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Mathematical modeling of Black Sea level change for the last 20000 years



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ABSTRACT

A mathematical model of the Black Sea level change for the last 20,000 years is presented in this paper, based on the equations of viscous incompressible fluid motion according to Navier–Stokes, Sen–Venans, and the equation of water balance in the sea. The solution of the differential equations shows that the Black Sea level has been generally rising due to glacier melting and increase of freshwater balance in the late Pleistocene. Water which was flowing into the Black Sea did not have time to fully flow into the Marmara Sea because water was flowing through the long and very narrow canyon, which the Bosphorus strait was then. Water accumulating in the Black Sea increased the sea level, and increased the slope of the surface of river in the strait towards the Marmara Sea. When the level of the world ocean rose above river level, the strait began to form. Increasing depth caused excessive water to flow from the Black Sea. As a result of regression, the Black Sea level approached the Marmara Sea level, and the global oceanic transgression of the two seas began, continuing to the present.

When the depth of the strait was 12–15 m, the penetration of the Marmara Sea's dense water into the Black Sea occurred. Water discharge in the counterflow was increasing when the ocean level was rising.

The calculations showed that secondary fluctuations of the Black and Marmara Seas levels greater than 50 cm were theoretically impossible during the Holocene, as the deep strait allowed the possibility of intensive flow toward either the Black or Marmara Sea. When the value of the freshwater balance is positive, the regression of the Black Sea level below Marmara Sea level is theoretically impossible.

Vertical movement of Earth crust can modify the shape of the eustatic sea level curves. A method allows calculation of the eustatic sea level course using the known local curves. The eustatic Mediterranean Sea level change and world ocean level change are calculated using nine local Mediterranean Sea level change curves. Tectonic components of the nine local curves were calculated. An assessment of the method's accuracy was made.

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1. Introduction

One of the fundamental paleoceanological tasks is study of glacioeustatic sea level changes and changes of the inland sea levels caused by this process. Several issues are crucial for long-term climate forecasting and for understanding the processes occurring in the Pleistocene. One is the question of whether the Black Sea could be drying during any periods, if evaporation was greater than the volume of water entering in the sea. Some articles have presented the hypothesis that the Black Sea dried during the glacial period, during the melting of glaciers, and even 8300 years ago.

However, this seems doubtful, because a large excess volume of freshwater (200 km³ per year) goes into the sea. The Bosphorus counterflow has been bringing the same volume of saline Marmara water throughout the Holocene.

The second unsolved problem is the question about numerous multi-meter secondary level fluctuations of the Black Sea during the Holocene, in the modern climate and the current situation in the Bosphorus Strait, presented by many authors. This seems unlikely because water can flow freely in opposite directions through the strait, thus aligning the levels of the neighboring seas. In this regard, the following questions arise: If such fluctuations occurred during the Holocene, why did they not occur 500 years in the past? What caused these fluctuations, if they stopped quite unexpectedly? If the level of the Black Sea begins to oscillate in the future, it may have oscillated in the Holocene as many authors suggest, and

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the impact of the sea on human society will be clearly more disastrous than was expected. However, in our opinion, Black Sea level rise due to greenhouse effect is unlikely.

A third unsolved issue is the following: Was the Black Sea level able to fall below the level of the Marmara Sea, if it has shown numerous fluctuations during the Holocene? Theoretically, if the freshwater balance of the Black Sea was positive, its level would always be above the Marmara Sea. The level of the Black Sea could be below only if the freshwater balance of the sea was negative.

Another problem is the study of the Black Sea freshwater balance. Its sign and absolute value determine the basic processes associated with changes of the sea level. Esin (1987) showed that it has been positive during the last 20000 years. This conclusion established the basic hydrodynamic and lithodynamic processes in the Black Sea and the Bosphorus.

In this paper, a mathematical model of the Black Sea basin filling by water during a transgression of the ocean level is presented. This model describes the course of the eustatic sea level. The numerical solution of differential equations shows how the level of the Black Sea was changing over the last 20,000 years.

A method of calculating the eustatic change of the sea level using the local (relative) curves is offered. The course of the world ocean level and the Black Sea level during the Holocene has been calculated using this method. The inverse calculation of the local curves allows objective evaluation of the credibility of this method by comparing the theoretical local curves with the actual local curves.

Significant progress in studying the problem of world ocean fluctuations has been made from 2005 to 2011 during the implementation of projects IGCP521-INQUA501. During this time the results of studies in the Black and Marmara seas and in the Bosphorus were presented and analyzed, and some theoretical generalizations were made. New results obtained relate to: the changes of the Black Sea water balance for the last 20,000 years; the lithodynamic processes in the Bosphorus Strait; the process of the Bosphorus and Dardanelles development during the Pleistocene; and the tectonic processes in the sea, which are greatly distorting the curve of eustatic level changes.

2. Estimation of the Black Sea freshwater balance value and sign during the Late Pleistocene–Holocene

The value and sign of the Black Sea freshwater balance are the main parameters which determine the change of its level during the period when the world ocean level was below the level of the Black Sea. If the freshwater balance was negative, then the Black Sea was drying up, its level was below the Bosphorus floor and the basin held an enclosed lake. If the balance was positive, then the sea level was above the Bosphorus floor, and the excess of freshwater flowed from the Black Sea to the Marmara Sea. As a result of this, desalination of Marmara and Aegean seas waters was occurring.

Some conclusions about the value and sign of the Black Sea freshwater balance during the glacial period can be done through estimation of modern freshwater balance values. These parameters which had been calculated by various authors are presented in Dzhoashvili (2003). The freshwater balance is estimated from +137 to +300 km³ per year. According to eighteen authors, the average balance is 212 km³. This is approximately equal to the Danube River runoff. According to earlier generalizations by Ovchinnikov et al. (1976), the freshwater balance equals 180 km³ per year, and according to our calculations (Zhiliyev and Esin, 1990), 190 km³ per year. Thus, all known values of the freshwater balance are similar. New data indicate that the Black Sea freshwater balance is 300 km³ per year, and the total flow of water to the Marmara Sea through the Bosphorus Strait is 612 km³ per year (Unluata et al., 1990).

Measurements of current velocity and other parameters in the strait over six months have shown that water circulation is much more complicated than previously believed (Jarosz et al., 2011). For this reason, the question about the Black Sea freshwater balance and water discharge during low periods seems to be unsolved. Calculations based on 50-year observation give an average water balance for this period. Freshwater balance in the Black Sea is equal to 200–212 km³ per year, and such a volume of water is brought by bottom backflow in the strait.

Through mathematical models, Kislov and Toropov (2005, 2006 a, b) showed that the river flow was 175.5 km³ per year, and the evaporation and precipitation was less than their present values by 20–30% and 30–40% respectively during the glacial period. This allows us to calculate the freshwater balance of the Black Sea during the glacial period, when its level was approximately –100 m, and the area of the sea was less than today by 24%. For this purpose, we take the average balance as described above, and decrease the values of evaporation and precipitation by 24% initially, and then by 30% and 40% respectively. We obtain: evaporation of 176 km³ per year, and precipitation of 98 km³. Taking into account the river flow (rounded to 176 km³ per year), the freshwater balance is 98 km³ (Esin et al., 2010b). The balance is positive and very significant and is roughly equal to the total runoff of three rivers with the volume of the Don.

There is geological evidence that a river flowed along the Bosphorus during the glacial period and late Pleistocene (Hiscott et al., 2002). Moreover, the Bosphorus and the Dardanelles were developed as rivers, with channels created by erosion, as shown by geological and geomorphological studies (Yilmaz and Gokasan, 2009).

Important evidence of incoming freshened waters in the Marmara Sea from the Black Sea during the glacial period is the formation of sapropels in the Marmara Sea. This requires that the upper layer of water had been freshened for a long time (Thunell et al., 1983, 1984). Desalinated water was coming from the Black Sea. Other sources of fresh water have never been present.

Parameters of the Black Sea freshwater balance which are presented characterize the hydrological situation in the Bosphorus Straits in the case of absence of the bottom counterflow. The volume of freshwater runoff plus compensatory bottom counterflow began to arrive into the Marmara Sea in the Holocene, when the bottom counterflow was formed. Currently, the average value of runoff into the Marmara Sea is 412 km³. Taking into account the freshwater balance, the bottom counterflow runoff is 200 km³. In this case, the boundary surface of upper flow and counterflow is the bottom (floor) of the upper flow of water, where the velocity of flow equals zero.

3. Geological structure of the Bosphorus Strait and evolution of its base during the Pleistocene

Fig. 1 shows a geological longitudinal cross section of the Bosphorus Strait (Algan, 2001). The base of the strait is composed by Quaternary sediment lying on the solid rocks of the Paleozoic. The base of the northern part of the strait is almost horizontal. The depth there is approximately 70 m. South of the southern threshold, the thickness of sediments increases, and the depth of the strait is reduced to 36 m.

The surface of the bedrock is aligned with three “outliers” (hills). The north-aligned surface is situated at a depth of approximately 100 m, close to the Black Sea level during the glacial period. It is likely that this surface was formed by erosion and denudation.

A roughly similar situation is observed in the Marmara Sea. There, the aligned surface is situated at a depth of 125 m, and it corresponds to the position of ancient shorelines of the Marmara

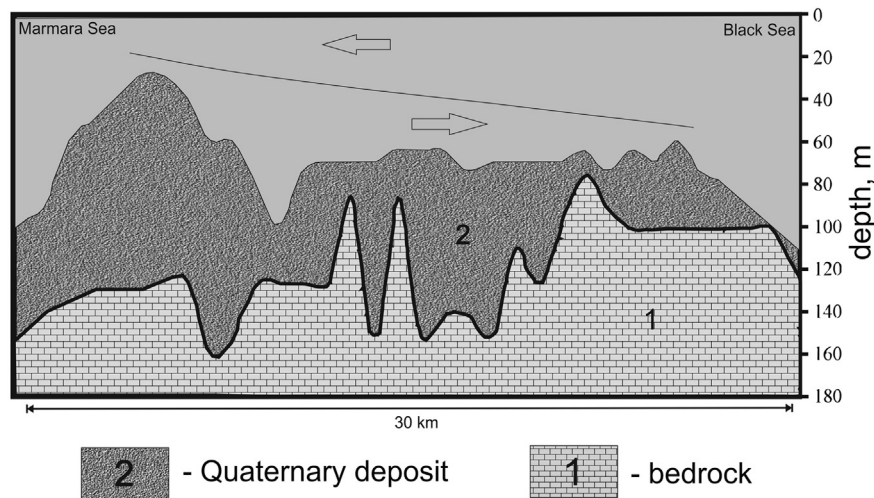


Fig. 1. Longitudinal geological cross section of the Bosphorus Strait according to [Algan \(2001\)](#). 1 – Bedrock (Paleozoic), 2 – Quaternary sediments.

