# CMSC 330: Organization of Programming Languages

Context-Free Grammars

## Review

- Why should we study CFGs?
- What are the four parts of a CFG?
- How do we tell if a string is accepted by a CFG?
- What's a parse tree?



## Review

A *sentential form* is a string of terminals and non-terminals produced from the start symbol

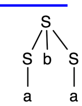
Inductively:

- The start symbol
- If $\alpha A \delta$ is a sentential form for a grammar, where ($\alpha$ and $\delta \in (N|\Sigma)^*$), and $A \rightarrow \gamma$ is a production, then $\alpha\gamma\delta$ is a sentential form for the grammar
  - In this case, we say that $\alpha A \delta$ *derives* $\alpha\gamma\delta$ in one step, which is written as $\alpha A \delta \Rightarrow \alpha\gamma\delta$



## Leftmost and Rightmost Derivation

- Example: S → a | SbS String: aba




Leftmost Derivation
S ⇒ SbS ⇒ abS ⇒ aba
At every step, apply production to leftmost non-terminal

Rightmost Derivation
S ⇒ SbS ⇒ Sba ⇒ aba
At every step, apply production to rightmost non-terminal

- Both derivations happen to have the same parse tree
- A parse tree has a unique leftmost and a unique rightmost derivation
- Not every string has a unique parse tree
- Parse trees don't show the order productions are applied

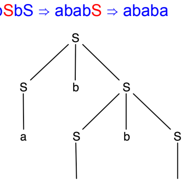


## Another Example (cont'd)

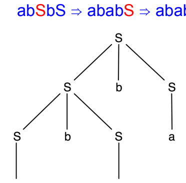
S → a | SbS

- Is ababa in this language?

A leftmost derivation
S ⇒ SbS ⇒ abS ⇒ abSbS ⇒ ababS ⇒ ababa



Another leftmost derivation
S ⇒ SbS ⇒ SbSbS ⇒ abSbS ⇒ ababS ⇒ ababa





## Ambiguity

- A string is *ambiguous* for a grammar if it has more than one parse tree
  - Equivalent to more than one leftmost (or more than one rightmost) derivation
- A grammar is *ambiguous* if it generates an ambiguous string
  - It's can be hard to see this with manual inspection
- Exercise: can you create an unambiguous grammar for S → a | SbS ?

## Are these Grammars Ambiguous?

(1) S → aS | T
T → bT | U
U → cU | ε

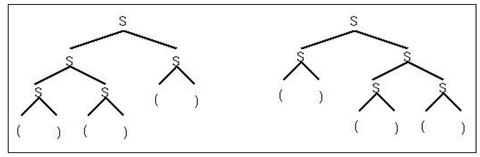
(2) S → T | T
T → Tx | Tx | x | x

(3) S → SS | () | (S)



## Ambiguity of Grammar (Example 3)

- 2 different parse trees for the same string: ()()()
- 2 distinct leftmost derivations :
  - S ⇒ SS ⇒ SSS ⇒()SS ⇒()()S ⇒()()()
  - S ⇒ SS ⇒ ()S ⇒()SS ⇒()()S ⇒()()()



- We need unambiguous grammars to manage programming language semantics



## More on Leftmost/Rightmost Derivations

- Is the following derivation leftmost or rightmost?
  - S ⇒ aS ⇒ aT ⇒ aU ⇒ acU ⇒ ac
  - There's at most one non-terminal in each sentential form, so there's no choice between left or right non-terminals to expand
- How about the following derivation?
  - S ⇒ SbS ⇒ SbSbS ⇒ SbabS ⇒ ababS ⇒ ababa



## Tips for Designing Grammars

1. Use recursive productions to generate an arbitrary number of symbols

   A → xA | ε  Zero or more x's

   A → yA | y  One or more y's

2. Use separate non-terminals to generate disjoint parts of a language, and then combine in a production

   G = S → AB
   A → aA | ε
   B → bB | ε
   L(G) = $a^*b^*$



## Tips for Designing Grammars (cont'd)

3. To generate languages with matching, balanced, or related numbers of symbols, write productions which generate strings from the middle

