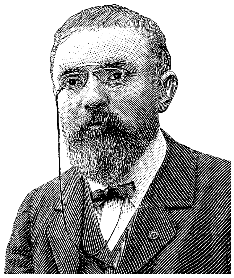

Volume 57, Number 3, 2021
ISSN 0246-0203

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

57/3

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

Vol. 57, No. 3 (August, 2021) 1203–1810

ANNALES DE L'INSTITUT HENRI POINCARÉ
PROBABILITÉS ET STATISTIQUES

Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques (ISSN 0246-0203), Volume 57, Number 3, August 2021. Published quarterly by Association des Publications de l'Institut Henri Poincaré.

POSTMASTER: Send address changes to Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques, Dues and Subscriptions Office, PO Box 729, Middletown, Maryland 21769, USA.

Président et directeur de la publication : Cédric Villani
Périodicité : 4 n^{os} / an

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1203–1228
https://doi.org/10.1214/20-AIHP1067

The mean-field quantum Heisenberg ferromagnet via representation theory

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Abstract. We use representation theory to write a formula for the magnetisation of the quantum Heisenberg ferromagnet. The core new result is a spectral decomposition of the function $\alpha_k 2^{\alpha_1+\cdots+\alpha_n}$ where α_k is the number of cycles of length k of a permutation. In the mean-field case, we simplify the formula further, arriving at a closed-form expression for the magnetisation, which allows to analyse the phase transition.

Résumé. À l'aide de la théorie des représentations, nous donnons une formule pour la magnétisation du ferro-aimant de Heisenberg quantique. La nouveauté-clé est une décomposition spectrale de la fonction $\alpha_k 2^{\alpha_1+\cdots+\alpha_n}$ où α_k est le nombre de cycles de longueur k d'une permutation. Dans le cas à champ moyen, nous simplifions encore la formule, ce qui donne une expression fermée pour la magnétisation qui permet d'analyser la transition de phase.

MSC2020 subject classifications: 82C22; 60B15; 20C30

Keywords: Quantum heizenberg ferromagnet; Interchange process; Random walk; Symmetric group; Magnetisation; Phase transition

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1229–1254
https://doi.org/10.1214/20-AIHP1116

Conformal welding for critical Liouville quantum gravity

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Abstract. Consider two critical Liouville quantum gravity surfaces (i.e., γ-LQG for $\gamma = 2$), each with the topology of $\mathbb{H}$ and with infinite boundary length. We prove that there a.s. exists a conformal welding of the two surfaces, when the boundaries are identified according to quantum boundary length. This results in a critical LQG surface decorated by an independent SLE_4. Combined with the proof of uniqueness for such a welding, recently established by McEnteggart, Miller, and Qian (2018), this shows that the welding operation is well-defined. Our result is a critical analogue of Sheffield's quantum gravity zipper theorem (2016), which shows that a similar conformal welding for subcritical LQG (i.e., γ-LQG for $\gamma \in (0, 2)$) is well-defined.

Résumé. Considérons deux surfaces de gravité quantique de Liouville critiques (c'est-à-dire des surfaces γ-LQG avec $\gamma = 2$), chacune ayant la topologie de $\mathbb{H}$ et avec une longueur de bord infinie. Nous montrons qu'il existe p.s. une soudure conforme des deux surfaces quand les bords sont identifiés selon leur longueur de bord quantique. Ceci résulte en une surface critique LQG décorée d'un SLE_4 indépendant. En combinant ceci avec le résultat d'unicité d'une telle soudure, démontré récemment par McEnteggart, Miller, et Qian (2018), nous montrons que l'opération de soudure est bien définie. Notre résultat est un analogue dans le cas critique du théorème de la « fermeture éclair quantique » de Sheffield (2016), qui démontre le résultat similaire que l'opération de soudure conforme pour la LQG sous-critique (c'est-à-dire avec $\gamma \in (0, 2)$) est bien définie.

MSC2020 subject classifications: 60J67; 60G57; 60G60; 60D05; 30C20

Keywords: Conformal welding; Critical Liouville quantum gravity; Schramm–Loewner evolutions; Quantum zipper

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1255–1280
https://doi.org/10.1214/20-AIHP1112

$\mathcal{U}$-bootstrap percolation: Critical probability, exponential decay and applications

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Abstract. Bootstrap percolation is a wide class of monotone cellular automata with random initial state. In this work we develop tools for studying in full generality one of the three 'universality' classes of bootstrap percolation models in two dimensions, termed subcritical. We introduce the new notion of 'critical densities' serving the role of 'difficulties' for critical models (Bollobás et al.), but adapted to subcritical ones. We characterise the critical probability in terms of these quantities and successfully apply this link to prove new and old results for concrete models such as DTBP and Spiral as well as a general non-trivial upper bound. Our approach establishes and exploits a tight connection between subcritical bootstrap percolation and a suitable generalisation of classical oriented percolation, which will undoubtedly be the source of more results and could provide an entry point for general percolationists to bootstrap percolation.

Furthermore, we prove that above a certain critical probability there is exponential decay of the probability of a one-arm event, while below it the event has positive probability and the expected infection time is infinite. We also identify this as the transition of the spectral gap and mean infection time of the corresponding kinetically constrained model. Finally, we essentially characterise the noise sensitivity properties at fixed density for the two natural one-arm events.

In doing so we answer fully or partially most of the open questions asked by Balister, Bollobás, Przykucki and Smith (*Trans. Amer. Math. Soc.* **368** (2016) 7385–7411) – namely we are concerned with their Questions 11, 12, 13, 14 and 17.

