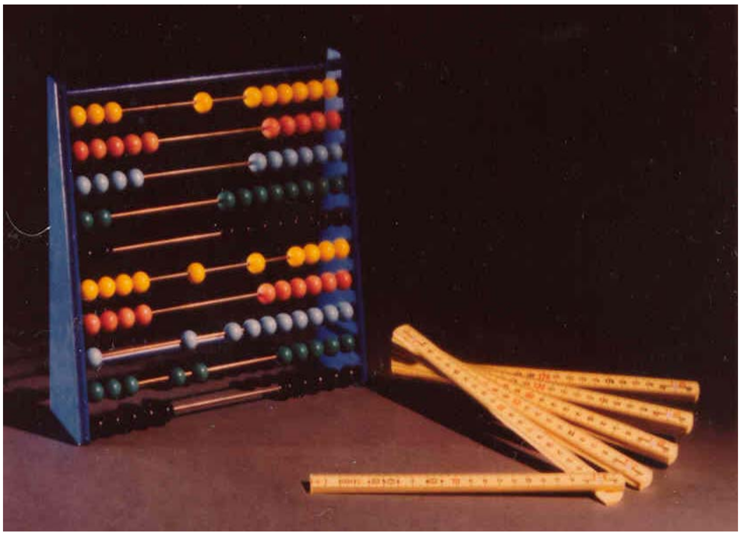
SURVEY COMPUTATIONS

This is a live document that keeps changing and improving, with contributions by students welcome

Version: 11/01/2019



Calculations and measurements. Photo: Wild Instruments

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AUSTRALIA

- We acknowledge the contributions of former staff and students of our school to this book.
- We acknowledge that some of the CAD material is derived from the CivilCAD software and training manuals.

Contents

About this book

There is more to Surveying, Spatial Information Systems, and Geomatic Engineering than just measurements and calculations, but this book is about our basic calculations. Calculations and plan drawing are a traditional part of surveyors' work and many fields of surveying involve data collection, calculations and presentation of results using computers. Computers make our work easier and usually better quality.

Survey Computations has been a subject or course at UNSW for many years. CAD has also been taught here for many years, in a separate subject to survey computations, and with a variety of software products and versions. Over the years many lecturers have taught these topics, including: M. Maughan, S. Ganeshan, F. Brunner, B. Harvey, V. Janssen, C. Roberts, and Y. Zhou. Some taught for longer than others, Ganeshan probably for the longest. Various lecturers over the years have added to the lecturing material and tutorial questions. We now have a collection of material some of which has unknown sources. Did the lecturers take it from some book for internal educational use only; did they generate it themselves, which lecturer provided which material? We are not sure. I, Bruce Harvey, know that I wrote a considerable portion. Some of the CAD material is based on CivilCAD software and training manuals. I continue to modernise the material, adding and deleting where appropriate. I share the material with you in a way that I believe is in accordance with copyright laws.

Survey Computations includes the elementary survey calculations like radiations, intersections, traverses and resections. They form the basis of many traditional field surveys and are still required knowledge for some surveyors today. Using computer aided drafting (e.g. CivilCAD, AutoCAD etc) software to process surveying data for design and plan production purposes is an important and essential skill for surveying graduates. This book introduces surveying/civil CAD packages commonly used in engineering surveying. Instructions are given in data entry, contouring and plan drawing for detail survey, subdivision and road design.

Most of the problems in this book assume the earth is flat, there is no atmosphere and measurements are perfect. Of course the assumptions are not true, but you will learn in other surveying courses how to cater for those aspects e.g. correct the observations for refractive effects and how to work on a map projection coordinate system. Then many of the calculations we do in this course can be used. This course is currently taught to second year students at UNSW who have studied one previous surveying course. A course in Least Squares adjustments follows this course on survey computations to better cope with measurement errors.

The aims and methods for this book are:

- To present the information necessary to do Survey Computations calculations, with many examples and worked solutions to enable students to solve plane survey computation problems and to be able to learn to use any of the currently available surveying CAD packages or those developed in the future.
- To be available at the lowest cost to students, so an electronic version is on the web for free download. [I would prefer students not to print it out onto paper; look after our natural resources. I prefer other teachers use it with appropriate acknowledgement.]
- To use the best of many years of collected teaching materials and add new material.
- To add new methods and a new approach.
- To update it regularly, it is live and never final, so I put information in quickly and polish it later. So I do not refer to page numbers or figure numbers to make it easier to update.
- To allow students to contribute to the book for their own benefit and the benefit of other students. Students will contribute to this book with additions or improvements, including summaries, colour figures, worked solutions, computer source code, spreadsheets, step by step guides through CAD, calculator steps etc. Contact me if you wish to improve parts of the book.
- But the aim is **not** for you to acquire a vast knowledge of all the options/steps available in CAD nor is it to remember all the equations used in plane survey computations.

It would be very tempting for me to keep adding more and more material to this book and try to tell you everything I know about the topic. But that would be too much for one course. For example, I used to include some computer programming for survey computations in this book, but there is not sufficient time in our course to teach it. So it is no longer included in the book. I have been guided by: *"Make everything as simple as possible, but not simpler" - Albert Einstein* and by *"Simplicity hinges as much on cutting nonessential features as on adding helpful ones." - Walter Bender.* I try to make each new version simpler rather than more complex, to keep the best of the old ways and to add the best of new ways with the latest technology.

How to study this book

If you have the pdf version of this book you can use the FIND feature to locate key words, so we don't need an index. You can also use the Pages and Bookmarks functions in the navigation pane if you are using Acrobat Reader or similar to read the file.

Downloading – READ ME

Apparently downloading files is addictive and there is a similar problem with photocopying.

The amount of class and reference material that students can read is now enormous. Most people will download files or photocopy things because if they have the file or the paper, they can read it at any time … so they don't have to read it now. So hardly anyone reads anymore and people only collect files or printouts or photocopies and store them away. Is getting copies of all the material the only way you can keep up?

Have you tried an alternative? It's an old fashioned process where you place the pages in front of your eyes and read, you let it go through into the brain and it is much better than a download or a photocopy. Perhaps I should put lots of files on my web site with no significant or useful content, just random gibberish, and see how many times the files are downloaded. ☺

Paraphrased by Bruce Harvey from an article on Neuroxing (source unknown)

Even better: Students should do the calculations in the questions and the worked examples, don't just read them, and certainly don't just download the file and do nothing with it.

I hope you enjoy learning from this book, practise the material by doing the questions and the worked examples. How much you learn (and whether you pass the course or not), depends more on how many problems you solve than on how much you read or try to memorise the book. I know that it takes a lot of time and effort to work through all the problems. The reward is that you learn by doing, not just by reading.

Copies of the data, spreadsheets and programs used in some questions and examples, can be made available from the School of Surveying and Spatial Information Systems, UNSW, via the web site.

Students are not a homogeneous group and different people learn in different ways. Some have asked me to keep the notes as short as possible, to be concise. Others have asked for worked examples with full details. I have tried to please both groups. You have to find the learning style that suits you best. For example, a former student said: "Once an understanding of what is happening is made, it becomes more interesting. I've also discovered that reading the textbook after I tried something on the computer was a lot more beneficial than attempting the reverse." I hope you will also try to do the problems and gain some experience and understanding, rather than just trying to memorise the book.

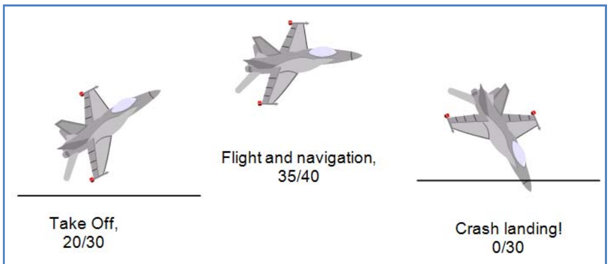
Don't just use your old calculator for every problem in this book. Move out of your comfort zone and learn something new. It is important to do some of the problems in a spreadsheet or in CAD, or with some other software e.g. Matlab. It is only by using different "machines" that you learn the advantages and disadvantages of each.

I have given examples and answers for many of the questions, and will add more in the future.

However, one problem with giving worked solutions is that it is easy to superficially read them and think you understand or have learnt something. You should force yourself to do the calculations and mechanics of the problem, don't just read my solution. Also, think about how the problem was solved. What are the assumptions, steps, decisions and procedures? If you had a different problem to solve what aspects of the examples might be applicable? On the other hand, if you can't solve a problem then the answers and worked solutions may help. The answers also serve as extra examples for those who need them. My solutions may also point out errors in your work (calculations, assumptions or methods) which you might not otherwise have been aware of.

Another disadvantage of including answers is that you become accustomed to looking to the 'back of the book' to check your work. Then students don’t learn the importance of self checking their own work. In the real world there are rarely solutions and answers provided (if the answers are already available who would pay you to do the work?). You have to be able to decide, and check for yourself, that your work is correct. You have to learn how to check your methods and calculations. Giving you the answers (spoon feeding) does not help you learn this important aspect. Though, I am sure that after being spoon fed for a while you will want to solve problems on your own. Incidentally, I don't guarantee the answers in this book are correct. You can check them and tell me about any errors. Some answers are not complete. If you need more help with them or the questions that are not answered, then consult your lecturer, tutor, or peers. If you have a better solution for a question or find an error please tell me.

In a course based on this book it may be possible to pass by memorising a series of steps, but you may forget it later. I hope you will try to understand this subject. Practice doing the calculations. Think about the derivations. This is a practical course, the more practice and experience you get the better you will understand the topic and the faster you will be able to solve problems.



Does the pilot pass the exam? (55/100)

What to do if you can’t understand the book? Ask me or your teacher, ask other students, read other books in the library or on the web.

Sources of these figures unknown.

After this book:

Harvey, B.R. (2016) **Practical Least Squares and Statistics for Surveyors**, Monograph 13, Third Edition reprinted, Surveying, CVEN, UNSW. 332 + x pp. ISBN 0-7334-2339-6.

The book has recently been reprinted, now in colour, still with spiral binding for ease of use. At the lowest possible cost, but not as a free pdf. I retain copyright to the book. To order the book, go to UNSW Bookshop or : https://www.bookshop.unsw.edu.au/details.cgi?ITEMNO=9780733423390

1. REVISION of CALCULATIONS

The aim of this section is to show important principles and methods of carrying out survey computations using calculators and computers. This chapter is an introduction to the calculations for plane surveying. I assume you have prior knowledge of surveying measurements and include this material here for your revision.

Why should you learn this material? Calculations are a traditional part of surveyors' work. Computers and calculators make the work easier. Even though machines (computers & calculators), software and programming languages will change in the future, once you convince yourselves (and us) that you can learn this subject then you should be able to learn new machines and languages successfully in the future.

Parts of two movies will be shown in lectures to help you think about some principles of calculation. The original Star Wars movie (a section near the end of the movie) shows that computer software does not always work and sometimes we should use our experience to guide us. The Rainman movie has a section that shows some clever calculations being done but no understanding of the answers.

History

Much of our course in survey computations is geometry. From Wikipedia: Geometry (Greek γεωμετρία; geo = earth, metria = measure) is a part of mathematics concerned with questions of size, shape, and relative position of figures and with properties of space. Geometry is one of the oldest sciences. Initially a body of practical knowledge concerning lengths, areas, and volumes, in the third century B.C. geometry was put into an axiomatic form by Euclid, whose treatment set a standard for many centuries to follow. … Introduction of coordinates by Descartes and the concurrent development of algebra marked a new stage for geometry, since geometric figures, such as plane curves, could now be represented analytically. … The visual nature of geometry makes it initially more accessible than other parts of mathematics, such as algebra …

Should we put the following sign above the door to our comps lab, and allow students of survey computations to enter too?

ΜΗΔΕΙΣ ΑΓΕΩΜΕΤΡΗΤΟΣ ΕΙΣΙΤΩ

It means “Let no-one ignorant of Geometry enter” and was the sign over Plato’s door. Another translation for the inscription on Plato's door: "Let no one enter who does not know geometry." Another translation: The Greek "Medis Ageometretos Eisito" was taken from the inscription over the portal of Plato's Academy. The literal meaning is "Let no one without skill in geometria enter" the primary meaning of the Greek work geometria being "measurement of Earth." It is used in the crest and the registered trade mark of the Institution of Surveyors, NSW:

Plato (c.427-347 B.C.E.) … pursuit of truth through questions, answers, and additional questions. Plato's most prominent student was Aristotle. Plato founded an Academy in Athens, often described as the first university. Plato loved geometry. He wrote: "*[Geometry is] . . . pursued for the sake of the knowledge of what eternally exists, and not of what comes for a moment into existence, and then perishes, ... [it] must draw the soul towards truth and give the finishing touch to the philosophic spirit."*

Over the years the computing ‘machines’ available to students and surveyors have improved considerably. The history of computing is well documented elsewhere but the history of Surveying Computing includes: Logarithm tables, slide rules, calculating machines e.g. Facit, calculators with useful functions and some programmable (e.g. HP 45, 29c, 41 …), instrument on board calculations e.g. electronic theodolite total stations with coordinate calculation, digital levels for level run calculations, GPS… For photos see our on line museum: http://www.ssis.unsw.edu.au/info-about/our-school/historical-artefacts/surveying-instrument-collection

Layout of Calculations

Calculation techniques depend to a certain extent on the individual person, but some general rules for layout of calculations have been found to be useful when writing the calculations on paper. Some are also relevant for layout of calculations on spreadsheets.

Get used to doing your hand or calculator calculations neatly and orderly, cross out any mistakes and continue. Don't do a quick, rough, messy job and then write up a neat copy of the calculations. Writing the calculations once leads to less chance of making mistakes because of untidy work, and there are no errors caused by mistakes when copying to a second version. As a general rule, the length of time taken over a computation is of little importance compared with the necessity for obtaining a correct answer.

A good layout makes it easier, and more reliable, for other people to read your calculations or even for you to read them yourself some time later. Calculations can often be laid out in table form (i.e. with rows and columns) in a neat, logical way and include comments. The layout of output from computer programs should be neat, easily readable and understandable with suitable headings and comments, echo all input, and not give insignificant digits in the results.

It is advisable to include details of steps of the calculating sequence, especially to state the formulas used. A summary of the results of the calculations should be given, or the results highlighted.

As much as possible of the intermediate results of calculations should be stored in the machine's memories. This reduces errors due to transferring figures incorrectly between paper and machine. Quite often it is helpful to record intermediate results so that any errors may be easily traced. This means just record them and do not transfer them in and out of the machine. Extreme care has to be exercised when transferring numbers from the worksheet to the calculating machine or vice versa, or from one page to another.

Hand written mistakes are corrected by drawing a line through the wrong number and inserting the corrected value above the wrong one e.g. $23\overset{1}{\not{6}}$. Do not overwrite on the original e.g. 236 because it is often not clear what the correct value is. When writing numbers containing many digits it is preferable to group the digits in sets of three, in both directions from the decimal point e.g. 5 236 978.645 23

For cadastral surveys in NSW the Surveying Regulation 2006 specifies the method of recording angles and bearings in field notes: “All angles and bearings must be observed and recorded in degrees, minutes and seconds, and all bearings must be reckoned and expressed clockwise from zero to 360 degrees.”

When angles (and bearings, latitudes etc) contain minutes or seconds less than 10 it is preferable to include additional zeros. For example write 36° 12' 05" instead of 36° 12' 5", the latter may be misinterpreted (especially by machines) as 36° 12' 50". Similarly write 21° 03' 36.4" instead of 21° 3' 36.4".

Mental Arithmetic

- Why is the ability to do mental arithmetic calculations useful?
- Build your confidence with practice

Useful constants for mental arithmetic	Metric conversion
π = 3.14 actually π = 3.1415926535897932…	1 foot = 0.3048 m (exactly) = 12 inches
1 radian = 206264.8… ” ≈ 200000 ”	1 chain = 100 link = 66 feet = 22 yards
1’ subtends ≈ 0.03 per 100 $\ell = r\theta$	1 mile = 5280 feet
1” subtends ≈ 0.005 per 1000 or 1mm per 200m	1 acre = 160 perch ≈ 64 m x 64 m ≈ 0.4 ha
1” of latitude ≈ 30 m (& longitude near equator)	1 hectare = 10 000 m² = 100 m x 100 m
1 ppm = 1 mm per 1 km	

Try the following questions without using calculator or computer. If possible do the intermediate working in your head rather than on paper. Some answers require approximate values, often 1 or 2 significant figures are enough. Note the time taken

Practice Questions:

1. 180° + 82° 33' 44" = ? (useful for checking FL & FR on theodolite directions, and back bearings)
2. 360° - 82° 33' 44" = ? (useful for checking FL & FR on theodolite ZA)
3. 360° - (94°23'45" + 265°35'33") = ?
4. What is half of 1'24" ?
5. Reverse bearing of 135° 33' 55" = ?
6. $2 * \pi * 7 = ?$
7. Convert 50 feet to metres
8. $\sqrt{(0.03)^2+(0.05)^2} = ?$
9. At a scale of 1:500 what does 3m on the ground plot as on the plan?
10. What is 3ppm over 4 km?
11. An offset of 6cm over 100m represents an angle of ?
12. What offset does 2" cause over a distance of 1km?
13. What length (metres) on the earth's surface is spanned by 0.03" of latitude?

Answers: *262° 33' 44", 277° 26' 16", 42", 42", 315° 33' 55", 44, 15, .06, 6mm, 12mm, 2', 1cm, 0.9m*

Integers and real numbers in surveying

Measurements (e.g. distances and angles) are estimates of real numbers and are part of a continuous sample. Measurements are never perfectly accurate they always contain some error, so the measured value is an approximate representation of the true value. For example the distance between two marks may be 43.64372364590278658910... metres. If we use a measuring tape that is graduated every millimetre our value for the distance (if we are careful and skilful) will be 43.644 m. Now if we estimated the fraction between the 4 and 5 mm marks on the tape we might say the distance is about 43.6437 m. The number of decimal places you record for the measurement depends on how accurate you can measure. But you will always only approximate the true value. You will never know what the exact value is!

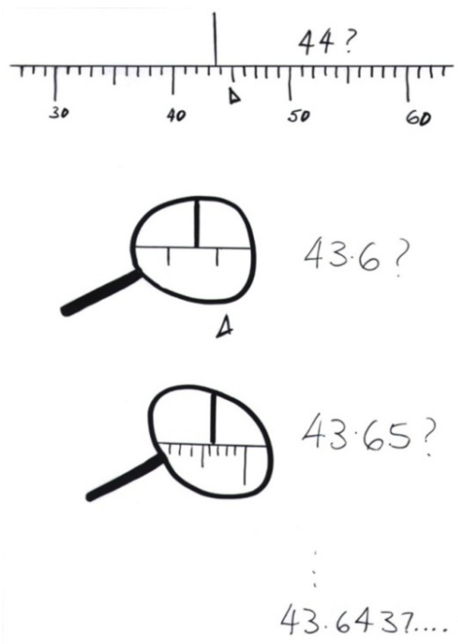


Figure at right. The more precise our measurement equipment the more decimal places or significant figures we can see.

On the other hand there are some numbers in surveying that are exact or in other words discrete integer values. Integer values exist when we count something, for example, how many marks in the survey, or how many times you repeated a measurement.

When we do calculations in surveying the equations usually involve combinations of integers, constants and real number measurements and occasionally other values e.g. π. Even some of the constants in equations (e.g. $\sqrt{2}$) cannot be expressed exactly in a fixed number of decimal places.

In computers, integers are stored accurately in binary form, and use less storage space, and calculations are quicker and more are accurate (except division) than with real numbers. Real numbers are converted to scientific notation and to binary and have round off errors in these binary conversions and calculations. So if you compare a real number with an integer in an IF statement inside a computer program or spreadsheet you may get some unexpected answers. For example, does 3 = 3.0?

You will experience an example of this computing problem in the conversion of bearings, angles, latitudes etc, between DMS (degrees minutes seconds) and degrees with a decimal component. The calculation of integer D M S from decimal degrees can sometimes be wrong on a computer. More is said about this problem later.

Rounding numbers and expressing results

As you know, rounding 123.234567 to two decimal places gives 123.23. Some people always round trailing 5s up to the next value so their values are always slightly too large. So if the part to be rounded-off exactly equals 5 then round to the nearest even value, e.g. again to two decimal places: 123.235 becomes 123.24 and 123.285 becomes 123.28. If you always round to an even number it can be 'cleanly' divided by 2 whereas odd numbers generate further decimal places and trailing fives. Rounding to the even value means that sometimes the new value will be slightly too large and sometimes slightly too small this helps to reduce the effect of round-off errors accumulating when a long series of calculations are involved. Beware of problems of rounding an already rounded number, e.g. 869.79749 when rounded to 4 places becomes 869.7975 if then rounded to 3 places becomes 869.798 but if you rounded to 3 places from the original number you would get the correct result = 869.797.

Test out your calculator and your computer software to see how the rounding is done when you tell the machine to round off to a certain number of digits. Does it use the above convention, does it round trailing 5s up, or does it just truncate trailing digits?

A common student mistake is to write down too many digits in the answers from their calculations. How many digits should we show in the results of our calculations? A common approach in surveying is to determine the accuracy of the results and then decide how many digits to display based on this accuracy. For example, if you do some calculations to find the distance between two points and the accuracy is about ±3mm then you would round off the distance to the nearest mm and show no further digits. You will learn in other courses how to determine the accuracy of your measurements and results. Until then you will have to rely on intuition, experience and your lecturers' advice.

Don't show more decimal places in your results than you can be sure of. If you quote your results in the form x ± y where x is the mean and y is the standard deviation then show your value of y to one or two significant figures. We usually use one significant figure if the first digit is greater than 2, e.g. 0.3", 0.09cm, and we usually use two significant figures if the first digit is 1, e.g. 0.14m, 12", 0.19, 0.0013. Round off the parameter to the same number of decimal places as the standard deviation or y value. Sometimes the number of digits is defined by law.

When calculating a mean and standard deviation of a set of measurements write the standard deviation to 1 or 2 significant figures and the mean to no more than the number of decimal places in the standard deviation. eg mean = 138.12412... standard deviation = ± 0.006234... should be written as mean = 138.124 and standard deviation = ± 0.006.

If possible you should do all your calculations with the numbers as you obtain them stored in a calculator or computer without rounding off intermediate steps and then round off the answer. If you round off each measurement and then do your calculations your answer might be wrong because of round off error.

Calculators

Survey calculations used to be done by using special tables of logarithms and trig functions. In some cases where only a few decimal places were required slide rules could be used to give approximate answers. Mechanical machines were also common in the past. Special computation methods were devised to make their use easier. You will see old survey text books in libraries which discuss computation methods specifically designed for these machines. This chapter will look at methods designed for efficient and reliable use of the current machines in use, i.e. calculators and computers.

A common question: I am planning on buying one of those scientific surveying type calculators for my work as well as uni. I was wondering if you could tell me if there are any requirements as to what calculator we are allowed to have within our surveying courses, or if we are allowed to use HPs in uni in the surveying courses and exams.

My answer: I used to like my old HP calculator for field use and I liked programming it, but I use computers instead now. What you can use in uni exams depends on the course involved so see the course outlines. Generally calculators that can be programmed or display lots of text (like a full keyboard) are not allowed in most exams at UNSW. When setting exams most lecturers prefer to find what the student understands and knows, rather than lots of detailed calculations. So usually a high school calculator is enough. For assignments you can usually use a computer. Many survey instruments now do a lot of calculations "on board". So there is not a lot of need for a good calculator at uni. If you need one for work then I suggest you talk to the people at work to see what they recommend. However I do know students who have spent hundreds of dollars on calculators because work suggested it, then found they hardly ever used them.

For fieldwork our calculation options are calculators, laptop computers, on board functions on survey instruments, PDAs with spreadsheets, and smart phones with suitable apps. In the office computers are the most common way to do survey computations with spreadsheets, CAD software, custom software and programs like Matlab (which you learn about in a later course).

The following figure shows worked examples of surveying calculations on one 'high school' calculator. *Write your own version of these instructions to match your calculator.*

Some instructions for use of Casio fx-911W Calculator

Setting number of decimal places in display to eg 3 [MODE] [MODE] [MODE] 1 3

Storing and recalling memories eg 123 [STO] [A] [RCL] [A]

D M S conversion eg 14°25'36" to D.D 14 [° ' "] 25 [° ' "] 36 [° ' "] [=] [° ' "] gives 14.427

Reverse calculation 14.42666667 [=] [SHIFT] [° ' "] gives 14°25°36.

To make sure you are working in degrees mode use [MODE] [MODE] [1]

Trig calcs eg cos(63°52'41") [cos] 63 [° ' "] 52 [° ' "] 41 [° ' "] [=] gives 0.44...

eg $\cos^{-1}(0.61)$ [SHIFT] [$\cos^{-1}$] 0.61 [=] [SHIFT] [° ' "] gives 52°24°37.79

Polar to Rectangular conversion eg distance = 20.5 and bearing = 60°23'34"

[SHIFT] [Rec(] 20.5 [,] 60 [° ' "] 23 [° ' "] 34 [° ' "] [)] [=] gives ΔN 10.128

[RCL] [F] gives ΔE 17.823

Memory E contains ΔN and memory F contains ΔE

Rectangular to Polar conversion eg ΔN = 60.5 and ΔE = 30.4

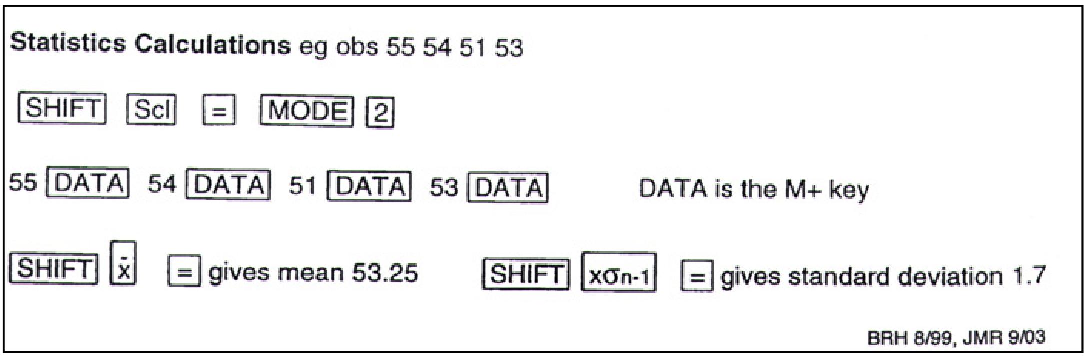
IMPORTANT: If the ΔE or ΔN is negative, type [(−)] before the number!

[Pol(] 60.5 [,] 30.4 [)] [=] gives distance 67.708

[RCL] [F] gives bearing {if <0 then RCL F [=] + 360 =} then [° ' "] gives bearing in dms 26°40°42.9

Note the order ΔN then ΔE and distance then bearing

Memory E contains distance and memory F contains bearing


Statistics Calculations eg obs 55 54 51 53

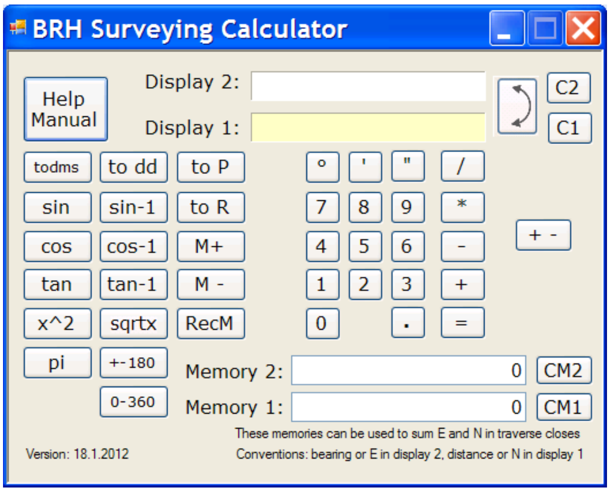
SHIFT Scl = MODE 2

55 DATA 54 DATA 51 DATA 53 DATA DATA is the M+ key

SHIFT $\bar{x}$ = gives mean 53.25 SHIFT $x\sigma_{n-1}$ = gives standard deviation 1.7

BRH 8/99, JMR 9/03

I wrote a computer program to work as a calculator specifically designed for basic surveying calculations. You might have access to it. It looks like this:

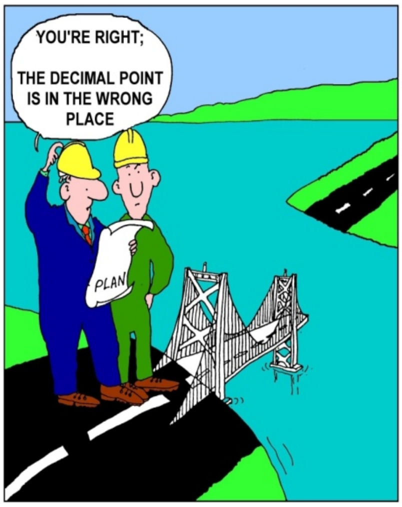


If you search the web you will find computer based versions that are excellent replicas of the famous HP calculators, many are free. You will also find their manuals. Seek and enjoy.

Accuracy of calculations

In surveying we take measurements and perform calculations to arrive at some result. However the results themselves are only half the answer, we also require some estimate of the accuracy of the results. The accuracy depends on errors due to the measurement techniques and errors in the calculations such as round-off error. Naturally mistakes could also occur and we hope to avoid these with appropriate checking methods. Errors due to the measurements and how they propagate into the final results will be covered in other subjects; here we concentrate on the calculation errors.

You must consider all sources of error in your calculations and take appropriate action to minimise their effects. Calculation errors can occur in a number of ways:



- Calculators and computers have limits and they are not perfectly accurate so sometimes the answers are wrong!

- Round-off errors because intermediate values are expressed to a finite number of significant figures.

- Errors in the calculation model, e.g. using equations that are only approximately correct.

- Mistakes.

Here are some of the things that can cause wrong answers:

- You enter the data incorrectly - eg typographical errors.

Cure: be more careful, do the calculation more than once to see if the answer stays the same.

- You enter the wrong key sequence on the calculator. Cure: same as above.
- The program in the machine is incorrect. Cure: test it thoroughly - we will cover how to do that later in this subject.
- Using the wrong program or equations for the particular problem. Cure: investigate the problem carefully.

Even if none of the above apply the answers could still be wrong because calculators and computers can only work with a finite number of significant figures. I will show you this by examples. What does your calculator give for $(\sqrt{2})^2 - 2$? The answer should be 0 (shouldn't it?) but my old calculator gives 1×10^{-9}. What happens if you take the cosine of 1" and then the inverse cosine function, do you get exactly 1"? Calculate $(111,111,111)^2$, the correct answer is 12,345,678,987,654,321.

The next example comes from F. Gruenberger, Scientific American (252, 1984, p 10). Enter the number 1.000 000 1 and press the square function (x^2) 27 times. Compare your answer with 674 530.47 . . . Most calculators get only the first 3 digits correct! This shows you how round off error can accumulate in a series of calculations.

Computing machines can handle very small numbers (such as 1.5×10^{-98} on many calculators) but they can only carry a certain number of significant figures (e.g. 10 on calculators and 16 on computers). So your machine might not be able to tell the difference between 9.1234560000 and 9.1234560001 for example.

If the effect of calculation-induced errors on the final results is less than the effect of inaccuracies in the measurements then you have nothing to worry about. It is up to you to use techniques to avoid calculation induced errors as much as possible. For example use your calculator's function to convert 23° 46' 53.1234" to decimal degrees (23.7814..). Now repeat the calculation using a modified method: (53.1234 / 3600) + (46 / 60) then add 23 when you write down the answer thus saving some space in your calculator display for the last few digits.

We will cover the accuracy of trigonometric functions in machines later.

Thorough checking of your calculations and results can help find some of the above mistakes and using special techniques you can often avoid the machine precision problem.

Checking Calculations

Surveyor's Tip of the Month *Col Murray, NSW Azimuth, Feb 2007*:

Leave the job knowing it's correct, not thinking it's correct.
… (and I add: or hoping its correct)

As accuracy is of paramount importance and since everyone is liable to make mistakes, it is essential that the calculation should be checked. When checking, you should not take the original calculation and check it step by step as there is always a great danger of accepting as correct a figure already written down even when it is wrong, that is, making the same mistake again. Checks should be carried out as far as possible by:

- Get an independent calculation by **another person** and compare the results of the calculations. Investigate and rectify any discrepancies. (Hopefully the two people don't make the same mistake!)
- Do a second calculation but use a **different mathematical process** so that the tendency to repeat an error in the original calculation is eliminated. For example in trigonometry you could solve a triangle with one set of equations then check it by using different equations.
- If possible, **reverse the calculation.** That is, use your results to calculate the input data. Examples are given later but a simple case is where you convert data eg feet to metres then check the calculation by converting your values in metres back to feet.

- Some survey problems have **built-in checks**; they are one of the best checks. For example after calculating the angles in a triangle they should total 180°.

- Check that data has been correctly entered. For example get the computer to “echo” i.e. display what it has read in.

- Estimate the likely magnitude of the answer prior to the calculation. Then check that the result from the calculation is close to the original estimate.

- Use test data from text books as into data into the program or spreadsheet, or when manually calculating with a calculator to check you are following the equations and steps correctly.

The best methods of calculation are those with the most built-in checks but it is not always easy to devise these in a way which does not involve a lot of extra calculation. When only a partial checking system is used, it is essential for you to realise which parts of the computation are unchecked, so that you can treat these parts with special care. Obvious sources of errors are:- illegible figures, untidy arrangements (particularly when figures are arranged in columns), unsystematic work, trying to do too many steps at once, transcription errors, and typographical errors...

You must also ensure that the data is correct. Whilst this may seem obvious, experience has shown that in a fair percentage of student calculations that give wrong answers, the data has been incorrectly entered into the machine and the subsequent calculation effort has been completely wasted. Similarly your measurement techniques should be designed to avoid, minimise or locate any mistakes.

Another useful check is to make an estimate of the likely magnitude of your answer before doing the calculations. This estimate can be based on approximate mental calculations, or from experience or intuition. You can even make estimates of the intermediate results of the calculations so that checks can be made of each step as you go along. To avoid typographical errors when putting data into a calculator you could store the input data in memories, recall the memories to check the values are correct and then do the calculations by recalling the appropriate memory at the required steps. A similar thing can be done to check input to computer programs but we will leave the details until later. Another check - which is called a stability test - is to change all your measurements by a small amount and use this new data in the same calculations or program. The answers should be only slightly different to those with the real (original) data.

If you are using a program to do your calculations you should check the program first. One way is to use test data. This is data which has already been calculated and you know the results are accurate. Run this data in the program and see if you get the correct results. Either use examples from textbooks or generate simple data which is easy to calculate by hand (e.g. use rounded data with say one significant figure in each value). Another check of a program is to compare the results with those from a program independently written by someone else.

You will, as a surveyor, do many calculations. Sometimes you will make a mistake; that is almost inevitable. I don’t know any surveyors who have never made a mistake, but the good ones find them and fix them before any ‘damage’ is done. It is important that you check your work and discover all mistakes **before** you report your results to your employer or client. If you don’t do checks you won’t detect errors.

Finally, don’t be **overconfident** in your survey measurements or calculations. Many times I have seen students take measurements or do exam question calculations and they are certain their work is correct, only to sadly find out later that there were mistakes that they didn’t know about.

Calculating means and standard deviations

Some of the most common and basic of survey calculations are calculating means and standard deviations, and calculating bearing and distance between the coordinates of two points (and the reverse). We will look at some special aspects of computing means and standard deviations here. Bearing and distance calculations are covered in a later section.

SAMPLE MEAN: Basically, add up all measurements and divide by the number of measurements. There are two equations for calculating the mean:

$$\bar{x} = \frac{\Sigma x_i}{n} \quad \text{or} \quad \bar{x} = x_a + \frac{\Sigma(x_i - x_a)}{n}$$

The second version (where x_a is an arbitrary constant) is often better because fewer digits have to be entered. Thus it is quicker and there is less chance of transcription and typographical errors. This applies to hand calculations, using calculators and when entering data to computer programs.

For example, calculate the mean of distances: 195.436, 195.437, 195.435,
or instead just calculate the mean of: .436, .437, .435, and add 195. to your answer, or even better calculate mean of : 6, 7, 5, and add 195.43 to your answer.

SAMPLE STANDARD DEVIATION of a single observation is calculated from:

$$s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\Sigma v_i^2}{n-1}} = \sqrt{\frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1}} \qquad \text{where} \quad v_i = \bar{x} - x_i$$

Check that your calculator's in-built standard deviation function divides by (n-1) and not n, where n is the number of observations. To use the first two versions of the equations

$\left(s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\Sigma v_i^2}{n-1}} \right)$ you have to enter or go through the data twice, once to calculate the mean then again to get the standard deviation. In the last version of the equation,

$s_x = \sqrt{\frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1}}$, each observation is only entered once.

So if you have to write a calculator or computer program for standard deviation use the last version.

Standard deviation can also be found by using an arbitrary origin x_a, i.e. subtract a constant from all the observations. (Try it yourself on a set of data to convince yourself, and prove it theoretically.) The use of an arbitrary origin may be important for a practical reason. Note that when calculating standard deviation many numbers are squared. If you have numbers with many significant figures for example, distance in kilometres measured to a few millimetres e.g. 36 134.567, then you may lose some accuracy when values are squared, if your calculator or computer only works with a certain number of digits. So you should take a constant part off each measurement before calculating standard deviation. In a computer program one easy way to do this is to subtract the first observation from all the observations, thus giving new values for the observations, note the first observation will then equal zero. Alternatively, just use decimal parts of distances or the seconds part of directions.

Spreadsheet software - basics

Use the “Right Tool for the Job”. For each survey calculation you should consider whether to use a calculator, a computer, or the instrument itself. If you use a computer decide which software is appropriate. For example the Microsoft Office Suite includes: Word - word processing, Excel – spreadsheet, PowerPoint – presentation, Access – database, or you might use Visual Basic to create a specialized, customized application. In this course I expect students to have access to Word and Excel at home. If you don't then you might wish to buy them, there are some educational/academic prices for students. Of course there are competitor products. An alternative is to use a free replacement for Microsoft Office such as OpenOffice (www.openoffice.org). It includes word processor, spreadsheet etc. If you do use them, please send me some feedback on any differences, difficulties, or even examples that we can include in the class text book.

Is it appropriate to always use the same software, because it is your favourite you are fast that way, or you don’t want to change or learn something new? If you are comfortable with Excel you might want to use it for some problems that would be better solved with a custom program or a surveying module in a CAD program. Use MS Word (or similar) for word processing, but not for presentations or calculations. Use MS Excel or other spreadsheets for calculations where you want to see the intermediate steps and the data is in small quantities, especially when you want to enter the data by typing, and when you may have to use the same equations with subsequent data. But don’t use spreadsheets for large database purposes. Use MS PowerPoint (or similar) for presentations but not for report writing. Use databases like MS Access to store large amounts of data and to query or search the data. Use a programming language to create a specialized and customized applications including calculations and graphics. Use CAD programs for drawing and calculating especially Survey or Civil customised CAD applications for survey computations and plans. Spreadsheets can be transferred to PDAs and includes many but not all functions and features.

These notes are intended as only a very brief introduction for beginners with Excel. If you already have some experience with Excel just skim the following pages for anything new. Note that all the instructions here also work in the OpenOffice spreadsheet software.

Spreadsheets are very suitable for doing all the traditional survey calculations done on programmable calculators. They naturally favour the table style of calculations common for 'ancient' pre-programmable traverse adjustments, angle reductions, level books, etc. They are probably the easiest way on a computer to do a simple calculation or enter data into equations. Also, spreadsheets are easier to program than many calculators. So spreadsheets could well replace office use of calculators for basic survey calculations.

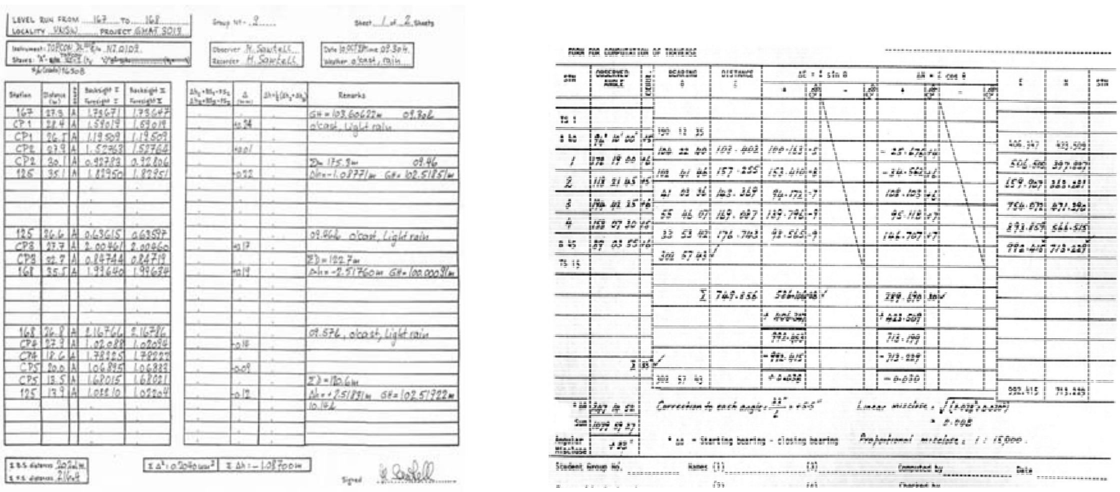


Fig. Traditional surveying computations in table form

Remember to build in check calculations where possible and to test the *acuracy* (sic) of your sheet.

Spreadsheet Advantages: user friendly interface (GUI), have many built-in mathematical and statistical functions, you can easily see what equations are used and can change the program as you wish, a matrix looks like a matrix, equations are 'hidden' but available under the numbers, *etal*. Excel now works with about 15 digits of accuracy (old spreadsheets used to be less precision) so it is accurate enough for most survey calculations.

Spreadsheet Disadvantages: you can make copies of worksheets but it is not easy to vary data set size, it is often slow for large data sets, and it requires the Master program (e.g. Excel), *etal*.

Adding numbers and using inbuilt functions in spreadsheets

Open Excel application (double click on the Excel icon in windows). Put the cursor in cell A3 (i.e. column A row 3), click the mouse, and type a number in the cell A3 and press enter. Enter similar numbers in cells A4 to A8 so you have e.g.:

	A
2	
3	65.6232
4	65.6220
5	65.6218
6	65.6194
7	65.6204
8	65.6199
9	

Click in A9, type in the function command: = A3+A4+A5+A6+A7+A8
OR = SUM(A3,A4,A5,A6,A7,A8) OR = SUM(A3:A8)

Press enter and watch the computer calculate and display the answer. This will add all the numbers between and including A3 to A8. Note the equivalent methods of referring to the cells.

Similarly click in A10, type in the command: = A9 / 6 this will divide the number stored in A9 by 6. Press enter and watch the computer calculate and display the answer.
Click in A11, type in the function command: = STDEV(A3:A8) this will give you the standard deviation of the numbers between and including A3 to A8. Press enter and watch the computer calculate and display the answer.
Choose Save As under menu File, and give your file a name.

You might like to write some words in the cells beside your answers explaining what they represent. Your work may now look like:

	A	B
7	65.6204	
8	65.6199	
9	393.73	Sum
10	65.621	Mean
11	0.0015	Std dev

Click on one of your input numbers, change it, press enter and watch the computer re calculate.

The basic equation operators are: + - / * % ^ ( )
For example = SQRT((M6-M8)^2+(M9-M7)^2)
Multiplications are not implicit so use * e.g. =A3 * (B4 + C5)

Be careful of cell references when copy/pasting groups of cells including functions, Excel changes them (usually appropriately). We can also give a cell or group of cells a name (Insert, Name or use name box)

B3 is an example of relative reference to a cell, so if you copy the formula to another cell it will change B3.
D4 is an example of absolute reference to a cell, so if you copy the formula to another cell it still uses D4.
A4:D6 refers to all cells between the top LH corner and bottom RH corner specified

Spreadsheet Formatting

Design and format your worksheet for better use, more efficient, easier for other users and less error prone. Experiment with changing column widths, row heights, changing number formats, turning grid lines off, removing row & column headings, using borders, patterns and colours, e.g. user input cells in different colour. Try hiding columns or rows where appropriate. If the worksheet is large split the screen (horizontal and or vertical) so there is not too much scrolling. Insert | Comments on some cells and or use text boxes. Use Format | Cells | Number | Custom to add leading zeros to angle minutes i.e. format as 00, and to seconds as 00.0

Advanced: Try adding a degree symbol too e.g. 23° or units for distances e.g. 23.465 m.

D M S ↔ Degrees & data entry

Angles expressed in Degrees Minutes and Seconds (DMS) are the traditional way to record theodolite readings (angles, directions and zenith angles) in the English speaking world. The form is also often used for Latitude and Longitude. An example of an angle in D M S form is: 12° 34' 56". There are a variety of possible data entry methods:

1) D M S e.g. 12, 34, 56
   Entry into Excel or Computer program takes 3 cells/numbers,
   Integer, Integer, Real (types of number storage)

2) D.MS e.g. 12.3456
   This is the old calculator entry style, with entry as one number. It is still used in CivilCAD software and on some total station instruments. This method can be confusing for people not familiar with the format / jargon but is quicker, easier entry. But calculations are also required, e.g. Int(12.3456) = 12 and Int(100*Frac(12.3456)) = 34 etc. MS Excel doesn't support the Frac function so use =INT(100*MOD(12.3456,1)) for the minutes in Excel. If the 1st or 3rd decimal place is 6 or more, then clearly the angle is in D.D format, otherwise it is not self-evident. So use custom units in MS Excel e.g. 12.3456 d.ms or a heading D.MS

3) D.D e.g. 12.582222…
   Modern electronic instruments such as theodolites and GPS, can easily be set to display this decimal format and we can book it or record it for direct entry to computers. There is one cell or number to enter, no extra calculations are needed. Four decimal places of a degree provide more precision than D M S. The European gons/Grad system is also a decimal system with millions being a common surveying measure of angular precision. (400 gons = 360°).
   Question: Which is the smallest angle of the following: 0.0001° or 1" or 0.1mgon?

4) dddmmss e.g. 1234512
   This format was described to us by student Robert Dicker. It allows us to put the D M S information all in one cell and is similar to method (2) above. It is used in navigation standards. The only decimal point is if there are places of a second e.g. 1234512.678 Make sure leading zeros in mins and secs are there if <10 e.g. 1230512 not 123512. In Excel, we can enter the data as dddmmss (e.g. 1234512) then use custom formatting: 0##°##'##\" → 123°45'12".

So why do we still use DMS? Think about it and discuss it with others.

DMS Conversion calculations in spreadsheets

D M S (as in 1 above) to D.D (as in 3 above) is simple, accurate and reliable "D.D" = D + M/60 + S/3600

But D.D to D M S involves several steps or a custom function. Unfortunately the D.D to D M S process is not always reliable in a computer because of conversions between real, binary and integers. Sometimes the result is 60' or 60" instead of 00' or 00". For example, take the angle 85° 21' 00" and convert it to D.D (≈85.4) then convert it back to D M S. You may find you get 85°20'60" in Excel! Another tricky one is 350° 16' 00".

Here are some methods to calculate DMS. An example Excel spreadsheet is shown below with some columns hidden. This is done so that column names match the contents and make the equations easier to understand. If a bearing in degrees and decimals (D.D) is in cell B5, then one version of the Excel equations to calculate D M and S and put D into D5, M into M5, and S into S5 is:

	A	B	D	M	S
5	Brg	201.25	201	15	1.82

Degs, in D5 =INT(B5)
Mins, in M5 =INT(B5*60-D5*60) **this version usually avoids the 60" problem mentioned above**
Secs, in S5 =(B5-D5)*3600-(M5*60) or =((B5-D5)*60-M5)*60

Where the INT function calculates the lower integer value, i.e. INT(3.6) = 3 and INT(-3.6) = -4. Note that calculating minutes using INT(B5-D5)*60 seems OK but does not always give correct answers. An alternative Excel formula for minutes is =INT(MOD(B5,1)*60)

If you are doing calculations with negative angles (e.g. southern latitudes in deg min sec) you may prefer to use TRUNC instead of INT, because TRUNC(-3.6) = -3 The TRUNC(cell, decimal places) function allows you to cut off any unwanted decimal places. If no decimal places are required you can omit ,0 Be careful when using the TRUNC(cell or number, decimal places), INT(cell or number), and ROUND(cell or number, decimal places) functions. Although these are similar functions there are subtle differences. You might also wish to experiment with the ROUNDUP, ROUNDDOWN Excel functions, which offer other subtleties.

To convert a dddmmss angle to D.D or radians do some additional calculations e.g. if cell C1 contains dddmmss then "D.D" =TRUNC(C1/10000)+MOD(TRUNC(C1/100),100)/60+MOD(C1,100)/3600. You can write other versions of this and even modify it for negative angles (esp latitudes). To convert D.D to dddmmss in one step use e.g. =10000*TRUNC(G4)+100*TRUNC(60*(G4-TRUNC(G4)))+100*MOD(36*G4,1) where the D.D value is in cell G4.

Revision of Trigonometry

Many survey computations involve knowledge of plane trigonometry, which students will already have acquired at school, but a list of the more important formulae is given below for reference purposes. A clear understanding of the trigonometric functions and their application is essential for many surveying calculations. Many older survey computations textbooks include equations with the trig functions cot, sec and cosec etc. They were frequently used when calculations were done with tables and logarithms before calculators and computers became available. These functions are probably not familiar to modern students and because they are rarely available as functions on calculators or computers we will mostly avoid them in this book.

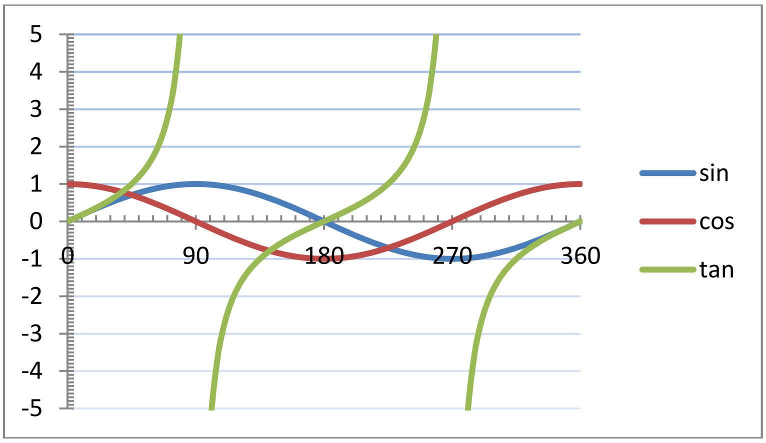


Figure. Trig function values

Calculators and computers usually produce the result of $\sin^{-1}$ etc in the ranges -180° to + 180°. To convert these to the survey convention (usually 0° ≤ α ≤ 360°) simply add 360° to any negative angle.

Summary of trigonometric formulae:

$$\sec A = \frac{1}{\cos A} \qquad \text{cosec } A = \frac{1}{\sin A} \qquad \cot A = \frac{1}{\tan A} \qquad \tan A = \frac{\sin A}{\cos A}$$

$$\cos^2 A + \sin^2 A = 1 \qquad \sec^2 A = 1 + \tan^2 A \qquad \text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \qquad \cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\sin(A - B) = \sin A \cos B - \cos A \sin B \qquad \cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$\sin(90°+A) = \cos A$ $\qquad$ $\cos(90°+A) = -\sin A$
$\sin(90°-A) = \cos A$ $\qquad$ $\cos(90°-A) = \sin A$
$\sin(180°-A) = \sin A$ $\qquad$ $\cos(180°-A) = -\cos A$ $\qquad$ $\tan(180°-A) = -\tan A$
$\sin 2A = 2 \sin A \cos A$ $\qquad$ $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

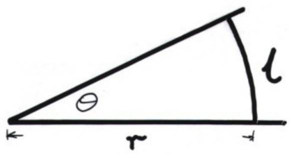
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin A + \sin B = 2 \sin[(A + B)/2] \cos[(A - B)/2]$ $\qquad$ $\sin A - \sin B = 2 \cos[(A + B)/2] \sin[(A - B)/2]$
$\cos A + \cos B = 2 \cos[(A + B)/2] \cos[(A - B)/2]$ $\qquad$ $\cos A - \cos B = 2 \sin[(A + B)/2] \sin[(B - A)/2]$

Conversion of degrees to radians

Since computers often use radians as the base for trigonometric calculations you need to know how to convert between degrees and radians. It is also useful to know the conversion for other applications, e.g. mental arithmetic and road curves. The ratio of arc length L (subtended by an angle θ) to the radius of the circle r is called the angle in radians,

$L = r\,\theta$



So one radian equals the angle subtended by an arc length equal to the circle radius (≈ 57°). To convert angular measure in degrees into radians we use the fact that a complete circle subtends 360° and has a circumference of $2\pi r$. Now in radians the angle of one complete revolution is $\theta = 2\pi r / r = 2\pi$. So $2\pi = 360°$ or $\pi = 180°$.

So to convert an angle in degrees to radians: $\theta^c = \theta° \frac{\pi}{180}$ Where we use ^{c} to indicate radians.

Similarly to convert an angle in seconds to radians: $\theta^c = \theta'' \frac{\pi}{(180 \times 3600)} \approx \frac{\theta''}{206264.8}$

[History: Before surveyors had calculators and computers they often used the constant 206265 for converting angles to and from radians. The exact value of the conversion constant is $180*60*60/\pi$ and it can also be found approximately as 1/sin1" so θ in radians ≈ (sin1") * θ"]

For very small angles you will find that: $\tan a \approx a^c \approx \sin a$
In fact, if a is less than about 1° then the above equation is exact for 5 significant figures. (Check this yourself by using the series expansion of sin and tan, and by using trial values in your machine.)

Surveyors in some European countries use instruments which measure in gons or grads where a circle is divided into 400, instead of 360 as in degrees. The conversion of these angles to degrees or radians is simple, try it yourself.

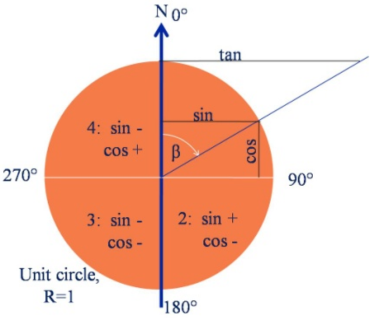
In MS Excel the built in function to convert degrees to radians is e.g. =RADIANS(B3) . The reverse calculation is e.g. = DEGREES(F40). When programming you write your own source code or function including e.g. Radians = (d + m/60 + s/3600) * pi /180 and you must previously define pi, if is not "built in". If π , pi is not built in use pi =3.14159265358979… or get your program to calculate π from pi = 4 * Atn(1) because tan(45°) = tan(π/4) = 1
The Radians to Degrees constant is: * 180/ π or * 45 / Atn(1)

Trig and inverse trig functions

Beware: the inverse trig functions determined by computers and calculators only gives one of the possible answers. For example, find a if cos a = -0.871 357. a = $\cos^{-1}$(-0.871 357) so a = 150°37' or 209°23'.
Since there are **2** possible answers you need more information to decide which one is correct.

You are expected to remember the signs of the trig functions in each quadrant (see figure):
A In the first quadrant, 0° to 90°, sin, cos, and tan all yield positive results.
S In the second quadrant, 90° to 180°, sin yields positive results.
T In the third quadrant, 180° to 270°, tan yields positive results.
C In the fourth quadrant, 270° to 360°, cos yields positive results.

(ASTC = All Stations To Central and tells you which function is positive)



In many survey applications where $\tan^{-1}$ is involved the calculation is of the form $\tan^{-1}\left(\frac{a}{b}\right)$ where a and b are known values. Think of a and b as being the x and y coordinates of a vector. If you know whether the x and y vectors are positive or negative it is easy to say which quadrant the vector or angle is in.

Since $\tan D = \left(\frac{\sin D}{\cos D}\right)$ then use the signs as if $\tan\left(\frac{a}{b}\right)$ was $\frac{\sin a}{\cos b}$. If you are in doubt draw a diagram.

For example: $\tan^{-1}$(0.3) = 16.7° or 196.7°, but $\tan^{-1}\left(\frac{3}{7}\right)$ = 23.2° because 3 is positive and sine is positive in 1st and 2nd quadrants also 7 is positive and cosine is positive in 1st and 4th quadrants. The only common quadrant here is the 1st so the angle is between 0° and 90°.

$\tan^{-1}\left(\frac{-1.1}{3.4}\right)$ = -17.9° + 360° = 342.1° because numerator is negative and sin is negative in 3rd and 4th quadrants also denominator is positive and cos is positive in 1st and 4th quadrants. The only common quadrant here is the 4th so the angle is between 270° and 360°.

$\tan^{-1}\left(\frac{-1.1}{-3.4}\right)$ = 17.9° + 180° =197.9° because numerator is negative and sin is negative in 3rd and 4th quadrants also denominator is negative and cos is negative in 2nd and 3rd quadrants. The only common quadrant here is the 3rd so the angle is between 180° and 270°.

It is easier to use ATAN2 (in excel) or R→P (rectangular to polar on calculators) instead of $\tan^{-1}$ (atan) if you want to find $\tan^{-1}\left(a/b\right)$ because they place the angle in the correct quadrant. This is possible because the software uses the signs of both a and b to determine the correct quadrant. Think about it, do you understand how this works? More details later.

Examples:

acos (0.8) = $\cos^{-1}$ (0.8) = 36.9° or 323.1°
$\cos^{-1}$ (-0.7) = 134.4° or 225.6°

$\sin^{-1}$ (1.3) = not possible
$\sin^{-1}$ (0.6) = 36.9° or 143.1°
$\tan^{-1}$ (0.3) = 16.7° or 196.7°
$\tan^{-1} = \left(\frac{3}{7}\right)$ = 23.2° (1st quad) $\tan^{-1} = \left(\frac{-1.1}{3.4}\right)$ = 342.1° (4th quad) $\tan^{-1} = \left(\frac{-1.1}{-3.4}\right)$ = 197.9° (3rd quad)

Accuracy of trigonometric functions

If you have angular values near the cardinals (0° 90° 180° 270°) then there are a few precautions you should take to get the most significant figures possible and to avoid division by nearly zero.

If 0° < a < 45° i.e. tan a < 1 try to arrange your calculations so you use the tan function rather than the cot function and vice versa: if 45° < a < 90° ie tan a > 1 try to arrange your calculations so you use the cot function rather than the tan function. Similar rules apply to the other cardinals, determine them yourself.

If sin or cos ≈ 1 e.g. 0.9999932... then in current machines you will lose some accuracy. It is better to have an answer like 1 - 0.68... x 10^{-5} because more decimal places can be obtained. When sin, cos or tan < 0.1 they are represented as e.g. 0.7897 x 10^{-3} and so values close to 0 are much more accurate than those close to 1, because machines can only hold so many decimal places. How close to 0° 90° 180° or 270° do you have to be to worry about this? Avoid taking cosines (or $\cos^{-1}$) of small angles (close to 0° and 180°) and sines (or $\sin^{-1}$) of angles close to 90° and 270° - try to use other equations. For example, cos 5" = 1 on some machines.

Plane triangles

You should already know the material in this section; it is included here just for revision and completeness. Derivations of the equations will not be given since you will have already seen them. The convention used in these notes is to represent the side lengths by a, b and c and the angles opposite them by A, B, and C respectively, as shown in figure below. The angle at A is sometimes written as ∠BAC, meaning at A and measured clockwise from B to C. Sometimes it is drawn with an arc symbol on a figure: ∡. A square symbol ∟at a corner indicates a right angle of 90°.

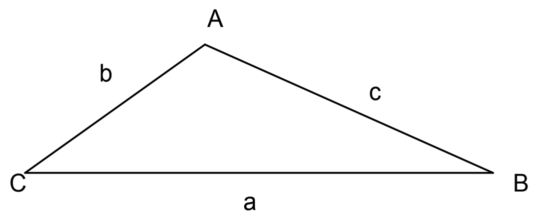


Figure: Triangle naming convention

A + B + C = 180°

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Note the cyclic nature of this, and many other trigonometric equations, simply cycle around a triangle and insert the appropriate values in an equation so you only have to remember one version of an equation.

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ and $b^2 = c^2 + a^2 - 2ca \cos B$ and $c^2 = b^2 + a^2 - 2ab \cos C$
Rearranging the terms gives:
$\cos A = (b^2 + c^2 - a^2)/2bc$ $\cos B = (a^2 + c^2 - b^2)/2ac$ $\cos C = (a^2 + b^2 - c^2)/2ab$

Projection rule: a = b cosC + c cosB
and similarly with cyclic changing: b = c cosA + a cosC and c = a cosB + b cosA

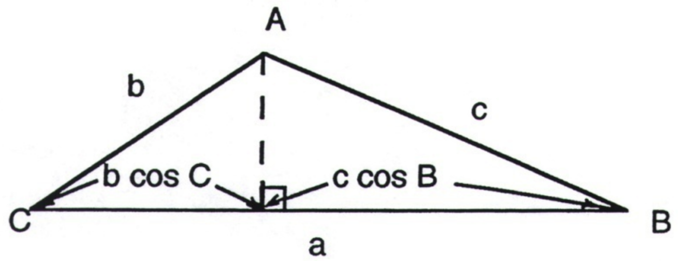


Figure: Triangle derivation of projection rule. Project A at 90° to the line CB

The above equations are sufficient to solve any plane triangle. In the past many other equations were generated by algebraic manipulation of the above equations. These other equations, which you might find in old surveying textbooks, were designed to make the calculations simpler when the multiplications etc had to be solved by using logarithmic tables.

Area of Triangle $= \frac{bh}{2} = \frac{bc\sin A}{2} = \frac{a^2 \sin B \sin C}{2\sin A} = \sqrt{s(s-a)(s-b)(s-c)}$ where s = (a+b+c)/2

The version with side lengths only (and s) is called Hero's formulae.
Remember to use cyclic changing (go around the triangle) to obtain other equations for area.

Solution of plane triangles

The three angles of a plane triangle are dependent, because their sum must always be 180° exactly. There are three different cases for the solution of an oblique plane triangle: 2 angles and 1 side are given; 1 angle and 2 sides are given; 3 sides are given. If we are given three angles only then the shape but not the size (or scale) of a plane triangle can be determined. There is often more than one method to solve any particular triangle but we will look at just one way for each case.

Given: Two angles and one side

Given B, C and a the third angle can be calculated from: A = 180° - (B + C)
Then b and c can be calculated from the sine rule.

A suitable check calculation follows. Choose the longest side and project the other two sides onto it. The two angles adjacent to the longest side of a plane triangle are always the smallest angles, this increases the significance of this check (see fig). Using the projection to one of the smaller sides might fail to detect an error in the side being checked.

Example
Remember that you should carry out as many calculation steps as possible in the machine, without transferring the numbers. The intermediate values are given here merely to aid you.

Given: B = 48°30'22" C = 45°47'48" a = 304.250m
then A = 85°41'50" from A = 180° - (B + C) and
b = 228.5354 and c = 218.7242 from the sine rule (note one more decimal place than final answer).

Check using the projection rule:
Since a is the longest side use a = b cosC + c cos B = 304.2499 which is acceptable as results will be rounded to mm level (because that is what a is given to) so then A = 85°41'50", b = 228.535 m, and c = 218.724 m.

Given: One angle and two sides

There are two cases here; one where the known angle is included between the two sides and the other where it is not.

a) Given two sides and the included angle i.e. A, b, and c.
Side a can be found by using the cosine rule. Then B and C can be found by using the sine rule.
Check calculations could be to see that the angles sum to 180° and use the projection rule.

Example
Given: A = 61°40'30", b = 375.44m and c = 278.20m
Then a = 345.305m, B = 73°09'18", and C = 45°10'12" check using the projection rule on b.
The value of b will disagree from the given value (after a certain number of digits). What is important is that this difference is due to rounding off and is not a mistake.

b) Given two sides and an opposite angle e.g. A, a, and c.
The difficulty with this case is that you don't know whether one of the other angles is greater than or less than 90° so there are two possible answers - see figure below. Unless you have some other information e.g. a plan or map of the triangle, then you have to give both answers. One way to solve the triangle is to use the sine rule and you will then find two values that satisfy the inverse sine, one in the first quadrant and one in the second. Then the cosine rule could be used for each possibility to find two values for side b. Naturally you should then apply check calculations using equations that you have not already used in the solution.

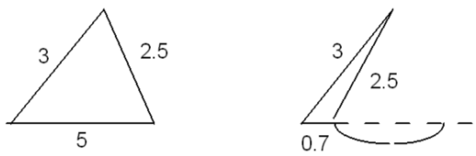


Figure. An example of two similar triangles.

<u>Given: three sides</u>

Use the cosine rule to solve for the angles. Then check that their sum is 180°.

Example:
a = 250.04m b = 360.72m c = 432.18m

Then A = 35°19'39"
B = 56°32'02"
C = <u>88°08'19"</u>
Σ = 180°00'00" ✓

Another way to solve some plane triangles is to use coordinate geometry. This will be discussed in the next section.

Coordinates, Bearings and Distances

Q: What are alternative methods to coordinates for describing where a place is or how to get there?

Most of the calculations in plane surveying (assumes a flat earth) can involve coordinates and these coordinates will be denoted by E (eastings) and N (northings). Bearings will be measured by the clockwise angle from the North direction from 0° to 360°. Note the difference to the mathematical system of x and y with angles measured anti-clockwise from the x direction (see figure below). If you treat the N axis as equivalent to the x axis of mathematics and the E axis as equivalent to the y axis of mathematics then you will not have any troubles when using calculators of computers designed for the mathematical system. When coordinates are quoted in these notes, the Easting coordinate will be given first and the Northing coordinate second. This is a widely adopted convention.

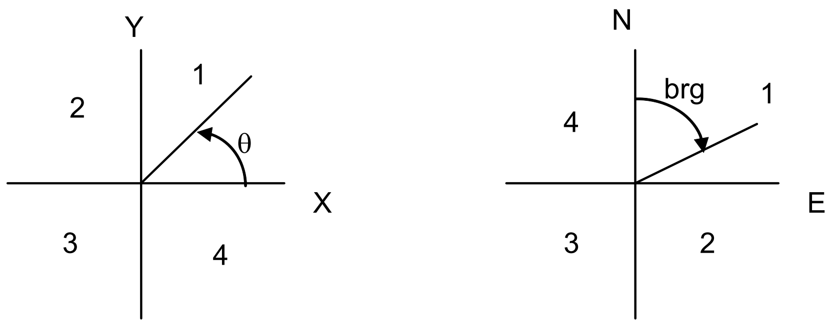


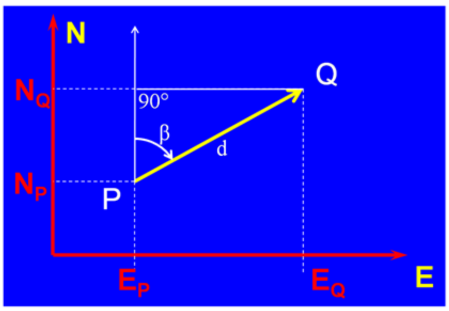
Fig. x y system of maths and E N system of surveying.

There are two basic computations in problems involving 2D coordinate systems in surveying. They are often called radiation and join.

Radiation

Given the coordinates of a point and the bearing and distance of a line, find the coordinates of the new point.

When dealing with computations involving a surveyed line PQ, the triangle used in the calculation is the right angled triangle which the line makes with the northing line through P and the easting line through Q. The orientation of the triangle will depend on which quadrant the line PQ is in. The length of the line is always taken as positive but the trigonometrical functions of the bearing may be either positive or negative.



Let the length of the line be d and the bearing β. The convention we adopt is that the line PQ means from P to Q and for the differences in coordinates you take 'to' site minus 'from' site.

Difference in Northings $\Delta N = N_Q - N_P = d_{PQ} \cos \beta_{PQ}$

Difference in Eastings $\Delta E = E_Q - E_P = d_{PQ} \sin \beta_{PQ}$

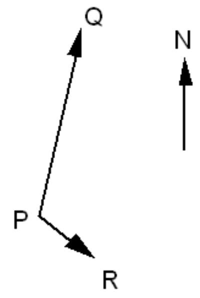
The coordinates of Q are given by: $E_Q = E_P + d_{PQ} \sin \beta_{PQ}$ and $N_Q = N_P + d_{PQ} \cos \beta_{PQ}$

Check:
The easiest check with modern machines is to calculate the coordinates of Q and then to do the reverse calculation (join, shown below) to see if you obtain the original bearing and distance.

Example:
Given P = (177 413.0, 446 111.0), β_{PQ} = 12°30'15" and d_{PQ} = 6235.42
then

$E_Q = E_P + d_{PQ} \sin \beta_{PQ}$ = 177 413.0 + 6235.42 sin(12°30'15") = 178 763.04
$N_Q = N_P + d_{PQ} \cos \beta_{PQ}$ = 446 111.0 + 6235.42 cos(12°30'15") = 452 198.52

so Q = (178 763.04, 452 198. 52)



If β_{PR} = 125°33'46" and d_{PR} = 2345.63
then $E_R = E_P + d_{PR} \sin \beta_{PR}$ = 177 413.0 + 2345.63 sin(125°33'46") = 179 321.12
and $N_R = N_P + d_{PR} \cos \beta_{PR}$ = 446 111.0 + 2345.63 cos(125°33'46") = 444 746.79

so R = (179 321.12, 444 746.79)

Spreadsheet: Radiations, coordinate calculations

Open a new sheet by clicking on a TAB at bottom of sheet. Double clicking the TAB at the bottom of a worksheet allows you to change its name to something meaningful. Enter headings, data and coordinates like this:

	A	B	C	D	E
5	Radiation				
6	E from				4062.455
7	N from				9769.092
8	Brg	14	23	27.00	
9	Dist				245.620
10	E to				
11	N to				

$$E_{to} = E_{from} + dist \sin(Brg) \qquad N_{to} = N_{from} + dist \cos(Brg)$$

Then enter equations into the appropriate cells to calculate coordinates. To do trigonometry the angles need to be converted from D M S to decimal degrees and then to radians because trig functions in Excel work in radians not degrees. The equations will need to look like (but cell names depend on where you put the data, and remember the = sign):
click in E8, i.e. where you want to put answer, then type: =B8+(C8/60)+(D8/3600)
similarly click in E10: =E6+E9*SIN(E8*PI()/180) OR = E6+E9*SIN(RADIANS(E8))
similarly click in E11: =E7+E9*COS(RADIANS(E8))

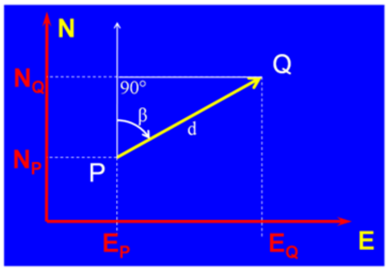
Answers:

	A	B	C	D	E
5	Radiation				
6	E from				4062.455
7	N from				9769.092
8	Brg	14	23	27.00	14.39
9	Dist				245.620
10	E to				4123.500
11	N to				10007.005

Now change some of your data (by clicking in a cell and entering new values) and watch the computer recompute the new answers.

Join

This is the reverse problem to radiation. Given the coordinates of two points find the bearing and distance of the line between them.



$$\beta_{PQ} = \tan^{-1}\left(\frac{E_Q - E_P}{N_Q - N_P}\right) = \tan^{-1}\left(\frac{\Delta E}{\Delta N}\right) \quad \text{or} \quad \beta_{PQ} = \cot^{-1}\left(\frac{N_Q - N_P}{E_Q - E_P}\right)$$

$$d_{PQ} = \sqrt{(E_Q - E_P)^2 + (N_Q - N_P)^2} = \sqrt{\Delta E^2 + \Delta N^2}$$

It should be noted that as tan X = tan (180° + X) the equations for β will give two values for β, i.e. it will give the bearing PQ and the bearing QP. A simple diagram will show which is correct.

Checks:

The length d could also be calculated from the formulae: $d_{PQ} = \frac{(E_Q - E_P)}{\sin \beta_{PQ}} = \frac{(N_Q - N_P)}{\cos \beta_{PQ}}$ provided β has already been calculated. This provides a check of formula for β and d above. If β is near north-south i.e. 0° or 180°, which equation for β should be used and which equation for d? Similarly, if β is near east-west i.e. 90° or 270°, which equations should you use? How important is it? (For some answers see the NSW Lands Dept ISG manual p43-45, but try to work it out yourself first).

Join Example 1:
Given S = (36 894.00, 45 321.00) and T = (37 375.904, 45 357.047)

then ΔE = 37 375.904 - 36 894.00 and ΔN = 45 357.047 - 45 321.00

so β_{ST} = 85°43'20" and d_{ST} = 483.25

Join Example 2:
Given S = (36 894.00, 45 321.00) and U = (36 657.868, 45 302.566)

then ΔE = 36 657.868 - 36 894.00 and ΔN = 45 302.566 - 45 321.00

so β_{SU} = 265°32'10" and d_{SU} = 236.85

Spreadsheet: Bearings and distances, coordinate calculations

An example of a join (B&D) calculation in MS Excel is shown below. Data has been placed in columns appropriate to the data and some columns have been hidden to make the equations easier to understand. Symbols and leading zeros are entered by custom formatting.

The equations used are:

	A	B	C	D	E	M	N	S
1	point 1				1000.000		3000.000	
2	point 2				1091.816		2975.52	
3								
4	Bearing 12	104.93		104°		55'		33.9"
5	Distance 12		95.02					
6								

Brg =MOD(DEGREES(ATAN2((N2-N1),(E2-E1))),360)
Distance =SQRT((E2-E1)^2+(N2-N1)^2)
Bearing, D =INT(B4)
Bearing, M =INT(B4*60-D4*60)
Bearing, S =(B4-D4)*3600-M4*60

Where:
SQRT calculates square root.
MOD(a,360) converts the value a into the range 0 to 360, if a is negative it adds 360, if a > 360 it subtracts 360.
DEGREES converts radians to degrees as the trig functions in Excel work in radians mode.
The INT function has been explained previously.

To calculate $\tan^{-1}$ you can use either ATAN(ΔE/ΔN) or ATAN2(ΔN, ΔE). Note the change in order of N and E in the Excel Atan2 function and the comma. ATAN2 is better because it places the bearing in the correct quadrant ATAN may not.

Using R → P and P → R on calculators

The Polar to rectangular coordinate conversion (P → R) and the rectangular to polar coordinate conversion (R → P) on many scientific calculators make survey calculations much easier. There are fewer steps to the solution which is therefore quicker and there is less chance of typographical error.

As we have said before, when you calculate bearing from $\tan^{-1}\left(\frac{\Delta E}{\Delta N}\right)$ it may not be in the correct quadrant. An important point to remember is that when the calculator function R → P is used the resultant bearing **is** in the correct quadrant. However the bearing is usually in the range -180° to + 180°. To put the bearing in the surveying convention of 0° to 360° you simply add 360° to any negative bearing.

If you enter the E and N coordinates (or X and Y) in the wrong order when using the →P function then the resultant distance will be correct but your bearing will be wrong.

The necessary steps on an RPN calculator and on an algebraic calculator are given in the table below. The values in *italics* are those output by the machine. The keystrokes may be different on your own calculator, so make sure you know how to use yours!

R → P on an RPN calculator ΔE Enter ΔN R→P *distance* x↔y *bearing*	Algebraic Casio fx 911w calculator R→P Pol(ΔN , ΔE) = *distance* RCL F *bearing* (add 360° if –ve) Note: Memory E contains d, while F contains θ.
P → R on an RPN bearing Enter distance P→R *ΔN* x↔y *ΔE*	P→R Rec(d , θ) = *ΔN* RCL F *ΔE* Note: Memory E contains ΔN, while F contains ΔE.

In the above steps bearings are input and output in degrees (on some calculators check that degree mode has been set). If you want output bearing in degrees, minutes, seconds you have to convert them.

There are other applications of P→R and R→P in surveying. For example the calculation of horizontal distance and height difference given slope angle and slope distance. Determine the keystrokes yourself.

Example: *Bearing and Distance From Coordinates (by Adam Long and Bruce Harvey)*

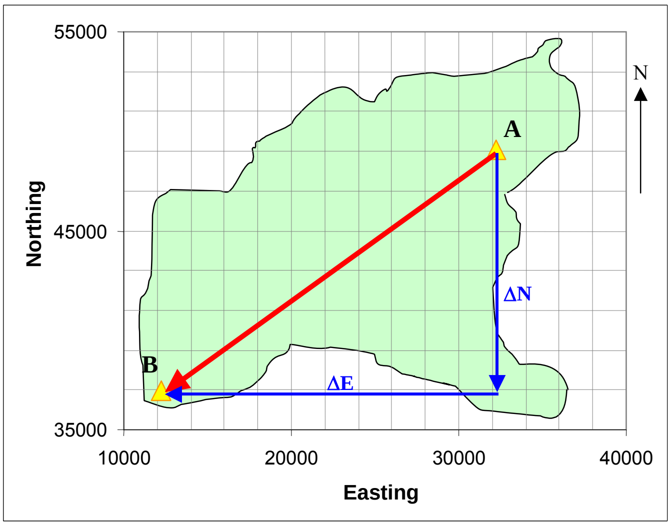
Two points have the following coordinates:

Point	Easting (m)	Northing (m)
A	32255.751	49076.286
B	12231.864	36939.667

Calculate the bearing A to B and plan distance AB.

Solution:

Visualise the problem, draw a figure.



Now find the ΔE and ΔN:

Note: To find the bearing of A to B we take B coordinates minus A

$\Delta E = E_B - E_A$ = 12231.864 - 32255.751 = -20023.887 m

$\Delta N = N_B - N_A$ = 36939.667 - 49076.286 = -12136.619 m

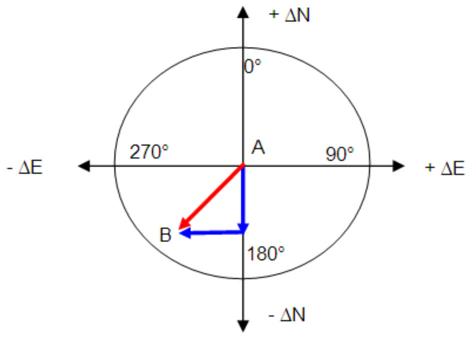
Using Pythagoras's Theorem to solve d_{AB}:

$$d_{AB}^2 = \Delta E^2 + \Delta N^2 = -20023.887^2 + -12136.619^2 \qquad d_{AB} = \sqrt{-20023.887^2 + -12136.619^2}$$

d_{AB} = 23 414.815 m

Now for the Bearing β: $\beta = \tan^{-1}\left(\frac{\Delta E}{\Delta N}\right)$

The signs of ΔE and ΔN will determine the quadrant of the bearing. The diagram shows that our bearing will be in the 3rd quadrant, between 180° and 270°



$$\beta = \tan^{-1}\left(\frac{-20023.887}{-12136.619}\right) = 58.77965125°$$

This is in the wrong quadrant and we have to add 180°

58.77965125° + 180° = 238.77965125°

To convert to degrees, minutes and seconds:

The degrees are just the integer of 238.77965125:

Degrees = 238°

Minutes = (238.77965125 - 238) × 60 = 46.779075 (take the integer of this) = 46'

Seconds = $(238.77965125 - 238 - \frac{46}{60}) \times 3600$ = 46.7"

Therefore: **β = 238°46'46.7"**

Using a calculator

This problem can easily be solved using the rectangular to polar function on a CASIO fx-911W calculator.

1. Enter [Pol(] ΔN [,] ΔE [)] [=]:

   [Pol(] -12136.619 [,] -20023.887 [)] [=]

Note: Distance is stored in memory E and bearing is stored in memory F

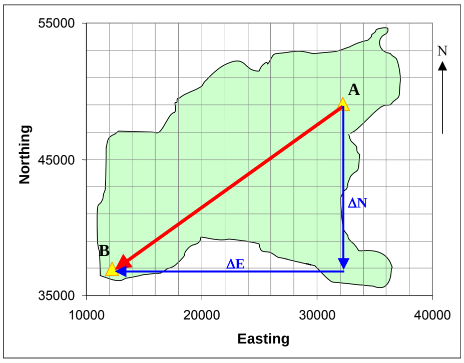
2. The first number to be displayed is the distance: 23414.815 m

3. Now press [RCL] and [F] to display the bearing: -121.2203487 °
When using this function the bearing is always in the correct quadrant.

4. If your bearing is negative just add 360°. -121.2203487 + 360 [=] 238.77965125 °

5. To display the bearing in degrees, minutes and seconds press [SHIFT] [° ’ ”]: 238°46’46.7”

Check by Reverse Solution:



We can check our result by working backwards starting at point A and calculating the coordinates of B using the derived bearing and distance.

$$\Delta E_{AB} = d_{AB} \sin \beta_{AB}$$

= 23414.815 × sin(238°46’46.7”)
= -20023.884 m

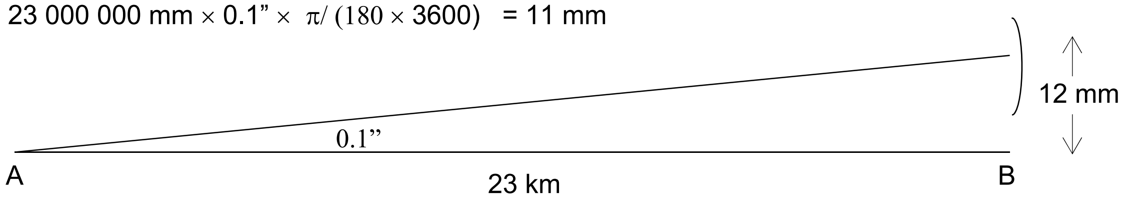
$$\Delta N_{AB} = d_{AB} \cos \beta_{AB}$$

= 23414.815 × cos(238°46’46.7”) = -12136.623 m

Coordinates of B:
E_B = 32255.751 + -20023.884 = 12231.867 m
N_B = 49076.286 + -12136.623 = 36939.663 m

Now you will notice that the easting and northing of B differs by 3 mm and 4 mm respectively. This is due to the rounding of our bearing (0.1” error over our 23 km line represents about 11 mm).

$\ell = r \times \theta$
$= 23\,000\,000 \text{ mm} \times 0.1'' \times \pi / (180 \times 3600) = 11 \text{ mm}$



Also consider that A and B are 23 km apart, which is too far to assume the earth is flat and too far to use plane trigonometry. To improve this result we should use more advanced calculations, such as spherical trigonometry, with the coordinates in latitude and longitude. Other methods include ellipsoidal geometry or map projection system coordinate calculations. These are covered in other subjects.

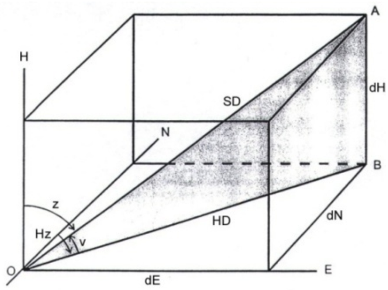
Excel Solution:

	A	B	C	D	E	F
1	E_A	32255.751				
2	E_B	12231.864				
3	N_A	49076.286				
4	N_B	36939.667				
5	ΔE_{AB}	-20023.887				
6	ΔN_{AB}	-12136.619				
7	d_{AB}	23414.815				
8	β_{AB}	238.7796513	=	238	46	46.7

=B2-B1				
=B4-B3				
=SQRT(B6*B6+B5*B5)				
=MOD(DEGREES(ATAN2(B6,B5)),360)	=	=INT(B8)	=INT((B8-D8)*60)	=(B8-D8-E8/60)*3600

3D Calculations

Calculations in 3D i.e. XYZ or East, North, Height are a simple extension of the above concepts. Slope distance is the distance between two points.



Slope distance = $\sqrt{\Delta E^2 + \Delta N^2 + \Delta H^2}$

Horizontal distance = $\sqrt{\Delta E^2 + \Delta N^2}$

Bearing is calculated with the same equations as for 2D above.

Slope angle (from a horizontal plane)

$$= \tan^{-1}\frac{\Delta H}{\text{horizontal distance}} = \sin^{-1}\frac{\Delta H}{\text{slope distance}}$$

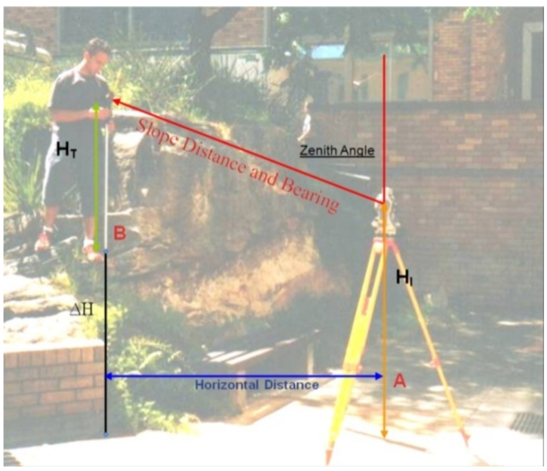
Zenith angle (from the vertical) $= \tan^{-1}\frac{\text{horizontal distance}}{\Delta H} = \cos^{-1}\frac{\Delta H}{\text{slope distance}}$

R →P and atan2 functions can be useful for ZA calculations, as well as for bearings. The zenith angle formula that divides by ΔH can be unstable. Horizontal lines (with ZA near 90°) have ΔH = 0! So for lines nearly horizontal, which are quite common, use the alternative cosine formula.

Some of the tutorial problems below give you practice at 3D calculations. These 3D calculations assume a plane coordinate system - you will learn more about real world coordinate systems in later subjects.

Example: *3D Detail Survey Calculations (by Adam Long and Bruce Harvey)*

A detail survey was carried out using a number of radiations from a total station set up on Station A to a target placed on an area of interest. Station A has the following coordinates, in metres on a local plane datum:



Point	Easting	Northing	Height
A	255.751	176.286	42.623

The instrument is set up at a height of 1.565m (h_i) and the target pole is set at a height (h_t) of 1.690m. A radiation was made to a feature labelled B:

Bearing (β): 240°25'20"
Slope Distance (SD): 11.682 m
Zenith Angle (ZA): 93°22'30"

Calculate the coordinates of B including its height.
Visualise the Problem →

Calculation

The first thing to do is calculate the Easting and Northing of B using the methods described in previous example "Bearing and Distance From Coordinates". Here's how we do it:

Firstly we have to convert the slope distance into a horizontal plane distance:

HD = SD × sin (ZA) = 11.682 × sin (93°22'30")
= 11.662 m

Now calculate the ΔE and ΔN using your calculator (the key strokes may be different on your calculator):

6. Enter [Rec(] HD [,] β [)] [=]:

   [Rec(] 11.662 [,] 240 [° ' "] 25 [° ' "] 20 [° ' "] [)] [=]

7. The first number to be displayed is the ΔN: -5.756

8. Now press [RCL] and [F] to display the ΔE: -10.142

Do these values seem correct? Look at the bearing 240°25'20", this is in the SW quadrant which has both negative ΔE and ΔN, so yes it does seem to be correct.

Coordinates of B:

E_B = 255.751 + (-10.142) = 245.609 m
N_B = 176.286 + (-5.756) = 170.530 m

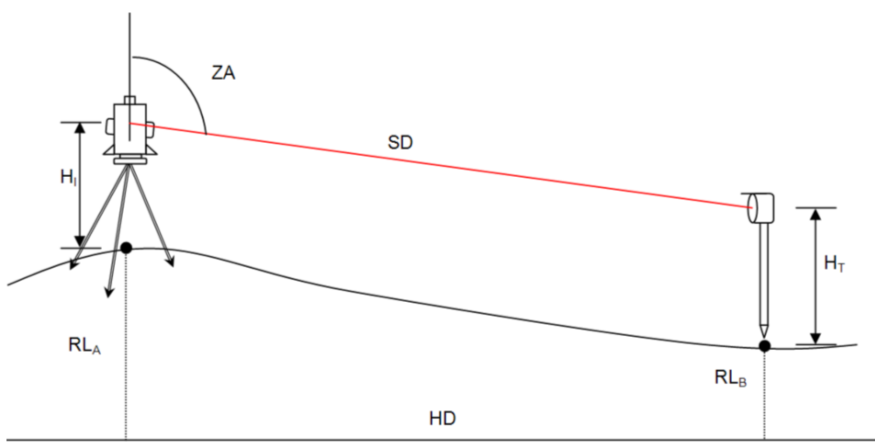
Now we have to calculate the reduced Level (RL) of B. The formula used to calculate this is as follows:

$$RL_B = RL_A + H_i + SD \times \cos(ZA) - H_t$$

Where:

RL_B is the reduced level of B
RL_A is the reduced level of A
H_i is the height of the instrument at A
H_t is the height of the reflector target at B
SD is the slope distance from A to B; and
ZA is the Zenith Angle

Picture the side on view showing all components:



Now we just substitute our values into the equation:

$RL_B = RL_A + H_i + SD \times \cos(ZA) - H_t$ = 42.623 + 1.565 + 11.682 × cos(93°22'30") - 1.690 = 41.812 m

Does this seem correct?

Look at the zenith angle, it is greater than 90° which means that the instrument is observing downhill, thus the RL of B is less than the RL of A. This is not true in all cases though. Consider that the height of target could be 2 metres and the instrument only 1.5 metres, on level ground the instrument will be observing at an inclination.

Therefore the coordinates of B in metres are:

Point	Easting	Northing	Height
	245.609	170.530	41.812

Check

The usual way to check the coordinates of a point is to measure to it twice, from different stations. Then a join is calculated between the two versions of the measured point. The join distance should be small. To check the horizontal coordinates for this example we will just calculate the join between A and B and check that against the given bearing and distance. For the vertical plane we will substitute our calculated RL of B into the formula and compare it to the given RL of A.

Firstly the horizontal plane. The join from A to B. $\Delta N_{AB} = N_B - N_A = -5.756$

Enter [Pol(] ΔN [,] ΔE [)] [=] i.e. [Pol(] -5.756 [,] -10.142 [)] [=]

> On the calculator we used: Distance is stored in memory E and bearing is stored in memory F

The first number to be displayed is the distance: 11.662 m

Now press [RCL] and [F] to display the bearing: 240.4233162°
(When using this function the bearing is always in the correct quadrant)

To display the bearing in degrees, minutes and seconds press [SHIFT] [° ' "]: 240°25'23"

You can see that the bearing checks within 3 seconds, however the distance is out by 20mm. This is because we haven't converted it into a slope distance.

$$SD = \frac{HD}{\sin(ZA)} = \frac{11.662}{\sin(93°22'30")} = 11.682 \text{ m.}$$ Which is correct.

For another check we could also calculate SD from the coordinates of A and B as follows:

$$SD = \sqrt{(E_B - E_A)^2 + (N_B - N_A)^2 + (H_B + h_t - H_A - h_i)^2}$$

Now substitute the RL of B into the equation below:

$$RL_B = RL_A + H_i + SD \times \cos(ZA) - H_t$$
$$41.812 = RL_A + 1.565 + 11.682 \times \cos(93°22'30") - 1.690$$
$$RL_A = 41.812 - 1.565 - 11.682 \times \cos(93°22'30") + 1.690 = 42.623 \text{ m}$$ Which is correct.

Excel Example:

	A	B	C	D	E	F	G	H	J	K	L
1											
2		Instrument Station			Measurements				Target		
3											
4						°	'	"			
5					Bearing	240	25	20			
6		Coordinates of A			Zenith Angle	93	22	3	Cordinates of B		
7		E	N	H					E	N	H
8		255.751	176.286	42.623	Slope Distance	11.682			245.609	170.530	41.812
9					HI	1.565					
10					HT	1.690					
11					Horizontal Distance	11.662					

Formulas in the calculated cells:

I5 =F5+G5/60+H5/3600
I6 =F6+G6/60+H6/3600
F11 =F8*SIN(RADIANS(I6))
J8 =B8+F11*SIN(RADIANS(I5))
K8 =C8+F11*COS(RADIANS(I5))
L8 =D8+F9+F8*COS(RADIANS(I6))-F10

Computer programming in surveying

Computer programming is not a part of this course, but a few comments are included here. Current students are welcome to submit source code or example programs for any of the survey computations problems in this course.

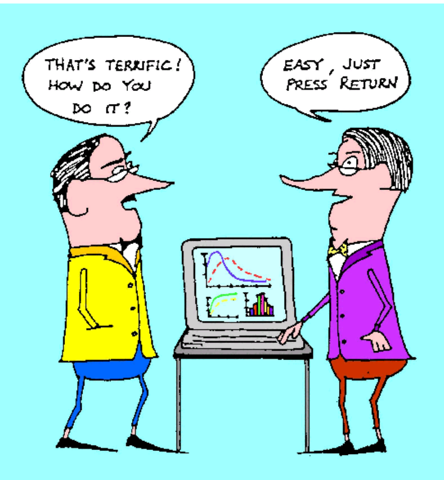
Why should we learn computer programming?

- To write custom made programs to do survey calculations and plots when there are no other programs on the market to do the job, e.g. a special or unusual job / task / survey.
- To write macros (sub programs) in Excel or in GIS or web pages
- To modify data files on computer (eg instrument download) without manual editing and transcription errors.
- You become more confident when using computer software written by others
- So that you know exactly what equations, functions and assumptions are used in the software.
- So that you can customise the program for your own data types or formats or procedures, so easier to use.
- Can debug your own program, can't debug someone else's if an error is found.
- Having a debugged program reduces the risk of mistakes that may happen in one off calculations.
- An input form ensures all data is booked/recorded/entered.
- Can produce graphics (eg plans) as well as calculations – calculators can't.
- To have copyright and to sell program. An extra income source. Diversification of firm or individual.
- Even if you never need your own software then learning programming helps problem solving skills, needed in many jobs. It also builds your abilities in thoroughness, rigour and attention to detail that is required e.g. in cadastral surveying and project management. If your job and your expertise is mainly report writing, presentations, discussion etc then you might "waffle over" or avoid some things but in programming you learn the need to get the details correct and not to omit required content.

Edited Comments from a graduate, Stephen Mitchell in 2002

I developed programs because of what I saw as a need at work to automate more tasks. I think that we could benefit dramatically by fully customising our software to suit our particular needs. Programmed routines can drastically improve CAD productivity by simplifying repetitive and complex tasks. I have developed these routines to be used by others in the office and to make their job easier. My CAD skills have greatly improved since writing these routines and I now spend part of my time training others in the office.

I have written a program to calculate ridge heights. This program is actually a series of functions and subroutines which use a MS Excel front end. The repetitive nature of the calculations makes it an ideal task to be automated using a program. Using a program reduces the risk of making a mistake in the calculations. Care still needs to be taken ensuring the data is entered correctly. The program input page is used as a form in the field to record observations. Forms help to ensure that all the required information is booked, and introduces a standardised way of booking observations. It also reduces the possibility of forgetting to book a vital component such as the height of the instrument etc. This is then transferred to the same form in the spreadsheet. Then by simply clicking "Calculate Ridges" all the remaining data including the ridge RL's are calculated. A program also makes it possible for non-surveying staff to perform the calculations. It is still the surveyor's responsibility to ensure that the data has been transferred from the form to the program correctly.



Summary

• Calculations should be set-out in a neat and logical way.

• Extreme care has to be exercised when transferring numbers e.g. from paper into a calculator or computer or vice versa, or from one page to another.

• Include sufficient explanation / comments in your calculations so they can be interpreted by another person and also by yourself at any time later.

• Measurements are estimates of real numbers and are never perfectly accurate.

• If possible you should not round off intermediate values in your calculations (store them in machine's memory), just round off the answer. Do not give insignificant digits in the results.

• When calculating a mean and standard deviation of a set of measurements write the standard deviation to 1 or 2 significant figures and the mean to no more than the number of decimal places in the standard deviation.

• Calculators and computers have underflow, overflow, accuracy and speed limits.

• To convert any negative angle to the survey convention, simply add 360°.

• To convert an angle in degrees to radians: $\theta^{c} = \theta^{\circ} \dfrac{\pi}{180}$

• To convert an angle in seconds to radians: $\theta^{c} = \theta^{\circ} \dfrac{\pi}{(180 * 3600)} \approx \dfrac{\theta''}{206264.8}$

• For inverse trig functions there are two possible answers, calculators and computers only give one of the possible answers.

• For $\tan^{-1}\left(\dfrac{E}{N}\right)$ where a and b are known positive or negative you can say which quadrant the vector or angle is in. In spreadsheets use the function atan2(N,E)

• When solving triangles you should apply check calculations using equations that you have not already used in the solution

• In these notes the coordinates are in the order E,N. The line PQ means from P to Q and for the differences in coordinates you take 'to' site minus 'from' site.

• Bearings are measured by the clockwise angle from North, 0° to 360°.

• Radiation: Equations for bearing and distance → coordinates.

• Join: The reverse problem to radiation. Equations for coordinates of two points → bearing and distance of the line between them. The spreadsheet equations used are:

Brg (in degs)	=MOD(DEGREES(ATAN2((N2-N1),(E2-E1))),360)
Distance	=SQRT((E2-E1)^2+(N2-N1)^2)
Bearing, D	=INT(B4)
Bearing, M	=INT(B4*60-D4*60)
Bearing, S	=(B4-D4)*3600-M4*60

• The easiest check of radiations and joins is to use the reverse calculation to obtain the original data.

• The P → R and R → P on many calculators and ATAN2, MOD & DEGREES functions in MS Excel, make the radiation and join calculations much simpler. When the calculator function R → P or Excel's ATAN2 function is used the resultant bearing is in the correct quadrant.

1. Tutorial Problems

Do the following problems by various 'machines': calculator, spreadsheet, phone apps, by writing your own program, maybe even slide rule or log tables for some. Please push beyond your comfort zone and don't just use your favourite calculator for every problem. Learn something new! Later you may use CAD or total station on board calculations. Form opinions about which is the best machine/software for different problems. You may choose to submit word documents with worked solutions for some questions. Another option is to go through this chapter and add links to your first year surveying textbook.

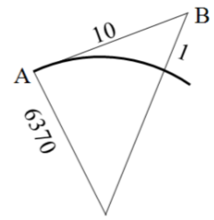
Q1. Calculate the mean and standard deviation of the angles Z_A and Z_B.

Z_A	Z_B
88° 46' 25.6"	92° 59' 58.7"
88° 46' 24.3"	92° 59' 57.6"
88° 46' 25.2"	92° 59' 59.1"
88° 46' 25.4"	92° 59' 58.9"
88° 46' 26.3"	92° 59' 59.2"
88° 46' 26.5"	92° 59' 59.7"
88° 46' 27.1"	93° 00' 00.3"
88° 46' 25.5"	93° 00' 00.2"
88° 46' 27.0"	93° 00' 00.5"

Do calculations with arbitrary origins (e.g. subtract 88°46' from ZA) and then repeat without the origins (i.e. with the full D M S value for each measurement. Calculate with both Excel and your calculator and compare results. Use the formulas given above and compare with the results from the built-in functions.

Q2. Given Δx = 9 546 379.218 Δy = -14 502.736 Δz = 905 684.842

Find $D = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$. Is it seriously affected by machine precision in your calculator or spreadsheet? Why? If the problem exists how can you avoid it?



Q3. In the figure at right point A is at sea level and point B is inland at a height of 1km above sea level. The straight line distance between A and B is 10km. If the radius of the earth is 6370 km, then what is the arc length at sea level from A to the point directly below B? Assume the earth is spherical. The figure at right is not to scale and the angle at A is not 90°.

Q4. DMS input and conversion. Theodolite measurement to a target was FL: 37°16' 20" and FR: 217°15' 50". Calculate the 'mean' direction using (FL + FR ± 180)/2. Do it by mental arithmetic, then on your calculator and by Excel. In Excel do it twice, firstly enter D M S in three separate cells, secondly enter in D.MS format. In both cases in Excel you need to convert the directions to D.D format, and then convert the answer (mean) to D M S and/or D.MS format.

Trigonometry

Q5. a) Convert 5" into radians. b) Convert 5° into radians.

Q6. Solve for side a in the triangle that has: A = 5", b = 23.456, c = 19.234

Q7. To determine the width of a river, zenith angles (Z_T and Z_B) were measured from the point A on one side of a river, to the top and base of a tower on the other side of the river. Calculate the width of the river (W). Check your solution. Data: Height of tower TB = 47.30 m Z_T = 61°20'30" Z_B = 71°40'25"

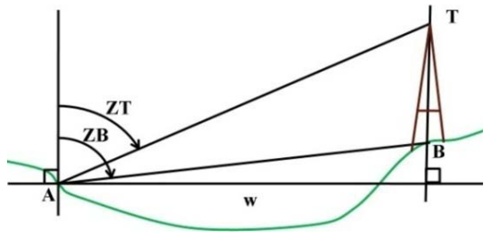
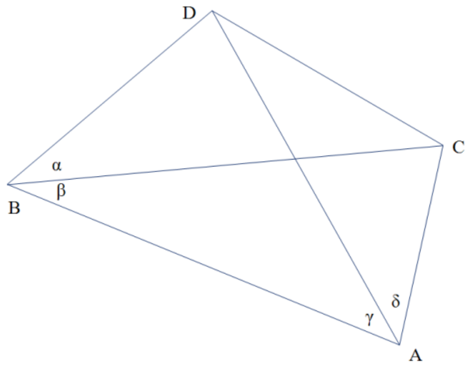


Fig. Cross section / elevation view:

Q8A. *Braced Quadrilateral.* Four of the angles in a braced quadrilateral were measured and the distance BC was calculated from coordinates. Calculate the distance AD to the nearest cm.

Data: BC = 483.06 m α = 52°41'24" β = 34°19'27"
γ = 42°18'42" δ = 41°42'47"



Q8B. (challenging) The Hansen Problem. Four of the angles in a braced quadrilateral were measured and the distance DC was calculated from coordinates. Calculate the other distances.

Data: DC = 438.50 m α = 35°04'36" β = 27°14'02"
γ = 37°56'44" δ = 41°49'12"

Coordinates

Independently check your results in this section; you should not need to rely on the answers given below.

Q9. Given that the coordinates of three points A, B and C are

	E(m)	N(m)
A	2614.162	1086.215
B	1932.783	1399.554
C	2174.368	1206.173

Find the bearings and distances AB, AC, BC and the internal angles of the triangle ABC.

Q10. The bearing of the line OA is 318°16' and the included angle, measured clockwise from OA, is angle AOB = 331°14'. The coordinates of O are (100.00, 200.00) and the distance OB is 250.00m. Calculate the bearing of OB and the coordinates of B.

Q11. Given: Bearing BA = 15°25'35"
Distance AB = 123.455 m
Angle BAC = 271° 17' 51"
Distance AC = 100.00 m.

Assuming B to be the origin of the coordinate system calculate:

(a) bearing of AC
(b) coordinates of A and C
(c) bearing and distance of BC check your results by another method.

Q12. Given:

Station	Eastings(m)	Northings(m)	Height(m)
P	52381.72	12381.91	72.81
Q	52712.11	12757.55	250.81

Compute: Bearing of P from Q. Horizontal distance PQ. Slope distance PQ. Slope angle from P to Q

Q13. Two points have the following 3D coordinates:

Point	Easting	Northing	Height
A	100 032.751	50 076.286	225.973
B	100 121.864	49 939.667	185.401

For the line A to B, calculate the bearing, horizontal (plan) distance, zenith angle, and slope distance.

Q14. A detail survey was carried out using a number of radiations from a total station set-up on Station A to a reflector target placed on an area of interest. Station A has the following coordinates (E, N, H) in metres on a local plane datum: (301.245, 299.215, 35.214). The instrument is set up at a height of 1.565 m (h_i) and the reflector target is set at a height (h_t) of 1.690 m. A radiation was made to a feature

labelled B:
Bearing (β): 160°25’20”   Slope Distance (SD): 15.162 m   Zenith Angle (ZA): 101°44’30”
Calculate the coordinates of B, including its height. Check your answer.

Spreadsheet

Q15: Calculate the grey values from the measured black values in the following levelling calculations. Use conditional formatting to colour red the negative numbers in the Rise/Fall column.

BS	IS	FS	Rise / Fall	RL	Remarks
2.857				45.568	BM
0.356		1.988	0.869	46.437	CP1
	0.65		-0.294	46.143	K
	2.11		-1.460	44.683	L
	3.04		-0.930	43.753	M
4.325		3.125	-0.085	43.668	N
	1.85		2.475	46.143	K
4.037		1.556	0.294	46.437	CP1
		4.907	-0.870	45.567	BM RL45.568
11.575		11.576	-0.001		Sum
		-0.001		-1E-03	Checks

Q16: Calculate the blue values from the measured black values in the following theodolite calculations. For direction calculations below you may need to add extra columns for intermediate steps that calculate the directions in decimal degrees, and then hide those columns. The formula for mean of FL and FR is like: =(E3 +MOD(I3-180,360))/2 where E3 is the FL direction in decimal degrees and I3 is the FR direction in decimal degrees.

Pt	FL			FR			Mean			Red mean			Grand mean		
Obs	d	m	s	d	m	s	d	m	s	d	m	s	d	m	s
Q	0	04	30	180	03	30	0	04	00	0	00	00	0	00	00
R	37	16	20	217	15	50	37	16	05	37	12	05	37	12	27
S	193	42	30	13	42	20	193	42	25	193	38	25	193	37	60
T	216	25	50	36	24	50	216	25	20	216	21	20	216	18	48
Q	90	15	40	270	14	10	90	14	55	0	00	00			
R	127	28	00	307	27	30	127	27	45	37	12	50			
S	283	53	00	103	52	00	283	52	30	193	37	35			
T	306	36	50	126	25	30	306	31	10	216	16	15			

Checking

Q17: Imagine a student setup an instrument on a tripod over a survey mark and measured the height of instrument with a metric tape as 1.712 m. To guard against transcription errors in reading, recording or entering the height into a computer for further computations, she also used a tape graduated in feet and inches. To be even more careful to guard against errors another student took a photo of the tape reading with his mobile phone camera, for later confirmation of the measurement. Use the part of the photo shown here to make your own reading. Then convert it to metric using two different conversion constants (one for inches to cm and the other for feet to metres). What is the difference in mm between the metric tape reading and the converted feet and inches reading?

[A big 3D control survey network or extensive detail survey with total station or with GPS can be ‘damaged’ by errors in height of instrument.]

Chapter 1. Tutorial Solutions

These solutions can be read AFTER you have made some attempt to solve the question.

Q1.
On my old calculator: mean ZA = 88°46'25.9", s_{ZA} = ±4.0" (which is wrong)
Using xa = 88°46' → mean ZA = 88°46'25.9", s_{ZA} = ±0.9" [note large difference in standard deviation!]
mean ZB = 92°59'59.4", s_{ZB} = ±0.0" (which is wrong)
Using xa = 92°59' (last three obs are: 60.3", 60.2", 60.5"): ZB = 92°59'59.4", s_{ZB} = ±0.9"
Eqn 5 without arbitrary origin xa gave incorrect answers on my calculator for this data set.

So be careful when using calculators for cases like these examples. The round off problem is overcome when we use a suitable constant.

In the current version of Excel, the answers are the same whichever method is used. There is no significant error due to round off. The spreadsheet calculations:

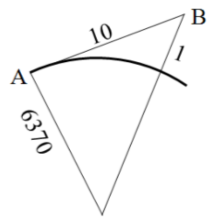
d	m	s	d.d	v in secs	v^2		d	m	s	d.d	new s	v in secs	v^2
88	46	25.6	88.77	-0.28	0.077		92	59	58.7	93.00	58.7	-0.66	0.430
88	46	24.3	88.77	-1.58	2.489		92	59	57.6	93.00	57.6	-1.76	3.082
88	46	25.2	88.77	-0.68	0.459		92	59	59.1	93.00	59.1	-0.26	0.065
88	46	25.4	88.77	-0.48	0.228		92	59	58.9	93.00	58.9	-0.46	0.208
88	46	26.3	88.77	0.42	0.178		92	59	59.2	93.00	59.2	-0.16	0.024
88	46	26.5	88.77	0.62	0.387		92	59	59.7	93.00	59.7	0.34	0.119
88	46	27.1	88.77	1.22	1.494		93	0	0.3	93.00	60.3	0.94	0.892
88	46	25.5	88.77	-0.38	0.143		93	0	0.2	93.00	60.2	0.84	0.713
88	46	27.0	88.77	1.12	1.259		93	0	0.5	93.00	60.5	1.14	1.310
count		9		6.716	6.716		count		9			6.842	6.842
S in “		0.9162	0.9162		**0.9162**					0.9248	0.9248		**0.9248**
88	46	25.878	88.77	Mean			92	59	59.36	93.00	59.36	Mean	

The spreadsheet with Formulas shown:

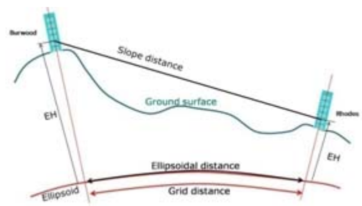
	A	B	C	D	E	F	G	H	I	J	K	L	M
2	d	m	s	d.d	v in secs	v sqd	d	m	s	d.d	new s	v in secs	v sqd
3	88	46	25.6	=A3+B3/60+C3/3600	=C3-C$14	=E3*E3	92	59	58.7	=G3+H3/60+I3/3600	=I3	=K3-K$14	=L3*L3
4	88	46	24.3	=A4+B4/60+C4/3600	=C4-C$14	=E4*E4	92	59	57.6	=G4+H4/60+I4/3600	=I4	=K4-K$14	=L4*L4
5	88	46	25.2	=A5+B5/60+C5/3600	=C5-C$14	=E5*E5	92	59	59.1	=G5+H5/60+I5/3600	=I5	=K5-K$14	=L5*L5
6	88	46	25.4	=A6+B6/60+C6/3600	=C6-C$14	=E6*E6	92	59	58.9	=G6+H6/60+I6/3600	=I6	=K6-K$14	=L6*L6
7	88	46	26.3	=A7+B7/60+C7/3600	=C7-C$14	=E7*E7	92	59	59.2	=G7+H7/60+I7/3600	=I7	=K7-K$14	=L7*L7
8	88	46	26.5	=A8+B8/60+C8/3600	=C8-C$14	=E8*E8	92	59	59.7	=G8+H8/60+I8/3600	=I8	=K8-K$14	=L8*L8
9	88	46	27.1	=A9+B9/60+C9/3600	=C9-C$14	=E9*E9	93	0	0.3	=G9+H9/60+I9/3600	=I9+60	=K9-K$14	=L9*L9
10	88	46	25.5	=A10+B10/60+C10/3600	=C10-C$14	=E10*E10	93	0	0.2	=G10+H10/60+I10/360	=I10+60	=K10-K$14	=L10*L10

										0			
11	88	46	27	=A11+B11/60+C11/3600	=C11-C$14	=E11*E11	93	0	0.5	=G11+H11/60+I11/3600	=I11+60	=K11-K$14	=L11*L11
12	count		=COUNT(C3:C11)		=SUMSQ(E3:E11)	=SUM(F3:F11)	count		=COUNT(I3:I11)			=SUMSQ(L3:L11)	=SUM(M3:M11)
13	S in "		=STDEV(C3:C11)	=3600*STDEV(D3:D11)		**=SQRT(F12/(C12-1))**				=3600*STDEV(J3:J11)	=STDEV(K3:K11)		**=SQRT(M12/(I12-1))**
14	88	46	=AVERAGE(C3:C11)	=AVERAGE(D3:D11)	Mean		92	59	=K14	=AVERAGE(J3:J11)	=AVERAGE(K3:K11)	Mean	

Q2.
If the data is put in spreadsheet columns B17, B18, B19 then the formula for D is =SQRT(B17*B17+B18*B18+B19*B19) and D = 9 589 256.047...
I got the same result with my calculator; there were no problems. I can check it by writing a program and doing the calculations with double precision. The squaring can be checked by multiplying by hand the value times itself (on paper, wow!) but the square root is not so easy to check. One way to check the square root is to reverse it. Does $D^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$? That is, take the answer for D above and square it and compare it to the sum of the x y z squares. It did OK for me in Excel, but it carries only about 15 places for each number so it is not perfect, but is good enough for this data.



