# The Analysis of Archaeoastronomical Orientations

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Version 7.0, September 2021

# Content

# 1 Introduction

Summarized in this publication are the most important data, as well as the theoretical bases, needed for the archaeoastronomical analysis of architectural lines [19]. Such are understood to be as different as e.g. alignment of stone rows, the main axes of buildings and temples. Further the orientation of gates, windows or openings, but also of graves and skeletons. The target group here are all those interested in Archaeoastronomy.

Of eminent importance for Archaeoastronomy are the four main directions *North, East, South, West*, which are therefore also called cardinal axes [1]. To mark these directions, a simple procedure, practiced in earlier times is presented. Further a commented inventory of solar and lunar *horizon azimuths* is presented, which have already been suggested in the archaeoastronomical literature. Hereafter the directions to the rising and setting points of stars are denoted as *horizon azimuths*. In most cases these are the northernmost and southernmost extreme values. In the case of the moon they are called *lunistices* or in the case of the sun *solstices*. With the sun, additional intermediate points are added, but in most cases limited to the *equinoxes*. Finally, this list is concluded with the azimuths of the so-called quarter days, whose dates are located in the middle between solstices and equinoxes and can play a role in connection with megalithic objects like stone rows etc. The dates of the so-called "Celtic calendar", which today are mainly popular in "esoteric circles", are located very close around these four days.

In the case of planetary azimuths, there are predominantly the so-called "Venus Extremes" which, with a few justified exceptions (e.g. Maya culture), are mainly proposed by laymen for the orientation of architectural lines. Here the complex pattern is demonstrated, how these extreme horizon azimuths really occur and which references exist for the strongly fluctuating values.

In the second part it is suggested how to proceed with azimuth differences influenced by different horizon heights. Furthermore it is shown how azimuths can be determined with obstructed bearings. Furthermore, methods for refraction and parallax correction of azimuths are presented. Finally, it is explained how the variable ecliptic obliquity is related to the rising and setting azimuths and can be used for calculations. With this equation set, the values listed in the first section can be reproduced or converted for other latitudes and horizon elevation angles. The tool *"Archaeoastronomical Calculations"* is Excel-based for solving the most important basic archaeoastronomical tasks [33]. It contains most of the equations presented here as an easy-to-use calculation program, supplemented with several examples.

Finally presented, for those interested in mathematics, are fundamental principles to celestial mechanics and astrometry, with a relation to archaeoastronomy. This includes an introduction to the equatorial coordinate system, definitions for solstices and lunistices, equations for calculating the azimuth of rising and setting, and for determining the frequently discussed angles of rising and setting of the stars. A crucial role play here necessarily the theorems of spherical trigonometry.

Finally I would like to thank Holger Filling [16], who supported me in compiling the equations in the mathematical part, and Wolfgang Tross (†), who always provided me with suitable literature from his extensive Megalithik library.

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# 2 Terms and Definitions

## 2.1 What is Archaeoastronomy?

Archaeoastronomy is the name of the scientific branch that deals with analyses at the interface between astronomy and archaeology. Archaeoastrometry would probably be the more precise term, because these cultures at that time could only deal with the apparent course and positions of the stars. The lack of light pollution, as well as the probably small offer of organized evening entertainment, probably substantially promoted the interest in the processes in the night sky. The scientific investigation of the celestial bodies remained inaccessible to them because the technical possibilities were simply lacking. In the foreground was the definition of the points of rising and setting on the horizon, whether for calendar or cultic purposes. In this context, the term *"Horizon Astronomy"* is also applied. Architectural lines worldwide have been oriented towards such points. In contrast to many historically interested astronomers, some archaeologists are still skeptical about the relevance of such azimuths.

Various cultures have documented the observation of bright comets, explosions and brightness eruptions of stars (supernovae and novae), as well as conspicuous conjunctions. Well-known examples are the numerous hypotheses about the "Star of Bethlehem" and the documented observation of the Supernova M1 by the Chinese in 1054 A.C., whose precisely known date is of eminent importance for modern stellar astronomy.

Conjunctions are apparently close coincidences or even occultation of two or more celestial bodies with the participation of sun, moon, or planets. Besides, however, also bright stars and conspicuous star clusters (e.g. the Pleiades) can be involved. The prediction of solar and lunar eclipses was regarded as a paramount discipline here, proven e.g. by the Greeks and Babylonians. The following amateur pictures from internet sources show conjunctions of Venus with the moon (left) and the star cluster of the Pleiades (right).

The Pleiades have been known to play an important role in several cultures, as this star cluster (M45) is conspicuous even by attentive laymen. It is also located just north of the ecliptic and is therefore often "traversed" by the moon or the planets. Most of the numerous interpretations also see it on the "Sky Disc of Nebra" (chapt. 4.7). A hint to his today's degree of popularity is his representation as emblem of the car brand Subaru, which in Japanese means "Pleiades"!

## 2.2 Definition of Astronomical Rise and Set for Sun and Moon

In the case of the sun and moon, the rising and setting points are defined as the locations on the real landscape horizon where the upper limb of the disks appears first or disappears last. Hereafter the direction to these points is called horizon azimuth. As a result of the atmospheric refraction, and influenced by parallax effects (chapt. 15), these apparent points are slightly shifted to the north compared to the directions calculated by celestial mechanics and related to the center of the celestial body. Therefore the term "apparent" is also used in this context (chapt. 15.4). With point-shaped appearing stars and planets, the definition as horizon-point simplifies, where the star apparently appears or disappears at the horizon-line. All azimuth values in this paper refer to the astronomical (mathematical) horizon.

This definition seems to have been applied to most prehistoric objects. According to Alexander Thom, in the case of sun and moon the lower limb of the disk was also decisive for the horizon azimuth [2]. He justifies this with the statistical distribution of the azimuths, which he examined on the British Isles at 244 Megalith objects. Calculated values of ephemeris and planetarium programs for sun and moon always refer to the center of the disk (until otherwise noted).

## 2.3 Calculation Basis and Application Range for Azimuth Values

All horizon azimuths have been calculated here with stellar declinations based on data and simulation runs with the NASA ephemeris calculator "Horizons" of the Jet Propulsion Laboratory (JPL) in Pasadena [28]. The term "simulation" is used here because many values cannot be calculated explicitly, but have to be "filtered out" from generated data lists over certain epochs. The apparent declination values, related to the equinox of the entered date, were determined in order to obtain the actual horizon levels at that time (chapt. 12.3). All azimuths given here are also corrected for refraction effects, refer here to the astronomical horizon, the latitude of 47° N (approx. Berne) and mostly to the time 2000 B.C. and 2000 A.C. For sun and moon the necessary parallax corrections have also been carried out (chapt. 15.7, 15.8).

NASA, JPL Horizons provides high-precision ephemeris for all conceivable objects in our solar system, based on the same algorithms NASA uses for planning and executing its space missions (self-declaration JPL). This is certainly a stroke of luck for archaeoastronomy because the program's scope of application now extends from 9998 B.C. to 9999 A.D. and can be freely operated via the Internet portal "HORIZONS Web Interface" with amazingly short response times. JPL Horizon's data is therefore often used as a reference in test reports on planetarium programs.

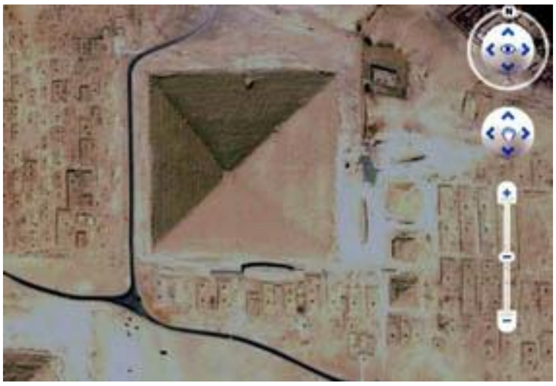
Important: Published theoretical archaeoastronomical azimuths may only be transferred to an object if the latitude is known as a reference value and approximately corresponds to the examined location! However, declination values can be used universally. The equations presented in chapt. 15 can be applied to calculate the horizon azimuth for any latitude and horizon elevation angle.

# 3 The Four Cardinal Directions

The four main points of the compass are archaeoastronomically highly relevant. In this context, we therefore also speak of cardinal axes [1]. The meridian runs in the north-south direction, above which all stars reach their highest level above the horizon.

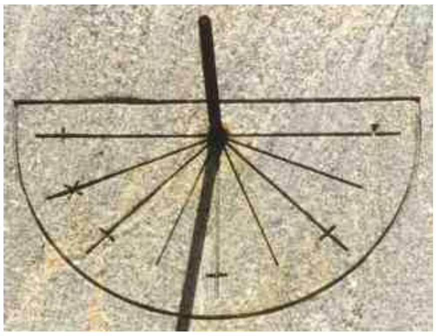
## 3.1 Examples of Orientation towards the four Main Points of the Compass

Various megalithic alignments are directed to the main points of the compass. The pyramid of Cheops in Giza is also aligned to N - S within a few arc minutes. Furthermore, the longitudinal axes of many Christian churches are oriented west-east, including St. Peter's Basilica in Rome (Google Earth with inserted compass rose). Some seem to have been aligned to the sunrises or sunsets of their patron saints. In some cases, e.g. at the Grossmünster in Zurich (azimuth 125°), there are also solstices represented, which in most cases is attributed to a pre-Christian precursor object.

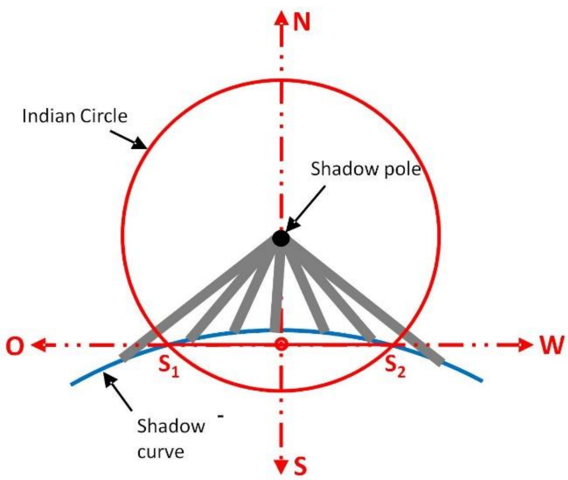



## 3.2 Staking out the Main Points of the Compass

The north-south direction can in principle be marked out with a so-called *"Gnomon"*. A vertical pole (or *"Gnomon"*) casts the shortest shadow in the course of the day at noon (highest position of the sun) and marks the meridian. Since the shadow curve is relatively flat, the minimum can only be determined rudimentarily. To make matters worse, there are also semi-shade effects, as the sun is not a point-shaped light source.



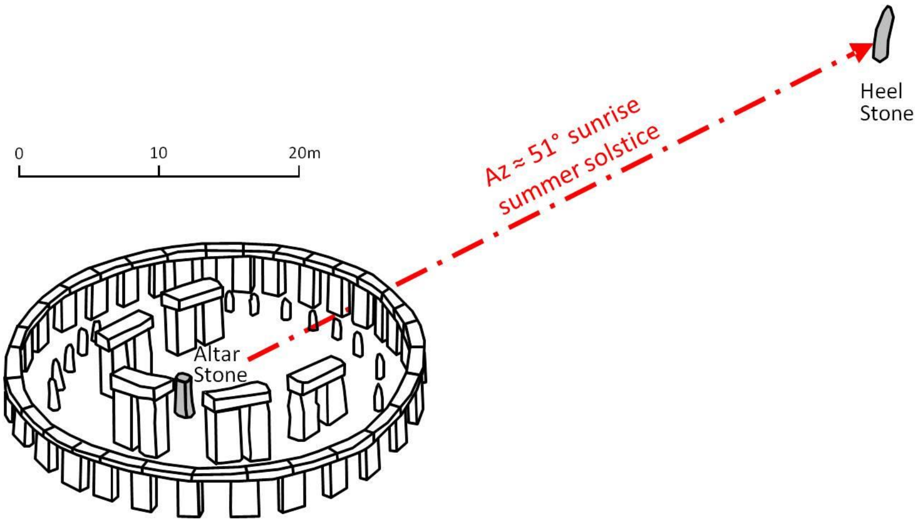
The precision of this stake out method can be significantly increased with the so-called *"Indian circle"* [1]. This is drawn centrically around the foot of the shadow pole. Here not the shortest shadow length is determined, but the two intersection points *S1* and *S2*, which the shadow curve generates before and after noon with the circle line. The corresponding chord between *S1* and *S2* marks the *East-West* direction. Now this chord just needs to be halved to set the second stake out point next to the shadow pole for the *North-South* direction.



# 4 Archaeoastronomical Solar Azimuths

## 4.1 The Main Solar Axes

Worldwide, numerous architectural lines have been aligned to the main solar axes. This is hardly surprising, since of all the celestial bodies, the sun has the most lasting influence on our everyday lives. The best known example is certainly Stonehenge, where the peripheral "Heel Stone" and the following, more than 500 m long so-called "Procession Road" are aligned with an accuracy of approx. 5 arc minutes to the sunrise of the summer solstice. The opposite direction aims to the sunset of the winter solstice.




In addition to the four main points of the compass, this azimuth category reaches the highest acceptance among experts. The six main solar azimuths are defined by the horizon points where the sun rises and sets at the solstices and equinoxes. They roughly divide the year into four sections. Midsummer solstice data are approx. 21 December (winter solstice) and 21 June (summer solstice); equinoxes are approx. 21 March (primary equinox) and 22 September (secondary equinox), for definition see Chapt. 14.

## 4.2 Celestial Mechanical Influences and Parallax Effects

In contrast to the fixed star and planetary azimuths, the precession of the Earth's axis has no influence on the solar and also lunar declination values, and thus also on their horizon azimuths (see Section 13.2). Minor, long-periodic inclination fluctuations of the Earth's axis (chapt. 13.3), however, cause the declination difference between the solstices of 2000 B.C. and 2000 A.C. It yields just about 0.5°, which corresponds to a mean, apparent sun- or moon diameter. The equinoctial azimuths are hardly affected by this effect, because at this time the sun is exactly on the celestial equator and the points of rising and setting are practically on the east-west axis. Due to the weakly eccentric, elliptical Earth orbit, the winter half-year is a few days shorter than the summer half-year. The slight shift of the two equinoctial azimuths to the W - O direction (to 90°, resp. 270°) is caused by the disk parallax and the atmospheric refraction due to the upper limb definition (Chapt. 15).

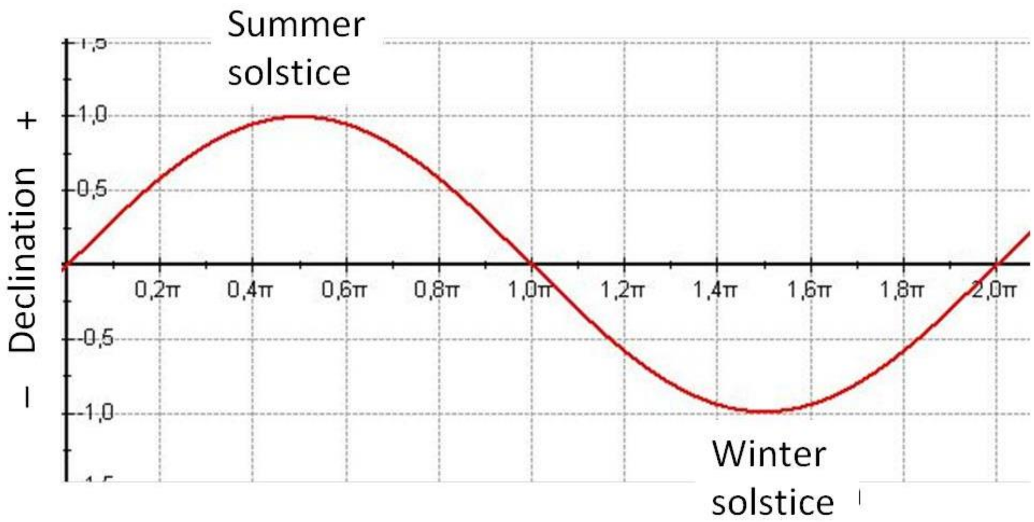
## 4.3 Solar Main Azimuths for 47°N

In the following table, the declination values δ of the sun are rounded to arc minutes and the azimuths, calculated for 47° north, are given in decimal degrees.

| Solar Main azimuths for 47° N | 2000 B.C. | | | 2000 A.C. | | |
|---|---|---|---|---|---|---|
| | Dec δ | Rising | Setting | Dec δ | Rising | Setting |
| Winter Solstice WS | −23° 55′ | 125.4° | 234.6° | −23°26′ | 124.6° | 235.4° |
| Equinoxes Eq | 0° 00′ | 89.1° | 270.9° | 0° 00′ | 89.1° | 270.9° |
| Summer Solstice SS | +23° 55′ | 52.4° | 307.6° | +23°26′ | 53.2° | 306.8° |

## 4.4 Effects in the Range of the Solstices

Since the solar horizon azimuths are closely linked to the seasons, they have the potential to be used for calendar purposes. For the following reasons the solstices are an exception. The term solstice for the solstices comes from Latin, which could be translated as "standstill of the sun". In fact, during these phases it reaches its greatest southern or northern distance from the celestial equator. There over several days it changes its declination value and thus also its horizon azimuth only in extremely small steps. The reason for this is the apparent solar orbit (ecliptic), whose projection on the celestial sphere is sinusoidal. The graph below shows the flat course of this curve in the upper and lower vertex areas.




