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COMPUTER PROGRAMS FOR SMOOTHING AND SCALING AIRFOIL COORDINATES

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SUMMARY

This report contains detailed descriptions of the theoretical methods and associated computer codes of a program to smooth and a program to scale arbitrary airfoil coordinates. The smoothing program utilizes both least-squares polynomial and least-squares cubic spline techniques to smooth iteratively the second derivatives of the y-axis airfoil coordinates with respect to a transformed x-axis system which unwraps the airfoil and stretches the nose and trailing-edge regions. The corresponding smooth airfoil coordinates are then determined by solving a tridiagonal matrix of simultaneous cubic spline equations relating the y-axis coordinates and their corresponding second derivatives. A technique for computing the camber and thickness distribution of the smoothed airfoil is also discussed.

The scaling program can then be used to scale the thickness distribution generated by the smoothing program to a specified maximum thickness which is then combined with the camber distribution to obtain the final scaled airfoil contour. Computer listings of the smoothing and scaling programs are included as appendices. A user-guide and sample input and output cases for both programs are also included as appendices. Both computer programs are available from COSMIC with identifications LAR-13132 for the airfoil smoothing program "AFSMO" and LAR-13133 for the airfoil scaling program "AFSCL".

INTRODUCTION

Since its early beginning, the NACA and the NASA have been actively involved in the design and testing of airfoil sections for a wide variety of applications. During the 1930's, 40's, and 50's, the airfoils developed by the NACA consisted of the well-known 4-digit-, 5-digit-, 1-, 6-, and 7-series airfoils. These airfoils were generated by combining thickness and camber distributions that were defined analytically by polynomial equations of various order, and, therefore, the surface coordinates of these airfoils are very smooth. A summary of many of the NACA airfoils and a detailed description of the equations used to generate their coordinates are presented in reference 1.

During the mid-1960's, the introduction of the supercritical airfoil concept by Dr. Richard Whitcomb of the Langley Research Center created a renewed interest in the development of an improved series of airfoils for applications at high subsonic and transonic flow conditions. Initial attempts to generate a series of supercritical airfoils from analytical expressions were unsuccessful because no theoretical methods were available to guide in the selection of adequate analytical expressions relating airfoil shape and the desired high-speed flow characteristics. During the early 1970's, Dr. Paul R. Garabedian of New York University developed a series of computer codes for the design and analysis of supercritical airfoils with no or very weak shocks. These codes, as described in reference 2, relied on a system of equations based on the method of complex characteristics in the hodograph plane and are solved numerically using conformal mapping and fast Fourier transform techniques.

During the mid- and latter-1970's, the NASA was also actively involved in the development of an improved series of subsonic airfoils for application to general aviation, glider, and commuter aircraft. Several computer codes were developed, such as the NASA/Lockheed-Georgia Multi-Component Airfoil Code (ref. 3) and the Eppler Low-Speed Airfoil Code (ref. 4) to aid in the design and analysis of these new airfoils. These codes utilize a variety of conformal mapping and distributed source- and vortex-singularity methods to obtain the potential flow characteristics of the airfoil and a variety of finite-difference and integral boundary-layer methods to obtain the viscous characteristics.

Both the subsonic and transonic airfoil codes have undergone extensive refinement and improvement in the past decade and are widely utilized by both the domestic and foreign scientific communities. The agreement between the theoretical and experimental characteristics of the airfoils designed using these codes has been generally excellent for airfoils with fully attached flow. The rapid development of the high-speed digital computer since the 1970's has greatly reduced the computer costs to design and analyze a new airfoil; therefore, it is no longer necessary to test a large number of airfoils to obtain one with the desired performance characteristics. The theoretical methods used in these computer codes are generally sensitive to the numerical techniques used and, as a result, often generate airfoils with wavy or unsmooth surface coordinates. The transonic airfoils have been shown to be particularly sensitive to coordinate smoothness both experimentally and theoretically.

The purpose of this report is to describe in detail the features of a computer code developed to smooth and scale airfoil coordinates. The smoothing code utilizes a variety of least-squares poynomial and cubic spline techniques to smooth the airfoil coordinates in the second derivative. The computer code has an internal Langley designation of "AFSMO" and consists primarily of a main controlling program and an input, a smoothing, a punch output, and plotting subroutines. Additional subroutines have been included to compute the camber and thickness distributions of the smoothed airfoil and to interpolate additional coordinates. The airfoil scaling program has an internal Langley designation of "AFSCL" and uses the camber and thickness distribution data generated by the AFSMO code to generate additional airfoil shapes with the same camber distribution and a scaled thickness distribution. The AFSCL code consists of a main controlling program, a subroutine to scale the coordinates, and a subroutine to fit a cubic spline through a set of input points. A detailed description of the smoothing and scaling methods used in these codes is presented in addition to a discussion of the possible applications of the codes. Appendices are included that describe the user input requirements, a sample input case, a sample output listing, sample plots, and tabulated listings for both programs.

SYMBOLS

a_i, b_i, c_i, d_i	polynomial coefficients
c	chord of airfoil
g	generalized cubic spline function
h	cubic spline interval
k	curvature
K	value of x/c where $\theta = \pi$
N	total number of upper and lower surface coordinates
t	thickness
S	least-squares cubic spline smoothing parameter
w	weighting factor
x,y	coordinates of airfoil
$\bar{x}$,$\bar{y}$	nondimensionalized x/c and y/c coordinates
$\hat{x}$,$\hat{y}$	coordinates in local camberline axis system
x_c,y_c	x/c and y/c coordinates of camberline
$\bar{y}'$	$d(y/c)/d\theta$
$\bar{y}''$	$d^2(y/c)/d\theta^2$
γ	local surface slope in $\hat{x}$- and $\hat{y}$-axis system
ϕ	local slope of camberline
θ	x-axis transformation function

Subscripts:

c	camber
i	iteration or element number
ℓ	lower surface
u	upper surface

DESCRIPTION OF SMOOTHING METHOD

Smoothing Criteria

The smoothness criteria used in the development of the smoothing method presented in this report is that the curvature distribution of the airfoil surface be continuous and smooth. The curvature, which is the reciprocal of the radius-of-curvature, is defined as

$$k = \frac{\left| d^2y/dx^2 \right|}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} \qquad (1)$$

The curvature distribution will be continuous, provided the airfoil contour is continuous with single-valued upper and lower surface coordinates. This can easily be determined by visual inspection of the initial input airfoil shape. The application of cubic spline functions to relate the smoothed y-axis airfoil coordinates to their smoothed second derivatives with respect to the x-axis will insure that the first derivatives are smooth and, consequently, that the curvature distribution is also smooth. Therefore, the smoothing method established is first to compute the second derivatives of the input airfoil coordinates, to smooth the second derivatives, and then to employ cubic spline functions to determine the new smoothed airfoil coordinates.

The second derivatives of the input y-coordinates are determined by fitting a least-squares polynomial to each coordinate and a specified number of points adjacent to the coordinate and then by analytical differentiation, computing the second derivative of the coordinate and its new y-value. This procedure is repeated for each y-coordinate until a new set of y-values are obtained which are then substituted for the previous set of y-values. The entire procedure is repeated and each time the sum of the squares of the differences between the current and prior second derivatives is computed. This iterative procedure continues until a specified number of iterations have been reached, or the sum of the squares quantity falls below a specified value or begins to oscillate.

X-Axis Transformation Function

Initial attempts to employ this least-squares polynomial technique to an input set of x- and y- coordinates resulted in large oscillations in the computed second derivatives and the new y-values from one iteration to the next. The oscillation was caused by the very rapid change in the curvature in the nose or leading-edge region which is characteristic of most airfoils. This problem was eliminated by utilizing an x-axis transformation function that stretches the axis in the nose region. One such transformation function used in the multi-component airfoil analysis code developed by Lockheed-Georgia (ref. 3) is

$$\bar{x} = \frac{1}{2}\left[1 - \cos(\theta)\right], \qquad (2)$$

where $0 \leq \theta \leq \pi$. However, this transformation function stretches the x-axis in both the leading- and trailing-edge regions. For application in the multi-component analysis code this stretching at both ends of the airfoil is necessary to ensure adequate definition of the maximum suction peak in the leading-edge region and to properly satisfy the Kutta flow condition in the trailing-edge region. To smooth an airfoil does not require as much stretching of the x-axis in the trailing-edge region as in the leading-edge region because the curvature is generally considerably less near the trailing edge of the airfoil. The hyperbolic functions behave in a manner similar to that for trigometric functions and, after considerable trial-and-error, the following transformation equation was found that reduced the amount of trailing-edge stretching and that could be mated with the trigonometric equation (2) for the leading edge:

$$\bar{x} = K \left\{ \tan^{-1} \left[\sinh (\theta - \pi/2) \right] + 1 \right\} , \qquad (3)$$

where $\pi/2 \leq \theta \leq \pi$. The constant K was determined by specifying that at θ equals π, the value of $\bar{x}$ is unity; therefore,

$$K = \frac{1}{\tan^{-1}\left[\sinh (\pi/2)\right] + 1} = 0.46278 \qquad (4)$$

By substituting the constant of 1/2 in equation (2) with the constant K from equation (3), the transformation equation for the leading-edge region becomes

$$\bar{x} = K \left[1 - \cos(\theta) \right] \qquad (5)$$

where $0 \leq \theta \leq \pi/2$.

The first and second derivatives of equation (3) are

$$\frac{d\bar{x}}{d\theta} = \frac{K}{\cosh (\theta - \pi/2)} \quad \text{and} \qquad (6)$$

$$\frac{d^2\bar{x}}{d\theta^2} = - \frac{K \sinh (\theta - \pi/2)}{\cosh^2 (\theta - \pi/2)} , \qquad (7)$$

respectively, and of equation (5) are

$$d\bar{x}/d\theta = K \sin (\theta) \quad \text{and} \qquad (8)$$

$$d^2\bar{x}/d\theta^2 = K \cos (\theta) , \qquad (9)$$

respectively. At $\theta = \pi/2$, the value of equations (3), (5), (6), and (8) is equal to K and the value of equations (7) and (9) is zero which verifies that the leading- and trailing-edge transformation equations are continuous at the matching point. A plot of the resultant transformation function and its first and second derivatives are presented in figure 1 and tabulated in table I.

The inverse of equation (3) is

$$\theta = \pi/2 + \sinh^{-1} \left[\tan \left(\frac{\bar{x}}{K} - 1 \right) \right] , \tag{10}$$

where $\sinh^{-1}(z) = \ln (z + \sqrt{z^2 + 1})$ and the inverse of equation (5) is

$$\theta = \cos^{-1} \left(1 - \frac{\bar{x}}{K} \right). \tag{11}$$

The first and second derivatives of the $\bar{y}$-coordinate with respect to $\bar{x}$ can be obtained from the derivatives with respect to the θ value using the following relationships:

$$d\bar{y}/d\bar{x} = \bar{y}' \frac{1}{d\bar{x}/d\theta} \tag{12}$$

$$d^2\bar{y}/d\bar{x}^2 = \frac{\bar{y}''(d\bar{x}/d\theta) - \bar{y}' \, (d^2\bar{x}/d\theta^2)}{(d\bar{x}/d\theta)^3} \tag{13}$$

Piecewise Least-Squares Polynomial Smoothing to Determine Second Derivative

The piecewise least-squares polynomial smoothing procedure requires that the independent variable increase monotonically to prevent simultaneous smoothing of upper and lower surface coordinates. This meant simply that the airfoil had to be unwrapped around the nose, which was easily accomplished by letting the lower surface transformation function run from 0 to $-\pi$ and the upper surface function run from 0 to +π. The remaining problem associated with computing the second derivatives using the least-squares polynomial procedure was to determine the number of points to include adjacent to the coordinate point and the degree of the polynomial. To determine these two quantities, the coordinates of the well-known NACA 0012 airfoil were input and various values were tried for each quantity until a combination was found that produced the best agreement between the calculated and theoretical values of the second derivatives. The number of points adjacent to the coordinate point was found to be 3 before and 3 after for a total of 7 points, and the degree of the polynomial was found to be 4. The computer code for the piecewise least-squares polynomial smoothing procedure is contained in subroutine LSQSMO.

Least-Squares Cubic Spline Smoothing of Second Derivative

After completion of the least-squares polynomial smoothing procedure, the resultant values of $\bar{y}''$ are input to subroutine CSDS which was formulated based on a method that fits a smooth cubic spline through a set of input data in a least-squares manner. The method defines a continuous cubic spline function in the form

$$g(\theta)_i = a_i h_i^3 + b_i h_i^2 + c_i h_i + d_i, \qquad (14)$$

where $h_i = (\theta - \theta_i)$ and $i = 1, 2, 3, \ldots, N-1$.

The coefficients a_i, b_i, c_i, and d_i are computed such that

$$\sum_{i=1}^{N} \left[\frac{g(\theta)_i - f_i}{\delta f_i}\right]^2 \leq S \qquad (15)$$

and $$\int_{\theta_1}^{\theta_N} \left[d^2g/d\theta^2\right]^2 d\theta \text{ is a minimum} \qquad (16)$$

where the smoothing parameter S is in the interval $(N - \sqrt{2N}) \leq S \leq (N + \sqrt{2N})$, N is the number of points, $f_i = \bar{y}_i''$, and δf_i is the allowable standard error deviation of f_i. A detailed description of the least-squares cubic spline method is presented in reference 5. After extensive application of the smoothing program to a wide range of airfoil shapes, the value of 10^{-4} was selected for standard error deviation and a conservative value of N was chosen for the smoothing parameter S.

Cubic-Spline to Compute New $\bar{y}$-Coordinate.

After obtaining the new smoothed second derivatives, the next step is to determine the corresponding smoothed $\bar{y}$-coordinate values that are also smooth and continuous in the interval between input points. The natural choice was a cubic spline which consists of defining the $\bar{y}$ coordinates between the interval end points with a third-order polynomial similar to equation (14) and solving for the coefficients so that the $\bar{y}$ coordinates and the first- and second-derivatives at the intersection with the adjacent interval are equal at each end. This ensures that the $\bar{y}$ coordinates, the slope, and the curvature are continuous and smooth. The cubic spline polynomial and its first- and second-derivatives are:

$$\bar{y}_i = a_i h_i^3 + b_i h_i^2 + c_i h_i + d_i \quad , \tag{17}$$

$$\bar{y}'_i = 3a_i h_i^2 + 2b_i h_i + c_i \quad , \tag{18}$$

and $$\bar{y}''_i = 6a_i h_i + 2b_i \tag{19}$$

where $h_i = (\theta - \theta_i)$.

At the two end points of the ith interval, the $\bar{y}$ coordinates are

$$\bar{y}_i = d_i \tag{20}$$

at $\theta = \theta_i$ and

$$\bar{y}_{i+1} = a_i h_i^3 + b_i h_i^2 + c_i h_i + d_i \tag{21}$$

at $\theta = \theta_{i+1}$, and the second derivatives are

$$\bar{y}_i'' = 2b_i \text{ or } b_i = \bar{y}_i''/_2 \tag{22}$$

at $\theta = \theta_i$ and

$$\bar{y}''_{i+1} = 6a_i h_i + 2b_i \text{ or } a_i = \frac{\bar{y}''_{i+1} - \bar{y}''_i}{6h_i} \tag{23}$$

at $\theta = \theta_{i+1}$.

Combining equations (20) through (23) and simplifying,

$$c_i = \left(\frac{\bar{y}_{i+1} - \bar{y}_i}{h_i}\right) - \left(\frac{\bar{y}''_{i+1} + 2\bar{y}''_i}{6}\right) h_i \tag{24}$$

At $\theta = \theta_i$, $\bar{y}''_i$ equals c_i and from the previous interval

$$\bar{y}'_i = 3a_{i-1}h^2_{i-1} + 2b_{i-1}h_{i-1} + c_{i-1} \tag{25}$$

where from a similar analysis,

$$a_{i-1} = \left(\frac{\bar{y}''_i - \bar{y}''_{i-1}}{6h_{i-1}}\right) , \tag{26}$$

$$b_{i-1} = \frac{\bar{y}''_{i-1}}{2} , \tag{27}$$

and

$$c_{i-1} = \left(\frac{\bar{y}_i - \bar{y}_{i-1}}{h_{i-1}}\right) - \left(\frac{\bar{y}''_i + 2\bar{y}''_{i-1}}{6}\right) h_{i-1}. \tag{28}$$

By substituting equations (26), (27), and (28) into equation (25) and setting equation (24) equal to (25), the following simplified form of the cubic-spline equation is derived:

$$\left(\frac{1}{h_{i-1}}\right) \bar{y}_{i-1} - \left(\frac{1}{h_i} + \frac{1}{h_{i-1}}\right) \bar{y}_i + \left(\frac{1}{h_i}\right) \bar{y}_{i+1} = \left(\frac{h_{i-1}}{6}\right) \bar{y}''_{i-1} + \left(\frac{h_{i-1} + h_i}{3}\right) \bar{y}''_i + \left(\frac{h_i}{6}\right) \bar{y}''_{i+1} \tag{29}$$

which represents a set of tridiagonal equations with i = 2, 3, 4, . ., N - 1. By specifying the desired $\bar{y}$ coordinates at the end points, the resultant N- by N-matrix equation can be solved with a simplified matrix inversion technique. The equations that define the $\bar{y}$ coordinates and the first- and second-derivatives in each interval are

$$\bar{y}(\theta) = \bar{y}''_i \left[\frac{(\theta_{i+1} - \theta)^3}{6h_i} - \frac{(\theta_{i+1} - \theta)\, h_i}{6} \right] +$$

$$\bar{y}''_{i+1} \left[\frac{(\theta - \theta_i)^3}{6h_i} - \frac{(\theta - \theta_i)}{6} h_i \right] + \left[\frac{\bar{y}_i(\theta_{i+1} - \theta) + \bar{y}_{i+1}(\theta - \theta_i)}{h_i} \right], \quad (30)$$

$$\bar{y}'(\theta) = \bar{y}''_i \left[\frac{h_i}{6} - \frac{(\theta_{i+1} - \theta)^2}{2h_i} \right] + \bar{y}''_{i+1} \left[\frac{(\theta - \theta_i)^2}{2h_i} - \frac{h_i}{6} \right] +$$

$$\left[\frac{\bar{y}_{i+1} - \bar{y}_i}{h_i} \right], \quad (31)$$

and

$$\bar{y}''(\theta) = \bar{y}''_i \left(\frac{\theta_{i+1} - \theta}{h_i} \right) + \bar{y}''_{i+1} \left(\frac{\theta - \theta_i}{h_i} \right) \quad (32)$$

where $h_i = (\theta_{i+1} - \theta_i)$. The computer code for this cubic spline method is contained in subroutine INVY in the airfoil smoothing program.

The initial application of the cubic spline method with the lower and upper trailing-edge $\bar{y}$ coordinates input for i=1 and N, produced airfoil shapes that did not generally pass through the nose

$\bar{y}$ coordinate computed during the previous least-squares smoothing step. This problem was partially overcome by first applying the cubic-spline method from the lower surface trailing edge to the nose and then from the nose to the upper surface trailing-edge coordinates. Although this procedure generated an airfoil shape that had the same $\bar{y}$ coordinate and second derivative at the nose when approaching from both the upper and lower surface, the first derivatives were not necessarily equal; therefore, the curvature was discontinuous at the nose. This additional problem was overcome by adding a small constant increment to the input second derivatives which would generate first derivatives at the nose that were more closely matched. The increment produced the same effect as a constant of integration, resulting in a very small global stretching or shrinking of the $\bar{y}$ coordinates. The value of the increment is determined iteratively using a simple Newton-Raphson technique which is very stable and generally converges in less than four iterations. The computer code for this iteration procedure is contained in subroutine YNEW in the airfoil smoothing program.

Camber and Thickness Distribution

By defining the smoothed airfoil shape with a cubic-spline function, the $\bar{y}$ coordinate and its derivatives can be computed at any desired θ-value with equations (30) through (32). Because of this capability, it was therefore possible to develop a method to compute a camberline and a thickness distribution for the smoothed airfoil. The equations for combining the camber and thickness distributions to obtain the upper surface coordinates of an airfoil are

$$x_u = x_c - t/2 \sin (\phi) \tag{33}$$

and $$y_u = y_c + t/2 \cos (\phi) \ , \tag{34}$$

and for the lower surfaces are

$$x_\ell = x_c + t/2 \sin (\phi) \tag{35}$$

$$y_\ell = y_c - t/2 \cos (\phi) \tag{36}$$

where x_c and y_c are the coordinates of the camberline, t is the local thickness, and ϕ is the local slope of the camberline. The airfoil generated with these equations will not be unique because a large number of other thickness and camber combining equations could be used to generate the same airfoil shape. However, given the shape of an airfoil, a unique camberline can be obtained which satisfies equations (33) through (36) by simply specifying that the absolute value of the slope at upper and lower points are equal in magnitude. The local slope is determined with respect to an axis system whose y-axis passes through the upper and lower surface points and whose x-axis passes through the mid-point of the line connecting the two points as illustrated in figure 2.

The equations for translating and rotating the input coordinates in the x- and y-axis system to the camberline $\hat{x}$- and $\hat{y}$-axis system are

$$\hat{x} = (\bar{x} - x_c) \cos(\phi) + (\bar{y} - y_c) \sin(\phi), \tag{37}$$

$$\hat{y} = (\bar{y} - y_c) \cos(\phi) - (\bar{x} - x_c) \sin(\phi). \tag{38}$$

The differentials with respect to $\bar{x}$ are

$$d\hat{x}/d\bar{x} = \cos(\phi) + \sin(\phi)\, d\bar{y}/d\bar{x} \tag{39}$$

$$d\hat{y}/d\bar{x} = \cos(\phi)\, d\bar{y}/d\bar{x} - \sin(\phi) \tag{40}$$

which combines to obtain the equation for the local slope

$$d\hat{y}/d\hat{x} = \frac{d\hat{y}/d\bar{x}}{d\hat{x}/d\bar{x}} = \frac{\cos(\phi)\, d\bar{y}/d\bar{x} - \sin(\phi)}{\sin(\phi)\, d\bar{y}/d\bar{x} + \cos(\phi)} , \tag{41}$$

where for a given set of upper and lower surface input points,

$$\phi = \tan^{-1}\left(\frac{\bar{y}_u - \bar{y}_\ell}{\bar{x}_u - \bar{x}_\ell}\right) . \tag{42}$$

To determine the camberline simply requires that for either an upper or lower surface input point, an opposite surface point be located which satisfies the criteria that

$$\left| d\hat{y}/d\hat{x} \right|_u = \left| d\hat{y}/d\hat{x} \right|_\ell \tag{43}$$

The computer code for the camberline technique is contained in subroutine CAMTK. The execution procedure in this subroutine starts the search for the camberline at the upper surface trailing edge and proceeds in a counterclockwise direction toward the nose of the airfoil. A simply linear interpolation procedure is used to locate the corresponding lower surface point which satisfies the camberline criteria. The search for the lower surface point is performed with an interpolation interval of 1/2000th of the chord. After locating the lower surface point, execution continues to the next upper surface point and the search for the lower surface point begins at the previously located point. This cycle continues until all of the upper surface points have been used. The leading-edge point of the camberline (where thickness equals zero) is computed by fitting a second-order polynomial to the three previous camberline points in the nose region and then extrapolating to determine the intersection of the camberline with the input airfoil contour. The only noteworthy problem that has occurred with the use of this technique has been difficulty locating the first few camberline coordinates for airfoils with reflexed (upward-turned) camberlines near the trailing edge. This problem can generally be overcome by simply reversing the input order of the upper and lower surface coordinates to the smoothing program which means that the search for the camberline will be reversed proceeding clockwise along the lower surface from the trailing edge to the nose.

DESCRIPTION OF COMPUTER PROGRAM

The airfoil smoothing computer program AFSMO consists of a main program, fifteen subroutines, and two function subprograms and is listed in Appendix A. The airfoil scaling computer program AFSCL consists of a main program and two subroutines and is listed in Appendix B. A description of the input data requirements for the airfoil smoothing program is presented in Appendix C and a corresponding description of the output for a sample case presented in Appendix D. Likewise, a description of the input data requirements for the airfoil scaling program is presented in Appendix E and the description of the output in Appendix F. The primary input and output quantities and execution sequence of each main program and subroutine are described in this section.

Program AIRSMO

The primary function of the main program AIRSMO is to control the overall execution of the airfoil smoothing process. After specifying and computing several global program constants, calls are made to subroutines PSEUDO and LEROY to initialize the plot vector file SAVPLT for subsequent postprocess plotting on a variety of plotters at Langley. The subroutine INPUT is then called which reads and prepares the user-supplied input data. The subroutine SMOXY is then called which smooths the input airfoil coordinates. If punched output data are desired by the user, subroutine PCARD is then called. All punched data are written on output file TAPE1 which can be disposed of in any manner the user desires.

If plots of the coordinates, first and second derivatives, and curvature of the smoothed airfoil are desired, calls are then made to subroutine PLOTAF and PLOTCK. If the user also desires to compute the camber and thickness distribution of the smoothed airfoil, subroutine CAMTK is then called. Then, if the user desires to interpolate additional smoothed airfoil coordinates, subroutine INTP is called. This entire execution procedure is repeated until all input cases have been input and smoothed. A call is then made to subroutine CALPLT to finalize the plot vector file.

The following arrays must be dimensioned and constants defined or checked in this program:

TITLE 80-column title for input case

XINT array containing $\bar{x}$ interpolation values

X,Y arrays containing reordered $\bar{x}$ and $\bar{y}$ coordinates

W array containing input weighing factors

YSMO array containing smoothed $\bar{y}$ coordinates

YPS array containing smoothed $\bar{y}'$ values

YPPS array containing smoothed $\bar{y}''$ values

THETA array containing θ-transformation values

PI value of π

RAD value of one radian $\pi/180$

CONS value of constant K defined by equation (4)

JREAD number of tape or file containing input data

JWRITE number of tape or file containing output data

IPRINT if equal to zero, the smoothing data generated during each iteration of the least-squares polynomial smoothing process in subroutine SMOXY and the interpolated data in PLOTAF and PLOTCK will be output

EPS convergence criteria used during least-squares polynomial smoothing process in subroutine SMOXY

DF standard deviation used during least-squares cubic spline smoothing process in subroutines SMOXY and CSDS

IERR if a nonzero value appears following a call to subroutine INPUT, it indicates that another case follows; and if it appears following a call to subroutine SMOXY, an error has occurred

Subroutine INTER

Subroutine INTER is a utility subprogram used to interpolate a y-value at a given x-value from an input table of x- and y-values. The interpolation can be performed using either a linear (straight line) or a weighted quadratic-equation fit of the y-values in the interpolation interval. The only restrictions are that the input table of x-values be single-valued and monotonically increasing or decreasing and that, for the weighted quadratic-equation fit, the input table of x-values contain at least four values. The initial execution step in this subroutine is a search to determine the x-interval containing the desired interpolation x-value ($x_{i-1} \leq x \leq x_i$). For the weighted quadratic-equation method, three y-values are interpolated:

(1) y_s by fitting a straight line between x_{i-1} and x_i,

(2) y_1 by fitting a quadratic equation between x_{i-2}, x_{i-1}, and x_i, and

(3) y_2 by fitting a quadratic equation between x_{i-1}, x_i, and x_{i+1}.

The deviations between the quadratic-equation and straight-line interpolated y-values are

$$\varepsilon_1 = \left|y_1 - y_s\right| \quad \text{and} \quad \varepsilon_2 = \left|y_2 - y_s\right| \tag{44}$$

The final interpolated y-value is obtained by linear weighting of the two deviations so that

$$y = w_1 y_2 + w_2 y_1 \quad , \tag{45}$$

where $w_1 = \dfrac{\Delta_1}{\Delta_1 + \Delta_2}$ and $w_2 = \dfrac{\Delta_2}{\Delta_1 + \Delta_2}$ (46)

$\Delta_1 = \varepsilon_1(x - x_{i-1})$ and $\Delta_2 = \varepsilon_2(x_i - x)$. (47)

For the linear interpolation method, the interpolated y-value is simply equal to y_s.

The following is a description of the parameters in the argument list for this subroutine:

XINT	input interpolation x-value
YINT	output interpolated y-value
N	number of values in input x and y arrays
X and Y	arrays containing input x- and y-values
JSTART	array index to begin search for interval containing XINT
JEND	array index of x-interval containing XINT
ICD	if equal to 0, the weighted quadratic-equation method is used, and, if equal to 1, the linear method is used

In the airfoil smoothing program, subroutine INTER is called by subroutine BADPT which checks for bad input airfoil coordinates and by subroutine SMOXY during the search for inflection points in the final smoothed airfoil contour.

Subroutine INPUT

The primary functions of subroutine INPUT are to read and print the input airfoil data and to prepare the input data in the proper format for input to the smoothing program. A detailed description of the required input airfoil data and the various options available for plotting and punching the output data is presented in the user-guide given in Appendix C. After reading the input data from the file JREAD and writing on output file JWRITE and if desired, the next execution step is to call subroutine BADPT to check the upper and lower surface coordinates for obvious bad points. If no errors occur during the check for bad points and again if desired, subroutine TRNSRT is called to translate and rotate the input airfoil to an axis system coincident with the longest chord of the airfoil.

The next execution step is to reorder the input coordinates, which are input from the leading edge to the trailing edge for each surface, from the lower surface trailing edge clockwise around the airfoil to the upper surface trailing edge. The reordered coordinates are also nondimensionalized by the chord length and, at the same time, the equivalent transformation θ-values computed using equations (10) and (11). If, instead of x and y coordinates, the $\bar{y}$ coordinates, $\bar{y}'$ values, or $\bar{y}''$ values as a function of θ are input, the equivalent $\bar{x}$ values are computed using equations (3) and (5).

The following input quantities are defined in this subroutine:

ITER	allowable number of smoothing iterations
IPLOT	plotting option
IPUNCH	punch output option
IOP	input airfoil coordinate option
ICAMTK	camber and thickness distribution option
INTR	interpolation option
IBAD	bad coordinate check option
ITRN	translation and rotation option
YLTE, YNOSE, YUTE	input desired $\bar{y}$ coordinates at the lower surface trailing edge, the nose, and the upper surface trailing edge, respectively

NINT number of input interpolation $\bar{x}$ values

CNEW desired chord of interpolated $\bar{y}$ coordinates (all $\bar{y}$ coordinates computed in subroutines INTP are multiplied by CNEW)

NP number of elements in output arrays X, Y, W, THETA, YPS, and YPPS

NOSE array index of nose point after reordering the coordinate

CHORD computed longest chord length

IERR if not equal to zero, the last input case has been read or an error occurred during the calls to subroutine IBAD

TITLE input 80-column title

X output array containing reordered $\bar{x}$ coordinates

Y output array containing reordered $\bar{y}$ coordinates for IOP=0 or 1

W output array containing reordered weighing factors

THETA output array containing equivalent θ values

YPS output array containing $\bar{y}'$ values for IOP=2

YPPS output array containing $\bar{y}''$ values for IOP=3

The following arrays and constants are used internally in this subroutine:

XL array containing input lower surface x coordinates if IOP=0 and θ-values if IOP=0.

YL array containing input lower surface y coordinate if IOP=0, $\bar{y}$ coordinates if IOP=1, $\bar{y}'$ values if IOP=2, and $\bar{y}''$ if IOP=3

WL array containing input lower surface weighting factors

XU,YU,WU same as XL, YL, and WL except for upper surface

NL number of elements in XL, YL, and WL arrays

NU number of elements in XU, YU, and WU arrays

ITRMAX maximum number of allowable smoothing iterations

TOLR allowable deviation between input and interpolated $\bar{y}$ coordinate in subroutine BADPT

NMAX maximum number of NU or NL values

Subroutine TRNSRT

The function of subroutine TRNSRT is to translate and rotate the input airfoil coordinates to an axis system coincident with the longest chord. The longest chord is defined as the distance from the trailing-edge bisector to the farthest input coordinate in the nose region of the airfoil. The translation and rotation equations are identical to equations (37) and (38) where x_c and y_c are the nose coordinates and ϕ is the angle between the longest chord and the input x-axis. After the input coordinates have been translated and rotated, the input coordinate and weighing factor arrays are reloaded with the newly defined transformed values. The following parameters are used internally in this subroutine:

ANGLE computed angle of longest chord and input x-axis

XNOSE, YNOSE computed nose coordinate of longest chord

XTE, YTE computed coordinates of trailing-edge bisector of longest chord

Subroutine BADPT

The function of subroutine BADPT is to identify and possibly to correct input $\bar{y}$ coordinates whose corresponding interpolated values exceeds a specified tolerance. The user may execute a call to this subroutine by specifying a nonzero value for the parameter IBAD in subroutine INPUT; however, the call should be made only if the user has a concern about possible bad points or excessive waviness in the input coordinates. Following entry to this subroutine the θ equivalent of each input $\bar{x}$ coordinate is computed for use during the interpolation process. Then for each input $\bar{y}$ coordinate, a corresponding interpolated value is obtained using the weighted quadratic-equation method of subroutine INTER with input arrays loaded with the remaining $\bar{y}$ coordinates and θ values. (Note that the input $\bar{y}$ coordinate itself is not loaded.) If the deviation between the input and interpolated $\bar{y}$ coordinate exceeds a specified tolerance, the interpolated $\bar{y}$ coordinate is flagged as being out-of-tolerance, the interpolated value substituted, and then execution continues to the next point. If, however, during this interpolation process, two consecutive points are found to be out-of-tolerance, an error flag is set which will terminate the execution of the particular input case. The following additional parameters are used in this subroutine:

X,Y input arrays containing either upper or lower surface $\bar{x}$ and $\bar{y}$ coordinates

ISURF if equal to 1, indicates upper surface coordinates input, and, if equal to 2, lower surface

TI work array containing all θ values except value at desired interpolation point

YI work array containing all $\bar{y}$ coordinates except value at the desired interpolation point

YN temporary array containing interpolated $\bar{y}$ coordinates

IERR if output with a nonzero value, two adjacent points are out-of-tolerance

Subroutine SMOXY

The primary function of subroutine SMOXY is to perform the iterative smoothing process and is, therefore, the most important subroutine in the entire airfoil smoothing program. The basic inputs to this subroutine are the initial $\bar{x}$ and $\bar{y}$ coordinates, either $\bar{y}'$ or $\bar{y}''$, the transformed θ values, weighting factors for each input point, and the input option parameter IOP which specifies the type of input data. If either $\bar{y}'$ or $\bar{y}''$ are input instead of the $\bar{y}$ coordinates, the desired trailing edge and nose $\bar{y}$ coordinates must also be input.

After entry to the subroutine, the input option parameter is checked to determine the type of input data. If the first derivatives $\bar{y}'$ are input (IOP = 2), two sets of second derivatives $\bar{y}''$ are computed. One set is computed using the least-squares polynomial smoothing method (subroutine LSQSMO) and the second set, using the least-squares cubic-spline method (subroutine CSDS). Each set of second derivatives and the desired trailing-edge and nose $\bar{y}$ coordinates are then input to subroutine YNEW which computes a corresponding set of $\bar{y}$ coordinates. These $\bar{y}$ coordinates and their corresponding second derivatives are then used to compute a new set of first derivatives using the spline equation (31). The $\bar{y}$ coordinates and the sum-of-the-squares of the difference between the original input and computed first derivatives are then computed for each set and the set with the smallest sum is chosen for subsequent smoothing.

If the second derivatives $\bar{y}''$ and the desired trailing-edge and nose $\bar{y}$ coordinates are input (IOP=3), a corresponding set of $\bar{y}$ coordinates are computed with subroutine YNEW and a set of first derivatives computed with spline equation (31). Then, regardless of the input option, program execution proceeds to the iterative smoothing process. Prior to the start of this iteration cycle, a search is made of the upper and lower surface $\bar{y}$ coordinates to determine the maximum upper surface and minimum lower surface values. During each smoothing cycle, these two coordinates are heavily weighted in an attempt to insure that the maximum thickness of the final smoothed airfoil is reasonably close to that of the original input airfoil.

As discussed in the method section of this report, the initial step in the smoothing process is to determine the smoothed second derivatives of the input $\bar{y}$ coordinates using an iterative piecewise least-squares polynomial smoothing method. During this iteration process, each call to subroutine LSQSMO produces a new set of $\bar{y}$ coordinates and their corresponding first and second derivatives. The next step in the iteration process is to compute the sum-of-the-squares of the difference between the current and previous set of second derivatives and then to check the sum to insure that the current value is less than the previous value. This will determine whether or not the iteration process is converging. If the process is diverging, the iteration cycle is terminated, an appropriate error message printed, and execution proceeds to the next step. If the process is converging, the next iteration input $\bar{y}$ coordinates for subroutine LSQSMO are computed using the following weighting procedure:

$$\text{If} \quad \Delta_{i-1} = \left[\bar{y}_N - \bar{y}_I\right]_{i-1}$$

$$\text{and} \quad \Delta_i = \left[\bar{y}_N - \bar{y}_I\right]_i \qquad (48)$$

and if the sign or magnitude of Δ_i equals Δ_{i-1}, then

$$(\bar{y}_N)_{i+1} = \frac{1}{2}(\bar{y}_I + \bar{y}_N)_i \tag{49}$$

and, if not, the Newton-Raphson formula

$$(\bar{y}_N)_{i+1} = (\bar{y}_I)_{i-1} - \left(\frac{\Delta_{i-1}}{\Delta_i - \Delta_{i-1}}\right) \cdot \left[(\bar{y}_I)_i - (\bar{y}_I)_{i-1}\right] \tag{50}$$

is used, where i is the iteration number, I indicates input value, and N indicates new value computed by LSQSMO. After computing the new weighted coordinates, the sum-of-the-squares difference of the second derivatives is checked to see if it is less than the specified convergence value EPS. However, if the value of the difference sum is greater than the convergence value, the iteration cycle is repeated. If the value has converged or the iteration cycle begins to diverge, program execution proceeds to the next step which is to smooth the second derivatives one additional time using the least-squares cubic-spline method of subroutine CSDS. The additionally smoothed second derivatives and the final trailing-edge and nose $\bar{y}$ coordinate from the piecewise least-square polynomial smoothing process are then input to subroutine YNEW which computes a corresponding final set of smoothed $\bar{y}$ coordinates.

The final smoothed coordinates are then checked for relative smoothness by another call to LSQSMO with all the coordinate weighting factors set equal to 1.0. The next execution step is to compute a corresponding set of final smoothed first derivatives using spline equation (31). Then the final smoothed first and second derivatives with respect to $\bar{x}$ and the curvature are computed and printed in addition to the original input and final smoothed coordinates and the final smoothed first and second derivatives $\bar{y}'$ and $\bar{y}''$.

Following the detailed printout step, a check is made for negative thickness or crossover between the upper and lower surface near the trailing edge of the airfoil. During the least-squares polynomial smoothing process, the input weighting for the trailing-edge coordinates are multiplied by a factor of 7 to help ensure that the final smoothed airfoil has the same trailing-edge thickness as the original input airfoil. In spite of this additional weighting, the final smoothed airfoil will often have negative trailing-edge thickness; especially if the input airfoil has zero or a very small trailing-edge thickness. If a crossover is discovered during this step, an error message is printed, an error flag set, and execution returned to the calling program.

If no crossover is discovered, the next and final step is to determine the location of all inflection points (i.e. $\bar{y}' = 0$) in the final smoothed airfoil. This step is accomplished by checking each θ-interval of the final airfoil for θ locations where the first derivative spline equation (31) is equal to zero. This equation can be written as the quadratic equation

$$a\theta^2 + b\theta + c = 0 \tag{51}$$

with

$$\begin{aligned} a &= \left(\frac{\bar{y}''_i - \bar{y}''_{i+1}}{2h_i}\right) \\ b &= \left(\frac{\bar{y}''_{i+1}\theta_i - \bar{y}''_i\,\theta_{i+1}}{h_i}\right) \\ c &= \left(\frac{\bar{y}''_i\,\theta^2_{i+1} - \bar{y}''_{i+1}\theta^2_i}{2h_i}\right) + \frac{h_i}{6}\,(\bar{y}''_{i+1} - \bar{y}''_i) - \left(\frac{\bar{y}_{i+1} - \bar{y}_i}{h_i}\right) \end{aligned} \tag{52}$$

where $h_i = \theta_{i+1} - \theta_i$. The real solutions to this equation which lie within the θ-interval are the inflection points. All inflection point locations and the results of the final smoothness check are then printed and control returned to the calling program.

A description of the parameters in the argument list for this subroutine is presented in the description of program AIRSMO and the subroutine INPUT. The following parameters are used internally:

WT	multiplier for weighting of maximum thickness coordinates
YPP and YPPU	work arrays containing current values of $\bar{y}''$
YUSMO and YN	work arrays containing current values of $\bar{y}$
WK, A, and DUM	internal work arrays
SUMY	array containing sum-of-squares differences from least-squares polynomial smoothing process
JMAXL and JMAXU	array index values for the minimum lower surface $\bar{y}$ and for the maximum upper surface $\bar{y}$, respectively
GP and GPP	$d\bar{x}/d\theta$ and $d^2\bar{x}/d\theta^2$
DYDX and DY2DX	$d\bar{y}/d\bar{x}$ and $d^2\bar{y}/d\bar{x}^2$
CURV	curvature k
RLE	leading-edge radius (1/k at nose)

Subroutine YNEW

The primary function of subroutine YNEW is to control the iterative procedure that computes a set of new $\bar{y}$ coordinates from an input set of second derivatives and desired trailing edge and nose coordinates. The new set of coordinates can be computed using two different solution approaches. For the first approach (IPT = 0), the resultant simultaneous cubic-spline equations solved are generated using the combined upper and lower surface second derivatives and settinq the end conditions eq気ual to the leading- and trailing-edge coordinates. The value of the first derivative at the nose will, of course, be the same whether approached from either the upper or lower surface; however, the $\bar{y}$-coordinate at the nose may differ from the desired input value. The desired input nose coordinate can be obtained by adding a small constant incremental value to the input second derivatives. This small value acts the same as a constant of integration resulting in a small stretching or shrinking of the computed $\bar{y}$ coordinates. The incremental value is determined in this subroutine using the simple iterative Newton-Raphson equation

$$\Delta x_{i+1} = \Delta x_i - \frac{f(\Delta x_i)}{f'(\Delta x_i)} \qquad (53)$$

where Δx represents the incremental value, $f(\Delta x)$ the difference between the desired and computed nose coordinates, $f'(\Delta x)$ the slope of the difference curve (determined using simple differencing), and i the iteration number.

For the second approach (IPT = 1), the resultant simultaneous cubic-spline equations solved are generated in a piecewise manner first using the lower surface second derivatives and setting the end conditions equal to the trailing-edge and nose coordinates, and then using the corresponding quantitites for the upper surface. This approach ensures, of course, that the resultant airfoil will have the desired nose coordinate; however, the slope at the nose may differ when approached from the upper and lower surfaces. Here again, like the first approach, a better match can be obtained by adding a small incremental value to the input second derivatives. This increment is determined using the same iterative Newton-Raphson equation as that used for the first approach except the Δx represents the difference between upper and lower surface first derivatives at the nose. Both approaches should theoretically produce the same incremental values; however, experience has shown that the convergence of the second approach is generally quicker and more stable.

The following additional parameters are used internally in this subroutine:

DUM and WK	internal work arrays
DELTA	incremental value added to second derivatives

Subroutine INVY

The function of this subroutine is to compute a set of $\bar{y}$ coordinates from an input set of second derivatives and desired $\bar{y}$ coordinates at the start and end of the set. The input second derivatives and transformation θ-values are used to compute a matrix of simultaneous equations using the cubic-spline equation (29). The resultant matrix is tridiagonal with two less equations than unknowns and relates the second derivatives $\bar{y}''$ and the corresponding $\bar{y}$ coordinates. The two remaining unknowns are specified as the desired $\bar{y}$ coordinates at the start and end of the set. The solution of the resultant matrix is greatly simplified because only the diagonal elements d_i and the two adjacent elements e_i and f_i differ from zero. Using the Crout reduction method described in reference 6, the solution becomes a simple back substitution

$$\bar{y}_N = \bar{c}_N \text{ for } i=N$$

and $\bar{y}_i = \bar{c}_i - \bar{f}_i \bar{y}_{i+1}$ for $i = N-1, N-2, \ldots 1$ (54)

where

$$\bar{d}_i = d_i - e_i \bar{f}_{i-1}$$

$$\bar{f}_i = f_i / \bar{d}_i \tag{55}$$

and $\bar{c}_i = \dfrac{c_i - e_i \bar{c}_{i-1}}{\bar{d}_i}$.

The tridiagonal terms from equation (29) are

$$e_i = 1/h_{i-1}$$

$$d_i = -1/h_{i-1} - 1/h_i$$

$$f_i = 1/h_i \tag{56}$$

and $c_i = \left(\frac{h_{i-1}}{6}\right) \bar{y}''_{i-1} + \left(\frac{h_{i-1} + h_i}{3}\right) \bar{y}''_i + \left(\frac{h_i}{6}\right) \bar{y}''_{i+1}$

At the ends the coefficient terms are

$$d_1 = 1, \; f_1 = 0, \; c_1 = \bar{y}_1 \tag{57}$$

and

$$e_N = 0, \; d_N = 1, \; c_N = \bar{y}_N \; .$$

The following is a description of the parameters in argument list for this subroutine:

X	input array containing θ values
YPP	input array containing $\bar{y}''$ values
NS	index of start element
NE	index of end element
Y	output array containing $\bar{y}$ coordinates
YSTART	desired $\bar{y}$ coordinate at start
YEND	desired $\bar{y}$ coordinate at end
A	internal work array

Subroutine LSQSMO

The function of this subroutine is to smooth and compute the second derivatives of an input set of $\bar{y}$ coordinates using the piecewise least-squares polynomial method described in the previous method section. The subroutine smooths each coordinate by fitting a least-squares polynomial of the 4th degree through the input coordinate and six adjacent coordinates. If possible, the six coordinates used are the three coordinates just prior to and the three just after the input coordinate; otherwise, six consecutive coordinates are used. Prior to the execution of the smoothing process, a check is made of the three corresponding upper and lower surface coordinates adjacent to the nose coordinate to determine whether or not the input airfoil is symmetric about the θ-axis in the nose region. If the airfoil is symmetric in the nose, the smoothing process is performed in the clockwise direction for the upper surface and counterclockwise for the lower surface; otherwise, it is performed clockwise for both surfaces.

