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MATSCIENCE REPORT 18

COLLECTED SEMINAR LECTURES
ON
ELEMENTARY PARTICLES

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MADRAS - 4 (India)

COLLECTED SEMINAR LECTURES ON ELEMENTARY PARTICLES

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SOME REMARKS ON RECENT EXPERIMENTAL DATA AND TECHNIQUES

(An informal talk)

By

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Notes by

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Some Remarks on Recent Experimental Data and Techniques

By

Prof. E. Segrè

I shall give you a brief account of some recent experimental results in elementary-particle physics.

To begin with, I shall say something about the resonances; I shall confine myself mainly to polypion resonances, and just mentio an experiment assigning the spin of the K^*.

The possible sources of information on the poly-pion resonance are the following:

1. Electron-positron colliding beam experiments.
2. Photoproduction of these resonances.
3. Nucleon form factors.
4. Peripheral pion-nucleon collisions, esp. the reaction

$$\pi + N \rightarrow 2\pi + N$$

and

5. $N\bar{N}$ annihilation experiments.

1) is a conceivable experiment but has not yet been carried out. 2) also has not yet been extensively done*. As regards 3), attempt have been made to fit nucleon form factors with the resonances. The ρ-meson fits very well, but apparently not the ω-meson.

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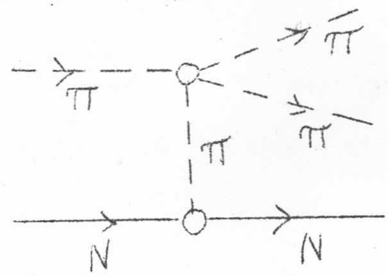
*NOTE: However, there are double photoproduction experiments which have shown the ρ resonance; also, evidence for the ABC anomaly has been found recently in a photoproduction experiment.

Most of the information, then, has come from 4) and 5). Method 4), that is, of peripheral pion-nucleon collisions, is probably the best so far, owing to the ease of getting pion beams of different energies. The Chew-Low extrapolation method has been used in connection with the study of peripheral pion-nucleon collisions leading to double pion production. The defeat of this method is that it gives an exact equality only in a limit (i.e. in the limit that the exchaged pion is on the mass shell); this limit cannot be attained, as it lies in the unphysical region. (Measurement of effective-mass distributions and missing-mass distributions gives information on the mass and lifetime of the resonances; the extrapolation method gives the $\pi\pi$ cross-section, according to the formula



$$\frac{\partial^2\sigma}{\partial p^2\,\partial\omega^2} = \frac{f^2}{2\pi}\,\frac{(p/2m)^2}{p^2+m^2}\,\frac{\omega\left(\frac{\omega^2}{4}-m^2\right)^{1/2}}{q^2}\,\sigma_{\pi\pi}$$

where the limit $p^2 \to -m^2$ is to be taken.)

<u>Quantum Numbers of the Resonances.</u>

The quantum numbers to be found out are

1) Mass (m)

2) Lifetime (τ)

3) Iso-spin (I)

4) Spin (J)

5) Parity (Π)

6) G-parity (G), where relevant.

1) The mass is perhaps the most easily fixed quantum number; it is given by the position of a peak.

2) As the lifetime (τ) is very short, it cannot be measured directly. What is measured is the half width (Γ), which is related to the lifetime by the uncertainty principle $\Gamma \approx \hbar/\tau$

The accuracy to which a width can be measured at present is of the order of 10 Mev. Thus the observed widths of all the resonance except the ρ resonance are instrumental, as they are of the order of or less than 10 Mev.

3) The isospin (I) can be determined fairly easily and unambiguously; this is done by observing in what charge-states the resonance occurs. For instance, if a resonance with zero strangeness occurs only in a state with total charge $Q=0$ and not in $Q=1,2$, states, then it may be assigned an isospin $I=0$.

The Spin, Parity, and G-parity.

The spin and parity of a resonance may be determined by two methods:

(i) The Dalitz method: This may be used when the resonance is observed to decay into 3 particles. The observed decays are plotted on the traingular Dalitz plot; the distribution of points observed is compared with that expected for different spin assignments.

A requisite for this method to be applicable is that the resonance must decay purely through strong interactions. When electromagnetic interactions are also involved in the decay process, as in the decay of the η meson, normal methods of analysing the decay do not apply.

The G-parity of a resonance may also be obtained from the Dalitz plot if purely strong interactions are involved in the decay.

(ii) The Adair Method: Here the spin of a resonance is deduced from its 2-body decay (into spinless particles) by observing the distribution of the decay products with respect to the momentum of the incident particle in the reaction in which the resonance is produced. Very recently the spin of the K^* has been established to be 1^- by this method.

Table 1 below gives some of the pseudoscalar mesons and resonances; the first row indicates that a PS meson (0^-) has the same spin-parity as a ($N\bar{N}$) pair in a $^1S_0$ state, while a vector meson (1^-) has the same spin-parity as a ($N\bar{N}$) pair in a $^3S_1$ state. (One may think of the Fermi-Yang model where the pion is a bound $^1S_0$ state of a nucleon and anti-nucleon.) The K and K^* have of course a strangeness of ± 1; for them G-parity is not a useful concept. The η, K , and π cannot decay via purely strong interactions.

TABLE 1*.

I	J^{PG} $PS \leftrightarrow N\bar{N}$ in $^1S_0$	J^{PG} $V \leftrightarrow N\bar{N}$ in $^3S_1$
0	η 0^{-+} $\rightarrow 3\pi$ $\rightarrow 2\gamma$	ω 1^{--} $\Rightarrow 3\pi$
$\frac{1}{2}$	K 0^- $\dashrightarrow 3\pi$, etc.	K^* 1^- $\Rightarrow K+\pi$
1	π 0^{--} $\rightarrow 2\gamma$ $\dashrightarrow \mu+\nu$	ρ 1^{-+} $\Rightarrow 2\pi$

* (For notation, see next page)

Notation: $\Rightarrow$ denotes a strong decay,

$\longrightarrow$ denotes a decay involving electromagnetic interactions, and

$\dashrightarrow$ denotes a decay via weak interactions.

3 recent experiments relating to polypion resonances are:

1) The decay $\eta \to 2\gamma$ has been observed recently at Frascati. (Re Mencuccini et al (preprint)).

2) A resonance at above 1250 Mev with $I = 0$ and decaying into 2π to have been observed. If confirmed, this could be the J = 2 resona predicted by the Regge pole hypothesis (by a straight line extrapola tion of the Pomeranchuk trajectory).

3) The ABC anomaly has again been observed in a double-photoprodu tion experiment at Frascati.*

Table II gives an up-tp-date list of the various resonances (as Sep. 15, 1962).** I may just remark that the existence of the ζ -mes cannot be considered to be established.

The other data I shall mention are

1) The β-decay of the π-meson, i.e. the decay

$$\pi^+ \to \pi^0 + e^+ + \nu$$

predicted by the Gell-Mann - Feynman Theory, has been observed. It a branching ratio of about 10^{-8}.

2) The two-neutrine hypothesis has received good support from a Br haven experiments, where it was observed that the neutrines obtaine the decay mode $\pi \to \mu + \nu$ produced muons but no electrons. (Ref. Rev.Lett. 9, 36 (1962) by G.Danby et al). Further experiments on th are planned at CERN.

Finally, I shall mention an important advance in experimental nique, viz. the production of a target containing polarized hydroger

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*Note: Ref. also an experiment by Richter. (Phys.Rev.Lett. 9, 217 (1962) by B.Richter)

** Table II is a copy of "Tentative Data on Strongly Inte Particles", Sept. 1962, by A.H.Rosenfeld. (UCRL)

This is work done by Chamberlain, Jeffries, Schultz, and Shapir The hydrogen nucleus in a sample of $La_2 \; Mg_3 \, (NO_3)_2 . 24 \, H_2O$ with 1% of Mg replaced by a paramagnetic impurity (Nd^{142}) was polarized by a dynamical method. The sample was cooled to a temperature of 1.5°K in a magnetic field of 9160 gauss, and a microwave voltage at 34.3 k Mc/sec was applied. The total mass of the sample was 20 gm; this contained 0.6 gm. of hydrogen. The elctron-proton system has four levels, corresponding to the different spin orientations; the applied microwave voltage saturates one of the transiti The polarization obtained was 20%. Only one proton in thirty was polarized; however, with coincidence methods (e.g. for pp scattering), even this would be useful. (It is a free proton that is polarized. As scattering of an incident proton by a free target proton is characterised by a definite angle ($\simeq$ 90°) between the final protons, it can be distinguished from the scattering on the unpolariz (bound) protons).

This is only a beginning; a much larger degree of polarization is expected to be achieved with improvements. The availability of a polarized target would open up a world of new possibilities. It would be possible to do experiments in which a polarized proton beam is scattered off a polarized target of protons, which would directly give the spin dependence of phase shifts. Also, by observing the polarization of Σ's produced by π mesons incident on polarized nucleons, the ($K \Sigma$) relative parity could be established with definiteness. The cross-section for this is given by

$$\sigma = \sigma_0 \left(1 \pm \vec{P} \cdot \vec{P}_0 \right)$$

where $\mathcal{P}_o$ is the polarisation of the target proton and $\mathcal{P}$ is the polarisation of the Σ produced on an unpolarized target (at the same energy and angle). The two signs are for different relative parities. Various other experiments can also be carried out with polarized targets.

DISCUSSION:

Q. Could you please explain the details of the experiment assigining the spin of the K^*?

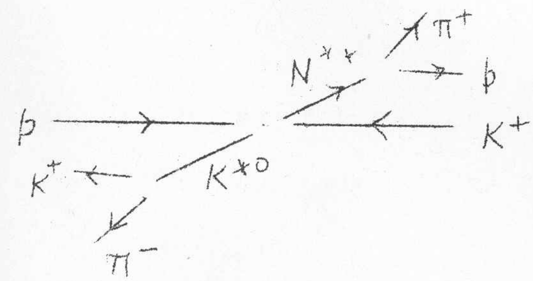
Ans. The experiment is by W. Chinowsky, G.Goldhaber, S.Goldhaber W.Lee, and T.O'Halloran. The reaction studied is

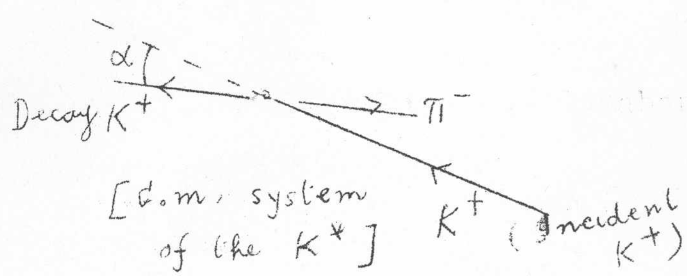
$$K^+ + p \rightarrow K^{*o} + N_{33}^{*++}$$

$$\qquad\qquad\quad \downarrow \qquad\qquad \downarrow$$

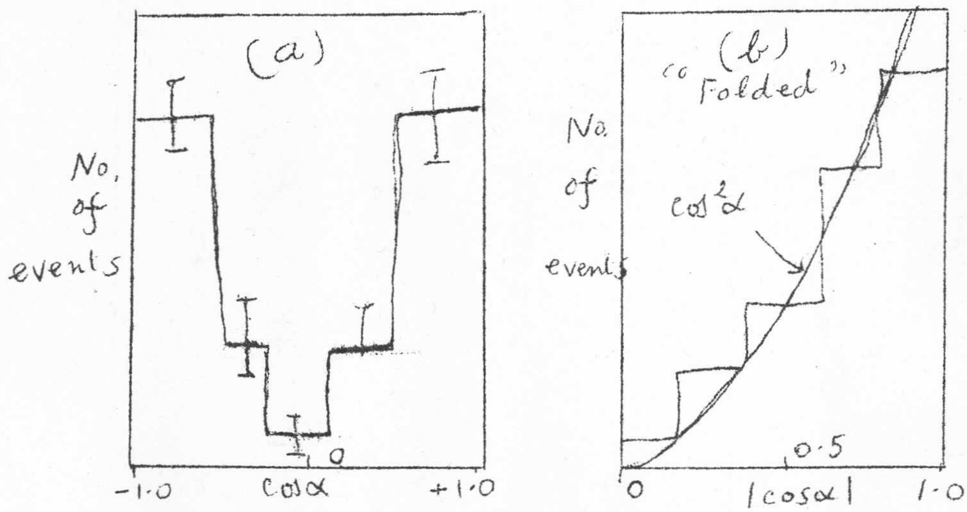
$$\qquad\qquad (K^+ + \pi^-) \quad (p + \pi^+)$$

All the particle produced are observed (in a bubble chamber). From the momenta of the final nucleon and pion, the momentum of the N^*_{33} in the centre-of-mass system (and hence that of the K^*) is inferred. Selecting events in which the K^* is emitted close to the forward direction, the distribution in the angle between the final K^+ in the c.m.s. of the K^* and the incident K^+ direction is observed.





The observed $\cos^2\alpha$ distribution is shown in the figure below; it indicates a spin ≥ 1. Alston's earlier experiments showed $J < 2$; hence we conclude that the K* spin is 1.



Q. What about the ($K\pi$) resonance at 730 Mev ?

Ans. Referring to Table II, we find that all the assignments for this resonance have question marks. Thus the existence of this is doubtful.

Particle	Established Quantum Number $I(J^{PG})$	Possible assignment Quantum No. $I(J^{PG})$	Possible assignment Regge Trajectory	Mass (Mev)	(Mev)	$Mass^2$ $(Bev)^2$	Dominant decays Mode	%	Q (Mev)	P or P max. (Mev/c)
Vacuum ?	-	$0(2^{++})$	$+\omega_\alpha$	-	-	-	(Even no. of π) $\bar{K}$ K etc.			
η	$0\ (0^{-+})$		$\mp\omega_\beta$	548	10	.30	Neutrals	75	-	-
							$\pi^+\pi^-\pi^0$	25 ± 4	136	175
ω	$0\ (1^{--})$		$-\omega_\gamma$	782	15	.62	$\pi^+\pi^-\pi^0$	86	368	326
							$\pi^0\gamma$	14 ± 4	647	379
π π^0	$1(0^{--})$		$-\pi_\beta$	135	0	.018	$\pi^0\to 2\gamma$	100	135	67
$\pi^\pm$				140	0	.02	$\pi^\pm\to\mu\nu$	58	34	30
ρ	$1\ (1^{-+})$		$+\pi_\gamma$	750	100	.56	$\pi\pi$ (p-wave)	100	471	348
ζ	1(?).	$1(0^+)$	$-\pi_\alpha$	560	15	.31	$\pi\pi$	?	290	245
K K^0	$1/2(1^-)$		K_β	498	0	.24	$K_1^0\to\pi^+\pi^-$	$2/3K_1$	219	206
$K^\pm$				494	0		$K^\pm\to\mu\nu$	58	388	236
$K^*_{1/2}$ (888)	$1/2\ (1^-)$		K_γ	888	50	.78	$K\pi$ (p wave)	100	$251(K^0\pi)$	283
$K^*_{1/2}$ (730)	1/2(?)	?	?	730	20	.53	$K\pi$	?	$101(K^-\pi^0)$	161

Table 2.

TENTATIVE DATA ON STRONGLY INTERACTING PARTICLES (A.H.Rosenfeld)

$N \begin{cases} n \\ p \end{cases}$	1/2(1/2+)	N_α	940	0	.88	$pe^-\bar{\nu}$	100	.78	1.2
			938			-	-	-	-
$N^*_{1/2}$("900" Mev)	1/2 (5/2 +)	N_α	1688	100	2.84	$N\pi$ (f wave)	?	610	572
$N^*_{1/2}$("600" Mev)	1/2 (3/2 -)	N_γ	1512	150	2.28	$N\pi$(d wave)	?	434(p)	450
$N^*_{3/2}$(isobar)	3/2 (3/2+)	Δ_δ	1238	100	1.53	$N\pi$(p wave)	100	160(p)	233
$N^*_{3/2}$	3/2(J ≥ 5/2) ; 3/2(7/2 +)	Δ_δ	1920	200	3.69	$N\pi$+ other	?	824(p)	722
Λ	0(1/2+)	Λ_α	1115	0	1.24	$\pi^- p$	67	38	100
Y_0^*	0(j . 5/2) 0(5/2 +)	Λ_α	1815	120	3.29	KN+ others	?	383(p)	541
Y_0^*	0(?) 0(½+) 0(½ -)	Λ_β	1405	50	1.97	$\Sigma\pi$ / $\Lambda 2\pi$	{100	{69 / 10	144 / 69
Y_0^*	0 (3/2-)	Λ_γ	1520	15	2.31	$\Sigma\pi$(d wave)	60	194	267
						$\bar{K}N$(d wave)	30	88	244
						$\Lambda 2\pi$	10	125	253
Σ^0	1 (½ +)	Σ_α	1191	0	1.42	$\Lambda\gamma$	100	76	74
Σ^-			1196	0	1.42	$n\pi^-$	100	117	192
Σ^+			1189	1	1.42	$n\pi^+$	50	110	185
Y_1^*	1(J . 3/2) 1(3/2 +)	Σ_δ	1385	50	1.92	$\Lambda\pi$	98	135($\Lambda\pi^0$)	210
						$\Sigma\pi$	2 ± 2	49 ($\Sigma^-\pi^+$)	119
Y_1^*	1 (?) ?	?	1685?	?	2.85 ?	$\Lambda\pi$+others.	?	435	459

Table 2 (contd.)

Ξ	Ξ^0	1/2(?)	$\frac{1}{2}(\frac{1}{2}+)$	Ξ_α	1311	0	1.72	$\Lambda\pi^0$	-	61	131
	Ξ^-				1321			$\Lambda\pi^-$	-	66	138
Ξ^*		1/2(?) ?		?	1530	<7	2.34	$\Xi\pi$	100	74	148

Table 2 . (concluded)

THE INSTITUTE OF MATHEMATICAL SCIENCES

MADRAS - 4 (India)

ON THE NEW RESONANCES

Two Seminar Lectures

by

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Madras (May 1962)

Notes prepared by
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ON THE NEW RESONANCES

Generally we take the states of matter characterized by I-spin, spin J, parity and a well defined energy ($\equiv$ mass) and do not make any distinction between resonances and particles. In particular we shall deal here with bosons; such "states of matt are called the 'New Mesons' or resonances. One of the first indications for the existence of the resonances came from the nucleon-antinucleon annihilation experiments

$$p + \bar{p} \rightarrow N\pi$$

The statistical model predicts an average particle multiplicity ($\langle N \rangle$) of order 3. However it was observed that

$$\langle N \rangle = 5.1 \pm 0.3$$

To reconcile this with the statistical model, it was suggested that the average $\langle N \rangle$ may be due to some three bodies in the final system rather than three π-mesons so that

$$\begin{aligned} p + \bar{p} \rightarrow\ & \pi + \pi + (\pi\pi\pi) \\ \text{or}\ & \pi + (2\pi) + (2\pi) \end{aligned}$$

which are in agreement with the experimental $\langle N_\pi \rangle \approx 5$ and thus the ~~xxx~~ existence of short-lived 3π or 2π particle states would reconcile the statistical model with experiment.

<u>The ρ meson</u> was the first 'resonance' that was observed. It was studied in the reactions[1-3]

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1. Pickup et. al. Bull. Am. Phy. Soc., <u>6</u>, 301 (1961)
2. Anderson et. al., Phys. Rev. Lett. <u>6</u>, 365 (1961)
3. Erwin et. al., Phys. Rev. Lett. <u>6</u>, 624 (1961)

$$\pi + N \rightarrow \pi + \pi + N$$

and its mass and width were found to be

$$M_\rho \approx 765 \text{ Mev}$$

$$\Gamma_{1/2} = 100 \text{ Mev} \quad \text{(half width)}$$

and it was assigned the quantum numbers

$$I = 1 \;, \quad J = 1^{-} \;.$$

It was first predicted by Frazer and Fulco[1] to explain the isovector form factor of the nucleon. The dominant decay mode is

$$\rho^0 \rightarrow \pi^+ + \pi^- \;, \quad \rho^{\pm} \rightarrow \pi^{\pm} + \pi^0$$

Some recent experiments seem to indicate a fine structure for the ρ^0 meson and the original broad resonance seems to be due to two closely-spaced narrow resonances ρ_1 and ρ_2 each with a width $\leq$ 30 Mev.

(see fig. 1)

<u>ω meson.</u>

Nambu[2] had predicted the existence of an isoscalar vector meson which would explain the isoscalar form factor of the nucleon. The form factor of the nucleon charge distribution is given by $F = F^S + F^V$. The isovector part changes its sign going from proton to neutron, the scalar one remains unchanged. Thus, the mean square radius is given by

$$\left.\begin{aligned} r^2 &= (r^V)^2 + (r^S)^2 = 8 \text{ for Proton} \\ &= (r^V)^2 - (r^S)^2 = 0.0 f \text{ for Neutron} \end{aligned}\right)$$

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1) W.R. Frazer and J.R. Fulco, Phys. Rev. Letter, 2, 365 (1959)
2) Y. Nambu, Phys. Rev., 106, 1366 (1957).

from Hofstadter's experiments.[3] (V and S indicate isovector and isoscalar respectively). Thus one concludes that $r^V \simeq r^S$ and since ~~the~~ dispersion theory gives

$$r^2 = 6/m^2_{res}$$

it follows that

$$m^V_{res} \simeq m^S_{res}$$

Thus the isoscalar particle of Nambu[2] was expected to have nearly the same mass as the ρ meson. The mass of the ρ estimated by Frazer and Fulco was 400-500 Mev. In that case, the ω should decay as

$$\omega \rightarrow \pi^0 + \gamma$$

and should be found in

$$\gamma + p \rightarrow \omega + p \rightarrow \pi^0 + p + \gamma$$

However no high energy photon could be detected. m_ρ was later estimated to be 765 Mev; and Sudarshan predicted that $m_\rho < m_\omega < m_\rho + m_\pi$. It was suspected that it should be possible to observe the ω in

$$p + \bar{p} \rightarrow N\pi$$

by studying the effective mass distribution of the triplets of pions.

The effective mass distribution of the triplets of pions was estimated[3] using the relation

1. Hofstadter et. al. Rev. of Modern Physics, 30, 482 (1958), Phys. Rev. 110, 552 (1958); Phys. Rev. 111, 934 (1958).
2. A neutral isoscalar meson had been earlier suggested by Teller and Duerr (1956) in connection with the saturation of nucleon forces and nucleon-antinucleon phenomena. Teller & Duerr, Phys. Rev. 101, 494 (1956).
3. Maglic et. al. Phys. Rev.Lett., 7 , 178 (1961).

$$M_3^2 = (E_1+E_2+E_3)^2 - (\vec{p}_1+\vec{p}_2+\vec{p}_3)^2$$

in the reaction

$$p+\bar{p} \rightarrow \pi^+ + \pi^+ + \pi^- + \pi^- + \pi^0$$

(see fig. 2)

and the graphs are given. For triplets with different total charge, namely,

$|Q| = 0$ $\pi^+\pi^-\pi^0$ 4 combinations

$|Q| = 1$ $\pi^\pm\pi^\pm\pi^\mp$ 4 combinations

$|Q| = 2$ $\pi^\pm\pi^\pm\pi^0$ 2. combinations

(see Fig. 3)

In Fig. 3, (A) and (B) which correspond to $|Q| = 1$ and 2 do not have any peak in contrast to the peak at 787 Mev for the $|Q| = 0$ case.[1] This determines the isospin of ω to be 0 . (i.e.) $I_\omega = 0$, $M_\omega = 787$ Mev $\Gamma_{1/2} \leq 15$ Mev
The angular distribution of the π^- and π^+ were also studied for $\bar{p}$ annihilation in flight and were found to have the forward and backward peaking respectively. This could be expected from the Koba-Takeda model as well as from any other theoretical picture of annihilation, which takes into account the structure of the nucleon. To our knowledge this is ~~xx~~ so far the most striking confirmation of the mesic cloud in the nucleon predicted by Yukawa.

(see Fig. 5)

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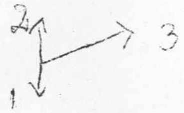
1. Wong and Lynch, Phys. Rev. Lett., 7, 327 (1961).

Determination of the spin of ω .

1. A $T = 0$ state of a 3π system must be antisymmetric in all pairs



Let the 3π system be split into single pion (3) and a dipion (1), (2) systems and since $T_3 = 1$ and $T_\omega = 0$ we have $T_{12} = 1$. Hence the space wave function of 1 , 2 should also be antisymmetric. Let L be the orbital angular momentum of 1 , 2 in the dipion rest system. Then the lowest value of $= 1$ since it has to be odd. Let ℓ be the orbital angular momentum of particle 3 in the 3π rest system. Then the different spin parity assignments for ω are

"Meson" Type, J		$\vec{\ell}$, $\vec{L}$	Matrix elements. Type J		Form	vanishes at
V,	1^-	1, 1	A,	1^+	$E_-(p_0 \times p_+) + E_0(p_+ \times p_-) + E_+(p_- \times p_0)$	whole boundary
PS	0^-	1, 1	S,	0^+	$(E_- - E_0)(E_0 - E_+)(E_+ - E_-)$	a, c, e b d, f.
A,	1^+	0, 1	V,	1^-	$E_-(\vec{p}_0 - \vec{p}_+) + E_0(\vec{p}_+ - \vec{p}_-) + E_+(\vec{p}_- - \vec{p}_0)$	b, d, f only

where spin 2 has been ruled out. The above matrix elements are of the simplest form possible. It is then convenient to make a Dalitz plot as shown in Fig. 6.

(see Fig. 6).

In the nonrelativistic limit, the conservation of energy and momentum restricts the allowed region to a circle. For extreme relativistic case it is a Δ . The allowed region for the energy involved in the experiment is shown in the figure as abcdef . The size of the figure is proportional to

$$T_1 + T_2 + T_3 = Q = m_\omega - \left(2m_{\pi^\pm} + m_{\pi^0}\right)$$

All the three matrix elements are antisymmetric. They vanish whenever one of the pions has its maximum kinetic energy (d, f , b). However the more striking feature of the peak region plot is the depopulation when any $\vec{p}_\pi = 0$ (a, c, e). This suggests an angular momentum barrier ($\ell > 0$) and constitues evidence against an Axial vector meson. The medians correspond to equal energies of two of the pions. The scalar matrix element will have to vanish on these lines and hence if ω were 0^- there would be depopulation along these lines, which is not observed. The whole boundary of the Dalitz plot corresponds to parallel and amtiparallel configurations for the pions and so the $|(\text{matrix element})|^2$ should vanish at the boundary for a vector meson which indeed seems to be the case in Fig. (6).

The Fig. 7 represents the number of events per unit area of the Dalitz plot for the peak region and for the control region versus the distance from the centre of the Dalitz plot; ~~is given and~~ the curves expected for 1^+, 1^- and 0^- mesons are also drawn.[1, 2]

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1. N.H.Xuong and G.R.Lynch, Phys. Rev. Lett. 7, 327 (1961)
2. M.L.Stevenson, L.W. Alvarez, B.C.Maglic and A.H. Rosenfeld, UCRL - 9856.

(see fig. 7)

The experimental data agree well with the curve predicted by a matrix element of a vector meson (1^{-}) and not at all with the prediction of a 0^{-} or 1^{+} mesons.[1]

However these analyses were based on the assignment of G parity -1 to ω . The Dalitz plot for a 0^{-} particle with $G = +1$ should be flat, which is excluded by our data

The effective mass distribution for the neutral ρ^0 show two peaks at 720 Mev ρ^0_1 and 780 Mev ρ^0_2 . The latter is very near the ω meson and suggests the possibility of a G violating ω decay

$$\omega(G=-1) \rightarrow \pi^+ + \pi^- + \pi^0 \; (G=-1) \quad (\text{G - allowed})$$

$$\rightarrow \pi^+ + \pi^- \; (G=+1) \quad (\text{G - forbidden})$$

Feld was first to indicate the possibility that $\omega \rightarrow \pi^+\pi^-$ electromagnetically.

<u>η meson.</u>

Pevsner et. al.[2] observed two peaks in the histogram of the effective case of 3π system in the reactions

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1. Heisenberg and Duerr showed, taking into account G-parity, that only the assignments 0^{-+} and 1^{--} were compatible with the data available on the ω meson; they pointed out that the observed narrow width of the ω meson favoured slightly 0^{-+} while the Dalitz plot was in favour of the assignment 1^{--}.

2. Pevsner et. al., Phys. Rev. Letters, 7, 421 (1961).

$$\pi^+ + d \rightarrow \pi^+ + \pi^- + \pi^0 + p + p$$

(see Fig. 8).

at 550 Mev and 770 Mev respectively. The large peak near 770 Mev is clearly identifiable as the ω^0. The other peak is named η meson. The mass and width calculated are

$$M_\eta = 546 \text{ Mev} \qquad \text{and} \qquad \Gamma/2 \leq 25 \text{ Mev}$$

and the quantum numbers tentatively assigned were $1^{-(-)}$ (G parity (-) . Bastien et. al.[1] have observed the η in the reactions $K^- + p \rightarrow \Lambda + \eta^0$

(see Fig. 9)

with $\eta^0 \rightarrow \pi^+ + \pi^- + \pi^0$

and $\eta^0 \rightarrow$ neutrals

and have deduced the ratio

$$\eta^0_{ch}/\eta_{neut} \text{ at } 760 \text{ Mev/c} = 0.31 \pm 0.11$$

and favour 0^{-+} for the η though 1^{--} is also not ruled out.* Pickup, Robinson and Salant also get a flat Dalitz plot which probably establishes 0^- for $G = +1$. It is noteworthy that if η is 0^-, 30 % of the η decay seem to violate G parity while in the case of ω only < 10 % violate G parity.