Sea. Possibly, this surface was also created by erosion and denudation.

The northern threshold of the strait, which is formed by bedrock, is situated at a depth of approximately 80 m, and the top of the southern threshold is situated a few meters below. This indicates that the ancient river flowed from the Black Sea to the Marmara Sea in the late Pleistocene. The slope of the southern slope of the southern threshold is 0.06. The cross-section of the strait ([Fig. 2](#)) shows that the width of the river flowing through the strait at low sea levels is 150–200 m. It can be assumed that the width of the canyon is even smaller in the area of a narrow strait. Consequently, the canyon formed by erosion of the river ([Yilmaz and Gokasan, 2009](#)) was deep and extremely narrow.

[Fedorov \(1978\)](#) showed that the four last transgressive–regressive cycles of the Black Sea level changes associated with glacioeustatic fluctuations of ocean level had occurred in approximately the same scenario during the Pleistocene ([Fig. 3](#)). This scenario allows us to describe the events in the interval from 10 to 20 ka BP. A situation close to the modern condition had been present in the strait by the end of Karangatsk transgression: freshwater balance of the sea had been positive, and unconsolidated sediments had accumulated in the southern part of the strait. When the next regression of the world ocean began, with strait depth decreasing, the strait began to transform into a river which was flowing into the Marmara Sea. As ocean level decreased, the river eroded the cover of unconsolidated sediment. The river

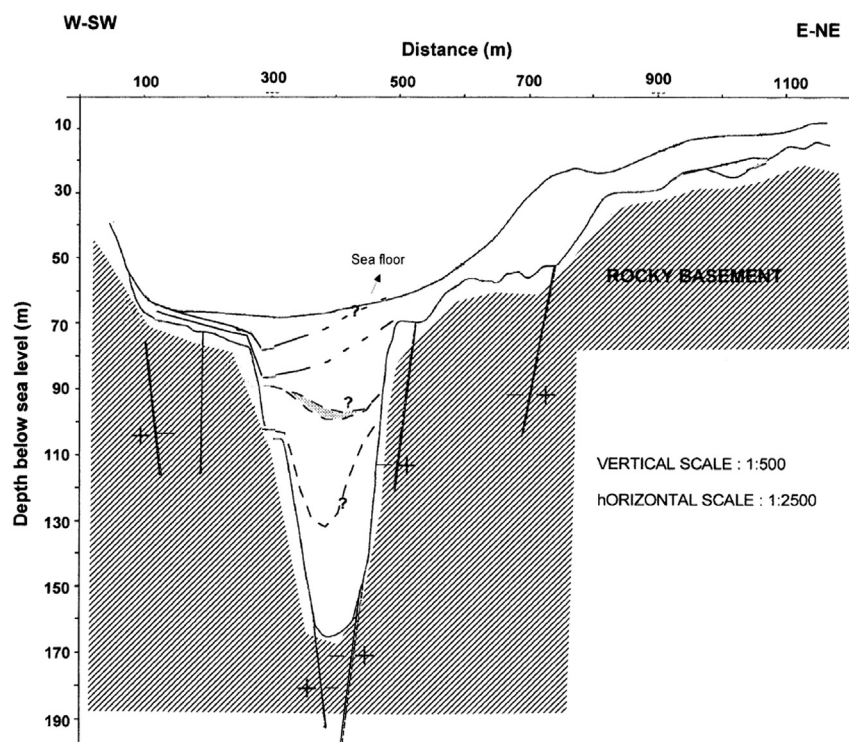


Fig. 2. Section across the centre of Bosphorus according to [Algan \(2001\)](#).

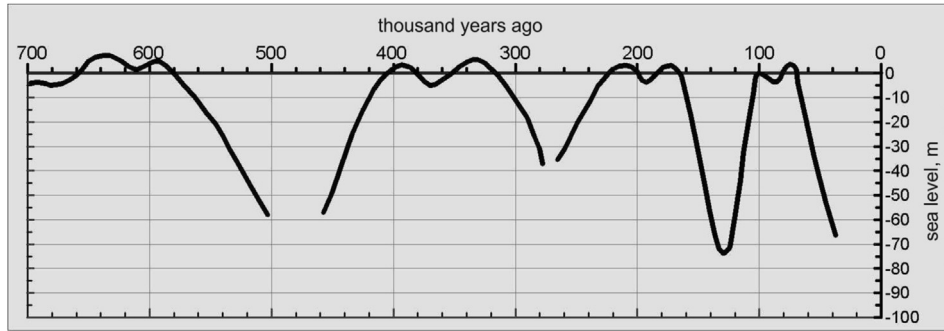


Fig. 3. Black Sea level Change during the Pleistocene according to Fedorov (1978).

flowed through a channel which had been controlled by solid bedrock during the late Pleistocene (Fig. 4). During ocean transgression, the ocean level rose above the river level, and the process of the Bosphorus Strait formation began.

4. Mathematical model of water flow in the Bosphorus Strait

During low levels of the Marmara Sea, the river which flowed from the Black Sea first flowed in a slightly sloped riverbed, and then through the much steeper southern area. It is possible to describe the flow through a slightly inclined plane, and then through an inclined plane, and to splice these solutions.

4.1. Flow of water along a slightly inclined horizontal plane

We use the Navier–Stokes equations (Kochin et al., 1948) for describing this flow. In the case of the plane problem with coordinate system as shown in Fig. 5, for the slightly inclined part of the river we can write:

$$g \sin \alpha_1 - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} = 0 \tag{1}$$

$$\frac{\partial p}{\partial z} = -\rho g \cos \alpha_1 \tag{2}$$

$$\int_0^h u dz = \frac{Q}{l} \tag{3}$$

where, g – is the acceleration of gravity, ρ – is the density of water, p – is the hydrodynamic pressure, u – the velocity of the water flow along the axis x , Q – is the water discharge in the river, H – is the level of the Black Sea relative to a given coordinate system, $h = h(x)$ – is the depth of the river (in case $x = 0, h = H$), ν – is the coefficient of the kinematic viscosity, l – is the wide of the river, α – is the angle of the river bottom to the plane of the horizon.

In the Navier–Stokes equations were taken into account that the vertical velocity of the water is three orders of magnitude smaller than the horizontal velocity. Equations (1)–(3) describe the current in linear approximation.

The boundary condition for the resulting system of equations: when $z = 0, u = 0$; when $z = 0, u = 0$; when $z = h, \partial u / \partial z = 0$, when $z = h, p = 0$. The solution of the equations system is written as:

$$p = \rho g \cos \alpha_1 (h - z);$$

$$U = \frac{g}{2\nu} \cos \alpha_1 \left(\frac{dh}{dx} - \text{tg} \alpha_1 \right) z(z - 2h); \tag{4}$$

$$Q = \frac{lg}{3\nu} \cos \alpha_1 \left(\text{tg} \alpha_1 - \frac{dh}{dx} \right) h^3; \frac{dh}{dx} < 0. \tag{5}$$

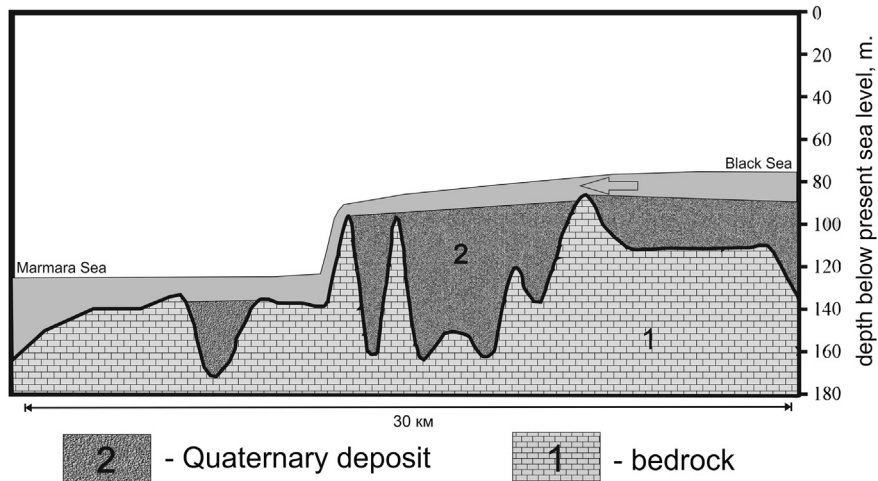


Fig. 4. Water flow in the Bosphorus during low Marmara Sea level.

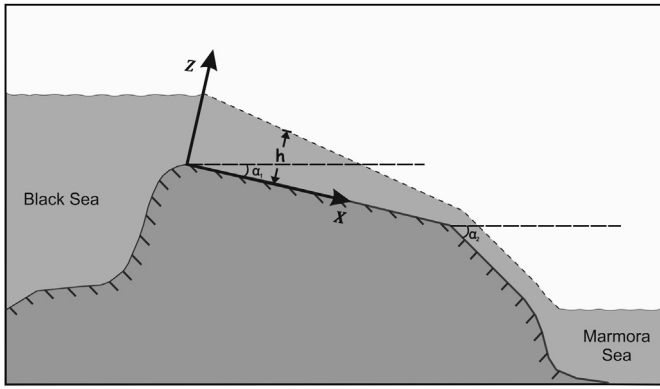


Fig. 5. Water flow along an inclined plane with a fracture.

Equation (5) was used to solve the problem of the maximum possible increase of the Don River level during the spring floods (Esin and Zhiliyaev, 1971). Solution of the differential Equation (5) can be achieved for a given value of Q and the maximum level of the receiving basin (the Azov Sea). In this formulation, the problem of calculation of the Don River water surface exactly corresponds to the problem of calculation of the river water surface in the Bosphorus Strait.