During the smoothing process, each coordinate is given the specified input weighting factor and the six adjacent coordinates are given a weighting of 1.0. The maximum and minimum thickness coordinates are also given an additional weighting equal to the parameter WT times the input value. In a similar manner, the upper and lower surface trailing-edge coordinates are given an additional weighting of 7 times the input value. After computing the coefficients of the least-squares polynomial for each coordinate, a new $\bar{y}$-coordinate value, the first-, and the second-derivatives are computed using equation (17), (18), and (19), respectively.

The following is a description of the parameters in the argument list and the internally used arrays and constants:

X input array of θ values

Y input array of $\bar{y}$ values

W input array of weighting factors

YN output array of smoothed $\bar{y}$ coordinates

YP output array of first derivatives $\bar{y}'$

YPP output array of second derivatives $\bar{y}''$

N number of input coordinates

IMAX and JMAX array index of maximum and minimum thickness coordinates

NOSE array index of nose coordinate

WT additional weighting factor for maximum and minimum thickness coordinate

EPS allowable deviation between corresponding upper and lower surface θ and $\bar{y}$ values in the nose region

ISYM if equal to zero, input airfoil is symmetric in nose region

XI, YI, WI arrays containing 7 consecutive values of θ, $\bar{y}$, and w

A array containing elements of symmetric least-squares matrix

B array containing coefficients of resultant 4th order least-squares polynomial

Subroutine CSDS

The function of subroutine CSDS is to fit a least-squares cubic spline through a set of input θ values and either the $\bar{y}$ coordinates or the second derivative $\bar{y}''$. A very detailed description of theory and computer coding associated with this subroutine is presented in reference 5 and, therefore, will not be presented in this report. This subroutine is also a part of the standard math-library subprogram package on the Langley CDC computer system and is identified by the same call name and parameter list. A complete description of the input and output parameters are presented at the beginning of the listing of the subroutine in Appendix A.

Subroutine PCARD

The function of subroutine PCARD is to write the final smoothed data on an output file (TAPE1) for postprocess disposal to a desired output device. The case title is written on the output file initially and is followed by a card image containing the value of the input option (IOP parameter) corresponding to the output option

(IPUNCH parameter). Then for the upper and lower surface, the number of coordinates is written on the output file followed by one of four types of smoothed output data as specified by the value of the output option parameter IPUNCH. The four types of output data are as follows:

IPUNCH = 1 x-coordinate, smoothed y-coordinate, and weighting

IPUNCH = 2 θ-value, smoothed $\bar{y}$ coordinates, and weighting

IPUNCH = 3 θ-value, smoothed $\bar{y}'$, and weighting

IPUNCH = 4 θ-value, smoothed $\bar{y}''$, and weighting

If IPUNCH equals 3 or 4, the $\bar{y}$ coordinates of the lower surface trailing edge, the nose, and the upper surface trailing edge are also written on the output file. All data are written on the output file in a format suitable for input to the airfoil smoothing program. Except for the IPUNCH parameter, all other parameters in the argument list are fully defined in the description of subroutine INPUT.

Subroutine PLOTAF

The function of subroutine PLOTAF is to plot the input and smoothed $\bar{y}$ coordinates, smoothed $\bar{y}'$, and smoothed $\bar{y}''$ versus the θ values (IPLOT=1) and to plot the input and smoothed $\bar{y}$ coordinates versus the input $\bar{x}$ coordinates (IPLOT=2). All plots are scaled for postprocess plotting on the Langley 33-inch CALCOMP drum plotters. The called subroutines CALPLT, NOTATE, AXES, PNTPLT, LINE, and NFRAME are all part of the Langley plotting subroutine package and are available by attaching the CALCOMP direct-access library file. Prior to plotting the smoothed $\bar{y}$ and $\bar{x}$ coordinates and the smoothed $\bar{y}'$ values, additional values are interpolated at each degree of θ from -180 to +180 degrees. The ordinate axes are automatically scaled to insure that all input values will be plotted. A sample of the two types of plots generated by this subroutine is presented in figure 3 for IPLOT=1 and in figure 4 for IPLOT=2. Except for the IPLOT parameter, all other input parameters are fully defined in the description of subroutine INPUT.

Subroutine PLOTCK

The function of subroutine PLOTCK is to plot the square root of the local smoothed curvature versus the θ-transformation value (IPLOT=3). Prior to plotting the curvature, additional values are interpolated at each one-half degree of θ from -180 to +180 degrees. By plotting the square root of the curvature rather than just the curvature, the very large curvature peaks in the nose region of the airfoil are reduced and the normally low curvatures in the trailing-edge regions are increased and, as a result, a more evenly proportioned plot is generated. A sample of the type of plot generated by this subroutine is presented in figure 5. All input argument parameters are fully defined in the description of subroutine INPUT.

Subroutine CAMTK

The function of subroutine CAMTK is to compute the camber and thickness distribution of the final smoothed airfoil. A detailed explanation of the method used to compute the camberline is presented in the method section of this report. The first execution step in the subroutine is to load the x and y coordinates and y values into separate arrays for the upper and lower surfaces from the nose to the trailing edge. The first derivatives dy/dx are then computed at each input point on the upper surface.

The next execution step is the search for the camberline. As previously stated in the method section, the search begins at the upper surface trailing-edge point and proceeds counterclockwise along the upper surface to the nose point. At each upper surface point, a simple linear interpolation procedure is used to locate the corresponding lower surface point that satisfies the camberline criteria of equal magnitudes of the local upper and lower surface slopes with respect to an axis system aligned with the local camberline. The search for the lower surface point is performed with an interpolation interval of 1/2000th of the chord. After locating the lower surface point, execution continues to the next upper surface point and the search begins on the lower surface at the previously located point and proceeds clockwise toward the nose point.

After completing the camberline search for each point on the upper surface, the next execution step is to locate the intersection of the camberline with the airfoil leading edge which is the location of zero thickness. This intersection is found by fitting a second-order polynomial to the previous three camberline coordinates and then extrapolating to find the intersection with the nose region which is defined with cubic-spline functions. The upper surface coordinates, corresponding lower surface coordinates, camberline coordinates, thickness, and slope of the camberline are printed at each step during the search for the camberline and the nose intersection points. An error term is also printed for each point and represents the absolute value of the difference between the local slopes of the upper and lower surface camberline search points with respect to the local camberline-axis system.

The next execution step is to write the camber and thickness distribution data on an output file (TAPE1) for possible input to the airfoil scaling program AFSCL. This execution step is activated only if the value of the IPUNCH input parameter equals 5. The final execution step, if the value of the input KPLOT parameter is nonzero (IPLOT = 4, 8, 9, or 10), is to plot the camber and thickness distribution data. A sample of the type of plot generated is presented in figure 6. The camberline coordinates are plotted at the bottom part of the figure, the half-thickness distribution at the center, and the upper and lower surface search points at the top part of the figure.

A description of the parameters in the argument list for this subroutine is presented in the description of program AIRSMO and subroutine INPUT. The following parameters are used internally:

TU and TL — temporary arrays containing input upper and lower surface θ-values from nose to trailing-edge points.

YU and YL — temporary arrays containing input upper and lower surface smoothed $\bar{y}$ coordinates

YPPU and YPPL — temporary arrays containing input upper and lower surface $\bar{y}''$ values

DYXU — array containing $d\bar{y}/d\bar{x}$ values for upper surface

XLS and YLS arrays containing $\bar{x}$ and $\bar{y}$ coordinates of lower surface camberline search points

TH array containing value of slope of camberline

XC and YC arrays containing x_c and y_c coordinates of camberline

TK array containing the half-thickness values

NM number of interpolated points allowed on the lower surface

NT number of camberline coordinates

DU and DL slope of the upper and lower surface search points with respect to the local camberline axis system

Subroutine INTP

The function of subroutine INTP is to interpolate additional smoothed airfoil coordinates. This subroutine is called if the user specifies a value of 1 or 2 for the parameter INTR read by subroutine INPUT. If the value of INTR equals 1, the interpolation is is performed at a standard set of 57 $\bar{x}$ values loaded internally in the subroutine and defined as follows:

$\bar{x}$ = 0.0, 0.00025, 0.0005, 0.00075, 0.001, 0.0015, 0.002, 0.0025, 0.005, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1, 0.125, 0.15, 0.175, 0.2, 0.225, 0.25, 0.275, 0.3, 0.325, 0.35, 0.375, 0.4, 0.425, 0.45, 0.475, 0.5, 0.525, 0.55, 0.575, 0.6, 0.625, 0.65, 0.675, 0.7, 0.725, 0.75, 0.775, 0.8, 0.825, 0.85, 0.875, 0.9, 0.925, 0.95, 0.97, 0.98, 0.99, 1.0.

If the value of INTR equals 2, the desired $\bar{x}$ values are input by the user and may include up to 100 values as specified by the parameter NINT. The interpolation is performed for the upper and then the lower surfaces using the cubic-spline equations (30), (31), and (32). The derivatives $d\bar{y}/d\bar{x}$ and $d^2\bar{y}/d\bar{x}^2$ and the curvature are also computed and printed for each $\bar{x}$ value. The user must also input a value for the parameter CNEW which is the desired value of the chord. The $\bar{x}$ and $\bar{y}$ interpolated coordinates are multiplied by CNEW and printed as x and y coordinates. If the value of the parameter IPUNCH equals 6, the interpolated x and y coordinates are written on the output file (TAPE1) for postprocess disposal to a desired output device. A description of the parameters in the argument list for this subroutine is presented in the description of program AIRSMO and subroutine INPUT.

Subroutine COORD

The function of subroutine COORD is to interpolate a value for for $\bar{y}$, $d\bar{y}/d\bar{x}$, $d^2\bar{y}/d\bar{x}^2$, and the curvature at a specified value of θ using the cubic-spline equations (30), (31), and (32). The following subroutine constants are used internally:

TI	input θ value
YI	interpolated $\bar{y}$-coordinate
DYDX	interpolated first derivative $d\bar{y}/d\bar{x}$
DY2DX	interpolated second derivative $d^2\bar{y}/d\bar{x}^2$
CURV	interpolated curvature

Function Subprograms SINH and COSH

The function of these two function subprograms is to compute the hyperbolic sine and cosine in terms of the exponential function. The relationships are

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2} \tag{58}$$

and

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2} , \tag{59}$$

respectively.

Program SCALE

The primary function of program SCALE is to read the input data and control the execution of the airfoil scaling process. The camber and thickness distribution data input to this program are generated by the subroutine CAMTK in the airfoil smoothing program AFSMO. After specifying and computing several global program constants, the first execution step is to read the input data. A detailed description of the input data and the required formats are discussed in the user-guide presented in Appendix E. After reading the input data, calls are made to subroutines PSEUDO and LEROY to initialize the plot vector file SAVPLT for subsequent postprocess plotting on a variety of plotters at Langley. The input x_c coordinates of the input camberline are then checked to insure monotonically increasing order. The equivalent θ value for each camberline x_c coordinate is then computed.

The next execution step is to compute the x_c location and the magnitude of the maximum value of the input half-thickness distribution. A cubic spline is fit through the input thickness data and then all locations and corresponding thickness values where the first derivative of the spline function equals zero are computed using equations (51) and (52). The location of the maximum value is then determined and printed on the output file. If the value of the input parameter IOP equals 1, the slope of the camberline coordinates are then computed using spline equation (31). The angular value of the slope is then obtained by computing the arctangent of the value of the first derivative.

The next step is to call the scaling subroutine SCTK to generate first the coordinates of the airfoil with the input maximum thickness-chord ratio and then the coordinates of the airfoil with each of the desired scaled maximum thickness-chord ratios. The final execution step is to call subroutine CALPLT to finalize the plot-vector file SAVPLT.

The following arrays must be dimensioned and constants defined in this program:

XC and YC   arrays containing input x_c and y_c coordinates of the camberline

TK   array containing input half-thickness distribution t/c/2

TH   array containing input camberline slopes ϕ

THETA   array containing computed θ values

YPP   array containing computed second derivatives

$$\frac{d^2y_c}{dx_c^2}$$

TKNEW   array containing input values of desired maximum thickness-chord ratios

TITLE   80-column title for input case

VAR and WK   work arrays

JWRITE   number of tape or file containing output data

JREAD	number of tape or file containing input data
NTMAX	maximum number of allowable elements in TKNEW array
PI	value of π
RAD	value of one radian $180/\pi$
CONS	value of constant K defined by equation (4)
NT	number of elements in XC, YC, TK, and TH arrays
IOP	camberline slope option
IPLOT	plotting option
IPUNCH	punch output option
LT	number of desired input maximum thickness values
TKMAX	value of the maximum thickness-chord ratio of the input thickness distribution
DELTA	x_c location of TKMAX
IERR	if nonzero, error occurred during generation of scaled airfoil in subroutine SCTK

Subroutine SCTK

The function of subroutine SCTK is to scale the coordinates of an input airfoil from the input maximum thickness-chord ratio to a new desired maximum thickness-chord ratio. The first execution step is to generate the coordinates of the baseline airfoil by combining the input camber and the scaled thickness distributions using equation (33) and (34) for the upper surface and equations (35) and (36) for the lower surface. Each scaled thickness distribution is obtained by multiplying the input thickness distribution by the ratio of the desired-to-input maximum thickness-chord ratio. This procedure is simple; however, several problems may occur which require special handling.

If the value of the input camber distribution is nonzero in the trailing-edge region, the airfoil generated may not have either an upper or lower surface $\bar{y}$ coordinate at the trailing-edge location where $\bar{x}$ equals 1.0. To eliminate this problem, a second-order polynomial is fit to the last three computed coordinates near the trailing edge on each surface and a new $\bar{y}$ coordinate either extrapolated or interpolated at $\bar{x}$ equals 1.0. Also, if the camber distribution is nonzero in the nose region, the airfoil generated may have $\bar{x}$ coordinates that are less than 0.0. This problem is eliminated by translating and stretching or shrinking the coordinates of the airfoil so that the nose of the adjusted airfoil is at $\bar{x}$ equals 0.0 and the trailing edge at $\bar{x}$ equals 1.0. The only other problem that may occur is the possible generation of either upper or lower surface $\bar{x}$ coordinates that are not monotonically increasing from nose to trailing edge. This particular problem cannot be eliminated; therefore, a check is made to see if it occurred and, if so, an error message is printed, an error flag set, and execution returned to program SCALE.

The upper and lower surface $\bar{x}$ and $\bar{y}$ coordinates are multiplied by the value of the parameter CNEW and then loaded into separate arrays from the nose to the trailing edge. The coordinates, input camber distribution, and scaled thickness distributions are then printed. If the IPUNCH parameter is nonzero, the scaled airfoil coordinates are then written on the output file TAPE1 in a format suitable for input to the smoothing program. If the IPLOT parameter is nonzero, the next and final execution step is to plot the scaled airfoil and its corresponding camber and thickness distributions as illustrated in figure 7. A description of the parameters in the argument list for this subroutine is presented in the description of program SCALE.

Subroutine CUBSPL

The function of subroutine CUBSPL is to fit a cubic spline through an input set of x and y values. The input data are used to compute a matrix of simultaneous equations using the cubic spline equation (29) with the unknowns being the second derivatives at each input point. This tridiagonal matrix has two less equations than unknowns; therefore, the second derivative at end points of the data set must be specified. In this subroutine second derivatives at the end points are computed by fitting a second-order polynomial of the form

$$y = ax^2 + bx + c \tag{60}$$

to each end point and its two adjacent points and then differentiating to determine the second derivative which is

$$d^2y/dx^2 = 2a \tag{61}$$

The Crout reduction method, which is discussed in the description of subroutine INVY, is used to solve the matrix for the remaining second derivative. The tridiagonal matrix terms are

$$\begin{aligned} e_i &= h_{i-1}/6 \\ d_i &= \frac{h_{i-1} + h_i}{3} \\ f_i &= h_i/6 \end{aligned} \tag{62}$$

and

$$c_i = \left(\frac{y_{i+1} - y_i}{h_i}\right) - \left(\frac{y_i - y_{i-1}}{h_{i-1}}\right) .$$

The following parameters are used in this subroutine:

X and Y	array containing input x and y values
YPP	array containing computed second derivatives d^2y/dx^2
N	number of elements in X Y, and YPP arrays
A	work array dimensioned by 2 times N in the calling program

DISCUSSION OF PROGRAM APPLICATION AND RESULTS

The airfoil smoothing program was formulated to smooth the coordinates of airfoil-type contours which are characteristically round in the front and sharp or blunt in the rear. Several users in the past have attempted to use this program to smooth nonairfoil shapes such as internal contours of engine nacelles or wind tunnels. These attempts have been generally unsuccessful because of the effects of the θ-transformation function which was formulated to stretch the x-axis in the leading- and trailing-edge regions. The smoothing program can be used successfully to smooth nonairfoil contours by redefining the θ-transformation function as

$$\theta = \pm\pi\bar{x} \qquad (63)$$

and making the appropriate changes in the computer code.

An airfoil contour may be input into the smoothing program in several forms. The most widely used form is, of course, as x and y coordinates (IOP = 0) which have been obtained from actual measurements of an existing airfoil or from theoretical computations.

Regardless of the source of the coordinates, the user should strive to input a proportionally larger number of coordinates in regions of higher curvature which is generally the nose region for most airfoils. The user may input as many as 100 coordinates for each airfoil surface; however, it is recommended that no more than 35 to 40 coordinates be input for each surface because, in general, the more dense the coordinate spacing the more restricted the smoothing process will become. If the user desires to limit the extent of smoothing in a particular region, it is suggested that a few highly weighted coordinates be input rather than a large number of closely spaced coordinates.

The question often arises as to the number of smoothing iterations (ITER parameter) the user should specify. It is recommended that zero iterations be specified for the initial run of a new airfoil case. The plots generated during the initial run can then be examined to establish the initial smoothness of the airfoil, the suitability of the input x-coordinate spacing, and the possible existence of bad input y coordinates. During all subsequent runs, it is recommended that the maximum of 300 iterations be specified. The convergence criteria for this smoothing program is rather stringent; however, the smoothing process should converge or be near convergence in less than 100 iterations for most airfoils. If the process has not conveyed in 300 iterations, the resultant coordinates can be written on the output file TAPE1 in the form of either x and y coordinates or θ and $\bar{y}$ values and then input again into the smoothing program for another 300 iterations. If, during the initial smoothing attempt, the process begins to oscillate, it is suggested that fewer coordinates be selected in the region where the oscillation occurs and the case be resubmitted. The oscillatory region can be located by setting the IPRINT parameter in program AIRSMO equal to 0 which will generate a summary print of the computed second derivatives for each iteration.

The airfoil contour may also be input in the form of $\bar{y}$ coordinates and the corresponding θ-values (IOP = 2). This form is often used to resubmit a set of coordinates that required adjustment due to either bad or poorly defined nose $\bar{y}$ coordinates that are often revealed during the initial run of a new airfoil. The stretching effect of the θ-transformation function will highlight any coordinate discrepancies in the nose region of the airfoil.

Two additional input forms are available to modify or smooth an airfoil contour and are less direct than the previous two forms discussed. The two additional input forms consists of inputting the first $\bar{y}'$ (IOP = 3) or second $\bar{y}''$ (IOP = 4) derivatives as a function of the θ-transformation value. The corresponding $\bar{y}$ coordinates are obtained by solving a tridiagonal matrix of simultaneous cubic spline equations; therefore, local changes in the input derivatives have a less localized and more global effect in the computed $\bar{y}$ coordinates. Great care should be exercised when using either of these two input forms; especially the second derivative, because seemly small changes in the derivatives will very often result in rather large changes in the $\bar{y}$ coordinates. In spite of its sensitivity, these two input forms provide a very easy and direct method to reduce or eliminate waviness in the curvature of the final smoothed airfoil.

The airfoil smoothing program has been used extensively at Langley for the past several years and has worked successfully for a wide range of airfoil shapes. A comparison between the unsmoothed and smoothed first and second derivatives for a typical airfoil is presented in figure 8. The corresponding changes in the $\bar{y}$ coordinate are very small and are not distinguishable on a page-size plot of the airfoil contours. As illustrated in figure 9, the improvement in the smoothness of the curvature distribution is excellent.

Only two problems have occurred persistently during the past several years of program utilization. The first problem occurs when attempting to smooth airfoils with very sharp or zero-thickness trailing edges. Although the trailing-edge coordinates are heavily weighted, the smoothing process will often result in a small shift in the upper and lower surface trailing-edge coordinates. Many times the shift will be in the opposite direction and a negative trailing-edge thickness will occur. As previously discussed, the program checks for negative thickness and, if detected, will print an error message and proceed to the next input case. The most practical solution to this problem is simply to terminate the input coordinates very near the trailing edge at a point with small finite thickness. The second problem, as noted in the method section of this report, is a difficulty in locating the first few camberline coordinates of an airfoil with a reflexed (upward-turned) camberline near the trailing edge. This problem can generally be overcome by simply reversing the input order of the coordinates so that lower surface coordinates are input first, followed by the upper surface coordinates. This will not affect the smoothing process, but will cause the camberline search procedure to reverse surfaces.

CONCLUDING REMARKS

The airfoil computer programs AFSMO and AFSCL described in this report have been used successfully at Langley for several years to smooth and scale a wide variety of airfoil shapes generated by various theoretical methods or measured from existing airfoil models and wing panels. The smoothing process is very stable and generally converges in less than a hundred iterations. The smoothing program user-supplied input requirements are very simple and consist of basically specifying the title, input/output options, and the upper and lower surface coordinates. The camberline search procedure in the smoothing program generates the basic camber and thickness distribution data needed as input to the scaling program. The only additional user-supplied input for the scaling program are a title, input/output option, and the number of and the values for the desired maximum thickness-chord ratios.

The output plots generated by the smoothing program are very helpful during the analysis and possible modification of the smoothed airfoil. After several years of extensive use by Langley personnel, no appreciable execution errors have occurred or airfoil shape limitations been revealed. The use of the AFSMO program to smooth nonairfoil shapes should not be attempted without redefinition of the x-axis transformation function. Both programs were coded for use on the Langley CDC CYBER computers. No specialized system software is needed to execute either program and all required subroutines are listed in this report except for several basic CALCOMP plotting subroutines which are unique to the Langley computers. Both programs have been successfully converted for use on other computer systems; however, double-precision accuracy was necessary for the conversion of the smoothing program because of its very stringent convergence criteria.

APPENDIX A

COMPUTER LISTING OF AIRFOIL SMOOTHING PROGRAM AFSMO

This appendix contains a computer listing of the airfoil smoothing program AFSMO which consists of a main program, fifteen subroutines, and two function subprograms.

CARD NO.

```
      PROGRAM AIRSMO(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1)
C
C     THIS PROGRAM PRESENTS A TECHNIQUE FOR SMOOTHING AIRFOIL
C     COORDINATES USING LEAST SQUARES POLYNOMIAL AND CUBIC SPLINE
C     METHODS
C
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
      DIMENSION TITLE(8), XINT(100), X(200), Y(200), W(200), YSMO(200),
     1YPS(200), YPPS(202), THETA(202)
C
      COMMON /HLM/ DUMMX(2000)
C
      COMMON /SMY/ DUMMY(2130)
C
      COMMON /BLK1/ PI,PI2,RAD,CONS
C
      COMMON /INOUT/ JREAD,JWRITE,IPRINT
C
C     SINH(X)=0.5*(EXP(X)-EXP(-X))
C
C     INITIALIZE PROGRAM CONSTANTS
C
      PI=ACOS(-1.)
      PI2=PI/2.
      RAD=180./PI
      CONS=1./(1.+ATAN(SINH(PI2)))
      JREAD=5
      JWRITE=6
      IPRINT=1
      EPS=1.E-6
      DF=1.E-4
      REWIND 1
C
C     INITIALIZE PLOTTING DEVICE
C
      CALL PSEUDO
      CALL LEROY
C
C     READ INPUT DATA
```

CARD NO.

```
C                                                                        AS  41
1     CALL INPUT (TITLE,ITER,IPLOT,IPUNCH,IOP,ICAMTK,INTR,YLTE,YNOSE,YUT AS  42
     1E,NINT,XINT,CNEW,NP,X,Y,W,THETA,YPS,YPPS,NOSE,CHORD,IERR)          AS  43
      IF (IERR-1) 2,1,5                                                  AS  44
C                                                                        AS  45
C     SMOOTH AIRFOIL COORDINATES                                         AS  46
C                                                                        AS  47
2     CALL SMOXY (THETA,X,Y,W,YSMO,YPS,YPPS,NP,NOSE,YLTE,YNOSE,YUTE,EPS, AS  48
     1DF,ITER,TITLE,IOP,IERR)                                            AS  49
      IF (IERR.NE.0) GO TO 1                                             AS  50
C                                                                        AS  51
C     PUNCH OUTPUT DATA                                                  AS  52
C                                                                        AS  53
      IF (IPUNCH.GE.1.AND.IPUNCH.LE.4) CALL PCARD (IPUNCH,X,Y,W,THETA,YS AS  54
     1MO,YPS,YPPS,NOSE,NP,CHORD,TITLE)                                   AS  55
C                                                                        AS  56
C     PLOT SMOOTHED AND UNSMOOTHED Y/C, SMOOTHED YPS, AND SMOOTHED       AS  57
C     YPPS VERSUS THETA. ALSO PLOT SMOOTHED AND UNSMOOTHED Y/C VERSUS    AS  58
C     X/C                                                                AS  59
C                                                                        AS  60
      IF (IPLOT.EQ.0.OR.IPLOT.EQ.4) GO TO 4                              AS  61
      IF (IPLOT.EQ.3) GO TO 3                                            AS  62
C                                                                        AS  63
      CALL PLOTAF (THETA,Y,YSMO,YPS,YPPS,NP,TITLE,IPLOT)                 AS  64
C                                                                        AS  65
      IF (IPLOT.EQ.5.OR.IPLOT.EQ.1) GO TO 4                              AS  66
      IF (IPLOT.EQ.6.OR.IPLOT.EQ.7) GO TO 3                              AS  67
      IF (IPLOT.EQ.10) GO TO 3                                           AS  68
      GO TO 4                                                            AS  69
C                                                                        AS  70
C     PLOT SMOOTHED CURVATURE VERSUS THETA                               AS  71
C                                                                        AS  72
3     CALL PLOTCK (THETA,YSMO,YPS,YPPS,NP,TITLE)                         AS  73
C                                                                        AS  74
4     KPLOT=0                                                            AS  75
      IF (IPLOT.EQ.4.OR.IPLOT.GE.8) KPLOT=1                              AS  76
C                                                                        AS  77
C     COMPUTE THICKNESS AND CAMBER DISTRIBUTION                          AS  78
C                                                                        AS  79
      IF (ICAMTK.EQ.1) CALL CAMTK (THETA,YSMO,YPPS,NOSE,NP,EPS,KPLOT,IPU AS  80
```

CARD NO.

```
     1NCH,TITLE)
C
C      INTERPOLATE NEW COORDINATES
C
       IF (INTR.GT.0) CALL INTP (THETA,X,YSMO,YPPS,NP,NOSE,CHORD,TITLE,NI
     1NT,XINT,CNEW,INTR,IPUNCH)
C
C      RETURN AND READ NEXT CASE
C
       GO TO 1
C
C      FINALIZE PLOTTING DEVICE
C
5      CALL CALPLT (0.,0.,999)
       WRITE (JWRITE,6)
       END FILE 1
       REWIND 1
       STOP
C
6      FORMAT (1H1////48X,38H-- THE LAST CASE HAS BEEN PROCESSED --)
       END
```

```
      SUBROUTINE INTER (XINT,YINT,N,X,Y,JSTART,JEND,ICD)
C
C     INTERPOLATION ROUTINE
C
C        ROUTINE SOURCE -- NORTH AMERICAN ROCKWELL L. A. DIVISION  1973
C
C        ICD=0  WEIGHTING METHOD USED
C        ICD=1  LINEAR INTERPOLATION
C
      DIMENSION X(N), Y(N)
C
C     CHECK TO SEE IF XINT IS OUTSIDE BOUNDS OF X-ARRAY
C
      JEND=JSTART
      IF (JSTART.EQ.N) GO TO 12
C        CHECK TO SEE IF X ARRAY IS INCREASING OR DECREASING
      SGN=1.
      IF (X(N).LT.X(JSTART)) SGN=-1.
      D1=SGN*(XINT-X(N))
      IF (D1.GE.0.0) GO TO 12
      D1=SGN*(XINT-X(JSTART))
      IF (D1.LE.0.0) GO TO 13
      IF (ICD.EQ.1) GO TO 14
C     WEIGHTING METHOD REQUIRES AT LEAST 4 VALUES IN X AND Y ARRAYS
      IF (N.LT.4) GO TO 14
C
C     WEIGHTING METHOD
C
C        DETERMINE X-ARRAY INDICES FOR TWO POINTS FORWARD (J,L) AND TWO
C        POINTS AFT (K,M) OF XINT
      DO 1 L=JSTART,N
      J=L
      D1=SGN*(X(J)-XINT)
      IF (D1) 1,2,3
1     JEND=J
2     YINT=Y(J)
      RETURN
3     IF (J.LE.2) GO TO 5
      IF (J.EQ.N) GO TO 4
      JJ=3
      GO TO 6
    4 JJ=2
      J=N-1
      GO TO 6
    5 JJ=1
      J=3
    6 K=J-1
      M=J-2
      L=J+1
C        INTERPOLATE A YINT VALUE (YSL) BY FITTING A STRAIGHT LINE
C        BETWEEN K AND J
      D1=XINT-X(M)
      D2=XINT-X(K)
      D3=XINT-X(J)
      D=(XINT-X(K))/(X(J)-X(K))
      YSL=D*Y(J)+(1.0-D)*Y(K)
C        INTERPOLATE A YINT VALUE (YP1) BY FITTING A QUADRATIC BETWEEN
C        M, K, AND J
      C1=D3*D2/((X(M)-X(K))*(X(M)-X(J)))
      C2=D1*D3/((X(K)-X(M))*(X(K)-X(J)))
      C3=D2*D1/((X(J)-X(M))*(X(J)-X(K)))
      YP1=C1*Y(M)+C2*Y(K)+C3*Y(J)
C        INTERPOLATE A YINT VALUE (YP2) BY FITTING A QUADRATIC BETWEEN
C        K, J, AND L
      D4=XINT-X(L)
      C1=D4*D3/((X(K)-X(J))*(X(K)-X(L)))
      C2=D2*D4/((X(J)-X(K))*(X(J)-X(L)))
      C3=D3*D2/((X(L)-X(K))*(X(L)-X(J)))
      YP2=C1*Y(K)+C2*Y(J)+C3*Y(L)
C
      IF (JJ-2) 7,8,9
    7 YP2=YP1
      D=(XINT-X(1))/(X(2)-X(1))
      YSL=D*Y(2)+(1.0-D)*Y(1)
      GO TO 9
    8 YP1=YP2
      D=(XINT-X(N-1))/(X(N)-X(N-1))
      YSL=D*Y(N)+(1.0-D)*Y(N-1)
C        COMPUTE DEVIATION BETWEEN LINEAR AND QUADRATIC YINT VALUES
    9 DEV1=ABS(YP1-YSL)
```

CARD NO.

```
      DEV2=ABS(YP2-YSL)
      IF (DEV1+DEV2) 10,10,11
   10 YINT=YSL
      RETURN
C        COMPUTE WEIGHTING FACTORS
   11 WT2=(DEV1*D)/(DEV1*D+(1.0-D)*DEV2)
      WT1=1.0-WT2
C        COMPUTE FINAL YINT
      YINT=WT2*YP2+WT1*YP1
      RETURN
   12 YINT=Y(N)
      JEND=N
      RETURN
   13 YINT=Y(JSTART)
      RETURN
C
C     LINEAR INTERPOLATION METHOD
C
   14 DO 15 L=JSTART,N
      J=L
      D1=SGN*(X(J)-XINT)
      IF (D1) 15,2,16
   15 JEND=J
   16 YINT=Y(J-1)+(Y(J)-Y(J-1))*(XINT-X(J-1))/(X(J)-X(J-1))
      RETURN
      END
```

CARD NO.

```
      SUBROUTINE INPUT (TITLE,ITER,IPLOT,IPUNCH,IOP,ICAMTK,INTR,YLTE,YNO
     1SE,YUTE,NINT,XINT,CNEW,NP,X,Y,W,THETA,YPS,YPPS,NOSE,CHORD,IERR)
C
C     ROUTINE TO READ INPUT DATA FOR AIRFOIL SMOOTHING PROGRAM
C
C         CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
C*********************************************************************
C*                                                                   *
C*           DESCRIPTION OF INPUT CARDS FOR SMOOTHING PROGRAM        *
C*                                                                   *
C* CARD NUMBER                     DESCRIPTION                       *
C*                                                                   *
C*...................................................................*
C*   1    FORMAT(8A10)                                               *
C*             TITLE CARD                                            *
C*...................................................................*
C*   2    FORMAT(8F10.0)                                             *
C*             ITER - MAXIMUM NUMBER OF SMOOTHING ITERATIONS         *
C*             IPLOT - PLOTTING OPTION                               *
C*                     0 - NO PLOTS                                  *
C*                     1 - PLOT SMOOTHED AND UNSMOOTHED Y/C, SMOOTHED *
C*                         YPS, AND SMOOTHED YPPS VS THETA           *
C*                     2 - PLOT SMOOTHED AND UNSMOOTHED Y/C VS X/C   *
C*                     3 - PLOT SMOOTHED CURVATURE VS THETA          *
C*                     4 - PLOT CAMBER AND THICKNESS DISTRIBUTION    *
C*                     5 - PLOT OPTIONS 1 AND 2                      *
C*                     6 - PLOT OPTIONS 1 AND 3                      *
C*                     7 - PLOT OPTIONS 1, 2, AND 3                  *
C*                     8 - PLOT OPTIONS 1 AND 4                      *
C*                     9 - PLOT OPTIONS 1, 2, AND 4                  *
C*                    10 - PLOT OPTIONS 1, 2, 3, AND 4               *
C*             IPUNCH - PUNCH OUTPUT OPTION                          *
C*                      0 - NO PUNCHED OUTPUT                        *
C*                      1 - SMOOTHED (X,Y,W) PUNCHED                 *
C*                      2 - SMOOTHED (THETA,Y/C,W) PUNCHED           *
C*                      3 - SMOOTHED (THETA,YPS,W) PUNCHED (YLTE,    *
C*                          YNOSE, AND YUTE ALSO PUNCHED)            *
C*                      4 - SMOOTHED (THETA,YPPS,W) PUNCHED (YLTE,   *
C*                          YNOSE, AND YUTE ALSO PUNCHED)            *
```

```
C*                    5 - THICKNESS AND CAMBER DISTRIBUTION (X/C,       * IU  41
C*                        Y/C, T/C/2, AND SLOPE) PUNCHED                * IU  42
C*                    6 - INTERPOLATED COORDINATES PUNCHED              * IU  43
C*           IOP - INPUT DATA OPTION                                    * IU  44
C*                   0 - (X,Y,W) INPUT                                  * IU  45
C*                   1 - (THETA,Y/C,W) INPUT                            * IU  46
C*                   2 - (THETA,YPS,W) INPUT                            * IU  47
C*                   3 - (THETA,YPPS,W) INPUT                           * IU  48
C*           ICAMTK - THICKNESS AND CAMBER DISTRIBUTION OPTION          * IU  49
C*                      0 - DO NOT COMPUTE THICKNESS AND CAMBER         * IU  50
C*                      1 - COMPUTE THICKNESS AND CAMBER                * IU  51
C*           IBAD - BAD COORDINATE CHECK OPTION                         * IU  52
C*                    0 - DO NOT CHECK FOR BAD COORDINATES              * IU  53
C*                    1 - CHECK FOR BAD COORDINATES                     * IU  54
C*           ITRN - INPUT COORDINATE TRANSLATION AND ROTATION OPTION    * IU  55
C*                    0 - DO NOT TRANSLATE AND ROTATE                   * IU  56
C*                    1 - TRANSLATE AND ROTATE SO THAT X-AXIS           * IU  57
C*                        CORRESPONDS TO THE LONGEST CHORDLINE          * IU  58
C*           INTR - COORDINATE INTERPOLATION OPTION                     * IU  59
C*                    0 - NO INTERPOLATION DESIRED                      * IU  60
C*                    1 - INTERPOLATE NEW COORDINATES USING STANDARD 57 * IU  61
C*                        X/C COORDINATES DEFINED IN SUBROUTINE INTP    * IU  62
C*                    2 - INTERPOLATE NEW COORDINATES AT INPUT X/C      * IU  63
C*                        VALUES  (0.0 .GE. X/C .LE. 1.0)               * IU  64
C*...................................................................* IU  65
C*   3   FORMAT(10.0)                                                   * IU  66
C*           NU - NUMBER OF UPPER SURFACE INPUT COORDINATES             * IU  67
C*...................................................................* IU  68
C*   4   FORMAT(3F10.0)                                                 * IU  69
C*           XU,YU,WU - UPPER SURFACE INPUT COORDINATES AND WEIGHTING * IU  70
C*                        (NU CARDS ARE INPUT)                          * IU  71
C*                     IF IOP=0, XU=X AND YU=Y COORDINATES              * IU  72
C*                     IF IOP=1, XU=THETA AND YU=Y/C                    * IU  73
C*                     IF IOP=2, XU=THETA AND YU=YPS                    * IU  74
C*                     IF IOP=3, XU=THETA AND YU=YPPS                   * IU  75
C*                     FOR ALL IOP, WU=WEIGHTING FACTOR                 * IU  76
C*...................................................................* IU  77
C*   5   FORMAT(10.0)                                                   * IU  78
C*           NL - NUMBER OF LOWER SURFACE INPUT COORDINATES             * IU  79
C*...................................................................* IU  80
```

```
C*    6   FORMAT(3F10.0)                                                * IU  81
C*              XL,YL,WL - LOWER SURFACE INPUT COORDINATES AND WEIGHTING * IU  82
C*                        (NL CARDS ARE INPUT)                          * IU  83
C*                     IF IOP=0, XL=X AND YL=Y COORDINATES              * IU  84
C*                     IF IOP=1, XL=THETA AND YL=Y/C                    * IU  85
C*                     IF IOP=2, XL=THETA AND YL=YPS                    * IU  86
C*                     IF IOP=3, XL=THETA AND YL=YPPS                   * IU  87
C*                     FOR ALL IOP, WL=WEIGHTING FACTOR                 * IU  88
C*.....................................................................* IU  89
C*    7   FORMAT(3F10.0)        SKIP IF IOP=0 OR 1                      * IU  90
C*              YLTE,YNOSE,YUTE - LOWER SURFACE TRAILING-EDGE, NOSE,    * IU  91
C*                                AND UPPER SURFACE TRAILING-EDGE       * IU  92
C*                                Y/C COORDINATES                       * IU  93
C*.....................................................................* IU  94
C*    8   FORMAT(F10.0)         SKIP IF INTR=0 OR 1                     * IU  95
C*              NINT - NUMBER OF INTERPOLATION X/C COORDINATES          * IU  96
C*.....................................................................* IU  97
C*    9   FORMAT(8F10.0)        SKIP IF INTR=0 OR 1                     * IU  98
C*              XINT - INTERPOLATION X/C COORDINATES (NINT VALUES INPUT) * IU  99
C*.....................................................................* IU 100
C*   10   FORMAT(F10.0)         SKIP IF INTR=0                          * IU 101
C*              CNEW - DESIRED CHORD LENGTH OF INTERPOLATED COORDINATES * IU 102
C*.....................................................................* IU 103
C*                                                                      * IU 104
C*       RESTRICTIONS:                                                  * IU 105
C*              ITER NOT GREATER THAN 300                               * IU 106
C*              NU OR NL NOT GREATER THAN 100                           * IU 107
C*              NINT NOT GREATER THAN 100                               * IU 108
C*                                                                      * IU 109
C*********************************************************************** IU 110
C                                                                         IU 111
      DIMENSION VAR(8), TITLE(8), XINT(1), X(1), Y(1), W(1), THETA(1), Y IU 112
     1PS(1), YPPS(1)                                                      IU 113
C                                                                         IU 114
      COMMON /SMY/ XU(100),YU(100),WU(100),XL(100),YL(100),WL(100)        IU 115
C                                                                         IU 116
      COMMON /BLK1/ PI,PI2,RAD,CONS                                       IU 117
C                                                                         IU 118
      COMMON /INOUT/ JREAD,JWRITE,IPRINT                                  IU 119
C                                                                         IU 120
```

```
C     SINH(X)=(EXP(X)-EXP(-X))/2.
C
C     INITIALIZE ROUTINE CONSTANTS
C
      ITRMAX=300
      NMAX=100
      TOLR=1.E-2
      IERR=0
C
C     READ AND PRINT INPUT DATA
C
C        READ AND WRITE TITLE
      READ (JREAD,27) TITLE
      IF (EOF(JREAD)) 25,1
1     WRITE (JWRITE,28) TITLE
C        READ AND WRITE OPTIONS
      READ (JREAD,29) VAR
      ITER=IFIX(VAR(1))
      IPLOT=IFIX(VAR(2))
      IPUNCH=IFIX(VAR(3))
      IOP=IFIX(VAR(4))
      ICAMTK=IFIX(VAR(5))
      IBAD=IFIX(VAR(6))
      ITRN=IFIX(VAR(7))
      INTR=IFIX(VAR(8))
C              CHECK LIMITS OF OPTIONS
      IF (ITER.GT.ITRMAX) ITER=ITRMAX
      IF (IPLOT.GT.10) IPLOT=0
      IF (IPUNCH.GT.6) IPUNCH=0
      IF (IOP.GT.3) GO TO 23
      IF (ICAMTK.NE.0) ICAMTK=1
      IF (IBAD.NE.0) IBAD=1
      IF (ITRN.NE.0) ITRN=1
      IF (INTR.GT.2) INTR=0
      WRITE (JWRITE,30) ITER,IPLOT,IPUNCH,IOP,ICAMTK,IBAD,ITRN,INTR
C        READ AND WRITE NUMBER OF UPPER SURFACE INPUT POINTS
      READ (JREAD,29) VAR(1)
      NU=IFIX(VAR(1))
      IF (NU.GT.NMAX) GO TO 22
      WRITE (JWRITE,31) NU
```

```
C        READ AND WRITE UPPER SURFACE INPUT POINTS AND WEIGHTING
      READ (JREAD,32) (XU(I),YU(I),WU(I),I=1,NU)
      DO 2 I=1,NU
      IF (WU(I).LT.1.0) WU(I)=1.0
    2 CONTINUE
      IF (IOP.EQ.0) WRITE (JWRITE,33) (XU(I),I=1,NU)
      IF (IOP.NE.0) WRITE (JWRITE,34) (XU(I),I=1,NU)
      IF (IOP.LT.2) WRITE (JWRITE,35) (YU(I),I=1,NU)
      IF (IOP.EQ.2) WRITE (JWRITE,36) (YU(I),I=1,NU)
      IF (IOP.EQ.3) WRITE (JWRITE,37) (YU(I),I=1,NU)
      WRITE (JWRITE,38) (WU(I),I=1,NU)
C        READ AND WRITE NUMBER OF LOWER SURFACE INPUT POINTS
      READ (JREAD,29) VAR(1)
      NL=IFIX(VAR(1))
      IF (NL.GT.NMAX) GO TO 22
      WRITE (JWRITE,39) NL
C        READ AND WRITE LOWER SURFACE INPUT POINTS AND WEIGHTING
      READ (JREAD,32) (XL(I),YL(I),WL(I),I=1,NL)
      DO 3 I=1,NL
      IF (WL(I).LT.1.0) WL(I)=1.0
    3 CONTINUE
      IF (IOP.EQ.0) WRITE (JWRITE,40) (XL(I),I=1,NL)
      IF (IOP.NE.0) WRITE (JWRITE,41) (XL(I),I=1,NL)
      IF (IOP.LT.2) WRITE (JWRITE,42) (YL(I),I=1,NL)
      IF (IOP.EQ.2) WRITE (JWRITE,43) (YL(I),I=1,NL)
      IF (IOP.EQ.3) WRITE (JWRITE,44) (YL(I),I=1,NL)
      WRITE (JWRITE,45) (WL(I),I=1,NL)
C        READ AND WRITE TRAILING-EDGE COORDINATES
      IF (IOP.LE.1) GO TO 4
      READ (JREAD,29) YLTE,YNOSE,YUTE
      WRITE (JWRITE,46) YLTE,YNOSE,YUTE
C        READ AND WRITE NUMBER OF INTERPOLATION COORDINATES
    4 IF (INTR.EQ.0) GO TO 6
      IF (INTR.NE.2) GO TO 5
      READ (JREAD,29) VAR(1)
      NINT=IFIX(VAR(1))
      IF (NINT.GT.NMAX) GO TO 24
      WRITE (JWRITE,47) NINT
C        READ AND WRITE INTERPOLATION COORDINATES
      READ (JREAD,29) (XINT(I),I=1,NINT)
      WRITE (JWRITE,48) (XINT(I),I=1,NINT)
C        READ AND WRITE NEW CHORD OF INTERPOLATED COORDINATES
    5 READ (JREAD,29) CNEW
      WRITE (JWRITE,49) CNEW
C
C     CHECK UPPER SURFACE COORDINATES FOR BAD POINTS
C
    6 IF (IOP.NE.0) GO TO 7
      IF (IBAD.EQ.1) CALL BADPT (XU,YU,NU,TOLR,1,IERR)
      IF (IERR.NE.0) GO TO 26
C
C     CHECK LOWER SURFACE COORDINATES FOR BAD POINTS
C
      IF (IBAD.EQ.1) CALL BADPT (XL,YL,NL,TOLR,2,IERR)
      IF (IERR.NE.0) GO TO 26
C
C     TPANSLATE AND ROTATE THE INPUT COORDINATES SO THAT THE X-AXIS
C     CORRESPONDS TO THE LONGEST CHORDLINE OF THE AIRFOIL
C
      IF (ITRN.EQ.1) CALL TRNSRT (XU,YU,WU,NU,XL,YL,WL,NL,TITLE)
C
C     LOAD X, Y, THETA, YPS, AND YPPS ARRAYS
C
    7 IF (IOP) 8,8,15
C        IF IOP=0, COMPUTE THETA FROM INPUT X
C             COMPUTE THETA FOR LOWER SURFACE
    8 CHORD=XL(NL)-XL(1)
      DELTA=XU(NU)-XU(1)
      IF (DELTA.GT.CHORD) CHORD=DELTA
      NP=0
      DO 11 I=1,NL
      NP=NP+1
      J=NL+1-I
      W(NP)=WL(J)
      DELTA=(XL(J)-XL(1))/CHORD
      IF (DELTA.LE.CONS) GO TO 9
      DELTA=TAN(DELTA/CONS-1.)
      THETA(NP)=-PI2-ALOG(DELTA+SQRT(DELTA*DELTA+1.))
      GO TO 10
    9 THETA(NP)=-ACOS(1.-DELTA/CONS)
```

```
   10 X(NP)=XL(J)/CHORD
   11 Y(NP)=YL(J)/CHORD
      NOSE=NP
C             COMPUTE THETA FOR UPPER SURFACE
      J=1
      IF (XL(1).EQ.XU(1).AND.YL(1).EQ.YU(1)) J=2
      DO 14 I=J,NU
      NP=NP+1
      W(NP)=WU(I)
      DELTA=(XU(I)-XU(1))/CHORD
      IF (DELTA.LE.CONS) GO TO 12
      DELTA=TAN(DELTA/CONS-1.)
      THETA(NP)=PI2+ALOG(DELTA+SQRT(DELTA*DELTA+1.))
      GO TO 13
   12 THETA(NP)=ACOS(1.-DELTA/CONS)
   13 X(NP)=XU(I)/CHORD
   14 Y(NP)=YU(I)/CHORD
      GO TO 20
C        IF IOP=1, 2, OR 3, COMPUTE X/C FROM INPUT THETA
C             COMPUTE X/C FOR LOWER SURFACE
   15 CHORD=1.0
      NP=0
      DO 17 I=1,NL
      NP=NP+1
      J=NL+1-I
      W(NP)=WL(J)
      IF (IOP.EQ.1) Y(NP)=YL(J)
      IF (IOP.EQ.2) YPS(NP)=YL(J)
      IF (IOP.EQ.3) YPPS(NP)=YL(J)
      THETA(NP)=XL(J)/RAD
      DELTA=ABS(THETA(NP))
      IF (DELTA.GT.PI2) GO TO 16
      XL(J)=CONS*(1.-COS(DELTA))
      GO TO 17
   16 XL(J)=CONS*(ATAN(SINH(DELTA-PI2))+1.)
   17 X(NP)=XL(J)
      NOSE=NP
C             COMPUTE X/C FOR UPPER SURFACE
      XU(1)=XL(1)
      DO 19 I=2,NU
```

```
            NP=NP+1                                                     IU 281
            W(NP)=WU(I)                                                 IU 282
            IF (IOP.EQ.1) Y(NP)=YU(I)                                   IU 283
            IF (IOP.EQ.2) YPS(NP)=YU(I)                                 IU 284
            IF (IOP.EQ.3) YPPS(NP)=YU(I)                                IU 285
            THETA(NP)=XU(I)/RAD                                         IU 286
            DELTA=ABS(THETA(NP))                                        IU 287
            IF (DELTA.GT.PI2) GO TO 18                                  IU 288
            XU(I)=CONS*(1.-COS(DELTA))                                  IU 289
            GO TO 19                                                    IU 290
   18       XU(I)=CONS*(ATAN(SINH(DELTA-PI2))+1.)                       IU 291
   19       X(NP)=XU(I)                                                 IU 292
   C                                                                    IU 293
   C        PRINT SUMMARY OF INPUT DATA                                 IU 294
   C                                                                    IU 295
   20       WRITE (JWRITE,50) TITLE                                     IU 296
            DO 21 I=1,NP                                                IU 297
            DELTA=THETA(I)*RAD                                          IU 298
            IF (IOP.LE.1) WRITE (JWRITE,51) I,X(I),Y(I),DELTA,W(I)      IU 299
            IF (IOP.EQ.2) WRITE (JWRITE,52) I,X(I),DELTA,YPS(I),W(I)    IU 300
            IF (IOP.EQ.3) WRITE (JWRITE,53) I,X(I),DELTA,YPPS(I),W(I)   IU 301
   21       CONTINUE                                                    IU 302
            WRITE (JWRITE,54) CHORD                                     IU 303
            GO TO 26                                                    IU 304
   C                                                                    IU 305
   C        PRINT ERROR MESSAGES                                        IU 306
   C                                                                    IU 307
   22       NN=IFIX(VAR(1))                                             IU 308
            WRITE (JWRITE,55) NN                                        IU 309
            GO TO 25                                                    IU 310
   23       WRITE (JWRITE,56) IOP                                       IU 311
            GO TO 25                                                    IU 312
   24       WRITE (JWRITE,57) NINT                                      IU 313
   C                                                                    IU 314
   C        NO ADDITIONAL INPUT DATA                                    IU 315
   C                                                                    IU 316
   25       IERR=2                                                      IU 317
   C                                                                    IU 318
   C        RETURN TO CALLING PROGRAM                                   IU 319
   C                                                                    IU 320
```