1. Bastein et. al., Phys. Rev. Letters, 8, 114 (1962).

* Recently Shaw and Wong have given reasons for favouring the assignment 0^{--} for the η meson; they suggest that a small amount of isospin violation distorts the distribution of points on the Dalitz plot from that expected for a particle with $G = -1$.

ζ meson.

The ζ meson was observed in the reaction

$$\pi^+ + p \rightarrow \pi^+ + \pi^0 + p$$

by (1) French group at Saclay,[1]

(2) the Michigan group, and

(3) in the reaction

$$p + p \rightarrow D + \pi^+ + \pi^0$$

B.S.Zorn at Brookhaven.[2.]

(see Fig. 10)

It seems that only in experiments with large momentum transfer the ζ meson is observed and hence may be related to the core.[3.] The mass, width and I spin are found to be

$$M_\zeta = 575 \text{ Mev} \pm 15 \text{ Mev}$$

(No peak was observed in the reaction

$$\pi^+ + p \rightarrow \pi^+ + \pi^+ + n$$

which favours $I = 1$ for ζ)

Pais indicates that ζ could be a 0^+ meson with $I = 1$ and hence $\zeta \rightarrow \pi^+ + \pi^0$ (the dominant decay mode) will also violate G parity.

The ABC particle.[4.]

It has been observed only in two events and is expected to be 0^+ with $I = 0$ if it exists and with a mass of about 280 Mev.

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1. Roland Barloutand et. al. Phys. Rev. Lett. 8, 326 (1962)
2. B.S.Zorn, Phys. Rev. Lett. 8, 282 (1962)
3. However no trace of ζ was found by the Yale group.)
4. Abashian et. al., Phys. Rev.Lett., 5, 258 (1960).

<u>α meson.</u>

Pickup, Robinson and Galant observed a peak in the re-[1] action

$$p + p \rightarrow p + n + \pi^+ + \pi^+ + \pi^- \qquad \text{(i)}$$

in addition to those in

$$p + p \rightarrow p + p + \pi^+ + \pi^- + \pi^0 \qquad \text{(ii)}$$

In (ii) the peak may be due to ω or η with $Q = 0$. However in (i) we have a $|Q| = 1$ (3π) resonance (i.e. I = 1) at M_α = 625 Mev with $\Gamma_{1/2}$ = 20 Mev. (Fig. 11) called the α meson and ~~itxxx~~, ~~it~~ is expected to be a 1^+ meson It was predicted by Primakoff (at 4.3 m_π) as the particle which will explain the strangeness ~~axxxx~~ conserving part of axial vector form factor in K decay.[2]

<u>K^* and M meson.</u>

Don Miller et. al.[3] observed this resonance in the reaction

$$\pi^- + p \rightarrow \Lambda + K^+ + \pi^-$$
$$\rightarrow \Lambda + K^0 + \pi^0$$

- as peaks in the $K\pi$ system; a broad one at 880 Mev. with width 60 Mev and a very narrow one at 730 Mev. The first is

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1. Pickup.et.al., Phys. Rev. Lett. 8, 329 (1962).
2. P. Dennery and H. Primakoff (to be published. The only neutral decay mode of α has to be $3\pi^0$ mesons since
   $\alpha \rightarrow 2\gamma$ is forbidden by angular momentum conservation
   $\rightarrow 2\pi^0$ " Bose statistics
   $\rightarrow \gamma + 2\pi^0$ " by charge conjugation invariance.
3. Don Miller et. al. UCRL 10195.

called the K* meson and the second the M meson. Since it is narrow, the M resonance is unlikely to be an S wave resonance and hence M is expected to be 1^- while K* may be 0^+. The dominant decay mode is $K^* \to K + \pi$, $M \to K + \pi$

χ meson.

Lynch and Xuong* have an indication of a peak in the 4π effective mass analysis in the reaction

$p + \bar{p} \to 3\pi^+ + 3\pi^-$ corresponding to $+ + - -$ $Q = 0$

and $\to 3\pi^+ + 3\pi^- + 2\pi^0$ $+ - 0\,0$ $Q = 0$

at $M_\chi = 1.05$ Bev.

(Fig.

Guiragossian et. al.,** have observed a peak again at 1 Bev in the reaction

$$\pi^- + p \to n + \pi^+ + \pi^-$$

This may be the spin 2 particle of Chew's theory[1].

A 420 Mev Meson

At $M_\beta = 420$ Mev Schwartz et. al.*** observed a hump in the reaction

$$\pi^- + p \to n + \pi^+ + \pi^- \quad \text{with} \quad Q = 0$$

This might be due to some final state πN interaction if it is only a hump as observed also in the experiment of Perez-Mendez et. al.****

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1. Chew and Frautschi, Phys. Rev.Lett. 8, 41 (1962).
* Lynch and Xuong, Bull.Am.Phys.Soc. 7, 281 (1962) and N.Y.Xuong UCRL - 10129 (Ph.D. thesis)
** Guiragossian, Powell, White, Bull.Am.Phys.Soc.7,281(1962).
*** Schwartz, Kirt, Tripp, Bull.Am.Phys.Soc. 7, 282 (1962).
**** Kurtz, Barish, Perez-Mendez, Solomon, Bull.Am.Phys.Soc. 7, 280 (1962).

Thus the May 1962 table of Mesons is

T \ J	0^-	1^-	0^+	1^+	2^+
1	π	ρ	ζ	(α)	
1/2	K	M	K*		
0	η	ω	η'		χ

However Peysner et. al. seem to have observed it as a peak in the reaction

$$\pi^+ + d \rightarrow p + p + \pi^+ + \pi^-$$

The peak was observed in a $Q = 0$ state. While "hump" could be produced by many causes, it is very unlikely that a real peak could arise from a final state πN interaction. We shall refer to this as to η' .

A view to the future ; the sonic spark chamber and the missing mass spectrometer

There is no a priori reason why there should not exist, apart from the observed 2π and 3π resonance, 4π , 5π and even 10π resonances, corresponding to higher values of J and higher masses. An instrument is needed which would enable us to observe such massive pionic states, if they exist.

The earlier method of detecting resonances was based on observing the distribution of events as a function of the "effective masses" of groups of n particles in the final state of the reaction under study (if the resonance being looked for was an n particle resonance) e.g. in the reaction

$$A + B \rightarrow C + D + E + F$$

one would look for a three-particle resonance in the system ~~one would look for the~~ (DEF) by plotting the number of events observed against the effective mass $(M_{eff})_3$ given by

$$(M_{eff})_3^2 \equiv (E_D + E_E + E_F)^2 - (\vec{p}_D + \vec{p}_E + \vec{p}_F)^2$$

If a resonance existed in the (DEF) system, it would appear as a peak at some value of M_{eff}.

However, this method involves all the uncertainties occurring in the measurement of three vector momenta. A simpler method is the method of studying the distribution of the "missing mass" $(M_m)_c$ defined by

$$(M_m)_c^2 = (E_{A+B} - E_c)^2 - \vec{p}_c^{\,2}$$

$(M_m)_c$ is the same as (M_{eff}) ; the advantage in the missing mass method is that in this only one vector momentum angle the angle of emission is measured. The spark chamber is convenient for the analysis of missing mass distributions.

A future programme comprises the study of the missing mass distributions in reactions of the type

$$\pi + p \rightarrow p + n\pi \qquad n = 1, 2, \ldots, 10$$

using the sonic spark chamber. All the resonances in systems with upto ten pions would be located with much less labour than involved in the earlier bubble chamber work.

The spark chamber had been earlier used for recording the track of high-energy particles by a photographic method.

A recent innovation* of an acoustic method of recording perhaps promises to make the spark chamber one of the fastest devices for use in elementary-particle work. Its advantages are:

(i) Angular measurements can be made with it to an accuracy of $\pm (1/20)^{\circ}$;

(ii) it has a time resolution; particles appearing earlier or later in time can be distinguished;

(iii) it is a very fast device, as the necessary information can be deduced by an automatic device without the scanning etc. to be done in bubble-chamber work.

A particle passing through a spark chamber triggers a spark between oppositely charged electrodes; the times of flight of the emergins shock wave to two (piezo-electric) microphones locate the position of the spark. The track of the particle is found out from the measured positions of the two sparks.

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* B. Maglic', F. Kirsten, Acoustic Spark Chamber, UCRL - 10057 (submitted to Nucl. Instr. and Mathks).

Meson	Mass Mev	(half width) (Mev)	I	J	Decay Products
ρ^0	725(ρ^0)	40 or	1	1^-	$\pi^+\pi^-$, $\pi^\pm\pi^0$
$\rho^\pm$	750				
ω	780	$\lesssim 12$	[illegible]	1^-	$\pi^+\pi^-\pi^0$ $\pi^+\pi^-$ (neutrals
η	546	$\lesssim 12$	0	0^-	$\pi^+\pi^-\pi^0$ (neutrals
ζ	575	< 70	1	[illegible]$^+$	$\pi^\pm\pi^0$
(ABC)	280		[illegible]	0^+	$\pi^+\pi^-$
α	625	20	1	1^+	$\pi^\pm\pi^\pm\pi^\mp$
η'	430		0	0^+	
(χ)	1 Bev	or	0 0	2^+ 1^+	$\pi^+\pi^-$
K^*	885	~ 60	1/2	0^-	$K\pi$
M	730	$\lesssim 12$	1/2	1^-	$K\pi$

Name	Mass (Mev)	Width	(Mass)^2 (m_π)	I	J parity	Orbital wave	Products	Branching fraction.	Q (Mev)	Ref.
N*	1238	90	78	3/2	3/2 +	p	$N+\pi$	100%	159	c
	1510	60	117	1/2	3/2	d	$N+\pi$ + (others)	?	430	c
	1680	100	145	1/2	5/2	d, f	$N+\pi$ + (others)	?	600	c
	1900	200	185	3/2	7/2	?		?	-	d
Y*	1385	50	97	1	3/2 ?	?	$\Lambda+\pi$	96%	130	e
							$\Sigma+\pi$	4%	45	
	1405	40	101	0	?		$\Sigma+\pi$	100%	6.9	f
							$\Lambda+2\pi$		10	
	1525	16	119	0	3/2	d	$\Sigma+\pi$	5	189	g
							$\Lambda+2\pi$	1	130	
							$\bar{K}^0+n$	3	84	
						d	K^-+p			
	1685	50	[illegible]	1	?	*				
	1815	120	109	0	$\geq 3/2$	?	many	-	-	h

* Alexander, Jacobs, Kallfleisch, Miller, Smith, Schwartz, UCRL - 10286.

c. B.J.Moyer, Rev.Mod.Phys. 33, 367 (1961). The entry at 1680 Mev is probably not a simple resonance.

d. Paul Falk-Vairant and G.Valladas; 1960 Rochester Conference, May not be a resonance.

e. M.H.Alston and M.Ferro-Luzzi: Rev.Mod.Phys., 33, 416 (1961) UCRL 9587, Ely et al: Phys.Rev.Lett. 7, (Dec.15).

f. Alston et. al. Phys.Rev.Lett. 6, 698 (1961). Bastian et. al.: Phys.Rev. Lett. 6, 702 (1961).

g. M.Ferro-Luzzi, R.D.Tripp, & M.Watson, Phys.Rev.Lett.(to be published)

h. Chamberlain et. al. (Phys.Rev. to be published); L.T.Kerth, Rev.Mod.Phys. 33, 389 (1961). This hump may not turn out to be a resonance.

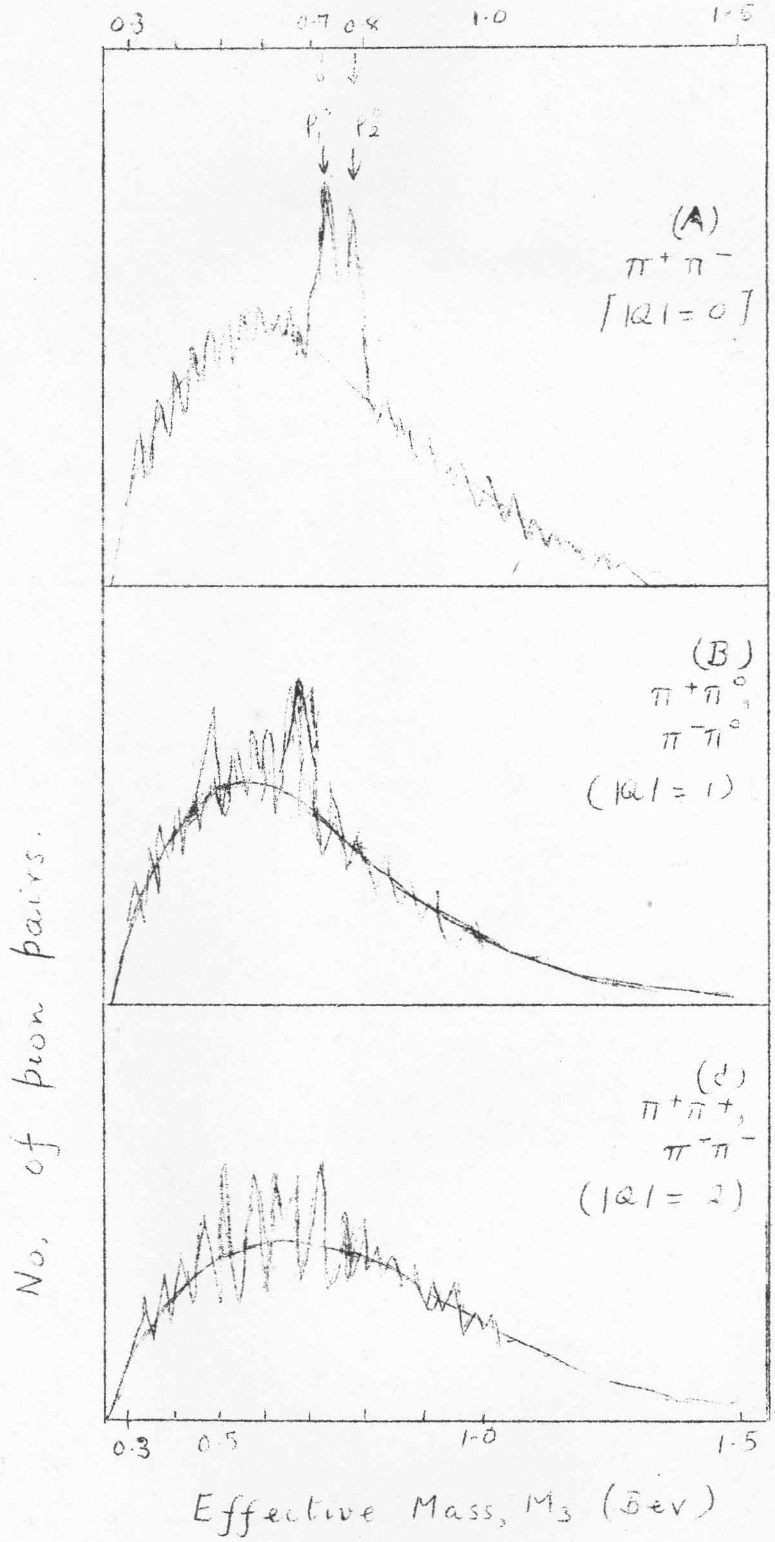
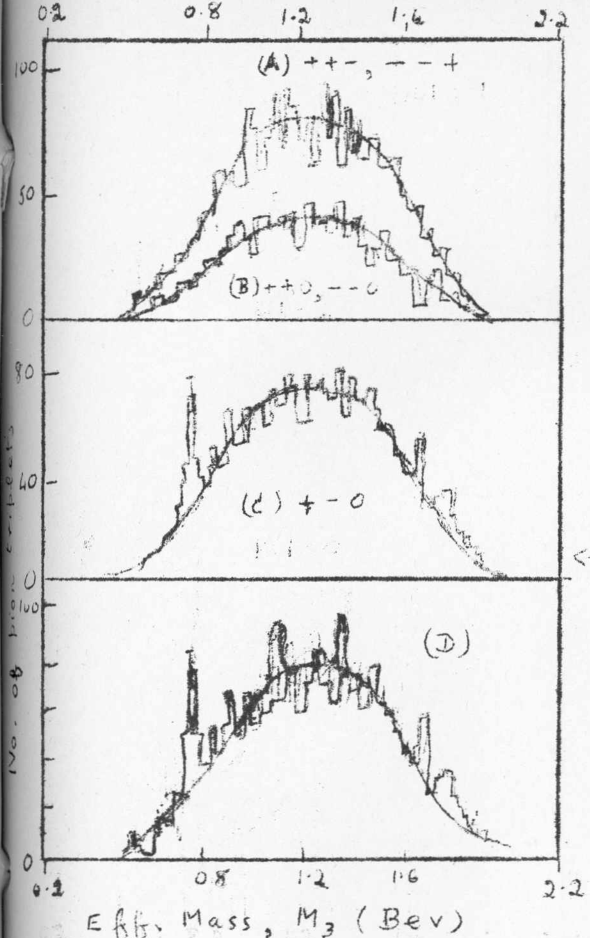


Fig. 1

Fine Structure of the ρ^0 meson (Pevsner et al).

[The splitting of the peak occurs only for the $|Q|=0$ pairs.]



No. of pion triplets vs. Eff. Mass (M_3) of the triplets for the reaction

$$\bar{p}+p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$$

In (D), the combined distributions (A) and (B) (smooth line) are contrasted with distribution (C).

Fig. 2

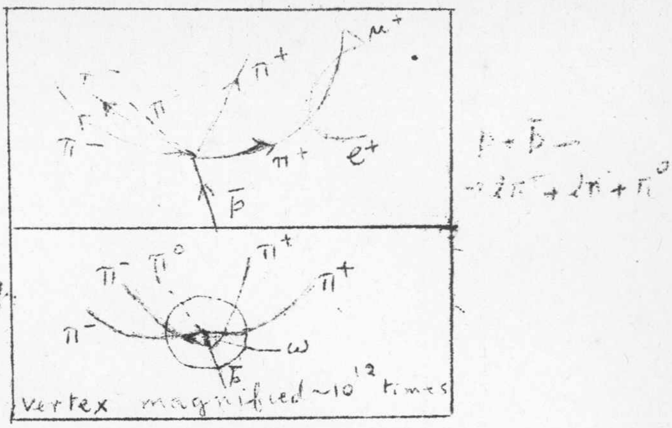
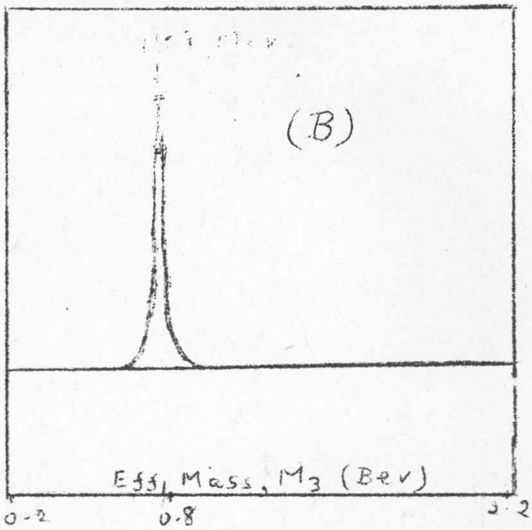
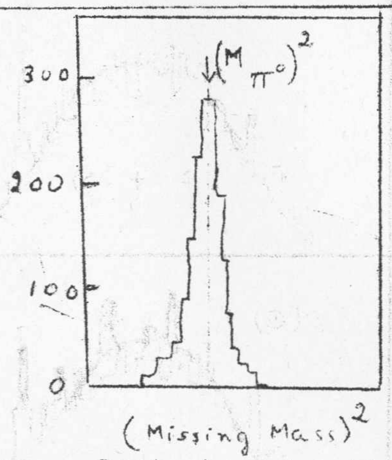


Fig. 3



Resonance Curve drawn through the peak at 787 Mev in the M_3 spectrum of the neutral π's in distribn. C (see fig on left), with smooth background subtracted. Half-width of peak = 15 Mev.

Fig 4



Missing Mass Distribn. in the reaction

$$p+\bar{p} \rightarrow 2\pi^+ + 2\pi^- + \pi^0$$

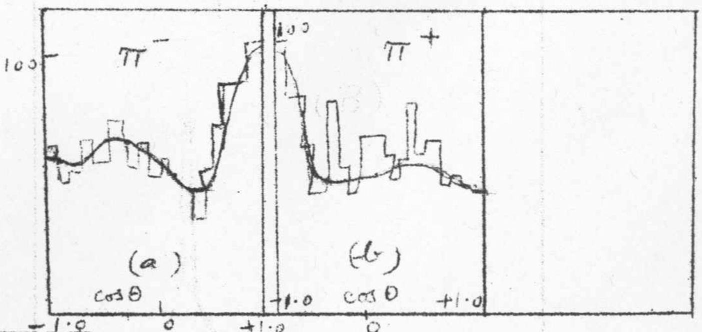


Fig. 5 π^- and π^+ angular distribns in $p\bar{p}$ annihil. in flight.

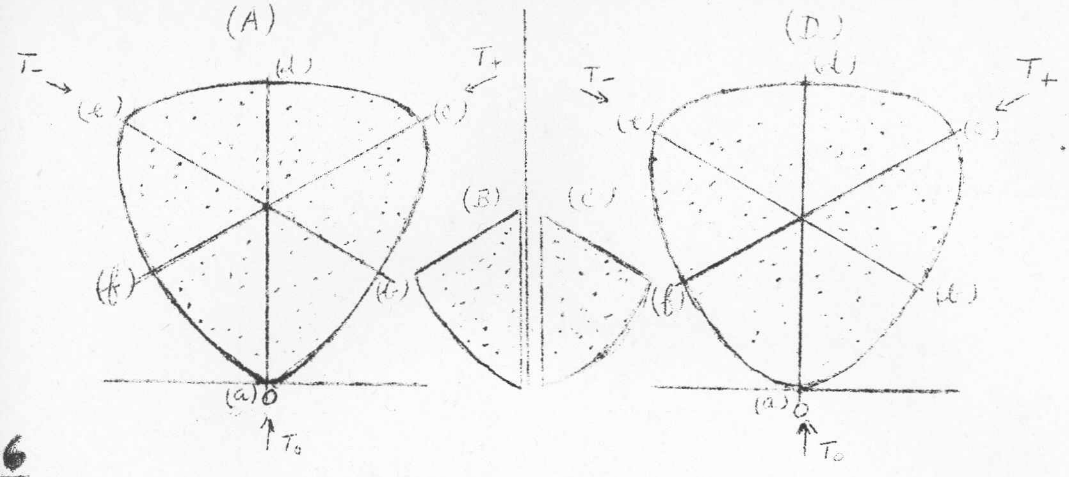


Fig. 6

(A) Dalitz plot of triplets from the control region.

(B) Folded control region plot.

(D) Dalitz plot for triplets in the peak region, 43 % of which are due to ω mesons.

(C) Folded peak region plot.

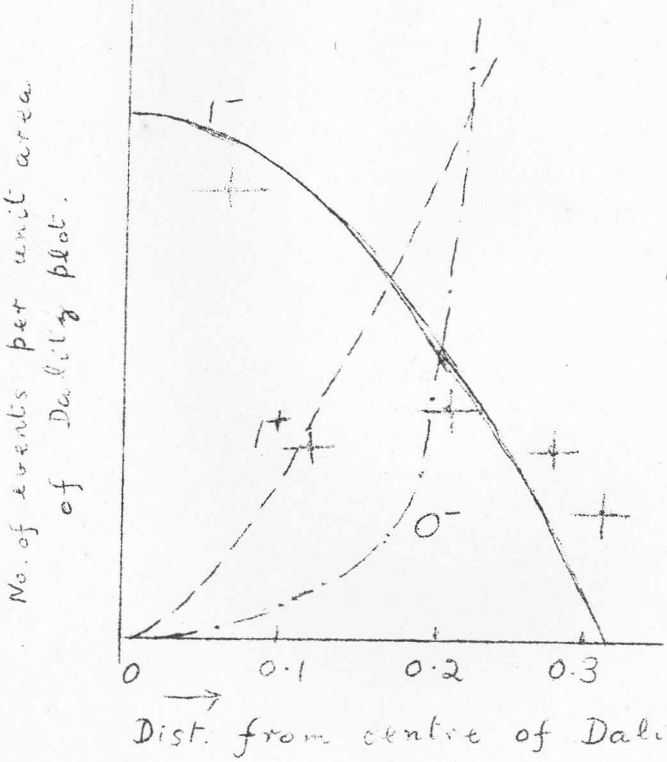
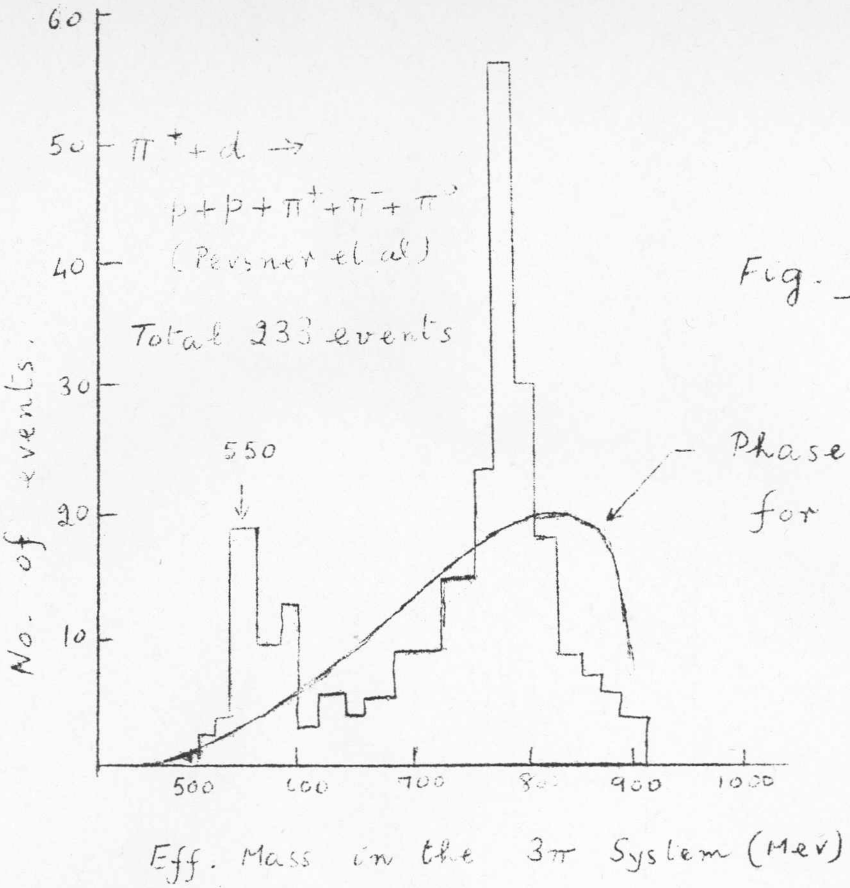


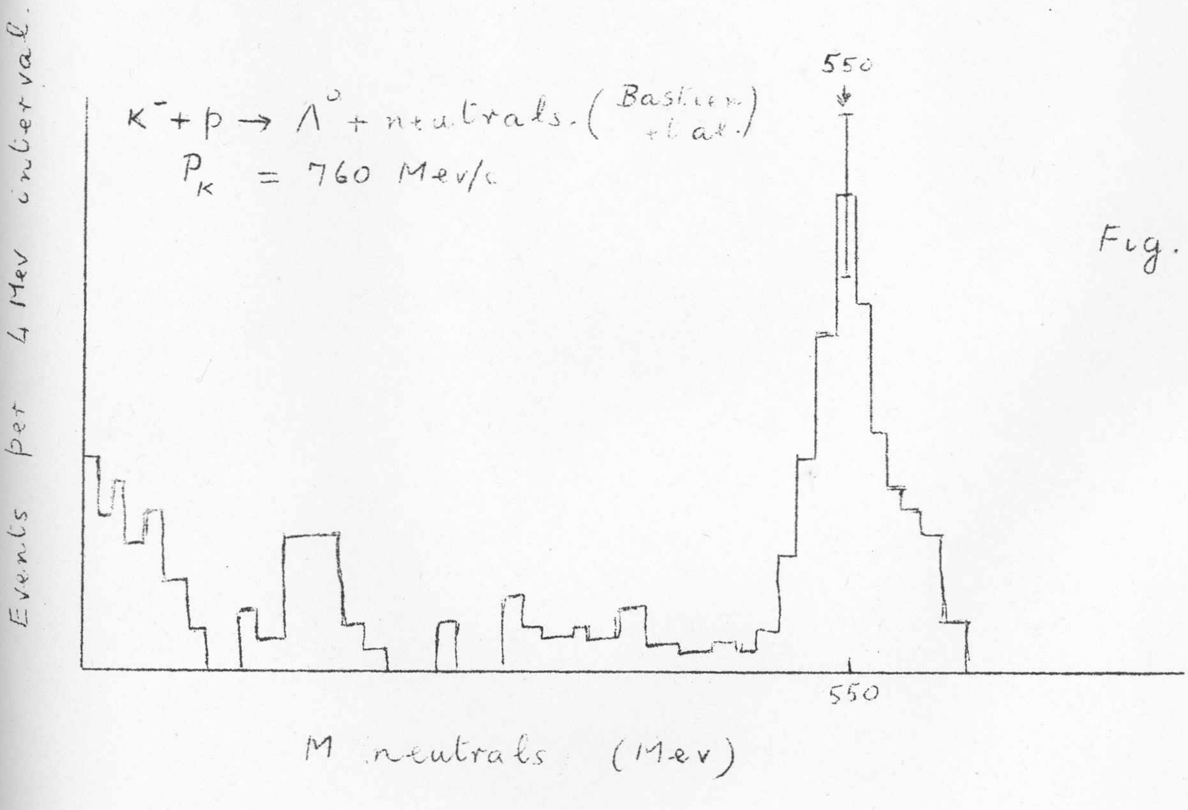
Fig. 7 [Ref. Xuong & Lynch PHYS. REV. LETT 7, 327]

