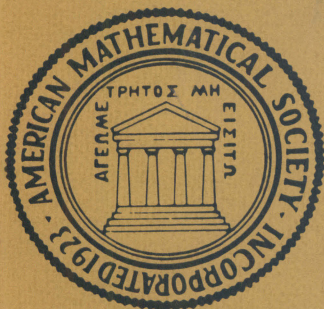


# Notices

of the  
American Mathematical Society



January 1979, Issue 191

Volume 26, Number 1, Pages 1–82, A-1–A-190

**Providence, Rhode Island USA**

ISSN 0002-9920

Handy OK

7490

## CALENDAR OF AMS MEETINGS

Theme B  
1/1/81  
13

THIS CALENDAR lists all meetings which have been approved by the Council prior to the date this issue of the NOTICES was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

ABSTRACTS OF CONTRIBUTED PAPERS should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts of papers to be presented at the meeting in person must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

MEETING NUMBER	DATE	PLACE	ABSTRACTS	
			DEADLINE	for ISSUE
764	March 30-April 1, 1979	Honolulu, Hawaii	JANUARY 30	February
765	April 19-20, 1979	New York, New York	FEBRUARY 27	April
766	April 27-28, 1979	Iowa City, Iowa	FEBRUARY 27	
767	June 15-16, 1979	Vancouver, Canada	APRIL 24	June
768	August 21-25, 1979 (83rd Summer Meeting)	Duluth, Minnesota	JUNE 12	August
	November 2-3, 1979	Kent, Ohio		
	November 16-17, 1979	Riverside, California		
	January 3-7, 1980 (86th Annual Meeting)	San Antonio, Texas		
	March 28-29, 1980	Boulder, Colorado		
	April 25-26, 1980	Davis, California		
	January 7-11, 1981 (87th Annual Meeting)	San Francisco, California		

Programs of the meetings will appear in the issues indicated above. First and second announcements of the meetings will have appeared in earlier issues. The deadline for abstracts is also the deadline for other information intended for publication in the same issue: personal items, news items, entries for the Special Meetings Information Center. Deadlines for receipt in Providence of abstracts of papers presented to the Society for publication by title (rather than for presentation in person at a meeting) are as follows:

<i>Theme based</i> ISSUE	<i>Theme based</i> DEADLINE
February 1979	JANUARY 23
April 1979	FEBRUARY 20
June 1979	APRIL 17
August 1979	JUNE 5

## OTHER EVENTS SPONSORED BY THE SOCIETY

		<i>Theme</i> ANNOUNCEMENT APPEARED
March 27-30, 1979	Symposium on the Geometry of the Laplace Operator, University of Hawaii, Honolulu, Hawaii	January 1979 p. 49
June 19-30, 1979	Seminar/Workshop on Algebraic and Geometric Methods in Linear Systems Theory, Harvard University, Cambridge, Massachusetts	January 1979 p. 52
June 25-July 20, 1979	Summer Research Institute on Finite Group Theory, University of California, Santa Cruz, California	January 1979 p. 53

# Notices

## of the American Mathematical Society

### EDITORIAL COMMITTEE

Ed Dubinsky, Robion C. Kirby,  
Arthur P. Mattuck, Barbara L.  
Osofsky, Everett Pitcher (Chairman),  
Scott W. Williams

### MANAGING EDITOR

Lincoln K. Durst

### ASSOCIATE EDITOR FOR QUERIES

Hans Samelson

### SUBSCRIPTION ORDERS

NOTICES is published eight times a year (January, February, April, June, August, October, November, and December). Subscription for Vol. 26 (1979), \$19.00 list, \$9.50 member. The subscription price for members is included in the annual dues. Subscriptions and orders for AMS publications should be addressed to the American Mathematical Society, P.O. Box 1571, Annex Station, Providence, R. I. 02901.

*All orders must be prepaid.*

### ADVERTISING & INQUIRIES

The NOTICES publishes situations wanted and classified advertising, details of which can be found in those departments in each issue, and display advertising for publishers and academic or scientific organizations. Requests for information:

*Abstracts:* Josephine Faria

*Advertising:* Elaine Lauble

*Change of address or*

*subscriptions:* Ellen Hopkins-Heiser

To avoid interruption in service please send address changes four to six weeks in advance. It is essential to include the member code which appears on your address label with all correspondence regarding subscriptions.

American Mathematical Society,  
P.O. Box 6248, Providence, RI 02940  
(Telephone 401-272-9500).

Second class postage paid at  
Providence, RI, and additional  
mailing offices; U. S. Postal Service  
Publication No. 398520.

Copyright © 1979 by the  
American Mathematical Society.

Printed in the United States of America.

Volume 26, Number 1, January 1979

- 2 MEETINGS OF THE SOCIETY
  - Biloxi, *January 24*, 2 (Abstracts A-20)
  - Special Sessions and Invited Speakers, 48
  - Hawaii, *March 30*, 49
  - New York City, *April 19*, 51
  - Iowa City, *April 27*, 52
  - Summer Seminar/Workshop, *June 19*, 52
  - Summer Research Institute, *June 25*, 53
- 54 QUERIES
- 55 POSTDOCTORAL POSITIONS
  - A CEEP Data Subcommittee Report
- 61 LETTERS to the EDITOR
- 66 NEWS & ANNOUNCEMENTS
- 71 SPECIAL MEETINGS INFORMATION
- 73 NEW AMS PUBLICATIONS
- 75 MISCELLANEOUS
  - Personal Items, 75; Deaths, 75; New Doctorates, 76; Visiting Mathematicians, 78
- 79 AMS REPORTS & COMMUNICATIONS
  - Recent appointments, 79
- 80 AMS BOOKS ON COMBINATORICS
- A-1 ABSTRACTS
  - Errata to Abstracts, A-153
- A-154 ADVERTISEMENTS
  - Classified Advertising, A-154
  - Room Reservation Form (N. Y. C. Meeting) A-190

# Program for the 85th Annual Meeting

*Convention Center, Biloxi, Mississippi, January 24–27, 1979*

The eighty-fifth annual meeting of the American Mathematical Society will be held in Biloxi, Mississippi, from Wednesday, January 24, through Saturday, January 27, 1979. Sessions will be held primarily in the Convention Center, although some will be held in adjacent hotels.

The fifty-second Josiah Willard Gibbs Lecture will be presented at 8:30 p. m. on Wednesday, January 24, by MARTIN D. KRUSKAL of the Department of Astrophysical Sciences at Princeton University. The title of his lecture is "What are solitons and inverse scattering anyway, and why should I care?"

There will be one series of four Colloquium Lectures presented by PHILLIP A. GRIFFITHS of Harvard University. The title of his lecture series is "Complex analysis and algebraic geometry." The lectures will be given at 1:00 p. m., daily, Wednesday through Saturday, January 24–27.

The Bôcher Memorial Prize and the Leroy P. Steele Prizes will be awarded at a session at 3:20 p. m. on Thursday, January 25.

By invitation of the Program Committee, there will be eight invited one-hour addresses as follows: 9:00 a. m., Wednesday, MICHAEL ARTIN, Massachusetts Institute of Technology, "Some applications of algebraic geometry to ring theory"; 10:30 a. m., Wednesday, JULIUS L. SHANESON, Rutgers University, "Manifolds and submanifolds"; 3:30 p. m., Wednesday, JOHN E. FORNAESS, Princeton University, "Proper holomorphic mappings"; 9:00 a. m. Thursday, CHARLES S. PESKIN, Courant Institute of Mathematical Sciences, New York University, "The heart valve problem of cardiac fluid dynamics and its numerical solution"; 10:30 a. m., Thursday, JACOB FELDMAN, University of California, Berkeley, "Time-change in flows"; 2:10 p. m., Thursday, ABRAHAM H. TAUB, University of California, Berkeley, "Spacetimes with distribution valued curvature tensors"; 2:15 p. m., Friday, HEINZ-OTTO KREISS, California Institute of Technology, "Problems with different time scales and their numerical solution"; and 3:30 p. m., Friday, BHAMA SRINIVASAN, Clark University, "Representations of classical groups."

By invitation of the same committee, there will be thirteen special sessions of selected twenty-minute papers. The titles of these special sessions, and the names of the mathematicians arranging them, are as follows: Number theory and its applications, STEFAN BURR; Operator theory, DOUGLAS N. CLARK; Differential geometry and general relativity, PAUL EHRLICH; Ordinary differential equations: Oscillations and asymptotic behavior, JOHN R. GRAEF and A. G. KARTSATOS; Mathematical psychology, WILLIAM C. HOFFMAN; Modular and automorphic functions in a single complex variable, MARVIN KNOPP; Diffusion reaction systems in biology, SIMON A. LEVIN; Game theory, WILLIAM LUCAS; Global differential geometry, RICHARD MILLMAN; Constructive mathematics, RAY MINES and

FRED RICHMAN; Integral equations with emphasis on Fredholm and Hammerstein equations, M. Z. NASHED; Summability and related topics, B. E. RHOADES; and Nonacademic mathematical research, ROBERT J. THOMPSON.

There will be sessions for contributed ten-minute papers Wednesday morning, Wednesday afternoon, Thursday morning, Thursday evening, Friday afternoon, and Saturday afternoon.

Rooms where special sessions and contributed-paper sessions will be held will be equipped with an overhead projector and screen. Blackboards will not be available.

At 4:45 p. m. on Friday, January 26, the AMS Committee on Science Policy will sponsor a panel discussion on "Mathematicians' views of government support of research." Richard D. Anderson will be the moderator. Among the speakers will be Joseph J. Kohn and Saunders Mac Lane.

The Society's Committee on Employment and Educational Policy (CEEP) has scheduled a meeting of heads of Ph. D.-granting departments on Wednesday, January 24, from 2:30 p. m. to 4:30 p. m. The discussion, under the direction of Lowell J. Paige, University of California, Berkeley, will be devoted to graduate education in mathematics. Other members of the panel are Edwin E. Floyd, University of Virginia; Wendell H. Fleming, Brown University; and Gail S. Young, University of Rochester.

CEEP will also present a panel discussion at 7:00 p. m. on Friday evening, January 26, entitled "The mathematician outside the university." This panel discussion, which is cosponsored by the Mathematical Association of America, will be devoted to career opportunities for mathematicians outside of academia, with emphasis on graduate programs designed to prepare mathematicians for such work. Attention will be given to academic preparation for work in industry at both the masters level and the doctorate level. The program is under the direction of William E. Boyce and Richard C. DiPrima of Rensselaer Polytechnic Institute; the panel members are Gary McDonald of General Motors Research Laboratories, Edward C. Posner of the Jet Propulsion Laboratory, California Institute of Technology, Albert C. Williams of Mobil Research and Development Corporation, and Norman D. Winarsky of RCA Laboratories.

In addition to these activities, a special session on Nonacademic Mathematical Research has been arranged by Robert J. Thompson at the suggestion of CEEP. (See list of special sessions on this page.)

At 11:30 a. m. on Saturday, January 27, the Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics will cosponsor a progress report by Edmund G. Lee concerning the work of a Congressional Fellow.

## Game Theory and its Applications January 22–23, 1979

The American Mathematical Society will present a one and one-half day short course entitled "Game Theory and its Applications," on Monday and Tuesday, January 22 and 23, 1979, in the Holiday Inn in Biloxi, Mississippi.

This area of applied mathematics is unusual in the nature of its main areas of motivation and use (the behavioral sciences rather than the physical or engineering sciences), in the varied flavor of its technical methods (often lacking the usual dominance of calculus and its descendants), and in the broad intellectual appeal of the problems with which it deals. The topic provides both an attractive addition to the mathematics curriculum, and a vehicle for joint research with scholars studying decision processes from the viewpoint of other disciplines; the course is being planned with these attributes explicitly in mind.

Game theory is a collection of mathematical models designed to study situations involving conflict and/or cooperation. It allows for a multiplicity of decision makers who may have different preferences and objectives. Such models involve a variety of different solution concepts concerned with strategic optimization, stability, bargaining, compromise, equity and coalition formation. This short course will be primarily concerned with the  $n$ -person theory ( $n \geq 3$ ). It will emphasize the cooperative models, but the fundamental aspects of the noncooperative approach will also be covered. The applications will include auctions, bidding, and market equilibria in economics and measurements of power in political science, as well as some multiperson and equity considerations in operations research.

The program is under the direction of William F. Lucas of the Center for Applied Mathematics and School of Operations Research and Industrial Engineering at Cornell University. The short course was recommended by the Society's Committee on Employment and Educational Policy, whose members are Lida K. Barrett, Alan J. Goldman, Arthur P. Mattuck, Hugo Rossi, Martha K. Smith, and Robert J. Thompson. The short course series is under the direc-

tion of the CEEP Short Course Subcommittee, whose members are Alan J. Goldman, Ronald L. Graham, Cathleen S. Morawetz, and Harold M. Stark.

The program will consist of six seventy-five minute lectures. Three lectures (one by William F. Lucas, one by Lloyd S. Shapley of the Rand Corporation, and one by Robert J. Weber of the Cowles Foundation for Research in Economics and the School of Organization and Management, Yale University) will present the basic concepts and many of the fundamental theorems from multiperson game theory. The other three lectures will be devoted to important areas of application: to political science (Lloyd S. Shapley), to operations research (William F. Lucas), and to economic markets (Louis J. Billera of the School of Operations Research and Industrial Engineering, and the Department of Mathematics, Cornell University).

Abstracts for the talks and accompanying reading lists appeared on pages A-671–A-673 of the October issue of the NOTICES. A basic knowledge of undergraduate mathematics will be presumed. A few specific theorems of a more specialized nature, e.g., Brouwer's fixed point theorem, will be employed, but prior knowledge of these is not essential. Those who wish to get the most benefit from the course should consult the excellent (but dated) elementary survey Games and Decisions by R. Duncan Luce and Howard Raiffa, Wiley and Sons, 1958, or a basic text such as Game Theory by Guillermo Owen, Saunders Company, 1968 (especially Chapters 1, 7 to 10, and the appendix). The reading lists also give a variety of sources for study prior to the course.

Note that a special session on game theory is also being planned for the January meeting, and some recent developments of a more advanced nature will be presented there.

The short course is open to all who wish to participate upon payment of the registration fee. There are reduced fees for students and unemployed individuals. Please refer to the section entitled MEETING REGISTRATION for details.

### COUNCIL AND BUSINESS MEETING

The Council of the Society will meet in the Vogue Room of the Broadwater Beach Hotel at 2:00 p. m. on Tuesday, January 23. The Business Meeting of the Society will take place at 5:00 p. m. on Thursday, January 25, in the Coliseum at the Convention Center. The secretary notes the following resolution of the Council: "Each person who attends a Business Meeting of the Society shall be willing and able to identify himself as a member of the Society." In further explanation, it is noted that "each person who is to vote at a meeting is thereby identifying himself as and

claiming to be a member of the American Mathematical Society." For additional information on the Business Meeting, refer to the box on the next page.

### OTHER ORGANIZATIONS

The Mathematical Association of America (MAA) will hold its annual meeting on January 26–28, Friday–Sunday, in conjunction with this meeting of the Society. Some sessions of the Association on Sunday, January 28, will be held jointly with the National Council of Teachers of Mathematics (NCTM). For a more detailed listing of

### Committee on the Agenda for Business Meetings

The Society has a Committee on the Agenda for Business Meetings. The purpose is to make Business Meetings orderly and effective. The committee does not have legal or administrative power. It is intended that the committee consider what may be called "quasi-political" motions. The committee has several possible courses of action on a proposed motion, including but not restricted to

- (a) doing nothing;
- (b) conferring with supporters and opponents to arrive at a mutually accepted amended version to be circulated in advance of the meeting;
- (c) recommending and planning a format for debate to suggest to a Business Meeting;
- (d) recommending referral to a committee;
- (e) recommending debate followed by referral to a committee.

There is no mechanism that requires automatic submission of a motion to the committee. However, if a motion has not been submitted through the committee, it may be thought reasonable by a Business Meeting to refer it rather than to act on it without benefit of the advice of the committee.

The committee consists of Barbara L. Osofsky, David A. Sanchez, Michael Taylor, and Guido L. Weiss, with the secretary as chairman.

In order that a motion for the Business Meeting of January 25, 1979 receive the service to be offered by the committee in the most effective manner, it should have been in the hands of the secretary by December 22, 1978.

Everett Pitcher, Secretary

the activities of the Association, see the TIME-TABLE beginning on page 9.

The Association for Symbolic Logic (ASL) will hold its annual meeting on Wednesday and Thursday, January 24-25. The TIMETABLE provides more details on the ASL program.

The Association for Women in Mathematics will present a panel discussion on "ERA and Bakke" at 4:00 p.m. on Saturday, January 27. Judith Roitman will moderate.

The Conference Board of the Mathematical Sciences (CBMS) will sponsor a panel discussion at 2:15 p.m. on Friday, January 26, on "Mathematics today." Panel members include Felix E. Browder, Peter Freund, Saunders Mac Lane (moderator), and J. Ian Richards. The discussion will present some of the ideas arising in the volume *Mathematics Today*, just published by the Joint Projects Committee for Mathematics (Hirsh Cohen, chairman). CBMS will also sponsor a symposium on "Computer science and the mathematical sciences: Interfaces and overlaps," at 7:30 p.m. on Thursday, January 25. The symposium is under the direction of E. P. Miles, Jr., of Florida State University.

National Science Foundation (NSF) staff members will be available in the exhibit area to provide counsel and information on NSF programs of interest to mathematicians from 9:00 a.m. to 5:00 p.m., Thursday through Saturday, January 25-27. NSF will also sponsor a discussion of the project announcement of the Mathematical Sciences Research Institute at noon on Friday, January 26.

### MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The Employment Register will be in session on Thursday, Friday, and Saturday, January 25-27, in the Foyer of the Convention Center. A short (optional) orientation session will be held

by the AMS-MAA-SIAM Joint Committee on Employment Opportunities at 9:00 a.m. on Thursday, January 25. The purpose of this session is to familiarize participants with the operation of the Register and with registration procedures. Registration for the Register will begin at 9:30 a.m. on Thursday, and interviews will begin at 9:30 a.m. on Friday and Saturday. Interview request cards must be turned in to the code clerk before 4:00 p.m. on the day prior to the interview.

Provision has been made for scheduling of interviews in half-day modules. This allows for four half-days of interviews; Friday a.m. and p.m., and Saturday a.m. and p.m. There will be no interviews scheduled for Thursday.

On Saturday afternoon, an "employers' choice" session has been scheduled. For this session, interviews will be scheduled with applicants requested by employers. Applicants may not submit interview request forms for this session. Requests for interviews must be submitted by the employer on Friday prior to the deadline of 4:00 p.m. in order to receive a schedule for Saturday afternoon.

Applicants and employers should be sure to indicate in the appropriate place on the forms exactly what times they will be available for interviews. Applicants and employers are asked not to duplicate their interview requests for both morning and afternoon schedules on the same day; applicants and employers should also be advised that the program will NOT automatically reschedule a morning appointment to an afternoon session, if it could not be scheduled when requested for the morning. Interview requests should not be submitted, of course, unless the individual requested has indicated availability during the time period desired. Morning schedules will be distributed on Friday and Saturday at 8:45 a.m.; the afternoon schedules will be distributed at 9:00 a.m. on the same days.

Applicants should be aware of the fact that

interviews arranged by the Register are only an initial contact with employers, and hiring decisions are not always made immediately after the interview.

Lists of preregistered employers and applicants will be distributed in Biloxi free of charge to those who preregistered. Other participants may obtain copies of the printed lists at the meeting for \$1 each.

All participants in the Register are required to register for the Joint Mathematics Meetings. For applicants there is no additional fee for participation in the Register. For employers, additional fees are \$10 if paid at time of preregistration, or \$15 if paid at the meeting.

#### EXHIBITS

The book and educational media exhibits will be located in the Exhibit Area of the Convention Center from Thursday through Saturday, January 25-27. The exhibits will be open from 9:00 a. m. to 5:00 p. m.

All participants are encouraged to visit the exhibits during the meeting.

#### BOOK AND AUDIO TAPES SALES

Books published by the Society and the Association, and audio tapes of AMS invited addresses, will be sold for cash prices somewhat below the usual prices when these same books and tapes are sold by mail. The book sales will be located in the Exhibit Area of the Convention Center.

#### MEETING REGISTRATION

Participants who wished to preregister for the meetings should have completed the preregistration form and submitted it to Providence by December 22, 1978. Preregistrants will be able to pick up their badges and programs after 2:00 p. m. on Tuesday, January 23. Complete instructions on making hotel reservations are given in the section titled HOTEL ACCOMMODATIONS.

Meeting preregistration and registration fees partially cover expenses of holding the meetings. The preregistration fee does not represent an advance deposit for lodgings.

Please note that separate fees are required for the Short Course and the Joint Mathematics Meetings. These fees are as follows:

#### AMS Short Course Game Theory and its Applications

	At Meeting
Student/unemployed	\$ 5
All other participants	20
One-day fee for second day only	10

#### Joint Mathematics Meetings

	At Meeting
Members of AMS, ASL, MAA, and NCTM	\$25
Nonmembers	35
Student/unemployed	3

There will be no extra charge for members of the families of registered participants, except that all professional mathematicians who wish to attend sessions must register independently.

Students are considered to be only those currently working toward a degree who do not receive compensation totaling more than \$7,000 from employment, fellowships, and scholarships.

The unemployed status refers to any person currently unemployed, actively seeking employment, and who is not a student. It is not intended to include persons who have voluntarily resigned or retired from their latest position.

A fifty percent refund of the preregistration fee will be made for all cancellations received in Providence no later than January 21. There will be no refunds granted for cancellations received after that date, or to persons who do not attend the meetings.

Registration for the short course only will begin on Monday, January 22, at noon and will be located in the lobby of the Holiday Inn. Participants who are not attending the short course are advised that no general meeting information or registration material will be available prior to the time listed below for the Joint Mathematics Meetings registration. The Joint Mathematics Meetings registration desk will be located in the Exhibit Area of the Convention Center. The desks will be open during the hours listed below:

#### AMS Short Course on GAME THEORY AND ITS APPLICATIONS Holiday Inn Lobby

Monday, January 22	noon - 5:00 p. m.
Tuesday, January 23	8:00 a. m. - 2:00 p. m.

#### JOINT MATHEMATICS MEETINGS Exhibit Area, Convention Center

Tuesday, January 23	2:00 p. m. - 8:00 p. m.
Wednesday, January 24	8:00 a. m. - 5:00 p. m.
Thursday, January 25	} 8:00 a. m. - 4:00 p. m.
Friday, January 26	
Saturday, January 27	
Sunday, January 28	8:30 a. m. - 2:30 p. m.

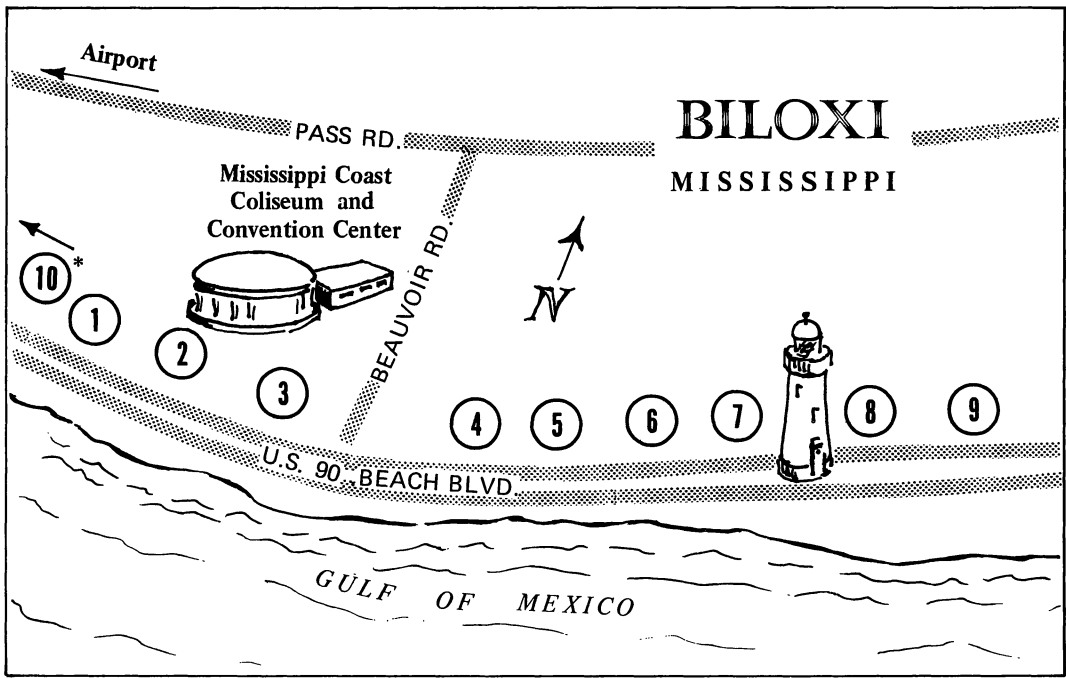
#### HOTEL ACCOMMODATIONS

The form for requesting accommodations in advance should have been submitted so as to arrive no later than December 22, 1978. Participants wishing hotel reservations who did not request them before the deadline must deal directly with the hotels listed below.

Participants wishing to share accommodations should be aware that the Broadwater Beach has some cottages which sleep four and some that sleep ten. These cottages have light kitchen facilities.

In all cases "single" refers to one person in one bed; "double" refers to two persons in one bed; and "twin" refers to two persons in two beds. A rollaway cot for an extra person can be added to double or twin rooms only.

Please make all reservation changes with the Housing Bureau in Providence prior to January 20. After that date, changes and cancellations must be made directly with Mrs. Nicey Lewis of the Mississippi Gulf Coast Convention Bureau (601-388-8000). The rates quoted here are sub-



- |                     |                     |                       |
|---------------------|---------------------|-----------------------|
| 1. Howard Johnson's | 4. Broadwater Beach | 7. Rodeway Inn        |
| 2. Admiral Benbow   | 5. Biloxi Hilton    | 8. Ramada Inn         |
| 3. Holiday Inn      | 6. Sheraton-Biloxi  | 9. La Linda Motor Inn |
- \* 10. Biloxi Beach Motor Inn— $1\frac{1}{2}$  miles west of ①

ject to a seven percent sales tax; all hotels are in Biloxi, Mississippi 39531. The number in a circle after the name of the hotel refers to the number it carries on the map. Shuttle bus service will be provided to and from the Convention Center and all hotels listed except the Admiral Benbow, Biloxi Beach Motor Inn, Holiday Inn, and Howard Johnson's Motor Lodge. The Biloxi Beach Motor Inn will provide shuttle service for participants using its own vehicles.

All hotels listed are on the beach and overlook the Gulf of Mexico.

#### THE BROADWATER BEACH (headquarters) ④

West Beach Boulevard

Telephone: Toll-free (800) 647-3964 or  
(601) 388-2211

Single	\$32, \$40
Double	\$36, \$44
Triple	\$41, \$49
Quadruple	\$46, \$54

#### Executive Cottage (sleeps 10)

(1-6 people)	\$150
(7 people)	\$155
(8 people)	\$160
(9 people)	\$165
(10 people)	\$170

#### Single Cottage (sleeps 4)

(1-2 people)	\$50
(3 people)	\$55
(4 people)	\$60

#### ADMIRAL BENBOW INN ②

West Beach Boulevard

Telephone: (601) 388-1000

Single	\$17
Double	\$22
Twin Double	\$24, \$26
Triple	\$28, \$30, \$32
Quadruple	\$32

#### BILOXI BEACH MOTOR INN ⑩

West Beach Boulevard

Telephone: (601) 388-3310

Single	\$21
Double	\$24
Twin Double	\$24
Triple	\$28
Quadruple	\$32
Quintuple	\$36
Parlor Suite	\$75
(one bedroom)	

#### BILOXI HILTON ⑤

3580 West Beach Boulevard

Telephone: (601) 388-7000

Single	\$26, \$29, \$33, \$38, \$40, \$42
Double	\$34, \$37, \$41, \$46, \$48, \$50
Twin Double	\$34, \$37, \$41, \$46, \$48, \$50
Triple	\$42, \$45, \$49, \$54, \$56, \$58
Quadruple	\$50, \$53, \$57, \$62, \$64, \$68
Suites: 1 bedroom	\$75, \$110, \$150
2 bedroom	\$150, \$186



**HOLIDAY INN** ③  
 92 West Beach Boulevard  
 Telephone: (601) 388-3551  
 Single \$24  
 Twin Double \$30  
 Triple \$34  
 Quadruple \$38  
 Quintuple \$42

**HOWARD JOHNSON'S MOTOR LODGE** ①  
 3920 West Beach Boulevard  
 Telephone: (601) 388-6310  
 Single \$22  
 Twin Double \$28  
 Triple \$32, \$36  
 Quadruple \$36

**LA LINDA MOTOR INN** ⑨  
 3410 West Beach Boulevard  
 Telephone: (601) 388-4621  
 Single \$20  
 Double \$20  
 Twin Double \$24  
 Triple \$24, \$28, \$32  
 Quadruple \$32  
 Quintuple \$36

**RAMADA INN** ⑧  
 3719 West Beach Boulevard  
 Telephone: (601) 388-5512  
 Single \$21  
 Twin Double \$23, \$25  
 Triple \$27, \$29  
 Quadruple \$31, \$33  
 Parlor Suite \$60  
 (one bedroom)

**RODEWAY INN** ⑦  
 West Beach Boulevard  
 Telephone: (601) 388-3131  
 Single \$21  
 Twin Double \$23, \$28  
 Triple \$27, \$32, \$36  
 Quadruple \$31, \$36  
 Quintuple \$40

**SHERATON-BILOXI MOTOR INN** ⑥  
 3634 West Beach Boulevard  
 Telephone: (601) 388-4141  
 Single \$36  
 Twin Double \$42  
 Triple \$46, \$50  
 Quadruple \$50  
 Quintuple \$54  
 Parlor Suite  
 (one bedroom) \$120  
 (two bedroom) \$160

**ENTERTAINMENT AND LOCAL INFORMATION**

Biloxi, Mississippi, is located midway between New Orleans, Louisiana, and Mobile, Alabama, on a beautiful 26-mile-long white sand beach. There is a wide range of outdoor recreational activities available including boating, tennis, swimming, fresh and salt water fishing, strolling along the beach or just plain lazin' in the sun. In addition, the area has such attractions as Beauvoir (Jefferson Davis' restored

plantation estate), and theme parks like Marine Life, Deer Ranch and Eight Flags. Shopping, antiquing, scenic drives, and tours are available. Gourmet dining on delicious seafood and a choice of exciting "after dark" entertainment offer interesting night life. A tour of beautiful Bellingrath Gardens near Theodore, Alabama, may be of interest, or perhaps an excursion to old New Orleans for a delightful evening. Detailed information concerning these activities will be available at the Local Information Section of the registration desk.

The Local Arrangements Committee has planned a social for 9:00 p.m. on Friday, January 26, at the Broadwater Beach. A cash bar will be available, and a jazz band and other entertainment will be provided. Also, a bus trip to New Orleans is being organized for Saturday evening, January 27. Additional details will be available at the Local Information Section of the registration desk.

Addison-Wesley will sponsor a foot race for interested participants early Friday morning. For further details, please visit the Addison-Wesley booth in the Exhibit Area.

**CHILD CARE**

All the hotels being used for the meeting have lists of qualified babysitters who will come to the hotel. Prices range from \$2 to \$3 per hour. Several licensed day care centers are close to the hotels and Convention Center:

A B C Nurseries, 650 Forrest Avenue, Biloxi, 39531, twenty-four hours. \$0.95/hr. one child, \$1.60/hr. two children. Telephone (601) 435-4735.

Smith's Day Care & Kindergarten, 405 West Division Street, Biloxi, 39531, twenty-four hours. \$1/hr. one child, \$1.50/hr. two children. Telephone (601) 432-7304.

Kinderworld, Pass Road, Biloxi, 39531, twenty-four hours. \$0.75/hr. one child, \$0.90/hr. two children, \$1.10/hr. three children. Telephone (601) 388-2748.

**MAIL AND MESSAGE CENTER**

All mail and telegrams for persons attending the meetings should be addressed in care of Joint Mathematics Meetings, Mississippi Coast Coliseum and Convention Center, Biloxi, Mississippi 39531. Mail and telegrams so addressed may be picked up at the meeting registration area in the Convention Center.

A telephone message center will be located in the same area to receive incoming calls for all participants. The center will be open from January 23 through January 28, during the same hours as the Joint Meetings registration desk. Messages will be taken down and the name of any individual for whom a message has been received will be posted until the message has been picked up at the message center. The telephone number of the center is (601) 388-8003.

## TRAVEL

The George Smith Travel Service of Gulfport has offered to assist participants in obtaining airline transportation to the Biloxi meeting at the lowest possible cost, including group and other fares. Mr. Smith will also arrange ground transportation from the New Orleans and Gulfport airports. All participants arranging their air transportation through Smith will receive colored luggage tags, which should be attached to the participant's luggage before it is loaded onto the aircraft at point of origin. When participants arrive at the airport, they should go to the Joint Mathematics Meetings transportation desk set up in the baggage claim area, and present their luggage claim tickets. Luggage will be claimed for the participant, put on a bus to the appropriate hotel in Biloxi, and delivered to the participant's room (provided the participant has preregistered and obtained a hotel room through the Housing Bureau). The participant can then purchase a ticket at the transportation desk for the ground travel (\$15 from New Orleans, \$5 from Gulfport, including tips). A lounge area will be provided at New Orleans, where a cash bar will be set up, and where participants can await the departure of the bus to Biloxi in comfort.

Also available is the Mississippi Coast Limousine Service (telephone 601-432-2649 or 864-7660), which operates by reservation only, and provides service from New Orleans six times daily. The trip takes about one hour and twenty minutes and, again, the cost is \$15, but not including tips. All major car rental agencies maintain desks at the New Orleans International Airport.

Participants who have not purchased their airline tickets through Smith, but who have obtained a room through the Housing Bureau, may purchase a ground travel ticket at the airport transportation desk, turn over their luggage claim tickets, and indicate the hotel to which the bags should be delivered. These services will, of course, be available to participants arriving on regularly scheduled flights; charter and special flights not arranged by Smith cannot be anticipated, however, and the transportation desk may not be open for these arrivals.

Participants who have neither purchased their airline tickets through Smith, nor obtained a room through the Housing Bureau, may be able

to obtain a hotel room assignment at the airport transportation desk, turn over their luggage claim tickets, and purchase a ground travel ticket, but a room cannot be guaranteed. Again, these services will be available only to participants arriving on regularly scheduled flights.

There will also be a transportation desk set up in the Joint Meetings registration area in the Convention Center so that participants may sign up for return airport transportation.

There will be shuttle bus service at a charge to and from the Convention Center and those hotels not immediately adjacent to it. The shuttle will also carry participants to and from area restaurants representing a wide range in both price and type of food during the evening mealtime hours. Information on the cost of this service and the schedule will be in the meeting registration packet.

AMTRAK provides daily service from Chicago and Los Angeles into New Orleans. Trailways and Greyhound buses serve Biloxi on a regular basis. There is regular city bus service available from the Biloxi bus terminal to the hotels, as well as taxi service.

Participants driving to the meeting can reach Biloxi from the interstate highway system from the north on the Interstates I25, I35, I45, I55, I65, I85, and I95, which connect with the Interstates I90 or I10 from the east or west.

## WEATHER

For the month of January in Biloxi the average daily maximum temperature is 16°C (60.7°F) and the average daily minimum is 6.3°C (43.4°F). The average January rainfall is 12 cm (4.76 inches). The highest temperature recorded in January is 24°C (75°F) and the lowest is -12°C (10°F); however, on the average, below freezing temperatures occur only 5 days in the month.

## LOCAL ARRANGEMENTS COMMITTEE

Thomas A. Atchison (chairman), John T. Baldwin, Frank T. Birtel (ex officio), Wendell Deer, Stephen A. Doblin, Roosevelt Gentry, James E. Keisler, William J. LeVeque (ex officio), Eldon L. Miller, Carol B. Ottinger, Charles S. Rees, David P. Roselle (ex officio), Robert A. Shive, Jr., and Billy R. Sneed.

All sessions are at the Convention Center, unless noted as follows:

BB - Broadwater Beach Hotel  
HI - Holiday Inn

## Timetable

(Central Standard Time)

AMERICAN MATHEMATICAL SOCIETY SHORT COURSE SERIES	
MONDAY, January 22	GAME THEORY AND ITS APPLICATIONS
noon - 5:00 p. m.	REGISTRATION (Short Course Only) - Lobby, HI
	<u>MISSISSIPPI ROOMS NORTH &amp; SOUTH, HI</u>
2:00 p. m. - 3:15 p. m.	Game theory: An overview and the n-person cooperative model William F. Lucas
3:45 p. m. - 5:00 p. m.	Valuation of games Lloyd S. Shapley
TUESDAY, January 23	
8:00 a. m. - 2:00 p. m.	REGISTRATION (Short Course Only) - Lobby, HI
	<u>MISSISSIPPI ROOMS NORTH &amp; SOUTH, HI</u>
9:00 a. m. - 10:15 a. m.	Noncooperative games Robert J. Weber
10:45 a. m. - noon	Economic market games Louis J. Billera
1:30 p. m. - 2:45 p. m.	Measurement of power in political systems Lloyd S. Shapley
3:00 p. m. - 4:15 p. m.	Some uses of game theory in operations research William F. Lucas
4:30 p. m. - 5:00 p. m.	General discussion
JOINT MATHEMATICS MEETINGS	
TUESDAY, January 23	American Mathematical Society
2:00 p. m. - 8:00 p. m.	REGISTRATION - Exhibit Area
2:00 p. m. - 8:00 p. m.	BOOK SALES - Exhibit Area
2:00 p. m.	COUNCIL MEETING Vogue Room, BB
WEDNESDAY, January 24	AMS
8:00 a. m. - 5:00 p. m.	REGISTRATION - Exhibit Area
8:00 a. m. - 5:00 p. m.	BOOK SALES - Exhibit Area
	SESSIONS FOR CONTRIBUTED PAPERS
8:00 a. m. - 11:55 a. m.	One and Several Complex Variables Room 5
8:00 a. m. - 9:55 a. m.	Linear and Multilinear Algebra, Matrix Theory (Finite and Infinite) Room 9
	SPECIAL SESSIONS
8:30 a. m. - 11:20 a. m.	Game Theory, I Room 1
8:30 a. m. - 12:20 p. m.	Integral Equations with Emphasis on Fredholm and Hammerstein Equations, I Room 6
8:30 a. m. - 11:20 a. m.	Global Differential Geometry, I Room 4
8:30 a. m. - 10:55 a. m.	Constructive Mathematics, I Room 11
8:30 a. m. - 11:10 a. m.	Diffusion Reaction Systems in Biology, I Room 8
	SESSIONS FOR CONTRIBUTED PAPERS
8:30 a. m. - 11:55 a. m.	Number Theory Room 2
8:30 a. m. - 11:55 a. m.	Operator Theory, I Room 7
9:00 a. m. - 10:00 a. m.	INVITED ADDRESS: Some applications of algebraic geometry to ring theory Michael Artin, Coliseum

TIMETABLE

WEDNESDAY, January 24	American Mathematical Society	Other Organizations
9:00 a.m. - 10:30 a.m.	SESSION FOR CONTRIBUTED PAPERS	Association for Symbolic Logic SESSIONS FOR CONTRIBUTED PAPERS Esquire & Vogue Rooms, BB
10:15 a.m. - 11:10 a.m.	Special Functions Room 12	
10:30 a.m. - 11:30 a.m.	INVITED ADDRESS: Manifolds and submanifolds Julius L. Shaneson, Coliseum	
10:45 a.m. - 11:45 a.m.		ASL - RETIRING PRESIDENTIAL ADDRESS A report on intuitionistic algebra Dana Scott, Vogue Room, BB
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURES: Complex analysis and algebraic geometry Lecture I Phillip A. Griffiths, Coliseum	
1:30 p.m. - 2:30 p.m.		ASL - INVITED ADDRESS Determinateness and the continuum problem John Steel, Vogue Room, BB
	SPECIAL SESSIONS	
2:15 p.m. - 4:40 p.m.	Game Theory, II Room 1	
2:15 p.m. - 6:30 p.m.	Integral Equations with Emphasis on Fredholm and Hammerstein Equations, II Room 6	
2:15 p.m. - 4:40 p.m.	Global Differential Geometry, II Room 4	
2:15 p.m. - 5:05 p.m.	Constructive Mathematics, II Room 11	
2:15 p.m. - 4:55 p.m.	Diffusion Reaction Systems in Biology, II Room 8	
	SESSIONS FOR CONTRIBUTED PAPERS	
2:15 p.m. - 4:55 p.m.	Real Functions, Measure and Integration Room 2	
2:15 p.m. - 4:40 p.m.	Harmonic Analysis and Integral Transforms Room 3	
2:15 p.m. - 4:55 p.m.	Operator Theory, II Room 7	
2:15 p.m. - 5:40 p.m.	Ordered Algebraic Structures and General Mathematical Systems Room 10	
2:30 p.m. - 4:30 p.m.	Committee on Employment and Educational Policy MEETING OF DEPARTMENT HEADS Coronet Room, BB	
2:45 p.m. - 4:00 p.m.		ASL - SPECIAL SESSION A talk on proof theory Daniel J. Leivant  Constructability and large cardinals William Mitchell Vogue Room, BB
	SESSION FOR CONTRIBUTED PAPERS	
3:15 p.m. - 5:25 p.m.	Statistics, Computer Science, Games and Economics Room 9	
3:30 p.m. - 4:30 p.m.	INVITED ADDRESS: Proper holomorphic mappings John E. Fornaess, Coliseum	
4:10 p.m. - 5:20 p.m.		ASL - SESSIONS FOR CONTRIBUTED PAPERS Esquire & Vogue Rooms, BB
	SESSION FOR CONTRIBUTED PAPERS	
5:00 p.m. - 6:10 p.m.	Nonassociative Rings and Algebras Room 5	
5:30 p.m. - 7:00 p.m.		ASL - COCKTAIL PARTY Crown Room, BB
8:00 p.m. - 11:00 p.m.		ASL - COUNCIL MEETING Esquire Room, BB
8:30 p.m. - 9:30 p.m.	JOSIAH WILLARD GIBBS LECTURE "What <u>are</u> solitons and inverse scattering anyway, and why should I care?" Martin D. Kruskal, Coliseum	

TIMETABLE

THURSDAY, January 25	American Mathematical Society	Other Organizations
8:00 a.m. - 4:00 p.m.	REGISTRATION - Exhibit Area	
8:00 a.m. - 5:00 p.m.	BOOK SALES - Exhibit Area	
	SESSION FOR CONTRIBUTED PAPERS	
8:00 a.m. - 12:10 p.m.	Combinatorics, I Room 9	
	SPECIAL SESSIONS	
8:30 a.m. - 12:20 p.m.	Integral Equations with Emphasis on Fredholm and Hammerstein Equations, III Room 6	
8:30 a.m. - 11:30 a.m.	Number Theory and its Applications, I Room 8	
8:30 a.m. - 11:20 a.m.	Summability and Related Topics, I Room 7	
8:30 a.m. - 11:45 a.m.	Modular and Automorphic Functions in a Single Complex Variable, I Room 1	
8:30 a.m. - 11:20 a.m.	Mathematical Psychology, I Room 11	
	SESSIONS FOR CONTRIBUTED PAPERS	
8:30 a.m. - 11:25 a.m.	Geometry Room 2	
8:30 a.m. - 11:55 a.m.	General Topology, I Room 12	
8:30 a.m. - 11:10 a.m.	Commutative Rings and Algebras Room 5	
8:30 a.m. - 12:10 p.m.	Ordinary Differential Equations, I Room 10	
8:40 a.m. - 9:50 a.m.		ASL - SESSIONS FOR CONTRIBUTED PAPERS Esquire & Vogue Rooms, BB
9:00 a.m. - 10:00 a.m.	INVITED ADDRESS: The heart valve problem of cardiac fluid dynamics and its numerical solution Charles S. Peskin, Coliseum	
9:00 a.m. - 5:00 p.m.	EXHIBITS - Exhibit Area	
9:00 a.m. - 9:30 a.m.	EMPLOYMENT REGISTER	ORIENTATION SESSION - Foyer
	SESSIONS FOR CONTRIBUTED PAPERS	
9:00 a.m. - 11:55 a.m.	Partial Differential Equations Room 3	
9:00 a.m. - 11:25 a.m.	History, Logic and Foundations Room 4	
9:00 a.m. - 4:00 p.m.		Mathematical Association of America BOARD OF GOVERNORS MEETING Coronet Room, BB
9:30 a.m. - 4:00 p.m.	EMPLOYMENT REGISTER REGISTRATION - Foyer	
10:00 a.m. - 11:20 a.m.		ASL - SPECIAL SESSION Model Theory, Vogue Room, BB  First order theories of modules Steven Garavaglia  Models of arithmetic Julia Knight
10:30 a.m. - 11:30 a.m.	INVITED ADDRESS: Time-change in flows Jacob Feldman, Coliseum	
	SESSION FOR CONTRIBUTED PAPERS	
11:25 a.m. - 12:20 p.m.	Category Theory, Homological Algebras Room 5	
11:30 a.m. - 12:30 p.m.		ASL - RETIRING PRESIDENTIAL ADDRESS New axioms for set theory Joseph R. Shoenfield, Vogue Room, BB
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURE II Complex analysis and algebraic geometry Phillip A. Griffiths, Coliseum	
2:10 p.m. - 3:10 p.m.	INVITED ADDRESS: Space-times with distribution valued curvature tensors Abraham H. Taub, Coliseum	

TIMETABLE

THURSDAY, January 25	American Mathematical Society	Other Organizations
3:20 p. m. - 4:50 p. m.	BŒCHER AND STEELE PRIZE SESSION Coliseum	
5:00 p. m. - 6:00 p. m.	BUSINESS MEETING Coliseum	
	SPECIAL SESSIONS	
7:00 p. m. - 10:00 p. m.	Number Theory and its Applications, II Room 8	
7:00 p. m. - 9:50 p. m.	Summability and Related Topics, II Room 7	
7:00 p. m. - 9:25 p. m.	Mathematical Psychology, II Room 11	
7:00 p. m. - 9:25 p. m.	Differential Geometry and General Relativity, I Room 6	
	SESSIONS FOR CONTRIBUTED PAPERS	
7:00 p. m. - 9:55 p. m.	Combinatorics, II Room 9	
7:00 p. m. - 9:10 p. m.	Algebraic Number Theory, Field Theory and Polynomials Room 2	
7:00 p. m. - 8:25 p. m.	Topological Groups, Lie Groups Room 3	
7:00 p. m. - 9:55 p. m.	Ordinary Differential Equations, II Room 10	
7:00 p. m. - 8:40 p. m.	Integral Equations, Calculus of Variations, Optimal Control Room 5	
7:00 p. m. - 9:10 p. m.	General Topology, II Room 12	
7:00 p. m. - 9:55 p. m.	Numerical Analysis and Information Theory Room 4	
7:00 p. m.		MAA - FILM PROGRAM Coliseum
		Unless noted otherwise, all films are in color
7:00 p. m. - 7:22 p. m.		Adventures in perception (Maurits Escher)
7:25 p. m. - 7:45 p. m.		Cycloidal curves or Tales from the Wanklenberg Woods (Carl B. Allendoerfer)
		Films of the Topology Films Project:
7:50 p. m. - 7:55 p. m.		Zooms
7:57 p. m. - 8:00 p. m.		Sierpinski's curve drawn as a function of time
8:01 p. m. - 8:20 p. m.		Regular homotopies in the plane, Part II
		A presentation of films made in collaboration with Charles Strauss
8:25 p. m. - 9:30 p. m.		Computer animation and the geometry of surfaces in 3- and 4-space, Thomas F. Banchoff
7:30 p. m. - 10:00 p. m.		Conference Board of the Mathematical Sci- ences - Symposium on Computer Science and the Mathematical Sciences: Interfaces and Overlaps - E. P. Miles, Jr. (presiding) Coronet Room, BB
		Keynote Address - Interplay between mathe- matics and computer science Jacob Schwartz
		Interactions in Specific Areas Stylitism, synergism, syncretism—the interface of computer science and operations research Richard Nance
		Computer science and recursion theory Marian Pour-El
		Computer science core curriculum and mathematics Alan Tucker
		Computer science and numerical analysis Robert Gregory

TIMETABLE

THURSDAY, January 25	American Mathematical Society	Other Organizations
8:00 p.m. - 10:00 p.m.		CBMS - Symposium (continued) Computer science and graduate education in applied mathematics Clayton Aucoin Complexity of computations A. Borodin Computer science and statistics Olin Johnson Computer related curricular changes in traditional mathematics E. P. Miles, Jr. Conclusion Summary and discussion E. P. Miles, Jr. ASL - COUNCIL MEETING Esquire Room, BB
FRIDAY, January 26	AMS	Other Organizations
8:00 a.m. - 4:00 p.m.	REGISTRATION - Exhibit Area	
8:00 a.m. - 5:00 p.m.	BOOK SALES - Exhibit Area	
9:00 a.m. - 5:00 p.m.	EXHIBITS - Exhibit Area	
9:00 a.m. - 5:30 p.m.	EMPLOYMENT REGISTER INTERVIEWS - Foyer	
9:00 a.m. - 10:20 a.m.		MAA - PANEL DISCUSSION: Innovations in the teaching of calculus M. Crampin Louis C. Leithold Jean Nunn R. M. Pengelly John Richmond (moderator) Coliseum
10:30 a.m. - 11:50 a.m.		MAA - INVITED ADDRESS Calculus with programmable calculators Harry P. Allen, Rooms 4, 6, 7 and 8
10:30 a.m. - 11:50 a.m.		MAA - PANEL DISCUSSION: The advanced mathematics test of the Graduate Records Examination Richard D. Anderson (moderator) Coliseum The test construction process J. R. Jefferson Wadkins The changing nature of the examination I. N. Herstein Item analysis—what is shown about collegiate mathematics Gloria C. Hewitt
noon - 1:00 p.m.		National Science Foundation - Discussion of project announcement of Mathematical Sciences Research Institute Room 1
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURE III Complex analysis and algebraic geometry Phillip A. Griffiths, Coliseum	
2:15 p.m. - 3:15 p.m.	INVITED ADDRESS: Problems with different time scales and their numerical solution Heinz-Otto Kreiss, Coliseum	
2:15 p.m. - 5:05 p.m.	SPECIAL SESSIONS Modular and Automorphic Functions in a Single Complex Variable, II Room 1	

TIMETABLE

FRIDAY, January 26	American Mathematical Society	Other Organizations
	SPECIAL SESSIONS	
2:15 p. m. - 5:30 p. m.	Differential Geometry and General Relativity, II Room 6	
2:15 p. m. - 4:15 p. m.	Operator Theory (General), I Room 8	
2:15 p. m. - 5:30 p. m.	Ordinary Differential Equations: Oscillation and Asymptotic Behavior, I Room 7	
	SESSIONS FOR CONTRIBUTED PAPERS	
2:15 p. m. - 4:25 p. m.	Algebraic Geometry Room 2	
2:15 p. m. - 5:55 p. m.	Group Theory and Generalizations, I Room 12	
2:15 p. m. - 5:25 p. m.	Functional Analysis, I Room 4	
2:15 p. m. - 6:25 p. m.	Algebraic Topology and Manifolds Room 11	
2:15 p. m. - 5:55 p. m.	Probability Theory and Stochastic Processes Room 5	
2:15 p. m. - 4:15 p. m.		CBMS - PANEL DISCUSSION Mathematics today Saunders Mac Lane (moderator) Coronet Room, BB  The relevance of mathematics Felix E. Browder  Mathematics and theoretical physics Peter Freund  Number theory J. Ian Richards
3:00 p. m. - 5:00 p. m.		MAA - Committee on Two-Year Colleges INFORMAL MEETING Green Room
3:30 p. m. - 4:30 p. m.	INVITED ADDRESS: Representations of classical groups Bhama Srinivasan, Coliseum	
	SESSION FOR CONTRIBUTED PAPERS	
3:30 p. m. - 4:55 p. m.	Biology, Behavioral Sciences and Fluid Mechanics Room 9	
4:45 p. m.	Committee on Science Policy PANEL DISCUSSION: Mathematicians' views of government support of research Richard D. Anderson (moderator) Joseph J. Kohn Saunders Mac Lane Coliseum	
7:00 p. m. - 9:00 p. m.		AMS Committee on Employment and Educational Policy/MAA PANEL DISCUSSION: The mathe- matician outside the university Gary McDonald Edward C. Posner Albert C. Williams Norman D. Winarsky Coliseum
9:00 p. m.		COCKTAIL PARTY
SATURDAY, January 27	AMS	Other Organizations
8:00 a. m. - 4:00 p. m.	REGISTRATION - Exhibit Area	
8:00 a. m. - 4:00 p. m.	BOOK SALES - Exhibit Area	
9:00 a. m. - 5:00 p. m.	EXHIBITS - Exhibit Area	
9:00 a. m. - 5:30 p. m.	EMPLOYMENT REGISTER INTERVIEWS - Foyer	
9:00 a. m. - 9:50 a. m.		MAA - PANEL DISCUSSION: National Science Foundation programs in mathematics education and research Alphonse Buccino Thomas A. Keenan William H. Pell (moderator) Alvin I. Thaler Coliseum



TIMETABLE

SATURDAY, January 27	American Mathematical Society	Other Organizations
10:00 a.m. - 10:50 a.m.		MAA - BUSINESS MEETING Coliseum
11:00 a.m. - 11:50 a.m.		MAA - PANEL DISCUSSION: Prospects in mathematics education in the 1980's: A report on a conference Gloria F. Gilmer Donald L. Kreider (moderator) James W. Vick Coliseum
11:30 a.m. - 12:30 p.m.		AMS-MAA-SIAM Progress report on the work of a Congressional Fellow Edmund G. Lee, Rooms 4 and 6
noon - 1:00 p.m.		MAA - POSTER SESSION Exhibit Area
noon - 1:00 p.m.		MAA - Louisiana-Mississippi Section BUSINESS MEETING Room 7
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURE IV Complex analysis and algebraic geometry Phillip A. Griffiths, Coliseum	
	SPECIAL SESSIONS	
2:15 p.m. - 3:50 p.m.	Operator Theory (Subsession on Subnormal Operators), II Room 8	
2:15 p.m. - 5:30 p.m.	Ordinary Differential Equations: Oscillation and Asymptotic Behavior, II Room 7	
2:15 p.m. - 4:40 p.m.	Nonacademic Mathematical Research Room 11	
	SESSIONS FOR CONTRIBUTED PAPERS	
2:15 p.m. - 5:25 p.m.	Associative Rings and Algebras Room 5	
2:15 p.m. - 4:40 p.m.	Group Theory and Generalizations, II Room 12	
2:15 p.m. - 4:40 p.m.	Sequences, Series and Approximations Room 2	
2:15 p.m. - 4:55 p.m.	Functional Analysis, II Room 4	
2:15 p.m. - 2:55 p.m.	Mathematical Education Room 6	
4:00 p.m. - 5:00 p.m.		Association for Women in Mathematics PANEL DISCUSSION: ERA and Bakke Judith Roitman (moderator) Room 1
7:00 p.m.		MAA - FILM PROGRAM Rooms 4, 6, 7 and 8  Unless noted otherwise, all films are in color
7:00 p.m. - 8:01 p.m.		Let us teach guessing (George Pólya)
8:05 p.m. - 8:12 p.m.		Newton's equal areas
8:15 p.m. - 8:24 p.m.		Linear programming
8:27 p.m. - 8:42 p.m.		The theorem of the mean—a film of the MAA Calculus Film Project
8:45 p.m. - 8:58 p.m.		Dihedral kaleidoscopes—a film of the College of Geometry Project
9:00 p.m.		BBC Broadcast as part of the Calculus Block of the Open University's Foundation Course in Mathematics
9:00 p.m. - 9:25 p.m.		Behaviour of functions
9:30 p.m. - 9:55 p.m.		Taylor polynomials
7:30 p.m. - 9:00 p.m.		MAA-CUPM - PANEL DISCUSSION: Mathematics appreciation courses Room 1

TIMETABLE

SUNDAY, January 28	Mathematical Association of America
8:30 a.m. - 2:30 p.m.	REGISTRATION - Exhibit Area
9:00 a.m. - 9:50 a.m.	INVITED ADDRESS Gödelian puzzles Raymond M. Smullyan, Coliseum
10:00 a.m. - 10:50 a.m.	MAA/NCTM - INVITED ADDRESS The NSF status surveys in school mathematics James T. Fey Jeremy Kilpatrick Rooms 4, 6, 7 and 8
10:00 a.m. - 10:50 a.m.	INVITED ADDRESS The Claremont program in applied mathematics Jerome Spanier, Coliseum
11:00 a.m. - 11:50 a.m.	MAA/NCTM - PANEL DISCUSSION Basic skills: Placement and pedagogy Thomas A. Carnevale Bernard L. Madison (moderator) Amber Steinmetz Rooms 4, 6, 7 and 8
11:00 a.m. - 11:50 a.m.	INVITED ADDRESS Intersection graphs, food webs, ecological phase space, and the boxicity conjecture Fred S. Roberts, Coliseum
1:30 p.m. - 2:20 p.m.	INVITED ADDRESS Mathematical problems in ESP research Persi W. Diaconis, Coliseum
2:30 p.m. - 3:20 p.m.	INVITED ADDRESS Curvature and arc length Andrew M. Gleason, Coliseum
3:30 p.m. - 4:20 p.m.	INVITED ADDRESS Sherlock Holmes in Babylon R. Creighton Buck, Coliseum