The following table demonstrates this effect. It shows the apparent declination values and the calculated rising azimuth in decimal degrees, during- and four days after the summer solstice (SS) of 21.6.2008 (Simulation with NASA, JPL horizons in 1h intervals).

| Date/Time [UT] | 2008 A.C. Azimuths for 47° N | | |
|---|---|---|---|
| | Decl. δ | Rising | Difference |
| 21.6. 18:00 UT, SS | +23° 26′ 26″ | 53.195° | |
| 22.6. 18:00 UT | +23° 26′ 14″ | 53.201° | 0.006° |
| 23.6. 18:00 UT | +23° 25′ 36″ | 53.218° | 0.017° |
| 24.6. 18:00 UT | +23° 24′ 35″ | 53.247° | 0.029° |
| 25.6. 18:00 UT | +23° 23′ 08″ | 53.287° | 0.040° |

This means that for more than about 12 days around this date, the daily azimuth difference remains in the range of only a few hundredths of a decimal degree. Restricted to optical measuring aids, such as Gnomon and Alignments, the solstices can thus only be marked, but the dates can never be determined this way. This fact is also an indication that e.g. Stonehenge was probably built as a sacral- rather than a calendar monument [1]. One could therefore describe the solstices as pure "cultic azimuths", while the remaining solar intermediate azimuths can certainly be used for calendar purposes.

### 4.5 Effects in the Range of Equinoxes

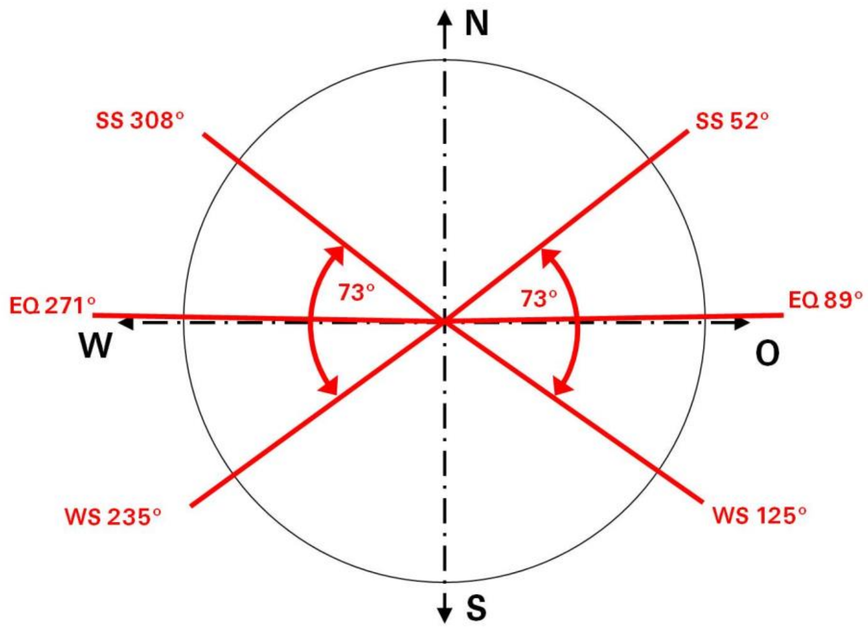
In contrast to the solstities, azimuth changes are most clearly visible during equinoxes. The daily declination difference of the sun is here approx. 24', which for 47°N corresponds to an azimuth difference of approx. 35' – or more than an apparent diameter of the solar disk! This point in time is therefore also suitable for generating shadow images, which should become visible exclusively on these dates.

Probably the most spectacular example worldwide is the pyramid of the god of the "Feathered Serpent Kukulcán" in Chichén Itzá, Mexico. On all four flanks stairs with 365 steps each lead to the temple, which is enthroned on top of the building. Exclusively around the dates of 21 March and 22 September, the nine-stepped edge of the pyramid casts a wavy shadow on one of the side walls of the northern staircase, which is the only one to end at the base of the pyramid in two stone snake heads. This creates the astonishing impression that the snake Kukulcán leaves its temple and winds its way down the pyramid. Picture: Wikipedia

Another example from Switzerland, discovered 2014 by Georg Brunner (†), is the Dolmen-like "Erdmanndlistein" near Wohlen [13] [14] [15]. Around these dates, in each case before sunset, approx. 16:30 to 17:00 o'clock, the sunlight penetrates between the capstone and the two carrier stones and projects a picture of wolf- or fox head on a laterally standing rock plate (white arrow). Interestingly, this impression is supported by the (modified?) surface structure of the rock.

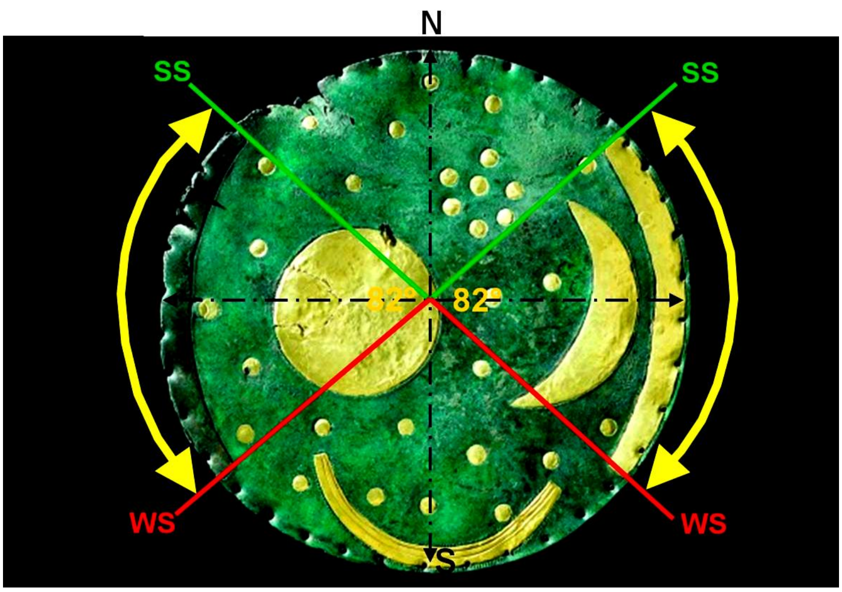
## 4.6 The Pendulum Angle

The following graphic shows the main solar axes with the rounded azimuths of rising and setting. The azimuth arc between SS and WS can also be called the *pendulum angle*. Between these extreme points the sun's rising and setting points oscillate within the course of a year. At 47° degrees of latitude and in 2000 B.C. the pendulum angle of the horizon arc was approx. 73°.



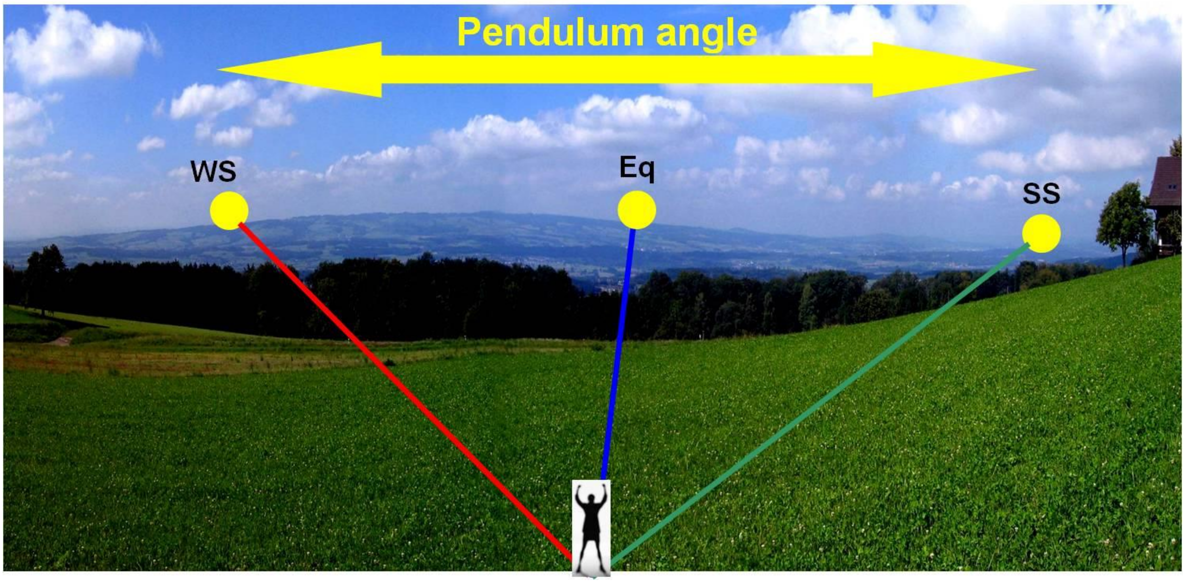

## 4.7 The Main Solar Azimuths and the Nebra Sky Disk

The pendulum angles or azimuth sectors, which define the area of sunrises in the east and sunsets in the west, are most likely also displayed on the "Nebra Sky Disk". The value of 82° roughly corresponds to the pendulum angle observable at that time on the latitude of northern Germany. The gold fitting of the western pendulum arch is missing on the disc. The circular star cluster with seven gold points in the upper right quadrant is today interpreted by many experts as the Pleiades (M45).



## 4.8 Pendulum Angle of Sunsets – Practical Example

The following figure shows the example of a pendulum angle of the sunsets over the western horizon.



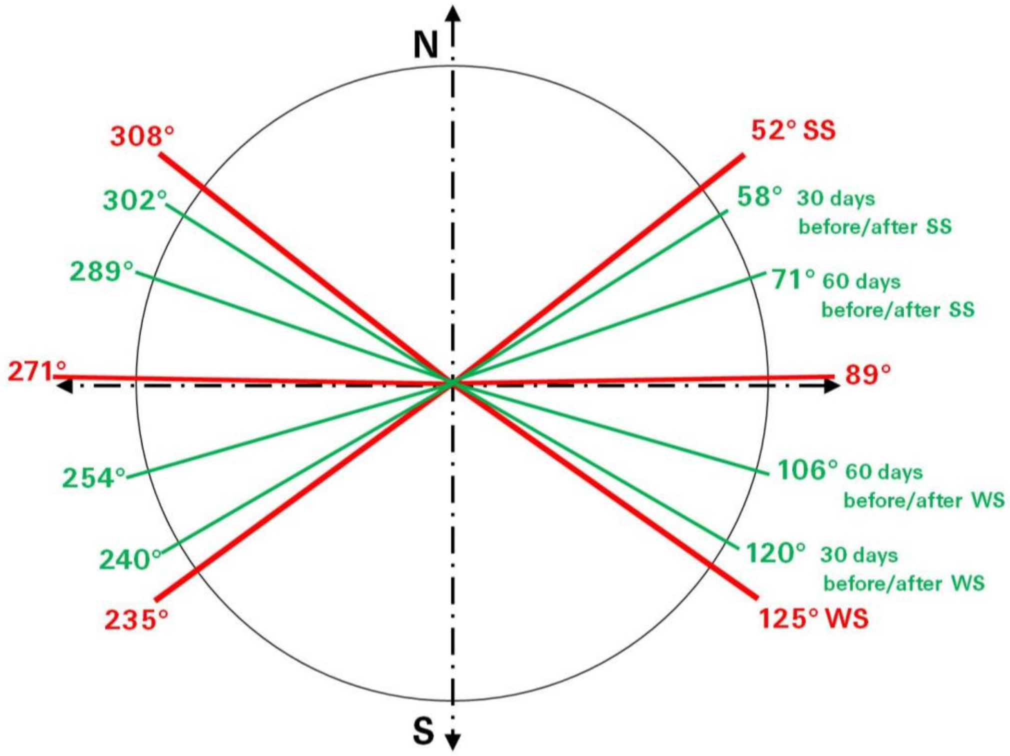

## 4.9 Solar Intermediate Azimuths

Ulrich and Greti Büchi [11] measured azimuths of megalithic rock formations in the Surselva (Switzerland), in France and in southern England. These azimuths are directed at the horizon points of the sun about 30 and 60 days before and after the winter and summer solstices. The author also noticed such orientations at various objects in the vicinity of Zurich [36]. Thus symmetrically to the two solstices it results a rough subdivision of the year into 12 sections, which can also be interpreted as a calendar. The 30-day interval was also used by the advanced Egyptian civilizations and is attributed by most sources to the originally used lunar calendar (chapt. 7), [5].

Due to the elliptical orbit of the earth, the intermediate azimuths are slightly asymmetrical between the winter and summer half-years. The declination values are rounded to arc minutes and the calculated azimuths are given in decimal degrees (JPL Horizons).

| Solar Intermediate Azimuths for 47° N | 2000 B.C. | | |
|---|---|---|---|
| Year divided in 12 sections | Dec. δ | Rising | Setting |
| Ca. 30 days before and after Winter Solstice | −20° 33′ | 119.9° | 240.1° |
| Ca. 60 days before and after Winter Solstice | −11° 37′ | 106.2° | 253.8° |
| Ca. 60 days before and after Summer Solstice | +12° 22′ | 70.7° | 289.3° |
| Ca. 30 days before and after Summer Solstice | +20° 40′ | 57.8° | 302.2° |

The following graphic shows the rounded solar main directions (red), supplemented with the intermediate azimuths (green):




### 4.10 Solar Azimuths of the Quarter Days or the "Celtic Calendar"

The Quarter Days mark the time mids between the solstices and the equinoxes. This can be easily determined by counting days according to the current annual calendar and corresponds approximately to the dates: 4 February, 6 May, 6 August and 6 November (approx. ± 1 day).

The declination of the sun corresponds on these days relatively exactly to the values which W. Schlosser designates in [19] as *±0.71ε declinations* or *"Beltaine azimuths"*. Thus he defines the solar declination δ of these holidays with 71% of the ecliptic obliquity ε, which may be equated for our purposes in good approximation with the maximum solar declination δ at the solstices (chapt. 13.5). This yields for 2000 B.C and 2000 A.C.

$$\delta_{-2000} = \pm 17.01° \qquad \delta_{+2000} = \pm 16.64°$$

Based on these declination values, the following refraction- and parallax-adjusted azimuths can be calculated for 47° N and the upper limb of the sun (Chapt. 15):

| Quarter days 47° N | 2000 B.C. | | 2000 A.C. | |
|---|---|---|---|---|
| | Rising | Setting | Rising | Setting |
| ~ 6 May and 6 August | 63.6° | 296.4° | 64.2° | 295.8° |
| ~ 6 Nov. and 4 February | 114.4° | 245.6° | 113.8° | 246.2° |

Several alignments of the megalithic site in Falera seem to have been aligned approximately to these data [11]. Based on C14 charcoal analysis, this site was dated to the Middle Bronze Age around 1500 - 1260 B.C. which suggests that such orientations had been important long before the Celtic period. Around these four relatively precisely defined dates, several holidays are grouped, which possibly have been passed on from prehistoric times to the cultures of later epochs.

### 4.11 The "Celtic" Holidays

The so-called four "Celtic" holidays *Imbolg, Beltaine, Lugnasad* and *Samhain* appear rather diffusely and scientifically rather weakly founded. The spelling, but partly also the etymological interpretation, differs according to source and language area. The names are usually associated with Celtic gods and festivals. The Walpurgis Night of 30 April, which may have been passed down from prehistoric times to medieval traditions, also appears in this context. These dates today mainly play a role for esoteric circles, especially from the Neopaganism scene. Popular sources such as the "Bauernkalender" (peasant's calendar) date them to the beginning of today's months.

*Imbolg:* 1 February, *Beltaine:* 1 May, *Lugnasad:* 1 August, *Samhain:* 1 November.

### 4.12 The Peasant Winter

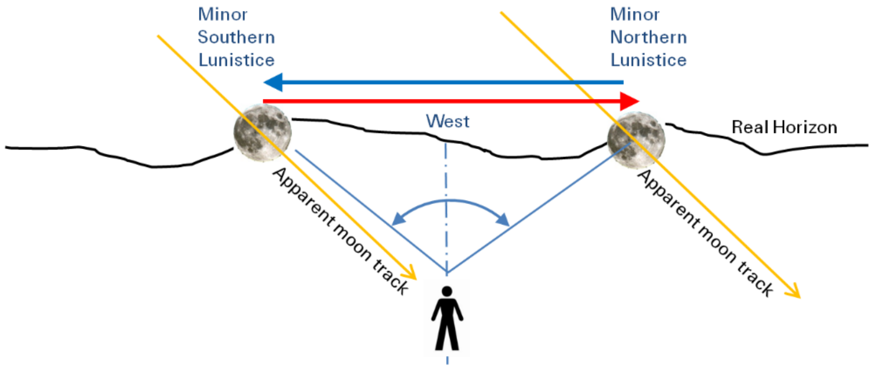
The beginning and end of the so-called peasant winter, which meant an interruption in the peasant year, seem to be relatively well accepted. With a difference of 42 days each, these dates are approximately symmetrical to the winter solstice and were later passed on to Christian traditions. Since the early Middle Ages, the peasant year begins with Candlemass on 2 February and ends with Martini on 11 November. On these days, lease and interest periods as well as salary periods began and ended. The other two quarter days in the summer half-year are also often assumed to be of agricultural origin. Here the deviation of these horizon azimuths to that of the quarter days is $<1°$.

