstone bridge made of two non-reactive arms of, say, 100 ohms each and two tuned arms having an effective resistance of 100 ohms at the fundamental frequency, are connected in series with the load resistance r. Across the bridge is a resistance of 100 ohms in the place of the conventional galvanometer. This resistance is in the form of a potentiometer well calibrated. A vacuum-tube voltmeter is supplied from the contact arm of the potentiometer and need not be calibrated. It may be made of any "all-purpose" tube with a microammeter M balanced for zero by means of a resistance R' when no alternating e.m.f. is impressed between grid and cathode.

It is very convenient to interpose a transformer of good frequency characteristics between the 100-ohm potentiometer and the tube voltmeter input, to increase very much the sensitivity of the voltmeter which is really acting as an ammeter of 100-ohm resistance. Now, with the switches SS' open, the load current will pass through the two non-reactive resistances of the bridge and the 100-ohm potentiometer, all of them in series with each other and with r, and by adjusting the arm of the potentiometer, a good indication will be obtained in the microammeter the absolute value of which is immaterial. It will be a measure of the r-m-s value of the load current. By closing SS', the bridge, which has been previously balanced for the fundamental, will exclude completely this frequency from the potentiometer and hence from the vacuum-tube meter. If the coils L and condensers C have a "Q" over 10, the bridge will act for all the harmonics as if switches SS' were open, since the reactance of the tuned arms will be large compared to 100 ohms. Consequently, there will be across the potentiometer an e.m.f. made up exclusively of harmonics. By adjusting the potentiometer to a new value such that the same deflection is obtained as when the fundamental was present, the percentage of harmonic contents in the voltage across or current through the load resistance will be determined by dividing the value of the setting over the first. For very accurate determinations, 300 ohms should be added to r in figuring out the load resistance. When the bridge switches are closed, this resistance is 200 ohms less for the fundamental, and additional resistance may be added to r to make up for the reduction although in most cases this is not necessary because r is several thousand ohms. When lower plate resistance tubes are to be tested, a 10-ohm bridge may be used instead of the 100-ohm bridge shown in Fig. 16 with a transformer between the potentiometer and the grid. A commercial microphone step-up transformer may be used very successfully in connection with a 10- to 100-ohm bridge.

It has been found that when most commercial tubes of average power rating are tested according to the methods discussed above, the harmonics amount to about 5 per cent of the total current through a non-reactive load of optimum value. The method used for this determination lends itself for further useful determination, such as the optimum value of load resistance for a given limit of harmonic "distortion." All that is needed is to vary r (Figs. 15 and 16) while measurements are made of total voltage or current and its harmonic content and plot a graph showing percentage of harmonics as abscissas and values of r as ordinates. The optimum values of steady grid voltage for all the other given conditions, such as plate voltage and load resistance, may be determined likewise. Care should be exercised to introduce a suitable input circuit simulating the actual conditions of the circuit when the tube is to be placed so as not to eliminate, during the test, the possibility of grid distortion. The gain per stage and also μ and r_p can be obtained in a similar manner.

20. Pentode in a Power Stage. This five-electrode tube has many advantages over the ordinary triode as a "power" amplifier (all tubes are power amplifiers) in spite of some distortion that exists in sets that use it.

It delivers about 30 mw per volt squared applied to the grid, while most tubes of the '71, '45, and '50 type deliver only about 2 mw per

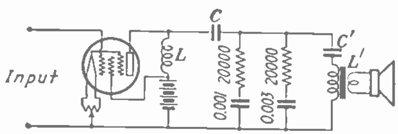


FIG. 17.—Pentode power tube with tone compensating network.

volt squared. A gain of about 15 may be obtained on a load resistance of approximately 7,500 ohms. The plate resistance is very high, about 50,000 ohms, and the dynamic characteristics of the pentode are such that the optimum load resistance for least harmonic development is about 7,500 ohms. Thus the pentode is more like a constant current device than like a constant voltage source, just the inverse case of ordinary "power" triodes. For this reason, and because loud-speakers have a rising frequency-impedance characteristic, high frequencies seem over-emphasized. Corrective networks have been devised but the simplest form consists of a 0.005-μf condenser with a 10^4-ohm resistance in series, both across the primary of the speaker transformer.

The bass tones seem to be very weak in pentode radio sets, but a great deal of this is due to insufficient baffle in the speaker cabinet. A bass compensating circuit and a better treble compensating network are shown in Fig. 17. The high frequencies are more gradually equalized by two successive steps at different starting points of frequency-amplitude reduction. The reactances of condenser C together with the effective inductance of the speaker transformer, combined in parallel with the inductive reactance of the choke L, will form a circuit which has a rising impedance at a given low frequency depending upon the constants of these elements. This frequency should be made of about 50 cycles.

The pentode has the advantage of running on moderate plate voltages and currents, while the power output is much larger than triodes working under the same conditions. For greater power output, the use of two pentodes of the '47 type in parallel offers the advantage of a lower-plate matching impedance, about 3,500 ohms, which is the value offered by most dynamic speakers equipped with commercial transformers suitable for operation from power triodes.

By choosing different values of plate impedance than those given for minimum distortion, the third harmonic may be reduced, but the second will increase. Push pulling will neutralize all even harmonics to some extent, and it is on this principle that push-pull transformers with pentodes are designed.

21. Power Tubes with Positive Grids. It is possible to obtain a considerably greater power from a triode for a given limit of harmonic distortion, and a given plate voltage limit by permitting the grid to become positive during a part of the cycle, *provided* that, in so doing, the impressed e.m.f. will not lose its shape by the fact that the grid is no longer a voltage controlled device but requires appreciable power to operate it. If we take a triode and connect its input or grid-cathode circuit to a low resistance source of sinusoidal e.m.f. and measure the input power and its harmonic contents as previously shown (see Figs. 15 and 16) and then do the same when the source has an internal resistance of a value close to the plate resistance of amplifying triodes with their coupling device, entirely different results will be obtained.

With optimum values of grid bias and plate resistance chosen after several tests, it is possible to get over five times the power in the first case with respect to the second for a given percentage of harmonics and plate voltage.

There are, in development, tubes that will operate with positive grids during a part of the cycle and will deliver about one watt at $E_B = 110$ volts and four watts at $E_B = 250$. Auxiliary elements in the tube furnish the power required to operate the grid while the input impedance is made of the usual grid-to-cathode capacity with negligible losses.

22. Push-pull Amplifiers. Any symmetrical system of impressed e.m.fs. or of impedances, if subject to non-linear laws of variation, will give rise to odd harmonics exclusively. As the second harmonic is preponderant in vacuum-tube distortion, by connecting two tubes in push pull this harmonic is balanced out. Much has been written pro and con regarding the merits of push-pull amplifiers. The net result seems to be boiled down to the following considerations:

Two triodes in push pull deliver almost the same power as three in parallel when best conditions of voltages and impedances are fulfilled for a given tolerance of distortion.

The output transformer has its iron core free of d-c saturation; hence, less iron will be required and a closer coupling available between windings as no gap is necessary.

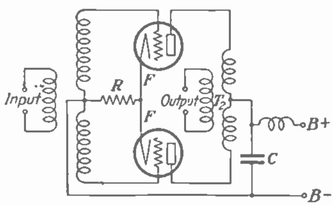


FIG. 18.—Push-pull amplifier.

The disturbances coming from the power pack, being introduced in phase on both tubes cancel each other in the output transformer. This is particularly true of hum. There is ordinarily eliminated one filter section in the power pack when a push-pull output stage is used. An additional filter section carrying considerably less power must be inserted between the supply line and the rest of the tubes outside the power stage.

Regeneration due to coupling through grid and plate return circuits is positive for one of the push-pull tubes and negative for the other, thereby cancelling itself. Care should be exercised, however, lest the powerful output currents develop appreciable modulation of the input e.m.f. as follows:

Strong even harmonic currents will cancel each other in the output transformer T_2 (Fig. 18), but they do not cancel each other in the circuit completing the plate-to-cathode return, comprising the filter condenser C, and the biasing resistor R. Now, if the grid-to-cathode return circuit is common in part to this circuit, as when the "biasing" resistor is not by-passed by large condensers between cathodes (filament) F and B, or ground, these harmonic currents will modulate the incoming e.m.f. if their amplitude is not sufficiently reduced by suitable filtering and separation of grid and plate circuits.

23. Class B Amplifiers.[1] Use of two tubes in a class B circuit, often called "push-push," for audio amplification offers the advantages over class A operation of marked economy of power supply, greater operating efficiency, and, from equivalent power supply, greater undistorted power output. In practice either the tubes are over-biased so that very little plate current flows, or tubes with a fairly high amplification factor are employed, operated at zero grid bias.

Design of class B circuits is somewhat more complicated than design of class A circuits. The characteristics of plate load, input transformer, and B supply are all interrelated. Since the grids of the tubes which are fed from an input push-pull transformer draw current, the stage feeding the class B tubes must furnish appreciable power. Again, because the grid takes current, the input circuit must have a resistance low in comparison with the grid-cathode resistance of the tubes. With the '46-type tube, especially designed for class B operation in radio receivers, this resistance is of the order of 1,000 ohms. The input transformer, therefore, steps down the voltage from the feeder stage, which may be a single '46 tube or other amplifier tube.

For a limited grid voltage to the two tubes, the greatest power output will be secured when the plate load is such that the product of the plate voltage and plate current swing is greatest. Distortion from such a circuit is greatest at low power output because of the curvature of the characteristic at low grid voltages. For an overall distortion of 5 per cent total harmonics, not more than 2 per cent second harmonic can be tolerated from the driver tube. The power output available from a typical circuit is shown in Fig. 19. Some radio-set manufacturers in 1932 were using step-down transformers, working into class A tubes, the idea being the ability to drive the grids of the tubes positive without the distortion ordinarily encountered and without the danger of increased distortion at low input signal voltages when using class B circuits.

Class B operation is especially valuable in amplifiers which must be fed from a high-cost source of power—batteries, for example, where the cost of supplying power is $10 per kilowatt hour. Since the plate current taken by the tubes is low at zero (or low) signal level and is only high on high modulation peaks, the user pays for only what he gets.

24. Power Supply to Tubes of an Amplifier. The power required is in the form of filament or heater current and B and C unidirectional, non-fluctuating voltages. The former is almost universally supplied at low voltage a.c., by means of twisted leads of very low resistance to neutralize the magnetic field which may otherwise induce hum. The B and C voltages are derived by taking the drop across a resistance

[1] By the Editor.

network through which rectified current passes after it has been filtered. It is of utmost importance that the internal impedance of the B and C sources, for even the lowest audio frequency, shall be negligibly small compared to the impedances of the tubes and of the apparatus attached

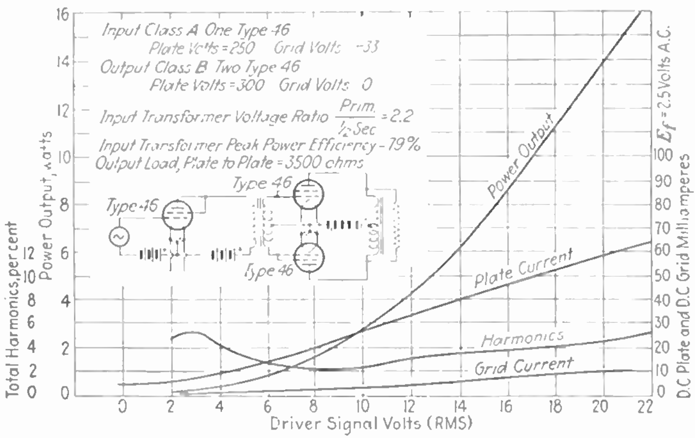


FIG. 19.—Class B circuit and characteristics.

to them. Therefore, by-pass condensers should be provided across the resistances the drop across which is to be utilized for B and C potentials. It is better to have individual resistors in series between cathode and

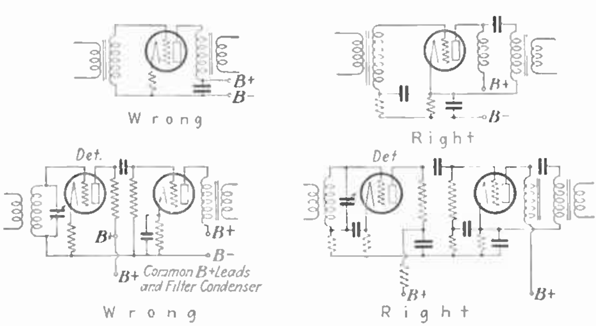


FIG. 20.—Methods of supplying voltages to amplifier tubes.

ground for each tube with a shunted condenser, so in such case, there will be no undesirable intercoupling between stages. Figure 20 shows diagrammatically an example of the right and the wrong way of feeding amplifiers with B and C potentials.

The simple filters when consisting of condensers and resistances in the grid and condensers and chokes in the plate serve admirably not only to stop intercoupling and regeneration, but also to permit considerable economy in the construction of the *B* voltage supply filter. There is only one caution, and it relates to the "time constant" of these circuits. If too small, the filters may be ineffective at low audio frequencies, and if too large, may give rise to motor boating. It is not easy to predict what values are the best, but a little experimentation will solve the problem much more easily. Start with plenty of capacity by-passing and resistance separation of circuits and cut down on the values or even eliminate some few filter sections here and there until a happy medium is reached. The maximum filtration will be required in amplifiers for "talkies" operated from photo cells, and the least is required in "midget" radio sets that have no audio stages.

SECTION 12

RADIO-FREQUENCY AMPLIFIERS

By R. S. Glasgow[1]

1. Class A Amplifier. Amplifiers are divided into three general classes, *A*, *B*, and *C*, depending on the type of service in which they are to be used.

A class *A* amplifier is one which operates so that the plate output wave shapes of current are practically the same as those of the exciting grid voltage.

This is accomplished by operating the tube with sufficient negative grid bias such that some plate current flows at all times, and by applying an alternating excitation voltage to the grid of such value that the dynamic operating characteristic is essentially linear. The grid must not go positive on excitation peaks and the plate current must not fall low enough at its minimum to cause distortion due to curvature of the characteristic.

The characteristics of class *A* operation are freedom from distortion and relatively low power output. Practically all a-f amplifiers are operated in this manner. Radio-frequency amplifiers of the type used in receiving sets to amplify the signal voltage prior to detection are also of this class.

Class *B* and *C* amplifiers will be discussed under Power Amplifiers.

2. Radio-frequency amplifiers for receiving sets are usually classified as to the type of coupling employed between stages. This coupling means can be either a resistance, an impedance, a transformer, or any combination of these elements. The circuit constants of the coupling means may be adjustable or fixed, giving rise to a further classification of a tuned or an untuned amplifier. In the latter the circuits are essentially the same as those employed for a-f amplifiers, and are in general unsatisfactory except for the lower radio frequencies.

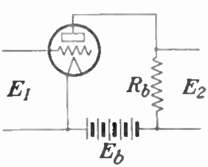


Fig. 1.—Resistance-coupled amplifier.

3. Resistance-coupled Amplifier. This type of amplifier is occasionally used where uniform amplification is desired over a moderate band in the lowest range of radio frequencies. In Fig. 1 the output voltage E_2 is given by

$$E_2 = \frac{\mu R_b}{r_p + R_b} E_1 \qquad (1)$$

[1] Associate professor of Electrical Engineering, Washington University, St. Louis.

where μ and r_p are respectively the amplification factor and plate resistance of the tube used. Defining the voltage amplification per stage G as the ratio of the output voltage to the input voltage, we have

$$G = \frac{E_2}{E_1} = \frac{\mu R_b}{r_p + R_b} \tag{2}$$

As R_b is made very large compared to r_p the value G approaches μ as a limit so that tubes having a large value of μ are necessary if reasonably high gain per stage is desired. Equation (2) presumes that the input impedance of the next stage which is shunted across R_b is enormously large, so that R_b is not appreciably reduced as a result of being shunted by this input impedance.

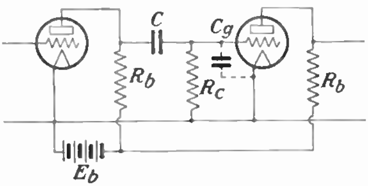


FIG. 2.—Cascade amplifier.

In a typical cascade amplifier as shown in Fig. 2, R_b is in effect shunted by the grid leak R_c in parallel with C_g, the input capacity of the tube. The reactance of the blocking condenser C in series with them is negligibly small in comparison. For frequencies lower than 500 kc with a pure resistance in its plate circuit C_g may be regarded as constant and independent of the frequency, and is given by

$$C_g = C_{gf} + C_{gp}\left(1 + \frac{\mu R_b}{r_p + R_b}\right) \tag{3}$$

where C_{gf} = capacity between grid and filament
C_{gp} = capacity between grid and plate

These interelectrode capacities will be from 4 to 7 $\mu\mu$f depending on the type of tube and socket used so that C_g may lie anywhere from 40 to 80 $\mu\mu$f. Thus at 1,000 cycles the input impedance of the tube alone will be about 3 megohms while at 100 kc it has dropped to about 30,000 ohms. As a result the gain per stage diminishes as the frequency increases due to the reduction of the effective value of R_b by the short-circuiting effect of C_g. In addition to these limitations, a-f disturbances and tube noises are readily amplified so that resistance-coupled amplifiers are usually not very satisfactory for radio frequencies.

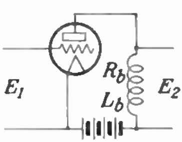


FIG. 3.—Impedance-coupled amplifier.

4. Impedance-coupled Amplifier. The simplest amplifier of this type merely employs a choke coil in the plate circuit as shown in Fig. 3. The voltage amplification per stage is given by

$$G = \frac{E_2}{E_1} = \frac{\mu\sqrt{R_b^2 + \omega^2 L_b^2}}{\sqrt{(r_p + R_b)^2 + \omega^2 L_b^2}} \tag{4}$$

where R_b and L_b are respectively the resistance and inductance (in henrys) of the choke coil and $\omega = 2\pi \times$ frequency. If the resistance of the coil is small compared to its reactance, ωL_b, and to the plate resistance r_p of the tube, the expression for the amplification becomes

$$G = \frac{\mu\omega L_b}{\sqrt{r_p^2 + \omega^2 L_b^2}} \tag{5}$$

If ωL_b is very large compared to r_p, G approaches μ of the tube as a limiting value, as was the case with the resistance-coupled amplifier. By choosing L_b large enough so that the reactance of the coil is large compared to the plate resistance of the tube at the lowest frequency we are interested in, the gain will be constant for all higher values of frequency. Owing to distributed capacity effects and the shunting of the coil by the input capacity of the next tube it is not possible to obtain uniform amplification as predicted above except at low frequencies. For high frequencies such as the present broadcast band the effect of this capacity is to produce a parallel resonant circuit whose impedance is high at the resonant frequency, but which drops off rapidly for frequencies higher than resonance. This results in a reduction of the gain for frequencies above resonance. To avoid this, it becomes necessary to use a value of choke-

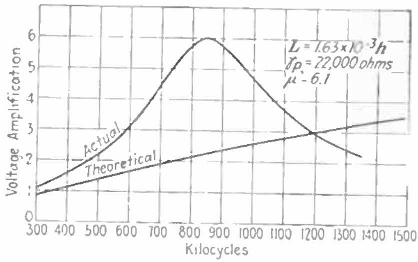


FIG. 4.—Amplification of a choke-coupled amplifier tube.

coil inductance such that resonance occurs somewhat below the highest frequency to be amplified. This value of inductance is governed chiefly by the input capacity of the next tube which may be of the order of 10 to 20 $\mu\mu$f, depending on the type of tube used and the nature of the load in its plate circuit. For this reason there is little to be gained by reducing the distributed capacity of the coil if it is already small compared to the tube input capacity. At broadcast frequencies the value of inductance thus obtained results in too low a reactance to give good amplification for frequencies much below resonance.

This is illustrated in Fig. 4. The coil used was a single layer solenoid closely wound with 173 turns of No. 28 wire having an inductance of 1.63×10^{-3} henry and about 10 ohms d-c resistance. The distributed capacity of the coil was 3.5 $\mu\mu$f. The curve shows the measured amplification[1] using a Western Electric 215-A "peanut" tube which had an amplification factor of 6.1 and a plate resistance of 22,000 ohms. The input capacity of the vacuum-tube voltmeter which used a tube of the same type was 18 $\mu\mu$f, including leads, which lowered the natural period of the choke coil to 850 kc. The lower curve shows the theoretical amplification that would be obtained if these shunting capacities were absent.

If C represents the total capacity shunting the coil in Fig. 3, the expression for the amplification becomes

$$G = \frac{\mu Z}{\sqrt{(r_p + R)^2 + X^2}} \tag{6}$$

[1] FRIIS and JENSEN, *Bell System Tech. Jour.*, 3, 187.

where

$$R = \frac{R_b}{\omega^2 C^2 R_b^2 + (\omega^2 L_b C - 1)^2} \tag{7}$$

$$X = \frac{L_b - C(R_b^2 + \omega^2 L_b^2)}{\omega^2 C^2 R_b^2 + (\omega^2 L_b C - 1)^2} \tag{8}$$

$$Z = \sqrt{R^2 + X^2} \tag{9}$$

The above expression for Z is the resultant impedance of the coil in the plate circuit when shunted by the capacity C under the assumption that this capacity has no appreciable resistance associated with it. The voltage amplification will be a maximum when Z is a maximum, which will occur at resonance. When Z is a maximum the apparent reactance as given by (8) becomes zero, and

$$Z_{\text{max.}} = \frac{R_b^2 + \omega^2 L_b^2}{R_b} \tag{10}$$

the expression for maximum gain thus becomes

$$G_{\text{max.}} = \frac{\mu}{1 + \dfrac{r_p R_b}{R_b^2 + \omega^2 L_b^2}} \tag{11}$$

If the shunting capacity C has an effective resistance R_c in series with it, the expressions for R and X in (7) and (8) become

$$R = \frac{\omega^2 C R_c[R_b(R_b + R_c) + \omega^2 L_b^2] + R_b}{\omega^2 C^2 (R_b + R_c)^2 + (\omega^2 L_b C - 1)^2} \tag{12}$$

$$X = \omega \frac{L_b - C[R_b^2 + \omega^2 L_b(L_b - C^2 R_c^2)]}{\omega^2 C^2 (R_b + R_c)^2 + (\omega^2 L_b C - 1)^2} \tag{13}$$

5. Use at Low Frequencies. At frequencies in the vicinity of 50 kc much higher values of inductance can be secured and while the distributed capacities of such coils will be greater, the total capacity shunted across

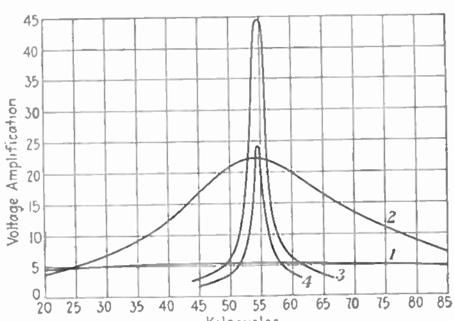


FIG. 5.—Amplification as a function of turn ratio.

the coil will not be more than two or three times the value that would obtain at broadcast frequencies. Since the voltage amplification will be approximately constant and equal to the μ of the tube as long as the impedance of the coil in the plate circuit is large compared to r_p, uniform amplification can be readily obtained for a wide band at the lower radio frequencies. This is shown by curve 1 in Fig. 5. In order to secure the high inductance needed at the lower frequencies without unduly increasing the distributed capacity a multilayer winding of "honeycomb" type is often used. Another method is to subdivide the coil into a number of thin, closely adjacent, random-wound pancake sections by using a coil form with a number of narrow grooves turned in it. Such coils can be made astatic by reversing the direction of the winding in each alternate section so that their magnetic fields are in opposition. This form of construction requires a greater number of turns to secure a given inductance, but it possesses the advantage of being immune from stray magnetic couplings with the other coils in the amplifier.

At the lower radio frequencies suitable iron cores can be profitably employed, enabling high values of inductance to be obtained with a nominal number of turns on the coil. The iron must be very well laminated to reduce the eddy currents, or an objectionable increase in the resistance of the coil will result. The iron commonly used for this purpose has a thickness of only one to two mils. Dust cores of iron and its magnetic alloys[1] are very satisfactory for this purpose. However, as the frequency increases the advantages of an iron core diminish. The resistance of the coil rapidly increases while the apparent permeability of the iron becomes less, so that at high frequencies the iron contributes comparatively little to the inductance. This results in a ratio of $\omega L/R$, which is lower than would be obtained with a suitable air core inductance. For this reason iron cores are seldom used for frequencies above 500 kc.

In 1931, Polydoroff developed a method for tuning a circuit by variations in the inductance by inserting a core of finely powdered iron. Because of the high specific resistance of the core material, the losses in the tuned circuit were kept low, and sufficient change in permeability of the inductance core was secured to tune a circuit over a 3 to 1 variation in frequency.

6. Transformer-coupled Amplifiers. The problems in an untuned transformer-coupled amplifier are much the same as those just discussed. The primary of the transformer is merely a choke coil in the plate circuit of the tube so that the secondary voltage may be obtained by multiplying the primary voltage by the ratio of transformation. The expression for the voltage amplification can then be obtained by multiplying (4) or (6) by this ratio.

If the impedance into which the transformer is working is enormously large so that the secondary may be assumed to be on open circuit the ratio of transformation is given by

$$a = \frac{M}{L_p} = K\sqrt{\frac{L_s}{L_p}} \tag{14}$$

where M = mutual inductance
L_p = primary inductance
L_s = secondary inductance
K = coefficient of coupling = $M/\sqrt{L_pL_s}$

From (14) it is seen that the ratio of transformation is only equal to the turns ratio if the coupling between primary and secondary is unity and if

[1] Shackelton and Barber, Compressed Powered Permalloy, *Trans. A.I.E.E.*, pp. 429–436, April, 1928.

the inductances are proportional to the square of the number of turns. These conditions are usually obtained only if an iron core is used.

The assumption that the secondary of the transformer is working into an open circuit is seldom valid due to the input capacity of the following tube. The effect of this capacity becomes more pronounced as the step-up ratio of the transformer is increased. At low ratios of transformation the response curve is relatively flat, but as the ratio is increased the tuning effect of the tube capacity across the secondary becomes quite pronounced resulting in a sharply defined resonance curve. This is illustrated in Fig. 5. The transformer used was a coil of 2,400 turns wound on an iron dust core which had an inductance of 0.33 henry. Taps were brought out so that the coil could be used as an auto transformer of adjustable ratio by connecting the plate of the amplifier tube across any portion of the total inductance. The same tube was used as in Fig. 4. Curve 1 was for a 1:1 ratio, the coil being used as an impedance-coupled amplifier. Curves 2, 3, and 4 are for step-up ratios of 1:4, 1:16 and 1:48 respectively.

The general vector expressions for the currents in the primary and secondary of a transformer-coupled amplifier are given by

$$i_p = \frac{\mu e_g(Z_s + Z_2)}{(r_p + Z_p)(Z_s + Z_2) - Z_m^2} \tag{15}$$

$$i_s = \frac{-\mu e_g Z_m}{(r_p + Z_p)(Z_s + Z_2) - Z_m^2} \tag{16}$$

The circuit is shown in Fig. 6, *a* being the actual connection and *b*, the electrical equivalent. The negative sign in (16) indicates that the secondary

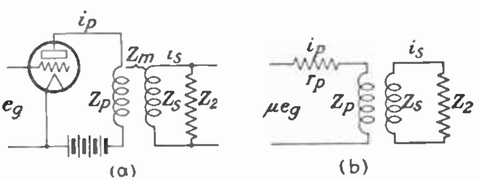


Fig. 6.—Transformer-coupled amplifier.

current is flowing in a direction opposite to that in the primary and can be disregarded if we are interested only in the magnitude of the current. Substituting the following vector expressions for the various impedances and assuming that Z_2 is a condenser having a capacity, C_2, we have

$$Z_p = R_p + j\omega L_p$$

$$Z_s + Z_2 = R_s + j\left(\omega L_s - \frac{1}{\omega C_2}\right)$$

$$Z_m = j\omega M$$

where R_s is the resistance of the transformer secondary and includes any resistance that may be associated with C_2. The vector expression for the secondary current is then

$$i_s = \frac{-\mu e_g j\omega M}{\left[(r_p + R_p)R_s + \frac{L_p}{C_2} - \omega^2(L_pL_s - M^2)\right] + j\left[r_p\left(\omega L_s - \frac{1}{\omega C_2}\right) + \omega L_p R_s\right]} \tag{17}$$

and the voltage amplification at any frequency is given by

$$G = \frac{i_s Z_2}{e_g} = \frac{\mu \frac{M}{C_2}}{\sqrt{\left[(r_p + R_p)R_s + \frac{L_p}{C_2} - \omega^2(L_pL_s - M^2)\right]^2 + \left[r_p\left(\omega L_s - \frac{1}{\omega C_2}\right) + \omega L_p R_s\right]^2}} \tag{18}$$

Equations (17) and (18) neglect the effects of distributed capacity of the primary and possible capacity coupling between primary and secondary. These items, if appreciable, will modify the expression for the gain as given by (18).[1]

The voltage amplification will be a maximum at resonance, or when the j term in (17) is zero. This will occur when

$$\omega = \frac{1}{\sqrt{C_2\left(L_s + L_p \frac{R_s}{r_p + R_p}\right)}} \tag{19}$$

and the gain will be given by

$$G_{\text{max.}} = \frac{\mu \frac{M}{C_2}}{(r_p + R_p)R_s + \omega^2\left(M^2 + L_1^2 \frac{R_s}{r_p + R_p}\right)} \tag{20}$$

At high frequencies the tube input capacity is a complex function of the frequency and of the constants in the output circuit. By using a coil in the output circuit whose natural period is slightly lower than the lowest frequency to be amplified, the input capacity of that tube can be made to increase as the frequency is lowered. Since C_2 in the above equations is composed largely of tube input capacity it is possible by proper design to have C_2 increase automatically as the frequency of the incoming signal decreases, and at the proper rate so as to tune the transformer secondary to approximate resonance for a reasonable range of frequencies. This automatic tuning effect results in a much broader and more uniform response curve than would be obtained if C_2 were fixed in value.

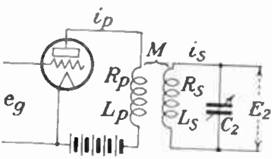


FIG. 7.—Typical tuned r-f transformer.

7. Tuned Amplifiers. The circuits discussed in the preceding sections are employed when it is desired to amplify a fixed band of frequencies. The width of this band and the uniformity of the amplification therein are governed by design limitations. The majority of receiving sets must be capable of amplifying a selected narrow band of frequencies and excluding all others. The *selectivity* of the receiving set is dependent upon how thoroughly this latter item is carried out. In receivers designed for entertainment purposes, the *fidelity* is also of importance and depends upon the uniformity of the amplification within the selected band. The type of detector used and the characteristics of the a-f amplifier also affect the fidelity.

[1] DIAMOND and STOWELL, Note on Radio-frequency Transformer Theory, *Proc. I.R.E.*, **16**, 1194, September, 1928.

A typical tuned r-f transformer connection is shown in Fig. 7. The current in the secondary and the voltage amplification per stage at any frequency with C_2 fixed in value are given by (17) and (18) respectively.

At resonance,

$$\omega\left(L_s + L_p\frac{R_s}{r_p + R_p}\right) = \frac{1}{\omega C_2} \tag{21}$$

and the gain will be a maximum and is given by (20). The resistance of the primary is usually negligible in comparison with r_p, and since $\frac{L_pR_s}{r_p + R_p}$ is also small, (21) becomes

$$\omega L_s = \frac{1}{\omega C_2} \tag{22}$$

and the expression for the gain at resonance to a sufficiently close degree of approximation becomes

$$G_{\max.} = \frac{\mu\omega M}{r_pR_s + \omega^2M^2}\omega L_s = \frac{\mu\frac{}{C_2}}{r_pR_s + \omega^2M^2} \tag{23}$$

If the mutual inductance M in (23) is adjusted to satisfy the condition

$$\omega M = \sqrt{r_pR_s} \tag{24}$$

the optimum value of voltage amplification will be obtained, and (23) reduces to

$$G_{\text{opt}} = \frac{\mu\omega L_s}{2\sqrt{r_pR_s}} \tag{25}$$

Equation (25) gives the maximum amplification it is possible to obtain with a given tube and coil.

When M is adjusted to its optimum value it will be noted that the figure of merit of the tube is $\mu/\sqrt{r_p}$. Therefore if two tubes have equal values of mutual conductance the one having the highest amplification factor will give the greatest gain. For this reason tetrodes, or screen-grid tubes, are capable of giving very high amplification. With M less than optimum the gain becomes more nearly proportional to the mutual conductance. When optimum coupling is employed the amplification is directly proportional to the ratio of the coil reactance to the square root of its resistance. Consequently, secondary coils using relatively small wire of comparatively high resistance can be used without seriously reducing the amplification. In this respect the r-f transformer differs from a coil aerial as in the latter the gain falls off directly with the coil resistance. For values of M considerably below optimum the gain will fall off at a rate more nearly proportional to the first power of the coil resistance. It is interesting to note that the turn ratio between primary and secondary does not enter into the expression for the amplification, the mutual inductance between them being the criterion. When optimum amplification is obtained the impedance looking into the primary of the transformer is equal to the plate resistance of the tube. This condition differs from the impedance-coupled amplifier in that in the latter optimum amplification is obtained only when the impedance in the plate circuit is enormous compared to r_p of the tube. The impedance looking into the primary of the circuit in Fig. 7 is

$$Z_p' = R_p + j\omega L_p + \frac{\omega^2 M^2}{R_s + j\left(\omega L_s - \frac{1}{\omega C_2}\right)} \tag{26}$$

8. Effect of Mutual Inductance. The effect of the magnitude of M on the resonant amplification for four different frequencies is shown in Fig. 8.[1] The secondary circuit resistance varied from 4 ohms at 500 kc to 25 ohms at 1,500 kc. It will be observed that a fixed value of mutual inductance of about 45 μh would give approximately optimum amplification for the entire range of frequencies included in the curves. There is

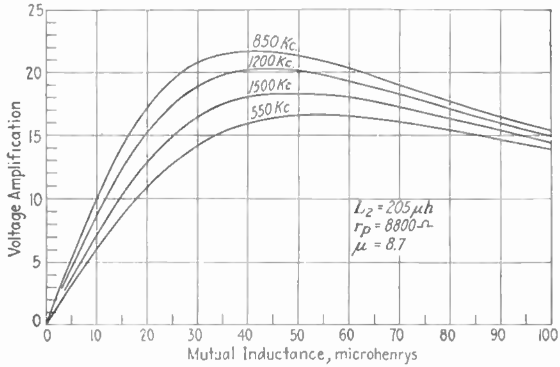


FIG. 8.—Importance of M on resonant amplification.

therefore little to be gained in sensitivity by adjusting the coupling in this type of amplifier for various frequencies providing sufficient coupling has been initially employed. There is, however, considerable advantage to be obtained by increasing M as the frequency is lowered in order to secure more uniform selectivity and better fidelity. The mechanical complications involved in automatically varying the amount of coupling with tuning have prevented its use in commercial receiving sets up to the present.

The effect of the value of M on selectivity is shown in Fig. 9, using the same tube and transformer as in Fig. 8. It will be noted that these curves have the same characteristics as those in Fig. 5, curve 3 of that figure having the proper turn ratio to produce the optimum value of M. If the ordinates of Fig. 9 are reduced to a percentage basis as in Fig. 10 the increased broadening of tuning with increased M is clearly apparent. As M approaches zero the response curve approaches the resonance curve of the secondary circuit. Therefore if good selectivity is desired M must be fairly small—usually well under its optimum value—so that some sacrifice must be made in sensitivity. A further difficulty presents itself

[1] GLASGOW, R. S., Tuned Radio-frequency Amplifiers, *Jour. A.I.E.E.*, p. 327, May, 1928.

when the selectivity at various frequencies is investigated, as illustrated in Fig. 11. At the lowest frequency the response curve is so sharp that the gain for side-band frequencies 5 kc off resonance is only 36 per cent of the resonant amplification. The fidelity is therefore impaired. At

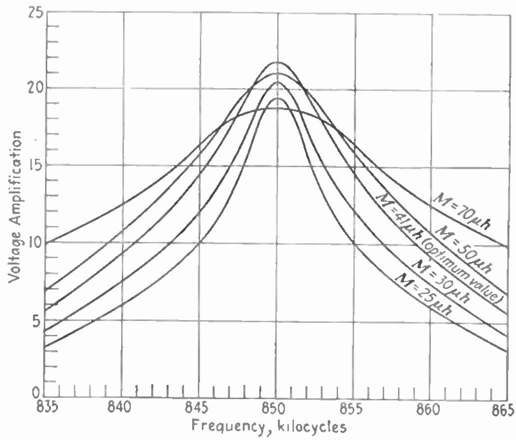


FIG. 9.—Effect of varying M on selectivity.

the highest frequency the fidelity is good but the selectivity is very poor. Reducing the value of M would sharpen the tuning at high frequencies but would cause it to become still sharper at the low frequencies with

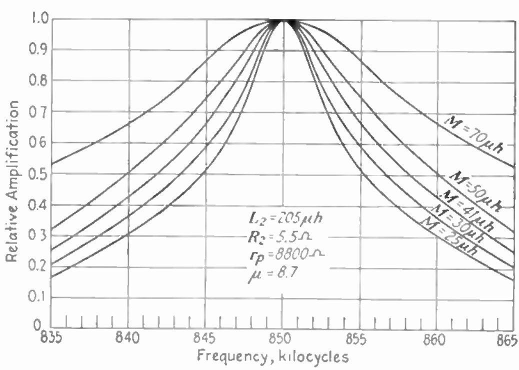


FIG. 10.

further impairment of fidelity in this region. This reduction in M would also cause a serious loss in sensitivity at the lower frequencies. Consequently the design of a tuned r-f receiving set for entertainment purposes represents a compromise between good fidelity and sensitivity at the long waves, and fair selectivity at the short waves. Any attempt to improve the performance at one end of the tuning range results in impaired performance at the other. This has resulted in the introduction of various modifications in the circuit of Fig. 7. These will be discussed later.

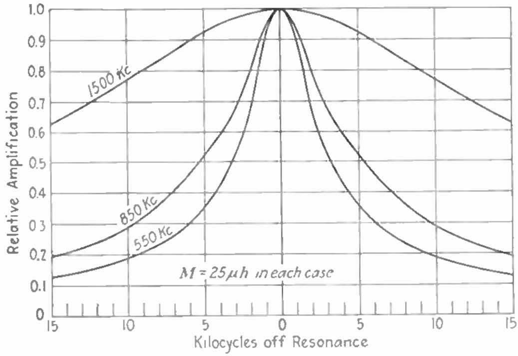


FIG. 11.—Selectivity as a function of frequency.

9. Cascade Amplifiers. If two or more identical stages of amplification are connected in cascade the over-all voltage amplification is given by

$$G = G^n \tag{27}$$

where n = number of stages
G = amplification per stage

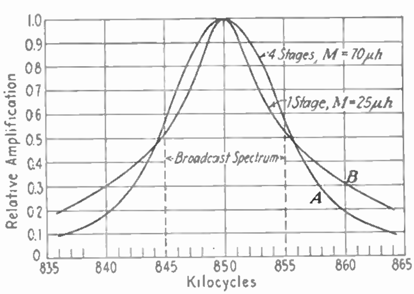


FIG. 12.—Increase in selectivity with cascading.

This expression presumes that the various stages do not react on each other, which is not always the case in practice due to small unavoidable couplings between input and output circuits. If the various stages are not all identical the over-all amplification will be the product of the individual values of G per stage. The response curve of a multi-stage amplifier composed of identical stages is readily obtained from the curve of an individual stage by raising its ordinates to the nth power, where n is the number of stages.

The use of several stages of cascade tuned r-f amplification enables both the selectivity and fidelity of the amplifier to be increased, provided the tuning of each stage is made broader as the number of stages are increased. This is illustrated in Fig. 12 where A is the response curve of a four-stage amplifier, each stage having the constants of the top curve of Fig. 10. Curve B is a single stage and is the bottom curve of this same figure. The necessity for broader tuning per stage in multi-stage amplifiers in order to avoid too great a sacrifice in fidelity permits the use of coils of rather compact dimensions wound with relatively small wire. The increased coil resistance thus produced will reduce the gain per stage but this can be offset if necessary by increasing the mutual inductance to more nearly the optimum value. At frequencies sufficiently remote from resonance such that the gain per stage becomes less than unity a cascade amplifier acts as an *attenuator* of the signal. An increase in the number of stages will therefore actually decrease the strength of interfering signals whose frequencies are above or below the band where the gain per stage is equal to or greater than one. All signals whose frequencies lie within this band will be strengthened by an increase in the number of stages. For this reason two types of selectivity may be recognized; the *adjacent-channel selectivity*, and the *distant-channel selectivity*. It is therefore possible in a comparative test of two amplifiers of equal sensitivity to find that the first will produce less interference from interfering signal of, say, 30 kc away from resonance than the second; while for a signal of, say, 60 kc away there may be more interference present than in the second amplifier.

The attenuation of signals remote from the resonant frequency requires that the amplifier be well shielded in order to prevent short portions of the lead wires and circuits of the output stage from acting as aerials and picking up energy. Thus a few inches of exposed wire running to the grid of the detector tube might have a voltage induced in it from an interfering powerful local station which is much greater in magnitude than these same signals after passing through the amplifier.

10. Band-pass Filters. A rectangular response curve would be ideal for the radio-frequency amplifier of a receiving set designed for entertainment purposes. The use of a pair of tuned circuits as a coupling means between stages results in a flatter response curve with steeper sides than can be obtained with a single tuned circuit. Such an arrangement is shown in Fig. 13 and the general appearance of the resultant response curves is given in Fig. 14. Due to the more uniform amplification obtained over a wider band of frequencies, these circuits are often referred to as band-pass filters.

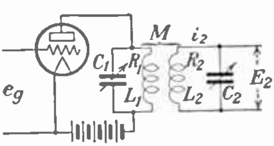


FIG. 13.—Transformer with primary and secondary tuned.

If the primary and secondary are both tuned to the same frequency, the width of the band depends on the magnitude of the coupling between them. A double-humped response curve results if ωM is made greater than $\sqrt{R_1 R_2}$ and as ωM is increased, the two peaks move farther apart and the hollow between them becomes deeper. The expression for the voltage amplification per stage is rather complicated and is given by

$$G = \frac{\mu M}{C_2\sqrt{A^2 + B^2}} \tag{28}$$

where

$$A = R_2[R_1 + R_p(1 - \omega^2 L_1 C_1)] - \omega(L_1 + r_p R_1 C_1)\left(\omega L_2 - \frac{1}{\omega C_2}\right) + \omega^2 M^2$$

$$B = \omega R_2(L_1 + r_p R_1 C_1) + [R_1 + r_p(1 - \omega^2 L_1 C_1)]\left(\omega L_2 - \frac{1}{\omega C_2}\right) + \omega^2 M^2 C_1 r_p$$

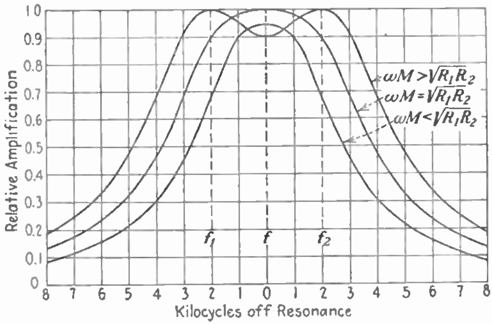


FIG. 14.—Response curves of doubly-tuned r-f stage.

If the primary is tuned so that $\omega^2 L_1 C_1 = 1$, the gain will be a maximum when A becomes zero, which will occur when

$$\frac{1}{\omega C_2} = \omega L_2 - \frac{\omega^2 M^2 + R_1 R_2}{\omega^2 L_1^2 + r_p R_1} \tag{29}$$

When these conditions are fulfilled (28) reduces to

$$G = \frac{E\dfrac{M}{C_2}}{(R_1 R_2 + \omega^2 M^2)\left(\dfrac{r_p}{\omega L_1} + \dfrac{R_1 L_1}{\omega^2 L_1^2 + r_p R_1}\right) + \omega L_1 R_2} \tag{30}$$

With ωM greater than $\sqrt{R_1 R_2}$ the amplification will be a maximum for two different values of frequency. If R_1 and R_2 are both small and $L_1C_1 = L_2C_2 = LC$, the approximate values of these frequencies are given by

$$\left.\begin{aligned} f_1 &= \frac{f}{1 + K} \\ f_2 &= \frac{f}{1 - K} \end{aligned}\right\} \tag{31}$$

where

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

With fixed coupling between primary and secondary the width of the band becomes greater as the frequency increases, causing a progressive reduction in selectivity in much the same manner as with a single tuned circuit. Although the selectivity becomes poorer at the higher frequencies it is still superior to that which can be obtained with an equal number of single tuned stages. Tuned coupled circuits are admirably suited for the intermediate frequency amplifier of a superheterodyne receiver. Here the frequency of the band to be amplified is fixed so that problem of varying selectivity is not encountered.

11. Coupled Circuits with Fixed Primary Tuning. It is possible to reduce the characteristic decrease in selectivity in the higher frequency range of a tuned amplifier by the use of a primary which is resonant to a frequency lower than the lowest frequency to which the amplifier will tune. This is accomplished by using a high-inductance primary having a large number of turns. The distributed capacity of the primary

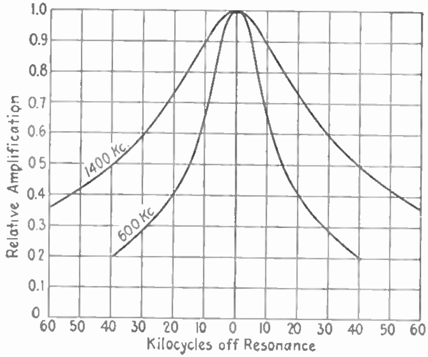


FIG. 15.—Variation in selectivity in t-r-f amplifier having high-inductance primary.

together with the plate-filament capacity of the tube results in a natural period below the lowest working frequency. Some form of honeycomb winding is usually employed. With this type of construction the effective secondary resistance is increased at the lower frequencies, tending to broaden out the tuning in this region where it is normally too sharp for good fidelity. At the higher frequencies the effective secondary resistance tends to become less. This would merely transpose the tuning characteristics and cause the response curve to become broader as the frequency diminished. However, the resistance of the secondary coil increases rapidly with frequency so that the change in the total effective resistance is much less than with the conventional primary coil of comparatively few turns. Figure 15 shows the response curves for frequencies of 600 and 1,400 kc. The selectivity ratio is approximately 2.5 to 1 whereas a ratio of 6 to 1 is about as good as can be obtained using low-inductance primary coils.

The conventional tuned r-f transformer usually has insufficient coupling at low frequencies so as to prevent broad tuning at the higher frequencies. This results in a reduction of amplification as the frequency is lowered. With the high-inductance primary, nearly uniform gain can be obtained throughout the entire frequency range. The amplification curves can be calculated from (28).

If this type of circuit is used with a three-element tube, means must be employed to counteract the effect of feed-back through the grid-to-plate capacity. Ordinarily this feed-back tends to produce oscillation but in this case the primary is operated at a frequency above its natural period so that the reactance in the plate circuit is capacitive. This results in a feed-back which tends to reduce the voltage applied to the grid of the tube so that the amplification will be reduced to a fraction of the value predicted by (28) unless this reaction is properly balanced out. Any of the neutralizing circuits described in the following section may be employed for this purpose.

12. Regeneration in Amplifiers. The three-electrode vacuum tube is not a perfect unilateral device but permits the amplified output energy to react upon the input circuit. The grid-to-plate capacity of the tube serves to electrostatically couple the input and output circuits as shown in Fig. 16. If some of the output voltage is fed back into the input circuit so as to be in phase with e_g the total, or regenerative amplification, may be expressed by

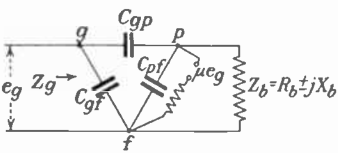


FIG. 16.—Interelectrode capacity network.

$$G_r = G\frac{1 - S}{1 - GS} \tag{32}$$

where S is the fraction of the output which is fed back into the input circuit and G is the gain of the amplifier if feed-back were absent. If the quantity GS is unity, the total amplification becomes infinite and a continuous oscillation will result. In addition to feed-back due to C_{gp} which almost always has to be balanced out to secure stability, feed-back due to coupling resulting from the use of a common B or C battery may be sufficient to cause instability. Small electrostatic or electromagnetic couplings between the input and output circuits of the amplifier can also give rise to oscillation even if each stage has been perfectly neutralized. For example, a four-stage amplifier having a gain of 10 per stage will oscillate if as much as 0.01 per cent of the output voltage succeeds in getting into the input circuit in the proper phase. Consequently multi-stage amplifiers of high over-all gain must be carefully shielded to avoid instability, particularly at the higher frequencies.

The oscillation of a single stage amplifier can occur only if the plate circuit is sufficiently inductive. If the impedance in the plate circuit is pure resistance or a condensive reactance, no oscillations can take place, although in the latter case anti-regenerative feed-back may occur of sufficient magnitude to greatly reduce the resultant gain. The effect of feed-back may be looked upon as being due to the input impedance Z_g of the grid-filament terminals of the tube. This impedance is of the form

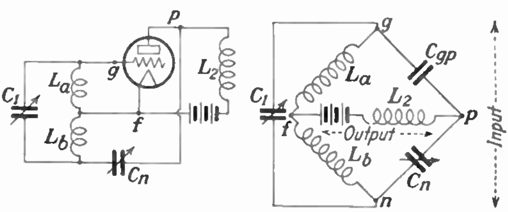
$$Z_g = \pm r_g - j\frac{1}{\omega C'_g} \tag{33}$$

When the plate circuit is inductive the sign of r_g is negative so that the tube is then capable of annulling part or all of the positive resistance of the associated input circuit. In the latter event, oscillations occur. The effect of the various circuit elements of Fig. 16 on Z_g is given by

$$Z_g = \frac{C_{gp} + C_{pf} - j\frac{1}{\omega}\left(\frac{1}{R_b \pm jX_b} + \frac{1}{r_p}\right)}{\frac{\mu C_{gp}}{r_p} + (C_{gf} + C_{gp})\left(\frac{1}{R_b \pm jX_b} + \frac{1}{r_p}\right) + j\omega(C_{gf}C_{gp} + C_{gp}C_{pf} + C_{pf}C_{gf})} \qquad (34)$$

When Z_b is capacitive and has sufficient resistance associated with it, r_g is positive and the tube may introduce rather large losses into the input circuit, even though the grid is biased sufficiently negative so that no conductive grid current flows.

13. Methods of Avoiding Oscillation. Circuits designed to combat the effects of regeneration are of two general types. Either sufficient resistance is introduced into the input circuit to offset the negative resistance introduced by the tube, or else a suitable network of circuit elements is employed so as to electrically isolate the input and output circuits by making them two pairs of opposite points of an a-c bridge. The most common method of the first mentioned group is to insert a



(a)

FIG. 17.—Rice neutralized amplifier.

resistance of several hundred ohms in series with the grid of the tube. In a tuned amplifier designed to cover a range of frequencies this resistance must be sufficiently large to secure stability at the highest frequency, which means that it is much larger than necessary at the lower frequencies. This results in loss of amplification at these frequencies. In a number of instances where this method was used in commercial receiving sets, only a part of the stability was secured in this fashion—the balance was obtained by utilizing some stray coupling between the parts so that a bridge circuit in effect was produced. Another, although rather inefficient method, applies an adjustable positive bias to the grid of the tube by connecting the grid return lead to the arm of a potentiometer connected across the filament-heating battery.

14. Neutralizing Circuits. One form of bridge circuit due to C. W. Rice is shown in Fig. 17 where are given the actual circuit and the electrical equivalent with the tube electrodes omitted. The filament terminal of the tube, instead of being connected to the lower end of the input circuit, is connected to an intermediate point which divides the inductance into two parts, L_a and L_b. The lower terminal n of the input circuit is connected to the plate through a small balancing condenser C_n. The terminals g and n of the input circuit and f and p of the output circuit constitute two pairs of opposite points of a bridge. An inspection of the latter figure indicates that no voltage can exist across the input terminals gn due to a voltage between fp if the arms are balanced.

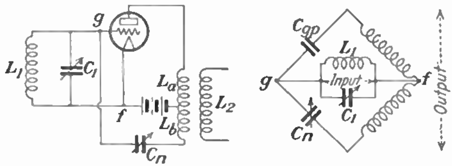


FIG. 18.—Hazeltine neutralized amplifier.

Hence the energy which is fed back through C_{gp} is opposed in phase by that which flows through C_n. The conditions for a balance are

$$\frac{L_a}{L_b} = \frac{C_n}{C_{gp}} \tag{35}$$

This balance is not entirely independent of frequency as (35) would indicate unless the coupling between L_a and L_b is substantially unity. This is because L_a is shunted by the input capacity of the tube. With certain arrangements a high-frequency parasitic oscillation may take place which will impair the performance of the amplifier at the frequencies for which it was designed. A small capacity of about the size of C_n shunted across L_2 will often prevent such parasites in receiving circuits.

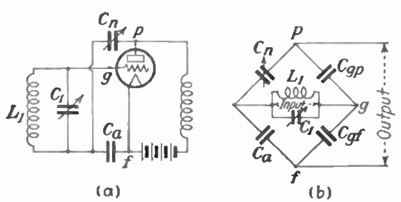


FIG. 19.—Capacity bridge-type neutralization of grid-plate capacity.

The Rice circuit is commonly used in neutralizing r-f power amplifier circuits in transmitting sets.

Another form of balancing circuit due to L. A. Hazeltine known as the *Neutrodyne* is shown in Fig. 18. This type of circuit applies the same principle to the output circuit as the previous method did to the input. The conditions for balance are the same as (35). The coupling between L_a and L_b should again be approximately unity if the circuit is to remain balanced for a wide range of frequencies with a fixed adjustment of C_n, as L_a is shunted by the output impedance of the tube. This circuit has the advantage over the Rice circuit for receiving sets in that one set of plates of the tuning condenser is at filament or ground potential. This enables the rotors of the condensers to be mounted directly on a common shaft without requiring insulating bushings or couplings. A modification of this circuit has the neutralizing condenser C_n connected to a tap at some intermediate point in L_2 thus dispensing with the coil L_b. Lack of tight coupling between L_a and L_2 with this arrangement makes it more difficult to secure complete neutralization for a wide range of frequencies.

A circuit wherein all four of the bridge arms are condensers is shown in Fig. 19. The grid-plate capacity as well as the grid-filament capacity of the tube is involved, these two capacities serving as a pair of ratio arms. The conditions for a balance are

$$\frac{C_n}{C_a} = \frac{C_{gp}}{C_{gf}} \tag{36}$$

The value of C_a is usually about 100$\mu\mu$f, which requires a value of C_n somewhat larger in size than the neutralizing condensers of the preceding circuits. In order to avoid the accumulation of a charge on the grid which may cause the tube to "block," C_a is usually shunted by a 250,000-ohm grid leak. The distributed capacity of a suitable choke coil whose natural frequency is below the frequency to be amplified can also be substituted for the condenser C_a.

Another form of circuit involving the principle of a mutual inductance bridge is illustrated in Fig. 20. The conditions for a balance are

$$\frac{M}{L_2} = \frac{C_{gp}}{C_{gp} + C_n} \tag{37}$$

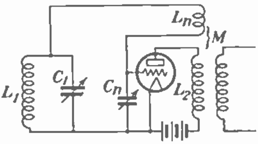


Fig. 20.—Mutual inductance bridge circuit.

Since C_n is in parallel with the grid-filament capacity of the tube it is possible to utilize C_{gf} in place of an actual neutralizing condenser, C_n, and balance by proper adjustment of the mutual inductance between L_n and L_2.

15. Neutralizing Adjustments. The most convenient method of neutralizing the above circuits is to tune the amplifier to a signal in the high-frequency range of the receiving set. The tube filament of the stage to be neutralized is then opened, usually by slipping a piece of paper between the filament pin and the filament terminal in the tube socket. This destroys the repeater action of the tube and converts that portion of the circuit into its equivalent electrical network. The neutralizing condenser is then adjusted until the signal disappears. The filament is then lighted and the procedure is repeated with the next stage.

When stray couplings are present the value of balancing capacity required may vary with the frequency so that when exact neutralization is obtained at one frequency the stage may be sufficiently unbalanced at some other frequency so that oscillations occur. In this case a compromise adjustment of C_n must be found which will hold the stage out of oscillation for the entire tuning range. This may not be possible if considerable stray coupling is present together with high gain per stage.

16. Neutralizing Power Amplifiers. Radio-frequency power amplifiers such as are used in transmitting sets where sufficient power is available can be neutralized by means of a suitable r-f ammeter in the output tank circuit. In these circuits provision is usually made to remove the plate voltage from the tube to be neutralized rather than to switch off the filament.

Figure 21 shows the last two stages of power amplification of a typical 1-kw broadcast transmitter. The first stage consists of two 75-watt screen-grid tubes in parallel which require no neutralization. The second stage is neutralized by means of the condenser C_n which connects to the input tank circuit L_1C_1 at the point shown. The principle is the same as that of Fig. 17. The turns to which the various taps on L_1 are connected are indicated by the numbers. A 30-ohm resistance R_2 is connected in series with C_n to secure a more exact phase balance, since C_{gp} of the tube will have some losses associated with it and will therefore have a phase angle of less than 90 degrees.

The neutralizing adjustment is made as follows: The switch S_1 is thrown to the top position inserting a low-range thermocouple Th_1 in the output tank circuit L_2C_2. At the same time the galvanometer A_4 is connected to the thermocouple and the plate circuit is opened by S_2 which is mechanically connected with S_1. With excitation applied to the grid the balancing

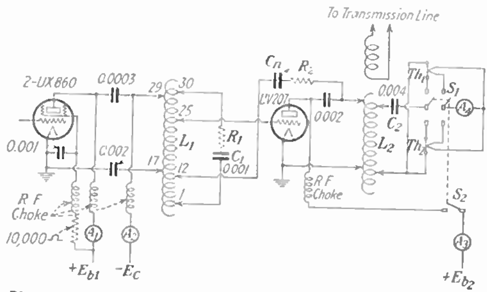


FIG. 21.—Broadcast transmitter power amplifier.

condenser C_n is then adjusted until A_4 reads zero. The switch S_1 is then thrown to the lower position, closing the plate circuit and inserting a high range thermocouple Th_2 in the tank circuit, and at the same time transferring A_4.

17. Screen-grid Tubes. Screen-grid tubes or tetrodes, are rapidly replacing three-element tubes in r-f amplifiers. The shielding effect of the second grid results in a reduction of C_{gp} to something of the order of 0.05 $\mu\mu$f as compared with a value of about 4 to 6 $\mu\mu$f in the conventional receiving tube. This reduction of the tube coupling capacity responsible for the instability of amplifiers enables high gain per stage to be obtained without the use of neutralizing circuits. Voltage amplifications of 30 to 50 per stage at 1,000 kc can be obtained without particular difficulty. Much higher gains than this are theoretically possible since the amplification factor of the 24-type tube commonly used in receiving sets is about 400 with a plate resistance in the vicinity of 400,000 ohms. For example, at 1,000 kc using as a secondary a coil of 200 μh having a resistance of 10 ohms, a voltage amplification of 125 per stage is given by (25). The required value of M would be 320 μh. In practice, a primary sufficiently large to obtain this required value of M would have considerable distributed capacity which would reduce the gain below that predicted by (25). Furthermore, this high a value of gain per stage would be sufficient in all probability to cause the tube to oscillate in spite of the small value of C_{gp}.

The foregoing equations for voltage amplification are all applicable to tetrodes as well as triodes. The higher values of plate resistance present in the former require larger values of M than are needed in triodes having the same value of mutual. Sensitivity by conductance control in sets using ordinary tetrodes in the r-f amplifier presents a difficulty not present to quite the same degree in amplifiers employing triodes. This is due to intermodulation or cross-talk effects from interfering stations produced by the curvature of the tube characteristic when the value of mutual conductance is lowered by increasing the negative bias or reducing the screen-grid potential—the usual method of volume control employed with these tubes. Distortion is also present when the volume control is turned down to obtain the desired output level from local stations. This necessitates the use of a "local-distance" switch which reduces the sensitivity of the amplifier for local reception by means other than increased negative bias or reduced screen-grid potential. Tetrodes having a variable value of μ have been designed, and are now used, which overcome these difficulties.[1]

Tetrodes are also employed for power amplification in transmitting circuits. Their chief advantage is that they do not require any neutrali-

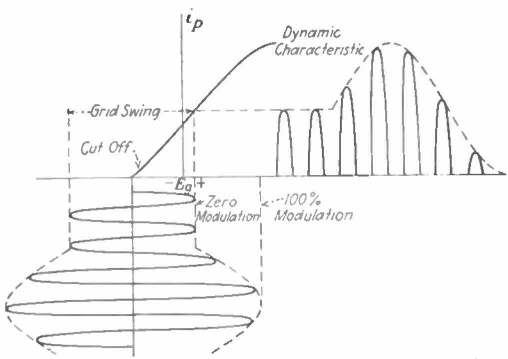


FIG. 22.—Characteristics of Class B amplification.

zation. The power amplification obtained is no better than can be secured with neutralized triodes.

18. Power Amplifiers. Class A amplifiers have a maximum possible plate efficiency of 50 per cent and a relatively low power output compared with the tube rating. For this reason they are hardly ever used as r-f power amplifiers. Much greater output and higher efficiency are obtainable under Class B or Class C operation.

19. Class B Amplifiers. A Class B amplifier is one which operates so that the power output is proportional to the square of the excitation

[1] BALLANTINE and SNOW, Reduction of Distortion and Cross-talk in Radio Receivers by Means of Variable-mu Tetrodes, *Proc. I.R.E.*, **18**, 2102, December, 1930.

grid voltage. This is accomplished by operating the tube with a negative grid bias such that the plate current is almost zero with no grid excitation. A grid excitation voltage of such magnitude is applied that essentially half-sine waves of plate current are produced on the least negative half-cycles of grid voltage. The grid usually swings positive on excitation peaks and therefore introduces harmonics into the output waves which have to be filtered from the output by suitable means.

The characteristics of Class B operation are medium efficiency and output, with a relatively low ratio of power amplification. A general idea of the characteristics of Class B operation will be obtained from Fig. 22. The impedance of the output circuit is adjusted so that the dynamic characteristic of the tube is essentially linear. For this reason, a Class B amplifier is frequently called a *linear amplifier*. The instantaneous peak output of the tube with 100 per cent modulation will be four times the power output when the grid excitation voltage is unmodulated. The continuous power output with 100 per cent modulation is 1.5 times the output at zero modulation. These power requirements must be taken into consideration by providing adequate tube capacity to take care of modulation peaks.

During the positive grid swings grid current will flow which will increase the load on the source of excitation during this portion of the cycle. For this reason, the preceding tank circuit is usually loaded with sufficient resistance so that the energy taken by the grid is reasonably small compared to the normal power being dissipated. This is the purpose of R_1 in Fig. 21. The amount of grid bias required for Class B operation is approximately the plate voltage of the tube divided by the amplification factor. The plate current wave shapes are distorted but due to the "flywheel" effect of the output tank circuit the oscillatory current in the tank is practically sinusoidal. A single impulse of varying magnitude is being received by the tank circuit during each cycle, which causes the tank current to rise and fall according to the modulation swings. Two tubes may be arranged in push pull in which case their output circuit will receive two impulses per cycle. Push-pull operation of Class B amplifiers is apt to be troublesome due to the production of parasitic oscillations unless considerable care is taken in the design and layout of the circuit. These parasites have the habit of disappearing when the grid excitation is removed, making it difficult to eliminate them.

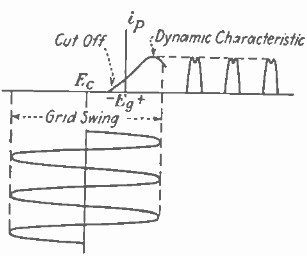


FIG. 23.—Class C operation.

A plate efficiency of about 70 per cent may be secured from a Class B amplifier with 100 per cent modulation and about 33 per cent when unmodulated.

20. Class C Amplifiers. A Class C amplifier is one in which high output is the primary consideration. The output varies as the square of the plate voltage within limits. This is accomplished by operating the tube with a negative grid bias of more than a sufficient value to reduce the plate current to zero with no grid excitation. An alternating grid excitation voltage is applied such that large amplitudes of plate current flow during a fraction of the least negative half-cycle of the grid excitation voltage. The grid usually swings sufficiently positive to cause saturation plate current to flow through the tube, and therefore causes harmonics to be present in the plate output waves. Means are usually provided to remove these harmonics from the output.

The characteristics of Class C operation are high efficiency and output, with a relatively low ratio of power amplification. Plate efficiencies of 85 per cent can be obtained. Since the power output varies as the square of the plate voltage this class of amplifier is frequently used as a modulated amplifier. When used in this capacity, the grid bias should be approximately twice the value required to reduce the plate current to zero. Class C amplifiers can also be used to amplify the output of a modulated amplifier. When used in this manner, a somewhat lower value of negative grid bias must be employed or over-modulation may take place. This class of amplifier is sometimes used to increase the percentage modulation. For example, if the maximum modulation present in the grid excitation voltage is less than 100 per cent it can be stepped up to 100 per cent in the output of the Class C amplifier by using a bias such that the plate current just falls to zero on minimum values of grid excitation.

21. Frequency Multipliers. Since the plate current wave shapes contain a relatively high percentage of harmonics, a power amplifier of this type may be readily employed to double or triple the excitation frequency. Crystal control of frequency is especially important at very high frequencies. Quartz crystals become increasingly fragile as their natural frequency is increased so that the problem can be solved by using an amplifier of this type as a frequency multiplier and using a lower frequency crystal. The coupling means employed in frequency multipliers is the same as illustrated in Fig. 21. The output circuit is merely tuned to the frequency of the harmonic it is desired to amplify. Several frequency multiplying stages can be used in cascade enabling very high frequencies to be thus obtained. Either tetrodes or triodes can be used. The efficiency of a frequency multiplier is somewhat lower than if input and output frequencies were the same. A Class C amplifier having an efficiency of 80 per cent would show an efficiency of about 70 per cent when used as a frequency doubler. The efficiency falls off as the multiplication becomes greater.

SECTION 13

RECEIVING SYSTEMS

BY G. L. BEERS, B.S.[1]

1. Classification. The following is a classification of radio receivers according to their operating principle.

1. Tuned-radio-frequency.
2. Superheterodyne.
3. Regenerative.
4. Superregenerative.

2. Tuned-radio-frequency Receivers. Tuned-radio-frequency (t-r-f) receivers are those which obtain their selectivity and r-f amplification through the use of circuits which function at the frequency of the incoming signal.

Modern t-r-f receivers use from three to six circuits which are tuned simultaneously by means of a single tuning control. A gang condenser which consists of several variable condensers assembled in a single unit is used to vary the frequency of the tuned circuits. The series resistance of a conventional tuned circuit, whose frequency is varied by means of a variable condenser, increases with frequency. The selectivity of t-r-f broadcast receivers varies in a ratio of about three to one from one end of the broadcast range to the other. One or two of the tuned circuits in a t-r-f receiver are generally used in the antenna-input system and the remainder are used to provide the coupling between the stages of the radio-frequency amplifier. Screen-grid tubes are used almost universally in the r-f amplifier. A grid-leak and condenser detector or negatively biased detector and one or two stages of audio-frequency amplification are used in the audio portion of the receiver. Tuned r-f receivers are best suited for use where the selectivity requirements are not extreme.

3. Superheterodyne Receivers. In the superheterodyne receiver the received voltage is combined with a voltage from a local oscillator and converted into a voltage of a lower or intermediate frequency which is then amplified and detected to reproduce the original signal wave.

The superheterodyne receiver utilizes the essential components of a t-r-f receiver, and in addition, a frequency converter and intermediate-frequency (i-f) amplifier. The frequency converter consists of a variable-frequency oscillator and a detector. The function of the frequency converter is to change the frequency of the received signal to the intermediate frequency. The oscillator and t-r-f circuits in superheterodyne receivers are usually tuned simultaneously by means of a gang con-

[1] Engineering Department, Research Section, RCA Victor Company, Inc.

denser. Through the use of a combination of fixed shunt and series condensers the oscillator is made to maintain a constant-frequency difference from the r-f circuits although the variable condensers for tuning each of these circuits are identical in capacity. The i-f amplifier uses two or three transformers, which usually contain two coupled circuits with the coupling adjusted to provide the so-called band-pass filter characteristics. The i-f amplifier provides the major portion of the amplification and selectivity. Since the characteristics of this amplifier are independent of the frequency to which the receiver is tuned, the sensitivity and selectivity of a superheterodyne receiver are usually very uniform throughout its tuning range. The t-r-f circuits are used primarily for eliminating certain types of interference which are common to this type of receiver. The performance of the superheterodyne receiver is in general superior to that of any other type of receiver in use today.

4. Regenerative Receivers. In a regenerative receiver the following action takes place: The received voltage is impressed on the grid of a vacuum tube. A portion of the resultant voltage which appears in the plate circuit of the tube is fed back to the grid circuit in the proper phase relation to increase the applied grid voltage. The effect of this action is to reduce the effective resistance of the resonant circuit to which the signal is applied and, thereby, provide considerable amplification of the received signal.

Regenerative receivers are usually provided with two controls, one for tuning the receiver and the other for controlling the amount of feed-back energy. If the feed-back is increased beyond a certain value, sustained oscillations are produced. It is common practice to tune regenerative receivers while sustained oscillations are being produced, as the beat frequency produced between the carrier wave of the transmitting station and the locally produced oscillations indicates when the receiver is properly tuned. This method of tuning is called the "zero-beat" method as the tuning of the receiver is adjusted so that the best note decreases in frequency till it is no longer audible. When a conventional regenerative receiver is tuned in this way interference is produced in nearby receivers which are tuned to the same station. A stage of tuned r-f amplification is sometimes used between the antenna and the regenerative circuit to reduce the possibility of producing this type of interference. The regenerative receiver is quite sensitive considering the number of tubes which are used. It is not very selective since only a single tuned circuit is generally used. They are now practically obsolete as broadcast receivers, although they are still used to a limited extent in short-wave work.

5. Superregenerative Receivers. A superregenerative receiver is a regenerative receiver in which sustained oscillations are prevented by the periodic variation of the effective resistance of the resonant circuit to which the received signal is applied.

In the superregenerative receiver oscillations are permitted to build up at a periodic rate in a resonant circuit tuned to the frequency of the received signal wave. Sustained oscillations in this circuit are prevented by the application of a quenching frequency potential to the grid of the superregenerative tube which periodically affects the tube characteristics in such a way as to stop the oscillations. The quenching frequency may be supplied either by a separate oscillator or by the superregenerative tube itself. The audio system of this type of receiver is usually provided with an a-f filter to remove the quenching frequency from the audio output. This type of receiver is used on short waves as it is quite sensitive for very high frequencies. The chief objection to this type of receiver is the difficulty of preventing radiation from the superregenerative circuit.

6. Method of Rating. Receiving sets are generally rated on the basis of the following characteristics:

1. Sensitivity.
2. Selectivity.
3. Fidelity.
4. Overload level.
5. Power consumed.

1. The *sensitivity* is that characteristic which determines to how weak a signal it is capable of responding. It is measured quantitatively in terms of the input voltage required to give a standard output.

2. The *selectivity* is the degree to which the receiver is capable of differentiating between the desired signal and signals of other carrier frequencies. This characteristic is not expressible by a single numerical value but requires one or more graphs for its expression.

3. The *fidelity* of a radio receiver is the degree to which it accurately reproduces at its output terminals the signal which is impressed upon it. As applied to a radio receiver, fidelity is measured by the accuracy of reproduction at the output terminals of the modulation of the received wave.

4. The *overload level* of a receiver is the maximum power output which can be obtained from it when the output voltage does not contain more than ten per cent of total harmonics.

7. Method of Testing. A standardized method of testing radio receivers has been established by the Institute of Radio Engineers and is described in detail in the Year Book of the Institute. The following is a brief summary of the procedure.

1. *Definition of Terms.*

a. Sensitivity, selectivity, fidelity and maximum undistorted output (see Method of Rating).

b. Normal test output: An a-f power output of 0.05 watt in a non-inductive resistor connected across the output terminals of the receiver is the normal test output for a broadcast radio receiver. The output resistor should have the value recommended by the tube manufacturer to obtain maximum undistorted output power for the type of output tube used.

c. Normal radio-input voltage: This term represents the r-m-s r-f voltage modulated 30 per cent at 400 cycles which results in normal test output at resonance.

d. Standard test frequencies: In the testing of a broadcast radio receiver, the five standard carrier frequencies are 600, 800, 1,000, 1,200, and 1,400 kc. When tests at only three carrier frequencies are required, the carrier frequencies of 600, 1,000, and 1,400 kc are used.

2. *Equipment Required.*

a. A signal generator: This consists of a shielded vacuum-tube oscillator whose frequency can be varied from 500 to 1,500 kc. An a-f oscillator is provided to modulate the r-f oscillator by a known amount at any frequency from 40 to 10,000 cycles. A calibrated resistance-type attenuator is used to impress a known potential on the standard antenna connected to the receiver. The attenuator system should be such as to allow a range of voltage impressed on the standard antenna unit from 1 μv to 200,000 μv.

b. Standard antenna: The standard antenna for a broadcast radio receiver not having a self-contained antenna is an antenna having in series a capacity of 200 $\mu\mu$f and a self-inductance of 20 μh and a resistance of 25 ohms.

c. Output-measuring circuit: This consists of a load resistor, output filter and vacuum-tube voltmeter. The output resistor should be adjustable to any desired value between 1 and 20,000 ohms and capable of dissipating 10 watts. An output filter is provided for preventing the flow of d.c. through the load resistor when testing sets which normally have d.c. in their output circuit. A vacuum-tube voltmeter or equivalent device is used for determining accurately the r-m-s voltage across the load resistor.

d. Harmonic-measuring circuit: For this purpose the instrument described in The Alternating-current Bridge as a Harmonic Analyzer is recommended. This article appeared in the *Journal of the Optical Society of America and Review of Scientific Instruments*, September, 1927.

3. *Tests.*

a. Sensitivity: The sensitivity is determined by impressing an r-f voltage, with 400 cycles, 30 per cent modulation, in series with a standard antenna and adjusting the intensity of the input voltage until normal test output is obtained for carrier frequencies between 550 and 1,500 kc.

b. Selectivity: The selectivity of a receiver is determined by tuning it to each test frequency in succession with the receiver in the same condition as in the sensitivity test and measuring the r-f input necessary to give normal test output at steps not greater than 10 kc at least up to 100 kc on either side of resonance or until the radio-input voltage has increased to ten thousand times or more if the measuring equipment permits.

d. Fidelity: This is determined by tuning the radio receiver to each standard test frequency in succession with the receiver in the same condition as in the sensitivity and selectivity tests, adjusting the impressed voltage to the normal radio-input voltage and then varying the modulation frequency from 40 to 10,000 cycles at 30 per cent modulation and constant r-f input voltage throughout, taking readings of relative output voltage at convenient modulation frequencies.

4. *Additional Tests.*

a. Determination of the overload level: This is determined by increasing in successive steps the r-f input to the receiver (with modulation adjusted to 30 per cent at 400 cycles) and measuring both the power output and the percentage harmonics. The overload level of the receiver is the maximum power output obtained from it when the output voltage does not contain more than 10 per cent of total harmonics.

b. Volume-control tests: This test is a determination of the effect of the volume control on the sensitivity, selectivity, and fidelity.

c. Test for hum: For determining the hum voltage, a filter is connected between the output of the receiver and the voltmeter. This filter has a characteristic which evaluates the various hum components according to their quantitative effect on the human ear.

8. Design of Receiving Systems. The majority of receiving sets in use today are broadcast receivers designed to cover the frequency range of from 550 to 1,500 kc. The essential electrical elements of a modern broadcast receiver may be classified as follows:

1. Radio-frequency system.
2. Audio-frequency system.
3. Volume-control system.
4. Power-supply system.
5. Loud-speaker.

9. Radio-frequency System. *Antenna-input Systems.* The antenna-input system transfers the signal wave intercepted by the antenna to the grid of the first tube in the receiver. The antenna-input system also contributes to the over-all performance as follows:

1. One or more t-r-f circuits in the antenna-input system provide selectivity for the separation of stations as well as the prevention of cross modulation.

2. A reduction in tube noise for a given sensitivity is obtained through the step-up in voltage provided by the use of tuned circuits in antenna-input systems.

A typical antenna-input system is illustrated in Fig. 1. Since there is considerable variation in the characteristics of receiving antennas used the value of the antenna-coupling inductance is chosen so that the antenna system is always tuned to a frequency below the tuning range of the receiver. If the antenna circuit becomes resonant in the tuning range of the receiver the first tuned circuit in a unicontrolled receiver will be thrown out of alignment with the remainder of the receiver and the over-all performance will be seriously affected. Figure 2 shows the voltage step-up between the antenna and the grid of the first tube which is obtained from such an arrangement. Two coupled tuned circuits are sometimes used between the antenna and the grid of the first tube. This reduces the voltage gain to approximately half that obtained with the single tuned circuit but increases the selectivity and therefore reduces the possibility of cross modulation in the first tube of the receiver.

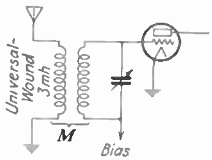


FIG. 1.—Antenna-input system.

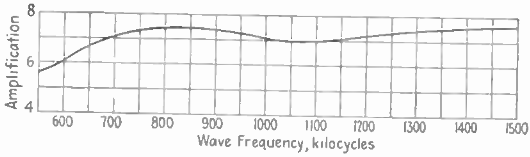


FIG. 2.—Amplification of input system of Fig. 1.

An antenna-input system is shown in Fig. 3, which provides considerably greater coupling between the antenna and the first tuned circuit. This arrangement, however, requires an adjustment to compensate for differences in antenna characteristics. The voltage gain obtained from this arrangement is approximately twice that obtained with the system shown in Fig. 1.

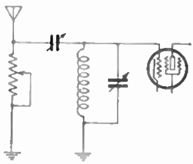


FIG. 3.—Closely coupled antenna-input system.

10. Radio-frequency Amplifiers. The types of r-f amplifiers in use in broadcast receivers may be classified as tuned, fixed tuned, and untuned.

Tuned r-f amplifiers are those which amplify a narrow band of frequencies and are provided with a control by which the position of this band of frequencies may be moved over a wide frequency range.

Untuned r-f amplifiers are not provided with a tuning control and are designed to amplify a wide band of frequencies.

Fixed-tuned r-f amplifiers are those which pass a narrow band of frequencies and whose resonant frequency is not varied with the tuning of the receiver. The intermediate-frequency amplifier of a superheterodyne receiver is an amplifier of this type.

11. Single-tuned Circuit T-R-F Amplifiers. The selectivity and amplification which can be obtained from a conventional t-r-f amplifier stage is a function of the effective resistance of the tuned circuit used in the interstage transformer. Since the selectivity provided by a t-r-f amplifier cannot be increased beyond a certain limit without serious attenuation of the high-modulation frequencies, the useful amplification which can be obtained from an amplifier stage is therefore limited. The selectivity and amplification which a t-r-f amplifier will provide can be calculated. From a practical standpoint of receiver design, however, it usually requires less time and is more accurate to determine the characteristics of a particular transformer experimentally by laboratory measurements since a determination of the effective resistance of the tuned circuit is necessary even if the characteristics of the transformer are to be calculated. It is likewise difficult to take into consideration the effects of regeneration and the proximity of shielding, etc., in a mathematical consideration of r-f transformer characteristics. The ratio of reactance to effective resistance or $\omega L/R$ of the tuned circuits used in r-f transformers for broadcast receivers is usually between 100 and 150 throughout the broadcast frequency range. The diameter of the coils used in the t-r-f circuits of broadcast receivers is usually from 1 to 2 in. and the size of the copper wire used for winding the coils is usually between Nos. 28 and 32 B. & S.

12. Neutralization. One of the major problems in the design of r-f amplifiers is the prevention of oscillations due to the effects of coupling between the grid and plate circuits of an amplifier. One of the principal sources of such coupling in t-r-f amplifiers using triodes is due to the

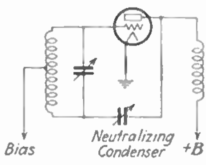


FIG. 4.—Rice system of neutralization.

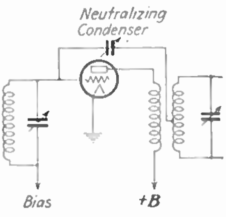


FIG. 5.—Hazeltine neutralization method.

grid-plate capacity of the tubes themselves. The grid-plate capacity of the conventional triode is of the order of 6 $\mu\mu$f. The coupling due to this capacity in a well designed amplifier is sufficient to produce sustained oscillations unless some means of stabilization is provided. Figures 4 and 5 show two of the most common methods of neutralizing the coupling due to the grid-plate capacity. Both methods use a bridge arrangement in which a small adjustable condenser is used to provide coupling equal and opposite to that of the tube capacity. The method shown in Fig. 4 requires that both the rotor and stator of the tuning condenser be insulated from ground. With the arrangement in Fig. 5 the rotors of the tuning condensers may be grounded, thus permitting the use of a gang condenser with the rotors on a common shaft.

13. Screen-grid Tubes. Practically all modern broadcast receivers employ screen-grid tubes in the r-f amplifiers and the following advantages are gained through their use:

1. No neutralization of grid-plate capacity is required, since the grid-plate capacity of the screen-grid tube is usually less than 0.01 $\mu\mu$f.
2. The high plate impedance (500,000 ohms) of this type of tube produces a negligible effect on the selectivity of the tuned circuits used in t-r-f amplifiers.
3. Higher amplification per stage can be obtained due to the high impedance and high mutual conductance of this type of tube.

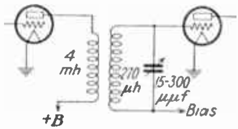


Fig. 6.—T-r-f interstage transformer.

Considerable shielding is required in screen-grid r-f amplifiers to prevent coupling between circuit elements and wiring which may likewise cause oscillations. It is common practice to locate the grid circuits and plate circuits associated with each tube in separate metal compartments to prevent coupling between them.

Figure 6 illustrates the type of t-r-f transformer which is used in the majority of broadcast receivers. The primary of the transformer is a small "universal-wound" coil which is either wound on a form of small diameter so that it can be mounted inside the secondary or is wound directly on the end of the same form as the secondary. The secondary is wound on a piece of tubing made of Bakelite or some similar material.

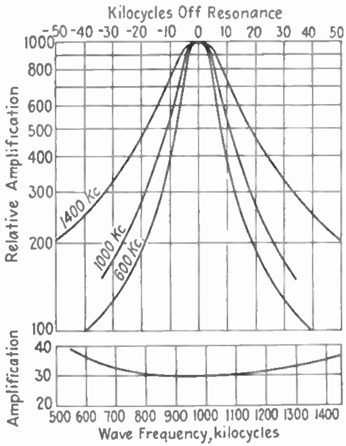


Fig. 7.—Characteristics of transformer in Fig. 6.

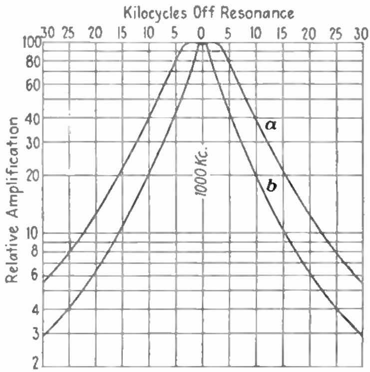


Fig. 8.—Selectivity comparison of single and coupled tuned circuits.

The primary is coupled electromagnetically to the secondary. The amplification and selectivity characteristics obtained with this transformer when used with a UY-224 tube are shown in Fig. 7.

14. Coupled Tuned-circuit T-R-F Amplifiers. A number of broadcast receivers use one or more transformers in which two tuned circuits are used. The two circuits are coupled near the point of critical coupling. The advantage obtained through the use of this type of transformer is that a considerable improvement is obtained in the shape of the selectivity characteristic. Figure 8 illustrates this improvement. Curve *a* shows the characteristic obtained with two coupled tuned circuits, and curve *b* shows the characteristic obtained with two similar tuned circuits in cascade. The width at the top of the resonance curve of a coupled tuned-circuit transformer depends on the coupling between the two circuits. The flatness of the top of the curve depends on the effective resistance of the tuned circuits. By using slightly greater than critical coupling at the low-frequency end of the broadcast range and less at the high-frequency end of the range, the selectivity of this type of transformer can be made more uniform over the broadcast range than one using a single tuned circuit. Figure 9 shows the selectivity characteristic obtained from a transformer of this type. The voltage gain provided by a coupled tuned-circuit t-r-f transformer is approximately one-half that which can be obtained from a transformer using a single tuned circuit.

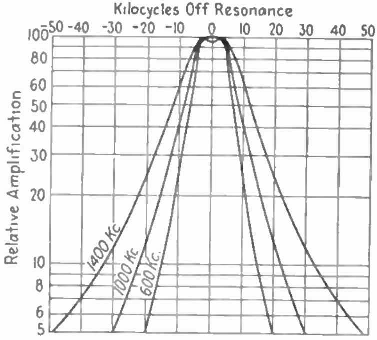


FIG. 9.—Selectivity characteristics of coupled tuned-circuit t-r-f transformer.

15. Untuned R-F Amplifiers. A stage of fixed-tuned r-f amplification is sometimes used in receivers where additional gain is desired without the need for the additional selectivity which would be provided by a stage of t-r-f amplification. A transformer of this type is frequently used to provide a receiver with a more uniform over-all sensitivity characteristic throughout its tuning range.

Figure 10 shows a fixed-tuned r-f transformer which provides a fair degree of amplification over the broadcast band. The transformer consists of a primary of 160 turns and secondary of 130 turns of No. 40 E.C. wire. Both windings are wound on a separate piece of ½-in. paper tubing. Each winding is assembled on an L-shaped core built up of 120 three-mil silicon-steel laminations. The air gap between the two sections of the core shown in Fig. 10 is 1/16 in. The secondary of the transformer is tuned by a capacity of 25 μμf. The voltage gain provided by a stage of fixed-tuned radio-frequency amplification using this transformer and a UY-224 tube is shown in Fig. 11.

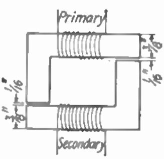


FIG. 10.—Untuned transformer.

16. Intermediate-frequency (I-F) Amplifiers. The i-f amplifier in a superheterodyne is the major factor in determining the receiver's sensitivity and selectivity.

The intermediate frequency used in the majority of modern superheterodyne receivers is between 150 and 200 kc. The usual i-f amplifier consists of two or three transformers and one or two tubes of the UY-224 type. Three transformers are used in the higher-priced sets and two are used in all the sets of the midget type. A typical i-f transformer consists of two universal-wound coils assembled on an insulating support such as a wooden rod or piece of Bakelite tubing. These two coils constitute the inductive elements of two-tuned coupled circuits. One of the tuned circuits is connected in the plate circuit of the amplifier tube and the other in the grid circuit of the succeeding tube. The electromagnetic coupling between these circuits is determined by the spacing between the coils. In some cases a copper ring is placed between the two coils so that the spacing between them may be reduced without producing excessive coupling between the two circuits. The tubing on which the coils are wound is mounted on a plate of insulating material, such as porcelain or isolantite. On this plate are also mounted the two small adjustable condensers which tune the two coupled tuned circuits. Care must be exercised in the design of these condensers to insure that the capacity of the condensers remains constant after they have been adjusted to their proper value. The entire transformer assembly is enclosed in a metal container which serves both to protect the unit and shield it electrically. The two adjustable condensers are so located that the screws for adjusting their capacity are accessible through holes in the top of the container.

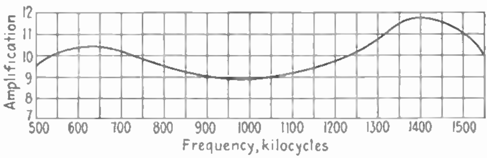


FIG. 11.—Amplification characteristic of untuned r-f transformer.

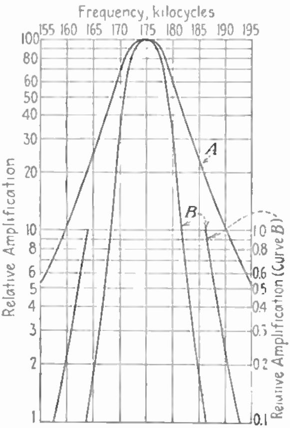


FIG. 12.—Intermediate-frequency selectivity characteristics: Curve A, one stage; curve B, three stages.