Q3.
By the cosine rule and then $\ell = r\theta$:
Let a = 10, b = 6370, c = 6371 then $b^2+c^2-a^2$ = 81166441 and 2bc = 81166540
So cos A = 81166441/81166540 = 0.99999878, then A = 0.001561867 radians (=0.089... degrees)
Length = 6370 * A in radians = 9.949094 km



An alternative method from geodesy follows. A sphere is a special case of an ellipse, where the radius is same in all directions.
Reduction of slope distance (Ds), to chord distance (D3):
$D3 = [(Ds^2 - (hA - hB)^2) / (1+ hA/R\alpha)(1 + hB/R\alpha)]^{½}$ = 9.949 093km
where hA = 0, hB = 1, Rα = 6370, Ds = 10k m
Then D3 is reduced to ellipsoidal distance De:
$De = D3[1 + (D3^2/24R\alpha^2 + 3D3^4/640R\alpha^4 + ...]$ = 9.949 094km
which is the same as by cosine rule method.

Q4.
37°16' 05" by mental arithmetic, in my head.

Excel numbers:

D	M	S		D.D	D.MS	D	M	S	D.MS
37	16	20		37.27222222	37.1620				
217	15	50		217.2638889	217.1550				
				37.26805556		**37**	**16**	**05.00**	37.1605

Excel Formulas

	A	B	C	D	E	F	G	H	I	J
27	37	16	20		=A27+B27/60+C27/3600	=A27+B27/100+C27/10000				
28	217	15	50		=A28+B28/60+C28/3600	=A28+B28/100+C28/10000				
29					=(E27+E28-180)/2		=TRUNC(E29)	=TRUNC(E29*60-G29*60)	=((E29-G29)*60-H29)*60	=G29+H29/100+I29/10000

For Q5 – 8, you should be able to check all your answers yourself either by two independent methods or by reversing the calculations (that is, use your answers to obtain the given data). Why did I round off my answers to these values?

Q5. a) 2.4×10^{-5} b) 0.087

secs into radians	5	2.42407E-05	degs	5	0.087266463
reverse		5	reverse		5

	C	D	E	F	G	H
32	secs into radians	5	=RADIANS(D32/3600)	degs	5	=RADIANS(G32)
33	reverse		=3600*DEGREES(E32)	reverse		=DEGREES(H32)

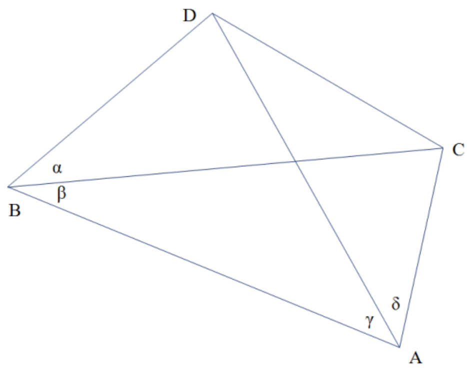
Instead of the Radians function we could use =D32*PI()/(180*3600) and =G32*PI()/180
Instead of the Degrees function we could use =E32*180*3600/PI() and =G32*180/PI()

Why round off? Calculate the radians value for 4.5" and 5.5", i.e. the comparable limits of round off, then round the radians value to an appropriate number of significant figures.

Q6.
Cosine rule as implemented in spreadsheets: a = SQRT(b^2+c^2-2*b*c*COS(RADIANS(A"/3600))) = 4.222
Check by reverse calculation A in " = 3600*DEGREES(ACOS((b^2+c^2-a^2)/(2*b*c)))
Check by alternative methods, sine rule to get angles B and C, then sum A + B + C = 180°?, or by traversing.

Q7.
In the triangle ATB the angle at T equals Z_T because we assume the vertical lines at A and B are virtually parallel (flat earth for these short distances). The angle TAB = $Z_B – Z_T \approx 10.3°$
Use the sine rule to calculate distance AB: AB = TB * $\sin(Z_T) / \sin(Z_B – Z_T)$ = 231.421
Call the point below B at the same level as A, C.
Now solve for w in the right angled triangle at the base of the figure (i.e. ABC). w = AB * $\cos(90° - Z_B)$ = 219.683
Check: Calculate AT in triangle ATB, and then calculate w from the right angled triangle ATC.
The angle ABT = $180° - Z_T - (Z_B – Z_T) = 180° - Z_B$
By sine rule AT = TB * $\sin(180° - Z_B) / \sin(Z_B – Z_T)$ = 250.353
Solve for w in the larger right angled triangle (i.e. ATC). w = AT * $\cos(90° - Z_T)$ = 219.683 OK.

Q8A.
Braced Quad with one diagonal distance known. Rename angles for spreadsheet use: alpha = a, beta = b, gamma = g, delta = d. Sides are AB, BC etc



BC = 483.06
a = 52 41 24 = 52.6900 °
b = 34 19 27 = 34.3242 °
g = 42 18 42 = 42.3117 °
d = 41 42 47 = 41.7131 °

Angle ADB = 180-a-b-g = 50.6742 °
Angle ACB = 180-b-g-d = 61.6511 °
Triangle ABC Sine rule: AB = BC sin(180-b-g-d)/sin(g+d) = 427.450
Triangle ABD Sine rule: AD = AB sin(a+b) / sin(180-a-b-g) = **551.829**
so to nearest cm AD = 551.83
Triangle ABC Sine rule: AC = BC sin(b)/sin(g+d) = 273.873
Triangle ABD Sine rule: BD = AD sin(g)/sin(a+b) = 371.976

Triangle BCD   Cosine rule: DC = √(BD^2+BC^2-2 BD BC cos a) = 392.284

Calc Other Angles using cosine rules of the type $\cos A = (b^2 + c^2 - a^2)/2bc$
Angle BCD   48.9548
Angle CDA   27.6810
Check   Sum of angles in quad = a + b + g + d + ADB + ACB + BCD + CDA = 360.0000   OK

Q8B.
This famous problem is named after Peter Hansen (1795–1874) who worked on the geodetic survey of Denmark in the 1800s. So it has been challenging students for many years ☺. When the side length opposite the given angles is provided as in this traditional Hansen problem the solution involving somewhat complicated trigonometry has been documented elsewhere e.g. in Wikipedia. Here we provide a new alternative solution that uses simple sine and cosine rules. Part A of this question is a similar problem that is much easier to solve than the Hansen problem because the length of one of the diagonals of the quadrilateral is known. The simpler problem, with a diagonal length known, requires only the use of sine and cosine rules of triangles. From the solution of the simpler problem an approach to develop a solution for the Hansen problem can be made, as follows.

Start with a guess for BC, calculate DC as above Q8a steps, change BC until DC is correct (this can be done directly by scaling all distance appropriately). Alternatively, use Microsoft Excel's Solver to change BC until DC is correct. More details about Solver later in this book.

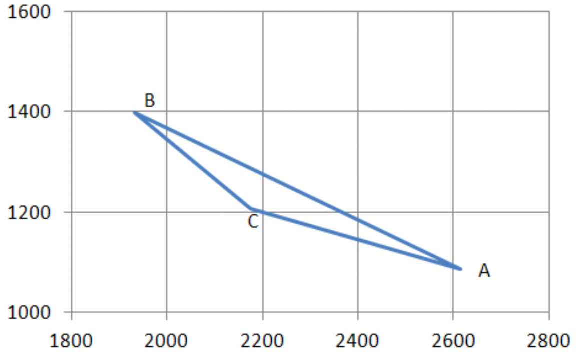
From the sums of angles in a triangle calculate:
Angle ADB = 180° – α – β – γ = 79° 44' 38" and Angle ACB = 180° – β – γ – δ = 73° 00' 02"
Calculate AB from triangle ABC by sine rule: $AB = BC \sin(180° - \beta - \gamma - \delta)/\sin(\gamma+\delta)$
Calculate AD from triangle ABD by sine rule: $AD = AB \sin(\alpha+\beta) / \sin(180° - \alpha - \beta - \gamma)$
Calculate AC from triangle ABC by sine rule: $AC = BC \sin(\beta)/\sin(\gamma+\delta)$
Calculate BD from triangle ABD by sine rule: $BD = AD \sin(\gamma)/\sin(\alpha+\beta)$
Calculate DC from triangle BCD by cosine rule: $DC = \sqrt{BD^2 + BC^2 - 2\, BD\, BC \cos\alpha}$
Calculate the other angles using the cosine rule and the side lengths. As a check, the sum of all angles in the corners of the quadrilateral should equal 360°.

Thus we can calculate DC if we know BC and the four angles. In our problem we have DC and the four angles but not BC. So estimate the length of BC as for example 700. The derived value of DC, called DC', will be 428.559 which is too small. Then simply scale all the distances in the network appropriately by the factor: DC / DC'.

The answers are: BD = 434.939, AD = 626.316, BC = 716.237, AC = 333.066, AB = 696.017

Another solution method that requires little thought is to use existing computer software for least squares solutions of survey control networks, even though there is no redundant data in the problem. The input consists of the given distance DC, the four observed angles and arbitrary fixed coordinates for a point and bearing for a line. Solve for coordinates of points and calculate the lengths of the sides.

Q9.



				E		N	
A				2614.162		1086.216	
B				1932.783		1399.554	
C				2174.368		1206.173	
Line	Brg	Dist	D		M		S
AB	294.696	749.972	294°		41'		44.7"
AC	285.257	455.860	285°		15'		24.2"
BC	128.676	309.450	128°		40'		34.1"
Interior angles							
ABC	13.980		13°		58'		49.4"
BCA	156.581		156°		34'		50.1"
CAB	9.439		9°		26'		20.5"

	A	B	C	D	E	M	N	S
3	A				2614.162		1086.216	
4	B				1932.783		1399.554	
5	C				2174.368		1206.173	
6	Line	Brg	Dist	D		M		S
7	AB	=MOD(DEGREES(ATAN2((N4-N3),(E4-E3))),360)	=SQRT((E4-E3)^2+(N4-N3)^2)	=INT(B7)		=INT(B7*60-D7*60)		=(B7-D7)*3600-M7*60
8	AC	=MOD(DEGREES(ATAN2((N5-N3),(E5-E3))),360)	=SQRT((E5-E3)^2+(N5-N3)^2)	=INT(B8)		=INT(B8*60-D8*60)		=(B8-D8)*3600-M8*60
9	BC	=MOD(DEGREES(ATAN2((N5-N4),(E5-E4))),360)	=SQRT((E5-E4)^2+(N5-N4)^2)	=INT(B9)		=INT(B9*60-D9*60)		=(B9-D9)*3600-M9*60
	Interior angles							
11	ABC	=B9-(B7-180)		=INT(B11)		=INT(B11*60-D11*60)		=(B11-D11)*3600-M11*60
12	BCA	=B8-B9		=INT(B12)		=INT(B12*60-D12*60)		=(B12-D12)*3600-M12*60
13	CAB	=B7-B8		=INT(B13)		=INT(B13*60-D13*60)		=(B13-D13)*3600-M13*60

Check: by looking at the graph / plot, I can see that it all looks reasonable. So no large error exists. But that doesn't check for small errors. So I compared answers with those from many former students too.

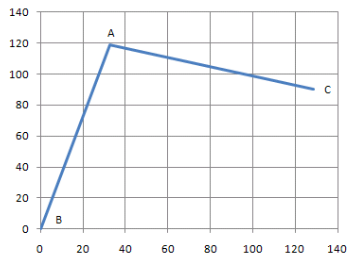
Q10.
289°30' (-135.66, 283.45)

	Brg	Dist	D	E	M	N
O				100		200
OA	318.2667		318		16	
AOB	331.2333		331		14	
OB	289.5	250	289°		30'	
B				-135.660		283.452

	A	B	C	D	E	M	N
19	O				100		200
20	OA	=D20+M20/60		318		16	
21	AOB	=D21+M21/60		331		14	
22	OB	=MOD(B20+B21,360)	250	=INT(B22)		=INT(B22*60-D22*60)	
23	B				=E19+C22*SIN(RADIANS(B22))		=N19+C22*COS(RADIANS(B22))

Check: I compared answers with those from former students.

Q11.



	Brg	Dist	D	E	M	N	S
B				0		0	
BA	15.4264	123.455	15		25		35
A				32.839		119.007	
Angle BAC	271.2975		271		17		51
AC	106.7239	100	106°		43'		26.0"
C				128.609		90.231	
BC	54.9468	157.105	54°		56'		48.5"

		B	C	D	E	M	N	S
26	B				0		0	
27	BA	=D27+M27/60+S27/3600	123.455	15		25		35
28	A				=E26+C27*SIN(RADIANS(B27))		=N26+C27*COS(RADIANS(B27))	
29	Angle BAC	=D29+M29/60+S29/3600		271		17		51
30	AC	=MOD(B27+180+B29,360)	100	=INT(B30)		=INT(B30*60-D30*60)		=(B30-D30)*3600-M30*60
31	C				=E28+C30*SIN(RADIANS(B30))		=N28+C30*COS(RADIANS(B30))	
32	BC	=MOD(DEGREES(ATAN2((N31-N26),(E31-E26))),360)	=SQRT((E31-E26)^2+(N31-N26)^2)	=INT(B32)		=INT(B32*60-D32*60)		=(B32-D32)*3600-M32*60

Check: Dist BC by cosine rule on triangle BAC
=SQRT(C27^2+C30^2-2*C27*C30*COS(RADIANS(360-B29))) = 157.105 OK
Check bearing by comparison with students results.

ans) 106°43' 26"; (32.839, 119.007), (128.609, 90.231); 54°56'48", 157.105m.

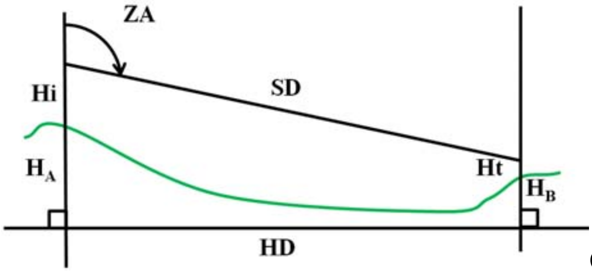
Q12.
221° 19' 58"; 500.263 m; 530.987 m; + 19° 35' 10"

Q13.
Calculate $\Delta E = E_B - E_A = 89.113$ $\Delta N = N_B - N_A = -40.57$ and $\Delta H = H_B - H_A = -89.113$

For bearing use R→P on calculator or ATAN2 on a spreadsheet, or use $\tan^{-1}$ and be careful to work out the quadrant for the bearing. If your bearing is negative simply add 360° before converting to D M S.
For plan (i.e. horizontal) distance use R→P on calculator as above, or use $D = \text{Sqrt}(\Delta E^2+\Delta N^2)$
For slope distance use $SD = \text{Sqrt}(\Delta E^2+\Delta N^2+\Delta H^2)$
For slope angle use $\sin^{-1}(\Delta H/SD)$ or $\tan^{-1}(\Delta H/HD)$. If the angle is negative it indicates sloping down, below horizontal.
For Zenith Angle use $\cos^{-1}(\Delta H/SD)$ but not $\tan^{-1}(HD/\Delta H)$ because ΔH might be zero or close to it.

Ans: 146°53’05”, 163.113, 103°58’05”, 168.083

Q14.



Cross section (elevation) view, not to scale.

The ZA is > 90° so the line slope is down. Let's first calculate the height difference from the instrument axis (not the ground mark at A) to the target above B.

$\Delta H_{ABIT} = -SD_{AB} \sin(ZA-90°) = SD_{AB} \cos(ZA) = -3.085$.

If using a computer convert the angle to RADIANS before taking sine or cosine. Perhaps you can think of another way to do this.

Calculate $H_B = H_A + Hi + \Delta H_{ABIT} - Ht = 35.214 + 1.565 - 3.085 - 1.690 = 32.004$

To calculate the E N of B we need the horizontal distance AB.

Calculate horizontal distance, $HD_{AB} = SD_{AB} \cos(ZA - 90°) = SD_{AB} \sin(ZA) = 14.845$

If using a computer convert the angle to RADIANS before taking sine or cosine.

$E_B = E_A + HD_{AB} \sin\beta_{AB} = 306.219$

$N_B = N_A + HD_{AB} \cos\beta_{AB} = 285.228$

Checks:

SD from instrument to target = √((EB-EA)^2+(NB-NA)^2+(HB+Ht-(HA+Hi))^2) = 15.162 OK

Bearing AB = MOD(DEGREES(ATAN2((NB-NA),(EB-EA))),360) = 160.422222 = 160° 25' 20.0" OK

ZA from instrument to target = DEGREES(ATAN2(HB+Ht-(HA+Hi),SQRT((EB-EA)^2+(NB-NA)^2))) = 101.741667 = 101° 44' 30.0" OK

Q15:

	I	J	K	L	M	N
40	BS	IS	FS	Rise / Fall	RL	Remarks
41	2.857				45.568	BM
42	0.356		1.988	=I41-K42	=M41+L42	CP1
43		0.65		=I42-J43	=M42+L43	K
44		2.11		=J43-J44	=M43+L44	L
45		3.04		=J44-J45	=M44+L45	M
46	4.325		3.125	=J45-K46	=M45+L46	N
47		1.85		=I46-J47	=M46+L47	K
48	4.037		1.556	=J47-K48	=M47+L48	CP1
49			4.907	=I48-K49	=M48+L49	BM RL45.568
50						
51	=SUM(I41:I48)		=SUM(K41:K49)	=SUM(L41:L49)		sum
52			=I51-K51		=M49-M41	Checks

Q16:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
56	Pt		FL			FR		Mean		Mean				Red mean				Grand mean	
57		d	m	s	d	m	s		d	m	s	d.d	d	m	s	dir	d	m	s
58	Q	0	4	30	180	3	30	=(B58+C58/60+D58/3600+MOD((E58+F58/60+G58/3600)-180,360))/2	=INT(H58)	=INT(H58*60-I58*60)	=(H58-I58-(J58/60))*3600	=0	=INT(L58)	=INT(L58*60-M58*60)	=(L58-M58-(N58/60))*3600	=(L58+L63)/2	=INT(P58)	=INT(P58*60-Q58*60)	=(P58-Q58-(R58/60))*3600
59	R	37	16	20	217	15	50	=(B59+C59/60+D59/3600+MOD((E59+F59/60+G59/3600)-180,360))/2	=INT(H59)	=INT(H59*60-I59*60)	=(H59-I59-(J59/60))*3600	=H59-H$58	=INT(L59)	=INT(L59*60-M59*60)	=(L59-M59-(N59/60))*3600	=(L59+L64)/2	=INT(P59)	=INT(P59*60-Q59*60)	=(P59-Q59-(R59/60))*3600
60	S	193	42	30	13	42	20	=(B60+C60/60+D60/3600+MOD((E60+F60/60+G60/3600)-180,360))/2	=INT(H60)	=INT(H60*60-I60*60)	=(H60-I60-(J60/60))*3600	=H60-H$58	=INT(L60)	=INT(L60*60-M60*60)	=(L60-M60-(N60/60))*3600	=(L60+L65)/2	=INT(P60)	=INT(P60*60-Q60*60)	=(P60-Q60-(R60/60))*3600
61	T	216	25	50	36	24	50	=(B61+C61/60+D61/3600+MOD((E61+F61/60+G61/3600)-180,360))/2	=INT(H61)	=INT(H61*60-I61*60)	=(H61-I61-(J61/60))*3600	=H61-H$58	=INT(L61)	=INT(L61*60-M61*60)	=(L61-M61-(N61/60))*3600	=(L61+L66)/2	=INT(P61)	=INT(P61*60-Q61*60)	=(P61-Q61-(R61/60))*3600
62																			
63	Q	90	15	40	270	14	10	=(B63+C63/60+D63/3600+MOD((E63+F63/60+G63/3600)-180,360))/2	=INT(H63)	=INT(H63*60-I63*60)	=(H63-I63-(J63/60))*3600	0	=INT(L63)	=INT(L63*60-M63*60)	=(L63-M63-(N63/60))*3600				
64	R	127	28	0	307	27	30	=(B64+C64/60+D64/3600+MOD((E64+F64/60+G64/3600)-180,360))/2	=INT(H64)	=INT(H64*60-I64*60)	=(H64-I64-(J64/60))*3600	=H64-E$9	=INT(L64)	=INT(L64*60-M64*60)	=(L64-M64-(N64/60))*3600				
65	S	283	53	0	103	52	0	=(B65+C65/60+D65/3600+MOD((E65+F65/60+G65/3600)-180,360))/2	=INT(H65)	=INT(H65*60-I65*60)	=(H65-I65-(J65/60))*3600	=H65-E$9	=INT(L65)	=INT(L65*60-M65*60)	=(L65-M65-(N65/60))*3600				
66	T	306	36	50	126	25	30	=(B66+C66/60+D66/3600+MOD((E66+F66/60+G66/3600)-180,360))/2	=INT(H66)	=INT(H66*60-I66*60)	=(H66-I66-(J66/60))*3600	=H66-E$9	=INT(L66)	=INT(L66*60-M66*60)	=(L66-M66-(N66/60))*3600				

---

17) The reading is 5 ft $7^{3}/_{8}$ in.
Convert to inches = 5*12+7+3/8 = 67.375 in. Convert to cm = 68.375 * 2.54 = 171.1325 cm → 1.711m.
Convert 5 ft $7^{3}/_{8}$ in, to feet = 5+(7+3/8)/12 ≈ 5.615 ft. Convert to metres = 5.615 * 0.3048 = 1.711325 m → 1.711m.
The difference between these converted values and the measured metric value is less than 1mm. This is less than the error in measurement due to the slight bending of the tape and the physical zero marker of the tape, and indicates there is no gross error. The next photo shows part of a tape with both metric and imperial units (this photo is a little distorted due to bending of the tape, but it provides another approximate check of our calculations).

2. INTERSECTIONS and RESECTIONS

Intersections are a method of finding the coordinates of a point from the directions or distances to it from two or more known points. In this course we consider two observations from two points to determine 2D coordinates. The directions to the point can be specified either by angles (α) or by bearings (β).

A three-point resection is a method of determining the coordinates of a point (P) from direction observations taken from that point to three points (A, B, C) that have known coordinates.

In this chapter we assume the earth is flat or that corrections (e.g. map projection) have already been made, and that the observations have been corrected for atmospheric effects etc.

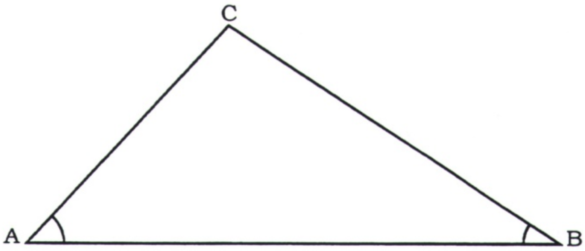
Some applications of intersections and resections include:

- To get coordinates of points by angle measurements by theodolites, this was common before EDM & total stations could measure distances easily.
- To get coordinates (or position) of inaccessible points e.g. high voltage, gas, heat, high above ground, nowhere to stand and hold prism
- Over short distances angle measurements give very accurate coordinates
- They provide alternatives to radiation from known points which requires distance measurement
- Intersection by distances, by Disto or tape, can give coordinates without the need for a tripod mounted theodolite or total station instrument.

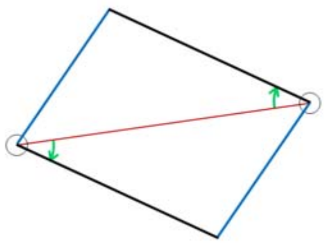
If there are more than two intersection observations or more than three resection observations then we use the least squares method described in a later course.

Intersection by Angles

Given: Coordinates of A and B, angles at A and B



Beware there are two possible answers to intersection by angles. C can be on either side of the line AB as in the diagram at right. In real applications we often have more information to help us decide which answer is the correct one.



Method 1, Solution:

1) Using the coordinates of A and B, calculate $\underline{\beta}_{AB}$ and d_{AB} (use R→P on calculators or atan2 on computers).

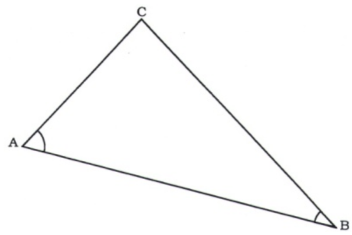
2) Using d_{AB} and angles A and B, solve triangle ABC for d_{AC} and d_{BC} by sine rule.

3) Using $\underline{\beta}_{AB}$ and angles A and B, calculate $\underline{\beta}_{AC}$ and $\underline{\beta}_{BC}$.

4) Using d_{AC} and $\underline{\beta}_{AC}$, calculate the coordinates of C from A (use P→R).

5) Similarly, calculate the coordinates of C from B and use it to check your result from 4) above.

Example:



Point C has been intersected from two control points A and B. What are the coordinates of C?

Given: coordinates of A = (2589.40, 6717.85)
B = (4717.77, 5625.10)
angle A = 63° 40' 28" angle B = 42° 02' 04"

Solution (assume C is to the left of AB):

1) Use coordinates of A and B to calculate (R→P): $\underline{\beta}_{AB}$ = 117° 10' 37" and d_{AB} = 2392.501m

2) Apply sine rule to triangle ABC:

$$\frac{2392.501}{\sin(A+B)} = \frac{d_{AC}}{\sin B} = \frac{d_{BC}}{\sin A}$$

note: C = 180° - (A+B) so sinC = sin(A+B)

Therefore d_{AC} = 1664.119m, d_{BC} = 2227.572m

3) Using $\underline{\beta}_{AB}$ and angles A and B, calculate:

$\underline{\beta}_{AC} = \underline{\beta}_{AB} - A$ = 117° 10' 37" - 63° 40' 28" = 53° 30' 09"

$\underline{\beta}_{BC} = \underline{\beta}_{AB} + 180° + B$ = 117° 10' 37" + 180° + 42° 02' 04" = 339° 12' 41"

4) Using d_{AC} and $\underline{\beta}_{AC}$, calculate coordinates of C from A (P→R):

ΔE_{AC} = 1337.757m, ΔN_{AC} = 989.798m C = (3927.157, 7707.648)

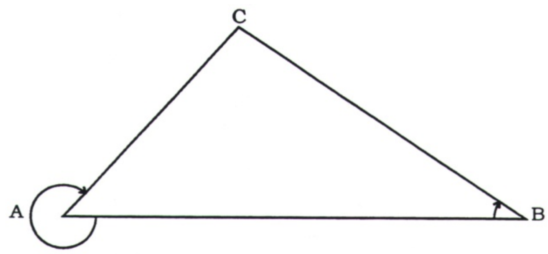
5) Using d_{BC} and $\underline{\beta}_{BC}$, calculate coordinates of C from B (P→R):

ΔE_{BC} = -790.612m, ΔN_{BC} = 2082.549m C = (3927.158, 7707.649)

The two sets of coordinates agree very closely with each other. This serves as a check of the calculations.

Method 2:

This method is more suitable for automated application with calculators and computers. Overcome the ambiguity as to which side of AB the point C lies by taking angles A and B measured clockwise from the baseline AB to the point to be fixed (i.e. C):



The coordinate differences are: $E_B - E_A = d_{AB} \sin \underline{\beta}_{AB}$ and $N_B - N_A = d_{AB} \cos \underline{\beta}_{AB}$

From the figure it can be seen: Angle C = 180° - B - (360° - A) = A - B - 180°

Also: $E_C - E_A = d_{AC} \sin (\underline{\beta}_{AB} + A)$ and $d_{AC} = \frac{d_{AB} \sin B}{\sin C}$ [sine rule]

Therefore:

$$E_C = E_A + \frac{d_{AB} \sin B}{\sin C}(\sin\beta_{AB} \cos A + \cos\beta_{AB} \sin A) = E_A + \frac{\sin B}{\sin C}(d_{AB} \sin\beta_{AB} \cos A + d_{AB} \cos\beta_{AB} \sin A)$$

$$= E_A - \frac{\sin B}{\sin(A - B)}\{(E_B - E_A)\cos A + (N_B - N_A)\ \sin A\} \qquad or = E_A - \frac{(E_B - E_A)\sin B \cos A + (N_B - N_A)\ \sin B \sin A}{\sin A \cos B - \cos A \sin B}$$

Dividing numerator and denominator by sinAsinB gives:

$$E_C = E_A - \frac{\sin B}{\sin(A - B)}\{(E_B - E_A)\cos A + (N_B - N_A)\ \sin A\} \qquad = E_A - \frac{\Delta E_{AB} \cot A + \Delta N_{AB}}{\cot B - \cot A}$$

A similar derivation gives:

$$N_C = N_A - \frac{\sin B}{\sin(A - B)}\{(N_B - N_A)\cos A - (E_B - E_A)\ \sin A\} \qquad = N_A - \frac{\Delta N_{AB} \cot A - \Delta E_{AB}}{\cot B - \cot A}$$

Similarly, the radiation from B gives a check: $E_C = E_B - \frac{\Delta E_{AB} \cot B + \Delta N_{AB}}{\cot B - \cot A}$

$$N_C = N_B - \frac{\Delta N_{AB} \cot B - \Delta E_{AB}}{\cot B - \cot A}$$

Note that the cot function is available in OpenOffice spreadsheets but not in Excel. So in Excel use the sine and cosine version of the formulas.

<u>Example</u>:

Given previous data, coordinates of A and B:
E_A = 2589.40m N_A = 6717.85m E_B = 4717.77m N_B= 5625.10m

Angles (note our defined convention):
angle A = 360° - 63° 40' 28" = 296° 19' 32" angle B = 42° 02' 04"

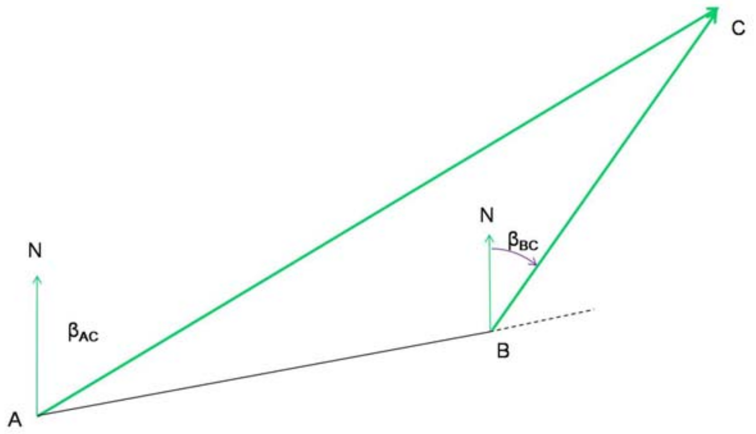
Solution:
Radiation from A using the derived formulae: ΔE_{AB}= 2128.37m and ΔN_{AB}= -1092.75m

$$E_C = E_A - \frac{\Delta E_{AB} \cot A + \Delta N_{AB}}{\cot B - \cot A} = 3927.157\text{m} \qquad N_C = N_A - \frac{\Delta N_{AB} \cot A - \Delta E_{AB}}{\cot B - \cot A} = 7707.648\text{m}$$

Radiation from B yields identical values and thus confirms the results.

Intersection by Bearings

Sometimes we can't see between the two known stations (i.e. A to B), but other points such as C can be seen from each of the known points. If the bearings from A to C and B to C can be observed using other known stations as reference objects, or otherwise obtained, then the coordinates of C can be calculated.



Given: Coordinates of A and B, bearings β_{AC} and β_{BC}

Method 1, Derivation of Solution:

Calculate d_{AC} using the sine rule for ABC and the angles A and B

$$\frac{d_{AC}}{\sin B} = \frac{d_{BC}}{\sin A} = \frac{d_{AB}}{\sin(A+B)} \rightarrow d_{AC} = \frac{d_{AB}\sin B}{\sin(A+B)}$$

Substituting A = β_{AB} - β_{AC}, B = 180° - (β_{AB} - β_{BC}), and A+B = 180° - (β_{AC} - β_{BC}) gives:

$$d_{AC} = \frac{d_{AB}\sin(\beta_{AB}-\beta_{BC})}{\sin(\beta_{AC}-\beta_{BC})} = \frac{d_{AB}\sin\beta_{AB}\cos\beta_{BC} - d_{AB}\cos\beta_{AB}\sin\beta_{BC}}{\sin(\beta_{AC}-\beta_{BC})} = \frac{\Delta E_{AB}\cos\beta_{BC} - \Delta N_{AB}\sin\beta_{BC}}{\sin(\beta_{AC}-\beta_{BC})}$$

Radiation from A to C:

$$E_C = E_A + d_{AC}\sin\beta_{AC} \quad = E_A + \frac{\Delta E_{AB}\cos\beta_{BC} - \Delta N_{AB}\sin\beta_{BC}}{\sin(\beta_{AC}-\beta_{BC})}\sin\beta_{AC}$$

Similarly:

$$N_C = N_A + d_{AC}\cos\beta_{AC} \quad = N_A + \frac{\Delta E_{AB}\cos\beta_{BC} - \Delta N_{AB}\sin\beta_{BC}}{\sin(\beta_{AC}-\beta_{BC})}\cos\beta_{AC}$$

Radiation from B to C (a check):

$$E_C = E_B + \frac{\Delta E_{AB}\cos\beta_{AC} - \Delta N_{AB}\sin\beta_{AC}}{\sin(\beta_{AC}-\beta_{BC})}\sin\beta_{BC} \qquad N_C = N_B + \frac{\Delta E_{AB}\cos\beta_{AC} - \Delta N_{AB}\sin\beta_{AC}}{\sin(\beta_{AC}-\beta_{BC})}\cos\beta_{BC}$$

Example:

Given: A = (E 2589.40, N 6717.85) B = (E 9307.04, N 3423.63) β_{AC} = 62°26'58" β_{BC} = 359°49'25"

Solution:
Coordinate differences from A to B: ΔE_{AB} = 6717.64m, ΔN_{AB} = -3294.22m
Radiation from A to C:

$$d_{AC} = \frac{\Delta E_{AB}\cos\beta_{BC} - \Delta N_{AB}\sin\beta_{BC}}{\sin(\beta_{AC}-\beta_{BC})} = 7553.260$$

$$E_C = E_A + d_{AC}\sin\beta_{AC} \quad = E_A + 6696.743 \quad = 9286.143\text{m}$$

$$N_C = N_A + d_{AC}\cos\beta_{AC} \quad = N_A + 3493.617 \quad = 10211.467\text{m}$$

Radiation from B to C gives the same results and serves as a check.

Method 2, Alternative formula for intersection by bearings

The ISG manual (NSW Dept Lands, 1976) gives the following formulae:

$$N_C = N_A + \frac{(N_B - N_A)\tan\beta_{BC} - (E_B - E_A)}{\tan\beta_{BC} - \tan\beta_{AC}} \qquad N_C = N_B + \frac{(N_B - N_A)\tan\beta_{AC} - (E_B - E_A)}{\tan\beta_{BC} - \tan\beta_{AC}}$$

$$E_C = E_A + (N_C - N_A)\tan\beta_{AC} \qquad E_C = E_B + (N_C - N_B)\tan\beta_{BC}$$

Their example data are: A (422 145.515, 1817 938.975) B (398 112.145, 1828 011.324)
β_{AC} = 237°14'21.6" β_{BC} = 165°53'42.8"

Answer: C (403 635.851, 1806 028.287) [You may use this to check your spreadsheet or program]

Intersection by Bearings: Calculations in spreadsheet programs

Label the points 1 2 3 instead of A B C, and store the Easting of 1 in cell E1, similarly store N1, E2, N2. These cells were chosen to make the equations easier to read. Then convert the bearings to decimal degrees and then to radians and store β_{13} in cell B1 and bearing β_{23} in B2, the excel equations for E3 and N3 are:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Brg 1_3:	1.090		E1:	2589.400			62	26	58			N1:	6717.850	
2	Brg 2_3:	6.280		E2:	9307.040			359	49	25			N2:	3423.630	
3		radians		E3:	**9286.143**			D	M	S			N3:	**10211.467**	
4															

E3: =E1 + (((E2-E1) * COS(B2) - (N2-N1) * SIN(B2)) / SIN(B1-B2)) * SIN(B1)
N3: =N1 + (((E2-E1) * COS(B2) - (N2-N1) * SIN(B2)) / SIN(B1-B2)) * COS(B1)

Or writing the terms in a different sequence:

E3: = E1 + (SIN(B1) * ((E2 - E1) * COS(B2) - (N2 - N1) * SIN(B2)) / (SIN(B1 - B2)))
N3: = N1 + (COS(B1) * ((E2 - E1) * COS(B2) - (N2 - N1) * SIN(B2)) / (SIN(B1 - B2)))

The spreadsheets available in the free OpenOffice software work similarly to Microsoft Excel, but also have a COT function.

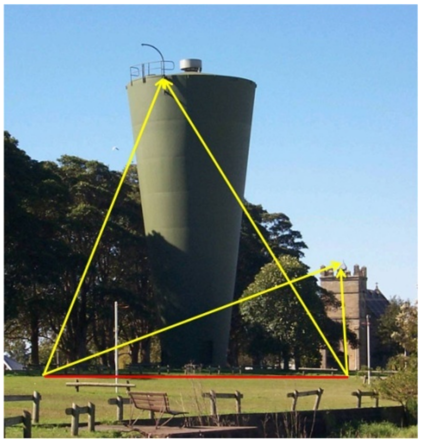
Intersection calculations in CAD

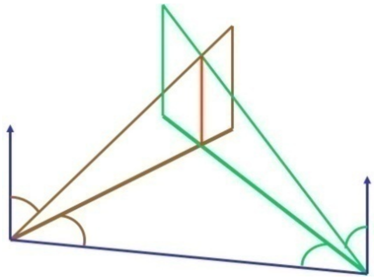
Intersection calculations in CAD will be learnt later in this course. Find the menu or button option for Brg/Dist Intersection. This is sometime in a section called COGO (meaning Coordinate Geometry). There are usually a group of similar calculations that create the answer point for each of the following types of intersections:

- intersection of a bearing or distance from one point and the bearing or distance from another point;
- intersection of a line and the bearing or distance from another point.
- intersection of a line and another line

Intersection by angles or bearings in 3D

Theodolites measure angles in a horizontal plane and bearings are also on a plane (or map projection) so intersection calculations in 3D can proceed as described for 2D above. Make sure the distances involved are horizontal distances not slope distances to the targets. E N coordinates are calculated; Heights are not involved. If zenith angles are also observed we can solve for 3D coordinates, but there is some redundancy so we will deal with that in a later course.

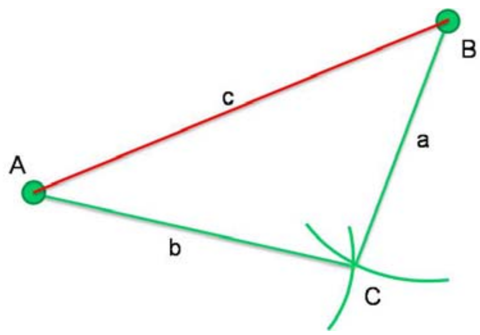




Intersection by Distances (Trilateration)

Trilateration, or intersection by distances, is a method of fixing the coordinates of a point using distances from two or more control points. Note that the distances can be measured in either direction from e.g. from A to C or from C to A. This technique does not need a theodolite; the distances can be measured by

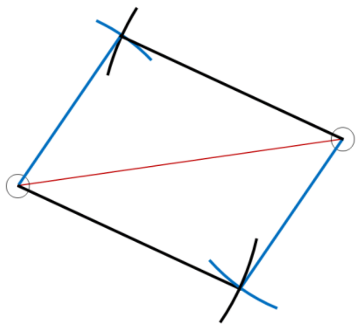
- Tape - don't need tripod
- Disto™ - don't need tripod
- Total station / EDM
- Laser scanner
- or by new technologies, e.g. Locata



Given: Coordinates of A and B, distances AC (b) and BC (a)

Problem: Find the coordinates of C

As with intersection by angles, there are two possible locations for C, one on either side of the line AB. Use other information to determine which C you want.



Method 1. Derivation of Solution by finding angles in the triangle:

1. Using the coordinates of A and B, calculate the bearing (β_{AB}) and distance (c) of AB (R→P or atan2).

2. Using the calculated distance AB (c) and the measured distances of AC (b) and BC (a), calculate the angles A and B from the cosine rule:

a2 = b2 + c2 – 2bc cosA → $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ b2 = a2 + c2 – 2ac cosB → $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

3. Assuming that C is to the right of AB (i.e. A, B and C are in clockwise order), calculate the coordinates of C from A (note $\beta_{AC} = \beta_{AB} + A$):

$$E_C = E_A + b\sin(\beta_{AB} + A) \qquad \text{----- (1)}$$

$$N_C = N_A + b\cos(\beta_{AB} + A) \qquad \text{----- (2)}$$

4. Calculate the coordinates of C from B as a check (note $\beta_{BC} = \beta_{AB} + 180° - B$):

$$E_C = E_B + a\sin(\beta_{AB} + 180 - B) = E_B - a\sin(\beta_{AB} - B) \qquad \text{----- (3)}$$

$$N_C = N_B + a\cos(\beta_{AB} + 180 - B) = N_B - a\cos(\beta_{AB} - B) \qquad \text{----- (4)}$$

Note: These equations can be used only if A, B and C are in clockwise order. If they are in anti-clockwise order, substitute negative values for the angles A and B.

Method 2:

Expanding equation (1): $E_C = E_A + b\sin(\beta_{AB} + A)$

Using sin (A + B) = sinA cosB + cosA sinB: $E_C = E_A + b\sin\beta_{AB}\cos A + b\cos\beta_{AB}\sin A$

But $c \sin \beta_{AB} = E_B - E_A$ and $c \cos \beta_{AB} = N_B - N_A$:

$$E_C = E_A + \frac{b}{c}[(E_B - E_A)\cos A + (N_B - N_A)\sin A] \qquad \text{---- (5)}$$

Similarly by expanding equations (2), (3) and (4) in turn, we get:

$$N_C = N_A + \frac{b}{c}[(N_B - N_A)\cos A - (E_B - E_A)\sin A] \qquad \text{---- (6)}$$

$$E_C = E_B - \frac{a}{c}[(E_B - E_A)\cos B - (N_B - N_A)\sin B] \qquad \text{---- (7)}$$

$$N_C = N_B - \frac{a}{c}[(N_B - N_A)\cos B + (E_B - E_A)\sin B] \qquad \text{---- (8)}$$

For example, expanding equation (4): $N_C = N_B - a\cos(\beta_{AB} - B)$

using cos (A - B) = cosA cosB + sinA sinB: $N_C = N_B - a\cos\beta_{AB}\cos B - a\sin\beta_{AB}\sin B$

But $c \sin \beta_{AB} = E_B - E_A$ and $c \cos \beta_{AB} = N_B - N_A$:

$$N_C = N_B - \frac{a}{c}[(N_B - N_A)\cos B + (E_B - E_A)\sin B] \qquad \text{---- (8)}$$

- Calculate coordinates of C using equations (5) and (6) and use equations (7) and (8) to check your results.
- These equations can be used only if A, B and C are in clockwise order. If they are in anti-clockwise order, substitute negative values for the angles A and B.

Method 3:

Let $s = (d_{AB} + d_{AC} + d_{BC}) / 2$ and $r = \sqrt{\dfrac{(s - d_{AC})(s - d_{AB})(s - d_{BC})}{s}}$

If A and B are the triangle's internal angles at points A and B respectively, then

$A = 2\tan^{-1}(\frac{r}{s-d_{BC}})$ and $B = 2\tan^{-1}(\frac{r}{s-d_{AC}})$

From a diagram and the known bearing from A to B simply calculate the bearings from A to C and B to C. Then calculate the coordinates of C by radiation from A or B.

Method 4. Algebraic Solution of equations of circles (without using trig functions):

Point C can be considered as the intersection point of a circle centred at A with radius b, and a circle centred at B with radius a. The coordinates of the intersection point can be found by solution of the equations of the two circles. Your CAD program may solve the problem this way. No trig functions are required and it is not necessary to determine the angles in the triangle. Derivations can be found in textbooks and websites, but are not included here. Students are welcome to search for the derivations or to derive them or contact me for my derivations.

We can start with the two circle equations:

$(E_C-E_A)^2 + (N_C-N_A)^2 = d_{AC}^2$

$(E_C-E_B)^2 + (N_C-N_B)^2 = d_{BC}^2$

Then we derive by a variety of methods a set of equations for the coordinates of C. There are a number of forms of these equations; here we give just one version. Of course there are usually two possible solutions for C. If the circles intersect they usually do so in two places.

Calculate two intermediate values:

$d^2 = d_{AB}^2 = (E_B-E_A)^2 + (N_B-N_A)^2$ and $K = \text{sqrt}(((r_A+r_B)^2-d^2)(d^2-(r_A-r_B)^2))/4$ = area of the triangle $= \text{sqrt}((s(s-r_A)(s-r_B)(s-d))$ where $s = (r_A + r_B + d)/2$

Then one solution point is

$E_C = (E_B+E_A)/2 + (E_B-E_A)(r_A^2-r_B^2)/2d^2 + 2(N_B-N_A)K/d^2$ $N_C = (N_B+N_A)/2 + (N_B-N_A)(r_A^2-r_B^2)/2d^2 - 2(E_B-E_A)K/d^2$

The other solution point is

$E_C = (E_B+E_A)/2 + (E_B-E_A)(r_A^2-r_B^2)/2d^2 - 2(N_B-N_A)K/d^2$ $N_C = (N_B+N_A)/2 + (N_B-N_A)(r_A^2-r_B^2)/2d^2 + 2(E_B-E_A)K/d^2$

Example:

EDM distances were measured from two known control points A and B to a new point C. Calculate the coordinates of C, if it lies to the left of AB.

Given: A = (E 1859.75, N 3722.63) d_{AC} = b = 1537.75m
B = (E 1078.37, N 2405.38) d_{BC} = a = 2487.56m

Solution, Method 1:

Using the coordinates of A and B, calculate the bearing (β_{AB}) and distance (c) of AB (R→P):

ΔE_{AB} = -781.38m ΔN_{AB} = -1317.25m → c = 1531.569m β_{AB} = 210° 40' 33.6" (1 extra digit)

Using the calculated distance AB (c) and the measured distances of AC (b) and BC (a), calculate the angles A and B from the cosine rule:

$\cos A = \frac{b^2+c^2-a^2}{2bc}$ → A = 108° 16' 53.8" $\cos B = \frac{c^2+a^2-b^2}{2ac}$ → B = 35° 56' 33.3"

If C lies to the left of AB (i.e. A, B and C are in anti-clockwise order), negative values need to be used for the angles A and B: A = -108° 16' 53.8" B = -35° 56' 33.3"

Calculate the coordinates of C from A:

$E_C = E_A + b\sin(\beta_{AB} + A) = 3361.660\text{m}$ $N_C = N_A + b\cos(\beta_{AB} + A) = 3392.568\text{m}$

Calculate the coordinates of C from B (check):

$E_C = E_B - a\sin(\beta_{AB} - B) = 3361.660\text{m}$ ✓ $N_C = N_B - a\cos(\beta_{AB} - B) = 3392.569\text{m}$ ✓

Solution, Method 2:

Calculate coordinates of C from A:

$$E_C = E_A + \frac{b}{c}(\Delta E_{AB}\cos A + \Delta N_{AB}\sin A) = 3361.660\text{m}$$

$$N_C = N_A + \frac{b}{c}(\Delta N_{AB}\cos A - \Delta E_{AB}\sin A) = 3392.568\text{m}$$

Calculate coordinates of C from B (check):

$$E_C = E_B - \frac{a}{c}(\Delta E_{AB}\cos B - \Delta N_{AB}\sin B) = 3361.660\text{m} \checkmark$$

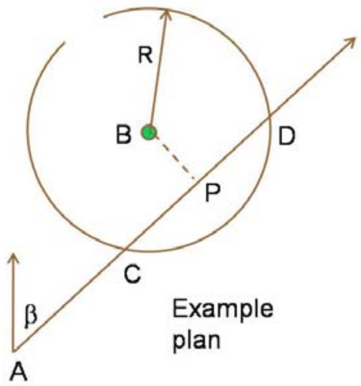
$$N_C = N_B - \frac{a}{c}(\Delta N_{AB}\cos B + \Delta E_{AB}\sin B) = 3392.568\text{m} \checkmark$$

Solution, Method 4: d^2_{AB} = 2345702.267 m^2, K = 1118147.452 m^2

➔ E 850.038, N 4882.439 not valid, and E 3361.660, N 3392.568 which is correct

Intersection by 1 bearing + 1 distance

One example of an intersection by a bearing and a distance occurs when a line of known bearing intersects with a circular curve i.e. a distance from a point. It is common in cadastral lot calculations with circular arc curved boundaries. Details of applications will be given in chapter 8 and another solution method in chapter 3. This section has been improved thanks to the help of Joel Haasdyk in 2008.



Intersection of bearing β from A and distance R from B can lie in two places, either C or D. One solution method is based on transforming (rotation, no scale) the vector AB to the bearing AD follows. It is an efficient method that has no $\sin^{-1}$ or divisions.

$$d_{AP}=(E_B-E_A)\sin\beta_{AD}+(N_B-N_A)\cos\beta_{AD}$$

$$d_{BP}=(E_B-E_A)\cos\beta_{AD}-(N_B-N_A)\sin\beta_{AD}$$

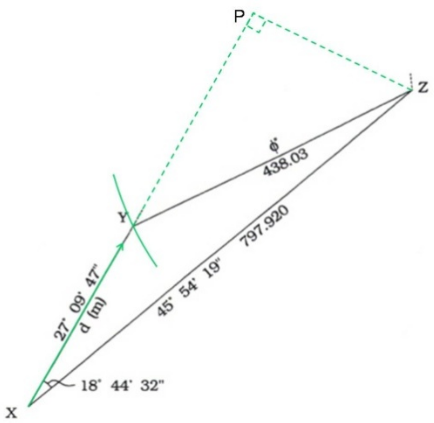
$$d_{CP} = d_{DP} = \sqrt{(R^2-d_{BP}^2)}$$

Then calculate the coordinates of C or D by radiation from A using β and $d_{AC} = d_{AP} - d_{CP}$ or $d_{AD} = d_{AP}+d_{DP}$

Another method is to solve for the distance AC by sine rule on triangle ABC. But beware $\sin^{-1}$ has two answers i.e. in the first and second quadrants.

Example 1:

Consider the triangle in the following figure, and solve the triangle using the sine rule. The bearing and distance between X and Z is available from coordinates. Find the location of Y given that it lies on a bearing 27°09’47” from X and distance 438.03 from Z.



Note that there is another point considerably further north of Y that satisfies the data but is not the point required.
The angle at Y is obtained from the sine rule:

$$\frac{797.92}{\sin Y} = \frac{438.03}{\sin 18°44'32"} \rightarrow Y = 35°49'28" \text{ or } 144°10'32"$$

For this problem we want the 144° angle for Y, according to the plan in the figure.

The bearing of YZ is then: ϕ = 27°09'47" + 180° – Y
= 62°59'14.9"

The angle at Z can now be determined: Z = ϕ – 45°54'19" = 17°04'55.9"

The distance XY is then obtained from the sine rule: $d = \dfrac{438.03 \cdot \sin Z}{\sin 18°44'32''} = 400.448$

The coordinates of Y can then be calculated by radiation from X with bearing and distance of 27°09'47" and 400.448

Example 2:

Solve the data in example 1 using coordinates and the given formulae. Set coordinates of X (100, 500). By radiation at 45°54'19" and 797.92, the coordinates of Z are (673.058, 1055.230). Consider Z at the centre of a circle with radius 438.03 that goes through Y.

The perpendicular distance from Z to the line XY is $d_{ZP} = (E_Z-E_X)\cos\beta-(N_Z-N_X)\sin\beta = 256.380$

And the distance along the line XY, of the pedal point is $d_{XP} = (E_Z-E_X)\sin\beta+(N_Z-N_X)\cos\beta = 755.609$
where β = 27°09'47" in both equations.

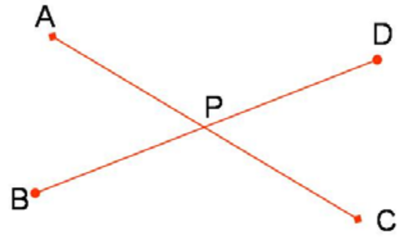
Then $d_{XY} = d_{XP} - \sqrt{(R^2-d_{ZP}^2)} = 755.609 - 355.161 = 400.448$ which is the same value as in example 1.

The coordinates of Y can then be calculated by radiation from X with bearing and distance of 27°09'47" and 400.448, as in example 1: (282.814, 856.283).

Check calculations:
Using the coordinates of X, Y and Z the bearing and distances of the triangle sides can be calculated.
Line XY is 27°09'47.0" 400.448 ✓ Line YZ is 62°59'14.9" 438.030 ✓

Intersection of two coordinated lines

If line AC intersects line BD at a point P and coordinates of A B C D are known, find the coordinates of P.



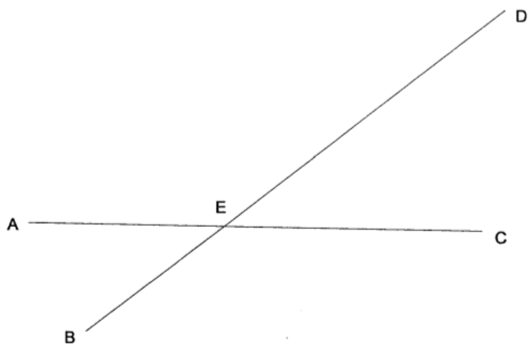
Let $$K = \frac{(E_B - E_A)(N_D - N_B) - (N_B - N_A)(E_D - E_B)}{(E_C - E_A)(N_D - N_B) - (N_C - N_A)(E_D - E_B)}$$

Then $E_P = E_A + K(E_C-E_A)$ and $N_P = N_A + K(N_C-N_A)$

No bearings and no trig functions needed! Of course, an alternative method is to calculate the bearings from coordinates then treat it as an intersection by bearings problem.

Example:
Using the data given, compute the coordinates of the intersection point E to the nearest mm.

	COORDINATES	
STATION	E (m)	N (m)
A	1101.61	1113.14
B	1134.86	1061.14
C	1334.91	1098.36
D	1358.31	1211.90

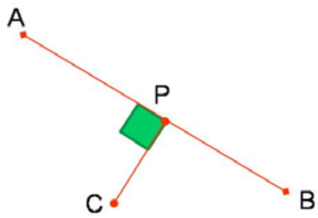


From formula above:
K = 0.432286
E: 1202.462, 1106.751

Check, bearings from coordinates:
BE 55° 59' 33.9" = BD 55° 59' 33.9" ✓ AE 93° 37' 29.8" = AC 93° 37' 29.8" ✓

Intersection of perpendicular lines

Find coordinates of a point P that is the shortest distance from point C to the line AB. Coordinates of A B C are known. Line CP is perpendicular to line AB.



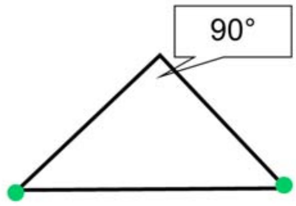
Let $$K = \frac{(E_C - E_A)(E_B - E_A) - (N_C - N_A)(N_B - N_A)}{(E_C - E_A)^2 - (N_C - N_A)^2}$$ then

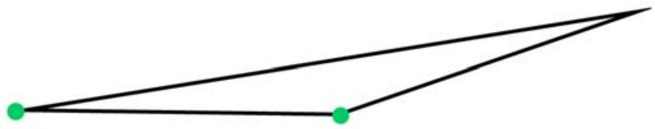
$E_P = E_A + K(E_B-E_A)$ and $N_P = N_A + K(N_B-N_A)$

The distances d_{PC}, d_{PA} and d_{PB} can then be calculated from the coordinates of P, if required.

Intersection Design Geometry

The angle of the intersection at point C affects the accuracy of the coordinates of C. You will learn more about that in a later course. Briefly, good design has intersection angles close to 90° and bad design has angles close to 0° or 180°.





Good intersection design

Bad intersection design

SUMMARY OF INTERSECTIONS

- Intersection: Finding the coordinates of a point (C) by measuring directions, angles, bearings or distances from two or more known points (A and B).
- Several mathematical expressions for the coordinates of the point C have been derived and can easily be used on calculators, in spreadsheets or in computer source code. Some of the equations follow.
- Intersection by 2 angles

$$E_C = E_A - \frac{\sin B}{\sin(A - B)} \left\{ (E_B - E_A)\cos A + (N_B - N_A)\sin A \right\}$$

$$N_C = N_A - \frac{\sin B}{\sin(A - B)} \left\{ (N_B - N_A)\cos A - (E_B - E_A)\sin A \right\}$$

Where A and B are the angles measured clockwise from the line AB, they are NOT the internal angles.

- Intersection by 2 bearings

$$d_{AC} = \frac{\Delta E_{AB} \cos \beta_{BC} - \Delta N_{AB} \sin \beta_{BC}}{\sin(\beta_{AC} - \beta_{BC})} \qquad E_C = E_A + d_{AC} \sin \beta_{AC} \qquad N_C = N_A + d_{AC} \cos \beta_{AC}$$

E3: =E1 + (((E2-E1) * COS(B2) - (N2-N1) * SIN(B2)) / SIN(B1-B2)) * SIN(B1)
N3: =N1 + (((E2-E1) * COS(B2) - (N2-N1) * SIN(B2)) / SIN(B1-B2)) * COS(B1)

- Intersection by 2 distances (also called trilateration)

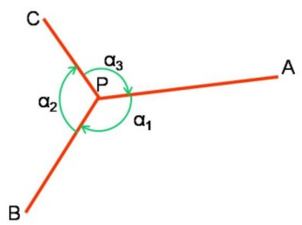
$$E_C = E_A + \frac{b}{c}(\Delta E_{AB} \cos A + \Delta N_{AB} \sin A) \qquad N_C = N_A + \frac{b}{c}(\Delta N_{AB} \cos A - \Delta E_{AB} \sin A)$$

- Is the new point C left or right of the line AB? Be careful with the equations.
- Draw a sketch to visualise the problem and to make clear which parameters are used in the equations and to get an idea what results to expect.
- Always check your results by using a second set of equations to solve for the coordinates of C, or by reversing the process, i.e. from the coordinates of A B and C calculate the distances and bearings of all lines and angles of the triangle.
- Practice the calculations!

RESECTION

A three-point resection is a method of determining the coordinates of a point (P) from direction (or angle) observations taken from that point to three points (A, B, C) that have known coordinates. Historically, Snellius found a solution around 1615 and many people have worked on this interesting challenge since then. Applications include surveys where no distances are available or possible, where marks with known coordinates can be seen and it is desired to set up not over a survey mark, on industrial or construction sites where marks on the ground get destroyed or lost but coordinated wall marks remain, and in a coordinated cadastre. It is an alternative to traversing if many control points can be seen. Sometimes it is called “free stationing” and many modern total station survey instruments do the calculations “on board” the instrument. Resection involves the observation of known points from an unknown point, sometimes distance and zenith angle observations are also included (but we leave that for a later course).

In these notes we choose any point as A then label the points clockwise from A to B to C, just as a theodolite measures directions from P. From direction observations the angles (α) can be calculated. Note that we do not know the bearings from P to the points A B or C. If we knew the bearings we would only need two targets (e.g. A and B) and could calculate P by intersection of bearings. Similarly if we have the distances from P we only need two targets (e.g. A and B) and could calculate P by intersection of distances.

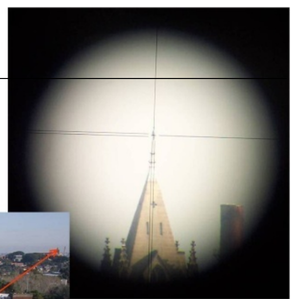


The minimum number of directions required for a solution is 3. This gives a unique solution of the point’s coordinates. The solution of resection problems with more than three target points is covered in a later course. There are a variety of problems similar to resections, sometimes called free stationing, where an instrument is placed in a new (uncoordinated) position and directions or distances to known coordinated points. Many modern instruments can do the calculations on-board. One example case is where an instrument at P observes directions to A and B and distances to A and B. Thus two distances and an included angle are observed and the distance between A and B can be calculated from their coordinates. Then the triangle can be solved by the sine rule, bearings calculated from the angles, then coordinates by radiation.

There are several methods of solving the three point resection problem, e.g.

- Collins method
- Tangent method (Blunt’s method)
- Cassini method
- Tienstra’s Barycentric method
- Willerdings formula
- Solver tool in Microsoft Excel
- Semigraphic method
- Least Squares

Graphical solution



In another course at UNSW a field exercise familiarises students with the measuring of horizontal directions using an electronic tacheometer. Students determine the coordinates of a pillar on the roof of a building by multiple resections with directions. The expected precision is a few millimetres. If you are doing, or have done a similar exercise then use your own observations for this exercise. Otherwise use the student data provided below.

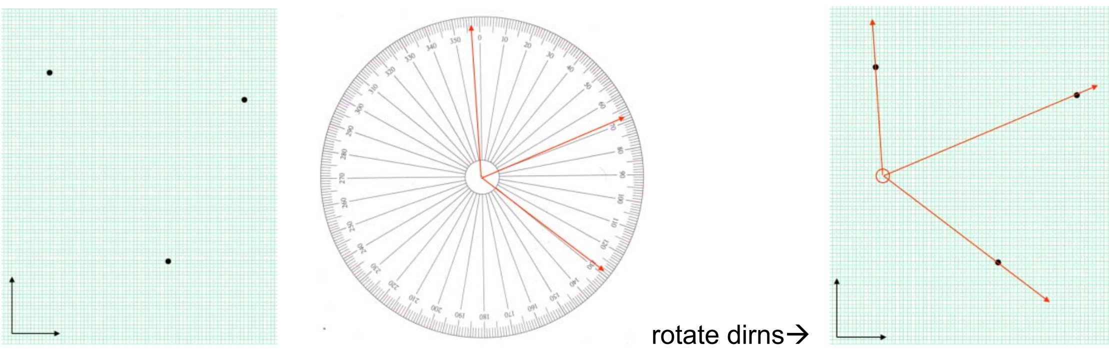
Sample student observations for a Resection

Target	Grand mean direction	E target	N target
LC801	119 59 57.8	336509.444	6245543.320
LC850	122 56 23.6	336613.812	6245516.370
TS840	122 47 27.6	336421.190	6245544.511
TS830	200 03 00.5	336290.609	6245444.209
TS820	303 15 42.1	336124.363	6245587.848
TS802	39 10 19.0	336283.771	6245576.098

1 Reduce the observations to give the grand mean directions to the observed targets, as above.

2 Carefully plot the coordinates of the target stations used at an appropriate scale (e.g. 1:25 000 or 1:10 000) on A4 graph paper. On a sheet of clear tracing paper draw (with a protractor) the measured rays. Then, slide and rotate the tracing paper, until *all rays* cross the corresponding targets on the graph paper. The origin of the rays can then be pinned through onto the graph paper. If you find that it is impossible to get a match on all rays, then the plotted coordinates or the measurements are wrong. If so, **find and correct the error before proceeding**. Record the graphically determined coordinates of your pillar. Discuss the fit. In the case of a poor fit report on any remedial action taken.

3 From the diagram in **2**, select three target stations, which are close and provide good angles for the numerical resection. Make sure that your resection point is not on the circumference of a common circle with the three selected targets (not on a straight line through the three points). Use the corresponding direction observations to compute the coordinates of the instrument station by three-point resections, using one of the methods described later.

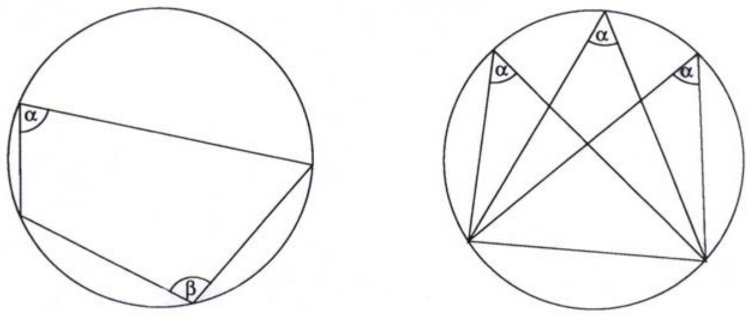


4 Repeat step 3 using a different group of three target stations. Compare the pillar coordinates obtained from the two resections and discuss any discrepancies. If the differences are larger than expected, **find** (and correct) **the error** in your computation (or field measurement). Consult a teacher if you cannot find an error!

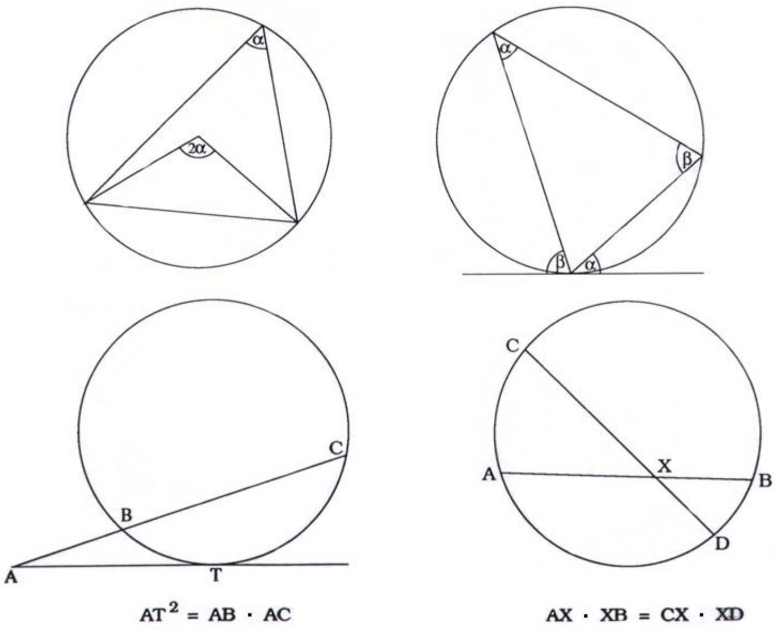
Some students who do the above task should scan each step of the graphics and send to me for inclusion in this book.

Some properties of circles

Revision from High School Maths:



$\alpha + \beta = 180°$

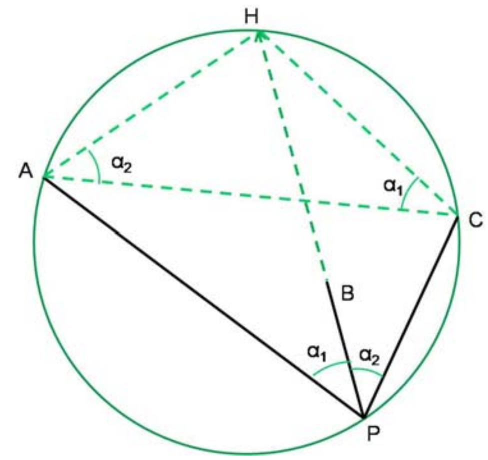


COLLINS' POINT METHOD for $\alpha_1+\alpha_2 < 180°$

The Collins' point method is an elegant method that reduces the resection problem into two intersections by creating an intermediate point. The intermediate point can be on either side of the control points depending on the geometry, as shown below. Here we start with the case where the sum of the two angles ($\alpha_1+\alpha_2$) is < 180°.

Given: Direction observations from P to three control points A, B and C.
Problem: Find the coordinates of P.

Solution:



- Construct a circle through the resected point (P) and any two of the control points (say A and C). Join P to the third control point B and extend it (if necessary) to meet the circle in H. H is known as the "Collins' Auxiliary Point".

- From the observed direction observations at P, calculate angles APB (α_1) and BPC (α_2).

- From the geometry of the circle (see 'high school' circle property figures previously):

  angle ACH = angle APH = α_1 angle CAH = angle CPH = α_2

- Computational steps:

  1) Using coordinates of A and C, calculate β_{AC} and d_{AC} (R→P or atan2).

  2) Using β_{AC}, α_1 and α_2, calculate bearings β_{AH} and β_{CH}.

  3) Using d_{AC}, α_1 and α_2, solve triangle ACH for distances d_{AH} and d_{CH} (sine rule).

  4) Using β_{AH} and d_{AH}, calculate coordinates of H from A (P→R).

  5) Similarly, using β_{CH} and d_{CH}, calculate coordinates of H from C and use it as a check (P→R).

6) Calculate β_{PBH} from coordinates of B and H (R→P or atan2).

7) Using β_{PBH}, α_1 and α_2, calculate bearings β_{PA} and β_{PC}.

8) Using β_{AC}, β_{PA} and β_{PC}, calculate the angles of triangle ACP.

9) Using d_{AC} and the angles of triangle ACP, solve the triangle for the distances d_{AP} and d_{CP} (sine rule).

10) Using β_{PA} and d_{AP}, calculate the coordinates of P from A (P→R).

11) Similarly, using β_{PC} and d_{CP}, calculate the coordinates of P from C and use it as a check (P→R).

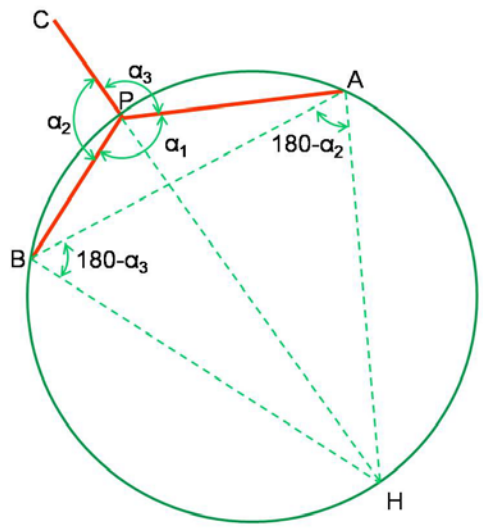
Notes:

- Steps 1-5 are equivalent to intersection of H from A and C (intersection by angles).
- Steps 8-11 are equivalent to intersection of P from A and C (intersection by bearings).
- Steps 1-5 are checked by the calculation in step 5.
- Steps 8-11 are checked by the calculation in step 11.
- The only calculations not checked by this method are steps 6 and 7.

Check of resection coordinates

A good check calculation for any of the resection methods is to calculate the bearings β_{PA}, β_{PB} and β_{PC} from the final coordinates of P. Then calculate $\alpha_1 = \beta_{PB} - \beta_{PA}$ and $\alpha_2 = \beta_{PC} - \beta_{PB}$ from these bearings and compare with given angles or directions.

COLLINS' POINT METHOD for $\alpha_1+\alpha_2 > 180°$



We treat this case by example.

Given: A = (E 18 597.25, N 113 722.63)
B = (E 10 783.71, N 111 405.38)
C = (E 10 186.97, N 116 800.92)
angle APB = α_1 = 140° 40' 23"
angle BPC = α_2 = 117° 00' 06"

Solution:

Plot A, B and C in their relative positions. Calculate bearing and distance between AB, BC and CA:

β_{AB} = 253° 28' 52.7" d_{AB} = 8149.911m
β_{BC} = 353° 41' 19.7" d_{BC} = 5428.439m
β_{CA} = 110° 06' 12.2" d_{CA} = 8955.930m

Computational steps:

1) Using coordinates of A and B, calculate β_{AB} and d_{AB}: β_{AB} = 253° 28' 52.7" d_{AB} = 8149.911m

2) Using $\beta_{AB,}$ α_1 and α_2, calculate bearings β_{AH} and β_{BH}:

α_1 = 140° 40' 23" α_2 = 117° 00' 06" α_3 = 360° - α_1 - α_2 = 102° 19' 31"

$\beta_{AH} = \beta_{AB} - (180° - \alpha_2)$ = 190° 28' 58.7"
$\beta_{BH} = (\beta_{AB} - 180°) + (180° - \alpha_3)$ = <u>151° 09' 21.7"</u>

3) Solve triangle ABH for distances d_{AH} and d_{BH} by sine rule:

$$\frac{d_{BH}}{\sin(180-\alpha_2)} = \frac{d_{AB}}{\sin(\alpha_2+\alpha_3-180)} = \frac{d_{AH}}{\sin(180-\alpha_3)}$$

$$\rightarrow \; d_{BH} = -\frac{d_{AB}\sin\alpha_2}{\sin(\alpha_2+\alpha_3)} = 11\,458.109\text{m} \qquad d_{AH} = -\frac{d_{AB}\sin\alpha_3}{\sin(\alpha_2+\alpha_3)} = 12\,563.525\text{m}$$

4) Using β_{AH} and d_{AH}, calculate coordinates of H from A:

β_{AH} = 190° 28' 58.7" d_{AH} = 12 563.525m → E_H = 16 311.401m N_H = 101 368.802m

5) Similarly, using β_{BH} and d_{BH}, calculate coordinates of H from B and use it as a check (P→R):
β_{BH} = 151° 09' 21.7" d_{BH} = 11 458.109m → E_H = 16 311.400m N_H = 101 368.802m □

6) Calculate β_{PCH} from coordinates of C and H: β_{PH} = 158° 21' 13.4" β_{PC} = 338°21'13.4"

7) Calculate bearings β_{PA} and β_{PB}:
$\beta_{PA} = \beta_{PC} + \alpha_3 = \beta_{PH} - (180 - \alpha_3)$ = 80° 40' 44.4" $\beta_{PB} = \beta_{PC} - \alpha_2$ = 221° 21' 07.4"

8) Using β_{AB}, β_{PA} and β_{PB}, calculate the angles of triangle ABP:
angle PBA = $\beta_{BA} - \beta_{BP}$ = 32° 07' 45.3" angle BAP = $\beta_{AP} - \beta_{AB}$ = 7° 11' 51.7"

9) Using d_{AB} and the angles of triangle ABP, solve the triangle for the distances d_{AP} and d_{BP} (sine rule):

$$\frac{d_{AP}}{\sin(PBA)} = \frac{d_{AB}}{\sin\alpha_1} = \frac{d_{BP}}{\sin(BAP)} \quad \rightarrow \quad d_{AP} = \frac{d_{AB}\sin(PBA)}{\sin\alpha_1} = 6839.308\text{m}$$

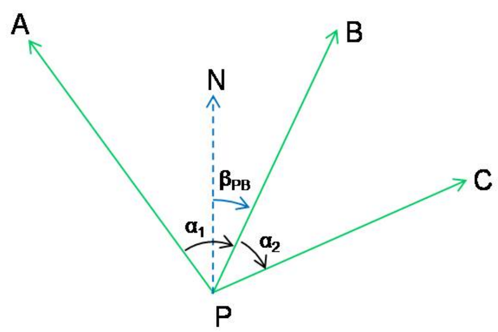
$$d_{BP} = \frac{d_{AB}\sin(BAP)}{\sin\alpha_1} = 1611.263\text{m}$$

10) Using β_{AP} and d_{AP}, calculate the coordinates of P from A:
β_{AP} = 260° 40' 44.4" d_{AP} = 6839.308m → E_P = 11 848.245m N_P = 112 614.898m

11) Similarly, using β_{BP} and d_{BP}, calculate the coordinates of P from B and use it as a check:
β_{BP} = 41° 21' 07.4" d_{BP} = 1611.263m → E_P = 11 848.246m N_P = 112 614.897m ✓

TANGENT METHOD (Blunt's Method)

This method is easier to program than Collins Point Method. Stations are labelled A, B, C from left to right (B is in the middle), i.e. clockwise from P. Angles α_1 and α_2 are calculated from the observations taken at P.



Our aim here is to find the bearing of the line PB, β_{PB}. The bearing is not observed (only directions and angles are observed) and the coordinates of P are not yet known. So we do some algebra and derive useful equations, as follows.

From the figure we could find β_{PB} if we knew the coordinates of P:

$$\tan\beta_{PB} = \frac{E_B - E_P}{N_B - N_P} \quad \rightarrow \quad E_P = E_B - (N_B - N_P)\tan\beta_{PB} \qquad \text{------ (1)}$$

Similarly for lines to A and C:

$$E_P = E_C - (N_C - N_P)\tan\beta_{PC} \qquad \text{------ (2)}$$

$$E_P = E_A - (N_A - N_P) \tan \beta_{PA} \qquad \text{------ (3)}$$

Equations (1) and (2) give:

$$E_B - (N_B - N_P) \tan \beta_{PB} = E_C - (N_C - N_P) \tan \beta_{PC}$$

$$\rightarrow N_P = \frac{E_B - E_C + N_C \tan\beta_{PC} - N_B \tan\beta_{PB}}{\tan\beta_{PC} - \tan\beta_{PB}} \qquad \text{------ (4)}$$

Similarly with equations (3) and (2), and (3) and (1) give:

$$N_P = \frac{E_A - E_C + N_C \tan\beta_{PC} - N_A \tan\beta_{PA}}{\tan\beta_{PC} - \tan\beta_{PA}} \qquad \text{----- (5)}$$

$$N_P = \frac{E_A - E_B + N_B \tan\beta_{PB} - N_A \tan\beta_{PA}}{\tan\beta_{PB} - \tan\beta_{PA}} \qquad \text{----- (6)}$$

From equations (4), (5) and (6), we can derive:

$$\tan\beta_{PB} = \frac{(E_A - E_B)\cot\alpha_1 + (E_C - E_B)\cot\alpha_2 - (N_C - N_A)}{(N_A - N_B)\cot\alpha_1 + (N_C - N_B)\cot\alpha_2 + (E_C - E_A)} \qquad \text{----- (7)}$$

- **Tips:** In this equation for bearing of PB, cot = 1/tan but it also equal cos/sin which may work better when the angle is one of the cardinal directions. When calculating β_{PB} it is necessary to look at the sign of the numerator and denominator to determine the correct quadrant. So, instead of using ATAN and doing the full division, calculate numerator and denominator separately (i.e. in two memory cells) then use ATAN2 and MOD functions (or use R→P on calculators).

Procedure:

1) Calculate β_{PB} using equation (7).
2) Calculate β_{PA} and β_{PC} using β_{PB}, α_1 and α_2.
3) Calculate N_P using equation (4), (5) or (6).
4) Calculate E_P using equation (1), (2) or (3).

Choose the equation which uses the smallest magnitude of the trig functions.

<u>Example</u>:

Given previous data: A = (E 18 597.25, N 113 722.63) α_1 = 140° 40' 23"
B = (E 10 783.71, N 111 405.38) α_2 = 117° 00' 06"
C = (E 10 186.97, N 116 800.92)

Solution:

1) Calculate β_{PB} using equation (7):

$$\tan\beta_{PB} = \frac{(E_A - E_B)\cot\alpha_1 + (E_C - E_B)\cot\alpha_2 - (N_C - N_A)}{(N_A - N_B)\cot\alpha_1 + (N_C - N_B)\cot\alpha_2 + (E_C - E_A)} = 221°21'07.4"$$

2) Calculate β_{PA} and β_{PC} using β_{PB}, α_1 and α_2:

$\beta_{PA} = \beta_{PB} - \alpha_1$ = 80° 40' 44.4" $\beta_{PC} = \beta_{PB} + \alpha_2$ = 338° 21' 13.4"

3) Calculate N_P using equation (4), (5) or (6):

Eq. (4) uses β_{PB} and β_{PC}. $|\tan \beta_{PA}| \approx 6.08$
Eq. (5) uses β_{PA} and β_{PC}. $|\tan \beta_{PB}| \approx 0.88$
Eq. (6) uses β_{PA} and β_{PB}. $|\tan \beta_{PC}| \approx 0.40$
Eq. (4) uses the smallest trig. function values, so:

$$N_P = \frac{E_B - E_C + N_C \tan\beta_{PC} - N_B \tan\beta_{PB}}{\tan\beta_{PC} - \tan\beta_{PB}} = 112\,614.898\text{m}$$

4) Calculate E_P using equation (1), (2) or (3):

Eq. (1) uses β_{PB}. $|\tan \beta_{PA}| \approx 6.08$
Eq. (2) uses β_{PC}. $|\tan \beta_{PB}| \approx 0.88$
Eq. (3) uses β_{PA}. $|\tan \beta_{PC}| \approx 0.40$

Eq. (2) uses the smallest trig function value (smaller tan function values are more reliable):

$E_P = E_C - (N_C - N_P) \tan \beta_{PC}$ = 11 848.246m

5) Check by using equation (1): $E_P = E_B - (N_B - N_P) \tan \beta_{PB}$ = 11 848.246m ✓

6) Independent Check, reverse the process:

- Calculate the bearings β_{PA}, β_{PB} and β_{PC} from the final coordinates of P:

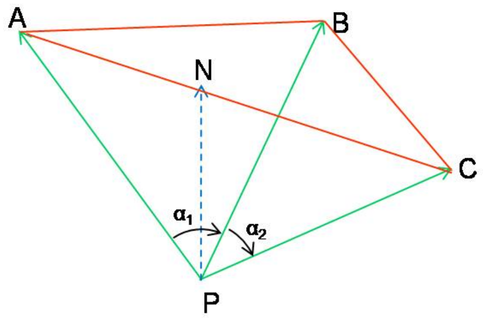
  β_{PA} = 80° 40' 44.4" β_{PB} = 221° 21' 07.4" β_{PC} = 338° 21' 13.4"

- Calculate α_1 and α_2 from these bearings and compare with given values:

  $\alpha_1 = \beta_{PB} - \beta_{PA}$ = 140° 40' 23" ✓ $\alpha_2 = \beta_{PC} - \beta_{PB}$ = 117° 00' 06" ✓

TIENSTRA'S METHOD

The calculations in Tienstra's method are similar to a weighted mean. It is easy to program. Also called the Method of Barycentric coordinates.
Reference: Porta J.M., Thomas F. Concise proof of Tienstra's formula, Journal of Surveying Engineering 135(4), 170–172, 2009.



Stations are labelled A, B, C from left to right and clockwise.
1) Angles α_1 and α_2 are observed or calculated from the direction observations taken at P. α_1 = dirPB –dirPA and α_2 = dirPC –dirPB. Calculate the third angle at P, α_3 : α_3 = 360° – (α_1 + α_2) or α_3 = dirPA –dirPC

2) Calculate the angles in the triangle ABC from their coordinates, via the bearings of the lines joining A, B and C as shown in figure. $A = \beta_{AC} - \beta_{AB}$ $B = \beta_{BA} - \beta_{BC}$ $C = \beta_{CB} - \beta_{CA}$

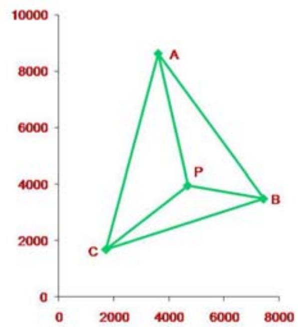
3) Calculate the "weights" (remember cot = 1/tan = cos/sin):

$$w_A = 1/(\cot A - \cot\alpha_2) \qquad w_B = 1/(\cot B - \cot\alpha_3) \qquad w_C = 1/(\cot C - \cot\alpha_1)$$

4) Then the coordinates of P are:

$$E_P = \frac{E_A w_A + E_B w_B + E_C w_C}{w_A + w_B + w_C} \qquad N_P = \frac{N_A w_A + N_B w_B + N_C w_C}{w_A + w_B + w_C}$$

5) Check: Calculate the bearings β_{PA}, β_{PM} and β_{PB} from the final coordinates of P. Calculate α_1 and α_2 from these bearings and compare with given angles or directions.



TIENSTRA Example:

A resection was used to fix point P from points A, B, and C, whose coordinates are listed below. Note that the observed directions are not bearings.

	E	N	Observed directions
A	3613.52	8609.71	PA 00° 00' 00"
B	7444.39	3487.16	PB 112° 34' 50"

C 1712.06 1693.38 PC 245° 43' 21"
Calculation, intermediate values:

angles at P from directions	Bearings from coordinates	ABC angles from bearings	weights
$\alpha_1 = dir_{PB} - dir_{PA}$ = 112.6º	β_{AB} = 143.2º	A = β_{AC}-β_{AB} = 52.2º	w_A = 0.583
$\alpha_2 = dir_{PC} - dir_{PB}$ = 133.1º	β_{BC} = 252.6º	B = β_{BA}-β_{BC} = 70.6º	w_B = 1.245
$\alpha_3 = dir_{PA} - dir_{PC}$ (+360)=114.3º	β_{AC} = 195.4º	C = β_{CB}-β_{CA} = 57.3º	w_C = 0.944
		Sum w:	2.772

$$w_A = 1/(\cot A - \cot\alpha_2) \qquad w_B = 1/(\cot B - \cot\alpha_3) \qquad w_C = 1/(\cot C - \cot\alpha_1)$$

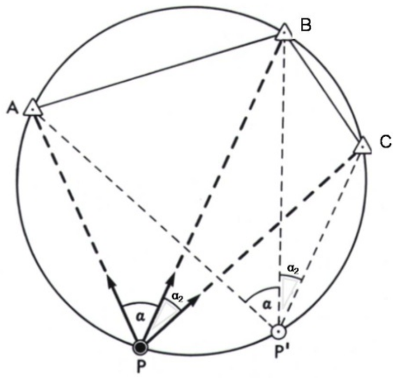
$$E_P = \frac{E_A w_A + E_B w_B + E_C w_C}{w_A + w_B + w_C} \qquad N_P = \frac{N_A w_A + N_B w_B + N_C w_C}{w_A + w_B + w_C}$$

P = 4685.680, 3954.316

Check bearings from coordinates:

	Bearings			Directions			Angles	
PA	347º	01'	50.3"	0º	00'	00"		
PB	99º	36'	40.3"	112º	34'	50"	112.6º	OK
PC	232º	45'	11.3"	245º	43'	21"	133.1º	OK

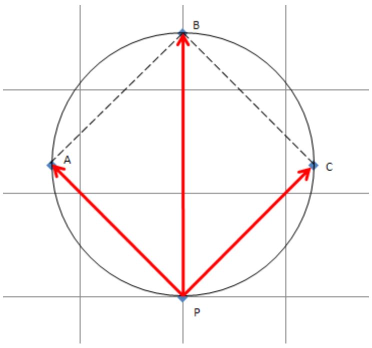
DANGER CIRCLE



Assume the unknown point P lies on the circle passing through the 3 given control points (A, B, and C). By the properties of circles, the angles at P and P' are the same. In this case the coordinates of P cannot be fixed by the angles α_1 and α_2! P could be anywhere on the circle and so there are an infinite number of solutions. Avoid locating your survey so that P lies on or near the circle that contains the target points.

How do we avoid the danger circle or know that we are near it? One method is to plot the points on a map or plan and to use your knowledge of the location of P. Another method is to calculate the sum of the angles, S:

S = α_1 + α_2 + angle CBA where angle CBA = $\beta_{BA} - \beta_{BC}$



If S is a multiple of 180° or close to it, then P is on or near the danger circle.

In the Tienstra Method example above (where P is clearly not on the danger circle), S ≈ 113º + 133º + 71º = 317º

Now look at some artificial data shown in the plot at right, where P was intentionally put on the danger circle. The data is given in tutorial question 5 later.

Here α_1 = 45°, α_2 = 45°, angle CBA = $\beta_{BA} - \beta_{BC}$ = 225° - 135° = 90°

So S = 45° + 45° + 90° = 180° and we are on the danger circle. P could lie anywhere on the circle.

The design and error analysis of resection geometry will be covered in more detail in a later course. But in general the closer the targets (A B C) are to P the better. Also the error in the calculated coordinates increases substantially when approaching the danger circle. Errors in coordinates of P are smaller if P is inside the circle formed by the three known points ($\alpha_1 + \alpha_2 > 180°$) rather than outside ($\alpha_1 + \alpha_2 < 180°$). This is because when the angles α_1 and α_2 are larger the solution becomes stronger. You will get a good determination of your position if you select 4 close targets, which are equally spaced around you. Adding distance measurements also strengthens a solution.

Willerding's formula

This is a new method using complex numbers that works well in MATLAB. For details read: Heindl G, Analysing Willerding's formula for solving the planar three point resection problem, J. Appl. Geodesy 2018. The paper gives a MATLAB function that I reproduce here and modify. Note the coordinates are entered here as N + Ei and directions in radians. You can modify and implement it yourself.

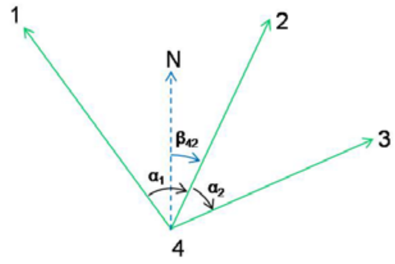
```
function willerding(p1,p2,p3,t1,t2,t3)
% p1,p2,p3 pairwise complex numbers representing the coordinates N + Ei
% t1,t2,t3 are the directions to p1, p2, p3 respectively in radians
% here direct entry of data
p1 = 1554.748 + 3370.049i;
p2 = 1077.877 + 3819.845i;
p3 = 821.397 + 3218.233i;
t1 = (0.0 + 0.0/60.0 + 0.0/3600.0)*pi/ 180.0;
t2 = (112.0 + 17.0/60.0 + 56.7/3600.0)*pi/ 180.0;
t3 = (240.0 + 32.0/60.0 + 59.1/3600.0)*pi/ 180.0;
%
c1 = conj(p3-p2)*exp(-i*2*t1);
c2 = conj(p1-p3)*exp(-i*2*t2);
c3 = conj(p2-p1)*exp(-i*2*t3);
% the ref paper uses symbol N for sum of c terms. Here we use S instead
S = c1+c2+c3;
absS = abs(S)
if absS > 1.0e-10 %suitably chosen lower bound
  p0 = (c1*p1+c2*p2+c3*p3)/S
  fprintf('N = %0.3f E = %0.3f', p0,imag(p0));
  %
  %Computing eps:
  eps1 = angle(exp(i*(angle(p1-p0)-t1)));
  eps2 = angle(exp(i*(angle(p2-p0)-t2)));
  eps3 = angle(exp(i*(angle(p3-p0)-t3)));
  eps = (eps1+eps2+eps3)/3;
  del = abs(eps-eps1)+abs(eps-eps2)+abs(eps-eps3)
  if del > 1.0e-10 %suitably chosen lower bound
    'data are incompatible'
  else
    eps
  end
else
   'p0,p1,p2,p3 are close to a circle or a line'
end
```

The output of this function are the N and E coordinates of the resected point, p0, or a warning message about danger circle. For this data P_E = 3500.123 and P_N = 1111.222

Excel Solver Solution of Resection

Label the known points 1, 2, and 3 running clockwise when viewed from the occupied / unknown point and call point 4, then there are two unknowns (E_4, N_4). Make two equations to solve for these unknowns using the two observed angles:

$\alpha_1 = \beta_{42} - \beta_{41}$ and $\alpha_2 = \beta_{43} - \beta_{42}$



Rearranging the terms and substituting the equations for bearing from coordinates, leads to two equations with two unknowns:

$$\tan^{-1}\frac{(E_2 - E_4)}{(N_2 - N_4)} - \tan^{-1}\frac{(E_1 - E_4)}{(N_1 - N_4)} - \alpha_1 = 0 \qquad \text{--- (1)}$$

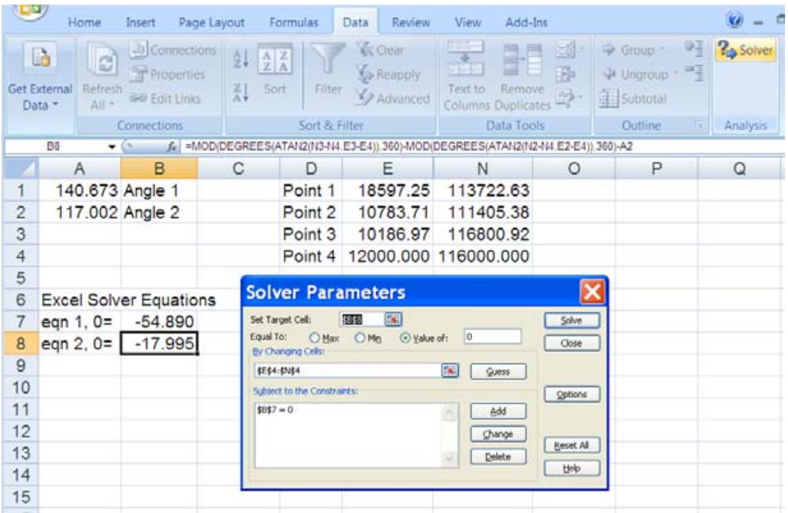
$$\tan^{-1}\frac{(E_3 - E_4)}{(N_3 - N_4)} - \tan^{-1}\frac{(E_2 - E_4)}{(N_2 - N_4)} - \alpha_2 = 0 \qquad \text{--- (2)}$$

Then we can use Microsoft Excel's Solver function to determine the coordinates of point 4. (If Solver is not installed in your version of Excel see the HELP section to add it.)

Demonstration of the process with one of the previous example's data:

Given: 1 = (E 18 597.25, N 113 722.63) α_1 = 140° 40' 23"
2 = (E 10 783.71, N 111 405.38) α_2 = 117° 00' 06"
3 = (E 10 186.97, N 116 800.92)

To make the Excel equations easier to read in this example store E coordinates in column E, Northings in N and angles in A, use rows to match point or angle number, and hide some columns.



In cell B7 type:
=MOD(DEGREES(ATAN2(N2-N4,E2-E4)),360)-MOD(DEGREES(ATAN2(N1-N4,E1-E4)),360)-A1

In cell B8 type:
=MOD(DEGREES(ATAN2(N3-N4,E3-E4)),360)-MOD(DEGREES(ATAN2(N2-N4,E2-E4)),360)-A2

Select cell B8 and choose Tools | Solver or Data | Analysis | Solver (depends on version of Excel).

The Solver options and constraints here are to vary the estimates E4 and N4 until both B7 and B8 = 0. Set one as the target cell and the other as a constraint.

Click SOLVE and the solution is placed in E4 and N4. In this data set starting at E4 and N4 = 0 i.e. blank does not lead to a good solution. The Solver tool needs a better starting point. An approximate value of the answer needs to be estimated. In the real world you could look at a site plan or topographic map to estimate the coordinates of point 4. Another way is to take the mean coordinates of points 1, 2, and 3. Another way is to graph the points and estimate point 4's coordinates. In this example I choose 12000 and 116000 as starting coordinates for point 4. Solver then gives the answer for 4 as: 11848.246, 112614.898 which is the same as that determined by our previous methods and has both equation cells as 0. As a check of the solution I would recommend calculating the bearings from Solver's coordinates for 4 to 1, 2, and 3. Then calculate angles between these bearings and see if they match the original angles.

If the North point from 4 lies between 1 and 2 or between 2 and 3 then we need to change the equations for the angles. Draw a picture for assistance. For example, if North lies between 1 and 2 then:

$$\tan^{-1}\frac{(E_2 - E_4)}{(N_2 - N_4)} + \left(360 - \tan^{-1}\frac{(E_1 - E_4)}{(N_1 - N_4)}\right) - \alpha_1 = 0 \qquad \text{--- (1)}$$

$$\tan^{-1}\frac{(E_3 - E_4)}{(N_3 - N_4)} - \tan^{-1}\frac{(E_2 - E_4)}{(N_2 - N_4)} - \alpha_2 = 0 \qquad \text{--- (2)}$$

Opinion: There are several solution methods inside Solver, and you probably don't know what they are doing. Usually it uses an iterative method to get closer and closer to an answer. When you run it you don't see the iterations or the step by step processes. It can be easy to use. But can be risky too. I include it here because it can sometimes help solve a tricky problem and because some students learn about it in year 1. Solver only works if you have a spreadsheet program or similar, and often it doesn't help you understand the problem. All the other traditional methods that I show are direct formula based solutions, not iterative. And you can see the full workings, which you could for example write into a program if you know how to "code".

Other calculation methods

1. Use a Total Station Instrument's "On board" software. This depends on your brand and model of instrument. See lectures for more details.
2. Use Least Squares e.g. program Fixit which is covered in another course at UNSW.
3. Use surveying CAD or CoGo programs. If resections are not directly available in your CAD software, but intersections are then it is possible to solve a resection using Collins point method with intersections.
4. Write your own computer program.

Summary of Resections

- Resection: Fixing the coordinates of a point (P) from direction observations taken from that point to 3 or more control points.

- Note that you have, or are given, directions from P or angles at P. You do **not** have the bearings from P in resection problems. If you knew the bearings you could solve the problem by intersection of bearings.

- Drawing a sketch clarifies the problem and gives you an indication of the approx. coordinates of P.

- Know the properties of circles and beware of the danger circle. $S = \alpha_1 + \alpha_2 + (\beta_{BA} - \beta_{BC}) = 180°*n$?

- Collins' point method. Tangent method. Tienstra's method. Excel Solver. Other methods…

  Tangent method equations: $$\tan\beta_{PB} = \frac{(E_A - E_B)\cot\alpha_1 + (E_C - E_B)\cot\alpha_2 - (N_C - N_A)}{(N_A - N_B)\cot\alpha_1 + (N_C - N_B)\cot\alpha_2 + (E_C - E_A)}$$

  then use angles to get bearings of other lines, then

  $$N_P = \frac{E_B - E_C + N_C\tan\beta_{PC} - N_B\tan\beta_{PB}}{\tan\beta_{PC} - \tan\beta_{PB}}$$ $$E_P = E_B - (N_B - N_P)\tan\beta_{PB}$$

- Check your calculations by calculating bearings from P to each of the control points. The differences between the bearings should equal the angles you started with.

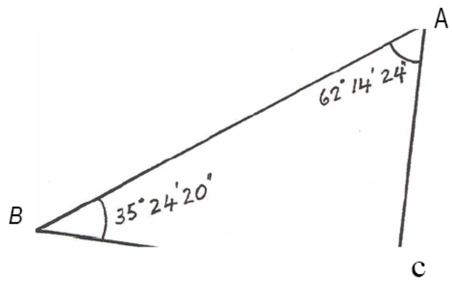
- Practice the calculations!

Tutorial Questions

Q1. From the following data, compute the coordinates of C using

(a) Solution of triangles method

(b) Formulae for intersection by angles (be careful to use clockwise angles from the line AB).



	COORDINATES	
POINT	E	N
A	45 328.172	26 985.030
B	44 626.185	26 616.600

Q2. Find the coordinates of the intersection of the two rays RX and SX details of which are given below

	E	N		Bearing
R	45328.172	26985.030	RX	234°48'20"
S	44626.185	26916.600	SX	126 11 30

Q3. With the following data, compute the coordinates of point C. (Note: C lies to the east of B)

	COORDINATES			
STN	E	N	Observed Distances	
A	1161.634	3941.286	AC	223.201
B	1099.689	4085.466	BC	216.014

Q4. A resection is used to fix point P from points A, B, and C, whose coordinates are listed below.

	E	N	Observed directions (not bearings)	
A	82 613.52	54 609.70	PA	00° 00' 00"
B	86 444.39	49 487.16	PB	112° 17' 56".7
C	80 712.06	47 693.38	PC	240° 32' 59".1

Calculate the coordinates of P by (a) by graphical solution, (b) Collins' Point Method, and (c) Tangent Method.

Q5. Prepare an MS Excel spreadsheet to calculate and plot a three-point resection using the tangent method. Design the spreadsheet so that users can easily enter the observed directions and known coordinates. The sheet then calculates the answers and provides check calculations. Test your sheet using the example data supplied below, and any other data you can devise. Note that the example data is designed to test if your 'program' works for a variety of different geometry data, not just for one data set.

Example Data 1: STN	E (m)	N (m)	Observed directions (not bearings)
A	18597.25	113722.63	PA: 0° 00' 00"
B	10783.71	111405.38	PB: 140° 40' 23"
C	10186.97	116800.92	PC: 257° 40' 29"

Solution:

α_1 = 140° 40' 23" α_2 = 117° 00' 06"

β_{PB} = 221° 21' 07.4" The quadrant of β_{PM} is resolved by noting the numerator and denominator are both negative.

$\beta_{PA} = \beta_{PB} - \alpha_1 = 80°40' 44.4"$ $\beta_{PC} = \beta_{PB} + \alpha_2 = 338°21'13.4"$

N_P = 112614.898 E_P = 11848.246

Check: Calculate the bearing from the derived coordinates of P to each of A B and C. Then calculate the reduced directions (or angles) from these bearings and compare them with the original directions (or angles). Using the derived coordinates: β_{PA} = 80°40'44.4", β_{PB} = 221°21' 07.4", β_{PC} = 338°21'13.4"

And directions, with PA set to 0:

PB = $\beta_{PB} - \beta_{PA}$ = 140° 40' 23.0"
PC = $\beta_{PC} - \beta_{PA}$ = 257° 40' 29.0"   Which are close enough to the original data.

Danger circle calculations: S =140°40'23" + 117°00'06" + 79°47'30" ≈ 337° so not on the danger circle.

Test Data 2 (PA north):	STN	E (m)	N (m)		Directions	*True P*
	A	10	100	PA	0° 00' 00"	*10,10*
	B	100	100	PB	45° 00' 00"	
	C	100	10	PC	90° 00' 00"	

Test Data 3 (danger circle):	STN	E (m)	N (m)		Directions	*True P*
	A	36.36	163.64	PA	0° 00' 00"	*100,100*
	B	100	227.279	PB	45° 00' 00"	
	C	163.64	163.64	PC	90° 00' 00"	

Test Data 4 (PB north):	STN	E (m)	N (m)		Directions	*True P*
	A	74.019	85	PA	0° 00' 00"	*100,100*
	B	100	130	PB	120° 00' 00"	
	C	125.981	85	PC	240° 00' 00"	

Test Data 5 (PB east):	STN	E (m)	N (m)		Directions	*True P*
	A	85	125.981	PA	0° 00' 00"	*100,100*
	B	130	100	PB	120° 00' 00"	
	C	85	74.019	PC	240° 00' 00"	

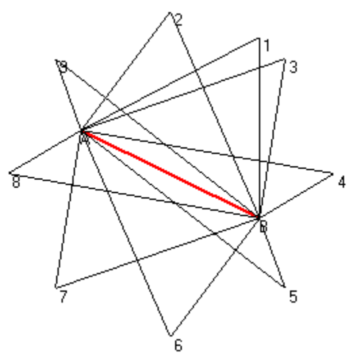
Q6. Intersection by bearings program

Write a computer program to calculate the coordinates of several target points by intersection of bearings, and draw a plan of their location. Alternatively, do the calculations in a spreadsheet.

High precision surveys in industry use two high quality electronic theodolites to observe directions to targets on an object such as the internal wall of a cylindrical silo several metres across to an accuracy of less than 1mm. Distances are not measured. Observed directions from two instrument sites (A and B) were downloaded electronically from the instruments and recorded in decimal degrees; not in degrees, minutes and seconds. The directions were converted to bearings. The coordinates are on a local plane datum.

Data set (observations inside a cylindrical silo) A (102.5894, 506.7179) B (109.3070, 503.4246)

```
Brg from A  Brg from B
  62.44944   359.82361
  36.84660   336.63967
  70.74688     9.26113
  99.84844    59.26759
 127.86516   160.05027
 156.63967   216.84660
 189.26113   250.74688
 239.26759   279.84844
 340.05027   307.86516
```



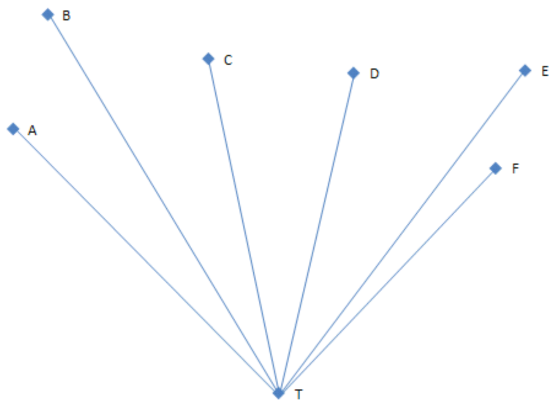
Q7. Phase Ambiguities Problem (challenging)

Many documents have been written over the years to explain GPS carrier phase ambiguities and solution methods. You will learn about that in a later course. Often it helps students to understand a concept if they actually do something. The following problem provides an opportunity to do some calculations on a similar problem.

EDM has a somewhat similar approach for measuring distances from phases and cycles of EM waves. Before GPS and EDM surveyors used a 100m steel band to measure precise distances. Metric steel bands often have brass markers every 2m and they have a 3m tape attached to the zero end of the band. The tape is called the reader and it is graduated in the reverse direction, usually every millimetre. So to measure a distance surveyors placed a brass marker on one end of the line and read the 3m tape at the other end of the line. The distance is the sum of the two readings. This is similar to a GPS carrier signal, but here the cycle length is the spacing between the brass markers. If we do not record the number at the brass marker then we have an ambiguity problem.

In this problem we work in 2D coordinates and have a 2 metre cycle length. The coordinates of our known points A to F (our satellites) are 'perfect' and are given in the table below.

Point	E	N	Distance from T:	Marker dist.	Dist. on reader
A	335.80	647.29		a	0.88
B	336.61	652.18		b	1.04
C	340.36	650.29		c	0.37
D	343.75	649.68		d	1.79
E	347.76	649.79		e	0.93
F	347.07	645.61		f	0.86



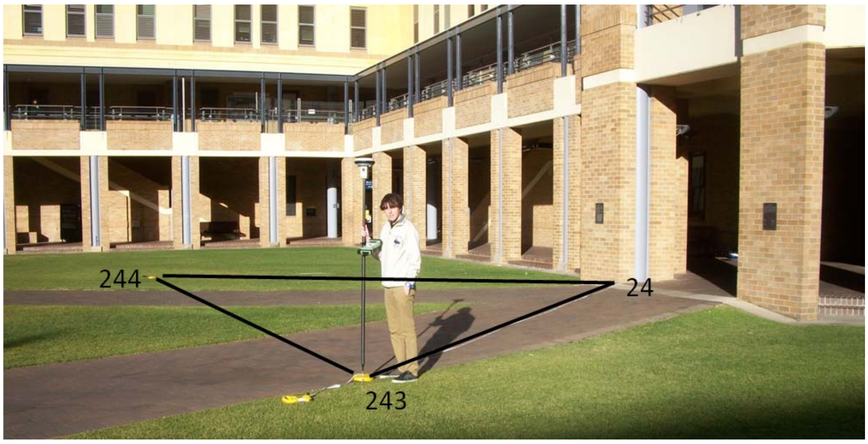
Sketch plan not to scale

From the table, the distance from T to A is a + 0.88 and a is an integer. Similarly, the table gives the ambiguous distances from T to the other known points. Find the integer values of a, b, c, d, e, and f and the coordinates of T. We know that T is south of all the points A to F and that all the distances are less than 20m.

You may use a computer and any method you wish. But how do you know if you have the correct answers? In this data set all distances are reliable to less than 1cm, so when you have correct coordinates the residuals from least squares, or the differences between the calculated and observed distances are less than 1cm for all lines. You could try every combination of integers from 0 to 20, but is there a better way to do it?

Q8. Hidden point by distances

For a detail survey plan we required the coordinates of the walls of the UNSW Quadrangle building using GPS. At a place on the brick wall under the sundial (labelled 24 in the photo below) half of the sky was not visible because of the high walls, so GPS could not measure coordinates accurately. Two temporary survey marks (243 and 244) were placed in positions where there was sufficient sky coverage to obtain coordinates to ± a few cm accuracy using RTK GNSS equipment on a pole. From these temporary survey marks distances were measured to point 24. The marks were placed in positions that created a good intersection angle of about 90°. Point 24 is south of the other two marks.



DATA
GNSS downloaded data: Pt, E, N
244, 336496.162, 6245622.976
243, 336481.605, 6245624.285

Horizontal distances were measured by steel tape (the grid scale factor is insignificant):
244 – 24 = 11.323, 243 – 24 = 9.976, 244 – 243 = 14.610

Calculate the E and N coordinates of 24 using the distance from 244 and the distance from 243.

A) 336487.22, 6245616.04
B) 336488.60, 6245631.40
C) 336489.17, 6245615.86
D) 336490.55, 6245631.23

Q9. Three point resection by directions

In 2014 a group of students measured the following resection observations from point 5 to targets placed on three control marks with known coordinates.

Aerial view of site of resection

Point	E(m)	N(m)
991	290000.504	6147540.603
926	289477.373	6147660.737
921	290192.164	6147882.102

From	To	° ' "
5	991	150 41 57
5	926	246 42 13
5	921	28 16 26

The approximate coordinates of point 5 derived from the aerial photo overlay are (289980, 6147690). The observed directions are the mean of FL and FR with a total station, they are not bearings; they are not orientated towards north.

a) Calculate the coordinates of the point P, to the nearest mm.

b) Show a separate, independent check of your resection answers.

[The above data is a subset of a real survey that also observed to more targets and distances to some targets. The calculations and analysis of the full data set is beyond this course; learn about that in a later course.]

Q10. Three point resection by directions

a) Describe with the aid of sketches what is a danger circle in 3 point resections and how can they be prevented.

b) At a construction site using a local coordinate system the following resection observations (mean of FL and FR) were recorded from point 61 to three points with known coordinates (points 91, 92, 93). **Calculate** using **Tienstra's method** the E and N coordinates of the instrument station at point 61, to the nearest mm. Estimates of the approximate coordinates of P61 are E 100, N 200. The observed directions are not orientated towards north, they are not bearings.

Point	E(m)	N(m)
91	124.753	245.860
92	131.384	214.126
93	150.394	202.921

From	To	Direction ° ' "	ZA ° ' "
61	91	0 41 57	88 23 56
61	92	46 42 13	93 02 19
61	93	62 16 26	91 45 12

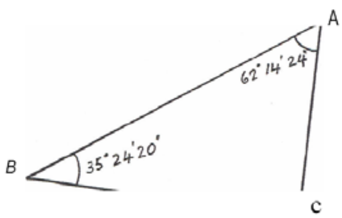
c) Show a separate, independent check of your resection answers.

Chapter 2. Tutorial Solutions – intersections and resections

These solutions can be read by AFTER you have made some attempt to solve the question.

Q1.

a) Solution of triangle, then radiate C



Calculate internal angles in triangle ABC
A = 360-Agiven = 62.2400
B = Bgiven = 35.406
C =180-(A+B) = 82.3544

From coordinates using spreadsheet or R → P on calculator
BrgAB = MOD(DEGREES(ATAN2(NB-NA,EB-EA)),360) = 242.3078
DistAB = SQRT((EB-EA)^2+(NB-NA)^2) = 792.797
Side b (i.e. AC) by sine rule = SIN(RADIANS(B))*DAB/SIN(RADIANS(C)) = 463.435
BrgAC = BrgAB – A = 180.0678

Radiate coordinates of C from A
EC = EA+b*SIN(RADIANS(BrgAC)) = 45327.624 NC = NA+b*COS(RADIANS(BrgAC)) = 26521.596

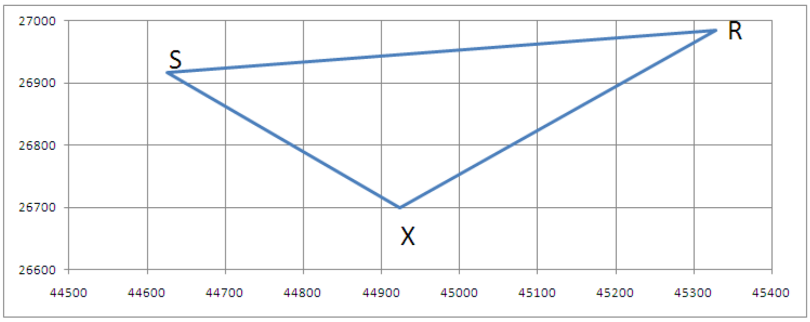
b) INTERSECTION BY ANGLES

		D		**M**		**S**
	Angle A	297		45		36
	Angle B	35		24		20
Angle A	Angle B					
297.7600	35.4056	degrees				
5.197	0.618	radians				
	Point		**Easting**		**Northing**	
	A		45328.172		26985.030	
	B		44626.185		26616.600	
		Diff	-701.987		-368.430	
ANSWER	**C**		45327.624		26521.596	

Spreadsheet formulas:

	A	B	C	D	E	M	N	S
1				**D**		**M**		**S**
2			Angl e A	297		45		36
3			Angl e B	35		24		20
4	Angle A	Angle B						
5	=D2+M2/60 +S2/3600	=D3+M3/60 +S3/3600	degr ees					
6	=RADIANS(A5)	=RADIANS(B5)	radi ans					
7		**Point**			**Easting**		**Northing**	
8		**A**			45328.172		26985.03	
9		**B**			44626.185		26616.6	
10				Diff	=E9-E8		=N9-N8	
11	**ANSWER**	**C**			=E8-((SIN(B6)/SIN(A6-B6))*(E10*COS(A6)+N10*SIN(A6)))		=N8-((SIN(B6)/SIN(A6-B6))*(N10*COS(A6)-E10*SIN(A6)))	

Q2.



	radians		E	D	M	S	N
Brg RX:	4.098	ER:	45328.172	234	48	20	26985.030
Brg SX:	2.202	ES:	44626.185	126	11	30	26916.600
		EX:	**44923.111**				**26699.350**

Spreadsheet formulas:

	B	E	H	I	J	N
21	=RADIANS(H21+I21/60+J21/3600)	45328.172	234	48	20	26985.03
22	=RADIANS(H22+I22/60+J22/3600)	44626.185	126	11	30	26916.6
		=E21+(((E22-E21)*COS(B22)-(N22-N21)*SIN(B22))/SIN(B21-B22))*SIN(B21)				=N21+(((E22-E21)*COS(B22)-(N22-N21)*SIN(B22))/SIN(B21-B22))*COS(B21)

Check with alternative formulas

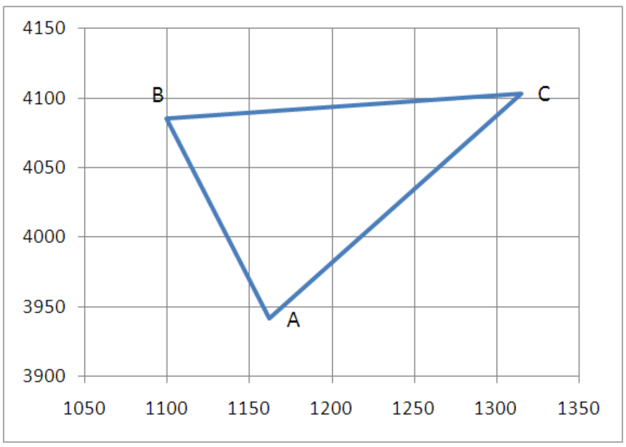
$$N_C = N_A + \frac{(N_B - N_A)\tan\beta_{BC} - (E_B - E_A)}{\tan\beta_{BC} - \tan\beta_{AC}} \qquad E_C = E_A + (N_C - N_A)\tan\beta_{AC}$$

Spreadsheet formulas:

	B (=Ec)	**C (= Nc)**
38	=E21+(C38-N21)*TAN(B21)	=N21+((N22-N21)*TAN(B22)-(E22-E21))/(TAN(B22)-TAN(B21))

Gives same answers as above.