+ Expt. points



60
50
40
30
20
10
No. of events.
$\pi^+ + d \rightarrow$
$p + p + \pi^+ + \pi^- + \pi^0$
(Pevsner et al)
Total 233 events
550
Phase space
for 1850 Mev.
500
600
700
800
900
1000
Eff. Mass in the 3π System (Mev)

Fig. 8



Events per 4 Mev interval.
$K^- + p \rightarrow \Lambda^0 +$ neutrals. (Bastien et al.)
$P_K = 760$ Mev/c
550
550
M neutrals (Mev)

Fig. 9

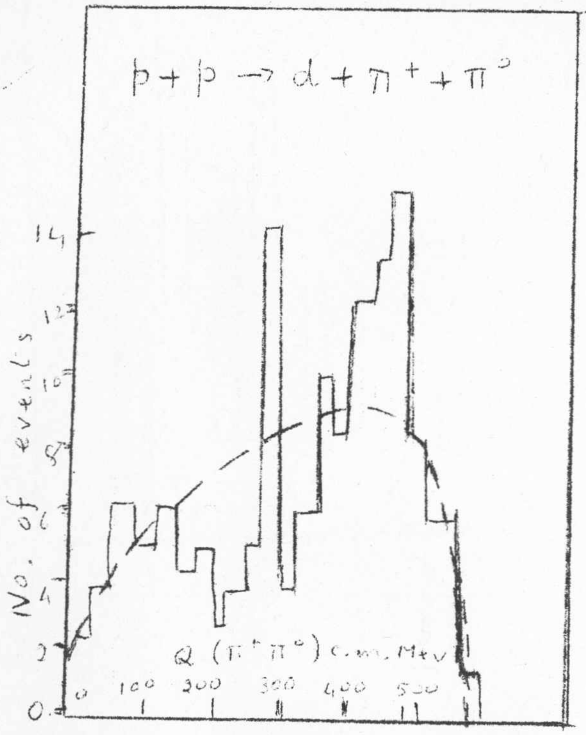


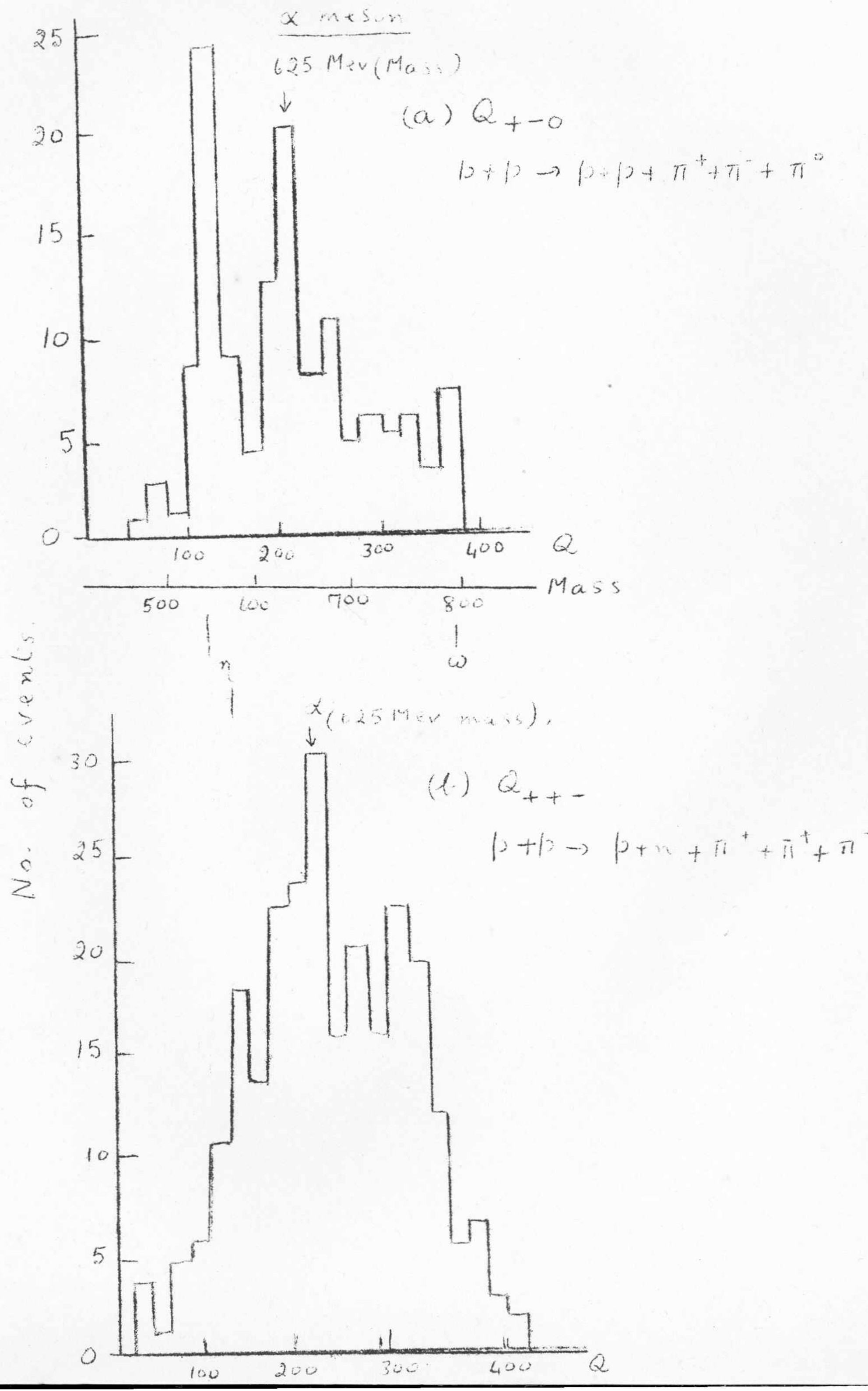
Fig. 10 T = 1 ππ resonances

[B. Sechi Zorn (Phys. Rev. Lett. 8, 282 (1962))]

Long dashed curve :- Predictions of statistical model.

Full line - Q ($\pi^+\pi^-$) value distribution (histogram in 25 Mev interval).

Fig. 11
3-pion mass distributions in the expts of Pickup, Robinson, and Salant. [Phys. Rev. Lett 8, 329 (1962)] Shows peaks corr. to the η, ω, and the α-meson (at ~625 Mev).



THE INSTITUTE OF MATHEMATICAL SCIENCES

MADRAS - 4 (India)

DETERMINATION OF SPINS, PARITIES AND ISOTOPIC SPINS OF 3π RESONANCES

Lecture
by

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Notes by
K. Venkatesan and V. Devanathan

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DETERMINATION OF SPINS, PARITIES AND ISOTOPIC SPINS OF 3 RESONANCES

We shall be mainly concerned about decays of the type $X \to 3\pi$ and the analysis of the spin, parity and isospin quantum numbers of the parent particle. Further we are interested in Regge recurrences., e.g., the -meson with mass 780 Mev. and a particle ϕ of mass 1030 (reported by UCLA) have the same spin-parity 1^-. So we can expect spin 3^- or 5^- for both ω and ϕ.. In the case of baryons we know that along with spin 1/2, we can have unstable particle with spin 5/2, 9/2, etc. (Spins upto 11/2 have been observed). But such a feature is not predicted by the Yukawa potential.

Now if π can be considered a bound state of other elementary particles these latter could be, in the logical order,

$$\pi = (\pi\rho), (K\bar{K}), (\rho\omega), (N\bar{N}), ----$$

Calculations show that if figure 1 is to be taken into consideration, so also should figure 2. The force between π and ρ would come from the exchange of a π or ω. Since ρ has spin 1, the pion has to be in a ρ-state to obtain a total angular momentum 0 for the intermediate pion. Similarly the $I=0$, ω-meson also is in a p-state so that if ω is a bound state of $\pi\rho$ then is not since both diagrams have the same strength of interaction. The situation regarding the $K\bar{K}$ and $\rho\omega$ combination may also be the same. So let us consider the $N\bar{N}$ state to

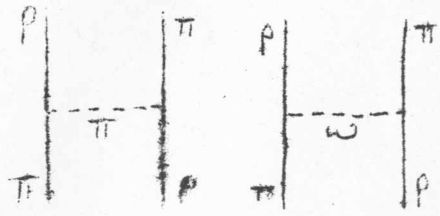

Fig. 1 Fig. 2

get bound into a pion of mass 140 Mev. with a large B.E. The same mechanism that binds $N\bar{N}$ may give other bound states in the same partial wave. There could be a π' with a spin-parity (0^-) and mass $m < 2$ Bev. which has the other quantum numbers the same as the pion. Presently experiments are being extended to this energy region Igi* has suggested that there is a second Pomeranchon with the same quantum numbers as the first pomeronchon. But a similar situation namely, different nucleon states of a nucleus with the same quantum numbers does not seem to exist). All this is speculation.

We shall now consider how to determine from experimental data the spin J, parity P and isotopic spin, I, of the parent particle in the decay.

$$X \longrightarrow \underset{(1)}{\pi} + \underset{(2)}{\pi} + \underset{(3)}{\pi} \tag{1}$$

where the numbers below the particles label them. The decay amplitude, M (J , I) is a tensor function of rank J. In the rest frame of X, a covariant relativistic description is not needed if one takes into account the conservation laws;

$$\begin{aligned} \vec{P}_1 + \vec{P}_2 + \vec{P}_3 &= 0 \\ \omega_1 + \omega_2 + \omega_3 &= m_x \end{aligned} \tag{2}$$

The most general form of the matrix element is

$$M = \Sigma M_I \, M_{JP} \, M_f \tag{3}$$

where M_I refers to the isotopic spin combination, M_{JP} the spin and parity of the decay particle and M_f refers to the energy dependent form factors.

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* K.Igi, Phys. Rev. Letters 9, 76 (1962).

Let us consider first the isotopic spins of the decay products. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ denote the isotopic spin vectors of the three pions. In terms of the charge state of the pions we have

$$a_+ = \frac{a_1 - i a_2}{\sqrt{2}}, \quad a_- = \frac{a_1 + i a_2}{\sqrt{2}}$$

and $a_0 = a_3$ represent respectively a π^+, π^- and π^0

We have
$$\vec{a} \cdot \vec{b} = a_+ b_- + a_- b_+ + a_0 b_0$$

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = -i \begin{vmatrix} a_+ & a_0 & a_- \\ b_+ & b_0 & b_- \\ c_+ & c_0 & c_- \end{vmatrix} \tag{4}$$

The most general scalar formed out of the energy and momenta of the particles is of the type $f(W_1, W_2, W_3)$ (Actually the dependence on the third variable is redundant). The dependence on all other scalars will reduce to this, e.g.,

$$2 \vec{p}_1 \cdot \vec{p}_2 = (\vec{p}_1 + \vec{p}_2)^2 - p_1^2 - p_2^2 = p_3^2 - p_1^2 - p_2^2$$

Because of the conservation laws any variable referring to the third pion can be reexpressed in terms of the other two. Let us now introduce the following notations.

E = function of energy-momentum which is completely symmetric in particle labels 1,2,3)

O = Function of energy-momentum which is completely anti symmetric.

A = A (2, 3) = A (3,2) (Symmetric function of the two variables 2, 3)

B = A (3, 1) = A (1,3)

C = A (1,2) = A (2,1) (5)

We also define the antisymmetric combinations

$$\begin{aligned}
\tilde{A} &= (2,\ 3) &&= -A\ (3,\ 2) \\
\tilde{B} &= &&= \tilde{A}\ (3,\ 1) = -A\ (1,3) \\
\tilde{C} &= &&= \tilde{A}\ (1,\ 2) = -A\ (2,1)
\end{aligned} \qquad (6)$$

In terms of these we can write the general form of the decay amplitude. Let us consider the various possibilities. From three isospin 1 values each with 3 components there are 27 states possible, viz., one $I = 0$ state, three $I = 1$ states, two $I = 2$ states and one $I = 3$ state.

<u>I = 0</u>, $M\ (I = 0) = (\vec{a} \times \vec{b})\ \vec{c}$ (7)

since there is only one combination of three isospins of value 1 which add up to a total 0 and this combination is antisymmetric The boson symmetry allows only the odd function 0.

<u>M (I = 1)</u> $= \vec{a}(\vec{b} \cdot \vec{c})A + \vec{b}(\vec{a} \cdot \vec{c})B + \vec{c}(\vec{a} \cdot \vec{b})C$ (8)

which is symmetric in any pair of a, b and c.

<u>M (I = 2)</u> Both symmetric and antisymmetric combinations are possible in this case. Let us define $M^{(a)}$ and $M^{(b)}$ to be antisymmetric and symmetric respectively under the change $1 \leftrightarrow 2$

i.e. $(1\ 2)\ M^{(a)} = -M^{(a)}\ ;\ (1\ 2)\ M^{(s)} = M^{(s)}$

or $$(1\ 2)\begin{pmatrix} M^{(a)} \\ M^{(s)} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} M^{(a)} \\ M^{(s)} \end{pmatrix} \qquad (9)$$

Interchange of 1 and 2 corresponds to a reflection in the line (3 4). $\begin{pmatrix} M^{(a)} \\ M^{(s)} \end{pmatrix}$ forms the two-dimensional irreducible representation group of three objects we are considering

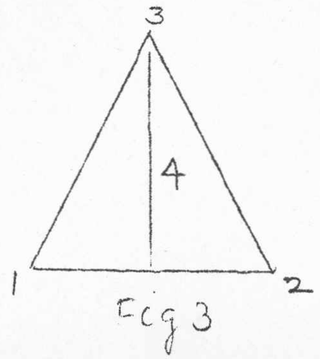


Fig 3

Similarly we have

$$13\begin{pmatrix} M^{(a)} \\ M^{(s)} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}\begin{pmatrix} M^{(a)} \\ M^{(s)} \end{pmatrix}$$

$$23\begin{pmatrix} M^{(a)} \\ M^{(s)} \end{pmatrix} = \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}\begin{pmatrix} M^{(a)} \\ M^{(s)} \end{pmatrix} \qquad (10)$$

Then the amplitude M (I = 2) can be written as

$$M(I=2) = \sqrt{3}M^{(a)}(A-B) + M^{(s)}(2C-A-B) = M^{(a)}(2\tilde{C}-\tilde{A}-\tilde{B}) + M^{(s)}(2C-A-B)$$

$$M(I=3) = M_{I=3}\,E(12)$$

which again involves a unique combination of the three isospin values.

To find out the quantum J, P and I for the decaying particle we have to look for the following quantitites.

(I) The branching ratios like $\frac{\gamma(2\pi^+\pi^-)}{\gamma(\pi^0\pi^0\pi^+)}$ or $\frac{\gamma(\pi^+\pi^-\pi^0)}{\gamma(\pi^0\pi^0\pi^0)}$; γ = rate $\propto |M|^2$

(2) Zero inregions in the Dalitz plot. The regions where the density of points of the plot vanish throw light on the spin-parity assignments.

(3) Angular correlations between the various particles (Adair-type analysis).

<u>The Branching Ratio:</u> The isotopic spin analysis gives us information about the branching ratios. e.g. If I = 0, $(\pi^+\pi^-\pi^0)$ is allowed while $(\pi^+\pi^0\pi^0)$ is forbidden. For I = 1 and I = 1, we have

$$M = a_+ (b_+ c_- + b_- c_+ + b_0 c_0) A + b_+ (a_+ c_- + a_- c_+ + a_0 c_0) B + c_+ (a_+ b_- + a_- b_+ + a_0 b_0) C$$

$$\therefore M(\pi^+ \pi^+ \pi^-) = A + B \qquad (13)$$

$$M(\pi^+ \pi^0 \pi^0) = C$$

and $$\Gamma = \frac{\gamma(\pi^+ \pi^+ \pi^-)}{\gamma(\pi^+ \pi^0 \pi^0)} = \frac{\Sigma_R |A+B|^2}{\Sigma_R |C|^2}$$

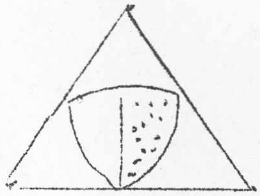


Fig 4

where by R we mean all the dots in the physical region of the Dalitz plot. (We need consider only the dots to the right of the vertical line of Fig.4 because of the presence of identical particles in the $\pi^+ \pi^+ \pi^-$ decay mode). In the case of K^+ decay, where the Q value is small so that A, B, C are constants the assumption A = B = C leads to the experimentally verified ratio $\Gamma = \frac{4}{1}$ If A + B + C = 0, the ratio would have been 1 : 1.

For the case I = 1, $I_z = 0$, we have the examples,

$$M(\pi^+ \pi^- \pi^0) = C \;; \quad M(\pi^0 \pi^0 \pi^0) = A + B + C$$

$$\frac{\gamma(+-0)}{\gamma(000)} = \frac{\sum_{\mathrm{I}+\mathrm{II}+\mathrm{III}+\mathrm{IV}+\mathrm{V}+\mathrm{VI}} |C|^2}{\Sigma_{\mathrm{I}} |A+B+C|^2}$$

$$= \frac{\Sigma_{\mathrm{I}}\, 2\left(|A|^2 + |B|^2 + |C|^2\right)}{\Sigma_{\mathrm{I}} |A+B+C|^2} \qquad (15)$$

If $A = B = C$ we have a ratio 2/3 as in the cases of the decay of the η - meson.

For $I = 2$ we have

$$M(\pi^+\pi^+\pi^-) = M(\pi^0\pi^0\pi^+) = 2C - A - B$$

$$M(\pi^+\pi^-\pi^0) = A - B$$

$$M(\pi^0\pi^0\pi^0) = 0$$

so that $$\frac{\gamma(++-)}{\gamma(000)} = 1 \tag{16}$$

Thus the branching ratio helps to determine the isotopic spin of the decaying particle.

<u>FORM FACTORS</u>: Let us now write down the various form factors which are symmetric or antisymmetric under exchange of the particle labels. Let us define $f_e = f_e(\omega_1, \omega_2, \omega_3)$ to be the completely symmetric function. We can then write the antisymmetric function as

$$\begin{aligned} f_o &= (\omega_1 - \omega_2)(\omega_2 - \omega_3)(\omega_3 - \omega_1) f_e \\ &= W_{123} f_e \end{aligned} \tag{17}$$

which defines W_{123}. We also define the even functions

$$\begin{aligned} f_1 &= f(\omega_2, \omega_3) = f(\omega_3, \omega_2) \\ f_2 &= f(\omega_1, \omega_3) = f(\omega_3, \omega_1) \\ f_3 &= f(\omega_1, \omega_2) = f(\omega_2, \omega_1) \end{aligned} \tag{18}$$

and the odd functions which can be formed from them like $(\omega_2 - \omega_3) f_1$ etc. which are antisymmetric function of two variables.

The vectors available for our problem are: $\vec{P}_1, \vec{P}_2, \vec{P}_3$

$$\vec{q} = \vec{P}_1 \times \vec{P}_2 = (\vec{P}_2 \times \vec{P}_3) = (\vec{P}_3 \times \vec{P}_1)$$

We also note that the three-pions have odd intrinsic parity. We shall now consider how to calculate tensors of angular momentum J from basic vectors.*

$J = 1 \qquad T_i \,, \qquad i = 1, 2, 3$

$J = 2, 1, 0 \qquad T_{i_1 i_2} \,, \qquad \sum_i T_{ii} = \text{Scalar}$

$\sum_{i,k} \epsilon^{ik} T_{ik} = \text{Vector}$

$T_{i_1 i_2}$: Symmetric and traceless combination corresponds to angular momentum 2.

In general, we can show that the tensor of angular momentum J has (2J 1) independent components. For this, we have to find the number of symmetric and traceless combinations of $T_{i_1 i_2 \cdots i_J}$

. Number of symmetric combinations of

$$T_{i_1 i_2 \cdots\cdots i_J} = \frac{(J+2)!}{J!\,2!}$$

$$= \frac{1}{2}\left(J^2 + 3J + 2\right)$$

If the trace with respect to first two indices vanishes i.e.

$\sum T_{kki_3 \cdots\cdots i_J} = 0$ then the trace with respect to any pair of indices will vanish since T is symmetric. Thus the number of constraints $= \frac{J(J-1)}{2} = \frac{J^2 - J}{2}$ Subtracting this from the number of symmetric combinations, we obtain the independent combinations.

$$\frac{1}{2}\left(J^2 + 3J + 2\right) - \frac{\left(J^2 - J\right)}{2} = 2J + 1$$

This is indeed the number of independent components of a Tensor of angular momentum J.

We shall now construct the spin-parity function for the decay X $\rightarrow \pi + \pi + \pi$

Case K: Normal parity : $(-)^{J+1}$

The additional factor (-1) is due to intrinsic odd parity of 3π system.

Case ii: Abnormal parity $(-1)^{J}$

To constrct normal parity spin functions, we need use only the basic momentum vector $\vec{P}_1, \vec{P}_2, \vec{P}_3$. To construct abnormal parity functions, we need use, in addition, $\vec{q}$ only once.

CONSTRUCTION OF NORMAL PARITY SPIN FUNCTIONS : M_{JP}

$$\begin{aligned}
0^- &= 1 \\
1^+ &= p_1,\ p_2,\ p_3 \\
2^- &= T(11),\ T(22),\ T(33) \\
3^+ &= T(111),\ T(222),\ T(112),\ T(122)
\end{aligned}$$

where

$$T_{ij}(12) = \frac{(p_1)_i (p_2)_j + (p_2)_i (p_1)_j}{2} - \frac{\delta_{ij}}{3}(\vec{P}_1 \cdot \vec{P}_2)$$

CONSTRUCTION OF ABNORMAL PARITY SPIN FUNCTIONS:

$$\begin{aligned}
0^+ &: \text{none.} \\
1^- &: q \\
2^+ &: T(1q),\ T(2q),\ T(3q) \\
3^- &: T(11q),\ T(22q),\ T(33q)
\end{aligned}$$

where

$$T_{ijk}(11q) = (P_1)_i (P_1)_j q_k + \begin{array}{l} i \leftrightarrow j \\ i \leftrightarrow k \\ j \leftrightarrow k \end{array}$$

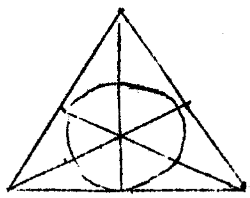
$$+ \delta_{ij} \left[\alpha (P_1)_k + \beta (q)_k \right]$$

$$+ \begin{array}{l} i \longleftrightarrow k \\ j \longleftrightarrow k \end{array}$$

From the vanishing of points on the Dalitz plot, we may beable to assign the isospin, spin and parity quantum numbers. We consider below the various possible situations.

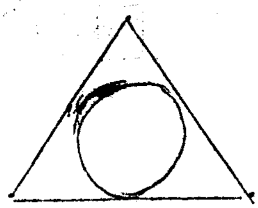
0^-

$$E = f_e$$
$$A = f_1$$
$$0 = W_{123} f_e$$
$$M(I=0) = W_{123} f_e$$



If the points on the Dalitz plot vanish along the median lines, the assignment omay be 0^-

$$M(I=1) = \pi^+ \pi^+ \pi^-$$
$$= f_1 + f_2$$



The vanishing of the points on the periphery corresponds to this case

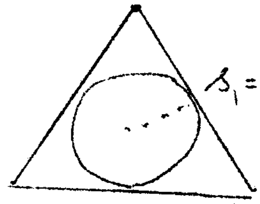
$$M(I=2) = \pi^+ \pi^- \pi^0$$
$$= f_1 - f_2$$
$$W_1, W_2, W_3 \geqslant m_\pi$$

If instead of W_1, W_2, W_3 we use $s_1 = W_1 - \frac{M_x}{3}$, $s_2 = W_2 - \frac{M_x}{3}$, $s_3 = W_3 - \frac{M_x}{3}$ such that $s_1 + s_2 + s_3 = 0$ the centre of the Dalitz plot is the point $s_1 = s_2 = s_3 = 0$

$$\frac{Q}{6} \leqslant s_3 \leqslant \frac{Q}{3} \quad \text{non rel.}$$

$$f_e\left(\frac{s_1}{m_\pi}, \frac{s_2}{m_\pi}, \frac{s_3}{m_\pi}\right) = \text{Const.}$$

$$\text{if } \frac{Q}{3m_\pi} \ll 1$$



$s_1 = s_2 = s_3 = 0$

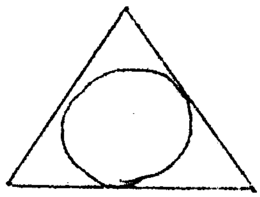
$s_3 = -\frac{Q}{3}$

where $Q = M_x - 3m_\pi$

If the Q value is small, the variation in the three numbers s_1, s_2, s_3 is small.

$\underline{\underline{1^-}}$

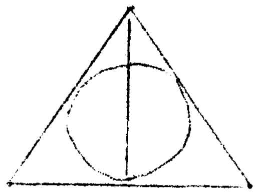
$$O = q f_e$$
$$E = q\, W_{123} f_e$$
$$A = q\,(s_2 - s_3) f_1$$
$$M(I=0) = q f_e$$



The edge of the Dalitz plot is that along which the momenta p_1 and p_2 are parallel

$$M(I=1, \pi^+\pi^+\pi^-) = q\left\{(s_2 - s_3) f_1 - (s_1 - s_3) f_2\right\}$$

If $s_1 = s_2$, $M = 0$ along the vertical line provided $f_1 = f_2$



$\underline{1^+}$

P_3 and $P_1 - P_2$ are independent vectors.

$$O : (P_1 - P_2) f_3 + (P_2 - P_3) f_1 + (P_3 - P_1) f_2$$

f_3 is even in 1 and 2.

$$E : P_3 f_3 + P_2 f_2 + P_1 f_1$$

$$A : h\,(32)\,\vec{P_3} + h\,(23)\,\vec{P_2}$$

$$B = h\,(31)\,\vec{P_3} + h\,(13)\,\vec{P_1}$$

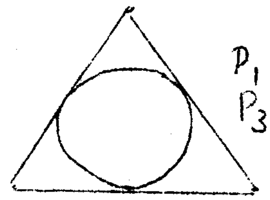
<u>I = 0</u>

$$|M|^2 = |0|^2$$

<u>I = 1</u>

$$\pi^+ \pi^+ \pi^-$$

$$M\,(I = 1) = A + B$$



$P_1 = P_2$
$P_3 = P_1 + P_2$
$= 2 P_1$

$P_3 = 0$
$P_1 = -P_2$
$s_1 = s_2$
$f_1 = f_2$

The interesting regions are edge, centre, head and foot of the medians.

All terms with q have vanishing Dalitz points at the edge.

I = 0, 2^- must vanish at the centre. ω cannot have 2^- because of this.

<u>Angular correlation:</u> If we include the production mechanism of the decaying particle X, the total matrix element can be written as

$$m = M_{ijk}\, Q_i\, P_j\, P_k$$

Consider the series of decays of π' with 0^- and I = 1

$$\pi' \rightarrow \pi + \rho \rightarrow 3\pi$$

$$m\,(\pi' \rightarrow \pi + \rho) = \vec{P} \cdot \vec{e}\,(\vec{a} \times \vec{d})$$

$$m\,(\rho \rightarrow 2\pi) = (P_2 - P_3)\,\vec{d}\,(\vec{b} \times \vec{c})$$

$$m\,(\pi' \rightarrow 3\pi) = [\vec{d} \times (\vec{b} \times \vec{c})]\,\vec{P} \cdot (\vec{P_2} - \vec{P_3})$$

$$|m|^2 = |\vec{P_i} \cdot (\vec{P_2} - \vec{P_3})|^2 = |P_2^2 - P_3^2|^2$$
$$= w_2^2 - w_3^2$$

By the symmetry of the problem we need consider only figure b instead of figure a.

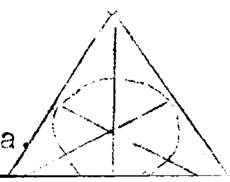


fig a

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HIGHER RESONANCES IN THE PION-NUCLEON SYSTEM *

by

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(Notes by K. Venkatesan and V. Devanathan).

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* This is a preliminary version of two lectures delivered by Prof. G. Takeda at the Institute of Mathematical Sciences in June. The lecturer has not looked into these notes.

THE HIGHER RESONANCES IN THE PION-NUCLEON SYSTEM

We shall discuss the experimental situation regarding the higher resonances in the pion-nucleon system first and then consider various theoretical approaches that have been made to explain them. The first resonance N_1 in the $J = 3/2$, $T = 3/2$ is well understood and we shall not speak about it here.

The following table summarises the parameters of the nucleon and the various pion-nucleon resonances that have been observed till now.

