CALCULATION COVER SHEET

BECHTEL

PROJECT Limerick JOB NO. 08031 DISCIPLINE Plant Design

SUBJECT Study Calculation for E-System Clamp on Elbows FILE NO. A-263

CALC. NO. SR8031-SS10

NO. OF SHEETS 35

RECORD OF ISSUES

NO.	DESCRIPTION	BY	DATE	CHKD	DATE	APPRD	DATE	
0	Stress Analysis on Core Spray Line	CKH	4-19-83	AYH CYC	4/19/83	MZK	4/19/83	

THIS CALCULATION HAS BEEN PERFORMED BY STAFF AND IS REVIEWED AND ACCEPTED BY

[signature] ON 4/22/83

(GROUP SUPERVISOR OR DESIGNEE) (DATE)

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STUDY CALCULATION

FOR

E-SYSTEM CLAMP ON ELBOWS

SR 8031-SS10

BY

BECHTEL POWER CO.

PLANT DESIGN STRESS STAFF

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M.Z. Khlafallah

April 18, 1983

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Table of Content

1. Introduction

The purpose of this calculation is to investigate the impact of the local stresses in the pipe wall due to the use of E-system clamp at the end of elbow. For Limerick project, the BOP piping systems which involve clamps attached on elbows are:

2 on Nuclear Class 1 Core Spray, Carbon steel Clamp, Stainless Steel Pipe.

1 on Nuclear Class 1 Feedwater, both the clamp and pipe are Carbon steel.

1 on Nuclear Class 3 MSRV lines inside drywell, both clamp and pipe are Carbon steel.

The local stresses on both core spray and Feedwater lines are evaluated. This calculation includes the detailed calculations of the primary stress and usage factor for Core spray line.

(a) Piping System

(i) Pipe Size & Material — Stainless Steel

$\alpha_p = 9.11 \times 10^{-6}$

$E_p = 28.3 \times 10^6$ psi

$OD_p = 12.75$ in

$t_p = 0.688$ in

$I = 475.8$ in^4

(ii) Load Condition

The load histogram specified in the design specification for Core Spray Lines is considered in this calculation

* Design Pressure 1250 psi

* Design Temperature 582 °F

The operating temperature at the clamp is 150°F

* 10 operating cycles of cold water injection with thermal transient from 150°F → 50°F at step change.

* 50 cycles of (OBE + SRV$_{\text{building response}}$)

* 7650 Cycles of $SRV_{building\ response}$

* 380 Cycles of Thermal (522°F) and Pressure (1000 psi) operating loads including start up & shut down

(b) Clamp (Sketch shown in attachment 1.)

(i) Size & Material – Carbon Steel

$\alpha_c = 6.07 \times 10^{-6}$

$E_c = 27.9 \times 10^6$ psi

$t_c = 1.25$ in

$O.D_c = 16.45$ in

ii) Load

The total load (including $OBE + SRV_{building\ response}$) from snubber is 14K. The snubber load due to SRV alone is 4.25K.

The local stresses induced by the presence of Clamp are

* preload

* Constraint of thermal expansion

* Constraint of Pressure expansion

* Snubber load

(C) Method of Calculation

Piping stresses are calculated in accordance with Code equations specified in ASME Section III NB 3650. The local stresses due to snubber loads, thermal and pressure constraints are calculated and added to piping stresses. The total stresses are compared with Nuclear Class 1 code allowables. Primary stress intensity is evaluated for Design condition. Primary plus Secondary stress intensity and Peak stress intensity are calculated for each loading condition and the total usage factor for 40 years of plant life is conservatively calculated. The following equations are used.

(i) Primary Stress

EQ. 9 $$B_1 \frac{PD}{2t} + B_2 \frac{MD}{2I} + (P_L + P_b)_\ell < 1.5\, S_m$$

(ii) Primary plus Secondary

EQ. 10 $$S_N = C_1 \frac{PD}{2t} + C_2 \frac{MD}{2I} + \frac{1}{2(1-\nu)} E\alpha|\Delta T_1| + C_3 E_{ab} |\alpha_a T_a - \alpha_b T_b| + S_{n\ell}$$

0

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(iii) EQ 11

$$S_p = K_1 C_1 \frac{PD}{2t} + C_2 \frac{MD}{2I} + \frac{1}{2(1-\nu)} E\alpha |\Delta T_1| + \frac{1}{1-\nu} E\alpha |\Delta T_2|$$

$$+ K_3 C_3 E_{ab} |\alpha_a T_a \cancel{- \alpha_b T_b}| + S_{p\ell}$$

$$0$$

$$S_{alt} = \frac{K_e}{2} S_p$$

where $(P_L + P_b)_\ell$ — Primary Local Membrane Stress Intensity

$S_{N\ell}$ — Local Primary plus Secondary stress Intensity

$S_{p\ell}$ — Local Peak Stress Intensity

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2. Local Stress Calculations

a. Clamp Preload Stresses

Although piping stresses due to clamp preload are nonrecurring secondary type in nature, the potential for pipe damage due to bolt overtorque is assessed.

Bolt Torque $T = 90$ ft-lb *

Bolt diameter $d = 1.25$, 8UN-2A

Assume lubricated bolt (No friction)

$T = Fd\sin\theta$, $\theta = 11.31°$. $\sin\theta = 0.196$

$$F = \frac{T}{d\sin\theta} = \frac{90 \times 12}{1.25(0.196)} = 4408\,lb$$

To calculate the pressure between the U-bolt and the pipe wall, static equilibrium equation is used.

* Note: The preload torque is given by Limerick Pipe Clamp installation Spec. 8031-P-143-30-4

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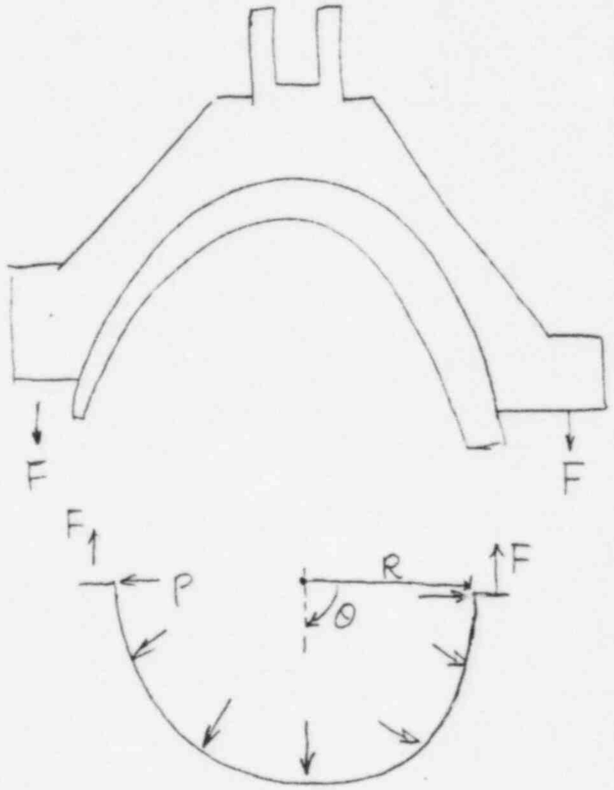

$$2F = \int_0^{\pi} p\,(R\,d\theta)(2b)\sin\theta$$

where : p = Clamp pre-load induced pressure

$$R = \frac{12.75}{2} = 6.375''$$

$2b$ = width of strap

(strap is used between U-bolt & pipe for snug fit)

$= 2.86$ in

$$\Rightarrow \quad 2F = \int_0^{\pi} pR(2b)\sin\theta\,d\theta$$

$$= (2b)\,pR\int_0^{\pi}\sin\theta\,d\theta$$

$$= 4bpR$$

$$\Rightarrow \quad p = \frac{F}{(2b)R} = \frac{4408}{(2.86)(6.375)} = 242\ psi$$

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The resulting longitudinal & hoop stresses can be calculated by using the equation presented by Roark.

From eq. 12, Page 301, 4th. Edition, "Formulas for Stress & Strain", it is obvious:

$$M = \frac{P}{2\lambda^2} e^{-b\lambda} \sin b\lambda$$

$$\lambda = \sqrt[4]{\frac{3(1-\nu^2)}{R_p^2 \ t_p^2}}$$

$\nu = 0.3$

$R_p = \frac{12.75}{2} = 6.375$

$t_p = 0.688$

$$\lambda = \sqrt[4]{\frac{3(1-0.3^2)}{(6.375 \times 0.688)^2}} = 0.614/in$$

$b = \frac{2.86}{2} = 1.43$

$e^{-b\lambda} = 0.416$

$\sin b\lambda = 0.769$

$$M = \frac{242}{2(0.614)^2}(0.416)(0.769) = 103 \text{ in-lb/in}$$

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Max. longitudinal stress

$$S_1 = \frac{6M}{t_p^2} = \frac{6 \cdot 103}{(0.688)^2} = 1306 \text{ psi}$$ (compression at outer surface, tension at inner surface)

Max hoop stress

$$S_2 = \frac{PR_p}{t_p}(1 - e^{-b\lambda} \cos b\lambda)$$

$$\cos b\lambda = 0.639$$

$$S_2 = \frac{242 \cdot 6.375}{0.688}(1 - 0.416 \times 0.639)$$

$$= 1648 \text{ psi}$$

It can be seen from next section that the effective area of the pipe resisting the radial load from the Clamp is greater than the U-Bolt, therefore the U-Bolt will yield before pipe. In addition, the deformation of U-Bolt will reduce the piping stresses somewhat.

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b. Differential Thermal Expansion stresses

This section calculates the stresses induced in the pipe wall due to the differential thermal expansion rate of a stainless steel pipe constrained by a carbon steel clamp.

Assumptions

a. The clamp represents a ring around the pipe causing a uniform external force on it.

b. Area of clamp that is acting on the pipe to be the cross section area of the U-Bolt.

c. Area of pipe upon which clamp is acting includes the reinforcement area.

d. Change in temperature = 582 - 70 = 512°F

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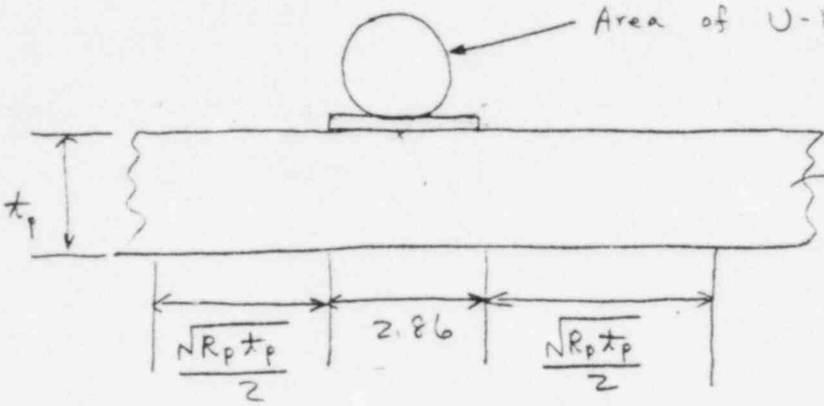

Area of U-bolt $= \frac{\pi}{4} t_c^2 = 1.227 \text{ in}^2$

$A_c = 1.227 \text{ in}^2$

$A_p = 0.688(2.86 + \sqrt{R_p t_p})$

$= 3.409 \text{ in}^2$

Geometry Compatibility requires:

Change in pipe diameter = Change in clamp diameter

$$\alpha_p \Delta T (O.D_p) + \frac{\sigma_p}{E_p} O.D_p = \alpha_c \Delta T (I.D_c) + \frac{\sigma_c}{E_c} (I.D_c)$$

$$9.11 \times 10^{-6} \times 512 (12.75) + \frac{\sigma_p}{28.3 \times 10^6} (12.75) = 6.07 \times 10^{-6} \times 512 \times 13.95 + \frac{\sigma_c}{27.9 \times 10^6} 13.95$$

$$59470.08 + 0.451 \sigma_p = 43354.368 + 0.5 \sigma_c$$

$$16115.712 = 0.5 \sigma_c - 0.451 \sigma_p \quad \text{------ (1)}$$

Force Compatibility yields:

$$\sigma_p A_p + \sigma_c A_c = 0$$

$$\Rightarrow 3.409 \sigma_p + 1.227 \sigma_c = 0 \quad \text{------ (2)}$$

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From (2)

$$\sigma_p = -\frac{1.227}{3.409}\sigma_c = -0.36\sigma_c$$

⇒ From (1)

$$16115.712 = 0.5\sigma_c + (0.451)\cdot(0.36)\,\sigma_c$$

$$\sigma_c = 24331 \text{ psi}$$

$$\sigma_c = \frac{P_c R}{t}, \quad t = \frac{A_c}{2b} = \frac{1.227}{2.86} = 0.429$$

$$R_c = \frac{16.45}{2} = 8.225$$

$$P_c = \frac{\sigma_c t}{R} = \frac{24331 \cdot 0.429}{8.225} = 1269 \text{ psi}$$

By the same egation from Roark as described in section A, one can find the max. longitudinal & hoop stresses

$$M = \frac{P}{2\lambda^2} e^{-b\lambda} \sin b\lambda$$

$$= 540 \text{ in-lb/in}$$

Max. longitudinal stresses

$$S_1 = \frac{6M}{t_p^2} = \frac{6 \cdot 540}{t_p^2} = 6845 \text{ psi}$$ [Comp. on outer surface, tension on inner surface]

Max. hoop stress

$$S_2 = \frac{PR}{t_p}(1 - e^{-b\lambda}\cos b\lambda) = 8640 \text{ psi}$$

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Temp. 522°F & 300°F are used to calculate the interaction pressure for later fatigue evaluations.

$$\alpha_p \Delta T (O.D_p) + \frac{\sigma_p}{E_p} O.D_p = \alpha_c \Delta T (I.D_c) \frac{\sigma_c}{E_c} (I.D_c)$$

T = 522 °F

$$9.11 \times 10^{-6} \times 452 (12.75) + \frac{\sigma_p}{28.3 \times 10^6} (12.75) = 6.07 \times 10^{-6} \times 452 \times 13.95 + \frac{\sigma_c}{27.9 \times 10^6} 13.95$$

$$\Rightarrow 14228 = 0.5 \sigma_c - 0.451 \sigma_p \quad \text{—— (3)}$$

$$\sigma_p A_p + \sigma_c A_c = 0$$

$$3.409 \sigma_p + 1.227 \sigma_c = 0 \quad \text{—— (4)}$$

From (3) & (4)

$$14228 = 0.5 \sigma_c + (0.451)(0.36) \sigma_c$$

$$\sigma_c = 21481 \text{ psi}$$

$$P_c = \frac{\sigma_c t}{R} = \frac{21481 \cdot 0.429}{8.225} = 1121 \text{ psi}$$

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$$T = 300°F$$

$$9.11 \times (300-70)(12.75) + \frac{\sigma_p}{28.3}(12.75) = 6.07 \times 230 \times 13.95 + \frac{\sigma_c}{27.9} \times 13.95$$

$$26715.075 + \frac{12.75}{28.3}\sigma_p = 19475.595 + \frac{13.95}{27.9}\sigma_c$$

$$7239.48 = 0.5\sigma_c - 0.451\sigma_p$$

$$\sigma_p = -0.36\sigma_c$$

$$\Rightarrow 7239.48 = 0.5\sigma_c + (0.451)(0.36)\sigma_c$$

$$\sigma_c = 10929.827$$

$$P_c = \frac{10929.827 \cdot 0.429}{8.225} = 570 \text{ psi}$$

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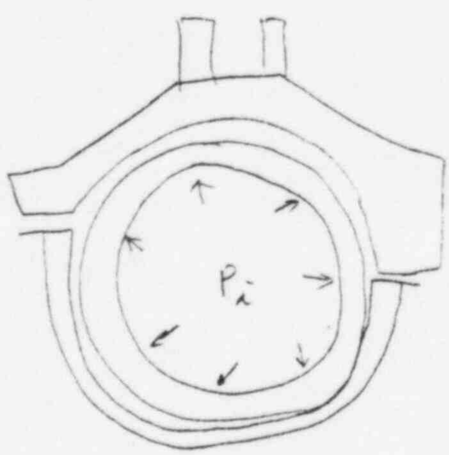
C. Constraint of Pressure Expansion stresses

When the pipe is pressurized it will expand. The longitudinal and hoop stresses in the pipe wall due to the restraint of free expansion by the clamp is evaluated.



Clamp constraint of pipe Expansion due to internal pressure

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Assumptions

a. $P_i = 1250$ psi

b. Clamp acting uniformly on the pipe

From equation 13, page 302 of Roark's Formulas:

$$M = \left(\frac{P_i}{2\lambda^2}\right)\left[\frac{A}{A + t_p(2b) + \frac{2t_p}{\lambda}}\right]$$

$P_i = 1250$

$\lambda = 0.614$

A = cross-sectional area of clamp $= 1.227$ in^2

$t_p = 0.688$

$2b = 2.86$

$$M = \frac{1250}{2 \cdot 0.614^2}\left[\frac{1.227}{1.227 + 0.688(2.86) + \frac{2 \cdot 0.668}{0.614}}\right]$$

$= 374$ in-lb/in

Max. longitudinal stress:

$$S_1 = \frac{6M}{t_p^2} = \frac{6 \cdot 383}{0.688^2} = 4744 \text{ psi}$$

$V_0 = 2M\lambda$

$$P = \frac{2V_0}{2b} = \frac{4M\lambda}{2b} = \frac{4 \cdot 374 \cdot 0.614}{2.86} = 321 \text{ psi}$$

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Max. hoop stresses

$$S_2 = \frac{PR}{t}(1 - e^{-bx}\cos bx)$$

$$= 2184 \text{ psi}$$

d. Stresses Induced by Pad Bearing

Most of the snubber load against the pipe will be resisted by the rectangular central pad under the snubber attachment point. The stresses can be calculated by the method outlined in WRC bulletin 107, which is computerized in Bechtel's in-house program ME-210.

Assumptions

1) 100% of snubber load is acting on central pad.

2) The pressure on central pad is uniform.

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3) Pad Area = $4.5 \times 5.01 = 22.55$ in^2

The stresses Calculated by computer program ME-210 (Ref. 2) based on 14K load are summarized in the following.

Location	Direction	STRESSES (Ksi)	
		Primary	Primary + Secondary
Outter Surface	longitudinal	4.25	13.70
	circumferencial	4.30	13.48
Inner Surface	longitudinal	4.25	7.54
	circumferencial	4.30	7.16

Table - 1

3. Primary Stress Intensity & Fatigue Evaluation

a) Calculation of stress Indices

From Table NB-3681(a)-1, NB-3000, ASME Code, 1980 Edition. For elbows:

$B_1 = 0.5$

$C_1 = \dfrac{2R - r}{2(R - r)}$

R = curved pipe radius

r = mean radius of x-section

$R = 1.5\,D = 1.5 \times 12 = 18$

$r = 6.031$

$C_1 = \dfrac{2 \cdot 18 - 6.031}{2(18 - 6.031)} = 1.252$

$C_2 = 1.95 / b_2^{2/3}$, but not less than 1.5

$B_2 = 0.75\,C_2$

$b_2 = \dfrac{tR}{r^2}$

t = wall thickness = 0.688.

$R = 18$

$r = 6.031$

$b_2 = \dfrac{0.688 \cdot 18}{6.031^2} = 0.340$

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$C_2 = 1.95 / \frac{3/3}{b_2} = 4.003$

$B_2 = 0.75\, C_2 = 3.002$

$K_1 = K_2 = K_3 = 1.0$

The stress indices C_2 & B_2 can be reduced at elbow ends based on following arguments.

i) L.K. Severud discussed, in the paper (Ref. 5) entitled "Experience with Simplified Inelastic Analysis of Piping Designed for Elevated Temperature Services", that stresses and strains at the elbow midsection were found by Markl to govern the fatigue life of pipe elbows. Toward the end of elbow, the stress intensification effect should be less than that at the elbow midsection. In the present study, the clamp is located at the end of the elbow.

From Figure 14d of Severud's paper, a reduction factor 0.50 can be applied on C_2 & B_2 due to "far away from midsection" effect.

* The finite element results given in Ref 7 also have the same conclusion.

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ii) In Markl's paper Ref. 6 entitled "piping - Flexibility Analysis", it is noted that stress intensification factor is reduced by the presence of a ring stiffner or flange at the end of an elbow. For $b_2 = 0.34$ the reduction factor is $\frac{(0.34)^{2/3}}{(0.34)^{1/2}} = 0.84$. This additional reduction factor is deemed appropriate for the problem under consideration.

Flexibility of the elbow will be reduced because of the use of the clamp. However, based on Markl's paper, the difference will be no more than 15% for this clamped elbow. The change of the flexibility for one elbow of the whole piping system will not have significant change in thermal expansion and seismic stresses. Therefore, it is reasonable that this flexibility effect is neglected.

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By i) & ii)

$$C_2 = 4.003 \times 0.5 \times 0.84 = 1.681$$

$$B_2 = 3.002 \times 0.5 \times 0.84 = 1.261$$

Because the clamp is not an integral part of the pipe, i.e. there is no geometry structural discontinuity, peak stress due to local stress effect is insignificant. Therefore, no modification for K_1, K_2 and K_3 is necessary, i.e., $K_1 = K_2 = K_3 = 1.0$

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(b) Primary Stress Intensity

Eq. 9 $B_1 \frac{PD_o}{2t} + B_2 \frac{D_o}{2I} M \leq 1.5\, S_m$

$S_m = 13990$ psi

$B_1 = 0.5$

$P = 1250$ psi

$D_o = 12.75$ in

$t = 0.688$ in

$I = 475.8$ in

$B_2 = 1.261$

$M = M_{wt} + (M_{OBE}^2 + M_{SRV}^2)^{1/2}$

$M_x = 4898 + 17153 = 22051$ ft-lb

$M_y = 846 + 15838 = 16684$ ft-lb

$M_z = 1315 + 6207 = 7522$ ft-lb

$M = \sqrt{22051^2 + 16684^2 + 7522^2} = 28656$ ft-lb

$= 343876$ in-lb

Eq. 9 $= (P_L + P_b)_\ell + B_1 \frac{PD_o}{2t} + B_2 \frac{D_o}{2I} M$

$(P_L + P_b)_\ell$ = Stresses due to snubber load

$= 4300$ psi

Eq. 9 $= 4300 + 5791 + 5810$

$= 15901 < 1.5\, S_m = 20985$ psi

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C). Fatigue Evaluation

Load on Pad = (Uniform Pressure) x (Pad area) x (Contact ratio)

Due to Snubber load = 14,000 lb

Due to Thermal discontinuity = 1269 x 22.5 x 1.3 = 37118 lb

Due to pressure discontinuity = 321 x 22.5 x 1.3 = 9389 lb

Total load = 14000 + 37118 + 9389 = 60507 lb

stresses induced by clamp $= \left(\frac{60507}{14000}\right) 13700 = 59210$ psi

$C_2 = 1.681$

$C_1 = 1.252$

$P_0 = 1120$ psi

Since $(M_{OBE} + M_{SAM} + M_{SRV}) > M_{TH}$

$M_x = 2[(M_{OBE})_x + (M_{SAM})_x + (M_{SRV})_x]$

$= 2[(11849) + 12120 + 6500]$

$= 60938$ ft-lb

$M_y = 2[(14140) + 4450 + 3109]$

$= 43398$ Ft-lb.

$M_z = 2[(5172) + 4599 + 1428]$

$= 22398$ Ft-lb

$M = \sqrt{60398^2 + 43398^2 + 22398^2}$

$= 77672$ Ft-lb

$= 932067$ in-lb

$C_1 \frac{P_o D_o}{2t} + C_2 \frac{D_o}{2I} M$ + Clamp induced local stresses

$= 1.252 \frac{1120 \cdot 12.75}{2 \cdot 0.688} + 1.681 \frac{12.75}{2(475.8)} 932067 + 59210$

$= 12993 + 20993 + 59210$

$= 93196$ psi $> 3S_m = 41970$ psi

Since eq. 10 > 3Sm, further consideration is required according to code.

First, eq (12) is considered

$$C_2\left(\frac{D_o}{2I}\right)M_i^{*} = 1.681\ \frac{12.75}{2(475.8)}M_i^{*}$$

$$M_i^{*} = \sqrt{7034^2+6302^2+2234^2}\ \times 12 = 116460 \text{ in-lb}$$

$$C_2\left(\frac{D_o}{2I}\right)M_i^{*} = 2623 \text{ psi} < 3S_m = 41970 \text{ psi}$$

Eq. 13

Since $\alpha_a T_a - \alpha_b T_b = 0$

$$C_1\frac{P_oD_o}{2t} + C_2\left(\frac{D_o}{2I}\right)M_i \le 3S_m$$

From the Code, the range of Primary plus secondary membrane plus bending stress intensity are to be included. It is noted that the membrane stresses induced by snubber load on clamp should be included, since it is secondary membrane stress.

$$\Rightarrow\ 4300 + C_1\frac{P_oD_o}{2t} + C_2\frac{D_oM}{2I},$$

$$= 4300 + 12993 + 7745$$

$$= 25038 \text{ psi} < 3S_m = 41970 \text{ psi}$$

Eqs 12 & 13 are satisfied.

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In view of the load histogram for the piping system under consideration, it is decided that eq. 11 can be calculated from four different load set pairs:

i) Cold Water Injection at Startup (load set ②), n=10 cycles

it is found from the load histogram that $P_o = 1000$ psi and $T = 522°F$ are appropriate.

then ⇒ Load = pressure x Pad area x contact ratio

Due to Thermal constraint $= 1121 \times 22.5 \times 1.3 = 32789$ lb

Due to pressure constraint $= 321 \times \frac{1000}{1250} \times 22.5 \times 1.3 = 7511$ lb

Total load $= 32789 + 7511 = 40300$ lb

local stresses $= \frac{40300}{14000} 13700 = 39436$ psi

$$S_n = C_1 \left(\frac{P_o D_o}{2t}\right) + C_2 \left(\frac{D_o}{2I}\right) M_i + \frac{1}{2(1-\nu)} E\alpha |\Delta T_1| + \text{local stresses}$$

$$= 11601 + 2623 + \frac{28.3 \times 9.11 \times 82}{2(1-0.3)} + 39436$$

$$= 68760$$

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$$K_e = 1.0 + \frac{1-n}{n(m-1)}\left[\frac{S_n}{3S_m} - 1\right]$$

$$= 1.0 + \frac{0.7}{0.3 \cdot 0.7}\left[\frac{68760}{3 \cdot 13990} - 1\right]$$

$$= 3.128$$

$$S_p = K_1 C_1 \left(\frac{P_i D_o}{2t}\right) + K_2 C_2 \left(\frac{D_o}{2I}\right) M_i + \frac{1}{2(1-\nu)} K_3 E\alpha |\Delta T_1| + \frac{1}{1-\nu} E\alpha |\Delta T_2|$$

+local stress

$$= 68760 + \frac{1}{1-0.3} 28.3 \times 9.11 \times 46$$

$$= 68760 + 16942$$

$$= 85702$$

$$S_a = \frac{1}{2} K_e S_p = \frac{1}{2}(3.128)(85702)$$

$$= 134038$$

$$N = 450$$

$$n/N = 0.022$$

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(ii) OBE+SRV Loadings (Load set $\triangle 12$ & $\triangle 13$), n=50 Cycles

$P = 760$ psi

Since $(M_{OBE}^2 + M_{SRV}^2)^{1/2} + M_{SAM} > M_{th}$,

$$M_i = 2\left[(M_{OBE}^2 + M_{SRV}^2)^{1/2} + M_{SAM}\right]$$

$$= 893265 \text{ in-lb}$$

$$S_n = C_1\left(\frac{P_o D_o}{2t}\right) + C_2\left(\frac{D_o}{2I}\right)M_i + \text{local stresses}$$

$$= 8817 + 1.621\left(\frac{12.75}{2 \cdot 475.8}\right)893265 + \text{local stresses}$$

$$= 8817 + 20119 + \text{local stresses}$$

For local stresses:

snubb load = 14000 lb

Thermal expansion constraint = 570 x 22.5 x 1.3 = 16675

(T = 300°F)

as in (i), pressure constraint = 7511

total load = 14000 + 16675 + 7511 = 38186 lb

local stresses = 38186/14000 x 13700 = 37368 psi

$$S_n = 8817 + 20119 + 37368$$

$$= 66304$$

$$K_e = 1.0 + \frac{1-n}{n(m-1)}\left[\frac{S_n}{3S_m} - 1\right] = 2.933$$

$$S_p = S_n$$

$$S_a = \frac{1}{2} K_e S_p = 97223$$

$$N = 1300$$

$$n/N = 0.0385$$

iii) SRV Loadings (Load set 14 & 15), n=7650 cycles

$S_n = C_2 \left(\frac{D_o}{2I}\right)(M_{SRV})_{range}$ + local stresses due to SRV

$M_{SRV} = 88145$ in-lb

$M_{OBE} + M_{SRV} = 289892$ in-lb

$$\frac{M_{SRV}}{M_{OBE} + M_{SRV}} = \frac{88145}{289892} = 0.304$$

The same ratio is used to calculate local stresses due to SRV

local stresses due to SRV $= 13700 \times 0.304$

$= 4165$ psi

$(M_{SRV})_{range} = 2 \cdot M_{SRV}$

$S_n = 1.681 \times \frac{12.75}{2 \cdot 475.8} 2(88145) + 4165$

$= 8135$

$S_p = S_n \qquad K_e = 1.0$

$S_a = \frac{1}{2} S_p = 4068$

$N > 10^6$

$n/N \approx 0$

iv) Thermal & Pressure Loadings for the rest of operating cycles (including startup & shut down)

$p = 1000$ psi, $T = 522°F$

$$S_n = C_1\left(\frac{P \cdot D_o}{2t}\right) + C_2\left(\frac{D_o}{2I}\right) M_{th} + \text{local stress due to thermal \& pressure}$$

$$\text{local stress } (th+p) = (32789 + 7511)/_{14000} \times 13700 = 39436$$

$$S_n = 11601 + 1.681 \ \frac{12.75}{2 \cdot 475.8} \ 116460 + 39436$$

$$= 53660$$

$S_p = S_n$

$$K_e = 1.0 + \frac{1-n}{n(m-1)}\left[\frac{S_n}{3S_m} - 1\right]$$

$$= 1.93$$

$$S_a = \frac{1}{2}(2.039)(53660)$$

$$= 51740$$

$N = 19000$

$$n/N = 380/19000 = 0.02$$

Usage factor $= 0.022 + 0.0385 + 0 + 0.02 = 0.0805$

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4. Conclusion

Based on the worst case, the calculation has shown that the calculated stresses meet the Class 1 code allowables. In addition, it should pointed out that the results are calculated very conservatively. Some of the conservatism are summarized in the following:

i) Based on projected clamp pad bearing area, only 60% of the snubber load will be applied on central pad. 100% load is used in the calculation.

ii) Stress indices can further be reduced if the method outlined on ORNL's report "End effects on elbows subjected to moment Loadings" or stress indices provided in Tables NB3685.1-1 and -2 is used.

iii) WRC bulletin 107 is used to calculate clamp induced local stresses. This method is for lug (integral attachment) on straight pipe, stresses should be lower for clamp (non-integral attachment) or elbows.