Q3.



	Brg deg		Dist	E		N
AC	43.3846	b (AC)	223.201	1161.634	A	3941.286
BC	85.2112	a (BC)	216.014	1099.689	B	4085.466
AB	336.7499	c (AB)	156.924	-61.945	Diff	144.18

		Angle	Radians	Degrees		
		A	1.1630	66.6347		
		B	1.2486	71.53867		
				1314.949	**C**	**4103.499**

Spreadsheet formulas:

	B	C	D	E	F	N
1	=MOD(B3+E5,360)	**b**	223.201	1161.634	A	3941.286
2	=MOD((B3-E6)-180,360)	**a**	216.014	1099.689	B	4085.466
3	=MOD(DEGREES(ATAN2((N3),(E3))),360)	**c**	=SQRT(E3^2+N3^2)	=E2-E1	Diff	=N2-N1
4			**Radians**	**Degrees**		
5		**A**	=ACOS((D1^2+D3^2-D2^2)/(2*D1*D3))	=DEGREES(D5)		
6		**B**	=ACOS((D2^2+D3^2-D1^2)/(2*D2*D3))	=DEGREES(D6)		
7				=E1+D1*SIN(RADIANS(B1))	C	=N1+D1*COS(RADIANS(B1))

Check, distance a (BC) from coordinates: =SQRT((E7-E2)^2+(N7-N2)^2) = 216.014   OK

Q4.

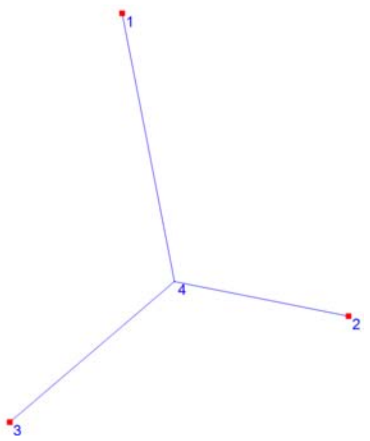
Coordinates of P =   E 83 499.009   N 50 073.254

a) Labelling points A B C P as 1 2 3 4 respectively:

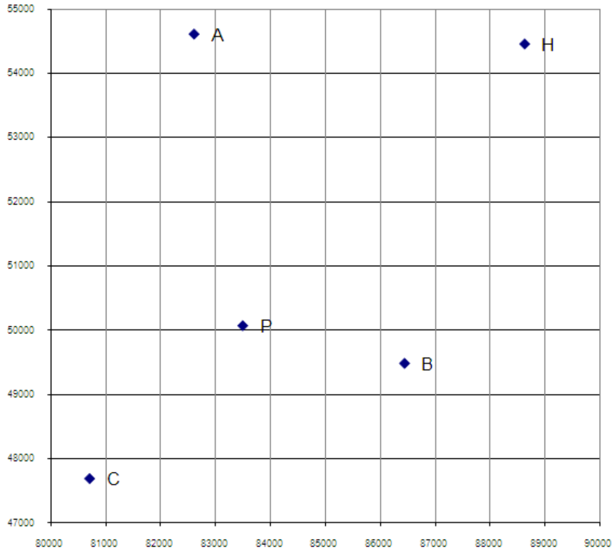
Coordinates:

Directions:

Directions rotated to fit coordinates:



(b) Collins' Point Method, with sum of angles > 180°



Pt	E	N	D	M	S	dd		
A	82613.520	54609.700	0	00	00.0	0.0		
B	86444.390	49487.160	112	17	56.7	112.3		
C	80712.060	47693.380	240	32	59.1	240.5		
			112	17	56.7	112.3	Alpha 1	dir 2-1
			128	15	02.4	128.3	Alpha 2	dir 3-2
			119	27	00.9	119.5	Alpha 3	
	d AB	6396.560	143	12	33.0	143.2	Brg AB	
	d AH	6020.234	91	27	35.4	91.5	Brg AH	
	d BH	5429.307	23	45	32.1	23.8	Brg BH	
H	**88631.800**	**54456.327**						
			229	30	17.3	229.5	brgHC=	brgPC
			281	15	14.9	281.3	Brg BP	
			168	57	18.2	169.0	Brg AP	
			41	57	18.1	42.0	angle PBA	
	dAP	4622.059						
P	**83499.009**	**50073.254**						

Check bearings from coords:

				brg	dirn	
PA	348	57	18.2	349.0 °	0.0 °	
PB	101	15	14.9	101.3 °	112.3 °	OK
PC	229	30	17.3	229.5 °	240.5 °	OK

Spreadsheet formulas:

	A	C	D	E	F	G	H
4	A	82613.52	54609.7	0	0	0	=E4+F4/60+G4/3600
5	B	86444.39	49487.16	112	17	56.7	=E5+F5/60+G5/3600
6	C	80712.06	47693.38	240	32	59.1	=E6+F6/60+G6/3600
9				=INT(H9)	=INT((H9-E9)*60)	=(H9-E9-(F9/60))*3600	=H5-H4
10				=INT(H10)	=INT((H10-E10)*60)	=(H10-E10-(F10/60))*3600	=H6-H5
11				=INT(H11)	=INT((H11-E11)*60)	=(H11-E11-(F11/60))*3600	=360-H9-H10
12		d AB	=SQRT((C5-C4)^2+(D5-D4)^2)	=INT(H12)	=INT((H12-E12)*60)	=(H12-E12-(F12/60))*3600	=MOD(DEGREES(ATAN2(D5-D4,C5-C4)),360)
13		d AH	=-D12*SIN(RADIANS(H11))/SIN(RADIANS(H10+H11))	=INT(H13)	=INT((H13-E13)*60)	=(H13-E13-(F13/60))*3600	=H12-(180-H10)
1		d BH	=-	=INT	=INT((H14	=(H14-E14-	=MOD(H12-180+(180-

4			D12*SIN(RADIANS(H10))/SIN(RADIANS(H10+H11))	(H14)	-E14)*60)	(F14/60))*3600	H11),360)
15	H	**=C4+SIN(RADIANS(H13))*D13**	**=D4+COS(RADIANS(H13))*D13**				
16				=INT(H16)	=INT((H16-E16)*60)	=(H16-E16-(F16/60))*3600	=MOD(DEGREES(ATAN2(D6-D15,C6-C15)),360)
17				=INT(H17)	=INT((H17-E17)*60)	=(H17-E17-(F17/60))*3600	=MOD(H16+180-H10,360)
18				=INT(H18)	=INT((H18-E18)*60)	=(H18-E18-(F18/60))*3600	=MOD(H16-(H9+H10)+180,360)
19				=INT(H19)	=INT((H19-E19)*60)	=(H19-E19-(F19/60))*3600	=H12+180-H17
20		dAP	=D12*SIN(RADIANS(H19))/SIN(RADIANS(H9))				
21	P	**=C4+D20*SIN(RADIANS(H18))**	**=D4+D20*COS(RADIANS(H18))**				

	E	F	G	H	I	J
23	=INT(H23)	=INT(H23*60-E23*60)	=(H23-E23-(F23/60))*3600	=MOD(DEGREES(ATAN2((D4-D8),(C4-C8))),360)	0	
24	=INT(H24)	=INT((H24-E24)*60)	=(H24-E24-(F24/60))*3600	=MOD(DEGREES(ATAN2((D5-D8),(C5-C8))),360)	=MOD(H24-H23,360)	=IF(ABS(I24-H5)<(1/3600),"OK","X")
25	=INT(H25)	=INT((H25-E25)*60)	=(H25-E25-(F25/60))*3600	=MOD(DEGREES(ATAN2((D6-D8),(C6-C8))),360)	=MOD(H25-H23,360)	=IF(ABS(I25-H6)<(1/3600),"OK","X")

(c) Tangent Method and Tienstra Method

Pt	E	N	D	M	S	dd	angle	cot(angle)			Brg from P	cot(Brgform P)
A	82613.52	54609.70	0	00	00.0	0.0 º			Top (M)	Bot (N)	349.0 º	-5.12
B	86444.39	49487.16	112	17	56.7	112.3 º	112.3 º	-0.410	13006.5	-2588.1	101.3 º	-0.20
C	80712.06	47693.38	240	32	59.1	240.5 º	128.3 º	-0.788			229.5 º	0.85
P	**83499.009**	**50073.254**	by tangent method				119.5 º					
Bearings						angles	w					
AB	143.2 º				A	52.2 º	0.639					
BC	252.6 º				B	70.6 º	1.090					
AC	195.4 º				C	57.3 º	0.949					
						sum	2.679					
P	**83499.009**	**50073.254**	by Tienstra method									

Spreadsheet formulas for Tangent method:

	A	B	C	D	E	F	G	H	I	J	K	L	M
96	A	82613.52	54609.7	0	0	0	=D96+E96/60+F96/3600			Top (M)	Bot (N)	=MOD(L97-H97,360)	=1/TAN(RADIANS(L96))
97	B	86444.39	49487.16	112	17	56.7	=D97+E97/60+F97/3600	=G97-G96	=1/TAN(RADIANS(H97))	=(B96-B97)*I97+(B98-B97)*I98-(C98-C96)	=(C96-C97)*I97+(C98-C97)*I98+B98-B96	=MOD(DEGREES(ATAN2(K97,J97)),360)	=1/TAN(RADIANS(L97))
98	C	80712.06	47693.38	240	32	59.1	=D98+E98/60+F98/3600	=G98-G97	=1/TAN(RADIANS(H98))			=MOD(L97+H98,360)	=1/TAN(RADIANS(L98))
99	P	=(C96-C97-B96*M96+B97*M97)/(M97-M96)	=C96+(B99-B96)*M96					=MOD(G96-G98,360)					

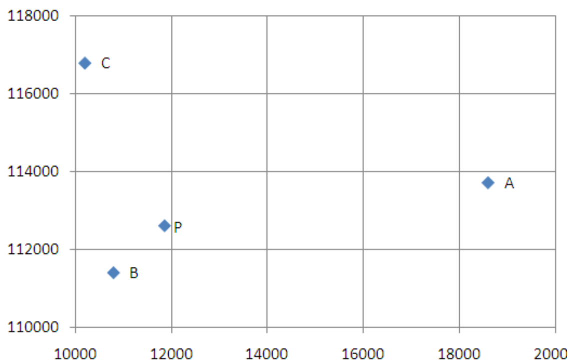
Spreadsheet formulas for Tienstra method:

	A	B	C	F	G	H
102	AB	=MOD(DEGREES(ATAN2((C97-C96),(B97-B96))),360)		A	=B104-B102	=1/((1/TAN(RADIANS(G102)))-(1/(TAN(RADIANS(H98)))))
103	BC	=MOD(DEGREES(ATAN2((C98-C97),(B98-B97))),360)		B	=180+B102-B103	=1/((1/TAN(RADIANS(G103)))-(1/(TAN(RADIANS(H99)))))
104	AC	=MOD(DEGREES(ATAN2((C98-C96),(B98-B96))),360)		C	=B103-B104	=1/((1/TAN(RADIANS(G104)))-(1/(TAN(RADIANS(H97)))))
105					sum	=SUM(H102:H104)
106	P	=(B96*H102+B97*H103+B98*H104)/(H105)	=(C96*H102+C97*H103+C98*H104)/H105			

Q5.

Example Data 1: STN	E (m)	N (m)	Observed directions (not bearings)
A	18597.25	113722.63	PA: 0° 00' 00"
B	10783.71	111405.38	PB: 140° 40' 23"
C	10186.97	116800.92	PC: 257° 40' 29"

Solution:



α_1 = 140° 40' 23" α_2 = 257° 40' 29" - 140° 40' 23" = 117° 00' 06"

M = (E1 - E2) * cot(α_1) + (E3 - E2) * cot(α_2) - N3 + N1 = -12311.3
N = (N1 - N2) * cot(α_1) + (N3 - N2) * cot(α_2) + E3 - E1 = -13988.1
tanPB = M/N = 0.88013
β_{PB} =MOD(DEGREES(ATAN2(N,M)),360)

β_{PB} = 221° 21' 07.4" The quadrant of β_{PM} is resolved by noting the numerator and denominator are both negative.

$\beta_{PA} = \beta_{PB} - \alpha_1$ = 80°40' 44.4" $\beta_{PC} = \beta_{PB} + \alpha_2$ = 338°21'13.4"

E_P = (N1 - N2 - E1 * cot(B1) + E2 * cot(B2)) / (cot(B2) - cot(B1)) = 11848.246

N_P = N1 + (E4 - E1) * cot(B1) = 112614.898

Check: Calculate the bearing from the derived coordinates of P to each of A B and C. Then calculate the reduced directions (or angles) from these bearings and compare them with the original directions (or angles). Using the derived coordinates: β_{PA} = 80° 40' 44.4" β_{PB} = 221° 21' 07.4"
β_{PC} = 338° 21' 13.4"

And directions, with PA set to 0:
PB = β_{PB} - β_{PA} = 140° 40' 23.0"

PC = β_{PC} - β_{PA} = 257° 40' 29.0" Which are close enough to the original data.

Danger circle calculations: S = $\alpha_1 + \alpha_2$ + angle CBA where angle CBA = $\beta_{BA} - \beta_{BC}$
S = <APB + <BPC + <CBA = 140°40'23" + 117°00'06" + 79°47'30" ≈ 337°
This is not close to 180 or 360, so it is not on the danger circle.

A	B	C	D	E	F	G	H	I
Pt	E	N	D	M	S	dd	angle	cot
A	18597.25	113722.63	0	0	0	=D27+E27/60+F27/3600		
B	10783.71	111405.38	140	40	23	=D28+E28/60+F28/3600	=G28-G27	=1/TAN(RADIANS(H28))
C	10186.97	116800.92	257	40	29	=D29+E29/60+F29/3600	=G29-G28	=1/TAN(RADIANS(H29))
M	=(B27-B28)*I28+(B29-B28)*I29-(C29-C27)				BrgPB	=MOD(DEGREES(ATAN2(B31,B30)),360)		=1/TAN(RADIANS(G30))
N	=(C27-C28)*I28+(C29-C28)*I29+B29-B27				BrgPA	=MOD(G30-H28,360)		=1/TAN(RADIANS(G31))
					BrgPC	=MOD(G30+H29,360)		=1/TAN(RADIANS(G32))
Ans:	=(C27-C28-B27*I31+B28*I30)/(I30-I31)	=C27+(B33-B27)*I31			BrgBA	=MOD(DEGREES(ATAN2((C27-C28),(B27-B28))),360)		
					BrgBC	=MOD(DEGREES(ATAN2((C29-C28),(B29-B28))),360)		
						=H28+H29+MOD(G33-G34,360)	danger circle	
Check brgs from coords:								
PA			=INT(G37)	=INT((G37-D37)*60)	=(G37-D37-(E37/60))*3600	=MOD(DEGREES(ATAN2((C27-C33),(B27-B33))),360)		
PB			=INT(G38)	=INT((G38-D38)*60)	=(G38-D38-(E38/60))*3600	=MOD(DEGREES(ATAN2((C28-C33),(B28-B33))),360)	=MOD(G38-G37,360)	=IF(ABS(H38-H28)<(1/3600),"OK","X")
PC			=INT(G39)	=INT((G39-D39)*60)	=(G39-D39-(E39/60))*3600	=MOD(DEGREES(ATAN2((C29-C33),(B29-B33))),360)	=MOD(G39-G38,360)	=IF(ABS(H39-H29)<(1/3600),"OK","X")

Solution by Excel Solver:

140.6731	Angle 1		Point 1	18597.25	113722.63
117.0017	Angle 2		Point 2	10783.71	111405.38
			Point 3	10186.97	116800.92
			Point 4	11848.246	112614.898
Excel Solver Equations					
eqn 1, 0=	0.000				
eqn 2, 0=	0.000				

	A	B	E	N
1	=140+40/60+23/3600	Angle 1	18597.25	113722.63
2	=117+0/60+6/3600	Angle 2	10783.71	111405.38
3			10186.97	116800.92
4			11848.246	112614.898
6	Excel Solver Equations			
7	eqn 1, 0=	=MOD(DEGREES(ATAN2(N2-N4,E2-E4)),360)-MOD(DEGREES(ATAN2(N1-N4,E1-E4)),360)-A1		
8	eqn 2, 0=	=MOD(DEGREES(ATAN2(N3-N4,E3-E4)),360)-MOD(DEGREES(ATAN2(N2-N4,E2-E4)),360)-A2		

Test Data 2 (PA north):	STN	E (m)	N (m)		Directions	*True P*
	A	10	100	PA	0° 00’ 00”	*10,10*
	B	100	100	PB	45° 00’ 00”	
	C	100	10	PC	90° 00’ 00”	

M and N both = 0 and the above Tangent Method spreadsheet crashes with divide by 0 trying to calculate bearings. S = 180.00!

Test Data 3 (danger circle):	STN	E (m)	N (m)		Directions	*True P*
	A	36.36	163.64	PA	0° 00’ 00”	*100,100*
	B	100	227.279	PB	45° 00’ 00”	
	C	163.64	163.64	PC	90° 00’ 00”	

The above spreadsheet gives correct answers for P and checks are OK. S = 180.00!

Test Data 4 (PB north):	STN	E (m)	N (m)		Directions	*True P*
	A	74.019	85	PA	0° 00’ 00”	*100,100*
	B	100	130	PB	120° 00’ 00”	
	C	125.981	85	PC	240° 00’ 00”	

M = 0 and the above spreadsheet crashes with divide by 0. S = 300

Test Data 5 (PB east):	STN	E (m)	N (m)		Directions	*True P*
	A	85	125.981	PA	0° 00’ 00”	*100,100*
	B	130	100	PB	120° 00’ 00”	
	C	85	74.019	PC	240° 00’ 00”	

The above spreadsheet gives correct answers for P and checks are OK. S = 300

Q6.

Note that the bearings are in degrees and decimals of degrees.

INSTRUMENT COORDINATES						
	(m)					
POINT	EASTING	NORTHING				
A	102.5894	506.7179				
B	109.3070	503.4246				
	BEARINGS					
	(Decimal Degrees)		radians	radians	E	N
POINT	BRG From A	BRG From B	BRG A	BRG B		
C	62.44944	359.82361	1.090	6.280	109.2861	510.2115
D	36.84660	336.63967	0.643	5.875	105.9485	511.2005
E	70.74688	9.26113	1.235	0.162	110.2821	509.4048
F	99.84844	59.26759	1.743	1.034	112.0762	505.0710
G	127.86516	160.05027	2.232	2.793	110.2828	500.7363
H	156.63967	216.84660	2.734	3.785	105.9479	498.9420
I	189.26113	250.74688	3.303	4.376	101.6143	500.7377
J	239.26759	279.84844	4.176	4.884	99.8202	505.0715
K	340.05027	307.86516	5.935	5.373	101.6136	509.4062

	A	B	C	D	E	F	G
	P	E	N				
5	A	102.5894	506.7179				
6	B	109.307	503.4246				
		D D		radians	radians	E	N
	PT	BRG A	BRG B	BRG A	BRG B		
11	C	62.44944	359.82361	=RADIANS(B11)	=RADIANS(C11)	= B$5 + (SIN(D11) * ((B$6 -B$5) * COS(E11) - (C$6 - C$5) * SIN(E11)) / (SIN(D11 - E11)))	= C$5 + (COS(D11) * ((B$6 - B$5) * COS(E11) - (C$6 - C$5) * SIN(E11)) / (SIN(D11 - E11)))
1	D	36.8	336.	=RADI	=RADI	= B$5 + (SIN(D12) * ((B$6 -B$5) *	= C$5 + (COS(D12) * ((B$6 - B$5) *

2		466	63967	ANS(B12)	ANS(C12)	COS(E12) - (C$6 - C$5) * SIN(E12)) / (SIN(D12 - E12)))	COS(E12) - (C$6 - C$5) * SIN(E12)) / (SIN(D12 - E12)))
13	E	70.74688	9.26113	=RADIANS(B13)	=RADIANS(C13)	= B$5 + (SIN(D13) * ((B$6 -B$5) * COS(E13) - (C$6 - C$5) * SIN(E13)) / (SIN(D13 - E13)))	= C$5 + (COS(D13) * ((B$6 - B$5) * COS(E13) - (C$6 - C$5) * SIN(E13)) / (SIN(D13 - E13)))
14	F	99.84844	59.26759	=RADIANS(B14)	=RADIANS(C14)	= B$5 + (SIN(D14) * ((B$6 -B$5) * COS(E14) - (C$6 - C$5) * SIN(E14)) / (SIN(D14 - E14)))	= C$5 + (COS(D14) * ((B$6 - B$5) * COS(E14) - (C$6 - C$5) * SIN(E14)) / (SIN(D14 - E14)))
15	G	127.86516	160.05027	=RADIANS(B15)	=RADIANS(C15)	= B$5 + (SIN(D15) * ((B$6 -B$5) * COS(E15) - (C$6 - C$5) * SIN(E15)) / (SIN(D15 - E15)))	= C$5 + (COS(D15) * ((B$6 - B$5) * COS(E15) - (C$6 - C$5) * SIN(E15)) / (SIN(D15 - E15)))
16	H	156.63967	216.8466	=RADIANS(B16)	=RADIANS(C16)	= B$5 + (SIN(D16) * ((B$6 -B$5) * COS(E16) - (C$6 - C$5) * SIN(E16)) / (SIN(D16 - E16)))	= C$5 + (COS(D16) * ((B$6 - B$5) * COS(E16) - (C$6 - C$5) * SIN(E16)) / (SIN(D16 - E16)))
17	I	189.26113	250.74688	=RADIANS(B17)	=RADIANS(C17)	= B$5 + (SIN(D17) * ((B$6 -B$5) * COS(E17) - (C$6 - C$5) * SIN(E17)) / (SIN(D17 - E17)))	= C$5 + (COS(D17) * ((B$6 - B$5) * COS(E17) - (C$6 - C$5) * SIN(E17)) / (SIN(D17 - E17)))
18	J	239.26759	279.84844	=RADIANS(B18)	=RADIANS(C18)	= B$5 + (SIN(D18) * ((B$6 -B$5) * COS(E18) - (C$6 - C$5) * SIN(E18)) / (SIN(D18 - E18)))	= C$5 + (COS(D18) * ((B$6 - B$5) * COS(E18) - (C$6 - C$5) * SIN(E18)) / (SIN(D18 - E18)))
19	K	340.05027	307.86516	=RADIANS(B19)	=RADIANS(C19)	= B$5 + (SIN(D19) * ((B$6 -B$5) * COS(E19) - (C$6 - C$5) * SIN(E19)) / (SIN(D19 - E19)))	= C$5 + (COS(D19) * ((B$6 - B$5) * COS(E19) - (C$6 - C$5) * SIN(E19)) / (SIN(D19 - E19)))

Q7. Answer to phase ambiguities.
a = 12, b = 16, c = 14, d = 12, e = 14, f = 10. T: E ≈ 342.00 N ≈ 636.00

One solution method is to calculate an intersection by distances (sometimes called two missing bearings) from two points. For those familiar with computer programming, an algorithm to explain the way it could be done is:

```
DO a = 2, 18, step = 2
  DO f = 2, 18, step = 2
    Calculate coordinates of T by intersection of distances from A and F
    Calculate distance from T to C
    If fractional part of dist TC agrees with the reader within a few cm, then check distances to other points, if all OK then you have the answer for T
    Otherwise your coordinates of T are not valid, try new values
  NEXT f
NEXT a
```

There is no solution if the distances don't intersect, so $d_{AT} + d_{FT}$ must be >= d_{AF}
We know T is south of the points A to F so that allows us to choose which side of the AF line the intersected point T lies on.

A cleverer way to search for the answers is to start with something close to the final answer perhaps with prior knowledge of the coordinates of T. In this case we are only given the range being limited to 0 to 20. So start trying a and f equal 10, and then move to nearby values.

Here are some of the calculations. Blank lines represent no intersection. The c? column is the estimate of c by subtracting the reader value for that line from d_{TC}, it should be close to an even integer. Similarly for the b?, d? and e? columns. Useful values are shaded yellow below. While some values of T look valid for distance to one of the 'satellites' only one value in the table below is suitable for all satellites.

Perhaps the table is too large for publication. GPS software has a considerable computation task to resolve ambiguities so clever solution methods are used.

a	f	d_{AT}	d_{FT}	ET	NT	d_{TC}	c?	d_{TB}	b?	d_{TD}	d?	d_{TE}	e?
2	2	2.88	2.86										
2	4	2.88	4.86										
2	6	2.88	6.86										
2	8	2.88	8.86	338.21	645.71	5.06	4.69	6.66	5.62	6.81	5.02	10.38	9.45
2	10	2.88	10.86	336.27	644.45	7.13	6.76	7.74	6.70	9.13	7.34	12.67	11.74
2	12	2.88	12.86	334.23	644.87	8.18	7.81	7.68	6.64	10.66	8.87	14.39	13.46
2	14	2.88	14.86										
2	16	2.88	16.86										

2	18	2.88	18.86										
4	2	4.88	2.86										
4	4	4.88	4.86										
4	6	4.88	6.86	340.22	645.22	5.07	4.70	7.84	6.80	5.68	3.89	8.81	7.88
4	8	4.88	8.86	338.53	643.25	7.28	6.91	9.14	8.10	8.28	6.49	11.31	10.38
4	10	4.88	10.86	336.67	642.49	8.63	8.26	9.69	8.65	10.09	8.30	13.28	12.35
4	12	4.88	12.86	334.58	642.57	9.65	9.28	9.83	8.79	11.61	9.82	15.03	14.10
4	14	4.88	14.86	332.31	643.88	10.29	9.92	9.35	8.31	12.83	11.04	16.54	15.61
4	16	4.88	16.86										
4	18	4.88	18.86										
6	2	6.88	2.86										
6	4	6.88	4.86	342.26	644.92	5.70	5.33	9.20	8.16	4.99	3.20	7.35	6.42
6	6	6.88	6.86	340.88	642.65	7.66	7.29	10.44	9.40	7.59	5.80	9.91	8.98
6	8	6.88	8.86	339.29	641.36	8.99	8.62	11.14	10.10	9.44	7.65	11.94	11.01
6	10	6.88	10.86	337.43	640.61	10.12	9.75	11.60	10.56	11.06	9.27	13.82	12.89
6	12	6.88	12.86	335.30	640.43	11.08	10.71	11.82	10.78	12.53	10.74	15.58	14.65
6	14	6.88	14.86	332.93	641.04	11.87	11.50	11.73	10.69	13.85	12.06	17.22	16.29
6	16	6.88	16.86	330.41	643.01	12.33	11.96	11.07	10.03	14.91	13.12	18.63	17.70
6	18	6.88	18.86										
8	2	8.88	2.86	344.33	644.81	6.77	6.40	10.67	9.63	4.91	3.12	6.05	5.12
8	4	8.88	4.86	343.30	642.54	8.29	7.92	11.74	10.70	7.15	5.36	8.51	7.58
8	6	8.88	6.86	342.03	640.96	9.48	9.11	12.46	11.42	8.89	7.10	10.53	9.60
8	8	8.88	8.86	340.45	639.72	10.57	10.20	13.03	11.99	10.49	8.70	12.44	11.51
8	10	8.88	10.86	338.57	638.85	11.58	11.21	13.47	12.43	12.00	10.21	14.29	13.36
8	12	8.88	12.86	336.40	638.43	12.50	12.13	13.75	12.71	13.44	11.65	16.06	15.13
8	14	8.88	14.86	333.97	638.60	13.32	12.95	13.83	12.79	14.78	12.99	17.76	16.83
8	16	8.88	16.86	331.31	639.63	13.99	13.62	13.62	12.58	15.99	14.20	19.34	18.41
8	18	8.88	18.86	328.52	642.21	14.34	13.97	12.84	11.80	16.97	15.18	20.68	19.75
10	2	10.88	2.86	345.82	643.04	9.07	8.70	12.97	11.93	6.95	5.16	7.03	6.09
10	4	10.88	4.86	344.87	641.28	10.08	9.71	13.68	12.64	8.48	6.69	8.99	8.06
10	6	10.88	6.86	343.59	639.70	11.08	10.71	14.30	13.26	9.98	8.19	10.92	9.99
10	8	10.88	8.86	342.00	638.35	12.05	11.68	14.84	13.80	11.47	9.68	12.81	11.88
10	10	10.88	10.86	340.09	637.29	13.00	12.63	15.29	14.25	12.92	11.13	14.66	13.74
10	12	10.88	12.86	337.88	636.61	13.90	13.53	15.62	14.58	14.33	12.54	16.47	15.54
10	14	10.88	14.86	335.39	636.42	14.73	14.36	15.81	14.77	15.68	13.89	18.21	17.28
10	16	10.88	16.86	332.65	636.88	15.47	15.10	15.81	14.77	16.95	15.16	19.88	18.95
10	18	10.88	18.86	329.69	638.29	16.06	15.69	15.52	14.48	18.10	16.31	21.42	20.49
12	2	12.88	2.86	347.90	642.87	10.58	10.21	14.63	13.59	7.97	6.18	6.92	5.99
12	4	12.88	4.86	346.90	640.75	11.56	11.19	15.38	14.34	9.47	7.68	9.08	8.15
12	6	12.88	6.86	345.58	638.91	12.52	12.15	16.02	14.98	10.92	9.13	11.09	10.16
12	8	12.88	8.86	343.95	637.32	13.46	13.09	16.58	15.54	12.36	10.57	13.04	12.11
12	10	12.88	10.86	342.01	636.00	14.38	14.01	17.05	16.01	13.79	12.00	14.94	14.01
12	12	12.88	12.86	339.76	635.03	15.27	14.90	17.43	16.39	15.18	13.39	16.79	15.86
12	14	12.88	14.86	337.22	634.49	16.11	15.74	17.70	16.66	16.54	14.75	18.58	17.65
12	16	12.88	16.86	334.40	634.49	16.89	16.52	17.83	16.79	17.84	16.05	20.32	19.38
12	18	12.88	18.86	331.34	635.21	17.57	17.20	17.77	16.73	19.07	17.28	21.96	21.03
14	2	14.88	2.86										
14	4	14.88	4.86	349.45	641.37	12.73	12.36	16.78	15.74	10.08	8.29	8.58	7.65
14	6	14.88	6.86	348.03	638.82	13.80	13.43	17.58	16.54	11.68	9.89	10.98	10.05
14	8	14.88	8.86	346.33	636.78	14.77	14.40	18.21	17.17	13.16	11.37	13.09	12.16
14	10	14.88	10.86	344.33	635.10	15.70	15.33	18.74	17.70	14.59	12.80	15.08	14.15
14	12	14.88	12.86	342.03	633.78	16.60	16.23	19.18	18.14	15.99	14.20	17.01	16.08
14	14	14.88	14.86	339.44	632.86	17.45	17.08	19.52	18.48	17.36	15.57	18.86	17.93
14	16	14.88	16.86	336.56	632.43	18.26	17.89	19.75	18.71	18.69	16.90	20.66	19.73
14	18	14.88	18.86	333.41	632.60	19.00	18.63	19.84	18.80	19.96	18.17	22.39	21.46
16	2	16.88	2.86										
16	4	16.88	4.86										
16	6	16.88	6.86	351.03	640.01	14.82	14.45	18.87	17.83	12.11	10.32	10.31	9.38
16	8	16.88	8.86	349.19	637.01	15.95	15.58	19.71	18.67	13.79	12.00	12.86	11.93
16	10	16.88	10.86	347.10	634.75	16.94	16.57	20.34	19.30	15.30	13.51	15.05	14.12
16	12	16.88	12.86	344.73	632.96	17.87	17.50	20.86	19.82	16.74	14.95	17.10	16.17
16	14	16.88	14.86	342.07	631.62	18.75	18.38	21.28	20.24	18.14	16.35	19.04	18.11
16	16	16.88	16.86	339.12	630.74	19.59	19.22	21.59	20.55	19.50	17.71	20.92	19.99
16	18	16.88	18.86	335.90	630.41	20.37	20.00	21.78	20.74	20.81	19.02	22.72	21.79
18	2	18.88	2.86										
18	4	18.88	4.86										
18	6	18.88	6.86										

18	8	18.88	8.86	352.61	638.70	16.87	16.50	20.92	19.88	14.11	12.32	12.11	11.18
18	10	18.88	10.86	350.35	635.26	18.05	17.68	21.80	20.76	15.86	14.07	14.76	13.83
18	12	18.88	12.86	347.87	632.78	19.06	18.69	22.44	21.40	17.40	15.61	17.02	16.09
18	14	18.88	14.86	345.13	630.88	19.99	19.62	22.94	21.90	18.85	17.06	19.10	18.16
18	16	18.88	16.86	342.11	629.50	20.87	20.50	23.34	22.30	20.25	18.46	21.07	20.14
18	18	18.88	18.86	338.82	628.65	21.69	21.32	23.63	22.59	21.60	19.81	22.95	22.02

So a = 12 and f = 10 gives values that fit with all the satellites. The answer for T in the table above is calculated from intersection from two points only. When a least squares best fit solution is obtained from all the points, the coordinates are slightly different. The main purpose (of this question in this course) is to determine the ambiguities not the coordinates of T.

Q8. Answer is not supplied here. Do your own checks so that you know if your answer is correct. I hope you get a very good answer.

Q9. Here I label the points 991, 926, 921, 5 as A, B, C, P respectively. Intermediate and final results are shown below, rather than full details.

Solution by the Tangent method:
Angle APB = 96°00'16.0" Angle BPC = 141º34'13.0"
tan PB = -1297.4 / -74.7 so brg PB = 266º42'17.1"
Coordinates of P (point 5) = 289976.130, 6147689.454

Check, calculate bearings from coordinates:
PA = 170º42'01.1"
PB = 266º42'17.1"
PC = 48º16'30.1"
From these bearings we calculate the angles APB and BPC to see if they are the same as above. We can also calculate the difference between bearings and the observed directions to see if they are all the same.
PA: 170º42'01.1" - 150°41'57" = 20º00'04.1"
PA: 266º42'17.1"- 246°42'13" = 20º00'04.1"
PA: 48º16'30.1"- 28°16'26" = 20º00'04.1" OK ☺

Solution by the Teinstra method
Angles between the observed directions: APB = 96.0º, BPC = 141.6º, CPA = 122.4º
Bearings from coordinates: AB = 282.9º, BC = 72.8º, AC = 29.3º
Angles in the triangle formed by the control points at A = 106.4º, B = 30.1º, C = 43.5º
W_A = 1.035, W_B = 0.424, W_C = 0.863, $W_A + W_B + W_C$ = 2.321

P = 289976.130, 6147689.454 OK

These coordinates match the tangent method, and we can do the same checks with bearings from P as above.

Solution by the Collins Point Method
This solution is set up for when P is in between all 3 of ABC i.e. in the circle ABC, and B is chosen as the middle point to create the Collins Point, H.
From coordinates distance AC = 391.606 and bearing AC = 29°18'09.0"
Bearing AH = bearing AC – angle BPC = 247°43'56.0"
Bearing CH = bearing CA + angle APB = 305°18'25.0"
By intersection by bearings from A and C: H = 290427.487, 6147715.441
From coordinates: brgBH = brgPB = 86°42'17.1"
From brgPB and observed angles: brgCP = 48°16'30.1" and brgAP = 170°42'01.1"
By intersection by bearings from A and C:

P = 289976.130, 6147689.454 OK

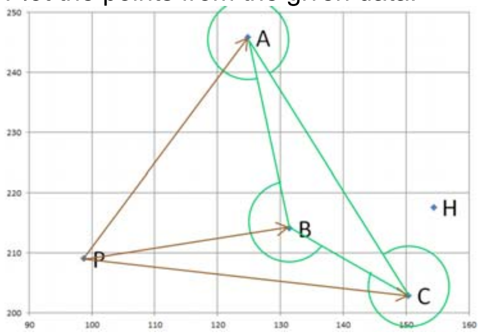
These coordinates match both above methods, and we can do the same checks with bearings from P as above.

Q10. a) see text page 2-20

b) For consistency with equations in this book, label the points as follows.
91 = A, 92 = B, 93 = C, 61 = P.

The ZA observations given in the question are not involved in the solution. Sometimes we have more data than we need and students and surveyors decide which of the available data is required.

Plot the points from the given data:



The Collins' point H is also shown.

Note B is closer to P than the line AC, so the angles A B C are not the internal angles of the triangle.

angles at P from directions	Bearings from coordinates	ABC angles from bearings (mod 360)	weights
$\alpha_1 = dir_{PB} - dir_{PA} = 46.0°$	β_{AB} = 168.2°	A = β_{AC}-β_{AB} = 341.0°	w_A = -0.154
$\alpha_2 = dir_{PC} - dir_{PB} = 15.6°$	β_{BC} = 120.5°	B = β_{BA}-β_{BC} = 227.7°	w_B = 0.689
$\alpha_3 = dir_{PA} - dir_{PC}$ (+360) = 298.4°	β_{AC} = 149.2°	C = β_{CB}-β_{CA} = 331.4°	w_C = -0.358
			Σw = 0.177

Where $w_A = 1/(\cot A - \cot\alpha_2)$ $\quad w_B = 1/(\cot B - \cot\alpha_3)$ $\quad w_C = 1/(\cot C - \cot\alpha_1)$

$$E_P = \frac{E_A w_A + E_B w_B + E_C w_C}{w_A + w_B + w_C} \qquad N_P = \frac{N_A w_A + N_B w_B + N_C w_C}{w_A + w_B + w_C}$$

P = 98.766, 209.123

c) Check bearings from coordinates:

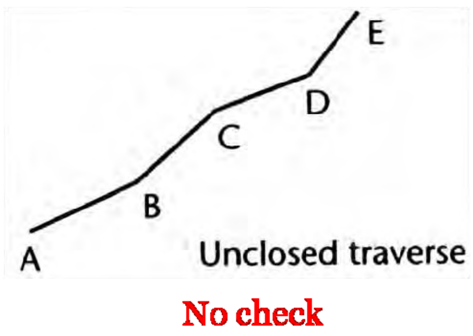
	Bearings			Angles	
PA	35°	16'	29.6"		
				46.0°	OK
PB	81°	16'	45.6"		
				15.6°	OK
PC	96°	50'	58.6"		
				298.4°	OK

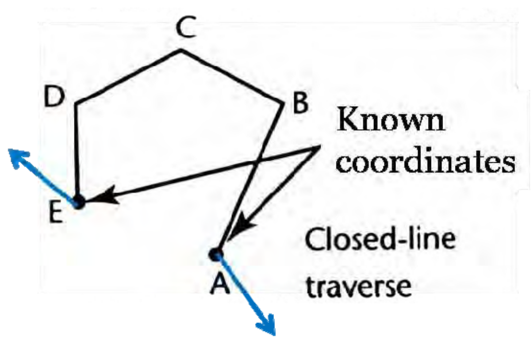
3. TRAVERSES and LOOP CLOSES

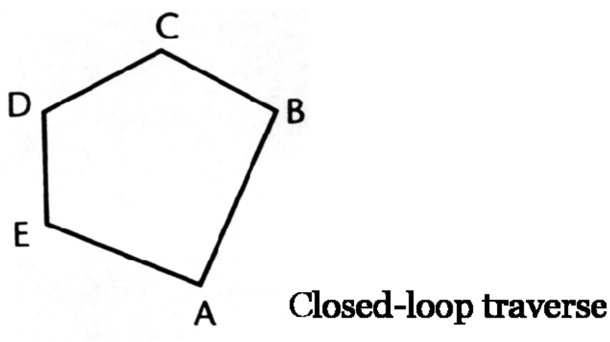
The problems in this chapter are sometimes called Missing Data Problems. They typically use traverse close calculations to find out a distance, bearing or angle from a loop. These problems, in this chapter, assume there are no errors in the data, i.e. no small measurement precision errors and no gross mistakes.

First, let's revise traverses and then the process of calculating around a traverse loop, by looking at examples.

Definition: Traversing is a method of control surveying. A series of control points (each inter-visible with its adjacent stations) are joined by traverse lines (vectors). In a traverse we measure angles between successive lines, or bearings, or directions of each line, and the length of each line.







Note that unclosed traverses are not recommended because they do not include misclose calculations, so there are no checks. Traverses between points with known coordinates are better if they include observations to other targets along known bearings as shown in the example figure above with blue arrow lines.

Traverse surveys are made for many purposes and types of projects, e.g.

- To determine the positions of existing cadastral boundary marks.
- To establish the position of boundary lines.
- To determine the area encompassed within the confines of a boundary.
- To determine the position of arbitrary points from which data may be obtained for preparing various types of maps, i.e. to establish control for mapping.
- To establish ground control for photogrammetric mapping.
- To establish control for locating railroads, highways and other construction work.

In many modern applications traverses are more complicated than simple linear or loop traverses. The surveys can include radiations and links with or connections to other traverse lines. The survey control then forms a network. Network adjustments are beyond the scope of this course, and will be covered in a later course.

The various stages of traversing may be grouped as follows:

1. Reconnaissance and layout
2. Station marks and targets
3. Angular observations and distance measurements
4. Computations and adjustments

RECONNAISSANCE & DESIGN

The main issues in selecting route of a traverse:

- Make individual legs as long as possible and as equal in length as possible.
- Avoid very short legs of < 10m.
- Avoid grazing rays, that is, lines of sight very close to the ground or to walls, because the line of sight might be bent or curved – refraction.
- Select lines that will avoid heavy clearing of trees or damage to crops or property.

STATION MARKS AND TARGETS

- The marks placed must be durable, stable, recognisable, and situated to accord with anticipated future use.
- Site selection can be considered as the most important part of the whole operation.
- Design of traverse is dictated by topographical features and structures, such as hills, vegetation, roads and buildings.
- In urban and fringe urban areas, traverses usually follow the street pattern.
- In non-urban areas, traverse stations will mostly be in the road reservation, but the sight lines can usually cross the open country alongside the road.
- The number of stations between junction points and the length of the traverse legs will vary according to the road pattern and topographical detail.

- There are several factors which must be considered when choosing the site for a traverse station:
  - Permanent marks will be used as instrument stations.
  - Intervisibility between stations is to be maintained.
  - The sight lines should not be obscured.
  - Lines of sight should clear of all obstructions by at least one metre.
  - Adjoining traverse legs are to be of equal length as much as possible.
  - Marks should be permanent and should not be placed in areas where they could be disturbed.
  - Permanence of all emplaced marks is essential to the concept of integrated surveys.
  - All government and local government authorities responsible for local services, roadways and utilities must be consulted regarding future plans for development and any relocation of services.

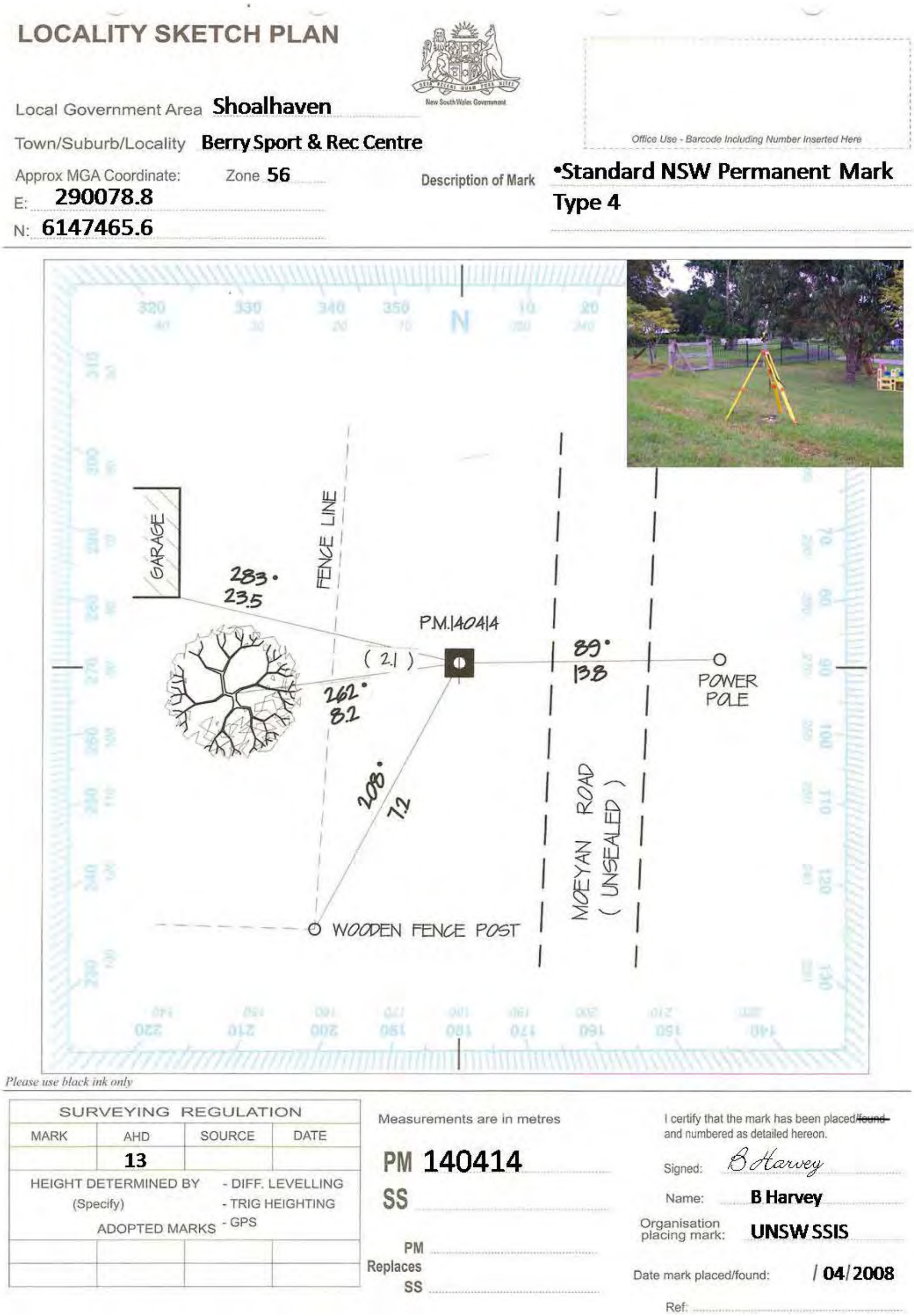
- Final site selection is necessarily a compromise in most cases, but due consideration must be given to all design aspects before making that selection.
- For this reason traverse design, planning, reconnaissance and selection of station sites is to be undertaken by experienced survey personnel.
- The construction and type of station depends on the requirements of the survey.
- For general purpose traverses, wooden (or poly) pegs can be used which are hammered into the ground until the top of the peg is almost flush with ground level. A nail should be tapped into the top of the peg to define the exact position of the station.
- A more permanent station would normally require marks set in concrete. These have to be placed with the permission of the land owners as subsurface concrete blocks placed in the field could do considerable damage to farm machinery.

Student task: Search the web for the latest information and pictures of survey marks.

The purpose of a witness mark is to verify the stability of the traverse station and to provide an alternative coordinated point.

LOCALITY SKETCH PLAN

A Locality Sketch plan on the appropriate form is to be prepared for each State Control Survey Mark. The plan is usually oriented to north and show the magnetic bearing and distance to any witness marks or nearby survey marks. To allow the mark to be readily found, connection should also be made to sufficient surrounding features (e.g. buildings, fences, gates, kerbs, hydrant boxes, manholes, road signs). In recent years it is recommended to record approximate coordinates from, for example, handheld GPS receivers. A digital photograph of the mark's surrounds can also be useful.



LOCALITY SKETCH PLAN

New South Wales Government

Local Government Area Shoalhaven

Town/Suburb/Locality Berry Sport & Rec Centre

Approx MGA Coordinate: Zone 56

E: 290078.8

N: 6147465.6

Office Use - Barcode Including Number Inserted Here

Description of Mark •Standard NSW Permanent Mark Type 4

Please use black ink only

SURVEYING REGULATION			
MARK	AHD	SOURCE	DATE
	13		
HEIGHT DETERMINED BY (Specify)		- DIFF. LEVELLING - TRIG HEIGHTING - GPS	
ADOPTED MARKS			

Measurements are in metres

PM 140414

SS

Replaces PM

SS

I certify that the mark has been placed/~~found~~ and numbered as detailed hereon.

Signed: B Harvey

Name: B Harvey

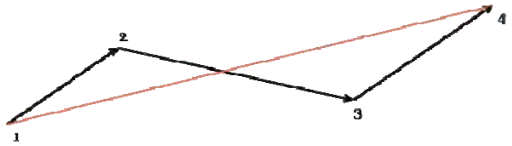
Organisation placing mark: UNSW SSIS

Date mark placed/found: / 04/ 2008

Ref:

Traverses and Vector arithmetic

Traverses can be thought of as a series of vectors, or a series of radiations. Some calculators have a Σ+ key (or M+ with two variables) which enables you to add up a series of Δx and Δy or ΔE and ΔN coordinates. See your calculator manual for more details and examples. It is quite common in plane surveying to have a series of lines - called a traverse - as shown in the following figure:



The total value of coordinate differences for the series of the lines (vectors) from 1 to 4 can then be easily calculated and then converted to one bearing and distance for the line 1 to 4. If the vectors form a loop then the join between the start and end of the loop (called the misclose) is a useful measure of the accuracy of the measurements.

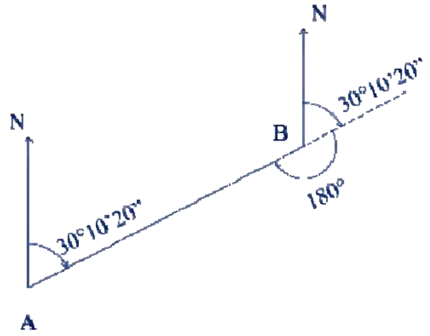
Bearing from A to B is called β_{AB}, the reverse bearing is $\beta_{BA} = \beta_{AB} \pm 180°$. If you have trouble understanding the concept of adding or subtracting 180° to get a reverse (also called back) bearing, do the following. Draw the line and add north direction vectors at both ends. Bearing is angle between the north point and the line.

<u>Example:</u>

Bearing AB = 30°10'20"
Bearing BA = 30°10'20" + 180° = 210°10'20".

If an angle is measured at B from A to C (always clockwise in surveying) = 160° 21'34"
Bearing BC = 210°10'20" + 160° 21'34" = 370°31'54"

But this is greater than 360° so subtract 360° gives 10°31'54".



Choice of origin of coordinates

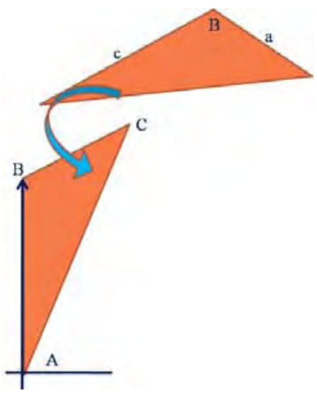
If you are doing a survey where you assign coordinates arbitrarily then it is a good idea (in order to make data input easier and more reliable) to:

a) Choose coordinate axes so that all points are in the first quadrant, all coordinates are positive, then the origin is SW of the net. Don't have any more digits than necessary. If for example, your net is only 1km in extent then you want coordinates like 836.457 not 1234 675.536.

b) Choose the origin so that all E (or y) coordinates have the same first digit and all N (or x) coordinates have another first digit. So for example, 2346.57 and 2129.64 are E and 5116.01 and 5964.37 are N. This way you are less likely to enter an E when an N is required, and vice versa.

Alternative method to solve plane triangles

Traverse calculations (by vector arithmetic) can be used as an easy way to solve some triangles without needing the sine or cosine rules. Simply assign an arbitrary value to the bearing of one line (e.g. 0°) and calculate the ΔE and ΔN of two lines, the ΔE and ΔN of the third line can then simply be calculated. From this ΔE and ΔN the bearing and distance can be calculated and any angles found by subtraction of appropriate bearings.

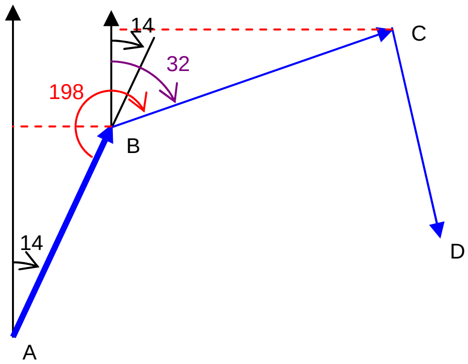


In the example shown in figure at right, you are given a, B, and c. Assign the coordinates of A as (0, 0) and the bearing of AB = 0°. Then the coordinates of B can be found from 0° and c (e.g. use →R and Σ+). Then bearing BC = 180° - B. Then the coordinates of C can be found from B_{BC} and a (e.g. use →R and Σ+). Now from the coordinates of A and C the bearing and distance of AC can be found (e.g. use Σ- and →P). The angle A = 180° - B_{CA}

= B_{AC} and angle C = B_{CB} - B_{CA}.

Excel Task: Traverse coordinates

An example of a traverse:



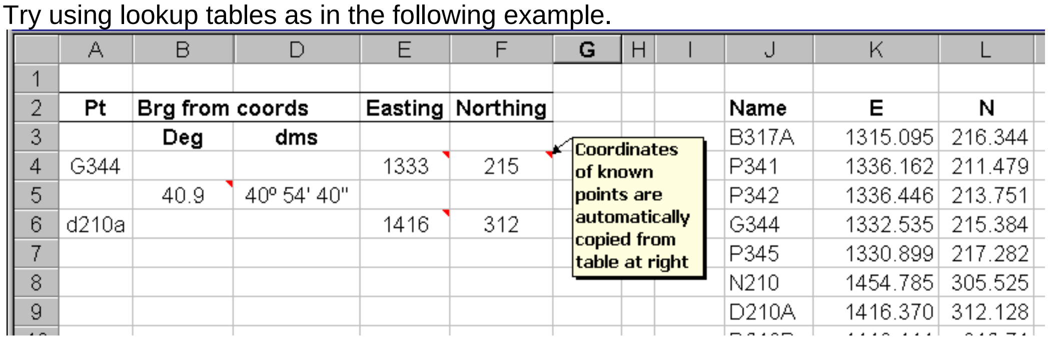
Not to scale

Set up and calculate the red values in the following survey calculations. Then modify the input data and see if your equations are still valid. Experiment with the IF or MOD functions in your calculations.

Point / Line		Angle		Dec Deg		Brg		Distance	Easting	Northing
A					D	M	S		4123.459	9789.010
				14.39	14	23	28	245.620		
B									4184.504	10026.920
ABC	198	06	30	32.50	32	29	58	310.000		
C									4351.064	10288.373
BCD	284	01	30	136.52	136	31	28	480.340		
D									4681.560	9939.807
A to D				74.88	74	52	47.8	578.115		

Excel: Lookup tables for coordinates

Try using lookup tables as in the following example.



	A	B	D	E	F	G	H	I	J	K	L
1											
2	Pt	Brg from coords		Easting	Northing				Name	E	N
3		Deg	dms						B317A	1315.095	216.344
4	G344			1333	215				P341	1336.162	211.479
5		40.9	40º 54' 40"						P342	1336.446	213.751
6	d210a			1416	312				G344	1332.535	215.384
7									P345	1330.899	217.282
8									N210	1454.785	305.525
9									D210A	1416.370	312.128

Coordinates of known points are automatically copied from table at right

The next screen view was obtained by selecting MS Excel menu options   Tools | Options | √ Formulas

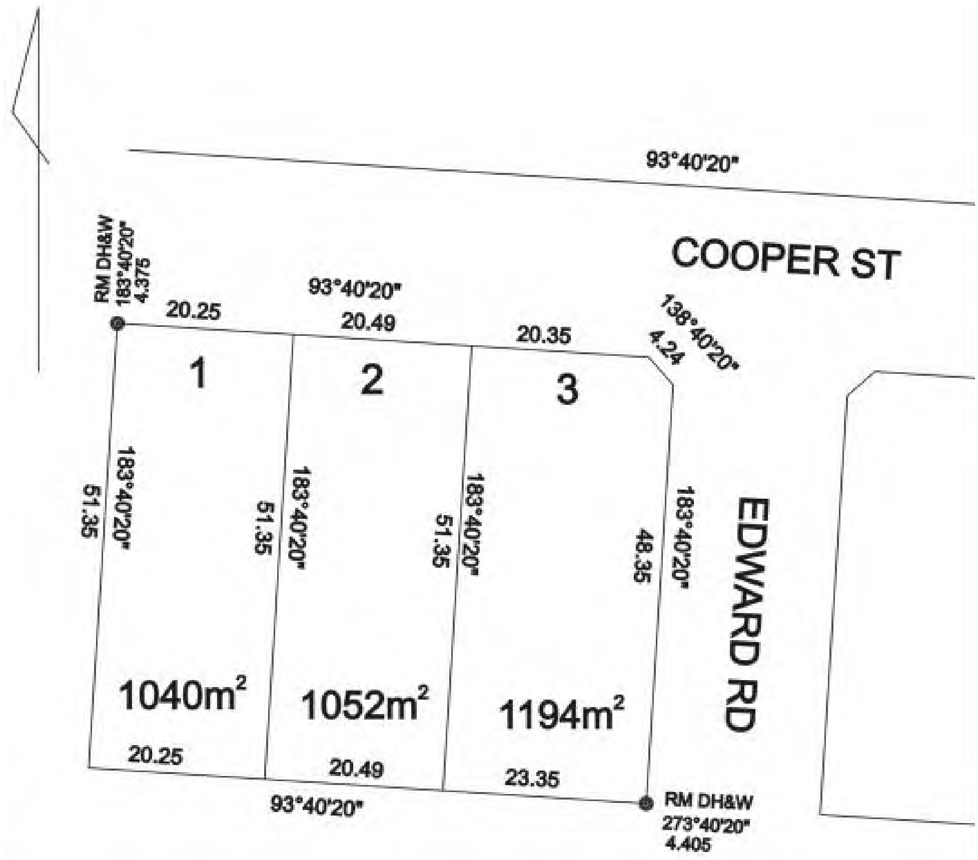
In this sheet the cells from J3 to L23 were NAMED CL, for coordinate list. The formula =VLOOKUP(A4,CL,2,FALSE) uses CL to refer to these cells, alternatively the formula could have been written as =VLOOKUP(A4,J3:L23,2,FALSE) In this example A4 refers to the “lookup” column, i.e. the value in cell A4 (the name of the point) is searched in the first column of the CL matrix, CL or J3:L23 refers to the matrix, 2 means to take the second column value (the Easting) in the same row as the value in A4.

	A	B	D	E	F
1					
2	Pt	Brg from coords		Easting	Northing
3		Deg	dms		
4	G344			=VLOOKUP(A4,CL,2,FALSE)	=VLOOKUP(A4,CL,3,FALSE)
5		=MOD(DEGREES(ATAN2((F6-F4),(E6-E4))),360)	=dms(C5)		
6	d210a			=VLOOKUP(A6,CL,2,FALSE)	=VLOOKUP(A6,CL,3,FALSE)
7					

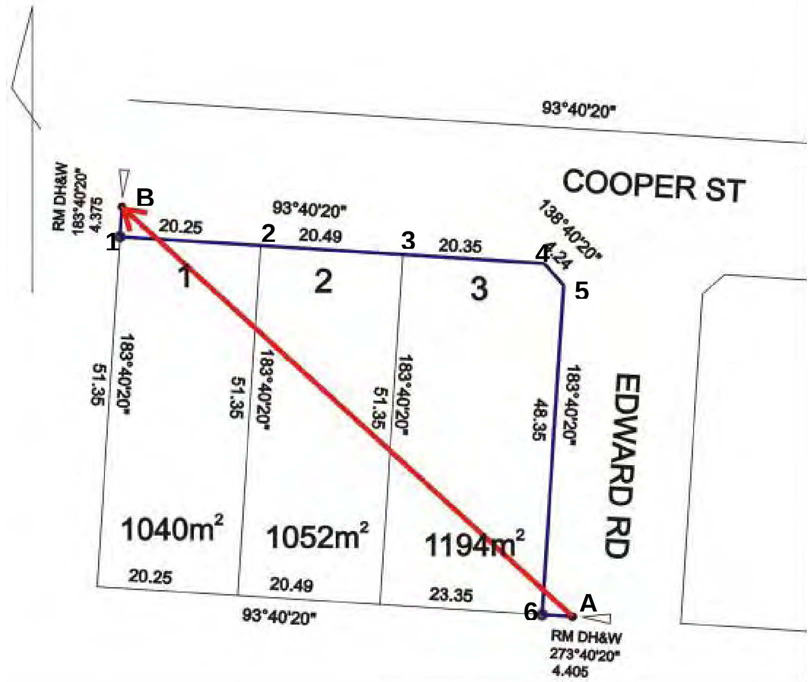
In column D the function dms( ) is a macro that I wrote to convert the bearing into a text with symbols. It is not a built-in function.

Example: Cadastral DP Calculations (by Adam Long and Bruce Harvey)

Calculate the bearing and distance from the reference mark (RM DH&W) at Lot 3 to the reference mark (RM DH&W) at Lot 1 using the data provided on the following Deposited Plan (DP):



Visualise the Problem:



We have to calculate the bearing and distance along the red line, from A to B, by adding up differences in Eastings and Northings along the blue line. Once we have the total difference in Eastings and Northings, a bearing and distance can be calculated using what you learnt in a previous question (for calculating bearings and distances from plane coordinates).

Calculation:

The way to calculate the bearing from A to B is to calculate the traverse starting at the drill hole and wing (DH&W) at B, and to work our way along the boundaries to the DH&W at A. When entering the bearings of the lines, be sure that the correct orientation is entered. Do this by checking the bearing against the north arrow and make sure that it is the forward bearing not the back bearing.

In the field a calculation like this could be done on a calculator with a "close program", but for this exercise an Excel spreadsheet is used. The formulas in the spreadsheet are the same as those used in a "close program", so it will give you a 'behind the scenes' view of the program. [Excel spreadsheets can also be used in the field with a laptop computer or Pocket PC.]

The spreadsheet shown below also keeps a cumulative bearing and distance to A so you can see how they change as we traverse along the blue lines in the figure above.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Point	Plan Bearing				Plan Distance	ΔE	ΔN	ΣΔE	ΣΔN	Bearing to B				Distance to B
2		d	m	s	dd	m	m	m	m	m	dd	d	m	s	m
3	B										0	0	0	0	0
4		183	40	20	183.6722	4.375									
5	1						-0.280	-4.366	-0.280	-4.366	3.67222	3	40	20.0	4.375
6		93	40	20	93.6722	20.25									
7	2						20.208	-1.297	19.928	-5.663	285.86356	285	51	48.8	20.717
8		93	40	20	93.6722	20.49									
9	3						20.448	-1.312	40.376	-6.975	279.80163	279	48	5.9	40.974
10		93	40	20	93.6722	20.35									
11	4						20.308	-1.303	60.684	-8.279	277.76850	277	46	6.6	61.246
12		138	40	20	138.6722	4.24									
13	5						2.800	-3.184	63.484	-11.463	280.23506	280	14	6.2	64.511
14		183	40	20	183.6722	48.35									
15	6						-3.097	-48.251	60.388	-59.713	314.67842	314	40	42.3	84.926
16		93	40	20	93.6722	4.405									
17	A						4.396	-0.282	64.784	-59.996	312.80257	312	48	9.3	88.297

Final Bearing and Distance to B

So the final bearing and distance from A to B is 312° 48' 09.3" and 88.297 m. Check this against the north point to make sure it is the right bearing.

Excel Formulae:

The formulas used in this example are based on material learnt previously, that is rectangular to polar, and polar to rectangular conversions. Here is what the Excel spreadsheet should have. Enter the data and convert bearings to decimal degrees:

	A	B	C	D	E	F
1	Point				Plan Bearing	Distance
2		d	m	s	dd	m
3	B					
4		183	40	20	=B4+C4/60+D4/3600	4.375
5	1					
6		93	40	20	=B6+C6/60+D6/3600	20.25
7	2					
8		93	40	20	=B8+C8/60+D8/3600	20.49
9	3					
10		93	40	20	=B10+C10/60+D10/3600	20.35
11	4					
12		138	40	20	=B12+C12/60+D12/3600	4.24
13	5					
14		183	40	20	=B14+C14/60+D14/3600	48.35
15	6					
16		93	40	20	=B16+C16/60+D16/3600	4.405
17	A					

Now calculate the ΔE and ΔN as well as ΣΔE and ΣΔN:

	G	H	I	J
1	ΔE	ΔN	ΣΔE	ΣΔN
2	m	m	m	m
3				
4				
5	=F4*SIN(RADIANS(E4))	=F4*COS(RADIANS(E4))	=G5	=H5
6				
7	=F6*SIN(RADIANS(E6))	=F6*COS(RADIANS(E6))	=I5+G7	=J5+H7
8				
9	=F8*SIN(RADIANS(E8))	=F8*COS(RADIANS(E8))	=I7+G9	=J7+H9
10				
11	=F10*SIN(RADIANS(E10))	=F10*COS(RADIANS(E10))	=I9+G11	=J9+H11
12				
13	=F12*SIN(RADIANS(E12))	=F12*COS(RADIANS(E12))	=I11+G13	=J11+H13
14				
15	=F14*SIN(RADIANS(E14))	=F14*COS(RADIANS(E14))	=I13+G15	=J13+H15
16				
17	=F16*SIN(RADIANS(E16))	=F16*COS(RADIANS(E16))	=I15+G17	=J15+H17

Now using ΣΔE and ΣΔN calculate the bearing and distance to B at each point:

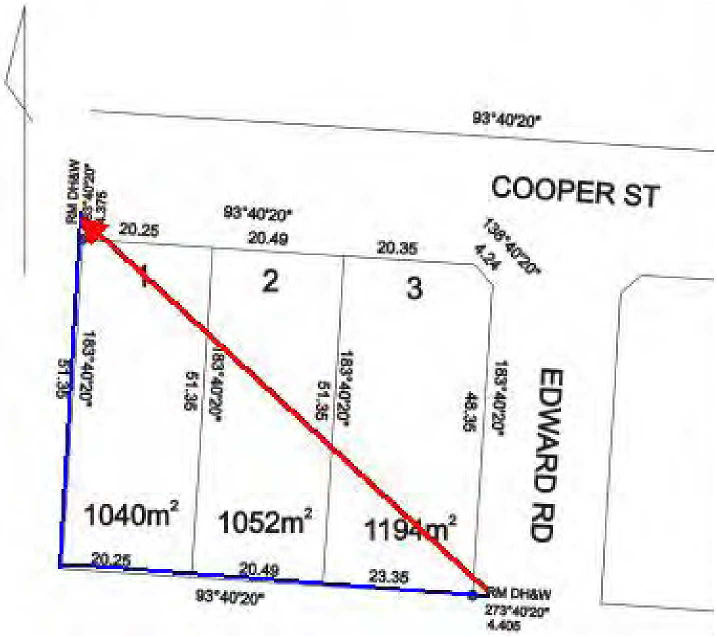
	K	L	M	N	O
1		Bearing to B			Distance to B
2	dd	d	m	s	m
3	0	0	0	0	0
4					
5	=(DEGREES(ATAN2(J5,I5)))+180	=INT(K5)	=INT((K5-L5)*60)	=(K5-L5-M5/60)*3600	=SQRT(I5^2+J5^2)
6					
7	=MOD(DEGREES(ATAN2(J7,I7)),360)+180	=INT(K7)	=INT((K7-L7)*60)	=(K7-L7-M7/60)*3600	=SQRT(I7^2+J7^2)
8					
9	=MOD(DEGREES(ATAN2(J9,I9)),360)+180	=INT(K9)	=INT((K9-L9)*60)	=(K9-L9-M9/60)*3600	=SQRT(I9^2+J9^2)
10					
11	=MOD(DEGREES(ATAN2(J11,I11)),360)+180	=INT(K11)	=INT((K11-L11)*60)	=(K11-L11-M11/60)*3600	=SQRT(I11^2+J11^2)
12					
13	=MOD(DEGREES(ATAN2(J13,I13)),360)+180	=INT(K13)	=INT((K13-L13)*60)	=(K13-L13-M13/60)*3600	=SQRT(I13^2+J13^2)
14					
15	=MOD(DEGREES(ATAN2(J15,I15)),360)+180	=INT(K15)	=INT((K15-L15)*60)	=(K15-L15-M15/60)*3600	=SQRT(I15^2+J15^2)
16					
17	=MOD(DEGREES(ATAN2(J17,I17)),360)+180	=INT(K17)	=INT((K17-L17)*60)	=(K17-L17-M17/60)*3600	=SQRT(I17^2+J17^2)

The last bearing and distance is the answer required.

Check and Exercise

If you want some more practice check this answer by calculating the bearing and distance from A to B by going around the western boundary of Lot 1 and along the back of Lots 1, 2 and 3. You will find some discrepancy, as the dimensions on a DP are usually rounded values.

Answer: the bearing is the same, the distance is 3 mm larger (this is due to the plan showing distances to only the nearest cm).



Loop Close (Missing Data) Problems

This section provides useful problem solving practice. It is a calculation method, it is not intended to be a means of overcoming poor survey design. Surveyors should measure the data reliably and not use the techniques described in this section to merely calculate values instead of measuring them.

Traverse computations are frequently used to solve problems involving missing data. These are generally computed in a loop traverse but the principle is also applicable to closed line traverses. The assumption is made that the known observations are accurate and that there is no misclose in the traverse. Thus there will be 2 equations, one equating the difference in Eastings of the traverse lines to the ΔE between the starting and closing stations (zero for a loop traverse). The other equation does the same for the Northings. Therefore, in general, 2 unknown elements can be found. Two 'simultaneous' equations with 2 unknowns can be solved be a number of methods. These notes show the 'long hand' manual substitution method. A second method, shown for some of the problems below, forms the equations into matrices and uses Matrix Algebra (e.g. Inversion and multiplication) in MS Excel. The third method is to use Excel's SOLVER. Alternatively a triangle with its sides representing the unknown elements and the misclose vector can be constructed and the required elements can then be found by solving this triangle.

If there are more than 2 unknown elements then these methods are not applicable. If there is only 1 missing distance or 1 missing bearing or angle then the loop misclose will reveal the value required.

In a traverse we measure angles (→bearings) and distances. The following cases of missing data will be dealt with:

1) Bearing and distance of one line missing.

2) Distances of two lines missing.

3) Bearings of two lines missing.

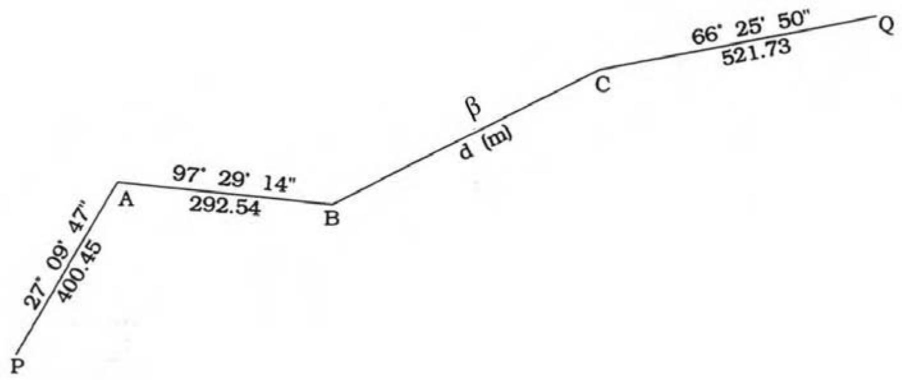
4) Bearing of one line and distance of another missing.

Checks: In each of these cases, after the 'missing' values are determined, then calculate the traverse miscloses. The miscloses should be zero (or at least smaller than the last decimal place in the data).

In the following cases the same example traverse data is used for each case. It is a traverse between two points labelled P and Q. The methods also work with loop traverses. Of course, in a loop traverse the total $\Delta E = 0$ and total $\Delta N = 0$ instead of using the ΔE_{PQ} and ΔN_{PQ} values as in the following examples.

BEARING AND DISTANCE OF ONE LINE MISSING (β and d)

Consider the following traverse where the bearing (β) and distance (d) of BC are missing:



The coordinates of P and Q are given: P (365.30, 699.37), Q (1706.61, 1425.10)

So ΔE_{PQ} = 1341.31 and ΔN_{PQ} = 725.73

Computing the ΔE and ΔN of the traverse lines we get:

Line	ΔE	ΔN
PA	182.815	356.285
AB	290.046	-38.119
BC	d sinβ	d cosβ
CQ	478.205	208.619
Sum	951.066 + d sinβ	526.785 + d cosβ

The sum of the difference in Eastings of all traverse lines is equal to ΔE between the terminals (i.e. P and Q):

$$951.066 + d \sin\beta = 1341.31 \quad \rightarrow \quad d \sin\beta = 390.244 \qquad \text{--- (1)}$$

The sum of the difference in Northings of all traverse lines is equal to ΔN between the terminals (i.e. P and Q):

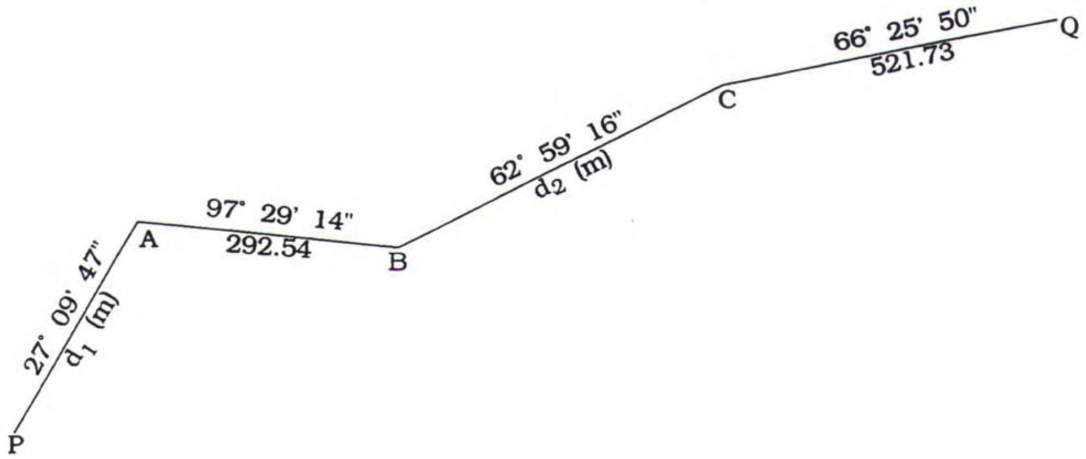
$$526.785 + d \cos\beta = 725.73 \quad \rightarrow \quad d \cos\beta = 198.945 \qquad \text{--- (2)}$$

Equations (1) and (2) can be solved for d and β using R→P on your calculator. Treat 390.244 as ΔE and 198.945 as ΔN.

The missing bearing and distance of the line BC are then: β = 62° 59' 16" d = 438.03 m

DISTANCES OF TWO LINES MISSING (2d)

Consider the following traverse where the distances d_1 and d_2 of PA and BC are missing:



The coordinates of P and Q are given: P (365.30, 699.37), Q (1706.61, 1425.10)

So $\Delta E_{PQ} = 1341.31$ and $\Delta N_{PQ} = 725.73$

<u>Method 1</u>

Compute the ΔE and ΔN of the traverse lines:

Line	ΔE	ΔN
PA	d_1 sin27°09'47"	d_1 cos27°09'47"
AB	290.046	-38.119
BC	d_2 sin62°59'16"	d_2 cos62°59'16"
CQ	478.205	208.619
Sum	$0.456524\, d_1 + 0.890910\, d_2 + 768.251$	$0.889711\, d_1 + 0.454181\, d_2 + 170.500$

∑ΔE of all traverse lines is equal to ΔE between the terminals (i.e. P and Q):

$$0.456524 \cdot d_1 + 0.890910 \cdot d_2 + 768.251 = 1341.31 \quad \rightarrow 0.456524 \cdot d_1 + 0.890910 \cdot d_2 = 573.059 \quad \text{--- (3)}$$

∑ΔN of all traverse lines is equal to ΔN between the terminals (i.e. P and Q):

$$0.889711 \cdot d_1 + 0.454181 \cdot d_2 + 170.500 = 725.73 \quad \rightarrow 0.889711 \cdot d_1 + 0.454181 \cdot d_2 = 555.230 \quad \text{--- (4)}$$

"Long hand" solution:

Dividing equations (3) and (4) by their respective coefficients of d_2, we get:

$$0.512424 \cdot d_1 + d_2 = 643.229 \quad \text{--- (5)}$$

$$1.958934 \cdot d_1 + d_2 = 1222.486 \quad \text{--- (6)}$$

Subtracting equation (5) from equation (6) gives: $1.446510 \cdot d_1 = 579.257$ → $d_1 = 400.451$

Substituting the value of d_1 in equation (5) gives: $d_2 = 438.028$

Therefore the missing distances are: PA = d_1 = 400.45 m BC = d_2 = 438.03 m

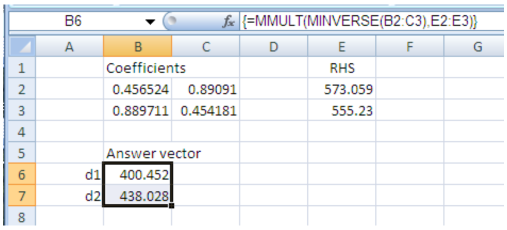
Check calculations by substituting these values in equations (3) & (4). Or better still; recalculate the close of the traverse using these values.

Matrix Algebra solution:

The two equations, one for eastings and one for northings, (3 and 4 above) can be rewritten in matrix form, A x = b:

$$\begin{pmatrix} 0.456524 & 0.890910 \\ 0.889711 & 0.454181 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 573.059 \\ 555.230 \end{pmatrix}$$

Then $x = A^{-1}b$



If, for example, we enter the A matrix into cells B2:C3 in an Excel spreadsheet and b vector into cells E2:E3, then the Excel equation to calculate the x vector is =MMULT(MINVERSE(B2:C3),E2:E3). As usual with matrix equations in Excel you select all the cells that you want the answer vector or matrix to occupy, type in the equation, and then press CTRL Shift Enter keys simultaneously.

The result is $\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 400.452 \\ 438.028 \end{pmatrix}$ which is the same as the 'long hand' solution above.

<u>Method 2</u> (based on solution of triangle)

If the traverse is computed excluding the lines with missing information, it will give a large misclose which will represent the resultant of the vectors representing these lines. Therefore a triangle can be drawn with two of its sides representing the two lines with missing information, while the third side represents the large traverse misclose calculated above. This triangle can be solved for the missing data.

Compute the traverse excluding the lines with missing information:

Line	ΔE	ΔN
PA		
AB	290.046	-38.119
BC		
CQ	478.205	208.619
Sum	768.251	170.500

The traverse misclose is then:

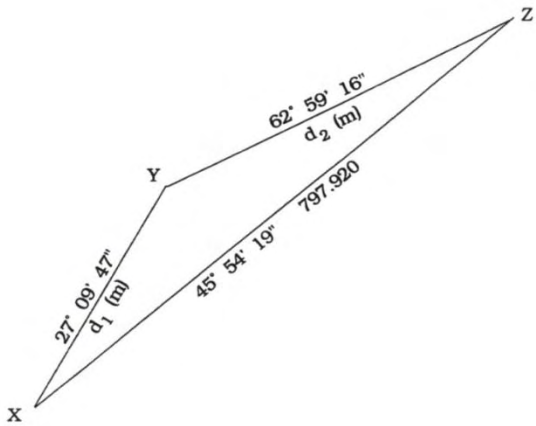
m_E = 1341.31 – 768.251 = 573.059

m_N = 725.73 – 170.500 = 555.230

The misclose vector (direction from calculated to given position) is calculated (using R→P or atan2):

β = 45° 54' 19"   d = 797.920 m

The misclose vector and the two lines with the missing information can be represented by the three sides of the following triangle:

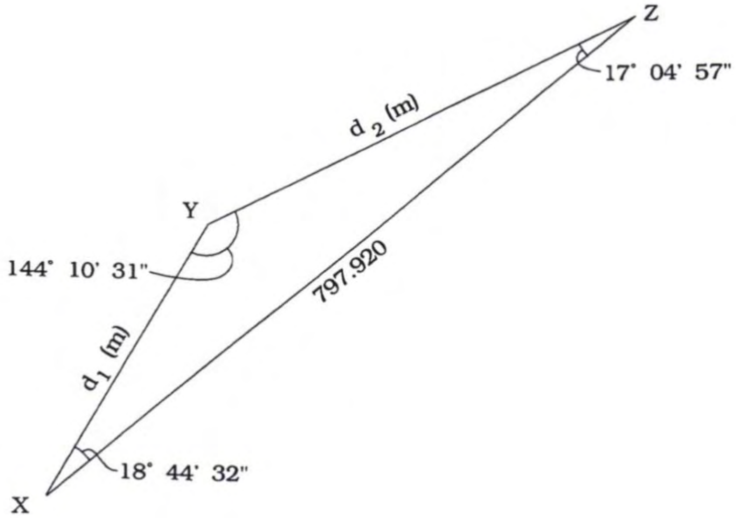


XY represents the line PA, YZ represents the line BC and XZ represents the traverse misclose. Triangle of one known distance and three known angles can be solved for the missing distances.

Calculate the angles from the given bearings:

Angle X = 45° 54' 19" – 27° 09' 47" = 18° 44' 32"
Angle Z = 62° 59' 16" – 45° 54' 19" = 17° 04' 57"
Angle Y = 180° + 27° 09' 47" – 62° 59' 16" = 144° 10' 31"
X + Y + Z = 180° (check)

The triangle becomes then:



Calculate the missing distances using the sine rule: $\frac{d_1}{\sin 17°04'57"} = \frac{d_2}{\sin 18°44'32"} = \frac{797.920}{\sin 144°10'31"}$

→ $d_1 = \frac{797.920 \cdot \sin 17°04'57"}{\sin 144°10'31"} = 400.452$

→ $d_2 = \frac{797.920 \cdot \sin 18°44'32"}{\sin 144°10'31"} = 438.027$

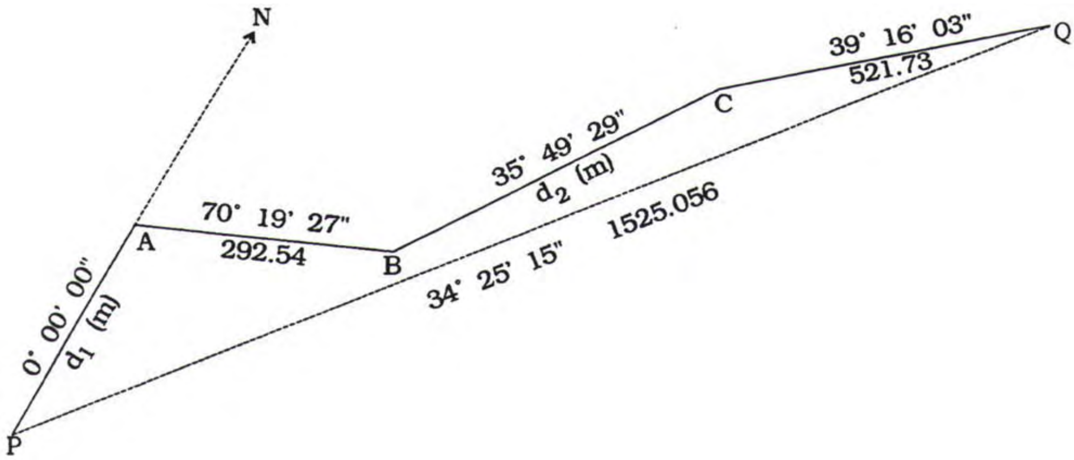
Therefore the missing distances are:

PA = 400.45 m BC = 438.03 m

Method 3 (Cardinal direction method)

The entire traverse is rotated such that one of the lines with missing data points either in N-S or E-W direction. The rotated traverse is then computed to obtain the missing data.

First calculate the bearing and distance of PQ from the known coordinates: β = 61° 35' 02" d = 1525.056 m. Then rotate the coordinate axes such that one axis is parallel to the direction of one of the lines with missing data (this is our selected cardinal direction). For example, rotate the traverse so that North is parallel to PA (all bearings have to be reduced by 27° 09' 47"):



From this point onwards the procedure is the same as for method 1. $\sum\Delta E$ of all traverse lines is equal to ΔE between the terminals (i.e. P and Q):

$$0 + 292.54 \sin 70°19'27" + d_2 \sin 35°49'29" + 521.73 \sin 39°16'03" = 1525.056 \sin 34°25'15"$$

This one equation with only one unknown gives: $d_2 = 438.026$

Similarly, after substituting the value of d_2 calculated above, $\sum\Delta N$ of all traverse lines is equal to ΔN between the terminals (i.e. P and Q):

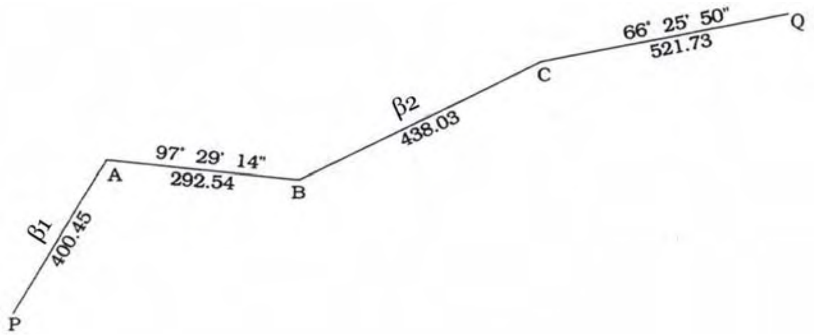
$$d_1 + 292.54 \cos 70°19'27" + 438.026 \cos 35°49'29" + 521.73 \cos 39°16'03" = 1525.056 \cos 34°25'15"$$

This one equation with only one unknown gives: $d_1 = 400.454$

Therefore the missing distances are: PA = 400.45 m BC = 438.03 m

BEARINGS OF TWO LINES MISSING (2β)

Consider the following traverse where the bearings β_1 and β_2 of PA and BC are missing:



The coordinates of P and Q are given: P (365.30, 699.37), Q (1706.61, 1425.10)
So $\Delta E_{PQ} = 1341.31$ and $\Delta N_{PQ} = 725.73$

Method 1

Computing the ΔE and ΔN of the traverse lines we get:

Line	ΔE	ΔN
PA	400.45 $\sin\beta_1$	400.45 $\cos\beta_1$
AB	290.046	-38.119
BC	438.03 $\sin\beta_2$	438.03 $\cos\beta_2$
CQ	478.205	208.619
Sum	400.45 $\sin\beta_1$ + 438.03 $\sin\beta_2$ + 768.251	400.45 $\cos\beta_1$ + 438.03 $\cos\beta_2$ + 170.500

∑ΔE of all traverse lines is equal to ΔE between the terminals (i.e. P and Q):

$400.45 \sin\beta_1 + 438.03 \sin\beta_2 + 768.251 = 1341.31 \rightarrow 400.45 \sin\beta_1 = 573.059 - 438.03 \sin\beta_2$ --- (7)

∑ΔN of all traverse lines is equal to ΔN between the terminals (i.e. P and Q):

$400.45 \cos\beta_1 + 438.03 \cos\beta_2 + 170.500 = 725.73 \rightarrow 400.45 \cos\beta_1 = 555.230 - 438.03 \cos\beta_2$ --- (8)

Squaring and adding equations (7) and (8) gives:

$(400.45)^2 = (573.059)^2 + (555.230)^2 + (438.03)^2 - 2 * 573.059 * 438.03 \sin\beta_2 - 2 * 555.230 * 438.03 \cdot \cos\beta_2$

When simplified, this equation gives:

$573.059 \sin\beta_2 + 555.230 \cos\beta_2 = 762.718$ --- (9)

There are several ways of solving equations of the form: C sinβ + D cosβ = K, but they are quite long and involved and the details are not included here.

Solution by Excel Solver
If we take the equations 7 and 8 above and rearrange the terms we can use MS Excel's Solver. If it is not installed in your version of Excel see the HELP section to add it.

∑ΔE of all traverse lines is equal to ΔE between the terminals (P and Q):
$400.45 \sin\beta_1 - 573.059 + 438.03 \sin\beta_2 = 0$

∑ΔN of all traverse lines is equal to ΔN between the terminals (P and Q):
$400.45 \cos\beta_1 - 555.230 + 438.03 \cos\beta_2 = 0$

In the example screen below, initially leave cells B8 and B9 blank.
In cell B10 type: =400.45*SIN(B8)-573.059+438.03*SIN(B9)

In cell B11 type: =400.45*COS(B8)-555.23+438.03*COS(B9)

	A	B	C	D	E	F	G	H	I
1	**Example data set 2 missing bearings**								
2	ΣΔE of all traverse lines:	400.45 $\sin\beta_1$ = 573.059 – 438.03 $\sin\beta_2$							
3									
4	ΣΔN of all traverse lines:	400.45 $\cos\beta_1$ = 555.230 – 438.03 $\cos\beta_2$							
5									
6									
7	Use Excel Solver	rads		degs	d	m	s		
8	Brg1	1.13		64.6	64	38	52		
9	Brg2	0.50		28.8	28	49	22		
10	eqn for ΔE = 0:	1.4E-07							
11	eqn for ΔN = 0:	1.5E-07							

To avoid round off errors the above numbers e.g. 400.45, could be replaced by the cell reference that holds the value from previous calculations on your spreadsheet.

Depending on your version of Excel, select cell B11 and choose Tools | Solver or Data | Analysis | Solver

The solver options and constraints here are to vary / estimate B8 and B9 until both B10 and B11 = 0. You set one as the target cell and the other as a constraint. I chose to work in radians and then convert bearings to d.d and d m s, but you could work in degrees e.g. use cos(radians(B9)) etc.

Our answer is one of the two possible answers found by the other methods, but with much quicker and easier calculations. You may find other ways to use Solver. Send me a description of your versions.

Method 2 (based on solution of triangle)

Just like in the case of two missing distances, compute the traverse excluding the lines with missing information:

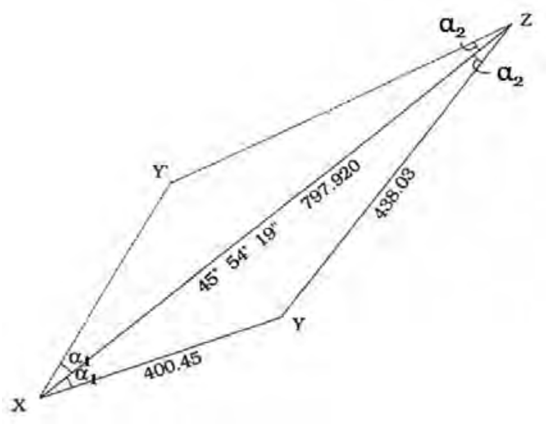
Line	ΔE	ΔN
PA		
AB	290.046	-38.119
BC		
CQ	478.205	208.619
Sum	768.251	170.500

The traverse misclose is then:

m_E = 1341.31 – 768.251 = 573.059 and m_N = 725.73 – 170.500 = 555.230

The misclose vector (direction from calculated to given position) is calculated:
β = 45° 54' 19" d = 797.920 m

The misclose vector and the two lines with the missing information can be represented by the three sides of the following triangle:



XY represents the line PA, YZ represents the line BC and XZ represents the traverse misclose. Note that Y and Y' indicate two possible positions of Y.

The angles α_1 and α_2 can be obtained using the cosine rule:

$$\cos\alpha_1 = \frac{(400.45)^2 + (797.92)^2 - (438.03)^2}{2 \cdot 797.92 \cdot 400.45} = 0.946972 \qquad \alpha_1 = \pm\ 18^\circ\ 44'\ 33''$$

$$\cos\alpha_2 = \frac{(438.03)^2 + (797.92)^2 - (400.45)^2}{2 \cdot 797.92 \cdot 438.03} = 0.955882 \qquad \alpha_2 = \pm\ 17^\circ\ 04'\ 57''$$

Calculate the missing bearings from α_1 and α_2:

$\beta_1 = \beta_{PA} = \beta_{XY} = \beta_{XZ} \pm \alpha_1 = 45^\circ\ 54'\ 19'' \pm 18^\circ\ 44'\ 33'' = 64^\circ\ 38'\ 52''$ or $27^\circ\ 09'\ 46''$

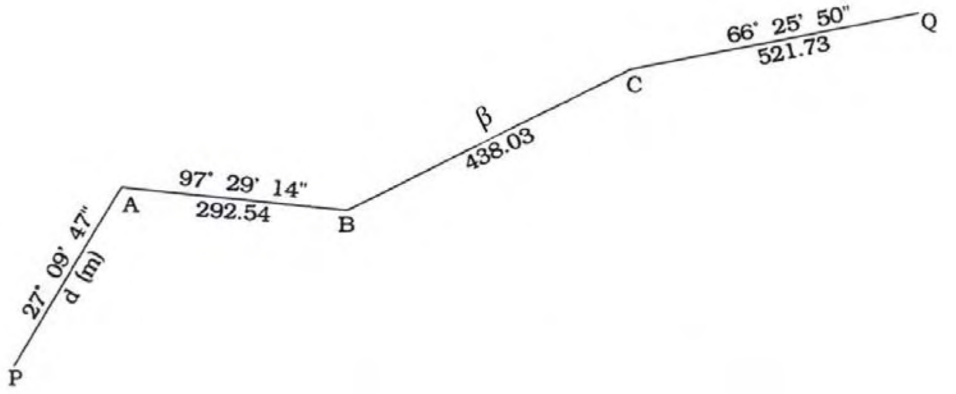
$\beta_2 = \beta_{BC} = \beta_{YZ} = \beta_{XZ} \mp \alpha_2 = 45^\circ\ 54'\ 19'' \mp 17^\circ\ 04'\ 57'' = 28^\circ\ 49'\ 22''$ or $62^\circ\ 59'\ 16''$

There are two possible answers. The actual (unique) value can only be obtained if additional information is available (e.g. the approx. bearing of PA).

The Cardinal direction method cannot be used for this problem since the two missing data are bearings. The rotation necessary to set one of the lines with missing data to the cardinal direction is not known.

BEARING of a line and DISTANCE of another line missing (β and d)

Consider a traverse where the bearing (β) of BC and the distance (d) of PA are missing:



The coordinates of P and Q are given: P (365.30, 699.37), Q (1706.61, 1425.10)
So ΔE_{PQ} = 1341.31 and ΔN_{PQ} = 725.73

Method 1

Computing the ΔE and ΔN of the traverse lines we get:

Line	ΔE	ΔN
PA	d_1 sin27°09'47"	d_1 cos27°09'47"
AB	290.046	-38.119
BC	438.03 sinβ	438.03 cosβ
CQ	478.205	208.619
Sum	d sin27°09'47" + 438.03 sinβ + 768.251	d cos27°09'47" + 438.03 cosβ + 170.500

∑ΔE of all traverse lines is equal to ΔE between the terminals (i.e. P and Q):

d sin27°09'47" + 438.03 sinβ + 768.251 = 1341.31

→ d = (573.059 – 438.03 sinβ) / sin 27°09'47" = 1255.265 – 959.489 sinβ --- (10)

∑ΔN of all traverse lines is equal to ΔN between the terminals (i.e. P and Q):

d cos27°09'47" + 438.03 cosβ + 170.500 = 725.73

→ d = (555.230 – 438.03 cosβ) / cos 27°09'47" = 624.057 – 492.328 cosβ --- (11)

Equating equations (10) and (11) gives:

959.489 sinβ – 492.328 cosβ = 631.208 --- (12)

This equation is of the form: C sinβ + D cosβ = K, and is not further described here.

Method 2 (based on solution of triangle)

Compute the traverse excluding the lines with missing information:

Line	ΔE	ΔN
PA		
AB	290.046	-38.119
BC		
CQ	478.205	208.619
Sum	768.251	170.500

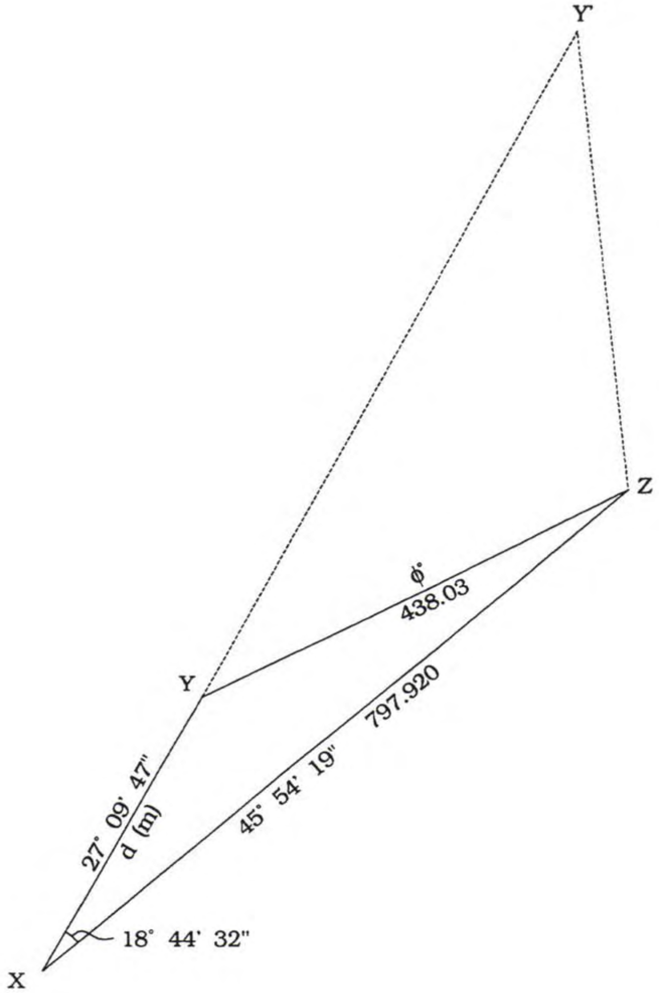
The traverse misclose is then:

m_E = 1341.31 – 768.251 = 573.059 and m_N = 725.73 – 170.500 = 555.230

The misclose vector (direction from calculated to given position) is calculated:

β = 45° 54' 19" d = 797.920 m

The misclose vector and the two lines with the missing information can be represented by the three sides of the following triangle:



XY represents the line PA, YZ represents the line BC and XZ represents the traverse misclose. Note that Y and Y' indicate two possible positions of Y. Triangle of two known distances and one known angle can be solved for the missing data. Alternatively, the method of intersection of bearing and distance given in Chapter 2 can be used.

The angle at Y is obtained from the sine rule: $\frac{797.92}{\sin Y} = \frac{438.03}{\sin 18°44'32"}$

→ Y = 35° 49' 28" or 144° 10' 32"

The missing bearing of YZ is then: β = 27° 09' 47" + 180° – Y = 171° 20' 19" or 62° 59' 15"

The angle at Z can now be determined: Z = β – 45° 54' 19" = 180° – 18° 44' 32" – Y = 125°26'00" or 17°04'56"

The missing distance is then obtained from the sine rule:

$d = \frac{438.03 \cdot \sin Z}{\sin 18°44'32"} = 1110.771$ or 400.448

Therefore the missing data are

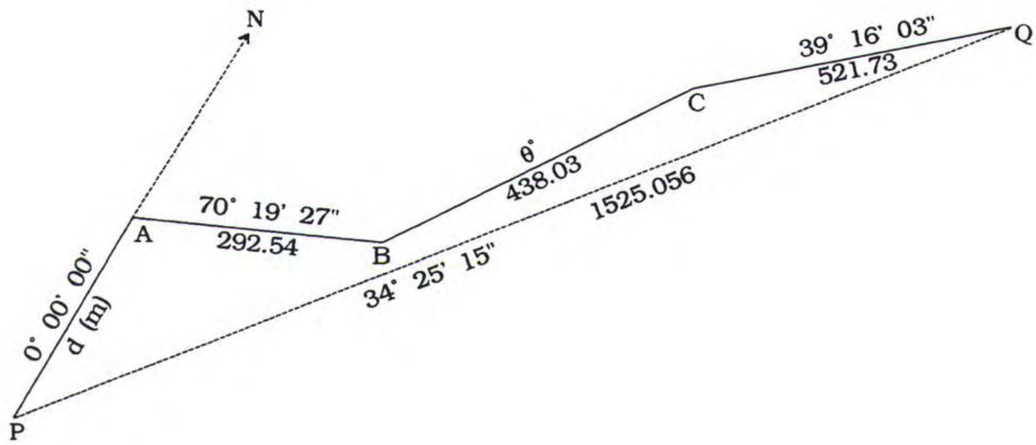
either: β_{BC} = 171° 20' 19" d_{PA} = 1110.77 m or: β_{BC} = 62° 59' 15" d_{PA} = 400.45 m

Additional information is needed to find correct set.

Method 3 (Cardinal direction method)

First calculate the bearing and distance of PQ from the known coordinates:
β = 61°35'02" d = 1525.056 m

Choose the North cardinal direction along the line PA. The original bearing of PA was 27° 09' 47", so all bearings have to be reduced by 27° 09' 47". Adding or subtracting a constant to all traverse lines has the same effect as rotating the entire traverse. So the entire traverse is then rotated anti-clockwise by 27° 09' 47" (note that θ = β – 27° 09' 47"):



$\sum\Delta E$ of all traverse lines is equal to ΔE between the terminals (i.e. P and Q):

0 + 292.54 sin70°19'27" + 438.03 sinθ + 521.73 sin39°16'03" = 1525.056 sin34°25'15"

This 1 equation with only 1 unknown gives:

438.03 sinθ = 1525.056 sin34°25'15" – 292.54 sin70°19'27" – 521.73 sin39°16'03"

→ sinθ = 0.585302 → θ = 35° 49' 28" or 144° 10' 32"

Similarly, $\sum\Delta N$ of all traverse lines is equal to ΔN between the terminals (i.e. P and Q):

d + 292.54 cos70°19'27" + 438.026 cosθ + 521.73 cos39°16'03" = 1525.056 cos34°25'15"

This one equation with only one unknown gives:

d = 1525.056 cos34°25'15" – 292.54 sin70°19'27" – 438.026 cosθ – 521.73 cos39°16'03"

= 400.449 if θ = 35°49'28" (i.e. β = 62°59'15")

= 1110.771 if θ = 144°10'32" (i.e. β = 171°20'19")

Therefore the missing data are

either: β_{BC} = 62° 59' 15" d_{PA} = 400.45 m

or: β_{BC} = 171° 20' 19" d_{PA} = 1110.77 m

Additional information needed to find correct set.

SUMMARY

- For missing data problems the assumption is made that the known observations are accurate and that there is no misclose in the traverse. They are useful for determining bearings and distances between points on survey plans.
- The following cases of missing data have been dealt with:

  1) Bearing and distance of one line missing.

  2) Distances of two lines missing.

  3) Bearings of two lines missing.

  4) Bearing of one line and distance of another missing.

- The following methods have been used:

  1) The $\sum\Delta E$ (ΔN) of all traverse lines is equal to ΔE (ΔN) between the terminals.

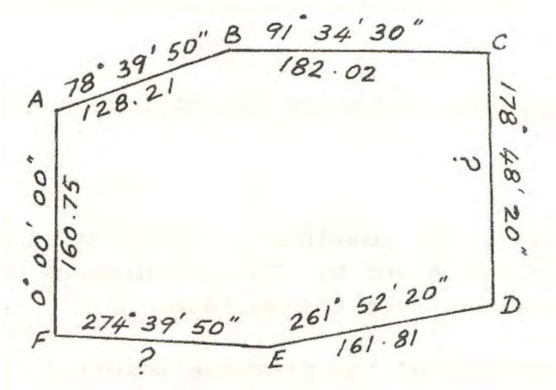
  2) Solution based on a triangle.

  3) Cardinal direction method.

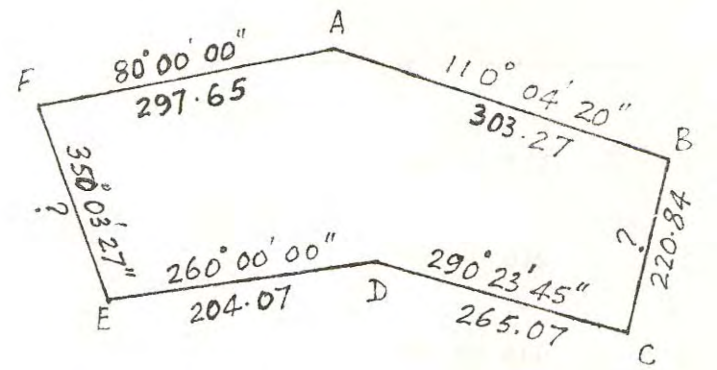
TUTORIAL: MISSING DATA PROBLEMS

The data for these questions is available in a separate spreadsheet file to save data entry time.

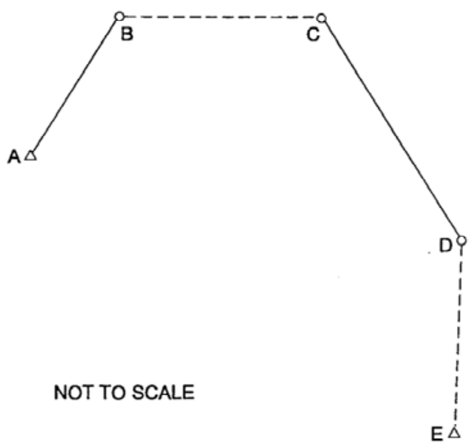
Q1. In the traverse shown below, the distances CD and EF are missing. Calculate these missing distances.



Q2. In the figure below, bearing BC and distance EF are missing. Calculate these missing elements.



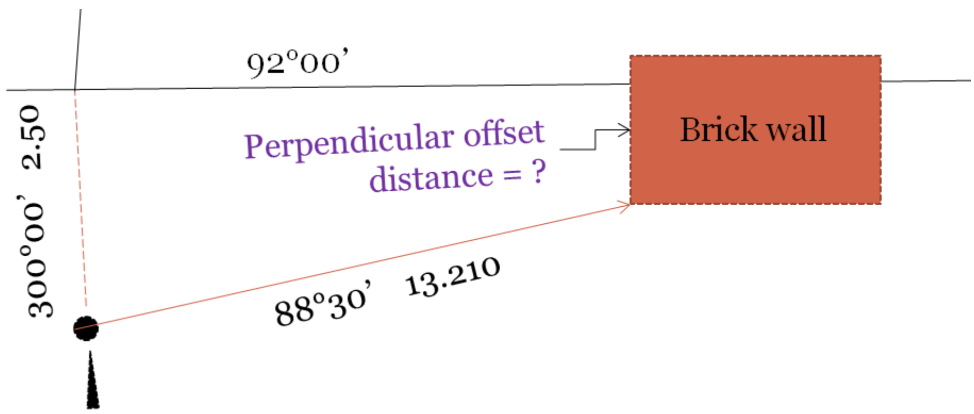
Q3. With the following data compute the missing bearings of BC and DE.



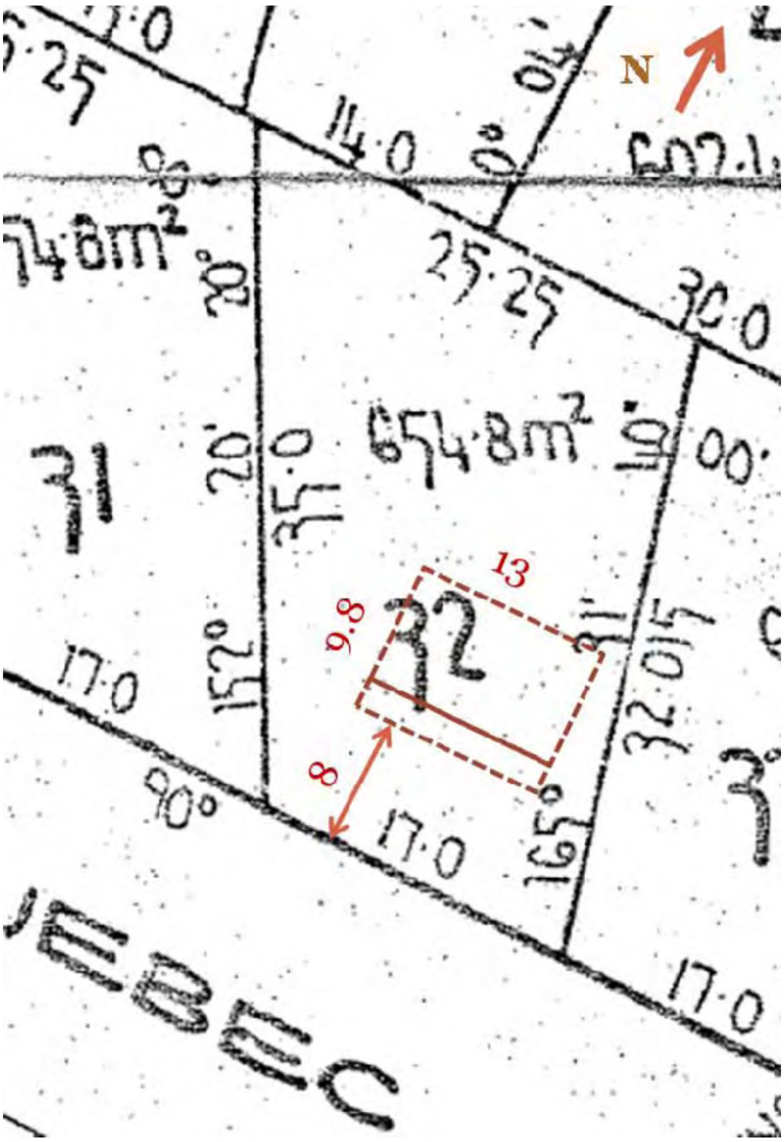
COORDS	E	N
A	1616.36	3948.21
E	1980.95	3836.99

LINE	BEARING	DISTANCE
AB	46°10'30"	161.34
BC	?	210.33
CD	131 36 50	88.34
DE	?	161.92

Q4. Calculate the offset of the brick wall from the boundary.