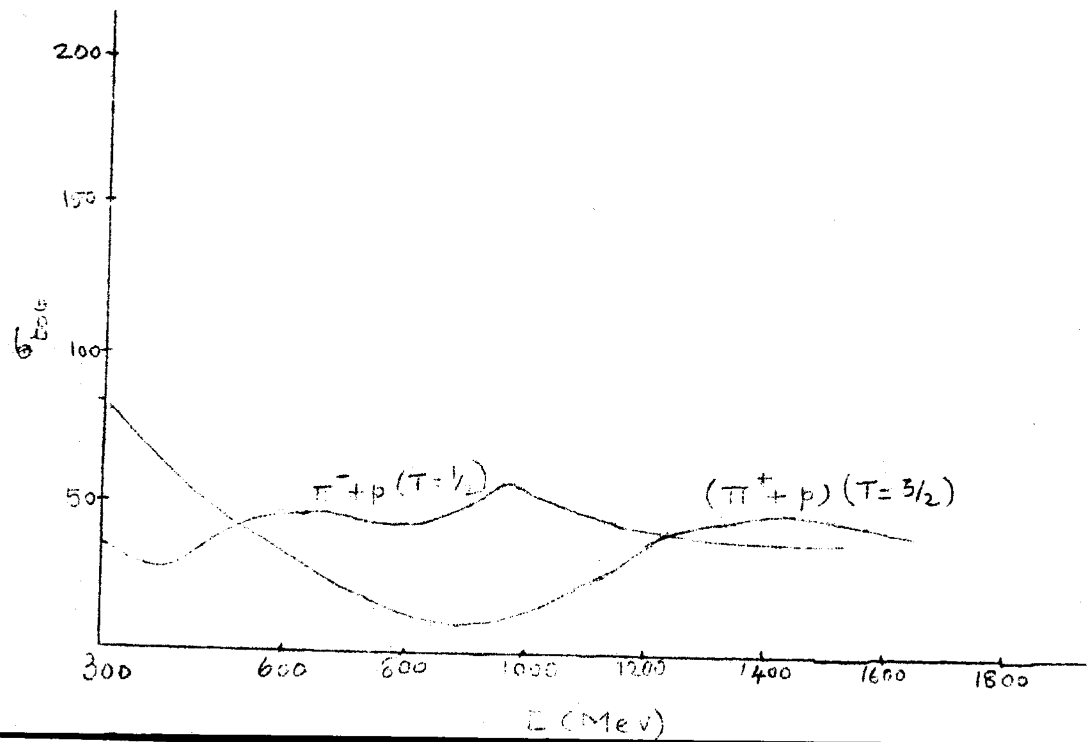
Particle or Resonance	Mass Mev	J	l	T	Γ (Mev
p, n	940	1/2	p	1/2	0
N_1	1240	3/2	p	3/2	~ 100
N_2	1510	3/2	d	1/2	~ 150
N_3	1690	5/2	f	1/2	~ 100
N_4	1920	(7/2)	(f)	3/2	≤ 200
N_5	~ 2220	$\leq 5/2$	(f)	1/2	(~ 200)
N_6	~ 2340	$\leq 3/2$	-	3/2	(~ 200)

In addition to the higher resonances N_2 and N_3 in the T = 1/2 state ($\pi^- - p$ system) and N_4 in the T = 3/2 state ($\pi^+ - p$ system) two new resonances N_5 and N_6 have been observed recently* the first in ($\pi^- p$) scattering and the second in $\pi^+ p$ scattering

* A.N. Diddens et al, Phys. Rev. Letters 10, 262 (1963)

The figures inside brackets in the above table are not definit

Let us now discuss the likely J and T values for the resonances N_5 and N_6 using the experimental data on cross-sections. For this purpose let us recall briefly the expressions for the elastic and inelastic and total cross-sections in terms of the phase shifts. If we have an incoming plane wave (corresponding to the incident particles), then in the case of two scattering there will be a phase ($e^{2i\delta} - 1$) between the ingoing plane wave and outgoing spherical waves which together represent the scattered wave. If there is also absorption of part of the incident wave the phase difference will be ($\eta e^{2i\delta} - 1$) with $\eta \leqslant 1$. η will depend on the angular momentum J as is the case for the phase shift δ. The elastic (σ_{el}) inelastic (σ_{inel}) and total (σ_{tot}) cross-sections are then given by the following expressions.



$$\sigma_{el} = \sum_J{}' \pi (2J+1) \lambda\!\!\!^{-2} \left| 1 - \eta_J e^{2i\delta} \right|^2$$

$$\sigma_{inel} = \sum_J \pi (2J+1)^2 \lambda\!\!\!^{-2} (1 - \eta_J^2)$$

$$\sigma_{tot} = \sum{}' 2\pi (2J+1) \lambda\!\!\!^{-2} (1 - \eta_J \cos 2\delta)$$

If we have a resonance, then $\delta = \frac{\pi}{2}$ & we have

$$\sigma_{el} = \sum{}' \pi (2J+1) \lambda\!\!\!^{-2} (1+\eta_J)^2$$

$$\sigma_{tot} = \sum{}' 2\pi (2J+1) \lambda\!\!\!^{-2} (1+\eta_J)$$

So that $\dfrac{\sigma_{el}}{\sigma_{tot}} = \dfrac{1+\eta_J}{2}$

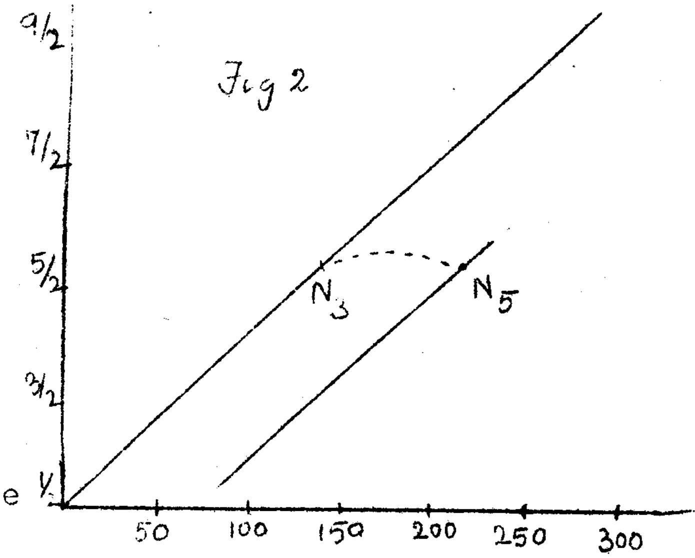
and $$\sigma_{tot}^{resonance} = \sum_J{}' 4\pi \lambda\!\!\!^{-2} (2J+1) \frac{\sigma_{el}}{\sigma_{tot}}$$

or $$\sigma_{tot}^{J} \simeq 4\pi \lambda\!\!\!^{-2} (2J+1) \left(\frac{\sigma_{el}}{\sigma_{tot}} \right)_J$$

$$\leq 4\pi \lambda\!\!\!^{-2} (2J+1)$$

Experimentally it has been observed that for N_5 $(\sigma^J_{tot}) \leq 10$ mb and for N_6 , $\sigma^J_{tot} \leq 5$ mb. . So we should expect the the resonance N_5 to have J value >1/2 and for N_6, J ≧ 1/2.

A way of determining the spins of these resonances is to draw the Regge curves. It has been observed that if the squares of the masses of the resonances (in units of the square of the mass of the pion) are plotted against spins, then N and N_3 lie on a straight line making a slope of 1/50 with the x-axis.



Now if we assume the spin of N_5 to be 5/2, we see from Fig.2 that the line bends i.e. the spin is coming down with increasing mass. Now Wignes* has shown that for potential scattering $d\delta/dp$ where δ is the phase shift and p the momentum of the incident particle is a measure of the time of collision. He has further shown that $\frac{d\delta}{dp} > -a$ where a is the distance beyond which the potential vanishes. Sakita and Bincer** have extended Wigner's treatment using dispersion relations technique and found that

$$\frac{d\delta}{dp} > -\left(3/8 + n\right)\frac{1}{2\mu} \quad ;$$

* E.P.Wigner, Physics, Rev. 98, 145 (1955)

** A.M.Bincer and B.Sakita, Phys. Rev. 129, 1905 (1963) where n is the number of zero points on the left-hand cut in the complex energy plane for π-N scattering

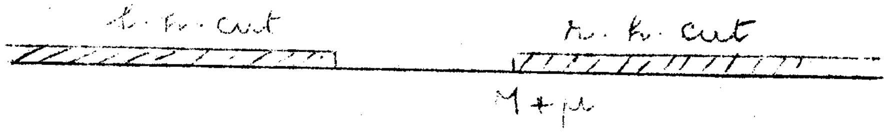


Fig.3

The width of the resonance is found to be quite big: $\Gamma \gtrsim 1.5$ Bev in gross disagreement with the experimental value of $\sim$ .2 Bev. So this mode of approaching the point N_5 onthe Regge curve is ruled out.

The second possibility is to allow the N_5 on parallel to the NN_3 line which would then necessitate a resonance in the p 1/2 state. Some experiment evidence has been adduced for its existence.

We shall now turn from experimental considerations to what the theorists have done by way of explaining these resonances. In the S-matrix apporach one is interested in the location and strength of the singularities in the complex energy (or momentum transfer) plane. In the complex W $(= \sqrt{s})$ plane the position of the poles and branch cuts would appear as shown in Fig.4. The matrix element can be written as

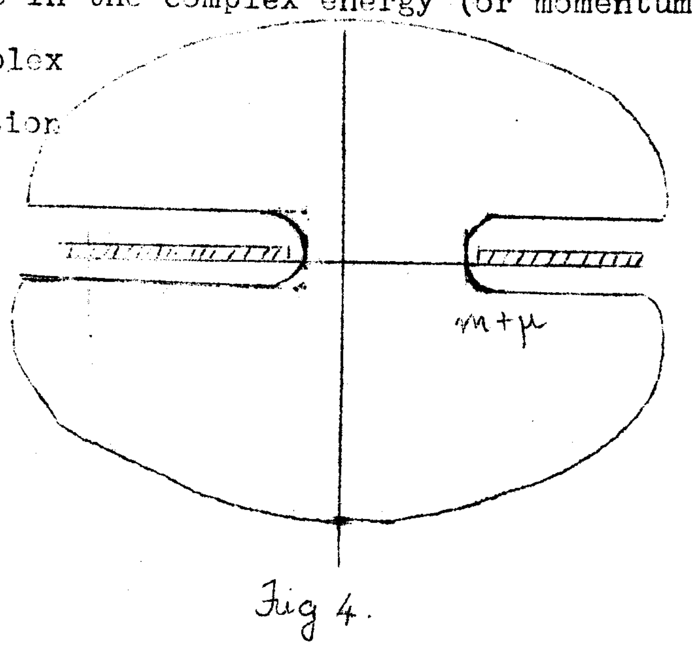


Fig 4.

$$f(W) = \frac{c}{W-m} + \frac{1}{\pi}\int_{m+\mu}^{\infty} \frac{f(W')}{W'-W}\, dW'$$

The unitarity condition gives as

$$\mathrm{Im}\, f = f^* \rho f$$

$$\mathrm{Im}\,(1/f) = \mathrm{Im}\left(\frac{f^*\rho}{f^*\rho f}\right) = -i\,\frac{\mathrm{Im}\, f\, \rho}{f^* \rho f} = -i\rho$$

ρ is the phase space factor which for the pion-nucleon intermediate state is proportional to the centre of mass momentum q i.e.

$$\propto \sqrt{W - (m+\mu)}$$

Now if we write

$$\frac{1}{f} = \frac{1}{A} - i\rho \qquad \text{or} \qquad f = \frac{A}{1 - i\rho A}$$

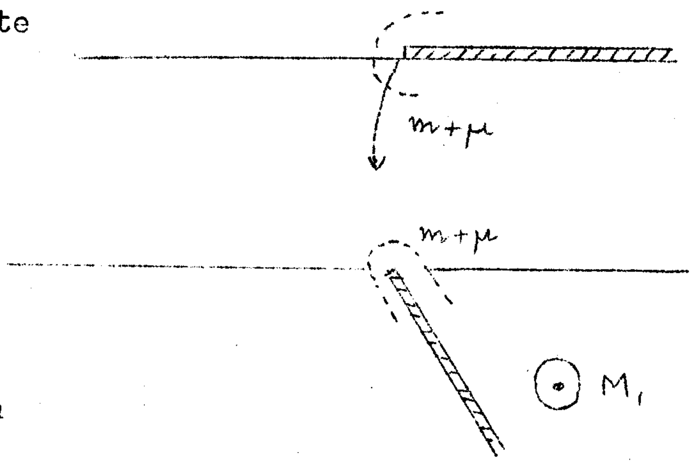
where A is any smooth function of W, then both the right-hand cut and the unitarity condition are taken care of. Near the threshold $M+\mu$ A can be expanded in the form

$$A = A_0 + A_1 q^2 + A_2 q^4 + \cdots$$

Below the threshold $\rho = i|q|$; Hence if $1 - i\rho A = 1 + |q| A_0 = 0$ which will be the case for $A_0 < 0$ we have a pole corresponding to a bound state.

To obtain the singularities corresponding to the resonances we have to pass through the right-hand cut into the second Riemann sheet. As is well-known poles in these sheets correspond to resonances and branch cuts to some peculiar anomalies in the cross-sections discussed by Tripp and Perez-Mender (see below)

The procedure to locate a complex pole in the second Riemann sheet is to swivel the branch cut which we had originally taken along the positive real axis about the branch point $(W = M + \mu)$



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until it crossed a (possible) pole as M_1. (See Fig.5) Now $\rho \to -\rho$ so that the partial wave amplitude in the second sheet is

$$f = \frac{A}{1 + i\rho A}$$

If $1 + i\rho A = 0$ for $W = M_1$ (and M_1^*) then

$$1 + i\rho A \propto (W - M_1)$$

Hence near the complex pole we can write the cross-section as

$$\sigma \propto \frac{1}{|W - M_1|^2} = \frac{1}{[W - W_R]^2 + \left(\frac{\Gamma}{2}\right)^2}$$

where W_R and $\Gamma/2$ are the real and imaginary parts of M_1, $M_1 = W_R - i\Gamma/2$. This is of the familiar Breit-Wigner form so that M_1 indeed represents the position of a resonance.

The above discussion is a simplified picture of the realistic case. In the case of pion-nucleon scattering we have to consider in addition to the graphs corresponding to the poles and cuts in the s-variable (Fig.6) also the poles and cuts in the u variable (Fig.7) where

$$u = 2m^2 + 2\mu^2 - s - t$$

t being the momentum transfer.

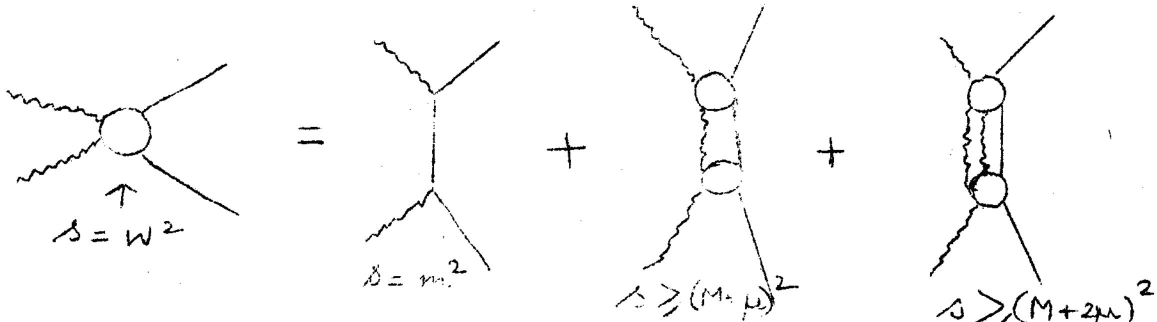

Fig 6

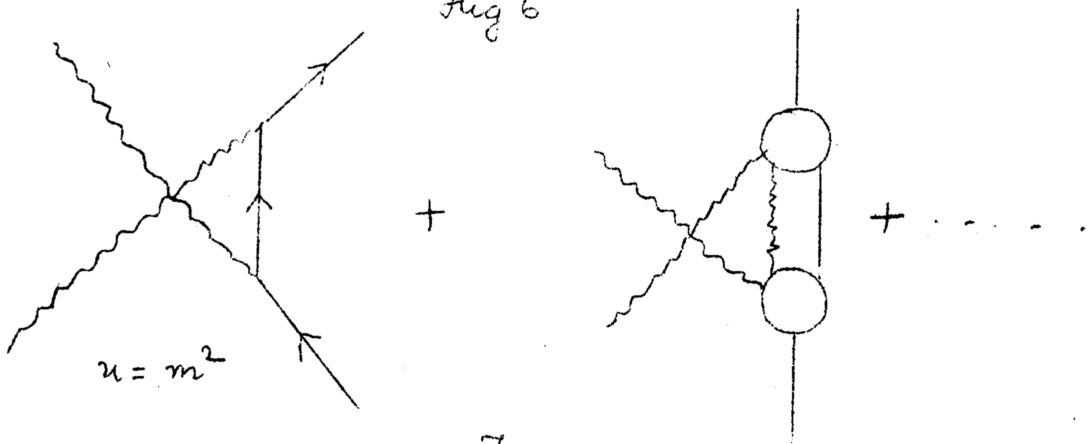

Fig 7.

The singularities of the partial wave amplitudes for pion-nucleon scattering have been discussed by Frazer and Fulco* and by Frautschi and Walecka** among others. In addition to the unitarity cuts from $m+\mu$ to ∞ (the cut from $-m-\mu$ to $-\infty$ can be re-expressed in terms of the positive cut using a symmetry relation between the partial wave amplitudes) there will be other singularities arising from the vanishing of the denominators containing the and t variables such as short cuts from $-(m^2+2\mu^2)^{1/2} \leq W \leq -m+\frac{1}{m}$

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*W.R.Frazer and J.R.Fulco, Phys. Rev. 119, 1420 (1960)

**S.C.Frautschi and J.D.Walecka, Phys.Rev.120, 1486 (1960) and $-m+\mu \leqslant W \leqslant m-\mu$. cut along the entire imaginary axis and a circular cut about the sign . having a radius $r = \sqrt{m^2 - \mu^2}$ (see Fig.8)

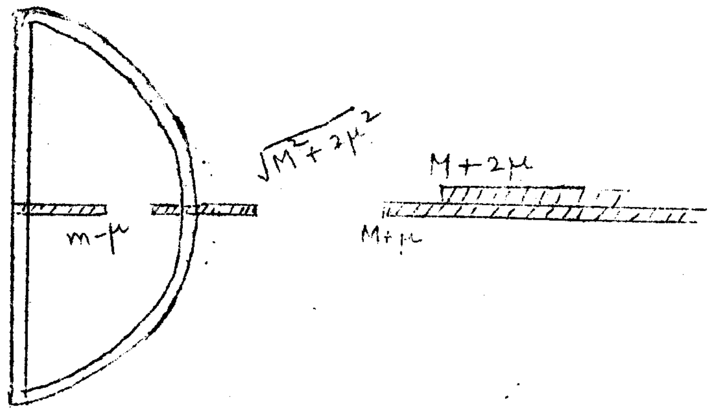

Fig.8

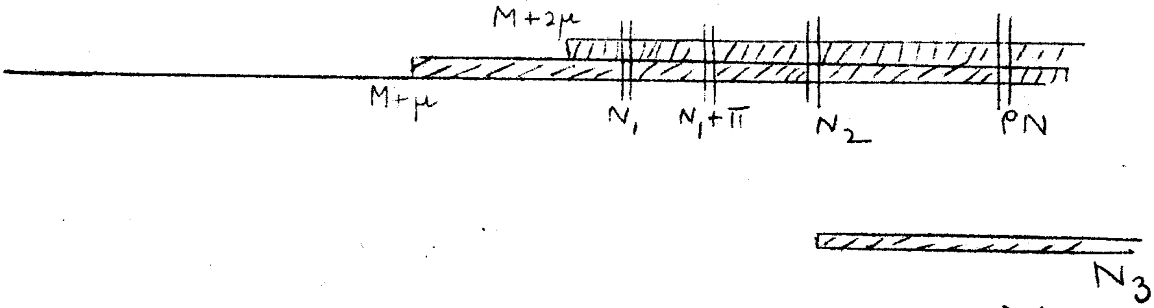

Fig.9

In addition to the above singularities there may be an additional pole and cut suggested by pe..rls.* If If we consider the process, $\pi + N_1 \rightarrow \pi + N_1$ where the resonance N_1 is taken to be an unstable particle, then consideration of Fig.9a shows us that isobar being unstable can decay into a pion and nucleon which subsequently absorbs another pion.

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* R.E.Peierls, Phys.Rev. Letters 6, 641 (1961).

This represents a real process and the corresponding pole shows up in the physical region for the momentum and scattering angle. The contribution from this pole shows a resonance-like behaviour which can produce a strong effect in pion-nucleon scattering via the reaction Fig. 9b.

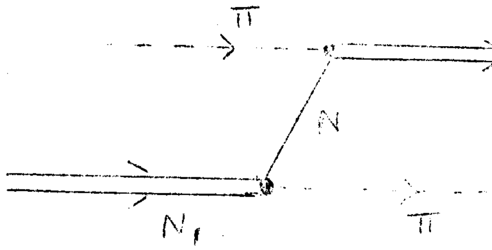


Fig. 9a

In the partial wave analysis the Peierls pole may lead to a cut which we have drawn roughly between the positions of the resonance N_2 and N_3. One might think of N_2 and N_3. as poles which

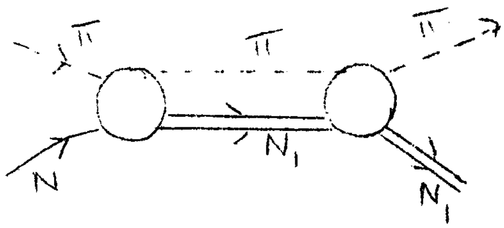


Fig 9b

replace the cut. If the PN channel (the threshold for which lies in this region.) contributes a negative A to the amplitude then there is the possibility of a pole.

The situation regarding the angular momentum state of N_2 is not clear. Recently the beam strength of the Berkeley bevatron has been increased by a factor 10 so that formation of the resonant state N_1 can be isolated and a study made of the reaction

$\pi \; N \rightarrow \pi \; N_1$ (Fig. 10). The angle χ is measured in the rest system of N_1. The angular distribution of πN_1 formation and the decay

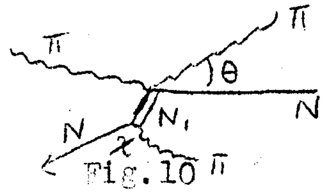


Fig. 10

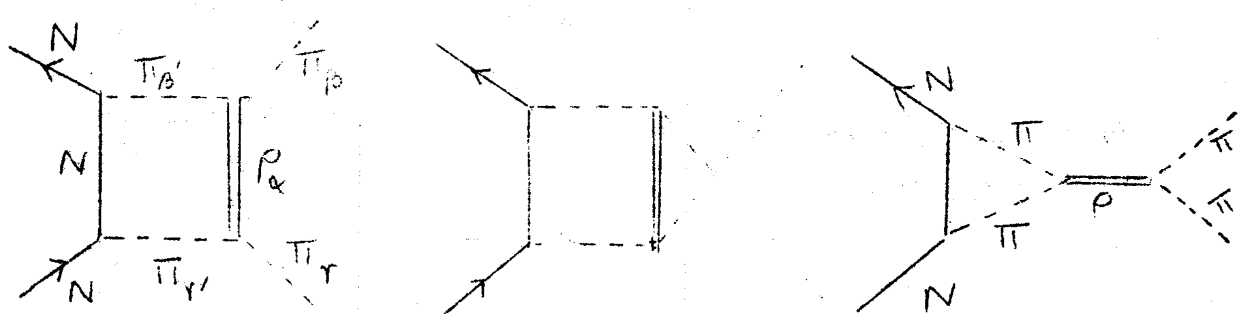
distribution of N_1 can be measured.

The following table should be helpful.

πN	πN_1	$d\sigma/d\Omega$	$d\sigma/d\chi$
$S_{1/2} \rightarrow$	$d_{1/2}$	const	$1 + 3\cos^2\chi$
$P_{1/2} \rightarrow$	$P_{1/2}$	"	"
$d_{3/2} \rightarrow$	$d_{3/2}$	"	$(1+3\cos^2\chi)(1+3\cos^2\theta) + 9\sin^2\chi\sin^2\theta$
$P_{3/2} \rightarrow$	$P_{3/2}$	$7 - 6\cos^2\theta$	
$f_{5/2} \rightarrow$	$F_{5/2}$	$1 + 2\cos^2\theta$	

This and a similar tables for the process $\pi N \rightarrow \rho N$ should be helpful to the experiments.

Now we shall describe briefly the work we* have done using perturbation theory which reproduces many of the features of the higher resonances in the pion-nucleon system. The main feature of this approach is the importance of the ρ meson-nucleon intermediate state for πN scattering (Fig. 11a)



11(a) 11(b) 11(c)

* K. Itabashi, M Kato, K. Nakagawa & G. Takeda, Progress of Theoret. Phys, 24, 529 (1960)

The $\rho\pi\pi$ interaction Hamiltonian can be written as

$$H_{\rho\pi\pi} = F \sum_{\alpha,\beta,\gamma}$$

where α β, γ are the isotopic spin indices and χ φ ... ρ and π field operators . F is the coupling constant and its large value ($F^2/4\pi \approx 5$) will ultimates found to be responsible for the N_2 and N_3 resonances. The procedure to find F is to calculate the pion-pion scattering amplitude in perturbation theory using the above Hamiltonian and comparing it with the one-level formula of Frazer and Fulco.

A calculation of the imaginary part of the p-wave scattering amplitude $f_{11}(p)$ in the center of mass system gives

$$\mathcal{I}_m \; f_{11}(b) = \frac{F^2}{6} \frac{p^3}{W} \delta(m_\rho^2 - W^2)$$

while the one-level formula, under the assumption that the width of the resonance is small gives

$$\mathcal{I}_m \; f_{11}\, p \approx \pi \Gamma W_R \delta(W_R^2 - W^2)$$
$$\approx \frac{8\pi \Gamma' p^3}{W} \delta(W_R^2 - W^2)$$

where according to Frazer and Fulo $\Gamma' \approx 0.4$ and $W_R \approx 600\,Mev$ We therefore have $F^2/4\pi \approx 12\,\Gamma' \approx 5$ and $W_R \approx m_\rho \approx 600\,Mev.$

We shall now show that a strong $\rho\pi\pi$ interaction $H_{\rho\pi\pi}$ gives a rather strong attractive potential of the pion-nucleon system in the T = 1/2 state for $W \approx m_\rho$ and the observed second and third resonances are identified as those in the T = 1/2, $d = 3/2$ and $T = 1/2$ f $5/2$ states respectively.

The scattering amplitude corresponding to Fig.11a becomes very large for $W \approx m_\rho$ since energies of intermediate ρN states can be very nearly equal to the initial energy W of the colliding system. For Figs. 11b and 11c however the energy differences between the intermediate and initial states are large for $W \gtrsim m_\rho$ and at least of order W so that they give only a small contribution to the πN force. Further if, identify the nature of the potential with the sign of the energy denominator, $W_i - W_n$ then we should expect an attractive force in the first case.

Now that we expect a large attractive force at $W = m_\rho$ to come out from Fig.11a, let us find out the isotopic spins and spins of the higher resonances. The isotopic spin dependence of the $\pi - N$ potential is given by

$$\langle \pi_\beta | V | \pi_\gamma \rangle \sim \sum_{\alpha, \beta', \gamma'} \epsilon_{\alpha\beta\beta'} \tau_{\beta'} \epsilon_{\alpha\gamma\gamma'} \tau_{\gamma'}$$

$$= 2\delta_{\beta\gamma} + \frac{1}{2}[\tau_\beta, \tau_\gamma] = 4P_{1/2} + P_{3/2}$$

where the P's are the isotopic spin projection operators. We thus see that the attractive $\pi - N$ potential is 4 times larger in the T = 1/2 state than in the T = 3/2 state even in the lowest order perturbation theory.

The angular momentum and parity of the large attractive forces can be decided by evaluating the $\pi N \to \rho N$ transition matrix element for ρN states of small $\vec{q}$ (Fig. 12)

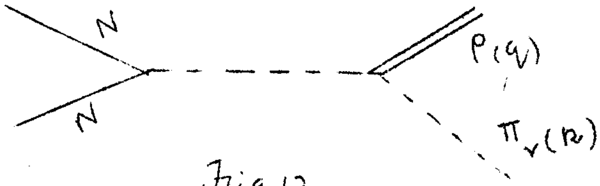


Fig 12.

For the $NN\pi$ vertex the static approximation can be used. The result is that if only terms independent of $\vec{q}$ are retained so that ρ is in an orbital angular momentum state $\ell_\rho = 0$ the amplitude is found to be proportional to

$$\left[P(s_{1/2} \to s_{1/2}) + P(d_{3/2} - s_{3/2})\right]$$

Similarly if terms linear in $\vec{q}$ are also retained so that ℓ_ρ can be 1, the strengths of the attractive $\pi - N$ potential is found to be in the ratio (8/3:1:10) for the states $f_{5/2}$, $p_{3/2}$ and $p_{1/2}$ going into ρN intermediate states. Thus it is natural to associate the N_2 resonance with $d_{3/2}$ and N_3 with $f_{5/2}$. The possibility of a $p_{1/2}$ resonance at about the same energy as N_3, which was mentioned earlier, is also explained.

Finally the broad bump in the total $\pi^+ - p$ cross-section observed at $W \approx$ 940 Mev could be interpreted as being due to a maximum in the inelastic cross-sections $\pi N \to \rho N \to 3\pi N$ due to final state interactions among the produced pions and nucleon. The isotopic spin dependence of the $\pi N \to \rho N$, transition amplitudes is independent of the details of the choice of the interaction, $H_{NN_1\pi}$ and the relative magnitudes of the various $\pi N \to \rho N_1$ amplitudes are

$$f(\pi^+ p \to \rho^+ N_1^+) : f(\pi^+ p \to \rho^0 N_1^{++}) : f(\pi^- p \to \rho^- N_1^+) : f(\pi^- p \to \rho^0 N_1^0) = \sqrt{2/3} : 1 : \sqrt{2/3} : \sqrt{1/2}$$

so that the inelastic scattering cross-section for $\pi N \to \rho N_1$ in T = 3/2 state is larger than that in T = 1/2 state by a factor 5/2, thus indicating that the maximum shows up only in $\pi^+ p$ collisions.