Since i-f transformers make use of coupled tuned circuits the selectivity characteristic provided by such a transformer approaches the flat-topped band-pass filter characteristic. The gain obtained from an amplifier stage using a transformer of this type is determined by the mutual conductance of the tube, the tube impedance, the resonant impedance of the tuned circuits and the coupling between them. The band of frequencies which such a transformer will pass is determined by the coupling between the two tuned circuits and the flatness of the top of the curve is determined by the effective resistance of the tuned circuits. Since the frequency to which the i-f amplifier is adjusted is not varied with the tuning of the receiver, it is the usual practice in the design of i-f transformers to use circuits with a comparatively high L to C ratio. The limit in this direction is determined by the point at which variations in the inter-electrode capacity of the amplifier tubes produce a serious effect on the alignment of the circuits.

Screen-grid tubes are used almost exclusively in i-f amplifiers, since with this type of tube, arrangements to neutralize the effect of the grid-plate capacity are not required. Since high-impedance circuits can readily be obtained in i-f transformers, it is possible to realize the comparatively high degree of voltage amplification which is inherent in this type of tube.

The selectivity characteristic provided by a typical i-f transformer is shown in curve *A* (Fig. 12). A voltage amplification of 100 can readily be obtained with a single i-f transformer and UY-224 tube. Curve *B* shows the selectivity provided by a three-transformer amplifier. The voltage gain for the usual i-f amplifier, consisting of three transformers and two amplifier tubes, when measured from the grid of the first detector to the grid of the second detector is usually from 15,000 to 30,000. The voltage gain in the amplifiers using two transformers and one amplifier tube is usually about 5,000. The amplification in the three-transformer amplifier is usually held considerably below the optimum value to prevent instability.

17. Frequency Converters. The change in frequency by which the received signal wave in the superheterodyne receivers is changed to a signal wave of an intermediate frequency is accomplished through the medium of a frequency converter, which consists of a detector and variable-frequency oscillator. The detector is generally called the first detector due to its position in the circuit. This detector is usually a negatively biased UY-224 and operates due to the curvature of the E_gI_p characteristic. The received signal voltage and a voltage from the local oscillator are both impressed on the grid of the detector. The beat-frequency potential produced by the rectification of these two currents is impressed on a tuned i-f circuit connected in the plate circuit of the detector.

The major problems in the design of the frequency converter for a unicontrolled superheterodyne receiver are:

1. To maintain a constant-frequency difference between the oscillator and radio-frequency circuits.
2. To minimize variations in the oscillator frequency with variations in the supply voltage and variations in tubes, etc.
3. To maintain a constant oscillator voltage on the detector grid throughout the tuning range of the receiver.
4. To minimize radiation from the oscillator in order to prevent interference in nearby receivers.

18. Methods of Maintaining Constant-frequency Difference. Three methods have been used to maintain a constant-frequency difference between the oscillator and first detector in unicontrolled superheterodyne receivers.

The *first method* makes use of straight-line-frequency condensers and requires that the oscillator rotor be displaced with respect to the radio-frequency circuit rotors by an amount sufficient to give the proper frequency difference. This arrangement has the disadvantage that the useful tuning range of the condensers is reduced by the amount that the rotors are displaced. For this reason this method cannot be used where the intermediate frequency is high.

The *second method* uses a gang condenser in which the oscillator condenser plates have a special shape. The problem of test and alignment for condensers of this type is somewhat complicated and more costly than condensers in which all the elements are alike.

The *third method* which is the one in general use makes use of condensers of equal capacity for both the t-r-f and oscillator circuits. The constant-frequency difference between the t-r-f and oscillator circuits is obtained through the use of a combination of shunt and series condensers in the oscillator circuit. The oscillator in superheterodyne receivers is generally tuned to a higher frequency than the t-r-f circuits, since a smaller percentage change in frequency is required and a smaller change in capacity is therefore necessary to produce the desired variation in the oscillator frequency. The oscillator tuning inductance is therefore smaller than that of the r-f circuits and its value is such that the correct frequency difference between the oscillator and t-r-f is obtained at the middle of the tuning range with equal capacity in each circuit. The combination of shunt and series condensers used in the tuned oscillator circuit maintains the frequency difference constant throughout the tuning range of the receiver.

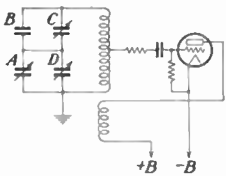


FIG. 13.—Typical superheterodyne oscillator circuit.

These condensers are shown in Fig. 13. Condenser A is the main tuning condenser. Condenser B is the fixed-series capacity. Condenser C is a small adjustable condenser for accurately adjusting the total series capacity. Condenser D is the small adjustable shunt condenser.

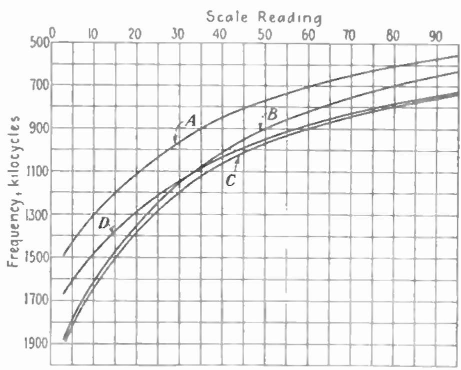


FIG. 14.—Effect of shunt and series condensers in oscillator circuit.

The values used in a commercial superheterodyne receiver are:

Main tuning capacity A	15 $\mu\mu$f to 350 $\mu\mu$f
T-r-f tuning inductance	270 μh
Oscillator tuning inductance	215 μh
Fixed series capacity B	750 $\mu\mu$f
Adjustable series capacity C	15 $\mu\mu$f to 70 $\mu\mu$f
Adjustable shunt capacity D	5 $\mu\mu$f to 40 $\mu\mu$f

The effect of these condensers is shown in Fig. 14. Curve A shows the relation between frequency and dial reading for the r-f circuits.

Curve B shows the same relations for the oscillator circuit without the shunt and series condensers. Curve C shows the effect of the shunt condenser and curve D shows the effect of both shunt and series condensers. Similar treatment can be applied to gang condensers of the straight-line-frequency, mid-line or straight-line-capacity types.

Figure 13 shows a typical oscillator circuit used in superheterodyne receivers. It will be noted that the tube is connected across only a portion of the tuned circuit so as to minimize the effect of tube variations on the oscillator frequency.

The oscillator is frequently coupled to the first detector through electromagnetic coupling between the inductive elements of the two-tuned circuits. A small amount of capacity coupling is sometimes used to oppose the electromagnetic coupling at the high-frequency end of the tuning range in order to maintain a constant oscillator voltage of approximately five volts on the first detector grid. The first detector is biased negatively so that its normal plate current is 0.5 ma. The oscillator voltages on the first detector grid cause an increase of approximately 1 ma in the average value of the first detector plate current. Sufficient bias, plate and screen potentials should be used on the first detector so that a signal voltage of three or four volts in addition to the oscillator potential is required on the grid of the first detector before the tube starts to draw grid current.

19. Audio-frequency Systems. Two functions are performed by the a-f portion of a broadcast receiver. The first function is the demodulation of the received carrier wave. This result is accomplished by means of a detector. The second function of the a-f system is the amplification of the a-f output of the detector until it is of a sufficient magnitude to actuate a loud-speaker. One or more stages of a-f amplification are used for this purpose.

20. Detectors. The detectors which are in use in the a-f systems of broadcast receivers may be classified as to their operating principle as follows:

1. Grid-leak and condenser or grid-circuit detectors.
2. Negatively biased or plate-circuit detectors.
3. Diode or Fleming valve detectors.

The grid-leak and condenser detector functions by virtue of the grid-voltage grid-current characteristic of the tube, and the a-f potential developed across the grid leak is then amplified by the tube.

The negatively biased detector depends on the curvature of the grid-voltage plate-current characteristic of a tube for its action.

The Fleming valve or two-element detector functions due to the fact that such a tube offers a very high impedance to the flow of current in one direction, while its impedance to the flow of current in the opposite direction is comparatively low.

21. Power Detectors. Detectors capable of providing sufficient audio output to feed a power-output tube without an intervening stage of a-f amplification have been called *power detectors*. A UY-227 tube used as a negatively biased detector with a plate voltage of 250 volts and a corresponding high-negative bias has been used most extensively as this type of detector. The general advantages secured from such a detector are as follows:

1. The amount of filtering required in an a-c operated receiver is reduced, due to the reduction in a-f amplification.
2. Distortion is reduced as the detector operates on a more linear characteristic and the distortion of one audio stage is eliminated.
3. Microphonic feed-backs produced by detector tubes are reduced by the reduction of the a-f amplification.
4. Problem of equipping a receiver with automatic volume control is simplified since larger r-f potentials are available to operate such a control.

22. Detector Characteristics. The characteristics of detectors which must be considered in the design of a radio receiving set are:

1. Sensitivity: Audio output for a given modulated r-f input.
2. Fidelity: Absence of frequency discrimination and wave-form distortion.
3. Output characteristic: Maximum audio output which the detector will supply.
4. Input impedance: Effect on associated radio-frequency circuits.

Considerable information on the sensitivity, output characteristics, and wave-form distortion of detectors can be obtained from curves such as shown in Fig. 15. These curves show the relation between the change in voltage across the external plate impedance and a-c input for a UY-227 tube used as a negatively biased detector with various load resistances. From a set of curves of this type the output of a detector for a given modulated r-f input can be determined. The change in voltage across the external plate resistance is determined for the change in carrier amplitude corresponding to the per cent modulation, and the audio output of the detector in r-m-s volts is then equal to this voltage divided by $2\sqrt{2}$. The divergence of these curves from a straight line is an indication of the distortion introduced by the detector. If sufficient input is applied to the detector, the curves will indicate the maximum output which the detector will supply. The relation between the output of a detector and the frequency of the modulation, as well as the effect of the detector on associated r-f circuits, can best be determined experimentally.

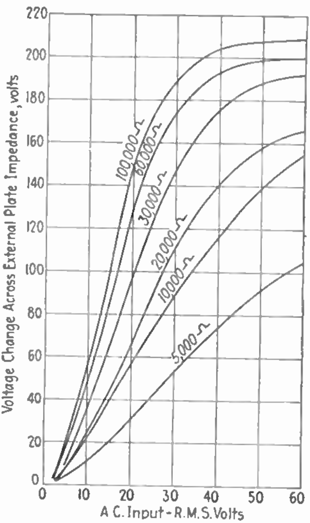


Fig. 15.—Input-output curves of negatively biased UY-227 detector.

23. Performance Characteristics of the Three Types of Detectors. The grid-leak and condenser detector is the most sensitive of the three types for weak signal inputs. For small signal inputs, the relation between modulated r-f input and a-f output for this type of detector follows the square law. When a detector is used which operates on the square law a second harmonic of the modulation frequency is present in the output of the detector which is equal to $\frac{1}{4}m^2$ where m is the per cent modulation of the received carrier. A detector in which the relation between modulated r-f input and audio output is linear does not introduce this distortion. The negatively biased detector is comparatively insensitive for small signal inputs and likewise operates on the square law. If high plate and bias voltages are used with such a detector it becomes reasonably sensitive to large signal inputs, and the relation between modulated r-f input and audio output then becomes approximately linear.

The Fleming valve detector is the least sensitive of the three types. The relation between modulated r-f input and a-f output for this detector is practically linear except for very small input voltages. If high values of resistance and capacity are used in the leak and condenser combination, considerable attenuation of the high-modulation frequencies will occur. Both the grid-leak and condenser and Fleming valve detectors have a comparatively low input impedance and produce a considerable broadening effect on the selectivity characteristic of a tuned circuit connected to such a detector. This effect can be minimized by connecting the detector across only a portion of the tuned circuit so as to provide a proper impedance match. The grid-leak and condenser detector is used where high gain is desired in the system. It is usually used in conjunction with a two-stage transformer coupled amplifier. The negatively biased detector is used most generally where a power detector is desired. The UY-227 type tube is usually used as a power detector and, as such, operates with a plate potential of 250 volts and a negative bias sufficient to reduce the normal plate current to 0.5 ma. This biasing potential is frequently obtained through a self-biasing resistor of approximately 50,000 ohms in the cathode lead. The output impedance of this type of detector is comparatively high, being approximately 50,000 ohms and therefore requires a transformer, having a primary inductance of several hundred henrys, in order to realize the full output of the detector and maintain a flat frequency-response characteristic. If some of the detector output can be sacrificed, an a-f transformer with a lower primary inductance may be used, provided the primary is shunted with a resistor to maintain a flat frequency-response characteristic. A UY-224 tube is sometimes used as a negatively biased detector. On account of the high plate impedance of this type of detector, an inductance of 1,000 henrys is required as an impedance-coupling element to realize the maximum output which such a detector will provide. Fleming valve detectors have been used to a limited extent in broadcast receivers. One receiver in which this type of detector is used employs two stages of a-f amplification in addition to a push-pull power-output stage to complete the a-f system.

AUDIO-FREQUENCY AMPLIFIERS

24. Transformer Coupled A-F Amplifiers. This type of amplifier makes use of an a-f transformer designed to provide uniform amplification over the a-f range. A conventional a-f transformer consists of a primary of approximately 5,000 turns and a secondary of 15,000 turns of No. 40 enameled copper wire assembled on a closed core of 14 mil silicon steel laminations. To realize maximum gain from an a-f stage, the reactance of the transformer primary should be at least twice the tube impedance for the frequency range over which the transformer is to operate. The step-up ratio usually used in interstage a-f transformer is from 3 to 6.

Transformer coupling is used in the majority of broadcast receivers. One of its chief advantages is that this type of coupling can be utilized readily in push-pull circuits.

The limiting factors in determining the maximum step-up ratio which can be used in an a-f transformer to provide a given characteristic are:

1. Primary inductance must be of sufficient value to provide satisfactory low-frequency amplification.
2. Distributed capacity of windings must not exceed a certain value to maintain satisfactory high-frequency response.
3. Size of wire is usually limited by its physical properties.
4. Saturation of the core.

25. Impedance Coupling. An impedance coupled a-f amplifier stage consists of a resistance or reactance in the plate circuit of the amplifier

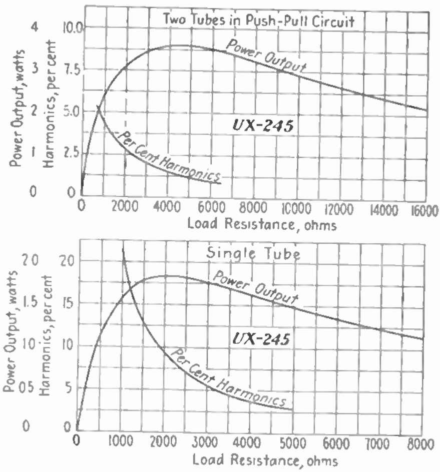


FIG. 16.—Relation between load resistance, power output, and distortion for single tube and push-pull a-f amplifiers.

tube, and the a-f voltage drop across this impedance is conveyed to the grid of the succeeding tube by means of a suitable coupling condenser. Resistance coupling is the form of impedance coupling which is most frequently used. This type of coupling provides a very uniform frequency-response characteristic over a wide frequency range. One of the disadvantages of this type of coupling is the additional voltage which must be provided by the plate potential source in order to take care of the d-c drop through the coupling resistor.

26. Power Amplifiers. The power-output stage in a broadcast receiver usually consists of two power-output tubes connected in a push-pull arrangement.

The advantages of the push-pull output stage as compared to the single output arrangement are:

1. Reduction in the amount of distortion, since the even harmonics which appear in the plate circuits of the tubes balance out in the output transformer.

2. Reduction in the amount of filtering and a-f by-passing required, since the load on the plate supply system remains practically constant as the current in one tube increases as the other tube decreases.
3. Smaller output transformer required due to the balancing of the d-c flux produced by the two halves of the winding.

The single output tube is used in a few receivers where space is limited and high-power output is not required. The relative distortion produced by the two types of output stages and the effect of load impedance on distortion and output are shown in Fig. 16 which appeared in the May, 1930, issue of *Electronics*.

The output rating of power-output tubes is usually given by the tube manufacturers as the maximum power output which can be obtained with normal voltages on the tube without exceeding a distortion of five per cent second harmonic. The approximate power output which can be obtained from an output tube for a given grid swing can be determined from the following equation:

$$P = \frac{\mu^2 E_g^2}{9R_p}$$

where P = power output in watts
μ = amplification constant of the tube
E_g = peak a-c voltage on the grid of the tube
R_p = tube impedance

The external load impedance is assumed as equal to $2R_p$. If a high-impedance circuit is used to feed the grid of the output tubes, the peak a-c voltage on the grid cannot exceed the bias voltage without the introduction of considerable distortion.

The power-output stage is connected to the loud-speaker through an output transformer in order to provide the proper impedance match. The electrodynamic type of loud-speaker, which is used in the majority of broadcast receivers, has a very low impedance and the output transformer, which feeds this type of speaker, usually has a step-down ratio of about 25 to 1.

27. Pentode Output Tubes. This type of tube has recently been used as the output tube in a number of broadcast receivers. The pentode-power output tube has considerably greater power sensitivity than the triode and therefore requires less grid swing for a given power output. Since the plate impedance of this tube is high compared with the load impedance which is normally used with the tube the current through the loud impedance remains substantially constant, even though the load impedance may vary over a wide range with frequency. With the triode-power output tube the load impedance is high compared with the tube impedance and the voltage across the load impedance therefore remains substantially constant with variations in the load impedance. When a pentode output tube is used with the conventional electrodynamic type of loud-speaker the increase in impedance of this type of load-speaker at the high-frequency end of the range and the impedance peak due to resonance at the low-frequency end of the range are likely to cause serious peaks in the load-speaker output unless some means is used to prevent them.

In 1932 the use of class B output tubes became common where large amounts of power output were desired (see Sec. 11, Art. 23).

28. Tone Control. A considerable number of broadcast receivers are equipped with a tone control, which is a device which enables the user of a receiver to vary the over-all fidelity characteristic of the receiver. The usual tone control operates on some portion of the a-f system in such a manner as to vary the high-frequency response. Figure 17 shows the most general method of accomplishing this result.

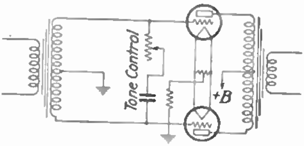


FIG. 17.—Tone control circuit.

The advantages of a tone control are:

1. Noise encountered when receiving distant stations can be reduced considerably by decreasing the high-frequency response of a receiver through the use of a tone control.
2. All broadcast transmitters do not have the same fidelity characteristics and a tone control permits the user to compensate for some of these variations.
3. The frequency-response characteristic of the ear varies with the intensity of the sound. A tone control compensates for this characteristic.

Acoustically Compensated Volume Control. A volume-control arrangement has been used in a number of broadcast receivers in which the

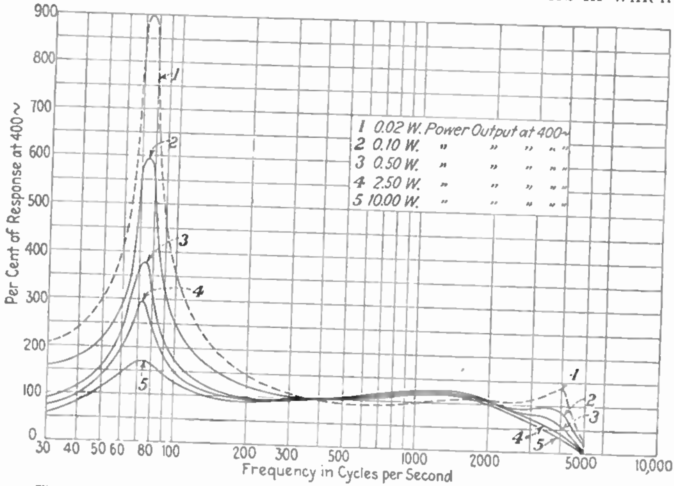


FIG. 17a.—Variation of low-frequency response with volume.

over-all frequency-response characteristic of the receiver varies with the audio output level. This type of volume control has been called an acoustically compensated volume control and is intended to compensate for the variation in the frequency-response characteristic of the ear with amplitude. Reducing the audio output of a receiver to a low value with a typical volume-control system gives the listener the impression that the very low and high frequencies have been attenuated and the middle frequency range has been correspondingly accentuated. The acoustically compensated volume control was devised to correct this effect. Figure 17*b* shows one of the arrangements which has been used to accomplish this result. This volume-control system makes use of a resonant circuit which attenuates the middle frequency range more than the high and low frequencies when the audio output is reduced. The effect of this type of control is illustrated by the curves in Fig. 17*a*, which show the relation between the audio output and frequency-response characteristic of the receiver. The low-frequency compensation shown by these curves was used not only to compensate for the variation in the frequency-response characteristic of the ear with amplitude, but also to correct for the acoustic deficiencies of the cabinet in which the receiver was installed. Since a definite relation should exist between the audio output level and the frequency-response characteristic of a receiver equipped with an acoustically compensated volume control, it is necessary that the audio output for a given setting of the volume control be independent of the strength of the received signal. Some form of a.v.c. is necessary to meet this requirement.

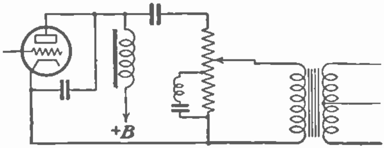


Fig. 17*b*.—Circuit for varying tone with volume.

29. Volume-control System. The two types of volume control which are used in broadcast receivers are manual and automatic.

The two methods by which volume control is accomplished are:

1. Variation in the mutual conductance of the amplifier tubes by varying the control-grid or screen-grid bias, etc.
2. Variation of the coupling between two circuits.

The advantages of the first method over the second are:

a. Volume control can be applied to a number of tubes simultaneously from a single potentiometer or variable resistor and thereby obtain a wide range of control.

b. When the control-grid potential is varied the volume-control system does not have to supply power which is a prerequisite of any practical automatic volume-control system.

c. Minimum hiss and tube noise is secured, since the gain after the grid of the first tube is varied.

The only serious disadvantage of system 1 as compared to system 2 is the distortion and cross modulation encountered under certain volume-control conditions. This distortion usually occurs when local stations are being received and the amplifier tubes are biased near the point of plate-current cut-off. The wave form of the receiver output is distorted to such an extent under some conditions as to be unintelligible. This distortion and cross modulation are functions of the third and higher derivatives of the E_gI_p characteristic of the tube. A local-distance switch has been provided on a number of receivers which permits the reduction of the coupling between the antenna and the receiver when local stations are being received. This makes it unnecessary to use the portion of the tube characteristic which introduces the distortion.

The recently developed "variable-mu" or "exponential-type" screen-grid tube reduces the amount of distortion due to the control-grid bias type of volume control. A comparison of the E_gI_p characteristics of this type of tube and the standard UY-224 is shown in Fig. 18. It will be seen that the shape of the curve of the "variable-mu" tube near the cut-off point is considerably different from that of the standard tube.

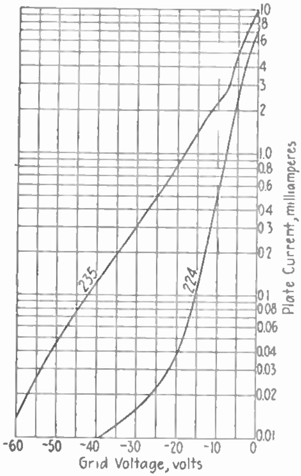


FIG. 18.—$E_g I_p$ characteristics of screen-grid and exponential tubes.

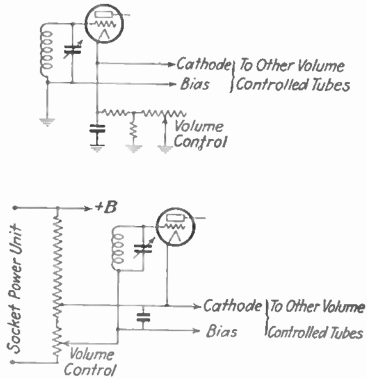


FIG. 19.—Volume-control circuits.

This change in characteristic has been produced by changing the control-grid and screen-grid structure for the purpose of reducing cross modulation and volume-control distortion.

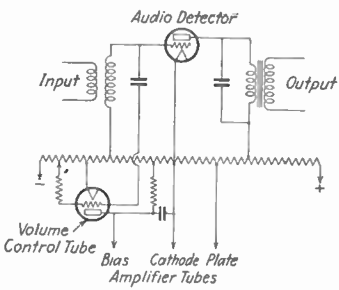


FIG. 20.—Automatic volume-control circuit.

The disadvantages of the tube from the standpoint of receiver design are:

1. A greater variation in control-grid bias is required to produce a given range of volume control. This characteristic reduces the effectiveness of an automatic volume-control system, since the volume-control bias is determined entirely by the type of automatic volume-control system which is being used and the strength of the received signal.

2. An additional type of tube is required, as the variable-mu or exponential tube is not universal in its application, since it cannot be used efficiently as a detector.

The control-grid bias method of volume control is the type of volume control used in the majority of receivers. The bias potential for the manual type of control is usually obtained from a potentiometer connected in the negative end of the plate potential source. A variable resistor in the cathode circuits of the amplifier tubes is also frequently used. A fixed current must be passed through this resistor from a bleeder resistor or other constant load; otherwise, it will be impossible to bias the tubes to the cut-off point. These two volume-control arrangements are shown in Fig. 19.

30. Automatic Volume Control. Automatic volume control (a.v.c.) is used in a number of modern receiving sets. It has the advantage that practically the same audio output is obtained from the receiver irrespective of the input. This is an advantage in tuning from one station to another where a considerable difference exists in the relative field strength of the stations. It also has the advantage of compensating for some of the more serious effects of fading. Automatic volume control also makes the actual control of volume in a receiving set less critical since the entire range of the manual control is used only to vary the actual audio output. With the manual type of volume control only a small fraction of the total variations of the control may be required to vary the sound output for a given station from minimum to maximum. The manual type of control is therefore likely to be very critical to adjust.

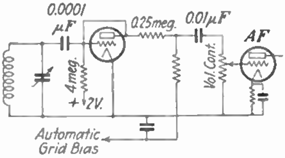


Fig. 21.—Combination detector—volume-control tube circuit.

Figures 20 and 21 show two a.v.c. arrangements. In each arrangement the d-c component of the rectified output of a detector is used as additional control-grid bias on the r-f amplifier tubes. In the first arrangement a separate volume-control tube functions as a rectifier to provide the additional bias voltage. In the second arrangement a single tube

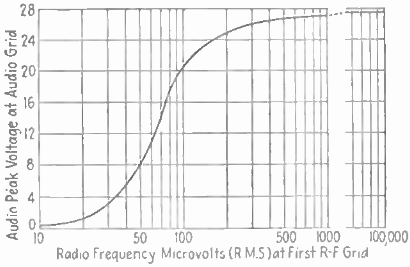


Fig. 22.—Automatic volume-control characteristic.

performs the dual function of providing the control-grid bias and demodulating the received signal. In the first arrangement the output level is controlled by varying the bias on the control tube. In the second arrangement the output level is controlled by varying the audio amplification. A typical control characteristic for an automatic volume control is shown in Fig. 22.

Noise Suppressor or Tuning Silencer. Two of the objectionable characteristics of a receiver equipped with a conventional a.v.c. system are the accentuation of noise when tuning between stations and the seeming lack of selectivity when a station is being tuned in. Several arrangements have been devised to overcome these objections. One of the systems is illustrated in Fig. 23. This a.v.c. system makes use of a noise-suppressor tube *A*, the bias of which is controlled by the detector and a.v.c. tube *B*. The d-c drop across a resistor in the plate circuit of tube *A* is used to control the bias on the a-f amplifier tube *C*. When the receiver is tuned between stations, the bias on tube *A* is such that sufficient plate current flows through the resistor in its plate circuit to increase the negative bias on tube *C* to the point where the amplifier is inoperative. When a signal of a predetermined strength is tuned in, tube *B* increases the negative bias on tube *A* so that its plate current is greatly reduced and the bias on the audio amplifier tube *C* is restored to

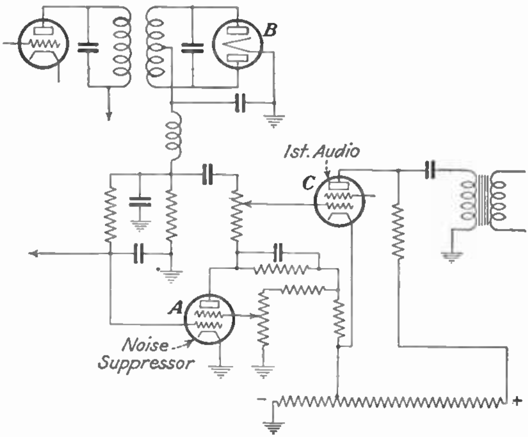


FIG. 23.—Circuit for keeping between-carrier noise out of a receiver.

its normal value and the amplifier then functions in the conventional manner. The signal level at which the receiver will respond is controlled by varying the screen-grid voltage on the noise-suppressor tube. Another scheme of this type makes use of a very selective circuit which controls the noise-suppressor tube. This selective circuit is tuned to the center of the frequency band passed by the intermediate frequency amplifier. A separate control tube that causes the noise-suppressor tube to operate is connected to the i-f amplifier through this selective circuit. Through the use of the additional tube and selective circuit the receiver is made to respond to received signals only when tuned to almost exact resonance. This type of arrangement, therefore, makes it impossible to tune a receiver so as to give the disagreeable distortion which is obtained when the carrier wave of a station is being received on the side of the receiver resonance curve.

POWER-SUPPLY SYSTEMS

31. Alternating-current Operated Receiver Power Supply. A schematic diagram of a typical power-supply system for an a-c operated receiver is shown in Sec. 15, Fig. 1.

The essential elements of the system are:

1. *A power transformer:* This is used to supply the filament voltages for all the tubes and the proper potential to the rectifier tube. The power transformer follows the conventional design for small transformers, the main requirements from the standpoint of radio-receiver design being freedom from vibration of parts.

2. *A rectifier tube:* The UX-280 and type-82 full-wave rectifier tubes are used almost universally in broadcast receivers. The maximum ratings of the UX-280 are:

a.	A-c voltage per plate (volts r-m-s)	350
	D-c output current (maximum ma)	125
b.	A-c voltage per plate (maximum volts r-m-s)	400
	D-c output current (maximum ma)	110
c.	A-c voltage per plate (maximum volts r-m-s)	550
	D-c output current (maximum ma)	135
	This rating is permissible only with filter circuits having an input choke of at least 20 h. If desired a condenser of not more than 0.1 μf may be used across the input of the filter.	

The type-82 mercury-vapor rectifier is used where good voltage regulation is of prime importance. The voltage drop in this tube is approximately 15 volts and is practically independent of the current within the emission limits of the filament.

3. *A filter for smoothing the pulsating d-c output of the rectifier tube:* The filter which is used in the majority of receivers is usually a two-section filter of the "brute-force" type. The field coil of the electrodynamic loud-speaker is frequently used as one of the choke coils. The inductance of the other choke coil is usually 20 henrys. The recent development of electrolytic condensers of both the dry and wet types which can be obtained for a few cents a microfarad have made it more economical to use a higher-capacity and lower-inductance filter. This type of filter minimizes the voltage drop in the filter and reduces the likelihood of audio feed-backs, flutters, etc.

4. *A voltage divider:* A resistance network is used to provide the proper plate and bias potentials to the tube. The trend in the design of voltage-supply circuits is to have as many of the tubes as possible supply their own bias through resistors in their cathode circuits. Bleeder resistors are used to by-pass a certain amount of current from the plate supply source through a resistor which provides the bias for volume-control purposes. In the case of automatic volume control it is desirable to have the fixed bias for the amplifier tubes which are controlled to be as independent of their cathode current as possible, as any degree of self-biasing for these tubes reduces the effectiveness of the automatic volume control. The voltage-divider arrangements vary considerably between different receivers, due to the number and type of tubes which are used.

32. Direct-current Operated Receivers. Figure 24 shows a typical schematic arrangement for operating a broadcast radio receiver from a d-c power source. The heaters of all the tubes are connected in series and supplied through a resistor directly from the line. The plate and bias potentials are supplied directly from the line through a single-section filter to remove variations due to commutator ripple, etc. The output tubes are frequently two UX-245's in push pull, and their plate potential is obtained directly from the line ahead of the filter.

33. Complete Receiving System. The usual broadcast radio receiver consists of the following elements, which may be assembled in either separate or combination units:

1. The *receiver chassis:* This contains all the r-f circuits, detector and a-f circuits except the power-output tubes.
2. The *socket-power unit:* The power-output stage is usually mounted on this unit in addition to the rectifier and filter.
3. The *loud-speaker.*
4. The *cabinet.*

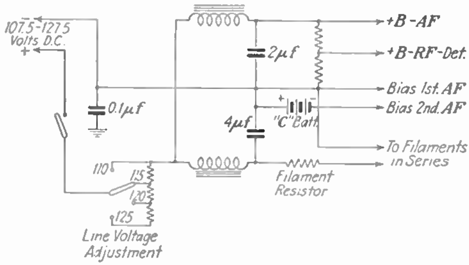


FIG. 24.—Circuit for obtaining receiver supply from d-c lines.

There are a number of receivers, particularly those of the midget type in which the receiver chassis and socket-power unit are combined in a single unit. The chief advantage of this arrangement is the simplification in wiring due to the elimination of a cable and terminal board. This advantage is offset to a considerable extent in the larger receivers by the difficulty of handling the larger and heavier units in the factory. The receiver chassis in the majority of console sets is mounted above the loud-speaker in order to place the tuning controls in a convenient location.

The receiver chassis of broadcast radio receivers which are capable of producing considerable power output are usually flexibly mounted to prevent acoustic feed-backs. Sound vibrations, produced by the loud-speaker, are transmitted through the cabinet to the receiver chassis and are likely to cause the tuning condenser plates or detector tube to vibrate and thus produce a feed-back which will cause the receiver to howl. Flanges on the base of the receiver chassis are sometimes inserted in blocks of sponge rubber to provide a suitable flexible mounting to prevent this type of feed-back.

34. Single-dial Tuning Problem. One of the major problems in the design of a unicontrolled broadcast receiver is the maintenance of the proper alignment of the tuned circuits throughout the broadcast frequency range. To maintain such alignment normally requires that the inductances and variable condensers be made very uniform. It is common practice in a number of factories to sort the coils in groups so that the variation in inductance between coils is less than 0.5 per cent. Receivers are likewise equipped with gang condensers in which an outside plate on each rotor is slotted into a number of segments. In the process of the alignment of the circuits, these segments are bent so as to compensate for variations in both the coils and variable condensers. One type of receiver, which has been produced commercially, made use of a cam arrangement by which the position of one element of the tuning capacitor could be adjusted with relation to the other element at a number of points in the tuning range of the receiver. Such an arrangement provides greater compensation for variations in the inductance of the coils and capacity of the variable condensers than can be obtained with the slotted-end plate condensers.

35. Tuned-radio-frequency Receivers. The general performance characteristics of a t-r-f receiver can be determined readily from the characteristics of the various components. Figure 25 shows the voltage gain between the antenna and the grid of each tube in a t-r-f receiver. The gain in the detector is determined for a carrier modulated 30 per cent of sufficient amplitude to produce a receiver output of 0.05 watt across the normal output impedance. Figure 26 shows the circuit diagram of the receiver. It utilizes four t-r-f circuits.

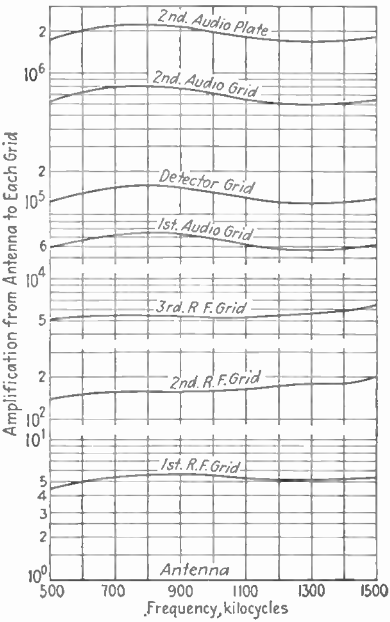


FIG. 25.—Sensitivity curves of voltage gain in t-r-f receiver.

36. Over-all Selectivity. Figure 27 illustrates a graphic method of determining the over-all selectivity of the receiver from the selectivity characteristics of the individual tuned circuits. The curves in this figure show the selectivity contributed by the tuned circuits in the receiver from the antenna to the grid of each tube. To obtain these curves the selectivity curves of the individual circuits are plotted to the same scale on logarithmic coordinates. The over-all selectivity-characteristic curves are then obtained by laying off for each frequency a distance which is equal to the sum of the distances which represent the ordinates of the individual selectivity characteristics for the same frequency. This procedure may be reversed and the selectivity characteristics which a given number of individual circuits must have to give a particular over-all selectivity characteristic can be determined. Such a determination is made by dividing the distance which represents the ordinate for a given frequency on the over-all selectivity characteristic by the number of tuned circuits. The distances obtained in this way then determine the ordinates of the individual selectivity-characteristic curve. In this case it is assumed that the selectivity characteristics of all the tuned circuits are alike.

The attenuation of the r-f system for the high-frequency side bands at 1,000 kc is shown in Fig. 28A. This is simply one-half of the top

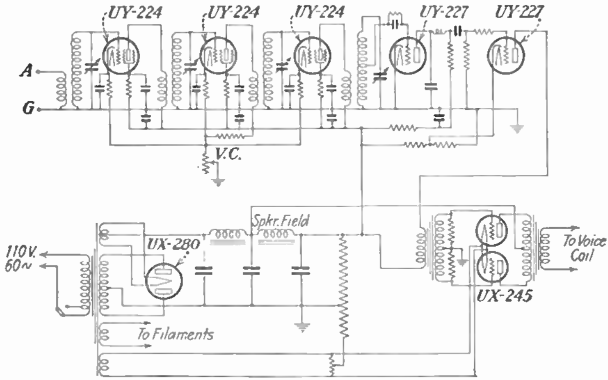


FIG. 26.—Typical t-r-f receiver.

of the selectivity-characteristic curve of the receiver plotted on an enlarged scale. The frequency-response characteristic of the detector and a-f system is shown in Fig. 28B. The ordinates for each frequency in the over-all fidelity characteristic curve of the receiver is obtained by multiplying the corresponding ordinates of these two curves. The over-all fidelity characteristic of the receiver is shown in Fig. 28C. Compensation for the attenuation of the higher modulation frequencies in the r-f system by a corresponding accentuation of the high-frequency response of the audio system is occasionally used. This method of obtaining a flat over-all fidelity characteristic in a t-r-f receiver is not entirely satisfactory due to the difference in the high-frequency side-band attenuation at the high-frequency and low-frequency ends of the broadcast range.

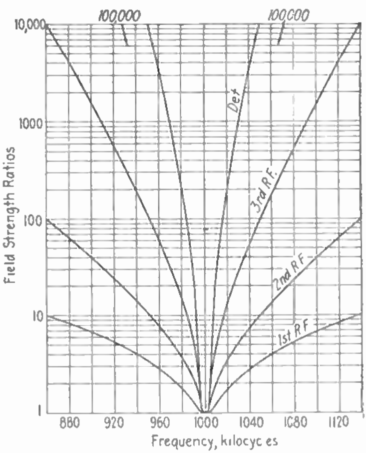


FIG. 27.—Over-all selectivity of t-r-f receiver.

37. Superheterodyne Receivers. The case of obtaining high amplification and a high degree of selectivity with a minimum of shielding allows considerable flexibility in the design of a superheterodyne receiver. Sufficient amplification can be obtained in the r-f and i-f circuits so that a power detector and single stage of a-f amplification are sufficient to provide the desired sensitivity. The general tendency in the design

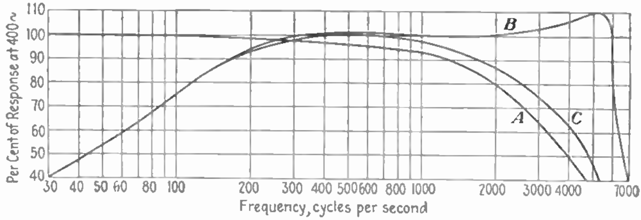


FIG. 28.—*A*, Side-band attenuation due to r-f circuits; *B*, frequency-response characteristic of detector and a-f amplifier; *C*, over-all frequency-response characteristic.

of superheterodyne receivers has been to take advantage of the high degree of selectivity which this type of receiver can provide at a corresponding sacrifice in fidelity. The superheterodyne receiver, however, lends itself just as well to the design of a high-fidelity receiver since the advantages of coupled tuned circuits can readily be realized in this type of receiver.

38. Superheterodyne Characteristics. The adjacent-channel selectivity, and fidelity of a superheterodyne receiver can be determined readily from the characteristics of the individual components of the receiver.

Figure 29 illustrates a method of determining the sensitivity and shows the gain from the antenna to the grid of each tube. Figure 30 shows similar curves giving the total selectivity contributed by the tuned circuits between the antenna and the grid of each tube. These curves are determined in the same manner as that described under the design of t-r-f receivers. From these two sets of curves it is possible to determine the voltage on the grid of each tube from a local station when the receiver is tuned to a distant station on an adjacent channel. Such a determination is frequently desirable in this type of receiver where the

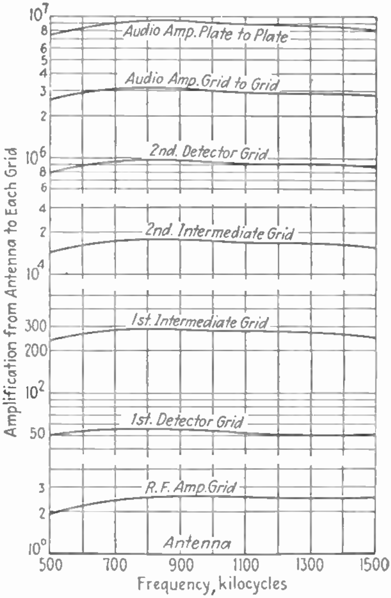


FIG. 29.—Voltage gain in superheterodyne receiver.

selectivity contributed by the circuits between each tube is not uniform. This relation between gain and selectivity between each tube must be properly proportioned; otherwise, the signal from a local station may be sufficient to draw grid current on one of the tubes even if the over-all selectivity of the receiver is sufficient to separate the signals from the local and distant stations before they reach the second detector. Figure 31 shows (*A*) the side-band attenuation in the radio-frequency circuits of the receiver and (*B*) the over-all fidelity characteristic.

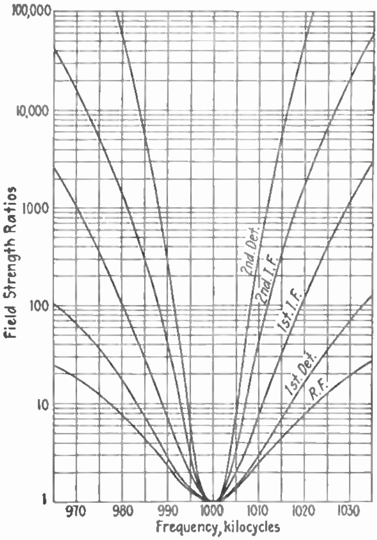


FIG. 30.—Superheterodyne selectivity characteristics.

39. Superheterodyne Interference Problems. The selectivity of a superheterodyne receiver as determined in Fig. 30 is not a true indication of the actual selectivity of the receiver under all conditions, as this type of receiver is susceptible to certain types of interference which are not encountered with a t-r-f receiver. The susceptibility to these interferences is a result of converting the received signal to an intermediate frequency. The following classification gives the more important possible sources of interference common to a superheterodyne receiver in which the intermediate frequency is lower than any frequency in the tuning range of the receiver.

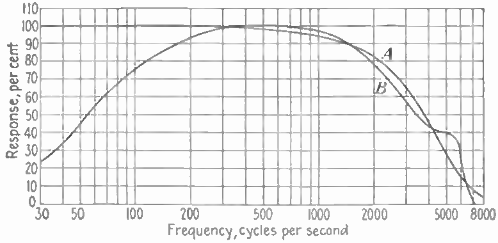


FIG. 31.—*A*, Side-band attenuation due to r-f circuits of superheterodyne; *B*, over-all fidelity characteristic.

1. *Image-frequency interference:* If f is the oscillator frequency in a superheterodyne and IF the intermediate frequency, signals impressed on the first detector, having frequencies of either $f + IF$ or $f - IF$, will be heterodyned to the intermediate frequency and pass through the receiver. It is therefore

necessary to prevent one of these signals from reaching the first detector; otherwise, what has been called "image-frequency interference" will be the result. R-f circuits, tuned to the signal which it is desired to receive, are the usual arrangement for preventing image-frequency interference. Since the oscillator in superheterodyne receivers is usually tuned to a higher frequency than the radio-frequency circuits, a signal which can produce image-frequency interference must have a frequency of $f_1 + 2IF$ where f_1 is the frequency of the desired station.

2. *Interference due to harmonics of the oscillator heterodyning undesired stations:* If a signal having a frequency of $2f \pm IF$ is impressed on the first detector, it will cause interference with the signal being heterodyned by the fundamental oscillator frequency f. Tuned r-f circuits ahead of the first detector likewise reduce the possibility of this type of interference.

3. *Interference due to stations which are separated by the intermediate frequency:* Combinations of signals are sometimes encountered which are separated by the intermediate frequency and if such signals are permitted to reach the first detector, interference will result. Tuned r-f circuits ahead of the first detector are also used to prevent this type of interference.

4. *Interference due to harmonics of the intermediate frequency produced by the second detector:* When the intermediate frequency is lower than any frequency in the tuning range of the receiver, certain harmonics of the intermediate frequency fall in the broadcast frequency band. If these harmonics, which are produced by the second detector, are of sufficient amplitude and are fed back to the input system of the receiver, they will cause interference when a station is received whose frequency is equal to a particular harmonic of the intermediate frequency. With an intermediate frequency of 175 kc this type of interference is likely to be encountered at 700, 875, 1,050, 1,225, and 1,400 kc. This type of interference is eliminated by careful shielding of the second-detector circuits.

40. Choice of the Intermediate Frequency. The choice of the intermediate frequency for a superheterodyne receiver is a compromise between the following factors:

1. With a given t-r-f system ahead of the first detector, the possibility of encountering image-frequency interference is reduced as the intermediate frequency is increased.
2. Under the above conditions, the possibility of interference due to two stations separated by the intermediate frequency is also reduced as the intermediate frequency is raised.
3. The possibility of interference due to harmonics of the intermediate frequency being fed back from the second detector to the input of the receiver increases as the intermediate frequency is raised, since lower harmonics appear in the broadcast band and the amplitude of the harmonics which can cause interference is therefore increased.
4. The difficulty of obtaining a high degree of selectivity and amplification in an i-f amplifier is increased as the intermediate frequency is raised.

The intermediate frequency which is used at the present time in the majority of broadcast receivers is 175 kc. With this frequency three t-r-f circuits ahead of the first detector are sufficient to eliminate image-frequency interference, except under the most extreme receiving conditions. With an intermediate frequency of 175 kc, the fourth harmonic is the first to appear in the broadcast range.

41. Tuned-radio-frequency Circuits. The t-r-f circuits ahead of the first detector in a superheterodyne receiver are used primarily for eliminating certain types of interference common to the superheterodyne type of receiver. Figure 32 shows the attenuation of one, two, and three t-r-f circuits for frequencies up to 800 kc off resonance when tuned to 600 kc. From curves of this type it is possible to obtain the image-frequency ratio for any given r-f system which may be used ahead of the first detector. *Image-frequency ratio* has been termed the ratio between the field strength necessary to produce standard output from a super-

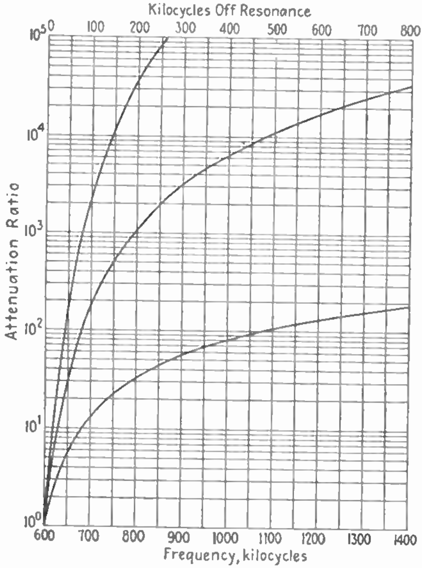


FIG. 32.—Attenuation of one, two, and three t-r-f circuits.

heterodyne at the image frequency and that necessary to produce standard output at the frequency to which the receiver is tuned. An image-frequency ratio of 100,000 : 1 is considered satisfactory for all except the

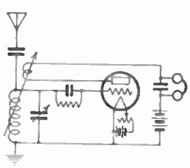


FIG. 33.—"Tickler"-type regenerative receiver.

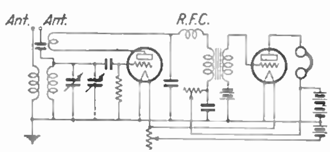


FIG. 34.—Regenerative circuit with resistance control.

most extreme receiving conditions. Care must be exercised in the design of a superheterodyne receiver to use sufficient shielding so that the actual selectivity of the t-r-f circuits is realized. If a reasonable amount of shielding is not used, signals which will cause image-frequency interference may be picked up directly on the first detector circuits and the benefit of the t-r-f circuits between the antenna and this detector will be lost.

42. Regenerative Receivers. Two typical regenerative-receiver circuits are shown in Figs. 33 and 34. In the first arrangement the regeneration is controlled by varying the coupling between the "tickler" coil, which is connected in the plate circuit of the regenerated tube, and the inductance of the tuned grid circuit. A variable resistance is used in the plate circuit of the regenerated tube in the second receiver. This variable resistance is used to vary the plate potential on the tube and thereby control the regeneration. The coupling between the tickler coil and the inductance of the tuned circuit in the second receiver is fixed. This arrangement is generally used in receivers which make use of plug-in coils to cover a wide frequency range since the tickler coil can then be wound on the same form as the tuned circuit inductance.

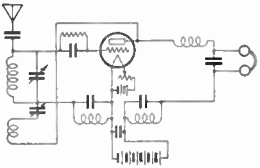


FIG. 35.—Single-tube superregenerator.

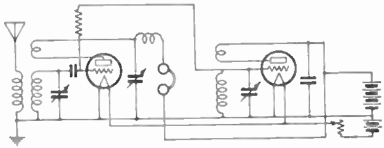


FIG. 36.—Superregenerative circuit with separate quenching tube.

43. Superregenerative Receivers. Two typical superregenerative circuits are shown in Figs. 35 and 36. Figure 35 shows a single-tube arrangement in which the quenching frequency is produced by the same tube which provides the superregeneration. In the circuit shown in Fig. 36 a separate quenching tube is used to provide the quenching frequency which is usually between 5,000 and 20,000 cycles. A filter is generally used in the output circuit of the superregenerative tube to eliminate the quenching frequency so that it does not appear in the receiver output.

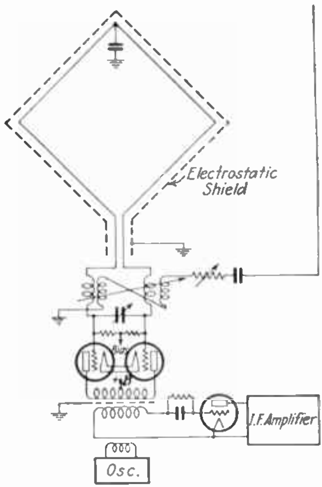


FIG. 37.—Direction-finder circuit.

SPECIAL RECEIVERS

44. Commercial Receivers. The principles underlying the design of commercial receivers are the same as those employed in the design of broadcast receivers.

Ruggedness and reliability are among the chief considerations in the design of commercial receivers, since such receivers must usually remain in continuous operation for long periods of time. Simplicity of tuning is not so important in this type of receiver as in broadcast radio receivers, since commercial receivers are generally used by skilled operators. Commercial radio receivers are generally designed to use battery-operated tubes. The plate potential for such receivers is supplied from either batteries or a motor generator. In some transatlantic receiving systems, three complete receiver and antenna combinations are used to overcome the effects of fading. In an installation of this type the antennas are separated by several wave lengths. An automatic volume-control arrangement is provided so that only the output of the receiver which is receiving the strongest signal is used.

45. Direction Finders. The directional property of a loop antenna is utilized in direction finders to determine the plane in which the radio transmitter and the direction finder are located. The circuit diagram of a typical finder is shown in Fig. 37. The loop antenna in this receiver is enclosed in an electrostatic shield. The center tap on the loop is grounded. These precautions are taken to eliminate the electrostatic effect of the loop antenna. If this effect is present, a broad minimum is obtained as the loop antenna is rotated and it is impossible to obtain an accurate bearing. The diagram shows an arrangement for compensating for the effect of a nearby metal object which might distort the field around the loop. A small antenna having characteristics as similar to the metal object as possible is erected and connected through a resistor to the variometer shown in the diagram. By proper adjustment of the variometer the signals introduced by the nearby metal object and the compensating antenna and variometer arrangement are made to balance so that they produce no effect on the inherent directional properties of the loop antenna. The superheterodyne circuit is usually employed in direction finders. Both the loop antenna and oscillator circuits are tuned through the use of a single control. Bearings can be determined to within about 1 deg.

46. Television Receivers. The major difference between television receivers and standard broadcast receivers for providing aural entertainment is in the relative width of the frequency bands which the two receivers must amplify. A flat frequency-response characteristic from 30 to 5,000 cycles is generally considered satisfactory for an a-f amplifier. A picture-frequency amplifier must amplify a band of frequencies several times this width. The highest frequency which the picture-frequency amplifier in a television receiver must amplify is equal to

$$\frac{len}{2}$$

where l = number of lines in each picture
e = number of elements in each line
n = number of pictures per second.

To reproduce a 60-line picture having a 5 by 6 ratio of height to width and 20 pictures per second, the picture-frequency amplifier in a television receiver should provide uniform amplification for all frequencies from 20 to 43,200 cycles. The r-f circuits in the receiver should pass a band of frequencies approximately 90 kc wide. Resistance coupling and screen-grid tubes are used in the picture-frequency amplifiers. Coupled tuned circuits are generally used in the r-f system to amplify the desired band of frequencies and still provide a reasonable degree of selectivity. The flatness of the top of the characteristic of such tuned circuits is controlled by varying the resistance of the tuned circuits. The television receivers

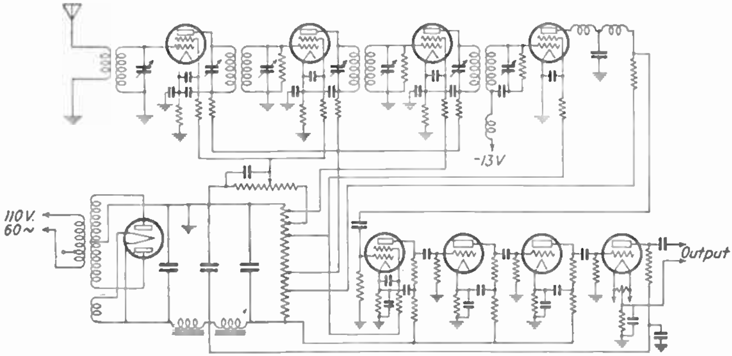


FIG. 38.—Typical television receiver circuit.

on the market at the present time are designed to cover a tuning range of 2,000 to 3,000 kc per second. The two sources of light which are used with television receivers are neon tubes and cathode-ray tubes. Neon tubes are usually operated directly in the plate circuit of a power-output tube. The variation in light intensity with the cathode-ray tube is

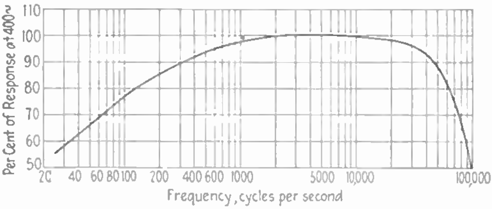


FIG. 39.—Characteristic suitable for 60-line, 20-picture television receiver.

obtained by varying the grid potential of the tube, and a power tube is, therefore, not necessary to operate this device. The circuit diagram of a typical television receiver is shown in Fig. 38. The frequency-response characteristic of a picture-frequency amplifier designed for a 60-line, 20-picture per second television image is shown in Fig. 39.

SECTION 14

BROADCASTING

BY C. W. HORN[1]

Definition. Radio broadcasting is a form of radio transmission intended for general reception; in its usual form it must be a prearranged schedule service on a daily basis.

1. Essential Elements in Broadcasting. Broadcasting of speech or music involves the following essential steps:

1. The production of the original sound.
2. Conversion of the acoustical energy into electrical energy by means of a microphone.
3. Amplification of the audio-frequency output of the microphone by means of vacuum-tube amplifiers, generally termed "speech-input amplifiers."
4. Transmission by wire of the amplified audio-frequency energy to the radio transmitter.
5. Amplification of the audio-frequency currents at the radio transmitter to the point where they may be impressed upon the modulator tubes.
6. Modulation at audio frequency of the radio-frequency carrier current.
7. The radiation of the modulated radio-frequency currents into space by means of an antenna.

2. Audio Frequencies Involved in Broadcasting. Although it is generally agreed that a wider range is desirable, most broadcasting stations transmit at present audio frequencies between about 80 and 6,000 cycles with good fidelity; the restriction of the upper frequencies is due largely to the characteristics of the wire lines rather than the transmitting equipment itself. The charts (Figs. 1 and 2) show what frequency ranges are required for various degrees of fidelity; it will be noted that for very high quality the range should extend from 30 up to about 10,000 or 11,000 cycles.

3. Volume Range. Table I below gives the peak power of various musical instruments playing triple *forte*. A violin playing very softly has an output of about 4 μw so the power ranges from 70 watts (full orchestra) down to 4 μw, an intensity range of about 73 db. Due to limitations of the broadcasting circuits, this volume range must be compressed within the limits which can be handled by the wire lines and associated equipment.

4. Microphones. A microphone is an electroacoustic transducer actuated by power in an acoustic system and delivering power to an electric system, the wave form in the electric system corresponding to the wave form in the acoustic system. Two types of microphones are used in broadcasting, the double-button carbon type, and the condenser type. Also a new type known as the dynamic microphone is being experimented with.

[1] General Engineer, National Broadcasting Company.

Note	Cycles per second	Organ pipe	Remarks
C^8	32,768		Beyond limit of audibility for average person.
C^7	16,384		Telephone silent with 40 volts on receiver terminals.
	10,000		Considered ideal upper limit for perfect transmission of speech and music.
C^6	8 192	¾ in.	Highest note on fifteenth stop.
	5 000		Considered as satisfactory upper limit for high quality transmission of speech and music.
C^5	4 096		Highest note of pianoforte.
E^4	2 560		Approximate resonant point of ear cavity.
G^4	3 072		
	3 000		Considered as satisfactory upper limit for good quality transmission of speech.
C^4	2 048		
	2 000		Maximum sensitivity of ear.
	1 500		Mean speech frequency from articulation standpoint.
A^2	850		
$A\flat^2$	800		Representative frequency telephone currents.
E^2	600		
A^1	426⅔		Orchestral tuning. See note below.
C^1	256		
	200		Considered as satisfactory lower limit for good quality transmission of speech.
C^0	128		
	100		Considered as satisfactory lower limit of high quality transmission of speech and music.
E_0	80		Lower note of man's average voice.
C_0	64	8 ft.	Lowest note of 'cello.
B_1	60		
C_1	32	16 ft.	Lowest note of average church organ.
	30		Considered ideal lower limit for perfect transmission of speech and music.
A_2	27		Lowest note of pianoforte.
G_2	25		
C_2	16	32 ft.	Lowest audible sound. Longest pipe in largest organ.

Notes of the "Gamut"	C	D	E	F	G	A	B	C
Vibration frequencies proportional to	1	$9/8$	$5/4$	$4/3$	$3/2$	$5/3$	$15/8$	2
Intervals between successive notes	$9/8$	$10/9$	$16/15$	$9/8$	$10/9$	$9/8$	$16/15$	

NOTE: Nearest note is indicated. Scale based on Middle C^1 (Physical Pitch) = 256 cycles.

FIG. 1.—Frequencies to be transmitted on high-quality system.

TABLE I.—PEAK POWER OF MUSICAL INSTRUMENTS
(Fortissimo Playing)

Instrument	Peak Power, Watts
Heavy orchestra	70
Large bass drum	25
Pipe organ	13
Snare drum	12
Cymbals	10
Trombone	6
Piano	0.4
Trumpet	0.3
Bass saxophone	0.3
Bass tuba	0.2
Bass viol	0.16
Piccolo	0.08
Flute	0.06
Clarinet	0.05
French horn	0.05
Triangle	0.05

5. Condenser Microphones. A condenser microphone involves a variation in electrostatic capacity produced by a sound wave. It consists essentially of a thin metal diaphragm under tension, separated by a small distance from a metal plate, the plate and the diaphragm forming the two electrodes of an air condenser. Figure 3 gives a cross-sectional view of the RCA UZ4083A, also called 4AA, condenser microphone. The diaphragm is usually two or three inches in diameter and

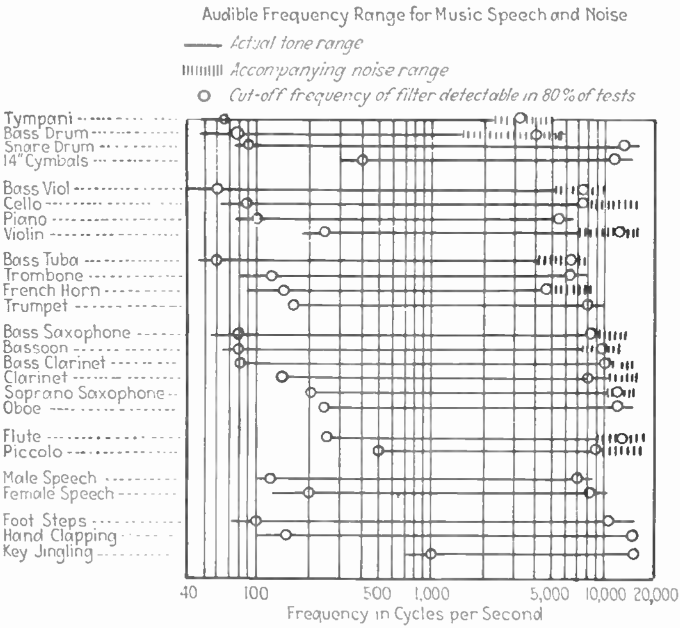


Fig. 2.—Frequency range of musical sounds.

has a capacity of about 200 $\mu\mu$f. When affected by sound waves, the vibration of the diaphragm alters the electrostatic capacity by an amount in the order of 0.01 per cent and the slight variation in voltage thereby produced can be impressed on the grid of a vacuum tube and amplified.

Because of the low sensitivity of the condenser microphone, it did not assume a position of importance among acoustical instruments until suitable amplifiers had been developed. In 1917, E. C. Wente published an account of the work he had done on an improved condenser microphone having a stretched diaphragm and a back plate so located that in addition to serving as one plate of the condenser, it added sufficient air damping greatly to reduce the effect of diaphragm resonance. Most of the condenser microphones in use today embody the essential features of the Wente microphone.

Early types of condenser microphones utilized thin sheets of steel for the diaphragm, but its relatively large mass and the high stiffness it required to secure the desired resonant frequency has caused it to be replaced by aluminum alloys. The microphone illustrated has a diaphragm of aluminum alloy 0.001 in. in thickness. The edges are clamped between threaded rings, the requisite stiffness being obtained by advancing the stretching ring until the desired resonant frequency, usually about 5,000 cycles, is obtained.

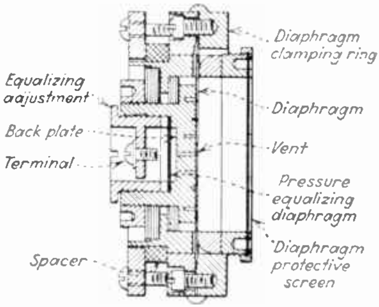


FIG. 3.—Condenser microphone

In determining the response characteristics of a condenser microphone use has frequently been made of the *thermophone* method, the thermophone consisting of two strips of gold foil mounted on a plate and fitted into the recess in the front of the microphone, the recess being entirely enclosed and filled with hydrogen. A direct current on which is superimposed an alternating current is passed through the foil and causes fluctuations in the temperature of the foil and in the gas immediately surrounding it. These fluctuations in temperature cause changes in

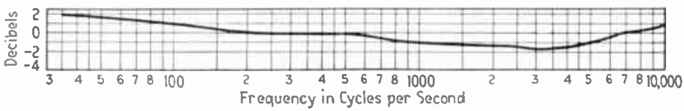


FIG. 4.—Pressure calibration of microphone.

the pressure on the microphone diaphragm and the magnitude of the pressure developed on the diaphragm can be computed from the constants of the system. A thermophone calibration is often referred to as a "pressure" calibration, since it depends entirely upon the actual pres-

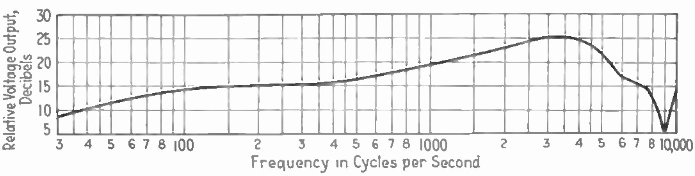


FIG. 5.—Microphone response for sounds normal to diaphragm.

sure developed on the diaphragm and hence does not take into account any effects which may occur when the microphone is used for actual pickup purposes. The response obtained by placing the instrument in a sound field of constant pressure is termed a "field" calibration. Figure

4 shows a thermophone or "pressure" calibration of a representative-type microphone. Figure 5 shows field calibrations for sounds approaching normal to the diaphragm.

The effect of the diffusion of the sound field and the tendency for most acoustic materials to be more absorbant at high frequency appears to

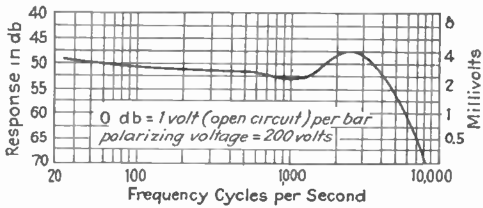


FIG. 6.—Studio characteristics of microphone.

cause the response of such a microphone under studio conditions to conform closely to the characteristics shown in Fig. 6. This perhaps accounts in part at least for the instances in which a corrective network designed to compensate for the field calibration normal to the diaphragm failed to effect a noteworthy improvement in quality.

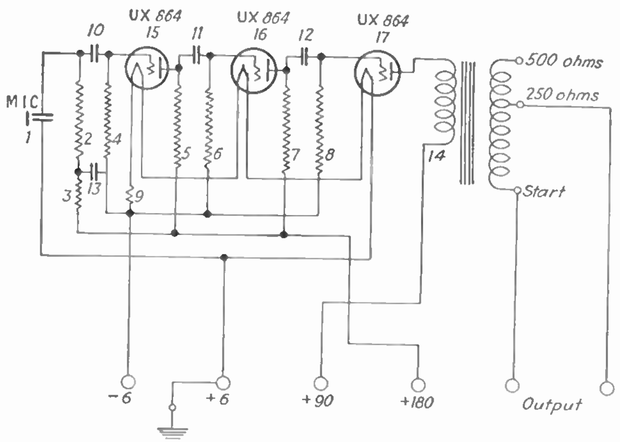


FIG. 7.—Amplifier for condenser microphone.

Since the condenser microphone is inherently a high-impedance device, it is usual to incorporate an amplifier in the microphone housing so as to reduce to a minimum the length of lead between the microphone and the grid of the first amplifier tube. Sometimes a compact amplifier is placed on the floor alongside of the microphone, the two being connected with a length of low-capacity cable. The condenser transmitter operates with a constant d.-c. polarizing voltage which may be as high as 500 volts but which is more commonly set at about 180 volts. Figure 7 gives the circuit of an amplifier designed for use with a condenser microphone.

6. Carbon Microphones. A carbon microphone is one utilizing the variation resistance of carbon granules. A typical example of the present day "double-button" carbon microphone is shown in Fig. 8; this

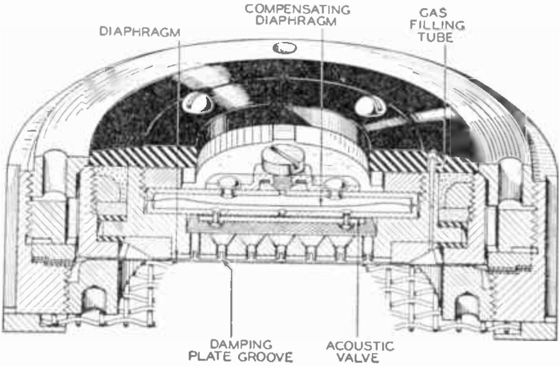


FIG. 8.—Carbon microphone.

gives a cross-sectional view of the type-387 Western Electric carbon microphone. The diaphragm of this microphone is made from duralumin 0.0017 in. in thickness and is clamped securely around its outer edge. The stretching of the diaphragm to give the desired resonant frequency, usually about 5,700 cycles, is done in two steps by means of two stretching rings. In order to insure uniformly low contact resistance, the portions

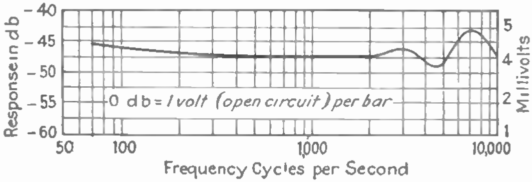


FIG. 9.—Response of air-damped duralumin diaphragm.

of the diaphragm which are in contact with the granular carbon are covered with a thin film of gold deposited by cathode sputtering. The size of the carbon granules is such that they will pass through a screen having 60 meshes per inch but will be retained on a screen having 80 meshes per inch. Each button contains about 0.06 cc of carbon corresponding to about 3,000 granules.

Referring to Fig. 9, it will be noted that the use of an air-damped stretched duralumin diaphragm has resulted in a carbon microphone having a substantial uniform response over a wide range of frequencies.

The operation of a carbon microphone may be effected by cohering (sometimes called "caking") of the granules. Severe cohering causes a large reduction in resistance and sensitivity which persists for an extended period unless the instrument is tapped so as to agitate mechanically the granules. One of the common causes of cohering is breaking the circuit when current is flowing through the microphone. Experience has shown that the use of a simple filter consisting of two 0.02 μfd condensers and three coupled coils, each having a self-inductance of 0.0014 henry, will effectively protect the microphone button without introducing an appreciable transmission loss; a potentiometer switch also serves to prevent caking.

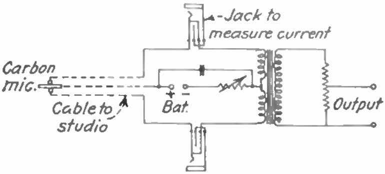


FIG. 10.—Carbon microphone connections.

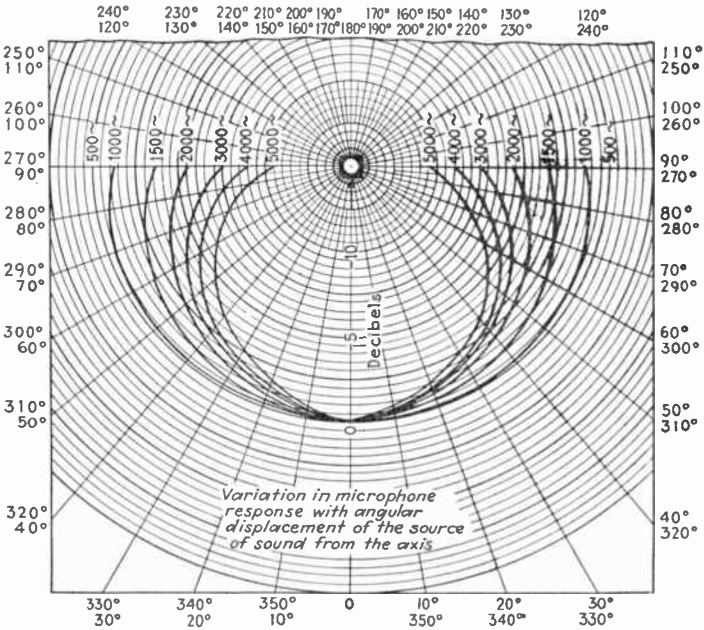


FIG. 11.—Directional characteristic of microphone.

The quality of transmission obtained with a double-button carbon microphone compares favorably with that secured with a condenser microphone; the carbon microphone has the disadvantage however of a high noise level or "microphone hiss." Figure 10 shows the manner in which the carbon microphone is connected to its associated amplifier. The jacks are used to measure the current flowing through each button, these currents usually being in the order of 10 or 20 ma.

7. Microphone Technique. During the first years of broadcasting it was the rule rather than exception to use more than one microphone to pick up a program. This arrangement had the disadvantage that the outputs from the several microphones were not in the proper phase relation, and distortion resulted therefore when the outputs were combined and fed into a common amplifier. These difficulties were especially apparent when using carbon microphones because their high background noise made it necessary to place the microphone close to the source of sound.

Considerable improvement was possible when using the condenser microphone due to its very low background noise which permitted its placement at a greater distance from the source of sound and made it possible to obtain reasonably good acoustical balances with a single microphone. However, as the distance from the source of sound to the microphone is increased, the acoustical characteristics of the studio or auditorium become more apparent.

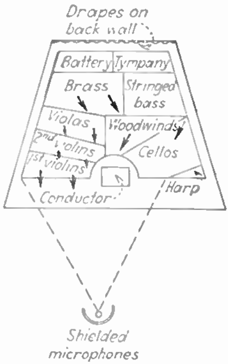


Fig. 12.—Set-up of large symphony orchestra.

The characteristics of any diaphragm type of microphone depend upon the relative positions of the microphone and the source of sound. When the sounds approach at right angles to the plane of the microphone diaphragm a flat response over the desired frequency range might be obtained. But if the sounds approach from any other point, it will be found in general that the response will fall off with frequency. This characteristic is illustrated by Fig. 11 which indicates how response varies with the angular displacement of the source of sound from the axis of the microphone. It will be noted that there is a serious loss at the high frequencies for high angular displacements. Since the majority of musical instruments depend for their quality or timbre upon the presence of overtones, it is obvious that if these overtones are discriminated against the quality will be changed materially. If, in considering this loss in high frequencies with angular displacement, we apply the limitation that the loss at 5,000 cycles shall not be more than 2 db, then Fig. 11 indicates that using a single microphone all of the instruments should be kept within an angle of 30 deg. either side of the microphone axis.

A typical set-up of a large symphony orchestra is shown in Fig. 12. It will be noted that the instruments are so placed not only to obtain the desired balance for theater work but also to obtain the proper harmonic balance allowing for the microphone's directional characteristics on the higher frequencies. The microphone is acoustically shielded to prevent reverberation from the studio or auditorium behind the microphone.

In field work it is frequently necessary to deviate from the general practice of using but one microphone since the conditions are far from ideal. For example, in picking up the operatic performances from the old Chicago Opera House in 1926 eighteen carbon microphones were used, and during most of the performance at least nine microphones were connected in the circuit. Microphones had to be switched in and out of the circuit as the performance moved from one part of the stage to another.

8. Parabolic Reflector or Directional Microphone. These problems were simplified by the development during 1930 of a more sensitive microphone with pronounced directional characteristics, the former characteristic making it possible to place it sufficiently far from the source of sound to obtain the proper balance and the directional characteristic making it possible to swing the microphone and its reflector as one would a search light to follow the action on the stage. Curve *B* (Fig. 13) shows the directional characteristic at 1,000 cycles of an ordinary camera-type condenser microphone, while curve *A* shows the

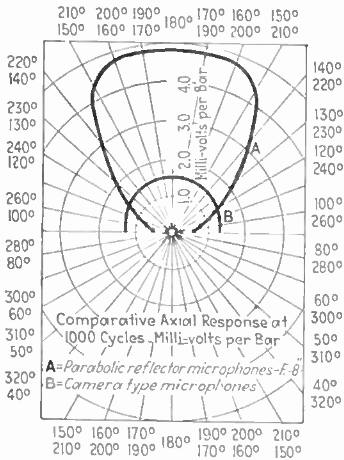


FIG. 13.—Parabolic microphone characteristic.

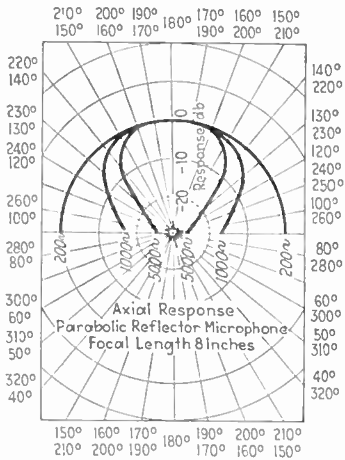


FIG. 14.—Frequency characteristic of microphone of 8-in. focal length.

response of a condenser microphone with a parabolic reflector. It will be noted that there is an increase in sensitivity along the line of axis of about four to one due to the use of the parabolic reflector.

Since the reflector increases the sensitivity and makes it possible to locate the microphone at a greater distance from the source of sound, it is desirable that the output of the microphone fall off rapidly if the sound originates at a point displaced by more than about 30 deg. from the axis of the microphone; if this characteristic is obtained, reverberation and reflections in the studio or auditorium will have very little effect. Figure 14 shows the frequency characteristic with the microphone placed on an 8-in. focal length. It will be seen that uniform response is obtained over a range of about 30 deg. either side of the microphone axis. Figure 15 shows how the frequency response at the high-frequency end can be altered by changing the position of the microphone in the reflector. By this arrangement the response can be made sensibly flat up to 7,000 cycles or if, desired, the high-frequency response may be increased by as much as 15 db over the response at low frequencies. In certain instances where the high-frequency absorption is considerable, the ability to accentuate the highs by refocusing proves very helpful. The directional microphone can be placed at a point sufficiently far from an orchestra so that it is essentially equidistant from all the instruments, and the problems of balance and volume control are thereby greatly reduced.

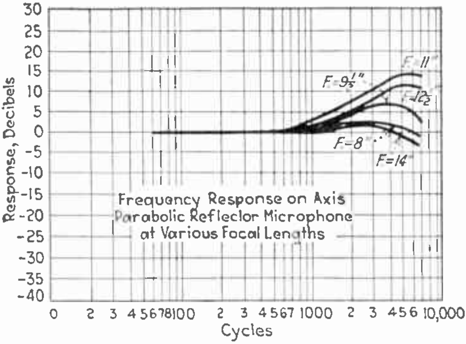


FIG. 15.—Variation of high-frequency response by changing microphone in reflector.

Another distinct advantage of the directional microphone is its ability largely to disregard the acoustics of the room as it responds only to the sounds upon which it is directly focused. In some instances this effect may be so marked as to make it necessary to use another microphone without any reflector to pick up some of the reverberation and make the reproduction more realistic.

9. Speech-input Amplifiers. Speech-input amplifiers, sometimes termed microphone amplifiers, are here considered to comprise the necessary apparatus to convert the microphone output into electrical energy of a kind and amount suitable to impress on the input system of a broadcast radio transmitter. Figure 16 is a block diagram showing the usual arrangement of microphones and speech-input amplifier equipment. The speech-input amplifier equipment comprises microphone controls, amplifiers, volume indicators, monitoring amplifiers, and relay systems.

Speech-input equipment is designed usually to have a uniform frequency characteristic from about 30 to 10,000 cycles. The frequency characteristic of a speech input amplifier is given in Fig. 17, and Fig. 18 gives the circuit diagram of a typical three-stage amplifier. The gain of the amplifier is usually adjusted to give an output of about 12.5 ma which corresponds to zero level. The maximum gain from input to output is usually about 70 db, and the maximum output without overloading about plus 16 db above 12.5 ma.

For conveying general information intercommunicating telephones are generally used to connect the control room with the studios. A Morse telegraph circuit generally connects the control room with the transmitter station; if the distance is short, this circuit may be phantomed onto the program line. The necessary switching of circuits is usually

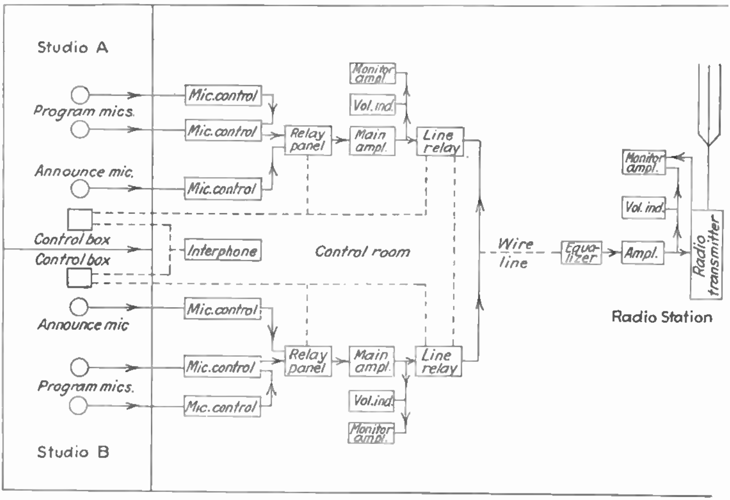


FIG. 16.—Arrangement of microphones and amplifier equipment.

done by the control-room operator and the announcer through interlocking relays. These relays control lamp signals and continuously indicate the circuit setup.

In the control room a monitoring amplifier and loud-speaker keep the operator in touch with the program. Sometimes a second monitoring amplifier supplies loud-speakers at other points.

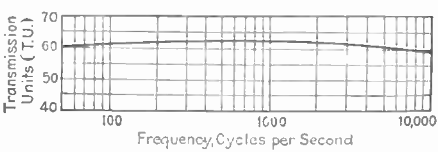


FIG. 17.—Speech-input amplifier characteristic.

Since the telephone lines are limited in the amount of energy they can handle and since transmitter equipment is designed generally to operate at an input power level of 12.5 ma, it is necessary to employ volume indicators at the control room to control the level delivered to the line and at the transmitter to indicate the level supplied to the transmitter. These units, which are vacuum-tube voltmeters, give visual indication of the signal level, especially the peak voltages, and allow the operator to adjust the gains of the amplifiers to the proper values.

10. Wire Lines. Wire telephone systems are employed almost exclusively for the national distribution of programs to the various stations connected on a network. As of June, 1932, programs were

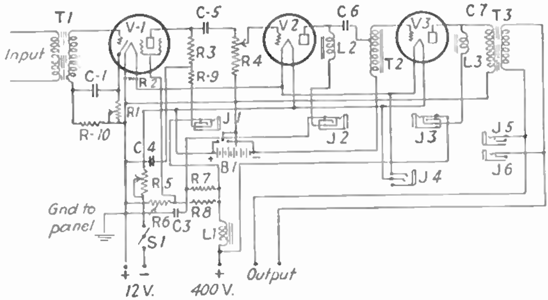


FIG. 18.—Typical speech amplifier.

distributed over five basic networks to which ten additional groups of stations are added as occasion demands. These networks involve some 179 stations which are connected together by approximately 35,000 miles of wire or twice this number of wire miles for programs alone. Telegraph circuits for interstation connections involve other thousands of miles of wire circuits. One of groups involves a radio link to the Hawaiian Islands for re-broadcasting American programs.

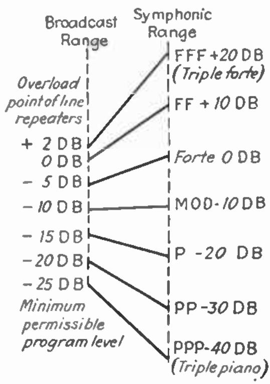


FIG. 19.—Compression of dynamic range in broadcasting system.

The frequency band which is transmitted over long-distance program circuits extends from about 100 cycles to about 5,000 cycles; to transmit music with improved fidelity a wider band than the above is desirable. A few circuits are at present available which extend the band down to 30 or 50 cycles and extend the higher range by 2,000 or 3,000 cycles. Program transmission circuits must be designed to handle wide ranges of volume. At present the volume range is limited to some 25 or 30 db, from about plus 2 or 4 db down to about minus 25 db. Obviously, since the dynamic range of a symphony orchestra is about 60 db, the wire line circuit necessitates some compression of the dynamic range. The chart of Fig. 19 indicates the manner in which the dynamic range of a symphony orchestra is compressed within the range that can be handled by the line.

It should be pointed out, however, that the listener may obtain an impression of greater dynamic range than is indicated by the above figures due to the fact that the harmonic content of the notes produced by various musical instruments varies with the loudness of the tone and since these harmonic frequencies are transmitted, the listener gets the impression of volume from the character of the sound as well as from the actual volume. Figures 20 and 21 show respectively the frequency

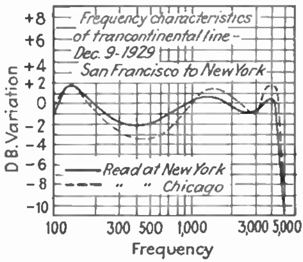


FIG. 20.—Transcontinental line as of 1929.

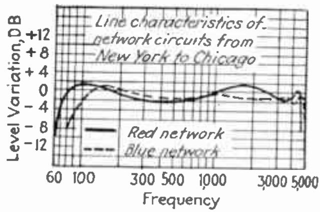


FIG. 21.—New York to Chicago circuit characteristic.

characteristics of the transcontinental line and the New York to Chicago circuit.

11. Broadcast Transmitter. The broadcast transmitter comprises the following essential components: the audio-frequency amplifiers, the modulators, the crystal-controlled r-f oscillator and r-f amplifiers, antenna system, and power-supply systems.

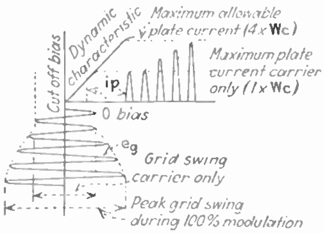


FIG. 22.—Class B amplifier operation.

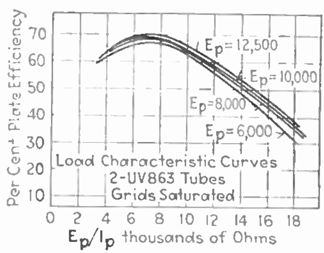


FIG. 23.—Plate-circuit efficiency—Class B.

The audio amplifier at the transmitter comprises the circuits between the point where the signal is picked off the wire line and the grids of the modulator tubes. The gain of the amplifier is sufficient, in modern transmitter circuits, so that with an input signal of about 0 db, the output voltage will be large enough to impress the maximum permissible voltage on the grids of the modulators.

Practically all broadcast transmitters use the Heising or constant current system of modulation. In the early types of transmitters, the output of the modulators was impressed upon the oscillator which fed energy directly into the antenna, or upon the final stage of r-f amplification.[1]

The power amplifier tubes in the transmitter may be operated as class B amplifiers; *i.e.*, the grid bias is such that with no signal on the grid, the plate current is reduced practically to the cut-off point. The operation of a class B amplifier is indicated by Fig. 22 which shows that the plate current drawn by the tube is very closely a linear function of the extent of the grid swing. The associated circuits are designed so that the tube operates under conditions which give maximum plate-circuit efficiency. Some curves showing how the plate-circuit efficiency varies with the plate-circuit resistance are given in Fig. 23.

The position of the crest of these curves depends upon the characteristics of the tube and upon the power factor of the circuit to which it is connected. The curves shown are for typical circuits at broadcast frequencies.

The output of such a tube is proportional to the square of the grid swing and hence the peak output under conditions of complete modulation is four times the output when the modulation is zero. The steady output under complete modulation is 1.5 times the output at zero modulation. An important consideration, then, is that the tubes in the transmitter must be capable of supplying peak powers four times greater than the nominal rating of the transmitter, assuming that the transmitter is designed for 100 per cent modulation.

12. Advantage of Complete Modulation. During recent years it has been demonstrated that a considerable reduction in background noise can be brought about by completely modulating the transmitter carrier; actually the stray-noise level is not reduced but its ratio to the received signal is decreased. Also interstation interference due to heterodyning carriers may be a serious cause of interference over much greater areas than that covered by the modulation components of an incompletely modulated carrier. The figure of merit for a transmitter may be defined as the ratio of the area over which it produces a satisfactory signal to the area over which it is capable of causing interference. The interference area remains constant for a given carrier amplitude whereas the signal carrier is proportional to some power of the modulation. Therefore the signal area of a completely modulated carrier more closely approaches that of the interference area. Furthermore if a transmitter is capable of being modulated 100 per cent the resulting side bands will have twice the amplitude of those produced by a transmitter capable of only 50 per cent modulation. To produce equivalent side bands with only 50 per cent modulation requires that the carrier amplitude be doubled or the carrier power multiplied by four.

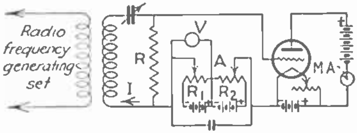


Fig. 24.—Peak voltmeter for checking modulation.

The percentage modulation of a carrier can be checked by several means. A method which checks for distortion as well as percentage modulation involves the use of a linear rectifier feeding into an oscillo-

[1] Low-power transmitters, such as 100, 250, and 1,000 watt, are what is called *output-stage modulated.* On transmitters from 5 up to 50 kw, tube economies are such that linear r-f amplifiers are very desirable.

graph element, the input to the rectifier being supplied with the modulated r-f currents.

Modulation percentage can also be checked by means of a peak-reading voltmeter of the type shown in Fig. 24. If the peak values of the modulated and unmodulated currents are determined, then the percentage modulation is

$$\frac{(I_m - I_r) \times 100}{I_r}$$

where I_m = peak value of the modulated current.
I_r = peak value of the unmodulated current.