PRESENTERS OF PAPERS

Following each name is the number corresponding to the speaker's position on the program

o Invited one-hour lecturers	* Special session speakers
*Abe, K #112	Bardwell, M. A. #156
Abian, A. #309	Barnard, R. W. #8
Adams, M. E. #158	*Batchelder, W. H. #231
Adu, D. I. #452	Batten, L. M. #242
Ahn, H. J. #286	*Bayliss, A. #534
Ahuja, M. #143	Bear, H. S. #571
Akiyama, J. #352	Beard, J. T. B., Jr., #358
Al-Bassam, M. A. #291	Becker, L. C. #381
*Alexander, S. #44	*Beem, J. K. #425
Aliprantis, C. D. #467	Beineke, L. W. #356
Amayo, R. #180	Belding, R. #157
Anderson, E. H. #493	Bell, W. C. #134
*Anderson, J. #430	Bellenot, S. F. #471
Andre, P. P. #444	*Berg, G. #50
Andrew, A. D. #575	*Berger, C. A. #433
Anellis, I. H. #306	*Berndt, B. C. #225
*Arms, J. M. #426	Bevis, J. H. #197
*Aronson, D. G. #54	*Bharucha-Reid, A. T. #200
oArtin, M. #85	*Bird, C. #93
Ash, J. M. #569	Birkenmeier, G. F. #549
Ashkenazi, M. #287	Birman, J. S. #492
Asimow, L. #468	*Bishop, R. L. #109
Astin, J. T. #584	Blanton, J. D. #494
*Atkin, A. O. L. #415	*Blavais, A. S. #236
*Atkinson, F. V. #436	*Bloom, G. S. #321
Attia, F. A. #500	*Boothby, W. M. #110
Aull, C. E. #260	*Borwein, D. #329
Baily, A. M. #365	*Boyd, J. P. #335
Bakke, V. L. #391	*Bressoud, D. M. #324
Baldwin, S. #312	Brewer, D. W. #385
Barbanel, J. #305	*Bridges, D. S. #52
	Brook, C. H. #131
	Brown, J. E. #3
	Brown, R. C. #380
	*Brown, S. W. #521
	Bruck, R. E. #71
	*Bryant, R. L. #428
	*Buntinas, M. #220
	Buoni, J. J. #82
	*Burton, T. A. #525
	Button, R. W. #399
	Butz, J. R. #145
	Cain, G. L., Jr., #83
	Campbell, S. L. #17
	*Cantor, M. #342
	*Carey, R. W. #520
	Carlson, B. C. #86
	*Case, J. #30
	Castro, A. #277
	*Cecil, T. E. #41
	Chae, S. B. #259
	Chambless, L. #543
	*Chan, Y. K. #53
	Chao, J.-A. #135
	Chase, K. #454
	Chen, K.-H. #516
	Chiou, K.-L. #278
	Choo, S. A. #129
	Christensen, M. J. #498
	Chu, C. H. #564
	Chudnovsky, D. V. #297
	Chudnovsky, G. V. #69
	Clark, B. #488
	Cochran, A. C. #398
	Cohen, J. #139
	*Cohn, H. #224
	Colvin, M. R. #486
	Comer, S. D. #163
	Comerford, L. P., Jr., #458
	*Conway, J. B. #523
	*Cowan, T. M. #232
	Creekmore, J. #580
	Daffer, P. Z. #503
	Davis, E. H. #239
	Davis, P. W. #299
	Davitt, R. M. #457
	*Dawson, D. F. #330
	Debnath, L. #518
	Dechéne, L. I. #451
	Defranza, J. #563
	Deloff, E. D. #249
	Derr, L. J. #407
	Deyo, R. C. #273
	*Diaconis, P. #210
	*Diamond, H. R. #97
	Dickman, R. F., Jr., #254
	*Dodwell, P. C. #233
	*Dolph, C. L. #99
	Doran, R. S. #576
	Downey, C. P. #409
	Downing, D. J. #148
	Duchamp, T. #389
	Dwyer, T. A. W., III, #477

Ebanks, B. R. #175  
 \*Eberlein, P. #46  
 Edwards, B. H. #20  
 Edwards, C. C. #460  
 Efrogmson, G. A. #443  
 Engl, H. W. #77  
 Entringer, R. C. #188  
 \*Erdds, P. #320  
 Eslinger, R. C. #583  
 Espelie, M. S. #394  
 Estes, D. R. #268  
 Ezell, C. L., Jr., #185  
 Faber, V. #357  
 Farmer, K. B. #238  
 Fay, T. H. #557  
 \*Federowicz, A. J. #536  
 \*Fegan, H. D. #43  
 Feingold, A. J. #177  
 oFeldman, J. #313  
 Ferguson, H. R. P. #64  
 \*Fife, P. #55  
 Fitzgerald, C. H. #11  
 Fitzgibbon, W. E. #386  
 Fitzpatrick, W. J. #496  
 \*Flaherty, F. #341  
 Flanigan, J. A. #174  
 Fleming, R. J. #153  
 Flinn, P. #572  
 Forcade, R. W. #63  
 Ford, N. S. #265  
 oFornaess, J. E. #176  
 Foulkes, J. B. #255  
 Frank, M. J. #127  
 Franke, W. T. #473  
 \*Freedman, A. R. #217  
 Freedman, M. A. #147  
 Frey, G. T. #141  
 \*Fridy, J. #222  
 Fryant, A. J. #87  
 Fulton, C. T. #379  
 Fulton, J. D. #18  
 \*Gaffney, T. #111  
 Gaglione, A. M. #552  
 Gagola, S. M., Jr., #558  
 Gajendragadkar, D. S. #459  
 Galambos, J. #168  
 Gale, D. #456  
 Gallian, J. A. #187  
 \*Gardner, R. B. #108  
 Gasper, G. #570  
 Gibson, P. M. #24  
 Gimbel, J. G. #349  
 \*Glasser, M. L. #323  
 Glenn, P. #316  
 Goldsmith, D. L. #190  
 \*Goode, J. J. #207  
 Goodman, J. E. #196  
 Goodman, V. #497  
 Grabiner, S. #72  
 Graham, G. #367  
 \*Graham, R. L. #322  
 Graham, S. #59  
 \*Grammatikopoulos, M. K. #442  
 \*Greenleaf, N. #48  
 Gregory, J. #401  
 Griggs, J. R. #189  
 \*Grimmer, R. #439  
 \*Grinstein, G. #228  
 \*Gromoll, D. #45  
 Gruenhagen, G. #397  
 Guggenheimer, H. #244  
 \*Gupta, C. P. #33  
 Gurak, S. J. #366  
 Haaland, P. #167  
 Hagler, J. #474  
 Hale, M. P., Jr., #241  
 \*Hallam, T. G. #524  
 \*Halmos, P. R. #434  
 Hanlon, P. #199  
 Harary, F. #353  
 Harris, G. A. #267  
 Harris, S. G. #246  
 Hartwig, R. E. #23  
 \*Hayashi, Y. #100  
 Heatherly, H. #546  
 Heiberg, C. #124  
 Hennefeld, J. #152  
 Herod, J. V. #146  
 Herring, J. R. #247  
 Herrmann, R. A. #251  
 Hewitt, G. C. #317  
 Higgins, J. #554  
 Hill, T. P. #501  
 Hindman, N. #192  
 \*Hinton, D. #441  
 Hitt, L. R. #490  
 Ho, A. #472  
 \*Hoffman, W. C. #336  
 Hooker, J. W. #383  
 Hopkins, M. R. #555  
 Hostetler, R. P. #582  
 \*Howard, L. N. #56  
 \*Hsiao, G. C. #205  
 Hsieh, J.-T. #465  
 Hu, Thakyn #151  
 Huard, J. G. #362  
 \*Hubbard, E. #38  
 Huff, R. E. #476  
 \*Hulme, B. L. #533  
 \*Humi, M. #339  
 Hunter, R. H. #560  
 Indlekofer, K.-H. #562  
 Infante, R. P. #360  
 Infantozzi, C. A. #161  
 Jacobowitz, H. #248  
 Jajodia, S. #485  
 \*James, D. A. #417  
 Jamison, J. E. #154  
 Janos, L. #257  
 \*Jeroslow, R. G. #214  
 John, D. J. #269  
 \*Johnson, D. L. #47  
 Johnson, G. W. #130  
 \*Johnson, G. G. #333  
 Johnston, E. H. #1  
 \*Jones, G. D. #529  
 Jones, J., Jr., #22  
 Jones, M. F. #550  
 Jordan, G. S. #387  
 Joseph, J. E. #261  
 Juberg, R. K. #296  
 \*Julian, W. H. #49  
 Kadekawa, S. #301  
 Kaftal, V. #469  
 \*Kailath, T. #104  
 \*Kalaba, R. E. #201  
 \*Kannan, R. #106  
 Kannappan, P. #412  
 Kaplan, A. #369  
 Karlof, J. #410  
 \*Kartsatos, A. G. #440  
 Kaufman, W. E. #76  
 Kazarinoff, N. D. #513  
 Keagy, T. A. #561  
 Keenan, D. E. #350  
 Keeton-Williams, J. #162  
 Keisler, M. #133  
 Kemperman, J. H. B. #170  
 \*Kendall, R. P. #535  
 \*Kilgour, D. M. #26  
 Kim, J. H. #264  
 King, J. P. #565  
 King, S. C. #559  
 Kitchen, J. W. #573  
 \*Kleinman, R. E. #98  
 Ko, H.-P. #198  
 \*Kochen, M. #235  
 \*Kolodner, I. I. #32  
 Komkov, V. #284  
 Kon, M. A. #81  
 Kramarz, L. #404  
 oKreiss, H.-O. #414  
 \*Kreith, K. #530  
 Kuo, H.-H. #508  
 \*Kusano, T. #437  
 Lachance, M. #568  
 Ladde, G. S. #283  
 \*Laetsch, T. #34  
 \*Lakshmikantham, V. #204  
 Langenhop, C. E. #384  
 \*Lardy, L. J. #39  
 \*Larson, R. G. #325  
 La Salle, M. M. #314  
 Laskar, R. C. #354  
 Lau, A. #252  
 \*Lee, D. A. #532  
 Lee, S. J. #279  
 Leela, S. #295  
 Legg, D. A. #78  
 \*Lehner, J. #230  
 \*Lerner, D. #345  
 Leung, A. #514  
 \*Leung, D. S. P. #429  
 Leung, Y. #5  
 Levi, M. #288  
 Lewis, G. N. #403  
 Lewis, P. W. #578  
 Lewis, W. #263  
 Lifschitz, V. #172  
 Lightbourne, J. H., III #378  
 \*Lin S. #537  
 \*Lindman, H. #337  
 Lindner, C. C. #351  
 \*Lindquist, A. G. #105  
 Lomonaco, S. J., Jr., #487  
 Long, C. T. #61  
 Lubbers, D. #144  
 \*Lucas, W. F. #234  
 Ludington, A. L. #57  
 Lutzer, D. J. #256  
 \*Magid, M. A. #427  
 Magill, K. D., Jr., #453  
 \*Mahfoud, W. E. #528  
 Malone, J. J. #545  
 \*Mandelkern, M. #51  
 Margulies, W. #73  
 Marin, S. P. #405  
 \*Marsden, J. E. #422  
 Masih, S. #484  
 Mathis, F. H. #406  
 Matijevic, J. R. #270  
 \*Matula, D. W. #213  
 Mauldin, D. #125  
 \*Mayes, V. M. #328  
 McCann, R. C. #373  
 McCleary, J. #480  
 McCoy, P. A. #293  
 McEnnis, B. W. #54  
 McGibbon, C. A. #482  
 McKee, T. A. #116  
 McLaughlin, J. R. #281  
 McLoughlin, A. M. #184  
 Meeker, L. D. #390  
 \*Metzger, T. A. #418  
 Miller, D. E. #395  
 Miller, J. G. #302  
 Miller, M. #450  
 Milne, S. C. #39  
 \*Mines, R. #119  
 Moak, D. S. #88  
 Molnar, R. K. #540  
 \*Moreno, C. J. #421  
 Morgan, E. J. #194  
 Morley, L. J. #462  
 Morley, T. D. #75  
 Morris, P. #479  
 Morrison, J. A. #499  
 Morrison, T. #361  
 Mullen, G. L. #359  
 \*Myerson, G. #209  
 Nagle, R. K. #298  
 \*Nashed, M. Z. #101  
 Nathanson, M. B. #67  
 \*Nerode, A. #115  
 \*Newman, M. #223  
 \*Nicholls, P. J. #419

Nichols, W. #547  
 Niedzwecki, G. P. #466  
 \*Nohel, J. A. #203  
 \*Nordgren, E. A. #431  
 Nummela, E. C. #371  
 \*Nussbaum, R. D. #37  
 Nyikos, P. #396  
 O'Brien, B. #464  
 O'Connor, T. A. #509  
 \*Odlyzko, A. M. #212  
 \*Olin, R. #522  
 Oliver, R. K. #368  
 \*Onose, H. #527  
 \*O'Sullivan, J. J. #423  
 \*Owen, G. #31  
 \*Packel, E. W. #96  
 Panchal, C. D. #275  
 \*Parameswaran, M. R. #221  
 Park, C. #505  
 \*Parker, P. E. #344  
 \*Parson, L. A. #226  
 \*Parthasarathy, T. #29  
 Passell, N. #126  
 Passty, G. B. #79  
 Patula, W. T. #531  
 Payne, D. A. #294  
 Pennline, J. A. #517  
 Pesek, J. D., Jr., #138  
 oPeskin, C. S. #290  
 Peterson, A. #375  
 \*Pimbley, G. H. #36  
 \*Pincus, J. D. #206  
 Plaster, J. #9  
 Pollington, A. D. #60  
 Porter, J. F. #166  
 Poucher, W. B. #195  
 Powder, C. R. #377  
 Powell, W. B. #160  
 Pranger, W. #10  
 \*Prenter, P. M. #103  
 Prichett, G. D. #65  
 \*Purzitsky, N. #420  
 Qualls, D. D. #502  
 Quine, J. R. #448  
 Quinn, F. #495  
 \*Raghavan, T. E. S. #28  
 Ramanujan, M. S. #581  
 Randell, R. #489  
 Rankin, S. M., III #376  
 \*Raphael, L. A. #331  
 \*Rapoport, A. #237  
 \*Razar, M. J. #229  
 Reed, G. M. #393  
 Rehmer, K. O. #132  
 \*Reilly, R. C. #113  
 Reznick, B. #62  
 \*Rhoades, B. E. #327  
 Rice, R. E. #300  
 Richardson, G. #169  
 Richman, D. J. #346  
 \*Richman, F. #120  
 Richoux, A. #548  
 Richter, G. #491  
 Riley, J. H., Jr., #470  
 Ringeisen, R. #193  
 Ritter, G. A. #511  
 Robbins, N. #70  
 Robinson, J. E. #155  
 Rodemich, E. #6  
 Rogers, D. D. #74  
 \*Rogers, L. #116  
 Rose, B. I. #308  
 \*Rosen, K. H. #227  
 Rosenthal, J. W. #363  
 Roth, R. #181  
 Rubin, A. L. #310  
 \*Ruckle, W. H. #27  
 \*Russell, D. C. #218  
 \*Saari, D. G. #338  
 Sadowsky, J. #173  
 \*Salinas, N. #432  
 Sampson, G. #136  
 Sanders, W. M. #165  
 \*Sather, D. #35  
 Saunders, K. W. #307  
 Savage, T. R. #544  
 Sawey, B. #128  
 \*Šćedrov, A. #114  
 Schep, A. R. #478  
 Schoen, A. H. #240  
 Schreiner, E. A. #159  
 Schwartz, C. F. #446  
 Scott, B. M. #262  
 \*Segel, L. A. #121  
 \*Seidenberg, A. #117  
 Seifert, C. J. #579  
 \*Sember, J. J. #216  
 oShanerson, J. L. #90  
 Shapiro, H. #21  
 Shaw, J. K. #280  
 \*Shawyer, B. L. R. #332  
 \*Sheingorn, M. #416  
 Sheng, L. T. #408  
 \*Shenoy, P. P. #92  
 Shiflett, R. C. #19  
 Shonkwiler, R. #515  
 Shores, T. #274  
 Silverman, H. #2  
 Silvia, E. #12  
 \*Simmons, G. J. #211  
 Simms, J. C. #311  
 Sine, R. C. #80  
 Singh, B. #382  
 Singkofer, K. #150  
 Sizer, W. S. #463  
 \*Sloan, I. H. #102  
 Sloane, N. J. A. #461  
 Smith, B. #137  
 \*Smith, D. A. #334  
 Smith, H. F. #179  
 Smith, K. C. #542  
 Smith, R. C. #449  
 \*Smoller, J. #122  
 Snow, D. R. #347  
 Snyder, H. H. #7  
 Sobczyk, A. #178  
 \*Solderitsch, J. #118  
 Solomon, J. L. #400  
 Soong, C.-S. #510  
 Speiser, R. #445  
 \*Spikes, P. W. #438  
 oSrinivasan, B. #512  
 Stahl, S. #183  
 Stahl, S. H. #304  
 \*Staikos, V. A. #526  
 Stanton, R. O. #553  
 Staton, W. #191  
 Stevens, T. C. #370  
 Stokes, J. F. #142  
 \*Straffin, P. D., Jr., #95  
 Su, L.-Y. #566  
 Suffridge, T. J. #15  
 Suh, S. #140  
 Sullivan, F. #475  
 Sumner, D. P. #355  
 Sun, T. C. #507  
 Surowski, D. B. #455  
 \*Swaminathan, S. #219  
 Swanson, L. #506  
 Szeto, G. #541  
 Taft, E. J. #539  
 Tarabay, A. A. #14  
 oTaub, A. H. #319, #424  
 Thomas, B. S. #372  
 Thomas, B. #447  
 Thompson, R. L. #411  
 Tinberg, N. B. #551  
 \*Tipler, F. J. #343  
 Tonne, P. C. #66  
 Triantaphyllou, D. #364  
 Trotter, W. T., Jr., #348  
 Tupitsyn, V. G. #68  
 \*Turbay, G. J. #94  
 Turett, B. #577  
 Turner, J. C. #171  
 \*Uhlenbeck, K. K. #340  
 Unell, A. D. #481  
 Ungar, A. #292  
 Usmani, R. A. #402  
 \*Utz, W. R. #435  
 Vaaler, J. D. #58  
 Vatsala, A. S. #388  
 Vaughan, J. E. #250  
 Vaughan, N. #266  
 Vaughn, R. L. #276  
 Vo Van, T. #243  
 Walters, T. J. #285  
 Wang, S. S. #272  
 Ward, J. R. #282  
 Wardlaw, W. P. #164  
 Warren, R. H. #258  
 Waymire, E. #504  
 Weakley, W. D. #271  
 \*Weber, R. J. #25  
 Weston, K. W. #556  
 White, E. #315  
 \*Whitt, L. #42  
 Wicke, H. H. #392  
 Wilkerson, R. W. #4  
 Williams, L. R. #149  
 Williams, S. W. #253  
 Williamson, F., Jr., #303  
 \*Winfree, A. T. #123  
 \*Wing, G. M. #202  
 Wong, R. W. #538  
 Wright, L. C. #374  
 Wright, S. #574  
 Wu, C.-M. #483  
 Young, E. C. #289  
 Zame, W. R. #16  
 Zemyan, S. M. #13  
 Ziegler, L. R. #567  
 Zoltek, S. M. #245

## PROGRAM OF THE SESSIONS

### SHORT COURSE ON GAME THEORY AND ITS APPLICATIONS

All Sessions in Mississippi Rooms North and South, Holiday Inn

MONDAY, 2:00 P. M. – 5:00 P. M.

- 2:00- 3:15 p. m. Game theory: An overview and the n-person cooperative model. WILLIAM F. LUCAS, Cornell University, Ithaca, New York (SC 79-1)
- 3:45- 5:00 p. m. Valuation of games. LLOYD S. SHAPLEY, The Rand Corporation, Santa Monica, California (SC 79-2)

TUESDAY, 9:00 A. M. – 5:00 P. M.

- 9:00-10:15 a. m. Noncooperative games. ROBERT JAMES WEBER, School of Organization and Management, Yale University, New Haven, Connecticut (SC 79-3)
- 10:45- noon Economic market games. LOUIS J. BILLERA, Cornell University, Ithaca, New York (SC 79-4)
- 1:30- 2:45 p. m. Measurement of power in political systems. LLOYD S. SHAPLEY, The Rand Corporation, Santa Monica, California (SC 79-5)
- 3:00- 4:15 p. m. Some uses of game theory in operations research. WILLIAM F. LUCAS, Cornell University, Ithaca, New York (SC 79-6)
- 4:30- 5:00 p. m. General discussion

---

### THE EIGHTY-FIFTH ANNUAL MEETING

---

The time limit for each contributed paper in the general sessions is ten minutes. In the special sessions the time varies from session to session and within sessions. To maintain the schedule, the time limits will be strictly enforced.

WEDNESDAY, 8:00 A. M.

#### Session on One and Several Complex Variables, Convention Center, Room 5

- 8:00- 8:10 (1) The boundary modulus of continuity of harmonic functions. Dr. ELGIN H. JOHNSTON, Iowa State University (763-30-5)
- 8:15- 8:25 (2) Extreme points of subclasses of close-to-convex functions. H. SILVERMAN\*, College of Charleston, and D. N. TELAGE, University of Kentucky (763-30-7)
- 8:30- 8:40 (3) Geometric properties of a class of support points of univalent functions. JOHNNY E. BROWN, University of Michigan, Ann Arbor (763-30-8)
- 8:45- 8:55 (4) Linear maps of holomorphic functions on commutative algebras. Professor RALPH W. WILKERSON, Winthrop College (763-30-9)
- 9:00- 9:10 (5) Integral means of derivatives of some univalent functions. Mr. Y. LEUNG, University of Hawaii, Honolulu (763-30-10)
- 9:15- 9:25 (6) An inequality for univalent functions with real coefficients. Dr. EUGENE RODEMICH, Jet Propulsion Laboratory, Pasadena, California (763-30-11)
- 9:30- 9:40 (7) An extended Cauchy integral formula on certain associated algebras. Professor H. H. SNYDER, Southern Illinois University, Carbondale (763-30-12)
- 9:45- 9:55 (8) On coefficient bounds of  $f$  in  $S$  when  $f'$  is univalent. Preliminary report. Professor ROGER W. BARNARD\*, Texas Tech University and Professor TED SUFRIDGE, University of Kentucky (763-30-13)
- 10:00-10:10 (9) A geometric criterion for determining non-Bazilevichness. Preliminary report. Mr. JOHN PLASTER, Texas Tech University (763-30-14) (Introduced by Roger W. Barnard)
- 10:15-10:25 (10) Bounded analytic functions. Preliminary report. WALTER PRANGER, De Paul University (763-30-15) (Introduced by Professor Jerry Goldman)

---

\*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

- 10:30-10:40 (11) Conditions for a function on an arc to have a bounded analytic extension. Professor CARL H. FITZGERALD, University of California, San Diego (763-30-16)
- 10:45-10:55 (12) Convex families of starlike functions. Preliminary report. HERB SILVERMAN, College of Charleston and EVELYN SILVIA\*, University of California, Davis (763-30-17)
- 11:00-11:10 (13) A minimal outer area problem in conformal mapping. Preliminary report. Professor STEPHEN M. ZEMYAN, Georgia Institute of Technology (763-30-18)
- 11:15-11:25 (14) Local and global analysis on branched analytic covers. Dr. AJAJ A. TARABAY, Texas Tech University (763-32-1)
- 11:30-11:40 (15) Holomorphic idempotents in  $C^n$ . Professor L. F. HEATH, University of Texas at Arlington and Professor T. J. SUFFRIDGE\*, University of Kentucky, Lexington (763-32-3)
- 11:45-11:55 (16) Primary ideals in rings of holomorphic germs. Professor JOSEPH A. BECKER and Professor WILLIAM R. ZAME\*, State University of New York at Buffalo (763-32-4)

WEDNESDAY, 8:00 A. M.

Session on Linear and Multilinear Algebra, Matrix Theory (Finite and Infinite), Convention Center, Room 9

- 8:00- 8:10 (17) Continuity of the Drazin inverse. Dr. STEPHEN L. CAMPBELL, North Carolina State University, Raleigh (763-15-1)
- 8:15- 8:25 (18) Generalized inverses of matrices over fields of characteristic two. Professor JOHN D. FULTON, Clemson University (763-15-2)
- 8:30- 8:40 (19) A Birkhoff theorem for doubly stochastic matrices with vector entries. Preliminary report. Professor RAY C. SHIFLETT, California State University, Fullerton (763-15-3)
- 8:45- 8:55 (20) Split Clifford algebras. Preliminary report. Professor BRUCE H. EDWARDS, University of Florida (763-15-4)
- 9:00- 9:10 (21) The characteristic polynomial of a pencil generated by a pair of Hermitian matrices. Ms. HELENE SHAPIRO, California Institute of Technology (763-15-5)
- 9:15- 9:25 (22) Solutions of the Lyapunov matrix equation. JOHN JONES, JR., Air Force Institute of Technology (763-15-6)
- 9:30- 9:40 (23) Unit and unitary solutions to the matrix equation  $AX = B$ . Preliminary report. Dr. R. E. HARTWIG, North Carolina State University (763-15-7) (Introduced by Professor John Franke)
- 9:45- 9:55 (24) Permanent polytopes of doubly stochastic matrices. Professor PETER M. GIBSON, University of Alabama in Huntsville (763-15-8)

WEDNESDAY, 8:30 A. M.

Special Session on Game Theory. I, Convention Center, Room 1

- 8:30- 8:50 (25) Recent results on the valuation of games. Professor ROBERT JAMES WEBER, Yale University (763-90-13)
- 8:55- 9:15 (26) Equilibrium points of infinite stationary truels. Dr. D. MARC KILGOUR, Wilfrid Laurier University (763-90-14)
- 9:20- 9:40 (27) On the constructability of solutions to two person games. Dr. WILLIAM H. RUCKLE, Clemson University (763-90-3)
- 9:45-10:05 (28) Completely mixed games and M. matrices. Professor T. E. S. RAGHAVAN, University of Illinois at Chicago Circle (763-90-4) (Introduced by Professor William F. Lucas)
- 10:10-10:30 (29) Markov games. Preliminary report. Dr. T. PARTHSARATHY, University of Kansas (763-90-5) (Introduced by Professor William F. Lucas)
- 10:35-10:55 (30) A game of N buyers in comparative shopping. Preliminary report. Dr. JAMES CASE, Federal Trade Commission, Washington, D. C. (763-90-7)
- 11:00-11:20 (31) A game-theoretic approach to a problem of cost allocation. Dr. GUILLERMO OWEN\*, Instituto SER, Calle, Bogota, Colombia and Dr. NANCY JOHNSON, Jones School of Administration, Rice University (763-90-8) (Introduced by Professor William F. Lucas)

WEDNESDAY, 8:30 A. M.

Special Session on Integral Equations with Emphasis on Fredholm and Hammerstein Equations. I,  
Convention Center, Room 6

- 8:30- 8:50 (32) Equations of Hammerstein type—a survey of recent results. Professor IGNACE I. KOLODNER, Carnegie-Mellon University (763-45-23)
- 8:55- 9:15 (33) Some recent results on Fredholm and Hammerstein equations and applications. Professor CHAITAN P. GUPTA, Northern Illinois University (763-45-4)
- 9:20- 9:40 (34) Minimax principles for nonlinear eigenvalue problems. Preliminary report. THEODORE LAETSCH, University of Arizona (763-45-17)
- 9:45-10:05 (35) Applications of group representations to nonlinear Hammerstein equations. Preliminary report. Professor GEORGE H. KNIGHTLY, University of Massachusetts, Amherst and Professor D. SATHER\*, University of Colorado, Boulder (763-45-15) (Introduced by Professor M. Z. Nashed)
- 10:10-10:30 (36) Nonoccurrence of secondary bifurcation for certain nonlinear operators in Banach space. Preliminary report. Dr. GEORGE H. PIMBLEY, National Science Foundation, Washington, D. C. and Los Alamos Scientific Laboratory, Los Alamos, New Mexico (763-47-6)
- 10:35- 10:55 (37) A quadratic integral equation. Preliminary report. Dr. ROGER D. NUSSBAUM\* and Dr. MICHAEL MOCK, Rutgers University, New Brunswick (763-45-24)
- 11:00-11:20 (38) Finite dimensional approximation and  $\beta$ -convergence for solutions to problems with nonunique minima. Preliminary report. Ms. ELAINE HUBBARD, Kennesaw College (763-45-12)
- 11:25-11:45 (39) An analysis of a numerical method for Hammerstein equations. Preliminary report. Professor L. J. LARDY, Syracuse University (763-65-1)
- 11:50-12:20 (40) Open questions, informal discussion, Desirable Directions for Research

WEDNESDAY, 8:30 A. M.

Special Session on Global Differential Geometry. I, Convention Center, Room 4

- 8:30- 8:50 (41) Tight and taut immersions into hyperbolic space. Professor THOMAS E. CECIL\*, Holy Cross College and Professor PATRICK J. RYAN, Indiana University at South Bend (763-53-4)
- 8:55- 9:15 (42) Isometric homotopies and codimension two isometric immersions of spheres into Euclidean space. Professor LEE WHITT, Texas A & M University (763-53-7)
- 9:20- 9:40 (43) Macdonald's identities and the heat equation. HOWARD D. FEGAN, Rice University (763-35-3)
- 9:45-10:05 (44) Geodesics in Riemannian manifolds-with-boundary. Professor RALPH ALEXANDER and Professor STEPHANIE ALEXANDER\*, University of Illinois. Urbana (763-53-16)
- 10:10-10:30 (45) Rigidity of positively curved manifolds with large diameter. DETLEF GROMOLL\*, State University of New York at Stony Brook, and KARSTEN GROVE, University of Copenhagen, Denmark (763-53-20)
- 10:35-10:55 (46) Isometry groups. SU-SHING CHEN, Georgia Institute of Technology, and PATRICK EBERLEIN\*, University of North Carolina, Chapel Hill (763-53-12)
- 11:00-11:20 (47) Curvature and Euler characteristic for six-dimensional Kähler manifolds. Dr. DAVID L. JOHNSON, Texas A & M University (763-53-2)

WEDNESDAY, 8:30 A. M.

Special Session on Constructive Mathematics. I, Convention Center, Room 11

- 8:30- 8:50 (48) What Cantor did and didn't prove—a constructive evaluation. NEWCOMB GREENLEAF, Naropa Institute, Boulder, Colorado (763-04-1)
- 8:55- 9:15 (49) The Jordan curve theorem. WILLIAM H. JULIAN, New Mexico State University, Las Cruces (763-54-4)
- 9:20- 9:40 (50) Constructive dimension theory. Dr. GORDON BERG, Motorola Inc., Mesa, Arizona (763-54-2) (Introduced by Professor Ray Mines)
- 9:45-10:05 (51) Located sets. Professor MARK MANDELKERN, New Mexico State University, Las Cruces (763-26-1)

- 10:10-10:30 (52) Constructive aspects of approximation theory. Preliminary report. Dr. DOUGLAS S. BRIDGES, University College, Buckingham, England (763-41-1) (Introduced by Professor Fred Richman)
- 10:35-10:55 (53) Constructive probability theory and related topics. Preliminary report. Dr. Y. K. CHAN, Boeing Computer Services Company, Tukwila, Washington (763-60-3) (Introduced by Professor Ray Mines)

WEDNESDAY, 8:30 A. M.

Special Session on Diffusion Reaction Systems in Biology. I, Convention Center, Room 8

- 8:30- 9:10 (54) Density dependent diffusion. Professor DONALD G. ARONSON, University of Minnesota, Minneapolis (763-92-13)
- 9:30-10:10 (55) Modeling with reaction-diffusion systems. Professor PAUL FIFE, University of Arizona (763-92-12)
- 10:30-11:10 (56) Some oscillatory reaction-diffusion wave structures in finite domains. Professor LOUIS N. HOWARD, Massachusetts Institute of Technology (763-92-19)

WEDNESDAY, 8:30 A. M.

Session on Number Theory, Convention Center, Room 2

- 8:30- 8:40 (57) A bound on Kaprekar constants. Professor ANNE L. LUDINGTON, Hamilton College (763-10-6)
- 8:45- 8:55 (58) Inequalities for the volume of certain convex bodies with applications to linear forms. Professor JEFFREY D. VAALER, University of Texas, Austin (763-10-14)
- 9:00- 9:10 (59) On the greatest prime factor of the integers in an interval. Dr. SIDNEY GRAHAM, California Institute of Technology (763-10-15)
- 9:15- 9:25 (60) On generalized arithmetic and geometric progressions. Mr. A. D. POLLINGTON, Illinois State University (763-10-19) (Introduced by Professor L. C. Eggan)
- 9:30- 9:40 (61) A limited arithmetic on simple continued fractions. III. CALVIN T. LONG, Washington State University and Clemson University (763-10-20)
- 9:45- 9:55 (62) Nonnegative symmetric quartic forms. Professor M. D. CHOI, University of Toronto, Professor T. Y. LAM and Professor BRUCE REZNICK\*, University of California, Berkeley (763-10-21)
- 10:00-10:10 (63)  $\mathbb{Z}$ -linear dependence algorithms, Part I. Preliminary report. Professor H. R. P. FERGUSON and Dr. R. W. FORCADE\*, Brigham Young University, Provo (763-10-23)
- 10:15-10:25 (64)  $\mathbb{Z}$ -linear dependence algorithms, Part II. Preliminary report. Professor H. R. P. FERGUSON\* and Dr. R. W. FORCADE, Brigham Young University, Provo (763-10-24)
- 10:30-10:40 (65) Terminating cycles for iterated difference values of five digit integers. Professor GORDON D. PRICHETT, Hamilton College (763-10-25)
- 10:45-10:55 (66) When is  $3^N$  congruent to 1 or 2 modulo N? Preliminary report. Dr. PHILIP C. TONNE, Emory University, Atlanta (763-10-26)
- 11:00-11:10 (67) Sumsets contained in infinite sets of integers. Professor MELVYN B. NATHANSON, Southern Illinois University, Carbondale (763-10-27)
- 11:15-11:25 (68) Hamming's metric and T-orbits in a dynamical system. Professor V. G. TUPITSYN, Southern Illinois University, Carbondale (763-10-28) (Introduced by Professor Melvyn B. Nathanson)
- 11:30-11:40 (69) The bound for linear forms in elliptic logarithms for the complex multiplication case. G. V. CHUDNOVSKY, Columbia University, New York (763-10-29)
- 11:45-11:55 (70) On Fibonacci numbers which are powers. Dr. NEVILLE ROBBINS, University of San Francisco (763-10-32)

WEDNESDAY, 8:30 A. M.

Session on Operator Theory. I, Convention Center, Room 7

- 8:30- 8:40 (71) Asymptotic behavior of certain second-order equations involving monotone operators. Professor RONALD E. BRUCK, University of Southern California (763-47-2)



- 8:45- 8:55 (72) Operator ranges and invariant subspaces. Professor SANDY GRABINER, Pomona College (763-47-4)
- 9:00- 9:10 (73) Numerical Range. Preliminary report. Professor WILLIAM MARGULIES, California State University, Long Beach (763-47-11)
- 9:15- 9:25 (74) On Fuglede's theorem and operator topologies. Professor DONALD D. ROGERS, U. S. Naval Academy (763-47-12)
- 9:30- 9:40 (75) The geometric mean and the Asplund average of positive operators. W. N. ANDERSON, JR., West Virginia University, T. D. MORLEY\*, University of Illinois, Urbana, and G. E. TRAPP, West Virginia University (763-47-13)
- 9:45- 9:55 (76) Semiclosed operators in Hilbert space. WILLIAM E. KAUFMAN, University of Houston, Houston (763-47-15)
- 10:00-10:10 (77) Measurability of outer inverses of linear random operators. Dr. HEINZ W. ENGL\*, Kepler-Universität, Linz, Austria and University of Delaware and Professor M. ZUHAIK NASHED, University of Delaware (763-47-16)
- 10:15-10:25 (78) Hermitian liftings in Orlicz sequence spaces. Professor G. D. ALLEN and Professor J. D. WARD, Texas A & M University, and Professor D. A. LEGG\*, Indiana-Purdue University, Fort Wayne (763-47-17)
- 10:30-10:40 (79) Ergodic convergence to a zero of the sum of monotone operators. Preliminary report. Professor GREGORY B. PASSTY, University of Southern California (763-47-18)
- 10:45-10:55 (80) Fixed points for certain nonlinear contraction semigroups. ROBERT C. SINE, University of Rhode Island (763-47-19)
- 11:00-11:10 (81) Asymptotic probability distributions of observables in canonical ensembles. Dr. MARK A. KON, Massachusetts Institute of Technology (763-47-20)
- 11:15-11:25 (82) Joint Browder spectrum. Preliminary report. Dr. J. J. BUONI\*, Youngstown State University and Dr. BHUSHAN L. WADHWA, Cleveland State University (763-47-21)
- 11:30-11:40 (83) Fixed points and stability of condensing multi-functions on random normed spaces. Professor GEORGE L. CAIN, JR., Georgia Institute of Technology (763-47-22)
- 11:45-11:55 (84) The distortion coefficient of an operator with bounded characteristic function. Dr. BRIAN W. McENNIS, Ohio State University, Marion (763-47-23)

WEDNESDAY, 9:00 A. M.

Invited Address, Convention Center Colliseum

- (85) Some applications of algebraic geometry to ring theory. Professor MICHAEL ARTIN, Massachusetts Institute of Technology (763-16-15)

WEDNESDAY, 10:15 A. M.

Session on Special Functions, Convention Center, Room 12

- 10:15-10:25 (86) Integrals of the square root of a rational function. Preliminary report. Professor B. C. CARLSON, Ames Laboratory-DOE, Iowa State University (763-33-1)
- 10:30-10:40 (87) Ultraspherical expansions and pseudoanalytic functions. Professor ALLAN J. FRYANT, U. S. Naval Academy (763-33-2)
- 10:45-10:55 (88) The  $q$ -gamma function. Preliminary report. Mr. DANIEL S. MOAK, University of Wisconsin, Madison (763-33-3)
- 11:00-11:10 (89) A multiple series transformation of the very well poised  ${}_{2k+4}\Psi_{2k+4}$ . Preliminary report. Professor STEPHEN C. MILNE, Texas A & M University (763-33-4)

WEDNESDAY, 10:30 A. M.

Invited Address, Convention Center Colliseum

- (90) Manifolds and submanifolds. Professor JULIUS L. SHANESON, Rutgers University, New Brunswick (763-55-7)

WEDNESDAY, 1:00 P. M.

Colloquium Lectures, Lecture I, Convention Center, Coliseum

- (91) Complex analysis and algebraic geometry. Professor PHILLIP A. GRIFFITHS, Harvard University

WEDNESDAY, 2:15 P. M.

Special Session on Game Theory, II, Convention Center, Room 1

- 2:15- 2:35 (92) A reframing of two person nonzero-sum games. Preliminary report. PRAKASH P. SHENOY\* and P. L. YU, University of Kansas, Lawrence (763-90-10) (Introduced by Professor William F. Lucas)
- 2:40- 3:00 (93) On the formation of coalitions in infinite player games. Professor EDWARD AZOFF, University of Georgia and Dr. CHARLES BIRD\*, General Motors Research Laboratories, Warren, Michigan (763-90-9)
- 3:05- 3:25 (94) A duality analysis of balanced sets. Dr. GABRIEL J. TURBAY, Drake University (763-90-15) (Introduced by Professor William F. Lucas)
- 3:30- 3:50 (95) Game theory and the Tennessee Valley Authority. Professor PHILIP D. STRAFFIN, JR. \*, Beloit College and Professor JAMES P. HEANEY, University of Florida (763-90-6)
- 3:55- 4:15 (96) A stochastic solution concept for n-person games. Professor EDWARD W. PACKEL, Lake Forest College (763-90-11) (Introduced by Ivan Filippenko)
- 4:20- 4:40 (97) Asymptotic equilibria in a class of symmetric N-person games on  $[0, T]$ . Professor HARVEY R. DIAMOND, West Virginia University (763-90-12)

WEDNESDAY, 2:15 P. M.

Special Session on Integral Equations with Emphasis on Fredholm and Hammerstein Equations, II, Convention Center, Room 6

- 2:15- 2:35 (98) Integral equation method for scattering by penetrable objects with corners. Professor RALPH E. KLEINMAN, University of Delaware (763-45-22)
- 2:40- 3:00 (99) Fredholm integral equations, scattering theory, and the singularity expansion method (SEM). Professor C. L. DOLPH, University of Michigan, Ann Arbor (763-45-1)
- 3:05- 3:25 (100) The Fredholm integral equation of the first kind, in relation to the Dirichlet problem for an open boundary. Professor YOSHIO HAYASHI, Nihon University, Tokyo, Japan and National Research Council, Ottawa, Canada (763-45-16)
- 3:30- 3:50 (101) Finite element regularization of ill-posed integral equations: smooth and non-smooth kernels. Professor M. ZUHAIR NASHED\*, University of Delaware, and Dr. MANSUK SONG, Daejeon, Korea (763-45-14)
- 3:55- 4:15 (102) Fredholm equations of the second kind by product integration via polynomial interpolation. Preliminary report. Dr. IAN H. SLOAN, University of Maryland, College Park and University of New South Wales, Australia (763-65-5) (Introduced by Professor M. Z. Nashed)
- 4:20- 4:40 (103) Regularization with differential operators. Professor JOHN LOCKER and Professor P. M. PRENTER\*, Colorado State University, Fort Collins (763-45-7)
- 4:45- 5:05 (104) Exploiting Toeplitz-like structure for Fredholm integral equations. Professor THOMAS KAILATH, Stanford University (763-45-10)
- 5:10- 5:30 (105) Fredholm integral equations in stochastic estimation theory. Professor ANDERS G. LINDQUIST, University of Kentucky (763-93-2) (Introduced by Professor M. Z. Nashed)
- 5:35- 5:55 (106) Approximate solutions of nonlinear operator equations. Professor R. KANNAN, University of Texas at Arlington (763-47-35)
- 6:00- 6:30 (107) Open questions, informal discussion, Desirable Directions for Research

WEDNESDAY, 2:15 P. M.

Special Session on Global Differential Geometry, II, Convention Center, Room 4

- 2:15- 2:35 (108) Applications of the derived flag of Pfaffian systems. Preliminary report. Professor ROBERT B. GARDNER, University of North Carolina, Chapel Hill (763-50-1)

- 2:40- 3:00 (109) Cylindricity of infinitesimal deformation of hyperplanes. RICHARD L. BISHOP, University of Illinois, Urbana (763-53-15)
- 3:05- 3:25 (110) On compact, homogeneous, symplectic and contact manifolds. Professor WILLIAM M. BOOTHBY\* and Dr. PHILIP B. ZWART, Washington University, St. Louis (763-53-1)
- 3:30- 3:50 (111) Singularities of mappings and orthogonal projections of spaces. Preliminary report. Professor TERENCE GAFFNEY\* and Ms. MARIA RUAS, Brown University (763-53-23)
- 3:55- 4:15 (112) Some structures on the boundary of an analytic set. Preliminary report. Professor KINETSU ABE, University of Connecticut, Storrs (763-53-5)
- 4:20- 4:40 (113) Geometric applications of the solvability of certain boundary-value problems. Professor ROBERT C. REILLY, University of California, Irvine (763-53-9) (Introduced by Professor Richard Millman)

WEDNESDAY, 2:15 P. M.

Special Session on Constructive Mathematics, II, Convention Center, Room 11

- 2:15- 2:35 (114) On an extension of the intuitionistic propositional calculus. Preliminary report. Mr. JOHN MYHILL and Mr. ANDREJ ŠČEDROV\*, State University of New York at Buffalo (763-02-2)
- 2:40- 3:00 (115) Recursive vs. constructive mathematics. Professor ANIL NERODE, Cornell University (763-02-1)
- 3:05- 3:25 (116) Basic subgroups from a constructive viewpoint. Dr. LAUREL ROGERS, University of Colorado, Colorado Springs (763-20-2)
- 3:30- 3:50 (117) On the Lasker-Noether decomposition theorem. A. SEIDENBERG, University of California, Berkeley (763-12-5)
- 3:55- 4:15 (118) Class groups of quadratic number fields whose p-Sylow subgroups require three generators. Dr. JAMES SOLDERITSCH, Villanova University (763-12-3)
- 4:20- 4:40 (119) Theory of fields. RAY MINES, New Mexico State University, Las Cruces (763-12-4)
- 4:45- 5:05 (120) Finite dimension algebras. Preliminary report. FRED RICHMAN, New Mexico State University, Las Cruces (763-16-1)

WEDNESDAY, 2:15 P. M.

Special Session on Diffusion Reaction Systems in Biology, II, Convention Center, Room 8

- 2:15- 2:55 (121) Examples of spatio-temporal patterns in biophysics, cell biology, and ecology. LEE A. SEGEL, Weizmann Institute, Rehovot, Israel and Rensselaer Polytechnic Institute (763-92-22)
- 3:15- 3:55 (122) Some questions in the theory of reaction-diffusion systems. Professor JOEL SMOLLER, University of Michigan, Ann Arbor (763-35-5)
- 4:15- 4:55 (123) A 2-dimensional continuum of circadian oscillators. Dr. ARTHUR T. WINFREE, Purdue University (763-92-21) (Introduced by Dr. Murray Gerstenhaber)

WEDNESDAY, 2:15 P. M.

Session on Real Functions, Measure and Integration, Convention Center, Room 2

- 2:15- 2:25 (124) Extrema for functions of several variables. Professor CHARLES HEIBERG, U. S. Naval Academy (763-26-2)
- 2:30- 2:40 (125) The set of continuous nowhere differentiable functions. Preliminary report. DAN MAULDIN, North Texas State University (763-26-3)
- 2:45- 2:55 (126) Generalized Cantor functions. Preliminary report. Dr. NICHOLAS PASSELL, University of South Florida, New College, Sarasota (763-26-4)
- 3:00- 3:10 (127) Results on iteration of piecewise-monotonic functions. Preliminary report. Professor M. J. FRANK, Illinois Institute of Technology (763-26-5)
- 3:15- 3:25 (128) Pseudo-metric spaces from the power set of a metric space. Preliminary report. Dr. BENNETT SAWEY, Morningside College (763-26-6)
- 3:30- 3:40 (129) Separability of a space of continuous vector-valued functions. Professor SEKI A. CHOO, Manchester College (763-28-1)

- 3:45- 3:55 (130) Scale-invariant measurability in Wiener space. Professor GERALD W. JOHNSON\* and Professor DAVID L. SKOUG, University of Nebraska-Lincoln (763-28-2)
- 4:00- 4:10 (131) Vector measures and projections in the universal measure space. Professor CECILIA H. BROOK, Goucher College (763-28-3)
- 4:15- 4:25 (132) A completion of a space of molecular measures. Preliminary report. Dr. KARL O. REHMER, University of Missouri-Rolla (763-28-4)
- 4:30- 4:40 (133) Conditional extensions of quasi-measures. Preliminary report. Dr. MICHAEL KEISLER, Arkansas Tech University (763-28-5)
- 4:45- 4:55 (134) Unbounded uniformly absolutely continuous sets of measures. Dr. WAYNE C. BELL, Murray State University (763-28-6)

WEDNESDAY, 2:15 P. M.

Session on Harmonic Analysis and Integral Transforms, Convention Center, Room 3

- 2:15- 2:25 (135) Hardy spaces of  $q$ -martingales. Preliminary report. Professor J.-A. CHAO, University of Texas, Austin (763-42-1)
- 2:30- 2:40 (136) On weak restricted estimates and endperiod problems for convolutions with oscillating kernels (I). Preliminary report. W. B. JURKAT, Syracuse University and G. SAMPSON\*, State University of New York at Buffalo (763-42-3)
- 2:45- 2:55 (137) Finite abelian group cohesion. Professor P. ERDÖS, Hungarian Academy of Sciences. Budapest and B. SMITH\*, University of Kentucky (763-42-4)
- 3:00- 3:10 (138) Hardy spaces on  $R^n$  defined by anisotropic geometries. Preliminary report. JOHN D. PESEK, JR., University of Illinois. Urbana (763-42-5)
- 3:15- 3:25 (139) A multilinear singular integral in  $R^n$ . JONATHAN COHEN\* and JOHN GOSSELIN, University of Georgia (763-42-6)
- 3:30- 3:40 (140) Constructive proofs of the Rudin-Katzenelson theorem and the Kahane theorem. Professor SUNGWOON SUH, University of Missouri-Rolla (763-43-1)
- 3:45- 3:55 (141) Relatively dense and countably dense sets in semigroups. Preliminary report. Dr. G. THOMAS FREY. St. Bonaventure University (763-43-2)
- 4:00- 4:10 (142) A new glimpse at the Faltung integral. Professor JOSEPH F. STOKES, Western Kentucky University (763-44-1) (Introduced by Dr. James B. Barksdale)
- 4:15- 4:25 (143) Formally real rings of distributions. II. Preliminary report. Dr. MANGHO AHUJA, Southeast Missouri State University (763-44-2)
- 4:30- 4:40 (144) Some theorems on the Hankel spectrum of a function. Preliminary report. Dr. DAVID LUBBERS, Clemson University (763-44-3)

WEDNESDAY, 2:15 P. M.

Session on Operator Theory. II, Convention Center, Room 7

- 2:15- 2:25 (145) Hankel operators with Hilbert space range. Professor JEFFREY R. BUTZ, University of Oklahoma, Norman (763-47-24)
- 2:30- 2:40 (146) A Hille-Hosida theory for evolutions. Professor JAMES V. HEROD\*, Georgia Institute of Technology and Professor R. W. MCKELVEY, University of Montana (763-47-25)
- 2:45- 2:55 (147) Integral equations and evolution operations. Preliminary report. Mr. MICHAEL A. FREEDMAN, Georgia Institute of Technology (763-47-26)
- 3:00- 3:10 (148) Some coincidence theorems in metric and Banach spaces. Preliminary report. Dr. D. J. DOWNING, Oakland University (763-47-27)
- 3:15- 3:25 (149) Equality of essential spectra of certain quasisimilar seminormal operators. Professor LAWRENCE R. WILLIAMS, University of Texas, Austin (763-47-28)
- 3:30- 3:40 (150) Solvability of nonlinear equations using  $K$ -monotonicity. Preliminary report. KAREN SINGKOFER, University of South Florida (763-47-29) (Introduced by R. Kent Nagle)
- 3:45- 3:55 (151) An extension of Markov-Kakutani theorem. THAKYIN HU, Tamkang College of Arts and Sciences, Tamsui, Taipei, Taiwan, Republic of China (763-47-30) (Introduced by Professor Frank T. Birtel)
- 4:00- 4:10 (152) Smooth compact operators. Professor JULIEN HENNEFELD, Brooklyn College (763-47-31)

- 4:15- 4:25 (153) Hermitian operators on  $C(X, E)$ . Preliminary report, Professor RICHARD J. FLEMING\* and Professor JAMES E. JAMISON, Memphis State University (763-47-32)
- 4:30- 4:40 (154) The isometries of  $\mathfrak{sp}$ . Preliminary report, Professor RICHARD J. FLEMING and Professor JAMES E. JAMISON\*, Memphis State University (763-47-33)
- 4:45- 4:55 (155) A comparison of shifts on Banach spaces. Preliminary report, Mr. JAMES E. ROBINSON, LeMoyné-Owen College (763-47-34) (Introduced by Professor Richard J. Fleming)

WEDNESDAY, 2:15 P. M.

Session on Ordered Algebraic Structures and General Mathematical Systems, Convention Center, Room 10

- 2:15- 2:25 (156) Right-ordered groups and regular ordered permutation groups. Dr. MAUREEN A. BARDWELL, St. Norbert College (763-06-1)
- 2:30- 2:40 (157) Structures characterizing partially ordered sets, and their automorphism groups. Dr. RUSSELL BELDING, IIT Research Institute, Annapolis, Maryland (763-06-2)
- 2:45- 2:55 (158) Frattini sublattices in varieties of lattices. Dr. M. E. ADAMS\* and Dr. J. SICHLER, University of Manitoba (763-06-3)
- 3:00- 3:10 (159)  $K(L)$ , the lattice of complete ideals of  $L$ . Professor A. A. BISHOP, Western Illinois University, and Professor E. A. SCHREINER\*, Western Michigan University (763-06-4)
- 3:15- 3:25 (160) Projectives in a variety of lattice-ordered modules. Dr. WAYNE B. POWELL, University of Kansas (763-06-5)
- 3:30- 3:40 (161) Ordered fields: their completion by contiguous classes. Professor CARLOS A. INFANTOZZI, Inst. de Estudios Superiores, Montevideo, Uruguay (763-06-6)
- 3:45- 3:55 (162) Categorical structure theory. Dr. JAMES KEETON-WILLIAMS, Bowling Green State University (763-08-1)
- 4:00- 4:55 (163) The countable chain condition in quasi-primal varieties. Professor STEPHEN D. COMER, The Citadel, Charleston (763-08-2)
- 5:00- 5:10 (164) Associativity in finite groupoids. Preliminary report. Professor WILLIAM P. WARDLAW, U. S. Naval Academy (763-08-3)
- 5:15- 5:25 (165) Nondesarguesian finite projective planes derived from desarguesian planes. Preliminary report. Professor W. M. SANDERS, James Madison University (763-08-4)
- 5:30- 5:40 (166) Representation spaces for Banach lattices. W. A. FELDMAN and J. F. PORTER\*, University of Arkansas at Fayetteville (763-46-23)

WEDNESDAY, 3:15 P. M.

Session on Statistics, Computer Science, Games and Economics, Convention Center, Room 9

- 3:15- 3:25 (167) Characterization of certain partitioning methods with application to statistical monitoring systems. PERRY HAALAND, University of Miami (763-62-1)
- 3:30- 3:40 (168) A statistical test for extreme value distributions. Professor JANOS GALAMBOS, Temple University, Philadelphia (763-62-2)
- 3:45- 3:55 (169) Application of convergence structures to statistics. Preliminary report. Mr. GARY RICHARDSON, East Carolina University, Greenville (763-62-3)
- 4:00- 4:10 (170) On the norm of an unbiased estimator. Professor J. H. B. KEMPERMAN, University of Rochester (763-62-4)
- 4:15- 4:25 (171) A stochastic algorithm for the design of multi-spectral sensors. Preliminary report. Dr. CHRIS P. TSOKOS, University of South Florida, and Dr. JOHN C. TURNER\*, U. S. Naval Academy (763-62-5)
- 4:30- 4:40 (172) The analysis of a partition algorithm. Dr. VLADIMIR LIFSCHITZ\*, Brigham Young University and Dr. LEON PESOTCHINSKY, University of California, Santa Barbara (763-68-1)
- 4:45- 4:55 (173) Addresses in computer data files which are ordered as lattices. Preliminary report. Mr. JOHN SADOWSKY, U.S. Bureau of the Census, Washington, D. C. (763-68-2)

- 5:00- 5:10 (174) Selective and shortened selective sums of loopy partizan games. Preliminary report. JIM A. FLANIGAN, University of California, Los Angeles (763-90-16)
- 5:15- 5:25 (175) A condition for a utility function to be separable. Dr. BRUCE R. EBANKS, Texas Tech University (763-90-17)

WEDNESDAY, 3:30 P. M.

Invited Address, Convention Center Coliseum

- (176) Proper holomorphic mappings. Professor JOHN E. FORNAESS, Princeton University (763-32-2)

WEDNESDAY, 5:00 P. M.

Session on Nonassociative Rings and Algebras, Convention Center, Room 5

- 5:00- 5:10 (177) Hyperbolic GCM Lie algebras and Fibonacci numbers. Preliminary report. Professor ALEX JAY FEINGOLD, Drexel University (763-17-1)
- 5:15- 5:25 (178) Octonionic linear algebra. Preliminary report. Professor ANDREW SOBCZYK, Clemson University (763-17-2)
- 5:30- 5:40 (179) On nilpotency in certain varieties. Preliminary report. Dr. HARRY F. SMITH, Iowa State University (763-17-3)
- 5:45- 5:55 (180) Curtis algebras. Dr. RALPH AMAYO, Southern Illinois University, Carbondale (763-17-4)
- 6:00- 6:10 (181) The catalogue of Hall triple systems of small order. ROBERT ROTH, Ohio State University, Columbus (763-17-5)

WEDNESDAY, 8:30 P. M.

Josiah Willard Gibbs Lecture, Convention Center Coliseum

- (182) "What are solitons and inverse scattering anyway, and why should I care?" Professor MARTIN D. KRUSKAL, Princeton University

THURSDAY, 8:00 A. M.

Session on Combinatorics, I, Convention Center, Room 9

- 8:00- 8:10 (183) Permutation-partition pairs. Preliminary report. Dr. SAUL STAHL, University of Kansas (763-99-1)
- 8:15- 8:25 (184) The covering radius of the  $(m-3)$ rd order Reed-Muller codes and a lower bound on the  $(m-4)$ th order Reed-Muller codes. Dr. AILEEN M. McLOUGHLIN, Trinity College, Dublin, Ireland (763-05-2)
- 8:30- 8:40 (185) Observations on the construction of covers using permutation voltage assignments. Dr. CLOYD L. EZELL, JR., Stetson University (763-05-4)
- 8:45- 8:55 (186) A quantifier for matroid duality. Dr. TERRY A. McKEE, Wright State University (763-05-5)
- 9:00- 9:10 (187) Cyclotomic polynomials and nonstandard dice. Professor JOSEPH A. GALLIAN\*, University of Minnesota-Duluth and Mr. DAVID J. RUSIN, University of Chicago, (763-05-6)
- 9:15- 9:25 (188) Minimum graphs with complete  $k$ -closure. L. CLARK and R. C. ENTRINGER\*, University of New Mexico and D. E. JACKSON, Eastern New Mexico University (763-05-7)
- 9:30- 9:40 (189) On chains and Sperner  $k$ -families in ranked posets. Preliminary report. Dr. JERROLD R. GRIGGS, California Institute of Technology (763-05-8)
- 9:45- 9:55 (190) A sufficient condition for equality of edge-connectivity and minimum degree of a graph. Professor DONALD L. GOLDSMITH\*, Western Michigan University and Professor ROGER C. ENTRINGER, University of New Mexico (763-05-9)
- 10:00-10:10 (191) Bipartite subgraphs of cubic graphs. Preliminary report. WILLIAM STATON, University of Mississippi (763-05-10)
- 10:15-10:25 (192) Partitions and sums and products—two counterexamples. NEIL HINDMAN, California State University, Los Angeles (763-05-11)
- 10:30-10:40 (193) Crossing #s of permutation graphs of cycles. Preliminary report. Professor R. RINGEISEN, Wright-Patterson A. F. B., Ohio (763-05-12)

- 10:45-10:55 (194) On  $3 \times 3$  integer matrices with constant row and column sum. Preliminary report. Dr. ELIZABETH J. MORGAN, University of Queensland, Australia (763-05-13) (Introduced by Dr. Anne Penfold Street)
- 11:00-11:10 (195) A note on intersection preserving embeddings of partial  $(n, 4)$ -PBDs. Preliminary report. Professor WILLIAM B. POUCHER, Abilene Christian University (763-05-14)
- 11:15-11:25 (196) Configurations in the plane: isotopy and combinatorial equivalence. Preliminary report. Professor JACOB E. GOODMAN, City University of New York, City College (763-05-15)
- 11:30-11:40 (197) Determinants for variable adjacency matrices. Preliminary report. Professor JEAN H. BEVIS, Georgia State University (763-05-16)
- 11:45-11:55 (198) A multiplier theorem of group ring. Professor HAI-PING KO\*, Oakland University and Professor DIJEN K. RAY-CHAUDHURI, Ohio State University, Columbus (763-05-17)
- 12:00-12:10 (199) The enumeration of 3-connected graphs. Professor R. W. ROBINSON, University of Newcastle, New South Wales, Australia and PHIL HANLON\*, California Institute of Technology (763-05-18)

THURSDAY, 8:30 A. M.