In the calculation, interaction of the Born term like Fig.13. are included Fig.14 represents the total cross-sections curve drawn on the basis of three theoretical values.

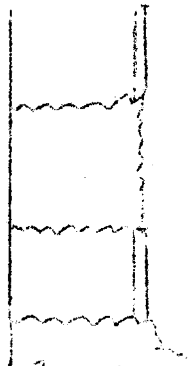


Fig 13

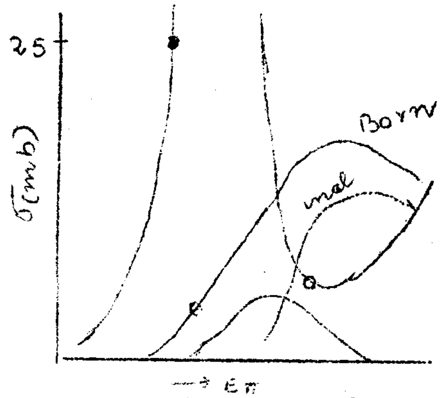


Fig 14

As is evident, the cross-section is heading for a maximum above the value, 25 mb. The phase shifts corresponding to various values of   are given in the table below.

E_π	3	4	5
δ	38.5°	174°	154°
η	0.94	0.96	0.95

Finally, we have only time to mention briefly an anomaly, in the cross-section for the process of pion production in pion-nucleon collision in the energy region $M(2\pi) = 400 - 500$ Mev. with I = 0. The anomaly was noticed by Tripp and Perez-Mendez independently. It is higher than the ABC (Anomaly (about 310 Mev) and perhaps swamps it. We can expect it to arise from an infinite anomalous singularity (discussed by Landshoff and Treiman*) corresponding to a triangle diagram of the kind, Fig 15

The Dalitz plot for the even is given in Fig. 16

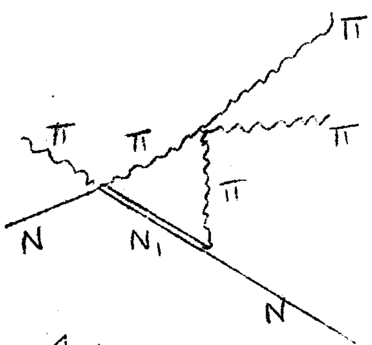


Fig 15

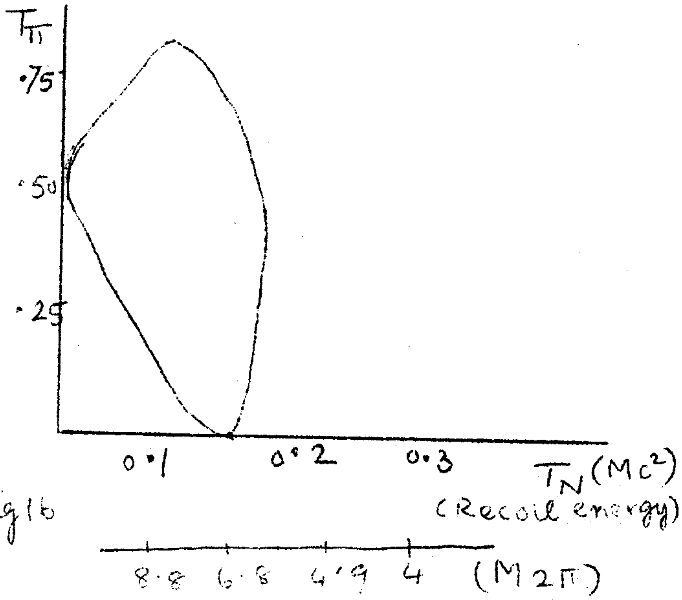


Fig 16

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* P.V.Landshoff and S.B.Treiman, Phys. Rev. 127, 649 (1962). See also R.Aaron, Phys. Rev. Letters 10, 32 (1963) for the possibility of a similar anomaly in the reaction

$$K^- + p \rightarrow \Sigma + \pi + \pi$$

An alternative explanation might be to consider the one pion exchange graph (Fig.) and the diagram

Fig.

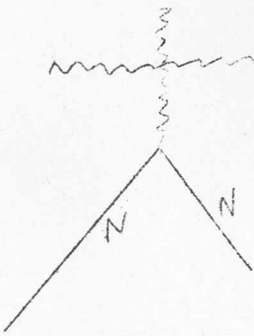


Fig 17

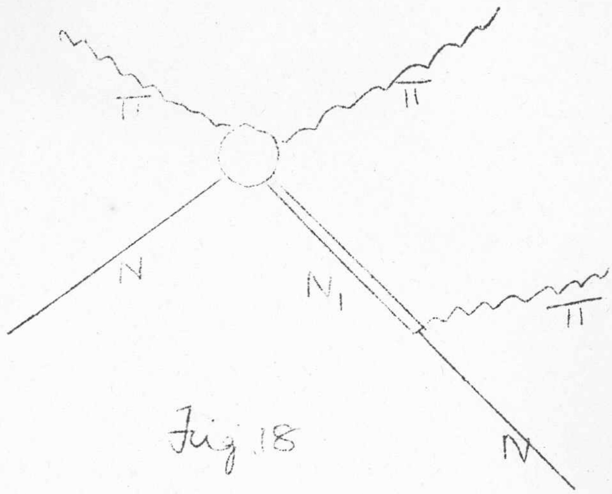


Fig 18

The corresponding matrix elements are respectively

$$M_1 \propto \frac{\sigma \cdot \Delta}{\Delta^2+\mu^2} = \frac{|\Delta|}{\Delta^2+\mu^2}$$

Spin nonflip $\propto \cos\theta_{lab}$

Spin-flip $\propto \sin\theta_{lab}$

$$M_2 \propto (\sigma_2\, q_2') - \frac{1}{2}(\sigma\, q^c) = \frac{2}{3}\sigma_\pi q_2^{\,c} - \frac{1}{3}\sigma_\pi q_1^{\,c}$$

Spin-nonflip $\propto \frac{2}{3}\cos\theta_c$

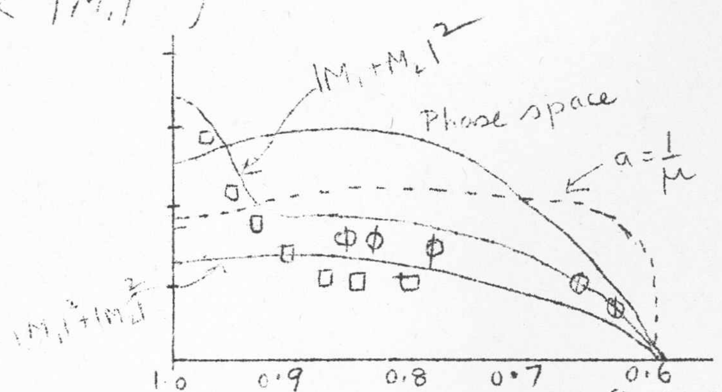
Spin-flip $\propto -\frac{1}{3}\sin\theta_c$

$d\sigma \propto |M_1|^2 + |M_2|^2 +$ Interference term.

where (Interference term) $\propto \cos\theta_{lab}\cos\theta_c$

$$- \frac{1}{2}\sin\theta_{lab}\sin\theta_c$$

The phase space $\left(|M_2|^2 < |M_1|^2\right)$ factors, $|M_1|^2 + |M_2|^2$ and $|M_1 + M_2|^2$ are plotted along with the experimental points.



ϕ corresponds to the reaction $\pi^- p \rightarrow \pi^+\pi^- n$

$\square$ corresponds to the reaction $\pi^- p \rightarrow \pi^0\pi^0 n$

None of the above explanations regarding the anomaly seem to be satisfactory.

THE INSTITUTE OF MATHEMATICAL SCIENCES

MADRAS - 4 (India)

SOME TOPICS IN THE THEORY OF WEAK INTERACTIONS

(Seminar Lectures)

By

K.Venkatesan*

*Member, The Institute of Mathematical Sciences, Madras-4.

CONTENTS.

SYMMETRIES OF STRONG AND WEAK INTERACTIONS

(Summary of Gursey's work on four-dimenstional isospin rotation)

Generalizing his earlier work on the Pauli-Gursey group, Gursey develops a scheme for rotation in four-dimensional isospin space such that the group G_4 associated with it together with the hypercharge gauge group H contains charge independence, strangeness selection rules and the empirical symmetries of weak interactions as various subgroups. The expectation is that if G_4 leaves (in an approximate way) the strong interactions invariant, the renormalization effects will be small for any interaction invariant under a subgroup of G_4 and hence also for weak interactions which are found to be invariant under such subgroups. The removal of the doublet symmetry by a doublet 'perturbation' (DP) should make itself felt both for strong and weak interactions.

To arrive at a four-dimensional isospin rotation as a product of two three-dimensional rotations it is necessary that we have in addition to the usual isospin rotation $e^{i\vec{\tau}\cdot\vec{\omega}}$ (for the pion-nucleon system) another rotation. Let us start with the new rotation defined by

$$G_3' : \quad \psi \rightarrow e^{i\gamma_5\vec{\tau}\cdot\vec{\omega}}\psi \qquad (1)$$

which has the same form as the Dyson-Foldy transformation which converts the pseudoscalar pion-nucleon coupling to a psendo-vector one , The free-nucleon equation is not left invariant under (1) if the mass $\neq 0$. The transformed equation is

$$\gamma_\mu \partial_\mu \psi = m \exp[2i\gamma_5 \vec{\tau}.\vec{\omega}]\psi \tag{2}$$

If we choose $\vec{\omega} = f\vec{\varphi} = \frac{g\vec{\varphi}}{2m}$ where f and g are the pseudo-vector and pseudo-scalar coupling constants respectively (if we identify φ with the pion field) and expand in powers of f we obtain

$$\begin{aligned}\gamma_\mu \partial_\mu \psi &= m \exp[2if\gamma_5 \vec{\tau}.\vec{\varphi}]\psi \\ &= m\psi + ig\gamma_5 \vec{\tau}.\vec{\varphi}\psi - \frac{g^2\varphi^2}{2m}\psi + \end{aligned} \tag{3}$$

which can be split up into the two equations

$$\begin{aligned}\gamma_\mu \partial_\mu \psi_L &= m \bar{\Phi} \psi_R \\ \gamma_\mu \partial_\mu \psi_R &= m \bar{\Phi} \psi_L\end{aligned} \tag{4}$$

where

$$\begin{aligned}\psi_L &= \tfrac{1}{2}(1+\gamma_5)\psi, \quad \psi_R = \tfrac{1}{2}(1-\gamma_5)\psi \\ \Phi &= \exp[2if\vec{\tau}.\vec{\varphi}], \quad \bar{\Phi} = \exp[-2if\vec{\tau}.\vec{\varphi}] \\ \Phi\bar{\Phi} &= \bar{\Phi}\Phi = 1\end{aligned} \tag{5}$$

Thus the operation (1) has led to a system of left-handed and righ handed nucleons and a generalized boson field. We can perform two independent isospin rotations on these thus~~,~~ defining two groups,

$$\begin{aligned}G_3^{(1)} :\ \psi_L &\to \exp[i\vec{\tau}.\vec{\mu}]\psi_L \\ \Phi &\to \Phi \exp[-i\vec{\tau}.\vec{\mu}], \quad \psi_R \to \psi_R\end{aligned} \tag{6}$$

and

$$G_3^{(2)}: \psi_R \to \exp[i\vec{\tau}\cdot\vec{\nu}]\psi_R, \quad \Phi \to \exp[i\vec{\tau}\cdot\vec{\nu}]\Phi; \quad \psi_L \to \psi_L \tag{7}$$

The product of these two defines the four-dimensional isospin rotation group,

$$G_4: \psi' = \exp\left[i\left\{\frac{1+\gamma_5}{2}\vec{\tau}\cdot\vec{\mu} + \frac{1-\gamma_5}{2}\vec{\tau}\cdot\vec{\nu}\right\}\right]\psi$$
$$\Phi' = \exp[i\,2f\,\vec{\tau}\cdot\vec{\varphi}'] = \exp[i\vec{\tau}\cdot\vec{\nu}]\,\Phi\,\exp[-i\vec{\tau}\cdot\vec{\mu}] \tag{8}$$

We readily see that the isospin rotation group (in 3 dimensions) G_3 is obtained by taking $\vec{\mu} = \vec{\nu} = \vec{\omega}$ and the subgroup G_3' (eqn. (1)) by taking $\vec{\mu} = -\vec{\nu} = \vec{\omega}'$. Further $G_3^{(1)}$ and $G_3^{(2)}$ do not commute with either the parity operator P (the γ_4 matrix) or the charge-conjugation operator C (γ_2) separately, but commute with CP. Thus we expect these subgroups to be associated with the weak interactions.

We can make the kinetic energy part of the pion field, $(\partial_\mu \varphi)^2$ invariant under G_4 by the substitution.

$$(\partial_\mu \varphi)^2 \to \frac{1}{4} f^2 \,\mathrm{Tr}\left[(\partial_\mu \Phi)(\partial_\mu \bar{\Phi})\right]^2 \tag{9}$$

since under (8) the right hand side of (9) will undergo a unitary transformation which leaves the trace invariant. But the mass term, φ^2 (we are taking the pion mass to be unity) is not left invariant under (8). Thus in the present doublet approximation the pion mass must be zero. But since DP is invariant under G_3, it can generate a pion mass which will be strongly renormalized by the pion-pion interactions.

The γ_5-transformation (applicable to fermions with non-zero mass also) arises naturally in this model. If we take the discrete subgroup of G_4 by choosing $\nu_3 = \pi$, $\nu_1 = \nu_2 = 0$ we obtain the transformations.

$$\psi \rightarrow \gamma_5 \psi \quad , \quad \Phi \rightarrow - \Phi \tag{10}$$

which leave the equations (4) invariant. Since in the doublet approximation other strong interactions will not disturb G_4 invariance, the invariance (10) will remain valid in this approximation. The effect of (DP) will then be to induce simultaneously a pion mass, a split in the baryon masses, separation of the $\Lambda - \Sigma$ system and correction to the $V-A$ form of the weak interaction.

We shall now proceed to give group representations to the other baryons and bosons; labelling the representation by two-numbers m,n corresponding to the parameters $\vec{\mu}, \vec{\nu}$ of the groups $G_3^{(1)}$ and $G_3^{(2)}$ respectively. The nucleons and the Φ-field have the representation

$$\psi_L : (\tfrac{1}{2}, 0) \; ; \; \psi_R : (0, \tfrac{1}{2}) \; ; \; \Phi : (\tfrac{1}{2}, \tfrac{1}{2}) \tag{11}$$

from (6) and (7). The cascade doublet has the representation

$$\Xi_L : (0, \tfrac{1}{2}) \; ; \quad \Xi_R : (\tfrac{1}{2}, 0) \tag{12}$$

so that it will interact with the pion according to the equation

$$\gamma_\mu \partial_\mu \Psi_\Xi = m_\Xi \exp[-2 i f \gamma_5 \vec{\tau}\cdot\vec{\varphi}] \Psi_\Xi \tag{13}$$

The Λ and Σ which are clubbed together in the doublet approximation are given the representation

$$Y_L : (\tfrac{1}{2}, \tfrac{1}{2}) \; ; \; Y_R = \mp \Lambda_R + i \vec{\tau}\cdot\vec{\Sigma}_R$$

$$\Lambda_R : (0,0) \quad , \quad \Sigma_R : (0,1) \tag{14}$$

so that they transform according to

$$Y_L \to \exp[i \vec{\tau}\cdot\vec{\mu}] Y_L \exp[-i \vec{\tau}\cdot\vec{\nu}]$$

$$Y_R \to \exp[i \vec{\tau}\cdot\vec{\nu}] Y_R \exp[-i \vec{\tau}\cdot\vec{\nu}] \tag{15}$$

and obey the equations

$$\gamma_\mu \partial_\mu Y_L = m_Y \Phi Y_R$$

$$\gamma_\mu \partial_\mu Y_R = m_Y \bar{\Phi} Y_L \tag{16}$$

which are left invariant under G_4 .

The K-meson doublet is assigned the representation $(0,\frac{1}{2})$ There are no representations corresponding to (1,0) for baryons and $(\frac{1}{2},0)$ for the bosons. They are missing components just as the neutrino with the other spin is absent, in the two-neutrino theory.

Since only the nucleon, cascade and K doublets have hypercharge quantum number different from zero; we can write the hypercharge gauge transformation as

$$H: \psi \to \exp[iu]\,\psi, \quad \Xi \to \exp[-iu]\,\Xi \qquad (17)$$
$$K \to \exp[iu]\,K$$

with the other fields unaffected.

For strong interactions we can enlarge the group G_4 to $G_4 \times H$ which has as subgroups H, G_3, $G_3^{(1)}$ and $G_3^{(2)}$. When (DP) is switched on, G_4 invariance ~~under~~ is lost and the strong interactions are invariant only under groups H and G_3 (which commute with C and P separately) whereas the weak interactions are invariant under only $G_3^{(1)}$ or $G_3^{(2)}$. There can be two kinds of weak interactions depending on whether they are invariant under or . Since charge must also be conserved we can enlarge $G_3^{(1)}$ and $G_3^{(2)}$ to 4 parameter unitary groups which will also describe charge conservation. Consider, e.g., the unitary subgroup $U^{(1)}$ of $(G_4 \times H)$ obtained by choosing $\nu_1 = \nu_2 = 0$, $\nu_3 = u$ ~~and~~ . The transformation properties of the various fields under $U^{(1)}$ are

$$
\begin{aligned}
U^{(1)}:\quad \psi_L &\to \exp[iu]\exp[i\vec{\tau}\cdot\vec{\mu}]\,\psi_L \\
\psi_R &\to \exp[i(1+\tau_3)u]\,\psi_R \\
\Xi_L &\to \exp[-i(1-\tau_3)u]\,\Xi_L \\
\Xi_R &\to \exp[-iu]\exp[i\vec{\tau}\cdot\vec{\mu}]\,\Xi_R \\
Y_L &\to \exp[i\vec{\tau}\cdot\vec{\mu}]\,Y_L\exp[-i\tau_3 u] \\
Y_R &\to \exp[i\tau_3 u]\,Y_R\exp[-i\tau_3 u]
\end{aligned}
$$

$$\bar{\Phi} \rightarrow \exp[i\vec{\tau}\cdot\vec{\mu}]\,\bar{\Phi}\,\exp[-i\tau_3 u]$$
$$K \rightarrow \exp[iu]\,\exp[i\tau_3 u]\,K \qquad (18)$$

If we take the baryon part of the leptonic decay processes to transform as in (18), we see that the leptonic current $j_\mu = (\bar{e}+\bar{\mu})\gamma_\mu \nu$ must transform as

$$U^{(1)}:\ j_\mu \longrightarrow \exp[2iu]\,j_\mu$$

We can construct various Lagrangians which are invariant under $U^{(1)}$. For instance

$$L_K = aGf^{-3}\,\mathrm{Tr}\{(\bar{\Phi}\partial_\mu\bar{\Phi})[\partial_\mu\theta + K_1^0(\bar{\Phi}\partial_\mu\bar{\Phi})]\} + h.c. \qquad (19)$$

where θ is the matrix $\begin{pmatrix} \bar{K}_0 & K^+ \\ -K^- & K^0 \end{pmatrix} = K_1^0 + i\vec{\tau}\cdot\vec{K}$

Expanding $\bar{\Phi}$ we obtain

$$L_K \approx 4aGf^{-1}\Big[K_1^0(\partial_\mu\varphi)^2 + \tfrac{1}{2}f^{-1}(\partial_\mu\vec{K})\cdot(\partial_\mu\vec{\varphi}) - (\partial_\mu\vec{K})(\vec{\varphi}\cdot\partial_\mu\vec{\varphi}) + \tfrac{1}{3}f(\partial_\mu\vec{K})\cdot(\varphi^2\partial_\mu\vec{\varphi} - 2\vec{\varphi}\,\partial_\mu\varphi^2)\Big] \qquad (20)$$

The first and fourth terms give the 2 pion decay of K_1^0 and the 3-pion decay modes of K_2^0 and K^+. The $\Delta I = \frac{1}{2}$ is also seen to hold.

Another interaction left invariant under $U^{(1)}$ is

$$L_Y = a' G f^{-2}\, \mathrm{Tr} \left\{ (\bar{\Phi}\, \partial_\mu \Phi)\, Y_L \begin{pmatrix} 0 & 0 \\ \bar{p}_L & \bar{n}_L \end{pmatrix} \gamma_\mu + (\bar{\Phi}\, \partial_\mu \bar{\Phi}) \left[Y'_L \begin{pmatrix} \bar{n}_L & 0 \\ -\bar{p}_L & 0 \end{pmatrix} + Y'_R \begin{pmatrix} 0 & 0 \\ \bar{p}_R & \bar{n}_R \end{pmatrix} + Y_R \begin{pmatrix} \bar{n}_R & 0 \\ -\bar{p}_R & 0 \end{pmatrix} \right] \gamma_\mu \right\} + h.c. \tag{21}$$

which an expansion gives

$$\begin{aligned} L_Y \approx 4 a' G f^{-2} \{ & (\sqrt{2}\, \bar{p}_L \gamma_\mu \Lambda_L \partial_\mu \varphi^+ - \bar{n}_L \gamma_\mu \Lambda_L \partial_\mu \varphi^0) + (\bar{n}_L \gamma_\mu \Sigma_L^0 \partial_\mu \varphi^0 \\ & - \sqrt{2}\, \bar{p}_R \gamma_\mu \Sigma_R^0 \partial_\mu \varphi^+) + \sqrt{2}\, \bar{p}_R \gamma_\mu \Sigma_R^+ \partial_\mu \varphi^0 \\ & + (\bar{n}_L \gamma_\mu \Sigma_L^- - \bar{n}_R \gamma_\mu \Sigma_R^-)\, \partial_\mu \varphi^+ \\ & + (\bar{n}_L \gamma_\mu \Sigma_L^+ + \bar{n}_R \gamma_\mu \Sigma_R^+)\, \partial_\mu \varphi^- \\ & + h.c. \} \end{aligned} \tag{22}$$

The first term which leads to the $\Lambda \to p + \pi^-$ decay reduces (as regards its spatial property) to the form $\gamma_\mu(1+\gamma_5)$ as also the term giving the cecay $\Sigma^+ \to p + \pi^0$. The combination of the two terms corresponding to $\Sigma^- \to n + \pi^-$ however gives a pure $\gamma_\mu \gamma_5$ i.e. axial vector interaction and similarly the two terms corresponding to $\Sigma^+ \to n + \pi^+$ give a pure γ_μ (vector) interaction. Thus the asymmetries observed namely that the first two decays mentioned show asymmetry whereas the lost two have almost complete parity purity is explained.

References

1. F. Gursey, Nuovo Cimento, 7, 411 (1958)

t being the momentum transfer.

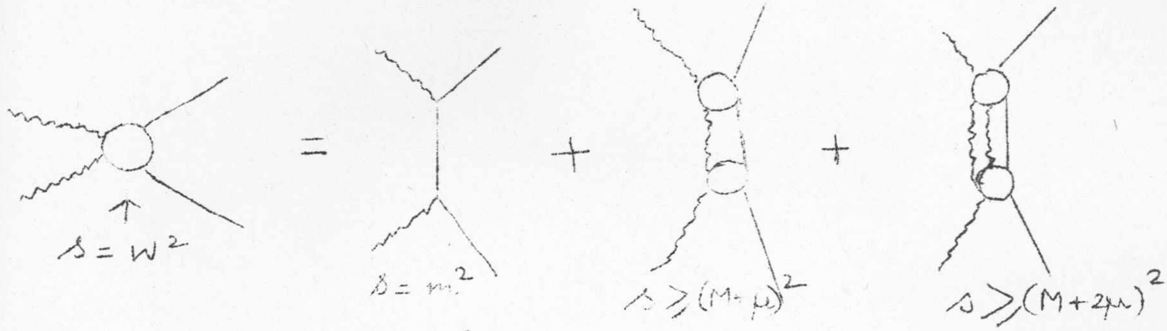


Fig 6

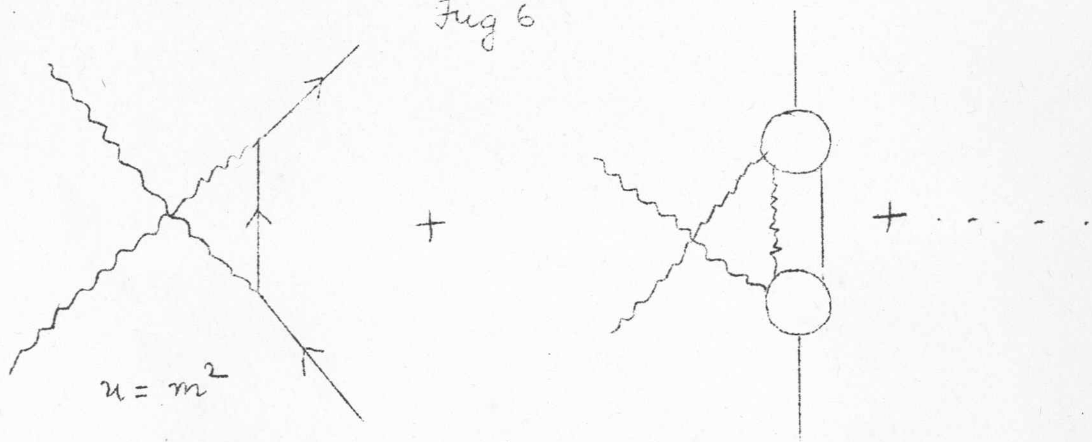


Fig 7.

The singularities of the partial wave amplitudes for pion-nucleon scattering have been discussed by Frazer and Fulco* and by Frautschi and Walecka** among others. In addition to the unitarity cuts from $m+\mu$ to ∞ (the cut from $-m-\mu$ to $-\infty$ can be re-expressed in terms of the positive cut using a symmetry relation between the partial wave amplitudes) there will be other singularities arising from the vanishing of the denominators containing the and t variables such as short cuts from $-(m^2+2\mu^2)^{1/2} \leq W \leq -m+\frac{1}{m}$

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*W.R.Frazer and J.R.Fulco, Phys. Rev. 119, 1420 (1960)

**S.C.Frautschi and J.D.Walecka, Phys.Rev.120, 1486 (1960) and $-m+\mu \leq W \leq m-\mu$. cut along the entire imaginary axis and a circular cut about the sign . having a radius $r = \sqrt{m^2 - \mu^2}$ (see Fig.8)

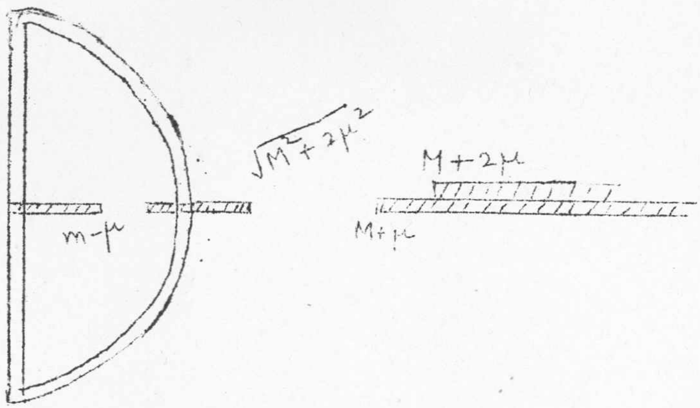

Fig.8

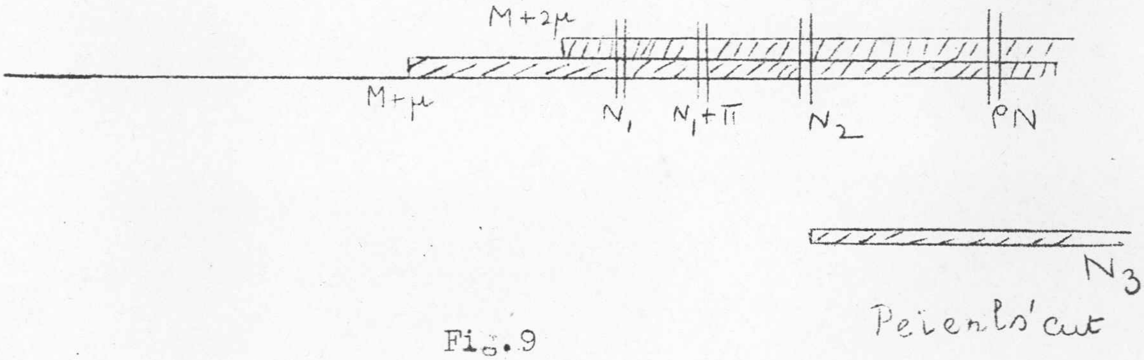

Fig.9

In addition to the above singularities there may be an additional pole and cut suggested by pe..rls.* If If we consider the process, $\pi + N_1 \rightarrow \pi + N_1$ where the resonance N_1 is taken to be an unstable particle, then consideration of Fig.9a shows us that isobar being unstable can decay into a pion and nucleon which subsequently absorbs another pion.

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* R.E.Peierls, Phys.Rev. Letters 6, 641 (1961).

This represents a real process and the corresponding pole shows up in the physical region for the momentum and scattering angle. The contribution from this pole shows a resonance-like behaviour which can produce a strong effect in pion-nucleon scattering via the reaction Fig.9b.