Imbolg on 1 February falls on the eve of Candlemass and is therefore mostly interpreted as the pagan forerunner of the Christian feast and thus also of the end of the peasant winter. Samhain could be connected with the beginning of the peasant winter, i.e. Martini on 11 November. The interpretation that this festival was also a pagan forerunner of the memorial days All Saints and All Souls or even the Anglo-Saxon Halloween, is under debate.

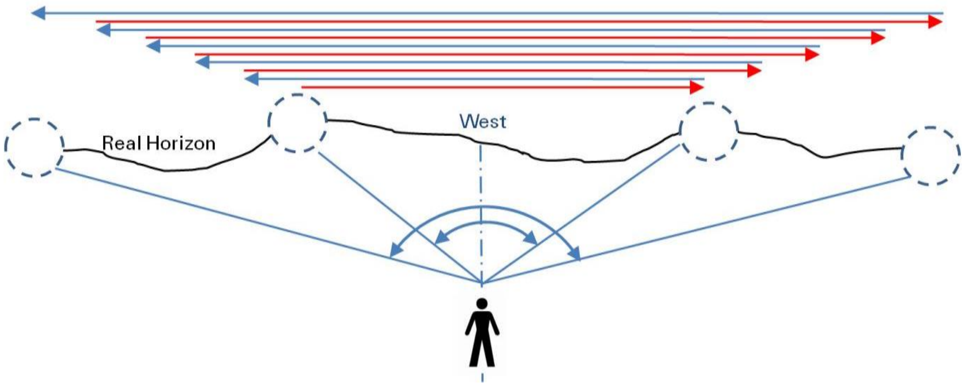
# 5 Archaeoastronomical Lunar Azimuths

## 5.1 The Lunistices

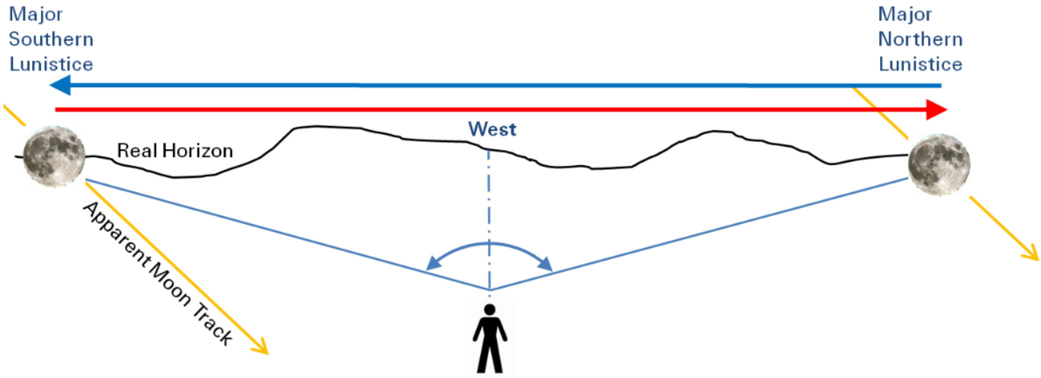
The apparent moon orbit is much more complicated compared to the solar orbit. The sun's rising and setting points are well-known to perform a simple, complete "pendulum motion" (South – North – South) once a year between a southern extreme value, the winter solstice, and a northern extreme point, the summer solstice (Chapt. 4.3). The moon performs a comparable, complete pendulum deflection at each sidereal moon orbit within about 27.32 days. Sidereal means considered relatively to the *extremely distant star background* and this motion takes place asynchronously to the lunar phase. During the *"Major Lunistice"* or the *"Major Standstill"* this pendulum deflection is *maximal*. In the case of the *"Minor Lunistice"* or the *"Minor Lunar Standstill"*, this apparent "pendulum deflection" is *minimal,* see following sketch.




After this minimum pendulum deflection has been completed, the amplitudes become larger and larger during the next rounds (illustration for the sunsets on the western horizon)




After approx. 9.3 years or about 125 orbits, the lunar azimuths reach the maximum values of the "Major Lunistice" or the "Major Lunar Standstill".



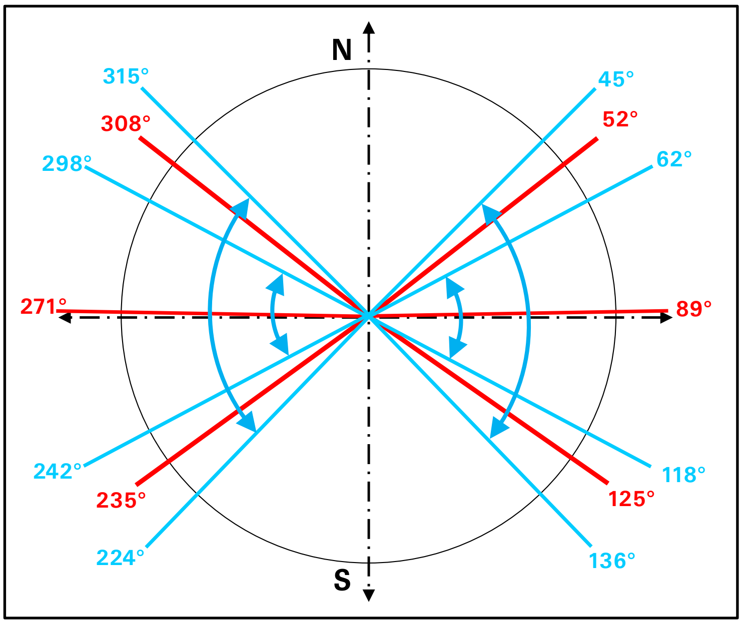
In the following 9.3 years the deflections become continuously smaller again, until after about 18.61 years finally the *Minor Lunistice* is reached again. This entire time span corresponds exactly to a full revolution of the lunar nodal line (see chapter 14.3).

## 5.2 Lunistice Azimuths for 47°N

The following declination values δ are rounded to arc minutes, the calculated azimuths of the Major and Minor Lunistices are given in decimal degrees. The azimuth differences between 2000 B.C. and A.C are also small for the moon-related azimuths and arise for the same reasons as explained in chapter 4.2. For instance a Minor Lunistice was observed in 1997, and a Major Lunistice in 2006.

| Horizon azimuths of the Lunistices for 47° N | Major Lunistice 2013 B.C.<br>Minor Lunistice 2004 B.C. | | | Major Lunistice 2006 A.C.<br>Minor Lunistice 1997 A.C. | | |
|---|---|---|---|---|---|---|
| | δ | Rising | Setting | δ | Rising | Setting |
| Major Northern Lunistice | +29° 13′ | 44.5° | 315.5° | +28° 43′ | 45.4° | 314.6° |
| Minor Northern Lunistice | +18° 39′ | 62.2° | 297.8° | +18° 08′ | 63.0° | 297.0° |
| Minor Southern Lunistice | −18° 39′ | 118.1° | 241.9° | −18° 09′ | 117.3° | 242.7° |
| Major Southern Lunistice | −29° 13′ | 135.9° | 224.1° | −28° 43′ | 135.0° | 225.0° |

Important: Compared to the solar azimuths, an additional uncertainty of the horizon azimuth of about ±1.5° must be taken into account when analyzing lunar alignments. Due to inclination oscillations of the strongly eccentric orbital ellipse (see chapter 14) and other orbital perturbations not considered here, larger maximum declination values may occur than those listed in the table above. Example: The declination during the 2006 Major Southern Lunistice on September 29, yielded 29° 37′! For more precise clarifications, the data from JPL Horizons (chapter 2.3) can be consulted. Anyway, as a dilemma, the decision for the date of a (pre-) historical lunar standstill would be required. For most archaeoastronomical investigations, however, the accuracy of the values above can be considered as sufficient. The following graph shows the rounded azimuths of the Lunistices (blue) for 2000 B.C. and 47° N, supplemented with the main solar azimuths (red). Also shown are the two pendulum arcs of the Major and Minor Lunistices.



## 5.3 The Phenomenon of the Lunar "Standstill Phase"

The Lunistices are exactly defined by celestial mechanics (chapt. 14.3). The following example of the Major Lunistice 2006 is simulated by NASA's JPL Horizons and in 1h-intervals. It clearly shows that the exact time of the lunistice cannot be determined just by the horizon azimuth. During these periods, the declination maxima of the individual lunar orbits approach the maximum values for more than a year by weak oscillation amplitudes of only a few arc minutes. Two secondary maxima usually occur, as shown here by the values of 4.4. (4th April) and 15.9. 2006 (15th September). In this context R. Müller [2] even mentions 2−3 "oscillations". Such effects arise due to the high complexity of the moon's orbit, especially due to the strong eccentricity of its orbital ellipse. In the literature this period of time is therefore also referred as the "lunar standstill" [2], analogous to the solstices. The following table shows the maximum northern (+) declination values for each lunar orbit and the corresponding rising azimuth for 47° N around the Major Northern Lunistice of 2006. The following data format is [date / declination / rising azimuth]. The two lines with the maximum +δ values and thus generating the northernmost horizon azimuths are written in red.

| | | |
|---|---|---|
| 12.1. / +28° 25′ / *46.0°* | 1.5. / +28° 37′ / *45.6°* | 18.8. +28° 39′ / *45.5°* |
| 8.2. / +28° 33′ / *45.7°* | 29.5. / +28° 29′ / *45.8°* | **15.9. +28° 43′ / *45.4°*** |
| 8.3. / +28° 42′ / *45.4°* | 25.6. / +28° 27′ / *45.9°* | 12.10. +28° 41′ / *45.4°* |
| **4.4. / +28° 43′ / *45.4°*** | 22.7. / +28° 31′ / *45.7°* | 8.11. +28° 32′ / *45.7°* |

The enormous duration of these lunar standstills had the advantage that the extreme horizon levels of the "Lunistice" could be observed during several cycles and the visibility of this event was less susceptible to meteorological caprices.

For earth dwellers, the moon is the second brightest celestial body after the sun, enables even the reading of newspapers during full moon and is sometimes also visible in the day sky. Its gravitational effect creates directly visible effects on the earth's surface, e.g. the ocean tides. Its constantly changing phases are also spectacular. So it seems understandable that also the extreme horizon azimuths of the moon are discussed for archaeoastronomical alignments. In the sighted literature relatively few of the world-famous archaeological objects with alignments to the moon extremes can be found. An exception is Stonehenge with a rather controversial interpretation of moon azimuth by the American astronomer G.S. Hawkins [2]. The following objects, with the exception of Yverdon Clendy (CH), represent just the Major Lunistice and then mostly the southern ones. The Minor Lunistices seem to have played a rather subordinated role.

## 5.4 Examples for Possible Alignments to the Lunistices

Callanish on the Isle of Lewis (Hebrides) is often referred to as the "Stonehenge of the North". Here a double row of menhirs is directed towards a depression within the horizon, which favoures the observation of the "standstill phase" around the Major Southern Lunistices [2]. The arrangement of the whole complex is cross-shaped.

Picture: *Callanish* (Wikipedia)

In connection with Lunistices W. Schlosser [1] mentions the megalithic complex on the Isle of Mull (GB) as well as the concentric circular ditch complex of Bochum-Harpen. Further B. Steinrücken [21] has detected such orientations in 42 of 50 investigated stone circles in Scotland, which often show a so called "Recumbent Stone" on the southwestern sector of the circle. Here the moon setted during the Major Southern Lunistices.

Picture: Loanhead of Daviot Stonecircle with lying altar stone (Wikipedia).

Within the scope of an expertise for the museum in Yverdons-les-Bains, the author has found a possible orientation of the famous Alignments of Yverdon-les-Bains Clendy towards the Major and Minor Southern Lunistices [35].

Picture: Northern Alignment in the megalithic site of Yverdons-les-Bains - Clendy

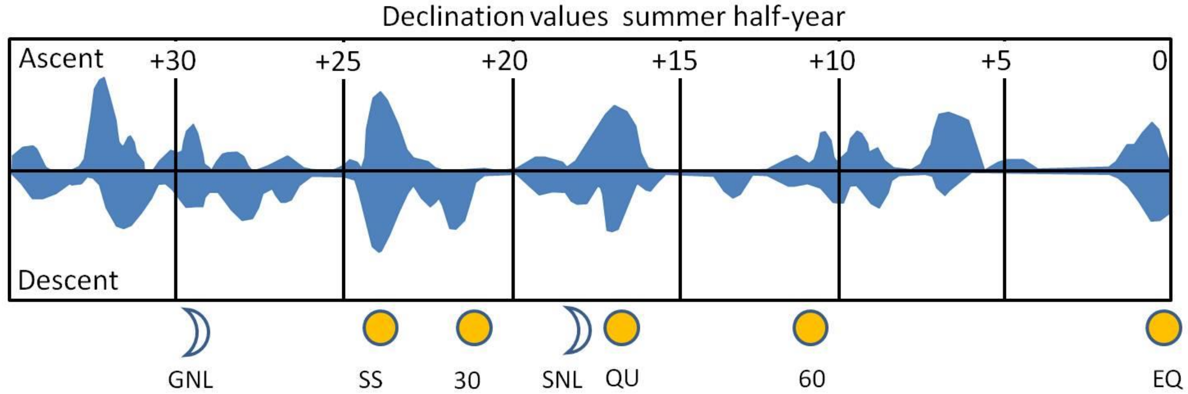
## 5.5 Lunistices – Cultic- or Calendar Function?

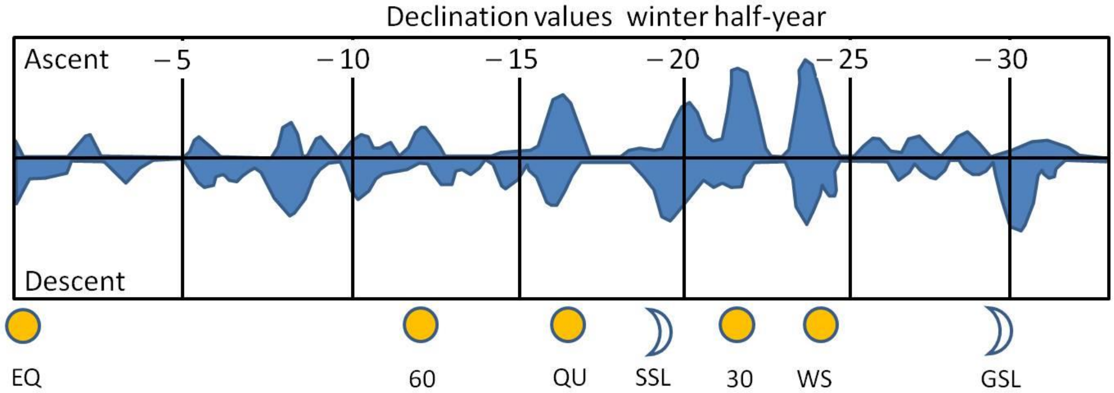
The phases of the moon form the striking indicator for the lunar calendar (Section 7.2) and are linked to the *synodal* orbits with an average duration of 29.53 days. However, the pendulum movements of the lunar horizon azimuths according to chapt. 5.1, are synchronized with the significantly shorter *sidereal* lunar orbits (approx. 27.32 days) and appear therefore completely independent from the lunar phases. Theoretically these periods could also serve as calendar cycles, as well as the very long time spans between the Lunistices. However, the latter are even more difficult to determine than those between the solstices, since the lunar standstill phases do not last just several days, but in each case approx. a whole year. Thus, a cultic meaning appears to be much more probable here, compared to a possible calendar function.

# 6 Distribution of Solar and Lunar Azimuths after A. Thom

The Scottish engineer Alexander Thom (1894 - 1985) statistically investigated the distribution of azimuths within some 300 megalithic objects. The area of investigation encompassed an area from the north of Scotland to Wales. The corresponding distribution of the declination values is shown in the following graph, schematically adapted after A. Thom and supplemented with the Major and Minor Lunistices as well as the solar azimuths, dividing the year into twelve sections. Here it becomes clear that many of the relevant sun and moon declinations are more or less clearly represented. Otherwise there are also concentrations recognizable, which e.g. at approx. 7° and 32°, are clearly outside of the range to generate the known solar or lunar horizon azimuths.

As an alternative, A. Thom has further proposed a dividing of the year into sixteen sections, with "month lengths" of approx. 22 - 23 days.




Caption:

Sun Moon

| | | | |
|---|---|---|---|
| GNL: | (Great) Major Northern Lunistice | WS: | Winter Solstice |
| SNL: | (Small) Minor Northern Lunistice | EQ: | Equinox |
| GSL: | (Great) Major Southern Lunistice | QU: | Quarter days |
| SSL: | (Small) Minor Southern Lunistice | 60: | 60 days before/after SS or WS |
| SS: | Summer Solstice | 30: | 30 days before/after SS or WS |

# 7 Archaeoastronomical Fixed Star Azimuths

In contrast to the apparent orbit of the sun and moon, the appearance of the fixed star sky is dramatically influenced over the millennia by the precessional motion of the earth's axis and, to a lesser extent, by the peculiar movements of the individual stars. Therefore the testing of the hypothesis whether an object was aligned to a stellar horizon azimuth, requires an independent age determination, to calculate the equatorial coordinates of that epoch and the dependent ascend and descend azimuths. Today, this task is accomplished best with appropriate computer programs. In cases where an independent age determination of a facility is impossible, such considerations remain inevitably highly speculative.