Q5. In the figure of a cadastral lot shown below, the bearing of the southern boundary of lot 32 is 90°00' and the bearing of the eastern boundary is 165°31'40". A rectangular house (13m wide and 9.8m deep) is to be built parallel to the boundary with Quebec St and as close as possible to both the road and the eastern boundary. The Local Council specifies houses must be no closer than 8m to the road and no closer than 1m to a side boundary. Calculate the "peg out" radiation from the SE corner of lot 32 to the SE corner of the house

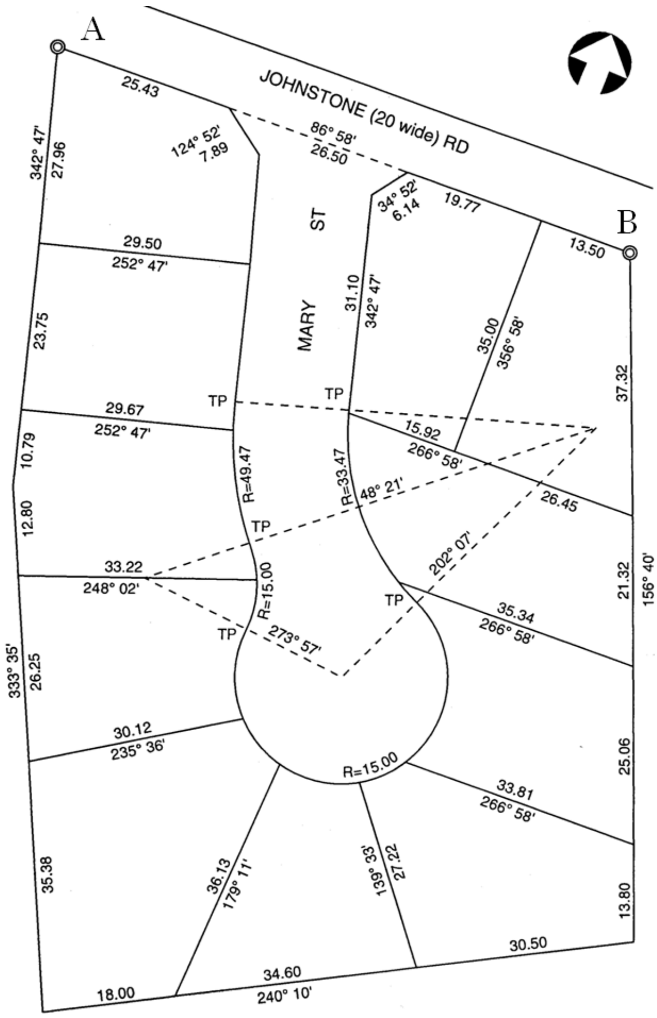


Q6. A determination of the distance between two points A and E is required, but it cannot be directly measured. Obtain the distance from the following notes of a traverse run from A to E:

Line	Bearing	Distance
AB	346 18	795.47
BC	73 57	194.40
CD	296 33	133.75
DE	18 21	385.45

Q7. Using the data in the subdivision plan shown below:

a) Calculate the bearing and distance from RM A to RM B, given references for A and B.
   A: RM DH&W 150°00' 2.23 B: RM GIP 165°30' 1.45
b) Calculate the misclose around the outer perimeter of this subdivision.
c) Calculate the bearings from the centre of the cul-de-sac circle to the lot corners that front that circle. Calculate chord distances of these “frontages”.



Original source: TAFE drafting exercise; redrawn by UNSW student.

3. Traverses & Loop Closes: Worked Solutions

These solutions can be read AFTER you have made some attempt to solve the question. Remember, you can check your answers by substituting them in the data sets and calculating the traverse miscloses.

Q1. Two missing distances.
Method 1, Sum ΔE and ΔN around loop

Line	Bearing	Distance	ΔE	ΔN
AB	78°39'50"	128.21	125.709	25.201
BC	91°34'30"	182.02	181.951	-5.003
CD	178°48'20"			
DE	261°52'20"	161.81	-160.185	-22.877
EF	274°39'50"			
FA	0°00'00"	160.75	0	160.750
		sum	147.475	158.072

Solve 2x2 equation by matrix algebra:

$\sin B_{CD}\ D_{CD} + \sin B_{EF}\ D_{EF} = -\text{sum}\Delta E$

$\cos B_{CD}\ D_{CD} + \cos B_{EF}\ D_{EF} = -\text{sum}\Delta N$

The coefficients are:

$0.021\ D_{CD} - 0.997\ D_{EF} = -147.475$

$-1.000\ D_{CD} + 0.081\ D_{EF} = -158.072$

So $D_{CD} = 170.430$ and $D_{EF} = 151.530$

Method 3
Rotate vectors so one missing line is cardinal. I chose to make line EF 270° so subtract 4° 39' 50" from each bearing. This will cause ΔN for EF is 0 regardless of the distance EF. The distance CD can then be found from the misclose in Northings.

	Bearing	Distance	ΔE	ΔN
Rotate -	4°39'50"			
AB	74°00'00"	128.21	123.243	35.339
BC	86°54'40"	182.02	181.756	9.808
CD	174°08'30"		17.396	
DE	257°12'30"	161.81	-157.794	-35.826
EF	270°00'00"			0.000
FA	-4°39'50"	160.75	-13.0706	160.218
		sum	151.530	169.540

$d_{CD} \cos(174°08'30") + 169.540 = 0$ So $d_{CD} = 170.4297$

Use this d_{CD} to calculate misclose in E and then $d_{EF} = -151.530/\sin(270)$ → $d_{EF} = 151.530$

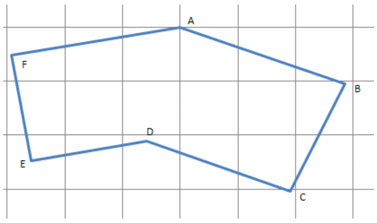
Check: Solution was done by two methods. Could also calculate loop misclose after entering distances.

Q2. Solution by rotation:
Rotate the figure until Line EF has bearing 0° so dis_{EF} is not part of sum of ΔE of new loop:

Swing = 360 – B_{EF} = + 9.94°

	New Bearing	Distance	ΔE
AB	120°00'53"	303.27	262.601
BC		220.84	
CD	300°20'18"	265.07	-228.771
DE	269°56'33"	204.07	-204.070
EF	0°00'00"		0.000
FA	89°56'33"	297.65	297.650
		sum	127.410

So to give a zero misclose for the loop the ΔE for line BC must = -127.410
ΔE = dis * sin brg so sin B_{BC} = -0.57693
B_{BC} = 324°45'54" OR 215°14'06"
Remove the swing to return to original bearings:
B_{BC} = 314°49'21" OR 205°17'33"



From the figure the likely answer is 205°17'33", otherwise the loop is completely different in appearance with CDE all north of FAB.

We can now proceed to find D_{EF} from the original bearings loop (we could also have done it in the rotated loop)

	Bearing	Distance	ΔE	ΔN
AB	110°04'20"	303.27	284.850	-104.084
BC	205°17'33"	220.84	-94.352	-199.670
CD	290°23'45"	265.07	-248.452	92.378
DE	260°00'00"	204.07	-200.970	-35.436
EF	350°03'27"			
FA	80°00'00"	297.65	293.128	51.686
		sum	34.204	-195.125

The bearing and distance of the misclose vector should match the line EF.
$D_{EF} = \sqrt{(misE^2+misN^2)}$ = 198.101
Bmisc = atan2(-misN,-misE) = 350°03'27" = B_{EF}

Check solution by calculating coordinates and misclose. Choose arbitrary coordinates for A, e.g. 300,500.

						E	N
	Bearing	Distance	ΔE	ΔN	A	300	500
AB	110°04'20"	303.27	284.850	-104.084	B	584.850	395.916
BC	205°17'33"	220.84	-94.352	-199.670	C	490.498	196.247
CD	290°23'45"	265.07	-248.452	92.378	D	242.046	288.625
DE	260°00'00"	204.07	-200.970	-35.436	E	41.076	253.188
EF	350°03'27"	198.101	-34.204	195.125	F	6.872	448.314
FA	80°00'00"	297.65	293.128	51.686	A	300.000	500.000

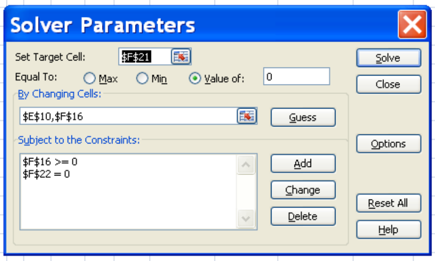
Solution by Excel Solver:
We could start this by entering the data and leaving the missing data (shaded yellow below) blank (ie E10 = 0 and F16 = 0). Try to use Excel Solver. Set up equations for misclose in E and N as follows.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Sample test														
2															
3															
4	**Solution by Solver:**														
5	DATA														
6	PT	d	m	s	brg dd	dist	E	N							
7	A						1000.000	3000.000							
8		110	4	20	110.072	303.27									
9	B											d	m	s	
10					205.293	220.84				#		205	17	33	
11	C														
12		290	23	45	290.396	265.07									
13	D														
14		260	0	0	260	204.07									
15	E														
16		350	3	27	350.058	198.101									
17	F														
18		80	0	0	80	297.65									
19	A						1000.000	3000.000							
20															
21	Eqn for E, 0=					-4E-07									
22	Eqn for N, 0 =					-8E-07									
23															

For Eastings, F21 is:
=G7+F8*SIN(RADIANS(E8))+F10*SIN(RADIANS(E10))+F12*SIN(RADIANS(E12))+F14*SIN(RADIANS(E14))+F16*SIN(RADIANS(E16))+F18*SIN(RADIANS(E18))-G19

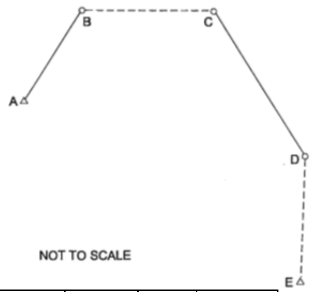
For Northings, F22 is:
=H7+F8*COS(RADIANS(E8))+F10*COS(RADIANS(E10))+F12*COS(RADIANS(E12))+F14*COS(RADIANS(E14))+F16*COS(RADIANS(E16))+F18*COS(RADIANS(E18))-H19

My first attempt at this problem in Excel's solver did not work, so add a constraint that the missing distance must be >= 0. That still did not work. Then enter approx values for the missing bearing and distance by looking at the figure, e.g. 200 degrees and 150 m. Run solver again and it comes up with sensible answers.



Q3.

A) Solution with solver: USE GOOD STARTING VALUES FOR BEARINGS From the figure I estimate Brg BC = 90° and Brg DE = 180°, or approx.: Brg BC = 1.6 and Brg DE = 3.1 radians as starting values. Then Solver gave the results shown below.



	E	N
A	1616.36	3948.21
E	1980.95	3836.99

			E	N						
LINE	Dist	BRG	1616.360	3948.210		answers				
AB	161.34	46°10'30.0"	1732.760	4059.931		radians	d.d	d	m	s
BC	210.33	?	1943.035	4055.115		1.5937	91.312	91	18	43.7
CD	88.34	131°36'50.0"	2009.081	3996.448						
DE	161.92	?	1980.950	3836.990		3.3162	190.005	190	00	18.2
		Solver target (≈0)→	-2.3E-10	3.3E-09						

Answer: β_{BC} = 91° 18' 43.7", β_{DE} = 190° 00' 18.2"

B) Solution by moving the vectors: Swap vector BC with CD, so traverse becomes

A			1616.36	3948.21
AB	46°10'30.0"	161.34		
B			1732.760	4059.931
BC'	131°36'50.0"	88.34		
C'			1798.806	4001.264

Then intersection by distances

from C' (=1798.806, 4001.264) dist C'P = 210.33
from E (=1980.95, 3836.99) dist EP = 161.92

Answer is (2009.081, 3996.448) and the bearings of the two lines are 91°18'43.7" and 190°00'18.2"

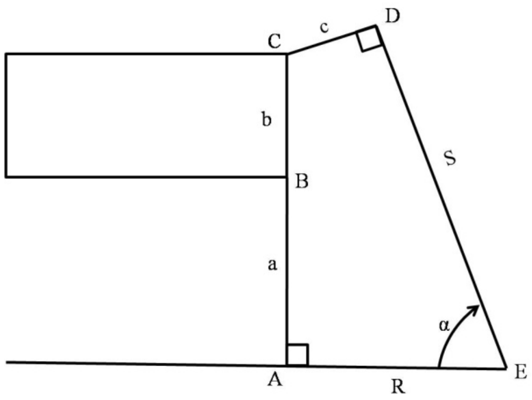
Q4. Answer = 0.367

Label the points starting at RM = 1, going clockwise

Data	E	N	Brg DD	Dist
1	200	500		
1 to 2			300	2.500
2 to 3			92	
3 to 4			182	
1 to 4			88.5	13.210
Method 1				
RM square offset to boundary:				1.174
RM line parallel to bdy, offset to wall:				0.806
		wall from bdy:		0.367
Method 2				
Join 2 to 4 , then simple triangle				
2	197.835	501.250		
4	213.205	500.346		
2 to 4			93.37	15.397
sine rule, wall offset:				0.367

	A	B	C	D	E
2		E	N	Brg DD	Dist
3	1	200	500		
4	1 2			300	2.5
5	2 3			92	
6	3 4			182	
7	1 4			88.5	13.21
9	**Method 1**				
10	RM square offset to boundary:				=E4*COS(RADIANS(362-D4))
11	RM line parallel to bdy, offset to wall:				=E7*SIN(RADIANS(D5-D7))
13			wall from bdy:		=E10-E11
15	**Method 2**				
16	Join 2 to 4 , then simple triangle				
17	2	=B3+E4*SIN(RADIANS(D4))	=C3+E4*COS(RADIANS(D4))		
18	4	=B3+E7*SIN(RADIANS(D7))	=C3+E7*COS(RADIANS(D7))		
19	2 4			=MOD(DEGREES(ATAN2(C18-C17,B18-B17)),360)	=SQRT((B18-B17)^2+(C18-C17)^2)
20	sine rule, wall offset:				=(E19*SIN(RADIANS(D19-D5)))/SIN(RADIANS(90))

Q5. Method 1. *Derivation of equations for Cadastral peg-out*
Label the dimensions as shown in figure below. If α is > 90° then we are more interested in the distance from the side boundary to B than to C. If α = 90° the problem is simple, S = a+b and R = c.



Rotate the figure (swing the bearings of the lines) until the line AC has a bearing (β_{AC}) of 0° and the road has a bearing (β_{AE}) of 90° (lines AC and AE are perpendicular to establish shortest distances). Then the bearing of line DE will be = 90° + α and the bearing of the line DC will be 90° + α + 90° = 180° + α. The bearing of the line CD will thus be α in this rotated system.

Write an equation for close of Northings around the loop. Since β_{AE} now = 90° (β_{EA} = 270°) the distance R will not be in the equation. Thus the sum of Northings starting at A and going clockwise:

(a+b) cos0° + c cosα + Scos(90°+α) + R cos270°= 0 [cos0° = cos270°= 1, cos(90°+α) = -sinα]

→ (a+b) + c cosα - Ssinα = 0

→ $$S = \frac{(a+b)+c\cos\alpha}{\sin\alpha}$$

Similar to the above approach, we now rotate the figure until line DE has a bearing of 180° so that the sum of Eastings won't include S. Then the new bearing of line EA will be β_{EA} =360°-α, which is the same as - α and the new bearing of line CD, β_{CD} = 90°. With some study of the figure you will see that new β_{AC} = 90°-α.

Write an equation for the loop close in Eastings starting at E and going clockwise:

Rsin(-α) + (a+b) sin(90°- α) + c sin(90°) = 0 [sin(-α) = -sinα, sin(90°-α) = cosα, and sin90° = 1]

→ -Rsinα + (a+b)cosα + c = 0

→ $$R = \frac{(a+b)\cos\alpha + c}{\sin\alpha}$$

For the special case when α = 90°, substitution in the above equations yields: S = a+b and R = c, which is the correct result.

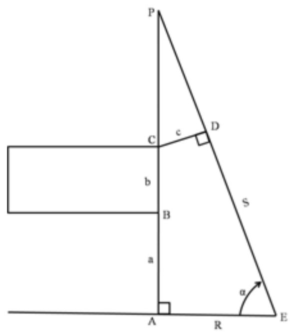
For the example data, angle α = 165°31'40" - 90° = 75°31'40", street offset a = 8, house depth b = 9.8 and side boundary clearance c = 1.

$S = \frac{(a+b)+c\cos\alpha}{\sin\alpha}$ = 18.641 and $R = \frac{(a+b)\cos\alpha + c}{\sin\alpha}$ = 5.627

After R is known it is a simple right angle triangle with R and a to determine the bearing and distance from the SE corner of lot 32 to the SE corner of the house (EB): 324°52'43" and 9.781m

A check of the answers is to use the bearings and distances of each line and calculate the misclose around the loop ABCDEA.

Another way to derive the equations, when α is less than 90°, is to extend the line AC until it intersects an extension of the line DE at P, as shown in figure →
There is then a right angle triangle CDP with angle PCD = α, and 90° at D.
So cosα = c/CP and sin α = PD/CP. Then CP = c/cosα and PD = c tanα
In the right angle triangle EAP the angle at E is α and the angle at A is 90°.
The three sides of this triangle EAP are a + b + c/cosα , S + c tanα and R.



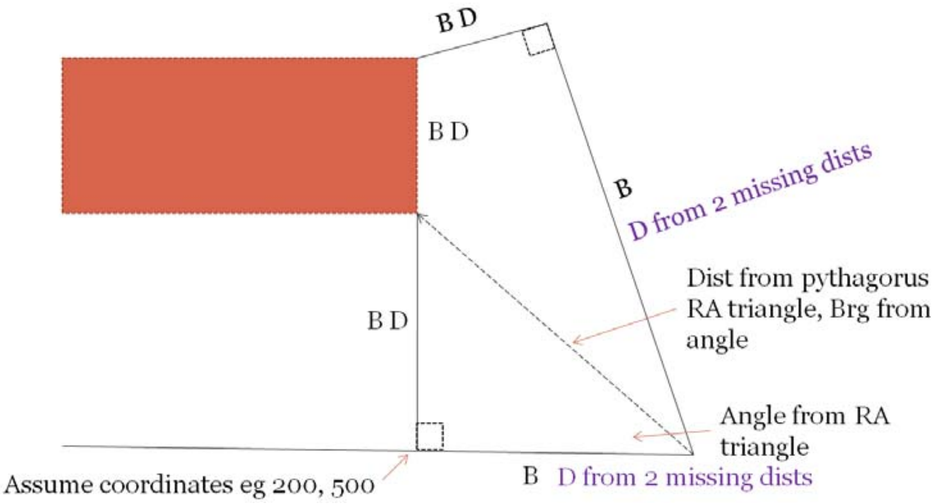
So, sinα = (a + b + c/cosα) / (S + c tanα) → S = ((a + b + c/cosα) / sinα) - c tanα

Algebraic rearrangement of terms and knowing that $\sin^2\alpha + \cos^2\alpha = 1$ leads to $S = \frac{(a+b)+c\cos\alpha}{\sin\alpha}$ as before.

Also from the triangle EAP: tanα = (a+b + c/cosα) / R

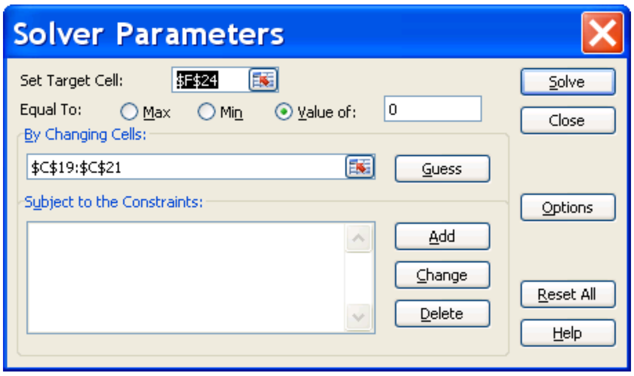
So, R = (a + b + c/cosα) / tanα $= \frac{(a + b)\cos\alpha + c}{\sin\alpha}$ as before.

Method 2 Solver



To use Solver (if it is add in to your Excel) set up the table below using the usual traversing calculations around a loop. The unknown values C19 and C21 start as 0. Cell F24 is the distance between the original coordinates of A and the values calculated by using the data around the loop. F24 =SQRT((D22-D23)^2+(E22-E23)^2). If you start with C19 and C21 as 0, then F24 will be about 18.1. Select Solver with settings as shown below and it will calculate the adjusted distances of 18.641 and 5.627. They agree with those from method 1 above.

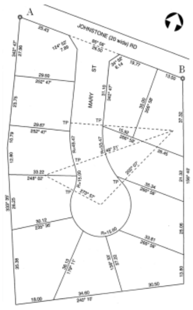
	A	B	C	D	E	F	G	H	I
11		Brg	Dist	E	N	brg rads			
12	A			200	500				
13	AB	0	8			0			
14	B			200	508				
15	BC	0	9.8			0			
16	C			200	517.8		d	m	s
17	CD	75.53	1			1.32	75	31	40
18	D			200.968	518.050				
19	DE	165.53	18.641			2.89	165	31	40
20	E		0	205.627	500.000				
21	EA	270	5.627			4.71			
22	A			200.000	500.000				
23	org A			200	500				
24	Diff			0.000	0.000	0.000			
25									
26	Brg & Dist								
27	EB	324.88	9.781			324.88	324	52	43
28	angle from main road			54.88					



Answer: Radiation EB = 324°52'43" and 9.781 m

Q6. This is simply a matter of calculating the misclose of the loop A B C D E A. One method is to assign arbitrary coordinates for A and then calculate the coordinates of E. From the coordinates of A and E calculate the distance AE.

Pt / Line	Bearing d	m	radians	Distance	E	N
A					1000	5000
AB	346	18	6.04	795.47		
B					811.602	5772.838
BC	73	57	1.29	194.4		
C					998.425	5826.585
CD	296	33	5.18	133.75		
D					878.779	5886.369
DE	18	21	0.32	385.45		
E					1000.127	6252.219
Ans				Dist AE	1252.219	



Q7. a) Bearing and distance from RM A to RM B. Treat this as an unclosed loop or traverse of vectors:
RMA to A 150°00', 2.23m → Δe, Δn = 1.115, -1.931.
A to B 86°58', (25.43+26.5+19.77+13.5 = 85.2) → Δe, Δn = 85.081, 4.509
B to RMB 345°30', 1.45 (i.e. reverse bearing so +180°) → Δe, Δn = -0.363, 1.404
Sum Δe, Δn = 85.833, 3.981
Brg & dist of join = 87° 20' 39.8", 85.925m

b) Misclose around the outer perimeter of this subdivision.
A to B 86°58', 85.2 (=25.43+26.5 +19.77 + 13.5) → Δe, Δn = 85.081, 4.509
B to C 156°40', 97.5 (=37.32+21.32+25.06+13.8) → Δe, Δn = 38.618, -89.526
C to D 240°10', 83.1 (=18+34.6+30.5) → Δe, Δn = -72.087, -41.340
D to E 333°35', 74.43 (=35.38+26.25+12.8) → Δe, Δn = -33.114, 66.658
E to A 342°47', 62.5 (=10.79+23.75+27.96) → Δe, Δn = -18.499, 59.700
Sum Δe, Δn = misclose in E and N = -0.002, 0.000
Brg & dist of misclose = 258°41'59.1", 0.002m

c) Bearings from the centre of the cul-de-sac circle to the lot corners that front that circle.
You are welcome to send me your solutions for this. Save me typing it up ☺
d) Arc and chord distances of these "frontages".
You are welcome to send me your solutions for this. Save me typing it up ☺

> **Note:** Just reading the solutions might give you a false confidence. They might look easy enough to follow, but if you were in an examination or doing a real survey project outside university would you be able to do similar problems correctly? You should think how to solve each problem. You should actually do the calculations – not just copy the steps shown above.

4. TRAVERSE CALCULATIONS & ADJUSTMENTS

This chapter continues traverse calculations with revision of material students have already learnt in a first year course and that is covered in many standard Surveying textbooks. It deals with calculation and adjustment of measured values. It includes material on using and interpreting miscloses and error detection. There are four parts to this chapter. Firstly, how to calculate traverse miscloses which, with modern equipment, should be small. That is revision. Secondly, how to interpret large miscloses and find errors that may have caused a large misclose. Thirdly, revision of Bowditch and similar traverse adjustment methods is covered. Finally, some related topics including swinging the traverse back onto azimuth of a line, and what to do with multiple reference targets (ROs) at traverse terminals.

Observations

Two methods of measuring the angles, directions and bearings of a traverse are:

1) Included Angle Method: Angles or rounds of directions are measured at each station and the bearings subsequently calculated from the measurements and the given bearing of the first line. The angle included by the traverse lines at each station is measured by the usual method for measuring horizontal directions with added refinements (such as repeating measurements with faces and zeros changed). If the survey is a high quality control network with more lines from each point than the two in a traverse then the rounds of directions method may be preferred.

2) Direct Bearing Method: The setting of the theodolite is so arranged to give direct readings of bearings. This method is suitable for short traverses where great precision is not required and the traverse is either a loop traverse or ends on a line with known bearing. This method is often called carrying the bearings and is useful for finding existing marks in some types of surveys. A related method with modern total stations is to carry the coordinates. Note that the bearings or coordinates carried in this mode are provisional and may change slightly when the survey is completed and adjusted. When carrying bearings remember to include the effects of differences between FL and FR observations.

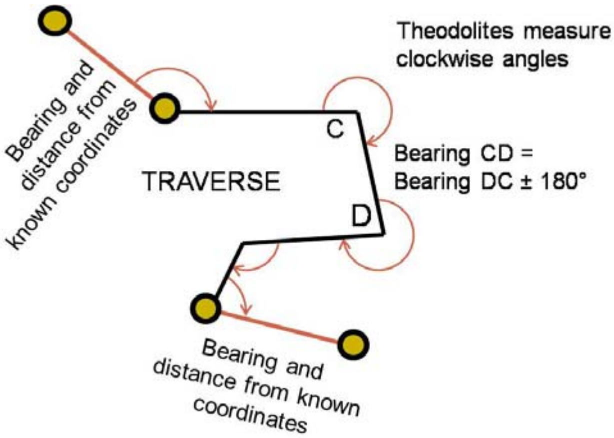
In the past accurately standardised invar or steel tapes were used for distance measurements. Now EDM is usually used. Several corrections need to be applied to traverse distance measurements. These are covered in other courses, but include corrections for: Temperature, Atmospheric pressure, (Tension & Sag for tapes), Slope, Sea-level (or height above ellipsoid). Also if map projection coordinates are used then the distances need to have a scale factor applied with them that depends mainly on the Easting coordinates of the line.

Calculations

In a traverse we need to calculate the coordinates of each station from observed angles and distances, e.g. closed-line traverse:

To calculate bearings from traverse angles:

- add angles to known bearings
- can add or subtract 360° to/from any bearing
- to reverse bearing + or - 180°



Computational steps:

1) Apply corrections to the linear (distance) measurements (e.g. temperature & pressure for EDM, possibly map grid corrections too to fit with coordinates on a map projection).

2) Compute the angular misclose of the traverse and the adjusted bearing of each line.

3) Calculate the coordinates of the stations, and the linear misclose (in coordinates) of the traverse.

4) If misclose is small enough then adjust the traverse to obtain the final coordinates. If misclose is not small enough then investigate to find the error.

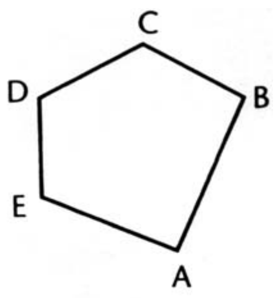
TRAVERSE MISCLOSE CALCULATION

ANGULAR MISCLOSE

Loop traverse with n points

If no errors in angles then:
Σ interior angles = (n-2) $\cdot$180°
Σ outer angles = (n+2) $\cdot$180°

Misclose = Σ internal angles - (n-2) $\cdot$180°

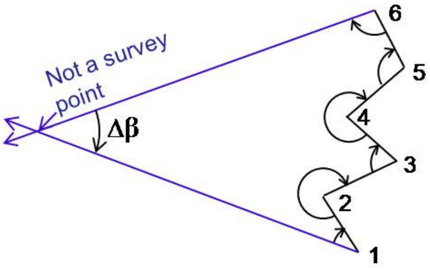


Closed-line traverse

Σ angles + $\Delta\beta$ = (n-2) $\cdot$180° (in this figure: n = 7 !)

where $\Delta\beta$ = starting bearing – closing bearing

Misclose = Σ angles + $\Delta\beta$ - (n-2) $\cdot$180°

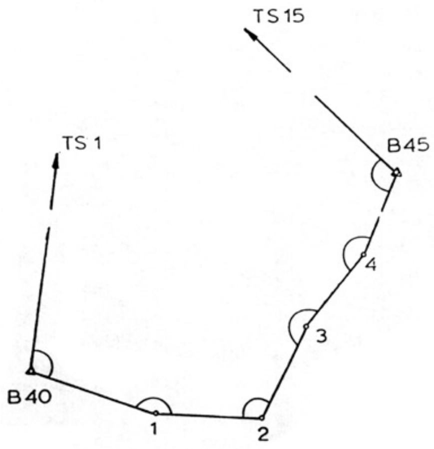


DISTRIBUTION of ANGULAR MISCLOSE

- If the angular misclose is acceptably small then it is usually distributed evenly to all observed angles.
- Correction to each angle: $\alpha_c = -\frac{\text{angular misclose}}{n}$ where n = number of observed angles
- The bearings are then calculated using the adjusted angles: $\beta_{\text{forward}} = \beta_{back} + adj\ angle \pm 180°$
- Alternatively, the bearings of lines calculated from unadjusted angles, can be corrected sequentially. i.e. first bearing gets + α, second bearing gets + 2* α, third bearing gets + 3* α, etc

Example 1: TRAVERSE with measured angles

A traverse was observed from B40 to B45 (which have known coordinates) as shown in the figure below. The angles were derived from mean of FL and FR directions and reduced to zero on the backsight. The distances have been corrected for slope, atmosphere etc and are the mean of forward and backward observations.



Observations:

	Angle	From - To	Dist m
B40	94° 10’ 00”	B40 – 1	103.402
1	178 19 00	1 – 2	157.225
2	118 21 45	2 – 3	143.369
3	194 42 25	3 – 4	169.087
4	158 07 30	4 – B40	176.743
B45	89 03 55		

Given data:

Coordinates (E, N)	Bearings from coordinates
B40: 406.347, 423.509	B40 – TS1: 10° 12’ 35”
B45: 992.415, 713.229	B45 – TS15: 302° 57’ 43”

Step 1 – Angular misclose and adjusted bearings

This step can be done by:

(A) summing the angles, distributing a portion of the misclose as a correction to each angle, then calculating the adjusted bearings, or

(B) calculating bearings of each line, then the misclose, then distribute a sequentially increasing portion of the misclose to the bearings.

We show both methods here and assume the misclose is acceptably small. If the misclose is large then the angles and bearings should not be adjusted and the error should be found. More details on how to do that later.

(A) Adjusting Angles:

- $\Delta\beta$ (starting brg – closing brg) = 67° 14’ 52” Note: bearings <u>from</u> terminal <u>to</u> target control station.
- For a loop traverse, $\Delta\beta$ is zero.
- Angular misclose $m_\alpha = \Sigma$ angles - (n-2) ·180° with n = number of angles.
  = 832° 44’ 35” + 67° 14’ 52” – (7-2) * 180° here n=7 (6 traverse stations + $\Delta\beta$)
  = 899° 59’ 27” - 900° → m_α = -33”
- Correction to angles = $-m_\alpha$ / n = +5.5”
- Bearings from corrected angles: $\beta_i = \beta_{i-1} + \alpha_i \pm 180°$

Pt	Angle	Cor	Adj Angle	Adj Brg
TS1				
				190°12’35” Known
B40	94°10’00”	5.5”	94°10’05.5”	
				104 22 41
1	178 19 00	5.5	178 19 05.5	
				102 41 46

2	118 21 45	5.5	118 21 50.5		
				41 03 37	
3	194 42 25	5.5	194 42 30.5		
				55 46 07	
4	158 07 30	5.5	158 07 35.5		
				33 53 42	
B45	89 03 55	5.5	89 04 00.5		
				302 57 43	
TS15					
	Misc -33”	33.0		**302 57 43**	Known

- Check: last bearing calculated = given bearing

(B) Adjusting Bearings Method:

Bearings are calculated from unadjusted angles. Then each bearing is corrected by a sequentially increasing component of the misclose.

Correction to i^{th} bearing $= -\frac{i \cdot \text{ang. misclose}}{n} = i \cdot \frac{33"}{6}$

n = number of bearings to be corrected = number of observed angles.

So in this example the first bearing is increased by 5.5”, then next bearing by 2*5.5 = 11”, then next by 3*5.5, etc

Pt	Angle	Brg	Cor	Adj Brg
TS1				
				190°12’35” Known
B40	94°10’00”			
		104°22’35”	5.5”	104°22’41”
1	178 19 00			
		102 41 35	11.0	102 41 46
2	118 21 45			
		41 03 20	16.5	41 03 36
3	194 42 25			
		55 45 45	22.0	55 46 07
4	158 07 30			
		33 53 15	27.5	33 53 42
B45	89 03 55			
		302 57 10	33.0	302 57 43
TS15				
		Misc -33”		**302 57 43** Known

COMPUTATION of COORDINATES

- The coordinates of the traverse stations are calculated from the adjusted bearings and distances (P→R):

  $\Delta E = d \sin\beta$ and $\Delta N = d \cos\beta$

- The sum of the ΔE and ΔN is compared to the given values obtained from the known coordinates of the start and end control station:
  misclose in ΔE: $m_E = \Sigma\Delta E$ – given ΔE
  misclose in ΔN: $m_N = \Sigma\Delta N$ – given ΔN

- Or, calculate coordinates along the traverse and compare last station coordinates with given values:
  misclose in ΔE: m_E = Calc E – given E (of last point)

misclose in ΔN: m_N = Calc N – given N (of last point)

- The linear misclose is calculated from: $m = \sqrt{m_E^2 + m_N^2}$

- The proportional misclose is obtained by: $m_p = 1 : (D / m)$

where D = total traverse length, by simply summing the observed distances for each line.

BLUNDER DETECTION in TRAVERSES

Remember: Preventing errors is much better than trying to find and correct errors later!

During a traverse gross errors can occur in the measurement of distances and/or angles. These errors may be due to a booking mistake, e.g.

- Transferring the wrong number from the display onto the field form.
- The booker mishearing the value that the observer is calling out.
- Swapping numbers (e.g. book 68.123 instead of 86.123).

Other error sources include errors in centring and levelling of the instrument, observing to a target other than the intended target (or using wrong target ID), setting scale factors, additive constants and atmosphere corrections incorrectly. There are even more possible error sources. Some methods to prevent or detect measurement errors as they occur are taught in other surveying courses. Methods include using instruments that allow direct download of measurements from instruments to computers, or doing calculations within the instrument. Another method is to check measurement consistency of the individual readings, e.g. comparison of two faces or two arcs of an angle, or comparison of forward and backward distance measurement of each traverse line.

A large misclose indicates that a gross error occurred in the traverse. How do we discover where the gross error occurred if we have one distance error or one angle error? Hopefully, you haven't made more than one error! Large errors are usually easier to find than small errors.

What is a large misclose? That depends on the equipment and procedures used. Generally any misclose that is significantly larger than the specified or required value or, by experience significantly larger than the values obtained in similar previous traverses. Specified or required values for traverses can be set by clients or for example by state legislation for cadastral surveys or national standards for control surveys. The standards for surveys governed by the NSW **Surveying Regulation 2006** (www.legislation.nsw.gov.au) are:

- Angular misclose < 20" +10√n or 2'
- Linear misclose < 15mm + 100ppm
- Measure lengths to 10mm + 15ppm or better at a 95% confidence interval.

In the past it was customary in NSW to apply tests the misclose ratio. The linear misclose, was not to exceed a specified value depending on the type of survey, e.g. Misclose < S / 8000, S = traverse length [km]. For a cadastral traverse in NSW the required accuracy used to be better than 1/8000, for example. For further reading on Traverse Design and Specifications such as mark types, accuracy etc see: www.lands.nsw.gov.au/publications/guidelines/surveyor_generals_directions (SG Directions 1, 2 and 11).

In Australia the Standards and Practices for Control Surveys, frequently referred to as Special Publication 1 (SP1), is produced by the Intergovernmental Committee on Surveying and Mapping. It is available for free download at http://www.icsm.gov.au/icsm/publications/sp1/sp1.html

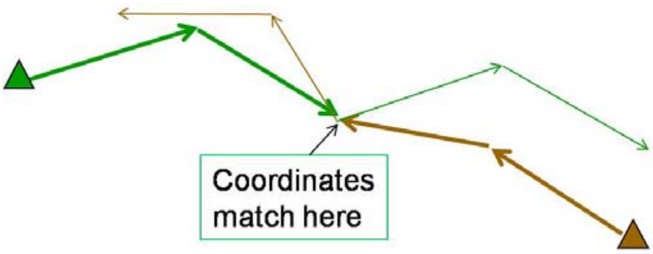
Student traverses at UNSW usually have miscloses less than 20" and 15mm.

ANGULAR ERROR

A large error in the measurement of an angle will cause a large angular misclose in a traverse.

Method 1

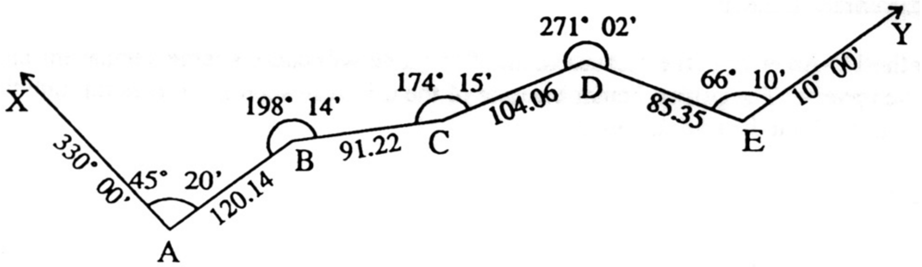
Compute the traverse coordinates from both ends of the traverse without adjusting the angular misclose. The station at which the coordinates from both computations agree closely (usually within a few cm) is the station at which the gross angular error occurred. This is because the erroneous angle has not been used to determine coordinates of points up to and including the point with the angle error. This method can be applied to closed line traverses or loop traverses. An angular error can also occur at the start or end point of the traverse, this method can detect that too.



A spreadsheet for traverse calculations can be modified to do these calculations. Alternatively, a surveying CAD program can do the calculations and provide a plan for visual identification of the likely error point. In the CAD program enter the control points by their coordinates, then enter two traverses one from each end.

Example:

Find the station at which a gross angular error occurred in the following traverse. The coordinates of control points A and E are known, A is (100.00, 100.00) and E is (309.50, 331.80). Figure below is not to scale.

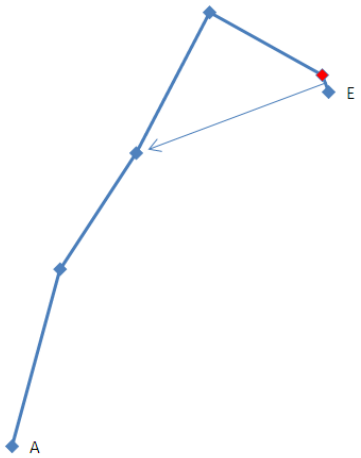


Solution:

Forward Computation | Backward Computation

Station	Bearing	Distance	Easting	Northing		*Station*	Bearing	Distance	Easting	Northing
A			**100.00**	**100.00**		E			**309.50**	**331.80**
	15° 20'	120.14					303° 50'	85.35		
B			131.77	215.86		D			238.60	379.32
	33° 34'	91.22					212° 48'	104.06		
C			182.21	291.87		C			182.23	291.85
	27° 49'	104.06					218° 33'	91.22		
D			230.76	383.91		B			125.39	220.51
	118° 51'	85.35					200° 19'	120.14		
E			305.52	342.72		A			83.67	107.85
	5° 01'						334° 59'			
Y						X				

The angular misclose is about 5°. If only one angle was wrong then the angular error occurred at C because the two directions of computation yield differences of about 2cm in coordinates for C. The coordinates of the other points differ by 100s of metres. The angle at C should have been about 179°15'.



Method 2

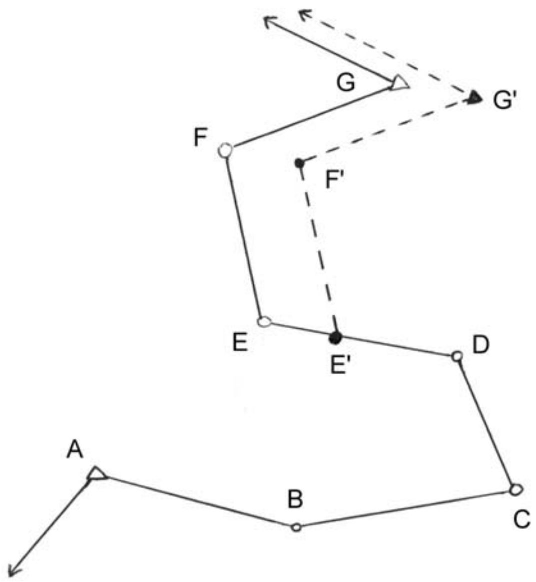
The perpendicular bisector of the misclose vector will pass through the station at which the error occurred. This method is not reliable if the misclose is small.
Computational steps:

1) Calculate the coordinates of each traverse station.
2) Determine the bearing of the misclose vector.
3) Calculate the mid-point M of the misclose.
4) Calculate the bearings from M to all traverse stations.
5) The bearing that is approx perpendicular to the misclose vector indicates at which station the error occurred.

DISTANCE ERROR

I suggest you look for angular errors before distance errors, because angle errors are indicated by the angle misclose and distances are not involved. If the angular misclose is small then to detect distance errors we use the coordinate miscloses which depend on angles / bearings as well as distances.

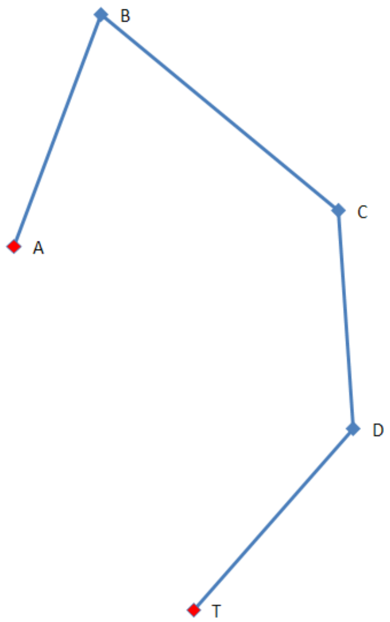
An error in the distance measurement of a traverse line will cause a larger linear misclose. The bearing of the misclose will be approximately the same as the bearing of the line containing the error. This enables the erroneous line to be identified. This method can be applied to closed line traverses or loop traverses. In the figure below, an error in the distance DE will create an error in the coordinates of G the bearing of the misclose G'G will be close to the bearing of the line DE if the distance error is large.



Example:

The bearing and distance of each traverse line has been measured. The coordinates of the control points on either end (A and T) are known.

	East	North
A	1043.216	2117.910
T	1112.492	1972.858



Line	Bearing	Distance
AB	20° 01' 30"	98.231
BC	130° 20' 40"	120.556
CD	176° 15' 50"	78.141
DT	220° 33' 40"	95.213

Computational steps:

1) Calculate the coordinates of each traverse station.
2) A large linear misclose (between the calculated and given coordinates of T) indicates a gross error.
3) Calculate the bearing and distance of the misclose (i.e. the line T'T).

4) The bearing of the misclose will identify the erroneous line and the distance will indicate the magnitude of the error.

Solution:

T' (calculated):	1111.916	1981.846
T (given):	1112.492	1972.858

→ line T'T: $\Delta E = 0.576m$ $\Delta N = -8.988m$ → $\beta = \tan^{-1}(\Delta E/\Delta N) = 176°19'59"$ $d = 9.006m$

This indicates that the error is probably in line CD that was β = 176°15'50", d = 78.141m.
The length of this line should probably be d = 87.141m.

Other blunders

Error in control point coordinates:

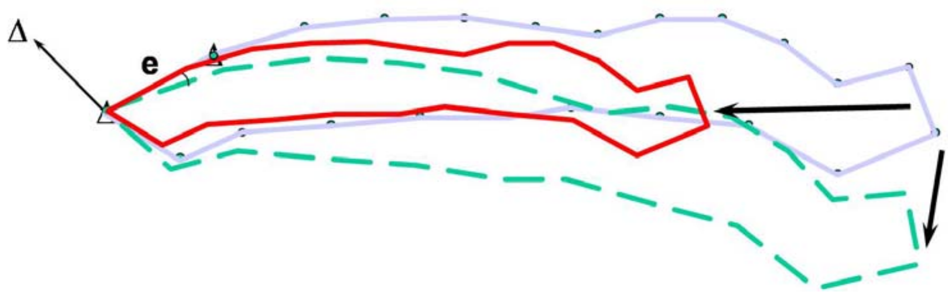
A large misclose could perhaps be caused by an error in one or more of the coordinates of the control points. In this case the bearing of the misclose will be close to 0/180° or 90/270°, if only one coordinate is wrong. If both E and N are incorrect (for example using the wrong survey mark) then the misclose could be large and the bearing may not be parallel to any of the traverse lines.

Scale error in distances:

There could be a scale error in your distances due to calculation errors or incorrect instrument calibration. For example, a map scale factor is wrong, or not applied or applied in opposite direction, so that lines are longer when they should have been shorter. A scale error in distances in a traverse between control points will cause a misclose.

Loop traverses:

Long loop traverses are risky because errors propagate and there is no warning about, or check for, scale errors or azimuth control in the miscloses.



DISTRIBUTION of MISCLOSE

The distribution of angular misclose has been discussed earlier. **IF** the misclose in E and N is small then it can be adjusted (i.e. distributed) using the Bowditch method or Transit or other methods. The Bowditch and Transit methods are described below and in many surveying textbooks. The adjusted ΔE and ΔN are then used to obtain the final coordinates of the traverse stations.

Bowditch's method causes the already adjusted bearings to be altered to a much greater extent than does the Transit method (particularly for N-S and E-W lines). The Transit method alters distances more than Bowditch. The Bowditch method has traditionally been more popular in NSW. A Least Squares adjustment is a much better way to adjust a traverse, but is taught in a following course not in this course.

Bowditch method

This method assumes errors in distance and angle cause equal displacement of a point.
The values of the adjustment are directly proportional to the length of the traverse lines.

$$\text{correction to } \Delta E = -\frac{\text{total error in } \Delta E}{\text{total length of traverse}} \cdot \text{length of side} = -\frac{m_E}{D} d_i$$

$$\text{correction to } \Delta N = -\frac{\text{total error in } \Delta N}{\text{total length of traverse}} \cdot \text{length of side} = -\frac{m_N}{D} d_i \qquad \text{where } D = \Sigma d_i$$

Transit method

The values of the adjustment are proportional to the values of ΔE and ΔN for each traverse line. The lengths of the traverse lines are not included in the calculations.

$$\text{correction to } \Delta E = -\frac{\text{total error in } \Delta E}{\text{sum of the } |\Delta E|} \cdot |\Delta E| \text{ of } i^{th} \text{ line} = -\frac{m_E}{\Sigma|\Delta E|}|\Delta E_i|$$

$$\text{correction to } \Delta N = -\frac{\text{total error in } \Delta N}{\text{sum of the } |\Delta N|} \cdot |\Delta N| \text{ of } i^{th} \text{ line} = -\frac{m_N}{\Sigma|\Delta N|}|\Delta N_i|$$

Example traverse 1 – Linear misclose and adjusted coordinates

Here we continue with the example traverse given near the start of this chapter, from B40 to B45.
Use the horizontal distances and adjusted bearings and to calculate coordinate differences.
Either use a calculator's P→R or computer's: $\Delta E = d \sin\beta$ $\quad \Delta N = d \cos\beta$

Calculate misclose in Easting and Northing: $m_E = \Sigma\Delta E - \text{given } \Delta E$ $\quad m_N = \Sigma\Delta N - \text{given } \Delta N$

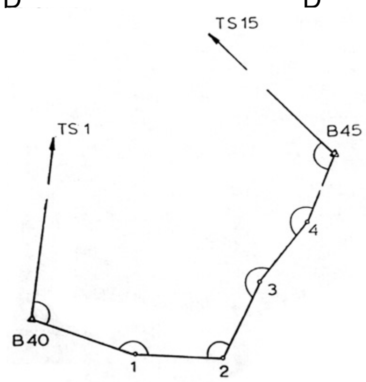
Use Bowditch's rule to correct ΔE and ΔN: $\text{corr } \Delta E = -\frac{m_E}{D} d_i$ $\quad \text{corr } \Delta N = -\frac{m_N}{D} d_i$

Check: $\Sigma \text{ corr } \Delta E = -m_E$ and $\Sigma \text{ corr } \Delta N = -m_N$

Check: last coordinates calculated = given coordinates

Linear misclose: $m = \sqrt{m_E^2 + m_N^2}$

Proportional misclose $m_p = 1 : N$ where $N = \frac{D}{m}$



Pt	Adj Brg			Dist	DE	cor E	Easting	DN	cor N	Northing
B40	°	‘	“				**406.347**			**423.509**
	104	22	41	103.402	100.163	-0.001		-25.676	0.003	
1							506.509			397.836
	102	41	46	157.225	153.381	-0.002		-34.555	0.005	
2							659.888			363.286
	41	03	36	143.369	94.172	-0.002		108.103	0.004	
3							754.058			471.393
	55	46	07	169.087	139.796	-0.002		95.118	0.005	
4							893.852			566.516
	33	53	42	176.743	98.565	-0.002		146.707	0.006	
B45							992.415			713.229
						Given:	**992.415**		Given:	**713.229**
			Σ	749.826	586.078			289.697		
				Known:	586.068			289.720		
					0.010	-0.010		-0.023	0.023	
				Linear Misclose:		0.025		Ratio 1:	29643	

Table. Bowditch traverse adjustment example. Distances and coordinates are in metres.

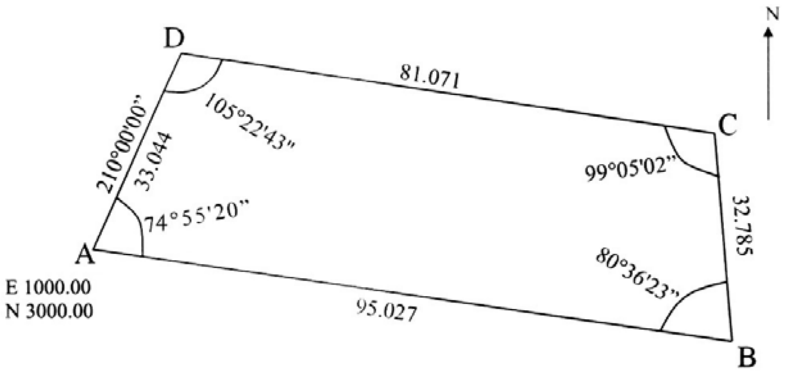
Example 2: Loop Traverse

A loop traverse ABCD was observed with the following data. Calculate the coordinates (easting and northing) of the points B, C, and D, of the traverse from the following data:

	Angle	Line	Distance (m)
A	74° 55’ 20”	AB	95.027
B	80° 36’ 23”	BC	32.785
C	99° 05’ 02”	CD	81.071
D	105° 22’ 43”	DA	33.044

Given coordinates of A: E 1000.00, N 3000.00 Given bearing of line D to A: 210°00’00”.

1) Visualise the problem:



Calculator Calculations

2) Calculate the angular misclose: $m_\alpha = \Sigma$ angles $+ \Delta\beta - (n-2)\cdot 180°$
with n = number of angles (incl. $\Delta\beta$) For loop traverse $\Delta\beta = 0$.

3) Correct each angle for the angular misclose: $corr = -\frac{m_\alpha}{n}$ Check: Σ corr = - angular

misclose

4) Calculate bearings from corrected angles: $\beta_i = \beta_{i-1} + \alpha \pm 180^\circ$ Check: last bearing calculated = given bearing

5) Calculate ΔE, ΔN for each line using β_i and d_i: ΔE = d sinβ and ΔN = d cosβ (or P→R)

6) Calculate misclose in ΔE, ΔN: For loop traverse: $m_E = \Sigma\Delta E$, $m_N = \Sigma\Delta N$

7) Bowditch adjustment to correct ΔE_i and ΔN_i: corr ΔEi $= -\frac{m_E}{D} d_i$ and corr ΔNi $= -\frac{m_N}{D} d_i$

Check: Σ corr ΔE = $-m_E$ and Σ corr ΔN = $-m_N$

8) Calculate coordinates of traverse stations using known coordinates of first station and corrected ΔE & ΔN of each line. Check: last coords calculated = given coords

9) Calculate linear misclose: $m = \sqrt{m_E^2 + m_N^2}$

10) Calculate proportional misclose: m_p = 1 : N where $N = \frac{D}{m}$

FORM FOR COMPUTATION OF TRAVERSE

STN	OBSERVED ANGLE	CORR.	BEARING θ	DISTANCE l	ΔE = l sin θ +	COR	ΔE = l sin θ -	COR	ΔN = l cos θ +	COR	ΔN = l cos θ -	COR	E	N	STN
D															
A	74°55'20"	+8	210°00'00"	(adopted	bearing)								1000.00	3000.00	A
B	80°36'23"	+8	104°55'28"	95.027	91.821	-5			-24.474	-1			1091.816	2975.525	B
C	99°05'02"	+8	5°31'59"	32.785	3.161	-2			32.632	0			1094.975	3008.157	C
D	105°22'43"	+8	284°37'09"	81.071	-78.446	-5			20.462	-1			1016.524	3028.618	D
A			210°00'00" ✓	33.044	-16.522	-2			-28.617	-1	← actually -0.4mm		1000.000 ✓	3000.000 ✓	A
			Σ	241.927	Σ 0.014	-14 ✓			Σ 0.003	-3 ✓					
					misclose				misclose						
	Σ	32" ✓													
* Δθ	0°00'00"														
Sum	359°59'28"														
Angular Misclose	-32"														

Angular misclose = Σ angles − (n−2)·180°
Correction to each angle = $-\frac{m_\alpha}{n} = \frac{32''}{4} = +8''$
Bearings $\theta_i = \theta_{i-1} + \alpha \pm 180^\circ$
* Δθ = Starting bearing - closing bearing = 0 for a loop traverse

Bowditch: $corr_i = -\frac{m_{E/N}}{D} \cdot d_i$
Linear misclose: $m = \sqrt{m_E^2 + m_N^2} = \sqrt{(0.014)^2 + (0.003)^2} = 0.024$ m
Proportional misclose = $1 : \left(\frac{D}{m}\right) = 1 : \left(\frac{241.927}{0.024}\right) = 1 : 10{,}000$

Student Group No. ____________ Names (1) __________________ (3) __________________ Computed by V. Janssen Date 6 Aug 2003

(2) __________________ (4) __________________ Checked by ________________

Prepared by S. Ganeshan
21st March 1978

Excel Calculation

The calculations involved in a loop traverse are easily done within a spreadsheet. There are four main steps to follow:

1. Enter the data and calculate bearings
2. Remove the angular misclose
3. Perform a Bowditch adjustment
4. Perform a transformation (shown later in this chapter)

It is suggested that you enter the formulas into a spreadsheet as you work through this example.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2																
3	Data Entry and Bearing Calcs								Removing Angular Misclose						Bowditch Adjustment	
4	Pt	Angle			Bearin	Bearing			Cor	Adj Brg	Adj Brg			Dist	ΔE	cor E
5		d	`	"	dd	d	`	"	"	dd	d	`	"	m	m	m
6	D															
7		Adopted Brg:			**210.0**	**210**	**00**	**00.0**		210.0						
8	A	74	55	20.0					08							
9					104.9	104	55	20.0		104.9	104	55	28.0	95.027	91.821	-0.006
10	B	80	36	23.0					08							
11					5.5	5	31	43.0		5.5	5	31	59.0	32.785	3.161	-0.002
12	C	99	05	02.0					08							
13					284.6	284	36	45.0		284.6	284	37	09.0	81.071	-78.446	-0.005
14	D	105	22	43.0					08							
15					210.0	209	59	28.0		210.0	210	00	00.0	33.044	-16.522	-0.002
16	A															
17																
18			Misclose:		-0.0089			-32.0	32	0.000				241.927	0.014	-0.014
19									sum					sum	misc	sum

	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
1															
2															
3	Bowditch Adjustment				Transformation										
4	Easting	ΔN	cor N	Northin	Bearing	Bearing			Dist	swung	bearing			E	N
5	m	m	m	m	dd	d	`	"	m	dd	d	`	"	m	m
6															
7															
8	1000.000			3000.000										1000.000	3000.000
9		-24.474	-0.001		104.9261	104	55	33.9	95.022	104.9236	104	55	25		
10	1091.816			2975.525										1091.817	2975.529
11		32.632	0.000		5.5298	5	31	47.2	32.784	5.5273	5	31	38		
12	1094.975			3008.157										1094.975	3008.161
13		20.462	-0.001		284.6176	284	37	03.2	81.075	284.6150	284	36	54		
14	1016.524			3028.617										1016.523	3028.618
15		-28.617	0.000		210.0025	210	00	09.1	33.045	210.0000	210	00	00		
16	1000.000			3000.000										1000.000	3000.000
17															
18		0.003	-0.003	swing:	-0.0025			-9.1							
19		misc	sum												

The first thing to do is to enter the measured data. Enter the measured angles as shown below; the distances will be entered later. The next step is to adopt a bearing or azimuth for a leg of the traverse. In this example the leg D to A is adopted to be 210°00'00" and held fixed. Now we can calculate the bearing of the other legs of the traverse shown below:

Enter the clock wise angle measured at each point

	A	B	C	D	E	F	G	H
2								
3								
4	Pt	Angle			Bearing	Bearing		
5		°	`	"	dd	°	`	"
6	D							
7		Adopted Brg:			**210.0**	**210**	**00**	**00.0**
8	A	74	55	20.0				
9					104.9	104	55	20.0
10	B	80	36	23.0				
11					5.5	5	31	43.0
12	C	99	05	02.0				
13					284.6	284	36	45.0
14	D	105	22	43.0				
15					210.0	209	59	28.0
16	A							

Adopted Bearing

Adopted Bearing plus the angle at A

Formulas for calculating the bearings:

	A	B	C	D	E	F	G	H
3	Data Entry and Bearing Calcs							
4	Pt	Angle			Bearing	Bearing		
5		d	`	"	dd	d	`	"
6	D							
7		Adopted Brg			**=F7+G7/60+H7/3600**	**210**	**0**	**0**
8	A	74	55	20				
9					=MOD(E7+B8+C8/60+D8/3600+180,360)	=INT(E9)	=INT(60*(E9-F9))	=3600*(E9-F9)-60*G9
10	B	80	36	23				
11					=MOD(E9+B10+C10/60+D10/3600+180,360)	=INT(E11)	=INT(60*(E11-F11))	=3600*(E11-F11)-60*G11
12	C	99	5	2				
13					=MOD(E11+B12+C12/60+D12/3600+180,360)	=INT(E13)	=INT(60*(E13-F13))	=3600*(E13-F13)-60*G13
14	D	105	22	43				
15					=MOD(E13+B14+C14/60+D14/3600+180,360)	=INT(E15)	=INT(60*(E15-F15))	=3600*(E15-F15)-60*G15
16	A							
17								
18			Misclose:		=E15-E7			=3600*E18

Note that for cell G9, a better formula is =INT(E9*60-F9*60), similarly for the other rows in column G.

2. Removing the Angular Misclose

Because of errors in the angular measurements, a small misclose is created. As you can see our final bearing for the leg D to A is 32” too small. Thus we have a -32” angular misclose. What we do to remove this misclose is divide it evenly amongst each angle. So we add 8” to all four of the measured angles resulting in a total correction of 32 seconds, and an angular misclose of zero.

This is the correction applied to each bearing (H18 / 4)

Also note that it is positive because the final leg A to D was less than 210°

	A	B	C	D	E	F	G	H	I	J	K	L	M
2													
3													
4	**Pt**		**Angle**		**Bearing**	**Bearing**			**Cor**	**Adj Brg**	**Adj Brg**		
5		**d**	**`**	**"**	**dd**	**d**	**`**	**"**	**"**	**dd**	**d**	**`**	**"**
6	D												
7			Adopted Brg:		**210.0**	**210**	**00**	**00.0**		210.0			
8	A	74	55	20.0					08				
9					104.9	104	55	20.0		104.9	104	55	28.0
10	B	80	36	23.0					08				
11					5.5	5	31	43.0		5.5	5	31	59.0
12	C	99	05	02.0					08				
13					284.6	284	36	45.0		284.6	284	37	09.0
14	D	105	22	43.0					08				
15					209.99	209	59	28.0		210.0	210	00	00.0
16	A												
17													
18			Misclose:		-0.0089			-32.0	32	0.000			
19									sum				

This is the adjusted bearing (E9 + I8)

As a check this should be exactly 210°

This is the angular misclose E15-E7 in decimal degrees

This is the angular misclose in seconds (E18 * 3600)

Formulas for removing the misclose:

	H	I	J	K	L	M
1						
2						
3		Removing Angular Misclose				
4		**Cor**	**Adj Brg**	**Adj Brg**		
5		**"**	**dd**	**d**	**`**	**"**
6						
7			=E7			
8		=-H18/4				
9			=E9+I8/3600	=INT(J9)	=INT(60*(J9-K9))	=3600*(J9-K9)-60*L9
10		=-H18/4				
11			=E11+2*I10/3600	=INT(J11)	=INT(60*(J11-K11))	=3600*(J11-K11)-60*L11
12		=-H18/4				
13			=E13+3*I12/3600	=INT(J13)	=INT(60*(J13-K13))	=3600*(J13-K13)-60*L13
14		=-H18/4				
15			=E15+4*I14/3600	=INT(J15)	=INT(60*(J15-K15))	=3600*(J15-K15)-60*L15
16						
17						
18	=3600*E18	=SUM(I8:I14)	=J15-J7			
19		sum				

Again, note that for cell L9, a better formula is =INT(J9*60-K9*60), similarly for the other rows in column L.

3. Bowditch Adjustment

If you were to plot this traverse as it is, the coordinates where you started will not be the coordinates where you finish. So, we need to adjust the distances and bearings between points to make it geometrically correct. One way to do this is using a Bowditch adjustment which distributes the misclose amongst the ΔE and ΔN of each leg, proportional to the leg's distance. These are the formulas used:

$$\text{Cor E}_{AB} = -\frac{\Delta E \text{ Misclose} \times \text{Distance}_{AB}}{\Sigma\text{Distance}} \qquad \text{Cor N}_{AB} = -\frac{\Delta N \text{ Misclose} \times \text{Distance}_{AB}}{\Sigma\text{Distance}}$$

Once the corrections are calculated they are added to the ΔE or ΔN for each respective leg. The coordinates of each station are then calculated using the known coordinates of A.

	N	O	P	Q	R	S	T
3		Bowditch Adjustment					
4	Dist	ΔE	cor E	Easting	ΔN	cor N	Northing
5	m	m	m	m	m	m	m
6							
7							
8				1000.000			3000.000
9	95.027	91.821	-0.006		-24.474	-0.001	
10				1091.816			2975.525
11	32.785	3.161	-0.002		32.632	0.000	
12				1094.975			3008.157
13	81.071	-78.446	-0.005		20.462	-0.001	
14				1016.524			3028.617
15	33.044	-16.522	-0.002		-28.617	0.000	
16				1000.000			3000.000
17							
18	241.927	0.014	-0.014		0.003	-0.003	swing:
19	sum	misc	sum		misc	sum	
20							
21		Lin				Ratio 1:	16691

This is where the measured distances from the traverse are entered.

This is the total length of the traverse.

This is the misclose in the ΔE and ΔN. Expect these to be close to zero.

These are the corrections applied, calculated from the Bowditch adjustment formulae.

Formulas for Bowditch Adjustment:

	N	O	P	Q	R	S	T
3				Bowditch Adjustment			
4	Dist	ΔE	cor E	Easting	ΔN	cor N	Northing
5	m	m	m	m	m	m	m
6							
7							
8				1000			3000
9	95.027	=N9*SIN(RADIANS(J9))	=-O$18*N9/N$18		=N9*COS(RADIANS(J9))	=-R18*N9/N$18	
10				=Q8+O9+P9			=T8+R9+S9
11	32.785	=N11*SIN(RADIANS(J11))	=-O$18*N11/N$18		=N11*COS(RADIANS(J11))	=-R18*N11/N$18	
12				=Q10+O11+P11			=T10+R11+S11
13	81.071	=N13*SIN(RADIANS(J13))	=-O$18*N13/N$18		=N13*COS(RADIANS(J13))	=-R18*N13/N$18	
14				=Q12+O13+P13			=T12+R13+S13
15	33.044	=N15*SIN(RADIANS(J15))	=-O$18*N15/N$18		=N15*COS(RADIANS(J15))	=-R18*N15/N$18	
16				=Q14+O15+P15			=T14+R15+S15
17							
18	=SUM(N9:N15)	=SUM(O9:O15)	=SUM(P9:P15)		=SUM(R9:R15)	=SUM(S9:S13)	
19	sum	misc	sum		misc	sum	
20							
21			Linear Misclose:	=SQRT(P18^2+S18^2)			

Example 3: Traverse with carried bearings

This example is based on observations and calculations of a 2D traverse between MGA coordinated points along Willis Street near UNSW. The traverse was observed by a group of students in 2007: Sian Elliott, Matthew Cooper, Simon Chow, and Keirnan Smithson. The traverse starts on a control point, observes to two control targets, traverses via two new points to a final control point and again observes to two control targets. Along the traverse a point is radiated from both of the traverse stations. The method of carrying bearings was used in the field.

Fig. Plan of traverse. Traverse and radiations = Red Lines, Control target Sightings =Yellow lines.

Since the control point coordinates are on the MGA coordinate system, the mean horizontal distances were converted to MGA distances.

Line	**MGA Distances**
D210B to Station A	62.7694
Station A to BG361	38.8733
Station A to Station B	44.2769
Station B to BG361	5.9125
Station B to D811	87.6276

Bearings between control points calculated from coordinates:

From	To	°	'	"
D210B	TS133	295	15	17.1'
D210B	LC154	299	35	57.4
D811	LC153	334	32	16.6
D811	LC170	292	58	50.3

Observed directions: (I have modified the observations for educational purposes.) In this traverse students set the bearings on a "backsight" and carried bearings in the field observations as follows.

From	To	FL			FR			Mean			RM			Comment
		°	'	"	°	'	"	°	'	"	°	'	"	
D210B	TS133	295	15	04	115	15	07	295	15	06	295	15	17	
	LC154	299	35	54	119	36	03	299	35	59	299	36	10	13" error, OK
	ST A	279	22	56	99	22	57	279	22	57	279	23	08	
ST A	D210B	99	23	23	279	23	22	99	23	22	99	23	08	
	BG361	216	09	45	36	09	33	216	09	39	216	09	25	
	ST B	212	49	51	32	49	54	212	49	52	212	49	38	
ST B	ST A	32	49	51	212	49	58	32	49	54	32	49	38	
	BG361	10	25	32	190	25	51	10	25	42	10	25	26	
	D811	179	44	25	359	44	20	179	44	22	179	44	06	
D811	ST B	359	44	30	179	44	37	359	44	34	359	44	06	
	LC 153	334	32	48	154	32	58	334	32	53	334	32	25	8" error, OK
	LC 170	293	00	36	113	01	07	293	00	50	293	00	22	92" error, !!

The observed FL values are close to bearings if the calculations are carried correctly in the field. Experienced surveyors with this modern equipment would have the FL readings closer to the actual calculated bearings than is shown above. For the purposes of this practical we choose to use the bearing to one reference target at the start of the traverse and use the observation to the other reference target as a check. Similarly, at the end of the traverse, we use one target's observations to calculate the misclose, and the other reference target in this example has an error. We learn how to use all observations simultaneously when we do a least squares adjustment.

The angular misclose in this traverse is + 8", so we subtract sequentially 8/4 = 2" from each bearing. Thus the adjusted bearings are:

From	To	°	'	"
D210B	ST A	279	23	06
ST A	BG361	216	09	23
ST A	ST B	212	49	34
ST B	BG361	10	25	22
ST B	D811	179	44	00
D811	LC 153	334	32	17

Next, use adjusted bearings and grid distances from above, and known coordinates of start and end points to calculate miscloses.

Bowditch Adjustment:

Point	ΔE	ΔN	E	N	adj ΔE	adj ΔN	E	N
D210B			**336633.044**	**6245395.354**			**336633.044**	**6245395.354**
ST A	-61.93	10.236	336571.115	6245405.590	+0.0066	+0.0009	336571.121	6245405.591
ST B	-24	-37.21	336547.113	6245368.383	+0.0047	+0.0006	336547.124	6245368.384
D811	0.4078	-87.63	336547.521	6245280.756	+0.0092	+0.0012	336547.541	6245280.759
D811			**336547.541**	**6245280.759**			**336547.541**	**6245280.759**
					Σ 0.0205	Σ 0.0027		
Misclose			-0.0205	-0.0027				

Length of traverse = 194.674
Misclose in Eastings = -20 mm, Misclose in Northings = -3mm. The linear misclose ratio is 1: 9,400

The coordinates of BG361 where calculated from Station A and then calculated from station B. The values from the two radiations differ by 2mm in Eastings and by 5mm in Northings.

Miscellaneous Problems in Traversing

This section discusses some problems that might be encountered during the survey of a traverse. However we place less emphasis on these topics compared to many years ago. The material is included here for your reference in the future, if you ever need it. Some of the calculations can be done better by using Least Squares.

Transformation / Rotation of Traverse

If a Bowditch adjustment of a loop traverse has been done then the bearing between the adjusted coordinates of each line will change. In some applications it is useful to have the final bearing of one line in the traverse fixed at a certain value (e.g. datum azimuth in a cadastral survey).

To illustrate this calculation we continue with the data from Example 2: Loop Traverse, given previously.

Step 4. Perform a Transformation

Once the Bowditch adjustment is done, you will find (by performing a join between D and A) that the traverse has rotated from the adopted bearing. This is due to the change in coordinates caused by the Bowditch method. To fix this problem we find out how much the traverse has swung off the adopted bearing and then swing the whole traverse back, as explained below. We calculated bearings and distances for each traverse line from the adjusted coordinates. Then we calculate the rotation (swing) angle of the line of interest by subtracting the current bearing from the desired value of the bearing. In this case the difference is about 9". We then apply this value to the bearing of each traverse line. We use the new bearings and the distances calculated from the adjusted coordinates to calculate new transformed coordinates.

	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
3		Joins					Transformation				
4	**Bearing**	**Bearing**			**Dist**	**swung**	**bearing**			**E**	**N**
	dd	**d**	**`**	**"**	**m**	**dd**	**d**	**`**	**"**	**m**	**m**
										1000.000	3000.000
9	104.9261	104	55	33.9	95.022	104.9236	104	55	25		
10										1091.817	2975.529
11	5.5298	5	31	47.2	32.784	5.5273	5	31	38		
										1094.975	3008.161
	284.6176	284	37	03.2	81.075	284.6150	284	36	54		
										1016.523	3028.618
	210.0025	210	00	09.1	33.045	210.0000	210	00	00		
16										1000.000	3000.000
17	swing:										
18	-0.0025										

These are the bearings calculated from the Bowditch adjusted coordinates.

Here we find that the traverse has swung -0.0025° or −9.1" ($\beta - \beta_{DA}$)

Now add -9.1" to each bearing.

These are the known coordinates of A

Now calculate the coordinates again using the swung bearings and the adjusted distances

Formulas to perform the transformation:

First calculate the joins between the coordinates.

	U	V	W	X	Y
1					
2					
3			Transformation		
4	**Bearing**		**Bearing**		**Dist**
5	**dd**	**d**	**`**	**"**	**m**
6					
7					
8					
9	=MOD(DEGREES(ATAN2((T10-T8),(Q10-Q8))),360)	=INT(U9)	=INT(60*(U9-V9))	=3600*(U9-V9)-60*W9	=SQRT((Q8-Q10)^2+(T8-T10)^2)
10					
11	=MOD(DEGREES(ATAN2((T12-T10),(Q12-Q10))),360)	=INT(U11)	=INT(60*(U11-V11))	=3600*(U11-V11)-60*W11	=SQRT((Q10-Q12)^2+(T10-T12)^2)
12					
13	=MOD(DEGREES(ATAN2((T14-T12),(Q14-Q12))),360)	=INT(U13)	=INT(60*(U13-V13))	=3600*(U13-V13)-60*W1	=SQRT((Q12-Q14)^2+(T12-T14)^2)
14					
15	=MOD(DEGREES(ATAN2((T16-T14),(Q16-Q14))),360)	=INT(U15)	=INT(60*(U15-V15))	=3600*(U15-V15)-60*W1	=SQRT((Q14-Q16)^2+(T14-T16)^2)
16					
17					
18	=E7-U15			=3600*U18	

Now swing the traverse and calculate new coordinates.

	Z	AA	AB	AC	AD	AE
1						
2						
3				Transformation		
4	**swung**	**bearing**			**E**	**N**
5	**dd**	**d**	**`**	**"**	**m**	**m**
6						
7						
8					=Q8	=T8
9	=U9+U$18	=INT(Z9)	=INT(60*(Z9-AA9))	=3600*(Z9-AA9)-60*AB9		
10					=AD8+Y9*SIN(RADIANS(Z9))	=AE8+Y9*COS(RADIANS(Z9))
11	=U11+U$18	=INT(Z11)	=INT(60*(Z11-AA11))	=3600*(Z11-AA11)-60*AB11		
12					=AD10+Y11*SIN(RADIANS(Z11))	=AE10+Y11*COS(RADIANS(Z11))
13	=U13+U$18	=INT(Z13)	=INT(60*(Z13-AA13))	=3600*(Z13-AA13)-60*AB13		
14					=AD12+Y13*SIN(RADIANS(Z13))	=AE12+Y13*COS(RADIANS(Z13))
15	=U15+U$18	=INT(Z15)	=INT(60*(Z15-AA15))	=3600*(Z15-AA15)-60*AB15		
16					=AD14+Y15*SIN(RADIANS(Z15))	=AE14+Y15*COS(RADIANS(Z15))

So the final coordinates, in metres, are:

Point	E	N
A	1000.000	3000.000
B	1091.817	2975.529
C	1094.975	3008.161
D	1016.523	3028.618

If you have entered the above example and its formulas into an Excel spreadsheet, then it is easy to try using data from another four station loop traverse:

Point	Angle	Line	Distance (m)
A	91°18'10"	AB	20.080
B	90°15'55"	BC	51.222
C	91°26'25"	CD	21.507
D	86°58'45"	DA	51.879

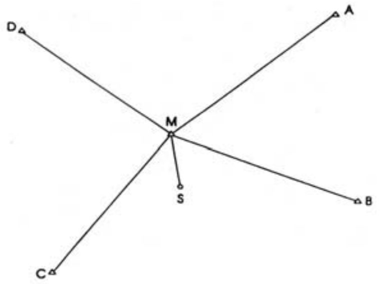
Coordinates of A (m): E 1500.000, N 2500.000   Assume the bearing of 185°30'40" for the line D to A.

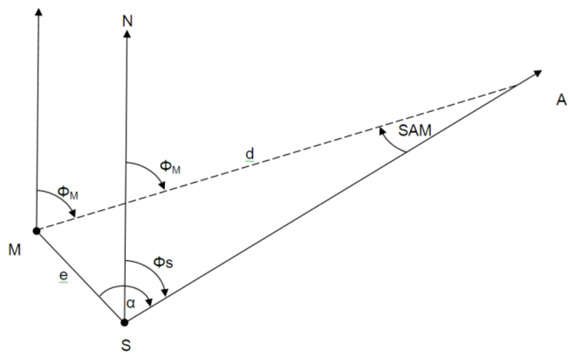
Answers:	Point	E	N
	A	1500.000	2500.000
	B	1519.937	2497.618
	C	1526.252	2548.449
	D	1504.982	2551.639

ECCENTRIC DIRECTIONS

Problem: Sometimes it is not possible to set up a survey instrument over a control mark. For example, observations are required at a trig station which is marked by a beacon supported by a large cairn of stones. It is not convenient to dismantle the cairn in order to set up the theodolite over the mark. Another example is where a mark is on the side of a wall.

Solution: Set up the theodolite a short distance from the control mark (M) at a ‘satellite station’ (S), observe to the required points and, in addition, to M. Because the sight to M is a very close one, the angle need only be recorded approximately (eg to the nearest minute of arc). Don’t be so close that you can’t focus onto M. The distance between the satellite station and the ‘trig’ station must be measured (approximate distances to the other points must also be available, e.g. by scaling from a map). It is then possible to calculate what the equivalent observations would have been from the main required to reduce the observations from the eccentric satellite station onto the TS.





S = satellite station, M = main station (nearby TS),
A, B, C, D = distant stations

Consider the observations to target A:

e = measured eccentric horizontal distance SM

α = angle MSA measured clockwise from M

d = hor. distance from M to distant station. d is obtained from coordinates or calculated from the distance SA and use the cosine rule.

See the diagram above right. $\Phi_M = \Phi s$ – angle SAM

In this case the angle at A (SAM) can be calculated from

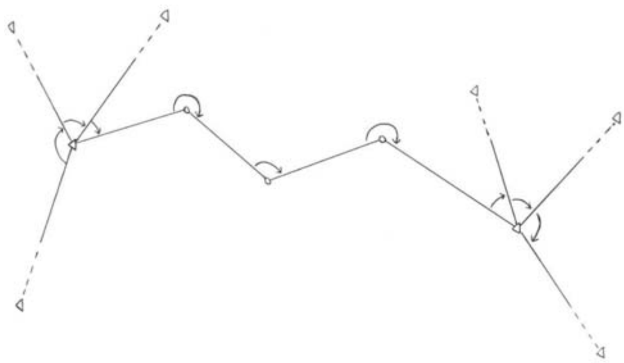
$$\frac{e}{\sin(SAM)} = \frac{d}{\sin(\alpha)} \quad \textit{therefore} \quad \sin(SAM) = \frac{e}{d}\sin(\alpha)$$

Calculate the direction or bearing β_{MA} from the observed β_{SA} using: $\phi_{MA} = \phi_{SA} + \sin^{-1}\left(\frac{e}{d}\sin\alpha\right)$

Do similar calculations for the other distant targets.

ORIENTATION OF ARCS AT TRAVERSE TERMINALS

At the starting and ending point of a traverse you know that it is necessary to observe to targets at known bearings. This is so that the bearings of the traverse lines can be calculated from the observed angles and the misclose in the angles checked. However you should, if possible, observe to more than one known target at each end of the traverse. I would suggest observing to 2 or 3 known targets at each terminal station if possible (see fig). This checks that you have got the correct coordinates for the control marks (people sometimes look them up or copy them incorrectly). It also checks that you are looking at the right target/mark (i.e. guards against misidentification of target), e.g. you have coordinates for a church spire but observe to the wrong spire or a different church, or you observe to a tree at a long distance that just happens to look like a trig station.



This also checks that you are set up on the right control mark at the traverse end, or that your coordinates for it are OK (3+ targets give a resection), or that the mark hasn't moved by a large amount (you probably can't detect small movements).

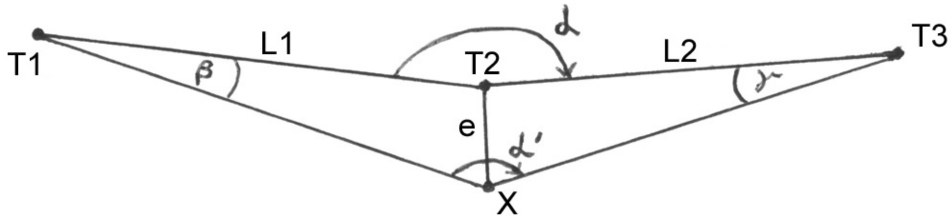
So, observing to more than one terminal station guards against gross errors. It also improves the precision/accuracy of the bearings of the traverse lines because your estimate of the misclose is more accurate and the bearings of the first and last lines in the traverse are better.

Some ways to do the calculations:

- Use the best target for the calculations, use the other targets just to check for gross errors
- Calculate the mean bearing. Calculate the bearing of the first traverse line from each of the control targets. If 3 targets, you get 3 values for the bearing. Take the mean of these values. Do a similar thing at the other end of the traverse. Then adjust the angular misclose through the traverse leaving the first and last lines fixed, i.e. adjust only the angles at the other (inner) traverse stations.
- "Orientation of arcs of directions" method. (See lecturer if you want more details).
- Least Squares network adjustment (as taught later).

EFFECT OF CENTRING ERRORS

Centring errors can cause large errors in the observed angles, especially for short traverse lines. Consider a centring error e at traverse station T2, it causes an error in the angle α ($\Delta\alpha = \alpha' - \alpha$) as follows.



$$\beta'' = \frac{e}{L1} \cdot \rho'' \qquad \text{and} \qquad \gamma'' = \frac{e}{L2} \cdot \rho''$$

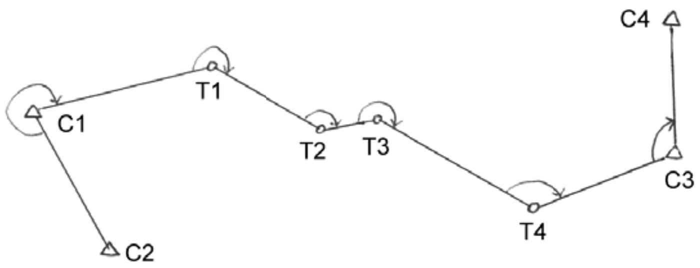
Where ρ" ≈ 206264.8… and represents the number of seconds in a radian. It is used to convert units from radians to seconds.

If e = 2mm, L1 = L2 = 100m: $\Delta\alpha = \beta + \gamma = \frac{0.002}{100} \cdot \rho'' + \frac{0.002}{100} \cdot \rho''$ = 4" + 4" = 8"

If e = 2mm, L1 = 100m, L2 = 10m: $\Delta\alpha = \beta + \gamma = \frac{0.002}{100} \cdot \rho'' + \frac{0.002}{10} \cdot \rho''$ = 4" + 41" = 45" (!!!)

SHORT TRAVERSE LINES

It is desirable to keep traverse lines approximately equal in length. However, in practice, short traverse legs cannot always be avoided. Centring and focus (moves LoS) errors are most severe on short traverse lines.



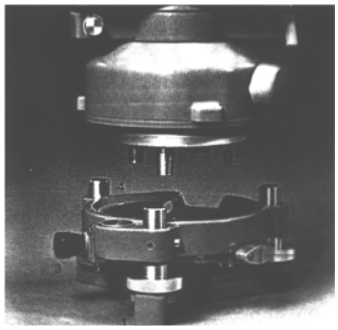
Solutions:

- Constrained (forced) centring using theodolite or total stations and targets that have compatible removable tribrachs.
- Sight to distant control stations from each end of the short leg.
- Place the short leg station on line to any distant point (coordinates of that point are not essential).

CONSTRAINED CENTRING

When the lines of a traverse are short, the centring errors become prominent. Forced (or constrained) centring is used to minimise these errors. The equipment used consists of a theodolite, 3 or more tripods with tribrachs, and at least 2 special targets which fit into the tribrachs and are interchangeable with the theodolite. The theodolite is set up at a station, and a tripod incl. tribrach and target at both the rear and the forward station.

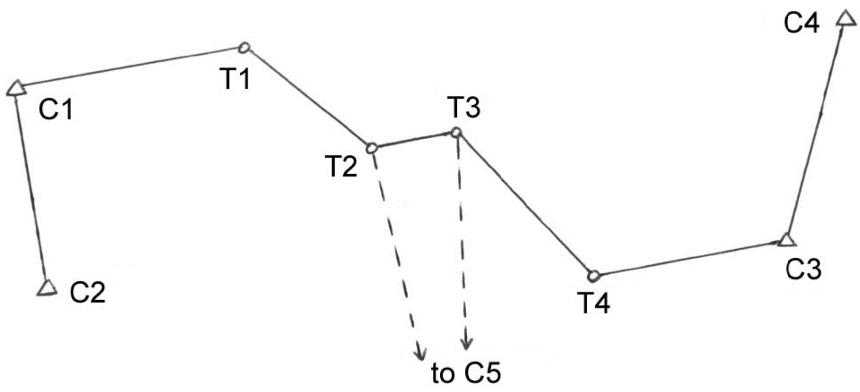
Constrained centring in tribrach GDF6:



When observations at this station are complete, the theodolite is taken off its tribrach (without disturbing the latter) and moved and placed over the tribrach marking the forward station, from which the target has been removed. The target at the rear station is transferred to the original theodolite tribrach, the forward target being moved on and placed on a tripod at the next forward station. In this way, centring errors are reduced to a minimum since the vertical axes of the theodolite and targets always occupy the same position on the tripods.

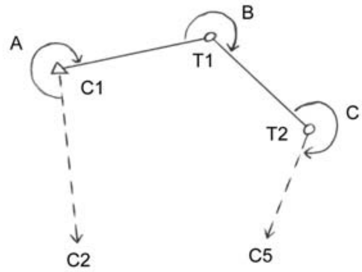
SIGHT TO DISTANT CONTROL TARGETS

At T2 and T3, sight to distant control station C5.



Compute the traverse as follows:

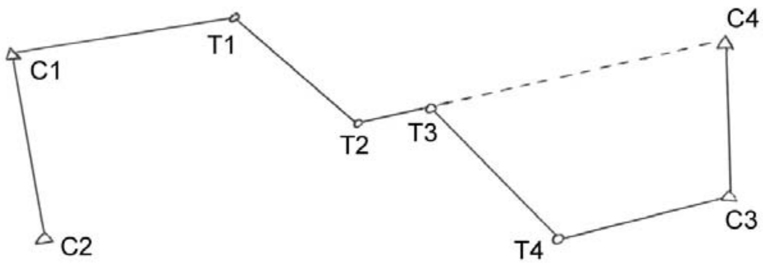
1) Calculate the traverse C1 T1 T2 T3 T4 C3 in the usual way to provide provisional coordinates for T2 and T3.
2) Using the provisional coordinates of T2 and T3, calculate the bearings T2 C5 and T3 C5.
3) Using the bearings C1 C2 and T2 C5 as initial and final bearings, calculate the angular misclose for this section:



4) Adjust the angles A, B, C by applying 1/3 of the misclose to each angle.
5) Calculate new bearings C1 T1 and T1 T2 from the adjusted angles.
6) Repeat steps 1-5 for the section C5 T3 T4 C3 C4 to obtain the bearings T3 T4 and T4 C3.
7) Bearing T2 T3 can be calculated from T2 or T3 but is in any case not critical.

PLACE SHORT LEG ON LINE TO DISTANT POINT

Angle measurements to a distant target are more accurate than to a short target.



Computational steps:

1. Place the short traverse leg T2 T3 on line to the (uncoordinated) distant point C4.
2. At T2 measure the angle T1 T2 C4, which is equal to the required angle T1 T2 T3.
3. At T3 measure the angle C4 T3 T4.
4. Calculate the required angle at T3:   angle T2 T3 T4 = angle C4 T3 T4 - 180°

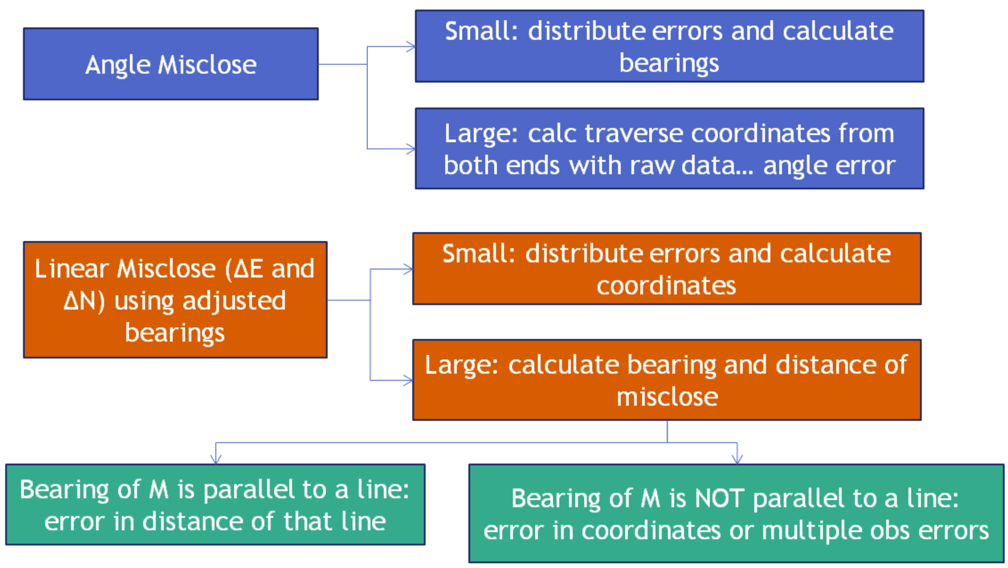
ERROR PROPAGATION in TRAVERSES

Error sources in the measurement of traverse angles and traverse distances are covered in another course at UNSW. The statistical method of propagation of variances can be used to determine the quality of the traverse coordinates. This topic is covered in Least Squares Adjustments in another course at UNSW.

Generally the weakest point in a traverse is at the point midway between the known terminals, or half way around the loop.

Summary and Final Points

- Closed-line traverse and Loop traverse computations are similar, when coordinates are used.
- Visualise the problem with a sketch plan.
- Misclose is an error and it has the opposite sign (+ -) to the corrections to be applied.
- If a traverse has large miscloses, do not adjust it, do try to find the source of the error.
- Calculate angular misclose, if it is small then adjust the angles or bearings, if it is large search for the source of the error.
- Preventing errors in traversing, by using good field and office techniques, is better than trying to fix a traverse that contains errors.
- Calculate the coordinates of each station from bearings and distances, and the linear misclose.
- If linear misclose is small then adjust the coordinates (by e.g. Bowditch method or Transit method). If the misclose is large search for the source of the error.
- Practice the calculations.
- Traverse calculations can effectively be done by calculator (though if you enter a value incorrectly you may have to re-enter many values), or on an Excel spreadsheet, or in surveying CAD programs.
- Blunder detection in traversing:
    - Linear error: Bearing of linear misclose approx. same as bearing of line containing error.
    - Angular error: Forward and backward computation: Station where coordinates from both computations agree is station at which gross error occurred.
    - Error in control point coordinates or scale error in distances.

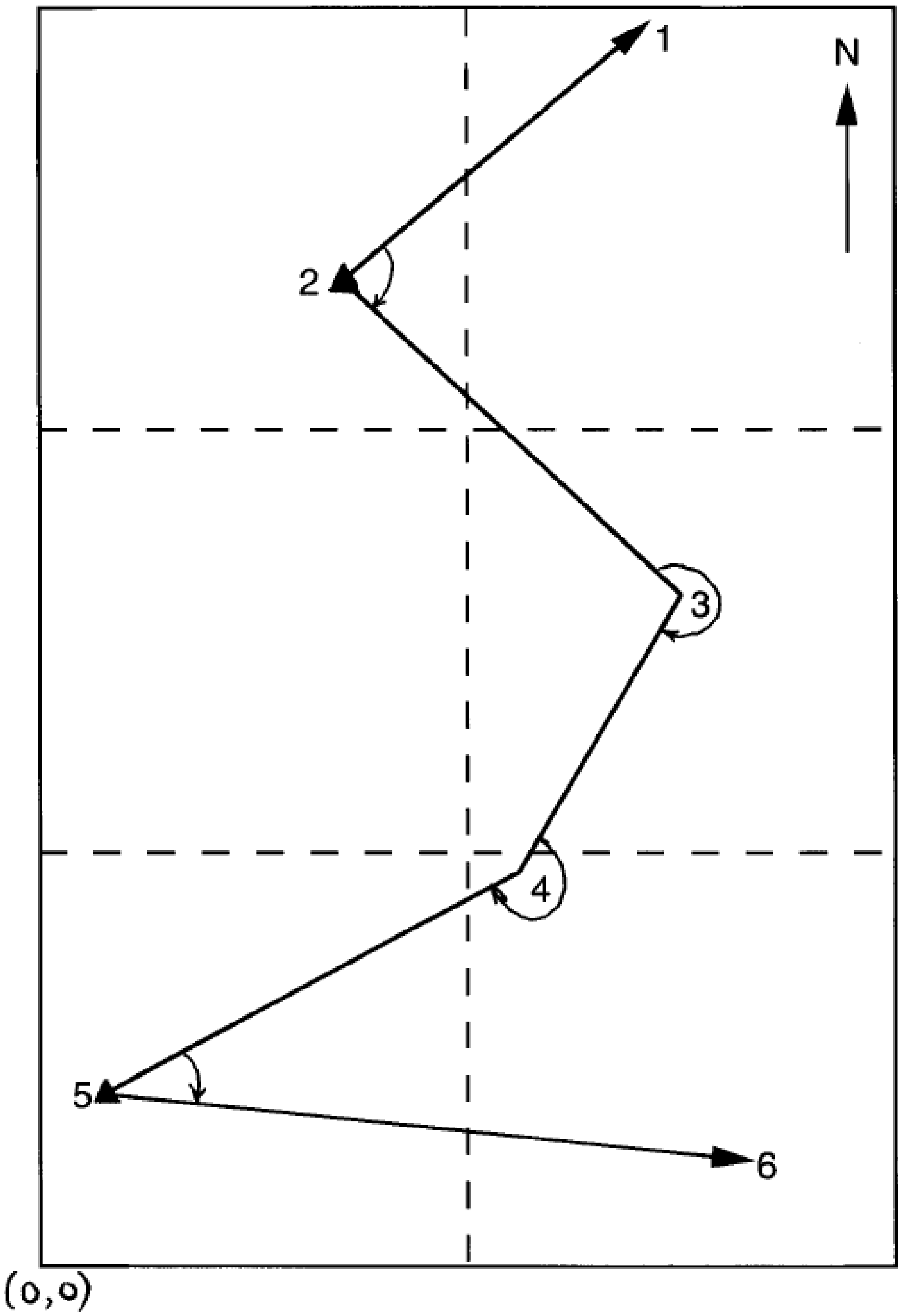


- Least squares adjustment is a better way to adjust traverses, especially if they are part of a network or have radiations attached to them. However, even with access to Least Squares, it may still be necessary to calculate traverse miscloses as part of the quality control and error detection processes.

TUTORIAL PROBLEMS - TRAVERSE

A separate spreadsheet file is available that contains the data in these questions, so you don't have to type them in. Ask the lecturer which of the following questions are required for assessment and which are optional for study purposes. Some of the questions are supplied as revision of your year 1 surveying course.

Q0. This question aims to help you understand the basic concepts of converting angles to bearings, and bearings and distances to coordinates. If you are comfortable with these basics skip this question.



Print out this page.

Measure bearings (angle from North) of 2-1 and 5-6 with protractor.
Measure angles to the nearest degree at 2, 3, 4, and 5 with a protractor. Set 0 on the backsight, for each angle.
Calculate bearings, from your measurements of angles. Check answers by measuring bearings directly.
Calculate misclose in bearings.
Measure distances at a scale of 1:1000.
Measure coordinates of 2 and 5, by scaling with a ruler and using the grid lines. Origin of coordinates is shown.
Calculate coordinates of 3, 4 and 5 and misclose.
Check coordinates by measuring them directly.
There may be gross errors in the following problems. If there are any, you are expected to identify them, estimate the most probable value and contact your tutor.

Q1. Calculate the provisional coordinates of the points B, C, D, E and F of the traverse ABCDEF and the misclose from the following data:

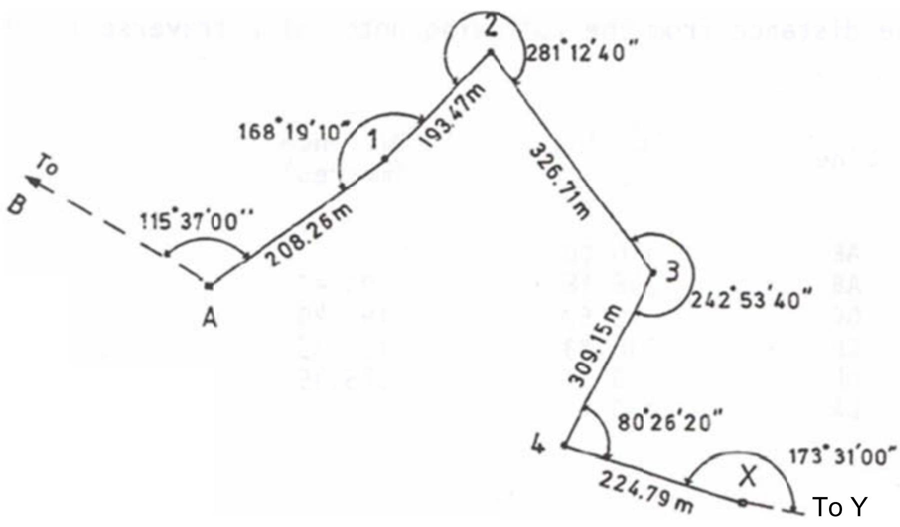
Line	Bearing ° ' ''	Distance (metres)
AB	360 00 00	52.965
BC	84 19 00	36.429
CD	163 42 00	46.833
DE	227 35 00	26.217
EF	309 10 00	43.388
FA	170 24 30	21.645

Coordinates of A (metres): E 400.000 N 600.000

Q2. A closed traverse was run between stations A and X as shown in the diagram. The coordinates of the control stations at the ends of the traverse are as follows:-

	E(m)	N(m)
A	1769.15	2094.72
B	1057.28	2492.39
X	2334.71	1747.32
Y	2995.85	1616.18

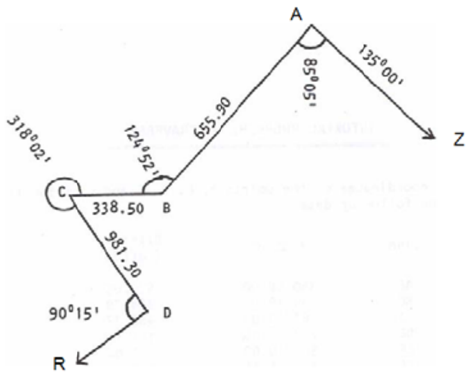
Calculate the angular misclose and provisional coordinates of stations 1, 2, 3 and 4.



Q3. Given, Fixed Plane Coordinates:

Point	E	N
A	3000.00	3000.00
D	2906.48	1809.00
R	2175.20	1127.07

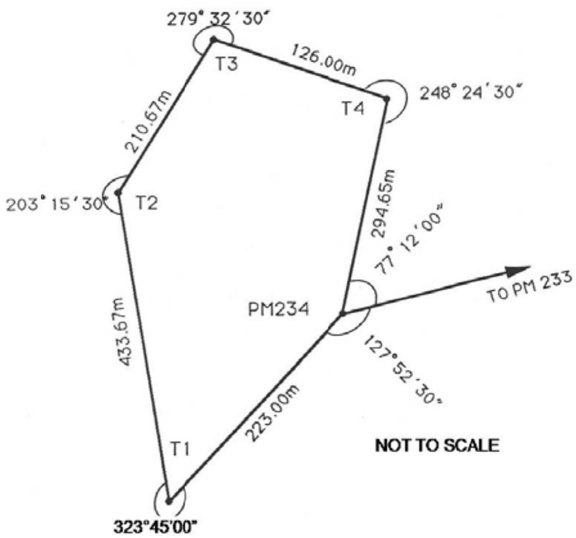
Compute the adjusted coordinates of points B and C and briefly comment on the limitations of any assumptions made, given that Z, A, D and R are ground control points of fixed plane coordinates and that B and C are uncoordinated points connected to A and D by traverse.



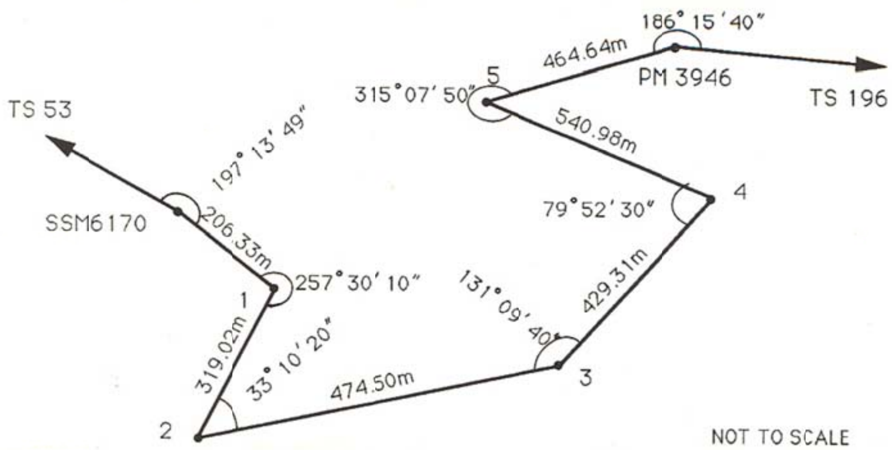
Q4. Using the information given of a loop traverse, calculate the miscloses. If the miscloses are small then calculate the adjusted coordinates of the traverse stations T1, T2, T3, and T4.

Known coordinates:

STN	E	N
PM 233	323 780.189	1245 841.133
PM 234	321 640.675	1245 679.240



Q5. A traverse was observed from SSM6170 to PM3946. The observed angles and horizontal distances are shown in the figure below.

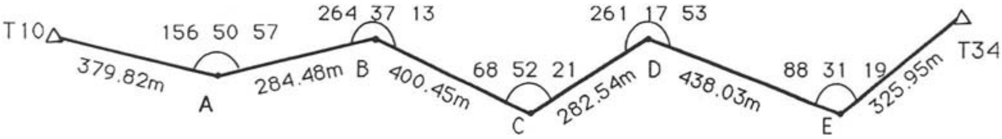


The coordinates of the known points are:

	E	N
TS53	455.14	490.45
SSM6170	598.63	275.28
PM3946	1182.71	575.08
TS196	2046.83	613.93

Calculate the miscloses. If the miscloses are small, then calculate adjusted coordinates of the traverse stations.

Q6. The following observations were taken during a traverse survey. Sketch is not to scale.



At the terminal stations the following observations were taken.

	At STN. T 10 DIRECTION		At STN. T34 DIRECTION
T 12	0° 00' 00"	T30	0° 00' 00"
A	106° 22' 00"	T33	35° 19' 39"
T11	114° 59' 34"	T35	141° 13' 01"
PM 108	139° 33' 40"	E	289° 22' 18"

The coordinates of the control stations are:

STN	E	N
T10	2570.390	5624.352
T11	2916.228	5547.611
T12	2476.261	6049.742
T30	3614.254	5679.190
T33	3850.302	5868.865
T34	4120.805	5546.894
T35	4478.896	5707.409
PM 108	2782.509	5464.030

Calculate the angular misclose and then the coordinates of A, B, C, D and E.

Q7. The following notes were taken during a theodolite traverse. Bearing of line AB 14° 48' 00"
The convention used here for angles: ABC means set up at B backsight to A and read clockwise angle to foresight at C. Calculate the bearing from A to G and the distance between A and G.

Angle observed		Length (metres)	
ABC	198° 06' 30"	AB	245.62
BCD	284° 01' 30"	BC	310.00
CDE	200° 12' 30"	CD	480.34
DEF	271° 33' 30"	DE	709.27
EFG	268° 01' 30"	EF	430.14
		FG	607.12

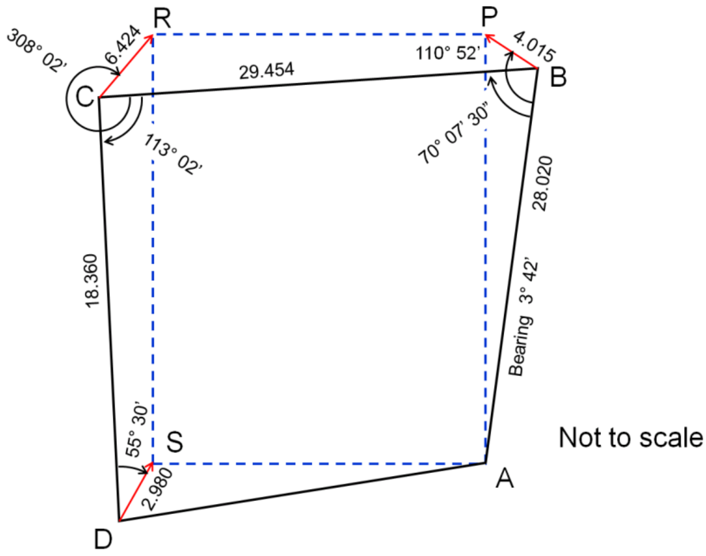
Q8. The following traverse is run from A to E, between which there occur certain obstacles.

Line	Distance (m)	Bearing
AB	142.00	38° 20'
BC	172.83	347° 55'
CD	203.15	298° 12'
DE	143.32	29° 46'

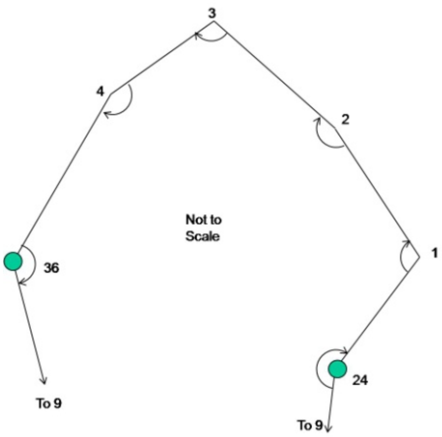
It is required to mark the point exactly midway between A and E. Calculate the length and bearing of a line from station C to the required point.

Q9. This diagram represents part of a property survey. Some information from the original diagram has been removed to make it suitable for this tutorial. The bearing of AB is given as 3° 42'. Using the data given on the diagram, compute the bearings and distances of the property lines (dashed).

Hints. (a) Compute bearings of BC and CD. (b) Assuming any arbitrary coordinates for A, compute the coordinates of B, C & D. (c) Compute coordinates of P, R & S from B, C & D respectively and then compute the bearings and distances of the property lines.



Q10. A traverse was observed by students at Morpeth, NSW and a field sketch plan is given below. Note carefully the direction of the angle measurements. Distances and coordinates are on a local datum, there is no scale factor. Three versions of their data are given below. Treat each data set as a separate problem; do not simply compare the numbers to find any errors.



For each data set:

- Calculate the traverse miscloses.
- Comment on whether the miscloses are acceptable (within the standards) or not. The required standards are: angular misclose < 20" +10√n and linear misclose < 15mm.
- If the miscloses are acceptable then calculate adjusted coordinates of the traverse stations using Bowditch. If the miscloses are not acceptable then determine where the error may have occurred and estimate the likely correct values of erroneous data.

a) DATA A

KNOWN COORDINATES

		E	N
9	RO	132.08	1981.69
24	START BM	171.94	2339.56
36	END BM	20.58	2392.58

Angles

BS	AT	FS	° ′ ″
9	24	1	211 52 52
24	1	2	106 25 56
1	2	3	168 43 16
2	3	4	101 26 53
3	4	36	155 43 06
4	36	9	134 15 33

Horizontal Distances

24 - 1	63.575
1 - 2	72.854
2 - 3	57.832
3 - 4	61.556
4 - 36	84.805

b) DATA B

Known Coordinates

		E	N
24	START BM	171.94	2339.56
36	END BM	220.58	2392.58

Known Bearings

From	To	° ' "
24	9	186 21 20
36	9	164 49 04

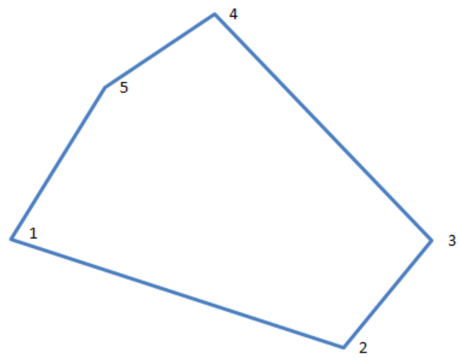
Angles				Horizontal Distances	
BS	AT	FS	° ' "		
9	24	1	211 52 52	24 - 1	63.575
24	1	2	106 25 56	1 - 2	72.854
1	2	3	168 43 16	2 - 3	75.832
2	3	4	101 26 53	3 - 4	61.556
3	4	36	155 43 06	4 - 36	84.805
4	36	9	134 15 33		

c) DATA C

KNOWN COORDINATES		E	N
9	RO	1132.08	6981.69
24	START BM	1171.94	7339.56
36	END BM	1020.58	7392.58

Angles				Horizontal Distances	
BS	AT	FS	° ' "		
9	24	1	211 52 52	24 - 1	63.575
24	1	2	106 25 56	1 - 2	72.854
1	2	3	186 43 16	2 - 3	75.832
2	3	4	101 26 53	3 - 4	61.556
3	4	36	155 43 06	4 - 36	84.805
4	36	9	134 15 33		

Q11. A loop traverse was observed and the data and a field sketch plan is given below. Distances and coordinates are on a local datum, there is no scale factor. Note carefully the direction of the angle measurements, they are clockwise internal angles.
Calculate the traverse mistcloses. Comment on whether the miscloses are acceptable (within the standards) or not. The required standards are: angular misclose < 20" +10√n and linear misclose < 15mm. If the miscloses are acceptable then calculate adjusted coordinates of the traverse stations using Bowditch. If the miscloses are not acceptable then determine where the error may have occurred and estimate the likely correct values of erroneous data.



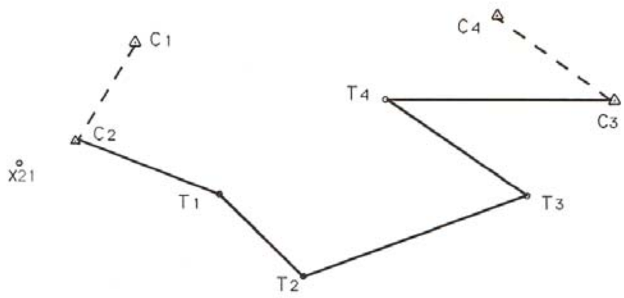
KNOWN COORDINATES		
Pt	E	N
1	123.330	398.750

KNOWN BEARING
1 to 2 = 109°00′00″

Interior clockwise Angles		Horizontal Distances	
At	° ′ ″		
1	78 44 43	1 - 2	160.335
2	108 56 12	2 - 3	65.575
3	99 54 56	3 - 4	147.610
4	96 41 13	4 – 5	91.556
5	155 43 06	5 – 1	84.805

Q12. You have just started your first full-time job with a small surveying consulting firm. While working on a seven leg closed polygon traverse you notice that one of the angles was not measured, but keep quiet because you don't want to cause any trouble or look stupid. While on your way back to the office the party chief tells you to sum up the measured interior angles in the traverse. You do this and say 843°18'22". The party chief next tells you to write an angle 56°41'24" into the field book where the missing angle should have been written. You are then told to adjust the angles and start a preliminary traverse computation. What is your most ethically correct response? [Adapted from *Geomatics Engineering: A Practical Guide to Project Design by Clement Ogaja CRC Press 2011.*]

Q13. The sketch plan at right shows a traverse from C2 to C3 in which the control station C2 was not accessible. Station X21 was used as an auxiliary station to overcome the problem. Station T4 could not be occupied, so observations were taken at T4 (ecc). Using the observations below, compute all the angles in the traverse adjust the angular misclose and calculate the bearings of all the lines.



From	To	Direction	Distance
X21	C1	259° 26' 12"	
X21	C2	281° 58' 17"	
X21	T1	323° 46' 24"	316.93
T1	X21	55° 26' 33"	
T1	C2	73° 51' 40"	
T1	T2	273° 41' 14"	
T2	T1	260° 01' 17"	
T2	T3	3° 43' 50"	
T3	T2	116° 59' 42"	
T3	T4	183° 34' 20"	
T4 (ecc)	T4	352° 37' 00"	1.11
T4 (ecc)	C3	128° 59' 37"	475.21
T4 (ecc)	T3	177° 38' 29"	284.15
C3	T4	226° 17' 58"	
C3	C4	273° 36' 38"	

The coordinates of the control stations are:

Stn	E	N
C1	2341.71	1580.11
C2	2230.89	1417.88
C3	3204.05	1375.74
C4	3008.91	1600.14

FIELDWORK 1: Loop Traverse, local datum

Students at UNSW will be given more specific instructions for this field class, including details about instrument use. Readers not at UNSW are supplied with the following notes so they can see the type of exercise UNSW students do in this course. The exercise could be modified to suit other campuses.

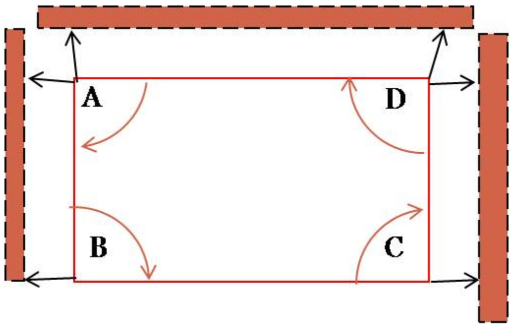
This practical exercise is similar to many cadastral and small-scale engineering surveys to measure the location of ground marks and features such as walls. Our aim for this exercise is to practice measurements and calculations of a loop traverse on a local plane coordinate system and to determine if the column walls supporting the balconies of the UNSW Quadrangle building have been aligned at 90° to each other.