   $\{a^nb^n \mid n \geq 0\}$ (not a regular language!)
   S → aSb | ε
   Example: S ⇒ aSb ⇒ aaSbb ⇒ aabb

   $\{a^nb^{2n} \mid n \geq 0\}$
   S → aSbb | ε



## Tips for Designing Grammars (cont'd)

$\{a^nb^m \mid m \geq 2n, n \geq 0\}$
S → aSbb | B | ε
B → bB | b

The following grammar also works:
S → aSbb | B
B → bB | ε

How about the following?
S → aSbb | bS | ε

## Tips for Designing Grammars (cont'd)

$\{a^n b^m a^{n+m} \mid n \geq 0, m \geq 0\}$

Rewrite as $a^n b^m a^m a^n$, which now has matching superscripts (two pairs)

Would this grammar work?

$S \rightarrow aSa \mid B$ Doesn't allow m = 0

$B \rightarrow bBa \mid ba$

Corrected:

$S \rightarrow aSa \mid B$

$B \rightarrow bBa \mid \varepsilon$

The outer $a^n a^n$ are generated first, then the inner $b^m a^m$



## Tips for Designing Grammars (cont'd)

4. For a language that's the union of other languages, use separate nonterminals for each part of the union and then combine

$\{ a^n(b^m|c^m) \mid m > n \geq 0\}$

Can be rewritten as

$\{ a^n b^m \mid m > n \geq 0\} \cup$

$\{ a^n c^m \mid m > n \geq 0\}$



## Tips for Designing Grammars (cont'd)

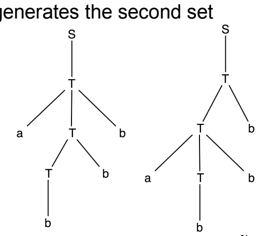
$\{ a^n b^m \mid m > n \geq 0\} \cup \{ a^n c^m \mid m > n \geq 0\}$

$S \rightarrow T \mid U$

$T \rightarrow aTb \mid Tb \mid b$ T generates the first set

$U \rightarrow aUc \mid Uc \mid c$ U generates the second set

- What's the parse tree for string abbb?
  - Ambiguous!






## Tips for Designing Grammars (cont'd)

$\{ a^n b^m \mid m > n \geq 0\} \cup \{ a^n c^m \mid m > n \geq 0\}$

Will this fix the ambiguity?

$S \rightarrow T \mid U$

$T \rightarrow aTb \mid bT \mid b$

$U \rightarrow aUc \mid cU \mid c$

- It's not amgiguous, but it can generate invalid strings such as babb



## Tips for Designing Grammars (cont'd)

$\{ a^n b^m \mid m > n \geq 0\} \cup \{ a^n c^m \mid m > n \geq 0\}$

Unambiguous version

$S \rightarrow T \mid V$

$T \rightarrow aTb \mid U$

$U \rightarrow Ub \mid b$

$V \rightarrow aVc \mid W$

$W \rightarrow Wc \mid c$



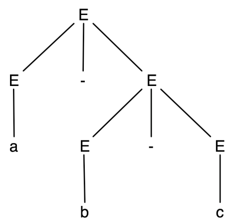
## CFGs for Languages

- Recall that our goal is to describe programming languages with CFGs
- We had the following example which describes limited arithmetic expressions

  $E \rightarrow a \mid b \mid c \mid E{+}E \mid E{-}E \mid E{*}E \mid (E)$
- What's wrong with using this grammar?
  - It's ambiguous!