A practical application here is the search for fixed stars, which could come into question for a given prehistoric object orientation. First the declination of the hypothetical "target star" is calculated with the given azimuth of rising or setting. For this purpose equation {14} in chapt. 15.2 or with [33] can be applied. With a planetarium program like e.g. Stellarium, it is now possible to search for bright stars, which are suitable for the presumed or archaeologically determined epoch. Very useful here is also a table of B. Steinrücken [18] about the prehistoric equatorial coordinates of the brightest fixed stars visible at that time from 5000 to 2500 B.C. Tables and diagrams for determining prehistoric fixed star coordinates can also be found in Schlosser/Cierny [1].

# 8 Archaeoastronomical Planetary Azimuths

## 8.1 Preliminary Remarks

It is highly controversial which role played in archaeoastronomy the extreme rising and setting points of the planets. Like the fixed star azimuths, these are also subject to the precession effect – albeit to a much lesser extent. However, all planets circle on elliptical orbits of various eccentricities around the sun, which are just weakly but different different angles inclined to the ecliptic. Thus the extreme values of their rising and setting azimuths show short as well as long-periodic fluctuations which further depend on the respective epoch. These complex correlations also explain why precise prehistoric planetary positions must be determined with computer-aided simulation runs and can hardly be calculated explicitly with simple equation sets.

## 8.2 Archaeoastronomical Venus Azimuths

The archaeoastronomical interest here is concentrated almost exclusively on the hereinafter called "Venus Extremes". Well known example for such orientations is the ancient observatory El Caracol in the Mayan city of Chichén Itzá in Mexico [20].

Amateur researchers generally tend quite quickly to bring Venus extremes into play when suggesting interpretations, mostly without any awareness of the complex form in which this phenomenon really occurs! It seems therefore justified to have a closer look at this difficult topic. Venus actually occupies a special position among the planets for the following reasons, which may explain the fascination of earlier civilisations.

1. After the sun and moon, it appears in certain phases as the third brightest celestial body. It is the only planet that can theoretically cast a visible shadow at an extremely dark location!
2. By instructed persons and during its brightest phases, it can also be found in the day sky.
3. Seen from Earth, Venus never seems to be far away from the Sun, as it circles around on an inner orbit. Thus it appears either as a "morning" or "evening" star. The same also applies to Mercury. Due to its narrow solar orbit, however, the latter usually remains

hidden in the dusk or dawn and is only inconspicuously visible during the phases of greatest elongation (angular distance to the sun).

4. The orbital ellipse of Venus is inclined about 3° 23' to the ecliptic and has by far the lowest eccentricity of all planetary orbits. It therefore runs almost on a circular orbit around the sun [6].

5. Due to a celestial mechanical coincidence, Venus, seen from Earth and related to any date, reaches eight years later almost exactly the same position! This can be easily reproduced with any planetarium program. By chance, the time span for five synodic Venus orbits is just ca. 2.32 days shorter than eight tropical Earth years [4]. To say it somewhat casually, Venus overtakes the Earth five times on its "inner orbit" within almost eight years. Thus also the horizon azimuths of Venus, near the southern and northern extreme positions, can be observed just about every eight years.

## 8.3 Basic Cycle of Venus Extremes

This approx. 8-year basic cycle is demonstrated here by the annual northern declination maxima +δ max between 1995 and 2007 and the directly dependent azimuths of rising and setting (chapt. 15). The apparent δ values are rounded to arc minutes the calculated azimuths are given in decimal degrees. The rows with maximum declination values are written in red, those with minimum values in blue (based on NASA JPL Horizon's 1h interval simulation).

| Date | δ max | Rising | Setting |
|---|---|---|---|
| 8.7.1995 | +23° 23′ | 53.65° | 306.35° |
| **5. 5.1996** | **+27° 47′** | **46.03°** | **313.97°** |
| 7.6.1997 | +24° 23′ | 51.97° | 308.03° |
| 23.7.1998 | +22° 48′ | 54.62° | 305.38° |
| 11.5.1999 | +25° 57′ | 49.28° | 310.72° |
| 22.6.2000 | +23° 54′ | 52.78° | 307.22° |
| **6.8.2001** | **+21° 54′** | **56.10°** | **303.90°** |
| 23.5.2002 | +25° 01′ | 50.89° | 309.11° |
| 8.7.2003 | +23° 24′ | 53.62° | 306.38° |
| **5.5.2004** | **+27° 49′** | **45.97°** | **314.03°** |
| 6.6.2005 | +24° 24′ | 51.94° | 308.06° |
| 23.7.2006 | +22° 49′ | 54.59° | 305.41° |
| 10.5.2007 | +26° 00′ | 49.19° | 310.81° |

Comment:

1. Within this short time frame the annual northern δ max values vary considerably between +21° 54' and +27° 49'! The range of the directly dependent horizon azimuths is correspondingly large.
2. All declination values, and thus also the horizon azimuths, show nearly the same values after about eight years.
3. Thus "Venus extremes" also repeat themselves in an eight-year cycle (here 1996 and 2004).

### 8.4 Long-period Cycle of Venus Extremes

For archaeoastronomical purposes, the evolution of these cycles must be followed over very long periods of time. Therefore the period of ±3000 years was searched for the most extreme, apparent Venus declinations, which refer to the equinox of the date (chapter 12.3). As expected, due to the elliptical and oblique Venus orbit the periodic eight-year extremes, proved to be as not constant. The following table shows, reduced to a few representative data points, such a period, which lasts here 251 years, i.e. from 2012 to 2263 A.D. It becomes obvious, how the northern +δ max values of the base cycles change within this period and thus one must also abandon the idea of fixed permanent Venus Extremes with regard to long-term time frames! They decrease by about 1.4° until 2092, which is almost three times the apparent diameter of the solar disk! Afterwards they rise again, up to the next maximum 2263. The apparent δ values are rounded to arc minutes and the calculated azimuths are given in decimal degrees (basis: NASA, JPL Horizons, simulation in 1h intervals). The rows at the beginning and end of this period are marked in red, the minimum value between in blue.

| Date | δ max | Rising | Setting |
|---|---|---|---|
| 5.5.2004 | +27° 47' | 46.0° | 314.0° |
| **5.5.2012** | **+27° 49'** | **46.0°** | **314.0°** |
| 28.4.2060 | +27° 22' | 46.8° | 313.2° |
| 24.4.2076 | +26° 58' | 47.5° | 312.5° |
| **21.4.2092** | **+26° 26'** | **48.4°** | **311.6°** |
| 7.5.2103 | +26° 36' | 48.2° | 311.8° |
| 7.5.2127 | +26° 47' | 47.8° | 312.2° |
| 7.5.2151 | +27° 00' | 47.4° | 312.6° |
| 6.5.2191 | +27° 23' | 46.8° | 313.2° |
| 8.5.2223 | +27° 42' | 46.2° | 313.8° |
| 7.5.2247 | +27° 53' | 45.9° | 314.1 |
| 6.5.2255 | +27° 55' | 45.8° | 314.2° |
| **6.5.2263** | **+27°57'** | **45.7°** | **314.3°** |
| 5.5.2271 | +27° 57' | 45.7° | 314.3° |
| 4.5.2279 | +27° 55' | 45.8° | 314.2° |

Here it becomes apparent that the eight-year basic cycles dissolve after several decades and slowly change into new ones. This must be taken into account when searching for maximum or minimum values in simulation runs! The two maximum declination values of 2012 and 2263 A.D. already show here the trend that with decreasing date the maximum declination values become smaller on a long-term basis (difference here −8').

## 8.5 The long-periodic Venus Cycles from 3000 B.C. to 3000 A.D.

The following tables finally show the long-period, northern and southern maximum values of Venus declinations over the period of ±3000 years and related to the equinox of the date. From this, the rising and setting azimuths of the Venus Extremes were calculated for 47° N. The apparent δ values are rounded to arc minutes and the azimuths are given in decimal degrees. The dates before 1582 show the change from the Julian to the Gregorian calendar. They are indicated here according to historical convention, which counts the year 0. In a comparison with NASA, JPL Horizons data it must therefore be considered that ephemeris programs calculate without the year 0 and thus according to the "B.C./A.D. system". This results in a difference of one year for the negative "pre-Christian" years given here. Examples: Year 0 ≙ 1B.C. Year −1 ≙ 2 B.C. Year 1 ≙ 1 A.D.

| Northern Venus Extremes | | | |
|---|---|---|---|
| Date | δ max | Rising | Setting |
| 6.5.2773 | +28° 12′ | 45.3° | 314.7° |
| 6.5.2522 | +28° 04′ | 45.5° | 314.5° |
| 6.5.2263 | +27° 57′ | 45.7° | 314.3° |
| **5.5.2012** | **+27° 49′** | **46.0°** | **314.0°** |
| 4.5.1761 | +27° 42′ | 46.2° | 313.8° |
| 25.4.1502 | +27° 35′ | 46.4° | 313.6° |
| 26.4.1251 | +27° 28′ | 46.6° | 313.4° |
| 27.4.1000 | +27° 22′ | 46.8° | 313.2° |
| 29.4.741 | +27° 15′ | 47.0° | 313.0° |
| 1.5. 482 | +27° 09′ | 47.2° | 312.8° |
| 3.5.223 | +27° 02′ | 47.4° | 312.6° |
| 4.5.−28 | +26° 56′ | 47.6° | 312.4° |
| 5.5.−287 | +26° 50′ | 47.7° | 312.3° |
| 7.5.−546 | +26° 45′ | 47.9° | 312.1° |
| 9.5.−805 | +26° 39′ | 48.1° | 311.9° |
| 10.5.−1064 | +26° 34′ | 48.2° | 311.8° |
| 12.5.−1323 | +26° 29′ | 48.4° | 311.6° |
| 14.5.−1582 | +26° 24′ | 48.5° | 311.5° |
| 17.5.−1849 | +26° 20′ | 48.6° | 311.4° |
| 18.5.−2108 | +26° 15′ | 48.8° | 311.2° |
| 20.5.−2367 | +26° 11′ | 48.9° | 311.1° |
| 26.5.−2650 | +26° 07′ | 49.0° | 311.0° |
| 26.5.−2909 | +26° 03′ | 49.1° | 310.9° |

| Southern Venus Extremes | | | |
|---|---|---|---|
| Date | δ max | Rising | Setting |
| 8.11.2878 | −28° 31′ | 133.6° | 226.4° |
| 9.11.2627 | −28° 25′ | 133.4° | 226.6° |
| 7.11.2376 | −28° 18′ | 133.2° | 226.8° |
| 6.11.2125 | −28° 11′ | 133.0° | 227.0° |
| 6.11.1874 | −28° 05′ | 132.8° | 227.2° |
| 5.11.1623 | −27° 58′ | 132.6° | 227.4° |
| 26.10.1372 | −27° 52′ | 132.4° | 227.6° |
| 28.10.1121 | −27° 45′ | 132.2° | 227.8° |
| 29.10.870 | −27° 38′ | 132.0° | 228.0° |
| 30.10.619 | −27° 31′ | 131.8° | 228.2° |
| 31.10.368 | −27° 25′ | 131.6° | 222.4° |
| 1.11.109 | −27° 18′ | 131.4° | 228.6° |
| 3.11.−142 | −27° 12′ | 131.3° | 228.7° |
| 4.11.−393 | −27° 05′ | 131.1° | 228.9° |
| 5.11.−652 | −26° 59′ | 130.9° | 229.1° |
| 7.11.−911 | −26° 53′ | 130.8° | 229.2° |
| 8.11.−1162 | −26° 47′ | 130.6° | 229.4° |
| 10.11.−1421 | −26° 41′ | 130.4° | 229.6° |
| 11.11.−1680 | −26° 35′ | 130.2° | 229.8° |
| 13.11.−1931 | −26° 29′ | 130.0° | 230.0° |
| 15.11.−2198 | −26° 24′ | 129.9° | 230.1° |
| 17.11.−2465 | −26° 19′ | 129.8° | 230.2° |
| 19.11.−2724 | −26° 14′ | 129.6° | 230.4° |
| 21.11.−2983 | −26° 09′ | 129.5° | 230.5° |

Comment:

1. The maximum declination values of the long-period Venus cycles decrease steadily in the direction of the past over the entire period examined of ± 3000 years, whereas this tendency flattens somewhat towards the year −3000.

2. The azimuth difference over the entire time span is >3°. This forbids applying horizon azimuth of current Venus Extremes to prehistoric periods! A corresponding interpretation therefore requires at least a credible hypothesis or an archaeological reference for the age of the site.

3. The apparent δ values are strongly influenced by long period oscillations with a length of 243, 251 or 259 years.

The 243-year period, based on the series of Venus Transits, which has already been suggested for the search for such values [20], is not confirmed here by the *apparent* JPL coordinates. However, if these maximum values are searched for *astrometric* coordinates, reduced to J2000.0, in fact a period length of 235 or 243 years is shown! However as explained in chapter 12.3, coordinates "reduced" to a fixed equinox of the present are normally useless for archaeoastronomy.

Interesting in this context is the list of Venus Transits by Fred Espenak from the NASA Goddard Space Flight Center. By colleagues Fred is also jokingly called "Mr. Eclipse" [30], [31]. The positive declination values of all Venus transits show an even stronger decrease in the direction of the past than the maximum +δmax values. They decrease relatively steadily from δ = +23.21° in the year 2984 A.D. to δ = +16.16° in the year 1892 B.C. These values are consistent with the apparent NASA, JPL horizons data, which also correctly reflect the Transits of Venus.

## 8.6 Extreme Horizon Azimuths and Maximum Elongation

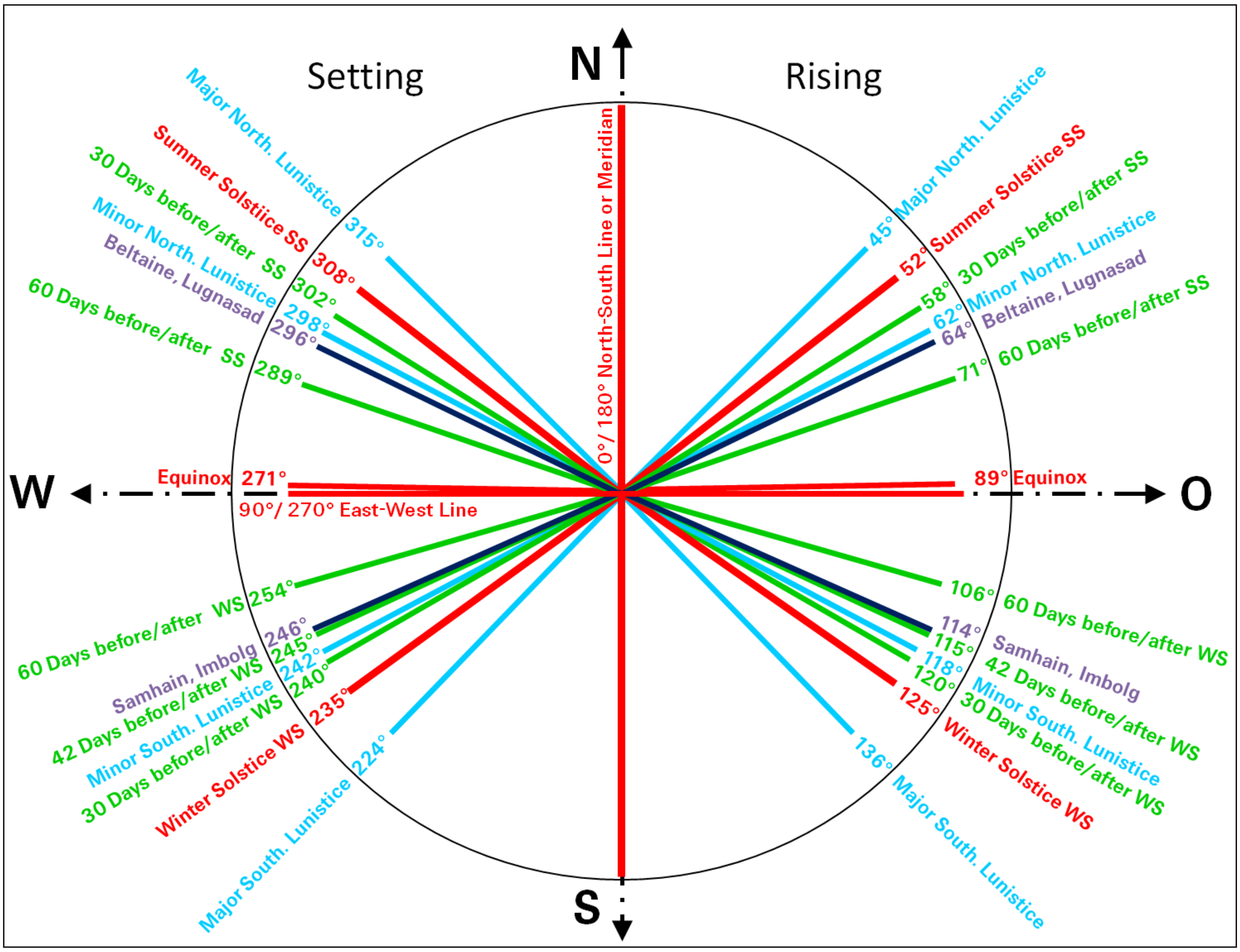
In publications not listed here, amateurs have already equated erroneously the extreme rising and setting azimuths of Venus with their largest apparent angular distance to the sun of about 48° each (maximum eastern and western elongation)! Apart from that, Venus positions in maximum elongation hardly ever coincide with their largest declination values. Example: On 5.5.2012 Venus reached the largest northern declination of this cycle and thus for several centuries its relatively most extreme northern horizon azimuths. Their eastern elongation was then just about 37°!