CARD NO.

```
      26    RETURN                                                        IU 321
      C                                                                   IU 322
      27    FORMAT (8A10)                                                 IU 323
      28    FORMAT (1H1,57X,14H--INPUT DATA--//5X,7HTITLE--,2X,8A10)      IU 324
      29    FORMAT (8F10.5)                                               IU 325
      30    FORMAT (/5X,6HITER =,I4,3X,7HIPLOT =,I3,3X,8HIPUNCH =,I3,3X,5HIOP IU 326
           1=,I3,3X,8HICAMTK =,I3,3X,6HIBAD =,I3,3X,6HITRN =,I3,3X,6HINTR =,I3 IU 327
           2)                                                             IU 328
      31    FORMAT (/5X,4HNU =,I4)                                        IU 329
      32    FORMAT (3F10.5)                                               IU 330
      33    FORMAT (/5X,3HXU=,8E15.6/(8X,8E15.6))                         IU 331
      34    FORMAT (/5X,3HTU=,8E15.6/(8X,8E15.6))                         IU 332
      35    FORMAT (/5X,3HYU=,8E15.6/(8X,8E15.6))                         IU 333
      36    FORMAT (/4X,4HYPU=,8E15.6/(8X,8E15.6))                        IU 334
      37    FORMAT (/3X,5HYPPU=,8E15.6/(8X,8E15.6))                       IU 335
      38    FORMAT (/5X,3HWU=,8E15.6/(8X,8E15.6))                         IU 336
      39    FORMAT (/5X,4HNL =,I4)                                        IU 337
      40    FORMAT (/5X,3HXL=,8E15.6/(8X,8E15.6))                         IU 338
      41    FORMAT (/5X,3HTL=,8E15.6/(8X,8E15.6))                         IU 339
      42    FORMAT (/5X,3HYL=,8E15.6/(8X,8E15.6))                         IU 340
      43    FORMAT (/4X,4HYPL=,8E15.6/(8X,8E15.6))                        IU 341
      44    FORMAT (/3X,5HYPPL=,8E15.6/(8X,8E15.6))                       IU 342
      45    FORMAT (/5X,3HWL=,8E15.6/(8X,8E15.6))                         IU 343
      46    FORMAT (/3X,6HYLTE =,E15.6,5X,7HYNOSE =,E15.6,5X,6HYUTE =,E15.6)  IU 344
      47    FORMAT (/3X,6HNINT =,I4)                                      IU 345
      48    FORMAT (/3X,5HXINT=,8E15.6/(8X,8E15.6))                       IU 346
      49    FORMAT (/3X,6HCNEW =,F10.3)                                   IU 347
      50    FORMAT (1H1,29X,25H--SUMMARY OF INPUT DATA--//5X,9HTITLE--  ,8A10/ IU 348
           1/9X,1HI,10X,3HX/C,12X,3HY/C,12X,5HTHETA,10X,3HYPS,12X,4HYPPS,14X,1 IU 349
           2HW)                                                           IU 350
      51    FORMAT (I10,2F15.6,F15.2,30X,F15.2)                           IU 351
      52    FORMAT (I10,F15.6,15X,F15.2,F15.6,15X,F15.2)                  IU 352
      53    FORMAT (I10,F15.6,15X,F15.2,15X,F15.6,F15.2)                  IU 353
      54    FORMAT (/5X,7HCHORD =,F15.6)                                  IU 354
      55    FORMAT (//5X,28HINPUT CARD ERROR  NU OR NL =,I4)              IU 355
      56    FORMAT (//5X,23HINPUT CARD ERROR  IOP =,I4)                   IU 356
      57    FORMAT (//5X,24HINPUT CARD ERROR  NINT =,I5)                  IU 357
            END                                                           IU 358-
      SUBROUTINE TRNSRT (XU,YU,WU,NU,XL,YL,WL,NL,TITLE)
C
C     ROUTINE TO TRANSLATE AND ROTATE THE INPUT AIRFOIL COORDINATES SO
C     THAT THE X-AXIS CORRESPONDS TO THE LONGEST CHORDLINE
C
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
      DIMENSION XU(1), YU(1), WU(1), XL(1), YL(1), WL(1), TITLE(8)
C
      COMMON /HLM/ X(200),Y(200),W(200)
C
      COMMON /BLK1/ PI,PI2,RAD,CONS
C
      COMMON /INOUT/ JREAD,JWRITE,IPRINT
C
C     PRINT INPUT COORDINATES
C
      WRITE (JWRITE,13) TITLE
      J=NU
      IF (NL.GT.NU) J=NL
      DO 1 I=1,J
      IF (I.LE.NU.AND.I.LE.NL) WRITE (JWRITE,14) I,XU(I),YU(I),XL(I),YL(
     1I)
      IF (I.LE.NU.AND.I.GT.NL) WRITE (JWRITE,14) I,XU(I),YU(I)
      IF (I.GT.NU.AND.I.LE.NL) WRITE (JWRITE,15) I,XL(I),YL(I)
1     CONTINUE
C
C     COMPUTE LONGEST CHORDLINE
C
C        LOAD LOWER SURFACE COORDINATES
      N=0
      DO 2 I=1,NL
      J=NL+1-I
      N=N+1
      W(N)=WL(J)
      X(N)=XL(J)
2     Y(N)=YL(J)
      J=1
      IF (XL(1).EQ.XU(1).AND.YL(1).EQ.YU(1)) J=2
C        LOAD UPPER SURFACE COORDINATES
```

```
            DO 3 I=J,NU
            N=N+1
            W(N)=WU(I)
            X(N)=XU(I)
      3     Y(N)=YU(I)
      C        COMPUTE MIDPOINT OF TRAILING-EDGE BASE
            XTE=0.5*(X(1)+X(N))
            YTE=0.5*(Y(1)+Y(N))
      C        FIND MOST FORWARD LEADING-EDGE POINT AND LONGEST CHORD
            CHORD=0.0
            DO 5 I=1,N
            DIST=SQRT((X(I)-XTE)**2+(Y(I)-YTE)**2)
            IF (DIST-CHORD) 5,5,4
      4     CHORD=DIST
            NOSE=I
            XNOSE=X(I)
            YNOSE=Y(I)
      5     CONTINUE
      C
      C     TRANSLATE AND ROTATE AIRFOIL
      C
            IF (CHORD.LE.0.0) GO TO 6
            COSA=(XTE-XNOSE)/CHORD
            SINA=(YTE-YNOSE)/CHORD
            ANGLE=ATAN(SINA/COSA)*RAD
            GO TO 7
      6     COSA=0.0
            SINA=0.0
            ANGLE=0.0
      7     DO 8 I=1,N
            DIST=X(I)
            X(I)=(DIST-XNOSE)*COSA+(Y(I)-YNOSE)*SINA
      8     Y(I)=(Y(I)-YNOSE)*COSA-(DIST-XNOSE)*SINA
      C
      C     REDEFINE LOWER AND UPPER SURFACE COORDINATES
      C
            DO 9 I=1,NOSE
            J=NOSE+1-I
            WL(I)=W(J)
            XL(I)=X(J)
```

```
    9     YL(I)=Y(J)
          NL=NOSE
          DO 10 I=NOSE,N
          J=I+1-NOSE
          WU(J)=W(I)
          XU(J)=X(I)
   10     YU(J)=Y(I)
          NU=J
    C
    C     PRINT NEW AIRFOIL COORDINATES
    C
          WRITE (JWRITE,16) TITLE
          J=NU
          IF (NL.GT.NU) J=NL
          DO 11 I=1,J
          IF (I.LE.NU.AND.I.LE.NL) WRITE (JWRITE,14) I,XU(I),YU(I),XL(I),YL(
         1I)
          IF (I.LE.NU.AND.I.GT.NL) WRITE (JWRITE,14) I,XU(I),YU(I)
          IF (I.GT.NU.AND.I.LE.NL) WRITE (JWRITE,15) I,XL(I),YL(I)
   11     CONTINUE
          WRITE (JWRITE,12) XNOSE,YNOSE,ANGLE
          RETURN
    C
   12     FORMAT (/5X,7HXNOSE =,F15.6,5X,7HYNOSE =,F15.6,5X,7HANGLE =,F8.3)
   13     FORMAT (1H1,32X,21H--INPUT COORDINATES--//5X,7HTITLE--,2X,8A10//9X
         1,1HI,11X,2HXU,13X,2HYU,13X,2HXL,13X,2HYL)
   14     FORMAT (5X,I5,4F15.6)
   15     FORMAT (5X,I5,30X,2F15.6)
   16     FORMAT (1H1,21X,38H--TRANSLATED AND ROTATED COORDINATES--//5X,7HTI
         1TLE--,2X,8A10//9X,1HI,11X,2HXU,13X,2HYU,13X,2HXL,13X,2HYL)
          END
```

```
      SUBROUTINE BADPT (X,Y,NP,TOLR,ISURF,IERR)
C
C     ROUTINE TO EDIT BAD POINTS FROM X AND Y INPUT COORDINATES
C
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
      DIMENSION X(1), Y(1), SURF(2)
C
      COMMON /HLM/ TI(100),YI(100),YN(100),THETA(100)
C
      COMMON /BLK1/ PI,PI2,RAD,CONS
C
      COMMON /INOUT/ JREAD,JWRITE,IPRINT
C
      DATA SURF(1)/5HUPPER/,SURF(2)/5HLOWER/
C
C     IF TOLERANCE IS ZERO OR NEGATIVE RETURN
C
      IERR=0
      IF (TOLR.LE.0.0) RETURN
C
C     COMPUTE LOCAL CHORD
C
      CHORD=X(NP)-X(1)
C
C     INITIALIZE ITERATION PARAMETERS
C
      ICD=0
      IPTP=0
      N1=NP-1
      NMAX=0
      TOLC=TOLR*CHORD
C
C     COMPUTE THETA EQUIVALENT OF X
C
      DO 2 I=1,NP
      DELTA=(X(I)-X(1))/CHORD
      IF (DELTA.LE.CONS) GO TO 1
      DELTA=TAN(DELTA/CONS-1.)
      THETA(I)=PI2+ALOG(DELTA+SQRT(DELTA*DELTA+1.))
      GO TO 2
    1 THETA(I)=ACOS(1.-DELTA/CONS)
    2 CONTINUE
C
C     LOOP TO SEARCH FOR BAD POINTS
C
    3 NMAX=NMAX+1
      JSTART=1
C        COMPUTE NEW Y VALUE BY INTERPOLATION
      DO 5 I=2,N1
      K=0
C                LOAD TI AND YI ARRAY - OMIT THE I(TH) INPUT DATA POINT
      DO 4 J=1,NP
      IF (I.EQ.J) GO TO 4
      K=K+1
      TI(K)=THETA(J)
      YI(K)=Y(J)
    4 CONTINUE
C                INTERPOLATE I(TH) DATA POINT
      CALL INTER (THETA(I),YN(I),K,TI,YI,JSTART,JEND,ICD)
      JSTART=JEND
    5 CONTINUE
C        CHECK TOLERANCE OF INTERPOLATED POINTS
      IPT=0
      ERRMAX=0.
      DO 7 I=2,N1
      ERRMIN=0.
      ERR=ABS(YN(I)-Y(I))
      IF (ERR.GE.TOLC) ERRMIN=ERR
      IF (ERRMIN-ERRMAX) 7,7,6
    6 IPT=I
      ERRMAX=ERRMIN
    7 CONTINUE
      IF (IPT.EQ.0) RETURN
C        PRINT COORDINATES OF BAD POINTS
      IF (NMAX.EQ.1) WRITE (JWRITE,9) SURF(ISURF),TOLC
      WRITE (JWRITE,10) IPT,X(IPT),Y(IPT),YN(IPT)
C        REPLACE BAD POINT WITH INTERPOLATED VALUE
      Y(IPT)=YN(IPT)
C        CHECK TO SEE IF THIS BAD POINT IS ADJACENT TO THE PREVIOUS BAD
```

```
C        POINT -- IF IT IS, PRINT A WARNING MESSAGE AND TERMINATE
C        PROGRAM EXECUTION
      IF ((IPTP.EQ.IPT-1).OR.(IPTP.EQ.IPT+1)) GO TO 8
      IF (IPTP.EQ.IPT) GO TO 8
      IPTP=IPT
      IF (NMAX.GE.NP) RETURN
C
C     RETURN TO START OF LOOP AND SEARCH FOR NEXT BAD POINT
C
      GO TO 3
C
C     WARNING MESSAGE PRINT STATEMENT
C
    8 WRITE (JWRITE,11)
      IERR=1
      RETURN
C
    9 FORMAT (1H1//1X,44HWARNING -- BAD POINTS HAVE BEEN FOUND ON THE,1X
     1,A5,1X,37HSURFACE BASED ON AN EDIT TOLERANCE OF,F10.6/)
   10 FORMAT (1X,15HBAD POINT AT I=,I4,5X,4HX = ,F10.6,5X,4HY = ,F10.6,5
     1X,18HREPLACED WITH Y = ,F10.6/)
   11 FORMAT (1X,93HADJACENT BAD POINTS HAVE BEEN FOUND -- PLEASE CORREC
     1T YOUR INPUT DATA AND RESUBMIT THIS CASE.)
      END
      SUBROUTINE SMOXY (THETA,X,Y,W,YSMO,YPS,YPPS,NP,NOSE,YLTE,YNOSE,YUT
     1E, EPS,DF,ITER,TITLE, IOP,IERR)
C
C     THIS SUBROUTINE PRESENTS A TECHNIQUE FOR SMOOTHING Y INPUT
C     COORDINATES USING LEAST SQUARES POLYNOMIAL AND CUBIC SPLINE
C     METHODS
C
C     IF IOP=0 OR 1, COMPUTE YPPU (UNSMOOTHED SECOND DERIVATIVES) FROM
C     LEAST SQUARES POLYNOMIAL FITTING OF Y VS THETA. THEN COMPUTE
C     YPPS (SMOOTHED SECOND DERIVATIVES) FROM LEAST SQUARES CUBIC
C     SPLINE FITTING OF YPPU VS THETA. FINALLY COMPUTE YSMO (SMOOTHED Y
C     COORDINATES) USING INVERSE CUBIC SPLINE METHOD.
C
C     IF IOP=2, COMPUTE SECOND DERIVATIVES FROM INPUT FIRST DERIVATIVES.
C     THEN COMPUTE UNSMOOTHED Y COORDINATES FROM SECOND DERIVATIVES AND
C     FOLLOW SAME PROCEDURES AS OUTLINED ABOVE FOR IOP 0 OR 1.
C
C     IF IOP=3, COMPUTE UNSMOOTHED Y COORDINATES FROM INPUT SECOND
C     DERIVATIVES. THEN FOLLOW SAME PROCEDURES AS OUTLINED ABOVE FOR
C     IOP 0 OR 1.
C
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
C        DIMENSION THETA, X, Y, W, YSMO, YPS, AND YPPS BY NP IN CALLING
C        PROGRAM
      DIMENSION TITLE(8), THETA(1), X(1), Y(1), W(1), YSMO(1), YPS(1), Y
     1PPS(1)
C
      COMMON /HLM/ WK(200,10)
C
      COMMON /SMY/ YPP(200),YUSMO(200),DUM(200),A(200,4),YN(200),YPPU(20
     10),SUMY(300),LTER(30)
C
      COMMON /BLK1/ PI,PI2,RAD,CONS
C
      COMMON /INOUT/ JREAD,JWRITE,IPRINT
C
      DATA LMX/200/,WT/100./
C
C     SINH(X)=(EXP(X)-EXP(-X))/2.
```

```
C     COSH(X)=(EXP(X)+EXP(-X))/2.
C
      IERR=0
      IF (IOP.EQ.0.OR.IOP.EQ.1) GO TO 13
      IF (IOP.EQ.2) GO TO 1
      IF (IOP.EQ.3) GO TO 11
C
C        IF IOP=2, COMPUTE SECOND DERIVATIVES FROM INPUT FIRST
C        DERIVATIVES. THEN COMPUTE INITIAL Y/C COORDINATES FROM SECOND
C        DERIVATIVES.
C
C               COMPUTE SECOND DERIVATIVES USING CSDS
1     DO 2 I=1,NP
2     DUM(I)=1.0
      T1=0.0
      CALL CSDS (LMX,NP,THETA,YPS,DUM,T1,-1,A,WK,IERR)
      IF (IERR.NE.0) GO TO 71
      DO 4 I=1,NP
      IF (I.EQ.NP) GO TO 3
      YPPS(I)=A(I,2)
      GO TO 4
3     DELTA=THETA(I)-THETA(I-1)
      YPPS(I)=(3.*A(I-1,4)*DELTA+2.*A(I-1,3))*DELTA+A(I-1,2)
4     CONTINUE
C               COMPUTE SECOND DERIVATIVES USING LSOSMO
      DELTA=1.
      CALL LSOSMO (THETA,YPS,W,DUM,YPP,YUSMO,NP,1,NP,NOSE,DELTA,EPS,IERR
     1)
      IF (IERR.NE.0) RETURN
C               COMPUTE Y/C COORDINATES
      CALL YNEW (THETA,YPPS,Y,NOSE,NP,YLTE,YNOSE,YUTE,EPS,DUM,WK,JWRITE,
     10)
      CALL YNEW (THETA,YPP,YUSMO,NOSE,NP,YLTE,YNOSE,YUTE,EPS,DUM,WK,JWRI
     1TE,0)
C               COMPUTE NEW FIRST DERIVATIVES AND COMPARE WITH INPUT
C               FIRST DERIVATIVES
      WRITE (JWRITE,73) TITLE
      SUM1=0.0
      SUM2=0.0
      DO 7 I=1,NP
      IF (I.EQ.1) GO TO 5
      DELTA=THETA(I)-THETA(I-1)
      YN(I)=YPPS(I-1)*DELTA/6.+YPPS(I)*DELTA/3.+(Y(I)-Y(I-1))/DELTA
      DUM(I)=YPP(I-1)*DELTA/6.+YPP(I)*DELTA/3.+(YUSMO(I)-YUSMO(I-1))/DEL
     1TA
      GO TO 6
5     DELTA=THETA(2)-THETA(1)
      YN(1)=-YPPS(1)*DELTA/3.-YPPS(2)*DELTA/6.+(Y(2)-Y(1))/DELTA
      DUM(1)=-YPP(1)*DELTA/3.-YPP(2)*DELTA/6.+(YUSMO(2)-YUSMO(1))/DELTA
6     T1=YPS(I)-YN(I)
      T2=YPS(I)-DUM(I)
      SUM1=SUM1+T1*T1
      SUM2=SUM2+T2*T2
7     WRITE (JWRITE,74) I,YPS(I),YN(I),T1,DUM(I),T2
      WRITE (JWRITE,75) SUM1,SUM2
C               SELECT OUTPUT FROM EITHER CSDS OR LSQSMO
      DO 10 I=1,NP
      IF (SUM2.LT.SUM1) GO TO 8
      YPP(I)=YPPS(I)
      GO TO 9
8     Y(I)=YUSMO(I)
      YN(I)=DUM(I)
9     YSMO(I)=Y(I)
10    YUSMO(I)=Y(I)
      IF (SUM2.GE.SUM1) WRITE (JWRITE,76)
      IF (SUM2.LT.SUM1) WRITE (JWRITE,77)
      IF (ITER.EQ.0) GO TO 48
      GO TO 13
C
C         IF IOP=3, COMPUTE INITIAL Y/C FROM INPUT SECOND DERIVATIVES
C         AND Y/C COORDINATES AT THE UPPER AND LOWER SURFACE TRAILING
C         EDGE AND NOSE
C
11    CALL YNEW (THETA,YPPS,Y,NOSE,NP,YLTE,YNOSE,YUTE,EPS,DUM,WK,JWRITE,
     10)
C               COMPUTE FIRST DERIVATIVES
      DO 12 I=1,NP
      YSMO(I)=Y(I)
      YUSMO(I)=Y(I)
      IF (I.EQ.1) GO TO 12
      DELTA=THETA(I)-THETA(I-1)
      YN(I)=YPPS(I-1)*DELTA/6.+YPPS(I)*DELTA/3.+(Y(I)-Y(I-1))/DELTA
   12 YPP(I)=YPPS(I)
      DELTA=THETA(2)-THETA(1)
      YN(1)=-YPPS(1)*DELTA/3.-YPPS(2)*DELTA/6.+(Y(2)-Y(1))/DELTA
      IF (ITER.EQ.0) GO TO 48
C
C        INITIALIZE ARRAYS
C
   13 DO 14 I=1,NP
      YUSMO(I)=Y(I)
      IF (IOP.LT.2) YPP(I)=0.0
      YSMO(I)=THETA(I)*RAD
   14 DUM(I)=1.
      IF (ITER.GT.0) GO TO 17
C
C        IF IOP=0 OR 1 AND NO SMOOTHING DESIRED (I.E. ITER=0) , COMPUTE
C        SECOND DERIVATIVE FROM CUBIC SPLINE SUBROUTINE
C
      CALL CSDS (LMX,NP,THETA,Y,DUM,0.0,-1,A,WK,IERR)
      IF (IERR.NE.0) GO TO 71
C        COMPUTE Y AND SECOND DERIVATIVE
      DO 16 I=1,NP
      IF (I.EQ.NP) GO TO 15
      YSMO(I)=A(I,1)
      YN(I)=A(I,2)
      YPP(I)=2.*A(I,3)
      GO TO 16
   15 DELTA=THETA(I)-THETA(I-1)
      YSMO(I)=((A(I-1,4)*DELTA+A(I-1,3))*DELTA+A(I-1,2))*DELTA+A(I-1,1)
      YN(I)=(3.*A(I-1,4)*DELTA+2.*A(I-1,3))*DELTA+A(I-1,2)
      YPP(I)=6.*A(I-1,4)*DELTA+2.*A(I-1,3)
   16 CONTINUE
      GO TO 48
C
C        FIND MAXIMUM INPUT Y VALUE AND ITS LOCATION FOR UPPER AND
C        LOWER SURFACES
C             LOWER SURFACE
   17 YMAX=0.0
      JMAXL=1
```

CARD NO.

```
      DO 19 I=1,NOSE
      J=NOSE+1-I
      IF (ABS(Y(J)).GT.YMAX) GO TO 18
      GO TO 19
   18 YMAX=ABS(Y(J))
      JMAXL=J
   19 CONTINUE
C              UPPER SURFACE
      YMAX=0.0
      JMAXU=1
      DO 21 I=NOSE,NP
      IF (ABS(Y(I)).GT.YMAX) GO TO 20
      GO TO 21
   20 YMAX=ABS(Y(I))
      JMAXU=I
   21 CONTINUE
C
C        COMPUTE UNSMOOTHED SECOND DERIVATIVE USING LEAST
C        SQUARES POLYNOMIAL METHOD
C
      J1=0
      ICON=0
      MTER=0
      J=ITER
      KTI=0
      IF (IPRINT.NE.0) WRITE (JWRITE,78) TITLE
      DO 23 I=1,30
      KTI=KTI+1
      LTER(I)=10
      J=J-10
      IF (J) 22,24,23
   22 LTER(I)=10+J
      GO TO 24
   23 CONTINUE
   24 DO 39 LL=1,KTI
      N1=LTER(LL)
      DO 34 I=1,N1
C              CALL LEAST SQUARES POLYNOMIAL SMOOTHING ROUTINE
      CALL LSQSMO (THETA,YUSMO,W,YN,DUM,YPPU,NP,JMAXL,JMAXU,NOSE,WT,EPS,
     1IERR)
```

```
      IF (IERR.NE.0) RETURN
C              COMPUTE ERROR TERM
      SUMY(I)=0.0
      DO 25 J=1,NP
   25 SUMY(I)=SUMY(I)+(YPPU(J)-YPP(J))**2
      J1=J1+1
      IF ((I.LE.3).AND.(LL.EQ.1)) GO TO 26
      IF (I.EQ.1) GO TO 26
C              CHECK FOR OSCILLATIONS IN CONVERGENCE OF ERROR TERM
      IF (SUMY(I)-SUMY(I-1)) 26,26,32
C              LOAD ARRAYS FOR NEXT ITERATION
   26 DO 31 J=1,NP
      WK(J,I)=YPPU(J)
      IF (LL.EQ.1.AND.I.EQ.1) YPPS(J)=YPPU(J)
      YPP(J)=YPPU(J)
      CC=YUSMO(J)
      IF (J1-2) 29,28,27
   27 AA=YN(J)-YUSMO(J)
      BB=A(J,1)-A(J,2)
      T1=SIGN(1.,AA)
      T2=SIGN(1.,BB)
      IF (T1.EQ.T2.OR.AA.EQ.BB) GO TO 28
      YUSMO(J)=A(J,2)-BB*(YUSMO(J)-A(J,2))/(AA-BB)
      GO TO 30
   28 YUSMO(J)=0.5*(YUSMO(J)+YN(J))
      GO TO 30
   29 YUSMO(J)=YN(J)
   30 A(J,1)=YN(J)
   31 A(J,2)=CC
      GO TO 33
   32 NTER=I-1
      ICON=2
      GO TO 36
   33 NTER=I
C              CHECK FOR CONVERGENCE BASED ON INPUT EPS
      IF (SUMY(I).LE.EPS) GO TO 35
   34 CONTINUE
      GO TO 36
   35 ICON=1
C
```

```
C           PRINT SECOND DERIVATIVES GENERATED DURING SMOOTHING PROCESS
C
36    IF (IPRINT.NE.0) GO TO 38
      WRITE (JWRITE,80) TITLE
      DO 37 J=1,NP
37    WRITE (JWRITE,81) J,YSMO(J),(WK(J,I),I=1,MTER)
      WRITE (JWRITE,82) (SUMY(I),I=1,NTER)
38    IF (IPRINT.NE.0) WRITE (JWRITE,79) LL,(SUMY(I),I=1,NTER)
      MTER=MTER+NTER
      IF (ICON.NE.0) GO TO 40
39    CONTINUE
40    IF (ICON.EQ.0) WRITE (JWRITE,83) MTER
      IF (ICON.EQ.1) WRITE (JWRITE,84) MTER
      IF (ICON.EQ.2) WRITE (JWRITE,85) MTER
C
C           COMPUTE SMOOTHED SECOND DERIVATIVE USING LEAST SQUARES
C           CUBIC SPLINE
C
      DO 41 I=1,NP
41    DUM(I)=DF
C                 CALL LEAST SQUARES CUBIC SPLINE ROUTINE
      CALL CSDS (LMX,NP,THETA,YPPU,DUM,FLOAT(NP),-1,A,WK,IERR)
      IF (IERR.NE.0) GO TO 71
C                 COMPUTE SECOND DERIVATIVE
      SUM=0.0
      DO 44 I=1,NP
      IF (I.EQ.NP) GO TO 42
      YPP(I)=A(I,1)
      GO TO 43
42    DELTA=THETA(I)-THETA(I-1)
      YPP(I)=((A(I-1,4)*DELTA+A(I-1,3))*DELTA+A(I-1,2))*DELTA+A(I-1,1)
43    SUM=SUM+(YPPU(I)-YPP(I))**2
44    YPPU(I)=YPPS(I)
      WRITE (JWRITE,88) SUM
C
C           COMPUTE NEW Y COORDINATES FROM SMOOTHED SECOND DERIVATIVES
C
      CALL YNEW (THETA,YPP,YSMO,NOSE,NP,YUSMO(1),YUSMO(NOSE),YUSMO(NP),E
     1PS,DUM,WK,JWRITE,1)
C
```

```
C        CHECK NEW Y COORDINATES FOR SMOOTHNESS
C
C              CALL LEAST SQUARES POLYNOMIAL ROUTINE
      DO 45 I=1,NP
   45 A(I,1)=1.0
      CALL LSQSMO (THETA,YSMO,A,YN,DUM,YPPS,NP,1,NP,NOSE,WT,EPS,IERR)
      IF (IERR.NE.0) RETURN
C              COMPUTE ERROR TERMS
      SUM1=0.0
      SUM2=0.0
      DO 46 I=1,NP
      A(I,1)=YSMO(I)-YN(I)
      A(I,2)=YPP(I)-YPPS(I)
      SUM1=SUM1+A(I,1)**2
   46 SUM2=SUM2+A(I,2)**2
C
C        COMPUTE FIRST DERIVATIVE FROM SMOOTHED SECOND DERIVATIVE
C
      N1=NP-1
      DO 47 I=1,N1
      DELTA=THETA(I+1)-THETA(I)
   47 YN(I)=-YPP(I)*DELTA/3.-YPP(I+1)*DELTA/6.+(YSMO(I+1)-YSMO(I))/DELTA
      DELTA=THETA(NP)-THETA(N1)
      YN(NP)=YPP(N1)*DELTA/6.+YPP(NP)*DELTA/3.+(YSMO(NP)-YSMO(N1))/DELTA
C
C        PRINT SUMMARY OF SMOOTHED AND UNSMOOTHED DATA
C
   48 WRITE (JWRITE,86) TITLE
      DO 53 I=1,NP
      YPS(I)=YN(I)
      IF (THETA(I).LE.0.) YN(I)=-YN(I)
      T1=ABS(THETA(I))
      IF (T1.GT.PI2) GO TO 49
      GP=CONS*SIN(T1)
      GPP=CONS*COS(T1)
      GO TO 50
   49 DIF=COSH(T1-PI2)
      DELTA=SINH(T1-PI2)
      GP=CONS/DIF
      GPP=-CONS*DELTA/(DIF*DIF)
   50 IF (I.EQ.NOSE) GO TO 51
      DYDX=YN(I)/GP
      DY2DX=(YPP(I)*GP-YN(I)*GPP)/(GP**3)
      CURV=ABS(DY2DX)/(SQRT(1.+DYDX**2)**3)
      GO TO 52
   51 DYDX=0.1E99
      DY2DX=0.1E99
      CURV=CONS/(YN(I)**2)
      RLE=1./CURV
   52 DELTA=THETA(I)*RAD
      DIF=Y(I)-YSMO(I)
      YPPS(I)=YPP(I)
   53 WRITE (JWRITE,87) I,DELTA,X(I),Y(I),YUSMO(I),YSMO(I),DIF,YPS(I),YP
     1P(I),DYDX,DY2DX,CURV
      WRITE (JWRITE,89) RLE
C
C     CHECK FOR INTERSECTION OF UPPER AND LOWER SURFACES
C
C        DEFINE ITERATION INTERVAL
      KRT=1001
      N1=2*KRT
      TE=THETA(NP)
      TN=-THETA(1)
      IF (TN.LT.TE) TE=TN
      DIF=TE/FLOAT(KRT-1)
      BB=0.5*DIF
      AA=0.85*TE
      YL1=YU1=YSMO(NOSE)
      TP=TN=0.0
      J1=NOSE
      J2=2
C        DO-LOOP TO SEARCH FOR INTERSECTION
      DO 59 I=2,N1
      IF (TP.LE.AA) TN=TN+DIF
      IF (TP.GT.AA) TN=TN+BB
      IF (TN.GT.TE) GO TO 61
      TI=TN
C        FIND UPPER SURFACE Y-COORDINATE AT THETA = TN
      DO 54 K=J1,NP
      J=K-1
      IF (TI.GE.THETA(J).AND.TI.LE.THETA(J+1)) GO TO 55
54    CONTINUE
55    DELTA=THETA(J+1)-THETA(J)
      T2=THETA(J+1)-TI
      T1=TI-THETA(J)
      YU2=YPPS(J)*(T2**3/(6.*DELTA)-T2*DELTA/6.)+YPPS(J+1)*(T1**3/(6.*DE
     1LTA)-T1*DELTA/6.)+(YSMO(J)*T2+YSMO(J+1)*T1)/DELTA
      J1=J
      IF (J1.LT.NOSE) J1=NOSE
C        FIND LOWER SURFACE Y-COORDINATE AT THETA = TN
      TI=-TN
      DO 56 K=J2,NOSE
      J=NOSE+1-K
      IF (TI.GE.THETA(J).AND.TI.LE.THETA(J+1)) GO TO 57
56    CONTINUE
57    DELTA=THETA(J+1)-THETA(J)
      T2=THETA(J+1)-TI
      T1=TI-THETA(J)
      YL2=YPPS(J)*(T2**3/(6.*DELTA)-T2*DELTA/6.)+YPPS(J+1)*(T1**3/(6.*DE
     1LTA)-T1*DELTA/6.)+(YSMO(J)*T2+YSMO(J+1)*T1)/DELTA
      J2=NOSE+1-J
      IF (J2.LT.2) J2=2
C        COMPUTE THETA FOR INTERSECTION OF STRAIGHT LINE SEGMENTS THRU
C        LAST TWO POINTS ON EACH SURFACE
      CC=(YU2-YU1-YL2+YL1)/(TN-TP)
      IF (ABS(CC).LT.1.E-10) GO TO 58
      T1=(YL1-YU1)/CC+TP
      IF (I.EQ.2) GO TO 58
C        CHECK TO SEE IF INTERSECTION THETA IS BETWEEN THIS TN-VALUE
C        AND THE PREVIOUS TN-VALUE
      IF (T1.GE.TP.AND.T1.LE.TN) GO TO 60
C        CONTINUE TO NEXT TN-VALUE
58    YU1=YU2
      YL1=YL2
      TP=TN
59    CONTINUE
      GO TO 61
60    IF (T1.GE.TE) GO TO 61
C        IF INTERSECTION OCCURS WRITE ERROR MESSAGE AND RETURN TO
C        CALLING PROGRAM
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```
            T1=T1*RAD
            WRITE (JWRITE,72) T1
            IERR=1
            RETURN
C
C               FIND LOCATIONS WHERE DY/DX=0.
C
   61       KRT=0
            N1=NP-1
            DO 66 I=1,N1
            DELTA=THETA(I+1)-THETA(I)
            AA=(YPP(I)-YPP(I+1))/(2.*DELTA)
            BB=(YPP(I+1)*THETA(I)-YPP(I)*THETA(I+1))/DELTA
            CC=(YPP(I)*THETA(I+1)**2-YPP(I+1)*THETA(I)**2)/(2.*DELTA)+(YPP(I+1
          1)-YPP(I))*DELTA/6.-(YSMO(I+1)-YSMO(I))/DELTA
            GP=BB*BB-4.*AA*CC
            IF (GP) 66,62,62
   62       GP=SQRT(GP)
            T1=(-BB+GP)/(2.*AA)
            T2=(-BB-GP)/(2.*AA)
            IF (T1.GE.THETA(I).AND.T1.LE.THETA(I+1)) GO TO 63
            GO TO 64
   63       KRT=KRT+1
            WK(KRT,1)=T1
   64       IF (T2.GE.THETA(I).AND.T2.LE.THETA(I+1)) GO TO 65
            GO TO 66
   65       KRT=KRT+1
            WK(KRT,1)=T2
   66       CONTINUE
            IF (KRT.EQ.0) GO TO 70
C                     FIND X/C AND Y/C WHERE DY/DX=0.
            DO 69 I=1,KRT
            CALL INTER (WK(I,1),WK(I,2),NP,THETA,X,1,KTI,0)
            DO 67 J=1,N1
            J1=J
            J2=J+1
            IF (WK(I,1).GE.THETA(J).AND.WK(I,1).LE.THETA(J+1)) GO TO 68
   67       CONTINUE
   68       AA=THETA(J2)-WK(I,1)
            BB=WK(I,1)-THETA(J1)
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      WK(I,1)=WK(I,1)*RAD                                                SO 441
      DELTA=THETA(J2)-THETA(J1)                                          SO 442
   69 WK(I,3)=YPP(J1)*(AA**3/(6.*DELTA)-AA*DELTA/6.)+YPP(J2)*(BB**3/(6.* SO 443
     1DELTA)-BB*DELTA/6.)+(YSMO(J1)*AA+YSMO(J2)*BB)/DELTA                SO 444
   70 CONTINUE                                                           SO 445
      IF (KRT.GT.0) WRITE (JWRITE,90) (WK(I,2),WK(I,3),WK(I,1),I=1,KRT)  SO 446
C                                                                        SO 447
C         PRINT RESULTS OF SMOOTHNESS CHECK                              SO 448
C                                                                        SO 449
      IF (ITER.EQ.0) RETURN                                              SO 450
      WRITE (JWRITE,91) TITLE,DF                                         SO 451
      WRITE (JWRITE,92) (I,A(I,1),A(I,2),I=1,NP)                         SO 452
      WRITE (JWRITE,93) SUM1,SUM2                                        SO 453
      RETURN                                                             SO 454
C                                                                        SO 455
C         PRINT WARNING MESSAGE IF ERROR OCCURRED IN CALL TO CSDS        SO 456
C                                                                        SO 457
   71 WRITE (JWRITE,94) IERR                                             SO 458
      RETURN                                                             SO 459
C                                                                        SO 460
   72 FORMAT (/5X,108HERROR MESSAGE  ---  SMOOTHING PROCESS RESULTED IN  SO 461
     1AN INTERSECTION OF THE UPPER AND LOWER SURFACES AT THETA =,F10.3)  SO 462
   73 FORMAT (1H1,1X,7HTITLE--,2X,8A10//12X,62H--CHECK OF FIRST DERIVATI SO 463
     1VES GENERATED FROM IOP=2 INPUT DATA--///9X,1HI,5X,12HDY/DT(INPUT), SO 464
     24X,11HDY/DT(CSDS),8X,3HDIF,6X,13HDY/DT(LSQSMO),8X,3HDIF/)          SO 465
   74 FORMAT (5X,I5,5(5X,F10.6))                                         SO 466
   75 FORMAT (/25X,16HSUM OF SQUARES =,4X,F10.6,20X,F10.6)               SO 467
   76 FORMAT (/10X,25HOUTPUT FROM CSDS SELECTED)                         SO 468
   77 FORMAT (/10X,27HOUTPUT FROM LSQSMO SELECTED)                       SO 469
   78 FORMAT (1H1,1X,7HTITLE--,2X,8A10//30X,53H--SUM OF SQUARES GENERATE SO 470
     1D DURING SMOOTHING PROCESS--)                                      SO 471
   79 FORMAT (/1X,I5,10F12.7)                                            SO 472
   80 FORMAT (1H1,1X,7HTITLE--,2X,8A10//30X,67H--SECOND DERIVATIVES W/R  SO 473
     1THETA GENERATED DURING SMOOTHING PROCESS--/4X,1HI,5X,5HTHETA,10(5X SO 474
     2,6HDY2/DT)/)                                                       SO 475
   81 FORMAT (I5,F10.2,10F11.6)                                          SO 476
   82 FORMAT (/1X,14HSUM OF SQUARES,(10F11.6))                           SO 477
   83 FORMAT (/3X,41HSMOOTHING PROCESS HAS NOT CONVERGED AFTER,I4,1X,10H SO 478
     1ITERATIONS)                                                        SO 479
   84 FORMAT (/3X,33HSMOOTHING PROCESS CONVERGED AFTER,I4,1X,10HITERATIO SO 480
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     1NS)                                                               SO 481
   85 FORMAT (/3X,41HSMOOTHING PROCESS BEGAN OSCILLATING AFTER,I4,1X,10H SO 482
     1ITERATIONS)                                                       SO 483
   86 FORMAT (1H1,1X,7HTITLE--,2X,8A10//48X,28H--SMOOTHING OUTPUT SUMMAR SO 484
     1Y--//4X,1HI,5X,5HTHETA,5X,3HX/C,7X,3HY/C,7X,4HYT/C,5X,6HYSMO/C,4X, SO 485
     25HDELTA,7X,3HYPS,6X,4HYPPS,8X,5HDY/DX,7X,11HD(DY/DX)/DX,6X,       SO 486
     19HCURVATURE/)                                                     SO 487
   87 FORMAT (I5,F10.2,7F10.6,3E15.6)                                   SO 488
   88 FORMAT (/3X,58HSUM OF SQUARES FROM LEAST SQUARES CUBIC SPLINE SMOO SO 489
     1THING =,E12.4)                                                    SO 490
   89 FORMAT (/3X,22HLEADING-EDGE RADIUS/C=,F10.6)                      SO 491
   90 FORMAT (/3X,16HDY/DX=0. AT X/C=,F10.6,5X,4HY/C=,F10.6,5X,6HTHETA=, SO 492
     1F10.3)                                                            SO 493
   91 FORMAT (1H1,1X,7HTITLE--,2X,8A10//12X,29HCHECK OF SMOOTHED COORDIN SO 494
     1ATES,3X,3HDF=,F10.6//9X,1HI,5X,20H(YSMO/C-CHECK VALUE),7X,        SO 495
     218H(YPPS-CHECK VALUE)/)                                           SO 496
   92 FORMAT (5X,I5,10X,F10.6,15X,F10.6)                                SO 497
   93 FORMAT (/5X,15HSUM OF SQUARES=,F10.6,15X,F10.6)                   SO 498
   94 FORMAT (/3X,21HINPUT ERROR -- POINT ,I3,18H IS NOT INCREASING/)   SO 499
      END                                                               SO 500-
      SUBROUTINE YNEW (THETA,YPP,Y,NOSE,NP,YLTE,YNOSE,YUTE,EPS,DUM,WK,JW
     1RITE,IPT)
C
C     ROUTINE TO COMPUTE NEW Y/C COORDINATES USING AN ITERATION
C     PROCEDURE THAT INSURES A DESIRED Y/C COORDINATE AT THE NOSE
C     (IPT=0) OR THAT INSURES CONTINUITY OF THE FIRST DERIVATIVE W/R TO
C     THETA AT THE NOSE (IPT=1)
C
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
C        DIMENSION THETA, YPP, Y, AND DUM BY NP AND WK BY 2*NP IN
C        CALLING PROGRAM
      DIMENSION THETA(1), YPP(1), Y(1), DUM(1), WK(1)
C
C     INITIALIZE ITERATION PARAMETERS
C
      NMAX=20
      N1=-1
      DELTA=0.
      T1=THETA(NOSE)-THETA(NOSE-1)
      T2=THETA(NOSE+1)-THETA(NOSE)
      DO 1 I=1,NP
    1 DUM(I)=YPP(I)
C
C     ITERATION LOOP TO COMPUTE INCREMENTAL ADJUSTMENT TO SECOND
C     DERIVATIVE TO INSURE THAT THE DESIRED CONVERGENCE OPTION AT
C     THE NOSE IS OBTAINED
C
    2 N1=N1+1
      IF (N1.GT.NMAX) GO TO 11
      IF (IPT.EQ.1) GO TO 3
C        IF IPT=0, COMPUTE UPPER AND LOWER SURFACE Y/C COORDINATES
C        CONCURRENTLY
      CALL INVY (THETA,DUM,1,NP,Y,YLTE,YUTE,WK)
C        COMPUTE DIFFERENCE BETWEEN OUTPUT AND DESIRED Y/C COORDINATE
C        AT THE NOSE
      DIF=Y(NOSE)-YNOSE
      GO TO 4
C        IF IPT=1, COMPUTE UPPER AND LOWER SURFACE Y/C COORDINATES
C        CONSECUTIVELY
```

```
3      CALL INVY (THETA,DUM,NOSE,NP,Y,YNOSE,YUTE,WK)
       CALL INVY (THETA,DUM,1,NOSE,Y,YLTE,YNOSE,WK)
C         COMPUTE DIFFERENCE BETWEEN UPPER AND LOWER SURFACE FIRST
C         DERIVATIVES AT THE NOSE
       AA=-DUM(NOSE)*T2/3.-DUM(NOSE+1)*T2/6.+(Y(NOSE+1)-Y(NOSE))/T2
       BB=DUM(NOSE-1)*T1/6.+DUM(NOSE)*T1/3.+(Y(NOSE)-Y(NOSE-1))/T1
       DIF=AA-BB
C         CHECK FOR CONVERGENCE
4      IF (ABS(DIF).LE.EPS) GO TO 9
C         COMPUTE ADJUSTMENT VALUE TO SECOND DERIVATIVE
       IF (N1.EQ.0) GO TO 6
       IF (DIF.EQ.DIFP) GO TO 5
       SP=(DELTA-DELTAP)/(DIF-DIFP)
       DELTAP=DELTA
       DIFP=DIF
       DELTA=DELTA-DIF*SP
       GO TO 7
5      DELTA=0.5*(DELTA+DELTAP)
       GO TO 7
6      DELTAP=DELTA
       DIFP=DIF
       DELTA=DELTA+DIF
C         ADD ADJUSTMENT VALUE TO SECOND DERIVATIVE
7      DO 8 I=1,NP
8      DUM(I)=YPP(I)+DELTA
C         CONTINUE TO ITERATE
       GO TO 2
C
C      PRINT CONVERGENCE MESSAGE
C
9      WRITE (JWRITE,14) N1,DELTA
C         REDEFINE THE SECOND DERIVATIVE
       DO 10 I=1,NP
10     YPP(I)=DUM(I)
       IF (IPT.EQ.1) GO TO 12
       GO TO 13
C
C      PRINT NON-CONVERGENCE MESSAGE
C
11     N1=N1-1
```