A comparison of the numerical solution of equation (5) with a water surface of the Don River obtained based on the instrumental measurements at different elevations of the water level in Taganrog Bay is shown in Fig. 6. The water surface profiles calculated with $\nu=3 \cdot 10^{-3} \text{ m}^2/\text{s}$ are very close to the real profiles. The error will not exceed a few centimeters in the calculations with increasing ν . Thus, the adaptation of the model obtains solutions with accuracy of a few centimeters throughout 60 km of the river.

A model given as equations (1)–(3) describes the water surface of the river rather well. It can be used for the description of the river water surface the Bosphorus Strait when the ocean level is low.

In the case of a given value Q the changes of the flow depth $h(x)$ can be found by solving equation (5), with the following boundary conditions: at $x = L$, $h = h_0$, where L – is length of the slightly inclined section of the channel, h_0 – is depth of flow at the end of this site.

In the case of horizontal bottom ($\alpha_1 = 0$) we obtain from (5):

$$Q = -\frac{gl}{3\nu} \frac{dh}{dx} h^3 \quad (6)$$

If the river flows along the inclined bottom, without change of the depth ($dh/dx = 0$), then

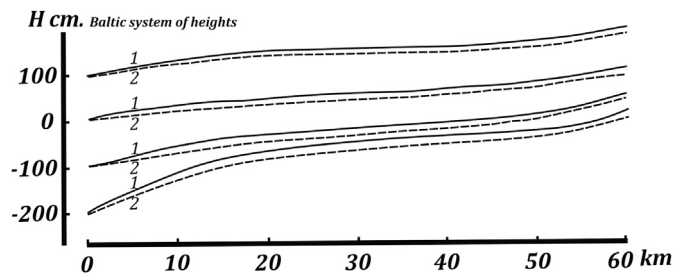


Fig. 6. The calculated profiles of the water surface in the mouth of the Don River in cases of different positions of the sea level. The profiles were obtained theoretically (2), and constructed according to the actual observations at gauges (1) (Esin and Zhiliyaev, 1971).

$$Q = \frac{gl}{3\nu} \sin \alpha_2 h_0^3 \quad (7)$$

where α_2 – is angle of the river bottom.

Hence, we find the value h_0 in the end of the slightly inclined riverbed

$$h_0 = \sqrt[3]{\frac{3\nu Q}{gl \sin \alpha_2}} \quad (8)$$

Thus, equation (5) describes the depth of the water flow in the strait, and the formula (8) describes the depth of the water flow on the southern slope of the Bosphorus southern threshold.

If we neglect the very small slope of the river bottom in the strait and take $\text{tg} \alpha_1 = 0$, then it is possible to obtain the solution of the equation (6) as:

$$h^4 - h_0^4 = \frac{12\nu Q}{gl} (L - x) \quad (9)$$

The water flow along the horizontal plane can be “spliced” with the water flow along the inclined plane as follows.

Because in this situation $h = H$, where $x = 0$, we find from (9):

$$H^4 - h_0^4 = \frac{12\nu QL}{gl} \quad (10)$$

or:

$$Q = \frac{gl}{12\nu L} (H^4 - h_0^4) \quad (11)$$

Equating the expression for Q from (7) and (11), we obtain

$$H^4 = h_0^3 (h_0 + 4L \sin \alpha_2) \quad (12)$$

Because in this situation $h_0 \ll 4L \sin \alpha_2$, we find

$$h_0 = H \left(\frac{H}{4L \sin \alpha_2} \right)^{\frac{1}{3}} \quad (13)$$

The discharges of the water flows along the horizontal and inclined planes will be the same with h_0 . It is determined by the depth in the beginning of the river, the level of the Black Sea.

$$Q = \frac{glH^4}{12\nu L} \left[1 - \left(\frac{H}{4L \sin \alpha_2} \right)^{\frac{4}{3}} \right] \quad (14)$$

The previous section presented two formulas for calculating of the water discharge flowing along horizontal and inclined planes. In reality, the water flowed in the Bosphorus in the channel, and it suffered friction with the sidewalls. It seems reasonable to estimate the effect of lateral friction on the water flow in the channel, using formula (14) to calculate the flow of water along the plane. It is interesting to compare the formulas for the water flow on the plane and in the channel.

4.2. Exact solution of the water flow in the channel

Water flow in a channel with sloping bottom without friction on the side walls is defined by (7). The problem about water flow with friction on the vertical walls can be stated as follows. If we direct the X axis along the bottom of the channel, the Y axis across the channel, and the Z axis perpendicular from the bottom of the channel, the system of Navier–Stokes equations takes the following form:

$$\begin{cases} g\sin(\alpha) - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right) = 0; \\ -\frac{1}{\rho} \frac{\partial P}{\partial y} = 0; \\ -g\cos(\alpha) - \frac{1}{\rho} \frac{\partial P}{\partial z} = 0. \end{cases} \quad (15)$$

We obtain the Poisson equation, by excluding the pressure P :

$$\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = -\frac{g}{\nu} \sin(\alpha). \quad (16)$$

The boundary conditions for it have the next form:

$$\begin{aligned} \frac{\partial U}{\partial z} \Big|_{z=h} &= 0 \text{ shear stress on the liquid surface is equal to zero;} \\ U(y,0) &= 0 \text{ sticking to the bottom;} \\ U(0,z) &= 0 \text{ sticking to a wall;} \\ U(l,z) &= 0 \text{ sticking to a wall.} \end{aligned}$$

To reduce the problem to the solution of Laplace's equation, we make the change: $U(y,z) = V(y) + W(y,z)$. Thus we find: $V(y) = -1/2g/\nu \sin(\alpha)y^2 + C_1y + C_2$, and obtain for $W(y,z)$ Laplace's equation:

$$\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2} = 0 \quad (17)$$

and boundary conditions:

$$\begin{cases} W_z(y, h) = 0 \\ W(y, 0) = -V(y) \\ W(0, z) = 0 \\ W(l, z) = 0 \end{cases}$$

After application of the method separation of the variables and performing a series of transformations, as well as satisfying the boundary conditions, we find for the function $W(y,z)$ solution in the form of an infinite convergent series:

$$W(y, z) = -\sin(\alpha) \frac{2gl^2}{\nu\pi^3} \sum_{n=1}^{\infty} \left(\frac{1 - \cos\pi n}{n^3(1 + e^{-2\pi n h})} \right) \cdot \left(e^{\frac{\pi n}{l}(z-2h)} + e^{-\frac{\pi n}{l}z} \right) \cdot \sin\left(\frac{\pi n}{l}y\right) \quad (18)$$

The water discharge is obtained by next formula $Q = \int U ds$, where S – is area of the live flow cross-section. We find:

$$Q = \frac{ghl^3}{12\nu} \sin(\alpha) - \frac{8gl^4}{\nu\pi^5} \sin(\alpha) \sum_{k=2n-1}^{\infty} \frac{1}{k^5} \cdot th\left(\frac{\pi k}{l}h\right), n = 1, 2, 3... \quad (19)$$

The calculation of values Q by use the formulas (3) and (12) is presented in Table 1

Table 1

The values of water discharge in the channel, taking into account the lateral friction, and without it, with a various number of members of the series and various values of h and α .

h, l	α	Water discharge without friction on the side walls of the channel (m^3/sec)	Water discharge considering friction on the side walls of the channel (m^3/sec)		Error in Equation (3) (%)
			$n = 200$	$n = 1000$	
$h = 3 M$	$A = 1^\circ$	5393.0	5364.5	5364.3	0.5
$l = 700 M$	$\alpha = 2^\circ$	10784.4	10726.7	10726.7	0.5
	$\alpha = 3^\circ$	16172.5	16086.5	16086.5	0.5
$h = 10 M$	$\alpha = 1^\circ$	199742.7	196146.6	196146.5	1.8
$l = 700 M$	$\alpha = 2^\circ$	399424.7	392233.4	392233.3	1.8
	$\alpha = 3^\circ$	598985.0	588200.6	588200.4	1.8

Calculations were made with kinematic viscosity coefficient $\nu=2 \cdot 10^{-1} M^2/c$. Table 1 shows that the kinematic viscosity coefficient must being changed for different flow regimes, it is necessary for saving the maximum flow velocity under reasonable limits. After comparison of the values which were obtained for the water discharge, at the same flow parameters, the correction because of the water friction on the sides of the channel is not more than 1.8%. The value l was significantly less than its current size during the melting of glaciers, and the value of the correction was higher than 1.8%. Hence the real sea level rise should be greater than the calculated values in the case of water flow in a narrow canyon.

5. Mathematical model of the Black Sea level change under climate warming and world ocean level rise

We use the balance equation of the water flowing into the reservoir and then flowing out from it, for a mathematical model of the Black Sea level change:

$$S \frac{dH}{dt} = W - Q \quad (20)$$

where $S = S(H)$ is the area of the reservoir on the vertical mark H , $H = H(t)$ is the reservoir level change depending on the balance of water coming in and flowing out, W is the value of the freshwater balance (in the more general case, the difference between the total volume of all incoming water in the reservoir and evaporation from the reservoir), Q is the river outflow (discharge of water flowing from the Black Sea). In the coordinate system shown in Fig. 5, we can write:

$$S \frac{dH}{dt} = W - \frac{glH^4}{12\nu L} \left(1 - \left(\frac{H}{4L \sin \alpha_2} \right)^{4/3} \right) \quad (21)$$

In the initial period of the glacial melting the freshwater balance of the Black Sea was close to its value during the glacial period. Then, the intensity of the ice melting increased with warming. Freshwater balance of the sea began to decrease after reaching its maximum value. As a first approximation, its change can be written as follows:

$$W = A + B \left(1 - \cos \frac{2\pi t}{T} \right) + f(t) \quad (22)$$

where $A = 100 \text{ km}^3/y$ (or $3150 \text{ m}^3/s$) (freshwater balance during the glacier melting), $f(t)$ – secondary fluctuations of the freshwater balance, which had a period from 700 to 550 years according to results of geological research. The main harmonic of the freshwater balance is presented in the form $B(1 - \cos 2\pi t/T)$, where

$B = (2200–2500) \text{ km}^3/\text{y}$ and $T=17,000 \text{ y}$. Therefore, if the glaciers started to melt 20000 years ago, the maximum of the freshwater balance was 11 500 years ago. The accepted expression for W in the form of (13) corresponds to the geological studies (Izmailov, 2005; Balabanov, 2007).