Such a method can also be used to give a continuous indication of modulation percentage, if the tube voltages are adjusted so that the plate current just falls to zero with the unmodulated signal applied to the input. The meter reading then would increase with the percentage of modulation and the meter could be calibrated in terms of this quantity.

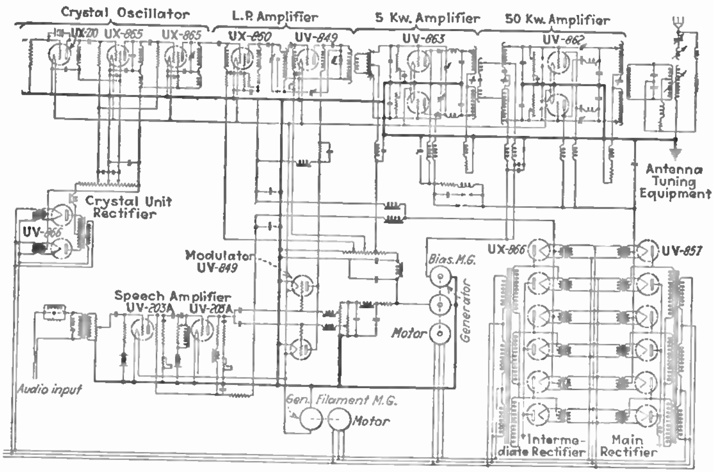


FIG. 25.—Circuit diagram of modern 50-kw transmitter.

The circuit of Fig. 25 gives the simplified circuit of a 50-kw transmitter of modern design; transmitters of lower power rating are essentially the same except, of course, for the lower ratings of the tubes and associated equipment. In this transmitter low-level modulation is used, the modulating amplifier being a 350-watt tube operating at 1,500 volts through a voltage reducer from 3,000 volts and supplied with audio-frequency modulating power from the final audio-power stage which employs two 350-watt tubes operating at 3,000 volts. This ratio of audio to r-f power permits complete modulation (100 per cent) without distortion.

The last two stages of r-f power amplification are push-pull stages with cross neutralization; the final stage employs a push-pull arrangement of two tubes feeding into an r-f transmission line. To reduce harmonic radiation, two shunt circuits adjusted to the frequency of the second harmonic are connected between ground and each side of the input to the transmission line. During "warming up" periods and for transmitter tests which do not require actual radiation into the antenna, the output of the final stage can be switched from the real antenna circuit to an artificial antenna circuit which consists of coils and condensers and a resistor capable of dissipating 75 kw of r-f energy.

The transmitter is crystal controlled, and with proper maintenance the frequency can be kept continuously constant within 50 cycles. The quartz plates are ground to approximately the proper frequency and then final frequency adjustments are made by varying the temperature; the temperature coefficient of the quartz plate varies from 30 to 100 parts in a million per degree Centigrade.

Considerable progress has been made in the method of transferring power to the antenna accomplished by means of high-frequency transmission lines which permit the building of the transmitter enclosure at a considerable distance from the antenna itself; thereby enabling the transmitter buildings to be located well out of the immediate field of the antenna. The transmission line behaves exactly like a low-frequency transmission line. A radio-frequency transmission line 1,000 ft. long operating at 790 kc exhibits all the phenomena of a 60-cycle power transmission line 2,500 miles long, except for the fact that the efficiency of the radio-frequency line is about 99 per cent, whereas it is certain that there would be very little power left at the end of the 2,500-mile line operated at 60 cycles.

13. Harmonics in Broadcast Transmitters. Since broadcast transmitters are necessarily expensive, good engineering economy lies in the direction of overloading rather than in underloading the tubes. When this is done, the tubes generate a considerable amount of energy at harmonic frequencies, and means must therefore be used to suppress radiation of this harmonic energy which would cause interference on some broadcast channel. For example a station broadcasting on 600 kc would, if second-harmonic radiation were permitted, produce interference with other stations operating on 1,200 kc. It is therefore necessary to use some scheme for by-passing and suppressing these harmonics whose production the engineer cannot economically prevent. The extent to which harmonics are eliminated is a matter of compromise between their practically complete elimination and the cost of equipment which can be justified. Figure 26 shows circuit arrangements used in a typical 50-kw transmitter to suppress harmonics. Although the instantaneous peak power in the antenna circuit of such a transmitter may be as high as 200 kw, the harmonic radiation can be reduced to less than 0.005 watt.

The Committee on Broadcasting in the January, 1930, *Proceedings of the Institute of Radio Engineers* recommends (see vol. 18, No. 1) that all transmitters be so designed as to limit the field intensities at one mile, of all components which they produce outside of the licensed frequency band, to not less than 0.05 per cent of the fundamental or 500 μv per meter.

A complete discussion of the problem of harmonic suppression and circuit arrangement for its accomplishment will be found in an article entitled The Suppression of Radio-frequency Harmonics in Transmitters by J. W. Labus and Hans Roder (*Proceedings of the Institute of Radio Engineers*, vol. 19, No. 6, June, 1931).

14. Water-cooled Tubes. For tubes of low power artificial cooling during operation is usually not necessary, radiation into the air being sufficient. For the larger tubes, however, artificial cooling is usually accomplished by means of a circulating water system which causes a sheet of water to pass over the anode surface at very high velocity.

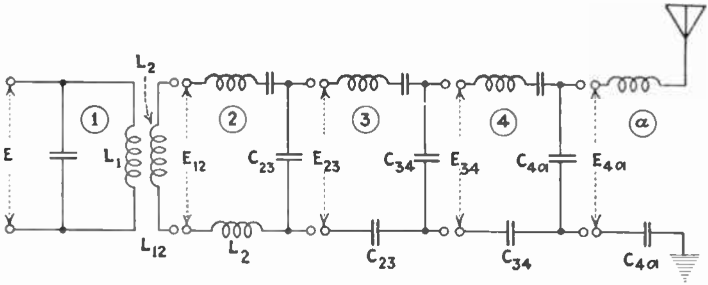


FIG. 26.—Harmonic suppression circuit.

To restrict leakage of current from the anodes to the grounded pipes of the water system, connection is made between the anodes and the water system through a long length of coiled hose. This interposes between the anode and ground columns of water long enough to make the electrical resistance to ground very high; as much of several hundred feet of coiled hose may be used giving resistances to ground in the order of 0.5 up to several megohms.

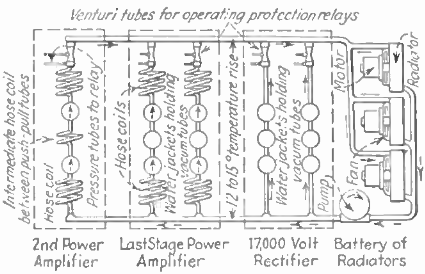


FIG. 27.—Water-cooling and circulation system.

In many cases distilled water is used, the water being maintained at a satisfactory temperature by an artificial cooler since for economical reasons it is desirable that the same water be used indefinitely.

The water cooling and circulating system is automatically started when the transmitter is turned on and the transmitter is automatically turned off in the event of any failure in the water-cooling system. One method of doing this is shown in Fig. 27, where the water system contains a Venturi tube whose inlet and output orifices are connected to a device containing two opposed metallic bellows operated by the difference in pressure established between the two orifices by the flow of water. If the flow is interrupted or falls below its normal value, the bellows at once open a contactor and, through additional relays, cause the power supply to be disconnected.

Sometimes a milliammeter is provided on the transmitter panel which indicates the magnitude of the current leaking through one of the closed coils, the amount of current serving to indicate the relative purity of the water, and indicating when it is advisable to change the water supply.

Table II below indicates the chemical contents of cooling water used at four different radio stations for cooling tubes. The first three samples are unsatisfactory and would form scale. Tube prices being what they are, it is best to use a closed circulatory system with distilled water. Figure 28 is a diagram of a water-cooled tube.

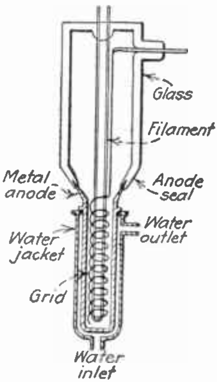


FIG. 28.—Water-cooled tube.

TABLE II

Substance	Grains per gallon			
	1	2	3	4
Calcium carbonate	4.73	0.26	3.85	None
Calcium sulphate	None	2.76	Trace	0.45
Sodium carbonate	3.91	None	None	None
Sodium sulphate	3.09	0.28	0.24	None
Sodium chloride	1.02	1.02	0.85	0.67
Sodium and potassium nitrates	0.41	None	None	None
Magnesium carbonate	1.37	0.13	1.68	None
Magnesium sulphate	None	0.76	0.19	0.47
Aluminum and iron oxides	0.36	Trace	Trace	Trace
Silica	1.22	0.17	0.81	0.15
Total	16.11	5.38	7.62	1.74

15. Power Supply. Plate-voltage supply for transmitters may be obtained from d-c generators, high-vacuum tube rectifiers, mercury-arc rectifiers, or hot-cathode mercury-vapor rectifiers. Direct-current generators are generally used for moderate voltages, but hot-cathode mercury-vapor tubes for rectification of high voltages have found rapidly increasing favor because of their reliability of operation and performance.

The hot-cathode mercury-vapor rectifier is one of the newest and evidently the best method of supplying high voltages to transmitter plate circuits. The operation of the hot-cathode mercury-vapor rectifier is similar in several respects to that of a mercury-vapor rectifier.

The most striking difference between mercury-vapor tubes and high-vacuum tubes is the internal-voltage drop between plate and cathode. In the high-vacuum tube, the voltage drop may vary from a few volts to several thousand volts, depending upon the current, element spacing, etc. In the mercury-vapor tube, the space charge is limited by the arc-drop of the vapor which is practically constant at values between 12 and 17 volts regardless of the current.

Table III below gives a direct comparison of the relative efficiency of a high-vacuum tube and two mercury-vapor tubes. Note that the mercury-vapor tubes give very low internal-voltage drop and have considerably higher efficiencies.

TABLE III.—COMPARISON OF HIGH-VACUUM AND MERCURY-VAPOR TUBE RECTIFIERS*

No. of tubes	Radiotron	Circuit	D-c output			Tube-drop		Losses, kilowatts		Efficiency, per cent
			Volts	Amperes	Kilowatts	Volts	At amperes	Filament	Tube-drop	
6	UV-214	3ϕ double Y	15,000	12	180	1,560	6	6.9	18.7	87.5
6	UV-857	3ϕ full wave	15,000	12	180	15	12	1.8	0.36	98.8
†6	UV-857	3ϕ full wave	19,100	20	382	15	20	1.8	0.6	99.4

* I. R. E., Vol. 18, No. 1, January, 1930.
† Maximum rating.

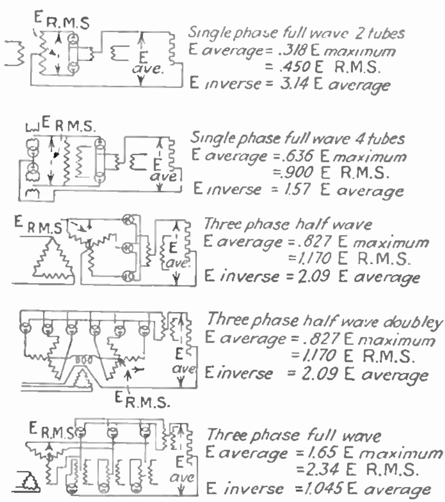


FIG. 29.—Hot-cathode mercury-vapor power circuits.

There are two fundamental limits which determine the power output that can be obtained from any number of tubes operated in any type of circuit. These ratings are (*a*) the maximum peak inverse voltage at which the tube can operate without flashing back and (*b*) the maximum peak plate current which the cathode can supply with a reasonably long life.

The maximum peak inverse voltage which can exist across a tube in any of the usual types of circuits is equal to the line-to-line peak or crest voltage of the power transformer less the voltage drop of the conducting tube.

The peak plate current depends upon the type of circuit, tube, filter, and load. In a single-phase full-wave circuit, each tube must carry the full load current for half the time. In the three-phase half- and full-wave circuit, each tube carries the load current for one third of the time. If the rectifier feeds into an inductance, square blocks of current are drawn from the rectifier and the peak plate current approaches the d-c value. If the rectifier feeds into a capacity load plate, current is drawn for only a part of each half cycle and the peak current may reach values of from three to five times that of the d-c load current.

Table IV below gives data on several typical hot-cathode mercury-vapor tubes designed for radio-power supply purposes. The circuits most commonly used with these types of tubes are shown in Fig. 29. The single-phase full-wave and the three-phase half-wave circuits are quite generally used. The three-phase full-wave circuit was suggested by D. C. Prince as being particularly applicable to the half-wave mercury-vapor tube, since it gives a peak inverse voltage whose magnitude is only 4.5 per cent greater than the average output voltage; the wave form is that of a six-phase rectifier.

TABLE IV.—HOT-CATHODE MERCURY-VAPOR TUBE RATINGS

Radiotron	Filament		Peak inverse voltage	Peak anode current, amperes
	Volts	Amperes		
UX-866	2.5	5	5,000	0.6
UV-872	5	10	5,000	2.5
UV-869	5	20	20,000	5.0
UV-857	5	60	20,000	20.0

SECTION 15

RECTIFIERS AND POWER-SUPPLY SYSTEMS

By R. C. Hitchcock, M.A.[1]

1. Power-supply Design. The main elements in a unit whose input is a.c. and whose output is d.c., in the order of current flow are: (1) transformer, (2) rectifier, (3) filter, and (4) voltage divider. The filter and voltage divider are also used in *B* and *C* supply units from

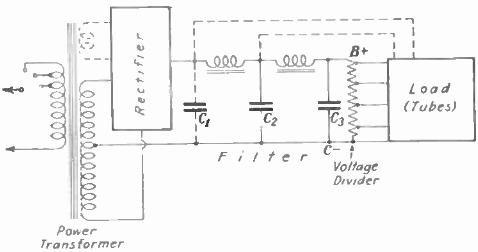


Fig. 1.—Typical a-c powered unit for furnishing *B* and *C* voltages.

d-c supply lines, in which case the rectifier and transformer are not needed. Although the main application of the design of the above items 1 to 4 is for *B* and *C* power supply, they may also be applied to filament *A* supply systems.

It is usually advisable to start the design with the tubes (or with the d-c load requirement), to determine what plate and grid voltages are needed, and then to take up the design of the filter, its rectifier and transformer. It is also possible to start the solution by beginning with the rectifier, thereby determining the maximum output voltages which are reasonable with good filtering and regulation. If any one particular item (1 to 4 above) must be used, the other three can be designed to work with that item.

Beginning with the load is quite logical, because the filter-choke design requires the knowledge of the load current, and the ratio of choke inductance to the filter capacity depends on the load resistance. If, in starting with the desired voltages, it is found when the rectifier is reached that no standard tube will supply a sufficiently high voltage or current, it may be advisable to make the necessary percentage cut in voltage or current, and to use a standard rectifier, thus reducing the

[1] Electronics Section, Meter Engineering, Westinghouse Electric and Manufacturing Company, Newark, N. J.

output by the percentage mentioned. It is usually easy to design a suitable power transformer, the rating being increased by using a thicker stack of standard punchings, the spool size being increased accordingly.

A voltage divider usually is designed for a definite load on each of several voltage taps. If the load is increased on a certain tap, that voltage decreases and all the other voltages, too, are decreased, the amount depending on the amount of the increase in load, and the fraction of total voltage supplied to that certain tap.

2. Parallel Voltage Divider. The simplest form of parallel voltage divider is shown in Fig. 2. To start with, the resistance of the coupling devices between the tube plates and the B supply will be neglected. In the series voltage divider these will be taken up in detail.

The voltage applied from $B+$ to $C-$ comprises the total B, plus C, voltage that needs to be supplied to the load. For example, a UX-245 with a B

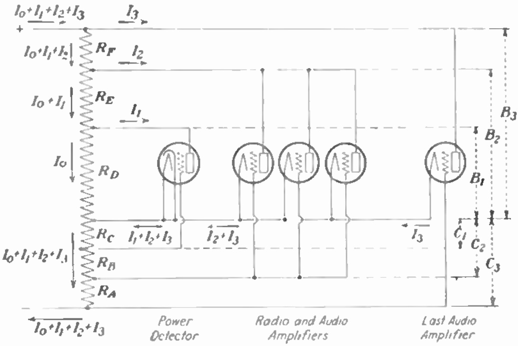


FIG. 2.—Parallel voltage divider.

voltage of 250 means that the plate is 250 volts positive with respect to the filament. The corresponding grid voltage is 50, meaning that the grid is 50 volts negative with respect to the filament. Thus, the filament can be considered as at an intermediate potential between $B+$ and $C-$. On the voltage divider as shown in Fig. 2, $B+$ will be on the upper part of the resistor, $B-$ lower down, and $C-$ still lower down than $B-$.

The tube plates take current from the voltage divider, and the use of Ohm's law gives the resistor values. The grids take no appreciable current, so from $B-$ to $C-$ no allowance need be made for currents entering or leaving, the total current passing through the voltage divider from $B-$ to the lowest $C-$ used. The filament returns are schematic, without showing the potentiometers which adjust for minimum hum. As shown there are three B voltages, noted at the extreme right of the figure, B_1 for the detector, B_2 for the radio and audio amplifiers, and B_3 for the last audio tube. The subscripts on the C voltages correspond with those on the B voltages. For instance, C_3 is the grid voltage for the power tube using B_3 plate volts.

The power detector takes B_1 plate volts and C_1 grid volts. If the grid-leak type of detector is used there is no grid voltage required, and $C_1 = 0$. The plate currents are I_1, I_2, and I_3. If, as in the case of I_2, there are several tubes being supplied with the same voltage, the individual plate currents are multiplied by the number of tubes. For example, I_2 in Fig. 2 is made up of three equal parts, one part supplying each of three identical tubes. The current I_0, flowing in parallel with I_1, is known as the circulating or *waste* current.

The resistance of the voltage divider has been divided into six sections denoted by the letter R, with subscripts A to F, to avoid confusion by using the same numerical subscripts on B and C voltages. Thus, the voltage drops from B_3 to B_2 through the resistor R_F, from B_2 to B_1 through R_E, etc.

The currents and their directions are shown by arrows, the amounts being indicated by the letter I with appropriate subscripts.

In calculating the resistors of Fig. 2, the equations are:

$$R_F = \frac{B_3 - B_2}{I_0 + I_1 + I_2} \tag{1}$$

$$R_E = \frac{B_2 - B_1}{I_0 + I_1} \tag{2}$$

$$R_D = \frac{B_1}{I_0} \tag{3}$$

$$R_C = \frac{C_1}{I_0 + I_1 + I_2 + I_3} \tag{4}$$

$$R_B = \frac{C_2 - C_1}{I_0 + I_1 + I_2 + I_3} \tag{5}$$

$$R_A = \frac{C_3 - C_2}{I_0 + I_1 + I_2 + I_3} \tag{6}$$

The equivalent resistance of the voltage divider and tube load is used in calculating the filter, and for Fig. 2 the equivalent resistance is equal to the total voltage divided by the total current:

$$R_{\text{equiv.}} = \frac{B_3 + C_3}{I_0 + I_1 + I_2 + I_3} \tag{7}$$

In this equation, as in Eqs. (1) to (6) the absolute values of the B and C voltages are used.

3. Voltage Regulation. The parallel voltage divider has good voltage regulation when the circulating current I_0 is large compared to the other currents. For reasons of economy, however, it is inadvisable to make I_0 very large. First, the increase in I_0 causes the equivalent load resistance to go down, which requires (see Fig. 6) larger filter condensers. The extra I_0, in the second place, causes extra heat in the voltage divider which in turn means that the power transformer must be supplied a heavy current and the operating cost is increased.

The formula for determining the $B + C$ voltage is, to a good approximation:

$$B + C = E - (I_0 + I_1 + \cdots)R_t \tag{8}$$

where $B + C$ are the voltages across the load and voltage divider, E the r-m-s voltage of one high-voltage winding of the transformer, and R_t the combined resistance of the chokes, transformer winding and tube resistance.

From Eq. (8) it will be seen that, if any of the individual I's increase, the total $B + C$ voltage decreases, and from Eqs. (1) to (7), written in the form

$$R_F(I_0 + I_1 + I_2) = B_3 - B_2 \text{ rewriting (1)} \tag{9}$$

it is seen that the voltage drop between taps is greater; for example, less voltage is supplied at B_2 if I_1 is increased. For good regulation it is essential that the R_t of Eq. (8) be small compared to the load resistance, and that the circulating current I_0 be large. For reasons of economy in manufacture as well as operating cost I_0 should be small so that the filter can be cheaper.

4. Series-parallel Voltage Divider. By putting $I_0 = 0$ (R_D = open circuit) in Fig. 2 a series-parallel voltage divider is made. The C

voltages are, as before, taken from resistors R_A, R_B, and R_C. The equations of (1) to (8) still hold by putting $I_0 = 0$ wherever it appears.

5. Series Voltage Divider. Figure 3 shows the series voltage divider, the currents and their directions being shown by arrows. The currents for the tubes are the same as for Fig. 2, I_1 being the power-detector plate current, B_1 and C_1 the plate and grid voltages for the power-detector tube, etc.

The series voltage divider is simpler to figure than the parallel, because there is no circulating current, each B voltage has only one set of tube currents flowing through it, and the grid-bias resistors are separate for each group of tubes. A further simplification results in the calculation of the plate resistors, as the resistance of the coupling device is easily

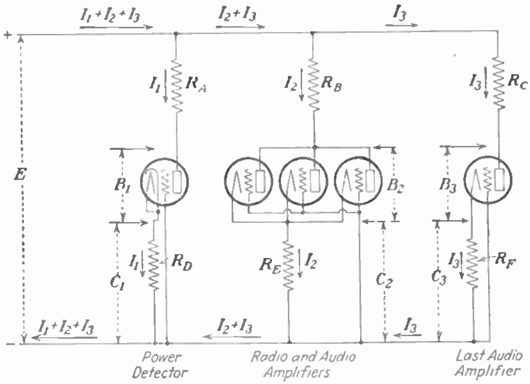


FIG. 3.—Series voltage divider.

included in the plate-resistor calculation. That is, the resistance of the transformer-coil choke or coupling resistor between $B+$ and a particular tube plate is considered as incorporated in the resistor incorporated in the plate circuits of Fig. 3. The last audio stage requires the most voltage and therefore the B and C voltage which it requires is the main factor in determining the maximum B and C voltages needed. The total voltage from the filter is not applied to the plate directly but through a coupling device, as indicated by the resistor R_C. This resistor is not a means of decreasing the voltage to a proper value but represents the coupling device only.

The total voltage supplied at the left of Fig. 3 is larger than the B_3 plus C_3 used by the last audio tube by an amount equal to the drop in the coupling device. The voltage at the end of the filter is

$$E = I_3R_C + B_3 + C_3 \tag{10}$$

R_C determines the supply voltage; if R_C is large, the required E is large, and *vice versa*. The equations for the other resistors are:

$$R_A = \frac{E - B_1 - C_1}{I_1} \tag{11}$$

$$R_D = \frac{C_1}{I_1} \tag{12}$$

$$R_B = \frac{E - B_2 - C_2}{I_2} \tag{13}$$

$$R_E = \frac{C_2}{I_2} \tag{14}$$

$$R_F = \frac{C_3}{I_3} \tag{15}$$

In the power detector and radio and audio amplifiers the R_A and R_B include the resistance values of the coupling devices. For example, if a choke coil of 1,000 ohms feeds the plate of the power detector then the actual resistance to use will be (R_A − 1,000 ohms).

6. Voltage Divider with Graded Filter. A voltage divider of a graded form is shown in Fig. 4. The output stage comprises two similar tubes

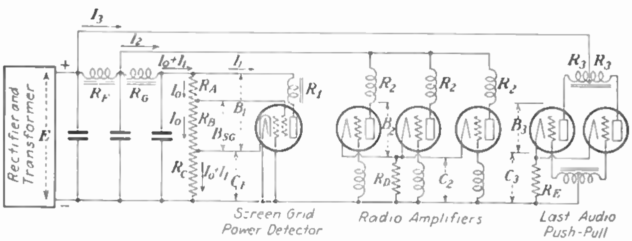


FIG. 4.—Voltage divider with graded filter.

in push pull, an arrangement which requires a minimum of filtering. The B_3 comes directly from the positive side of the rectifier. Note the filter condenser across from B_3 to the negative terminal. Sometimes, as in Fig. 1, the first condenser shown dotted is omitted. When taking the B voltage for a push-pull stage directly from the receiver it is necessary that this first condenser be retained.

The "grading" of this arrangement is apparent; the output stage receives the least filtering, push pull requiring practically no filter when the tubes are matched; the radio amplifiers have one stage of filtering in their B supply; and the power detector has two stages of filtering for its B. This grading is economical; the heavy output-tube currents do not flow through any of the filter chokes. This reduces heat losses and allows the design of a smaller choke, or a better one of the same physical size. The voltage drop of the choke is eliminated, thus the power-transformer voltage can be lower. Many of the feed-back difficulties are also avoided. When common resistors supply several tubes which are in cascade, it is often found that a small variation in the output-tube current will change the grid voltage of a preceding tube, which by virtue of its amplification and coupling, causes a larger variation in the output-tube circuit. This process may be cumulative and continue to build up and "howl" or "motor boat." This feed-back is avoided by separate resistors, and is especially well eliminated by having a full stage of filtering between the output tube, radio amplifiers, and detector, as in Fig. 4.

It should again be noted that only with a push-pull output stage is it possible to supply $B+$ directly from a filter. If a single output tube is used, the plate voltage must have at least one stage of filtering (*i.e.*, a series choke and its accompanying condenser).

Figure 4 gives the coupling units as R_1 for the detector, R_2 for each radio amplifier, and R_3 for each half of the push-pull output choke. The voltage for the screen grid of the detector tube is furnished by a parallel connection across the B resistor after the second stage of the filter. The screen-grid current is small and may be neglected if I_0, the circulating current, is, say, 10 to 20 ma.

The chokes of Fig. 4 may have a very high resistance, over 1,500 ohms each for R_F and R_G, because no very heavy currents flow through them. In fact, it may be desirable to have some high series resistance in the circuit to reduce the voltage from that required by the last audio stage to that needed by the radio amplifiers and detector. For this reason, it should be borne in mind that the values of R_F and R_G for the chokes and R_1 and R_2, the plate-coupling units for detector and radio tubes, should be considered as including any voltage-reducing resistor which is needed.

As before, I_1 is the detector plate current, I_2 includes three-tube plate currents, and I_3 now has two plate currents, the equations for Fig. 4 being:

$$R_A = \frac{B_1 - B_{SG}}{I_0} \tag{16}$$

$$R_B = \frac{B_{SG}}{I_0} \tag{17}$$

$$R_C = \frac{C_1}{I_0 + I_1} \tag{18}$$

$$R_D = \frac{C_2}{I_2} \tag{19}$$

$$R_E = \frac{C_3}{I_3} \tag{20}$$

The equations determining the choke and coupling resistors are:

$$B_1 + C_1 = E - R_F(I_2 + I_1 + I_0) - R_G(I_0 + I_1) - I_1R_1 \tag{21}$$

$$B_2 + C_2 = E - R_F(I_2 + I_1 + I_0) - \frac{I_2R_2}{3} \tag{22}$$

$$B_3 + C_3 = E - \frac{I_3R_3}{2} \tag{23}$$

By substituting values either for the choke resistances or for the coupling coils, Eqs. (21) to (23) simplify into useful working formulas.

If screen-grid radio tubes are used, their screen-grid voltages may be obtained from the same supply as shown in Fig. 4 for the detector.

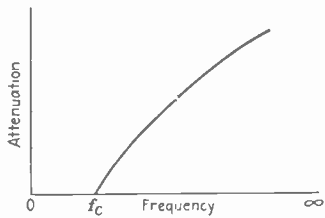
The equivalent resistances of the first and second filter load are not the same, due to the different load currents. For the first filter section the equivalent load is:

$$R_{\text{equiv.}} = \frac{E}{I_2 + I_1 + I_0} - R_F \tag{24}$$

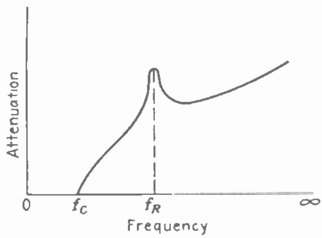
and that for the second is:

$$R_{\text{equiv.}} = \frac{E}{I_1 + I_0} - R_F\frac{(I_2 + I_1 + I_0)}{I_1 + I_0} - R_G \tag{25}$$

7. Filters. The filters used to give d.c. from rectified a.c. are known as low-pass filters.[1] Low-pass filters are divided into two classes, tuned and untuned filters. The tuned filter offers a maximum impedance or attenuation to the frequency of the supply, but the impedance at nearby higher or lower frequencies, is not quite so great (see Fig. 5b), although the general trend of the curve is a rising attenuation as the frequency increases.



(a)



(b)

FIG. 5.—(a) Low-pass filter. (b) Tuned low-pass filter.

The usual form of untuned low-pass filter is that of Figs. 1 and 4, using three condensers and two chokes. This filter (Fig. 5a) has a continuously rising curve of impedance as the frequency increases. To obtain good filtering with this filter it is desirable to choose f_c, the frequency at which attenuation begins, as low as possible. The equations for determining the proper inductance and capacity for this filter are:

$$C = \frac{1}{\pi f_c R} = \frac{0.3183}{f_c R} \text{ farads} \tag{26}$$

$$L = \frac{R}{\pi f_c} = \frac{0.3183R}{f_c} \text{ henrys} \tag{27}$$

where f_c = frequency at which attenuation begins
C = capacity in farads
R = resistance in ohms
L = inductance in henrys

As this is an often-used type of filter, Fig. 6 is devised to give the data of Eqs. (26) and (27) in a convenient chart form. The four columns from left to right are f_c in cycles per second; L in henrys; R in load ohms; and C in microfarads. Thus with any two of the factors fixed, the corresponding two are determined from this chart by a straightedge across the two known factors. For use on 60 cycles half-wave rectification, it is necessary that f_c be below 60, and for the double-wave rectifier f_c should be below 120 cycles, and the lower the f_c the better will be the filtering at the desired frequency, as shown by the rising attenuation curve of Fig. 5a.

The third column R is the usual starting place for finding the filter values, when the voltage divider and tube load have been calculated

[1] The theory of filters is admirably covered in the following books: K. S. Johnson and T. S. Shea, "Transmission Circuits for Telephone Circuits"; G. W. Pierce, "Electric Waves and Oscillations."

first. When the point on the R column is fixed, and f_c, say, 50 cycles per second, the values of L and C are quickly determined. It is seen

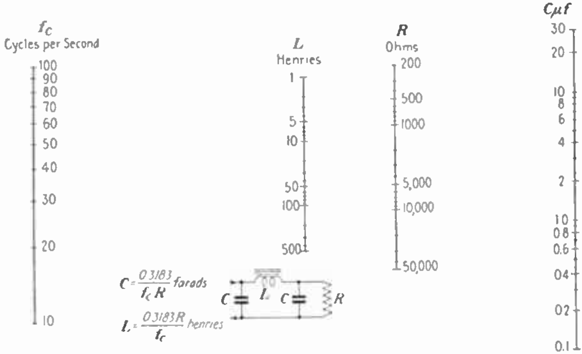


FIG. 6.—Low-pass filter design chart, Pi section.

from Fig. 6 that, for a given cut-off frequency f_c, as the load resistance increases the L increases, while the C value goes down. Very high load resistances require chokes of large inductance values, but as high-resistance loads mean small currents, the use of large inductances is feasible.

By assuming that the load resistance, R, does not affect the values of L or C, a useful approximation[1] can be secured, concerning the amount of filtering needed in each stage for the circuit shown in Fig. 4. Suppose the output stage is supplied with plate power which is filtered x per cent, so that its hum is reduced to x per cent of its unfiltered value, and at this value it gives no noticeable hum in the loud-speaker. Suppose further that the amplification between the plate of this last tube, and the preceding tube plate is A. Then the preceding stage must have its power supply filtered x/A per cent. This means that the ripple in the plate supply of the next to the output stage must be $1/A$ as much as the output stage, because of its amplification. Figure 7 gives this relation in useful graphic form. If a stage of amplification has a gain of 25, it is essential that the

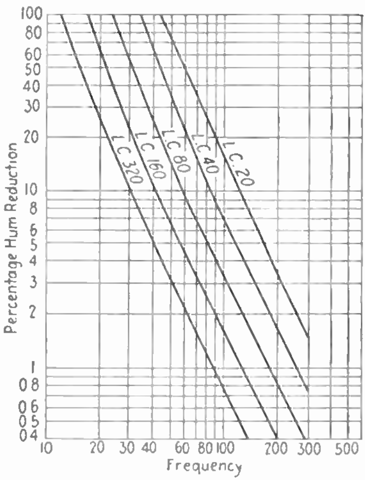


FIG. 7.—Smoothing effected by various products of inductance (henrys) and capacity (microfarads).

[1] COCKING, W. T., *Wireless World*, Nov. 19, 1930, pp. 565–568.

preceding tube be supplied with plate power with one twenty-fifth the ripple, or 4 per cent. An LC product of 56 will give this degree of filtering at 100 cycles, according to Fig. 7, and this means a 28-henry choke and a 2-μf condenser which are close to standard values.

A similar circuit to Fig. 4, using resistors instead of chokes, is frequently used to provide an extra degree of filtering for stages preceding a power stage (see Fig. 8). This is especially useful when the output stage requires a high voltage, and the voltage for the other stages must be materially reduced. The reason chokes are used is that they have high impedance to the unwanted rectified a.c., but low resistance to the desired d.c. Now if the amount of d.c. is no great object, a resistance of as great a value as the impedance can be employed, and this is quite useful in some cases where the voltage is to be reduced. If, as in Fig. 8, two stages of choke and condenser filtering are used, the additional resistance and condenser filter stages simply increase the amount of filtering, without the extra cost of chokes which are more expensive than resistors. The RC values and the degree of filtering are given in Fig. 9 and the use is the same as that of Fig. 7. The circuit of Fig. 8 is quite similar to Fig. 4, in eliminating the undesired feed-back effects.

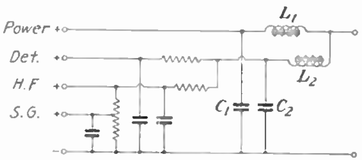


FIG. 8.—Circuit which minimizes feed back.

The use of the chart (Fig. 6), based on Eqs. (26) and (27), gives very satisfactory results, but the experimental curves[1] showing the effects of load, and

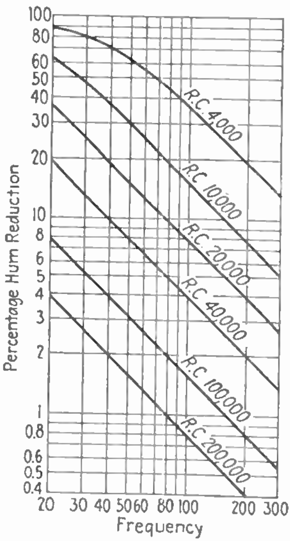


FIG. 9.—Filtering effected by resistance (ohms)-capacity (microfarads) circuit.

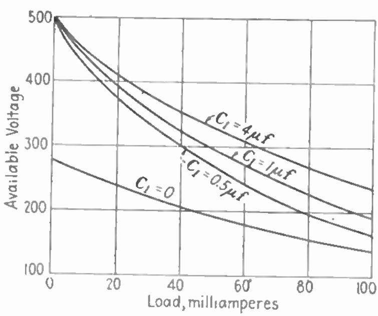


FIG. 10.—Effect of C_1 on voltage available.

[1] The curves (Figs. 10 to 13) are experimental curves taken from the *Aerovox Res. Worker*, articles by Sidney Fishberg, research engineer.

different condenser values are quite interesting, and will give a clearer idea of the validity of the chart.[1]

8. First Filter Condenser. The effect of the first filter condenser, shown dotted in Fig. 1, is to raise the available output voltage. Figure 10 gives the output voltage available as the first condenser C_1 is changed, as a function of the load current.

Figure 11 gives the per cent ripple in output as the capacity of C_1 is varied. This curve shows that the use of a single condenser C_1 can never reduce the ripple much below 10 per cent with a reasonable value of capacity. Much less than one-half of 1 per cent is needed in a good filter, and as at least two condensers must be used to provide a single filter section, Fig. 11 agrees with the theory.

9. Second and Third Filter Condensers. Figure 12 gives the per cent ripple as a function of C_2 and C_3 for a given current drain. It will be seen that when $C_2 = C_3$ the most economical filter results. For example, supppose the ripple permissible to be 0.1 per cent. This can be supplied with $C_2 = 0$ if $C_3 = 5$ μf, a total of 5 μf. But this can also be met with $C_2 = 2$ μf, and $C_3 = 2$ μf, a total of only 4 μf. The dotted line gives the ripple value where C_2 and C_3 are equal. The per cent ripple figures of course apply only to a specific filter, but the relations between the condenser values hold for similar filter circuits.

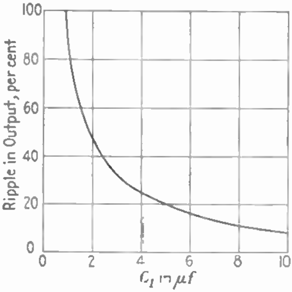


FIG. 11.—Effect of C_1 on ripple in output.

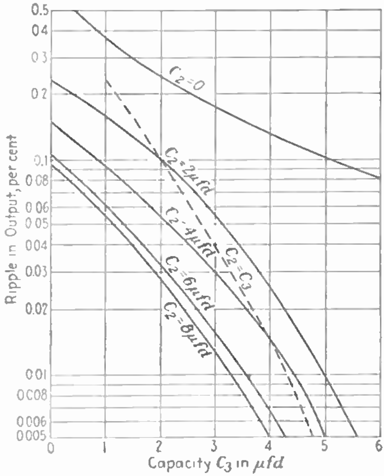


FIG. 12.—Percentage ripple as a function of C_2 and C_3.

Figure 13 gives the percentage hum as a function of the current drain. This shows that the higher the values of C_2 and C_3 the lower the percentage hum. It should be remembered that increasing current means a decreasing load resistance. From Fig. 6, assuming f_c is constant, the capacity should increase and the inductance decrease as the load resistance decreases. Thus, as Fig. 13 was taken using the same inductance coils throughout, larger values for C_2 and C_3 are needed as the current

[1] A theoretical calculation of the effects of C_1, C_2, and C_3 on the output voltage is given in *Gen. Elec. Rev.*, **19**, 177, 1916.

drain increases. It is almost certain that the inductance values of the chokes decreased as the current through them increased. To a certain extent this inductance decrease does not interfere with the filtering, especially if the capacity is increased, as, referring again to Fig. 6, when the resistance decreases to half a certain value, the capacity should be doubled, while the inductance need be only half its former value, if f_c be kept the same. Thus in Fig. 13, as in the other figures, the experimental facts agree with the theoretical chart (Fig. 6) and Eqs. (26) and (27) for this type of filter.

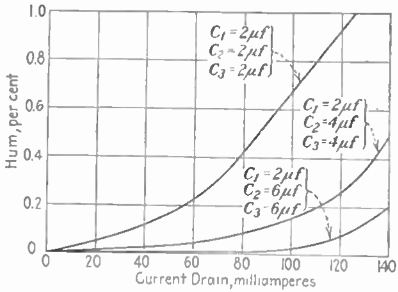


FIG. 13.—Percentage hum as a function of current drain.

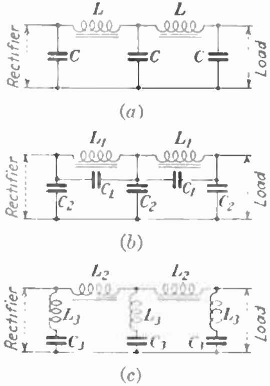


FIG. 14.—(a) Low-pass filter. (b) and (c) Tuned low-pass filters.

10. Tuned Low-pass Filter. Two tuned low-pass filter circuits are given in Fig. 14, *b* and *c*, whose attenuation characteristics were given in Fig. 5*b*. For comparison, Fig. 14*a* gives the ordinary low-pass filter.

For the tuned filter of Fig. 14*b*, having the series chokes shunted by small condensers, the equations are

$$C_1 = \frac{1}{4\pi f_c Ra\sqrt{a^2-1}} = \frac{0.07858}{f_c Ra\sqrt{a^2-1}} \text{ farads} \tag{28}$$

$$C_2 = 4C_1(a^2-1) \text{ farads} \tag{29}$$

$$L = R^2C \text{ henrys} \tag{30}$$

$$a = \frac{f_R}{f_c} \tag{31}$$

For the tuned filter of Fig. 14*c* having small chokes in series with the condensers, the equations are

$$C_3 = \frac{\sqrt{a^2-1}}{f_c Ra} = \frac{0.3183\sqrt{a^2-1}}{f_c Ra} \text{ farads} \tag{32}$$

$$L_2 = R^2C_3 \text{ henrys} \tag{33}$$

$$L_3 = \frac{L_2}{4(a^2 - 1)} \text{ farads} \tag{34}$$

$$a = \frac{f_R}{f_c} \tag{35}$$

If wide variations in the supply frequency were likely to occur, this type of filter would not be advisable. As a rule, the frequency of most power companies is now kept constant enough to run synchronous electric clocks, and this is quite good enough for this type of tuned circuit. However, the values of C_1, L_1, and C_3, L_3 have to be accurately maintained in order fully to secure the advantages of the tuned filter. Due to these closer manufacturing limits, the use of the tuned filter is not so wide in large production as its advantages would seem to warrant. A combination of tuned low-pass filter and the regular-type filter is sometimes used with very good results.

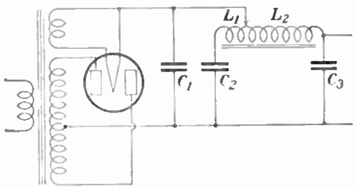


FIG. 15.—Tapped choke-filter circuit.

11. Filter Chokes Having Mutual Inductance. An interesting type of filter is one in which the first and second choke are magnetically

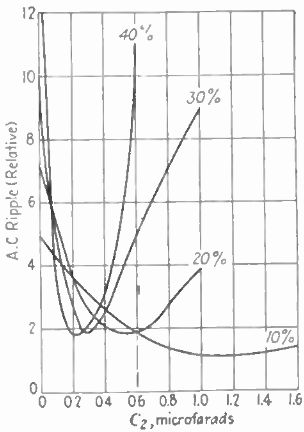


FIG. 16.—Tapped choke. Percentage of total turns (Fig. 15) used in L_1.

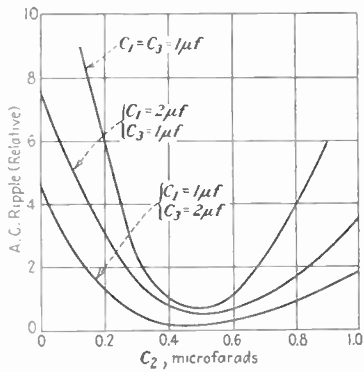


FIG. 17.—Condenser values for tapped choke filter (Fig. 15).

coupled.[1] Figure 15 shows a tap on the first choke[2] to which the positive rectifier lead and a filter condenser are connected. The a-c component,

[1] This type of filter is manufactured under Clough patents and is sold by Silver Marshall under the trade name Unichoke.

[2] *Proc. I.R.E.*, January, 1930, p. 161, from which Figs. 15 to 17 are taken.

flowing through the L_1 section of the choke, neutralizes to a large degree the a-c component of L_2 so that the output ripple is reduced. Figure 16 shows the relative a-c output ripple with a variable C_2 as the tap on the choke is changed so that L_1 uses from 10 to 40 per cent of the total turns of the choke.

Figure 17 shows how the values of C_1 and C_3 affect the relative a-c ripple as a function of C_2. These curves indicate that the best C_2 value is fairly independent of C_1 and C_3.

12. Design of Filter Chokes. It is important that the filter choke be designed to carry the desired direct current and at the same time to offer the necessary reactance to the a-c component. A direct method of design[1] has been derived using both the normal and incremental permeability curves for the core material.

The derivation gives two working equations:

$$\frac{LI^2}{V} = \frac{B^2\left(\frac{1}{\mu} + \frac{a}{l}\right)^2 \times 10^{-8}}{0.4\left(\frac{1}{\mu_\Delta} + \frac{a}{l}\right)} \tag{36}$$

$$\frac{NI}{l} = \frac{B}{0.4\pi}\left(\frac{1}{\mu} + \frac{a}{l}\right) \tag{37}$$

where L = henrys
I = d-c amperes
V = core volume in cubic centimeters (cm³)
N = turns
l = magnetic path in centimeters
a = air gap in centimeters
B = steady flux density on iron and air gap in gausses
μ = normal permeability B/H
μ_Δ = incremental permeability $\Delta B/\Delta H$ for a minor hysteresis loop

The original curves were plotted with a/l as a parameter, LI^2/V being the ordinate, and NI/l as the abscissa for both 4 per cent silicon steel and hipernik. Figures 18 and 19 are alignment charts which include the data of the original curves. LI^2/V is the left column, and NI/l and a/l are on the right column. A straightedge passing through a given LI^2/V and tangent to the curve in the central part of the chart will cut the right column at the corresponding value of NI/l and a/l. The reverse procedure, beginning with NI/l, is also possible.

Figure 19*a* gives typical permeability curves for three grades of magnetic material which is commercially available.[2] A chart for calculating chokes, using Armco Radio 4 is Fig. 19*b*, the values of LI^2/V and NI/l being the same as for Figs. 18 and 19. In Fig. 19*b* either the desired value of LI^2/V is followed over the curve and then down to NI/l or the reverse procedure can be followed. The gap ratio a/l shown opposite the curve has exactly the same significance as before.

13. Designing a Choke to Carry D-c. A small choke to carry 80 ma and have 14 henrys is desired. The left column of Fig. 18 is LI^2/V, and this is calculated first. L is 14 henrys, I is 0.08 amp.; I^2 is 64×10^{-4} amp.² V is the volume of the core, which was calculated to be 83.6 cm.[3]

[1] HANNA, C. R., *Jour. A.I.E.E.*, **46**, p. 128, February, 1927.
[2] These curves were supplied by C. W. Rust, electrical engineer, American Rolling Mill Company, Middletown, Ohio.

$$\frac{LI^2}{V} = \frac{14 \times 64 \times 10^{-4}}{83.6} = 10.7 \times 10^{-4} = 0.00107$$

Lining up this value with a straightedge which is tangent to the central curve (Fig. 18) the value of NI/l is found to be 18. The core used has $l = 14$ cm so $N = 18 \times l/I = 18 \times 14/0.08 = 3{,}150$ turns. Thus to get 14 henrys, 3,150 turns are wound on the core given. To have this inductance at 80 ma an air gap is needed, as shown in Fig. 18, the a/l (gap ratio) being 0.0021. As l is 14 cm, $a = l \times 0.0021$ or $14 \times 0.0021 = 0.029$ cm (equivalent to $0.029/2.54 = 0.011$ in.). This required air gap is made by inserting paper sheets of the proper thickness between the punchings, and then clamping them firmly in position.

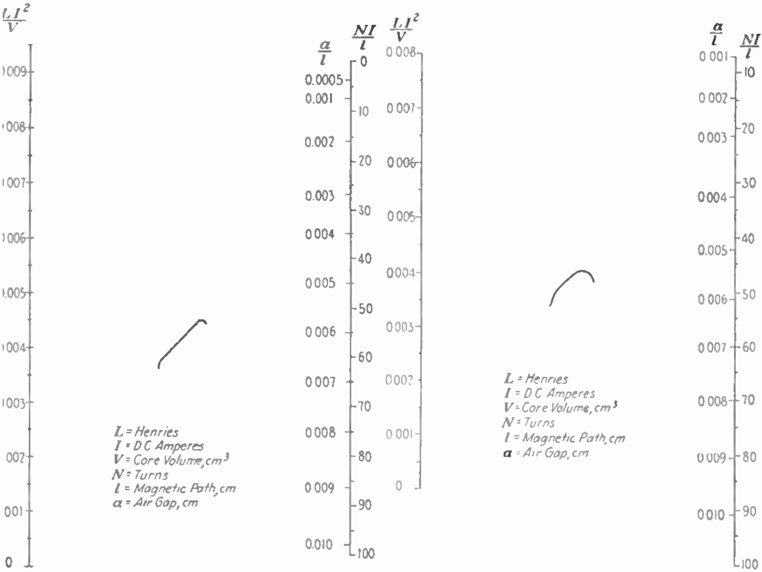


Fig. 18.—Choke design; 4 per cent silicon core.

Fig. 19.—Choke design; hipernik core.

The inductance of a choke depends to some degree on the frequency. For use with low frequencies in a filter circuit the inductance remains practically constant. Both the hysteresis loss and eddy-current loss are of importance in choosing a core material for chokes and transformers. The hysteresis loss is directly proportional to the frequency if the maximum flux density remains constant; and to the 1.6 power of the maximum flux density if the frequency remains constant.

The eddy-current loss can be kept low by using thin sheets of core material. A usual standard thickness is 0.014 in., and this is quite satisfactory for filter choke and transformers for 60 cycles. The insulation between laminations does not need to be very thick, the usual oxide layer on the sheet being sufficient.

14. Filter-condenser Ratings. Some rectifiers begin supplying rectified voltage before the tubes in the load heat up sufficiently to take their rated currents. (This is especially true of the slow indirect-heated tubes.) For this reason it is often desirable, especially from a factor of safety viewpoint, to use peak voltages in calculating all condenser ratings.

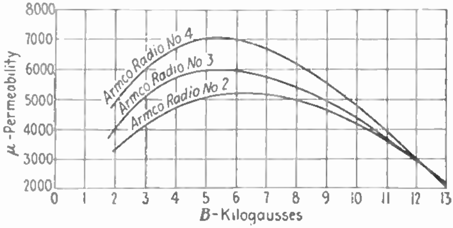


FIG. 19*a*.—Typical permeability curves of radio grades of Armco iron.

The first condenser should, then, be able to stand the peak voltage of the power-transformer secondary. For a 400-volt secondary the peak is 564 volts. For reliable continuous use, the rating of the first filter condenser should be 564 volts. If no current flows, the voltage

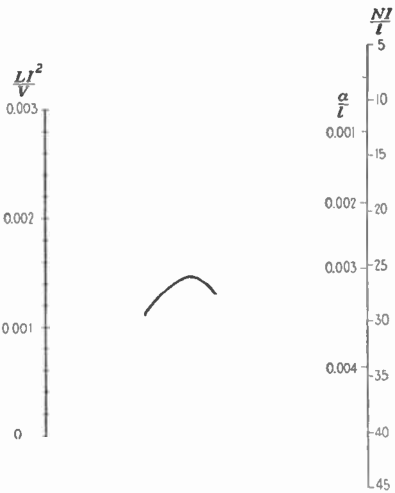


FIG. 19*b*.—Choke design; Armco Radio 4.

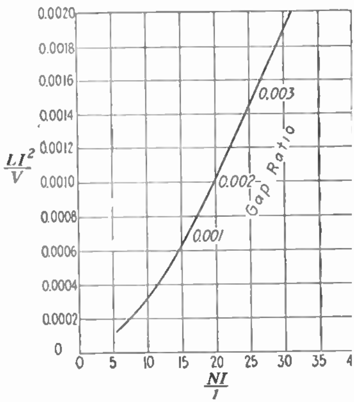


FIG. 19*c*.—Typical reactor design curve; V in cubic centimeter and l in centimeter. Armco Radio 4.

on both the second and third condensers will also be within a few per cent of the peak value 564 volts.

Assuming that an appreciable percentage of the total load current flows in the voltage divider as a "waste" or "circulating" current

(I_0 in Figs. 2 and 4) the second and third condenser ratings do not have to be as high as that of the first condenser, by the amount of the voltage drop in the chokes. This drop is figured by the usual $E = IR$ formula, where the circulating current is I and the resistance is that of the respective chokes.

If an appreciable load of resistors, or fast-heating tubes is always in the circuit, the IR drop through the chokes can be subtracted from the voltage applied to the first condenser. For instance, if a current of 60 ma flows through the first choke having $R = 400$ ohms, the voltage drop is $0.06 \times 400 = 24$ volts. Assuming the r-m-s voltage (neglecting the tube drop), the first condenser is 400 volts, the steady voltage component at the second condenser is $400 - 24 = 376$ volts.

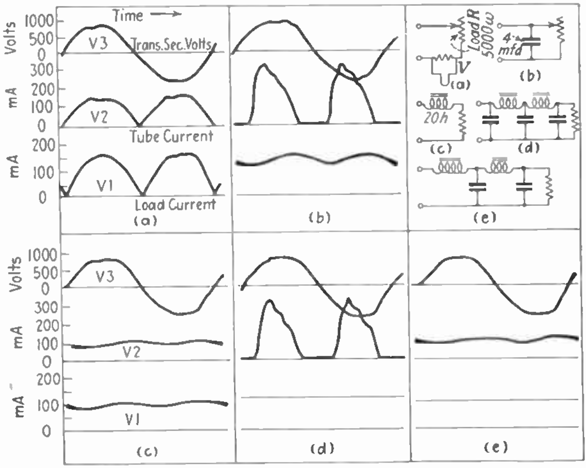


Fig. 20.—Half-wave rectifier, different load circuits.

To this should be added 10 per cent to allow for the ripple, so that $376 \times 1.1 = 413.6$ volts should be the d-c rating for the second condenser.

It is true that a good filter condenser will stand, for a time, voltages greater than its d-c rating, but the practice of applying these higher voltages is seldom advisable.

15. Rectifiers. The general types of rectifiers are:

Vacuum-tube rectifier with filament cathode.
Gas-filled tube with filament cathode.
Glow rectifier, gas filled.
Dry-disk metal rectifier.
Electrolytic rectifier.

Any of these can be used as either half- or full-wave rectifiers, although some of the units are half-wave devices, two being required to give full-wave rectification.

16. Vacuum-tube Rectifier with Filament Cathode. This type of rectifier is used in nearly all a-c powered radio receivers, and has numerous applications in higher powered units. Some oscillograms[1] showing the effect of different load circuits are given in Figs. 20 and 21. In both figures, the letters *a* to *e* refer to similar load circuits, *a* being a simple resistor load, *b* a 4-μf condenser across the resistance, *c* a 20-henry choke in series with the resistor, *d* a standard three-condenser, two-choke filter with load resistance, *e* the same as *d* with the first condenser omitted.

For each load three factors are shown, the *V* letters denoting the oscillograph vibrators, the transformer secondary voltage being V_3, the tube current V_2, and the load current V_1. The curves of special interest are those of *d* and *e* in Figs. 20 and 21. In both figures *d* shows a severe load current being drawn from the rectifier tube, the peak current from the half-wave tube being 540 ma, and the output

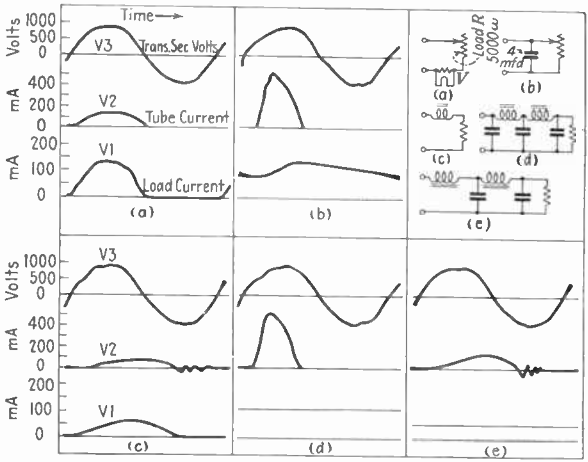


FIG. 21.—Full-wave rectifier, different load circuits.

current 102 ma, a ratio of 5.3:1. The full-wave tube peak current is 290 ma, while the output current is 118 ma, a ratio of 2.5:1. For the *e* section of these two figures, the half-wave tube peak current is 130 ma and the load 45 ma, a ratio of 2.9:1 while the full-wave peak current is 110 ma, and the load 96 ma, a ratio of 1.5:1. In all these curves the power transformer was the same, and an idea of the relative output voltages and currents can be secured by comparing the desired circuits of Figs. 20 and 21.

From the standpoint of the rectifier tube, these figures show that the omission of the first filter condenser will decrease the high periodic loads which are required by the standard filter having an input condenser. By referring to Fig. 10 it will be seen that the omission of C_1 decreases the available voltage, and this is verified by the curves in Figs. 20 and

[1] WISE, ROGER, *Radio Broadcast*, April, 1929, pp. 394–395.

21, as the same transformer supplied the voltages to both *d* and *e* circuits in turn.

Figure 22[1] gives the load current, through several cycles, for several forms of filter. The letters are made the same as for Figs. 21 and 20 wherever possible for convenient reference. Curve *B* of Fig. 22 corresponds to the *b* curve of the full-wave rectifier of Fig. 21 while *B′* is the same as *b* with the condenser capacity approximately six times as large. *B″* is the same as *B′*, for a half-wave rectifier, and *B‴* has about six times as much capacity as *B″* but is otherwise the same. Curve *C* corresponds to the regular *c* of the former figures, and *C′* is the same as *c* with a 2.13 mfd condenser across the rectifier side of the choke. Curve *C″* is like *C″* with the condenser increased to nearly six times its original value. Curve *D* resembles the *d* of the former figures, except that it comprises only one filter section instead of two as in Fig. 21.

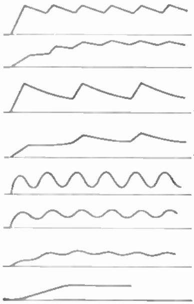


Fig. 22.—Load currents for several forms of filter.

17. Characteristics of Rectifiers for Receivers. The UX-280 full-wave rectifier and UX-281 half-wave rectifier are largely used in receivers, and in addition to their characteristics as given in Table I, the curves of Figs. 23 to 25 give their available voltage output as a function of the r-m-s transformer secondary voltage and load current.

Table I.—Vacuum-type Rectifiers, Filament Cathode

Type	Filament		Maximum a-c supply, r-m-s voltage	Maximum d-c load current, milliamperes	Cooling
	Volts	Amperes			
UX-280[1]	5.0	2.0	350[2]	125[3]	Air
UX-281	7.5	1.25	700[4]	85[5]	Air
UV-217A	10.0	3.25	1,500	200	Air
UV-217C	10.0	3.25	3,000	150	Air
UV-1651	11.0	14.75	4,000	250	Air
			Maximum peak inverse voltage	D-C amperes	
UX-218	11.0	14.75	50,000	0.75	Air
UV-856	11.0	16.75	50,000	0.85	Air
UX-219	22.0	24.5	50,000	2.5	Air
UV-855	14.5	52.0	50,000	5.0	Water
UV-214	22.0	52.0	50,000	7.5	Water

[1] UX-280 is the only full-wave rectifier in this table.
[2] 350 volts r-m-s maximum per plate.
[3] Both plates.
[4] 650 volts is recommended.
[5] 65 ma is recommended.

18. Hot-cathode Mercury-vapor Rectifier. The hot-cathode mercury-vapor rectifier[2] differs from the mercury-arc tube in two respects. First, it operates at a relatively low temperature, so that the vapor pressure

[1] Kuhlman and Barton, *Jour. A.I.E.E.*, January, 1928, p. 17.
[2] Pike and Maser, *QST*, February, 1929, p. 20.

is low. This low mercury pressure gives a useful characteristic, a high breakdown voltage in the inverse direction. Second, the electrons are emitted from the filament and not from a pool of mercury. In the second respect this tube resembles the vacuum-tube rectifier, but the

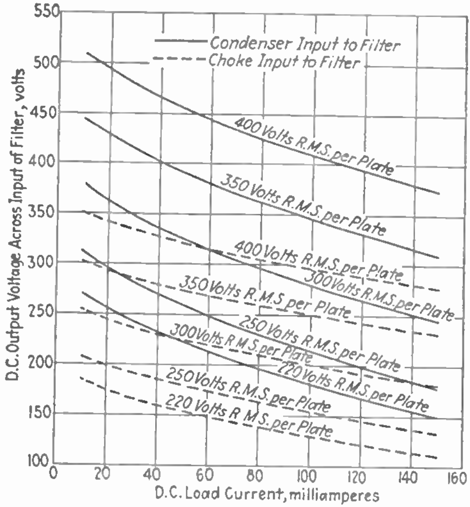


FIG. 23.—Output characteristics. UX-280.

difference lies in the much lower potential drop due to the neutralizing of the filament space charge by the positively charged mercury ions.

The filament-to-plate drop of the mercury-vapor tube is about 15 volts, and is practically independent of the load current. This low drop

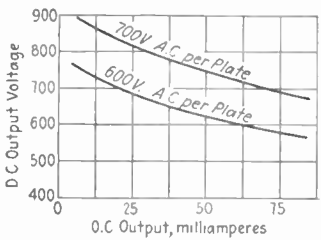


FIG. 24.—Output of UX-281.

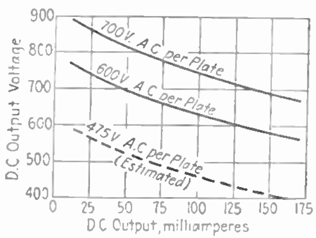


FIG. 25.—Full-wave rectifier (two UX-281).

helps regulation as well as increasing the available d-c output. This tube is self-igniting, and does not require the starting mechanism of the mercury-arc rectifier. Table 11 gives tube characteristics.

TABLE II.—HOT-CATHODE MERCURY-VAPOR RECTIFIERS

Type	Filament		Maximum peak inverse, volts	Maximum peak plate current, amperes	Approximate volts drop, in tube
	Volts	Amperes			
RCA-82	2.5	3.0	1,400	0.4	15
RCA-83	5.0	3.0	1,400	0.8	15
RCA-871	2.5	2.0	5,000	0.3	15
RCA-866	2.5	5.0	7,500	0.6	15
UV-872	5.0	10.0	5,000	2.5	15
UV-869	5.0	20.0	20,000	5.0	15
UV-857	5.0	20.0	20,000	20.0	15

19. Rectigon Bulbs. Although this section is primarily devoted to the supplying of high voltages and low currents, there is a definite field for a low-voltage high-current rectifier. The argon-filled, tungsten-filament Rectigon bulbs fill this need. The use is largely that of charging storage batteries and no filter is needed for this application.

EDITOR'S NOTE. The Tungar rectifier is a similar tube made by the General Electric Company.

Filters have been designed for use with these rectifiers, so that the output can be fed directly to the filaments of d-c radio tubes. To design a proper low-pass filter for d-c tube filament currents, Eqs. (26) and (27) should be used, as the chart of Fig. 6 does not cover this range. The condenser has to have a large capacity, and the low voltage "dry" electrolytic condensers are often used. In using these condensers it is important to connect the correct polarity to the rectifier.

TABLE III.—RECTIGON-BULB CHARACTERISTICS

Number	Filament		Load, amperes	Approximate drop in tube, volts	d-c load, volts
	Volts	Amperes			
289415*	2.75	17	2	10	75
289416†*	2.75	19	6	10	75†

* Style number of Westinghouse Electric and Manufacturing Company.
† Often used up to 100 volts.

20. Mercury-arc Rectifiers. Formerly, mercury-arc rectifiers were most used[1] in the field lying between the Rectigon and the filament-vacuum rectifier. With the introduction of the mercury-vapor tubes, however, many of the advantages of the mercury-arc rectifier—low voltage drop, high efficiency—were duplicated. The voltage drop of a typical mercury-arc rectifier tube is about 12 volts, as compared with 15 for the mercury-arc rectifier. The mercury-arc tube requires a starting electrode, and usually a mechanical tilting device for starting.

[1] PRINCE and VOGDES, "Principles of Mercury Arc Rectifiers and Their Circuits," p. 23, McGraw-Hill Book Company, Inc.

21. Glow Rectifier. The Raytheon tube uses a cold cathode, with two rod-shaped anodes, giving full-wave rectification. Helium gas at low pressure fills the bulb. With both electrodes cold, a small current flows in the reverse direction in Raytheon tubes. Another characteristic is the abrupt current rise of the output of this type of tube. No current

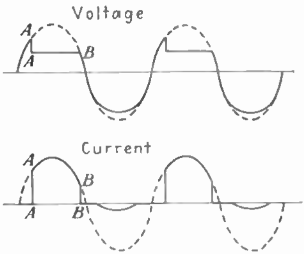


FIG. 26.—Voltage and current waves. Raytheon gaseous rectifier.

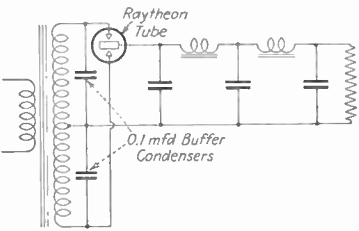


FIG. 27.—Raytheon rectifier circuit.

flows until the glow discharge takes place, and this requires a voltage of several hundred volts, the actual value depending on the electrode shape, spacing, material, gas, and pressure. When the current starts, it does so abruptly until the steady-glow voltage is reached (see Fig. 26). Filter circuits using the Raytheon tube have a buffer condenser from each anode to cathode (see Fig. 27).

TABLE IV.—RAYTHEON RECTIFIER TUBES

Type	Rated output, volts	Rated output d-c milliamperes	Maximum rated r-m-s volts per plate
BH	300	125	350
BA	200	350	350

These tubes are more sensitive to voltage overload than to current overload,[1] the high voltages causing insulation troubles. The guaranteed life at full load is 1,000 hours, but this is frequently more than doubled in practice. The maximum efficiency of the tube itself is about 55 per cent.[1]

It is usually desirable to use filter condensers with a large safety factor of voltage rating, as the glow rectifier supplies *B* and *C* voltages before tube filaments can heat up sufficiently to take their rated plate currents.

22. Dry-contact Rectifiers. The crystal detector is a familiar example of a dry-contact rectifier. The most important dry-contact rectifier for power supply is the copper-cuprous oxide Rectox type. The theory of these rectifiers is not completely understood, and the assembly of a unit which has long life and good properties requires special engineering information. Cooling is especially important in these rectifiers, the rating being increased by the use of cooling fins, or by drafts of air directed on the unit. The proper current densities are important in securing a desirable ratio of forward to inverse resistance.

[1] KRYSTER, *QST*, May, 1929, p. 33.

The usual rating of a 1½-inch diameter Rectox disk is ¼ amp. and 3 volts, but in some cases where the operating conditions are favorable, the current and voltage can be nearly doubled.

These rectifiers allow an appreciable reverse current to pass, this current increasing with temperature rise. The operating temperature should be preferably below 100°F. The leakage current is usually of the order of milliamperes, when the load current is amperes. A typical current-voltage curve is Fig. 28.[1]

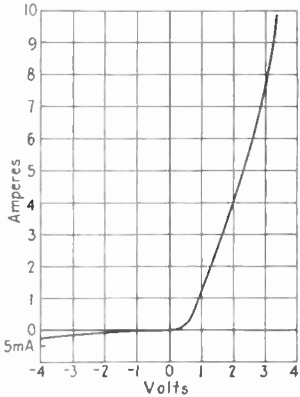


Fig. 28.—Typical Rectox current-voltage curve.

23. Circuits for Dry-contact Rectifiers. One large use of dry rectifiers is battery chargers. Even for this low voltage several disks in series have to be used. With this in mind, a simplification of the transformer can be made; *i.e.*, no center tap need be supplied. Figure 29 shows the circuit, a bridge form. This circuit is not used with tube rectifiers, as it requires four rectifying arms, which would mean four half-wave tube rectifiers. With the Rectox this is no disadvantage, as the series disks are simply grouped in four sections.

In Fig. 29 when A is positive, the path of the current is $ADBC$; and when C is positive, the current path is $CDBA$. During the time that A is positive, nearly the entire transformer voltage is applied across the bridge arm DC and across AB. It is this voltage which is the limiting factor—the 3 volts per disk, as mentioned previously.

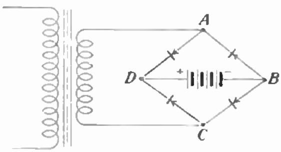


Fig. 29.—Bridge circuit for Rectox rectifier.

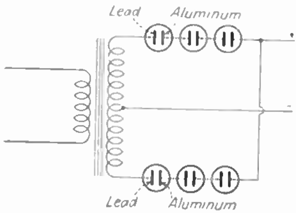


Fig. 30.—Electrolytic rectifier connections.

24. Electrolytic Rectifiers. Electrolytic (or chemical) rectifiers are bulky and, due to their liquid, are likely to be messy. (A layer of oil on top of the liquid will prevent evaporation and creeping of the solution.) These rectifiers are economical and especially well suited to amateur use. The circuit for electrolytic rectifiers can be a bridge connection like that of Fig. 30 where the arrows representing the direction of current indicate several cells in series. Another circuit for electrolytic rectifiers is given in Fig. 30.

[1] *Wireless World*, Dec. 19, 1928, p. 825.

The theory of electrolytic rectification[1] is as follows: "On the anode a solid oxide film is formed which increases in thickness with the passage of the current; at the same time a thin film of gas is formed on the solid film which further increases the resistance of the cell. The action of rectification is attributed, therefore, to the ease with which free electrons, which are present on the surface of the anode, can penetrate the oxide and gas layer owing to the high potential gradient, and traverse the electrolyte to the cathode; whereas the heavier cations are more or less completely held up by the film on account of their greater mass. This results in the production of a high counter e.m.f. or e.m.f. of polarization, which opposes the passage of a reverse current."

The (pure) aluminum-lead rectifier, in a dilute sodium bicarbonate, baking soda, solution, is an economical rectifier[2] for fairly low currents, as for instance as a *B* supply for power tubes.

The tantalum-lead rectifier in a sulfuric acid solution is better adapted to heavier load currents, especially for battery chargers.

Duriron and duralumin[3] also can be made into a good electrolytic rectifier.

The current flow in the usual sense, opposite to the electron flow, for these three rectifiers, is given in Fig. 31.

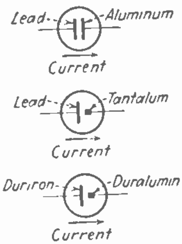


FIG. 31.—Typical electrolytic rectifier cells.

25. Current Density, Aluminum Lead. The usual current density allowed for an aluminum-lead rectifier is 40 ma per square inch of aluminum sheet, and a safe voltage to use is 40 volts per cell. As these rectifiers are especially used on high voltages, several cells are used in series, as shown in Fig. 30.

When newly assembled, the cells will not stand full voltage, and a forming process is necessary. During this forming, the cells should not be allowed to pass more than their rated current. To do this, less than the final rated voltage must be applied, and it is usual to use a lamp bank or other resistance in series with the power transformer primary. As the aluminum plates become dull white, the current will decrease, and the lamp bank can be progressively reduced in resistance, until the cells are being supplied full operating voltage at the rated current.

26. Duriron-duralumin Rectifiers. The duralumin anode has the composition 94.66 per cent aluminum, 3.93 per cent copper, 0.56 per cent manganese, 0.50 per cent magnesium, 0.33 per cent silicon, and 0.02 per cent carbon, and when it has become formed it offers a more stable rectifying film than pure aluminum. Another advantage is that the electrolyte of this cell has a lower resistance than the soda solution of the aluminum-lead cell. The duriron-duralumin cell uses as electrolyte a solution of 93 parts by volume of a 20 per cent solution of diammonium hydrogen phosphate, 3 parts of a 10 per cent solution of potassium dichromate, and 4 parts of an 8 per cent solution of oxalic acid. Potassium dichromate is here used as a depolarizer to decrease the internal resistance of the cell. The duriron electrode is a brittle iron-silicon alloy containing about 13 per cent silicon, which resists the electrolytic corrosion.

[1] WOLDMAN, *QST*, October, 1928, p. 45.
[2] "Radio Amateur's Handbook," 4th ed., p. 107.
[3] WOLDMAN, *loc. cit.*

In the reference given, a 3/16-in. duralumin rod is enclosed in a hard rubber tube, so that only 3/8 in. of the rod protrudes. The exposed area, 0.01 sq. in. can rectify 30 to 40 ma, and at this load the temperature of the cells never rises above 40°C.

27. Transformers. The design of a reliable power transformer having high efficiency, requires fairly elaborate calculations,[1] and to take into account the d.c. which flows in a transformer secondary when a half-wave rectifier is used, some interesting equations have been derived.[2]

A simple approximate-design method will be given here, for the construction of single-phase low-powered transformers up to 180 volt-amp., or 180 watts for approximately unity power factors. This design is especially suited to transformers which supply a full-wave rectifier and filament energy to an a-c powered radio receiver, three factors making it possible to secure a satisfactory transformer without complicated design methods, these factors being:

1. There is no urgent need for high efficiency. An 80 per cent efficient transformer which takes 60 watts to supply 48 output watts is fairly satisfactory, if it can radiate the heat which it generates.

2. These transformers are operated at a fairly constant load. This improves the maintenance of the various output voltages as each secondary winding will have a constant IR drop.

3. The load on the transformer secondary is nearly of unity power factor. The filament power load is essentially a resistance load, with unity power factor. The current supplied to the filter has slightly less than unity power factor, but this can be disregarded in low-powered transformers. The indirect heated receiving tubes, UY-227 and UY-224, require less than half as much d-c power in their plate and grid circuits, as that which is needed to heat their cathodes. This would mean a unity power-factor heater supply and (assuming a series voltage divider) less than half as many additional watts for plate and grid supply, at a lower power factor. It is true that a power tube, such as UX-250 at its maximum rating, uses slightly over three times the wattage in its $B + C$ circuit than in its filament. It is rare, however, to have more than two power tubes in a receiver, and the assumption that the power factor of the secondary is unity is usually not over 20 per cent off. This means that the wire of the high-voltage secondary and of the primary should be increased to allow for this added current.

28. Small Transformer Details. Economy in a transformer is secured when the winding encloses a maximum of core area with a minimum of wire, and the magnetic path should be as short as possible.

The core form of a small transformer can be of several shapes, but it is usual to use standard punchings shaped like capital letter E's. As a rule, two punchings are used, one having longer legs than the other so that the magnetic circuit "breaks joints" in stacking the iron. Another convention usually followed in small transformers is the use of a single-winding form, all secondaries and primary being on the middle leg of the E core.

The spool form is usually an insulating tube, and side pieces may be fitted on which terminals are placed, or, if the coil is to be machine wound

[1] "Standard Handbook for Electrical Engineers," McGraw-Hill Book Company, Inc., by Charles L. Fortescue.

[2] Harder, E. L., *Elec. Jour.*, October, 1930, p. 601.

with interwoven cotton, the side pieces can be omitted, and flexible leads provided.

29. Ten Steps in Designing a Small Power Transformer. 1. *Determine the Volts and Amperes Needed for Each Secondary.*

a. Find the total maximum secondary watts = $W_s = E_1I_1 + E_2I_2 + \cdots$

b. Find the total watts needed for primary = W_p

Assuming 90 per cent efficiency $W_p = W_s/0.9$

c. Find primary amperes assuming 90 per cent power factor

$$I_p = \frac{W_p}{E_p \times 0.9} = \frac{W_s}{0.81E_p}$$

and for $E_p = 110$ volts, $I_p = W_s/89.1$ amp.

2. *Size of Wire.* Knowing the current for each winding, the wire size is determined by the circular mils per ampere which it is desired to use. A safe rule is to use 1,000 cir. mils per ampere for transformers under 50 watts, and 1,500 cir. mils per ampere for higher powers.

3. *Core Considerations.* A curve showing core areas for different powers is Fig. 32 which shows the area for 40 watts to be 1 sq. in., 70 watts 1.5 sq. in.,

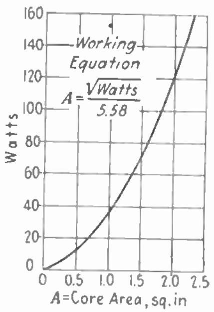


FIG. 32.—Small power transformer core area as a function of watts.

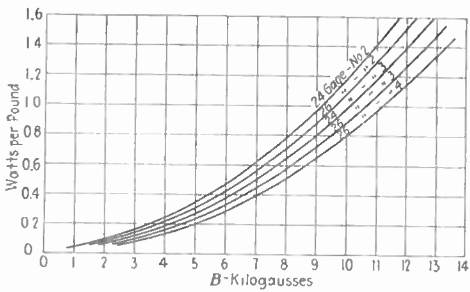


FIG. 33.—Core-loss curves Armco Radio grades (60 v).

2 sq. in. for 120 watts. The area of the core is the same as the inside dimensions of the spool, making a 10 per cent allowance for stacking; for example, a spool 1 by 2 in. inside would enclose 2 sq. in., but, allowing for a 10 per cent loss, only 90 per cent or 0.9 × 2 = 1.8 sq. in. is the net core area. The core area is needed to determine the turns per volt.

4. *Core Loss and Induction.* The flux density at which the core is to be worked determines the iron (core) loss. Figure 33 gives several curves of different core materials, watts per pound being plotted against flux densities in kilolines per square inch. Sixty-five kilolines per square inch is an average value of the induction. The making of a curve such as Fig. 33 depends largely on experimental data, not directly on a theoretical basis. For this reason, no definite value of the core loss can be given; it depends on the quality of core material which is available. It should be noted that better and better core material is constantly being made, having lower loss per pound, so that the use of higher flux densities is becoming possible. Up to 14 kilolines is not uncommon, but unusual for this application. The core loss increases with frequency, a typical curve being Fig. 34.

5. *Induced-voltage Equation, Turns per Volt.* The elementary definition, that 10^8 magnetic lines cut, per second, will induce one volt pressure, is the basis of the equation

$$E = \frac{BANf}{10^8} \times 4.44$$

where E is the voltage, A the area of the core, B the flux density in the same units as A, f the cycles per second, and N the number of turns. A more useful working equation for small power transformers is obtained by solving for N/E in turns per volt:

$$\frac{N}{E} = \frac{10^8}{BAf4.44}$$

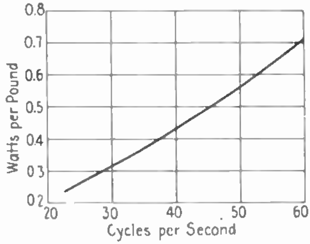


FIG. 34.—Core loss *vs.* frequency B = 10,000.

Figure 35 is an alignment chart of this equation. The left column is B the flux density, in both kilolines per square inch and kilogausses (kilolines per square centimeter), the center column in the net core area in both square inches and square centimeters, the right column giving the turns per volt for both 25 and 60 cycles per second.

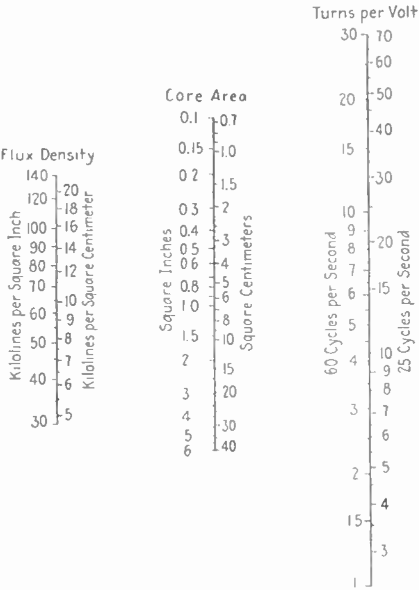


FIG. 35.—Transformer design chart based on $E = \frac{BANf \times 4.44}{10^8}$.

Using a flux density of 65 kilolines per square inch and the net core area mentioned in step 3 (1.8 sq. in.), the turns per volt for 60 cycles are found to be 3.1 turns per volt. Thus for each volt on the transformer, there must be 3.1 turns. It is customary to change the turns per volt to an even number so that the proper center taps can be made. In this case, by using 4 turns per volt, with the same core area, the induction will be lower, with a corresponding lower core loss. It is also quite possible, and sometimes advisable, to change the core area so that an even number of turns per volt is given. For example, by increasing the core area to 2.8 sq. in. 2 turns per volt could be used, or decreased to 1.4 sq. in. so that 4 turns per volt would be used. The reason for desiring the even numbers of turns per volt is to supply the ½-volt steps for receiving tubes, such as 7½ volts, which would require an integral number of turns when the turns per volt are used.

The voltage drop in the transformer winding should be mentioned here, and it will be again taken up in detail in the example. For instance, the load voltage at a tube filament is lower than the no-load voltage by the amount of IR drop in the winding and the connecting wires to the tube. Thus, it may be that to secure 7½ volts at the tube filament, the transformer no-load voltage will have to be 8. In this case, any integral number of turns per volt, either odd or even, will suit the design.

6. *Turns for Each Winding.* In step 1 the desired voltages were given, E_1, E_2, etc. Using the value of turns per volt in step 5, the total turns for each winding are found. For example, with 4 turns per volt, a 110-volt winding should have $4 \times 110 = 440$ turns.

7. *Winding Space Required.* From the total turns for each winding, and the wire size, the total area of winding space is calculated. Different wires and insulations have definite turns per square inch. The method of insulation, however, may have these values vary by factors of as much as three to one. That is, a 900-turn coil wound in layers with enamel wire may take up one square inch of cross-section area. By interleaving thin insulating paper between layers, only 600 turns can be wound in a square-inch area; and by using a certain size of cotton interwoven between turns, only 400 turns can be wound in a square inch. Thus, the space of winding depends to a large degree on the kind and thickness of insulation. Double cotton-covered wire takes up considerably more space than enameled wire. Yet, if the extra-needed insulating space for the interlayer protection is considered, the space ratio may not be so great.

After adding up the winding space of all the windings the area should be compared with that of the core. If the winding will go in the core space, this part of the design is finished.

If the wires will not go in the available space, the winding may be redesigned, or the core area increased. Using thinner coverings for wire, fewer secondaries or fewer circular mils per ampere will decrease the space needed for the wire. A larger iron size or a thicker stack of the same sized iron will increase the core area and allow a smaller number of turns per volt, thus decreasing the cross section of the winding.

8. *Copper Loss.* *a.* Find the length of the mean (average) turn in feet.

b. Find the length of each winding in feet by multiplying the number of turns by the mean turn length.

c. From wire tables find the ohms per 1,000 ft. for the size wire used, and then from 8-*b* the actual ohms for this length.

d. Multiply the current squared for each winding by the ohms for that winding.

e. Add the I^2R's for each winding to get the copper loss L_1.

9. *Core Loss.* The core loss in watts L_2 is found from the weight of the core and the flux density and kind of core used in step 4. A useful factor is that 4 per cent silicon steel weighs 0.27 lb. per cubic inch.

10. *The approximate percentage efficiency* is $\dfrac{W_s \times 100}{W_s + L_1 + L_2}$, W_s being the secondary watts (see step 1).

NOTE. If step 10 shows about 90 per cent efficiency, the design is complete. If much less than 90 per cent, step 1*a* must be modified, a new larger value of I_p being used in finding a larger primary wire. This will not change the efficiency, but will prevent overloading the primary winding due to its carrying a greater current than that for which it was designed.

It is desirable, as a rule, to keep the efficiency above 90 per cent, and this can be done by reducing L_1 and L_2, by using larger wires, or larger cores.

30. Typical Small Transformer Design. This transformer gives a full-wave rectifier supply, filament supply for rectifier and receiver, and works on a primary voltage of 110, at a frequency of 60 cycles.

1. The desired secondary voltages and currents are:

E, volts	I, amperes	Use	Watts = EI
330	0.05	B and C supply	16.5
330	0.05	B and C supply	16.5
5.0	2.0	Rectifier filament	10.0
2.5	2.5	Filament	6.25
2.5	3.0	Filament	7.5

a. Total secondary watts W_s 55.75
b. Primary watts $W_p = W_s/0.9 = 55.75/0.9 = 61.9$
c. Primary amperes $I_p = W_s/89.1 = 55.75/89.1 = 0.69$

2. This transformer is over 50 watts, so 1,500 circ. mils per ampere is the current density to use in finding the proper-sized wire. The wire sizes, with the identifying current and voltages, are listed in table on p. 401. The use of larger wires of even numbers keeps the IR drop lower than when using a smaller wire. However, if the use of these larger wires makes too large a winding cross section, smaller wires must be used.

3. The core area available is $1\frac{3}{8} \times 2$ inches, the net area being $1\frac{3}{8} \times 2.0 \times 0.9 = 2.48$ in. This is larger than necessary as shown by Fig. 35, but allows the design, in this case, of a transformer with good efficiency and good regulation.

Volts	Amperes	Size wire
110	0.69	20
330	0.05	30
330	0.05	30
5	2.0	14
2.5	2.5	14
1.5	3.0	12

4. The flux density used is 65 kilolines per square inch, and 4 per cent silicon iron with a loss of 0.6 watt per pound.

5. The turns per volt for 65 kilolines per square inch and core area of 2.48 sq. in. give three turns per volt.

6. The turns for each winding are:

Volts	Turns
110	330
330	990
330	990
5	15
2.5	7.5(8)*
1.5	4.5(5)

* It is usual to add ½ turn to filament windings for 2.5, and 1.5 to allow for the IR drop in the winding and leads to the tube filaments. An even number like 8 also makes taps easier.

7. Winding space, in square inches, using enamel wire:

Turns	Size wire	Turns per square inch	Actual space, square inch
330	20	590	0.56
990	30	4,000	0.25
990	30	4,000	0.25
15	14	190	0.08
8	14	190	0.04
5	12	135	0.04
Total.	. .	. . . . .	1.22

a. The mean turn is 11 in. = $1\frac{1}{12}$ ft.
b. The space needed is 1.2 sq. in. and the space available is 1 × 2 = 2 sq. in., so the extra space can be used for the spool and for insulation between windings and layers.
c.

Turns	Feet	Ohms per 1,000 ft.	Actual ohms	IR volts drop	I^2R, watts
330	320	10	3.02	. . . . .	0.14
990	906	105	95	. . . . .	0.25
990	906	105	95	. . . . .	0.25
15	13.7	2.6	0.035	0.07	0.14
8	7.3	2.6	0.019	0.05	0.12
5	4.6	1.8	0.008	0.024	0.07
Total.	. . . . .	. . . . .	. . . . . .	. . . . .	0.97

d. The copper loss L_1 is 0.97 watt.

8. The core weighs approximately 5 lb., which at 0.6 watt per pound gives 5 × 0.6 = 3.0 watts = L_2.
9. Watts output = 55.75 = W_s

$$\text{Losses} = L_1 + L_2 = 3.0 + 0.97 = 3.97 \text{ watts}$$

$$\text{Per cent efficiency} = \frac{55.75 \times 100}{55.75 + 3.97} = \frac{5{,}575}{59.72} = 93 \text{ per cent}$$

NOTE. It is seen that the copper losses are about one-third the iron loss; this means that a smaller core could be used. A higher efficiency could be obtained, if there were enough winding space, by using higher induction with more turns per volt, thus decreasing the core loss, without increasing the copper loss very much.
10. *Volts Drop.* It is seen by the IR column that the drop in the winding is not serious.

SECTION 16

LOUD-SPEAKERS AND ACOUSTICS

By Irving Wolff, B.S., Ph.D.[1]

1. Symbols.

a Instantaneous displacement of air particle in sound wave.
b Flare factor of exponential horn.
c Velocity of sound in air.
d Diameter of disk.
e Napierian base
j $\sqrt{-1}$
l Length of conductor; length of horn.
m Mass.
p Sound pressure.
r Distance from center of a sphere.
s Stiffness.
t Time
u Instantaneous velocity of particle in sound wave.
v Voltage.
x Distance from an axis of coordinates.
z Mechanical impedance.
z_r Mechanical resistance.
z_1 Impedance per unit area at throat of horn.
z_2 Impedance per unit area at mouth of horn.

A Strength of small sound source.
E Energy density in sound field.
H Magnetic or electric field strength.
H_0 Polarizing magnetic or electric field strength.
I Current.
J Energy flux density in sound wave.
M Vector force factor or electromechanical coupling coefficient.
R Electrical resistance.
R_s Electrical resistance of supply source.
S Surface or cross-sectional area.
S_1 Cross-sectional area of throat of horn.
S_2 Cross-sectional area of mouth of horn.
T Reverberation time.
U Maximum velocity of air particles in sound wave.
V Volume of an enclosure.
W Acoustic power emission of sound source.
Z Electrical impedance.

$\bar{\alpha}$ Average absorption coefficient $\bar{\alpha} = \dfrac{S_1\alpha_1 + S_2\alpha_2 \cdots S_n\alpha_n}{S_1 + S_2 \cdots S_n}$

γ Angle between line joining point of observation to the center of a sound radiator and line perpendicular to the radiator.
dS Small surface element.

λ Wave length.
ρ_0 Density of air when undisturbed.
θ Temperature, degrees centigrade.
μ Magnetic permeability.
ω 2π frequency.

[1] In charge of General Physics and Acoustics Section, Research Division, R C A Victor Company, Inc.

2. Sound Waves in Air. A sound wave in air is a compressional wave which is characterized by a to-and-fro motion of the air particles and an increase and decrease of sound pressure above and below atmospheric in the path of the wave. In the interest of conciseness when the term "pressure of a sound wave" is used, the difference in pressure between atmospheric and the pressure which occurs when the sound wave is present is what is referred to.