CARD NO.

```
               WRITE (JWRITE,15) N1                                          YW  81
        C                                                                    YW  82
        C      COMPUTE NEW UPPER AND LOWER SURFACE Y/C COORDINATES CONCURRENTLY  YW  83
        C                                                                    YW  84
        12     CALL INVY (THETA,YPP,1,NP,Y,YLTE,YUTE,WK)                     YW  85
        C                                                                    YW  86
        C      RETURN TO CALLING PROGRAM                                     YW  87
        C                                                                    YW  88
        13     RETURN                                                        YW  89
        C                                                                    YW  90
        14     FORMAT (/3X,88HITERATION PROCEDURE TO COMPUTE INCREMENTAL ADJUSTME YW  91
              1NT TO SECOND DERIVATIVE CONVERGED IN ,I3,23H ITERATIONS AND DELTA  YW  92
              2=,E12.4)                                                      YW  93
        15     FORMAT (///10X,40HWARNING THE FOLLOWING ERROR HAS OCCURRED//3X,95H YW  94
              1ITERATION PROCEDURE TO COMPUTE INCREMENTAL ADJUSTMENT TO SECOND DE YW  95
              2RIVATIVE DID NOT CONVERGE IN ,I3,11H ITERATIONS)              YW  96
               END                                                           YW  97-
```

CARD NO.

```
      SUBROUTINE INVY (X,YPP,NS,NE,Y,YSTART,YEND,A)
C
C     THIS ROUTINE COMPUTES Y VALUES FROM KNOWN SECOND DERIVATIVES AND
C     END CONDITIONS
C
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
C        IN CALLING PROGRAM DIMENSION X, YPP, AND Y BY NE AND A BY 2*NE
C
      DIMENSION X(1), YPP(1), Y(1), A(NE,2)
C
C     SET END CONDITIONS
C
      Y(NS)=YSTART
      Y(NE)=YEND
C
C     PERFORM FORWARD ELIMINATION
C
      A(1,1)=YSTART
      A(1,2)=0.0
      N=NE-NS+1
      N1=N-1
      DO 1 I=2,N1
      J=NS+I-1
      H1=X(J)-X(J-1)
      H2=X(J+1)-X(J)
      C=(H1*YPP(J-1)/6.+(H1+H2)*YPP(J)/3.+H2*YPP(J+1)/6.)*H1*H2
      D=-H2*(A(I-1,2)+1.)-H1
      A(I,2)=H1/D
    1 A(I,1)=(C-H2*A(I-1,1))/D
C
C     PERFORM BACK SUBSTITUTION
C
      J=NE
      DO 2 I=2,N1
      J=J-1
      N=N-1
    2 Y(J)=A(N,1)-A(N,2)*Y(J+1)
C
C     RETURN TO CALLING PROGRAM
```

```
LISTING OF DECK: INVY                                                                          PAGE   2

CARD NO.

   41          C                                                                                IV  41
                     RETURN                                                                     IV  42
                     END                                                                        IV  43-
```

CARD NO.

```
      SUBROUTINE LSQSMO (X,Y,W,YN,YP,YPP,N,IMAX,JMAX,NOSE,WT,EPS,IERR)
C
C     THIS SUBROUTINE IS USED TO SMOOTH X AND Y BY CONSECUTIVELY FITTING
C     A LEAST SQUARES POLYNOMIAL OF DEGREE 4 THRU 7 POINTS AT A TIME
C
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
      DIMENSION X(1), Y(1), W(1), YN(1), YP(1), YPP(1)
C
      DIMENSION XI(7), YI(7), WW(7), A(5,6), B(5)
C
      COMMON /INOUT/ JREAD,JWRITE,IPRINT
C
C     CHECK NOSE REGION FOR SYMMETRY
C
      ISYM=1
      DO 1 I=1,3
      IF (ABS(X(NOSE-I)+X(NOSE+I)).GT.EPS) ISYM=0
      IF (ABS(Y(NOSE-I)+Y(NOSE+I)).GT.EPS) ISYM=0
    1 CONTINUE
      IERR=0
C
C     FIT A LEAST SQUARES POLYNOMIAL OF DEGREE 4 THRU 7 POINTS
C
      DO 14 I=1,N
C        LOAD 7 POINTS FOR LEAST SQUARES POLYNOMIAL FIT
      IF (I.LT.4) GO TO 2
      IF (I.GT.N-3) GO TO 3
      J1=I-3
      J2=I+3
      GO TO 4
    2 J1=1
      J2=7
      GO TO 4
    3 J1=N-6
      J2=N
    4 KK=0
      IF (ISYM.EQ.0) GO TO 7
      IF (I.GT.NOSE-3.AND.I.LE.NOSE) GO TO 5
      IF (I.LT.NOSE+3.AND.I.GT.NOSE) GO TO 6
      GO TO 7
    5 J1=NOSE-6
      J2=NOSE
      GO TO 7
    6 J1=NOSE
      J2=NOSE+6
    7 DO 8 L=J1,J2
      J=L
      IF (I.LE.NOSE) J=J1+J2-L
      KK=KK+1
      WW(KK)=1.0
      IF (I.EQ.J) WW(KK)=W(I)
      IF (J.EQ.IMAX.OR.J.EQ.JMAX) WW(KK)=WT*W(J)
      XI(KK)=X(J)
    8 YI(KK)=Y(J)
      IF (I.LE.4) WW(7)=7.*W(1)
      IF (I.GE.N-3) WW(7)=7.*W(N)
C        COMPUTE LEAST SQUARES MATRIX
      DO 9 L=1,5
      DO 9 J=1,6
    9 A(L,J)=0.
      DO 11 K=1,7
      T1=1.
      DO 11 J=1,5
      T2=T1
      DO 10 L=1,5
      A(J,L)=A(J,L)+T2*WW(K)
   10 T2=T2*XI(K)
      A(J,6)=A(J,6)-YI(K)*T1*WW(K)
   11 T1=T1*XI(K)
C        SOLVE FOR COEFFICIENTS OF LEAST SQUARES POLYNOMIAL
      DO 12 K=1,4
      DO 12 J=K,4
      T1=A(J+1,K)/A(K,K)
      DO 12 L=K,6
   12 A(J+1,L)=A(J+1,L)-A(K,L)*T1
      B(5)=-A(5,6)/A(5,5)
      DO 13 L=2,5
      K=6-L
      B(K)=-A(K,6)/A(K,K)
```

CARD NO.

```
            K1=K+1                                                        LM  81
            DO 13 J=K1,5                                                  LM  82
   13       B(K)=B(K)-B(J)*A(K,J)/A(K,K)                                  LM  83
   C           COMPUTE NEW Y , FIRST , AND SECOND DERIVATIVE              LM  84
            YN(I)=(((B(5)*X(I)+B(4))*X(I)+B(3))*X(I)+B(2))*X(I)+B(1)      LM  85
            YP(I)=((4.*B(5)*X(I)+3.*B(4))*X(I)+2.*B(3))*X(I)+B(2)         LM  86
            YPP(I)=(12.*B(5)*X(I)+6.*B(4))*X(I)+2.*B(3)                   LM  87
   14       CONTINUE                                                      LM  88
            IF (ISYM.EQ.0) RETURN                                         LM  89
            YN(NOSE)=0.0                                                  LM  90
            YPP(NOSE)=0.0                                                 LM  91
            YP(NOSE)=1.0                                                  LM  92
            RETURN                                                        LM  93
            END                                                           LM  94-
```

```
      SUBROUTINE CSDS (MAX,IX,X,F,DF,S,IPT,COEF,WK,IERR)                 CS    1
C*********************************************************************** CS    2
C*                                                                     * CS    3
C*    PURPOSE:                                                         * CS    4
C*                 SUBROUTINE CSDS FITS A SMOOTH CUBIC SPLINE TO A     * CS    5
C*                 UNIVARIATE FUNCTION.  DATA MAY BE UNEQUALLY SPACED. * CS    6
C*                                                                     * CS    7
C*        USE:                                                         * CS    8
C*                 CALL CSDS(MAX,IX,X,F,DF,S,IPT,COEF,WK,IERR)         * CS    9
C*                                                                     * CS   10
C*         MAX     INPUT INTEGER SPECIFYING THE MAXIMUM NUMBER OF DATA * CS   11
C*                 POINTS FOR THE INDEPENDENT VARIABLE.                * CS   12
C*                                                                     * CS   13
C*          IX     INPUT INTEGER SPECIFYING THE ACTUAL NUMBER OF DATA  * CS   14
C*                 POINTS FOR THE INDEPENDENT VARIABLE.  IX@MAX.       * CS   15
C*                                                                     * CS   16
C*           X     ONE-DIMENSIONAL INPUT ARRAY DIMENSIONED AT LEAST    * CS   17
C*                 IX IN THE CALLING PROGRAM.  UPON ENTRY TO CSDS,     * CS   18
C*                 X(I) MUST CONTAIN THE VALUE OF THE INDEPENDENT      * CS   19
C*                 VARIABLE AT POINT I.                                * CS   20
C*                                                                     * CS   21
C*           F     ONE-DIMENSIONAL INPUT ARRAY DIMENSIONED AT LEAST    * CS   22
C*                 IX IN THE CALLING PROGRAM.  UPON ENTRY TO CSDS,     * CS   23
C*                 F(I) MUST CONTAIN THE VALUE OF THE FUNCTION AT      * CS   24
C*                 POINT X(I).                                         * CS   25
C*                                                                     * CS   26
C*          DF     ONE-DIMENSIONAL INPUT ARRAY DIMENSIONED AT LEAST    * CS   27
C*                 IX IN THE CALLING PROGRAM.  UPON ENTRY TO CSDS,     * CS   28
C*                 DF(I) MUST CONTAIN AN ESTIMATE OF THE STANDARD      * CS   29
C*                 DEVIATION OF F(I).                                  * CS   30
C*                                                                     * CS   31
C*           S     A NON-NEGATIVE INPUT PARAMETER WHICH CONTROLS THE   * CS   32
C*                 EXTENT OF SMOOTHING.  S SHOULD BE IN THE RANGE      * CS   33
C*                 (IX-(2*IX)**.5)<S<(IX+(2*IX)**.5).                  * CS   34
C*                                                                     * CS   35
C*         IPT     INPUT INITIALIZATION PARAMETER.  THE USER MUST      * CS   36
C*                 SPECIFY IPT=-1 WHENEVER A NEW X ARRAY IS            * CS   37
C*                 INPUT.  THE ROUTINE WILL ALSO CHECK TO INSURE THAT  * CS   38
C*                 THE X ARRAY IS IN STRICTLY INCREASING ORDER.        * CS   39
C*                                                                     * CS   40
```

```
      C*        COEF     A TWO-DIMENSIONAL OUTPUT ARRAY DIMENSIONED (MAX,4)  * CS  41
      C*                 IN THE CALLING PROGRAM.  UPON RETURN, COEF(I,J)     * CS  42
      C*                 CONTAINS THE J-TH COEFFICIENT OF THE SPLINE FOR     * CS  43
      C*                 THE INTERVAL BEGINNING AT POINT X(I).  THE          * CS  44
      C*                 FUNCTIONAL VALUE OF THE SPLINE AT ABSCISSA X1,      * CS  45
      C*                 WHERE X(I)<X1<X(I+1), IS GIVEN BY:                  * CS  46
      C*                    F(X1)=((COEF(I,4)*H+COEF(I,3))*H+COEF(I,2))*H    * CS  47
      C*                    +COEF(I,1)                                       * CS  48
      C*                 WHERE H=X1-X(I)                                     * CS  49
      C*                                                                     * CS  50
      C*          WK     A ONE-DIMENSIONAL WORK AREA ARRAY DIMENSIONED AT    * CS  51
      C*                 LEAST (7*IX+9) IN THE CALLING PROGRAM.              * CS  52
      C*                                                                     * CS  53
      C*        IERR     OUTPUT ERROR PARAMETER:                             * CS  54
      C*                 =0   NORMAL RETURN.  NO ERROR DETECTED.             * CS  55
      C*                 =J   THE J-TH ELEMENT OF THE X ARRAY IS NOT IN      * CS  56
      C*                      STRICTLY INCREASING ORDER.                     * CS  57
      C*                 =-1  THERE ARE LESS THAN FOUR VALUES IN THE X ARRAY.* CS  58
      C*                                                                     * CS  59
      C*                 UPON RETURN FROM CSDS, THIS PARAMETER SHOULD BE     * CS  60
      C*                 TESTED IN THE CALLING PROGRAM.                      * CS  61
      C*                                                                     * CS  62
      C*                                                                     * CS  63
      C*                                                                     * CS  64
      C*    REQUIRED ROUTINES                  -NONE                         * CS  65
      C*                                                                     * CS  66
      C*    SOURCE                             IMSL ROUTINE ICSSMU MODIFIED BY * CS  67
      C*                                       COMPUTER SCIENCES CORPORATION * CS  68
      C*                                                                     * CS  69
      C*    LANGUAGE                           -FORTRAN                      * CS  70
      C*                                                                     * CS  71
      C*    DATE RELEASED                      SEPTEMBER 5, 1973             * CS  72
      C*                                                                     * CS  73
      C*    LATEST REVISION                    MARCH 1975                    * CS  74
      C********************************************************************** CS  75
      C                                                                        CS  76
      C                                                                        CS  77
            DIMENSION X(1), F(1), DF(1), COEF(MAX,4), WK(1)                    CS  78
      C                                                                        CS  79
      C                                SET UP WORKING AREAS                    CS  80
```

```
C
      IERR=0
      IF (IPT.NE.-1) GO TO 4
      IPT=0
      IF (IX.LT.4) GO TO 1
      GO TO 2
    1 IERR=-1
      RETURN
    2 IX1=IX-1
      DO 3 I=1,IX1
      IF (X(I+1)-X(I).GT.0) GO TO 3
      IERR=I+1
      RETURN
    3 CONTINUE
      NP1=IX+1
      IB1=NP1
      IB2=IB1+NP1
      IB3=IB2+NP1+1
      IB4=IB3+NP1
      IB5=IB4+NP1
      IB6=IB5+NP1+1
      WK(1)=0.
      WK(2)=0.
      WK(IB2)=0.
      WK(IB3)=0.
      IJK2=IB2+NP1
      WK(IJK2)=0.
      IJK5=IB5+1
      WK(IJK5)=0.
      IJK5=IB5+2
      WK(IJK5)=0.
      WK(IB6)=0.
      IJK5=IB5+NP1
      WK(IJK5)=0.
    4 CONTINUE
      P=0.
      H=X(2)-X(1)
      F2=-S
      FF=(F(2)-F(1))/H
      IF (IX.LT.3) GO TO 10
```

```
              DO 5 I=3,IX                                               CS 121
              G=H                                                       CS 122
              H=X(I)-X(I-1)                                             CS 123
              E=FF                                                      CS 124
              FF=(F(I)-F(I-1))/H                                        CS 125
              COEF(I-1,1)=FF-E                                          CS 126
              IJK3=IB3+I                                                CS 127
              WK(IJK3)=(G+H)*.66666666666667                            CS 128
              IJK4=IB4+I                                                CS 129
              WK(IJK4)=H/3.                                             CS 130
              IJK2=IB2+I                                                CS 131
              WK(IJK2)=DF(I-2)/G                                        CS 132
              WK(I)=DF(I)/H                                             CS 133
              IJK1=IB1+I                                                CS 134
              WK(IJK1)=-DF(I-1)/G-DF(I-1)/H                             CS 135
      5       CONTINUE                                                  CS 136
              DO 6 I=3,IX                                               CS 137
              IJK1=IB1+I                                                CS 138
              IJK2=IB2+I                                                CS 139
              COEF(I-1,2)=WK(I)*WK(I)+WK(IJK1)*WK(IJK1)+WK(IJK2)*WK(IJK2)  CS 140
              COEF(I-1,3)=WK(I)*WK(IJK1+1)+WK(IJK1)*WK(IJK2+1)          CS 141
              COEF(I-1,4)=WK(I)*WK(IJK2+2)                              CS 142
      6       CONTINUE                                                  CS 143
      C                                                                 CS 144
      C                             NEXT ITERATION                      CS 145
      C                                                                 CS 146
      7       IF (IX.LT.3) GO TO 10                                     CS 147
              DO 8 I=3,IX                                               CS 148
              IJK1=IB1+I-1                                              CS 149
              IJK0=I-1                                                  CS 150
              WK(IJK1)=FF*WK(IJK0)                                      CS 151
              IJK2=IB2+I-2                                              CS 152
              IJK0=I-2                                                  CS 153
              WK(IJK2)=G*WK(IJK0)                                       CS 154
              IJK0=I                                                    CS 155
              IJK3=IB3+I                                                CS 156
              WK(IJK0)=1./(P*COEF(I-1,2)+WK(IJK3)-FF*WK(IJK1)-G*WK(IJK2))  CS 157
              IJK5=IB5+I                                                CS 158
              IJKN=IJK5-1                                               CS 159
              IJK0=IJKN-1                                               CS 160
      WK(IJK5)=COEF(I-1,1)-WK(IJK1)*WK(IJKN)-WK(IJK2)*WK(IJKO)
      IJK4=IB4+I
      FF=P*COEF(I-1,3)+WK(IJK4)-H*WK(IJK1)
      G=H
      H=COEF(I-1,4)*P
    8 CONTINUE
      DO 9 I=3,IX
      J=IX-I+3
      IJK5=IB5+J
      IJK6=IJK5+1
      IJK7=IJK6+1
      IJK1=IB1+J
      IJK2=IB2+J
      WK(IJK5)=WK(J)*WK(IJK5)-WK(IJK1)*WK(IJK6)-WK(IJK2)*WK(IJK7)
    9 CONTINUE
   10 E=0
      H=0
C
C                                     COMPUTE U AND ACCUMULATE E
C
      DO 11 I=2,IX
      G=H
      IJK5=IB5+I
      H=(WK(IJK5+1)-WK(IJK5))/(X(I)-X(I-1))
      IJK6=IB6+I
      WK(IJK6)=(H-G)*DF(I-1)*DF(I-1)
      E=E+WK(IJK6)*(H-G)
   11 CONTINUE
      G=-H*DF(IX)*DF(IX)
      IJK6=IB6+NP1
      WK(IJK6)=G
      E=E-G*H
      G=F2
      F2=E*P*P
      IF (F2.GE.S.OR.F2.LE.G) GO TO 14
      FF=0.
      IJK6=IB6+2
      H=(WK(IJK6+1)-WK(IJK6))/(X(2)-X(1))
      IF (IX.LT.3) GO TO 13
      DO 12 I=3,IX
      G=H
      IJK6=IB6+I
      H=(WK(IJK6+1)-WK(IJK6))/(X(I)-X(I-1))
      IJK1=IB1+I-1
      IJK2=IB2+I-2
      G=H-G-WK(IJK1)*WK(I-1)-WK(IJK2)*WK(I-2)
      FF=FF+G*WK(I)*G
      WK(I)=G
   12 CONTINUE
   13 H=E-P*FF
      IF (H.LE.0) GO TO 14
C
C                                  UPDATE THE LAGRANGE MULTIPLIER P
C                                     FOR THE NEXT ITERATION
C
      P=P+(S-F2)/((SQRT(S/E)+P)*H)
      GO TO 7
C
C                                  IF E LESS THAN OR EQUAL TO S,
C                                  COMPUTE THE COEFFICIENTS AND RETURN.
C
   14 DO 15 I=2,NP1
      IJK6=IB6+I
      COEF(I-1,1)=F(I-1)-P*WK(IJK6)
      IJK5=IB5+I
      COEF(I-1,3)=WK(IJK5)
   15 CONTINUE
      DO 16 I=2,IX
      H=X(I)-X(I-1)
      COEF(I-1,4)=(COEF(I,3)-COEF(I-1,3))/(3.*H)
      COEF(I-1,2)=(COEF(I,1)-COEF(I-1,1))/H-(H*COEF(I-1,4)+COEF(I-1,3))*
     1H
   16 CONTINUE
      RETURN
      END
      SUBROUTINE PCARD (IPUNCH,X,Y,W,THETA,YSMO,YPS,YPPS,NOSE,NP,CHORD,T PH   1
     1ITLE)                                                              PH   2
C                                                                        PH   3
C     ROUTINE TO PUNCH OUTPUT DATA   (TAPE 1 IS PUNCH FILE)              PH   4
C                                                                        PH   5
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982          PH   6
C                                                                        PH   7
      DIMENSION TITLE(8), X(1), Y(1), W(1), THETA(1), YSMO(1), YPS(1), Y PH   8
     1PPS(1)                                                             PH   9
C                                                                        PH  10
      COMMON /HLM/ DX(200),DY(200),DW(200)                               PH  11
C                                                                        PH  12
      COMMON /BLK1/ PI,PI2,RAD,CONS                                      PH  13
C                                                                        PH  14
      COMMON /INOUT/ JREAD,JWRITE,IPRINT                                 PH  15
C                                                                        PH  16
      IF (IPUNCH.LE.0.OR.IPUNCH.GE.5) RETURN                             PH  17
C                                                                        PH  18
C     PUNCH TITLE CARD                                                   PH  19
C                                                                        PH  20
      WRITE (JWRITE,10) IPUNCH,TITLE                                     PH  21
      WRITE (1,11) TITLE                                                 PH  22
C                                                                        PH  23
C     DETERMINE OUTPUT PUNCH OPTION                                      PH  24
C                                                                        PH  25
      IOP=0                                                              PH  26
      IF (IPUNCH.EQ.2) IOP=1                                             PH  27
      IF (IPUNCH.EQ.3) IOP=2                                             PH  28
      IF (IPUNCH.EQ.4) IOP=3                                             PH  29
      WRITE (JWRITE,12) IOP                                              PH  30
      XI=FLOAT(IOP)                                                      PH  31
      WRITE (1,13) XI                                                    PH  32
C                                                                        PH  33
C     PUNCH UPPER SURFACE QUANTITIES                                     PH  34
C                                                                        PH  35
      J=KP=0                                                             PH  36
      DO 1 I=NOSE,NP                                                     PH  37
      J=J+1                                                              PH  38
      DW(J)=W(I)                                                         PH  39
      IF (W(I).GT.1.0) KP=1                                              PH  40
      IF (IOP.EQ.0) DX(J)=X(I)*CHORD
      IF (IOP.NE.0) DX(J)=THETA(I)*RAD
      IF (IOP.EQ.0) DY(J)=YSMO(I)*CHORD
      IF (IOP.EQ.1) DY(J)=YSMO(I)
      IF (IOP.EQ.2) DY(J)=YPS(I)
      IF (IOP.EQ.3) DY(J)=YPPS(I)
    1 CONTINUE
      WRITE (JWRITE,14) J
      XI=FLOAT(J)
      WRITE (1,15) XI
      IF (IOP.EQ.0) WRITE (JWRITE,16) (DX(I),I=1,J)
      IF (IOP.NE.0) WRITE (JWRITE,7) (DX(I),I=1,J)
      WRITE (JWRITE,17) (DY(I),I=1,J)
      IF (KP.EQ.1) WRITE (JWRITE,21) (DW(I),I=1,J)
      DO 3 I=1,J
      IF (IOP.NE.0) GO TO 2
      IF (DW(I).GT.1.0) WRITE (1,22) DX(I),DY(I),DW(I)
      IF (DW(I).LE.1.0) WRITE (1,18) DX(I),DY(I)
      GO TO 3
    2 IF (DW(I).GT.1.0) WRITE (1,8) DX(I),DY(I),DW(I)
      IF (DW(I).LE.1.0) WRITE (1,9) DX(I),DY(I)
    3 CONTINUE
    C
    C PUNCH LOWER SURFACE QUANTITIES
    C
      J=KP=0
      DO 4 I=1,NOSE
      J=J+1
      K=NOSE+1-I
      DW(J)=W(K)
      IF (W(K).GT.1.0) KP=1
      IF (IOP.EQ.0) DX(J)=X(K)*CHORD
      IF (IOP.NE.0) DX(J)=THETA(K)*RAD
      IF (IOP.EQ.0) DY(J)=YSMO(K)*CHORD
      IF (IOP.EQ.1) DY(J)=YSMO(K)
      IF (IOP.EQ.2) DY(J)=YPS(K)
      IF (IOP.EQ.3) DY(J)=YPPS(K)
    4 CONTINUE
      WRITE (JWRITE,19) J
      XI=FLOAT(J)
      WRITE (1,15) XI
      IF (IOP.EQ.0) WRITE (JWRITE,16) (DX(I),I=1,J)
      IF (IOP.NE.0) WRITE (JWRITE,7) (DX(I),I=1,J)
      WRITE (JWRITE,17) (DY(I),I=1,J)
      IF (KP.EQ.1) WRITE (JWRITE,21) (DW(I),I=1,J)
      DO 6 I=1,J
      IF (IOP.NE.0) GO TO 5
      IF (DW(I).GT.1.0) WRITE (1,22) DX(I),DY(I),DW(I)
      IF (DW(I).LE.1.0) WRITE (1,18) DX(I),DY(I)
      GO TO 6
    5 IF (DW(I).GT.1.0) WRITE (1,8) DX(I),DY(I),DW(I)
      IF (DW(I).LE.1.0) WRITE (1,9) DX(I),DY(I)
    6 CONTINUE
C
C     PUNCH YLTE AND YUTE
C
      IF (IOP.LE.1) RETURN
      YLTE=YSMO(1)
      YNOSE=YSMO(NOSE)
      YUTE=YSMO(NP)
      WRITE (JWRITE,20) YLTE,YNOSE,YUTE
      WRITE (1,18) YLTE,YNOSE,YUTE
C
C     RETURN TO CALLING PROGRAM
C
      RETURN
C
    7 FORMAT (/3X,4HTH =,8F10.5/(7X,8F10.5))
    8 FORMAT (F10.5,F10.6,F10.2)
    9 FORMAT (F10.5,F10.6)
   10 FORMAT (1H1,10X,36HTHE FOLLOWING DATA HAVE BEEN PUNCHED,5X,7HIPUNC
     1H=,I4//3X,8A10)
   11 FORMAT (8A10)
   12 FORMAT (/3X,5HIOP =,I4)
   13 FORMAT (30X,F10.2)
   14 FORMAT (/3X,4HNU =,I4)
   15 FORMAT (F10.2)
   16 FORMAT (/3X,4HDX =,8F10.6/(7X,8F10.6))
   17 FORMAT (/3X,4HDY =,8F10.6/(7X,8F10.6))
   18 FORMAT (3F10.6)
```

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CARD NO.

  121       19    FORMAT (/3X,4HNL =,I4)                                                  PH 121
            20    FORMAT (/3X,6HYLTE =,F10.6,5X,7HYNOSE =,F10.6,5X,6HYUTE =,F10.6)       PH 122
            21    FORMAT (/3X,4HDW =,8F10.2/(7X,8F10.2))                                  PH 123
            22    FORMAT (2F10.6,F10.2)                                                   PH 124
  125             END                                                                     PH 125-
```

CARD NO.

```
      SUBROUTINE PLOTAF (THETA,Y,YSMO,YPS,YPPS,NP,TITLE,IPLOT)
C
C     THIS ROUTINE PLOTS INPUT AND SMOOTHED Y/C, SMOOTHED YPS, AND
C     SMOOTHED YPPS VERSUS THETA. ALSO PLOTS INPUT AND SMOOTHED Y/C
C     VERSUS X/C.
C
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
      DIMENSION TITLE(8), THETA(1), Y(1), YSMO(1), YPS(1), YPPS(1)
C
      COMMON /HLM/ XI(363),YI(363),TI(363)
C
      COMMON /SMY/ YPSI(363)
C
      COMMON /BLK1/ PI,PI2,RAD,CONS
C
      COMMON /INOUT/ JREAD,JWRITE,IPRINT
C
      DATA NM/361/,SIZ/.40/,ISIZ/3/
C
C     SINH(X)=(EXP(X)-EXP(-X))/2.
C
C        INTERPOLATE NM SMOOTHED COORDINATES Y/C AND YPS VALUES
C
      YMAX=0.0
      DP=(THETA(NP)-THETA(1))/FLOAT(NM-1)
      YP=THETA(1)-DP
      M=2
      DO 5 I=1,NM
      YP=YP+DP
      IF (YP.LT.THETA(1)) YP=THETA(1)
      IF (YP.GT.THETA(NP)) YP=THETA(NP)
      TI(I)=YP*RAD
      IF (M.LT.2) M=2
      TP=ABS(YP)
      IF (TP.LE.PI2) GO TO 1
      XI(I)=CONS*(ATAN(SINH(TP-PI2))+1.)
      GO TO 2
    1 XI(I)=CONS*(1.-COS(TP))
    2 DO 3 K=M,NP
```

```
      J=K-1
      IF (YP.GE.THETA(J).AND.YP.LE.THETA(K)) GO TO 4
    3 CONTINUE
    4 M=J
      DELTA=THETA(J+1)-THETA(J)
      X2=THETA(J+1)-YP
      X1=YP-THETA(J)
      YI(I)=YPPS(J)*(X2**3/(6.*DELTA)-X2*DELTA/6.)+YPPS(J+1)*(X1**3/(6.*
     1DELTA)-X1*DELTA/6.)+(YSMO(J)*X2+YSMO(J+1)*X1)/DELTA
      YPSI(I)=YPPS(J)*(DELTA/6.-X2*X2/(2.*DELTA))+YPPS(J+1)*(X1*X1/(2.*D
     1ELTA)-DELTA/6.)+(YSMO(J+1)-YSMO(J))/DELTA
      IF (ABS(YI(I)).GE.YMAX) YMAX=ABS(YI(I))
    5 CONTINUE
    C
    C    PRINT INTERPOLATED Y/C-COORDINATES
    C
      IF (IPRINT.NE.0) GO TO 6
      WRITE (JWRITE,15) TITLE
      WRITE (JWRITE,16) (I,TI(I),XI(I),YI(I),I=1,NM)
    C
    C    DETERMINE SCALING FACTOR FOR Y/C AXIS
    C
    6 YSCALE=0.1
      IF (YMAX.LE.0.06) YSCALE=0.01
      IF ((YMAX.GT.0.06).AND.(YMAX.LE.0.12)) YSCALE=0.02
      IF ((YMAX.GT.0.12).AND.(YMAX.LE.0.24)) YSCALE=0.04
      IF ((YMAX.GT.0.24).AND.(YMAX.LE.0.30)) YSCALE=0.05
      YMIN=-6.*YSCALE
      YSAV=YSCALE
    C
    C    DRAW AND LABEL Y/C AND THETA AXIS
    C
      IF (IPLOT.EQ.2) GO TO 11
      CALL CALPLT (2.,1.,-3)
      CALL NOTATE (0.,0.,SIZ,TITLE,0.,80)
      CALL AXES (0.,2.,0.,36.,-180.,10.,-2.,1.,10HTHETA,DEG.,SIZ,-10,0)
      CALL AXES (0.,2.,90.,12.,YMIN,YSCALE,-1.,0.,3HY/C,SIZ,3,2)
      CALL NOTATE (1.0,13.1,.4,2,0.,-1)
      CALL NOTATE (1.5,12.9,SIZ,8HSMOOTHED,0.,8)
      CALL NOTATE (1.0,13.7,.4,3,0.,-1)
      CALL NOTATE (1.5,13.5,SIZ,5HINPUT,0.,5)
      CALL CALPLT (0.,8.,-3)
C
C        PLOT INPUT Y/C-COORDINATES VS THETA
C
      DO 7 I=1,NP
      TP=THETA(I)*RAD/10.+18.0
      YP=Y(I)/YSCALE
      CALL PNTPLT (TP,YP,22,ISIZ)
    7 CONTINUE
C
C        PLOT SMOOTHED Y/C-COORDINATES VS THETA
C
      TI(NM+1)=-180.0
      TI(NM+2)=10.0
      YI(NM+1)=0.
      YI(NM+2)=YSCALE
      CALL LINE (TI,YI,NM,1,0,0,0.)
C
C        DETERMINE SCALING FACTOR FOR FIRST DERIVATIVE AXIS (YP AXIS)
C
      YMAX=0.0
      DO 8 I=1,NM
      IF (ABS(YPSI(I)).GT.YMAX) YMAX=ABS(YPSI(I))
    8 CONTINUE
      CSCALE=.1
      IF ((YMAX.LE.0.30).AND.(YMAX.GT.0.24)) CSCALE=.05
      IF ((YMAX.LE.0.24).AND.(YMAX.GT.0.12)) CSCALE=.04
      IF ((YMAX.LE.0.12).AND.(YMAX.GT.0.06)) CSCALE=.02
      IF ((YMAX.LE.0.06).AND.(YMAX.GE.0.00)) CSCALE=.01
      CMIN=-6.*CSCALE
C
C        DETERMINE SCALING FACTOR FOR SECOND DERIVATIVE AXIS (YPP AXIS)
C
      YMAX=0.0
      DO 9 I=1,NP
      IF (ABS(YPPS(I)).GT.YMAX) YMAX=ABS(YPPS(I))
    9 CONTINUE
      YSCALE=1.
      IF ((YMAX.LE.3.00).AND.(YMAX.GT.2.40)) YSCALE=.5
      IF ((YMAX.LE.2.40).AND.(YMAX.GT.1.20)) YSCALE=.4
      IF ((YMAX.LE.1.20).AND.(YMAX.GT.0.60)) YSCALE=.2
      IF ((YMAX.LE.0.60).AND.(YMAX.GT.0.30)) YSCALE=.1
      IF ((YMAX.LE.0.30).AND.(YMAX.GT.0.24)) YSCALE=.05
      IF ((YMAX.LE.0.24).AND.(YMAX.GT.0.12)) YSCALE=.04
      IF ((YMAX.LE.0.12).AND.(YMAX.GT.0.06)) YSCALE=.02
      IF ((YMAX.LE.0.06).AND.(YMAX.GE.0.00)) YSCALE=.01
      YMIN=-6.*YSCALE
C
C        DRAW AND LABEL YP, YPP, AND THETA AXIS
C
      CALL CALPLT (0.,8.,-3)
      CALL AXES (0.,0.,0.,36.,-180.,10.,-2.,1.,10HTHETA,DEG.,SIZ,-10,0)
C             DRAW AND LABEL YPS AXES
      CALL AXES (0.,0.,90.,12.,CMIN,CSCALE,-1.,0.,3HYPS,SIZ,3,2)
C             DRAW AND LABEL YPPS AXES
      CALL AXES (36.,0.,90.,12.,YMIN,YSCALE,-1.,0.,4HYPPS,SIZ,-4,2)
      CALL NOTATE (1.0,11.1,.4,3,0.,-1)
      CALL NOTATE (1.5,10.9,SIZ,4HYPPS,0.,4)
      CALL NOTATE (1.0,11.7,.4,2,0.,-1)
      CALL NOTATE (1.5,11.5,SIZ,3HYPS,0.,3)
      CALL CALPLT (0.,6.,-3)
C
C        PLOT SMOOTHED FIRST DERIVATIVES YP VS THETA
C
      YPSI(NM+1)=0.0
      YPSI(NM+2)=CSCALE
      CALL LINE (TI,YPSI,NM,1,0,0,0.)
C
C        PLOT SMOOTHED SECOND DERIVATIVES YPP VS THETA
C
      THETA(NP+1)=-PI
      THETA(NP+2)=10./RAD
      YPPS(NP+1)=0.0
      YPPS(NP+2)=YSCALE
      CALL LINE (THETA,YPPS,NP,1,0,0,0.)
      DO 10 I=1,NP
      TP=THETA(I)*RAD/10.+18.0
      YP=YPPS(I)/YSCALE
      CALL PNTPLT (TP,YP,22,ISIZ)
```

```
10    CONTINUE
      CALL NFRAME
C             CHECK PLOT OPTION
      IF (IPLOT.EQ.1.OR.IPLOT.EQ.6) RETURN
      IF (IPLOT.EQ.8) RETURN
C
C        DETERMINE SCALING FACTOR FOR Y/C AXIS
C
11    IF (YSAV.EQ.0.01) YMAX=8
      IF (YSAV.EQ.0.02) YMAX=12
      IF (YSAV.EQ.0.04) YMAX=20
      IF (YSAV.EQ.0.05) YMAX=24
      YMIN=-0.0125*YMAX
C
C        PLOT INPUT AND SMOOTHED Y/C-COORDINATES VS X/C
C
C        DRAW AND LABEL Y/C AND X/C AXIS
C
      CALL CALPLT (2.,2.,-3)
      CALL NOTATE (0.,0.,SIZ,TITLE,0.,80)
      CALL CALPLT (0.,2.,-3)
      CALL AXES (0.,0.,0.,40.,0.,.025,-2.,1.,3HX/C,SIZ,-3,2)
      CALL AXES (0.,0.,90.,YMAX,YMIN,0.025,-2.,1.,3HY/C,SIZ,3,2)
      YP=YMAX-0.9
      CALL NOTATE (1.0,YP,SIZ,2,0.,-1)
      YP=YMAX-1.1
      CALL NOTATE (1.5,YP,SIZ,8HSMOOTHED,0.,8)
      YP=YMAX-.3
      CALL NOTATE (1.0,YP,SIZ,3,0.,-1)
      YP=YMAX-.5
      CALL NOTATE (1.5,YP,SIZ,5HINPUT,0.,5)
      YP=0.5*YMAX
      CALL CALPLT (0.,YP,-3)
C
C        PLOT INPUT Y/C-COORDINATES
C
      DO 14 I=1,NP
      TP=ABS(THETA(I))
      IF (TP.LE.PI2) GO TO 12
      XP=CONS*(ATAN(SINH(TP-PI2))+1.)/.025
      GO TO 13
12    XP=CONS*(1.-COS(TP))/.025
13    YP=Y(I)/.025
      CALL PNTPLT (XP,YP,22,ISIZ)
14    CONTINUE
C
C        PLOT SMOOTHED Y/C-COORDINATES
C
      XI(NM+1)=YI(NM+1)=0.0
      XI(NM+2)=YI(NM+2)=.025
      CALL LINE (XI,YI,NM,1,0,0,0.)
      CALL NFRAME
C
C        RETURN TO CALLING PROGRAM
C
      RETURN
C
15    FORMAT (1H1,1X,7HTITLE--,2X,8A10//49X,28H--INTERPOLATED COORDINATE
     1S--/10X,1HI,3X,5HTHETA,5X,3HX/C,7X,3HY/C,12X,1HI,3X,5HTHETA,5X,3HX
     2/C,7X,3HY/C,12X,1HI,3X,5HTHETA,5X,3HX/C,7X,3HY/C/)
16    FORMAT (3(7X,I4,F8.2,2F10.6))
      END
```

CARD NO.

```
      SUBROUTINE PLOTCK (THETA,YSMO,YPS,YPPS,NP,TITLE)
C
C     ROUTINE TO PLOT SQUARE ROOT OF SMOOTHED CURVATURE VERSUS THETA
C
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
      DIMENSION THETA(1), YSMO(1), YPS(1), YPPS(1), TITLE(8)
C
      COMMON /HLM/ TI(723)
      COMMON /SMY/ CURV(723)
      COMMON /BLK1/ PI,PI2,RAD,CONS
      COMMON /INOUT/ JREAD,JWRITE,IPRINT
C
      DATA NM/721/,SIZ/.40/,ISIZ/3/
C
C     SINH(X)=0.5*(EXP(X)-EXP(-X))
C     COSH(X)=0.5*(EXP(X)+EXP(-X))
C
C        INTERPOLATE NM CURVATURE POINTS
C
      IF (IPRINT.NE.0) GO TO 1
      WRITE (JWRITE,15) TITLE
    1 DP=(THETA(NP)-THETA(1))/FLOAT(NM-1)
      TDEL=THETA(1)-DP
      M=2
      DO 8 I=1,NM
      TDEL=TDEL+DP
      IF (TDEL.LT.THETA(1)) TDEL=THETA(1)
      IF (TDEL.GT.THETA(NP)) TDEL=THETA(NP)
      TI(I)=TDEL*RAD
      TP=TDEL
      IF (M.LT.2) M=2
      DO 2 K=M,NP
      J=K-1
      IF (TP.GE.THETA(J).AND.TP.LE.THETA(K)) GO TO 3
    2 CONTINUE
    3 M=J
      DELTA=THETA(J+1)-THETA(J)
      T2=THETA(J+1)-TP
      T1=TP-THETA(J)
      YI=YPPS(J)*(T2**3/(6.*DELTA)-T2*DELTA/6.)+YPPS(J+1)*(T1**3/(6.*DEL
     1TA)-T1*DELTA/6.)+(YSMO(J)*T2+YSMO(J+1)*T1)/DELTA
      YPI=YPPS(J)*(DELTA/6.-T2*T2/(2.*DELTA))+YPPS(J+1)*(T1*T1/(2.*DELTA
     1)-DELTA/6.)+(YSMO(J+1)-YSMO(J))/DELTA
      YPPI=(YPPS(J)*T2+YPPS(J+1)*T1)/DELTA
      DELTA=YPI
      IF (TP.LE.0.0) DELTA=-DELTA
      TP=ABS(TP)
      IF (TP.GT.PI2) GO TO 4
      GP=CONS*SIN(TP)
      GPP=CONS*COS(TP)
      XI=CONS*(1.-COS(TP))
      GO TO 5
    4 T1=COSH(TP-PI2)
      T2=SINH(TP-PI2)
      XI=CONS*(ATAN(T2)+1.)
      GP=CONS/T1
      GPP=-CONS*T2/(T1*T1)
    5 IF (TP.LE.0.0.OR.GP.EQ.0.0) GO TO 6
      DYDX=DELTA/GP
      DY2DX=(YPPI*GP-DELTA*GPP)/(GP**3)
      CURV(I)=ABS(DY2DX)/(SQRT(1.+DYDX**2)**3)
      GO TO 7
    6 DYDX=0.1E99
      DY2DX=0.1E99
      CURV(I)=CONS/(DELTA*DELTA)
    7 IF (IPRINT.NE.0) GO TO 8
      WRITE (JWRITE,16) I,TI(I),XI,YI,YPI,YPPI,DYDX,DY2DX,CURV(I)
    8 CURV(I)=SQRT(CURV(I))
C
C        DETERMINE SCALING FACTOR FOR CURVATURE AXES
C
      CMAX=0.0
      DO 9 I=1,NM
      IF (CURV(I).GT.CMAX) CMAX=CURV(I)
    9 CONTINUE
      M=IFIX(CMAX)+1
      CMAX=FLOAT(M)/20.
C
C        DRAW AND LABEL CURVATURE AND THETA AXES
```

```
C
      CALL GRIDCK
      CALL CALPLT (2.,2.,-3)
      CALL NOTATE (0.,0.,SIZ,TITLE,0.,80)
      CALL CALPLT (0.,2.,-3)
      CALL AXES (0.,0.,0.,36.,-180.,10.,-2.,1.,10HTHETA,DEG.,SIZ,-10,0)
      CALL AXES (0.,0.,90.,20.,0.,CMAX,-2.,1.,15HSQRT(CURVATURE),SIZ,15,
     12)
C
C        PLOT INTERPOLATED CURVATURE POINTS
C
      TI(NM+1)=-180.0
      CURV(NM+1)=0.0
      TI(NM+2)=10.
      CURV(NM+2)=CMAX
      CALL LINE (TI,CURV,NM,1,0,0,0.0)
C
C        COMPUTE AND PLOT CURVATURE AT INPUT THETA POINTS
C
      DO 14 I=1,NP
      DELTA=YPS(I)
      IF (THETA(I).LE.0.0) DELTA=-DELTA
      TP=ABS(THETA(I))
      IF (TP.GT.PI2) GO TO 10
      GP=CONS*SIN(TP)
      GPP=CONS*COS(TP)
      GO TO 11
   10 T1=COSH(TP-PI2)
      T2=SINH(TP-PI2)
      GP=CONS/T1
      GPP=-CONS*T2/(T1*T1)
   11 IF (TP.LE.0.0.OR.GP.EQ.0.0) GO TO 12
      DYDX=DELTA/GP
      DY2DX=(YPPS(I)*GP-DELTA*GPP)/(GP**3)
      T1=ABS(DY2DX)/(SQRT(1.+DYDX**2)**3)
      GO TO 13
   12 T1=CONS/(DELTA*DELTA)
   13 T2=THETA(I)*RAD/10.+18.0
      T1=SQRT(T1)/CMAX
      CALL PNTPLT (T2,T1,22,ISIZ)
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CARD NO.