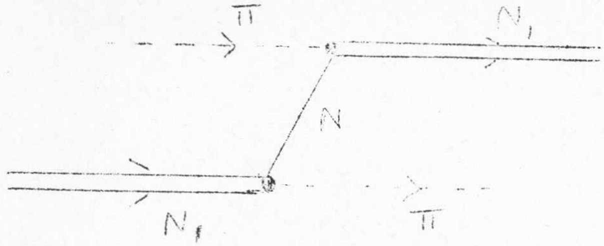


Fig.9a

In the partial wave analysis the Peierls pole may lead to a cut which we have drawn roughly between the positions of the resonance N_2 and N_3. One might think of N_2 and N_3. as poles which replace the cut. If the PN channel (the threshold for which lies in this region.) contributes a negative A to the amplitude then there is the possibility of a pole.

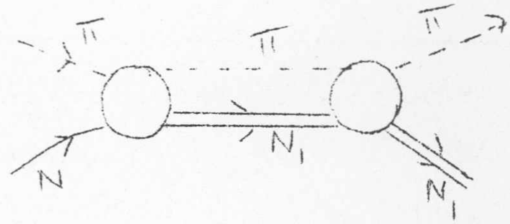


Fig 9b

The situation regarding the angular momentum state of N_2 is not clear. Recently the beam strength of the Berkeley bevatron has been increased by a factor 10 so that formation of the resonant state N_1 can be isolated and a study made of the reaction $\pi N \rightarrow \pi N_1$ (Fig.10). The angle χ is measured in the rest system of N_1. The angular distribution of πN_1 formation and the decay

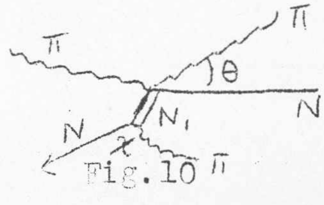


Fig.10

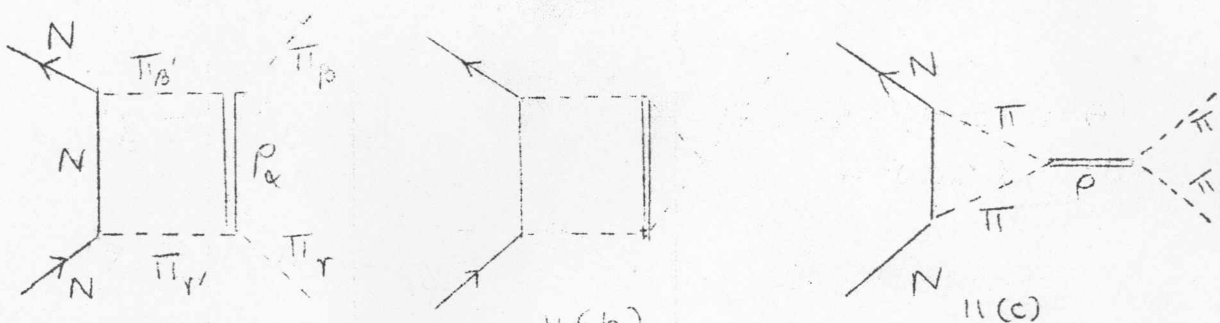
distribution of N_1 can be measured.

The following table should be helpful.

πN	πN_1	$d\sigma/d\Omega$	$d\sigma/d\chi$
$S_{1/2} \rightarrow$	$d_{1/2}$	const	$1 + 3\cos^2\chi$
$P_{1/2} \rightarrow$	$P_{1/2}$		"
$d_{3/2} \rightarrow$	$d_{3/2}$		$(1+3\cos^2\chi)(1+3\cos^2\theta) + 9\sin^2\chi\sin^2\theta$
$P_{3/2} \rightarrow$	$P_{3/2}$	$7 - 6\cos^2\theta$	
$f_{5/2} \rightarrow$	$f_{5/2}$	$1 + 2\cos^2\theta$	

This and a similar tables for the process $\pi N \rightarrow \rho N$ should be helpful to the experiments.

Now we shall describe briefly the work we* have done using perturbation theory which reproduces many of the features of the higher resonances in the pion-nucleon system. The main feature of this approach is the importance of the ρ meson-nucleon intermediate state for πN scattering (Fig. 11a)



11 (a) 11 (b) 11 (c)

* K. Itabashi, M. Kato, K. Nakagawa & G. Takeda, Progress of Theoret. Phys, 24, 529 (1960)

The $\rho \pi \pi$ interaction Hamiltonian can be written as

$$H_{\rho \pi \pi} = F \sum_{\alpha, \beta, \gamma}$$

where α β, γ are the isotopic spin indices and χ φ ... , and π field operators . F is the coupling constant and its large value ($F^2/4\pi \simeq 5$) will ultimates found to be responsible for the N_2 and N_3 resonances. The procedure to find F is to calculate the pion-pion scattering amplitude in perturbation theory using the above Hamiltonian and comparing it with the one-level formula of Frazer and Fulco.

A calculation of the imaginary part of the p-wave scattering amplitude $f_{11}(p)$ in the center of mass system gives

$$\mathrm{Im}\, f_{11}(p) = \frac{F^2}{6} \frac{p^3}{W} \delta(m_\rho^2 - W^2)$$

while the one-level formula, under the assumption that the width of the resonance is small gives

$$\mathrm{Im}\, f_{11}\, p \simeq \pi \Gamma W_R \delta(W_R^2 - W^2)$$

$$\simeq \frac{8\pi \Gamma' p^3}{W} \delta(W_R^2 - W^2)$$

where according to Frazer and Fulo $\Gamma' \simeq 0.4$ and $W_R \approx 600\,\mathrm{Mev}$ We therefore have $F^2/4\pi \simeq 12\,\Gamma' \approx 5$ and $W_R \approx m_\rho \approx 600\,\mathrm{Mev}$.

We shall now show that a strong $\rho \pi \pi$ interaction $H_{\rho \pi \pi}$ gives a rather strong attractive potential of the pion-nucleon system in the T = 1/2 state for $W \simeq m_\rho$ and the observed second and third resonances are identified as those in the T = 1/ $J = 3/2$ and $T = 1/2$ J $5/2$ states respectively.

The scattering amplitude corresponding to Fig.11a becomes very large for $W \approx m_\rho$ since energies of intermediate ρN states can be very nearly equal to the initial energy W of the colliding system. For Figs. 11b and 11c however the energy differences between the intermediate and initial states are large for $W \approx m_\rho$ and at least of order W so that they give only a small contribution to the πN force. Further if, identify the nature of the potential with the sign of the energy denominator, $W_i - W_n$ then we should expect an attractive force in the first case.

Now that we expect a large attractive force at $W = m_\rho$ to come out from Fig.11a, let us find out the isotopic spins and spins of the higher resonances. The isotopic spin dependence of the $\pi - N$ potential is given by

$$\langle \pi_\beta | V | \pi_\gamma \rangle \sim -\sum_{\alpha,\beta',\gamma'} \epsilon_{\alpha\beta\beta'} \tau_{\beta'} \epsilon_{\alpha\gamma\gamma'} \tau_{\gamma'}$$

$$= 2\delta_{\beta\gamma} + \frac{1}{2}[\tau_\beta \ \tau_\gamma] = 4P_{1/2} + P_{3/2}$$

where the P's are the isotopic spin projection operators. We thus see that the attractive $\pi - N$ potential is 4 times larger in the T = 1/2 state than in the T = 3/2 state even in the lowest order perturbation theory.

The angular momentum and parity of the large attractive forces can be decided by evaluating the $\pi N \rightarrow \rho N$ transition matrix element for ρN states of small $\vec{q}$ (Fig. 12)

For the $NN\pi$ vertex the static approximation can be used. The result is that if only terms independent of $\vec{q}$ are retained

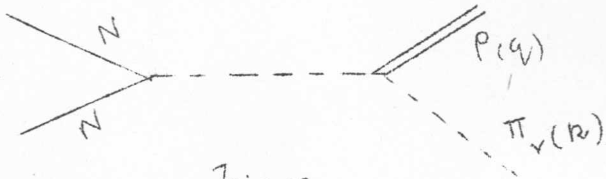


Fig 12.

so that ρ is in an orbital angular momentum state $\ell_\rho = 0$ the amplitude is found to be proportional to

$$\left[P(s_{1/2} \to s_{1/2}) + P(d_{3/2} - s_{3/2})\right]$$

Similarly if terms linear in $\vec{q}$ are also retained so that ℓ_ρ can be 1, the strengths of the attractive π - N potential is found to be in the ratio (8/3:1:10) for the states $f_{5/2}$, $p_{3/2}$ and $p_{1/2}$ going into ρN intermediate states. Thus it is natural to associate the N_2 resonance with $d_{3/2}$ and N_3 with $f_{5/2}$. The possibility of a $p_{1/2}$ resonance at about the same energy as N_3, which was mentioned earlier, is also explained.

Finally the broad bump in the total π^+ - p cross-section observed at $W \approx$ 940 Mev could be interpreted as being due to a maximum in the inelastic cross-sections $\pi N \to \rho N \to 3\pi N$ due to final state interactions among the produced pions and nucleon. The isotopic spin dependence of the $\pi N \to \rho N$, transition amplitudes is independent of the details of the choice of the interaction, $H_{NN,\pi}$ and the relative magnitudes of the various $\pi N \to \rho N$, amplitudes are

$$f(\pi^+p \to \rho^+N_1^+) : f(\pi^+p \to \rho^0N_1^{++}) : f(\pi^-p \to \rho^-N_1^+) : f(\pi^-p \to \rho^0 N_1^0) = \sqrt{2/3} : 1 : \sqrt{2/3} : \sqrt{1/3}$$

so that the inelastic scattering cross-section for $\pi N \to \rho N_1$ in T = 3/2 state is larger than that in T = 1/2 state by a factor 5/2, thus indicating that the maximum shows up only in $\pi^+ p$ collisions.

In the calculation, interaction of the Born term like Fig.13. are included Fig.14 represents the total cross-sections curve drawn on the basis of three theoretical values.

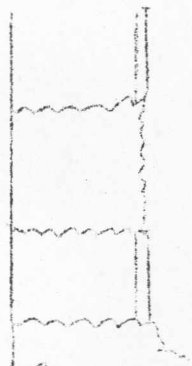


Fig 13

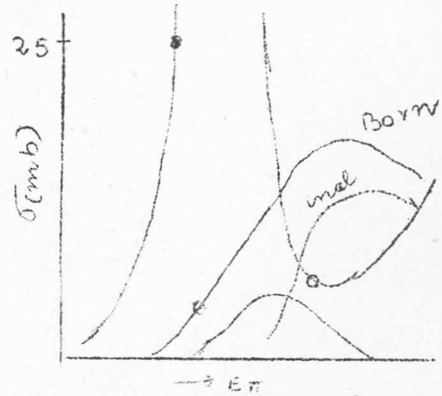


Fig 14

As is evident, the cross-section is heading for a maximum above the value, 25 mb. The phase shifts corresponding to various values of   are given in the table below.

E_π	3	4	5
δ	38.5°	174°	154°
η	0.94	0.96	0.95

Finally, we have only time to mention briefly an anomaly, in the cross-section for the process of pion production in pion-nucleon collision in the energy region $M(2\pi) \simeq 400 - 500$ Mev. with I = 0. The anomaly was noticed by Tripp and Perez-Mendez independently. It is higher than the ABC (Anomaly (about 310 Mev) and perhaps swamps it. We can expect it to arise from an infinite anomalous singularity (discussed by Landshoff and Treiman*) corresponding to a triangle diagram of the kind, Fig 15

The Dalitz plot for the even is given in Fig. 16

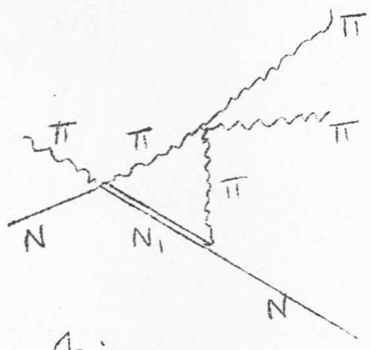


Fig 15

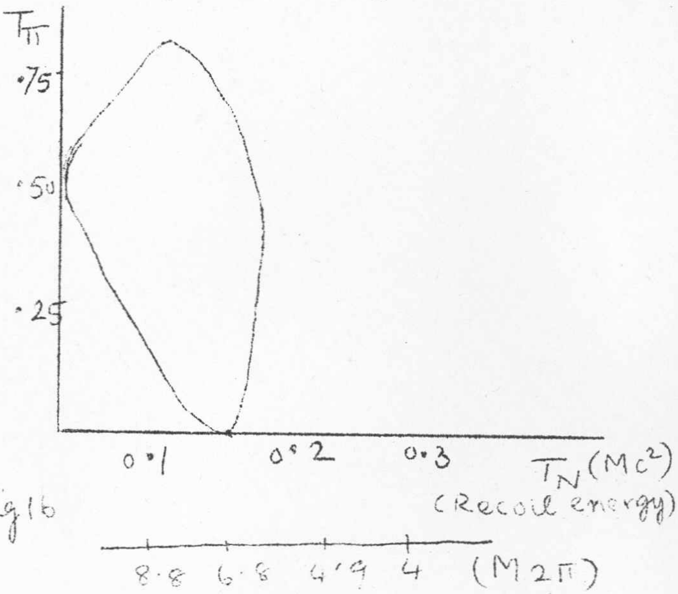


Fig 16

* P.V.Landshoff and S.B.Treiman, Phys. Rev. 127, 649 (1962). See also R.Aaron, Phys. Rev. Letters 10, 32 (1963) for the possibility of a similar anomaly in the reaction

$$K^- + p \rightarrow \Sigma_1 + \pi + \pi$$

An alternative explanation might be to consider the one pion exchange graph (Fig.) and the diagram

Fig.

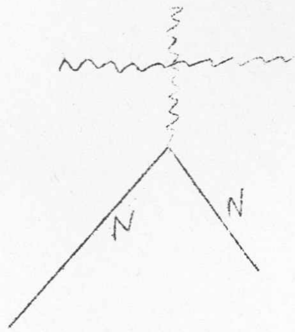


Fig 17

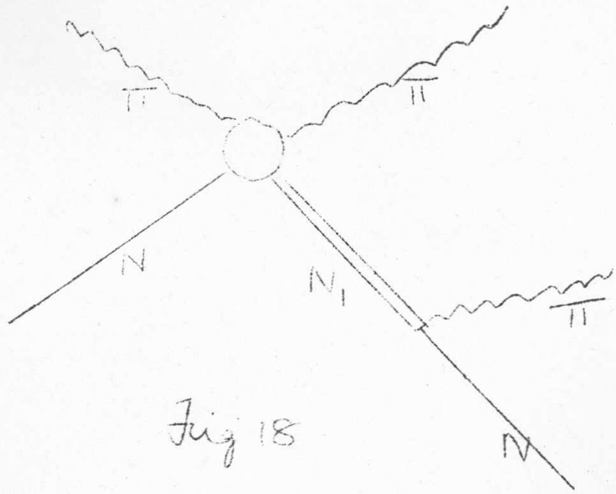


Fig 18

The corresponding matrix elements are respectively

$$M_1 \propto \frac{\sigma \cdot \Delta}{\Delta^2 + \mu^2} = \frac{|\Delta|}{\Delta^2 + \mu^2}$$

Spin nonflip $\propto \cos\theta_{lab}$

Spin-flip $\propto \sin\theta_{lab}$

$$M_2 \propto (\sigma_2 \cdot q_2') - \frac{1}{2}(\sigma \cdot q^c) = \frac{2}{3}\sigma_\pi q_2^c - \frac{1}{3}\sigma_\pi q_1^c$$

Spin-nonflip $\propto \frac{2}{3}\cos\theta_c$

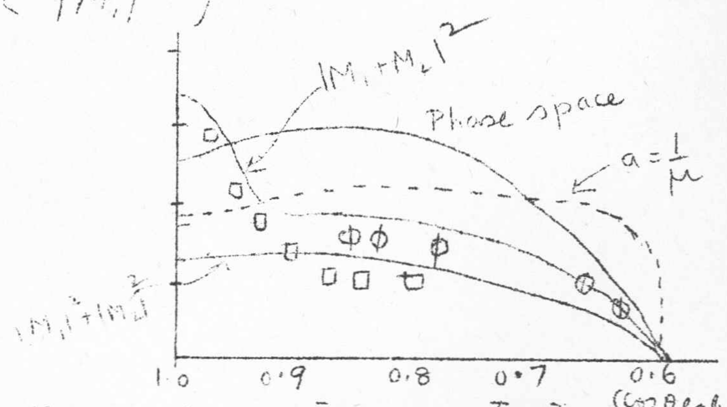
Spin-flip $\propto -\frac{1}{3}\sin\theta_c$

$$d\sigma \propto |M_1|^2 + |M_2|^2 + \text{Interference term.}$$

where (Interference term) $\propto \cos\theta_{lab} \cos\theta_c$

$$- \frac{1}{2} \sin\theta_{lab} \sin\theta_c$$

The phase space $\left(|M_2|^2 < |M_1|^2 \right)$ factors, $|M_1|^2 + |M_2|^2$ and $|M_1 + M_2|^2$ are plotted along with the experimental points.



ϕ corresponds to the reaction $\pi^- p \rightarrow \pi^+ \pi^- n$

$\square$ corresponds to the reaction $\pi^- p \rightarrow \pi^0 \pi^0 n$

None of the above explanations regarding the anomaly seen to be satisfactory.

THE INSTITUTE OF MATHEMATICAL SCIENCES
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SOME TOPICS IN THE THEORY OF WEAK INTERACTIONS

(Seminar Lectures)

By

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CONTENTS.

SYMMETRIES OF STRONG AND WEAK INTERACTIONS

(Summary of Gursey's work on four-dimenstional isospin rotation)

Generalizing his earlier work on the Pauli-Gursey group, Gursey develops a scheme for rotation in four-dimensional isospin space such that the group G_4 associated with it together with the hypercharge gauge group H contains charge independence, strangeness selection rules and the empirical symmetries of weak interactions as various subgroups. The expectation is that if G_4 leaves (in an approximate way) the strong interactions invariant, the renormalization effects will be small for any interaction invariant under a subgroup of G_4 and hence also for weak interactions which are found to be invariant under such subgroups. The removal of the doublet symmetry by a doublet 'perturbation' (DP) should make itself felt both for strong and weak interactions.

To arrive at a four-dimensional isospin rotation as a product of two three-dimensional rotations it is necessary that we have in addition to the usual isospin rotation $e^{i\vec{\tau}\cdot\vec{\omega}}$ (for the pion-nucleon system) another rotation. Let us start with the new rotation defined by

$$G_3' : \quad \psi \rightarrow e^{i\gamma_5\vec{\tau}\cdot\vec{\omega}}\psi \tag{1}$$

which has the same form as the Dyson-Foldy transformation which converts the pseudoscalar pion-nucleon coupling to a psendo-vector one . The free-nucleon equation is not left invariant under (1) if the mass $\neq 0$. The transformed equation is

$$\gamma_\mu \partial_\mu \psi = m \exp[2i\gamma_5 \vec{\tau}.\vec{\omega}]\,\psi \qquad (2)$$

If we choose $\vec{\omega} = f\vec{\varphi} = \frac{g\vec{\varphi}}{2m}$ where f and g are the pseudo-vector and pseudo-scalar coupling constants respectively (if we identify φ with the pion field) and expand in powers of f we obtain

$$\begin{aligned}\gamma_\mu \partial_\mu \psi &= m \exp[2if\gamma_5 \vec{\tau}.\vec{\varphi}]\,\psi \\ &= m\psi + ig\gamma_5 \vec{\tau}.\vec{\varphi}\psi - \frac{g^2\varphi^2}{2m}\psi + \end{aligned} \qquad (3)$$

which can be split up into the two equations

$$\begin{aligned}\gamma_\mu \partial_\mu \psi_L &= m\bar{\Phi}\psi_R \\ \gamma_\mu \partial_\mu \psi_R &= m\Phi\psi_L\end{aligned} \qquad (4)$$

where

$$\begin{aligned}\psi_L = \tfrac{1}{2}(1+\gamma_5)\psi, &\quad \psi_R = \tfrac{1}{2}(1-\gamma_5)\psi \\ \Phi = \exp[2if\vec{\tau}.\vec{\varphi}], &\quad \bar{\Phi} = \exp[-2if\vec{\tau}.\vec{\varphi}] \\ \Phi\bar{\Phi} &= \bar{\Phi}\Phi = 1\end{aligned} \qquad (5)$$

Thus the operation (1) has led to a system of left-handed and righ handed nucleons and a generalized boson field. We can perform two independent isospin rotations on these thus~~,~~ defining two groups,

$$\begin{aligned}G_3^{(1)} ;\ \psi_L &\to \exp[i\vec{\tau}.\vec{\mu}]\psi_L \\ \Phi &\to \Phi \exp[-i\vec{\tau}.\vec{\mu}], \quad \psi_R \to \psi_R\end{aligned} \qquad (6)$$

and

$$G_3^{(2)} ;\ \psi_R \rightarrow \exp[i\vec{\tau}.\vec{\nu}]\psi_R,\ \Phi \rightarrow \exp[i\vec{\tau}.\vec{\nu}]\Phi ; \quad \psi_L \rightarrow \psi_L \tag{7}$$

The product of these two defines the four-dimensional isospin rotation group,

$$G_4 ;\ \psi' = \exp\left[i\left\{\frac{1+\gamma_5}{2}\vec{\tau}.\vec{\mu} + \frac{1-\gamma_5}{2}\vec{\tau}\vec{\nu}\right\}\right]\psi$$
$$\Phi' = \exp[i\,2f\,\vec{\tau}.\vec{\varphi}'] = \exp[i\vec{\tau}.\vec{\nu}]\,\Phi\,\exp[-i\vec{\tau}.\vec{\mu}] \tag{8}$$

We readily see that the isospin rotation group (in 3 dimensions) G_3 is obtained by taking $\vec{\mu} = \vec{\nu} = \vec{\omega}$ and the subgroup G_3^{4} (eqn. (1)) by taking $\vec{\mu} = -\vec{\nu} = \vec{\omega}'$. Further $G_3^{(1)}$ and $G_3^{(2)}$ do not commute with either the parity operator P (the γ_4 matrix) or the charge-conjugation operator C (γ_2) separately, but commute with CP. Thus we expect these subgroups to be associated with the weak interactions.

We can make the kinetic energy part of the pion field, $(\partial_\mu \varphi)^2$ invariant under G_4 by the substitution.

$$(\partial_\mu \varphi)^2 \rightarrow \frac{1}{4} f^2 \,Tr\left[(\partial_\mu \Phi)(\partial_\mu \bar{\Phi})\right]^2 \tag{9}$$

since under (8) the right hand side of (9) will undergo a unitary transformation which leaves the trace invariant. But the mass term, φ^2 (we are taking the pion mass to be unity) is not left invariant under (8). Thus in the present doublet approximation the pion mass must be zero. But since DP is invariant under G_3, it can generate a pion mass which will be strongly renormalized by the pion-pion interactions.

The γ_5-transformation (applicable to fermions with non-zero mass also) arises naturally in this model. If we take the discrete subgroup of G_4 by choosing $\nu_3 = \pi, \nu_1 = \nu_2 = 0$ we obtain the transformations.

$$\psi \to \gamma_5 \psi \quad , \quad \Phi \to -\Phi \tag{10}$$

which leave the equations (4) invariant. Since in the doublet approximation other strong interactions will not disturb G_4 invariance, the invariance (10) will remain valid in this approximation. The effect of (DP) will then be to induce simultaneously a pion mass, a split in the baryon masses, separation of the $\Lambda - \Sigma$ system and correction to the $V-A$ form of the weak interaction.

We shall now proceed to give group representations to the other baryons and bosons; labelling the representation by two-numbers m,n corresponding to the parameters $\vec{\mu}, \vec{\nu}$ (of the groups $G_3^{(1)}$ and $G_3^{(2)}$ respectively. The nucleons and the Φ-field have the representation

$$\psi_L : \left(\tfrac{1}{2}, 0\right) \; ; \; \psi_R : \left(0, \tfrac{1}{2}\right) \; ; \; \Phi : \left(\tfrac{1}{2}, \tfrac{1}{2}\right) \tag{11}$$

from (6) and (7). The cascade doublet has the representation

$$\Xi_L : \left(0, \tfrac{1}{2}\right) \; ; \; \Xi_R : \left(\tfrac{1}{2}, 0\right) \tag{12}$$

so that it will interact with the pion according to the equation

$$\gamma_\mu \partial_\mu \Psi_\Xi = m_\Xi \exp[-2if\gamma_5 \vec{\tau}\cdot\vec{\varphi}]\Psi_\Xi \tag{13}$$

The Λ and Σ which are clubbed together in the doublet approximation are given the representation

$$Y_L : (\tfrac{1}{2},\tfrac{1}{2})\ ;\ Y_R = \Lambda_R + i\vec{\tau}\cdot\vec{\Sigma}_R$$

$$\Lambda_R : (0,0)\ ,\ \Sigma_R : (0,1) \tag{14}$$

so that they transform according to

$$Y_L \rightarrow \exp[i\vec{\tau}\cdot\vec{\mu}]\,Y_L\,\exp[-i\vec{\tau}\cdot\vec{\nu}]$$

$$Y_R \rightarrow \exp[i\vec{\tau}\cdot\vec{\nu}]\,Y_R\,\exp[-i\vec{\tau}\cdot\vec{\nu}] \tag{15}$$

and obey the equations

$$\gamma_\mu \partial_\mu Y_L = m_Y \Phi Y_R$$

$$\gamma_\mu \partial_\mu Y_R = m_Y \bar{\Phi} Y_L \tag{16}$$

which are left invariant under G_4.

The K-meson doublet is assigned the representation $(0,\frac{1}{2})$ There are no representations corresponding to $(1,0)$ for baryons and $(\frac{1}{2},0)$ for the bosons. They are missing components just as the neutrino with the other spin is absent, in the two-neutrino theory.

Since only the nucleon, cascade and K doublets have hypercharge quantum number different from zero; we can write the hypercharge gauge transformation as

$$H: \quad \psi \rightarrow \exp[iu]\,\psi, \quad \Xi \rightarrow \exp[-iu]\,\Xi \qquad\qquad (17)$$
$$K \rightarrow \exp[iu]\,K$$

with the other fields unaffected.

For strong interactions we can enlarge the group G_4 to $G_4 \times H$ which has as subgroups $H, G_3, G_3^{(1)}$ and $G_3^{(2)}$. When (DP) is switched on, G_4 invariance ~~under~~ is lost and the strong interactions are invariant only under groups H and G_3 (which commute with C and P separately) whereas the weak interactions are invariant under only $G_3^{(1)}$ or $G_3^{(2)}$. There can be two kinds of weak interactions depending on whether they are invariant under or . Since charge must also be conserved we can enlarge $G_3^{(1)}$ and $G_3^{(2)}$ to 4 parameter unitary groups which will also describe charge conservation. Consider, e.g., the unitary subgroup $U^{(1)}$ of $(G_4 \times H)$ obtained by choosing $\nu_1 = \nu_2 = 0, \nu_3 = u$ ~~and~~ . The transformation properties of the various fields under $U^{(1)}$ are

$$\begin{aligned}
U^{(1)}: \quad \psi_L &\rightarrow \exp[iu]\exp[i\vec{\tau}\cdot\vec{\mu}]\,\psi_L \\
\psi_R &\rightarrow \exp[i(1+\tau_3)u]\,\psi_R \\
\Xi_L &\rightarrow \exp[-i(1-\tau_3)u]\,\Xi_L \\
\Xi_R &\rightarrow \exp[-iu]\exp[i\vec{\tau}\cdot\vec{\mu}]\,\Xi_R \\
Y_L &\rightarrow \exp[i\vec{\tau}\cdot\vec{\mu}]\,Y_L \exp[-i\tau_3 u] \\
Y_R &\rightarrow \exp[i\tau_3 u]\,Y_R \exp[-i\tau_3 u]
\end{aligned}$$

$$\bar{\Phi} \to \exp[i\vec{\tau}\cdot\vec{\mu}]\,\bar{\Phi}\,\exp[-i\tau_3 u]$$
$$K \to \exp[iu]\,\exp[i\tau_3 u]\,K \tag{18}$$

If we take the baryon part of the leptonic decay processes to transform as in (18), we see that the leptonic current $\jmath_\mu = (\bar{e}+\bar{\mu})\gamma_\mu\nu$ must transform as

$$U^{(1)} : \jmath_\mu \to \exp[2iu]\,\jmath_\mu$$

We can construct various Lagrangians which are invariant under $U^{(1)}$. For instance

$$L_K = aGf^{-3}\,Tr\{(\bar{\Phi}\partial_\mu\bar{\Phi})[\partial_\mu\theta + K_1^0(\bar{\Phi}\partial_\mu\bar{\Phi})]\} + h.c. \tag{19}$$

where θ is the matrix $\begin{pmatrix} \bar{K}_0 & K^+ \\ -K^- & K^0 \end{pmatrix} = K_1^0 + i\vec{\tau}\cdot\vec{K}$

Expanding $\bar{\Phi}$ we obtain

$$L_K \approx 4aGf^{-1}\Big[K_1^0(\partial_\mu\varphi)^2 + \tfrac{1}{2}f^{-1}(\partial_\mu\vec{K})\cdot(\partial_\mu\vec{\varphi}) - (\partial_\mu\vec{K})(\vec{\varphi}\cdot\partial_\mu\vec{\varphi}) + \tfrac{1}{3}f(\partial_\mu\vec{K})\cdot(\varphi^2\partial_\mu\vec{\varphi} - 2\,\vec{\varphi}\,\partial_\mu\varphi^2)\Big] \tag{20}$$

The first and fourth terms give the 2 pion decay of K_1^0 and the 3-pion decay modes of K_2^0 and K^+. The $\Delta I = \frac{1}{2}$ is also seen to hold.