Thus, the equation of the Black Sea level changes can be written as follows:

$$S \frac{dH}{dt} = A + B \left(1 - \cos \frac{2\pi t}{T} \right) + f(t) - \frac{glH^4}{12\nu L} \left[1 - \left(\frac{H}{4L \sin \alpha_2} \right)^{4/3} \right] \quad (23)$$

6. Adaptation of the model to the conditions of the Bosphorus Strait

Adaptation of the mathematical model to the real conditions is carried out from the following considerations. Currently, about $400 \text{ km}^3/\text{year}$ of water (freshwater balance plus $200 \text{ km}^3/\text{year}$ of water brought by the bottom countercurrent of the Bosphorus Strait into the Black Sea) or $12,670 \text{ m}^3/\text{s}$ flow into the Marmara Sea through the Bosphorus Strait. The theoretical calculations (Oguz et al., 1990) showed that in the condition when the water discharge through the strait equals $12,350 \text{ m}^3/\text{s}$, the surface of the counterflow is situated at the depth of 22 m and thickness of the bottom counterflow layer is 14 m in the most shallow part. The difference between the levels of the Black and Marmara seas currently is 0.3 m. These data allow us to determine the value of the coefficient ν , where in the case of the channel depth equaling 22 m, width l and the difference between levels of 0.3 m, $12,350 \text{ m}^3/\text{s}$ of the water will flow. Thus, we are able to adapt the model to specific conditions. For this, we find from (6):

$$\nu = \frac{(H^4 - h_0^4)gl}{12QL} \quad (24)$$

Inserting $H = 22.3 \text{ m}$, $h_0 = 22 \text{ m}$, $g = 9.81 \text{ m/s}^2$, $l = 700 \text{ m}$, $Q = 12350 \text{ m}^3/\text{s}$, $L = 30000 \text{ m}$, we obtain the value $\nu = 2 \cdot 10^{-2} \text{ m}^2/\text{s}$. That is the value of ν for a model which describes the relatively slow movement of the water flow on the smooth bottom, which is happening in present time, where the smooth bottom is a surface of the bottom counterflow. But during the glacial melting, the river was flowing over the uneven bottom of the strait with much greater speed and greater intensity of turbulence, and therefore the kinetic energy loss due to friction was greater. For this reason, the value of the kinematic viscosity coefficient was greater during the time of most intensive ice melting, when the freshwater balance W_i was equal to $1480 \text{ km}^3/\text{y}$ (Chepalyga, 2005 a). We have taken $\nu = 8 \cdot 10^{-1} \text{ m}^2/\text{s}$. Siddall et al. (2004) took $\nu = 10^{-1} \text{ m}^2/\text{s}$ to describe approximately the same flow of water through the Bosphorus Strait. In the equations of the technical fluid mechanics, this value ν corresponds the Chezy coefficient equal to $50–60 \text{ m}^{1/2}/\text{sec}$, which is accepted for the calculation of the river flow where the bottom is composed of fine pebbles. Thus, it is expedient to construct the dependence between the coefficient of the kinematic viscosity and the freshwater balance of the Black Sea for this model, as the character of the water flow varies in the river depending on the water discharge. The linear approximation gives the following expression:

$$\nu = \frac{W(t) - W_{\min}}{W_{\max} - W_{\min}} (\nu_{\max} - \nu_{\min}) + \nu_{\min}, \quad (25)$$

where $W_{\min} = 12,350 \text{ m}^3/\text{s}$, $\nu_{\min} = 2 \cdot 10^{-2} \text{ m}^2/\text{s}$, $W_{\max} = 1480 \text{ km}^3/\text{year} = 46,930 \text{ m}^3/\text{s}$, $\nu_{\max} = 8 \cdot 10^{-1} \text{ m}^2/\text{s}$.

We assume that the bottom of the Strait has been rising with average velocity $n=3 \text{ mm/y}$ for the considered time interval. The solution is performed relative to the fixed coordinate system, which at the initial moment of the time interval coincides with a moving coordinate system which is connected with the moving bottom of the strait. That is, we make the replacement from H to $H_1 - nt$ everywhere in the balance equation (25). Taking into account (14) we get:

$$S(H_1 - nt) \frac{d(H_1 - nt)}{dt} = W(t) - \frac{gl}{12 \left(\frac{W(t) - W_{\min}}{W_{\max} - W_{\min}} (\nu_{\max} - \nu_{\min}) + \nu_{\min} \right) L} (H_1 - nt)^4 \left[1 - \frac{H_1 - nt}{4L \sin \alpha} \left(\frac{H_1 - nt}{4L \sin \alpha} \right)^{1/3} \right] \quad (26)$$

Since the moment of time t_0 when the sea level rose to the level of river surface and began to grow with velocity m , the balance equation takes the following form:

$$S(H_1 - nt) \frac{d(H_1 - nt)}{dt} = W(t) - \frac{gl}{12 \left(\frac{W(t) - W_{\min}}{W_{\max} - W_{\min}} (\nu_{\max} - \nu_{\min}) + \nu_{\min} \right) L} \left[(H_1 - nt)^4 - \left((H_1(t_0) - nt_0) \cdot \sqrt[3]{\frac{H_1(t_0) - nt_0}{4L \sin \alpha}} + (m - n)(t - t_0) \right)^4 \right] \quad (27)$$

The theoretical curve of the Black Sea level change which is obtained by numerical solution of the equation (27) is shown in Fig. 7. This correlates with the Izmailov (2005) curve obtained by geological research. Some difference at the time interval 19–16,000 years ago is explained as follows. We assume that the glacier melting started 20–18,000 years ago. After sea level regression at 11.5–10,000 years ago, transgression began which was caused by world ocean level increase.

A mathematical model shows that the Upper Pleistocene transgression had two peaks. The first was due to a very large increase of the freshwater balance and a narrow strait during the glacier melting period, and the second was due to the transgression of the world ocean. All other transgressions of the Black Sea during the Pleistocene had two peaks (Fig. 3). This indicates that all transgressive–regressive cycles in the Pleistocene occurred according to one scenario, where the main factor was the flow of the river along the narrow canyon which connects the Black Sea and Marmara Sea.

7. Evaluation of the Black Sea secondary fluctuations during the Holocene

One of discussion questions is, how many secondary-level fluctuations occurred in the Black Sea during the Holocene, and what were the magnitudes of the changes? According to some researchers, their number could equal from 15 to 20, with oscillation from 5 to 15 m (Chepalyga, 2002; Izmailov, 2005; Balabanov, 2009).

To calculate the scope of the fluctuations of the Black Sea during the Holocene in cases of different depths in the strait, we use the following expression for W :

$$W = 400 \cdot 10^9 \text{ m}^3/\text{year} - 300 \cdot 10^9 \sin \left(\frac{2\pi t}{550} \right) \text{ m}^3/\text{year}.$$

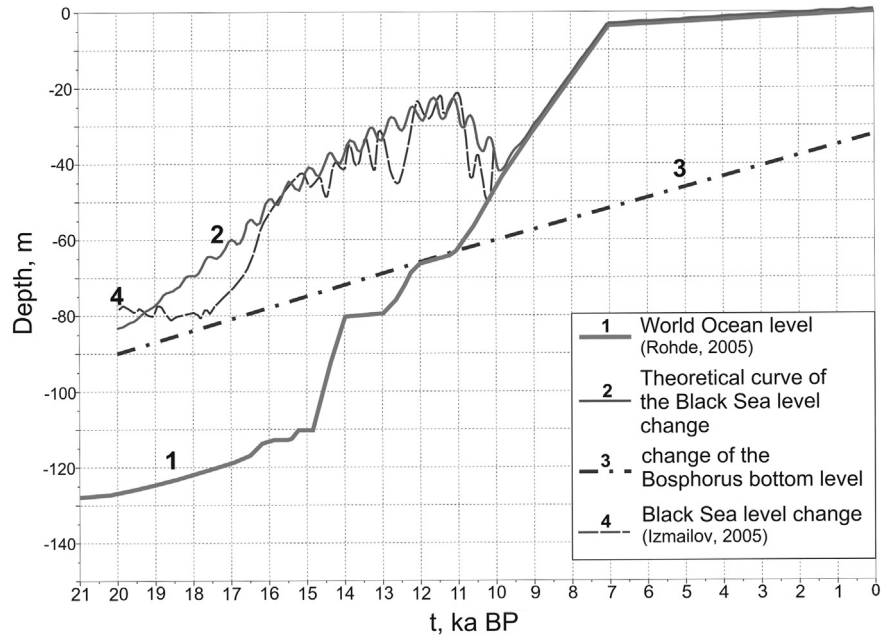


Fig. 7. Theoretical curve of the Black Sea level changes during the Late Pleistocene and the curve obtained based on geological research (Izmailov, 2005).

Assuming that $l = 700$ m, $L=20,000$ m, $\nu = 2 \cdot 10^{-2}$ m²/s, the period of secondary fluctuations equal 550 years, and their scope from 100 to 700 km³/y. For the depth of the river end h_0 we considered values of 5 m, 10 m, 15 m, and 20 m. The depth of the flow through the strait was changing as follows. At the end of the Late Pleistocene regression of the Black Sea (about 10000 years ago) the value h_0 was 5 m. At this time the ocean level was rising at 13.3 mm/y, therefore the depth of the strait increased by 13.3 m for a thousand years, and its depth was equal to 18.3 m. Thus, the depths of 5, 10, 15 m in the strait could occur only during the first thousand years after connection. It is highly probable that the strait had been deepening before 10000-7000 years ago, when the velocity of the water flow in the strait decreased, and the process of sedimentation began. After that, the depth in the Strait began to decrease as a result of sedimentation.

The calculations showed the following. The scope of the Black Sea level fluctuations could be 4.5 m if the depth of the strait was 5 m; if depth equaled 10 m, 2.7 m; if depth equaled 15 m, 1.2 m; and if depth equaled 20 m, 0.45 m. Thus, even at the extremely high (100–700 km³ per year) and perhaps unrealistic scope of fluctuations of waters entering the Black Sea waters, the scope of the sea level fluctuations (over the changing sea level) is not greater than 0.45 m. Consequently, Black Sea level fluctuations of a few meters during the Holocene suggested by different authors are physically impossible because the water flow through the strait from the adjacent sea. In order to have 700 km³/y of water flow through the strait, the difference between levels was approximately 45–50 cm. A decrease in Black Sea level to below the Marmara Sea is theoretically impossible because the freshwater balance has been positive.