## 8.7 Concluding Summary to Venus Extremes and Archaeoastronomy

For an architectural line, in addition to its orientation, further first-class arguments must be available in order to be allowed to seriously propose these very complex Venus Extremes. For example, corresponding archaeological finds should be available or the culture involved should have had a proven connection to Venus (example Maya, Chichén Itzá). In addition, the architectural line in question must be sharply defined and not merely diffusely marked. Furthermore, at least one plausible hypothesis is needed for the approximate age of the complex. To associate any row of stones with Venus extremes, without any further indications, must not only be denoted as highly speculative, but simply as non-serious.

# 9 Summarized Presentation of the Azimuths for 47° N

The following chart summarizes all azimuth categories that have been presented so far, based on 2000 B.C. and rounded to 1°. Such a display has advantages and disadvantages. On the one hand, the enormous concentration of potentially relevant azimuths can be demonstrated and effectively warned against the risk of misinterpretation or overinterpretation. Thus it also becomes obvious which azimuths are closely adjacent and thus susceptible to confusion and misinterpretation. On the other hand, there is the danger that this "template" is now placed on any object layouts and then a crazy mix of interpretations is uncritically brewed from numerous azimuth categories. The problem is increased by the fact that the relevant azimuths are not evenly distributed, but appear to be bundled symmetrically to the north-south axis within four narrow sectors. Applying correspondingly generous tolerances, almost any direction could be archaeoastronomically "interpreted" here. A way out of this dilemma seems to be an object-specific weighting of the individual azimuth categories, i.e. in many cases a clear prioritization of the main solar azimuths and the main points of the compass. The following chart further shows that orientations in the northern sector between 45° and 315°, and in the southern sector between 136° and 224°, can just affect rising and setting points of stellar objects or have no astronomical relevance. An exception is the north-south line, marking the meridian.



Such considerations usually trigger the question about the probability with which certain azimuths could bee intentionally directed on astronomical targets. In general, the probability increases with:

1. the number of marked, archaeoastronomically relevant azimuths
2. the accuracy of the alignment
3. the clarity the direction is marked
4. a smallest possible number of involved azimuth categories (e.g. only solar main- or cardinal azimuths)

# 10 Excursus to the Calendar Types

After the introduction of the solar and lunar azimuths follows a short and in no way complete outline of the different calendar types. For further information refer to the relevant specialised literature.

## 10.1 Solar Calendar

The time interval for the group of solar calendars is defined by the Earth's orbit around the Sun, which takes place within a so-called solar year and, as a decisive advantage, results in synchronization with the seasons. The determination of the important key dates, e.g. equinoxes and solstices, requires at least staked out lines of sight for the bearing of the corresponding solar rising and setting azimuths. The determination of the intermediate time intervals, however, also requires the counting of days and thus arithmetic capabilities. Therefore, such solar calendars already belong to the advanced group of arithmetic calendars.

## 10.2 Lunar Calendar

The Lunar calendar, which is generally accepted as the origin today, belongs to the simplest category of astronomical calendars. Their cycles are based purely on observation and do not require any auxiliary constructions or calculations. With the lunar calendar just the observation of the lunar phases is necessary, i.e. the synodic moon orbits or lunations. The moon is the only celestial body, which shows a clearly visible, daily change by its phase. With the help of its phase quarters, the individual weeks can easily be determined and, with a little practice, the days can also be estimated on the basis of the phase details, with the exception of the Full Moon, which in terms of time is even more difficult to determine than the New Moon phase.

The period between two New Moon phases is called Lunation and varies between 29,272 and 29,833 days. It is therefore significantly shorter than the average calendar month of today. Therefore pure lunar calendars inevitably run asynchronously to the solar calendars and thus also to the seasons. Typical representative is the Islamic calendar. This determines still today the fasting month Ramadan which shifts every year.

## 10.3 Stellar Calendar

These simple basic forms compete with more complex concepts, operating with leap days or leap months. The Egyptians had already divided the year into 12 x 30 = 360 days. However, this grid was stellar and not solar oriented. The first visible rise of Sirius at dawn, after having remained invisible for a long time, defined the cycle of this calendar and at the same time also the New Year's Day. This event is also called the *Heliacal Rise* and announced the beginning of the Nile Flood. The missing five days to 365 were declared as leap days and called Epagomene. They were not assigned to any month and celebrated as birthdays of the gods [2], [5]. The opposite pole to the Heliacal Rise is the *Acronychic Fall*, i.e. the last visibility of a star at dusk.

The Egyptians had already noticed – among other things by the progressively delayed Nile flood – that this correction was not enough. The reason: The duration of the so-called "Tropical year", defining the cycle of the solar calendar, is with 365,242 days somewhat longer than 365. Ptolemaios III intended to solve this problem. In 237 B.C. he prevailed, however only for a short time, against the resistance of the priesthood and introduced an additional leap day every 4 years. This was then definitively implemented much later by Julius Caesar (Julian calendar 45 B.C.) and then on decree of Emperor Augustus, 26 B.C.

### 10.4 The Gregorian Calendar Reform

The way to today's solar calendar still led via the calendar reform of Pope Gregory VIII in 1582 A.D. At this time the observed spring equinox had already been delayed by 10 calendar days. Pragmatically therefore the 10 days between the 4th and 15th October 1582 were simply skipped and additionally a set of rules with further leap days were introduced. These skipped days must be considered e.g. with the patrocinium research, if the examined church was established before this date. This is relevant, for example, when clarifying whether the longitudinal axis of a church could have been aligned to the sunrise on the patron saint's name day [33].

In the age of atomic clocks, precise synchronization with the tropical year is achieved unspectacularly with additional leap seconds. This requires international agreements but no more papal decrees.

### 10.5 Lunisolar Calendar

The *Lunisolar Calendars* are inevitably complicated, since they "synchronize" the pure *Lunar calendar* with the solar year by inserting leap months [4]. A typical representative. still in use today. is the *Jewish Calendar*, which strives for this alignment with complex switching rules.

### 10.6 Other Forms of Lunar, Solar and Planetary Calendars

M. Kerner [4] postulates further calendar types for the Bronze Age, some of them very complex and somewhat speculative, mostly based on analyses of notch and scratch patterns on archaeological finds (stone axes, notch woods, etc.). As calendar cycles he identifies various astronomical phenomena which occur periodically in the solar system:

1. As intervals for the *Planetary Calendars* the periodically occurring opposition and conjunction patterns, with the participation of the bright planets Venus, Mars, Jupiter and Saturn.
2. As interval with the *Lunar Node Calendars* the rotation time of the lunar nodal line (approx. 18.61 years).
3. With the *Luni Solar Planetary Calendars* the combination of not less than three different "interval generators", i.e. the synodic moon orbits (lunations) and the solar orbits of Earth and Venus. For the "synchronization" of these different clock lengths he postulates long-periodic oscillations [4].

Such complex systems certainly had never practical significance in everyday life and may at best have been of interest for a very small and highly specialised elite.

# 11 Aspects of Surveying Techniques

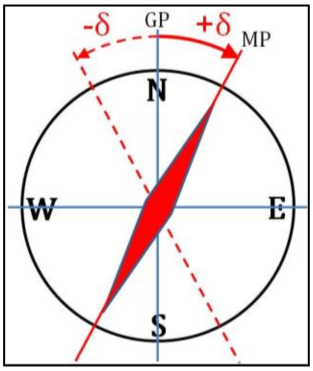
Here some simple methods are presented, which are used for typical surveying tasks in archaeoastronomy.

## 11.1 The Measurement of Azimuths

The word "azimuth" comes from Arabic *"as sumut"*, which means "the ways". Important for archaeoastronomy is the cartographic or nautical azimuth system, which today is also applied worldwide in astronomy, aviation and nautics, meteorology etc. It is also called "north azimuth" and is the clockwise measured angle between geographic north and any direction, starting from North 0°, East 90°, South 180°, West 270°, and back to North 0° (360°).

Today most amateur researchers measure azimuths with special *prismatic bearing compasses*, such as those manufactured by the companies Suunto or Thommen. This is mostly adequate for the measurement of megalithic objects. Before each reading, it must be ensured that the scale disc can oscillate freely in the oil bath by slightly pivoting it around the bearing point. By mounting the compass on a tripod the accuracy can be increased. In addition, the iron measuring rule applies: *One measurement is no measurement!* Several averaged readings are ideal here. If more precise measurements are required, a theodolite or tachymeter must be applied. For such purposes ordinary hiking compasses are generally too inaccurate.

Measurements within settlement areas, but also in the vicinity of power and railway lines, can be massively disturbed by magnetic influences. This is especially important when measuring churches, where e.g. roof drains, iron gates and electrical installations can massively falsify the results. In such cases a bearing from a greater distance along a façade or over the gable of the building often helps. Google Earth is suitable for the rough examination of such an azimuth but only of limited use for a real measurement.



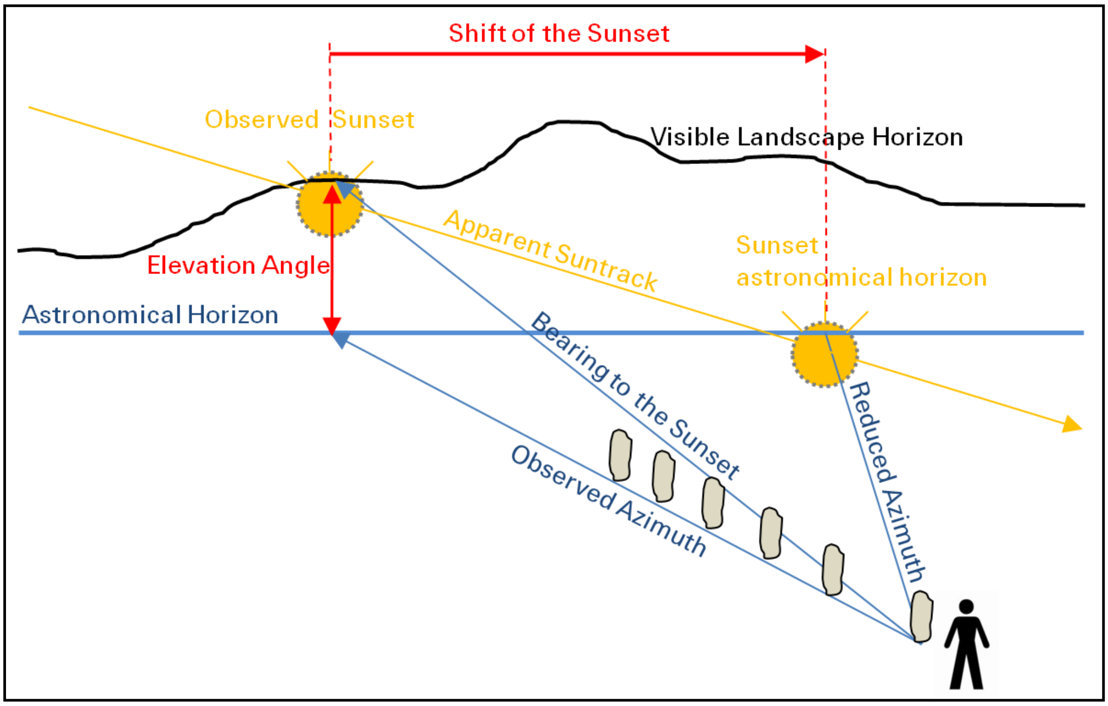

The deviation of a magnetic azimuth measurement between the geographical northpole GP (true north) and the magnetic pole MP is called *magnetic declination* ***δ*** and must never be confused with the declination *δ* of a celestial body! Depending on the geographic position this value can strongly vary, irrespective of the above-mentioned interferences. As a result of the continuous migration of the magnetic pole, both for Switzerland and Germany, increasingly *eastern* or *positive* declination values are to be expected in the near future, which in this case must be *added* to the magnetic azimuth measurement.

In certain areas, stronger local anomalies can also influence the measurement result. In case of doubt it is therefore recommended to measure a control azimuth between two points with known coordinates. For Switzerland declination values can be found under [24] and for Germany under [25]. The NOAA website provides a global overview of this phenomenon and allows the determination of the current declination for any coordinates worldwide. [26].

An alternative to these methods is to determine the time when the bearing to the sun, or the direction of an associated shadow, coincides with the line to be measured. In addition, the geographic coordinates and the altitude of the location are needed to determine the azimuth (and elevation) of the sun e.g. by a planetarium program.

## 11.2 Influence of the Visible Landscape Horizon on the Azimuths

In order for the practically measured azimuths to be comparable with the theoretical azimuths, they must first be converted or "reduced" from the visible landscape horizon to the astronomical (or mathematical) horizon. The Shift of the horizon azimuth is calculated here according to [33] and normally takes place in a northerly direction for both, the rising and setting points. With the exception of the seashore, the astronomical horizon remains usually invisible for topographic reasons. It is roughly defined for the corresponding location by the horizontal tangent plane to a virtual globe with a featureless surface and a perfect spherical shape.




In some cases the landscape horizon may even appear below the astronomical horizon, e.g. when looking from a high mountain to a very large plain or to the sea. Due to the lowered appearing horizon the correction has to be done in the opposite direction. In astronavigation such values below the astronomical horizon are called "Dip of horizon". Finally, a further important question for archaeoastronomy is whether and by what amount an observed horizon height might have changed until today (due to vegetation, erosion). The easiest way to calculate this shift of the horizon azimuth is to apply the calculation tool for Archaeoastronomy, Tab. 6 [33].

## 11.3 Measurement of Lengths and Heights

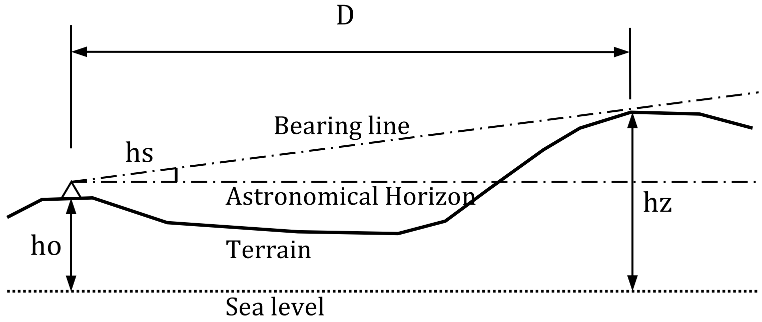
Larger distances and altitudes can, for most archaeoastronomical applications with sufficient accuracy, be measured from large scale topographic maps (e.g. 1:25'000). With the measuring tools of today's online maps, mostly the corresponding azimuth can also be determined, or even combined with a distance measurement. For shorter distances, ordinary measuring tapes or laser devices can be applied.

## 11.4 Azimuth and Elevation Angle of the Landscape Horizon

The knowledge of the apparent, i.e. real observable horizon height, here expressed as vertical angle ***hs***, is essential for many archaeoastronomical tasks. It can be measured e.g. with theodolites, sextants or clinometers (inclinometers). Alternatively, this value can also be calculated from the topographic map. The following values must be obtained:

- the map altitude (or altimeter reading) at the object location ***ho***
- the map altitude (or altimeter reading) at the object location ***hz***
- The horizontal distance ***D***

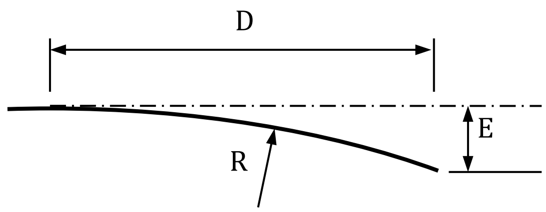
$$hs = \tan^{-1}\frac{hz - ho}{D} \qquad \{1\}$$




For most archaeoastronomical applications, at short distances <10 km, both the lowering of the target point by the earth's curvature and the small lift due to atmospheric refraction can be neglected (altitude error <7 m). For longer distances, however, it is becoming increasingly important to take these influences into account, since the apparent lowering of the horizon **E** increases exponentially with the square of the distance. For the entire W – E length of Switzerland of about 349 km, **E** however amounts to astonishing 8300 m! If, for example, Mt. Everest would be located in Geneva, from Lake Constance just the top few hundred metres of the summit could be seen. The Federal Office of Topography [29] proposes the following approximation equation, which additionally reduces the error due to atmospheric refraction by a factor of 0.87.

***R*** is the earth radius with 6'370 km

$$E = \frac{0.87 \cdot D^2}{2R} \qquad \{2\}$$




This approximation equation applies only for $D \ll R$

The apparent lowering of the horizon E must now be subtracted from ***hz*** for this case and thus reduces the apparent horizon angle hs. This subtraction does not take into account that the plumbline directions at the object- and at the horizon point do not run parallel, since both aim approximately at the center of the earth. However, as an approximation this effect can easily be neglected for our purposes.

Thus for hs [°] applies:

$$hs = tan^{-1}\frac{hz - ho - \frac{0.43D^2}{6'370'000}}{D} \qquad \{3\}$$

All variables in this equation must be inserted in [m]! "$\tan^{-1}$" is the modern form of notation for the inverse function Arcus Tangens.