Special Session on Integral Equations with Emphasis on Fredholm and Hammerstein Equations. III,  
Convention Center, Room 6

- 8:30- 8:50 (200) Numerical solutions of random Fredholm equations. Preliminary report. Professor ALBERT T. BHARUCHA-REID\*, Wayne State University and Professor MARK J. CHRISTENSEN, Georgia Institute of Technology (763-45-5)
- 8:55- 9:15 (201) Cauchy system for implicit nonlinear integral equations. Professor ROBERT E. KALABA\*, University of Southern California, Los Angeles and Professor ELENA ZAGUSTIN, California State University, Long Beach (763-45-6)
- 9:20- 9:40 (202) An integral equation of transport theory and some generalizations. Professor G. MILTON WING, Southern Methodist University (763-45-3)
- 9:45-10:05 (203) A class of nonlinear hyperbolic Volterra equations. Professor C. M. DAFERMOS, Brown University and Professor J. A. NOHEL\*, University of Wisconsin-Madison (763-45-11)
- 10:10-10:30 (204) Equations with unbounded delay: A survey. Professor C. CORDUNEANU, University of Tennessee, Knoxville and Professor V. LAKSHMIKANTHAM\*, University of Texas at Arlington (763-45-2)
- 10:35-10:55 (205) Finite-element approximations of integral equations with logarithmic kernels. Professor GEORGE C. HSIAO\*, University of Delaware and Professor WOLFGANG L. WENDLAND, Technische Hochschule Darmstadt, Germany (763-45-9)
- 11:00-11:20 (206) Symmetric Wiener-Hopf equations. JOEL D. PINCUS, State University of New York, Stony Brook (763-45-18)
- 11:25-11:45 (207) The Frank-Wolfe algorithm for infinite dimensional problems with nonunique solutions. Preliminary report. Dr. JAMIE J. GOODE\*, Georgia Institute of Technology and Ms. ELAINE HUBBARD, Kennesaw College (763-45-13)
- 11:50-12:20 (208) Formal discussion

THURSDAY, 8:30 A. M.

Special Session on Number Theory and its Applications. I, Convention Center, Room 8

- 8:30- 8:50 (209) Number theory and numerical integration. Dr. GERALD MYERSON, State University of New York at Buffalo (763-10-17)
- 8:55- 9:15 (210) The analysis of computer algorithms and probabilistic number theory. Professor PERSI DIACONIS, Bell Laboratories, Murray Hill, New Jersey (763-10-22)
- 9:20- 9:40 (211) Some number theoretic questions arising in asymmetric encryption techniques. Dr. GUSTAVUS J. SIMMONS, Sandia Laboratories, Albuquerque, New Mexico (763-94-1) (Introduced by Dr. Stefan A. Burr)
- 9:45-10:05 (212) Recursive linear digital filters and the values of cyclotomic polynomials at roots of unity. R. P. KURSHAN and A. M. ODLYZKO\*, Bell Laboratories, Murray Hill, New Jersey (763-12-2)

- 10:10-10:30 (213) Using finite precision fractions for computer arithmetic. Preliminary report. Professor DAVID W. MATULA, Southern Methodist University (763-10-12) (Introduced by Dr. Stefan A. Burr)
- 10:35-10:55 (214) Linear Diophantine inequalities and integral programming. Professor R. G. JEROSLOW, Georgia Institute of Technology (763-10-10)
- 11:00-11:30 (215) Problem session

THURSDAY, 8:30 A. M.

Special Session on Summability and Related Topics, I, Convention Center, Room 7

- 8:30- 8:50 (216) The unrestricted sections of sequences. Professor JOHN J. SEMBER, Simon Fraser University (763-40-14)
- 8:55- 9:15 (217) Densities and summability. ALLEN R. FREEDMAN, Simon Fraser University (763-40-16)
- 9:20- 9:40 (218) Beta-duals of matrix fields. Professor DENNIS C. RUSSELL\*, York University and Professor AMNON JAKIMOVSKI, Tel-Aviv University, Israel (763-40-7)
- 9:45-10:05 (219) Superreflexivity and summability. Professor S. SWAMINATHAN, Dalhousie University, Halifax, Nova Scotia (763-46-4)
- 10:10-10:30 (220) Approximation by Abel means and Tauberian theorems in  $BK$ -spaces. Dr. MARTIN BUNTINAS, Loyola University of Chicago (763-40-13)
- 10:35-10:55 (221) The modified reduction principle in summability and some applications. Professor M. R. PARAMESWARAN, University of Manitoba (763-40-5) (Introduced by Professor H. C. Finlayson)
- 11:00-11:20 (222) Tauberian theorems via block dominated matrices. Professor JOHN FRIDY, Kent State University, Kent (763-40-1)

THURSDAY, 8:30 A. M.

Special Session on Modular and Automorphic Functions in a Single Complex Variable, I, Convention Center, Room 1

- 8:30- 8:50 (223) The subgroup method in the theory of the elliptic modular functions. Professor MORRIS NEWMAN, University of California, Santa Barbara (763-10-2)
- 8:55- 9:15 (224) Complex multiplication on Fricke's torus-covering. Professor HARVEY COHN, City University of New York, City College (763-10-5)
- 9:20- 9:40 (225) Chapter 14 of Ramanujan's second notebook. Professor BRUCE C. BERNDT, University of Illinois, Urbana (763-10-1)
- 9:45-10:05 (226) A class of nonlinear functional equations connected with modular functions. Preliminary report. Professor L. ALAYNE PARSON, Ohio State University, Columbus (763-10-7)
- 10:10-10:30 (227) The enumeration of lattice points in certain higher-dimensional tetrahedra and a conjecture of Rademacher. Professor KENNETH H. ROSEN, Ohio State University, Columbus (763-10-11)
- 10:35-10:55 (228) Functional equations similar to those of Eisenstein series. Preliminary report. Dr. GEORGES GRINSTEIN, Auburn University at Montgomery (763-10-18)
- 11:00-11:20 (229) Eichler cohomology and Eisenstein series. MICHAEL J. RAZAR, University of Maryland, College Park (758-10-3)
- 11:25-11:45 (230) Diophantine approximation on the Hecke groups. Preliminary report. Professor JOSEPH LEHNER, University of Pittsburgh, Pittsburgh (763-30-4)

THURSDAY, 8:30 A. M.

Special Session on Mathematical Psychology, I, Convention Center, Room 11

- 8:30- 8:50 (231) Some problem foci in mathematical psychology. Preliminary report. Dr. WILLIAM H. BATCHELDER, University of California, Irvine (763-92-14) (Introduced by Professor William C. Hoffman)
- 8:55- 9:15 (232) Topological analysis of impossible figures. Dr. THADDEUS M. COWAN, Kansas State University (763-92-3) (Introduced by Professor William C. Hoffman)
- 9:20- 9:40 (233) Mapping the visual field onto the brain. Preliminary report. Professor PETER C. DODWELL, Queen's University at Kingston (763-92-15) (Introduced by Professor William C. Hoffman)



- 9:45-10:05 (234) On what others receive in a fair division. Preliminary report. Professor WILLIAM F. LUCAS, Cornell University (763-92-11)
- 10:10-10:30 (235) Induction in adaptive learning. Professor MANFRED KOCHEN, University of Michigan, Ann Arbor (763-92-5)
- 10:35-10:55 (236) A model of color vision of trichromatic mammals based on a group-theoretic approach. Dr. A. S. BLAVAIS, Institute of Higher Nervous Activity, Moscow, USSR (763-92-10) (Introduced by Professor William C. Hoffman)
- 11:00-11:20 (237) Separating and assessing motivation pressures in  $2 \times 2$  experimental games. Professor ANATOL RAPOPORT, University of Toronto (763-92-7)

THURSDAY, 8:30 A. M.

Session on Geometry, Convention Center, Room 2

- 8:30- 8:40 (238) Geometries containing dual affine planes. Preliminary report. Professor K. BOLLING FARMER\* and Professor MARK P. HALE, Jr., University of Florida (763-50-2)
- 8:45- 8:55 (239) Translation planes of order 25 with nontrivial X-OY perspectivities. Professor ELWYN H. DAVIS, Pittsburg State University (763-50-3)
- 9:00- 9:10 (240) Loxodromic polyhedra with plane quadrangular faces. Professor ALAN H. SCHOEN, Southern Illinois University, Carbondale (763-50-4) (Introduced by Professor Melvyn B. Nathanson)
- 9:15- 9:25 (241) Synthetic problems in spaces with mixed planes. Preliminary report. Professor MARK P. HALE, Jr., University of Florida (763-50-5)
- 9:30- 9:40 (242) Embeddability of linear spaces with  $p = f(n)$  points and line range a subset of  $\{n-1, n, n+1\}$ . Dr. L. M. BATTEN, University of Winnipeg (763-50-6)
- 9:45- 9:55 (243) On the kahlerity of strongly pseudoconvex manifolds. Mr. TAN VO VAN, University of Massachusetts, Boston (763-53-3)
- 10:00-10:10 (244) Differential invariants that withstand catastrophies. Preliminary report. H. GUGGENHEIMER, Polytechnic Institute of New York, Brooklyn (763-53-14)
- 10:15-10:25 (245) A normal form for a special class of 5-dimensional manifolds. Dr. STANLEY M. ZOLTEK, Wright State University, Dayton (763-53-17)
- 10:30-10:40 (246) Geodesic triangles in a Lorentz manifold. Preliminary report. Mr. STEVEN G. HARRIS, University of Chicago (763-53-18)
- 10:45-10:55 (247) The Jacobi equation on a Lie group. JOHN R. HERRING\* and JOHN J. O'SULLIVAN, Pennsylvania State University, University Park (763-53-19)
- 11:00-11:10 (248) Surfaces in real projective space. HOWARD JACOBOWITZ, Rutgers University, Camden (763-53-21)
- 11:15-11:25 (249) Naturally reductive homogeneous Riemannian manifolds with nonpositive curvature. Preliminary report. Mr. EDWARD D. DELOFF, Rutgers University, New Brunswick (763-53-22)

THURSDAY, 8:30 A. M.

Session on General Topology. I, Convention Center, Room 12

- 8:30- 8:40 (250) A countably compact, first countable, non-normal  $T_2$ -space. Professor JERRY E. VAUGHAN, University of North Carolina at Greensboro (763-54-1)
- 8:45- 8:55 (251) A nonstandard approach to pseudotopological compactifications. Professor ROBERT A. HERRMANN, U. S. Naval Academy (763-54-3)
- 9:00- 9:10 (252) Certain local homeomorphisms of continua are homeomorphisms, II. Dr. AUGUST LAU, North Texas State University (763-54-5)
- 9:15- 9:25 (253) A note on co-absolutes of Stone-Čech remainders. Preliminary report. Dr. SCOTT W. WILLIAMS, Ohio University and State University of New York at Buffalo (763-54-6)
- 9:30- 9:40 (254) A proof of a conjecture of A. H. Stone. Professor R. F. DICKMAN, Jr., Virginia Polytechnic Institute and State University (763-54-7)
- 9:45- 9:55 (255) Convexity and the  $J_\epsilon$ -function. Preliminary report. Mr. JOHN B. FOULKES, University of Delaware (763-54-8)
- 10:00-10:10 (256) Second category function spaces. Professor D. J. LUTZER\*, Texas Tech University and Professor R. A. McCOY, Virginia Polytechnic Institute and State University (763-54-9)

- 10:15-10:25 (257) A linearization of semiflows in the Hilbert space  $\ell_2$ . Embeddability of semiflows into flows. LUDVIK JANOS, Mathematical Reviews, Ann Arbor (763-54-10)
- 10:30-10:40 (258) Topological spaces with a unique compatible uniformity. Dr. RICHARD H. WARREN, University of Nebraska at Omaha (763-54-11)
- 10:45-10:55 (259) Remote points. Professor SOO BONG CHAE, University of South Florida, New College (763-54-12)
- 11:00-11:10 (260) C-embedding in functionally normal and related spaces. C. E. AULL, Virginia Polytechnic Institute and State University (763-54-13)
- 11:15-11:25 (261) Some remarks on  $\theta$ -rigidity. Mr. JAMES E. JOSEPH, Howard University (763-54-14)
- 11:30-11:40 (262) Spaces in which points have bilinearly ordered bases. Preliminary report. Dr. BRIAN M. SCOTT, Cleveland State University (763-54-15)
- 11:45-11:55 (263) Periodic homeomorphisms on chainable continua. Dr. WAYNE LEWIS, Texas Tech University (763-54-16)

THURSDAY, 8:30 A. M.

Session on Commutative Rings and Algebras, Convention Center, Room 5

- 8:30- 8:40 (264) R-automorphisms of  $R[t] [[X_1, \dots, X_n]]$ . Preliminary report. Dr. J. H. KIM, East Carolina University (763-13-2)
- 8:45- 8:55 (265) Inertial subalgebras of algebras possessing finite automorphism groups. Dr. NICHOLAS S. FORD, Pennsylvania State University, Uniontown (763-13-3)
- 9:00- 9:10 (266) An integral domain with an almost division algorithm. Preliminary report. Professor NICK VAUGHAN, North Texas State University (763-13-4)
- 9:15- 9:25 (267) A nonconstant rank implicit function theorem. Preliminary report. Dr. GARY A. HARRIS, Texas Tech University (763-13-5)
- 9:30- 9:40 (268) Determinants of Galois automorphisms of maximal commutative rings of  $2 \times 2$  matrices. DENNIS R. ESTES, University of Southern California (763-13-7) (Introduced by Professor G. Pasty)
- 9:45- 9:55 (269) The word problem is solvable in  $N_0$ . Preliminary report. Dr. DAVID J. JOHN\*, Valdosta State College and Dr. MARY F. NEFF, Emory University, Atlanta (763-13-8)
- 10:00-10:10 (270) Local-global criteria for outer product rings. DENNIS R. ESTES and JACOB R. MATIJEVIC\*, University of Southern California (763-13-9)
- 10:15-10:25 (271) A structure theory for modules satisfying chain conditions over commutative rings. Preliminary report. Mr. WILLIAM D. WEAKLEY, Northwestern University (763-13-10)
- 10:30-10:40 (272) Extension of derivations. Preliminary report. Professor STUART S. WANG, Oakland University (763-13-11)
- 10:45-10:55 (273) Universal trace classes. Preliminary report. Mr. RODERIC C. DEYO, University of California, San Diego (763-13-12)
- 11:00-11:10 (274) Torsion theories with BSP. Mr. FREDERICK CALL and Professor THOMAS SHORES\*, University of Nebraska, Lincoln (763-13-13)

THURSDAY, 8:30 A. M.

Session on Ordinary Differential Equations, I, Convention Center, Room 10

- 8:30- 8:40 (275) A differential equation for undamped forced nonlinear oscillations. Preliminary report. Dr. CHAMPAK D. PANCHAL, University of North Florida (763-34-37)
- 8:45- 8:55 (276) Existence and comparison results for differential equations of Sobolev type. Preliminary report. Professor A. S. VATSALA, University of Texas at Arlington and Professor R. L. VAUGHN\*, Texas Christian University (763-34-38)
- 9:00- 9:10 (277) A two point BVP with jumping nonlinearities. Dr. ALFONSO CASTRO, CIEA del IPN, Mexico (763-34-39)
- 9:15- 9:25 (278) Stability criteria for equation  $P(D)x(t) + f(t)q(D)x(t) = 0$ . Professor KUO-LIANG CHIOU, Wayne State University (763-34-40)
- 9:30- 9:40 (279) Operators with countably many generalized boundary conditions. SUNG J. LEE, University of Guelph, Ontario, Canada (763-34-3)

- 9:45- 9:55 (280) Oscillation properties of singular Sturm-Liouville operators. Preliminary report. Dr. J. K. SHAW, Virginia Polytechnic Institute and State University (763-34-4)
- 10:00-10:10 (281) A constructive technique for solving a second order inverse eigenvalue problem. Preliminary report. Professor GEORGE H. HANDELMAN and Professor JOYCE R. McLAUGHLIN\*, Rensselaer Polytechnic Institute (763-34-5)
- 10:15-10:25 (282) Existence of periodic solutions for nonlinearly perturbed conservative systems. Professor JAMES R. WARD, Pan American University (763-34-8)
- 10:30-10:40 (283) Stabilizing and oscillating time-delays. Preliminary report. Professor GANGARAM S. LADDE, State University of New York, College at Potsdam (763-34-9)
- 10:45-10:55 (284) An embedding technique for a class of stability problems. VADIM KOMKOV, Mathematical Reviews, Ann Arbor (763-34-10)
- 11:00-11:10 (285) A characterization of positive linear functionals and oscillation criteria for matrix differential equations. Dr. TERRY J. WALTERS, Pan American University (763-34-13)
- 11:15-11:25 (286) Vibrations of a pendulum consisting of a bob suspended from a wire: The method of integral equations. Dr. HYUN JOON AHN, Indiana State University (763-34-14)
- 11:30-11:40 (287) Periodic solutions of a class of functional differential equations containing a small parameter. Dr. MAX ASHKENAZI, Michigan State University (763-34-15)
- 11:45-11:55 (288) Qualitative analysis of the periodically forced relaxation oscillations. Preliminary report. MARK LEVI, Northwestern University (763-34-16)
- 12:00-12:10 (289) Comparison theorems for a coupled system of hyperbolic differential inequalities. Preliminary report. Professor C. Y. CHAN and Professor E. C. YOUNG\*, Florida State University (763-35-7)

THURSDAY, 9:00 A. M.

Invited Address, Convention Center Coliseum

- (290) The heart valve problem of cardiac fluid dynamics and its numerical solution. Professor CHARLES S. PESKIN, Courant Institute of Mathematical Sciences (763-65-11)

THURSDAY, 9:00 A. M.

Session on Partial Differential Equations, Convention Center, Room 3

- 9:00- 9:10 (291) On partial differential equations associated with Euler-Poisson-Darboux equation. Professor M. A. AL-BASSAM, Kuwait University, Kuwait (763-35-1)
- 9:15- 9:25 (292) A new operational calculus method. Dr. ABRAHAM UNGAR, Rhodes University, Grahamstown, South Africa (763-35-2)
- 9:30- 9:40 (293) Global characterizations of generalized biaxially symmetric potentials via best local approximates. Professor PETER A. McCOY, U. S. Naval Academy (763-35-4)
- 9:45- 9:55 (294) Bäcklund transformations in several variables. Preliminary report. Dr. DEAN A. PAYNE, Illinois State University, Normal (763-35-6)
- 10:00-10:10 (295) Comparison theorems for reaction-diffusion equations in abstract cones. Preliminary report. Professor V. LAKSHMIKANTHAM and Professor A. S. VATSALA, University of Texas at Arlington and Professor S. LEELA\*, State University of New York at Geneseo (763-35-8)
- 10:15-10:25 (296) Stabilization for certain parabolic problems. Preliminary report. Professor R. K. JUBERG, University of California, Irvine (763-35-9)
- 10:30-10:40 (297) Infinite component nonlinear Schrödinger and modified Korteweg-deVries equations. Preliminary report. Dr. DAVID V. CHUDNOVSKY, Columbia University, New York (763-35-10)
- 10:45-10:55 (298) Nonlinear elliptic boundary value problems at resonance. Preliminary report. Professor R. KENT NAGLE, University of South Florida (763-35-11)
- 11:00-11:10 (299) Application of comparison principles to systems arising in combustion theory. Preliminary report. Dr. JAGDISH CHANDRA, U. S. Army Research Office, Research Triangle Park, North Carolina and Professor PAUL W. DAVIS\*, Worcester Polytechnic Institute (763-35-12)
- 11:15-11:25 (300) Fractional iterates of functions on restricted domains. Preliminary report. Professor RICHARD E. RICE, Union College, Schenectady (763-39-2)

- 11:30-11:40 (301) Asymptotic behavior of solutions for the nonlinear Schrödinger equation. Preliminary report. Mr. SHIGEMITSU KADEKAWA, Indiana University at Bloomington (763-81-1) (Introduced by R. Glassey)
- 11:45-11:55 (302) Pseudospherically symmetric tachyon fluids. Dr. J. G. MILLER, Texas A & M University (763-83-9)

THURSDAY, 9:00 A. M.

Session on History, Logic and Foundations, Convention Center, Room 4

- 9:00- 9:10 (303) Richard Courant and the finite element method; A further look. Professor FRANK WILLIAMSON, Jr., Ramapo College of New Jersey (763-01-1)
- 9:15- 9:25 (304) The Borel letters on the theory of sets. Dr. STANLEY H. STAHL\*, Mount Holyoke College and Smith College and Dr. CATHERINE STAHL, Westfield State College (763-01-2)
- 9:30- 9:40 (305) Results on supercompact cardinals. Preliminary report. Mr. JULIUS BARBANEL, State University of New York at Buffalo (763-02-3)
- 9:45- 9:55 (306) The Löwenheim-Skolem theorem, theories of quantification, and Beweistheorie. Dr. IRVING H. ANELLIS, University of Florida (763-02-4)
- 10:00-10:10 (307) A modal logic treatment of "probable". Dr. KEVIN W. SAUNDERS, CEMREL Education Laboratory, St. Louis, Missouri (763-02-5) (Introduced by Joel Schneider)
- 10:15-10:25 (308) Prime quantifier eliminable rings. Dr. BRUCE I. ROSE, University of Notre Dame (763-02-6)
- 10:30-10:40 (309) Various methods of evaluating negation and the corresponding logics. ALEXANDER ABIAN, Iowa State University (763-02-7)
- 10:45-10:55 (310) On the cardinality of sets of incomparable cardinals. Preliminary report. Dr. ARTHUR L. RUBIN, Jet Propulsion Laboratory, Pasadena, California (763-04-2)
- 11:00-11:10 (311)  $\Sigma_1^0(\Pi_1^1)$ -determinacy. Preliminary report. Mr. JOHN CARSON SIMMS, Texas Tech University (763-04-3)
- 11:15-11:25 (312) A normal form for some coherent sequences of ultrafilters. Preliminary report. Mr. STEWART BALDWIN, University of Colorado, Boulder (763-04-4) (Introduced by Professor W. N. Reinhardt)

THURSDAY, 10:30 A. M.

Invited Address, Convention Center Coliseum

- (313) Time-change in flows. Professor JACOB FELDMAN, University of California, Berkeley (763-28-7)

THURSDAY, 11:25 A. M.

Session on Category Theory, Homological Algebras, Convention Center, Room 5

- 11:25-11:35 (314) Construction of equivalent categories. Preliminary report. Professor MARGARET M. LA SALLE, University of Southwestern Louisiana (763-18-1)
- 11:40-11:50 (315) Projective dimension of valuated vector spaces. Professor PAUL HILL and Mrs. ERRIN WHITE\*, Auburn University, Auburn (763-18-2)
- 11:55-12:05 (316) Cohomology groups and first-order theories. Preliminary report. Dr. PAUL GLENN, Union College, Schenectady, New York (763-18-3)
- 12:10-12:20 (317) On  $\aleph$ -Noetherian conditions. G. C. HEWITT, University of Montana (763-18-4)

THURSDAY, 1:00 P. M.

Colloquium Lectures, Lecture II, Convention Center Coliseum

- (318) Complex analysis and algebraic geometry. Professor PHILLIP A. GRIFFITHS, Harvard University

THURSDAY, 2:10 P. M.

Invited Address, Convention Center Coliseum

- (319) Space-times with distribution valued curvature tensors. Professor ABRAHAM H. TAUB, University of California, Berkeley (763-83-8)

THURSDAY, 3:20 P. M.

Bôcher and Steele Prize Session, Convention Center Coliseum

THURSDAY, 5:00 P. M.

Business Meeting, Convention Center Coliseum

THURSDAY, 7:00 P. M.

Special Session on Number Theory and Its Applications, II, Convention Center, Room 8

- 7:00- 7:20 (320) Some number-theoretic problems in combinatorics and geometry. Preliminary report. Professor PAUL ERDÖS, Hungarian Academy of Sciences, Budapest, Hungary (763-10-16)
- 7:25- 7:45 (321) Numbering graphs for fun and profit. Preliminary report. Dr. GARY S. BLOOM, City University of New York, City College (763-10-30)
- 7:50- 8:10 (322) On a diophantine equation arising in graph theory. RONALD L. GRAHAM, Bell Laboratories, Murray Hill, New Jersey (763-10-9)
- 8:15- 8:35 (323) An application of number theory to crystal physics. M. LAWRENCE GLASSER, Clarkson College of Technology (763-10-8) (Introduced by Dr. Stefan A. Burr)
- 8:40- 9:00 (324) Partition theory and q-orthogonal polynomials. Dr. DAVID M. BRESSOUD, Pennsylvania State University, University Park (763-10-3)
- 9:05- 9:25 (325) Construction of Knut Vik designs. Professor A. O. L. ATKIN, Professor L. HAY and Professor R. G. LARSON\*, University of Illinois at Chicago Circle (763-05-3)
- 9:30-10:00 (326) Problem session

THURSDAY, 7:00 P. M.

Special Session on Summability and Related Topics, II, Convention Center, Room 7

- 7:00- 7:20 (327) Some properties of the Leininger generalized Hausdorff matrix. Professor VIVIENNE M. MAYES, Baylor University and Professor B. E. RHOADES\*, Indiana University, Bloomington (763-40-8)
- 7:25- 7:45 (328) Classes of means which are common to Gronwall means. Preliminary report. Professor VIVIENNE M. MAYES, Baylor University (763-40-10)
- 7:50- 8:10 (329) Weighted means, generalised Hausdorff matrices and the Borel property. Professor DAVID BORWEIN\* and Professor F. P. CASS, University of Western Ontario (763-40-4)
- 8:15- 8:35 (330) Summability factors for certain sequence spaces. Preliminary report. Professor DAVID F. DAWSON, North Texas State University (763-40-3)
- 8:40- 9:00 (331) Stieltjes summability and application. Professor LOUISE A. RAPHAEL, Clark College, Atlanta (763-40-2)
- 9:05- 9:25 (332) Maximal maximum modulus theorems for sequence spaces. Professor BRUCE L. R. SHAWYER, University of Western Ontario (763-40-15) (Introduced by Professor B. E. Rhoades)
- 9:30- 9:50 (333) Moments and differential equations. Professor GORDON G. JOHNSON, JSC and University of Houston, Houston (763-40-6)

THURSDAY, 7:00 P. M.

Special Session on Mathematical Psychology, II, Convention Center, Room 11

- 7:00- 7:20 (334) Differential models for perception of optical illusions. Professor DAVID A. SMITH, Duke University (763-92-8)
- 7:25- 7:45 (335) Theory and data as adjoints in a topos. Dr. JOHN PAUL BOYD, University of California, Irvine (763-92-2) (Introduced by Professor William C. Hoffman)
- 7:50- 8:10 (336) The simplicial category as a model for cognitive information processing. Dr. WILLIAM C. HOFFMAN, Oakland University, Rochester (763-92-4)
- 8:15- 8:35 (337) A non-Euclidean model of visual illusions. Preliminary report. Dr. HAROLD LINDMAN, Indiana University, Bloomington (763-92-6) (Introduced by Professor William C. Hoffman)

- 8:40- 9:00 (338) A dynamical model for cognitive development. Preliminary report. Professor DONALD G. SAARI, Northwestern University (763-92-9)
- 9:05- 9:25 (339) Principles and quantitative measurements of visual illusions. Professor S. P. DIAZ and Dr. M. HUMI\*, Worcester Polytechnic Institute (763-92-1)

THURSDAY, 7:00 P. M.

Special Session on Differential Geometry and General Relativity. I, Convention Center, Room 6

- 7:00- 7:20 (340) Zermelo's navigation problem and null geodesics. Professor KAREN K. UHLENBECK, University of Illinois at Chicago Circle (763-49-2)
- 7:25- 7:45 (341) Maximal hypersurfaces in Lorentzian manifolds. FRANK FLAHERTY, Oregon State University, Corvallis (763-53-10)
- 7:50- 8:10 (342) Recent progress in the existence of asymptotically flat spacetimes. Dr. MURRAY CANTOR, University of Texas, Austin (763-83-3)
- 8:15- 8:35 (343) On the nonexistence of time periodic and almost time periodic spacetimes. Dr. FRANK J. TIPLER, University of California, Berkeley (763-83-5) (Introduced by Professor Paul Ehrlich)
- 8:40- 9:00 (344) Some new directions in relativity. PHILLIP E. PARKER, Syracuse University (763-83-2)
- 9:05- 9:25 (345) Self-dual solutions to Einstein's equations. Professor DAVID LERNER, University of Kansas (763-83-10) (Introduced by Professor Paul Ehrlich)

THURSDAY, 7:00 P. M.

Session on Combinatorics. II, Convention Center, Room 9

- 7:00- 7:10 (346) Cycle structure of strong GM-graphs. Professor DANIEL J. RICHMAN\*, Wake Forest University and Professor JOHN S. MAYBEE, University of Colorado, Boulder (763-05-19)
- 7:15- 7:25 (347) A functional inequality arising in combinatorics. Dr. DONALD R. SNOW, Brigham Young University (763-05-20)
- 7:30- 7:40 (348) A combinatorial problem for matrices. Dr. WILLIAM T. TROTTER, Jr. \*, and Mr. TED MONROE, University of South Carolina, Columbia (763-05-21)
- 7:45- 7:55 (349) An algorithm for determining whether a graph is homogeneously traceable. Preliminary report. Mr. JOHN G. GIMBEL, Western Michigan University (763-05-22) (Introduced by Professor Yousef Alavi)
- 8:00- 8:10 (350) On some cyclic configurations. Preliminary report. Dr. D. E. KEENAN, Texas A & M University (763-05-23) (Introduced by Dr. William Rundell)
- 8:15- 8:25 (351) A technique for embedding cyclic orthogonal arrays. Professor CHARLES C. LINDNER, Auburn University, Auburn (763-05-24)
- 8:30- 8:40 (352) Packing and covering in graphs. III: cyclic and acyclic invariants. Professor JIN AKIYAMA\*, Nippon Ika University, Kawasaki, Japan and Professor FRANK HARARY, University of Michigan, Ann Arbor (763-05-25)
- 8:45- 8:55 (353) The graphs for which all strong orientations are hamiltonian. Dr. MARTIN GRÖTSCHHEL, Universität Bonn, Germany and Professor FRANK HARARY\*, University of Michigan, Ann Arbor (763-05-26)
- 9:00- 9:10 (354) Path numbers of multipartite digraphs. Dr. WILLIAM G. FRYE and Professor RENU C. LASKAR\*, Clemson University (763-05-27)
- 9:15- 9:25 (355) Randomly matchable graphs. Professor DAVID P. SUMNER, University of South Carolina, Columbia (763-05-28)
- 9:30- 9:40 (356) On the cochromatic index of a graph. Professor LOWELL W. BEINEKE\* and Professor RICHARD D. RINGEISEN, Indiana University-Purdue University at Fort Wayne and Professor H. JOSEPH STRAIGHT, State University of New York at Fredonia (763-05-29)
- 9:45- 9:55 (357) Projective  $(2n, n, \lambda, 1)$ -designs. P. ERDÖS, V. FABER\*, and F. JONES, University of Colorado, Boulder (763-05-30)

THURSDAY, 7:00 P. M.

Session on Algebraic Number Theory, Field Theory and Polynomials, Convention Center, Room 2

- 7:00- 7:10 (358) Toward nonsplitting unitary perfect polynomials over each  $\text{GF}(p)$ . Professor JACOB T. B. BEARD, Jr. \*, Emory University and Ms. MICKIE SUE HARBIN, University of Texas at Arlington (763-12-1)
- 7:15- 7:25 (359) Equivalence of sets of functions over a finite field. Preliminary report. Professor GARY L. MULLEN, Pennsylvania State University, Sharon (763-12-6)
- 7:30- 7:40 (360) On the inverse problem in the Galois theory of difference fields. Professor RONALD P. INFANTE, Seton Hall University (763-12-7)
- 7:45- 7:55 (361) A characterization of induced space of derivations in the  $p$ -adic Galois theory. Preliminary report. Dr. NICKOLAS HEEREMA and Mr. TIM MORRISON\*, Florida State University (763-12-8)
- 8:00- 8:10 (362) The minimal index of a cyclic cube field. Dr. JAMES G. HUARD, Southern Illinois University, Carbondale (763-12-9)
- 8:15- 8:25 (363) On the lattice of algebraically closed subfields of an algebraically closed field of characteristic 0. II. Dr. CHRIS ASH, Monash University, Clayton, Victoria, Australia, and Dr. JOHN W. ROSENTHAL\*, Ithaca College (763-12-10)
- 8:30- 8:40 (364) Invariants of finite groups acting nonlinearly on rational function fields. Dr. DEBORAH TRIANTAPHYLLOU, University of California, Santa Cruz (763-12-11)
- 8:45- 8:55 (365) On the density of discriminants of quartic fields. Dr. ANDREW M. BAILY, University of Michigan, Dearborn (763-12-12)
- 9:00- 9:10 (366) The Hasse norm principle in nonabelian extensions. Dr. STANLEY J. GURAK, University of San Diego (763-12-13) (Introduced by Roderic Deyo)

THURSDAY, 7:00 P. M.

Session on Topological Groups, Lie Groups, Convention Center, Room 3

- 7:00- 7:10 (367) A Lie algebra for certain local differentiable monoids. Mr. GEORGE GRAHAM, University of Houston, Houston (763-22-1)
- 7:15- 7:25 (368) On Bieberbach's analysis of discrete Euclidean groups. Dr. R. K. OLIVER, Pittsburgh, Pennsylvania (763-22-2)
- 7:30- 7:40 (369) A class of nilpotent Lie groups with analytically hypoelliptic sublaplacians. Professor AROLDO KAPLAN, University of Massachusetts, Amherst (763-22-3)
- 7:45- 7:55 (370) Weakening the topology of a Lie group. Preliminary report. Ms. T. CHRISTINE STEVENS, Mount Holyoke College (763-22-4)
- 8:00- 8:10 (371) Topological group rings. Preliminary report. Professor ERIC C. NUMMELA, St. Cloud State University (763-22-5)
- 8:15- 8:25 (372) Free topological groups are (almost) never locally invariant. Preliminary report. Dr. BARBARA SMITH THOMAS\*, Memphis State University and Dr. TEMPLE H. FAY, New Mexico State University, Las Cruces (763-22-6)

THURSDAY, 7:00 P. M.

Session on Ordinary Differential Equations. II, Convention Center, Room 10

- 7:00- 7:10 (373) Asymptotically stable dynamical systems are linear. ROGER C. McCANN, Mississippi State University (763-34-17)
- 7:15- 7:25 (374) A comparison theorem for even-order, vector-matrix differential equations. Professor ROGER T. LEWIS, University of Alabama, Birmingham and Ms. LYNNE C. WRIGHT\*, University of Alabama, University (763-34-18)
- 7:30- 7:40 (375) Focal Green's functions for fourth order differential equations. Professor ALLAN PETERSON, University of Nebraska, Lincoln (763-34-19)
- 7:45- 7:55 (376) A remark on cosine families. Professor SAMUEL M. RANKIN III, West Virginia University, Morgantown (763-34-20)
- 8:00- 8:10 (377) On the asymptotic behavior of a fundamental set of solutions. Dr. CHARLES R. POWDER, University of Dayton (763-34-21)
- 8:15- 8:25 (378) Semilinear functional differential equations via the variation of constant formula. Preliminary report. JAMES H. LIGHTBOURNE III, Pan American University (763-34-23)

- 8:30- 8:40 (379) Numerical methods for a singular eigenvalue problem with eigenparameter in the boundary conditions. Professor CHARLES T. FULTON\*, Pennsylvania State University, University Park, and Professor STEVEN PRUESS, University of New Mexico (763-34-31)
- 8:45- 8:55 (380) Adjoint and extension theory of abstract boundary value problems. Professor RICHARD C. BROWN, University of Alabama, University (763-34-32)
- 9:00- 9:10 (381) Some results for Volterra integrodifferential equations. Preliminary report. Mr. LEIGH C. BECKER, Southern Illinois University, Carbondale (763-34-33)
- 9:15- 9:25 (382) Necessary and sufficient condition for eventual decay of oscillations in general functional equations with delays. Professor BHAGAT SINGH, University Center-Manitowoc County, Wisconsin (763-34-34)
- 9:30- 9:40 (383) On regular systems of ordinary differential equations. I. Preliminary report. Professor JOHN W. HOOKER\* and Professor C. E. LANGENHOP, Southern Illinois University, Carbondale (763-34-35)
- 9:45- 9:55 (384) On regular systems of ordinary differential equations. II. Preliminary report. Professor C. E. LANGENHOP\* and Professor JOHN W. HOOKER, Southern Illinois University, Carbondale (763-34-36)

THURSDAY, 7:00 P. M.

Session on Integral Equations, Calculus of Variations, Optimal Control, Convention Center, Room 5

- 7:00- 7:10 (385) The asymptotic stability of a nonlinear functional differential equation of infinite delay. Professor DENNIS W. BREWER, University of Arkansas at Fayetteville (763-45-8)
- 7:15- 7:25 (386) Representation and approximation of solutions to semilinear Volterra equations with delay. Preliminary report. Dr. W. E. FITZGIBBON, University of Houston, Houston (763-45-19)
- 7:30- 7:40 (387)  $L^1$  resolvents of Volterra integrodifferential equations. Professor G. S. JORDAN, University of Tennessee, Knoxville (763-45-20)
- 7:45- 7:55 (388) Existence and comparison results for a class of Volterra integral equations of Sobolev type. Preliminary report. Professor V. LAKSHMIKANTHAM and Professor A. S. VATSALA\*, University of Texas at Arlington, and Professor R. L. VAUGHN, Texas Christian University (763-45-21)
- 8:00- 8:10 (389) On the existence of global variational principals. I. M. ANDERSON and T. DUCHAMP\*, University of Utah (763-49-1)
- 8:15- 8:25 (390) Local time-optimal feedback control of nth-order two-input systems. Professor LOREN D. MEEKER, University of New Hampshire (763-49-3)
- 8:30- 8:40 (391) Optimal fields for problems with delays. Preliminary report. Professor VERNON L. BAKKE, University of Arkansas at Fayetteville (763-49-4) (Introduced by Professor James F. Porter)

THURSDAY, 7:00 P. M.

Session on General Topology. II, Convention Center, Room 12

- 7:00- 7:10 (392) A covering property which implies isocompactness. I. Professor J. M. WORRELL, Jr., and Professor H. H. WICKE\*, Ohio University at Athens (763-54-17)
- 7:15- 7:25 (393) Q-sets and Shelah's principle. Professor GEORGE M. REED, Institute for Medicine and Mathematics, Ohio University, Athens (763-54-18)
- 7:30- 7:40 (394) Properties of  $\theta$ -closure. Ms. M. SOLVEIG ESPELIE, Howard University (763-54-19)
- 7:45- 7:55 (395) Borel selectors for separated quotients. Professor DOUGLAS E. MILLER, University of Illinois at Chicago Circle (763-54-20)
- 8:00- 8:10 (396) Countable small rank and cardinal invariants. II. Professor PETER NYIKOS, Ohio University at Athens (763-54-21)
- 8:15- 8:25 (397) On the product of closed images of metric spaces. Professor GARY GRUENHAGE, Auburn University, Auburn (763-54-22)
- 8:30- 8:40 (398) A topology series for a convergence space. Preliminary report. Professor ALLAN C. COCHRAN, University of Arkansas at Fayetteville (763-54-23)



- 8:45- 8:55 (399) The maximal  $\mathcal{L}$ -compactification. Dr. ROBERT WARREN BUTTON, Ithaca College (763-54-24)
- 9:00- 9:10 (400) Structure of the set of subsequential limit points. J. L. SOLOMON, Mississippi State University (763-54-25)

THURSDAY, 7:00 P. M.

Session on Numerical Analysis and Information Theory, Convention Center, Room 4

- 7:00- 7:10 (401) The double eigenvalue problem. Preliminary report. Professor JOHN GREGORY\*, Southern Illinois University, Carbondale and Professor RALPH WILKERSON, Winthrop College (763-65-3) (Introduced by Professor Ronald Kirk)
- 7:15- 7:25 (402) Spline solutions of a boundary value problem. Dr. RIAZ A. USMANI, University of Manitoba (763-65-4)
- 7:30- 7:40 (403) Predicting resonance in turning point problems by numerical methods. Preliminary report. Dr. GILBERT N. LEWIS, Michigan Technological University (763-65-6)
- 7:45- 7:55 (404) A two-mesh collocation method for nonlinear two-point boundary value problems. Preliminary report. Dr. LUIS KRAMARZ, Emory University (763-65-7)
- 8:00- 8:10 (405) Variational procedures for a class of diffraction problems. Preliminary report. Professor R. C. MacCAMY, Carnegie-Mellon University and Dr. SAMUEL P. MARIN\*, General Motors Research Center, Warren, Michigan (763-65-8)
- 8:15- 8:25 (406) Convergence of the Ritz-Treffitz method for optimal control problems. Dr. F. H. MATHIS\*, Auburn University and Professor G. W. REDDIEN, Vanderbilt University (763-65-9)
- 8:30- 8:40 (407) Derivation of algebraic equations associated with Runge-Kutta formulas relative to systems of ordinary differential equations. Professor LEROY J. DERR\*, Professor CURTIS L. OUTLAW and Professor DIRAN SARAFYAN, University of New Orleans (763-65-10)
- 8:45- 8:55 (408) On the measurable solutions of a nonadditive functional equation. Professor LILLIAN T. SHENG\* and Professor PUSHPA N. RATHIE, State University of Campinas, Campinas, Brazil (763-94-2)
- 9:00- 9:10 (409) Gaussian channel codes from abelian groups. Professor CHARLES P. DOWNEY, University of Nebraska at Omaha (763-94-3)
- 9:15- 9:25 (410) Optimal  $(M, 3)$  group codes for the Gaussian channel. Professor JOHN KARLOF, University of Nebraska at Omaha (763-94-4)
- 9:30- 9:40 (411) Information and random automata. Dr. RICHARD L. THOMPSON, Bhaktivedanta Institute, Atlanta, Georgia (763-94-5)
- 9:45- 9:55 (412) On some symmetric recursive inset measures of randomized systems of events. PL. KANNAPPAN, University of Waterloo (763-94-6)

FRIDAY, 1:00 P. M.

Colloquium Lectures, Lecture III, Convention Center Coliseum

- (413) Complex analysis and algebraic geometry. Professor PHILLIP A. GRIFFITHS, Harvard University

FRIDAY, 2:15 P. M.

Invited Address, Convention Center Coliseum

- (414) Problems with different time scales and their numerical solution. Professor HEINZ-OTTO KREISS, California Institute of Technology (763-35-13)

FRIDAY, 2:15 P. M.

Special Session on Modular and Automorphic Functions in a Single Complex Variable. II, Convention Center, Room 1

- 2:15- 2:35 (415) Identical formulae for modular forms on the full modular group. Professor A.O.L. ATKIN, University of Illinois at Chicago Circle (763-10-13)
- 2:40- 3:00 (416) Bound for the norm of  $\theta_{\mathfrak{q}}$ . Preliminary report. Professor MARK SHEINGORN, University of Illinois, Urbana (763-30-2)
- 3:05- 3:25 (417) Meromorphic factors of automorphy. Preliminary report. Dr. DAVID A. JAMES, University of Michigan, Dearborn (763-10-4)

- 3:30- 3:50 (418) The limit set of a Fuchsian group and the absolute convergence of Poincaré series. Preliminary report. THOMAS A. METZGER, University of Pittsburgh, Pittsburgh (763-30-3)
- 3:55- 4:15 (419) The orbital counting function for a Fuchsian group of infinite area. Professor PETER J. NICHOLLS, Northern Illinois University, DeKalb (763-30-6)
- 4:20- 4:40 (420) On Fricke polygons, signatures, and canonical forms for infinitely generated Fuchsian groups. Professor NORMAN PURZITSKY, York University (763-30-1) (Introduced by Professor Marvin Knopp)
- 4:45- 5:05 (421) The Riemann-Siegel formula for automorphic L-functions. Preliminary report. Professor CARLOS J. MORENO, University of Illinois, Urbana (763-10-31)

FRIDAY, 2:15 P. M.

Special Session on Differential Geometry and General Relativity, II, Convention Center, Room 6

- 2:15- 2:35 (422) Supergravity for mathematicians. Preliminary report. Professor JERROLD E. MARSDEN, University of California, Berkeley (763-83-7)
- 2:40- 3:00 (423) Lagrange tensors and Ricci curvature. Dr. J.-H. ESCHENBURG, University of Münster, Federal Republic of Germany and Professor J. J. O'SULLIVAN\*, Pennsylvania State University, University Park (763-53-8)
- 3:05- 3:25 (424) Accelerating masses in Robertson-Walker space-times. Professor ABRAHAM H. TAUB, University of California, Berkeley (763-83-6)
- 3:30- 3:50 (425) Lorentzian distance and Lorentzian geometry. Professor JOHN K. BEEM\* and Professor PAUL E. EHRLICH, University of Missouri, Columbia (763-83-1)
- 3:55- 4:15 (426) Coupled gravitational and gauge fields. Dr. JUDITH M. ARMS, University of Utah (763-83-4)
- 4:20- 4:40 (427) Some problems involving indefinite Riemannian submanifolds. Preliminary report. MARTIN A. MAGID, Wellesley College (763-53-11)
- 4:45- 5:05 (428) Pfaffian derived types with no local invariants. Mr. ROBERT L. BRYANT, University of North Carolina, Chapel Hill (763-53-6)
- 5:10- 5:30 (429) On the local existence and deformations of submanifolds with constant mean curvature in a Riemannian manifold. Dr. DOMINIC S. P. LEUNG, U. S. Naval Academy (763-53-13)

FRIDAY, 2:15 P. M.

Special Session on Operator Theory (General), I, Convention Center, Room 8

- 2:15- 2:35 (430) Pathology in the Calkin algebra. Professor JOEL ANDERSON, Pennsylvania State University, University Park (763-47-8)
- 2:40- 3:00 (431) Lomonosov's theorem; extensions and limitations. Professor ERIC A. NORDGREN, University of New Hampshire (763-47-9)
- 3:05- 3:25 (432) C\*-algebras associated with free products of groups. Professor WILLIAM L. PASCHKE and Professor NORBERTO SALINAS\*, University of Kansas (763-47-5)
- 3:30- 3:50 (433) A fully explicit index theory for the C\*-algebra generated by two Toeplitz isometries. Professor CHARLES A. BERGER\* and Professor LEWIS A. COBURN, Yeshiva University, New York (763-47-14)
- 3:55- 4:15 (434) Closure and continuity, weak and strong. Professor PAUL R. HALMOS, Indiana University, Bloomington (763-47-7)

FRIDAY, 2:15 P. M.

Special Session on Ordinary Differential Equations; Oscillation and Asymptotic Behavior, I, Convention Center, Room 7

- 2:15- 2:35 (435) Properties of solutions of the differential equation  $xx'' - kx'^2 + f(x) = 0$ . Professor W. R. UTZ, University of Missouri, Columbia (763-34-11)
- 2:40- 3:00 (436) Subgroups of the reals and asymptotic stability for second-order nonlinear oscillations. Professor F. V. ATKINSON, University of Toronto (763-34-22)
- 3:05- 3:25 (437) Nonlinear oscillation of higher-order functional differential equations with deviating arguments. Professor TAKAŠI KUSANO, Hiroshima University, Hiroshima, Japan (763-34-24) (Introduced by Professor John R. Graef)

- 3:30- 3:50 (438) Nonoscillation and asymptotic properties of solutions of functional differential equations. Professor JOHN R. GRAEF and Professor PAUL W. SPIKES\*, Mississippi State University (763-34-27)
- 3:55- 4:15 (439) Nonoscillatory solutions of higher order delay equations. Preliminary report. Dr. KENT FOSTER, Emory University, and Dr. RONALD GRIMMER\*, Southern Illinois University, Carbondale (763-34-7)
- 4:20- 4:40 (440) Comparison theorems for equations with middle terms of order  $n-1$ . Professor ATHANASSIOS G. KARTSATOS\* and Professor JORGE GONZALES TORO, University of South Florida (763-34-12) (Introduced by Professor M. N. Manouagian)
- 4:45- 5:05 (441) Oscillation theory at a finite singularity. Professor DON HINTON\*, University of Tennessee, Knoxville and Professor ROGER LEWIS, University of Alabama, Birmingham (763-34-1)
- 5:10- 5:30 (442) Oscillation theorems for second order ordinary differential inequalities and equations with alternating arguments. Preliminary report. Dr. M. K. GRAMMATIKOPOULOS, University of Ioannina, Ioannina, Greece (763-34-26)

FRIDAY, 2:15 P. M.

Session on Algebraic Geometry, Convention Center, Room 2

- 2:15- 2:25 (443) Sums of squares on planar Nash rings. GUSTAVE A. EFROYMSON, University of New Mexico (763-14-1)
- 2:30- 2:40 (444) Conjugacy classes of split subgroups. PETER P. ANDRE, U. S. Naval Academy (763-14-2)
- 2:45- 2:55 (445) Formal meromorphic functions on rational varieties. Preliminary report. Professor ROBERT SPEISER, University of Minnesota, Minneapolis (763-14-3) (Introduced by Professor Joel Roberts)
- 3:00- 3:10 (446) Specialization to torsion of a rational point on an elliptic curve over  $\mathbb{Q}(t)$ . Mr. MICHAEL J. KENT, Rutgers University, New Brunswick, and Dr. CHARLES F. SCHWARTZ\*, Texas Tech University (763-14-4)
- 3:15- 3:25 (447) Singular points and Weierstrass points of algebraic plane curves. Mr. BEN THOMAS, University of North Carolina, Chapel Hill (763-14-5)
- 3:30- 3:40 (448) Plücker relations for  $p(e^{i\theta})$ . Professor J. R. QUINE, Florida State University (763-14-6)
- 3:45- 3:55 (449) The degree of the Prym map in dimension five. ROY C. SMITH, University of Georgia (763-14-7)
- 4:00- 4:10 (450) Self-duality of rank 2 reflexive modules. Mr. MATTHEW MILLER, University of Illinois, Urbana (763-13-1)
- 4:15- 4:25 (451) Adjacent extensions of rings. Preliminary report. Dr. LUCY I. DECHÉNE, Fitchburg State College (763-13-6)

FRIDAY, 2:15 P. M.

Session on Group Theory and Generalizations, I, Convention Center, Room 12

- 2:15- 2:25 (452) Green's relations on the semigroup  $\mathcal{S}_X$ . Preliminary report. Dr. DAVID I. ADU, University of Lagos, Akoka-Yaba, Nigeria (763-20-1)
- 2:30- 2:40 (453) Automorphism groups of some semigroups are Schützenberger groups of others. Preliminary report. Professor K. D. MAGILL, Jr., State University of New York at Buffalo (763-20-3)
- 2:45- 2:55 (454) Maximal groups in semigroups of binary relations. Dr. KAREN CHASE, University of Oklahoma, Norman (763-20-4) (Introduced by Carlton J. Maxson)
- 3:00- 3:10 (455) Permutation characters of finite groups of Lie type. Professor DAVID B. SUROWSKI, Kansas State University (763-20-5) (Introduced by John E. Maxfield)
- 3:15- 3:25 (456) Extensions of  $Z$  by  $Z(p^\infty)$ . Professor DEBORAH GALE, Montana State University (763-20-6)
- 3:30- 3:40 (457) On the automorphism group of a finite  $p$ -group with a small central quotient. Professor RICHARD M. DAVITT, University of Louisville (763-20-7)
- 3:45- 3:55 (458) Quadratic equations over free products of groups. Professor LEO P. COMERFORD, Jr. \*, University of Wisconsin-Parkside, and Professor CHARLES C. EDMUNDS, Mount Saint Vincent University, Halifax, Nova Scotia (763-20-8)

- 4:00- 4:10 (459) A characteristic class of characters of finite  $\pi$ -separable groups. Professor D. S. GAJENDRAGADKAR, University of Texas, El Paso (763-20-9)
- 4:15- 4:25 (460) F-orthodox semigroups. Dr. C. C. EDWARDS, Indiana University-Purdue University at Fort Wayne (763-20-10)
- 4:30- 4:40 (461) Invariants of finite groups and applications. Dr. N.J.A. SLOANE, Bell Laboratories, Murray Hill, New Jersey (763-20-11)
- 4:45- 4:55 (462) The nilpotency class of extensions of certain p-groups. Dr. LARRY J. MORLEY\*, Western Illinois University, and Dr. MANLEY PERKEL, Wright State University, Dayton (763-20-12)
- 5:00- 5:10 (463) Group retractions induced by a total order. Dr. WALTER S. SIZER, Southern Illinois University, Carbondale (763-20-14)
- 5:15- 5:25 (464) On abelian groups which are pure-injective, as modules over their endomorphism rings. Ms. BARBARA O'BRIEN, Texas Tech University (763-20-15) (Introduced by Dr. R. Barnard)
- 5:30- 5:40 (465) Fusion on p-elements in symmetric groups. Preliminary report. Mr. JOHN-TIEN HSIEH, University of Florida (763-20-16)
- 5:45- 5:55 (466) Abelian groups projective over their endomorphism rings. Preliminary report. Mr. G. P. NIEDZWECKI\* and Professor J. D. REID, Wesleyan University (763-20-17)

FRIDAY, 2:15 P. M.

Session on Functional Analysis, I, Convention Center, Room 4

- 2:15- 2:25 (467) Minimal topologies and  $L_p$ -spaces. Professor C. D. ALIPRANTIS\* and Professor O. BURKINSHAW, Indiana University-Purdue University at Indianapolis (763-46-14)
- 2:30- 2:40 (468) Superharmonic interpolation in subspaces of  $C_G(X)$ . L. ASIMOW, Syracuse University (763-46-15)
- 2:45- 2:55 (469) A weak convergence of vectors relative to a semifinite von Neumann algebra. Mr. VICTOR KAPTAL, Tel Aviv University, Israel (763-46-16)
- 3:00- 3:10 (470) Subalgebras of  $H^\infty$  and the corona property. Preliminary report. Mr. JOHN H. RILEY, Jr., University of Connecticut, Storrs (763-46-17)
- 3:15- 3:25 (471) Basic sequences in non-Schwartz Fréchet spaces. Professor STEVEN F. BELLENOT, Florida State University (763-46-18)
- 3:30- 3:40 (472) Trees and extreme points in Banach spaces. Preliminary report. Dr. AGGIE HO, Claremont Graduate School (763-46-19)
- 3:45- 3:55 (473) Some Banach spaces of distributions. Dr. WILLIAM TADD FRANKE, Emory University (763-46-20)
- 4:00- 4:10 (474) Smoothness and weak\* sequential compactness. JAMES HAGLER\* and FRANCIS SULLIVAN, Catholic University of America (763-46-21)
- 4:15- 4:25 (475) Nearly smooth norms for Banach spaces. Professor FRANCIS SULLIVAN, Catholic University of America (763-46-22)
- 4:30- 4:40 (476) Spaces which are nearly uniformly convex. Preliminary report. Dr. ROBERT E. HUFF, Pennsylvania State University, University Park (763-46-24)
- 4:45- 4:55 (477) Convolution equations in nonmetric and non-dual-metric domains. Preliminary report. Professor J. F. COLOMBEAU and Professor B. PERROT, Université de Bordeaux-I, France and Professor T.A.W. DWYER III\*, Northern Illinois University (763-46-25)
- 5:00- 5:10 (478) Generalized Carleman operators. Dr. ANTON R. SCHEP, California Institute of Technology (763-46-26)
- 5:15- 5:25 (479) A non-uniquely minimal Fourier projection. Dr. PETER MORRIS, Pennsylvania State University, University Park (763-46-27)

FRIDAY, 2:15 P. M.

Session on Algebraic Topology and Manifolds, Convention Center, Room 11

- 2:15- 2:25 (480) Realization of algebraic homomorphisms by continuous maps. Dr. JOHN McCLEARY, Bates College (763-55-1)

- 2:30- 2:40 (481) A non-embedding result for certain exotic spheres. ALAN D. UNELL, Northwestern University (763-55-2)
- 2:45- 2:55 (482) Stable properties of rank 1 loop spaces. Preliminary report. Dr. C. A. MCGIBBON, University of Pennsylvania (763-55-3)
- 3:00- 3:10 (483) On generators of the  $Z_5$  bordism rings. Dr. CHING-MU WU, University of West Florida and Tamkang College, Tamsui, Taiwan (763-55-4) (Introduced by James R. Weaver)
- 3:15- 3:25 (484) Fixed point index of weighted maps. Preliminary report. Dr. SAMUEL MASIH, Albany State College (763-55-5)
- 3:30- 3:40 (485) Classification problem for Lens spaces for one-relator groups with torsion. Dr. SUSHIL JAJODIA, University of Oklahoma (763-55-6)
- 3:45- 3:55 (486) Multi-valued mappings of closed and bounded subsets of a normed linear space—A mapping degree. Preliminary report. MICHAEL R. COLVIN, Louisiana State University, Baton Rouge (763-55-8)
- 4:00- 4:10 (487) The homotopy groups of knots, III. The k-invariant of knots. Preliminary report. Professor JOHN R. HARPER, University of Rochester and Professor S. J. LOMONACO, Jr. \*, State University of New York at Albany (763-57-1)
- 4:15- 4:25 (488) The genus of 3-manifolds obtained by surgery on cable links. Preliminary report. Professor BRADD CLARK, University of Southwestern Louisiana (763-57-2)
- 4:30- 4:40 (489) Some topology of cyclic multiple planes. Professor RICHARD RANDELL, University of Michigan, Ann Arbor (763-57-3)
- 4:45- 4:55 (490) Characterizations of ribbon n-knots. Professor L. RICHARD HITT, University of South Alabama (763-57-4)
- 5:00- 5:10 (491) Spanning arcs of patched disks. Dr. GARY RICHTER, Southwestern University (763-57-5)
- 5:15- 5:25 (492) Fibered knots and branched coverings of  $S^3$ . Preliminary report. Professor JOAN S. BIRMAN, Columbia University, Barnard College (763-57-6)
- 5:30- 5:40 (493) Uncomplicated group presentations for simply-connected three-manifolds. Preliminary report. E. H. ANDERSON, Mississippi State University (763-57-7)
- 5:45- 5:55 (494) On the Morse characteristic of fibre bundles. Preliminary report. Professor JOHN D. BLANTON, St. John Fisher College (763-57-8)
- 6:00- 6:10 (495) Resolutions of homology manifolds. FRANK QUINN, Virginia Polytechnic Institute and State University (763-57-9)
- 6:15- 6:25 (496) Convergent solutions of singular Pfaffian systems. Preliminary report. Professor WILLIAM J. FITZPATRICK, University of Southern California (763-58-1)

FRIDAY, 2:15 P. M.