8. Seasonal fluctuations of the Black Sea level during the Late Pleistocene

In analyzing the processes occurring during the late Pleistocene, one should bear in mind that the regime of water flow into the sea and the character of its level change had their own features which

distinguished them from their current processes. When the glaciers melted, the main volume of the water came from them. In this case, the volume of the glacial melt was proportional to the air temperature: the higher was the temperature, the higher was the flow of rivers. It was highest in July and August. In autumn and winter glacial melting stopped, and the river outflow was reduced to a small value. Thus, the sea was filled by annual pulsed flow of water during the late Pleistocene. River runoff greatly increased during that time.

The evaluation of the Black Sea seasonal fluctuations scope is very important for understanding of the lithodynamic processes. It is possible to calculate the seasonal fluctuations for the case of a water pulse coming into the sea, by taking the freshwater balance in following form:

$$W = A_0 + \left[A \left(1 - \cos \frac{\pi}{T_0} t \right) + B \left(1 - \cos \frac{\pi}{T_0} t \right) \sin \frac{2\pi t}{550} \right] \times (1 - \cos 2\pi t).$$

where A_0 – is freshwater balance of the sea during the cold time of year. In this case the curve $A(1 - \cos\pi/T_0t) + B(1 - \cos\pi/T_0t)\sin 2\pi t/550$ is the envelope for the seasonal fluctuations with a period of 1 year, described by the curve $(1 - \cos 2\pi t)$. The value of A is chosen so that the annual freshwater balance of the sea was the same as in previous calculations.

We obtain the following equation (Esin et al., 2010a):

$$S \frac{dH}{dt} = A_0 + \left[A \left(1 - \cos \frac{\pi}{T_0} t \right) + B \left(1 - \cos \frac{\pi}{T_0} t \right) \sin \frac{2\pi t}{550} \right] \times (1 - \cos 2\pi t) - \left(H^4 - h_0^4 \right) \frac{gl}{12\nu L}. \tag{28}$$

The numerical solution of this equation shows the following (Fig. 8). The scope of the level seasonal fluctuations was increased from 0 to 1 m with time (in the period of most intensive glacial melting 13–11 thousand years ago, it was 1 m). The large scope of the seasonal fluctuations is accounted for because in a short period

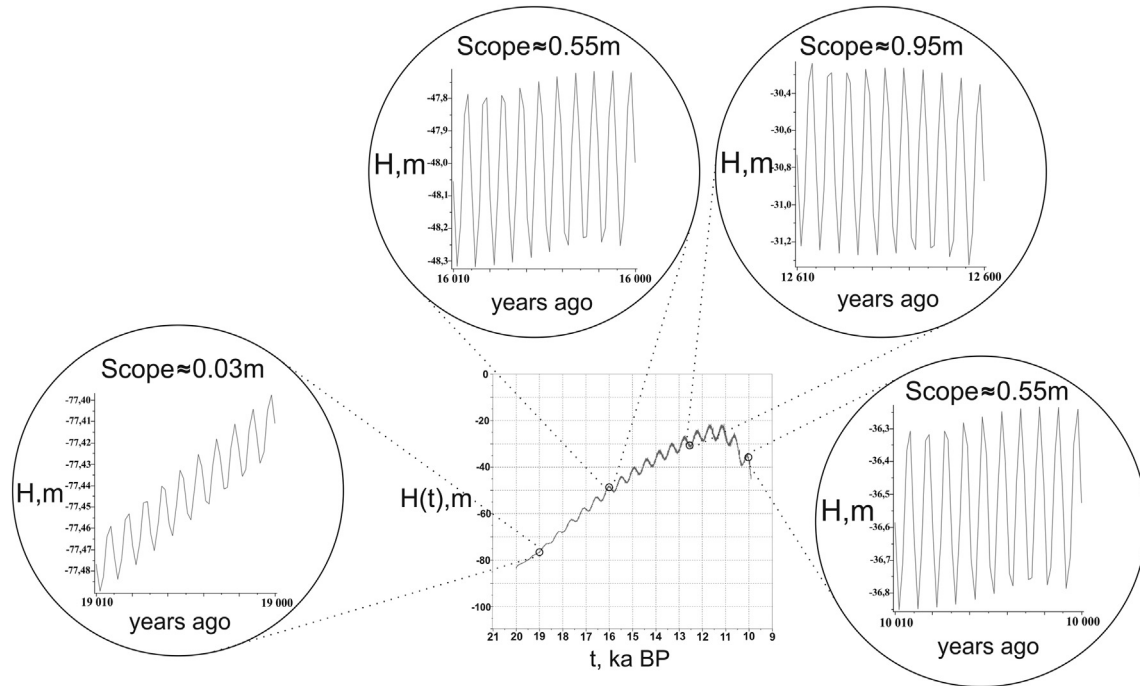


Fig. 8. Amplitudes of the Black Sea level seasonal fluctuations at the various stages of the Late Pleistocene transgression.

of time a large volume of water enters into the sea, much greater than the volume which outflows through the strait.

9. Analysis of a river flowing in the Bosphorus Strait with the use of equations of technical hydromechanics

Analysis of a river flowing in the Bosphorus Strait under low world ocean levels seems to be solved efficiently with the use of simple equations of technical hydromechanics. The most perfect and often used are the equations of Sen-Vensan. They were used for the description of the Kuban riverflood (Baranenko and Semenshi, 2010) and for the control of runoff (Hublaryan, 2009).

Sen-Vensan's combined equations are:

$$\frac{\partial y}{\partial x} + u \frac{\partial y}{\partial x} + g \frac{\partial h}{\partial x} + g \frac{u^2}{C^2 R} + q \frac{u}{hl} = g \operatorname{tg} \alpha; \tag{29}$$

$$u = \frac{Q}{lh}, \tag{30}$$

where C is Chezy coefficient, q – river afflux (increase of its runoff due to river branches). In this case $q = 0$, R – is a hydraulic radius of the strait (for rivers $R \approx h$).

The Chezy coefficient is introduced in the equation for the description of kinetic energy loss due to all kinds of friction in water flow. The Chezy coefficient in contrast to kinematic viscosity factor ν can be calculated subject to bottom roughness. A large collection of formulae is proposed for this (Yablonskiy, 1961).

Here, $q = 0$, $\partial u / \partial x$ is 10^{-10} m/s² and it may be neglected. Other terms of equations 29 and 30 are of the order $10^{-3} - 10^{-4}$ m/s². Taking this into account, we can write these equations in total derivatives:

$$\frac{dh}{dx} \left(g - \frac{Q^2}{h^3 l^2} \right) = -g \left(\operatorname{tg} \alpha - \frac{Q^2}{h^3 c^2 l^2} \right). \tag{31}$$

In the case of horizontal bottom of the channel ($\alpha = 0$) from (31) we get

$$\frac{dh}{dx} \left(g - \frac{Q^2}{h^3 l^2} \right) = -\frac{Q^2}{h^3 c^2 l^2}. \tag{32}$$

The following function is the solution of this equation when boundary conditions are $h = h_0$ and $x = L$:

$$h^4 = h_0^4 + \frac{4Q^2}{g l^2} (h - h_0) + \frac{4Q^2}{c^2 l^2} (L - x). \tag{33}$$

In this case a value of h_0 means the depth of the flow running along the southern slope of the strait southern threshold. On the transitional boundary between the flows running along the horizontal channel and inclined one the flow depth is the same and is equal to h_0 . It can be defined when $dh/dx = 0$ is put into (31). Then from (31) we get:

$$h_0 = \sqrt[3]{\frac{Q^2}{C^2 l^2 \operatorname{tg} \alpha}}. \tag{34}$$

Solution of (33) allows estimation of the contribution of the nonlinear term of the input equation into the formation of river surface of the strait at the northern sill, the Black Sea level. When $x = 0$:

$$H^4 = h_0^4 + \frac{4Q^2}{g l^2} (H - h_0) + \frac{4Q^2 L}{c^2 l^2} \tag{35}$$

Assume, for example, $Q = 16,000$ m³/s, $l = 400$ m, $C = 60$ m^{1/2}/s (river bottom is composed of fine pebble), $\operatorname{tg} \alpha = 6 \cdot 10^{-2}$. Then, according to formula (34) we can find $h_0 \approx 2$ m. If $Q = 40,000$ m³/s, and the other parameters are the same, $h_0 \approx 3.7$ m.

Calculation using formula (8) without the nonlinear term and with water discharge 16,000 m³/s gives $H = 14.5$ m, and when water discharge is 40,000 m³/s, it is 23 m.

Calculation of the equation root (35) when water discharge is 16,000 m³/s and 40,000 m³/s gives respectively $H = 14.9$ m and $H = 24.56$ m. Thus, the nonlinear term increases the Black Sea level for 0.4 m, when water discharge is 16,000 m³/s, and 1.5 m when water discharge is 40,000 m³/s. Hence, the nonlinear term may be neglected when solving a geological problem of the Black Sea level change in conditions of the world ocean level rise.

In the linear version solution (without udu/dx) of Sen-Vensan's equation is written as

$$H^4 = h_0^4 + \frac{4Q^2(L-x)}{C^2l^2} \quad (36)$$

That is identical to the solution of Navier–Stokes equation (9). A similar solution can be obtained from Chezy theory. Thus, two equations where a physical meaning of the coefficient, which regulates kinetic energy loss, is different, give similar formulae for water running along a horizontal plane. When equating coefficients when $(L-x)$, we get the dependence of ν (C):

$$\nu = \frac{gQ}{3C^2l} \quad (37)$$

Obtained dependence of ν coefficient from water discharge is important and physically grounded. When the velocity increases, the intensity of turbulent pulsations increases, and hence kinematic energy loss increases.

Several formulae are offered for C. The most prevalent is Manning's formula:

$$C = \frac{1}{n_0} \sqrt[6]{\frac{lh}{l+2h}} \quad (38)$$

For the open channel and river bottom composed of fine pebble $1/n_0 = 33.3$

For the river with the depth in the strait being 24 m (the depth of upper stream) we get $C \approx 57 \text{ m}^{1/2}/\text{s}$.

