Problem Set 2

2-3. Electron, $mc^2 = .511$ MeV, $u = .6c$, $\gamma = 1.25$

a) $\underline{\gamma = 1.25}$ b) $p = \gamma m v = (1.25)(.511 \frac{MeV}{c^2})(.6c)$

$$= \underline{.383 \text{ MeV}/c}$$

c) $E = \gamma mc^2 = (1.25)(.511) = \underline{.638 \text{ MeV}}$

d) $E_K = E - mc^2 = .638 - .511 = \underline{.127 \text{ MeV}}$.

2-7. a) Distance to moon ~ 250(3) mi $= 3.8 \times 10^8$ m.

So $v = d/t = 2.53(8)$ m/s $= .84\,c$.

b) $m_p c^2 = 938$ MeV, so, since $\gamma = 1.86$

$E = \gamma m_p c^2 = 1745$ MeV,

$E_K = E - m_p c^2 = 807$ MeV

c) $m = \gamma m_p = 1745 \text{ MeV}/c^2$ $(= E/c^2)$

d) E_K (classical) $= \frac{1}{2} mv^2 = \frac{1}{2}(mc^2)(v/c)^2$

$= 331$ MeV factor of 2.4 (240%?)

(Silly expression!)

2-10. a) $mc^2 = (10^{-3})(3(8))^2 = 9(13)$ J

b) 1 kwh $= (10^3)(3600) = 3.6(6)$ J, so

$9(13)$ J $= 2.5(7)$ kwh $\Rightarrow$ $\underline{\$2.5 \times 10^6}$ @ 10¢/kwh

c) 2.5×10^7 kwh / 0.1 kW $= 2.5 \times 10^8$ h

$= \underline{2.8 \times 10^4 \text{ yr.}}$

2-19. a) From table on p. AP1,

$m_d = 2.014102$ amu, $m_{He} = 4.002602$

So $\Delta m = \underline{2.56(-2)\,\text{amu}} = \underline{23.8\,\text{MeV}}$ (931.5 MeV/amu)

b) This is the energy release.

c) In 1 sec, need 1 J of energy @ $23.8 \times 1.6 \times 10^{-19} \times 10^{6}$

$= \underline{2.6 \times 10^{11}\ \text{reactions}}$.

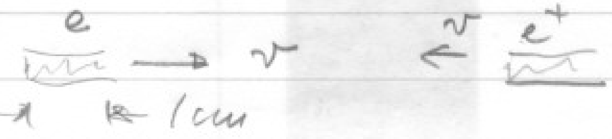
2-29. $p^2c^2 + (mc^2)^2 = E^2$, so

$(mc^2) = \sqrt{(1746)^2 - (500)^2} = \underline{1673\ \text{MeV}}$.

$\gamma = E/mc^2 = 1746/1673 = 1.043 = 1/\sqrt{1-\beta^2}$

$\beta = \sqrt{1 - 1/\gamma^2} = \underline{.286 = v/c}$,

$\underline{v = 8.58 \times 10^7\ \text{m/s}}$.

2-39. 

$m_e c^2 = .511$ MeV

a) $\gamma = E/mc^2 = (50 \times 10^9 + .51 \times 10^6)/(.511 \times 10^6)$

$\underline{\gamma = 9.78(4)}$.

Bunch length is length contracted to 1 cm,

So "proper length" is $\gamma \times 10^{-2}$ m

$= \underline{978\ \text{m}}$. It is still 10 μm in diameter.

b) In lab. or accelerator frame, 1 cm will do it. But in electron frame, need accelerator to be

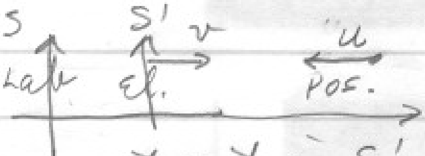
$(978\,\text{m})(\gamma) = 9.6 \times 10^4\ \underline{\text{km}}$!, since it sees

accelerator to be length contracted.