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(iv) The operating temperature at the clamp is 150°F. However in calculating thermal constraint stresses the higher temperatures (i.e., 582°F, 522°F and 300°F) are used.

(V) The sign of local stresses is ignored and the local stresses are added absolutely. This is very conservative in the evaluation of stress intensities.

Comparison between the result without clamp effect and that with the clamp effect is summarized in the following table.

Core Spray Line

	W/O Clamp	W/ Clamp
Primary stress	15370*	15901
Usage Factor	0.0032	0.0805

The resuts for Feedwater line are also shown:
(The detailed calculation is not included here)

	W/O Clamp	W/ Clamp
Primary Stress	8104*	7130
Usage Factor	0.0036	0.049

* B_2 values at mid of mitered elbow were used. These values are higher than the B_2 values at the end of elbow.

5. References:

1. Roark R. J. "Formulas For Stress and Strain", McGraw Hill, 4th Edition.

2. WRC Bulletin 107, "Local Stresses In Spherical and Cylindrical Skills due to External Loading".

3. "ASME Boiler and Pressure Vessel Code", Section III

4. Rodabaugh E. C., Iskander S. K. and Moore S. E., "END Effects on Elbows Subjected to Moment Loadings", March, 1978, ORNL. (Attachment 2)

5. Severud L. K., "Experience with Simplified" Inelastic Analysis of Piping Designed for Elevated Temperature Services", ASME Century 2 Nuclear Engineering Conference, August 19-21, 1980.

6. Markl A. R. C., "Piping-Flexibility Analysis", ASME Transactions, 1955 (Attachment 3)

7. T. Kano, K. Iwata, J. Asakura, and H. Takeda, "Stress Distributions of an Elbow with Straight Pipes", F 1/5 Transactions of the 4th International Conference on SMIRT. (Attachment 4)

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8. Load histogram - Core Spray for Limerick

CALCULATION SHEET

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CALC. NO. SK8031-SS10 REV. NO. 0

ORIGINATOR S. K. Hu DATE 4-18-83 CHECKED AYH DATE 4/19/83

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SUBJECT Clamp Induced Stresses SHEET NO. Attachment 1 1 of 2

From page 9, it is noted that the stresses in pipe wall induced by specified clamp preload is insignificant. Although this stress is one time nonrecurring load (i.e., no effect on fatigue life), the stress on central pad will be incorporated into eq. 9 to assess the effect of this preload.

By using eqs on page 23 & note

Preload on central pad = (uniform pressure) x (pad area) x (contact ratio)

$$= 242 \times 22.5 \times 1.3$$

$$= 7079 \text{ lb}$$

Preload stress on central pad area $= \frac{7079}{14000} \cdot 4300 = 2175$ psi

Eq. 9 $= 2175 + 15901$

$$= 18075 < 1.5 S_m = 20985 \text{ psi}$$

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Attachment 2

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THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
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Experience with Simplified Inelastic Analysis of Piping Designed for Elevated Temperature Services

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Screening rules and preliminary design of FFTF piping were developed in 1974 based on expected behavior and engineering judgment, approximate calculations, and a few detailed inelastic analyses of pipelines. This paper provides findings from six additional detailed inelastic analyses with correlations to the simplified analysis screening rules. In addition, simplified analysis methods for treating weldment local stresses and strains as well as fabrication induced flaws are described. Based on the FFTF experience, recommendations for future Code and Technology work to reduce design analysis costs are identified.

NOMENCLATURE

- a = Neuber equivalent grain half-length
- C_2 = Stress index = $1.95/\lambda^{2/3}$
- $\bar{C}_2$ = Weld shrinkage stress index
- D_o = Outer diameter of pipe
- E = Young's modulus
- h = Notch depth
- K_N = Fatigue strength reduction factor
- K_2 = Local stress index
- K_t = Elastic stress concentration factor
- K_T = Factor applied to peak thermal strain component
- K_ε = Elastic strain concentration factor
- N = Number of applied cycles
- N_d = Number of design-allowed cycles
- P = Primary stress intensity
- Q = Secondary stress intensity
- R = Bend radius of elbow
- r = Mean pipe radius
- $\bar{S}$ = Screening stress limit
- $3\bar{S}_m$ = Allowable design stress intensity range limit
- S_{mc} = Stress limit at cold end of stress range
- S_{rh} = Stress limit at hot end of stress range
- T = Time of applied stress
- T_D = Allowable time for design
- ΔT_1 = Linear thermal gradient temperature range through wall
- t = Pipe wall thickness
- α = Coefficient of thermal expansion
- β = Carry over factor
- λ = Pipe factor = $\frac{tR}{r^2}$
- ν = Poisson's ratio
- θ = Flow shape parameter
- σ_{TE} = Maximum thermal expansion secondary bending stress
- $\sigma_{\Delta T1}$ = Maximum radial gradient thermal stress
- σ_{DW} = Dead weight stress
- σ_p = Pressure stress
- σ_{eff} = Effective stress
- ε_{eff} = Effective strain
- ε_T = Total effective strain
- ε_e = Elastic strain
- ε_p = Plastic strain
- ε_F = Peak thermal strain

INTRODUCTION

The Fast Flux Test Facility (FFTF) piping design activities have progressed from the preliminary design in the early 1970's through detailed ASME Section III code analyses including detailed inelastic analyses for elevated temperature operation. Design activities concluded in 1979 with final as-built reconciliation of stress reports for construction and installation modifications. An overview of the flow of these activities is provided by Figure 1. As described in 1975 [1]*, significant project cost and schedule benefits can be obtained if screening rules and simplified analyses can be used to confidently identify a pipeline configuration that will pass detailed stress analysis code limits. The screening rules for preliminary design [1] used on the FFTF project have served very well. Detailed ASME Code analyses using elastic methods [2] and inelastic methods [3 - 5] have demonstrated that all Code design rules and limits are met. Accordingly, a correlation of the detailed inelastic analysis findings with the simplified analysis screening rules will be presented.

Contributed by the Nuclear Engineering Division of the American Society of Mechanical Engineers for presentation at the Century 2 Nuclear Engineering Conference, San Francisco, Calif., August 19-21, 1980. Manuscript received at ASME Headquarters March 17, 1980.

*Numbers in brackets indicate references

Before presenting the correlations, a short overview of the screening rules and background will be given. After the comparisons, simplifications in the detailed inelastic analyses and supplementary simplified inelastic analyses will be discussed. Finally, the paper concludes with recommendations for future code and technology work to reduce design analysis costs.

SCREENING RULES AND BACKGROUND

The preliminary design screening rules and limits [1] for primary stresses, shown in Figure 2, were easy to meet due to the low design pressures of the FFTF piping. These low primary stress levels helped control ratcheting and stress rupture damage at low levels.

Screening limits for primary plus secondary stress ranges (See Figure 3) considered limits associated with creep ratchet, creep fatigue, and shakedown. These limits were applied to the very simple screening equation of:

$$\sigma_{TE} + \sigma_{\Delta T_1} \leq \bar{S} \qquad (1)$$

where:

σ_{TE} = Maximum secondary bending stress in pipeline, usually at an elbow.

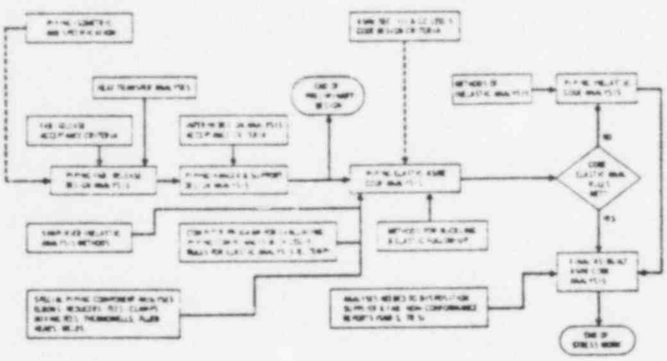

FIGURE 1. General Ingredients and Flow of Piping System Design Analysis, ASME III, I.

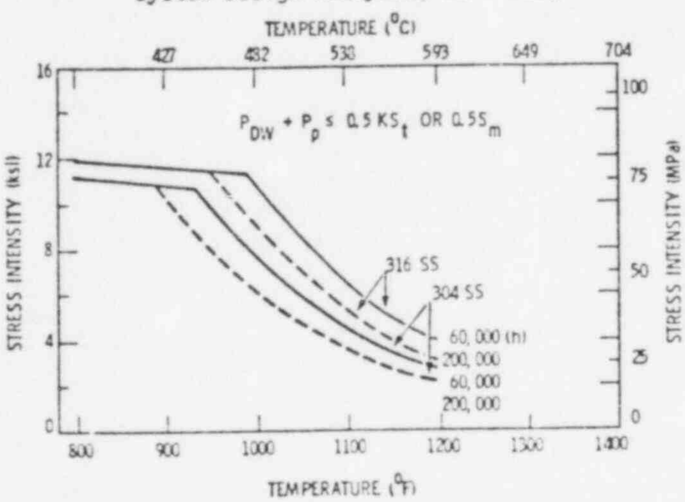

FIGURE 2. Preliminary Design Limits for Pressure and Weight Stresses Using 50% of Code Case 1311-8 Primary Stress Limits.

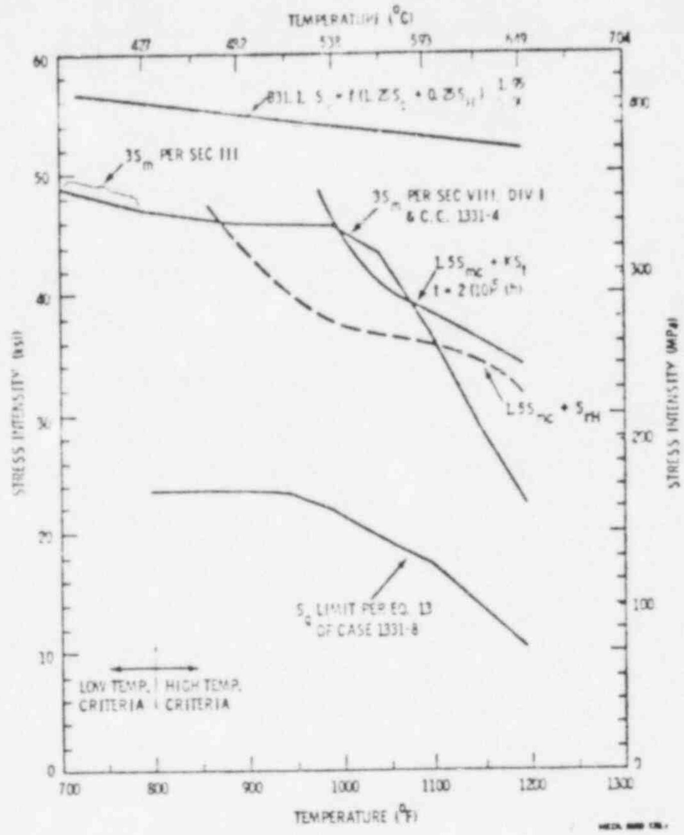

FIGURE 3. Primary Plus Secondary Stress Limits for 316 SS.

$\sigma_{\Delta T_1}$ = Maximum thermal shock radial gradient stress considering all of the plant thermal transient and equal to

$$\frac{E\,\alpha\,(\Delta T_1)}{2(1-\nu)}$$

$\bar{S}$ = An allowable stress intensity considering creep fatigue, creep ratcheting, and experience factors.

A major feature of the screening rules and limits is the shakedown and relaxation of stress during the hold-time providing the transient stress range is always less than the "elastic action" range. This is depicted in Figure 4. "Elastic action" range for primary plus secondary stress range P + Q is defined by:

$$3\,\bar{S}_m = 1.5\,S_{mc} + S_{rh} \qquad (2)$$

where the standard ASME Code terminology of the ASME Code Case 1592, Paragraph T-1325 Test No. 4 [18] is used.

If the $(P + Q)_R$ exceeds $3\,\bar{S}_m$, then the relaxation of stress could be as shown in Figure 5 and not similar to monotonic relaxation and not affected by the transients as shown in Figure 4.

Based on approximate calculations, expected behavior, engineering judgment, and a few detailed inelastic analyses of pipelines [9],[3], it was deemed important to provide enough flexibility into the piping isometric designs to satisfy Equation (1) and keep the $(P + Q)_R$ less than 3 $\bar{S}_m$. Results from inelastic analyses of pipelines is given in Figures 6 and 7.

Additional background is presented in Reference [1].

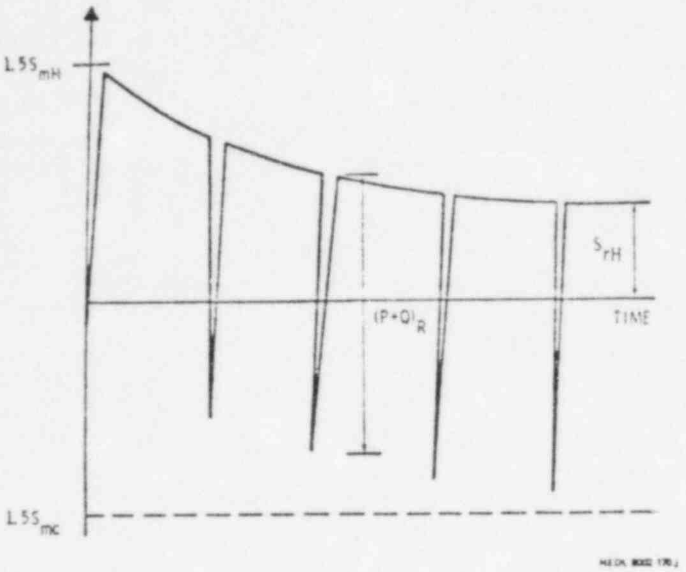

FIGURE 4. Typical History for Pipeline Primary Plus Secondary Stress When $(P+Q)_R \leq 1.5S_{mc} + S_{rH}$.

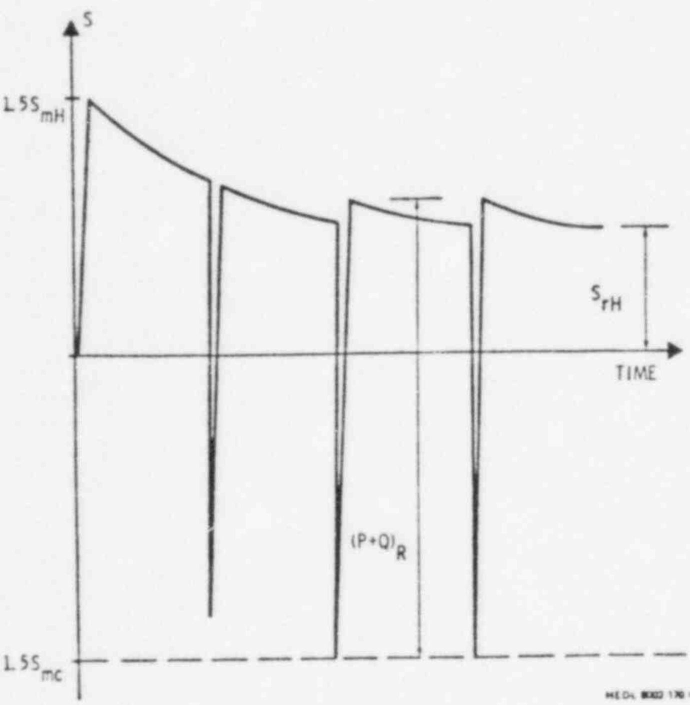

FIGURE 5. Typical History for Pipeline Primary Plus Secondary Stress When $(P+Q)_R \leq 1.5S_{mc} + S_{rH}$.

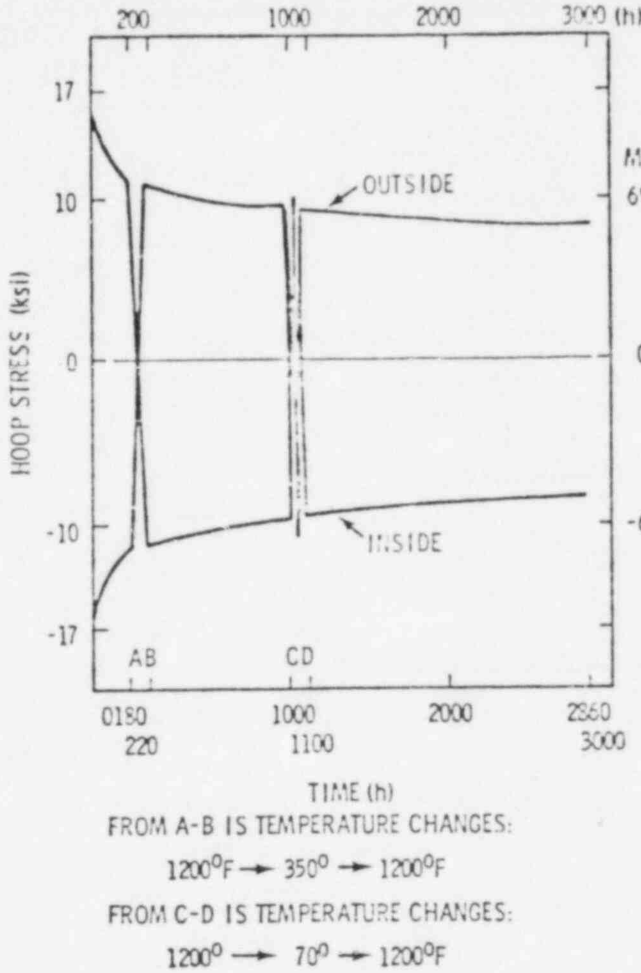

FIGURE 6. CLS Pipeline Inelastic Analysis Results (Ref. 3).

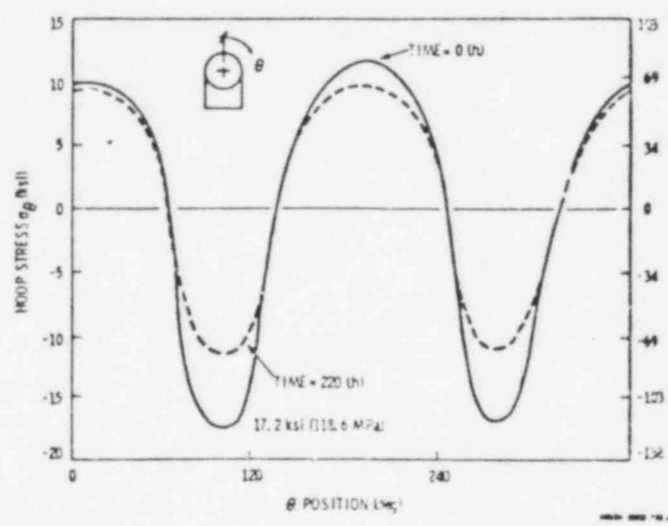

FIGURE 7. Hoop Stress Distribution in Criteria Elbow Inside Surface (Ref. 3).

CORRELATIONS OF DETAILED INELASTIC ANALYSIS FINDINGS WITH SIMPLIFIED ANALYSIS SCREENING RULES

Table 1 presents the correlation data. The detailed inelastic analyses include pipelines of simple, short runs as depicted in Figure 8 and

TABLE 1. CORRELATION OF DETAILED INELASTIC ANALYSIS FINDINGS WITH SIMPLIFIED ANALYSIS SCREENING RULES

Pipeline Description*			Stress Analysis					Detailed Inelastic Analysis				
			Stress Intensity Values (ksi)**					Inelastic Strain (%)***		Creep-Fatigue Damage****		
Diam (in.)	Temp (°F)	Ref (No.)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Avg	Linearized	Peak	Creep	Fatigue
3	1200	3	1.0	1.0	---	---	36.7	<0.19	<0.19	0.19	0.42	[illegible]
[illegible]	1200	5	1.0	1.0	6.4	20.8	27.6	0.15	0.22	0.75	0.17	[illegible]
4	1195	---	1.2	1.1	25.1	12.6	[illegible]	0.10	0.31	0.33	0.24	[illegible]
8	945	5	1.3	3.3	25.6	29.2	48.2	0.36	0.17	0.40	0.31	[illegible]
16	1050	4	2.1	4.8	10.8	17.0	25.8	0.18	0.22	0.29	0.21	0.007
28	1050	---	2.4	6.5	8.3	10.0	18.3	0.33	0.32	---	0.19	0.001

* All pipelines are 316 SS except for the 8 inch pipeline, which is 304 SS.

** Screening limits [illegible] = 4.0 ksi @ [illegible], [illegible] h, 316 SS
= 7.0 ksi @ [illegible], 133,000 h, 304 SS
= 7.0 ksi @ [illegible], 133,000 h, 316 SS
[illegible] = 24.0 ksi @ 1200°F
= 25.7 ksi @ [illegible]
= 30.0 ksi @ [illegible]

*** Code limits on strain are 1%, 2% and 5% for average, linearized and peak, respectively.

**** Code limits on creep-fatigue is D, which approaches 1.0 due to relatively small fatigue in comparison to creep damage. The creep-fatigue damage for the 16-inch and 28-inch pipelines [illegible] at the outer surface of the elbow end welds. All other pipelines had creep-fatigue damage governed by the elbow and midsection stresses and strains.

Conversion Factors: 1 ksi = 6.9 MPa, 1200°F = 704°C, [illegible], 1050°F = 566°C,
1 in. = 25.4 mm

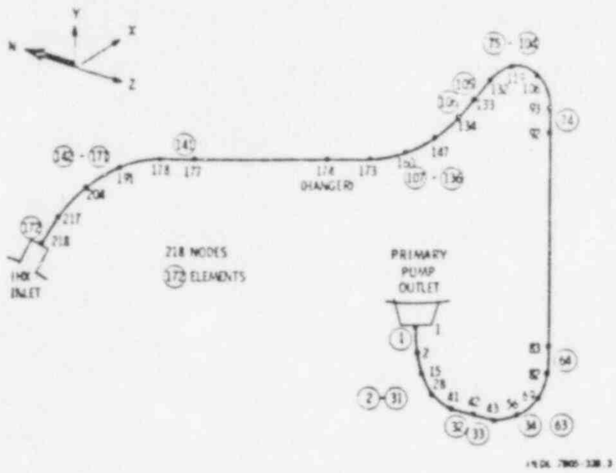

FIGURE 8. Primary Crossover Piping Mesh (Ref. 4).

fairly complex, long runs as depicted in Figures 9 and 10.

The simplified analysis screening values for the pipelines of Table 1 are cross-plotted on the screening rule limit curves in Figures 11 and 12.

The pipelines chosen for inelastic analyses had the highest simplified analysis screening values and the least design margins. The design margins were identified by detailed elastic ELTEMP [2] and simplified inelastic analyses such as the full relaxation Bree [7] and the O'Donnell-Porowski [8] methods. Accordingly, since even the most severely loaded FFTF pipelines were demonstrated to meet all the code requirements by inelastic analyses, many other FFTF pipelines with similar but less severe loads and thermal transients are also qualified. However, for these pipelines, some of the very conservative code elastic analysis limits (such as the S_q limits) were exceeded. See Figure 12.

SIMPLIFICATIONS IN DETAILED INELASTIC ANALYSES

Detailed inelastic analyses of piping do require some simplifying in modeling and analysis procedures to keep the analysis costs within reasonable limits. These simplifications are technically justified by the satisfaction of ad hoc rules and conservative modeling. The simplifications in modeling include:

- Use of constant bending elbow elements (Type 17, of the MARC computer program [6])
- Extrapolating elbow midsection stresses and strains to those for the elbow end weldments by use of "carry-over" factors and indices to account for nonuniformities introduced during the fabrication and welding of an elbow to a straight pipe section
- Use of indices and fracture mechanics crack-growth models to assess local peak stresses and strains

In general, detailed inelastic analyses of a pipeline system provide primary and secondary stress effects. Substructuring techniques or use of indices are needed to account for peak stresses and strains. The simplification in inelastic analysis procedures include:

- Enveloping and lumping of thermal transients
- Extrapolating ratchet and elastic followup strains to end of design life

The technical bases for some of the simplified methods identified above will be discussed in the following paragraphs.

Constant Bending Elbow Elements

The technical justification for accepting use of the constant bending elbow elements depends on each pipeline analysis application. Considerations include findings from prior elastic analyses of the pipeline such as the level of stress expected, the ratio of in-plane to out-of-plane bending on each elbow, and the number of elbow element segments used to model each elbow. Hibbitt [9] and Pan and Jetter [3] discuss the limitations of the constant bend element. Figure 13 shows a typical model of a 90° elbow using three 30° segments, 16 elements around the circumference and 11 layers through the wall.

A basic step in justifying the number of segments, number of elements around the circumference, and the number of through-wall layers to model each elbow in a pipeline was the comparison of elastic stress levels, moments, forces and displacements computed using the inelastic pipeline model to those computed using conventional elastic analysis pipeline models typical of analyses to NB-3600 of Section III of the ASME B&PV Code. Correlations to within 5% to 10% were judged acceptable. Moreover, for large diameter, thin wall elbows typical of breeder reactor plant piping, comparisons of elbow detailed shell finite element or finite difference elastic analysis findings with constant bending elbow element findings were also utilized. As seen in Figure 14a, the degree of ovalization varies around the elbow arc. Ovalization of 50% to 60% of the maximum at the elbow midsection exists at the junction of the straight tangent pipe. However, the elbow net elastic flexibility has been shown by tests [10 - 12] to be adequately predicted by use of the simple formula of $k = 1.65/h$ given in the ASME Code, NB-3600 [13]. This formula neglects local flexibility distribution and varying ovality along the elbow arc. This ovality also penetrates one to two diameters into the tangent straight pipe portion (Figure 14b & 14c). The code approach involves the flexibility factor as a constant factor applied to the elbow arc portion (Figure 14d). Accordingly, it is an "effective" flexibility factor for modeling the total elbow and effective tangent pipe flexibility for use in the total pipeline system flexibility analysis.

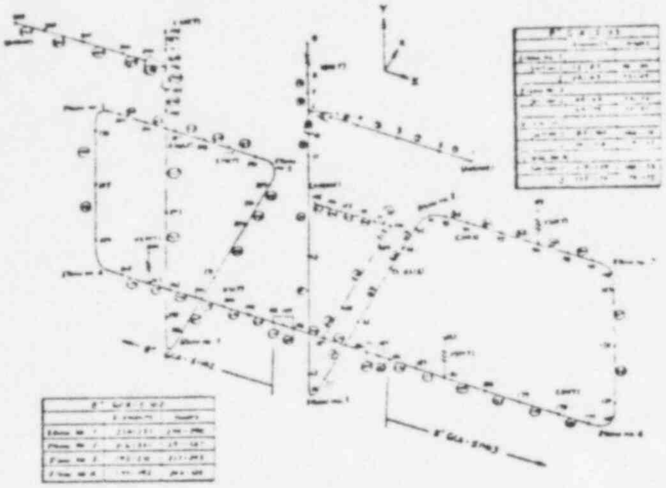

FIGURE 9. 8-inch SHL Inelastic Analysis Finite Element Model (Ref. 5).

As previously noted, test data [10 - 12] have been used to develop and have confirmed the simplified flexibility methods and models for elbows with straight tangents. The highest elastic stress indice in the elbow midsection has also been shown to be adequately predicted by use of the Code formula [13] of $C_2 = 1.95/\lambda^{2/3}$. Therefore, by assuring that the constant bending elbow elements used in the pipeline model provide numerical values of stress and deformation for elastic loading that correspond with sufficient accuracy to those computed using the standard Code formula and flexibility methods, the model is deemed adequate for use in the inelastic analyses, provided plasticity and creep effects are limited as discussed below.

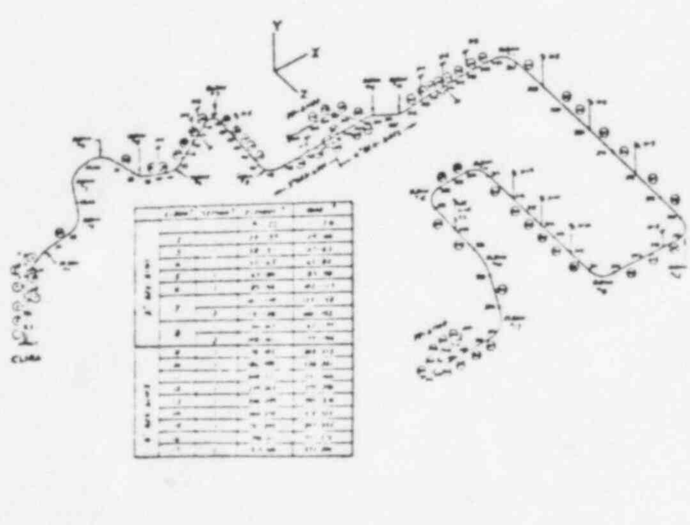

FIGURE 10. Analytical Model of System 61 Primary Hot Leg Loop No. 1 (Ref. 5).