1. EQUIPMENT per group of 2, 3 or 4 students

1 Electronic Tacheometer / Total Station, 3 EDM Target prisms with tribrach, 4 tripods, 1 thermometer, 1 barometer, 1 small tape measure, stick on targets and ground marks supplied by supervisor

2. EXERCISE

2.1 On the Quadrangle lawn (or balcony in wet weather), place four temporary traverse marks at points forming a quadrangle with sides about 50m long. Label these points A, B, C, D going **anti-clockwise**. From each corner a few detail points (such as corner of building, corner of fence, survey mark etc) can be seen. Set up tripods over points A, B and D.



Loop traverse with radiations to Quad building walls

2.2 Set up instrument at A, and targets at B and D. Check instrument settings. Enter temperature and pressure readings for correction of EDM distances. Do not use a map projection scale factor.

2.3 Measure one horizontal arc of directions at A to D, B and detail point features (e.g. Quad Building wall marks). Results should be recorded and reduced in the field book in a tabulated form and draw a sketch showing details. Back sight circle setting can be 0, or the back bearing, or random (you choose). Do not observe the diagonal lines of this loop because in many real surveys you may not be able to see across the loop.

Example of booking:

AT	TO	*F.L.*	*F.R.*	MEAN	REDUCED MEAN
A	D	*0° 03' 24"*	*180° 03' 30"*	0° 03' 27"	0° 00' 00"
	B	*97° 56' 37"*	*277° 56' 46"*	97° 56' 42"	97° 53' 15"
	Wall1	*193° 42' 00"*	*13° 42' 10"*	193° 42' 05"	193° 38' 38"

Numbers in italics are measurements. Non italics are calculated values.

2.4 Measure and book the horizontal distances AB, AD in FL and in FR.

2.5 Where possible also observe horizontal distances and directions (radiations) to the detail points on the walls. Measure horizontal distances for these radiations with pocket tape and plumb bob (consult the supervisor for advice on technique) and also measure the distance by reflectorless EDM. Other check measurements for the radiations may be possible, e.g. horizontal distance between two wall marks, or slope distance and ZA from instrument to wall mark, if in doubt ask supervisor.

Each group member is to observe at least one corner of the quadrangle.

2.6 Use the constrained centring method. Move the instrument to B, one target to A and one target to C, and perform similar observations as at A. Then move the instrument to C and repeat similar observations. Finally, move the instrument to D and repeat similar observations to finish the traverse.

2.7 Sum the angles and enter the misclose in the field notes.

Usually the fieldwork takes < 2½ hours.

3. REPORT

3.1 Make a table of the grand mean angles and the distance measurements.

3.2 Assume a bearing of 30°00'00” for the line from A to D, and coordinates of A are: E = +1000.00m and N = +3000.00m

3.3 Correct measured angles for any misclose in the sum of the angles, and calculate bearings for each line.

3.4 Using these bearings and the **mean** horizontal distances, calculate the ΔE and ΔN of AB, BC and CA. If the misclose in ΔE or ΔN exceeds about 30 mm, check the calculation for errors. Tabulate traverse calculations as shown in the spreadsheet example →.

3.5 When satisfied that miscloses are not due to arithmetical errors, adjust by Bowditch Method and calculate coordinates of B, C and D. Then calculate the coordinates of the radiated points.

3.6 Calculate the bearings and distances between the main detail points at each corner. For example, the bearings and distances of the sides of the building. “Swing” the bearings of all lines until the main east west wall of the quad (or substitute wall) becomes 90°00'00”. Are the sides of the building parallel? Are the corners 90°?

Loop traverse calculations on local datum

Data: M. Waud 1999

												Bowditch Adjustment						Joins				Transformation				
Pt	Mean Angle			Bearing			Cor	Adj Brg			Mean Dist	DE	cor E	Easting	DN	cor N	Northing	Bearing			Dist	bearing			E	N
	d	'	"	d	'	"	"	d	'	"	m	m	m	m	m	m	m	d	'	"	m	d	'	"	m	m
D																										
	Adopted Brg			210	00	00.0																				
A	74	55	20.0				08							1000.000			3000.000								1000.000	3000.000
				104	55	20.0		104	55	28.0	95.027	91.821	-0.006		-24.474	-0.001		104	55	33.9	95.022	104	55	25		
B	80	36	23.0				08							1091.816			2975.525								1091.817	2975.529
				5	31	43.0		5	31	59.0	32.785	3.161	-0.002		32.632	0.000		5	31	47.2	32.784	5	31	38		
C	99	05	02.0				08							1094.975			3008.157								1094.975	3008.161
				284	36	45.0		284	37	09.0	81.071	-78.446	-0.005		20.462	-0.001		284	37	03.2	81.075	284	36	54		
D	105	22	43.0				08							1016.524			3028.617								1016.523	3028.618
				209	59	28.0		210	00	00.0	33.044	-16.522	-0.002		-28.617	0.000		210	00	09.1	33.045	210	00	00		
A														1000.000			3000.000								1000.000	3000.000
	Misclose:					-32.0	32				241.927	0.014	-0.014		0.003	-0.003		swing:		-9.1						
							sum				sum	misc	sum		misc	sum										
												Linear Misclose:		0.014		Ratio 1:	16691									

© Version: 15 Apr 15

Booking Sheet: Traversing

	Date:			**Location:**		**Group No.:**	
	Instrument:			**Serial No.:**			
Group Members:							
Weather:							
Instrument Settings:							
Horizontal Directions							
At	**To**	**FL**	**FR**	**Mean**	**Red. Mean**	**Grand Mean**	**Comment**
Distances							
At	**To**	**Hor Dist FL**	**Hor Dist FR**	**Mean Dist**	**Slope Dist**	**ZA**	

FIELDWORK 2: 2D Traverse between MGA coordinated points

Students at UNSW will be given more specific instructions for this field class, including details about instrument use and about the existing survey marks. Readers not at UNSW are supplied with the following notes so they can see the type of exercise UNSW students do in this course. The exercise could be modified to suit other campuses.

1. AIM

To determine with an accuracy of a few millimetres the horizontal coordinates of some survey marks using EDM constrained-centring traversing.

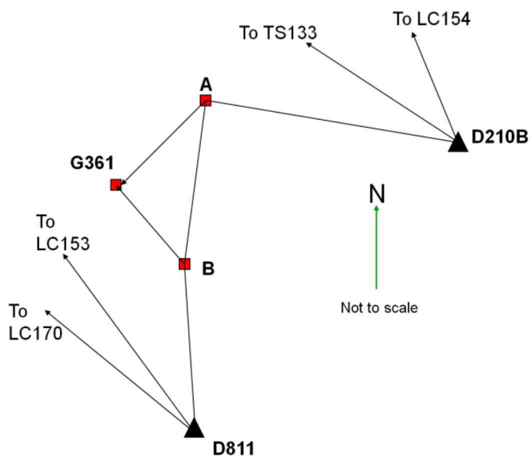
2. METHOD

The traverse starts and ends on control points near the UNSW Kensington Campus. Each group is allocated a starting and an endpoint and must place at least two intermediate points. The group must also observe to (but not occupy) mark **G361** from two of the intermediate traverse stations. Orient the traverse to two known control points at the starting and endpoints. Calculate the bearings of these lines from coordinates, before the fieldwork day. This traverse will “carry bearings”. The instrument can carry coordinates, but we will not do so in this practical exercise. Careful configuration of the instrument, proper levelling and centring of the total station, and careful observations will reduce the incidence of gross errors. Be very careful when crossing roads.

3. SEQUENCE OF OBSERVATIONS

Plan, by reconnaissance, the location of at least two intermediate points (A and B) taking into account that you will have to observe to a prism located over a mark **G361** for which you will later need to calculate the coordinates. *No set-ups on roads.* Place all traverse (dumpy) pegs before taking any measurements.

If working on a concrete footpath is unavoidable, ensure that the tripod legs are firmly placed (by pressing down on them with your feet). Where possible, locate the points of the tripod feet in cracks in the path. The supervisor also has a chisel that you may borrow to make small holes for the tripod legs. Take *extreme care* not to bump the legs from then on.



Read the operation instructions for your instrument. If necessary, ask the supervisors to check the instrument settings and to check your bookings before moving to another instrument station.

On a field form, draw a sketch of your traverse, similar to the sample shown. You should have at least two ROs (reference objects = targets with known coordinates) at each end of the traverse.

At the *starting station (e.g. D210B)*:

- Set up the instrument at the starting point and the reflector at the first intermediate station A. Carefully centre and level the instrument before turning the instrument on.
- Check the instrument settings that are relevant. In particular the correct reflector constant must be set and the level sensor/compensator must be switched on. The electronic tacheometer should be set to display direction, horizontal distance and height difference.
- Measure and book the pressure (in hPa) and the temperature (˚C). Enter pressure and temperature into the instrument.
- Measure one are of horizontal **directions** to two orientation points (RO) and the first intermediate station. Set calculated bearing to one of the RO, and use the other as a check. After setting bearing to an RO, turn the cross hairs away from the target then back on to the target and book your new reading; it may be a few seconds different to your setting. The aim of keeping your FL readings close to the actual bearing of a line is to make the calculations (mental arithmetic) easy and to indicate your misclose at the end of the traverse.

Example Booking of Horizontal Directions (setting and carrying bearings)

AT	TO	*F.L.*	*F.R.*	MEAN	REDUCED MEAN
D210B	TS133	*9° 03' 24"*	*189° 03' 30"*	9° 03' 27"	9° 03' 24"
	LC154	*27° 56' 37"*	*207° 56' 46"*	27° 56' 42"	27° 56' 39"
	A	*193° 42' 00"*	*13° 42' 10"*	193° 42' 05"	193° 42' 02"
A	D210B	*13° 42' 02"*	*193° 42' 10"*	13° 42' 06"	13° 42' 02"
	B	*147° 53' 30"*	*327° 53' 47"*	147° 53' 38"	147° 53' 34"
	G361	*313° 38' 48"*	*133° 39' 00"*	313° 38' 54"	193° 38' 50"

- Measure the **horizontal distance** to the first station in FL and in FR.

Example Booking of Horizontal Distances

FROM	TO	FL	FR	Mean	MGA Distance #
D210B	A	*92.347*	*92.349*	92.3480	92.3404
A	D210B	*92.348*	*92.348*	92.3480	92.3404
A	B	*85.739*	*85.740*	85.7395	85.7325

The mean horizontal distances were reduced directly into MGA'94 distances by using the (average) MGA'94 correction factor for this site, namely K = 0.9999182.

- Check all observations for consistency. Re-observe if necessary.

- At the starting point, replace the instrument with a reflector taking care not to disturb the tripod and the tribrach. Level this reflector and point to the new instrument station. At the first intermediate station remove the reflector and replace it with the instrument without disturbing the tripod and the tribrach. Set up (centre and level) the second reflector at the next intermediate station.

At all *intermediate stations (A, B and if necessary C)*:

- Level the instrument. Measure and book the pressure and the temperature. Input the pressure and temperature into the instrument.
- Measure one arc of horizontal directions. Set the back bearing calculated from the previous station observations, to the backsight.
- Measure the horizontal distances to the previous and to the next station, and to any radiation point, each in FL and FR.
- Check all observations for consistency. Re-observe if necessary.
- Replace the instrument at this intermediate station with a reflector (and level the reflector). Replace the reflector at the next station with the instrument. Remove the tripod and the reflector from the previous point and set them up (and centre and level it) at the next station.
- At the two intermediate stations that are most appropriate for observing **G361**, include the directions to **G361** in your two sets of horizontal directions and measure the horizontal distance to **G361** twice each. Check all observations for consistency and re-observe if necessary. **You are not allowed to occupy point G361 with the total station.** If there is a standard prism on G361 you need to carefully turn the prism located over **G361** to suit your observation needs, after checking that another group is not observing to it. If there is a 360° prism on G361 note that on your booking sheet because it may have a different prism constant.

At the *end station (e.g. D811)*:

- Level the instrument. Measure and book the pressure and the temperature. Input the pressure and temperature into the instrument.
- Measure one arc of horizontal **directions** to the last intermediate point and to the two end orientation points.
- Measure the **horizontal distance** to the previous intermediate station, in FL and FR.
- Check that the forward and backward (horizontal) distances are in reasonable agreement.
- Compute the angular misclose. The tolerance for the angular misclose is 60" of arc (for beginners ☺).
- Book the makes, types and serial numbers of the instrument, prisms, thermometer and barometer.

4. COMPUTATIONS

Obtain the grid distances in the map projection system (Map Grid of Australia 1994, MGA'94) by multiplying the (mean) measured horizontal distances by the (approximate) combined sea level and grid (Map Grid of Australia 1994, MGA'94) correction factor K = 0.9999182 for this site. [K for this site was computed with E = 336556 m, an AHD height (H) of 47.7 m, a bearing of 45°, a GDA'94 latitude of -33.92° and a geoid-ellipsoid separation (N) of 22.5 m. See another course for an explanation of these terms.]

Obtain the MGA'94 coordinates for the intermediate stations by completing the traverse calculations, including an adjustment by the Bowditch rule. Calculate the angular (in ") and the linear misclose (in mm). The tolerances are 60" for the angular and 25 mm for the linear misclose.

Calculate the MGA'94 coordinates of **G361 from** the **two** observing intermediate stations, after first having adjusted the traverse. Compare the two solutions and take the mean, if no gross errors are evident.

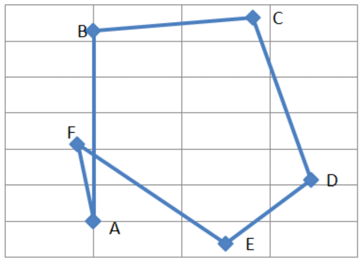
Prepare an input file for a least squares network program (FIXIT at UNSW) for your data. Example FIXIT input file:

```
TITLE     Example Willis St Traverse
COMMENT          Dis       ppm       Dir      cent
DSD              5.0       0.0      10.0       0.0
COORDINATE    1 EN  D210B               336633.044    6245395.354
COORDINATE    2     A                   336571.       6245406.
COORDINATE    3     B                   336547.       6245369.
COORDINATE    4 EN  D811                336547.541    6245280.759
COORDINATE    5     Target (B/G361)     336548.       6245374.
COORDINATE    6 EN  TS133               335477.610    6245940.409
COORDINATE    7 EN  LC154               336498.902    6245471.555
COORDINATE    8 EN  LC 153              336482.351    6245417.666
COORDINATE    9 EN  LC 170              336428.784    6245331.121
Comment     Mean MGA distances
H DISTANCE    1    2             62.7694
H DISTANCE    2    3             44.2769
H DISTANCE    2    5             38.8733
H DISTANCE    3    5              5.9125
H DISTANCE    3    4             87.6276
Comment     Reduced means of directions, carried bearings
DIRECTION     1    6    1 295 15 17.00
DIRECTION     1    7    1 299 36 10.00
DIRECTION     1    2    1 279 22  8.00
DIRECTION     2    1    1  99 22  8.00
DIRECTION     2    5    1 216  8 24.00
DIRECTION     2    3    1 212 48 38.00
DIRECTION     3    2    1  32 48 38.00
DIRECTION     3    5    1  10 24 25.00
DIRECTION     3    4    1 179 43  6.00
DIRECTION     4    3    1 359 43  6.00
DIRECTION     4    8    1 334 31 25.00
DIRECTION     4    9    1 292 59 24.00
```

4. Traverse Calculations & Adjustments: Worked Solutions

These solutions can be read AFTER you have made some attempt to solve the question.

Q1. The coordinates of a point e.g. B are found using $E_B = E_A + D*\sin(B)$ and $N_B = N_A + D*\cos(B)$ or use P→R on calculator to get the ΔE and ΔN components, and add them to previous point's E and N respectively. →



A	400	600
B	400.000	652.965
C	436.250	656.573
D	449.394	611.622
E	430.039	593.938
F	396.400	621.341
A'	400.007	599.999

Misclose E = $E_{finish} - E_{start}$ = 0.007 Misc N = -0.001
Misclose dist = sqrt(MiscloseE^2 + MiscloseN^2) = 0.007
Ratio 1: sum distances / misclose dist = 1: (2275/0.007) = 1:33,000

Q2.

Pt	Angle				Cor	Adj Brg	Dist	ΔE	cor E	Easting	ΔN	cor N	Northing
	D	M	S	Dec Deg	"	Dec Deg	m	m			m	m	
B										1057.280			2492.390
	Brg from coords:			119.189		119.189							
A	115	37	0	115.617	19.86					1769.150			2094.720
			Brg:	54.806		54.811	208.26	170.202	0.0138		120.0148	0.0100	
1	168	19	10	168.319	19.86					**1939.366**			**2214.745**
			Brg:	43.125		43.136	193.47	132.282	0.0128		141.1813	0.0092	
2	281	12	40	281.211	19.86					**2071.660**			**2355.935**
			Brg:	144.336		144.353	326.71	190.405	0.0216		-265.491	0.0156	
3	242	53	40	242.894	19.86					**2262.087**			**2090.460**
			Brg:	207.231		207.253	309.15	-141.564	0.0205		-274.833	0.0148	
4	80	26	20	80.439	19.86					**2120.543**			**1815.642**
			Brg:	107.669		107.697	224.79	214.152	0.0149		-68.3326	0.0107	
X	173	31	0	173.517	19.86					2334.710			1747.320
			Brg:	101.186		101.219			Given:	2334.710		Given:	1747.320
Y							1262.38	565.476		2995.850	-347.461		1616.180
	Brg from coords:			101.219		101.219		565.56			-347.4		
	Misclose:			-0.0331	-119.2	0		-0.0838	0.0838		-0.0606	0.0606	

So the angular misclose is -119" and the miscloses in E and N are -84mm and -61mm. Those miscloses are very large by the standards of current instruments, so the data above would need to be checked for errors before using the coordinates on a real site. But they are good numbers for students to practice calculations on (if all the miscloses were about 1" and 1mm you wouldn't see much to adjust).
1 (1939.366, 2214.745) 2 (2071.661, 2355.935) 3 (2262.087, 2090.459) 4 (2120.543, 1815.641)

Q3.
Bearing AZ is 135° 00'. Calculate the bearing DR from coordinates. Calculate the traverse miscloses. Note angles need to be clockwise from BS to FS to calculate bearings.
Ang Misc -04', Correction per angle +01', Misc E 0.276 Misc N -0.136 Total Misclose Dist 0.31

With this question you need to remember that bearings in a traverse are calculated from bearing to backsight + clockwise angle to foresight. In this data as shown in the figure, the angles at B C and D are on the 'wrong' side. The angle we want is 360° – the value shown. The traverse bearings are:

AZ given	135°	00'
A	85°	05'
AB	220°	05'
B	124°	52'
BC	275°	13'
C	318°	02'
CD	137°	11'
D	90°	15'
DR	226°	56'
DR from coords	227°	00'
Ang Misc		-04'
corr per ang		01'

Adjusted bearings		Dist	E	N
AZ given	135°00'			
A			3000.00	3000.00
AB	220°06'	655.90		
B			2577.52	2498.29
BC	275°15'	338.50		
C			2240.44	2529.26
CD	137°14'	981.30		
D			2906.76	1808.86
		Misc	0.276	-0.136

Total misclose distance = 0.31m. This misclose is very large by the standards of current instruments

Q4.

	Brg/angle	°	‘	“	Adj Brg	Dist	E	Raw E	N	Raw N
PM 233							323780.189		1245841.133	
	265.67				265.67					
PM234	127.88	127	52	30			321640.675	321640.675	1245679.240	1245679.240
	213.55				213.54	223.00				
T1	323.75	323	45	00				321517.456		1245493.374
	357.30				357.29	433.67				
T2	203.26	203	15	30				321496.927		1245926.558
	20.56				20.54	210.67				
T3	279.54	279	32	30				321570.841		1246123.836
	120.10				120.08	126.00				
T4	248.41	248	24	30				321679.877		1246060.692
	188.51				188.48	294.65				
PM234	77.20	77	12	00				321636.435		1245769.262
	85.71				85.67					
Misclose	120”				0.00	1287.99				
corr / angle	-20”					Sum dist	misc m	-4.240	misc m	90.022

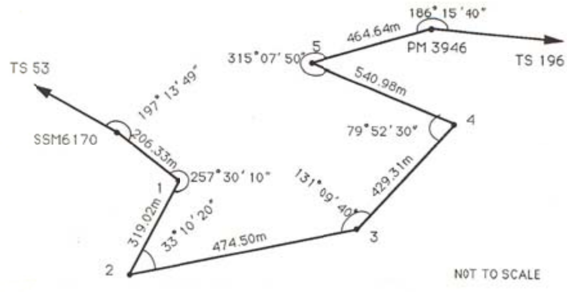
The angular misclose is 20” per angle which is reasonable given the round off in the input angle data (i.e. nearest 30”). But the total linear misclose is 90.122 m! The bearing of the misclose is 357.3° which is very close to the bearing of the line T1 - T2. It looks like T1-T2 should be 343.67 not 433.67. If it was 343.67 then the misclose would be 125 mm which is OK.

Since the coordinate misclose is **not** small, due to an apparent distance error, we do not calculate adjusted distances or adjusted coordinates.

Q5.

The calculations for the miscloses and adjusted coordinates of the traverse stations are shown in the table below. There is a huge angular misclose, 2847” which is about 50’ more than we would expect from data of this quality. So we don’t adjust the bearings or coordinates. But we can try to find the likely error, if there is only one wrong angle. Calculate raw coordinates using unadjusted bearings starting at SSM6170 and head east. Then do likewise starting at PM3946 and head west. Note the angles used when travelling west are 360° - angle in figure above.

Compare the coordinates of each point form the two sets of calculations and it appears that the angle at 3 is wrong because it has similar coordinates from both ends. That is because the coordinates are calculated up to that point without use of the erroneous angle. I added 5000m to northings in my solution below to avoid negative coordinates, but that is not essential – the process still works the same.



						E	N
TS53						455.14	5490.45
SSM6170						598.63	5275.28
PM3946						1182.71	5575.08
TS196						2046.83	5613.93
Forward Calc							
TS53	Dec °				dist	455.14	5490.45
	146.30						
SSM6170	197.23	197°	13'	49"		598.63	5275.28
	163.53				206.33		
1	257.50	257°	30'	10"		657.120	5077.414
	241.04				319.02		
2	33.17	33°	10'	20"		378.004	4922.920
	94.21				474.5		
3	131.16	131°	09'	40"		851.225	4888.109
	45.37				429.31		
4	79.88	79°	52'	30"		1156.739	5189.719
	305.24				540.98		
5	315.13	315°	07'	50"		714.916	5501.892
	80.37				464.64		
PM3946	186.26	186°	15'	40"		1173.013	5579.588
	86.64						
PM3946						1182.710	5575.080
	87.43						
TS196						2046.830	5613.930
Misc "	2847						
Reverse calc							
TS196						2046.83	5613.93
	267.43						
PM3946	173.74					1182.71	5575.08
	261.16				464.64		
5	44.87					723.583	5503.713
	126.03				540.98		
4	280.13					1161.056	5185.473
	226.16				429.31		
3	228.84					851.409	4888.108
	275.00				474.5		
2	326.83					378.714	4929.447
	61.83				319.02		
1	102.50					659.935	5080.073
	344.32				206.33		
SSM6170	162.77					604.182	5278.728
	327.09						
SSM6170						598.630	5275.280
	326.30						
TS53						455.140	5490.450
Misc "	-2847						

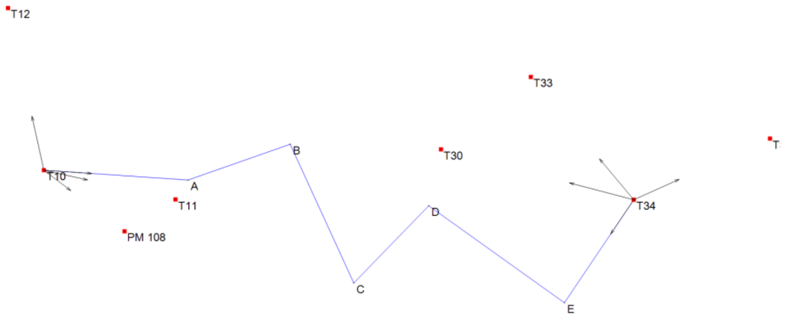
	Compare coordinates with forward run		
Pt	Diff E	Diff N	RSS
5	8.668	1.821	8.857
4	4.318	-4.246	6.056
3	0.184	-0.001	0.184
2	0.710	6.526	6.565
1	2.815	2.660	3.873

So there is a large angle error at point 3. The angle should be close to 132°

Q6.

One solution method is to get mean orientations at each end of traverse and hold bearings of end lines fixed.

You are welcome to send me your solution. Here is part of mine:



We need to rotate the direction observations to bearings. So bearing = direction + R

Calculate the bearing from coordinates for each of the directions to known targets, and then calculate R. At each end point calculate the mean R, and then apply that to the direction observation along the traverse.

at T10	Bearing		R
T12	347°31'22.1"		347°31'22.1"
T11	102°30'40.1"		347°31'06.1"
PM 108	127°04'56.7"		347°31'16.7"
A	93°53'15.0"	Mean:	347°31'15.0"
at T34			
T30	284°38'13.2"		284°38'13.2"
T33	319°57'53.3"		284°38'14.3"
T35	65°51'20.2"		284°38'19.2"
E	214°00'33.5"	Mean:	284°38'15.5"

Next calculate bearings through traverse and then calculate angle misclose.

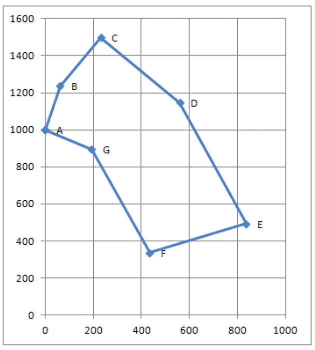
Brg T10 to A	93	53	15
angle A	156	50	57
	70	44	12
angle B	264	37	13
	155	21	25
angle C	68	52	21
	44	13	46
angle D	261	17	53
	125	31	39
angle E	88	31	19
Brg E to T34	34	02	58
Brg by rotation calcs	34	00	34
misclose			144

The misclose is 28.9” per angle, which is large for current instruments.

Q7.

Ans. 118°29'17" 220.755

I solved this problem by choosing arbitrary coordinates for A.



Pt	Bearing or angle	Dist	Easting	Northing
A			0	1000
	14°48'00"	245.62		
B	198°06'30"		62.743	1237.471
	32°54'30"	310.00		
C	284°01'30"		231.165	1497.729
	136°56'00"	480.34		
D	200°12'30"		559.164	1146.812
	157°08'30"	709.27		

E	271°33'30"		834.683	493.242
	248°42'00"	430.14		
F	268°01'30"		433.925	336.993
	336°43'30"	607.12		
G			194.025	894.705
Brg from coords:	118°29'17.3"	220.755	Dist from coords	

Q8.

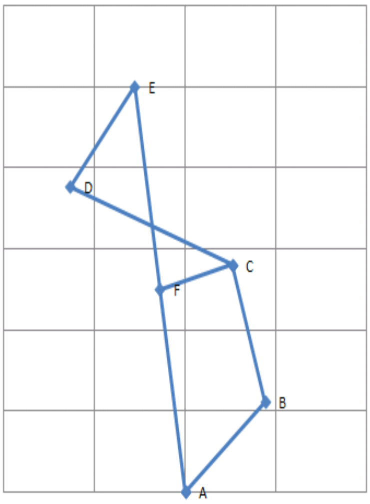
I solved this problem by choosing arbitrary coordinates for A.

Pt	Bearing	Dist	Easting	Northing
A			200	0
	38°20'00"	142.00		
B			288.073	111.387
	347°55'00"	172.83		
C			251.894	280.388
	298°12'00"	203.15		
D			72.857	376.387
	29°46'00"	143.32		
E			144.011	500.796

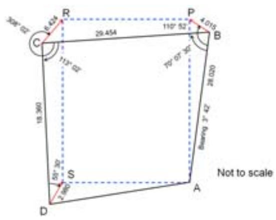
F is midway AE which is simply $E_F = (E_A+E_E)/2$ and similarly for NF

F			172.006	250.398

Brg CF from coordinates = 249°25'27"
Dist CF from coordinates = 85.332



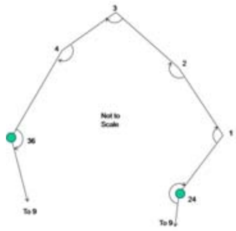
Q9.
Note this is not a fully observed traverse. The closing angles and distances are not supplied, so we can't calculate a misclose or adjust a loop traverse. So the loop data can't be checked, and neither can the radiations!
(a) Compute bearings of BC and CD.
(b) Assuming arbitrary coordinates for A, compute the coordinates of B, C & D.
(c) Compute coordinates of P, R & S from B, C & D respectively and then compute the bearings and distances of the property lines.



At	brg/ang	D	M	S	Dist	E	N
A						300	100
	3.70	3	42		28.02		
B	70.13	70	07	30		301.808	127.962
	253.83				29.454		
C	113.03	113	02			273.520	119.757
	186.86				18.36		
D	55.50	55	30			271.328	101.528
	62.36				2.98		
S						273.968	102.910
B	110.87	110	52				
P	294.57				4.015	298.157	129.631
C	308.03	308	02				
R	21.86				6.424	275.912	125.719
	Joins						
AP	356.44	356°	26'	24.6"	29.688		
PR	260.03	260°	01'	31.8"	22.586		
RS	184.87	184°	52'	20.9"	22.891		
SA	96.379	96°	22'	45.2"	26.195		

Q10.

a) The angle misclose = -8". So the angles are OK, no error in angles or given bearings. There is an 18m linear misclose parallel to line 2-3 so we suspect an error in distance 2-3. My estimate of the correct distance 2-3 = 75.832m. Linear misclose is not acceptable, so do not adjust the coordinates. I would investigate the field data to see if distance was measured from 2 to 3 and from 3 to 2 and what the raw distances were. Perhaps only one of them is wrong or the mean distance has been incorrectly recorded or entered into the calculations.



b) The angle misclose = -8". So the angles are OK, no error in angles or given bearings. There is a 200m misclose in E, and 28 mm in N. The linear misclose is not acceptable. The misclose is not parallel to any line, so probably not an error in a single distance. Error is most likely to be in the Easting coordinates of the known control points. The likely correct estimates are E 36 = 20.58, or E 24 = 171.94 – 200m. I would check the coordinates of the control (known) points.

c) The angle misclose = 17.998°. This is not acceptable. Calculate bearings and coordinates of the traverse forward, i.e. from 24 to 1 etc through to 36.

```
Pt   Angle     Bearing  Dist    Easting  Northing
     d  m  s   d  m  s   m         m        m
9                               132.08   6981.69
               6 21 20  (Bearing from coordinates)
24 211 52 52                    171.94   7339.56
              38 14 12  63.575
1  106 25 56                    211.287  7389.496
             324 40 08  72.854
2  186 43 16                    169.156  7448.932
             331 23 24  75.832
3  101 26 53                    132.844  7515.505
             252 50 17  61.556
4  155 43 06                     74.029  7497.341
             228 33 23  84.805
36 134 15 33                     20.58   7392.58
             182 48 56
9                               132.08   6981.69

Bearing 36-9 from coordinates:  164 49 04
```

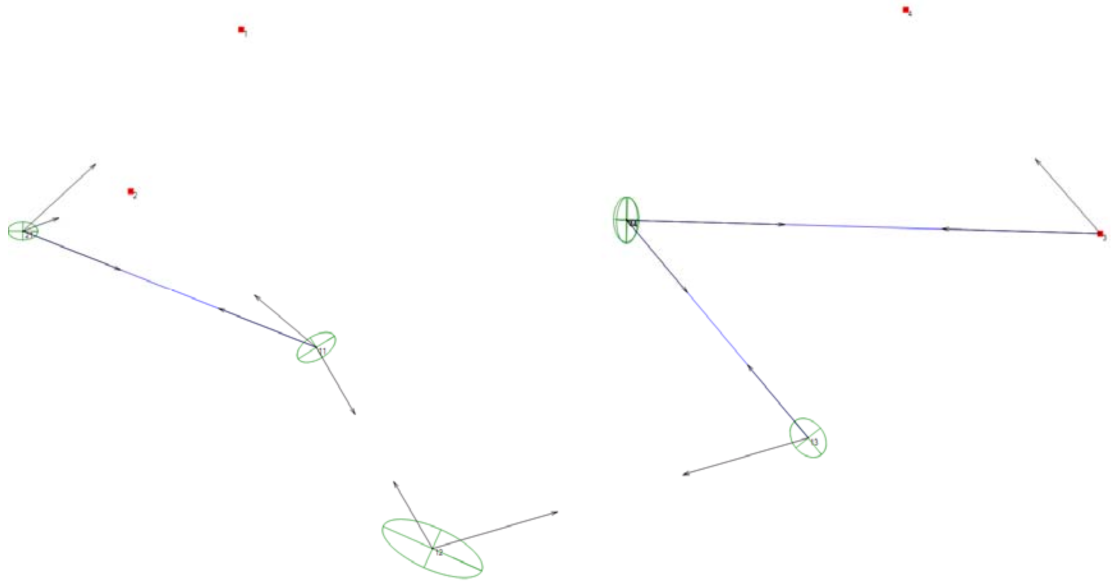
Calculate Reverse direction traverse from 36 to 24 and be careful calculating bearings because angles are 'backwards'. Compare coordinates of Forward and Reverse traverse calculations.

```
Pt    Angle     Bearing   Dist  Easting  Northing   Diff from forward run
     d  m  s   d  m  s     m       m         m          ΔE m     ΔN m
9                              132.08   6981.69
             344 49 04 (Bearing from coordinates)
36 134 15 33                    20.58   7392.58
              30 33 31  84.805
4  155 43 06                    63.696  7465.606     -10.332   -31.734
              54 50 25  61.556
3  101 26 53                   114.021  7501.054     -18.822   -14.451
             133 23 32  75.832
2  186 43 16                   169.126  7448.958      -0.029     0.027
             126 40 16  72.854
1  106 25 56                   227.561  7405.448      16.274    15.953
             200 14 20  63.575
24 211 52 52                   171.94  7339.56
             168 21 28  Bearing from coordinates: 186 21 20
9                              132.08  6981.69
```

The difference in coordinates between forward and reverse runs show that only point 2 has coordinates that are similar in both directions. So the angle error is probably at point 2. A correction of - 17.998° to the angle at 2 will remove misclose. 186° -18° = 168° i.e. a reversal of digits. My next step would be to investigate the raw observations, at the level of FL and FR readings (if I hadn't already done so when I saw the large angular misclose!).

Q11.
The angle misclose = +10", and is acceptable. There is a 20m linear misclose parallel to line 4-5, which is not acceptable. There is probably an error in the distance 4-5. It should be about 61.5m. I suggest investigate the forward and reverse measurements of distance of that line – if the raw field data is available.

Q13.
You are welcome to send me your solution. I have solved this problem with my least squares program (It is easier, but I haven't taught you how to that yet.) As an incentive to learn about least squares I give my solution.



If I select standard deviations of the observations as:
Distances 4.00 mm + 1.00 ppm
Directions 3.00 sec and 1.00 mm centring std dev at inst and at target
Given:

Point	E m	N m
C1	2341.71	1580.11
C2	2230.89	1417.88
C3	3204.05	1375.74
C4	3008.91	1600.14

Then the Adjusted Coordinates are:

Point	E m	N m
T1	2417.066	1261.105
T2	2533.668	1059.743
T3	2910.783	1170.478
T4	2728.240	1389.679
T4 ecc	2729.022	1388.891
X21	2122.557	1378.191

And the corrections (adjustments) to the observations are all small.

5. Introduction to CAD for Surveying

This chapter provides an introduction to the type of CAD software used by surveyors in NSW, a discussion on choosing which CAD package to use, and details with a practice session and worked solution on how to communicate between survey instruments and CAD (i.e. data upload and download). The associated lectures will demonstrate one or two commercial CAD packages.

Computer Assisted Drafting (CAD) is computer software that enables engineers and architects to design everything from furniture to airplanes. Surveying and Civil Engineering versions of CAD are specialised applications designed to do topographic map drafting (plan drawing), survey calculations and road design etc. The types of Plane Surveying that we will use CAD for in this course are:

- Topographic / Detail Surveys to determine locations & elevations and draw plans of features of the landscape;
- Cadastral surveys and lot subdivision, legal boundary dimensions and lot areas.
- Engineering surveys such as Road Design and setting out of buildings.

There are many other applications in drainage and sewer design, mining, dam surveys, hydrographic, highway, rail and tunnel surveys.

In the past surveyors took measurements in the field and recorded them on paper. Later they did calculations and drew their plans. Now we have automatic field mapping systems where observations are recorded on the instrument and later downloaded to computer software that does calculations and draws plans. We can use theodolite/EDM total stations, some have robotic tracking of a prism on a pole, which is ideal for detail surveying, or we can use GPS. Some total stations have computer screen map displays, but that has not been successful yet.

In the future (and I have been waiting a long time for this) we will collect the detail survey data in the field and at the same time process it and draw the surveying plans etc. The office post-processing stage can then be eliminated. This will allow us to see the contours and the plan location of buildings etc in the field as we survey the site and helps us spot errors, missing data, etc.

Why use CAD?

	CAD system	**Traditional drafting**
Survey Computations	Built in calculations	Calculations separate
Speed	Fast	Slow
Accuracy	Accurate	Good
Repeatable	Yes	No
Editing	Easy	Difficult
Archive	Electronic & paper	Paper

What is the difference between GIS and CAD? GIS and CAD initially look similar but CAD geometry contains many horizontal and vertical lines, GIS doesn't. In CAD, circular arcs and curves are essential, in GIS they are rare. In CAD a typical polygon has few vertices; in GIS a polygon may have many thousands of vertices. Databases in CAD are often peripheral to the main task. In GIS, the database is the most important aspect of the system. With future software development it is likely that CAD and GIS will include features and aspects of each other.

CAD systems

- Easy to handle locations and shapes of geographic features
- Easy to calculate distances, areas, and volumes
- Do not have attribute data

Desktop mapping

- Most of the capabilities of CAD systems
- Rudimentary linkages between location data and attribute data
- Simple queries based on single attribute files

Geographic information systems

- Capabilities of CAD and desktop mapping systems
- RDBMS-based attribute (or even object) management
- Query and identify locations or routes matching multiple criteria
- Support various spatial analysis to transform geographic data into geographic information

CAD Lab Introductory Exercise

- Familiarize yourself with your chosen CAD software's interface, menus, toolbars, etc and frequently used functions along with an instructed lab exercise. Refer to online or built in help or user manuals.
- If there is an introductory tutorial for your software, do it.

Detail Survey Principles

- Decide where to put your control marks if the survey is being measured by total station, so that you can obtain the coordinates of the instrument and can see (to measure by radiations) the features of the site. With RTK GPS this may not be necessary. Calculate coordinates and height of marks and features.
- Survey trees (note trunk and spread radius, tree type etc.)
- Description of object / feature should be recorded. Often use codes to identify detail point types e.g. building, drain, fence, road, tree; and strings to draw symbols and lines in the plan 'automatically'.
- Be aware of scale of finished plan (1:200 implies 1mm on plan is 0.2m on ground)
- Spacing of natural surface points must reflect scale of plan and nature of terrain
- Pick up enough points to define curves
- Use check measurements
- Particular attention around buildings (remember corners are usually 90°)
- Check where possible for blunders, bumped instrument, etc.
- Better to have too many points than too few!

Should we take FL and FR theodolite observations? Consider:

- 2 faces gives repeat observations, their mean reduces random errors, their differences help detect gross errors
- 2 faces reduce systematic errors: axis tilt, collimation error, etc
- 2 faces do not reduce vertical axis (levelling) tilt or centring error
- 1 face observations are faster and since plotting requires low precision may be precise enough for detail surveys

So generally we do traverse control observations with 2 faces, and radiations of detail features with 1 face.

Checks, if observation is in 1 face only: Does the plan look OK? Do buildings have 90° corner? Are trees etc in the correct position? There should not be unexpected holes or mountains in contours. We might consider radiating some points from two positions, and compare building side lengths with tape measurements.

CAD for Detail Survey

Most CAD software for surveying can do:

- Survey data input and reduction
- Survey computations such as radiation, bearing and distance, intersection of lines
- Traverse calculation and adjustment
- Points and string-line plotting
- Draw and edit points, symbols, lines, polygons, arcs, circles, text etc
- Layer control and management
- Terrain modelling
- Contour plotting with break lines
- Volume Calculations
- Perspective views, and perhaps 3D fly-through animations
- Plan plotting / printing
- Data output to other CAD or GIS data formats.

Feature codes with the survey measurements lead to automated map production by drawing features with appropriate symbols e.g. trees, power poles, and linking linear topographic features (strings) e.g. building outline, fence line, road edge.

Layers are imaginary transparent surfaces in a CAD drawing that help to organize CAD drawings and makes editing much easier. CAD software allows users to turn layers on/off to view or edit specific objects.

Data editing can be done to account for errors in data coding (e.g. delete spurious detail points), to change detail point types (e.g. pole to tree), or to change line types (e.g. fence to road). Plans or the source data set can also be edited to make aesthetic changes (e.g. change colours from their default settings) and add non-surveyed information (e.g. building names and road names). The non-surveyed information can be positioned by coordinates and stored along with the surveyed information or positioned graphically (e.g. by mouse) as independent text objects.

Terrain Modelling

A Digital Terrain Modelling (DTM) takes the heights of detail points on the ground and uses appropriate algorithms to create a 3D mathematical terrain. Using a DTM, it is possible to produce contour maps, produce cross-sections, calculate volumes, calculate cut and fill, create 3D visual models (views of the landscape) and fly-throughs. The DTM can be in the form of a TIN or a DEM; more about those later.

CAD for Road Design

Most CAD software for Road Design can do:

- Long and cross sections
- Intersection design
- Urban and rural road design with super-elevation
- Cut and fill volumes calculation
- Cross sections with table drains, batters and boxing
- Facilities for road reconstruction plans and calculations

Which CAD Software?

At UNSW we currently teach CAD with CivilCAD. There are a few, mostly historical, reasons for the use of CivilCAD. We don't have to have the best software, and how do you define that anyway? We do not have time to teach more than one package and we do not try to teach all aspects of the software. We aim to make students aware of some features of CAD and to gain some experience. Further training can often be obtained during students work experience e.g. summer employment. The lab exercises and assessment tasks in this course will use CivilCAD. Students who already have experience and access to other CAD software may use it, but staff may not be able to assist with specific technical and debugging style questions.

There are many “brands” of surveying CAD software available in Australia and around the world. Some are listed here in no particular order (apologies if I omit your favourite ☺): CivilCAD (Topcon Australia), Civil 3D (AutoDesk), LandMark, Geocomp, LISCAD, MOSS (MX ROAD), 12d Model, Civil 3D, Pythagoras, Microsurvey, Microstation, AutoCAD + many more …

We could do a survey to find out which CAD software is most common amongst local surveyors. But the most common CAD software is not necessarily the best. I suspect surveyors don't like changing their CAD because of training time and other reasons. Some surveyors use one type of CAD for the survey processing and design and another for the plan drawing and export, i.e. for ‘polishing’ plans or for sending plans to clients.

Many surveyors wonder if the software package they are using to reduce their observations, draw plans, design roads and calculate volumes etc. is the best package for them. Surveyors currently using CAD should consider:

- Is there a more suitable package out there?
- Will changing products be too expensive?
- Will changing products disrupt plan production?
- How do I know which packages to consider?

One way to determine this is to have an in depth review of the other packages available on the market and compare them to what you are currently using. Or learn and use them all! One of our former students, Candice Lowe, did her thesis in 2006 titled “A Critical Review of Surveying CAD Packages”. A brief overview of her thesis is on our class web site, or contact me if you wish to read the full thesis. She compared several CAD packages in terms of cost, ease of use, support provided, time required to learn how to use the software, quality of the output produced, and compatibility with instruments.

Communications with Surveying Equipment

Transfer (download) of survey data from the field, whether measured by Theodolite/EDM Total Stations or GPS, is now usually done by electronic transfer from data stored on the instrument to the computer software. Though it is still possible to write the field data on paper in the field and type it into a computer software later, that is a tedious time consuming and error prone task that is now history except for very small surveys or for educational purposes.

Similarly, coordinates of traverse stations or road set out marks that have been calculated or designed in CAD can be electronically transferred (upload) to survey instruments. In the field we can then refer to point numbers and the instruments recall the coordinates and guide the set out.

Some of the methods to get data in and out are cables, USB ports, data cards and Bluetooth.

Website References (you are welcome to update this list):

Trimble, “GeoExplorer Series: Communicating with External Devices”
http://trl.trimble.com/docushare/dsweb/Get/Document-172246/MGIS_SprtFAQ_GeoExplorerSeries_CommunicationExternalDevices.pdf
Trimble, “Bluetooth Connections with Microsoft Windows CE.NET”,
http://trl.trimble.com/docushare/dsweb/Get/Document-154598/SprtNote_GeoCE_CE.NET_Bluetooth_Connections.pdf
http://www.usb.org/home
http://www.usb.org/developers/usb20/developers/whitepapers/usb_20g.pdf
http://www.pcmcia.org/pccardstandard.htm
http://www.accurite.com/PCMCIAprimer.html

Total Station ←→ CAD

This section gives instructions for a Lab exercise: Electronic “Field to Finish” Detail Surveying. The purpose of this exercise is to gain experience with computer aspects of a surveying instrument – including uploading data, using the uploaded data, collecting measurement data and entering descriptions (codes) of the detail survey points for use in CAD, using menus on the instrument, and downloading the results. Our intention here is not to make an accurate survey (in the lab). We do *not* expect you to remember every step in this process because there are a lot of small steps (given below). Those skills will be developed with practice later. Various instruments and software work differently, but many aspects are similar, so after learning our one specific type (e.g. Sokkia / CivilCAD) of procedures you may find it easier learning alternative software with future employers.

A detail survey can also be done by laser scanning or by GNSS observations. We don’t describe the procedures here but they are not difficult. It is a matter of observing coordinates of points and the points’ descriptions or codes, and transferring them to CAD.

Manual Coordinate Entry and Detail Survey

Turn on total station and if necessary to get to the instruments main status screen.

Setup a new JOB
This step may include ‘Compensator’ levelling, setting over a mark, checking configuration, entering prism constant, and temperature / pressure corrections, entering map projection scale factor, etc.

Enter coordinates of known points
Type in the Easting, Northing and Height of a point e.g. 100.000, 300.000, 20.000 and its point number e.g. 901

Type in coordinates of more points, especially a backsite target e.g. 80.00, 350.00, 25.00 point 999.

Do some basic detail survey measurements from this coordinated point:
Usually this involves specifying which point number you are set up over, entering height of instrument, then specifying which point number you are going to use as a backsite target (to orientate you directions) and the height of the target above that point. Then point cross hairs to the target, measure and record.
[There are other ways to start a detail survey, e.g. free stationing / resection.]

Take measurements to other points by pointing cross hairs to a prism (or using reflectorless EDM to some other target, or using automatic target recognition to point to a prism), measuring and recording the observation or derived coordinates. It may be necessary to change target height for some points.

Computer Coordinate Upload and Detail Survey

Usually it is possible to create a computer file containing the coordinates of the known points of the survey site and to upload the file to a Total Station using the CAD program or with other transfer methods and software to do this task. Sometimes you need to set communication variables such as baud rate etc.

Example file, in this case it contains point number, E, N, H, description

```
    11,  89966.000,  7480.000,  ,  Start Road
    12,  90015.282,  7553.063,  ,  IP1
    13,  89968.153,  7728.949,  ,  IP2
    14,  90020.613,  7836.507,  ,  End Road
    900,89856.675,7404.574,0.000,SSM153000 Off
    910,90232.272,7846.408,0.000,PM140410
    914,90078.808,7465.591,0.000,PM140414 MOEYAN
    921,90192.161,7882.104,0.000,PIN IN C.B.
    944,90258.648,7816.903,33.614,SSM79544 HILL
```

I suggest you view the data in the total station after transfer, before commencing field work.

FEATURE CODES

Depending on the project there will probably be a set of codes to describe the common features that you will measure in the survey. Sometimes you may create your own codes. The table below gives a small example of feature codes. Some CAD or projects use numbers as codes others allow alphanumeric. Codes are a convenient way to abbreviate your descriptions of the things you are measuring. They also allow the CAD program to automatically place appropriate symbols on points and 'join the dots' for line features as required.

	POINT FEATURES		**STRING FEATURES** (add string number suffix e.g. BLDG01)
Code	*Description*	*Code*	*Description*
PM	Permanent Survey Mark	TOP	Top cut/fill/bank/drain/pondage etc.
SM	Survey Mark (General, new or temporary)	TOE	Toe cut/fill/bank
NS	Natural surface	VEGN	Group of trees/shrubs or garden
TREE*S	Single tree, add radius of spread in m	BOK	Bottom of Kerb
DRN	Drainage pit	TOK	Top of Kerb &/or Channel
SIGN	Sign	EB	Edge of Bitumen
EL	Light Pole (No overhead wires)	EF	Edge of Formation/Shoulder
EP	Electricity Pole	PATH	Pedestrian path
GATE	Gate	BLDG	Major building
UN	Unknown or other point feature	RWALL	Retaining Wall (Crib wall)
		VER	Veranda/awning etc.
		FENC	Fence
		US	Unknown or other line feature

There are other aspects of using codes that can save time with the CAD drawing, such as:

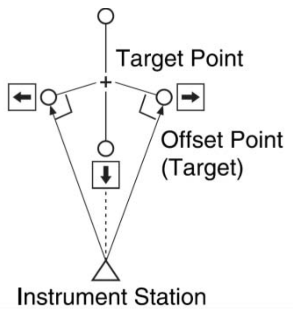
- When one point represents two features you can often join the codes e.g. light pole at edge of path is PATH01*EL, and a fence that meets a building is BLDG02*FENC01.

- Arcs can be drawn in CAD as arcs instead of as straight line segments by assigning appropriate codes. For example: TOK*AS, TOK, TOK*AE.

- Joining / closing lines for example around buildings so that all four sides are drawn by CAD: on the last corner use a special code so that CAD will draw a line from this point to the first point in the string.

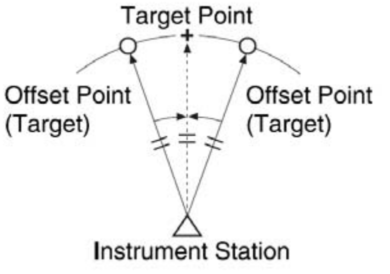
Tree measurement

When trying to coordinate the centre of a large tree or similar large objects it is not possible to hold the prism at the centre so offset measurements are needed. This is not necessary for detail surveys of small objects or small offsets if the offset is smaller than a millimetre or so, on the final plan.



Sokkia has at least two methods of doing this. Try to keep the prism as close to the desired point as possible and with base of pole at the height desired for contours.

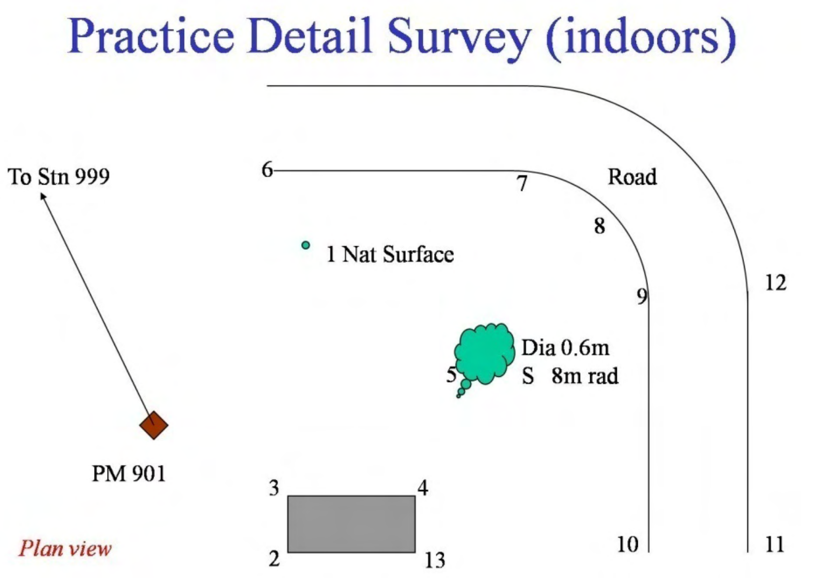
a) Place the prism beside or in front of the desired point, measure distance only to the prism and enter an estimate of the distance to the desired point e.g. 2m, as shown in the figure. Also enter the direction left, right, forward or backward.



b) Place the prism beside the desired point at the same distance as the desired point, measure distance only to the prism and then move cross hairs to centre of tree by turning horizontally (don't tilt the telescope up or down), as shown in the figure. Select one side of the tree, left or right, you don't have to do both sides.

Practice Survey

Here we do a practice detail survey with a total station. Use electronic field recording with codes, then transfer the data to CAD to produce a plan. The plan of the practice survey and detailed step by step instructions are also given in an accompanying PowerPoint file.



This exercise can be done indoors or outdoors. Here I give an overview of the survey in the above plan, and then below I explain the steps in more detail. Set up the total station as though you were on survey mark PM901, but don't worry about actually being on a mark. Observe angle to a backsight (stn 999). Measure to a prism placed arbitrarily nearby in the room (if indoors) as though it was on a prism pole on the natural ground surface at point 1 in the plan above. Move the prism to new points 2 the corner of a building, 3, 4, ... for each observation but don't worry if it is not in accordance with figure above. We are not going to use the data, just practice recording it with correct codes and then download it. Imagine you are observing to a prism at the points in the figure above, in numerical order.

Point numbers. For Control points (existing survey marks and your own traverse stations) we will use Station Point numbers from 900-999, and for Station ID we will use real names e.g. PM 1432 or T6. For detail survey radiation points we will start at 1 and increment by 1 (stop < 900!)

Codes can be entered by number or alphanumeric. We will use a set of alpha codes.

STEP BY STEP instructions for the Practice survey:

Note that some total stations will do things slightly differently to the generic instructions given below.

Select at new 'job' or project on the total station. Enter data such as temp and pressure, prism type and constant. For this exercise use a scale factor of 1.000.

Manually enter coordinate data for points 901 and 999 if it was not uploaded. Enter the point codes and height of instrument or target. Code = PM.

To commence the survey, tell the total station you are set up over point 901 (occupied).
Point cross hairs to a target (no prism necessary); record the angles at the point number 999.

To take measurements to other points:

1) Point to prism at position 1 in the figure, the natural surface point, enter code (= NS), observe and record.

Continue collection for other points in the figure 2 to 12; move the prism. Your software may complain if you don't move the prism for each observation because the data is not changing. You might prefer to leave the prism in the same place for each point and to then ignore messages that complain about measurements remaining the same.

2) At point 2 observe pole at building. If height of target has not changed there is usually no need to enter the target height. Set Code to BLDG01 and record. This string 01 is our first BLDG, later if we have another building we can call it 02.

3) At point 3 observe pole at next corner of building. Usually the previous code keeps getting used until you enter a new one. So if several consecutive points have the same code you don't have to keep entering the code each time. Point number will be 3 and HT is still the same.

Take a break from measuring to view what you have recorded. This is useful to do occasionally to ensure your data is being recorded correctly.

If you make a mistake with a code or a measurement, just do the observation and code again, then a new line will be added to the file. Write something in your field notes to remind you of this. Later on a computer you can delete the erroneous lines or edit the contents. This might reduce the stress levels of beginners, but as you gain experience there should not be much need to edit the file.

4) At point 4 observe pole at third corner of building. We don't have to measure all corners of the building now, but we should measure the corners in the order we want the strings to be drawn. Here lines 2 - 3 then 3 – 4 is good. If we observed corner 2 then 4 then 3 we would get a diagonal line in a building! Later, perhaps when set up at another point where we can see the back corner of the building (point 13) we use code BLDG01 again and point 4 is joined to 13. We can also enter a code (e.g. BLDG01*C) to close the string so that the line 13 to our starting point (2) is drawn in CAD.

To measure to a small (depending on the scale of the plan and its requirements) tree we place the prism near the centre of the tree. To measure to a tree with a large trunk like point 5, or any other object where we can't get the prism to the centre, then:
5) At point 5 observe prism pole beside the tree and measure the distance, then move cross hairs to centre of tree by turning horizontally (don't tilt the telescope up or down) then record those angles with the previous distance. The actual 'button press' steps depend on the instrument's on board software. The code we will us is TREE*S8 (this tells the CAD software that the leaf spread has 8m radius).

6) At point 6 observe pole on edge of road bottom of kerb. Our code is BOK01. Note that this string 01 is our first BOK, it doesn't matter that we have a BLDG01 string already.

We are going to do this survey with our prism person walking along one side of the road and then returning along the other side. This makes it easier for the person doing the coding and may be safer in the field, but requires more walking than zigzagging backwards and forwards with multiple road crossings and alternating codes between one side of the road (string 01) and the other side of the road (string 02).

7) Point 7 on next edge bottom of kerb is start of an arc. Our code is BOK01*AS. In our CAD software the * in the code is a way to enter an additional parameter. Here we are still on BOK01 string but we tell the CAD software that we are starting a three point arc. Then CAD will join the three points with a circular arc instead of straight lines. Other CAD software may have alternative ways to code such requirements.

8) At point 8 on arc our code is BOK01.

9) At point 9 end of arc: Code is BOK01*AE
10) At point 10 end of road: Code is BOK01
11) At point 11 other side of road: Code BOK02

12) At point 12 we need a new pole height to see over an obstacle. Enter HT = 1.600. This is an example of entering a new pole height, which you may have to do to see over a bush etc. Note that pole height will then be used for all subsequent points unless you change it again. So if the pole is extended for one observation only, then returned to its normal height remember to enter a new pole height when you return staff to its usual height.

View your recorded data before turning off the instrument and leaving the field.

Downloading total station data to computer

There are often several download options. Basically it involves setting the communication parameters in the CAD software and instrument. Connect the instrument and computer by cable, or Bluetooth, or insert the instruments data card or USB memory into the computer. Run the software and commence the download. Often there is a choice of the format you wish to store the data in. Some formats are easily to read with a text editor, others are designed for efficient use by the CAD software.

Fieldwork and CAD

This section includes my opinions and general advice on doing topographic detail surveys (which have various other names around the world) and processing them in CAD to produce plans for a client. Students at UNSW will gain more experience with this topic at survey camps.

Field reconnaissance and prior control surveys might be useful. It is possible to start a detail survey without known coordinates of points; however I do not recommend it for students. Your processing in CAD will be easier if you start with good coordinates for the point you are setup over and for a backsight. Of course Detail surveys can be done by GPS or other techniques, not just by total stations.

In the field, students should draw a sketch plan with point numbers for features you have measured, string numbers for lines, and include feature names and codes and perhaps some other comments. This helps debug problems that might occur when processing data. In the field get into the habit of storing the codes and strings with the data. Don't join the dots later in the CAD software because that can cause errors as it is based on memory. Some students do too much drafting / editing of their plans.

The survey should be accurate enough to produce a plan the required scale. For example, at a scale 1:200 objects 200mm across are only 1mm on the plan. Small objects will be shown as symbols, and it is not necessary to measure their 'corners'. Remember to measure *height of instrument* (if you want to produce contours) and to check back site bearings.

How much detail to 'pick up'? Measure to all the major features, it is better to have too many points than too few, but many points are costly to collect. For a linear feature such as a fence or road, don't take too many points along the line unless you need them for contouring. Remember to get some natural surface points, but only where slope changes significantly; there is no need for a saturation

coverage or grid of a paddock. Measure the location of large trees and note the spread size. If there are many trees close together just observe a few positions around their perimeter and use a vegetation string, unless the client specifically requires more information about the trees.
If the edge of a road or creek is vague you decide on an appropriate place and consider the scale of the plan. When a building has large eaves or veranda, measure the corner of walls and corners of the overhanging structure. If it is not possible to radiate some object consider measuring sufficient distances by tape to locate the point, e.g. obscured corners around a building.

Detail surveys with theodolite based total stations usually only take observations in FL because that is much faster when there are many points to measure and because plotting requires low precision coordinates. Since observations are taken in one face only consider the following checks:

- Does the plan look OK?
    - buildings usually have 90° corners
    - trees etc in correct position
    - no unexpected holes or mountains in contours
- Radiate some points from two positions
- Compare building side lengths with tape measurements
- Check where possible for blunders, bumped instrument, etc by occasionally reobserving a control mark (or backsight).

Modifying Records

I prefer students to try to get the coding right in the field; next preference is to edit the downloaded text. I discourage a lot of draftsman style editing of the plan in CAD later in the office because:

- A correct(ed) data file can be processed almost automatically in the office by an assistant if the surveyor has ‘got it right’ in the field, in the field you can see what is around you – where the path is, what is path what is fence etc. So try to get the coding and strings right in the field. Include names of buildings, roads etc. I still hope that soon we will have CAD plans drawn in the field.
- Office editing of CAD files relies too much on memory and blunders can happen that way,
- Adjoining area surveys or additional surveys can be added to a data file more seamlessly than trying to add two or more CAD drawings together. This happens on survey projects where your site grows into adjoining areas, or when new construction occurs on your site and you are asked (paid) to do an updated survey plan.
- Consider the contours and the TIN, they won’t have to be regenerated as many times if the data file is correct before entry into CAD.

But, especially for beginners, sometimes it may be necessary to modify one or more of your data records. If something was observed or recorded wrongly e.g. wrong code or wrong HT, simply do the measurement and recording again in the field. I suggest you make notes about this in the field while you are collecting the data. Then in the “office” edit the text file of your downloaded data appropriately, by e.g. deleting the wrong data line or changing a code. I suggest it is better for student (beginners) to correct your data this way than to try to edit the data in the field or to edit the plan in CAD later.

When you graduate I am not aiming for you to be experts in CAD, but I do want you to be able to do a detail survey and process it in a CAD and I want you to be able to compare and decide between different CAD software. If you use another software later at work then you can compare it with the CAD you used at university. I also want you to be able to use CAD in an ongoing learning way. That is to be able to keep up to date with new versions or new brands, to use new features (if they are relevant). I don’t want you to spend years doing things the same old way all the time.

I realise that not all of you will do detail surveys in your future career, but you can still learn useful things from this exercise such as data management, field (or any other data collection) to computer pathways and things to watch out for when managing such projects. If you employ someone to do a detail survey or similar for you then you should know what to expect from them, how to check what they have given you and how much effort it would have taken to produce. I am not interested in students memorising where to look, or which box to tick, in the menu system in a current version of a particular CAD.

Sometimes you have to do produce several versions of the plans of a site, perhaps at different scales. Producing more than one plan lets you see if there are things that you are doing manually for each plan that would be nice to have done automatically, or once only.

Consider this:
Imagine you are supervising several survey parties in your business or subcontracting some work on the site to other surveyors. Your task may be to produce a detail survey plan of the entire site and it is a large site that is undergoing construction and development. You are required to produce new up to date plans of the site on a regular basis. Users of your plans want an MGA grid overlaid on the plan so they can see where the new designs will "fit in", or so they can navigate around the site. They want contours on the plan. How would you join the data sets from your field crews, at the data file level or at the CAD plan level? A cut and paste of pdf images is not going to be good enough. You want the contours from one group to join up with another group. Would you want some groups using different CAD software to the others? Would you issue instructions to the field crews before their surveys about how they should code their data?

Digital file submission:
Some surveyors supply clients with digital versions of detail surveys because some clients want more than a printed plan. Traditionally you could give them a printout or pdf version of the plan. Now you may also be asked to supply files in a particular format that can be read by many CAD programs, or files formatted for particular software. Some points to consider:

- If you give a client a CAD file and they import it into their software they see and get more than if you just provided a printed plan. If you have your plan plus other things in your file then they can see the other things too, and the file might even open to the last screen view you looked at. For example, you do some calculations like a simple intersection in local coordinates off the side of your plan, or you have a few draft versions of road designs or lot layouts. This might lead the client to mistakenly using the wrong data.

- Some clients might want to create 3D models based on your data, not just to have a contour plan. If you designed your survey and CAD work to produce just a contour plan it might not be suitable as a basis for 3D model work. Your data set might have "holes" where some of your plan's line work doesn't have a height value or assumes a height of zero. You might need to be careful about setting the heights for all information that doesn't have heights such as cadastral boundaries, text etc. If you know your client wants the data to produce 3D models then you design your survey and the data delivered, appropriately.

- Does your work appear on their computer the way it does on yours? Are you assuming some default values without setting them explicitly? For example, the data in your survey is probably in units of metres and some things might be set as unit less. If an architect imports your data into their software where they use default values of mm then anything that you had as unit less will be treated as though it is in mm e.g. a building 10m across may appear as 10mm across in their system.

6. DETAIL SURVEYS and CoGo in CAD

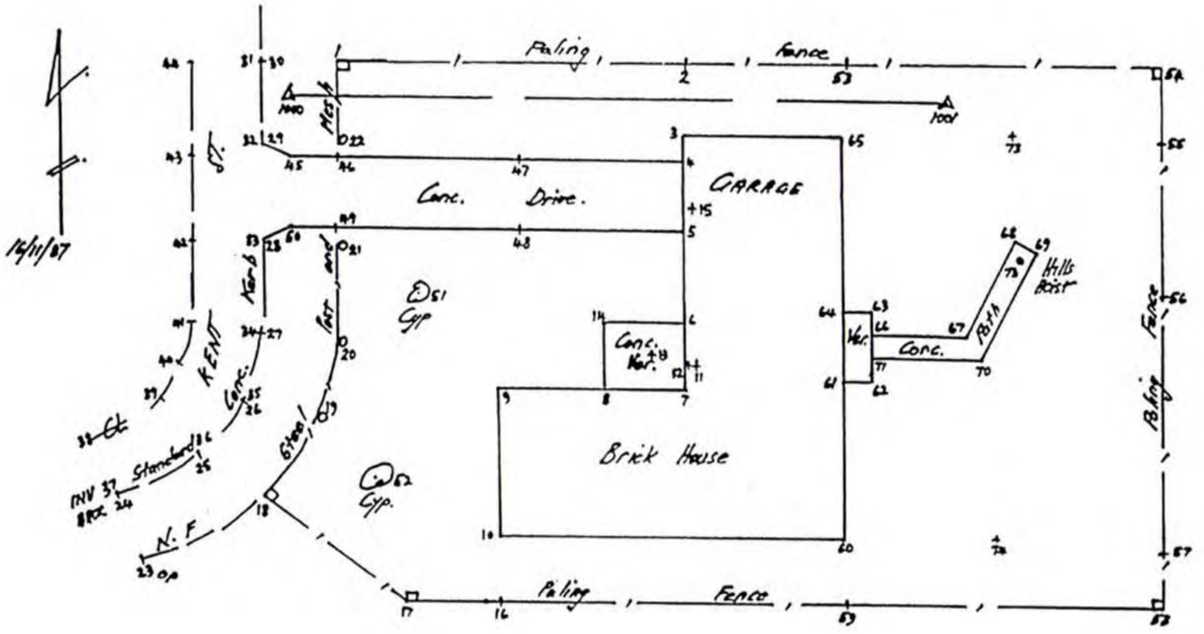
CoGo refers to coordinate geometry calculations. Detail surveys map the natural and artificial features at a site:

- Natural features e.g. vegetation, rivers
- Artificial features e.g. buildings, roads, walls and other constructed features
- Overhead details e.g. power and phone lines
- Underground details e.g. water pipes, sewer runs
- Contours and spot heights are often also determined as part of a topographic detail survey.

Currently detail surveys are usually based on 3D radiation observations with Total Stations (electronic theodolites with EDM) or by direct measurement with GPS (e.g. RTK GPS). When total stations are used they are set up at points with known coordinates that are part of a control network and can be established by Traversing, Free stationing (Resection) or GPS. Laser Scanning and photogrammetry can also be used for detail surveys – but they are topics covered in other courses.

The data recorded in a detail survey includes **measurements**: either coordinates or angles, distances, offsets, height differences, height of instrument, height of reflector etc. and **codes** describing the features: such as TREE, FENCE line and diameter of trees and dimensions of structures (e.g. manholes, drains).

Even with modern measuring and recording equipment it is often good practice to have some paper based field notes for sketches or additional notes. It may also be useful to take photographs of parts of the site. Other useful information includes: names of survey personnel, location of survey, weather conditions, job identification or purpose of survey, date (and time) of survey, main equipment used (including box or serial numbers), and coordinates and datum information. Some of this information can be recorded electronically. A sketch plan annotated with observation point numbers, string numbers, building names etc is advisable, especially for beginners. An example is shown below.



FEATURE CODES

Many types of feature coding are used in Australia and around the world, ranging from simple to comprehensive. Some are widely used amongst large survey organisations or for specific Instruments or specific CAD software, others are developed for individual use. Some are numeric, some are alphanumeric. Generally part of the code specifies the type of feature eg TREE, part specifies whether the feature is a string (i.e. the CAD software joins the dots). The code may also specify whether the points contourable or not. Remember to take some spot height measurements (e.g. Natural Surface) if you want contours

A simple feature code system requires heavy reliance on field notes to provide the additional information. In a more comprehensive feature code system, the field notes are only used to show details that have not been able to be described with a code.

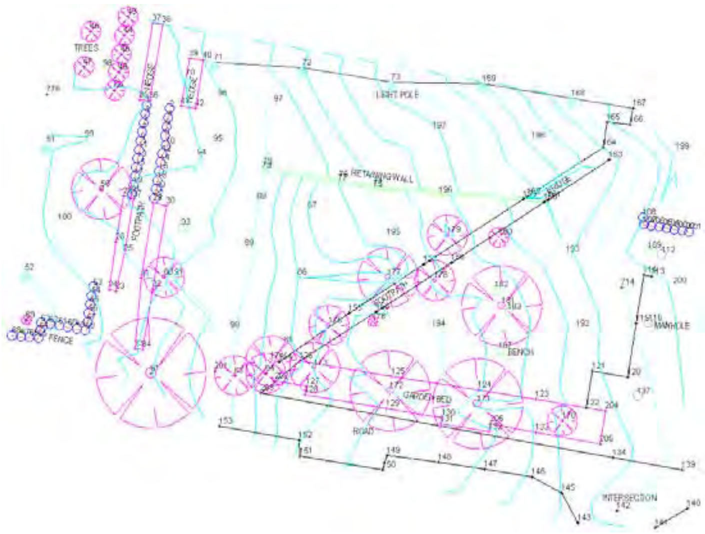
After an introductory lecture, the following lab exercises are designed to help you step by step through the process of drawing detail survey plans of three sites. The first two sites are the UNSW Physics Lawn and the Morpeth Conference Centre where we used to run survey camps. Both the data sets supplied were measured and coded by students. The third data set was provided by CivilCAD.

We do not expect students to memorise all the steps. We expect some students will perform the tasks more quickly than others, but that most students will experience some of the features of surveying CAD software. A summary of the main steps is given at the end of this chapter. If you do not like the step by step learning method as given below, you can read the summary, read the problem then explore the menu items (and Help) in the software and find your own solution. This later method may be preferable if you have some prior experience with CAD.

6.1 Detail Surveys - Manual Data Entry

This lab exercise's objective is to go through the Detail Survey data processing procedure in CAD, using supplied survey data, without doing the actual fieldwork. Photographs below show the surveyed area, it is part of the Physics Lawn on UNSW Kensington campus. Use any surveying CAD program you wish to draw a plan of this site.

The following plan of the whole site was generated and plotted by a group of students in 2006.
The line or symbol colours are not correctly presented due to the image capture from screen.



Since most students do a similar exercise to this in a year 1 course, we enter only a few lines of data as revision instead of entering all data. We then proceed to another Lab exercise, given later, that uses data in a computer file downloaded from a total station, thus avoiding manual data entry.

The detail survey control points used in the exercise are:

Control Point No.	Easting	Northing	Height	NOTE
718	1160.066	342.786	30.090	STN (Instrument Station)
982	1087.180	358.130		BS (Backsight)

Detail surveys points measured with electronic tacheometer (total station):

Instrument Stn. No. :718				Backsight Stn. No. :982		
Instrument Height :1.700				Backsight (angle) Direction : 28.1508		
POINT NO.	**DIRECTION**	**HORIZ. DISTANCE**	**VERT. DISTANCE**	**REFL. HEIGHT**	**CivilCAD POINT CODE**	**NOTE**
1	107°21'22"	24.592	0.315	1.700	ELP	Light Pole
6	64°26'07"	38.861	-0.566	1.700	ELP	Light Pole
10	50°54'56"	57.441	-1.187	1.700	ELP	Light Pole
15	44°02'27"	77.686	-1.754	1.700	ELP	Light Pole
28	38°50'16"	68.349	-1.887	1.700	NS	Natural surface
31	43°44'35"	52.412	-1.371	1.700	TREE*S1.5	Tree R=1.5m
33	48°10'40"	42.605	-1.092	1.700	TREE*S1.5	Tree R=1.5m
35	58°08'45"	29.797	-0.656	1.700	TREE*S1.5	Tree R=1.5m
38	69°31'50"	22.844	-0.421	1.700	NS	Natural surface
40	88°13'32"	14.682	-0.404	1.700	01PATH	Foot Path
etc						

Our Physics Lawn example uses the following codes: SM, NS, DRN, 01RTW, PP, 02RTW, 01PATH, 01PATH*PP, 02PATH and others. 01PATH*PP means a power pole is on the edge of the path. 01FCE*01BLD signifies a common point, or intersection, of 2 strings.

Worked Solution

Here we described some of the steps, though not all software does things in the same order or the same method.

- Start a new project and enter the Control Points data.
- Enter the observations / measurements data and point codes for the detail points.
- Usually, the reflector height only needs to be entered for the first observation. CAD keeps

using the same value until you enter a new one. If you change prism height for an observation then you enter the new height.

- 'Reduce' the data, that is, get the software to calculate coordinates of all the points.

For example:

```
civilcad 6 - H, D and Ht Difference Reduction Report     Apr 24, 2008  12:01PM        Page:   1

MANUAL_PHYS.ccx

Occupying stn        :         718
Coord  (E, N, H)     :    1160.066      342.786       30.090  Database
Code                 :
Instrument ht        :       0.000
Back sight pt        :         982
Coord  (E, N, H)     :    1087.180      358.130               Database
Code                 :
Azimuth              :  281°53'18"
Back sight angle     :   28°15'08"
Swing angle          :    0°00'00"
Correction factor    :       1.000
```

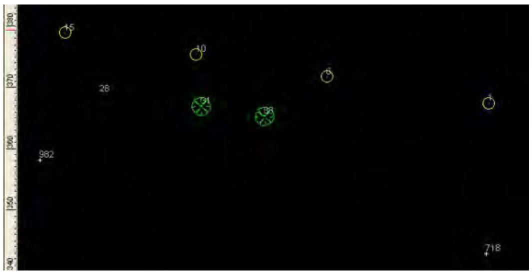
Pt No	Hor Ang	Hor Dst	Ht Diff	Tar Ht	Code	Eastings	Northings	Heights
1	107°21'22"	24.592	0.315	1.700	ELP	1160.492	367.374	28.705
6	64°26'07"	38.861	-0.566	1.700	ELP	1134.099	371.698	27.824
10	50°54'56"	57.441	-1.187	1.700	ELP	1112.757	375.364	27.203
15	44°02'27"	77.686	-1.754	1.700	ELP	1091.269	378.870	26.636
28	38°50'16"	68.349	-1.887	1.700	NS	1096.908	368.913	26.503
31	43°44'35"	52.412	-1.371	1.700	TREE*S1.5	1113.525	366.889	27.019
33	48°10'40"	42.605	-1.092	1.700	TREE*S1.5	1123.862	365.246	27.298

etc

Easting, Northing and Height were calculated for all the detail points based on the observations. It is wise to check this and to correct any errors before proceeding further.

The reduced points should look like part of the plan shown here.

I am sure you will agree that this manual method of data entry is laborious, time consuming and error prone, so we learn better methods in the following section.



6.2 Detail Survey with Downloaded data

Most surveying CAD packages can read survey data from most of the common survey instruments. In this section there are two examples. The first is not instrument specific, and can be created from most total stations, GPS and other coordinate measuring instruments. The second example is more instrument specific.

6.2.1 Data import from an xyz file

Instead of entering theodolite style observations manually as above, ENH coordinates may be a better way to enter data. A comma separated (CSV), ASCII text file can be created with point numbers, ENH coordinates and codes. GPS or theodolite data can be entered in this coordinate format. Either the theodolite data can be copy/pasted into MS Excel and ENH coordinates calculated for each radiated point, or (if possible) we can set the theodolite in the field to display ENH coordinates and book those values. The textfile format is usually one line per point, such as:

PointNo, Easting, Northing, Height, Code

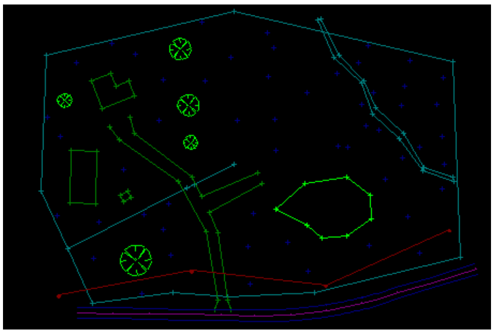
The Code is used to read a symbol from the software Library, and apply appropriate properties to the created entities. If string numbers are included with the codes the points will be automatically connected with lines. In this data set the string numbers are the 2 digit prefix.

A sample data set provided by CivilCAD in this format, has the following first few lines:

```
    1,          921.889,       4915.960,        48.250,       01F
    2,          967.024,       4921.680,        49.390,       01F
   42,          991.021,       4920.125,        50.320,       01F
   13,          997.129,       4919.734,        50.320,       01F
    3,         1045.510,       4921.584,        52.090,       01F
```

A text file for the full data set above, SURVTUT.XYZ, is available from the author. Note that the point numbers don't have to be sequential. It is possible to edit the file. Usually strings are drawn in the order they appear in the input data file, not according to point numbers.

Open a new project. Import the data file. Display the detail plan that looks like:



Examine your detail survey plan, symbols, line types, point numbers, point codes, point properties, line properties, etc. If you see the points (e.g. crosses) and point symbols (e.g. trees) but not the string lines then you probably need to change some settings

A later section of this chapter describes how to edit a plan and how to add contours and text annotations.

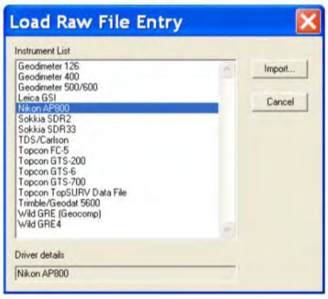
6.2.2 Nikon AP8 Format Data

A set of detail survey raw data has been downloaded from a NIKON DTM810 total station. The file is DETAIL6.AP8 (available from the author). Some photos of the site are included below. Open the file with a text editor and have a look at its contents. After viewing the data use CAD to draw a plan of the site with lines, symbols and contours. In this data the string number part of codes e.g. 1 in PATH1 is after the word, in some data sets it is before the word e.g. 01PATH.

The first part of the data file is DETAIL6.AP8:

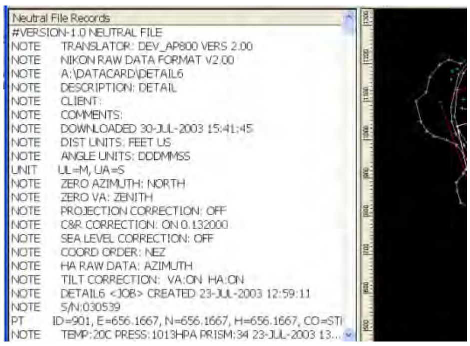
```
CO,Nikon RAW data format V2.00
CO,a:\datacard\detail6
CO,Description: DETAIL
CO,Client:
CO,Comments:
CO,Downloaded 30-Jul-2003 15:41:45
CO,Dist Units: Feet US
CO,Angle Units: DDDMMSS
CO,Zero azimuth: North
CO,Zero VA: Zenith
CO,Projection correction: OFF
CO,C&R correction: ON 0.132000
CO,Sea level correction: OFF
CO,Coord Order: NEZ
CO,HA Raw data: Azimuth
CO,Tilt Correction:  VA:ON  HA:ON
CO, DETAIL6 <JOB> Created 23-Jul-2003 12:59:11
CO,S/N:030539
MC,901,,656.1667,656.1667,656.1667,STN
CO,Temp:20C Press:1013hPa Prism:34 23-Jul-2003 13:12:16
CO,23-Jul-2003 13:12:16
ST,901,,980,,4.9869,0.00000,0.00000
F1,980,5.1509,392.6108,0.00000,91.09410,13:12:16
CP,980,,5.1509,392.6108,0.00000,91.09410,13:12:16,
CO,Temp:20C Press:1013hPa Prism:34 23-Jul-2003 13:18:18
CO,23-Jul-2003 13:18:18
ST,901,,980,,4.9869,0.00000,0.00000
F1,980,5.1509,392.6108,0.00000,91.09440,13:18:18
CP,980,,5.1509,392.6108,0.00000,91.09440,13:18:18,
CO,BS IN TOLERANCE
CO,HDm=119.643 BS Zm=197.524 DELTAS: HD=0.000 Z=0.002
SS,1,4.2651,27.8149,345.57410,91.20220,13:23:52,FENC1
```

```
SS,2,4.2651,87.3292,333.45010,89.09510,13:24:56,FENC1
SS,3,4.2651,138.5365,25.26510,90.26220,13:26:58,NS
SS,4,4.2651,85.7643,355.05310,90.26070,13:27:42,NS
SS,5,4.2651,55.6331,11.18480,90.38380,13:28:14,NS
SS,6,4.2651,98.7990,330.32480,89.00490,13:30:18,TOP1
SS,7,4.2651,153.9269,331.21020,89.15460,13:30:58,TOP1
SS,8,4.2651,163.6808,341.28200,89.41460,13:31:30,TOP1
SS,9,4.2651,174.1565,359.55100,90.10480,13:32:26,TOE1
SS,10,4.2651,153.9695,349.43230,90.09330,13:33:18,TOE1
SS,11,4.2651,140.5115,335.56210,90.05490,13:33:42,TOE1
SS,12,4.2651,102.2964,338.08370,90.11240,13:34:14,TOE1
SS,13,4.2651,126.0890,358.47170,90.15470,13:34:52,NS
SS,14,4.2651,209.2089,357.57190,90.19460,13:36:00,PATH1
SS,15,4.2651,220.3211,356.59450,90.19420,13:36:18,PATH1
SS,16,4.2651,190.5902,346.52370,89.50260,13:37:00,PATH1
SS,17,4.2651,169.9997,328.43400,88.49400,13:37:46,PATH1
SS,18,4.2651,162.4505,328.41190,88.48370,13:38:04,PATH1
SS,19,4.2651,172.7359,343.24430,89.39140,13:38:34,PATH1
SS,20,4.2651,170.7575,328.18210,88.43210,13:41:06,VER1
SS,21,4.2651,107.5096,328.08500,87.53310,13:42:46,VER1
SS,22,4.2651,109.2452,318.41350,87.52360,13:43:32,VER1*C
SS,23,4.2651,108.7137,318.36400,88.52320,13:44:08,NS
SS,24,4.2651,107.2340,328.06460,88.55470,13:44:28,NS
SS,25,4.2651,171.3284,328.24290,89.17330,13:45:22,NS
SS,26,4.2651,163.8448,325.25290,88.36390,13:46:54,BLDG1
SS,27,4.2651,115.9151,324.05580,88.00310,13:48:20,BLDG1
SS,28,4.2651,117.0798,319.15410,88.00270,13:49:02,BLDG1
SS,29,4.2651,92.4637,316.43470,88.56090,13:49:40,BLDG1
SS,30,4.2651,91.7649,270.23490,89.31260,13:51:50,BLDG
SS,31,9.0223,115.8331,263.13460,87.05460,13:55:08,BLDG
SS,32,4.2651,88.8942,261.54340,88.59550,13:57:12,TREE*S15
CO,H O/S:32,1.300,27.095,260.3751,88.5955,13:57:12
```
Etc



Solution: Nikon AP8 Format Data

Open a new project in your CAD.
Check the Library or Code Settings to make sure the string number is read as a suffix like PATH1 not a prefix like 01PATH.
Import your data file while selecting “Nikon AP800” in an instrument list (there is usually a long list of many instrument and format types) and view its contents as seen in the CAD program.
‘Reduce’ the data and draw a plan.



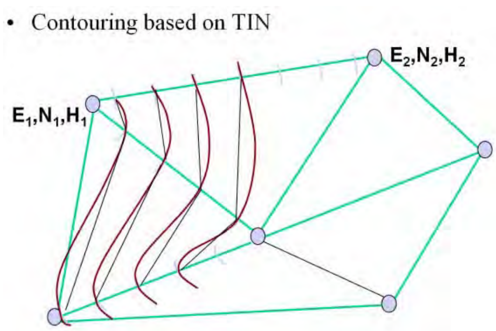
6.2.3 Sokkia and Neutral Data Formats

Some students observed a detail survey in 2009 with Sokkia SET530K total stations and downloaded their data in the Sokkia SDR format and CivilCAD’s neutral format. For comparison purposes parts of the files are included below and some lines of the real data have been removed. It is not intended for students to become experts in any of these formats, but you should be able to see that it is not hard to interpret the files and to check your data has been recorded correctly.

Neutral format file	SDR33 format
#VERSION-1.0 NEUTRAL FILE	00NMSDR33 V04-04.02 29-Jul-09 00:00 113121
NOTE TRANSLATOR: SDR33 VERS 1.01	10NMJOB2 121111
UNIT UL=M, UA=S	06NM1.00014000
UNIT UL=M, UA=S	01NM:SET530RK3 V33-03168280SET530RK3 V33-0316828031
JOB JOB2	0.000
SCALE 1.00014000	08KI 801289960.320 6147409.457 12.454
PT ID=801,E=289960.320,N=6147409.457,H=12.454,CO=	08KI 802290040.679 6147349.462 8.694
PT ID=802,E=290040.679,N=6147349.462,H=8.694,CO=	08KI 803289943.655 6147112.727 6.762
PT ID=803,E=289943.655,N=6147112.727,H=6.762,CO=	…
…	08KI 900289856.680 6147404.575 12.056
PT ID=900,E=289856.680,N=6147404.575,H=12.056,CO=	08KI 914290078.813 6147465.593 12.579
PT ID=914,E=290078.813,N=6147465.593,H=12.579,CO=	08KI 926289477.373 6147660.737
PT ID=926,E=289477.373,N=6147660.737,H=,CO=	02TP 801289960.320 6147409.457 12.454 1.610 SM
STN ID=801, HI=1.610, CO=SM	08KI 927290040.679 6147349.462 8.694
XYZ ID=801,E=289960.320,N=6147409.457,H=12.454	07TP 801 927126.74444 126.74444
PT ID=927,E=290040.679,N=6147349.462,H=8.694,CO=	03NM1.485
BKB ID=927,AZ=126.4440,HA=126.4440	09F1 801 927 92.22250 126.74444 SM
BS ID=927,HA=126.4440,VA= 92.1321,SD=0.000,HT=1.485	02TP 801289960.320 6147409.457 12.454 1.610 SM
STN ID=801,HI=1.610,CO=SM	08KI 802290040.679 6147349.462 8.694
XYZ ID=801,E=289960.320,N=6147409.457,H=12.454	07TP 801 802126.74444 126.74444
PT ID=802,E=290040.679,N=6147349.462,H=8.694,CO=	09F1 801 802 92.22222 126.74444 SM
BKB ID=802,AZ=126.4440,HA=126.4440	08TP 928290040.689 6147349.450 8.687 SM
BS ID=802,HA=126.4440,VA= 92.1320,SD=0.000,HT=1.485	08TP 001290040.688 6147349.451 8.688 SM
PT ID=928,E=290040.689,N=6147349.450,H=8.687,CO=SM	03NM1.650
PT ID=001,E=290040.688,N=6147349.451,H=8.688,CO=SM	08TP 002289974.656 6147394.946 11.949 BLDG01
PT ID=002,E=289974.656,N=6147394.946,H=11.949,CO=BLDG01	08TP 003289974.294 6147398.443 12.333 PATH01
PT ID=003,E=289974.294,N=6147398.443,H=12.333,CO=PATH01	08TP 004289973.987 6147399.469 12.339 PATH01
PT ID=004,E=289973.987,N=6147399.469,H=12.339,CO=PATH01	08TP 005289930.316 6147384.822 12.034 PATH01
PT ID=005,E=289930.316,N=6147384.822,H=12.034,CO=PATH01	08TP 006289930.611 6147383.874 12.017 PATH01
PT ID=006,E=289930.611,N=6147383.874,H=12.017,CO=PATH01	08TP 007289964.707 6147395.275 12.249 PATH01
PT ID=007,E=289964.707,N=6147395.275,H=12.249,CO=PATH01	08TP 008289974.299 6147398.428 12.343 TOK01
PT ID=008,E=289974.299,N=6147398.428,H=12.343,CO=TOK01	08TP 009289974.060 6147398.339 12.419 TOK01
PT ID=009,E=289974.060,N=6147398.339,H=12.419,CO=TOK01	08TP 010289959.188 6147393.397 12.279 TOK01
PT ID=010,E=289959.188,N=6147393.397,H=12.279,CO=TOK01	…
…	08TP 015289974.300 6147398.358 12.353 TOK01
PT ID=015,E=289974.300,N=6147398.358,H=12.353,CO=TOK01	08TP 016289965.720 6147395.471 12.329 PATH02
PT ID=016,E=289965.720,N=6147395.471,H=12.329,CO=PATH02	08TP 017289964.750 6147395.144 12.330 PATH02
PT ID=017,E=289964.750,N=6147395.144,H=12.330,CO=PATH02	08TP 018289965.208 6147393.862 12.291 PATH02
PT ID=018,E=289965.208,N=6147393.862,H=12.291,CO=PATH02	08TP 019289965.220 6147393.834 12.131 PATH02
PT ID=019,E=289965.220,N=6147393.834,H=12.131,CO=PATH02	…
…	08TP 028289931.799 6147380.601 11.938 BLDG01
PT ID=028,E=289931.799,N=6147380.601,H=11.938,CO=BLDG01	08TP 029289937.443 6147364.294 11.820 BLDG01
PT ID=029,E=289937.443,N=6147364.294,H=11.820,CO=BLDG01	08TP 030289980.215 6147378.625 11.846 BLDG01
PT ID=030,E=289980.215,N=6147378.625,H=11.846,CO=BLDG01	08TP 031289980.419 6147378.900 11.555 FENC01
PT ID=031,E=289980.419,N=6147378.900,H=11.555,CO=FENC01	08TP 032289996.878 6147383.675 11.617 FENC02
PT ID=032,E=289996.878,N=6147383.675,H=11.617,CO=FENC02	08TP 033289995.357 6147397.296 12.616 FENC02
PT ID=033,E=289995.357,N=6147397.296,H=12.616,CO=FENC02	08TP 034289993.958 6147410.040 12.881 FENC02
PT ID=034,E=289993.958,N=6147410.040,H=12.881,CO=FENC02	08TP 035289992.614 6147422.833 12.835 FENC02
PT ID=035,E=289992.614,N=6147422.833,H=12.835,CO=FENC02	03NM1.300
PT ID=036,E=289991.749,N=6147431.140,H=12.488,CO=FENC02	08TP 036289991.749 6147431.140 12.488 FENC02
PT ID=037,E=289994.519,N=6147433.252,H=12.487,CO=FENC02	08TP 037289994.519 6147433.252 12.487 FENC02
PT ID=038,E=290008.026,N=6147434.851,H=12.522,CO=FENC02	03NM1.650
PT ID=039,E=290018.040,N=6147436.122,H=12.736,CO=FENC02	08TP 038290008.026 6147434.851 12.522 FENC02
PT ID=040,E=289959.402,N=6147391.446,H=12.208,CO=TREE*5	08TP 039290018.040 6147436.122 12.736 FENC02
SS ID=41,HA=180.1320,VA= 90.3903,SD=18.030,HT=1.650,CO=TREE*S5	08TP 040289959.402 6147391.446 12.208 TREE*5
SS ID=42,HA=117.1224,VA= 89.5922,SD=20.215,HT=1.650,CO=TREE*S8	09F1 801 04118.030 90.65083 180.22222 TREE*S5
SS ID=43,HA= 96.3636,VA= 89.1401,SD=20.753,HT=1.650,CO=TREE*S8	09F1 801 04220.215 89.98944 117.20667 TREE*S8
SS ID=44,HA= 96.2346,VA= 89.2204,SD=29.727,HT=1.650,CO=TREE*S5	09F1 801 04320.753 89.23361 96.61000 TREE*S8
SS ID=45,HA= 88.5003,VA= 89.2222,SD=29.070,HT=1.650,CO=TREE*S4	09F1 801 04429.727 89.36778 96.39611 TREE*S5
SS ID=46,HA= 74.3937,VA= 89.2326,SD=29.037,HT=1.650,CO=TREE*S4	09F1 801 04529.070 89.37278 88.83417 TREE*S4
…	09F1 801 04629.037 89.39056 74.66028 TREE*S4
SS ID=51,HA=288.1818,VA= 91.0219,SD=43.413,HT=1.650,CO=TREE*S9	…
PT ID=052,E=289930.022,N=6147410.767,H=12.130,CO=BLDG02	09F1 801 05143.413 91.03861 288.30500 TREE*S9
PT ID=053,E=289935.391,N=6147394.702,H=12.186,CO=BLDG02	08TP 052289930.022 6147410.767 12.130 BLDG02
PT ID=054,E=289945.325,N=6147408.405,H=12.312,CO=NS	08TP 053289935.391 6147394.702 12.186 BLDG02
PT ID=055,E=289907.453,N=6147373.502,H=11.812,CO=BLDG03	08TP 054289945.325 6147408.405 12.312 NS
PT ID=056,E=289930.927,N=6147381.322,H=11.879,CO=BLDG03	08TP 055289907.453 6147373.502 11.812 BLDG03
…	08TP 056289930.927 6147381.322 11.879 BLDG03
PT ID=393,E=289896.807,N=6147360.567,H=11.915,CO=BLDG04	…
PT ID=394,E=289896.759,N=6147365.533,H=12.061,CO=BLDG04	08TP 393289896.807 6147360.567 11.915 BLDG04
PT ID=395,E=289856.234,N=6147332.070,H=10.697,CO=NS	08TP 394289896.759 6147365.533 12.061 BLDG04

6.3 Generate Digital Terrain Model (DTM)

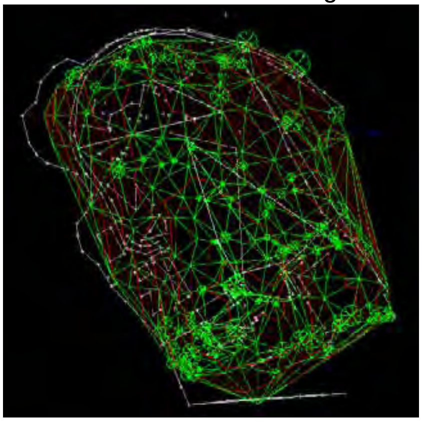
DTM's are typically formed using a triangulated irregular network (TIN). A TIN is a series of 'best fit' triangles, with a detail point at each node (triangle corner). It is usually possible to manually override (change) the TIN mesh of triangles to suit your site.



With a DTM, it is possible to produce contour maps; produce cross-sections; calculate volumes; calculate cut and fill; carry out road design; and generate perspective views of the ground surface.

Again, the order of steps and their details varies from one CAD software to another. Here is a guide.

- Select the option to create a Surface or DTM.
- Set the contour intervals and colours that you require.
- Form the triangle mesh. (shown in green →)
- Modify the mesh if or as required.
- Draw contours (perhaps with smoothing). (brown →)
- Edit the plan if necessary to eliminate errors.



6.4 Plan Editing and Plotting

Previous sections have enabled us to draw basic detail survey plans. This section explores some of the finer detail and more advanced aspects of improving a plan. The topics that you can explore are: choice of scale, contour interval, break-lines and removing contours inside buildings, adding a coordinate grid, changing or choosing appropriate symbols and line types, text annotation of roads and buildings, add a title block and a north indicator.

6.4.1 FAQ for Detail Survey Processing Using CAD

Here we learn from the experience of previous students who have made mistakes or had problems while learning.

If a survey has been measured and the data file contains point numbers, E, N, H and codes in a text file, then generally it is better to not do too much "fiddling" with the graphical editor in CAD, instead fix the input file, save it and re-import the data.

Q: Point in wrong place

A: Double click the point, look at its coordinates in properties, typo? If so edit the coordinates. If not then perhaps make it non contourable and replace symbol with a dot or cross.

Q: Symbols or lines don't show

A: Check your setup of code libraries and other settings. If the point symbols appear but not the line symbols check whether your data uses prefix string numbers or not and that Library Survey Code settings are correct.

Q: Tree symbol does not appear.

A: Perhaps you are using the wrong code or the tree radius was not correctly coded into the data file. Edit the XYZ file and start again.

Q: DRN (Drain) symbols don't show (or similar for other symbols)

A: Turn on the DRN and default layers, turn off all other layers. Investigate the properties of the points.

Q: Order of points is wrong e.g. string zig zag around building or path

A: The point number is not used by some software, so strings are drawn in the order the points were surveyed and listed in the input file. Edit the XYZ data file. Move the data lines into the correct sequence to draw the string in the correct order, usually no need to change point numbers. Then rerun CAD.

Q: How do I put two symbols on one point? e.g. edge of garden and edge of path intersection or Light Pole on edge of path.

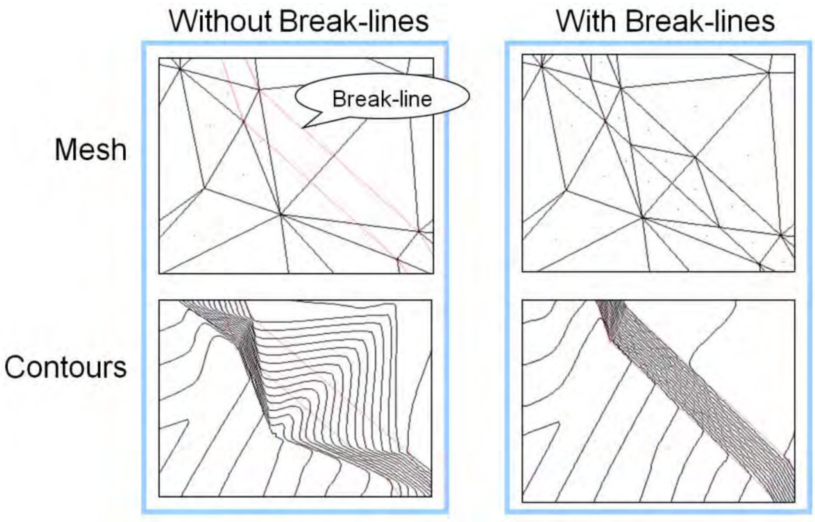
A: In CivilCAD use both symbols and join them with * e.g. 01PATH*PP. How is this done in your software?

Q: Contours show an unexpected mountain or hole on your site.

A: Select the central point causing the problem, check its height for typo error. If there is no typo then select non-contourable for that point (e.g. beside a tree the pole may not have been on surrounding ground level.

6.4.2 Editing a DTM: Breaklines and contours

Breaklines are linear features that define and control the surface behavior of a TIN in terms of smoothness and continuity. In areas such as along ridge lines, at the upper and lower edges of a steep embankment or cliff, along drain lines, and in the vicinity of constructed cuts and fills, the points can be supplemented by breaklines that indicate a sharp change in slope. Generally we set the properties of strings along retaining walls, kerbs and paths as a breakline. But it is possible to add extra breaklines or to change the mesh lines that join points in the TIN to better reflect the slope of the site. Triangles in the TIN are formed to run along the edge of breakline. This aids the correct contours to be drawn. If triangle sides 'cut across' sharp changes in slope the incorrect contours will be drawn. The following figure shows the effects of breaklines on contours on a site with sudden changes of slope.



Generally we need two "parallel" breaklines for breaklines to actually affect contours, e.g. two sides of path or two sides of road or two sides of retaining wall or top and bottom of cliff. But a single breakline does work for the centreline of a creek.

If the field survey measured points at top and bottom of retaining wall then ensure the lines joining the appropriate points are breaklines. If the field survey only measured the top (or vice versa) of a wall you need to insert parallel breaklines with small offset between them.

For some projects we do not want to show contours on parts of the site. For example if we survey the four corners of a building we don't usually want contours to appear inside the building. This is partly because the ground slope inside the building is probably not the same as outside, and the building (and its name in text perhaps) is more prominent on the plan without contours. Other examples of parts of the site we may not want contours are roads and paths (sometimes depending on the purpose of the survey) other man made features on the site, and outside the site or survey boundaries. If the boundaries of the site are somewhat irregular triangles might be formed near but outside the boundary. One way to remove contours in retaining walls or footpaths or buildings etc is to remove the offending triangles from the mesh or to change the property of these triangles to non contourable. Other methods to delete or clip contours involve creating boundaries in a separate layer, and then not producing contours in that boundary layer.

6.4.3 More Plan Editing

In this exercise work with the SURVTUT data set again and practice further editing of the plan. Edit the detail survey plan so that it meets the following specifications:

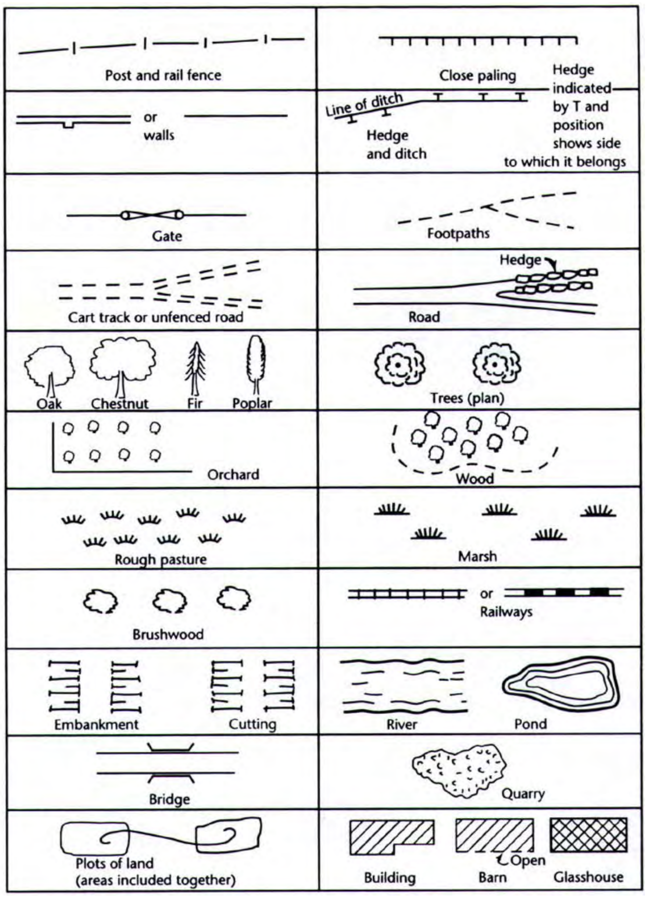
Scale: 1:1000
Contour interval: major = 2.5 m, minor = 0.5. Contours are not to be shown inside roads and buildings.
Labelled grid at 100 m interval in blue colour.
Control points, power poles and trees to be shown with appropriate symbols.
Fences, roads and vegetation boundary to be shown with appropriate line types.
Add bearings and distances along traverse lines
Add a fence between points 8 and 65.
Make the 02F (i.e. fence) layer blue. Similarly, change other entities' colours.
Use **purple** colour for survey marks, **green** for trees and vegetation, **blue** for fences, **black** for other details, **red** for minor contours, and **brown** for major contours.
Select the line between points 55 and 56 change its property to remove the breakline. Do same for line 54-55. Update the DTM. Make the two lines breaklines again, update the DTM and see what happens.
Remove the triangles and thus the contours inside the buildings and road, and outside the fence boundary.
Show crosses only on trees and spot heights in appropriate colour.
Roads, buildings, vegetation beds to be annotated with black text, in a separate layer.
Create and add a title block and a north indicator.
Print the final plan to paper or to a pdf file.

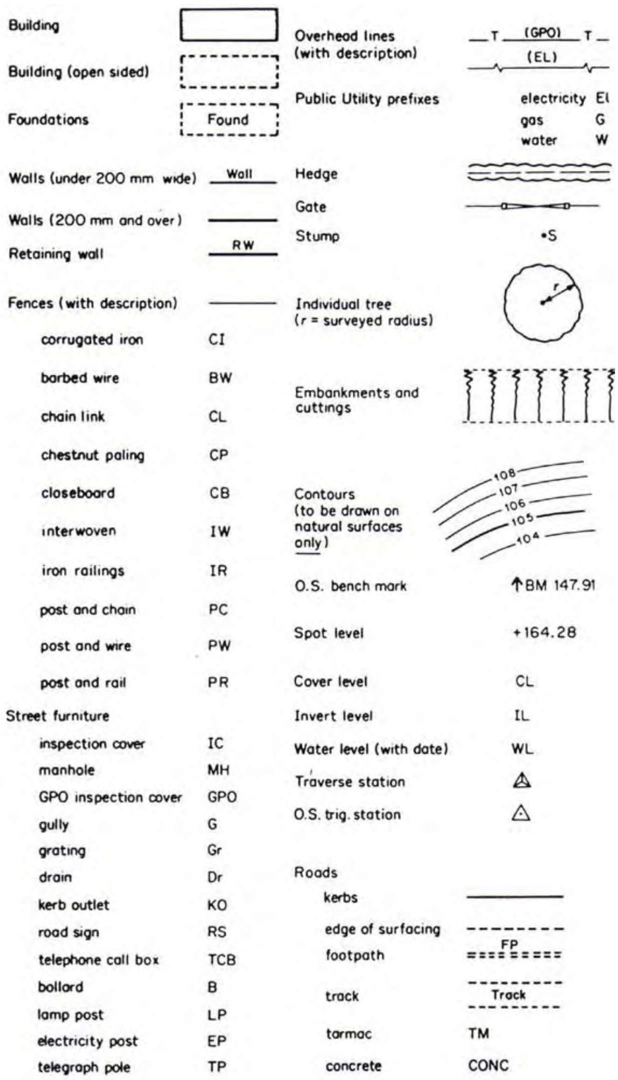
6.5 Traditional Survey Drafting

The process of hand drawing survey plans is extremely time consuming and requires considerable skill levels to achieve a high standard of presentation. This used to be overcome by surveyors passing the observations, which were booked manually into field notes, to drafts'men' who then drew plans drawn by hand. Nowadays computer CAD software is used to draw plans very efficiently. The following notes present the traditional methods, some of it is still relevant for modern surveys.

The amount and type of detail picked up varies enormously with the scale and intended use of the plan. The accuracy of the detail survey depends on the scale of the plan and the accuracy with which it can be plotted. Assuming 0.5mm plotting accuracy, this will correspond to 5 cm at 1:100 scale, and 50 cm at 1:1000 scale.

Many types of symbols were used to represent detail:



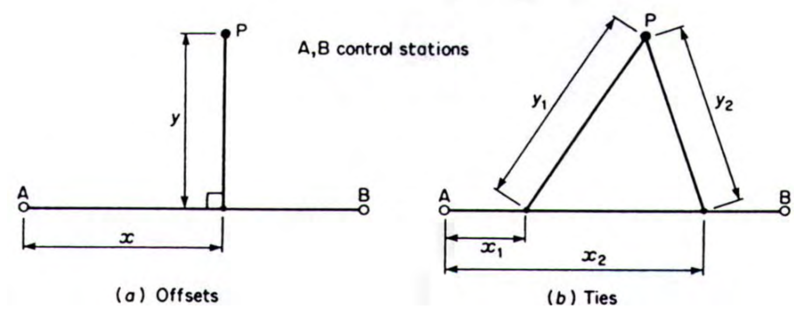


For contours the vertical interval depends on the scale of the plan. Suitable vertical intervals are:

Scale:	1:50	1:100	1:200	1:500	1:1000
Interval:	0.05m	0.1m	0.25m	0.5m	1m

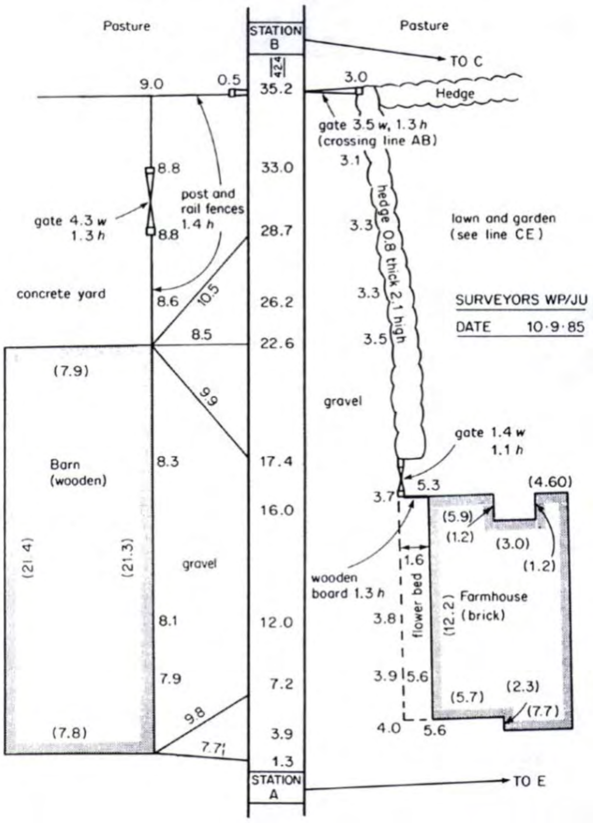
Usually spot levels (spot heights) on natural surface are recorded to 0.05m and on hard surfaces are recorded to 0.01m.

While detail points are usually coordinated by 3D radiation (or coordinated directly by GPS) they can also be located from a control network framework by other methods, e.g. Offsets and ties. Offsets at right angles to the lines between control points, or ties from two (or more) points are used to locate P:



In practice, a tape can be laid along the control line to measure x, the offsets are then measured using a second tape (or use total station). For best results with a tape, offsets should be kept below 10m. For larger offsets, (usually two) ties should be used. Note that by tape detail can only be located in 2D. Spot heights are required for a 3D fix. Dimensions of certain features (e.g. buildings, width of paths, tree spread) should be recorded.

Example Booking, offsets and ties:



When the basic control network has been plotted, the detail for each control line can be added by marking off the distances along each line corresponding to the points on the tape at which offsets and ties were taken.

6.6 Summary of Detail Surveys in CAD

Data input from imported text file of coordinates with PointNo, Easting, Northing, Height, Code; or by manual point entry of instrument coordinates and radiation measurements e.g. direction, hor distance and ΔH. Example codes for points and strings: 01FCE (fence string number 1), 05BLD (number 5 building string), TREE*S6 (tree with 6m radius spread), 01FCE*01BLD (the * means point is intersection of 2 strings where a fence is against a building).

CAD then draws a plan, inspect it. Zoom in and out. If plan is not correct, check the point codes in your original data file edit the file and re import, or edit the points' features in CAD.

Plan Editing. Use options for deleting, inserting or editing points, lines or arcs as necessary to tidy up data. Save file frequently.

Create DTM. A TIN is displayed as a triangular mesh. Contours are produced from the TIN. Modify settings and display. Inspect for points with incorrect heights. Some points made non-contourable (e.g. use position of point but not its height). After changes: Update DTM. View and edit the triangular mesh. Delete long thin triangles or triangles in buildings. Add triangles or breaklines if necessary to change the triangles in the mesh. Save file frequently.

Text Annotation. Insert text to name roads, paths, buildings, control points etc. Move or rotate text, or change its properties / parameters.

6.7 COGO in CAD

In earlier chapters we learnt Survey Computations with calculator and spreadsheet solutions of: radiation, join, intersections, resections, missing data, traverse miscloses and Bowditch. Then we used CAD for plan drawing. Now we use CAD for Survey Computations that are sometimes called COordinate GeOmetry (COGO) calculations. COGO functions in CAD are used to enter survey data, calculate precise locations and boundaries, define curves, etc. Later we will use CAD for design.

Most surveying CAD packages have similar COGO functions to enquire, compute or modify:
Area, Angle, Bearing/Distance, Offset Distance, Set Point Height, Bearing/Distance Intersection, Insert Point into Line, Delete Point on Line, Extend/Trim Line, Segment Line/Arc, Offset Line, Fit Line, Fit Arc, Convert 3-Point Arc, Create Parallel Figure, Move Point/Line etc.