Résumé. La percolation bootstrap constitue une classe étendue d'automates cellulaires aux conditions initiales aléatoires. Dans ce travail on développe des outils pour étudier en toute généralité l'une des trois classes « d'universalité » de modèles de percolation bootstrap en deux dimensions appelée souscritique. On introduit la nouvelle notion de « densités critiques » qui jouent le rôle des « difficultés » pour les modèles critiques (Bollobás et al.), mais adaptée aux modèles souscritiques. On charactérise la probabilité critique en termes de ces quantités et on emploie ce lien pour démontrer de résultats nouveaux et d'autres déjà connus sur des modèles concrets tels que DTBP et Spiral, aussi bien qu'une borne supérieure non-triviale générale. Notre approche établit et exploite un lien étroit entre la percolation bootstrap souscritique et une généralisation convenable de la percolation orientée classique, qui ne manquera pas à servir davantage et pourrait constituer un point d'entrée dans la percolation bootstrap pour les percolationistes généraux.

De plus, on montre qu'au dessus d'une certaine probabilité critique il y a décroissance exponentielle de la probabilité d'un évènement à un bras, tandis qu'en dessous l'évènement a une probabilité positive et le temps d'infection moyen est infini. On identifie cette transition comme celle du trou spectral et du temps d'infection moyen du modèle cinétiquement contraint associé. Enfin, on caractérise essentiellement les propriétés de sensibilité au bruit à densité fixée pour les deux évènements à un bras naturels.

En ce faisant, on apporte une réponse complète ou partielle aux questions ouvertes posées par Balister, Bollobás, Przykucki et Smith (*Trans. Amer. Math. Soc.* **368** (2016) 7385–7411), notamment leurs Questions 11, 12, 13, 14 et 17.

MSC2020 subject classifications: Primary 60K35; secondary 60C05; 82B43; 82C22

Keywords: Bootstrap percolation; Oriented percolation; Kinetically constrained spin models

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1281–1305
https://doi.org/10.1214/20-AIHP1117

The TAZRP speed process

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Abstract. In (*Ann. Probab.* **39** (4) (2011) 1205–1242) Amir, Angel and Valkó studied a multi-type version of the totally asymmetric simple exclusion process (TASEP) and introduced the TASEP speed process, which allowed them to answer delicate questions about the joint distribution of the speed of several second-class particles in the TASEP rarefaction fan. In this paper we introduce the analogue of the TASEP speed process for the totally asymmetric zero-range process (TAZRP), and use it to obtain new results on the joint distribution of the speed of several second-class particles in the TAZRP with a reservoir. These is a close link from the speed process to questions about stationary distributions of multi-type versions of the TAZRP; for example we are able to give a precise description of the contents of a single site in equilibrium for a multi-type TAZRP with continuous labels.

Résumé. Dans (*Ann. Probab.* **39** (4) (2011) 1205–1242) Amir, Angel et Valkó ont étudié une version multi-type de processus d'exclusion simple totalement asymétrique (TASEP), et introduit le processus de vitesse du TASEP, ce qui leur avait permis de résoudre des questions délicates sur la loi jointe de la vitesse de plusieurs particules de seconde classes dans l'éventail de raréfaction du TASEP. Dans cet article, nous introduisons l'analogue du processus de vitesse du TASEP pour le processus de portée zéro totalement asymétrique (TAZRP), et nous l'utilisons pour obtenir de nouveaux résultats sur la loi jointe de la vitesse de plusieurs particules de seconde classe dans le TAZRP avec un réservoir. Il existe un lien étroit entre le processus de vitesse et la loi stationnaire de versions multi-types du TAZRP ; par exemple, nous sommes en mesure de donner une description précise du contenu d'un site donné à l'équilibre pour un TAZRP multi-type avec étiquettes continues.

MSC2020 subject classifications: 82C22; 60K35

Keywords: Speed of a second class particle; Zero range process; Speed process

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1306–1335
https://doi.org/10.1214/20-AIHP1127

Cutoff for polymer pinning dynamics in the repulsive phase

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Abstract. We consider the Glauber dynamics for model of polymer interacting with a substrate or wall. The state space is the set of one-dimensional nearest-neighbor paths on $\mathbb{Z}$ with nonnegative integer coordinates, starting at 0 and coming back to 0 after L ($L \in 2\mathbb{N}$) steps and the Gibbs weight of a path $\xi = (\xi_x)_{x=0}^L$ is given by $\lambda^{\mathcal{N}(\xi)}$, where $\lambda \geq 0$ is a parameter which models the intensity of the interaction with the substrate and $\mathcal{N}(\xi)$ is the number of zeros in ξ. The dynamics proceeds by updating ξ_x with rate one for each $x = 1, \dots, L-1$, in a heat-bath fashion. This model was introduced in (*Electron. J. Probab.* **13** (2008) 213–258) with the aim of studying the relaxation to equilibrium of the system.

We present new results concerning the total variation mixing time for this dynamics when $\lambda < 2$, which corresponds to the phase where the effects of the wall's entropic repulsion dominates. For $\lambda \in [0, 1]$, we prove that the total variation distance to equilibrium drops abruptly from 1 to 0 at time $(L^2 \log L)(1 + o(1))/\pi^2$. For $\lambda \in (1, 2)$, we prove that the system also exhibits cutoff at time $(L^2 \log L)(1 + o(1))/\pi^2$ when considering mixing time from "extremal conditions" (that is, either the highest or lowest initial path for the natural order on the set of paths). Our results improve both previously proved upper and lower bounds in (*Electron. J. Probab.* **13** (2008) 213–258).