Session on Probability Theory and Stochastic Processes, Convention Center, Room 5

- 2:15- 2:25 (497) Quadratic convergence and almost sure convergence of sums of independent random variables. KENNETH ALEXANDER, University of Washington and VICTOR GOODMAN\*, Indiana University and University of Wisconsin, Madison (763-60-1)
- 2:30- 2:40 (498) A theorem on the average roots of random algebraic polynomials. Preliminary report. Professor MARK J. CHRISTENSEN, Georgia Institute of Technology (763-60-2) (Introduced by Professor A. T. Bharucha-Reid)
- 2:45- 2:55 (499) Distribution of waiting times for two discrete-time queues in tandem. Preliminary report. Dr. JOHN A. MORRISON, Bell Laboratories, Murray Hill, New Jersey (763-60-4)
- 3:00- 3:10 (500) On the probability distribution of level crossings by a stochastic process. Preliminary report. Dr. FARAG A. ATTIA, Kuwait University, Kuwait (763-60-5)
- 3:15- 3:25 (501) On reaching a goal quickly in a finite state gambling problem. Professor THEODORE P. HILL, Georgia Institute of Technology (763-60-6)
- 3:30- 3:40 (502) A potential theory for geometrically ergodic infinite interacting particle systems. Dr. DEBRA D. QUALLS, University of Arkansas at Fayetteville (763-60-7)

- 3:45- 3:55 (503) On compact convex subsets of  $D[0,1]$ . P. Z. DAFFER, Louisiana Tech University (763-60-8)
- 4:00- 4:10 (504) Zero-range interaction at Bose-Einstein speeds under a positive recurrent single particle law. Dr. ED WAYMIRE, University of Mississippi (763-60-9)
- 4:15- 4:25 (505) Evaluations of absorption probabilities for Wiener process on large intervals. Professor CHULL PARK\* and Professor FREDERICK J. SCHUURMANN, Miami University, Oxford, Ohio (763-60-10)
- 4:30- 4:40 (506) An automorphism all of whose  $n$ -fold Cartesian products are loosely Bernoulli. Preliminary report. LAIF SWANSON, Texas A & M University (763-60-11)
- 4:45- 4:55 (507) A finite element method for random differential equations with random coefficients. T. C. SUN, Wayne State University (763-60-12)
- 5:00- 5:10 (508) Central limit theorem for  $\Phi^1$ -valued random variables. Professor HUI-HSIUNG KUO, Louisiana State University, Baton Rouge (763-60-14)
- 5:15- 5:25 (509) Infinitely divisible distributions similar to class  $L$  distributions. THOMAS A. O'CONNOR, University of Louisville (763-60-15)
- 5:30- 5:40 (510) A central limit theorem for exchangeable sequences in Banach spaces. Preliminary report. Dr. CHI-SHANG SOONG, Villanova University (763-60-16) (Introduced by Professor J. Galambos)
- 5:45- 5:55 (511) Sums of independent random variables conditioned to stay bounded. Professor GRANT A. RITTER, University of Florida (763-60-17)

FRIDAY, 3:30 P. M.

Invited Address, Convention Center Coliseum

- (512) Representations of classical groups. Professor BHAMA SRINIVASAN, Clark University (763-20-24)

FRIDAY, 3:30 P. M.

Session on Biology, Behavioral Sciences and Fluid Mechanics, Convention Center, Room 9

- 3:30- 3:40 (513) Control of oscillations in haematopoiesis. Professor N. D. KAZARINOFF\*, State University of New York at Buffalo and Professor PAULINE VAN DEN DRIESSCHE, University of Victoria (763-92-16)
- 3:45- 3:55 (514) Conditions for global stability concerning a prey-predator model with delay effects. Preliminary report. Professor ANTHONY LEUNG, University of Cincinnati (763-92-17)
- 4:00- 4:10 (515) A model for common source epidemics. Professor R. SHONKWILER\*, Georgia Institute of Technology and Professor MAYNARD THOMPSON, Indiana University at Bloomington (763-92-20)
- 4:15- 4:25 (516) The diffusion approximation to models of cultural evolution. Preliminary report. Dr. K.-H. CHEN, Stanford University Medical School (763-92-23)
- 4:30- 4:40 (517) An integral formulation of a reaction-diffusion equation. Preliminary report. JAMES A. PENNLINE, Virginia Commonwealth University (763-92-24)
- 4:45- 4:55 (518) Capillary-gravity waves against a vertical cliff. Dr. LOKENATH DEBNATH\*, East Carolina University and Dr. UMA BASU, University of Calcutta, India (763-76-2)

SATURDAY, 1:00 P. M.

Colloquium Lectures, Lecture IV, Convention Center Coliseum

- (519) Complex analysis and algebraic geometry. Professor PHILLIP A. GRIFFITHS, Harvard University

SATURDAY, 2:15 P. M.

Special Session on Operator Theory (Subsession on Subnormal Operators), II, Convention Center, Room 8

- 2:15- 2:35 (520) Principal functions and real valued Cauchy transforms. Preliminary report. RICHARD W. CAREY, University of Kentucky (763-47-36) (Introduced by Professor Douglas N. Clarke)

- 2:40- 3:00 (521) Banach algebras that are dual spaces. Preliminary report. SCOTT W. BROWN, University of California, Santa Barbara (763-47-1)
- 3:05- 3:25 (522) Algebras of subnormal operators. Mr. ROBERT OLIN\* and Mr. JAMES THOMSON, Virginia Polytechnic Institute and State University (763-47-3)
- 3:30- 3:50 (523) A result on quasi-similar subnormal operators. Preliminary report. Professor JOHN B. CONWAY, Indiana University, Bloomington (763-47-10)

SATURDAY, 2:15 P. M.

Special Session on Ordinary Differential Equations: Oscillation and Asymptotic Behavior. II, Convention Center, Room 7

- 2:15- 2:35 (524) Asymptotic behavior of Lotka-Volterra systems. Professor THOMAS G. HALLAM, University of Tennessee, Knoxville (763-92-18)
- 2:40- 3:00 (525) Stability theory for Volterra equations. Professor T. A. BURTON, Southern Illinois University, Carbondale (763-34-6)
- 3:05- 3:25 (526) Differential inequalities and comparison results in oscillation theory. Preliminary report. Professor V. A. STAIKOS, University of Ioannina, Ioannina, Greece (763-34-30)
- 3:30- 3:50 (527) Further results on the oscillation of functional differential equations. Preliminary report. Professor HIROSHI ONOSE, Ibaraki University, Mito, Japan (763-34-25)
- 3:55- 4:15 (528) Comparison theorems for delay differential equations. Professor W. E. MAHFOUD, Murray State University (763-34-29)
- 4:20- 4:40 (529) Oscillation properties of nth order linear differential equations. Professor GARY D. JONES, Murray State University (763-34-28)
- 4:45- 5:05 (530) Nonlinear relationships between oscillation and asymptotic behavior. Professor ALLAN EDELSON and Professor KURT KREITH\*, University of California, Davis (763-34-2)
- 5:10- 5:30 (531) Comparison theorems for second-order linear difference equations. Preliminary report. Professor WILLIAM T. PATULA, Southern Illinois University, Carbondale (763-39-1)

SATURDAY, 2:15 P. M.

Special Session on Nonacademic Mathematical Research, Convention Center, Room 11

- 2:15- 2:35 (532) System identification: Examples of regularizable ill-posed problems. Dr. DAVID A. LEE, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio (763-93-1) (Introduced by Dr. Robert J. Thompson)
- 2:40- 3:00 (533) Graph-theoretic path problems arising in facility safeguards studies. Preliminary report. Mr. BERNIE L. HULME, Sandia Laboratories, Albuquerque, New Mexico (763-90-2) (Introduced by Dr. Robert J. Thompson)
- 3:05- 3:25 (534) Boundary conditions for the dynamic solution of acoustic equations in an unbounded region. Dr. ALVIN BAYLISS\*, ICASE, Hampton, Virginia and Dr. ELI TURKEL, Courant Institute of Mathematical Sciences (763-65-2) (Introduced by Dr. Robert J. Thompson)
- 3:30- 3:50 (535) Self-adaptive Galerkin methods for one dimensional miscible and immiscible displacements. Preliminary report. Dr. RICHARD P. KENDALL\*, Exxon Production Research Co., Houston, Texas, Professor JIM DOUGLAS, Jr., University of Chicago, Professor MARY F. WHEELER, and Mr. BRUCE L. DARLOW, Rice University (763-76-1)
- 3:55- 4:15 (536) Asymptotic solutions to engineering design problems. Dr. ALEXANDER J. FEDEROWICZ, Westinghouse R & D Center, Pittsburgh (763-90-1) (Introduced by Dr. Robert J. Thompson)
- 4:20- 4:40 (537) Construction of balanced switching networks. Dr. FRANK K. HWANG and Dr. SHEN LIN\*, Bell Laboratories, Murray Hill, New Jersey (763-05-1)

SATURDAY, 2:15 P. M.

Session on Associative Rings and Algebras, Convention Center, Room 5

- 2:15- 2:25 (538) Finitely generated ideals in free group algebras. Dr. ROMAN W. WONG, Washington & Jefferson College (763-16-2)

- 2:30- 2:40 (539) Bialgebras with left antipodes. Preliminary report. Professor EARL J. TAFT, Institute for Advanced Study and Rutgers University, New Brunswick (763-16-3)
- 2:45- 2:55 (540) Hopf algebras and tensor products of semisimple representations. Preliminary report. Dr. RICHARD K. MOLNAR, Oakland University (763-16-4)
- 3:00- 3:10 (541) On a splitting theorem of Azumaya algebras. Preliminary report. Professor GEORGE SZETO, Bradley University (763-16-5)
- 3:15- 3:25 (542) Simple near-ring centralizers of finite rings. Professor KIRBY C. SMITH\* and Professor CARLTON J. MAXSON, Texas A & M University (763-16-6)
- 3:30- 3:40 (543) N-dimension and N-critical modules. Preliminary report. LLOYD CHAMBLESS, University of North Carolina, Chapel Hill (763-16-7)
- 3:45- 3:55 (544) Reduction of matrices over von Neumann regular rings. Preliminary report. THOMAS R. SAVAGE, St. Olaf College (763-16-8)
- 4:00- 4:10 (545) A nonabelian 2-group whose endomorphisms generate a ring, and other examples of E-groups. Professor J. J. MALONE, Worcester Polytechnic Institute (763-16-9)
- 4:15- 4:25 (546) H-monogenic near-rings. Preliminary report. Dr. HORACE OLIVIER and Professor HENRY HEATHERLY\*, University of Southwestern Louisiana (763-16-10)
- 4:30- 4:40 (547) The Kostant structure theorems for  $K/k$  Hopf. WARREN NICHOLS, Florida State University (763-16-11)
- 4:45- 4:55 (548) Derivations and commutativity of rings. L. O. CHUNG, JIANG LUH and A. RICHOUX\*, North Carolina State University (763-16-12)
- 5:00- 5:10 (549) Reduced and densely nil right ideals. Preliminary report. Dr. GARY F. BIRKENMEIER, Southeast Missouri State University (763-16-13)
- 5:15- 5:25 (550)  $f$ -projectivity and flat epimorphisms. Dr. MARSHA F. JONES, Claremont Graduate School (763-16-14)

SATURDAY, 2:15 P. M.

Session on Group Theory and Generalizations, II, Convention Center, Room 12

- 2:15- 2:25 (551) Some indecomposable modules of groups with split  $(B, N)$ -pairs. Preliminary report. Dr. NALSEY B. TINBERG, Southern Illinois University, Carbondale (763-20-13)
- 2:30- 2:40 (552) A generalization of a theorem of P. Hall on nilpotent products. Preliminary report. Professor ANTHONY M. GAGLIONE, U. S. Naval Academy (763-20-18)
- 2:45- 2:55 (553)  $S$ -groups. Dr. ROBERT O. STANTON, St. John's University (763-20-19)
- 3:00- 3:10 (554) Topologies induced in free semigroups. Preliminary report. Professor J. HIGGINS, Brigham Young University (763-20-20)
- 3:15- 3:25 (555) On groups of Ree type. Dr. MARK R. HOPKINS, Drake University (763-20-21)
- 3:30- 3:40 (556) On the elementary theory of a certain class of groups. Preliminary report. KENNETH W. WESTON, University of Wisconsin-Parkside (763-20-22)
- 3:45- 3:55 (557) Torsion and tensor products for valuated groups. Professor TEMPLE H. FAY, New Mexico State University, Las Cruces (763-20-23)
- 4:00- 4:10 (558) Real characters and the Schur multiplier. Preliminary report. Dr. SIDNEY C. GARRISON and Dr. STEPHEN M. GAGOLA, Jr. \*, Texas A & M University (763-20-25) (Introduced by Kirby Smith)
- 4:15- 4:25 (559) Small subgroups. Mr. STEPHEN C. KING, California State University, Los Angeles (763-20-26)
- 4:30- 4:40 (560) Global Warfield groups. ROGER H. HUNTER\* and FRED RICHMAN, New Mexico State University, Las Cruces (763-20-27)

SATURDAY, 2:15 P. M.

Session on Sequences, Series and Approximations, Convention Center, Room 2

- 2:15- 2:25 (561) Summability of matrix transforms of subsequences. Dr. THOMAS A. KEAGY, Texas Eastern University (763-40-9)
- 2:30- 2:40 (562) On Euler- and Sonneschein-matrices. Professor KARL-HEINZ INDLEKOFER, University of Paderborn, Federal Republic of Germany (763-40-11)



- 2:45- 2:55 (563) Nonnegative  $\lambda$ - $\lambda$  Nörlund means. Preliminary report. Mr. JAMES DEFranza, Kent State University, Kent (763-40-12)
- 3:00- 3:10 (564) Some examples of integer approximation. Dr. CECELIA H. CHU, Department of Energy, Washington, D. C. (763-41-2)
- 3:15- 3:25 (565) Probabilistic interpretation of some positive linear operators. Mr. J. P. KING, Lehigh University (763-41-3)
- 3:30- 3:40 (566) On best two-point local approximation. Preliminary report. Professor CHARLES K. CHUI and Mr. LO-YUNG SU\*, Texas A & M University (763-41-4)
- 3:45- 3:55 (567) Norm and zero asymptotics for extremal polynomials. Dr. LYNN R. ZIEGLER, Ohio State University, Columbus (763-41-5)
- 4:00- 4:10 (568) Inequalities for polynomials with a prescribed zero. Mr. M. LACHANCE\* and Professor E. B. SAFF, University of South Florida and Professor R. S. VARGA, Kent State University, Kent (763-41-6)
- 4:15- 4:25 (569) Approximations of the second derivative. Preliminary report. Dr. J. M. ASH\* and Dr. R. L. JONES, DePaul University (763-41-7)
- 4:30- 4:40 (570) Multipliers for Fourier series and Jacobi series. Preliminary report. Professor GEORGE GASPER\*, Northwestern University and Professor WALTER TREBELS, Technische Hochschule Darmstadt, Federal Republic of Germany (763-42-2)

SATURDAY, 2:15 P. M.

Session on Functional Analysis. II, Convention Center, Room 4

- 2:15- 2:25 (571) Approximate identities and pointwise convergence. Professor H. S. BEAR, University of Hawaii, Honolulu (763-46-1)
- 2:30- 2:40 (572) A characterization of  $M$ -ideals in  $B(\ell_p)$  for  $1 < p < \infty$ . Preliminary report. PATRICK FLINN, Ohio State University, Columbus (763-46-3)
- 2:45- 2:55 (573) Sectional representation of Banach modules. Professor J. W. KITCHEN\*, Duke University and Professor D. A. ROBBINS, Trinity College, Hartford (763-46-5)
- 3:00 3:10 (574) Compact and weakly compact derivations of  $C^*$ -algebras. Preliminary report. CHARLES A. AKEMANN, University of California, Santa Barbara and STEVE WRIGHT\*, Oakland University (763-46-6)
- 3:15- 3:25 (575) On James' quasi-reflexive Banach space as a Banach algebra. Professor ALFRED D. ANDREW\* and Professor WILLIAM L. GREEN, Georgia Institute of Technology (763-46-7)
- 3:30- 3:40 (576) Essential subspaces. Preliminary report. Professor R. S. DORAN, Texas Christian University (763-46-8)
- 3:45- 3:55 (577) Basically scattered measures. Dr. N. J. KALTON, University of Missouri, Columbia, Dr. B. TURETT\*, Oakland University and Dr. J. J. UHL, Jr., University of Illinois, Urbana (763-46-9)
- 4:00- 4:10 (578) Uniform strong additivity as a lattice property. Professor RUSSELL G. BILYEU and Professor PAUL W. LEWIS\*, North Texas State University (763-46-10)
- 4:15- 4:25 (579) The Banach-Saks property in Lebesgue-Bochner function spaces. C. J. SEIFERT, University of Kansas (763-46-11)
- 4:30- 4:40 (580) Type and cotype in Lorentz  $L_{pq}$  spaces. Preliminary report. JOSEPH CREEKMORE, Kent State University (763-46-12) (Introduced by Professor Joseph Diestel)
- 4:45- 4:55 (581) Subspaces of some nuclear sequence spaces. Dr. M. ALPSEYMEN and Professor T. TERZIOGLU, METU, Ankara, Turkey and Professor M. S. RAMANUJAN\*, University of Michigan, Ann Arbor (763-46-13)

SATURDAY, 2:15 P. M.

Session on Mathematical Education, Convention Center, Room 6

- 2:15- 2:25 (582) The old math, the new math, and the aftermath. Professor ROBERT P. HOSTETTLER, The Behrend College of the Pennsylvania State University, Erie (763-97-1) (Introduced by Professor Roland E. Larson)

2:30- 2:40 (583) A program of undergraduate mathematical research. Professor ROBERT C. ESLINGER, Hendrix College (763-98-1)

2:45- 2:55 (584) Considerations in beginning a mathematics assistance center. Preliminary report. Professor JANICE T. ASTIN\* and Professor JULIA P. KENNEDY, Georgia State University (763-98-2)

New Orleans, Louisiana

Frank T. Birtel  
Associate Secretary

## Organizers and Topics of Special Sessions

Names of the organizers of special sessions to be held at meetings of the Society are listed below, along with the topic of the session. Papers will be considered for inclusion in special sessions, if their abstracts are submitted to the Providence office by the deadlines given below. These deadlines are three weeks earlier than those for abstracts for regular sessions of ten-minute contributed papers. The most recent abstract form has a space for indicating that the abstract is for a special session. If you do not have a copy of this form, be sure your abstract is clearly marked "For consideration for special session (title of special session)." Papers not selected for special sessions will automatically be considered for regular sessions unless the author gives specific instructions to the contrary.

<u>764th Meeting</u>	<u>Honolulu, Hawaii, March 1979</u>	<u>Deadline: January 9</u>
Christopher J. Allday, Hugh M. Hilden, and Bob Little	Geometric topology	
Ronald P. Brown and Thomas C. Craven	Quadratic forms	
William P. Hanf and Dale W. Myers	Countable models	
Nobuo Nobusawa and Arthur A. Sagle	Nonassociative algebras and applications	
L. Thomas Ramsey and Benjamin B. Wells, Jr.	Commutative harmonic analysis	

<u>765th Meeting</u>	<u>New York, New York, April 1979</u>	<u>Deadline: February 6</u>
Harold M. Hastings	Homotopy theory	
Heisuke Hironaka and George R. Kempf	Algebraic geometry	
Gangaram S. Ladde	Mathematical modeling	
Gerard J. Lallement	Algebraic and topological semigroups	
Harry Kesten	Probability theory	
Louis F. McAuley	Monotone and open mappings	

<u>766th Meeting</u>	<u>Iowa City, Iowa, April 1979</u>	<u>Deadline: February 6</u>
Daniel D. Anderson	Commutative ring theory	
Kent R. Fuller	Noncommutative ring theory	
William H. Jaco	Three-dimensional manifold theory	
James P. Kuelbs and Walter V. Philipp	Probability on Banach spaces	
Richard P. McGehee	Dynamical systems	
Paul S. Muhly	Operator theory	
John C. Polking	Several complex variables	

<u>767th Meeting</u>	<u>Vancouver, Canada, June 1979</u>	<u>Deadline: April 3</u>
Priscilla E. Greenwood	Probability	
Stanley S. Page	Representations and Ring Theory	
Lon M. Rosen	Mathematical physics	

## Invited Speakers at AMS Meetings

The individuals listed below have accepted invitations to address the Society at the times and places listed. For some meetings, the lists of speakers are incomplete.

<u>Honolulu, Hawaii, March 1979</u>		<u>Iowa City, Iowa, April 1979</u>	
Henry A. Dye	William A. Harris, Jr.	Charles G. Conley	Judith D. Sally
		Wolfgang R. G. Haken	B. A. Taylor
<u>New York, New York, April 1979</u>		<u>Kent, Ohio, November 1979</u>	
Richard W. Dudley	H. Hironaka	Kyung W. Kwun	Albert Marden
Eugene B. Dynkin	George R. Kempf	Daniel R. Lewis	Paul H. Rabinowitz
Richard F. Gundy			

# Announcement of the 764th Meeting

*University of Hawaii, Honolulu, March 30–April 1, 1979*

The seven hundred sixty-fourth meeting of the American Mathematical Society will be held at the University of Hawaii at Manoa in Honolulu, Hawaii, from Friday, March 30, through Sunday, April 1, 1979.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited one-hour addresses. HENRY A. DYE of the University of California, Los Angeles, will lecture at 11:00 a. m. on Friday; the title of his address will be given in the February issue of the NOTICES. WILLIAM A. HARRIS, Jr., of the University of Southern California will speak at 11:00 a. m. on Saturday. The title of his talk is "Laplace integrals and factorial series in singular differential and difference equations." Both lectures will be in the Kuykendall Auditorium.

By invitation of the same committee, there will be five special sessions of invited twenty-minute papers. All of the organizers are at the University of Hawaii at Manoa except for Arthur A. Sagle, who is at the University of Hawaii at Hilo. CHRISTOPHER J. ALLDAY, HUGH M. HILDEN, and BOB LITTLE are organizing a special session on Geometric topology; the list of speakers includes Neal E. Brand, Ronald A. Fin-tushel, Cameron Gordon, S. Halperin, Dennis Johnson, Kenneth C. Millett, Robert Oliver, Peter Sie Pao, Nobuyuki Albert Sato, Martin G. Scharleman, John R. Stallings, and Ronald J. Stern. RONALD P. BROWN and THOMAS C. CRAVEN are organizing a special session on Quadratic forms; the tentative list of speakers includes Lawrence Berman, Craig M. Cordes, Andrew G. Earnest, Richard S. Elman, J. S. Hsia, Donald G. James, Jerrold L. Kleinstein, Tsit-Yuen Lam, Murray Marshall, Bernard R. McDonald, Takashi Ono, Paul Ponomarev, Alex Rosenberg, Daniel B. Shapiro, Olga Taussky-Todd, Adrian R. Wadsworth, and Roger P. Ware. WILLIAM P. HANF and DALE W. MYERS are arranging a special session on Countable models; the list of speakers will include Miroslav Benda, Nigel Cutland, Harvey M. Friedman, Edgar G. K. Lopez-Escobar, Michael Makkai, Arnold W. Miller, Anil Nerode, Kenneth Schilling, John S. Schlipf, and Robert L. Vaught. NOBUO NOBUSAWA and ARTHUR A. SAGLE are organizing a special session on Non-associative algebras and applications and the tentative list of speakers is Georgia M. Benkart, Morton L. Curtis, Stephen Joseph Doro, Erwin Kleinfeld, J. Marshall Osborn, Earl J. Taft, Gregory P. Wene, and David J. Winter. L. THOMAS RAMSEY and BENJAMIN B. WELLS, Jr., are arranging a special session on Commutative harmonic analysis; the list of speakers includes Aharon Atzmon, John J. F. Fournier, Colin C. Graham, Henry Helson, Edwin Hewitt, Louis Pigno, Daniel G. Rider, and Alberto Torchinsky.

There will be sessions of contributed ten-minute papers. Abstracts should be sent to the

American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, so as to arrive by the deadline of January 30, 1979. Late papers will be accepted for presentation at the meeting, but will not be listed in the printed program.

## SYMPOSIUM ON THE GEOMETRY OF THE LAPLACE OPERATOR

With the anticipated support of the National Science Foundation, a symposium on the Geometry of the Laplace Operator is scheduled to take place Tuesday through Friday, March 27–30. This topic was selected by the Committee to Select Hour Speakers for Far Western Sectional Meetings, whose members are Paul C. Fife (chairman), David M. Goldschmidt, Robert Osserman, Rihhak Ree, and Kenneth A. Ross.

The purpose of the symposium is to bring together people working in several related areas of geometry and analysis. The common thread is the Laplace operator, whose eigenvalues and eigenfunctions have been found in recent years to contain an unsuspected wealth of information on the geometry and topology of the underlying domain. There are also links to physics through potential theory, the basic equations of wave motion and heat flow or diffusion, and quantum mechanics. These links have proven fruitful in both directions.

It is planned to have eight half-day sessions devoted to various subtopics, including the following: The spectrum of the Laplacian (distribution of eigenvalues, relations to periodic geodesics and global geometry); Riemann surfaces (Selberg trace formula, dependence of spectrum on genus, conformal type, etc.); bounds on eigenvalues (influences of curvature, topology, etc.); nonlinear problems (harmonic mappings, prescribed Gauss or scalar curvatures); group theory (the Laplacian on Lie groups and homogeneous spaces); and applications (physics, chemistry).

The Organizing Committee for the symposium includes David Bleecker, University of Hawaii, Honolulu, and Robert Osserman, Stanford University (co-chairmen); Victor Guillemin, Massachusetts Institute of Technology; Henry P. McKean, Jr., Courant Institute of Mathematical Sciences; Karen Uhlenbeck, University of Illinois at Chicago Circle; and Joel Weiner, University of Hawaii, Honolulu.

Some of the speakers, and the titles of their lectures, are: M. V. BERRY, H. H. Wills Physics Laboratory, Bristol, England, "Morphology of wave functions in integrable and non-integrable systems"; PETER BUSER, University of Bonn, Federal Republic of Germany, "On Cheeger's inequality  $\lambda_1 \geq h^2/4$ "; ISAAC CHAVEL, CUNY, City College, "Lowest-eigenvalue inequalities"; JEFF CHEEGER, SUNY, Center at Stony Brook, "The Laplacian on spaces with singularities"; S.-Y. CHENG, Princeton University, "Some remarks on harmonic maps";

HAROLD DONNELLY, University of California, Berkeley, "Expansions associated to clean intersections"; DENNIS HEJHAL, University of Minnesota, "Some Dirichlet series whose poles lie along critical lines" (tentative); DAVID KAZHDAN, Harvard University, "Spectrum of Laplace operators on manifolds of the negative curvature"; JOHAN KOLK, University of California, Los Angeles, "Spectra of compact locally symmetric spaces" (tentative); H. BLAINE LAWSON, Jr., University of California, Berkeley and SUNY, Center at Stony Brook, "Connections with harmonic curvature" (tentative); ELLIOTT H. LIEB, Princeton University and Kyoto University, Japan, "Bounds on the number of eigenvalues of  $-\Delta + V(x)$ "; RICHARD MELROSE, Massachusetts Institute of Technology, "The Laplacian on a geodesically convex manifold with boundary"; RICHARD SCHOEN, New York University, Courant Institute of Mathematical Sciences, "Geometric bounds on the low eigenvalues" (tentative); I. M. SINGER, University of California, Berkeley, "Determinants in quantum field theory"; EUGENE B. TRUBOWITZ, New York University, Courant Institute of Mathematical Sciences, "Action angle variables for ordinary differential operators"; ALAN WEINSTEIN, University of California, Berkeley, and Rice University, "Nonlinear stabilization of quasi-modes" (tentative); HAROLD WIDOM, University of California, Santa Cruz, "Asymptotics of compressions to spectral subspaces of the Laplacian"; and S.-T. YAU, Stanford University, "On the estimate of the first eigenvalue of a compact Riemannian manifold". Other lecturers, the titles of whose talks are not yet available, are: PIERRE H. BERARD, CNRS, Paris, France; JAMES EELS, Mathematics Institute, Coventry, England; JERRY KAZDAN, University of Pennsylvania; and FRANK WARNER, University of Pennsylvania.

#### REGISTRATION

The registration desk will be located in Kuykendall Hall, and will be open from 8:30 a. m. until noon, and 1:00 p. m. to 4:30 p. m. on Tuesday, Wednesday, and Thursday; on Friday and Saturday the desk will be open from 8:00 a. m. to noon, and from 1:00 p. m. to 4:00 p. m.; on Sunday the hours will be from 9:00 a. m. until noon. Tourist information will be available at the registration desk.

Eugene, Oregon

The registration fees for the meeting and symposium are:

	<u>Meeting Only</u>	<u>Symposium Only</u>	<u>Meeting and Symposium</u>
Nonmember	\$5	\$3	\$8
Member	3	2	5
Student/Unemployed	1	1	2

#### TRAVEL AND ACCOMMODATIONS

The meeting coincides with a very busy tourist season in Hawaii, so participants should make their plans as soon as possible! Some flights are already booked; it is imperative that plans be made NOW. Reservations for air travel and hotel accommodations can be made through Hawaii Conference Planners, 2222 Kalakaua Avenue, P. O. Box 8519, Honolulu, Hawaii 96815. They will provide a choice of hotels, group air fare rates, and optional tours and activities. The deadline for receipt of hotel reservations is January 26, 1979. Participants communicating with HCP should mention that they are attending the mathematics meeting. For your convenience an information request form can be found on page A-690 of the October issue of the NOTICES. You may prefer to consult your own travel agent for assistance, or to determine whether any special air fares or travel packages would apply from your point of origin at that time.

#### FOOD SERVICE

The East-West Center Cafeteria in Jefferson Hall and the Campus Center Cafeteria will both be open. In addition, several restaurants and fast food establishments on University Avenue are within easy walking distance of campus.

#### LOCAL TRANSPORTATION AND PARKING

The bus system in Honolulu is quite good. To go from Waikiki to the University of Hawaii, take bus #4 Nuuanu-Dowsett. For the reverse trip, take bus #4 University-Waikiki. Buses run every fifteen minutes and require 25¢ in exact change.

Persons driving to the campus may take the Lunalilo Freeway to University Avenue, proceed north to Dole Street (just prior to Founder's Gate) and turn right onto Dole Street. Free parking will be available in the parking structure south of Dole Street. Parking on the upper campus will cost 50¢ per hour.

Kenneth A. Ross  
Associate Secretary

# First Announcement of the 765th Meeting

*Biltmore Hotel, New York City, April 19–20, 1979*

The seven hundred sixty-fifth meeting of the American Mathematical Society will be held at the Biltmore Hotel, Madison Avenue at 43rd Street, New York City, on Thursday and Friday, April 19 and 20, 1979.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be five invited one-hour addresses and six special sessions of twenty-minute papers. Three of the invited addresses and a special session will be devoted to the theme "Probability theory and applications to contiguous branches of mathematics." These invited addresses will be presented by RICHARD M. DUDLEY, Massachusetts Institute of Technology; EUGENE B. DYNKIN, Cornell University; and RICHARD F. GUNDY, Rutgers University. HARRY KESTEN of Cornell University, and DONALD DAWSON of Carleton University, are organizing a related special session on Probability theory. The second theme is "Algebraic geometry," for which the invited speakers will be HEISUKE HIRONAKA of Harvard University, and GEORGE R. KEMPF of Johns Hopkins University. They are also organizing a related special session on Algebraic geometry. Titles of the lectures will be announced in a later issue of the NOTICES.

Four additional special sessions are being organized. The titles of these special sessions and the names of the organizers are: Homotopy theory, HAROLD M. HASTINGS, University of Georgia; Mathematical modeling, GANGARAM S. LADDE, SUNY, College at Potsdam; Algebraic and topological semigroups, GERARD J. LALLEMENT, Pennsylvania State University; and Monotone and open mappings, LOUIS F. McAULEY, SUNY, Center at Binghamton.

Anyone submitting an abstract for the meeting who feels that his or her paper would be particularly suitable for one of these special sessions should indicate this clearly on the abstract, and mail it so as to reach the office of the American Mathematical Society in Providence by February 6, 1979, three weeks before the normal deadline for abstracts of contributed papers.

There will also be sessions for contributed ten-minute papers. The abstract deadline is February 27, 1979. If necessary, late papers will be accepted for presentation at the meeting, but it will not be possible to list these in the printed program.

The Council of the Society will meet on Wednesday evening, April 18, at the Biltmore Hotel.

University Park, Pennsylvania

## REGISTRATION

The registration desk will be located in the Vanderbilt Suite on the first floor of the Biltmore Hotel, and will be open from 8:00 a. m. to 4:30 p. m. on Thursday, and from 8:00 a. m. to 3:00 p. m. on Friday. The registration fees will be \$5 for nonmembers, \$3 for members, and \$1 for students or unemployed mathematicians.

## ACCOMMODATIONS

A block of rooms has been set aside at the Biltmore Hotel for use by participants attending the meeting. Persons planning to stay at the Biltmore should make their own reservations with the hotel and should be sure to mention the mathematics meeting, in order to obtain the special rates. For your convenience, a reservation form and list of the room rates will be found on page A-190 of these NOTICES. The deadline for receipt of reservations is April 4, 1979.

## TRAVEL

The Biltmore Hotel is located on Madison Avenue at 43rd Street, on the east side of New York City. Walkways from Grand Central Station connect with the hotel, and signs are posted directing persons to the hotel lobby.

Those arriving by bus at the Port Authority Bus Terminal may take the Independent Subway System, a taxi, or bus to the hotel. There is shuttle bus service directly to Grand Central Station from LaGuardia, Kennedy, and Newark Airports, and starters will direct passengers to the correct bus.

Persons arriving by car will find several parking garages in the area, in addition to the garage at the Biltmore Hotel. Parking service can be arranged through the hotel doorman. The present rate for parking in the hotel garage is \$12.50 for each 24-hour period, and there is an additional charge for extra pickup and delivery service if it is required. The parking fee is subject to New York City taxes.

## MAIL ADDRESS

Registrants at the meeting may receive mail addressed to them in care of the American Mathematical Society Meeting, Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York 10017.

Raymond G. Ayoub  
Associate Secretary

# First Announcement of the 766th Meeting

*University of Iowa, Iowa City, April 27–28, 1979*

The seven hundred sixty-sixth meeting of the American Mathematical Society will be held on Friday and Saturday, April 27–28, 1979 at the University of Iowa, Iowa City, Iowa. Most of the sessions of the meeting will be held in the Iowa Memorial Union, which will also be the hotel headquarters for the meeting.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be four invited one-hour addresses. The speakers will be: CHARLES C. CONLEY, University of Wisconsin, at 11:00 a.m. on Friday; WOLFGANG R. G. HAKEN, University of Illinois, at 1:45 p.m. Friday; JUDITH D. SALLY, Northwestern University, at 11:00 a.m. on Saturday; and B. A. TAYLOR, University of Michigan, at 1:45 p.m. on Saturday. All four addresses will be presented in the Illinois Room of the Iowa Memorial Union.

By invitation of the same committee, there will be seven special sessions of selected twenty-minute papers. The organizers and topics of these special sessions are: DANIEL D. ANDERSON, University of Iowa, Commutative ring theory; KENT R. FULLER, University of Iowa, Noncommutative ring theory; WILLIAM H. JACO, Rice University, Three-dimensional

Urbana, Illinois

manifold theory; JAMES P. KUELBS, University of Wisconsin, and WALTER V. PHILIPP, University of Illinois, Probability on Banach spaces; RICHARD P. McGEHEE, University of Minnesota, Dynamical systems; PAUL S. MUHLY, University of Iowa, Operator theory; and JOHN C. POLKING, Rice University, Several complex variables. Most of the talks to be presented in these special sessions will be by invitation. However, anyone submitting an abstract for the meeting who feels that his or her abstract would be particularly suitable for one of these special sessions, should indicate this clearly on the abstract, and mail it in time to reach the office of the American Mathematical Society in Providence by February 6, 1979, three weeks before the normal deadline for abstracts of contributed papers.

There will also be sessions for contributed ten-minute papers as needed. The abstract deadline is February 27, 1979.

Iowa City is within 300 miles driving distance from such cities as Chicago, Kansas City, Milwaukee, Minneapolis, Omaha, and St. Louis. Further information about travel and accommodations will appear in the February issue of these NOTICES.

Paul T. Bateman  
Associate Secretary

## 1979 SUMMER SEMINAR/WORKSHOP

### Algebraic and Geometric Methods in Linear Systems Theory

*Harvard University, Cambridge, Massachusetts, June 19–30, 1979*

The Society will sponsor a seminar/workshop on Algebraic and Geometric Methods in Linear Systems Theory to be held June 19–30, 1979 at Harvard University, Cambridge, Massachusetts. The cochairmen of the conference are Roger Brockett, Harvard University; C. I. Byrnes, Harvard University; and Clyde Martin, Case Western Reserve University. The conference will bring together researchers in control and its advanced developments, and will be held with the financial support of Ames Research Center of the National Aeronautics and Space Administration, and the NATO Advanced Study Institute Programme.

About three-quarters of the time will be devoted to a series of high level pedagogical lectures on the mathematics of linear systems theory, intended as an introduction to some of the advanced techniques in linear systems. The tentative list of speakers includes H. Rosenbrock who will lecture on developments in transfer function techniques; M. Wonham who will give a

series of lectures on "System design via controllability subspace techniques"; Y. Rouchaleau and C. I. Byrnes will share responsibility for a series of lectures on "Systems over rings"; Professor Byrnes will also share responsibility with M. Hazewinkel and C. Martin for a set of lectures on "Linear systems and algebraic geometry"; P. Fuhrmann will give a set of lectures on infinite dimensional systems; and J. C. Willems will expand the scope of the lectures by discussing linear stochastic systems. The remaining time will be devoted to presentation of research results and needs by other participants.

Funds for participant support will be limited, and it is hoped that a number of participants who wish to attend will obtain their own sources of support. Those wishing to take part in the seminar/workshop and/or be considered for financial assistance should write to Clyde Martin, Department of Mathematics, Case Western Reserve University before March 1, 1979. Those who wish financial support should write as soon as possible.

# 1979 SUMMER RESEARCH INSTITUTE

## Finite Group Theory

University of California, Santa Cruz, June 25–July 20, 1979

The twenty-seventh Summer Research Institute sponsored by the American Mathematical Society will be devoted to finite group theory, and will take place at the University of California, Santa Cruz, for a period of four weeks from June 25–July 20, 1979. The Organizing Committee consists of Jonathan L. Alperin, Michael Aschbacher, Nicholas Burgoyne, Bruce Cooperstein, Daniel Gorenstein (chairman), and Geoffrey Mason. It is anticipated that the institute will be supported by a grant from the National Science Foundation.

The tremendous progress in finite group theory in recent years promises to change the entire field. After more than a quarter of a century of intensive effort by several hundred people, the classification of all finite simple groups appears to be rapidly nearing conclusion. One can safely predict that by 1980 the general theory of simple groups will have been fully developed and that if the classification is not complete by then, all that will remain to analyze will be a very few specialized problems. More and more of the practitioners are beginning to look beyond simple groups and to ask what the significant questions concerning finite groups are going to be in the "post simple group" era.

In view of these developments, the time is perfect to hold a large scale conference on finite group theory, having as its subject two main themes:

- A. The classification of finite simple groups
- B. Future directions in finite group theory

Housing accommodations for those attending the institute will be provided in a complex of residence houses on the campus, and meals will be served in a nearby dining hall. In the early spring a brochure of information will be available, which will contain details about the program, room and board rates, physical facilities, registration fees, and a reservation form for accommodations.

Funds for participant support will be limited, and it is hoped that a number of participants who wish to attend will obtain their own support. Those wishing to take part in the institute and/or be considered for financial assistance should write to Dr. William J. LeVeque, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, before April 16, 1979. The committee will then consider these requests, and applicants will be informed of its decision shortly thereafter.

### LECTURES IN APPLIED MATHEMATICS

#### *MODERN MODELING OF CONTINUUM PHENOMENA*

edited by Richard C. DiPrima

The articles contained in this volume follow the pattern of the lectures presented at the Ninth Summer Seminar on Applied Mathematics, sponsored jointly by the American Mathematical Society and the Society for Industrial and Applied Mathematics, held at Rensselaer Polytechnic Institute from July 7 to July 18, 1975. The articles are more detailed and include generalizations that could not be presented during the lectures.

The purposes of the seminar and therefore of this volume are (i) to introduce the participants to selected mathematical research areas of high current interest and relevance, (ii) to present the

underlying fundamental laws of continuum model building, and (iii) to present selected mathematical topics particularly useful in solving modern mathematical problems of continuum phenomena.

The table of contents is: *An introduction to continuum theory* by Lee A. Segel; *Perturbation theory* by Donald S. Cohen; *Introduction to the asymptotic analysis of stochastic equations* by George C. Papanicolaou; *Lectures in population dynamics* by G. Oster; *Amoeboid motions* by Garrett M. Odell; and *Earthquake sources* by Leon Knopoff and John O. Mouton.

Volume 16

251 pages

List price \$30.80; member price \$23.10

ISBN 0-8218-1116-9; LC 77-9041

Publication date: October 31, 1977

To order, please specify LAM/16

NEW INDIVIDUAL  
MEMBER PRICE \$15.40  
PRICE \$23.10

Prepayment is required for all American Mathematical Society publications.

Send for the book(s) above to: AMS, P. O. Box 1571, Annex Station, Providence, RI 02901

# QUERIES

Edited by Hans Samelson

QUESTIONS WELCOMED from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning published or unpublished conjectures.

REPLIES from readers will be edited, when appropriate, into a composite answer and published in a subsequent column. All answers received will ultimately be forwarded to the questioner.

QUERIES AND RESPONSES should be typewritten if at all possible and sent to Professor Hans Samelson, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940.

## QUERIES

**165. Burnett Meyer** (Department of Mathematics, University of Colorado, Boulder, Colorado 80309). I vaguely remember reading, perhaps in an expository paper, of extensions of the Mean Value Theorem to the complex plane. Can anyone give references?

**166. James T. Bruening** (Department of Mathematics, Baker University, Baldwin City, Kansas 66006). I am attempting to publish a proof that every inverse of a rectangular matrix of full rank can be written as a linear combination of matrix extensions of the adjoints of square submatrices of the original matrix. The originality of my work has been questioned, but I can find no existence of such a theorem. Has anyone published such a result before?

**167. Herbert E. Salzer** (941 Washington Avenue, Brooklyn, New York 11225). Does the following result appear in the literature? Write

$$P(x) = q(x, x_i) \prod_1^m (x - x_i) + r(x, x_i)$$

where  $P$  is a polynomial of degree  $n$ ,  $m < n$  and the  $x$ -degree of  $r$  is  $< m$ ; then  $q$  is symmetric in all  $m + 1$  variables.

**168. Maurice M. Mizrahi** (Center for Naval Analyses of the University of Rochester, 1401 Wilson Boulevard, Arlington, Virginia 22209). Is there a known procedure for finding the coefficients  $f_n$  in

$$\sum_0^\infty e^{-z} f_n = g(z),$$

where  $g$  is a meromorphic function? Existence and uniqueness statements are also welcome.

where the  $\varphi_i$  alternate between  $f$  and  $g$ , equals  $x$  or  $cx + d$  with  $c \neq 0$ , and  $\varphi_n$  is  $g$ ; take the second derivative, and show that  $\varphi_1^{r_1} \circ \dots \circ \varphi_{n-1}^{r_{n-1}}$  equals a linear function (note  $(g^n)' = a^n$ ); if  $\varphi_n$  is  $f$ , consider  $\varphi_n^{r_n} \circ \varphi_1^{r_1} \circ \dots \circ \varphi_{n-1}^{r_{n-1}}$  instead. Finally note that  $\varphi_1^{r_1} \circ \varphi_2^{r_2}$  can never be linear. (Contributed by Ulrich Abel)

**157. (vol. 25, p. 424, October 1978, Baer)**. The expected number of extreme points of the convex hull of  $n$  points chosen at random in the unit square is  $h(n) = (8/3)(\log n + \gamma) + O(1)$ ,  $\gamma =$  Euler's constant. For this and related facts see A. Rényi and R. Sulanke, *Z. Wahrscheinlichkeitstheorie und Verw. Gebiete* 9(1968), 146–157; H. Raynaud, *J. Appl. Probability* 7(1970), 35–48; L. A. Santaló, *Encyclopedia of Mathematics and its Applications*, Vol. 1, Addison-Wesley, Reading, Massachusetts, 1976. (Contributed by William F. Eddy; Albert A. Mullin)

**159. (vol. 25, p. 424, October 1978, Wilansky)**. A polynomial  $f \in \mathbb{Z}[x]$  with leading term  $m \cdot x^d$  whose values at integers are all divisible by  $n$  exists iff  $n \mid m \cdot d!$  (If: Take  $f(x) = m \cdot x \cdot (x-1) \cdots (x-d+1) = md! \binom{x}{d}$ ). Only if: The  $d$ th difference of  $f$  is  $md!$ ) For more general facts see (1) Problem A-6, 35th Putnam Competition, December 7, 1974; solution in *Amer. Math. Monthly* 82(1975), 910; (2) D. Singmaster, *Math. Mag.* 30(1966), 103–107, and *J. Number Theory* 6(1974), 345–352. (Contributed by G. M. Bergman, David M. Bressoud, David G. Cantor, Harley Flanders, Christopher Landauer, David Singmaster)

**160. (vol. 25, p. 424, October 1978, Mercier)**. (The term  $A \times g(x)$  in the problem should have been  $A \cdot x \cdot g(x)$ .) For any real valued function  $g(x)$  there exists  $A > 0$  such that

$$G(x) \equiv \sum_{n \leq x} (g(x) - \sum_{p|n} p)^2 \geq A \cdot x \cdot g(x)^2$$

for all  $x$ . (Contributed by Gerd H. Fricke). P. Erdős notes that  $G(x) > c \cdot x^3 / \log x$ ; for related results see K. Alladi and P. Erdős, *Pacific J. Math.* 71 (1977), 275–294.

## RESPONSES

The replies below have been received to queries published in recent issues of the NOTICES. The editor would like to thank all who reply.

**110 (vol. 24, p. 82, January 1977, Friedman)**. In fact, if  $f$  is any monotone polynomial of degree  $> 1$  and  $g$  is  $ax + b$  with  $a \neq 0$ , then  $f$  and  $g$  generate a free group under composition. For the proof suppose a “product” of “powers”  $\varphi_1^{r_1} \circ \dots \circ \varphi_n^{r_n}$ ,



# Postdoctoral and Other Nontenure Track Appointments in the Mathematical Sciences

In order to study research support for new doctorates and, in particular, to obtain information on the number of positions available at the postdoctoral level, the Data Subcommittee of the Society's Committee on Employment and Educational Policy conducted a survey in November 1978 of U. S. departments which offer doctoral degrees in the mathematical sciences.

Each department was asked to provide data on positions of the following types: postdoctoral research fellowships (with no teaching duties), specifically designated or named research instructorships or assistant professorships (with teaching and research but no other duties), and other nontenure track or visiting instructorships or assistant professorships for doctorates (which may include duties other than research and teaching, such as advising, registration, committee assignments, etc.).

The responses submitted by departments are recorded on the following pages. Departments are grouped in five categories: Group I includes the 27 departments of mathematics ranked highest by Roose and Andersen in their 1969 ACE study of U. S. graduate education\*. Group II includes the other 38 mathematics departments rated in the same study. Group III consists of the 90 other U. S. departments of mathematics with doctoral programs. Group IV consists of 66 U. S. departments of statistics and biostatistics and Group V contains 117 other U. S. departments with doctoral programs in the mathematical sciences (applied mathematics, computer science, operations research, history of mathematics, etc.).

This report supplements information published in the special issue of the NOTICES, Assistantships and Fellowships in the Mathematical Sciences, December 1978. Readers should note that the list which follows is not intended to provide information regarding possible job openings. For detailed, current listings of open positions, including salary information, readers are urged to consult Employment Information in the Mathematical Sciences, a periodical which appears six times during the academic year and is published jointly by the

American Mathematical Society and the Mathematical Association of America. (For subscription information, see pages A-679, A-680 of the October 1978 issue of the NOTICES.)

Response rates are summarized in the following table:

Group	Number of Departments	Number of Responses	Number reporting no positions of these types
I	27	19	2
II	38	21	9
III	90	58	31
IV	66	30	18
V	117	18	10

The following comments were prepared by Richard D. Anderson:

Each year some 500 U. S. citizens receive doctorates in pure or classical applied mathematics. Extrapolation of the data reported below suggests that annually there are about 140 new temporary positions in Group I and II departments which average two years duration and carry teaching loads of about six hours per week. In Groups II and III there are an additional ninety new temporary positions of similar duration, but with teaching loads of around 9 or 10 hours per week. Some of these positions are filled by visitors from abroad. Other data, collected earlier by the AMS, suggest that, in Groups I, II and III combined, some 150 to 200 tenure-track positions become available per year; and that roughly half of the holders of these positions eventually receive tenure. It therefore appears that there are research-oriented temporary faculty positions for about half of the new doctorates each year. The number of temporary positions may be considered adequate if one believes that about half of the new doctorates have research potential, but long term research-oriented faculty positions available for young doctorates are increasingly in short supply.

---

\*Kenneth D. Roose and Charles J. Andersen, A Rating of Graduate Programs, American Council on Education, Washington, D. C., 1969, 115 pp. The section on mathematics was reprinted in the NOTICES, February 1971, pp. 338-340.

GROUP I	AVERAGE NO. PER YEAR		NORMAL LENGTH OF APPT.	YEARS SINCE PH.D. FOR TYPICAL RECIPIENT	SOURCE OF FUNDS	TEACHING LOAD (HRS./WK.)	APPL. DEADLINE
	NEW APPTS.	TOTAL					
<i>Top 27 Ranked Mathematics Departments</i>							
<u>CALIFORNIA</u>							
California Institute of Technology Bateman Research Instructorship	2	4	2	0	---	3	Jan
University of California, Berkeley Lecturer	4	8	2	0-2	---	3-6*	
University of California, Los Angeles Hedrick Assistant Professorship	2	4	2	1	---	3 for 2 qtrs. 6 for 1 qtr.	Jan
University of California, San Diego Visiting Assistant Professor	1	**	1	0	---	6*	
<u>ILLINOIS</u>							
Northwestern University Postdoctoral Research Fellowship Other Nontenure Track	1	2	1	1	---	---	May
University of Illinois at Urbana-Champaign Visiting Lecturer	2	4	2	1	---	5*	
University of Illinois at Urbana-Champaign Visiting Lecturer	5	10	2	0-2	---	6-8*	
<u>INDIANA</u>							
Indiana University, Bloomington Hlavaty Research Assistant Professor	1	3	3	0-2	---	3 first year 6 remaining 2 years	Jan
Purdue University, Lafayette Visiting Assistant Professor	5	10	2	0	---	6*	
<u>MASSACHUSETTS</u>							
Massachusetts Institute of Technology Research Associate	1-2	-	1	2-5	MIT	---	
Moore Instructor	5-6	10-12	2	0	---	4.5***	Jan
Applied Mathematics Instructor	2-3	4-6	2	0	---	5***	
Instructor	2-3	4-6	2	0	---	6***	
Lecturer	1-2	3-4	1-2	2-3	---	6***	
<u>MICHIGAN</u>							
University of Michigan, Ann Arbor Hildebrandt Research Assistant Professor	2	6	3	0-2	---	5	Jan
Assistant Professor	2	6	3	0-3	---	7*	
<u>MINNESOTA</u>							
University of Minnesota, Minneapolis Visiting Assistant Professor or Instructor	1-2	1-2	1	1	---	7-8*	
<u>NEW JERSEY</u>							
Princeton University Assistant Professor	2	11	3	2	---	6#	
Instructor	5	12	2	0	---	6*	
<u>NEW YORK</u>							
Columbia University Ritt Assistant Professor	3	12	4	1	---	6##	Jan
Assistant Professor	1	3	5	4	---	6##	Jan
New York University Visiting Membership	6	7	1	1	govt.	-	Jan
Courant Institute Instructorship	2	4	2	1	---	4	Jan
<u>RHODE ISLAND</u>							
Brown University Tamarkin Assistant Professorship	1	3-4	3	0-2	---	6	Jan
<u>WASHINGTON</u>							
University of Washington Acting Assistant Professor	2	4	2	1	---	6*	
<u>WISCONSIN</u>							
University of Wisconsin, Madison Van Vleck Assistant Professorship	4	8	2-3	1	---	6	Dec
Visiting Assistant Professor	1	1	1	3	---	6*	

\*No duties, other than teaching or research.

\*\*Only one in five years.

\*\*\*Research and teaching only formal duties. However, in recent years, due to decreasing resources, have been forced to begin involvement in certain committee, advising and other departmental responsibilities. Also forced to increase teaching loads for all postdoctoral people except Moore Instructors.

#Some advising.

##Duties: Advising and committee assignments.

No response was received from the department of mathematics at each of the following institutions: Stanford, Yale, Chicago, Johns Hopkins, Brandeis, Harvard, Rockefeller, Pennsylvania.

The department of mathematics at each of the following institutions reported it had no positions of these types: Cornell, Virginia.

GROUP II	AVERAGE NO. PER YEAR		NORMAL LENGTH OF APPT.	YEARS SINCE PH.D. FOR TYPICAL RECIPIENT	SOURCE OF FUNDS	TEACHING LOAD (HRS./WK.)	APPL. DEADLINE
	NEW APPTS.	TOTAL					
<i>Other ACE Rated Mathematics Departments</i>							
<u>COLORADO</u>							
University of Colorado, Boulder							
Special Assistant Professor	1.5	3	2	0	---	6-8	Feb/Mar
<u>INDIANA</u>							
University of Notre Dame							
Visiting Assistant Professor	2	2	1	0-2	---	6-7*	
<u>KANSAS</u>							
University of Kansas							
Instructor	5.5	11	2	0	---	7.5*	
<u>MICHIGAN</u>							
Michigan State University							
Research Associate	2	4	2	0	---	0-5	Feb
Instructor	5	10	2	0-1	---	8-10*	
<u>MISSOURI</u>							
Washington University							
Visiting Assistant/Associate/Professor	1	1	1	no policy	---	6*	
<u>NEW YORK</u>							
Rensselaer Polytechnic Institute							
Visiting Assistant/Associate Professor	1	1.5	1	1-2	---	6-7*	
Syracuse University							
Visiting Assistant Professor	1	1	1	0-3	---	6*	
<u>NORTH CAROLINA</u>							
University of North Carolina, Chapel Hill							
Visiting Lecturer	1	2	2	1-3	---	6	Feb
<u>OHIO</u>							
Ohio State University							
Lecturer	2	4	1**	0	---	6-8*	
Lecturer (Remedial)	3	6	1#	2	---	5-10##	
<u>TEXAS</u>							
Rice University							
Evans Instructorship	1-2	3	2	0-1	---	4.5	Jan
University of Texas at Austin							
Moore Instructors	1	2	2-3	0	---	7.5	Feb
Instructor	2	7	2-3	0	---	7.5*	
<u>UTAH</u>							
University of Utah							
Instructorship	5	16	3	1-2	---	6	Mar

\*No duties, other than teaching or research.

\*\*Can be renewed for a total of three years.

#Can be renewed for a second year.

##Provide and develop remedial instruction.

The department of mathematics at each of the following institutions reported it had no positions of these types: Florida, Iowa State, Tulane, Wayne State, SUNY (Buffalo), Yeshiva, Duke, Oregon, Pennsylvania State.

No response was received from the from: Arizona, California (Riverside), Southern California, Florida State, Georgia, Illinois Institute of Technology, Iowa, Kentucky, Louisiana State, Maryland (College Park), Massachusetts (Amherst), North Carolina (Chapel Hill), Case Western, Oregon State, Rutgers, Carnegie-Mellon, Lehigh.

GROUP III	AVERAGE NO. PER YEAR		NORMAL LENGTH OF APPT.	YEARS SINCE PH.D. FOR TYPICAL RECIPIENT	SOURCE OF FUNDS	TEACHING LOAD (HRS./WK.)	APPL. DEADLINE
	NEW APPTS.	TOTAL					
<i>Unrated Mathematics Departments</i>							
<u>ALABAMA</u>							
Auburn University							
Visiting Assistant Professor	1	2	1	2-3	---	10	Jan
Instructor	2	4	2	0-1	---	13*	
<u>CALIFORNIA</u>							
Claremont Graduate School							
Visiting Assistant/Associate Professor	1	1	1	2-7	---	8**	
Retraining Postdoctorates	4	2-4	1	1-3	---		
University of California, Santa Barbara							
Lecturer	1	2	2	2	---	6*	
University of California, Santa Cruz							
Assistant Professor in Residence	-	3	3	1	---	7	

**GROUP III**

(Continued)

	AVERAGE NO. PER YEAR		NORMAL LENGTH OF APPT.	YEARS SINCE PH.D. FOR TYPICAL RECIPIENT	SOURCE OF FUNDS	TEACHING LOAD (HRS./WK.)	APPL. DEADLINE
	NEW APPTS.	TOTAL					
<u>COLORADO</u>							
Colorado School of Mines Adjunct Assistant Professor	-	1	1	not required	---	11*	
Instructor	-	1	1	not required	---	11*	
<u>DELAWARE</u>							
University of Delaware Visiting Assistant Professor	3	3	1-2	0-2	---	6-7*	
<u>FLORIDA</u>							
University of South Florida Visiting Assistant Professor	2	2	1	0-1	---	8*	
<u>GEORGIA</u>							
Emory University Assistant Professor	1.5	3	2	1	---	8*	
Georgia Institute of Technology Visiting Assistant Professor	2	4	2	0	---	8***	
<u>ILLINOIS</u>							
Illinois State University Assistant Professor	2-3	5-7	3	0-1	---	8-12*	
<u>LOUISIANA</u>							
University of Southwestern Louisiana Assistant Professor	1	3-4	3	0-2	---	9-10	
<u>MARYLAND</u>							
University of Maryland, Baltimore County Research Fellowship	-	1	1	0	grant	---	Mar
<u>MASSACHUSETTS</u>							
Northeastern University Instructor or Assistant Professor	1	1	1	0-2	---	12**	
<u>MISSOURI</u>							
St. Louis University Instructor	1	1	1	3	---	3*	
<u>NORTH CAROLINA</u>							
North Carolina State University Visiting Instructor	3	5	1-2	0-1	---	9-10*	
<u>NEW HAMPSHIRE</u>							
Dartmouth College Young Research Instructorship	2	4	2	0-2	---	5-6	Feb
<u>OKLAHOMA</u>							
University of Oklahoma Instructor	2-3	3-4	1-2	0-1	---	9-12*	
Visiting Assistant Professor	1-2	2-3	1-2	0-3	---	9 fall/ 6 spring	
<u>PENNSYLVANIA</u>							
Drexel University Postdoctoral Instructor	1	2	2	0	---	8-10	Mar
Visiting Assistant Professor	1	2	2	0	---	8-10	Mar
Visiting Assistant Professor	1	1	1	0	---	12*	
Temple University Lawton Lectureships	3	6	2	recent	---	12*	
University of Pittsburgh Mellon Postdoctoral Fellowship	1	1	1	1	endowed	---	Jan
<u>RHODE ISLAND</u>							
University of Rhode Island Replacements for Sabbatical or Leaves	0-2	-	1	0-2	---	7.5*	
<u>SOUTH CAROLINA</u>							
University of South Carolina Visiting Assistant Professor	2	2	1	1	---	6-8*	
<u>TEXAS</u>							
Texas A&M University Visiting Assistant Professor	2	2	2-3	0	---	6-10*	
Texas Christian University Assistant Professor	1	1	1-2	0-2	---	9-10*	
Texas Tech University Visiting Lecturer	3	6	2	0	---	9*	

GROUP III	AVERAGE NO. PER YEAR		NORMAL LENGTH OF APPT.	YEARS SINCE PH.D. FOR TYPICAL RECIPIENT	SOURCE OF FUNDS	TEACHING LOAD (HRS./WK.)	APPL. DEADLINE
	NEW APPTS.	TOTAL					
<i>(Continued)</i>							
<u>WISCONSIN</u>							
University of Wisconsin, Milwaukee							
Lecturer							
	5	12	3	0	---	10	July

\*No duties other than teaching or research.  
 \*\*Direct two math clinic teams.  
 \*\*\*Registration.