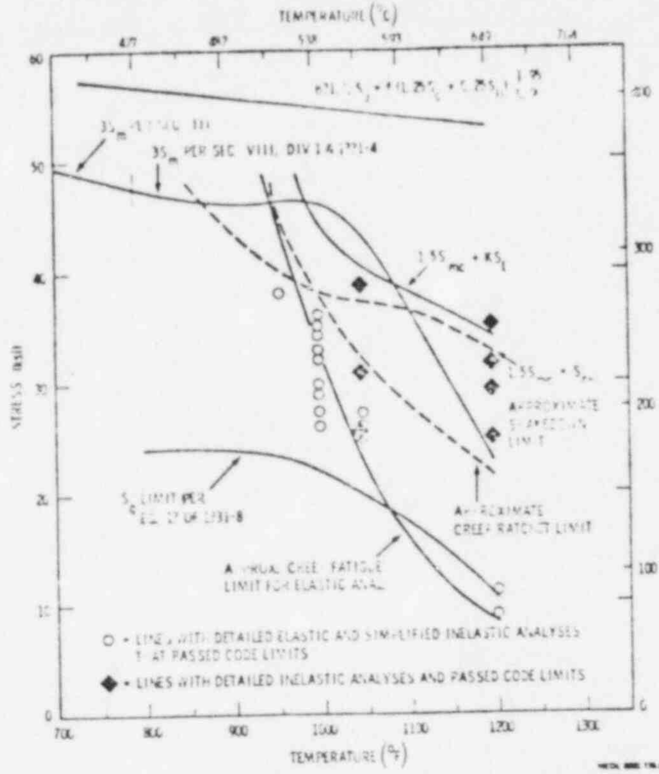

FIGURE 11. Primary Plus Secondary Stress Range Limits for Preliminary Design of 316 SS Pipelines.

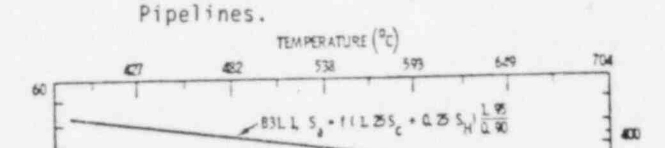

FIGURE 12. Preliminary Design Primary Plus Secondary Stress Limits for 316 SS Pipelines.

The thermal expansion and thermal radial gradient maximum elastically-calculated stresses imposed on the elbows were, for FFTF, kept less than the 3 S_m level. This limited the plasticity and creep effects to local and small portions of the pipeline wall.

Thus, many elbow element segments necessary to capture stress redistribution associated with gross plasticity and creep throughout the elbow were not needed. In addition, as the pipeline response was expected to "shakedown" (Figures 4 and 6), good margins between the Code limits and the calculated values of accumulated inelastic strain and creep-fatigue damage were expected to offset possible limitations associated with the approximate elbow model.

Some details of the elbow models used in FFTF inelastic analyses are given in Table 2. Generally, the FFTF analyses [3, 4, 5] found two or three elbow element segments with 14 to 18 elements around the circumferences and 10 or 11 layers through the wall were sufficient at reasonable cost.

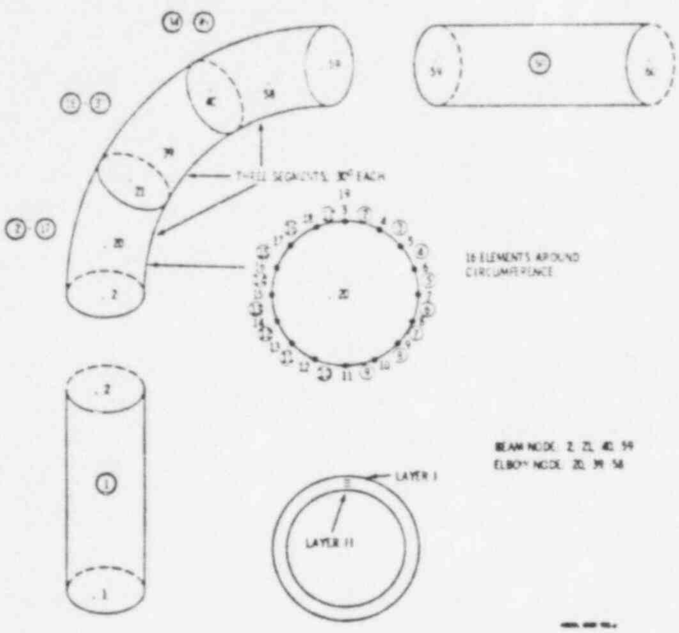

FIGURE 13. Typical Constant Bending Elbow Elements.

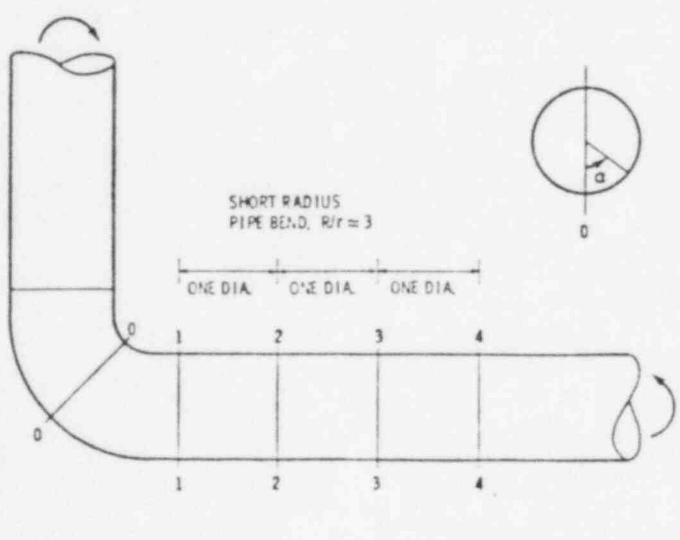

FIGURE 14a. Typical Variations of Stress and Ovalization Along Pipe for In-Plane Bending.

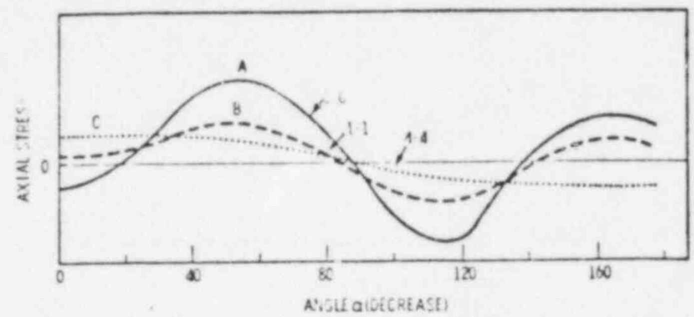

FIGURE 14b. Typical Distribution of Axial Surface Along Pipe for In-Plane Bending Load.

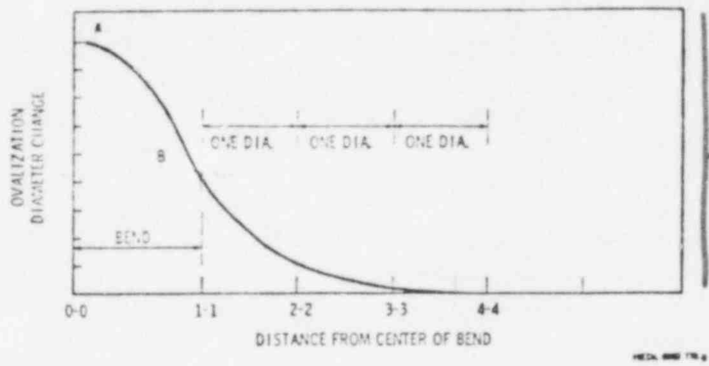

FIGURE 14c. Typical Variation of Ovalization Along Pipe for In-Plane Bending Load.

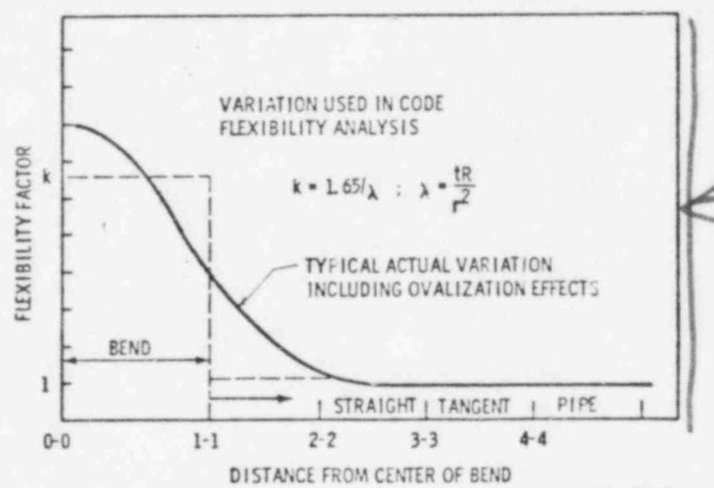

FIGURE 14d. Flexibility Factors Along Pipe for In-Plane Bending Load.

Stresses and Strains at Elbow End Weldments

Elbows are often attached to straight portions of piping by girth weldments located at the junction of the elbow torus and the tangent straight pipe. Stresses and strains at the elbow midsection were found by Markl [14, 15] to govern the fatigue life of pipe elbows tested below temperatures where creep effects are significant. Accordingly, the ASME Code [13] in NB-3600 provides stress indices for butt welding elbows that are based on midsection stresses, but a $D_o/t \leq 100$ is required. For FFTF, the largest D_o/t is 64. As the D_o/t ratio gets larger, the stresses at the elbow and weldments

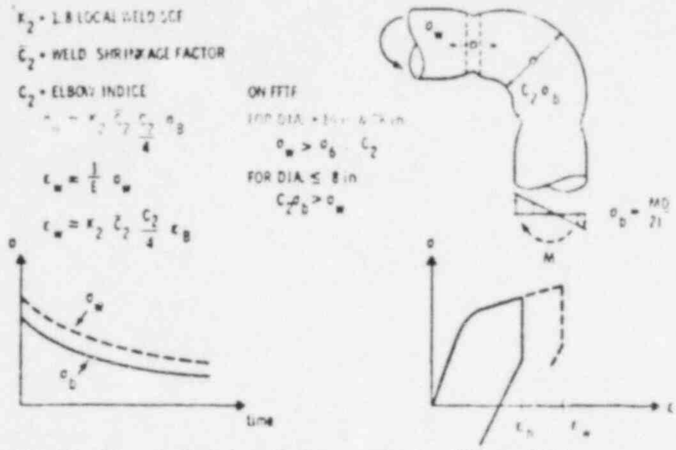

FIGURE 15. Relating Weld σ and ε to Elbow Midsection σ and ε Due to Moment Loading.

TABLE 2. ELBOW MODEL DESCRIPTIONS

Diam (in.)	Wall t (in.)	(mm)	Bend Rad R (in.)	h	Angle of Bends	No. of Circum Elements	No. of Layers Through Wall	[illegible]
3	0.216	5.5	4.5	0.43	90	10	---	2
3	0.216	5.5	4.5	0.43	90	14	11	2 elbows had 2 6 elbows had 1
4	0.237	6.0	6.0	0.36	45 to 90	14	11	8 elbows had 1
4	0.237	6.0	6.0	0.36	90 to 180	18	11	3 elbows had 2 6 elbows had 1
8	0.322	8.2	12.0	0.28		16	11	4 elbows had 2 4 elbows had 1
16	0.375	9.5	24.0	0.14	45 to 90	10	11	3
28	0.375	9.5	42.0	0.08	90	18	10	2 elbows had 1 3 elbows had 2 7 elbows had 1

1 in. = 25.4 mm

may become larger than at the elbow midsection. Considering the limitations of Markl's test data, it was decided to also calculate stresses and strains at the elbow end weldments, in addition to those at the elbow midsection for the FFTF.

Detailed inelastic analysis of pipelines using the constant bending elbow element (No. 17 of the MARC finite element computer program) [6, 9] do not account for the secondary stresses due to fabrication mismatch or radial shrinkage at the welds. In addition, peak stresses and strains due to weld surface irregularities, etc. are not directly included in the pipeline inelastic model. Accordingly, a simplified method of evaluation was devised and conceptually described in Figure 15.

The simplified method consisted of using the pipeline system model to predict inelastic response for primary and secondary stresses excluding fabrication mismatch and weld local effects. Stresses and strains at the elbow end weld joints were then approximated using the values computed for the elbow midsection combined with carryover and shrinkage factors.

The carryover factors were determined from detailed shell finite element, finite difference analyses of the FFTF elbow designs, and from consideration of the experimental data [10, 11, 12] on ovality distribution such as shown in Figures 14b and 14c. For the FFTF applications, a carryover factor of 1/2 was found conservative. However, higher factors may be needed for larger and thinner-walled pipe elbows.

Radial welds on FFTF to join the seamless pipe and machined elbows were done by an automatic welding machine. The weld reinforcement and surface irregularities were much milder than typical manual welds. Fabrication alignment and mismatch tolerances and the welds were all kept within Code limits. Accordingly, the Code stress indices tied to the fabrication limits were considered appropriate to account for local stress concentrations and fatigue strength reduction factors. Thus, a local indice for the girth welds based on the Code [13] was taken to be 1.8 in magnitude. The Code does not have a factor for radial weld shrinkage effects.

Discontinuity stresses of the type depicted in Figure 16 and due to radial shrinkage in thinwall piping, were approximated by elastic analysis of a number of shell shrinkage distributions and $R/_t$ ratios. Based on these findings and code indices for girth welds, the special indices of Figure 17 were adopted. These indices were intended for use in predicting the maximum stresses and strains at welds in the pipe axial direction because the K_2 = 1.8 local factor was considered an axial fatigue strength reduction factor. An appropriate K_2 value relative to the pipe/elbow hoop direction was judged to be ~1.1 to 1.2.

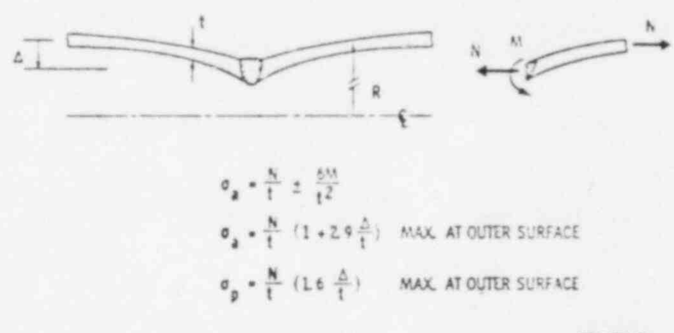

FIGURE 16. Discontinuity Stress Due to Radial Shrinkage in Welds.

To obtain approximate values of elbow and weld maximum stresses and strains for comparing to the elbow midsection stresses, the method was as depicted in Figure 15. The 1/4 factor is based on the 1/2 carryover factor and the maximum axial stress of ~1/2 of the maximum hoop stress. Table 3 shows combined indices for the various FFTF pipe sizes for both elbow midsections and elbow end weldments. Note that as the diameter gets larger, the weld indices are larger than for the bend midsection. These indices were used with the elastic flexibility analyses of the pipelines and are based on shrinkage and mismatch data of Table 4.

To obtain stresses and strains at elbow end weldments for use with the inelastic analysis code evaluation, a simplified method was used. The hoop and axial maximum stress and strain values computed for the elbow midsections, using the inelastic pipe analysis, were first examined. The values at the elbow end weldments, exclusive of weld shrinkage and configuration peak stress effects, were taken as 1/2 of the midsection values. The radial weld shrinkage produces shell bending under axial membrane load. One approach is to apply the stress indices of Figure 17 as multipliers with weld peak stress indices to the calculated effective stress and strain, exclusive of weld effects. That is:

$$\text{weld } \sigma_{eff} = \left(\tfrac{1}{2}\, \beta\, \bar{C}_2\, K_2\right) \sigma_{eff} \text{ at elbow midsection} \quad (3)$$

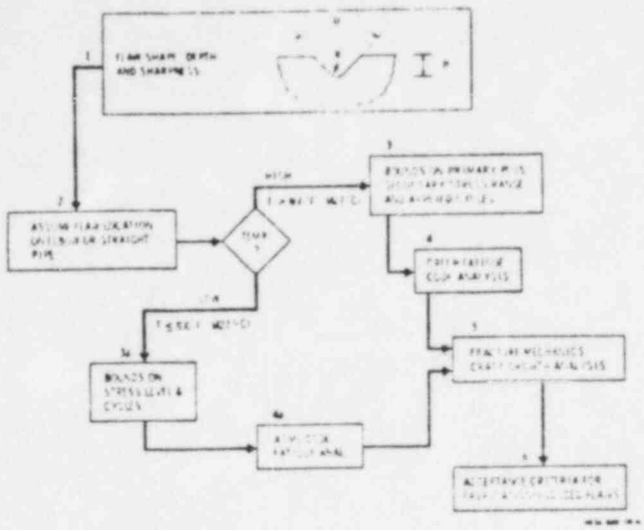

FIGURE 20. Analysis Program for Developing Acceptance Criteria for Fabrication-Induced Surface Flaws.

Crack-Growth Analysis

In the design of FFTF piping, the range of primary plus secondary stresses in pipe fittings such as elbows and tees, were limited to a value of 3 $\overline{S}_m$ as given in Table 5. Due to the radial shrinkage of the weldments joining the fittings to the straight pipe sections, the welds also represent location of increased stresses. Outside the fittings on straight pipe section surfaces, the maximum applied stress range is about half that of the fitting (i.e., $1/2 \cdot 3\ \overline{S}_m$).

The evaluation of the flaws were divided into two stages; crack initiation and crack propagation. Normally, a non-flawed smooth surface will require many cycles of stressing before a small crack will develop. However, a notched or flawed surface can initiate a crack very early in the part life and then the question shifts to how fast will the crack grow. Figure 22 shows the threshold flaw size calculated for the crack to grow under various applied stress ranges. For maximum allowable design stresses the threshold sizes are given in Table 6.

The crack-growth fracture mechanics analyses were accomplished conservatively assuming that the surface flaw, which is normally not as sharp as a crack, to be a crack. The determination of the applied stress intensity, ΔK, was based on the methods of Section XI of the ASME Boiler and Pressure Vessel Code. Other formula based on the work of Hsu and Liu [24] and Shah and Kobayashi [25] were also employed for further insight.

The crack-growth analyses indicate the growths are fairly sensitive to stress level (see Figure 23) but very little growth is expected below 800°F (427°C). See Figure 24.

The crack-growth rate and threshold stress intensity data (see Figure 25 and 26) used were based on work by James [22, 23]. To account for long-time high-temperature effects, an environmental rate acceleration factor (Figure 27) was obtained by extrapolation.

TABLE 5. PIPING STRESS RANGE BOUNDS

Location	Oper Temp (°F)	Oper Temp (°C)	$P_L + P_B + Q_R$ Max Value (ksi)	$P_L + P_B + Q_R$ Max Value (MPa)	Max Cycles
Elbow Midsec.	800	427	50	345	843
Elbow Midsec.	1050	566	35	240	843
Elbow Midsec.	800 & 1050	427 & 566	4	28	10^9
Elbow Ends	800	427	25	172	843
Elbow Ends	1050	566	18	124	843
Elbow Ends	800 & 1050	427 & 566	2	14	10^9
Straight Pipe	800	427	18	124	843
Straight Pipe	1050	566	14	97	843
Straight Pipe	800 & 1050	427 & 566	2	14	10^9

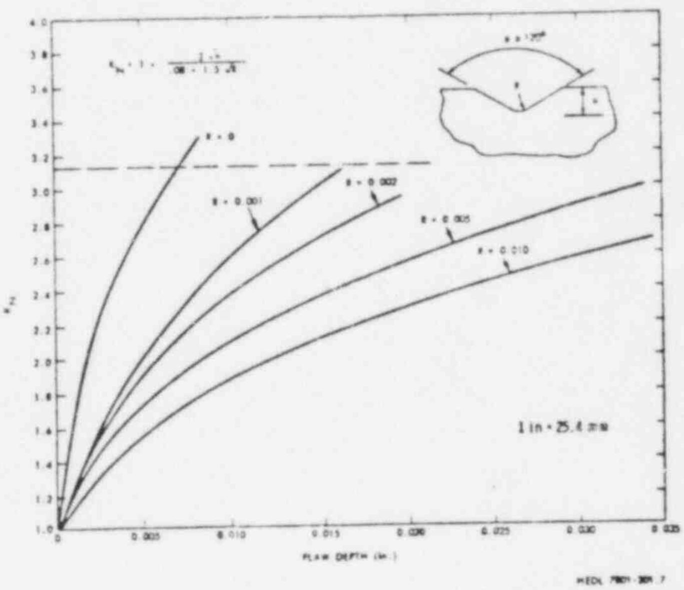

FIGURE 21. Typical Fatigue Strength Reduction Factors K_N for Drill-Induced Flaw Shapes.

An example of the creep-fatigue and crack-growth analyses findings is given in Table 7. As shown in Table 7, the high-temperature elbow midsections are located where such flaws may cause non-satisfaction of the creep-fatigue criteria. However, the high-temperature elbow ends and straight pipe sections do have adequate creep-fatigue margins. When a drill-induced 0.010-inch (0.025-mm) deep blemish, which is not as sharp as a crack, was assumed a crack, its growth was predicted at ~ 0.042 inches (0.10 mm) for the high temperature

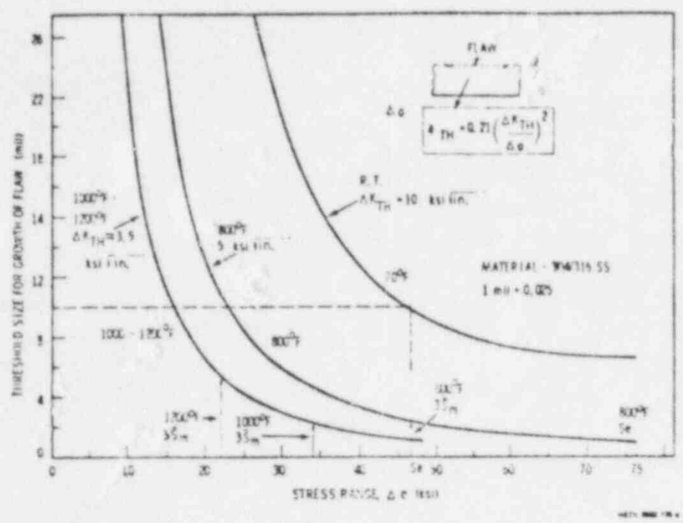

FIGURE 22. Estimated Thresholds for Flaw Growth.

TABLE 6. THRESHOLD FLAW SIZE FOR CRACK GROWTH

Stress, $\Delta\sigma = 3\ \bar{S}_m$, Elbow, Tee, etc. (°F)	(mil)*	Stress, $\Delta\sigma = 1/2 \cdot 3\ \bar{S}_m$, On Pipe Straight Section (mil)*
70	6	25
800	2.5	10
1000	2.0	8
1200	5.0	18

* 1 mil = 0.025 mm

elbow midsection. The other locations had very small growth, < 0.001 inches. The wall thickness, in this case, was ~3/8 inch (9.5 mm).

The high-temperature crack-growth results are fairly uncertain due to the very rapid change in growth rate as the applied stress and effective stress intensity change (see Figures 25 and 26) and the cyclic time change. This time-dependent effect is often referred to as a "frequency" or "hold-time" effect (see Figure 27). Thus, although the crack extension for a 0.010-inch (0.025 mm) crack-like flaw in a high-temperature high-stressed elbow midsection is calculated to be 0.042 inch (0.10 mm), it could actually be much larger or much smaller. A 50% increase in stress level would increase the predicted crack growth to 0.13 inch (3.3 mm). A 50% increase in depth, 0.010 inch (0.025 mm) to 0.015 inch), (0.037 mm) results in a predicted crack growth of 0.27 inch (6.9 mm).

Above 800°F (427°C), creep effects can greatly enhance the crack-growth rates and reduce the low-cycle fatigue life. If the maximum allowable code design stress is developed during operation, a very small flaw, as little as 2 to 4 mil deep, may grow during the design cyclic life to unacceptable levels. Thus, elbows, which do have local areas stressed to the Code limits, should have all surface flaws removed. In straight pipe sections, the operating stresses are usually less than half those in the elbows. Round bottom flaws up to 10 mil deep may be tolerated with no significant adverse effects on the piping fatigue integrity, provided all operating vibratory induced stresses are as low as expected.

For operation below 800°F (427°C), where creep effects are insignificant, the crack growth is slow and the low-cycle fatigue life for a given cyclic stress level is greatly increased. Thus, low temperature (below 800°F) piping flaws anywhere on the piping, up to 10 mil in depth, could be tolerated.

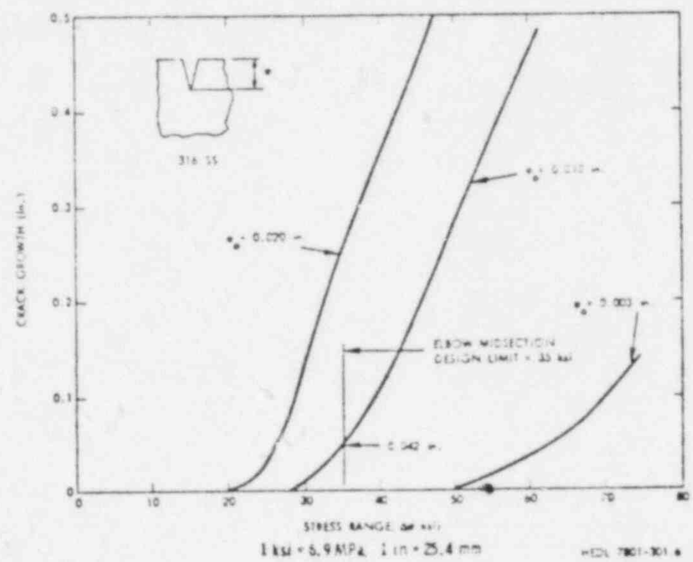

FIGURE 23. Crack-Growth Rate vs Stress for 1050°F.

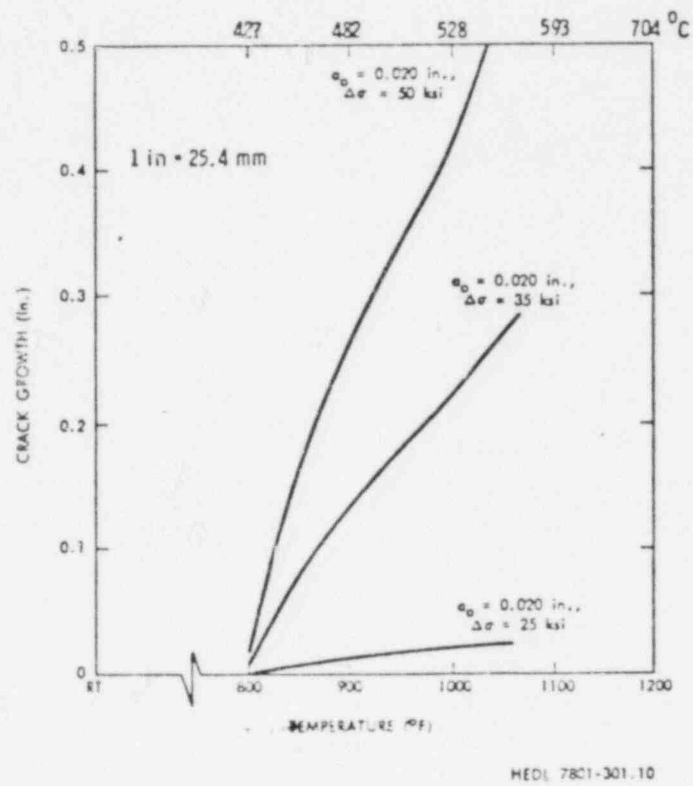

Figure 24. Crack Growth vs Temperature.

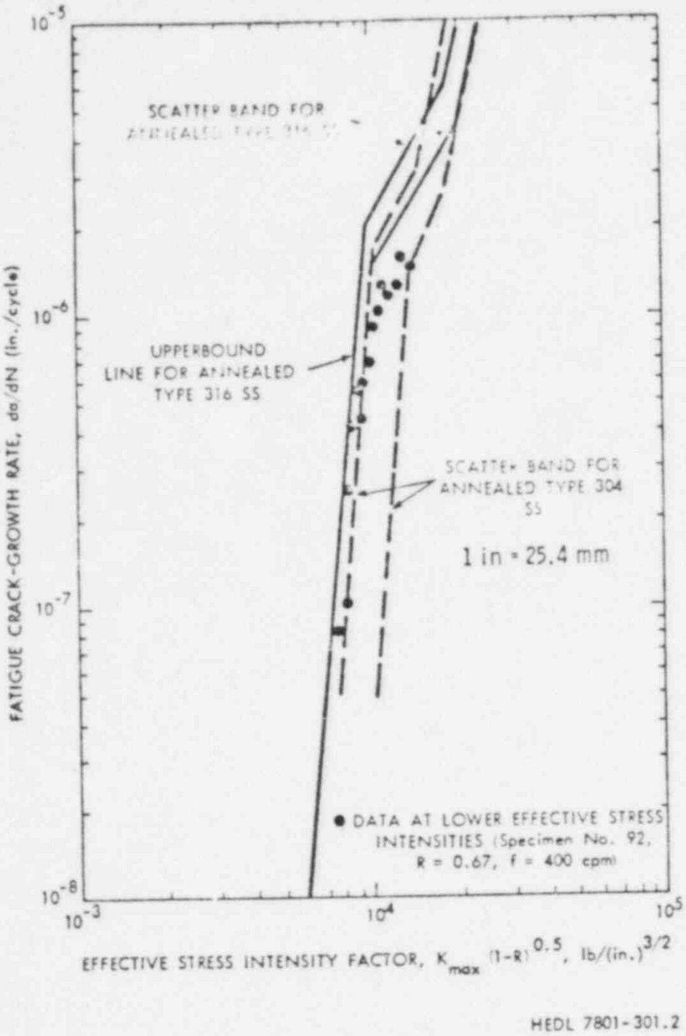

FIGURE 25. Upper Bound for Fatigue-Crack Propagation Behavior of Annealed 316 SS in an Air Environment at 1000°F for the Full Range of Effective Stress Intensities.