Résumé. Nous considérons la dynamique de Glauber pour un modèle de polymère interagissant avec un substrat ou mur. L'espace d'états est l'ensemble des chemins sur $\mathbb{Z}_+$ avec incréments ± 1, commençant en 0 et revenant à 0 après L pas ($L \in 2\mathbb{N}$). Le poids de Gibbs d'un chemin est donné par $\lambda^{\mathcal{N}(\xi)}$, où $\lambda \geq 0$ est un paramètre qui modélise l'intensité de l'interaction avec le substrat et $\mathcal{N}(\xi)$ est le nombre de zéros du chemin ξ. La dynamique procède en mettant à jour ξ_x avec taux un pour chaque $x = 1, \dots, L-1$ à la manière d'un bain de chaleur. Ce modèle a été introduit dans (*Electron. J. Probab.* **13** (2008) 213–258) avec le but d'étudier la relaxation à l'équilibre du système. Nous présentons des nouveaux résultats concernant le temps de mélange de cette dynamique pour la distance en variation totale lorsque $\lambda < 2$. Ce régime correspond à la phase où les effets de répulsion entropique de la paroi dominent. Pour $\lambda \in [0, 1]$, nous prouvons que la distance de variation totale à l'équilibre chute brusquement de 1 à 0 au temps $(L^2 \log L)(1 + o(1))/\pi^2$. Pour $\lambda \in (1, 2)$, nous prouvons que le système présente également un "cutoff" au temps $(L^2 \log L)(1 + o(1))/\pi^2$ en considérant le temps de mélange à partir des conditions extrêmales (c'est-à-dire le chemin initial le plus élevé ou le plus bas pour l'ordre naturel sur l'ensemble des chemins). Nos résultats améliorent les limites supérieures et inférieures déjà prouvées dans (*Electron. J. Probab.* **13** (2008) 213–258).

MSC2020 subject classifications: Primary 82D60; secondary 60K35; 82C05

Keywords: Polymers; Glauber dynamics; Mixing time; Cutoff

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1336–1368
https://doi.org/10.1214/20-AIHP1113

On a stochastic representation theorem for Meyer-measurable processes

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Abstract. In this paper we study a representation problem first considered in a simpler version by Bank and El Karoui (*Ann. Probab.* **32** (2004) 1030–1067). A key ingredient to this problem is a random measure μ on the time axis which in the present paper is allowed to have atoms. Such atoms turn out to not only pose serious technical challenges in the proof of the representation theorem, but actually have significant meaning in its applications, for instance, to irreversible investment problems. These applications also suggest to study the problem for processes which are measurable with respect to a Meyer-σ-field that lies between the predictable and the optional σ-field. Technically, our proof amounts to a delicate analysis of optimal stopping problems and the corresponding optimal divided stopping times.

Résumé. Dans cet article, nous étudions un problème de représentation considéré en premier dans une version plus simple par Bank and El Karoui (*Ann. Probab.* **32** (2004) 1030–1067). Un ingrédient clé de ce problème est une mesure aléatoire μ sur l'axe de temps qui, dans l'article présent, est permis d'avoir des atomes. Il se trouve que non seulement tels atomes posent des difficultés sérieuses dans la preuve du théorème de représentation, mais ils ont aussi un rôle important dans les applications, par exemple, aux problèmes d'investissement irréversibles. Ces applications suggèrent d'étudier le problème de représentation avec des processus mesurables par rapport à une tribu de Meyer qui est située entre la tribu prévisible et la tribu optionelle. Techniquement, notre preuve revient à une analyse délicate des problèmes d'arrêt optimal et des temps d'arrêt divisés optimaux correspondants.

MSC2020 subject classifications: 60G07; 60G40; 60H30; 93E20

Keywords: Stochastic representation theorem; Meyer-σ-fields; Divided stopping times; Optimal stochastic control; Optimal stopping

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1369–1387
https://doi.org/10.1214/20-AIHP1121

Non-uniqueness for reflected rough differential equations

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Abstract. We give an example of a reflected differential equation which may have infinitely many solutions if the driving signal is rough enough (e.g. of infinite p-variation, for some $p > 2$). For this equation, we identify a sharp condition on the modulus of continuity of the signal under which uniqueness holds. Lévy's modulus for Brownian motion turns out to be a boundary case. We further show that in our example, non-uniqueness holds almost surely when the driving signal is a fractional Brownian motion with Hurst index $H < \frac{1}{2}$. The considered equation is driven by a two-dimensional signal with one component of bounded variation, so that rough path theory is not needed to make sense of the equation.

Résumé. Nous donnons un exemple d'équation différentielle avec réflexion qui peut avoir une infinité de solutions si le signal sous-jacent est suffisamment irrégulier (par exemple de p-variation infinie, pour un $p > 2$). Pour cette équation, nous identifions une condition sur le module de continuité du signal sous laquelle il y a unicité. Le module de Lévy pour le mouvement brownien se trouve être un cas limite. Nous montrons de plus que, dans notre exemple, la non-unicité a lieu presque sûrement quand le signal est un mouvement brownien fractionnaire d'indice de Hurst $H < \frac{1}{2}$. L'équation que nous considérons est conduite par un signal bi-dimensionnel dont une composante est à variation bornée, on peut donc lui donner un sens san avoir besoin de la théorie des trajectoires rugueuses.

MSC2020 subject classifications: 60L20; 60H10

Keywords: Reflected differential equations; Rough differential equations; Fractional Brownian motion

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1388–1410
https://doi.org/10.1214/20-AIHP1124

Stochastic processes on surfaces in three-dimensional contact sub-Riemannian manifolds

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Abstract. We are concerned with stochastic processes on surfaces in three-dimensional contact sub-Riemannian manifolds. Employing the Riemannian approximations to the sub-Riemannian manifold which make use of the Reeb vector field, we obtain a second order partial differential operator on the surface arising as the limit of Laplace–Beltrami operators. The stochastic process associated with the limiting operator moves along the characteristic foliation induced on the surface by the contact distribution. We show that for this stochastic process elliptic characteristic points are inaccessible, while hyperbolic characteristic points are accessible from the separatrices. We illustrate the results with examples and we identify canonical surfaces in the Heisenberg group, and in $\mathrm{SU}(2)$ and $\mathrm{SL}(2,\mathbb{R})$ equipped with the standard sub-Riemannian contact structures as model cases for this setting. Our techniques further allow us to derive an expression for an intrinsic Gaussian curvature of a surface in a general three-dimensional contact sub-Riemannian manifold.