No response was received from the department of mathematics at each of the following institutions: Alabama (Tuscaloosa), Arizona State, Arkansas (Fayetteville), California (Davis), California (Irvine), Denver, Connecticut, Georgetown, Southern Illinois, Illinois (Chicago Circle), Boston University, Clark, Tufts, Western Michigan, Mississippi, Missouri (Rolla), New Hampshire, Stevens Institute of Technology, New Mexico State, New Mexico, Clarkson, Bowling Green State, Kent State, Cincinnati, Toledo, Oklahoma State, Bryn Mawr, Memphis, Texas (Arlington), Texas (Dallas), Vermont, Virginia Polytechnic Institute and Southern University.

The department of mathematics at each of the following institutions reported it had no positions of these types: Colorado State, Northern Colorado, Wesleyan, American, Catholic University of America, George Washington, Miami (Coral Gables), Hawaii, Idaho State, Idaho, Kansas State, Louisiana Tech, Missouri (Kansas City), Montana State, Montana, Nebraska (Lincoln), Adelphi, CUNY Graduate Center, Fordham, Polytechnic Institute of New York, SUNY (Albany), SUNY (Binghamton), SUNY (Stony Brook), Ohio (Athens), Clemson, Tennessee, Vanderbilt, North Texas State, Houston, Washington State, Wyoming.

GROUP IV	AVERAGE NO. PER YEAR		NORMAL LENGTH OF APPT.	YEARS SINCE PH.D. FOR TYPICAL RECIPIENT	SOURCE OF FUNDS	TEACHING LOAD (HRS./WK.)	APPL. DEADLINE
	NEW APPTS.	TOTAL					
<i>Statistics, Biostatistics, and Biometry Departments</i>							
<u>STATISTICS, BIOSTATISTICS, BIOMETRY</u>							
<u>FLORIDA</u>							
Florida State University							
Department of Statistics							
Postdoctorate in Environmental Health							
Measurement & Statistics							
	1	2	2	0-5	NIH	---	Mar
<u>ILLINOIS</u>							
University of Chicago							
Department of Statistics							
Research Associate							
	1	2	2	0-2	---	0-3*	
<u>INDIANA</u>							
Purdue University, Lafayette							
Department of Statistics							
Visiting Assistant Professor							
	1	1	1	0-1	---	6**	
<u>KANSAS</u>							
Kansas State University							
Department of Statistics							
Temporary Assistant Instructor							
	1	1	1	0	---	6**	
Temporary Instructor							
	1	1	1	0	---	9**	
<u>MINNESOTA</u>							
University of Minnesota, Minneapolis							
Department of Biometry, School of Public Health							
Biometry Fellowship							
	1	2	2	3	NIH	---	Mar
<u>MISSOURI</u>							
University of Missouri, Columbia							
Department of Statistics							
Visiting Assistant Professor							
	1	1	1	0	---	---	
<u>NEW JERSEY</u>							
Princeton University							
Department of Statistics							
Assistant Professor							
	2	4	3	0-2	---	4**	
Research Associate							
	2	2	2	0-10	---	4**	
<u>NORTH CAROLINA</u>							
University of North Carolina, Chapel Hill							
Department of Biostatistics							
Visiting Scholars							
	7	14	1-3	1-11	NIH	---	none
Research Assistant Professor							
	1	4	1-3	1	---	---	none
<u>OHIO</u>							
University of Cincinnati							
Department of Epidemiology and Biostatistics							
Postdoctoral Fellowship in Environmental							
Health Biostatistics							
	1-2	2	1-2	0	NIH	---	May
<u>PENNSYLVANIA</u>							
Temple University							
Department of Statistics							
Visiting Assistant Professor							
	1	1	2 sem.	varies	---	9**	

**GROUP IV**

(Continued)

WASHINGTON

	AVERAGE NO. PER YEAR		NORMAL LENGTH OF APPT.	YEARS SINCE PH.D. FOR TYPICAL RECIPIENT	SOURCE OF FUNDS	TEACHING LOAD (HRS./WK.)	APPL. DEADLINE
	NEW APPTS.	TOTAL					
University of Washington Department of Biostatistics Senior Fellowship	1	4	2	0-5	NIH	---	Jan
Research Assistant Professor	.5	4	indef.	0-1	---	varies	Jan
Assistant Professor	.5	4	indef.	0-1	---	varies	Jan

WISCONSIN

University of Wisconsin, Madison Department of Statistics Lecturer	2	4	1	1	---	3***	
--	---	---	---	---	-----	------	--

\*Consulting  
 \*\*No duties other than teaching or research.  
 \*\*\*Consulting, Department Committee duties.

18 departments reported they had no positions of these types; 36 departments did not respond.

**GROUP V**

Other Mathematical Sciences Departments

OTHER MATHEMATICAL SCIENCES DEPARTMENTS

ALABAMA

University of Alabama Department of Biomathematics Postdoctoral Fellowship	.33	.33	1	1-3	state	---	none
--	-----	-----	---	-----	-------	-----	------

CALIFORNIA

University of California, Santa Cruz Department of Information Science Lecturer	1	-	1	0-3	---	5*	
---	---	---	---	-----	-----	----	--

DISTRICT OF COLUMBIA

George Washington University Department of Operations Research. Visiting Assistant Professor	.33	.33	1	3-5	---	3**	
--	-----	-----	---	-----	-----	-----	--

NEW YORK

SUNY at Buffalo Department of Computer Science Research Associate Professor	1	1	3	14	NIH	---	
Research Assistant Professor	2	5	2	2	---	3**	
University of Rochester Department of Computer Science Research Associate	1	1	2	varies	grants	---	
Research Associate & Part-time Instructor	1	1	2	varies	---	3**	
Visiting Assistant Professor	1	1	1	varies	---	3*	

NORTH CAROLINA

North Carolina State University Operations Research Department Visiting Assistant Professor	1	1	.50	2	---	3**	
---	---	---	-----	---	-----	-----	--

OHIO

Ohio State University Department of Computer and Information Science Visiting Assistant Professor	-	1	1	1-2	---	7**	
---	---	---	---	-----	-----	-----	--

RHODE ISLAND

Brown University Division of Applied Mathematics Visiting Assistant Professor of Research	-	2	1	varies	univ. & contracts		Feb
---	---	---	---	--------	-------------------	--	-----

\*Some advising.  
 \*\*No duties, other than teaching or research.

10 departments reported they had no positions of these types; 99 departments did not respond.

# LETTERS TO THE EDITOR

Editor, the NOTICES

In the dialogue in the NOTICES concerning the federal funding of research there are several long term factors affecting our research effort which, in my opinion, have not been adequately stressed. The purpose of this letter is to call some of these to the attention of the mathematical community. The comments below refer to core mathematics only; the situations in the other mathematical sciences have many other and somewhat different components.

(1) The primary funding source for research in core mathematics since the war has been the academic community through the Ph. D. -granting departments of mathematics. My own preliminary estimates of the current level of this research support is of the order of \$50 to \$75 million per year as compared to about \$16-\$18 million in NSF support for core mathematics per year. The university estimate is based on teaching loads, average salaries, overhead expenses, etc. with most of the data coming from published AMS reports. The figures given here for academic research support are "soft" and subject to some revision. However, the relative orders of magnitude appear right.

(2) Historically, any substantial federal funding of research in mathematics has developed only since the war and has concentrated on summer support of those judged by peer evaluations to be most productive or promising. This pattern has developed at a time of evident great improvement of, and success in our national research effort; any substantial change in the pattern of federal research should not be undertaken lightly or without good cause. However, the pattern of funding has developed to meet the set of conditions existing over the past twenty-five years. If, as noted below, conditions important for research vitality over the next fifteen years can be expected to be radically different from those of the past, there is obvious need for a serious re-assessment of overall federal levels and mechanisms of support.

(3) There has been a changing pattern of research training in mathematics. Whereas in the 1960s very many or most of the research-oriented young Ph. D. 's could and did get initial academic research-oriented positions with good prospects of permanency, that situation has now changed. Most now appear to get temporary positions at important research centers or near active individual research mathematicians in their field and then to seek prospective permanency following several years of what is tantamount to postdoctoral experience. This newer pattern is potentially healthy for the profession in that it broadens research training and lets those responsible for hiring have a postdoctoral track record on which to judge applicants. But to be really healthy there must also be a pattern whereby research-oriented positions are available for those who are four or five years past their

doctorates and who have shown the most research promise. However, more and more university departments following the lead of the most prestigious ones appear to be developing a new pattern of employment whereby they keep most nontenured positions as strictly temporary and have an almost fixed number of tenured faculty.

(4) There is currently, an almost fully tenured faculty (of the order of 75%) in the Ph. D. -granting departments (which have a total faculty of about 5,000). There are known demographic phenomena which insure that the 18-year-old population will drop by 25%-30% in the years from 1979 to 1991 and then will stay almost level till 1995. This has important implications for future overall faculty size. There are known age distributions of the mathematics faculty that produce a combined retirement and death rate of only about 1% per year now with a gradual increase to about 2% per year over the next 10 to 15 years, but this is complicated by the new retirement laws. (By the beginning of the next century the rate goes to 5% or 6%.) Any additional non-federal funding for higher education is in a state of limbo at best. Thus, there is a virtual certainty that the customary university departmental modes of support for anything like the traditional number of promising young mathematicians past their early postdoctoral experience will simply not be available. This fact and its implications for the health of our subject are serious and should not be ignored.

(5) There is a growing pattern of very promising young mathematicians after some postdoctoral experience taking positions in the college, not university, sector of academia with inadequate research libraries, high teaching loads, little institutional research emphasis, and little or no access to day-by-day contact with other active research people. This development has serious implications for the mechanisms of federal funding of research over the next fifteen years and is complicated by the likely demographically induced reduction in the number of new similar positions available for young mathematicians in that period. We may need partial academic year support for some people in this area.

(6) It is the author's view (based in part on detailed observation of the field of geometric topology and in part on more general observations) that much and probably a very substantial part of the really important research is done by people who have had their degrees from 5 to 15 years (with a few people doing important work prior to being five years out). If this is, in fact, a general phenomenon, then the mathematical community should be particularly concerned with the development and encouragement of mathematicians to the five-year post-doctoral level and with the mechanisms of research support for them for their next ten years of activity. It is precisely the support in research environments of people 5 to 15 years out that is threatened by the demographic factors

affecting academia over the next generation. My own preliminary estimates are that by 1990 the number of mathematicians who have had their degrees 5 to 15 years and are employed in Ph. D.-granting departments will only be from  $\frac{1}{4}$  to  $\frac{1}{2}$  the present number of such people.

(7) The author believes, but it may well be disputed by others, that really important research is now being done in a much broader spectrum of locales than was the case 10 or 20 years ago. For example, in geometric topology, brilliant and definitive recent results have been obtained by young mathematicians at Kentucky, UCLA, VPI, Warsaw, and Wisconsin and one could mention half a dozen other places almost as deserving of recognition. A recurrent theme in much of the recent activity is the interplay of many peoples' ideas and methods and the utility of frequent short and long-term contacts among research mathematicians. Apparently, the system of research effectively utilizes the broad base of involvement of many people.

(8) The roles of frequent conferences and symposia, the ready availability of preprints, and the frequent short and long term visits of active research mathematicians have undoubtedly markedly accelerated the progress of research. A corollary of this observation and of the developing dispersion of research-oriented people to isolated environments is the need for programs which keep isolated people current and active. Their research is likely to be significant only if the individuals have frequent short and occasional prolonged contact with leaders in their fields. Continuing and probably sizably expanded support for activities directed to meet these issues would appear to be very much in order.

Conclusion: The levels as well as the mechanisms of federal support of mathematical research need to be reexamined in light of the significantly changing needs of the research community. In particular, finding adequate means for the support and encouragement of our active young mathematicians when at the heights of their powers is a developing and long-term problem for our community.

R. D. Anderson  
Louisiana State University  
AMS Committee on Science  
Policy

Editor, the NOTICES

This letter is to call the attention of all our colleagues to an important point not brought out clearly in the November NOTICES in the material on pages 481-494 about the establishment of a mathematical sciences research institute. Most of the material there presented was prepared before the meeting of the National Science Board on September 21, 1978. At that Board meeting Dr. James A. Krumhansl, Assistant Director for Mathematical, Physical, and Engineering Sciences, said that the project announcement for the research institute as then drafted in NSF 78-80 and as published in the November NOTICES, would have added to it a statement to the following effect:

(F) "Proposals for such research institutes will not only be compared with each other, but will be compared with other options for the possible additional support of mathematical research and, in particular, for the encouragement and development of young mathematicians. These other options may consist of, but are not restricted to, more postdoctoral fellowships, group support of postdoctoral fellowships, peripatetic institutes, special research years, and the mathematical institute itself."

We might list these options in more detail about as follows (much as in an earlier memo written by one of us):

- (1) Added postdoctoral positions provided on NSF grants;
- (2) Group or departmental grants to cover postdoctoral visitors;
- (3) A formal postdoctoral fellowship program;
- (4) Support of "special years" at University X, subject Y;
- (5) Peripatetic Research Institutes, such as the Postgraduate Research Institute as recommended in 1976 by the AMS Committee on Science Policy;
- (6) An NSF-supported research institute in mathematical sciences;
- (7) More grants of the present type for the support of research. (There is in particular a new possibility of "group grants.")

Comments:

Ad (3) Any organization now experienced in making grants for postdoctoral fellowships in mathematics could apply for NSF support for such grants.

Ad (2) Such proposals as (2) could now be submitted.

Ad (4) and (5) Departments can now (and have in the past, as of (4)) submitted proposals.

Ad (6) An institute need not be as large as suggested in the Project Announcement.

We hope that our colleagues will carefully consider the merits of these options and the possibilities of corresponding proposals to the Science Foundation.

Saunders Mac Lane  
University of Chicago  
Elias Stein  
Princeton University

Editor, the NOTICES

Several times in the last several months I have encountered references in the NOTICES, and elsewhere, to the recent decline in the number of Ph. D. degrees awarded annually in the United States. That a reduction in the number has occurred is a fact, and I suppose that it is natural to appeal to this fact if one is arguing, for example, that the NSF should increase its support of mathematical research. However, I am uneasy at the thought that such arguments could be seen as calling, even implicitly, for an increase in our annual production of Ph. D.'s in mathematics.

The over-supply of mathematicians which has been with us for nearly a decade has been a disaster whose effects are all too well-known: the



demeaning scramble for college and university positions, the years of professional and economic insecurity as one approaches the ever-higher tenure hurdle, the almost total absence of mobility for those who do have tenure, and in extreme cases, blighted careers and even personal tragedies.

If I may refer to my own experiences, I have seen capable undergraduates go on to achieve their doctorates from excellent universities and then spend years drifting from one university to another in search of a permanent position. I have watched several younger colleagues give up and leave the profession, not because they were un-talented, but because they were caught in a combination of the job squeeze and the hardening attitudes of administrators toward granting tenure. (One college dean of my acquaintance speaks of "making hay while the sun shines," the sunshine in this case being the oversupply of mathematicians.) Similar experiences are commonplace in the mathematical community as a whole.

The relative ease with which our present unhappy situation was brought about is disturbing. In a letter to the November 1978 issue of the NOTICES Professor Saunders Mac Lane remarked that "A report of the President's Science Advisory Committee, 'Meeting Manpower Needs in Science and Technology' (the Gilliland Report), issued by The White House in December, 1962, called for a massive increase (4-fold to 8-fold) in the number of Ph.D.'s to be trained in mathematics. Enthusiasts in the profession pressed for such an increase and there was an increase of perhaps 3-fold." (With a little effort one can imagine the appalling fix we could now be in if the 8-fold increase had actually been achieved!)

The thing that really worries me is that it is at least conceivable that on some desk in some office in Washington there might even now exist a recommendation to the effect that "the current alarming drop in the number of doctorates awarded annually in mathematics be reversed by appropriate programs, with a goal of increasing the annual number of Ph.D.'s awarded to 1,800, etc." A program aimed at increasing our current over-supply of mathematicians would be an absurdity, of course, but absurdities do occasionally occur in government. With the experience of the recent past in mind, I sincerely hope that the members of the mathematical community will be alert for, and resist resolutely, any pressures toward the end of increasing Ph.D. production in mathematics in the foreseeable future.

Patrick Shanahan  
Holy Cross College

Editor, the NOTICES

Knowing few chemists or physicists, I cannot dispute Professor Hoffman's claim (as reported in the November 1978 NOTICES, p. 493) that mathematicians are no more greedy than they. Moreover, from my personal experience I am reasonably certain that mathematicians as a group are no more greedy than insurance salesmen or realtors, either. And I used to know a bookmaker who was probably greedier than any

ten mathematicians in the world put together. Nonetheless, I do find it disturbing that at a time when some of the best mathematicians in this country are living with their mothers and collecting unemployment benefits, the National Science Foundation is bestowing upon "senior statesmen" and "well established" professors an additional 2/9 of their already \$30,000+ salaries as "summer support". But this is an old scandal, and I find I can no longer get very excited about it. In these same November 1978 NOTICES, however, I read of a proposal to further exalt and reward these grand savants with \$35,000-\$45,000 per year positions at a new Mathematics Institute, under the guise of helping the junior members of the profession by creating a few (low paid and very temporary) positions for them, too. I find this proposal more than just disturbing: I find it obscene.

The single greatest need of the profession at this time is for additional tenure track positions; the proposed new Mathematics Institute will not create any of these. This is why funding for it should not be sought at the present time (and not because funding it might deprive some esteemed genius of his summer support). I would hope that the great senior wizards of the profession would use their influence to have the NSF fund, either directly or indirectly, additional tenure track positions; but since these geniuses would derive neither money nor fame from such a proposal, I suppose I should know better.

Both Professors Mac Lane and Kohn take for granted that great benefits are bestowed upon young mathematicians working in the presence of senior statesmen. Not being a senior statesman, I am less convinced of this. I am very sure, however, that if, after working for two years with various exalted members of the profession, a young mathematician then finds himself on a bread line, he will be bitter, unhappy, and resentful.

K. L. Fields  
Rider College

Editor, the NOTICES

An Open Letter from the  
Mathematics Department  
of Yeshiva University

On September 1, 1978, Yeshiva University dismissed three tenured members of the Mathematics Faculty. This action was unanimously condemned by the Division of Natural Science and Mathematics at a meeting on September 5. Those fired were Associate Professors Jonathan Ginsberg, Charles Patt, and Norman S. Rosenfeld. They have served the Mathematics Department for periods of from 10 to 15 years.

In June 1977 the President of Yeshiva announced his intention of dismantling the Belfer Graduate School of Science (which consisted of the Mathematics and Physics Departments), thus ending the graduate programs in science at Yeshiva, this to be accomplished in one year. The plan was essentially carried out. In Spring 1978 the Administration made it known that in the

following year there would be eight positions in undergraduate Mathematics, with need for two or three additional persons to teach the Information Science Department. Moreover, nine members of the Mathematics Faculty received letters during the spring declaring the need for their services and "re-establishing" their positions. Two of these have since resigned, and one was transferred to the Information Science Department. Professors Ginsberg, Patt, and Rosenfeld did not receive "re-affirmation" letters last spring, and on September 1 they were dismissed. With two resignations, one transfer, and three dismissals, the Mathematics Faculty is now reduced to six. The final decision to undertake this action was made, apparently at the end of August, by a budget committee of Deans and Vice Presidents. This committee has cancelled classes to accommodate the reduced faculty size.

Those dismissed had been scheduled to teach some Information Science courses. We now find that nontenured and part-time faculty have been hired in the Information Science Department. This is the first year that any appointments have been made in that department. Formerly all of its courses were taught by the faculties of Mathematics and Physics.

Far from seeking to avoid this crisis, we note that the Administration has denied requests from several faculty members for sabbatical and unpaid leaves.

The faculty was of course not consulted in this undertaking, and we note in this connection that the Yeshiva Administration has recently argued successfully in a Federal Court, in an action concerning the status of a faculty union, that the faculty has managerial status.

Lewis A. Coburn  
 Leopold Flatto  
 Adam Koranyi  
 Arnold Lebow  
 Martin Schechter

Editor, the NOTICES

This letter is addressed to mathematicians who may receive soon an opinion survey, "The 1979 Survey of the American Professoriate," directed by E. C. Ladd and S. M. Lipset. They should be aware that a previous 1977 version of this survey did reach some of our mathematical colleagues and has been severely criticized by them. The present draft of the new (1979) version of the survey does not take adequate account of these criticisms.

Serge Lang, one of those who was asked to answer the 1977 survey, recorded his criticisms in an article in the *New York Review of Books* for May 18, 1978. Neal Koblitz independently wrote some of his observations to Professor Lipset and received a wholly inadequate reply. Jack Goldhaber received a copy of the survey, responded, and subsequently regretted that he had responded. Because of other interests in survey technique, I had occasion to study the survey. I found 38 of its questions deficient and reported these points to Professors Ladd and Lipset. They have not yet taken adequate steps to remove these defi-

ciencies. I have discussed this type of opinion survey with a number of sociologists and other experts on survey technique. They have pointed out other possible disadvantages. Some questions may not have been sufficiently pretested (before circulation by mail) or sufficiently validated (to verify that opinions as to behavior match actual behavior). Moreover, questions asked of the professoriate or other such groups may tend to stereotype those groups.

Under these circumstances, I do recommend to mathematicians who may receive copies of the new survey that they look carefully at the accuracy and appropriateness of the questions before they take the trouble to answer them.

Saunders Mac Lane  
 University of Chicago

Editor, the NOTICES

I bring to your attention the following resolution of sentiment, passed overwhelmingly at the AWM meeting held on August 18, 1978 at the International Congress of Mathematicians in Helsinki, Finland:

We note the absence of women from the list of invited speakers at the 1978 ICM, from the IMU general assembly, and from the IMU committees, despite the large number of internationally distinguished women mathematicians. We urge that this situation be rectified by the 1982 ICM.

I am personally dismayed that such a situation is still possible, and hope that readers of these NOTICES will join me in efforts to ensure that it does not happen again.

Judith Roitman  
 President of the Association  
 for Women in Mathematics

Editor, the NOTICES

Enclosed is a blank registration form for the annual meeting on January 24-28, 1979, in Biloxi, Mississippi. I would love to bask in the Gulf Coast sun in Biloxi, Mississippi next January. However, I feel that I must boycott the meeting because of its location in a state which has failed to ratify the so-called ERA 'amendment'. I put the word amendment in quotes because it is really not an amendment at all. The Declaration of Independence contains the words-

"WE HOLD THESE TRUTHS TO BE SELF EVIDENT, THAT ALL MEN ARE CREATED EQUAL, THAT ---"

The ERA simply attempts to spell out the clause 'THAT ALL MEN ARE CREATED EQUAL'. WHAT DOES THE WORD, 'MEN', MEAN HERE? Does it mean 'males', does it mean 'white males', does it mean 'white christian males', or does it mean 'humanity'-equal under the law?

Finally, I feel that this is not a political issue, i. e., that it transcends politics. There are thousands of women mathematicians and their lot has not been an easy one. This has been due not to the mathematics community, but to the

ambient space, i. e., the society in which we live and the environment it produces.

Helen F. Cullen  
University of Massachusetts  
Amherst

Editor, the NOTICES

Here is one more letter about ERA.

Once we set a precedent with the proposed boycott, the gates are open. Next we will boycott every city whose City Council does not include a woman and every university whose top administration does not include a Spanish surname; and we will have to boycott all Western International hotels because there is one in Johannesburg. After that we will be boycotting all states that do not enforce the 55 mph speed limit, all cities that do not provide park trails for their joggers, and all hotels whose newsstands sell cigarettes. All these efforts, however worthy, are orthogonal to the purposes of the Society.

Leonard Gillman  
University of Texas  
Austin

Editor, the NOTICES

Professor Konrad Jacobs of the Mathematics Institute at Erlangen maintains a collection of black and white photographs of mathematicians, for posterity, as a complement to the written history of mathematics. A set of copies of his photographs is on display at the Mathematical Research Institute in Oberwolfach.

I have arranged to lend him copies of my photographs of mathematicians, from which he will have black and white copies made to add to his collection. For the most part, these are pictures I have taken for the Berkeley Mathematics Department's photographic display, begun in 1968 and soon expanded to include not only regular faculty members but also visitors staying with us more than three months. (My collection includes copies of some but not all of the photographs

taken by others for this display while I was away in 1969-1971.) They also include a few pictures of some other mathematical friends and acquaintances that I have taken over the years, here and there, with their knowledge.

Anyone whom I have photographed who does not want his or her pictures to be included in Prof. Jacobs' collection should contact me, and I will remove these from the collection I send. If I hear from you after I send the pictures in (which will probably be in early 1979 for people no longer at Berkeley), I will write Jacobs to remove the picture from his collection. (Since not everyone reads this letters column, I hope people will mention this letter to colleagues who they know have been visitors or faculty members at Berkeley in the past ten years.)

Conversely, if any mathematicians would like to send any good pictures, recent or otherwise, of themselves or other mathematicians for copying, they would be gratefully received by Prof. Dr. Konrad Jacobs, 852 Erlangen, Bismarkstrasse 1 $\frac{1}{2}$ , Mathematisches Institut der Universität Erlangen, Federal Republic of Germany.

George M. Bergman  
University of California,  
Berkeley

---

Letters submitted for publication in the NOTICES are reviewed by the editorial committee whose task is to determine which ones are suitable for publication. The publication schedule requires from two to four months between receipt of the letter in Providence and the publication of the earliest issue of the NOTICES in which it could appear. The committee adopted a policy that the NOTICES does not ordinarily publish complaints about reviews of books or articles, although, following an instruction from the Council, rebuttals and correspondence concerning reviews in the *Bulletin* will be considered for publication. Letters submitted for consideration by the editorial committee should be mailed to the Editor of the NOTICES, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940.

## MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

*BOUNDS ON TRANSFER PRINCIPLES  
FOR ALGEBRAICALLY CLOSED  
AND COMPLETE DISCRETELY VALUED FIELDS*  
by Scott Shorey Brown

This memoir begins with a unified treatment of computationally efficient decision procedures for the additive group of rational numbers, the ordered additive group of integers, and the ordered field of real numbers. Analogous procedures are developed for algebraically closed fields and complete discretely valued fields when the characteristic of the field is zero or is large compared to the length of the statement whose truth is to be determined. Bounds for the size of  $p$  required to apply the classical transfer principles between algebraically

closed fields of characteristics 0 and  $p$  or between complete discretely valued fields of characteristics 0 and  $p$  follow and yield a bound for the exceptional primes in the Artin Conjecture. Finally, we derive a bound on the characteristic of the base field of a complete discretely valued field which ensures that a power series in one variable over the valued field which represents an algebraic function will converge up to the nearest singularity of the algebraic function.

Number 204

96 pages

List price \$7.60; member price \$5.70

ISBN 0-8218-2204-7; LC 78-9121

Publication date: July 31, 1978

To order, please specify MEMO/204

NEW INDIVIDUAL  
MEMBER PRICE \$3.80  
PRICE \$5.70

Prepayment is required for all American Mathematical Society publications.

Send for the book(s) above to: AMS, P. O. Box 1571, Annex Station, Providence, RI 02901

# NEWS AND ANNOUNCEMENTS

## SALEM PRIZE

The Salem Prize for 1978 was awarded to Dr. Björn E. J. Dahlberg, of the University of Gothenburg, for his work on harmonic functions and harmonic measures. The prize, established in 1968, is given every year to a young mathematician who is judged to have done outstanding work in Fourier series and related topics, the fields of interest of Raphaël Salem (1898-1963).

Dr. Dahlberg received the prize for showing the equivalence of harmonic measure and Lebesgue measure for Lipschitz domains in  $\mathbb{R}^n$  and the solution of Dirichlet's problem for  $L^2$  boundary-functions for such domains. The relevant paper is "Estimates of harmonic measure" in the Archive for Rational Mechanics and Analysis 65 (1977), no. 3, 275-288.

The 5000 franc award has been made possible by gifts provided by the family of Salem. The selection committee for 1978 consisted of Lennart Carleson, Jean-Pierre Kahane, Charles Pisôt, Elias M. Stein, and Antoni Zygmund.

Previous recipients have been Nicholas Varopoulos in 1968, Richard Hunt in 1969, Yves Meyer in 1970, Charles Fefferman in 1971, Thomas Körner in 1972, E. M. Nikišin in 1973, Hugh Montgomery in 1974, William Beckner in 1975, Michael R. Herman in 1976, and S. V. Bočkarëv in 1977.

## OLGA TAUSKY-TODD

Olga Taussky-Todd, Emeritus Professor of Mathematics at California Institute of Technology, has been awarded the highest scientific honor of the Austrian government. The Austrian Cross of Honour for Science and Art was presented to her by Dr. Heimo Kellner, the Austrian Consul General for Los Angeles, in a ceremony at Caltech on October 4, 1978. She was given the award for her outstanding achievements in mathematics.

"With the Cross of Honour for Science and Art, Austria honors persons who have gained general recognition and international reputation through outstanding creative achievements in the field of science or art," according to a statement by the Austrian Consul General. "The Cross of Honour was created in 1956 and has two grades. Professor Todd is awarded the Cross of Honour first class, which is the highest grade, by the Federal President of Austria on suggestion of the Minister of Science and Technology," said the statement.

## INTERNATIONAL SYMPOSIUM TO HONOR S. S. CHERN

An International Symposium in Differential Geometry will be held at the University of California, Berkeley from Monday, June 25 to Friday, June 29, 1979, in honor of Shiing-shen Chern on the occasion of his retirement. This

Symposium will review recent major developments in differential geometry and its relation to such areas as nonlinear partial differential equations, complex analysis, algebraic geometry and theoretical physics.

The scientific program will consist of ten to fifteen invited one-hour lectures. To date, the following have accepted the invitation to speak: M. F. Atiyah, R. Bott, E. Calabi, P. A. Griffiths, N. H. Kuiper, J. Moser, L. Nirenberg, R. Osserman, Wu Wen-Tsun, C. N. Yang, and S. T. Yau.

Participants may choose to organize their own seminars during the Symposium.

No funds for travel or living expenses are available. However, arrangements for housing in the university dormitories can be made. Those who plan to attend the Symposium should send their name, affiliation, address, and approximate dates of arrival and departure, clearly indicating whether they need housing information to: Ms. Nora Lee, Department of Mathematics, University of California, Berkeley, California 94720, USA.

## ABRAHAM ROBINSON CHAIR

As a tribute to the late Abraham Robinson, Yale University invites contributions toward an endowed professorship in the Department of Mathematics. The chair honoring the memory of this distinguished mathematician will be held by an individual whose interests and ideals reflect Professor Robinson's. Members of the scientific community who wish to join in this memorial should write or call Walter Feit, Chairman, Department of Mathematics, Yale University, 218 Leet Oliver Memorial Hall, 12 Hillhouse Avenue, New Haven, Connecticut 06520. Telephone: 203-432-4731.

## NEW COVER DESIGNS

Beginning with the January 1979 issues, new cover designs will appear on AMS journals. The new designs, which were created by Phoebe Murdock of the AMS staff, are coordinated so that journals published by the Society may be recognized at a glance by their distinctive designs. On the whole, the traditional colors of the covers have been retained. One exception is the Bulletin-New Series, whose cover is a lighter shade of green than the old Bulletin's.

The journals, whose new covers appear this month, are the following:

- Bulletin-New Series
- Current Mathematical Publications
- Mathematical Reviews
- Mathematics of Computation
- Memoirs of the AMS
- Notices of the AMS
- Proceedings of the AMS
- Transactions of the AMS

## RESEARCH ANNOUNCEMENTS FOR BULLETIN—NEW SERIES

Research announcements which have not been published in the Bulletin since March 1978 as a result of a decision of the Council to revise the journal's format, will again appear when the first issue of the Bulletin—New Series is published shortly. A report of the changes being made in the Bulletin appeared in the February 1978 issue of the NOTICES, on pp. 126-127.

Research announcements provide a means for quick publication of new and significant results of interest to mathematicians. As many as fifteen papers are to be chosen for each issue of the new journal by the Editorial Board for Research Announcements.

To submit research announcements, authors should send five copies of their manuscript (to speed the decision process) to I. M. Singer, Executive Editor for Research Announcements, whose address is: Department of Mathematics, Evans Hall, University of California, Berkeley, California 94720. Announcements are limited to 125 typed lines of 70 spaces each. Authors are requested to include an introductory paragraph intelligible to the nonexpert, stating the nature and significance of their results. Sketches and/or ideas of proof must be included. The deadlines for submission are February 1 for the July issue, April 1 for the September issue, and June 1 for the November issue.

## INDIVIDUAL MEMBER DISCOUNTS ON AMS BOOKS

In December, the Board of Trustees authorized a special fifty percent discount rate for individual members of the Society on most of the books published by the AMS. The new rate applies to sales subsequent to January 1, 1979. The former twenty-five percent member's discount will continue to apply to books purchased by institutional members.

Production costs have increased markedly in recent years, forcing a steady increase in prices for AMS publications. Two steps have been taken recently in an effort to increase the distribution of Society publications among individual members. One was the decision to issue more books in soft-cover editions, which helps to reduce costs; the other is the recent decision to offer larger discounts to individuals than in the past.

Prices quoted in the NEW PUBLICATIONS section and in the AMS advertisements in this issue of the NOTICES, show these new discounts.

## 1978 WORLD DIRECTORY OF MATHEMATICIANS

The sixth edition (1979) of the World Directory of Mathematicians has been scheduled for production in August 1979. It is being published under the auspices of the International Mathematical Union (IMU) with the cooperation of the National Committees of Mathematics. Prepara-

tion of the new edition, which contains about 20,000 names and addresses of mathematicians from all over the world, was begun by Professor O. Frostman, carried on by his widow and her son, and is being completed under the supervision of Professor M. Nagata of the Mathematical Society of Japan.

The price of the new Directory will be US \$20 per copy (¥ 3,500 inside Japan). However, prepublication prices are available before March 1, 1979. To order before March 1, 1979:

### With payment enclosed

US \$13 per copy plus US \$2 per check  
(¥ 2,500 inside Japan).

### With payment later

US \$14 per copy plus US \$2 per check  
(¥ 2,700 inside Japan).

Copies will be shipped upon receipt of payment.

Copies of the World Directory of Mathematicians may be ordered by those in Japan from: Bureau of the W. D. M., Department of Mathematics, Kyoto University, Kyoto 606 (Japan). Payments from outside Japan should be made by a check payable to: I. M. U. - WORLD DIRECTORY ACCOUNT, No. 0862/656205-82 in US Dollars, Schweizerische Kreditanstalt, Zürich-Rigiplatz, Universitätstrasse 105 - 8033 Zürich, Switzerland. All correspondence should be sent to: BUREAU OF THE WORLD DIRECTORY OF MATHEMATICIANS, Mathematics Department, Faculty of Sciences, Kyoto University, Kyoto 606, Japan.

## KEY WORD INDEX NOW AVAILABLE THROUGH AMS

It is now possible to order the comprehensive key word index to the AMS(MOS) Subject Classification Scheme (1970)<sup>1</sup> described in the August 1978 issue of the NOTICES (p. 333), directly from the Society.

The Index was prepared by the Mathematical Library of the University of Ljubljana, Yugoslavia, and is published by the Society of Mathematicians, Physicists, and Astronomers of SR Slovenia. The 96-page booklet includes each significant word used in the classification scheme, along with the subject number and the full text of the line in which it appears.

Copies may now be ordered prepaid at \$15 each (order code: SUBJINDEX) from the American Mathematical Society, P. O. Box 1571, Annex Station, Providence, R.I. 02901.

<sup>1</sup>The AMS(MOS) Subject Classification Scheme (1970) was prepared with support from the National Science Foundation under GN-875.

## A NEW CANADIAN JOURNAL

A new Canadian journal began publication in November 1978. Mathematical Reports of the Academy of Science is published by the Royal Society of Canada and edited by J. Aczél, H. S. M. Coxeter, N. S. Mendelsohn, P. Ribenboim, and G. de B. Robinson. The journal is produced by photographic reproduction to provide rapid

publication of articles (up to four pages in length) summarizing important completed research.

Subscriptions may be requested from the Editorial Office of Mathematical Reports, University of Waterloo, Faculty of Mathematics, Waterloo, Ontario N2L 3G1. Detailed information on submission of papers may be obtained from Professor P. Ribenboim, Department of Mathematics, Queen's University, Kingston, Ontario K7L 3NG, Canada.

**EDITOR'S NOTE:** The information above replaces information previously given on subscriptions and submission of papers to this journal (NOTICES, October 1978, p. 433).

#### COMBINATORIAL MATHEMATICS SOCIETY OF AUSTRALASIA

A new organization, the Combinatorial Mathematics Society of Australasia (CMSA) was formed recently. The society will promote understanding of combinatorial mathematics and disseminate information about combinatorics and combinatoricists, and will conduct an annual conference. The annual subscription is A\$4. The rate for students (not in receipt of significantly more than a Commonwealth Postgraduate Scholarship living allowance) is A\$2. All membership subscriptions or enquiries should be sent to Professor W. D. Wallis, Department of Mathematics, University of Newcastle, New South Wales, 2308, Australia. Checks may be made payable to "C. M. S. A."

#### LECTURE SERIES IN APPLIED MATHEMATICS

The Chicago Area Applied Mathematics Consortium has announced its 1978-1979 Distinguished Lecture Series in Applied Mathematics. The lectures for this academic year will be given by Harold Grad, Joseph B. Keller and Peter D. Lax. The Consortium is jointly sponsored by the Department of Engineering Sciences and Applied Mathematics of Northwestern University, the Department of Mathematics of the University of Chicago and the Applied Mathematics Division of Argonne National Laboratory. More detailed information may be obtained by writing Bernard J. Matkowsky, Northwestern University, Evanston, Illinois, 60201.

#### NEW DEPARTMENT CREATED

Bowling Green State University has created a Department of Applied Statistics and Operations Research. The Department sponsors undergraduate areas of concentration in statistics and in operations research and sponsors jointly with the newly named Department of Mathematics and Statistics a Master of Science in Applied Statistics. Information concerning any of these programs can be obtained from Professor Robert A. Patton, Chair, Department of Applied Statistics and Operations Research, Bowling Green State University, Bowling Green, Ohio 43403.

#### VISITING MATHEMATICIANS AVAILABLE FOR LECTURES

Among the visiting Fulbright-Hays scholars from abroad each year, there are many who welcome opportunities to participate in programs and meet colleagues on campuses other than those where they are officially located. Each year the Council for International Exchange of Scholars prepares a Directory of these scholars who may be invited directly to participate in professional conferences or to give occasional lectures or seminars as their programs permit it. The following Fulbright scholars listed in the Directory in the fields of mathematics, computer science and statistics are currently in the United States: Bernd Aulbach, Matania Ben-Artzi, Phillippe Boulanger, Harold Garth Dales, Gokulananda Das, Iurii Vladimirovich Egorov, Ivor O. Grattan-Guinness, Dusan Kodek, Witold Lipski, Clement H. Lutterodt, Anna Maria Mantero, Do Le Minh, Djamchid Parvizi, Francis Pastijn, Visay D. Pathak, Hans Sheerer, Karl Sigmund, and Lucas Vienne.

Complete information on these scientists' home country, place and length of academic visit, and field of special interest may be found in the lists of Visiting Mathematicians in the October and November 1978 issues and on p. 78 of this issue of the NOTICES. Further details and copies of the Directory are available from Mrs. Mary Ernst, Faculty Fulbright Advisor, CIES, 11 Dupont Circle, N. W., Washington, D. C. 20036.

#### EDUCATION EXCHANGE BETWEEN U.S. AND CHINA

Agreement has been announced on the general framework of an educational exchange between the United States and The People's Republic of China (PRC). The agreement was reached during discussions held between delegations from the two sides October 12-20, 1978.

Groundwork for these discussions was laid during talks in Peking in July 1978 between Frank Press, Science Adviser to the President, and Fang Yi, Chairman of the PRC Commission on Science and Technology. The Chinese delegation was headed by Chou Pei-yuan, Acting Chairman of the Science and Technology Association, PRC, and the US delegation by Richard C. Atkinson, Director of the US National Science Foundation.

The educational exchange program will involve students, scientists, and visiting scholars of both sides. The Chinese side has expressed interest in studies and research primarily in the fields of science and technology, including physical and biomedical sciences, engineering, and selected fields of applied technology. The US expects to send students and scholars in the social sciences, and humanities including history, philosophy, language and literature, archaeology, and art history; and in various aspects of natural and physical science, such as earth sciences, medicine, and agricultural science. The sending side will pay the costs associated with its participants.

The Chinese delegation indicated it wished to send some 500-700 persons under this program in the 1978-1979 academic year. The first Chinese students are expected to arrive in the United States early in 1979.

The US plans to provide for about 60 students and scholars in 1979 under a US Government-financed program. It is anticipated that other American students under separate arrangements would also be studying in China. An organization will be selected shortly to coordinate selection and placement of US students and scholars to go to China.

In closing negotiations Dr. Atkinson said, "We all agree that this visit of the Chinese Education Delegation has been a success. We have worked in the spirit of the Shanghai Communiqué and of the friendship which has been traditional between our two peoples. We can now look forward to the smooth development of an exchange of our students, scientists and visiting scholars, an exchange which can only benefit both of us. In our discussions, we have reached an understanding of what is to be involved in this exchange. We have talked about the principle of mutual benefit. We have agreed that both sides will exert their best efforts to develop the program in such a manner as to ensure success for 1978-1979 and to lay the groundwork for an expanded program in future years."

NSF News Release

#### VISITING WOMEN SCIENTISTS

Last year NSF supported a pilot Visiting Women Scientists Program in which 40 women scientists visited 110 high schools across the country to encourage young women to consider careers in science and technology. Based on the success of the pilot program, a number of schools have requested lists of women scientists who might be willing to meet with their students. Women scientists who wish to be included in such a roster to be released next fall to schools should send the following information by January 31, 1979 to Ms. Carol Place, Research Triangle Institute, Box 12194, Research Triangle Park, North Carolina 27709: 1. name, 2. mailing address, 3. telephone number, 4. type of science (biological, physical, social science, engineering or mathematics), 5. specific science field (e.g. bacteriology, mechanical engineering), 6. highest degree earned, 7. type of employment (academic, nonprofit organization, profitmaking organization, government), 8. race or ethnic background. Respondents should omit any information they do not wish to have released.

#### A NATIONAL REGISTRY OF WOMEN IN SCIENCE AND ENGINEERING

The Association for Women in Science (AWIS) has announced the availability of its AWIS Registry for employers and advisory groups seeking candidate lists of qualified women sci-

entists. The Registry is a computerized file of several thousand women in all areas of science and engineering, organized by scientific specialty. This resource provides employers and administrators a broader base for the selection of the most qualified person for job openings.

AWIS is a nonprofit, tax-exempt organization affiliated with the American Association for the Advancement of Science and the Federation of Organizations for Professional Women. It was founded in 1971 when the growing awareness of sex discrimination in the professions indicated the need to assist women to reach their full career potentials.

The Registry is a nonprofit service financed primarily through search fees paid by employers seeking candidate lists. To obtain a list of qualified women scientists for a particular opening, the prospective employer should send a purchase order for \$50 per position to be searched with a detailed set of qualifications to: Mary Lee Schneiders, Registrar, AWIS, 1346 Connecticut Avenue, N.W., Suite 1122, Washington, D.C. 20036 (202-833-1998).

#### MINIMAL MATHEMATICS FOR COLLEGE GRADUATES

The Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America has established a Panel to consider the question: "What should every graduate of an American college or university know of mathematics?" It is hoped that the ultimate recommendations of this Panel will provide welcome guidance to colleges, universities, and such bodies as state boards of education, many of which are already actively considering such questions in the current wave of renewed interest in core curricula and general education.

The Panel, relying extensively on surveys of informed opinion, wishes to arrive at a list of minimum mathematical competencies for all college graduates, where "mathematical" is meant to include statistics, computing, etc., as well as mathematics in the narrow sense. Its report should contain, besides this list, a reasoned statement about why every college graduate should have acquired some understanding of mathematical thought; suggestions about courses in which the minimal competencies might be acquired; and general observations about such matters as inter-institutional coordination in furthering mathematical literacy.

The Panel has begun to collect information and opinions on the problem, and will welcome contributions from readers of this announcement. Facts about other local, regional, or national efforts (past or present) in the area of the Panel's charge, personal views about the general issue or specific aspects, and copies of or references to pertinent documents are among the things the Panel would be glad to receive. They may be sent to the Panel in care of its chairman, D. Bushaw, Department of Pure and Applied Mathematics, Washington State University, Pullman, Washington 99164.

## HEW FELLOWS PROGRAM

The Fellows Program of the U.S. Department of Health, Education and Welfare has been established in order to open the processes of government to fresh ideas, to identify, encourage and develop leadership ability, and to promote public understanding of government programs and problems. It offers the Fellow both formal and informal educational opportunities. Lectures and seminars are given by recognized national leaders and opportunities are available for interchange of ideas and experiences with participants from similar programs (White House Fellows, Congressional Fellows, etc.).

The Fellows Program provides a one-year assignment to the office of one of several key officials of the Department of HEW, depending on the Fellow's special competencies and the needs of the department. These offices are located in Atlanta, Boston, Chicago, Dallas, Denver, Kansas City, New York, Philadelphia, San Francisco, and Seattle, as well as in Washington, D.C.

To qualify for the HEW Fellows Program, candidates must be citizens of the United States, have a record of demonstrated interest in community service and the nation's social problems, possess a combination of education and specialized managerial or administrative experience, and qualify in the GS range of 11-15 of the US Civil Service Commission. The GS-11 range includes a minimum of three years of general experience and three years of specialized experience of a managerial type. HEW is, of course, an equal opportunity employer.

To apply for consideration as an HEW Fellow, the candidate must submit Standard Form 171 (Personal Qualifications Statement), a listing indicating community service participation, and a statement describing the applicant's reasons for wishing to be an HEW Fellow. The application deadline for the September 1979-1980 Fellowships is January 19, 1979.

All inquiries and requests for application blanks should be addressed to: The Director, HEW Fellows Program, 330 Independence Avenue, S.W., Washington, D.C. 20201 (Telephone: 202-245-6087).

## GUIDE TO NSF PROGRAMS

The National Science Foundation's newly published Guide to Science Education Programs provides summary information about NSF programs for the fiscal year 1979. It is intended as a source of general guidance for institutions and individuals interested in participating in these programs. Listings describe the principle characteristics and basic purpose of each activity, eligibility requirements, closing dates where applicable, and the address from which more detailed information, brochures, or application forms may be obtained. Single copies of the Guide NSF 78-45, are available on request from the National Science Foundation, 1800 G Street, N.W., Washington, D.C. 20550 (Attn: Mail Security and Records Section). Additional copies are for sale from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402.

# 1979 MATHEMATICAL SCIENCES administrative directory

- PROFESSIONAL MATHEMATICAL ASSOCIATIONS
- ACADEMIC INSTITUTIONS – DEPARTMENTS IN THE MATHEMATICAL SCIENCES AND OTHER FACILITIES in the U.S., Canada, Central America, and the Caribbean, coded according to the highest degree offered
- NONACADEMIC ORGANIZATIONS
- HEADS AND KEY PERSONNEL of a selected group of government agencies
- EDITORS OF JOURNALS in the mathematical sciences
- OFFICERS AND COMMITTEE MEMBERS in more than 25 professional mathematical organizations
- AN INDEX OF COLLEGES AND UNIVERSITIES

\$6.00

Publication date: March 15, 1979

Code: ADMDIR/79 – Prepayment Required

american mathematical society  
p. o. box 1571, annex station, providence, r.i. 02901



# SPECIAL MEETINGS INFORMATION CENTER

**THIS CENTER** maintains a file on prospective symposia, colloquia, institutes, seminars, special years, and meetings of other associations, helping the organizers become aware of possible conflicts in subject matter, dates, or geographical area. The printed lists contain announcements of meetings of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings or symposia devoted to specialized topics. The lists also contain announcements of regularly scheduled meetings of national or international mathematical organizations.

**AN ANNOUNCEMENT** will be published in the **NOTICES** if it contains a call for papers, and specifies the place, date, subject (when applicable), and the speakers; a second full announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in each issue until it has been held and a reference will be given in parentheses to the month, year and page of the issue in which the complete information appeared.

**IN GENERAL**, SMIC announcements of meetings held in the United States and Canada carry only date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadline dates for abstracts or contributed papers, and name of person to write for further information. Meetings held outside the North American area may carry more detailed information. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society in Providence.

**DEADLINES** are the same as the deadlines for abstracts. They are listed on the inside front cover of each issue.

1978-1979. **Academic Year devoted to Algebraic Geometry and the Geometry of Banach Spaces**, The Mittag-Leffler Institute, Djursholm, Sweden. (January 1978, p. 62)

1978-1979. **Special Year in Harmonic Analysis**, University of Maryland. (April 1978, p. 192; see also February 1978, p. A-339)

## JANUARY 1979

January 1-December 22, 1979. **Mathematisches Forschungsinstitut Oberwolfach** (Weekly Conferences), Federal Republic of Germany. (November 1978, p. 503)

3-8. **AAAS Annual Meeting**, Houston, Texas. (November 1978, p. 504)

4-5. **Assessment in Mathematics**, University of Birmingham, Birmingham, England. (October 1978, p. 440)

15-27. **Computer Science School (Escola de Computação)**, São Paulo, Brazil. (November 1978, p. 505)

15-February 9. **Australian Mathematical Society Nineteenth Summer Research Institute**, Macquarie University, New South Wales, Australia. (October 1978, p. 440; November 1978, p. 505)

22-26. **Forty-ninth ANZAAS Congress**, Auckland, New Zealand.

*Information:* Honorary Secretary, 49th ANZAAS Congress, University of Auckland, Private Bag, Auckland, New Zealand.

29-30. **Short Conference on Differential Equations**, Mississippi State University, Mississippi State, Mississippi.

*Program:* Sessions of papers on various topics will be scheduled. Efforts will be made to arrange transportation from Biloxi to Mississippi State.

*Information:* Brief abstracts indicating the length of time needed for the talk and requests for further information should be sent to John R. Graef, Department of Mathematics, Mississippi State University, Mississippi State, Mississippi 39762.

29-31. **Sixth ACM SIGACT-SIGPLAN Symposium on Principles of Programming Languages**, San Antonio, Texas. (August 1978, p. 329)

31-February 3. **Second Paderborn Conference on Functional Analysis**, Gesamthochschule Paderborn, Paderborn, Federal Republic of Germany.

*Program:* There will be 15 invited lectures on various topics of current research.

*Speakers:* E. Alfsen (Oslo), R. Aron (Dublin), J. Diestel (Kent), S. Dineen (Dublin), K. Floret (Kiel), S. Goldberg (Maryland), R. Haydon (Oxford), H. König (Bonn), M. A. Kaashoek (Amsterdam), M. Neumann (Saarbrücken), H. J. Petzsche (Düsseldorf), L. Tzafriri (Jerusalem), H. v. Weizsäcker (Regensburg).

*Information:* K. D. Bierstedt and B. Fuchssteiner, Organizers, Gesamthochschule Paderborn, D 4790 Paderborn, Federal Republic of Germany.

## FEBRUARY 1979

4-8. **Australian Mathematical Society Applied Mathematics Conference**, Leura Motel, New South Wales, Australia. (October 1978, p. 440)

*Information:* R. I. Tanner, Department of Mechanical Engineering, Sydney University, New South Wales 2006, Australia.

9-10. **1979 Virginia Polytechnic Topology Conference**, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.

*Guest Lecturer:* R. Engelking.

*Program:* There will be several 15-minute papers presented.

*Information:* J. C. Smith, Director, Mathematics Department, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061.

13-16. **Fifth Interamerican Conference on Mathematical Instruction**, Campinas, São Paulo, Brazil. (August 1978, p. 330)

## MARCH 1979

5-9. **NSF-CBMS Regional Research Conference on Special Functions and their Relations with the Theory of Representations of Lie Groups**, East Carolina University, Greenville, North Carolina.

*Principal Lecturer:* Jean Dieudonné, University of Nice, France.

*Program:* The lectures will cover broad research-expository topics. Some additional invited lectures will also be included in the conference program.

*Support:* There is a limited number of spaces available with limited support for travel and lodging.

*Deadline for Applications:* Applications for participation and possible support should be made on or before January 15, 1979. Late applications will be considered as space permits. Inclusion of a short resume with emphasis on current research and publications would be helpful.

*Information and Applications:* Lokenath Debnath, Mathematics Department, East Carolina University, Greenville, North Carolina 27834.

12-16. **Annual Lecture Series in Mathematics: Some Aspects of the Theory of Riesz Spaces**, University of Arkansas, Fayetteville, Arkansas.

*Program:* W. A. J. Luxemburg (California Institute of Technology) will present a sequence of five lectures. In addition, there will be sessions for contributed papers;

abstracts should be received by February 15, 1979.

*Information:* W. H. Summers, Department of Mathematics, University of Arkansas, Fayetteville, Arkansas 72701.

- 14-16. **Twelfth Annual Simulation Symposium**, Causeway Inn, Tampa, Florida.

*Program:* There will be presented papers, and talks will be given by Paul Roth (National Bureau of Standards) and Tuncer Oren (University of Ottawa). The Symposium will be preceded, on March 12-13, by two workshops given by the IEEE and the ACM.

*Information:* Sudesh Kumar, NCR Corporation, 4045 Sorrento Valley Boulevard, San Diego, California 92121.

- 15-17. **Algebra and Ring Theory Conference**, University of Oklahoma, Norman, Oklahoma. (November 1978, p. 505)

- 18-23. **Conference on Geometry (Foundations) and Differential Geometry**, University of Haifa, Haifa, Israel. (October 1978, p. 440)

- 26-28. **Workshop on Continuous Lattices III**, University of California, Riverside, California.

*Program:* (Tentative). The primary concern of this workshop will be the applications of continuous lattices and similar structures to other areas of mathematics and computer science. Dana Scott will lecture on the lattice theoretic approach to the theory of computation. Louis S. U. Tulane will speak on recent research on the topological and algebraic structure of continuous lattices. Contributed papers are invited.

*Organizing Committee:* Albert Stralka, Dana Scott and Karl Hofmann.

*Information:* Albert Stralka, Chairman, Department of Mathematics, University of California, Riverside, California 92521.

- 28-30. **1979 Conference on Information Sciences and Systems**, Baltimore, Maryland. (October 1978, p. 440)

#### APRIL 1979

- 2-6. **Tenth Southeastern Conference on Combinatorics, Graph Theory and Computing**, Florida Atlantic University, Boca Raton, Florida.

*Program:* Instructional lectures will be given by Paul Erdős (Hungarian Academy of Sciences), Ronald L. Graham (Bell Telephone Laboratories), Marshall Hall, Jr. (California Institute of Technology), William H. Mills (Institute for Defense Analyses), Ronald C. Mullin (University of Waterloo), Crispin St. J. A. Nash-Williams (University of Reading), John L. Selfridge (Mathematical Reviews), and Ralph G. Stanton (University of Manitoba).

*Contributed Papers:* There will be sessions for fifteen-minute presentations of contributed papers. Abstract deadline is March 20, 1979.

*Information:* Frederick Hoffman, Director, Tenth Southeastern Conference, Florida Atlantic University, Boca Raton, Florida 33432.

- 3-5. **1979 ACM SIGNUM Meeting on Numerical Ordinary Differential Equations**, Department of Computer Science, University of Illinois at Urbana-Champaign, Urbana, Illinois. (October 1978, p. 440)

- 3-5. **IEEE Specifications of Reliable Software Conference**, Cambridge, Massachusetts. (October 1978, p. 441)

- 16-20. **Conference on Several Complex Variables**, Princeton University, Princeton, New Jersey.

*Information:* John Erik Fornæss, Mathematics Department, Fine Hall, Box 37, Princeton University, Princeton, New Jersey 08540.

- 30-May 2. **Eleventh Annual ACM Symposium on Theory of Computing**, Atlanta, Georgia. (November 1978, p. 505)

#### MAY 1979

- 4-6. **Conference in Analysis**, Purdue University, West Lafayette, Indiana 47907.

*Program:* Conference to be held in honor of Casper Goffman. Invited participants include A. Bruckner, L. Cesari, L. W. Cohen, H. Federer, J. Serrin, D. Waterman.

*Sponsor:* School of Science and Department of Mathematics, Purdue University.

*Support:* Limited funds for local lodging and main banquet.

*Information:* D. Drasin, W. Gorman or C. J. Neugebauer, Department of Mathematics, Purdue University, West Lafayette, Indiana 47907.

- 19-26. **International Conference on Functional-Differential Systems and Related Topics**, Białeżyńko, Poland. (November 1978, p. 505)

- 21-25. **Coxeter Symposium**, Toronto, Canada.

*Purpose:* The mathematical work of H. S. M. Coxeter has long had great influence within the field of geometry. In recent years its influence is felt more widely—because of new and diverse applications of packing problems and other geometric inequalities, because of the increased importance of polytopes, and because of rapid development in the study of groups in terms of generators and relations. This Symposium is intended to allow those engaged in research in such areas to take stock of their progress jointly and to assess future prospects.

*Program:* Invited speakers will include: John Horton Conway, H. S. M. Coxeter, L. Fejes Tóth, Branko Grünbaum, William M. Kantor, P. McMullen, C. A. Rogers, J. J. Seidel, G. C. Shephard, Jacques L. Tits, and W. T. Tutte. There will also be sessions for contributed papers.

*Information:* Chandler Davis, Department of Mathematics, University of Toronto, Toronto M5S 1A1, Canada.

- 23-25. **Optimization Days 1979**, McGill University, Montreal, Canada. (November 1978, p. 505)

- 24-25. **Symposium on Mathematical Programming with Data Perturbations**, George Washington University, Washington, D. C.

*Call for Papers:* Contributed papers are solicited in the following areas: sensitivity and stability analysis for optimization and/or equilibrium problems; solution methods for implicitly defined problem functions; solution methods for problems involving parameters; solution approximation techniques. In addition to the above areas, "clinical" presentations, perhaps describing problems encountered in real applications, are also solicited.

*Instructions for Authors:* Abstracts must be received by February 5, 1979. Abstracts should not exceed 500 words, and should be sent in duplicate to Stephen M. Robinson, Department of Industrial Engineering, University of Wisconsin-Madison, 1513 University Avenue, Madison, Wisconsin 53706.

*Information:* Anthony V. Fiacco, George Washington University, Washington, D. C. 20052.