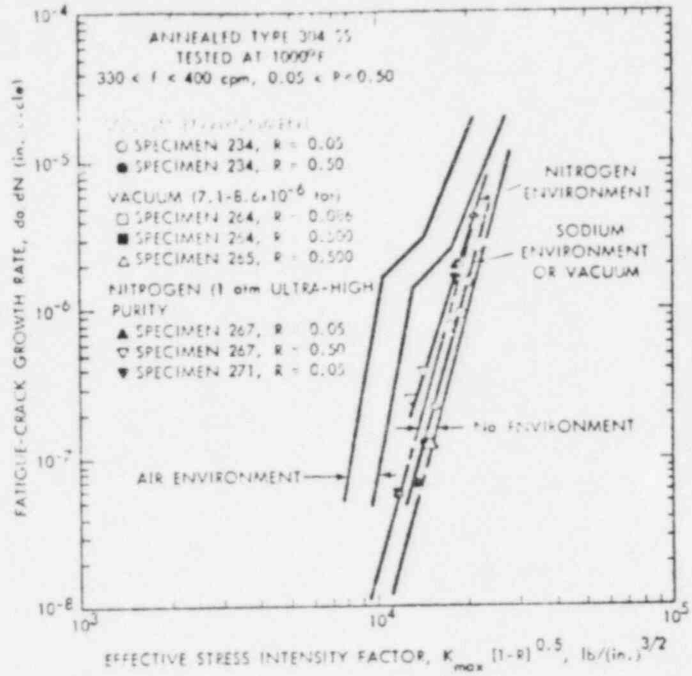

FIGURE 26. Fatigue-Crack Growth Behavior of Annealed 304 SS in Sodium Vaccuum Nitrogen, and Air Environment at 1000°F.

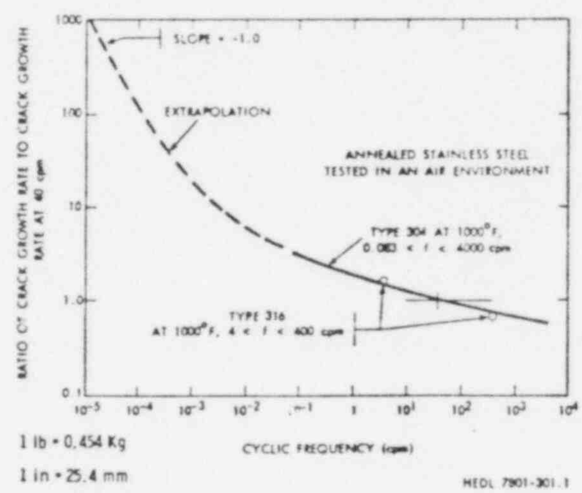

FIGURE 27. Frequency Effects on Crack Growth for Stress Intensities Over 15000 lb/(in.)$^{3/2}$

From the findings presented and from other analyses and considerations, it was concluded that, in general, high-temperature straight pipe will tolerate drill holes and surface flaws on the order of 0.015-inch (0.037 mm) deep. Low-temperature (below 800°F) large pipe and elbows will tolerate 0.025-inch (0.64-mm) deep drill holes with no need for blending.

Based on the creep-fatigue and crack growth fracture mechanics stress analyses, limits were developed that depend on whether the flaw is located on a piping fitting (such as an elbow) or on a straight section of piping, and on the intended operating temperature. The acceptance criteria developed called for all of the following surface defects to be blended out:

1) For low temperature piping with operating temperatures 800°F (427°), or below

   a) Any surfaces defects over 0.010 inch (0.025 mm) in depth.

   b) Any arc strikes or weld splatter.

2) For high temperature piping with operating temperatures above 800°F (427°C),

   a) Any surface defects of any perceptable depth in elbows or fittings.

   b) Any surface defects with sharp bottoms of any perceptable depth in pipe surfaces.

   c) Any smooth bottom defect over 0.010 inch (0.025 mm) in depth in pipe surfaces

   d) Any arc strikes or weld splatter.

TABLE 7. SUMMARY OF CREEP-FATIGUE AND CRACK-GROWTH ANALYSIS RESULTS

Location	Temp (°F)	Stress Range (ksi)	Max Cycles	C-F Usage Factor	C-F Factor	ΔK* (ksi√in.)	da/dN	Freq Factor	Crack Growth (in.)
Elbow Midsection	800	50	843	0.10	1.0	11	5×10^{-7}	5	0.002
Elbow Midsection	1050	35	843	0.34 to 2.5	0.73	7.7	4×10^{-7}	175	0.042**
Elbow Midsection	800 & 1050	4	10^9	0	0.73	0.9	0***	---	---
Elbow Ends	800	25	843	0.001	1.0	5.5	2×10^{-8}	5	0.00008
Elbow Ends	1050	18	843	0.45	0.77	4.0	5×10^{-9}	125	0.0005
Elbow Ends	800 & 1050	2	10^9	0	0.77	0.4	0***	---	0
Straight Pipe	800	18	843	0	1.0	4.0	0***	---	0
Straight Pipe	1050	14	843	0.4	0.78	3.1	0***	---	0
Straight Pipe	800 & 1050	2	10^9	0	0.78	0.4	0***	---	0

*ΔK = 2.2 (Δσ)$\sqrt{a}$ where a_0 = 0.010 inch.

**The value of 0.042 in. (0.10 mm) is based on 0.010 in. (0.025 mm) initial flaw depth. For an initial flaw depth of 0.015 in. the crack growth is 0.21 inch.

***No crack growth because ΔK is below threshold.

The depth of blend was also controlled so that the residual wall thickness was adequate to meet primary and secondary stress limits. Defects greater than 0.025 inch (0.64 mm) in depth were given case-by-case evaluation and repair treatment.

The 10-mil limit in the acceptance criteria was a conservative limit chosen with the recognition that considerably larger flaws could be tolerated. But the field inspection technique was too crude to allow the limit to approach the maximum calculated capability any closer. Moreover, most of the flaw types experienced previously in construction were less than 10 mil deep. Flaws greater than 10 mil, but not greater than 25 mil deep, are blended out. Flaws deeper than 25 mil are given special evaluation and the appropriate action determined on a case-by-case basis.

High-temperature piping elbows should be free of defects as they are generally the most vulnerable locations for effects of surface flaws on piping integrity. This is due to the uncertainty in the time-dependent crack-growth rates, and the elbows are locations of maximum stresses, and there is potential for vibration-induced high-cycle stresses. As it is difficult to accurately predict system vibratory response, measurements and inspection for pipe vibratory motion have been taken and will continue during FFTF plant startup testing. This will assure that the piping has sufficient margins against high-cycle fatigue.

CONCLUSIONS AND RECOMMENDATIONS

Simplified rules and preliminary design limits developed for FFTF piping in 1974, based on expected behavior, engineering judgment, approximate calculations, and detailed inelastic analyses of three pipelines have served very well. All designs based on these simplified rules and limits have been confirmed by detailed code and inelastic analyses.

Six additional FFTF pipelines have had detailed inelastic analyses and comparisons to simplified analysis finding have been performed. Accordingly, detailed system inelastic analyses of pipelines are practical for primary and secondary stress/strain evaluations. However, simplified analysis methods were needed and developed for weldment radial shrinkage and local surface stresses and strains. The use of K indices, as in elastic analysis, seems to be the only practical way to treat weldments.

Simplified analyses and elastic analyses provide significant insight for designing a pipeline and they help provide valuable data useful for comparing with detailed inelastic analysis results. After adequate comparisons of detailed inelastic analysis response for lines limited to $P + Q \leq 3\,\bar{S}_m$, where the temperature hold-time relaxation continues monotonically unaffected by the thermal transient, the development of less conservative elastic analysis rules should be attempted by ASME Code bodies. Moreover, it is our experience that by keeping the pipeline primary plus secondary stresses in the range where shakedown in creep occurs, the (i.e, $P + Q_R \leq 3\,\bar{S}_m$) creep-fatigue life will be governed by the stress-time history and very little usage will be consumed by the cycle fraction related to the strain range. That is, the N/N_d fraction is small and the T/T_D is designed so that elastic followup is not significant and the P + Q stress ranges are less than the elastic shakedown range. Then the creep damage will correspond to monotonic relaxation during the service life and be acceptably low. Of course, elbow end weldment radial shrinkage, mismatch and configuration must be controlled or the weld will become design controlling.

Scratches, dings, chisel marks, etc. inadvertently get imposed on piping and equipment while the plant is under construction. Accordingly, considering that such flaws can reduce the operational fatigue capabilities, an acceptance criteria should be developed for identifying what flaws can be tolerated and what flaws should be removed prior to insulating and placing the pipe into operation. For future designs, to provide a sacrificial layer of material that could be blended off, it is recommended that 0.025-inch (0.64-mm) allowance be applied in the design like a corrosion allowance or wall thickness tolerances. Moreover, more high temperature cycle fatigue and crack-growth data are desired in the ASME code to assess fabrication induced flaws and vibratory stresses. In particular, threshold ΔK and da/dN crack-growth rates up to 1200°F are desired. Smooth bar high-cycle fatigue data to 10^9 cycles are also desired.

Significant advances in methods and technology for elevated temperature piping design have occurred in recent years but improvements are still expected and desired to reduce design costs and to enhance the reliability of the piping.

ACKNOWLEDGEMENT

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Piping-Flexibility Analysis

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A steadily expanding literature bears testimony to the growing recognition of the importance of the problem of providing flexibility in piping and to the many difficulties besetting efforts at establishing a simple rational approach for its solution. This paper aims to outline the various phases of the problem, with particular emphasis on the phenomena of plastic flow and fatigue which distinguish the behavior of piping systems under thermal expansion from the ordinary room-temperature steady-state structural problem and lead to the concept of a limiting-stress range rather than an allowable stress as the criterion of the adequacy of a layout. In treating his subject, the author has sought to present the consensus of the Task Force on Flexibility charged with reformulating Chapter 3 of Section 6 of the Code for Pressure Piping. Their Proposed Rules are included as a focal point on which it is hoped broad discussion will center.

Introduction

In the course of a general review and revision of the Code for Pressure Piping, initiated in 1951, a Task Force on Flexibility[2] was appointed by ASA Sectional Committee B31.1 to study and report on the adequacy of the current provisions of the chapter on "Expansion and Flexibility" included in Section 6 of the Code. Two subgroups[3] were formed, one to deal with stresses and their allowable limits, and the second to digest available information on physical properties entering into piping-flexibility analysis.

The former group, with the findings of which this paper is solely concerned, came to the conclusion that a complete reformulation of this chapter was desirable to improve its clarity and, more importantly, to bring its clauses into accord with advanced theoretical concepts, new research results, and accumulated experience. A working group[4] was charged with the task of devising rules which would realize this objective and still be readily understandable and easy to apply. A draft effecting a satisfactory compromise between scientific truth and the simplicity so essential to any body of rules destined for wide application was produced and accepted by the Sectional Committee.

However, since the Proposed Rules depart appreciably from past practice in several respects, primarily by their open recognition of the concept of stress range, it was thought desirable to publish them first in nonmandatory form as a Task Force Report (121)[5] to permit piping engineers at large to familiarize themselves with them, test their suitability by application to their individual problems, and assist in arriving at a final formulation assuring uniform interpretation and intelligent enforcement. At the same time, the author was invited to prepare a paper to explain the basic philosophy and scientific background underlying the Proposed Rules.

The Problem

The objective of piping-flexibility analysis is to assure safety against failure of the piping material or anchor structure from overstress, against leakage at joints, and against overstrain of connected equipment, without waste of material. While expansion joints of various types in some instances prove useful for this purpose, by far the more common and generally preferable practice is to provide for thermal expansion by utilizing the inherent flexibility of the pipe run itself acting as a spring in bending or torsion.

Piping-flexibility analysis resolves itself into the following:

1 The calculation of the forces, moments, and stresses (and desirably also, displacements) at all significant locations in a tubular structural frame under the influence of thermal expansion.

2 Their comparison with allowable limits.

The frame can be in one or more planes. The number of redundants will vary with the number of branch lines or intermediate restraints (guides, braces, and so on). For a space system, there will be six unknown reaction components (three forces and three moments, or three forces and their lever arms) for each anchor point in excess of one; intermediate restraints introduce a lesser number of unknowns.

As compared with the parallel structural problem, the evaluation of the reactions, stresses, and deformations in a piping system under thermal expansion involves a number of additional considerations, of which the following are the most important:

1 Piping components other than straight pipe, notably elbows and bends, exhibit peculiar stress-and-strain behavior under bending which generally reflects itself in increased flexibility, usually accompanied by intensification of stresses.

2 Piping systems are not intended to behave elastically in their entirety. As a result of local creep (at high temperatures) or local yielding (even at ordinary temperatures) relaxation may take place whereby the reactions and stresses in the operating condition are lowered and substantially equivalent reactions and stresses are made to appear in the cold or off-stream condition. This process can be anticipated by cold springing.

3 Owing to the cyclic nature of the operation of all piping systems, fatigue becomes a factor requiring consideration, particularly where the fluid carried is corrosive to any degree.

The General Process of Solution

In the flexibility analysis of any system of given line size, configuration, and material, with a predetermined amplitude and number of temperature cycles, the following steps are involved:

1 The significant physical properties of the material, such as expansion coefficient, modulus of elasticity, Poisson's ratio, yield stress, creep and relaxation stress, and endurance strength have to be determined. This paper will not concern itself with the

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[2] For membership, see source (121). Numbers in parentheses refer to the Bibliography at the end of the paper.

[3] Under the chairmanship of H. C. E. Meyer and R. Michel, respectively.

[4] With S. W. Spielvogel as chairman, and N. Blair, H. V. Wallstrom, and the author as members.

[5] The original formulation by Subgroup 1 is transcribed in Appendix 1; alternate clauses introduced later in deference to a dissenting viewpoint are given separately in Appendix 2.

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first three constants, for which the values and the basis for their selection have been covered by a separate paper by the chairman of the second subgroup of the Task Force (124). The way in which the strength properties enter into the solution of the problem under the Proposed Rules, on the other hand, will be discussed in detail at the appropriate point in this development.

2 Assumptions have to be made regarding the dimensions of the piping, notably those associated with the cross section. For simplicity, the Proposed Rules disregard dimensional tolerances and the uncertain and erratic changes in thickness caused by corrosion or erosion and permit use of the nominal dimensions throughout.

3 Conditions of end restraint have to be assessed. The Proposed Rules give no prescriptions in this respect, but general practice is to take the ends as fully fixed in the absence of detailed analysis of the rotations and deflections of vessel shells, pump or turbine casings, pipe anchors, or other structures to which the line may be connected. However, equipment expansions must be taken into account since they may cause increased forces, moments, or stresses.

4 The significance of different forms of intermediate restraints has to be appraised. Major restrictions to free movement of the line due to guides, solid hangers, or braces are usually taken into account in calculations or other forms of analysis. Secondary restraints, such as unbalanced spring forces or frictional forces at supports, usually are ignored; however, caution should be exercised in extending this practice to systems whose weight is great in relation to their stiffness, a condition often encountered in pump or turbine leads because of manufacturers' limitations upon thrusts.

5 A method of analysis suitable to the importance of the system must be selected. The solution can be approached by analytical, graphical, chart, or model-test methods, or even by comparison with past successful layouts, and may involve various degrees of approximation. However, for approximate solutions an allowance for the probable error should be included.

6 Finally, a comparison of the results has to be made with allowable limits. These are clearly established in the Proposed Rules for the stresses, but left to the designer's judgment and consultation with manufacturers of equipment in the case of reactions, because of the diversity in shape and design of connected structures.

Flexibility Factor

It has been stated earlier that the calculation of the reactions and stresses in a piping system is complicated by peculiarities of stress-and-strain distribution in certain piping components under bending, one of the effects of which is to endow such fittings with (usually) greater flexibility than would be predicted from the ordinary beam theory.

In calculations, this is commonly taken into account by the application of a so-called flexibility factor. This can be defined as the ratio of the rotation per unit length of the part in question produced by a moment, to the rotation per unit length of a straight pipe of the same nominal size and schedule or weight produced by the same moment. It is applied either as a multiplier of the length of the part, or as a divisor of its nominal moment of inertia (the moment of inertia of the matching pipe) or of the elasticity modulus. Available information on its magnitude for different types of fittings will be discussed briefly in the text following.

Curved Elbows or Bends. These are by far the most significant group of piping components from the standpoint of providing increased flexibility. At the same time, they constitute the only group for which flexibility factors have been derived theoretically and confirmed by an adequate amount of testing.

The increased flexibility of curved tubular members results from their flattening along one or the other axis under bending.

The flexibility factor k in common use in this country was developed by von Kármán (3) in 1911, from a first approximation of an assumed Fourier-series solution. It was redeveloped on a different basis and experimentally checked by Hovgaard (11) in 1926. It is usually given as

$$k = \frac{12h^2 + 10}{12h^2 + 1} \qquad [1]$$

where $h = tR/r^2$ is the so-called flexibility characteristic which depends on the pipe-wall thickness t, its mean radius r, and the radius of curvature R of the center line of the pipe.

Originally this factor was used only for correcting the deflection of curved members bent in the plane of their curvature. This practice continued until Vigness (70), in 1942, demonstrated that it applied equally to transverse or out-of-plane bending.

The first-approximation Kármán-Hovgaard factor has been used generally for both types of loading until Beskin (77), among others, pointed out the need for using more terms in von Kármán's Fourier series for bend proportions where the characteristic h falls below 0.3. The following close approximation suggested in Beskin's development commends itself by its general validity[6] and startling simplicity

$$k = \frac{1.65}{h}, \quad \geq 1 \qquad [2]$$

This formula strictly applies only to the central portion of a curved tube of relatively large arc under bending, and does not consider the effects of internal pressure or end restraints.

The effect of ordinary steam pressures on the flexibility of 6-in. and 12-in. bends has been investigated by Wahl (12). He found the tendency toward restoration of the circular form to be of a low order, and as a result it has become customary to neglect this effect. This conclusion may need modification for thin-wall short-radius elbows of large diameter.

With regard to end restraints, it is obvious that even straight tangents will tend to reduce ovalization of the curved pipe and therewith impair its flexibility. The restraining influence of end tangents has been demonstrated by diameter measurements reported by the author (87) and more thoroughly explored by Pardue and Vigness (99). Its effect, however, was found to be relatively minor for arcs 90 deg or greater. For smaller arcs, the reduction in flexibility would be expected to be more pronounced, but since it is known to be accompanied by a commensurate reduction in stress intensification, it is ignored in the interest of keeping calculations reasonably simple.

The effect of the attachment of stiff rings or flanges to the ends of curved pipe, on the other hand, was found to be quite marked in the tests conducted by Pardue and Vigness; each flange appeared to cancel the influence of approximately 30 deg of arc of the bend. In the Proposed Rules, these data have been used to derive simple empirical-correction factors $h^{1/6}$ and $h^{1/3}$ designed to reduce flexibility factors in the range below $h = 1$ to

$$k' = \frac{1.65}{h^{1/6}} \qquad [2a]$$

and

$$k'' = \frac{1.65}{h^{1/3}} \qquad [2b]$$

[6] The more generally known formulas for the flexibility factor of curved pipe are discussed briefly in Appendix 3. It will be noted that Equation [2] closely approximates von Kármán's third approximation and Jenks' proposed formula as originally given in the discussion of Shipman's paper (17).

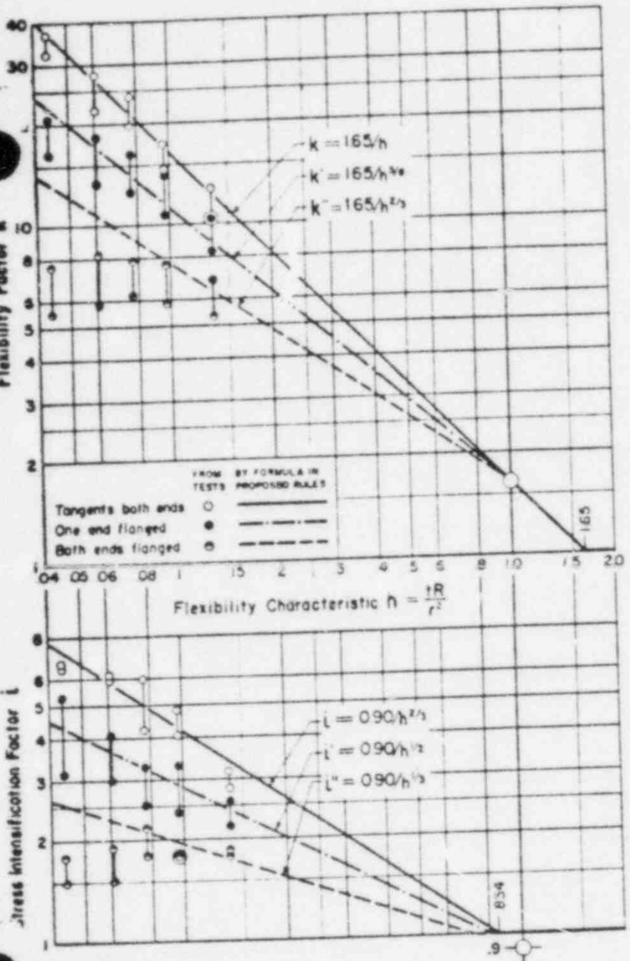

Fig. 1 Flexibility and Stress-Intensification Factors for Curved Pipe With and Without Flanges

(In upper graph, upper pairs of connected test points are averages of k_{NB} and $k_{T\theta}$; lower points are averages of $k_{N\theta}$ and k_{TB}, as given in Tables 2, 3, and 4 of reference (99); in lower graph, upper and lower test points are 60 per cent of values for in-plane and transverse bending, respectively, from Figs. 12 and 16 of the same source.)

for 90-deg elbows flanged at one and both ends, respectively.

A comparison of the test data with Equations [2], [2a], and [2b] is given in the upper chart in Fig. 1. It will be noted that Equations [2] and [2a] are in satisfactory accord with the results of these specific tests, while Equation [2b] overestimates the flexibility factor for low values of h. A study of the lower chart leads to similar observations with respect to the corresponding stress-intensification factors, which are obtained by the application of the same correction factors. In view of the limitation of available test data to a single pipe size, bend radius, and flange type, attempts at a more refined correlation appear unwarranted at the present time. The corrections are to be regarded as no more than first crude approximations, defensible on the basis that inaccuracies in evaluation of both flexibility and stress-intensification factors tend to cancel each other, at least with respect to stress calculations.

Mitre Bends. On the basis of isolated test data and service experience, these piping components are known to possess increased flexibility approaching that of curved bends, particularly where both mitre spacing and mitre angle are small so that the mitre bend comes to resemble a curved elbow. In-plane bending test data on 4-mitre quarter-bends with tangents of various lengths followed by flanges at each end, on which Zeno reports in a discussion of Pardue and Vigness' paper (99), are of particular interest in this connection. The stiffening effect of flanges placed close to the ends of the bend is equally evident as in the case of curved bends, but as the tangents are lengthened to approximately two pipe diameters, the flexibility factor asymptotically approaches 80 per cent of that computed for a corresponding curved bend.

In the absence of a theoretical development, the sparse available test data on mitre bends, including the results of unpublished load-deflection tests secured in connection with fatigue tests reported by the author (114), have been evaluated conservatively in the Proposed Rules as

$$k = \frac{1.52}{h^{5/6}} \ldots\ldots\ldots\ldots\ldots\ldots [3]$$

The characteristic h herein is as defined under Equation [1], except that an equivalent radius R_e is used which is given as

$$R_e = \frac{s}{2} \cot \alpha \text{ for } s \leq r\,(1 + \tan \alpha) \ldots\ldots [3a]$$

and

$$R_e = \frac{r}{2}\,(1 + \cot \alpha) \text{ for } s \geq r\,(1 + \tan \alpha) \ldots\ldots [3b]$$

where s is the mitre spacing at the center line, r is the pipe-wall radius, and α is one half the angle between adjacent mitre axes (or the angle defining the out-of-squareness of the mitre cut). It should be noted that for wide spacing, Equation [3b], the mitre bend is to be taken as consisting of a number of arcs with intervening tangents.

Corrugated Pipe. Straight or curved corrugated pipe and creased bends are the only other shapes to which increased flexibility is assigned under the Proposed Rules. The limited test data available on these types of components are summarized in a paper by Rossheim and Markl (55) on the basis of which a uniform flexibility factor $k = 5$ is suggested as a first approximation. This should be used with caution, since the flexibility of corrugated and creased pipe may be expected to vary with diameter, thickness, and bend radius of the pipe, and height, pitch, and contour of the corrugations. The effect of some of these variables has been demonstrated theoretically for idealized shapes by Donnell (26) and Hetényi (80). It also has been found experimentally. Dennison (45), working with 6-in. standard-weight pipe, reports a value of 5 for the flexibility of creased bends and between 6.4 and 7.2 for that of corrugated tangents and bends, and tests reported by Rossheim and Markl gave values of 3.7 and 2.9 for specially made 2-in. standard-weight corrugated tangents and bends.

Other Components. Forged or fabricated tees or screwed or flanged connections comprise some of the components which may exhibit increased or decreased flexibility as compared with straight pipe, depending upon their individual dimensions and contours. Because of the lack of a sound basis for even a crude empirical formulation, the Proposed Rules assign unit flexibility to all such parts; the error incurred by so doing will never become critical since such fittings usually constitute only a small part of the line.

It may be worth while to draw attention to the fact that the Proposed Rules do not make it mandatory to use the specific flexibility (and stress-intensification) factors given therein for any of the piping components. This represents a tacit admission of the tentative nature of the evaluation of existing data made by the Task Force, and points up the desirability of a more thorough theoretical and experimental exploration of the field.

Stress-Intensification Factors

In discussing stress-intensification factors for piping components, it is necessary to distinguish between formulas or values derived from theory or static strain-gage tests and such obtained from full-scale fatigue tests. The primary difference between them lies in the point of reference. Theory refers to an ideal homogeneous notch free material, while the results of fatigue tests of commercial products preferably are related to parallel results on commercial pipe joined by butt welding, which itself contains stress raisers in the form of surface imperfections. This change in reference point, and possibly also the redistribution and attendant relief of peak stresses occurring under cyclic loading, accounts for the observation that stress-intensification factors derived from fatigue tests are generally lower than those predicted by theory or measured in strain-gage tests.

Since pipe is the primary constituent of piping systems and service failures of piping are almost always associated with the effects of cyclic loading (generally aggravated by corrosive influences) the stress-intensification factor will be defined here as the ratio of the bending moment producing fatigue failure in a given number of cycles in a straight pipe of nominal dimensions, to that producing failure in the same number of cycles in the part under consideration.

This definition implies that the curves of failure stress versus number of cycles to failure parallel each other for straight pipe and other piping components. While this is not strictly true, test data conform reasonably well to a law expressed by

$$iSN^{0.2} = C \qquad [4]$$

where i designates the stress-intensification factor, S the nominal endurance strength (cyclic moment[7] applied at point of failure divided by section modulus of matching pipe, rather than fitting), N the number of stress reversals to failure, and C a materials constant.

From Rossheim and the author's tests (55), later confirmed by tests run by the author's present company (114), a value C = 245,000 was established as suitable for Grade B carbon steel at room temperature. From additional unpublished test data in the author's company's files, a tentative value C = 281,000 was deduced for stainless steel, type 316, at room temperature. Finally, Stewart and Schreitz's tests (116) suggested a value C = 183,500 for stainless steel, type 347, at 1050 F.

In view of the all too common misconception that fatigue is always associated with a large number of loading cycles, it appears pertinent to point out that the author has found Equation [4] to be as valid for the determination of stress-intensification factors at 20 as at 2,000,000 cycles. The author has observed no evidence of leveling off of the S-N curve at either end, except in the case of straight pipe which to some extent tends to follow the trend of polished-bar tests. The endurance limit of commercial piping components is not reached as soon as in the case of polished bars. The thought suggests itself that possibly the number of cycles defining the knee in the S-N curve is the higher, the higher the stress-intensification factor.

The foregoing gives the general approach used in setting stress-intensification factors. In the following the detailed sources are given from which the values of i published in the Proposed Rules are taken. At the same time, isolated additional test data are adduced from the fatigue-test files of the author's company to round out the picture.

Fittings for Directional Changes. These can be treated as a group because of their striking similarities in behavior under bending fatigue. In the course of evaluating and correlating fatigue tests on mitre bends, forged and fabricated tees, similarities in crack location and direction obtruded themselves upon the author's observation. It seemed as if all these fittings conformed to some extent to the behavior of curved elbows or bends. This led the author to suggest a common empirical expression for the stress-intensification factor (114) which is

$$i = \frac{0.9}{h_e^{2/3}};\ \geq 1 \qquad [5]$$

where

$h_e = c\,(t_e R_e/r^2)$ = effective flexibility characteristic (dimensionless)

$c = (t_e/t)^{1.5}$ = section-modulus correction factor (dimensionless)

= 1 wherever fitting has same thickness as matching pipe

t_e = effective fitting thickness, in.