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      14    CONTINUE                                                     PC 121
      C                                                                  PC 122
      C         ADVANCE TO NEXT FRAME AND RETURN                         PC 123
      C                                                                  PC 124
            CALL NFRAME                                                  PC 125
            RETURN                                                       PC 126
      C                                                                  PC 127
      C                                                                  PC 128
      15    FORMAT (1H1,1X,7HTITLE--,2X,8A10//36X,26H--INTERPOLATED CURVATURE- PC 129
           1-/3X,1HI,6X,5HTHETA,5X,3HX/C,7X,3HY/C,6X,5HDY/DT,5X,6HDY2/DT,7X,5H PC 130
           2DY/DX,7X,11HD(DY/DX)/DX,5X,9HCURVATURE/)                     PC 131
      16    FORMAT (I5,F10.2,4F10.6,3E15.6)                              PC 132
            END                                                          PC 133-
      SUBROUTINE CAMTK (THETA,YSMO,YPPS,NOSE,NP,EPS,KPLOT,IPUNCH,TITLE) CK   1
C                                                                       CK   2
C     THIS SUBROUTINE COMPUTES THE THICKNESS AND CAMBER DISTRIBUTIONS   CK   3
C     OF THE SMOOTHED AIRFOIL                                           CK   4
C                                                                       CK   5
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982         CK   6
C                                                                       CK   7
      DIMENSION TITLE(8), THETA(1), YSMO(1), YPPS(1)                    CK   8
C                                                                       CK   9
      COMMON /SMY/ TU(100),YPPU(100),TL(100),YPPL(100),DYXU(100),LX(101) CK  10
     1,XLS(101),YLS(101),TH(101),XU(102),YU(102),XL(102),YL(102),XC(103) CK  11
     2,YC(103),TK(103)                                                  CK  12
C                                                                       CK  13
      COMMON /BLK1/ PI,PI2,RAD,CONS                                     CK  14
C                                                                       CK  15
      COMMON /INOUT/ JREAD,JWRITE,IPRINT                                CK  16
C                                                                       CK  17
      DATA NM/2001/,SIZ/.40/,ISIZ/3/                                    CK  18
C                                                                       CK  19
C     COSH(X)=0.5*(EXP(X)+EXP(-X))                                      CK  20
C     SINH(X)=0.5*(EXP(X)-EXP(-X))                                      CK  21
C                                                                       CK  22
      F(X1,X2,X3,X4,X5,X6,X7,X8,X9)=X1*(X5*X9-X6*X8)+X2*(X6*X7-X4*X9)+X3 CK  23
     1*(X4*X8-X5*X7)                                                    CK  24
C                                                                       CK  25
C        LOAD THETA, X/C, Y/C, AND SECOND DERIVATIVES INTO SEPARATE     CK  26
C        ARRAYS FOR UPPER AND LOWER SURFACES                            CK  27
C                                                                       CK  28
      J=0                                                               CK  29
      NU=NP-NOSE+1                                                      CK  30
      DO 2 I=NOSE,NP                                                    CK  31
      J=J+1                                                             CK  32
      TU(J)=THETA(I)                                                    CK  33
      YU(J)=YSMO(I)                                                     CK  34
      TP=ABS(THETA(I))                                                  CK  35
      IF (TP.GT.PI2) GO TO 1                                            CK  36
      XU(J)=CONS*(1.-COS(TP))                                           CK  37
      GO TO 2                                                           CK  38
    1 XU(J)=CONS*(ATAN(SINH(TP-PI2))+1.)                                CK  39
    2 YPPU(J)=YPPS(I)                                                   CK  40
```

```
      NL=NOSE
      J=NOSE+1
      DO 4 I=1,NOSE
      J=J-1
      TL(J)=THETA(I)
      YL(J)=YSMO(I)
      TP=ABS(THETA(I))
      IF (TP.GT.PI2) GO TO 3
      XL(J)=CONS*(1.-COS(TP))
      GO TO 4
    3 XL(J)=CONS*(ATAN(SINH(TP-PI2))+1.)
    4 YPPL(J)=YPPS(I)
C              COMPUTE FIRST DERIVATIVES OF UPPEP SURFACE
      DO 5 I=2,NU
      DELTA=TU(I)-TU(I-1)
      DYXU(I)=YPPU(I)*DELTA/3.+YPPU(I-1)*DELTA/6.+(YU(I)-YU(I-1))/DELTA
      IF (TU(I).LE.PI2) DYXU(I)=DYXU(I)/(CONS*SIN(TU(I)))
      IF (TU(I).GT.PI2) DYXU(I)=DYXU(I)*COSH(TU(I)-PI2)/CONS
    5 CONTINUE
      DYXU(1)=0.1E99
C
C         COMPUTE THICKNESS AND CAMBER DISTRIBUTIONS BY FINDING LOWER
C         SURFACE COORDINATE (XLS,YLS) CORRESPONDING TO INPUT UPPER
C         SURFACE COORDINATE (XU,YU)
C
      NT=0
      KSAVE=1
      NS=1
      NL1=NL-1
      NM1=NM-1
      A1=PI/FLOAT(NM1)
      DEL=1./(FLOAT(NM1)**2)
      DO 12 I=1,NU
C              LOAD XU AND YU
      IJ=NU+1-I
      XXU=XU(IJ)
      YYU=YU(IJ)
      DYU=DYXU(IJ)
      NN=1
C              FIND XLS
```

```
      DO 9 K=NS,NM                                                      CK  81
      TP=A1*FLOAT(NM-K)                                                 CK  82
      IF (K.EQ.1) TP=ABS(TL(NL))                                        CK  83
      IF (K.EQ.NM) TP=ABS(TL(1))                                        CK  84
      IF (TP.LE.PI2) XXL=CONS*(1.-COS(TP))                              CK  85
      IF (TP.GT.PI2) XXL=CONS*(ATAN(SINH(TP-PI2))+1.)                   CK  86
      IF (NN.EQ.NL) NN=NL1                                              CK  87
      DO 6 J=NN,NL1                                                     CK  88
      J2=NL-J                                                           CK  89
      J1=J2+1                                                           CK  90
      IF (TP.GE.ABS(TL(J2)).AND.TP.LE.ABS(TL(J1))) GO TO 7              CK  91
    6 CONTINUE                                                          CK  92
    7 DELTA=TL(J2)-TL(J1)                                               CK  93
      T1=-TP-TL(J1)                                                     CK  94
      T2=TL(J2)+TP                                                      CK  95
      YYL=YPPL(J1)*(T2**3/(6.*DELTA)-T2*DELTA/6.)+YPPL(J2)*(T1**3/(6.*DE CK  96
     1LTA)-T1*DELTA/6.)+(YL(J1)*T2+YL(J2)*T1)/DELTA                     CK  97
      DYL=YPPL(J1)*(DELTA/6.-T2*T2/(2.*DELTA))+YPPL(J2)*(T1*T1/(2.*DELTA CK  98
     1)-DELTA/6.)+(YL(J2)-YL(J1))/DELTA                                 CK  99
      IF (TP.LE.PI2) DELTA=CONS*SIN(TP)                                 CK 100
      IF (TP.GT.PI2) DELTA=CONS/COSH(TP-PI2)                            CK 101
      IF (TP.LE.0.0) DYL=0.1E99                                         CK 102
      IF (TP.GT.0.0) DYL=-DYL/DELTA                                     CK 103
      NN=NL+1-J1                                                        CK 104
      D=SQRT((XXL-XXU)**2+(YYL-YYU)**2)                                 CK 105
      IF (I.EQ.1.AND.D.LE.DEL) GO TO 10                                 CK 106
      IF (D.LE.DEL) GO TO 9                                             CK 107
      COST=(YYU-YYL)/D                                                  CK 108
      SINT=(XXL-XXU)/D                                                  CK 109
      IF (DYU.NE.0.1E99) DU=(COST*DYU-SINT)/(SINT*DYU+COST)             CK 110
      IF (DYU.EQ.0.1E99.AND.SINT.NE.0.0) DU=COST/SINT                   CK 111
      IF (DYU.EQ.0.1E99.AND.SINT.EQ.0.0) DU=0.1E99                      CK 112
      IF (DYL.NE.0.1E99) DL=-(COST*DYL-SINT)/(SINT*DYL+COST)            CK 113
      IF (DYL.EQ.0.1E99.AND.SINT.NE.0.0) DL=-COST/SINT                  CK 114
      IF (DYL.EQ.0.1E99.AND.SINT.EQ.0.0) DL=-0.1E99                     CK 115
      IF (K.EQ.NS) GO TO 8                                              CK 116
      DKL=(DL-DLP)/(XXL-XP)                                             CK 117
      DKU=(DU-DUP)/(XXL-XP)                                             CK 118
      IF (DKU.EQ.DKL) GO TO 8                                           CK 119
      XK=XP+(DLP-DUP)/(DKU-DKL)                                         CK 120
      IF (XK.LE.XP+DEL.AND.XK.GE.XXL-DEL) GO TO 11
    8 KSAVE=K
      XP=XXL
      DUP=DU
      DLP=DL
    9 CONTINUE
      IF (I.GT.1) GO TO 12
   10 XK=XL(NL)
      KSAVE=NS
   11 NT=NT+1
      LX(NT)=IJ
      XLS(NT)=XK
      NS=KSAVE
   12 CONTINUE
C               COMPUTE YLS FOR EACH XLS AND PRINT RESULTS
      WRITE (JWRITE,44) TITLE
      DO 19 I=1,NT
      IJ=LX(I)
      DELTA=XLS(I)
      IF (DELTA.GT.1.) DELTA=1.
      IF (DELTA.LE.CONS) GO TO 13
      DELTA=TAN(DELTA/CONS-1.)
      TP=PI2+ALOG(DELTA+SQRT(DELTA*DELTA+1.))
      GO TO 14
   13 TP=ACOS(1.-DELTA/CONS)
   14 DO 15 J=1,NL1
      J2=NL-J
      J1=J2+1
      IF (TP.GE.ABS(TL(J2)).AND.TP.LE.ABS(TL(J1))) GO TO 16
   15 CONTINUE
   16 DELTA=TL(J2)-TL(J1)
      T1=-TP-TL(J1)
      T2=TL(J2)+TP
      YYL=YPPL(J1)*(T2**3/(6.*DELTA)-T2*DELTA/6.)+YPPL(J2)*(T1**3/(6.*DE
     1LTA)-T1*DELTA/6.)+(YL(J1)*T2+YL(J2)*T1)/DELTA
      YLS(I)=YYL
      XC(I)=(XU(IJ)+XLS(I))/2.
      YC(I)=(YU(IJ)+YYL)/2.
      TK(I)=0.5*SQRT((XU(IJ)-XLS(I))**2+(YU(IJ)-YYL)**2)
      IF (YU(IJ).EQ.YYL) TH(I)=0.0
```

```
            IF (YU(IJ).NE.YYL) TH(I)=ATAN((XLS(I)-XU(IJ))/(YU(IJ)-YYL))         CK 161
            IF (TK(I).LE.0.0) GO TO 17                                           CK 162
            DYL=YPPL(J1)*(DELTA/6.-T2*T2/(2.*DELTA))+YPPL(J2)*(T1*T1/(2.*DELTA   CK 163
           1)-DELTA/6.)+(YL(J2)-YL(J1))/DELTA                                    CK 164
            IF (TP.LE.PI2) DELTA=CONS*SIN(TP)                                    CK 165
            IF (TP.GT.PI2) DELTA=CONS/COSH(TP-PI2)                               CK 166
            IF (TP.LE.0.0) DYL=0.1E99                                            CK 167
            IF (TP.GT.0.0) DYL=-DYL/DELTA                                        CK 168
            COST=(YU(IJ)-YYL)/(2.*TK(I))                                         CK 169
            SINT=(XLS(I)-XU(IJ))/(2.*TK(I))                                      CK 170
            DU=(COST*DYXU(IJ)-SINT)/(SINT*DYXU(IJ)+COST)                         CK 171
            DL=(COST*DYL-SINT)/(SINT*DYL+COST)                                   CK 172
            T2=ABS(ABS(DU)-ABS(DL))                                              CK 173
            GO TO 18                                                             CK 174
   17       T2=0.0                                                               CK 175
   18       T1=TH(I)*RAD                                                         CK 176
            WRITE (JWRITE,45) I,XU(IJ),YU(IJ),XLS(I),YYL,XC(I),YC(I),TK(I),T1,   CK 177
           1T2                                                                   CK 178
   19       CONTINUE                                                             CK 179
   C                                                                             CK 180
   C           COMPUTE STARTING LOCATION OF CAMBER DISTRIBUTION (I.E.            CK 181
   C           THICKNESS = 0) BY FITTING SECOND ORDER CURVE TO LAST THREE        CK 182
   C           COMPUTED CAMBER LINE COORDINATES AND THEN DETERMINING             CK 183
   C           INTERSECTION OF THAT CURVE WITH AIRFOIL SURFACE                   CK 184
   C                                                                             CK 185
            ISYM=1                                                               CK 186
            DO 20 I=1,5                                                          CK 187
            IF (ABS(XU(I)-XL(I)).GT.EPS) ISYM=0                                  CK 188
            IF (ABS(YU(I)+YL(I)).GT.EPS) ISYM=0                                  CK 189
   20       CONTINUE                                                             CK 190
            IF (ISYM.EQ.1) GO TO 30                                              CK 191
            IF (XC(NT).LE.DEL) GO TO 31                                          CK 192
            X1=XC(NT)**2                                                         CK 193
            X2=XC(NT-1)**2                                                       CK 194
            X3=XC(NT-2)**2                                                       CK 195
            D=F(X1,XC(NT),1.,X2,XC(NT-1),1.,X3,XC(NT-2),1.)                      CK 196
            A1=F(YC(NT),XC(NT),1.,YC(NT-1),XC(NT-1),1.,YC(NT-2),XC(NT-2),1.)/D   CK 197
            A2=F(X1,YC(NT),1.,X2,YC(NT-1),1.,X3,YC(NT-2),1.)/D                   CK 198
            A3=YC(NT)-A1*X1-A2*XC(NT)                                            CK 199
            NM1=NM/4                                                             CK 200
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```
      D=XC(NT)/FLOAT(NM1)                                               CK 201
      X=0.0                                                             CK 202
      XP=X                                                              CK 203
      YYUP=YU(1)                                                        CK 204
      YYLP=YL(1)                                                        CK 205
      YYCP=(A1*X+A2)*X+A3                                               CK 206
      NM1=NM1+1                                                         CK 207
      DO 27 I=2,NM1                                                     CK 208
      X=X+D                                                             CK 209
      IF (X.GT.CONS) GO TO 27                                           CK 210
      TP=ACOS(1.-X/CONS)                                                CK 211
      DO 21 K=2,NU                                                      CK 212
      K1=K-1                                                            CK 213
      K2=K                                                              CK 214
      IF (TP.GE.TU(K1).AND.TP.LE.TU(K2)) GO TO 22                       CK 215
   21 CONTINUE                                                          CK 216
   22 DELTA=TU(K2)-TU(K1)                                               CK 217
      T1=TP-TU(K1)                                                      CK 218
      T2=TU(K2)-TP                                                      CK 219
      YYU=YPPU(K1)*(T2**3/(6.*DELTA)-T2*DELTA/6.)+YPPU(K2)*(T1**3/(6.*DE CK 220
     1LTA)-T1*DELTA/6.)+(YU(K2)*T1+YU(K1)*T2)/DELTA                     CK 221
      DO 23 J=2,NL                                                      CK 222
      J2=J-1                                                            CK 223
      J1=J                                                              CK 224
      IF (TP.GE.ABS(TL(J2)).AND.TP.LE.ABS(TL(J1))) GO TO 24             CK 225
   23 CONTINUE                                                          CK 226
   24 DELTA=TL(J2)-TL(J1)                                               CK 227
      T1=-TP-TL(J1)                                                     CK 228
      T2=TL(J2)+TP                                                      CK 229
      YYL=YPPL(J1)*(T2**3/(6.*DELTA)-T2*DELTA/6.)+YPPL(J2)*(T1**3/(6.*DE CK 230
     1LTA)-T1*DELTA/6.)+(YL(J1)*T2+YL(J2)*T1)/DELTA                     CK 231
      YYC=(A1*X+A2)*X+A3                                                CK 232
      DKC=(YYC-YYCP)/(X-XP)                                             CK 233
      DKU=(YYU-YYUP)/(X-XP)                                             CK 234
      IF (DKU.EQ.DKC) GO TO 25                                          CK 235
      XKU=XP+(YYCP-YYUP)/(DKU-DKC)                                      CK 236
      IF (XKU.GE.XP.AND.XKU.LE.X) GO TO 28                              CK 237
   25 DKL=(YYL-YYLP)/(X-XP)                                             CK 238
      IF (DKL.EQ.DKC) GO TO 26                                          CK 239
      XKL=XP+(YYCP-YYLP)/(DKL-DKC)                                      CK 240
      IF (XKL.GE.XP.AND.XKL.LE.X) GO TO 29                               CK 241
   26 XP=X                                                               CK 242
      YYLP=YYL                                                           CK 243
      YYUP=YYU                                                           CK 244
      YYCP=YYC                                                           CK 245
   27 CONTINUE                                                           CK 246
      GO TO 31                                                           CK 247
   28 NT=NT+1                                                            CK 248
      LX(NT)=0                                                           CK 249
      XLS(NT)=XKU                                                        CK 250
      XC(NT)=XKU                                                         CK 251
      DU=(A1*XKU+A2)*XKU+A3                                              CK 252
      TK(NT)=0.                                                          CK 253
      TH(NT)=ATAN(2.*A1*XKU+A2)                                          CK 254
      TP=ACOS(1.-XKU/CONS)                                               CK 255
      DELTA=TU(K2)-TU(K1)                                                CK 256
      T1=TP-TU(K1)                                                       CK 257
      T2=TU(K2)-TP                                                       CK 258
      YYU=YPPU(K1)*(T2**3/(6.*DELTA)-T2*DELTA/6.)+YPPU(K2)*(T1**3/(6.*DE CK 259
     1LTA)-T1*DELTA/6.)+(YU(K2)*T1+YU(K1)*T2)/DELTA                      CK 260
      YLS(NT)=YYU                                                        CK 261
      YC(NT)=YLS(NT)                                                     CK 262
      D=ABS(ABS(DU)-ABS(YC(NT)))                                         CK 263
      T1=TH(NT)*RAD                                                      CK 264
      WRITE (JWRITE,45) NT,XLS(NT),YLS(NT),XLS(NT),YLS(NT),XC(NT),YC(NT) CK 265
     1,TK(NT),T1,D                                                       CK 266
      GO TO 31                                                           CK 267
   29 NT=NT+1                                                            CK 268
      LX(NT)=0                                                           CK 269
      XLS(NT)=XKL                                                        CK 270
      XC(NT)=XKL                                                         CK 271
      DL=(A1*XKL+A2)*XKL+A3                                              CK 272
      TK(NT)=0.                                                          CK 273
      TH(NT)=ATAN(2.*A1*XKL+A2)                                          CK 274
      TP=ACOS(1.-XKL/CONS)                                               CK 275
      DELTA=TL(J2)-TL(J1)                                                CK 276
      T1=-TP-TL(J1)                                                      CK 277
      T2=TL(J2)+TP                                                       CK 278
      YYL=YPPL(J1)*(T2**3/(6.*DELTA)-T2*DELTA/6.)+YPPL(J2)*(T1**3/(6.*DE CK 279
     1LTA)-T1*DELTA/6.)+(YL(J1)*T2+YL(J2)*T1)/DELTA                      CK 280
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```
      YLS(NT)=YYL
      YC(NT)=YLS(NT)
      D=ABS(ABS(DL)-ABS(YC(NT)))
      T1=TH(NT)*RAD
      WRITE (JWRITE,45) NT,XLS(NT),YLS(NT),XLS(NT),YLS(NT),XC(NT),YC(NT)
     1,TK(NT),T1,D
      GO TO 31
   30 IF (LX(NT).EQ.1) GO TO 31
      NT=NT+1
      LX(NT)=1
      XC(NT)=0.0
      YC(NT)=YU(1)
      XLS(NT)=0.0
      YLS(NT)=YL(1)
      TK(NT)=0.0
      TH(NT)=0.0
      D=0.0
      WRITE (JWRITE,45) NT,XC(NT),YC(NT),XLS(NT),YLS(NT),XC(NT),YC(NT),T
     1K(NT),TH(NT),D
C
C        PUNCH CAMBER AND THICKNESS DISTRIBUTIONS
C
   31 IF (IPUNCH.NE.5) GO TO 33
      WRITE (1,46) TITLE
      WRITE (JWRITE,41) IPUNCH,TITLE,NT
C
      D=FLOAT(NT)
      WRITE (1,42) D
C
      DO 32 I=1,NT
      J=NT+1-I
      WRITE (JWRITE,43) XC(J),YC(J),TK(J),TH(J)
      WRITE (1,47) XC(J),YC(J),TK(J),TH(J)
   32 CONTINUE
C
C        PLOT CAMBER AND THICKNESS DISTRIBUTIONS
C
   33 IF (KPLOT.EQ.0) RETURN
C              PLOT CAMBER
      CALL CALPLT (4.,2.,-3)
      CALL NOTATE (0.,0.,SIZ,TITLE,0.,80)                               CK 321
      CALL CALPLT (0.,2.,-3)                                            CK 322
      CALL AXES (0.,0.,0.,20.,0.,.05,-2.,1.,3HX/C,SIZ,-3,1)             CK 323
      DU=0.0                                                            CK 324
      DO 34 I=1,NT                                                      CK 325
      IF (ABS(YC(I)).GT.DU) DU=ABS(YC(I))                               CK 326
   34 CONTINUE                                                          CK 327
      D=.1                                                              CK 328
      IF (DU.LE.0.2.AND.DU.GT.0.08) D=.05                               CK 329
      IF (DU.LE.0.08.AND.DU.GT.0.04) D=.02                              CK 330
      IF (DU.LE.0.04) D=.01                                             CK 331
      DL=-4.*D                                                          CK 332
      CALL AXES (0.,0.,90.,8.,DL,D,-1.,0.,3HY/C,SIZ,3,2)                CK 333
      CALL CALPLT (0.,4.,-3)                                            CK 334
      XC(NT+1)=YC(NT+1)=0.0                                             CK 335
      XC(NT+2)=.05                                                      CK 336
      YC(NT+2)=D                                                        CK 337
      DO 35 I=1,NT                                                      CK 338
      XU1=XC(I)/.05                                                     CK 339
      YU1=YC(I)/D                                                       CK 340
      CALL PNTPLT (XU1,YU1,22,ISIZ)                                     CK 341
   35 CONTINUE                                                          CK 342
      CALL LINE (XC,YC,NT,1,0,0,0.)                                     CK 343
C               PLOT THICKNESS                                          CK 344
      CALL CALPLT (0.,6.,-3)                                            CK 345
      CALL AXES (0.,0.,0.,20.,0.,.05,-2.,1.,3HX/C,SIZ,-3,1)             CK 346
      DU=0.0                                                            CK 347
      DO 36 I=1,NT                                                      CK 348
      IF (ABS(TK(I)).GT.DU) DU=ABS(TK(I))                               CK 349
   36 CONTINUE                                                          CK 350
      D=.1                                                              CK 351
      IF (DU.LE.0.06) D=.01                                             CK 352
      IF (DU.GT.0.06.AND.DU.LE.0.12) D=.02                              CK 353
      IF (DU.GT.0.12.AND.DU.LE.0.24) D=.04                              CK 354
      IF (DU.GT.0.24.AND.DU.LE.0.30) D=.05                              CK 355
      CALL AXES (0.,0.,90.,6.,0.,D,-1.,0.,5HT/C/2,SIZ,5,2)              CK 356
      TK(NT+1)=0.0                                                      CK 357
      TK(NT+2)=D                                                        CK 358
      DO 37 I=1,NT                                                      CK 359
      XU1=XC(I)/.05                                                     CK 360
      YU1=TK(I)/D                                                        CK 361
      CALL PNTPLT (XU1,YU1,22,ISIZ)                                      CK 362
   37 CONTINUE                                                           CK 363
      CALL LINE (XC,TK,NT,1,0,0,0.)                                      CK 364
C          PLOT INPUT AIRFOIL AND AIRFOIL GENERATED BY COMBINING         CK 365
C          THICKNESS AND CAMBER DISTRIBUTIONS                            CK 366
      CALL CALPLT (0.,8.,-3)                                             CK 367
      CALL AXES (0.,0.,0.,20.,0.,.05,-2.,1.,3HX/C,SIZ,-3,1)              CK 368
      CALL AXES (0.,0.,90.,8.,-.2,.05,-2.,1.,3HY/C,SIZ,3,1)              CK 369
      CALL CALPLT (0.,4.,-3)                                             CK 370
      XU(NU+1)=YU(NU+1)=0.0                                              CK 371
      XU(NU+2)=YU(NU+2)=.05                                              CK 372
      CALL LINE (XU,YU,NU,1,0,0,0.)                                      CK 373
      XL(NL+1)=YL(NL+1)=0.0                                              CK 374
      XL(NL+2)=YL(NL+2)=.05                                              CK 375
      CALL LINE (XL,YL,NL,1,0,0,0.)                                      CK 376
      DO 40 I=1,NT                                                       CK 377
      IJ=LX(I)                                                           CK 378
      IF (IJ.EQ.0) GO TO 38                                              CK 379
      XU1=XU(IJ)/.05                                                     CK 380
      YU1=YU(IJ)/.05                                                     CK 381
      XL1=XLS(I)/.05                                                     CK 382
      YL1=YLS(I)/.05                                                     CK 383
      GO TO 39                                                           CK 384
   38 XU1=XL1=XLS(I)/.05                                                 CK 385
      YU1=YL1=YLS(I)/.05                                                 CK 386
   39 CONTINUE                                                           CK 387
      CALL PNTPLT (XU1,YU1,22,ISIZ)                                      CK 388
      CALL PNTPLT (XL1,YL1,22,ISIZ)                                      CK 389
   40 CONTINUE                                                           CK 390
      CALL NFRAME                                                        CK 391
      RETURN                                                             CK 392
   41 FORMAT (1H1,5X,47HTHE FOLLOWING CAMBERLINE DATA HAVE BEEN PUNCHED, CK 393
     15X,7HIPUNCH=,I4//5X,8A10//5X,4HNT =,I4//9X,3HX/C,7X,3HY/C,5X,5HT/C CK 394
     2/2,5X,5HSLOPE)                                                     CK 395
   42 FORMAT (F10.2)                                                     CK 396
   43 FORMAT (5X,4F10.6)                                                 CK 397
   44 FORMAT (1H1,1X,7HTITLE--,2X,8A10//32X,37H--THICKNESS AND CAMBER DI CK 398
     1STRIBUTION--//4X,1HI,5X,4HXU/C,6X,4HYU/C,6X,4HXL/C,6X,4HYL/C,6X,3H CK 399
     2X/C,7X,3HY/C,6X,5HT/C/2,5X,5HSLOPE,10X,5HERROR/)                   CK 400
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 401          45     FORMAT (I5,7F10.6,F10.4,5X,F10.6)                                                  CK 401
              46     FORMAT (8A10)                                                                      CK 402
              47     FORMAT (4F10.6)                                                                    CK 403
                     END                                                                                CK 404-
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      SUBROUTINE INTP (THETA,X,YSMO,YPPS,NP,NOSE,CHORD,TITLE,NINT,XINT,C
     1NEW,INTR,IPUNCH)
C
C     ROUTINE TO INTERPOLATE ADDITIONAL UPPER AND LOWER SURFACE
C     COORDINATES
C
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
      DIMENSION TITLE(8), THETA(1), X(1), YSMO(1), YPPS(1), XINT(1)
C
      DIMENSION XSAV(57)
C
      COMMON /INOUT/ JREAD,JWRITE,IPRINT
C
      COMMON /BLK1/ PI,PI2,RAD,CONS
C
      COMMON /HLM/ XU(100),YU(100),XL(100),YL(100),TLS(100)
C
C          STANDARD X/C COORDINATE INTERPOLATION VALUES
      DATA (XSAV(I),I=1,57)/0.0,.00025,.0005,.00075,.001,.0015,.002,.002
     15,.005,.01,.02,.03,.04,.05,.06,.07,.08,.09,.1,.125,.15,.175,.2,.22
     25,.25,.275,.3,.325,.35,.375,.4,.425,.45,.475,.5,.525,.55,.575,.6,.
     3625,.65,.675,.7,.725,.75,.775,.8,.825,.85,.875,.9,.925,.95,.97,.98
     44,.99,1.0/
C
C     IF INTR EQUAL 1, LOAD STANDARD X/C COORDINATE VALUES
C
      IF (INTR.EQ.0) RETURN
      IF (INTR.EQ.2) GO TO 2
      NINT=57
      DO 1 I=1,NINT
1     XINT(I)=XSAV(I)
C
C     INTERPOLATE UPPER SURFACE COORDINATES
C
2     WRITE (JWRITE,7) TITLE
      XUP=X(NP)*CHORD
      XNOSE=X(NOSE)*CHORD
      XLO=X(1)*CHORD
      RATIO=CNEW/CHORD
      DO 5 I=1,NINT                                                     IT  41
      XU(I)=XINT(I)*CHORD+XNOSE                                         IT  42
      XL(I)=XU(I)                                                       IT  43
      IF (XU(I).GT.XUP) XU(I)=XUP                                       IT  44
      IF (XL(I).GT.XLO) XL(I)=XLO                                       IT  45
      XU(I)=(XU(I)-XNOSE)*RATIO                                         IT  46
      XL(I)=(XL(I)-XNOSE)*RATIO                                         IT  47
      DELTA=XINT(I)                                                     IT  48
      IF (DELTA.LE.CONS) GO TO 3                                        IT  49
      DELTA=TAN(DELTA/CONS-1.)                                          IT  50
      TU=PI2+ALOG(DELTA+SQRT(DELTA*DELTA+1.))                           IT  51
      GO TO 4                                                           IT  52
    3 TU=ACOS(1.-DELTA/CONS)                                            IT  53
    4 TL=-TU                                                            IT  54
      IF (TL.LT.THETA(1)) TL=THETA(1)                                   IT  55
      IF (TU.GT.THETA(NP)) TU=THETA(NP)                                 IT  56
      TLS(I)=TL                                                         IT  57
      CALL COORD (THETA,YPPS,YSMO,NP,TU,YU(I),DYDX,DY2DX,CURV)          IT  58
      YU(I)=YU(I)*CNEW                                                  IT  59
      WRITE (JWRITE,8) I,XU(I),YU(I),DYDX,DY2DX,CURV                    IT  60
    5 CONTINUE                                                          IT  61
      WRITE (JWRITE,9) CNEW                                             IT  62
    C                                                                   IT  63
    C INTERPOLATE LOWER SURFACE COORDINATES                             IT  64
    C                                                                   IT  65
      WRITE (JWRITE,10) TITLE                                           IT  66
      DO 6 I=1,NINT                                                     IT  67
      TL=TLS(I)                                                         IT  68
      CALL COORD (THETA,YPPS,YSMO,NP,TL,YL(I),DYDX,DY2DX,CURV)          IT  69
      YL(I)=YL(I)*CNEW                                                  IT  70
      WRITE (JWRITE,8) I,XL(I),YL(I),DYDX,DY2DX,CURV                    IT  71
    6 CONTINUE                                                          IT  72
    C                                                                   IT  73
    C PUNCH COORDINATES                                                 IT  74
    C                                                                   IT  75
      IF (IPUNCH.NE.6) RETURN                                           IT  76
      WRITE (JWRITE,11) CNEW,TITLE                                      IT  77
      WRITE (1,12) TITLE                                                IT  78
      WRITE (JWRITE,13) NINT                                            IT  79
      XNT=FLOAT(NINT)                                                   IT  80
      WRITE (1,14) XNT
      WRITE (JWRITE,15) (XU(I),I=1,NINT)
      WRITE (JWRITE,16) (YU(I),I=1,NINT)
      WRITE (1,17) (XU(I),YU(I),I=1,NINT)
      WRITE (JWRITE,18) NINT
      WRITE (1,14) XNT
      WRITE (JWRITE,19) (XL(I),I=1,NINT)
      WRITE (JWRITE,20) (YL(I),I=1,NINT)
      WRITE (1,17) (XL(I),YL(I),I=1,NINT)
C
C     RETURN TO CALLING PROGRAM
C
      RETURN
C
    7 FORMAT (1H1,5X,9HTITLE--  ,8A10//26X,42H--UPPER SURFACE INTERPOLAT
     1ED COORDINATES--//9X,1HI,10X,2HXU,13X,2HYU,11X,5HDY/DX,6X,11HD(DY/
     2DX)/DX,6X,9HCURVATURE)
    8 FORMAT (I10,2F15.6,3E15.6)
    9 FORMAT (/10X,7HCHORD =,F10.6)
   10 FORMAT (1H1,5X,9HTITLE--  ,8A10//26X,42H--LOWER SURFACE INTERPOLAT
     1ED COORDINATES--//9X,1HI,10X,2HXL,13X,2HYL,11X,5HDY/DX,6X,11HD(DY/
     2DX)/DX,6X,9HCURVATURE)
   11 FORMAT (1H1,10X,50HTHE FOLLOWING DATA HAVE BEEN PUNCHED FOR A CHOR
     1D =,F10.6//3X,9HTITLE--  ,8A10)
   12 FORMAT (8A10)
   13 FORMAT (5X,4HNU =,I4)
   14 FORMAT (F10.2)
   15 FORMAT (5X,4HXU =,8F10.6/(9X,8F10.6))
   16 FORMAT (5X,4HYU =,8F10.6/(9X,8F10.6))
   17 FORMAT (2F10.6)
   18 FORMAT (5X,4HNL =,I4)
   19 FORMAT (5X,4HXL =,8F10.6/(9X,8F10.6))
   20 FORMAT (5X,4HYL =,8F10.6/(9X,8F10.6))
      END
      SUBROUTINE COORD (THETA,YPPS,YSMO,NP,TI,YI,DYDX,DY2DX,CURV)
C
C     ROUTINE TO COMPUTE THE Y COORDINATE, DY/DX, D(DY/DX)/DX, AND
C     CURVATURE AT A GIVEN VALUE OF THETA
C
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982
C
      DIMENSION THETA(1), YPPS(1), YSMO(1)
C
      COMMON /BLK1/ PI,PI2,RAD,CONS
C
C     COSH(X)=(EXP(X)+EXP(-X))/2.
C     SINH(X)=(EXP(X)-EXP(-X))/2.
C
      DO 1 K=2,NP
      J=K-1
      IF (TI.GE.THETA(J).AND.TI.LE.THETA(K)) GO TO 2
1     CONTINUE
2     DELTA=THETA(J+1)-THETA(J)
      T2=THETA(J+1)-TI
      T1=TI-THETA(J)
      YI=YPPS(J)*(T2**3/(6.*DELTA)-T2*DELTA/6.)+YPPS(J+1)*(T1**3/(6.*DEL
     1TA)-T1*DELTA/6.)+(YSMO(J)*T2+YSMO(J+1)*T1)/DELTA
      YPI=YPPS(J)*(DELTA/6.-T2*T2/(2.*DELTA))+YPPS(J+1)*(T1*T1/(2.*DELTA
     1)-DELTA/6.)+(YSMO(J+1)-YSMO(J))/DELTA
      YPPI=(YPPS(J)*T2+YPPS(J+1)*T1)/DELTA
      DELTA=YPI
      IF (TI.LE.0.0) DELTA=-DELTA
      TP=ABS(TI)
      IF (TP.GT.PI2) GO TO 3
      GP=CONS*SIN(TP)
      GPP=CONS*COS(TP)
      GO TO 4
3     T1=COSH(TP-PI2)
      T2=SINH(TP-PI2)
      GP=CONS/T1
      GPP=-CONS*T2/(T1*T1)
4     IF (TP.LE.0.0.OR.GP.EQ.0.0) GO TO 5
      DYDX=DELTA/GP
      DY2DX=(YPPI*GP-DELTA*GPP)/(GP**3)
```

CARD NO.

```
                  CURV=ABS(DY2DX)/(SQRT(1.+DYDX**2)**3)                       CD  41
                  RETURN                                                      CD  42
          5       DYDX=0.1E99                                                 CD  43
                  DY2DX=0.1E99                                                CD  44
                  CURV=CONS/(DELTA*DELTA)                                     CD  45
                  RETURN                                                      CD  46
                  END                                                         CD  47-
```

```
LISTING OF DECK: SINH
                                                                   PAGE   1
CARD NO.

                FUNCTION SINH(X)
          C     HYPERBOLIC SINE                                      SH    1
                SINH=0.5*(EXP(X)-EXP(-X))                            SH    2
                RETURN                                               SH    3
                END                                                  SH    4
                                                                     SH    5-
```

```
CARD NO.

                      FUNCTION COSH(X)                                          CH    1
                C     HYPERBOLIC COSINE                                         CH    2
                      COSH=0.5*(EXP(X)+EXP(-X))                                 CH    3
                      RETURN                                                    CH    4
                      END                                                       CH    5-
```

APPENDIX B

COMPUTER LISTING OF AIRFOIL SCALING PROGRAM AFSCL

This appendix contains a computer listing of the airfoil scaling program AFSCL which consists of a main program and two subroutines.

```
      PROGRAM SCALE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1)          SC   1
C                                                                         SC   2
C     THIS PROGRAM PRESENTS A TECHNIQUE FOR SCALING THE COORDINATES OF    SC   3
C     AN AIRFOIL FROM ITS INPUT MAXIMUM THICKNESS RATIO TO A DESIRED      SC   4
C     OUTPUT MAXIMUM THICKNESS RATIO                                      SC   5
C                                                                         SC   6
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982           SC   7
C                                                                         SC   8
C*********************************************************************** SC   9
C*                                                                     * SC  10
C*               DESCRIPTION OF INPUT CARDS FOR SCALING PROGRAM        * SC  11
C*                                                                     * SC  12
C*.....................................................................* SC  13
C*                                                                     * SC  14
C* CARD NUMBER                   DESCRIPTION                           * SC  15
C*                                                                     * SC  16
C*.....................................................................* SC  17
C*    1                                 FORMAT(8A10)                   * SC  18
C*               TITLE CARD                                            * SC  19
C*.....................................................................* SC  20
C*    2                                 FORMAT(4F10.0)                 * SC  21
C*               NT - NUMBER OF INPUT CAMBER, THICKNESS, AND SLOPE     * SC  22
C*                    POINTS                                           * SC  23
C*               IPLOT - PLOT OPTION                                   * SC  24
C*                       0 - NO PLOTS DESIRED                          * SC  25
C*                       1 - PLOTS DESIRED                             * SC  26
C*               IPUNCH - PUNCH OUTPUT OPTION                          * SC  27
C*                        0 - NO PUNCHED OUTPUT DESIRED                * SC  28
C*                        1 - PUNCH COORDINATES OF SCALED AIRFOIL      * SC  29
C*               IOP - SLOPE OF CAMBERLINE OPTION                      * SC  30
C*                     0 - SLOPES INPUT ON CARD 3                      * SC  31
C*                     1 - SLOPES COMPUTED BY PROGRAM                  * SC  32
C*.....................................................................* SC  33
C*    3                                 FORMAT(4F10.0)                 * SC  34
C*               XC - X/C COORDINATES OF CAMBERLINE                    * SC  35
C*               YC - Y/C COORDINATES OF CAMBERLINE                    * SC  36
C*               TK - T/C/2 THICKNESS DISTRIBUTION                     * SC  37
C*               TH - SLOPE OF CAMBERLINE IN RADIANS                   * SC  38
C*                    NOTE -- CARD 3 IS READ NT TIMES                  * SC  39
C*.....................................................................* SC  40
```

```
C*   4                            FORMAT(F10.0)                        *
C*            LT - NUMBER OF DESIRED OUTPUT MAXIMUM THICKNESS RATIOS   *
C*.....................................................................*
C*   5                            FORMAT(F10.0)                        *
C*            TKNEW - DESIRED OUTPUT MAXIMUM THICKNESS RATIO           *
C*                 NOTE -- CARD 5 IS READ LT TIMES                     *
C*.....................................................................*
C*                                                                     *
C*       RESTRICTIONS:                                                 *
C*              NT NOT GREATER THAN 101                                *
C*              LT NOT GREATER THAN 10                                 *
C*              XC MUST BE MONOTONICALLY INCREASING                    *
C*                                                                     *
C***********************************************************************
C
      DIMENSION XC(101), YC(101), TK(101), TH(101), THETA(101), YPP(101)
     1, TKNEW(10), TITLE(8), VAR(4)
C
      COMMON /HLM/ WK(404,3)
C
      COMMON /BLK1/ PI,PI2,RAD,CONS
C
      COMMON /INOUT/ JREAD,JWRITE,IPRINT
C
      SINH(X)=0.5*(EXP(X)-EXP(-X))
C
C        INITIALIZE PROGRAM CONSTANTS
C
      JWRITE=6
      JREAD=5
      IPRINT=0
      NTMAX=101
      PI=ACOS(-1.)
      PI2=PI/2.
      RAD=180./PI
      CONS=1./(1.+ATAN(SINH(PI2)))
C
C        READ AND PRINT INPUT DATA
C
1     READ (JREAD,26) TITLE
```

```
      IF (EOF(JREAD)) 25,2
    2 READ (JREAD,27) VAR
      NT=IFIX(VAR(1))
      IF (NT.GT.NTMAX) GO TO 24
      IPLOT=IFIX(VAR(2))
      IF (IPLOT.NE.0) IPLOT=1
      IPUNCH=IFIX(VAR(3))
      IF (IPUNCH.NE.0) IPUNCH=1
      IOP=IFIX(VAR(4))
      IF (IOP.NE.0) IOP=1
      WRITE (JWRITE,28) TITLE,NT,IPLOT,IPUNCH,IOP
      READ (JREAD,29) (XC(I),YC(I),TK(I),TH(I),I=1,NT)
      WRITE (JWRITE,30) (XC(I),I=1,NT)
      WRITE (JWRITE,31) (YC(I),I=1,NT)
      WRITE (JWRITE,32) (TK(I),I=1,NT)
      IF (IOP.EQ.0) WRITE (JWRITE,33) (TH(I),I=1,NT)
      READ (JREAD,34) VAR(1)
      LT=IFIX(VAR(1))
      IF (LT.LE.0) GO TO 1
      IF (LT.GT.10) LT=10
      READ (JREAD,34) (TKNEW(I),I=1,LT)
      WRITE (JWRITE,35) LT,(TKNEW(I),I=1,LT)
C
C        INITIALIZE PLOTTING DEVICE
C
      CALL PSEUDO
      CALL LEROY
C
C        CHECK FOR INCREASING XC
C
      DO 3 I=2,NT
      IF (XC(I).LE.XC(I-1)) GO TO 4
    3 CONTINUE
      GO TO 5
    4 WRITE (JWRITE,36)
      GO TO 1
C
C        FIND MAXIMUM THICKNESS RATIO OF INPUT AIRFOIL
C
C             COMPUTE THETA EQUIVALENT OF XC
```

```
    5 CHORD=XC(NT)-XC(1)
      DO 7 I=1,NT
      DELTA=(XC(I)-XC(1))/CHORD
      IF (DELTA.LE.CONS) GO TO 6
      DELTA=TAN(DELTA/CONS-1.)
      THETA(I)=PI2+ALOG(DELTA+SQRT(DELTA*DELTA+1.))
      GO TO 7
    6 THETA(I)=ACOS(1.-DELTA/CONS)
    7 CONTINUE
C             FIT CUBIC SPLINE THRU TK VS THETA
      CALL CUBSPL (THETA,TK,YPP,NT,WK)
C             FIND LOCATIONS WHERE D(TK)/D(THETA) = 0.0
      KRT=0
      N1=NT-1
      DO 12 I=1,N1
      DELTA=THETA(I+1)-THETA(I)
      AA=(YPP(I)-YPP(I+1))/(2.*DELTA)
      BB=(YPP(I+1)*THETA(I)-YPP(I)*THETA(I+1))/DFLTA
      CC=(YPP(I)*THETA(I+1)**2-YPP(I+1)*THETA(I)**2)/(2.*DELTA)+(YPP(I+1
     1)-YPP(I))*DELTA/6.-(TK(I+1)-TK(I))/DELTA
      GP=BB*BB-4.*AA*CC
      IF (GP) 12,8,8
    8 GP=SQRT(GP)
      T1=(-BB+GP)/(2.*AA)
      T2=(-BB-GP)/(2.*AA)
      IF (T1.GE.THETA(I).AND.T1.LE.THETA(I+1)) GO TO 9
      GO TO 10
    9 KRT=KRT+1
      WK(KRT,1)=T1
   10 IF (T2.GE.THETA(I).AND.T2.LE.THETA(I+1)) GO TO 11
      GO TO 12
   11 KRT=KRT+1
      WK(KRT,1)=T2
   12 CONTINUE
      IF (KRT.EQ.0) GO TO 16
C             COMPUTE XC LOCATIONS WHERE D(TK)/D(THETA) = 0.0
      DO 15 I=1,KRT
      T1=ABS(WK(I,1))
      IF (T1.LE.PI2) WK(I,2)=CONS*(1.-COS(T1))
      IF (T1.GT.PI2) WK(I,2)=CONS*(ATAN(SINH(T1-PI2))+1.)
```

```
            DO 13 J=1,N1                                                      SC 161
            J1=J                                                              SC 162
            J2=J+1                                                            SC 163
            IF (WK(I,1).GE.THETA(J).AND.WK(I,1).LE.THETA(J+1)) GO TO 14       SC 164
   13       CONTINUE                                                          SC 165
   14       AA=THETA(J2)-WK(I,1)                                              SC 166
            BB=WK(I,1)-THETA(J1)                                              SC 167
            DELTA=THETA(J2)-THETA(J1)                                         SC 168
   15       WK(I,3)=YPP(J1)*(AA**3/(6.*DELTA)-AA*DELTA/6.)+YPP(J2)*(BB**3/(6.*SC 169
           1DELTA)-BB*DELTA/6.)+(TK(J1)*AA+TK(J2)*BB)/DELTA                   SC 170
   16       CONTINUE                                                          SC 171
   C                  COMPUTE AND PRINT MAXIMUM THICKNESS RATIO               SC 172
            IF (KRT.EQ.0) GO TO 23                                            SC 173
            TKMAX=0.0                                                         SC 174
            DO 18 I=1,KRT                                                     SC 175
            IF (WK(I,3).GE.TKMAX) GO TO 17                                    SC 176
            GO TO 18                                                          SC 177
   17       N1=I                                                              SC 178
            TKMAX=WK(I,3)                                                     SC 179
   18       CONTINUE                                                          SC 180
            TKMAX=2.*TKMAX                                                    SC 181
            DELTA=WK(N1,2)*CHORD+XC(1)                                        SC 182
            WRITE (JWRITE,37) TKMAX,DELTA                                     SC 183
            IF (TKMAX.LE.0.0) GO TO 1                                         SC 184
   C                                                                          SC 185
   C           IF IOP=1, COMPUTE SLOPES OF CAMBERLINE                         SC 186
   C                                                                          SC 187
            IF (IOP.NE.1) GO TO 21                                            SC 188
            CALL CUBSPL (XC,YC,YPP,NT,WK)                                     SC 189
            DO 20 I=1,NT                                                      SC 190
            IF (I.EQ.NT) GO TO 19                                             SC 191
            DELTA=XC(I+1)-XC(I)                                               SC 192
            TH(I)=-YPP(I)*DELTA/3.-YPP(I+1)*DELTA/6.+(YC(I+1)-YC(I))/DELTA    SC 193
            GO TO 20                                                          SC 194
   19       DELTA=XC(NT)-XC(NT-1)                                             SC 195
            TH(I)=YPP(NT-1)*DELTA/6.+YPP(NT)*DELTA/3.+(YC(NT)-YC(NT-1))/DELTA SC 196
   20       TH(I)=ATAN(TH(I))                                                 SC 197
   C                                                                          SC 198
   C           COMPUTE AND PRINT COORDINATES OF INPUT AIRFOIL                 SC 199
   C                                                                          SC 200
   21 CALL SCTK (XC,YC,TK,TH,NT,TITLE,TKMAX,TKMAX,IPUNCH,IPLOT,IERR)
      IF (IERR.NE.0) GO TO 1
C
C        COMPUTE AND PRINT COORDINATES OF SCALED AIRFOILS
C
      DO 22 I=1,LT
      CALL SCTK (XC,YC,TK,TH,NT,TITLE,TKNEW(I),TKMAX,IPUNCH,IPLOT,IERR)
      IF (IERR.NE.0) GO TO 1
   22 CONTINUE
C
C        READ NEXT CASE
C
      GO TO 1
C
C        PRINT ERROR MESSAGE
C
   23 WRITE (JWRITE,38)
      GO TO 1
   24 WRITE (JWRITE,39) NTMAX
      GO TO 1
C
C        FINALIZE PLOTTING DEVICE
C
   25 CALL CALPLT (0.,0.,999)
      STOP
C
   26 FORMAT (8A10)
   27 FORMAT (4F10.6)
   28 FORMAT (1H1,57X,14H--INPUT DATA--//5X,7HTITLE--,2X,8A10//5X,3HNT=,
     1I3,5X,6HIPLOT=,I3,5X,7HIPUNCH=,I3,5X,4HIOP=,I3)
   29 FORMAT (4F10.6)
   30 FORMAT (/4X,4HX/C=,8E15.6/(8X,8E15.6))
   31 FORMAT (/4X,4HY/C=,8E15.6/(8X,8E15.6))
   32 FORMAT (/2X,6HT/C/2=,8E15.6/(8X,8E15.6))
   33 FORMAT (/2X,6HSLOPE=,8E15.6/(8X,8E15.6))
   34 FORMAT (F10.2)
   35 FORMAT (/2X,3HLT=,I3,5X,9HNEW T/C =,10F12.6)
   36 FORMAT (/5X,40HXC ARRAY IS NOT MONOTONICALLY INCREASING)
   37 FORMAT (/5X,28H(T/C)MAX FOR INPUT AIRFOIL =,F10.6,2X,8HAT X/C =,
     1F10.6)
```

CARD NO.