As usual with our study of CAD in this course we do not aim to train you in great detail in every function available in any specific program. Rather we give you exercises that allow you to get some experience with some of the more common or more useful features. You can learn more details and learn about other CAD programs in your own time and perhaps during vacation employment or after you graduate.

We will use some of these functions in this chapter, but also in later chapters for cadastral calculations and subdivision and road design.

Point Coordinates

Open a new project and insert a point. A point may be a surveyed point, a designed point, a construction point to aid computation, or a feature position. It has coordinates (easting, northing or x, y). It may have an elevation (height or z). Each point has a unique alphanumeric "number" ID. It has properties, e.g. Contourable - the point will be included in the creation of a DTM, or symbol, etc.

Lines

A line represents a join between 2 points. Options to create a line:

- enter coordinates for points at ends of line, or
- enter bearing and distance from a known point, or
- join existing points.

Enter a line by the methods of bearing and distance from the known point above.

Bearing and Distance

It is possible to enquire or calculate the bearing and distance between 2 points, or of a line. When points have elevations, change in height and slope details are also displayed. To select the line or the points use the mouse or type the 2 point numbers in the appropriate entry box.

Intersections

It is possible to calculate an intersection point using either bearings or distances from 2 points. As we learnt in Chapter 2, some intersections have two possible solutions. CAD will usually calculate both and let you decide which one you want.

Not all CAD solve traditional 3 point resections, but if necessary we can solve them using Collins Point Method and two intersections by bearing.

Traverse Adjustment

CAD survey computations usually also include the following options;
Translate/Rotate, Traverse Adjustment, LS Transformation, Volumes, Subdivision, Alignments, Cross Sections etc. Explore them. Better yet, test them. Perhaps your CAD is not giving correct answers due to certain assumptions it makes?

6.8 Lab exercises

DETAIL SURVEY

Q1. Work through the detail survey examples in this chapter.

Q2. Produce a data file for a detail survey, similar to the practice survey in Chapter 5.

COGO

Use CAD to re calculate one tutorial question from previous classes on: Radiation, Join between 2 points, Intersection by bearings, Intersection by distances, Traverse adjustment as shown below.

Q1. Radiation.
Given P = (741.30, 4611.10), β_{PQ} = 12°30'15" and d_{PQ} = 623.542

Ans: Q = (876.304, 5219.852)

Q2. Join between 2 points.

Given	Station	Eastings(m)	Northings(m)	Height(m)
	P	52381.72	12381.91	72.81
	Q	52712.11	12757.55	250.81

Compute: (i) Bearing of P from Q.
(ii) Horizontal distance PQ.
(iii) Slope distance PQ (not all CAD software will calculate the 3D slope distance)

Q3. Intersection by bearings

Given: A = (E 2589.40, N 6717.85) B = (E 9307.04, N 3423.63)

β_{AC} = 62° 26' 58" β_{BC} = 359° 49' 25"

Ans: $E_C = 9286.143$ m $N_C = 10211.467$m

Q4. Intersection by distances.

Given: A = (E 1859.75, N 3722.63) B = (E 1078.37, N 2405.38)

d_{AC} = 1537.75m d_{BC} = 2487.56m

Ans: $E_C = 3361.660$m $N_C = 3392.568$m

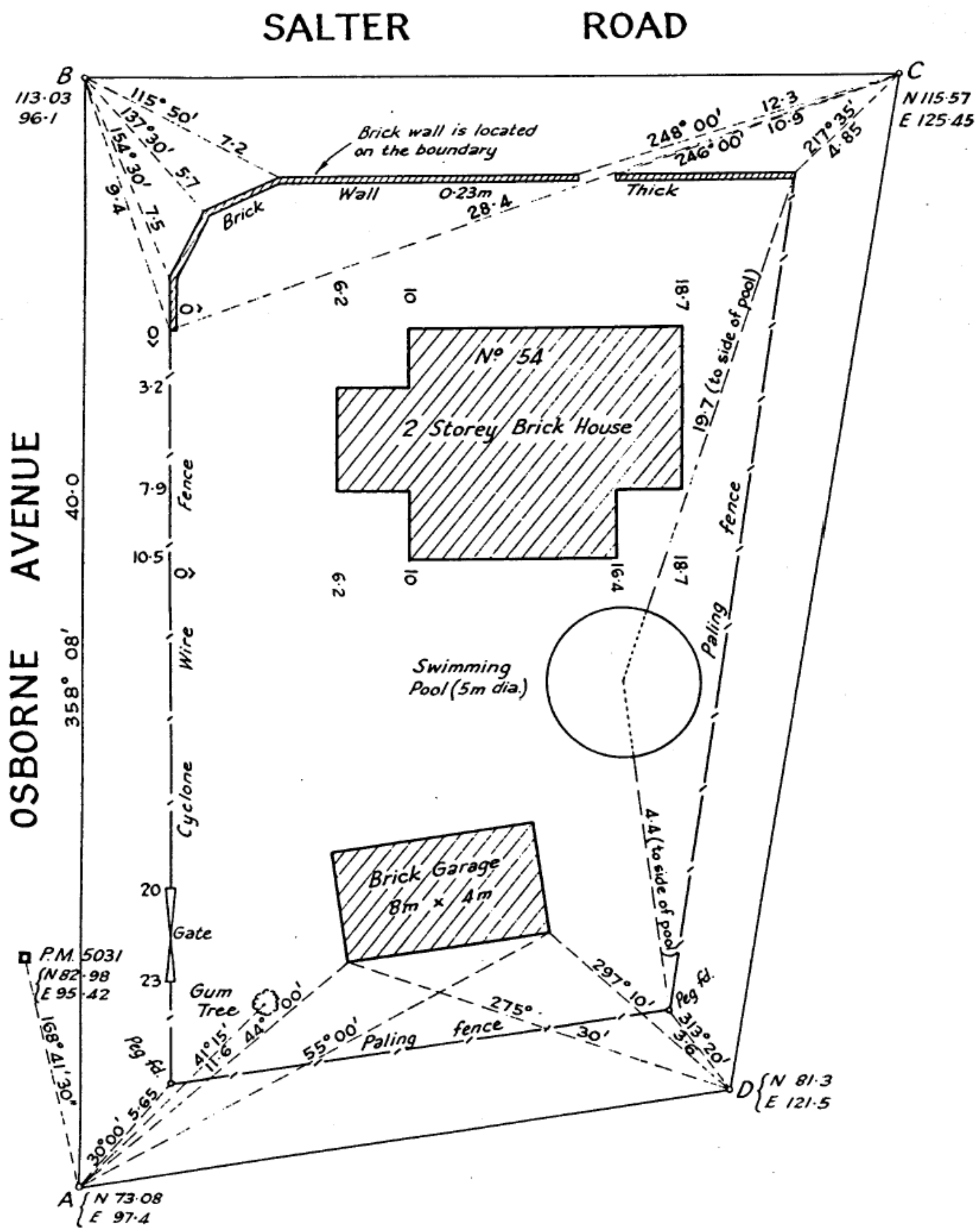
Q5. Traverse adjustment. The angular misclose has already been distributed in the following data. Use Bowditch adjustment to calculate traverse coordinates.

		Brg		Dist	Easting	Northing
B40					**406.347**	**423.509**
B40-1	104°	22'	41"	103.402		
1 - 2	102	41	46	157.225		
2 - 3	41	03	36	143.369		
3 - 4	55	46	07	169.087		
4 – B45	33	53	42	176.743		
B45					**992.415**	**713.229**

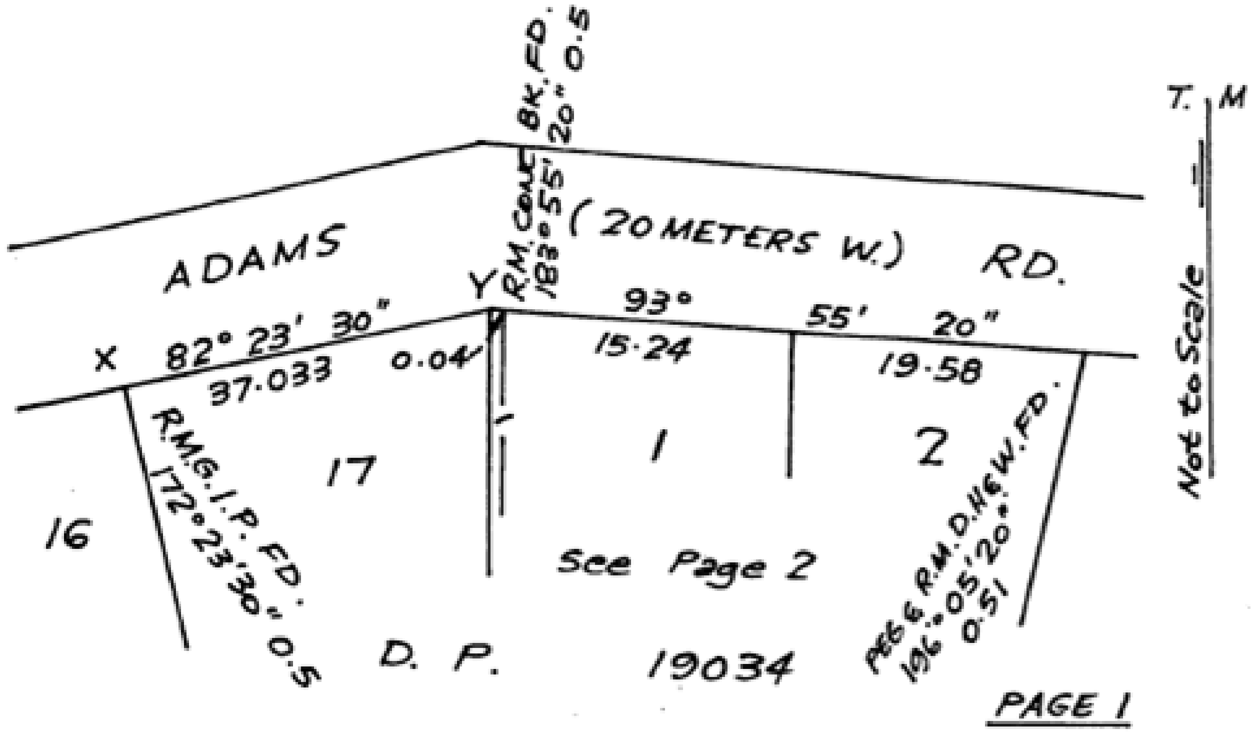
Survey Drafting Questions

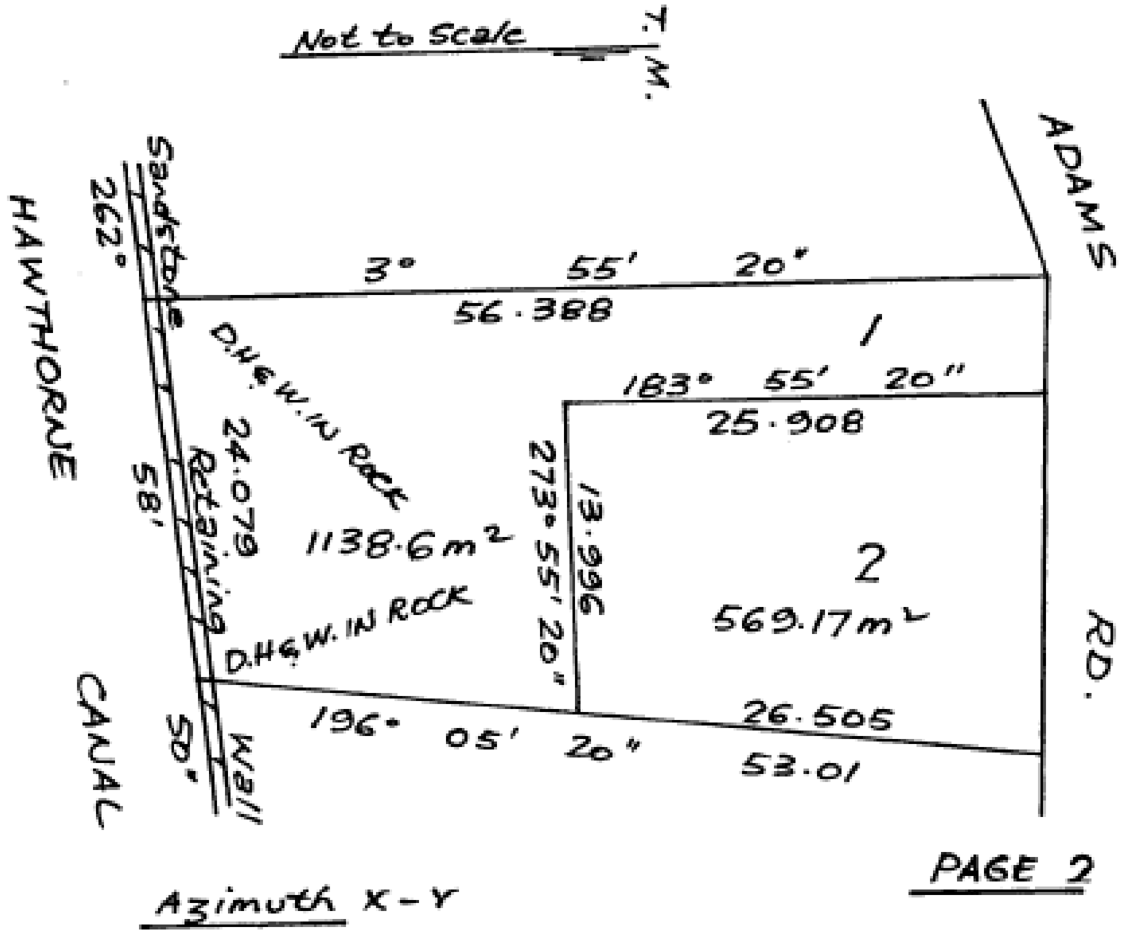
History: Many years ago students had courses and examinations in SURVEY DRAFTING. I looked through my archives of old exam papers from the late 1970s to late 1980s. Typically at least one question needed to be drawn in ink on drawing paper, and "lettering guides or stencils must **not** be used". Lettering guides and stencils made the text and numbers easier for beginners. These questions may now be drawn by students using CAD perhaps more easily, of better quality and in colour). They are reproduced here as tutorial exercises, with only minor modification to the wording.

Q1. This sketch represents a surveyor's field notes for a survey of No. 54 Salter Road. You are required to accurately plot to a scale of 1:250, the information on paper within a border area of 300mm x 300mm. Draw a 10 metre grid over the drawing area and neatly finish your plan. (Modern students with CAD may print in colour.)


SALTER ROAD
OSBORNE AVENUE
B
113·03
96·1
C
N 115·57
E 125·45
D
N 81·3
E 121·5
A
N 73·08
E 97·4
P.M. 5031
N 82·98
E 95·42
168° 41' 30"
40·0
358° 08'
Brick wall is located on the boundary
Brick Wall 0·23m Thick
Nº 54
2 Storey Brick House
Swimming Pool (5m dia.)
Brick Garage 8m × 4m
Cyclone Wire Fence
Gate
Gum Tree
Peg fd.
Paling fence
19·7 (to side of pool)
4·4 (to side of pool)
115° 50'
137° 30'
154° 30'
7·2
5·7
7·5
9·4
248° 00'
246° 00'
12·3
10·9
217° 35'
4·85
28·4
297° 10'
275° 30'
313° 20'
3·6
55° 00'
41° 15'
44° 00'
11·6
30° 00'
5·65
3·2
7·9
10·5
20
23
18·7
16·4
10
6·2

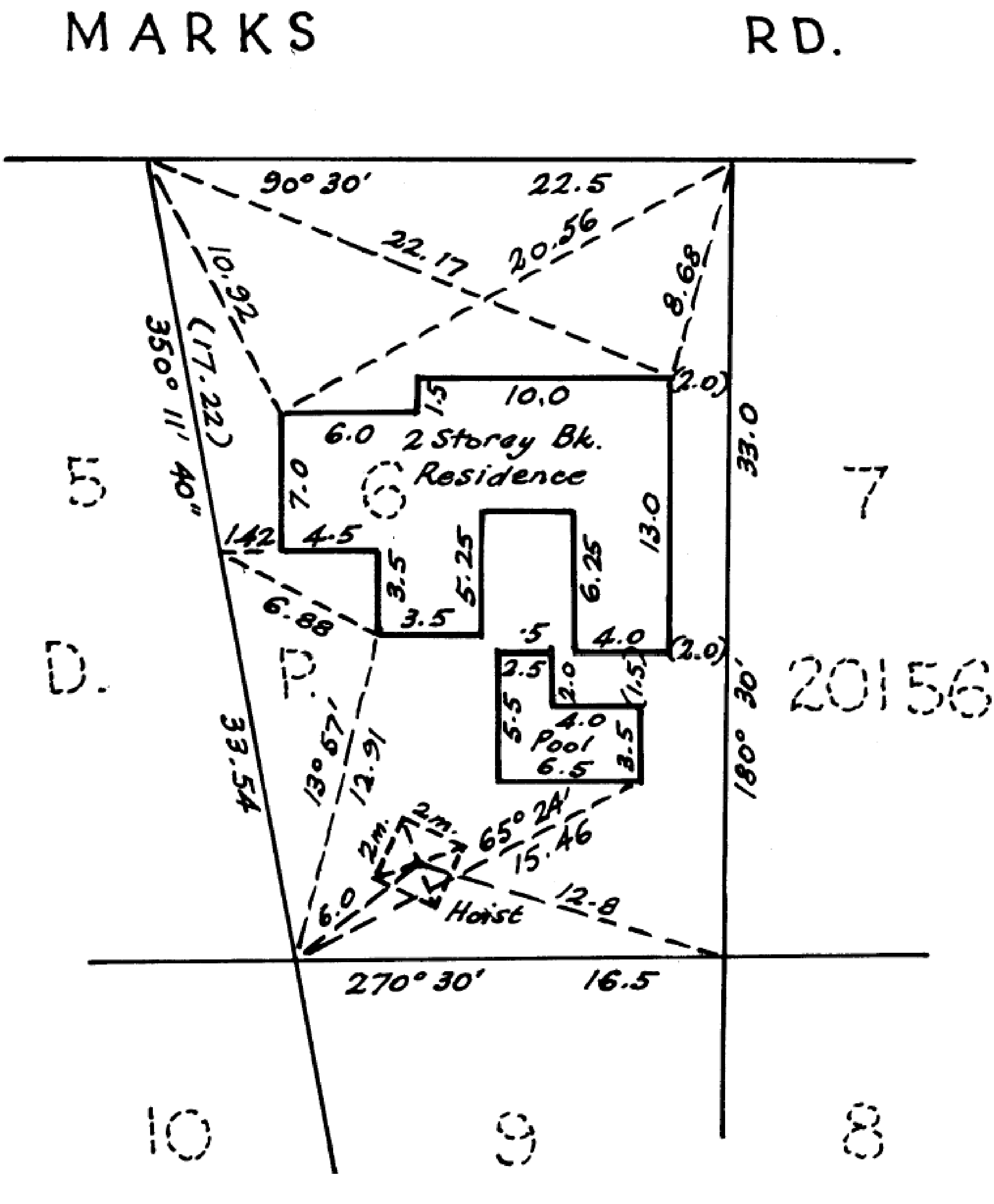
Q2. The following field notes (sketches) represent a survey of lots 1 and 2 which is a subdivision of lot 18 DP 19034. You are required to prepare a single plan of subdivision, to a scale of 1:500. Your plan should be prepared within a plan drawing area of 225 mm x 175 mm. Students are reminded that the method of presenting information in field notes is not necessarily suitable for plan preparation.



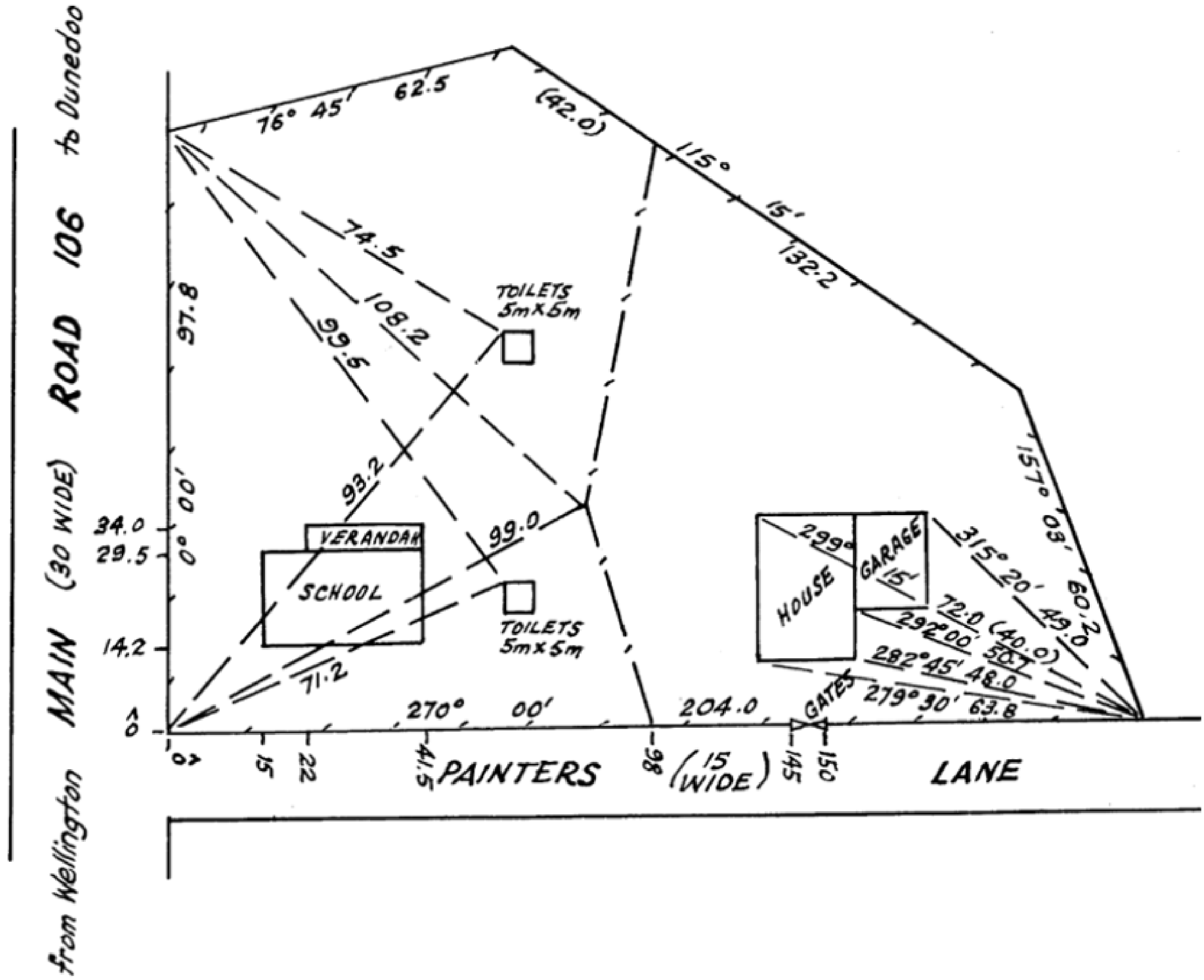


Q3. The field sketch on below illustrates the traverse stations A, B, C and D located at the corners of Lot 6, D.P. 20156 and other measurements fixing detail by radiation, ties or offsets. Prepare a plan at a scale of 1 : 200 of lot 6 showing the relationship of detail to the surveyed boundaries. Complete the plan to the required scale with particular reference to:

(a) Illustrating the lot boundaries and detail with the lot. (Do not record measurements extraneous to the boundary definition).
(b) Locating the drawing within borders 210 mm by 297 mm, and giving neat presentation of the street name, lot numbers and other necessary information.

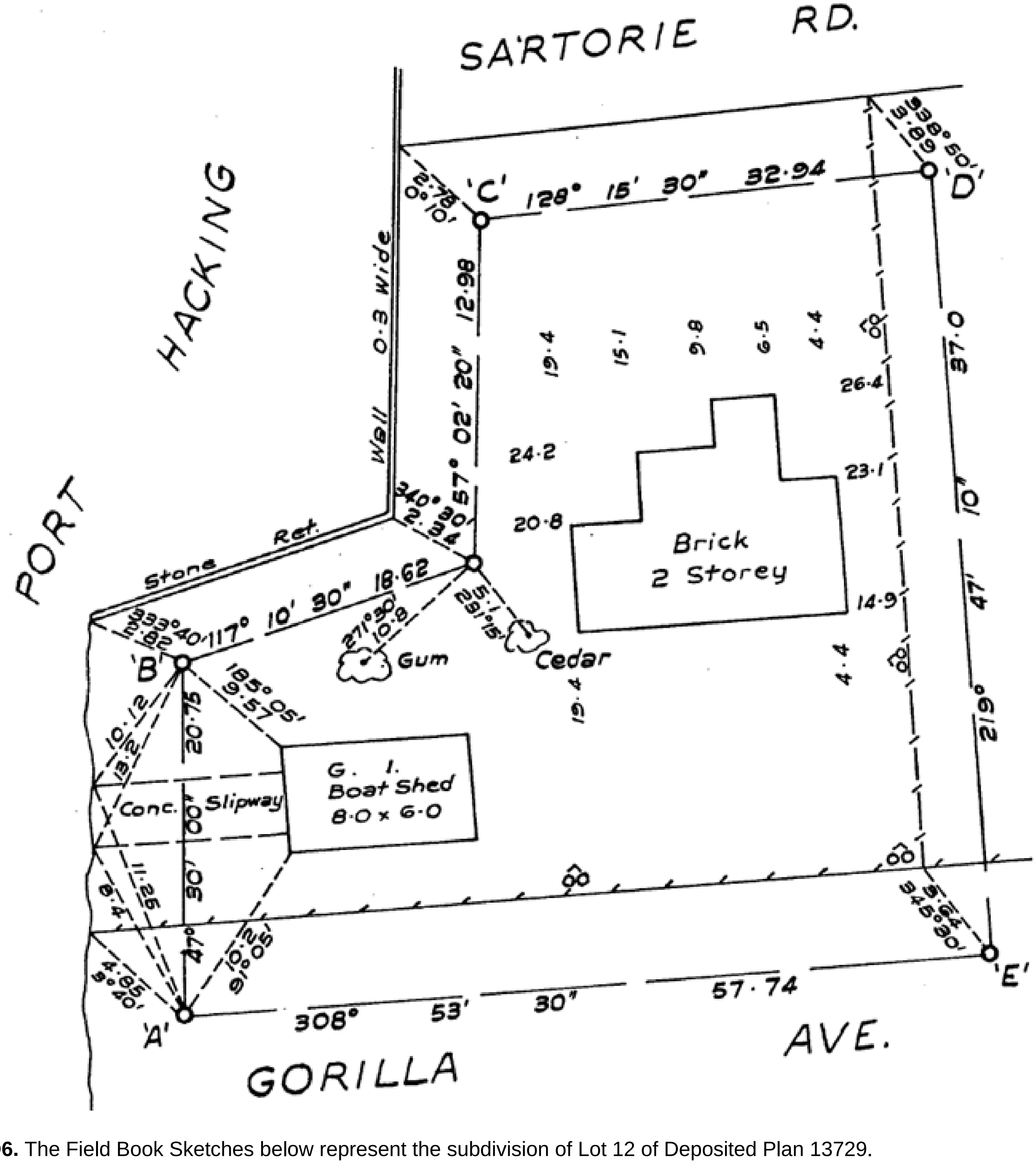


Q4. The figure below shows the survey information for Saxa Crossing School Site. From the information plot a plan of the site to a scale of 1 : 1000.

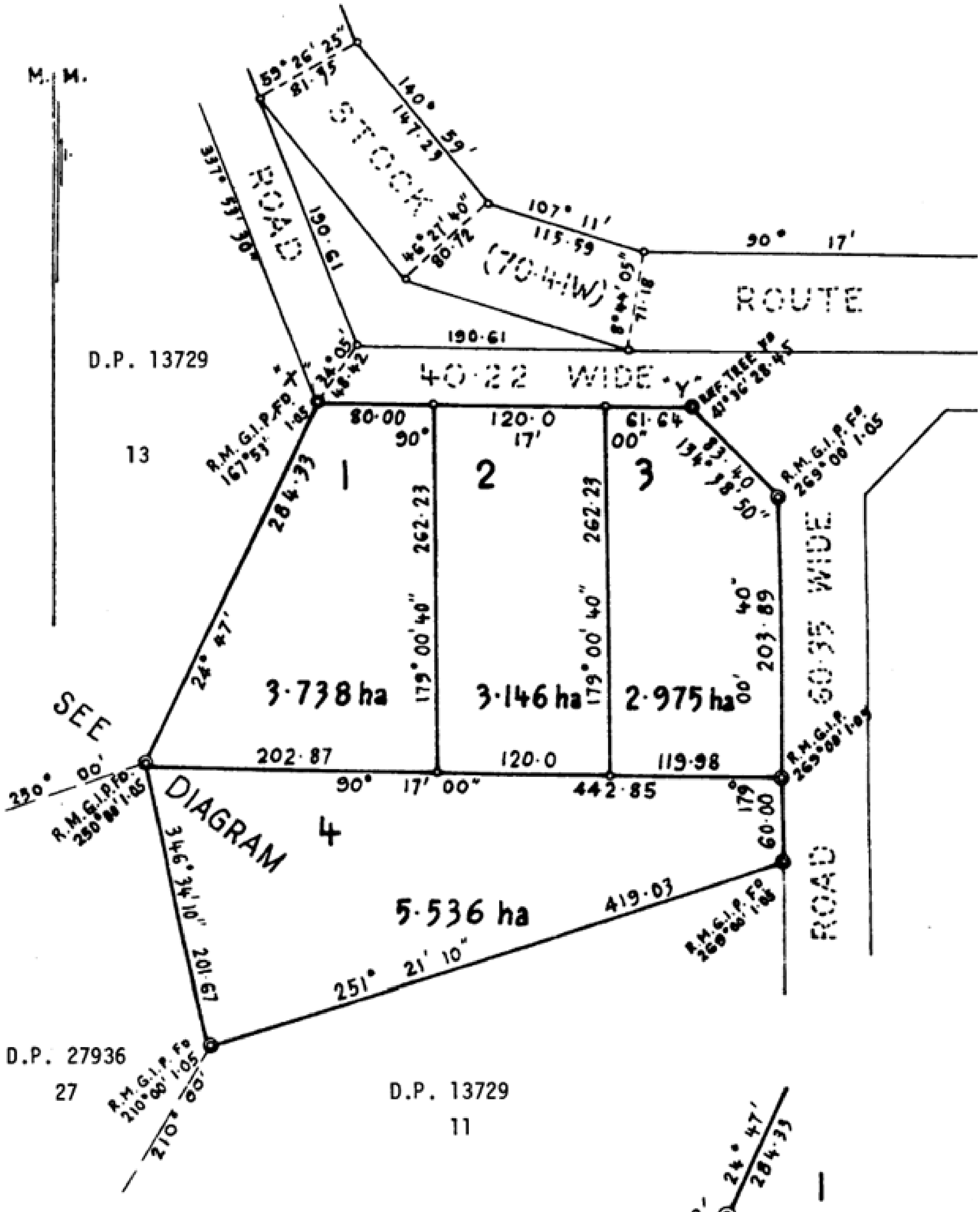


Q5. You are required to prepare a plan at a scale of 1:250 of a parcel of land (Lot 17, D.P. 21064) showing the relationship of buildings, etc., to the surveyed boundaries.
The surveyor's field sketch below illustrates -
(a) The traverse stations: A, B, C, D and E.
(b) The corners of the lot are located by radiations from the traverse stations.
(c) The positions of buildings, etc., determined by radiations, ties and offsets.

Complete the plan to the required scale with particular reference to -
(a) Precise plotting of the major traverse radiation and offsets, etc.,
(b) Neat presentation, in the appropriate style and size, of street names, lot numbers and other necessary miscellaneous information.



Q6. The Field Book Sketches below represent the subdivision of Lot 12 of Deposited Plan 13729. Prepare a plan of subdivision suitable for lodgement at the Registrar General's (Land Titles Office / LPI / Dept of Lands) at a scale of 1:5000 on a suitable cadastral plan form.



SUBDIVISION OF LOT 12 D.P. 13729
COUNTY OF CLAIRE, PARISH OF SLOUGH.

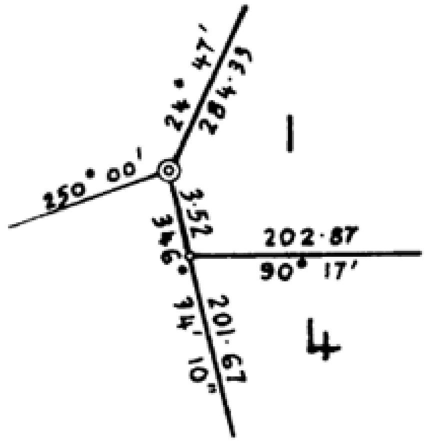


DIAGRAM
Not to Scale

7. CADASTRAL CALCULATIONS

In previous sections of this book we have calculated loop closes around cadastral lots. In this section we include a brief revision of the calculation of areas of cadastral lots. Then we cover the calculations for comparisons between survey measurements and cadastral dimensions (PO comparisons), then cadastral road intersections, and then subdivision of cadastral lots (area calculation aspects, not subdivision design).

A primary role of many surveyors in Australia deals with the definition or redefinition of the legal boundaries of land parcels. This is more broadly called *cadastral surveying*. A Registered Surveyor (called a Licensed Surveyor in some states) has the professional authorisation to divide land thereby creating new *Certificates of Title* over the newly created Freehold land parcels. The registered or licensed surveyor must adhere to the rules and regulations set out in the Surveyors Act. Each state in Australia has its own Surveyors Act, with varying names, each with its own requirements.

A general principle of cadastral surveying is that a registered surveyor must seek to set out, to the best of their ability, the original boundaries as defined at the time of the first survey. Modern day surveyors are therefore confronted with a myriad of problems as a result of the changing accuracy of measuring technologies, loss of some original marks, and changing legal systems. Problems stem from inaccuracies in original compass and chain surveys, lost or destroyed survey marks, conflicting orientations between adjacent plans from subsequent surveys, issues with riparian rights (i.e. land abutting rivers) as well as legal decisions made over land parcels. A cadastral surveyor must be both a detective and quasi-land-lawyer. Cadastral surveying will be covered in much more detail in other courses.

7.1 AREA CALCULATIONS

You are expected to have learnt about this topic in a previous course, so a revision summary is included here without derivations and without all the details. Many basic Surveying textbooks cover this topic well.

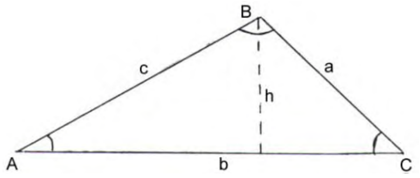
Surveyors calculate areas for a number of applications including cadastral surveying, e.g. for land parcels and lots. We deal here with the calculation of area of figures that have regular (i.e. straight line) boundaries. Areas to irregular boundaries (e.g. creeks and rivers) are dealt with elsewhere.

Unit conversions for lengths and areas:

1 inch = 25.4 mm exactly
1 foot = 0.3048m exactly = 12 inches
1 chain = 100 link = 66 feet = 22 yards (cricket pitch length)
1 mile = 5280 feet = 1.609…km
1 acre = 4 rood = 160 perch ≈ 64m * 64m (10 sq chain)
1 hectare (ha) = 10,000 m^2 = 100m * 100m

AREA OF A TRIANGLE

Several equations give the area of a triangle.



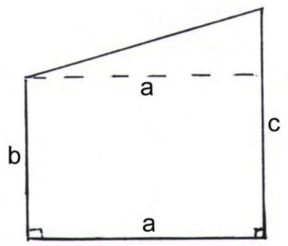
$$Area = \frac{1}{2}\,b\,h = \frac{1}{2}\,a\,b\,\sin C = \frac{1}{2}\,a\,c\,\sin B = \frac{1}{2}\,b\,c\,\sin A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)} \quad \text{where} \quad s = \frac{a+b+c}{2}$$

A figure can be divided into many triangles. The area of each triangle is calculated. All areas are then added up to give the total area. This is very laborious, hence not often used.

AREA OF A TRAPEZIUM

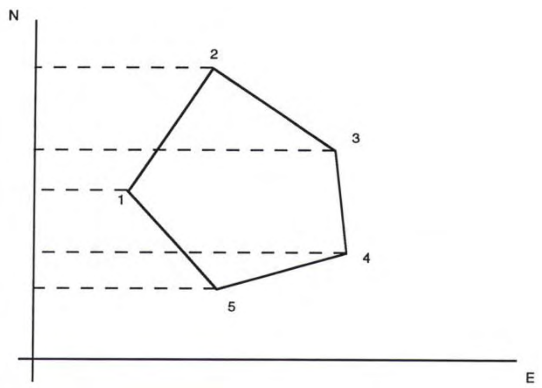
The area can be calculated from the sum of a rectangle and a triangle:

$$Area = ab + \frac{1}{2}\,a\,(c-b) = a\left(\frac{b+c}{2}\right)$$



AREA FROM COORDINATES

There are several versions of the equations for area of a polygon.



Area = Σ positive trapezia – Σ negative trapezia

Area can be obtained from the full coordinates (zigzag method), beware of round-off errors!:

$$\text{Area} = \left|\ \frac{1}{2}[\,E_1N_2 - E_2N_1 + E_2N_3 - E_3N_2 + E_3N_4 - E_4N_3 + E_4N_5 - E_5N_4 + E_5N_1 - E_1N_5\,]\ \right|$$

Worked example, very easy to set up in a spreadsheet:

Area by coordinates

E	N	EiNi+1	NiEi+1
100.000	300.000		
96.896	307.373	30737.3	29068.7
147.678	328.755	31854.9	45392.2
160.106	395.813	58452.8	52635.7
181.738	391.804	62730.2	71934.2
170.201	329.558	59893.2	66685.5
100.000	300.000	51060.4	32955.8
		294728.7	298672.0

Area = 1971.6

The equations can be extended for a figure with n corners, and rearranged into different forms:

Area = ½ | $[(N_1E_2 + N_2E_3 + \ldots + N_nE_{n+1}) - (E_1N_2 + E_2N_3 + \ldots + E_nN_{n+1})]$ |

Beware of round-off error!!

$$\text{Area} = \left|\frac{1}{2}[N_1(E_2 - E_n) + N_2(E_3 - E_1) + \ldots + N_n(E_1 - E_{n-1})]\right| = \left|\frac{1}{2}\left[\sum_{i=1}^{n} N_i(E_{i+1} - E_{i-1})\right]\right|$$

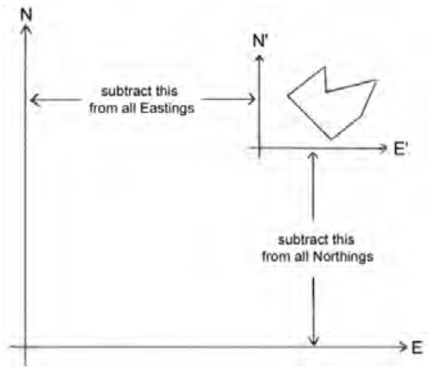
$$\text{Area} = \left|\frac{1}{2}[E_1(N_2 - N_n) + E_2(N_3 - N_1) + \ldots + E_n(N_1 - N_{n-1})]\right| = \left|\frac{1}{2}\left[\sum_{i=1}^{n} E_i(N_{i+1} - N_{i-1})\right]\right|$$

$$\text{Area} = \left|\frac{1}{2}\left[\sum_{i=1}^{n}(N_{i+1} - N_i)(E_{i+1} + E_i)\right]\right|$$

$$\text{Area} = \left|\frac{1}{2}\left[\sum_{i=1}^{n}(N_{i+1} + N_i)(E_{i+1} - E_i)\right]\right|$$

Note: To use these formulas consider $N_0 = N_n$, $E_0 = E_n$, $N_{n+1} = N_1$, and $E_{n+1} = E_1$

If some of the coordinates are negative, just put them into the equation with their correct sign. If the coordinates have many digits, subtract a constant term to avoid decreased accuracy due to accumulated round-off errors.



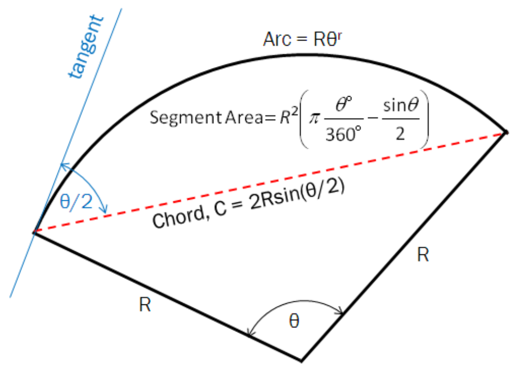
Circular curved boundaries

Some cadastral lot boundaries are circular arcs. In this case calculate the area of the polygon where the arc boundary is replaced with its chord. Then calculate the area of the segment of the circle and add or subtract it to the polygon area as appropriate.

If chord length is known: $\theta = 2\sin^{-1}\left(\frac{C}{2R}\right)$

$$\text{Total Area} = \pi R^2\left(\frac{\theta^\circ}{360^\circ}\right)$$

Area of a triangle is ½ ab sinC, so Area of the segment



between the curve and the chord is: $\text{Segment Area} = R^2\left(\pi\frac{\theta^\circ}{360^\circ} - \frac{\sin\theta}{2}\right)$

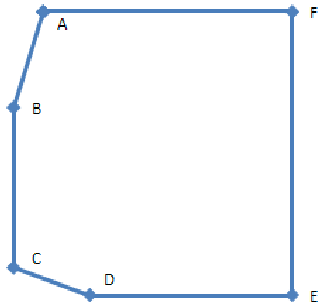
7.2 PO Comparisons

PO stands for "per original". Original survey plan dimensions can be compared with your new survey measurements between marks found. If the plan is old then distances on the plan could be in different units or could be less accurate than your modern survey. Some very old cadastral plans in NSW intentionally contained scale errors. Over the years a variety of datums have been used for bearings. Some surveys relate bearings to Magnetic North, or True North, or one of the grid norths: AMG, ISG or MGA.

PO comparisons can be used to determine scale (distance) and swing (bearing) values if two marks can be found that were measured in the original and new surveys.

The following worked example is based on a problem published in Azimuth (Institution of Surveyors NSW, magazine, Aug 2009) but modified for our purposes. Points A to E lie along a road and F is in a farmer's paddock with no fences nearby. The old survey plan of the lot shows:

Line	Dist links	Bearing
AB	1000	197° 00'
BC	1600	180° 00'
CD	800	110° 00'
DE	2000	90° 00'
EF	?	0° 00'
FA	?	270° 00'



Survey marks were found at A and E but not at B C or D. A new survey measured the position of A and E as MGA coordinates (E, N) A = 289856.680, 6147404.575 and E = 290353.969, 6146837.380. Calculate a PO comparison between A and E, then determine MGA coordinates of F so that we can search for a mark there.

Solution: 100 links = 66 feet and 1 foot = 0.3048 metres, so 100 links = 20.1168 metres.
Calculate the join AE using the plan 'traverse' data we can calculate the 'close' AE as 3749.268 links = 754.233 m and 139°00'26".
Calculate the join AE from the MGA coordinates gives AE as 754.326 m and 138°45'26".

So the PO comparison yields:
754.326 - 754.233 = +93mm as the distance difference, and
138°45'26" - 139°00'26" = -15' 00" as the swing.
We add these values to the old plan to give the 'new' survey values. This distance difference expressed as a scale ratio is 1.000123 to scale plan metres to MGA, or 0.2011927 to scale plan links to MGA.

This distance EF and AF are missing from the plan but they can be calculated by intersection of bearings or two missing distance / close calculations that we learnt in previous chapters. In this case the lines are exactly NS and EW, so that makes it even easier to determine. If you do the calculations you will see that the plan distances are
EF = 2829.92 and AF = 2459.38 links. These can be converted to MGA distances using our scale factor above, e.g. AF = 2459.38 * 0.2011927 = 494.810 m.

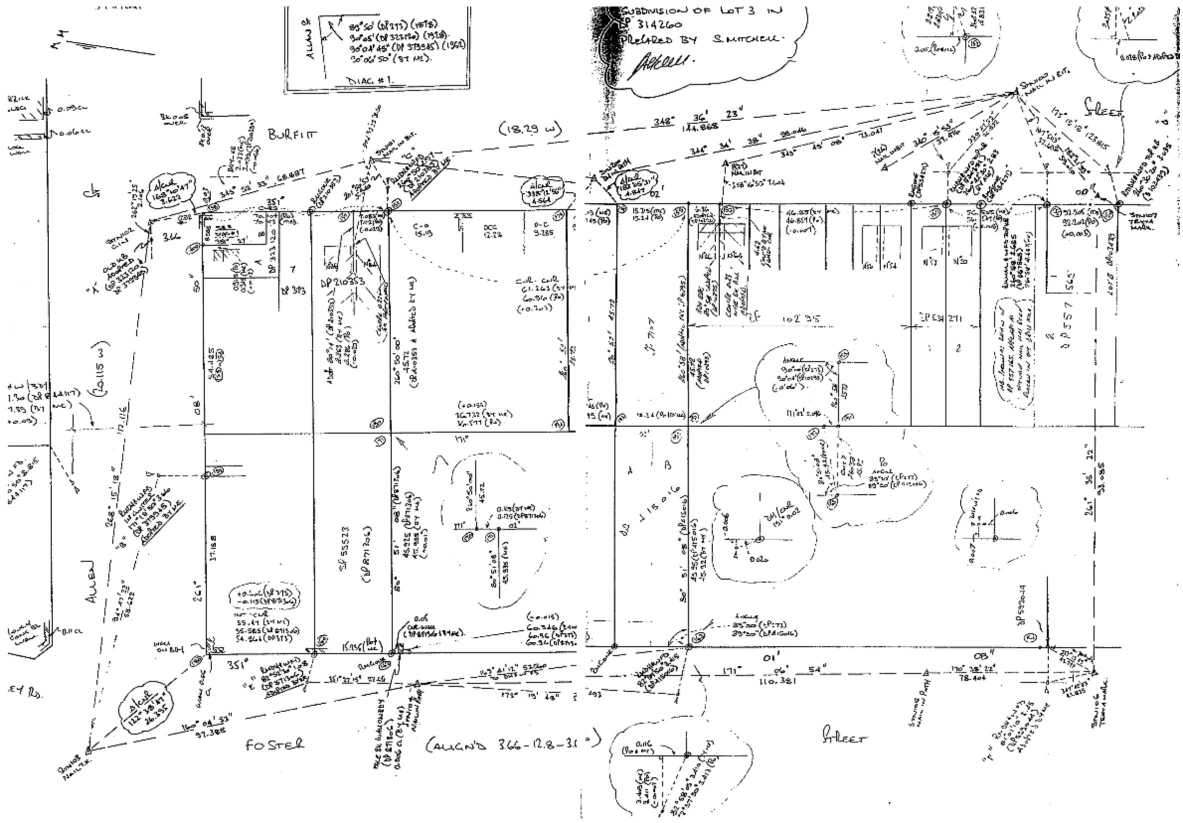
The MGA bearing of lines AF and EF can be calculated by adding our swing (-15' as above) to the plan bearings, e.g. AF = 90°00' – 15' = 89°45'.

The coordinates of F can then be calculated by radiation from A or from E. Use one of them to check the other.
A = 289856.680, 6147404.575, bearing and distance AF = 89°45’ and 494.810m, so F = 290351.485, 6147406.734

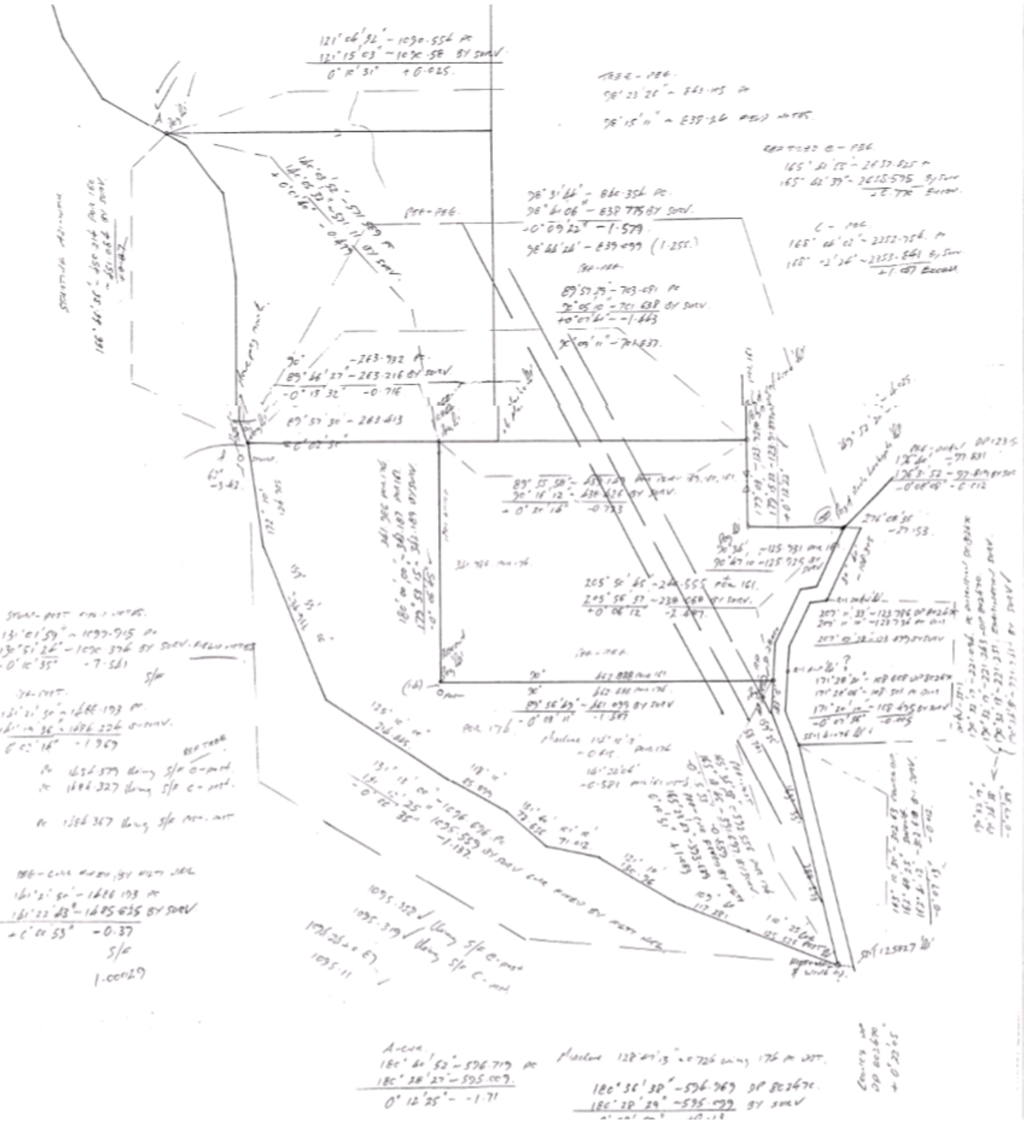
Tutorial question 3 below also aims to help you to learn about this process.

‘Calc’ sheets

Cadastral surveyors doing boundary redefinition surveys need to consider the comparisons between their measured data and that of previous survey plans for many boundary lines and for join lines between marks that they have found. Traditionally surveyors have drawn plans called ‘calc sheets’ where they write onto the plan the comparison values. Here we try to introduce the concepts; you may learn more about ‘calc sheets’ and their usefulness in later courses or during work experience. Some examples are shown below, look at their style; you don’t need to be able to read the figures in detail.



It is possible to do the calculations on computer and to annotate computer drawn plans with these values. Traditionally the calculations were done by calculator and the surveyor drew the plan and annotated it on paper. Whether it is done on paper or on computer doesn’t matter. I believe it is still very useful to produce these plans in modern times and not to rely solely on numbers. The calc sheet plans help surveyors make decisions about boundary definition. The plans provide a graphical view of the survey and surveyors look for trends such as similar scale factors or distance differences on a few lines, or similar swings in bearings of lines. For example the bearing of lines by your survey may be different to those of a previous plan but perhaps the angle at a corner is similar. The PO comparisons on a plan can also help the surveyor decide if there is a problem with a particular mark.



With current boundary definition surveys in NSW, the distances and angles are the main concern not the coordinates of points. But coordinates can be useful for finding marks particularly with GPS and perhaps in future years if the cadastral system moves to a coordinated system.

2D TRANSFORMATIONS

Transformation calculations can also be used to find the relationship between two different coordinate systems, e.g. a local network of points can be transformed into a national reference frame. 2D transformations can be used like the PO comparisons described above, but work with coordinates of points. In PO comparisons we achieve the same results but work with bearings and distances of lines between points.

In later courses 2D transformations will be covered for cases where two points have known coordinates. If there are more than 2 known points it is better to use a least squares estimation of the transformation parameters (that is covered in a later course). In linear transformations the connection between two coordinate systems can be written in the form:

$$\begin{bmatrix} E' \\ N' \end{bmatrix} = \begin{bmatrix} p_1 & q_1 \\ p_2 & q_2 \end{bmatrix} \begin{bmatrix} E \\ N \end{bmatrix} + \begin{bmatrix} t_E \\ t_N \end{bmatrix}$$

where (E',N') are the coordinates in the new system corresponding to coordinates (E,N) in the old system, the p and q parameters can be used to determine the rotation of axes and a scale change, and the t parameters represent the shift (translation) in origin.

7.3 ROAD INTERSECTIONS

In this section we will learn about the calculations for straight sided road cadastral boundaries and their intersections. This topic is different to the Road Design calculations that we will learn about later where road centrelines follow curves that you are familiar with on freeways.

It is estimated that 10 – 20% of the Australian land mass is covered by road reserve. Therefore many surveyors and spatial scientists will deal with the location of road reserve at some time in their professional career.

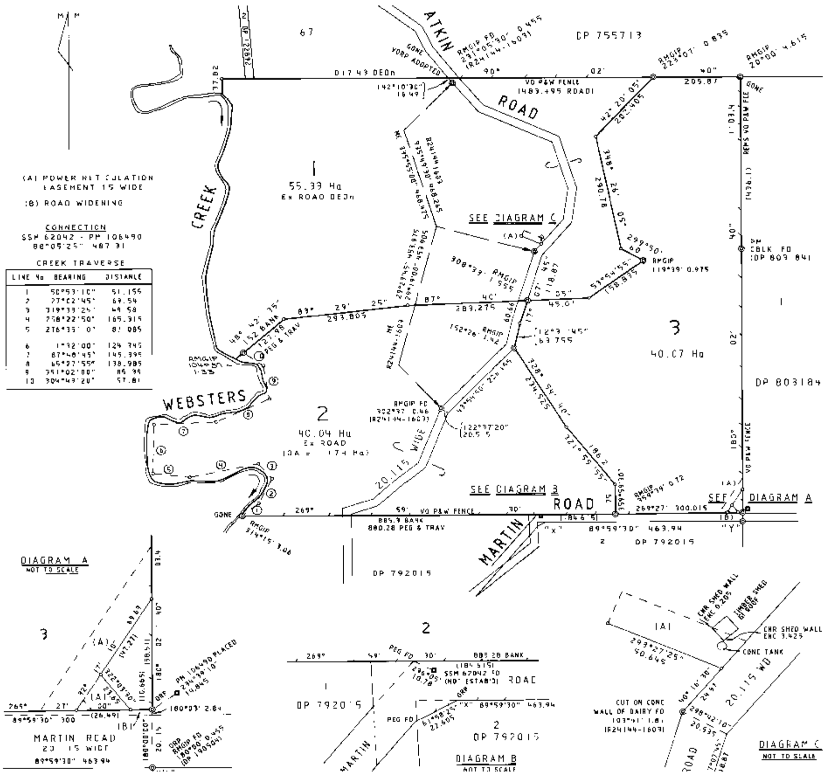


Figure: Part of a rural cadastral plan

A Road reserve is a legal boundary, usually delineated by a surveyor in a land subdivision. Loosely speaking, the legal boundary of a road is where fences should be placed, it is not the kerb. Again, loosely speaking, road reserves include the road surface, kerbs and gutters and footpaths. New roads can be *opened*, old roads can be *closed* and existing roads can be widened or narrowed.

Historically, during the time of settlement, some plans were drawn in isolation with no regard for the local topography. These plans were simply drafted on a map and the poor surveyor was required to peg this new road reserve in the field as required. Sometimes the road reserve headed straight over a cliff – hardly a suitable location for a road. These roads were not always required to be pegged or built and some still exist today as “paper roads”.

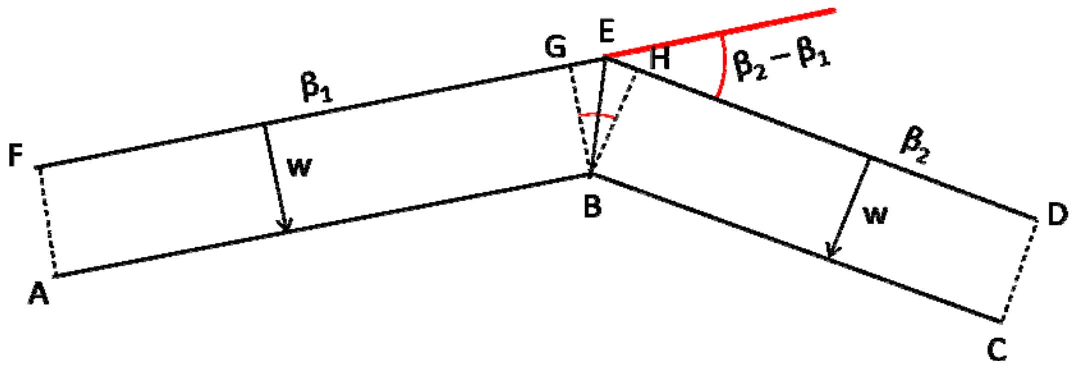
Traditionally country roads were given a standard width of one-chain. A chain is a standard length measuring tool used by surveyors and comprises 100 links. One link is equivalent to 0.201168 m. A one-chain road therefore is 20.117 m wide. Wider country or metropolitan roads were given a width of two-chain (40.24m) or sometimes three-chain (60.35m). Nowadays, metric widths are applied to new land subdivisions and some curved (circle arc) boundaries are used.

In later Cadastral and Land Law courses at UNSW students will learn about the New South Wales Legislation in the form of Acts of Parliament and associated Regulations that cover the legal aspects relating to land and roads. A registered surveyor is required to define the new boundaries after opening or closing of a road reserve or after road widening/narrowing has been undertaken.

A common problem in cadastral surveying is where roads have to be constructed through lots to join with existing roads or boundaries. Our solutions involve elementary trigonometry. We will consider both: intersection of roads of equal width; and intersection of roads of unequal width (usually called variable width in NSW).

7.3.1 ROADS OF EQUAL WIDTH

Consider the following situation where the two sides of the road are parallel, the road 'bends' at B and has a constant width of w. The dotted lines in the figure below are at 90° to the road.



We want to calculate the bearing β_{BE} and the distance d_{BE} at the join / bend / road angle.

<u>Derivation / Solution</u>:

Calculate the half angle: $GBE = \frac{1}{2}\,GBH = (\beta_2 - \beta_1)/2$

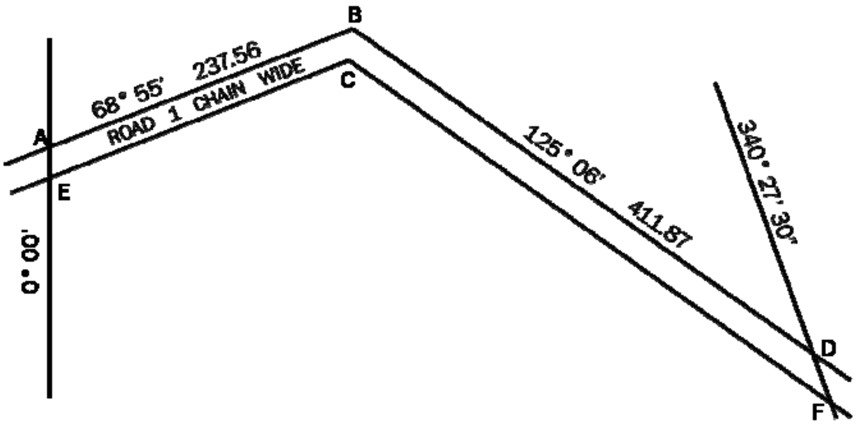
The bearing β_{BE} is then:

$$\beta_{BE} = \beta_2 - 90° - (\beta_2 - \beta_1)/2 \quad \text{or} \quad = \beta_1 - 90° + (\beta_2 - \beta_1)/2 \quad \text{or} \quad = (\beta_1 + \beta_2)/2 \pm 90°$$

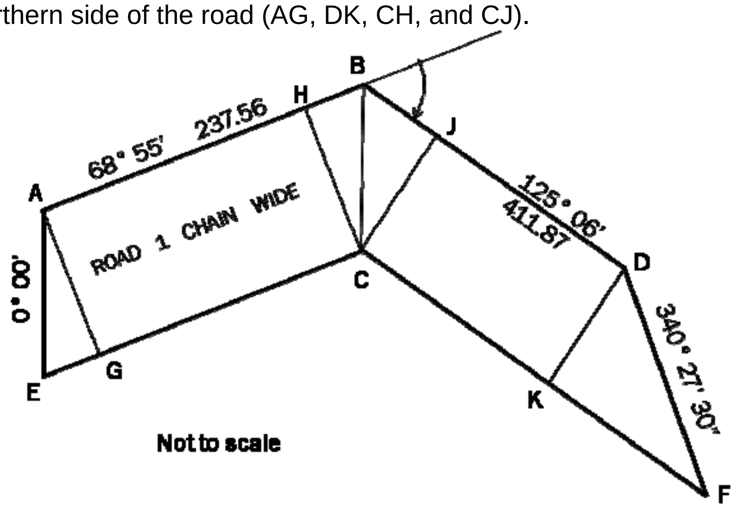
The distance d_{BE} can be calculated from: $\cos(GBE) = \frac{w}{d_{BE}}$ → $d_{BE} = w / \cos\left(\frac{\beta_2 - \beta_1}{2}\right)$

Alternative method:
Assign coordinates to point A, do traverse close calculations using bearings and distances to generate coordinates for F, E, D, C. If distances FE or ED are unknown, select arbitrary values. Then calculate the coordinates of B by intersection of bearings from A and C. Then calculate the join BE, i.e β_{BE} and d_{BE}

<u>Example</u>:
A cadastral survey plan shows a 1 chain road. 1 chain = 66 feet ≈ 20.117 metres. The convention is that both sides of the road are parallel. The distances on this plan have already been converted to metres. Calculate the other bearings and distances of interest for this road. Also, calculate the area of this part of the road.



Solution:
Construct perpendicular lines from A and D to the southern side of the road, and from C to both directions of the northern side of the road (AG, DK, CH, and CJ).



The angles CHB and CJB are both 90° and BC is common to both triangles.
Also, CH = CJ = w = 20.117m
Deflection angle of the road is calculated from the given bearings:
$\Delta\beta = \beta_{BD} - \beta_{AB}$ = 125°06' – 68°55' = 56° 11'

Therefore we get: angle HCB = angle BCJ $= \frac{1}{2}(\beta_{BD} - \beta_{AB}) = \frac{1}{2}\Delta\beta = 28°\,05'\,30''$

Bearing β_{CB} can then be obtained:

$\beta_{CB} = \beta_{AB} - 90° + HCB$ = 68° 55' 00" – 90° + 28° 05' 30" = 7° 00' 30"

or from $\beta_{BC} = \frac{(\beta_{AB} + \beta_{BD})}{2} \pm 90°$

→ $\beta_{BC} = \beta_{CB} + 180°$ = <u>187° 00' 30"</u>

Distance d_{BC} is calculated from: d_{BC} = w / cos (HCB) = 20.117 / cos 28°05'30" = <u>22.803m</u>

Calculate the distance AE:

angle EAG = $\beta_{AE} - \beta_{AG} = \beta_{AE} - (\beta_{AB} + 90°)$ = 180° 00' - 158° 55' = 21° 05'

d_{AE} = w / cos (EAG) = 20.117 / cos 21°05' = <u>21.560m</u>

Calculate the distance DF:

angle KDF = $\beta_{DK} - \beta_{DF} = \beta_{DK} - (\beta_{BD} + 90°)$ = 160° 27' 30" - 215° 06' 00" = 54° 38' 30"

d_{DF} = w / cos (KDF) = 20.117 / cos 54°38'30" = <u>34.763m</u>

Calculate the distance EC:

$d_{EC} = d_{EG} + d_{AB} - d_{HB} = d_{AE}\sin(EAG) + d_{AB} - d_{BC}\sin(HCB)$ = 7.756+237.56-10.738 = <u>234.578m</u>

Calculate the distance CF:

$d_{CF} = d_{BD} - d_{BJ} + d_{KF} = d_{BD} - d_{HB} + d_{DF} \cdot \sin(KDF)$ = 411.87 - 10.738 + 28.351 = <u>429.483m</u>

Can you think of other ways to solve the above problem? How would you do it with CAD?

Calculate the area of the road:

Area = triangle AEG + trapezium AGCB + trapezium BCKD + triangle DKF

$$= \frac{d_{EG} \cdot w}{2} + \frac{d_{AB} + d_{GC}}{2} \cdot w + \frac{d_{BD} + d_{CK}}{2} \cdot w + \frac{d_{KF} \cdot w}{2}$$

$$= \frac{20.117}{2}(7.756 + 237.56 + 226.822 + 411.87 + 401.132 + 28.351) \quad = \underline{13212 \text{ m}^2}$$

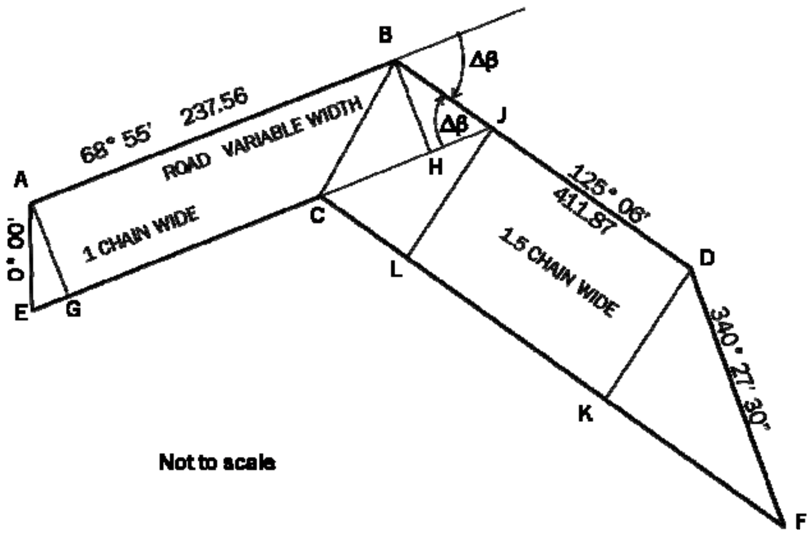
Alternatively, assign coordinates to the points and then calculate area.

7.3.2 ROADS OF VARIABLE (UNEQUAL) WIDTH

Each problem of roads changing width at a bend is different, but can be solved by similar methods. I treat this topic by way of example because I see no advantage in deriving and applying formulas for this topic. This example is similar to the previous example but the eastern road is now 1.5 chains wide (≈30.175m). It is required to calculate: bearing and distance of BC, distances AE and DF, distances EC and CF, and the area of the road.

Solution Method 1:

Construct perpendiculars from A and D to the southern side of the road (AG, DK). Extend EC to intersect BD at J. Then drop a perpendicular from B to intersect CJ at H. Drop a perpendicular from J to intersect CF at L.



The deflection angle of the road is calculated from the given bearings:

$\Delta\beta = \beta_{BD} - \beta_{AB} = 125°\ 06' - 68°\ 55' = 56°\ 11'$

Calculate the distance d_{CJ}: $d_{CJ} = \frac{w_2}{\sin \Delta\beta} = \frac{30.175}{\sin 56°11'} = 36.321\text{m}$

Calculate the distance d_{HJ}: $d_{HJ} = w1 / \tan \Delta\beta = 20.117 / \tan 56°11' = 13.476\text{m}$

Calculate the distance d_{CH}: $d_{CH} = d_{CJ} - d_{HJ} = 36.321 - 13.476 = 22.845\text{m}$

The distance d_{BC} is then: $d_{BC} = \sqrt{d_{CH}^2 + w_1^2} = \underline{30.440\text{m}}$

Calculate the angle BCH: $\alpha_{BCH} = \tan^{-1}\left(\frac{w_1}{d_{CH}}\right) = \tan^{-1}\left(\frac{20.117}{22.845}\right) = 41°\ 22'$

The bearing β_{CB} is then: $\beta_{CB} = \beta_{AB} - \alpha_{BCH} = 68°55' - 41°22' = 27°33' \rightarrow \beta_{BC} = \beta_{CB} + 180° = \underline{207°33'}$

Calculate the distance AE:

$\alpha_{EAG} = \beta_{AE} - \beta_{AG} = \beta_{AE} - (\beta_{AB} + 90°) = 180° - 158°55' = 21°\ 05'$

$d_{AE} = w_1 / \cos(\alpha_{EAG}) = 20.117 / \cos 21°05' = \underline{21.560\text{m}}$

Calculate the distance DF:

$\alpha_{KDF} = \beta_{DK} - \beta_{DF} = \beta_{BD} + 90° - \beta_{DF} = 125°06'00" + 90° - 160° 27' 30" = 54° 38' 30"$

$d_{DF} = w_2 / \cos(\alpha_{KDF}) = 30.176 / \cos 54°38'30" =$ <u>52.146m</u>

Calculate the distance EC:

$d_{EC} = d_{EG} + d_{AB} - d_{CH} = d_{AE} \cdot \sin(\alpha_{EAG}) + d_{AB} - d_{CH} = 7.756 + 237.56 - 22.845 =$ <u>222.471m</u>

Calculate the distance CF:

$d_{CF} = d_{BD} - d_{BJ} + d_{CL} + d_{KF} = 411.87 - w_1 / \sin\Delta\beta + w_2 / \tan\Delta\beta + d_{DF} \cdot \sin(\alpha_{KDF})$

$= 411.87 - 24.213 + 20.214 + 42.528 =$ <u>450.399m</u>

Calculate the area of the road: The area of the road can be computed by dividing the road into a series of triangles, rectangles and/or trapezia, and then adding the areas of each.

Area = triangle AEG + trapezium AGCB + trapezium BCKD + triangle DKF

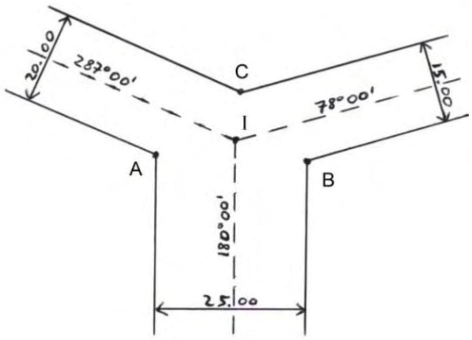
$$= \frac{d_{EG} \cdot w_1}{2} + \frac{d_{AB} + d_{GC}}{2} \cdot w_1 + \frac{d_{BD} + d_{CK}}{2} \cdot w_2 + \frac{d_{KF} \cdot w_2}{2}$$

$$= \frac{20.117}{2}(7.756 + 237.56 + 214.715) + \frac{30.175}{2}(411.87 + 450.399) = \underline{17637\ m^2}$$

<u>Alternative Method</u>: Calculate a traverse close and determine the missing distances.

7.3.3 MULTIPLE ROAD INTERSECTIONS

Example:



Generally it is required to calculate the bearing and distance of the lines between the centreline intersection and the corners of the road (IA, IB, IC) and the area of the road. This problem can be treated as a number of simple road intersections.

General procedure: Draw in centre line. Drop perpendiculars from each corner to each centre line. Then solve the triangles.

7.3.4 SUMMARY OF ROAD INTERSECTIONS

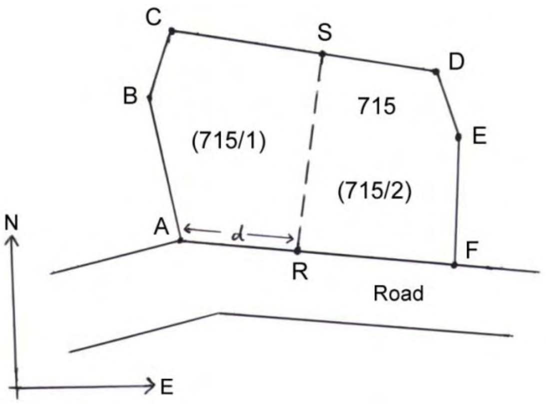
- Road intersections are a common problem in rural cadastral surveying. Generally, the bearing and distance at the join need to be calculated.
- Solutions involve elementary trigonometry or traverse closes and intersections by bearings.
- The area of the road can be obtained as the sum of several triangles, rectangles and/or trapezia, or by coordinates.
- Multiple road intersections can be treated as a number of simple road intersections.
- Practice the calculations!

7.4 SUBDIVISION COMPUTATIONS

Registered Surveyors may be asked by a client to subdivide their land into two or more parcels. The surveyor is guided by the planning regulations set out in the local council area which may prescribe certain constraints over the proposed new land parcels. These constraints may include a minimum area or a minimum street frontage. The client may require a right-of-way along one side of the property or one boundary perpendicular or parallel to another. Other considerations deal with good building sites, an efficient street layout (for larger subdivisions) and suitable drainage.

Surveying software packages aid the computations necessary for subdivision design but cannot always solve every problem. These notes provide some examples of subdivision computations carried out long-hand to give the student an appreciation of the constraints that must sometimes be satisfied in order to compute a satisfactory subdivision.

There is a great variety of subdivision problems. Consider the subdivision of the block of land 715 bounded by the points ABCDEFA:



The new subdivision line RS should cut the parcel 715 into two blocks 715/1 and 715/2, where R and S are on the boundary lines AF and CD respectively. There are three different boundary line types:

a) RS should start at a point R, which is *d* metres from A on the line AF.
b) RS should be perpendicular to a line, e.g. AF.
c) RS should be parallel to a line, e.g. AC.

The line RS could be determined by a condition for the areas of the new blocks of land, e.g.

1) A fixed area for one of the blocks of land.
2) A given ratio between the two new blocks of land.
3) No change in the areas of two adjoining properties, but a complicated (zig-zag) boundary line between them should be replaced by one straight line (i.e. boundary regulation).
4) Subdivisions according to the values of the properties, if certain areas of the concerned properties have different values of the soil.

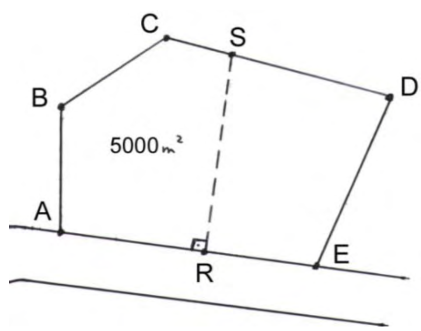
There are other types of subdivision too. For example in a “battle axe” subdivision the line RS above is not a single straight line. Usually the solutions are found by solving triangles and trapezia using plane trigonometry. It is then required to calculate the coordinates of the new boundary points.

Subdivision Example 1:

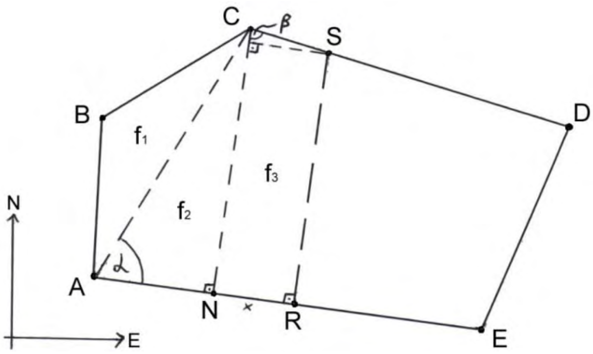
Consider the following subdivision where the new boundary RS is to be perpendicular to AE and the area cut off in the western part is to be 5000m^2.

The coordinates of the corners of the property are:

Point	**E [m]**	**N [m]**
A	2016.27	4040.38
B	2019.17	4084.17
C	2086.28	4107.36
D	2174.37	4065.28
E	2125.38	4011.26



Task: Calculate the coordinates of the new boundary points R and S, and their offsets along the lines AE and CD respectively.



Solution:

Divide the desired area ABCSRA (which should equal 5000m^2) into three parts f_1, f_2 and f_3:

Calculate area f_1 from the coordinates of A, B and C:

$f_1 = |0.5[E_A N_B - N_A E_B + E_B N_C - N_B E_C + E_C N_A - N_C E$
$= 1435.748 \text{ m}^2$

Calculate area f_2 of the right-angled ΔACN:

$\Delta E_{AC} = 70.01\text{m} \quad \Delta N_{AC} = 66.98\text{m} \quad \rightarrow \beta_{AC} = 46° 16' 02" \quad AC = 96.890\text{m}$
$\Delta E_{AE} = 109.11\text{m} \quad \Delta N_{AE} = -29.12\text{m} \quad \rightarrow \beta_{AE} = 104° 56' 35" \quad AE = 112.929\text{m}$

$\rightarrow \alpha = \beta_{AE} - \beta_{AC} = 58° 40' 33"$

$AN = AC \cdot \cos\alpha = 96.890 \cdot \cos 58°40'33" = 50.371\text{m} \quad CN = AC \cdot \sin\alpha = 96.890 \cdot \sin 58°40'33" = 82.767\text{m}$

$$\rightarrow \quad f_2 = \frac{AN \cdot CN}{2} = \frac{50.371 \cdot 82.767}{2} = 2084.528 \text{ m}^2$$

The remaining area f_3 is CSRN: $f_3 = 5000 - (f_1 + f_2) = 5000 - (1435.748 + 2084.528) = 1479.724 \text{ m}^2$

In order to determine the new property corners R and S, the distances NR (=x) and CS need to be calculated. The area f_3 (trapezium CSRN) can be calculated as:

$$f_3 = \frac{CN + RS}{2} \cdot x \qquad \text{where } RS = CN - x / \tan\beta$$

So: $2f_3 = (CN + CN - x \cot\beta) \cdot x \quad = 2CN \cdot x - \cot\beta \cdot x^2 \qquad \rightarrow \cot\beta \cdot x^2 - 2CN \cdot x + 2f_3 = 0$

This is a quadratic equation in x and has the solution:

$$\rightarrow \quad x = \frac{2CN \pm \sqrt{4CN^2 - 4 \cdot \cot\beta \cdot 2f_3}}{2\cot\beta} = \frac{CN \pm \sqrt{CN^2 - 2f_3 \cdot \cot\beta}}{\cot\beta} = \tan\beta \cdot \left(CN \pm \sqrt{CN^2 - 2f_3 \cdot \cot\beta} \right)$$

The angle β can be calculated from the given bearings:

$\beta = \beta_{CN} - \beta_{CD} = \beta_{AE} + 90° - \beta_{CD}$ where ΔE_{CD} and ΔN_{CD} $\rightarrow \beta_{CD} = 115° 32' 01"$ CD = 97.625m

$\rightarrow \beta = 104°56'35" + 90° - 115°32'01" = 79°24'34"$

The distance NR (=x) can now be calculated:

$$x = \tan\beta \cdot \left(CN \pm \sqrt{CN^2 - 2f_3 \cdot \cot\beta} \right)$$
$$= \tan 79°24'34" \cdot \left(82.767 \pm \sqrt{82.767^2 - 2 \cdot 1479.724 \cdot \cot 79°24'34"} \right)$$
$$= 867.076\text{m} \quad \text{or} \quad 18.254\text{m}$$

From the figure and the already calculated distances it is obvious that the correct value is:
NR = x = 18.254m

The distance CS is calculated from: $CS = \frac{x}{\sin\beta} = \frac{18.254}{\sin 79°24'34"} = 18.570\text{m}$

The coordinates of R and S are then calculated by radiation:

$\beta_{AR} = \beta_{AE}$ = 104° 56' 35" and AR = AN + x = 50.371 + 18.254 = 68.625m
$\beta_{CS} = \beta_{CD}$ = 115° 32' 01" CS = 18.570m

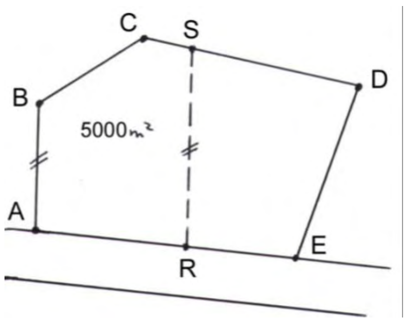
→ E_R = 2082.574m N_R = 4022.684m
→ E_S = 2103.036m N_S = 4099.356m

Check – Calculate area ABCSRA from coordinates:

$$Area = \left|\frac{1}{2}[E_A N_B - N_A E_B + E_B N_C - N_B E_C + E_C N_S - N_C E_S + E_S N_R - N_S E_R + E_R N_A - N_R E_A]\right| = 4999.964\text{m}^2 \checkmark$$

The offsets along the lines AE and CD are: AR = 68.625m CS = 18.570m

Subdivision Example 2:

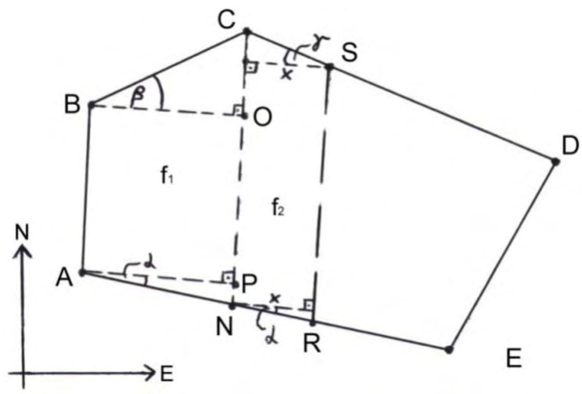


Consider a subdivision where the new boundary RS is to be parallel to AB and the area cut off in the western part is to be 5000m². The coordinates of the corners of the property are the same as in Subdivision example 1.

Task: Calculate the coordinates of the new boundary points R and S, and their offsets along the lines AE and CD respectively.

Solution:

Divide the desired area ABCSRA into two parts f_1 and f_2:



Calculate bearing and distance of AB and BC (from coordinates) and then angle β = 22° 51' 06"
Calculate distances BO and CO:

BO = BC cosβ = 71.004 cos 22°51'06" = 65.431m
CO = BC sinβ = 71.004 sin 22°51'06" = 27.574m

Calculate bearing & distance of AE from coordinates, then angle
$\alpha = \beta_{AE} - (\beta_{AB} + 90°)$ = 11° 09' 15"

Calculate the distances AN and CN:

AN = BO · sec α = 65.431 · sec 11°09'15" = 66.691m
PN = BO · tan α = 65.431 · tan 11°09'15" = 12.901m
→ CN = CO + AB + PN = 27.574 + 43.886 + 12.901 = 84.361m

Calculate area f_1 of trapezium ABCNA: $f_1 = \frac{AB + CN}{2} \cdot BO = \frac{43.886 + 84.361}{2} \cdot 65.431 = 4195.665\ \text{m}^2$

Calculate the angle $\gamma = \beta_{DC} - (\beta_{AB} - 90°)$ = 21° 44' 41"
The remaining area f_2 must be equal to: $f_2 = 5000 - f_1 = 5000 - 4195.665 = 804.335\ \text{m}^2$

The area f_2 (trapezium NCSRN) can be obtained from: $f_2 = \frac{CN + RS}{2} \cdot x$

where RS = CN – x tan γ + x tan α
So: $2f_2 = (CN + CN - x\tan\gamma + x\tan\alpha)\cdot x = 2CN\cdot x - \tan\gamma\cdot x^2 + \tan\alpha\cdot x^2$ → $(\tan\alpha - \tan\gamma)\cdot x^2 + 2CN\cdot x - 2f_2 = 0$

This is a quadratic equation in x and has the solution: x = 9.644m or 826.944m
From the figure and the already calculated distances it is obvious that the correct value is: x = 9.644m
Calculate the offsets CS and AR of S and R:

$CS = x / \cos\gamma = 9.644 / \cos 21°44'41" = 10.383m$
$NR = x / \cos\alpha = 9.644 / \cos 11°09'15" = 9.830m$
→ AR = AN + NR = 66.691 + 9.830 = 76.521m

The coordinates of R and S are then calculated by radiation:

$\beta_{AR} = \beta_{AE} = 104° 56' 35"$ AR = 76.521m → $E_R =$ 2090.203m $N_R =$ 4020.648m
$\beta_{CS} = \beta_{CD} = 115° 32' 01"$ CS = 10.383m → $E_S =$ 2095.649m $N_S =$ 4102.885m

Check – Calculate area ABCSRA from coordinates:

$$Area = \left| \frac{1}{2} [E_A N_B - N_A E_B + E_B N_C - N_B E_C + E_C N_S - N_C E_S + E_S N_R - N_S E_R + E_R N_A - N_R E_A] \right| = 4999.9m^2$$

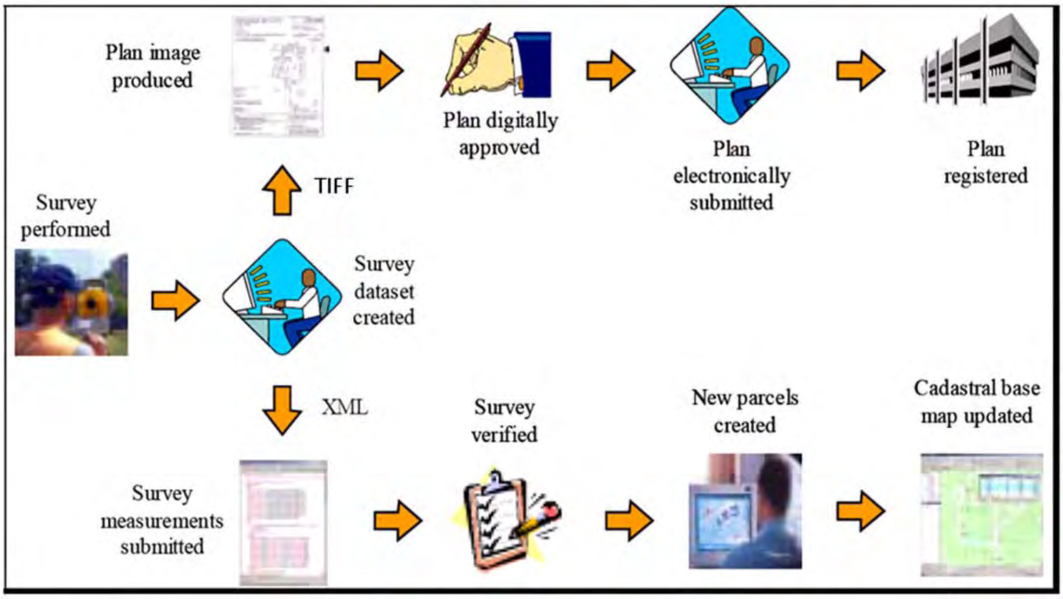
The offsets along the lines AE and CD are: AR = 76.521m CS = 10.383m

7.5 Electronic Deposited plans

Refs: Azimuth Feb 2009, Byrnes' UG thesis UNSW SSIS 2007

Cadastral Deposited Plans (DPs) lodged at the NSW Land's Department (which has had several different names over the years) were traditionally hand drawn plans by draftsmen on paper, linen or film. Some years ago Surveyors began using CAD to draw plans and print them on paper / film for lodgement at the Lands Dept. A few years ago under the ePlan scheme, surveyors were able to submit tiff files of their plans by for example email. In 2008 about 33% of plans were submitted as tiff files. A tiff file is like a scanned image you see the lines and numbers but can't actually use the numbers unless some one reads them and types them in. Commencing in about 2009, surveyors will be able to draw their plan in CAD, SaveAs LandXML format files, and submit the LandXML file. A landxml is similar in concept to a html file that is used to produce a web page. A program that can read a LandXML file can produce a plan, but also has the data (numbers) that can be used directly for calculations. So a GIS program that can 'read' the data can be used to produce graphics and calculations.

Byrnes (2007) produced the following diagram of Plan Registration in NSW using the electronic lodgement methods.

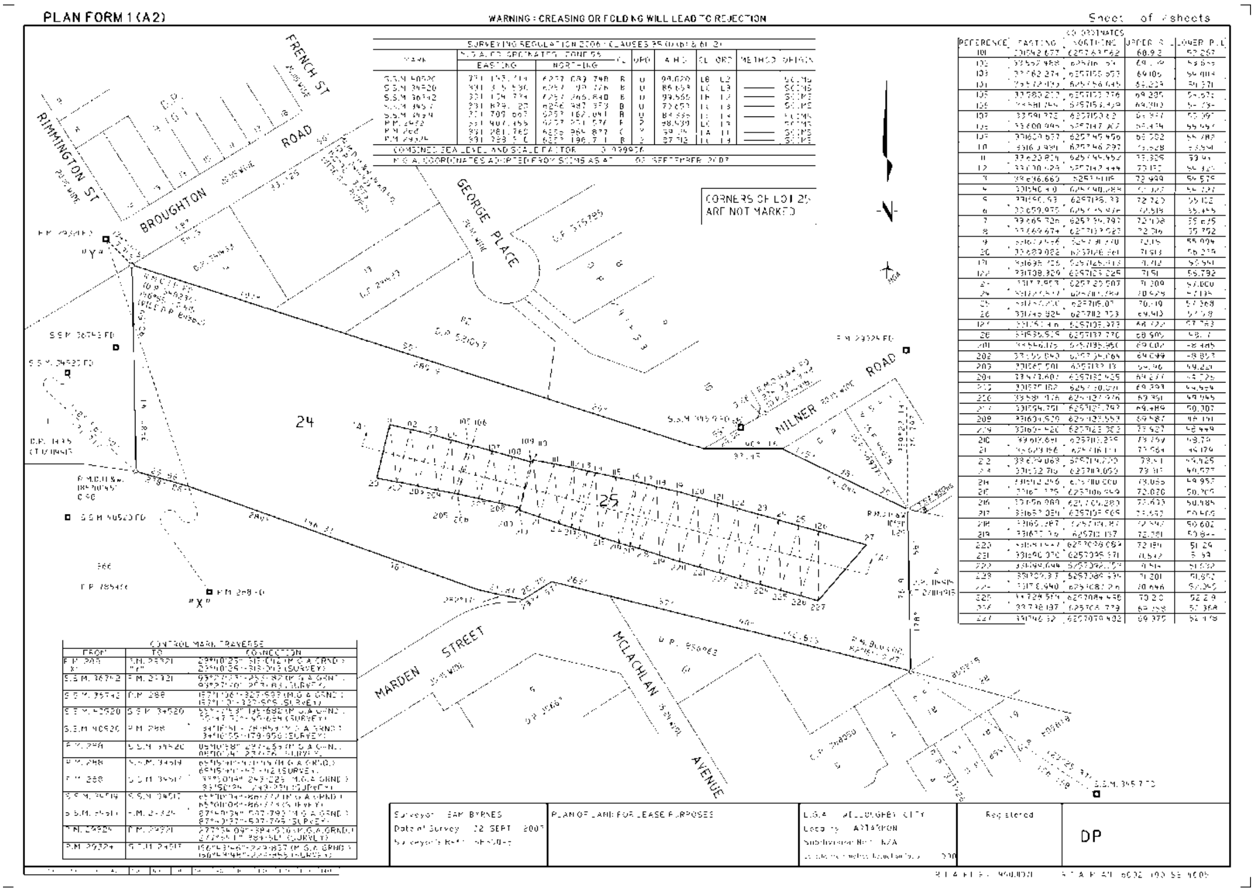


LandXML is an XML schema, a set of rules which define structure and allowed data types, specifically designed to standardise civil engineering and surveying data. LandXML was first released in 2001 and ratified it as an acceptable standard in 2002 by Australian and NZ authorities, and it has been further developed since then.

In NSW the EPlan system will be lodging digital LandXML files in an adaptation of the national schema. It is hoped that by embracing this system it will lead to faster registration of plans as well as faster updating of the DCDB (GIS coordinated cadastre).

The following example material was presented by Byrnes (2007) while working on his thesis and with surveyors at the RTA, for part of the Lane Cove Tunnel. The traditional plan was drafted to the standards required by the Department of Lands using the Liscad software.

Example, traditional plan part of LandXML File:

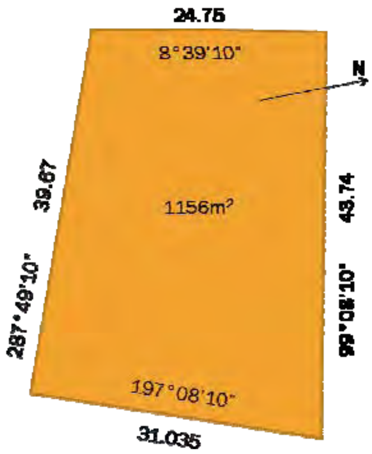


Example, part of LandXML File:

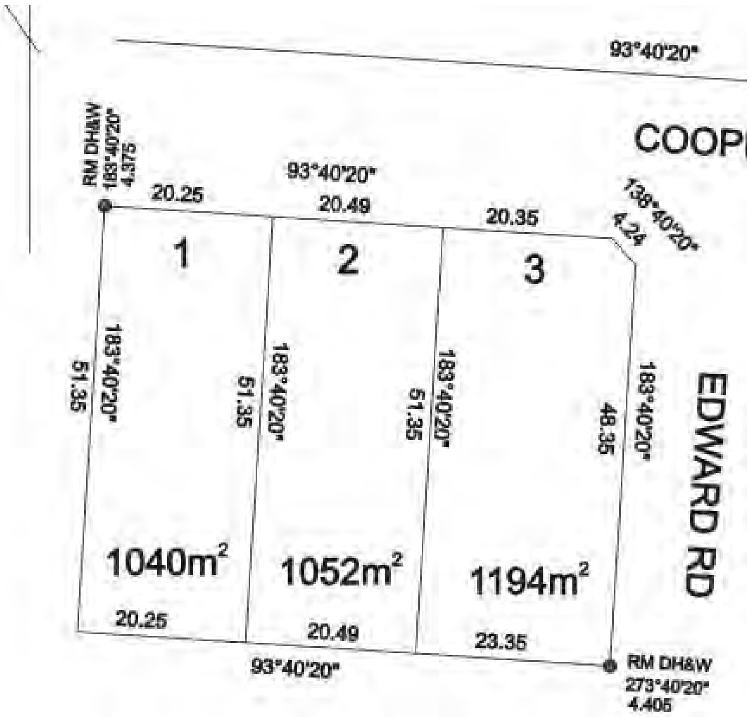
<?xml version="1.0" encoding="iso-8859-1" ?>
<LandXML xsi:schemaLocation="**http://www.landxml.org/schema/LandXML-1.0 http://www.landxml.org/schema/LandXML-1.0.xsd**" version="**1.0**" date="**2007-10-25**" time="**23:11:50**" readOnly="**false**" language="**english**" xmlns:xsi="**http://www.w3.org/2001/XMLSchema-instance**" xmlns="**http://www.landxml.org/schema/LandXML-1.0**">
<Application name="**ListechStyleSheet**" version="**1, 0, 0, 0**" manufacturer="**LISTech Pty Ltd**" manufacturerURL="**www.liscad.com**">
<Author createdBy="**Listech Development Team**" />
</Application>
<Project name="**LISCAD**" desc="" />
<Units>
<Metric linearUnit="**meter**" areaUnit="**squareMeter**" volumeUnit="**cubicMeter**" temperatureUnit="**celsius**" pressureUnit="**HPA**" />
</Units>
<CgPoints>
<CgPoint name="**B110**" code="**DEFAULT**">**6256955.520000 331326.994000**</CgPoint>
<CgPoint name="**B214**" code="**DEFAULT**">**6257116.253000 331216.161000**</CgPoint>

7.6 Tutorial Questions

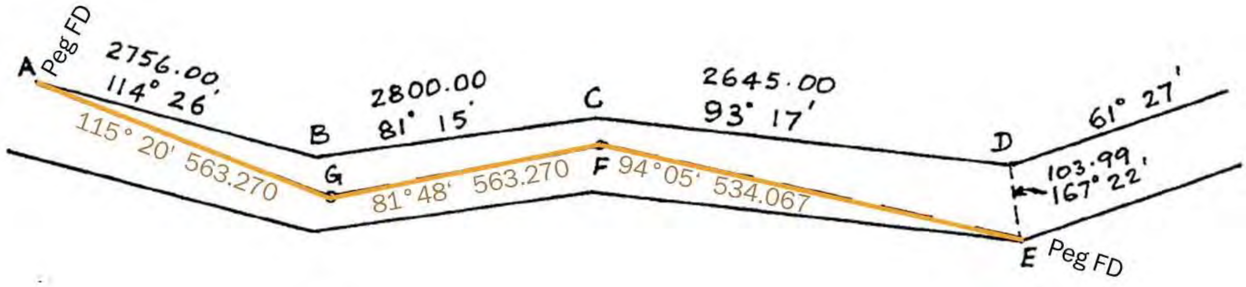
Q1. The figure at right shows the dimensions of a lot as shown on a survey plan. Has there been a typo or drafting error? Calculate misclose and area.



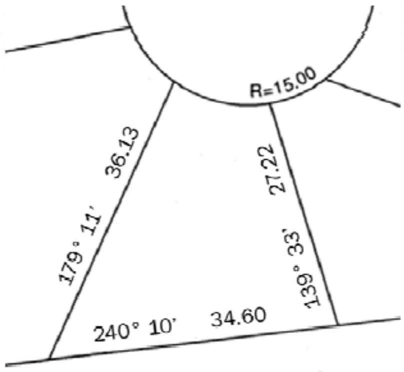
Q2. Assume that the figure at right is part of a cadastral DP. The MGA coordinates of the DH&W in the North West corner of lot 1 are (289571.797, 6146779.731). An azimuth swing of 2°10'20" has to be added to all bearings in the plan to align them with MGA and distances in the plan have to be multiplied by 1.000140 to convert them to grid distances. Calculate the MGA coordinates of all lot corners shown in the figure.



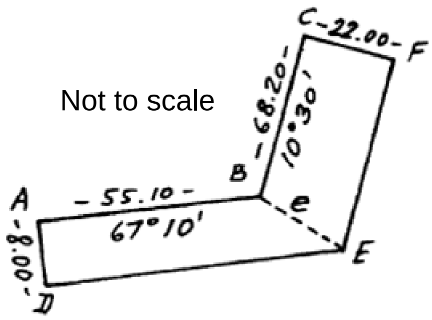
Q3. The figure below shows an old road survey of 100 links wide. [100 link = 66 * 0.3048 metres exactly] Pegs on this survey were found at A and E. A traverse, on an assumed bearing, was run along A G F E, with AG = 115° 20' for 563.270 m, GF = 81°48' for 563.270 m and FE = 94°05' for 534.067 m. Find the correction to be made to the adopted bearing of the traverse to bring it onto the same azimuth (bearing) as the road survey. Also, state the comparison in the distance AE between the original and the new traverse.



Q4. Calculate the missing chord (bearing and distance), then calculate the area of the lot in the figure below. Do not assume the side boundaries are in line with the centre of the arc. Radius = 15.00

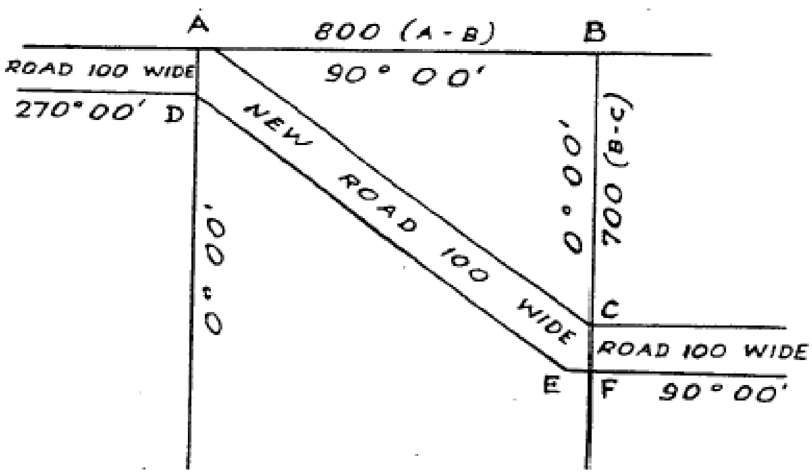


Q5: A road, 8.00 m wide and on a bearing of 67°10' meets a second road which has a bearing of 10°30' and a road width of 22.00 m (see figure). The distances: AB = 55.10 m, and BC = 68.20 m



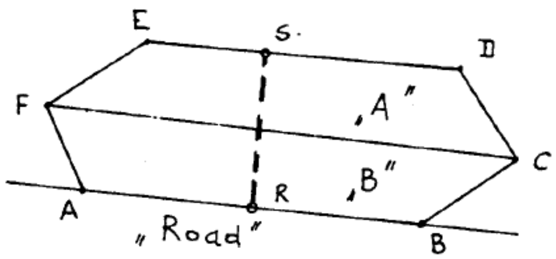
a) Calculate the length (e) and the bearing of the road crossing BE (β_{BE}) if both sections of the road were 8m wide.
b) Calculate the length (e) and the bearing of the road crossing BE (β_{BE}) using road widths as shown in the figure above.
c) Check your results for e and β_{BE} using an independent method.
d) Calculate the area of the road A B C F E D (A),

Q6. It is required to join existing roads 100 wide with a new road 100 wide as shown on diagram below. Measurements are in links. From the data supplied calculate the bearings and distances of DE and EF and the area of the new road. [This question is derived from page 13 of the Survey Computations book by R B Horner, 2nd Ed reprinted 1962. The first edition was published in 1948, and students have been challenged by this question ever since ☺.]



Q7. The coordinates of the corners are:

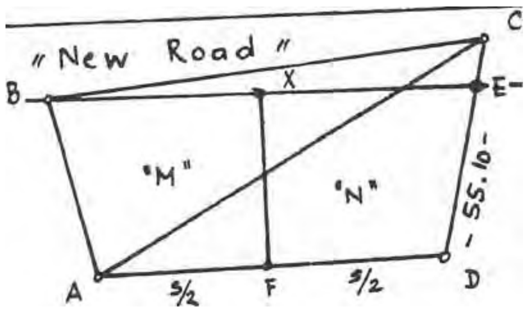
Pt	E	N
A	2 016.27	4 040.38
B	2 125.38	4 011.26
C	2 162.17	4 031.64
D	2 174.37	4 065.28
E	2 066.28	4 107.36
F	2 019.17	4 084.17



Divide the parcel [A F E D C B (A)] so that there is no change in the areas of the properties and the new boundary line now runs from R to S, and R divides the road line AB in two equal parts.

After the subdivision the owner A will get the new parcel [A F E S R (A)]. Work out the coordinates of R and S and the distances: AB, RS, ES, SD, RB.

Q8. The coordinates of the corners A, B, C and D of two adjoining properties are stated below. "M" owns A B C (A) and "N" owns A C D (A). A new road design cuts the two properties along the line BE, such that E is 55.10 m from D. The remaining total area B E D A (B) is to be subdivided by the line FX in such a way that "M" and "N" will lose equal amounts to the new road and that F is the midpoint of AD. Work out the coordinates of the end points of the new subdivision lines F, X and E. Give the areas of the blocks of land after the subdivision to the nearest m^2. How much is the contribution in m^2 of "M" and "N" to the new road area. Calculate and list the following distances: BX, XE, CE, AF, FO, FX.



1. THEORETICAL SOLUTION
State the steps including the necessary formulae you intend to take for the theoretical solution of the given problem. Give detailed information about your solution and the check calculations to be applied.

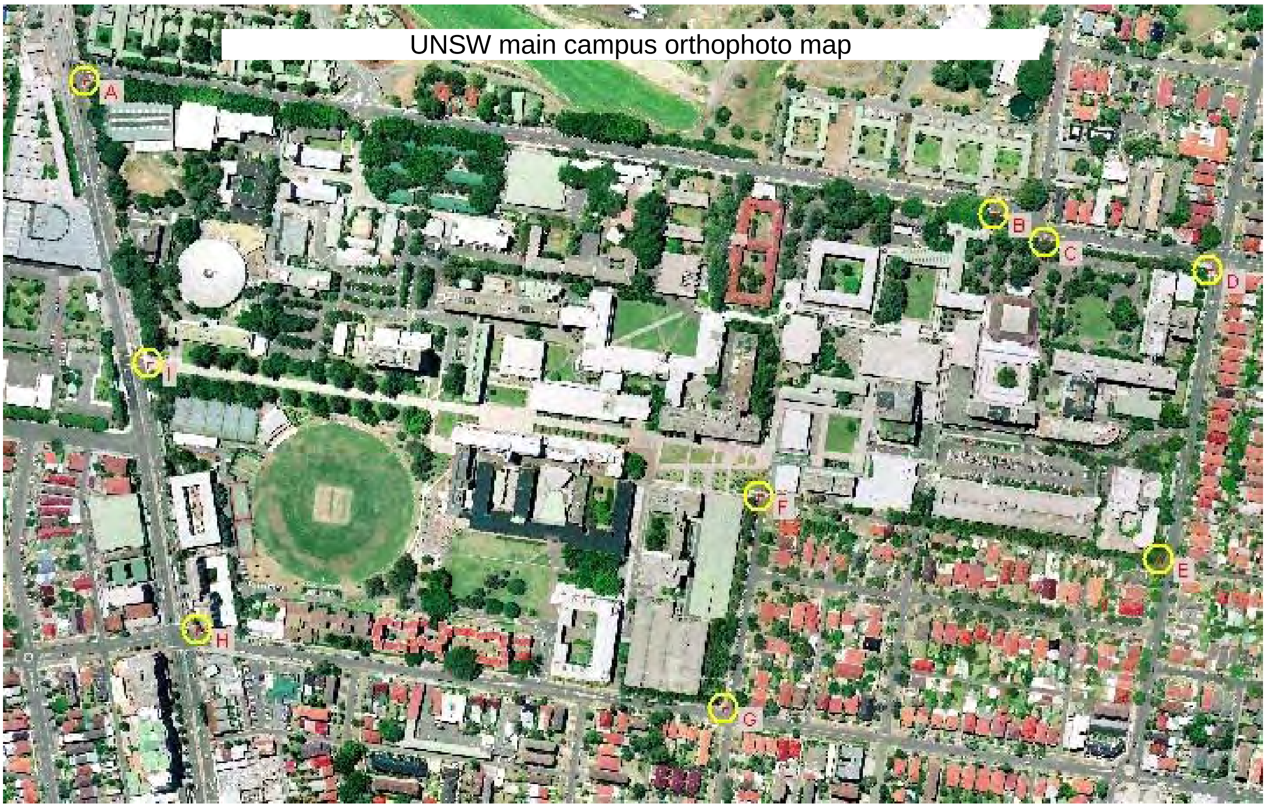
2. NUMERICAL SOLUTION
Carry out the necessary calculations of your theoretical solution with the following data:
The coordinates of the corners are:

Pt	E	N
A	2 016.28	4 040.38
B	2 036.28	4 107.46
C	2 144.34	4 095.26
D	2 125.76	4 020.00

Q9. UNSW Campus Area

Background: Land parcels (lots) have boundaries that are legally defined and are determined by suitably qualified land surveyors. The area of a lot is important, as well as the dimensions of the boundary lines and their location. Areas of lots are used in conjunction with real estate valuations, land taxes, council rates etc. So an error in area, or your calculation of the area, has considerable $ implications. To ensure we get an accurate answer we will calculate area by two independent calculation methods.

Data: For the purposes of this question the coordinates of the corners of the UNSW lot are given in Table 1 below (and shown in Figure 1). We will assume that the lot boundaries are straight lines between each of these corners.



About the Figure:
An orthophoto map is made from special high quality aerial photographs combined with ground survey data and mathematically corrected so that while it looks like a photo it is as accurate as a map. The orthophoto was made by AAM and kindly given to the School. The company employs some of our Surveying and SIS graduates and current students. Date of Photography 8/1/2002. The files supplied to the school were a 24 bit colour tiff orthoimage with a resolution (ground sample distance of 0.125 m) and a file to geocode the orthoimage. The geocode file gives the ground distance of each pixel's width and height and the MGA coordinates of the top left (NW) corner. North is up the page. The image in figure 1 is a low resolution screen capture of the actual image supplied.

Table 1. MGA (Map Grid of Australia) coordinates of UNSW campus corners

Corner	Easting (m)	Northing (m)
A	335 968	6 245 859
B	336 777	6 245 739
C	336 822	6 245 715
D	336 968	6 245 690
E	336 924	6 245 432
F	336 568	6 245 487
G	336 536	6 245 299
H	336 067	6 245 368
I	336 024	6 245 606

Step 1: Enter the coordinate data into a spreadsheet. How can you check that you have entered the data correctly? i.e. that there are no typo / transcription errors. One way is to do a sum calculation of your entered data. Add up all the Easting (E) coordinates. The sum should equal 3 028 654 exactly, with no round off errors. If you do not get this value then you have an error (or I have an error in the question ☺). Similarly the sum of Northing (N) coordinates should equal 56 210 195. There is an optional extra step that can be done from here, it is described later.

Step 2: Calculate the area using a triangle method. Divide the campus into a network of triangles by drawing lines connecting corners, so that the entire campus is covered and each part is a triangle. Certainly there is more than one pattern of triangles that could be selected. Then calculate the side lengths of all triangles. Each length is simply calculated from the formula for distance between two coordinated points, i.e. $D_{12} = \sqrt{(E_2 - E_1)^2 + (N_2 - N_1)^2}$

When calculating distances from coordinates you may wish to use the vlookup function in excel so that you merely enter the labels of the two corners e.g. C and E, and the program finds the Eastings and Northings for you from the originally entered data.

Once we have the three side lengths (a, b, c) of a triangle we can calculate the area of the triangle using the formula: $Area = \sqrt{s\,(s - a)\,(s - b)\,(s - c)}$ *where* $s = ½\,(a + b + c)$

Then simply add the areas of all triangles to find the total area of campus. Convert the area from m^2 to hectares. [1 ha = 10000 m^2].

Step 3: Calculate the area using an area of polygon formula.
There are several varieties of formula for the area of a polygon, one version is:

Area = $½ \,|\, [(N_1E_2 + N_2E_3 + ... + N_nE_{n+1}) - (E_1N_2 + E_2N_3 + ... + E_nN_{n+1})] \,|$

i.e. $$Area = \left| \frac{1}{2} \left[\sum_{i=1}^{n} (N_i E_{i+1}) - (E_i N_{i+1}) \right] \right|$$ where $N_{n+1} = N_1$, $E_{n+1} = E_1$

Alternative versions of the formula are:

$$Area = \left| \frac{1}{2} \left[\sum_{i=1}^{n} N_i (E_{i+1} - E_{i-1}) \right] \right| = \left| \frac{1}{2} \left[\sum_{i=1}^{n} E_i (N_{i+1} - N_{i-1}) \right] \right| = \left| \frac{1}{2} \left[\sum_{i=1}^{n} (N_{i+1} - N_i)(E_{i+1} + E_i) \right] \right|$$

Note: to use these formulas consider $N_0 = N_n$, $E_0 = E_n$, $N_{n+1} = N_1$, and $E_{n+1} = E_1$
In these formulas the points around the polygon loop are labeled from 1 to n. Progressing around the loop clockwise will give an area with opposite sign to that if the calculations are done by progressing around the loop in an anti-clockwise direction. We treat area as a positive number, thus the absolute value in the equations. Remember to divide by 2.

Calculate your area and convert it to hectares

Step 4: Compare your area answers in step 2 and step 3 above, and think about how to round off the area i.e. how many significant figures you should report as your answer. Think about, and comment on, how easily your spreadsheet could be modified

a) If more accurate values for the coordinates were available, and
b) If a different site with more or fewer corners was to be processed.