= average of crotch and side-wall thickness, for welding tees[8]

= pipe-wall thickness increased by one-half excess thickness provided in either run or branch, by use of thicker piping or pad or saddle, for reinforced fabricated intersections

= t for welding elbows, curved or mitre bends, or unreinforced fabricated intersections of a thickness equal to that of matching pipe

t = thickness of matching pipe, in.

r = mean radius of matching pipe, in.

R_e = effective bend radius, in.

= R = radius to center line of curvature for elbows or smooth bends

= $r + r_c$ for welding tees,[8] where r_c designates crotch radius

= r for single-mitre bends and unreinforced and reinforced fabricated 90-deg branch intersections

$\leq \frac{s}{2}\cot\alpha;\ \leq \frac{r}{2}(1 + \cot\alpha)$ for multiple-mitre bends, where s designates mitre spacing at center line, in., and α designates one-half angle between adjacent mitre axes, deg

The condensed information given in the Proposed Rules is directly derived from reference (114). The correction factors $h^{1/6}$ and $h^{1/3}$ proposed to account for the effect of end flanges on the stress-intensification factor for curved or mitre bends have been discussed already under the heading Flexibility Factor. Note that the higher of the stress intensifications for the flanged elbow and the flange itself must be used.

Corrugated Pipe. This type pipe and corrugated or creased bends have been assigned a stress-intensification factor of 2.5 in the Proposed Rules. This substantially follows the recommendation given by Rossheim and Markl (55) in a paper evaluating the information available up to the year 1939; the value selected is that for "noncyclic" service since correction for definitely cyclic service is effected under the Proposed Rules by the application of a stress-reduction factor. Considering the important influence of the diameter-to-thickness ratio of the pipe, as also the shape, thickness variation, height, and pitch of the corrugations of any specific manufactured product of this type, the assignment of a constant stress-intensification factor obviously represents a gross oversimplification. A more thorough theoretical and experimental exploration of this type of construction appears urgently needed, if it is to be used in severe services.

Bolted Flanged Connections. These present a dual problem from the standpoint of piping-flexibility analysis. It is necessary not only to guard against ultimate failure by rupture but also against disablement of the joint through leakage across the

[7] Where the stress amplitude applied in the tests exceeded the yield strength in bending, a fictitious moment based on a straight-line extension of the elastic moment-deflection curve was computed to conform with usual calculation practice.

[8] For welding tees conforming to ASA Standard B16.9, assumption of R_e = 1.35 r and t_e = 1.60 t usually will produce conservative estimates of i on the basis of representative measurements.

gasket. The values of the stress-intensification factors shown in the Proposed Rules are taken from a paper by Markl and George (97) and serve to predict ultimate rupture in joints bolted to about 40,000 psi stress. With lower bolt stresses there is the possibility of premature leakage. Although there are no published data on the subject, the author judges from isolated test runs conducted by his company that freedom from leakage can be assured by application of a factor i of the order of 1.5 regardless of the type of flange, except in the active creep range where periodical retightening may become necessary.

Pipe Joints. These joints when made by butt welding form the basis for comparison for all other fittings, and hence take a factor of 1. Fillet-welded and screwed joints are assigned the same values as single-fillet-welded and screwed flanges, since the failure of a flanged connection of a ductile material usually occurs in the attachment to the pipe.

Tapered Transitions. Components such as are used for connecting pipe of different wall thicknesses or for the hub ends of flanges or valves can be given the following approximate stress-intensification factors on the basis of isolated unpublished tests run by the author's company:

15-deg taper, $i = 1.1$
30-deg taper, $i = 1.2$
45-deg taper, $i = 1.3$

Only the small end of the hub need be considered in such an analysis, since possible higher stress intensifications at the large end, of course, are compensated by the relatively lower stress level corresponding to the increased thickness (which also explains why ASA welding neck flanges always fail at the attachment end, never at the root of the hub). Incidentally, it will be noted that the factor of 1.3 for a 45-deg taper is the same as for a fillet weld, the two representing the same geometrical shape.

Other Components. Components such as reducing elbows and tees, box-type fittings, anchor structures, and the like, in the absence of directly applicable data must be evaluated by analogy with fittings for which factors are available.

It already has been pointed out that neither the flexibility nor the stress factors given in the Proposed Rules are made mandatory. While the formulas and values given are based on the best available information, they are by no means to be taken as scientific fact. The prime purposes served by their publication are to call attention to the existence of such stress intensifications, to provide standardized assumptions in place of complete chaos, and, finally, to stimulate further research by all connected with the piping industry.

Primary Analysis

For the purposes of a brief study of available methods of analysis of piping systems under thermal expansion, let it be assumed that the system be installed with 100 per cent cold spring, i.e., that members be cut short by the full amount of their anticipated expansion and then pulled into line, Fig. 2. Let it be assumed further that the proportional limit of the material should not be exceeded at any point during this initial prestressing. It follows from these assumptions that the system will be free of expansion

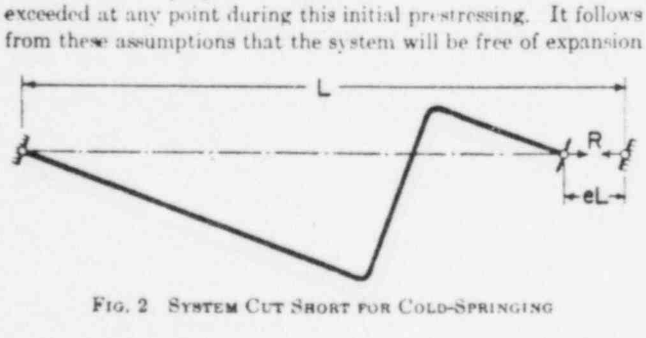

Fig. 2 System Cut Short for Cold-Springing

stress in its hot or operating condition, and will not undergo inelastic action leading to relaxation, the consideration of which will be discussed in another section of this paper.

The evaluation of the forces, moments, and stresses existing in the initial prestressed cold condition evidently reduces to the analysis of a tubular-frame structure under the influence of given end and intermediate displacements and rotations. This is a standard structural problem, but for the need of correcting the deflection and computed stresses of certain members by the application of the flexibility and stress-intensification factors discussed in the text preceding.

The initial cold reaction[9] R at the line terminals and the controlling stress S_E in the line for 100 per cent cold spring are given by the following generalized expressions

$$R = eE_cI\,F_r = \frac{ZF_r}{iF_s}\,S_E = FS_E \qquad [6a]$$

$$S_E = eE_cI\,\frac{iF_s}{Z} \qquad [6b]$$

where e is the unit expansion from installation temperature to maximum operating temperature upon which the amount of cold spring was based; E_c is Young's modulus at the installation temperature; I and Z are, respectively, the moment of inertia and section modulus of the pipe; i is the stress-intensification factor at the controlling point; F_r and F_s are shape factors expressing the over-all effect of line configuration and axial dimensions, including flexibility factors; and F is a composite factor relating the reaction to the controlling stress.

While general solutions of this problem have long been available, their application to piping-flexibility analysis has been restricted because of the specialized knowledge and formidable expenditure of time required to carry out a calculation. Equations [6a] and [6b] are deceptively simple, but the shape factors F_r and F_s, appearing therein, in themselves generally represent extremely complex mathematical expressions. To reduce their computation to practical limits, some of the foremost piping-stress analysts have expended considerable effort and ingenuity in devising simplifications consisting either of preorganization of parts of the solution without affecting accuracy, or of making approximations of greater or lesser validity.

Among devices of the first kind applied primarily to strictly mathematical solutions, the following are the most important:[10]

1 Preintegration of recurring shape coefficients (17).

2 Introduction of virtual center of gravity or elastic center (38).

3 Introduction of conjugate axes (41).

4 Application of principle of cyclic permutation of co-ordinates[11] to reduce multiplane problem to single-plane problem (61).

5 Exploitation of symmetry of simultaneous equations to reduce number of operations required (61).

6 Application of matrix method to provide clearer visualization of components entering into the problem (117).

[9] For clarity, the developments in this and the following section refer to a single end force which is all that is needed in the case of a single-plane bend with two hinged ends. The definition of R can be expanded to relate to the $3(n-1)$ force components and $3(n-1)$ moment components created by cold springing or thermal expansion of a space system with n points of fixation without loss of validity of the conclusions derived.

[10] The scope of the paper permits no more than a brief enumeration of various approaches. To enable interested readers to acquaint themselves with them, parenthetical references are given to the better-known sources employing them without, however, implying that they are necessarily the original proponents.

[11] First application of this concept is credited to G. W. Watts and W. R. Burrows.

Among approximate assumptions leading to a variable degree of accuracy, the following are probably the most common:

1 Subdivision of line into short elements, the mass of which is concentrated at their mid-points (51); if the elements selected are short enough, this method is practically precise.

2 Substitution of square corners for curved members; this widely used approximation ignores the increased flexibility of elbows and leads to an overestimate of reactions and either an over or underestimate of the stresses depending upon whether the stress-intensification factors are considered or ignored.

3 Correction of developed square-corner length of system by addition of a virtual length representing the excess flexibility of all the elbows (105); in effect, this distributes the excess elbow flexibility uniformly over the entire system. The accuracy of the results is considerably improved as compared with the foregoing approach.

4 Concentration of the excess flexibility of each elbow in a single point located at the intersection of its two tangents (120); this modification of the square-corner solution is somewhat more complex and more accurate.

5 Introduction of two or more weighted points for each elbow; this further refinement of the square-corner solution leads to almost precise results with proper selection of the weights assigned to the points.

6 Assumption that neutral axis parallels line connecting anchors (20); this produces precise results for symmetrical cases, but the accuracy very rapidly diminishes as the shape departs from symmetry or becomes antisymmetrical.

7 Assumption that neutral axis connects the anchors directly; this, in effect, assumes a hinged system, and may lead to major error where the ends are rigidly anchored.

8 Assumption that bending and torsional rigidity are identical (30); taking the shear modulus equal to one half the elasticity modulus in tension simplifies the solution of space problems without leading to excessive error.

9 Assumption that the stress-intensification factors are identical for in-plane and out-of-plane bending (114); use of the higher of the two for either condition leads to a conservative error not in excess of 20 per cent for elbows and common full-size intersections. The suggested stress-intensification factors tabulated in the Proposed Rules utilize this assumption.

In addition to purely mathematical approaches (to which the preceding text primarily refers), there are graphoanalytical methods (76) of equal range of accuracy, in which the moments are built up from one end to the other with the aid of precalculated solutions for each element of the line. Furthermore, a number of chart solutions have been published which represent more or less complete precalculations of entire systems of unit stiffness and displacement (33, 37, 105); the latter obviously are restricted to simple configurations.

While many methods are theoretically suitable for application to systems with any number of terminal and intermediate restraints, the computation work increases rapidly with the number of redundants. For this reason mathematical and semimathematical methods rarely have been applied to systems with more than three points of fixation. To supply the need for a means of evaluating the flexibility of lines with many branches or intermediate restraints, such as guides or wind braces, model-testing methods have been devised wherein the reactions caused by given end displacements are measured by either springs (52, 56, 60) or electric strain gages (78). Of late, memory-endowed electronic or other computing devices have been utilized by at least two companies (123). Once the operations are coded properly, which is a time-consuming task for experts in this field, these machines are capable of solving any problem of the same type and thus serve to expand greatly the number of systems which can be calculated within a given time.

Self-Spring and Cold-Spring Effects

In order to focus the reader's attention on "methods" of analysis, the problem of calculating forces, moments, and stresses in the foregoing has been reduced to a familiar structural problem by imposing special conditions. In what follows, the scope of the investigation will be broadened to embrace all conceivable conditions of installations and temperature or stress which might be encountered in actual practice.

Let it be assumed that a system be installed cut short an arbitrary amount, so that a gap ceL is left between the end of the line and one terminal. This problem is identical with that shown in Fig. 2 and is solved in general terms by Equations [6a] and [6b], except for the introduction of the so-called cold-spring factor c which ranges from zero (for no cold spring) to unity (for 100 per cent cold spring). Equations [6a] and [6b] are based on $c = 1$. Obviously, both the initial cold reaction and the initial cold stress for the more general case will differ from those given by these equations by a factor c. Actually, since reaction and stress are interrelated by a factor F, which is constant for any line of given shape and dimensions, a study of the behavior of the system can be restricted to a study of the controlling stresses created therein by changing temperature conditions. The initial cold stress is

$$S_c' = cS_E \qquad [7]$$

As the line is brought up to temperature this stress decreases, becoming zero when the line has expanded by the amount ce per unit length. Upon further expansion by the amount $(1 - c)e$ remaining to give a total of e, a stress of reversed sign[12] is produced (initial hot stress)

$$S_h' = (1 - c)\frac{E_h}{E_c} S_E \qquad [8]$$

where E_h is Young's modulus at the hot or maximum operating temperature.

In the absence of yielding or creep in either the cold or hot condition, the controlling stress thereafter will alternate between the two limits given by Equations [7] and [8] during successive cycles of cooling down and heating up. This is generally true of moderately stressed lines operating at temperatures at which the metal is not subject to creep, and also of lines operating at elevated temperatures which have been cold sprung sufficiently to keep the initial hot stress below the creep limit.

Where these conditions are not met, initially high stresses, particularly in the hot condition, will relax with time until they reach a level which can be maintained indefinitely; this phenomenon is illustrated by the recordings traced in Fig. 3 which are taken from laboratory tests of a small-scale expansion loop which was alternately heated to approximately 950 F and allowed to cool to atmospheric temperature. It will be noted that the controlling hot stress (and therewith also the hot reaction) dropped off to a constant level after the first few cycles; the line has sprung itself, whence the designation "self-spring." If the asymptotic value toward which the hot stress tends be designated as S_r, then the ultimate hot stress after adjustment becomes

$$S_h'' = S_r \qquad [9]$$

[12] All expressions are shown as absolute values.

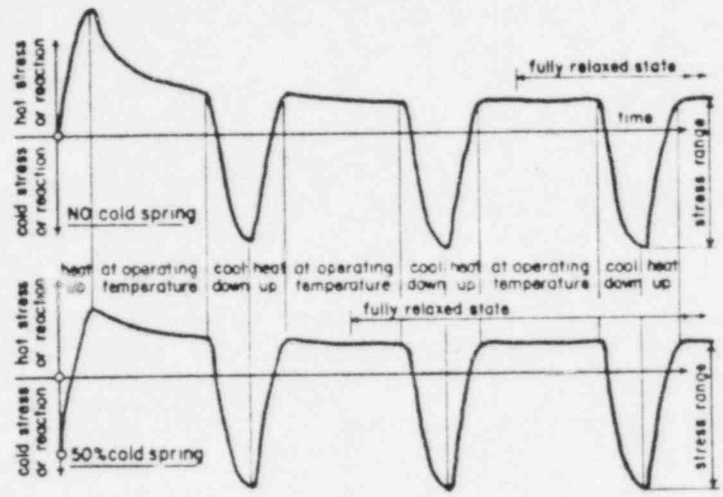

Fig. 3 Effect of Relaxation Upon Reactions and Stresses

Upon cooling down, each unit length of the line contracts again by an amount ϵ, the stress reverses, and the ultimate cold stress becomes

$$S_c'' = S_E - \frac{E_c}{E_h} S_r \quad \text{[10]}$$

The preceding four equations fully circumscribe the extreme stress conditions encountered during the service life of a system, whether it be installed with cold spring or not, and whether it be subject to relaxation or not.

Now, as Stromeyer (6) pointed out in 1914, and Dennison (45) re-emphasized more recently, service failures are associated with cyclic, rather than static-stress application. Fatigue, with corrosion usually an important contributory factor, must be accepted as the primary cause of failure. Resistance to fatigue is measured by the so-called endurance limit (fully reversed stress supported over an indefinite number of cycles, in the millions) or by the endurance strength (stress supported over a given number of cycles), the latter being of more direct significance to the present problem, since even in the process industries the number of major temperature cycles rarely exceeds six per day, corresponding to approximately 40,000 over a 20-yr life. Actually, the stresses usually are not fully reversed in actual piping installations, but since the mean stress is indeterminate, particularly where relaxation occurs, and of subordinate importance to the stress range, the latter is taken as the sole criterion in the Proposed Rules. For simplicity, these set the value of the "calculated-stress range" equal to the stress S_E produced by 100 per cent cold spring; that this is a reasonably correct or at least conservative assumption will be shown in the text that follows.

For the initial condition (which is maintained throughout where no adjustment occurs), the stress range is given by the summation of the stresses given by Equations [7] and [8]

$$S_c' + S_h' = \frac{E_h}{E_c} S_E + \left(1 - \frac{E_h}{E_c}\right) c S_E \quad \text{[11]}$$

For the ultimate condition in the case where relaxation does occur, it is given by the summation of Equations [9] and [10]

$$S_c'' + S_h'' = S_E + \left(1 - \frac{E_c}{E_h}\right) S_r \quad \text{[12]}$$

As one limit, applicable to lines of small temperature change, set $E_h = E_c$; then

$$S_c' + S_h' = S_E \quad \text{[11a]}$$

$$S_c'' + S_h'' = S_E \quad \text{[12a]}$$

As a second approximate limit, for hot lines where the relaxation limit S_r is small, set $E_h = {}^2/_3 E_c$; then

$$S_c' + S_h' = \frac{2 + c}{3} S_E \quad \text{[11b]}$$

$$S_c'' + S_h'' = S_E - \frac{1}{2} S_r \quad \text{[12b]}$$

Note that at one limit the stress range equals S_E; i.e., is constant and independent of the amount of cold spring, and that at the other it is lower than S_E and affected only to a minor extent by the values of c and S_r.

In the following four equations the corresponding reactions are given as obtained by multiplying the right-hand terms of Equations [7] to [10] by R/S_E

$$R_c' = cR \quad \text{[13]}$$

$$R_h' = (1 - c) \frac{E_h}{E_c} R \quad \text{[14]}$$

$$R_h'' = \frac{S_r}{S_E} R \quad \text{[15]}$$

$$R_c'' = \left(1 - \frac{E_c}{E_h} \frac{S_r}{S_E}\right) R \quad \text{[16]}$$

Detailed discussion of the relaxation limit S_r has been deferred to this point because, under the Proposed Rules, it is considered to affect only the computation of reactions. With the establishment of the approximate stress range S_E as the primary criterion of the flexibility of the piping system proper, individual stresses at any one time during the temperature cycle have come to be ignored. In the case of the reactions, on the other hand, the extreme values in the hot and cold conditions are taken to control directly; the reason is that strain-sensitive equipment, such as pumps or turbines, can be seriously damaged by a single overload, even though this may be promptly relaxed as a result of yielding or creep somewhere in the system.

The relaxation limit S_r can be defined as the asymptotic value toward which the stress in a prestressed structure with a fixed distance between its terminals tends as the material flows as a result of yielding or creep. It is not possible to assign an accurate value to this property, at least under bending (the predominant type of loading introduced by thermal expansion) where higher stresses are necessary to produce flow than under tension (the type of loading for which most of the yield and creep data have been developed). However, it appears conservative for the present purposes to set its value equal to the lesser of the tensile yield strength and 160 per cent of the stress producing 0.01 per cent creep in 1000 hr at the given temperature; this corresponds to $S_r = 1.6\ S_h$, where S_h is the allowable S-value at operating metal temperature. The selection of S-values in the Power Piping Section[13] is based on the rule given under Table P-7 of Section I of the ASME Boiler Construction Code, which states that the S-value equals the lesser of 25 per cent of the tensile strength, $62^1/_2$ per cent of the yield strength, 100 per cent of the stress producing 0.01 per cent creep in 1000 hr, and 60 per cent of the average or 80 per cent of the minimum stress producing rupture in 100,000 hr.

Actually, the Proposed Rules rest on a much more conservative basis; in effect, they assume $S_r = S_h$. In addition, they credit only two thirds of the designed cold spring in the computation of the initial hot reaction, while requiring the use of the full amount of the cold spring in computing the corresponding cold reaction. The ultimate hot reaction is, of course, ignored, since it is never greater than the initial hot reaction. This leads to the following equations:

1 Extreme hot reaction, paralleling Equation [14]

$$R_h = \left(1 - \frac{2}{3}c\right)\frac{E_h}{E_c}R \qquad [17]$$

2 Extreme cold reaction, greater of values given by Equations [13] and [16] after substituting $S_r = S_h$, with the further proviso

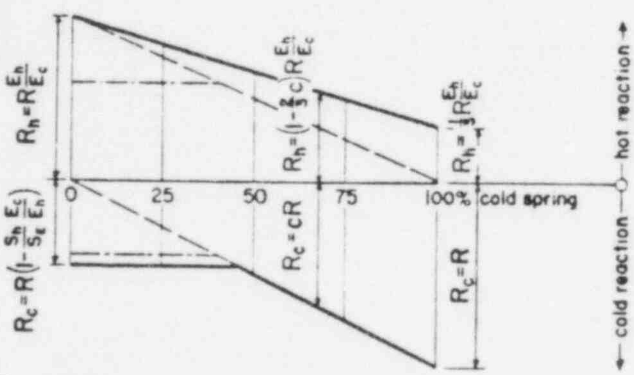

FIG. 4 RELATION OF REACTIONS COMPUTED BY PROPOSED RULES TO THEORETICAL REACTIONS

that $(S_h/S_E)(E_c/E_h)$ not be taken greater than unity (reaction otherwise would obtain same sign as R_h which always is higher)

$$R_c = cR \text{ or } \left(1 - \frac{E_c}{E_h}\,\frac{S_h}{S_E}\right)R \qquad [18]$$

Fig. 4 gives a qualitative comparison of the reactions computed by the Proposed Rules (heavy solid lines) and the corresponding theoretical values; the dash lines indicate the magnitude of the reactions in the absence of relaxation, while the dash-dot lines illustrate the modification of the latter as a result of relaxation.

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[13] The author would prefer basing the expansion stresses for all services on the S-values in this section.

ALLOWABLE STRESS RANGE

It has been suggested earlier that $S_r = 1.6\ S_h$ represents a conservative estimate of the stress at which flow starts under a bending moment at elevated temperature. By the same token, $S_r = 1.6\ S_c$, where S_c is the S-value at the minimum or (usually) installation temperature, might be taken to express a suitable condition for flow at the minimum temperature. The sum of these two limiting stresses, or

$$S_{av} = 1.6\ (S_c + S_h) \qquad [19]$$

then could be considered the maximum stress range S_{av} to which a system could be subjected without producing flow at either limit.

In the Proposed Rules, the allowable range of the expansion stresses by themselves has been established tentatively as follows

$$S_A = f\ (1.25S_c + 0.50S_h) \qquad [20]$$

Herein f is a stress-range reduction factor for cyclic conditions, varying from $f = 1$ for $N \leq 7000$ cycles, to $f = 0.5$ for $N \geq 250{,}000$ cycles, as shown in Fig. 5. The variation roughly follows the law

$$fN^{0.2} = 6 \qquad [21]$$

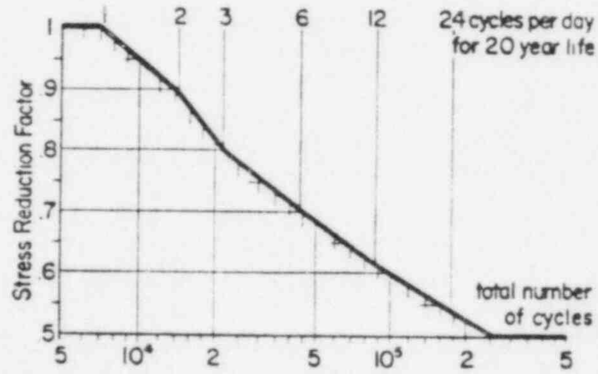

FIG. 5 PLOT OF STRESS-REDUCTION FACTOR CONTAINED IN PROPOSED RULES

which parallels the correlation of fatigue-test data on piping components suggested by the author (87) in 1946; see also Equation [4]. The motive for selecting 7000 cycles as the starting point for the application of the factor f was to free the calculation of everyday systems from this added complication; 7000 cycles roughly conform to a cycle per day over a period of 20 years, which is more than most systems are subjected to.

To obtain the maximum combined-stress range, the allowance $S_{PW} = 0.75\ S_h$ set aside in the Proposed Rules for pressure and weight stresses has to be added to the allowable expansion-stress range given by Equation [20]; this is done here on the assumption that f is unity, which covers the usual range of conditions

$$S_A + S_{PW} = 1.25\ (S_c + S_h) \qquad [22]$$

By comparison with Equation [19], it will be noted that the Proposed Rules as written utilize at most 78 per cent of the available stress range S_{av} deduced in the opening paragraph of this section; however, selection of a proper value for the factor on the right side of the equation is open for discussion and review.

An estimate of the average safety factor against rupture inherent in the Proposed Rules in the range between 7000 and 250,000 cycles is derived in Table 1 from the limited experimental data

TABLE 1 ESTIMATE OF SAFETY FACTOR FOR A LIFE OF 7000 CYCLES

Material	Carbon steel		Stainless steel	
Grade	Grade B		Type 316	Type 347
Constant test temperature	Room		Room	1050 F
Tests conducted by	Author's company (87)		Author's Co. (unpublished)	Stewart and Schreitz (116)
Factor C in formula $SN^{0.2} = C$	245000		281000	183500
Average stress range $S_A' = 2C/7000^{0.2}$ to produce failure under reversed bending in 7000 cycles	83400		95660	62470
Section of Code for Pressure Piping	Power piping	Oil piping	Oil piping	Oil or power
Allowable stress $S_c = S_h$ psi at given temperature under given section of Code for Pressure Piping	15000	20000	18750	13100
Allowable stress range $S_A + S_{PW} = 1.25\,(S_c + S_h)$ psi per Proposed Rules[a]	37500	50000	46875	32750
Safety factor in terms of stress $= S_A'/(S_A + S_{PW})$	2.22	1.67	2.04	1.91
Safety factor in terms of life $= [S_A'/(S_A + S_{PW})]^5$	54	13	35	25

[a] See Equation [20].

available. In terms of stress, the safety factor is found to be of the order of 2; in terms of cyclic life, it is of the order of 30. The very least safety factor available, considering the 25 per cent spread encountered between individual test data, might be estimated as 1.25 in terms of stress and 3 in terms of life. This emphasizes the need for making a conservative estimate of the number of cycles of major temperature change a system is likely to undergo.

In the range below 7000 cycles, the safety factor provided in the Proposed Rules increases. For example, for one cycle per week over 20 years, or a total of approximately 1000 cycles, the safety factor in terms of stress would increase by roughly 50 per cent. The minimum safety factor probably would be close to 2, which would be more than ample, provided the actual stresses are evaluated properly.

As far as the zone from 250,000 cycles upward is concerned, no estimate of the safety factor will be ventured, since the proportionality between the moment supported by pipe and fittings will be progressively lost. Fortunately, this zone has little practical significance with regard to expansion problems.[14]

A note of caution is in order. The provisions of the Proposed Rules do not take into account corrosion which would lower the endurance strength an unpredictable amount.

Allowable Reactions

The degree of flexibility required in a piping system is often controlled by the forces and moments the connected equipment can sustain without becoming inoperative or requiring excessive maintenance. Most frequently, the problem of setting allowable reactions arises in connection with equipment containing moving parts such as pumps or turbines, but it sometimes also requires consideration for other strain-sensitive equipment, such as large-diameter, thin-wall pressure vessels or exchanger shells with removable tube bundles fitting with close clearances.

With good logic, piping-stress analysts expect to be able to turn to equipment manufacturers for guidance in this matter on the premise that the latter should be in a position to advise what provisions have been made for absorbing piping reactions in the design of individual parts of their units and the completed assembly. The attitude often encountered in the past, that piping strains are no direct concern of the equipment designer, is fast disappearing, and it is becoming more and more recognized that lines connecting pumps or turbines or similar equipment would present no more of a problem than other lines, but for the fact that the piping has to absorb not only the expansion of the line, but also to protect the equipment from the effects of its own expansion. If this were not the case, the piping engineer could very simply discharge his task by rigidly anchoring his line adjacent to the equipment.

Unfortunately, only a few of the major manufacturers publish allowable thrusts and moments for their standard units (106, 115) or are prepared to advise whether the reactions computed by their customers for a specific installation can be tolerated. In general, even they are inclined to understate the capacity of their equipment, primarily because of a fear of discrepancies between the results of calculations based on simplifying assumptions and the reactions imposed upon the unit in actual service. It would appear that a change in policy toward permitting more liberal allowances would be contingent upon the following developments:

1 More general adoption of assumptions and methods of analysis of proved accuracy or conservatism; to foster this is one of the purposes of the Proposed Rules.

2 Improved understanding of the necessity of realizing the assumed design conditions in the actual installation. This implies proper specification and supervision of cold spring; also, a clear realization of the fact that calculations based on the assumption of a weightless system and frictionless supports can grossly underestimate reactions caused by thermal expansion where these are small in relation to the weight of piping supported.

3 Publication of information on the order of magnitude of the various components of piping reactions expected to be produced in well-designed piping systems leading to and from strain-sensitive pieces of equipment.[15]

Until better information becomes available, piping designers will be forced to continue to resort to rules of thumb to guide them in preparing their layouts. Some of these are given in the form of blanket limits upon thrusts; as an example, Baggerud and Jernstrom (51) suggested 3000 lb as an upper limit for ships' turbines. Others provide limits for both the resulting thrust and the resulting moment, the moment in foot-pounds often being taken equal to the thrust in pounds. Others consider the components of the reactions separately for different directions, higher limits usually being assigned to downward loads than to lateral thrusts. All of the rules cited seemingly disregard the size of the unit, although they are actually intended to apply to conditions customarily encountered in specific fields of engineering.