- 31-June 6. **Seventh Conference of Analytic Functions**, Wisła (Carpathian Mountains, Province Bielsko-Biala), Poland. (October 1978, p. 441)

#### JUNE 1979

- 18-22. **International Conference on the Global Theory of Dynamical Systems**, Northwestern University, Evanston, Illinois.

*Speakers:* (Tentative). C. Conley, M. Hirsch, J. Palis, C. Pugh, D. Ruelle, M. Shub, S. Smale, F. Takens, J. Yorke and E. C. Zeeman.

*Support:* Some support for travel and subsistence will be available.

*Organizing Committee:* John Franks, Clark Robinson, Bob Williams.

*Information:* Secretary, Dynamical Systems Conference, Department of Mathematics, Northwestern University, Evanston, Illinois 60201.

- 20-22. **1979 International Symposium on Fault-Tolerant Computing**, Madison, Wisconsin. (October 1978, p. 441)

- 20-22. **Third IMACS International Symposium on Computer Methods for Partial Differential Equations**, Lehigh University, Bethlehem, Pennsylvania. (October 1978, p. 441)
- 25-29. **1979 International Symposium on Information Theory**, Grignano, Italy. (October 1978, p. 441)

### JULY 1979

- 1-20. **London Mathematical Society Conference on Aspects of Contemporary Complex Analysis**, Collingwood College, Durham University, England.
- Sponsors:* London Mathematical Society and the North Atlantic Treaty Organization Scientific Affairs Division.
- Topics:* Brownian motion, approximation in the plane, conformal mapping.
- Speakers:* J. L. Doob, J. Korevaar, P. L. Duren, D. L. Burkholder, A. M. Chollet, J. Becker, A. Baernstein, A. Stray, C. Pommerenke, W. K. Hayman, J. G. Clunie, D. A. Brannan, M. Heins, and W. J. Schneider.
- Support:* Partial support is available for citizens of NATO countries.
- Information:* J. G. Clunie (Ref.: IC/79), Department of Mathematics, Imperial College, London SW7, England.

- 16-20. **Sixth International Colloquium on Automata, Languages and Programming**, Graz, Austria.
- Information:* A. H. Maurer, Institut für Informationsverarbeitung, Techn. Universität Graz, Steyrergasse 17, A-8010 Graz, Austria.

### AUGUST 1979

- 23-September 8. **IXème Ecole d'Été de Calcul des Probabilités de Saint-Flour**, Université de Clermont, France.
- Speakers:* Peter J. Bickel (University of California, Berkeley); Nicole El-Karoui (Faculté des Sciences du Mans); Marc Yor (Laboratoire de Calcul des Probabilités de l'Université Paris VI).
- Information:* P. L. Hennequin, Département de Mathématiques Appliquées, Université de Clermont, Boîte Postale 45, 63170 Aubière, France.
- 27-31. **Tenth International Symposium on Mathematical Programming**, Montreal, Canada.
- Sponsor:* Mathematical Programming Society.
- Call for Papers:* Papers are invited on all theoretical, computational and applicational aspects of mathematical programming. Abstracts of up to 200 words with minimal notation should be submitted in English to the address below.
- Deadline for Abstracts:* March 1, 1979.
- Information:* Symposium Secretariat, Tenth International Symposium on Mathematical Programming, 772 Sherbrooke Street West, Montreal, Quebec, Canada H3A 1G1.

### SEPTEMBER 1979

- 4-9. **Ninth IFIP Conference on Optimization Techniques**, Warsaw, Poland. (November 1978, p. 505)
- 19-21. **Sixteenth Midwestern Mechanics Conference**, Kansas State University, Manhattan, Kansas. (November 1978, p. 506)

## NEW AMS PUBLICATIONS

### PROCEEDINGS OF SYMPOSIA IN APPLIED MATHEMATICS

(ISSN 0160-7634)

#### NUMERICAL ANALYSIS

edited by Gene H. Golub and Joseph Oliger

This is the collection of texts prepared by the lecturers for the Numerical Analysis Short Course given at the AMS meeting in Atlanta, Georgia, in January 1978. The subject matter was chosen to emphasize prominent research areas and attitudes in numerical analysis. These are introductory lectures on the subject matter for presentation to an audience of scientists from other areas or disciplines. Typically, there is an introduction to a given problem area and to techniques used, an application to applied problems, and a discussion of current research questions or directions.

The papers given here are mainly of a mathematical nature. Several trends in modern numerical analysis are discussed in these lectures. These texts should be useful to the practicing users of numerical methods, programmers, scientists, and engineers who would like to know what progress is being made on the theoretical and developmental side of the subject. The papers and the bibliographies of current work should be useful in general to those who wish to understand the nature of numerical analysis and its current problems of interest.

The papers included are:

- Cleve B. Moler, *Three Research Problems in Numerical Linear Algebra*
- J. E. Dennis, Jr., *A Brief Introduction to Quasi-Newton Methods*
- Carl de Boor, *The Approximation of Functions and Linear Functionals: Best vs. Good Approximation*
- James M. Varah, *Numerical Methods for the Solution of Ordinary Differential Equations*

Joseph E. Oliger, *Methods for Time Dependent Partial Differential Equations*

George J. Fix, *Variational Methods for Elliptic Boundary Value Problems*

Number 22

135 + viii pages

List price \$10.00 (soft cover); \$14.00 (hard cover);

Member price \$7.50 (soft cover); \$10.50 (hard cover)

ISBN 0-8218-0122-8; LC 78-11096

Publication date: November 30, 1978

To order, please specify PSAPM/22;

(Soft cover; PSAPMS/22)

### LECTURES ON MATHEMATICS IN THE LIFE SCIENCES

(ISSN 0075-8523)

#### SOME MATHEMATICAL QUESTIONS IN BIOLOGY. IX

edited by Simon A. Levin

This volume contains lectures given at the Eleventh Symposium on Some Mathematical Questions in Biology, held in Denver, Colorado, on February 24-25, 1977, in conjunction with the Annual Meeting of the American Association for the Advancement of Science. The Symposium was supported by the National Institutes of Health and cosponsored by the Society for Industrial and Applied Mathematics and the AMS.

The contents of the volume are as follows:

- John Guckenheimer, *Comments on Catastrophe and Chaos*
- J. F. G. Auchmuty, *Qualitative Effects of Diffusion in Chemical Systems*

D. Marr, *Representing Visual Information*  
Ronald M. Shymko, *Control of Mitosis and Tissue  
Growth in Three Dimensions*

Stuart A. Kauffman, Ronald M. Shymko and Kenneth  
Trabert, *Control of Sequential Compartment Forma-  
tion in Drosophila*

Volume 10

244 + ix pages

List price \$11.20; Institutional member price \$8.40;  
Individual member price \$5.60

ISBN 0-8218-1160-6; LC 77-25086

Publication date: December 15, 1978

To order, please specify LLSCI/10

## MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

(ISSN 0065-9266)

### *MATHEMATICAL FOUNDATIONS OF QUANTUM SCATTERING THEORY FOR MULTIPARTICLE SYSTEMS*

I. M. Sigal

The work is devoted to basic mathematical questions of scattering theory for nonrelativistic quantum systems. Systems under consideration are short range, i.e. potentials between their particles decay at infinity faster than  $|x|^{-2}$ .

The main result of the work is a theorem on existence of complete sets of channel wave operators for quantum systems in question. This theorem, conjectured a long time ago, is basic for a mathematical foundation of quantum mechanical scattering theory.

The proof of the theorem proceeds in two steps. First, the problem is reduced to the proof of some properties of the resolvents of  $n$ -body Hamiltonians (Schrodinger operators) near the continuous spectra. Then the required information about behavior of the resolvents near the real axis is obtained by deriving certain operator equations for them and studying these equations in appropriate Banach spaces. The method used in the proof being constructive provides basic elements for further physics treatment of the problem. It can also be used for studying other mathematical problems of quantum mechanics of many body systems.

The work is essentially self-contained.

Number 209

147 + iii pages

List price \$8.40; Institutional member price \$6.30;  
Individual member price \$4.20

ISBN 0-8218-2209-8; LC 78-10154

Publication date: November 1, 1978

To order, please specify MEMO/209

## PROCEEDINGS OF THE STEKLOV INSTITUTE

(ISSN 0081-5438)

### *INVESTIGATIONS IN THE GENERAL THEORY OF STATISTICAL DECISIONS*

A. S. Holevo

This monograph is devoted to the "noncommutative" theory of statistical decisions, which permits one to include, along with Wald's classical scheme, an essentially new class of statistical problems arising in the theory of quantum measurement and quantum communication channels. It is a matter of optimal measurement of the state parameters of a system (classical quantum measurement). An apparatus for integral representation of functionals, to the minimization of which the problem of optimal measurement reduces, is worked out: the problem of existence of an optimal measurement is studied and conditions for optimality are established. Applications to the problem of optimal measurement of the mean value of a boson field are considered.

Number 124(1976)

140 + vi pages

List price \$40.00; institutional member price \$30.00;  
individual member price \$20.00

ISBN 0-8218-3024-4; LC 78-16140

Publication date: September 1978

To order, please specify STEKLO/124

### *C-TYPES OF $n$ -DIMENSIONAL LATTICES AND 5-DIMENSIONAL PRIMITIVE PARALLELOHEDRA (WITH APPLICATION TO THE THEORY OF COVERINGS)*

S. S. Ryškov and E. P. Baranovskii

This monograph is devoted to one of the domains of discrete geometry. A new approach is given to the classification problem for  $n$ -dimensional parallelohedra as well as a solution of this problem for  $n=5$ . As an application, the problem of least dense lattice covering of five-dimensional Euclidean space by equal spheres is solved.

Number 137(1976)

140 + iv pages

List price \$40.00; institutional member price \$30.00;  
Individual member price \$20.00

ISBN 0-8218-3037-6; LC 78-21923

Publication date: December 15, 1978

To order, please specify STEKLO/137

## Personal Items

DAVID ALEXANDER BRANNAN of Queen Elizabeth College, London, has been appointed to the Chair of Pure Mathematics at The Open University, Milton Keynes, England.

LAMBERTO CESARI of the University of Michigan has been elected a member of the Academy of Science and Letters of Milan (Istituto Lombardo).

CHARLES M. CHAMBERS of George Washington University has been appointed staff associate and legal advisor for the Council on Post-secondary Accreditation, Washington, D. C.

HAROLD SCOTT MACDONALD COXETER of the University of Toronto has been elected an honorary member of the London Mathematical Society.

CHRISTOPHER CROKE of the University of Chicago has been appointed a lecturer at the University of California, Berkeley.

LEONARD E. DOR of the University of Illinois, Urbana, has been appointed to an assistant professorship at Wayne State University.

RICHARD G. GIBBS of Indiana University has been appointed to an assistant professorship at Rose-Hulman Institute of Technology.

SERGIU KLAINERMAN of New York University has been awarded a fellowship by the Miller Institute for Basic Research in Science at the University of California, Berkeley.

YUDELL L. LUKE of the University of Missouri-Kansas City has been designated by the University of Missouri Board of Curators as a Curators' Professor.

JOHN RAWNSLEY of the Dublin Institute for Advanced Studies has been appointed to a visiting assistant professorship at the University of California, Berkeley.

RODOLFO R. ROSALES of the California Institute of Technology has been appointed a lecturer at the University of California, Berkeley.

ALEX ROSENBERG of Cornell University has been appointed to a visiting professorship at the University of California, Berkeley.

VICTOR SAKS of Windham College and the University of Costa Rica has been appointed to an associate professorship at Daemen College.

STANLEY SAWYER of Yeshiva University has been appointed to a professorship at Purdue University.

SEYMOUR SCHUSTER of Carleton College has been selected as a visiting lecturer and consultant to the Mathematical Association of America's program on visiting lecturers and consultants for 1978-1979.

PETER TOMAS of the University of Chicago has been appointed to an assistant professorship at the University of Texas at Austin.

HALE TROTTER of Princeton University has been appointed to a visiting professorship at the University of California, Berkeley.

### PROMOTIONS

To Director of the Center for Pure and Applied Mathematics, University of California, Berkeley: MURRAY H. PROTTER.

To Chairperson, Department of Mathematics, University of California, Berkeley: SHOSHICHI KOBAYASHI.

To Vice Chairperson for Graduate Appointments and Nonacademic Personnel, Department of Mathematics, University of California, Berkeley: JOHN B. WAGONER.

To Chairman and Associate Professor, Department of Mathematics, Fairfield University: BENJAMIN FINE.

To Professor, Rose-Hulman Institute of Technology: GARY J. SHERMAN; University of California, Berkeley: GEORGE M. BERGMAN, RALPH N. MCKENZIE, JOHN B. WAGONER; University of Texas at Austin: RALPH E. SHOWALTER.

To Associate Professor, University of California, Berkeley: MICHAEL J. KLASS; Fairfield University: JOSEPH MACDONNELL.

To Assistant Professor, University of California, Berkeley: JENNY HARRISON.

## Deaths

Mr. CORNELIUS (NEIL) PAUL ABOFF of Rose-Hulman Institute of Technology died in July, 1978, at the age of 24. He was a member of the Society for 5 years.

Professor H. DAVID BLOCK of Cornell University died on October 6, 1978, at the age of 58. He was a member of the Society for 31 years.

Professor RUFUS BOWEN of the University of California, Berkeley, died on July 31, 1978, at the age of 31. He was a member of the Society for 6 months.

Dr. WALTER H. GAGE of the University of British Columbia died on October 3, 1978, at the age of 73. He was a member of the Society for 51 years.

Professor PAUL J. HESSLER of Wittenberg University died on November 4, 1978, at the age of 41. He was a member of the Society for 17 years.

Dr. GEORGE LOULLIS of the University of Crete died on September 26, 1978, at the age of 28. He was a member of the Society for 4 years.

Professor E. K. McLACHLAN of Oklahoma State University died on October 27, 1978, at the age of 54. He was a member of the Society for 23 years.

Professor ZEEV NEHARI of Carnegie-Mellon University died on September 1, 1978, at the age of 63. He was a member of the Society for 30 years.

Dr. JAMES ROBERT OVERMAN of Bowling Green State University died on May 23, 1978, at the age of 90. He was a member of the Society for 55 years.

Dr. ROBERT C. SEBER of Western Michigan University died on September 6, 1978, at the age of 51. He was a member of the Society for 28 years.

Professor WALTER C. STRODT of St. Lawrence University died on October 19, 1978, at the age of 63. He was a member of the Society for 41 years.

## Doctorates Conferred in 1977–1978 – *Supplementary List*

The following are among those who received doctorates in the mathematical sciences and related subjects from universities in the United States and Canada during the interval July 1, 1977–June 30, 1978. This is a supplement to the list printed in the October 1978 issue of the NOTICES. The numbers appearing in parentheses after each university indicate the following: the first number is the number of additional degrees listed for that institution; the next seven numbers are the number of degrees in the categories of 1. Pure Mathematics (i.e., algebra, number theory, geometry, topology, analysis, functional analysis, logic, or probability), 2. Statistics, 3. Operations Research, 4. Computer Science, 5. Applied Mathematics, 6. Mathematics Education, 7. Other. Each entry contains the dissertation title. There are fourteen universities listed with a total of 63 individual names. This total, added to the previous list, includes doctorates from 148 universities with a total of 1,015 individual names; 236 departments granting doctorates.

### CALIFORNIA

UNIVERSITY OF SOUTHERN CALIFORNIA  
(2;2,0,0,0,0,0,0,0,)

#### Mathematics

- Passty, Gregory B.  
Asymptotic behavior of an implicit differencing scheme associated with accretive operators in Banach spaces
- Wolfe, David S.  
On the structure of a ring and its subring fixed by a finite automorphism group

### FLORIDA

FLORIDA STATE UNIVERSITY (6;2,2,2,0,0,0,0)

#### Statistics

- Boos, Dennis D.  
The differential approach in statistical theory and robust inference
- Conlon, John Charles  
G-ordered functions with applications in statistics
- Reiland, Susan  
Stochastic models for carrier-borne epidemics
- Reiland, Thomas William  
Contributions to duality theory in continuous nonlinear programming
- Wong, Derek Chung-Fat  
Second order duality and complementarity in continuous time programming
- Yeh, Ching-Ming  
Designs with a polynomial trend in complete or incomplete blocks

### IOWA

UNIVERSITY OF IOWA (4;0,0,0,4,0,0,0)

#### Computer Science

- Critcher, Adrienne Pendery  
Function schemata
- Eckstein, Denise Marie  
Parallel graph processing using depth-first and breadth-first search
- Limaye, Ranjan Sharatchandra  
Analysis of string patterns using a procedure-type model and formal languages
- Liu, Ken-Chih  
An efficient algorithm for string pattern matching

### KANSAS

UNIVERSITY OF KANSAS (1;0,0,0,1,0,0,0)

#### Computer Science

- Meyers, Leonard D.  
N speed homomorphisms, simulations and parallel states of automata

### LOUISIANA

UNIVERSITY OF SOUTHWESTERN LOUISIANA  
(3;0,2,0,0,1,0,0)

#### Mathematics and Statistics

- Magoun, Dale  
Piecewise linear regression techniques
- Neal, Bill Larry  
Hybrid techniques for the numerical solution of stiff systems of differential equations
- Paul, Joey  
Minimum risk joint confidence regions for parameters of normal populations

### MASSACHUSETTS

BRANDEIS UNIVERSITY (7;7,0,0,0,0,0,0)

#### Mathematics

- Chang, Haichau  
On algebraic surfaces of general type with  $e_1^2 = 2Pg - 2$
- Cohen, Ralph  
On odd primary stable homotopy theory
- Drager, Lance  
On the intrinsic symbol calculus for pseudo-differential operators on manifolds
- Glynn, Robert  
A characterization of projective space by its Hilbert polynomial
- Gutkin, Eugene  
Holomorphically induced representations and Hilbert spaces with reproducing kernels
- Sato, Nobuyuki Albert  
Algebraic invariants of links of codimension two
- Yang, Su-Win  
Order of canonical vector bundles on  $C_n(k)/\Sigma_k$

HARVARD SCHOOL OF PUBLIC HEALTH (1;0,0,0,0,0,0,1)

#### Biostatistics

- Eng, Jamie Pearl  
The effect of changes in mortality over time on several demographic measures
- HARVARD UNIVERSITY (9;9,0,0,0,0,0,0)
- Mathematics
- Breiner, Moshe  
Essential singularities of entire analytic varieties
- Brooks, Robert  
On the smooth cohomology of groups of diffeomorphisms
- Gabber, Ofer  
Some theorems on Azumaya algebras
- Gillet, Henri  
The applications of algebraic K-theory to intersection theory
- Gross, Benedict  
Elliptic curves with complex multiplication
- Lang, William  
Quasi elliptic surfaces in characteristic three
- Morrison, Ian  
Projective stability of ruled surfaces
- Offner, Carl  
Zeros and growth of entire functions of order one and maximal type with application to the random signs problem
- Peterson, Dale  
Geometry of the adjoint representation of a complex semisimple Lie algebra

## NEW YORK

CORNELL UNIVERSITY (13;0,0,13,0,0,0,0)

### Operations Research and Industrial Engineering

- Barton, Russell R.  
Exoffender post release performance evaluation with related theoretical topics in variable selection and followup intervals for regression models using grouped censored survival data
- Borges, Wagner  
Extreme value theory in triangular arrays with an application to the reliability of fibrous materials
- Engelbrecht-Wiggans, Richard  
On the fair and efficient allocation of indivisible commodities
- Garg, Udai K.  
A queuing network model of multiprogrammed time sharing virtual memory system for performance evaluation
- Hooper, Jeffrey H.  
Selection procedures for ordered families of distributions
- March, Salvatore T.  
Models of storage structures and the design of database records based upon a user characterization
- Phoha, Rajesh C.  
A sequential two sample test for multivariate distributions and asymptotic Bayesian inference in some nonstandard cases
- Provan, John S.  
Decompositions, shellings, and diameters of simplicial complexes and convex polyhedra
- Raanan, Joseph  
The value of the non-atomic game arising from a rate-setting application and related problems
- Shanker, Kripa  
An analysis of two-echelon inventory system for recoverable items
- Shenoy, Prakash  
On game theory and coalition formation
- Tamari, Robert  
Combinatorial algorithms for certain classes of binary matroids
- Weber, Glenn M.  
A solution technique for binary integer programming using matchings on graphs

NEW YORK UNIVERSITY, (3;0,0,3,0,0,0,0)

### Operations Research and System Analysis

- Marpert, Mark  
The application of time series analysis to capital allocation under risk
- Rosenberg, David  
Monopoly inventory models
- Yablon, Marvin  
On the strict monotonicity of minimum Bayes risk and the expected losses as functions of certain separability measures

POLYTECHNIC INSTITUTE OF NEW YORK

(1;0,0,1,0,0,0,0)

### Operations Research and System Analysis

- Koutras, Dionysios  
Bayesian reliability analysis of the alpha distribution with applications

YESHIVA UNIVERSITY (3;2,0,0,0,0,0,1)

### Mathematics

- Goldring, Thomas  
Decomposition theorems for differential forms on Hilbert spaces

- Linden, Orin M.  
Fatou theorems for eigenfunctions of the Laplace-Beltrami operator
- Rusinek, Roza  
Results on the stepping stone model

## PENNSYLVANIA

CARNEGIE-MELLON UNIVERSITY (2;0,0,1,0,1,0,0)

### Mathematics

- Buckwalter, Jeff T.  
Extensions of Fourier's treatment of linear inequality systems
- Marin, Samuel P.  
A finite element method for problems involving the Helmholtz equation in two dimensional exterior regions

TEMPLE UNIVERSITY (3;0,3,0,0,0,0,0)

### Applied Statistics

- Huang  
Some decision theoretic models in clinical trials
- Kim, Jung-Ik  
Randomised response techniques for surveying human populations
- Schneider, Bruce Edward  
Kolmogorov-Smirnov test statistics for the gamma distribution with unknown parameters

## WASHINGTON

WASHINGTON STATE UNIVERSITY (1;1,0,0,0,0,0,0)

### Pure and Applied Mathematics

- Uong, Juhn-Hsiung  
On analysis of the life length of multiple-component coherent structures

## CANADA

McMASTER UNIVERSITY (6;6,0,0,0,0,0,0)

### Mathematics

- Lai, Chee-Chong  
Localization in non-Noetherian rings
- Lee, Sang Shin  
Relative adjointness and preservation of non-existing limits
- Lopez, Manuel  
Matrix and B\*-algebra eigenfunction expansions
- Ong, Eng Meng  
Continuity of positive functionals and representations
- Richards, Ronald  
Noetherian prime rings of Krull dimension one
- Watson, Saleem  
Topological algebras with bases

## 1976—1977 — Supplement

The name below is an addition to the list originally published in the October 1977 NOTICES, and to its supplement on page 56 of the January 1978 NOTICES.

## TEXAS

UNIVERSITY OF HOUSTON (1;1,0,0,0,0,0,0)

### Mathematics

- Sorenson, Peter  
Sums of integral squares in certain cyclotomic number fields

## Visiting Mathematicians – *Supplementary List*

The list of visiting mathematicians includes both foreign mathematicians visiting in the United States and Canada, and U.S. and Canadian mathematicians visiting abroad. Note that there are two separate lists. These supplement the listings in the October and November 1978 issues of the NOTICES.

### Mathematicians Visiting Abroad

<u>Name and Home Country</u>	<u>Host Institution</u>	<u>Field of Special Interest</u>	<u>Period of Visit</u>
Binding, Paul A. (Canada)	University of Warwick, England	Differential Equations and Control Theory	7/78 - 6/79
Flath, Daniel (U.S.A.)	University of Paris VII, France	Automorphic Representations	9/78 - 6/79
Grassmann, E. G. (Canada)	Technical University of Berlin, Federal Republic of Germany; ETH, Zürich, Switzerland	Numerical Techniques of Conformal Mapping	7/78 - 6/79
Lappan, Glenda (U.S.A.)	Chelsea College, University of London, England	Mathematics Education	9/78 - 6/79
Lappan, Peter A. (U.S.A.)	Imperial College of Science and Technology, England	Complex Analysis	9/78 - 6/79
Libera, Richard J. (U.S.A.)	Marie Curie-Skłodowska University, Poland; Freie University of Berlin, Federal Republic of Germany	Complex Analysis	
Milner, E. C. (Canada)	Oxford University, England; Hungarian Academy of Sciences, Hungary; Tel Aviv University, Israel	Finite Matroid Theory	8/78 - 7/79
Sahney, B. N. (Canada)	St. Andrews, Scotland; Birmingham University, England; Sauger University, India	Approximation Theory	7/78 - 6/79
Schassberger, R. (Canada)	Technical University of Berlin, Federal Republic of Germany	Stochastics	7/78 - 6/80
Westbrook, D. R. (Canada)	Brunel University, England; University of Dundee, Scotland	Moving and Free Boundary Problems	8/78 - 7/79
Winter, D. L. (U.S.A.)	Westfield College, England	Group Theory	9/78 - 5/79
Winter, M. J. (U.S.A.)	Chelsea College, University of London, England	Mathematics Education	9/78 - 5/79

### Visiting Foreign Mathematicians

Abramovici, F. (Israel)	University of Calgary	Mathematical Geophysics	9/78 - 5/79
Cowsik, R. C. (India)	Michigan State University	Algebraic Geometry	9/78 - 6/79
Das, Gokulananda (India)	University of California, Santa Barbara	Sequence Spaces, Banach Limits and Absolute Albert Consequence	9/78 - 7/79
Edelstein, M. (Israel)	Michigan State University	Functional Analysis	3/79 - 6/79
Engl, Heinz W. (Austria)	University of Delaware	Nonlinear Functional Analysis	9/78 - 6/79
Gopalsamy, G. (India)	University of Calgary	Applied Mathematics	8/78 - 4/79
Grammatikopoulos, M. K. (Greece)	Mississippi State University	Oscillatory and Nonoscillatory Solutions of Ordinary Differential Equations	
Grattan-Guinness, Ivor O. (United Kingdom)	Institute for Advanced Study	History of Mathematical Physics	1/79 - 4/79
Kodek, Dusan (Yugoslavia)	Princeton University	Digital Signal Processing	9/78 - 6/79
Lipski, Witold (Poland)	University of Illinois	Mathematical Foundations of Data Bases	9/78 - 6/79
Lutterodt, Clement H. (Ghana)	University of South Florida	Approximations of Functions	3/79 - 7/79
Pathak, Visay D. (India)	University of California, Santa Barbara	Algebras of Continuous Functions on Locally Compact Hausdorff Space	9/78 - 9/79
Rodman, L. (Israel)	University of Calgary	Analysis of Operator-Valued Functions	8/78 - 7/79
Schaal, W. (Federal Republic of Germany)	Michigan State University	Number Theory	9/78 - 3/79
Shastri, A. R. (India)	University of Calgary	Topology	8/78 - 4/79
Singh, B. M. (India)	University of Calgary	Elasticity	4/78 - 3/79
Taylor, P. J. (Scotland)	University of Calgary	Numerical Analysis; Partial Differential Equations	1/79 - 6/79
Wehrhahn, Karl H. (Australia)	University of Calgary	Statistics	7/78 - 2/79



# AMS REPORTS AND COMMUNICATIONS

## RECENT ~~Appointments~~ APPOINTMENTS

President R. H. Bing has appointed Murray Gerstenhaber, R. James Milgram, and Cleve B. Moler to the Committee on the Publication Program. Other members of the committee are Donald W. Anderson (chairman), William J. LeVeque and Everett Pitcher.

Robert Gilmer has been appointed to the Committee on Summer Institutes by President R. H. Bing. Continuing members are Robion C. Kirby, Peter E. Ney, Ralph E. Showalter, and Joseph L. Taylor (chairman).

An ad hoc Committee on the Current Yeshiva Situation has been appointed by President R. H. Bing. Its members are Murray Gerstenhaber, Kenneth Hoffman, George D. Mostow, and Shlomo Sternberg (chairman).

A new standing committee, a Committee on Membership, has been appointed by President R. H. Bing. Its members are Philip T. Church, A. Bruce Clarke, Edward R. Fadell (chairman), Paul C. Fife, Frederick W. Gehring, and Jack K. Hale. Its purpose is to study the question of how to increase membership in the group of persons who naturally should belong to the Society but do not.

Victor Kac has been appointed by President R. H. Bing to the AMS Subcommittee of the joint AMS-IMS Committee on Translations. Other members of the subcommittee are Felix Albrecht,

Ivo Babuška, Israel Berstein, Ralph P. Boas, S. S. Chern, Ronald G. Douglas, Eugene B. Dynkin, David Ebin, N. D. Kazarinoff, James King, and Melvyn Nathanson.

A Committee on Policy for Obituaries has been appointed by President R. H. Bing. Its members are Felix E. Browder, Andrew M. Gleason, Paul R. Halmos (chairman), Gustav A. Hedlund, and Everett Pitcher.

John A. Nohel has been appointed by President R. H. Bing to serve as Acting Chairman of the Committee on Human Rights of Mathematicians while Chairman Lipman Bers is on leave. Members of the committee are Lipman Bers, Chandler Davis, Murray Gerstenhaber, Mary W. Gray, John A. Nohel, Vera S. Pless, and Eduardo D. Sontag.

Tellers in the election of 1978 have been appointed by President R. H. Bing. They are W. Wistar Comfort and James D. Reid.

President R. H. Bing, in consultation with President-elect Peter D. Lax, has appointed Calvin C. Moore to be chairman of the Nominating Committee of 1979. Continuing members of the committee are Creighton Buck, Wendell Fleming, and Barbara L. Osofsky. The members elected in the 1978 election are Richard D. Anderson, Judy Green, Paul R. Halmos, and Victor L. Klee, Jr.

## MATHEMATICAL SURVEYS

### *VECTOR MEASURES*

by J. Diestel and J. J. Uhl, Jr.

In this survey the authors endeavor to give a comprehensive examination of the theory of measures having values in Banach spaces. The interplay between topological and geometric properties of Banach spaces and the properties of measures having values in Banach spaces is the unifying theme.

The first chapter deals with countably additive vector measures, finitely additive vector measures, the Orlicz-Pettis theorem and its relatives. Chapter II concentrates on measurable vector valued functions and the Bochner integral.

Chapter III begins the study of the interplay among the Radon-Nikodým theorem for vector measures, operators on  $L$  and topological properties of Banach spaces. A variety of applications is given in the next chapter.

Chapter V deals with martingales of Bochner integrable functions and their relation to dentable subsets of

Banach spaces. Chapter VI is devoted to a measure-theoretic study of weakly compact, absolutely summing and nuclear operators on spaces of continuous functions.

In Chapter VII a detailed study of the geometry of Banach spaces with the Radon-Nikodým property is given. The next chapter deals with the use of Radon-Nikodým theorems in the study of tensor products of Banach spaces. The last chapter concludes the survey with a discussion of the Liapounoff convexity theorem and other geometric properties of the range of a vector measure.

Accompanying each chapter is an extensive survey of the literature and open problems.

Number 15

322 pages

List price \$35.60; member price

ISBN 0-8218-1515-6; LC 77-9625

Publication date: 8-31-77

To order, please specify SURV/15

Prepayment is required for all American Mathematical Society publications.

Send for the book(s) above to: AMS, P. O. Box 1571, Annex Station, Providence, RI 02901

# RELATIONS BETWEEN COMBINATORICS AND OTHER PARTS OF MATHEMATICS

Proceedings of a Symposium in Pure Mathematics

Held at the Ohio State University in March 1978

Edited by Dijen K. Ray-Chaudhuri

*The idea of this symposium was not merely to have another conference on combinatorics, but rather to have a wider-based symposium dealing with the important role combinatorics plays in other areas of mathematics. The Symposium, indeed, fulfilled its designated role very well. Invited speakers brought into focus interconnections between combinatorics on the one hand and geometry, group theory, number theory, special functions, lattice packings, logic, topological embeddings, games, experimental designs, sociological and biological applications on the other hand.*

## CONTENTS

- GEORGE E. ANDREWS, *Connection coefficient problems and partitions*  
DAVID W. BARNETTE, *Path problems and extremal problems for convex polytopes*  
JAMES E. BAUMGARTNER, *Independence proofs and combinatorics*  
RAJ C. BOSE, *Combinatorial problems of experimental design. I: Incomplete block designs*  
FRANCIS BUEKENHOUT, *The geometry of diagrams*  
PETER J. CAMERON, *A combinatorial toolkit for permutation groups*  
JOEL E. COHEN, JÁNOS KOMLÓS, and THOMAS MUELLER, *The probability of an interval graph, and why it matters*  
H.S.M. COXETER, *On R. M. Foster's regular maps with large faces*  
CHARLES F. DUNKL, *Orthogonal functions on some permutation groups*  
PAUL ERDÖS, *Combinatorial problems in geometry and number theory*  
DOMINIQUE FOATA and ADRIANO M. GARSIA, *A combinatorial approach to the Mehler formulas for Hermite polynomials*  
A. M. GARSIA and J. REMMEL, *On the raising operators of Alfred Young*  
BRANKO GRÜNBAUM and G. C. SHEPARD, *Incidence symbols and their applications*  
A. J. HOFFMAN, *Linear programming and combinatorics*  
J. M. GOETHALS and J. J. SEIDEL, *Spherical designs*  
NEIL J. A. SLOANE, *Self dual codes and lattices*  
LOUIS SOLOMON, *Partially ordered sets with colors*  
ALAN P. SPRAGUE, *Incidence structures with specified planes*  
RICHARD P. STANLEY, *Combinatorics and invariant theory*  
JOHN PHILIP HUNEKE, *On the genus of a graph*  
RICHARD M. KARP, *Probabilistic analysis of a canonical numbering algorithm for graphs*

## ORDERS MUST BE PREPAID

1979, 392 pages. List \$23.20; institutional member \$17.40; individual member \$11.60.

Order Code: PSPUM/34

American Mathematical Society

P. O. Box 1571, Annex Station, Providence, Rhode Island 02901

# BOOKS ON COMBINATORICS FROM THE AMS BACKLIST

## CBMS REGIONAL CONFERENCE SERIES IN MATHEMATICS

### ARRANGEMENTS AND SPREADS

Branko Grünbaum

This survey deals mostly with rather elementary mathematics, so elementary in fact that most of its results and problems are (or at least should be) understandable to undergraduates. It was written out of the conviction that many neglected aspects of elementary geometry deserve a wider dissemination because of their inherent beauty and interest, and for the inspiration and understanding they can impart to students and mathematicians. Throughout the survey, many conjectures are explicitly stated, and many additional problems are hinted at. The subject obviously offers extremely varied opportunities for research. Due to the elementary nature of the topic, the solutions of many of the open problems will probably require more inspiration than erudition, but that is a hallmark of beauty in mathematics.

1972, 111 pages; list \$6.80; individual \$5.10  
(LC 71-38926; ISBN 0-8218-1659-4). Code: CBMS/10

## COLLOQUIUM PUBLICATIONS

### THEORY OF GRAPHS

Oystein Ore

New fields of application, such as game theory and programming, communications theory, electrical networks and switching circuits, and problems from biology and psychology, have given an intense stimulus to the development of graph theory in the past twenty years. This volume gives an almost complete treatment of the basic concepts and the results of particular systematic interest; the second volume will deal with such topics as the four-color conjecture, the theory of flow, electrical networks, and games.

The fifteen chapters of the first volume present, in a well-organized setting, the results of Cayley, Ramsey, Frucht, Hall, Mann, Ryser, Dirac and many other outstanding researchers in the subject.

This volume is reprinted without changes from the first edition of 1962.

1962; reprinted 1974, 270 pages; list \$14.80; institutional member \$11.10; individual member \$7.40  
(LC 61-15687; ISBN 0-8218-1038-3). Code: COLL/38

## PROCEEDINGS OF SYMPOSIA

### IN APPLIED MATHEMATICS

#### COMBINATORIAL ANALYSIS

Edited by Richard Bellman and Marshall Hall, Jr.

##### Contents

- Marshall Hall, Jr., *Current studies on combinatorial designs*  
R. H. Bruck, *Quadratic extensions of cyclic planes*  
D. R. Hughes, *On homomorphisms of projective planes*  
A. A. Albert, *Finite division algebras and finite planes*  
L. J. Paige and C. B. Tompkins, *The size of the  $10 \times 10$  orthogonal Latin square problem*  
R. P. Dilworth, *Some combinatorial problems on partially ordered sets*  
R. J. Walker, *An enumerative technique for a class of combinatorial problems*  
A. L. Whiteman, *The cyclotomic numbers of order ten*

A. J. Hoffman, *Some recent applications of the theory of linear inequalities to extremal combinatorial analysis*

A. W. Tucker, *A combinatorial equivalence of matrices*

H. W. Kuhn, *Linear inequalities and the Pauli principle*

H. J. Ryser, *Compound and induced matrices in combinatorial analysis*

Marvin Marcus and Morris Newman, *Permanents of doubly stochastic matrices*

A. M. Gleason, *A search problem in the n-cube*

D. H. Lehmer, *Teaching combinatorial tricks to a computer*

J. D. Swift, *Isomorph rejection in exhaustive search techniques*

Olga Taussky and John Todd, *Some discrete variable computations*

R. E. Gomory, *Solving linear programming problems in integers*

Richard Bellman, *Combinatorial processes and dynamic programming*

Murray Grerstenhaber, *Solution of large scale transportation problems*

Robert Kalaba, *On some communication network problems*

J. D. Foulkes, *Directed graphs and assembly schedules*

E. N. Gilbert, *A problem in binary encoding*

M. M. Flood, *An alternative proof of a theorem of König as an algorithm for the Hitchcock distribution problem*

1960, 311 pages; list \$13.60; institutional member \$10.20; individual member \$6.80  
(ISBN 0-8218-1310-2). Code: PSAPM/10

## PROCEEDINGS OF SYMPOSIA

### IN PURE MATHEMATICS

#### COMBINATORICS

Edited by T. S. Motzkin

##### Contents

- A. O. L. Atkin and H. P. F. Swinnerton-Dyer, *Modular forms on noncongruence subgroups*  
R. C. Bose, *Selfconjugate tetrahedra with respect to the Hermitian variety  $x_0^3 + x_1^3 + x_2^3 + x_3^3 = 0$  in  $PG(3, 2^2)$  and a representation of  $PG(3, 3)$*   
M. S. Cheema and T. S. Motzkin, *Multipartitions and multipermutations*  
Henry H. Crapo and Gian-Carlo Rota, *Simplicial geometries*  
P. Erdős, *Problems and results in combinatorial analysis*  
Basil Gordon, *Multirowed partitions with strict decrease along columns (notes on plane partitions. IV)*  
R. L. Graham and B. Rothschild, *Rota's geometric analogue to Ramsey's theorem*  
Alfred W. Hales, *Combinatorial representations of abelian groups*  
Marshall Hall, Jr., *Designs with transitive automorphism groups*  
Haim Hanani, *Truncated finite planes*  
Alexander Hurwitz, *Homogeneous 0-1 matrices*  
Victor Klee, *The greedy algorithm for finitary and cofinitary matroids*  
Daniel Kleitman, *Collections of subsets containing no two sets and their union*  
N. S. Mendelsohn, *A combinatorial method for embedding a group in a semigroup*  
Leo Moser, *Asymptotics of tournament scores*

Theodore S. Motzkin, *Sorting numbers for cylinders and other classification numbers*  
 E. T. Parker, *Pathological Latin squares*  
 Richard Rado, *Some problems in the partition calculus*  
 D. K. Ray-Chaudhuri and Richard M. Wilson, *Solution of Kirkman's schoolgirl problem*  
 Bruce Rothschild, *A generalization of Ramsey's theorem*  
 E. G. Straus, *Nonaveraging sets*  
 J. D. Swift, *On  $(k, l)$ -coverings and disjoint systems*  
 Olga Tausky, *(1, 2, 4, 8)-Sums of squares and Hadamard matrices*  
 W. T. Tutte, *Dichromatic sums for rooted planer maps*  
 1971, 255 pages; list \$24.00; institutional member \$18.00; individual member \$12.00  
 (LC 74-153879; ISBN 0-8218-1419-2). Code: PSPUM/19

**MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY**

**ON THE ENUMERATION OF NON-PLANAR MAPS**

*William G. Brown*

A map is a linear graph imbedded in a closed surface so that the components of its complement are simply connected. The Memoir considers problems of enumerating by the number of edges several classes of maps on the real projective plane and the torus. The techniques developed generalize those used by W. T. Tutte and the author in the enumerations of various classes of maps on the sphere.

1966, 42 pages; list \$4.40; institutional member \$3.30; individual member \$2.20  
 (ISBN 0-8218-1265-3). Code: MEMO/65

**THE UNDECIDABILITY OF THE DOMINO PROBLEM**

*Robert Berger*

A domino set consists of a finite number of different kinds of square plates (with an unlimited number of each kind), to be assembled on an infinite plane according to a set of adjacency rules. If the plane can be covered, the domino set is called solvable. The Domino Problem is that of deciding, for an arbitrary domino set, whether or not it is solvable. In this book the Domino Problem is shown to be undecidable. The proof consists mainly of a method of constructing, for any given member of a certain set of Turing machines with an undecidable halting problem, a domino set which is solvable if and only if the Turing machine never halts.

1966, 72 pages; list \$4.40; institutional member \$3.30; individual member \$2.20  
 (ISBN 0-8218-1266-1). Code: MEMO/66

**PROPERTIES OF PLANAR GRAPHS WITH UNIFORM VERTEX AND FACE STRUCTURE**

*J. Malkevitch*

**Contents**

1. Definitions and introduction
2. Planar graphs with almost uniform vertex and face structure

3. Coded paths in planar graphs

Appendix I. Tables

1970, 116 pages; list \$4.40; institutional member \$3.30; individual member \$2.20  
 (ISBN 0-8218-1299-8). Code: MEMO/99

**ORDERED STRUCTURES AND PARTITIONS**

*Richard P. Stanley*

A general theory is developed for the enumeration of order-reversing maps of partially ordered sets  $P$  (usually finite) into chains. This theory comprehends many apparently disparate topics in combinatorial theory, including (1) ordinary partitions, (2) ordered partitions (compositions), (3) plane and multidimensional partitions, with applications to Young tableaux, (4) the Eulerian numbers and their refinements, (5) the tangent and secant numbers (or Euler numbers) and their refinements, (6) the indices of permutations, (7) trees, (8) stacks, and (9) protruded partitions, with applications to the Fibonacci numbers. The main tool used is that of generating functions. In particular, the influence of the structure of  $P$  on the form of the generating functions under consideration is studied. As an application, new combinatorial relationships between a finite partially ordered set  $P$  and its distributive lattice of order ideals are derived.

1972, 104 pages; list \$4.00; institutional member \$3.00; individual member \$2.00  
 (ISBN 0-8218-1819-2). Code: MEMO/119

**FACING UP TO ARRANGEMENTS: FACE-COUNT FORMULAS FOR PARTITIONS OF SPACE BY HYPERPLANES**

*Thomas Zaslavsky*

The topic of this Memoir is the usefulness of matroid theory (combinatorial incidence geometry) in the study of arrangements of hyperplanes in Euclidean and projective space. The number of faces of each dimension of an arrangement is found to be a function of the (incomplete) matroid lattice of intersections of the hyperplanes, expressible in terms of the Möbius function of that lattice. Both topological and combinatorial proofs are given. A similar function for the number of bounded faces of a Euclidean arrangement is proved combinatorially. Variants of the problem and solution are compared, including: the classical enumerations of faces, edges, and vertices of an arrangement of lines; Winder's degeneracy approach; partitions of a point set by hyperplanes; and enumerations of the faces of a zonotope. It is shown that the number of bounded regions of a Euclidean arrangement is independent of projective transformation. It equals Crapo's beta invariant, whose properties imply two geometric criteria for the existence of bounded regions and the simple dimensionality of the union of all bounded regions and the simple dimensionality of the union of all bounded faces.

1975, 102 pages; list \$5.60; institutional member \$4.20; individual member \$2.80  
 (ISBN 0-8218-1854-6). Code: MEMO/154

**ORDER FOR BOOKS IN COMBINATORICS**

Please place your mailing label or name and address here

PSPUM/34	\$ _____	MEMO/65	\$ _____
CBMS/10	_____	MEMO/66	_____
COLL/38	_____	MEMO/99	_____
PSAPM/10	_____	MEMO/119	_____
PSPUM/19	_____	MEMO/154	_____
		Total enclosed \$	_____

Prepayment is required for all American Mathematical Society publications.

Send for the book(s) above to: AMS, P. O. Box 1571, Annex Station, Providence, RI 02901

# ABSTRACTS

The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form and are based on the AMS (MOS) Subject Classification Scheme (1970). Abstracts for which the author did not indicate a category are listed under miscellaneous.

Abstracts are printed from typed copy submitted by the authors, on special forms available from most mathematics departments and from the headquarters office of the Society. The reproduction material for this section is thus produced on many different typewriters. Authors are urged to submit abstracts in *black* type, preferably using a reasonably *new* BLACK ribbon (please avoid gray ribbons), in order to improve the appearance of the abstract section; when type is gray rather than black, the printing is light and some characters may print only in part or not at all.

\* Indicates that preprints are available from author. • Indicates invited addresses.

Abstracts of papers for	Appear on page
<i>Presentation by title</i>	A-7
<i>Past meetings (late papers)</i>	A-19
<i>85th Annual Meeting in Biloxi, January 24–27, 1979</i>	A-20
<b>Subjects represented†</b>	<b>Page</b>
<i>A Algebra and Theory of Numbers</i>	A-7
<i>B Analysis</i>	A-8
<i>C Applied Mathematics</i>	A-12
<i>D Geometry</i>	A-15
<i>E Logic and Foundations</i>	A-15
<i>F Statistics and Probability</i>	A-16
<i>G Topology</i>	A-17
<i>H Miscellaneous</i>	—

†In addition, abstracts of papers presented at annual or summer meetings are published in order according to the subject classification scheme which appears with that meeting's abstracts.

## Abstracts of Papers Presented to the Society

The abstracts printed in this section were accepted by the American Mathematical Society for written presentation. An individual may present only one abstract by title in any one issue of the NOTICES, but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for the same issue.

### *Algebra and Theory of Numbers (05, 06, 08, 10, 12–18, 20)*

\*79T-A1 V. SITA RAMAIAH, Andhra University, Waltair 530003, India. The number of semi-A-k-free divisors of m which are prime to n.

Let  $k \geq 2$ ,  $u \geq 1$  and  $n \geq 1$  be integers with  $(u, n) = 1$ . Let  $m \geq 1$  be an integer variable and  $x$  a real variable  $\geq 3$ . Let  $A$  be any regular convolution. An integer  $m$  is called semi-A-k-free if  $p^k \in A(m)$  for no prime  $p$ . Let  $\tau_{A,k}^{S^*}(m;n)$  denote the number of semi-A-k-free divisors of  $m$  which are prime to  $n$ . In this paper, we obtain an asymptotic formula for the sum  $\sum_{m \leq x; (m,u)=1} \tau_{A,k}^{S^*}(m;n)$  with error term  $E_{k,u}(x;n)$ ,

where  $E_{k,u}(x;n) = O(x^{1/k} \delta(x) \sigma_{-\alpha}^*(u) \sigma_{-\alpha}^*(n) \sigma_{-1+\epsilon}^*(n))$  or  $O(x^\alpha \sigma_{-\alpha}^*(u) \sigma_{-\alpha}^*(n) \sigma_{-1+\epsilon}^*(n))$  according as  $k = 2, 3$  or  $k \geq 4$ ;  $\epsilon$  is any positive real,  $\delta(x) = \exp\{-H \log^{3/5} x (\log \log x)^{-1/5}\}$ ,  $H$  being a positive constant,  $\alpha$  is the number appearing in the Dirichlet divisor problem and  $\sigma_s^*(m)$  is the sum of the  $s$ th powers of square-free divisors of  $m$ . We further improve the above error term on the assumption of the Riemann hypothesis.

(Received June 15, 1978.) (Author introduced by Professor D. Suryanarayana).

79T-A2 RONSON J. WARNE, University of Alabama in Birmingham, Birmingham, Alabama 35294. Strongly regular semigroups. I. Preliminary report.

Is  $X$  a semigroup, let  $E(X)$  denote the set of idempotents of  $X$ . A regular semigroup  $S$  s.t.  $e, f, g \in E(S)$ ,  $e \geq f$  and  $e \geq g$  imply  $fg = gf$  is termed a locally inverse semigroup. If, furthermore, for each  $e \in E(S)$ , there exists a subgroup  $G_e$  of  $S$  with identity  $e$  and  $\cup(G_e : e \in E(S))$  is a subsemigroup, we term  $S$  a strongly regular semigroup. Let  $Y$  be a semilattice. Let  $B = \{(k, s, n) : k, s, n \in Y, k \geq n, s \geq n\}$  and define  $(k, x, n) \cdot (r, v, m) = (k, v, nm)$ . Let  $(T, *)$  be a locally inverse semilattice  $Y$  of completely simple semigroups  $(T_y : y \in Y)$  s.t.  $E(T) \supseteq B$  and  $B \cap T_y \neq \emptyset$  for each  $y \in Y$  and s.t., for each  $b, c \in B$ ,  $b \cdot c$  is the identity of the maximal subgroup of  $(T, *)$  containing  $b * c$ . Let  $(V, \circ)$  be a regular semigroup s.t.  $E(V) = B$ . Suppose  $T \cap V = \cup(G_b : b \in B)$  where  $(G_b, \circ) ((G_b, *))$  is a subgroup (maximal subgroup) of  $(V, \circ) ((T, *))$  with identity  $b$  and assume  $b \circ c = b * c$  for  $b, c \in T \cap V$ . Let  $l_a(r_a)$  denote the  $\mathcal{L}$ -class ( $\mathcal{R}$ -class) of  $(T, *)$  containing  $a \in T$ . Let  $W = \{(i, b, j) : b \in V, i \in l_a, j \in r_a\}$ . Define an equivalence relation  $\sim$  on  $W$  by the rule  $(i, b, j) \sim (p, c, q)$  if and only if there exists  $u, v \in V$  s.t.  $u \in r_{c \circ c^{-1}} \cap l_{b \circ b^{-1}}$ ,  $v \in r_{b^{-1} \circ b} \cap l_{c^{-1} \circ c}$ ,  $i = p * u$ ,  $j = v * q$  and  $u \circ b \circ v = c$ . Let  $[i, b, j]$  denote the  $\sim$  class containing  $(i, b, j)$ . Let  $(Y, T, V)$  denote  $W/\sim$  under the multiplication  $[i, b, j][r, c, s] = [i * ((b \circ c) \circ (b \circ c)^{-1}), b \circ (j * r) \circ c, ((b \circ c)^{-1} \circ (b \circ c)) * s]$ . Theorem.  $(Y, T, V)$  is a strongly regular semigroup and, conversely, every strongly regular semigroup is isomorphic to some  $(Y, T, V)$ . (Received September 18, 1978.)

79T-A3 O. M. KOSHELEVA, USSR 192123 Leningrad D-123 poste restante. On a general notion of boundedness. Preliminary report.

A definition of boundedness for concrete categories containing the set of all reals  $R$  is proposed: object  $A$  is called bounded iff for any morphism  $f : A \rightarrow R$ ,  $f(A)$  is bounded; object  $B \subset A$  is called bounded in A iff for any morphism  $f : A \rightarrow R$  the set  $f(B)$  is bounded. This definition is justified by the following theorems: Set (sets and functions)  $A$  is bounded  $\Leftrightarrow$  it is finite;  $A$  is bounded in  $B$  iff  $A$  is finite. Top (normal separable topological spaces & continuous mappings)  $A$  is bounded  $\Leftrightarrow$  it is compact;  $A$  is bounded in  $B \Leftrightarrow$  it is relatively compact in  $B$  (i.e. closure  $A$  is compact). Ord (linearly ordered separable sets & monotone functions):  $A$  is bounded  $\Leftrightarrow$  it has maximal & minimal elements;  $A$  is bounded in  $B \Leftrightarrow \exists b, b' \in B \forall a \in A (b < a < b')$  (i.e.  $A$  is bounded in usual sense). (Received September 18, 1978.) (Author introduced by V. Ja. Kreĭnovič).

\*79T-A4 RALPH FREESE, Vanderbilt University, Nashville, Tennessee 37235. Free modular lattices.

The modular lattice freely generated by a five element set has an unsolvable word problem. The proof uses the techniques used in showing the variety of modular lattices is not generated by its finite members and some results from ring theory. (Received September 21, 1978.)

\*79T-A5 RALPH FREESE AND J. B. NATION, Vanderbilt University, Nashville, Tennessee 37235. The generalized word problem for finitely presented lattices.

We show that that the generalized word problem for finitely presented lattices is solvable. This is done by showing that if  $F$  is a finitely presented lattice,  $U$  is a sublattice of  $F$ , and  $v \in F - U$ , then there is a homomorphism  $f : F \rightarrow L$  with  $L$  finite such that  $f(v) \notin f(U)$ . For background on these problems, see T. Evans, Word problems, Bulletin AMS, 84(1978), 789-802. (Received September 21, 1978.)

\*79T-A6 JOHN ROYLE, McMaster University, Hamilton, Ontario, Canada. P.I. Algebras and Finite Generation of Ideals.

Let  $R = k[x_1, \dots, x_m]_{n \times n}$  denote the  $k$ -algebra of  $m$  generic  $n \times n$  matrices, over a field  $k$ . Are the prime ideals of  $R$  finitely-generated? L. Small asked this question, in his article in the "Proceedings of the Conference on Ring Theory", Park City, Utah, 1971. It is well-known that the maximal ideals are

finitely-generated, even as one-sided ideals. Theorem: Suppose  $P$  is a prime ideal of  $R$  such that  $R/P$  is one-dimensional. Then  $P$  is finitely-generated. The fact that, for such primes  $P$ ,  $R/P$  is a finite module over its center was observed by W. Schelter and also by Mme. M.P. Malliavin-Brameret. So the theorem follows from the following: Proposition: Let  $S = A\{x_1, \dots, x_m\}$  be any finitely-generated algebra, over a commutative Noetherian ring  $A$ , and  $I$  an ideal of  $S$  such that  $S/I$  is a finite module over its center. Then  $I$  is finitely-generated. (Received September 21, 1978.) (Author introduced by Professor Bruno J. Mueller).

\*79T-A7 BRADLEY W. JACKSON, The Pennsylvania State University, University Park, Pennsylvania 16802. Embeddings of Complete Partite Graphs.

Using a device called a flow diagram it is sometimes possible to derive a triangular embedding for the complete partite graph  $K(n_1, n_2, \dots, n_i)$  from a triangular embedding for  $K(n_1, n_2, \dots, n_i)$  when  $n$  is odd. Triangular embeddings exist for the complete graph  $K(1, 1, \dots, 1)$  or  $K_i$  when  $i$  is congruent to zero or one modulo three. It can easily be shown that triangular embeddings of  $K(n, n, \dots, n)$  or  $K_{n(i)}$  are not possible unless  $i$  is congruent to zero or one modulo three or if  $n$  is divisible by three. Triangular embeddings are given for  $K_{n(i)}$  for each  $i = 12m+1, 12m+3, 12m+4, 12m+6, 12m+7, 12m+9, 12m+10$  and either for  $n$  equal to all odd numbers or all odd numbers relatively prime to three. From triangular embeddings of  $K_{2(i)}$  by Jungerman and Ringel it is possible to derive triangular embeddings of  $K_{2(i)}$  for some even numbers  $n$ . The author also derives some embeddings of  $K_{3(i)}$  and uses them to derive embeddings for  $K_{n(i)}$  when  $n$  is divisible by three and  $i$  is arbitrary. Finally triangular embeddings are derived for  $K((i-2)n, n, n, \dots, n)$  whenever  $n$  is odd and  $i$  is not congruent to two modulo four. (Received September 21, 1978.)

\*79T-A8 DUNCAN A. BUELL, Bowling Green State University, Bowling Green, Ohio 43403 and KENNETH S. WILLIAMS, Carleton University, Ottawa, Ontario, Canada K1S 5B6.

An octic reciprocity law of Scholz type.

The authors (Amer. Math. Monthly 85 (1978), 483-484) have conjectured that if  $p$  and  $q$  are distinct primes satisfying  $p \equiv q \equiv 1 \pmod{8}$ ,  $(p|q)_4 = (q|p)_4 = +1$ , then

$$\left(\frac{p}{q}\right)_8 \left(\frac{q}{p}\right)_8 = \begin{cases} \left(\frac{\epsilon_p}{q}\right)_4 \left(\frac{\epsilon_q}{p}\right)_4, & \text{if } N(\epsilon_{pq}) = -1, \\ (-1)^{h(pq)/4} \left(\frac{\epsilon_p}{q}\right)_4 \left(\frac{\epsilon_q}{p}\right)_4, & \text{if } N(\epsilon_{pq}) = +1, \end{cases}$$

where  $\epsilon_p$  is the fundamental unit of  $\mathbb{Q}(\sqrt{p})$ ,  $N(\epsilon_{pq})$  denotes the norm of the unit  $\epsilon_{pq}$ , and  $h(pq)$  is the class number of  $\mathbb{Q}(\sqrt{pq})$ . Using some results of Bucher (Mitt. Naturforsch. Ges. Luzern 14 (1943), 1-18), the authors have been able to prove this conjecture. (Received September 22, 1978.)

\*79T-A9 Kenneth S. Williams, Carleton University, Ottawa, Ontario, Canada K1S 5B6.

Remark on the divisibility of the class number of  $\mathbb{Q}(\sqrt{-p_1 \dots p_n})$  by  $2^{n+2}$ .

Let  $q = p_1 \dots p_n$ , where  $n$  is 2 or an odd number and the  $p_i$  are primes congruent to 1 modulo 8 such that  $(p_i | p_j) = -1$  ( $1 \leq i < j \leq n$ ). Classical results of Dirichlet and Tano guarantee that the norm of the fundamental unit  $\epsilon_q$  of  $\mathbb{Q}(\sqrt{q})$  is  $-1$ , so  $\epsilon_q = T + U\sqrt{q}$ , where  $T \equiv 0 \pmod{4}$ . Further, an old result of Plancherel asserts that the class number  $h(-q)$  of  $\mathbb{Q}(\sqrt{-q})$  satisfies  $h(-q) \equiv 0 \pmod{2^{n+1}}$ . Recently Kaplan (J. Reine Angew. Math., 283|284

(1976), 313-363: Prop. B'  $\frac{1}{16}$  and Thm. 2) has shown that  $(2|q)_4 (q|2)_4 = (-1)^{h(-q)} / 2^{n+1}$ .

Applying Kaplan's argument (Acta Arith., 32 (1977), 239-244: p. 240) to  $q$  instead of  $p$ , we obtain  $(2|q)_4 (q|2)_4 = (-1)^{T/4}$ . Putting these together we obtain the elegant result

$$h(-q) \equiv 0 \pmod{2^{n+2}} \iff T \equiv 0 \pmod{8},$$

of which the case  $n = 1$  is due to Lehmer (J. Reine Angew. Math., 250 (1971), 42-48: Thm. 2A). (Received September 22, 1978.)

\*79T-A10 BRIAN J. DAY, Department of Pure Mathematics, University of Sydney, N.S.W. 2006, Australia. Presentable objects in a closed category.