The following equations give the relations between amplitude of motion of the air particles, velocity of motion of the particles, and pressure in a *plane* sound wave, where u represents the instantaneous velocity of an air particle, U is the maximum velocity, λ is the wave length of the sound wave in air, c is the velocity of propagation of the sound wave, x is a coordinate taken in the direction of propagation of the plane sound wave, a is the displacement of the air particles, p is the pressure of the sound wave, and ρ_0 is the density of air. If the wave is simple harmonic,

$$u = U \cos \frac{2\pi}{\lambda}(ct - x) \tag{1}$$

$$a = \frac{\lambda U}{2\pi c} \sin \frac{2\pi}{\lambda}(ct - x) \tag{2}$$

$$p = c\rho_0 U \cos \frac{2\pi}{\lambda}(ct - x) \tag{3}$$

The next equations give the same relations for a *spherical* sound wave where all symbols are the same as above; A represents the strength of a small source considered to be at the center of the sphere as represented by the maximum rate of emission of air, and r is the distance from the center of the sphere.

$$u = \frac{A}{2\lambda r} \cos \frac{2\pi}{\lambda}(ct - r) + \frac{A}{4\pi r^2} \sin \frac{2\pi}{\lambda}(ct - r) \tag{4}$$

$$a = \frac{A}{4\pi rc} \sin \frac{2\pi}{\lambda}(ct - r) - \frac{\lambda A}{8\pi^2 r^2 c} \cos \frac{2\pi}{\lambda}(ct - r) \tag{5}$$

$$p = \frac{c\rho_0 A}{2\lambda r} \cos \frac{2\pi}{\lambda}(ct - r) \tag{6}$$

In a plane sound wave of simple harmonic type the maximum of pressure takes place at the same time that the velocity of the particles (air molecules) is a maximum in the direction of propagation of the wave. That is, suppose we are observing at a certain position in space a sound due to a source which is vibrating with simple harmonic motion at some distance from us so that the wave front is almost a plane. If we were just able to observe pressure at this position in space where we are stationed, we would note that the pressure varies in a simple harmonic fashion about atmospheric pressure. If we were just able to follow velocity, we would see the air molecules moving back and forth. If, however, both factors can be observed at the same time, we shall note that as the air particles move forward the pressure becomes greatest at the time that the air particles are moving fastest. In analytical terms, the pressure and velocity are in phase.

On the other hand, the equations show, that in a spherical sound wave, the pressure and velocity are no longer in phase except when the radius of the sphere is very great and the wave approximates to a plane wave. When the radius of the sphere is very small, the pressure and velocity are almost 90 deg. out of phase. It will also be noted that although the pressure dies down at a rate inversely proportional to the distance away from the source or center of the spherical wave, the velocity reduces at a much greater rate when a point is taken close to the center of the spherical wave. It is only in plane waves that the pressure and velocity are in phase and always proportional to each other in magnitude. In the plane wave, the pressure is always equal to $\rho_0 c$ times the velocity. In the spherical wave it is equal to $\rho_0 c\sqrt{1 + \frac{\lambda^2}{4\pi^2 r^2}}$ times the velocity in absolute magnitude. In other shapes of waves still different relations hold which will not be considered here.

Imagine a large plane sheet vibrating and sending out a sound wave. Work has to be done to move this sheet back and forth against the air resistance. This work generates a sound wave which gives a means for transferring the energy from the sheet to some distant point. The energy transfer through any small area in space is determined by taking the product of the root-mean-square force and the velocity. In addition to the transfer of energy due to the sound wave (the value of this transfer through a square centimeter of surface is called the *energy flux density*) there also exists, due to presence of the sound wave, a certain amount of kinetic and potential energy in any small region in the path of the wave. This energy is known as the *energy density* in the wave. By means of the relations connecting pressure and velocity in the sound wave given in the previous equations the energy density and flux can be expressed in terms of the pressure alone. The equations connecting energy flux and density and sound pressure, and which hold for both plane and spherical sound waves, are:

$$J = \frac{p^2}{\rho_0 c} \tag{7}$$

$$\text{and } E = \frac{p^2}{\rho_0 c^2} \tag{8}$$

where J is the energy flux and E is the energy density.

3. Velocity of Sound in Some Common Materials. (From International Critical Tables) Velocity of sound in air at different temperatures is given by means of the equation:

$$C = 330.6\sqrt{1 + 0.003707\theta - 1.256\theta^2 10^{-7}}$$

where θ is expressed in degrees centigrade and the velocity is in meters per second. The density of air is 0.00129 g/cc at 0°C. and 760-mm mercury pressure. The velocity of sound in some other materials is given in the table shown on page 406.

Velocity of Sound in Some Materials

Material	Temperature, degrees centigrade	Meters per second	Material	Temperature, degrees centigrade	Meters per second
Aluminum	..	5,105	Rubber (vulcanized, black)	0	54
Argon	..	308	Rubber (vulcanized, black)	50	30.7
Beeswax	16	863	Silver	20	2,678
Brass	..	3,479	Slate	..	4,510
Brick	..	3,652	Steel	..	4,990
Bromine	..	135	Tin	13	2,490
Cadmium	..	2,307	Woods:		
Chlorine	..	208	Ash (parallel to grain)	..	4,670
Cobalt	..	4,724	Ash (across grain)	..	1,260
Copper	20	3,560	Beech	..	3,340
Cork	..	430–530	Cedar	..	3,975
Ebonite	15	1,573	Cherry	..	4,410
Gelatin	..	1,364	Elm (parallel to grain)	..	4,120
Gold	20	1,743	Elm (across grain)	..	1,013
Glass	16	5,202	Fir	..	5,256
Granite	..	3,950	Mahogany	..	4,135
Helium	..	971	Maple	..	4,110
Iodine	..	108	Oak	..	3,381
Iron	..	5,130	Pine	..	3,320
Lead	18	1,229	Poplar	..	4,280
Magnesium	..	4,602	Sycamore	..	4,460
Marble	..	3,810	Walnut	..	4,781
Nickel	..	4,973	Zinc	13	2,681
Nitrogen	..	338			
Oxygen	..	316			
Palladium	10	3,074			
Paraffin	6	1,522			
Paraffin	35	250			
Platinum	20	2,690			

4. Electrical, Mechanical, and Acoustical Impedance. In electrical engineering, the concept of electrical impedance is very useful. The impedance of any part of an electrical circuit is the complex quantity obtained by taking the complex quotient of the voltage in the circuit to the current flowing through it. In acoustical and mechanical work, analogous concepts are equally useful. The Institute of Radio Engineers has defined the mechanical impedance of a mechanical system as follows: "The *mechanical impedance* of a mechanical system is the complex quotient of the alternating force applied to the system by the resulting alternating linear velocity in the direction of the force at its point of application." Furthermore, in analogy to the electrical quantities, the *mechanical resistance* has been defined as the real component of the mechanical impedance and the *mechanical reactance* as the imaginary component of the mechanical impedance.

Another useful concept is the *acoustic impedance* of a sound medium. The definition given by the Institute of Radio Engineers for this quantity is as follows: "The acoustic impedance of a sound medium on a given surface is the complex quotient of the pressure (force per unit area) on the surface by the flux (volume velocity or linear velocity multiplied by the area) through that surface. The acoustic impedance may be expressed in terms of mechanical impedance, acoustic impedance being equal to the mechanical impedance divided by the square of the area of the surface considered." The *acoustic resistance* has been defined as the real component of the acoustic impedance and the *acoustic reactance* as the imaginary component.

The apparently conflicting definitions for mechanical and acoustic impedance may at first seem needlessly confusing, but practice has found the definitions which are given to be the most practical. In mechanical systems, we deal with the motion under the influence of certain forces and, therefore, the force and motion have been taken as the quantities in terms of which the impedance is to be defined. In electrical systems, we deal with voltage and current. The acoustic systems are quite analogous to electrical systems. The analogous quantities are pressure and total flow, and it is found that the consideration of complex acoustic circuits is simplified by the use of these quantities.

5. Acoustics of Rooms. When a source of sound is in a room or auditorium the sound waves leaving the source are reflected many times by the walls before they are absorbed. These successive reflections of the sound are known as *reverberation.* Architects and designers of theatres and auditoriums have found the reverberation characteristics of the room of great importance in reference to its effect on the quality of music and intelligibility of speech.

To have a quantitative measure of this reverberation, Wallace Sabine defined the *reverberation time* of a room as the time necessary for the average sound energy in the room to drop to one-millionth of its original value after all sources of sound are shut off. He also determined by numerous psychological experiments the values of the reverberation time which observers found most pleasant. This time was found to depend on the size of the auditorium, a longer time being permissible in a larger room. A curve giving the relation between optimum reverberation time and room size is shown in Fig. 1.

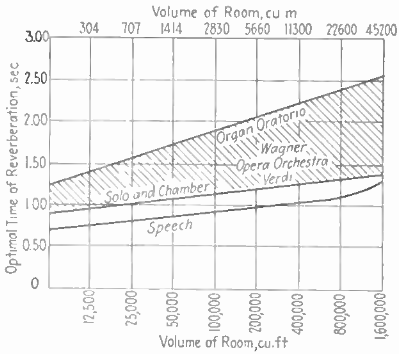


FIG. 1.—Optimal reverberation times as a function of room size and type of sound as given by V. O. Knudsen.

The factors which influence the reverberation time can be determined both theoretically and experimentally. A very good check has been found between experiment and theory. The fundamental equation governing the building up of the average sound energy density in a room due to a source having a power emission W started at time $t = o$, where $\bar{\alpha}$ is the average absorption of surfaces and objects in the room obtained by summing all the products of absorption coefficients and areas and where the absorption of the air itself is neglected is:

$$\bar{\alpha} = \frac{S_1\alpha_1 + S_2\alpha_2 S_3\alpha_3 + \cdots}{S_1 + S_2 + S_3 + \cdots}$$

S is the total absorbing area.

$$S = S_1 + S_2 + S_3$$

(Text continued on page 412)

6. Absorption Coefficient for Common Materials.

Material	Absorption coefficients for frequency							Auth.	Date
	64	128	256	512	1,024	2,048	4,096		
Open window, Theoretical	1.0	1.0	1.0	1.0	1.0	1.0	1.0		
Masonry, Plasters, and Tiles:									
Brick wall, 18 in., unpainted	0.021	0.024	0.025	0.081	0.042	0.049	0.070	WS	06
18 in., painted	0.011	0.012	0.013	0.017	0.020	0.023	0.025	WS	06
Concrete, porous breeze, 2-in. blocks, set in 1:3 cement-sand mortar	.....	0.15	0.21	0.43	0.37	0.39	0.51	BR	26
Plaster, Akoustolith, ½ in., on ¼-in. thick lime water	.....	0.21	0.24	0.29	0.33	0.37	0.42	CS	23
Ambler, sound absorbing	.....	0.03	0.06	0.14	0.17	0.19	0.11	FW	26
Calacoustic, ¼ in	.....	0.12	0.13	0.16	0.21	0.35	.....	VK	27
Gypsum, on hollow tile	0.012	0.013	0.015	0.020	0.028	0.040	0.050	WS	14
Lime, on wood lath	0.048	0.020	0.024	0.034	0.030	0.028	0.043	WS	14
Lime, on wood lath with finishing coat	0.036	0.012	0.013	0.018	0.045	0.028	0.055	WS	14
Sabinite, ½ in., acoustical plaster	.....	.....	0.166	0.214	0.29	0.34	.....	PS	27
Sabine, ¾ in., fixed as tiles	.....	0.07	0.07	0.23	0.43	0.27	0.41	BR	26
1 in. (as modified by BRS), trowel applied	.....	0.11	0.11	0.29	0.47	0.29	0.38	BR	26
Tile, Akoustolith, 1 in	.....	0.06	0.12	0.36	0.52	0.52	0.36	WS	14?
Rumford, 1 in	.....	0.09	0.18	0.29	0.34	0.34	0.30	WS	14?
Sabine, acoustical	0.064	0.068	0.12	0.19	0.25	0.26	0.22	WS	14
Sabine, West Point (ceramic tile)	0.012	0.013	0.018	0.029	0.040	0.048	0.053	WS	14
Building boards and panel:									
Acoustex, 1 in	.....	0.10	0.25	0.55	0.73	0.64	0.56	PS	
1½ in	.....	0.14	0.34	0.68	0.82	0.63	0.52	PS	
Armstrong corkboard, 1 in., 0.875 lb. per sq. ft	.....	.....	0.08	0.30	0.31	0.28	.....	FW	27
1 in., sprayed with cold-water paint	.....	.....	0.07	0.30	0.28	0.29	.....	FW	27
2 in., 1.6 lb. per sq. ft	.....	.....	0.17	0.35	0.27	0.34	.....	FW	27
Acoustic Zenitherm, 1.14 in., cork granules cemented into a porous, stiff title	.....	0.03	0.13	0.33	0.42	0.42	0.15	FW	26
Acousti-Celotex, Type A, 13⁄16 in., 1.11 lb. per sq. ft., perforated with 441 small holes per sq. ft. on back of material	.....	0.13	0.28	0.25	0.23	0.23	0.23	CEL	29
Type B, as above, with perforations exposed on front, painted or unpainted	.....	0.22	0.28	0.47	0.53	0.62	0.62	CEL	29
Type BB, 1¼ in., 1.67 lb. per sq. ft., perforated on front as above, painted or unpainted	.....	0.28	0.42	0.70	0.74	0.77	0.77	CEL	29

Type C, 3/8 in., 0.48 lb. per sq. ft., completely perforated, painted or unpainted	.....	0.14	0.16	0.30	0.45	0.57	0.55	CEL	29
Celotex, ½ in., standard building board	.....	0.17	0.18	0.20	0.20	0.19	0.19	CEL	29
Cork (coarse), 1 in., slab 3¼ ft. by 1 ft., 7½ in., 1 in. from wall, framed in wood	.....	0.14	0.25	0.40	0.25	0.34	0.21	BR	26
Insulite, ½ in., standard building board	.....	0.23	0.26	0.28	0.29	0.32	.....	VK	28?
Acoustile, single layer	.....	0.24	0.26	0.30	0.36	0.38	.....	VK	27?
Acoustile, double layer, ¾-in. air space	.....	0.30	0.31	0.34	0.37	0.40	.....	VK	27?
Masonite, 7/16 in., building board, bare	.....	0.19	0.25	0.32	0.36	0.36	.....	VK	28
7/16 in., on 2 by 1-in. furring	.....	0.17	0.24	0.28	0.295	0.30	0.28	PS	27
7/16 in., on 2 by 4-in. studding	.....	0.18	0.245	0.31	0.34	0.30	0.24	PS	27
Wood, sheathing, 0.8-in. pine	0.064	0.098	0.11	0.10	0.081	0.082	0.11	WS	06
3-ply teak panels, 3 ft. by 2 ft. 2 in., 1 in. from wall, framed in wood	.....	0.09	0.17	0.17	0.15	0.15	0.15	BR	26
Felts and membranes:									
Asbestos felt, 3/8 in., 33 per cent of volume is solid material	0.06	0.06	0.14	0.32	0.25	0.19	0.18	WS	12
3/8 in. felted to asbestos cloth	0.07	0.08	0.17	0.35	0.30	0.23	0.20	WS	12
Balsam wool, ½ in., paper and cloth covering, weight 0.20 lb. per sq. ft	.....	0.05	0.22	0.41	0.58	0.52	0.39	PS	24
1 in., paper and cloth covering, 0.265 lb. per sq. ft	.....	0.06	0.30	0.56	0.70	0.58	0.46	PS	24
1 in., paper on under side, other side bare, 0.233 lb. per sq. ft	.....	0.09	0.24	0.45	0.64	0.55	0.42	PS	24
1 in., loosely felted quilt of wool fiber, 0.26 lb. per sq. ft.	.....	.....	0.18	0.44	0.62	0.62	.....	FW	27
1 in. covered with steel tile perforated with 64 1/16-in. holes per sq. in., 0.93 lb. per sq. ft	.....	.....	0.19	0.47	0.64	0.66	.....	FW	27
Cabot quilt, 3 ply, 2 layers 1½ in. from wall plus canvas cover 1 in. distant	.....	0.22	0.42	0.74	0.77	0.69	0.44	PB	26
Flax-li-num, 9/16 in	.....	0.08	0.14	0.31	0.54	0.51	0.45	PS	27
1-in. felted flax fibers, 1.17 lb. per sq. ft., bare	.....	.....	0.49	0.61	0.67	0.66	.....	FW	27
1 in., with unpainted decorative membrane (0.1 lb. per sq. ft., mesh 10 per in.) mounted ¾ in. distant	.....	.....	0.30	0.61	0.60	0.55	.....	FW	27
Hair and asbestos felt, 19% volume solid	0.04	0.05	0.11	0.38	0.55	0.46	0.39	WS	12
Hair felt, 12% volume is solid 1 in. thick	0.09	0.10	0.20	0.52	0.71	0.66	0.44	WS	12
Same covered with burlap attached with silicate of soda	0.13	0.13	0.33	0.74	0.76	0.49	0.18	WS	12
Same with light membrane (0.87 oz. per sq. ft.) stretched near surface	0.17	0.20	0.40	0.65	0.27	0.14	0.11	WS	12

6. Absorption Coefficient for Common Materials. *(Continued)*

Material	Absorption coefficients for frequency 64	128	256	512	1,024	2,048	4,096	Auth.	Date
Same with heavy membrane (2.58 oz. per sq. ft.) stretched near surface	0.25	0.29	0.41	0.32	0.19	0.11	0.08	WS	12
1 in., in contact with wall	0.09	0.10	0.23	0.58	0.72	0.66	0.46	WS	12
1 in., spaced 2 in. from wall	0.10	0.11	0.26	0.62	0.73	0.66	0.45	WS	12
1 in., spaced 4 in. from wall	0.11	0.13	0.30	0.66	0.74	0.66	0.45	WS	12
1 in., spaced 6 in. from wall	0.12	0.15	0.35	0.68	0.75	0.66	0.45	WS	12
1 in., located at center of room with barrel ceiling	0.14	0.15	0.32	0.96	1.27	1.02	0.62	WS	12
1 in., located at sides of room with barrel ceiling	0.11	0.20	0.25	0.54	0.43	0.48	0.20	WS	12
Jute felt, ½ in.	0.038	0.049	0.076	0.17	0.48	0.52	0.51	WS	06
1 in.	0.12	0.15	0.22	0.54	0.63	0.57	0.52	WS	06
1½ in.	0.19	0.24	0.38	0.63	0.65	0.57	0.52	WS	06
2 in.	0.27	0.34	0.50	0.69	0.67	0.58	0.52	WS	06
2½ in.	0.34	0.43	0.59	0.75	0.67	0.58	0.52	WS	06
3 in.	0.40	0.50	0.66	0.77	0.68	0.58	0.52	WS	06
J-M asbestos akoustikos felt, ½ in. bare	.....	0.07	0.14	0.31	0.51	0.51	0.43	PS	28
¾ in. bare	.....	0.08	0.23	0.45	0.65	0.56	0.46	PS	28
1 in. bare	.....	0.11	0.31	0.59	0.68	0.58	0.46	PS	28
1½ in. bare	.....	0.13	0.41	0.73	0.73	0.58	0.46	PS	28
2 in. bare	.....	0.21	0.46	0.79	0.75	0.58	0.46	PS	28
3 in. bare	.....	0.33	0.56	0.79	0.77	0.58	0 46	PS	28
J-M Nashkote, Type AX, ½ in. (J-M asbestos Akoustikos felt with batiste membrane cemented to felt, surface painted with one coat of No. 3000 paint)	.....	0.10	0.22	0.34	0.41	0.32	0.17	PS	28
Type AX ¾ in.	.....	0.13	0.24	0.38	0.45	0.35	0.17	PS	28
1 in.	.....	0.15	0.38	0.43	0.40	0.29	0.18	PS	28
1½ in.	.....	0.22	0.38	0.41	0.39	0.29	0.20	PS	28
2 in.	.....	0.34	0.38	0.44	0.4	0.30	0.23	PS	28
3 in.	.....	0.40	0.47	0.48	0.45	0.31	0.24	PS	28
Membrane, light, 0.87 oz. per sq. ft	0.01	0.01	0.04	0.10	0.07	0.02	0.01	WS	12
Heavy, 2.58 oz. per sq. ft	0.05	0.06	0.16	0.16	0.10	0.07	0.06	WS	12
Canvas, 6 in. from wall	.....	0.10	0.12	0.25	0.33	0.15	0.35	BR	26

Rock wool, banner, 1 in.	.....	0.35	0.49	0.63	0.80	0.83	.....	VK	28
2 in.	.....	0.44	0.59	0.68	0.82	0.84	.....	VK	28
4 in., granulated, packed loosely between 2 by 4 in., 16 in. o.c.	.....	0.43	0.52	0.57	0.65	0.64	.....	VK	28
1 in., 2 layers separated by 1¾-in. air space	.....	0.53	0.64	0.71	0.87	0.90	.....	VK	28
Rock wool, Gimco, 1¼ in., silicate fibers felted between metal lath, 1.44 lb. per sq. ft.	.....	.....	0.46	0.57	0.56	0.72	.....	FW	27
Slagbestos, 1½-in. slabs, ¾ in. from wall	.....	0.32	0.38	0.65	0.73	0.30	0.29	BR	26
1½ in., plus canvas 1 in. distant	.....	0.42	0.49	0.80	0.78	0.47	0.42	BR	26
Floor coverings:									
Carpet, 0.4-in. pile, on concrete	.....	0.09	0.08	0.21	0.26	0.27	0.37	BR	26
0.4-in. pile, on felt, ⅛ in. on concrete	.....	0.11	0.14	0.37	0.43	0.27	0.25	BR	26
0.4-in. pile, on felt, on ¾ in. polished cork on concrete	.....	0.17	0.14	0.35	0.42	0.23	0.34	BR	26
0.4-in. pile, on felt, on ¾ in. pitch pine blocks on concrete	.....	0.11	0.13	0.38	0.45	0.29	0.29	BR	26
Amritza, 0.43 in., on concrete	.....	0.09	0.06	0.24	0.28	0.11	0.21	BR	26
Cardinal Batala, 0.43 in. on concrete	.....	0.12	0.10	0.28	0.42	0.21	0.33	BR	26
3/16 in., rubber on concrete	.....	0.04	0.04	0.08	0.12	0.03	0.10	BR	26
3/16 in., rubber on polished cork on concrete	.....	0.09	0.04	0.15	0.11	0.10	0.04	BR	26
Cork, ¾-in. flooring slabs, glued down	.....	0.08	0.02	0.08	0.18	0.21	0.22	BR	26
¾-in. flooring slabs, waxed and polished	.....	0.04	0.03	0.05	0.11	0.07	0.02	BR	26
Wood, ¾-in. pine blocks, laid in mastic	.....	0.05	0.03	0.06	0.09	0.10	0.22	BR	26
¾-in. Gurjan, laid in mastic	.....	0.03	0.04	0.07	0.14	0.09	0.15	BR	26
Individual objects:									
Audience, per person	1.7	3.6	4.3	4.7	4.7	5.0	5.0	WS	06
Cushions, cotton, 2¾ sq. ft., under canvas and short nap plush	0.99	1.7	1.9	2.0	2.8	2.0	1.3	WS	06
Hair, 2¾ sq. ft., under canvas and plush	0.86	0.99	1.1	1.8	1.7	1.4	0.91	WS	06
Hair, under canvas and thin leatherette	0.67	1.1	1.3	1.9	1.3	0.73	0.43	WS	06
Vegetable fiber, under canvas and damask	0.64	0.75	1.0	1.5	1.6	1.4	1.2	WS	06
Chairs, bent ash	0.15	0.15	0.16	0.17	0.18	0.20	0.23	WS	06

The abbreviations employed are: Auth., authority; W.S., W.C. Sabine; P.S., P.E. Sabine; F.W., F.R. Watson; V.K., V.O. Knudsen; B.R., Building Research Station, England; CEL, average of results by P.S., F.W. and V.K.

V is the volume of the room and c is the velocity of sound:

$$E = \frac{4W}{cS\bar{\alpha}}\left(1 - \epsilon^{\frac{cS \log_\epsilon (1 - \bar{\alpha})t}{4V}}\right) \tag{9}$$

Inspection of the above equation shows that for large t its value approaches $4W/cS\bar{\alpha}$, which is the steady-state sound energy.

For the decay of the sound energy after the source has been shut off,

$$E = \frac{4W}{cS\bar{\alpha}} \epsilon^{\frac{cS \log_\epsilon (1 - \bar{\alpha})t}{4V}} \tag{10}$$

From the latter equation the reverberation time as defined by Sabine may be calculated. Evaluating all constants,

$$T = \frac{0.16V}{-S \log_\epsilon (1 - \bar{\alpha})} \text{ if all dimensions are in meters} \tag{11}$$

It has been customary to express absorption coefficients of objects in square feet of perfect absorption in this country, and a great many measurements of rooms are given in English units. The following equation gives the reverberation time when feet are used for all measurements:

$$T = \frac{0.05V}{-S \log_\epsilon (1 - \bar{\alpha})} \tag{12}$$

If the average absorption is less than 0.5 Eqs. (11) and (12) may be simplified approximately to:

$$T = \frac{0.16V}{S\bar{\alpha}} \tag{13}$$

and

$$T = \frac{0.05V}{S\bar{\alpha}} \tag{14}$$

Using Eq. (12), (14) (14*a*) or (14*b*) and the tables (pp. 408–411) giving absorption coefficients of some common materials and objects, the reverberation time of a room can be calculated.

Equations (9) to (14) assume that the energy density in the room averaged over regions large compared to the wave length is uniform during the decay and that all directions of sound energy flux at each point in the space are equally probable. In most rooms of regular shape these conditions are fulfilled. Certain rooms, however, are of such shape as to cause peculiar concentrations and directions of sound, and the equations will not apply.[1]

At the higher frequencies the absorption of the air itself in rather reverberant rooms may be important, particularly in dry atmosphere. An experimental determination of the absorption to be expected has been made by Knudsen.[2] The general reverberation time equation corrected for the absorption of the air is:

$$T = \frac{0.16\ V}{-S \log_\epsilon (1 - \bar{\alpha}) + 4KV} \text{ in meters} \tag{14(a)}$$

[1] For examples and a discussion of some special cases see K. Schuster, and E. Waetzmann, *Ann. Physik*, **1**, 5, 671, 1929.

[2] *Jour. Acoustical Soc. America*, **3**, 126, 1931.

$$T = \frac{0.05\,V}{-S\log_e(1-\alpha) + 4KV} \text{ in feet} \qquad 14(b)$$

A table showing the experimentally determined values of K at 21°C. for use in these equations follows:

Relative humidity, %	K per foot				K per meter			
	2,000~	3,000~	4,000~	6,000~	2,000~	3,000~	4,000~	6,000~
20	0.00065	0.00125	0.00270	0.00480	0.000215	0.000410	0.000885	0.00160
30	0.00055	0.00115	0.00240	0.00420	0.000180	0.000375	0.000785	0.00140
40	0.00045	0.00105	0.00210	0.00365	0.000150	0.000340	0.000690	0.00120
50	0.00040	0.00095	0.00185	0.00315	0.000130	0.000315	0.000605	0.00105
60	0.00035	0.00090	0.00165	0.00275	0.000115	0.000295	0.000540	0.000900
70	0.00035	0.00085	0.00150	0.00240	0.000115	0.000280	0.000490	0.000785

CHARACTERISTICS OF THE EAR

7. Frequency and Intensity Limits. The normal ear recognizes tonal qualities in sounds varying in frequency from 16 to 20,000 cycles and with pressures from 0.0004 to 3,000 bars. These limits are of course approximate and vary from ear to ear. Above this pressure a sensation of feeling sets in. The minimum value of the sound pressure which gives a sensation of tone is called the *threshold of audibility*, the minimum value which will stimulate a sensation of feeling is called the *threshold*

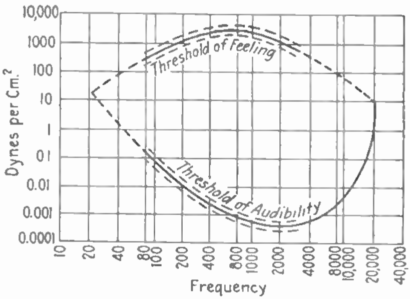


FIG. 2.—Sound pressure applied to a single ear corresponding to thresholds of audibility and feeling at different frequencies as given by H. Fletcher.

of feeling. Both thresholds vary with the frequency. The values of these thresholds as functions of frequency are shown in Fig. 2.

8. Relation between Loudness and Intensity. The loudness of a sound is measured by the psychological effect of the sound on the ear, while the intensity is measured in physical units. Experiments have been performed with pure tones which show that equal additions of physical intensity do not increase the loudness by the same amount at all intensities, but that the *minimum change in intensity which is noticeable is* *roughly proportional to the intensity.* For this reason a logarithmic scale for expressing the physical quantities which affect the ear has been found

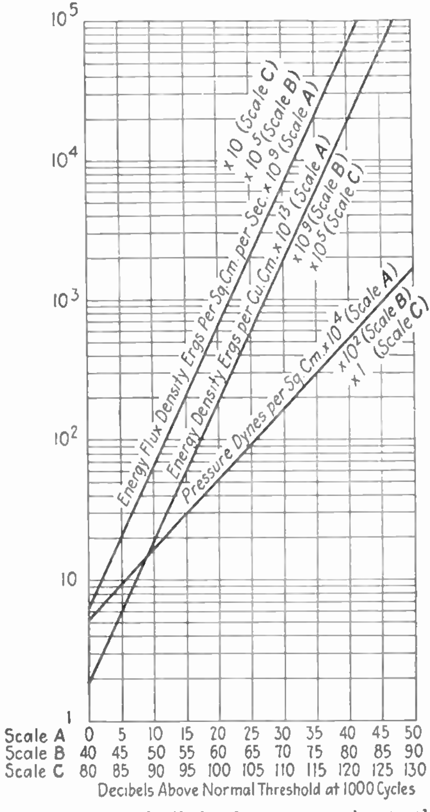


FIG. 3.—Relation between decibels above approximate threshold of audibility at 1,000 cycles, pressure energy density and energy flux density in sound wave.

A zero level of 0.0005 bar has been used in a great many measurements of noise and other aural quantities and is therefore given as reference level for these curves. This zero level corresponds closely to the threshold at 1,000 cycles for direct pressure on a single ear. The threshold for listening in a free progressive sound wave coming direct to the observer is closer to 0.0002 bar. Using this zero level, all decibel readings should be increased 8 db. It is probable that the latter scale will be in more prominent use in the future.

convenient, and the *sensation level* of a sound has been defined as the logarithm of the ratio of the physical intensity of the sound to the intensity of sound at the threshold of audibility. It is usually expressed in decibels. The relations between sensation level at 1,000 cycles, pressure, energy flux, and energy density for a free sound wave are shown in Fig. 3.

9. Relation between Loudness and Frequency. As has been noted above the threshold of audibility varies with the frequency. The physical intensity which will cause the same loudness effect on the ear varies in an analogous manner. The effective loudness of a pure tone of any frequency has been determined by comparing it with a 1,000-cycle tone on a throw-over test. The results of these experiments are shown in Fig. 4. The low tones require a much smaller increase in intensity for an equivalent increase in loudness than do the tones of higher frequencies.

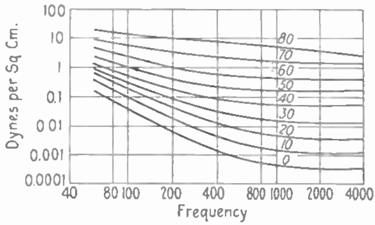


FIG. 4.—Sound pressures corresponding to equal loudness at different frequencies for 10 db steps above threshold of audibility at 1,000 cycles as given by B. A. Kingsbury.

LOUD-SPEAKERS

10. Desirable Characteristics. The loud-speaker should, when used in conjunction with an ideal microphone and transfer system (which may contain electrical lines and a number of stages of recording), reproduce the sound wave which would have reached the ear of the listener if he had been present in the place where the original music or speech was produced. There is a certain amount of argument as to just how much allowance should be made in the psychology of listening for the acoustics of the place where the original should be imagined to take place and the room in which the observer listened to the reproduction. No definite evidence has been brought forward to give a definite decision as to just what weight should be given to these factors, although the preponderance of opinion among broadcasters and those connected with the recording of motion pictures is that the attempt should be made in the reproduction to reproduce as accurately as possible the acoustic characteristics of the place in which the sound was picked up, and the loud-speaker should be designed to fit the location in which it will be placed, so that it will balance whatever acoustic peculiarities there are in the reproducing room.

On this basis, the loud-speaker should give a constant sound pressure output at the position of the listener in the room in which the loud-speaker is to be used for a sound wave having the same pressure at the microphone at all frequencies. Assuming that the remainder of the system does its work properly, the loud-speaker should reproduce constant pressure under the conditions specified above for constant voltage input to the last tube of the audio amplifier.

Loud-speakers can be designed so as to radiate sound in a beam similar to that sent out by a headlight of an automobile or to radiate sound uniformly in all directions. Whether the uniform radiation or the beam radiation is desired will depend on the condition of use. The loud-speaker which is used in the radio set or phonograph at home should not be too sharply directional, as the listeners would normally be scattered over a wide angle in front of the loud-speaker, and those not directly in front will suffer under conditions of directional radiation. Radiation in the form of a hemisphere in front of the radio set is most desirable for this use. When loud-speakers are used in theaters, the reverberation characteristics of the theater are usually injurious to the best intelligibility and various devices have been resorted to, such as cutting off the low frequencies to improve the results. The best effects have been obtained by making the loud-speaker radiation directional and pointing it toward the audience, so that the maximum radiation will strike them directly before reflections from any other surfaces.

The loud-speaker must handle whatever energy it is required to reproduce without distortion. Distortion noticeable in loud-speaker reproduction is probably most often due to overloading in the electrical system but can be due to non-linear effects in the loud-speaker or more often to rattles and buzzes due to parts which are set into vibration when the loud-speaker is subject to violent motion. The buzz due to loud-speaker rattle is so similar to that noticed due to amplifier overloading that the listener should always be careful to make sure that no distortion is taking place in the electrical or recording system before blaming the loud-speaker.

11. Calculation of Loud-speaker Efficiency. At first sight it might seem most useful to define loud-speaker efficiency in the same manner as the efficiency of other generators is defined, *viz.*, in terms of the ratio of the power delivered to the power which is supplied to it. Due to the conditions under which a loud-speaker is used, however, another definition of efficiency which has been called the *absolute efficiency* has been found of more value.

In practice, a loud-speaker is supplied from either a vacuum tube or a transformer attached to a vacuum tube. The impedance of this vacuum tube or the effective impedance of the transformer when placed in the circuit is very nearly a pure resistance independent of the frequency. If the loud-speaker motor has a large reactive component of impedance, or if its impedance varies greatly as the frequency is changed, it will be impossible to supply electrical power to the loud-speaker which is equal to that which could be delivered to a resistance having the same resistance as the supply source (the condition for maximum power transfer from a supply source to an external unit). Even though the loud-speaker might, therefore, have a high efficiency in the usual sense, under the conditions of use, it would not be possible to deliver a large amount of power to it and it would, therefore, from a practical standpoint, not be an efficient loud-speaker.

The definition of loud-speaker efficiency which has been adopted by the Institute of Radio Engineers has been worded so as to allow for the ability of the loud-speaker to absorb energy from the supply source as well as the ability of the loud-speaker to convert that electrical energy into acoustic output.

The general definition which the Institute of Radio Engineers has given for the absolute efficiency of electro-acoustic apparatus when applied to loud-speakers can be interpreted as follows:

The *absolute efficiency* of a loud-speaker for a given circuit condition is the *ratio of the acoustic output of the loud-speaker to the maximum power which can be drawn from the supply source.*

Based on this definition, the absolute efficiency of a loud-speaker not of the relay type is given by the following formula:

$$\text{Eff.} = \frac{\frac{4z_r}{|z^2|}\,|M^2|\,R_s}{\left|\frac{M^2}{z} + Z + R_s\right|^2} \tag{15}$$

where z_r is the mechanical resistance due to acoustic radiation, z is the total mechanical impedance, including reactance due to air reaction and masses and stiffnesses in the drive system; also resistance due to radiation and any other energy losses, M is the vector force factor, *i.e.*, complex quotient of force developed in the mechanical system per unit current in the electrical system, Z is the impedance of the electrical system excluding the impedance due to the motion of the mechanical system which is included in the M^2/z term, R_s is the electrical impedance of the supply source, and the bars indicate absolute values. When using the formula in the form in which it stands, all mechanical quantities must be expressed in c.g.s. absolute units; and electrical quantities in absolute electromagnetic units when the force action is electromagnetic and in absolute electrostatic units when the force action is electrostatic.

12. Sample Calculation of Efficiency of a Dynamic Loud-speaker on a Large Baffle. In the next succeeding paragraphs an illustration showing how formula (15) can be used to calculate the efficiency of a loud-speaker is given. The determination of a number of the quantities which are required for the calculation is discussed in the succeeding sections, and reference will be made to these sections as required.

It is usually not possible to calculate the efficiency of a loud-speaker with mathematical precision, but information may be obtained by making approximations, which allow an analysis to be made of the factors that are important in determining its efficient operation, and which permit the engineer to determine the most economical manner in which he can improve the design.

To simplify the illustration, a dynamic cone loud-speaker will be chosen having the following characteristics:

An 8-in. diameter paper cone, with ½-in. suspension. Mass of cone plus coil 14.7 g. Mechanical system, due to the stiffness of the suspension and the centering means, resonant at 100 cycles. Diameter of the air gap 4 cm. Number of turns in the coil 120. Flux in the gap 9,000 gauss. The transformer feeding the loud-speaker is so designed that it reflects the output tube impedance into the secondary as 11 ohms.

To simplify the calculation, the assumption can be made that the radiation from the front and rear is equal and the same as that from a vibrating disk in an infinite baffle.

Referring to Eq. (15) we must first calculate the mechanical impedance due to radiation z_r and the total impedance z. z_r is obtained directly from curve 8*a* by multiplying the values given on that curve by the area of the disk. The diameter of the disk plus the vibrating part of the suspension is approximately 22 cm, giving an area of 380 sq cm. The values of z_r for a series of frequencies are shown in column 3 of the table (p. 420). The frequencies are given in column 1, and values of d/λ corresponding to each frequency, where d is the diameter of the disk and λ is the wave length of sound at that frequency, are shown in column 2. The total mechanical reactance is made up of a mass component due to the mass of the cone plus drive coil, a stiffness component due to the stiffness of suspension and centering device, and an additional mass component due to reaction of the air, the value of which is obtained by referring to curve 8*b* and multiplying by the area of the disk. The values of the latter at a series of frequencies are shown in column 4, while the component due to the mass of the cone itself, which is equal to ω times the mass, is shown in column 5.

Since the system is resonant at 100 cycles the total mass component must be equal to the total stiffness component at that frequency and the stiffness component therefore equals 13.4×10^3 mechanical ohms at 100 cycles, and has values inversely proportional to the frequency, as shown in column 6 for the other frequencies. The total reactive component of the mechanical impedance, which is obtained by subtracting the total stiffness component from the total mass component, is shown in column 7. It will be noted that any frictional or heat losses in the vibrating system have not been included as they are negligibly small compared to the other quantities. Columns 3 and 7 determine the total vector mechanical impedance.

We next require the force factor M. A discussion of the determination of the force factor for a dynamic loud-speaker is given in paragraph 16 under the discussion of Moving Conductor Motor, and it is equal to the product of the length of conductor in the gap times the magnetic field strength. In the case of the loud-speaker under discussion, this is equal to $\pi \times 4 \times 120 \times 9{,}000$ and is the same at all frequencies. Its value is shown in column 8.

The supply impedance must be expressed in electromagnetic absolute units and is equal to 11×10^9 abohms, as shown in column 9. The electrical impedances of the system, as measured with the mechanical system clamped so that it cannot vibrate, and expressed in abohms are shown in columns 10 and 11; 10 gives the resistive component and 11 the reactive component. The efficiencies calculated by means of formula 15 and the values which have been given in the preceding columns of the table are shown in column 12. Care must be taken in using formula 15 to use absolute values and components in the proper place as indicated by the double bars.

The calculation of the efficiency has been carried to only 1,600 cycles, as the simple assumptions which have been made no longer hold for frequencies above this value. The fact that the vibrating body is a cone rather than a disk affects the radiation at frequencies where the depth of the cone becomes comparable with the wave length. The cone also fails to vibrate as if it were moving all in phase at the higher frequencies so that the assumption of a vibrating piston is no longer valid. The calculation of the efficiency where the more complicated phenomena take place is beyond the scope of this simple example and reference can be made to an article by M. J. O. Strutt[1] for additional information. The effect of the use of a finite baffle has also been excluded as this calculation usually involves a consideration of the cabinet which is used to surround the loud-speaker and must be considered as a separate problem.

The response of the loud-speaker in any direction may also be obtained by means of the efficiency values which have just been calculated and the directional radiation curves shown in Fig. 10. The response of a

[1] *Ann. Physik*, **11**, 129, 1931.

loud-speaker as defined by the Institute of Radio Engineers[1] is expressed in terms of the quantity $\frac{p}{v/\sqrt{R}}$, where p is the resultant sound pressure in the medium expressed in bars, R is a resistance equal to that of the source to which the loud-speaker is designed to be connected expressed in ohms, and v is the voltage supplied to the loud-speaker in series with a resistance R. The calculation of the response as thus defined by means of the efficiency values and the directional curves is made in the following manner:

The absolute efficiency has been defined in the paragraph preceding Eq. (15) of this section. The acoustic output of the loud-speaker expressed in ergs per second may be obtained by integrating the sound-energy flux density over a sphere with the loud-speaker as center. The energy flux density through any small area dS is equal to JdS, where J is the energy flux density through that area. Expressed in terms of solid angle, this is $Jr^2d\Omega$, where r is the radius of the sphere with the loud-speaker as center and $d\Omega$ is the solid angle subtended by the area dS. Referring to Eq. (7), the energy flux density may be expressed in terms of the pressure produced by the loud-speaker. The product of the density of air by the velocity of sound in air in the denominator of this equation is approximately equal to 40, and the energy flux density through the area dS is therefore equal to $\frac{p^2r^2}{40}d\Omega$. The total energy flux is obtained by integrating this over the surface of the sphere. If p_0 equals the sound pressure directly in front of the loud-speaker, the sound pressure at any other point at the same distance from the loud-speaker is equal to $p_0\phi$, where ϕ is the relative pressure, as shown on Fig. 11. The total sound energy flux is thus equal to $\frac{p_0{}^2r^2}{40}\int\phi^2d\Omega$. The maximum power which can be drawn from the supply source is $v^2/4R \times 10^7$ expressed in ergs per second.
The efficiency is therefore

$$\frac{\frac{p_0{}^2r^2}{40}\int\phi^2d\Omega}{v^2/4R\times 10^7} = \frac{p_0{}^2}{v^2/R}\times 10^{-8}\times r^2\int\phi^2d\Omega$$

and

$$\frac{p_0}{v/\sqrt{R}} = \frac{10^4}{r}\sqrt{\frac{\text{efficiency}}{\int\phi^2d\Omega}}$$

An expression is thus given for the response directly in front of the loud-speaker in terms of the efficiency and the integral of ϕ^2 taken over the whole sphere. The values of this integral, as determined from Fig. 10, integrating by quadrature, are given in column 13, and the values of the response directly in front of the loud-speaker, calculated by means of the

[1] See Section 12 on "Acoustic Measurements" in Report of the Committee on Standardization of the I.R.E." sections on Definitions of Electro-acoustic Devices and Tests of Electro-acoustic Devices, 1931.

Calculation of Loud-speaker Efficiency

(1) Frequency	(2) d/λ	(3) z_r	Reactive component of mechanical impedance				(8) M
			(4) Air loading	(5) Mass of cone	(6) Stiffness of suspension	(7) Total	
50	0.032	0.16×10^3	2.0×10^3	4.6×10^3	-26.8×10^3	-20.2×10^3	13.6×10^6
75	0.049	0.35×10^3	3.0×10^3	7.0×10^3	-17.9×10^3	-7.9×10^3	13.6×10^6
100	0.065	0.62×10^3	4.1×10^3	9.3×10^3	-13.4×10^3	0	13.6×10^6
200	0.13	2.5×10^3	8.2×10^3	18.5×10^3	-6.7×10^3	$20. \times 10^3$	13.6×10^6
400	0.26	9.5×10^3	16.4×10^3	37.0×10^3	-3.3×10^3	50.1×10^3	13.6×10^6
800	0.52	26.8×10^3	23.0×10^3	74.0×10^3	-1.6×10^3	95.4×10^3	13.6×10^6
1,600	1.04	32.6×10^3	4.6×10^3	148.0×10^3	-0.8×10^3	$152. \times 10^3$	13.6×10^6

Frequency	(9) R_s abohms	Electrical impedance $X + jY$		(12) Absolute efficiency, per cent	(13) $\int \phi^2 d\Omega$	(14) Response directly in front $\frac{p_0}{v/\sqrt{R}}$
		(10) X abohms	(11) Y abohms			
50	11×10^9	10.7×10^9	1.0×10^9	0.6	12.5	$220 \times 1/r$
75	11×10^9	10.7×10^9	1.5×10^9	4.7	12.5	$610 \times 1/r$
100	11×10^9	10.8×10^9	2.0×10^9	12.7	12.5	$1,010 \times 1/r$
200	11×10^9	11.0×10^9	3.6×10^9	9.9	12.3	$900 \times 1/r$
400	11×10^9	12.0×10^9	7.0×10^9	5.6	11.4	$700 \times 1/r$
800	11×10^9	13.9×10^9	12.6×10^9	2.9	7.8	$610 \times 1/r$
1,600	11×10^9	17.6×10^9	22.0×10^9	0.9	2.5	$600 \times 1/r$

Note: r expressed in centimeters

last equation, are given in column 14. It will be noted that the response is more uniform as a function of frequency than the efficiency due to the fact that the radiation is encompassed in a smaller solid angle as the frequency is increased. The response in any other direction may be determined by reference to Fig. 11 and the values in column 14 of the table.

This discussion illustrates some of the methods which can be employed to evaluate theoretically, with a fair degree of accuracy, the results which can be expected from a loud-speaker. As in the case which was chosen for illustration, it is usually necessary to make simplifying assumptions in order to limit the complexity of the problem and reduce the variables which affect the response and the efficiency to the simplest terms. Even though an exact solution is not obtained, methods similar to the above will be found very useful in efficient design of loud-speakers.

The calculations which are shown above have been checked experimentally in a number of instances, showing that the dynamic cone is not a very efficient loud-speaker. The efficiency of a dynamic driving mechanism may be considerably improved by the use of a directional baffle or a horn and air chamber. By these means the efficiency can be increased to 30 per cent or more.[1]

THE LOUD-SPEAKER MOTOR

13. Determination of Force Factors. Three types of loud-speaker motors have been in common use for obtaining the motion required to produce a sound wave from an electrical wave. The first loud-speakers

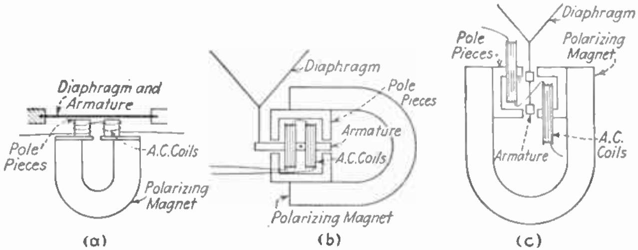


FIG. 5.—Types of magnetic-armature loud-speaker motors: (*a*) Bipolar; (*b*) balanced; (*c*) fringing flux.

which were built were of the *magnetic armature* type. More recently these have practically been superseded by the *moving conductor* drive (principally electrodynamic) and the larger portion of all loud-speakers in use today are of this type. The **condenser loud-speaker** is one of the oldest forms, having been proposed when singing condensers were first noticed and has a number of ardent exponents at this time but has never reached the popular favor that the magnetic types have assumed. Other types of drive, such as *magneto-striction* and *piezo-electric* have been proposed, but have never had wide usage.

[1] WENTE, E. C., and A. L. THURAS: *Bell System Tech. Jour.*, **8**, p. 140, 1928; H. F. OLSON, *Jour. Acoustical Soc. America*, **2**, 485, 1931.

14. Magnetic Armature Motor. Magnetic armature motors are characterized by a ferromagnetic armature or diaphragm in a polarizing magnetic field and some means for superposing an alternating field on the fixed field. The polarizing field is always required if non-linear distortion is to be avoided, since the force due to a magnetic field is proportional to the square of the field strength with the result that the use of an alternating field alone would lead to total absence of fundamental reproduction.

Several types of unit are shown in Fig. 5. Type *a* was quite common in older loud-speakers and in telephone receivers, due to its simplicity and low cost. Type *b* is called a *balanced unit*, since the magnetic circuit is such as to balance the magnetic flux in the armature when in its rest position. This driver has been very popular, as it permits the use of a very light armature and efficient magnetic circuit in which the variable magnetic flux does not have to pass through the higher-reluctance fixed magnetic field circuit. Type *c* is a *fringing flux type* and has the advantage of having the armature motion parallel to the pole piece faces, thus eliminating the possibility of the armature striking the pole faces on high amplitude of motion. It has the disadvantage of being wasteful of magnetic flux.

The force developed by a magnetic field on a portion of a ferromagnetic armature in air in which the magnetism is induced is

$$\frac{H^2}{8\pi}\left(1 - \frac{1}{\mu}\right)dS \tag{16}$$

where H is the component of field strength in the air perpendicular to the armature surface, μ is the permeability of the armature material and dS is the surface element. This formula assumes that the iron is not saturating so that the permeability can be taken as constant throughout the armature. In all practical loud-speakers, μ is so large compared to 1 that $1/\mu$ can be neglected.

In types *a* and *b* the total magnetic flux entering the armature can be considered without much error as parallel to the direction of motion and therefore useful in developing force. In type *c* considerable flux exerts force components on the armature which are not in the direction of motion.

15. Necessity for Polarizing Field. A consideration of the formula given for the force on the armature shows the necessity for polarizing field. Calling the polarizing field strength H_0 and the instantaneous value of the alternating field $H \sin \omega t$, the force on a small armature section moving a negligible distance in the air gap is

$$\frac{1}{8\pi}(H_0^2 + 2H_0H \sin \omega t + H^2 \sin^2 \omega t)dS \tag{17}$$

The first term exerts a steady pull which, it will be seen, tends to attract the armature to the pole piece, the second term leads to a force proportional to the product of fixed field and alternating field, and the third term leads to second harmonic production. To make the second harmonic distortion negligible, H must always be kept small compared to H_0 particularly in units of type *a* where the tendency to balance the second harmonic distortion as in types *b* and *c* is not present. The second term in the above expression is the one which determines the alternating force and therefore the force factor. Since H is proportional to the current through the voice-coil winding, the computation of the relation between H and the current, as explained in the chapter on magnetic circuits, is all that is required for the force-factor determination.

Due to the change in air-gap length as the armature vibrates, with resultant change in reluctance of the magnetic circuit, an additional alternating force in phase with the displacement is set up which for small displacements is proportional to the displacement of the armature from its equilibrium position and in the direction to increase the displacement. This force has the general characteristics of a stiffness but is opposite in sign and is therefore called a negative stiffness. In magnitude it is equal to $\frac{1}{4\pi}H_0{}^2dS$ times the relative change in reluctance. It is subtracted from the mechanical stiffness in determining the mechanical impedance of the unit. Contrary to popular belief, the balanced units types b and c do not reduce this negative stiffness. As a matter of fact, it is twice as large due to the reduction of force on one side while that on the other side is increased.

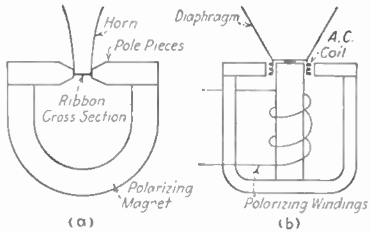


FIG. 6.—Types of moving-conductor loud-speaker motors. (a) Ribbon; (b) dynamic.

16. Moving Conductor Motor. In the moving conductor motor a non-magnetic conductor is placed in a magnetic field whose lines of force are transverse to the direction in which motion is desired. In the dynamic type shown in Fig. 6 the conductor takes the form of a coil of one or more turns of cylindrical shape to which a diaphragm is attached in a ring-shaped air gap. In the ribbon type shown in Fig. 6

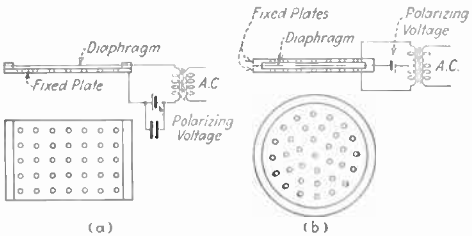


FIG. 7.—Condenser loud-speakers. (a) Unilateral; (b) bilateral.

the conductor is usually a thin strip of aluminum which acts at the same time as diaphragm. Numerous other modifications have been proposed, all of which operate by the force developed in a conductor through which a current is flowing in a transverse field.

The force in dynes developed on the conductor when current flows through it is IlH_0 where I is the current in absolute electromagnetic units, l is the total length of conductor and H_0 is the polarizing field.

The force factor for the moving conductor type of unit is thus lH_0. If the field is not perpendicular to the conductor in the air gap, the component of magnetic field perpendicular to the conductor should be taken for H_0. If the field is non-uniform, the integral of lH_0 over the length of the conductor is taken.

17. Condenser Loud-speaker. In the condenser loud-speaker the force on the diaphragm is developed by direct electrostatic attraction. It has the theoretical advantage of the possibility of driving the diaphragm with a uniform force at all points on the surface. Both unidirectional and balanced electrostatic fields have been proposed and tried on the condenser loud-speaker as is shown diagrammatically in Fig. 7.

The attractive force in dynes per square centimeter of diaphragm pulling the diaphragm toward the fixed electrode is $H^2/8\pi$ with air as dielectric, where H is the electric field strength in electrostatic units. In constructing electrostatic loud-speakers, a polarizing electric field is required similar to the polarizing magnetic field which is used in the magnetic armature type. Without the presence of this polarizing field, no fundamental reproduction is obtained. Calling the strength of the polarizing field H_0 and the variable field which is superposed on this $H \sin \omega t$, the total attractive force per square centimeter becomes

$$\frac{1}{8\pi}(H_0^2 + 2H_0H \sin \omega t + H^2 \sin^2 \omega t) \tag{18}$$

By making the polarizing field strong compared to the alternating field, the second harmonic distortion which is included in the last term can be made negligible compared with the fundamental reproduction due to the second term, which is

$$\frac{H_0H \sin \omega t}{4\pi} \tag{19}$$

The force factor is determined by obtaining the ratio of electric field strength to current through the loud-speaker and then by the use of the last equation determining the ratio of force to current. Since the force on the diaphragm, electric field, and voltage are all in phase and in quadrature with the current, the force factor will, in general, be multiplied by j, and M^2 in Eq. (15) will be negative.

Similarly to the magnetic armature type, the electrostatic loud-speaker has a negative stiffness component due to the tendency of the diaphragm to be attracted more strongly towards the fixed electrode as it approaches it. It is equal to $H_0^2/4\pi$ times the relative change in air-gap length, per square centimeter of surface. In the balanced unit, although the static forces due to the polarizing field are balanced out approximately, the negative stiffness force which arises due to motion away from the equilibrium position is double as large as for the single-sided type, due to the decrease in attractive force on one side corresponding to the increase in attractive force on the other.

18. Mechanical Impedance of Loud-speaker Elements. The mechanical impedance of loud-speakers is due to the masses and stiffnesses in the loud-speaker armatures, coils, connecting links, and diaphragms and the loading due to air. The loading due to the air will be considered below. Assuming that a force is applied to a simple system consisting of a mass attached to a spring and that the mass is in some viscous material, such as oil, which damps its motion, the mechanical impedance due to the mass is $jm\omega$, where m is the mass in grams, ω is 2π times the frequency and j is the square root of -1; that due to the stiffness is equal to $s/j\omega$, where s is the stiffness in dynes per centimeter of displacement. The total impedance of the system is $jm\omega + \frac{s}{j\omega} + z_r$, where z_r is the mechanical resistance. These impedances are all in series since all parts of the system are moving with the same velocity which corresponds to having the same current in electrical circuits. When the same force is applied to a number of mechanical impedances which move with velocities determined by the force, then the same equations hold as if they were impedances in parallel in electrical circuits.

In the common forms of loud-speakers, the impedances of the motor, diaphragm and air loading are usually in series when the frequency of agitation is low enough so that the flexing of the members may be neglected. At the higher frequencies, where the flexing must be taken into account, the relations become more complicated and must be worked out for each individual case.

LOUD-SPEAKER RADIATOR

19. One Single Diaphragm. The simplest type of loud-speaker radiator to consider, and one which is closely approximated in many

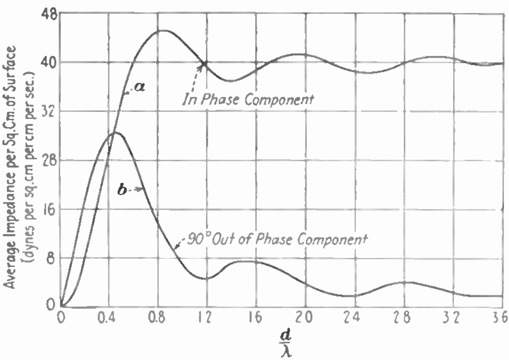


FIG. 8.—Load on a vibrating circular diaphragm set in an infinite baffle.

loud-speakers, is a piston vibrating back and forth in an infinite wall. The sound radiation from a source of this kind has been considered by Lord Rayleigh and completely solved in mathematical terms. At the lower frequencies, where the wave length is comparable with the size of the piston, it is not a very efficient radiator of sound waves. A curve showing the force developed per unit area of a piston per unit velocity (mechanical impedance per unit area) as a function of the ratio of piston size to wave length is given in Fig. 8. The force developed may be divided into two components, one of which is in phase with the velocity, and the other one in quadrature with the velocity and of such sign as to act as if the mass of the piston were increased. At the higher frequencies, the quadrature component becomes negligible compared to the component in phase with the velocity which approaches a value of approximately 41 dynes per square centimeter per centimeter per second of velocity.

20. A System of Diaphragms. By making the diaphragm larger, the efficiency of low-frequency radiation can be increased but other defects arise which make this procedure impractical for most purposes. As the diaphragm is made larger, it becomes necessary to make it thicker to obtain sufficient rigidity. The added mass which is thus introduced makes reproduction of high frequencies difficult. It has, therefore, been found most practical to use a number of small diaphragms placed adjacent to each other when good reproduction of low frequencies is desired. By means of this procedure, each vibrating diaphragm reacts

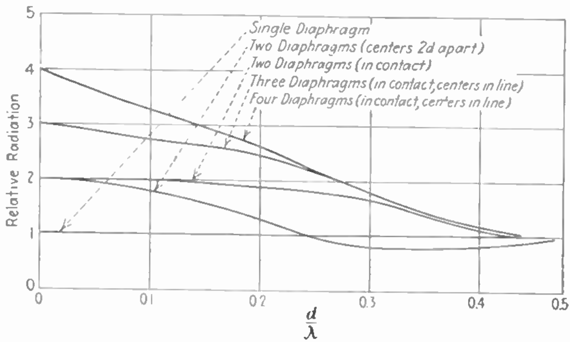


FIG. 9.—Radiation from a system of vibrating diaphragms all vibrating in phase, compared with that from a single diaphragm, as a function of diaphragm size and wave length.

on the others to increase the resistive low-frequency loading with consequent increase in radiation. A curve showing the relative radiation from a system of one, two, three and four diaphragms placed adjacent to each other and in line is shown in Fig. 9. It will be noted that at low enough frequencies the radiation is proportional to the number of diaphragms which are used, while at high frequencies the increase in the number of diaphragms does not improve the efficiency of radiation. For a more detailed consideration of the radiation from a combination of diaphragms reference can be made to the original articles.

21. Horns. A second method which has been widely used for many centuries for obtaining increased low-frequency radiation is the device known as a horn. Up to recent times a conical horn and some type of flaring horn have been employed.

At present, the exponential-type horn is used almost exclusively, that is, one whose cross-sectional area is given by a formula of the type:

$$S = S_1 \epsilon^{bx}$$

where S_1 is the cross-sectional area of the throat, ϵ is the Napierian base and

b is a constant which determines the rate of flare. By means of a horn having a certain length and flare, it is theoretically possible, excluding frictional losses, to load a diaphragm so that a resistance loading of 41 mechanical units per square centimeter can be obtained at any frequency. The formula for the resistive force per square centimeter on a diaphragm for unit velocity (resistive impedance per square centimeter) for an exponential horn of infinite length is given in Eq. (20).

$$z_r = \rho_0 c \sqrt{1 - \frac{b^2c^2}{4\omega^2}} \tag{20}$$

When $bc/2\omega$ is greater than z_r, that is, for frequencies less than $bc/4\pi$, the expression above becomes imaginary and no energy is radiated (the impedance is entirely reactive).

22. Effect of Flare and Length. It will be noted that the flare of the horn, as determined by the constant b, is the factor which determines to how low a frequency the loading on the diaphragm caused by the horn becomes effective. From a practical standpoint, there is a limit to the use of a very small flare, since it is found necessary to make the mouth opening of the horn of the same order of magnitude as the wave length of sound being radiated in order to secure efficient radiation. To obtain the large mouth opening with a very small flare, the horn length becomes excessive and impractical. It is this factor of size which places a practical limit on the low-frequency radiation possible from horns.

When the horn is finite in length, it is necessary to make a correction for the reflection from the open end in order to obtain the radiation characteristic. The formula for the loading per unit area due to a finite horn is as follows:

$$z_1 = \rho_0 c \left[\frac{z_2 \cos(ql - \phi) + j\rho_0 c \sin(ql)}{\rho_0 c \cos(ql + \phi) + z_2 \sin(ql)} \right] \tag{21}$$

where z_1 is the impedance per square centimeter due to the horn, z_2 is the impedance per square centimeter at the mouth of the horn, l is the length of the horn,

$$q \text{ is } \frac{1}{2}\sqrt{\frac{16\pi^2}{\lambda^2} - b^2}$$

b is the flare constant, and

$$\phi = \tan^{-1}\left[\frac{-1}{\sqrt{\frac{16\pi^2}{\lambda^2 b^2} - 1}}\right]$$

In choosing the impedance at the open end, a value sufficiently accurate for most purposes is obtained by assuming that the loading at the open end is the same as for a piston of size equal to the mouth opening of the horn (see Fig. 8).

23. Increasing Loading to Obtain Greater Efficiency. In the preceding paragraph, the assumption has been made that the horn throat opening is the same size as the diaphragm. As has been stated, under these conditions the maximum loading of the diaphragm by the air is approximately 41 dynes per square centimeter per centimeter per second velocity. It is very often desirable to increase this loading to obtain more efficient loud-speaker action. See Eq. (15) for loud-speaker efficiency. This increased loading can be obtained by using a so-called *air chamber* adjacent to the diaphragm and using a horn mouth opening which is smaller than the diaphragm size. Assuming that the chamber is small enough so that the compression of the air can be neglected, the impedance per unit area on the diaphragm for any fixed velocity is increased by the ratio of the area of the diaphragm to the area of the mouth opening of the horn. Further details for the case where the compression of the air in the chamber cannot be neglected can be found in original articles.

24. Directional Characteristics of Loud-speakers. The directional characteristics of loud-speakers are very important in determining their performance. For loud-speakers to be used in small rooms (home entertainment), a non-directional characteristic is to be desired. For

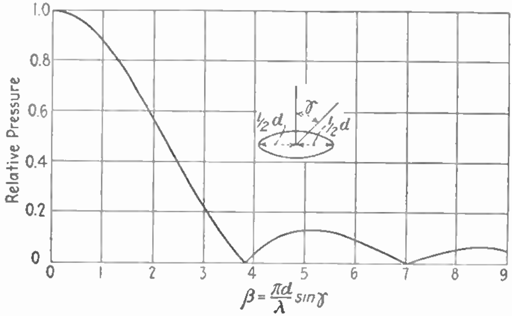


Fig. 10.—Directional radiation characteristics of vibrating disk in infinite baffle.

loud-speakers to be used in auditoriums, a rather sharply defined directional characteristic, such that the loud-speaker sprays sound over the audience and nowhere else, is most desirable for maximum intelligibility. For loud-speakers to be used out of doors, a directional characteristic in which the loud-speaker directs the sound toward the places where it is to be heard and nowhere else is of great importance in order to obtain maximum efficiency.

A non-directional characteristic is obtained from a source of sound (single radiator) which is small compared to the wave length. When the source is a double radiator, such as a loud-speaker on a baffle, a maximum of radiation is obtained directly in front and in back with zero intensity in the plane of the baffle where the sound wave from the front and rear interferes.

The directional radiation characteristic due to a circular disk is shown in Fig. 10. Along the ordinate axis is plotted the ratio of the pressure at any point in space to the pressure at a point the same distance away from the center of the disk but on the axis. Along the axis of abscissas is a function β defined by

$$\beta = \left(\frac{\pi d}{\lambda}\right) \sin \gamma$$

where d is the diameter of the disk, λ is the wave length of sound being radiated, and γ is the angle between the perpendicular to the disk at its center and the direction under consideration.

This directional characteristic will be found useful in making approximate estimates of the directivity of numerous loud-speakers. When the loud-speaker is of the cone type, the diameter of the base of the cone can be taken as the diameter of the disk up to frequencies where the wave length of radiated sound becomes smaller than the base diameter. Above that frequency, the cone shape must be taken into account as well as the fact that most cones no longer vibrate with their whole surface in phase at the high frequencies. Experiment has shown that the directional characteristic is more accurately represented by taking that of a disk with diameter three-fourths to one-half that of the cone base at the higher frequencies.

When the loud-speaker is of the horn or directional baffle type, the mouth opening of the horn can be taken as the diameter of the disk. In the case of these loud-speakers, the directional characteristic as thus determined will usually be somewhat too sharp, particularly at the higher frequencies, due to the fact that the sound intensity over the mouth of the horn is not uniform and the wave which leaves the mouth is more spherical than plane. A somewhat more accurate picture is obtained by taking the value approximately three-fourths the diameter of the mouth opening with further reduced size as the frequency is increased.

ACOUSTIC MEASUREMENTS

25. Loud-speaker Measurements. To determine the performance of a loud-speaker, the following quantities should be known:

1. The frequency response and directional characteristics of the loud-speaker.
2. The power-handling capacity of the loud-speaker.
3. The efficiency of the loud-speaker.

The frequency response and directional characteristics of the loud-speaker are usually measured by actuating the loud-speaker with a simple-harmonic current and measuring the sound output by means of some form of calibrated microphone-amplifier system. The measurements must be carefully made, and attention must be paid to all details so that the results may not be in error or influenced by surroundings which are not connected with the loud-speaker. For details of methods which have been found most suitable for making these measurements, reference should be made to the 1931 report of the Institute of Radio Engineers Standardization Committee in the chapter on "Performance Indexes and Tests on Electro-acoustic Devices" and to the references which are given at the end of this chapter.

The response of the loud-speaker is measured in terms of a quantity $\dfrac{p}{v/\sqrt{R}}$, where p is the resultant sound pressure in the medium (at the specified frequency) at a specified point or the average of the resultant pressure at specified points relative to the loud-speaker, R is a resistance equal to that of the source to which the loud-speaker is designed to be connected, and v is the voltage (at the specified frequency) supplied to the loud-speaker in series with the resistance R. This quantity gives a measure of the pressure developed at the specified point in space by the loud-speaker, taking into account its ability to draw energy from the electric circuit as well as its ability to convert such energy into sound waves.

When the loud-speaker is to be used in the home, a fair approximation to the performance to be expected can be obtained by taking a frequency-response characteristic with the microphone placed almost directly in front of the loud-speaker, and another one with the microphone at an angle 45 deg. to the axis of the loud-speaker. When it is to be used in auditoriums or outdoors, it is important that the directional as well as the frequency response characteristic be obtained, and the response should be measured at a number of positions around the loud-speaker.

26. The determination of the power-handling capacity is somewhat difficult due to the psychological factors involved in determining when the overload point has been reached. The most important factors in determining overload are the production of extraneous tones which were not present in the original input. The annoyance of such tones will be a function of the frequency. It is known that the higher-pitched tones are much more unpleasant and a greater source of annoyance than those of lower pitch having the same physical intensity or even the same loudness.

The most common source of overloading is rattles in the loud-speaker, due to vibration of loose parts, or very often the so-called "oil-can" effect, an unstable impulsive vibration due to excessive amplitudes. It may also happen that the loud-speaker diaphragm motion is not a linear function of the impressed voltage, leading to production of harmonics, sum tones, and difference tones in the sound output similar to the distortion present in an overloaded vacuum tube.

Whether the loud-speaker is able to handle sufficient energy is best determined by a listening test. Precautions must be taken to be sure that the electric wave which is impressed on the loud-speaker is undistorted, as it is very difficult to distinguish between rattles in the loud-speaker and non-linear distortion in a vacuum tube. A variety of musical selections should be used for test purposes. Rattles are particularly likely to show up on impulse tones such as are generated in a piano or on complicated waves like those produced by an orchestra having a large number of instruments. After it has been determined that rattles are present in the loud-speaker, the output from a continuously variable oscillator having sufficient power should be impressed on the voice coil, while a listening test is made to determine at what frequency the buzz is the most prominent. This will usually disclose the reason for the rattle and steps can be taken to attempt to eliminate it.

27. The effective efficiency of a loud-speaker also depends on psychological factors. No two loud-speakers of different design have the same frequency response, and without a definite weighting system to be applied to the individual frequencies an estimate of the relative efficiencies of the two loud-speakers is not possible. In practice, a listening test in which the loudness of the loud-speakers is compared when listened to by an observer in a position relatively the same with respect to the loud-speakers gives a good indication of their relative efficiencies. The louder-speaker should have its output attenuated until the apparent volume of the two is the same. The amount of the attenuation introduced gives a measure of the increased efficiency of the more intense loud-speaker over the less sensitive one.

References

General:

AIGNER: "Unterwasserschalltechnik."
CRANDALL: "Theory of Vibrating Systems and Sound."
GEIGER, H., and KARL SCHEEL: "Handbuch der Physik," (8, Akustik), 1927.
LAMB: "Dynamic Theory of Sound."
RAYLEIGH, J. W. S. LORD: "Theory of Sound."
RICHARDSON, E. G.: "Sound."
STEWART and LINDSAY: "Acoustics."
WAGNER, K. W.: "Die Wissenschaftlichen Grundlagen des Rundfunkempfangs." Chaps. 1–4.

Acoustics of Rooms:

Articles on Sound Transmission and Measurement of Sound-transmission Coefficients, *Jour. Acoustical Soc. America*, **1**, January, 1930.
DAVIS and KAYE: "The Acoustics of Buildings."
EYRING, CARL F.: Reverberation Time in "Dead" Rooms, *Jour. Acoustical Soc. America*, **1**, 217, 1930.
KNUDSEN, V. O.: Acoustics of Music Rooms, *Jour. Acoustical Soc. America*, **2**, 435, 1931.
———"Architectural Acoustics."
SCHUSTER, K. and E. WAETZMANN: Reverberation in an Enclosed Space, *Ann. Physik*, **1**, 5, 671, 1929.
WATSON, F. R.: "Acoustics of Buildings."

Characteristics of the Ear:

Bell System Tech. Jour., **3**, 375, 1925.
FLETCHER, H.: "Speech and Hearing."
KINGSBURY, B. A.: Direct Comparison of the Loudness of Pure Tones, *Phys. Rev.*, **29**, 588, 1927.

Loud-speakers:

MONCH, W.: "Mikrophon und Telephon." Also certain chapters in references Stewart and Lindsay, Crandall, Wagner, Geiger and Scheel.
RICE and KELLOGG: Dynamic Loud-speaker *Jour. A.I.E.E.*, **44**, 982, 1925.
Ribbon Loud-speaker:
GERLACH, E.: *Physik. Zeits.*, **25**: 675, 1924.
SCHOTTKY, W.: *Physik. Zeits.*, **25**: 672, 1924.
Condenser Loud-speakers:
EDELMAN, P. E.: Condenser Loud-speakers with Flexible Electrodes, *Proc. I.R.E.*, **19**, 256, 1931.
GREAVES, V. F., L. W. KRANZ, and W. D. KROZIER: The Kyle Condenser Loud-speaker, *Proc. I.R.E.*, **17**: 1142, 1929.
HANNA, C. R.: Theory of the Electrostatic Loud-speaker, *Jour. Acoustical Soc. America*, **2**, 143, 1930.
Vogt Electrostatic Loud-speaker, *Wireless World*, **24**, 553, 1929.
Loud-speaker Characteristics:
WOLFF, IRVING: Sound Measurements and Loud-speaker Characteristics, *Proc. I.R.E.*, **16**, 1729, 1928.
BOSTWICK, L. G.: *Jour. Acoustical Soc. America*, **2**, 242, 1930.
HANNA, C. R.: *Jour. Acoustical Soc. America*, **2**, 150, 1930.
Horn Loud-speakers:
HANNA and SLEPIAN, *Trans. A.I.E.E.*, **43**, 393, 1924.
Sound Radiation from a System of Vibrating Diaphragms:
WOLFF and MALTER: *Phys. Rev.*, **33**, 1061, 1929.
Directional Characteristics of Loud-speakers:
STENZEL, H.: *Elektrische Nachr. Tech.*, June, 1927, May, 1929.
WOLFF and MALTER: *Jour. Acoustical Soc. America*, **2**, 201, 1930.

Loud-speaker Measurements:

KENNELLY: "Electrical Vibration Instruments."
"Report of the Committee on Standardization of the I.R.E.," sections on Definition of Electro-acoustic Devices and Tests of Electro-acoustic Devices, 1931.
Loud-speaker Testing:
BOSTWICK, L. G.: *Bell System Tech. Jour.*, **8**, 135, 1929.
KELLOGG, E. W.: *Jour. Acoustical Soc. America*, **2**, 157, 1930.
MEYER, ERWIN: Elektrische Nach. Tech., **3**, 290, 1926; **4**, 86, 203, 1927; *Zeits. Tech. Physik.*, **10**, 309, 1929.

OLNEY, BENJAMIN: *Proc. I.R.E.*, **19**, 1113, 1931.
WOLFF and A. RINGEL: *Proc. I.R.E.*, **15**, 363, 1927.

Calibration of Microphones for Loud-speaker Measurements:

BALLANTINE, STUART: Effect of Diffraction around the Microphone on Sound Measurements, *Phys. Rev.*, **32**, 988, 1928; *Proc. I.R.E.*, December, 1928.

———: Theoretical and Experimental Determination of the Correction Due to the Resonance in the Microphone Cavity and Obstacle Effect to be Made in Comparing Free Wave and Pressure Calibrations of a Condenser Microphone, *Contributions from Radio-frequency Laboratories*, No. 18.

BARNES, E. J., and WEST: Calibration and Performance of Rayleigh Disk, *Jour. I.E.E.*, **65**, 871.

OLIVER, D. A.: An Improved Condenser Microphone for Sound Pressure Measurements, *Jour. Sci. Instruments*, **7**: 113, 1930.

SECTION 17

HIGH-FREQUENCY TRANSMISSION AND RECEPTION

By Albert Hoyt Taylor, Ph.D.[1]

1. Historical. High-frequency transmission may be said to have begun with the laboratory work of Heinrich Hertz in 1886. He was dealing with damped waves of the order of 1 to 3 m in wave length produced by spark discharge and detected by similar means, namely, micrometer spark set. Naturally with such crude methods of detection the range of the transmissions was not beyond the limits of the laboratory in Berlin. Nevertheless, the essential nature of the phenomena as an electromagnetic wave motion was demonstrated and the facility with which these very short waves permitted themselves to be reflected, focussed by parabolic mirrors, etc., was clearly proved. In spite of the later introduction of thermocouples in receiving gear, little progress had been made with short-wave transmission and reception outside of laboratory experiments at still a much later date. It was very natural, after the discoveries of Lodge and Marconi as to the action of elevated antennae, that the size of the radiating structure should be increased, which meant a corresponding increase in wave length. This was all the more certain to happen because in the search for means of producing energy at higher levels, the physical dimensions of the circuits had to be increased, which also tended to raise the wave length or lower the frequencies with which experiments were undertaken. So, in the early part of this century, we witnessed the steady progress toward longer and longer waves with higher and higher power levels. It was not until after the advent of vacuum-transmitting tubes of considerable power that it was possible to produce such high frequencies as, say 100,000-kc or 3-m waves, with any great amount of energy and even yet at this date extremely high energies have not been produced on those frequencies.

By the year 1920 there were long-wave stations in existence with frequencies as low as 12 kc and powers running into many hundreds of kilowatts. Nevertheless, it is worthy of note that some of Marconi's experiments in 1899 showed the transmission of intelligible signals through space over a distance of 1¾ miles using a directive system for waves of 1 m in length. In 1916 Marconi and Franklin demonstrated the feasibility of using high-frequency waves between 2 and 15 m in length (150,000 to 20,000 kc) up to distances of about 100 miles. Again here the great utility of reflectors or other devices for concentrating the energy was proven. The same two investigators in 1923 carried out similar experiments for frequencies in the neighborhood of 3,000 kc with a power of 12 kw in the aerial, giving a daylight range of 1,250 miles. In the

[1] Commander, U. S. Naval Reserve; superintendent, Radio Division, Naval Research Laboratory; past president, Institute of Radio Engineers.

meantime other investigators, too numerous to mention individually by name, had entered into the picture in this and other countries. The noteworthy contributions of American amateurs to the field of high-frequency transmission and reception cannot, however, be passed by without notice, because these amateurs brought about results which had been generally believed to be impossible without the use of extraordinary equipment, and yet they did the work with the simple means available to amateur experimenters of very limited financial resources. Naturally, the government agencies (particularly the United States Navy) took an important part in this program and developed the first high power, crystal-controlled set in the high-frequency band in 1924. Since 1923 there has been a steady increase in the practically useable portions of the radio-frequency spectrum. Prior to 1923 there were very few stations making any practical use of frequencies higher than 3,000 kc. Now there are hundreds of stations using frequencies above 3,000 kc, and practical uses are being found for frequencies as high as 100,000 kc and in 1931 successful telephone conversations were carried on across the English Channel in a wave length of 18 cm (1,670,000 kc).

2. Peculiarities of High Frequencies. It may seem strange that so many years elapsed after the introduction of vacuum tubes for transmitters and receivers before much practical use was made of the upper frequencies. One reason for this was the fact that very extensive investigation had been carried out on the attenuations of various radio frequencies in transmissions of considerable distance and it was known to be the general rule that attenuation increased rapidly as the frequency was raised, especially for daylight communication. It was recognized that freak transmissions could occur during the dark hours when abnormally high level signals were received. They were, however, erratic and unreliable. The work of Marconi in 1916, working over distances of the order of 100 miles, had apparently created little impression upon other engineers, probably for the reason that the success of these experiments (as far as they went) was ascribed to the use of concentrating (reflecting) devices at the transmitting and receiving end. Certainly it was not recognized by anyone at that time that frequencies above 3,000 kc showed properties radically different from these at the lower frequencies. Some use of the higher frequencies between 3,000 and 7,000 kc was made during the world war in connection with simple types of spark transmitters of very low power with crude crystal detectors in corresponding receivers. These sets were used either for trench work over very limited distances or for airplane spotting. When Westinghouse commenced their experiments in the 3,000-kc band and when, a little later, amateurs on low power working between 2,700 and 3,000 kc succeeded in spanning the Atlantic to some degree of regularity, engineers everywhere began to realize that the higher frequencies followed an entirely different attenuation law from the lower frequencies.

From 1923, progress was very rapid, but it brought with it some noteworthy perplexities, the most important of these being the fact that signals above 6,000 kc in frequency would frequently dwindle away to an extremely low level a few miles away from the transmitting station, skip a zone of territory, and come in again later on receivers located hundreds of miles distant. Investigation of this phenomenon by Reinartz and other amateurs, and by the Naval Research Laboratory, resulted in the determination of skip distances for many frequencies, for both night-and daytime conditions, and with some regard to seasonal variations which were even then recognized to be of importance. The most striking peculiarity of the high frequencies is their extraordinary carrying power compared with the energy of the transmitter. The second most important peculiarity is the *skip-distance* effect just referred to. The fact that the skip distance was much greater at night than in the daytime at first led to erroneous conclusions that some of the higher frequencies traveled better in the daytime than they did at night; that is, were less attenuated. This, however, is not the case. Skip distance is merely increased during the dark hours. Correlated with this effect is a third and equally important one which is known as the *limiting-frequency effect.* For a given atmospheric condition, which will vary with the time of year and time of day, there is a limiting frequency beyond which reception over long distances is no longer possible.

3. Status of Facts and Theory in 1925. About the year 1925 or 1926, the known facts were about as follows:

Frequencies between 10 *kc and about* 2,300 *kc* followed, during the daytime at least, a very regular law of attenuation especially in transmission over water, with the attenuation a function of the wave length such that the extrapolation of the formula to still higher frequencies indicated that it was hopeless to expect these frequencies to be useful in long-haul communication.

Frequencies above 3,000 *kc* showed a value in long-haul communication (especially in the daytime) which rapidly increased with the frequency instead of decreasing, and which gave observed values of received signal thousands of times greater than that which could be calculated from extrapolation of the long-wave formula.

Frequencies above 6,000 *kc* or thereabouts showed a skip-distance effect which was greater at night than it was in the daytime and greater in the winter than it was in the summer time.

The limiting frequency, or highest frequency, useful for long-haul communication, was distinctly lower at night than it was in the daytime and in general (with certain exceptions which will be noted later on) lower in the winter time than it was in the summer time. It was therefore apparent that: (1) the attenuation theory was all wrong, or (2) it was applicable only to the lower frequencies, or (3) the higher frequencies did not follow the ordinary law of radio communication at all.

This aspect of affairs caused a great many investigators in the field of pure science to become interested in wave-propagation phenomena and led to a development which showed that the second of these possibilities was more nearly correct. As shown by publications of A. Hoyt Taylor and E. O. Hulburt in *Physical Review,* February, 1926, the long-wave theory could be amended in such a way as not to damage its usefulness in the long-wave field and yet could take account in a fairly reasonable way of the properties of the higher frequencies.

4. Theoretical Considerations—Kennelly-Heaviside Layer. Since radio waves are known to be electromagnetic in character and are of the same general properties as the waves of radiant heat and light, there is no reason why they should not travel in a straight line with no more than the customary deviations due to diffraction, reflection, and refraction. One of the earliest problems in radio theory after the success of Marconi's first transoceanic signal in 1901–1902 was to account for the manner in which the waves emanating from Clifden, Ireland, progressed to Glace

Bay, Nova Scotia, where Marconi picked them up. One had three choices here:

1. To assume that the wave penetrated through the earth's crust.
2. To assume that the diffraction effects caused them to bend around the surface and follow the curvature of the globe.
3. To assume that something happened in the upper layers of the earth's atmosphere to refract the waves back to the earth's surface. A ray of energy sent out tangentially from Clifden, for instance, would otherwise pierce the sky in a straight line and be lost as far as further usefulness was concerned.

The first of these possibilities was easily thrown out. The electrical constants of average earth and sea water were well known. The absorption of a wave which would be obliged to travel through the earth's crust would be so enormous that there would be no possibility of getting across the Atlantic. Calculations made on the pure diffraction effect indicated that this also is not sufficient to account for the bending of the waves around the contour of the globe. Kennelly, in this country, had suggested a reflecting medium in the upper layers of the earth's atmosphere, and simultaneously Heaviside, in England, had made the same suggestion with the additional idea that the reflecting medium was ionized, that is, contained positive and negative ions. The theoretical possibilities of such a layer were analyzed by a great number of different investigators and a satisfactory explanation for the return of the rays to the earth was arrived at when it was found that the properties of such a layer could easily be such as to cause an advance in the phase of velocity of that portion of the wave front which extended up into the upper regions of the earth's surface. This caused a change in direction of this advancing wave, which continually bent it back again towards the earth. In case the bending is more than sufficient to equal the earth's curvature, the ray will return to the earth at some point at a distance from the transmitter which will depend upon the frequency. The higher the frequency, the more difficult it will be for the ionized layer to bend the ray, and therefore the farther it will travel before coming down to the earth. After coming down to earth the ray is no doubt reflected again and proceeds upward where it encounters the Kennelly-Heaviside layer at a certain time and is returned to the earth at a point approximately twice as far away from the transmitter as the point of its first return.

Considering any single ray then, we would have possible points of reception at regular distances at recurring intervals, from the transmitter outward, assuming for the purposes of argument a similar condition in the Kennelly-Heaviside layer over a considerable stretch of territory. Actually, however, we do not have points of reception but rather zones of reception, which we familiarly refer to as first, second, third, etc., zones of reception, corresponding to the first, second, third, etc., regions where the rays are returned from the layer. Excepting at the limiting frequency, there is always a cone of rays available for communication purposes. The ray which is most nearly horizontal will strike the Kennelly-Heaviside layer at the flattest angle and will therefore be most certain to be turned down, but it will (from the nature of its path) travel long distances before coming down again. A ray which more nearly approaches the vertical will, if returned at all, come back much closer to the transmitter, but if the frequency is high enough it will have such penetrating power and so little deviation that it will never be returned at all.

For any given frequency then, there is, under ideal conditions, a limiting angle of uptake from the transmitting antenna above which radiation is no longer useful because it penetrates the layer and does not return to the earth. Now, as the frequency is increased and the rays become more penetrating and less easily deviated, they have to strike the layer at flatter and flatter angles in order to have any chance of returning to earth at all, and therefore as the frequency is increased the rays which angle sharply upward are eliminated as far as useful communication is concerned and only the very low-angle rays are of any use to us, until finally, the useful cone of radiation, which is included between the horizontal ray and the ray proceeding upwards to the critical angle with the horizontal, has become a very small angle indeed and with still further increase of frequency this angle vanishes altogether. Thus, for a given condition of the layer there is a critical frequency above which it is not possible to get long-distance communication. Immediately below this critical frequency we get a very narrow cone of rays available, perhaps up to 3 or 4 deg. above the horizontal ray, and naturally when these are turned back down to earth they come back a long distance away from the transmitter, thus showing very large skip-distance effect and after they rebound again from the earth and are turned back a second time there is a second zone of reception, but between the two zones there is a wide region over which reception is not feasible and this region (the existence of which was experimentally verified in the spring and summer of 1926 by the Naval Research Laboratory) has been called the *secondary skip-distance region.* Occasional instances of tertiary skip distances have also been found.

5. Changes in Layer Height. If the effective height or density of the Kennelly-Heaviside layer is altered, the whole picture changes. If the layer is higher or less dense in electrons, the picture will be essentially the same except that all frequencies will be somewhat lower than when dealing with the low layer or a very dense layer. Thus, even at fairly moderate frequencies secondary skip zones may open up at night and, as is well known, critical frequencies are much lower at night in general than they are in the daytime. Obviously the frequencies most suitable for general communication purposes are those for which a fairly wide cone of rays is available, so that the first zone of reception is wide enough to overlap with the second zone and the second with the third zone, etc., in order that there may be no missing regions intervening other than the first skip-distance zone.

It is far more necessary to understand the properties of wave propagation when attacking practical communication problems with the aid of high frequencies than it is even in the case of low frequencies. There are marked seasonal variations, as far as optimum frequencies are concerned, in those circuits which pass through the regions of the earth's surface subject to wide differences in climate from summer to winter. This is particularly true in such circuits as pass close to the polar regions. During summer the polar regions are exposed to long periods of sunlight, and the presumption is that the production of electrons is at a high rate. The equivalent height of the layer is low, and successful communication is best obtained with relatively high frequencies. This has been more or less substantiated by work with polar and Alaskan expeditions. During the long polar night, however, the situation is reversed and the effective height of the layer is very high, making it necessary to use much lower frequencies than can be used in the summer time. On the other hand, the low frequencies thus used in the winter time are not satisfactory also in the summertime because of the high absorption at that period. Aside from what may be called geographical variations and seasonal variations, there is a possibility that there is a connection between general radio conditions, especially in the very high frequency band, and sun-spot activities. If that be true, we may expect a long period of variation in perhaps an eleven-year cycle corresponding somewhat to the rise and fall of sun-spot activities. This idea has some foundation in fact, but observations have not yet been continued long enough to make the matter certain. Theoretically, it seems very plausible that sun spots should certainly be the source of very intense ultra-violet radiation. It is well known that on the upper frequencies, particularly above 12,000 kc the effect of magnetic disturbances is extremely violent.

Magnetic disturbances in general have a very disastrous effect upon east and west communication and a somewhat less disastrous effect upon north and south communication. In general, it seems necessary that the circuit in question must be exposed to the magnetic storm during the daylight hours; otherwise it will not be seriously affected.