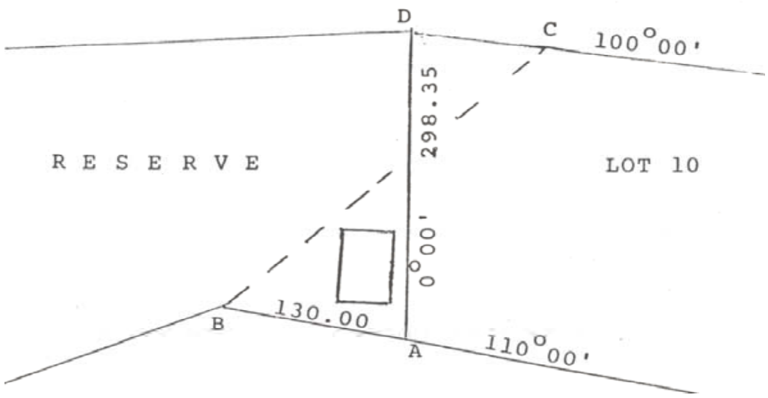
Step 5: The supplied coordinates are on the MGA projection with a combined scale factor (SF) for this site of 0.9999182. Grid / projection distance = SF * horizontal (ground level) distance. The area you have calculated in the previous steps is on the projection surface, what is the area at ground level?

Optional step: Solve for the area by entering the coordinates into CAD.

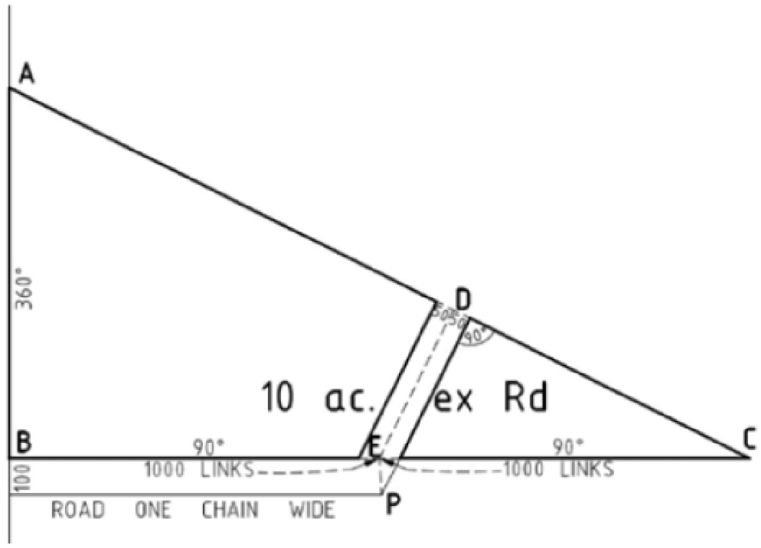
Optional step: Once you have entered the coordinates it is an easy step to calculate the mean E and mean N. This is one way to describe the 'centre' of the campus. Next, try to plot this point on figure 1 and see where it is on campus and which building is closest to it. Think about how to plot it accurately on your printout of the plan, there are several ways to do that.

Optional step: Calculate the bearing and distance of each boundary line, i.e. A to B, B to C, etc.

Q10. (A 1983 ☺ exam question) The owner of Lot 10 in the figure at right has built his house on the reserve by mistake. It is required to place a new boundary BC for Lot 10 to include the house but leaving the original lot area unchanged. Find the distance DC and the bearing and distance of BC.



Q11. This is a question from the March 1909 Surveyors Registration Exam. It was published in Azimuth magazine of July 2009. In the figure below compute lengths AB, ED, angle ACB, and the bearing and distance of EP.



7. CADASTRAL CALCULATIONS: Worked Solutions

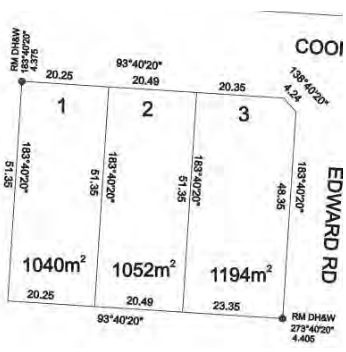
These solutions can be read AFTER you have made some attempt to solve the question.

Q1. Assign coordinates to any point. Here, top left corner is given (0, 0).

Traverse								Zig Zag method				ΔN form
Pt	Dist	Bearing										
		d	m	s		ΔE	ΔN	E	N		E + E	Cumulative
1								**0.000**	**0.000**			0
	24.75	8	39	10		3.724	24.468					
2								**3.724**	**24.468**		3.724	91.1087
	43.74	99	08	10		43.185	-6.945					
3								**46.909**	**17.523**		50.632	-260.535
	31.035	197	08	10		-9.144	-29.657					
4								**37.764**	**-12.134**		84.673	-2771.71
	39.67	287	49	10		-37.767	12.140					
1								**-0.002**	**0.006**		37.762	-2313.29
							Additives		-503.727			
							Subtractives		1809.56			
							Area		**1156.643**			1156.643

Distances on plan are rounded to 5 mm or 1cm and directions to 10" so the misclose here is acceptable. There is no significant typo error in boundary dimensions. Area was calculated by two different methods. My value for area is slightly different to the plan. The surveyor might have calculated area from the boundary values before they were rounded off. That would explain our small difference.

Q2.

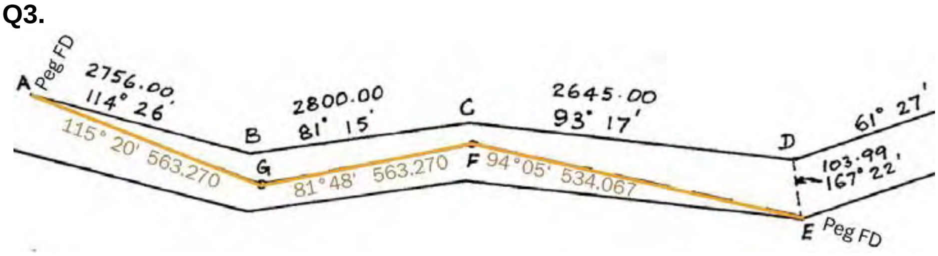


swing	2	10	20	2.172222				
scale				1.00014				
Point	desc	brg d	m	s	MGA brg rads	DP dist	E	N
1	DHW NW						289571.797	6146779.731
2	NW lot 1	183	40	20	3.24	4.375	289571.351	6146775.378
3	NE lot 1	93	40	20	1.67	20.25	289591.499	6146773.316
4	SE lot 1	183	40	20	3.24	51.35	289586.269	6146722.226
5	SW lot 1	273	40	20	4.81	20.25	289566.122	6146724.288
2	NW lot 1	3	40	20	0.10	51.35	289571.351	6146775.378

Point 2 closes OK. Do similar calculations for other corners. Spreadsheet formulas:

	M	N	O	P	Q	R	S	T	U
2	swing	2	10	20	=N2+O2/60+P2/3600				
3	scale				1.00014				
4	Point	desc	brgd	m	s	MGA brg rads	DP dist	E	N
5	1	DHW NW						289571.797	6146779.731
6	2	NW lot 1	183	40	20	=RADIANS(Q2+O6+P6/60+Q6/3600)	=4.375	=T5+S6*Q3*SIN(R6)	=U5+S6*Q3*COS(R6)
7	3	NE lot 1	93	40	20	=RADIANS(Q2+O7+P7/60+Q7/3600)	20.25	=T6+S7*Q3*SIN(R7)	=U6+S7*Q3*COS(R7)
8	4	SE lot 1	183	40	20	=RADIANS(Q2+O8+P8/60+Q8/3600)	51.35	=T7+S8*Q3*SIN(R8)	=U7+S8*Q3*COS(R8)
9	5	SW lot 1	273	40	20	=RADIANS(Q2+O9+P9/60+Q9/3600)	20.25	=T8+S9*Q3*SIN(R9)	=U8+S9*Q3*COS(R9)
10	2	NW lot 1	3	40	20	=RADIANS(Q2+O10+P10/60+Q10/3600)	51.35	=T9+S10*Q3*SIN(R10)	=U9+S10*Q3*COS(R10)

Q3.



Solution based on work by Brad Jackson. 100 links = 66 * 0.3048 = 20.1168 metres

OLD Traverse							NEW Traverse					
				E links	N links						E m	N m
line	dist links	brg					line	dist m	brg			
A		d	m	10000	5000		A		d	m	10000	5000
AB	2756	114	26				AG	563.27	115	20		
B				12509.18	3860.02		G				10509.10	4758.986
BC	2800	81	15				GF	563.27	81	48		
C				15276.59	4285.97		F				11066.61	4839.325
CD	2645	93	17				FE	534.067	94	05		
D				17917.25	4134.48		E				11599.32	4801.295
DE	103.99	167	22									
E				17940.00	4033.01							

From the above coordinates,
Join AE from old traverse (PO): dist = 7998.663 links = 1609.075 metres, brg = 96° 56’ 37.35”
Join AE from new traverse: dist = 1611.622 metres, brg = 97° 04’ 56.31”

Bearing difference (swing) = 97°04’56” - 96°56’37” = 08’18.96”

So, to convert the new traverse bearings to PO subtract 08’19” from them. Alternatively, to convert the PO bearings to the new traverse orientation add 08’19” to the PO bearings. Multiply PO distances by 1611.622 / 7998.663 to convert them to the new traverse scale. The difference in distances is 1611.622 - 1609.075 = 2.547 m (or ≈ 1:600) which is large!

line	Scaled dist	Swung brg		
AB	555.296	114	34	19
BC	564.162	81	23	19
CD	532.931	93	25	19
DE	20.953	167	30	19
Converted coordinates				
pt	E	N		
A	10000.000	5000.000		
B	10505.009	4769.088		
C	11062.811	4853.561		
D	11594.792	4821.751		
E	11599.325	4801.295		

	Check				
Join	dist	brg	d	m	s
AE	1611.622	97.08231	97	04	56.31

If coordinates of A are set to e.g. 10000, 5000 (metres) then it is possible to calculate coordinates of B C D E G and F, as shown above. Then calculate bearings and distances for radiations: GB, FC and ED. The radiations could be set out in the field to help find old marks or to place new marks at B, C and D. For these radiation calculations, the coordinates of B come from the converted coordinates calculated with swung bearings and scaled distances, and the coordinates of G come from the 'new' traverse.

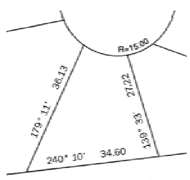
Setout Radiations from traverse marks to old corners:

		Bearing		
	Dist (m)	d	m	s
GB	10.900	337	56	27
FC	14.736	345	02	37
ED	20.953	347	30	19

Former student Daniel Jung notes that dimensions DE shown in the figure are rounded values. More precise values for the join DE can be calculated using the 'half angle' equations. We have not implemented that in the above solution. It is a small effect compared to the 2m difference in overall distances.

Q4.

One method is to assign arbitrary coordinates (could be 0,0) to one corner then determine the coordinates of the other corners, then calculate the area using a polygon coordinate formula, then subtract the area between the arc and chord. How many significant figures should we round it off to? M. Cooper's answers: 82°02', 12.995, 682.8 (polygon area = 695.8, segment = 12.96)

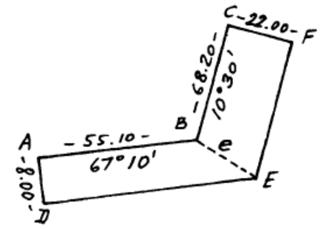


Point	Distance	Bearing				
		d	m	s	E	N
1					1000	2000
	36.13	179	11	00		
2					1000.515	1963.874
	34.60	60	10	00		
3					1030.530	1981.086
	27.22	319	33	00		
4					1012.870	2001.800
	Chord length 1-4		12.995			
	Bearing		82.0376	degrees		
	Angle theta		51.3374	degrees	=DEGREES(2*ASIN(D34/30))	
			0.89601	radians		
	Segment Area		12.956		=15^2*(D37/2-0.5*SIN(D37))	

Polygon Area					
	Additives		8034634		
	Subtractives		8033243		
	Area		695.7765		
Total Area (Polygon Area - Segment Area)					
	Area =	682.820			

Q5:

a) β_{BE} = 128° 50' BE, e = 9.089 b) β_{BE} = 87° 57' 28" BE, e = 22.538 d) Area of road = 1971.6 m^2



One method is to assign coordinates to D, then calculate coordinates of A B C F. Then calculate coordinates of E by intersection of bearings from D and F.

Another method is to use the road half angle formulae to get distance e and bearing BE.

A check of the solution is to calculate misclose around the loop A B C F E D (A).

Part A equal road widths

	A	B	C	D	E	F	G	H	I	J	K
2	Data	d	m	dist	dd				dd	d	m
3	DA	337	10	8	337.167		Half angle bearing		128.833	128	50
4	AB	67	10	55.1	67.167		Half angle dist		9.089		
5	BC	10	30	68.2	10.500						
6	CF	100	30	8	100.500						

Excel formula: I3 =(E4+E5)/2 +90, I4 =D3/COS(RADIANS((E4-E5)/2))

Part B unequal road widths

Data	d	m	dist
DA	337	10	8
AB	67	10	55.1
BC	10	30	68.2
CF	100	30	22

		E	F					
		E	N					
17	D	100	300					
18	A	96.896	307.373					
19	B	147.678	328.755					
20	C	160.106	395.813					
21	F	181.738	391.804					
22		BrgDE rads	1.17					
23		BrgFE rads	3.325		dist	d	m	
24	E	170.201	329.558	BE	22.538	87	57	

The Excel formulas for the coordinates of E are:
E = E17+(((E21-E17)*COS(F23)-(F21-F17)*SIN(F23))/SIN(F22-F23))*SIN(F22)
N =F17+(((E21-E17)*COS(F23)-(F21-F17)*SIN(F23))/SIN(F22-F23))*COS(F22)

The answers check OK with those in part A when we replace 22 by 8.

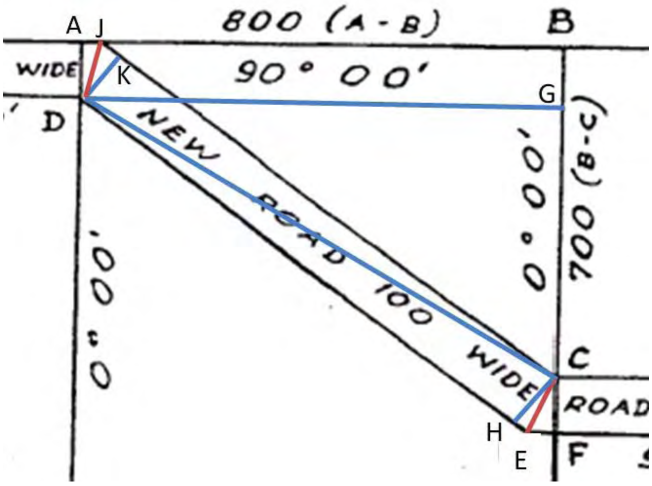
Area by coordinates			
E	N	EiNi+1	NiEi+1
100.000	300.000		
96.896	307.373	30737.3	29068.7

147.678	328.755	31854.9	45392.2
160.106	395.813	58452.8	52635.7
181.738	391.804	62730.2	71934.2
170.201	329.558	59893.2	66685.5
100.000	300.000	51060.4	32955.8
		294728.7	298672.0
	Area =	1971.643	

Answers by student Cooper, different methods

Section	Length	Bearing					
		d	m	dd			
AB	55.1	67	10	67.16667			
BC	68.2	10	30	10.5			
	Bearing difference		56.67	(alpha)			
	d(BG)		26.332				
	d(JG)		5.262				
	d(BJ)		21.070				
	d(BE)		22.538				
	Angle Beta		20.79		d	m	s
	Bearing of BE		87.96		87	57	27.2
Area within road							
	CH		53.73				
Areas							
	ABEF		525.1				
	BJE		84.28				
	EJG		21.05				
	BHG		159.2				
	CDGH		1182				
	TOTAL AREA		**1972**				

Q6. Hints: A is not at the half angle of the road. Draw a larger and clearer sketch.



Method 1:
Draw in several right angle triangles and solve them. Calculate bearing and distance of DC from right angle triangle DGC. DG is 800, GC is 700-100 = 600 So DC = $\sqrt{(800^2+600^2)}$ = 1000. [It is actually a 3-4-5 triangle so we could have done that in our heads ☺]

AB = DG = 800
GC = 600
DC = SQRT(DG^2+GC^2) = 1000
W, road width = 100

Then solve a right angle triangle DHC, using DC and the road width, to give bearing DE.
DH = SQRT(DC^2-W^2) = 994.987

Calculate angles in RA triangles then bearings
Angle GDC = ATAN(GC/DG)) = 36° 52’ 11.6”
Angle CDH = ASIN(W/DC)) = 5° 44’ 21.0”
β DE = 90 + GDC + CDH = 132° 36’ 32.6”
β EC = (β DE +90)/2-90 = 21° 18’ 16.3”
Half angle = (β DE -90)/2 = 21.3
HE = EF = AJ = JK = W * TAN(Half angle) = 38.997
Half angle dist = W/COS(Half angle) = 107.335
DE = DH + HE = 1033.985

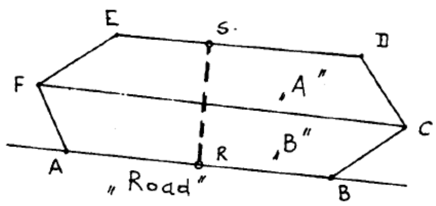
Area = 1 rectangle plus 4 equal RA triangles = DH * W + 4 * (½ * HE * W)
= 994.987 * 100 + 4 * (½ * 38.997 * 100) = 107298 sq. links

Method 2: Set up coordinates and solve the problem by closes etc and area from coordinates.

Method 3: Enter points and lines into CivilCAD and solve with COGO functions.

Q7.

R is mid way between A and B, so its coordinates are the mean of A and B:
R = 2070.825, 4025.82
Dist AB = 112.929, Dist AR = 56.465, Dist RB = 56.465
Area owner A (FCDEF)
Zig zag subtotals: 34300352, 34290412 Area = 4970.191
Use solver to find coordinates of S so that ARSEFA = Area of owner A
Solver didn't work on first attempt. Start with an estimate of S as midpoint of DE instead of zero. S start 2120.325 4086.32, still didn’t work, so add a constraint to solver so brgSE-brgDE = 0 i.e. S lies on the line DE



A	2016.27	4040.38
R	2070.825	4025.82
S	**2110.939**	**4089.974**
E	2066.28	4107.36
F	2019.17	4084.17
A	2016.27	4040.38

subtotals 41854399 41844458
New Area owner A = 4970.191 Diff areas = 0
Check S is on line DE by comparing bearings
Brg SE = 291.2712 Brg DE = 291.2712 diff brgs = 1.34E-07
Distances from coordinates:
AB = 112.929
AR = 56.465
RB = 56.465
DS = 68.069
SE = 47.923
RS = 75.663
DE = 115.992 = DS + SE = 115.992 check OK

Another method instead of solver?

Q8. I haven't typed this one yet. You are welcome to send me your solution.

Q9. UNSW Campus Area

Triangle areas

Points									side lengths				
			E1	N1	E2	N2	E3	N3	a	b	c	s	Area
A	B	I	335968	6245859	336777	6245739	336024	6245606	817.9	764.7	259.1	920.8	98978.5
I	B	F	336024	6245606	336777	6245739	336568	6245487	764.7	327.4	556.9	824.5	80979.5
I	F	H	336024	6245606	336568	6245487	336067	6245368	556.9	514.9	241.9	656.8	62177.5
H	F	G	336067	6245368	336568	6245487	336536	6245299	514.9	190.7	474.0	589.8	45190
B	C	F	336777	6245739	336822	6245715	336568	6245487	51.0	341.3	327.4	359.9	8178
C	D	F	336822	6245715	336968	6245690	336568	6245487	148.1	448.6	341.3	469.0	19819
F	D	E	336568	6245487	336968	6245690	336924	6245432	448.6	261.7	360.2	535.3	47134
												SUM	362456.5

Sum of the triangle areas = 362456.5 m^2 = 36.25 ha

Repeating the Triangle areas, as in the table above, but scaling the side lengths:

Points			side lengths				side lengths at ground level				
			a	b	c	s	a	b	c	s	Area
A	B	I	817.9	764.7	259.1	920.8	817.9	764.7	259.1	920.9	98994.69
I	B	F	764.7	327.4	556.9	824.5	764.7	327.4	556.9	824.5	80992.75
I	F	H	556.9	514.9	241.9	656.8	556.9	515.0	241.9	656.9	62187.67
H	F	G	514.9	190.7	474.0	589.8	515.0	190.7	474.1	589.9	45197.39
B	C	F	51.0	341.3	327.4	359.9	51.0	341.3	327.4	359.9	8179.338
C	D	F	148.1	448.6	341.3	469.0	148.1	448.6	341.3	469.0	19822.24
F	D	E	448.6	261.7	360.2	535.3	448.6	261.7	360.3	535.3	47141.71
											362515.8

Sum of the triangle areas = 362515.8 m^2 = 36.25 ha

If you used the coordinate formula method for area, a simple way to convert that area to the equivalent ground area (for a site where one value of scale factor applies to the whole site) is:

Ground area = Projection Coordinate area / SF^2 = 36.25158 ha

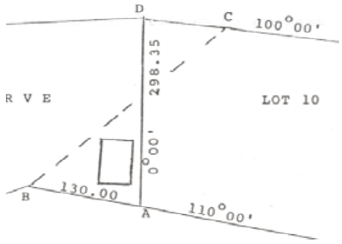
Think of it this way; an area of 36 ha is equivalent to a square with sides √362456.5 = 602.0436 m. Convert that 602... distance to ground by / SF, = 602.0929 and then squaring the value to give area = 362515.8 m^2

Mean coordinates: 336517.11 6245577.22

Bearing and distance of each boundary line, i.e. A to B, B to C, etc.

			E1	N1	E2	N2		DIST	D	M	S
A	B		335968	6245859	336777	6245739		817.851	98	26	14.0
B	C		336777	6245739	336822	6245715		51.000	118	04	21.0
C	D		336822	6245715	336968	6245690		148.125	99	43	00.1
D	E		336968	6245690	336924	6245432		261.725	189	40	41.7
E	F		336924	6245432	336568	6245487		360.224	278	46	56.8
F	G		336568	6245487	336536	6245299		190.704	189	39	35.6
G	H		336536	6245299	336067	6245368		474.049	278	22	09.9
H	I		336067	6245368	336024	6245606		241.853	349	45	31.4
I	A		336024	6245606	335968	6245859		259.124	347	31	09.0

Q10.



Ans: DC = 134.368 BC = 47°49'29"   343.393   Area ABD = Area BCD = 18223.22

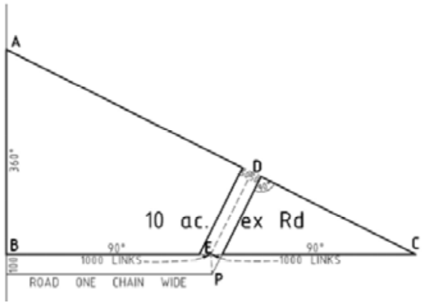
Area of triangle ABD = Area of triangle BDC
Can work in coordinates or by triangle formulas, eg get Distance BD from cosine rule, and get area of triangle from 2 sides and an angle, or 3 sides. Here we do it in CAD (for a change).

Enter point 1 (A) at 300,500.
Point 2 (B) by radiation from 1. = 177.840, 544.463
Point 3 (D) by radiation from 1. = 300.000, 798.350
Insert lines to join the points.
Inquiry, Bearing & Distance DB = 205.41.42 281.748
Inquiry, area (inside ABD) = 18223.224
Add a point along the line DC but further than C, by estimate say 200 distance.
Point 4 (E) by radiation from 3. = 496.962, 763.620
Insert lines DE and BE
Make the triangle BDE a lot: Design | Subdivision | Insert Lot   into triangle BDE , call it lot 1
Now subdivide the lot BDE.
Design | Subdivision | Lot Layout (min Area)
Select the line BD, pivot on point B and Area = 18223.224 from above, create the lot.
Get the properties of the new point (C) to see coordinates = 432.327, 775.017

Inquiry, Bearing & Distance DC = 100.00.00 134.368 (note the bearing checks)
Inquiry, Bearing & Distance BC = 47.49.29 343.393

Q11.

It might be useful to discuss this triangle problem with the road in it. Note how difficult these problems were to calculate for surveyors 100 years ago, with no calculators and log tables, before computers, calculators with trig functions, spreadsheets, CAD etc existed.
But I think it is also useful to have real data and real modern problems. Without machines we would spend more time writing the equations and doing algebraic rearrangements of terms, now with CAD some surveyors solve these problems with a bit of trial and error. I did this triangle one without CAD and without trial and error. Sometimes I use the lot subdivision features in CAD for area problems.



A comment about survey jargon - the area was 10 acres excluding the road. But what does that mean? Experienced cadastral surveyors know what that means but students may not. Sometimes I need to point to the figure and talk not just write, but the worked solution below might help your understanding. The lot of land is bordered by the bold lines in the figure; it has two parts separated by a road. The lot's area is 10 acres. The full triangle ABC that includes the road and both parts of the lot has an area larger than 10 acres.

Now to this triangle question: I did it in Excel, not CAD. I worked entirely in units of links so as not to induce any conversion errors that might occur in e.g. acres to hectares or m^2. This course teaches students how to check their calculations. One way is to have independent people do the problem, four surveyors are quoted in Aug Azimuth as having independent solutions. Another check of calculations is to reverse the problem, that is take the answers and see if you generate the original data eg 3 pt resection coordinates, get bearings, get angles, see if they match the original angles. So let's start with the answer published in Azimuth, C = 27° 37' 03"

10 acres is exactly 1,000,000 square links, with no round off errors.
1 acre = 4840 sq yards. 100 links = 22 yards = 1 chain. A **chain** is a unit of length; it measures 66 feet or 22 yards or 4 rods or 100 links (20.1168m). An acre is the area of 10 square chains (that is, an area of one chain by ten chains).

Using right angle triangle ABC and our C gives AB (=BCtanC) as 1046.353 links.
The area of the triangle ABC (ignoring the road) is thus 1,046,353 sq links using half base * height.
With D at the centreline of the road, solve the right angled triangle CDE.

Using our C gives DC = 886.062
The length of the eastern side of the road (nearest C) is 497.018 using (DC - 50) tan C
The length of the western side of the road is 549.335 using (DC + 50) tan C
Now the area of the road is simply the area of one RA triangle (using western side of road and C) minus another (eastern side and C), and half base * height rule. So area of road is 46,357 sq links.
The area of the LOT is area of triangle ABC - area of road: 1,046,353 – 46,357 = 999,996 which is close enough to 1,000,000 sq links i.e. 10 acres.
If the angle C is 27 37 03.3 you get 10 acres, and it is certainly reasonable to round off the angle and the area.
So the published C and AB are correct.

The Solver function in Excel can do it this way directly - saves using trial and error.

Azimuth July 2009 question from 1909 exam. Working in units of links and square links then convert answers to metric, to avoid round off and transcription errors

Start with an estimate of C, assume you know what C is, then work through equations. Then use solver to get the value of C in degs, and then convert to DMS.

Angle at C ?		27.61759	degs	this by solver		= 27° 37' 03.3"			
Area	10	acres =	1,000,000	sq links	0.201168		27	37	03
	Azim	27.61750							
BC	2000			402.336	m				
EC	1000								

Area ABC = 1,000,000 sq links - road area   plus or minus road?
AB = 1046.357   BC tan C   210.494 m
Area ABC =   1046357   half base * height of RA triangle = 1/2 * BC * AB

DC = 886.0613 =   EC cos C
half road width = 50
east side of road = 437.409   = (DC - 50) tan C
west side of road = 489.727   = (DC + 50) tan C
Road area = One triangle from C - another triangle from C. 1/2 base * Height (west) - half base * area of road = 46357
area of road = 46357
Height (east)
Area ABC lot   1,000,000   (1/2 AB * BC) - road area

target :   1,000,000   -   0

Area full ABC triangle   1046356.8   includes road

8. ROAD “CENTRELINE” PROBLEMS

This chapter includes a summary of formulas, without derivations, for horizontal circular curves, transition spirals, and parabolic vertical curves. The formulas are followed by a worked example of horizontal road alignment calculations in CAD. The remainder of the chapter is tutorial questions. These questions were prepared by previous lecturers in our School in previous courses as tutorial and exam questions. You may solve the problems using a calculator, spreadsheet, or CAD, or some combination. It is recommended that you gain some experience with each of these methods. Of course, solving all the following problems is the ideal learning experience. However, to ease your workload it is recommended that you solve at least some of the problems. You are welcome to submit typed worked solutions, but that is not compulsory. Your solutions may be added to this text for the benefit of future students.

Further information on this topic can be obtained from reading the many excellent introductory Surveying textbooks with chapters on road calculations.

Summary of Formula

Horizontal Curves

Arc Length, TPU $A = R\,\vartheta^c$

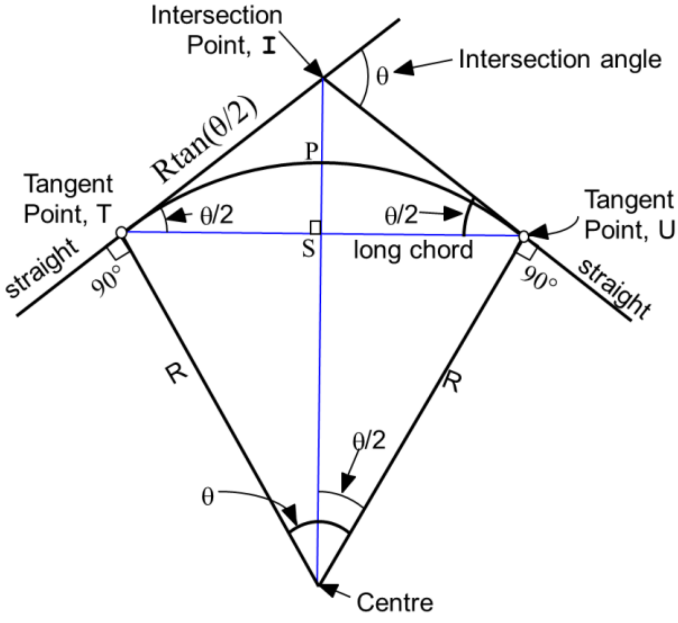
Tangent length, TI, IU $T = R\tan\theta/2$

Distance PI $= R\left(\frac{1}{\cos\theta/2} - 1\right)$

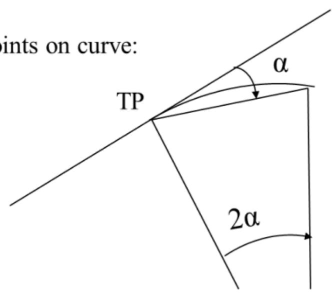
Chord Length, TSU

$$C = 2R\sin\theta/2 = 2T\cos\theta/2$$

Distance PS $= R\left(1 - \cos\theta/2\right)$



Points on curve:



$$\alpha^\circ = \frac{arc}{2R}\cdot\frac{180}{\pi}$$

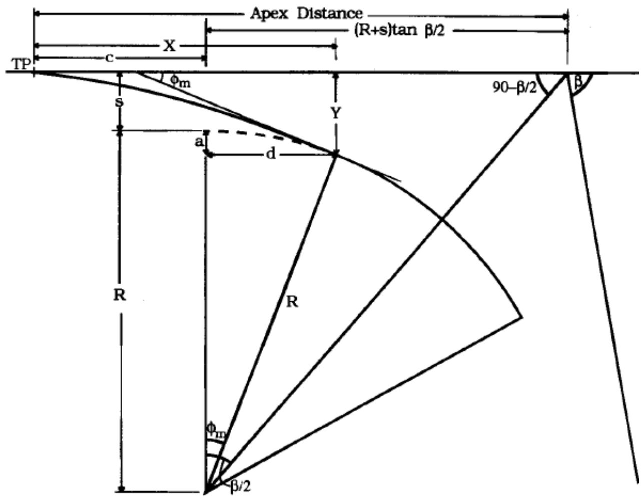
$$chord = 2R\sin\alpha$$

$$\approx arc = R.2\alpha$$

Deflection angle $\alpha^c = \frac{arc\_length}{2R}$

Area of Segment $Area = \pi R^2 \times \frac{\theta^\circ}{360}$

Transition Curves



Transition curve $rl = K$ Centrifugal force $F = \frac{Mv^2}{R}$

Length of Transition $L_T = \frac{v^3}{aR} = \frac{v^3}{3.6^3 aR}$

Max allowable SE $SE = \frac{Bv^2}{282.8R}$

Clothoid equation $\phi = \frac{l^2}{2RL_T}(radians)$ &

$\phi_{max} = \frac{L_T}{2R}(radians)$

$x = \left[l - \left(\frac{l^5}{40R^2L^2}\right) + \left(\frac{l^9}{3456R^4L^4}\right) - \ldots\right]$ $y = \left(\frac{l^3}{6RL}\right) - \left(\frac{l^7}{336R^3L^3}\right) + \left(\frac{l^{11}}{42{,}240R^5L^5}\right) - \ldots$

Long Chord $C = \sqrt{x^2 + y^2}$ Short Chord $c = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}$

Apex distance $Apex = c + (R + s)\tan\left(\frac{\beta}{2}\right)$

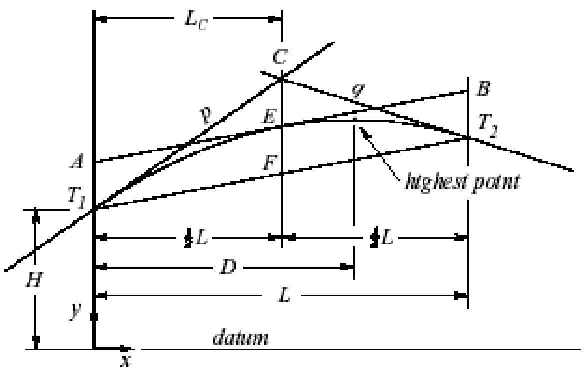
Shift of circular arc from straight $s = \frac{L^2}{24R} - \frac{L^4}{2688R^3} + \ldots$ Shift of tangent point from IP

$c = \frac{L}{2} - \frac{L^3}{240R^2} + \ldots$

Cubic parabola $x = \frac{y^3}{6RL_T}$ & $\delta = \frac{\phi}{3}$

Vertical Curves

Convention: Chainage increases left to right, p is first slope, q is second slope. Uphill (left to right) is positive, downhill is negative. In following equations a slope of 3% is entered as 0.03



Equation of Vertical Curve $y = \left(\frac{q-p}{2L}\right)x^2 + px + H$

$RL_F = \frac{(q+p)L}{4} + H$ $RL_E = \frac{(q+3p)L}{8} + H$ $CE = \frac{CF}{2}$

Distance to lowest/highest point on curve $D = \left(\frac{-p}{q-p}\right)L$

Vertical distance from grade to curve $h = \left(\frac{q-p}{2L}\right)x^2$

Length of curve (vertical acceleration) $L = \frac{(q-p)V^2}{12.96a}$ $a = \frac{v^2}{r}$

Stopping Distance $D_S = D_R + D_B = R_T . \frac{V}{3.6} + \frac{V^2}{254(f \pm 0.01G)}$

Length of summit vertical curve for stopping distance:

$L > D$: $L = \frac{D^2(p-q)}{2\left(\sqrt{h_1}+\sqrt{h_2}\right)^2}$ $L = D$: $L = \frac{2\left(\sqrt{h_1}+\sqrt{h_2}\right)^2}{p-q}$ $L < D$: $L = 2D - \frac{4(h_1+h_2)}{p-q}$

Length of sag vertical curve for stopping distance D and clearance height H_C

$L > D$: $L = \frac{D^2(q-p)}{2\left(\sqrt{H_C-h_1}+\sqrt{H_C-h_2}\right)^2}$ $L=D$: $L = \frac{2\left(\sqrt{H_C-h_1}+\sqrt{H_C-h_2}\right)^2}{q-p}$ $L < D$:

$$L = 2D - \left(\frac{4}{p-q}\right)\left[2H_C - (h_1+h_2)\right]$$

Length of sag vertical curve for headlight sight distance S $L > S$: $L = \frac{S^2(q-p)}{1.5+0.035S}$ $L \leq S$:

$$L = \frac{2(h+S\theta)}{q-p}$$

8.1 Horizontal Road alignment calculations in CAD

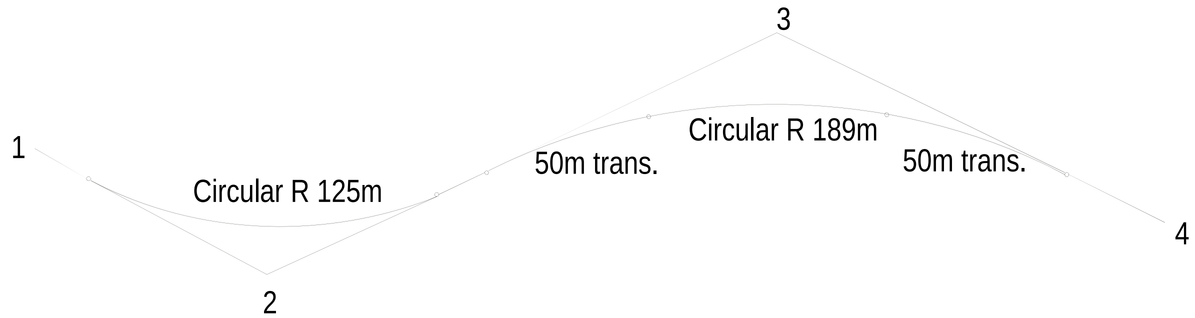
Our aim is for students to experience the process of drawing and calculating roads in CAD. You will see some of what can be done in CAD. We do not expect students to memorise the steps, but we do expect you to gain some experience and understanding of the steps involved. Our objective is to get you to a standard that allows you to be able to process your own data in future, with the assistance of these and other notes.

In this section an example is supplied with several key steps. If the instructions are not clear to you, or if it is difficult to implement in your CAD software then seek assistance from an experienced user. As you proceed through the exercise write your own notes that include step by step instructions. That should be useful for you when you process other data later.

Exercise: Proposed Morpeth Road

This exercise was part of a survey camp at Morpeth. The task is to determine coordinates of points at even 20m chainages along the centreline of the road shown in the figure below.

ROAD ALIGNMENT DESIGN PLAN:



- Open a new project in your CAD.
- Enter the details of given design points 1 to 4 and then join the straight lines.

Point No	E	N
1	58223.93	77326.71
2	58193.64	77243.95
3	58279.82	77083.54
4	58254.34	76969.69

- Add coordinates of nearby survey points with known coordinates to be used in the setting out. For example, in the area near this road the following control points exist:

COE Belfry	58358.272	77381.928
R.C.	59287.787	77396.101
T3	58270.275	77440.289
T4	58296.258	77356.073
T73	58240.896	77328.202
T73A	58246.730	77328.167

- Assign unique point numbers for all points.
- View the points and lines, to ensure no gross errors.
- The process varies from CAD to CAD. Here we create an 'alignment' from the lines, to signify that the lines are the centreline of a road. Then edit the alignment.
- Select point 2 and enter radius as 125
- Select point 3 and enter radius as 189, entry spiral (transition curve) length as 50, and exit spiral length as 50.
- Enter start chainage at as 0.0 and on some software the end chainage (e.g. 373.749 or 400).
- Enter the chainage spacing required as 20.

CAD then calculates the details and displays the alignment report, save it to file. The report should include a list of coordinates of points to be set out. These points can then be exported to a text file for upload to a survey instrument (total station or GPS). Within CAD the points can also be used for calculations of bearing and distance joins etc to nearby survey control marks, if required.

The next step is to create a file that can be "uploaded" to your Total Station or GPS in a suitable format, so that the marks can be set out in the field.

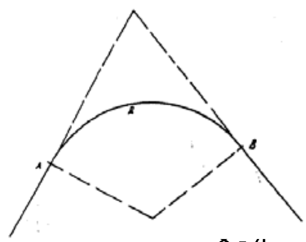
Your calculations should yield:

58223.930, 77326.710 chainage 00, at point 2
58217.056, 77307.928 chainage 20
58212.925, 77296.643 chainage 32.018, tangent point
58210.423, 77289.064 chainage 40
58206.317, 77269.512 chainage 60
58205.378, 77249.556 chainage 80
etc

8.2 Horizontal Curve Questions

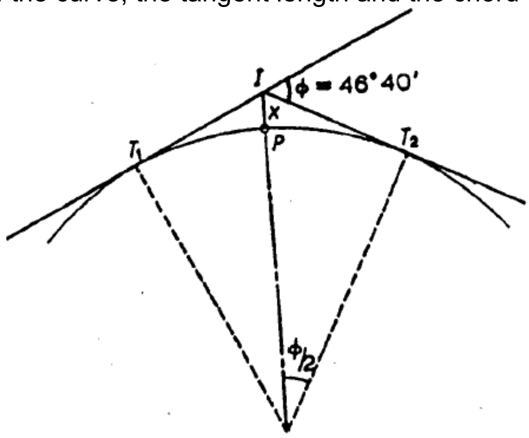
Question 1
A circular curve of radius 250.450m and an intersection of 62°12' 20" (the internal angle in this figure, not θ) is required. Calculate the

(i) arc length
(ii) chord distance
(iii) tangent distance



Question 2

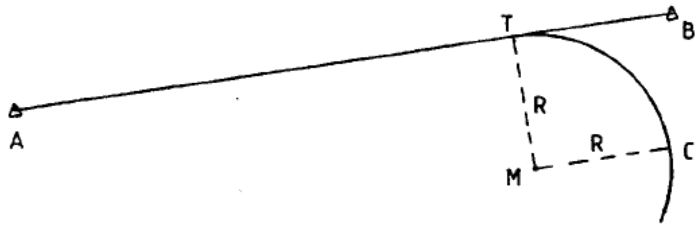
A circular railway curve is to be set out to pass through a point which is 40m from the intersection of the two straights and equidistant from the two tangent points. The straights are deflected through 46°40'. Calculate the radius of the curve, the tangent length and the chord distance.



Question 3

Two points A and B define the centre line of a road. From a point T on this centre line starts a circle of unknown radius R tangential to the road AB and passes through the point C. The distance AT is 57.32 m.

pt	E	N
A	100.72	120.32
B	200.18	192.37
C	250.38	158.12



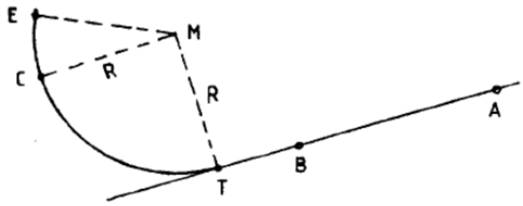
Work out the radius R and the coordinates of the centre M of the circle.

Question 4

From the data given below work out the coordinates of T, E and M, the distance BT, the length of the curve and the deflection angle of the curve. AB is tangential to the curve at T.

Coordinates of the points:

pt	E	N
A	934.82	379.85
B	705.48	403.10
C	524.91	463.84



Radius R = 274.00 m   Arc length CE : 87.23 m

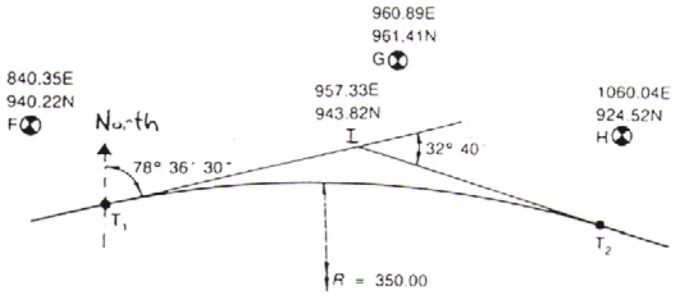
Question 5

A road begins at A (E 1000.000 m, N 1000.000 m) and runs in a direction 33°50'00" for 350 m before entering clockwise into a circular curve at a point P for another 120 m. After which it continues from point Q tangential to the curve along a straight line. If the intersection point (I) lies directly north of the centre of the circular curve, determine:

(a) The deflection angle of the two straights.

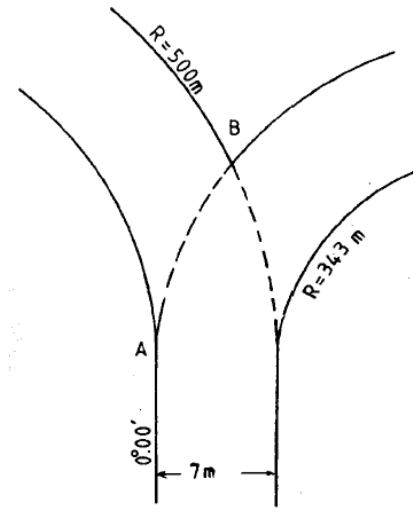
(b) The radius of the circular curve.

Question 6
The centre line of a road is to be set out as part of a new development. The road includes a circular curve of radius 350 m, deflecting to the right through 32° 40'. The diagram below illustrates the relationship with existing control points. The chainage of the intersection point I is 1029.35 m. Derive the bearings and distances from G for setting out the first two points on the curve using 20 m chainages.

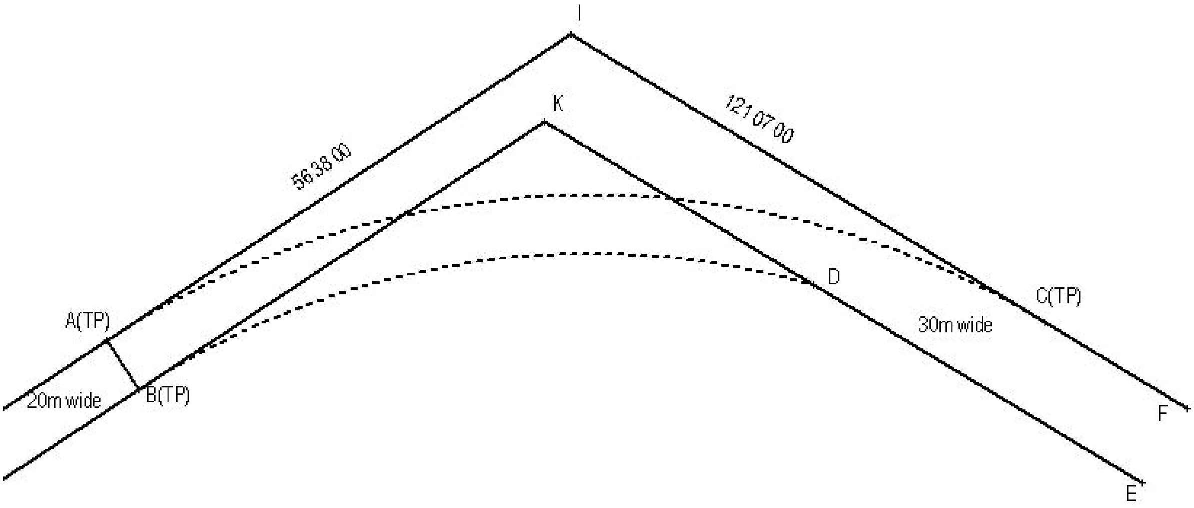


Question 7
Two straights intersect at B. Straight AB has a bearing of 115° and straight BC a bearing of 135°. The straights are to be connected by a circular curve which passes through a point D which lies 45.72 m from B on a bearing of 285°. Calculate the radius of the curve, the tangent length, the length of the curve and the deflection angle for a 20 m chord. (Hint: If using a calculator, hold all decimal places of accuracy in your calculator when doing this calculation don't just write a few places onto paper and re-enter into the calculator.)

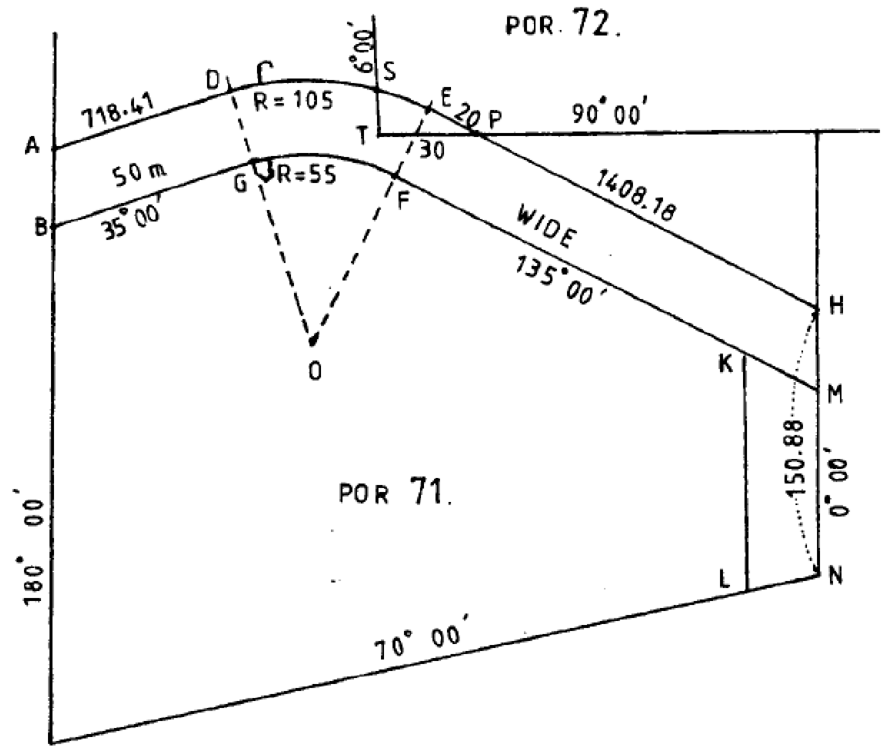
Question 8
You have to locate B from A in setting out the intersection of two circular curves, where A is the common tangent point. From the data supplied, find the bearing and distance from A to B.



Question 9
It is proposed to replace the bend in the road AICDKB with a circular curve, outer radius 300 m, inner radius 280 m. The bearing and distance of IK was calculated to be 196°28'01" and 31.007 m respectively.
Calculate the distance KD, so that D can be located from K. (D is intersection of inner arc with KE).



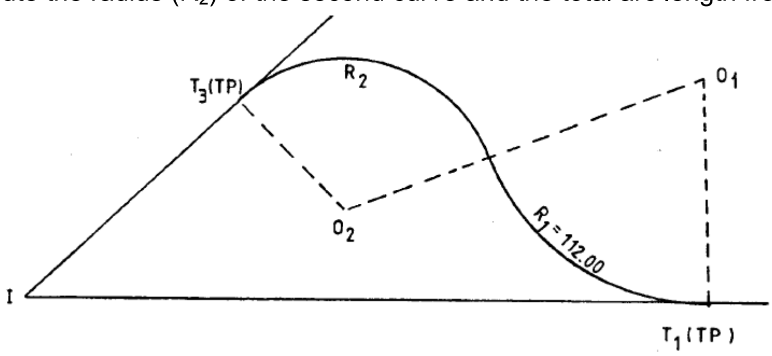
Question 10
A proposed road 50 m wide is to be resumed from Portion 71 and Portion 72 as shown on the diagram. Calculate
(a) The dimensions of the parcel to be resumed
(b) The area of the parcel to be resumed from Portion 72
NOTE: All arc lengths and bearings and distances of all chords should be calculated.



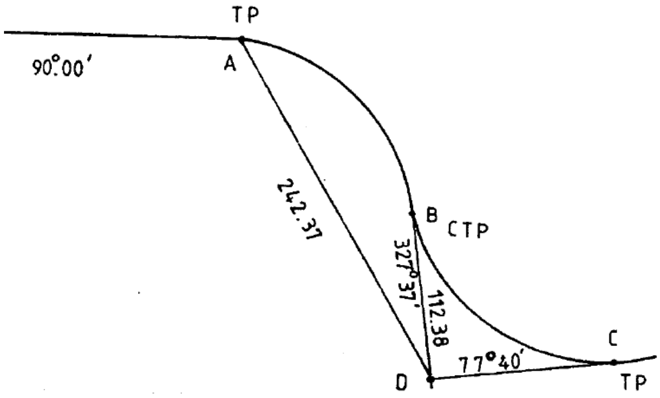
Question 11
With the following data, compute the radius (R_2) of the second curve and the total arc length from T_1 to T_3.

R_1 = 112.00

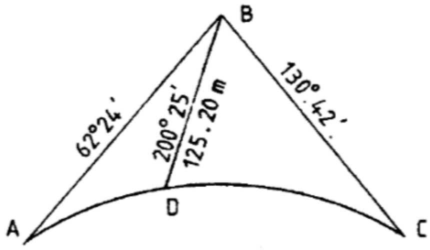
STN	E	N
T1	450.15	100.00
I	101.64	100.00
T3	201.81	194.38



Question 12
From the data given on diagram find the radii of the curves AB and BC, together with the length of the arc BC. D is the intersection of tangents from B and C.



Question 13
Calculate the radius of the curve which is tangential to AB and BC, and passes through D.



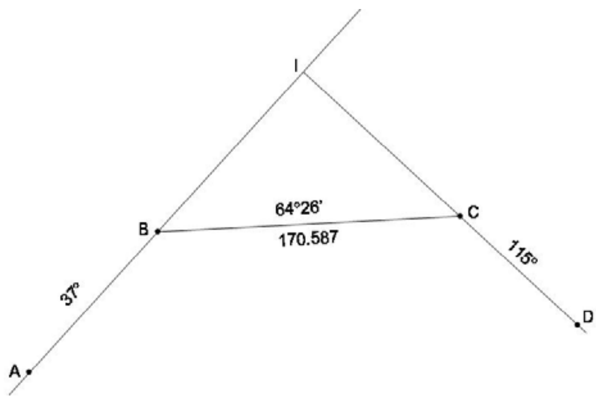
A similar question to this was given in a 1909 exam and has reappeared in the May 2009 NSW Surveyors Azimuth magazine! The 1909 data was in links with AB 65°, BC 154°30', and BD 183°10' 650.0 and has an answer of R = 1498.3 = 301.41 metres.

Question 14
A circular curve having a radius of 900 m is to be set out between two straight road centre lines LM and MN. The intersection point M is inaccessible and two points P and Q have been set up on LM and MN respectively. The length of PQ was found to be 627.8 m and angles MPQ and MQP measured 16° 27' and 12° 33' respectively.

(a) Calculate the distance to be measured from P and Q to set out the tangent points for the curve.
(b) Determine the deflection angles for setting out the first three points on the curve using 20 m chords.
(c) Explain how the remainder of the curve would be set out if the line of sight from the tangent point was obstructed beyond the third point on the curve.

Question 15
Two straights, ABI and ICD are tangent to a proposed circular curve of radius 250m. The chainage of point B is 1205.670m. Note that B and C are not necessarily the tangent points of the curve. Calculate:

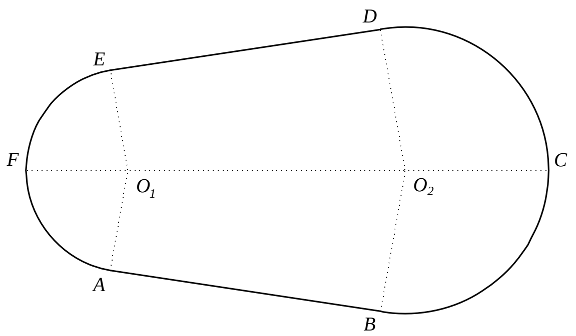
a) The chainages of both tangent points.
b) The bearing and distance to the mid-point of the curve from the first tangent point.
c) The distance from the intersection point to the mid-point of the curve.



Question 16
A running track is to be set out as shown below. The track has two straights *AB* and *ED* joining circular arcs *BCD* and *EFA*. The straights diverge by an angle of 10° each from the line *FC* and the

centres O_1 and O_2 lie on the line *FC*. If the radius of the smaller circular arc *EFA* is to be 40 m and the perimeter of the track *ABCDEFA* is to be 500 m, calculate the following.

(a) The length of the straight *AB*. Express your answer to the nearest mm.
(b) The radius of the larger circular arc *BCD*. Express your answer to the nearest mm.

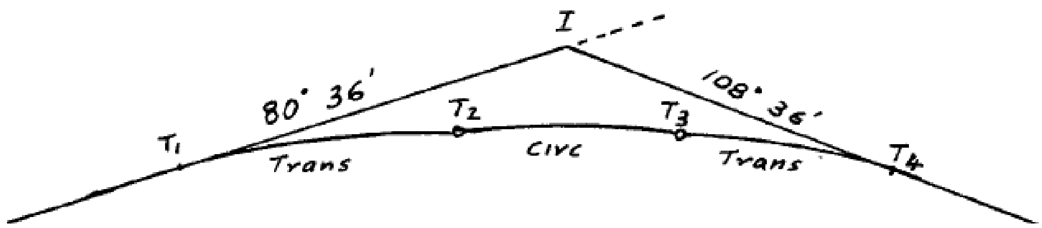


8.3 Transition Curves, example and questions

This section on transition curves has been significantly reduced from past versions. Students interested in understanding more about transition curves, which are also called spirals or clothoids, are advised to search and read elsewhere. The insertion of a transition curve between a straight and a circular arc results in a shift of the circular arc away from the straight i.e. if the circular arc is continued it is no longer tangent to the straight. The tangent point of transition curve and straight is further away from the intersection point than that for a circular curve.

Transition Curve Example

Data: A circular curve of radius of 200 m is to be set out at even 10 m chainages between two straights bearing 80°36' and 108°36'. Transition curves are to be inserted at entry and exit of the circular curve. The transition curves are to be clothoids.
Radius of Circular Curve = 200 m; Design Velocity = 50 km/hour (= 13.89 m/sec);
Chainage of IP = 1536.890 m; Maximum rate of change of radial acceleration = 0.3 m/sec^3



Calculations:

1. Length of Trans Curve $L = \frac{v^3}{aR} = \frac{13.89^3}{0.3 \times 200}$ = 44.65 m (minimum). Adopt L = 50 m.

2. Max. Tangential Angle $\Phi_m = \frac{L}{2R} = \frac{50}{2 \times 200}$ = 0.125 = 7°09'43"

3. Shift $S = \frac{L^2}{24R} - \frac{L^4}{2688R^3}$ = 0.5208 - .0003 = 0.520 m [So this 50m transition shifts the road ½ a metre.]

4. $C = \frac{L}{2} - \frac{L^3}{240R^2}$ = 25.0 - 0.013 = 24.987 m

5. Apex Distance $IT_1 = IT_4$ = C + (R+S) tan β/2 = 24.987 + 200.52 tan 28/2 = 74.982

6. Length of Circular Arc = R (β - 2 Φm) = 200 (28 – 2(7°09'43")) Radians = 47.738 m

7. Chainages of TP's $T_1 = I - IT_1$ = 1536.890 - 74.982 = 1461.908 m
$T_2 = T_1 + L$ = 1461.908 + 50 = 1511.908
$T_3 = T_2 +$ Arc $T_2 T_3$ = 1511.908 + 47.738 = 1559.646 m
$T_4 = T_3 + L$ = 1559.646 + 50 = 1609.646 m

8. Chainages to be marked every 10 m

Trans T_1T_2	Circ Curve T_2T_3	Trans Curve T_3T_4
1461.908	1520.0	1559.646
1470.0	1530.0	1560.0
1480.0	1540.0	1570.0
1490.0	1550.0	1580.0
1500.0		1590.0
1510.0		1600.0
1511.908		1609.646

9. Information for Setting Out (bearings and distances from TPs, could be then used to calculate coordinates)

Deflection angles for transition curves: $\alpha'' \approx 0.206265\left(\frac{\lambda^2}{6RL} - \frac{\lambda^6}{2835R^3L^3} - K\right)$ Chord ≈ arc

since arc < R/20

Deflection angles for circular curves - $\alpha'' \approx 0.206265\left(\frac{S}{2R}\right)$ where S = arc length

Table to set out Trans Curve T_1T_2 from T_1:

Chainage	ℓ	Chord	Deflection Angle	Bearing	Remarks
1461.908	0	0	0	80°36'00"	At T1 to ℓ
1470.0	8.092	8.092	03'45"	80 39 45	
1480.0	18.092	10.000	18 45	80 54 45	
1490.0	28.092	10.000	45 13	81 21 13	
1500.0	38.092	10.000	1 23 08	81 59 08	
1510.0	48.092	10.000	2 12 30	82 48 30	
1511.908	50.000	1.908	2 23 13	82 59 13	T2

Table to set out Circular Curve T_2T_3 from T_2:

Chainage	Arc(s)	Chord	Deflection Angle	Bearing	Remarks
1511.908	0	0	0	87°45'43"	Bearing of tangent at T2
1520.0	8.092	8.091	1°09'33"	88 55 16	
1530.0	10.000	9.999	2 35 29	90 21 12	
1540.0	10.000	9.999	4 01 26	91 47 09	
1550.0	10.000	9.999	5 27 23	93 13 06	
1559.646	9.646	9.645	6 50 17	94 36 00	T_3

Table to set out Trans Curve T_4T_3 from T_4:

Chainage	ℓ	Chord	Deflection Angle	Bearing	Remarks
1609.646	0	0	0	288°36'00"	At T_4 to ℓ
1600.0	9.646	9.646	05' 20"	288 30 40	
1590.0	19.646	10.0	22 07	288 13 53	
1580.0	29.646	10.0	50 21	287 45 39	

1570.0	39.646	10.0	1 30 03	287 05 57	
1560.0	49.646	10.0	2 21 12	286 14 48	
1559.646	50.0	0.354	2 23 13	286 12 47	T_3

Transition Curves Questions

Question 1

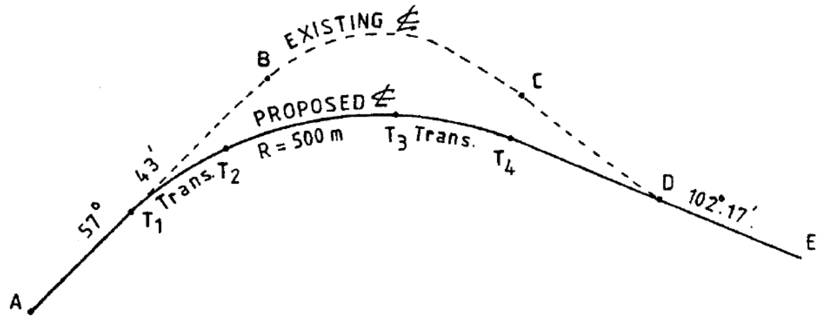
Two road straights which intersect at an angle of 25° are to be joined by a circular curve of radius 800 metres with transition curves at either end.

If the design speed is 100 kmh and acceleration due to gravity = 9.8 m/sec^2, calculate:

- i) The super elevation required for zero side-thrust on the circular curve.
- ii) The length of the transition curve given the rate of change of radial acceleration is 0.3m/sec^2.
- iii) The length of the circular curve.

Question 2

The existing reverse curve BCD is to be replaced by a new alignment T_1 T_2 T_3 T_4 D, where $T_1T_2T_3T_4$ is a 500 m radius curve with clothoid transitions at each end and T_4D is the prolongation of DE.



Pt	E	N	RL
B	4870.294	1146.262	815.804
D	5558.023	1050.386	813.380

Line AB Bearing 57°43'00" Grade + 2% (existing)
Line DE Bearing 102°17'00" Grade + 3% (existing)
Design Velocity: 100 km/hour   Max Rate of Change of Radial Acceleration 0.5 m/sec^3

Calculate:

1. The chainages of points T_1 T_2 T_3 T_4 and D
2. Coordinates of points T_1 T_2 T_3 T_4
3. The deflection angles to set out the even 25 m chainages and the coordinates of the points
4. The superelevation at these chainages.

N.B. (1) Chainages are to commence (00) from the new tangent point T_1 .
(2) Round up to the nearest 10m the calculated lengths of the transition curves before using them in further calculations.

Question 3

A compound reverse curve, which is wholly transitional, connects three straights, AB, BC and CD whose whole circle bearings are 78° 00', 90°00' and 80°00' respectively. If the design speed is 70 km/h and the rate of change of radial acceleration is 0.3m/s^3, determine the length of common tangent BC.

Question 4

a) Your survey firm is asked to provide expert advice on a vehicle crash scene investigation. You measure the distance from the start of the tyre marks found on the road to the crash site to be 33.4 m and the slope of the road to be an even -2%. The driver claims to have been doing the speed limit of 60 km/h. The standard RTA values from the Road Design Guide, March 1988:

f = (assumed coefficient of longitudinal friction demand for design speed) = 0.47 (at 60 km/h)
RT = Reaction Time = 1.5 secs (at 60 km/h)

Was the driver speeding? Show all calculations and then give a professional assessment based on your results.

b) Briefly explain the difference between stopping sight distance, overtaking sight distance and intermediate sight distance.

c) Why would intermediate site distance be used? What else could be done for safety?

d) When designing a vertical curve list four separate items that influence the length of curve.

Question 5

I received an email in April 2011; an edited part of it follows.

... I was studying with you in the ... survey course last semester. Now I am working at a construction company in [SE Asian country]. My duty is to write a program (Visual Basic and Matlab) that calculates the offset of any interested point to the designed alignment of the new sky train route.

According to the topic of this question, I now have a serious problem of how to calculate the offset of the point which is not on the spiral curve to the spiral curve. (...I have the coordinate of that point, and almost all the data of the spiral curve such as Ls, X, Y etc.). So, can you help guide me about how to solve this problem?

Your tutorial question is to answer this real world problem. You don't have to write a computer program, just describe a method of calculation.

8.4 Vertical Curves Questions

Question 1

Design a vertical curve 200 m long, connecting a rising gradient of 1 in 50 with a falling gradient of 1 in 75, which meet in a summit of RL 30.35 m and chainage 2752 m. Give offsets at 20 m even chainages. Calculate the chainage of the highest point on the curve.

Question 2

A parabolic (crest) vertical curve connects two straights with gradients p=+2.0% and q=-1.7% respectively. The two straights intersect at an elevation of 144.50 m at chainage 5023.00 m. The curve was designed such that its highest point is 109.58 m from the first tangent point. Determine the chainages and elevations of the two tangent points.

Question 3

A falling gradient of 4% meets a rising gradient of 5% at chainage 2450 m and R.L. 216.42 m. At chainage 2350 m the underside of a bridge has a R.L. of 235.54 m. The two grades are to be joined by a vertical parabolic curve giving 14 m clearance under the bridge.

a) Draw a diagram of the problem
b) Compute the length of the vertical curve.
c) Calculate the levels at 50 m intervals along the curve.
d) Calculate the chainage and reduced level of the lowest point on the curve
e) If the length of the curve was reduced, what effect would this have on the clearance?

Question 4

An uphill gradient of 1% meets a downhill gradient of 0.5% at a point where the chainage is 122.880m and the reduced level is 126.000m. If the rate of change of gradient is to be -1.2×10^{-4} per metre, calculate:

(a) the length of the vertical curve.
(b) the chainage and reduced levels of the tangent points.

(c) the chainage and reduced level of the highest point on the curve.

Question 5
A parabolic sag vertical curve connects a down gradient AB of 1 in 20 with a second down gradient BC of 1 in 50. Intersection point B has a chainage of 1459.00 m and its reduced level is 198.48 m above datum. In order to allow for clearance at a bridge, the reduced level of a point on the curve at chainage 1470.00 m is to be 198.56 m.

(a) Determine the total length of the vertical curve, and the rate of change of grade.
(b) The beams given out by the head lamps of a certain vehicle are parallel to the longitudinal axis of the vehicle. Calculate the sighting distance at night, given that the headlamps are 0.70 m above road level.

Question 6
On a straight portion of a new road, an upward gradient of 1 in 100 was connected to a downward gradient of 1 in 150 by a vertical parabolic summit curve of length 150 m. A point P, at chainage 5910.0m, on the first gradient, was found to have a reduced level of 45.12m, and a point Q, at chainage 6210.0m on the second gradient, of 44.95 m.

(a) Find the chainages and reduced levels of the tangent points to the curve.
(b) Tabulate the reduced levels of the points on the curve at intervals of 20 m from P and of its highest point.

Find the minimum sighting distance to the road surface for each of the following cases. Take the sighting distance as the length of the tangent from the driver's eye to the road surface.
(c) The driver of a car whose eye is 1.05 m above the surface of the road;
(d) The driver of a lorry for whom the similar distance is 1.80 m.

Question 7
A sag vertical curve PQ is continuous with a summit vertical curve QR, the relevant tangents being PA, AQB and BR respectively, with PA and BR falling to the right. The gradients of FA, AQB and BR are 1 in 40, 1 in
50 and 1 in 30 respectively, and curve PQ has a length of 200 m. If the difference in level between the lowest point on curve PQ and the highest point on curve QR is 2.09 m, determine the length of curve QR to the nearest metre, and thence the reduced level of the curve point midway between P and R. The reduced level of tangent point P is 80.00 m above datum.

Question 8
A parabolic vertical curve of length 156.00 m is to connect a 2.5% down grade to a 3.5% up grade. The reduced level and chainage of the intersection points of the grades are 59.34 m and 617.49 m respectively and in order to meet particular site conditions, the chainage of the entry tangent point is to be 553.17 m.

Calculate
(1) The reduced levels of the tangent points.
(2) The reduced levels at even 20 m through chainages along the curve.
(3) The chainage and reduced level of the lowest point on the curve.

Question 9.

Curve design problem by Craig Roberts. Randwick Council would like to design a crazy new waterslide for the residents. They have set aside a parcel of land 33m x 33m in size. The water slide will start from a tower 15m high. The tower is located in the parcel as per the diagram. The slide must have a slope of at least 1:10 but not more than 1:6. At least three horizontal curves and one reverse curve should be included in the design. Sliders exit into a plunge pool as per the diagram. The slide is 1 metre in width. The council have engaged you to design a water slide to maximize the length of the slide as per the constraints given. What is the maximum length of your design? Please show a plan diagram with all relevant dimensions included. Remember, the plan could show some overlaps as the design will be three dimensional yet drawn in 2D only.

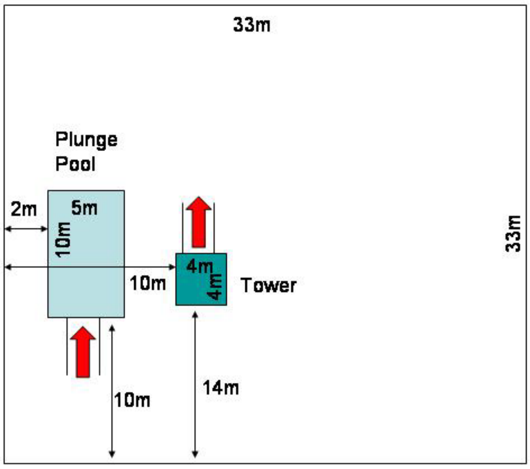


Diagram not to scale

Question 10.

A road grade question by B. Harvey using GPS data collected on his bicycle ☺. A part of Lawrence Hargrave Drive Stanwell Park, south of Bald Hill, is known to be steep - but how steep is it, and is the grade uniform? A GPS instrument that includes a reliable barometer recorded coordinates of points while riding up the hill, as shown in the blue dotted line on the map below. From the E and N coordinates the distance between data points was calculated, and converted to a chainage value by simply cumulative adding of the segment distances. The associated Height coordinate for each data point was obtained from the combined barometer and GPS readings. The chainage and height data, in meters, is shown in the table below and is available in a spreadsheet. Plot the long section of the road and calculate the average grade of the road.

Chainage	H	Chainage	H	Chainage	H	Chainage	H	Chainage	H	Chainage	H
0	183	302	163	559	135	909	103	1186	73	1350	55
32	181	307	161	571	134	944	100	1196	71	1360	55
46	181	324	160	584	132	963	96	1217	71	1370	54
91	180	336	158	596	131	972	93	1228	68	1380	53
124	178	343	158	633	130	982	92	1239	67	1390	53
131	177	350	157	657	127	1004	91	1250	66	1401	52
138	176	357	157	682	125	1015	89	1261	64	1412	52
146	176	390	156	695	122	1062	88	1272	64	1422	51
154	175	418	152	730	121	1074	84	1283	62	1432	51
162	174	447	149	742	117	1086	83	1294	61	1442	50
188	174	480	147	754	116	1110	81	1304	60	1451	49
205	171	515	143	777	115	1144	79	1313	59	1503	48
228	170	526	140	813	112	1154	76	1322	58	1514	45

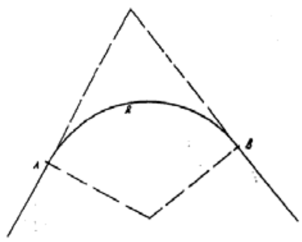
242	167	537	138	854	108	1164	75	1331	57	1535	44
276	166	548	137	868	104	1175	74	1340	56	1544	43

8. ROAD “CENTRELINE” PROBLEMS – Answers and some worked solutions

These solutions can be read AFTER you have made some attempt to solve the question.

Horizontal Curve Questions

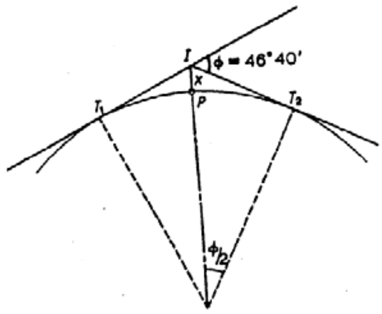
Question 1



R = 250.45
Angle = 62 12 20 = 62.20556 °
Theta = 180 - angle = 117.794 °
Arc length = R*RADIANS(theta) = 514.900 $A = R.\vartheta^{c}$
Chord = 2*R*SIN(RADIANS(theta /2)) = 428.892 $C = 2R\sin\frac{\theta}{2} = 2T\cos\frac{\theta}{2}$

Tangent = R*TAN(RADIANS(theta /2)) = 415.130 $T = R\tan\frac{\theta}{2}$

Question 2

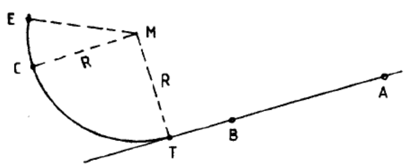


449.09, 193.72, 355.75 by Nic Woodward

Question 3

R: 93.342 M = E 201.900 N78.35

Question 4

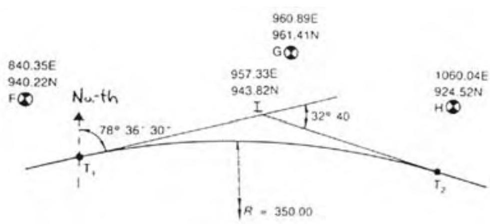


T = E 666.034 N 407.099 E = E 465.824 N 527.510
BT = 39.648 Curve = 241.358 Defl ang 50° 28' 12"

Question 5

112°20'00", 61.206 m

Question 6



Answers: Ch 940: 248° 43’ 10”, 97.770, Ch 960: 245° 15’ 46”, 78.484.

A step by step CivilCAD worked solution:

Insert Point XYZ, for I F G H as points 1 2 3 4 respectively.
Insert Line Traverse I to chainage 00 at brg = 78.3630 + 180 and dist = 1029.35
Insert other tangent line from I at brg = 78.3630 + 32.4 and dist = 500 (for example)
Survey Alignments Create Alignment click at LH point 5 (i.e. 00) then IP (pt 1) then RH end point 6. Right click.
Survey Alignments Edit IP: Radius = 350, super and spirals = 0
Survey Alignments List Alignment: Include TPs, Spacing 20 on straights and 20 on curves
Read and Save the report file.
Survey Alignments Create Centreline Points
File Export save as ASCII.pts

The first straight has chainage points every 20m with their E N coordinates. The first TP is at chainage 926.782, with coordinates 856.782, 923.561. So the first even chainage after the TP is at 940, its coordinates are 869.786, 925.927. The next point is at chainage 960, its coordinates are 889.608, 928.568.

Cogo Brg&Dist (or Insert Line Join 2 pts) and get the bearing and distance from G (pt 3) to 940 (pt 56?)
Similar for line G to 960 (pt 57?)
How do we save the bearing and distances of these set out lines, other than writing them down on paper? If we have the coordinates of points in a file we can copy and paste etc them into MS Excel and do the calculations or upload the coordinates to a total station (or GPS system) and use the instrument to set out the points without calculating the B & D.

Question 7

Answers: 950.402 m or 950.688?, 167.632, 331.753 m or 331.853?, 0°36'10"

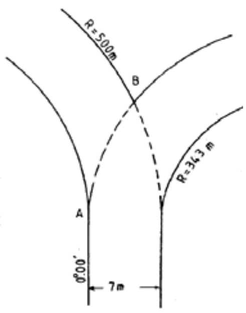
Solution method for questions 7 and 13
These questions are about designing a road to go through a point, typically to miss an obstruction. Determine the radius of a curve that passes through point D.

Method 1. Draw a triangle from B to D (distance known), D to centre of circle O (distance = R), O to B (distance = R/cosθ) This triangle's angle at B is also known since the bearing BO = mean of two tangent's bearings. Use cosine rule that yields a quadratic equation that can be solved to find R.

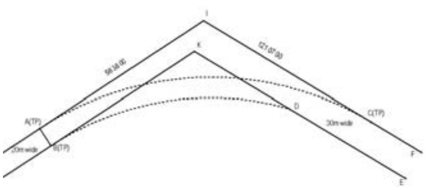
Method 2. Form a triangle B - D - TP1. The angle in the triangle at TP1 is the deflection angle α. The side length of the triangle TP1 to D is 2R sinα. Distance TP1 to B = Rtan(θ/2). Triangle angle at B can be found from bearings. Set up equations using triangle rules, for R and α. Possibly use Excel's Solver function to solve two equations with two unknowns.

Question 8

Brg AB = 4° 24' 05" dist AB = 53.720 m



Question 9

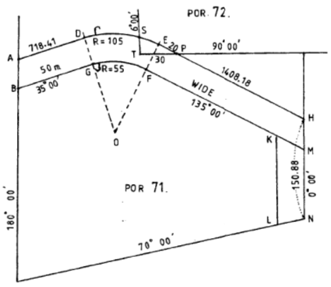


KD = 107.221 m

Question 10

You are welcome to send me your solution. One set of answers that haven't been independently checked are:

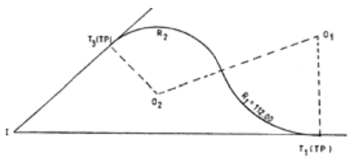
A) Find dimensions of the resumed road i.e. ADSEPHMFGBA. Answers to nearest minute and distances to 2 places as is given data. A 35°00' 718.41 D 80°17' 149.20 (arc len 165.94) S 130°17' 17.30 (arc len 17.32) E 135°00' 20.00 P 135°00' 1408.18 H 180°00' 70.71 M 315°00' 1478.18 F 265°00' 84.26 (arc len 95.99) G 215°00' 789.82 B 0°00' 87.17 A



B) Area of resumed part of 72. Area of polygon SEPTS by coordinates (and chords) = 394.1 Area of segment SE = 4.115. Total area resumed from por 72 = 4.1 + 394.1 = 398.2

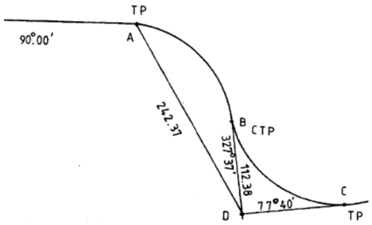
Question 11

R_2 = 91.732, arc len = 302.332 by Dean Gardiner



Question 12

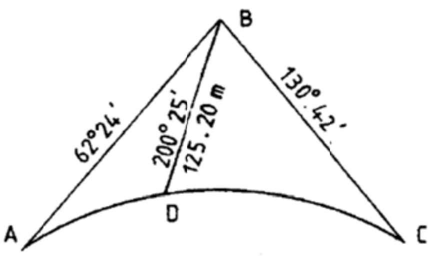
Radius BC = 160.644 Radius AB = 142.451



Question 13

Answer: R=579.65 See method for question 7

Hint: form a triangle BDO where O is centre of the circle that forms arc ADC, and side BO's bearing and dist from formulas.



One method: Get angles BOC (half of deflection angle≈34°) and DBO (≈14°, difference in bearings BD given and BO, from mean of bearings of tangents).
We know length BO = R/cosBOC.
Apply Sine Rule to triangle BDO where:
sinODB = BO x sinDBO/R
By substitution for BO: sinODB = R /cosBOC x sinDBO/R
The R cancels out, so sinODB = sinDBO / cosBOC
Remember when you calculate ODB by $\sin^{-1}$ the answer can be in two quadrants, chose the one near 180°. i.e. 180° - the value from the calculator or spreadsheet. ODB ≈ 163°
Angle BOD can now be computed by 180 ° - the other two angles in the triangle. ≈ 3°
Use Sine rule to determine R = BD x SinDBO/SinBOD =579.653

Another method: In the triangle BDO, set a = R, b = R/cos(BOC), c = side IP to D given, and cosA = cosDBO, Calculate DBO and BOC as above. Solve the cosine rule to determine the only unknown i.e. R. That can be done as the solution of a quadratic equation, or by using Excel's solver with target cell $0 = b^2 + c^2 - 2bc \cos A - a^2$ The results are:

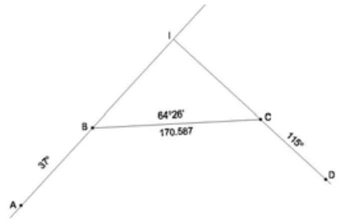
579.653	a = R
700.427	b = R/cos(deflangle/2)
125.200	c = side IP D, given
0.97	cos A = angle at IP

Question 14

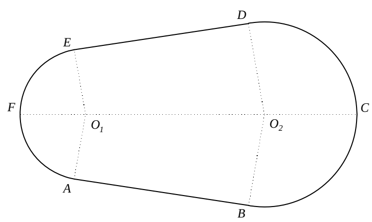
PT_1 = 48.624, QT_2 = 133.944

Question 15

a) 1137.923, 1478.262, b) 56°30’00”, 166.903, c) 71.690



Question 16

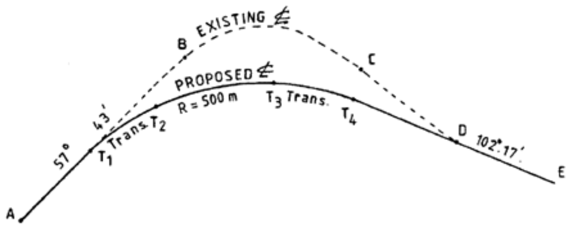


You are welcome to send me your solution.

Transition Curves Questions

Question 1
ANS:- 0.098, 89.308, 259

Question 2



ANS:- By former teacher, check these with CAD.

1) 00, 90, 388.918, 478.918, 867.71
2) (4722.212, 1052.709) (4799.678, 1098.458) (5089.689, 1149.595) (5178.131, 1133.100)

3) & 4)

Ch	d	m	s		Ch	d	m	s		Ch	d	m	s		Ch	d	m	s
00	0	00	00		90	0	00	00		478.918	0	00	00		00	0	0	0
25	0	07	57.3		100	0	34	22.6		475	0	00	11.4		25	2	30	16.6
50	0	31	49.5		125	2	00	19.2		450	0	16?	38.5?		50	4	59	59.0
75	1	11	37.1		150	3	26	15.8		425	0	37	00.5		75	7	28	20.0
90	1	43	07.5		175	4	52	12.4		400	1	19	17.5		90-388.918	8?	56	56.0
					200	6	18	09.0		388.918	1	43	07.5		400	7	51	42.2
					225	7	44	05.6							425	5	23	21.3
					250	9	10	02.2										
					275	10	35	58.8										
					300	12	01	55.4										
					325	13	27	52.0										
					350	14	53	48.6										
					375	16	19	45.1										
					388.918	17	07	35.9										

Question 3
ANS: 137.34 m.

Question 4
You are welcome to send me your solution

Question 5.
My first thoughts / quick reply answer to the email question about offsets to spiral curves:

If you have the spiral specifications then it shouldn’t be too difficult for you to calculate the coordinates of points along the curve. Now if you calculated points at a small chainage e.g. every 1 metre, then you can calculate the offset for your point of interest (let’s call it Pt A) as follows.
Calculate distance from A to each of your chainage points. The point with the shortest distance (let’s call it B) is close to being the offset to the curve. Now near the point on the curve having the shortest distance (B) you can calculate coordinates of some other points on the curve with a much smaller step

in chainage, e.g. every 0.1m for a metre either side of B. Similar to before, find the point with the shortest distance to A. This point's chainage will be close to B, and it is even closer to giving you the offset you require. Continue the concept until the accuracy is sufficient for your needs.

Not difficult to program this as a loop in a program. I suggest you test the idea with sample data and look at a plan view (e.g. in CAD). It also matters whether the curve is bending left or right and whether your point is on the left or right of the curve. Also, the offset (shortest perpendicular distance) may be to a point on the centreline of the rail that is before or after the curve.
Bruce

The reply:
Dear Dr. Bruce
thank you very much for your perfect suggestion. it really help me a lot. I am now trying to solve my problem follow your advise and it seems working well. thank you very much

---

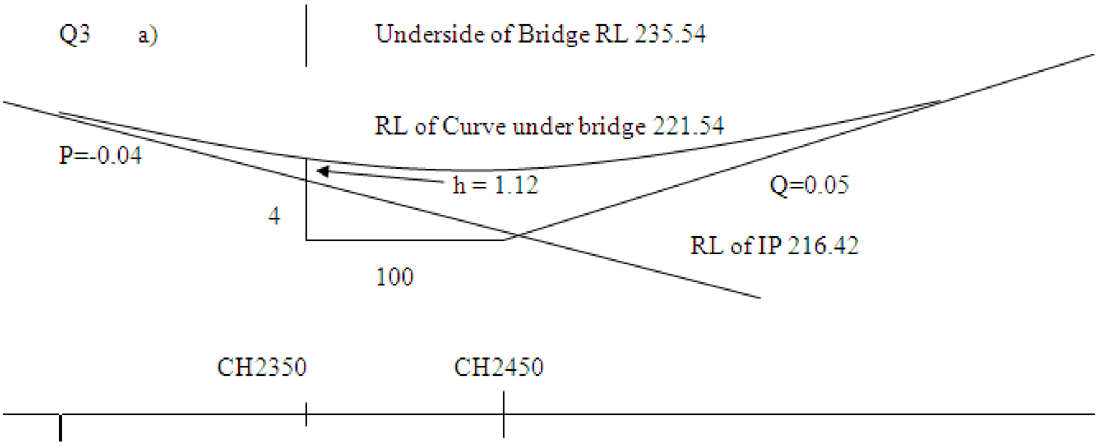
If you have a better solution you are welcome to send it to me.

Vertical Curves Questions

Question 1
Ch2652 28.35 m, Ch2660 28.51 m, Ch2680 28.84 m, Ch2700 29.12 m, Ch2720 29.32 m, Ch2740 29.46 m, Ch2760 29.54 m, Ch2780 29.55 m, Ch2800 29.49 m, Ch2820 29.36 m, Ch2840 29.17 m, Ch2852 29.02 m; 2772.12 m @ 29.55

Question 2: TP start Ch 4922 m, 142.47 m; TP end Ch 5124 m, 142.78 m

Question 3
L = 399.407, $RL_{0=2350} \equiv H$ = 224.408, RL_{50} = 222.690, RL_{100} = 221.535, RL_{150} = 220.943,...etc, $RL_{399.407}$ = 226.405 Check by calculating grade to TP2 = 226.405.
Hint: Solve quadratic equation for L using data under Bridge: x = (L/2)-100 (1)
and h = ((q-p)/2L)x^2 (2)
substitute (1) in (2) and h = 1.12 so (0.09/8)L^2 -1.12L-4.5L+450 = 0 etc)
Solution provided by Nic Gowans & edited by B Harvey**:**



b) Using $h = ((q-p)/(2L))*x^2$ and x = (L/2) - 100
$h = ((q-p)/(2L))* [(L/2) – 100]^2$
So $hL = ((q-p)/2)* [(L/2) – 100]^2$
$0 = ((q-p)/2)*(L^2/4—100L+10000) –hL = (0.09L^2/8) – 50L(0.09) + 5000(0.09) – 1.12L$
$0= 0.01125L^2 – 5.62L + 450$
Solve the quadratic to find L = 399.407 or 100.148. Take first answer. Check it by back substitution in the equations above.

c) Calc RLs at 50m intervals.

At CH2350 RL = 235.54 (given)
Chainage of TPs = 2450 ± 399.407/2 = 2250.2965 and 2649.7035
RL TP1 = 216.42 + 0.04 * (399.407/2) = 224.408
Use RL = (q-p)*x^2/2L + px + RL of TP1 to set up a spreadsheet to calc RLs given Chainage:

CH 2300.2965	RL 222.690
CH 2350.2965	RL 221.535
CH 2400.2965	RL 220.943
CH 2450.2965	RL 220.915
CH 2500.2965	RL 221.450
CH 2550.2965	RL 222.548
CH 2600.2965	RL 224.210
CH 2649.7035	RL 226.405

d) Lowest Point on curve at RL at CH = CH TP1 + (-p/(q-p)) * L = 2427.8107
Use spreadsheet to find RL CH 2427.8107 = 220.858
e) Using RL = (q-p)/2L*x^2 + px + RL TP1, we see that decreasing L will increase the RLs, reducing clearance under the bridge.

Question 4: *L = 125.000, TP1 60.38, 125.375, TP2 185.38, 125.688*

Question 5: *(a) 119.965 m, 2.5x10^{-4} (b) 74.822 m*

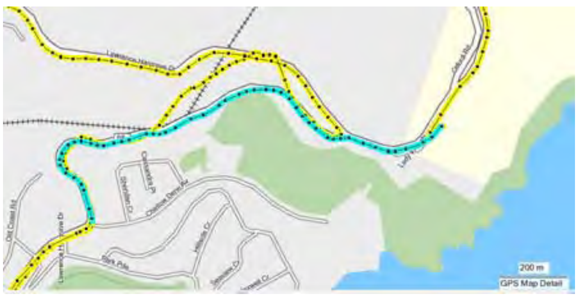
Question 6: *(a) Chainages 5944.8, 6094,8; (b) RL 45.92; (c) 137.48 m; (d) 183.0 m*

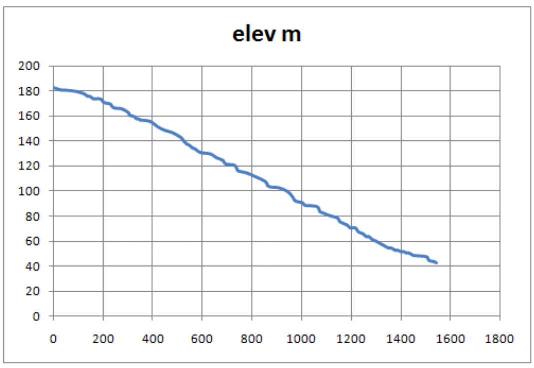
Question 7: *320m, 80.40.*

Question 8
(1) 60.95, 62.55 (2) 60.95, 60.792, 60.476, 60.380, 60.476, 59.502, 60.775, 61.155, 61.644, 62.240, 62.550 (3) 598.774, 60.378

Question 9: You are welcome to send me your solution

Question 10
If you are not familiar with the area south of Sydney you can see the road in Google Earth, or similar, near 34°13'22"S 153°59'31"E. The graph below shows that the slope of the road is fairly uniform even though the road curves in plan view. A line of best fit by linear regression has a slope of 0.099, which is close to 10% for the 1.5km climb.





9. SUBDIVISION and ROAD DESIGN in CAD

This chapter includes exercises based on a CivilCAD training manual. We have not included step by step instructions but a summary of the main steps is included. If the instructions are not clear to you ask a tutor for assistance or ask other students in the class, then write your own notes to clarify the process for future reference. Often it is useful to read the whole step before starting it.

Our aim is for students to experience the process of drawing and calculating subdivisions in CivilCAD. We do not expect students to memorise the steps, but we do expect you to gain some experience and understanding of the steps involved.

9.1 Subdivision Design in CAD

Exercise 1: Road Outline (Edited from Civilcad 6 Training Manual – Survey)

This exercise may seem as though you are just making a drawing, but you are actually calculating the coordinates of all the points and the bearings, distances of lines etc. The orientation and detail of the road outline are all taken from the following illustration.

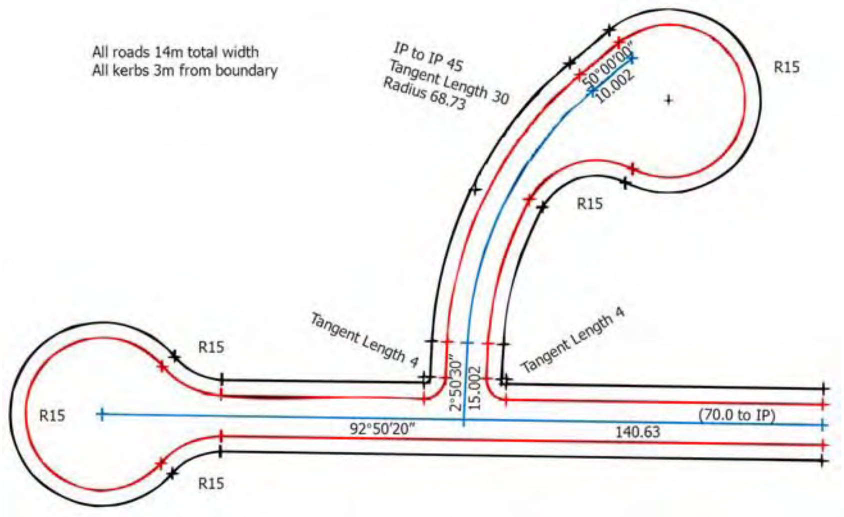
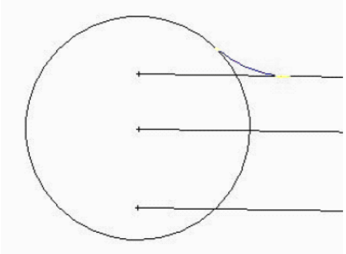


Figure. Road outline

1. Create a new project in CAD.
2. Create a new layer and call it SUBD, set colour and line type to black and solid line respectively.
3. Create the centreline of the major (east-west) road by entering arbitrary starting point coordinates for point 1. Then enter the Bearing 90°50’20” and Distance 140.63 to the second point.
4. Create the kerb lines, which are at an offset of 4m from the centreline of the major road.

5. Draw the circle for the cul-de-sac head with radius 15m at the end of the main straight. The centre point of the circle is probably your point 1.

6. A joining arc is needed to neatly connect the cul-de-sac circle with the straights of the road. Use an arc radius of 15m. Apply similar arcs to both the northern and southern kerb.



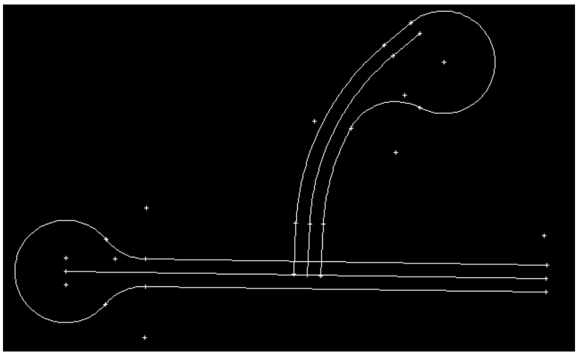
7. Create the point where the major and minor roads intersect, at a distance of 70m from the Eastern end (right hand end) of the road centreline.

8. Draw the centreline of the minor road that heads north. Draw two lines. Line 1 Bearing 2°50'30" Distance 45, and line 2 Bearing 50°00'00: Distance 40. Now insert an arc of radius 68.73 tangential to the existing straights.

9. Create the left and right sides of the minor road. Some CAD software has an option for making parallel lines or curves. Use that option if you can find it; otherwise create the kerb lines for the minor road individually.

10. Draw the second cul-de-sac circle. It has a radius of 15m and the arc angle is 270°.

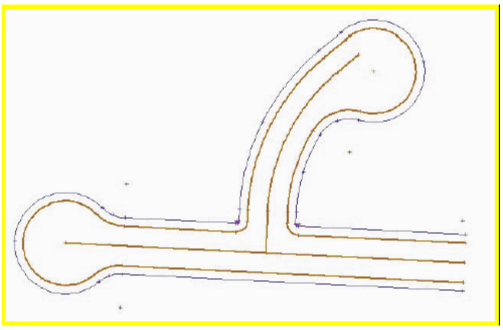
11. A joining arc is needed to neatly tie the cul-de-sac arc and the kerb of the minor road. The radius of the fillet arc is 15m. You may, or may not, need to trim some 'left over' lines.



12. The kerb lines of the junction of major and minor roads cross each other as seen in the figure. However your CAD software may not treat these line crosses, intersections, as points. You should create points at these line intersections, by appropriate calculations or functions (don't just click the mouse on the spot!)

13. Clean up your drawing by deleting un-wanted points and lines (e.g. the kerb lines across the road intersections). You may need to segment or break the original lines into sections so that some sections can be deleted.

14. Draw the curved corners at the intersection of the two roads, and define the arc using a tangent distance of 4m.



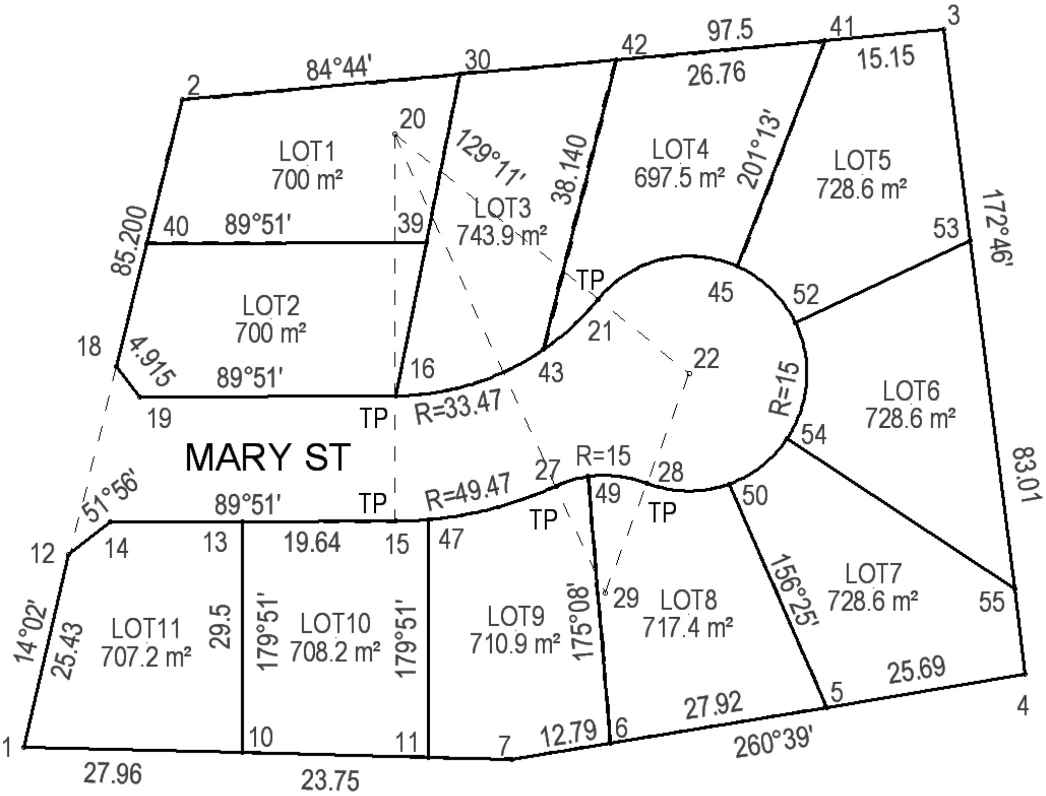
15. Create the lot boundaries (i.e. where owners might place their fences) of the two roads.

16. Clean up your road outline plan. DO NOT display point numbers.

17. Print out your final plan as a pdf file or on A4 size paper.

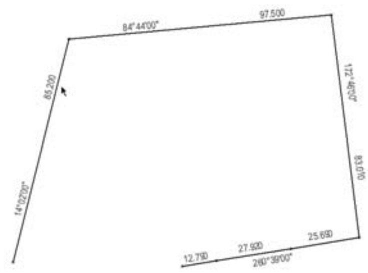
Exercise 2 - Subdivision

Again, in this exercise you are not just making a drawing, but you are actually calculating the coordinates of all the points, the bearings, distances of lines and the lot areas. In this exercise the orientation and detail of the proposed subdivision are all taken from the following illustration. The point numbers shown here are for instruction only. Do not show the point numbers on your final subdivision plan.



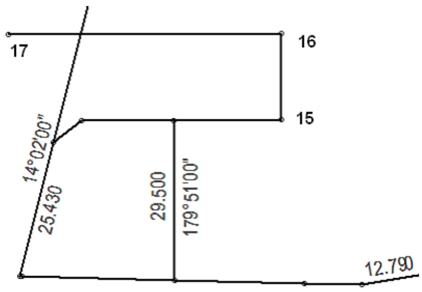
This figure and the point numbers are used through these instructions. North is up the page.

Procedure:

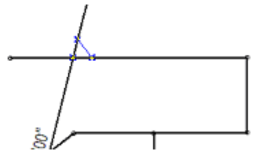


1. Create a new job; set layer, colour and line type as you did in exercise 1.
2. Insert lines from point 1 to 7. The first point can be arbitrary (anywhere on the screen), or you may choose to give it some coordinates e.g. 100, 300.
3. Join points 1 and 7.
4. Select line 1-7. Segment the line to create points 10 and 11. Then do similar to create points 12, 41 & 42.
5. Insert line 10-13. From point 10 the bearing is 359°51' to 13 (i.e. reverse bearing to that shown on plan).
6. Insert point 14 by intersection. We select point 12 and enter bearing 51°56', then select 13 and bearing 269°51'.
7. Insert lines 12-14 and 14-13 by joining the points.
8. Insert line 13-15 based on its bearing and length (point 15 is a TP).
9. Insert point 16 (TP) by insert dummy line 15-16 based on its bearing and length (Need some calculations, hint 15 and 16 are tangent points (TP). What angle are tangents at to the centre? That will give you the bearing. What is the difference between the two curve radii, i.e. the road width.).

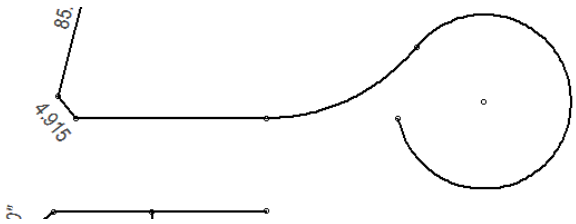
10. Insert line from point 16 with bearing 269°51’ (the plan bearing + 180°) and arbitrary distance, say 50. Or intersect line from 16 with a line from 12.



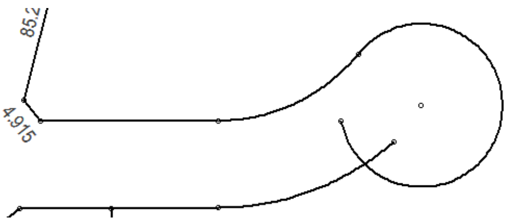
11. Create the ‘Spray Corner’ at the cross of line 1-2 and line 16-17, (it may be different in your case) as shown below. Type 4 in the **Tangent Distance** and then press Enter. 4m is the distance along the main road (we calculated it for you). There are other ways to enter this line, but we are showing you some of the options available in CivilCAD.



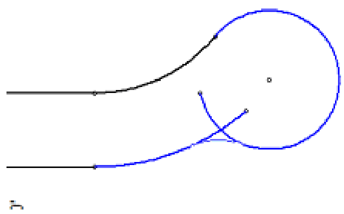
12. Insert arc 16-21 tangential to the line. Select line 16-19. Calculate the angle 16-20-21 from the bearings 20-21 (shown on plan) and 20-16 (90° to tangent).
13. Insert arc 21-28. The arc angle is not given and it must be large enough for later trimming.



14. Insert arc 15-27. The arc angle is not given and it must be large enough for later trimming.

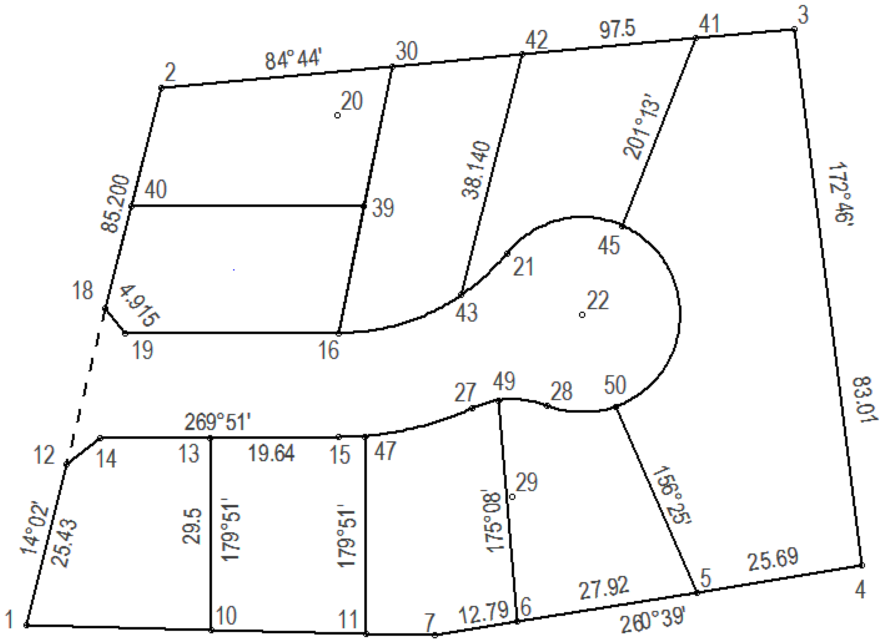


15. Insert arc 27-28 and truncate the ‘left over’ curves.



16. *This step is an important part of subdivision design work in CAD.* There may be several steps to follow here before you see the result. Create a LOT 16-19-18-2-30 (i.e. total of lots 1 and 2) using the existing lines on the plan. This turns the lines into a polygon (Lot) and creates point 30. Use the line 16-19 as the starting edge for the new lot you are creating and enter the area as 1400 (=700m^2 + 700m^2). Get the CAD software to create the line 16-30 pivoting at point 16 and giving the required total area.
17. Create Lot 2 by requiring line 39-40 to be parallel to line 16-19, and the area to be 700. CAD will calculate the coordinates of points 39 and 40 and insert the line and the lot as a polygon. Save the remaining area as Lot 1. Probably a good time to save your file, if you haven’t already.
18. Intersect point 43 (i.e. 43 in our figure, it may have a different number in your CAD file) using the two distances from 42 (=38.14) and 20 (=33.47, the radius of arc 16-21).
19. Use point 43 to segment the arc 16-21 and link points 42 and 43 with a line. This is important to form an expected lot 16-30-42-43.
20. Intersect point 45 using point 41 bearing = 201°13’; and arc 21-28. This will create a point on the arc. There are two possible answers, select the one we want and discard the other.

21. Segment arc 21-28 at point 45 and then link points 41 and 45 with a line. Similarly, intersect points 47, 49 and 50. Be careful with bearings, they might be 180° reverse to that shown on the plan. Insert the points onto their arcs, segment the arcs, and draw the join lines, as above.



22. Create lots 5, 6 and 7. Your CAD may have a way to create three equal size lots within the points 3-4-5-50-45-41-3. Or you can calculate the area of 3-4-5-50-45-41-3 e.g. area = 2185.915m^2 (your value may be different). Divide your value for area by 3 because you want three equal Lots (each area ≈ 728.638m^2) and no left over space. Then create lots with these areas using methods described before.

23. Assign LOT numbers which are the same as the ones on our figure, in case CAD has given them different lot numbers. If you haven't already done so, you can delete the line 15-16.

24. Look at the properties of each Lot. It may contain much information about the Lot you are creating such as House No, Street name and so on.

25. Lot Annotation. Find the method in your CAD to display in each lot the lot number, area and bearings and distances of the boundaries of the lot. You can probably select the font, style, roundoff etc.

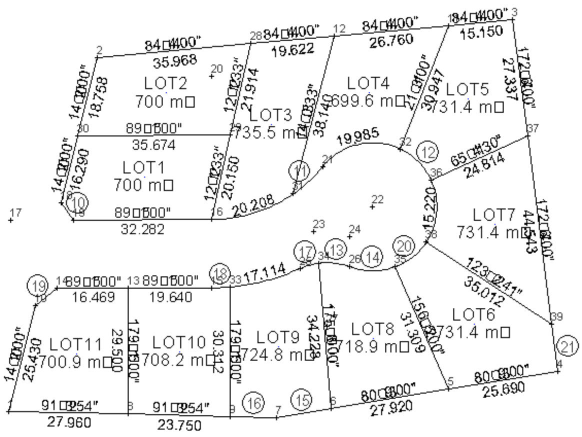
26. You might also want to create a separate table in your plan that includes details of the lots or of the short boundary lines such as those on the arcs.

27. **Export** the properties of all LOTs including point coordinates and bearings and distances of lines to a text report or file for the subsequent set out survey.

28. Print out your final plan to a pdf file (A3 page) with the Line and Arc tables. Lot areas should be in square meters and displayed to 1 decimal place only.

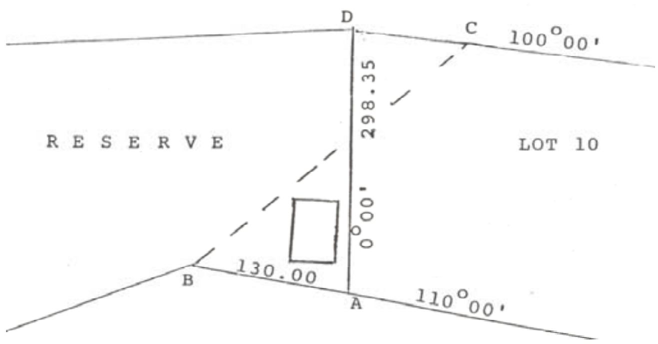
Number	Bearing	Distance
10	141□56'30"	4.915
15	80□39'00"	12.790
16	91□23'54"	10.991
19	51□56'00"	6.655
21	172□46'00"	11.130

Number	Radius	Arc Length	Angle	Chord Bearing	Chord Distance
11	33.470	9.477	16□13'22"	227□08'41"	9.445
12	15.000	10.438	39□52'13"	315□18'23"	10.229
13	15.000	7.421	28□20'46"	96□41'13"	7.346
14	15.000	10.867	41□30'37"	90□06'18"	10.631
17	15.000	4.514	17□14'29"	73□53'36"	4.497
18	49.470	4.106	4□45'20"	87□28'20"	4.105
20	15.000	9.417	35□58'19"	231□21'50"	9.263



Question:

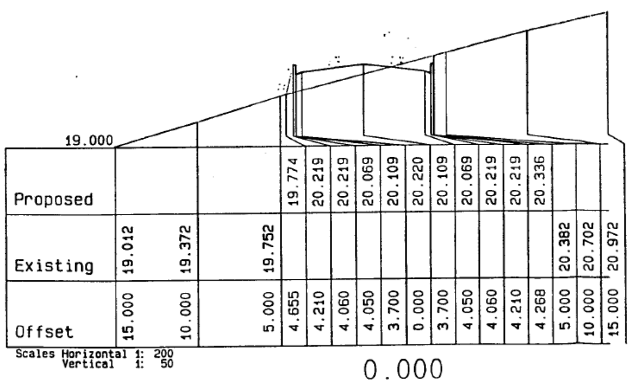
Solve Q10 from the previous chapter using the subdivision routines in CAD. “The owner of Lot 10 in the figure at right has built his house on the reserve by mistake. It is required to place a new boundary BC for Lot 10 to include the house but leaving the original lot area unchanged. Find the distance DC and the bearing and distance of BC. “



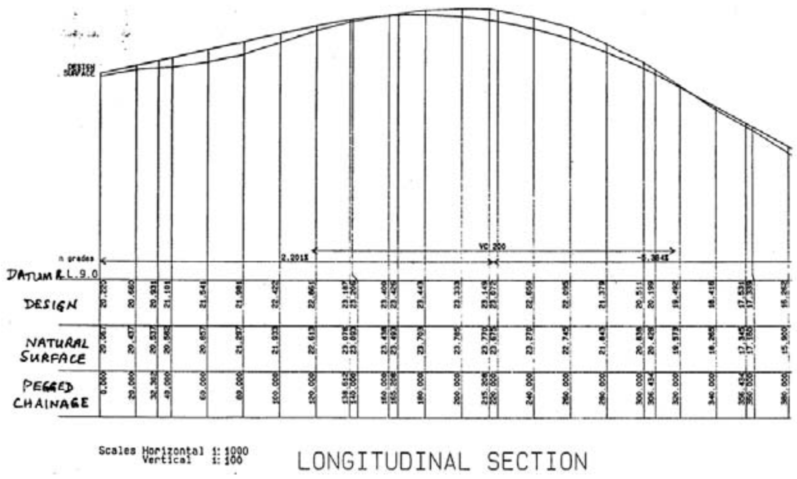
9.2 ROAD DESIGN

This section commences with a short section on Road design, then describes step by step instructions for Road calculations, design and drafting in CAD. The final section describes 3D views and fly-throughs in CAD. We do not have sufficient time to cover the interesting topics of road design in the current course including the consideration of forces, speed, safety, wear, drainage, sight distances, stopping distances, intersections and comfort. Many standard surveying textbooks have good chapters on this material and National Roads and Standards Associations (or equivalent) often publish specifications or guidelines related to these matters. It is your responsibility as a graduate to find and follow the latest versions.