Let  $V$  be a symmetric monoidal closed category which is complete and cocomplete and contains an object  $P$  which is projective with respect to strong epimorphisms of the form  $X_P \rightarrow C$  in  $V$ . Then the category of  $P$ -presentable objects is a full coreflexive subcategory of  $V$  and is consequently symmetric monoidal closed with respect to the tensor structure induced by  $V$ , provided  $I \in V$  is  $P$ -presentable (here an object  $C \in V$  is called  $P$ -presentable if there exists a coequaliser diagram  $Y_P \rightrightarrows X_P \rightarrow C$ ). (Received September 27, 1978.)

79T-A11 E.T. SCHMIDT, Mathematical Institute of the Hungarian Academy of Sciences H-1053 Budapest, Reáltanoda u.13-15. On splitting modular lattices.

Some necessary conditions are given for a modular lattice to be splitting in the variety of all modular lattices. Theorem 1. Let  $(v, x, y, z, u)$  be an isometric diamond of a splitting modular lattice  $S$ . If  $y$  is double-irreducible then the quotients  $x/v$  and  $z/v$  are not projective in the sublattice  $S_y = S \setminus \{y\}$ . Corollary (A. Day, C. Hermann and R. Wille)  $M_4$  is not splitting modular. Theorem 2. Let  $A$  be an ideal and let  $B$  be a filter of the finite subdirectly irreducible modular lattice, such that  $A \cap B = C$  is a chain. If  $C$  contains two prime quotients which are projective in  $A$  and in  $B$  then  $L$  is not finitely projected. Theorem 3. A planar subdirectly irreducible modular lattice  $L$  is a splitting modular if and only if  $L$  does not contain a sublattice isomorphic to  $M_4$  or a diamond circle. (Received September 27, 1978.) (Author introduced by Professor A. F. Pixley).

\*79T-A12 PAMELA A. FERGUSON, University of Miami, Coral Gables, Florida 33124. Finite Complex Linear Groups of Degree  $\frac{q-1}{2}$ .

Assume  $G$  is a finite group and  $P > 1$  is a Sylow  $p$  subgroup of  $G$  such that  $C(x) = C(P)$  for all  $x \in P^\#$ . Let  $|P| = q$ . If  $G$  has a faithful complex representation of degree less than  $\frac{q-1}{2}$ , then Sibley has shown that  $P \triangleleft G$ . In this paper it is shown that if  $\frac{N_G(P)}{C_G(P)}$  is cyclic and  $G$  has a faithful complex representation of degree  $\frac{q-1}{2}$  then either  $P \triangleleft G$  or  $\frac{G}{Z(G)} \cong \text{PSL}(2, q)$ . (Received September 27, 1978.)

79T-A13 WILLIAM C. POWELL, Ohio State University, Columbus, OH 43210. Representation of distributive complete Heyting algebras by topologies of complete Boolean algebras.

We show that any distributive complete Heyting algebra (cHa) is isomorphic to a topology of a complete Boolean algebra (cBa). Since any Heyting algebra has a distributive completion, the representation of Heyting algebras by topological Boolean algebras follows. Our proofs do not appeal to any form of the axiom of choice. Furthermore, these results may be reformulated in a classically equivalent form so that the proofs do not require the assumption of the law of excluded middle.

A topology on a cHa  $\mathbb{H} = \langle H, \leq \rangle$  is a subset of  $H$  containing the maximum element of  $\mathbb{H}$  and closed under finite meets and arbitrary joins. Any topology with the induce ordering is again a cHa.  $\mathbb{H}$  is distributive if  $a \vee \bigwedge B = \bigwedge \{a \vee b : b \in B\}$  for  $a \in H$  and  $B \subseteq H$ . We say that the pseudo complement  $-$  of a cHa is set theoretic if for some set  $I$ ,  $H \cong \mathcal{P}(I)$  and  $-a = \{i \in I : i \notin a\}$  for  $a \in H$ . We say equality is stable for a cHa if  $\forall a, b \in H (\neg \neg a = b \rightarrow a = b)$ .



Theorem: Any topology on a cBa with a set theoretic complement is a distributive cHa with stable equality. Conversely, any distributive cHa with stable equality is isomorphic to a topology on a cBa with a set theoretic complement. (Received September 28, 1978.)

\*79T-A14 Abraham A. Klein, Tel-Aviv University, Tel-Aviv, Israel. A commutativity theorem.

Herstein and independently Annan'in and Zyabko have proved that if  $R$  is a ring in which for any two elements  $a, b \in R$  there exist positive integers  $m = m(a,b)$ ,  $n = n(a,b)$ , such that  $a^m b^n = b^n a^m$ , then  $R$  has a nil commutator ideal. Linda Neumann has proved, in her Ph.D. thesis, that if  $R$  is a ring with 1 in which for any three elements  $a, b, c \in R$  there exists a positive integer  $m = m(a,b,c)$ , such that  $[[a^m, b^m], c^m] = 0$ , then  $R$  has a nil commutator ideal. For  $k \geq 2$ , we define inductively:  $[x_1, x_2, \dots, x_k] = [[x_1, \dots, x_{k-1}], x_k]$ . Our result is: Let  $R$  be a ring with 1 and  $k$  a fixed integer  $\geq 2$ . If for any  $a_1, a_2, \dots, a_k \in R$ , there exist  $m_i = m_i(a_1, \dots, a_k)$ ,  $i = 1, \dots, k$ , such that  $[a_1^{m_1}, \dots, a_k^{m_k}] = 0$ , then  $R$  has a nil commutator ideal. (Received September 28, 1978.)

79T-A15 SAAD MOHAMED, Kuwait University, Kuwait and SURJEET SINGH, Guru Nanak Dev University, Amritsar, India. Weak q-rings with zero singular ideal.

A ring  $R$  is called a (right) weak q-ring if every right ideal not isomorphic to  $R_R$  is quasi-injective. Let  $R$  be a ring with zero singular ideal. The main theorem proved is:  $R$  is a weak q-ring iff (i)  $R$  is a q-ring or (ii)  $R = \begin{bmatrix} D & D \\ 0 & D \end{bmatrix}$  for some division ring  $D$  or (iii) every right ideal of  $R$ , not isomorphic to  $R_R$  is completely reducible. It is further shown that every weak q-ring with zero singular ideal is right hereditary. (Received September 28, 1978.)

\*78T-A16 C. VINSONHALER and W. WICKLESS, University of Connecticut, Storrs, Connecticut 06268. Injective hulls of p-groups. Preliminary report.

Let  $G$  be a separable p-group,  $R = \text{End } G$  and  $I$  be the injective hull of  $R^G$ . Let  $\bigoplus_{\alpha \in J} \langle b_\alpha \rangle$  be a basic subgroup of  $G$  and  $e_\alpha$  be the projection of  $G$  onto  $\langle b_\alpha \rangle$ . Then, as an abelian group,  $I \cong \prod_{\alpha \in J} \langle b_\alpha \rangle \oplus A$ , where  $A = \{x \in I \mid e_\alpha x = 0, \forall \alpha \in J\}$ . As an abelian group,  $A \cong \text{Hom}_{\mathbb{Z}}[RSR/RS, G]$  where  $S = \{e_\alpha \mid \alpha \in J\}$ . If  $G$  is infinite,  $|A| = 2^{2^{|J|}}$ .  $A$  is torsion iff  $G$  is bounded.  $A$  is torsion free iff  $G$  is unbounded with all Ulm invariants finite. (Received October 5, 1978.)

79T-A17 DAVID ZEITLIN, 1650 Vincent Ave. North, Minneapolis, Minnesota, 55411. Parametric solutions for two equal sums of  $3n+2$  squares.

For each  $n$ , let  $P_1 > 0$ ,  $B_1 > 0$ ,  $i = 1, 2, \dots, n$ , be a set of  $2n$  distinct integers so that  $S = \sum_{i=1}^n P_i^2 - \sum_{i=1}^n B_i^2 \neq 0$ . Let  $W_{k+2} = W_{k+1} + SW_k$ ,  $k = 0, 1, \dots$ , where  $W_0$  and  $W_1$  are integers. Then, for  $k = 0, 1, \dots$ , the two equal sums of  $3n+2$  squares are given by

$$(*) \sum_{i=1}^n (SP_i W_k)^2 + \sum_{i=1}^n (P_i W_{k+1})^2 + \sum_{i=1}^n (B_i W_{k+2})^2 + W_{k+1}^2 + W_{k+2}^2 = \sum_{i=1}^n (SB_i W_k)^2 + \sum_{i=1}^n (B_i W_{k+1})^2 + \sum_{i=1}^n (P_i W_{k+2})^2 + ((S+1)W_{k+1})^2 + W_{k+2}^2.$$

Moreover, the two sums of the base variables are equal for all  $n$  and  $k$ . Ex. 1. For  $n=1$ ,  $k=0$ ,  $(*)$ , with  $S = P^2 - B^2$ , gives the identity (1)  $((P^3 - PB^2)W_0)^2 + (PW_1)^2 + (BW_1 + (B^2 - B^3)W_0)^2 + W_1^2 + ((1+P^2 - B^2)W_1 + (P^2 - B^2)W_0)^2 = ((B^3 - B^3)W_0)^2 + (BW_1)^2 + (PW_1 + (P^3 - PB^2)W_0)^2 + ((1+P^2 - B^2)W_1)^2 + (W_1 + (P^2 - B^2)W_0)^2$ .

REMARK. Chapter 24 in L.E. Dickson, *History of the Theory of Numbers, Vol. 2, Diophantine Analysis*, Chelsea, 1952, gives extensive references to papers on identities similar, but not identical, to  $(*)$ . An additional result on two equal sums of squares is forthcoming. For two equal sums of cubes, see these NOTICES 25(1978), A-480, A-229; 24(1977), A-610, A-528, A-414, A-7; 23(1976), A-274. For two equal sums of biquadrates, see these NOTICES 23(1976), A-642, A-574, A-481, A-423, A-353. For two equal sums of fifth powers, see these NOTICES 24(1977), A-367, A-233. (Received October 6, 1978.)

\*79T-A18 ALBERT A. MULLIN, 475-B Cooke Dr., Redstone Arsenal, AL 35808. On prime-power factors of consecutive integers.

Lemma 1. There exist infinitely many sextuples of consecutive positive integers such that each canonical prime-power factorization of the members of each such sextuple has a prime-power factor with exponent exceeding 1; a fortiori, this result holds for  $n$ -tuples,  $2 \leq n < 6$ . E.g., consider the sextuple  $\{30923, \dots, 30928\}$ . Clearly, the resulting prime-power factors cannot be either all consecutive integers (difference 1) or all equal (difference 0). Indeed, Lemma 2. Each such sextuple satisfies the condition that no two resulting prime-power factors (exponents exceeding 1) are equal. Conjecture. There does not exist even one triple (a fortiori,  $n$ -tuple,  $n > 3$ ) of consecutive integers all of whose resulting prime-power factors (exponents exceeding 1) are in arithmetic progression. Briefly, such resulting prime-power factors are progression-free; quickly proved for the case of common difference 1. (Received October 10, 1978.)

\*79T-A19 Bill Jacob, Princeton University, Princeton N.J. 08540. On the Structure of Pythagorean Fields.

Let  $K$  be a Pythagorean field. For each fan  $T$  of  $K$  we give an explicit construction of certain valuations compatible with  $T$ . Necessary and sufficient conditions for these valuations to be 2-Henselian are discussed. In terms of M. Marshall's theory for spaces of orderings we obtain: 1) If the space of  $K$  is a proper group extension (i.e. the Witt ring of  $K$  is a group ring), then  $\#(K^*/K^{*2}) \leq 4$  or  $K$  carries a non trivial 2-Henselian valuation. 2) For each direct summand of the space of  $K$  there is a valuation of  $K$  whose 2-Henselization has that space of orders. Other applications include: 3) Suppose  $K^*/K^{*2}$  is finite and that  $K^{*4} + K^{*4} = K^{*4}$ . Then  $K$  is superpythagorean. 4) Let  $R$  be a finitely generated reduced Witt ring. Call a field  $K$   $R$ -formal if there is a surjective ring homomorphism from  $W(K)$  to  $R$ . Then the class of  $R$ -formal fields has a decidable model companion. (Received October 12, 1978.)

79T-A20 R. BIRKENHEAD, N. SAUER, and M.G. STONE, University of Calgary, Alberta, Canada. The Algebraic Representation of Semigroups and Lattices; Representing Lattice Extensions.

A monoid  $S$  and a lattice  $L$  are *jointly algebraic* if there is a universal algebra  $\mathcal{O}$  with  $S \cong \text{End } \mathcal{O}$  and  $L \cong \text{Su } \mathcal{O}$  (cf. these *Notices* 758-08-1 August 1978). If  $H$  is a lattice an ideal  $J \subseteq H$  is *compactly embedded* in  $H$  provided the natural map  $\Pi : H \rightarrow J$  given by  $\Pi(x) = \bigvee_{\substack{j \in J \\ j \leq x}} j$  preserves both joins and compactness. We have

Theorem 1 If  $S$  and  $L$  are jointly algebraic and  $L \cong J$  for some ideal  $J \subseteq H$  which is compactly embedded in  $H$ , then  $S$  and  $H$  are jointly algebraic as well.

For lattices  $T$  and  $L$  we denote by  $T \dot{+} L$  the ordinal sum of  $T$  and  $L$  in which the  $1 \in T$  is identified with  $0 \in L$ .

Theorem 2 If  $S$  and  $L$  are jointly algebraic then  $S$  and  $T \dot{+} L$  are also jointly algebraic for any compactly generated lattice  $T$ . (Received October 13, 1978.)

\*79T-A21 C.Y. CHAO, University of Pittsburgh, Pittsburgh, PA 15260 and E.G. WHITEHEAD, JR., University of Pittsburgh, Pittsburgh, PA 15260. Chromaticity of Self-Complementary Graphs.

A graph is said to be self-complementary if it is isomorphic to its complement. We show that  $m^2$  is the maximal number of vertices for a self-complementary graph having chromatic number  $m$ . By construction, we show that these maximal graphs exist. Furthermore, for each integer  $m \geq 2$ , we construct

a self-complementary graph with diameter 3 having chromatic number  $m$ . Also, for each integer  $m \geq 3$ , we construct a self-complementary graph with diameter 2 having chromatic number  $m$ . We also obtain the chromatic polynomial for each graph in these two families of self-complementary graphs. (Received October 13, 1978.)

\*79T-A22 Robert Gilmer, Florida State University, Tallahassee, Florida 32306 and Raymond C. Heitmann, University of Texas, Austin, Texas 78712. On  $\text{Pic}(R[X])$  for  $R$  Seminormal.

Let  $R$  be a commutative ring with identity and let  $T$  be a set of indeterminates over  $R$ . There is a natural injection  $\phi: \text{Pic}(R) \rightarrow \text{Pic}(R[T])$ , and this paper is concerned with the problem of determining conditions under which  $\phi$  is surjective. For  $R$  a Noetherian reduced ring with finite integral closure, C. Traverso [Annali Scuola Norm. Sup.-Pisa 24(1970), 585-595] proved that  $\phi$  is surjective if and only if  $R$  is seminormal. We extend Traverso's result in two directions. First, for an integral domain  $R$ , we prove that  $\phi$  is surjective if and only if  $R$  is seminormal. Moreover, if  $R$  is a Noetherian reduced ring, then surjectivity of  $\phi$  is equivalent to seminormality of  $R$ . There is an example of a reduced seminormal ring  $S$  such that  $\phi$  is not surjective. (Received October 16, 1978.)

\*79T-A23 E.J. Cockayne, University of Victoria, B.C. Canada and S.T. Hedetniemi, University of Oregon, U.S.A. A Conjecture Concerning Broadcasting in  $m$ -dimensional Grid Graphs.

#### Abstract

Broadcasting in a graph is the spreading of information known by one vertex initially, subject to the rules (i) a vertex may only call an adjacent vertex, (ii) each call takes one unit of time and (iii) a vertex can participate in only one call per unit of time.

A lower bound is obtained for  $g(m,n)$ , the maximum number of vertices of the  $m$ -dimensional grid graph which may be informed in  $n$  time intervals. (Received October 16, 1978.)  
(Authors introduced by Dr. G. G. Miller)

\*79T-A24 DRAGOMIR Ž. DJOKOVIĆ, University of Waterloo, Waterloo, Ontario, Canada, N2L 3G1. On the Existence of Maximal Subgroups in Amalgamated Free Products.

Let  $G = A *_U B$  where  $U$  is a proper subgroup of  $A$  and of  $B$ . An element  $w \in G$  is cyclically reduced if  $w = a_1 b_1 \dots a_n b_n$  or  $w = b_1 a_1 \dots b_n a_n$  where  $n \geq 1$ ,  $a_i \in A-U$ , and  $b_i \in B-U$ . If  $G$  has a cyclically reduced element  $x$  such that  $x$  normalizes  $U \cap xUx^{-1}$ , we show that  $G$  has a maximal subgroup, say  $H$ , such that  $x \notin H$ . As a consequence we obtain the following result: If each of the normalizers  $N_A(U)$  and  $N_B(U)$  contains  $U$  properly then  $G$  has a maximal subgroup. (Received October 16, 1978.)

79T-A25 RONALD J. EVANS, University of California at San Diego, La Jolla, California 92093. Rational reciprocity laws. Preliminary report.

Rational  $2^r$ -th power reciprocity laws are given for each  $r \geq 2$ , generalizing, e.g., the classical law of quadratic reciprocity, K. Burde's quartic law [J. Reine Angew. Math. 235 (1969), 175-184], and K. S. Williams' octic law [Pac. J. Math. 63 (1976), 563-570]. The laws are unambiguously expressed in terms of the parameters occurring in the quadratic partitions of primes  $p \equiv 1 \pmod{2^r}$  described on p.329 of a paper of Giudici, Muskat, and Robinson [T.A.M.S. 171 (1972), 317-347]. The choice of parameters is governed by certain side conditions which can be formulated by purely rational considerations. An important tool in the proof is Stickelberger's prime ideal factorization of Jacobi sums  $(\text{mod } p)$ . (Received October 16, 1978.)

79T-A26 PETER JOFFE, 210 East 88th Street, New York, New York 10028. Graphs with spanning trees containing no 2-valent vertices. Preliminary report.

A graph will be called a "(2') graph" or "(2')", if it is spanned by a tree with no 2-valent vertices. For a graph  $G$  (no loops or multiple edges), we denote by  $E(G)$  and  $V(G)$  its edge and vertex sets respectively.

Theorem 1. A cubic graph  $G$  is (2') if and only if it contains a collection,  $M$ , of disjoint circuits (with  $(1/2)|V(G)| + 1$  edges) such that each circuit of  $G$  contains an edge in  $M$ . (If, also,  $G$  is a plane graph, then the circuits of  $M$  must be faces of  $G$ .) Theorem 2. No bipartite cubic graph is (2'). Theorem 3. The truncation of a 4-valent plane graph cannot be a (2') graph. Theorem 4. If, for any two nonadjacent vertices  $x$  and  $y$ ,  $((\text{degree of } x) + (\text{degree of } y)) \cong |V(G)| + 1$ , then  $G$  is a (2') graph. A plane, 3-connected, graph  $G$  is called a "skirted-tree" if the removal of all edges of the exterior face leaves a tree. Theorem 5.  $G$  is a skirted-tree with regular valence 3. Then,  $G$  contains at least two (2') trees if and only if  $G$  contains a set,  $M$ , of disjoint interior faces (with total edges =  $(1/2)|V(G)| + 1$ ) such that each face of  $G$  contains an edge of  $M$ . Theorem 6. If  $G$  is connected, then  $G^2$  is a (2') graph. Lastly, for skirted-tree graphs of a certain type we obtain Eberhard-type theorems with respect to the interior faces. This author thanks Professor Joseph Malkevitch of York College (CUNY). (Received October 18, 1978.)

\*79T-A27 C. Y. Chao, University of Pittsburgh, Pittsburgh, PA 15260. König's problem on groups and graphs.

In 1938, R. Frucht solved the abstract group version of König's problem. Here the solution of the permutation group version of König's problem is presented. Theorem. Let  $G$  be a permutation group on  $n$  letters. Then there exists a graph  $X$  (directed or undirected, with or without loops) with  $n$  vertices such that its group of automorphisms  $G(X)$  is  $G$  if and only if the following conditions are satisfied: (a)  $PA(X) = A(X)P$  for all  $P \in G^*$  where  $A(X)$  is the adjacency matrix of  $X$  and  $G^*$  is the permutation matrix group corresponding to  $G$ , and (b) the number of isomorphic graphs of  $X$  is equal to the index of  $G$  in the symmetric group on  $n$  letters. Also, an algorithm is presented for determining whether the given group has a graph, and if the given group has graphs, all of them can be constructed. (The algorithm is practically the one used in "On groups and graphs", Trans. AMS., 118 (1965) written by the author). (Received October 18, 1978.)

\*79T-A28 C. J. MOZZOCHI and R. BALASUBRAMANIAN, Mittag-Leffler Institute, Auravagen 17, S-182 62, Djursholm, Sweden. Analytic sufficiency conditions for Goldbach's conjecture.

The notation here is the same as that in Modern prime number theory by T. Estermann. For each  $n$  let  $E_n$  be those points in  $[x_0, x_0 + 1]$  which are not in any closed neighborhood of radius  $x_0$  about any rational number  $h/q$  where  $(h, q) = 1$  and  $q \leq F(n)$ . Let  $A(n)$  be the integral of  $t^2(x, n) e(-nx)$  over  $E_n$ . By the weak form of the prime number theorem we know that  $A(n) = O(n \log^{-1} n)$ . Theorem 1. Let  $F(n) = n^e$  where  $e$  is positive and sufficiently small. Then under the generalized Riemann hypothesis we have that  $A(n) = o(n \log^{-2} n)$  implies that every sufficiently large even integer is the sum of two primes. Theorem 2. Let  $F(n) = \log^{15} n$  and  $(q, n) = 1$ . Then  $A(n) = o(n \log^{-2} n)$  implies that every sufficiently large even integer is the sum of two primes. Conjecture. The assumption of the G. R. H. in Theorem 1 can be eliminated. Remark. These results are among those contained in our Institute Report Some comments on Goldbach's conjecture, copies of which are available from the first author at Box 1315, Hartford, Connecticut 06101. (Received October 10, 1978.)

## *Analysis (26, 28, 30–35, 39–47, 49)*

79T-B1 S. ZAIDMAN, Université de Montréal, Montréal, Québec, Canada. Some new inequalities for pseudodifferential operators.

We consider nonhomogeneous symbols  $a(x, \xi)$  and associated pseudodifferential operators  $\mathcal{A}(x, D)$  as in our previous work (Ann. Scuola Norm. Sup. Pisa, 1967 and J. Math. Anal. Appl., 1970). We prove the following. Theorem. Let  $t$  be a real number and let  $s$  be real such that  $s + t + 1 \geq 0$ . Let there be given  $\epsilon > 0$ . There exists a positive constant  $C_{\epsilon, s}$  such that  $|\mathcal{A}(x, D)U(v)| \leq C \|U\|_{H^{-s-1}} \|v\|_{H^s}$  for any couple  $U \in H^t(\mathbb{R}^n)$

and  $v \in \mathcal{S}'(\mathbb{R}^n)$  having supports at distance  $> \epsilon$ . (Here  $(T)(v)$  means duality between  $T \in \mathcal{S}'^1$  and  $v \in \mathcal{S}'$ .) This is a result connected with Theorem A.3 in Kohn-Nirenberg's fundamental research on pseudodifferential operators (Comm. Pure Appl. Math. 17(1965), 269-305) and is proved by a similar method. (Received June 5, 1978.)

\*79T-B2 STEVE WRIGHT, Department of Mathematics, Oakland University, Rochester, Michigan 48063  
Compact Derivations of C\*-algebras. Preliminary report.

Theorem 1. Let  $A$  be a C\*-algebra,  $\delta: A \rightarrow A$  a compact derivation. There exists an  $A$ -faithful central projection  $z \in A^{**}$  (i.e.,  $az = 0 \Rightarrow a = 0, \forall a \in A$ ) such that  $z\delta^{**}$  is the uniform limit (on  $A^{**}z$ ) of finite-rank derivations.

Theorem 2. Let  $A$  be a C\*-algebra with no finite-dimensional irreducible representations. Then  $A$  admits no nonzero compact derivation.

Corollary 1. No infinite-dimensional irreducible C\*-algebra admits a nonzero compact derivation.

Corollary 2. No infinite-dimensional simple C\*-algebra admits a nonzero compact derivation. (Received September 15, 1978.)

\*79T-B3 WILLIAM R. ALLAWAY, Lakehead University, Thunder Bay, Ontario, Canada P7B 5E1,  
On q-Orthogonal Polynomial Sets whose weight functions are related to the Jacobi Theta Function, Preliminary Report.

In 1894, Roger's, in his study of infinite products, introduced an orthogonal  $q$ -polynomial set  $\{A_n(x; q)\}$  defined by  $A_0(x; q) = 1, A_1(x; q) = 2x$  and  $A_n(x; q) = 2xA_{n-1}(x; q) - (1 - q^n)A_{n-2}(x; q)$ . In this paper we find some of the special function properties of  $\{A_n(x; q)\}_{n=0}^{\infty}$ . We show that  $\{A_n(\cos z; q)\}_{n=0}^{\infty}$  is orthogonal on  $[0, \pi]$  with respect to  $\theta_1(z, q^{1/2})$ , where  $\theta_1(z, q^{1/2})$  is the Jacobi Theta function defined by  $\theta_1(z, q) = 2 \sum_{n=0}^{\infty} (-1)^n q^{(n+1/2)^2} \sin(2n+1)z$ . We find some of the other special function properties of  $\{A_n(x, q)\}$  such as the generating function, orthogonality relation, fourier sine series of  $A_n(\cos z; q)\theta_1(z, q^{1/2})$  and how  $A_n(\cos z; q)$  is related to other known polynomial sets. One of the more interesting formulas we find is

$$A_n(x; q)A_m(x; q) = \sum_{r=0}^{\min(m,n)} \begin{bmatrix} m \\ r \end{bmatrix} \begin{bmatrix} n \\ r \end{bmatrix} [r]! A_{m+n-2r}(x; q) \text{ where } [r]! = \frac{(1-q)(1-q^2)\dots(1-q^r)}{(1-q)^r}$$

and  $\begin{bmatrix} m \\ r \end{bmatrix} = \frac{[m]!}{[m-r]![r]!}$ . (Received September 18, 1978.)

\*79T-B4 JAMES GUYKER, SUNY College at Buffalo. At present: Purdue University, West Lafayette, Indiana, 47907. A Structure Theorem for Operators with Closed Range.

Let  $T$  be an operator on Hilbert space, and suppose that the range of  $T$  is closed. Theorem. A necessary and sufficient condition that the kernels of  $T^j$  and  $T^{*j}$  are invariant under  $TT^*$  and  $T^*T$  respectively for every  $j = 1, 2, \dots, N$  is that  $T = T_1 \oplus T_2 \oplus \dots \oplus T_N \oplus V$  where  $T_j$  is a weighted truncated shift of index  $j$  with invertible operator weights and  $V$  is an operator with closed range such that  $VV^*(\ker V^j) = \ker V^j$  and  $V^*V(\ker V^{*j}) = \ker V^{*j}$  for every  $j = 1, 2, \dots, N$ . Moreover, the representation is unique, and a projection  $P$  commutes with  $T$  if and only if  $P = P_1 \oplus P_2 \oplus \dots \oplus P_N \oplus Q$  where  $P_j$  and  $Q$  are projections which commute with  $T_j$  and  $V$  respectively ( $j = 1, 2, \dots, N$ ). A corollary is that a hypothesis (that partial isometries have no isometric part) is unnecessary in previous work (these Notices 22, #75T-B225; Pacific J. Math 62 (1976), 419-433). (Received September 18, 1978.)

Let  $A$  be a von Neumann algebra, let  $\sigma$  be a strongly continuous representation of the locally compact abelian group  $G$  as  $*$ -automorphisms of  $A$ . Let  $M(\sigma)$  be the Banach algebra of bounded linear operators on  $A$  generated by  $\int \sigma_t \mu(t)$  ( $\mu \in M(G)$ ). Then it is shown that  $M(\sigma)$  is semisimple whenever either (i)  $A$  has a  $\sigma$ -invariant faithful, normal, semifinite, weight, (ii)  $\sigma$  is an inner representation or (iii)  $G$  is discrete and each  $\sigma_t$  is inner. It is shown that the Banach algebra  $L(\sigma)$  generated by  $\int f(t) \sigma_t dt$  ( $f \in L^1(G)$ ) is semisimple if  $\sigma$  is an integrable representation. Furthermore if  $\sigma$  is an inner representation with compact spectrum, it is shown that  $L(\sigma)$  is embedded in a commutative, semisimple, regular Banach algebra  $L$  with isometric involution that is generated by projections. This algebra is contained in the ultraweakly continuous operators on  $A$ . Also the spectral subspaces of  $\sigma$  are given in terms of projections of  $L$ . (Received September 25, 1978.)

S. MAZUR posed the following problem: "Does there exist a 3-dimensional Banach space with the property that every 2-dimensional Banach space is isometric to a subspace of it?" The above problem is equivalent to the following problem: Does there exist a 3-dimensional convex body  $K$  with a center  $O$ , such that any plane, centrally symmetric convex body  $V$  is affinely equivalent to the intersection of  $K$  with a suitable plane through  $O$ ? The purpose of this paper is to give a negative answer to MAZUR'S problem. (Received September 26, 1978.)

In this paper we establish a maximum principle of the Cauchy problem for the equation  $L[U] = K(t)$  ( $U_{xx} - U_{yy} + U_{tt} + a(x,y,t) \cdot U_x + b(x,y,t) \cdot U_y + c(x,y,t) \cdot U_t + \delta(x,y,t) \cdot U = f(x,y,t) \in C^0(\bar{D})$ ). We observe that such a maximum principle gives an analogous generalization to the results of WEINBERGER and SATHER (Arch. Rat. Mech. Analysis 18(1965), 4-26; Ann. Math. 64(1956), 505-513) for a more general class of equations, by applying the integral methods of DOUGLIS (Comm. Pure Appl. Math. 7(1954), 271-295).

THEOREM. Suppose that  $V = V(r, \theta, t) \in C^2(\bar{D})$  satisfies the differential inequality  $L[V] \leq K(t) \cdot (V_{rr} - \frac{1}{2} V_{\theta\theta}) + \frac{1}{2} K(t) \cdot (V_r + V_{tt} + a(r, \theta, t) \cdot V_r + b(r, \theta, t) \cdot V_\theta + c(r, \theta, t) \cdot V_t + \lambda(r, \theta, t) \cdot V = F(r, \theta, t) \leq 0$ , where  $K \in C^2(\bar{D})$  and  $a, b, c, \lambda$  are continuously differentiable functions of variables  $r, \theta, t$  in the domain  $DCR^3$ , such that  $K(t) < 0$  for  $t < 0$ . Let us assume the Cauchy data  $V(r_0, \theta, t) = 0$ ;  $V_r(r_0, \theta, t) = v(\theta, t) < 0$ , where  $v \in C^0(\bar{D})$ ,  $r_0 = \text{constant} (> 0)$ . Let us assume the following conditions:  $\delta = \frac{1}{2} \frac{d}{dt} (\sqrt{-K}) \cdot \frac{\partial}{\partial r}$ .

$$c = \frac{\partial}{\partial t} \left( \int_0^r \frac{K}{r} \cdot dr \right) - \left( a - \int_0^r a_r \cdot dr \right) \cdot [-K]^{-1/2}; \quad \frac{1}{2} \int_0^r \frac{(a)}{r} \cdot dr + (b)_\theta - \lambda \geq 0;$$

$$\frac{d}{dr} \left\{ W \cdot \left[ \frac{K'}{2\sqrt{-K}} + \left( a - \int_0^r a_r \cdot dr \right) \right] \right\} - \left\{ \frac{d}{dt} [\delta^+(w)] \right\} \cdot (\sqrt{-K}) \geq 0,$$

where  $w = w(r, \theta, t)$  is a special function. Then  $V(r, \theta, t) \leq 0$  in  $D$ . (Received September 26, 1978.) (Author introduced by Themistocles M. Rassias).

It is proved that any function  $f(x)$ , measurable and finite almost everywhere on the  $n$ -torus  $T^n$ , can be represented by an  $n$ -fold trigonometric series, summed by rectangles. More precisely, we show that there exists a function  $F$  continuous on  $T^n$  such that the rectangular sums of the multiple trigonometric series  $\sum_m (i) \prod_{j=1}^n \sum_{m_j} \hat{F}_{m_j} e^{im_j x_j}$  converge to  $f(x)$  almost everywhere, where  $m = (m_1, m_2, \dots, m_n)$  is an integer lattice point of  $R^n$  and  $\hat{F}_m$  the  $m$ -th Fourier coefficient of  $F$ . This result

generalizes a representation problem, posed by Lusin and solved by Men'shov, from finite almost-everywhere measurable functions on  $T$  to functions on  $T^n$ . Also it extends our previous work to the effect that any finite almost-everywhere measurable function on  $T^n$  can be represented by a multiple trigonometric series, summed by squares. (Received September 28, 1978.)

\*79T-B9 RAUL E. CURTO, State University of New York at Stony Brook, Stony Brook, New York 11794.

Fredholm and invertible tuples of bounded linear operators.

Let  $I(H)$  = commuting invertible tuples of operators on a Hilbert space  $H$ ,  $D(H)$  = doubly commuting invertible tuples and  $N(H)$  = commuting invertible tuples of normal operators (invertible means nonsingular in the sense of J. L. Taylor, A joint spectrum for several commuting operators, J. Functional Analysis 6(1970)). Theorem.  $D(H)$  and  $N(H)$  are arcwise connected; if  $\dim H$  is finite, so is  $I(H)$ . We conjecture that  $I(H)$  is always arcwise connected. Let  $F(H)$  = almost commuting tuples, invertible in the Calkin algebra. Proposition. Let  $B$  be any  $C^*$ -algebra (or  $H$ ),  $0 \leq n_k \in \mathbb{Z}$ ,  $n_k = 0$  for  $k < 0$ ,  $B_k = B \otimes C^{n_k}$  and  $D_k \in L(B_k, B_{k-1})$  be a matrix over  $B$  (or  $D_k \in L(H_k, H_{k-1})$ ) with  $D_k D_{k+1} = 0$ . Then the complex:  $\dots \rightarrow B_k \xrightarrow{D_k} B_{k-1} \rightarrow \dots$  is exact iff  $D_k^* D_k + D_{k+1} D_{k+1}^*$  is invertible (all  $k$ ). Corollary.  $T \in I(H)$  ( $T \in F(H)$ ) iff  $\hat{T} = \begin{pmatrix} d_1 & & \\ & d_2^* & \\ & & d_3 \dots \end{pmatrix} \in L(H \otimes C^{2n-1})$  is invertible (Fredholm), where  $d_k$  is the  $k$ th boundary map in the Koszul complex for  $T$ . Corollary. For a commuting tuple  $T$ ,  $\text{Sp}(T, H) = \text{Sp}(T, L(H))$ . For  $T \in F(H)$ ,  $\text{index}(T) = \text{index}(\hat{T})$  is continuous, invariant under compact perturbations, onto  $\mathbb{Z}$ . Does index identify the arcwise components of  $F(H)$ ? Theorem. Two Fredholm pairs with same index and a semi-Fredholm coordinate lie in the same path-component. Theorem. An essentially normal  $n$ -tuple with all commutators in trace class has index zero ( $n \geq 2$ ). A natural generalization of Atkinson's theorem holds. We also get Theorem. Let  $B$  be a  $C^*$ -algebra,  $A$  be a  $C^*$ -subalgebra of  $B$  and  $a = (a_1, \dots, a_n)$  be a commuting tuple of elements of  $A$ . Then  $\text{Sp}(a, A) = \text{Sp}(a, B)$ . (Received October 13, 1978.)

\*79T-B10 PEI YUAN WU, National Chiao Tung University, Hsinchu, Taiwan, Republic of China.  
 $C_{11}$  contractions are reflexive. Preliminary report.

It is shown that a completely non-unitary  $C_{11}$  contraction defined on a separable Hilbert space with finite defect indices is reflexive. The proof is based on the functional model for contractions developed by B. Sz.-Nagy and C. Foiaş. Results in the author's paper "Bi-invariant subspaces of weak contractions" (these Notices, 25 (1978), A - 429) are also used. (Received October 13, 1978.)

\*79T-B11 Philip W. McCartney and Richard C. O'Brien, Northern Kentucky University, Highland Heights, Kentucky 91711. A separable Banach space having the Radon-Nikodym Property which is not isomorphic to a subspace of a separable dual space.

A Banach space is constructed which has the properties stated in the title. This provides a negative answer to a question first raised by J. J. Uhl (see, e.g., Vector Measures by J. Diestel and J. J. Uhl). (Received October 16, 1978.)

79T-B12 H. DIEKHANS, Indiana State University, Terre Haute, Indiana 47809  
Measurable Sets in an Abstract Space. Preliminary Report.

Assume a measure space with its  $\mathcal{V}$ -algebra containing a family  $\mathcal{F}$  of sets with finite and positive measure  $\mu$ . Assume  $\mathcal{F} = \bigcup_n \mathcal{F}_n$ , where  $\mathcal{F}_n \subset \mathcal{F}_{n+1}$ , for each positive integer  $n$ . Suppose that the measure of the union of sets in each sub-family  $\mathcal{F}_n$  is finite.  $\Delta(\mathcal{F}_n)$  will denote the set of points  $x$  such that there is a sequence of sets  $E_n$  from  $\mathcal{F}_n$ , each containing  $x$ , and with  $\mu(E_n) \rightarrow 0$ .  $K(\mathcal{F}_n)$  will denote the class of sets  $H, \subset \Delta(\mathcal{F}_n)$ , such that there is a sub-family  $\mathcal{F}'_n$  of  $\mathcal{F}_n$  of sets which intersect  $H$  and with  $H = \Delta(\mathcal{F}'_n)$ . Absolutely continuous functions and measurable functions are defined with respect to  $K(\mathcal{F}_n)$ . Some of the theorems relate measurable sets in the space to sets of a class  $K(\mathcal{F}_n)$ , equal in measure a.e. or differing in measure by an arbitrary positive number. (Received October 18, 1978.)

\*79T-B13 BRIAN W. MCENNIS, The Ohio State University, Marion, Ohio 43302.  
Purely contractive analytic functions and characteristic functions of non-contractions. II

This abstract corrects an error in the author's abstract #752-47-29 that appeared on page A-129 of the January, 1978 issue of the "Notices". The notation of that abstract will be assumed here.

A self-adjoint unitary operator  $J$  on a Krein space is called a fundamental symmetry if the inner product  $(x, y) = [Jx, y]$  is a Hilbert space inner product.  $\Theta$  is called fundamentally reducible if there are fundamental symmetries on  $\mathcal{D}$  and  $\mathcal{D}_*$  that commute with  $\Theta(0)*\Theta(0)$  and  $\Theta(0)\Theta(0)^*$ , respectively.

Theorem  $\Theta$  coincides with the characteristic function  $\Theta_T$  of some operator  $T$  if and only if  $\Theta$  is purely contractive and fundamentally reducible.

The extra condition of fundamental reducibility is implicit in the cited work of J. Ball. (Received October 18, 1978.)

## Applied Mathematics

(65, 68, 70, 73, 76, 78, 80-83, 85, 86, 90, 92-94)

79T-C1 ETHELBERG N. CHUKWU, University of Jos, Jos, Nigeria. Euclidean Controllability of Nonlinear, Functional differential systems of neutral type. II.

In this paper the problem of Euclidean null-controllability of the linear system

$$\dot{x}(t) = \sum_{i=1}^p A_i(t) x(t-h_i) + \sum_{i=1}^p B_i(t) u(t-h_i)$$

with constrained delayed controls and delayed state is considered. The notion of a proper system is introduced. Explicit criteria for a system being proper are stated. If the uncontrolled system is uniformly asymptotically stable and if the control equation is proper, the system is Euclidean null-controllable. The controls are constrained to lie on a compact subset of Euclidean space. (Received May 31, 1978.)

79T-C2 V. JA. KREINOVIC and O. M. KOSHELEVA, USSR 196140 Leningrad, Pulkovo Special Astrophysical Observatory. Quantum derivation of Einstein's equations. Preliminary report.

Let us consider gravity as a spin-2 field  $h_{ij}$  in flat space whose source is a total energy-momentum tensor  $T_{ij}$  (including the energy-momentum tensor of that very field). Spin equations for any spin-2 field with sources  $t_{ij}$  have the form  $\square h_{ij} + h^{pq}{}_{,pq} \eta_{ij} - h^p{}_{,i} \eta_{ij} - h^p{}_{,j} \eta_{ij} = t_{ij}$ . Let us denote its left-hand side by  $(Dh)_{ij}$ . Assume  $L_0 = L_0(\mathcal{L}_A, \mathcal{L}_{A,k})$  is any lagrangian. By  $T_{kl}(L)$  we understand symmetric energy tensor, obtained from the lagrangian  $L$  by a standard procedure (e.g., Hilbert's). Theorem. For any  $L_0$  there exists 1 and only 1 (modulo complete div.) lagrangian  $L = L(h_{ij}, h_{ij,k}, \mathcal{L}_A, \mathcal{L}_{A,k})$  such that (1)  $L = L_0(\mathcal{L}_A, \mathcal{L}_{A,k})$ , when  $h_{ij} = h_{ij,k} = 0$ , (2) its variational equations  $\delta L / \delta h_{ij} = 0$  are equivalent to  $(Dh)_{kl} = KT_{kl}(L)$ . This  $L$  coincides with  $\sqrt{-g}(R + L_0^*)$ , where  $g_{ij}$  is connected with  $h_{ij}$  by  $h_{ij} = K(g^{ij} \sqrt{-g} - \eta_{ij})$  and  $L_0^*$  is obtained from  $L_0$  by changing the (flat metric)  $\eta_{ij}$  to  $g_{ij}$  and usual derivatives to covariant ones. Authors are thankful to G. Rumer and K. S. Thorne for valuable discussions. (Received September 18, 1978.)

\*79T-C3 R. GLASSEY, Indiana University, Bloomington, In. 47401 and W. STRAUSS, Brown University, Providence, R.I. 02912. Decay of a Yang-Mills Field coupled to a scalar Field. Preliminary Report.

Recently we proved that the energy of a classical solution of the Yang-Mills equations "radiates out" along the light cone at the rate  $O(t^{-2})$  as  $t \rightarrow \infty$ . Now consider

$$L = \int (\text{tr}(F^2 + (D\phi)^2) + V(\phi)) d^4x$$



where the first term is the Yang-Mills Lagrangian,  $\phi$  is a (real) scalar field and  $D$  is the covariant derivative. Theorem 1: The same decay is true for solutions to the Euler-Lagrange equations of  $L$ , provided  $V$  satisfies the inequality  $0 \leq 4V(s) \leq s \cdot V'(s)$ . In case  $V(s) = \lambda |s|^4$  ( $\lambda > 0$ ) the equations are conformally invariant. If  $V(\phi)$  includes a linear mass term with coefficient  $m > 0$ , we still get the following result. Theorem 2: The local energy is integrable in time provided  $0 \leq 2V(s) \leq s \cdot V'(s)$ . (Received September 27, 1978.)

79T-C4 J. C. Lagarias, Bell Laboratories, Murray Hill, N. J. 07974. On the computational complexity of determining the solvability of the Diophantine equation  $X^2 - DY^2 = -1$ . Preliminary Report.

This paper examines the set  $\mathcal{S} = \{D \mid X^2 - DY^2 = -1 \text{ solvable in integers}\}$  from the viewpoint of computational complexity. Theorem 1. For each  $D$ , there exists a verification of either the solvability or unsolvability of  $X^2 - DY^2 = -1$  requiring  $\ll (\log D)^6$  bit operations to check on a computer. These verifications are "certificates" in the sense of V. Pratt [Siam J. Comput. 4 (1975) 214-220]. Theorem 2. The set  $\mathcal{S}$  is in both NP and CO-NP. In particular, this suggests  $\mathcal{S}$  is not NP-complete. These results contrast with extensive numerical evidence suggesting that it may require on the order of  $\sqrt{D}$  binary bits to write down the minimal solution of  $X^2 - DY^2 = -1$  (when it exists) for an infinite set of squarefree  $D$ . The method of proof uses the theory of binary quadratic forms. (Received September 25, 1978.)

79T-C5 Mostafa A. Abdelkader, 25 Sh. Champollion, Alexandria, Egypt. Exact Solutions for Rayleigh's equation with variable mass.

The equ. of motion of the particle is  $d(xM)/dt + ax + bx^3 + cz = 0$ , where velocity  $x = dz/dt$ , and mass  $M = m + nx^2 + px^4$  (which may approximate that of a relativistic particle). Exact expressions for the phase-plane trajectories are given for four cases (in each of which four of the six coefficients of the equ. are arbitrary) containing the sub-cases of critical, over- and under-damping. An oscillatory example: For  $a, b, c, m$  arbitrary, and  $3an = 4bm$ ,  $5pa^2 = 3mb^2$ , the trajectories are given in terms of a parameter  $u$  by:  $z = (k \cos u + (a^2/2cm^2) \sin u) \exp v$ , and  $ax + bx^3 = (a^2/2cm) (s \cos u - \sin u) \exp v$ , where  $v = C - su$ ,  $k = (2cm - a^2)/4mrc^2$ ,  $r = ((cm/a^2) - (1/4))^{1/2}$ ,  $2rs = 1$ , and  $C$  is a constant of integration. (Received October 10, 1978.)

79T-C6 KARL LIEBERHERR, University of New Mexico, Dept. of Comp. and Info. Science, Albuquerque, NM 87131. Optimal Heuristics for the Graph Coloring Problem.

Theorem (i) Each graph with edge weights has a coloring  $C_m$  with  $m$  colors so that the coloring condition is satisfied for at least the fraction  $(m-1)/m$  of the weighted edges ( $m \geq 2$ ). (ii)  $C_m$  can be found by a polynomial algorithm  $COLOR_m$ . The time complexity of  $COLOR_m$  for a graph with  $n$  nodes and  $e$  edges is in  $O(e \cdot n^2)$ . (iii) If  $P \neq NP$ , the algorithms  $COLOR_m$  ( $m \geq 3$ ) are optimal in the class of polynomial algorithms in the following sense: The set of graphs having a coloring  $C_m$  which satisfies the coloring condition for at least the fraction  $c$  ( $c > (m-1)/m$ ,  $c$  rational) of the weighted edges, is NP-complete.

The proof uses a symmetrization method introduced in: Lieberherr K., Specker E., Interpretations of 2-satisfiable Conjunctive Normal Forms, Notices of the AMS, Feb. 78, Vol. 25(2), page A-295. (Received September 22, 1978.)

79T-C7 C. MUSÈS, c/o Math. Ed. Offices, 844 San Ysidro Ln, Santa Barbara, CA 93108.  
The implications of phase in the Schrödinger wave function. Preliminary report.

The current derivations of the probability-wave equation of Schrödinger are reductive and omit the phase term that Schrödinger had found in his original paper [*Ann.d.Physik*, 79(1926)] but unfortunately discarded. The full Schrödinger equation reads  $\nabla^2\psi = [8\pi m(E-V)/h^2]\psi - (2\pi i/h)E(\nabla^2\tau)\psi$ , and it is the last term that contains the expression for the phase of a probability wave. It can be shown that  $\nabla^2\tau = \nabla(1/w)$  where  $w$  is the wave or phase velocity of the associated de Broglie wave. More explicitly, the phase term coefficient of  $-i\psi$  is  $(2\pi E\sqrt{2m/h})[\partial/\partial x (\sqrt{E_x - V_x}/E_x) + \partial/\partial y (\sqrt{E_y - V_y}/E_y) + \partial/\partial z (\sqrt{E_z - V_z}/E_z)]$ , whence it is apparent that variations in phase could occur through variations in the potential energy  $V$  alone:  $V \rightarrow V + \delta V$ , since the Energy  $E$  of the stable state, and  $\pi$  and  $h$ , are constant. Such variations would take time to be equilibrated by counterchanges in  $E$ , and meanwhile the phase of the wave would be altered, causing consequent and appropriate changes in the probabilities of the corresponding events by (anti)resonance effects among the other probability waves involved. Thus, the phase term allows us to understand how a shift (say, of awareness or consciousness) that affects potential energy only, can alter event patterns and change physical history by probability-phase triggering. A new science of qualitative time emerges: the value of  $t$ , the fundamental independent variable for all phenomena, thus contributes ineluctably to the energy structure of any situation; and interactions among probability waves guide outcomes on all levels. Quantum physics so leads into chronotopology. (Received October 17, 1978.)

(Author introduced by K. Demys)

79T-C8 A.E. KONTRAROU, T.M. RASSIAS, and G.M. RASSIAS, Research Center for National Defense, K.E.E.Θ.A., Galatsi-Athens, GREECE. Singularities of the n-Body Problem of Celestial Mechanics.

One of the most significant open questions in the n-body problem of celestial mechanics is that of determining what constitutes a catastrophe. If we let a catastrophe be a singularity of the solution of the n-body problem, then the question can be rephrased as asking what physical events can cause a singularity at time  $t=0$ . In the present paper we investigate the problem of singularities due to collisions. We assume that the center of mass of the n point masses is fixed at the origin of an inertial coordinate system, and we denote the mass, position, and velocity of the  $i^{\text{th}}$  mass respectively by  $m_i$ ,  $r_i$ ,  $v_i$ . Define  $I = \frac{1}{2} \sum_{i=1}^n m_i v_i^2$ . In 1908, Von Zeipel published a statement to the effect that if the system remains bounded as  $t \rightarrow 0$ , then a singularity at time  $t_0$  is due to collisions. Unfortunately his proof is wrong as it contains several gaps and errors. After we have corrected and completed the proof of Von Zeipel's result we can state the following theorem "If there exists a singularity at  $t=0$  and if  $I$  is 'slowly varying' as  $t \rightarrow 0$ , then the singularity is due to collisions." We conclude by stating the following theorems. "In the linear n-body problem all singularities are due to collisions." The methods used to prove the above theorems are in the spirit of the Morse theory. (Received October 16, 1978.)

79T-C9 ALOIS GLANC, California State University, Northridge, California 91330.  
On Mechanization of Investigation of the Decision Problems in Predicate Logics.  
Preliminary Report.

In [1] we have suggested that the idea of Friedman's [2] semidecision procedure can be used for the following "research simulating" computer system: Generate certain classes of formulas of predicate logic classified according to their prefixes and matrices, find causes for conflicts in the matrices, form from those causes corresponding semidecision procedures, then try to prove that the semidecision procedure is a decision procedure for the given class. This kind of proofs can be automated because of their uniform character.

- [1.] Glanc, A., "Contribution to Applied Logic and Methodology," VLVDU Hradec Kralove, Vol. 77, 1968.
- [2.] Friedman, J., "A Semi-decision Procedure for Functional Calculus," Journal ACM 10, pp. 1-24, 1963. (Received October 18, 1978.)

## Geometry (50, 52, 53)

79T-D1 V. JA. KREINOVIC, USSR 196140 Leningrad Pulkovo Special Astrophysical Observatory. Note on billiards. Preliminary report.

In a previous article together with R. Sine (*Notices*) Amer. Math. Soc. 25(1978), p. A-132) we proved that a convex smooth body in  $R^n$  ( $n \geq 3$ ) is a ball iff any orbit of a correspondent billiard is contained in a 2-dimensional plane. Now we generalize it to nonconvex bodies. Theorem 1. Let  $K$  be a smooth ( $C^3$ ) body (not necessarily convex) in  $R^n$ ,  $n \geq 3$ . Then  $K$  is a ball iff any billiard orbit is contained in a 2-dimensional affine subset of  $R^n$ . Theorem 2. Let  $K$  be a smooth ( $C^3$ ) surface in  $R^n$  ( $n \geq 3$ ) such that any geodesics on it is contained in a 2-dimensional affine subset of  $R^n$ . Then  $K$  is either part of a sphere or part of a hyperplane. (Received September 18, 1978.)

\*79T-D2 PATRICK SHANAHAN, Holy Cross College, Worcester, Massachusetts 01610. On the Signature of Grassmannians.

The Atiyah-Bott fixed point theorem is used to calculate the signature of the grassmannian manifolds. For the grassmannian  $G_{n,k}$  of  $k$ -dimensional subspaces of  $R^n$  the signature is zero except when  $n$  and  $k$  are even and  $k(n-k)$  is divisible by 8; in this case

$$\text{Sign}(G_{n,k}) = \begin{pmatrix} \lfloor \frac{n}{4} \rfloor \\ \lfloor \frac{k}{4} \rfloor \end{pmatrix}.$$

For the manifolds  $G_{n,k}(\mathbb{C})$  of  $k$ -dimensional subspaces of  $\mathbb{C}^n$  the signature is zero unless  $k(n-k)$  is even; in this case

$$\text{Sign}(G_{n,k}(\mathbb{C})) = \begin{pmatrix} \lfloor \frac{n}{2} \rfloor \\ \lfloor \frac{k}{2} \rfloor \end{pmatrix}.$$

(The latter formula corrects a computation of Connolly and Nagano.) (Received September 25, 1978.)

## Logic and Foundations (02, 04)

79T-E1 ERROL P. MARTIN, Philosophy Department, RSSS, Australian National University, Canberra, A.C.T. 2600, Australia. Solution to the Belnap conjecture for P-W. Preliminary report.

The relevant propositional logic P-W is formulated in the usual manner, with  $\rightarrow$  as sole connective, and has all instances of (1)  $A \rightarrow A$ , (2)  $B \rightarrow C \rightarrow A \rightarrow B \rightarrow A \rightarrow C$ , (3)  $A \rightarrow B \rightarrow B \rightarrow C \rightarrow A \rightarrow C$  as axioms, together with Modus Ponens as sole rule. For details see §8.11 of A.R. Anderson and N.D. Belnap, Jr., Entailment: The Logic of Relevance and Necessity, Princeton, N.J., 1975 (where the system is called  $T_{\rightarrow}$ -W). Let the system S be formulated similarly, but without the axiom (1) of identity. Theorem: No instance of  $A \rightarrow A$  is provable in S. This answers a conjecture of Anderson and Belnap, viz., that in P-W, if  $A \rightarrow B$  and  $B \rightarrow A$  are both theorems, then  $A$  and  $B$  are the same formula, or rather, answers a reduction of the problem due to L. Powers (also reported in §8.11 of Entailment). The theorem, which shows that non-question-begging systems of logic are by no means out of reach, uses a semantical analysis of S and P-W developed by R.K. Meyer in conjunction with the author. (Received September 22, 1978.) (Author introduced by R. K. Meyer).

79T-E2 ROBERT I. SOARE, University of Chicago, Chicago, Illinois 60637. Relative enumerability.

We distinguish those degrees recursively enumerable (r.e.) in a degree  $\underline{a}$  from those which are actually r.e.

Theorem. For every nonzero r.e. degree  $\underline{a}$  there is a degree  $\underline{b} \geq \underline{a}$  such that  $\underline{b}$  is r.e. in  $\underline{a}$  but  $\underline{b}$  is not r.e. Since the proof relativizes to any degree  $\underline{d}$ , this negatively answers a conjecture of Cooper (Notices A.M.S. 1972, Abstract 72T-E4, p.A-20) as to whether for every high degree  $\underline{d}$  (or even for any high degree  $\underline{d}$ ) every degree  $\underline{a} \geq \underline{0}'$  which is r.e. in  $\underline{0}'$  is r.e. in  $\underline{d}$ . For any degree  $\underline{a}$  let  $R(\underline{a}) = \{\underline{b} : \underline{b} \geq \underline{a} \text{ and } \underline{b} \text{ r.e. in } \underline{a}\}$ .

Question. For every r.e. degree  $\underline{a}$  is  $R(\underline{a})$  isomorphic to  $R(\underline{0})$  under the usual ordering relation? (Received September 15, 1978.)

79T-E3 O. M. KOSHELEVA and V. JA. KREINOVIC, USSR 196140 Leningrad Pulkovo Special Astrophysical Observatory. Infinitary-language version of NF is inconsistent. Preliminary report.

Attempts to obtain a complete theory lead logicians to the investigation of theories with denumerably long formulas. For standard theories (arithmetic, ZF, etc.) we indeed obtain (to this or that extent) more complete theories. We will show that for Quine's NF it is not so: NF with denumerably long formulas is inconsistent.

Proof.  $A = \{x \mid \neg \exists s_1 \dots s_n \dots (\dots \& s_{n+1} \in s_n \& \dots \& s_2 \in s_1 \& s_1 \in x)\}$ —the set of all founded sets—is itself founded, hence  $A \in A$ , and  $\dots A \in A \dots \in A$ , so  $A$  is not founded—a contradiction. (Received September 20, 1978.)

\*79T-E4 DREW MCDERMOTT, Yale University, New Haven, Connecticut 06520 and JON DOYLE, Artificial Intelligence Laboratory, Massachusetts Institute of Technology, 545 Technology Square, Cambridge, Massachusetts 02139. Non-monotonic logic I.

"Non-monotonic" logical systems are logics in which the introduction of new axioms can invalidate old theorems. Such logics are very important in modeling the beliefs of active processes which, acting in the presence of incomplete information, must make and subsequently revise predictions in light of new observations. We present the motivation and history of such logics. We develop model and proof theories, a proof procedure, and applications for one important non-monotonic logic. In particular, we prove the completeness of the first-order non-monotonic predicate calculus and the decidability of the non-monotonic sentential calculus. We also discuss characteristic properties of this logic and its relation to stronger logics, logics of incomplete information, and truth maintenance systems. Please direct reprint requests to the second author. (Received September 25, 1978.)

79T-E5 HEINRICH HERRE, HELMUT WOLTER, Academy of Sciences, Inst. of Math. and Mech., 108 Berlin, Humboldt University, GDR. Decidability of the theory of linear orderings with cardinality quantifier  $Q_\alpha$ .

Let  $L(Q_\alpha)$  be the logic with the quantifier "there exist at least  $\omega_\alpha$  many",  $T < (Q_\alpha)$  the theory of linear order in the logic  $L(Q_\alpha)$ .

Theorem 1 (ZFC)  $T < (Q_\alpha)$  is decidable for all cardinals  $\omega_\alpha$

Theorem 2  $T < (Q_\alpha) = T < (Q_\beta)$  if and only if one of the following conditions is satisfied:

1.  $\alpha=0, \beta=0$ , 2.  $\alpha, \beta > 0$  and  $\omega_\alpha, \omega_\beta$  are successor cardinals or weakly inaccessible,
3.  $\omega_\alpha, \omega_\beta$  are strongly inaccessible cardinals, 4.  $\omega_\alpha, \omega_\beta$  are strongly singular cardinals,
5.  $\omega_\alpha, \omega_\beta$  are weakly singular cardinals.

(Received October 6, 1978.) (Author introduced by Professor Kenneth Bowen).