## 11.5 Determination of the Horizon Elevation Angle with Software Support

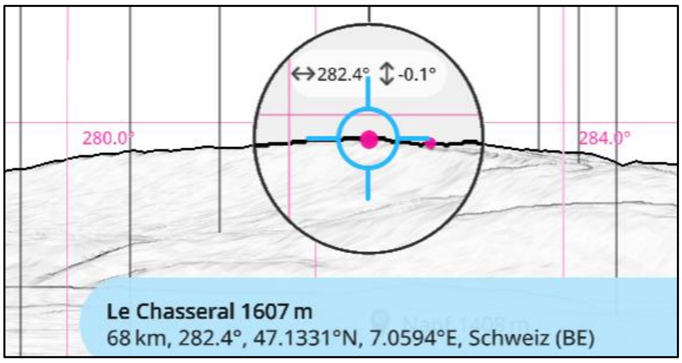
Today, these values can also be determined comfortably with software support. The "Peakfinder" by Peakfinder GmbH of the Swiss software developer and outdoor expert Fabio Soldati was tested here. Based on digital terrain models, the local landscape horizon, labelled with the most important summit names, can be displayed, with few exceptions, for any location worldwide and without considering the vegetation. This highly appreciated and widely used application was developed primarily for mountain climbers to identify peaks and can be downloaded to a smartphone for a small amount. The language can be switched between German, English, French and Italian.

The "Desktop" version for PC and laptops [27] is free of charge and has been tested by the author, anyway limited to Switzerland, for its suitability for archaeoastronomy [38].

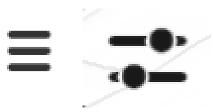
- For freely selectable, and even forested or overbuilt locations, the theoretically visible landscape horizon can be displayed.
- The panorama angle can be zoomed from approx. 110° – 7°.
- Azimuth and elevation can be determined for any horizon point using a measuring tool. The tested high accuracy shows, that both the earth's curvature and refraction effects have been taken into account.
- In addition to the astronomical horizon, also the apparent sun and moon track can be displayed not only of the current day, but also for past epochs [38].

The figure below shows the strongly zoomed view of the horizon with the summit of the Chasseral, viewed from the Napf summit.

The thin violet line indicates here the astronomical horizon. The measuring tool "digital telescope" displayed by clicking on the horizon line, shows here an azimuth of 282.4° for the Chasseral summit and an elevation of −0.1° below the astronomical horizon.




Under thefollowing icons, the location can be selected and additional functions be displayed,



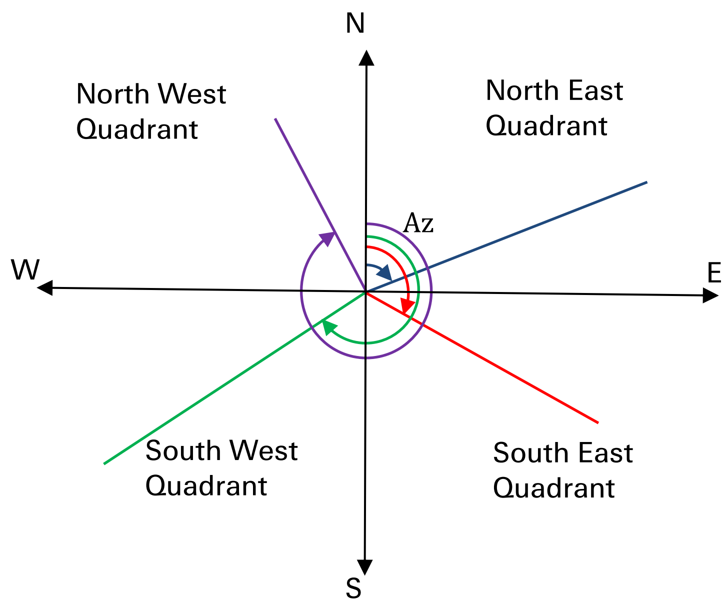
## 11.6 Determination of Azimuths for Obstructed Lines of Sight

In practice, we often have to determine azimuths whose lines of sight are today distorted by trees, buildings, etc. Therefore it is shown here how to calculate the azimuth - just based on the coordinates of the location and the target. This procedure is recommended just for distances above several hundred meters, since inaccuracies in the determination of coordinates then only have a reduced effect on the calculated azimuth. Therefore, it is strongly recommended to avoid any azimuth calculations between two points in the short distance range whose coordinates have been determined just with ordinary GPS devices.

First, the coordinate differences of these points have to be calculated in W - E direction = ΔY, and in N - S direction = ΔX. In contrast to the mathematical convention, the Swiss Grid designates here:

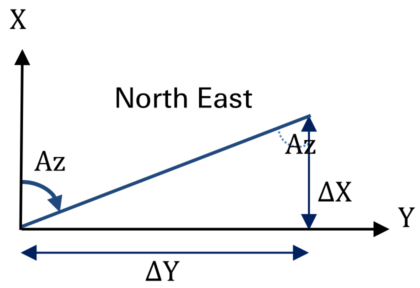
- Y as the "horizontal", W - E running axis,
- X as the „vertical", S - N running axis.

Further, the quadrant in which the line of sight is running must be determined (Fig. right).



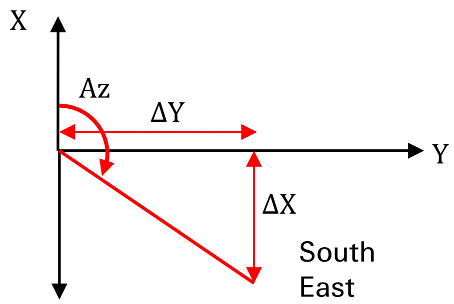

Case 1: NE Quadrant:

$$Az_{NE} = \tan^{-1}\frac{\Delta Y}{\Delta X} \quad \{4\}$$




Case 2: SE Quadrant:

$$AZ_{SE} = \tan^{-1}\frac{\Delta X}{\Delta Y} + 90° \quad \{5\}$$



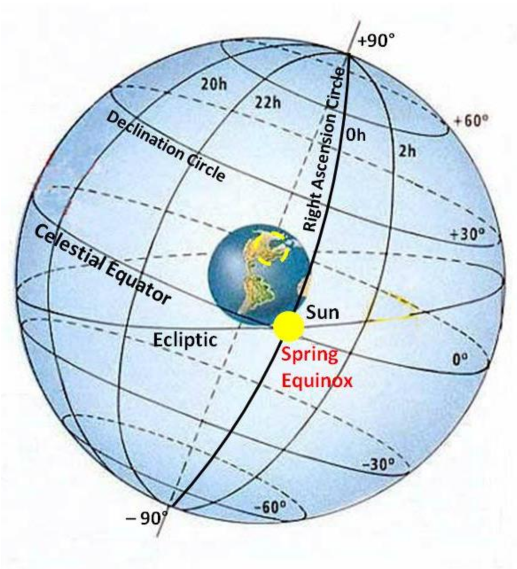

Case 3: SW Quadrant: = Calculation analog equation {4} (NE quadrant) + 180°

Case 4: NW Quadrant: = Calculation analog equation {5} (SE Quadrant) + 180°

# 12 The Equatorial Coordinate System

## 12.1 Introduction and Terms

This coordinate system also plays a decisive role for archaeoastronomy, for the understanding of the solstices and lunistices, and is an indispensable basis for the calculation methods, which will be explained in the next chapters.




The simplest way to imagine is to first project the pattern of the terrestrial coordinate system with its longitudes and latitudes onto the celestial sphere. This is an imaginary sphere with an infinitely large radius and the earth as its center (see graph). Thus already the pattern of the equatorial coordinate system is obtained. However the designation of the individual elements, as well as of the reference points of this system, is different:

1. The terrestrial equator becomes the *celestial equator*
2. The terrestrial longitude becomes here the *Right Ascension (RA or α)* and is measured, instead of degrees, with hours, minutes and seconds. 360° ≙ 24h and 15° ≙ 1h.
3. The terrestrial reference point of Greenwich's longitude cannot be applied here because the Earth rotates constantly with respect to the fixed starry sky. It is replaced by the spring equinox, at which the sun crosses the celestial equator from south to north on its apparent orbit, the ecliptic.
4. The right ascension of the vernal equinox corresponds to RA = 0h. The RA is measured from here in east direction.
5. The counterpart to the terrestrial latitude is called declination or δ. The declination of the celestial equator corresponds to 0°, analogous to the latitude of the terrestrial equator. From here the declination values are measured, in northern direction positive (+), in southern direction negative (-). The declination for the celestial north pole (close to the polar star) is +90° and for the celestial south pole -90°.

## 12.2 The Standard Equinox J2000.0

The spring equinox, as the origin of the coordinate system, is not stationary with respect to the fixed star background. As a result of the precession effects (chapter 13), it is shifting continuously. Equatorial coordinates are therefore defined for star maps and catalogs with a "normalized" reference date, which is designated as *"equinox"* in reference to the spring equinox. The current standard equinox is based on the January 1, 2000, 12h UT [7]. The designation for it is ICRS J2000.0 (International Celestial Reference System). In this system, the coordinates, observed at a certain period of time = "Epoch", are "reduced" to the date of the standard equinox or converted into the "astrometric coordinates" by a certain procedure [7]. This shift is made with a *standardized* precession value which does not correspond to the *real* precession (Section 13.2). In the settings of the NASA, JPL Horizons this convention is called "Astrometric RA & DEC". It is needed if e.g. planetary positions are to be plotted on a celestial map of the standard equinox [7].

## 12.3 The Equinox of the Date

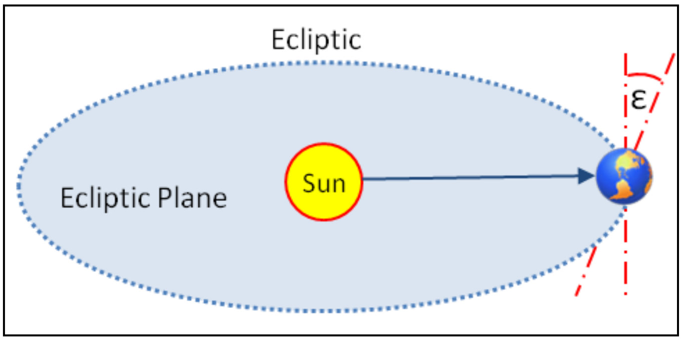
For most archaeoastronomical investigations, however, positions are required in relation to the coordinate system corresponding to the equinox of the observation time or epoch under investigation. These coordinates are also called *"apparent coordinates"* [7], not because they are "wrong", but correspond just to the *real observable values* in a certain epoch. This convention is therefore suitable for archaeoastronomical investigations because only this way the horizon positions of the celestial bodies at that time can be determined. In the settings of the NASA, JPL Horizons this convention is called *„Apparent RA & DEC"*.

## 12.4 The Astronomical Term "Apparent"

"Apparent" in the astronomical sense is a technical term which emphasizes the real observable nature of a phenomenon and distinguishes it from mathematical calculated positions, just considering certain physical parameters. Example: The apparent sunrise.

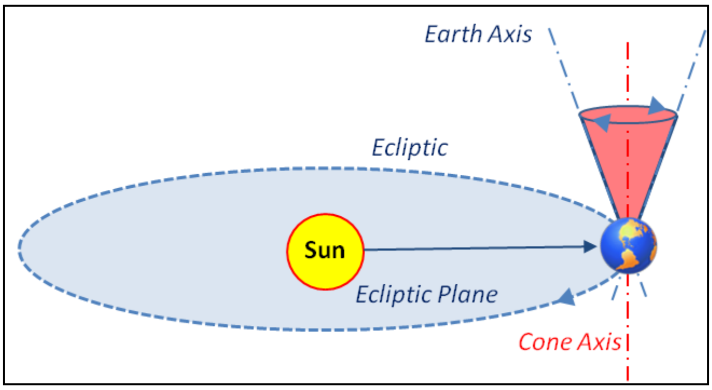
# 13 Precession and Oscillation of the Ecliptic Obliquity

## 13.1 Introduction and Terms




Viewed from Earth, the ecliptic is the projection of the apparent solar orbit onto the celestial sphere. Observed from space the Earth orbits around the Sun and spans this way the ecliptic plane. The rotation axis of the Earth is inclined by the angle ε, relative to the perpendicular on the ecliptic plane. This angle is also called *ecliptic obliquity **ε***. It is variable, if considered on very long periods of time, and amounts currently about 23.44°.

## 13.2 Lunisolar Precession




This effect is caused by the wobbling motion of the earth's axis, which within approx. 25,800 years describes the shape of a cone mantle. It is called "Lunisolar" because this pattern of motion is mainly caused by the gravitational influence of the sun and moon [6]. It has no influence on the declination values and thus also on the horizon azimuths of sun and moon, because the tip of the precession cone lies in the earth's center and its axis is perpendicular to the ecliptic plane. However, this effect dramatically changes the visibility of the fixed star background and, to a much lesser extent, of the planets orbiting the Sun on asymmetric and differently inclined elliptical orbits.

A standardized form of this slightly variable type of precession is applied to reduce equatorial coordinates of a certain epoch to the date of a standard equinox (see chapter 12.2). Thereby a constant ecliptic obliquity ε is assumed.

## 13.3 Nutation and Planetary Precession

The term nutation refers primarily to very weak, short-period oscillations of the earth's axis caused by the moon's orbits and the orbits of the moon's nodal line. They can be neglected for our purposes. The ecliptic obliquity ε does not remain constant over very long periods of time, but oscillates periodically between approx. 21° 55' and 24° 18' within approx. 41'050 years. This fluctuation is very important for us, because it also changes the declination values δ of sun and moon and the associated azimuth of rising and setting by small amounts. Main cause are gravitational influences of the planets, which is why this effect is often called planetary precession. Various sources, e.g. [7] but also NASA JPL Horizons subsume this under *"Nutation Effects"*.

## 13.4 General precession

The sum of lunisolar and planetary precession is called general precession. This combined effect leads to a movement of the equinox of about 50.256" per year.

## 13.5 Ecliptic Obliquity and Archaeoastronomical Calculations

The term ecliptic obliquity ε was already introduced in Chapter 13.1. For our purposes ε corresponds with sufficient accuracy to the absolute value of the maximum solar declination δmax, i.e.

$$\varepsilon \approx \left| \delta max_{Sun} \right| \quad \{6\}$$

So if we can determine ε for an earlier epoch, we also know the maximum solar declination at that time, the key for the calculation of the extreme horizon azimuths!

In addition, with this ε value for a certain epoch, the maximum and minimum moon declination and the associated extreme horizon azimuths of the Major and Minor Lunistices can approximately be calculated (see chap. 14.4 and 14.5). This effect is also applied in the tool for *Archaeoastronomical Calculations* Tab. 8 [33].

The oscillation of the ecliptic obliquity has been observed since very long time. No less than the mathematicians Euler, Laplace, Lagrange and Le Verrier have already dealt with the mathematical description of this oscillation in the 18th and 19th centuries. The constant effort to increase their accuracy continues until today. The following equation is taken from [3] and [7]. It provides with a 3rd degree polynomial the mean inclination of the ecliptic, adopted by the IAU (International Astronomical Union). Holger Filling converted it (decimal degrees, powers of 10) in such a way that it becomes practicable for pocket calculators.

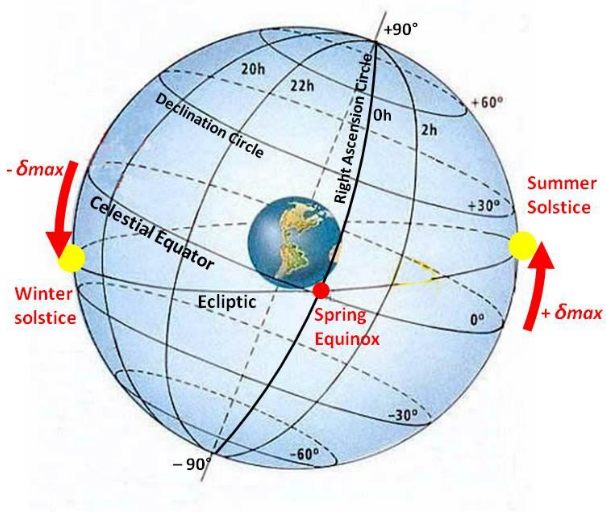
$$\varepsilon = 23.43929111° - 0.01300416° * T - 1.638° \cdot 10^{-7} * T^2 + 5.0361° \cdot 10^{-7} * T^3 \quad \{7\}$$

Here the value for T must be entered in "Julian centuries", measured from the epoch J2000.0. For example, for 2000 B.C. this would be −40. The Julian century is often used in the context of ephemerides and can for our purposes be equated with 100 tropical years [7].

Caution: The algebraic signs for T must be taken into account and applied correctly in the equation! Prehistoric ε values can also be taken from diagrams of various publications [1].