Another interaction left invariant under $U^{(1)}$ is

$$L_Y = a' G f^{-2}\, \mathrm{Tr}\Big\{ (\bar{\Phi}\, \partial_\mu \Phi)\, \gamma_L \begin{pmatrix} 0 & 0 \\ \bar{p}_L & \bar{n}_L \end{pmatrix} \gamma_\mu + (\Phi\, \partial_\mu \bar{\Phi}) \Big[\gamma'_L \begin{pmatrix} \bar{n}_L & 0 \\ -\bar{p}_L & 0 \end{pmatrix} + \gamma'_R \begin{pmatrix} 0 & 0 \\ \bar{p}_R & \bar{n}_R \end{pmatrix} + \gamma_R \begin{pmatrix} \bar{n}_R & 0 \\ -\bar{p}_R & 0 \end{pmatrix} \Big] \gamma_\mu \Big\} + h.c. \qquad (21)$$

which an expansion gives

$$\begin{aligned} L_Y \approx 4 a' G f^{-2} \Big\{ & (\sqrt{2}\, \bar{p}_L \gamma_\mu \Lambda_L \partial_\mu \varphi^+ - \bar{n}_L \gamma_\mu \Lambda_L \partial_\mu \varphi^0) + (\bar{n}_L \gamma_\mu \Sigma_L^0 \partial_\mu \varphi^0 \\ & - \sqrt{2}\, \bar{p}_R \gamma_\mu \Sigma_R^0 \partial_\mu \varphi^+) + \sqrt{2}\, \bar{p}_R \gamma_\mu \Sigma_R^+ \partial_\mu \varphi^0 \\ & + (\bar{n}_L \gamma_\mu \Sigma_L^- - \bar{n}_R \gamma_\mu \Sigma_R^-)\, \partial_\mu \varphi^+ \\ & + (\bar{n}_L \gamma_\mu \Sigma_L^+ + \bar{n}_R \gamma_\mu \Sigma_R^+)\, \partial_\mu \varphi^- \\ & + h.c. \Big\} \end{aligned} \qquad (22)$$

The first term which leads to the $\Lambda \to p + \pi^-$ decay reduces (as regards its spatial property) to the form $\gamma_\mu(1+\gamma_5)$ as also the term giving the cecay $\Sigma^+ \to p + \pi^0$. The combination of the two terms corresponding to $\Sigma^- \to n + \pi^-$ however gives a pure $\gamma_\mu \gamma_5$ i.e. axial vector interaction and similarly the two terms corresponding to $\Sigma^+ \to n + \pi^+$ give a pure γ_μ (vector) interaction. Thus the asymmetries observed namely that the first two decays mentioned show asymmetry whereas the lost two have almost complete parity purity is explained.

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SYMMETRIES SHARED BY STRONG AND WEAK INTERACTIONS -II

K.Venkatesan

In the previous lecture we saw how, starting from the doublet approximation, we can explain the curious pattern of the decay asymmetries of the Σ particle as well as the large asymmetry parameter for Λ^0 decay if the terms corresponding to $\Sigma^+ \rightarrow p + \pi^0$ and $\Lambda^0 \rightarrow p + \pi^-$ involve only one kind of the baryons (either left-handed or right-handed) while the terms corresponding to $\Sigma^+ \rightarrow n + \pi^+$ and $\Sigma^- \rightarrow n + \pi^-$ involve both right-handed and left-handed baryons. We shall now consider a different set of transformations, due originally to Treiman and Pais which in the doublet approximation leads to correct predictions regarding the asymmetries of non-leptonic decay modes of the strange-particles as well as certain relations between the matrix element for these processes. In the approximation of neglecting the $\Sigma - \Lambda$ mass difference and assuming the $\Sigma \Lambda$ parity to be even we can form the following four baryon doublets

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix} ; \quad N_2 = \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix} ; \quad N_3 = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix} ; \quad N_4 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix} \qquad (1)$$

where

$$Y^0 = \frac{\Lambda^0 - \Sigma^0}{\sqrt{2}} \quad ; \quad Z^0 = \frac{\Lambda^0 + \Sigma^0}{\sqrt{2}}$$

Considering only the baryon-pion interactions for the present we can write the strong interaction Hamiltonian as

$$H_{strong} = i G \left[\overline{N}_1 \vec{\tau} \gamma_5 N_1 + \overline{N}_2 \vec{\tau} \gamma_5 N_2 + \overline{N}_3 \vec{\tau} \gamma_5 N_3 + \overline{N}_4 \vec{\tau} \gamma_5 N_4 \right] \cdot \vec{\pi} \qquad (2)$$

Following Treiman we assume, at first, the weak interaction hamiltonian, which is the product of a $\Delta S = 0, T = 1$ current j and $\Delta S = 1, T = \frac{1}{2}$ current s to be given by

$$H_W(j s) = H_W^{ch}(j s) + H_W^{0}(j s)$$

$$H_W^{ch}(j s) = j^+ s^- + j^- s^+$$

$$H_W^{0}(j s) = (1/\sqrt{2})\, j^0 (s^0 + \bar{s}^0) \qquad (3)$$

where

$$\vec{j} = \sum_{i=1}^{4} \bar{N}_i \vec{\tau} N_i \; ; \; j^{\pm} = \frac{1}{\sqrt{2}}(j_1 \pm i j_2) \; ; \; j_0 = j_3$$

and

$$s = \begin{pmatrix} s^- \\ s^0 \end{pmatrix} = \begin{pmatrix} \bar{N}_3 N_1 + \bar{N}_4 N_2 \\ \bar{N}_2 N_1 - \bar{N}_4 N_3 \end{pmatrix}$$

By expanding s^- and s^0 in terms of the particles (p, n, Λ, Σ etc.) we can verify that it does not involve currents like $\bar{\Sigma}^+ n$ which are $T = 3/2$, $\frac{\Delta Q}{\Delta S} = -1$ currents. Both (2) and (3) are invariant under the set of transformations

$$N_1 \longleftrightarrow N_2 , \; N_3 \longleftrightarrow N_4 , \; \vec{\pi} \longrightarrow \vec{\pi} \qquad (4)$$

From (4) we have $\langle Y^0 | p \pi^- \rangle = \langle n | \Sigma^+ \pi^- \rangle$ (5) and if we assume that there is no final state interaction the latter is equal to $\langle \Sigma^+ | n \pi^+ \rangle$

(2) and (3) are also invariant under the transformations.

$$N_1 \longrightarrow i\tau_2 N_3 , \quad N_2 \longrightarrow i\tau_2 N_4$$

$$N_3 \longrightarrow -i\tau_2 N_1 , \quad N_4 \longrightarrow -i\tau_2 N_2$$

$$\pi^{\pm,0} \longrightarrow -\pi^{\mp,0} \qquad (6)$$

which yields

$$\langle \Sigma^0 | p\pi^- \rangle = \langle n | \Sigma^- \pi^+ \rangle = \langle \Sigma^- | n\pi^- \rangle \quad (7)$$

Adding (5) and (7) we obtain the relation

$$\sqrt{2} \langle \Lambda | p\pi^- \rangle = \langle \Sigma^+ | n\pi^+ \rangle + \langle \Sigma^- | n\pi^- \rangle \quad (8)$$

Similarly for the Ξ decay we get the relation

$$\sqrt{2} \langle \Xi^- | \Lambda\pi^- \rangle = \langle \Sigma^+ | n\pi^+ \rangle + \langle \Sigma^- | n\pi^- \rangle \quad (9)$$

which means that $\langle \Xi^- | \Lambda\pi^- \rangle = \langle \Lambda | p\pi^- \rangle$

$$\text{and} \quad \alpha_\Xi = \alpha_\Lambda \quad (10)$$

where α's are the asymmetry parameters for the two decays.

It is experimentally observed that there is maximum parity violation in the Σ decay mode $\Sigma^+ \rightarrow p + \pi^0$ whereas the modes $\Sigma^+ \rightarrow n + \pi^+$ and $\Sigma^- \rightarrow n + \pi^-$ are nearly parity pure. We can derive a relation between these matrix elements by using the $|\Delta I| = \frac{1}{2}$ rule observed to be obeyed in these decays. We can define the following reduced matrix elements

$$A = \langle \pi N (I = \tfrac{1}{2}) \| J_W^{1/2} \| \Sigma (I = 1) \rangle$$

$$B = \langle \pi N (I = \tfrac{3}{2}) \| J_W^{1/2} \| \Sigma (I = 1) \rangle \quad (11)$$

where $J_W^{1/2}$ represents a "spurian" current representing the fact that there is a change of ½ in the total isotopic spin, and the double bars indicate that the matrix element depends only on I (and not I_z). For the Σ^- decay only the $I = \frac{3}{2}$ channel is open since $I_z = -3/2$ in this case for the decay products. For the other two decays both channels are available. From the Wigner-Eckart theorem, viz.,

$$\langle I, m | T^{I_2}_{m_2} | I_1, m_1 \rangle = (I\, m\, I_1 I_2 | I_1\, m_1\, I_2\, m_2) \times \langle I \| T^{I_2} \| I_1 \rangle \tag{12}$$

we obtain

$$\begin{aligned}
\langle \pi^- n | J_W^{1/2} | \Sigma^- \rangle &= B \\
\langle \pi^+ n | J_W^{1/2} | \Sigma^+ \rangle &= \frac{2A + B}{3} \\
\langle \pi^0 p | J_W^{1/2} | \Sigma^+ \rangle &= \sqrt{2/3}\,(A - B)
\end{aligned} \tag{13}$$

from which we have immediately

$$\sqrt{2} \langle \pi^0 p | \Sigma^+ \rangle = \langle \pi^+ n | \Sigma^+ \rangle - \langle \pi^- n | \Sigma^- \rangle \tag{14}$$

8, 9 and (14) together imply that

$$\alpha_\Lambda \simeq -\alpha_0 \simeq \alpha_\Xi \tag{15}$$

where α_0 is the asymmetry parameter for the $\Sigma^+ \to p + \pi^0$ decay. Experimentally it has been observed that $\alpha_\Lambda \simeq -\alpha_0$ but $\alpha_\Lambda \simeq -\alpha_\Xi$ We observe that the latter result can be arrived at without altering any of the other predictions if we keep the transformation (5) as it is, but change (6) to

$$\begin{aligned}
&N_1 \to i\gamma_2 N_3, \quad N_2 \to -i\gamma_2 N_4, \\
&N_3 \to -i\gamma_2 N_1, \quad N_4 \to i\gamma_2 N_2 \\
&\pi^{\pm,0} \to -\pi^{\mp,0}
\end{aligned} \tag{16}$$

There is however a difficulty in the whole procedure in that the specific form of the weak interaction Hamiltonian chosen does not allow the $\Sigma^+ \rightarrow n + \pi^+$ decay to proceed at all, i.e $\langle \Sigma^+ | n \pi^+ \rangle = 0$. This can be seen by expanding the currents j and s in terms of the particles,

$$H_W^{ch}(js) = (\bar{n}p + \bar{Y}_0\Sigma + \bar{\Sigma}^- Z_0 + \quad)$$
$$\times (\bar{p} Z_0 + \bar{n}\Sigma^- + \quad)$$

$$H_W^0(js) = (\bar{p}p - \bar{n}n + \bar{\Sigma}^+\Sigma^+ + \cdots)$$
$$\times (\bar{p}\Sigma^+ + \bar{\Sigma}^+ p + \quad) \qquad (17)$$

We see that H_W^{ch} will never lead to the decay $\Sigma^+ \rightarrow n + \pi^+$ so that it is enough if we consider the mutilated interaction

$$H' = H_{strong} + H_W^0(js)$$

which is invariant not only under the transformation (5) and (6) but also under

$$N_1 \rightarrow i\gamma_2 N_2 \, ; \quad N_3 \rightarrow i\gamma_2 N_4$$
$$N_2 \rightarrow -i\gamma_2 N_1 \, , \quad N_4 \rightarrow -i\gamma_2 N_3 \qquad (18)$$
$$\pi^{\pm,0} \rightarrow -\pi^{\mp,0}$$

($H_W^{ch}(js)$ is not invariant under (18)). From (18) we immediately get

$$\langle \Sigma^+ | n\pi^+ \rangle = - \langle \Sigma^+ | n\pi^+ \rangle$$

thus forbidding the decay. One way out of this difficulty suggested by Pais is to take G form of doublet symmetry i.e. assume H strong to be

$$H_{strong} = iG\left[\overline{N}_1 \vec{\tau} \gamma_5 N_1 + \varepsilon\left(\overline{N}_2 \vec{\tau} \gamma_5 N_2 + \overline{N}_3 \vec{\tau} \gamma_5 N_3\right) + \overline{N}_4 \vec{\tau} \gamma_5 N_4\right] \cdot \vec{\pi}$$

where $\varepsilon = \pm 1$. The transformations (5), (6) and (18) are also modified regarding the pion part such that under (5) $\vec{\pi} \rightarrow \varepsilon \vec{\pi}$, under (6) $\pi^{\pm,0} \rightarrow -\varepsilon \pi^{\mp,0}$ and under (18) $\pi^{\pm,0} \rightarrow -\varepsilon \pi^{\mp,0}$. The modified (18) then gives

$$\langle \Sigma^+ | n\pi^+ \rangle = -\varepsilon \langle \Sigma^+ | n\pi^+ \rangle$$

which turns out to be a trivial statement for $\varepsilon = -1$, instead of being catastrophic as in the case $\varepsilon = +1$.

Another possibility suggested Pais to avoid the above difficulty is to welcome the result $\langle \Sigma^+ | n\pi^+ \rangle = 0$ as a blessing in disguise. This is done by considering the weak interaction Hamiltonian to have another term $H_W(J t)$ formed from the isotopic vector current $\vec{J}$ and the current defined by

$$\vec{t} = \frac{1}{\sqrt{2}}\left(\vec{t}_{12} + \vec{t}_{21} + \vec{t}_{34} + \vec{t}_{43}\right) \qquad (19)$$

where

$$\vec{t}_{ik} = \overline{N}_i \vec{\tau} N_k$$

It is readily seen that $H_W(Jt)$ is also invariant under the transformations (5) and (6). (It can be shown that the transformations (5) and (16) leave not only H strong as defined above but also with terms corresponding to the K interaction, $H_W(J\sigma)$ and $H_W(Jt)$ with slight modifications and terms representing weak vertices involving pions and kaons). Further by expanding $\vec{j}$ and $\vec{t}$ we see that $H_W(jt)$ allows $\Sigma^+ \to n + \pi^+$ but forbids $\Sigma^- \to n + \pi^-$. This circumstance enables use to introduce the idea of a parity clash between $H_W(j\Lambda)$ and $H_W(jt)$. We know from neutron beta decay that the current j has both a vector (V) and an axial vector (A) part. There is no such definite information regarding the strangeness violating currents. If we choose s and t to be pure currents (both of them either pure V or pure A), and further assume that the j's occurring in $H_W(jo)$ and $H_W(jt)$ are also pure but opposite (i.e. if one is j_A the other is j_V), then we have the situation that the decay modes $\Sigma^+ \to n + \pi^+$ and $\Sigma^- \to n + \pi^-$ (each of which is allwed to proceed through by only one term of the weak interaction Hamiltonian) will be parity conserving whereas the mode $\Sigma^+ \to p + \pi^0$ which is allowed by both $H_W(jo)$ and $H_W(jt)$ will be parity violating. The justification for choosing s and t to be pure comes from the low observed rates of hyperon β - decay. In the non-relativistic limit, both $\gamma_\mu \gamma_5$ and γ_μ have matrix elements nearly equal to one another so that $\frac{R(V)}{R(V-A)} = \frac{1}{4}$ where R denotes the decay rate. Further $\frac{R(V)}{R(V-1.2A)} = \frac{1}{5.3}$. Thus the choice of pure V or A for the s and t icurrents can explain the smallness of the hyperon β-decay compared to the neutron β-decay (which is V-1.2 A)

In the considerations above we have not taken up the question of the relative parity of the nucleon and cascade particles. This parity is defined only with respect to the K part of the strong interaction. Now if the S-violating currents s and t are pure (V or A) then the relative parity of K and π in the leptonic decay $K^+ \to \pi^0 + e^+ + \nu$ is well defined so that either this decay mode or $K^+ \to l^+ + \nu\ (l = \mu \text{ or } e)$ would be forbidden. Since this is not the case the relative Ξ-nucleon parity must be odd.

If we supplement the set of transformations (5) with the following additions

$$J^{\pm}_{lept} \longrightarrow J^{\pm}_{lept}$$

and the transformations (16) with

$$J^{\pm}_{lept} \longrightarrow J^{\mp}_{lept}$$

we obtain the following relations between the leptonic decay modes of the strange particles in which decays involving $\frac{\Delta Q}{\Delta S} = -1$ currents have also been retained in view of recent experiments

$$\langle \Xi^- | \Lambda l^- \bar{\nu}_l \rangle \sqrt{2} = \langle \Sigma^- | n l^- \bar{\nu}_l \rangle + \langle \Sigma^+ | n l^+ \nu_l \rangle$$

$$\langle \Lambda | p l^- \bar{\nu}_l \rangle = \langle \Xi^0 | \Sigma^+ l^- \bar{\nu}_l \rangle + \langle \Xi^0 | \Sigma^- l^+ \nu_l \rangle$$
$$= \langle \Sigma^+ | n l^+ \nu_l \rangle - \langle \Sigma^- | n l^- \bar{\nu}_l \rangle$$

$$\langle K^+ | \pi^0 l^+ \nu_l \rangle = \langle K^- | \pi^0 l^- \bar{\nu}_l \rangle,$$

$$\langle K^0 | \pi^- l^+ \nu_l \rangle = -\langle \bar{K}_0 | \pi^+ l^- \bar{\nu}_l \rangle$$

$$\langle K^0 | \pi^+ l^- \bar{\nu}_l \rangle = -\langle \bar{K}_0 | \pi^- l^+ \nu_l \rangle.$$

Finally we may mention that the product of transformations (5) and (6) or (5) and (16) extended to include K) mesons are R transformations, i.e. transformations under which $Y \to -Y$ and $Q \to -Q$ where Y and Q are the hypercharge and charge quantum numbers.(15) and (16) together give the following perfect 'octet' symmetry between the baryons and bosons.

$$p \longleftrightarrow -\Xi^- \qquad n \longleftrightarrow \Xi^0 \qquad \Sigma^{\pm,0} \longleftrightarrow -\bar{\Sigma}^{\mp,0}$$
$$K^+ \longleftrightarrow -K^- \qquad K^0 \longleftrightarrow \bar{K}_0 \qquad \pi^{\pm,0} \longleftrightarrow -\pi^{\mp,0}$$

$$\Lambda^0 \longleftrightarrow \Lambda^0$$
$$\pi^{0'} \longleftrightarrow \pi^{0'} \tag{20}$$

where $\pi^{0'}$ represents an isosinglet pion.

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SYMMETRIES SHARED BY STRONG AND WEAK INTERACTIONS-III.

In the previous lecture, the idea of a parity clash between the γs and γt couplings was introduced to explain the Σ-decay asymmetries. We shall now consider a more general scheme of this nature of which the γs and γt couplings are particular examples.

The first point we notice about the doublet approximation scheme is that $\vec{T}$ is the isotopic spin for N_1 and N_4 but not for Σ and Λ which have been artificially redistributed to form the doublets N_2 and N_3. We shall call the isotopic spin of the doublets as the doublet spin, I. For the pion, the actual isotopic spin T = I = 1. We are free to rotate in the (N_2, N_3) plane so that we can unite N_2 and N_3 to form the super-doublet $N_{23} = \begin{pmatrix} N_2 \\ N_3 \end{pmatrix}$ and assign to this doublet a spin called the K-spin. K = 1/2 with $K_3 = 1/2$ for N_2 and $-1/2$ for N_3. The relation between $\vec{T}$, $\vec{I}$, and $\vec{K}$ is $\vec{T} = \vec{I} + \vec{K}$. For the K-meson I = 0, T = K = 1/2.

We notice that H strong (equation (2) of last lecture) not only conserves baryons, but also conserves individual doublets. Thus in the doublet approximation transitions like $\Sigma^{\pm} \rightarrow \Sigma^{\mp}$ + pions is forbidden. If these transitions were allowed we would have the following difficulty. The fact that $\Sigma^+ \rightarrow n + \pi^+$ (Σ_+^+) and $\Sigma^- \rightarrow n + \pi^-$ (Σ_-^-) have nearly zero asymmetry parameters while the $p\pi^0$ mode shows definite asymmetry, together with the T = 1/2 rule implies that Σ^+ and Σ^- have opposite parities with respect to the pion-nucleon system. Thus if Σ_+^+ is pure s (or p) wave, Σ_-^- is pure p (or s) wave. But if it were possible for Σ^- to be part of the time in the virtual form $\Sigma^+ + 2\pi^-$, then Σ_-^- would develop an s (or p) component.

We have seen that the strong interactions (H strong) does not mix the doublets so that I = 0 in this case. If we assign a ΔI to a weak interaction the latter should be expressible in terms of doublets, in so far as the baryons are concerned so that the doublet symmetry is shared between the strong and weak interactions. All the nonleptonic decays (of Λ, Σ and Ξ) have $\Delta K_3 = 1/2$ and $\Delta K = 1/2$. Since the $T = 1/2$ rule is also known to be obeyed, it follows that $I = 0$ or 1 according to the vector addition $\Delta \vec{T} = \Delta\vec{I} + \Delta \vec{K}$. Thus the most general weak interaction Hamiltonian H_W is of the form.

$$H_W = H^{(o)} + H^{(1)} \qquad \cdot\cdot \qquad \cdot\cdot \qquad (1)$$

where $H^{(o)}$ and $H^{(1)}$ correspond to $\Delta I = 0$ and $\Delta I = 1$ respectively.

For the Σ decays we have $|\Delta I_3| = 1$ for Σ_-^- and $\Delta I_3 = 0$ for Σ_+^+ and Σ_o^+. Thus $H^{(o)}$ allows Σ^+ decays but forbits Σ^-. $H^{(1)}$ allows all three decays. We can write $H^{(1)} = H_{+1}^{(1)} + H_{-1}^{(1)} + H_o^{(1)}$ where the subscripts refer to the I_3 value. It looks as if we need only the term $H^{(1)}$ to explain the Σ decay asymmetries We could choose the parity structure of $H_o^{(1)}$ to differ from $H_{\pm 1}^{(1)}$ in such a way that Σ_+^+ (which proceeds via $H_o^{(1)}$) has parity opposite to Σ_-^- (which proceeds via $H_{-1}^{(1)}$.) But then Σ_o^+ would also be parity conserving. Instead of the Σ triangle we would have two amplitudes aligned along the s (or p) axis, the third aligned along the p (or s) axis in the (s, p) plane.

If however we had a prescription to make $H^{(1)}$ contribute only to Σ_o^+ (and not to Σ_+^+) then Σ_+^+ has to proceed through $H^{(o)}$ and Σ_-^- through $H^{(1)}$, while Σ_o^+ can proceed through both $H^{(o)}$ and $H^{(1)}$. Thus if $H^{(o)}$ and $H^{(1)}$ are parity-pure a clash between them could give the asymmetry of the Σ_o^+ decay.

It does not affect $H^{(o)}$ if we subject $H^{(1)}$ to an additional symmetry argument if the latter shares it with H strong. $H^{(1)}$ and $H^{(o)}$ as also $H \pm 1^{(1)}$ and $H_o^{(1)}$ do not interfere since the weak interaction are considered to first order only. Now let $H_o^{(1)}$ shares with H strong invariance under the following transformations:

$$N_1 \to i\ \epsilon_2\ \gamma_2\ N_2, \quad \pi^{\pm} \to -\epsilon\, \pi^{\mp}$$
$$N_2 \to i\ \epsilon_1\ \gamma_2\ N_1, \quad \pi^{o} \to =\epsilon\pi^{o} \qquad (2)$$

where ϵ's are phase factors equal to $\pm$ 1. H strong is left invariant provided $N_1 - N_2$ mass difference is neglected while $\epsilon = \pm 1$ for $G_{\pm}$ symmetries. For $\epsilon = -1$ (but not for $\epsilon = +1$) we must apply $N_3 \Leftrightarrow N_4$. If $H_o^{(1)}$ shares invariance under (2) then

$$< \Sigma^+_+ 1 n \pi^+ > \; = \epsilon_1\ \epsilon_2\ \epsilon < N \mid \Sigma^+\ \pi^-) \quad ..(3)$$

We require $\epsilon_1\ \epsilon_2\ \epsilon = -1$ to forbid this reaction.

If Σ_+^+ and Σ_-^- conserve parity, then using the selection rule $\Delta I = o, 1$; We shall now prove that the $\Lambda \to p + \pi^-$ decay does not conserve parity. For transformation (2) implies that

$$< Y^o \mid p\pi^- > \; = \epsilon_1\, \epsilon_2\, \epsilon < p \mid Y^o \pi^+ >$$
$$= o \text{ if } \epsilon_1\, \epsilon_2\, \epsilon = -1 \qquad (4)$$

Thus $Y_o \to p + \pi^-$ proceeds only via $H^{(o)}$. But $Z_o \to p + \pi^-$ is a $\Delta I_3 = -1^*$ transition and hence can proceed only via $H_{-1}^{(1)}$. Thus adding the two to obtain the $\Lambda \to p + \pi^-$ decay, we have a parity clash between $H^{(o)}$ and $H^{(1)}$.

To arrive at the general structure of $H^{(o)}$ and $H^{(1)}$ let us start with an enumeration of the various types of baryon currents. The most general form for the $\Delta S = 0$, $T = 1$ current is

$$\alpha_1 \bar{N}_1 \vec{\gamma} N_1 + \alpha (\bar{N}_2 \vec{\gamma} N_2 + \bar{N}_3 \vec{\gamma} N_3) + \alpha_4 \bar{N}_4 \vec{\gamma} N_4 + \alpha' \vec{j}' . \quad (5)$$

where

$$j^{+'} = \bar{N}_2 N_3 \sqrt{2} ; \quad j^{-'} = \bar{N}_3 N_2 \sqrt{2} ; \quad j_3 = -(\bar{N}_2 N_2 - \bar{N}_3 N_3) \quad ..(6)$$

By defining the matrices $\rho_1 = \sigma_1$; $\rho_2 = \sigma_2$ and $\rho_3 = -\sigma_3$ to act on the K spin components of N, we can write $\vec{j}' = \bar{N}_{23} \vec{\rho} N_{23}$. Thus the first three terms of (5), have I = 1, K = 0. The j' term has I = 0. K = 1. In general (i) any bilinear baryon current can only have I = 0 or 1; (ii) S — conserving currents have K = 0 or 1 and (iii) $|\Delta S| = 1$ currents have K = 1/2. We can therefore have the following classification of the baryon currents.

$\Delta S = 0$:-- T = 0, I = k = 0; $\rho = \eta_1 \bar{N}_1 N_1 + \eta \bar{N}_{23} N_{23} = \bar{N}_4 N_4$ ..(7)

I = K = 1; $\vec{\rho}' = \bar{N}_{23} \vec{\rho} \vec{\gamma} N_{23}$ ....(8)

T = 1; I = 1, K-0: $\vec{j} = \alpha_1 \bar{N}_1 \vec{\gamma} N_1 + \alpha \bar{N}_{23} \vec{\gamma} N_{23} + \alpha_4 \bar{N}_4 \vec{\gamma} N_4$ ...(9)

, I = 0, K = 1; $\vec{j}' = \bar{N}_{23} \vec{\rho} N_{23}$ ..(10)

T = 2; I = K = 1: The different currents are ϑ^{T_3} where

$$\vartheta^2 = \bar{N}_{23} \rho^- \tau^- N_{23} , \quad \vartheta^1 = -\frac{1}{\sqrt{2}} \bar{N}_{23} (\rho^- \tau_3 + \rho_3 \tau^-) N_{23}$$

$$\vartheta^0 = \sqrt{\tfrac{2}{3}} \bar{N}_{23} [\rho_3 \tau_3 - \tfrac{1}{2} (\rho^- \tau^+ + \rho^+ \tau^-] N_{23}$$

$$\vartheta^{-1} = \frac{1}{\sqrt{2}} \bar{N}_{23} (\rho^+ \tau_3 + \rho_3 \tau^+) N_{23}$$

$$\vartheta^{-2} = \bar{N}_{23} \rho^+ \tau^+ N_{23} \quad (11)$$

$|\Delta S| = 1$:- $T = \frac{1}{2}$, $I = 0$, $K = \frac{1}{2}$: $s = \beta_1 s_1 + \beta_2 s_2$

$I = 1$, $K = \frac{1}{2}$; The different currents are U^{T_3} where $U^{T_3} = \xi_1 u_1^{T_3} + \xi_4 u_4^{T_3}$

$$u_1^{3/2} = -\bar{N}_3 \tau^- N_1 , \quad u_4^{3/2} = -\bar{N}_4 \tau^- N_2$$
$$u_1^{1/2} = \sqrt{2/3}\left[\bar{N}_3 \tau_3 N_1 - \frac{1}{\sqrt{2}} \bar{N}_2 \tau^- N_1\right]$$
$$u_4^{\frac{1}{2}} = \sqrt{2/3}\left[\bar{N}_4 \tau_3 N_2 + \frac{1}{\sqrt{2}} \bar{N}_4 \tau^- N_3\right]$$
$$u_1^{1/2} = \sqrt{2/3}\left[\bar{N}_2 \tau_3 N_1 + \frac{1}{\sqrt{2}} \bar{N}_3 \tau^+ N_2\right]$$
$$u_4^{-\frac{1}{2}} = \sqrt{2/3}\left[-\bar{N}_4 \tau_3 N_3 + \frac{1}{\sqrt{2}} \bar{N}_4 \tau^+ N_2\right]$$
$$u_1^{-\frac{3}{2}} = \bar{N}_2 \tau^+ N_1 \; ; \; u_4^{-3/2} = -\bar{N}_4 \tau^+ N_3 \quad (12)$$
$$s_1 = \begin{pmatrix} \bar{N}_3 N_1 \\ \bar{N}_2 N_1 \end{pmatrix} ; \; s_2 = \begin{pmatrix} \bar{N}_4 N_2 \\ -\bar{N}_4 N_3 \end{pmatrix} ; \; s_3 = \begin{pmatrix} \bar{N}_3 \tau_3 N_1 + \bar{N}_2 \tau^- N_3 \sqrt{2} \\ -\bar{N}_2 \tau_3 N_1 + \bar{N}_3 \tau^+ N_1 \sqrt{2} \end{pmatrix}$$
$$s_4 = \begin{pmatrix} -\bar{N}_4 \tau_3 N_2 + \bar{N}_4 \tau^- N_3 \sqrt{2} \\ -\bar{N}_4 \tau_3 N_3 - \bar{N}_4 \tau^+ N_2 \sqrt{2} \end{pmatrix} ; \; s = \begin{pmatrix} s^- \\ s_0 \end{pmatrix} ; \; \bar{s} = \begin{pmatrix} s^+ \\ s_0 \end{pmatrix} \quad (13)$$

In the light of the above possible baryon currents let us look at the structure of $H^{(0)}$. Let (I_0, K_0) be the (I, K) values of $\Delta S = 0$ currents and $(I_1, K_1,)$ those for $\Delta S = 1$ currents. For $H^{(0)}$ for which the I value is o, we can have $I_0 = I_1 = 0$ or 1. In either case K_0 can be 0 or 1. Thus there are four possibilities.