6. Multiple Reflections. One curious result of the ability of high-frequency waves to reach a distant point by a series of alternate reflections from the earth's surface and refractions from the Kennelly-Heaviside layer is that a given receiver at a distance may be affected simultaneously by several different waves originating at the same transmitter but arriving over entirely different paths. For instance, in getting across the north Atlantic, one wave at a low angle may make the trip in three hops, touching the surface of the Atlantic only twice on the way over, whereas another one from the same transmitter at a higher angle (provided it does not exceed the critical angle) may make a much larger number of hops and still arrive with sufficient energy. Eckersley, in England, has shown six possible signals from telephone transmitters on the American side which may arrive at the English receiving stations over six different paths of different lengths. A splendid chance for interference results from this condition, and a certain dragging out of the signal even in moderate speed-code work is often noticeable from this cause. In telephony it might be quite disastrous, especially as the Kennelly-Heaviside layer is known not to be at rest but usually in the grip of uneasy movement which can quite rapidly alter the path conditions of the rays and shift their relative phases on arriving at the receiver.

7. Fading. Fading at frequencies high enough to produce audible modulation has frequently been observed. With the aid of highly directive receiving gear and by confining its attention (so to speak) only to a limiting cone of arriving rays, this effect can naturally be somewhat reduced, but it is still at times exceedingly troublesome. Another form of high-frequency interference which has an interesting theoretical bearing and which does not occur on the lower frequencies, is the round-the-world signal; and under certain conditions, depending upon the time of year, time of day, and geographical location of the station in question, a station *A* may communicate with station *B* either by the direct great-circle path or reversed one, although the reversed one may be much longer than the direct path.

8. Theory of High-frequency Wave Propagation. For purposes of reducing the problem to a more elementary form, the refractions in the upper layer of the earth's atmosphere may be theoretically replaced by reflections at a height which may be designated as the equivalent height of the Kennelly-Heaviside layer. Thus the problem may be treated as a reflection problem with a fair degree of accuracy, always keeping in mind, however, that the real process is not an abrupt reflection but a more gradual turning down brought about by refraction. Still, it often simplifies matters to think of things in this way.

Figure 1 shows how matters would be represented on a basis of the reflection theory. The transmitter is at T and the case represented is for an effective layer height of 500 miles. We see that the tangent ray comes back to earth at R_2 and that rays of higher elevation than the tangent ray are turned down closer in, until finally we reach the limiting ray, which is the last one turned down, and it reaches the earth again at R_1. The first zone of reception is therefore a region between R_1 and R_2, the skip distance is the region between T and R_1 (neglecting the short ground-wave range out from T), and the cone of rays actually useful to communication purposes is contained between the tangent ray, (which is ultimately reflected to R_2) and that other ray which is ultimately reflected to R_1. It is interesting to note that even the tangent ray, although

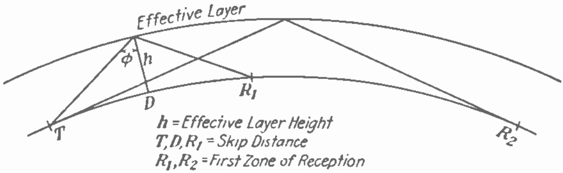


FIG. 1.—Reflection of a wave from ionized layer.

making a zero angle at the transmitter with the earth's surface, cuts the Kennelly-Heaviside layer at an appreciable angle and that even this angle may be too great or may exceed the critical angle if the frequency is sufficiently high. In that case the ray penetrates the layer and is not reflected down from it.

Figure 2 shows the difference in behavior of two radiations starting at W, one at 30 m or 10,000 kc, and the other at 15 m or 20,000 kc. At 30 m the critical angle may be much larger and therefore a relatively large cone of rays is available. This cone of rays first reaches the earth at 500 miles from the transmitter W, this 500 miles being the skip distance. The tangent ray reaches the earth at 2,000 miles for this particular case, where the Kennelly-Heaviside layer equivalent height is assumed to be 150 miles, a winter daytime average. The first zone of reception therefore lies between 500 and 2,000 miles for this frequency. After a second reflection from the layer a new zone is begun at 1,000 miles which extends from 1,000 to 2,500 miles. Again a third zone, after a third reflection from the layer, is begun at 1,500 so that between 0 and 500 miles we have no waves except a feeble ground wave which only goes a few miles out from the transmitter; between 500 and 1,000 miles we have rays which have suffered a single reflection. Between 1,000 and 1,500 miles we have rays of two sorts, some of which have suffered one reflection and some two. Between 1,500 and 2,000 miles we have rays which have suffered one, two, and three reflections. Thus, we see that the different zones of reception

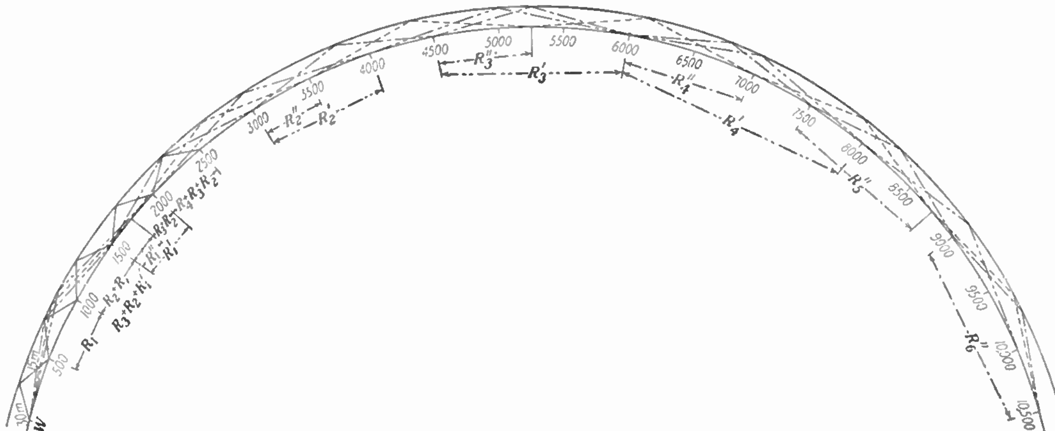


FIG. 2.—Successive reflections of a 15-m and a 30-m wave.

overlap. Also, it is evident that there is ample chance for interference patterns to develop in zones at a moderate distance. At great distances, however, so many zones overlap that likelihood of a complete fade-out due to interference is not so great. Now, if we consider the 15-m or 20,000-kc wave, we see that the first zone of reception due to the narrower cone of rays available (the critical angle being much smaller) will be between 1,500 and 2,000 miles, with a skip distance of 1,500 miles. This region is marked R_1'. Now, after a second reflection this wave comes down again between 3,000 and 4,000 miles giving a second zone of reception marked R_2' but these two zones do not overlap, there being a gap between 2,000 and 3,000 miles. The third zone marked R_3' also shows a gap between it and the second zone of 500 miles, but after the third zone there are no more gaps, the fourth zone marked R_4' meeting the third zone, and all the zones thereafter (not shown in the diagram) overlapping. If now we considered a still higher frequency with a still smaller critical angle we would find a first zone of reception similar to that marked R_1''. The corresponding second zone is marked R_2'', the third zone R_3'', the fourth R_4'', the fifth R_5'', and the sixth R_6''. So that we see, even if the waves have traveled halfway around the earth, there are still missing skip distances of a higher order than the first. They are, however, gradually closing up. It is easy to see that if the frequency is increased further, the zones of reception rapidly diminish to the vanishing point. Of course, this picture does not fit the actual case but is for the ideal case of a perfectly uniform layer distribution together with perfectly uniform reflections from the surface of the earth. It does, however, give us a general guide to what we may expect.

9. Use of Ultra-high Frequencies. The ranges of different high frequencies and their serviceability for different purposes are given in Table

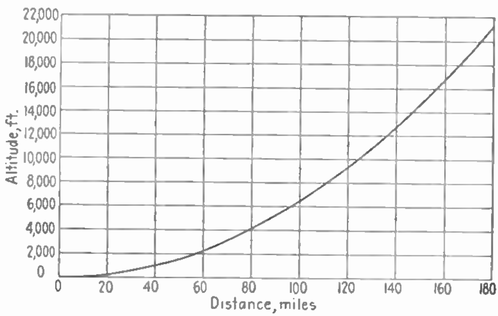


FIG. 3.—Necessary altitude above earth to receive direct ray from ground station.

I, which also gives approximate or average skip-distance effects based on data of various sorts accumulated over a period of years. Frequencies too high to be useful for long-distance communication may nevertheless be extremely valuable for certain other types of work where the points between which communication is to be established are of perhaps some altitude or are close enough together so that the curvature of the earth does not intervene and cut off the direct rays. This type of communica-

Table I.—Skip-distance and Range Table[1]
(For frequencies between 1500 and 30,000 kc.)

Frequency, kilocycles	Approximate wave length, meters	Range of ground wave	Skip distance, Summer, Day	Skip distance, Summer, Night	Skip distance, Winter, Day	Skip distance, Winter, Night	Maximum reliable range, Summer, Day	Maximum reliable range, Summer, Night	Maximum reliable range, Winter, Day	Maximum reliable range, Winter, Night	Services (International Radiotelegraph Convention)	Remarks
1,500– 1,575	200–175	100	...	...	...	...	100	100	150	300	Mobile	Police, television, aviation, etc.
1,715– 2,000	175–150	90	...	...	...	...	120	175	170	600	Mobile—Fixed—Amateur	U. S. Entirely amateur.
2,000– 2,250	150–133	85	...	...	...	...	130	250	200	750	Mobile—Fixed	U. S. 2002 to 2300 Experimental visual broadcast.
2,250– 2,750	133–109	80	...	...	...	...	150	350	220	1,500	Mobile	2398 Experimental.
2,750– 2,850	109–105	70	...	...	...	...	170	500	300	2,500	Fixed	2750 to 2850 Experimental visual broadcast.
2,850– 3,500	105– 85	65	...	...	...	...	200	900	350	3,000	Mobile—Fixed	Aviation, government, etc.
3,500– 4,000	85– 75	60	...	...	...	...	250	1,500	400	4,500	Mobile—Fixed—Amateur	Amateurs and government.
4,000– 5,500	75– 54	55	...	...	...	...	300	4,000	500	7,000	Mobile—Fixed	Point-to-point, etc.
5,500– 5,700	54.0 –52.7	50	...	...	...	...	400	4,000	600	8,000	Mobile.	
5,700– 6,000	52.7 –50.0	50	50	50	50	60	450	5,000	650	8,000	Fixed.	
6,000– 6,150	50.0 –48.8	50	60	70	60	90	500	5,500	700	8,000	Broadcast.	
6,150– 6,675	48.8 –45.0	45	70	115	80	175	550	6,500	750	8,000	Mobile.	
6,675– 7,000	45.0 –42.8	45	80	185	100	290	650	7,000	820	8,000	Fixed.	
7,000– 7,300	42.8 –41.0	45	90	220	115	360	700	7,500	900	8,000	Amateurs.	
7,300– 8,200	41.0 –36.6	40	140	290	175	465	750	8,000	1,100	8,000	Fixed.	
8,200– 8,550	36.6 –35.1	40	160	370	200	570	800	8,000	1,300	8,000	Mobile.	
8,550– 8,900	35.1– 33.7	40	170	420	230	630	900	8,000	1,460	8,000	Mobile—Fixed.	
8,900– 9,500	33.7 –31.6	40	200	485	270	710	950	8,000	1,680	8,000	Fixed.	
9,500– 9,600	31.6 –31.2	40	220	530	280	740	1,000	8,000	1,820	8,000	Broadcast.	
9,600–11,000	31.2 –27.3	35	260	625	325	860	1,100	8,000	2,140	8,000	Fixed.	
11,000–11,400	27.3 –26.3	35	300	750	380	1,000	1,200	8,000	2,460	8,000	Mobile.	
11,400–11,700	26.3 –25.6	35	315	800	400	1,080	1,300	8,000	2,700		Broadcast.	
11,700–11,900	25.6 –25.2	35	335	835	420	1,120	1,500	8,000	2,800		Fixed.	
11,900–12,300	25.2 –24.4	30	350	870	430	1,170	1,550	8,000	3,000		Fixed.	

Notes

Mobile: Ships and coastal stations, aircraft, railroad stock, etc.

Fixed: Permanent stations handling point-to-point traffic.

Skip Distance: Shortest distance beyond the ground wave at which communication is possible, or the point where the sky wave first comes to earth. On certain frequencies and at certain seasons communication is possible within the skip distance due to echoes and around-the-world signals.

TABLE I.—SKIP-DISTANCE AND RANGE TABLE.[1]—(*Continued*)

Frequency, kilocycles	Approximate wave length, meters	Range of ground wave	Skip distance				Maximum reliable range				Services (International Radiotelegraph Convention)	Remarks
			Summer		Winter		Summer		Winter			
			Day	Night	Day	Night	Day	Night	Day	Night		
12,300–12,825	24.4 –23.4	30	370	940	460	1,240	1,600	8,000	3,200	Not useful	Mobile.	The table was obtained from the general average of a large number of observations. For the night ranges given it is assumed that the greater part of the path between the transmitting and receiving stations is in darkness. As the distances given in this table are general averages many discrepancies may be found in practice due to seasonal changes, sun-spot activities, geographical location, local weather conditions, etc.
12,825–13,350	23.4 –22.4	30	390	1,000	485	Not reflected	1,700	8,000	3,440		Mobile—Fixed.	
13,350–14,000	22.4 –21.4	30	420	1,075	510		1,800	Not useful	3,660		Fixed.	
14,000–14,400	21.4 –20.8	30	440	1,150	545		1,950		4,060		Amateurs.	
14,400–15,100	20.80–19.85	30	460	1,230	580		2,200		4,360		Fixed.	
15,100–15,350	19.85–19.55	30	475	1,300	610		2,300		4,640		Broadcast.	
15,350–16,400	19.55–18.30	30	500	1,370	640		2,500		5,060		Fixed.	
16,400–17,100	18.30–17.50	25	550	Not reflected	700		3,000		5,600		Mobile.	
17,100–17,750	17.50–16.90	25	580		740		3,500		6,200		Mobile—Fixed.	
17,750–17,800	16.90–16.85	25	600		755		4,000		6,450		Broadcast.	
17,800–21,450	16.85–14.00	20	660		835		5,000		7,000		Fixed.	
21,450–21,550	14.00–13.90	20	750		1,050		6,000		7,000		Broadcast.	
21,550–22,300	13.90–13.45	20	780		1,090		7,000		7,000		Mobile.	
22,300–23,000	13.45–13.10	20	835		1,130		7,000		7,000		Mobile—Fixed.	
23,000–28,000	13.10–10.70	15	900		1,200		Unknown		Unknown		Not reserved.	Good only for few hours during daylight.
28,000–30,000	10.70–10.00	10	1,000		1,400		Unknown		Unknown		Amateurs.	

[1] Prepared by L. C. Young, Naval Research Laboratory.

tion would be called communication by the direct ray. It has some marked advantages and naturally some disadvantages also. Figure 3 shows the altitude at which an airplane would have to fly in order to get direct-ray communication with a ground station. Especially this figure shows that a plane at an altitude of 2,000 ft. could expect direct-ray communication up to about 60 miles, and at 10,000 ft. up to about 125 miles, the range increasing to 175 miles at 20,000-ft. altitude.

Among the advantages of these very high frequencies, say from 40 megacycles up, may be mentioned the following:

1. They can be confined into concentrated beams like a searchlight, since the necessary antenna or radiating structures, being commensurate with the wave length, are relatively small and therefore not too costly. These radiating structures are of many different forms, but they all depend more or less on what might be called, roughly, the diffraction-grating theory. In other words, they depend upon simultaneous in-phase radiation from a large number of properly spaced conductors. Such beams on various frequencies, from 3,000 to 75,000 kc, are finding much use for both telegraphy and telephony. Usually a network similar to the radiating network is placed one-fourth wave length behind it, that network acting as a tuned reflector, thus more or less effectively cutting out the radiation to the rear of the beam in question.

2. Waves are possible in the upper-frequency bands where no ground waves exist on account of high absorption and no sky waves on account of critical angle. The direct ray is free from fading, which is a very great advantage indeed.

3. These upper frequencies are practically free from atmospheric disturbances.

4. Where it is desired to modulate the carrier wave with a very wide band of frequencies, in television for instance, the ultra-high frequencies lend themselves very well to the solution of the problem, since the percentage variation from the carrier is, even with 200- or 300-kc modulation frequency, not very great.

The disadvantages of these upper frequencies are as follows:

1. It is difficult to build transmitters of high power and receivers with a high degree of sensitivity due to limitations within the vacuum tubes used for these purposes. These difficulties will be rapidly overcome. Certainly much progress has been made even in the last two years along these lines, and frequencies as high as 1,500 mc have actually been produced in the laboratory, although to date not in any great amount of energy. Suitable receivers for such frequencies are as yet undeveloped. Practically satisfactory transmitters are available, however, up to 100 mc (perhaps higher), and fairly satisfactory receivers up to about the same point.

2. Man-made radio disturbances, particularly from ignition systems on automobiles and airplane engines, as well as induction from transmission lines, telephone lines, X-ray machines, trolleys, etc., are very annoying on these upper frequencies. They do not, however, up to at least 100,000 kc constitute an insuperable barrier to progress.

3. These higher frequencies are very easily reflected from buildings and other objects near or on the path of transmission, resulting sometimes in complicating interference patterns or standing waves (shadows). Experiments, particularly with television in our larger cities, are showing peculiarities of this nature.

10. Band Width Required. For any frequency to be useful for practical communication purposes, the agency operating it must be allowed a certain channel or band within which to operate. The width of this channel depends upon the type of service in question. Television, for instance, requires a very wide band; high-grade program broadcasting, a fairly wide band; satisfactory speech, a somewhat narrower band; while high-speed telegraphy is notably narrower than speech telephony, and low-speed telegraphy a very narrow band indeed. Facsimile transmission requires a band, depending upon the speed of transmission, which generally lies somewhere between high-speed telegraphy and telephony. In addition to the band of frequencies which the station must actually transmit to accomplish its mission, a certain tolerance for the frequency stability must be allowed. This is an extremely important matter. The closer a group of stations can stay on their assigned frequencies the more likely they are to operate without mutual interference. It has been proved possible to operate a station day after day within a few parts in a million of its assigned frequency, but it is naturally difficult for many stations in the word, on account of financial and patent limitations, to come up to the same standard. Moreover, the same standard cannot very well be required of an airplane as would be required for a first-class fixed station where initial costs are not so important.

As the art stands today (1932), a station which holds its frequency to better than one part in 10,000 of its assigned frequency is doing very well indeed. One which does not hold its frequency to better than 10 parts in 10,000 is doing very poor work. There is every indication that the tolerance permitted on frequency stability will be rapidly tightened up by international agreement. In addition, then, to the actual band of frequencies necessary for service, we must add a small tolerance band and then a small guard band to be sure that the neighboring assignments do not result in interference. These, then, are the basic principles underlying allocations of frequency by practically all civilized nations.

11. Attainment of Frequency Stability. The wonderful development in piezo-electrically controlled circuits has set a standard for frequency stability of a very high order and is still indicating further possibilities of refinements. It is not to be overlooked, however, that there are many promising self-oscillating circuits of a very fair degree of stability, which, when handled with exactly the proper precaution, are capable of giving a very high order of stability.

12. Allocation of Frequencies. The national aspect of frequency allocation in this country is handled by the Federal Radio Commission. Internationally it is handled by such conferences as the Washington Conference of 1927 and the Madrid conference held late in 1932, assisted by the work of the International Technical Consulting Committee (C.C.I.R.) which has had one meeting at the Hague and another meeting at Copenhagen.

The present international allocation of frequencies for various services is herewith appended as Table II. No doubt some slight rearrangements of this table will be made in the near future. At the present time, it being not considered safe to allocate frequencies generally closer together than one-tenth of 1 per cent, it can be seen that in spite of the tremendous spread of frequencies opened up for public use by the exploitation of the high bands of frequencies, there are after all only a limited number of frequencies for the use of the air. Fortunately many of these frequencies can be duplicated in different parts of the world for simultaneous operation, if due account is taken of the time of day and geographical location of the stations. There is no harm for instance, in operating a station in the 6,000-kc band in this country on a frequency used in Europe provided that operation is not carried on later in the day than within an hour or two of the time when total darkness covers the path between here and Europe. During the dark hours, of course, interference will result, but during the hours of full daylight this frequency will not cross the Atlantic with sufficient intensity to cause interference. The super-frequencies, or limited-range frequencies, of course, can even be duplicated within our own country from city to city. Such duplications as these, however, have generally to be worked out through regional agreements such as we have with Canada at the present time, as Canada is close enough so that interference might result if the frequencies were not properly divided between the two countries. Any scheme that will permit a greater exploitation of the given channels or bands assigned to a station is worth developing. A good many such schemes have been tried out, and some of them have had a considerable degree of success. Most of them depend either upon the operation of a number of frequencies within a very narrow band all very accurately held in place, or on the multiple modulation at audio frequencies of a single carrier.

TABLE II

Frequency band, kilocycles	Services as given in Washington General Regulations, Art. 5	Frequency band, kilocycles	Services as given in Washington General Regulations, Art. 5
550 to 1,300	Broadcasting	9,600 to 11,000	Fixed
1,300 to 1,500	*a.* Broadcasting *b.* Maritime mobile, waves of 1,365 kc exclusively	11,000 to 11,400	Mobile
1,500 to 1,715	Mobile	11,400 to 11,700	Fixed
1,715 to 2,000	Mobile, fixed, and amateurs	11,700 to 11,900	Broadcasting
2,000 to 2,250	Mobile and fixed	11,900 to 12,300	Fixed
2,250 to 2,750	Mobile	12,300 to 12,825	Mobile
2,750 to 2,850	Fixed	12,825 to 13,350	Mobile and fixed
2,850 to 3,500	Mobile and fixed	13,350 to 14,000	Fixed
3,500 to 4,000	Mobile, fixed, and amateurs	14,000 to 14,400	Amateur
4,000 to 5,500	Mobile and fixed	14,400 to 15,100	Fixed
5,500 to 5,700	Mobile	15,100 to 15,350	Broadcasting
5,700 to 6,000	Fixed	15,350 to 16,400	Fixed
6,000 to 6,150	Broadcasting	16,400 to 17,100	Mobile
6,150 to 6,675	Mobile	17,100 to 17,750	Mobile and fixed
6,675 to 7,000	Fixed	17,750 to 17,800	Broadcasting
7,000 to 7,300	Amateurs	17,800 to 21,450	Fixed
7,300 to 8,200	Fixed	21,450 to 21,550	Broadcasting
8,200 to 8,550	Mobile	21,550 to 22,300	Mobile
8,550 to 8,900	Mobile and fixed	22,300 to 23,000	Mobile and fixed
8,900 to 9,500	Fixed	23,000 to 28,000	Not reserved
9,500 to 9,600	Broadcasting	28,000 to 30,000	Amateurs and experimental
		30,000 to 56,000	Not reserved
		56,000 to 60,000	Amateurs and experimental
		Above 60,000	Not reserved

TECHNICAL ITEMS PECULIAR TO HIGH-FREQUENCY DEVELOPMENTS

13. Transmitters. In the case of transmitters for telegraphic purposes, especially continuous wave telegraphy involving beat-tone reception, it is necessary for the transmitter frequency to have a very high degree of stability that there shall not be too much variation in tones of received signal. If, for instance, beat-tone reception is employed on a frequency of 20,000 kc and this tone is not allowed to vary more than 100 cycles, it will require that the transmitters be held at a frequency constant to within one part in 200,000. It is, of course, extraordinarily difficult to do this without the use of master-oscillator circuits which are followed by suitable amplifiers to bring the power up to the required level. In case piezo-electric control is used, it is highly desirable to keep the controlling crystal at a constant temperature, generally chosen at 50°C., and since it is not economical to grind crystals accurately for frequencies higher than 6,000 kc it is necessary to follow the crystal master with amplifiers which act as frequency multipliers.

It is, for instance, common practice to design a 1-kw set with a tube line-up somewhat as follows:

1. Master oscillator 7½ watt, worked at low power in order not to overheat the crystal.
2. Buffer stage on same frequency as master, also 7½-watt screen grid, worked at approximately full power.
3. 75-watt screen-grid stage which also acts as a multiplier, doubling or trebling the frequency.
4. A pair of 75-watt screen-grid tubes which may single, double, or treble the frequency as the case may be, and which in turn feed into:
5. A pair of 500-watt screen-grid tubes which always act as amplifiers without frequency multiplication in the best type of design.

It is always desirable to avoid frequency multiplication in the last stage in order to prevent emission on sub-harmonics and to reduce as far as possible harmonic emission, since the singling operation can be carried out without the use of the excessive negative C which is necessary for frequency multiplication. The tube is thus worked more nearly on the linear portion of the characteristic, giving less harmonic development. Unless such transmitters have their individual stages carefully shielded, the stages prior to the final amplifier are extremely liable to have sufficient coupling to the antenna through the set to give off strong sub-harmonies which may cause objectionable interference. In fact, in spite of the most careful shielding a certain amount of this emission always does seem to take place so that it is sometimes necessary to add a tunable tank circuit or some kind of filtering system between the last stage and the radiating structure. Moreover, to reduce the negative bias in the last stage too far would result in an abnormally low efficiency of the final power amplifier, so that a compromise has to be effected between efficiency and the tendency to produce harmonics. In some cases it is better to work the tubes with a somewhat higher negative bias and at higher efficiency and add the tank circuit.

14. Telephone Transmitters. In the case of transmitters intended for telephony the situation is far more complicated. It is not possible to work at very high efficiency. Otherwise distortion will result. Here the same general rules apply as for any high-grade broadcast transmitter, except that it is extremely inadvisable to modulate in the earlier stages subject to frequency multiplication because here we will certainly have distortion occurring. While the necessity for frequency control would not appear to be so great for telephonic communication, yet in reality it is of the same order of magnitude, since it has been found that wobbling in frequency gives rise to abnormal fading effects which have a very disastrous effect in distorting the received signal. Where the master oscillator is not piezo-electrically controlled or controlled by some mechanical or magneto-striction oscillator of similar precision, a self-oscillating master circuit may be used provided suitable precautions are complied with.[1] Circuits must be chosen which are inherently stable and which show the smallest variation in frequency with changes in filament and plate voltage. Such master oscillators must have their supply voltages for filament, plate, and negative *C* (if such be used) very carefully filtered to avoid frequency modulation by such effects. Moreover, special precautions must be taken to hold all these voltages very constant. However, when these things are rigidly complied with it is possible to get excellent results almost comparable with those obtained from piezo-electric control. Such circuits, however, generally require much more careful attention and more supervision and checking than piezo-electrically controlled master circuits.

15. Peculiarities of High-frequency Transmitters. For very high frequencies, operation of amplifiers in the push-pull arrangement is very effective. This is indeed absolutely imperative at the present time for the last stage of a high-power transmitter since it is necessary to have this amplifier work on the same frequency as the preceding stage. It must be balanced or neutralized against the preceding stages. Such balance or neutralization at very high frequencies is extremely unsatisfactory unless a symmetrical push-pull arrangement is used. Automatically neutralized (screen-grid) tubes are now commercially available up to 500 watts in this country, and some experimental screen-grid tubes in the water-cooled class have been made but are not yet commercially available.

Another peculiarity of h-f transmitters is the fact that it is much easier to design such transmitters with high-impedance tubes which have relatively low grid capacity than it is to design them for low voltage, low impedance tubes with high inter-element capacity. It is easier to get excitation and efficiency with a high-impedance tube and is much easier to carry out balancing operations. Such tube layouts as are commonly used, for instance in many broadcast transmitters, would be almost impossible to work with at very high frequencies.

In the case of transmitters having to cover a wide range of frequencies another difficulty is encountered; namely, dead-end effects in variable-coil systems. These must be carefully taken care of by short-circuiting certain turns or cutting them out. These dead-end effects may even be troublesome when the coil in question is disconnected entirely from the transmitter but is in the immediate vicinity of the particular coil that happens to be used for the transmission at that time.

Another peculiarity of the h-f transmitter is the fact that it is required to work into radiating systems of very unusual impedances. In the case of transmitters working on one or two fixed frequencies and into a transmission line, this difficulty is not so important since a transmission line of fixed characteristic impedance of, say, 600 to 800 ohms, can be generally adopted and adhered to, but in cases where they work more directly into an antenna (especially if that antenna be fixed for a wide range of frequencies), the impedance of the radiating system may vary from a few ohms up to several thousand. Therefore, it is often necessary to provide for an extremely wide range in coupling between the last stage of the transmitter and its radiating system. This does not hold where the transmitter is designed for one frequency and operating into a stand-

[1] Gunn, Ross, A New Frequency-stabilized Oscillator System, *Proc. I.R.E.*, September, 1930, p. 1560.

ard transmission line, feeding a radiating structure. For frequencies above 2,000 kc it is not only unnecessary to use Litzendraht for coils but is a disadvantage, hence the almost universal use of either solid conductor or water-cooled hollow conductor for coil windings. It is also necessary to avoid many forms of insulation that are perfectly adequate at lower frequencies, such as those used in the broadcast band. Many insulators which give excellent service at low frequencies show rapid development of heat and breakdowns within a few seconds when used in high-power h-f transmitters. Fortunately there have been developed within the last few years a number of types of insulators, some of which seem to be suitable even for frequencies as high as 100,000 kc and perhaps higher. Among them are Pyrex, Isolantite, Mycalex, Victron, and Silimanite.

The perfect insulator for these frequencies, namely, an insulator which has the requisite electric properties combined with strength and machineability, has not yet been discovered.

16. Receivers. Many h-f receivers are more or less of the same general type as those of low frequencies. They may have one or two stages of screen-grid amplification followed by a detector which can regenerate or oscillate, as needs be, and then by suitable audio amplification. The difficulty with this type of receiver is that the gain in r-f amplification per tube is very small as the frequency gets very high, until in many cases the gain is actually negative. The use of new insulating bases for tubes and better tube design is gradually extending the frequency range of such receivers. In the case of a receiver for continuous wave reception it is necessary to have a beat oscillator which may be a separate heterodyne or the detector itself may oscillate. The push-pull arrangement for both r-f amplification and oscillating detection has here some marked advantages. The amplification per stage will hold up better at higher frequencies with push pull than without it, and the detector will oscillate at much higher frequencies in the push-pull arrangement than in the single-tube arrangement.

Receivers for very high frequencies often have considerable trouble from tube noises and differ radically from ordinary receivers in one particular, namely, susceptibility to microphonic disturbances. The receiver box must have absolutely no loose contacts between bits of metal anywhere, and the tubes themselves must be non-microphonic if possible. Many a receiver which is sufficiently sensitive for its purpose fails utterly because of its microphonic properties. This is particularly true for receivers designed for shipboard work and still more so for aircraft receivers. It is also extremely good practice to put r-f filters in all supply leads, telephone cords, etc., to such receivers to keep signals from coming in by the wrong channel. When such receivers are provided with automatic volume control, they have a marked advantage in the presence of fading signals. Fading is, of course, one of the greatest drawbacks to h-f work.

17. High-frequency Superheterodynes. Another well-known type of receiver is the superheterodyne. It differs from the ordinary superheterodyne familiar to the broadcast listener in two respects: First, transfer frequency is usually much higher than that of the broadcast receiver. It may be anywhere from 100 to 1,500 kc depending on the frequency range to be covered. Second, the h-f superheterodyne seldom has high sensitivity, unless the first or h-f detector tube is regenerative.

One very sensitive type of receiver which is used to a considerable extent at the present time is a combination of these first two types. It uses two or three stages of r-f amplification preceding the first detector which is usually made regenerative. The rest of the receiver is of the superheterodyne type. The use of ganged controls is possible at fairly high frequencies but naturally more difficult. It is highly desirable, especially in aviation work where simplicity of operation is essential. For extremely high frequencies, say above 60,000 kc, the first type of receiver, namely, r-f, detector, and audio, is of very little use and the superheterodyne is far less effective than at somewhat lower frequencies. The fact that it is impossible to get adequate r-f amplification ahead of the detector is no doubt the reason for the ineffectiveness of both receivers. Nevertheless, with specially constructed tubes some progress is being made in this field with superheterodynes. The most sensitive receiver for these very high frequencies would seem to be the super-regenerative receiver, but it has the great drawback that it is not relatively as effective for continuous wave signals as for modulated signals. For modulated signals, however, it is extremely sensitive but unfortunately not any too selective. It is, however, largely used for work in these very high bands. For still much higher frequencies, that is, well above 100,000 kc, there is no known receiver that has at the present time any great amount of sensitivity. In fact, for experimental work we see the investigators turning back to the old crystal detector followed by audio amplification.

18. Magnetron Oscillator. On the whole it is clear that the transmitting art is (barring the fact that much still has to be done on the suppression of harmonics and sub-harmonics), very much in advance of the receiver art. By the use of magnetron tubes, frequencies have been produced corresponding to waves only a few centimeters in length. When we remember that 10 cm correspond to a frequency of three million (3,000,000) kc it can be appreciated that there is a tremendous range of frequencies available for exploration and exploitation. Of course, the magnetron oscillations are not very accurately controlled in frequency, but there is so much room in this portion of the spectrum, that if suitable receivers can be developed for these upper ranges, these frequencies will no doubt come into useful service to mankind, especially for limited-range communication.

19. Television and High Frequencies. It may be well to say a few words about the relation of h-f transmission to television. On account of the fact that television transmission requires the transmission of a very wide band of frequencies for clarity of results, the high and super-frequencies seem to be peculiarly fitted for this work. To put television in the lower band would wastefully consume for a single channel a large number of frequencies that could be adequately used for long-haul communication, but there is plenty of room for local work in the very high bands. For television reception the receiver has of course a special amplification system following the detector, which is capable of going to very high frequencies in order to speed up the response. It would seem, however, that in anything but the very high bands fading at both audible and sub-audible frequencies would seriously distort the pictures. Indeed, this is known to be the case. Just how far automatic volume control and diversity reception may go towards remedying the situation is still a matter of conjecture, but there seems little doubt that very interesting local work will before long be accomplished in the super-frequency band where the principal difficulty will be interference patterns, or shadows produced by local reflections from neighboring buildings and other structures.

20. Super-short-wave Oscillators. Within the last few years interest has been revived in the so-called Barkhausen-Kurz and Gill-Morrell oscillations. These oscillations are of very high frequencies corresponding to wave lengths often materially less than one m. They are characterized by the fact that the oscillation frequency is not uniquely determined by the electrical constants of tube and circuit, but is quite largely dependent upon plate and filament voltage, particularly plate voltage. They are also characterized by the fact that when the vacuum tube is used for the production of such oscillations, the grid is used with a high positive voltage, whereas the plate is either at ground potential or somewhat negative.

The periodicity of these oscillations is directly associated with the time required by the electrons to move from filament to grid. In other words, we are dealing with such very high frequencies and such short time intervals that the actual speed of motion of electrons within the tube is largely a determining factor in deciding the frequency of the oscillation. The oscillation will be self-sustained if the circuit losses are low enough. Some of these electrons will work through the grid and travel on toward the plate, but will be decelerated after passing the grid and will return toward the grid, especially if the plate voltage is negative. During this oscillation charges are produced on the grid or the plate with resultant development of the power at this very high frequency. If the phase relations are correct and this power is sufficient to overcome the circuit losses, this action will sustain itself. Naturally, an oscillation of this type is not susceptible to very accurate frequency control or capable of maintaining an extremely pure wave form. Furthermore, the efficiency of a tube thus used is naturally not high, nor can it be heavily loaded, otherwise the grid will be destroyed. However, in spite of the limited power thus available some very practical and interesting uses have been made of these oscillations.

Antennas of such small dimensions as corresponding to these frequencies can readily be provided with large parabolic metallic reflectors and a very high degree of directivity can be obtained in the resulting beam.

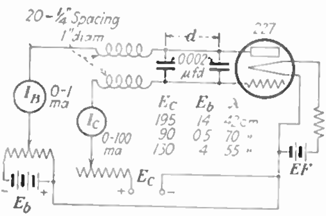


FIG. 4.—Barkhausen oscillator.

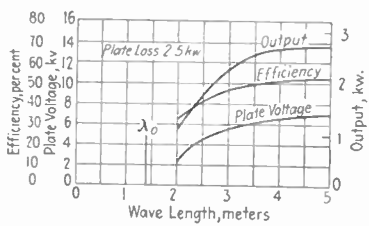


FIG. 5.—Characteristics of tube for very short waves RCA-846.

21. Immediate Problems of High-frequency Work. The following are the most urgent problems to be overcome in advancing the development of the upper frequencies:

1. Reduction of harmonics and sub-harmonics in transmitters.
2. Further precision of frequency control. (NOTE. This is well under way.)
3. The development of receivers of higher-frequency range.
4. Further development of diversity reception for reduction of fading.

With the solution of some of these problems the way will open for adding a vast number of new frequencies to the radio spectrum.

The recent experiments across the English Channel on wave lengths of approximately 18 cm, which are reported to have given duplex telephonic communication, were carried out by the aid of concentration by reflectors in both receiver and transmitter. The receiver circuits are quite similar to the transmitter circuits, except that the super-regenerative principle may be employed for extra sensitivity. Naturally, radiation of this kind must be handled as a searchlight is handled. There is very little bending around structures and very little tendency to follow the curvature of the earth.

For certain special cases of limited-range communication these applications may hope to be quite successful, since the very low power in the oscillations is apparently more than offset, by virtue of the fact that they may be concentrated in a very narrow pencil of radiation. On account of the difficulty of exact control of frequency, most of the work so far done has dealt with modulated waves.

SECTION 18

CODE TRANSMISSION AND RECEPTION

BY JOHN B. MOORE, B.S.[1]

1. Radio communication, as distinguished from radio broadcasting of educational and entertainment programs, is carried on chiefly by means of some one of the recognized telegraph codes. Radiotelegraph signals are, therefore, made up of short and long periods of constant signal strength separated by idle periods of proper duration to correspond to the combinations of dots, dashes, and spaces comprising the characters of the code being used. The design of the entire system must be such that the lengths of the dots, dashes, and spaces in the signal supplied to the receiving operator are substantially the same as they were made by the transmitting operator. In a simple system operated at slow speeds no special difficulties are encountered in meeting this requirement. Present day commercial systems, however, which utilize remote control from a central traffic office, and which are operated at high keying speeds, impose severe requirements on all of the equipment used.

2. Standard Codes. In international communication the International Morse Code is used. Specially marked and accented letters such as are used in German, French, and the Scandinavian languages have special characters which are used when working a station in the same country or its possessions. When communicating with a foreign station these letters are either replaced by a combination of unaccented letters or in some cases the unaccented letter is transmitted alone. Some countries such as Japan and Egypt having alphabets differing radically from the Latin alphabet use special codes for working within the country or to ships. Nationals of such countries desiring to transmit a message in their own language to a foreign country must spell out the sounds of their words in one of the languages using the Latin alphabet.

3. Business Codes. Business concerns that have a large volume of telegraph communication use so-called five-letter or ten-letter codes. Standard codes for such use are available and consist of groups of letters arranged alphabetically; each group standing for a complete sentence or part of a sentence. Special and private codes are also used, and large concerns often have a department for the coding and decoding of coded telegraphic messages.

4. Printing telegraph equipment has found a very limited use in the radio communication systems of the world. On short-distance circuits where the received signals are strong and steady, and where atmospheric disturbances are well below the signal level, such equipment can be operated satisfactorily.

[1] Research receiving engineer, R.C.A. Communications, Inc.

A ·—	Period ·· ·· ··
B —···	Semicolon —·—·—·
C —·—·	Comma ·—·—·—
D —··	Colon ———···
E ·	Interrogation ··——··
F ··—·	Exclamation point ——··——
G ——·	Apostrophe ·————·
H ····	Hyphen —····—
I ··	Bar indicating fraction —··—·
J ·———	Parenthesis —·——·—
K —·—	Inverted commas ·—··—·
L ·—··	Underline ··——·—
M ——	Double dash —···—
N —·	Distress Call ···———···
O ———	Attention call to precede every transmission —·—·—
P ·——·	General inquiry call —·—· —·——
Q ——·—	From (de) —·· ·
R ·—·	Invitation to transmit (go ahead) —·—
S ···	Warning - high power ——··——
T —	Question (please repeat after)— interrupting long messages ··——··
U ··—	Wait ·—···
V ···—	Break (Bk.) (double dash) —···—
W ·——	Understand ···—·
X —··—	Error ········
Y —·——	Received (O.K.) ·—·
Z ——··	Position report (to precede all position messages) — ·—·
Ä (German) ·—·—	End of each message (cross) ·—·—·
Á or Å (Spanish-Scandinavian) ·——·—	Transmission finished (end of work) (conclusion of correspondence) ···—·—
CH (German-Spanish) ————	
É (French) ··—··	
Ñ (Spanish) ——·——	
Ö (German) ———·	
Ü (German) ··——	
1 ·————	
2 ··———	
3 ···——	
4 ····—	
5 ·····	
6 —····	
7 ——···	
8 ———··	
9 ————·	
0 —————	

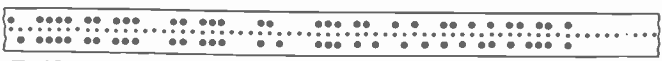
FIG. 1.—The Continental code.

One type of printer equipment operates from the regular telegraph code; a tape being perforated by a machine which is actuated from the incoming signal, and this tape then being fed into the actual printer. This types the letters on a paper tape which is cut and pasted on message blanks.

Another type of automatic printer equipment utilizes a special code in which six impulses comprise the total number of elements in any one character. A different number or combination of impulses, up to the maximum of six consecutive ones, is used for each character.

5. Character Formation. The unit used in code characters, and in figuring speeds of transmission, is the dot. Present practice, based on automatic transmitting equipment, is to speak of dots per second. On this basis the time required to transmit one dot includes the duration of the space separating the dot from the next element of the character. As the duration of the dot itself and of the following space are equal, they constitute a cycle. Keying speeds are, therefore, commonly stated in dots, or (square) cycles, per second. The equivalent time required for the transmission of the other elements of the code are: a dash, two dots; space between letters, one dot; space between words, three dots. For traffic purposes speeds are generally stated in words per minute. The ratio of words per minute to dots or cycles per second is generally accepted as being 2.5:1 for usual commercial traffic, 100 words per minute being equivalent to 40 cycles keying frequency.

In the Baudot code used for printing telegraph equipment, the duration of the character is divided into five equal periods. For any one of these periods either a marking, or a spacing (no current or reverse current) impulse may be transmitted. One impulse is required between letters, and in the non-synchronous type of equipment an additional impulse is required at the start of each character to set the receiving mechanism in motion. The total number of elements per character is, then, either



T H I S I S A S A M P L E

six or seven depending on the type of equipment used. The space between words is a full-length character. The code consists of a different combination of marking and spacing impulses for each character, there being a total of 32 possible combinations for the five periods utilized. For calculation of keying frequency the single period or element, which is the shortest impulse required to be transmitted, corresponds to the marking portion of a dot in the Morse Code. This is one half cycle. For the non-synchronous printer equipment each letter requires, for its transmission, seven half cycles or three and one-half full cycles. On the basis of five letters per word, and a space between words, the ratio of words per minute to keying cycles per second is 2.86 to 1. This is the figure realizable with automatic tape transmission. Where the impulses go directly from the keyboard-operated machine to the line, the dot speed will remain unchanged, but the number of words per minute that can be transmitted will be reduced on account of the unavoidable irregularities in the speed of the typist.

6. Required Frequency Range. A square wave shape such as a succession of dots, where the value of the current or voltage rises instantly to a steady value at which it remains for one half cycle and then instantly drops to zero, can be analyzed into the fundamental and all of its odd harmonics. The equation of the voltage wave is:

$$e = \frac{4E}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right) \qquad (1)$$

which holds for values of x between $-\pi$ and $+\pi$. For most practical telegraphic purposes it is only necessary for the system to pass the fundamental, third, and fifth in their proper intensity and phase, as terms of higher order do not add sufficiently to the fidelity to warrant building the equipment to handle them. The frequency range required by a sufficient number of higher order harmonics to give appreciable improvement can often be used to better advantage for additional channels.

For any service where the received signal strength rises to the same maximum value on every dot and dash it is not necessary to pass even the third harmonic of the keying frequency. A system which will pass the second harmonic of the fundamental keying frequency is satisfactory. The receiving equipment can be adjusted to operate at a fairly definite level on the building up and decaying of the current or voltage wave so as to give characters which are neither too heavy (long) nor too light (short) as compared to the spaces. However, in a system where the received signal may vary by 2:1 or more in intensity at fairly short and frequent intervals it is necessary to have quite a steep rise and fall of the received signal at make and break in order to obtain a constant "weight" of keying. This applies particularly to automatic reception, where the signal operates a recording device either directly from amplifiers or through a relay of either the mechanical or vacuum-tube types. For aural reception it is desirable to retain the harmonics of the keying frequency as the signal then sounds cleaner cut, and more definite, making it easier to read.

Cases of interference, in both the radio and the land-line portions of a system, are sometimes encountered where it is necessary slightly to round off the sharp, square envelopes of the dots, in order to reduce or eliminate the interference or cross talk caused by the too sudden rise and fall of current.

Where the exact effect of a given circuit on the shape of a square input wave is desired, the range of frequencies passed by the system must be considered as a continuous band rather than dealing with only odd harmonics of the keying frequency.

The usual modulation and sideband theory of radio telephony is applied to code transmission by considering the fundamental keying frequency, and such of its harmonics as are passed, to modulate the carrier 100 per cent. The total band width required to be passed by the entire system is equal to twice the frequency of the highest harmonic of the keying speed that it is desired to retain. (See Arts. 30 to 33 for actual values.)

7. Speeds Attainable. Speeds of transmission range from about 15 up to 300 words per minute; the corresponding keying frequencies being 6 to 120 square cycles per second. Work with ships, and with aircraft, is carried on mainly at speeds up to about 35 words per minute. Transmission is by means of a manually operated telegraph key. Reception is by ear. In point-to-point service, such as transoceanic, traffic speeds normally range from 30 up to 250 words per minute depending upon the type of equipment used, transmission conditions and the amount of traffic to be handled. Keying is done by machine almost entirely, hand-operated keys being used only for minor service communications. Reception is generally by means of an ink recorder, the telegraphic characters on the tape being transcribed on a typewriter by the operator. Aural reception is resorted to only under adverse conditions.

8. Fidelity of the mark-to-space ratio, while important at all speeds, requires special attention when automatic operation at speeds in excess of 100 words per minute is to be maintained. Where the duration of the mark portion of a dot is only one-eightieth of a second, or less, factors that are disregarded at slow speeds become of primary importance. Automatic transmitters, relays, and electrical circuits should be fast enough so that the signal supplied to the recording equipment will not be heavier than 60/40 or lighter than 40/60 in mark-to-space ratio at the highest speed used. At 200 words per minute, which is not exceptional in present-day short-wave work, this means a variation of not more than 1.25 millisec. in the duration of a dot. While it is sometimes possible to compensate for heavy or light keying characteristics by means of relay adjustments in another portion of the system, this should not be depended upon for obtaining the desired over-all fidelity. Each unit of the system should be capable of giving the required fidelity at a speed in excess of the maximum operating speed, the margin required depending on the number of elements in the over-all system and the fidelity of each.

9. Checking the keying characteristics of portions of, and of the entire, system is done by means of keying wheels which send out either a single word over and over, or a succession of dots of 50/50 mark-to-space ratio. For speeds up to about 100 words per minute the usual high-speed ink recorder can be used for checking character formation quite satisfactorily. For accurate information, especially at higher speeds, some form of oscilloscope or oscillograph must be used. The low-voltage type of cathode-ray oscilloscope is admirably suited to this work where photographic records are often not required. Associated amplifiers must have a fidelity considerably better than that of the equipment being tested.

10. Requirements for Facsimile. Facsimile service requires control and radio equipment capable of handling keying frequencies up to about 500 square dots per second. This speed is possible only on short-wave equipment and requires a band width of about 5,000 cycles.

RADIOTELEGRAPHIC SERVICES

11. Services. Code-communication channels and equipment can be classified, according to the type of service rendered by them, under the general headings of transoceanic, shorter distance point-to-point, ship-to-shore, aircraft, special mobile services, and military.

12. Transoceanic, or long-distance point-to-point, traffic and broadcasts were, prior to 1928, handled almost exclusively on frequencies ranging from about 14 to about 30 kc. Great-circle distances covered on such commercial circuits range from 2,000 to 5,000 miles, roughly. To cover distances greater than this with commercial reliability requires so much power to be radiated from the transmitter that it becomes uneconomical.

Approximate values of signal strength to be expected are calculated from the Austin-Cohen transmission formula

$$E = 120\pi \frac{HI}{\lambda D} \sqrt{\frac{\theta}{\sin \theta}} \times e^{-u} \tag{2}$$

where

$$u = \frac{0.0014D}{\lambda^{0.6}}$$

HI = effective height times current for transmitting antenna in meter amperes
λ = wave length in kilometers
D = great-circle distance in kilometers
θ = arc of great circle between transmitter and receiver
E = received field strength in microvolts per meter

or the slightly different expression

$$E \text{ in } \frac{\mu V}{m} = \frac{377HI}{\lambda D} e^{-u} \tag{3}$$

where

$$u = \frac{0.005D}{\lambda^{1.25}}$$

which is derived from data taken on the New York to London circuits at frequencies ranging from 17 to 60 kc.

13. Field Strength Required. For successful operation of a circuit the received field strength must be sufficiently above the level of atmospheric disturbances and other local sources of noise to give fully readable signals. Automatic recording requires a signal to noise ratio of at least two to one. This is based on the general, or average, noise level. Moderately severe atmospheric disturbances such as "crashes" and "clicks" will be from several to perhaps ten times as strong as a normally satisfactory signal. Field strengths obtained on transoceanic circuits range from 10 or less up to 250 μv per meter. A value of 20 is about the minimum for satisfactory communication under average conditions. Modern high-powered transmitting stations have an antenna input power of from 40 to 500 kw with output ratings up to some 130,000 meter amp.

14. Short Wave. During the last few years "short waves" have assumed increasing importance in long-distance radio communication of all types. Frequencies used range from about 7,500 to 23,000 kc, depending upon distance, season of year, time of day, and path traversed. Proper choice of frequency allows of reliable communication between any two points on the earth with transmitters of modern design. Power output of the equipment ranges from 1 to 40 kw. Due to the extreme variations in transmission conditions encountered at these frequencies it is necessary to have available at least 10 kw output from the transmitters, for high-speed automatic operation over the longer distances. Even with the maximum output of present transmitters, and with directive antennas for both transmission and reception, communication is slowed down or even stopped, at times, by severe disturbances in transmission conditions. Normal field strengths obtained at the receiving antennas range from 0.1 up to 100 or more μv/m, depending on transmitter radiation, path, and transmission conditions. The minimum signal required for reliable, commercial operation depends partly on the noise level at the receiving point. Atmospheric disturbances (static), while troublesome at times, are not so serious as in the case of long waves. Fading requires the use of a greater signal-to-noise ratio on short waves. Utilization of either space, frequency, polarization, or time diversity of fading will overcome, to a great extent, the bad effects of this phenomenon and permit successful operation on a much weaker signal than would otherwise be usable. A very rough estimate of the minimum field strength ordinarily required for code communication, with automatic recording, is 5 μv per meter. Slow-speed aural reception can be carried on with field strengths of as low as 0.1 μv per meter.

15. Short Waves *versus* Long Waves. Advantages of short waves for transoceanic code communication are: (1) lower first cost of equipment and antennas, (2) smaller power consumption, (3) higher keying speeds of which the equipment is capable, (4) less trouble from static, (5) directive transmission, (6) greater distances can be covered with a reasonable and practicable transmitter power. Disadvantages are: (1) interruption of service due to severe magnetic disturbances, (2) effects of fading, (3) necessity of having several frequencies, a separate antenna being required for each, for 24-hr. service the year round.

Advantages of long-wave operation are: (1) freedom from interruption of service by magnetic disturbances, (2) comparative reliability and steadiness of signal strengths.

Types of transmitting equipment used for long-wave work are: timed spark (nearly obsolete), arc, alternator, tube sets. Tube transmitters, only, are used for short-wave operation.

16. Point-to-point communication for distances up to some 2,000 miles is carried on at frequencies ranging from approximately 30 kc up to 100 kc. These stations are used for domestic service and also for the shorter international circuits. Certain bands in the 6,000-kc to 23,000-kc portion of the spectrum are also used for these shorter circuits.

Types of equipment used for 30- to 100-kc work include: spark (obsolete), arc, frequency multipliers, and tube transmitters. For short-wave operation tube transmitters are used exclusively.

17. Ship-to-shore and ship-to-ship communication is an entirely different class of service, in all respects, from point to point. Except at the larger coastal stations, and on a very few ships, transmission is entirely by hand and copying is by ear. This is because of the nature of the service; a coast station usually having not more than ten to twenty messages for one ship at a time, and *vice versa*. Automatic transmission and reception are used only when traffic on hand amounts to some forty messages or more. The same operator generally handles both transmission and reception, which is not the case in point-to-point work. Due to the great number of ships, and to the intermittent nature of their traffic, the marine frequency bands must be shared by all ships. This creates interference and traffic handling problems that are not encountered in point-to-point work. A marine operator must be located at the receiving equipment. Remote control is used only on the transmitters of coastal stations, the transmitting and receiving stations being separated by distances of up to 50 miles to permit of simultaneous transmission and reception.

Frequencies utilized lie within the 100- to 550-kc band; those around 150 kc being used for long-distance work to the larger ships, while those from 400 kc to 550 kc are for shorter-distance work mainly to the smaller ships, and for distress calls (500 kc). Coastal stations using efficient 5- to 10-kw transmitters and directive reception can normally work ships about 1,500 miles and up to 3,000 miles under favorable conditions, at the lower frequencies. Operation in the 400- to 550-kc band is more variable, a 5-kw transmitter having a normal daytime range of around 500 miles and a night range of several thousand under favorable conditions.

Spark (obsolete), arc, and tube transmitters are used at the lower frequencies. On the higher frequencies tube sets are replacing the old spark equipment. These operate either cw or icw as desired.

Short waves are coming into some use for the handling of ship-to-shore traffic and for special brokerage service to some of the larger ships. Short-wave equipment in the present state of the art is not so well adapted for general marine use as is the 100- to 550-kc equipment. For small craft, where space for equipment and antennas is limited, such equipment has the advantage of being able to work over long distances with comparatively small power.

TRANSMITTING SYSTEMS AND EQUIPMENT

18. The high-frequency alternator is one of the most used types of transmitter for long-wave transoceanic-code communication. The Alexanderson alternator used in this country is a high-speed inductor-type machine having a large number of poles so that frequencies up to 30 kc and higher may be obtained directly. These machines have an output of 200 kw and are driven by a 600-hp. two-phase induction motor through a set of gears to give the desired alternator speed. The stator is built in sections to facilitate dismantling for repairs and maintenance and has 64 separate windings which are connected to separate windings on the antenna-input transformer. One winding is used to supply a tuned circuit the output of which is rectified and used for automatic speed control. Forced lubrication and water cooling are used on account of the high speed and relatively high losses as compared with commercial power-frequency machinery. Such an alternator intended for operation at 27,200 cycles is driven at a speed of 2,675 r.p.m., has 1,220 poles and requires a field current of 2 amp. at about 120 volts.

To maintain the frequency constant to approximately 0.1 per cent and to have it the same under conditions of full load and practically no load, elaborate compensating means are provided as shown on the schematic diagram. Primary compensation saturation transformers each have an a-c and a d-c winding so connected that the voltage at the motor depends upon the impedance of these transformers which, in turn, depends upon the value of current in the d-c winding. Connected to the slip rings of the wound rotor are two banks of liquid rheostats, the "running" bank being connected at all times and the compensation bank being thrown on or off by the contactors. These contactors, and the contactor in the primary compensation d-c control circuit, are operated from a master relay which is controlled from the central traffic office. Compensation adjustments are made to maintain the machine at the same speed with the control key open or closed.

19. Method of Keying. Keying the output is accomplished by means of a magnetic modulator which is a special transformer having an a-c winding and a differentially connected d-c saturation winding. When the control key is open, a relay closes this d-c circuit, and the resulting drop in impedance of the a-c winding detunes the antenna and reduces the alternator output voltage so that practically no current circulates in the antenna circuit. For key closed the d-c winding is deenergized and the antenna circuit now becomes resonant to the alternator frequency so that normal antenna current is obtained. Due to the low frequency

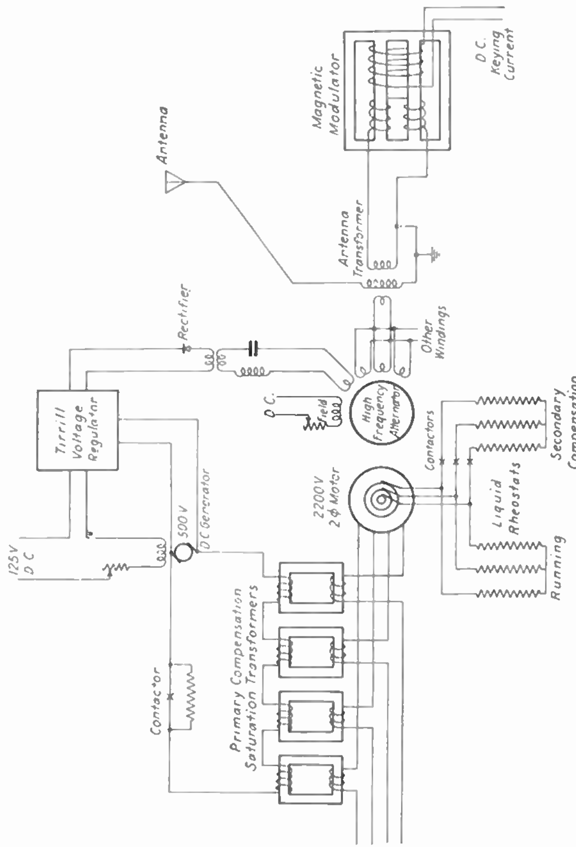


FIG. 2.—Alexanderson alternator equipment.

of the system and the low resistance of the antenna circuit, also on account of the large contactors required in the compensation circuits, keying speeds are limited to about 120 words per minute on long-wave transmitters.

20. Goldschmidt Alternator. Another type of high-frequency machine that has been used to some extent is the Goldschmidt alternator. The fundamental frequency generated is usually one-fourth of that desired. This is then changed successively to the second, third, and fourth multiples by utilizing the e.m.f. generated in one winding by the rotating field due to current of the next lower order frequency which is flowing in the other winding. The heavy circulating currents are obtained by tuning the respective windings, the output circuit being arranged to deliver energy to the antenna at the desired multiple frequency. The object of this method of obtaining radio frequencies is to use a comparatively low-speed machine rather than to attempt direct generation at the desired frequency, which requires the use of a high-speed machine having a large number of poles.

21. Static Frequency Multipliers. Present practice favors the use of static frequency multipliers where it is desired to use an alternator of comparatively low frequency. Two general methods, both of which depend upon the use of special transformers having d-c saturation windings, are employed. The first utilizes either two or three transformers connected in such a manner that the second or the third harmonic of the fundamental is in phase in the several output windings. The second may utilize but a single transformer, with a d-c saturation winding. The output winding is tuned to the desired harmonic frequency and receives its energy by "shock excitation." This is accomplished by so adjusting the d-c and a-c supply currents that voltage is induced in the secondary winding for only a small portion of a cycle of the supply frequency. In this manner harmonics of the fifth, and higher, orders may be obtained.

22. Arc transmitters are used, to some extent, for long-wave transoceanic work. There have been two main objections, however, to the

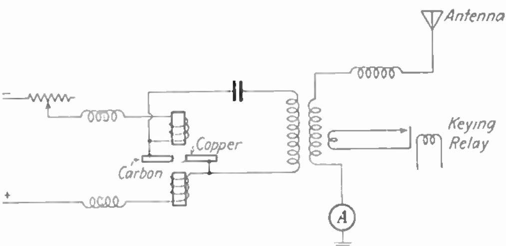


Fig. 3.—Arc transmitter.

use of such equipment. Most arc transmitters emit two frequencies, one for mark and the other for space. As there must be a sufficient frequency difference between these to allow of their being separated in the receiving equipment, one such transmitter really requires two communication channels for its operation. The other objection has been that most arc sets emitted strong harmonics. These can, however, be prevented from radiating strongly by proper shielding and the use of properly arranged circuits for feeding the antenna. Elimination of the space wave or "back wave" is rather difficult in transmitters of this type, especially when the output may be as high as 1,000 kw in large installations. The actual power output of the arc can not be keyed as the arc, to be stable, must draw a fairly constant current while in operation. Keying is generally accomplished by changing the inductance of the resonant circuit associated with the arc, thereby changing the frequency of the emitted wave. This is done by short-circuiting a few turns that are coupled to the main tuning inductance.

Methods have been proposed for shifting the output of the arc to a dummy antenna, or absorbing circuit for keying the actual power radiated on but one frequency. Such methods have not come into general use.

The arc is operated from a direct-current source, usually motor generators, at a voltage of from 300 to 3,500 volts depending upon the power rating of the unit. It burns in an atmosphere rich in hydrogen, which is supplied by gas or by the vaporization of some such liquid as alcohol which is fed into the arc chamber. For the efficient production of undamped oscillations the arc must burn in a transverse magnetic field. This is supplied by a large electromagnet the poles of which are respectively above and below the arc chamber and the coils of which are energized by passing the arc current through them. The intensity of magnetic field required for optimum results is inversely proportional to wave length and also depends upon the material used to furnish the hydrogenous atmosphere in the arc chamber. Values normally range from about 2 to 20 kilo-gausses. A water-cooled copper anode is used with a carbon cathode which is slowly rotated by means of a motor while the arc is in operation. A current-limiting resistor, normally used while striking the arc, is shorted out when the arc is running.

23. Tube transmitters have been used but little at frequencies between 14 and 30 kc for long-distance communication. Tubes to handle the power required have not been available until quite recently. This meant that a number of tubes had to be operated in parallel in the power-amplifier stage. Such transmitters have rated outputs of from 40 to 500 kw and are of the usual master-oscillator, power-amplifier type.

24. Long-wave antennas of the various familiar types such as the T, inverted L, and umbrella have been used. Masts for these structures have, in some cases, been as high as 1,000 ft. Ordinarily they range from 400 to 800 ft. high. The technical problem is to get as many amperes in an antenna of as great an effective height as possible with a given power input. Voltages from antennas to ground may easily be 100 kv or more so that corona and insulation considerations place a limitation on the design. Of the total power supplied to the antenna the useful portion is that radiated. The remainder is accounted for by conductor losses, coil losses, leakage and corona (if present), and by loss in the resistance of the ground-return path. In a structure where most of the capacity is from the flat top to earth, and where the dimensions are considerably less than a wave length, the radiation resistance is given approximately by the relation $R = 1600\frac{H^2}{\lambda^2}$ where H is the effective height of the antenna and λ the length of the radiated wave. Approximate calculation of H is possible in simple cases by summing up the products HI for all sections of the structure and dividing by the total current. This is done by calculating the capacities to earth of the various sections, and by measurement of the total value. Experimental methods of determining the capacity from small-size models are described by Lindenblad and Brown.[1]

[1] Lindenblad, N., and W. W. Brown, Main Consideration in Antenna Design, *Proc. I. R. E.*, June, 1926.

25. The multiple-tuned antenna, consists of a long, flat top supported by towers and having downleads at a number of points which pass through tuning inductances to earth. The total antenna current is the sum of all the currents measured at the base of the tuning coils. A system of buried wires, and overhead conductors connected to them through current-equalizing coils is laid out to give a uniform distribution of current in the earth under the antenna. This is approximately the condition for minimum earth resistance. This uniform distribution is sometimes altered, by experiment, to still further reduce the losses. Such antenna and ground systems often have a total resistance of less than ½ ohm. Total antenna currents of 700 amp. and more are obtained, by this means, from a transmitter output of 200 kw. For N tuning points the inductance of each downlead and coil is approximately N times that which would resonate with the total antenna capacity at the desired

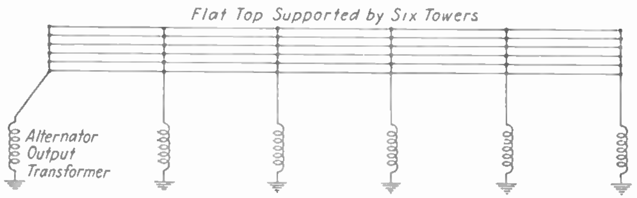


FIG. 4.—Multiple-tuned antenna.

frequency. The physical length of such an antenna for operation at 17 kc, or thereabouts, may be a mile or mile and a half, with as many as six tuning points.

26. Removal of Ice. In climates where sleet is experienced the antenna wires should be counterweighted, rather than solidly anchored, in order to lessen the chances of breakage. A heavy coating of sleet on the wires, with the attendant increase in sag, throws the antenna out of tune as well as endangering it mechanically. When this becomes serious it is necessary to melt the sleet from the wires in order to get normal antenna current. For this purpose break insulators and by-pass condensers are so arranged in the antenna wires that a series circuit of all (or part) of the wires is obtained at the low power-supply frequency. Special transformers supply power at about 2,000 volts for the purpose. This is sent through the antenna conductors just long enough to heat them sufficiently to melt off the sleet or ice.

27. Marine Transmitters. For marine work tube transmitters are replacing the older spark and arc equipment. The radiated energy is confined more to a single frequency, which is essential for reducing interference, and systems for simultaneous transmission and reception, for break-in operation, and for remote control are much more easily built up by the use of tube transmitters. With a well-filtered plate supply the beat note obtained by use of a heterodyne or autodyne receiver is fairly pure, and its pitch can be changed at will by the receiving operator to suit conditions. For attracting the attention of ships standing by on a calling wave, or for working ships not equipped for heterodyne reception, the radiated energy can be interrupted at an a-f rate.

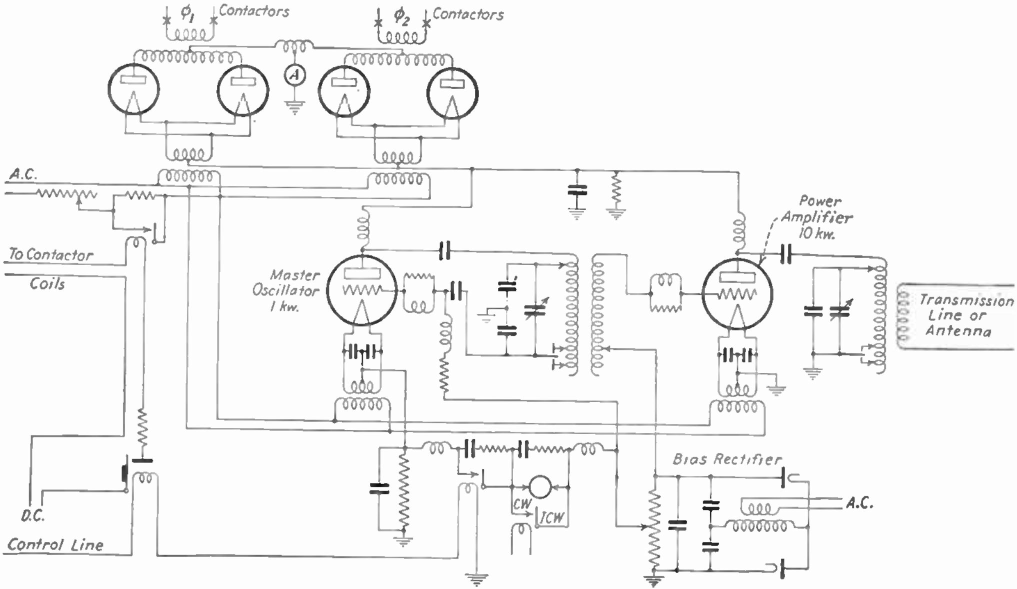


FIG. 5.—Marine coastal transmitter.

Transmitters for coastal stations usually have an output of from 5 to 10 kw. An air-cooled 1-kw tube functions as master oscillator and drives the 10-kw power-amplifier tube, which is of the water-cooled type. Plate supply is obtained from a full-wave kenotron rectifier, the output of which is filtered to some extent. Bias voltages are normally obtained from a small rectifier, to eliminate as much rotating machinery as possible. Filament supply is alternating current from step-down transformers. Because of the nature of the service, interruptions due to equipment trouble must be reduced to a minimum. For this reason two power-amplifier tubes are mounted so that either one can be used. Cooling water systems are provided in duplicate and equipped with pressure- or flow-operated relays which will shut down the transmitter in case of water failure. In some cases it is advisable to locate the antenna at a distance from the transmitter proper. A two-wire transmission line is used for this purpose, being matched to the power-amplifier and antenna-circuit impedances at its ends by means of air-core transformers.

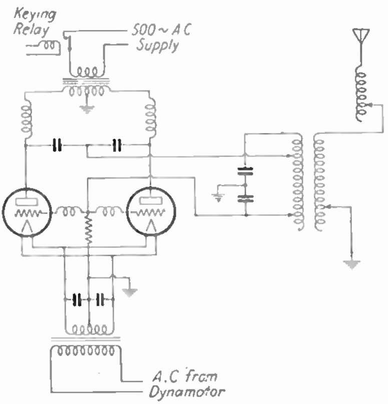


FIG. 6.—Essential circuit of icw marine transmitter with a-c plate supply.

To make the transmitter instantly available the tube filaments are operated at reduced voltage, with plate supply off, when not in actual use. The "starting" relay operates contactors which apply full voltage to the filaments and close the low-voltage circuit to the plate-supply transformers. For remote control, the starting and keying relays can be operated from a single line by using double-current keying with a polar "keying" relay and a neutral line relay with weighted armature for "starting." The chopper, for production of icw, may also be relay operated. Wave change can be arranged by relay-operated contactors which change taps on the tuning inductances, these contactors being operated by a polar relay controlled from the operator's table.

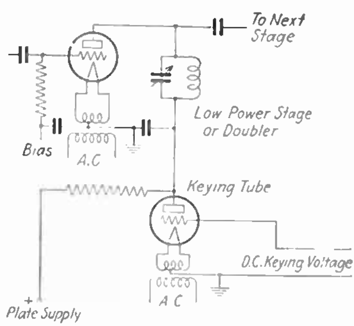


FIG. 7.—Tube keyer for transmitter.

28. Transmitters for shipboard use are generally of smaller power output than are those for coastal stations. Cost and space requirement are also important factors which must be kept down. The usual equipment is, therefore, more simple and compact than that treated above.

The master-oscillator power-amplifier arrangement with direct-current plate supply, or alternating current at a frequency of 350 cycles, meets the requirements very well in the intermediate frequency bands. The master oscillator holds the frequency steady regardless of changes in antenna capacity due to rolling of the ship, and the elimination of a separate rectifier saves space. Where space permits, a high-voltage direct-current generator is used for plate supply. Medium power tubes require about a 2,000-volt supply. Change of wave is accomplished by changing taps on the tuning inductances. Choice of several frequencies in the band is provided by means of a multi-point switch operated from the front of the panel. The normal power-supply mains being direct current, a motor generator is required to furnish the plate-supply voltage. Another machine may furnish alternating current for the filaments. On small transmitters satisfactory keying can be effected in the low-voltage alternating-current plate supply by means of a relay controlled from the operator's key.

29. Short-wave Technique. In the 6,000- to 23,000-kc band the demand for channels, and the comparatively narrow band required for a communication channel, has necessitated the use of transmitting equipment which will maintain its assigned frequency within very small limits. Channel spacing of one-tenth of 1 per cent requires a maximum tolerance of only 0.025 per cent or 1 part in 4,000 under all operating conditions of temperature and power supply. To do this requires the use of either a carefully stabilized and compensated tube oscillator or some control device, such as a quartz crystal, which, when kept at a constant temperature, will maintain the desired frequency within small limits. Crystal control has found most favor in this country, to date.

Commercial short-wave code transmitters used for long-distance communication have an output of from 20 to 40 kw. The crystal is kept at a constant temperature, and operates at one-eighth or one-fourth of the final frequency desired. The oscillator stage is followed by a screen-grid "buffer" stage, to isolate it from feed-back and detuning effects, then by two or three frequency-doubling stages before the first amplifier stage operating at the signal frequency. Screen-grid tubes used in these stages, with proper shielding of tubes and circuits, and filtering of supply leads, eliminate troublesome feed-back effects without the use of neutralization. Water-cooled triodes used in the final power amplifier must be employed in a balanced stage with proper neutralization of feed-back through the tube capacities. The tank circuit of the power amplifier is coupled either directly, or through a transmission line, to the antenna.

For high-speed telegraphic operation the voltage regulation of all plate and bias supplies must be small. If poor regulation exists the envelope shape of the characters will be triangular or irregular, instead of rectangular. (A small amount of lag may be introduced intentionally, in some cases, to round off the corners in order to eliminate trouble from keying clicks in nearby receivers.) For this reason hot-cathode mercury-vapor rectifiers are used for supplying the high direct-current potentials required. These tubes, together with the high-voltage transformers, have very good voltage regulation at high values of output voltage.

For continued operation at keying speeds up to 250 words per minute (100 cycles per second) it is inadvisable to use a system of keying which employs electromechanical relays. A vacuum-tube keying stage is therefore used to key one of the low-power stages of the transmitter.

Where a plate supply having good regulation is not available the load on it can be held constant by using two power amplifiers one of which supplies the antenna and the other a resistance load. Keying is accomplished by shifting the load from the main amplifier to the absorbing tube by biasing the amplifier grids below cut-off and bringing the absorbing tube grid bias up to such a value that the load drawn from the plate supply is the same as when the amplifier is supplying energy to the antenna.

For receiving systems which rely partly upon frequency diversity of fading it is desirable to modulate the wave radiated from the transmitter at an audio frequency of something under 1,000 cycles per second.

RECEIVING SYSTEMS AND EQUIPMENT

30. Long-wave Receivers. Long-wave receiving equipment must be designed to reduce trouble from static to a minimum, and to separate transmitters differing in frequency by only about 200 cycles, which is the approximate spacing of assigned channels. The use of four efficient tuned circuits provides the required selectivity together with moderate ease of handling. For commercial work it has been the practice to obtain the h-f selectivity ahead of an aperiodic amplifier, then to go to a heterodyne detector of either the single-tube or balanced-modulator type which is followed by as much a-f amplification as is required. The final selectivity may, if necessary, be obtained by the use of narrow a-f band-pass filters. For complete separation of signals on adjacent channels this is often necessary. Due to the difficulty of obtaining complete shielding, at these comparatively low radio frequencies it is generally advisable to use astatic pairs of coils in all tuned circuits, couplers, oscillators, etc., in addition to the use of a reasonable amount of shielding. Transformers and couplers are built with electrostatic shields to prevent capacity coupling, where this is undesirable.

In a multiplex receiving station, where it may be necessary to receive from ten to twenty signals from approximately the same direction, a single aperiodic antenna system is the most economical and practical. The individual receivers are fed by means of "coupling tubes" operated from a common, or from individual, antenna-output transformers. All tuning is done beyond these coupling tubes so that operation of the individual receivers is entirely independent of all others.

31. Directional Antennas. Reduction of static is accomplished by the use of directive-antenna systems. Arrays of large loops, or of loop and vertical combinations, are one means of obtaining directivity. Where the nature of the soil is such as to produce a considerable tilt of the wave front, the Beverage wave antenna is used to advantage. This antenna consists of one or two wires strung on poles at a height of about 20 ft. and extending in the direction of the desired signal for a distance of approximately one wave length. The antenna is highly directional, and small signal voltages obtained from stations to the rear can be compensated for by feeding into the signal circuit a small voltage of proper amplitude and phase obtained from the damping resistance connected between antenna and ground, or by setting up reflections in the antenna itself.

As keying speeds on long-wave transoceanic circuits seldom exceed 100 words per minute (40 cycles/per second), and signal strengths are steady, such a channel requires only a total band width of about 160 cycles. Frequency variations of the transmitters can be kept within

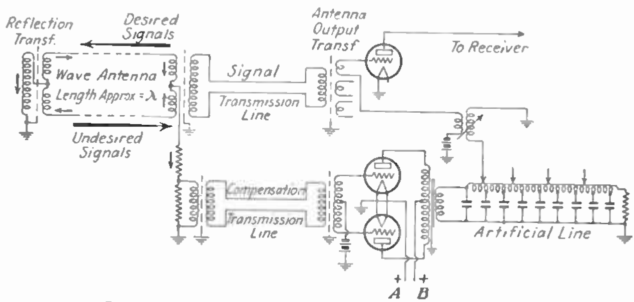


FIG. 8.—Wave antenna and output circuits.

about 0.1 per cent or 20 cycles in 20,000, and heterodyne oscillators used for reception should have as good stability.

32. Ship-to-shore Receivers. Receiving equipment for ship-to-shore service must cover the frequency range of 500 down to 14 kc in order to operate in the regular marine bands and also to receive broadcasts and time signals from high-powered long-wave stations. Receivers for shipboard use are of the autodyne type embodying a tuned antenna circuit coupled to the oscillating detector, which latter has a "tickler coil" for regeneration control, and generally two stages of audio-frequency amplification. By means of tapped inductances the receiver may tune from about 1,000 down to 60 kc. For the lower frequencies a set of loading inductances is used. One of the chief requirements is ease of operation and rapidity of tuning. Regeneration control allows the receiver to be operated oscillating for cw reception or non-oscillating for reception of spark, icw or modulated signals. Provision is made for disconnecting the receiver from the antenna when transmitting.

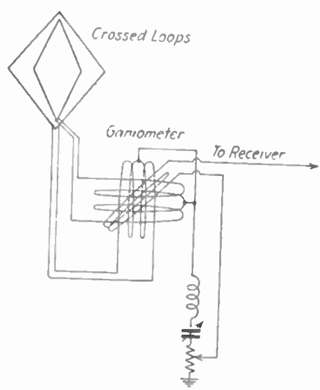


FIG. 9.—Loop-vertical antenna for directive reception.

Important coastal stations have separate receivers to cover the lower and higher frequency marine bands of approximately 115 to 171 kc and 375 to 500 kc respectively. Such receivers should have but a single tuning control and, to obtain the required selectivity, should be of the superheterodyne type. An intermediate frequency oscillator, which can be used at will by the operator, must be provided for cw reception.

The over-all selectivity should be such that a total band width of not more than 1 kc is passed at 80 per cent peak response.

As in long-wave reception, reduction of static and interference is accomplished by the use of directive antennas. For the lower frequency band the Beverage wave antenna has the advantage of relatively large pick-up, good directivity with compensation, and the ability to supply a number of receivers operating at the same or different frequencies. Where reception from all directions is required, and for the higher-frequency bands where the wave antenna is unsuitable for night reception, antennas of the flat top, inverted L, T, vertical, or loop types are employed. The loop and vertical combination, giving a cardioid directive diagram, can be arranged with crossed loops and a goniometer so that the operator can rotate his antenna-reception diagram at will.

33. Short-wave receiving equipment for code reception comprises two general classes, namely, (*a*) long distance point-to-point and (*b*) marine or mobile.

For point-to-point service the receiving equipment must deliver a signal which is as nearly perfect as is possible. This requires a high degree of frequency stability, very sharp over-all selectivity, and means for reducing the effects of fading to a minimum. With channels spaced

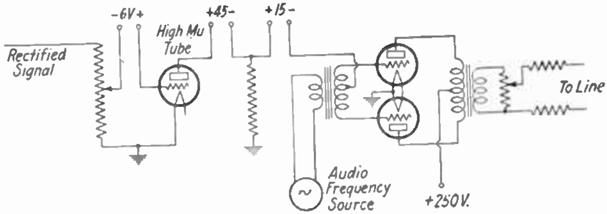


FIG. 10.—Tone keyer for receivers.

0.1 per cent the receiver should have a total band width such that, at the lowest frequency it will be required to operate at, the response at a total width of 0.1 per cent of this frequency will be at least 30 db below the response at mid band. At the highest frequency worked, the selectivity must not be so great that an undue amount of attention is required to keep the signal fairly well centred in the band of the receiver. With well stabilized transmitter and receiving oscillator frequencies, present-day operation up to some 23,000 kc requires uniform response over a 3-kc band. This selectivity can be obtained by means of audio-frequency filters or by the use of multiple detection employing one or more successively lower intermediate frequencies. In the latter the "image" signal, differing in frequency from that of the desired signal by twice the intermediate frequency, must be attenuated at least 40 db. Higher ratios than this are to be desired, a figure of 50 being quite a safe compromise. The multiple-detection receiver, with final rectification at a relatively low intermediate frequency, is to be preferred to the a-f filter type of equipment. The former utilizes the entire band passed; whereas the latter is only utilizing that half of its total band lying on one side of zero beat, at a given time.

In equipment used for high-speed automatic operation the signal is amplified, beat down to a lower frequency, and then rectified. The rectified output, consisting of short and long pulses of direct current, is used to operate a relay of either the electromechanical or vacuum-tube type. The former operates into a simplex, duplexed, or quadruplexed direct-current telegraph line to the central traffic office. The tube relay, or "keyer," controls the signal fed to the tone line from a local a-f source. The receiving operator is thus supplied with an audio signal of constant frequency and intensity regardless of any changes in the actual radio signal which are not great enough to make it drop out of the receiver. By means of a-f filters six or more keyed tones of this sort may be handled over a single two-wire tone line.

To minimize the effects of fading, receiving equipment is arranged to take advantage of the diversity of fading existing, at a given instant, either on slightly different frequencies at the same location or on the same frequency at points separated 10 wave lengths or more apart. Frequency diversity, in practice, is most economically obtained by modulating the carrier with an audio frequency of not higher than 1,000 cycles. This results in radiation on the carrier and on an upper and a lower frequency. If the band width of the receiver is sufficient to pass these three frequencies, and if the normal signal strength on any one of these frequencies is sufficient to operate the keying device, considerable diverse fading on the several frequencies received can be tolerated. In spite of the fact that a lesser peak voltage can be obtained from a modulated signal than from a pure cw signal considerable improvement is obtained, under practical conditions of fading, by its use. Where space diversity is utilized a pure, unmodulated signal is to be preferred. In this case two or three separate receivers are fed from separate directive antennas spaced ten wave lengths or more apart. The rectified outputs from these receivers are combined and made to operate the keying device. Confining the radiated energy to a single frequency means greater signal strength for a given transmitter power, and combination after rectification eliminates the consideration of instantaneous phase relations which might be such as to cancel rather than add.

Short-wave receivers for marine and mobile use generally consist of but one or two stages of tuned r-f amplification followed by an autodyne detector and one or two stages of a-f amplification.

34. Use of Limiting Circuits. Under conditions of high signal-to-noise ratio and violent fading, the use of considerable limiting in the receiving equipment is desirable. This should be done following the final selectivity, and must be in a system having small enough time constants so that the decaying transients occurring after each overload do not occupy an appreciable portion of the interval between characters. In order to use such limiting successfully it is essential, as stated before, to pass up to about the fifth harmonic of the keying frequency. If this is not done, wide variations in mark-to-space ratio of the final signal will occur as the degree of limiting varies with the signal strength.

Character formation can be maintained, in some cases of overloaded systems, by the use of a so-called "sliding bias" on the rectifier. The signal may be amplified up to some 30 or 40 volts maximum value and applied to the grid circuit of a rectifier tube which begins to take grid current at a relatively low applied signal voltage. By proper choice of grid- and plate-circuit resistors, and the use of a condenser across the grid-circuit resistor to give a relatively large time constant, only the tops of the character envelopes will be effective. In using such a system, however, reliance must be placed upon some form of diversity reception to prevent splitting of characters, and drop-outs, due to rapid fading.

35. Transmission Lines for Receivers. In a large radiotelegraph receiving station for long-distance communication there may be from 10 to 100 individual receivers installed and intended for simultaneous operation. To do this requires that each unit be effectively shielded and that all battery-supply leads be well filtered for the frequencies at which the respective units operate. High-frequency equipment must also be protected from low-frequency voltages which might be present on the battery supply busses, as such voltages may cause undesirable modulation of signals if allowed to get to the tube circuits. Transmission lines, where used, must be of a type which has negligible stray pick-up and radiation. Satisfactory types of line, depending upon the equipment with which it is to be used, are (*a*) the balanced four-wire line, (*b*) the two-wire transposed line, and (*c*) the concentric-pipe line. The first consists of four wires arranged at the corners of an imaginary square, diagonally opposite wires being connected together at both ends of the line. The four-wire and two-wire types are used where the system is to be kept balanced with respect to earth. Antenna systems which operate against earth generally use the concentric-pipe line in which the outer pipe is grounded.

To obtain the full benefits of good shielding stray feed-back through the battery-supply leads must be eliminated by means of properly proportioned, and located, filter circuits. This is of especial importance in short-wave equipment, and in medium-wave equipment for marine coastal station use.

36. Power supply for commercial receiving equipment must be absolutely reliable and not subject to interruption. Storage batteries operated either on a floating, or on a charge and discharge, basis are used for this service.

Charging equipment consists of motor-generator sets for filament batteries, where relatively heavy currents are required, and either motor generators or rectifiers for batteries of smaller rating such as used for plate and bias supply. Where receiving antennas may be located fairly close to the building that houses the charging equipment, this must be located in a specially shielded room to prevent direct radiation into the antennas. Equipment used for floating batteries that are in service must be provided with effective filtering between it and the battery and load bus.

CONTROL METHODS AND EQUIPMENT

37. Central Office. In commercial radiotelegraphic systems the transmitters are controlled from a central traffic office, and received signals are conveyed to this central office from the receiving station, by land lines. Transmitting and receiving stations are, in some cases, as much as 500 miles distant from the central office. The tendency, however, is to keep this distance below 100 miles to reduce initial and maintenance costs, or rentals, of land lines. Long control and tone lines are justified only if a distant location of the transmitter will effect a considerable saving in the power required to obtain satisfactory service, or if the distant receiving site is considerably superior to nearby ones in signal-to-noise ratio. In long-wave transoceanic and medium-wave marine work the use of long land lines is often well worth while. In short-wave work the over-all results are not so dependent upon geographical location. Suitable sites are generally available within 100 miles of the city to be served.

38. Automatic Transmitters. In "automatic" operation of code circuits a tough paper tape is perforated by means of a machine which has a keyboard similar to that of standard typewriters. This tape is then fed through the "automatic transmitter" in which two cam-operated steel rods come up against the tape at every point where a perforation might exist. Where one is, the rod goes on through, and a contact operated by a lever on the lower end of the rod is closed. These two rods controlling the "make" and "break" contacts alternate in coming against the tape and are sufficiently offset in the direction of travel of the tape so that perforations in the upper (make) and lower (break) rows, when opposite the same center hole, give a dot and when opposite adjacent center holes give a dash.

The two contacts supply current, in opposite directions, to a polar relay which, in turn, keys the control circuit going to the transmitting station. For speeds much above 100 words per minute it is desirable to have as few mechanical relays as possible between this main polar relay and the keying circuit of the radio transmitter. The time required for a relay armature to travel from one contact to the other, while short, becomes important when the duration of a dot is less than 0.010 sec.

39. Zone-control Circuits. Where only a few transmitters are to be controlled from one point, direct-current double-current keying is the most economical and satisfactory. A complete metallic circuit is to be preferred to a single wire with ground return, although the latter is entirely satisfactory in many cases.

In a large central-office system the number of control lines required can be greatly reduced by the use of multiplex tone, or "voice-frequency carrier," control. By the use of a number of different frequencies, and band-pass filters at both ends of the circuit, as many as ten channels can be obtained on a two-wire line which will pass frequencies from about 400 cycles up to 2,500 cycles with approximately equal attenuation. In one such type of equipment the audio-frequency supply is a multi-frequency inductor-type alternator having a separate winding and rotor for each frequency. Energy from this machine is keyed by means of either electromechanical or vacuum-tube relays which are controlled by the automatic tape transmitter and supply current to the control line. Band-pass filters in the individual control channels reduce the harmonic content of the signal supplied to the line to a low value and also round off the corners of the square keying envelopes.

The band width required in filters for tone-control work depends (1) upon the maximum keying speed which must be handled and (2) upon the fidelity of envelope shape required for the particular application. Where great fidelity is not required, or where the over-all transmission gain of line and associated equipment does not vary more than about 20 per cent, it is sufficient to pass the second harmonic of the keying frequency. This means a total band width of four times the keying frequency. To obtain fairly square envelope shape, with a mark-to-space ratio of about 60/40, it is necessary to pass up to the third harmonic or a total band of six times the keying frequency, at least.

For the lengths of line normally used between central offices and outlying stations, and present-day code keying speeds, the matter of phase distortion due to the line is of relatively small importance.

40. Control equipment used at transmitting stations may be of either the d-c or tone-operated type, depending upon the system used at the central office. In a double-current d-c system, the conventional polarized telegraph relay is used as a main-line relay for speeds up to some hundred words per minute. Where normal operating speeds run much above one hundred words per minute special high-speed relays of the polarized type must be used. Large keying and compensation relays and contactors used in long-wave transmitters are controlled by the line relay or a heavier intermediate relay. In tube sets—especially short-wave equipment—higher keying speeds are possible and require the use of a minimum number of mechanical relays. For d-c control the main line relay may operate directly into a tube keyer incorporated in the transmitter.

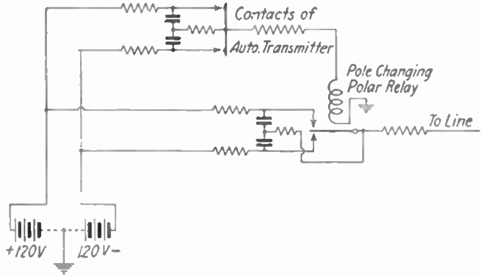


FIG. 11.—Double current-control circuits.