Résumé. On considère des processus stochastiques sur des surfaces dans des espaces tridimensionnels de contact sous-riemanniens. En utilisant des approximations riemanniennes définies par le champ de Reeb, on obtient un opérateur différentiel du second ordre sur la surface comme limite des opérateurs de Laplace–Beltrami correspondants. Le processus stochastique associé à l'opérateur limite est défini le long de la foliation caractéristique induite sur la surface par la distribution de contact. Nous montrons que pour ce processus stochastique, les points elliptiques sont inaccessibles alors que les points hyperboliques sont accessibles à travers les séparatrices. On discute les résultats sur des exemples et on identifie des surfaces canoniques dans le groupe de Heisenberg, ainsi que dans les groupes $\mathrm{SU}(2)$ et $\mathrm{SL}(2,\mathbb{R})$ équipés de leurs structures sous-riemanniennes de contact canoniques, jouant le rôle de cas modèles dans ce contexte. Ces techniques nous permettent de plus de dériver une expression de la courbure gaussienne intrinsèque d'une surface générale dans une variété sous-riemannienne de dimension trois.

MSC2020 subject classifications: 53C17; 58J65; 60J60

Keywords: Sub-Riemannian geometry; Contact manifold; Stochastic process; Model space; Bessel process

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1411–1441
https://doi.org/10.1214/20-AIHP1114

Persistence exponents in Markov chains

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Abstract. We prove the existence of the *persistence exponent*

$$\log \lambda := \lim_{n\to\infty} \frac{1}{n} \log \mathbb{P}_\mu(X_0 \in S, \dots, X_n \in S)$$

for a class of time homogeneous Markov chains $\{X_i\}_{i\geq 0}$ taking values in a Polish space, where S is a Borel measurable set and μ is an initial distribution. Focusing on the case of $\mathrm{AR}(p)$ and $\mathrm{MA}(q)$ processes with $p, q \in \mathbb{N}$ and continuous innovation distribution, we study the existence of λ and its continuity in the parameters of the AR and MA processes, respectively, for $S = \mathbb{R}_{\geq 0}$. For AR processes with log-concave innovation distribution, we prove the strict monotonicity of λ. Finally, we compute new explicit exponents in several concrete examples.

Résumé. Nous démontrons l'existence de *l'exposant de persistance*

$$\log \lambda := \lim_{n\to\infty} \frac{1}{n} \log \mathbb{P}_\mu(X_0 \in S, \dots, X_n \in S)$$

pour une classe de chaines de Markov $\{X_i\}_{i\geq 0}$ homogènes en temps avec valeurs dans un espace Polonais, où S est un ensemble Borélien et μ est une distribution initiale. En nous concentrant sur le cas de processus de type $\mathrm{AR}(p)$ ou $\mathrm{MA}(q)$ avec $p, q \in \mathbb{N}$ et une distribution d'innovation continue, nous étudions l'existence de l'exposant λ et sa continuité par rapport au paramètres des processus AR et MA, pour $S = \mathbb{R}_{\geq 0}$. Pour des processus AR ayant une distribution d'innovations qui est log-concave, nous démontrons la monotonicité stricte de λ. Finalement, nous calculons explicitement les exposants dans quelques exemples concrets.

MSC2020 subject classifications: Primary 60J05; 60F10; secondary 45C05; 47A75

Keywords: ARMA; Eigenvalue problem; Integral equation; Large deviations; Markov chain; Persistence; Quasi-stationary distribution

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1442–1475
https://doi.org/10.1214/20-AIHP1125

Piecewise deterministic Markov processes and their invariant measures

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Abstract. Piecewise Deterministic Markov Processes (PDMPs) are studied in a general framework. First, different constructions are proven to be equivalent. Second, we introduce a coupling between two PDMPs following the same differential flow which implies quantitative bounds on the total variation between the marginal distributions of the two processes. Finally two results are established regarding the invariant measures of PDMPs. A practical condition to show that a probability measure is invariant for the associated PDMP semi-group is presented. In a second time, a bound on the invariant probability measures in V-norm of two PDMPs following the same differential flow is established. This last result is then applied to study the asymptotic bias of some non-exact PDMP MCMC methods.

Résumé. Nous établissons dans cette étude quelques propriétés générales des processus de Markov déterministes par morceaux (PDMP). Tout d'abord, plusieurs constructions de tels processus sont décrites. Ensuite, nous introduisons un couplage entre deux PDMP suivant la même flot différentiel impliquant des bornes quantitatives en variation totale entre les distributions marginales des deux processus. Enfin, nous nous intéressons aux PDMP ergodiques. Nous introduisons des conditions permettant de montrer que certaines classes de fonctions sont des cores pour le générateur d'un PDMP, ce qui permet de vérifier aisément qu'une distribution est stationnaire pour le PDMP considéré. De plus, nous établissons sous certaines hypothèses une borne quantitative sur la V-norme entre les mesures stationnaires de deux PDMP suivant le même flot différentiel. Ce dernier résultat est ensuite appliqué à l'étude du biais asymptotique de certaines méthodes de Monte Carlo par Processus de Markov inexactes.