Jarosoz et al. (2011) give values of ν in the northern and southern parts of the strait. Influenced by the current velocity and other factors, ν can vary from 10^{-3} to $10^{-2} \text{ m}^2/\text{s}$, and may be more than $10^{-2} \text{ m}^2/\text{s}$. Values of ν were calculated from (37) for an average water discharge in the strait $Q = 12,350 \text{ m}^3/\text{s}$ and width $l = 3500 \text{ m}$. Taking $C = 60 \text{ m}^{1/2}/\text{s}$, we get $\nu = 3.6 \cdot 10^{-3} \text{ m}^2/\text{s}$. Earlier we got for the narrowest part of the strait the value of $\nu = 2 \cdot 10^{-2} \text{ m}^2/\text{s}$. The slightly increased value of ν can be explained by the fact that at this section of the strait current velocity is noticeably greatly than that of the strait's ends, where its width is several times more than in the narrowest place (700 m). Calculation made using formula (37) for other strait depth and water discharge values have shown that ν values also varies from 10^{-3} to $10^{-2} \text{ m}^2/\text{s}$.

During glacier melting, water discharge in the strait reached 40,000 m³/s, and the strait width in a V-shaped cut was not more than 300 m. In this case:

$$C = 33,3 \cdot \sqrt[6]{\frac{300 \cdot 30}{300 + 60}} \approx 57 \text{ m}^{1/2} / \text{s}$$

$$\nu = 4 \cdot 10^{-1} \text{ m}^2 / \text{s}$$

This value of ν is close to $10^{-1} \text{ m}^2/\text{s}$, which was taken in the calculation of disastrous water flow running along the Bosphorus strait from the Marmara Sea into the Black Sea (Siddall et al., 2004). Approximation of $\nu = F(Q)$ with the help of formula (25), where the maximum of ν is $t10^{-1} \text{ m}^2/\text{s}$, unexpectedly was confirmed. Hence, the calculation of water surface was made with real values of ν .

The ν value taken for calculation is in the interval of real values obtained as a result of direct measuring of flow velocity in the strait. Transferring from the Chezy coefficient to the kinematic coefficient ν according to formula (37) gives a real value of ν .

10. Influence of the vertical crustal movements on the results of a study of the eustatic sea-level changes

The results of many years research of the Black Sea level changes during the Late Pleistocene–Holocene are shown in Fig. 9. There are numerous curves and they are very different, not allowing establishment of a qualitative pattern of the sea level change. All curves were obtained by using the same methods, but they gave essentially different results. There is no universally recognized curve of the Black Sea level change.

The reason for many difference descriptions of the same process consists in the fact that the traces of eustatic sea-level changes in the form of the age and the vertical position of the coastal sediment for thousands of years have been moved to the other horizons by the vertical crustal movements. The identification of the vertical position of the modern sediment with its ancient position is incorrect. The variety of geological conditions at the different coasts has led to a variety of sea level curves. Consequently, the curves were presented by researchers as the “course of the sea level,” in fact they are local (relative) curves, as they include the vertical movement of the earth's crust, as well as the action of the random processes.

The problem is that the eustatic sea-level change is selected from these known curves. In this paper we propose a method of calculating of the eustatic sea-level curve from several local curves.

The investigation of the world ocean level influence on the level of the Black Sea during the Holocene is the other problem. The course of the Black Sea level is considered to be independent from the course of the ocean level according to many Russian authors. Our (Esin et al., 2010 a, b) theoretical studies have shown that the average level of the Black Sea repeated the change of the ocean level, in excess by ~10–80 cm during the last ~8000–9000 years.

11. Transformation of the curve of the eustatic sea level course to the local curves

The difference between the conditions in which different parts of the coast were developed explains the diversity of curves for the Mediterranean Sea which are available in numerous articles (Brückner et al., 2010). From the analysis of the vertical crustal movements role, it follows that most of the curves are actually local curves, which are reflecting the combined effect of the eustatic changes of the basin level and the vertical displacement of the coastal area relative to sea level as a result of neotectonic movements and/or erosion-accumulative processes. Another technique of paleogeographic data processing is required to obtain the eustatic level.

The kinematic relationship between the eustatic and relative (local) changes in sea level in conditions of uniform crustal movements can be written as $H(t) = F(t) + ut + \zeta(t)$, where $H(t)$ – is relative sea level curve; $F(t)$ – is eustatic sea level curve; t – is age of the object (sediment, residues of fauna or flora, landforms, etc.) from which the position of the coastline was defined; u – is average velocity of gradual vertical crustal movements; $\zeta(t)$ – is the contribution of random factors in the local curve. In general $H(t) = F(t) + G(t)$, where $G(t)$ is the crust displacement for the taken period of time t . Uneven movement of the earth's crust, catastrophic geomorphic effects (earthquakes, movement of the shoreline as a result of erosion-accumulative processes etc.) are random and difficult to take into account. Catastrophic processes cause serious distortion in the shape of the local curves and,

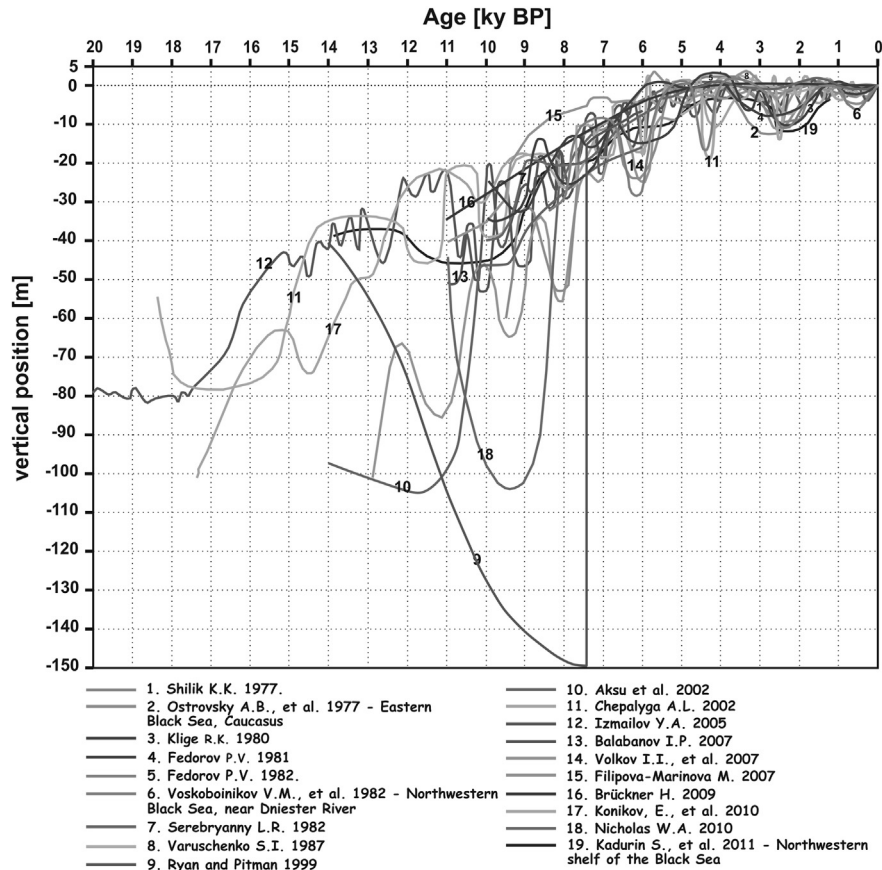


Fig. 9. The results of many years' research of the Black Sea level changes during the Late Pleistocene–Holocene.

consequently, in the calculation of sea level. Therefore, the local curves obtained for parts of the coast with gradual and unidirectional vertical neotectonic movements should be chosen for the calculations. A similar kinematic relationship between eustatic and tectonic components is presented in Lambeck (1996).

We show an example of transformations made by tectonic activity in the curve of the eustatic sea-level movements. In Fig. 10 are presented a curve (which was taken only as an example) of eustatic sea level rise with two fluctuations, as well as the calculated local curves corresponding to a given eustatic sea level course in case of coastal uplift with velocities of 1.5, 1, 0.5 mm/y and the subsiding coast with velocities of -0.5 , -1 , -1.5 mm/y.

Analysis of the local curves shows that the vertical crustal movements can distort beyond recognition the course of the eustatic sea level. Two transgressive–regressive cycles with the scope of the fluctuations of 2 m are transformed into continuous transgression of the sea with a variable velocity on the coast which is subsiding 1.5 mm/y. The value of the eustatic regression decreases and the value of the eustatic transgression increases in the local curves in the case of increasing subsidence.

The situation is reversed in the case of an uplifting coast. In general the eustatic transgression with two transgressive–regressive cycles is converted, if the velocity v about 2 mm/y, to almost continuous regression with variable velocity in the local curve. Herewith, the values of the velocities of the level eustatic regressions are increased in the local curves and the values of the velocities of the level eustatic transgressions are decreased in the local curves in case of increasing uplift. Thus, the total level eustatic transgression with two transgressive–regressive cycles can be represented (in the local curves) as opposite level movements on uplifting and sinking coasts.

In the case of increasing velocity of the tectonic movements from -1.5 to 1.5 mm/y, the beginning of each regression is shifted to the right, i.e. it lags in time (Fig. 10). At the same time in each overlying local curve the regression begins earlier than in the underlying curve. The transgression in each overlying curve is later than in the underlying. As a result, the duration of the regression is increased from bottom to top, and the duration of the transgression is decreased. Consequently, almost symmetric fluctuation of the eustatic sea level course is converted to asymmetric fluctuations by the vertical movements of the earth's crust. The displacement of the beginning and end of the transgressions and regressions occur in the local curves with respect to the eustatic curve.

Vertical crustal movements can very significantly distort the eustatic level change. As well as, the time of initiation of the transgressions and regressions varies depending on the velocity and the sign of vertical crustal movements. Consequently, the local curve by itself contains little information about the eustatic changes. It gives some information about changes of the level only if we know the velocity of the coastal tectonic movements.

12. Calculation of the Mediterranean Sea level change during the Holocene

Local sea level curves are obtained by the addition of $u_0 t + (t)$ (with its sign) to the ordinate of the eustatic sea level. In order to obtain the eustatic level from local curves we should perform the reverse action. In this chapter we show a technique which obtains the eustatic curve from the existing local curves of the Mediterranean Sea.