# 14 Definition of Solstices, Equinoxes and Lunistices

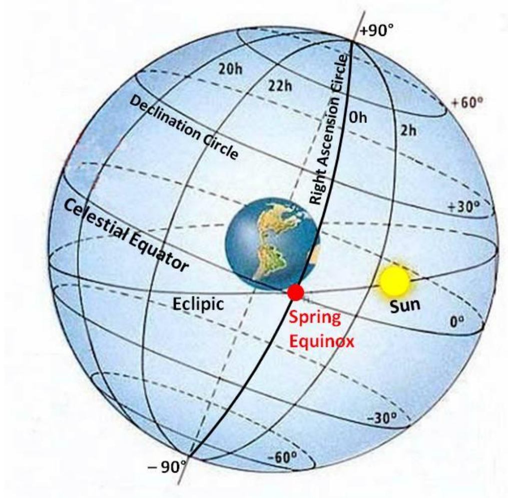
## 14.1 Astronomical Definition of Solstices




At the time of the summer solstice (approx. 21 June) the solar declination δ reaches exactly its largest northern (+) value of currently approx. +23° 26'. In the northern hemisphere, this date also determines for the sun the highest elevation above the horizon.

At the time of the winter solstice (approx. 21 December) the solar declination *δ* reaches its largest southern (−) value of currently approx. −23° 26'. In the northern hemisphere this date determines for the sun the lowest elevation above the horizon.

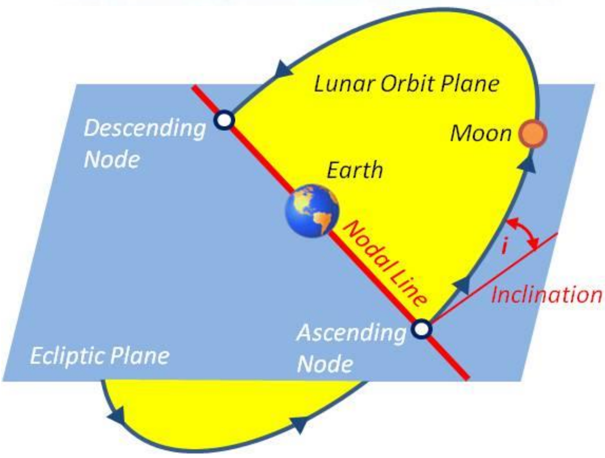
## 14.2 Astronomical Definition of Equinoxes




At the time of the equinoxes or equinoxes (lat. aequus = equal and nox = night), the sun on the ecliptic crosses the celestial equator, i.e. in spring (approx. 21 March) from south to north and in autumn (approx. 22 September) from north to south. The autumn point is invisible on the graphic, because it is covered by the globe.

Thus the sun declination for the equinoxes in spring and autumn yields δ = 0°.

## 14.3 Astronomical Definition of Lunistices




As a first, rather pragmatic approach for the determination of the lunistices, the time could be determined at which the most extreme horizon positions are observed during a sidereal moon orbit. In chapter 5.3, however, it was demon-strated how time-diffuse these constellations really appear.

The celestial mechanical definition of the Lunistices is complex [2], [22]. With help of the following graphics these connections shall be clarified. The intersection line between the planes of the moon's orbit and the ecliptic is called *nodal line*. The point at which the orbit intersects the ecliptic plane from south to north is called *Ascending Node* and the corresponding intersection point in south direction *Descending Node*. "North" in the solar system means the north direction of the earth's axis. The angle between the planes of the moon's orbit and the ecliptic is called *inclination i* with an average value of approx. 5.1452° [8]. This oscillates with a period of 173 days by about ±0.15°, which, however, in addition to other minor orbital perturbations, is not considered within the scope of this publication (chapter 5.2).

The alignment of the nodal line does not remain constant, because the plane of the moon's orbit performs a precessional movement, comparable to the motion figure of a gymnastics hoop, falling on the ground. The vertical axis of this motion figure is perpendicular to the ecliptic plane and the inclination angle ***i*** remains constant with high accuracy. About every 9.3 years and after a rotation of 180°, the nodal line coincides with the intersection line between the ecliptic and the celestial equator. In this case, one of the two nodes (ascending or descending) coincides with the spring equinox and the moon's orbit reaches extreme positions with respect to the celestial equator, allowing thisway extreme declination values of the moon [2], [6], [22].

General Definition for the Lunistices

The coincidence of the *moon's orbit* with the *equinoxes* determines the theoretical time for either the *Major* or *Minor Lunistices* and occurs *independently* of the current orbital position of the moon.

The moon will then be located somewhere along its orbit, but hardly at that point which would really allow reaching the absolute possible maximum declinations for this constellation. An entire orbit of the nodal line lasts 18.61 years and must not be confused with the somewhat shorter Saros period of 18.03 years, which plays a role in connection with eclipse cycles of sun and moon.

General Remark to the Precession:

The precessional behaviour of the Earth's moon is unique in the solar system. With just one other exception – Saturn's moon Japetus – the orbital planes of all other large moons are not precessing perpendicular to the ecliptic plane but on the *equatorial plane of their planets*!

## 14.4 Major Northern and Southern Lunistices

Major Northern Lunistice

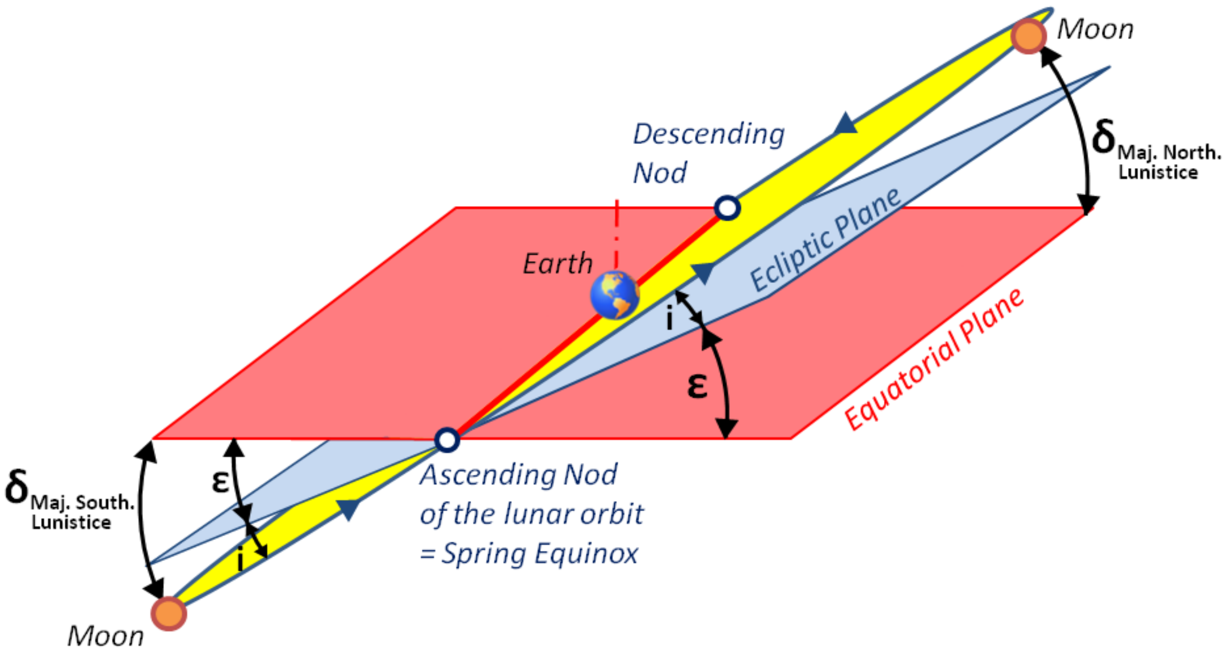
When the *ascending node* of the moon's orbit coincides with the *spring equinox*, then north of the celestial equator the full positive (+) amounts of the moon's orbit-inclination ***i*** will add up to the ecliptic obliquity ***ε***. This constellation allows the moon to reach during this orbit its theoretically *largest northern declination maximum +δmax*. However, this would require that the corresponding orbit-point would be reached exactly at the time of Lunistice, what hardly ever will be the case. Therefore, the observed northern declination maximum is always slightly smaller. This constellation is called *Major Northern Lunistice*. At this point the moon reaches its highest possible culmination point, generating thisway extremely northern horizon azimuths.

$$\delta_{Maj.\ North.\ Lunistice} \approx +\varepsilon + i \qquad \{8\}$$

Major Southern Lunistice

During the same orbit, but south of the celestial equator, the full negative (−) amounts of ***ε*** and ***i*** add up at the exact Lunistice time. This constellation enables the moon to reach during this orbit its theoretically *largest southern declination maximum −δmax*. For analogous reasons as with the Major Northern Lunistice, this is observed as slightly smaller. This constellation is called the *Major Southern Lunistice*. At this point the moon reaches its lowest possible culmination point, generating thisway extremely southern horizon azimuths.

$$\delta_{Maj.\ South.\ Lunistice} \approx -\varepsilon - i \qquad \{9\}$$




Definition for the Major Lunistices

The coincidence of the *ascending node* on the moon's orbit with the *spring equinox* determines the theoretical time for the *Major Lunistices* and occurs independently of the current orbital position of the moon.

## 14.5 Minor Northern and Southern Lunistices

Minor Northern Lunistice

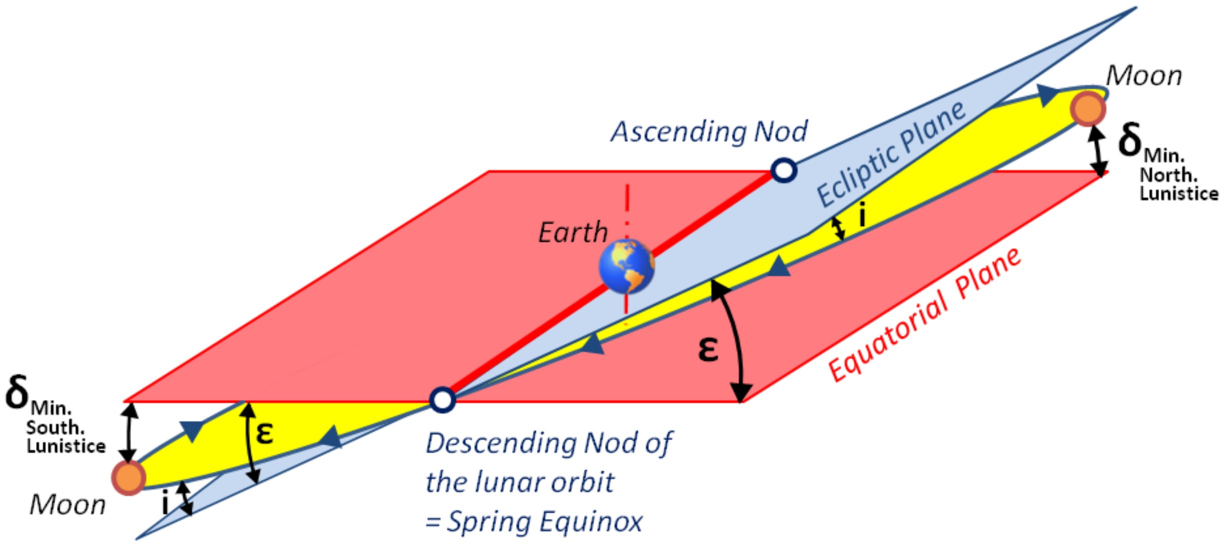
When the *descending node* of the moon's orbit coincides with the *spring equinox*, then north of the celestial equator the full amount of the moon's orbit-inclination ***i*** will be *subtracted* from the ecliptic obliquity ***ε***. This constellation enables the moon to reach during this orbit its theoretically *smallest northern declination maximum*. However, this would require that the corresponding orbit-point would be reached exactly at the time of Lunistice, what hardly ever will be the case. Therefore, this smallest declination maximum is observed as slightly higher. This constellation is called the *Minor Northern Lunistice*. During this orbit the moon reaches its *lowest maxima* of the northern horizon azimuths.

$$\delta_{Min.\ North.\ Lunistice} \approx +\varepsilon - i \qquad \{10\}$$

Minor Southern Lunistice

During the same orbit, but south of the celestial equator the full amount of the ecliptic obliquity ***ε*** will be *subtracted* from the moon's orbit-inclination ***i***. This constellation enables the moon to reach during this orbit its theoretically *smallest southern declination maximum.* For analogous reasons as with the Minor Northern Lunistice, this is observed as slightly higher. This constellation is called the *Minor Sourthern Lunistice*. During this orbit the moon reaches its *lowest maxima* of the southern horizon azimuths.

$$\delta_{Min.\ South.\ Lunistice} \approx -\varepsilon + i \qquad \{11\}$$




Definition for the Minor Lunistices

The coincidence of the *descending node* on the moon's orbit with the *spring equinox* determines the theoretical time for the *Minor Lunistices* and occurs independently of the current orbital position of the moon.

# 15 Calculation of the Rising and Setting Azimuths

## 15.1 Basic Spherical Trigonometric Equations for Azimuth Calculation

The spherical trigonometric equation for calculating the eastern rising azimuth *Aa* can, like the horizon angle *hw*, be derived as a basic task from the nautical triangle (Chap. 16).

$$Aa = \cos^{-1} \frac{\sin\delta - \sin\varphi \cdot \sin hw}{\cos\varphi \cdot \cos hw} \quad \{12\}$$

The western setting azimuth *Au* can be calculated with the same equation by subtracting 360°.

$$Au = 360° - \cos^{-1} \frac{\sin\delta - \sin\varphi \cdot \sin hw}{\cos\varphi \cdot \cos hw} \quad \{13\}$$

$\varphi$ = Latitude

$hw$ = true horizon elevation angle of the celestial body above (+) or below (−) the astronomical horizon at the setting or rising point.

$\delta$ = Declination of the celestial body (important: + for northern values, − für southern).

*cos*$^{-1}$ is the modern form of notation for the inverse function Arcus Cosinus.

## 15.2 Calculation of the Stellar Declination based on an Observed Horizon Azimuth

Equation {12} was rearranged here, to enable the calculation of the star declination *δ,* based on a given horizon azimuth *Haz:*

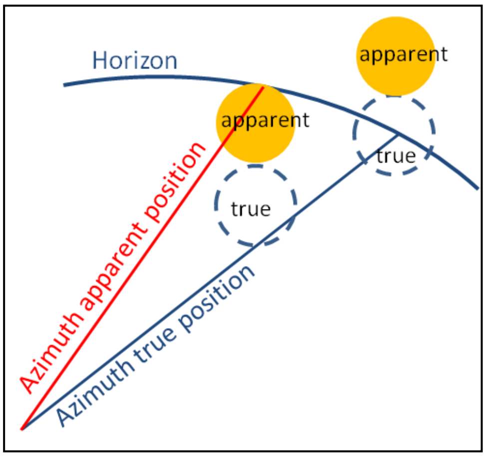
$$\delta = \sin^{-1} \left(\cos Haz \cdot \cos\varphi \cdot \cos hw + \sin\varphi \cdot \sin hw\right) \quad \{14\}$$

Here it does not matter whether for *haz* the rising azimuth *Aa* or the setting azimuth Au is inserted.

## 15.3 The Declination of the Star as a Key Value for its Horizon Azimuths

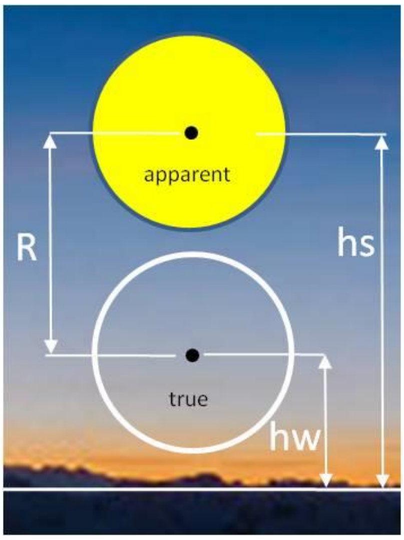
These equations show that (besides the variables *φ* and *hw*) the *celestial declination δ* is the real key value for determining the horizon azimuths and the largest declination values must necessarily yield the most extreme azimuths. If "senseless" δ values are entered − i.e. for circumpolar stars which never set, or those located too southerly and therefore never rise, undefined arcus cosinus values (*cos*$^{-1}$) will result. A computer responds this by default with an "Error" message.

## 15.4 Apparent and True Horizon Elevation Angles




The equations {12} and {13} provide the horizon azimuths, related to the center of the celestial bodies and the *true* horizon elevation angles ***hw***. However, we measure the azimuths of the apparent, i.e. really observed, positions, which are still affected by refraction and parallax effects (upper or lower limb observation). In order to be allowed to calculate with {12} and {13}, these apparent horizon elevation angles *hs* must therefore be corrected or "reduced" to the true elevation ***hw***.

## 15.5 Influence of the Atmospheric Refraction




Due to the refraction effect in the Earth's atmosphere, we can just at the zenith see where a celestial body really stands. This means that only there the true elevation angle or "height above the astronomical horizon" ***hw,*** corresponds to the apparent ***hs***. ($hw = hs$). For all other points, the true star position is lower by the refraction amount R, compared to the really observed location:

$$hw = hs - R \qquad \{15\}$$

Remark: With sun and moon and according to equations {12} and {13}, the true elevation angle ***hw*** usually refers to the disk center. However, in Archaeoastronomy the observed apparent elevation ***hs*** can refer either to the center, the upper- or the lower limb of the celestial body. Since here primarily rising and setting azimuths are to be calculated, the designation ***hs*** is used for both, the apparent elevation angle of the horizon- or of the celestial body.