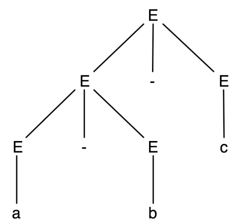
## Example: a-b-c

E ⇒ E-E ⇒ a-E ⇒ a-E-E ⇒ a-b-E ⇒ a-b-c




Corresponds to a-(b-c)

E ⇒ E-E ⇒ E-E-E ⇒ a-E-E ⇒ a-b-E ⇒ a-b-c




Corresponds to (a-b)-c



## The Issue: Associativity

- Ambiguity is bad here because if the compiler needs to generate code for this expression, it doesn't know what the programmer intended
- So what do we mean when we write a-b-c?
  - In mathematics, this only has one possible meaning
  - It's (a-b)-c, since subtraction is *left-associative*
  - a-(b-c) would be the meaning if subtraction was *right-associative*



## Another Example: If-Then-Else

<stmt> ::= <assignment> | <if-stmt> | ...
<if-stmt> ::= if (<expr>) <stmt> |
if (<expr>) <stmt> else <stmt>

- (Here <>'s are used to denote nonterminals and ::= for productions)

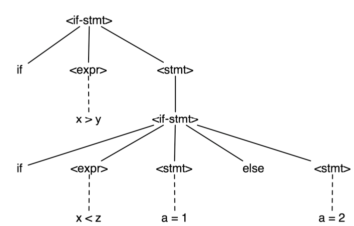
- Consider the following program fragment:

```
if (x > y)
 if (x < z)
  a = 1;
 else a = 2;
```

- Note: Ignore newlines



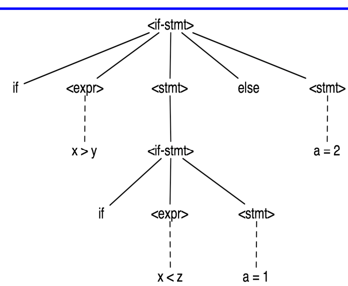
## Parse Tree #1




- Else belongs to inner if



## Parse Tree #2




- Else belongs to outer if

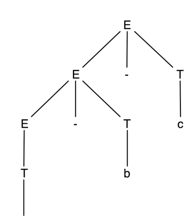


## Fixing the Expression Grammar

- Idea: Require that the right operand of all of the operators is not a bare expression

  E → E+T | E-T | E*T | T
  T → a | b | c | (E)

- Now there's only one parse tree for a-b-c
  - Exercise: Give a derivation for the string a-(b-c)



## What if We Wanted Right-Associativity?

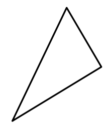
- Left-recursive productions are used for left-associative operators
- Right-recursive productions are used for right-associative operators
- Left:
  - E → E+T | E-T | E*T | T
  - T → a | b | c | (E)
- Right:
  - E → T+E | T-E | T*E | T
  - T → a | b | c | (E)



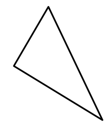
## Parse Tree Shape

- The kind of recursion/associativity determines the shape of the parse tree

left recursion



right recursion

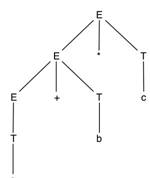


– Exercise: draw a parse tree for a-b-c in the prior grammar in which subtraction is right-associative



## A Different Problem

- How about the string a+b*c ?
  - E → E+T | E-T | E*T | T
  - T → a | b | c | (E)




- Doesn't have correct precedence for *
  – When a nonterminal has productions for several operators, they effectively have the same precedence
- How can we fix this?



## Final Expression Grammar

| | |
|---|---|
| E → E+T \| E-T \| T | lowest precedence operators |
| T → T*P \| P | higher precedence |
| P → a \| b \| c \| (E) | highest precedence (parentheses) |

- Exercises:
  – Construct tree and left and and right derivations for
    - a+b*c a*(b+c) a*b+c a-b-c
  – See what happens if you change the last set of productions to P → a | b | c | E | (E)
  – See what happens if you change the first set of productions to E → E +T | E-T | T | P