The most representative local curves of the Mediterranean Sea level change are presented in Brückner et al. (2010) a (Fig. 11). The

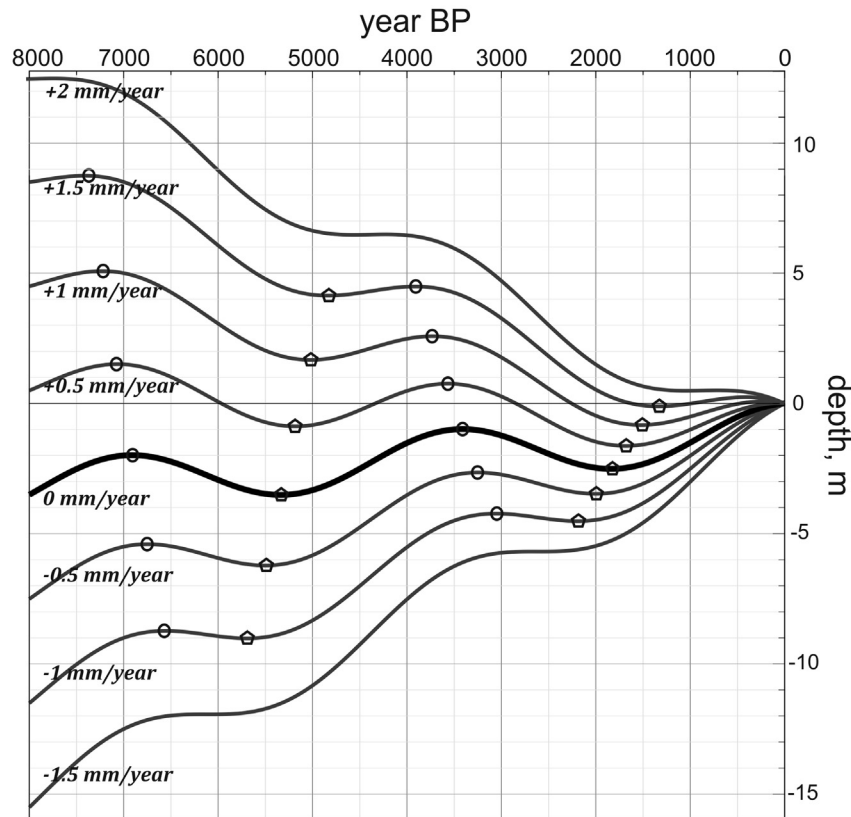


Fig. 10. An example of transformation of the given curve of the eustatic sea level changes ($U = 0$) to the local curves by vertical crustal movements. \circ – is the beginning of the regression; \triangle – is the beginning of the transgression. The velocities of the vertical crustal movements are indicated by the numbers. Sign (+) corresponds to uplift, the sign (–) subsidence.

eustatic change of sea level in this sea was calculated with the addition of two more smooth curves.

The relative curves are quite diverse because the vertical crustal movements were different in the various areas. They could be closer, if the value of the sediment displacement caused by tectonic movements of the earth's crust is deducted from their coordinates. For this, focusing on the world ocean level change (to which, of course, the Mediterranean Sea level change corresponded) according to Rohde (2007), we assume that 5000 years ago Mediterranean Sea level stood at the mark of -1.5 m, where the same age sediment was formed. This point is called a point of the level identification. Then for 5000 years, the sediment have been raised or lowered by tectonic movements on the current mark. The average velocity of the earth's crust movement can be defined as the quotient of the difference between the 1.5 m mark and the mark of the current sediment (which is shown on the local curve) divided at the age of sediment i.e. at 5000 years. Then, multiplying the velocity of vertical crustal movements at the age of the sediment which was formed at different times, for each curve we obtain the value of sediment displacement at different times from 7000 years ago to present time (the shift of the modern sediment is equal to 0). If the velocity of the tectonic movements had not changed over time and other processes of the sediment vertical displacement had not occurred and the calculations and measurements had been performed perfectly then all the relative curves would have been aligned on one line to describe the eustatic sea level change. Because of the processes and measurements were not ideal (uneven movement of the earth's crust, inaccurate determination of the vertical mark and the sediment age, etc.), the relative curves have not aligned to

each other after transformation, but are much closer together (Fig. 12).

The errors caused by tectonics have been reduced in the corrected curves, but the errors caused by effects of random processes remain. This effect can be significantly weakened by vertical averaging of the corrected curves. The resulting line will describe, with some degree of accuracy, the eustatic course of the Mediterranean Sea level change, and the world ocean change. After completing of the operation of averaging, we obtain a curve (Fig. 13). To obtain the exact solution at the present stage of research is impossible because of actions of random processes. The proposed method brings the known local curves closer together. They are entered into some certain corridor of the random processes' action, and then this corridor is compressed by averaging.

The Rohde (2007) curve of the world ocean level change, the Lambeck and Purcell (2005) curve of the Mediterranean Sea level change, and the curve obtained by us are shown on Fig. 13. The Rohde curve and ours almost exactly coincide in the time interval from 7 to 5000 years ago and they have the same shape and differ at less than 1 m in the interval from 5000 years ago until present time. Our curve shows a slight regression of the ocean in the time interval from 4.5 to 3000 years ago, and the Rohde curve shows unchanging position of the ocean level.

It is possible to estimate the accuracy with which our curve of the Mediterranean Sea level change has been determined. For this, we will perform the inverse operation, by calculating the local curves for the same parts of the Mediterranean coast, assuming that the curve obtained by us is the eustatic sea level. The theoretical and the local "geological" curves of the sea-level change are very close to each other. The same calculations were performed for

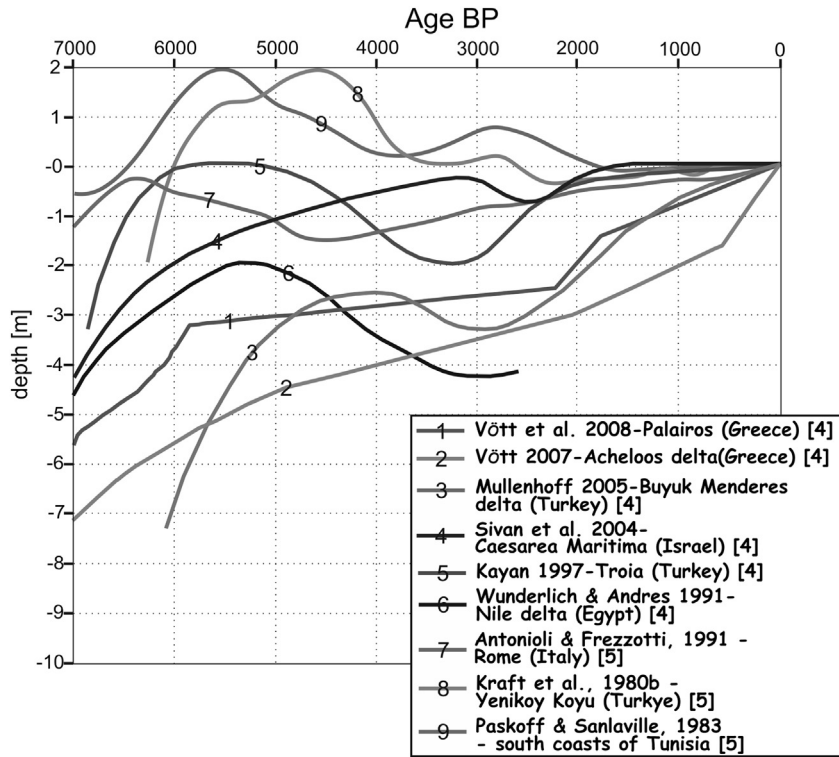


Fig. 11. Relative (local) curves of the Mediterranean Sea level change during the Holocene.

the Rohde curve. A comparison of the real local curves and their theoretical images allows the following conclusions. The curve obtained by us more accurately describes the course of the ocean level (in two cases we have obtained the almost exact coincidence of theoretical and “geological” local curves) than the Rohde and Lambeck curves. Three independent methods of research have given two very similar curves of the Mediterranean Sea and world ocean levels changes.

13. Substantiation of the method of local curve division for tectonic and eustatic components

Considering local curves in general form, for n local curves:

$$H_i(t) = F_i(t) + G_i(t) - \zeta_i(t), \quad i = 1, 2, \dots, n.$$

Here $H_i(t)$ are local curves, $F_i(t)$ are eustatic curves, $G_i(t)$ is vertical movements of the Crust, $\zeta_i(t)$, are random errors. Velocity of

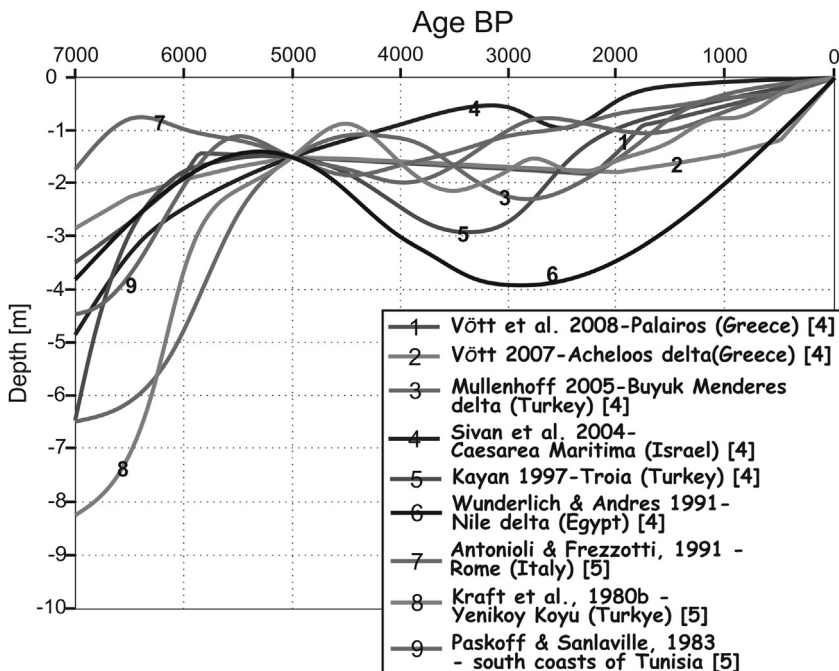


Fig. 12. Curves of the Mediterranean Sea level change corrected taking into account the vertical crustal movements.