MSC2020 subject classifications: Primary 60J25; 35R09; secondary 35B40; 60J75

Keywords: PDMP; Generator; Synchronous coupling; Invariant measure; Bouncy particle sampler

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1476–1494
https://doi.org/10.1214/20-AIHP1126

On consistent and rate optimal estimation of the missing mass

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Abstract. Given n samples from a population of individuals belonging to different types with unknown proportions, how do we estimate the probability of discovering a new type at the $(n+1)$th draw? This is a classical problem in statistics, commonly referred to as the missing mass estimation problem. Recent results have shown: (i) the impossibility of estimating the missing mass without imposing further assumptions on type's proportions; (ii) the consistency of the Good–Turing estimator of the missing mass under the assumption that the tail of type's proportions decays to zero as a regularly varying function with parameter $\alpha \in (0,1)$; (iii) the rate of convergence $n^{-\alpha/2}$ for the Good–Turing estimator under the class of $\alpha \in (0,1)$ regularly varying P. In this paper we introduce an alternative, and remarkably shorter, proof of the impossibility of a distribution-free estimation of the missing mass. Beside being of independent interest, our alternative proof suggests a natural approach to strengthen, and expand, the recent results on the rate of convergence of the Good–Turing estimator under $\alpha \in (0,1)$ regularly varying type's proportions. In particular, we show that the convergence rate $n^{-\alpha/2}$ is the best rate that *any* estimator can achieve, up to a slowly varying function. Furthermore, we prove that a lower bound to the minimax estimation risk must scale at least as $n^{-\alpha/2}$, which leads to conjecture that the Good–Turing estimator is a rate optimal minimax estimator under regularly varying type proportions.

Résumé. Etant donné un échantillon de taille n dans une population d'individus appartenant à différents types dont les proportions sont inconnues, comment estimer la probabilité de découvrir un nouveau type au $(n+1)$-ième tirage ? C'est un problème classique en statistique, souvent appelé le problème de l'estimation de la masse manquante. Des résultats récents on montré : (i) l'impossibilité d'estimer la masse manquante sans imposer des hypothèses sur les proportions des types ; (ii) la convergence de l'estimateur de la masse manquante de Good–Turing sous l'hypothèse que la queue des proportions des types décroît vers 0 comme une fonction régulière de paramètre $\alpha \in (0,1)$; (iii) la vitesse de convergence $n^{-\alpha/2}$ pour l'estimateur de Good–Turing pour la classe de probabilités à variation régulière $\alpha \in (0,1)$. Dans cet article, nous proposons une preuve alternative, et remarquablement plus courte, de l'impossibilité de l'estimation de la masse manquante sans hypothèse sur la distribution. Au delà de son intérêt propre, cette preuve alternative suggère une approche naturelle pour améliorer et étendre les résultats de vitesse de convergence de l'estimateur de Good–Turing sous l'hypothèse de proportions à variation régulière $\alpha \in (0,1)$. En particulier, nous montrons que la vitesse de convergence $n^{-\alpha/2}$ est la meilleure que peut atteindre un estimateur, à une fonction à variation bornée près. De plus, nous montrons qu'une borne inférieure à l'estimation du risque minimax est au moins d'échelle $n^{-\alpha/2}$, ce qui amène à la conjecture que l'estimateur de Good–Turing est l'estimateur minimax de vitesse optimale sous une hypothèse de proportions à variation régulière.

MSC2020 subject classifications: 62G05; 62C20

Keywords: Good–Turing estimator; Minimax rate; Missing mass; Optimal rate of convergence; Regular variation; Two-parameter Poisson–Dirichlet

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1495–1520
https://doi.org/10.1214/20-AIHP1128

Adaptive regression with Brownian path covariate

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Abstract. This paper deals with estimation with functional covariates. More precisely, we aim at estimating the regression function m of a continuous outcome Y against a standard Wiener coprocess W. Following Cadre and Truquet (*ESAIM Probab. Stat.* **19** (2015) 251–267) and Cadre et al. (*ESAIM Probab. Stat.* **21** (2017) 138–158) the Wiener–Itô decomposition of $m(W)$ is used to construct a family of estimators. The minimax rate of convergence over specific smoothness classes is obtained. A data-driven selection procedure is defined following the ideas developed by Goldenshluger and Lepski (*Ann. Statist.* **39** (2011) 1608–1632). An oracle-type inequality is obtained which leads to adaptive results.

Résumé. Cet article traite de l'estimation en présence de covariables fonctionnelles. Plus précisément, nous souhaitons estimer une fonction de régression m entre une variable réponse réelle Y et un coprocessus de Wiener standard W. En nous inspirant de Cadre et Truquet (*ESAIM Probab. Stat.* **19** (2015) 251–267) et Cadre et al. (*ESAIM Probab. Stat.* **21** (2017) 138–158), nous utilisons la décomposition de Wiener–Itô de $m(W)$ afin de construire une famille d'estimateurs. Nous obtenons des vitesses minimax sur des classes de régularité spécifiques et mettons en place une procédure de sélection dépendante des données basée sur les idées développées par Goldenshluger et Lepski (*Ann. Statist.* **39** (2011) 1608–1632). Enfin, nous obtenons une inégalité de type oracle permettant d'obtenir des résultats adaptatifs.

MSC2020 subject classifications: 62G08; 62H12

Keywords: Functional regression; Wiener–Itô chaos expansion; Oracle inequalities; Adaptive minimax rates of convergence

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1521–1548
https://doi.org/10.1214/20-AIHP1132

Spike and slab Pólya tree posterior densities: Adaptive inference

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Abstract. In the density estimation model, the question of adaptive inference using Pólya tree–type prior distributions is considered. A class of prior densities having a tree structure, called spike-and-slab Pólya trees, is introduced. For this class, two types of results are obtained: first, the Bayesian posterior distribution is shown to converge at the minimax rate for the supremum norm in an adaptive way, for any Hölder regularity of the true density between 0 and 1, thereby providing adaptive counterparts to the results for classical Pólya trees in Castillo (*Ann. Inst. Henri Poincaré Probab. Stat.* **53** (2017) 2074–2102). Second, the question of uncertainty quantification is considered. An adaptive nonparametric Bernstein–von Mises theorem is derived. Next, it is shown that, under a self-similarity condition on the true density, certain credible sets from the posterior distribution are adaptive confidence bands, having prescribed coverage level and with a diameter shrinking at optimal rate in the minimax sense.