To take care of the size effect, some rules are given in terms of pounds thrust per diameter inch or peripheral inch of the nozzle. Paul (79), for example, suggests 100 lb per peripheral inch of turbine nozzle as a reasonable thrust. Another rule of this character, which has been proposed by Wolosewick (102), relates the thrust to the sum of the nominal diameters of the suction and discharge piping and at the same time differentiates with respect to the anchorage of the unit. Rules expressed in terms of kilowatt rating or equipment weight attempt to accomplish the same purpose.

[14] The rules are not intended to cover transmitted vibrations or pressure pulsations.

[15] The tabulation of average reactions against pumps on p. 453 of the paper by D. B. Rossheim and the author (55) is indicative of the type of information desired.

Finally, there are a number of advocates of expressing the limitation in terms of the piping stress at the terminal. For example, Heath (90) suggests a nominal bending stress of 9000 psi (with no credit for cold spring) as a satisfactory design basis; other experienced stress analysts have established individual limits depending upon the type of equipment connected.

The foregoing recital of different approaches has been given with the thought of stimulating discussion by those who are more familiar with the subject than the author can claim to be. It is his thought that reasonably conservative empirical rules of some form will always be necessary as a first general guide to a piping designer; if the reactions obtained therefrom should be exceeded in a specific layout of visually adequate proportions, consultation with the manufacturer is advised, at least in the case of important units.

What Systems Require Analysis

The foregoing review of the theoretical considerations and experimental data underlying the Proposed Rules inescapably leads to the conclusion that, even after considerable simplification and idealization with resultant loss in accuracy, the flexibility analysis of any but the simplest piping system presents a formidable task, and that accordingly it would be unreasonable to demand that each line be analyzed by the most precise approach available. Approximations must be permitted, provided their effect can be at least roughly evaluated and compensated for. This is not enough; in many instances, perhaps in the majority of cases, appraisal of the flexibility by visual inspection or comparison with similar layouts with satisfactory service performance must be accepted in lieu of a mathematical analysis or tests.

The group formulating the Proposed Rules has attempted to reflect this point of view in the following general clauses contained in paragraphs *620(a)* and *620(b)*:

1 Formal calculations or model tests shall be required only where reasonable doubt exists as to the adequate flexibility of a system.

2 Each problem shall be analyzed by a method appropriate to the conditions.

3 Where simplifying assumptions are used in calculations or model tests, the likelihood of attendant underestimates of forces, moments, and stresses shall be taken into account.

These clauses admittedly are vague and offer no concrete guidance toward arriving at a decision whether analysis is necessary in any specific case, what degree of approximation will be acceptable, and how it is to be compensated for; furthermore, they do not indicate whose judgment in this matter is to be accepted, the engineer's, the customer's, or the inspection authority's. The formulating group devoted earnest consideration to these questions, but came to the conclusion that the variables involved in flexibility analysis are too numerous, and their individual effect too unpredictable, to permit the establishment of a simple set of explicit rules, observance of which would assure protection to life, health, and investment without imposing an impossible burden of work on piping engineers.

Variables fall into three major classifications:

1 Material and temperature-dependent physical properties.

2 Cross-sectional properties.

3 Shape factors, i.e., properties associated with the dimensions and configuration of the line axis.

In the first group the expansion coefficient and the elasticity modulus assume primary importance as measures, respectively, of the amount of strain introduced into the system and the elastic resistance opposed by the material. Yield and creep strength reflect modifying influences of plastic flow upon the resistance, and at the same time provide important yardsticks for the determination of the allowable stress range, which is further conditioned on the endurance strength of the material.

Among the cross-sectional properties, the moment of inertia and section modulus of the pipe similarly provide measures of the forces and moments generated and the resistance of the pipe thereto; the influence of the latter is modified by any stress intensifications present.

While the foregoing properties enter piping-flexibility calculations more or less directly as factors, the dimensions and configuration of the line axis and the shape of its components (as reflected in their flexibility factors) exert a much more complex effect on the forces and moments, and therewith the stresses.

It will be apparent from the foregoing that any rule or formula intended to provide a demarcation line between flexible and stiff, or understressed and overstressed layouts must contain factors representative of the material, the temperature, and the line size, length, and shape. The first three major variables can be taken care of readily, but attempts at reducing the effects of line length and configuration to a simple and reasonably accurate shape factor meet with almost insuperable difficulties.

The most promising approach toward a first approximation is to express this factor in terms of the ratio of the developed

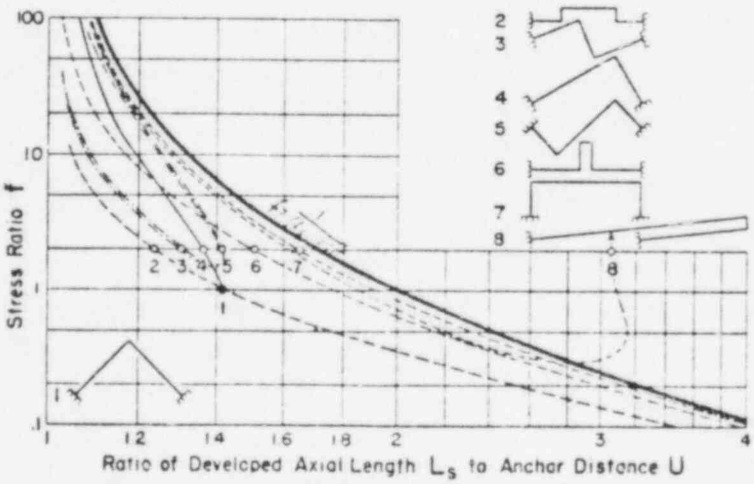

Fig. 6 Shape Factors for Simple Single-Plane Configurations

line length L_s to the distance U between anchors. What can be accomplished by this approach is shown in Fig. 6 which was developed from a study published by the author in one of his company's bulletins (105).[16] This is based on square-corner assumptions and embraces almost all conceivable proportions of single-plane configurations of the L-, Z-, U-, and expansion U-types. The abscissas are ratios L_s/U; the ordinates read the ratio f of the controlling stress in a bend of any of the shapes investigated to that in a square L-bend of equal anchor distance, pipe size and material, and temperature change. It will be noted that

$$f = \frac{1}{(L_s/U - 1)^2} \quad \text{[23]}$$

roughly describes the upper boundary of the entire family of curves except that applying to uncommon proportions of a U-bend with unequal legs, for which it may produce a gross underestimate of the stresses. As a rule, however, the stresses will be overestimated. For example, a stress ratio of the order of 6 is obtained for the square L-bend ($L_s/U = \sqrt{2}$), whereas by definition this should be unity. Obviously, the criterion is too insensitive to predict even the results of a square-corner solution with any degree of reliability. Since the latter itself often provides no more than a crude first approximation, it becomes evident that a formula of this simple character will not serve to provide a reliable means of distinguishing systems which must be calculated from those for which calculation can be waived.

This same criticism applies to the formula given in the alternate version of paragraph *620(c)* of the Proposed Rules[17] which assigns a definite limiting value to the stress ratio f

$$0.03\frac{U^2}{DY} \geq \frac{1}{(L_s/U - 1.05)^{1/2}} \quad \text{[24]}$$

where U and L_s, respectively, again designate anchor distance and developed line length (ft), and D and Y are the nominal pipe size and the resultant of the restrained thermal expansion and net linear terminal displacements (in.). The left-hand term in this case also contains approximations; specifically, it assumes a constant relationship between the allowable stress range and the modulus of elasticity.

Assuming that it would be possible to establish a criterion enabling the piping designer to eliminate amply flexible systems from consideration, the next problem is that of distinguishing the remaining systems with respect to the accuracy required in their calculation. Systems carrying flammable, noxious, or otherwise dangerous fluids, or failure of which would entail a major financial loss, obviously are more in need of precise analysis than those where a break is merely inconvenient and readily repaired. In the latter instances the application of approximate methods would appear economically justified from a standpoint of time saving; the use of approximations also may be necessary for more critical piping systems involving branch lines or intermediate restraints.

Wherever approximate methods are used, the question immediately arises how to compensate for the attendant error. Again, no simple rule can be advanced. The only advice which can be offered is to compare the results obtained by the approximate method it is proposed to use, with those of precise calculations for a sufficient number of cases covering the extreme conditions it is expected to encounter, and to derive correction factors therefrom. In some methods, such as those published by the author's company (105), such a check has already been made by the proponent of the method.

[16] See "Study of Shape Factor."

[17] Transcribed in Appendix 2.

Conclusion

The Proposed Rules present an attempt by some of the country's leading piping engineers gathered as a task force operating under Sectional Committee ASA B31.1 to reduce the complex problem of providing adequate flexibility in a piping system to a few simple guide lines reflecting the latest advances in theoretical understanding and accumulated practical experience. It has been the author's assignment to assemble the factual evidence underlying this document and explain certain concepts, such as stress-intensification factor, stress range, self-spring, which have been inherent in past formulations of the chapter on "Expansion and Flexibility," but are more openly referred to in the new draft.

On reviewing the evidence, numerous gaps in our knowledge of the magnitude of certain properties entering into the problem have become apparent. On the other hand, not all the present knowledge available on certain phases could be utilized in framing the Code Rules because of the need for keeping them simple. This has necessitated a weighing of the significance of the various factors and their effect on the over-all accuracy of the prediction of reactions and stresses.

While the Proposed Rules represent the group's best effort, the interpretation of the facts given therein is not necessarily the only one possible. Publication of the thought processes leading to their adoption is intended to provoke discussion by engineers at large, to uncover additional data not available to the group, and ultimately to lead to an improved formulation, particularly with regard to the clauses intended to promote uniformity of practice and intelligent enforcement.

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Appendix 1

Text of Proposed Rules for Chapter 3 of Section 6 of Code for Pressure Piping as Formulated by Task Force on Flexibility

(Note: The provisions of this chapter are not applicable to gas, air, and oil cross-country transmission (underground) piping.)

617 Preamble. (*a*) Piping systems are subject to a diversity of loadings creating stresses of different types and patterns, of which only the following more significant ones need generally be considered in piping stress analysis:

1 Pressure, internal or external.

2 Weight of pipe, fittings and valves, contained fluid and insulation.

3 Thermal expansion of the line.

The first two loadings produce sustained stresses which are evaluated by conventional methods. The stresses due to thermal expansion, on the other hand, if of sufficient initial magnitude will be relaxed as a result of local flow in the form of yielding or in the form of creep. The stress reduction which has taken place will appear as a stress of reversed sign in the cold condition. This phenomenon is designated as self-springing of the line and is similar in effect to cold springing. The amount of such self-springing will depend on the magnitude of the initial hot stress and the temperature. Accordingly, while the hot stress tends to diminish with time, the sum of the hot and cold stresses during any one cycle will remain substantially constant. This sum is referred to as the stress range. The fact that the stress range is the determining factor leads to the selection of an allowable combined stress (range) in terms of the sum of the hot and cold S-values.

(b) The beneficial effect of judicious cold springing in assisting the system to attain its most favorable condition sooner is recognized. Inasmuch as the life of a system under cyclic condition depends primarily on the stress range rather than the stress level at any one time, no credit for cold spring is warranted with regard to stresses. In calculating end thrusts and moments acting on equipment containing moving or removable parts with close clearances, the actual reactions at any one time rather than their range are significant and credit accordingly is allowed for cold spring in the calculations of thrusts and moments.

618 Materials. (a) This chapter applies to all classes of materials permitted by the Code.

(b) The thermal expansion range e shall be determined from Table–[18] as the difference between the unit expansion shown for the maximum normal-operating metal temperature and that for the minimum normal operating metal temperature (for hot lines, this may usually be taken as the erection temperature). For materials not included in this table, reference shall be made to authoritative source data, such as publications of the National Bureau of Standards.

(c) The cold and hot moduli of elasticity, E_c and E_h, respectively, shall be taken from Table –[18] as the values shown for the minimum and maximum normal-operating metal temperatures, respectively. For materials not included in this table, reference shall be made to authoritative source data, such as publications of the National Bureau of Standards.

(d) Poisson's ratio may be taken as 0.3 for all ferrous materials at all temperatures. (Elsewhere in the Code there will be found tables of values of Poisson's ratio for various materials which tables are given for general information.)

(e) The S-values, S_c and S_h at the minimum and maximum operating metal temperatures, respectively, to be used for determining the allowable expansion-stress range S_A shall be taken for the type of piping system involved from the applicable tables in the respective sections of the Code. In the case of welded pipe, the longitudinal-joint efficiency may be disregarded.

619 General. (a) Piping systems shall be designed to have sufficient flexibility to prevent thermal expansion from causing 1—failure from overstress of the piping material or anchors, 2—leakage at joints, or 3—detrimental distortion of connected equipment resulting from excessive thrusts and moments.

(b) Flexibility shall be provided by changes of direction in the piping through the use of bends, loops, and off-sets; or provision shall be made to absorb thermal strains by expansion joints of the slip joint or bellows types.[19] If desirable, flexibility may be provided by creasing or corrugating portions or all of the pipe.

(c) In order to modify the effect of expansion and contraction, runs of pipe may be cold sprung. Cold spring may be taken into account in the calculation of the reactions as shown in paragraph *621(d)* provided an effective method of obtaining the designed cold spring is specified and used.

620 Basic Assumptions and Requirements. (a) Formal calculations or model tests shall be required only where reasonable doubt exists as to the adequate flexibility of a system. Each problem shall be analyzed by a method appropriate to the conditions.

(b) Standard assumptions and requirements are given in paragraphs (d) to (g). Where simplifying assumptions are used in calculations or model tests, the likelihood of attendant underestimates of forces, moments, and stresses shall be taken into account.

(c) In calculating the flexibility of a piping system between anchor points, the system shall be treated as a whole. The significance of all parts of the line and of all restraints such as solid hangers or guides, shall be considered.

(d) Calculations shall take into account stress-intensification factors found to exist in components other than plain straight pipe. Credit may be taken for the extra flexibility of such components. In the absence of more directly applicable data, the flexibility factors and stress-intensification factors shown in Chart I may be used.

(e) Dimensional properties of pipe and fittings as used in flexibility calculations, shall be based on nominal dimensions.

(f) The total expansion range from the minimum to the maximum normal-operating temperature shall be used in all calculations, whether piping is cold sprung or not. Not only the expansion of the line itself, but also linear and angular movements of the equipment to which it is attached, shall be considered.

(g) Flexibility calculations shall be based on the modulus of elasticity E_c at room temperature.

621 Stresses and Reactions. (a) Using the above assumptions, the stresses and reactions due to expansion shall be investigated at all significant points.

(b) The expansion stresses shall be combined in accordance with the following formula

$$S_E = \sqrt{S_b^2 + 4S_t^2}$$

where

$S_b = i\,M_b/Z$ = resultant bending stress, psi
$S_t = M_t/2Z$ = torsional stress, psi
M_b = resultant bending moment, lb/in.
M_t = torsional moment, lb/in.
Z = section modulus of pipe, in.^{3}
i = stress-intensification factor

(c) The maximum computed expansion stress, S_E, shall not exceed the allowable stress, S_A, where

$$S_A = f\,(1.25\,S_c + 0.5\,S_h)$$

subject to the limitations of paragraph *622(b)*
where

S_c = allowable stress (S-value) in the cold condition
S_h = allowable stress (S-value) in the hot condition
S_c and S_h are to be taken from tables in the applicable sections of the Code.
f = stress-range reduction factor for cyclic conditions to be applied; in the absence of more applicable data the values of f shall be taken from the following table:

Total no. of full temp cycles over expected life	Stress-reduction factor f
7000 and less	1.0
14000	0.9
22000	0.8
45000	0.7
100000	0.6
250000 and over	0.5

[18] Tables of these properties will be provided upon adoption of these rules. In the meantime, data published in Piping Handbooks or catalogs may be used.

[19] In this case, anchors or ties of sufficient strength and rigidity shall be installed to provide for end forces due to fluid pressure and other causes.

(d) The reactions (forces and moments) R_h and R_c in the hot and cold conditions, respectively, shall be obtained as follows from the reactions R derived from the flexibility calculations

$$R_h = \left(1 - \frac{2}{3}c\right)\frac{E_h}{E_c}R$$

$$R_c = cR$$

or

$$R_c = \left(1 - \frac{S_h}{S_E}\frac{E_c}{E_h}\right)R$$

whichever is greater, and with the further conditions that $(S_h/S_E)(E_c/E_h)$ is less than 1,

where

- c = cold spring factor varying from zero for no cold spring to one for 100 per cent cold spring
- S_E = maximum computed expansion stress
- E_c = modulus of elasticity in the cold condition
- E_h = modulus of elasticity in the hot condition
- R = range of reactions corresponding to the full expansion range based on E_c

R_c and R_h represent the maximum reactions estimated to occur in the cold and hot conditions, respectively.

(e) The reactions so computed shall not exceed limits which the attached equipment can safely sustain.

622 Supports. (a) Pipe supports and restraints not expressly considered in flexibility calculations shall be designed to minimize interference with the thermal expansion of the line.

(b) The design and spacing of supports shall be checked to assure that the sum of the longitudinal stresses due to weight and pressure does not exceed S_h. Where this sum exceeds $^3/_4$ S_h but does not exceed S_h, the amount in excess of $^3/_4$ S_h shall be subtracted from S_A.

Appendix 2

Alternate Clauses for Chapter 3 of Section 6 of Code for Pressure Piping as Proposed by The M. W. Kellogg Company

620 Basic Assumptions and Requirements. (a) Formal analysis or model tests shall be required for pipe lines which simultaneously satisfy the following conditions:

Maximum normal operating metal temperature over 800 F.
Nominal pipe diameter over 6 in.
Rated service pressure over 15 psi.

The method of investigation shall be appropriately selected to conform with the condition of the problem under examination.

(b) The requirements for analysis shall be considered satisfied for duplicate units of successfully operating installations or for replacements of piping systems with a record of satisfactory service.

(c) It is recognized that for operating conditions not satisfying concurrently the provisions of paragraph (a), an analysis for each piping system is economically impractical. An analysis is, therefore, mandatory only if the following approximate criterion is not satisfied

$$\frac{DY}{U^2(R - 1.05)^{2/3}} \leqq 0.03$$

The constants 1.05 and 0.03, as well as the exponent of $^2/_3$, represent only approximate values, which will be subject to further investigation and correction as needed.

In the foregoing equation

- D = nominal pipe size, in.
- Y = resultant of restrained thermal expansion and net linear terminal displacements, in.
- U = anchor distance (length of straight line joining anchors), ft.
- R = ratio of developed pipe length to anchor distance, dimensionless.

(d) Standard assumptions and requirements are given in paragraphs (f) to (i).

(e) In calculating the flexibility of a piping system between anchor points, the system shall be treated as a whole. The significance of all parts of the line and of all restraints such as solid hangers or guides, shall be considered.

(f) For calculations made in conformity with paragraph (a), stress-intensification and flexibility factors may be omitted if the piping system is not subject to more than 2000 stress cycles during its expected life. For lines subject to more than 2000 stress cycles, calculations shall take into account stress-intensification factors found to exist in components other than plain straight pipe. Credit may be taken for the extra flexibility of such components. In the absence of more directly applicable data, the flexibility factors and stress-intensification factors shown in Chart I may be used.

(g) For thermal-expansion analysis, dimensional properties of pipe and fittings shall be based on nominal dimensions.

(h) The total expansion range from the minimum to the maximum normal operating temperature shall be used in all calculations, whether piping is cold sprung or not. Not only the expansion of the line itself, but also linear and angular movements of the equipment to which it is attached, shall be considered.

(i) Flexibility calculations shall be based on the modulus of elasticity E_c at room temperature.

621 Stresses and Reactions. (c) The maximum combined pressure and expansion stress shall not exceed 0.75 times the rated ultimate tensile strength of the annealed material at room temperature. The maximum computed expansion stress S_E shall not exceed the following allowable value

$$S_A = f\,(1.25\,S_c + 0.25\,S_h)$$

where

- S_c = allowable stress (S-value) in the cold condition
- S_h = allowable stress (S-value) in the hot condition

(S_c and S_h are to be taken from the tables in the applicable sections of the Code.)

- f = stress-reduction factor to be applied for cyclic service; in the absence of more applicable data the values of f shall be taken from the following table:

Total no. of full temp cycles over expected life	Stress-reduction factor, f
7000 and less	1.0
14000	0.9
22000	0.8
45000	0.7
100000	0.6
250000 and over	0.5

If the sum of longitudinal pressure and weight stresses is less than S_h, the difference between S_h and the sum of these stresses may be added to S_A.

622 Supports. (b) The design and spacing of supports shall be checked to assure that the sum of the longitudinal stresses due to weight and pressure does not exceed S_h. The analysis for pressure stresses shall be based on the eroded dimensions of the pipe.

CHART 1 FLEXIBILITY AND STRESS-INTENSIFICATION FACTORS FOR PIPING COMPONENTS

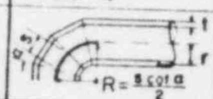
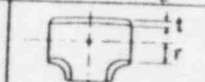
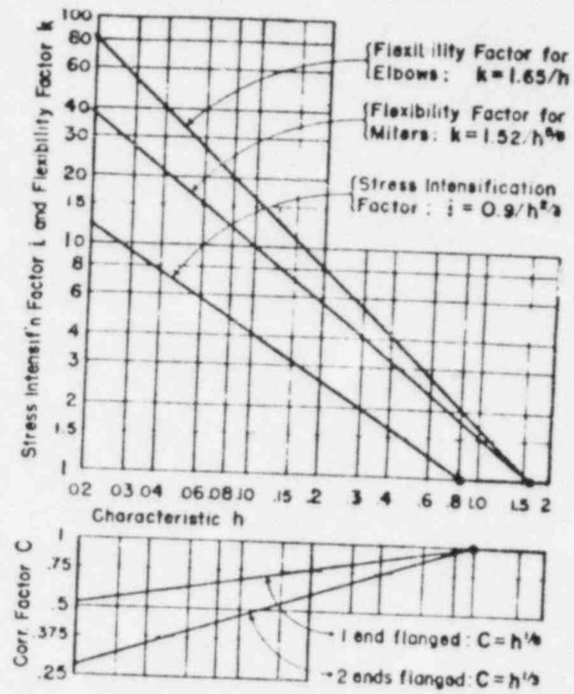
Description	Flexibility Factor k	Stress Int. Factor i	Flexibility Characteristic h	Sketch
Welding Elbow, [1,2,3] or Pipe Bend	$\frac{1.65}{h}$	$\frac{0.9}{h^{2/3}}$	$\frac{tR}{r^2}$	
Closely spaced Mitre Bend, [1,2,3] $s < r(1 + \tan\alpha)$	$\frac{1.52}{h^{5/6}}$	$\frac{0.9}{h^{2/3}}$	$\frac{\cot a}{2}\ \frac{ts}{r^2}$	
Widely spaced Mitre Bend, [1,2,4] $s \geq r(1 + \tan\alpha)$	$\frac{1.52}{h^{5/6}}$	$\frac{0.9}{h^{2/3}}$	$\frac{1 + \cot a}{2}\ \frac{t}{r}$	
Welding Tee [1,2] per ASA B16.9	1	$\frac{0.9}{h^{2/3}}$	$4.4\frac{t}{r}$	
Reinforced Fabricated Tee, [1,2] with pad or saddle	1	$\frac{0.9}{h^{2/3}}$	$\frac{(t + \frac{1}{2}T)^{5/2}}{t^{3/2}\, r}$	
Unreinforced Fabricated Tee [1,2]	1	$\frac{0.9}{h^{2/3}}$	$\frac{t}{r}$	
Butt Welded Joint, Reducer, or Welding Neck Flange	1	1.0		
Double-Welded Slip-On or Socket Weld Flange	1	1.2		
Fillet Welded Joint, or Single-Welded Socket Weld Flange	1	1.3		
Lap Joint Flange (with ASA B16.9 lap joint stub)	1	1.6		
Screwed Pipe Joint, or Screwed Flange	1	2.3		
Corrugated straight Pipe, or Corrugated or Creased Bend	5	2.5		

CHART 1a PLOT OF FLEXIBILITY AND STRESS-INTENSIFICATION FACTORS



[1] The flexibility factors k and stress intensification factors i in the Table apply to fittings of the same nominal weight or schedule as the pipe used in the system and shall in no case be taken less than unity. They apply over the effective arc length (shown by heavy center lines in the sketches) for curved and mitre elbows, and to the intersection point for tees.

[2] The values of k and i can be read directly from Chart 1a, entering with the characteristic h computed from the formulas given, where:

R = bend radius of welding elbow or pipe bend
r = mean radius of matching pipe
t = wall thickness of matching pipe
α = one-half angle between adjacent mitre axes
s = mitre spacing at center line
T = pad or saddle thickness

[3] Where flanges are attached to one or both ends, the values of k and i in the Table shall be corrected by the factors C given below, which can be read directly from Chart 1a, entering with the computed h:

One end flanged: $h^{1/6}$. Both ends flanged: $h^{1/3}$.

[4] Also includes single-mitre joint.

Appendix 3

Flexibility Factors for Curved Pipe

In the following, the three successive approximations of von Kármán's flexibility factor are shown both in the form in which they are usually found in literature (117) and in a reformulation by the author which makes them easier to compare

$$k_1 = \frac{12h^2 + 10}{12h^2 + 1}$$

$$= 1 + \frac{9}{12h^2 + 1}$$

$$k_2 = \frac{105 + 4136h^2 + 4800h^4}{3 + 536h^2 + 4800h^4}$$

$$= 1 + \frac{9 + 0.255000/h^2}{12h^2 + 1.3400 + 0.007500/h^2}$$

$$k_3 = \frac{252 + 73{,}912h^2 + 2{,}446{,}176h^4 + 2{,}822{,}400h^6}{3 + 3280h^2 + 329{,}376h^4 + 2{,}822{,}400h^6}$$

$$= 1 + \frac{9 + 0.300306/h^2 + 0.00105867/h^4}{12h^2 + 1.4004 + 0.013946/h^2 + 0.00001276/h^4}$$

In discussing Shipman's paper (17), Jenks gave the following formulation as reflecting the nth approximation of the von Kármán flexibility constant

$$k_n = \frac{12h^2 + 10 - j}{12h^2 + 1 - j} = 1 + \frac{9}{12h^2 + 1 - j}$$

where j is a complex function of h which has the following values:

h	0	0.05	0.1	0.2	0.3	0.4	0.5	0.75	1.00
j	1	0.7625	0.5684	0.3074	0.1764	0.1107	0.07488	0.03526	0.02026

The flexibility factors obtained from the preceding four formulas are compared with those obtained from Equation [2]

$$k_B = \frac{1.65}{h}, \geqq 1$$

for four values of h covering the normal useful range. It will be observed that this simple approximation gives values closely comparing with the more precise of the other formulations:

Flexibility characteristic h =	0.05	0.1	0.5	1
von Kármán first approximation, k_1 =	9.74	9.04	3.25	1.69
von Kármán second approximation, k_2 =	26.4	16.6	3.29	1.69
von Kármán third approximation, k_3 =	34.0	17.3	3.29	1.69
Jenks nth approximation, k_n =	34.6	17.3	3.29	1.69
Approximation based on Beskin, k_B =	33.0	16.5	3.30	1.65

Discussion

John E. Brock.[20] This is one of the most important papers which has ever been written on the subject of piping. The entire industry is indebted to the members of the Task Force and of the working group for the study and inventive effort that is represented by the May 4, 1953, Report, and the present paper goes beyond this in presenting not only the results but also the rationale. Further, it should be remarked that the manner of presenting not only this paper but also the Task Force Report is in keeping with the excellent tradition established in connection with the development of the ASME Boiler and Pressure Vessel Code and carried on in the ASME-sponsored American Standard Code for Pressure Piping—a tradition of orderly development incorporating contributions by as many interested persons as possible, with full and sympathetic study being given to all minority viewpoints, and with active solicitation of all kinds of criticism. A special word of praise is due to the author of this paper, for although the Task Force Report represents a joint effort, it is evident that at least in the preparation of this excellent paper, the author has gone far beyond the call of duty.

The writer's further comments will be given under the headings of the section titles to which they pertain.

Flexibility Factors and Stress-Intensification Factors. The practical piping engineer is not in a position to select from all the theoretical and experimental data which have appeared on the subject of flexibility factors and stress-intensification factors in piping components. The selection represented by the simplified Formulas [2] through [6] in the text of the paper and in Chart 1 of the Report will be of great convenience. While it is not unlikely that further research in years to come may suggest modification of some of these formulas, it may be fairly stated that they represent as good a selection as may presently be made and they are a great improvement over previous formulas in simplicity and convenience. It is particularly striking that, with proper interpretation of the quantities involved, Formula [5] is valid for so many different piping components, and the author is to be thanked for having introduced this formula in his paper, reference (114).[21]

The author mentions that the bend-flexibility factor given by Formula [1] or [2] or other similar formulas was used only for "in-plane" bending before Vigness (70) showed that it also should be used for "out-of-plane" bending. Unfortunately, the appearance of the Vigness paper was not sufficient to change a rather firmly established practice. The Task Force Report does not emphasize the applicability of this factor to both types of bending, and many analysts continue to ignore the increased out-of-plane flexibility. In the writer's opinion, the Report should forcibly direct the reader's attention to this development.

One other remark is in order concerning stress-intensification factors. The second paragraph of the section headed Stress-Intensification Factors adequately defines these factors for the purposes of the paper. However, there are other types of stress-intensification factors applying to pipe loadings other than those of interest in piping-flexibility analysis. For example, the factor

$$i = \frac{4R_{\text{bend}} - D_{\text{pipe}}}{4R_{\text{bend}} - 2D_{\text{pipe}}}$$

gives the intensification of hoop stress due to internal pressure which takes place at the throat of a pipe bend or elbow (118).