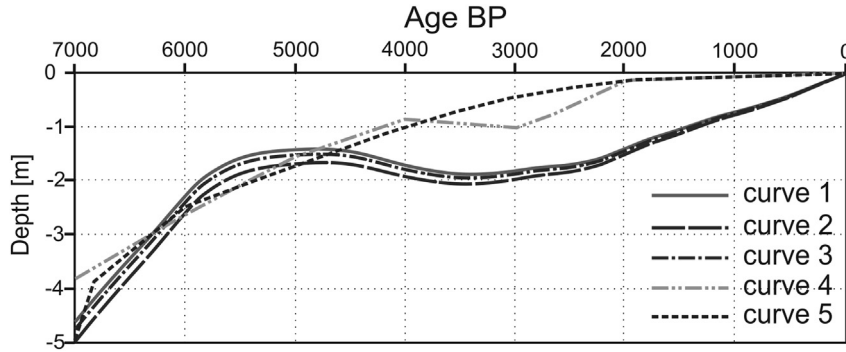


Fig. 13. The curves of the Mediterranean Sea level change, obtained by different methods. 1–2 are obtained by the above mentioned method, 3 – is obtained by least-squares method, 4 – is Rohde curve of the world ocean level change, 5 – is Lambeck curve of the Mediterranean Sea level change.

tectonic movements can be calculated for every i -curve having divided vertical displacement of the crust for periods of time, as above. We get a set of velocities u_i . By subtracting of vertical displacements from local curves, we get

$$H_i = F(t) + G_i(t) - u_i \cdot t + i(t)$$

$H(t)$ crosses an identification point (earlier identification point was (5000 years, 1, 5 m).

Then we summarize modernized local curves $H_i(t)$:

$$\sum_{i=1}^n H_i(t) = nF(t) + \sum_{i=1}^n (G_i(t) - u_i \cdot t) + \sum_{i=1}^n i(t)$$

and average the obtained result:

$$\frac{1}{n} \sum_{i=1}^n H_i(t) = F(t) - \frac{1}{n} \sum_{i=1}^n (G_i(t) - u_i \cdot t) + \frac{1}{n} \sum_{i=1}^n i(t)$$

For random processes (t) positive and negative values are equiprobable ones. For this reason the $\sum_{i=1}^n i(t)$ will decrease owing to cancellation of items with unlike signs. Thus, the sum divided for n will be small and it can be neglected.

For the difference $\frac{1}{n} \sum_{i=1}^n (G_i(t) - u_i \cdot t)$ in conditions of the uniform motions of the crust, its displacements can be written as: $G_i(t) = V_i \cdot t$, and $V_i = u_i$. For such sections $\frac{1}{n} \sum_{i=1}^n (G_i(t) - u_i \cdot t) = 0$ we may obtain:

$$F(t) = \frac{1}{n} \sum_{i=1}^n H_i(t)$$

If the values of u and V are similar, then the difference will be slightly less than $G_i(t)$ and $u_i(t)$. In general, the function $F(t)$ will cross the identification point, and the difference $(G_i(t) - u_i(t))$ will change sign, while crossing it. As far as the probability of the sign change for $(-)$ or $(+)$ for $(+)$ or $(-)$ is equal, then from the right and from the left members with unlike signs will be summed. They will decrease when the value of n increases. Now the solution can be written as:

$$F(t) = \frac{1}{n} \sum_{i=1}^n H_i(t) - \frac{1}{n} \sum_{i=1}^n (G_i(t) - u_i \cdot t)$$

The second member in the right part of the equation is an error, caused by irregular motions of the crust. Nevertheless, this member will be much less when crustal rise or lowering is more uniform. So, for the calculation of eustatic curves, we should choose coasts with more or less uniform tectonics.

14. Evaluation of accuracy

There is the possibility to estimate not only the accuracy of eustatic curve definition, but the relative accuracy of the method. Curves 1, 2, 3, 4 (Fig 13) are designated as $A(t), B(t), C(t), D(t)$. Using curve $A(t)$, we calculate a theoretical local curve for Palairos coast (Greece), for which we have a real local curve (Vött, 2007) (Fig. 11). We find an average velocity of a vertical motion of the crust u_1 by dividing the difference between ordinates of $(-5000; -3.1)$ point of the Vött curve and $(-5000; -1.3)$ point of our curve for the age of these level marks, i.e. for 5000 years. Then, multiplying it for t , we get a value of displacement in different times from 7000 years ago to nowadays. Adding $u_i(t)$ to curve $A(t)$, we obtain a theoretical local curve for Palairos coast (Greece), $A_1(t)$. Having done the same for each of the given curves of eustatic level of the Mediterranean Sea and for every coast section, for which real local curves are known, we get four sets of theoretical local curves $A_i(t), B_i(t), C_i(t), D_i(t)$, $i = 1..9$. Consider which of the four sets of theoretical local curves is the closest to real local curves. For this we find an average dilatation of each set of obtained curves from their real prototype:

$$\frac{\sum_{i=1}^9 \int_{-7000}^0 |A_i(t) - f_i(t)| dt}{9 \cdot 7000} = 0.57m;$$

$$\frac{\sum_{i=1}^9 \int_{-7000}^0 |B_i(t) - f_i(t)| dt}{9 \cdot 7000} = 0.62m;$$

$$\frac{\sum_{i=1}^9 \int_{-7000}^0 |C_i(t) - f_i(t)| dt}{9 \cdot 7000} = 0.58m;$$

$$\frac{\sum_{i=1}^9 \int_{-7000}^0 |D_i(t) - f_i(t)| dt}{9 \cdot 7000} = 0.83m$$

(here, local curves of the Mediterranean Sea level change showed in Fig 11 are marked through $f_i(t)$). The best results are for the curve which was obtained by us with the least-squares method. The largest average discrepancy (or an average error) with real local curves was with the Rohde curve. It is almost one and a half time more than the discrepancy obtained with the help of the second method.

Such estimation leads to the following conclusions. Our curve more precisely describes the Mediterranean Sea (ocean) level change, than does that of Rohde. Its small discrepancy with the

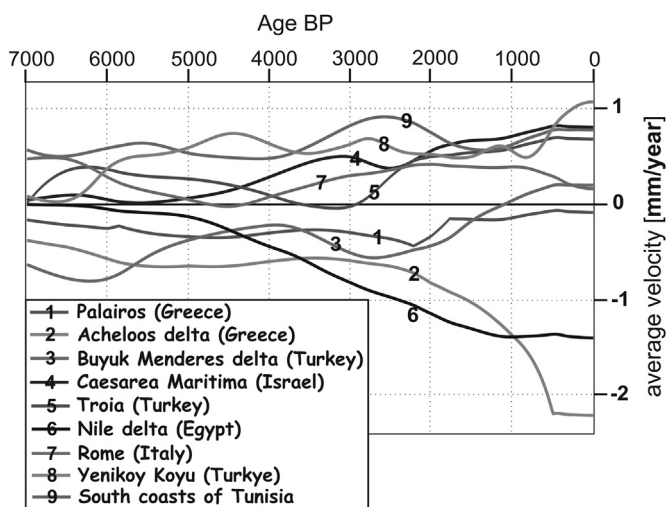


Fig. 14. Diagrams of average velocity of vertical movements of coastal sections.

curve obtained by the first method, is evidence that the identification point chosen by us (–5000 years; 1.5 m) very likely is close to a real level mark 5000 years ago. All diagrams presented in Fig 13 differ by not more than 1.3 m. The average difference of theoretical and real local curves for different methods is within the limits from 57 to 83 cm.

The level changes obtained by us (Fig. 13, line 1) describe local curves, taking into consideration complex processes. Calculation errors mainly are caused by the fact that neotectonic motions of the crust were irregular.

15. Extraction of tectonic component from local curves

Calculation of the eustatic change of the Mediterranean Sea level allows isolation of the tectonic component from local curves. To determine time changes of the vertical motions of the crust, it is necessary to subtract the eustatic curve from the local curve of sea level change over time. Diagrams of average velocity of vertical movements of coast sections are presented in Fig. 14.

Global analysis of obtained material shows that velocity of vertical movements changes rather fluently in time. The amplitude of fluctuation of all curves (except for curves 2 and 6) does not exceed ± 0.5 mm/y. Curves generally lack sizeable deviations which can be considered as the influence of chance factors. Fig. 14 indicates that tectonic velocities vary from 1 to –2 mm/y. Comparison of theoretical study and the results of field observations shows that understanding of hydrodynamic, geological and lithodynamic process mechanisms, taken as a basis of the theory, explains the multiformity of sea level change local curves obtained by geological methods.

16. Conclusions

1. A mathematical model of the Black Sea basin water filling was created and the calculation of the Black Sea level change during the latest 20,000 years made. The mechanism of two transgressions of the Black Sea (sea and ocean) during this time period is explained. The impossibility of fluctuations of the Black Sea on the order of metres in the Holocene is shown.
2. Calculations given in this paper have shown that freshwater balance in the Black Sea was positive both in warm and cold climates. Water flow out of the Bosphorus Strait made water of the Marmara and Aegean Seas fresher. This predetermined all

main hydrophysical and lithodynamic processes in the Black Sea and in the Bosphorus Strait, namely: bottom erosion in the strait to the level of the northern and southern threshold during glacial periods; glacial erosion of thresholds; the long existence of the Black Sea as a lake in conditions of water freshening; water level rise in the strait and in the Black Sea during the period of glacier melting due to the narrow canyon along which a river ran; regression of the level during the period of the river transformation into the strait; oceanic transgression of the level; origination of counterflow in conditions of the strait deepening; and alignment of neighbouring sea level by the Bosphorus.

3. A method of eustatic curve calculation of the sea level change on the local curves and division of local curves into eustatic and tectonic components is proposed. Calculation of eustatic changes of the Mediterranean Sea level change and vertical motions of some sections of its coast in Holocene is fulfilled. In the Holocene, the Black Sea level reflected the changes of world ocean level.
4. Taking into consideration the results of Fedorov (1978), all Pleistocene transgressions occurred on the same scenario in conditions of gradual widening and deepening of the Bosphorus and Dardanelles.
5. The Bosphorus and Dardanelles were developed as river channels. When the ocean level was low their bottoms were eroded by moving water, and when the ocean level was high their bottoms were covered by sediments.

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