In tone-control systems the equipment at the transmitting station comprises band-pass filters and amplifier-rectifier units. Two stages of transformer coupled amplification, with manual volume control, suffice. The rectified output may be used to operate either electromechanical relays or tube keyers. Where such equipment is used at a large, high-powered transmitting station it must be thoroughly protected from the stray fields of the transmitters, transmission lines, and antennas. It is generally necessary to locate it in a well-shielded room and effectively to filter all control lines and power-supply cables that enter or leave the room.

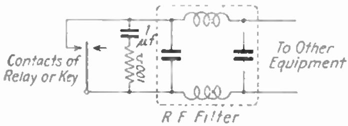


FIG. 12.—Spark absorber and disk filter.

Tube keyers, while more elaborate than the usual mechanical relays, are capable of operating at practically any speed desired. They also eliminate relay maintenance and adjustment. In the simpler arrangements the control tone is amplified, rectified by either a two-element or a three-element tube rectifier, then passed through a smoothing circuit or low pass filter. The d-c pulses thus obtained are applied to the control elements of the keying-stage tube or tubes. Grid-glow tubes have found some favor in such work due to the fact that, once triggered off, the plate current remains at a constant value until the plate voltage is removed, or reduced to a low value. This permits the squaring up of badly distorted envelope shapes without resorting again to mechanical relays.

41. Received Signal Transfer. Systems for transferring signals from the receiving station to the central office are similar to the transmitter-control systems. In short-wave work the actual radio signal, after heterodyne detection, is amplified and rectified and applied to a tube keyer. This may be arranged to supply direct current, or tone, for transfer to the traffic office. Audio-frequency filters, of the same type used for tone control, allow a number of channels to be handled over one line.

Where tone lines are long enough to require the use of one or more repeaters, care must be taken that the sum of the voltages of all channels is not high enough to cause any overloading of the repeaters. If this takes place, intermodulation between channels will be caused, which results in mutilated signals at the central office. With repeatered lines, and the usual band-pass filters, it is essential that all channels be kept at approximately the same signal level. A maximum difference of 2/1 between any two channels should not be exceeded. Large differences in channel levels are apt to cause interference on the weaker ones.

In medium-wave and short-wave receiving stations the contacts of all telegraph keys and relays must be prevented from sparking, and the wires to and from the contacts must be properly filtered. If these precautions are not taken serious click interference will be experienced in the receiving equipment. The same applies to commutator-type electric motors. Circuit breakers should preferably be located in a shielded room.

TRANSCRIBING METHODS AND EQUIPMENT

42. High-speed Reception. As the average operator copies at a rate of only about 40 words per minute, aural reception must be replaced by some method in which a record is made of the signal, on the high-speed circuits, the recorded signal then being copied off at a slower speed by one or more operators. The older dictaphone and photographic methods of recording were not entirely satisfactory. Most systems now use some form of "ink recorder" in which the movement of a pen is controlled by the incoming signal and makes short and long characters on a moving paper tape.

Reception by tape has the double advantage of speed and of there being a record to which the operator may refer or which may be looked up later in case any question arises.

43. Ink Recorder. One commonly used type of ink recorder consists of a small coil suspended in a strong unidirectional magnetic field supplied by an electromagnet. The signal is amplified and rectified and the d-c pulses sent through the recorder coil which, in turn, moves the pen arm up against an upper stop. With no signal current flowing the pen is held against the lower stop by the spring of the pen arm and coil suspension. To improve the action of the device at high speeds the coil is suspended midway between the stops and double current used, in place of pulsating direct current, to operate the coil. This is obtained from a pole-changing relay operated by the rectified signal, or from a

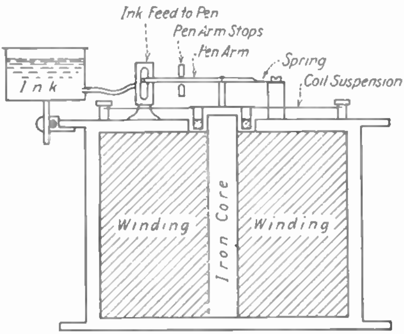


FIG. 13.—Ink recorder. Paper tape and tape guide not shown.

special amplifier-rectifier unit which gives an output direct current in opposite directions for "mark" and "space."

T H I S I S A S A M P L E

References

AUSTIN, L. W.: *Proc. I. R. E.*, June, 1926.

BEVERAGE, RICE, and KELLOGG: The Wave Antenna, a New Type of Highly Directive Antenna, *Proc. A. I. E. E.*, March, 1923.

BYRNES and COLEMAN: 20- to 40-kw High-frequency Transmitters, *Proc. I. R. E.*, March, 1930.

CARSON, JOHN R.: "Electric Circuit Theory and the Operational Calculus," McGraw-Hill Book Company, Inc., New York.

ESPENSCHIED, ANDERSON, and BAILEY: Transatlantic Radio Telephone Transmission, *Proc. I. R. E.*, February, 1926.

FULLER, L. F.: The Design of Poulsen Arc Converters for Radio Telegraphy, *Proc. I. R. E.*, October, 1919.

SHEA, T. E.: "Transmission Networks and Wave Filters," D. Van Nostrand Company, New York.

STEINER and MASER: Hot-cathode Mercury-vapor Rectifier Tubes, *Proc. I. R. E.*, January, 1930.

SECTION 19

TELEVISION

By C. H. W. Nason[1]

1. Statement of the Problem. By looking at an ordinary newspaper half tone it may be perceived that were the mesh of the structure refined there would be a definite point at which the improvement in detail with increasing number of elements per unit area would cease. A study of the interrelation between physiological and practical aspects of television, carried out by a number of investigators, has set upon a degree of definition such as might be obtained by the use of three hundred elements along the side of a reproduced image as being the minimum for a satisfactory reproduction. In order completely to remove the grain of the image structure this number must be set by the size of the image desired.

To transmit an image over an electric circuit, some means for transferring light values into electrical impulses of relative amplitude must be devised. Conversely the re-formation of the image requires a similar change from electrical impulses to light, and such a means of distribution as will reconstruct the transmitted scene at the receiving point. The photoelectric effect serves to convert the variations in light intensity into time variations of an electric current which may readily be amplified and transmitted over a wire or radio channel. The conversion of the electric current back into light variations may be obtained through the use of a high-intensity light source which passes a beam through some form of light valve or by the modulation of the light source itself.

The process of breaking the scene up into elemental units is termed *scanning*, and its effect is that of breaking up the image into a continuous strip which is passed before a sensitive photoelectric device so that an electric wave train is generated which corresponds to the light variations along the strip. In order that no effect of "flicker" may be observed it is necessary that this complete process be repeated at least fifteen times per second (in 1932 general practice was to use 20 frames per second). This process may be summed up as a rapid form of picture transmission. Although many ideas have been advanced concerning the possibilities of direct vision it is at present outside the limitations of the engineering art to devise an instantaneous process which will do away with the necessity for scanning the scene in some manner which is the equivalent of that described.

2. Scanning or Exploration Methods. The major proportion of the systems now in use employ the scanning disk of Nipkow, patented in Germany in 1884, or some similar mechanical or optical equivalent. By this system a series of spirally arranged apertures in a rotating disk

[1] Consulting engineer.

serves to break up the image into a series of strips which provide a continuous scanning of the scene. This may be done by illuminating the object and forming an image on the disk through which the light falls on

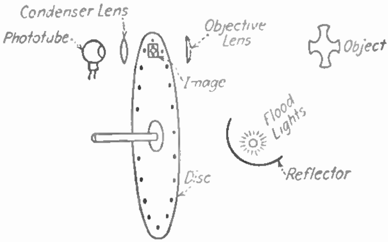


FIG. 1.—Scanning an illuminated object.

a single phototube as is shown in Fig. 1 or by scanning the scene with a pencil of light ("flying spot") from an intense source and picking up the reflected light with several phototubes as in Fig. 2. The theoretical

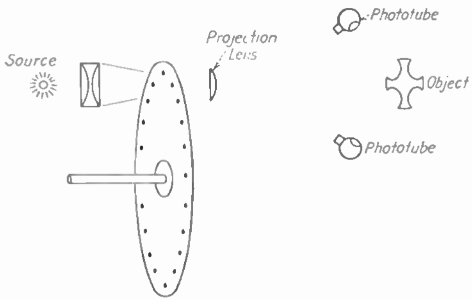


FIG. 2.—"Flying spot" method of scanning.

considerations are alike, although the first case presents serious practical difficulties owing to the extremely small amount of light available at the phototube.

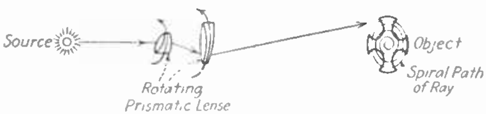


FIG. 3.—Spiral scanning by offset prisms.

Zworykin has used a pair of offset prisms rotating at differing speeds and ground so as to effect a spiral scanning of the subject as shown in Fig. 3. A difficulty encountered here which calls for a great deal of study is the fact that the speed of scanning at the periphery of the scanned area is much greater than that at the origin of the spiral. This may be corrected by careful study of the optical problems.

Many workers—Zworykin[1] among them—look to the cathode ray as the answer to the problem. Farnsworth[2] has developed an extremely interesting tube for the transmitting end, which epitomizes the work done along these lines. This device appears in cross section in Fig. 4. An optical image of the scene is projected through a window *A* at the end of the tube and is formed on the photo-sensitive surface of the device

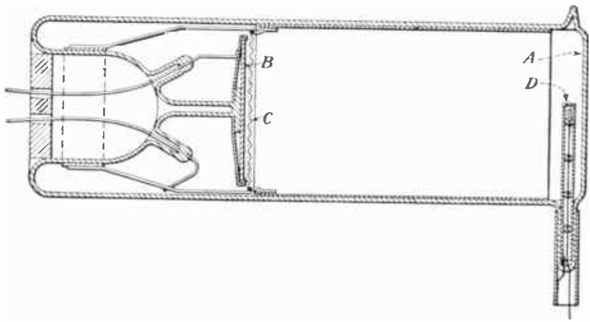


Fig. 4.—Farnsworth cathode-ray tube.

at the cathode *B*. Over this cathode but a short distance removed from it is a grid of very fine wires *C* corresponding to the desired structure of the image—a 200-line image requiring a grid of 200 wires. This grid is given a relatively high positive potential so as to accelerate the electrons given off by the cathode under the influence of the image. We now have a beam of electrons the size of the image directed back toward the window. The cross section of this beam provides an "electron" image of the scene. Two sets of coils are now brought to bear on this beam causing it to move bodily in such a manner that it scans a minute target *D*. Scanning of this target is accomplished by the use of two independent oscillations having a saw-tooth wave form as shown in Fig. 5. The one oscillation provides vertical scanning at a rate of twelve times per second, the other provides the horizontal component at a frequency of 2,400 cycles. The target is carefully shielded except for one minute opening. In some of the Farnsworth tubes the target is made from an electron emitting material so that the photoelectric current is enhanced by a secondary effect. The receiving tube used is termed the "oscillite."

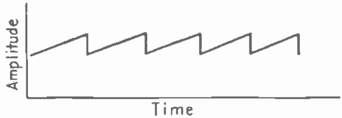


Fig. 5.—Saw-tooth wave form for scanning.

Exploration of the scene may be accomplished in many ways which are either optically or mechanically the equivalent of the Nipkow disk. It will be noted in Fig. 2 that the light available at any instant is relatively small, being the amount passed through one scanning aperture. Jenkins

[1] *Television News*, **1**, No. 1, p. 58.
[2] *Television News*, **1**, No. 1, p. 48.

has evolved a lens disk which allows the entire illumination available at the source to be brought to bear in the scanning operation. This is done by inserting a number of lenses in the disk which focus the light at the image. Horizontal scanning is provided by the rotation of the

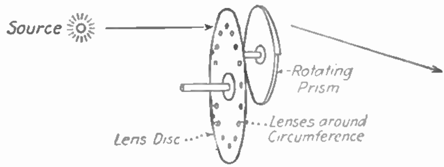


FIG. 6.—Scanning by rotating prism.

disk, while the vertical component may be achieved by giving each lens the proper offset from the horizontal or by placing a rotating prism between the lens disk and the scene as shown in Fig. 6.

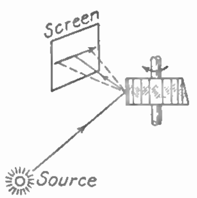


FIG. 7.—Weiller wheel.

In the Weiller wheel, which has been employed to some extent in Germany, the light from the source is focused on a rotating mirror of polygonal form. The various planes of the wheel are offset in such a manner as to provide the vertical component of the scanning motion while the horizontal component is provided by the rotational motion of the wheel. A simple explanatory sketch appears in Fig. 7.

3. Scanning Element. While the general effect is not the same, the detail of a television image may be described as the equivalent of a newspaper half-tone of a certain "screen." The fact that the scanning lines are not definitely broken up into dots gives a softening effect that serves to give an impression of still greater detail.

A 60-line image having an aspect ratio of 1.2, would be 60 by 72 elements, and would have a total number of elements equal to the product, or 4,320 elements. Although it would seem that the full detail implied would be available with a square aperture one-sixtieth the image height on a side, certain effects render this untrue. We assume a square aperture for economy of light, as the round aperture passes but 0.7854 the light admitted by a square aperture of equivalent dimension.

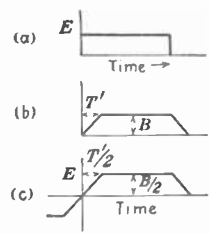


FIG. 8.—Effect of scanning a square-top wave (*a*); showing blurring (*b*).

We have already assumed the conversion of light into a time variation of an electrical current by means of the scanning system and the photoelectric effect. Should the scanning element pass over a section in the scene corresponding to a sharp variation in light density we would expect a resulting wave form in the electrical circuit similar to that shown in Fig. 8*a*. In practice this is not so, as the form is altered to that shown in Fig. 8*b* by the use of a finite aperture or scanning element. Here T represents the time represented by the change in density, and T' the time required by the scanning element in passing completely over the line of demarcation at the origin of the change in light intensity. A Fourier analysis of the a-c wave form in Fig. 8c gives

$$e = \frac{E}{\pi}\int_0^\infty 1/\omega\left(\frac{\sin\omega T'/2}{\omega T'/2}\right)\sin\omega t\,d\omega$$

This differs from the instantaneous value of the rectangular wave form by the factor within the parenthesis. This corresponds to a distortion from the ideal of amplitude *versus* frequency. It is necessary to correct for the expression

$$\frac{\sin\omega T'/2}{\omega T'/2}$$

A more detailed discussion has been given by Horton, Gray, and Mathes[1] and later by Horton[2] alone. In the latter reference it is stated that no noticeable distortion will take place if the value T' is not more than one-third the time required to traverse the smallest complete-cyclic variation in light density involved. This points to the fact that the correction may be achieved either mechanically, through the use of a slender scanning element, or through the employment of correcting networks in the amplifier circuits. In figuring the distortion involved it is necessary that we take account of the scanning-element distortion at the receiving end of the circuit as well as at the transmitting point. In other words the effect is twofold in any circuit.

4. Transmission Methods. The television signal may be considered as traversing an electric wave channel without consideration of whether that channel be radio or wire line. The amplification and transmission problems differ from those of speech and music in magnitude only.

The frequency band required is dependent upon the number of picture elements and the frequency of repetition. This last must be greater than fifteen times per second if flicker is to be avoided. The calculation for the frequency band required is

$$f = \frac{N}{2} \times n$$

where N is the number of picture elements, and n the frequency of repetition. For a 60- by 72-element image employing a scanning speed of 1,200 r.p.m. the calculation is

$$f = \frac{4{,}320}{2} \times 20 = 43{,}200 \text{ cycles}$$

The factor 2 enters the equation through the fact that a complete cyclic variation in light value is required for the production of a single a-c cycle.

It is essential that the low-frequency limit of the channel be not higher than the frequency of repetition. Thus we may assume the band required for fidelity in the image to be from 20 to 43,200 cycles. It has been found[3] that the excellence of the image does not deteriorate

[1] Horton, Gray, and Mathes, *Bell System Tech. Jour.*, **6**, 551–652.
[2] Horton, J. W., *Proc. I.R.E.*, **17**, 9, 1540–1563.
[3] Horton, Gray, and Mathes; *Bell System Tech. Jour.*, **6**, 551–652.

when the over-all response does not vary over a range of plus or minus 2 db over the range. Correction for phase displacement must be carried out over the entire range so that the slope of the phase-shift characteristic does not vary over a range of more than plus or minus 10 to 20 microseconds over the range except at the extreme low-frequency end where the phase shift may vary from the linear by a considerable amount without affecting the excellence of the image. A figure such as that used in judging the excellence, visually, of a given channel is shown in Fig. 9. The degree obtained through the channel is evidenced by the relative sharpness of the angles formed by the black and white portions near the tip of the figure. Blurring of these angles is the equivalent of a loss in the higher frequencies. Deviation from a linear phase displacement is also evidenced by distortion of this figure; a twist of the whole figure at some point in its altitude being evidence of a sharp difference in the propagation time for the particular frequency or band of frequencies represented. Phase displacement or "envelope delay" may be corrected by means of specialized networks having an equal and opposite effect to that found in the channel proper.

FIG. 9.—Figure for judging television fidelity.

If both side bands are transmitted, the width of a television channel calculated to handle a 60-line image at 20 pictures per second is upward of 85 kc. This necessitates a strict attention to all portions of the transmission channel subsequent to the modulator. It is usual to employ coupled-circuit systems exhibiting a double-peaked response characteristic in all tuned circuits where the decrement is not naturally sufficient to result in the transmission of the side frequencies.

It is usual to transmit a "positive image"—which is to say that the maximum amplitude of the carrier wave corresponds to a maximum light value in the subject scanned. Technically speaking this is tantamount to a statement that the low-frequency signal on the grid of the modulator tube (Heising or "constant current" modulation being assumed) has its maximum negative value coincident with maximum light at the photoelectric cell. Since the signal experiences a phase shift of 180 deg. through each low-frequency stage the number of stages preceding the modulator tube must be chosen so as to meet this requirement. The actual number of stages depends upon the character of the input circuit, *i.e.*, whether the grid of the first tube goes more positive or more negative with increased light intensity.

5. Propagation Phenomena. In the reception of the television signal certain defects are evidenced from time to time which require explanation. Foremost among these is the effect of double images. Where the receiving point is quite close, reflection effects cause a second and often a third image to be seen—an image which is slightly displaced in azimuth and often in phase.

The radio signal has a plethora of components. The ground wave assumes a direct path between the transmitter and the receiver; a sky wave may either assume a direct path or a devious one depending upon the frequency of the signal and the effective height of the Kennelly-Heaviside or "reflecting" layer. The optical effects, evidenced when a light beam is caused to pass from one medium into another which is more or less dense, are well known. Since light and radio waves are both electromagnetic manifestations an analogy is not difficult of deduction. The components of the transmitted wave meet an ionized layer in the upper atmosphere which reflects or refracts them in a manner analogous to the optical effect noted above and in a degree dependent upon their frequency.

Echo signals make themselves apparent in the viewing field of a television receiver in a manner shown in Fig. 10. Knowing the rotational speed of the disk and the linear difference in the spacing of the two images and of a third image, should one be strong enough to be apparent, a time factor representing the delay may be evolved. Knowing that the speed of propagation is 300 km or 186 miles per millisecond we may readily obtain a relation between the distance travelled by the ground component and that travelled by the reflected waves. This is shown quite plainly in Fig. 11. Since we are not concerned with the scientific aspects of wave propagation we need go no further into the effects of the reflecting layer. Whether the secondary image appears in a positive or a negative sense is determined by the relative phasing of the two carrier components as has been demonstrated by E. L. Nelson in an explanation of the phenomena.[1]

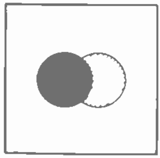


Fig. 10.—Echo signals caused by multiple-path transmission.

Effects as noted above may be cured by the use of antennas having corrected propagation as to the relative strength of the ground wave and the sky wave. In reception close to the transmitter the reflected wave is not in evidence at some frequencies and reception of the ground component alone is obtained. At an intermediate distance both may be received in such a manner as to render a perfect image impossible, whereas at long distances the ground wave drops out entirely and only the reflected wave is received. Experiments in Germany using shielded antennas to eliminate the reflected component entirely have resulted in good reception within a specified service area where no distant reception is desired. Commercial elimination of the effect will necessitate a correlated choice of frequency and type of radiator employed.

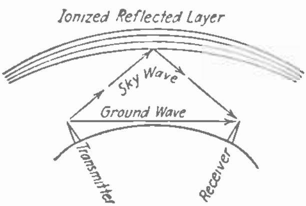


Fig. 11.—Paths traversed by radio wave.

Observations on metropolitan transmitters show that considerable difficulty will be experienced in commercial efforts in large cities because of reflection effects due to large masses of metallic structure. Continued study of antennas and choice of location should sooner or later render these effects innocuous.

6. Receiving Systems. Except in quantitative degree, the requirements of receiving systems do not differ greatly from those of sound receivers. Leaving sensitivity considerations behind, the specifications for a television receiver merely require that the relative phasing between detector input and power output be such that maximum carrier amplitude corresponds to maximum light intensity at the viewing point, that the fidelity be such that the response be flat within 2 db from about 15

[1] Horton, Gray, and Mathes; *Bell System Tech. Jour.*, 6, 551–652.

cycles out to some high frequency, and that the phase displacement be linear with respect to signal frequency. The first consideration is met by employing an even number of l-f stages where plate circuit detection is used and an odd number of stages with grid-current detection. The fidelity considerations may be met by the use of coupled-circuit systems together with carefully designed l-f amplifiers. The last demand is satisfied by avoiding resonant conditions in the l-f system. While transformers have been developed for use in apparatus covering so wide a range of frequencies, they are of such character as to be outside the abilities of all but the most capable designers.

Leaving aside "trick" systems where synchronizing signals are superposed upon the image variations or where attempts are made to convey sound and scene over the same channel, there are just two possible receiving systems. In the first place we have the system employing detection and l-f amplification as commonly employed in the reception of sound and in which a.c. corresponding to the image variations is applied to the light-producing device. In the second type receiver no detector is employed and r.f. is applied to the light-producing device, the variations in carrier amplitude being evidenced by increasing or decreasing brilliancy of the light source. Use of the second system demands a separate channel for the reception of sound from the television transmitter or a means for rectification of the signal during announcement periods.

Trouble has been experienced in many commercial television receivers through the use of untuned input stages or lack of selectivity prior to the first r-f stage. Although the harmonics of the broadcasters appear in the television band they are not of sufficient amplitude to swamp out the television signal. Use of untuned input stages results in a strong component of carrier frequency being produced across the grid of the first r-f stage by strong local broadcasters. Operation of the first r-f tube on an unfavorable portion of its characteristic curve, through this type of overloading, results in a strong harmonic of the broadcast signal in the plate circuit of the first tube—a harmonic that is amplified in the successive stages when close to the frequency of the television signal and causes added trouble. This is aside from the direct modulation of the television signal which occurs. Except for the fact that harmonic production in the plate circuit of the first tube may result in amplification of the broadcast harmonics, the selectivity considerations do not differ from those of a broadcast receiver.

7. Transducers and Light Valves. The transducers employed in television for the most part are gaseous discharge tubes employing a rectangular plate of the image size upon which the discharge glow forms. The other electrode or element of the tube consists of a wire ring or loop placed optically behind the plate. In most cases these elements are of nickel, although well-cleaned silicon steel has been found to give an even discharge. One make of discharge tube for television purposes employs two rectangular plates so that the discharge will appear on a flat surface without regard for the polarity of the d-c energizing potential.

The rectangular surface is the cathode element and the wire ring constitutes the anode. In the vacuum-tube circuit the tube acts as a resistance of rather high value as determined by its current and voltage characteristics. Its a-c characteristics are something quite different, however, the impedance of the tube being obtained from the slope of its characteristic curve. In Fig. 12 appears the characteristic of a neon tube of the type available for television experiment. The slope of the curve corresponds to an impedance of about 6,500 ohms, while the d-c resistance of the tube is about 26,000 ohms at 10 ma. The particular tube has excellent characteristics as far as its operation as the load for a vacuum-tube amplifier is concerned. The tubes now obtainable for use in experimental work have a d-c resistance of from 8,000 to 10,000 ohms and an impedance of from 500 to 1,000 ohms—not ideal for use in the output of a vacuum tube. It is necessary, because of the inadvisability of employing a transformer, to operate the output tube considerably under its rated capacity or to use several tubes in parallel the better to match the impedances.

An approximate match can be obtained by the use of a resistance in series with the neon tube in the plate circuit. This aids in matching the impedance of the load to that of the tube and does not incur a great loss in output power available, since the variations in light intensity are a function of the current through the neon tube.

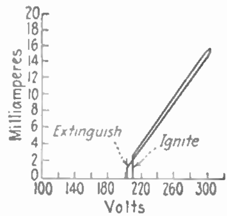


FIG. 12.—Characteristic of neon television tube.

The tubes mentioned above are for use with the normal type of scanning disk and the largest picture obtainable is therefore determined by the size of the plate upon which the discharge is formed. For optical scanning systems—Jenkins disks, Weiller wheels, and the like—a point source of light is desirable. The recording lamps common to the motion-picture industry may be used in this connection, although they are not capable of handling a great deal of power. Point sources are similar in their characteristics to the ordinary type of discharge tube described here. Tubes handling a large amount of power must have adequate cooling, and in some systems this is provided through the use of water jackets similar to those used in large transmitting tubes. The tubes used by the Bell laboratories have also provision for valving-in hydrogen as the tubes age, thus lengthening their useful life by removing the sluggishness in response, which develops as the gas remaining in the elements is expelled through heating. Naturally the valving-in of gas

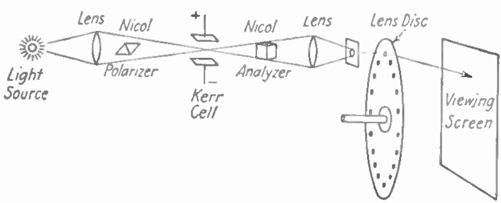


FIG. 13.—Nicol prism—Kerr cell light-valve system.

and water cooling are features beyond the limitations of home television apparatus.

Light valves of a mechanical character such as are employed on certain film-recording systems have been found incapable of handling the high frequencies of variation found in television but those applications of the Kerr effect which are common to the chemical engineering art are found well suited to the purpose. Here a powerful light source is intercepted by a nicol prism—a complex unit of Icelandic spar which has the property of polarizing light. A second nicol prism interposed between the first, and an optical scanning system in the manner shown in Fig. 13 will result in total extinction of the light, when the axis of the second prism is at right angles to that of the first. The Kerr cell is now interposed between the two prisms. This is an instrument which consists of two plates immersed in a dielectric of nitrobenzene, carbon bisulphide or some other transparent dielectric material. The dielectric has the property of rotating the plate of polarization of light passing through it when a potential difference exists between the two plates. The degree of light passing through the system will be found to vary in accordance with the potential applied across the plates of the Kerr cell in the manner shown in the curve in Fig. 14. This, it will be seen, is not linear in form, and a d-c bias such as to operate the Kerr cell over a linear portion of its characteristic is necessary. The biasing point is shown in the curve. There are other methods of polarizing light and other methods of rotating the plane of polarization by electrical means. The basis of all such systems is expounded above, and no comment on other possibilities is needed.

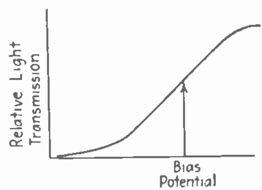


FIG. 14.—Characteristic of Kerr cell.

8. Analyzers at the Receiving Point. It is necessary to achieve some system of distribution at the receiving point corresponding to that of scanning at the transmitter. In the simplest apparatus the scanning disk of Nipkow is employed with a rectangular discharge tube placed behind the disk. With the disk rotating in exact synchronism with that at the transmitter, the variations in light intensity will correspond with the variations in illumination density in the scanned scene and an image of recognizable aspect will result—its quality and excellence depending upon the excellence of the channel as a whole. All of the scanning methods described in the first instance are applicable also in the reconstruction of the image at the receiver. The optical systems are to be preferred because they concentrate the total light available from the transducer at one point on the viewing screen, while the scanning disk is capable of no greater illumination than that available through a single aperture at any instant. By proper optical design the light efficiency of an optical scanning system can be advanced to a high degree. In these cases the size of the projected image at the viewing screen is limited only by the available light and by the degree of apparent definition required. The larger the image obtained the poorer will be the structure of the image.

The methods employed in reception by means of a cathode-ray tube are markedly similar. A tube is used with a fluorescent-viewing screen at one end and a source of electrons at the other. Elements for the concentration of the beam, for the variation of its intensity, and for varying its direction in two senses must be provided. Where the scanning at the transmitter is accomplished in the normal manner by traversing the scene from left to right and from top to bottom, the scanning of the cathode ray across the fluorescent-viewing screen must be in the same manner. This makes necessary the use of a deflecting influence having a wave form such as that shown in Fig. 5. This may be obtained mechanically by the use of a motor-driven potentiometer or by the use of a neon lamp as an oscillator. The general aspect of the cathode-ray tube as used in receiving a television image is as shown in

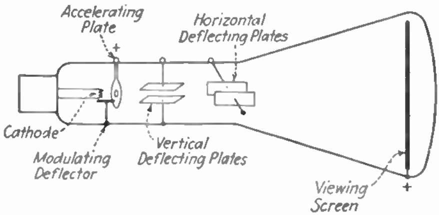


FIG. 15.—Cathode-ray tube for television.

Fig. 15. The systems of Zworykin, Farnsworth, and von Ardenne are similar in character.

9. Synchronization in Television. It is essential that complete synchronization obtain between the moving elements at both terminals of a television channel. By synchronization we do not mean the mere fact that the motor elements are rotating at a constant and uniform speed, but that they are in exact juxtaposition, element for element. This does not merely require that the frequency of the supply for transmitter and receiver remain identical and constant, but that the phase relations also remain intact. Where the two are fed from the same a-c supply source it is necessary that the relative phase alone be considered—the frequency remaining constant. In correcting for phase displacement it is essential that the motor frame or the scanning disk be rotatable to correct for the difference. It is possible to correct for an image out of frame in the vertical sense by switching the supply current on and off rapidly. Phase displacement evidenced by the image being out of frame in the horizontal sense may only be remedied by a mechanical means which permits of shifting the relations of the rotor and stator of the driving member while in rotation.

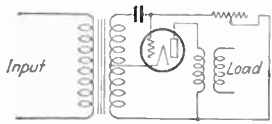


FIG. 16.—Thyratron method of phase adjustment for proper framing.

It is also possible to achieve this end electrically by means of a Thyratron circuit of the type shown in Fig. 16, where the phase rotation through the system is variable by adjusting the resistance. Adjustment of the frequency of two remote a-c circuits may also be carried out by means of the Thyratron so that a simple synchronous motor, together with Thyratron circuits for the adjustment of both frequency and phase, may be used in driving the scanning system. It is also possible to achieve synchronism by the use of a double-drive system in which a series-wound variable-speed motor is used to maintain the system in rotation while a second small synchronous motor maintains the speed constant. This synchronous motor may be driven by the output of a vacuum-tube amplifier. The characteristic frequency or scanning frequency of the television signal is obtained by multiplying the number of scanned lines by the picture frequency. In the case of the usual 60-line transmissions this results in a frequency of 1,200 cycles. A tuned amplifier receiving its input at any point in the l-f system of the television receiver will receive enough of the 1,200-cycle component to drive a small synchronous motor with sufficient power to maintain the system in synchronization. Such a motor need only consist of a pair of coils and a gear of magnetic steel—preferably laminated, the number of teeth depending upon the frequency of the supply, the desired speed, and whether d.c. is present in the windings. With d.c. present the number of teeth is half that required when a.c. alone is in the windings. With d.c. present the number is equal to the frequency divided by the number of revolutions per second desired. With no d-c excitation, twice this number is required. The form of such a motor is shown in Fig. 17.

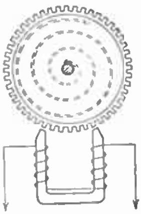


FIG. 17.—Synchronous motor.

There is a possible variation of this scheme known as the *thermionic brake* which has distinct possibilities. In this system the wheel shown in Fig. 18 operates as a generator and supplies plate voltage to a pair of tubes connected in push pull but with no applied grid or plate voltages. When the driving motor approaches the proper speed there will come an instant when the grid potential of one of the tubes and the plate potential will swing positive at the same instant. A degree of plate current dependent upon the magnitude of these voltages, but none the less high, will flow. At the correct speed, *i.e.*, when the plate and grid voltages are in synchronism, a corresponding effect will hold good in the very next

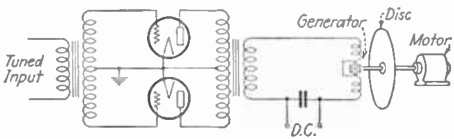


FIG. 18.—Thermionic brake system.

half cycle in the other tube of the system. Thus a continuous load will be placed on the driving motor such as to limit it to the desired speed. Should it fall below this speed, the braking effect will be instantly withdrawn and the speed will increase until the cycle described is repeated. A schematic version appears in Fig. 18.

References

BARTHELEMY, R.: *L'onde Electrique*, **10**, No. 109, 5–35.
NASON, C. H. W.: *Radio Broadcast*, **16**, No. 6, 318.

SECTION 20

FACSIMILE TRANSMISSION

By R. H. Ranger[1]

1. General Requirements. Elementary dots in a half-tone picture, printed in a newspaper, illustrate how a picture must be analyzed in photo units in order to transmit it electrically to a distance. There is no known way that a picture may be transmitted in its entirety electrically from one point to another; the process consists first in tracing across the original picture point by point in consecutive lines; second in sending a representation of the values of each such traced point over the communication system; and third in putting down the point representations in the proper places on the recording sheet to build up the picture at the receiving station.

The finer these picture elements are taken in the analysis the finer will be the resultant detail in the recorded picture as compared with the original. The number of such photo units to the picture therefore indicates the resolving power of the system.

It takes 50 dots in a line across a picture and 50 lines down a picture to represent a face in good shape. This means 2,500 dots or photo units total. Photo units to the number of 300,000 are required to get across a scene that would normally be required in newspaper work.

It makes no difference as to the size of the finished picture, the resolving power is entirely a question of the number of photo units transmitted in good shape. It takes just as many tiny photo units to represent a face well on a postage stamp as it takes to represent a face in larger photo units on a 10-ft. enlargement. Naturally it will be necessary to stand back from the large pictures to get the effect, but the detail will be equal in each case.

The maximum possible difference between two consecutive photo units would be that one was white and the next was dark. As the normal representations of this transition would be from a positive value of current for one and a negative or minimum value for the next, a complete electrical cycle would be represented by the two successive photo units. Therefore, the modulation impressed on the carrier in picture transmission is anything up to a frequency one-half the number of photo units being handled.

Photo units may be handled over normal telephone lines at the rate of 800 a second corresponding to a maximum modulation rate of 400 cycles. This will cover a 5-by-7 picture of a scene in 7 min. It must be borne in mind that a band width considerably in excess of the modulation rate should be allowed for picture transmission. The third harmonic of the

[1] Fellow, I. R. E., Royal Society of London; Member, A. I. E. E., American Physical Society, Optical Society, President, Rangertone; Inc.

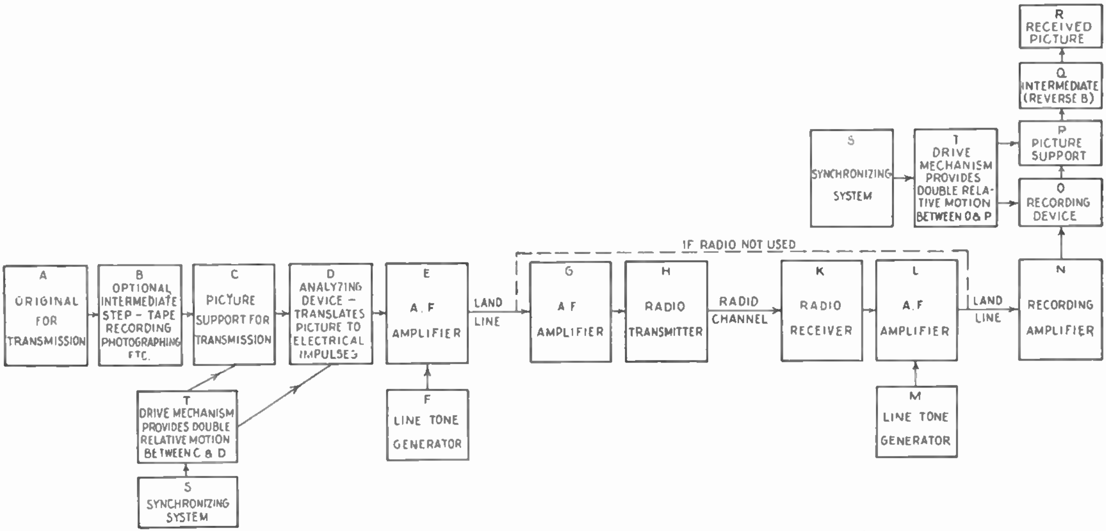


FIG. 1.—Block diagram of steps in picture transmission and reception.

modulation rate should be planned for; this means that a single side-band width of 1,200 cycles or a complete band width of 2,400 is generally necessary for the transmission of a 5-by-7 picture in 7 min. A carrier at least ten times the modulation rate is recommended.

Table of Units in Code and Facsimile

Average Values

(1 word = 5 letters + 1 space)

Code Transmission (Continental):

1 word requires 24 cycles modulation. $50 \frac{\text{words}}{\text{minute}}$ require $20 \frac{\text{cycles}}{\text{second}}$ modulation and therefore require at least a 200 cycle carrier and 60-cycle band width.

Facsimile Transmission:

6 lines typewriting to vertical inch
12 letters or 2 words to 1 in. of line
12 words per square inch
1 letter requires 60 photo units or 30 cycle modulation

1 word requires 360 photo units or 180 cycle modulation. $50 \frac{\text{words}}{\text{minute}}$ require $300 \frac{\text{photo units}}{\text{second}}$ or $150 \frac{\text{cycles}}{\text{second}}$ and thus require a band width of 450 cycles and a carrier of at least 1,500 cycles

Synchronism involves the timing of the tracing or scanning means at the transmitting and receiving points such that they are tracing corresponding points of the picture at the same time to prevent distortions between adjacent lines. This synchronism should be accurate to one half of a photo unit to prevent noticeable jiggles between lines; and between the start and finish of a picture it should be accurate to 1 part in 30,000. This means, for example, that in the traversing of the 5-by-7 picture, mentioned above, the tracing point will cover 3,500 linear inches in going over the picture line at a time, when those lines are 100 to the inch. And at the bottom of the picture, the accuracy of synchronism should be such that the final dot is no more than $\frac{1}{10}$ in. out. Further inaccuracy would objectionably skew the picture out of plumb.

As a matter of practice, each end of an independently synchronized picture station tries to hold its synchronism to 1 part in 100,000 so that the over-all synchronism will not be out more than 1 part in 50,000.

2. Scanning Methods. To pick up these elemental photo units that make up the transmitted picture, it is necessary to have some sort of a tracing pencil which goes over the entire picture. In the very first picture-transmission method of over 90 years ago an actual fine metal brush was suggested. This was fastened to the end of a swinging pendulum which therefore discribed an arc across the picture being transmitted. The picture consisted of a drawing in shellac on a piece of tin foil. At the end of each pendulum swing the table on which the foil was mounted would space down a small amount, corresponding to the next line of the picture, and the brush would then swing across again. This is one form of an analyzing head; but by far the most popular method has been some sort of a cylinder mounted in the manner of a lathe with the cylindrical drum carrying the picture to be sent and a point picking up the picture values, or, as has now become the method, of a fine point of light illuminating the picture, a point at a time. As the cylinder turns, the tracing point travels down the length of the cylinder, so that at the end of a certain time, the entire picture will have been traced.

All of these motions are relative; either the drum may rotate or the point of light, or one may be rotating and the other moving axially to accomplish the resolution at right angles to the first. In any event, the picture is covered line after line, and these lines may be curved or straight. Each method has certain values of its own.

Reciprocating motions may be used for the scanning, in which the tracing points move back and forth horizontally by means of a right and left worm, and the paper is advanced forward for each successive line. This means that the tracing is done first from left to right, and then from right to left. It has the advantage of working for continuous feed of paper, but it has the disadvantage of requiring greater accuracy in order that the successive lines may be very accurately framed so that the right and left scanning registers accurately at each end.

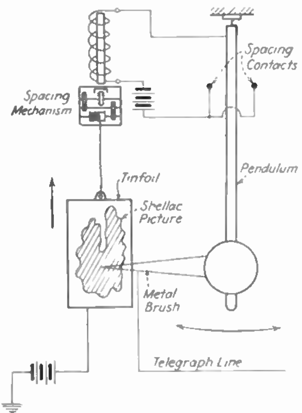


FIG. 2.—Alexander Bain's original picture apparatus (1843).

3. Inherent Accuracy. A word on inherent accuracy may here be given. This means that the system which does not require split thousandths for accuracy but instead a tendency to cancel out what errors there may be in the operation will have therefore by far the greater chance of successful operation. It is inherently accurate. At least its operation is such as to minimize inaccuracies in mechanical motion and electric timing, which are found to exist in any system.

A way of overcoming the inherent inaccuracy of the reciprocating motion, and still maintain the advantage of continuous motion, is provided in the spiral and bar recording equipment. The spiral rotates on one side of the paper, and the bar moves to and from the other side of the paper energized by the picture signals so that the point at which they will contact at successive intervals moves from one side of the sheet to the other, and then starts over again at the original side. It is used in carbon recording and chemical recording.

The lens disk accomplishes the same purpose at the transmitting station by moving a beam of light optically from one side to the other of a picture, and immediately starting again from the original side. It may be realized that it is inherently easier to get optical systems to register moving as a radius, as indicated for the stationary cylinders, than it is to have the beam cross a flat object.

In one method of scanning, the analysis is in two directions. First, a line is traced at an angle downward from left to right, and then a line downward from right to left starting at the same general height.

The net result is a cross-screen analysis of the picture. The analysis between successive lines of a picture is always far sharper than the analysis between successive points of the same line, due to the slower changes in value found in progressing from one line to the next. The cross screen takes advantage of this in that it tackles each photo unit from two different angles and therefore gives a better average definition between points. But it does require far greater machine precision for the actual structure. A simpler method of accomplishing the same result is to transmit the same picture twice and place it in the machine first so that the lines are vertical, and second so that they are horizontal. The pictures are then made into a composite print at the receiving point. This method averages out faults and provides pleasing results.

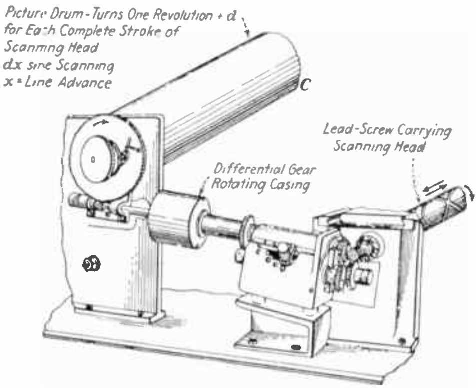


FIG. 3.—Cross-screen analyzing scanner.

4. Photo Analyzers. To pick up each photo element in terms of current value, so that it may be transmitted electrically, there must be some electric identification of the element. The first method was to construct the original picture in such form that it had in itself different electric characteristics. This was done by writing on tin foil with an insulating medium such as shellac. Then, when a metallic brush swept over the untouched part of the picture, electric contact was made between the metallic brush and the metal of the foil. When the brush came upon the shellac the circuit would be broken. This then constituted the differentiation between the two parts of the picture.

5. Photo-engraving Method. Instead of making the picture by drawing with shellac, it is possible to take advantage of the photo-engraving art in which a raised metal replica of the picture is made in dots which normally would take printer's ink. Instead of this, however, they are made to establish the electric contact for the tracing point. And instead of the usual dots of the half tone, it was proposed to use line screens in which the picture is etched in straight lines of varying width across the picture. The tracing point was then made to cut across these lines at right angles such that it made contact for greater or shorter duration as it crossed. This line method required much less accuracy of adjustment between the contact point and the lines of the picture than if the dots were used. The engraved lines of the picture were usually filled in with some insulating material so that the tracer rode across either the insulation or the live metal. The difficulties with this method were in the contact. Working with larger originals made the problem easier.

6. Potassium Bichromate Method. The engraving art was also adapted to the picture art in the use of bichromate prints. Potassium bichromate dissolved in a gelatine has the property of being light sensitive in such manner that where the light strikes the combination, the gelatine will be hardened and made quite impervious to water. Therefore, if a plate covered with such prepared gelatine be exposed to strong light through a negative (or positive) of the picture transmitted, the gelatine will be correspondingly hardened where the light passes through to greater or less degree. Subsequent treatment with water will make the parts less struck by light to rise. The result is that an impression in relief of the desired picture is produced. This relief print may then be placed under the scanning point which will rise and fall as it passes over the relief picture and either turn on and off a current for a black-and-white picture, or increase and decrease the current by a microphone contact for a photograph.

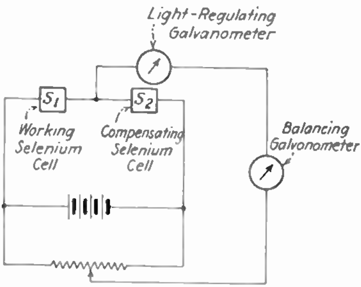


FIG. 4.—Compensating selenium cell lag. (*Korn.*)

Attempts have been made to interpret picture values electrically, by the variation in resistance of the different amounts of silver suspended in the usual photographic film, but the differences are not sufficiently regular to work out nicely, electrically.

7. "Pencil-of-light" Method. All of the above mechanical methods have been most carefully worked out and many pictures have been transmitted by them, but the inability of such means to pick up the fine detail of photographs has had to give way to the inherently easier methods of using a light pencil for the scanning device. But a pencil of light has limits of definition far beyond the requirements for picture transmission. This was early recognized, but the means for interpreting the light changes electrically were not so readily accomplished. The first useful tool in this direction was selenium, which by accident showed its

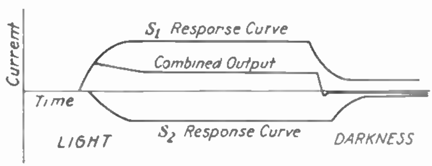


FIG. 5.—Compensation effect of circuit shown in Fig. 4.

change in resistance with change in illumination. This change is really quite large, but it has the drawback of not being immediately self-restoring. Some ingenious compensators have been arranged for this sluggishness which is a form of polarization. The compensation usually takes the form of two selenium cells working at a different time rate such that the quick one precedes the slow and establishes a new base line for subsequent changes by the fast one. Wonderful technique was established in this direction and excellent pictures were transmitted.

But again inherent simplification was welcomed in the true photoelectric cell in which the sluggishness is beyond the normal means of detection. Such cells are not so sensitive as selenium cells, so they in turn became of use only when the vacuum-tube art produced the efficient amplifier.

8. Photocell Amplifiers. Unless very excessive light values are used, the change in illumination of a normal photocell is not such as to give more than $\frac{1}{2}\mu$ a change from the black to white parts of the picture. Care must be taken to see that as much light change as possible comes from the picture to the photocell. This means that as great light efficiencies as possible are required. An easy rule is to keep the number of optical elements in the light system to a minimum. A good prism may reflect 95 per cent of the incident light; a metal mirror cannot do much better than 60 per cent; but both these percentages will be completely vitiated by a little dust on the surface. Every lens surface reflects wastefully some of the light supposed to pass through it; so inherently more light is passed by reducing the number of elements in the system.

9. Direct-current Amplification. Two general methods of amplification are valuable for photocells: so-called d-c amplification and a-c amplification. D-c amplification means that the coupling between the photocell and the successive stages of amplification will be such as to pass all changes of any frequency, from zero up. In other words, the average illumination of the picture will be effective through the amplifier. An a-c amplifier means one in which the coupling is such as to pass only changes which occur at a rate exceeding a certain minimum. D-c amplification is usually accomplished by means of some resistance or reactive and resistance coupling between stages. A-c amplification is accomplished by capacity coupling between stages or by transformer coupling. It should be pointed out that d-c amplification may still be accomplished as far as the picture is concerned if the original photocell current changes are made to modulate a tone which is then in turn amplified as a.c.

10. Modulated D-c Amplification. The more general way of accomplishing the same result is by means of a chopper wheel interposed somewhere in the optical system. A chopper wheel is a rotating disk with symmetrical holes to break up the light into individual pulses passing through the optical system, and giving rise to a characteristic tone throughout the electric system. The modulation for this tone should be proportional to the changes in density of the picture being transmitted as the successive elements are traced by the analyzing point of light. The changes in intensity of the tone should be linear with respect to the changes in light density of the picture. There are so many places where the transpositions are apt to follow the square law that care must be exercised to use inherently linear arrangements. It is obviously very bad if the square law happens in the same direction in two parts of the system making the resultant interpretations of the picture changes follow the fourth power of the original. The result of such distortion is to lose much of the detail of the picture; the eye is not capable of following many changes in density anyway, ten perhaps at most, so if some of these changes are crowded into either the light or dark parts of the picture by non-linearity in the operation of the system as a whole, the resolving power of the system is materially reduced.

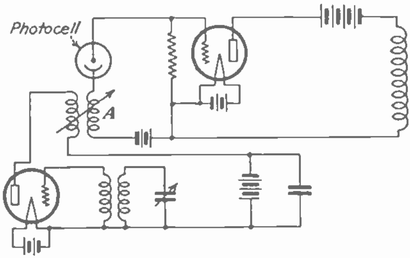


FIG. 6.—Tube-generated tone is fed into photocell circuit at *A* to permit a-c amplification of photo currents.

11. Necessary Accuracy. It may seem that such accuracy is only necessary for photograph transmission, and that for the black and whites of printed matter and diagrams such distortions may be tolerated. This is of course true where extreme simplicity may permit it, but generally speaking this is only an excuse, and good linear response is as advantageous in black and whites as it is on photographs. It means that truer resolution of the lines of black-and-white matter will be made by a certain density of photo units when the response is linear, than would be made by the same number of photo units per square inch of the paper if the response is not linear. Where the response is not linear, the reproduced picture will take on the built-up brick structure. It is of course likewise possible to reduce the effect of the square law in one part of the system by interposing another square law in the opposite direction somewhere else. But as usual no two wrongs can make a complete right. True design consists in replacing the square law by inherent linearity.

12. Balanced Circuits. In the same manner that the push-pull amplifier balances out some non-linearities in ordinary amplification, some of the defects in photocell amplification may be accomplished by balanced arrangements. One of these consists in using two photocells, which receive the light alternately from a mirrored chopper wheel. This reduces general interference from extraneous disturbances. One of its chief advantages lies in the fact that it balances out the actual light chopping by the picture elements.

This may seem a curious desideratum, but the reason is that the relative low-frequency impulses coming from the original picture will distort the beginning and end of the tone impulses. What is desired is a true modulation of the communication system in terms of the density of the picture. After the carrier has been so modulated it is not necessary to carry through the frequencies which caused such modulation, and to do so may easily cause distortion due to the fact that the frequencies are so far apart that they will be handled differently by all parts of the system, in both intensity and rate of response, and the result with all such intermodulations present is to widen out the markings on the picture, making them muddy and even producing shadows and ghost images, which look like echoes.

Take, for example, the 150-cycle modulations corresponding to 50-words-a-minute facsimile; assume that this is put on a 2,500-cycle carrier. Then the side bands will be 450 cycles either side of this, allowing for the third harmonic of the modulation frequency. This brings the lower side band down approximately to 2,000 cycles. Now all the essentials for getting the facsimile across are in the band from 2,000 to 3,000 cycles. If at the same time, the system attempts to handle all the straight modulation around 150 cycles, it is obvious that it will require special treatment to handle this in exactly the same manner that it does the higher frequencies. "Retlifs" may be used to correct phase lags, or filters may be used to pass only the desired upper band, but again the simpler method is inherently to cancel out the lower modulation frequencies before they are formed electrically. This is done by balancing the two photocells with their associated amplifier tubes such that they receive equal response from each half of the chopped light and by then connecting the two amplifier tubes in push pull.

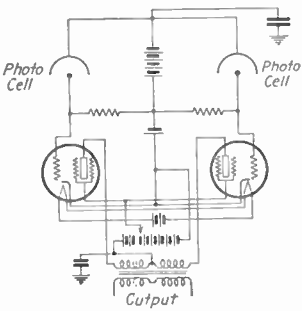


FIG. 7.—Balanced push-pull photocell amplifier.

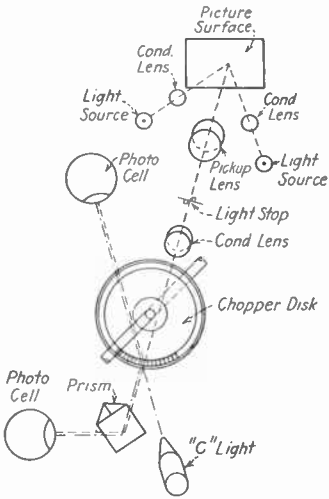


FIG. 8.—Balanced photocell optical system with chopper mirror and biasing C light.

13. Reproduction of Dark Portions. It is always harder to get good modulation in the darker parts of the picture. The photo currents are so low that the variations are around the threshold values of the cell and amplifier set-up. A convenient method of raising the tone above this noise level is to use a C light. This light acts to move the response to a more effective portion of the curve, just as a C battery does for an amplifier tube. In this case, it consists of a small light which is always shining through the chopper wheel to produce a tone with the photocells. Its light path must be adjusted so that it will be in phase with light projected from the picture. Under these conditions, it is seen that there is a definite minimum value to the tone in the push-pull output of the amplifier, and this tone is added to by the picture light values. This method is particularly useful if rectification of the tone is to be used later in the system, and the power capacity of the system is such as to handle the full value of the *C* tone plus the picture tone, for then the rectified response is moved well up on the square-law curve where it becomes quite linear for the picture changes. Likewise, the output is greatly increased by this process as the picture change has become a product function with the *C* light. For example, with a *C* light tone four times as strong as the average picture tone, the output will be eight times the output without the *C* light tone. The limit to such methods is the power capacity of the last tube in the amplification and rectification.

It may be noted that if the *C* light is adjusted 180 deg. out of phase with the picture tone, the output will be 180 deg. out of phase with the input; *i.e.*, stronger light from the picture decreases the tone. This is particularly useful in case the tone is to modulate a radio transmitter directly in which case the dark part gives maximum tones and the light part minimum, and even no tone for pure white if such adjustment is desired.

14. Use of Triggered Oscillators. A very interesting possibility in photocell amplification is the use of triggered oscillators. Of course straight modulation of an oscillator may be accomplished after d-c amplification of the photocell currents, but likewise a balanced system may be set up with two tubes as two arms of a Wheatstone balance. One of these tubes will be merely an adjustment for the other active one which is operated by the photocell currents. The tone may be introduced as normally in a Wheatstone bridge between the outer ends of the balance, or it may be introduced into the screens of both tubes using the new screen-grid tubes. In passing, it may be noted that suppressed carrier modulation may be obtained by acting on both tubes in the bridge push pull.

15. Electric Retouching. Finally, mention should be made of the use of electric retouching in photocell amplifiers. This consists in overemphasizing the intensity of quick changes. That is, where a sharp change in picture density occurs, the amplifier will overaccentuate the change. This is done by inserting resistance in a d-c amplifier stage and then shunting that resistance with a capacity. The same thing may be accomplished by reducing the gain in the center of the frequency band of the modulated carrier. The effect on the picture is to give a deeper black line at a quick change from white to black and a cleaner white line next to a change from black to white. This increases the snap to pictures which is especially desired by newspapers.

16. Changing Intensity to Time Values. There are two general methods by means of which the picture densities may be made to change electric values. One is to represent the density changes directly in current intensity changes. If the communication system is such as to maintain such intensity changes proportional throughout from transmitter to receiver, it is by far the best method. But a uniformity corresponding to 0.1 db is found necessary in order that objectionable streaks do not result in pictures. This is obviously difficult over any but the best lines, and quite impossible over present long-distance radio circuits. Fortunately, another method is possible; this consists in changing the picture density changes into interrupted current of uniform intensity. It is called the *telegraphic method*. Obviously a very short dot on the received picture, although it be printed absolutely black, will appear gray against a white background in the finished picture. Therefore the problem consists in changing the picture densities into dots or dashes of different lengths to make the appropriate imprint on the received picture. The method in which a tracer point went over a line engraving is one method of obtaining this change from intensity to dots and dashes. The half-tone screen has already interpreted intensity valuations into area valuations in the half-tone engraving process. The making of such an engraving before transmitting is expensive and time consuming. A more direct method builds up similar dots by adding some form of pulsating circuit to the electric response from the photocell. If a sensitive relay is included somewhere in the intensity-change electrical circuit, and this relay will turn on and off at well-defined limiting values, it is possible to add to the picture signal an alternating current preferably of a saw-tooth characteristic, and then the relay will trigger at times dependent upon the sum of the voltages from the alternating current and the picture currents.

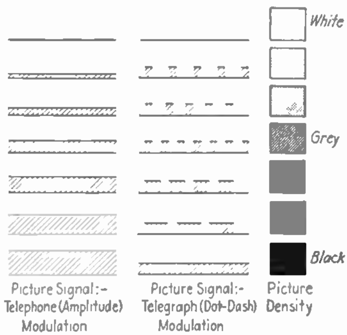


FIG. 9.—Telephonic and telegraphic interpretation of photo densities.

17. Use of Revolving Commutator. An old method is to make use of a revolving commutator which revolves once for every photo unit and is raised vertically by the picture changes. It has been suggested that even the relief picture would be sufficient to accomplish this. As the commutator rotates it will strike an inclined segment such that for raised parts of the picture the commutator will be closed around a longer arc of the commutator rotation.

18. Marginal-relay Method. Another simple method is the use of several marginal relays, the relays being set at successive values such that one will close at a low value of current from the picture, the next will close at a slightly higher value, and so on. Then a commutator picks up the relays in succession and will therefore deliver a longer impulse to the line if more of these relays are closed. Another method consists in making several black-and-white engravings of the picture. These engravings are purposely made to be either black or white at different densities of the original picture, by methods well known in the engraving art. Then these engravings are all mounted on a common axis on a cylinder and are rotated under separate tracing points, one for each separate plate. These points act exactly as the marginal relays mentioned above, in that a rotating commutator closes through each tracer point in succession and the more of these points that are in contact with their respective plates, the longer will be the contact on any given rotation of the commutator. In spite of the seeming complexity of this method, many successful pictures have been sent by cable in this manner.

19. Change of Marginal Setting. Another method has been to change the marginal setting of the photo current on each complete revolution of the drum carrying the single picture. Then every time the picture value crosses the marginal limit, the current is closed to the transmitter. The scanning lines are necessarily taken much closer together so that the picture is built up of what look, on enlargement of them, like very long relief sections of the picture.

The very fact that the successive lines of the latter method must be made one-fifth as wide in order that the individual lines will be unresolved by the eye and that a smooth series of tone values be given the picture show that it takes much more effort to give half-tone values than it does black and white. If modulation may be used as in the telephone method, it is not necessary to break up the picture into separate photo units, as the intensity changes are maintained over the complete communication system; but if such an intensity change has to be registered in time element changes, it is obvious that those time elements must be definite divisions of the photo unit. That is, if five different tone values are to be registered for each successive photo unit, the communication channel must be capable of differentiating to one-fifth the size of the photo unit. This means that the keying rate of the system will have to be able to modulate at five times the rate that it did for black-and-white alternate photo units. And only five different values is rather a limited possible valuation for true picture work. Ten would be a better figure, so that the modulation rate is anything from five to ten times more severe for picture work as it is for black and whites if the variable-size dot method of picture representation is to be used.

20. Dot-dash System. In the face of this dilemma, a new method of telegraphic modulation was devised known as the dot-dash method of building up the picture values synthetically. It consists of establishing a tie-up between the picture values and the marking and spacing lengths of a vibrating relay. Assume a relay that is oscillating back and forth in marking-to-spacing at around 300 cycles. This condition is to represent the middle value of the picture in the grays. Then for lighter values of the picture, the relay circuits will be changed such that the contact will stay on spacing for longer and longer periods although it will stay on the marking side for the same quite short period. The net result will be that for lighter parts of the picture, dots will be made farther and farther apart. Now for parts of the picture darker than the middle gray value, the spacing interval will be maintained the same, and the marking interval will be gradually increased in duration. This means that longer and longer dashes will be produced. These changes are accomplished in the electric circuits of the tube relay by the change of the rate of charge of condensers associated with the marking and spacing reactions respectively.

21. Double-modulation Method. Any of the picture-response methods may be improved in their operation by ways known as double modulation. This arises from the fact that none of the methods is very good at the extreme white or extreme black ends of the scale. Therefore, if the setting between the picture values and the current response is altered between successive lines, the black end will be brought down nearer the gray values on one traversal of the scanning, and on the next the white will be brought up into the gray setting where it will be given more faithful interpretation. The net result is a widening of the picture ranges over which the response will be more linearly faithful. The ways this may be accomplished are legion. One is to change a potentiometer affecting the gain of the photocell amplifier at the end of each scanning stroke, and then return it to the previous setting for the next after. A contact or commutator on the scanning drum makes this possible. Instead of making it at the end of every stroke it may be done at a rate slower than the modulation rate, say at 50 cycles a second, and this will give a definite line-up of the modulation setting at this 50-cycle rate.

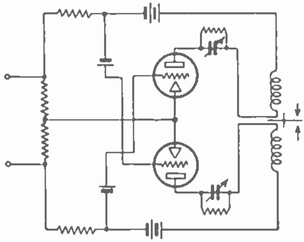


FIG. 10.—Timed push-pull relay for re-forming telegraphic impulses.

22. Intermediate Records. Instead of building up the finished picture directly from the current values obtained by scanning the original picture, intermediary processes may be used, for example, separate engraved plates. Other means are to record the picture variations by the normal punched tape common in telegraph practice. The tape punches may be made to represent different picture values.

For example, five dots across the tape will represent a darker part of the picture than one dot would represent. Not only this, but the dots may be used in different sequence so that much wider range of picture values may be obtained. Likewise, it would be possible to represent the number of times a given picture value was to be repeated. The more complete the ability of the tape to convey intelligence the more quickly will the transmission be accomplished; but by the same token the more of a thinking machine the tape puncher and tape reproducer have to be to get pictures out of the combination. Therefore there is a practical limit to automatic artists. The punched tape is made by marginal relays setting the proper punches. The perforated tape is then sent over the regular telegraphic systems by land wire, cable, or radio, and a duplicate tape is reperforated at the receiving end. This tape is then placed before a photo-recording drum which rapidly puts down light, controlled through the dots, on sensitive paper. This tape method has the distinct advantage that it may be sandwiched in with regular telegraphic traffic and that it may even be handled in sections.

23. Radio-transmitter Keying. The picture modulations must be put on a communication system. In the case of the telephone method the only requirement is to maintain the gain constant throughout the picture to 0.2 db. Otherwise the picture will be streaked. The smoothness of the telephone method of intensity recording makes this even more imperative.

For telegraphic modulation, the requirements are less severe. It is essential that the characteristics of the signals be kept as near their original form as possible. As they start as square waves, they should be kept near this form. This requirement is met fairly well if the third harmonic of the modulation rate is allowed to pass. These signals may be put over a regular telegraph line in what are termed direct-current pulses. This is the best way to do it for short distances over open wires, especially up to, say, 50 miles in length. But for greater distances, various serious distortions in the signals come in, due to the fact that the 150 cycles, for example, will not travel so fast as the 450 cycles. This is particularly noticeable where long intervals of solid white may be interposed between short sections of black, as in typewriting.

The steady stationary current is approached for the white, which far exceeds the value received in a normal modulation cycle. This effect may be reduced considerably by the use of the Reading condenser, familiar for years to telegraphic engineers. This consists of nothing more or less than a condenser shunted by a resistance in series with the line. Its position is at the receiving end of the line; but it helps at both ends. It means that short pulses are favored against long ones, as the short pass through the condenser more readily. A vacuum-tube arrangement has been devised which is of very great help in this direction especially where the lines are long and the received currents small. It is called the *push-pull relay* and consists generally of two tubes arranged in differential balance as regards signals coming over the line. They have Reading condensers in their plate circuits timed to the average rate of the signal pulses, and by this means have a tendency to reshape the incoming signals to their original square-cut characteristic.

24. Use of Neon Tubes. The introduction of neon tubes in such telegraphic signal circuits has also been of help. They require a certain minimum voltage impressed across them before they give any response. With such tubes inserted either directly in the line or more usually in the plate circuit of an amplifier of the line currents, (1) they will not respond to small line disturbances under the normal signal level, and (2) they will give a very sharp building up of the incoming waves when they do break down. They do not have this characteristic on the end of a signal; but by the use of two such tubes in differential arrangement, one for the positive current going on, and the other for the negative going off, the squaring action may be accomplished on both ends of the signals.

25. Carrier Systems. For long lines, the time of arrival of the signal components at the receiving end of the line becomes so different that the signals cannot be corrected by any general means. It is usual therefore to resort to tone carrier for the keying. This means that a tone preferably ten times the normal modulation rate should be used. If this modulation rate is 150 cycles, this would mean a carrier tone of 1,500 cycles. Now the band width, as previously pointed out, should include the third harmonic of the modulation. This means that a band width of 450 cycles should be passed either side of this 1,500 cycles. However, it is possible to use only one side band and the carrier. If the line, for example, does not pass signals up to 1,900 cycles nicely, no difficulty will be noticed if the carrier and perhaps the first harmonic above are passed. The only reason for suggesting that the upper side band also be retained is that some 25 per cent of the energy is in that side band. If it is desired to limit the band width of a carrier system, so that more channels may be crowded into the same total space, it is possible to insert filters after the carrier is modulated by the picture signals and before the tone is put on the line. This filter will pass the carrier and one set of side bands, say the lower.

26. Modulation Circuit. The actual modulation of a carrier by the modulation keying, is best accomplished in a balanced tube circuit. In this circuit, the tone is placed on two tubes in push-pull. The grids of these tubes however, are kept negative below cut-off during non-signal periods and the keying is then made to raise the grids in push-push to the operating point. This method balances out the objectionable click at the start and finish of such keying, if only one tube were used. The net effect of such clicks is to lengthen out the picture signals and produce muddy copies. As pointed out in the photocell modulation methods, this push-pull method is inherently better.

Finally, mention should be made of the phase correctors which are used to correct the phase of received signal frequency components. They consist of series-tuned circuits in the basic form, placed across the terminus of the line, a position in which they have the ability to retard the phase of the frequencies approaching the frequency for which the circuit is tuned.

At the end of a land line, if the signals are to be used to key a radio transmitter, care must be taken that lines are kept as free as possible from r-f pick-up as they come into the station. Shielding is of course the answer here. In practically all radio-transmitter keying circuits condensers are used in various portions. They must be kept to a minimum size in order that the transmitter may key quickly. The keying becomes quite an engineering problem in class *B* and class *C* transmitters especially.

27. Wave Lengths Used in Radio Transmission. Rigorous uniformity is demanded of the entire radio circuit to insure that streaks are not caused on radio pictures. For long waves the only general requirement is that the decrement of the circuit be not so low that the signals do not build up and decrease quickly. The long-wave antenna helps materially in giving directional selectivity to the signals reducing interference from static or other signals as well as giving much more energy on the desired signal. This means that the decrement of the tubing circuits may be worked at a higher value, thus giving faster keying to handle the picture signals.

The first long-wave transmission of pictures was handled on wave lengths of the order of 15,000 m, and 60-cycle modulation was generally the highest that could be handled well. Short waves opened up tremendous possibilities as to keying speeds. Keying speeds of 300 cycles may be successfully handled now over short waves, and higher speeds are being planned.

28. Necessity for Uniform Signal Strength. The great bugbear with short waves is the variation in intensity and actual fading out of the signal. It is obvious that telegraphic modulation is all that may be used where such fading exists. (For the very short wave lengths and searchlight distances telephonic modulation may well be used, due to the constancy of the signals.) One form of promoting constancy of signal strength is to have the incoming radio signals key audio oscillators. This keying is so arranged that a rectifier stage works down to cut-off in keying the audio oscillator.

One of the greatest developments is in *diversity reception* for short waves. In this, three antennas with their respective receivers control a common audio-tone carrier. If one signal fades, the chances are that one of the others will still be giving sufficient energy to key the full tone to the line. Directional set-up of the antennas themselves improves likewise the geographical selectivity of the system. The precaution to be made is that the time constants of the rectifiers accomplishing the keying of the audio oscillator by the radio signals shall be fast enough to follow the modulation.

29. Recording Methods. The photographic method exposes elementary areas of photographic paper to light in varying amounts corresponding to the modulated signals. It has the great advantage of the wonderful technique available coupled to the fact that no mechanical movements are necessary. Extremely light movements of mirrors or light valves may accomplish the exposure of the sensitive paper to light. The disadvantage is the requirement of dark room or box operation with no knowledge of the actual performance until the exposure and development of the complete picture are accomplished.

The basic principle of many photographic systems is to turn a light on and off. The usual filament light is slow in response but this response may be speeded up by cooling the filament with a gas in the bulb, such as hydrogen, and by placing the filament near the wall of the tube. Quicker

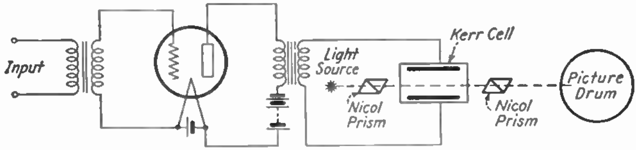


FIG. 11.—Photo recording by polarized light.

filament lighting may be accomplished by making it as fine as possible, or by using an auxiliary current which will keep the filament at incipient operation just below the heat value which would record on the paper, (this auxiliary current is better removed after the filament is lighted to speed up the cooling), or by the use of Reading condensers in series with the supply to the lamp such that the initial voltage will be higher than the working voltage. Much higher light operation is obtained by the use of a ribbon galvanometer.

30. Light Valves. A speedy method of light control is accomplished by electric bi-refringence in the Kerr cell, which alters polarized light so that it passes through a nicol prism system with current applied to

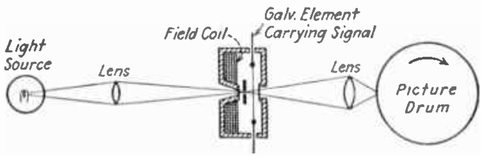


FIG. 12.—Photo recording with electro galvanometer.

the cell. The Kerr cell requires about 900 volts for good operation with a separation of the order of 1 mm between the plates in the nitrotoluol. Magnetic rotation of polarized light has also been tried, but the rather strong magnetic fields necessary, together with the slow reaction of such magnetic systems, has militated against their use. Mirror galvanometers have always been used, of course.

31. Glow Lamps. One of the simplest lights for recording is the gas-discharge tube in which ionization of a gas, such as neon, is quickly accomplished. When this discharge is limited to a crater, a very highly actinic beam is developed, efficient in power consumption and speed. Various gases have been used, perhaps the more successful being a mixture of helium with just a trace of argon. A pressure of about 15 mm with 0.1 per cent of argon gives an idea of the general values for such a tube. General voltage range for such a lamp is of the order of 200 volts.

32. Corona-discharge Exposure. An interesting method of recording on photographic paper is furnished by the corona discharge from a coil set-up of the Tesla variety in which a great step-up ratio is used. The discharge from a needle point to the sensitive paper gives interesting results when the discharge does not spread too much. It is particularly useful on half tones. As the discharge is virtually entirely in the ultraviolet, a paper especially sensitive to that region may be used. Yellow celluloid will make it possible to screen the operation and work in only a slightly darkened room.

33. Starch-iodine Paper. The oldest form of visible recording consists in utilizing the starch-iodine reaction, known for three-quarters of a century. Paper is moistened with a dilute solution of starch and potassium iodide. The cathode is the recording point, and iodine is liberated when very minute values of current pass from the cathode through the paper to the anode cylinder. Ferrocyanide solutions may likewise be used which have the advantage of greater sensitivity and greater permanence, but they always have the drawback of the poisonous characteristic of the solutions.

34. Mechanical and Liquid Recorders. Mechanical recorders have been in general use throughout the half century of picture development. The aim has always been to make them as light as possible so that they will be sensitive and respond accurately to the necessarily high frequencies.

A very free-flowing and instantly drying fluid is made of heated paraffin. It flows very readily when heated along metal, so that a very small stylus made about the size and shape of the bill of a bird may be mounted upon a very sensitive magnetic system. A string wick will carry the heated paraffin to the pen, which is likewise kept warm by being in the proximity of a small electric resistor. The paraffin is colored by an oil-soluble dye; the reds are particularly effective.

Carbon paper has been another great recording favorite. A stylus is used to record through the carbon paper into the white paper beneath. Heating the stylus makes it work even faster. One of the most ingenious suggestions in this direction was the thought of making an envelope of two pieces of paper with the carbon tissue inside and sealed before recording. The record was then made on the receiving instrument and the envelope delivered unopened to the addressee; in which case the latter is the first to see the traced message when he opens the envelope and removes the carbon paper.

35. Heat Methods. Heat has been used in many ways for recording. Air may be heated as it passes through an electrically heated tube leading to a fine jet. This heated jet of air is prevented from hitting the heat-sensitive paper by means of a very small electrically deflected vane. Or a jet of cold air may be used to deflect the hot air away from the paper, and the vane may act on the cold air. The vane is operated by the incoming signals.

Many forms of heat-sensitive paper have been made. Ordinarily, scorching of the paper requires too high temperatures. An endothermic reaction is generally used, in which two chemicals, non-reactive at normal temperatures, are broken down under the heat and then react with each other to produce a colored compound. For example, nickel sulphide is produced by the endothermic reaction of sodium hypo-sulphite and nickel sulphate. A thin coating of red mercuric iodide becomes white on the application of heat. One arrangement uses the heat to melt away a thin coating of paraffin laid on paper from a colloidal solution. Very little heat is required for this purpose. Then the paper so recorded may be inked with any color by rolling it up with a roller holding a water ink. By using a paint brush, such a wax record may be colored to correspond to the original.

Straight vapors may likewise be keyed by the vane. One such vapor is an alcoholic solution of an oil-soluble aniline dye, such as purple. This vapor will record very rapidly at a quarter of an inch distance from the nozzle, so that close operation is not essential. But at greater distance, the recording power falls off rapidly. This is a prime necessity, for otherwise the entire paper might be covered by the vapor. This is the general trouble with any of the gas reactions of, say, ammonia and a mercury salt in the paper. The entire paper at once becomes fogged.

36. Synchronizing Methods. Two general methods of synchronizing are extant: step by step and independent time control. The first consists

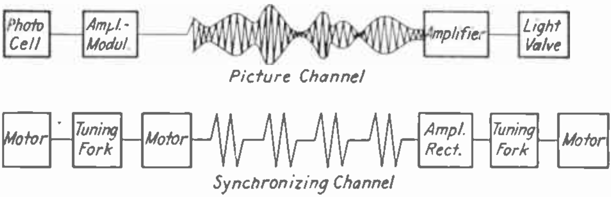


FIG. 13.—Superposition of motor control and picture signals on communication circuits.

in advancing the scanning at the receiver in step with the scanning at the transmitter by means of signals sent from the transmitter to the receiver. The signals may either be special signals on separate channels from the picture signals, or at the end of each line of scanning, or by the actual picture signals themselves. For the separate channel, it is usual to transmit a low frequency (40 cycles) either directly or by modulating a higher frequency, say 400 cycles, which in turn is transmitted over the channel. This frequency is obtained from some part of the transmitting apparatus, or it may be the frequency which likewise controls the transmitters.

Synchronous motors are generally used for the actual driving of the equipment. The motors are themselves synchronous or they are made into synchronous motors by having applied to them additional control energy which is synchronous to their rotation. For example, a direct-current motor may be turned into a synchronous motor, by putting an extra commutator on its shaft which has two segments and two slip rings on it. This commutator then makes alternate connections through the ensuing half revolutions of the shaft between the brush which bears against the segments and alternately with each of the slip-ring brushes which correspond to the two segments. By connecting the slip-ring brushes, to the two opposite contacts of a vibrating tuning fork, current may then be led from the fork prong through the contacts through the segments to the segment brush. This current may then be added to the field of the motor. If the motor is rotating synchronously with the fork, a large value of current will be carried through this hook-up, as the fork will change from one fork contact to the other, at the same time that the commutator segments change. This current will tend to slow down the motor. If the motor is exactly out of phase with the fork, so that its contacts change exactly opposite to the changes in the brushes, then no current will be added to the field of the motor. This will tend to speed up the motor again. The net result is that the motor will come to a position somewhere between these two values, depending upon its load. It is seen that the motor will not operate exactly in phase with the fork, but it will operate in synchronism, holding a position always at constant phase difference with the fork. Such phase difference means that the picture will be a little to the right or left of the exact correct position, but it will be perfectly straight up and down, if the motors remain synchronous. Such phase difference may be compensated by "framing" the picture. With such a device as just described, this is done by providing the brush which makes contact with the segments with a rotatable arm, which may be turned by hand until the picture is in exactly the proper position.

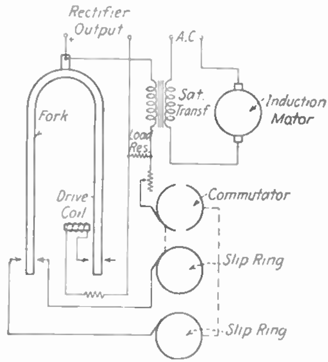


FIG. 14.—Fork control of induction motor speed, a-c supply.

37. Induction Motor-Tuning Fork System. The same synchronizing arrangement may be used to connect an induction motor with a tuning fork to make the motor synchronous with the fork. In this case, direct current obtained from a rectifier is led through the same contacts and segments as previously described, and this direct current operates to change the reactance in a saturation transformer. Such a transformer has two windings, one of high resistance for the direct current and the other of high reactance and low resistance for the a.c. it is to control. If the d.c. saturates the transformer, the reactance will be quite low, so that if the secondary of such a transformer is in series with the supply to an induction motor, this motor will receive more current and have a tendency to speed up.

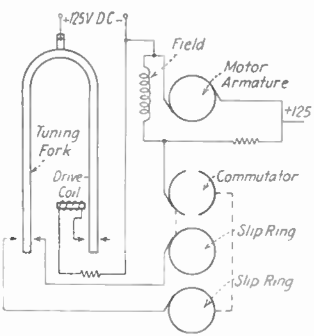


FIG. 15.—Fork control of shunt motor, d-c supply.

38. Motor-generator Control. Another method of control consists in directly connecting a motor and a generator mechanically. The generator has the same frequency as the fork which is to control it. In this case it is usual to use higher frequencies for control, say 500 cycles. The energy from the generator may be applied to the plates of two vacuum tubes, the grids of which are operated by the energy from an electric pick-up on the fork. If the generator is supplying voltage to the plates of the tubes, synchronous with the grid excitation from the fork, it will have to do more work; if it is exactly out of step it will have to do no work. The system adjusts to a middle position.

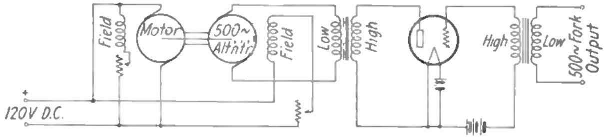


FIG. 16.—A-c thermionic brake on motor-generator drive.

39. Use of Thyratron Tubes. The advent of the Thyratron tube has made it possible to obtain alternating current of considerable size controlled by a small tuning fork in very compact form, sufficient to operate small synchronous motors. This reduces materially the special cost of equipment.

40. Accuracy of Synchronism. For permanent installations, the better practice seems to be to set up tuning forks of accuracies better than 1 part in 100,000 and adjust them with respect to each other from time to time. (The actual results on the pictures themselves furnish an excellent check.) It is then unnecessary to send special signals for synchronizing during the picture operation. It saves ether space and renders the entire operation much simpler.

For temporary installations or for mobile installations such as on airplanes, it is not generally possible to have the same care in set-up which would make such high accuracies possible. Under these conditions, it is generally wiser to lock the receiver to the transmitter by synchronizing signals. For example, there may be a characteristic tone of modulation set up by a small tone generator on the shaft of the motor driving the transmitter. This tone may be used for the picture carrier too, and when received it may be separated out and used to control the speed of the receiving motor. For this latter purpose it is usually better (unless practically perfect signals are available) to interpose some form of inertia between the speed signals and the motor. A tuning fork driven by the signals forms an excellent flywheel for this purpose, and then the driven fork is used to control the speed of the receiving motor. By this means the signals may cease for several seconds without the receiving motor losing control.

If synchronous dots are used for the transmission, they may be used for a step-by-step speed control, but, generally speaking, such methods have not continued in practice, first, owing to the fact that there is such a wide divergence in the size of the dots making it difficult for any relay system to hold to them and, second, owing to the fact that these dots are by no means always perfect.

41. Historical. The first reference to electrical picture transmission is but a short time after the original telegraphic developments. Alexander Bain in Scotland was working previous to 1841, as his first English patent was taken out in that year. He laid down the basic principle of start-stop synchronizing, scanning, and electrochemical recording.

Pendulums were his scanning arms, and a platen dropped at each terminus to present the next line for scanning at the end of each sweep of the arms. Shellac ink on tin foil constituted the original variable resistance. He also suggested raised-type letters for the transmission. Bakewell, in 1846, proposed now usual cylinders running under scanning points. The names of the workers since are legion. Outstanding has been the work of Amstutz, in the United States, who specialized in the use of the engraved line plate; Korn, in Germany, with the selenium cell; Belin, in France, with his high development of the relief method in the "telestereograph." T. Thorne Baker, in England, has been a leading exponent of developing the art for the amateur, using generally the Bakewell form of machine, with the line plate for photographs, and the development of especially light parts to simplify the general operation. Captain O. Fulton has added to this same type of equipment the use of a very neat start-stop release magnet equipment. C. Francis Jenkins applied his motion-picture development of the lens disk to picture transmission as a novel means of making the light spot traverse a flat surface, at both the transmitter and receiver.

42. Bell System. Outstanding perfection has been realized by H. E. Ives and his assistants in the Bell telephone system with the perfection of the transmission of pictures over telephone channels. Transparent film is used wrapped into a cylinder of itself, the photocell picks up the light transmitted from the outside of the cylinder to the center as the photocell moves slowly down the axis. The reception is in the form of an Einthoven galvanometer in which the moving element is a very thin, flat strip of silver. The sideways motion of this strip uncovers the light which passes through to a fine spot on the surface of the recording-film cylinder. First this strip was moved at right angles to the scanning motion, which made a variable-width picture in much the form of a single-line engraving. But this method gave pictures which, when supplied to the newspaper engravers for the production of an engraved plate, gave rise to an objectionable "Moret" between the line structure of the telephoto and the engraving screen. By timing the galvanometer so that the motion of the ribbon is in the same direction as the scanning, and by purposely fusing the edges of the recorded lines by the use of Iceland spar to blend the edges of the lines, a much smoother picture of excellent quality is obtained.

43. Bart-Lane System. Code methods of transmission have been developed by many in which an artist lays out the picture by squares according to a given plan, but it requires an artist and imagination at the receiving end to put life into these blocks. A far better code method is automatic, in which the picture elements set up their own code values. Outstanding in this development have been H. G. Bartholomew and M. L. D. MacFarlane, in England, with the "Bart-Lane" system. Their punched tapes have sent many pictures over the cables between London and New York.

44. Radio Systems. In radio transmission of pictures, the Telefunken Company has been leading in Germany, with Dr. Fritz Schröter and Professor A. Karolus giving emphasis to the Kerr-cell development. In England G. W. Wright has led the activities of the Marconi Company. With the R. C. A., in the United States, the author has been aided in the chemical developments by F. G. Morehouse, in the gas-tube field by

R. M. Williams, and had the advantage of radio-transmission and reception developments led by H. H. Beverage.

45. Picture-transmission Networks. There is a wide network of picture-transmission systems throughout Europe, England, the United States and Japan as well, all over wire lines. Photo-radio circuits have been operated from London to New York, commercially, since May 1, 1926, also between San Francisco and Honolulu; and by the Telefunken Company between Buenos Aires and Berlin.

SECTION 21

AIRCRAFT RADIO

By Harry Diamond, B.S., M.S.[1]

1. The success of any transportation system depends in a large measure upon the rigorous maintenance of safe scheduled operation. Probably nothing has contributed more to the safety and reliability of transportation systems than the associated communication systems. Radiotelegraph, radiotelephone, and the radio direction finder have been important elements to such safety in both sea and air transportation.

Radio serves as a communication means between airplanes and between airplane and ground, it furnishes the pilot with weather information, it tells him when he is on or off his course, helps him to land under conditions of poor visibility, and may prove to be of value in preventing collision with other planes or with fixed objects.

Either telegraphy or telephony may be used to communicate from the ground to the airplane or vice versa. One-way communication, such as transmission of weather conditions from the ground to the airplane requires the simplest type of equipment, two-way communication between airplanes or between airplane and ground requires heavier and more expensive equipment. The choice of apparatus is a compromise involving weight, expense, and convenience of operation.

Channels available are 235 to 500 kc for the government weather-broadcast stations and certain frequencies in the 1,600 to 6,500-kc region are for two-way communication.

2. Government Weather-broadcast Service. By October, 1932, there were 67 radio stations broadcasting weather information to pilots in flight and to airports along the route. A teletype system connects the radio stations, weather stations, and operations offices, providing a typewritten record on all machines. Transmission is at the rate of 40 words per minute. There were 60 weather broadcast stations and 13,600 miles of teletype in operation on the above date.

Before departing the pilot is advised as to weather conditions along his route and at his destination. After departure he is kept informed by radiotelephone broadcasts regarding landing and weather conditions, and if landing at the principal terminal points appears hazardous, he is told of alternate landing fields where a safe landing may be made. Pertinent information, such as ceiling heights, barometric pressure, temperature, wind direction and velocity is given the pilot en route.

In addition to the broadcast of weather information the radiotelephone transmitters provide facilities for the transmission of emergency messages essential to the safety of aircraft.

[1] Senior radio engineer, Bureau of Standards; chief of group on development of radio aids to air navigation.

3. Ground-station Equipment. The ground stations employ a transmitter of the master-oscillator, intermediate-amplifier, power-amplifier, type supplying 2 kw of radio-frequency power to a single-wire antenna 125 ft. high and 375 ft. long on frequencies from 100 to 550 kc.

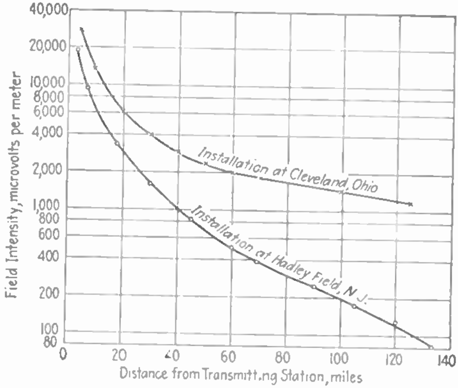


FIG. 1.—Useful service area of weather broadcast stations.

Either continuous wave (cw), interrupted continuous wave (icw), or telephone operation is provided. While radiotelephony only is employed for broadcasting to the pilot, radiotelegraphy may be used in emergencies, in case of failure of the teletype system and other means of communica-

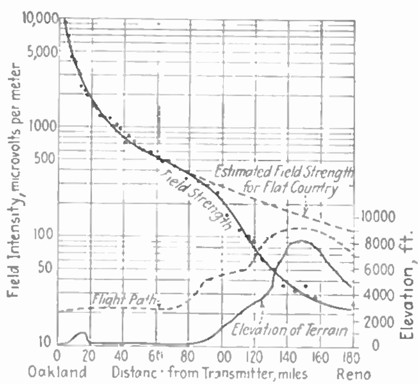


FIG. 2.—Effect of nature of terrain on field strength.

tion, and for interchanging the weather information from similar radio stations.

The transmitting circuit for the 2-kw weather-broadcast transmitter is conventional. Harmonic radiation is reduced so that the field intensity on any harmonic is not greater than 0.1 per cent of the fundamental field intensity.

An idea of the useful service area of the weather-broadcast stations may be had from a study of the graphs shown in Fig. 1. The graph of Hadley Field is more typical of average conditions in the United States, the values given in the other graph being unusually high. The measurements were taken on two installations employing identical transmitting sets and not greatly different transmitting antennas; the great difference in field intensities produced is, in all probability, due to the difference in the nature of the terrain along the transmission paths. Experience has shown that a field intensity of about 100 μv per meter is required to override the static level under the worst conditions occurring in winter, while a field intensity of about 500 μv per meter is necessary in the summer. This indicates a useful distance range of about 130 miles in the winter and 60 to 75 miles in the summer.

The effect of the nature of the terrain upon the field intensity is shown in Fig. 2. The presence of a mountain range considerably increases the attenuation.