$I_0 = I_1 = K_0 = 0$: $\quad \rho(s^0 + \bar{s}^0)$

$I_0 = I_1 = 0, K_0 = 1$: $\bar{N}_{23} \rho^+ N_{23} s^- - \frac{1}{\sqrt{2}} \bar{N}_{23} \rho_3 N_{23} s_0 + h.c.$

$I_0 = I_1 = 1, K_0 = 0$: $\vec{J}\,\vec{t}$ where $\vec{t} = \xi_1 (\vec{t}_{12} + \vec{t}_{21}) + \xi_4 (\vec{t}_{34} + \vec{t}_{43})$

$I_0 = I_1 = 1, K_0 = 1$: $\bar{N}_{23} \rho^+ \vec{\tau} N_{23} (\beta_1 \bar{N}_3 \vec{\tau} N_1 + \beta_2 \bar{N}_4 \vec{\tau} N_2)$
$$- \frac{1}{\sqrt{2}} \bar{N}_{23} \rho_3 \vec{\tau} N_{23} (\beta_1 \bar{N}_2 \vec{\tau} N_1 - \beta_2 \bar{N}_4 \vec{\tau} N_3) + h.c. \quad (14)$$

$H^{(1)}$ has an I -spin 1 and the symmetry shared between it and H strong is the product of an I -spin rotation and a $N_1 \longleftrightarrow N_2$ substitution. Since $\rho', \vec{J}$ and ϑ do not respect the transformation $N_1 \leftrightarrow N_2$ we have only three possibilities for $H^{(1)}$ each with $K_o = 0$.

$I_o = 1$, $I_1 = 0$: $\quad g^+ s^- - \frac{1}{\sqrt{2}} g^0 s^0 + h.c$

(gs coupling of Treiman) .. (15)

$I_o = 1$. $I_1 = 1$: $\quad g^+ s'^- - \frac{1}{\sqrt{2}} g^0 s'^0 + h.c$

(gs' coupling) (16)

$I_o = 0$, $I_1 = 1$: $\quad \rho (s'^0 + \bar{s}'^0$

($\rho s'$ coupling) .. (17)

Equations (15 - 17) have the properties of $H^{(1)}$ only if $\alpha_1 = \pm \alpha$ (gs coupling), $\alpha_1 = \mp \alpha$ (gs' coupling) and $\eta_1 = \pm \eta$ ($\rho s'$ coupling) for $\epsilon = \pm 1$ respectively. .. (18)

We are now ready to prove that the asymmetry parameters of $\Sigma^+ \to p + \pi^0$ and $\Lambda \to p + \pi^-$ are of opposite sign. We note that $H^{(o)}$ has $I = 0$ and hence satisfies doublet charge symmetry ($N_1 \leftrightarrow N_2$, $N_3 \leftrightarrow N_4$) by which

$$< Y^0 | p\pi^- > \; = \; < \Sigma^+ | n\pi^+ > \qquad (19)$$

$H^{(1)}_{-1}$ as defined by (15 - 18) shares with H strong invariance under $N_3 \to i\epsilon_1' \tau_2 N_1$; $N_1 \to i \epsilon_3' \tau_2 N_3$

$$\pi^{\pm 0} \to - \epsilon \pi^{\mp, 0}, \qquad \ldots \quad (20)$$

Provided $\epsilon_1' \epsilon_3' \epsilon = -1$, we have

$$< Z^0 | p\pi^- > \; = \; < \Sigma^- | n\pi^- > \qquad \ldots \quad (21)$$

(19) and (21) yield the desired result since by the $T = 1/2$ rule.

$$< \Sigma^+ | p\pi^0 > \sqrt{2} = < \Sigma^+ | n\pi^+ > - < \Sigma^- | n\pi^- > \qquad .. \quad (22)$$

.....

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ISOTOPIC SPIN STRUCTURE OF THE WEAK INTERACTIONS

To make a phenomenological analysis of the structure of weak interactions, we can split the weak interaction Lagrangian L_W into three parts, one involving only leptons $L_{W'}$, one involving both leptons and strongly interacting particles, $L_{W''}$ and the last $L_{W'''}$ involving only strongly interacting particles.

$$L_W = L_{W'} + L_{W''} + L_{W'''} \quad (1)$$

This division is made possible since the leptons do not have strong interactions. The only term of significance in (1) at present is

$$\frac{G}{\sqrt{2}} \left[\bar{\nu}_\mu \gamma_\alpha (1+\gamma_5) \mu \right] \left[e^- \gamma_\alpha (1+\gamma_5) \nu_e \right] + h.c. \quad (2)$$

There may be terms corresponding to weak νe or $\nu \mu$ scatterings butlonly experiments which are being done at present can settle the issue. But the muon-electron symmetry, viz. the fact that the leptonic currents in (2) which are both charged are coupled with the same strength to the current of strongly interacting particles with zero strangeness is taken into account We shall assume this to be true for strangeness-violating decays also. We can then write $L_{W''}$ as

$$\frac{G}{\sqrt{2}} (J_\mu + J_{A\mu})^+ \, l_\mu + h.c.$$
$$l_\mu = \bar{\nu}_e \gamma_\mu (1+\gamma_5) e + \bar{\nu}_\mu \gamma_\mu (1+\gamma_5) \mu \quad (3)$$

where the J's are currents involving strongly interacting particles, $J_{A\mu}$ represents the axial vector part involving $\gamma_5 \gamma_\mu$

For strong interactions isotopic spin, hypercharge and parity are good quantum numbers. Thus whatever the isospin, hypercharge and parity character of J_V and J_A will be maintained even in the presence of strong interactions.

To arrive at these properties of J_V and J_A, we observe that typical terms in the currents whichc contribute to β-decay $n \to p + e^- + \bar{\nu}_e$ are $(\bar{p}\gamma_\mu n)$ and $(\bar{p}\gamma_\mu\gamma_5 n)$ which which transform as the $I_3 = 1$ component of the isovector I = 1. Thus using the label (I, I_3) we can call these currents $J_V(1,1)$ and $J_A(1,1)$ which represent any transitions between two states involving strongly interacting particles differing by I = 1 and $I_3 = 1$ and with the same Y. Thus $J_{\gamma_5\gamma_\mu}(1,1)$ will also contribute to $\pi \to \ell + \nu$

We can now classify all the baryon currents which can lead to charged leptonic transitions (with typical examples) and the decays implied by them.

$\Delta S = 0, \Delta Q = 1$

$$J_A(1,1) : \bar{p}\gamma_\mu\gamma_5 n \qquad \pi^- \to \ell^- + \nu$$

$$J_V(1,1) : \bar{p}\gamma_\mu n \qquad n \to p + \ell^- + \bar{\nu}_e, \quad \Xi^- \to \Xi^0 + \ell^- + \bar{\nu}_\ell$$

$$J_V(2,1) : \bar{\Sigma}^0\gamma_\mu\Sigma^- + \bar{\Sigma}^+\gamma_\mu\Sigma^0 \qquad \pi^- \to \pi^0 + \ell^- + \bar{\nu}_\ell$$

$$J_A(2,1) : \bar{\Sigma}^0\gamma_\mu\gamma_5\Sigma^- + \bar{\Sigma}^+\gamma_\mu\gamma_5\Sigma^0 \qquad \Sigma^- \to \Sigma^0 + \ell^- + \bar{\nu}_\ell$$

$\Delta S = 1, \Delta Q = 1$

$$\mathcal{S}_A(\tfrac{1}{2}, \tfrac{1}{2}) : \bar{p}\gamma_\mu\gamma_5\Sigma^0 + \sqrt{2}\,\bar{n}\gamma_\mu\gamma_5\Sigma^- \qquad \bar{K}^+ \to \ell^- + \bar{\nu}_\ell$$

$$\mathcal{S}_V(\tfrac{1}{2}, \tfrac{1}{2}) : \bar{p}\gamma_\mu\Sigma^0 + \sqrt{2}\,\bar{n}\gamma_\mu\Sigma^- \qquad \bar{K}^+ \to \bar{\pi}^0 + \ell^- + \bar{\nu}_\ell, \quad \Sigma^- \to n + \ell^- + \bar{\nu}_\ell$$

$$\mathcal{S}_V(3/2, 1/2) : \sqrt{2}\,\bar{p}\gamma_\mu\Sigma^0 + \bar{n}\gamma_\mu\Sigma^- \qquad \bar{K}_0 \to \pi^+ + \ell^- + \bar{\nu}_\ell, \quad \Xi^0 \to \bar{\Sigma}^+ + \ell^- + \bar{\nu}_\ell$$

[illegible]

$\underline{\Delta S = 1, \Delta Q = -1}$

$$\left.\begin{array}{lll} s_V(3/2, -3/2) & : \bar{n}\gamma_\mu \Sigma^+ & \bar{K}_0 \to \pi^- + \ell^+ + \nu_\ell \\ s_A(3/2, -3/2) & ; \bar{n}\gamma_\mu\gamma_5 \Sigma^+ & \bar{K}_0 \to \pi^- + \pi^0 + \ell^+ + \nu_\ell \end{array}\right\} \begin{array}{l} \Xi^0 \to \\ \Sigma^- + \ell^+ + \nu_\ell \\ \Sigma^+ \to \\ n + \ell^+ + \nu_\ell \end{array}$$

$\Delta S = 2, \Delta Q = 1$

$$\left.\begin{array}{ll} t_V(0,0) & ; \bar{n}\gamma_\mu \Xi^- + \bar{p}\gamma_\mu \Xi^0 \\ t_A(0,0) & ; \bar{n}\gamma_\mu\gamma_5 \Xi^- + \bar{p}\gamma_\mu\gamma_5 \Xi^0 \\ t_V(1,0) & ; \bar{n}\gamma_\mu \Xi^- - \bar{p}\gamma_\mu \Xi^0 \\ t_A(1,0) & ; \bar{n}\gamma_\mu\gamma_5 \Xi^- - \bar{p}\gamma_\mu\gamma_5 \Xi^0 \end{array}\right\} \begin{array}{l} \Xi^0 \longrightarrow \\ p + \ell^- + \bar{\nu}_\ell \end{array} \qquad (4)$$

In the above K has been assumed to be pseudoscalar and that ΣN and ΞN parities are even. If the latter are not true make the change $\gamma_\mu \longleftrightarrow \gamma_\mu \gamma_5$ with regard to these particles. The brackets indicate that the enclosed currents can contribute to the decay. e.g., presence of $j_V(1,1)$ and $j_A(1,1)$ in β-decay is a consequence of the existence of both Fermi and Gamow-Teller transitions. The presence of $s_A(\frac{1}{2}, \frac{1}{2})$ is implied by the decay $K^+ \to \ell^+ + \nu_\ell$ and those of $s_V(\frac{1}{2}, \frac{1}{2})$ and $s_V(3/2, 1/2)$ by $K^+ \to \pi^0 + \ell^+ + \nu_\ell$. Recent experiments seem to indicate the presence of $s_V(3/2, -\frac{1}{2})$.

Information regarding $s_V(\frac{1}{2}, 1/2)$ and $s_V(3/2, \frac{1}{2})$ can be obtained as follows. We have

$$M(K^0 \rightarrow \pi^- + \ell^+ + \nu_\ell) = \sqrt{2/3}\, f(\tfrac{1}{2}, -\tfrac{1}{2}) + \sqrt{\tfrac{1}{3}}\, f(\tfrac{3}{2}, -\tfrac{1}{2})$$

$$M_+ = M(K^+ \rightarrow \pi^0 + \ell^+ + \nu_\ell) = \sqrt{\tfrac{1}{3}}\, f(\tfrac{1}{2}, -\tfrac{1}{2}) - \sqrt{\tfrac{2}{3}}\, f(\tfrac{3}{2}, -\tfrac{1}{2})$$

where the amplitudes f are

$$\sqrt{2/3}\, f(\tfrac{1}{2}, -\tfrac{1}{2}) = \langle \pi^- | s_\mu(\tfrac{1}{2}, -\tfrac{1}{2}) | K^0 \rangle \ell_\mu$$

$$\sqrt{1/3}\, f(3/2, -\tfrac{1}{2}) = \langle \pi^- | s_\mu(3/2, -\tfrac{1}{2}) | K^0 \rangle \ell_\mu \qquad (6)$$

If we define $M_{1,2} = M(K^0_{1,2} \rightarrow \pi^- + \ell^+ + \nu_\ell)$ then

$$y = \frac{f(3/2, -1/2)}{f(\tfrac{1}{2}, -\tfrac{1}{2})} = \frac{\sqrt{1/2}\left(1 + \frac{M_1}{M_2} - \frac{2M_+}{M_2}\right)}{\left(1 + M_1/M_2 + M_+/M_2\right)} \qquad (7)$$

If one of the amplitudes is zero $(y = 0 \text{ or } \infty)$ then we have $\frac{M_1}{M_2} = \frac{2M_+}{M_2} + 1$ or $-\left(1 + \frac{M_+}{M_2}\right)$. If we assume that the form factors describing the K_{ℓ_3} decays are independent of the pion energy (for which there is some evidence in the decay $K^+ \rightarrow \pi^0 + e^+ + \nu$ then M_1/M_2 can be determined from experiment ~~such~~

Also $\frac{M_+}{M_2} = \pm\sqrt{\frac{R_+}{R_2}}$ where R_+ and R_2 are the rates for the corresponding K_{ℓ_3} decays. Thus the absence of any of these two amplitudes would give rise to predictions that can be verified experimentally.

The matrix element for $\bar{K}^0_{\ell 3}$ decay can be written as

$$M(\bar{K}^0 \longrightarrow \pi^- + \ell^+ + \nu_0) = \langle \pi^- | \mathcal{S}_\mu(3/2, -3/2) | \bar{K}_0 \rangle \ell_\mu \quad (8)$$

If we now assume that the $\mathcal{S}_\mu(3/2, 1)$ are components of the same isotopic spin tensor, then,

$$\langle \pi^- | \mathcal{S}_\mu(3/2, -3/2) | \bar{K}_0 \rangle = \sqrt{3} \langle \pi^- | \mathcal{S}_\mu(3/2, 1/2) | K_0^+ \rangle \quad (9)$$

Then

$$Z = \frac{\sqrt{3}\,(1 - M_1/M_2)}{\left(2\frac{M_+}{M_2} - \frac{M_1}{M_2} - 1\right)} \quad (10)$$

If this happens to be a constant no definite conclusion can be drawn regarding the currents since the various form factors could be constant for purely dynamical reasons. But if experiment shows that this ratio is not a constant independent of the pion energy then $\mathcal{S}_\mu(3/2, 1/2)$ and $\mathcal{S}_\mu(3/2, -3/2)$ would not be components of the same isotopic tensor. The absence of the currents $\mathcal{S}_\mu(3/2, 3/2)$ implies $M_1 = M_2$

Let us now look at the recent experimental data regarding the leptonic decays of hyperons and K-mesons. If we denote by $\Gamma(L^\pm)$ the average over the rates of decays to $\mu^\pm$ and $e^\pm$ then

$$\frac{\Gamma_1}{\Gamma_2} = \frac{\Gamma_1(K_1^0 \longrightarrow \pi^\pm + L^\mp + \nu)}{\Gamma_2(K_2^0 \longrightarrow \pi^\pm + L^\mp + \nu)}$$

$$= 6\,6^{+6.0}_{-4.0} \qquad \text{(Alexander et al)}$$

$$= 3.5^{+3.9}_{-2.7}$$ (Crawford et al)

Also $$\frac{\Gamma_1(e^\pm)}{\Gamma_2(e^\pm)} = 11.9^{+7.5}_{-5.6}$$ (Ely et al))

Thus there seems to be good evidence for $\Delta Q = -\Delta S$ which is strengthened by the observation of a $\Sigma^+ \rightarrow n + \mu^+ + \nu$ event by Galtieri et al.

As regards the $\Delta I = \frac{1}{2}$ rule in leptonic decays. Alexander et al. obtain $\Gamma_2(L^\pm) = (9.31 \pm 2.49) \times 10^6$ sec^{-1} whereas the $\Delta I = 1/2$ rule would give

$$\Gamma_2(L^\pm) = 2\,\Gamma_{K^+}(\pi^0 L + \nu) = (16.5 \pm 1.18) \times 10^6 sec^{-1}$$

Thus this experiment gives a 50/1 odds against $\Delta I = \frac{1}{2}$ which in turn implies the presence of one or both of $\mathcal{S}(3/2, 1/2)$ and $\mathcal{S}(3/2, 3/2)$ currentss.

Another test for the presence of the $I = 3/2$ current would be to study the production crosssections for Σ particles in high energy neutrino experiments,

$$\nu_\mu + n \rightarrow \mu^+ + \Sigma^- \quad \text{and} \quad \nu_\mu + p \rightarrow \mu^+ + \Sigma^0$$

For a pure $\mathcal{S}(\frac{1}{2}, \frac{1}{2})$ current, the ratios of cross-sections in these two reactions is 2:1 while for a pure $I = 3/2$ current it is 1:2

For L_W''' there are two empirical rules which seem to hold fairly well. For $|\Delta S| = 1$ decays $|\Delta I| = 1/2$ while for all decays from L_W''' the rule is $|\Delta S| \leq 1$

The terms of the current-current int~~roduction~~eraction

$$\frac{G}{\sqrt{2}} I_\mu^+ I_\mu = \frac{G}{\sqrt{2}} (J_\mu + J_{A\mu} + l_\mu)^+ (J_\mu + J_{A\mu} + l_\mu) \tag{11}$$

which are not included in L_W are incompatible with $|\Delta I| = \frac{1}{2}$ as an exact rule in the absence of electromagnetic interactions since the product of the strangeness conserving and strangeness violating terms give rise to $|\Delta I| > \frac{1}{2}$. If the $\frac{\Delta Q}{\Delta S} = -1$ decay $K_0 \to \pi^- + l^+ + \nu$ exists and CP invariance holds, then these terms would give rise to a $K_1^0 - K_2^0$ mass difference much larger that that observed. Two ways are open to overcome this difficulty. We can add terms which will cancel off the $|\Delta I| = \frac{3}{2}$ part or rearrange the currents. If e.g, instead of $I_\mu^+ I_\mu$ we have a sum of terms $I_\mu^+(i) I_\mu(i)$, then in some cases it is possible to have both $|\Delta I| = \frac{1}{2}$ and $|\Delta S| \leq 1$. For example

$$I_\mu(1) = J_\mu(1) + \epsilon_1 l_\mu = a\{\delta_\mu(1,1) + \delta_{A\mu}(1,1) + \epsilon_1 l_\mu\}$$

$$I_\mu(2) = J_\mu(2) + J_{A\mu}(2) + \epsilon_2 l_\mu = b\{\lambda_\mu(\tfrac{1}{2},\tfrac{1}{2}) + \lambda_{A\mu}(\tfrac{1}{2},\tfrac{1}{2})\} + c\{t_\mu(0,0) + t_{A\mu}(0,0) + \epsilon_2 l_\mu\}, \tag{12}$$

a model due to d'Espagnat which satisfies the $|\Delta S| \leq 1$ rule since $J_\mu(2)$ carries $\Delta S = +1$ and $+2$ giving rise to terms having $\Delta S = 0, \pm 1$ in $J_\mu^+(2) J_\mu(2)$ and $I_\mu(1)$ carries $\Delta S = 0$ The $\Delta I = \frac{1}{2}$ rule for the $\Delta S = 1$ part of L_{WIII} arises since such terms are formed from $t^+(0,0)\,\lambda(\frac{1}{2},\frac{1}{2})$ which transforms like $(\frac{1}{2},\frac{1}{2})$ component of a spinor. $\Delta Q = -\Delta S$ decays are forbidden but $\Delta S = 2$ decays are permitted.

We can also have currents having 0 and -1 charge. Denoting them by $q = 0, \pm 1$ then:

$$L_W = \frac{G}{\sqrt{2}} \sum_q I_\mu^+(q) I_\mu(q) \tag{13}$$

The most general set of $I_\mu(q)$ which can be formed from $I = 0, \frac{1}{2}, 1, 3/2$ and 2 currents such that $|\Delta I| = \frac{1}{2}$ and $|\Delta S| \leq 1$ are

$$
\begin{aligned}
I_\mu(+2) &= \beta_0 \tilde{s}_\mu(3/2, 3/2) \\
I_\mu(+1) &= \rho_1 \tilde{j}_\mu(1,1) + \frac{\alpha}{\rho_1} \tilde{s}_\mu(\tfrac{1}{2}, 1/2) \\
&\quad + (\beta/\rho_1) \tilde{s}_\mu(3/2, 1/2) + \epsilon_1 l_\mu \\
I_\mu(0) &= \rho_2 \tilde{j}_\mu(1,0) + \frac{\alpha}{\sqrt{2}\rho_2} \tilde{s}_\mu(\tfrac{1}{2}, -\tfrac{1}{2}) + \frac{\sqrt{2}\beta}{\rho_2} \tilde{s}_\mu(3/2, -\tfrac{1}{2}) \\
I_\mu(-1) &= \rho_3 \tilde{j}_\mu(1,-1) + \frac{\sqrt{3}\beta}{\rho_3} s_\mu(3/2, -3/2) + \epsilon_2 l_\mu^+
\end{aligned} \tag{14}
$$

where $\sim$ denotes linear combinations of V and A currents. The constants ρ', α', β' are independent of m.

In the model (14) the $\Delta S = 0$ currents $j(I, I_3)$ are not supposed to exist for $I \neq 1$ as also the ~~currents~~ $\Delta S = 2$ currents $t(I, I_3)$ and the $\Delta S = 1$ currents $s(I, I_3)$ for $I \neq \frac{1}{2}$ or $3/2$

If instead of the current-current picture we introduce an intermediate boson, the weak Lagrangian can be written as

$$L_B = \sum_q g I_\mu^+(q) W_\mu(q) \tag{15}$$

Since the currents we have been using are not hermitian we have an octet of bosons corresponding to the 4 currents in(14). If $\beta_0 = 0$, the number of fundamental bosons is 6 as in the scheme of Lee and Yang or that of Takeda.

With $\beta = 0 = \beta_0$ only two independent currents are needed.

THE FOUR-COMPONENT NEUTRINO

K.Venkatesan

The effect of making the mass of a fermion zero on the solution of the corresponding Dirac equation can be described in two ways. First, we can make the mass term zero in the Dirac equation

$$(\vec{\alpha}\cdot\vec{p} + \beta m)\psi = -i\frac{\partial\psi}{\partial t} \qquad (1)$$

i.e. we no longer need four anticommuting matrices $\vec{\alpha}$ and β but only three of them. We have three such 2 x 2, matrices, namely, the Pauli spin matrices so that eqn.(1) reduces to the two component Weyl equation

$$i\frac{\partial\varphi_\nu}{\partial t} = -\vec{\sigma}\cdot\vec{p}\,\varphi_\nu \qquad (2)$$

where φ_ν is the two-component neutrino wave function. Thus the Hamiltonian H is given by

$$H = -\vec{\sigma}\cdot\vec{p} \qquad (3)$$

so that the eigenstates of energy are also eigenstates of helicity. For a given $\vec{p}$ only two solutions are possible, a positive energy particle with negative helicity (by the choice of sign in (3)) and a positive energy antiparticle which is right handed.

It can be shown that the above two-component neutrino theory is equivalent to the four component one. if we assume in the parity non-conserving β-decay Hamiltonian

$$H_{int} = \sum_{i=S,V,T,A,P}(\bar{\psi}_p\psi_n)\left(C_i\,\bar{\psi}_e\psi_\nu + C_i'\,\bar{\psi}_e\gamma_5\psi_\nu\right)$$

that the coefficients

$$C_S = \pm C_S' \,,\; C_V = \pm C_V' \,,\; etc.$$

for then the neutrino field always appears in Hint in the form $(1 \pm \gamma_5)\psi_\nu$, i.e. it is an eigenstate of chirality. Since the effect of the operator $1 \pm \gamma_5$ is to make the first two and last two components the same except perhaps for a sign, we can in effect use a two component form for the neutrino. This can be seen by taking the solutions of the Dirac equation for a particle and antiparticle with mass m

$$\psi_p = \begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E_p+m} \end{pmatrix} ; \quad \psi_{\bar{p}} = \begin{pmatrix} \dfrac{-\vec{\sigma}\cdot\vec{p}}{E_p+m} \end{pmatrix}$$

and putting the mass m = 0- If we choose the Z-axis as (5)
the direction of $\vec{p}$ we have the four solutions

$$\uparrow\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} ; \quad \downarrow\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} ; \quad \uparrow\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} ; \quad \downarrow\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

(6)

The operator $$1 + \gamma_5 = \begin{pmatrix} I & -I \\ -I & I \end{pmatrix}$$

where I is the 2 x 2 unit matrix projects out only the two middle spinors in (6) and ($1-\gamma_5$) the two end ones. Since the Golhaber experiment on K-capture in Europium showed that the neutrino was left handed, the first set of solutions were retained. But with the recent evidence for two types of neutrino, ν_e and ν_μ it would be interesting to see whether retaining all the four solutions of (6) can explain them.

Nishijima approached the problem by changing the usual formulation of β and μ decay theory with a single neutrino which corresponds to the transformations

$$\psi_\nu \rightarrow \gamma_5 \psi_\nu \, , \quad \psi_\mu \rightarrow \psi_\mu \, ; \quad \psi_e \rightarrow \psi_e \tag{7}$$

to

$$\psi_\nu \rightarrow \gamma_5 \psi_\nu \, ; \quad \psi_\mu \rightarrow -\psi_\mu \, ; \quad \psi_e \rightarrow \psi_e \tag{8}$$

An immediate consequence of this change is that reactions like.

$$\mu \rightarrow 3e \text{ or } e + \gamma \, , \quad \mu + p \rightarrow p + e \tag{9}$$

are forbidden. Also μ decay would be forbidden if we assume a neutrino-antineutrino pair to accompany the electron. But the decay will proceed if we can write

$$\mu^- \rightarrow e^- + \bar{\nu}_R + \bar{\nu}_L \tag{10}$$

which requires two antineutrinos with opposite helicity and thus is a 4-component neutrino theory. Further if e^- is a particle μ^- is an antiparticle if lepton conservation is to hold. Such an assumption ~~was~~ made earlier by Konopinski and Mahamoud.

Sokolov introduces the four component neutrino by constructing the projection operator

$$S_\epsilon = \frac{1}{2}\left(1 + s_\epsilon \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\right) \tag{11}$$

where

$$s_\epsilon = -\epsilon \tag{12}$$

and

$$S_\epsilon = +\epsilon \tag{13}$$

(12) corresponds to the solution with left handed helicity and eqn.(13) that with right-handed helicity. For mass zero particles,

$$S = \frac{1}{2}(1 \pm \gamma_5)$$

(For mass non-zero particles the operator S may be used to study the longitudinal polarisation if one remembers that in another Lorentz frame transverse polarisation may emerge). Sokolov associates s_ϵ and S_ϵ with the electonic and muonic neutrinos respectively ($\epsilon = +1$ and -1 corresponds to the particle antiparticle respectively in each case). The interaction Hamiltonian for β and μ decay is of the form

$$H = \sum_j G_j I_j^{+} I_j + h.c \tag{14}$$

where

$$I_j = \Psi_{e^-}^{+} \beta_j s_\epsilon \Psi_\nu + \Psi_\nu^{+} \beta_j S_\epsilon \Psi_{\mu^+} + \Psi_n^{+} \beta_j \Psi_p \tag{15}$$

β_J represents the Dirac matrices corresponding to $V-A$.

As in the theory of Nishijima, μ^+ has to be the particle and μ^- the antiparticles. An interesting consequence of this is that the decay of

$$\pi^- \rightarrow \mu^- + \nu_R'$$

is allowed wereas

$$\pi^- \longrightarrow e^- + \bar{\nu}_L$$

is strictly forbidden. In the ~~one~~ two-component neutrino theory both decays should have been forbidden if both e and μ were 100% polarized (for then they would come out with the wrong helicity).= The large branching ratio of the μ mode compared to the e mode was explained as due to the fact that the μ emitted would be less relativistic than e. Processes like (9) are also forbidden. Transition of muonium $(e^-\mu^+)$ to antimuonium $(e^+\mu^-)$ is not allowed.

References

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