```
      38     FORMAT (//5X,64H(T/C)MAX OF INPUT AIRFOIL WAS NOT FOUND -- CHECK Y SC 241
            1OUR INPUT DATA)                                                    SC 242
      39     FORMAT (//5X,35HINPUT CARD ERROR - NT GREATER THAN ,I4)            SC 243
             END                                                                SC 244-
```

CARD NO.

```
      SUBROUTINE SCTK (XC,YC,TK,TH,NT,TITLE,TKNEW,TKMAX,IPUNCH,IPLOT,IER SK   1
     1R)                                                                 SK   2
C                                                                        SK   3
C     THIS SUBROUTINE SCALES THE COORDINATES OF AN AIRFOIL FROM A BASIC  SK   4
C     MAXIMUM THICKNESS RATIO (TKMAX) TO A NEW MAXIMUM THICKNESS RATIO   SK   5
C     (TKNEW)                                                            SK   6
C                                                                        SK   7
C        CODED BY -- HARRY MORGAN  NASA/LARC/TAD/AAB       1982          SK   8
C                                                                        SK   9
      DIMENSION XC(1), YC(1), TK(1), TH(1), TITLE(8)                     SK  10
C                                                                        SK  11
      COMMON /HLM/ X(220),Y(220),XU(110),YU(110),XL(110),YL(110),XPRT(11 SK  12
     10),YPRT(110),TPRT(110)                                             SK  13
C                                                                        SK  14
      COMMON /BLK1/ PI,PI2,RAD,CONS                                      SK  15
C                                                                        SK  16
      COMMON /INOUT/ JREAD,JWRITE,IPRINT                                 SK  17
C                                                                        SK  18
C        SCALE THICKNESS AND COMPUTE UPPER AND LOWER SURFACE COORDINATES SK  19
C        OF NEW AIRFOIL                                                  SK  20
C                                                                        SK  21
      IERR=0                                                             SK  22
      DELT1=TKNEW/TKMAX                                                  SK  23
      DO 1 I=1,NT                                                        SK  24
      DELT2=COS(TH(I))                                                   SK  25
      DELT4=SIN(TH(I))                                                   SK  26
      XU(I)=XC(I)-TK(I)*DELT4*DELT1                                      SK  27
      YU(I)=YC(I)+TK(I)*DELT2*DELT1                                      SK  28
      XL(I)=XC(I)+TK(I)*DELT4*DELT1                                      SK  29
    1 YL(I)=YC(I)-TK(I)*DELT2*DELT1                                      SK  30
C                                                                        SK  31
C        LOAD SURFACE COORDINATES INTO X AND Y ARRAYS                    SK  32
C                                                                        SK  33
      DO 2 I=1,NT                                                        SK  34
      J=NT+1-I                                                           SK  35
      X(I)=XL(J)                                                         SK  36
    2 Y(I)=YL(J)                                                         SK  37
      N=NT                                                               SK  38
      M=1                                                                SK  39
      IF (XU(1).EQ.XL(1).AND.YU(1).EQ.YL(1)) M=2                         SK  40
      DO 3 I=M,NT
      N=N+1
      X(N)=XU(I)
    3 Y(N)=YU(I)
C
C        INTERPOLATE OR EXTRAPOLATE TRAILING EDGE COORDINATES
C
      IF (X(1)-X(N)) 4,6,5
    4 DELT1=X(2)-X(1)
      DELT2=X(3)-X(1)
      DELT3=Y(2)-Y(1)
      DELT4=Y(3)-Y(1)
      Y(1)=Y(1)+(X(N)-X(1))*((DELT3*DELT2-DELT4*DELT1)*(X(N)-X(1))+(DELT
     14*DELT1*DELT1-DELT3*DELT2*DELT2))/(DELT2*DELT1*DELT1-DELT1*DELT2*D
     2ELT2)
      X(1)=X(N)
      GO TO 6
    5 DELT1=X(N-1)-X(N-2)
      DELT2=X(N)-X(N-2)
      DELT3=Y(N-1)-Y(N-2)
      DELT4=Y(N)-Y(N-2)
      Y(N)=Y(N-2)+(X(1)-X(N-2))*((DELT3*DELT2-DELT4*DELT1)*(X(1)-X(N-2))
     1+(DELT4*DELT1*DELT1-DELT3*DELT2*DELT2))/(DELT2*DELT1*DELT1-DELT1*D
     2ELT2*DELT2)
      X(N)=X(1)
C
C        COMPUTE LONGEST CHORD
C
    6 CHORD=0.0
      DO 8 I=2,N
      DELT=X(1)-X(I)
      IF (DELT.GT.CHORD) GO TO 7
      GO TO 8
    7 CHORD=DELT
      NOSE=I
    8 CONTINUE
C
C        ADJUST COORDINATES FOR LONGEST CHORD
C
      DELT=X(NOSE)
```

CARD NO.

```
            DO 9 I=1,N                                                    SK  81
            X(I)=(X(I)-DELT)/CHORD                                        SK  82
      9     Y(I)=Y(I)/CHORD                                               SK  83
      C                                                                   SK  84
      C        CHECK UPPER AND LOWER SURFACE X VALUES TO DETECT CROSSOVER OF  SK  85
      C        PERPENDICULARS TO CAMBERLINE AND TO FIND NOSE POINT        SK  86
      C                                                                   SK  87
            DO 10 I=2,NOSE                                                SK  88
            IF (X(I)-X(I-1)) 10,20,20                                     SK  89
      10    CONTINUE                                                      SK  90
            J=NOSE+1                                                      SK  91
            DO 11 I=J,N                                                   SK  92
            IF (X(I)-X(I-1)) 20,20,11                                     SK  93
      11    CONTINUE                                                      SK  94
      C                                                                   SK  95
      C        LOAD COORDINATES INTO UPPER AND LOWER SURFACE ARRAYS       SK  96
      C                                                                   SK  97
            DO 12 I=1,NOSE                                                SK  98
            J=NOSE+1-I                                                    SK  99
            XL(I)=X(J)                                                    SK 100
      12    YL(I)=Y(J)                                                    SK 101
            DO 13 I=NOSE,N                                                SK 102
            J=I+1-NOSE                                                    SK 103
            XU(J)=X(I)                                                    SK 104
      13    YU(J)=Y(I)                                                    SK 105
            NL=NOSE                                                       SK 106
            NU=N-NOSE+1                                                   SK 107
      C                                                                   SK 108
      C        PRINT SCALED SURFACE COORDINATES                           SK 109
      C                                                                   SK 110
            WRITE (JWRITE,21) TITLE,TKNEW                                 SK 111
            J=NU                                                          SK 112
            IF (NL.GT.NU) J=NL                                            SK 113
            DO 14 I=1,J                                                   SK 114
            IF (I.LE.NU.AND.I.LE.NL) WRITE (JWRITE,22) I,XU(I),YU(I),XL(I),YL( SK 115
           1I)                                                            SK 116
            IF (I.LE.NU.AND.I.GT.NL) WRITE (JWRITE,22) I,XU(I),YU(I)      SK 117
            IF (I.GT.NU.AND.I.LE.NL) WRITE (JWRITE,23) I,XL(I),YL(I)      SK 118
      14    CONTINUE                                                      SK 119
      C                                                                   SK 120
```

CARD NO.

```
C         PRINT CAMBER AND THICKNESS DISTRIBUTIONS                      SK 121
C                                                                       SK 122
      WRITE (JWRITE,24) TITLE,TKNEW                                     SK 123
      DELT4=TKNEW/TKMAX                                                 SK 124
      DO 15 I=1,NT                                                      SK 125
      XPRT(I)=(XC(I)-DELT)/CHORD                                        SK 126
      YPRT(I)=YC(I)/CHORD                                               SK 127
      TPRT(I)=TK(I)*DELT4                                               SK 128
      DELT3=2.0*TPRT(I)                                                 SK 129
      DELT1=TH(I)*RAD                                                   SK 130
15    WRITE (JWRITE,25) I,XPRT(I),YPRT(I),DELT1,DELT3                   SK 131
C                                                                       SK 132
C         PUNCH DESIRED OUTPUT DATA                                     SK 133
C                                                                       SK 134
      IF (IPUNCH.EQ.0) GO TO 16                                         SK 135
      WRITE (JWRITE,26) (TITLE(I),I=1,6),TKNEW                          SK 136
      WRITE (1,27) (TITLE(I),I=1,6),TKNEW                               SK 137
      WRITE (JWRITE,28) NU                                              SK 138
      DELT1=FLOAT(NU)                                                   SK 139
      WRITE (1,29) DELT1                                                SK 140
      WRITE (JWRITE,30) (XU(I),I=1,NU)                                  SK 141
      WRITE (JWRITE,31) (YU(I),I=1,NU)                                  SK 142
      WRITE (1,32) (XU(I),YU(I),I=1,NU)                                 SK 143
      WRITE (JWRITE,33) NL                                              SK 144
      DELT1=FLOAT(NL)                                                   SK 145
      WRITE (1,29) DELT1                                                SK 146
      WRITE (JWRITE,34) (XL(I),I=1,NL)                                  SK 147
      WRITE (JWRITE,35) (YL(I),I=1,NL)                                  SK 148
      WRITE (1,32) (XL(I),YL(I),I=1,NL)                                 SK 149
16    IF (IPLOT.EQ.0) RETURN                                            SK 150
C                                                                       SK 151
C         PLOT AIRFOIL SHAPE AND CAMBER AND THICKNESS DISTRIBUTIONS     SK 152
C                                                                       SK 153
C                LABEL PLOT                                             SK 154
      CALL CALPLT (2.,0.,-3)                                            SK 155
      CALL NOTATE (0.,0.,.40,44HPLOT OF AIRFOIL GENERATED BY SCALING PRO SK 156
     1GRAM,0.,44)                                                       SK 157
      CALL NOTATE (16.0,0.,.40,10H(T/C)MAX =,0.,10)                     SK 158
      CALL NUMBER (20.0,0.,.40,TKNEW,0.0,3)                             SK 159
      CALL NOTATE (0.,1.,.40,TITLE,0.,80)                               SK 160
```

CARD NO.

```
C           PLOT AIRFOIL                                                SK 161
      CALL AXES (0.,4.,0.,20.,0.0,.05,-2.,1.,3HX/C,.40,-3,1)            SK 162
      CALL AXES (0.,4.,90.,8.,-.2,.05,-2.,1.,3HY/C,.40,3,1)             SK 163
      CALL CALPLT (0.,8.,-3)                                            SK 164
      X(N+1)=Y(N+1)=0.0                                                 SK 165
      X(N+2)=Y(N+2)=.05                                                 SK 166
      CALL LINE (X,Y,N,1,0,0,0.0)                                       SK 167
C           PLOT CAMBER DISTRIBUTION                                    SK 168
      CALL CALPLT (0.,6.,-3)                                            SK 169
      CALL AXES (0.,0.,0.,20.,0.,.05,-2.,1.,3HX/C,.40,-3,1)             SK 170
      DELT1=0.0                                                         SK 171
      DO 17 I=1,NT                                                      SK 172
      IF (ABS(YPRT(I)).GT.DELT1) DELT1=ABS(YPRT(I))                     SK 173
   17 CONTINUE                                                          SK 174
      DELT2=.1                                                          SK 175
      IF (DELT1.LE.0.2.AND.DELT1.GT.0.08) DELT2=.05                     SK 176
      IF (DELT1.LE.0.08.AND.DELT1.GT.0.04) DELT2=.02                    SK 177
      IF (DELT1.LE.0.04) DELT2=.01                                      SK 178
      DELT1=-4.*DELT2                                                   SK 179
      CALL AXES (0.,0.,90.,8.,DELT1,DELT2,-1.,0.,3HY/C,.40,3,2)         SK 180
      CALL CALPLT (0.,4.,-3)                                            SK 181
      XPRT(NT+1)=YPRT(NT+1)=0.0                                         SK 182
      XPRT(NT+2)=.05                                                    SK 183
      YPRT(NT+2)=DELT2                                                  SK 184
      DO 18 I=1,NT                                                      SK 185
      DELT3=XPRT(I)/.05                                                 SK 186
      DELT4=YPRT(I)/DELT2                                               SK 187
      CALL PNTPLT (DELT3,DELT4,22,3)                                    SK 188
   18 CONTINUE                                                          SK 189
      CALL LINE (XPRT,YPRT,NT,1,0,0,0.)                                 SK 190
C           PLOT THICKNESS DISTRIBUTION                                 SK 191
      CALL CALPLT (0.,6.,-3)                                            SK 192
      CALL AXES (0.,0.,0.,20.,0.,.05,-2.,1.,3HX/C,.40,-3,1)             SK 193
      CALL AXES (0.,0.,90.,7.,0.,.02,-1.,0.,5HT/C/2,.40,5,2)            SK 194
      TPRT(NT+1)=0.0                                                    SK 195
      TPRT(NT+2)=.02                                                    SK 196
      DO 19 I=1,NT                                                      SK 197
      DELT3=XPRT(I)/.05                                                 SK 198
      DELT4=TPRT(I)/.02                                                 SK 199
      CALL PNTPLT (DELT3,DELT4,22,3)                                    SK 200
```

```
 19   CONTINUE
      CALL LINE (XPRT,TPRT,NT,1,0,0,0.0)
      CALL NFRAME
      RETURN
C
C        PRINT ERROR MESSAGE
C
 20   WRITE (JWRITE,36) TITLE,TKNEW
      WRITE (JWRITE,37) (I,X(I),Y(I),I=1,N)
      IERR=1
      RETURN
C
 21   FORMAT (1H1,3X,7HTITLE--,2X,8A10//12X,33HSCALED COORDINATES FOR (T
     1/C)MAX =,F7.4//24X,5HUPPER,20X,5HLOWER//9X,1HI,10X,3HX/C,7X,3HY/C,
     212X,3HX/C,7X,3HY/C)
 22   FORMAT (5X,I5,5X,2F10.6,5X,2F10.6)
 23   FORMAT (5X,I5,30X,2F10.6)
 24   FORMAT (1H1,3X,7HTITLE--,2X,8A10//12X,49HCAMBER AND THICKNESS DIST
     1RIBUTIONS FOR (T/C)MAX =,F7.4//34X,6HCAMBER,22X,9HTHICKNESS//9X,1H
     2I,10X,3HX/C,12X,3HY/C,11X,5HSLOPE,10X,5HT/C/2)
 25   FORMAT (5X,I5,2(5X,F10.6),5X,F10.4,5X,F10.6)
 26   FORMAT (1H1,10X,36HTHE FOLLOWING DATA HAVE BEEN PUNCHED//5X,9HTITL
     1E--  ,6A10,10H(T/C)MAX =,F10.6)
 27   FORMAT (6A10,10H(T/C)MAX =,F10.6)
 28   FORMAT (/5X,4HNU =,I4)
 29   FORMAT (F10.2)
 30   FORMAT (/5X,4HXU =,8F10.6/(9X,8F10.6))
 31   FORMAT (/5X,4HYU =,8F10.6/(9X,8F10.6))
 32   FORMAT (2F10.6)
 33   FORMAT (/5X,4HNL =,I4)
 34   FORMAT (/5X,4HXL =,8F10.6/(9X,8F10.6))
 35   FORMAT (/5X,4HYL =,8F10.6/(9X,8F10.6))
 36   FORMAT (1H1,3X,7HTITLE--,2X,8A10//3X,38HATTEMPT TO SCALE AIRFOIL T
     10 (T/C)MAX =,F7.4,2X,55HFAILED DUE TO CROSSOVER OF PERPENDICULARS
     2TO CAMBERLINE//9X,1HI,9X,3HX/C,13X,3HY/C)
 37   FORMAT (5X,I5,5X,F10.6,5X,F10.6)
      END
```

CARD NO.

```
      SUBROUTINE CUBSPL (X,Y,YPP,N,A)
C
C     THIS SUBROUTINE FITS A CUBIC SPLINE TO A SET OF Y VS X INPUT
C     POINTS
C
C        CODED BY -- HARRY MORGAN      NASA/LARC/TAD/AAB      1982
C
C        IN CALLING PROGRAM DIMENSION X, Y, AND YPP BY N AND A BY 2*N
C
      DIMENSION X(1), Y(1), YPP(1), A(N,2)
C
C     COMPUTE SECOND DERIVATIVE AT END POINTS BY FITTING
C     Y=A*X**2+B*X+C TO THE LAST THREE POINTS AND SOLVE FOR A.
C     SECOND DERIVATIVE AT END POINT IS THEN EQUAL TO 2.*A
C
      H1=X(2)-X(3)
      H2=X(3)-X(1)
      H3=X(1)-X(2)
      YPP(1)=2.*(Y(1)*H1+Y(2)*H2+Y(3)*H3)/(H1*X(1)**2+H2*X(2)**2+H3*X(3)
     1**2)
      H1=X(N-1)-X(N)
      H2=X(N)-X(N-2)
      H3=X(N-2)-X(N-1)
      YPP(N)=2.*(Y(N-2)*H1+Y(N-1)*H2+Y(N)*H3)/(H1*X(N-2)**2+H2*X(N-1)**2
     1+H3*X(N)**2)
C
C     PERFORM FORWARD ELIMINATION
C
      A(1,1)=0.0
      A(1,2)=YPP(1)
      N1=N-1
      DO 1 I=2,N1
      H1=X(I)-X(I-1)
      H2=X(I+1)-X(I)
      H3=(Y(I+1)-Y(I))/H2-(Y(I)-Y(I-1))/H1
      D=H1*(2.-A(I-1,1))+2.*H2
      A(I,1)=H2/D
    1 A(I,2)=(6.*H3-H1*A(I-1,2))/D
C
C     PERFORM BACK SUBSTITUTION
```

CARD NO.

```
      C                                                                 CB  41
            J=N                                                         CB  42
            DO 2 I=2,N1                                                 CB  43
            J=J-1                                                       CB  44
      2     YPP(J)=A(J,2)-A(J,1)*YPP(J+1)                               CB  45
      C                                                                 CB  46
      C     RETURN TO CALLING PROGRAM                                   CB  47
      C                                                                 CB  48
            RETURN                                                      CB  49
            END                                                         CB  50-
```

APPENDIX C

DESCRIPTION OF INPUT FOR AIRFOIL SMOOTHING PROGRAM AFSMO

This appendix contains a description of the input requirements for the airfoil smoothing program AFSMO. All variables are input with a card format of 8F10.0, except the title card which has a format of 8A10.

CARD	VARIABLE	VALUE	DESCRIPTION
1	TITLE	-	80-column title
2	ITER	-	Maximum number of smoothing iteratives
2	IPLOT	0	No plots desired
		1	Plot smoothed and unsmoothed $\bar{y}$ and smoothed $\bar{y}'$ and $\bar{y}''$ versus θ
		2	Plot smoothed and unsmoothed $\bar{y}$ versus $\bar{x}$
		3	Plot smoothed curvature versus θ
		4	Plot camber and thickness distribution versus $\bar{x}$ (ICAMTK must equal 1)
		5	Plot combined options 1 and 2
		6	Plot combined options 1 and 3
		7	Plot combined options 1, 2, and 3
		8	Plot combined options 1 and 4
		9	Plot combined options 1, 2, and 4
		10	Plot combined options 1, 2, 3, and 4
2	IPUNCH	0	No punched output desired
		1	Punch smoothed x, y, and w
		2	Punch smoothed θ, $\bar{y}$, and w
		3	Punch smoothed θ, $\bar{y}'$, and w (YLTE, YNOSE, YUTE also punched)
		4	Punch smoothed θ, $\bar{y}''$, and w (YLTE, YNOSE, YUTE also punched)
		5	Punch x_c, y_c, t/c/2, and ϕ of camber and thickness distribution (ICAMTK must equal 1)
		6	Punch interpolated x and y coordinates (INTR must equal 1 or 2)
2	IOP	0	Upper and lower surface x, y, and w input

CARD	VARIABLE	VALUE	DESCRIPTION
		1	Upper and lower surface θ, $\bar{y}$, and w input
		2	Upper and lower surface θ, $\bar{y}'$, and w input
		3	Upper and lower surface θ, $\bar{y}''$, and w input
2	ICAMTK	0	Do not compute camber and thickness distribution
		1	Compute camber and thickness distribution
2	IBAD	0	Do not check for bad input coordinates
		1	Check for bad input coordinates
2	ITRN	0	Do not translate and rotate input coordinates
		1	Translate and rotate input coordinates so that x-axis corresponds to longest chordline
2	INTR	0	No coordinate interpolation desired
		1	Interpolate smoothed $\bar{y}$ coordinates for standard set of 57 $\bar{x}$ coordinates defined in subroutine INTP
		2	Interpolate smoothed $\bar{y}$ coordinates at input $\bar{x}$ coordinates (must specify NINT, XINT, and CNEW quantities)
3	NU	-	Number of input upper surface points
4	XU, YU, WU	0	Upper surface x, y, and w
		1	Upper surface θ, $\bar{y}$, and w
		2	Upper surface θ, $\bar{y}'$, and w
		3	Upper surface θ, $\bar{y}''$, and w (card 4 must be input NU times and x or θ runs from nose to trailing edge)
5	NL	-	Number of input lower surface points
6	XL, YL, WL	0	Lower surface x, y, and w
		1	Lower surface θ, $\bar{y}$, and w
		2	Lower surface θ, $\bar{y}'$, and w
		3	Lower surface θ, $\bar{y}''$, and w (card 6 must be input NL times and x or θ runs from nose to trailing edge)

CARD	VARIABLE	VALUE	DESCRIPTION
7	YLTE, YNOSE, YUTE	-	Lower surface trailing edge, nose, and upper surface trailing-edge $\bar{y}$ coordinates (Skip this card if IOP=0 or 1)
8	NINT	-	Number of desired interpolation $\bar{x}$ coordinates (Skip this card if INTR = 0 or 1)
9	XINT	-	Interpolation $\bar{x}$ coordinates (must be input NINT times with 8 values per card, but skip if INTR = 0 or 1)
10	CNEW	-	Desired chord length of interpolated $\bar{x}$ and $\bar{y}$ coordinates. (must be greater than zero, but skip if INTR = 0 or 1)

The primary restrictions on the input data are that the input value of the variables ITER not exceed 300 and the values of NU, NL and NINT not exceed 100. If the user desires to input a weighting value of 1.0 for any input point, the WU and WL columns may be left blank. The variables WU and WL are checked in subroutine INPUT to determine if the weighting value is less than 1.0 and, if so, a value of 1.0 is substituted. The coordinates and derivatives for the upper and lower surfaces must be input from the nose to the trailing edge for each surface and must be in monotonically increasing order.

APPENDIX D

DESCRIPTION OF OUTPUT FOR AIRFOIL SMOOTHING PROGRAM AFSMO

This appendix contains a description of the output for the airfoil smoothing program AFSMO. Presented in table II is the sample 12-page output for the smoothing program utilizing the plot, punch, camber and thickness, bad-point search, translation and rotation, and interpolation options.

A summary of the input data is printed on page 1 and all of the quantities printed are described in Appendix C. If the IBAD option is exercised and bad coordinates are found, the bad points and the corresponding replacement values will be printed on page 2. The allowable deviation (TOLR) and the surface identifier are printed at the top of page 2. If the ITRN option is exercised, pages 3 and 4 will be printed. Page 3 contains a listing of the input prior to translation and rotation and page 4 contains a listing after translation and rotation. On each page the upper surface coordinates are listed on the left and lower surface listed on the right. The coordinates of the leading edge of the longest chord (XNOSE and YNOSE) in the input axis-system and the angle (ANGLE) between the longest chord and the input x-axis are printed at the bottom of page 4. A summary of the input nondimensionalized $\bar{x}$ and $\bar{y}$ coordinates (X/C and Y/C), θ-transformation values (THETA), and weighting factors (W) are printed on page 5. All data are printed in the reordered format from the lower surface trailing-edge point clockwise around the airfoil to the upper surface trailing-edge point. If the IOP parameter equals 2, the input first derivative $\bar{y}'$ (YPS) will be printed instead of the $\bar{y}$ coordinate and, likewise if the IOP equals 3, the input second derivative $\bar{y}''$ (YPPS) will be printed. The value of the computed chord (CHORD) is printed at the bottom of page 5.

A summary of the results from the iterative smoothing process is printed on page 6. The sum-of-squares differences generated during the iterative least-squares polynomial smoothing process are printed initially. The differences are printed 10 to a line with iteration 1 to 10 on line 1, 11 to 20 on line 2, 21 to 30 on line 3, and so on. Immediately following the printout of the differences, a message is printed that states whether the smoothing process converged either within a specified number of iterations or tolerance, or began to oscillate during the smoothing process. The next message printed is the sum-of-squares difference for the least-squares cubic-spline smoothing process and should always be equal to the number of coordinates (NP) times the square of the allowable deviation (DF). The last line printed on page 6 is the result of the iteration procedure in subroutine YNEW to match the upper and lower surface slopes at the nose. The magnitude listed for DELTA is the incremental value added to all of the smoothed second derivative values.

A summary of the smoothed airfoil properties are printed on page 7. The quantities listed under the THETA, X/C, and Y/C headings are the θ-transformation values and the input $\bar{x}$ and $\bar{y}$ coordinates, respectively. The quantities listed under the YT/C heading are the partially smoothed $\bar{y}$ coordinates generated during the least-squares polynomial smoothing process and under the YSMO/C heading the final smoothed values following the solution of the cubic-spline matrix. The quantity listed under the DELTA heading are the differences between the input and final smoothed $\bar{y}$ coordinates (Y/C - YSMO/C). The quantities listed under the YPS, YPPS, DY/DX, D(DY/DX)/DX and CURVATURE headings are $\bar{y}'$, $\bar{y}''$, dy/dx, d^2y/dx^2, and k, respectively. The value of the leading-edge radius is printed next and is simply the reciprocal of the curvature at the nose. The locations of the upper and lower surface inflection points are printed at the bottom of page 7. A summary of the check of the final smoothed $\bar{y}$ and $\bar{y}''$ values is printed on page 8. The check values are obtained by making a call to the least-squares polynomial smoothing subroutine LSQSMO input with the final smoothed $\bar{y}$ coordinates and a uniform weighting factor of 1.0.

A summary of the desired punched data is printed on page 9. The upper surface quantities are listed first and then the lower surface quantities. The values listed adjacent to the DX heading are the x coordinates if IPUNCH equals 1 and the θ-values if IPUNCH is greater than 1. The values adjacent to the DY heading are y, $\bar{y}$, $\bar{y}'$, or $\bar{y}''$ if IPUNCH equals 1, 2, 3, or 4, respectively.

A summary of the camber and thickness distribution data is printed on page 10. The quantities listed under the XU/C and YU/C headings are the smoothed upper surface $\bar{x}$ and $\bar{y}$ coordinates input during the search for the camberline. The quantities listed under the XL/C and YL/C headings are the corresponding lower surface points located during the search. The quantities listed under the X/C, Y/C, T/C/2, and SLOPE headings are the x_c and y_c coordinates of the camberline, the local half thickness-chord ratio t/c/2, and the local slope of the camberline ϕ, respectively. The quantity listed under the ERROR heading are the absolute values of the difference between the local slopes of the upper and lower surface coordinates with respect to the local camberline-axis system.

The results of the interpolation process are printed on pages 11 and 12 for the upper and lower surfaces, respectively. The x and y coordinate values are listed under the XU and YU or XL and YL headings and are based on a chord equal to the value of the input parameter CNEW. The quantities listed under the DY/DX, D(DY/DX)/DX, and CURVATURE headings are dy/dx, d^2y/dx^2, and k, respectively.

APPENDIX E

DESCRIPTION OF INPUT FOR AIRFOIL SCALING PROGRAM AFSCL

This appendix contains a description of the input requirements for the airfoil scaling program AFSCL. All variables are input with a card format of 8F10.0, except the title card which has a format of 8A10.

CARD	VARIABLE	VALUE	DESCRIPTION
1	TITLE	-	80-column title
2	NT	-	Number of input thickness and camber points
2	IPLOT	0	No plots desired
		1	Plot scaled airfoil and its thickness and camber distributions
2	IPUNCH	0	No punched output desired
		1	Punch $\bar{x}$ and $\bar{y}$ coordinates of scaled airfoil
2	IOP	0	Slopes of camberline ϕ (TH array) are input on card 3
		1	Slopes of camberline to be computed by scaling program
3	XC, YC, TK, TH	-	y_c coordinates of camberline (YC), the half thickness distribution t/c/2 (TK), and slope of camberline ϕ (TH) versus x_c coordinate (XC). (Card 3 is input NT times)
4	LT	-	Number of scaled maximum thickness-chord ratios
5	TKNEW	-	Scaled maximum thickness-chord ratios (Card 5 is input LT times)

The input data restrictions are that the variable NT not exceed 101, the variable LT not exceed 10, and that the coordinates for the camberline and thickness distribution be input in a monotonically increasing order from nose to trailing edge.

APPENDIX F

DESCRIPTION OF OUTPUT FOR AIRFOIL SCALING PROGRAM AFSCL

This appendix contains a description of the output for the airfoil scaling program AFSCL. Presented in table III is a sample 3-page output for the scaling program. A summary of the input data is printed on page 1. A description of the input parameters is presented in Appendix E. The guantities listed adjacent to the X/C, Y/C, and SLOPE headings are the x_c and y_c coordinates and local slopes ϕ (XC, YC, and TH arrays) of the camberline and adjacent to the T/C/2 heading are the half thickness distribution values t/c/2 (TK array). The values listed adjacent to the heading NEW T/C are the desired scaled maximum thickness-chord ratios (TKNEW array). The value of the maximum thickness-chord ratio for the input airfoil and its $\bar{x}$ coordinate are printed on the last line of page 1.

Page 2 and 3 are then output for the input airfoil and each airfoil for a desired scaled maximum thickness-chord ratio. A summary of the upper and lower surface $\bar{x}$ and $\bar{y}$ coordinates of the scaled airfoil is presented on page 2 and the corresponding camber and thickness distributions on page 3. The slopes of the camberline in degrees are also printed on page 3.

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TABLE I. - TRANSFORMATION FUNCTION AND FIRST AND SECOND DERIVATIVES

θ, deg (+/-)	x/c	d(x/c)/dθ	$d^2(x/c)/d\theta^2$
0	0.00000	0.00000	.46278
1	.00007	.00808	.46270
2	.00028	.01615	.46249
3	.00063	.02422	.46214
4	.00113	.03228	.46165
5	.00176	.04033	.46101
6	.00254	.04837	.46024
7	.00345	.05640	.45933
8	.00450	.06441	.45827
9	.00570	.07239	.45708
10	.00703	.08036	.45574
11	.00850	.08830	.45427
12	.01011	.09622	.45266
13	.01186	.10410	.45091
14	.01375	.11196	.44903
15	.01577	.11978	.44701
16	.01793	.12756	.44485
17	.02022	.13530	.44255
18	.02265	.14301	.44013
19	.02521	.15066	.43756
20	.02791	.15828	.43487
21	.03074	.16584	.43204
22	.03370	.17336	.42908
23	.03679	.18082	.42599
24	.04001	.18823	.42277
25	.04336	.19558	.41942
26	.04684	.20287	.41594
27	.05044	.21010	.41234
28	.05417	.21726	.40861
29	.05802	.22436	.40475
30	.06200	.23139	.40078
31	.06610	.23835	.39668
32	.07032	.24523	.39246
33	.07466	.25205	.38812
34	.07912	.25878	.38366
35	.08369	.26544	.37908
36	.08838	.27201	.37439
37	.09319	.27851	.36959
38	.09810	.28491	.36467
39	.10313	.29123	.35964
40	.10827	.29747	.35451
41	.11351	.30361	.34926
42	.11887	.30966	.34391
43	.12432	.31561	.33845
44	.12988	.32147	.33289
45	.13554	.32723	.32723

TABLE I. - CONTINUED

θ, deg (+/-)	x/c	$d(x/c)/d\theta$	$d^2(x/c)/d\theta^2$
46	.14130	.33289	.32147
47	.14716	.33845	.31561
48	.15312	.34391	.30966
49	.15917	.34926	.30361
50	.16531	.35451	.29747
51	.17154	.35964	.29123
52	.17786	.36467	.28491
53	.18427	.36959	.27851
54	.19076	.37439	.27201
55	.19734	.37908	.26544
56	.20399	.38366	.25878
57	.21073	.38812	.25205
58	.21754	.39246	.24523
59	.22443	.39668	.23835
60	.23139	.40078	.23139
61	.23842	.40475	.22436
62	.24552	.40861	.21726
63	.25268	.41234	.21010
64	.25991	.41594	.20287
65	.26720	.41942	.19558
66	.27455	.42277	.18823
67	.28195	.42599	.18082
68	.28942	.42908	.17336
69	.29693	.43204	.16584
70	.30450	.43487	.15828
71	.31211	.43756	.15066
72	.31977	.44013	.14301
73	.32747	.44255	.13530
74	.33522	.44485	.12756
75	.34300	.44701	.11978
76	.35082	.44903	.11196
77	.35867	.45091	.10410
78	.36656	.45266	.09622
79	.37447	.45427	.08830
80	.38242	.45574	.08036
81	.39038	.45708	.07239
82	.39837	.45827	.06441
83	.40638	.45933	.05640
84	.41440	.46024	.04837
85	.42244	.46101	.04033
86	.43049	.46165	.03228
87	.43856	.46214	.02422
88	.44662	.46249	.01615
89	.45470	.46270	.00808
90	.46278	.46278	.00000

TABLE I. - CONTINUED

θ, deg (+/-)	x/c	d(x/c)/dθ	$d^2(x/c)/d\theta^2$
91	.47085	.46270	-.00807
92	.47893	.46249	-.01614
93	.48700	.46214	-.02418
94	.49506	.46165	-.03218
95	.50311	.46102	-.04013
96	.51115	.46025	-.04802
97	.51917	.45934	-.05584
98	.52718	.45830	-.06358
99	.53517	.45712	-.07122
100	.54314	.45582	-.07876
101	.55108	.45438	-.08618
102	.55900	.45281	-.09347
103	.56689	.45111	-.10063
104	.57474	.44930	-.10765
105	.58257	.44736	-.11451
106	.59036	.44530	-.12122
107	.59811	.44313	-.12775
108	.60583	.44084	-.13411
109	.61350	.43845	-.14029
110	.62113	.43595	-.14628
111	.62872	.43334	-.15208
112	.63626	.43064	-.15768
113	.64375	.42784	-.16308
114	.65119	.42495	-.16827
115	.65858	.42197	-.17326
116	.66592	.41890	-.17803
117	.67320	.41575	-.18260
118	.68043	.41253	-.18695
119	.68760	.40923	-.19108
120	.69472	.40586	-.19500
121	.70177	.40242	-.19871
122	.70876	.39892	-.20220
123	.71569	.39537	-.20548
124	.72256	.39175	-.20855
125	.72937	.38809	-.21140
126	.73611	.38437	-.21406
127	.74279	.38062	-.21650
128	.74940	.37682	-.21874
129	.75594	.37298	-.22079
130	.76241	.36911	-.22264
131	.76882	.36521	-.22430
132	.77516	.36128	-.22577
133	.78143	.35733	-.22706
134	.78764	.35336	-.22818
135	.79377	.34937	-.22911

TABLE I. - CONCLUDED

θ, deg (+/-)	x/c	d(x/c)/dθ	$d^2(x/c)/dθ^2$
136	.79983	.34536	-.22988
137	.80582	.34134	-.23049
138	.81175	.33732	-.23093
139	.81760	.33328	-.23123
140	.82338	.32925	-.23137
141	.82909	.32521	-.23137
142	.83473	.32117	-.23123
143	.84030	.31714	-.23096
144	.84580	.31311	-.23056
145	.85123	.30909	-.23004
146	.85659	.30508	-.22940
147	.86188	.30108	-.22865
148	.86710	.29710	-.22779
149	.87225	.29313	-.22683
150	.87733	.28918	-.22577
151	.88235	.28525	-.22462
152	.88729	.28134	-.22338
153	.89217	.27746	-.22206
154	.89698	.27359	-.22066
155	.90172	.26975	-.21919
156	.90639	.26594	-.21764
157	.91100	.26216	-.21604
158	.91554	.25840	-.21437
159	.92002	.25467	-.21264
160	.92443	.25098	-.21086
161	.92878	.24731	-.20904
162	.93307	.24368	-.20716
163	.93729	.24008	-.20525
164	.94145	.23652	-.20329
165	.94554	.23299	-.20131
166	.94958	.22949	-.19928
167	.95356	.22603	-.19724
168	.95747	.22261	-.19516
169	.96133	.21922	-.19306
170	.96512	.21587	-.19094
171	.96886	.21255	-.18881
172	.97254	.20928	-.18666
173	.97617	.20604	-.18449
174	.97973	.20284	-.18231
175	.98325	.19967	-.18013
176	.98670	.19655	-.17794
177	.99011	.19346	-.17575
178	.99346	.19041	-.17355
179	.99676	.18740	-.17135
180	1.00000	.18443	-.16915

TABLE II.- SAMPLE OUTPUT FOR AIRFOIL SMOOTHING PROGRAM

PAGE 1 OUTPUT

```
                                                  --INPUT DATA--

   TITLE--  //     GA(W)-1 AIRFOIL WITH BAD COORDINATE POINTS                                  //

   ITER = 100   IPLOT = 10   IPUNCH =  1   IOP =  0   ICAMTK =  1   IBAD =  1   ITRN =  1   INTR =  2

   NU =  40

   XU=   0.           .200000E-02   .500000E-02   .125000E-01   .250000E-01   .375000E-01   .500000E-01   .750000E-01
          .100000E+00  .125000E+00   .150000E+00   .175000E+00   .200000E+00   .250000E+00   .300000E+00   .350000E+00
          .400000E+00  .450000E+00   .500000E+00   .550000E+00   .575000E+00   .600000E+00   .625000E+00   .650000E+00
          .675000E+00  .700000E+00   .725000E+00   .750000E+00   .775000E+00   .800000E+00   .825000E+00   .850000E+00
          .875000E+00  .900000E+00   .925000E+00   .950000E+00   .975000E+00   .990000E+00   .995000E+00   .100000E+01

   YU=   0.           .130000E-01   .204000E-01   .307000E-01   .617000E-01   .496500E-01   .558900E-01   .655100E-01
          .730000E-01  .790000E-01   .840000E-01   .884000E+00   .920000E-01   .977000E-01   .101600E+00   .104000E+00
          .104910E+00  .104450E+00   .102580E+00   .991000E-01   .966800E-01   .937100E-01   .900600E-01   .859900E-01
          .813600E-01  .763400E-01   .709200E-01   .651300E-01   .590700E-01   .528600E-01   .464600E-01   .398800E-01
          .331500E-01  .263900E-01   .196100E-01   .128700E-01   .609000E-02   .200000E-02   .700000E-03  -.700000E-03

   WU=    .100000E+01  .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01
          .100000E+01  .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01
          .100000E+01  .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01
          .100000E+01  .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01
          .100000E+01  .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01

   NL =  40

   XL=   0.           .200000E-02   .500000E-02   .125000E-01   .250000E-01   .375000E-01   .500000E-01   .750000E-01
          .100000E+00  .125000E+00   .150000E+00   .175000E+00   .200000E+00   .250000E+00   .300000E+00   .350000E+00
          .400000E+00  .450000E+00   .500000E+00   .550000E+00   .575000E+00   .600000E+00   .625000E+00   .650000E+00
          .675000E+00  .700000E+00   .725000E+00   .750000E+00   .775000E+00   .800000E+00   .825000E+00   .850000E+00
          .875000E+00  .900000E+00   .925000E+00   .950000E+00   .975000E+00   .990000E+00   .995000E+00   .100000E+01