Résumé. Dans le modèle d'estimation de densité, la question de l'inférence adaptative est considérée, au moyen de lois a priori de type arbres de Pólya. Une classe de lois a priori à structure d'arbre, appelée "spike-and-slab Pólya trees", est introduite. Des résultats de deux types sont obtenus. D'une part, il est établi que la loi a posteriori bayésienne converge à vitesse minimax en terme de la norme-sup de façon adaptative pour toute régularité de Hölder entre 0 et 1, obtenant ainsi des versions adaptatives des résultats pour les arbres de Pólya classiques de Castillo (*Ann. Inst. Henri Poincaré Probab. Stat.* **53** (2017) 2074–2102). D'autre part, nous considérons le problème de la quantification de l'incertitude. Un théorème de Bernstein–von Mises nonparamétrique et adaptatif est obtenu. Puis, il est établi que, sous conditions d'auto-similarité, des régions de crédibilité bien choisies de la loi a posteriori forment des bandes de confiance adaptatives, de niveau de confiance pré-déterminé, et de diamètre de taille optimale au sens minimax.

MSC2020 subject classifications: Primary 62G20; secondary 62G07; 62G15

Keywords: Bayesian nonparametrics; Pólya trees; Supremum norm convergence; Bernstein–von Mises theorem; Spike-and-slab priors; Hierarchical Bayes

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1549–1568
https://doi.org/10.1214/20-AIHP1115

Strongly vertex-reinforced jump process on a complete graph

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Abstract. The aim of our work is to study vertex-reinforced jump processes with super-linear weight function $w(t) = t^\alpha$, for some $\alpha > 1$. On any complete graph $G = (V, E)$, we prove that there is one vertex $v \in V$ such that the total time spent at v almost surely tends to infinity while the total time spent at the remaining vertices is bounded.

Résumé. Le but de notre travail est d'étudier les processus de sauts renforcés par sites par une fonction de poids sur-linéaire $w(t) = t^\alpha$, avec $\alpha > 1$. Sur tout graphe complet $G = (V, E)$, on montre qu'il y a un sommet $v \in V$ tel que le temps total passé en v tend presque sûrement vers l'infini tandis que le temps total passé dans les sommets restants est borné.

MSC2020 subject classifications: 60J55; 60J75

Keywords: Vertex-reinforced jump processes; Nonlinear reinforcement; Random walks with memory; Stochastic approximation; Non convergence to unstable equilibria

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1569–1582
https://doi.org/10.1214/20-AIHP1119

Uniform spanning forests on biased Euclidean lattices

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Abstract. The uniform spanning forest measure (USF) on a locally finite, infinite connected graph with conductance c, is defined as a weak limit of uniform spanning tree measure on finite subgraphs. Depending on the underlying graph and conductances, the corresponding USF is not necessarily concentrated on the set of spanning trees. Pemantle (*Ann. Probab.* **19** (1991) 1559–1574) showed that on $\mathbb{Z}^d$, equipped with the *unit* conductance $c = 1$, USF is concentrated on spanning trees if and only if $d \le 4$. In this work we study the USF associated with conductances $c(e) = \lambda^{-|e|}$, where $|e|$ is the graph distance of the edge e from the origin, and $\lambda \in (0, 1)$ is a fixed parameter. Our main result states that in this case USF consists of finitely many trees if and only if $d = 2$ or 3. More precisely, we prove that the uniform spanning forest has 2^d trees if $d = 2$ or 3, and infinitely many trees if $d \ge 4$. Our method relies on the analysis of the spectral radius and the speed of the λ-biased random walk on $\mathbb{Z}^d$.

Résumé. La forêt couvrante uniforme (notée USF) sur un graphe connexe, infini et localement fini avec conductance $c(\cdot)$, est définie comme la loi limite de l'arbre couvrant uniforme de sous-graphes finis. Suivant le graphe et les conductances, la mesure USF ne porte pas nécessairement sur l'ensemble des arbres couvrants. Pemantle (*Ann. Probab.* **19** (1991) 1559–1574) a démontré que sur $\mathbb{Z}^d$ muni de la conductance *unité* $c = 1$, USF porte sur les arbres couvrants si et seulement si $d \le 4$. Dans ce travail, nous étudions USF associée aux conductances $c(e) = \lambda^{-|e|}$, où $|e|$ est la distance entre l'arête e et l'origine, et $\lambda \in]0, 1[$ est un paramètre fixé. Notre résultat principal montre que dans ce cas, USF porte sur un nombre fini d'arbres si et seulement si $d = 2$ ou 3. Plus précisément, nous prouvons que la forêt couvrante uniforme contient 2^d arbres si $d = 2$ ou 3, et contient une infinité d'arbres si $d \ge 4$. Notre approche s'appuie sur une analyse du rayon spectral et de la vitesse de la marche aléatoire λ-biaisée sur $\mathbb{Z}^d$.

MSC2020 subject classifications: Primary 60J10; 60G50; 05C81; secondary 60C05; 05C63; 05C80

Keywords: Uniform spanning forest; Biased random walk; Spectral radius; Speed

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Annales de l’Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1583–1635
https://doi.org/10.1214/20-AIHP1120

Testing degree corrections in stochastic block models

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Abstract. We study sharp detection thresholds for degree corrections in Stochastic Block Models in the context of a goodness of fit problem, and explore the effect of the unknown community assignment (a high dimensional nuisance parameter) and the graph density on testing for degree corrections. When degree corrections are relatively dense, a simple test based on the total number of edges is asymptotically optimal. For sparse degree corrections, the results undergo several changes in behavior depending on density of the underlying Stochastic Block Model. For graphs which are not extremely sparse, optimal tests are based on Higher Criticism or Maximum Degree type tests based on a linear combination of within and across (estimated) community degrees. In the special case of balanced communities, a simple degree based Higher Criticism Test (Mukherjee, Mukherjee, Sen (2021)) is optimal in case the graph is not completely dense, while the more complicated linear combination based procedure is required in the completely dense setting. The “necessity” of the two step procedure is demonstrated for the case of balanced communities by the failure of the ordinary Maximum Degree Test in achieving sharp constants. Finally for extremely sparse graphs the optimal rates change, and a version of the maximum degree test with a different rejection region is shown to be optimal.