4. Two-way Communication. The above system provides for the broadcasting of weather information or emergency messages, but does not afford complete facilities for continuous contact between the aircraft and ground. This need must be met by the establishment of h-f two-way communication. Transmitting stations on the frequencies assigned to this service are therefore being constructed at the principal airports and the airplanes are being equipped with the h-f receivers and transmitters in addition to the medium-frequency sets required in connection with government-operated radio aids.

Two distinct means for two-way communication are possible. The first requires the installation on the airplane of a low power (10 watt) radiotelephone transmitter, operating on 3,105 kc. The medium-frequency set, already on the airplane for receiving weather broadcasts, is sufficient for reception. Short-range two-way communication may then be obtained. The second requires a h-f transmitter of at least 50-watts power output when using telephony or 20 watts when using code and a h-f receiver in addition to the medium-frequency receiver. With the aeronautical chain stations located approximately 200 miles apart, constant contact with the ground is then feasible.

In the case of the itinerant flyer, the transmitter carried on the airplane is of about 10 watts power and transmits on the national calling and distress frequency, 3,105 kc. This transmitter together with the medium-frequency receiver is sufficient for short-range two-way communication. The pilot can receive on 278 kc from airport transmitters and from intermediate points where the airway keepers are supplied with 15-watt transmitters on this frequency. Since a watch on 3,105 kc is maintained at all radio stations, the flyer carrying a 3,105 kc transmitter of 10 watts is insured that distress or emergency messages will be heard practically at all times along his route.

The pilot of an airplane equipped with a medium-frequency receiving set, a h-f receiving set, and a h-f 50-watt radiotelephone capable of operation on the national calling wave and the day or night working wave can

1. Receive the weather-broadcast service and the radio range-beacon service.
2. Communicate on the assigned day or night frequency with aeronautical chain stations.

3. In the event of failure of 2, the pilot may transmit on 3,105 kc, being received by either government or aeronautical ground station. The message may then be relayed over the teletype system and a reply sent on the weather-broadcast transmitter.

4. Communicate at short range with airports and airways keepers on 3,105 kc requesting position, reporting service, or transmitting emergency messages, and receive acknowledgment on 278 kc.

5. Radio-wave Phenomena in 1,600- to 6,500-Kc Band. Figure 3 shows the average strength of daytime signals received in an airplane as a function of the distance from a 500-watt radiotelephone ground station on 1,510 kc. Figure 4 shows average reception from an airplane using a 50-watt radiotelephone transmitter of 1,625 kc. Figure 5 shows the effect of frequency upon the attenuation characteristic for a 500-watt ground station. Similar graphs for transmission during night, showing field strength as a function of distance are given in Fig. 6. (Communication on 1,608, 3,452, and 4,108 kc is generally satisfactory, while that on 5,690 kc is not satisfactory. The poor results obtained on 5,690 kc are mainly due to excessive fading and accompanying distortion effects.

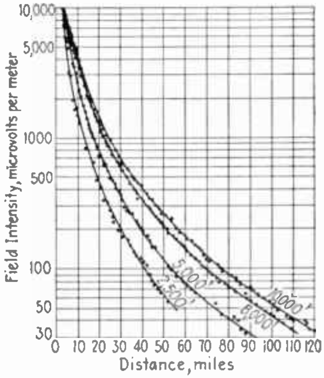


FIG. 3.—Average strength of daytime signals received in an airplane from 500-watt station on 1,510 kc. (Airplane at altitudes designated on graphs.)

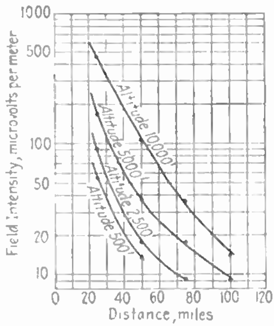


FIG. 4.—Reception from airplane using 50-watt transmitter on 1,625 kc. (Airplane at altitudes designated on graphs.)

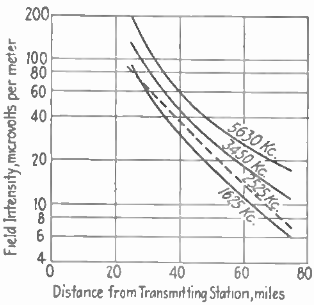


FIG. 5.—Effect of frequency on attenuation of 500-watt ground station.

Summarizing, satisfactory two-way radiotelephone communication between airplane and ground may be had in the 1,600- to 6,500-kc band using 500 watts of power on the ground and 50 watts on the airplane.

In general, the lower frequencies appear to give more satisfactory and reliable operation. However, the higher frequencies offer the advantage

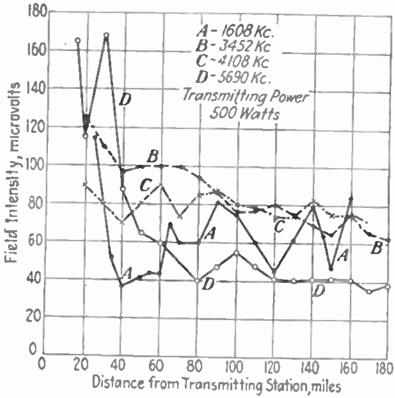


FIG. 6.—Night transmission phenomena.

of more efficient use on the airplane of fixed antennas of relatively small dimensions, thereby avoiding the use of the trailing-wire antenna which,

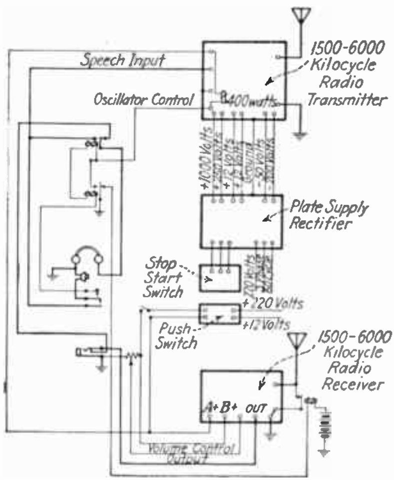


FIG. 7.—Terminal equipment of typical two-way radiophone system.

while being more satisfactory from an electrical viewpoint, introduces many mechanical disadvantages.

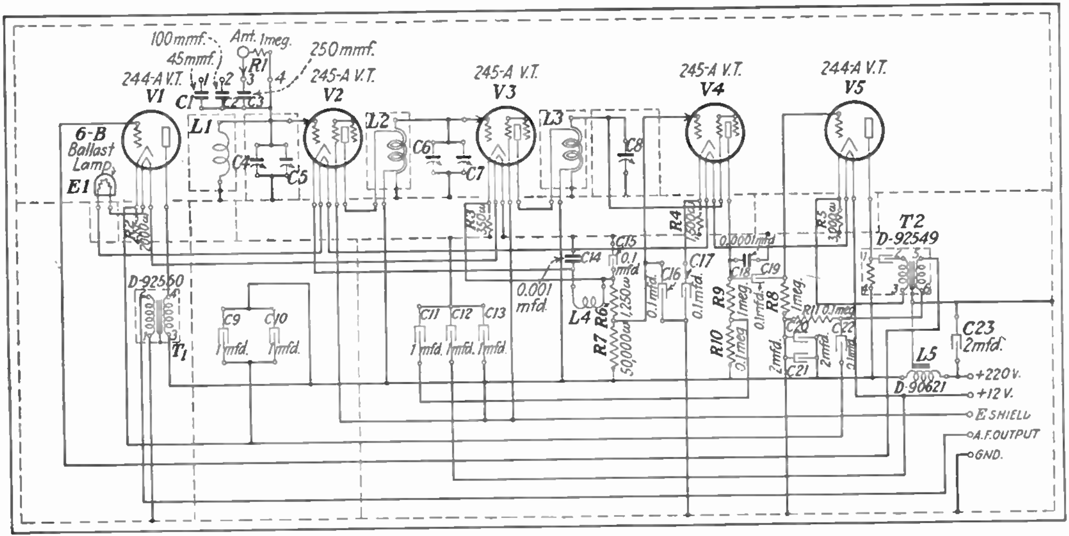


FIG. 8.—Circuit of receiver covering range 250 to 500 kc.

6. Ground Equipment for Two-way High-frequency Communication. The equipment at the fixed terminal of a typical two-way radiotelephone system is shown in Fig. 7. The same carrier frequency may be used both for transmission and for reception. The high-frequency receiving set may be identical with the high-frequency set for airplane use. The transmitter consists of a crystal-controlled oscillator, a frequency-doubler, a modulating amplifier, a power amplifier, associated speech-input and speech power-amplifying equipment, and the necessary power and control circuits. The set (400 watts) is capable of complete modulation.

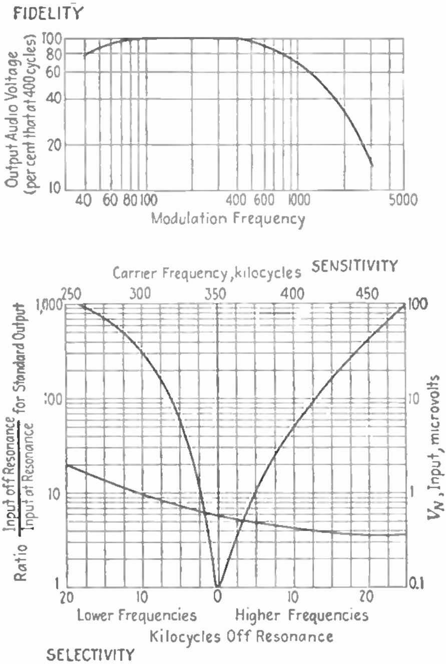


FIG. 9.—Performance of receiver of Fig. 8.

It may be adjusted to any frequency in the range of from 1,500 to 6,000 kc. By accurate temperature control the frequency of the oscillator in both the ground and airplane sets is held constant to within 0.025 per cent under all temperatures encountered. A total of about 3 kw of 60-cycle, 220-volt, three-phase power is required for the operation of the transmitting equipment.

AIRPLANE EQUIPMENT

7. Aircraft radio equipment must meet unusually severe requirements. Reliability and simplicity of operation are essential. The equipment must be constructed to withstand continued vibration and landing shocks without breakage or change in performance, and must operate under all conditions of weather encountered in flight. Space and weight must be kept down to a minimum. However, reductions in space and weight must not be obtained at the expense of reliability or accessibility for inspection and maintenance.

8. Medium-frequency Aircraft Receiving Set. A typical receiver covering the range 250 to 500 kc is shown in Fig. 8, and Fig. 9 gives its performance characteristics. The three tuning condensers are mounted on the same shaft to provide for unicontinuous tuning over the entire range. Heater-type tubes reduce microphonic noises.

Remote volume control is accomplished by locating the voltage divider, which supplies the screen-grid potential of the radio-frequency amplifier, in a special control unit which may be installed within reach of the pilot. A switch for turning the set on or off and an output jack for the head telephones are also incorporated in this control unit. Remote tuning is accomplished by a flexible shaft which turns in a casing. The shaft connects the tuning-control unit with the driving head on the receiving set, and operates at a speed 264 times that of the condenser shaft. This high ratio reduces the effect of the lost motion in the flexible shaft to a negligible amount. The tuning-control unit is mounted within reach of the pilot and is equipped with a calibrated luminous dial. The receiving set may be located as much as 40 ft. from the tuning-control unit and satisfactory tuning without appreciable backlash secured.

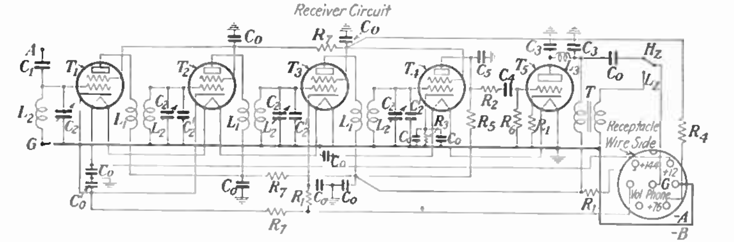
9. Power Requirements. The set requires for its operation 1.6 amp. d.c. at a voltage of approximately 12 and 25 ma d.c. at a voltage of 200. In airplanes having standard 12-volt storage-battery installations with charging generators, the low-voltage supply is obtained directly from the battery and a small dynamotor operating from the battery furnishes the 200-volt supply. In others, a small constant-speed wind-driven generator may be employed to supply both voltages. The propeller of this generator is of the self-regulating type maintaining constant speed within very close limits for all normal flying speeds of the airplane above 70 m.p.h. The only filtering required is to eliminate radio-frequency interference due to sparking at the commutators.

The antenna tuning coil and the interstage tuned r-f transformers are of the plug-in type. Special pins and spring clips are provided to insure that none of the coils or tubes works out of its socket due to vibration during flight. Extremely good shielding is necessary in a receiving set of this sensitivity, particularly since the dimensions of the set are reduced because of the limited space available for aircraft-radio equipment.

Tube noises are reduced to a negligible amount, chiefly because of the large voltage step-up secured in the tuned antenna circuit. The antenna circuit is arranged so that unituning control is possible with antennas of considerably differing constants. A high L/C ratio is used in the three tuned circuits in order that the necessary degree of selectivity may be secured. The two-stage audio amplifier provides ample power output for all possible uses of this set, an undistorted power output of 150 mw being available. The fidelity characteristic is extremely good at the low frequencies in order that the set may be used for the reception of signals from the visual-type radio range beacon

Cut-off for frequencies above 2,500 to 3,000 cycles is provided to reduce hissing and singing noises encountered in reception on aircraft.

10. High-frequency Aircraft Receiving Set. The circuit of a commercial high-frequency receiving set is given in Fig. 10. A very high degree of selectivity and sensitivity is obtained at any frequency in the



T_1 T_2 T_3 T_4-224-Heater Type Tubes
T_5 227-Heater Type Output Tube
A-Antenna Binding Post
G-Ground Binding Post
L_1 L_2-Plug-In R.F. Transformers
L_3-Filter Coil

T-Output Transformer
H_Z L_Z-Connections for High or Low Impedance Phones
C_o-Filter Condenser
C_1-Antenna Aligning Condenser
C_2-Gang Tuning

C_2'-Stage Aligning
C_3-0.005 mfd. Filter
C_4-0.005 mfd Grid
C_5-0.00025 mfd
R_1-1000 ohms Fixed Resistor
R_2-100,000 ohms Fixed Resistor

R_3-20,000 ohms Fixed Resistor
R_4-5,000 " "
R_5-200,000 " "
R_6-2 megohms "
R_7-550 ohms "

FIG. 10.—High-frequency (235 to 8,000 kc) receiver circuit.

range from 235 to 8,000 kc. Five sets of plug-in coils are used, each set having a range of frequencies slightly greater than 2 to 1. Unusual attention is paid to decoupling of different circuits by by-pass condensers and series resistors. The need for using the same tuning condensers

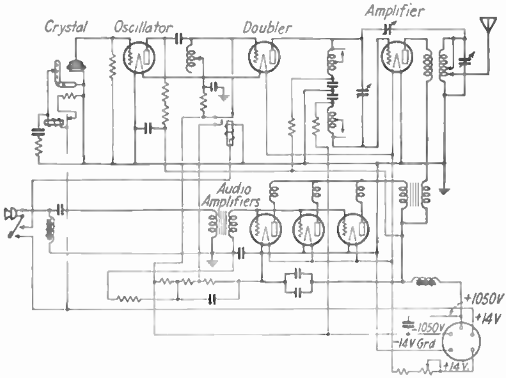


FIG. 11.—Airplane transmitter operating in 1,500 to 6,000 kc range and putting out 50 watts.

throughout the frequency range necessitates precise design to secure unituning control. A low L/C ratio is adopted, the condensers being 150 $\mu\mu f$ and shunted by 25 $\mu\mu f$ air-dielectric trimmers to offset small differences in stray coupling, etc. The antenna input uses a trimmer of 50 $\mu\mu$f in series.

An undistorted power output of 100 mw is available when rated plate voltage is applied. The filament circuit requires 1.75 amp. at 12 volts. The plate requirements are 7 to 12 ma at 130 to 145 volts.

11. High-frequency 50-watt Radiophone Transmitter. The carrier output for the airplane transmitter shown in Fig. 11 is 50 watts, and complete modulation is possible. It may be tuned to any frequency between 1,500 to 6,000 kc. Accurate temperature control of the quartz crystal enables the frequency to be stable within 0.025 per cent.

The crystal controls a 5-watt oscillator at one-half the required final frequency. A second 5-watt tube acts as frequency doubler. To span the entire frequency band two different output transformers are required. The final amplifier is modulated by the introduction into its plate-supply circuit of the speech-frequency output of three 50-watt tubes connected in parallel. D-c saturation of the modulation transformer is avoided by so arranging the windings that the magnetization due to the plate current of the modulating amplifiers tends to balance that produced by the plate current of the final r-f amplifier.

All power supply to the transmitter is fed through a removable plug (provided with a locking ring), and the speech input and the control circuits for starting and stopping the oscillator are connected to the transmitter through a three-conductor telephone plug. The transmitter measures about 9 by 12 by 15 in. and weighs, complete with crystal holder and vacuum tubes, 32 lb. The power-supply requirements for this transmitter are 15 amp. at 12 volts for the filament heating, and 0.4 amp. at 1,000 volts for the plate supply. The airplane storage battery usually serves as the low-voltage source while the high-voltage may be supplied from a dynamotor driven from the storage battery, an engine-driven generator or some other arrangement. The transmitter is designed for operation on any single frequency within 1,500 to 6,000 kc, no provision being made for changing frequency during flight.

12. Typical Arrangement of Airplane Equipment for Complete Radiotelephone Facilities. A schematic diagram showing the electrical arrangement of the complete equipment required on the airplane for fully utilizing the government-operated radio aids and in addition maintaining two-way radiotelephone communication with the high-frequency ground stations is shown in Fig. 12. The weights and dimensions of the component parts of the airplane radio equipment are given in Table I. Note that these figures do not include the primary source of power on the airplane, usually comprising a 65-amp., 12-volt storage battery and a d-c generator driven from the main engine for charging the battery.

13. High-frequency 20-watt Radiotelegraph Transmitting Set. The useful distance range of communication when a 20-watt airplane radiotelegraph transmitting set is employed is equivalent to that obtained with a 50-watt radiotelephone set. The reduction in weight is threefold: lower transmitting-set power rating, no modulating tubes required, and lower power-supply requirements for equivalent transmitter output rating. This reduction in weight is in small part offset by the fact that short-range radiotelephone communication is usually provided as part of the radiotelegraph transmitting equipment.

A 20-watt radiotelegraph transmitting set having also provision for short-range radiotelephony is commercially available. The weight of this trans-

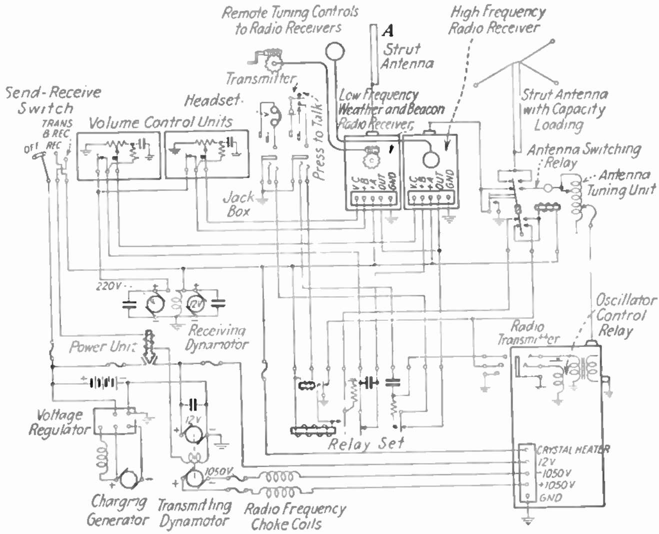


Fig. 12.—Complete receiver and transmitter for airplane.

Table I.—Weights for Complete Radiotelephone Installation on Aircraft

Equipment	Over-all approximate dimensions, inches	Approximate weight, pounds
Medium-frequency receiving set	11 × 6 × 11	18
High-frequency receiving set	11 × 6 × 11	18
Receiving dynamotor, control units, head telephones, etc		17
High-frequency 50-watt transmitting set	9 × 12 × 15	33
Transmitting dynamotor, starting relay, antenna-tuning unit		43
Wiring cables, antenna wire, terminal blocks, etc		21
Total		150

mitter is 11 lb., while the dynamotor required for supplying plate power, etc., weighs 13 lb. The total weight of 24 lb. is to be compared with a weight of 76 lb. for the 50-watt radiotelephone transmitter, together with its dynamotor and dynamotor starting relay. Moreover, the load on the airplane storage battery and charging generator system while the transmitter is in operation is considerably lower, of the order of 20 as compared with 80 amp.

14. High-frequency 10-watt Radiotelephone Transmitting Set for Itinerant Pilot. The circuit diagram for a 10-watt radiotelephone transmitting set for use by the itinerant pilot is given in Fig. 13. Since operation on a single frequency, 3,105 kc, is required, a very simple transmitting circuit may be employed. Provision for radiotelephony rather than radiotelegraphy is here requisite, since its operation involves untrained personnel at both the transmitting and receiving ends.

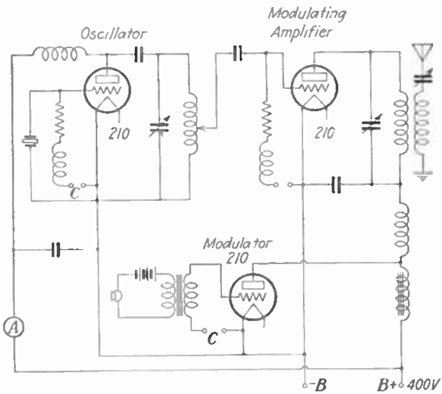


FIG. 13.—Ten-watt radiotelephone transmitter for itinerant pilot.

AIRCRAFT POWER EQUIPMENT

15. Five determining factors enter into the choice of the power system to be adopted: (1) reliability, (2) weight, (3) availability when main power plant of airplane is crippled, (4) electrical performance, and (5) maintenance required during service. Several distinct types of power-supply systems are available. The receiving-set power requirements are from 1 to 3.4 amp. d.c. at 12 volts and 10 to 50 ma d.c. at 135 to 200 volts. For 10-watt radiotelephone sets, approximately 4 amp. d.c. is required from the 12-volt battery and 100 ma d.c. from a 400-volt source. For 50-watt radiotelephone sets, 15 amp. d.c. at 12 volts and 400 ma d.c. at 1,000 volts is required. The general problem of power supply for receiving equipment has been treated above in connection with descriptions of medium-frequency and h-f sets.

16. Transmitting-set Power Supply. *Dynamotor System.* With this system the storage battery is used as the source of low-voltage supply and a dynamotor driven from the storage battery as the source of high-voltage supply. A low-voltage d-c generator driven from the main engine of the airplane and provided with a voltage regulator is used for keeping the battery charged during flight. A large and heavy battery (65-amp. hr. capacity) is required, since the total drain on the battery is about 80 amp. Approximately 30 min. emergency operation may be secured under average conditions of battery charge. The weights for a 700-watt system of this type as well as of other types to be described below are given in Table II.

Wind-driven Double-voltage Generator. The self-regulating propeller does away with the necessity of a voltage regulator. A variable-pitch propeller is used with a built-in centrifugal mechanism which adjusts the pitch to maintain very nearly constant speed with changes in slip-stream velocity, load, etc. The voltage regulation is excellent. The speed of the propeller is of the order of 4,000 r.p.m. The generator has a low-voltage d-c winding for charging the storage battery and a high-voltage winding for supplying plate voltage to the transmitter. Since power is derived only when a slip stream is provided by the airplane propeller, emergency operation after a forced landing is not usually feasible.

TABLE II.—WEIGHTS OF POWER-SUPPLY EQUIPMENT

	Dynamotor system, pounds	Wind-driven double-voltage generator	Combination wind-driven generator and dynamotor	Main engine-driven double-voltage generator	Auxiliary engine-driven double-voltage generator
65 amp.-hr. battery	70	.....	70		
33 amp.-hr. battery	.....	36	......	36	36
Cables	10	6	15	6	5
50-amp. charging generator and control box	47				
Dynamotor	26.5				
Double-voltage generator	.....	28	.....	50	
Generator dynamotor	.....	.....	36		
Gasoline engine, double-voltage generator unit	.....	.....	.....	....	100
Relay or control box	2	2	6	3	
Propeller	.....	5.5	6.25		
Average fuel	.....	.....	......	....	7.5
Actual weight	155.5	77.5	133.25	95	148.5
Equivalent weight	44.0	58.0	70.25	33.5	0.0
Effective weight	195.5	135.5	203.5	128.5	148.5

Combination Wind-driven Generator and Dynamotor. Several arrangements are feasible for securing emergency operation from a wind-driven generator. The most obvious arrangement is one in which the wind-driven generator is used as the battery-charging generator, a dynamotor driven from the storage battery serving as the source of high-voltage plate supply.

Main Engine-driven Double-voltage Generator. The use of a double-voltage generator directly coupled to the main engine shaft is the most efficient method for obtaining electrical power on aircraft. However, as in the case of the double-voltage wind-driven unit, no provision is made for emergency operation in case of failure of the main engine. In the usual application, the generator has a low-voltage and a high-voltage d-c winding. Because of the varying speed of the engine, a vibrating type of voltage regulator is employed. The generator itself is of conventional design, and since it operates at a fairly low speed, usually 2,200 r.p.m., is heavy for its output as compared with the wind-driven generator. The voltage regulation on the battery-charging winding is fixed by the regulator and is practically perfect when the regulator is operating correctly. The high-tension voltage regulation is usually worse than that of a wind-driven generator, but considerably better than that of a dynamotor. The equipment is probably less reliable than either the d-c wind-driven generator or d-c dynamotor.

Auxiliary Engine-driven Double-voltage Generator. The performance of an auxiliary-engine generator set is very good. A four-cycle engine is used so that the engine speed can be governed closely. Either a d-c or a-c generator can be used and no voltage regulator is required. The set can be made light-weight by operating at speeds as high as 4,000 r.p.m.

The reliability of this system depends upon that of the engine, which can be made of high order. The maintenance required is obviously very little. The engine will consume about 1½ lb. of gasoline per hour for an electrical output of 700 watts. For a 10-hr. flight, the average weight of fuel carried is 7½ lb.

AIRPLANE RECEIVING AND TRANSMITTING ANTENNAS

17. An aircraft antenna must have a good effective height, must be of sound aerodynamic design, and must be convenient to use under varying air-transport operation conditions. The trailing wire fulfils the first requirement, but is little used now because of its failure to fulfil the second and third requirements. These disadvantages have led to the use of fixed antennas. A pole extending 4 to 6 ft. vertically above the fuselage has an effective height of about 1 m, sufficient for use with sensitive receivers.

Errors in course indication or bearings taken are introduced unless the receiving antenna on the airplane is entirely non-directional. This restriction limits the antenna configuration to either the vertical-pole antenna, or to a vertical antenna with flat-top loading, the flat-top elements of which are so arranged that their horizontal effects neutralize each other. The symmetrical longitudinal or transverse T-antennas with vertical lead-in are examples of the latter type.

A typical transmitting antenna consists of two single-wire "T" units suspended between the two wing tips and the rudder post. If the lead-in wire were taken from the junction of the two flat-top wires at the rudder post, a "V" antenna would result. To increase the effective height, a mast approximately 6 ft. high is often used to raise the flat-top wires above the fuselage.

RADIO SHIELDING AND BONDING IN AIRCRAFT

18. Airplane-engine Ignition Shielding. Intense electrical disturbances are set up in the radio receiving circuits by the electrical ignition system of the airplane engine or engines unless ignition shielding is provided. To obtain effective shielding, it becomes necessary to enclose the entire electrical system of the engine ignition in a high-conductivity metallic shield. This requires the provision of suitable metallic covers for the magneto distributing heads, for the booster magneto, for the ignition distributing wires running from the magnetos to the spark plugs, for the spark plugs themselves, for the ignition switch, and for the switch and booster magneto leads.

With the low-power ground stations in present use, it is necessary to utilize field intensities not appreciably greater than the prevailing static level. Hence, very careful ignition shielding is essential.

Since the location of the antenna with respect to the interfering source obviously affects the signal to interference ratio, cases are often encountered where partial shielding of the ignition system proves sufficient. In the usual case, exposing even a few inches of high-tension lead or failing to ground the shield at frequent intervals is enough to introduce interference.

COURSE NAVIGATION AND POSITION DETERMINATION

19. Radio systems for guiding aircraft divide into two parts: (1) aids for aircraft flying the established airways, and (2) aids for aircraft flying over independent routes. The first is the more important in the United States. All commercial transport airplanes use fixed airways. The government's aids to air navigation are being provided with the primary view of serving aircraft flying these airways.

An ideal system suitable for use by aircraft flying either fixed airways or independent routes, on land or on sea, is such that:

1. The system shall give the pilot information to enable him to continue along a given route between any two points in a given service area when no landmarks or sky are visible. If he leaves the course, it should tell him how far off he is and to which side, should show him the way back to the course, and should inform him when he arrives at his destination.
2. The necessary directional service shall be available at all times and under all conditions, to all airplanes equipped to receive the service and flying within the area served.
3. The service shall be easily, positively, and quickly available to the pilot, with a minimum of effort on his part.
4. The radio equipment required on the airplane shall be simple, rugged, of light weight, and relatively inexpensive.
5. The ground equipment shall be as simple as possible. The radio frequencies, power, type of emission and location of ground transmitting stations shall be such as to serve the needs with maximum efficiency and conservation of the limited radio channels available.

20. Direction Finder on Airplane. One system employs a fixed-coil antenna the plane of which is perpendicular to the longitudinal axis of

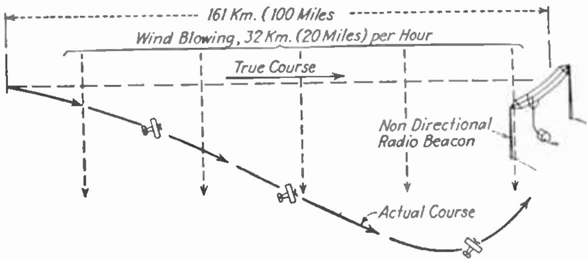


FIG. 14.—Circuitous path followed due to cross winds.

the airplane. Zero signal is obtained in the receiving-set output as long as the airplane is pointing to the ground transmitting station. This is essentially a "homing" system and is subject to the serious limitation that a circuitous path is followed if heavy cross winds prevail. This is illustrated in Fig. 14. Course corrections may be made periodically by observation of the compass indications; the course followed is still, however, not the most direct possible. The system also lacks means for giving the pilot the sense of deviation from the course, the signal increasing from zero whether the airplane deviates to the left or to the right. Moreover, the use of a zero-signal indication is difficult under conditions of severe atmospheric disturbances or interference from other services.

To obviate these difficulties, the *Robinson* direction-finding system was developed. In this system, two crossed-coil antennas are used, one coil having its plane along the longitudinal axis of the airplane and the second having the plane perpendicular to this axis. The signal due to the second or auxiliary loop antenna is alternately added to and subtracted from the signal due to the first- or main-loop antenna. When on the course, since no voltage

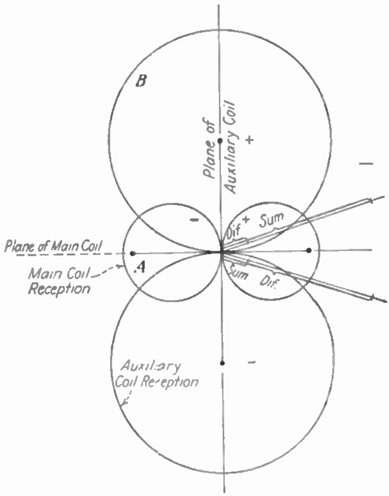


FIG. 15.—Crossed-coil direction-finding system.

is then induced in the auxiliary coil, the two signals are of equal intensities. This may be readily understood by reference to Fig. 15, where *A* and *B* correspond to the reception characteristics of the main and auxiliary coils, respectively. When off course to the left, assuming the phase relations indicated in Fig. 15, the sum of the two signals is greater than their difference, while when off course to the right their sum is less than their difference. By providing suitable switching so that the "additive" position precedes the "subtractive" position, the pilot knows that he is off course to the left when the first signal is louder than the second, while he is off course to the right when the reverse holds true. When on the course, the two signals are of equal intensity. To secure sharp off-course indications it is usual to make the auxiliary coil of about six times the effective height of the main coil.

The present trend is toward equipment giving visual indication of the position of the airplane relative to the course directed on the ground transmitting station. These are generally modifications of the Robinson direction finder such that the additive and subtractive positions are balanced against each other through an indicating instrument, for example, a zero-center pointer type.

To make full use of the possibilities of a direction finder aboard aircraft, automatic indication of the direction of the station tuned in is required.

21. Direction Finder on the Ground. One system of navigational aids to aircraft is a direction-finding system, but with the direction finder located on the ground. Every airplane utilizing this system carries a radiotelephone (or radiotelegraph) transmitter and receiver. Permanent direction-finding stations are located at ground stations at strategic points. When an airplane desires to learn its position, it transmits a request on the airplane transmitting set, whereupon two or more of the ground direction-finding stations each determines the direction by observations upon the radio waves transmitted from the airplane. Triangulation then gives the airplane's position, which information is transmitted to the airplane.

Five minutes is normally required between the time the request for a bearing is transmitted from an airplane and the time the bearing, as computed by two ground stations, is furnished the airplane. Obviously, the system is best suited to long-distance operation over routes not too heavily congested.

A simple loop antenna may be used in conjunction with the receiving set required with this system, thereby giving the pilot additional directional or "homing" service to supplement the bearings furnished by the ground station network. Even with this additional service, however, the airplane is not kept strictly on a given course at all times and is therefore not practicable where airplanes must fly over rigid airway routes.

22. Rotating Radio-beacon System. A method of furnishing navigational aid to a flyer is the rotating radio beacon developed in England. This method employs a transmitter located at an airport, which has a loop antenna rotating at a constant speed of one revolution per minute. A figure-of-eight pattern is thus rotated in space at a constant rate. A special signal indicates when the figure-of-eight minimum passes through north, and also when it passes through east. A pilot listening to the beacon signal in the output of his receiving set can start a stop watch when the north signal is received and stop it when the figure-of-eight minimum reaches him. The number of seconds multiplied by six gives him his true direction in degrees from north. The stop watch may be calibrated directly in degrees, so that the position of the second hand, when the minimum signal is received, gives the bearing directly. The east signal is provided to overcome the difficulty in receiving the north signal when the airplane is north or south of the beacon, as on that bearing the signal strength is a minimum.

The transmitter circuit is shown in Fig. 16. The keying of the circuit is automatically carried out by the rotation of the loop antenna. Since the rotation of the loop antenna is used as a basis of computation of bearings, close control of the speed of the driving motor is maintained. To secure as great a useful transmitting range as possible, the loop-antenna current is of the order of 70 amp. To reduce the losses in the transmitting circuit to a minimum an air-dielectric transmitting condenser is employed. The power input to the transmitting tube is approximately 2,000 watts.

The receiving antenna is of a non-directional type. The receiving set may be used in the reception of weather-broadcast messages and other communications when not employed in direction determination. The system is capable of giving simultaneous service to any number of airplanes in any direction. Drift may be checked by determining positions periodically, and correction may be employed. A number of disadvantages are, however, inherent in this system. The service is intermittent and somewhat slow, requiring at least 30 sec. for each bearing. The system is not suitable for guiding an airplane along a given fixed route. Since the determination of a minimum signal must be made, this system is particularly subject to interference and atmospheric disturbances. From the point of view of simplicity and reliability,

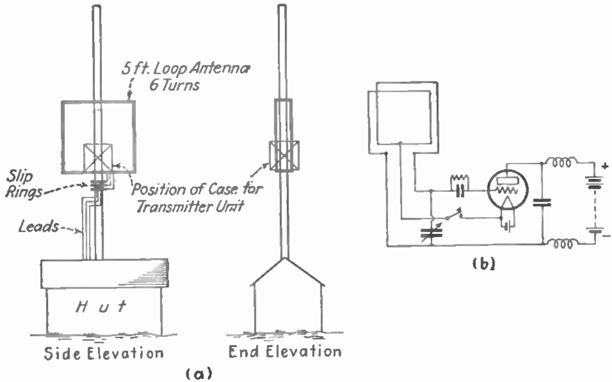


FIG. 16.—Transmitter for rotating beacon.

however, it is without doubt superior to the two other systems outlined in the foregoing text.

23. Radio Range-beacon System. The radio range-beacon transmitting station employs two loop antennas placed at right angles with each other. The antennas are triangular, the base being about 300 ft. long and the height 60 to 70 ft. The transmitting characteristics of the two antennas as a function of angular position with respect to the beacon station are given in Fig. 17. The intensities of the radio waves from the two antennas are equal along the lines *OA*, *OB*, *OC*, and *OD* which bisect the angles between the two antennas. Elsewhere, one of the two waves is stronger than the other. An airplane may therefore follow a course along the bisectors referred to if means are provided for distinguishing the two sets of radio waves from one another. A different signal is impressed on each set of waves for this purpose. The two types of radio range beacons developed differ mainly in the means employed for distinguishing the two sets of signals.

In the *aural-type* beacon two coded letters are used: an *N* (— .) and an *A* (. —). The r-f power fed to one antenna is supplied at time intervals corresponding to the characteristic N while that to the second antenna is furnished in accordance with the characteristic A. The two characteristics are interlocked, so that when received in equal intensities (*i.e.*, along the lines bisecting the two antennas) they merge into a long dash. Off course to one side of these lines the N predominates, while off course to the other side the A predominates. The interlocked signals are now sent in groups of four with a short coded signal provided between successive groups for identifying the different beacon stations of the airways network. To facilitate use by the pilot, the beacon space pattern is oriented, whenever possible, so that the A signal lies in the northeast and southwest quadrants while the N signal lies in the southeast and northwest quadrants.

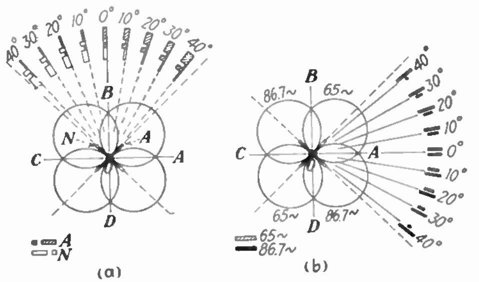


FIG. 17.—Transmitting characteristics of two-loop antenna system.

In the *visual-type* beacon, two low-frequency notes, usually 65 and 86.7 cycles, are employed. The r-f power in one antenna is modulated to 65 cycles, while that in the other antenna is modulated to 86.7 cycles. The modulated r-f is on the antennas continuously, instead of throwing from one to the other antenna as in the aural system. This permits the use of a continuously indicating instrument on the airplane. This instrument is connected in the output circuit of the receiving set employed and consists of two vibrating reeds mechanically tuned to the two modulation frequencies used at the beacon station. When the beacon signals are received the two reeds vibrate and, thus, may serve as a device for indicating relative intensities of the signals received from the two loop antennas. The tips of the reeds are white against a dark background so that when vibrating they appear as vertical white lines. At night, indirect lighting of the reed tips is provided.

The radio range beacon of either aural or visual type requires only a simple radio receiver aboard the airplane for its reception. Since directional transmission is used at the ground station, a non-directional antenna is employed on the airplane. The same receiving equipment is therefore suitable for receiving the government weather broadcasts.

Several methods have been developed for shifting the range-beacon courses from their 90 deg. relationship in order that they may be aligned with the airways. These are applicable to both the aural-type and the visual-type beacons. One consists of reducing the current in one of the two loop antennas. The effect secured is shown in Fig. 18*a*. A second method utilizes the circular radiation from a vertical antenna extending along the beacon tower, in addition to the normal figure-of-eight radiation due to each loop antenna. The vertical antenna may be excited to radiate one or both characteristic waves of the station. The corresponding effects secured are shown in Fig. 18*b* and *c*. Combinations of the two methods described are particularly useful. Figure 18*d* shows the results of one such combination in which the currents in the two-loop antennas are reduced while at the same time circular radiation is added to the normal figure-of-eight radiations of both antennas. To render the beacon system still more flexible and thus make it suitable for use at cities located at the junction of a large number of airways, a beacon transmitter of the visual type has been developed capable of serving 12 courses simultaneously. These courses are normally 30 deg. apart, but may be aligned with the airways by methods similar to those just outlined.

Research has resulted in the development of a transmitter for the simultaneous transmission of telephone and visual beacon signals on the

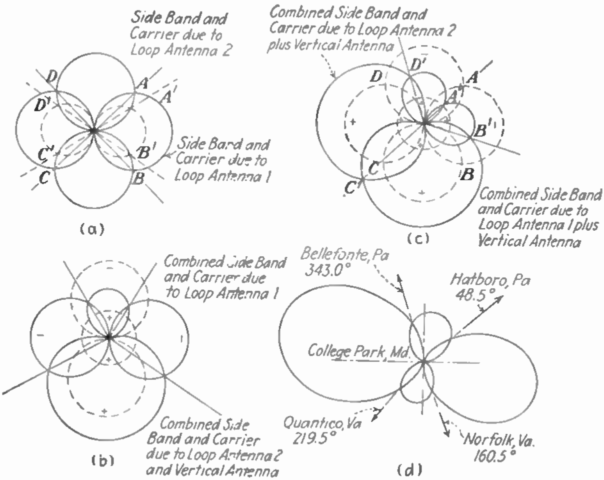


FIG. 18.—Methods of aligning beacon courses with airways.

same carrier frequency. The visual beacon modulation is below 150 cycles, while speech frequencies are above 250 cycles. The transmitting circuit and antenna are designed to set up circular radiation for the speech messages and figure-of-eight radiation for each of the two beacon waves. On the airplane an electrical filter circuit is employed in the receiving set output so that frequencies above 250 cycles are applied to the headphones and those below 250 cycles to the vibrating reed indicator.

24. Aural-type Radio Range Beacon. The master oscillator is a 50-watt tube employing the Colpitts circuit. The output from the master oscillator is amplified by a set of four 50-watt tubes operating in a push-pull cross-neutralized circuit. The final power amplifier consists of two 1-kw tubes. The tuned plate circuit is coupled by an untuned link circuit to the primaries of a goniometer, the goniometer secondary windings being connected each in series with one of the crossed loop antennas. Keying is accomplished in the link circuit by means of the keying cams. The goniometer is used for convenience in orienting the beacon space pattern and consists of two primary and two secondary windings. The primary windings are crossed at 90 deg., as are also the secondary windings, the two sets of windings being made concentric. One set of windings is fixed and the other set rotatable about the common axis. The angle between the primary and secondary windings may therefore be varied at will. Each primary winding, acting in conjunction with the two crossed secondary windings and the two crossed loop antennas, sets up a system which is electrically equivalent to a single loop antenna. The plane of this phantom antenna is dependent upon the relative coupling of the secondary coils to the primary coil under consideration. Since there are two primary windings, two such phantom antennas exist, the angle between their planes being equal to the angle between the primary windings. The two phantom antennas may therefore be rotated in space (thus changing the position of the equisignal zones or courses formed by their space patterns) by changing the relative position between the primary and secondary windings. Without the use of the goniometer it would be necessary mechanically to rotate the loop-antenna system to secure the same result. In practice, the rotation of the beacon space pattern is convenient in the first adjustment of the beacon, the goniometer being locked in position after this adjustment.

25. Visual-type Range Beacon. Three types of transmitters have been developed; the double-modulation or 2- and 4-course type, the triple-modulation or 12-course type, and the simultaneous radiotelephone and range-beacon type.

26. Double-modulation Type. The electrical-circuit arrangement for the double-modulation beacon is shown in Fig. 19. A 100-watt

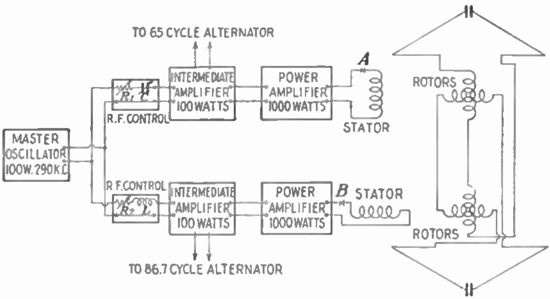


Fig. 19.—Double-modulation beacon.

master oscillator supplies r-f power to two 100-watt intermediate amplifiers which, in turn, supply power to two 1,000-watt power amplifiers. One amplifier branch is modulated to 65 cycles and the other amplifier branch to 86.7 cycles. One loop antenna therefore radiates a r-f wave modulated to 65 cycles while the other emits a r-f wave modulated to 86.7 cycles. Means are provided in the transmitter for adjusting the time-phase displacement between the carrier currents in the two antennas to either 0 or 90 deg. In the first case a 2-course beacon and in the second case a 4-course beacon is obtained.

27. Reed Indicator. The instrument employed for securing course indications may be either a reed-indicator or a reed-converter type course indicator. The latter type instrument is described in Art. 30. The reed indicator is very simple and rugged, being mounted on the instrument board in front of the pilot and electrically connected in the receiving-set output in place of, or in parallel with, the telephone receivers. It consists of a set of coils through which passes the audio output current of the receiving set. These coils actuate a pair of short steel strips or reeds which are mechanically tuned to the beacon-modulation frequencies 65 and 86.7 cycles per second. The reed indicator is very sensitive, requiring less than 2 mw for normal deflection of each reed, and weighs approximately 2 lb.

28. Triple-modulation Type. A circuit diagram of the transmitting arrangement for the triple-modulation or 12-course visual-type radio

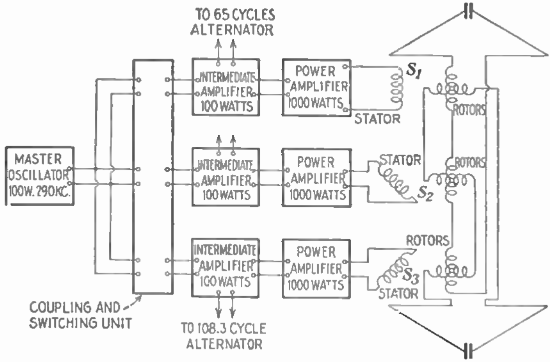


FIG. 20.—Triple-modulation range beacon.

range beacon is shown in Fig. 20. The modulation frequencies used are 65, 86.7, and 108.3 cycles respectively. A special goniometer is required for converting the two physical-loop antennas into three phantom-loop antennas crossed at 120 deg.

Since the stator coils are not at right angles to each other it is essential that but one stator be excited at any given time in order that coupling between the stators be avoided. Radio-frequency switching has been provided in the grid circuits of the intermediate amplifier tube for accomplishing this purpose. By means of a phase-splitting arrangement, the single-phase master oscillator is converted into a source of balanced three-phase supply.

The carriers in the three phantom antennas being 120 deg. out of phase both in time and in space, a revolving field is set up in space for the resultant carrier. Since the three sets of side bands are of different frequencies and do not combine, they set up three figures-of-eight crossed at 120 deg.

With the 12-course beacon as with the 4-course, the problem of adjusting the angles between courses arbitrarily to make them coincide with the airways must be solved. The methods for effecting this shift in course on the 4-course beacon may be employed in the 12-course system. In addition the stator windings may be displaced from their 120 deg. space relationship thereby modifying the space pattern radiated.

29. Simultaneous Radiotelephone-and-visual Type Radio Range Beacon. Simultaneous transmission of telephone broadcast and visual beacon on the same carrier frequency is possible chiefly because the beacon-modulation frequencies are all below 150 cycles, while intelligible speech does not require modulation frequencies below 250 cycles. At the transmitter, it is necessary to filter out frequencies below 250 cycles from the speech signals and also to suppress any existing harmonics of the beacon-modulation frequencies above 250 cycles which would produce undesirable noise in the head telephones at the receiving end. On the airplane an electric filter circuit is necessary in the output circuit of the receiving set so that frequencies above 250 cycles are applied to the head telephones and those below 250 cycles to the vibrating reed indicator. Other special problems of receiving-set design are involved.

The transmitting-circuit design must be so arranged that the normal beacon space pattern, consisting of two crossed figures-of-eight, is in no way affected by the transmission of speech signals.

The transmitting set employs a common master oscillator which supplies power of the same radio frequency to the radiotelephone and radio-beacon units. The radiotelephone unit is of conventional design, feeding approximately 2 kw to an open-type antenna, 125 ft. in height and provided with four flat-top elements for capacitive loading. The radio-beacon unit consists of two amplifier branches feeding the conventional radio range beacon loop-antenna system. Carrier suppression is effected in the radio-beacon unit so that the two loop antennas transmit the beacon side bands only. The open-type antenna is located centrally with respect to the loop-antenna system, care being taken that the voltages induced in the flat-top elements and lead-in wires by the loop antennas cancel out.

The open antenna transmits the carrier and the speech side bands, while the loop antenna transmits the beacon side bands. Hence the space pattern for the carrier and speech side bands is circular, while the space pattern for each set of beacon side bands is a figure-of-eight having its axis along the plane of the corresponding loop antenna. At the receiving end, the circular carrier beats with the circular speech side bands to give speech signals of equal intensity in all directions, while the circular carrier beats with the figure-of-eight beacon side bands to give the conventional beacon polar pattern.

To secure maximum operating efficiency it is necessary to provide a 90-deg. time-phase displacement between the carrier current in the open-type antenna and that which would be present in the loop antennas if no carrier suppression took place. This displacement serves to balance the 90-deg. difference in time-phase displacement in the radiation fields from an open-type and loop antenna and thus insures that the carrier and beacon side bands arrive in proper phase at the receiving end.

To provide field intensities for the radiotelephone and radio-beacon services comparable with those set up by existing radiotelephone and range-beacon stations, a ratio of peak-speech modulation to reed-frequency modulation of about 5 to 1 is required. The loop-antenna currents are therefore adjusted to values such that the carrier is modulated 15 per cent at each beacon frequency. The radiotelephone unit is designed to permit 75 per cent peak-speech modulation.

30. Reed-converter Type Course Indicator. This type of indicator makes possible securing the beacon-course indications on a pointer-type instrument instead of through the comparison of the relative amplitude vibrations of two reeds. The motion of two vibrating reeds is utilized to induce voltages in two pick-up or generating coils. These voltages are then rectified by means of copper-oxide rectifiers, and the rectified voltages applied in opposition to the terminals of the zero-center instrument. To preclude the possibility of the failure of either the transmitting or receiving set being unnoticed by the pilot, a volume indicator is employed. This may be, in the simplest case, a voltmeter recording the receiving-set output voltage. This indicator also facilitates tuning of the receiving set, controlling the volume, and also securing a sense of approach as the distance from the airplane to the beacon station decreases. Both a 4-course type and a 12-course type reed converter has been developed. With either arrangement a suitable selector switch is provided which the pilot sets according to the course he is to follow, and the direction of flight with respect to the beacon, "to" or "from." The pointer then deflects in the direction of deviation from the course.

31. Deviometer. One of the auxiliary devices developed to facilitate the use of the visual-type beacon is an instrument called a deviometer. By its use a pilot can follow any chosen course, within limits, on either side of the equisignal line for which the beacon transmitter is adjusted. The deviometer is essentially a device for changing the relative sensitivity of the two reeds of the course indicator, thereby permitting the pilot to fly courses (with equal reed deflections) along a line other than the equisignal line or zone set up by the beacon. It consists of a variable resistor connected to the reed-driving coils. A movement of the pointer to the right or left reduces the shunting resistance across one or another pair of reed-driving coils and thereby reduces the current through that pair of coils and consequently the sensitivity of the reed which they actuate. The scale over which the pointer moves may be calibrated in degrees off the equisignal zone and will be correct for all beacons having similar space characteristics. Tests have indicated that the deviometer may be advantageously employed to obtain new courses up to 15 deg. on either side of the equisignal zone.

32. Automatic Volume Control. An automatic volume-control device has been developed, primarily for use on receiving apparatus for the visual-type beacon, which relieves the pilot of excessive volume-control manipulation; in a flight he may experience field strengths in a ratio of 5,000 to 1.

A diagram of the circuit arrangement adopted, in one form, is given in Fig. 21. A portion of the receiving-set output voltage which operates the reed indicator is applied to the input terminals of a copper-oxide rectifier. The pulsating output voltage of this rectifier is then filtered and the resultant direct voltage (of negative polarity) applied to the control grid of one or more of the radio-frequency amplifying tubes of the receiving set. The manner of operation of the automatic volume control is, then, as follows. Any increase in the voltage across the reed-indicator terminals, due to an increasing input voltage to the receiving set, is accompanied by an increase in the voltage applied to the copper-oxide rectifier and consequently in an increasing negative direct voltage on the grids of the radio-frequency amplifying tubes. This results in a decrease of the receiving-set sensitivity, thereby tending to maintain substantially constant output voltage across the reed-indicator terminals. The reed-vibration amplitudes may thus be kept substantially constant regardless of the distance of the airplane from the radio range-beacon station, without any necessity for manipulation on the part of the pilot.

Referring to Fig. 21, the voltage divider *R* is provided for adjusting the magnitude of the negative voltage applied to the grids of the r-f tubes and may be used for securing any degree of automatic control. At the maximum setting of *R*, complete automatic control is effected, at the minimum setting manual control obtains. At an intermediate setting, semi-automatic volume control is secured. The performance graphs *A*, *B*, and *C* of Fig. 21 correspond to these three settings of the voltage divider. The automatic volume-control device may therefore be used in two ways.

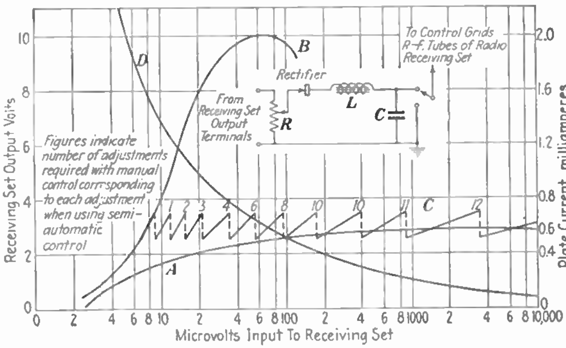


FIG. 21.—Automatic volume-control circuit and operation.

In the first method, completely automatic control is employed, no manual adjustment of the volume control being required on the part of the pilot. It is then necessary to provide some means whereby the pilot may be given sense of approach to the beacon station, which, when employing manual volume control, he obtains through the gradually increasing reed-vibration amplitudes and through the necessity for frequent volume-control adjustments as the distance between the airplane and the beacon station is decreased. A deflection instrument reading the d-c plate current of the radio-frequency amplifying tubes, on the grids of which the negative control voltage is applied, is used for this purpose.

The deflection of this instrument is an inverse function of the field intensity of the received radio wave and consequently of the distance from the beacon station. This instrument may therefore be calibrated approximately in miles from the station.

In the second method of utilizing the automatic volume-control device, the voltage-divider *R* is adjusted for semi-automatic control. The number of times the pilot needs to operate the manual volume control is then reduced to the order of one-fifth that required with normal manual-control operation.

RADIO AIDS TO BLIND LANDING OF AIRCRAFT

33. A radio system of blind landing aids developed at the National Bureau of Standards includes three elements to indicate the position of the landing airplane in three dimensions as it approaches and reaches the point of landing. Lateral position given for the purpose of keeping the airplane directed to and over the desired landing-field runway, is secured by a small directive beacon of the same type as the visual radio range beacon but lower in power and using small loop antennas.

Longitudinal position, to inform the pilot that he has arrived within the boundaries of the landing field, is given by a boundary-marker beacon. Vertical guidance is given by an inclined ultra-high-frequency radio beam. This landing beam operates on a frequency of about 100 megacycles and is directed at a small angle above the horizontal. It provides a gliding path for the landing airplane.

34. Method of Field Localization. Figure 22 is a three-dimensional view showing the location of the ground transmitting equipment for orienting a pilot along the desired landing runway, and illustrates the function of the landing beam when used in conjunction with the other elements of the system. Referring to Fig. 22, *A* is the 2-kw directive radiobeacon with large loop antennas, provided at terminal airports for point-to-point flying on the fixed airways. Utilizing the zero-signal

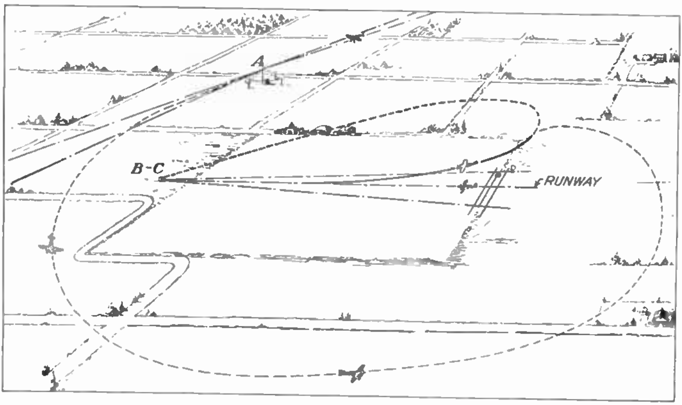


FIG. 22.—Model of landing runway and down-leading radio signal.

zone directly over the beacon tower, it is possible to locate this beacon to within 100 to 1,000 ft. depending upon the altitude of the airplane. Before reaching the beacon tower the pilot has learned the wind direction at the landing field either through the government weather broadcast or by two-way communication with the ground. Upon receiving the zero-signal indication over the tower of the main beacon, the pilot retunes his medium-frequency receiving set to the frequency of the low-power (200-watt) runway localizing beacon, located at *B*. This beacon, using small loop antennas located at one edge of the field without constituting an obstruction, directs a course along the runway most suitable for landing (under the particular wind conditions then existing). When crossing the boundary of the landing field a signal from the boundary-marker beacon *D*, operating on a radio frequency of about 10,000 kc, is obtained.

The medium-frequency set on the airplane together with a 2-tube marker-beacon set of fixed tuning is sufficient for receiving all these indications. If accurate indications of the absolute height of the airplane above ground are now secured, the complete information necessary for the blind landing of aircraft (in addition to that obtained from the flight instruments) becomes available.

35. Landing Beam. Figure 22 will show how the suitable indication of absolute height above ground is secured. The vertical space pattern of the inclined ultra-high-frequency landing beam located at C is clearly indicated. The polar pattern in the horizontal plane is of the same order of directivity. The airplane is therefore readily directed approximately along the horizontal axis of the beam by means of the course indications from the runway localizing beacon. It does not, however, fly along the inclined axis of the beam, but on a curved path whose curvature diminishes as the ground is approached. This path is of the line of equal intensity of received signal below the inclined axis of the beam. The diminution of intensity as the airplane drops below the inclined axis is compensated by the increase of intensity due to approaching the beam transmitter. Thus, by flying the airplane along such a path as to keep constant the received signal intensity, as observed on a microammeter on the instrument board, the pilot comes down to ground on a curved line suitable for landing. If the airplane rises above this line of equal intensity of received signal, the microammeter deflection increases, while if it drops below this line the microammeter deflection decreases.

36. Equipment Required on the Airplane. The same equipment carried for weather-broadcast and radio-beacon services is used for receiving the signals from the runway localizing beacon. The localizing beacon is of the visual type permitting the use of automatic volume control in its reception. This is quite essential, since the pilot, in making a landing, is concerned with so many things that the burden of close manual adjustment of receiving-set sensitivity must be eliminated.

The course indicator may be either the reed-indicator type or the zero-center pointer type operated by a reed converter. In the latter case, the instrument for securing runway-course indications may be combined with

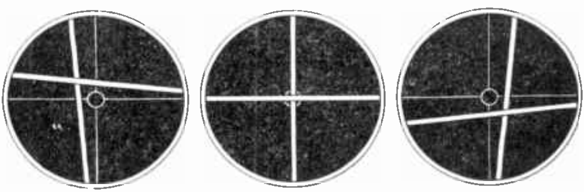


Fig. 23.—Course indicator showing: (*a*) Airplane on proper course; (*b*) Airplane too high and to the left of runway; (*c*) Airplane too low and to the right of runway.

the landing-beam indicator into a single instrument, which is much simpler to use than two separate instruments. Two perpendicular reference lines are provided on the face of the combined instrument, the vertical reference line corresponding to the position of the runway, and the horizontal reference line to the proper landing path. The pointers of the runway-course indicator and the landing-path indicator are arranged so that they cross each other, the former moving to the right or left of the vertical reference line and the latter above or below the horizontal reference line. The position of the point of intersection of the two pointers thus gives, through a single reading, the position of the airplane with respect to the runway and proper landing path. The instrument indications for several arbitrary positions of the airplane are given in Fig. 23. At *b* the airplane is to the left of the runway course and too high. At *a* the airplane is on the runway course and on the proper landing path. At *c*, the airplane is to the right of the runway course and too low.

37. Runway Localizing Beacon. This beacon is a 200-watt double-modulation beacon, employing small loop antennas so that it may be placed near the landing field without constituting an obstruction to flying. The two-course connection is employed (the antenna carrier currents being in time phase) so that, in circling the runway beacon the pilot will not be confused by the presence of courses at right angles to the runway. The modulation frequencies used at the beacon are 65 and 86.7 cycles, respectively. The antennas consist of seven turns of wire wound on frames 6 by 8 ft.

38. Boundary-marker Beacon. The boundary-marker beacon operates on a carrier frequency of about 10,000 kc. The transmitting antenna consists of a long horizontal wire 3 to 8 ft. above the ground and extending along the edge of the field at right angles to the runway. The signal is received about 100 ft. before the airplane reaches this antenna and is heard for about 100 ft. after the airplane has passed over the antenna. Since a vertical receiving antenna is employed on the airplane, zero signal is obtained when the airplane is directly over the antenna. This zero-signal zone, therefore, coincides with the landing-field boundary line. The transmitting set employs a 7.5-watt tube feeding an oscillatory circuit, coupled inductively to the horizontal antenna. The oscillator is modulated to about 500 cycles by means of an audio-frequency oscillator of about 7.5 watts.

39. Landing Beam. The landing beam consists of a horizontally polarized beam directed at a small angle above the horizontal, this angle and the degree of directivity being so adjusted that a predetermined line of constant field intensity will mark out just the proper gliding path, clearing all obstructions and convenient for landing. In the set-up at College Park, Md., the beam is transmitted on a frequency of 93,700 kc (3.2 m) and is oriented in the same horizontal direction as the course of the runway localizing beacon. On the airplane, a horizontal dipole antenna feeding a detector-amplifier-rectifier unit is employed for receiving the landing beam signal. The receiving equipment constitutes a vacuum-tube voltmeter for exploring the field intensity in different portions of the landing beam.

40. Theory of Operation. The glide curve or landing path may be derived from a proper combination of two graphs, one of which shows the inverse variation of field intensity with distance from the beam transmitter, and the other the polar curve showing the beam directivity in the vertical plane. A combination graph is given in Fig. 24 wherein the field intensities (in terms of the deflection of the landing-beam indicator) are plotted as abscissas and the altitude of the airplane as altitudes at each 1,000 ft. of distance from the landing-beam transmitter. A pilot coming in at an altitude of 1,000 ft. will observe half-scale deflection of his instrument (say, 250 μa) at a distance of approximately 9,000 ft. from the beam transmitter. If he then follows the line of constant field intensity corresponding to half-scale deflection on his instrument, he reaches an altitude of 10 ft. at a distance of 2,000 ft.

from the beam transmitter. This is actually the point of contact of the airplane with the ground, the receiving antenna being mounted on top of the airplane, 10 ft. from the ground.

41. Directive-transmitting Antenna System. An ultra-high frequency was chosen for the landing-beam transmitting system in order to secure the attendant reduction in size and simplicity of equipment. The

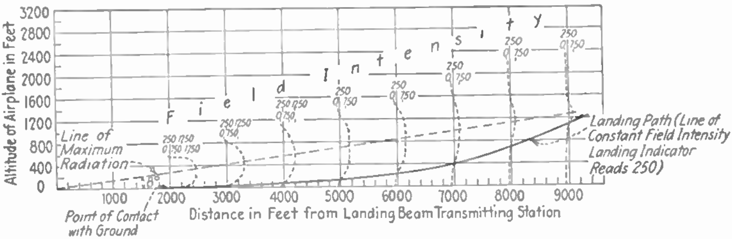


FIG. 24.—Field strength pattern of landing radio beam.

ultra-high-frequency source (93,700 kc) is coupled to a horizontal doublet (made of ⅛-in. copper tubing), which serves as the radiating antenna and is accurately tuned to the frequency of the source. About 0.8 m behind the radiating antenna is placed a reflecting antenna, also a horizontal doublet, tuned to a frequency somewhat lower than the frequency of the source. At approximately every meter in front of the radiating antenna, horizontal-doublet directing antennas are placed. These are tuned to a frequency somewhat higher than that of the source. This array of antennas is supported approximately 2.75 m above the ground, and pivoted on a vertical support. To obtain the desired vertical directive characteristic, the structure is tilted approximately 8 deg. above the horizontal. The necessary power output on the high frequency used is secured through the use of a 500-watt three-element tube (General Electric ZP-2) in the oscillatory circuit shown in Fig. 25. The small air condenser E is of a capacity about equal to the inter-electrode grid to plate capacity of the tube. The circuit is tuned to the desired frequency within narrow limits by moving one of the plates of condenser E by means of an insulated adjustment screw.

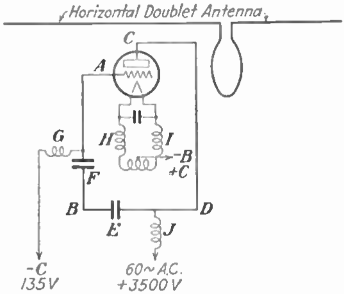


FIG. 25.—High-frequency transmitter for landing beam.

42. Receiving System. The receiving circuit arrangement (see Fig. 26) uses only two tubes without regeneration and requires no adjustments on the part of the pilot. Even the volume control is dispensed with, since the path followed during the use of the receiving set constitutes a line of constant field intensity of the directed beam. The detecting portion (within the dotted lines) is external to the airplane, mounted in a streamline weatherproof box about 14 in. above the top wing. The doublet antenna is in the form of two copper rods housed in wooden streamlined supports projecting from the streamlined detector box.

A 224-type tube is employed for the detector, to afford the necessary high amplification without undue microphonic noises. To obtain good efficiency the detector tube is connected directly in the center of the horizontal doublet antenna. The radio-frequency portion of the circuit is confined to the section above the four radio-frequency chokes. The

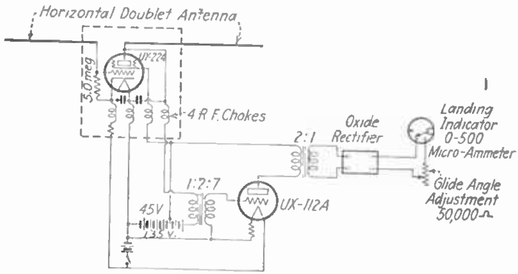


FIG. 26.—Receiving circuit for landing beam.

four leads running from the lower side of these chokes carry either direct current or the received audio modulation.

ABSOLUTE ALTIMETERS

43. Absolute altimeters may be used to replace the landing beam in the above system and may also be employed during point-to-point flight. Such altimeters fall into three classifications, the sonic altimeter, the capacity altimeter, and the reflection altimeter. In the sonic altimeter the time taken by sound to reach the ground and return to the airplane is measured. Knowing the velocity of sound, the height of the airplane above ground may be determined. In a model developed by the General Electric Company two horns are employed: one, motor driven, sends down the sound wave, and the other receives it back again after reflection from the ground. An instrument which is started by the emitted wave and stopped by the reflected wave records all heights above 50 ft., while a stethoscope connected to the receiving horn and adjustable to the pilot's ears, is used below 50 ft. At 50 ft. the echo comes back a tenth of a second after the emitted sound is sent out, at 5 ft. it comes back a hundredth of a second later. The whistle and the echo blend into one sound at some point above 5 ft. This indication may be used by the pilot in "leveling-off" for a landing.

44. In the capacity altimeter, the distance from the ground is measured by detecting the change in the electrical capacity between two plates on the airplane as the airplane approaches the ground. In one arrangement, this capacity is made a part of a resonant circuit, coupled to an extremely stable radio-frequency oscillator. A vacuum-tube voltmeter records the voltage developed across a portion of the resonant circuit. The circuit is adjusted so that the voltmeter-indicating instrument reads zero when the airplane is at any height above 100 ft. The gradual increase in capacity as the airplane approaches the ground serves to bring the resonant circuit into closer tune with the oscillator frequency, the voltmeter indication increasing accordingly. The indicating instrument, once calibrated, serves to indicate true height above ground. Since the capacity between the two plates is practically unchanged at altitudes greater than of the order of 100 ft., the field of usefulness of the capacity altimeter is limited to landing operations only.

45. The reflection altimeter utilizes the direct reflection of radio waves. Frequencies of the order of 10 to 30 megacycles have been found most useful for securing true reflection from the ground. The phase difference between the transmitted and reflected wave varies cyclically as a function of height above ground. Alexanderson has shown that this cyclic change of phase difference manifests itself in a corresponding change in frequency of the transmitting oscillator. He therefore employs two oscillators on the airplane, one tuned to, say, 30 megacycles, and the other to, say, 27 megacycles, which detect the beat frequency, 3 megacycles. This beat frequency changes cyclically as the altitude of the airplane is varied, passing through a maximum when the reflected wave tends to increase the frequency of the 30-megacycle oscillator at the same time as it decreases the frequency of the 27-megacycle oscillator. A little consideration will show that the maxima occur at 25 m (80 ft.), 75 m (240 ft.), 125 m (400 ft.), etc. Definite indications of true height above ground may therefore be secured at these points. By changing the difference frequency, different points may be obtained.

References

Dellinger, J. H., and H. Diamond: Radio Developments Applied to Aircraft, *Mech. Eng.*, **51**, 509, July, 1929.

———, ———, and F. W. Dunmore: Development of the Visual Type Airway Radiobeacon System, *Bur. Stand. Jour. Res., Res. Paper* 159, March, 1930. *Proc. I. R. E.*, **18**, 796, May, 1930.

——— and H. Pratt: Development of Radio Aids to Air Navigation, *Proc. I. R. E.*, **16**, 890, July, 1928.

Diamond, H.: Applying the Visual Double-modulation Type Radio Range to the Airways, *Proc. I. R. E.*, **17**, 2158, December, 1929; *Bur. Stand. Jour. Res., Res. Paper* 148, February, 1930.

——— and G. L. Davies: Characteristics of Airplane Antennas for Radio Range Beacon Reception, *Bur. Stand. Jour. Res.* May, 1931; *Proc. I. R. E.*, **20**, February, 1932.

——— and F. W. Dunmore: A Radiobeacon and Receiving System for Blind Landing of Aircraft, *Bur. Stand. Jour. Res., Res. Paper* 238, October, 1930.

——— and F. G. Kear: A 12-course Radio Range for Guiding Aircraft with Tuned-reed Visual Indication, *Bur. Stand. Jour. Res., Res. Paper* 154, March, 1930.

Doolittle, J. H.: Fog Flying, *S. A. E.*, preprint (St. Louis meeting), February, 1930.

Drake, F. H.: An Aircraft Radioreceiver for Use with Rigid Antenna, *Proc. I. R. E.*, **17**, 306, February, 1929; see also *Aero Digest*, **13**, 717, October, 1928.

——— and R. M. Wilmotte: On the Daylight Transmission Characteristics of Horizontally and Vertically Polarized Waves from Airplanes, *Proc. I. R. E.*, **17**, 2242, December, 1929.

Dunmore, F. W.: Design of Tuned Reed Course Indicators for Aircraft Radiobeacon, *Bur. Stand. Jour. Res., Res. Paper* 28, November, 1928.

———: A Tuned-reed Course Indicator for the 4- and 12-course Aircraft Radio Range, *Bur. Stand. Jour. Res., Res. Paper* 160, April, 1930; *Proc. I. R. E.*, **18**, June, 1930.

———: A Course Indicator of Pointer Type for the Visual Radio Range-beacon System, *Proc. I. R. E.*, **19**, September, 1931.

Engel, F. H., and F. W. Dunmore: A Directive Type of Radiobeacon and Its Application to Navigation, *Bur. Stand. Sci. Paper* 840, 1923.

Gill, T. H., and N. F. S. Hecht: Rotating-loop Radio Transmitters and Their Application to Direction Finding and Navigation, *Jour. I.E.E.*, **66**, March, 1928.

HYLAND, L. A.: The Constants of Aircraft Trailing Antennas, *Proc. I. R. E.*, **17**, 2230, December, 1929.
JACKSON, W. E., and S. L. BAILEY: Development of a Visual Type of Radio Range Having Universal Application to the Airways, *Proc. I. R. E.*, **18**: 2059, December, 1930.
MURPHY, W. H.: Space Characteristics of Antennae, *Jour. Franklin Inst.*, **201**, 411, 1926; **203**, 289, 1927.
——— and L. M. WOLFE: Stationary and Rotating Equi-signal Beacons, *Jour. S. A. E.*, **19**, 209, September, 1926.
NELSON, E. L., and F. M. RYAN: Provision of Radio Facilities for Aircraft Communications, *S. A. E.* preprint (St. Louis meeting), February, 1930.
PRATT, H.: Apparent Night Variations with Crossed-coil Radio Beacons, *Proc. I. R. E.*, **16**, 625, May, 1928.
WALLS, H. J.: The Civil Airways and Their Radio Facilities, *Proc. I. R. E.*, **47**, 2141, December, 1929.

SECTION 22

PHOTOCELLS

By H. C. Rentschler, Ph.D.[1]

1. Photoelectricity. Photoelectricity includes all such phenomena where a change in electrical behavior is produced by the action of light. These phenomena may be divided into three classes:

1. Photoelectric Emission.
2. Photoconductivity.
3. Photovoltaic Effect.

2. Photoelectric Emission. In 1888 Hallwachs[2] showed that a negatively charged zinc sphere, which had been freshly polished, was rapidly discharged when light from an arc was allowed to shine upon it; but that a positively charged sphere did not lose its charge unless a negatively charged body near by was irradiated at the same time. It has since been proved that this effect is due to the emission of electrons from the metal surface due to the light (especially light of short wave length), just as the filament of a thermionic device emits electrons when heated to the proper temperature. This emission of electrons from an illuminated metal is known as *photoelectric emission.* This photoelectric emission makes possible the current in an electrical circuit between the illuminated metal as cathode and a nearby anode.

3. Photoelectric Cell or Phototube. The photoelectric emission from a metal depends upon the condition of the metal surface (whether clean or covered by an oxide film, etc.). To obtain reproducible emission, the cathode and anode are now always mounted in a glass or quartz container which is either highly evacuated or contains an inert gas at a low pressure. Such devices are commonly known as photoelectric cells or phototubes.

4. Phototube Circuit Insulation. The current that can be passed through even the most sensitive phototubes is many times smaller than the current through a small thermionic-valve tube. Thus with the best commercial type of vacuum phototube with the cathode exposed to a 60-watt tungsten lamp at a distance of about 10 in., the maximum current is of the order of 10 to 20 μa.

Special care is necessary to insulate properly the parts of photoelectric tube circuits so as to avoid electrical leakage. Insulators generally used in electrical circuits even with thermionic devices are often useless for these circuits. For the more sensitive applications it is often necessary to use such insulators as amber, sulphur, or red sealing wax. Where

[1] Member, American Physical Society, American Association for the Advancement of Science, American Optical Society; director of research, Westinghouse Lamp Company.
[2] Hallwachs, *Ann. Physik*, **33**, 301, 1888.

phototubes are used for precision measurements or for the detection o very weak light intensities it is desirable to use tubes where the anode an cathode leads are not brought out through the same press.

5. Properties of Photoelectric Emission. The photoelectric emissio from a given cathode is practically instantaneous. Lawrence and Beams have shown that the interval between the incidence of the light and th full emission of electrons is less than 10^{-8} sec.

The photoelectric emission from a given cathode is independent o the temperature as long as there is no actual change in the cathode sur face and as long as its thermionic emission is negligible.

The photoelectric emission from a given cathode is strictly proportiona to the light intensity, provided the quality (color or wave length) of th light is not changed.

For such applications as strictly require one or all of these properties the phototube is superior to light-sensitive devices using either th principles of Photo-conductivity or the Photovoltaic effect, and it is inferior only in the magnitude of the response for a given intensity.

6. Color Sensitivity. All metals with clean surfaces are photoelectrically active when exposed to light of proper color or wave length. There

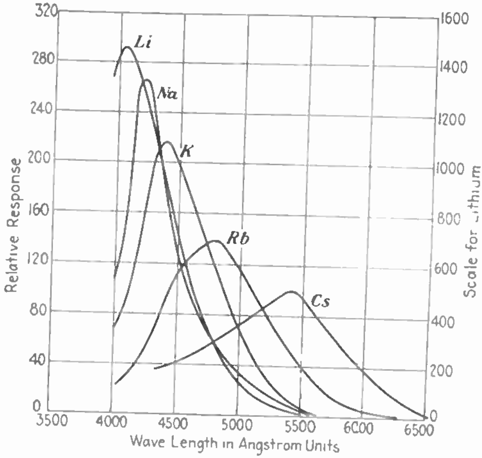


FIG. 1.—Color sensitivity for alkali metals.

is for each metal a longest wave length (called the *threshold wave length*) to which the metal responds. Thus the common metals such as iron, nickel, copper, etc., require very short ultra-violet radiation, while the alkali metals sodium, potassium, etc., are sensitive to visible light. The curves giving the emission or response of a cathode for equal energy of different wave lengths of radiation is called its *color-sensitivity curve*. Thus in Fig. 1 are shown the relative color sensitivities of different tubes containing the different alkali metals as measured by Miss Seiler.[2] Each

[1] LAWRENCE, E. O., and J. W. BEAMS, *Phys. Rev.*, **29**, 903, 1927.
[2] SEILER, *Astrophys. Jour.*, **52**, 129, 1920.

of the alkali metals has a definite wave length for which the response is a maximum.

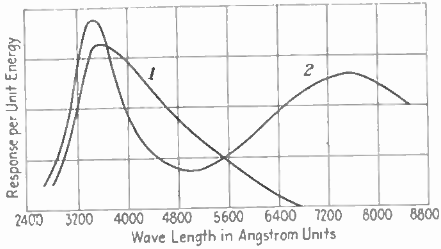


FIG. 2.—Color sensitivity for caesium on magnesium, curve 1. Color sensitivity for caesium on silver oxide specially processed, curve 2.

The color sensitivity of a metal is often dependent upon the thickness of the coating of the active metal deposited upon a second metal as a conducting backing and the treatment given to this coating. Thus in Fig. 2 is shown the relative color-sensitivity curve for a thin layer of caesium on magnesium (curve 1) and the similar response (curve 2) for a thin layer of caesium on silver oxide with a copper backing and processed so that it shows sensitivity for infra-red radiation.[1]

When photoelectric tubes are to be used for measuring or detecting

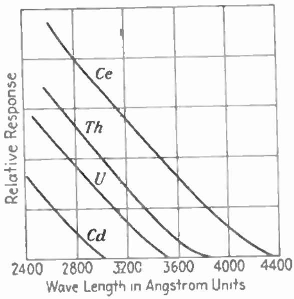


FIG. 3.

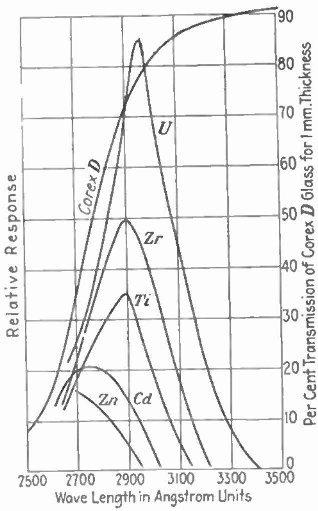


FIG. 4.

FIG. 3.—Sensitivity for vacuum phototubes in quartz bulbs with cathodes of cerium, thorium, uranium, and cadmium.

FIG. 4.—Sensitivity for phototubes in Corex D glass with uranium, zirconium, titanium, cadmium, and zinc cathodes.

ultra-violet radiation only, the elements cerium, thorium, uranium, and cadmium serve well for tubes having different threshold wave lengths.

[1] Details of the method used in the processing of cathodes to change the relative sensitivity are described by A. R. Olpin, *Phys. Rev.*, **36**, 251, 1930.

In Fig. 3 are shown the relative response curves for these metals as cathodes in quartz bulbs as vacuum-phototubes. The relative response curves for several metals as cathodes in bulbs of ultra-violet transmitting glass known as Corex D (wall thickness about 1 mm) are shown in Fig. 4. The peaks of maximum response for these tubes are not, as was the case with the alkali metals of Fig. 1, an inherent property of the cathode material. The short wave-length response is cut off by the absorption of the radiation by the glass container. These peaks may be shifted by varying the thickness and quality of glass used.

VACUUM AND GAS PHOTOTUBES.

7. Vacuum Tubes. The electrons liberated from the cathode under the influence of light come off with different velocities and in different directions. As the difference of potential between anode and cathode is raised, more of the electrons liberated from the cathode reach the anode, and saturation current is reached when all that leave the cathode are drawn directly to the anode as fast as they are produced. Because of the small emission from the cathode, the effect of space charge is negligible. Curve *a* (Fig. 5) shows the current-voltage relation for a vacuum tube with the specially processed caesium on silver oxide cathode. The slope of the curve and the potential necessary for saturation depend chiefly upon the spacing and shape of the electrodes.

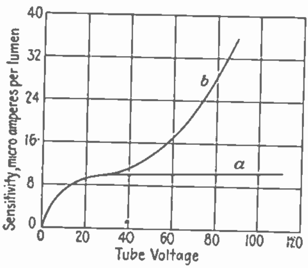


FIG. 5.—Characteristic for vacuum (*a*) and gas-filled (*b*) tubes.

8. Gas Tubes. Since the current obtainable by photoelectric action is very small, attention has always been given to ways for amplifying these currents. This may be accomplished by the use of devices such as three-electrode thermionic amplifier tubes. Another means often resorted to is the so-called "gas photoelectric tube." Here a small amount of an inert gas, such as argon, is introduced into the tube. The photoelectrons as they pass from the cathode to the anode, ionize the gas, and the ions so produced take part in carrying the current. Curve *b* of Fig. 5 is typical showing the current-voltage relation for a gas photoelectric tube which gives curve *a* when evacuated.

Commercial gas tubes usually have a pressure of about 100 μ or less and an amplification due to the gas of about 10. Higher amplifications are possible but result in greater instability of operation.[1]

The gas tube is not so linear in its response to light of varying intensity as is the vacuum tube. For most practical purposes, except for precision

[1] For more detailed discussion of the mechanism of amplification by gas ionization, the reader is referred to the excellent discussion by Campbell and Ritchie in their book on "Photoelectric Cells" (Sir Isaac Pitman & Sons, Ltd., 1929, Chaps. V and VI). Also to J. S. Townsend's book on "Electricity in Gases" (Clarendon Press, Oxford, 1915) for the general theory of ionization by collision.

Curves showing the effect of gas pressure on the maximum sensitivity of a photocell are given by Koller (*Jour. Optical Soc. America* and *Rev. Scientific Instruments*, **19**, 135, 1929).

measurements requiring high degree of accuracy and wide variations of intensity, its linearity is, however, quite good enough. But gas tubes require greater precautions in their use than do vacuum tubes.

For the protection of the tube and the rest of the apparatus in the circuit it is always well to insert a resistance of from 1,000 to 5,000 ohms in series with the tube. Such a resistance will prevent any damage which might result in the use of gas tubes if a glow developed due to too high an impressed voltage, and the danger of the glow breaking over into an arc. A glow in a phototube must never be permitted for any length of time. In some cases such a glow may result in increased sensitivity of the tube while in other cases it may result in permanent injury to the tube.

9. Choice of Phototube. It is always well to use vacuum tubes in preference to gas tubes whenever it is possible to do so. Vacuum tubes are simpler to handle and capable of giving more accurate and reliable results.

For most general applications, phototubes are operated from artificial-light sources. The most convenient source is the ordinary incandescent lamp. The maximum intensity of radiation from such a source is in the infra-red and falls off rapidly for the visible and ultra-violet. For such applications therefore the best tube to use is one that has as great a sensitivity in the visible and infra red as is obtainable. The phototube almost universally used at present for such applications is the one having the color sensitivity shown in curve 2 (Fig. 2).

Phototubes are now quite extensively used for the photometry of incandescent lamps. For such applications the tube must be very constant, and above all the cathode surface should be uniformly sensitive over the entire surface. The color-sensitivity curve should preferably be similar to that of the average human eye. In practical photometry of incandescent lamps, the general practice is to compare the radiation from the unknown lamp with that from a standard lamp having the same general radiation characteristic and which is operated at approximately the same temperature as the unknown lamp. For such applications the caesium-on-magnesium tube with a special green filter is quite satisfactory.

A similar problem is that of measuring ultra-violet radiation within a definite wave band. There is an ever increasing interest in the use of ultra-violet radiation for health, for medical treatment of certain ailments, and for photo-chemical reactions. Thus it has been fairly well established by the medical profession that radiations effective in the prevention of rickets extends from wave length of about 2,800 to about 3,200 Å. units.

The radiation of about 2,950 Å. units is the most powerful, and the beneficial effect falls off to a low value as the wave length of the radiation approaches either the longer limit of about 3,200 A. units or the shorter limit of about 2,800 Å. units. Referring to Fig. 4 it is evident that cells covering this range fairly well are now available. It is evident that the definite problem under investigation determines the cell best suited for the test.

10. Phototube Circuits. The practical use of phototubes for the various applications call for circuits to fit the particular use. Generally speaking, the amount of current obtainable in the particular case largely controls the choice of circuit. The simplest phototube circuit is that shown in Fig. 6.

Here the battery B sends a current through the phototube P when light falls on the cathode. This current is detected or measured by the galvanometer shown as G. If the phototube is relatively sensitive or the intensity of light is sufficiently great, the galvanometer G may be replaced by a microammeter. It is at once evident how variations in light intensity may be followed by changes in reading of the detecting instrument G.

In cases where the light intensity is low so that measurements require extreme sensitivity, an electrometer E and a high resistance R as shown in Fig. 7 are substituted for the galvanometer. In principle the electrometer has two highly insulated conductors A and C, and a movable element N which is maintained at a constant potential. As the potential

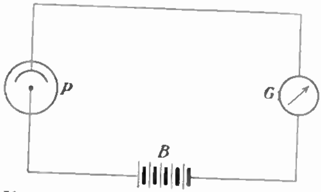


FIG. 6.—Simple circuit for measuring photoelectric current.

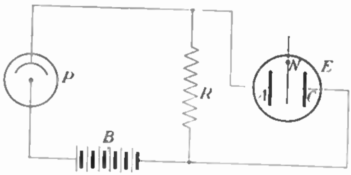


FIG. 7.—Circuit using a high resistance and an electrometer for measuring small photoelectric currents.

between A and C changes, the element N moves with reference to A and C. Thus the electrometer E of Fig. 7 is used to measure the potential drop across the resistance R.[1]

The resistance R is of the order of 10 to 1,000 megohms depending upon the sensitivity required.

Campbell and Ritchie[2] describe commonly known methods of making these high resistances. The writer[3] prefers to use the resistance of a special carbon deposit on a glass spiral sealed in an evacuated bulb. Such resistances having any value from a megohm or less to several hundred thousand megohms are easily made. In the use of this circuit it is essential that the lead connecting the phototube to the resistance R and the element A of the electrometer is very carefully insulated.

A simple circuit for measuring very small photoelectric currents is shown in Fig. 8.

Here the battery B sends a current i through the phototube P to charge a condenser K of capacity C to a potential V measured by the electrometer E, in the time t. The average current is given by the equation

$$i = \frac{CV}{t}$$

[1] Detailed instructions describing the uses of electrometers are found in Rutherford's "Radioactive Substances" (Cambridge University Press, 1912) or in any standard treatise on electricity and magnetism.

[2] CAMPBELL and RITCHIE, "Photoelectric Cells," pp. 126–129.

[3] RENTSCHLER and HENRY, *Rev. Scientific Instruments*, **3**, 91, 1932.

The phototube used in this method should be of the vacuum type, and the battery voltage should be sufficiently high, and the potential to which the condenser is charged in the observed time should be such that the tube operates with saturation current over the entire time. If these conditions do not hold, corrections must be made in the calculations of the photoelectric current.

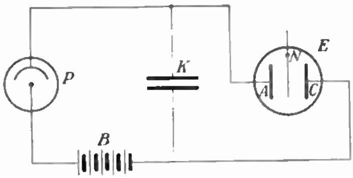


FIG. 8.—Measuring the photoelectric current by noting the rate at which it charges a known condenser.

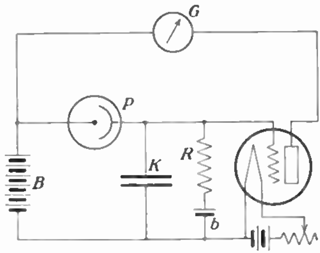


FIG. 9.—Amplifying the photoelectric current by the use of a thermionic tube.

Instead of using extra-sensitive instruments for measuring or detecting the small photoelectric currents, these currents may be amplified by the use of the three-electrode thermionic tubes. Thus the potential drop across the resistance R of Fig. 7 may serve as the control of the potential of the grid with reference to the cathode of a thermionic amplifier tube. In Fig. 9 is shown such a circuit using the same B battery in the plate-to-filament circuit and at the same time supplying the voltage to send the current through the phototube. The battery b is used to supply the proper grid bias. A condenser K represents the capacity of the phototube and its connections. This capacity need be considered only for such circuits where rapidly fluctuating light effects are to be recorded in the plate circuit.