Primary Analysis. The author's exposition is the first, of which the writer is aware, that lists the sources of inaccuracy in analytical evaluations of what he calls "shape factors." It is vitally important that these sources of error be recognized, for otherwise the analyst and those to whom the analysis is submitted cannot have a meeting of minds. That is definitely not to say, however, that the corresponding errors should be eliminated or outlawed; a good analyst is on the lookout for approximations which afford a considerable saving in effort at the expense of but a small and tolerable loss in accuracy. However, the approximation should not be hidden or denied; instead, it should be delineated fully and its effects on the final results should be evaluated.

Substitution of square corners for curved members is only one of the many ways that an actual piping configuration may be "idealized" preliminary to the actual mathematical or model analysis. Frequently it is justified to neglect relatively slight changes in direction, small offsets, and so on, in order to simplify the configuration actually subjected to analysis; however, the extent of such "idealization" should be made clear.

[20] Director of Research, Midwest Piping Company, St. Louis, Mo. Mem. ASME.

[21] Numbers in parentheses refer to the author's bibliography.

The "square-corner approximation" which is widely used still has never been evaluated fully. Life is short and problems are difficult so that this idealization will continue to be used, but it is to be hoped that further developments will enable us to make reliable estimates of its effects on calculated results and that still other developments will provide a truly simple method of correcting a square-corner analysis so as to obtain results definitely known to be within tolerable limits of accuracy. The writer is not particularly hopeful that these developments will be forthcoming in a matter of just a few years, though.

One indication of how deceptive this area may be, may be seen from the author's statement that the square-corner approximation "...leads to an overestimate of reactions..." This was an opinion which the writer shared until he was brought up short by a statement of Dr. E. G. Baker: "A serious objection (to the square-corner approximation) has been that square-corner errors are usually on the wrong side, that is, flexibility is over-estimated."[22] An attempt to reconcile the difference leads the writer to the conclusion that both statements are correct—if applied to the proper type of configuration. The writer's experience, and presumably also that of the author, have been mainly with stationary installations whereas that of Dr. Baker is with marine installations. A bend or elbow, as compared to a square corner, has two opposing effects on flexibility.

On the one hand, the Kármán-Hovgaard-Vigness-Beskin, etc., flexibility factor causes the bend to be more flexible than the corner; on the other hand, the bend or elbow "short-cuts" the corner and geometrically offers a stiffer path. In stationary installations, the bend radius is small compared to a representative dimension (say, the straight-line distance between end points) that characterizes the configuration, and consequently the increased flexibility is of greater influence than is the short-cut effect. On the other hand, marine installations are likely to be more confined and the bend radius is a larger fraction of the corresponding representative dimension so that the short-cut effect predominates.

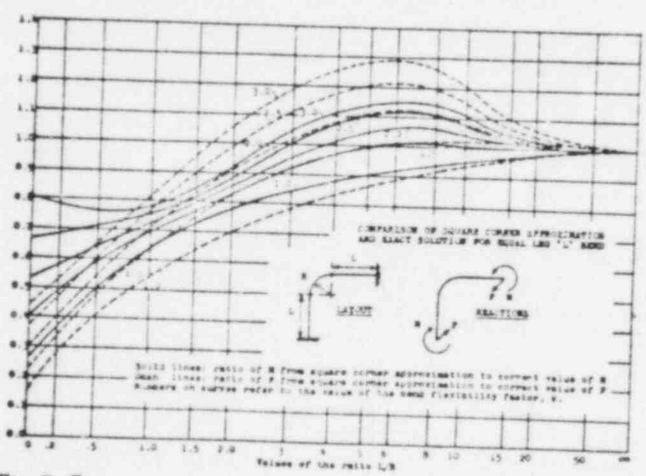


Fig. 7 Comparison of Square-Corner Approximation and Exact Solution for Equal Leg L-Bend

That the square-corner approximation can give values of reactive forces and moments which are sometimes too high and sometimes too low may be seen from Fig. 7 of this discussion, where the simplest possible case is analyzed—that of an equal leg L-bend. (In preparing Fig. 7, values were calculated only for $L/R = 0, 1, 4, 9$, and infinity.) If the tangents are short relative to the bend radius, the short-cut effect governs and the square-corner approximation underestimates reactions. For longer tangents, however, the increased flexibility of the bend governs and the reactions are overestimated in the square-corner approximation. Of course the effect depends upon the value of the bend-flexibility factor, k.

Another point should be raised concerning the nature of the approximations implicit in certain analytical solutions. In strictly two-dimensional cases, there is such a thing as a "neutral axis" and the resultant force system can be reduced to a single force passing through the "elastic centroid" of the configuration and lying along the neutral axis. In symmetrical cases the neutral axis does indeed parallel the line-connecting anchors, but as the author indicates, errors result from making this assumption in nonsymmetrical cases. In general three-dimensional cases, there is no such thing as a neutral axis. The resultant force system consists of three moment components and three force components all of which can be represented in various ways, namely, as a wrench, as a motor, as a combination of a line vector and a free vector, and so on.

Only in very special cases does the wrench or motor become singular, reducing to a line vector of force with the free-moment vector vanishing, and only in these cases is it possible to speak of a neutral axis. Attempts to represent a three-dimensional problem in terms of a neutral axis imply doing some violence to the fundamental principles of mechanics. Again, this is not to say that the attempts are not warranted if they succeed in affording a great simplification with but little loss in accuracy, but it does not appear that the question of how much the accuracy suffers has been explored adequately.

Finally, some inaccuracy is involved in neglecting secondary bending terms for bends and elbows.[23] In almost all cases the error is slight and the simplification great; however, for tight configurations involving short-radius thin-wall elbows, the secondary terms may become of great importance. In fairness it should be mentioned that no great difficulty is involved in taking the secondary terms into account in using the Piping Handbook method of analysis.

Self-Spring and Cold-Spring Effects. The analyses of the Report and of the paper assume that there is a single quantity c which characterizes the amount of cold spring in the system. In certain actual designs, however, different degrees of cold spring are employed in two or three of the co-ordinate dimensions. It is not clear what computational procedure may be implied in such cases by the rules incorporated in the Report.

The writer invites the author's further comments concerning the factor $^2/_3$ which appears in Formula [17]. The use of this factor theoretically results in an overestimate of the initial hot reaction and this fact, coupled with the extreme conservatism or ostrichlike attitude of many manufacturers of rotating or reciprocating equipment, may result in the rejection of some designs which actually are adequate. It is recognized that the $^2/_3$ limitation on the amount of cold spring that can be counted for certain computational purposes has been a part of the Code for several years. Does the Task Force have particular reason for reaffirming this attitude? Does the $^2/_3$ limitation reflect recognition of the fact that all computations and analyses are but imperfect ways of predicting physical behavior, and that no matter how carefully cold-springing is calculated and performed, actual reactions are not likely to be reduced to less than one third their calculated values without cold spring? If this is so, the writer would suggest that the formula for R_h become

$$R_h = \left(1 - \frac{2}{3}c\right)\frac{E_h}{E_c}R$$

or

[22] Private communication, April 29, 1953.

[23] Author's reference (75), p. 801.

$$R_h = \frac{E_h R}{3E_c}$$

(whichever is larger) so that cold spring in excess of 100 per cent might not be employed to reduce the theoretical value of R_h below what is reasonable.

The statement "... service failures are associated with cyclic, rather than static stress application..." appears in support of the stress-range concept. One may remark in passing that the service failures to which reference is made are clearly those in piping and connections and not in rotating or reciprocating mechanical equipment where a mal- or nonfunction is more usually the result of static overload. However, there are practical cases where static stress rather than stress range governs the integrity of the piping itself. In very "tight" configurations composed of very thin-wall piping and components, local crippling may govern. The phenomenon is one of instability rather than of strength and it is recognized that the Code cannot include rules for such exceptional cases.

Allowable Stress Range. The writer believes that the stress-range concept is useful and generally valid and is pleased to see it recognized by the proposed rules of the Task Force Report. However, he feels that there should be leeway for designer and user, if conditions call for it, to agree upon other criteria appropriate to the particular situation involved.

The last paragraph of this section of the paper should not escape attention and the Code rules themselves should contain an admonitory remark concerning the effect of corrosion.

Allowable Reactions. The writer would like to commend the author's evaluation of the lack of realism reflected in the limitations on allowable piping reactions presently established by manufacturers of the equipment to which the piping attaches and of the economies which would result from an upward revision of such limitations. The writer understands that this is a question under consideration by a group of users of major rotating equipment.

What Systems Require Analysis? It would be most helpful if the proponents of Formula [23] would present an evaluation of the applicability and accuracy of this formula and if others were to contribute to the subject. Formulas like this constitute, in effect, greatly simplified methods of analysis which, if their range of validity could be assessed, would be very useful for ranging shots and highly approximate analysis. However, the writer sides with the majority opinion of the Task Force in not wishing to raise this or a similar approximate formula to the dignity of a criterion for mandatory requirement for analysis.

Conclusion. The writer has submitted comments on the May 4, 1953, Report directly to the Task Force (and it should be hoped that all persons having an interest in the problem of piping flexibility will similarly contribute their comments). One point which was developed in these comments deserves mention here. The Code should definitely provide for alternate procedures of interpretation in certain exceptional cases where all interested parties can agree upon an answer based upon sound engineering principles even if this implies deviating from a strict and literal application of the Code rules. Otherwise, legal or contract technicalities, enforced by inspectors powerless to make exceptions, may require uneconomical design.

F. M. Kamarck.[24] Referring to the Preamble *617* of the Proposed Rules for Chapter 3, Section 6, of the Code for Pressure Piping, and also to the fourth and fifth paragraphs under Self-Spring and Cold-Spring Effects of the present paper, dealing with the relaxation of the first hot cycle of a piping system, the following comments are offered:

The first stress cycle indicates that the stress exceeded the elastic limit in one or more areas and must have produced localized deformation that later showed up as self-spring and stress when the line was allowed to become cold. This overstress is not desirable and neither is the change of grain structure that will take place in these deformed areas. This changed grain structure will have a higher creep rate from the rest of the pipe where the grain structure was not radically altered by overstress.[25,26] In other words, any piping system that shows a rapid relaxation (other than the normal creep characteristic for the designed stress) has been weakened in some area or areas and its strength is no longer certain. This, the writer submits, is not a good approach to the design of high-temperature piping.

The writer believes that it has been fairly well established that this localized deformation does not take place in the cantilever portions of the piping system (except possibly for a very small distance adjacent to bends and elbows when these contain overstress), and that this relaxing deformation occurs in the bends and elbows of the piping system with the present approach to design.

For most load-resisting members any permanent deformation or set that accompanies stresses below the yield point of the material does not damage the members seriously. It is the permanent deformation that occurs at the yield point that is dangerous.

The problem should be tackled at its source, bends, and elbows, and something should be done at these points. The writer also believes it to be incorrect for the proposed Code section to accept local overstress in any part of a piping system. Reinforcement would be added in any other place where it was known that high local stress was present. Why not at bends and elbows? It appears that the stress-intensification factor for bends and elbows requires serious re-evaluation and revision upward.

It is felt that the proposed Code section should require higher schedule thickness for bends and elbows (also possibly tees) than the required pipe thickness so as to eliminate or minimize local deformation caused by exceeding the elastic limit.

Such a requirement also will aid the more economical design of piping systems. At present too high a price is being paid for the false flexibility of bends and elbows by a stress-intensification factor that not only cancels out any apparent gain but causes an overdesign of the whole piping system in order to prevent the stress-intensification factor from carrying the stress in the elbows over the allowable stress limit. Thickening the elbows will tend to eliminate what is generally a minor source of flexibility of the piping system, but will bring the stress characteristic into line with the rest of the piping design. Relaxation, if it should occur, should be through slow creep and uniformly relative through the various stressed portions of the piping system.

H. C. E. Meyer.[27] To the author should go the thanks of the Society and industry in general for his most excellent paper.

Many years ago the writer discovered that when he became involved in the subject of stresses in piping as the result of thermal expansion he was entering a labyrinth from which he has not been able to extricate himself to this day.

That the subject is a most complex one is attested by the long Bibliography accompanying the paper, and some of the names that appear in this Bibliography are ones that will long be remembered. Such men as Prof. William Hovgaard, Mr. Sabin Crocker, Mr. D. B. Rossheim, Mr. A. R. C. Markl, and many, many others

[24] Staff Engineer, Burns and Roe, Inc., New York, N. Y.

[25] "Analysis of Basic Problems of High Temperature Creep," by O. D. Sherby and J. E. Dorn.

[26] "Metallurgical Aspects of Strength at High Temperature," by G. V. Smith.

[27] Gibbs & Cox, Inc., New York, N. Y.

have done much to throw light on the subject and provide means to calculate the stresses and reactions which may be encountered in service as a result of the expansion of piping.

The mere difficulties involved in computing the physical characteristics of elaborate piping systems are in themselves enormous, and we owe an everlasting debt of gratitude to Professor Hovgaard for the basic equations he developed.

On the surface it would appear that once we had methods for determining the stresses, all that would be necessary would be to set satisfactory limits on such stresses for various conditions, and that would be the end of the matter. However, in a paper by Mr. D. B. Rossheim and Mr. A. R. C. Markl in July, 1940, the conception of a stress range came into prominence.

When the writer was first asked to accept membership on the Subcommittee on Flexibility, he was still very much of the opinion that, by determining the stresses as accurately as practicable and establishing limits for these stresses, all would be well, as this method had given satisfactory results in the many Naval vessels where they were applied.

But then the stress-range concept began to enter the picture and, while it took the writer some time to get even a hazy view of the importance of this conception, it became a part of the proposed revision to the Code as the result of two very fortuitous circumstances.

The writer was required to spend considerable time in Europe a year ago, but before leaving asked a small group to work on a draft of the proposed revision. This group consisted of Messrs. Markl, Spielvogel, Blair, and Wallstrom, and the writer cannot refrain from stating once more his appreciation to these men for the wonderful work they did in coming up with a proposed draft, which, after the committee as a whole met, was submitted to the Executive Committee as a report, and has been circulated to the membership.

The second fortuitous circumstance is that for the past year the writer has been somewhat "under the weather" which perhaps has given him more time to think, and as a result has come to accept the concept of the stress range not merely with some reluctance but enthusiastically as a stroke of genius.

In order to come to this conclusion, he had to develop a certain few mental images which would make it clear what was involved in using the stress-range concept.

The first point was that in Professor Hovgaard's work he states that where a pipe is increased in length between two anchorage points as the result of temperature, the stresses are substantially the same as would occur if the anchorage points were moved mechanically by an external force which produced a displacement equal to the increase in length due to temperature, except for differences due to changes in moduli of elasticity.

The second point was that based on the foregoing statement: if we were to erect a piping system cold with 100 per cent cold spring in all directions, we could compute the stresses in the cold condition and when this system was heated up to the full temperature, the stress in the hot condition would be zero.

On the other hand, if the system were erected with zero cold spring, the stresses would be equal to those occurring in the cold condition with 100 per cent cold spring except that they would be opposite in sign and that they would be somewhat less because of the differences in moduli of elasticity.

Next, when establishing limits of stress for the stresses as computed for the cold condition with 100 per cent cold spring, the stress which can occur under any future conditions is limited, whether cold spring is used or not.

We have the phenomena of self-spring and relaxation to consider, and while these factors will not affect the stress range to which the pipe will be subjected, they will tend to relieve the maximum stresses in either the hot or cold condition.

Thus the use of cold spring in so far as stress is concerned becomes of little importance, but is of importance where the reactions and moments at attachments to equipment are concerned.

The proposed revision to the Code includes the necessary formulas for dealing with these reactions.

In considering this complex subject, the committee has kept strongly in mind three fundamental principles:

(*a*) Any requirements in a Code must be kept as simple as possible, since a Code is not a textbook, but an attempt to establish signposts as to when danger might exist.

(*b*) The greatest curse of regulations is that they regulate too much and, by so doing, cramp the freedom of the designer, and sometimes even result in freak designs being developed to circumvent unreasonable regulations.

(*c*) Since the whole subject is exceedingly complex, the determination, as to the method to be used for making analyses and as to when such computations are required, should be left in the hands of the designers who, on the other hand, should be prepared to provide necessary data, if and when a serious need for same is indicated.

In conclusion, the writer wishes to thank the author and all the members of the committee for what they have accomplished and state that it is his belief that the sooner this proposed Code can be adopted, the better it will be for industry.

There are many comments that still have to be digested but it is hoped that before long the committee can meet again and clean up the loose ends.

L. Pach.[28] The author's paper and the reports of the Task Force are valuable contributions to a better understanding and clarification of the many problems involved in the growing field of pipe-stress analysis.

This discussion is concerned only with the brief statements given by the author on approximate assumptions. Since detailed, correct pipe-stress calculations are very time-consuming, a large volume of work is done by using approximate assumptions. Therefore a more detailed comment on some of these assumptions seems worth while.

While the author's statements hold for the majority of pipe-stress problems, some exceptions may occur. Regarding the substitution of square corners for curved members, there are pipe bends having large radii ($R = 5D$, or larger), and heavy wall thicknesses, whose virtual lengths are smaller than those of the substituting square corners. This will be the case when the virtual bend length, $L' = 1.571\ KR$ is less than the length of the square corner, $L = 2R$, or for all bends, whose flexibility factors are less than

$$K = \frac{2}{1.571} = 1.273$$

While there are few bends with such low K-values, their existence, nevertheless, should be noted. Substitution of square corners for such bends will tend to "loosen up" rather than "tighten" the pipe.

Whether the substitution of square corners for bends and elbows will result in an over or underestimate of the stresses is difficult to predict. Besides the flexibility factor and stress-intensification factor, the proportion of curved member lengths to total pipe length, and the location of the curved members with respect to the neutral axis (thrust line) will also affect the end results. The neutral axis itself may shift considerably when the square-corner substitution is made, thus changing the moment arms as well as the forces. A shift of the point of maximum stress for the two assumptions also may result.

[28] Oil Refinery Division, Arthur G. McKee and Co., Cleveland, Ohio. Assoc. Mem. ASME.

Regarding the assumption that the neutral axis parallels the line connecting the anchors, this holds only for symmetry with respect to a line. For pipes which are symmetrical with respect to a point (such as a symmetrical Z-shape), this assumption would place the neutral axis as passing through the anchors. But such a position of the neutral axis would result in zero-bending moments and zero-bending stress at the anchors, which obviously is incorrect.

C. S. L. Robinson.[29] The stress range with emphasis on the anticipated cycling is certainly of greater physical significance than the maximum combined stress. It is better not to combine (as we have been doing) stress components like the pressure longitudinal stress and the weight-load stress with the thermal-expansion stress which may be relieved by yielding or by cold spring. The pressure longitudinal stress may be considered fully by conservative selection of pipe-wall thickness to accommodate the pressure circumferential stress. With shipboard piping the weight-load stresses are negligible because numerous supports are used to limit sway and vibration.

It also may be pointed out that the stress-range concept is useful where the movement is not entirely thermal. Such an example occurs in shipboard piping. If a pipe extends over a considerable length of the vessel, its flexibility may be increased to accommodate hull strains. Both this hull-movement stress and the thermal-expansion stress will be reduced by any plastic strain in the piping, and what are most serious about these stresses are their periodicities.

However, it is undesirable to state: "Formal calculations or model tests shall be required only where reasonable doubt exists as to the adequate flexibility of a system." This statement implies that the approximate thermal-expansion stresses are readily observable. Such is frequently not so. Furthermore, with higher temperatures (above, say, 800 F) detailed thermal-stress estimates are profitable not only because of the larger thermal movements but also because of the greater cost of the alloy piping and of its fabrication. With the current lessening of business activity more, and not less, attention could be directed to thermal-stress details. Our ASA Code may be too conservative but this should be corrected by increasing the allowable stress or stress-range values.

Ernest L. Robinson.[30] The writer wishes to emphasize the importance of recognizing and trying to evaluate and limit the maximum accumulated total strain in any worst location.

The paper is an excellent exposition of the proposed new section on flexibility of the Code for Pressure Piping. Certainly it is highly desirable to take cognizance of the stress range and prescribe a limitation for it. Certainly the initial stress does relax and tends to anneal away. But, by this very process, it does add to accumulated creep.

Creep is not uniform but it tends to be concentrated in the most highly stressed elbows or runs of pipe while the lower stressed lengths provide follow-up elasticity to multiply creep in critical regions. It would be desirable to try to evaluate the situation in these places and prescribe suitable limits.

The writer is somewhat less than satisfied with the comparisons embodied in the algebraic formulations given in the Appendix to the report of the Task Force. These formulations represent only one set of conditions whereas it would seem appropriate to give cognizance to at least four sets of conditions: (*a*) hot condition; (*b*) cold condition; (*c*) range of stress or strain; (*d*) maximum total local creep.

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[30] Structural Engineer, Turbine Division, General Electric Company, Schenectady, N. Y. Fellow ASME.

The Task Force reported that the amount of relaxation is unknown and cannot be judged reliably. If this were completely so, the writer would point out that this uncertainty would constitute a very good reason for taking steps to assure complete freedom from stress in the hot condition in order to minimize local creep due to relaxation. However, the writer would point out that either the relaxation properties of piping materials are well known or may be readily determined by well-known relaxation creep tests. If the proposed new section is to make no provision for estimating numerically the relaxation characteristics of a piping system, the writer recommends that it ought to give definite encouragement to the elimination of all need for relaxation during the early periods of operation by requiring installation to be such as will assure it to be free from stress when hot.

D. B. Rossheim[31] and E. F. Sheaffer.[32] The author is to be congratulated for the broadly comprehensive discussion he has presented, which more than fulfills the purpose of the paper in explaining the background of the Task Force Report. In this paper the author has documented carefully the accepted facts, and his well-thought-out conclusions are largely incontestable within the confines of present-day knowledge. Therefore this discussion will attempt to do no more than call attention to a few points on which we feel further progress is mostly to be desired.

We note that in describing the General Process of Solution, wherein the author has listed significant physical properties of the pipe material, he has omitted such properties as tensile strength and various measures of ductility, as well as impact values and transition temperature. While we do not have any specific proposals to offer at this time we should like to suggest that fracture in pipe materials often may be dependent upon properties which at present are largely unassessed, and that a fully dependable design basis awaits further fundamental research in the mechanism of fracture made under the co-operation of engineers and physical metallurgists.

Under the subject Flexibility Factor, we should like to call attention to what we believe is a noteworthy omission in not discussing the work of Clark and Reissner (author's reference 107), who succeeded in obtaining an asymptotic solution of the differential equations leading to the simple expression:

$$k = \sqrt{\frac{3\,(1-\nu^2)}{h}}$$

where

k = flexibility factor
ν = Poisson's ratio
h = flexibility characteristic

Using a value of $\nu = 0.29$ the flexibility factor is found to be

$$k = \frac{1.65}{h}$$

thus confirming by rigorous analysis the Beskin approximation given as the author's Formula [2].

On the subject Stress-Intensification Factors, it might be well to point out that there is some inconsistency in attempting carefully to evaluate local stresses and cyclic effects under thermal expansion of piping while ignoring them in other forms of loading. In both piping and vessel codes at present, local effects generally are taken care of by a margin in the allowable stresses, and definitely cyclic service is left to the responsibility of the designers. With piping calculations, stress intensifications at least have often

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[32] Mechanical Engineer, The M. W. Kellogg Company.

gone unassessed; thus past experience would not appear to support an extreme need for the detailed recognition of them which is proposed. Further supporting this contention it is also recalled that the author points out elsewhere that a safety factor of about 2 is available at the proposed allowable stress levels, even up to 7000 cycles.

Regarding the author's Formula [4], expressing a relation between failure stress and number of cycles, there appears to be a good possibility that the component 0.2 may vary considerably depending on the material and possibly upon its condition (i.e., cold-worked or heat-treated). Evidence of this appeared in cyclic tests of 18-8 corrugated expansion joints where an exponent of about 0.33 was indicated. Further data relating to this question have been presented by L. F. Coffin.

Attention is again directed to the efforts of Clark and Reissner from which a theoretical outer-surface circumferential stress-intensification factor of

$$\frac{0.813^3 \sqrt{12\,(1-\nu^2)}}{h^{2/3}} = \frac{1.80}{h^{2/3}}$$

may be obtained for pipe bends subjected to in-plane bending if Poisson's ratio is taken as 0.29. If the stress-intensification factor is related to the fatigue properties of straight pipe, following the author's definition, the foregoing relation should be divided by a factor of 2, representing the stress-intensification factor inherent in plain pipe as compared to polished bars. The operation yields

$$i = \frac{0.9}{h^{2/3}}$$

which offers partial substantiation of the author's Formula [5].

Regarding the section Self-Spring and Cold-Spring Effects, we believe that designers should be warned of certain practical aspects of applying cold spring. If the operation is to be fully effective, it is not usually sufficient to cut the pipe short and simply pull the ends together for the closing weld; counter-moments also should be applied when the last joint is made to arrest angular displacements of the adjoining parts (as would be required on the bend presented in Fig. 2, in addition to simply pulling the ends together through a distance eL). For the cold-springing of high-pressure turbine leads in space configurations, the writers' company has found it expedient to apply such counter-moments by suitably located forces, the location and magnitude of which are carefully calculated.

The author's remarks regarding the so-called relaxation limit invite some comments. At a temperature where viscous creep is significant, it would seem that the asymptotic value of residual stress would be zero. At lower temperatures, the process taking place consists of local yielding accompanied by the usual strain hardening. This leads eventually to a fully elastic action, and the whole operation would seem to be dependent more upon the shape of the part than the material of which it is made. We should be interested to have the author point out any evidence he has found to support the existence of the relaxation limit as a bona fide material property.

In his discussion of Allowable Reactions, the author directs attention to a problem which has been the source of a considerable waste of pipe material. We refer particularly to the case of pumps, turbines, and other equipment for which the manufacturers have been known to make it a condition of their warranty that no piping reactions be imposed whatsoever. As the author points out, such a requirement is quite impractical since the piping must usually absorb expansion of the equipment as well as its own expansion. It is hoped that continued emphasis of this point will induce equipment manufacturers to investigate and provide for reasonable limits of allowable piping reactions.

At some length the author has discussed What Systems Require Analysis. We concur with the author and others on the Task Force regarding the impossibility of formulating simple rules which will predict accurately the stresses in any piping system. We further agree that the vague guidance which the Task Force felt more or less compelled to retain is quite sufficient for a Standard of Good Practice. The salient point, however, is that the Piping Code may be considered no longer such a standard since it is rapidly being adopted as a Safety Code, its rules becoming mandatory and enforceable by law. Thus the proposed wording leaves the designer in a legally indefensible position unless he takes on a full burden of calculations. Furthermore, all permissible wording operates to the detriment of the responsible manufacturer who would be obliged to live up to the most stringent interpretation, whereas those who have no reputation to maintain would not hesitate to use the loophole afforded and prepare no calculations whatsoever.

The solution of Alt. Par. *620* proposed to the Task Force by our representative, Mr. Wallstrom, is admittedly not beyond improvement. Besides changes to the values given, additional criteria might incorporate such considerations as severity of cycling or hazard to personnel. Regardless of the exact rules, however, we are convinced that it is a step in the right direction to set up definite requirements stipulating that certain piping be calculated. The fact that a precise detection of every case of overstress cannot be had, short of making detailed calculations of each line, should not lead to the other extreme of requiring essentially no analysis at all. Even the most experienced piping-stress analysts often do not anticipate correctly the results of their calculations. Hence we conclude that if a criterion exists which for the average user of the Code will even moderately reduce the guesswork in this matter, such a criterion still must be adjudged a worth-while tool with which to encourage sounder engineering.

Author's Closure

The voluminous discussion of this paper is an encouraging token of the wide interest commanded by its subject. The author, the Task Force, and piping engineers at large owe a large debt of gratitude to the discussers who have given freely of their knowledge and experience, either to highlight the improvements made in the code formulation or direct attention to remaining shortcomings—the latter mostly the result of oversimplification in the interest of providing rules which could be followed by the average engineer.

In order not to add unduly to the length of this paper, the author's closing remarks will be confined primarily to those phases about which questions have been raised. It is gratifying to note that the general approach has met with unanimous approval and dissenting comments are largely of a cautionary nature, intended to warn against too implicit a reliance on the rules to the exclusion of good judgment.

The suggested flexibility and stress-intensification factors have been accepted generally as reflecting the best available information. In fact, Messrs. D. B. Rossheim and E. F. Sheaffer have gone further by demonstrating in detail how well the proposed values for curved members are confirmed by Clark and Reissner's analysis. Mr. J. E. Brock's suggestion, that equal applicability of these factors to out-of-plane bending be emphasized in the Proposed Rules, has since been acted upon by a change in Note 1 to Chart 1 shown in the last, April 1, 1954, draft of the Task Force Report.

Mr. F. M. Kamarck is alone in questioning the flexibility of elbows and bends and suggesting upward revision of the stress-intensification factors; the author confesses to difficulty in following his motivation. Messrs. Rossheim and Sheaffer appear to incline to the opposite view; while accepting the values suggested for the stress-intensification factors as proper, they question whether it is really necessary to include them in calculations, referring to pressure-vessel design practice where similar factors are tacitly absorbed in the safety factor. While conceding that a similar precedent exists for piping-stress analysis, the author believes it unsound to submerge calculable variables in the safety factor, its function being to take care of residual uncertainties. If the safety factor now embodied in the rules was felt to be too high—and there are experienced piping-stress analysts who incline to this view—it would appear more logical to correct this by expanding the allowable stress range, following Mr. C. S. L. Robinson's excellent advice.

Whether the rules are too conservative or not depends on their future interpretation by customers and inspection authorities. In the author's opinion, the stress range adopted is sufficiently conservative to allow the designer a reasonable amount of latitude, by which is meant that the error introduced by making approximations could run to something like 25 per cent without causing concern. In this phase of engineering, as in others of a complex nature, hard and fast rules never should be allowed to take precedence over sound engineering judgment; they should be used only to develop it or to supplement it. Both Mr. H. C. E. Meyer and Mr. J. E. Brock have warned against fettering the stress analyst by too strict a regulation or too literal an interpretation of any rules devised; the author would like to join them in a plea for enlightened enforcement, neither too strict nor too lenient. Perhaps the body of authoritative opinion encompassed in this discussion may help to bring this about.