   YL=   0.          -.930000E-02  -.138000E-01  -.205000E-01  -.269000E-01  -.319000E-01  -.358000E-01  -.421000E-01
         -.470000E-01 -.510000E-01  -.543000E-01  -.570000E-01  -.593000E-01  -.627000E-01  -.645000E-01  -.652000E-01
         -.649000E-01 -.635000E-01  -.610000E-01  -.570000E-01  -.540000E-01  -.508000E-01  -.469000E-01  -.428000E-01
         -.384000E-01 -.340000E-01  -.294000E-01  -.249000E-01  -.204000E-01  -.160000E-01  -.120000E-01  -.860000E-02
         -.580000E-02 -.360000E-02  -.250000E-02  -.260000E-02  -.400000E-02  -.570000E-02  -.670000E-02  -.800000E-02

   WL=    .100000E+01  .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01
          .100000E+01  .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01
          .100000E+01  .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01
          .100000E+01  .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01
          .100000E+01  .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01

 NINT =  17

 XINT=   0.           .100000E-02   .200000E-02   .500000E-02   .100000E-01   .500000E-01   .800000E-01   .100000E+00
          .200000E+00  .300000E+00   .400000E+00   .500000E+00   .600000E+00   .700000E+00   .800000E+00   .900000E+00
          .100000E+01

 CNEW =    10.000
```

PAGE 2 OUTPUT

```
WARNING -- BAD POINTS HAVE BEEN FOUND ON THE UPPER SURFACE BASED ON AN EDIT TOLERANCE OF   .010000

BAD POINT AT I=  12     X =    .175000     Y =    .884000     REPLACED WITH Y =    .088310

BAD POINT AT I=   5     X =    .025000     Y =    .061700     REPLACED WITH Y =    .041720
```

TABLE II. - CONTINUED

PAGE 3 OUTPUT

```
                         --INPUT COORDINATES--

TITLE--  //    GA(W)-1 AIRFOIL WITH BAD COORDINATE POINTS

     I          XU           YU           XL           YL
     1      0.000000     0.000000     0.000000     0.000000
     2       .002000      .013000      .002000     -.009300
     3       .005000      .020400      .005000     -.013800
     4       .012500      .030700      .012500     -.020500
     5       .025000      .041720      .025000     -.026900
     6       .037500      .049650      .037500     -.031900
     7       .050000      .055890      .050000     -.035800
     8       .075000      .065510      .075000     -.042100
     9       .100000      .073000      .100000     -.047000
    10       .125000      .079000      .125000     -.051000
    11       .150000      .084000      .150000     -.054300
    12       .175000      .088310      .175000     -.057000
    13       .200000      .092000      .200000     -.059300
    14       .250000      .097700      .250000     -.062700
    15       .300000      .101600      .300000     -.064500
    16       .350000      .104000      .350000     -.065200
    17       .400000      .104910      .400000     -.064900
    18       .450000      .104450      .450000     -.063500
    19       .500000      .102580      .500000     -.061000
    20       .550000      .099100      .550000     -.057000
    21       .575000      .096680      .575000     -.054000
    22       .600000      .093710      .600000     -.050800
    23       .625000      .090060      .625000     -.046900
    24       .650000      .085990      .650000     -.042800
    25       .675000      .081360      .675000     -.038400
    26       .700000      .076340      .700000     -.034000
    27       .725000      .070920      .725000     -.029400
    28       .750000      .065130      .750000     -.024900
    29       .775000      .059070      .775000     -.020400
    30       .800000      .052860      .800000     -.016000
    31       .825000      .046460      .825000     -.012000
    32       .850000      .039880      .850000     -.008600
    33       .875000      .033150      .875000     -.005800
    34       .900000      .026390      .900000     -.003600
    35       .925000      .019610      .925000     -.002500
    36       .950000      .012870      .950000     -.002600
    37       .975000      .006090      .975000     -.004000
    38       .990000      .002000      .990000     -.005700
    39       .995000      .000700      .995000     -.006700
    40      1.000000     -.000700     1.000000     -.008000
```

PAGE 4 OUTPUT

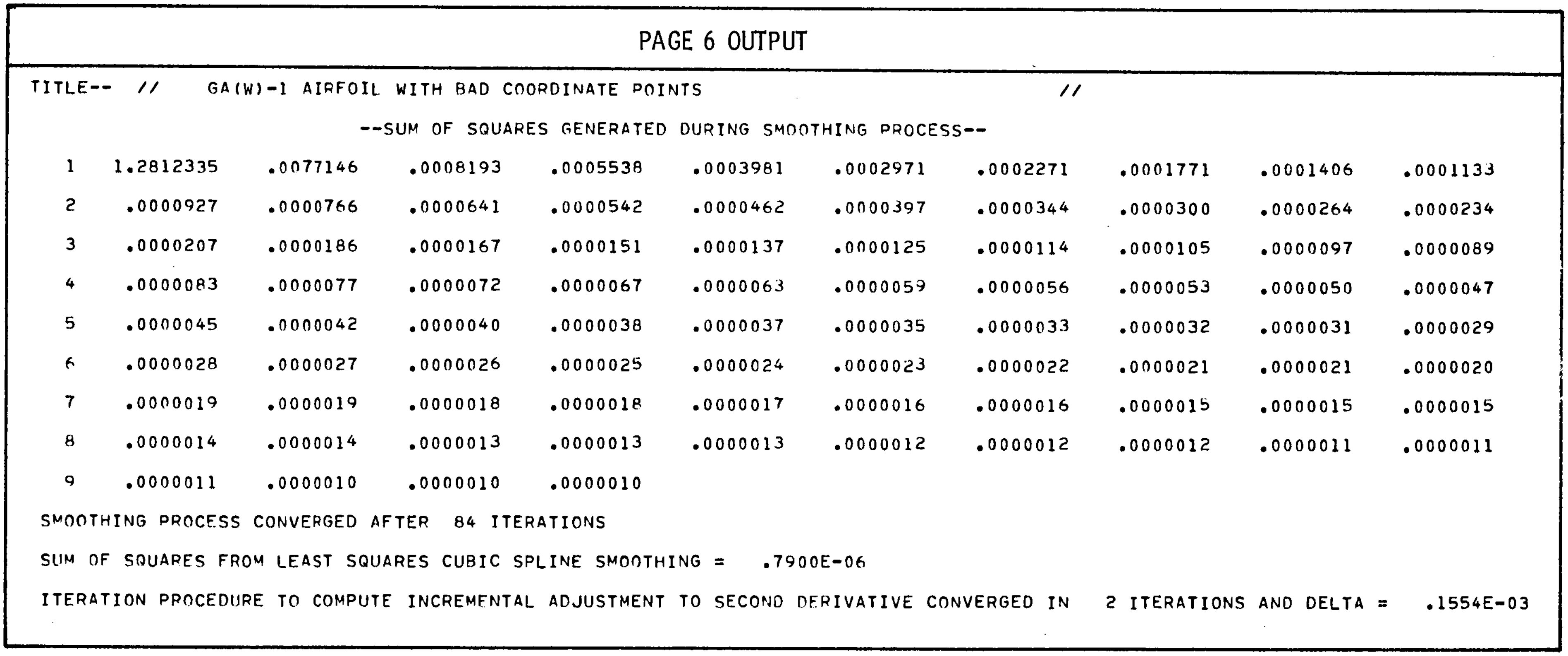
```
                 --TRANSLATED AND ROTATED COORDINATES--

TITLE--  //    GA(W)-1 AIRFOIL WITH BAD COORDINATE POINTS

     I          XU           YU           XL           YL
     1      0.000000     0.000000     0.000000     0.000000
     2       .001943      .013009      .002040     -.009291
     3       .004911      .020422      .005060     -.013778
     4       .012366      .030754      .012589     -.020445
     5       .024818      .041828      .025117     -.026791
     6       .037284      .049813      .037638     -.031737
     7       .049756      .056107      .050155     -.035582
     8       .074714      .065836      .075182     -.041773
     9       .099682      .073434      .100204     -.046565
    10       .124655      .079543      .125221     -.050456
    11       .149633      .084652      .150235     -.053647
    12       .174614      .089070      .175246     -.056238
    13       .199598      .092869      .200256     -.058429
    14       .249573      .098787      .250270     -.061612
    15       .299555      .102904      .300278     -.063194
    16       .349544      .105522      .350280     -.063677
    17       .399540      .106649      .400279     -.063159
    18       .449541      .106406      .450272     -.061542
    19       .499549      .104754      .500261     -.058824
    20       .549564      .101492      .550243     -.054607
    21       .574574      .099180      .575229     -.051498
    22       .599587      .096319      .600215     -.048190
    23       .624602      .092778      .625198     -.044181
    24       .649620      .088817      .650180     -.039972
    25       .674640      .084295      .675161     -.035463
    26       .699661      .079384      .700141     -.030955
    27       .724685      .074073      .725121     -.026246
    28       .749710      .068392      .750101     -.021637
    29       .774736      .062441      .775081     -.017029
    30       .799762      .056339      .800062     -.012520
    31       .824790      .050048      .825044     -.008411
    32       .849818      .043577      .850029     -.004902
    33       .874848      .036956      .875017     -.001994
    34       .899877      .030305      .900007      .000315
    35       .924906      .023634      .925002      .001524
    36       .949935      .017002      .950002      .001532
    37       .974964      .010331      .975008      .000241
    38       .989982      .006306      .990015     -.001393
    39       .994988      .005028      .995020     -.002372
    40       .999994      .003650     1.000025     -.003650

XNOSE =        0.000000     YNOSE =       0.000000     ANGLE =    -.249
```

TABLE II. - CONTINUED

PAGE 5 OUTPUT

```
                              --SUMMARY OF INPUT DATA--

 TITLE--  //    GA(W)-1 AIRFOIL WITH BAD COORDINATE POINTS                                   //

     I          X/C           Y/C          THETA          YPS            YPPS              W
     1       1.000000      -.003650      -180.00                                          1.00
     2        .994995      -.002372      -178.46                                          1.00
     3        .989990      -.001393      -176.97                                          1.00
     4        .974983       .000241      -172.67                                          1.00
     5        .949978       .001532      -166.10                                          1.00
     6        .924979       .001524      -160.12                                          1.00
     7        .899984       .000315      -154.63                                          1.00
     8        .874995      -.001994      -149.54                                          1.00
     9        .850008      -.004902      -144.77                                          1.00
    10        .825023      -.008411      -140.29                                          1.00
    11        .800042      -.012520      -136.03                                          1.00
    12        .775062      -.017028      -131.98                                          1.00
    13        .750082      -.021637      -128.10                                          1.00
    14        .725103      -.026245      -124.37                                          1.00
    15        .700124      -.030954      -120.77                                          1.00
    16        .675144      -.035463      -117.27                                          1.00
    17        .650164      -.039971      -113.86                                          1.00
    18        .625182      -.044180      -110.53                                          1.00
    19        .600200      -.048188      -107.27                                          1.00
    20        .575215      -.051497      -104.06                                          1.00
    21        .550229      -.054606      -100.89                                          1.00
    22        .500248      -.058823       -94.64                                          1.00
    23        .450261      -.061540       -88.45                                          1.00
    24        .400268      -.063158       -82.24                                          1.00
    25        .350271      -.063675       -75.93                                          1.00
    26        .300270      -.063193       -69.44                                          1.00
    27        .250264      -.061610       -62.66                                          1.00
    28        .200251      -.058428       -55.44                                          1.00
    29        .175242      -.056237       -51.59                                          1.00
    30        .150231      -.053646       -47.52                                          1.00
    31        .125217      -.050454       -43.16                                          1.00
    32        .100201      -.046563       -38.42                                          1.00
    33        .075181      -.041772       -33.12                                          1.00
    34        .050154      -.035581       -26.92                                          1.00
    35        .037637      -.031736       -23.27                                          1.00
    36        .025116      -.026790       -18.96                                          1.00
    37        .012589      -.020445       -13.39                                          1.00
    38        .005060      -.013778        -8.48                                          1.00
    39        .002040      -.009291        -5.38                                          1.00
    40       0.000000      0.000000         0.00                                          1.00
    41        .001943       .013008         5.25                                          1.00
    42        .004911       .020421         8.35                                          1.00
    43        .012366       .030753        13.28                                          1.00
    44        .024818       .041827        18.85                                          1.00
    45        .037283       .049811        23.16                                          1.00
    46        .049755       .056106        26.81                                          1.00
    47        .074712       .065834        33.01                                          1.00
    48        .099679       .073432        38.32                                          1.00
    49        .124652       .079541        43.06                                          1.00
    50        .149629       .084650        47.42                                          1.00
    51        .174610       .089068        51.49                                          1.00
    52        .199593       .092867        55.34                                          1.00
    53        .249566       .098784        62.57                                          1.00
    54        .299548       .102901        69.35                                          1.00
    55        .349535       .105519        75.84                                          1.00
    56        .399530       .106646        82.15                                          1.00
    57        .449530       .106404        88.36                                          1.00
    58        .499536       .104751        94.56                                          1.00
    59        .549550       .101489       100.81                                          1.00
    60        .574559       .099178       103.98                                          1.00
    61        .599571       .096317       107.19                                          1.00
    62        .624587       .092776       110.45                                          1.00
    63        .649603       .088814       113.79                                          1.00
    64        .674623       .084293       117.20                                          1.00
    65        .699644       .079382       120.70                                          1.00
    66        .724666       .074071       124.31                                          1.00
    67        .749691       .068390       128.04                                          1.00
    68        .774716       .062439       131.93                                          1.00
    69        .799742       .056338       135.99                                          1.00
    70        .824769       .050047       140.24                                          1.00
    71        .849797       .043576       144.73                                          1.00
    72        .874825       .036955       149.50                                          1.00
    73        .899854       .030304       154.61                                          1.00
    74        .924883       .023633       160.10                                          1.00
    75        .949911       .017002       166.08                                          1.00
    76        .974940       .010331       172.66                                          1.00
    77        .989957       .006306       176.96                                          1.00
    78        .994962       .005028       178.45                                          1.00
    79        .999968       .003650       179.99                                          1.00

 CHORD =        1.000025
```

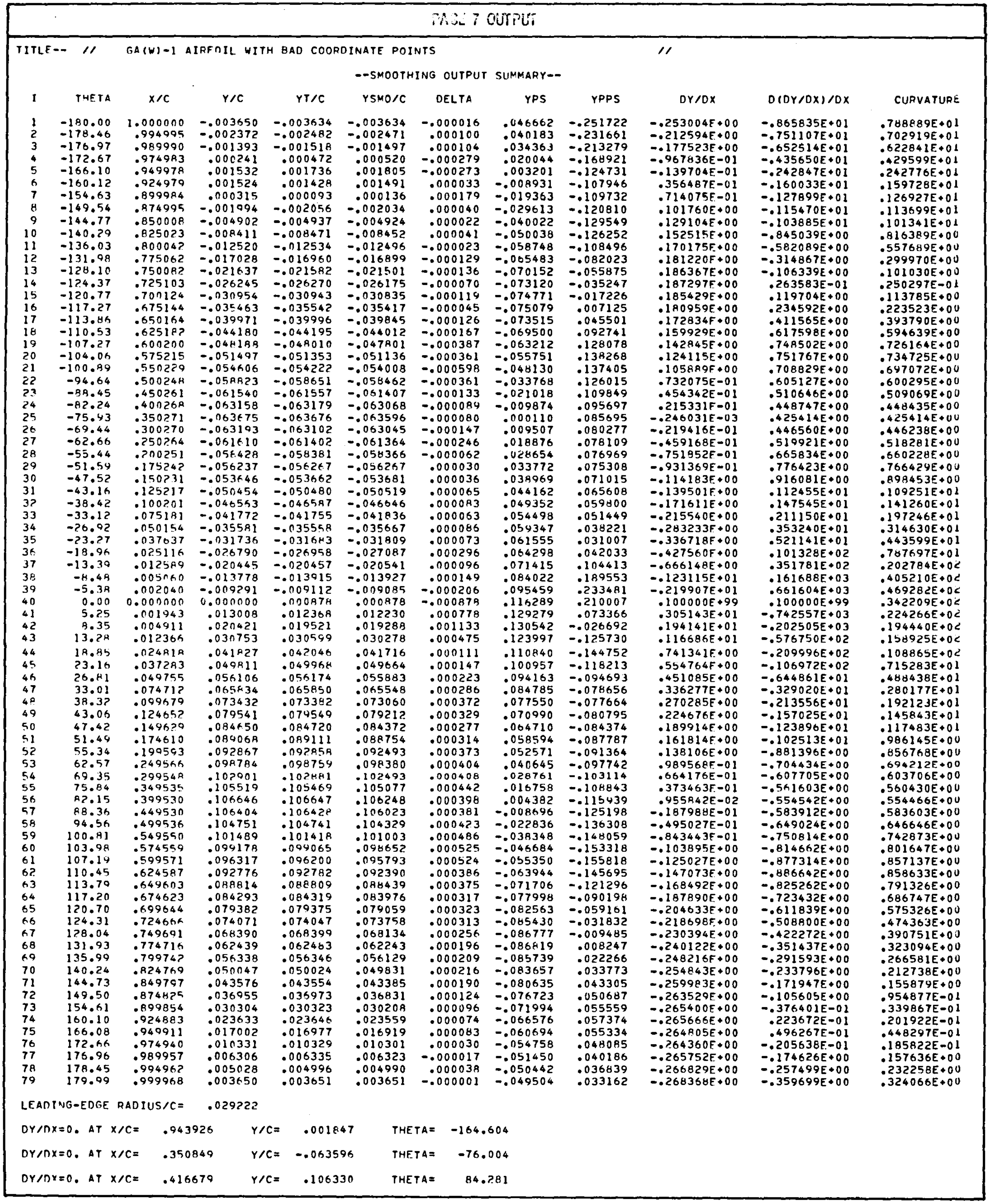
TABLE II. - CONTINUED



PAGE 6 OUTPUT

```
TITLE--  //    GA(W)-1 AIRFOIL WITH BAD COORDINATE POINTS                                   //

                              --SUM OF SQUARES GENERATED DURING SMOOTHING PROCESS--

   1  1.2812335   .0077146   .0008193   .0005538   .0003981   .0002971   .0002271   .0001771   .0001406   .0001133
   2   .0000927   .0000766   .0000641   .0000542   .0000462   .0000397   .0000344   .0000300   .0000264   .0000234
   3   .0000207   .0000186   .0000167   .0000151   .0000137   .0000125   .0000114   .0000105   .0000097   .0000089
   4   .0000083   .0000077   .0000072   .0000067   .0000063   .0000059   .0000056   .0000053   .0000050   .0000047
   5   .0000045   .0000042   .0000040   .0000038   .0000037   .0000035   .0000033   .0000032   .0000031   .0000029
   6   .0000028   .0000027   .0000026   .0000025   .0000024   .0000023   .0000022   .0000021   .0000021   .0000020
   7   .0000019   .0000019   .0000018   .0000018   .0000017   .0000016   .0000016   .0000015   .0000015   .0000015
   8   .0000014   .0000014   .0000013   .0000013   .0000013   .0000012   .0000012   .0000012   .0000011   .0000011
   9   .0000011   .0000010   .0000010   .0000010

 SMOOTHING PROCESS CONVERGED AFTER  84 ITERATIONS

 SUM OF SQUARES FROM LEAST SQUARES CUBIC SPLINE SMOOTHING =   .7900E-06

 ITERATION PROCEDURE TO COMPUTE INCREMENTAL ADJUSTMENT TO SECOND DERIVATIVE CONVERGED IN   2 ITERATIONS AND DELTA =   .1554E-03
```

TABLE II .- CONTINUED



PAGE 7 OUTPUT

```
TITLE--  //    GA(W)-1 AIRFOIL WITH BAD COORDINATE POINTS                                  //

                                              --SMOOTHING OUTPUT SUMMARY--

  I     THETA      X/C       Y/C       YT/C     YSMO/C     DELTA      YPS      YPPS        DY/DX         D(DY/DX)/DX      CURVATURE

  1   -180.00  1.000000  -.003650  -.003634  -.003634  -.000016   .046662  -.251722   -.253004E+00   -.865835E+01   .788889E+01
  2   -178.46   .994995  -.002372  -.002482  -.002471   .000100   .040183  -.231661   -.212594E+00   -.751107E+01   .702919E+01
  3   -176.97   .989990  -.001393  -.001518  -.001497   .000104   .034363  -.213279   -.177523E+00   -.652514E+01   .622841E+01
  4   -172.67   .974983   .000241   .000472   .000520  -.000279   .020044  -.168921   -.967836E-01   -.435650E+01   .429599E+01
  5   -166.10   .949978   .001532   .001736   .001805  -.000273   .003201  -.124731   -.139704E-01   -.242847E+01   .242776E+01
  6   -160.12   .924979   .001524   .001428   .001491   .000033  -.008931  -.107946    .356487E-01   -.160033E+01   .159728E+01
  7   -154.63   .899984   .000315   .000093   .000136   .000179  -.019363  -.109732    .714075E-01   -.127899E+01   .126927E+01
  8   -149.54   .874995  -.001994  -.002056  -.002034   .000040  -.029613  -.120810    .101760E+00   -.115470E+01   .113699E+01
  9   -144.77   .850008  -.004902  -.004937  -.004924   .000022  -.040022  -.129549    .129104E+00   -.103885E+01   .101341E+01
 10   -140.29   .825023  -.008411  -.008471  -.008452   .000041  -.050038  -.126252    .152515E+00   -.845039E+00   .816389E+00
 11   -136.03   .800042  -.012520  -.012534  -.012496  -.000023  -.058748  -.108496    .170175E+00   -.582089E+00   .557689E+00
 12   -131.98   .775062  -.017028  -.016960  -.016899  -.000129  -.065483  -.082023    .181220E+00   -.314867E+00   .299970E+00
 13   -128.10   .750082  -.021637  -.021582  -.021501  -.000136  -.070152  -.055875    .186367E+00   -.106339E+00   .101030E+00
 14   -124.37   .725103  -.026245  -.026270  -.026175  -.000070  -.073120  -.035247    .187297E+00    .263583E-01   .250297E-01
 15   -120.77   .700124  -.030954  -.030943  -.030835  -.000119  -.074771  -.017226    .185429E+00    .119704E+00   .113785E+00
 16   -117.27   .675144  -.035463  -.035542  -.035417  -.000045  -.075079   .007125    .180959E+00    .234592E+00   .223523E+00
 17   -113.86   .650164  -.039971  -.039996  -.039845  -.000126  -.073515   .045501    .172834E+00    .411565E+00   .393790E+00
 18   -110.53   .625182  -.044180  -.044195  -.044012  -.000167  -.069500   .092741    .159929E+00    .617598E+00   .594639E+00
 19   -107.27   .600200  -.048188  -.048010  -.047801  -.000387  -.063212   .128078    .142845E+00    .748502E+00   .726164E+00
 20   -104.06   .575215  -.051497  -.051353  -.051136  -.000361  -.055751   .138268    .124115E+00    .751767E+00   .734725E+00
 21   -100.89   .550229  -.054606  -.054222  -.054008  -.000598  -.048130   .137405    .105889E+00    .708829E+00   .697072E+00
 22    -94.64   .500248  -.058823  -.058651  -.058462  -.000361  -.033768   .126015    .732075E-01    .605127E+00   .600295E+00
 23    -88.45   .450261  -.061540  -.061557  -.061407  -.000133  -.021018   .109849    .454342E-01    .510646E+00   .509069E+00
 24    -82.24   .400268  -.063158  -.063179  -.063068  -.000089  -.009874   .095697    .215331E-01    .448747E+00   .448435E+00
 25    -75.93   .350271  -.063675  -.063676  -.063596  -.000080   .000110   .085695   -.246031E-03    .425414E+00   .425414E+00
 26    -69.44   .300270  -.063193  -.063102  -.063045  -.000147   .009507   .080277   -.219416E-01    .446560E+00   .446238E+00
 27    -62.66   .250264  -.061610  -.061402  -.061364  -.000246   .018876   .078109   -.459168E-01    .519921E+00   .518281E+00
 28    -55.44   .200251  -.056428  -.058381  -.058366  -.000062   .028654   .076969   -.751852E-01    .665834E+00   .660228E+00
 29    -51.59   .175242  -.056237  -.056267  -.056267   .000030   .033772   .075308   -.931369E-01    .776423E+00   .766429E+00
 30    -47.52   .150231  -.053646  -.053662  -.053681   .000036   .038969   .071015   -.114183E+00    .916081E+00   .898453E+00
 31    -43.16   .125217  -.050454  -.050480  -.050519   .000065   .044162   .065608   -.139501E+00    .112455E+01   .109251E+01
 32    -38.42   .100201  -.046563  -.046587  -.046646   .000083   .049352   .059800   -.171611E+00    .147545E+01   .141260E+01
 33    -33.12   .075181  -.041772  -.041755  -.041836   .000063   .054498   .051449   -.215540E+00    .211150E+01   .197246E+01
 34    -26.92   .050154  -.035581  -.035558  -.035667   .000086   .059347   .038221   -.283233E+00    .353240E+01   .314630E+01
 35    -23.27   .037637  -.031736  -.031683  -.031809   .000073   .061555   .031007   -.336718E+00    .521141E+01   .443599E+01
 36    -18.96   .025116  -.026790  -.026958  -.027087   .000296   .064298   .042033   -.427560E+00    .101328E+02   .787697E+01
 37    -13.39   .012589  -.020445  -.020457  -.020541   .000096   .071415   .104413   -.666148E+00    .351781E+02   .202784E+02
 38     -8.48   .005060  -.013778  -.013915  -.013927   .000149   .084022   .189553   -.123115E+01    .161688E+03   .405210E+02
 39     -5.38   .002040  -.009291  -.009112  -.009085  -.000206   .095459   .233481   -.219907E+01    .661604E+03   .469282E+02
 40      0.00  0.000000  0.000000   .000878   .000878  -.000878   .116289   .210007    .100000E+99    .100000E+99   .342209E+02
 41      5.25   .001943   .013008   .012368   .012230   .000778   .129279   .073366    .305143E+01   -.742557E+03   .224266E+02
 42      8.35   .004911   .020421   .019521   .019288   .001133   .130542  -.026692    .194141E+01   -.202505E+03   .194440E+02
 43     13.28   .012366   .030753   .030599   .030278   .000475   .123997  -.125730    .116686E+01   -.576750E+02   .158925E+02
 44     18.85   .024818   .041827   .042046   .041716   .000111   .110840  -.144752    .741341E+00   -.209996E+02   .108865E+02
 45     23.16   .037283   .049811   .049968   .049664   .000147   .100957  -.118213    .554764E+00   -.106972E+02   .715283E+01
 46     26.81   .049755   .056106   .056174   .055883   .000223   .094163  -.094693    .451085E+00   -.644861E+01   .488438E+01
 47     33.01   .074712   .065834   .065850   .065548   .000286   .084785  -.078656    .336277E+00   -.329020E+01   .280177E+01
 48     38.32   .099679   .073432   .073382   .073060   .000372   .077550  -.077664    .270285E+00   -.213556E+01   .192123E+01
 49     43.06   .124652   .079541   .079549   .079212   .000329   .070990  -.080795    .224676E+00   -.157025E+01   .145843E+01
 50     47.42   .149629   .084650   .084720   .084372   .000277   .064710  -.084374    .189914E+00   -.123896E+01   .117483E+01
 51     51.49   .174610   .089068   .089111   .088754   .000314   .058594  -.087787    .161814E+00   -.102513E+01   .986145E+00
 52     55.34   .199593   .092867   .092858   .092493   .000373   .052571  -.091364    .138106E+00   -.881396E+00   .856768E+00
 53     62.57   .249566   .098784   .098759   .098380   .000404   .040645  -.097742    .989568E-01   -.704434E+00   .694212E+00
 54     69.35   .299548   .102901   .102881   .102493   .000408   .028761  -.103114    .664176E-01   -.607705E+00   .603706E+00
 55     75.84   .349535   .105519   .105469   .105077   .000442   .016758  -.108843    .373463E-01   -.561603E+00   .560430E+00
 56     82.15   .399530   .106646   .106647   .106248   .000398   .004382  -.115939    .955842E-02   -.554542E+00   .554466E+00
 57     88.36   .449530   .106404   .106428   .106023   .000381  -.008696  -.125198   -.187988E-01   -.583912E+00   .583603E+00
 58     94.56   .499536   .104751   .104741   .104329   .000423  -.022836  -.136308   -.495027E-01   -.649024E+00   .646646E+00
 59    100.81   .549550   .101489   .101418   .101003   .000486  -.038348  -.148059   -.843443E-01   -.750814E+00   .742873E+00
 60    103.98   .574559   .099178   .099065   .098652   .000525  -.046684  -.153318   -.103895E+00   -.814662E+00   .801647E+00
 61    107.19   .599571   .096317   .096200   .095793   .000524  -.055350  -.155818   -.125027E+00   -.877314E+00   .857137E+00
 62    110.45   .624587   .092776   .092782   .092390   .000386  -.063944  -.145695   -.147073E+00   -.886642E+00   .858633E+00
 63    113.79   .649603   .088814   .088809   .088439   .000375  -.071706  -.121296   -.168492E+00   -.825262E+00   .791326E+00
 64    117.20   .674623   .084293   .084319   .083976   .000317  -.077998  -.090198   -.187890E+00   -.723432E+00   .686747E+00
 65    120.70   .699644   .079382   .079375   .079059   .000323  -.082563  -.059161   -.204633E+00   -.611839E+00   .575326E+00
 66    124.31   .724666   .074071   .074047   .073758   .000313  -.085430  -.031832   -.218698E+00   -.508800E+00   .474363E+00
 67    128.04   .749691   .068390   .068399   .068134   .000256  -.086777  -.009485   -.230394E+00   -.422272E+00   .390751E+00
 68    131.93   .774716   .062439   .062463   .062243   .000196  -.086819   .008247   -.240122E+00   -.351437E+00   .323094E+00
 69    135.99   .799742   .056338   .056346   .056129   .000209  -.085739   .022266   -.248216E+00   -.291593E+00   .266581E+00
 70    140.24   .824769   .050047   .050024   .049831   .000216  -.083657   .033773   -.254843E+00   -.233796E+00   .212738E+00
 71    144.73   .849797   .043576   .043554   .043385   .000190  -.080635   .043305   -.259983E+00   -.171947E+00   .155879E+00
 72    149.50   .874825   .036955   .036973   .036831   .000124  -.076723   .050687   -.263529E+00   -.105605E+00   .954877E-01
 73    154.61   .899854   .030304   .030323   .030208   .000096  -.071994   .055559   -.265400E+00   -.376401E-01   .339867E-01
 74    160.10   .924883   .023633   .023646   .023559   .000074  -.066576   .057374   -.265666E+00    .223672E-01   .201922E-01
 75    166.08   .949911   .017002   .016977   .016919   .000083  -.060694   .055334   -.264805E+00    .496267E-01   .448297E-01
 76    172.66   .974940   .010331   .010329   .010301   .000030  -.054758   .048085   -.264360E+00   -.205638E-01   .185822E-01
 77    176.96   .989957   .006306   .006335   .006323  -.000017  -.051450   .040186   -.265752E+00   -.174626E+00   .157636E+00
 78    178.45   .994962   .005028   .004996   .004990   .000038  -.050442   .036839   -.266829E+00   -.257499E+00   .232258E+00
 79    179.99   .999968   .003650   .003651   .003651  -.000001  -.049504   .033162   -.268368E+00   -.359699E+00   .324066E+00

 LEADING-EDGE RADIUS/C=    .029222

 DY/DX=0. AT X/C=    .943926      Y/C=    .001847      THETA=  -164.604

 DY/DX=0. AT X/C=    .350849      Y/C=  -.063596      THETA=    -76.004

 DY/DY=0. AT X/C=    .416679      Y/C=    .106330      THETA=     84.281
```

TABLE II. - CONTINUED

PAGE 8 OUTPUT

```
TITLE--  //    GA(W)-1 AIRFOIL WITH BAD COORDINATE POINTS

          CHECK OF SMOOTHED COORDINATES   DF=   .000100

      I      (YSMO/C-CHECK VALUE)        (YPPS-CHECK VALUE)

      1          -.000000                   -.000969
      2           .000000                   -.000343
      3           .000000                    .000447
      4          -.000000                    .001928
      5          -.000001                    .002463
      6          -.000000                    .001729
      7           .000002                    .000338
      8           .000003                   -.001214
      9           .000002                   -.002315
     10           .000001                   -.002145
     11          -.000000                   -.000833
     12          -.000001                    .000664
     13          -.000001                    .001267
     14          -.000000                    .000543
     15           .000001                   -.001177
     16           .000001                   -.002626
     17           .000000                   -.001807
     18          -.000001                    .002097
     19          -.000002                    .004562
     20          -.000004                    .003987
     21          -.000004                    .002889
     22          -.000003                    .001281
     23          -.000000                   -.000125
     24           .000000                   -.000594
     25           .000001                   -.000636
     26           .000000                   -.000322
     27          -.000001                    .000155
     28          -.000002                    .000774
     29          -.000001                    .000666
     30          -.000000                    .000042
     31           .000000                   -.000258
     32           .000000                    .000161
     33          -.000001                    .000756
     34           .000001                   -.000461
     35           .000002                   -.002933
     36           .000003                   -.005923
     37           .000001                   -.004470
     38          -.000006                    .004796
     39          -.000005                    .011671
     40          -.000005                    .014370
     41           .000002                    .000565
     42           .000001                   -.007354
     43           .000005                   -.011198
     44          -.000001                   -.003994
     45          -.000001                    .001782
     46          -.000002                    .003287
     47          -.000001                    .001379
     48           .000001                   -.000300
     49           .000001                   -.000540
     50           .000000                   -.000116
     51          -.000000                    .000129
     52          -.000000                    .000010
     53           .000000                   -.000202
     54           .000000                    .000024
     55          -.000000                    .000210
     56          -.000000                    .000311
     57           .000000                    .000179
     58           .000001                   -.000126
     59           .000001                   -.000797
     60           .000002                   -.001428
     61           .000001                   -.002166
     62           .000001                   -.002127
     63           .000000                   -.000790
     64          -.000000                    .000217
     65          -.000000                    .000592
     66          -.000000                    .000667
     67          -.000000                    .000620
     68          -.000000                    .000463
     69          -.000000                    .000286
     70          -.000000                    .000195
     71          -.000000                    .000218
     72          -.000000                    .000259
     73           .000000                    .000272
     74           .000000                    .000311
     75           .000000                    .000382
     76           .000000                    .000389
     77          -.000000                    .000238
     78          -.000000                    .000144
     79           .000000                    .000094

  SUM OF SQUARES=  .000000                   .000750
```

TABLE II.- CONTINUED

PAGE 9 OUTPUT

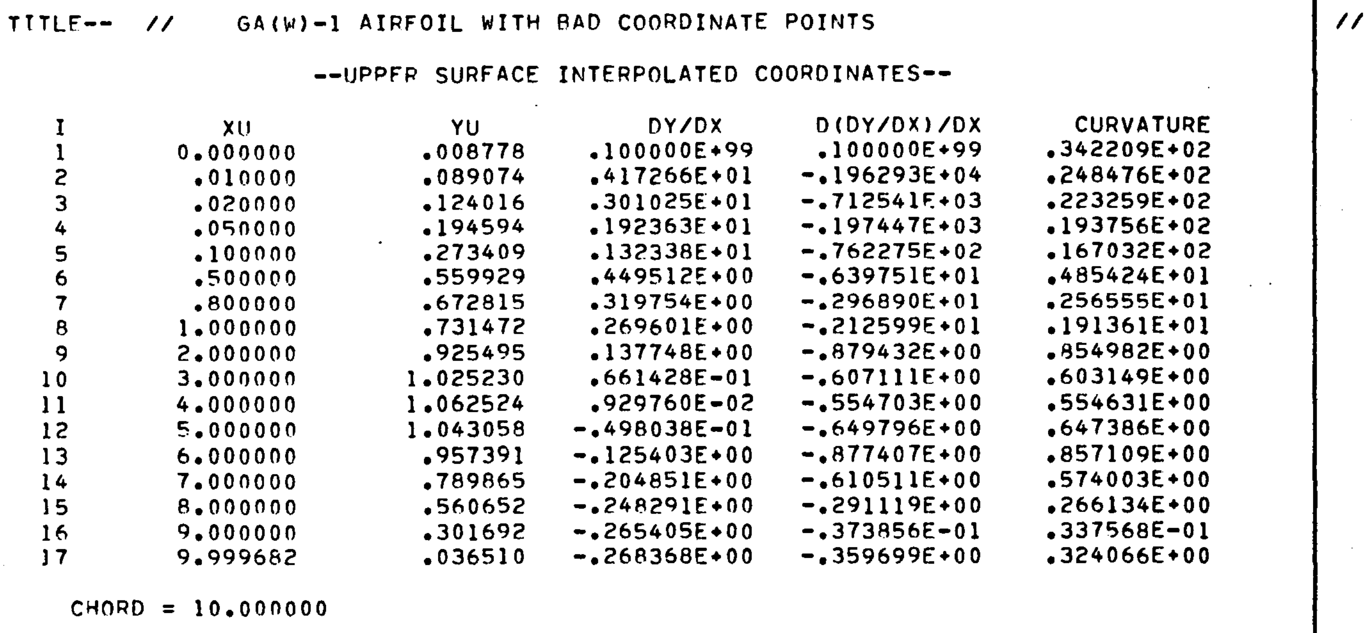
```
          THE FOLLOWING DATA HAVE BEEN PUNCHED      IPUNCH=   1

//    GA(W)-1 AIRFOIL WITH BAD COORDINATE POINTS                              //

IOP =    0

NU =   40

DX =   0.000000   .001943   .004911   .012366   .024818   .037284   .049756   .074714
        .099682   .124655   .149633   .174614   .199596   .249573   .299555   .349544
        .399540   .449541   .499549   .549564   .574574   .599587   .624602   .649620
        .674640   .699661   .724685   .749710   .774736   .799762   .824790   .849818
        .874848   .899877   .924906   .949935   .974964   .989982   .994988   .999994

DY =    .000878   .012230   .019288   .030279   .041717   .049666   .055884   .065550
        .073062   .079214   .084374   .088757   .092496   .098383   .102496   .105080
        .106251   .106026   .104331   .101005   .098655   .095795   .092392   .088442
        .083978   .079061   .073760   .068136   .062245   .056131   .049832   .043387
        .036832   .030209   .023560   .016920   .010301   .006323   .004991   .003651

NL =   40

DX =   0.000000   .002040   .005060   .012589   .025117   .037638   .050155   .075182
        .100204   .125221   .150235   .175246   .200256   .250270   .300278   .350280
        .400279   .450272   .500261   .550243   .575229   .600215   .625198   .650180
        .675161   .700141   .725121   .750101   .775081   .800062   .825044   .850029
        .875017   .900007   .925002   .950002   .975008   .990015   .995020  1.000025

DY =    .000878  -.009085  -.013927  -.020541  -.027087  -.031809  -.035668  -.041837
       -.046647  -.050521  -.053683  -.056268  -.058367  -.061366  -.063047  -.063597
       -.063070  -.061409  -.058463  -.054009  -.051138  -.047802  -.044014  -.039846
       -.035418  -.030836  -.026176  -.021502  -.016900  -.012497  -.008452  -.004924
       -.002034   .000136   .001491   .001805   .000520  -.001497  -.002472  -.003634
```

PAGE 10 OUTPUT

```
TITLE--  //    GA(W)-1 AIRFOIL WITH BAD COORDINATE POINTS                                 //

                             --THICKNESS AND CAMBER DISTRIBUTION--

  I     XU/C      YU/C      XL/C      YL/C      X/C       Y/C      T/C/2     SLOPE          ERROR

  1   .999968   .003651   .998234  -.003201   .999101   .000225   .003534  -14.2073        .000005
  2   .994962   .004990   .993303  -.002122   .994133   .001434   .003652  -13.1308        .000026
  3   .989957   .006323   .988331  -.001212   .989144   .002556   .003854  -12.1802        .000023
  4   .974940   .010391   .973251   .000681   .974095   .005491   .004883   -9.9589        .000039
  5   .949911   .016919   .947878   .001829   .948894   .009374   .007613   -7.6733        .000028
  6   .924883   .023559   .922434   .001395   .923658   .012477   .011149   -6.3035        .000019
  7   .899854   .030208   .897053  -.000078   .898453   .015065   .015208   -5.2838        .000014
  8   .874825   .036831   .871828  -.002362   .873326   .017234   .019653   -4.3739        .000012
  9   .849797   .043385   .846803  -.005343   .848300   .019021   .024410   -3.5160        .000001
 10   .824769   .049831   .821955  -.008924   .823362   .020454   .029411   -2.7419        .000005
 11   .799742   .056129   .797212  -.012980   .798477   .021574   .034578   -2.0968        .000006
 12   .774716   .062243   .772501  -.017364   .773609   .022439   .039819   -1.5938        .000000
 13   .749691   .068134   .747800  -.021927   .748745   .023104   .045040   -1.2025        .000005
 14   .724666   .073758   .723151  -.026541   .723909   .023609   .050155    -.8656        .000003
 15   .699644   .079059   .698614  -.031115   .699129   .023972   .055090    -.5352        .000004
 16   .674623   .083976   .674209  -.035586   .674416   .024195   .059781    -.1983        .000003
 17   .649603   .088439   .649866  -.039897   .649735   .024271   .064168     .1174        .000003
 18   .624587   .092390   .625454  -.043969   .625020   .024210   .068181     .3644        .000003
 19   .599571   .095793   .600862  -.047706   .600217   .024043   .071753     .5153        .000001
 20   .574559   .098652   .576102  -.051026   .575331   .023813   .074843     .5906        .000001
 21   .549550   .101003   .551259  -.053898   .550405   .023552   .077455     .6323        .000001
 22   .499536   .104329   .501520  -.058368   .500528   .022980   .081355     .6985        .000001
 23   .449530   .106023   .451823  -.061336   .450677   .022344   .083687     .7851        .000000
 24   .399530   .106248   .402236  -.063025   .400883   .021611   .084647     .9159        .000000
 25   .349535   .105077   .352752  -.063595   .351144   .020741   .084351    1.0926        .000000
 26   .299548   .102493   .303336  -.063111   .301442   .019691   .082823    1.3106        .000000
 27   .249566   .098380   .253936  -.061529   .251751   .018426   .079985    1.5654        .000000
 28   .199593   .092493   .204506  -.058680   .202049   .016907   .075626    1.8615        .000000
 29   .174610   .088754   .179774  -.056681   .177192   .016037   .072763    2.0338        .000000
 30   .149629   .084372   .155057  -.054222   .152343   .015075   .069350    2.2425        .000000
 31   .124652   .079212   .130394  -.051227   .127523   .013992   .065282    2.5205        .000000
 32   .099679   .073060   .105827  -.047589   .102753   .012736   .060403    2.9170        .000001
 33   .074712   .065548   .081471  -.043151   .078092   .011198   .054454    3.5578        .000002
 34   .049755   .055883   .057757  -.037724   .053756   .009079   .046974    4.8859        .000002
 35   .037283   .049664   .046504  -.034609   .041894   .007528   .042388    6.2449        .000001
 36   .024818   .041716   .036053  -.031268   .030435   .005224   .036922    8.7511        .000000
 37   .012366   .030278   .026086  -.027496   .019226   .001391   .029691   13.3584        .000003
 38   .004911   .019288   .018942  -.024216   .011927  -.002464   .022855   17.8758        .000003
 39   .001943   .012230   .014553  -.021786   .008248  -.004778   .018139   20.3388        .000002
 40  0.000000   .000878   .007551  -.016593   .003776  -.007858   .009517   23.3746        .000005
 41   .002056  -.009120   .002056  -.009120   .002056  -.009120  0.000000   36.7380        .000000
```

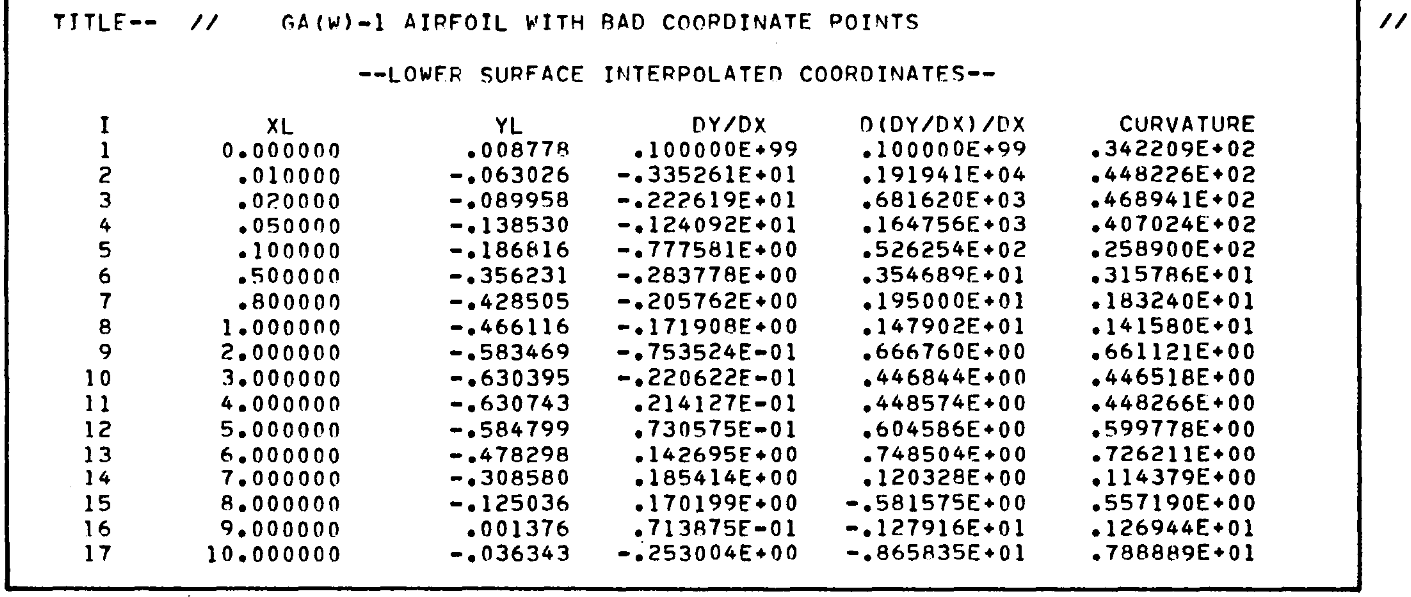
TABLE II. - CONCLUDED

PAGE 11 OUTPUT