It is at once evident how a relay may be used in place of the meter G, when it is desired to use this circuit for control purposes.

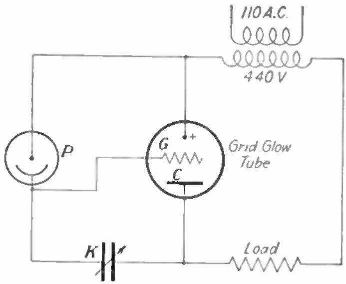


FIG. 10.—Circuit using photoelectric tube with Grid-Glow tube.

For such applications where a phototube is used for turning on or off a device, a simple gas-discharge tube known as the Grid-Glow tube[1] may replace the amplifier tube as is shown in Fig. 10. This tube is so designed that when voltage is impressed between the cathode and anode, the grid takes on a negative charge thus preventing a breakdown. If the grid is permitted to discharge as through a phototube, when light

[1] KNOWLES, *Elec. Jour.*, **26**, 176, 1928.

falls on the tube, a discharge is started in the anode-to-cathode circuit which is limited only by the impressed voltage and the load resistance. A condenser K is generally inserted as shown in Fig. 10 to control the sensitivity of the device.

11. Uses of Photoelectric Tubes. The practical uses of phototubes may be classified under three distinct groups.

1. For measurement of light intensities as:
   Photometry of lamps.
   Measurements of ultra-violet radiation.
   Measurements of light transmission through and reflection from different materials, etc.

For such application, circuits of Fig. 7 to Fig. 10 are suitable.

When a large number of similar measurements or tests are to be made as in photometry, the circuit used is preferably modified to supply the definite need for speed and simplicity of operation. An article by Dr. C. H. Sharp on Use of Photoelectric Cell in Photometry (*Electronics*, August, 1930, pp. 243–245) is an excellent illustration describing the modification of circuit of Fig. 9 for use in practical photometry.

2. For detection and control as:
   For counting objects by interrupting a light beam.
   For stopping of machinery when an object intercepts the light falling on a phototube.
   For turning on of lights when daylight falls below a certain level.
   For operating slave clocks from a master clock by having the pendulum interrupt the light falling on a phototube, etc.

For this class of applications, circuits of Figs. 9 and 10 are best modified to meet the definite requirements. Here a suitable relay replaces the meter of circuit Fig. 9 or the load of circuit Fig. 10.

3. Modulation of current by fluctuations of light intensities. This class includes such applications as:
   Facsimile transmission.
   Transmission of pictures by wire.
   Television.
   Conversion of sound films into speech, as talking movies, etc.

For these applications the circuit of Fig. 9 is adapted to meet the definite need. These circuits are discussed in greater detail in the various chapters covering these specific applications.

12. Photo-conductivity. This is a change in the electrical resistance of a material due to the action of light. This effect is particularly noticeable with the element selenium, which becomes a very much better electrical conductor in sunlight or under artificial illumination than in the dark.

13. Selenium Cell. In the construction of selenium cells (frequently called "*selenium bridges*"), the resistance of the selenium is so high that it is necessary to arrange the selenium so that the current passes a short distance through the selenium, and a large area is provided so that as much current as possible will pass through it for a given impressed voltage. Since only the exposed portion is affected by the light it is necessary to use a thin layer.

In one form of bridge these requirements are met by painting and heat treating two closely interpenetrating grids of gold or other metal on glass. These serve as the two leads between which the current flows through the selenium bridging between them. The selenium is spread over the surface in a thin layer and converted to the proper crystalline light-sensitive variety. To protect the active surface, this structure is generally sealed into a bulb which is exhausted or filled with an inert gas, thereby increasing the stability of the cell.

14. Properties of Selenium Cells. When the intensity of the light falling on a selenium cell is suddenly changed, there is an appreciable time lag before the current assumes a steady value. This time lag may be of the order of several minutes.

The conductivity depends upon the temperature of the cell.

For a fixed applied voltage the current is not strictly proportional to the intensity of the light.

The sensitivity to light of different wave lengths depends upon the crystalline form of the selenium, on the intensity of the light, on the duration of exposure, on the previous illumination, and upon the temperature.

The sensitivity extends well into the infra-red, through the visible into the ultra-violet, with a maximum sensitivity for yellow or red light depending upon the crystalline form of the selenium.

The dark current, that is the current when no light shines on the selenium, for commercial cells varies from a few to several hundred microamperes depending upon the design of the cell. The ratio of light current (that is the current for cell fully illuminated) to dark current is usually about 10 to 1, but cells may be made with a ratio as high as 1,000 to 1 or greater. These higher-ratio cells are usually far less reliable, and commercial cells usually use the smaller ratio. In the use of photo-conductive cells it is always desirable that the light be as uniformly spread over the active surface as is possible. For a detail discussion of the photo-conductive properties of selenium the reader is referred to Mellor's[1] "Inorganic Chemistry."

15. Thalofide Cell. This is a photo-conductive cell prepared by T. W. Case,[2] which uses the compound thallium oxysulphide in place of selenium. This cell has a maximum sensitivity at a wave length of about 10,000 Ångström units.

16. Photovoltaic Effect. The photovoltaic or Becquerel effect consists in creating an e.m.f. in a voltaic cell by illuminating either an electrode or the electrolyte. One commercial type has a cathode of a semi-cylindrical plate of copper coated with cuprous oxide. A heavy strip of lead serves as the anode, and a dilute solution of lead nitrate is used as the electrolyte. The circuit recommended is that shown in Fig. 11. These cells are often sensitive enough to operate small relays directly without the use of amplifiers or glow-discharge tubes. Like photoconductive devices these are not so well adapted where accuracy and reliability are essential, as are the less sensitive photoelectric emission tubes.

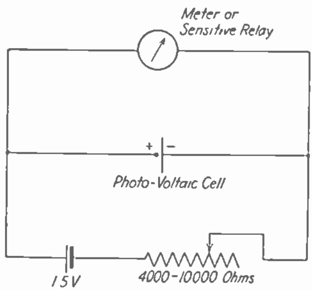


FIG. 11.—Circuit for use with a photovoltaic cell.

[1] MELLOR, "A Comprehensive Treatise on Inorganic and Theoretical Chemistry," vol. X, pp. 693–741.

[2] CASE, *Phys. Rev.*, **15**, 289, 1920; also *Jour. Optical Soc. America* and *Rev. Scientific Instruments*, **6**, 398, 1922.

17. Calculation of Voltage across Phototube Load. Characteristic curves similar to those used with vacuum tubes can be utilized to determine the voltage output of phototube circuits. For example, in Fig. 12 with a cell operated at 80 volts with a 5-megohm resistance, 60 volts appears across the cell (20 across the load) at a light flux of 0.3 lumen. In motion picture work, with the film out, the flux is about 0.2 lumen,

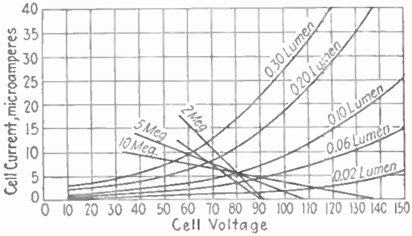


FIG. 12.—Method of calculating voltage across phototube load.

with the film running past the phototube the light coming through the film varies from 0.01 to 0.04 lumen (H. A. DeVry).

18. Commercial Light-sensitive Cells. A commercial selenium cell (FJ-31, General Electric Co.) has the following characteristics:

Average resistance at 100 foot candles	0.75 megohm
Average resistance in dark	6.0 megohms
Maximum voltage a.c. or d.c.	125 volts
Maximum current	0.5 ma

Its maximum sensitivity is around 7,000 Å., the response falling to 10 per cent at 6,000 and to 40 per cent at 8,000 Å. Measurements show that 90 per cent of the total change of resistance to light changes takes place in one-hundredth of a second. At 0.05 lumen per square inch the

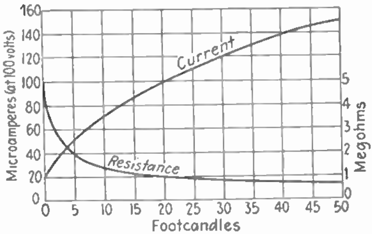


FIG. 13.—Characteristics of FJ-31 selenium tube.

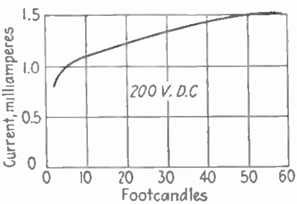


FIG. 14.—Burgess selenium cell characteristic.

relative response of the cell to light interrupted at 7 kc per second is less than one-fifth that at 500 cycles (see Fig. 13).

The Burgess selenium bridge consists of a layer of selenium about 2.5×10^{-3} cm thick on a thin glass base, on the face of which is a gold grid in the form of two interlocking combs. The surface exposed to light is approximately 25 by 50 mm. The selenium is placed in a glass envelope which is exhausted and then filled with an inert gas. The standard bridge has a dark resistance of about 4 megohms and will deliver about 100 to 150 μa output current. At 10 foot candles the ratio of dark to light resistance is at least 4 to 1 (see Fig. 14).

The Arcturus photolytic cell (probably photovoltaic) delivers considerable current at low voltage. Another voltaic cell put on the market in 1931 is the Weston photronic cell. It is a dry type of cell delivering about one microampere per lumen, has a linear output of current against light when worked into a low-resistance meter and has been applied successfully to illumination and density of film measurements.

References

ALLEN, H. STANLEY: "Photoelectricity," Longmans, Green & Co., 1925.
CAMPBELL and RITCHIE: "Photoelectric Cells," Sir Isaac Pitman & Sons, Ltd., 1929.
GUDDEN: "Lichtelektrische Erscheinungen," Julius Springer, Berlin, 1928.
HUGHES: "Photoelectricity," Cambridge University Press, 1914.
———: Report on Photoelectricity, *Bull. Nat. Res. Council*, 1921.
——— and DUBRIDGE: Photoelectric Phenomena, McGraw-Hill Book Company, Inc., 1932.
Report of the Committee on Standardization, *I.R.E.*, p. 57. 1928.
ZWORYKIN and WILSON: "Photoelectric Cells and Their Applications," John Wiley & Sons, Inc., 1930.

SECTION 23

SOUND MOTION PICTURES

By Franklin S. Irby, M.Sc., Ph.D.[1]

1. Introduction. The data given on the following pages are intended to cover a brief description of the principal apparatus and methods used in recording and reproducing sound motion pictures. The main amplifier equipment used in sound motion-picture work is similar to that used for public-address systems, electrical transcriptions, broadcasting, etc., with modifications to meet special requirements.

The equipment necessary for recording sound for motion pictures is much greater than that for reproducing. This compares with the amount of equipment required for a broadcasting station, as contrasted with a single receiver. Some of the portable sound-recording equipments have, however, been greatly simplified, and require no more units than a small theater reproducing system. The data furnished cover only those sound-recording systems in actual practical use.

While the various systems of recording differ in details, the principal parts of the apparatus are similar in purpose and design up to the recorder proper. Here, depending upon the method of recording (as described later) the recorders are different in construction and operation.

2. Methods of Recording. The principal methods of recording *sound on film* include: (1) variable-density method (which may be accomplished by using either a light-valve or glowlamp), (2) variable-area method (accomplished by using a galvanometer "vibrator"), and (3) film-engraving method (not used commercially).

The principal method of recording *sound on disk* involves using an electrical recorder similar to those used in making standard phonograph records.

Other methods of recording sound in synchronism with motion pictures includes the method proposed by Blattner for recording on a magnetic wire, and also a method using a magnetized tape.

1. *Variable-density Recording.* The *light-valve* method uses a light of constant intensity; the ribbons of the valve are modulated by the voice current, which causes a sound track of variable density to be recorded on the film. When using a *glowlamp* to produce a sound track, a light source, whose intensity is varied, is focused on a film through a slit of fixed dimensions. Sound tracks produced by these two methods are similar. A variable-density sound track is shown in Fig. 1*a*. The average density of the sound track in this case acts as a "carrier" on which the modulations of the sound waves are recorded in less or greater density variations than the "mean."

[1] Member, American Physical Society; Society of Motion Picture Engineers, U. S. Naval Institute; lieutenant commander, U. S. Naval Reserve.

2. *Variable-area Recording.* In general, this is accomplished by using a light of fixed intensity, which is modulated through the operation of a galvanometer, or vibrator, as this unit is called. This produces serrations on the sound-track area of the film, as shown in Fig. 1*b*.

3. *Recording with Kerr Cell.* In recording sound on film by this method, the light-valve unit or oscillograph unit is replaced by a Kerr cell. A simplified diagram of the Kerr-cell system of recording is shown in Fig. 2. The appearance of the sound track using the Kerr cell is similar to the variable-density sound track

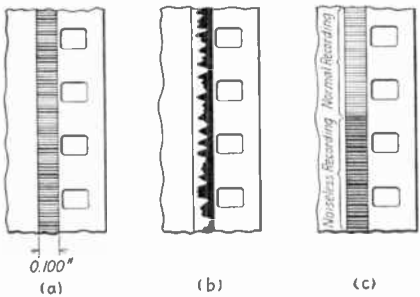


Fig. 1.—(*a*) Variable-density sound track produced by light-valve ribbons or glow lamp; (*b*) variable-area sound track produced by vibrating mirror; (*c*) noiseless recording showing greater density during periods of low modulation.

4. *Film-engraving System.* In this method of recording sound on film, an electric-cutting stylus actuated by a power amplifier is used to engrave the sound record directly on the edge of the film. The position of the sound track may be inside or outside the sprocket holes. The depth and shape of the groove as cut by this method are similar to those used for cutting disk records (*i.e.*, from 2 to 2.5 mils in depth, and 4 to 6 mils in width).

5. *Disk Recording.* In recording sound on disk in synchronism with the film record, it is the usual procedure to use soft wax records approxi-

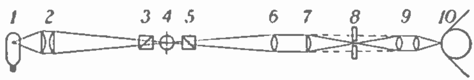


Fig. 2.—Kerr-cell recording system of Klangfilm. 3, Polarizer; 4, Kerr cell; 5, analyzer.

mately 17 in. in diameter and from 1 to 2 in. thick. These records are later processed to produce a hard record approximately 16 in. in diameter and ¼ in. thick.

The sound record is cut in the highly polished surface of the wax disk by means of an electromechanical recorder. The technique of cutting the wax records is similar to making standard electric phonograph records, except that for sound pictures, the procedure is to record from the center of the disk toward the outer edge, while for common phonograph records, it is the reverse. The standard speed for common phonograph records is 78 r.p.m., while for sound-picture records it is 33⅓ r.p.m. This speed, with a 16-in. disk, gives a playing time from 10 to 12 min.

a. Shape of Groove. The shape of the groove varies somewhat in commercial practice, but it is approximately 0.006 in. wide, and 0.0025 in. deep. The pitch of the groove is generally 0.010 to 0.011 in., leaving a space between grooves of about 0.004 in. With only this space available, the maximum safe amplitude is something less than 0.002 in., if the walls of the groove are not to be cut too thin.

b. Cutting Stylus. Aside from the recorder itself, the stylus must be of the correct shape and smoothness to perform properly. Synthetic ruby and similar materials having good wearing qualities are used for the cutting edge.

c. Playback Record. After the wax record has been cut, the sound may be reproduced directly without any further processing, by using a suitable pick-up. Such reproducers have to be carefully counterbalanced to prevent damage to the grooves in the soft wax records. This record may be played several times without injury. Under actual recording conditions, however, where two records are used for recording, only one is used as a "play-back," the other is used for processing.

3. Recording Apparatus. For recording sound pictures, the amount of apparatus required varies somewhat with the different systems and also whether it is designed for studio recording or portable news-reel work. Equipment for the larger studios is rather elaborate, and furnishes the necessary facilities for multiple-microphone recording when necessary, also additional equipment for play-back purposes, re-recording, etc. Portable equipment is reduced to the barest essentials, to eliminate weight and space required to make a sound film record with one or more cameras. Monitoring equipment in this case is generally accomplished by using a headphone only during recording operations. No play-back facilities, as a rule, are provided for portable use.

4. Microphones. The microphones used in sound-picture studios are similar to those used for broadcast studios, public address and similar uses. Generally more careful selection of units and adjustments is required, than for other than sound-picture work. Standard types of condenser microphones with a one- or two-stage amplifier assembled in the same case, are most generally used. The condenser transmitter with associated amplifier is furnished with a bail, in order to facilitate suspension in desired locations on the stage. Microphone booms, designed especially for this work, are in general use, allowing flexibility of the microphone during recording. A new form of dynamic microphone has been introduced for sound-picture recording. The principle of operation is somewhat similar to the Western Electric 555 receiver. An extremely light coil is attached to a thin diaphragm which actuates it in a magnetic field.

5. Ribbon Microphone. This is one of the special types of microphones developed by RCA-photophone for sound-picture work. The microphone consists of two extremely thin aluminum ribbons suspended between two magnets. The sound waves cause sufficiently slight movement to actuate the electrical circuit. Sound pick-up with such a microphone in the plane of the ribbons results in maximum volume output, while sounds coming from a position normal to the microphone face, are not picked up. This gives the microphone desirable directional characteristics.

6. Beam Microphones. These consist of various kinds of parabolic and other forms of reflectors designed for sound-concentration. The

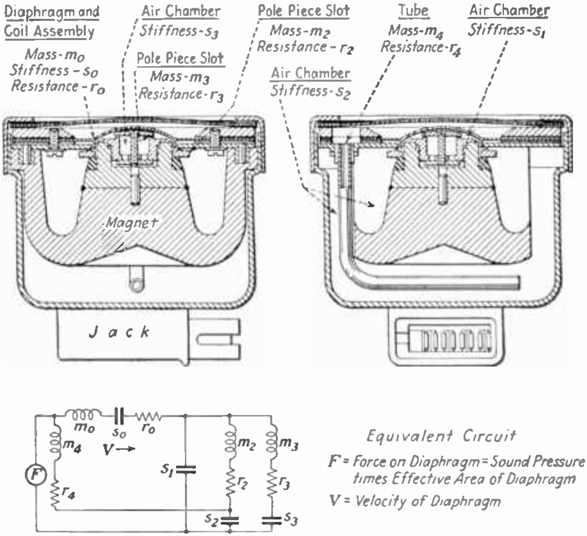


FIG. 3.—Cross-sectional view of dynamic microphone of Bell Telephone Laboratories.

microphone is placed at the focus of such reflectors, with the face of the microphone facing away from the source of sound. These reflectors have been used successfully in directional pick-up, especially in certain kinds of outdoor recording.

7. Sound-recording Channel. A schematic of a typical recording channel is shown in Fig. 5*a*. Reference to this diagram will assist in following the description of the principal amplifiers and recorder units given below. A typical transmission level diagram is shown in Fig. 5*b*, which indicates the various energy levels of the circuit in decibels from the point of pick-up to the recorder.

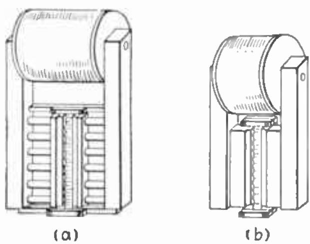


FIG. 4.—(*a*) Ribbon microphone in which the structure surrounding the ribbon is small; (*b*) microphone with baffle surrounding the ribbon.

8. Preliminary or Booster Amplifier. This amplifier is mounted between the mixer panel and the volume-control panel. It is used to amplify the output of the mixer before passing through the volume-control

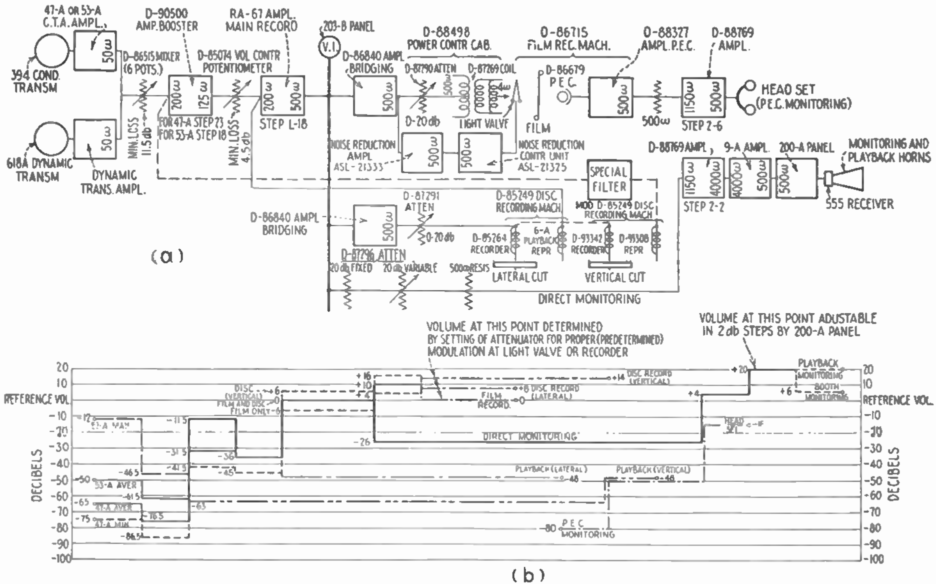


FIG. 5.—(*a*) Schematic of Western Electric recording system; (*b*) transmission-level diagram of system.

panel. Amplification is desired at this point to raise the recording level sufficiently high to prevent undesirable pick-up from stray electric currents or other sources entering the voice-transmission circuit. It also eliminates possible noise when operating the volume-control potentiometer. This amplifier differs in detail for various systems. In the Western Electric system, it is a three-stage resistance-coupled amplifier using three 239-A tubes.

9. Volume-control Panel. The outputs from the individual mixer panels are connected in parallel, and leads from them connected to the input of the preliminary or "booster" amplifier. The output from the preliminary amplifier is fed into a control potentiometer, which permits simultaneous adjustment of the total volume without changing the relative adjustments of individual mixer values. This panel also mounts an extension volume indicator to give a visible indication of the volume level maintained at the bridging bus.

10. Main Amplifier. This amplifier is so designated that it amplifies the output from the volume-control potentiometer, and delivers the amplified current to the bridging bus circuit (or in simpler installations, directly to the power-control panel and recording machine). It is the amplifier furnishing the largest gain in the recording channel. The main amplifier differs in details for the several recording systems. In the Western Electric system it is a three-stage impedance-coupled amplifier with input and output transformers. The first stage uses a Western Electric 102-type tube, and for the second and third stage, 205-type tubes. The total gain of this amplifier is approximately 70 db. The gain control of the amplifier is provided by a potentiometer in the input circuit.

The main amplifier in RCA-photophone system (PA-47), consists of four voltage-amplification resistance-coupled stages, and two push-pull power-output stages in parallel. Non-microphonic tubes are used in the voltage-amplification stages (UX-864). The output stages are UX-171A tubes. Each of the push-pull output stages is independent of the other, and two recorders may therefore be fed by the amplifier. The plate circuit in each stage has a resistance-capacity filter and the filament supply is filtered by a reactor. The input has been designed to operate from a 500-ohm line. The output may easily be altered to supply loads of 500, 250, or 167 ohms. The frequency characteristic is flat, to within plus or minus one decibel between 100 and 10,000 cycles, and is unaffected by the volume-control setting. The over-all amplification, with average tubes, is approximately 85 db, and each of the two output stages delivers 800 mw of undistorted power. The frequency characteristic of the two output stages shows a maximum deviation of 0.1 db from each other.

11. Bridging Amplifier. One of these amplifiers is required for each recording machine, its principal function being to prevent variation in individual recording circuits from introducing any loss or distortion to other circuits. It divides the electrical-circuit output from the main amplifier, depending upon the number of amplifiers connected to the bridging bus. It is essentially a power amplifier, with the input transformer arranged for a high input impedance, making the bridging of several of the amplifiers across the main bus practical.

The bridging amplifier outputs are connected to the film and wax recording machines in the recording room. The wax recorder requires approximately +8-db volume level, and the film recorder around +0 db.

12. Film Recorders. Up to the point of the recording machine, all methods of sound picture recording are essentially the same. The types of recorders differ in detail for different systems, depending upon whether they are designed for variable-area or variable-density recording. Where recording units are separate from the cameras, they are mounted on machines usually located in other parts of the studio, but connected to the same electrical motor system for maintaining synchronism with the cameras. In the case of sound-film cameras used in news-reel and similar work, recorders are mounted directly on the cameras.

13. The RCA-photophone recorder used for variable-area recording consists essentially of a sensitive galvanometer or vibrator, with an optical system for focusing the reflected light on the film sound track. The vibrator itself consists of a flat, wire ribbon 0.005 in. wide and 0.0005 in. thick, set in a vertical position over two bridges spaced approxi-

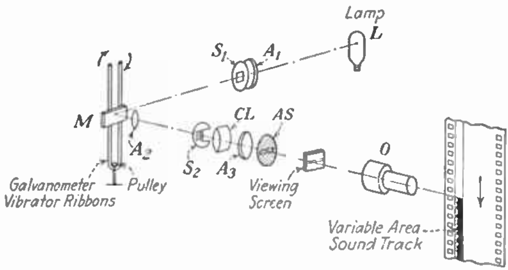


FIG. 6.—Schematic diagram RCA-photophone recorder. L, exposure lamp; A_1, spherical lens; S_1, light stop; A_2, galvanometer lens; M, mirror; S_2, scale; CL, cylindrical lens; A_3, spherical lens; AS, aperture slit; O, microscope objective.

mately 7⁄16 in. apart. This ribbon loop is placed under tension by means of a small spring attached to an ivory pulley, at the closed end of the loop. The ends of this ribbon, the two parts of which are spaced 0.01 in. apart, are attached to binding posts which in turn are connected to the output of the main amplifier.

At a point midway between the bridges, a tiny glass mirror is cemented to the ribbon loop. The vibrator is mounted between the poles of a permanent magnet, so that the ribbons are placed across the plane of greatest magnetic flux. The resonance period of this vibrator is approximately 6,000 cycles, its own response not being greater than 5 or 6 db. The whole vibrator unit is immersed in a container filled with a clear mineral oil to provide the necessary damping medium. Approximately 100 ma will give full-scale deflection to the vibrator. The sound track produced with such a recorder is shown in Fig. 1*b*.

14. The Western Electric light-valve recorder consists essentially of a duralumin ribbon suspended in a plane at right angles to a strong magnetic field. The ribbon is approximately 6 mils wide and ½ mil thick. This ribbon is stretched by means of an adjustable spring over a bridge having a narrow slit for passage of the light from the recording lamp through the optical system to the film.

Set screws are provided to center accurately the ribbon over the slot, which is approximately 8 mils wide and 250 mils long. The ribbons are spaced 1 mil apart for recording. A microscope is provided for checking this spacing.

The ribbon is tuned after proper spacing on the valve to approximately 8,500 cycles, so that its natural period will be outside the range of ordinary recording frequencies. A diagram of the optical system using a light valve for recording is shown in Fig. 7. The light source is provided by a special lamp having a horizontal filament. The lamp socket mounting is so adjustable that the filament can be focused properly on the light-valve slit. The sound track produced is shown in Fig. 1*a*.

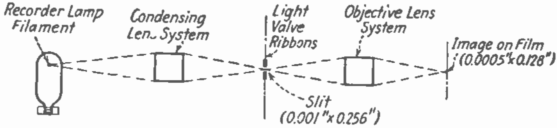


FIG. 7.—Optical system used in light-valve recording.

15. Glowlamp Recorder. This consists of a two-element gaseous-discharge tube which varies its illumination in accordance with the voice currents impressed on its circuit. This produces a variable-density sound track similar to the light-valve method. The Aeolight, used by Fox Films Corporation, is one of the recorders in this class. The lamp is not focused upon the film, but a portion of its illumination is allowed to pass through a quartz slit which is in contact with the film. The glowlamp is mounted in a holder with a base of quartz glass 0.2 in. square and 20 mils thick, having a silver coating. This silver coating is engraved, making a narrow slit approximately 0.01 in. long by 0.0008 in. wide. The base is then covered with a thin quartz glass which is only 1 mil thick at the point opposite the engraved slit. The slit is mounted on a floating metal shoe, which is a part of the lamp holder. The lamp holder is inserted in the camera base in contact with the film, so that the sound record is made at a predetermined distance from the picture aperture, this being a standard distance for all sound-picture recording.

The recording level for the Aeolight is approximately +12 db. All lamps have a steady d-c component impressed, which causes them to burn at a predetermined exposure. This exposure is modulated by an a-c component due to the introduction of voice currents from the recording amplifier. The resulting output is a variable-density sound track similar to that shown in Fig. 1*a*. The illumination from a glowlamp is approximately proportional to the amount of current flowing through it, within the normal recording range.

16. Portable Sound Cameras. All of the above recorders are adaptable for mounting directly on the camera itself. This method of mounting is used for news-reel and similar work requiring portable equipment. The portable cameras are essentially the same, the sound recorders differing only in details for mounting purposes.

MOTOR SYSTEMS

17. Synchronous Motor System. When recording sound for motion pictures, it is essential that the cameras and recording machines (if separate, as in studio installations) are run in exact synchronism and at the desired speed. The systems in general use to accomplish this differ considerably in details, but are standard as regards speed of film, which is 90 ft. per minute.

18. Interlocking Motor System. The Western Electric system consists essentially of a system of interlocking synchronous motors. Each piece of apparatus of the recording system, camera, film recorder, disk recorder, etc., is provided with a separate motor. These are controlled by a distributor which is in turn driven by a constant-speed motor using a vacuum-tube control circuit for maintaining accurate speed. Each motor of the synchronous system has a phase-wound rotor and a phase-wound stator. The three terminals of the stator windings of all motors,

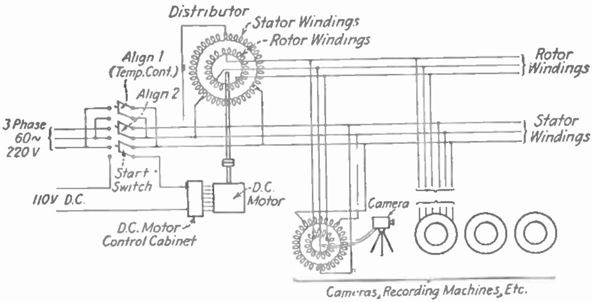


FIG. 8.—Interlocking motor system for driving camera and recorders in synchronism.

and also similar terminals of the distributor stator, are connected to a source of 220-volt, 50- or 60-cycle, three-phase power supply. The three terminals of the distributor rotor windings, which are brought out through slip rings, are similarly connected to the rotor windings of all motors in the system. The distributor is direct-coupled to a d-c motor, whose speed is regulated by a special control circuit. A simplified schematic of the interlocking motor system is shown in Fig. 8. Prior to starting the system, a definite synchronizing mark is made on the film by a punch mark in the cameras and recorders.

Alignment of the motors prior to starting is accomplished by closing align switch 1 and 2 in order, which places single-phase excitation on the system prior to actual starting. This will usually bring all motors in alignment prior to actual starting. Closing of the start switch will apply the other two phases of the stator winding, and also add d-c power to the distributor motor, simultaneously. All motors of the system thereafter run interlocked.

19. Non-interlocking System. This system involves the use of synchronous motors connected directly to a source of 110-volt a-c 50- or 60-cycle power supply. Each piece of the apparatus of the recording system is driven by a separate motor, as in the interlock system. No preliminary alignment, however, is made prior to starting. When ready to record, the main power-supply switch for the particular recording channel is closed, starting all motors together. When up to speed (a few seconds later), a fogging system is operated, to mark a definite synchronizing point upon the film in the cameras and the recording machine. This mark provides a means of matching the sound record with the picture at the proper point when they are combined later for printing.

20. Motor System for News-reel Outfits. Motors for this purpose consist of a small d-c motor of conventional design, connected directly to the camera in some cases, or through a flexible shaft. A source of direct current, usually supplied by storage batteries, furnishes the necessary motive power. A rheostat is provided in the motor circuit for speed adjustment, which is usually required due to temperature changes. A tachometer is connected to the motor shaft, to check the speed.

21. Motor System for Location Trucks. Motors for this purpose are usually of the interlocking type similar to those used in studio installations, though the system is greatly simplified by elimination of motor switching panels and other auxiliary apparatus.

SOUND-FILM PROCESSING

22. Film-recording Technique. This includes the various steps involved in adjusting the electrical recording circuits, recording lamps, printing machines and processing of film to obtain the desired results in the finished product, the whole object being to recreate the original sounds recorded in the best possible manner. Two distinct methods of processing sound film are in use, one for variable-density recording and the other for variable-area recording.

23. Photographic Analysis, Variable-density Recording. In this method of control the general problem is to obtain a sound-positive print which is proportional to the original negative-film exposure which it represents. This may be best explained in the following analysis: reference is made to the law of proportionality for film emulsions first described by Hurter and Driffield and generally referred to as the *H&D* curve of the emulsion. Typical *H&D* curves are shown in Fig. 9, in which the resulting density D is plotted against logarithm of exposure E. These curves indicate a curved "toe" in the region of underexposure, and a curved "shoulder" in the region of overexposure. The section between approaches a straight line the slope of which determines the gamma (γ) or contrast factor for the film. Up to a certain point, gamma increases for the time of development.

The extension of the straight-line portion of the curve intersects the *log* E axis, and determines the inertia i of the film.

The purpose of the proper photographic control is to obtain the relation between the gamma of the negative and positive film, so that the "over-all" gamma will equal unity.

The straight-line portion of the *H&D* curve, for a given gamma, may be represented by the equation

$$D_0 = \log \frac{1}{T_0} = \gamma(\log E_0 - \log i) \tag{1}$$

where

T_0 = transmission
D_0 = density

γ = slope of the straight-line portion of curve
$\log E_0$ = log of the exposure at any point E_0
$\log i$ = log of the inertia point of film.

Equation (1) may be written

$$T_0 = KE_0^{-\gamma} \tag{2}$$

This relation also holds for both negative and positive film, as (2) may thus be written

$$T_n = K_nE_n^{-\delta_n}$$
$$T_p = K_pE_p^{-\delta_p} \tag{3}$$

Where the subscripts n and p designate negative and positive film, and K factors are constants.

The process of printing consists in exposing the film to a constant light P, modulated by the transparencies of the interposed negative, which is in contact with the film during this operation. The following relation thus holds

$$E_p = PT_n \tag{4}$$

Substituting in the above equation we arrive at the expression which determines the over-all relation between the original exposure of the negative and the resulting transmission of the positive.

$$T_p = PE_n^{\gamma_n\gamma_p} \tag{5}$$

The exponent $(\gamma_n\gamma_p)$, or the product of the negative and positive gamma, should equal unity, if we are to have the required condition of proportionality between negative exposure and positive transmission. Within certain limits the over-all gamma should equal unity, and upon this relation depends the success of the variable-density recording. These limits have been determined in actual practice to lie somewhere between 0.8 and 1.2.

The values given below may be taken as averages in using the variable-density method of recording—

T_n = 18 – 25 per cent
T_p = 18 – 25 per cent
γ_n = 0.55 – 0.65
γ_p = 1.9 – 2.3

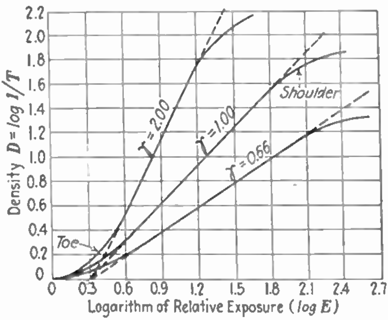


FIG. 9.—Typical $H\&D$ curves showing relation of log E to density to produce "gammas" of different values.

24. Toe Recording. This method of recording represents a variation in normal variable-density recording, by using the lower part or "toe" of the typical *H&D* curve for recording. This method has been found applicable where the original positive may be used for processing and later used for reproduction of the sound without making a print from it. Also in re-recording operations where improved quality is claimed due to greater density in the sound positive print, resulting in less ground noise.

25. Photographic Processing of Variable-area Recording. The only factors which may be altered appreciably by improper photographic treatment of variable-area sound films are the high-frequency response and the volume range which can be reproduced.

The volume range which can be reproduced is chiefly a function of the negative and positive densities. The limiting values of density between which a maximum volume range may be obtained are quite broad. Maximum response at high frequencies is secured by obtaining the maximum resolving power of the film. This depends upon the negative and positive densities, and upon the contrast of the sound track being recorded.

In the variable-area recorder, this latter factor is not critical. The image contrast is at least ten. With the various positive-film stocks now in use, which have been chosen for their high resolving power and contrast, all that is necessary to work for is a density of about 1.3 in both negative and print. This value will supply both a sufficiently high resolving power to minimize the high-frequency loss and a density of the opaque side of the track, together with a fog value of the clear side necessary to provide a maximum volume range.

High-frequency losses are a minimum with negative and print densities between 0.8 and 1.6. The high-frequency response is altered only very slightly in this region. Since a maximum volume range may be obtained with a negative density of 1.0 or higher, and a print density of 1.3, both requirements are therefore satisfied by the same values of density.

In cases where the negative density is not so high, if the print density is correspondingly reduced, the volume range lost will not be more than approximately 4 db. In no case will variations over even somewhat wider limits than those previously described cause non-linear distortion.

Variable-area sound track may be developed to practically any gamma as long as sufficient exposure is provided to insure a negative density within the range mentioned, that is, between 0.8 and 1.6. This allows the variable-area method to be employed either in cameras for simultaneously taking pictures on the same film or in separate film recorders. The latter procedure is also applicable in variable-density recording.

26. Kerr-cell Recording. The action of this cell is similar in effect to the light valve, in permitting light to pass through its narrow slit to the film behind, in varying quantities above and below a mean value, according to the signal fluctuations impressed upon it. The satisfactory working of the Kerr cell depends chiefly upon the accurate determination of its mean track value. The factors which go toward this, assuming a constant slit, are the d-c voltage across the electrodes, and the recorder-lamp brightness. The signal itself is superimposed upon this steady potential. If we assume a voltage of 700 across the cell, and an anode swing of 200 volts, the result of the complete cycle would be a rise to 900, back to 700, a drop to 500, and back again to 700 volts. This represents approximately the range to prevent overloading.

27. Noiseless Recording. A modification in recording technique provides a means of increasing the density of the sound track for variable-density recording, and a similar method, applicable to variable-area recording, for blocking off a portion of the sound track producing the same results.

For variable-density recording (using a light-valve), a portion of the voice-current energy is tapped off the system, and sent through a biasing amplifier, which in turn changes the spacing of the light-valve ribbons during the periods of low modulation. The normal spacing of the light-valve ribbon of 1 mil is thus reduced to approximately 0.3 mil. The average spacing of the ribbons follows the envelope of the modulation. The general density of the print is high during intervals of low modulation and less during intervals of high modulation. This resulting increase in the density of the sound positive print reduces ground noise about 12 db. This method also acts as a means of increasing the volume range of recording by approximately this amount. An example of the effects of noiseless recording is shown in Fig. 1*c*.

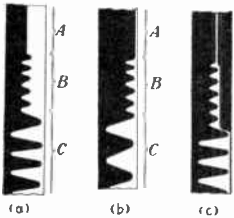


FIG. 10.—(a) Typical variable-area sound track; (b) and (c) methods of obtaining anti-ground noise recording by blanking off clear portion of track.

For variable-area recording, a portion of the voice-current energy is diverted and passed through a biasing amplifier which actuates the oscillograph obversely; that is, the serrations which are normally along the center of the sound track are moved over to the edge as shown in Fig. 10. A further modification in this method is made whereby the serrations always remain in the middle of the track but the clear portions of the print are matted out by an auxiliary light-blocking device.

DISK RECORDING

28. Necessary Equipment. Equipment necessary for disk recording consists essentially of a machine lathe especially designed to turn the wax record at a uniform speed, which is 33⅓ r.p.m. for motion-picture work. The carriage of the lathe is driven with a lead screw carefully machined to move the recorder holder at a predetermined rate while cutting the wax record. The lead screw is driven through a gear train which regulates the number of grooves cut per inch, usually 86, 92 or 98. A recorder holder provides the necessary support for the electrical recorder. The process of recording programs on wax disks for later modulating a radio transmitter, known as *electrical transcription*, is essentially the same as the process described here.

A horizontal turntable, driven through a vertical shaft, is provided for supporting the wax record. The vibration of the driving motor is eliminated on different lathes by various methods. The Western Electric lathe uses an oil dashpot placed below the lathe bench, and through which the vertical shaft of the turntable is driven. This dashpot provides the necessary damping to insure smooth recording on the record. The motor driving the turntable is run in synchronism with the camera motors.

A microscope, suitably mounted, is usually provided for observing the grooves of the wax during actual recording operations.

29. Disk Records. The grooves of a disk record are ordinarily spaced about 92 per inch. This allows about 0.011 in. from center to center of the groove, of which 0.006 in. is the width of the groove itself. The maximum lateral motion of the stylus is thus limited to about 0.0025 in. on either side. Generally, 0.002 in. should not be exceeded. Cutters generally used are designed as constant-velocity devices. In practice such cutters have this characteristic only above 200 cycles. Below this point, the amplitude is independent of frequency. If the maximum amplitude for a 200-cycle wave is equal to 0.002 in. on either side of the center, then a 1,000-cycle amplitude for the same electrical input level would be 0.0004 in.

30. Recorder Attenuator. This unit is usually provided for controlling the relative volume level of the voice currents actuating the electrical recorder. It is connected in the circuit between the output of the final amplifier and the terminals of the recorder or recorder-control box, thence to these terminals.

The recording machines on the market differ in details but consist essentially of the above units.

31. Determining the Starting Point. Disk records for sound pictures are cut from the inside out—just the reverse of regular phonograph records. To obtain a definite starting point for the records when in use, the first groove is spaced an appreciable distance from the rest of the cut. This is obtained by a coarse speed cam actuating the lead screw at the start of recording. As the lead screw makes its first complete revolution, it moves the recorder under the influence of the cam until the recorder is in its normal cutting position.

32. Electrical recorders provided for disk recording are generally designed so that the average linear velocity of the stylus (which may be expressed as the frequency times amplitude) is proportional, over a wide range of frequencies, to the impressed voltage. The method of damping the moving system varies with different records. The Western Electric recorder uses a rubber tube about ½ in. in diameter and 8 in. long, one end of which is fitted to the armature assembly and the other end free. Oil is sometimes used to damp the armature movement in other types of recorders.

33. Cutting stylus consists of a sapphire or other hard point fastened to the lower end of the stylus arm. One end of the sapphire has a rounded point about 0.002-in. radius, and a cutting angle between 86 and 88 deg. for the sides.

The advance ball is a small cylindrical sapphire, ground spherically at one end and held in an adjustable mounting attachment to the recorder. This ball supports the weight of the recorder and the arm being adjustable, permits regulation of the depth of the groove on the wax.

34. Play-back reproducer is provided to permit playing back the wax record immediately after it is cut for rehearsal work and test. This usually renders the wax unsuitable for processing, and for this reason, two wax records are usually provided for each recording channel, one of which can thus be used for play-back and the other for processing. The pressure of the needle on the wax is generally adjusted to between 15 and 20 gram.

A needle provided for playback from the soft wax is designed differently from the ordinary needle used for the finished hard record. The Western

Electric type has a point 0.003-in. radius. The needle is constructed on a mandrel, ground to a smooth finish, and the point given a chromium plate to improve wearing quality.

35. Checking Speed. The periphery of the turntable is usually divided with vertical lines, so that a neon lamp, operating from a 60-cycle source, may be used as a stroboscope to observe the turntable motion. The lines on a standard turntable are usually arranged so that with 60 cycles on the lamp, as the turntable rotates at exactly 33⅓ r.p.m., the lines will appear to be stationary. If faster than 33⅓ r.p.m., the lines will advance slowly, and if slower than 33⅓ r.p.m., the reverse will be the case. A check of the speed should be made with the wax record on the turntable.

36. Checking the Damping Action. A method of checking the instantaneous constant speed may also be used to check correct damping of the turntable. With the turntable rotating at normal speed, the oscillator for supplying 60-cycle source to the neon lamp may be adjusted until the vertical lines appear stationary. If the disk is now touched lightly by hand, the line or spot observed will appear to shift its position owing to momentary load. As soon as the hand is removed, the line or spot observed should come back to its original position. Observing the movement will determine whether the turntable has insufficient damping or too much damping.

37. Wax-suction Equipment. This equipment is provided to furnish a means of removing the shavings from the wax record during recording. The suction tube is so placed that the shavings thrown off by the stylus are carried away from the face of the wax. A central suction system is usually provided in studios having several recording channels. This usually consists of a turbine suction pump with pipe lines leading from a central suction point to a separator tank placed in each recording room. In some smaller installations, an individual bell jar, with a small suction motor, is used for each turntable.

38. Wax Preparation. Two types of waxes are generally used in sound recording, those having a working temperature of 75°F., and those with a working temperature about 90°F. Matthews type M, 75°F. working temperature, is perhaps most commonly used. It is considered good practice to maintain the room temperature for the type *M* wax around 75°F. when recording.

The procedure for preparing the wax consists briefly of the following steps:

1. At the center of the wax, which is usually indicated by a cross mark, a $\frac{9}{32}$-in. hole is drilled to a depth of ½ in.
2. A record cut is made for a depth of about ⅛ in. on one face of the wax and repeated as necessary to obtain a perfectly flat surface. The wax is later reversed, the first cut surface becoming the base for the finished wax.
3. On reversing the wax, a hole is cut from the other side to meet the hole drilled on the bottom.
4. A course cut is now made on the top surface and repeated where necessary to produce a smooth and flat surface. The wax is now ready for the final shaving or polishing cut, which is done with a sapphire or ruby cutting tool.
5. The face of the shaving knife is usually set at an angle of between 40 and 50 deg. to its line of travel, depending upon the particular design of the knife. Its rounded end is toward the center of the wax. The cutting face of the knife is set at an angle of 90 deg. to the surface of the wax. The turntable revolves in a counterclockwise direction.

6. The suction nozzle is placed close to the cutting knife, about ⅛ in. from the front face and 1/32 in. above the cutting edge.

7. The best finishing speed is usually determined by experience, but generally ranges from 150 to 160 r.p.m. The finished cut on the wax should give a perfectly polished surface free from ripples or blemishes of any kind.

39. Record Processing. Briefly, this consists of the various steps after obtaining the soft wax record, to produce the final hard record for commercial use. A complete description of each step would go beyond the limits of this chapter. The following are the essential steps in this process:

1. The surface of the soft wax is rendered conductive by spreading a very thin, extremely fine conducting powder, such as graphite, over its surface.
2. Electroplating of this record. The negative electroplate obtained is used to hot-press a molding compound, such as shellac, mixed with a finely ground filler. The first electroplate obtained is called a *master*.
3. Two test pressings are made from the first master, after which it is electroplated with a positive.
4. This *positive* is referred to sometimes as an *original*. From this positive a metal mold or *stamper* record is made.
5. From this record, duplicate originals may be made, and from them duplicate molds or stampers. By thus making a number of duplicates, it is possible to protect the original master from injury, or danger of destroying a valuable record.
6. From each stamper it is possible to obtain as many as 1,000 finished pressings.

SOUND-FILM STANDARDS

40. S.M.P.E. Standards. The dimensional standards for film adopted by the Society of Motion Picture Engineers are given complete in the transactions (see *Journal of the S.M.P.E.*, May, 1930). The dimensions given below refer in particular to those affecting sound film.

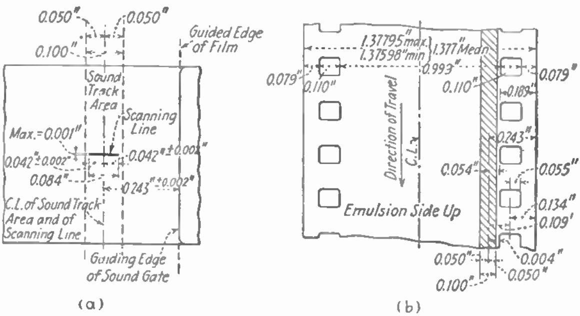


FIG. 11.—(*a*) Position and dimensions of scanning line; (*b*) standard dimensions of sound track on 35-mm. film.

1. *Taking speed* for standard 35-mm sound pictures is 24 pictures per second.
2. *Projection speed* for standard 35-mm sound pictures is 24 pictures per second.

3. *Scanning line* for combined sound and picture on 35-mm film is located at an average distance of 14.5 in. measured along the film, below the center of the picture gate. The transverse position, relative to the guided edge of the positive film, and dimensions of the scanning line are given in Fig. 11*a*.

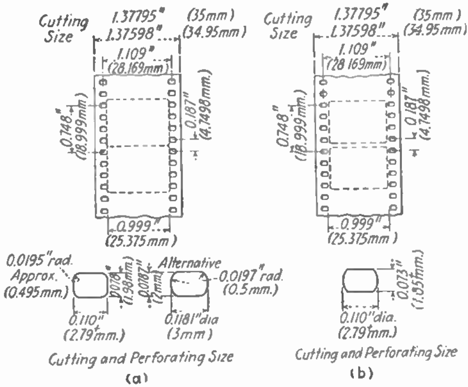


FIG. 12.

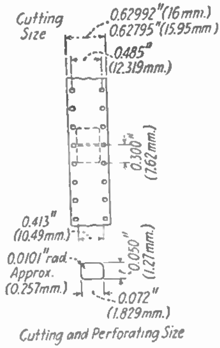


FIG. 13.

FIG. 12.—(*a*) Standard dimensions for 35-mm positive film; (*b*) dimensions for 35-mm negative film; variation is in dimensions of sprocket holes.
FIG. 13.—Standard dimensions for 16-mm positive and negative film.

The location and width of the sound track on combined sound and picture positives are shown in Fig. 11*b*.

The dimensional standards for standard negatives and positive 35-mm film are given in Fig. 12*a* and *b*. The dimensional standards for 16-mm positive or negative film are given in Fig. 13.

THEATER REPRODUCING EQUIPMENT

41. Sound Head. For reproducing sound in the theater, the projection machine is fitted with a "sound head" for sound-on-film reproduction, and a disk turntable, driven in synchronism with the picture film, when reproducing from the professional 33⅓-r.p.m. phonograph record. Owing to the improvements made recently in film recording, the practice of releasing sound films with accompanying records is rapidly disappearing. It is expected that shortly all sound pictures will be on film only.

The sound head for reproduction from films consists essentially of an exciting lamp to provide a strong light source, an optical system for focusing the lamp filament on the sound track, an aperture plate and tension pad to hold the film in focus while passing the sound gate, a photoelectric cell to register the light variations, and an associated PEC amplifier. All of this equipment is suitably housed and mounted just beneath the projector head. Typical sound heads are shown in Figs. 14 and 15.

The sound gate is placed 14½ in. from the picture frame and in advance of the picture. This is standard for all types of machines. From the relative

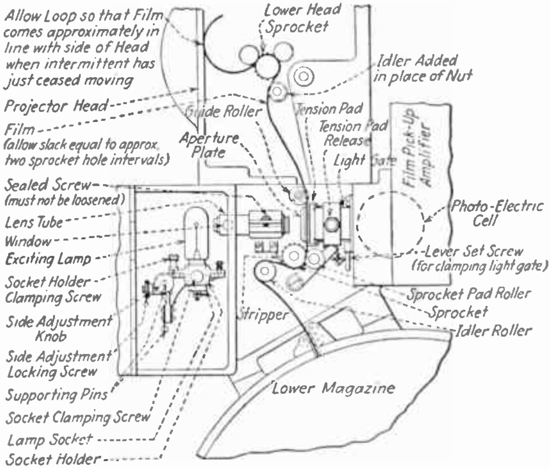


FIG. 14.—Schematic diagram of sound head (Western Electric), showing principal units required for reproducing sound from film.

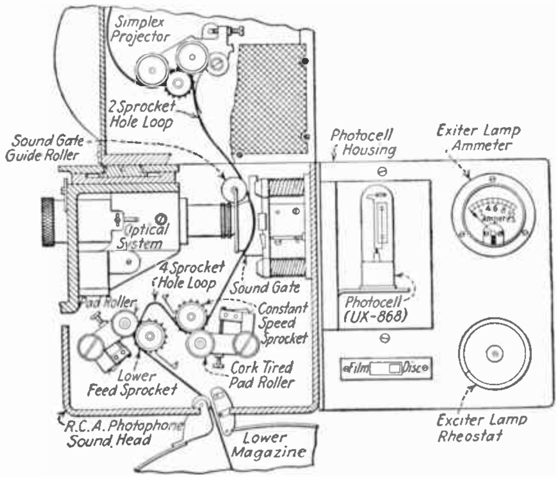


FIG. 15.—Schematic diagram (RCA-photophone) showing path of film through sound head.

position of the sound gate to the picture aperture, it is not possible to put the sound track opposite the corresponding picture frame. Furthermore, the picture moves intermittently before the projection lens, but the sound track must move at a constant speed in front of the sound gate. A certain amount of slack is thus necessary to allow for this continuous motion.

To prevent vibration from the projector, the PEC amplifier is usually suspended by springs in a special frame adjacent to the photoelectric cell. To overcome this vibration one sound equipment has a light beam reflected from the sound gate to the photoelectric cell and associated amplifier removed from the projection machine proper.

With the advent of a new type of caesium photocell having a high output, as well as certain liquid cells having a low impedance, it has been found practical to mount the associated PEC amplifier remote from the projection machine to avoid this vibration.

Identical equipment may be used for reproducing from a variable-area or variable-density sound track. The lamp filament and optical system must be adjusted so that the optical slit projected on the film is at 90 deg. to the motion of the sound track. The dimensions of the light slit focused on the sound track is 0.001 in. high by 0.080 in. wide. The sound-track standard width is 0.100 in.; therefore an allowance of 0.010 in. on each side is allowed for variation in the lateral position of the track in passing through the sound gate. This allowance has been found necessary in practice to prevent stray sounds from being picked up by light modulation from the edge and possibly outside the sound track.

42. Uniform speed, in reproduction, is extremely essential, as a variation greater than one-tenth of 1 per cent might be perceptible to the ear. The standard projection speed is 90 ft. per minute for sound film—the same as in recording. To maintain this constant speed, the usual projection motor is replaced by a special projector-drive motor and associated speed control units.

43. D-c Motor Drive. Where d-c motors are used for the projector drive, the speed is maintained constant by changing the resistance of the motor field circuit. For this purpose, some form of centrifugally operated moving contact is employed, which is mounted on the motor shaft. As the motor speeds up, the centrifugally operated weight moves out from the shaft and this allows the moving contact to approach the stationary contact. When the motor reaches the desired speed, the moving contact touches the stationary contact, which short-circuits the resistor in the field circuit. The resulting increase in field-current strength causes the motor to slow down.

When the motor slows a little, the contacts open, and this cuts out resistance in the field circuit. This causes the motor to speed up. This operation, occurring very rapidly, opening and closing the field-circuit contact, causes the motor to operate at a practically constant speed.

A more elaborate motor-control system has also been developed by Western Electric, in which a vacuum-tube control circuit is used to control the speed of the projection-drive motor.

44. Synchronous Motors. Various types of synchronous motors are used where a source of a.c. is available. Such motors are very satisfactory for driving sound projectors, because of the extremely uniform speed which they maintain. This is due to the limits maintained in the frequency of the power supply. Where such frequency is subject to a variation of only one cycle on either side of the standard 50- or 60-cycle supply, the change in frequency usually takes place so slowly, due to the mass of the generating equipment, that actual change in projection speed is hardly discernible.

To prevent too rapid acceleration of a-c motors when connected directly to the a-c line, an acceleration which would cause an undue strain on the projection equipment, resistor units are added in each line or in at least two of the lines where three-phase supply is used.

Different classes of synchronous motors are available for projector drives. A 220-volt, 60-cycle, three-stage synchronous motor, with a two- or four-pole construction, is one of the standard types used. Also, various types of single-phase motors have been adopted for the projector drive. These may be either the single-phase repulsion-induction type or the split-phase starting-induction motor type. The starting mechanism for a common type of repulsion-induction type consists of two brushes mounted so as to make contact with the commutator, with a resistor to prevent too rapid acceleration, and some form of centrifugal mechanism for shorting out the commutator after the motor has attained running speed.

45. Disk Sound Records. The records used for sound reproduction are played on a turntable, suitably damped to reduce vibrations of the driving mechanism and driven in synchronism with the film picture. Speed of rotation is 33⅓ r.p.m., the same as for recording, and corresponds to 90 ft. per minute of the film speed. Each disk record is plainly marked with a starting point on the first inside groove, which is placed an appreciable distance from the rest of the cut. Records will play approximately 11 to 12 min., and the accompanying film reel, made to show with this record, is cut to run the same length of time.

For synchronous reproduction of the sound, the record and picture must coincide at all times. Except for mishaps, such as the needle jumping a groove or breaking of the film, sound and picture will remain in synchronism, if started properly.

46. Reproducers used for disk reproduction are of the electromagnetic type, and generally of high quality, to insure good response over the desired frequency range. Oil-damped pick-ups are extensively used to overcome the effects of resonance. Recent improvements in pick-ups have tended to reduce the mechanical impedance at the needlepoint so that the undulations of the groove are followed more closely, without the necessity of heavy pressure at the bearing surface. A high-quality reproducer now available will give a fairly uniform response of 30 to 7,000 cycles.

47. Turntable. The same motor used to drive the projector mechanism is used to drive the disk turntable. This may be done by chain sprockets, belt drive, flexible shafting, or direct-shaft coupling. In all cases, some form of damping is necessary to overcome sudden and small variations in speed of the driving motor, which causes "wows" when playing a record. The damping mechanism varies for different systems. In some cases a flywheel is used, which is driven through a set of damping springs. In others, the driving force is communicated to the turntable through an oil dashpot mounted directly underneath the turntable.

48. Input Control Panels. In theater equipment, various pieces of apparatus are required to control the circuit between projectors and amplifiers. These include a film-disk transfer switch, a potentiometer for exciter-lamp control, control circuit for horns, faders for changing sound output from one projector to another during a continuous show, amplifier switches, etc. The film-disk transfer switch, which is generally provided on each projector, is for the purpose of changing the output from the film or disk record so that one or the other is always connected to the input of the main amplifiers.

49. Theater Horns. The size, type, and number of horns used for reproduction in the theater varies for different systems. In general, the volume of the auditorium in cubic feet governs the number of units required. Two types of horns have been most commonly used, the exponential type and the electrodynamic-cone directional baffle.

The receiver unit used with the various exponential horns have a thin diaphragm of the order of 0.002 in. thick to which is mounted a flat coil of wire or ribbon, wound on edge. The passage of voice currents through this coil interacts with the magnetic field and causes the diaphragm to move in and out. The diaphragm vibrates very much like a plunger. The receiver is attached to an exponential horn which isolates a column of air from the surrounding medium. This air column carries the necessary load of the receiver and also acts as an acoustic-coupling medium between the receiver and the mouth of the horn.

The loud-speakers are usually placed behind the picture screen to obtain the necessary illusion that the sound is coming from the action on the screen. This necessitates using a screen which will have the necessary acoustic properties for the passage of sound, as well as light-reflecting characteristics for the picture. Various forms of screens have thus been developed to meet these requirements. In general the screens are of a heavy opaque material covered with a diffusing reflective surface perforated with small holes to approximately 25 per cent of the total area.

As the screen perforations increase, the sound transmission increases, but picture clearness falls off; hence a satisfactory mean is struck between the two. No serious losses are experienced with good screens up from 6,000 to 7,000 cycles.

INDEX

A

Absorber, spark, 474
Absorption coefficient for common materials, 408
Acoustic impedance, 406
Acoustic measurements, 429
Acoustic reactance, 407
Acoustics, 403
Air gap, 64
Aircraft, antennas, 524
  blind landing, 535
  power equipment, 522
  radio, 511
  radio systems for guiding, 525
  radiotelephone, weights of, 521
  receivers, automatic volume control for, 534
  shielding and bonding in, 524
Airplane, direction finder, 525
  engine ignition shielding, 524
  equipment, 517
  radio, power requirements for, 518
Alternating-current circuits, general characteristics, 31
Alternator, Alexanderson, 460
  Goldschmidt, 461
Altimeter, absolute, 540
  capacity, 540
  reflection, 541
Ammeters, characteristics of a-c, 151
Ampere turns, 51
Amplification, calculation, 199
  d-c, 495
  factor, 198
  measurement, 199
Amplifier, audio, 281, 291
  bridging, 559
  cascade, 310
  Class A, 300
  Class B, 297, 319
  Class C, 320
  direct-coupled, 281
  d-c, 291
  impedance-coupled, 301
  intermediate-frequency (i-f), 329
  linear, 320
  Loftin-White, 292
  modulated, 269
  neutralized, 316
  photocell, 495
  power, 295–297, 336
  r-f, 300
  regeneration in, 314
  resistance-capacity coupled, 281, 300
  speech input, 363
  stage gain of a-f, 281
  three-electrode, 204
Amplifier, transformer-coupled, 281, 335
  transformer-coupled r-f, 304
  untuned r-f, 329
Anderson bridge, 180
Antenna, aircraft, 524
  Beverage wave, 468
  directional, 468
  directive-transmitting, 539
  harmonic frequencies, 37
  high-frequency reactance, 38
  input systems, 325
  long-wave, 463
  low-frequency reactance, 37
  magnetic field, 36
  multiple-tuned, 464
  resistance, 38
  vertical, 36
Anti-resonance, 111
Arc transmitters, 462
Arcturus photolytic cell, 553
Ardenne, Baron von, 487
Astatic galvanometer, 151
Attenuation constant, 131, 136
Attenuator, 311
  recorder, 567
Audibility, threshold of, 413
Audio-amplifier, stabilizing network, 281, 291
Audio-circuit, capacity-coupled, 288
  inductively coupled, 283
  resonance, 286
  time constant, 299
Austin-Cohen transmission formula, 458
Ayrton-Mather electrostatic voltmeter, 158

B

B-H curve, 34
Bain, Alexander, 492
Baker, T. Thorne, 509
Balanced circuits for facsimile, 496
Baldwin telephone, 153
Band-elimination filter, 142
Band-pass filter, 129, 143, 311
Band width, 444
  for modulation, 277
Barkhausen-Kurz oscillations, 451
Bart-Lane system of facsimile, 509
Bartholomew, H. G., 509
Beacon, aural-type, 528
  boundary-marker, 538
  double-modulation, 531
  runaway localizing, 538
  triple-modulation, 532
  visual-type, 529
Beam, landing, 537, 538
Beat frequency, 171

Becquerel effect, 551
Bell System facsimile, 509
Beverage, H. H., 509
Beverage wave antenna, 468
Boltzman law of radiation, 197
Bonding in aircraft, 524
Brake, thermionic, 508
Bridge, Anderson, 180
  Burgess selenium, 552
  Carey Foster, 173, 181
  Hay, 178
  Heaviside equal-arm, 181
  Kelvin double, 173
  Maxwell, 178
  measurement, errors in, 176
  Owen, 178
  resonance, 179
  Schering, 177
  Wien, 179
Broadcast transmitters, harmonics in, 369
Broadcasting, 354
Burgess selenium bridge, 552

C

C bias for detection, 263
C light, 497
Cables, large-gage, 132
Cameras, portable sound, 561
Capacitance, definition of, 23, 81
  grid-plate, 217
  interelectrode, 215, 301
  measurement, 99, 165
Capacity, 79
  calculation, 88
  chart, 18
  specific inductive, 81
Carborundum, 245
Carey Foster bridge, 173, 181
Carrier suppression, 272
Cascade amplifiers, 310
Case, T. W., 551
Cathode-ray tube, 161
  oscilloscope, 275
  study of transient phenomena by, 163
  timing axis for, 162
Cell, Arcturus photolytic, 553
  selenium, 550
  selenium FJ-31, 552
  Thalofide, 551
  Weston photronic, 553
Centimeter of capacity, 79
Characteristic impedance, r-f line, 134
  transmission line, 131
Chokes, imperfect, 237
Circuit, absorption modulation, 263
  audio-resonance, 286
  balanced facsimile, 496
  characteristics of a-c, 31
  compensation, 286
  coupled, 123
  design, oscillator, 222
  equations, 29
  harmonic-measuring, 325
  magnetic, 33
  magnetron oscillating, 241
  neutralizing, 315
  output-measuring, 325
  parallel, 116
  parameters, 29
  preselector, 129
Circuit, selector, 129
  series resonant, 113
  short-wave, 240
  split-tank, 121
  table of constants, 5
  time constant of audio, 299
  tone-control, 473
Circular mil foot, 41
Class B output tubes, 337
Coaxial conductor transmission line, 135
Code, business, 453
  Continental, 454
  frequency range, 455
  reception, 453
  standard, 453
  transmission, 453
Coefficient of coupling, 124
Coercivity, 34
Coil, universal wound, 328
Coil antenna, field, 37
  approximate wave lengths, 13
Collodion as binder, 69
Color code, R.M.A., 49
Color-sensitivity curve, 544
Colpitts oscillator, 233, 236
Communication band, authorized width, 18
Commutator, revolving, 499
Comparison measurements, 167
Complex notation, 104
Condenser, design equations, 94
  electrolytic, 91
  gang, 94
  imperfect, 237
  inductive and non-inductive, 90
  life tests, 102
  microphones, 356
  resistance measuring, 101
  trimming, 94
Conductance, grid, 202
  plate, 200
Conductors, materials as, 42
Constant, attenuation, 131, 136
  propagation, 131
  wave length, 131, 135
Constantin, 42
Contact potential, 187
Control panels, input, 573
Converter, reed, 533
Copper wire, tensile strength, 12
  turns per inch, tables, 12, 13
Coulomb's law, 20
Coupled circuits, 123
Coupling, capacitively, 281
  coefficient of, 124
  direct, 126
  galvanic, 281
  indirect, 126
  inductive, 127, 281
  stray, 130
Course indicator, 533
Critical damping, 146, 147
Cross-coil direction-finding system, 526
Cross modulation, 204
Cross-screen analyzing scanner, 493
Crystal control, 369
Crystal detector, 245
Crystal rectifier, 156
Current, circuits for measuring photo-electric, 548
  circulating, 388

Current, continuous and alternating, 24
direction of flow, 26
sensitivity, 147
standards, 166
steady-state, 57
stream of electrons, 184
transient, 29
Cutting stylus, 556

D

D'Arsonval galvanometer, 145
Decibel, 281
chart, 15
Decimal equivalents, 1
Demodulation, 244
of one signal by another, 256
Density, flux, 51
Detection, 244
C bias, 263
characteristic, 244
fidelity curve, 251
grid-current, 251
plate, 250
self-bias resistance for, 260
Detector, 333
biased, 335
crystal, 245
equations, 246
Fleming valve, 335
galena, 245
grid-leak and condenser, 334
heterodyne, 254
high-amplitude, 251
linear, 253
to measure modulation, 275
measurements, 250
oscillator, 263
Perikon, 245
power, 333
screen-grid tube as, 258
silicon, 245
triode as, 248
Deviometer, 534
Diamagnetic materials, 33
Dielectric absorption, 88
Dielectric constant, 81
measuring, 101
Dielectric losses, 40
Dielectric materials, 80
Dielectric strength, 87
Diodes, commercial, 194
Direct-coupled amplifiers, 281
Direct-current motor drive, 572
Direction finder, 352
on airplane, 525
crossed-coil, 526
Directional microphone, 362
Disk, recording sound on, 554
sound records, 573
Dissipation constant, 109
Distortion, measurement of, 293
modulation, 204
percentage, 248
Dot-dash system, 500
Double-button carbon microphone, 359
Double-modulation method, 500
Drysdale potentiometer, 168
Dushman, Saul, 191
Dushman's coefficient, 189
Dynamotor system, 522
Dynatron kink, 211
Dynatron oscillator, 171, 242

E

Ear, characteristics, 413
Echo signals, 483
Eckersley, P., 438
Eddy current, 40, 64, 284
Effective resistance, 26, 41
Effective value, 25
Efficiency, lines, 137
Einthoven string galvanometer, 150
Electric charge, nature, 20
Electric retouching, 498
Electrodyanometer, 150, 151
Electromagnetic field, electrons, 184
energy, 39
structures, 34
Electrometer, Kelvin absolute, 157
E.m.f., concept of, 22
Electron, 20, 182
affinity, 188
current between concentric cylinders, 192
between parallel plates, 191
in electromagnetic field, 184
in electrostatic field, 182
emission of, 184
Electrostatic voltmeters, 157
Emission, electron, 187
photoelectric, 185
secondary, 185
thermionic, 185
Emission characteristic, 196
Emissivity, thermal, 186
Energy in magnetic field, 23, 39
Energy density, 405
Equalizers, 116
Equations, circuit, 29
Equivalent impedance, 127

F

Facsimile, balanced circuits for, 496
Bell System, 509
requirements for, 455
scanning in, 489
synchronism in, 489
transmission, 489
use of Neon tubes, 502
Fading, 438
Farad, 79
Farnsworth cathode-ray tube, 479
Feeling, threshold, 413
Felici's mutual inductance, 180
Ferromagnetic materials, 33
Field, coil antenna, 37
energy of electromagnetic, 39
loop antenna, 37
Field localization, 536
Field strength, magnetic, 33
Filament, calculations, 188
design, 189
Filament characteristic, 196
Film, recorders, 560
recording sound on, 554
Filters, 380
band-elimination, 142
band-pass, 129, 143, 311
choke design, 386

Filters, chokes with mutual inductance, 385
composite, 138
condenser ratings, 387
constant-K structure, 140
design of wave, 138
graded, 378
high-pass, 142
L-type, 140
low-pass, 141
design chart, 381
tuned, 384
scratch, 116
uniform, 138
wave, 130, 138
Fleming valve detector, 335
Flux, magnetic, 33, 51
Flux density, 51
Flying spot, scanning, 477
"Flywheel" effect, 232, 320
Form factor, 25
Four-resistance network, 171
Fourier series, 253, 293
Frequencies, allocation of, 445
Frequency, band suppression, 288
converters, 331
intermediate, 349
inversion, 273
measurement, 165
meters, 166
modulation, 278
multipliers, 276, 321, 462
peculiarities of high, 434
regulation, use of resonant circuit, 115
separation, 19
stability, 445
standards of, 167
tolerance table, 19
use of ultra-high, 441
Fulton, Capt. O., 509

G

Galena detector, 245
Galvanometer, characteristics of a-c, 149
of d-c, 147
D'Arsonval, 145
differential, 148
Einthoven string, 150
moving-coil a-c, 150
vibration, 148
Gauss, 33
Generator, engine-driven double-voltage, 523
signal, 324
wind-driven double-voltage, 523
Gilbert, 33
Gill-Morrell oscillations, 451
Glowlamp, 554
Glowlamp recorder of Fox Films Corp., 561
Goldschmidt alternator, 461
Greek alphabet, 1
Grid, 194, 196
conductance, 202
phase-angle corrections, 235
suppressor, 211
Grid-current coefficients, 202
detection, 251
Grid-glow tube, 549
Grid-leak resistance, 229
Groove shape, 556
Ground-station equipment, 512
Grover method, 177

H

H & D curves, 563
Harmonic frequencies in antenna, 37
Harmonic measuring circuit, 325
Harmonics, 25
in broadcast transmitters, 369
Hartley circuit, 221, 233, 236, 270
Hay bridge, 178
Hazeltine neutralized amplifier, 316
Heater characteristic, 196
Heaviside equal-arm bridge, 181
Heising modulation, 279
Heising oscillator, 271
Henry, 52
Hertz, Heinrich, 433
Hertzian doublet, 37
Heterodyne detector, 254
Heterodyne frequency meter, 171
High-amplitude detector, 251
High-frequency reception, 433
High-frequency transmission, 433
High-pass filter, 142
Honeycomb coil data, 68
Horn, 426
flaring, 426
theater, 573
Horton, J. W., 481
Hot-wire meters, 154
Hulburt, E. O., 435
Hysteresis, 40, 284

I

Ice, removal, 464
Ignition shielding, airplane engine, 524
Image-frequency ratio, 350
Impedance, acoustical, 406
amplifier, 301, 336
complex or vector, 55
definition, 27, 104
equivalent, 127
input, 132, 136, 281
loss due to series, 110
due to shunt, 110
matching, 292
measurement, 164
of parallel resonance, 123
in parallel, 106
in series, 106
termination, 137
transformers, 137
transmission-line input, 132
Indicator, course, 533
reed, 532
Inductance, 51
bank winding, 66
basket-weave winding, 66
calculation, in air-core coils, 69
chart, 18
coil design, 67
definition, 23
and units, 52
duolateral, 66
Felici's mutual, 180
flat spiral, 66
formula for apparent, 65

Inductance, honeycomb winding, 66, 68
of iron-core coils, 62
layer-wound, 66
Litz wire, 68
measurement at high frequencies, 61
of iron-core, 60
at low frequencies, 58
multilayer, 66, 72
toroidal, 66
of various windings, 5
Induction, 33
Induction field, 36
Inductive power in circuit, 57
Inductive reactance, 54
Inductor, definition, 53
types, 65
Ink recorder, 475
Input control panels, 573
Input impedance, 132, 281
Insulators, power factor, 85, 87
Interelectrode capacities, 215, 216, 301
Interference, image-frequency, 348
Interference problems, superheterodyne, 348
Intermediate frequency, 349
Ion, negative and positive, 21
Iron and steel, magnetic properties of, 34
Ives, H. E., 509

J

Jenkins, C. Francis, 509
Joule, 23
heat, 38
Joule's law, 40

K

Karolus, A., 509
Kelvin absolute electrometer, 157
Kelvin double bridge, 173
Kennelly-Heaviside layer, 435, 436, 439
Kerr light cell, 485, 504, 555, 564
Keying, method of, 460
Kingsbury, B. A., 415
Kirchhoff's second law, 30
Knudson, V. O., 412
Korn, Arthur, 509

L

L-type filter, 140
Labus, J. W., 370
Lamps, glow, 504
Landing beam, 537, 538
Lead-loss correction, 189
Leakage current, 92
Light, polarized, 504
Linear amplifier, 320
Linear detector, 253
Lines, artificial, 133
open-wire, 132
Lissajous figures, 162
Litz wire, 68
standards, 75
Litzendraht, 151
"Local distance" switch, 319
Lodge, Sir Oliver, 433
Loftin-White amplifiers, 292
Logarithmic decrement, 146
Loop antenna, field, 37
Loud-speaker, 403, 415
calculation of efficiency, 416
condenser, 421, 424
directional characteristics, 428
effect of flare and length, 427
magnetic armature type, 421
magneto-striction, 421
measurements, 429
motor, 421
moving-conductor drive, 421
piezo-electric, 421
radiator, 425
Low-pass filter, 141

M

MacFarlane, M. L. D., 509
Magnetic armature motor, 422
Magnetic circuits, 33
Magnetic field, energy in, 23
strength, 33
vertical antenna, 36
Magnetic flux, 33, 51
Magnetic moment, 33
Magnetic properties of iron and steel, 34
Magnetic saturation, 34
Magnetic structure, design, 35
permanent-magnet type, 35
Magnetomotive force, 33
Magneto-striction, 167
Magnetron oscillator, 450
Manganin, 42
Marconi, Guglielmo, 433
Marginal-relay method, 499
Mark-to-space ratio, 455
Maxwell, 33
Maxwell's bridge, 178
Maxwell's equations, 36
Measurement, acoustic, 429
of amplification factor, 199
of capacitance, 99, 165
comparison, 167
of condenser resistance, 101
of dielectric constant, 101
of distortion, 293
frequency, 165
of high-frequency inductance, 61
of impedance, 164
of interelectrode capacitance, 216
of iron-core inductances, 60
of loud-speaker, 429
of modulation, 274
of mutual conductance, 202
of mutual inductance, 78
of parallel resonance impedance, 123
of plate resistance, 200
of power factor, 165
of Turner constant impedance, 61
voltmeter-ammeter, 163
Wheatstone bridge as detector, 250
Measuring instruments, 145
Megger, 163
Meter, copper-oxide rectifier, 157
frequency, 166
heterodyne-frequency, 171
hot-wire, 154
moving-diaphragm, 152
moving-iron, 152
rectifier, 156
soft-iron, 152
spark-gap, 158

Meter, thermocouple, 154
  tuned-circuit frequency, 171
Microphone, 354, 556
  beam, 557
  condenser, 356
  directional, 362
  double-button carbon, 359
  hiss, 361
  pressure calibration, 357
  ribbon, 556
  technique, 361
Mid-series termination, 134
Modulated amplifiers, 269, 272
Modulation, 244, 263
  absorption, 263
  analysis, 265
  change due to resonance characteristic, 276
    in percentage, 277
  complete, 367
  cross, 204, 277
  distortion, 204
  factor, 244
  Heising, 270
  iron saturation, 268
  measurement, 274
  percentage, 368
  plate-circuit, 269
  tubes, table, 272
Morehouse, F. G., 509
Motor, synchronous, 572
Motor drive, d-c, 572
"Motor boating," 290, 291
Motor system, interlocking, 562
  location trucks, 563
  news-reel, 563
  non-interlocking, 562
  sound recording, 562
Moving-conductor motor, 421
Moving-diaphragm meters, 152
Moving-iron meters, 152
Mu factor, plate grid, 203
Mutual characteristic, 195
Mutual conductance, 201
  calculation, 202
  inverse, 203
  measurement, 202
Mutual inductance, calculation, 78
  definition, 23
  measurement, 76

N

Navigation, course, 525
Nelson, E. L., 483
Neon tubes in facsimile, 502
Network, four-resistance, 171
  four-terminal, 173
  recurrent, 130
  six-impedance, 180
Neutralization, 327
Neutrodyne, 316
Nichrome, 42
Nicol prism, 485
Nipkow, 477
Noise suppressor, 341

O

Oersted, 33
Ohm, definition of, 41
Ohm's law, 22, 40
Ohmmeter, direct-reading, 163
Oscillation, control, 271
  output and efficiency, 225
Oscillations, parasitic, 240
Oscillator, 221
  calculations, 226
  circuit chart, 227
  circuit design, 222
  detector, 263
  dynatron, 171, 242
  efficiency, 222
  grid-circuit, 235
  Hartley, 221, 270
  Heising, 271
  magnetron, 241, 450
  modulator system, 270
  plate-circuit, 235
  triggered, 498
Oscilloscope, cathode-ray, 275
Output stage modulated, 367
Overload level, 324
Owen's bridge, 178

P

Parabolic reflector, 362
Paramagnetic materials, 33
Parameters, circuit, 29
Peak power of musical instruments, 355
Peltier effect, 187
"Pencil of light" method, 494
Pentode, 210
  compared with triode, 214
  power output, 212, 295, 337
  suppressor r-f, 214
Perikon detector, 245
Permeability, 33
  incremental, 63
Phase, 26
  angle, 26, 83
  difference, 83
  lagging, 26
  leading, 26
  relations, 239
Phosphor bronze, 42
Photo analyzers, 493
Photocells, 543
Photo-conductivity, 550
Photoelectric current, circuits for measuring, 548
Photoelectric emission, 185
Photoelectric tube, uses, 550
Photoelectricity, 543
Photo-engraving method, 493
Photographic analysis, 563
Photolytic cell, Arcturus, 553
Phototube, amplifiers, 495
  choice of, 547
  circuits, 547
  color sensitivity, 544
  gas, 546
  vacuum, 546
Photovoltaic effect, 551
Photronic cell, Weston, 553
Plate, characteristic, 196
  conductance, 200
  detection, 250
  grid mu factor, 203
  grid transconductance, 203
  resistance, 200, 201

Playback record, 556
Point-to-point communication, 459
Polarizing field, 424
Polydoroff, 304
Position determination, 525
Potassium bichromate method, 494
Potential, contact, 187
  nature of, 22
  units of, 24
Potentiometer, 167
Power, amplifiers, 336
  detectors, 333
  output, calculation, 213
    distortion, 213
  sensitivity, 283
  supply design, 374
  supply systems, 343, 374
  tubes, 296
    as modulators, 272
Power factor, in capacitive circuit, 83
  definition, 26
  in inductive circuit, 58
  measurement, 165
  values for insulators, 85, 87
Preselector circuits, 129
Processing, photographic, 564
  record, 569
  sound film, 563
Propagation, phenomena, 481
  velocity, 132, 135
Propagation constant, 131
Propagation factor, 132
Protons, 20
Push-pull amplifiers, 295, 296

Q

"Q," $L\omega/R$, 109
Quarter-deflection method, 170
Quartz crystals, 167, 321

R

Radiation, 40
  nature of, 36
Radiation field, 36
Radiator, loud-speaker, 425
Radio-beacon system, rotating, 527
Radio-frequency amplifiers, 300
Radio-frequency lines, efficiency of, 137
Radio range-beacon system, 528
  shielding and bonding in aircraft, 524
  symbols, 16
  systems for guiding aircraft, 525
Radiotelegraphic services, 457
Range table, 442
Raytheon tubes, 394
Reactance, 28
  acoustic, 407
  high-frequency antenna, 38
  inductive, 54
  low-frequency antenna, 37
  standards, 167
Receivers, automatic volume control for aircraft, 534
  d-c operated, 343
  fidelity, 324
  high-frequency, 449
  long-wave, 468
  medium-frequency aircraft, 518
  overload level, 324
Receivers, regenerative, 323, 351
  selectivity, 324
  sensitivity, 324
  ship-to-shore, 469
  superheterodyne, 322, 346
  superregenerative, 323, 351
  television, 352
Receiving systems, 322
Reception, diversity, 503
Record, playback, 556
Record processing, 569
Recorder, electrical, 567
  film, 560
  glowlamp, 561
  ink, 475
  RCA Photophone, 560
  Western Electric light-valve, 560
Recorder attenuator, 567
Recording, cutting stylus, 556
  disk, 555, 566
  film-engraving system, 555
  Kerr cell, 555, 565
  light-valve method, 554
  noiseless, 566
  sound, 554
    motor systems, 562
  sound channel, 557
  toe, 564
  variable area, 555
  variable density, 554
Records, characteristics of disk, 567
Rectification diagrams, 256
Rectifier, 374, 389
  copper oxide, 157, 246
  crystal, 156
  dry-contact, 394
  durion-duralumin, 396
  electrolytic, 395
  glow, 394
  hot-cathode mercury-vapor, 391
  mercury-arc, 393
  meters, 156
  tungar, 393
Rectigon bulbs, 393
Reed converter, 533
Reed indicator, 532
Reflections, multiple, 438
Regeneration in amplifiers, 314
Regenerative receivers, 351
Reinartz, John L., 434
Reluctance, 33
Retardation time, 132, 136
Richardson, O. W., 185
Richardson's temperature equation, 197
R.M.A. color code, 49
Roder, Hans, 370
Reproducer, playback, 567
Reproducing equipment, theater, 570
Resistance, 40
  antenna, 38
  apparent, 65
  balance, 177
  capacity-coupled amplifiers, 281
  critical damping, 147
  effect of load, 235
  effective, 26, 41
  equivalent, 84
  grid-leak, 229
  infinite, 26
  negative, 26
  plate, 200

Resistance, positive, 26
  self-bias for detection, 260
  specific, 41
  standards, 166
  temperature coefficient of, 41
Resistivity, volume, 41
Resistors, composition, 46
    rating, 48
  fixed, 45
  series and parallel, 42
  variables, 45
  wire wound, 46
Resonance, 29, 111
  anti-, 111
  bridge, 179
  characteristic, effect on modulation, 276
  circuit, 286
  parallel, 111
  phase, 111
  series, 111
Resonant circuit, design of parallel, 119
    of series, 113
  measurement of parallel impedance, 123
  use of, for frequency regulation, 115
Retentivity, 34
Reverberation time, 407

S

Saturation, magnetic, 34
Saw-tooth wave form, 479
Scanning, 477
  cross-screen analyzing, 493
  flying spot, 477
  line, 570
Schering bridge, 177
Schottky effect, 191
Schröter, Dr. Fritz, 509
Scrambling, 273
Screen-grid tube, 205, 318, 327
  detector, 258
Selectivity, 345
  adjacent-channel, 311
  distant-channel, 311
Selenium bridge, Burgess, 552
Selenium cell, 550
  FJ-31, 552
Sensation level, 414
Series, Fourier, 293
Service area of transmitter, 273
Sharp, C. H., 550
Ship-to-shore communication, 459
Short-wave technique, 433, 467
Signal generator, 324
Silicon detector, 245
Single-dial tuning, 344
Skin effect, 40, 43, 45, 151
Skip-distance, 435
  and range table, 442
  secondary region, 437
Soft-iron meters, 152
Solenoid, single layer, 66
Sound, on disk, recording, 554
  velocity of, in common materials, 405
Sound cameras, portable, 561
Sound film, processing, 563
  recording, 554
  standards of, 569
Sound head, 570
Sound motion pictures, 554
Sound recording channel, 557
Sound records, disk, 573
Sound wave, plane, 404
  spherical, 404
Sound waves in air, 404
Space charge, 184, 190
Space current, 197
Spark absorber, 474
Spark-gap meters, 158
Spark length in air, 14
Specific inductive capacity, 81
Speed, projection, 569
  taking, 569
Split-tank circuits, 121
Square-top waves, 278
Stability of oscillators, 241
Stage gain of a-f amplifiers, 281
Static characteristics, 195
Static frequency multipliers, 462
Steady-state current, 57
Stylus, cutting, 567
Supercontrol, 207
Superheterodyne, 449
Superheterodyne receivers, 346
Superheterodyne translation ratio, 262
Susceptibility, 33
Synchronizing methods, 506
  thyratron tubes in, 508

T

Tapered lines as impedance transformers, 137
Telegraph equipment, printing, 453
Telephone, 152
  Baldwin, 153
  dynamic, 153
Television, 450, 477
  receivers, 352
  synchronization, 485
Tetrode, 205
  screen-grid, 205
  space-charge grid, 210
Thalofide cell, 551
Theater horns, 573
Theater reproducing equipment, 570
Thermal emissivity, 186
Thermionic brake, 488, 508
Thermionic emission, 185
Thermocouple meters, 154
Thermophones, 154, 357
Threshold of audibility, 413
  of feeling, 413
Threshold wave length, 544
Thomson, J. J., 182
Thyratron tubes in synchronizing, 508
Time constant, inductive-circuit, 53
Toe recording, 564
Tone control, 116, 338, 473
Tone keyer, 470
Transconductance, plate-grid, 203
Transcription, electrical, 566
Transfer characteristic, 195
Transformer coupled amplifiers, 281
  design of small, 397
  impedance at various frequencies, 284
  winding distributed capacity, 284
Transient currents, 29, 57
Transient phenomena, study of, by cathode-ray, 163
Translation ratio, superheterodyne, 262

Transmission, facsimile, 489
 single side-band, 273
Transmission line, 121, 131, 135
Transmitter, arc, 462
 automatic, 473
 high-frequency, 448
 marine, 464
 service area of, 273
Triggered oscillators, 498
Trigonometric functions, 2
Triode, 194
 as detector, 248
 compared with pentode, 214
Tube, cathode-ray, 161
  timing axis for, 162
 Class B output, 337
 coupling amplifier, 281
 dome-top, 218
 grid-glow, 549
 parallel and push-pull, 240
 pentode, 210
 pentode work with compensating network, 295
 power-output pentode, 212, 337
 Raytheon, 394
 screen-grid, 318, 327
 thyratron in synchronizing, 508
 three-electrode, 194
 triple-grid, 218
 use of Neon in facsimile, 502
 uses of photoelectric, 550
 water-cooled, 370, 371
Tungar rectifier, 393
Tuning, single-dial, 344
Tuning silencer, 341
Turner constant-impedance method, 61
Turner voltmeter, 160
Turntables, 573

V

Vacuum tube, 182
 as rectifier, 157
 two-electrode, 192
 voltmeter, 158, 325
Valves, light, 484, 504
Variable area recording, 555
Variable mutube, 207
Variometer, 66
Vector diagrams, 104
Vertical antenna, 36
Vibrating reeds, 166
Voltage divider, parallel, 375
 series, 377
Voltage divider, series-parallel, 376
Voltage regulation, 376
Voltage standards, 166
Voltammeters, 148, 166
Voltmeter-ammeter measurement of resistance, 163
Voltmeter, amplifier-detector, 161
 Ayrton-Mather electrostatic, 158
 electrostatic, 157
 Turner, 160
 vacuum-tube, 158
  balanced, 160
  calibration, 160
  input resistance, 160
  sensitivity, 160
Volume control, acoustically compensated, 338
 for aircraft receivers, 534
 automatic, 341
 panel, 559
Volume resistivity, 41
Volume range in broadcasting, 354

W

Wagner earth connection, 174, 201
Wave filters, 130, 138
Wave form, 24
Wave length, 132
 chart, 18
 constant, 131, 135
 threshold, 544
Waves, square current, 241
 square-top, 278
Wax preparation, 568
Wax-suction equipment, 568
Weiller wheel, 480
Wente, E. C., 356
Weston cell, 22, 166
Weston photronic cell, 553
Wheatstone bridge, 171
 detector measurements, 250
Wien's bridge, 179
Williams, R. M., 509
Wire lines, 365
Work function, 187
Wright, G. W., 509

Z

"Zero beat," 323
Zworykin, V. K., 479