The following equation, presented by B. Steinrücken [17], provides the refraction values ***R*** in arc minutes ['], as a function of the *apparent elevation* ***hs*** in [°]. However, in practice, especially with inversion weather conditions, significant deviations from this theoretical value can be observed.

$$R\,['] = \frac{1'}{\tan\left(hs + \frac{7.31}{hs + 4.4}\right)} \qquad \{16\}$$

The equation shows that the R-value only becomes striking against lower elevation angles and finally reaches the maximum value of approx. 34.48 arc minutes or 0.5747 decimal degrees on the astronomical horizon (hs = 0°). Surprisingly, this corresponds to more than an apparent mean sun or moon disc diameter! In order to take this effect into account in the azimuth equations, the observed apparent elevation angles ***hs*** must be reduced to the true ***hw***. For this the following three cases must be distinguished, because for sun and moon additional parallax effects have to be considered.

## 15.6 Azimuth Reduction for Fixed Stars and Planets

This is the simplest case, which only requires a refraction correction. Therefore the above equation {15} can be used for these apparently "punctiform" stars without further corrections:

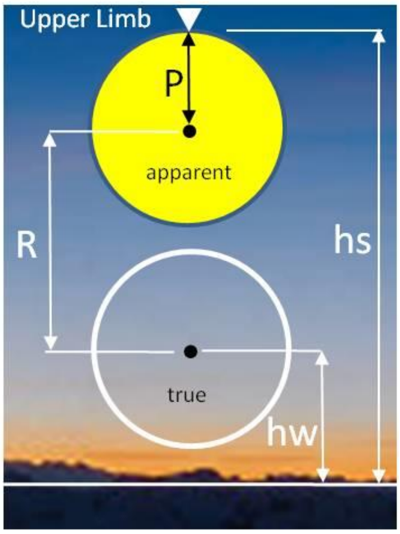
$$hw_{Star} = hs - R \qquad \{17\}$$

For calculations, related to the astronomical horizon, becomes $hs = 0°$ and

$$hw_{Star} = -R = -0.5747°$$

By this amount the star is already below the horizon while we are observing its set. Due to this effect both ***Au*** and ***Aa*** are shifted by a small amount northwards.

## 15.7 Azimuth Reduction for the Sun

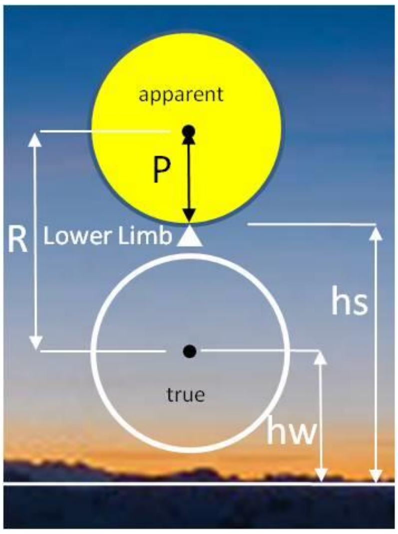



The calculated declination values always refer to the disk center of the sun. In most cases, however, we observe the *upper limb*. In this case as a correction value and in addition to the refraction, the parallax **P** must be subtracted, i.e. half of the apparent disc diameter P = ~16' or ~0.266°.

$$Correction\ hw_{Sun\ upper\ limb} = hs - P - R \quad \{18\}$$

For the special case of the astronomical horizon becomes $hs = 0°$.

$$Correction\ hw_{Sun\ upper\ limb} = -P - R = -0.8413°.$$




If the *lower limb* of the sun is considered, P must be added.

$$Correction\ hw_{Sun\ lower\ limb} = hs + P - R \quad \{19\}$$

For the special case of the astronomical horizon becomes $hs = 0°$.

$$Correction\ hw_{Sun\ lower\ limb} = +P - R = -0.308°.$$

## 15.8 Azimuth Reduction for the Moon

An additional correction step is necessary for the moon (see graph below).

1. <u>Special Case *Horizontal Parallax*: Reduction on the astronomical horizon</u>

Due to its relatively short distance to the earth, a fictive observer B in the center of the earth (calculated reference point) sees the moon significantly above the horizon, while observer A, watches it just rising or setting at the horizon,! This effect is called *Horizontal Parallax* **HP** with an average value of about +57' or +0.95°. It works exactly opposite to the refraction **R** and disk parallax **P**, which make the star to appear higher than it actually is. As already mentioned, for **P** both with the moon, and with the sun, approximately 16' or 0.2666° can be applied. For the special case of the astronomical horizon (hs=0°) and the *upper Limb* applies:

$$Corrction\ hw_{Moon\ upper\ limb} = -P - R + HP = +0.109°$$

Here the refraction **R** and the parallax **P** get overcompensated by the horizontal parallax **HP** and the correction value becomes even positive.

For lower limb considerations, **P** is here also added instead of subtracted. For the special case of the astronomical horizon the following applies:

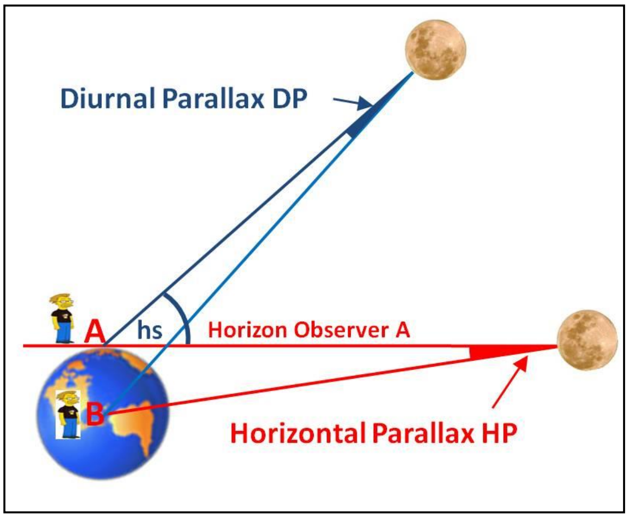
$$Correction\ hw_{Moon\ lower\ limb} = +P - R + HP = +0.642°$$

2. <u>General Case *Diurnal Parallax*, Reduction for any Moon Elevations above the Horizon</u>

For any moon elevation angles above the astronomical horizon, the special case of the *horizontal parallax* **HP** becomes the so-called *Diurnal Parallax* **DP**. It becomes smaller with increasing elevation angle **hs** and reaches finally at the zenith DP = 0°.

In general the following applies to the DP:

$$DP = HP \cdot cos\ (hs - R) \quad \{20\}$$




For upper limb*:*

$$Correction\ \ hw_{Moon\ upper\ limb} = hs - R - P + DP \quad \{21\}$$

For lower limb:

$$Correction\ hw_{Moon\ lower\ limb} = hs - R + P + DP \quad \{22\}$$

Remarks:

This parallax effect is also effective with any other celestial bodies, but due to the enormous distances it is so small that it can be neglected for our purposes.

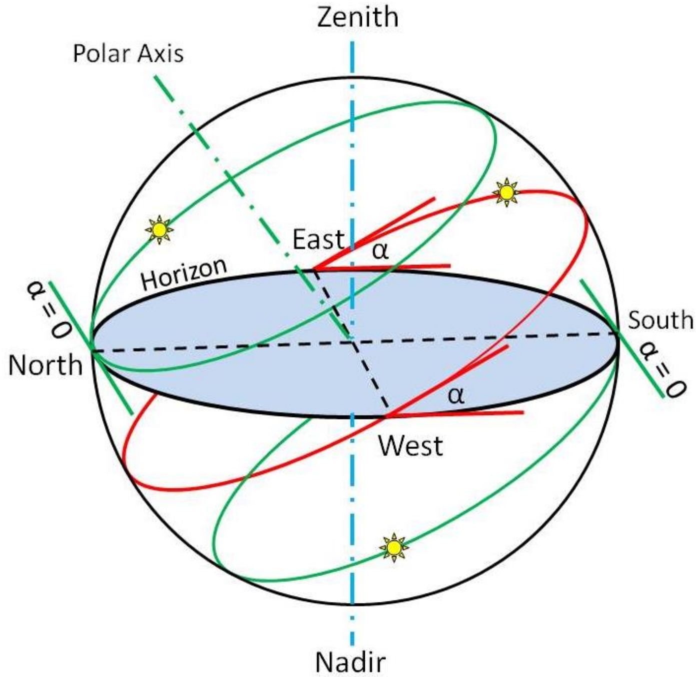
Planetarium programs usually calculate celestial positions in relation to the centre of the body, without taking these effects into account! If the software itself does not provide a correction option, the results have to be corrected afterwards!

# 16 Rising and Setting Angles of Celestial Bodies

## 16.1 Calculation of the Angles

In amateur circles, as well as in corresponding publications, this question is usually discussed controversially. This problem cannot be solved elementarily, because all celestial objects appear to move on the surface of the celestial sphere. Accordingly, all two-dimensional approaches fail in this model - probably the most frequent cause for misconceptions! Here the rules of spherical trigonometry must be applied. The most important results here in advance:

Relative to the astronomical horizon, the angles α of rising and setting for any celestial body remain the same for a given constant declination. Largest (steepest) they are exactly at the east and west point of the horizon and amount here to $\alpha = 90° - \varphi$ ($\varphi$ = Latitude). For the sun this is the case approximately at the time of the equinoxes, i.e. for 47° North $\alpha = 43°$.




Angle values for horizon points further north or south decrease symmetrically to the W - E direction, until they finally reach α = 0° at the north and south points. A corresponding celestial body "touches" the horizon here apparently just at one point, the tangents of both circles become overlapping and thus $\alpha = 0°$.

With formula {23} **α** can be calculated, related to the astronomical horizon, **φ** = latitude, **Az** = azimuth of the star, $\tan^{-1}$ stands for the inverse function arcus tangens. For elevated horizon points the formulas {24} and {26} must additionally consider the true horizon elevation angle hw.

$$\alpha = \tan^{-1}\frac{\sin Az}{\tan\varphi} \quad \{23\} \qquad \alpha = \tan^{-1}\frac{\sin Az}{\tan\varphi\cdot\cos hw - \sin hw\cdot\cos Az} \quad \{24\}$$

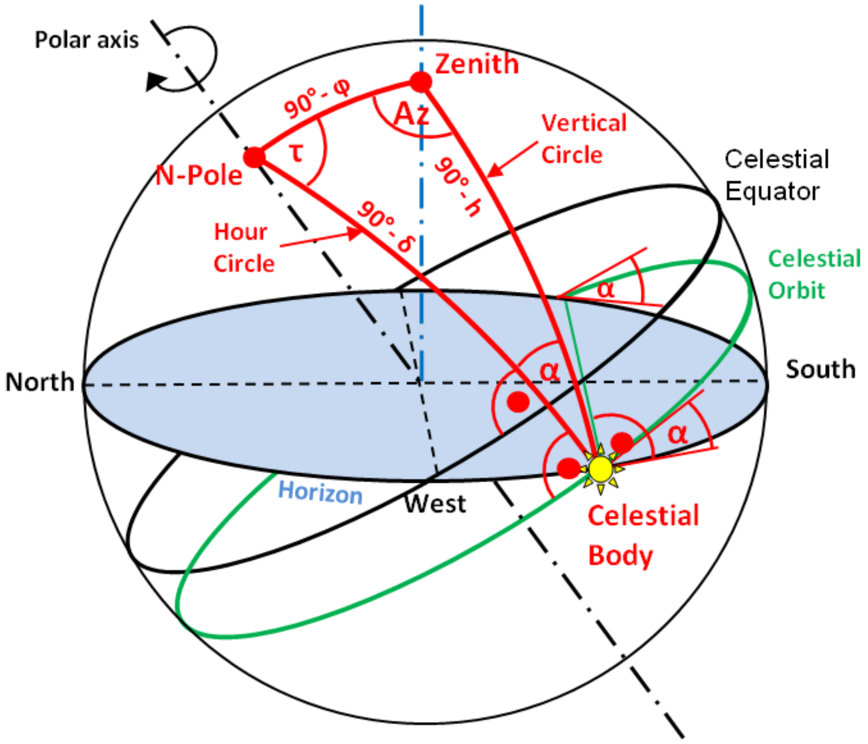
If instead of the azimuth **Az** the declination of the star **δ** is known, the following equations can be applied. $\cos^{-1}$ stands for the inverse function arcus cosinus.

$$\alpha = \cos^{-1}\frac{\sin\varphi}{\cos\delta} \quad \{25\} \qquad \alpha = \cos^{-1}\frac{\sin\varphi - \sin\delta\cdot\sin hw}{\cos\delta\cdot\cos hw} \quad \{26\}$$

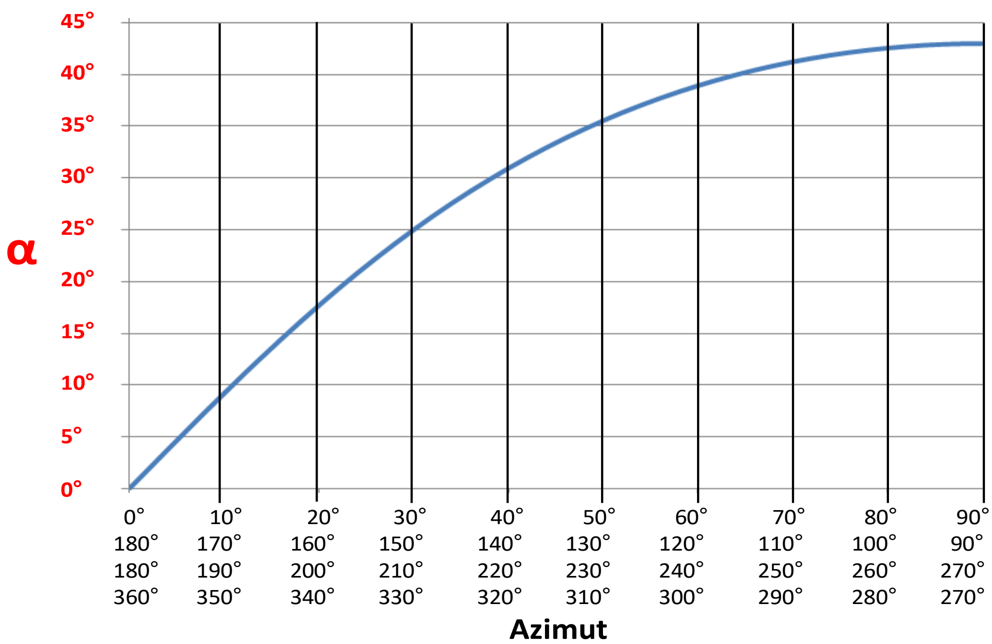
<u>Example:</u> The declination value of the sun at summer solstice yields ca. +23.4°, at winter solstice −23.4° and for the equinoxes by definition 0°, i.e. a sun position exactly on the celestial equator. For 47° North and $hw = 0°$ at the equinox horizon points results $\alpha = 43°$ and for both solstice points $\alpha = 37°$.

## 16.2 The Nautical Triangle

The so-called nautical triangle is defined by the following three points: *Zenith*, *Celestial North Pole (N pole)* and the *current position of the celestial body*. To simplify the drawing, the celestial body is drawn here at the horizon elevation ($hw = 0°$). The declination is negative here because it's located south of the celestial equator. Angle ***α*** at the celestial body is called *parallactic angle* and in contrast to the *hour angle* $\tau$, not needed for navigation calculations. However, it corresponds exactly to the angle of rising and setting α, since the hour circle through the celestial pole stands perpendicular to the celestial equator, as well as perpendicular to the parallel running celestial orbit. Furthermore the vertical circle, running through the zenith, is perpendicular to the horizon.




Because α is included in the nautical triangle, the formula can be derived directly from the *cosine rule for sides* theorem. The azimuth angle Az (or a supplement thereof) appears here at the zenith point of the triangle and can be derived with an analogous approach as for α. The following Excel graph shows the horizon angle α for a latitude of 47°N, in function of the azimuth and related to the mathematical horizon ($hw = 0°$).



# 17 Tool for Archaeoastronomical Calculations

Supplementary to this publication, the Excel-based archaeoastronomical calculation tool is available under [33]. This Excel Workbook contains programmed formulas and allows the simple solution of the most important basic tasks in the field of archaeoastronomy. The topics are structured using the spreadsheets, which can be called up at the bottom of the window. This workbook also contains a separate tab with practical calculation examples to demonstrate the capabilities of this program.

# 18 Literature and Internet

Most of the following publications are in German Language

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http://sternwarte-recklinghausen.de/astronomie/forschungsprojekt-vorzeitliche-astronomie/

[17] Burkard Steinrücken, *Tabellenwerk der extremalen Horizontstände von Sonne und Mond von – 3000 bis zur Zeitenwende*

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[19] Wolfhard Schlosser, *Eine universelle Funktion zur Bewertung alter Sonnenobservatorien*

[20] Thorsten Zipser, Burkard Steinrücken: *Zur astronomischen Orientierung von Bauwerken und Städten der Azteken, Mayas und Inkas im präkolumbischen Amerika*, u.a. mit Deutung möglicher Ausrichtungen auf die Venusextreme.

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The following publications about archaeoastronomy can be downloaded from the *Richard-Walkers-Page* (scroll down):
http://www.ursusmajor.ch/astrospektroskopie/richard-walkers-page/index.html

[32] Richard Walker: *The Analysis of Archaeoastronomical Orientations*

[33] Richard Walker: *Archaeoastronomical Calculations,.* Excel-based calculation tool for solving the most important archaeoastronomical basic tasks:

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