Résumé. Nous étudions les seuils optimaux de détection de la corrélation en degré dans des modèles par blocs stochastiques dans le cadre du problème de validité de l’ajustement, et explorons les effets de la non connaissance de l’affectation des communautés (qui est un paramètre de nuisance de grande dimension) et la densité du graphe en testant des corrections de degré. Quand les corrections de degré sont assez denses, un test simple basé sur le nombre total d’arêtes est asymptotiquement optimal. Pour les corrections de degré parcimonieuses, les résultats font apparaître plusieurs différences dans le comportement, dépendant de la densité du modèle par blocs stochastiques sous-jacent. Pour les graphes qui ne sont pas extrêmement parcimonieux, les tests optimaux sont basés sur des tests de haute criticité ou sur des tests du type du degré maximum portant sur une combinaison linéaire de chacun des degrés (estimés) des communautés. Dans le cas spécial des communautés équilibrées un simple test de haute criticité basé sur le degré (Mukherjee, Mukherjee, Sen (2021)) est optimal si le graphe n’est pas complètement dense, mais une approche plus compliquée basée sur des combinaisons linéaires est nécessaire si le graphe n’est pas complètement dense. La “nécessité” de cette approche en deux étapes est démontrée dans le cas des communautés équilibrées par l’échec du test classique du degré maximum à atteindre les constantes optimales. Enfin, pour les graphes extrêmement parcimonieux, les vitesses optimales changent, et une version du test du degré maximum avec une région de rejet différente est montrée être optimale.

MSC2020 subject classifications: 62G10; 62G20; 62C20

Keywords: Detection boundary; Stochastic block model; Sparse signals

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1636–1664
https://doi.org/10.1214/20-AIHP1118

Incompressible viscous fluids in $\mathbb{R}^2$ and SPDEs on graphs, in presence of fast advection and non smooth noise

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Abstract. The asymptotic behavior of a class of stochastic reaction-diffusion-advection equations in the plane is studied. We show that as the divergence-free advection term becomes larger and larger, the solutions of such equations converge to the solution of a suitable stochastic PDE defined on the graph associated with the Hamiltonian. Firstly, we deal with the case that the stochastic perturbation is given by a singular spatially homogeneous Wiener process taking values in the space of Schwartz distributions. As in previous works, we assume here that the derivative of the period of the motion on the level sets of the Hamiltonian does not vanish. Then, in the second part, without assuming this condition on the derivative of the period, we study a weaker type of convergence for the solutions of a suitable class of linear SPDEs.

Résumé. Le comportement asymptotique d'une classe d'équations stochastiques de réaction-diffusion-advection dans le plan est étudié. Nous montrons qu'à mesure que le terme d'advection sans divergence devient de plus en plus grand, les solutions de telles équations convergent vers la solution d'une EDP stochastique appropriée définie sur le graphe associé à l'Hamiltonien. Tout d'abord, nous traitons le cas où la perturbation stochastique est donnée par un processus de Wiener spatialement homogène singulier prenant des valeurs dans l'espace des distributions de Schwartz. Comme dans les travaux précédents, nous supposons ici que la dérivée de la période du mouvement sur les level sets de l'Hamiltonien ne s'annule pas. Puis, dans la seconde partie, sans supposer cette condition sur la dérivée de la période, nous étudions un type de convergence plus faible pour les solutions d'une classe appropriée de EDPS linéaires.

MSC2020 subject classifications: 60H15; 70K65; 60J25

Keywords: Averaging principle; Markov processes on graphs; Stochastic partial differential equations; Averaging principle; Hamiltonian systems

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1665–1701
https://doi.org/10.1214/20-AIHP1123

Exponential ergodicity for SDEs and McKean–Vlasov processes with Lévy noise

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Abstract. We study exponential ergodicity of a broad class of stochastic processes whose dynamics are governed by pure jump Lévy noise. In the first part of the paper we focus on solutions of stochastic differential equations (SDEs) whose drifts satisfy general Lyapunov-type conditions. By applying techniques that combine couplings, appropriately constructed L^1-Wasserstein distances and Lyapunov functions, we show exponential convergence of solutions of such SDEs to their stationary distributions, both in the total variation and Wasserstein distances. The second part of the paper is devoted to SDEs of McKean–Vlasov type with distribution dependent drifts. We prove a uniform in time propagation of chaos result, providing quantitative bounds on convergence rate of interacting particle systems with Lévy noise to the corresponding McKean–Vlasov SDE. Then, extending our techniques from the first part of the paper, we obtain results on exponential ergodicity of solutions of McKean–Vlasov SDEs, under general conditions on the drift and the driving noise.

Résumé. Nous étudions l'ergodicité exponentielle d'une large classe de processus stochastiques dont la dynamique est régie par un bruit de Lévy à sauts purs. Dans la première partie de l'article, nous nous concentrons sur les solutions d'équations différentielles stochastiques (SDEs) dont les dérives satisfont aux conditions générales de type Lyapunov. En appliquant des techniques qui combinent des couplages, des distances de L^1-Wasserstein et des fonctions de Lyapunov correctement construites, nous montrons une convergence exponentielle des solutions de telles SDE vers leurs distributions stationnaires, à la fois sous la distance de variation totale et de Wasserstein. La deuxième partie de l'article est consacrée aux SDEs de type McKean–Vlasov avec des dérives dépendant de la distribution. Nous prouvons un résultat de propagation de chaos uniformement en temps, en fournissant des bornes quantitatives sur le taux de convergence des systèmes de particules en interaction avec le bruit de Lévy vers le SDE deMcKean–Vlasov correspondant. Ensuite, en étendant nos techniques dans la première partie de l'article, nous obtenons des résultats sur l'ergodicité exponentielle des solutions de SDE de McKean–Vlasov, dans des conditions générales sur la dérive et le bruit.