```
TITLE--  //    GA(W)-1 AIRFOIL WITH BAD COORDINATE POINTS                                //

                          --UPPER SURFACE INTERPOLATED COORDINATES--

   I         XU             YU          DY/DX         D(DY/DX)/DX       CURVATURE
   1     0.000000        .008778     .100000E+99      .100000E+99      .342209E+02
   2      .010000        .089074     .417266E+01     -.196293E+04      .248476E+02
   3      .020000        .124016     .301025E+01     -.712541E+03      .223259E+02
   4      .050000        .194594     .192363E+01     -.197447E+03      .193756E+02
   5      .100000        .273409     .132338E+01     -.762275E+02      .167032E+02
   6      .500000        .559929     .449512E+00     -.639751E+01      .485424E+01
   7      .800000        .672815     .319754E+00     -.296890E+01      .256555E+01
   8     1.000000        .731472     .269601E+00     -.212599E+01      .191361E+01
   9     2.000000        .925495     .137748E+00     -.879432E+00      .854982E+00
  10     3.000000       1.025230     .661428E-01     -.607111E+00      .603149E+00
  11     4.000000       1.062524     .929760E-02     -.554703E+00      .554631E+00
  12     5.000000       1.043058    -.498038E-01     -.649796E+00      .647386E+00
  13     6.000000        .957391    -.125403E+00     -.877407E+00      .857109E+00
  14     7.000000        .789865    -.204851E+00     -.610511E+00      .574003E+00
  15     8.000000        .560652    -.248291E+00     -.291119E+00      .266134E+00
  16     9.000000        .301692    -.265405E+00     -.373856E-01      .337568E-01
  17     9.999682        .036510    -.268368E+00     -.359699E+00      .324066E+00

     CHORD =  10.000000
```

PAGE 12 OUTPUT



```
TITLE--  //    GA(W)-1 AIRFOIL WITH BAD COORDINATE POINTS                                //

                          --LOWER SURFACE INTERPOLATED COORDINATES--

   I         XL             YL          DY/DX         D(DY/DX)/DX       CURVATURE
   1     0.000000        .008778     .100000E+99      .100000E+99      .342209E+02
   2      .010000       -.063026    -.335261E+01      .191941E+04      .448226E+02
   3      .020000       -.089958    -.222619E+01      .681620E+03      .468941E+02
   4      .050000       -.138530    -.124092E+01      .164756E+03      .407024E+02
   5      .100000       -.186816    -.777581E+00      .526254E+02      .258900E+02
   6      .500000       -.356231    -.283778E+00      .354689E+01      .315786E+01
   7      .800000       -.428505    -.205762E+00      .195000E+01      .183240E+01
   8     1.000000       -.466116    -.171908E+00      .147902E+01      .141580E+01
   9     2.000000       -.583469    -.753524E-01      .666760E+00      .661121E+00
  10     3.000000       -.630395    -.220622E-01      .446844E+00      .446518E+00
  11     4.000000       -.630743     .214127E-01      .448574E+00      .448266E+00
  12     5.000000       -.584799     .730575E-01      .604586E+00      .599778E+00
  13     6.000000       -.478298     .142695E+00      .748504E+00      .726211E+00
  14     7.000000       -.308580     .185414E+00      .120328E+00      .114379E+00
  15     8.000000       -.125036     .170199E+00     -.581575E+00      .557190E+00
  16     9.000000        .001376     .713875E-01     -.127916E+01      .126944E+01
  17    10.000000       -.036343    -.253004E+00     -.865835E+01      .788889E+01
```

TABLE III.- SAMPLE OUTPUT FOR AIRFOIL SCALING PROGRAM

PAGE 1 OUTPUT

```
                                                  --INPUT DATA--

   TITLE--  *   GA(W)-1 SMOOTHED       *

   NT= 41      IPLOT=  1      IPUNCH=  0      IOP=  0

  X/C=     .182000E-02    .338100E-02    .791900E-02    .116860E-01    .192200E-01    .306300E-01    .420950E-01    .538920E-01
           .781350E-01    .102793E+00    .127608E+00    .152464E+00    .177313E+00    .202154E+00    .251836E+00    .301530E+00
           .351251E+00    .401010E+00    .450814E+00    .500657E+00    .550503F+00    .575409E+00    .600279E+00    .625084E+00
           .649822E+00    .674533E+00    .699266E+00    .724045E+00    .748863E+00    .773699E+00    .798548E+00    .823428E+00
           .848372E+00    .873407E+00    .898533E+00    .923728E+00    .948946E+00    .974128E+00    .989166E+00    .994153E+00
           .999118E+00

  Y/C=    -.843100E-02   -.739600E-02   -.455900E-02   -.239900E-02    .126700E-02    .502700E-02    .734200E-02    .889900E-02
           .109520E-01    .123760E-01    .135190E-01    .145050E-01    .153780E-01    .161540E-01    .174660E-01    .185130E-01
           .193480E-01    .200120E-01    .205450E-01    .209800E-01    .213350E-01    .214760E-01    .215800E-01    .216200E-01
           .215600E-01    .213760E-01    .210550E-01    .205940E-01    .199850E-01    .192040E-01    .182080E-01    .169490E-01
           .153830E-01    .134750E-01    .111980E-01    .850900E-02    .529300E-02    .127700E-02   -.174200E-02   -.288900E-02
          -.412100E-02

T/C/2=    0.              .902500E-02    .178730E-01    .226660E-01    .296660E-01    .370210E-01    .424880E-01    .470340E-01
           .544440E-01    .603680E-01    .652520E-01    .693270E-01    .727400E-01    .756000E-01    .799640E-01    .828210E-01
           .843640E-01    .846590E-01    .836790E-01    .813180E-01    .774050E-01    .748020E-01    .717340E-01    .681920E-01
           .642030E-01    .598240E-01    .551240E-01    .501760E-01    .450520E-01    .398330E-01    .346060E-01    .294590E-01
           .244720E-01    .197160E-01    .152540E-01    .111720E-01    .761800E-02    .488500E-02    .385800E-02    .365500E-02
           .353500E-02

SLOPE=     .592326E+00    .383967E+00    .337573E+00    .299424E+00    .228491E+00    .152664E+00    .108353E+00    .828360E-01
           .576090E-01    .462850E-01    .399860E-01    .355510E-01    .318050E-01    .284960E-01    .229570E-01    .184780E-01
           .148330E-01    .119300E-01    .972200E-02    .807400E-02    .650200E-02    .547000E-02    .388800E-02    .123000E-02
          -.277500E-02   -.780800E-02   -.133180E-01   -.190370E-01   -.252480E-01   -.326600E-01   -.419630E-01   -.533870E-01
          -.665680E-01   -.809090E-01   -.962880E-01   -.114138E+00   -.138752E+00   -.179363E+00   -.217802E+00   -.233989E+00
          -.252162E+00

LT=  2     NEW T/C =      .130000     .200000

   (T/C)MAX FOR INPUT AIRFOIL =    .169405  AT X/C =   .387925
```

TABLE III.- CONTINUED

PAGE 2 OUTPUT

```
TITLE--  *   GA(W)-1 SMOOTHED        *

        SCALED COORDINATES FOR (T/C)MAX =  .2000

                     UPPER                          LOWER

       I          X/C        Y/C             X/C        Y/C
       1       0.000000    .002481        0.000000    .002481
       2        .001540    .015339         .002428  -.008425
       3        .004400    .023152         .007977  -.017262
       4        .011888    .035353         .015506  -.024450
       5        .024575    .048189         .020174  -.027946
       6        .037252    .057165         .027742  -.032821
       7        .049869    .064188         .037858  -.038142
       8        .074987    .075064         .048093  -.042493
       9        .100029    .083506         .059051  -.046403
      10        .125043    .090425         .082383  -.053177
      11        .150050    .096227         .106619  -.058773
      12        .175058    .101134         .131197  -.063407
      13        .200067    .105290         .155864  -.067239
      14        .250087    .111761         .180515  -.070401
      15        .300103    .116186         .205149  -.073007
      16        .350115    .118846         .254418  -.076856
      17        .400120    .119861         .303713  -.079188
      18        .450118    .119240         .353067  -.080180
      19        .500107    .116891         .402503  -.079868
      20        .550096    .112631         .452037  -.078182
      21        .575094    .109702         .501656  -.074963
      22        .600098    .106187         .551283  -.069994
      23        .625114    .102049         .576059  -.066783
      24        .650142    .097283         .600756  -.063060
      25        .675175    .091932         .625312  -.058842
      26        .700204    .086063         .649722  -.054196
      27        .725225    .079760         .674073  -.049212
      28        .750239    .073100         .698472  -.043985
      29        .775248    .066155         .722971  -.038604
      30        .800256    .058983         .747555  -.033161
      31        .825259    .051639         .772179  -.027777
      32        .850250    .044177         .796831  -.022595
      33        .875225    .036647         .821550  -.017767
      34        .900182    .029101         .846409  -.013434
      35        .925129    .021596         .871465  -.009718
      36        .950069    .014189         .896722  -.006722
      37        .975017    .006946         .922126  -.004591
      38        .989999    .002703         .947583  -.003612
      39        .994998    .001308         .972961  -.004394
      40       1.000000   -.000079         .988032  -.006184
      41                                   .992999  -.007081
      42                                  1.000000  -.008667
```

TABLE III. - CONCLUDED

PAGE 3 OUTPUT

```
TITLE--  *   GA(W)-1 SMOOTHED       *

        CAMBER AND THICKNESS DISTRIBUTIONS FOR (T/C)MAX =  .2000

                              CAMBER                         THICKNESS

    I          X/C           Y/C           SLOPE           T/C/2
    1       .002428       -.008425        33.9378        0.000000
    2       .003988       -.007390        21.9997         .021310
    3       .008523       -.004555        19.3415         .042202
    4       .012287       -.002397        17.1557         .053519
    5       .019815        .001266        13.0916         .070048
    6       .031216        .005023         8.7470         .087414
    7       .042673        .007336         6.2082         .100323
    8       .054460        .008892         4.7462         .111057
    9       .078685        .010944         3.3008         .128554
   10       .103324        .012366         2.6519         .142541
   11       .128120        .013509         2.2910         .154073
   12       .152957        .014494         2.0369         .163695
   13       .177787        .015366         1.8223         .171754
   14       .202608        .016142         1.6327         .178507
   15       .252252        .017453         1.3153         .188812
   16       .301908        .018499         1.0587         .195558
   17       .351591        .019333          .8499         .199201
   18       .401312        .019997          .6835         .199897
   19       .451077        .020529          .5570         .197583
   20       .500882        .020964          .4626         .192009
   21       .550690        .021319          .3725         .182769
   22       .575576        .021459          .3134         .176623
   23       .600427        .021563          .2228         .169379
   24       .625213        .021603          .0705         .161015
   25       .649932        .021543         -.1590         .151597
   26       .674624        .021360         -.4474         .141257
   27       .699338        .021039         -.7631         .130159
   28       .724098        .020578        -1.0907         .118476
   29       .748897        .019970        -1.4466         .106377
   30       .773714        .019189        -1.8713         .094054
   31       .798544        .018194        -2.4043         .081712
   32       .823405        .016936        -3.0588         .069559
   33       .848329        .015371        -3.8141         .057783
   34       .873345        .013465        -4.6357         .046554
   35       .898452        .011189        -5.5169         .036018
   36       .923628        .008502        -6.5396         .026379
   37       .948826        .005289        -7.9499         .017988
   38       .973989        .001276       -10.2767         .011534
   39       .989015       -.001741       -12.4791         .009110
   40       .993998       -.002887       -13.4066         .008630
   41       .998960       -.004118       -14.4478         .008347
```

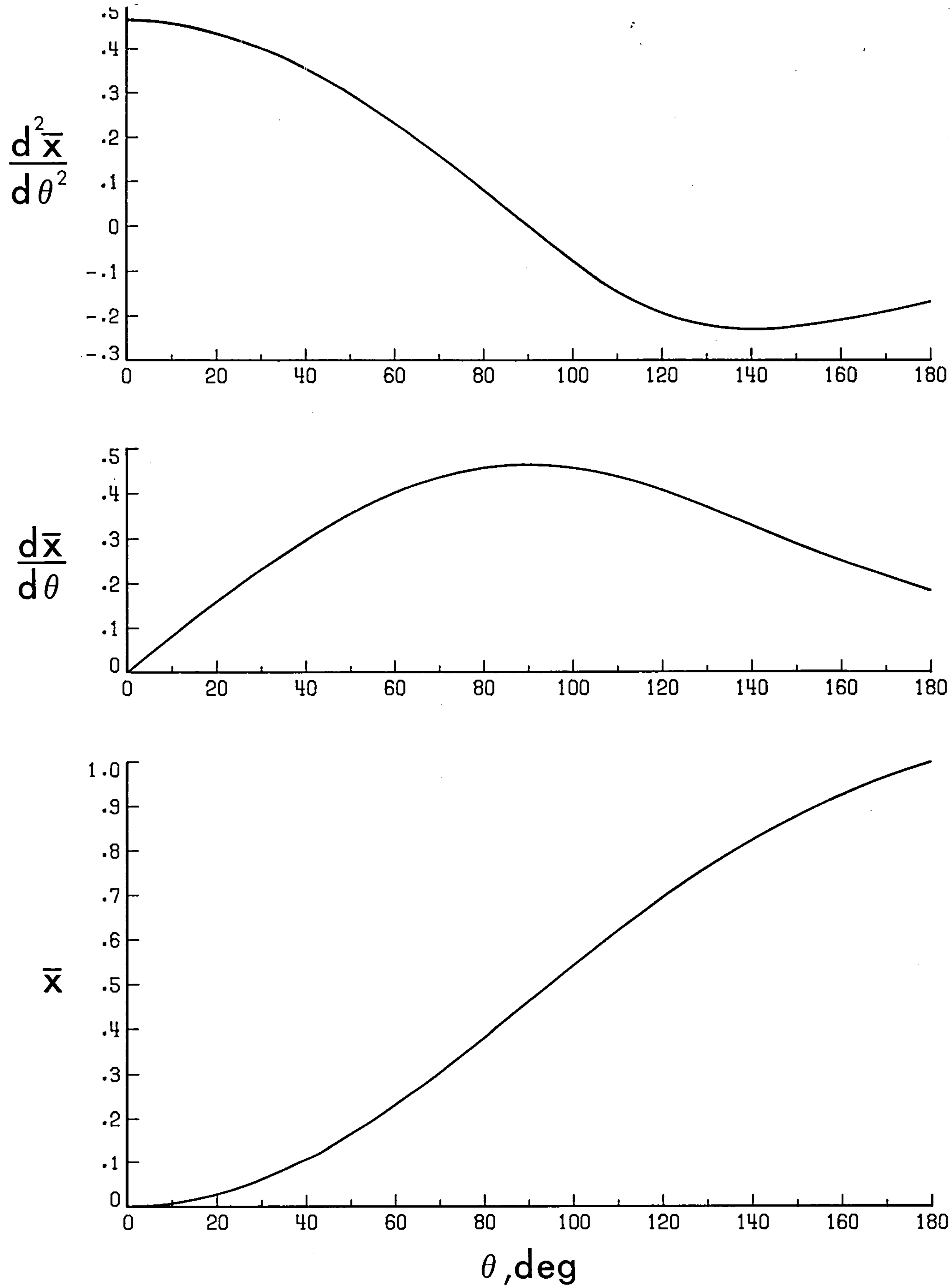


Figure 1. - Properties of θ-transformation function.

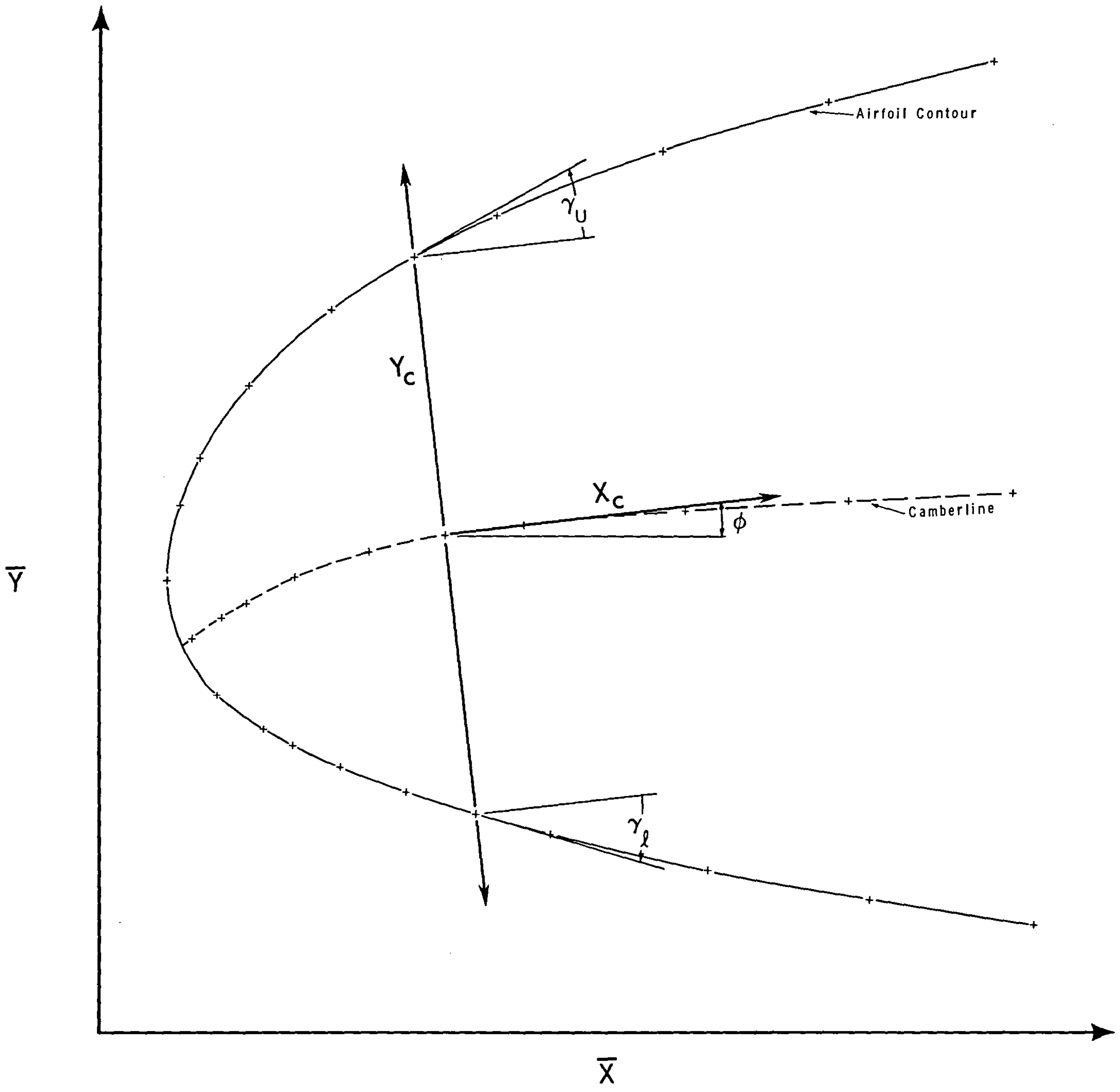


Figure 2. - Camberline axis system.

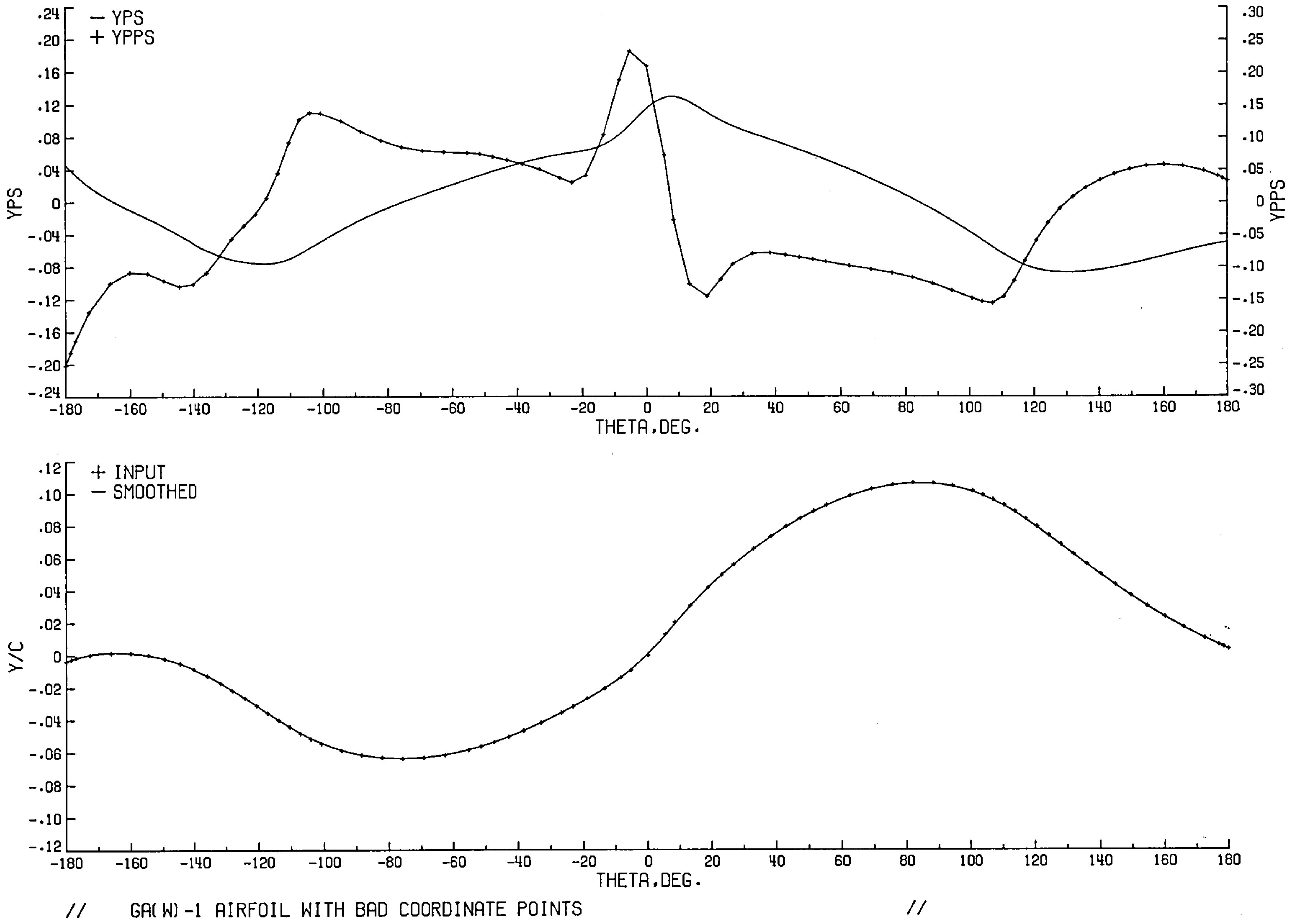


Figure 3.- Sample plot for airfoil smoothing program plotting option 1.

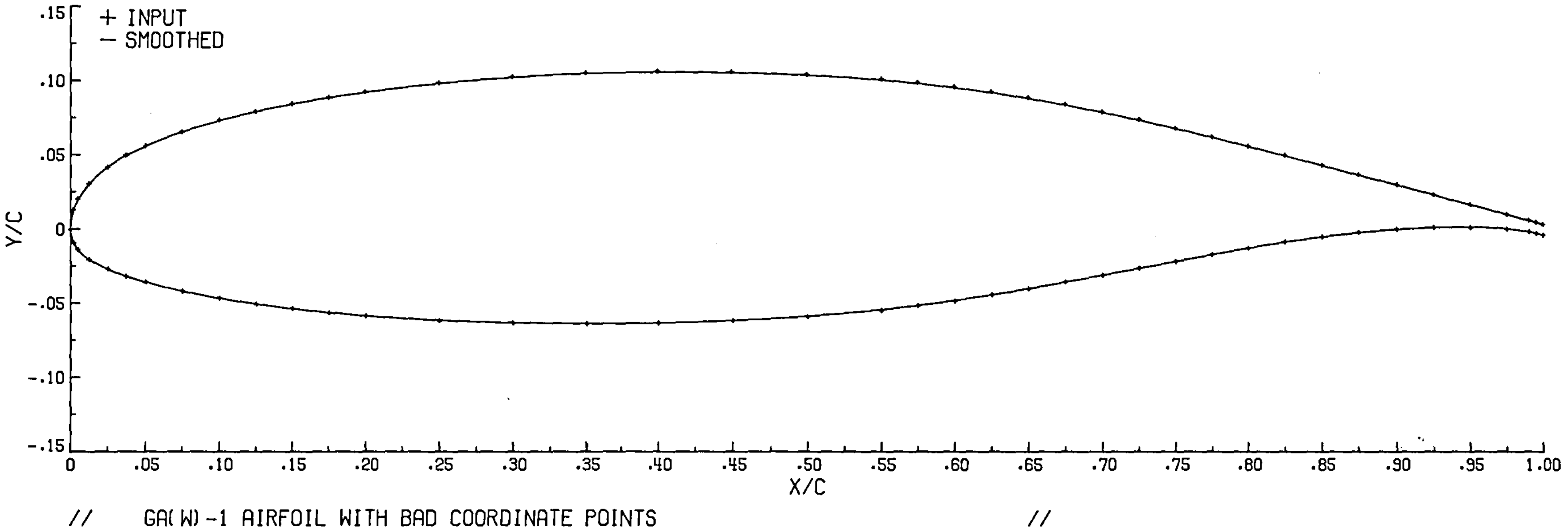


Figure 4. - Sample plot for airfoil smoothing program plotting option 2.

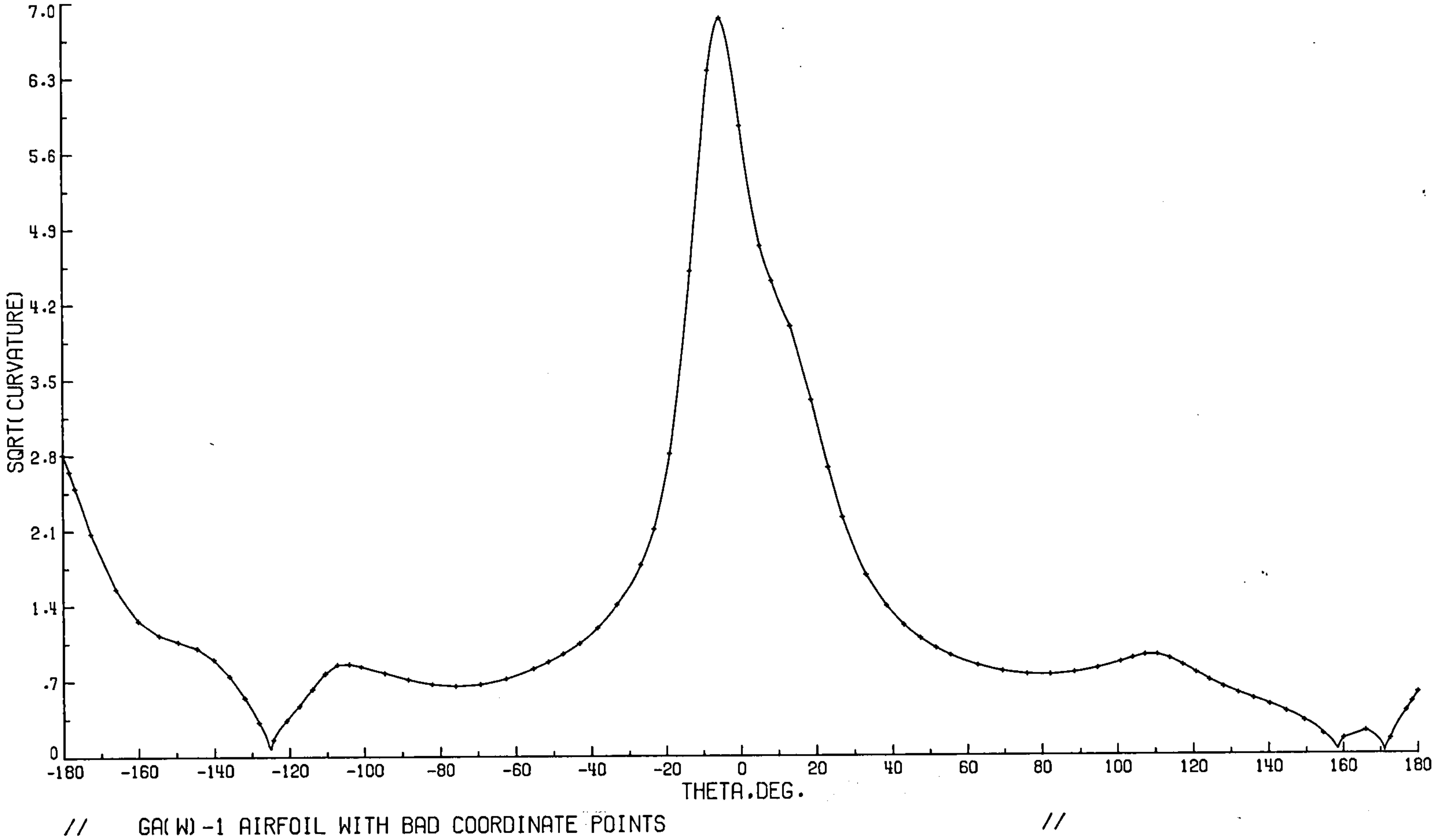


Figure 5. - Sample plot for airfoil smoothing program plotting option 3.

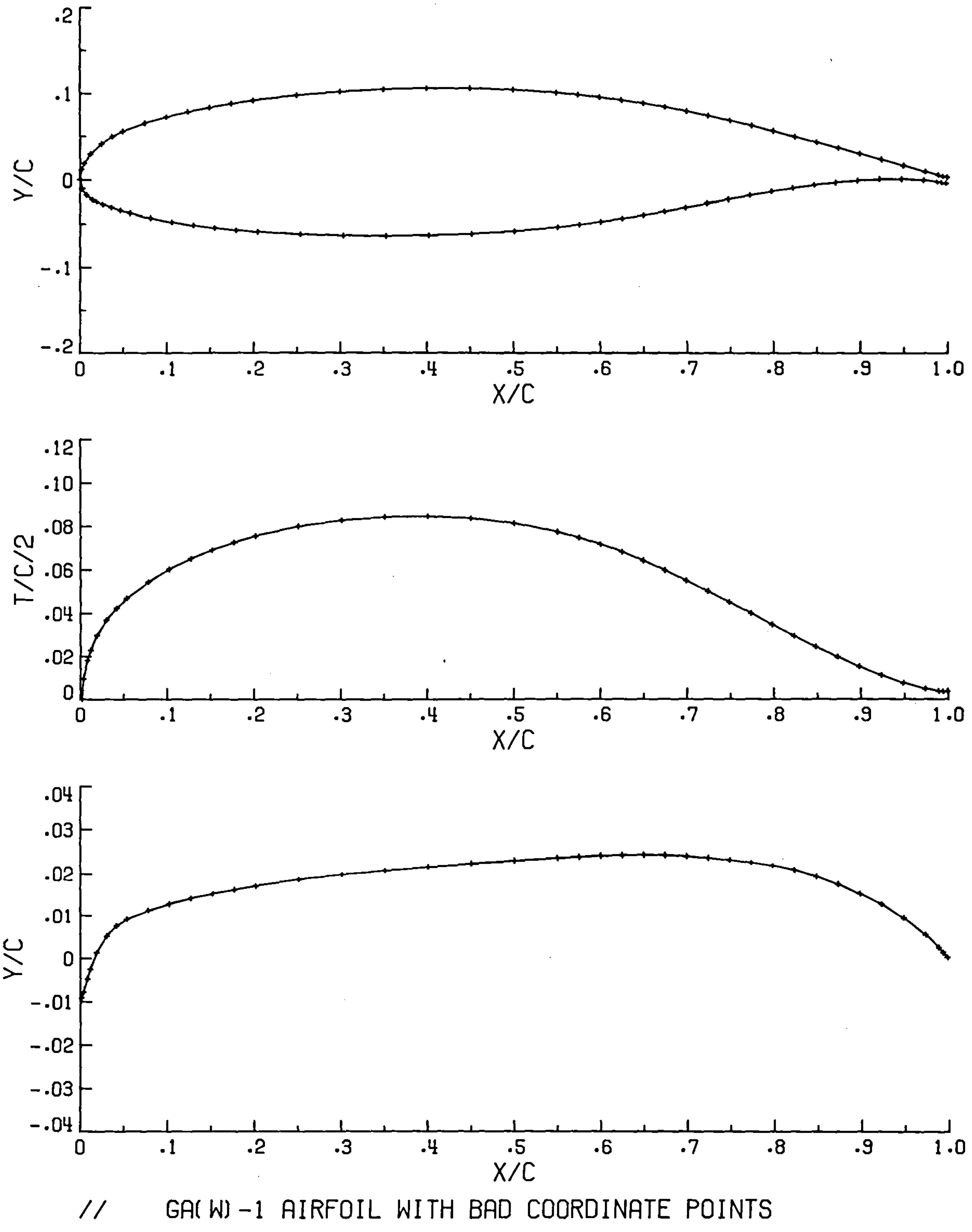


Figure 6.- Sample plot for airfoil smoothing program plotting option 4.

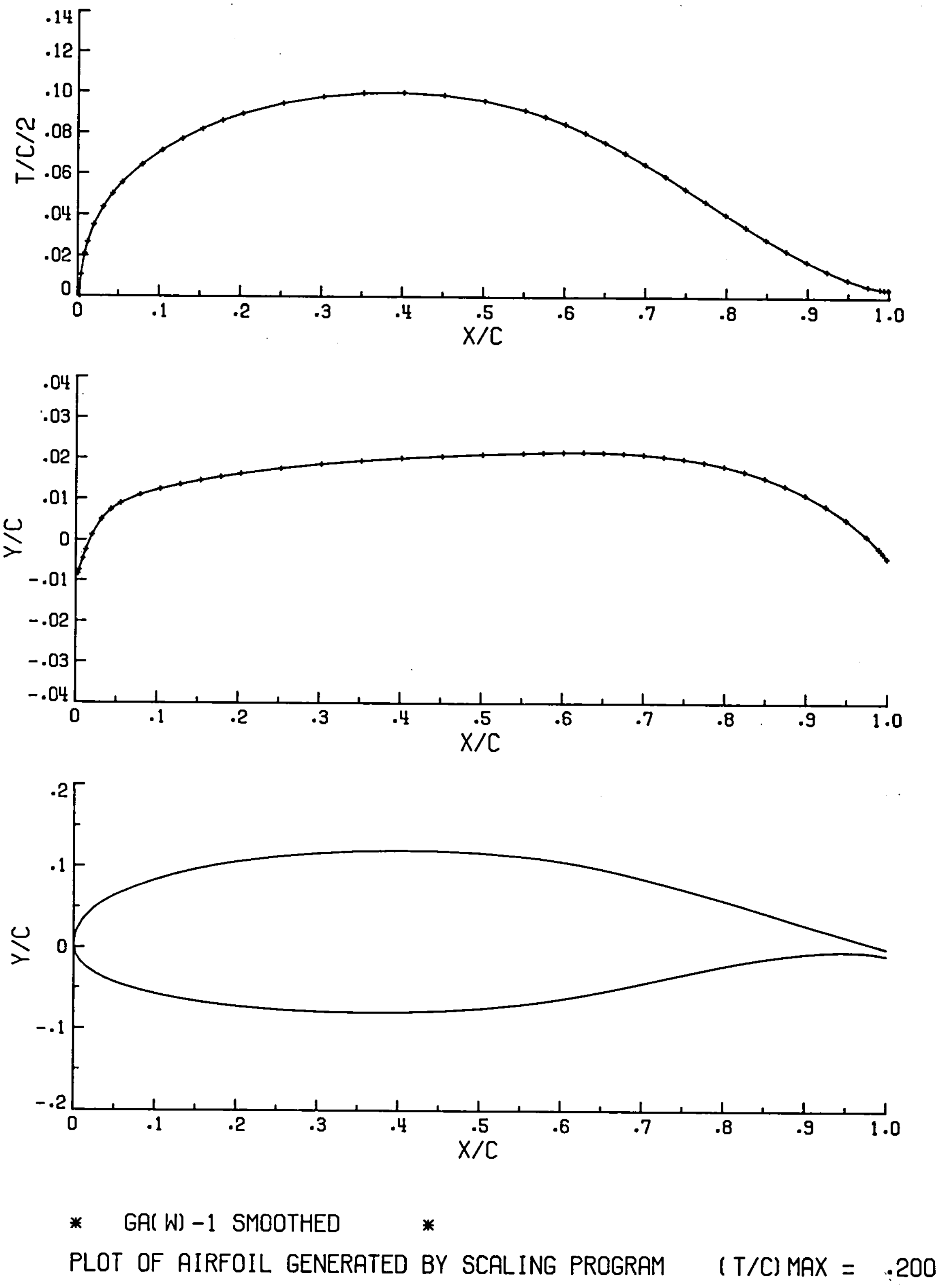
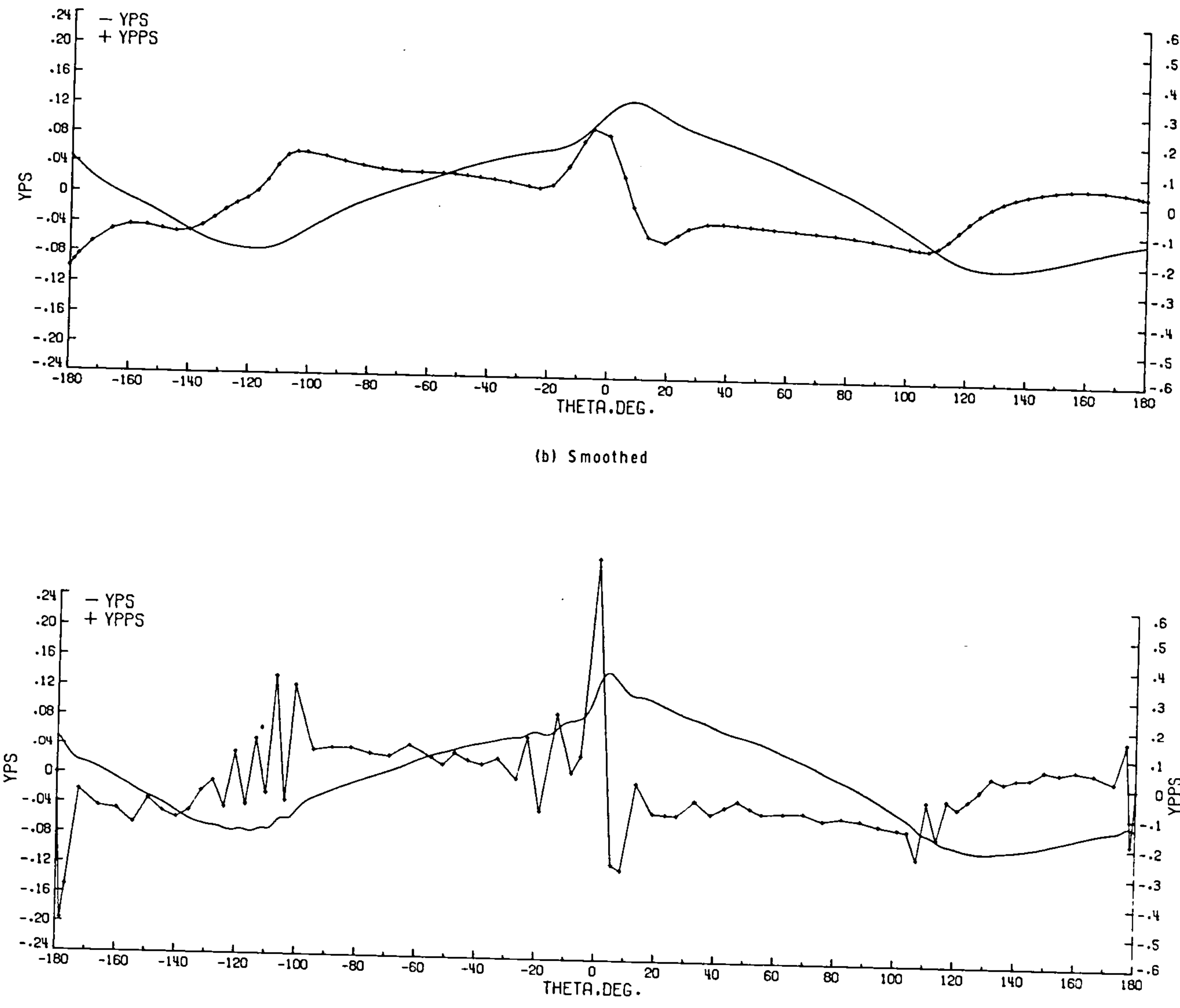


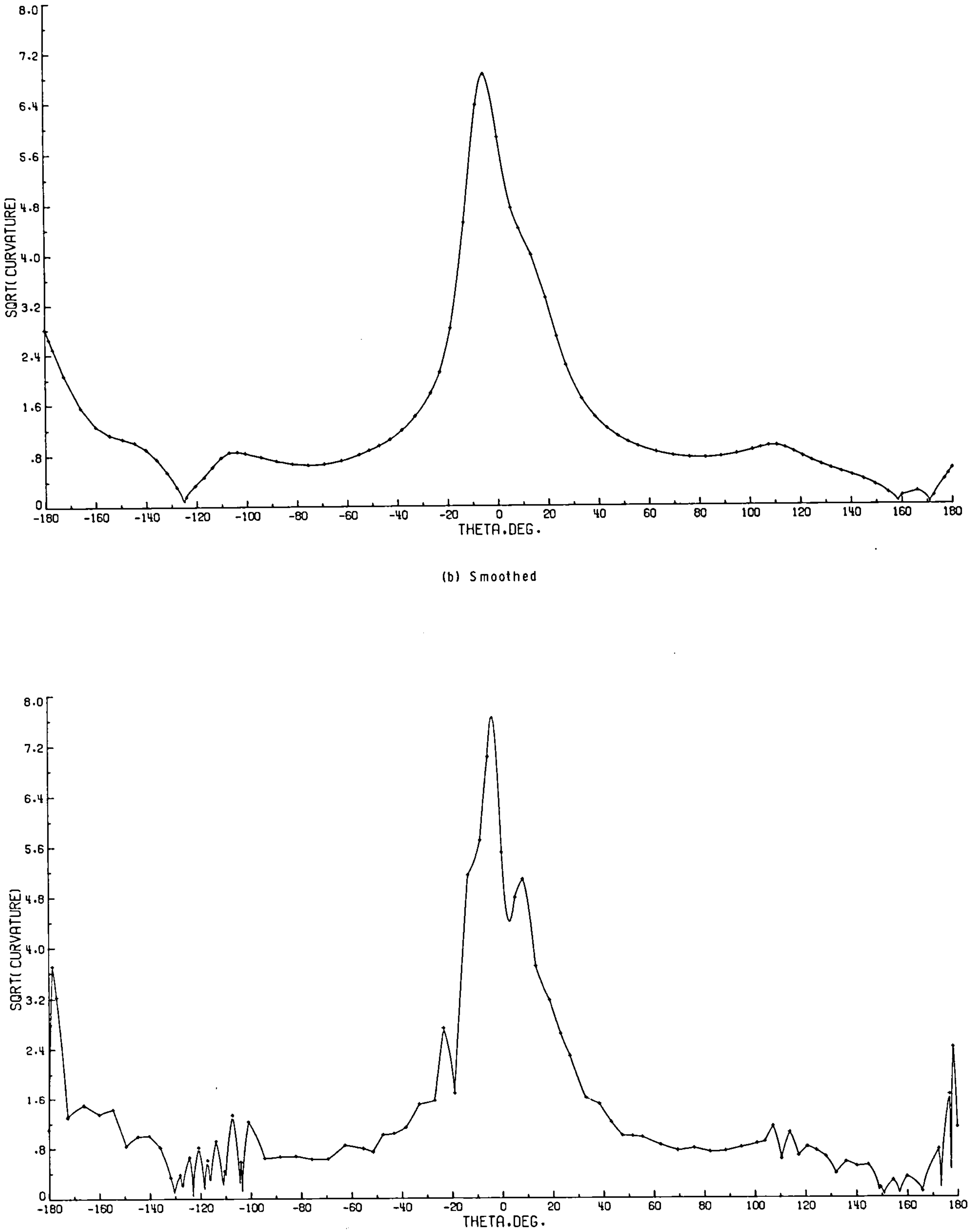
Figure 7. - Sample plot for airfoil scaling program.



(b) Smoothed

(a) Unsmoothed

Figure 8.- Comparison between unsmoothed and smoothed first (YPS) and second (YPPS) derivatives for a typical airfoil.



(b) Smoothed

(a) Unsmoothed

Figure 9. - Comparison between unsmoothed and smoothed square-root of curvature for a typical airfoil.

1. Report No.
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2. Government Accession No.

3. Recipient's Catalog No.

4. Title and Subtitle
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5. Report Date
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7. Author(s)

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16. Abstract

This report contains detailed descriptions of the theoretical methods and associated computer codes of a program to smooth and a program to scale arbitrary airfoil coordinates. The smoothing program utilizes both least-squares polynomial and least-squares cubic-spline techniques to smooth iteratively the second derivatives of the y-axis airfoil coordinates with respect to a transformed x-axis system which unwraps the airfoil and stretches the nose and trailing-edge regions. The corresponding smooth airfoil coordinates are then determined by solving a tridiagonal matrix of simultaneous cubic-spline equations relating the y-axis coordinates and their corresponding second derivatives. A technique for computing the camber and thickness distribution of the smoothed airfoil is also discussed.

The scaling program can then be used to scale the thickness distribution generated by the smoothing program to a specified maximum thickness which is then combined with the camber distribution to obtain the final scaled airfoil contour. Computer listings of the smoothing and scaling programs are included as appendices. A user-guide and sample input and output cases for both programs are also included as appendices. Both computer programs are available from COSMIC with identifications LAR-13132 for the airfoil smoothing program "AFSMO" and LAR-13133 for the airfoil scaling program "AFSCL".

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