While still on the subject of approximations and accuracy, the author wishes to signify his agreement with the conclusions reached by Mr. J. E. Brock and Mr. L. Pach relative to the effect of square-corner assumptions. It might be added that much greater deviations from the mathematically accurate results than are revealed in Fig. 7 of the paper would be obtained if the complete range of flexibility factors (up to 25) for long-radius welding elbows within the range of sizes and thicknesses of American Standard B36.10 were considered.

The only remaining issue of importance concerns the effects of local yielding or creep and the resultant relaxation. The author concurs with Mr. E. L. Robinson that a complete analysis of a piping system under thermal expansion should consider at least several stages or factors descriptive of its stress-and-strain history, and desirably should include the initial hot and cold stresses and strains, the ultimate (relaxed) hot and cold stresses and strains, the stress range and the mean stress (primarily for the ultimate condition), and finally the total strain. However, even if the knowledge and experience were available to set limits for each of these items, the complexity engendered would be prohibitive so that every single analysis would require the attention of an expert. To reduce the problem to a practical level, the stress range, the initial hot reaction, and the ultimate cold reaction were selected from this array as the most significant performance yardsticks; and the limits for the stress range, and the stress connoting the relaxation limit (an operating constant rather than a true physical property), were related to the allowable stresses established elsewhere in the Code for Pressure Piping.

No separate limitation on the total amount of creep was established, the reasoning being that the stress-range limitation would at the same time serve to control the total strain, and this view is still held by the Subgroup as applicable to the average piping system. However, Mr. E. L. Robinson's comments led to an investigation of less usual configurations characterized by small branches where relaxation would not be effective as a result of elastic follow-up from the larger, lower stressed portion of the line and the long-time ductility of the grain boundary (which could be as low as 1 per cent) could be exhausted; to cover these cases, a cautionary note has since been inserted in the preamble (see April 1, 1954, issue of the Task Force Report).

Mr. Kamarck, while approaching this topic from a different angle, appears to have been guided by the same fear as Mr. Robinson. However, his statement that any stressing beyond the elastic limit, even though it occurs only once, constitutes a dangerous weakening is not borne out by experience, in the piping field or elsewhere. It would condemn as unsafe the bulk of high-temperature piping installed in the past 20 years which has been designed to stress limits not much different from, often considerably higher than, those established in the Proposed Rules. This includes carbon-molybdenum steel piping, which is known for its low ductility under prolonged creep loading.

Of course, where legitimate doubt exists as to the ability of a material or system to absorb creep, 100 per cent cold spring remains a solution. However, as Messrs. Rossheim and Sheaffer point out, it is not as simple to cold-spring a system properly, as would appear at first glance. Incidentally, the $^2/_3$ factor in the formula for the hot reaction, about which Mr. Brock has raised questions, has been put there to allow for the uncertainty of attaining the designed cold spring in actual installation; the single formula given in the Proposed Rules, however, is sufficient, since the cold-spring factor c by definition is limited to unity as a maximum.

Attachment 4 CALC. ~~[illegible]~~ REV. 0
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STRESS DISTRIBUTIONS OF AN ELBOW WITH STRAIGHT PIPES

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SUMMARY

Elbows are the most flexible components in the thin-walled, large diameter coolant piping systems of a nuclear power plant. Prediction of the elbow behavior is very important in understanding the response of a piping system. Many theoretical and experimental studies have been reported on the stress and displacement analysis of the elbow. Recent development of the finite element method, in particular, enables one to solve even inelastic problems such as elastic-plastic and creep behavior of an entire piping system under complex loading conditions. Their proper analysis, however, requires an adequate selection of finite elements with respect to their characteristics, and a thorough understanding of elastic behavior of elbows under several basic conditions.

The present paper investigates three-dimensional elastic analyses of an elbow with straight pipes under in-plane or out-of-plane bending moment, using three different types of models in commercially available general purpose computer programs. A post-processor for output plotting is developed to assist intuitive understanding of the stress distributions over the surfaces of the elbow and straight pipes.

The elbow with straight pipes is fixed at one end, and, at the other end, subjected to loads corresponding to in-plane or out-of-plane bending moment. Three different models (quadrilateral thin shell elements in the ANSYS program, quadrilateral higher order solid elements in the ASKA program, and the combination of elbow and beam elements in the MARC program) were used to obtain reasonably accurate solutions of these problems and also to assess the validity of these approximations. Analyses were performed with well-refined meshes in all cases.

With respect to stress distributions, fairly good agreements were obtained between the shell and solid element idealizations. Computed stresses by the elbow and beam element model were relatively large compared with those obtained by the other two models, particularly near the connecting sections of the two segments. These differences may be ascribed to the fact that the tying condition of elbow and beam elements cannot satisfactorily simulate actual behaviors. However, the last model is sufficient for stress and strain evaluation at the central section of an elbow, and may provide an effective tool for elastic and/or inelastic analyses of the piping system at the present state-of-the-art. A post-processor was devised for a two-dimensional representation of the stress distribution of the entire pipe bend. With this plotting, the complex stress distribution caused by the interaction of the elbow and straight pipes can be understood completely. This post-processor is also valid for plotting the inelastic stress and strain distributions.

1. Introduction

As important structural components of piping systems, elbows are used widely in various industries. This type of piping structural component has so high a flexibility that it is conveniently used also for absorbing thermal expansions of a piping system. It is also widely practiced to adjust the locations and number of elbows to be used in an attempt to acquire a reasonable piping design.

It is very important to understand the stress distributions and deformation behaviors of elbows in the evaluation of the responses of such piping systems. Many theoretical and experimental studies have so far been made on the problem of the elbows subjected to bending moments or internal pressures. Such studies have lately been putting emphasis on the investigation of inelastic deformations rather than elastic deformations. In the analytical studies of such problems, inelastic analyses under complex loading conditions have also come to be performed particularly due to the advances in the finite element numerical method (1-5).

However, the elastic deformation behavior of elbows has not yet been fully understood. The known analysis results contain various assumptions concerning the geometry and boundary conditions. What are the true stress distributions and deformations after all such assumptions are eliminated? This is a problem we have to try to find answers from now on. This task involves many problems that need analytical studies in the future such as, for instance, whether the uneven wall thickness and initial imperfection of sectional geometry of elbows have important effects on stresses and deformations, what are the stresses in the joints of elbows, what are effects of the geometrical and material discontinuity of the welded joints, and what evaluation should be made of the flexibility factors and stress indices considering the interaction between elbows and straight pipes.

This paper is intended to obtain the entire picture of the stress distributions of the elbow with straight pipes and describes the analysis results by the finite element method in the cases where they are subjected to in-plane or out-of-plane loading. The analytical study uses three different types of finite element models, that is, three-dimensional solid elements, thin shell elements, and the combination of elbow and beam elements. The characteristics of thus obtained solutions are compared with one another, thereby to make reference to the validity of such approximations. The paper also describes the computer plotting that has been devised for intuitive understanding of the overall stress distributions of the elbow with straight pipes.

2. Elbow with Straight Pipes and its Finite Element Idealizations

The elbow with straight pipes that is treated in this paper is fixed at one end and subjected to in-plane or out-of-plane loading at the other end as illustrated in Fig.1. The finite element method is considered most effective in performing the detailed analysis of this problem, taking into consideration the interaction between the straight pipes and elbow. The stress distributions of the elbow has become fairly well known owing to the several analytical studies by the finite element method (1,4,5) and experimental results, but not to the extent that the entire picture thereof has been clarified. In the analysis results by the finite element method, for instance, localized disturbances of stresses are observed in the ends of the elbow in some cases and it is still not known whether they are real physical phenomena or the errors inherent in the finite element method arising due to the element characteristics and the employed finite element modeling. There are also few experimental and analytical works which have clarified the discontinuities of the stress distributions occurring in the elbow-straight pipe intersections. In this analysis, therefore, paying attention to the above point, solutions were obtained by use of the following three different types of finite element models, thereby to make a comparative study of thus obtained results. In performing the analysis, considerably fine meshes were used as compared with ordinary cases.

(a) Three-dimensional solid element

The three-dimensional solid element used was HEXEC-27 (6) of the ASKA program, that is, the hexahedral element formulated by the quadratic Lagrange interpolation function and 27 degrees of freedom per element. As seen from Fig. 1, the elbow with straight pipes was divided into sixteen sections in the circumferential direction, one section in the direction of thickness, and twenty-two sections in the axial direction (divided by the odd-numbered lines in the figure). The idealization by solid elements is generally uneconomical but it will be interesting to compare this type of idealization with the idealization by thin shell elements having some assumptions.

(b) Thin shell element

The shell element used in this analysis was STIF 43 (7) of the ANSYS program, that is, the 4-node quadrilateral flat plate element (20 degrees of freedom). The total structure was divided into thirty-two sections in the circumferential direction and forty-four sections in the axial direction as shown in Fig.2. The interpolation function of this element was bilinear for the in-plane displacements and incomplete bicubic for the out-of-plane displacement.

(c) Combination of elbow and beam elements

Some of the finite elements have been developed specially for the purpose of idealizing the geometry of the elbow. They include the elbow element (1) made up of a deformation mode of a torus that is an axisymmetric shell combined with a deformation mode of a curved beam, and the ring element (9) in which a ring sliced out of the elbow is assumed to be a finite element and the Fourier series expansion was used for the circumferential shape function and the Hermite interpolation for the axial one.

In this analysis was used the MARC program and the elbow structure was idealized by elbow elements (ELEMENT 17 (8)) and the straight pipes by beam elements (ELEMENT 14 (8)). The elbow elements were used partly for the purpose of confirming the fundamental characteristics of this element prior to the inelastic analysis of the complete piping system performed by the research group including the authors. Fig.3 shows the finite element idealization by use of the combination of elbow and beam elements. The elbow was divided into thirty-two sections in the circumferential direction and eight sections in the axial direction. In this type of idealization, the elbow elements are joined with the adjacent elbow and beam elements only via the nodes located at the centers of the cross sections. These nodes represent the beam mode deformations of elbow elements.

3. Analysis Results and Discussion

Figs 4 and 5 respectively show the stress distributions on the inner surface at the center of the elbow (Section B) and at the joint between elbow and straight pipe (Section C) in the case of an in-plane bending load. Solutions were obtained by the three finite element models. Figs 6 and 7 show the stress distributions in the case where these models were subjected to an out-of-plane bending load.

The stresses induced in the elbow subjected to the in-pane load (Fig. 4) are greatly different from those which are obtained by the curved beam theory. The elbow analyzed in this paper was a thin-walled, large diameter, long elbow (Fig.1), 8[illegible]3.9 mm in outside diameter (D), 1219.2 mm in radius of curvature (R) and 11.1 mm in thickness (t) and the pipe factor was $\lambda = Rt/r^2 = 0.28$. According to the curved beam theory, the main stress is the axial one σ_ϕ, the maximum compressive stress occurs at the location of $\theta = -90°$, and the maximum tensile stress at the location of $\theta = 90°$, but the calculated stress distributions were utterly different from these solutions, that is, they represented the behavior peculiar to a pipe elbow as is well known, when a bending load is applied to an elbow, unlike a beam whose end rotates due to the axial strain, the elbow is easily ovalized to cause the end to rotate, thus yielding the maximum stress in the circumferential direction.

Comparing the numerical solutions by the three types of finite element idealization, it was found that the maximum stress took the highest value in the case of elbow elements. This is presumably due to the fact that the elbow elements are joined with the adjacent elements only via the central positions of the sections and the angle of rotation, and the individual elbow elements are ovalized independently, thereby causing a rather larger degree of ovalization.

In contrast, when the shell elements were used, since the elements were connected with one another, the ovalization becomes continuous in the axial direction due to the restraints by all adjacent elements. The effects of the straight pipe that would not easily ovalized appeared in membrane stresses.

Elbow dimension

OD = 8[illegible]3.1 mm (= 35.2") thickness t = 11.1 mm (= .468")

R = 1219.2 mm = 1.48 × OD

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occurred near the centra

...ing and torsional moments of each shell element, and extended it near the typical section, the localization is weakened in this way, thereby slightly lower maximum stress than in the case of elbow elements.

The analysis results by the solid elements were fairly well in agreement with the results obtained by use of shell elements although the maximum stress took a value somewhat lower. In Fig.4 showing the stress distributions in the joint (Section C) between elbow and straight pipe, the elbow elements caused considerably high stresses as compared with other finite element idealizations. This tendency is also due to the similar reasons stated before. It is also presumably due to the fact that since the straight pipes were modeled by beam elements incapable of expressing the ovalization, the effects were further pronounced by the discontinuity of the stiffness properties in the joints. In these joints, when shell and solid elements were used, there occured considerably high shear stresses $\tau_{s\theta}$ and these values were well agreement between both cases. On the other hand, the elbow elements that were used here are incapable of expressing such shear stresses.

A similar tendency was observed when subjected to an out-of-plane load as when subjected to an in-plane load (Figs. 6 and 7).

From the above results, it can be said that the use of elbow elements is not necessarily sufficient for obtaining the stress distributions in the elbow with straight pipes but they show almost accurate solutions at the central section of the elbow and therefore this type of idealization will be useful in performing the elastic or inelastic analysis of the entire piping system in order to evaluate the stresses and strains in the central portions of the elbows.

4. Stress Distributions on the Surfaces of the Elbow with Straight Pipes

In the preceding section, we selected a typical section of the elbow and made a comparative study of the analysis results performed by use of three different types of finite element idealizations. Speaking only about the elbow with straight pipes, the shell element type can be said to be most effective in terms of calculation time, the expression of analysis results and accuracy of the obtained solutions although it is somewhat troublesome to generate the finite element mesh.

In order to investigate the overall stress distributions of the elbow with stright pipes on the basis of the results by the shell elements, we provided the postprocessor, that is, the surfaces of the elbow with straight pipes were expanded into a reactangular form to achieve a two-dimensional representation as shown in Fig.8 through 11 in order to explicitly show not only the stress distributions in specific sections but also the global stress distributions on the inner and outer surfaces.

For example, Fig. 8 shows the circumferential stress distribution on the outer surface due to the in-plane bending load. The maximum stress occurred near the central section and decreased sharply in the areas nearer the straight pipes. There occurred considerable stresses also in the straight pipes due to the ovalization influenced by the elbow.

Fig. 9 shows the shear stress distribution on the inner surface due to the in-plane bending load. The shear stresses $\tau_{\phi\theta}$ were large only in the areas near the elbow-straight pipe intersections and reached a peak in the neighborhood of $\theta=0$ and $\pm 90°$. These stresses are presumably caused due to the combination of structural elements having the different fundamental deformation modes.

Figs 11 and 12 show the stress distributions due to the out-of-plane load. This case was characterized by the fact that there are two peaks in the circumferential stress. Since the out-of-plane bending moments are necessarily applied accompanied with torsional moments, the total sum of shear stress in the section does not become zero and their distribution is not uniform and complicated. However, it was found that discontinuous stresses occurred only in the elbow-straight pipe intersections.

5. Conclusion

We performed the elastic stress analysis of the elbow with straight pipes by use of three different types of finite element idealizations. As a result of comparing thus obtained solutions, we have reached the following conclusion.

1) The analysis results by flat plate elements were fairly well in agreement with the results by higher-order solid elements, showing the effectiveness of this element in the stress analysis of the pipe elbows from an overall point of view including the calculation cost.
2) When elbow elements are used, the ovalization will be overestimated in the areas near the elbow-straight pipe intersections. With this element, there still remains a problem that torsional and shear stresses are not taken into consideration for the out-of-plane bending load. However, the overall stress distribution seems to be reasonably evaluated although the normal stresses in the central section of the elbow are somewhat overvalued.
3) There occur complicated discontinuous stresses in the elbow-straight pipe intersection. However, the maximum stress in the elbow subjected to the bending moment still occurs generally in the locations near the central section. This fact suggests that the stresses are evaluated in safety by an approximate method disregarding the interaction between the elbow and straight pipes.
4) The elbow-straight pipe intersection appears to be continuous but this is a location where discontinuous stresses occur as in the case of a pressure vessel consisting of a hemispherical shell connected with a cylindrical one. Such being the case, when performing detailed stress analysis of this particular portion, it is necessary to use a refined mesh, considering the discontinuity of this section.

5) The two-dimensiona
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Acknowledgement

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5. The two-dimensional representation of the stress distributions is effective to show explicitly the fundamental patterns of stress distributions in the elbow with straight pipes.

Acknowledgement

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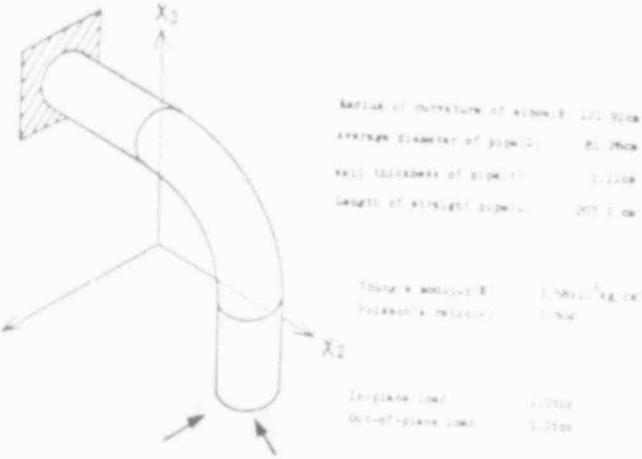

Fig. 1 Geometry of the Elbow with Straight Pipes

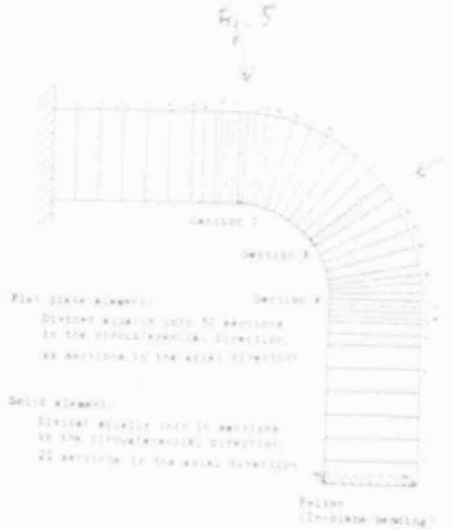

Fig. 2 Finite Element Idealization by Solid Elements or Flat Plate Elements

Fig. 3 Finite Element Idealization by Elbow and Beam Elements

Fig. 4 Stresses due to the In-P
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Fig. 5 Stresses due to the Out-o
Bending Load (Inner Surface of th

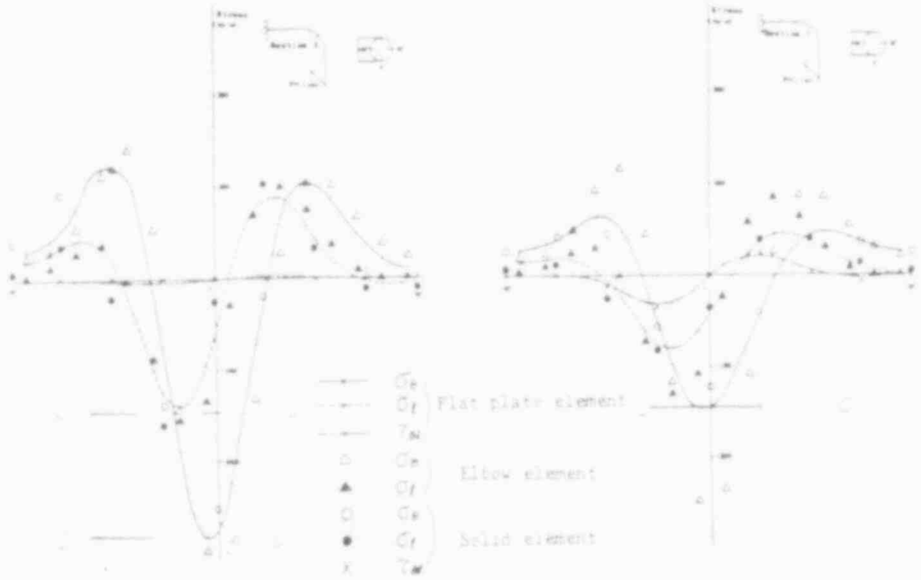

Fig. 4 Stresses due to the In-Plane Bending Load (Inner Surface of the Section B)

Fig. 5 Stresses due to the In-Plane Bending Load (Inner Surface of the Section C)

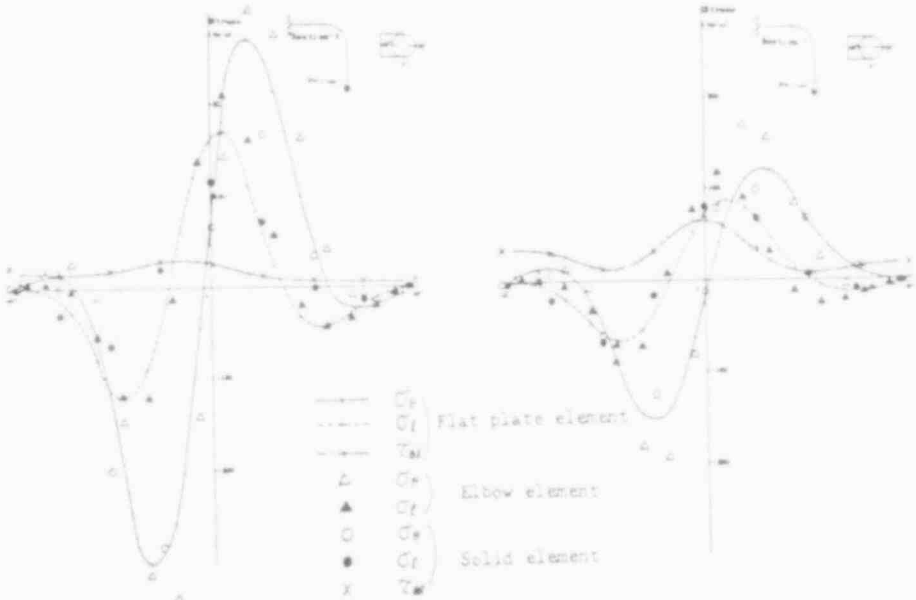

Fig. 6 Stresses due to the Out-of-Plane Bending Load (Inner Surface of the Section B)

Fig. 7 Stresses due to the Out-of-Plane Bending Load (Inner Surface of the Section C)

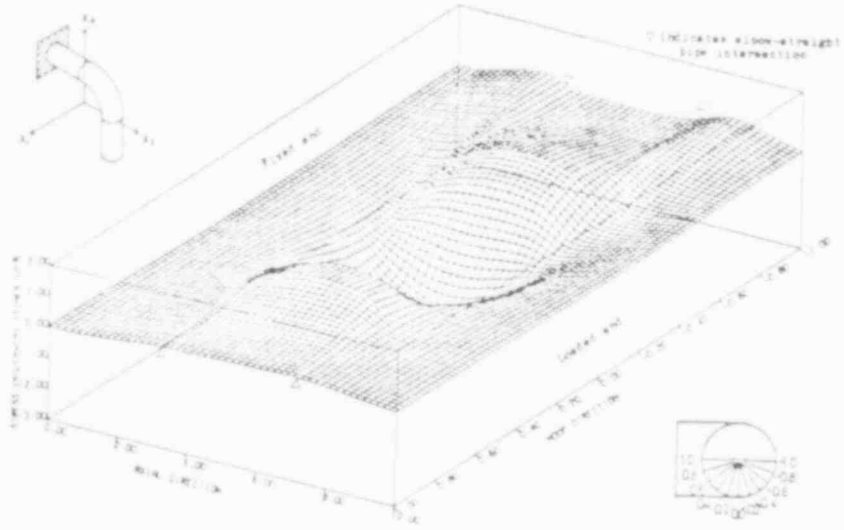

Fig. 8 Hoop Stress Distribution on the Inner Surface
due to the In-Plane Bending Load

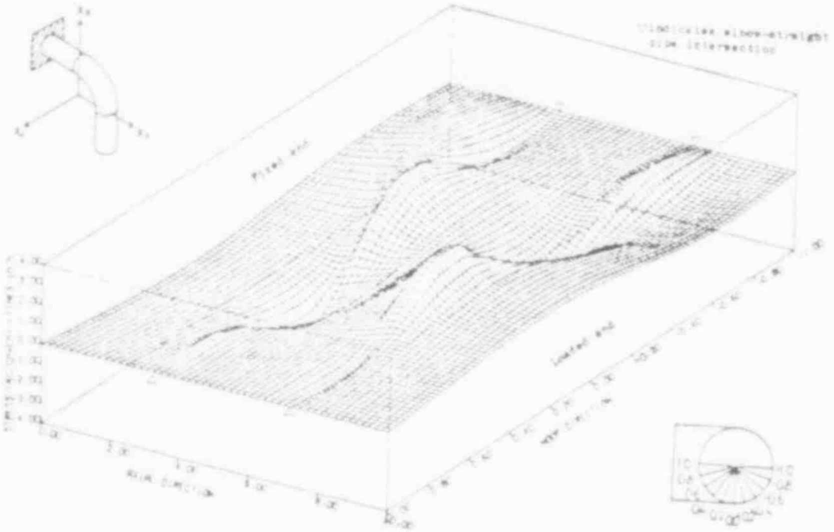

Fig. 9 Shear Stress Distribution on the Inner Surface
due to the In-Plane Bending Load

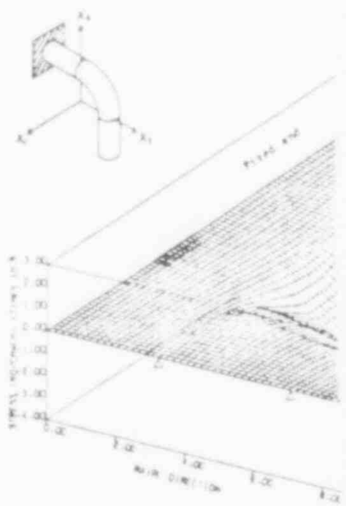

Fig. 10 Hoop Stres
due to the

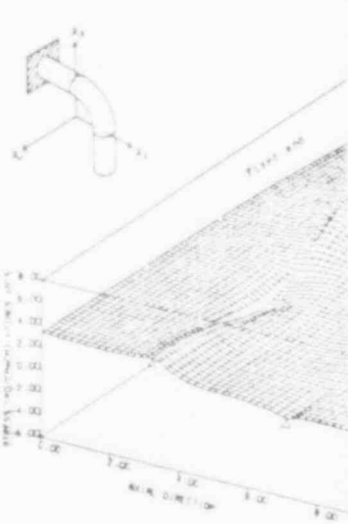

Fig. 11 Shear Stres
due to the

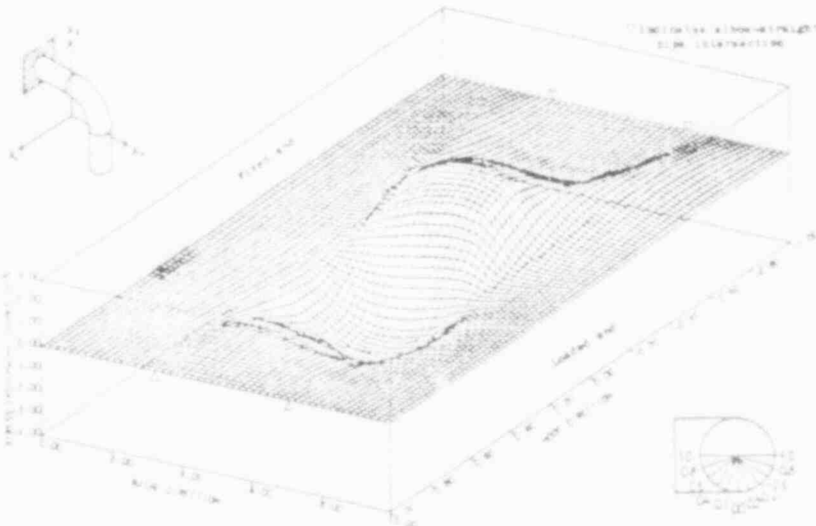

Fig. [illegible] Hoop Stress Distribution on the Inner Surface
Due to the Out-of-Plane Bending Load

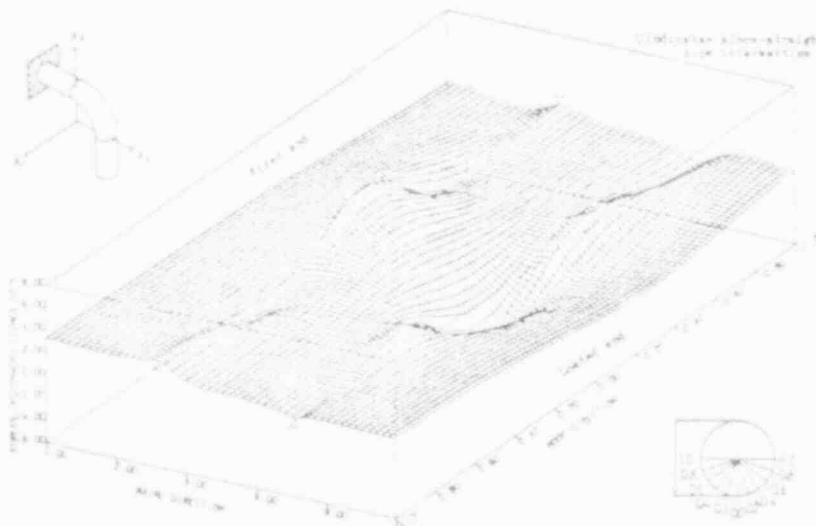

Fig. 21 Shear Stress Distribution on the Inner Surface
Due to the Out-of-Plane Bending Load