Example road cross section:



Example road long section:



In these exercises step by step instructions are supplied. If the instructions are not clear to you ask a teacher for assistance or ask other students in the class, then write your own notes to clarify the process for future reference. Often it is useful to read the whole step before starting it.

Our aim is for students to experience the process of drawing and calculating roads in CAD. You will see some of what can be done in CAD. We do not expect students to memorise the steps, but we do expect you to gain some experience and understanding of the steps involved. Our objective is to get you to a standard that allows you to be able to process your own data in future, with the assistance of these notes.

These notes were prepared based on information in CivilCAD's Road Manual.

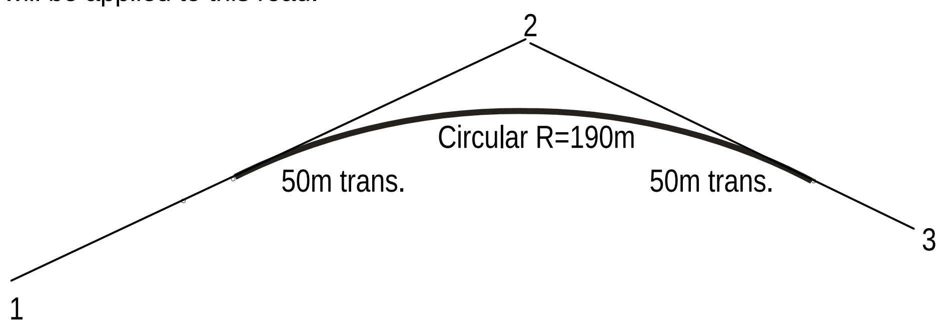
Exercise 1: Road on a surveyed site

In chapter 8 an exercise was given to calculate the horizontal alignment (centreline) of a proposed road. In this exercise we will create and define a horizontal alignment on a site that has already collected data in a detail survey so we can consider the topography of the site. This exercise uses a simple alignment of two straights and an arc, but most alignments will be more complex than this. We will then extract cross sections along this alignment from the triangular mesh (TIN) and DTM created from the survey data of the site.

The site for a proposed new road has been surveyed by GPS and the transformed E, N, H coordinates are supplied in a separate file (Road.pts). The point codes do not use a prefix. Part of the data file is shown below.

5502,290174.721,763.463,45.649,NS
3090,289976.081,523.526,25.387,NS
3089,289979.357,523.838,25.321,NS
3088,290008.050,527.588,26.222,NS
138,290151.426,749.132,41.039,NS
139,290116.717,747.572,35.894,NS
140,290085.135,745.775,32.462,NS
141,290048.885,740.225,29.237,NS
142,290018.373,739.609,27.084,NS
185,290021.393,467.465,30.976,NS
186,289998.516,457.847,31.051,NS
3263,290042.486,948.651,23.020,NS
3264,290029.555,935.555,22.756,NS
3265,290022.619,920.923,22.839,NS
3277,290134.215,667.349,33.399,NS
3278,290124.805,693.661,33.915,NS
3279,290141.984,710.046,37.032,NS
3280,290102.911,700.991,31.833,NS

The proposed road contains a circular curve and two spiral transition curves as shown in the figure below. No superelevation will be applied to this road.



The centre line of the road starts at point 1 (E = 290108.0, N = 598.0) with a chainage of 00. Point 2, the IP, is at (290077.0, 778.0) and point 3 is at (290138.0, 880.0).

1. Copy the survey data into your CAD data folder and the open the file from within your CAD as a new project.

2. Create the DTM, via mesh and contours.

3. Create the road alignment (see chapter 8) using: start point (first IP): 900, 5030; bearing and distance of the first line: 125° & 135.0m; bearing and distance of the second line: 62° & 140.0m; and start chainage at point 1 = 00.

4. Enter the road curve to the alignment.

5. Have a look at the plan with just the alignment and contours on display.

6. In this part of the exercise we will extract cross sections using the triangular mesh for the alignment previously created. The chainage interval will be 20m along both straight and curve and also include cross sections at the TPs. Your CAD may have a function or option labelled something like “EXTRACT CROSS SECTIONS FROM DTM”. Select the function and box define the cross sections to be 20m offset to left and right of the centreline, and enable a cross section being drawn at every TP (and if relevant a cross section drawn at every breakline).

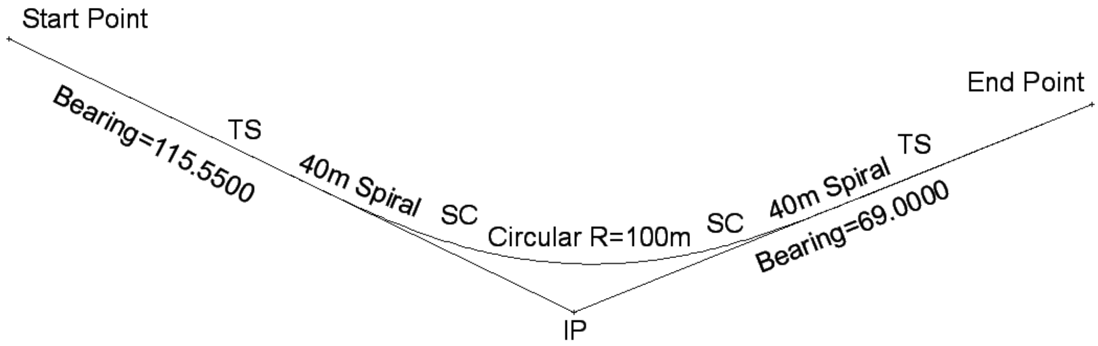
7. Look at the results.

Exercise 2: Long section and cross sections

This exercise will step through the procedures to create a road design surface that includes:

- Design template
- Fixed width batter
- Fixed slope batter
- Super-Elevation

You are required to design and plot the road as shown in the following figure.



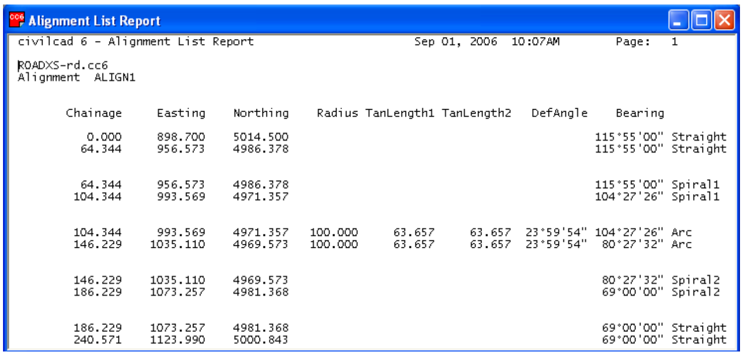
1. Use as separate survey data file as a basis for the natural surface. Create a new layer **ROAD** and make it current. Turn off all other layers.

Horizontal Alignment

2. The horizontal alignment is to begin at a point with coordinates **898.700E, 5014.500N** and end at coordinates **1123.990E, 5000.843N**. Insert the start and end points.
3. Intersect the intersection point (IP) with the bearings given in the above figure. Create the lines that link start point, IP and end point.
4. Create an alignment called **ALIGN1** from these lines. NOTE: The alignment direction must be from left to right.
5. Select the alignment and edit its IP properties with the values given below.

Design Speed:	50km/h
Circular curve radius:	100m
Entry spiral length:	40m
Exit spiral length:	40m
Other values:	default

6. Calculate and draw the alignment with specified spirals and arc.
7. List the details of the alignment, including the TPs in a report or file e.g. →
8. The report shows the straights, spirals and the circular curve parameters. Compare the values in my figure with yours. Any differences?



Alignment List Report

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ROADXS-rd.cc6
Alignment ALIGN1

Chainage	Easting	Northing	Radius	TanLength1	TanLength2	DefAngle	Bearing	
0.000	898.700	5014.500					115°55'00"	Straight
64.344	956.573	4986.378					115°55'00"	Straight
64.344	956.573	4986.378					115°55'00"	Spiral1
104.344	993.569	4971.357					104°27'26"	Spiral1
104.344	993.569	4971.357	100.000	63.657	63.657	23°59'54"	104°27'26"	Arc
146.229	1035.110	4969.573	100.000	63.657	63.657	23°59'54"	80°27'32"	Arc
146.229	1035.110	4969.573					80°27'32"	Spiral2
186.229	1073.257	4981.368					69°00'00"	Spiral2
186.229	1073.257	4981.368					69°00'00"	Straight
240.571	1123.990	5000.843					69°00'00"	Straight

Cross Section Extraction

9. Create the natural surface cross sections along the alignment ALIGN1 with spacing of 15m. Include TPs to create a cross section at each TPs.
10. List the alignment details again for ALIGN1.

Alignment List Report

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ROADXS-rd.cc6
Alignment ALIGN1

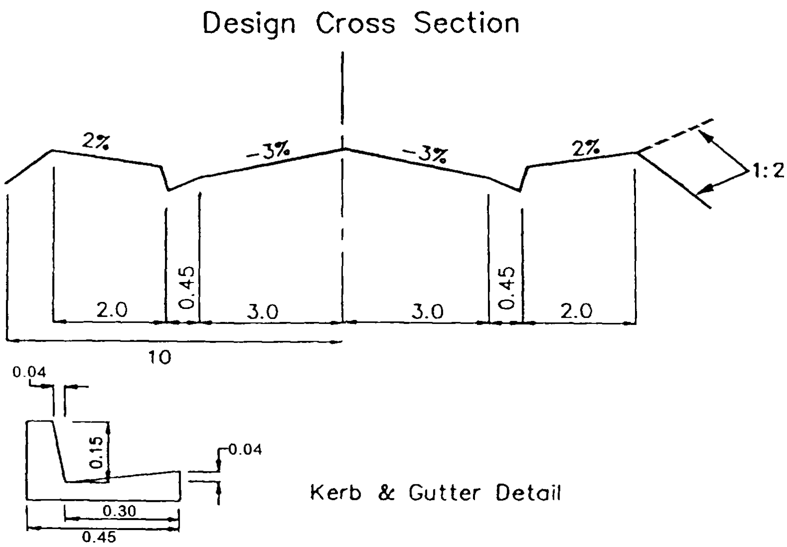
Chainage	Easting	Northing	Radius	TanLength1	TanLength2	DefAngle	Bearing	
0.000	898.700	5014.500					115°55'00"	Straight
15.000	912.191	5007.944					115°55'00"	Straight
30.000	925.683	5001.388					115°55'00"	Straight
45.000	939.174	4994.832					115°55'00"	Straight
60.000	952.666	4988.276					115°55'00"	Straight
64.344	956.573	4986.378					115°55'00"	Straight
64.344	956.573	4986.378					115°55'00"	Spiral1
75.000	966.179	4981.766					115°06'12"	Spiral1
90.000	979.941	4975.804					111°12'08"	Spiral1
104.344	993.569	4971.357					104°27'26"	Spiral1
104.344	993.569	4971.357	100.000	63.657	63.657	23°59'54"	104°27'26"	Arc
105.000	994.205	4971.195	100.000	63.657	63.657	23°59'54"	104°04'53"	Arc
120.000	1008.972	4968.648	100.000	63.657	63.657	23°59'54"	95°29'13"	Arc
135.000	1023.955	4968.337	100.000	63.657	63.657	23°59'54"	86°53'34"	Arc
146.229	1035.110	4969.573	100.000	63.657	63.657	23°59'54"	80°27'32"	Arc
146.229	1035.110	4969.573					80°27'32"	Spiral2
150.000	1038.817	4970.266					78°24'01"	Spiral2
165.000	1053.302	4974.135					72°13'39"	Spiral2
180.000	1067.438	4979.146					69°16'40"	Spiral2
186.229	1073.257	4981.368					69°00'00"	Spiral2
186.229	1073.257	4981.368					69°00'00"	Straight
195.000	1081.445	4984.512					69°00'00"	Straight
210.000	1095.449	4989.887					69°00'00"	Straight
225.000	1109.453	4995.263					69°00'00"	Straight
240.000	1123.457	5000.638					69°00'00"	Straight
240.571	1123.990	5000.843					69°00'00"	Straight

The report shows chainages from the start point to the end point in 15m interval and at all the TPs in the horizontal alignment. Compare your answers with those above.

<u>Design Cross Section</u>

11. Save your work.

12. Create a New cross section design surface template TEMPLATE1 using the values given in the figure below.



13. Nominate / select TEMPLATE1 to be used for the creation of the design surface of the road. 'Compute' the design. Set Vertical Exaggeration to 5 or 10 for a better view on the cross section profiles.

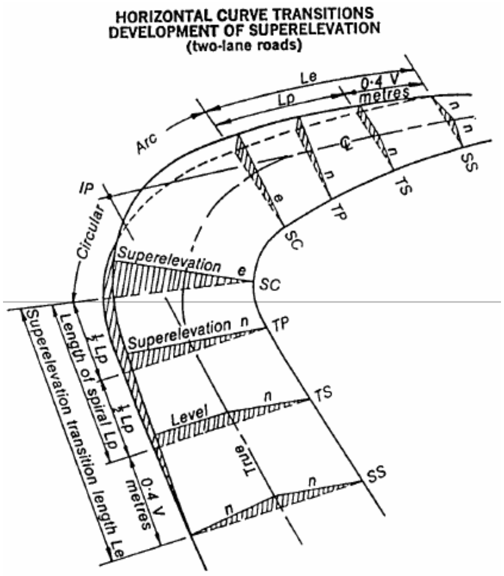
<u>Batter Template</u>

14. Batters on the left side are to extend to a width of 10m from the centre line. Re-Compute the road design. Then select each chainage (e.g. in the Long Section window) to view different cross section design surface profiles. You'll notice the changes at the left batter.

15. Batters on the right side are to be fixed slope at 1 in 2 for both cut and fill. Re-Compute the road design. Then Click on each chainage in the Long Section window to view different cross section design surface profiles. You'll notice the changes at the right batter.

Superelevation

16. The diagram below gives the specifications for the super-elevation to be applied. There is to be no widening of the road.



LEGEND

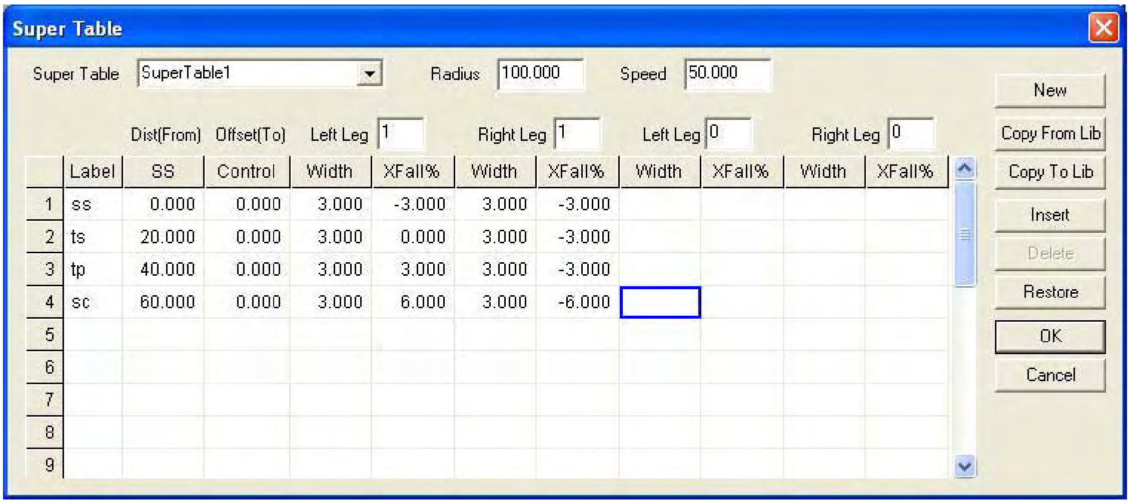
IP	Point of intersection of the main tangents
TS	Tangent spiral, common point of tangent and spiral
TP	Tangent point. Common point of tangent and circular curve
SS	Start of superelevation transition
SC	Spiral curve, common point of spiral and circular curve
L_p	Length of spiral between TS and SC
L_e	Length of superelevation transition
n	Normal pavement crossfall—tangent of angle
e	Pavement superelevation—tangent of angle
V	Design speed (km/h)

The superelevation transition commences at the point SS, $0{\cdot}4V$ m from the TS along the straight, and ends at the point SC. The pavement crossfall on the outer lane changes uniformly from normal, n, sloping towards the outside shoulder of the curve, to a superelevation n (the same crossfall as the normal section) sloping towards the inside shoulder of the curve, over the distance $0{\cdot}4V+\frac{1}{2}L_p$ from SS. For the remainder of the transition the crossfall changes uniformly on both traffic lanes to the maximum value e as calculated.

17. Set Super Elevation values: Radius = 100m and Speed = 50km/h.

Pt	Distance from SS	Note (distance calculation)
SS	00	
TS	20	= 0.4V = 0.4 x 50 (m) = 20 (m)
TP	40	= 0.4V+L/2 = 20+40/2 = 40 (m)
SC	60	= 0.4V+L = 20+40 = 60 (m)

e.g.



Why does the super elevation chainage start at 44.344? – The TS point is at 64.344 (see the alignment list report). The distance between SS and IS is 20m. So the SS point is at 44.344. The super elevation in the opposite of the arc starts from SC which is at 146.226 (see the alignment list report).

18. Re-Compute the road design. Then study each cross section profile.

Longitudinal Section

19. Create the longitudinal section design using the values given in the table below for the chainages and RLs (reduced level / heights) of the IPs. Use default values for other parameters.

Chainage	Reduced Level	Vertical Curve Length
0.000	49.755	0
124.213	49.020	60
207.824	53.109	50
240.571	55.711	0

20. Before computing the design, please look at the cross section window. Then compute the road design. Notice the changes of the gap between design and natural surfaces. Applying the vertical alignment is an important step in the overall road design.

Volume Calculation

21. Compute Volumes. The net volume (cut-fill) should be small enough to minimise the earth work, say < 5m^3. If it is not within this range, raise/lower an IP to adjust the net volume to be smaller. Then recompute the road design and recompute the volume. Repeat the procedure if necessary.

Plotting

22. Plot the Cross Sections and experiment with some of the style options.

23. Plot the Long Section and experiment with some of the style options.

24. A kerb return is a long section profile of a kerb usually around the intersection (corner) of two roads or the loop around the head of a cul-de-sac. It is used for drainage design, for example where to place gully pits. One way to create a kerb return profile is to create an alignment for the kerb line in addition to the alignment for the road centreline. Then produce a long section similar to the method used for the centreline.

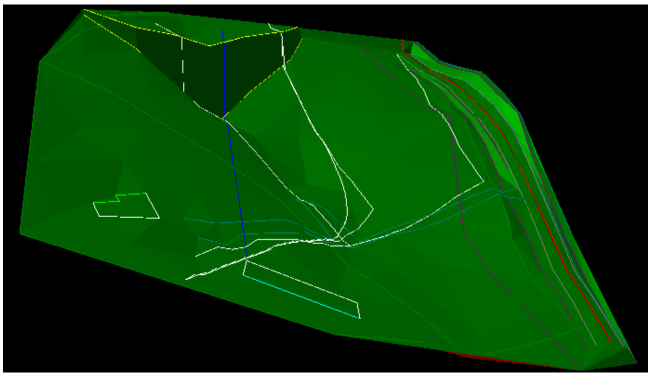
How do I design a kerb return? *By Michael Spiteri (one of our graduates). Source ISNSW Azimuth June 2007.*

The main aim of kerb returns is to make a smooth connection between two roads. The steps in designing a kerb return are:
1. From the design information determine the grades of the kerb in and out of the return and the levels of the kerb at the tangent points of the return.
2. Spilt the kerb return into 4 equal segments.
3. Draw a long section along the kerb return.
4. Fix the levels of the tangent points of the return as the levels determined in step 1.

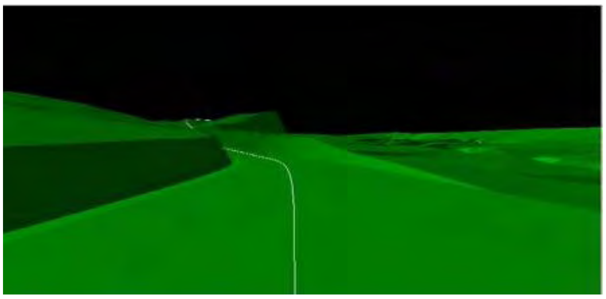
5. At the next segment points from either tangent point of the return, determine the levels of the kerb by extending the grades as calculated in step 1.
6. Create vertical curves over the two points in step 5 with their lengths equal to half the length of the total kerb return.
You will now have a kerb return design. This design may need to be modified to achieve low points at one of the tangent points of the return or to acquire minimum or maximum grades as stated by council in their design manual. This may be achieved by raising or lowering the levels of the segment points as determined in step 5 so that there is a maximum 1-1.5% difference between the grades calculated in step 1 and the new grades determined within the design. If it appears that this will not help it may be required to extend the kerb return design further along each road and insert a vertical curve before each tangent point or fit a bigger vertical curve for the whole kerb return. .

3D views and fly-throughs

Some CAD programs have a facility to see 3D views of a site that contains a DTM from a detail survey. If there is a road alignment on the site then it can provide a fly-through view along the road. Alternatively, the CAD data can be exported to other software packages specially designed for 3D views of a site from various positions or fly-throughs along a specified route e.g. a road alignment.



Changing the vertical exaggeration can be useful. This allows the user to increase the vertical scale and see the topography more clearly.



9.3 The main steps of ‘roads’ in CAD

To calculate the plan view and coordinates of a new road (e.g. for set out purposes):

1. Enter the coordinates of the main intersection points; join the points to form straights and tangents.
2. Convert the lines to a road alignment then add the circular and transition curves. (This process is different to adding curves to lines as done in the Subdivision CAD exercises.)
3. Add points at even chainages and at TPs along the road.
4. Export the point coordinates to a file that can be uploaded to a total station or GPS for road setout.

To design a road and prepare cross sections and longitudinal section plans, on a site that has already been ‘detail surveyed’:

1. Create the DTM (Mesh and Contours)
2. Create the alignment (straights, curve radius, superelevation, spiral length …)
3. Create the natural surface cross sections from the DTM, at the required chainages.
4. Design the road surface’s Cross Section, by creating a template with slopes and widths from the

centreline.

5. “Compute the Design” to see the cross sections with natural surface and road surface.
6. Apply superelevation and lane widening, if desired. Then “Compute the Design” again.
7. Design the long section by setting the chainage and height of IPs (i.e. changes of slope and lengths of vertical curves).
8. “Compute the Design” again, this will show you “cut and fill” gaps between natural and road surfaces in the cross section views.
9. Compute Volumes. If cut and fill not balanced then change the heights of road IPs or length of vertical curves or slopes of road centreline. Iterate this process.
10. Cross Section Plot. Set the text fonts, title block etc and produce the cross section drawing for each chainage onto final plan forms that can be saved or printed out.
11. Long Section Plot, similar process to above. It shows length and location of curves, grades of slopes and for each chainage the RL of the natural surface and the road surface.

In these exercises step by step instructions were supplied. At the successful completion of all the steps you might feel that you don’t understand, and can’t remember, what you did. However, our aim was for you to gain some experience, not complete mastery of the topic. In a later course students at UNSW use CAD to calculate the coordinates of points on a road and then set it out in the field. After setting out the road, you will measure a DTM of that site in the field. Then you will use CAD to produce cross section and long section plans. With the additional field experience and using these notes as a guide for your processing, we then expect your education of these matters to improve.

After using CAD for these tasks a number of times with different data sets, and if you take some initiative to explore other options in CAD and read some of the manuals, then you will be better educated. You should also then be able to learn how to use other surveying CAD software.

10. THE FINAL CHAPTER

This chapter commences with a discussion of some educational aspects of the course Survey Computations & CAD and an overview of the course. The next section includes problem solving methods. That is followed by a section on challenging survey computations problems with worked solutions to some of the problems.

We have considered how to teach CAD, knowing that different students prefer different styles. Should we give students detailed step by step instructions? Were too many steps described in earlier chapters? Should we give the problems, an overview and summary of the main steps of CAD, and let students work out how to do them? Over the last few years we have reduced the number of fine detailed steps and added summaries. We have also begun to explain the overall problems more at the start so that students can use alternative software if they wish.

Some previous students have been frustrated when some of their attempts using CAD have not been successful. Perhaps the software was at fault, or the way it was installed or setup, or perhaps they have omitted some steps in the instructions. In any case, good tutors with experience may be needed to help "debug" the problems.

If your learning in this course has been mainly from teacher input e.g. following step by step instructions, then many tasks may have been achieved quickly but perhaps not always understood and you feel dependent on teacher's material. This is easier for beginners.

Student led learning (e.g. use of reference books, websites, program Help functions and to explore menu options) usually yields less quantity but the quality of learning is better. There is more independence and an ability to continue to learn after the course ends, but it requires maturity, time and effort.

One method we have adopted is to use repetition and reinforcement education. Students use CAD for simple detail survey plans in a previous course to this one. Then students use CAD for road calculations and plots in a surveying camp following this course, and use CAD for subdivision design in another course. Our aim is for you to understand it better each time you do it.

Some technical aspects of surveying do require thorough, careful progress through many steps, not a 'waffle' essay or newspaper article. We hope you are becoming thorough and careful with your survey computations and CAD work.

Inventing the Future

We started the course with some history of survey computations, so at the end of the course it is appropriate to think about the future.

What will computing be like in your future work life?
How will you keep up to date?
Why do some people keep up to date but others don't? Find some role models.
How often will you upgrade hardware and software?

There are many famous predictions that have gone wrong. You can find plenty by a web search. Here are some:

- I think there is a world market for maybe five computers. Chairman of IBM, 1943
- Computers in the future may weigh no more than 1.5 tons. Popular Mechanics, 1949
- I have travelled …this country and talked with the best people, and I can assure you that data processing is a fad that won't last out the year. Editor of business books for Prentice Hall, 1957
- But what ... is it [the microchip] good for? IBM, 1968

- 640K ought to be enough for anybody. Bill Gates, 1981. Bill again, later: We always overestimate the change that will occur in the next 2 years and underestimate the change that will occur in the next 10.
- Everything that can be invented has been invented. Charles H. Duell Commissioner US Office of Patents,1899
- There is no reason for any individual to have a computer in their home. Chairman of Digital Equipment, 1977
- This “telephone” has too many shortcomings to be seriously considered as a means of communication. The device is inherently of no value to us. – Western Union, internal memo, 1876.

Errors in forecasting are caused by omission of factors in the environment, distortion of conclusions because of personal factors, and errors in some core assumptions.

“Quite frequently, the things I speculate about already exist and I just didn’t know about them. I call it ‘predicting the present’ and it’s a very common thing in today’s technological environment.” Bruce Sterling

To forecast the future of survey computing:

Invent it. Build your own career

Ask a panel of independent experts

Forecast the future by analogy with the past, but the influencing environments must be comparable.

Use growth curves from past and present, begin by determining future needs then identify the technology required by those future needs

Consider the needs, constraints and ‘environment’ of the future

Applying Forecasting Techniques:
What are the big picture issues that will shape future technology in the spatial information sciences?
What new capability and functionality do you predict will be available to practitioners?
What specific advances to existing technology, or what new technologies, do you anticipate in the short, medium, or longer term?
What affect will these new capabilities have on the existing surveying, mapping, GIS community?

Survey computations future:
“Computer technology is changing, and will continue to change the way Surveyors collect measurements, process them, and analyse their data.”
How can graduates cope with changes in computer technology and keep “up to date” during their careers?
How do other people keep up to date?
What motivates some people to keep up to date, and what scares some people?
Research it, then reflect (think!) on your future career

An example of future survey computations future?
Detail surveys by GPS or robotic total station, measures your pole coordinates (that can already be done). CAD draws your plan on your screen as you go (simple versions already exist). Someone else walks the field and transmits data via mobile phone to your computer screen…

10.1 Summary of Course Topics and Important Points

Principles of Survey Computations
- Know that your answer is right
- Do checks
- Be well organised

Intersection & Resections
- Intersection by solving triangles or by formula
- Coordinate based formula
- MS Excel formula

Missing Data & Traverse closes
- Data from plans etc, with 1 or 2 values 'missing'.
- Triangle solutions, or Traverse Misclose Coordinate equations, or …

Traverse adjustments
- Carry bearings or not?
- Check the miscloses are acceptable
- Then find mistakes or adjust the traverse

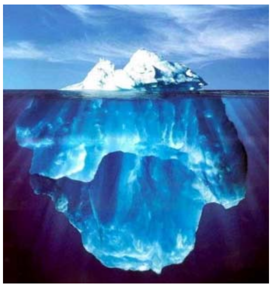
CAD
- Which brand?
- Upload coordinates to instrument
- Detail survey measurements + record codes
- Download and draw plans
- TIN, Contours, Breaklines
- CAD for surveying calculations

Subdivision design in CAD
- Design and draw roads and lots
- Bearings and distances, curves, areas

Road design and calculations
- Horizontal & Transition curves by formula or CAD
- Vertical Curves
- TIN → Cross section & Long section plots

Without using the detailed step by step instructions, can you use CAD to do a simple detail survey plan with contours, a subdivision plan, and a road alignment? I expect a few students can but not all. With detailed instructions you would be faster, but without the instructions most of you could do it given the software and enough time to explore the menu options.

What have you learnt by yourself?



What comes next? In this course our survey computations have used the minimum number of observations needed to calculate answers. We have learned about checking the calculations but not about taking more (redundant) observations as checks and to improve the quality of our answers. In later courses you will learn about computations using Least Squares with redundant data to get best fit answers and some information about the precision of your answers. You will also learn about calculations in the real world that is not flat earth and not a vacuum, things like map projections, in other courses. And there is plenty more after that too ☺. Perhaps we have just looked at the tip of the iceberg.

10.2 Problem Solving

In this section we summarise some of the problem solving methods that are used in computer programming and mathematics and then apply them to Survey Computations problems.

Problem resolution in the workplace: see cartoon in lecture.

Difference between Focusing on Problems and Focusing on Solutions - Case 1
When NASA began the launch of astronauts into space, they found out that the pens wouldn't work at zero gravity (ink wouldn't flow down to the writing surface). To solve this problem, it took them one decade and $12 million to develop a pen that worked at zero gravity, upside down, underwater, on practically any surface including crystal and in a temperature range from below freezing to over 300°C. And what did the Russians do...?? They used a pencil!

Moral: Always look for simple solutions. Devise the simplest possible solution that solves the problem. Focus on the solutions and NOT the problems.

Why learn computer programming?
Computers can make work easier and more efficient by: data collection, calculations, presentations. Learning to program helps: build computer confidence, understanding of commercial programs, develops a systematic approach to problem solving. Programming is a specialized form of problem solving. The best ways to learn programming in my opinion are practice and looking at textbook examples.

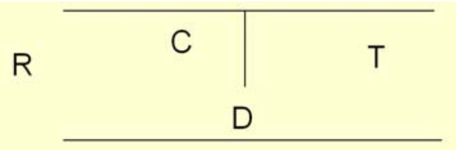
Problem solving process		Steps in Building a Computer Project
Understand the problem		Define the Problem
Devise a plan for solving the problem		Design and Plan
Carry out the plan		Build the Program
Evaluate the solution		Run the Program
Changes?		Test and Debug
		Document Your Program
		Compile and Distribute Program

Problem Solving by Subdivision
Problem
Major steps
Fine detail instructions
Algorithm

Problem Solving by Divide and Conquer
If you have a big problem, divide it into sections, look at one section at a time
Examples: fixing errors in programs; errors in survey networks
What has to be done to go from input to output?

In class group exercises:

Problem 1: Robot control
This is similar to measuring bearings and distances in surveying and similar to plotting on paper or computer screen to draw a map. Problem: write the instructions for a robot, robot to get cereal, take it through door and put on table, i.e. go from R to T via C and D.



Problem 2: Changing a car's flat tyre
Students work on this in pairs and then report results. The reason for working in pairs is to see how other people look at solving a problem. What did you learn about the process of problem solving while doing it? Fine detail steps out of order? Thought of extra things as you went along? Did you remember to stop the car?

Problem solving in survey computations
First try to understand the survey problem. Consider: Input available, Output desired, and then what has to be done to get from input to output.

Also consider:

- why should you solve the problem?
- what effect will it have on other people?
- are there alternative solutions or methods?

10.3 Surveying Geometry Problems

Questions 1 to 12 in this section are based on material supplied by George Baitch as edited by Bruce Harvey. George wrote several articles in the Azimuth magazine (ISA NSW) a few years ago. His articles were modern versions of material originally in a delightful book called "Survey Computations, A Compilation of Questions and Solutions" by R.B. Horner, MNZIS, FRGS, Malayan Survey Service, First published in by the Survey Department; Federation of Malaya, 1948 (now out of copyright). It contained 243 surveying puzzles that were posed and solved for the edification of Malayan cadet surveyors. The book is probably in the UNSW library, and I have a copy if you wish to look at it.

Part of Mr Horner's Introduction is shown below:

"*While many of the solutions supplied must appear rather elementary in conception, it should be explained that most were evolved during my own youthful training when rough bush camps formed one's only study and the great stillness one's only tutor. As the book is intended for trainees' use, it was felt that to leave solutions in their original form would best serve their present purpose. It cannot be too strongly recommended that users of this compilation should tackle each problem by their own methods in the first instance as most of those supplied are capable of several methods of solution, many of which will be found more direct than my early efforts.*

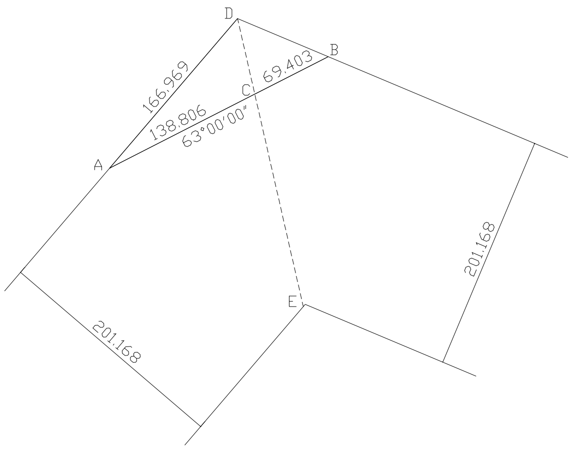
In most of the diagrams the firm lines represent the diagram as given in examination papers, the broken lines being my own constructional additions. Students should endeavour to work on each problem bearing this fact in mind, for often an apparent "teaser" is transformed into one of very easy solution by the addition of a few construction lines. An old Surveyor once advised me "if you cannot solve a problem, lad, plot it".

Not all of Horner's original problems 'puzzles' are included here. Those included are the ones that require the application of some mathematical, trigonometrical or geometric principle (or as Martin Gardner from the Scientific American used to call Mathemagical solutions). Note that Horner and his students did not have Excel or CAD software, or computers, or even calculators with trig functions. They would have used log and trig tables. We have converted the data from feet, links, chains and acres to equivalent metric values.

Question 1:

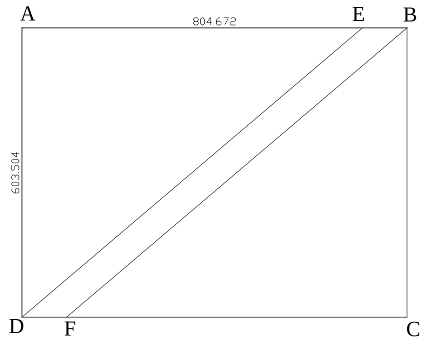
A boundary line runs across the road reserve as shown below. From the data supplied, find the bearings of the road sides and the length of BD.

After publication in Azimuth, Question 1 had a range of responses from all over Australia. All arrived at the correct answer, yet every one of them tackled the problem slightly differently.



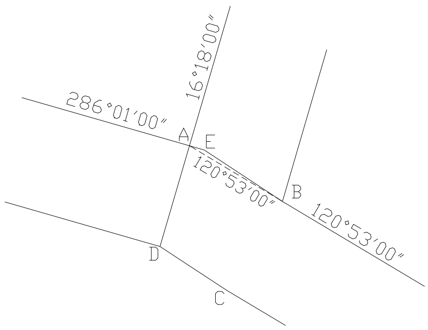
Question 2:

A road reserve, 60.350m in width, runs through the rectangular lot as shown on the diagram. From the data supplied (AB=804.672m, AD = 603.504m) compute the distances AE and ED.



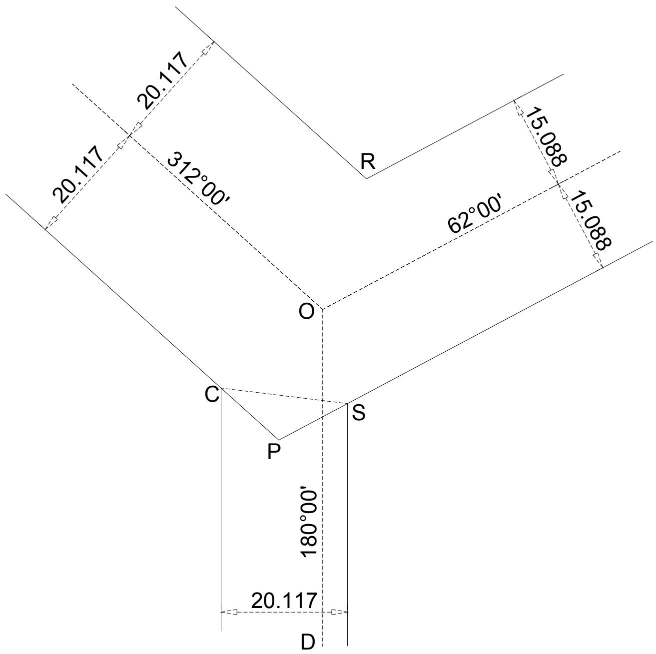
Question 3:

The road lines shown below form a reserve 20.115m wide in each case including EB and DC. Find that length of AE, the bearings and distances of EB and DC and the half angle lines ED and BC.



Question 4:

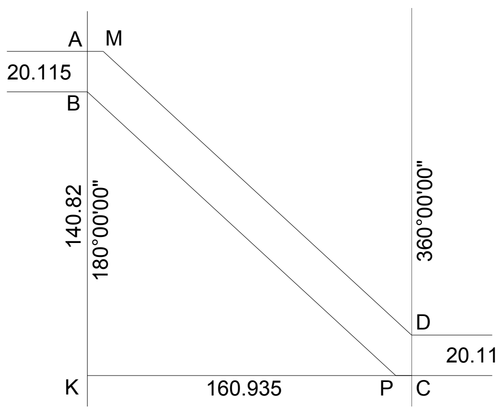
From the data on the diagram and given that the road reserve lines are in each case parallel to the



(dotted) traverse lines, find OC, OS, OR, CP, and the offsets from traverse-line OD. Given CP = PS.

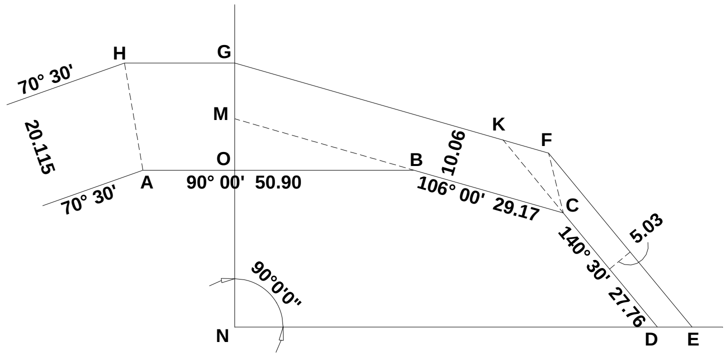
Question 5:

It is required to run a new 20.115m wide road reserve to connect the existing 20.115m wide road reserves at A B and C D where K B is 140.82m and K C is 160.935m. Find the bearing of the new lines and the lengths resulting.



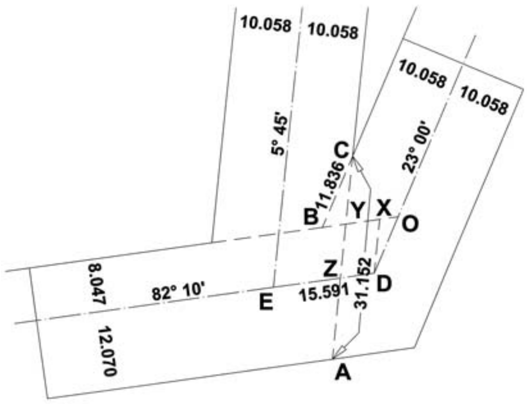
Question 6:

From the data on diagram, and assuming all opposite road sides to be parallel, compute the lengths of H G, G F, F E and the tie - line C F.
Check your answers carefully, as each solved triangle builds to produce the overall solution and cumulative errors can creep in. Question 6 is a particularly sensitive question to rounding effects.



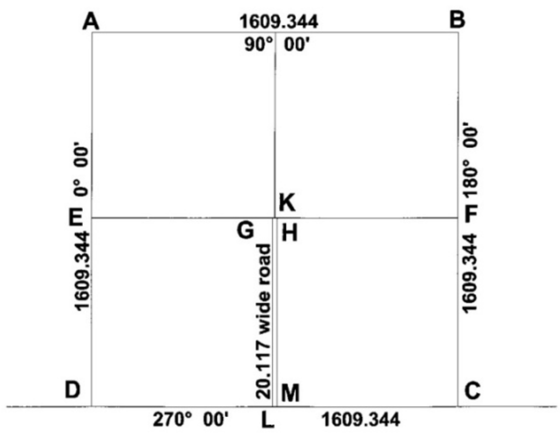
Question 7:

Three parallel sided road reserves meet as represented on the diagram. The two easterly reserves have centre traverses, while the other has its traverse 8.047m & 16.093m from the sides. Given that the measured distance of E D is 15.591m, find the distances of B C and A C.



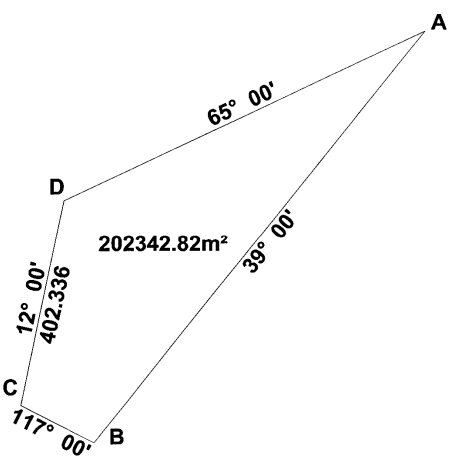
Question 8:

The block A B C D is to be subdivided into 4 equal areas by parallel lines, leaving the access road vertically from the centre of the road frontage to give access to the two northern lots. Find B F = A E and the area of each subdivision.



Question 9:

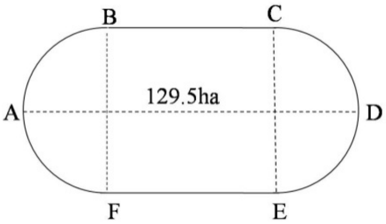
In the diagram, the figure A B C D contains 202,342.82m^2 (50 acres) and CD is 402.336m (was originally 2000 links). From the data supplied, find the lengths of the boundaries A D, A B and B C.



Question 10:

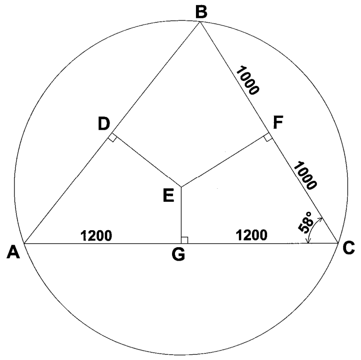
In the figure, A B C D E F represents a racecourse comprising two semicircles of equal radius and 2 sides of a square.

The area of the course is 129.5 hectares. Find the perimeter and length of extremities A - D.



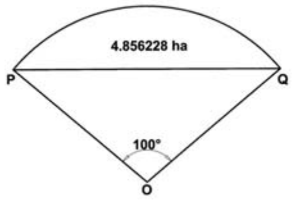
Question 11:

From the data on the diagram find A D and D E.



Question 12:

The segment shown in the diagram contains 4.856228ha (12 acres). From data shown, find the radius of the arc PQ.



The following questions are not from Horner's book. The methods to solve them are not explicitly given in this book. That is intentional. Enjoy the challenge.

Question 13:

A logic puzzle with a surveying theme was found at: *www.iowasurveyor.com/puzzles/Surveyor_Logic_Puzzle.pdf* I found it interesting and challenging, and solved it. So I reproduce it here for your educational benefit.

Five surveyors worked in a growing town. One was employed with the city while the other four worked for private companies. You could always tell who had surveyed a property because each surveyor preferred to use a different colour of ribbon on his stakes and also used different monuments. Determine where each surveyor was employed, which size & type of monument he used and his ribbon preference.

1. Charlie, who didn't work for the City, didn't use pipes. The Survey Solutions surveyor didn't use the ½-inch monument. Bob, who didn't work for Rocket Land Surveying, used rebar, but not the 1-inch size.
2. Ed, who didn't use rebar, used a larger monument than the surveyor who used blue ribbon.
3. The two ¾-inch size monuments were a pipe and the monument set by Blue Moon Surveying. Archie from Survey Solutions didn't use rebar. Doug used a ¾-inch monument, but it wasn't a pipe.
4. One of the surveyors who used rebar also used green ribbon. The City surveyor used rebar.
5. The two surveyors who used pipe were the one who also used orange ribbon and the Zodiac Engineering surveyor. One surveyor's rebar was the ½-inch size.
6. The surveyor who used pink ribbon also used rebar, but not the ½-inch size. The square bar wasn't set by the surveyor who used yellow ribbon.

Surveyor	Employer	Ribbon Colour	Monument Type	Monument Size
Archie				
Bob				
Charlie				
Doug				
Ed				

Question 14:

Determine the true length of the shortest distance between point C, and the line AB. Data:
C (30,30,0) [E N H in m], A is (0,0,10) and true length (slope distance) AB = 70m, bearing AB = 70°, height B = 45m.

Question 15:

Calculations are required for a hypothetical rescue of people, including the site's mine surveyor, who are trapped underground in a NSW south coast mine. For the purposes of this question assume we are to drill directly to their location with no offsets and that coordinates are in a local plane coordinates system with no map projection scale factors. People are located at P and the best estimate of their location, based on nearby wall markings and their last known location, are given below. There is a nearby 'tunnel' containing a centreline traverse with marks at A and B. The coordinates of A and B are also given in the table below.

Pt	East	North	Height
P	2164.	7793.	9223.
A	1982.214	7475.257	9209.691
B	2192.161	7882.103	9232.40

One option is to drill to P from the nearest point along the line AB, which is a point C. Determine the coordinates of C, assuming it is on line (in 3D) between A and B. What are the bearing, zenith angle and distance from C to P?

Question 16:

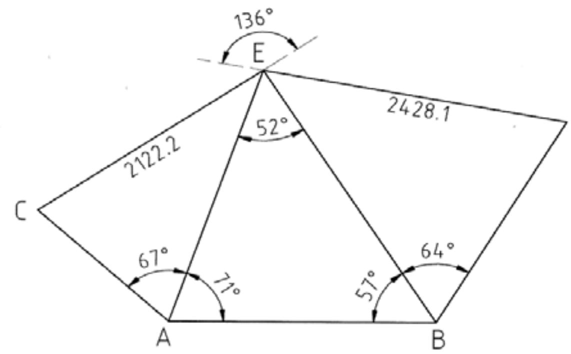
As part of the design of a new structure two straight pipes are to be placed near each other. It is important that the two pipes are not too close to each other. The coordinates of the two ends of the centreline of each pipe are given below.

	From			To		
	E	N	H	E	N	H
Pipe A	16.5	20.0	10.0	11.0	34.0	2.2
Pipe B	10.5	24.5	1.9	17.5	37.5	9.0

Calculate the shortest distance between the centrelines of the two pipes. Calculate the coordinates of the two end points of this shortest join line.

Question 17:

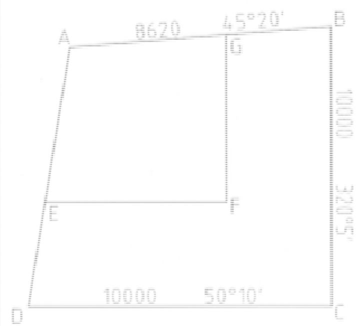
This question comes from the 1910 NSW Surveyors Registration exams. It was reprinted in NSW Azimuth Magazine, Feb 2010. Note that students in 1910 did not have electronic calculators or computers to assist them with this problem.



From the data shown above find the length of AB. (The units of distance here are links, but that is not an issue).

Question 18:

This question comes from the 1912 NSW Surveyors Registration exams. It was reprinted in NSW Azimuth Magazine, Apr 2012. Note that students in 1912 did not have electronic calculators or computers to assist them with this problem.



The figure shows the given bearing and distances (in links) AB, BC and CD. Given area of AGFEA = 320 acres. Given that line GF is parallel to BC and EF is parallel to DC. Require distance EF = FG. Find distances AG, GF, FE, EA

10.4 Solutions to Surveying Geometry Problems

Question 1:

This is a metric version of Horner's Q1 p 11. I show the results in both metric and original units, for comparison.

	Metric	Original
AD	166.969	830
AC	138.806	690
CB	69.403	345
Brg AB	63	63
w	201.168	1000

Angle ADE = Angle BDE

So AC/CB = AD/DB by Euclid

	metric	original
DB = AD * CB /AC =	83.4845	415

Cosine rule on triangle ADB because we now know all side lengths

cos A = $(b^2+c^2-a^2)/2bc$ = 0.924 A = 22°26'56.5"

brg AD = 40°33'03.5"

Cos B = 0.646 B = 49°47'36.5"

brg DB = 112°47'36.5" These answers agree with Horner's answers.

Analysis of Question 2:

Mr Horner prepared these questions before metrication, and had the advantage of expressing the distances as AD = 3000 links and AB = 4000 links (= 30 and 40 chains respectively). I do not apologise for not alerting you to this fact. However, in the event that you did notice this, it made the calculation of the diagonal *DB* almost trivial (a bit of Pythagorean magic!). The road width was 300 links.

Mr Horner said: *"A triangle having sides in the proportion 3:4:5 is a right angled triangle. Conversely, if a right angle is contained by sides which are in the proportion of 3:4, then the hypotenuse must be in proportion to 5. Thus in this case, by inspection, see that BD is 50 chains. (or in metres 1005.84m). Drop a perpendicular from D to BF produced at X. Then in the right angled triangle DBX we have two known sides. Compute the angle DBX = 3º 26' 23" and from the triangle DBC compute angle B = 53º 07' 48".*

Then the difference between these two angles is equal to angle *FBC* = 49º 41' 25", so that the angle *ABF* will be 40º 18' 35". The required distance may now be found by right angled triangles.

Results: AE = 711.383 and ED = 932.890 and EB = 93.289.

Our graduate Paul Kew presented a significantly different approach from Mr Horner as follows.

Draw a line from E perpendicular to line BF until it intersects at G.
EG = 60.350m (Road width)
DE is parallel to FB (Road reserve is a constant width)
∠AED = ∠ABF (Corresponding angles are equal)
Sin∠AED = AD / ED (Sine ratio in ∆DAE)
Sin∠AED = 603.504m / ED
Sin∠ABF = EG / EB (Sine Ratio in ∆EGB)

$\sin\angle ABF = 60.350m / EB$
$EB = 60.350m / \sin\angle ABF = 60.350m / \sin\angle AED$ ($\angle AED = \angle ABF$) = (60.350m) ED / 603.504m
AB = AE + EB
804.672m = AE + (60.350m) ED / 603.504m
AE = 804.672m - (60.350m) ED / 603.504m Equation 1
$ED^2 = AE^2 + AD^2$ (Pythagorean Theorem in ΔDAE)
$ED^2 = AE^2 + (603.504m)^2$ Equation 2
Substituting Equation 1 into Equation 2
$ED^2 = (804.672m)^2 – 2 (804.672m) (60.350m) ED / 603.504m + (60.350m)^2 ED^2 / (603.504m)^2 + (603.504m)^2$
$0.99 ED^2 + (160.933m) ED – 1,011,714.106m^2 = 0$
Solving the quadratic equation
$ED = \{ -160.933m \pm \sqrt{[(160.933m)^2 – 4 (0.99) (-1,011,714.106m^2)]} \} / [2 (0.99)]$
ED = -81.279m ± 1,014.169m = -1095.449m or 932.890m
ED = 932.890m (Distances are positive)
Substituting back into Equation 1
AE = 804.672m - (60.350m) (932.890m) / 603.504m = 711.384m
Substituting back into Equation 2
$ED^2 = AE^2 + (603.504m)^2$
$(932.890m)^2 = AE^2 + (603.504m)^2$
AE = ± 711.384m
AE = 711.384m (Check) (Distances are positive)

Solution to Question 3

Horner's published solution is: *A D is the cosecant distance (= 20.119). From the triangle A B D compute B D. Drop a perpendicular from D to E B. Then in the triangle D B X find the angle at B and from this deduce the bearing of B E (= D C). Take out secant half-angle distances E D and B C. Compute length of E B (from the right angled triangles B D X and E D X) from which, by applying tangents, D C may be found. The length of A E may be computed from either triangle E A D or E A B.*

Line	Bearing	Distance
AE		2.959
EB	123º 18' 30"	17.942
DC	123º 18' 30"	15.309
ED	204º 39' 45"	20.348
BC	212º 05' 45"	20.121

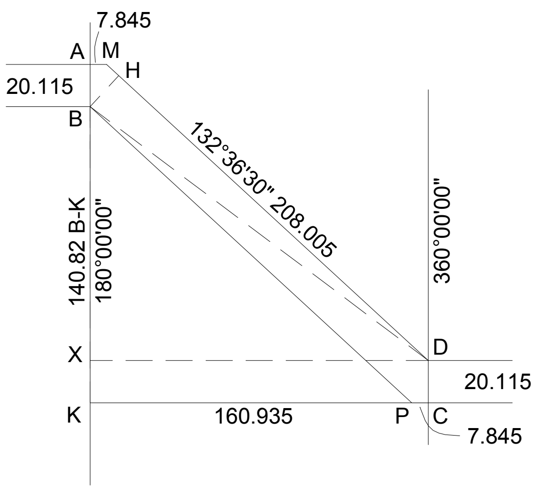
This question did not require as much mathemagics, rather an appreciation that the angle formed at A was not a right angle and formed the basis for most further computations.

Solution to Question 5

Horner's published solution is: Join B D and produce the old road line from D to meet B K at X. Then B X = 120.705.

Now in the triangle B D X, B X = 120.705 *(6 chains in Mr Horner's original puzzle)* & D X = 160.935 *(8 chains)* so that, being a right angled triangle, B D must be 201.170 *(10 chains). This is by observation being the Pythagorean 3 4 5 triangle for the more alert!*

The angle K B D is one whose tangent is $\frac{160.935}{120.705}$ or $\left(\frac{800}{600}\right)$ or 1.333 so that this angle is 53°07'50" and the bearing of B D = 128°52'10".



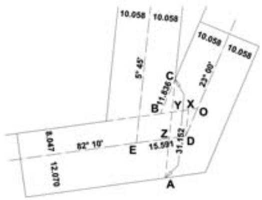
Drop a perpendicular from B to M D at H. Then the sine of the angle B D H $\frac{20.115}{201.170}$ or $\left(\frac{100}{1000}\right)$ = 0.10 = 5° 44' 20" so that the bearing of D M = 132° 36' 30" = B P.
The angle E M D = 137° 23' = angle B P C.
Now H D = $\sqrt{201.170^2 - 20.115^2}$ = 200.160 and A M = M H = 100 cotan 68° 41' 30" = 7.845.
Thus M D = 200.160 + 7.845= 208.005= B P. And A M = P C = 7.845.

George apologises for the imperfect conversion from links to metres. This is an artefact of rounding off.

Solution to Question 6:

Produce C B to meet O G in M and produce D C to meet F G in K.
Then M G = 10.06 Cosec 74° (= 10.465). Thus M O = 9.650.
Solve the right-angled triangle M O B for M B and O B. Then A O = 50.90 - O B. (= 17.248), and H G is equal to 17.248 + cotan 80° 15' for 20.115. Thus H G = 20.704.
Now C M = 29.17 + M B (= 35.008). Thus C M = 64.178.
In the triangle K C F, KC = Cosec 34°30' for 10.06 (= 17.761) and F K = Cosec 34°30' for 5.03 (= 8.881).
Solve the triangle K C F for C F direct (= 346°13'11" , 11.591).
Then G F = M C - cotan at C (= 5.757)+ cotan at G (= 2.885) = 61.306.
And F E = C D + cotan at D (= 4.146) + cotan at C (= 10.442) = 42.349.
The starting distance was quoted as 5.03 when in fact it was 5.029 (0.001 less). The next road width was shown as 10.06, but really 10.058 (0.002 less). Both these distances were affected by the acute cotangents creating multiplier effects of factors of three. The final road width was shown as 20.115 which in reality is 20.117 (0.002 greater!) [A direct conversion of Mr Horner's chain].

Solution to Question 7:



First draw a diagram to scale.
Draw D X parallel to A C. Then D X is cosecant of 76º 25' for 8.047m, and E Z is cosec of same angle for 10.058m. And Z D = 15.591 - E Z, and X Y = Z D. Compute O X from triangle D X O, having one side & angles.
B O = cosec of' 59º 10' for 10.058 and thus the distance B Y = B O - (O X + X Y).

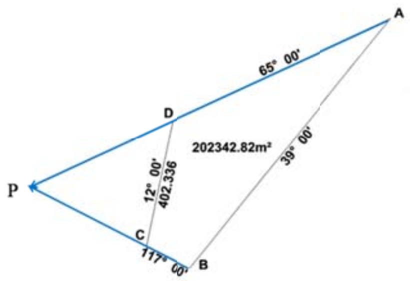
Solve triangle B C Y for B C and C Y.
Then A C = cosec A Y + C Y.
Results: B C = 11.836 and A C = 31.152

Solution to Question 8:

The block A B C D is to be subdivided into 4 equal areas by parallel lines, leaving the access road vertically from the centre of the road frontage to give access to the two northern lots. Find B F = A E and the area of each subdivision.

Let B F = x. Then F C = 1609.344 – x = E D. K F = 804.672.
Therefore H F = 794.614 = E G = D L = M C.
Then 804.672 x = 794.614 (1609.344 – x).
Thus 804.672 x + 794.614 x = 1278806.629.
Therefore 1599.286 x = 1278806.629.
Therefore x = 799.611.
So that B F = A E = 799.611.
And area of each subdivision area = 643424.7m².

Horner's solution to Question 9:



Extend the lines AD and BC to P. Calculate P by intersection of bearings, or trigonometry on triangle PDC

In ∆PDC	opp side by sine rule
P = 52°	p = CD = 402.336
C = 75°	c = PD = 493.174
D = 53°	d = PC = 407.761
	sum/2 = 651.636

$$Area_{PDC} = \frac{1}{2} c\, d \sin P = \sqrt{s(s-c)(s-d)(s-p)} \quad \text{where} \quad s = \frac{c+d+p}{2}$$

Area ∆PDC: 79,233.35 79,233.35 check by two formula

Area ∆PAB = given area + additional triangle = 202,342.82 + 79,233.35 = 281,576.17

Now, we know the area and all angles in the ∆PAB. If we call AB = q, PA = b and PB = a, then rearrange the formulas as follows

$$Area_{PAB} = \frac{1}{2} ab\sin P = \frac{1}{2} a^2 \sin B \sin P / \sin A \quad because \frac{b}{\sin B} = \frac{a}{\sin A}$$

$$So \quad a = \sqrt{\frac{2 Area_{PAB} \sin A}{\sin B \sin P}} \quad similarly \quad b = \sqrt{\frac{2 Area_{PAB} \sin B}{\sin A \sin P}} \quad and \quad q = \sqrt{\frac{2 Area_{PAB} \sin P}{\sin A \sin B}}$$

PB = a = 565.9338
CB = PB – PC = a – d = 158.173
AB = q = 1017.316
PA = b = 1262.781
DA = PA – PD = b – c = 769.607

These answers agree with Horner's original answers which were rounded off to 5 significant figures, converted to metric they are: AB = 1017.3m CB = 158.17m. Horner didn't have a computer or even an electronic calculator with trig functions!

Harvey's Excel solution to Q9:
This method uses MS Excel and establishes three equations to solve for three unknowns.
Label the 3 unknown distances: a = AB, b = BC, c = DA
Write one equation for area. It can be based on area by coordinates which includes a b and c terms, or split the figure into two triangles along a line DB and use the area = ½ absinC. Here we demonstrate the two triangle method. Our triangles have known angles at A and C.
The equation for area (remembering that angles must be converted to radians):

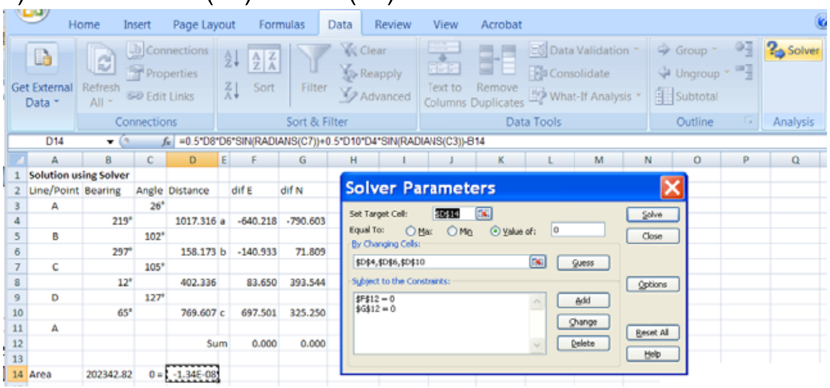
0 = 0.5*402.336*b*SIN(105) + 0.5*a*c*SIN(26) - 202342.82

Write another equation for misclose in E:

0 = a*sin(219) + b*sin(297) + 402.336*sin(12) + c*sin(65)

Write another equation for misclose in N

0 = a*cos(219) + b*cos(297) + 402.336*cos(12) + c*cos(65)



The equations are almost linear in a b c, (the first equation is not because of the a*c term). If the equations were linear we could solve them with matrix algebra with the coefficients of abc forming the A matrix, as $x=A^{-1}b$ where b = 0.

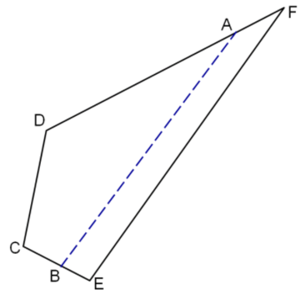
Our 3 equations with 3 unknowns can be solved using MS Excel's Solver and the results are shown.

The process in MS Excel is to enter the values as shown but to leave the cells containing the distances a b and c initially empty. Choose Solver (in the Data ribbon in version 2007). Set the target cell to be the area equation D14 (shown above as containing -1.3E-08) which is initially empty. The "changing cells" are the three empty cells for distances, D4, D6 and D10. Then add the constraints that sum of Eastings cell, F12 = 0 and sum of Northings cell, G12 = 0. After clicking the Solve button the values of a b and c appear in the cells as shown above.

Harvey's CAD solution to Q9

Concept: Create a lot that is bigger than required by placing a line parallel to AB but further from C. This requires an estimate of the distance CB, perhaps done graphically by measuring the figure if it is a scale drawing. Calculate the area of this large lot then subdivide the lot, parallel to the line AB to leave a lot of the required area. Then read the dimensions of the new lot.

Procedure: Enter point C at any coordinate, then line CD by bearing and distance. Add a line from C past B at 117° and at say 200m, call it E. Use COGO brg/dist intersection to create a point at the intersection of two bearings from D and E, call it F. Add the lines DF and EF by joining the points.



Create a lot CEFDC, its properties will include the enclosed area. If you set CE as 200m then the area is 245502.176. This means the area is 245502.176 - 202342.82 = 43159.356 too large.

Create a lot by subdivision to remove the surplus area. Select line EF so that the lot is parallel to this line. Enter the area of the surplus (43159.356 in this case) and a new lot will be created, save it and save the remainder as a lot too. This second, remainder lot has the required area of 202342.82.

Select the lines of interest and read their properties to obtain the missing distances, as follows:
CB = 158.173, BA = 1017.316, DA = 769.607.

Thus all three methods above have the same answers. They are independent methods will are useful checks, but also to show you there are sometimes diverse ways to solve a problem.

Horner's answer to Question 10:

Let the side BC = FE = BF = CE = x
Find the area of the circle (i.e.) two semi-circles, and that of the square in terms of x
Area of square = x^2
Area of 2 semi-circles = $\frac{x^2}{4}\pi$ = 0.7854 x^2
∴ Area of whole course in terms of' x = $x^2 + 0.7854x^2$ = $x^2(1 + 0.7854)$
∴ Total Area = 129.5 ha = $1.7854\ x^2$
∴ $x = \sqrt{\frac{1295000}{1.7854}}$ = 851.660m
∴ Radius of semicircle = r = 425.832m
Now Circumference of each semi-circle = πr = 1,337.788m
And total of two curves = $2\pi r$ = 2675.576m
Thus perimeter of course = 2675.576m + 2 x (851.660m) = 4378.900m
Length AD = 1703.324m

Answer to Question 11:

Since B F = F C = 1000, & E F is a perpendicular, then E B must equal E C.
Similarly E must be equidistant from A & C
Therefore A E = E C
Therefore E must be the centre of the circumscribing circle.
Therefore A D = D B= ½ A B
Now tan (B – A) / 2 = 4 / 44 cot 29° = 0.164004 = 9°18' 50"

And (A + B) / 2 = 61°
Therefore angle ABC = 70°18' 50" and angle BAC = 51° 41' 10"
Then AB = 2000 sin 58° x cosec 51° 41' 10" = 2161.662.
Therefore AD = DB = 1080.831.
Join BE. Then angle BEA is double BCA or 116°.
Hence angle EAB = EBA = 32°Thus DE = 1080.831 tan 32°= 675.378.

Though this puzzle appears very easy, a bit of quadratic mathematics and Pythagorean logic is necessary to find the same solution as Mr Horner. Please enjoy this small mental gymnastic.

Solution to 12:

Mr Horner solved this problem in a unique and very interesting way. He used a proportioning or trial method that is based on the principle of similar figures. When you are stuck trying to find a solution in the future, give this method a thought. It has great merit. His approach is shown below.

Let the required radius be 100.584m (5 chains). Then the area of the segment is equal to Area of Sector minus area of triangle O P Q.

The area of the sector = (100.584^2 x radian measure of 100°) / 2 = 8828.871m^2
The area of the triangle = (100.584^2 x sin 100°) / 2 = 3847.152m^2
Thus the area of segment = 3847.152m^2

Now:
$(\text{True Radius})^2 / (\text{Assumed Radius})^2 = (X^2/100.584^2) = (4856228\ m^2 / 3847.152 m^2)$
Therefore: (X / 100.584) = Square root (4856228 / 3847.152)
Thus *X* = 357.361m = Radius required.

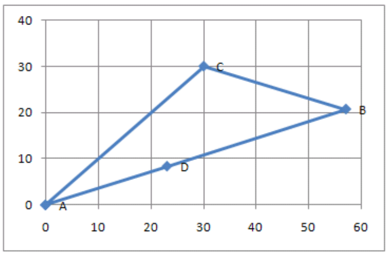
Solution to 13.

Surveyor	Employer	Ribbon	Monument	Size
Archie	Survey solutions	Orange	Pipe	¾ "
Bob	City	Green	rebar	½ "
Charlie	Rocket	Pink	rebar	1"
Doug	Blue Moon	Blue	Sq bar	¾ "
Ed	Zodiac	Yellow	Pipe	1"

Solution to 14.

There is more than one way to solve this problem. The main point of this question is to think of a solution method. My method follows. The triangle formed by the points ABC is an inclined plane in 3D space. The point on the line AB that is closest to C is called D.

Plan view →



First find the coordinates of B.
$Slopedist_{AB} = \sqrt{(Hordist_{AB}^2 + (H_B-H_A)^2)}$ so $Hordist_{AB}$ = 60.622
Then calculate the EN coordinates of B by radiation from A, using $Hordist_{AB}$ and bearing AB = 70 →
B: E = 56.966, N = 20.734 and we are given H = 45.

The side lengths of the triangle ABC are the slope distances which we calculate from ENH coordinates:
AB = 70 (given)
AC = 43.589
BC = 53.273

The angle in the inclined triangle ABC at A can be calculated from cosine rule and side lengths.
$\cos A = (b^2 + c^2 - a^2)/2bc = 0.649244$ So A ≈ 49.5°
In 3D space and on the surface of the triangle ABC the angle at D from the line AB to the DC is 90°.
That is, angle ADC = 90°

Calculate distance DC from right angle triangle ADC with known angle A and hypotenuse = length AC.

DC = sinA*AC = 33.153
AD (= slope dist AD) = AC*cosA = 28.300
$ZA_{AB} = \tan^{-1}(Hordist_{AB} /(H_B-H_A)) = 60.0° = ZA_{AD}$
$Hordist_{AD} = AD*\sin(ZA_{AD}) = 24.508$

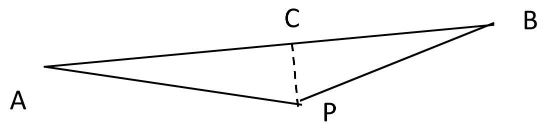
So coordinates D:
$E_D = E_A$+ Hor dist AD * sin(bearing AB) = 23.030
$N_D = N_A$+ Hor dist AD * cos(bearing AB) = 8.382
$H_D = H_A$+ AD * $\cos(ZA_{AD})$ = 24.150

Check slope dist CD = 33.153 (from coordinates)
Check slope dist AD = 28.300 (from coordinates)

Solution to 15.

This problem is similar to question 14.
Incidentally the heights of the points refer to a datum 10000m below to "sea level/geoid". So these points are about 800m underground.

Work with the triangle between ABP on an inclined plane with side lengths being the slope distances between the points. The angle at C is 90°. Sketch below is not to scale.



Use the ENH coordinates and the scale factor as in part B above to calculate slope distances for the three sides:
AB = 458.385, AP = 366.311, BP = 93.919

Calculate angle at A in inclined plane of triangle using $\cos A = (b^2 + c^2 - a^2)/2bc$
cosA 0.999… and A = 2.59…°.
Calculate the slope distance CP from the right angled triangle ACP, CP = sinA*AP = 16.554
Calculate the slope distance AC from the right angled triangle ACP, AC = cosA*AP = 365.937
Bearing AB from E N coordinates of AB = 27.29…° = brg AC
$ZA_{AB} = ZA_{AB} = \tan^{-1}(Hordist_{AB} /(H_B-H_A)) = ZA_{AC} = 87.16…°$
Hor dist AC = Slope dist AC * $\sin ZA_{AC}$ = 365.488
ΔH AC = Slope dist AC * $\cos ZA_{AC}$ = 18.129

Coordinates of C from A using hor dist, bearing, ΔH:
E = 2149.818, N = 7800.049, H = 9227.820

Join from C to P is determined directly from the 3D coordinates of C and P
Slope dist from ΔE, ΔN, ΔH = 16.554
Bearing from ΔE, ΔN = 116° 25' 49.3"
ZA from $\cos^{-1}(\Delta H/SD)$ = 106° 55' 39.4"

Answers to question 16.

I have solved this question by two independent methods, but I am reluctant to show full worked solutions. I want you to think about how to solve this, not just read a solution. If you do solve it then send me your solution and we can discuss if it is different to one of my methods. My answers are:

Distance = 0.373

End point on pipe A (E,N,H):	12.822	29.362	4.784
End point on pipe B (E,N,H):	13.106	29.340	4.544

Answer to question 17.

AB = 2101.895 links. The solution to this problem is not printed here because I want you to tackle it as a problem solving exercise. There are a few ways to solve the problem. One method uses equations that are included in Chapter 1; nothing else is required but it does take several steps. Some students say the question is too difficult, that if a solution was provided they could learn from it. However, if I wrote a solution and students read through it they would be able to follow the logic and the steps, and possibly think they have learnt something significant. A worked solution would assist them if they ever encountered a very similar problem, but it would not help them solve other challenging problems.

Another way to solve the problem is to use least squares adjustment, but that is beyond this course. Alternatively, you could use a trial and error approach and converge to a solution.

Answer to question 18.

DE = 3764.4, EA = 5613.8, FG = 5986, FE = 5986

You are welcome to send me any of your solutions to problems not solved above, or if you have novel solution methods.

A1. Sample examination papers

1.1 Assessment in the Course at UNSW

At UNSW we have small class sizes so we have considerable flexibility with assessment methods. For example tests can be conducted in our computer lab with all students present at one time. Computers used in tests don't have network or email access. We usually have three components to the total assessment, though portions vary from year to year:

Mid-session test	30%	See a sample below.
Lab and Prac work	30%	Based on the questions and fieldwork given in the book chapters.
Final Exam	40%	Held after the end of classes. See a sample below.

Each student is offered individual and detailed feedback on their exam or test paper soon after the exam has been marked. I go through their answers with them.

Students are required to demonstrate and explain all their tutorial and Lab exercises to a supervisor for comments and marking.

The tutorial/lab work is marked in the student's presence by viewing the students' notes or computer screens and immediate feedback is given. Students can be required to demonstrate and explain their work. There is no need to rewrite the work or to submit formal well written reports. Students are urged to manage their workload and make regular submissions during session. Students are expected to spend about 150 hours on this course.

The exams are set by the course convenor and reviewed by another staff member of the school. Teachers of similar courses at other universities are most welcome to send me copies of their test papers, solutions and marking schemes.

1.2 Sample Midsession test

The Midsession test is conducted in our computer lab. The marking criteria for this test place a strong emphasis on correct answers for calculation style questions. Students are expected to provide independent checks for their work. It is not sufficient to merely approach the problems with a valid method.

This test is given about mid way through the session after students have completed chapters 1 to 4. The questions change each year (of course). There are usually about 5 questions with no optional questions. In this sample I have included more than 5 questions so you can see the variety of questions used some years.

Solutions are given at the end of this sample paper

THE UNIVERSITY OF NEW SOUTH WALES

SAMPLE TEST PAPER

Answers at end of this paper

GMAT 2500 – SURVEYING COMPUTATIONS AND CAD

TIME ALLOWED - 1 HOUR 50 MINUTES

TOTAL NUMBER OF QUESTIONS – 3

Total Marks = 30

ANSWER **ALL** QUESTIONS

THIS PAPER MAY NOT BE RETAINED BY THE CANDIDATE

Students may use their own non-programmable calculator
A page of equations and instructions for a Casio *fx-911w* calculator are included.
A computer will be provided with MS Excel etc but no network or internet connections, no email and no access to class web site
A USB memory will be provided with an MS Excel file containing the input data for these questions. **Computer files should be saved regularly on the USB memory provided, not on the computer hard drives or desktop.**
Candidates may bring drawing instruments and /or rules.
Students may use calculator or MS Excel to solve the questions, but not CAD (e.g. MagnetOffice)

In numerical questions, calculator or computer may be used at the student's discretion. The calculation details of intermediate steps and results should be shown on this paper or in computer files, and the method of finding the answer clearly indicated.
Answers must be written in **ink on this paper**. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

Some instructions for use of Casio fx-911W Calculator:

Setting number of decimal places in display to e.g. 3 [MODE] [MODE] [MODE] 1 3

Storing and recalling memories e.g. 123 [STO] [A] [RCL] [A]

D M S conversion e.g. 14°25'36" to D.D 14 [° ' "] 25 [° ' "] 36 [° ' "] [=] [° ' "] gives 14.427

Reverse calculation 14.42666667 [=] [SHIFT] [° ' "] gives 14°25°36.

To make sure you are working in degrees mode use [MODE] [MODE] [1]

Trig calcs e.g. cos(63°52'41") [cos] 63 [° ' "] 52 [° ' "] 41 [° ' "] [=] gives 0.44...

e.g. $\cos^{-1}(0.61)$ [SHIFT] [$\cos^{-1}$] 0.61 [=] [SHIFT] [° ' "] gives 52°24°37.79

Polar to Rectangular conversion e.g. distance = 20.5 and bearing = 60°23'34"

[SHIFT] [Rec(] 20.5 [,] 60 [° ' "] 23 [° ' "] 34 [° ' "] [)] [=] gives ΔN 10.128

[RCL] [F] gives ΔE 17.823

Memory E contains ΔN and memory F contains ΔE

Rectangular to Polar conversion eg ΔN = 60.5 and ΔE = 30.4

[Pol(] 60.5 [,] 30.4 [)] [=] gives distance 67.708

[RCL] [F] gives bearing {if <0 then RCL F + 360 =} then [SHIFT] [° ' "]

gives bearing in dms 26°40°42.9
Note the order ΔN then ΔE and distance then bearing
Memory E contains distance and memory F contains bearing

Statistics Calculations e.g. obs: 55, 54, 51, 53
[SHIFT] [Scl] [=] [MODE] [2]

55 [DATA] 54 [DATA] 51 [DATA] 53 [DATA] DATA is the M+ key

[SHIFT] [$\bar{x}$] [=] gives mean 53.25 [SHIFT] [xsn-1] [=] gives standard deviation 1.7

Formula page

$$\bar{x} = \frac{\Sigma x_i}{n} \quad \text{or} \quad \bar{x} = x_a + \frac{\Sigma(x_i - x_a)}{n}$$

$$s_x = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\Sigma v_i^2}{n-1}} = \sqrt{\frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1}} \qquad v_i = \bar{x} - x_i$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$a = b \cos C + c \cos B$$

$$E_Q = E_P + d \sin\beta \quad \text{and} \quad N_Q = N_P + d \cos\beta$$

B in decimal degs, not negative: Degs, D =INT(B) Mins, M =INT(B*60-D*60) Secs, S =((B-D)*60-M)*60

$$\beta_{12} = \tan^{-1}\left(\frac{E_2 - E_1}{N_2 - N_1}\right) = MOD(DEGREES(ATAN2(N2-N1, E2-E1)),360)$$

$$d_{12} = \sqrt{(E_2 - E_1)^2 + (N_2 - N_1)^2}$$

$$E_C = E_A - \frac{\sin B}{\sin(A-B)}\{(E_B - E_A)\cos A + (N_B - N_A)\sin A\} \qquad N_C = N_A - \frac{\sin B}{\sin(A-B)}\{(N_B - N_A)\cos A - (E_B - E_A)\sin A\}$$

$$E_C = E_A + \frac{(E_B - E_A)\cos\beta_{BC} - (N_B - N_A)\sin\beta_{BC}}{\sin(\beta_{AC} - \beta_{BC})}\sin\beta_{AC} \qquad N_C = N_A + \frac{(E_B - E_A)\cos\beta_{BC} - (N_B - N_A)\sin\beta_{BC}}{\sin(\beta_{AC} - \beta_{BC})}\cos\beta_{AC}$$

$$E_C = E_A + \frac{b}{c}[(E_B - E_A)\cos A + (N_B - N_A)\sin A] \qquad N_C = N_A + \frac{b}{c}[(N_B - N_A)\cos A - (E_B - E_A)\sin A]$$

$$\tan\beta_{PB} = \frac{(E_A - E_B)\cot\alpha_1 + (E_C - E_B)\cot\alpha_2 - (N_C - N_A)}{(N_A - N_B)\cot\alpha_1 + (N_C - N_B)\cot\alpha_2 + (E_C - E_A)}$$

$$E_P = E_B - (N_B - N_P)\tan\beta_{PB} \qquad N_P = \frac{E_B - E_C + N_C\tan\beta_{PC} - N_B\tan\beta_{PB}}{\tan\beta_{PC} - \tan\beta_{PB}}$$

$$\alpha_1 = \text{dirPB} - \text{dirPA} \qquad \alpha_2 = \text{dirPC} - \text{dirPB} \qquad \alpha_3 = \text{dirPA} - \text{dirPC}$$

$$A = \beta_{AC} - \beta_{AB} \qquad B = \beta_{BA} - \beta_{BC} \qquad C = \beta_{CB} - \beta_{CA} \qquad \cot = 1/\tan = \cos/\sin$$

$$w_A = 1/(\cot A - \cot\alpha_2) \qquad w_B = 1/(\cot B - \cot\alpha_3) \qquad w_C = 1/(\cot C - \cot\alpha_1)$$

$$E_P = \frac{E_A w_A + E_B w_B + E_C w_C}{w_A + w_B + w_C} \qquad N_P = \frac{N_A w_A + N_B w_B + N_C w_C}{w_A + w_B + w_C}$$

Misclose = Σ internal angles - (n-2) ·180° or = Σ angles + ($\beta_{start} - \beta_{end}$) - (n-2) ·180°

$$\text{correction to } \Delta E = -\frac{\text{total error in } \Delta E}{\text{total length of traverse}} \cdot \text{length of side}$$

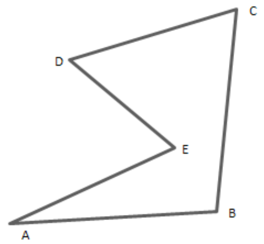
$$\text{correction to } \Delta N = -\frac{\text{total error in } \Delta N}{\text{total length of traverse}} \cdot \text{length of side}$$

If there are any other equations / formulae you would like to see listed on this page, contact the lecturer.

Q1. (a = 7 marks, b = 3 marks Total =10 Marks)

a) Calculate the distances of lines BC and CD from a loop shown on a survey plan with the following data. Figure below is not to scale. Show your answers to the nearest mm.

Line	Bearing	Distance (m)
AB	85° 21' 00"	202.655
BC	3° 58' 10"	?
CD	246° 22' 40"	?
DE	140° 10' 50"	160.425
EA	237° 14' 00"	197.397

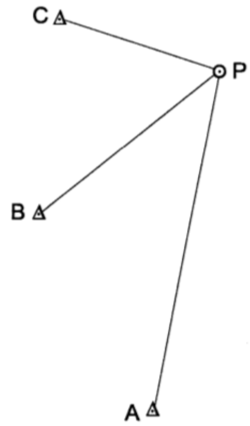


b) Show an appropriate check calculation of your solution of part (a).

Q2. (a = 7 marks, b = 3 marks Total = 10 Marks)

From the following resection observations, **calculate** the coordinates of the point P, to the nearest mm.

Point	E(m)	N(m)	Observed Directions	
A	9475.359	4206.716	A	0° 00' 00"
B	8647.509	5656.116	B	38° 30' 00"
C	8761.969	7126.176	C	98° 00' 00"



A site plan shows that the approximate coordinates of P are (9880, 6800) ± 20m. The observed directions are not orientated towards north.

a) Calculate the coordinates of P.

b) Describe a method to check resection answers. Then show a separate independent check of your answers.

Q3. (a = 2 mark, b = 5 marks, c = 3 Total =10 Marks)

A loop traverse was observed and the data and a field sketch plan is given below. Distances and coordinates are on a local datum, there is no scale factor. One of the observations (i.e. one angle or one distance) has been wrongly recorded. The survey was expected to meet the following specifications: angular misclose < 20" +10√n and linear misclose < 15mm.

a) Calculate the traverse angular misclose.

b) Find the incorrect observation, giving reasons.

c) Estimate the likely correct value of the erroneous observation

The sketch plan is not to scale.

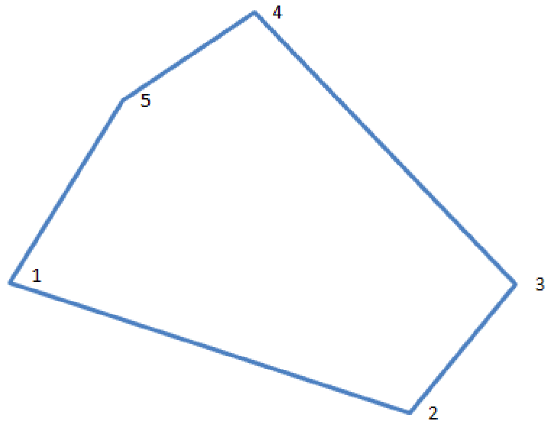
KNOWN COORDINATES

Pt	E	N
1	123.330	398.750

KNOWN BEARING
1 to 2 = 109°00′00″

Interior clockwise Angles

At	° ′ ″
1	78 44 43
2	108 56 12
3	99 54 56
4	96 41 13
5	155 43 06



Horizontal Distances

1 - 2	160.335
2 - 3	65.575
3 - 4	147.610
4 – 5	91.556
5 – 1	84.805

END of EXAM PAPER

MORE PAST EXAM QUESTIONS – you won't get this many in the exam.
To further assist your study more past exam questions are included below.

Q4. (5 marks) On five separate days the Zenith Angle from a trig station to a distant light house was measured:
92° 59' 59.2"
92° 59' 59.7"
93° 00' 00.3"
93° 00' 00.2"
93° 00' 00.5"

Calculate the mean and standard deviation. Display your answers to 2 decimal places of a second (").

Q5. (20 marks) From the following data, compute the coordinates of C.

	CO-ORDINATES	
POINT	E	N
A	45 328.172	26 985.030
B	44 626.185	26 616.600

Horizontal directions:

At A	FL	FR
To C	0° 00' 00"	180° 00' 02"
To B	62° 14' 20'	242° 14' 30"

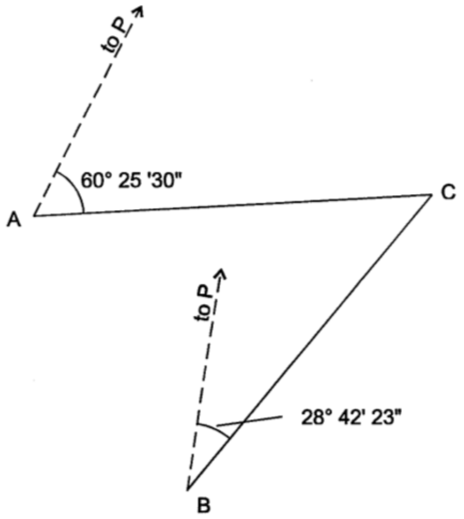
At B	FL	FR
To A	0° 00' 02"	180° 00' 04"
To C	35° 24' 18'	215° 24' 28"

Q6. (5 Marks)

a) A traverse is found to have a large angular misclose and it is suspected that only one angle is wrong. **Explain** a method of detecting which angle in the traverse is incorrect. You may use sketches.

b) A traverse is found to have a large linear misclose and it is suspected that only one distance is wrong. **Explain** a method of detecting which distance in the traverse is incorrect. You may use sketches.

Q7. A point C was intersected from two control points A and B. Horizontal angles were measured at A and B between a third control point P and the point C. The measured angles are shown in the diagram below. Using the coordinates of the control points given; **calculate** the coordinates of C to the nearest mm by any method you know.



COORDINATES

STATION	E (m)	N (m)
A	1881.61	1523.19
B	2134.86	1061.14
P	2339.91	2398.36

1.3 Sample Midsession test - solutions

Q1. A) BC = 283.563, CD = 172.844
Methods:
1 Assign and determine coordinates then intersection by bearings (or even by angles)
2 Misclose or join BD, then triangle solution
3 MS Excel Solver
4 Swing all bearings so that DC is 0 (or 90) then misclose in E or N gives one of the missing dists, similar for line BC.

B) Most common check is loop misclose. If used solver to get distances in part a then loop close is not a good check.

Q2.
A) 9876.543 6789.012 by any one of the standard methods in the text book.

Intermediate values by Tangent method

Pt	E	N	D	M	S	angle	cot	Brg from P	cot
A	9475.359	4206.716	0	00	00			188.83	6.4
B	8647.509	5656.116	38	30	00	38.5º	1.3	227.33	0.9
C	8761.969	7126.176	98	00	00	59.5º	0.6	286.83	-0.3
						Top =	-1811.3		
						Bottom =	-1669.6		
Ans P:	**9876.543**	**6789.012**				tanPB =	1.084861		
Check bearings from coords:									
PA			188	49	51.0				
PB			227	19	51.0	38.5º	OK		
PC			286	49	51.0	59.5º	OK		

Intermediate values by Teinstra method

Pt	E	N	D	M	S	α angle
A	9475.359	4206.716	0	0	0	
B	8647.509	5656.116	38	30	0	38.5º
C	8761.969	7126.176	98	00	00	59.5º
						262.0º
Bearings				angles	w	
AB	330.3º		A	16.0º	0.345	
BC	4.5º		B	145.8º	-0.620	
AC	346.3º		C	18.2º	0.560	
				sum	0.285	
P	**9876.543**	**6789.012**				

Intermediate values by Collins Point method

Pt	E	N		D	M	S	dd	
		d AC	3005.357	346	16	06.5	346.3	Brg AC
				286	46	06.5	286.8	Brg AH
				204	46	06.5	204.8	Brg CH
		d AH	1889.27				82.0	Angle AHC
		d CH	2614.952					
H	**7666.429**	**4751.779**						
				227	19	51.0	227.3	brgBH= brgPB
				106	49	51.0	106.8	Brg CP
				8	49	51.0	8.8	Brg AP
			angle ACP	59	26	15.5	59.4	
			angle APC				98.0	
		dAP	2613.274					
P	**9876.543**	**6789.012**						

B) By a different method to that used in (A) or by calculating bearings PA PB PC from coordinates then determining angles to see if they match the observed directions.

Q3. Angle misclose = +10 sec, so angles are OK. Large linear misclose is parallel to line 4-5, so error in that distance of about 20m. Distance 4-5 should be 61.5m. Do not calculate Bowditch adjustment of this data.

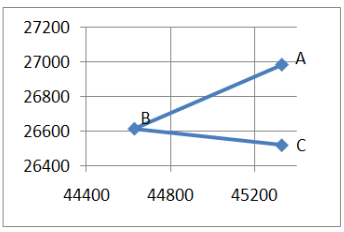
Q4. Mean 92° 59' 59.98" std dev ± 0.53”
Calculations can be done with seconds only, no need to enter D M S and convert to D.D then answer back to DMS. When working in seconds the values over 93° are entered as 60.3 etc.
How do you check these calculations? Calculate the residuals from the mean, their sum should be zero.

	v = mean-xi
	0.78
	0.28
	-0.32
	-0.22
	-0.52
sum	0.00

Sum v^2 = 1.108.
Calculate Standard deviation by formula from v and by excel function as a check
Standard deviation = $\sqrt{(\text{sum } v^2) / (n-1)} = \sqrt{(1.108/(5-1))} = 0.53$”

Q5. Draw a figure:
Can calculate bearings and use intersection by bearing formula or use intersection by angles:

$$E_C = E_A - \frac{\sin B}{\sin(A - B)}\{(E_B - E_A)\cos A + (N_B - N_A)\sin A\}$$



$$N_C = N_A - \frac{\sin B}{\sin(A - B)}\left\{(N_B - N_A)\cos A - (E_B - E_A)\sin A\right\}$$

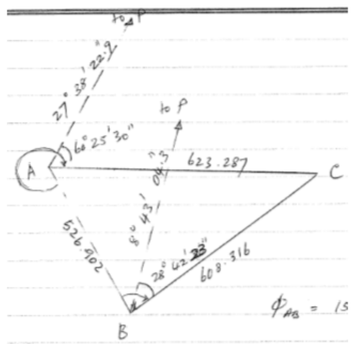
Horizontal directions:				
At A	Mean			Angle
To C	00°	00'	01"	
To B	62°	14'	25"	62.24°
At B				
To A	00°	00'	03"	
To C	35°	24'	23"	35.41°
angles clockwise from baseline AB		rads		
A	297.76°	5.2		
B	35.41°	0.6		

C coordinates: 45327.624 26521.596

Check:		Angles:				
bearing AB	242.30°					
bearing AC	180.07°	62.24°	62°	14'	24.000"	OK
bearing BC	97.71°	35.41°	35°	24'	20.000"	OK

Q6: See textbook

Q7. From coordinates:
βAP = 27 28 22.9 dAB = 526.902
βAB = 151 16 22.1
βBP = 8 43 04.3
Add the angles to get bearings:



βAC = 27 28 22.9 + 60 25 30 = 88 03 52.9
βBC = 8 43 04.3 + 28 42 23 = 37 25 27.3

Angle at A = 88 03 52.9 - 151 16 22.1 = 296 47 30.8

Angle at B = 37 25 27.3 – (180 + 151 16 22.1) = 66 09 05.2

Ec = 2504.541 Nc = 1544.239

1.4 Sample Final Exam

The **Final exam** is in the exam period in our computer lab. It involves written questions on the exam paper plus use of software on a computer. In previous years at UNSW students used CivilCAD software, think of it as an older version of Magnet Office CAD which is currently used. Students at other universities with access to other CAD packages may follow different pathways to similar answers.

SAMPLE Final Examination

Time allowed 2 hours
Total number of Questions: 4
Answer All Questions

Rules
The exam is not open book, but Excel and CivilCAD Help menus will be enabled, and some chapters of the Class text book's pdf files will be available on the exam computer.
Email and internet are not to be used; communication with others during the exam is not allowed.
Students will be given a special log in account for the exam and will not have access to other files.
Students will have access to Microsoft Office and to CivilCAD.
Students may bring their own calculators to the exam.
Prior files or programs on disk or on computer are not to be used.
If using a computer, save your work regularly and record details such as intermediate results on this exam paper.
When using the printer make sure that your name is printed as part of the document.

EQUATIONS

$$\text{Area} = \frac{1}{2} b\,h = \frac{1}{2} a\,b \sin C = \frac{1}{2} a\,c \sin B = \frac{1}{2} b\,c \sin A = \sqrt{s(s-a)(s-b)(s-c)} \qquad \text{where } s = \frac{a+b+c}{2}$$

Area = ½ | [($N_1E_2 + N_2E_3 + \ldots + N_nE_{n+1}$) - ($E_1N_2 + E_2N_3 + \ldots + E_nN_{n+1}$)] |

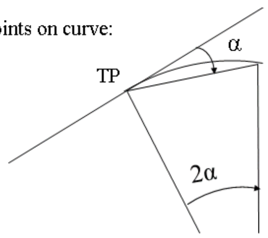
$$\text{Segment Area} = R^2\left(\pi\frac{\theta^\circ}{360^\circ} - \frac{\sin\theta}{2}\right) \qquad \theta = 2\sin^{-1}\left(\frac{C}{2R}\right)$$

$$d = w / \cos\left(\frac{\beta_2 - \beta_1}{2}\right)$$

Arc Length $A = R.\theta^c$ Tangent length $T = R\tan\frac{\theta}{2}$

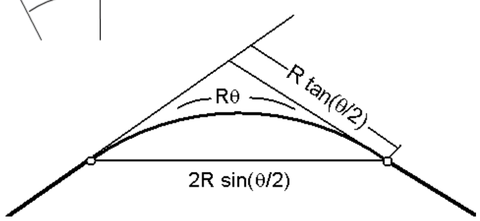
Chord Length $C = 2R\sin\frac{\theta}{2} = 2T\cos\frac{\theta}{2}$ Deflection angle $\alpha^c = \frac{arc\_length}{2R}$

Points on curve:



$$\alpha^\circ = \frac{\text{arc}}{2R}\cdot\frac{180}{\pi}$$

$$\text{chord} = 2R\sin\alpha \quad \approx \text{arc} = R.2\alpha$$



Q1

a) **List** and **briefly explain** several important principles of survey computations

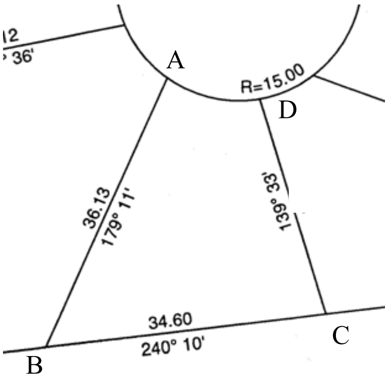
b) List three advantages of computer assisted drafting (CAD) compared with manual drafting.

c) Describe the advantages and disadvantages of using each of:
Calculators;
MS Excel spreadsheets;
Writing your own software (eg in Visual Basic), and
CivilCAD (or similar) software,

to calculate the coordinates of points on a road centreline that contains a horizontal curve.

Q2 Choose A or B

A) The lot drawn in the following figure is part of a new subdivision. The calculations in this question are part of the design process for the whole subdivision.



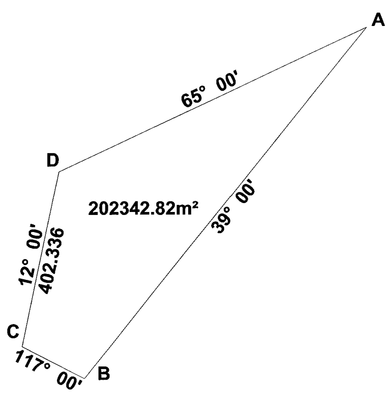
Figure, not to scale

Note, this question looks similar to a tutorial question, but it is slightly different.

The centre of the circle that forms arc AD is at a bearing of 359° 11' and distance 15.00m from A (i.e. along the line BA continued 'north')

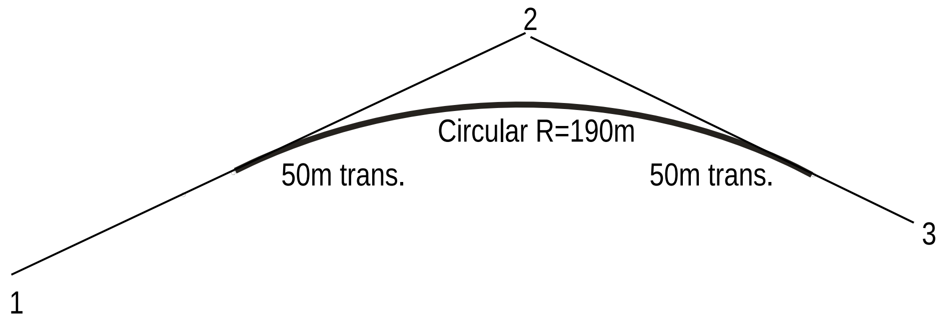
a) Calculate the distance CD and the area of the lot.
b) Use independent methods to check your answers from part A.
c) What role can mental arithmetic play in this problem? Give an example of its use.

B) In the diagram at right, the figure A B C D contains 202,342.82m^2. From the data supplied, find the lengths of the boundaries A D, A B and B C. Use mental arithmetic as an approximate check of your answers, explain your method.



Q3

The site for a proposed new road has been surveyed by GPS and the transformed E, N, H coordinates are supplied in file 13Exam8R.pts. The point codes do not use a prefix. The proposed road contains a circular curve and two spiral transition curves as shown in the figure below. No superelevation will be applied to this road.



The centre line of the road starts at point 1 (E = 290108.0, N = 598.0) with a chainage of 00. Point 2, the IP, is at (290077.0, 778.0) and point 3 is at (290138.0, 880.0).

a) Calculate the coordinates and chainage of the tangent points and of points on the centreline using even 20 m chainages. You may use a calculator, or spreadsheet or CAD, or a combination of these methods. What are the coordinates of the point at chainage 180?

b) What is the chainage of point 3?

c) What is the chainage of the last road centreline mark before point 3?

d) There is a nearby control mark at 290192.161, 882.103. Calculate the bearing and distance from this control mark to the last road centreline mark before point 3.

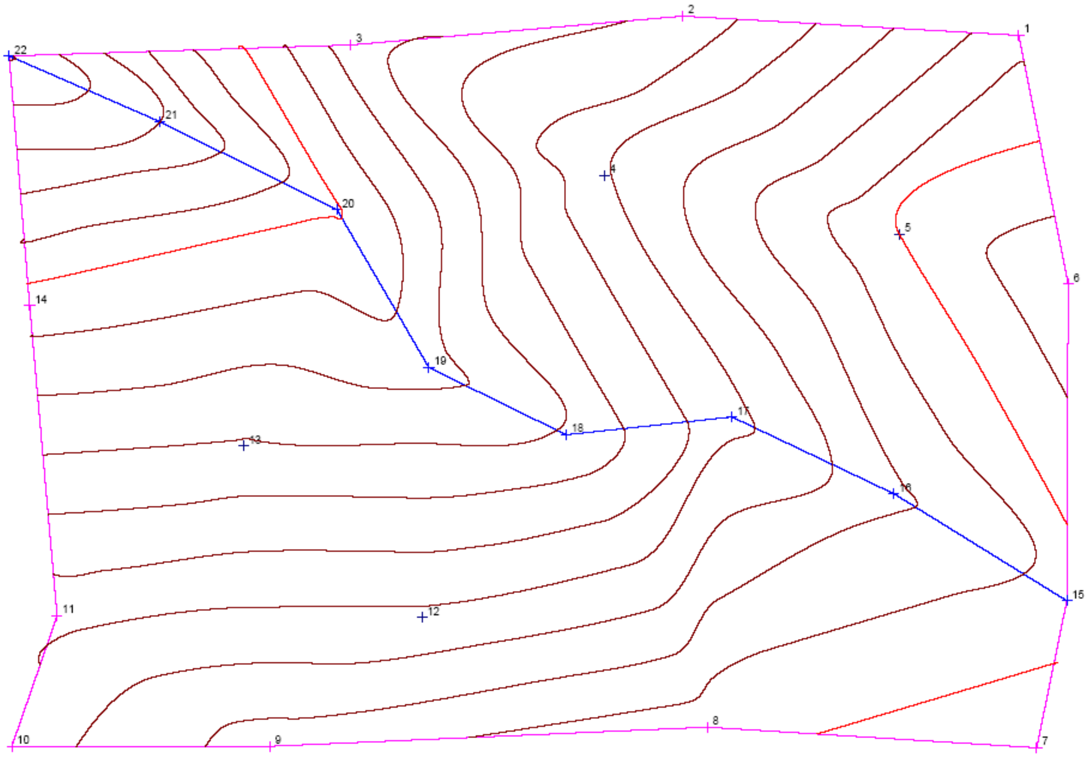
e) Produce the cross sections for the 20m chainage points, not for the tangent points. Use a left and right offset of 15 m. Construct a template for the road surface that falls 3% (i.e. -3) each side of the centre line for 6m. There is to be no kerb on this rural road. Batters on both sides of the road are to be fixed slope at 1 in 2 for both cut and fill. **Draw** (sketch) on this exam paper (or obtain a print out) the cross sectional view at the end of the road (point 3), with a vertical exaggeration of 5.

f) The road surface long section is designed to start (at point 1) and end (at point 3) at the same level as the current ground surface. Add a vertical curve of length 40m with IP at chainage 160 and reduced level equal to the current ground level.
What is the long section slope from chainage 00 to 160?
What is the long section slope from chainage 160 to the end of the road?

g) What is the reduced level of the designed road surface centreline at chainage 200?

h) After adding the vertical curve, **sketch** on this exam paper (or obtain a print out) the new cross section at chainage 180 with a vertical exaggeration of 5.

Q4   A detail survey of a farm site was measured. There is a fence around the property and a creek through it.



The data for this site has been entered into a CivilCAD file called 12Sample.pts that is available on the class website. The use of CivilCAD6 and/or this electronic file is recommended to answer this question. The format of the text data file is: Point number, Easting, Northing, Height, Code

Use CivilCAD software to produce a contour plan of the farm site with a 1 metre interval. Explain the methods used when editing or processing for each step.

Do not show the point numbers for the detail points on your final plan. Do not show contours or triangles (TIN) outside the fence boundary. To ensure correct contours near the creek there needs to be a triangle edge (in the TIN) along each segment of the creek.

If the fence lines are not drawn in the correct order (because the order they were surveyed in is different to the order required for plotting) then correct the fence lines. Describe two different methods for correcting the fence lines.

Add a proposed building to the site with (E, N) coordinates:

760.000	640.000
750.000	640.000
750.000	633.000
760.000	633.000

Add the words “Proposed Building” to the plan in an appropriate place, near or within the building. Ensure that contour lines do not go through the building.

Printout your plan at a scale of 1:4000, with a standard A4 title block/border, and your name, on the lab’s BW printer.

Additional Short Answer Questions to assist your study:

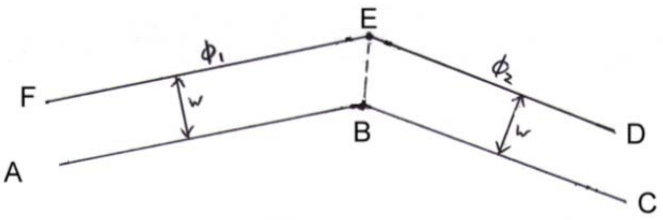
A) Explain the major difference between Insert/Text/Text and Survey/Annotation functions in CivilCAD6.

Ans: Inserted Text – It is an independent object. No link between an entity and a text inserted for it, e.g. inserted bearing/distance for a line is not linked to the line.

Annotated Text – linked to the entity. An annotated text is derived from the entity’s properties. E.g. the bearing/distance text is obtained from the line’s bearing/distance value

B) Explain the codes TREE*2 and 01FCE*01CREEK as used in CivilCAD. How do we deal with data sets that have string numbers prefix the code?

C) In the figure below, a rural road with parallel straights sides and width w = 20.115 m changes direction at E. The bearing FE = 73°00’ and the bearing ED = 115°00’. Calculate the bearing and distance from E to B.



E) Briefly explain the necessary procedures, commands or functions in CivilCAD to edit detail survey plans in each of the following ways:

1) Remove the detail point numbers for all points.

2) Insert breaklines to correct the contours which cross over bank boundaries;

3) Remove contours inside houses or roads;

4) Smooth the contours;

5) How do you display the contents of one particular layer only?

6) How can you display the properties of an entity e.g. a point?

7) What option would you use to find the length of a line?

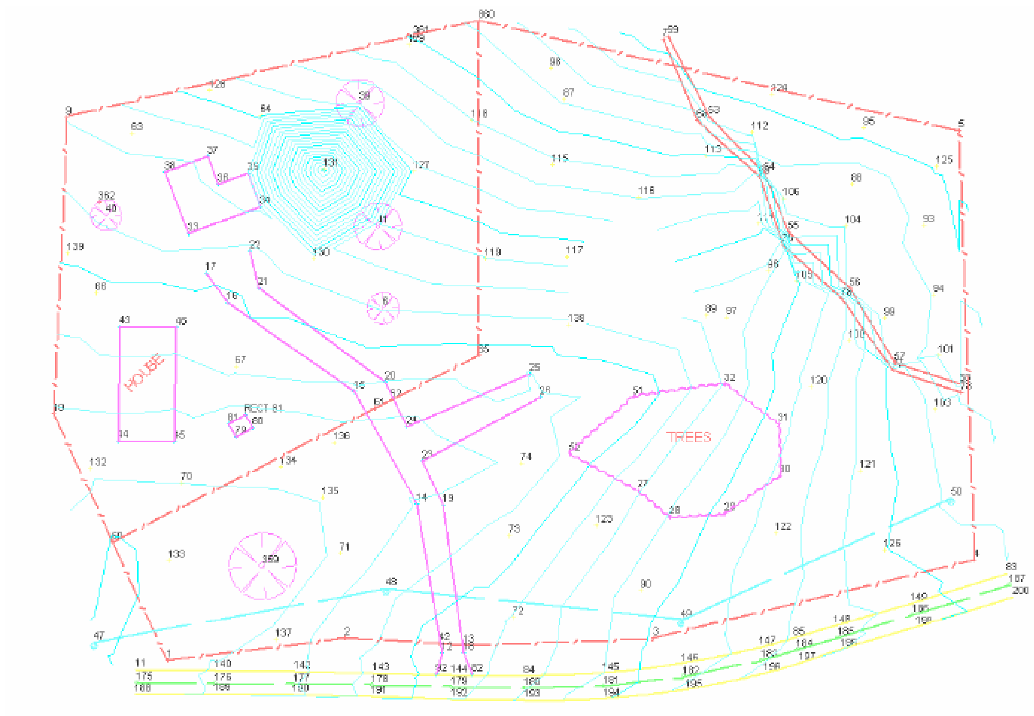
8) How can you tell which layer is current?

9) Which option would you use to add bearing and distance to a traverse line?

10) Which option would you use to add text to a plan?

11) If a Title Block file is available on your computer, which option would you use to add this Title Block to your plan?

12) Find some possible mistakes in the plan below. How would you correct those mistakes? Any suggestions on what needs to be improved and how to improve the plan?



Solutions

Q2A Dist CD = 30.817. Bearing D to centre of circle = 314°13'13". Area polygon ABCD = 715.5m^2

Segment area = 8.8 m2 and Area of lot = 706.7 m2. Can be solved using (centre of circle at M):

a) Excel's solver function with missing bearing of DM and missing dist CD, constraining misclose in E and N to both = 0.
b) Missing bearing and distance equations as in Chapter 3 or in CAD
c) Rotate the lines until CD is north (brg 0), then sum ΔE around loop, then find brg of DM from ΔE = 15sin(brgDM), then correct the bearing back to the original non-rotated system.
d) And other ways.

One approach is to select coordinates for a point and calculate the coordinates of the other points, for example starting at the centre of the circle O:

Point	Distance	Bearing						
		d	m	s	ΔE	ΔN	E	N
O							1000.000	2000.000
	15	179	11	00	0.214	-14.998		
A							1000.214	1985.002
	36.13	179	11	00	0.515	-36.126		
B							1000.729	1948.875
	34.6	60	10	00	30.015	17.2123		
C							1030.743	1966.088

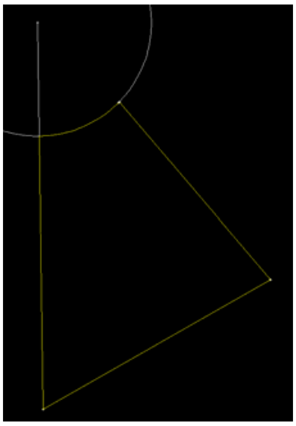
Then calculate the coordinates of D by intersection of distance from O and bearing from C, or by using MS Excel solver on the loop traverse back to O. This gives D (1010.750, 1989.539) and distance CD = 30.817.

The area of the polygon ABCDA using the coordinates is ½ (7973857 – 7972426) = 715.5

From that area we subtract the area of the segment of the curve. From coordinates of A and D, chord length A-D = 11.472, angle theta = 44.96 degrees and Segment Area = 8.8.

Total Area (Polygon Area - Segment Area) = 706.7

An independent check of the answers can be obtained by using a CAD program with surveying COGO module. Distance CD = 30.817

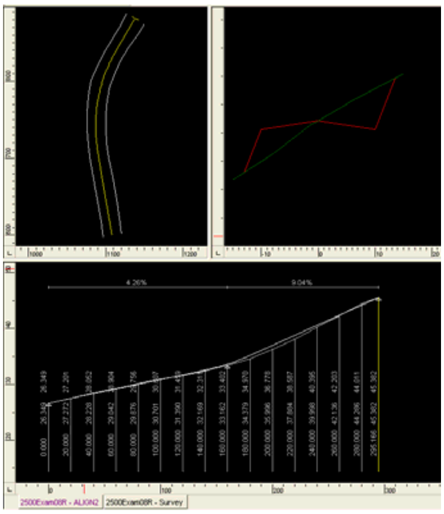


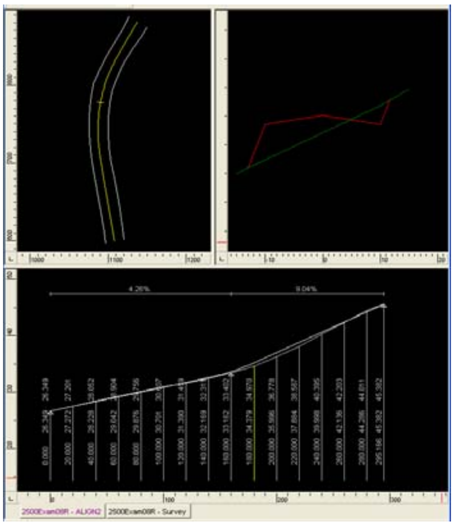
Mental arithmetic and a sketch plan drawn approximately to scale would yield CD as less than AB and BC, perhaps about 30m and AD about 10m. The area as similar to that of a rectangle of sides being the means of opposite sides of the lot: (36+30)/2 by (35+10)/2 = 740 m

Q2B
AB = 1017.316m   *CB = 158.173m*

Q3 solution

a) 290090.073 776.089
b) 295.166
c) 280
d) 256°17'02" (within 10" OK) 63.763
e) and h)





f) +4.26% (or 1 in 23.5) +9.04% (or 1 in 11)
g) 36.778 is design level