MSC2020 subject classifications: 60H10; 60J25; 60J75

Keywords: Exponential ergodicity; Lévy noise; Coupling; McKean–Vlasov process; Mean-field SDE; Propagation of chaos

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2021, Vol. 57, No. 3, 1702–1735
https://doi.org/10.1214/20-AIHP1129

Asymptotics of PDE in random environment by paracontrolled calculus

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Abstract. We apply the paracontrolled calculus to study the asymptotic behavior of a certain quasilinear PDE with smeared mild noise, which originally appears as the space-time scaling limit of a particle system in random environment on one dimensional discrete lattice. We establish the convergence result and show a local in time well-posedness of the limit stochastic PDE with spatial white noise. It turns out that our limit stochastic PDE does not require any renormalization. We also show a comparison theorem for the limit equation.

Résumé. Nous utilisons le calcul paracontrôlé pour étudier le comportement asymptotique de certaines EDP quasi-linéaires avec bruit régularisé, qui sont apparues initialement comme limites d'échelle spatio-temporelles de systèmes de particules en environnement aléatoire sur le réseau discret un-dimensionnel. Nous établissons un résultat de convergence et montrons que l'EDP stochastique limite est bien posée localement en temps avec un bruit blanc en espace. Il apparaît que notre EDP stochastique limite ne nécessite pas de renormalisation. Nous donnons aussi un théorème de comparaison pour l'équation limite.

MSC2020 subject classifications: 60H15; 35R60; 35S50

Keywords: Paracontrolled calculus; Quasilinear stochastic PDE; PDE in random environment

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1736–1767
https://doi.org/10.1214/20-AIHP1122

Probabilistic potential theory and induction of dynamical systems

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Abstract. We outline a version of a balayage formula in probabilistic potential theory adapted to measure-preserving dynamical systems. This balayage identity generalizes the property that induced maps preserve the restriction of the original invariant measure. As an application, we prove in some cases the invariance under induction of the Green–Kubo formula, as well as the invariance of a new degree 3 invariant.

Résumé. Nous développons dans cette article une version de l'identité de balayage, outil de la théorie probabiliste du potentiel, adaptée à l'étude de systèmes dynamiques préservant la mesure. Cette identité de balayage généralise la propriété selon laquelle les transformations induites préservent les restrictions de la mesure invariante d'origine. En application, nous démontrons, sous certaines hypothèses, que la formule de Green–Kubo est invariante par un procédé d'induction, de même qu'un nouvel invariant de degré 3.

MSC2020 subject classifications: Primary 37A05; 60F05; 60J45; secondary 37A40; 37A50

Keywords: Measure-preserving transformation; Potential theory; Green–Kubo formula

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1768–1785
https://doi.org/10.1214/20-AIHP1130

Longest common substring for random subshifts of finite type

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Abstract. In this paper, we study the behaviour of the longest common substring for random subshifts of finite type (for dynamicists) or of the longest common substring for random sequences in random environments (for probabilists). We prove that, under some exponential mixing assumptions, this behaviour is linked to the Rényi entropy of the stationary measure. We emphasize that what we establish is a quenched result.

Résumé. Dans cet article, nous étudions le comportement de la plus longue sous-chaîne commune pour des sous-shifts aléatoires de type fini (pour les dynamiciens) ou de la plus longue sous-chaîne commune pour des suites aléatoires en milieux aléatoires (pour les probabilistes). Nous prouvons que, sous des hypothéses de mélange exponentiel, ce comportement est lié à l'entropie de Rényi de la mesure stationnaire. Nous soulignons que ce que nous établissons est un résultat de type «quenched».

MSC2020 subject classifications: 60F15; 60K37; 37A50; 37A25; 37Hxx; 94A17; 92D20

Keywords: Longest common substring; Rényi entropy; Random dynamical systems; Random sequences in random environments; String matching

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2021, Vol. 57, No. 3, 1786–1810
https://doi.org/10.1214/20-AIHP1131

Derivative martingale of the branching Brownian motion in dimension $d \geq 1$

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Abstract. We consider a branching Brownian motion in $\mathbb{R}^d$. We prove that there exists a random subset Θ of $\mathbb{S}^{d-1}$ of full measure such that the limit of the derivative martingale exists simultaneously for all directions $\theta \in \Theta$ almost surely. This allows us to define a random measure on $\mathbb{S}^{d-1}$ whose density is given by the derivative martingale.

The proof is based on first moment arguments: we approximate the martingale of interest by a series of processes, which do not take into account particles that travelled too far away. We show that these new processes are uniformly integrable martingales whose limits can be made to converge to the limit of the original martingale.

Résumé. On considère un mouvement brownien branchant dans $\mathbb{R}^d$. Nous montrons qu'il existe presque sûrement un sous-ensemble aléatoire Θ de $\mathbb{S}^{d-1}$ de mesure pleine tel que la limite de la martingale dérivée existe simultanément pour toutes les directions $\theta \in \Theta$. Cela nous permet de définir une mesure aléatoire sur $\mathbb{S}^{d-1}$ dont la densité est donnée par la martingale dérivée.

La preuve est basée sur des arguments de premier moment : nous approchons les martingales d'intérêt par une série de processus, qui ne prennent pas en compte les particules qui ont voyagé trop loin. Nous montrons que ces nouveaux processus sont des martingales uniformément intégrables dont les limites convergent vers les limites des martingales d'origine.

MSC2020 subject classifications: Primary 60J80; 60G50; secondary 60G44; 60F17

Keywords: Branching Brownian motion; Brownian motion; Derivative martingale

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