c) This is really tricky. Can use $u_x' = \frac{u_x - v}{1 - u_x v/c^2}$

with $u_x = v = .9999\ldots\ldots c$, but will fall off end of calculator. On p. 78, text shows that $\gamma' = 1/\sqrt{1-u'^2/c^2} = \gamma\left(\frac{1 - v u_x/c^2}{\sqrt{1-u^2/c^2}}\right)$, where $\gamma = 1/\sqrt{1-v^2/c^2}$

and $\gamma' = 1/\sqrt{1-u'^2/c^2}$

where 

Let electrons move at v, at rest in S', positrons moving at $-v$, where $v \approx c$ (very large γ!). Then

$$\gamma' = \gamma\left(\frac{1-(c)(-c)/c^2}{\sqrt{1-v^2/c^2}}\right) = \gamma^2 (2)$$

$$= (9.78(4))^2 \times 2 = 1.91(10).$$

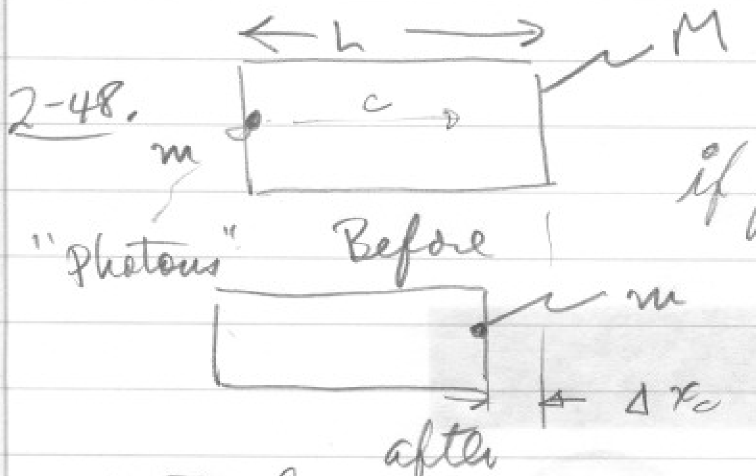
So in electron frame, positron bunch is $978\,m/\gamma' = \underline{5.1(-8)\,m}$ long!

d) Well, in frame of electrons, positrons have energy $E = \gamma' mc^2 = 9.76 \times 10^9$ MeV and

$$p \approx -E/c = -9.76 \times 10^9 \text{ MeV}/c.$$

Positrons see same of electrons.

2-48.



Idea is to show, if photons carry mass $\underline{m}$, the m is given by E_{photon}/c^2.

We use $E_{ph} = pc$, which is a classical result.

Center of mass of car moved $\Delta x =$ (time of flight)(Vel. of car during photon motion)

a) By momentum cons., $P_{car} = -P_{photons} \equiv P$, so
$|v_{car}| = P/M$

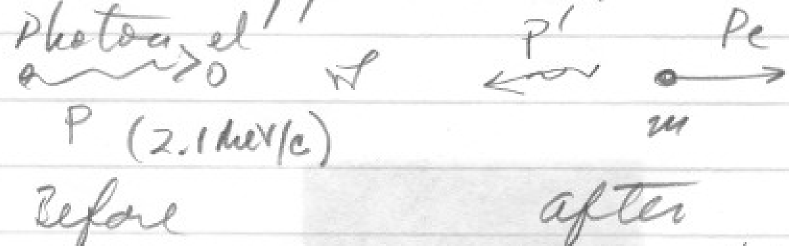
b) Car therefore moves $\Delta x_c = (P/M)\underbrace{(L/c)}_{\text{time photons move}}$

c) If center of mass is to remain fixed,
$\underbrace{M\Delta x_c}_{\text{car}}$ must $= \underbrace{m\,\Delta x_p}_{\text{photons}} \approx mL$

so $mL = (M)\left(\frac{P}{M}\right)\left(\frac{L}{c}\right)$ or, since $E(\text{photon}) = pc$

$$\boxed{m = P/c = \frac{E/c}{c} = E/c^2}.$$

PSet II, Supplementary



Momentum conservation: (Let $c = 1$)

$$p = -p' + p_e \qquad (1)$$

Energy conservation:

$$p + m = p' + \sqrt{p_e^2 + m^2} \qquad (2)$$

Solve for p_e, from which we get the (kinetic) energies of photon & electron after.

From (2) $p = p_e - p - m - \sqrt{p_e^2+m^2}$, i.e., eliminate p' into (1)

$$\cancel{p_e^2} + \cancel{m^2} = (2p - p_e + m)^2 = 4p^2 + \cancel{p_e^2} + \cancel{m^2} - 4pp_e + 4pm - 2p_e m$$

$$p_e(2p + m) = 2p^2 + 2mp$$

$$p_e = \frac{p+m}{(1 + m/2p)} = \frac{2.1 + .511}{(1 + .511/4.2)} = 2.327 \text{ MeV/c}$$

So $p' = p_e - p = 2.327 - 2.1 = .227$ MeV/c

So $\underline{E_{\text{photon after}}} = \underline{.227 \text{ MeV}}$

$\underline{K.E._{\text{electron}}} = \sqrt{2.327^2 + .511^2} - .511 = \underline{1.871 \text{ MeV}}$