79T-E6 WITHDRAWN

## Statistics and Probability (60, 62)

79T-F1 DEBORAH F. ALLINGER, MIT, Cambridge, Massachusetts 02139. Tensor Products and Abstract Wiener spaces. Preliminary Report.

It is shown that  $(i_1 \otimes i_2, H_1 \otimes H_2, E_1 \otimes E_2)$  is an Abstract Wiener space whenever  $(i_p, H_p, E_p)$  ( $p=1, 2$ ) are. Here,  $H_1 \otimes H_2$  represents the completed Hilbertian tensor product on  $H_p$ , ( $p=1, 2$ ), while  $E_1 \otimes E_2$  is the completed  $\Pi$ -crossnorm or the projective tensor crossnorm. In addition, the  $\mathcal{E}$ -crossnorm is briefly discussed along with examples of other tensor product spaces. (Received September 20, 1978.)

79T-F2 MAURY BRAMSON, Courant Institute of Mathematical Sciences, New York, New York 10012 and DAVID GRIFFEATH, University of Wisconsin, Madison, Wisconsin 53706. Asymptotic behavior of the d-dimensional biased voter model.

We investigate here the limiting behavior as  $t \rightarrow \infty$  of the continuous time biased voter model on  $Z^d$ ,  $d \geq 1$ . This behavior is the topic of two forthcoming papers. In the first, we show that conditional on its nonextinction, the privileged state of the voter model grows, and will (almost surely) eventually permanently occupy any finite  $A \subset Z^d$ . This strengthens more restrictive weak convergence results. In the second paper, we examine the rate of this growth, which we show to be linear. We apply a modification of the subadditivity techniques used by Richardson in his paper "Random growth in a tessellation". In this connection, it is relevant to note that the biased voter model may be interpreted as a model for the spread of cancer cells where recovery is permitted. (Received September 22, 1978.)

\*79T-F3 HARVEY R. DIAMOND, West Virginia University, Morgantown, WV 26506. Asymptotic Expectations for a Class of Functions of a Binomial Random Variable.

Let the random variable  $X$  have a binomial distribution  $B(N, p)$  where  $p = p(N)$ . Let  $f(x)$  be a locally bounded function defined for all  $x \geq 0$ . We are interested in the asymptotic behavior, as  $N \rightarrow \infty$ , of  $E[f(X)]$  under suitable restrictions on  $f(x)$  and  $p(N)$ . In particular we ask

- 1) Under what conditions does  $E[f(X)] \sim f(Np)$  hold?
- 2) If  $f(x)$  has derivatives, when does  $E[f(x)] \sim f(Np) + f''(Np)E[(X-Np)^2]/2! + \dots$  hold?

Definition: We say that  $f(x)$  has essentially zero asymptotic relative variation (EZARV) if the function  $g(x, a)$  defined by

$$g(x, a) = \sup_{|y| \leq a} \left| \frac{f(x+y\sqrt{x})}{f(x)} - 1 \right| \quad \text{satisfies } g(x, a) \rightarrow 0 \text{ as } x \rightarrow \infty \text{ for any fixed } a > 0.$$

Proposition: If  $f(x)$  is differentiable and  $f'(x)/f(x) = o(1/\sqrt{x})$  then  $f(x)$  has EZARV.

Theorem: If  $f(x)$  has EZARV and  $Np(N) \rightarrow \infty$  then  $E[f(X)] \sim f(Np)$

Theorem: Let  $k \geq 0$  be even and suppose  $f^{(k)}(x)$  exists, is locally bounded and has EZARV. If  $Np(1-p) \rightarrow \infty$  then

$$E[f(X)] = \sum_0^k \frac{f^{(i)}(Np)}{i!} s_i + \frac{f^{(k)}(Np)}{k!} s_k o(1) \quad \text{where } s_i = E[(X-Np)^i]$$

(Received October 16, 1978.)

## Topology (22, 54, 55, 57, 58)

\*79T-G1 Vo - Thanh - Liem, Department of Mathematics, Louisiana State University, Baton Rouge, Louisiana 70803. An  $\alpha$ -approximation theorem for  $Q^\infty$ -manifolds. Preliminary Report.

Given a  $Q^\infty$ -manifold  $M$  and an open cover  $\alpha$  of  $M$ , there is an open cover  $\beta$  of  $M$  such that every  $\beta$ -equivalence from a  $Q^\infty$ -manifold  $N$  to  $M$  is  $\alpha$ -close to a homeomorphism. (Received September 15, 1978.)

\*79T-G2 DARRYL McCULLOUGH, University of Oklahoma, Norman, Oklahoma 73019. Connected Sums of Aspherical Manifolds, Preliminary report.

Let  $M = M_1 \# M_2 \# \dots \# M_r$  be a connected sum of  $r \geq 2$  closed aspherical manifolds of dimension  $n \geq 3$ . Using obstruction theory with local coefficients, we prove there are exactly  $2^{r-1}$  homotopy classes of maps  $f: M \rightarrow M$  inducing the identity outer automorphism of  $\pi_1 M$ , and any such map is homotopic to a homeomorphism. (Received September 18, 1978.)

\*79T-G3 Richard E. Heisey, Vanderbilt University, Nashville, Tennessee 37235. Stability, Classification, and Open Embeddings of  $R$ -Manifolds. Preliminary report.

Let  $R^\infty = \text{dir lim } R^n$ ,  $R$  the reals. Let  $M, N$  denote paracompact, connected  $R^\infty$ -manifolds. Theorem 1. Given any open cover  $U$  of  $M$  there is a homeomorphism  $h: M \times R^\infty \rightarrow M$  which is  $U$ -close to the projection map. In particular,  $M \times R^\infty$  is homeomorphic to  $M$ . Theorem 2. There is an open embedding  $g: M \rightarrow R^\infty$ . Theorem 3. If  $f: M \rightarrow N$  is a homotopy equivalence then  $f$  is homotopic to a homeomorphism. Corollary 4. If  $M$  and  $N$  have the same weak homotopy type, then they are homeomorphic. (These results were announced at the Conference on Geometric Topology, Aug. 24-Sept. 2, 1978, in Warsaw.) (Received October 10, 1978.)

\*79T-G4 Vladimir N. Akis, University of California, Davis, California 95616. An example of a countable, Hausdorff and connected space with no dispersion points.

We have constructed a countable, Hausdorff space in which the closures of any two non-empty open sets, have a non-empty intersection, hence the space is connected. Furthermore, this space has no dispersion points. (The space consists of the natural numbers with the topology generated by the collection  $\beta = \{Z_p + r : p \text{ is any odd prime and } 1 \leq r \leq p-1\}$  where  $Z_p + r = \{kp + r : k=0,1,2,\dots\}$ .) (Received October 13, 1978.)

\*79T-G5 H. R. BENNETT, Texas Tech University, Lubbock, Texas 79409.  $\gamma$ -spaces on Certain GO-spaces. Preliminary report.

Theorem. Let  $(Y, \leq)$  be a separable LOTS and  $X$  a GO-space constructed on  $Y$ .  $X$  is a  $\gamma$ -space if and only if  $X$  is a quasi-metric space.

Definition. If  $X$  is a GO-space let  $R = \{x \in X \mid \{x\} \text{ is not open but } [x, \rightarrow) \text{ is open}\}$  and  $L = \{x \in X \mid \{x\} \text{ is not open but } (\leftarrow, x] \text{ is open}\}$ .

Theorem. Let  $(Y, \leq)$  be a separable LOTS and  $X$  a GO-space constructed on  $Y$ . If  $X$  is a quasi-metric space then  $R$  and  $L$  are  $F_\sigma$  subsets of  $R \cup L$  considered as a subspace of  $X$ . (Received October 16, 1978.) (Author introduced by D. J. Lutzer)

\*79T-G6 JUSTIN R. SMITH, University of Hawaii at Manoa, 2565 The Mall, Honolulu, HI 96822. Equivariant Eilenberg-MacLane Spaces. I - A multiplicative resolution.

This paper studies the following problem: given a group  $\pi$ , a module  $M$  over  $\mathbb{Z}\pi$  and a positive integer  $n$ , what is the chain-homotopy type of the singular chain complex of the space  $\{K(M, n) \times \tilde{K}(\pi, 1)\} / \pi = K_\pi(M, n)$ ? Here  $K(M, n)$  is equipped with a  $\pi$ -action compatible with the module structure of  $M$ ,  $\tilde{K}(\pi, 1)$  is the universal cover of a  $K(\pi, 1)$ , and the cartesian product has the diagonal  $\pi$ -action.

A formula is obtained that expresses this chain-complex as a twisted tensor product of complexes of type  $A(\mathbb{Z}^k, n)$  (see Eilenberg and MacLane, "On the groups  $H(\Pi, n)$ ," Ann. of Math., vol. 58 (1953), 55-106). This formula is a multiplicative analogue of a projective resolution of  $M$  and it reflects the homological properties of  $M$ . (Received October 16, 1978.)

79T-G7 August Lau, North Texas State University, Denton, Texas 76203. Dimension of the hyperspace of subcontinua. Preliminary Report.

Let  $X$  be a metric continuum and  $C(X)$  the collection of compact connected subsets of  $X$  with the Vietoris topology. The question is open whether a 2-dimensional continuum  $X$  always has an infinite dimensional hyperspace  $C(X)$ . Rogers in [Bull. Pol. Acad. Sci. vol. XX No. 2, 1972 p. 177-179] proved that if  $X$  is an arcwise connected 2-dim-

ensional continuum, then  $C(X)$  is infinite dimensional. A continuum is  $l$ -joinable if for every two points  $p$  and  $q$ , there exists a  $l$ -dimensional subcontinuum containing them. Theorem: A 2-dimensional  $l$ -joinable continuum has infinite dimensional hyperspace  $C(X)$ . (Received October 16, 1978.)

\*79T-G8 O.T. Alas, Universidade de São Paulo, Caixa Postal 20570 (Iguatemi), São Paulo, Brazil.  $\omega_\mu$ -metrizable spaces. Preliminary report.

Let  $\aleph_\mu$  be an uncountable cardinal number and let  $\omega_\mu$  be the first ordinal of cardinality  $\aleph_\mu$ . Theorem 1. A  $\omega_\mu$ -metrizable space  $X$  has a set of open coverings cofinal and totally pre-ordered if and only if the set of all non-isolated points of  $X$  is  $\aleph_\mu$ -compact. Theorem 2. Let  $X$  be a paracompact Hausdorff space with index  $\aleph_\mu$ . The diagonal of  $X \times X$  has an open basis of cardinality  $\aleph_\mu$  in  $X \times X$  if and only if  $X$  is  $\omega_\mu$ -metrizable and the set of all non-isolated points of  $X$  is  $\aleph_\mu$ -compact. For metrizable spaces the analogous results are due to the author (Atti Accad. naz. Lincei 50(1971), 1-3) and J. Ginsburg (Canad. Bull. Math. 20(1977), 513-514). (Received October 17, 1978.)

## LATE PAPERS — Presented at past meetings

759-B27 MARIO MARTELLI, University of California, Davis, California 95616. Hyperbolic problems : existence theorems and a counterexample. Preliminary report.

The solvability of the operator equation (1)  $L(x) = N(x)$  is studied in the framework of the so called hyperbolic problems. Existence theorems are obtained and a counterexample is given to show that the usual assumptions which ensures the existence of a solution of (1) in the case when  $L$  is Fredholm do not in general suffice in the present case. Boundary value problems for partial differential equations of hyperbolic type fit naturally into the given abstract setting. (Received September 28, 1978.) (Author introduced by Professor Stavros N. Busenberg).

\*759-F12 JACOB FELDMAN, University of California, Berkeley, California 94720.  $r$ -entropy and Macmillan's theorem for measure-preserving transformations and group actions.

Let  $T$  be a m.p.t. on the probability space  $(X, \mu)$ ,  $\mathcal{P}$  a finite measurable partition. One may define in a natural way an "approximate entropy"  $h_r(T, \mathcal{P})$ ,  $0 \leq r \leq 1$ , a continuous decreasing convex function of  $r$  which is 0 when  $r=1$ , and equals the "usual" entropy when  $r=0$ . There is a corresponding "second order" Macmillan theorem. The ideas may be generalized to m.p. actions of a large class of locally compact groups, leading to a definition of entropy in new situations, a meaningful "Macmillan theorem", and a possible generalization of the theory of Bernoulli shifts. Some basic questions remain unanswered. (Received September 14, 1978.)

760-B54 FON-CHE LIU, Institute of Mathematics, Academia Sinica, Taipei and Purdue University, West Lafayette, Indiana 47907. A Luzin type property of Sobolev functions. Preliminary Report.

Let  $G$  be a Lipschitz domain of  $R^n$  and  $W_p^{(\ell)}(G)$  be the Sobolev space with differentiability index  $\ell$  and integrability index  $p \geq 1$ . It is shown that any function  $f$  from  $W_p^{(\ell)}(G)$  can be approximated by  $C^\ell$ -functions in the following restricted sense: Given  $\epsilon > 0$ , there is a  $C^\ell$ -function  $g$  on  $G$  such that the measure of the set  $\{f \neq g\}$  is less than  $\epsilon$  and such that the Sobolev norms of  $f$  and  $g$  over  $\{f \neq g\}$  are both less than  $\epsilon$ . (Received October 5, 1978.) (Author introduced Professor D. Waterman).

S.J. Agronsky, R. Biskner and A.M. Bruckner proved in an unpublished paper that the characteristic function of a closed set in  $R = (-\infty, \infty)$  is the product of two derivatives and that an approximate derivative on  $R$  can be represented as  $f_1 + f_2 f_3 f_4 + f_5 f_6 f_7$ , where  $f_j$  are derivatives on  $R$ . These results can be improved as follows: 1. For each open set  $G \subset R$  let  $\mathfrak{F}(G)$  be the set of all functions  $F_1' F_2'$ , where  $F_j' \geq 0$  on  $G$ . Let  $A$  be closed in  $R$ ,  $B = R \setminus A$ ,  $f \in \mathfrak{F}(R)$ ,  $g \in \mathfrak{F}(B)$ ,  $g \leq f$  on  $B$ ,  $h = f$  on  $A$ ,  $h = g$  on  $B$ . Then  $h \in \mathfrak{F}(R)$ . 2. Let  $f_{ap}'$  exist on  $R$ . Then  $f_{ap}' = F_1' + F_2' F_3'$ ,  $F_j$  differentiable on  $R$ . (Received November 27, 1978.)

(Author introduced by Professor Daniel Waterman).

#### SUBJECT CLASSIFICATION SCHEME

The classification system used for abstracts of papers presented at annual and summer meetings consists of headings of the major sections of the AMS (MOS) Subject Classification Scheme (1970). A detailed listing of the scheme will be found in most of the index issues of *Mathematical Reviews*, volumes 38 (1969)–54 (1977).

00 General	32 Several complex variables and analytic spaces	62 Statistics
01 History and biography	33 Special functions	65 Numerical analysis
02 Logic and foundations	34 Ordinary differential equations	68 Computer science
04 Set theory	35 Partial differential equations	70 Mechanics of particles and systems
05 Combinatorics	39 Finite differences and functional equations	73 Mechanics of solids
06 Order, lattices, ordered algebraic structures	40 Sequences, series, summability	76 Fluid mechanics
08 General mathematical systems	41 Approximations and expansions	78 Optics, electromagnetic theory
10 Number theory	42 Fourier analysis	80 Classical thermodynamics, heat transfer
12 Algebraic number theory, field theory and polynomials	43 Abstract harmonic analysis	81 Quantum mechanics
13 Commutative rings and algebras	44 Integral transforms, operational calculus	82 Statistical physics, structure of matter
14 Algebraic geometry	45 Integral equations	83 Relativity
15 Linear and multilinear algebra, matrix theory	46 Functional analysis	85 Astronomy and astrophysics
16 Associative rings and algebras	47 Operator theory	86 Geophysics
17 Nonassociative rings and algebras	49 Calculus of variations and optimal control	90 Economics, operations research, programming, games
18 Category theory, homological algebra	50 Geometry	92 Biology and behavioral sciences
20 Group theory and generalizations	52 Convex sets and geometric inequalities	93 Systems, control
22 Topological groups, Lie groups	53 Differential geometry	94 Information and communication, circuits, automata
26 Real functions	54 General topology	96 Mathematical Education, Elementary
28 Measure and integration	55 Algebraic topology	97 Mathematical Education, Secondary
30 Functions of a complex variable	57 Manifolds and cell complexes	98 Mathematical Education, Collegiate
31 Potential theory	58 Global analysis, analysis on manifolds	99 None of the above
	60 Probability theory and stochastic processes	

## Abstracts for the 85th Annual Meeting

Convention Center, Biloxi, Mississippi, January 24–27, 1979

### 01 ► History and Biography

\*763-01-1 **FRANK WILLIAMSON, JR., Ramapo College of New Jersey, Mahwah, N.J. 07430**  
Richard Courant and the Finite Element Method: A Further Look

The finite element method has become, in recent years, a very popular numerical procedure for boundary value and eigenvalue problems. The present, variational form of the method, arising in the early 1960s, has been known since that time to have been conceived independently of a similar technique briefly suggested by Richard Courant in 1943. G. Strang and others have written to this effect. The present paper seeks to describe Courant's finite element work which led to the 1943 publication and shows that he used the finite element method as early as 1922 in connection with a Dirichlet principle existence proof. (Received October 12, 1978.)



763-01-2 Stanley H. Stahl, Mount Holyoke College, South Hadley, MA 01075 and Smith College, Northampton, MA 01060 and Catherine Stahl, Westfield State College, Westfield, MA 01085. The Borel Letters on the Theory of Sets.

In 1904 Zermelo published his proof that every set can be well-ordered; his proof used the recently isolated axiom of choice and, as is well known, engendered a great deal of controversy. In the same year Borel published a critique of Zermelo's proof and this was followed by an interchange of five letters on set theory and the axiom of choice between Borel, Hadamard, Lebesgue, and Baire. We discuss these letters from the perspective of the development of mathematics during the past 75 years. (Received October 16, 1978.)

## 02 ► Logic and Foundations

763-02-1 ANIL NERODE, Cornell University, Ithaca, New York 14850. Recursive vs. Constructive Mathematics.

There are constructivist traditions stemming from Kronecker in Algebra, Brouwer in Analysis which continue through such authors as Seidenberg, Bishop, Sanin. Recursive function theory, invented by Godel, Church, Kleene, Turing, Post, has been used to develop recursive analysis and algebra starting with Banach-Mazur and Frölich-Shepherdson respectively. Contemporary recursion theory has been used to analyze the effective content of various mathematical constructions by a number of authors, including the present one, Ersov, and co-workers. The constructivist results, interpreted classically, are necessary to justify the effectiveness of the atomic acts of the priority arguments for the recursive algebra and analysis. Conversely, the recursive function theoretic proofs often yield intuitionistic demonstrations of the absurdity of the assumption that certain constructions exist. We illustrate this with examples from field theory and from functional analysis. We also discuss the role of systems such as those of Myhill, Feferman, Friedman. (Received September 27, 1978.)

\*763-02-2 JOHN MYHILL, State University of New York at Buffalo, Buffalo, New York 14214 and ANDREJ ŠČEDROV, State University of New York at Buffalo, Buffalo, New York 14214. On an extension of the intuitionistic propositional calculus. Preliminary report.

Let  $PC^+$  be a system obtained from Heyting's propositional calculus by adjoining variables  $f_1, g_1, \dots$  for (one place) propositional connectives, quantifiers  $(\forall p), (\exists p), (\forall f), (\exists f)$  for both kinds of variables (with the usual rules for quantifiers) and the comprehension scheme. It is shown that  $PC^+ \vdash (\forall f)[(\forall pq)((fp \leftrightarrow fq) \leftrightarrow (p \leftrightarrow q)) \leftrightarrow (\forall p)(ffp \leftrightarrow p)] (*)$ . We also investigate the status of  $(\forall f)[(\forall pq)((fp \leftrightarrow fq) \leftrightarrow (p \leftrightarrow q)) \leftrightarrow (\forall p)(fp \leftrightarrow p)]$  in  $PC^+$  and its translation into the system CST described in JSL 40 (1975), 347-382. The set of all propositions  $\Omega$  is described as the power-set of a singleton, and Church's Thesis and Brouwer's Continuity Principle imply that  $\text{card}(\Omega)$  is incomparable with both  $\aleph_0$  and  $\text{card}(\mathbb{R})$ . (Received October 4, 1978.)

763-02-3 Julius Barbanel, State University of New York at Buffalo, Buffalo, New York 14214. Results on Supercompact Cardinals. Preliminary report.

Theorem 1: Let  $\kappa$  be a supercompact cardinal and  $\lambda > \kappa$  a strong limit cardinal. Then there is a collection  $\{G_\alpha : \alpha < \lambda\}$  such that: a. Each  $G_\alpha$  is a collection of  $2^{2^\lambda}$  normal ultrafilters on  $P_\kappa(\lambda)$ , b. The  $G_\alpha$ 's are pairwise disjoint, and c. If  $U_0, U_1 \in G_\alpha$  for some  $\alpha$ , then for each cardinal  $\eta$  with  $\kappa \leq \eta < \lambda$ ,  $U_0 \upharpoonright \eta = U_1 \upharpoonright \eta$ . This improves a result of C. DiPrisco [Advances in Math., v. 25, p. 253]. Theorem 2: Let  $\kappa$  be a supercompact cardinal. Define a function  $f$  on  $\kappa$  by  $f(0) = \kappa$ ,  $f(\alpha + 1) = 2^{f(\alpha)}$ , and  $f(\lambda) = [\sup_{\alpha < \lambda} f(\alpha)]^+$  if  $\lambda$  is a limit ordinal. Then, for

each  $\alpha < \kappa$ , there is a collection  $G_\alpha$  of  $2^{2^{f(\alpha)}}$  normal ultrafilters on  $P_\kappa(f(\alpha))$  such that for no normal ultrafilter  $U$  on  $P_\kappa(f(\alpha + 2))$  do we have  $U \upharpoonright f(\alpha) \in G_\alpha$ . Theorem 3: Let  $\kappa$  be a supercompact cardinal,  $\lambda \geq \kappa$ , and  $i$  the elementary embedding of the universe associated with a normal ultrafilter on  $P_\kappa(\lambda)$ . Then, for any limit cardinal  $\eta > 2^{\lambda^\kappa}$ ,  $i(\eta) = \eta$  iff  $[cf(\eta) < \kappa$  or  $cf(\eta) > \lambda^\kappa]$ . (Received October 19, 1978.)

763-02-4 IRVING H. ANELLIS, Department of Philosophy, University of Florida, Gainesville, Florida 32611.  
The Löwenheim-Skolem Theorem, theories of quantification, and Beweistheorie.

The theory of quantification is a "family of formal systems" whose members include Hilbert-type systems, Herbrand Quantification, Gentzen-type systems, and Natural Deduction (J. van Heijenoort, El desarrollo de la teoría de la cuantificación). Investigation strongly indicates that the technical developments in the Hilbert-type systems, and the rise of the other systems, were stimulated less by the Gödel Incompleteness results than by questions raised by the Löwenheim-Skolem Theorem on  $\aleph_0$ -satisfiability. Special reference can be made in this respect to the Beweistheorie for Hilbert-type systems. We also note, for example, that the fundamental expansion theorem of Herbrand is a special case of the infinite conjunction used by Löwenheim to obtain his results on  $\aleph_0$ -satisfiability. (Received October 23, 1978.)

\*763-02-5 KEVIN W. SAUNDERS, CEMREL Education Laboratory, St. Louis, Missouri 63139.  
A Modal Logic Treatment of "Probable".

The Kripke models for modal logic may be extended to models for modal treatments of probability-valued propositions allowing those propositions to be treated within a two-valued context. In such a treatment, a probability-valued proposition is interpreted as a statement regarding the relative frequency (treated as a limit) of elements of the base set of the model structure in which the proposition is true compared to the totality of the elements. Each element remains two-valued, while probability-valued propositions (and propositions regarding necessity and possibility) refer to the base set taken as a whole rather than to individual elements of that set. Through this approach, logical systems incorporating the operator "probable" may be examined. By considering various properties of the accessibility relation for the model structure, different modalizations are obtained. (Received October 23, 1978.) (Author introduced by Joel Schneider).

\*763-02-6 Bruce I. Rose, University of Notre Dame, Notre Dame, Indiana 46556.  
Prime quantifier eliminable rings.

The main result is that a prime ring with a finite center which admits elimination of quantifiers in the language of rings must be finite. This answers a question raised by the author and completes the classification of prime rings which admit elimination of quantifiers. That is, a prime ring admits elimination of quantifiers in the language of rings if and only if it is either an algebraically closed field, a finite field, or a  $2 \times 2$  matrix ring over a finite prime field. It is also shown that if  $R$  is a semiprime ring of characteristic  $p > 0$  and  $R$  admits elimination of quantifiers then  $R$  is either an algebraically closed field or is  $\aleph_0$ -categorical.

Other results about quantifier eliminable rings are obtained. (Received October 23, 1978.)

\*763-02-7 Alexander ABIAN, Department of Mathematics, Iowa State University, Ames, Iowa 50011  
Various methods of evaluating Negation and the corresponding Logics

Let  $S$  be a set and  $A$  a fixed subset of  $S$ . Let  $p, q, \dots$  be atomic formulas and  $\neg, \vee, \wedge, \rightarrow$  the connectives of a Propositional Language. To every atomic formula  $p$  a subset  $E(p)$  of  $S$  is assigned (by a fixed rule) as "the value of  $p$ ". Then for every formula  $\alpha$  and  $\beta$  the values  $E(\neg\alpha)$ ,  $E(\alpha \vee \beta)$ ,  $E(\alpha \wedge \beta)$ ,  $E(\alpha \rightarrow \beta)$  are defined in terms of  $E(\alpha)$  and  $E(\beta)$  in a variety of ways (giving rise to various Propositional Logics) subject to the following two conditions (in order to reflect some very basic tenets of our ordinary reasoning): for every formula  $\alpha$ ,

$$E(\alpha) \cap E(\neg\alpha) = \emptyset \quad \text{and} \quad E(\alpha) \cap A \neq \emptyset \quad \text{if and only if} \quad E(\neg\alpha) \cap A = \emptyset$$

For instance, after defining  $E(\neg\alpha)$  in compliance with the above conditions, often  $E(\alpha \vee \beta)$ ,  $E(\alpha \wedge \beta)$ ,  $E(\alpha \rightarrow \beta)$  can be defined respectively as:  $E(\alpha) \cup E(\beta)$ ,  $E(\alpha) \cap E(\beta)$ ,  $E(\neg\alpha) \cup E(\beta)$ .

The theorems of the corresponding Propositional Logics are formulas  $\tau$  such that  $E(\tau) \supseteq \Lambda$ . Generally in these Logics  $\alpha \rightarrow \alpha$  is not a theorem whereas  $\alpha \rightarrow \neg \neg \alpha$  is a theorem.

From each of these Logics, the Classical Propositional Logic is recovered by calling a formula  $\alpha$  "true" if and only if  $E(\alpha) \cap \Lambda \neq \emptyset$ .

An Example is provided by defining  $E(\neg p) = (S - A) - E(p)$  if  $E(p) \cap \Lambda \neq \emptyset$  and  $E(\neg p) = A$  if  $E(p) \cap \Lambda = \emptyset$ . Barring some exceptional cases (such as  $S$  being a two-element set) it is readily seen that in this Example  $\alpha \rightarrow \alpha$  is not a theorem whereas  $\alpha \rightarrow \neg \neg \alpha$  is a theorem.

The above Propositional Logics can be extended (in a usual way) to the corresponding Predicate Logics where, in compliance with the above conditions,  $E(\exists x)P(x)$  can be defined as  $\cup E(P(x))$ . (Received October 25, 1978.)

## 04 ► Set Theory

763-04-1 NEWCOMB GREENLEAF, Naropa Institute, 1111 Pearl Street, Boulder, Colorado 80302. What Cantor did and didn't prove—a constructive evaluation.

Paradox. The Cantor diagonal method, an eminently constructive procedure, is the essence of all proofs of higher cardinality, and the latter, when seen as implying the existence of undescrivable or uncomputable real numbers, is a key aspect of the nonconstructivity of classical mathematics. Various confusions that have surrounded this paradox will be examined, the viewpoints of Poincaré, Brouwer, and Weyl will be considered, and a simple restatement of Cantor's theorem, taken from recursive function theory, will be given. Definition. A set  $X$  is productive if for every sequence  $(x_n)$  in  $X$  an element  $x$  of  $X$  can be found which is different from every  $x_n$ . Cantor's Theorem. The real numbers are a productive set. Proofs of the existence of the undescrivable can now be seen to be circular, though a subtle analysis of the role of convergence in the Borel Lemma is necessary for the measure theory proof. Is it apt to use a quantitative metaphor, cardinality, to describe this difference between the natural and the real numbers? On one hand, relative productivity cannot be proved to be transitive ( $X$  is productive over  $Y$  if for every function from  $Y$  to  $X$  an element of  $X$  can be found which is outside of the image). On the other hand, the implications of the constructive theory of cardinality of Brouwer (1924) and Troelstra (1967) must be considered. (Received October 4, 1978.)

763-04-2 ARTHUR L. RUBIN, Jet Propulsion Laboratory, Pasadena, CA 91103. On the cardinality of sets of incomparable cardinals. Preliminary Report.

Let  $T^*(\mathfrak{p})$  denote "Any set of incomparable cardinals has cardinality less than  $\mathfrak{p}$ . Let  $\mathfrak{M}$  be the "standard" Fraenkel-Mostowski model of ZFU, obtained from a countable set of urelements  $U$ , the group of all permutations of  $U$ , and only allowing finite supports.

Theorem.  $\mathfrak{M} \models T^*((2^{\aleph_0})^+)$  &  $\neg AC$ .

If CH holds in the ground model, it holds in  $\mathfrak{M}$ , so that AC is independent of  $T^*(\aleph_2)$ .

Other results on the structure of the partially ordered class of cardinals will be presented, including the theorem of abstract 78T-E3. (Received October 23, 1978.)

763-04-3 John Carson Simms, Texas Tech University, Lubbock, Texas 79409.  $\Sigma_{\aleph_1}^0(\Pi_{\aleph_1}^1)$  - Determinacy. Preliminary report.

Definition. A set is  $\Sigma_{\aleph_1}^0(\Pi_{\aleph_1}^1)$  iff it is a countable union of finite Boolean combinations of  $\Pi_{\aleph_1}^1$  sets.

Theorem. Every  $\Sigma_{\aleph_1}^0(\Pi_{\aleph_1}^1)$  game is determined iff for all reals  $x$  there is a transitive inner model  $M$  of ZFC s.t. (1)  $x \in M$ , (2)  $\text{Ord} \in M$ , (3) the class of measurable cardinals in  $M$  is proper, and (4) there is a closed proper class of indiscernibles for  $\langle M, \varepsilon \upharpoonright M \times M, x \rangle$ . The appropriate relative effective form of this theorem holds, too. (Received October 23, 1978.)

763-04-4 STEWART BALDWIN, University of Colorado, Boulder, Colorado 80309. A Normal Form for some Coherent Sequences of Ultrafilters. Preliminary Report.

For definitions see W. Mitchell, Sets Constructible from Sequences of Ultrafilters, Journal of Symbolic Logic, vol. 39, pp. 57-66.

Def: For  $\alpha \in \text{OR}$ , define the classes  $A_\alpha$  as follows:  $A_0 = \text{OR}$ ,  $A_1 = \{\alpha : \alpha > \omega \ \& \ \alpha \text{ regular}\}$ ,  $A_\lambda = \bigcap_{\alpha < \lambda} A_\alpha$  if  $\lambda$  is a limit, and  $A_{\alpha+1} = \{\beta : A_\alpha \cap \beta \text{ is stationary in } \beta\}$ . Let  $B_\alpha = A_\alpha - A_{\alpha+1}$ .

For each ordinal  $\beta$  let  $b(\beta)$  be the unique  $\alpha$  s.t.  $\beta \in B_\alpha$ . For each  $(\alpha, \beta)$  with  $\beta < b(\alpha)$ , let  $U_0(\alpha, \beta)$  be the filter generated by all sets of the form  $x \cap B_\beta$ , with  $x$  closed and unbounded in  $\alpha$ . For each  $\mu, \nu$  define  $K_{[\mu, \nu]}$  by  $\text{dom } K_{[\mu, \nu]} = \nu$  and  $K_{[\mu, \nu]}(\alpha) = b(\alpha)$  if  $\alpha \geq \mu$  and 0 otherwise. Let  $U_{[\mu, \nu]} = U_0 \cap \{(\alpha, \beta) : \alpha < \nu, \beta < K_{[\mu, \nu]}(\alpha)\}$ .

**Theorem:** Assume there is a stationary class of cardinals  $\kappa$  s.t.  $\kappa \in B_\kappa$ . Let  $U$  be any coherent  $\kappa$ -sequence s.t.  $K(\alpha) \leq \alpha$  for all  $\alpha$ . Then there is a  $\lambda$  s.t. for all  $\mu \geq \lambda$  with  $\mu \in B_1$ , there is a  $\nu > \mu$  and an elementary embedding  $j : L[U] \rightarrow L[U_{[\mu, \nu]}]$ .

This result can be extended to measurable cardinals  $\kappa$  of degree greater than  $\kappa$  by assuming stronger Mahlo axioms, but at the expense of making the definition of the canonical filters  $U_0(\alpha, \beta)$  more difficult. (Received October 19, 1978.) (Author introduced by Professor William N. Reinhardt).

## 05 ► Combinatorics

763-05-1 FRANK K. HWANG and SHEN LIN, Bell Laboratories, 600 Mountain Avenue, Murray Hill, N.J. 07974. Construction of Balanced Switching Networks.

A balanced switching network is a multistage switching network where all channel graphs  $G(\alpha, \beta)$  (the union of all paths from an input switch  $\alpha$  to an output switch  $\beta$ ) are isomorphic. A switch is said to be of size  $n$  by  $m$  if it has  $n$  input lines and  $m$  output lines. The parameters of a multistage switching network are the number of switches and the sizes of the switches in each stage. (Switches in the same stage have the same size.) Given a set of parameters for a multistage switching network, the question whether a balanced switching network exists with these parameters, and if so, how can such a network be constructed, is a difficult combinatorial problem. We present here some answers to this problem. (Received October 2, 1978.)

\*763-05-2 AILEEN M. McLOUGHLIN, Department of Computer Science, Trinity College, Dublin, Ireland. The covering radius of the  $(m-3)$ rd order Reed Muller codes and a lower bound on the  $(m-4)$ th order Reed Muller codes.

A lower bound on the covering radius of  $RM(m, m-3)$  is obtained by exhibiting a coset and finding a lower bound on the weight of its leader. It is shown that this lower bound coincides with the upper bound given by Delsarte's theorem and is thus the exact covering radius. A lower bound on the covering radius of  $RM(m, m-4)$  is then obtained by exhibiting a coset leader and applying the result for  $RM(m, m-3)$ . (Received October 2, 1978.)

\*763-05-3 A.O.L. Atkin, L. Hay, and R.G. Larson, University of Illinois at Chicago Circle, Box 4348, 60680. Construction of Knut Vik Designs.

A Knut Vik Design of order  $n$  is an arrangement of  $n$  copies of the symbols 1 through  $n$  in the cells of an  $n$ -square such that every row, column, and (broken) diagonal, contains a permutation of the symbols 1 through  $n$ . Under this name they arise in the theory of design of experiments in statistics, although they enjoy an extensive previous literature, beginning with Euler, as Pandiagonal Latin Squares. The authors find all designs for  $n$  less than or equal to 13, and give some general methods of construction. (Received October 4, 1978.)

\*763-05-4 Cloyd L. Ezell, Jr., Stetson University, DeLand, Florida 32720. Observations on the Construction of Covers using Permutation Voltage Assignments.

In the usual treatment of covering spaces, the algebraic object associated with a cover is a conjugacy class of subgroups of the fundamental group of the target space. For finite degree covers, one could as easily choose the algebraic object to be a class of homomorphisms from the fundamental

group of the target space to the symmetric group on  $n$  letters, where  $n$  is the degree of the cover. A description of this representation of covering spaces is given in this paper. Then the permutation voltage assignment used in the permutation voltage construction of graph and surface covers is viewed as a means of defining one such homomorphism. Using this view, methods are given for counting the number of connected components in the constructed cover and for determining the orientability of the surface cover. In addition, it is observed that every branched cover can be constructed by permutation voltage assignments. (Received October 5, 1978.)

\*763-05-5 Terry A. McKee, Wright State University, Dayton, Ohio 45435.  
A quantifier for matroid duality.

A method is considered for taking a graph-theoretic statement properly expressed in terms of edges and cycles, and then finding its dual statement still in terms of edges and cycles.

Consider a quantifier on the elements of a matroid which says, for a fixed element  $e$ , "For all elements (except possibly  $e$  itself) of some circuit containing  $e$ , ...". The matroid dual of this quantifier happens to be its logical dual, and so a language utilizing this quantifier turns out to be just what is needed for such "cycle-logical" analysis. This quantifier also provides a rather elegant reformulation of Minty's self-dual axiomatization of matroids. (Received October 11, 1978.)

\*763-05-6 Joseph A. Gallian, University of Minnesota-Duluth, Duluth, Minnesota 55812 and David J. Rusin, University of Chicago, Chicago, Illinois 60637. Cyclotomic polynomials and nonstandard dice.

In this paper, we consider a broad generalization of a problem which first appeared in SCIENTIFIC AMERICAN, Vol. 238 (1978) pp 19-32. The original problem was to find all possible ways to label  $n$  cubes with positive integers so that the  $n$  cubes, when thrown simultaneously, will yield the same sum totals with the same frequency as  $n$  ordinary dice labelled 1 through 6. We investigate the analogous problem for  $n$  dice, each with  $m$  labels. A simple, purely algebraic characterization of solutions to this problem is given, and the problem is solved for certain infinite families of the parameter  $m$ . Several results on the general problem are included, and a number of avenues for further research are suggested. (Received October 12, 1978.)

\*763-05-7 L. CLARK, R.C. ENTRINGER, University of New Mexico, Albuquerque, NM 87131 and D.E. JACKSON, Eastern New Mexico University, Portales, NM 88131. Minimum graphs with complete  $k$ -closure.

The  $k$ -closure of a graph  $G$ , as defined by Bondy and Chvátal, is the graph obtained from  $G$  by recursively joining pairs of nonadjacent vertices with degree sum at least  $k$ . We determine the minimum number of edges  $G$  can have if its  $k$ -closure is complete. (Received October 16, 1978.)

\*763-05-8 Jerrold R. Griggs, California Institute of Technology, Pasadena, California 91125. On Chains and Sperner  $k$ -Families in Ranked Posets. Preliminary report.

A ranked poset  $P$  has the Sperner property if the sizes of the largest rank and of the largest anti-chain in  $P$  are equal. A natural strengthening of the Sperner property is condition S: for all  $k$ , the set of elements of the  $k$  largest ranks in  $P$  is a Sperner  $k$ -family.  $P$  satisfies condition T if for all  $k$  there exist disjoint chains in  $P$  which each meet the  $k$  largest ranks and which cover the  $k$ th largest rank. It is proven here that if  $P$  satisfies S, it also satisfies T, and that the converse, although in general false, is true for posets with unimodal Whitney numbers. The proof uses the result of Greene and Kleitman that every poset has a partition that is  $k$ - and  $(k + 1)$ -saturated. Conditions S and T and the Sperner property are compared here with two other conditions on posets concerning the existence of certain partitions of  $P$  into chains. (Received October 16, 1978.)

\*763-05-9 DONALD L. GOLDSMITH, Western Michigan University, Kalamazoo, Michigan 49008 and ROGER C. ENTRINGER, University of New Mexico, Albuquerque, New Mexico 87106. A sufficient condition for equality of edge-connectivity and minimum degree of a graph.

Let  $G$  be a graph of order  $p$ , with minimum degree  $\delta(G)$  and edge-connectivity  $\kappa_1(G)$ . It was proved by Chartrand that if  $\delta(G) \geq \lfloor p/2 \rfloor$ , then  $\kappa_1(G) = \delta(G)$ . The following theorem generalizes Chartrand's result.  $N(x)$  represents the neighborhood of the vertex  $x$ .

Theorem. Let  $G$  be a connected graph of order  $p \geq 2$ , with edge-connectivity  $\kappa_1(G)$  and minimum degree  $\delta(G)$ . If, for each vertex  $x$  of minimum degree we have

$$(*) \quad \sum_{w \in N(x)} \deg w \geq \begin{cases} \lfloor p/2 \rfloor^2 - \lfloor p/2 \rfloor & \text{for all even } p \text{ and for odd } p \leq 15, \\ \lfloor p/2 \rfloor^2 - 7 & \text{for odd } p \geq 15, \end{cases}$$

then  $\kappa_1(G) = \delta(G)$ . Furthermore, the lower bounds given in (\*) are best possible.

(Received October 18, 1978)

763-05-10 WILLIAM STATON, University of Mississippi, University, Mississippi 38677. Bipartite subgraphs of cubic graphs. Preliminary report.

Given a graph  $G$ , we call a bipartite spanning subgraph  $H$  large if  $H$  has as many edges as possible.

Theorem: If  $G$  is cubic and connected, then:

i) if  $G$  is not  $K_4$ , any large  $H$  contains at least  $7/9$  the edges of  $G$ . This lower bound is best possible.

ii) If  $G$  contains no triangle, then any large  $H$  contains at least  $11/14$  the edges of  $G$ .

(Received October 19, 1978.)

\*763-05-11 NEIL HINDMAN, California State University, Los Angeles, CA 90032. Partitions and sums and products -- two counterexamples.

A negative answer is provided to a question of Erdős. Specifically, a two celled partition of  $N$  is provided with the property that neither cell includes an infinite set together with all finite products and pairwise sums from that set. Also, a seven celled partition of  $N$  is provided with the property that no cell includes an infinite set together with all pairwise products and pairwise sums from that set. (Received October 19, 1978.)

763-05-12 R. RINGEISEN, AMRL/HEB, Wright-Patterson A.F.B., OH 45433 (on leave from Ind.-Purdue, Ft. Wayne), Crossing #'s of Permutation Graphs of Cycles, prelm. report.

For any permutation  $s$  in  $S_p$ , Chartrand and Harary define the  $s$ -permutation graph of a labeled graph with  $p$  vertices to be the union of two disjoint copies  $G_1$  and  $G_2$  of  $G$  together with the edges joining vertex  $v_i$  of  $G_1$  with  $s(v_i)$  in  $G_2$ . Let  $C(n,s)$  be the  $s$ -permutation graph of the cycle  $C_n$ . Thm.  $cr(C(n,s)) = 1$  if  $s$  is an  $n-1$  cycle of consecutive integers.

Thm. For  $n$  at least six,  $cr(C(n,s)) = 1$  or  $2$  if  $s$  is a 2-cycle of consecutive or nonconsecutive integers, respectively:  $cr(C(5,s)) = 1$  for all 2-cycles,  $s$ . We also obtain the crossing numbers of what are called generalized Petersen graphs. In addition other conjectures are discussed. (Received October 19, 1978.)

763-05-13 ELIZABETH J. MORGAN, University of Queensland, St. Lucia, Queensland 4067, Australia. On  $3 \times 3$  integer matrices with constant row and column sum. Preliminary report.

Let  $P = [p_{ij}]$  be a  $3 \times 3$  matrix with  $p_{ij} \in \{0, 1, 2, \dots, \mu-1\}$ , such that  $\sum_{i=1}^3 p_{ik} = \sum_{j=1}^3 p_{kj} = \mu$ , for  $k = 1, 2, 3$ . Any matrix which can be obtained from  $P$  by permuting rows and columns of  $P$  or by taking the transpose of  $P$  is said to be equivalent to  $P$ . The number,  $n(\mu)$ , of such inequivalent matrices for any  $\mu \geq 2$ , is given by  $n(\mu) = \sum_{i=1}^{\mu-2} \sum_{m=\max\{0, 2i-\mu\}}^{\lfloor i/2 \rfloor} \sum_{r=0}^{\lfloor i/2 \rfloor - m} c_{imr}$ , where  $c_{imr} = \mu - 2i + m + 1$  if  $m+r \neq \frac{1}{2}i$ , and  $c_{imr} = \lfloor \frac{1}{2}(\mu - 2i + m + 2) \rfloor$  if  $m+r = \frac{1}{2}i$ , and where  $\mu = 3\ell$ ,  $3\ell - 2$  or  $3\ell - 1$  according as  $\mu = 0, 1$ , or  $2 \pmod{3}$ . The problem of determining  $n(\mu)$  arose in connection with the enumeration of all non-isomorphic doubly balanced designs on eight elements with block size four. (Received October 20, 1978.)  
(Author introduced by Dr. Anne Penfold Street).

\*763-05-14 WILLIAM B. POUCHER, Abilene Christian University, Abilene, Texas 79601. A note on intersection preserving embeddings of partial  $(n, 4)$ -PBDs. Preliminary report.

An  $(n, K)$ -PBD is a pairwise balanced design of order  $n$  and block sizes from  $K$ . If  $|K| = 1$ , then we replace  $K$  with its single element. It has been shown by Lindner and Rosa for Steiner Triple Systems and by the author for all other  $(n, K)$ -PBDs where  $K \neq \{4\}$  that any collection of partial  $(n, K)$ -PBDs can be embedded in a collection of  $(v, K)$ -PBDs preserving initial blockset intersection. This paper solves the remaining problem for  $K = \{4\}$ . In addition it is shown THEOREM: Any partial SQS can be embedded in a  $3 \rightarrow 2$  resolvable partial SQS. (Received October 20, 1978.)

763-05-15 JACOB E. GOODMAN, City College (CUNY), New York, N.Y. 10031. Configurations in the plane: isotopy and combinatorial equivalence. Preliminary report.

A configuration  $(P_1, \dots, P_n)$  in  $R^2$  is nondegenerate if no three points are collinear and no lines joining pairs of points are parallel. Let  $C, C'$  be nondegenerate  $n$ -configurations in  $R^2$ .  $C$  is isotopic to  $C'$  if there is a continuous family of nondegenerate configurations joining  $C$  and  $C'$ . Each pair  $P_i, P_j$  in  $C$  determines a line  $L_{ij}$  through  $O$ , parallel to  $\overline{P_i P_j}$ ;  $C$  is combinatorially equivalent to  $C'$  if the cyclic ordering of this family of lines agrees in both configurations. A half-space of  $C$  is the subset lying on one side of some line. A basic problem in combinatorial geometry is that of finding necessary and sufficient conditions for a family of subsets of an abstract  $n$ -set to be the family of all half-spaces in some nondegenerate embedding of  $C$  in  $R^2$  (or in  $R^d$ , for that matter). We discuss the relation between combinatorial equivalence and isotopy, and state a conjectured solution of this problem involving sequences of permutations of the points of a configuration. (Received October 23, 1978.)

763-05-16 Jean H. Bevis, Georgia State University, Atlanta, Georgia 30303. Determinants for variable adjacency matrices. Preliminary report.

For a graph  $G=(V, E)$  the free idempotent commutative semiring  $F$  over  $E$  (considered as a set of labels or indeterminants) provides a natural setting for the study of path algebras over  $G$ . Although there are no additive inverses in  $F$ , a meaningful determinant function may be defined for matrices over  $F$ . This is obtained by using the symmetric difference of canonical representations of elements of  $F$ . Many of the results for determinants over the commutative ring of polynomials over  $E$  may be obtained in this new setting. In particular, versions of Harary's theorem for the variable adjacency matrix, and Kirchhoff's theorem for the variable incidence matrix are obtained for matrices over  $F$ . (Received October 23, 1978.)

763-05-17 HAI-PING KO, Oakland University, Rochester, Michigan 48063 and DIJEN K. RAY-CHAUDHURI, The Ohio State University, Columbus, Ohio 43210. A multiplier theorem of group ring.

Let  $G$  be an abelian group of exponent  $v^*$ . Write elements of the group ring  $Z[G]$  in the form  $\sum_{g \in G} a_g x^g$ ,  $a_g \in Z$ . For every subset  $D$  of  $G$ , define  $D(x) = \sum_{d \in D} x^d$ . An integer  $t$  is called a multiplier of a member,  $d(x)$ , of  $Z[G]$  if  $(t, v^*) = 1$  and  $d(x^t) = x^g d(x)$  for some  $g \in G$ . Theorem. Suppose  $d(x)d(x^\alpha) = a + bH(x) + cG(x)$  in  $Z[G]$  for some integers  $a, b, c$ , a subgroup  $H$  of  $G$ , and a homomorphism  $\alpha : G \rightarrow G$

such that  $H^\alpha = H$ . Let  $H$  have order  $n \geq 2$  and  $\sigma : G \rightarrow G/H$  be the endomorphism determined by  $\sigma(g) = H + g$ . Then  $t$  is a multiplier of  $d(x)$  if there exist positive integers  $k_1$  and  $k_2$  such that (M1)  $a = k_1 k_2$ ,  $(k_1, v^*) = 1$ ; (M2) for every prime divisor  $p$  of  $k_1$ , there exists a nonnegative integer  $t_p$  such that  $t \equiv p^{t_p} \pmod{v^*}$ ; (M3)  $t$  is a multiplier of  $d(x^\sigma)$  in  $Z[G/H]$ , and one of (T1), (T2) and one of (TT1), (TT2) defined as follows are satisfied. (T1)  $k_2 = 1$  and  $\alpha = -1$ ; (T2) all coefficients of  $d(x)$  are nonnegative,  $k_1 > b + c$  and  $k_1 > c$ ; (TT1)  $a + bn \neq 0$ ; (TT2)  $\alpha = -1$  and all coefficients of  $d(x)$  are 0 or 1. Here, (M3) may be replaced with conditions in terms of  $a, b, c$  only, by using the theorem itself. (Received October 23, 1978.)

\*763-05-18 Professor R. W. Robinson, University of Newcastle, New South Wales 2308, Australia  
 Phil Hanlon, California Institute of Technology, Pasadena, California 91125  
The Enumeration of 3-connected Graphs.

A connected graph  $G$  with  $k + 1$  or more points in  $k$ -connected if wherever  $k - 1$  distinct points are removed from  $G$ , the resulting graph is connected. W. T. Tutte has shown that given a 2-connected graph  $G$ , one can define a unique tree  $T(G)$  among whose points are the 3-connected components of  $G$ . A dissimilarity characteristic sum, taken over all such trees  $T(G)$  for  $G$  2-connected, yields an equation of the form

$$Z(H) - P(b_1, c_1, b_2, c_2, \dots) = f(Z(R))$$

where  $Z(H)$  is the line cycle index sum for 2-connected graphs, where  $f(Z(R))$  is a function of the line cycle index sum for 3-connected graphs and where  $P(b_1, c_1, b_2, c_2, \dots)$  is an easily computed member of  $Q[[b_1, c_1, b_2, \dots]]$ . Here  $n$ -cycles of lines which form diagonals of  $2n$  vertex cycles are indicated by  $c_n$ 's, all other  $n$ -cycles of lines by  $b_n$ 's. The above equation is inverted to the form

$$R(x) = Z(H)[b_n \rightarrow B(x^n), c_n \rightarrow C(x^n)] + Q(x)$$

where  $R(x)$  is the ordinary generating function for 3-connected graphs and where  $Q(x)$  is easily computed. To complete the computation of  $R(x)$ , the generating function  $Z(H)[b_n \rightarrow B(x^n), c_n \rightarrow C(x^n)]$  is evaluated using ideas R. W. Robinson used to count 2-connected graphs. (Received October 23, 1978.)

\*763-05-19 DANIEL J. RICHMAN, Wake Forest University, Winston-Salem, North Carolina 27109  
 and JOHN S. MAYBEE, University of Colorado, Boulder, Colorado 80309. Cycle Structure of Strong GM-graphs.

A strong GM-graph is a strongly connected, signed digraph such that every positive cycle contains all the vertices of any negative cycle with which it has at least one common vertex. The cycle structure of these graphs is examined and used to give a type of canonical form for them. Strong GM-graphs relate to a matrix problem investigated by James Quirk. Our methods include a new approach to his results as well as an extension of them. (Received October 23, 1978.)

\*763-05-20 DONALD R. SNOW, Dept. of Math., Brigham Young University, Provo, Utah 84602  
A Functional Inequality Arising in Combinatorics

This talk will discuss the functional inequality  $p(n+m) \leq \binom{n+m}{n} p(n) p(m)$  which arises in tournament theory and other places in combinatorics. A simple transformation removes the binomial coefficient and then the solution set divides naturally into three subsets called Classes I, II, and III. Class I is the set of all negative functions since the inequality puts no restriction on negative functions. The counting functions, i.e. the nonnegative solutions which are of interest in combinatorics, all lie in Classes II and III and satisfy easily obtainable growth estimates. Solutions of this type may be combined in various ways (e.g. sums and products) to give other such solutions so there is a structure on the set of nonnegative solutions. Various solutions and properties of solutions from Classes II and III may be obtained by introducing what the author calls a slack function. This technique and modifications of it can also be applied to other functional inequalities. For the present inequality it will be shown that this approach yields solutions that are associated in a special way with other solutions, sometimes even from a different solution class. (Received October 23, 1978.)



\*763-05-21 WILLIAM T. TROTTER, Jr. and TED MONROE, University of South Carolina, Columbia, South Carolina 29208. A Combinatorial Problem for Matrices.

Let  $\underline{n} = \{0, 1, 2, \dots, n-1\}$ . For an integer  $k \geq 1$ , define  $f(n, k)$  as the smallest integer  $t$  for which there exists a  $k \times t$  matrix  $M$  whose entries come from  $\underline{n}$  so that for each sequence  $x_1, x_2, \dots, x_k$  from  $\underline{n}$ , there exist  $k$  distinct integers  $j_1, j_2, \dots, j_k$  so that  $m_{ij_j} = x_i$  for  $i=1, 2, \dots, k$ . The definition of  $f(n, k)$  is motivated by the computation of the dimension of a family of posets which arises in the search for doubly irreducible posets, i.e., posets for which the removal of any two points decreases the dimension by two. The authors announced (Notices 25, October 1978, 761-A14) an exact formula for  $f(n, k)$  when  $n$  is large compared to  $k$ . The inequality  $f(n, k) \geq k + n - 1$  holds for all  $n$  and  $k$ . In this paper, the authors establish the following probabilistic inequality which holds for large  $k$ :  $f(n, k) < k + n \log n + n + o(n)$ . Further results on related extremal problems are also discussed. (Received October 23, 1978.)

763-05-22 John G. Gimbel, Esq., Western Michigan University, Kalamazoo, Michigan 49008. An algorithm for determining whether a graph is homogeneously traceable. Preliminary report.

In a graph  $G$  a path is said to be hamiltonian if it contains each vertex of  $G$ . If at each vertex of the graph there originates a hamiltonian path, the graph is said to be homogeneously traceable. An algorithm is presented which determines whether or not a graph is homogeneously traceable. The algorithm is then implemented, with the FORTRAN language, for computer usage. (Received October 23, 1978.) (Author introduced by Professor Yousef Alavi).

763-05-23 D. E. KEENAN, Texas A&M University, College Station, Texas 77840. On some cyclic configurations. Preliminary report.

Let  $S_1, \dots, S_n$  be  $n$  subsets of an  $n$ -set  $S$  with  $|S_i| = K$  and  $|S_i \cap S_j| \leq 1$ ,  $i \neq j$ . Suppose each subset has nonempty intersection with exactly  $n - 3$  of the remaining subsets. If  $A$  is the incidence matrix of such a configuration we show that the rows and columns of  $A$  may be permuted so that  $A$  is partitioned into cyclic submatrices and  $AA^T = A^T A$ . Restrictions on the sizes of the cyclic submatrices are also determined. (Received October 23, 1978.) (Author introduced by Dr. William Rundell).

\*763-05-24 Charles C. Lindner, Auburn University, Auburn, Alabama 36830. A technique for embedding cyclic orthogonal arrays.

A (partial) cyclic orthogonal array (COA) of strength  $k$  is a (partial) orthogonal array of strength  $k$  which is invariant under conjugation by the cycle  $\alpha = (123\dots k)$ . The main result in this paper is that any finite partial COA of strength  $k$  an odd prime can always be embedded in a finite COA of strength  $k$ . (Received October 25, 1978.)

\*763-05-25 Jin Akiyama, Nippon Ika University, Kawasaki, Japan and Frank Harary, University of Michigan, Ann Arbor, MI 48109. Packing and covering in graphs III: Cyclic and acyclic invariants.

It is possible to define many variations of packing and covering invariants for graphs which involve paths and cycles. These can be given terminology which is sufficiently intuitive that one can remember the definitions, e.g., arboricity, linear arboricity, point-arboricity, point-linear-arboricity, anarboricity, path number, cycle packing number, and cyclicity (the minimum number of cycles, not necessarily disjoint, whose union is the given graph). We investigate these concepts and mutual relations among them. In particular we determine these invariants for complete graphs, complete bipartite graphs, and their line graphs. (Received October 25, 1978.)

\*763-05-26 Martin Grötschel, Universität Bonn, D-5300 Bonn, Germany and Frank Harary, University of Michigan, Ann Arbor, MI 48109. The graphs for which all strong orientations are hamiltonian.

We show that the only graphs for which every strong orientation is hamiltonian are complete graphs and cycles. (Received October 25, 1978.)

\*763-05-27 WILLIAM G. FRYE AND RENU C. LASKAR, Clemson University, Clemson, South Carolina 29631. Path numbers of multipartite digraphs.

A path-factorization of a digraph  $D$  is an edge-disjoint collection of paths which cover the edges of  $D$ . The path number of  $D$ , denoted  $\Pi(D)$ , is the minimum cardinality of a path-factorization of  $D$ .

In this paper path numbers of some multipartite digraphs are given. (Received October 25, 1978.)

\*763-05-28 DAVID P. SUMNER, University of South Carolina, Columbia, South Carolina 29208. Randomly Matchable Graphs.

We define a connected graph to be randomly matchable if every matching of the graph can be extended to a perfect matching. This concept is analogous to the previously studied notions of randomly traceable and randomly hamiltonian graphs. We show that the only randomly matchable graphs are  $K_{2n}$  and  $K_{n,n}$  ( $n \geq 1$ ). (Received October 25, 1978.)

\*763-05-29 LOWELL W. BEINEKE and RICHARD D. RINGEISEN, Indiana University-Purdue University at Fort Wayne, Indiana 46805 and H. JOSEPH STRAIGHT, SUNY-College at Fredonia, New York 14063. On the cochromatic index of a graph.

Decompositions of the vertex-set of a graph into subsets which are homogeneous with respect to internal adjacencies (that is, each subset induces either a complete or null subgraph) were studied by Lesniak-Foster and Straight (Ars Combinatoria 3 (1977), 39-46). We consider the edge analogue of this concept and define the cochromatic index  $z'(G)$  of a graph  $G$  to be the minimum number of sets in a decomposition of the edge-set of  $G$  into mutually adjacent or independent sets of edges.

We observe that if  $\delta(G)$  and  $\Delta(G)$  denote the minimum and maximum degrees of  $G$ , then  $\delta(G) - 1 \leq z'(G) \leq \delta(G) + 1$ ; and we determine which triples of integers can be the triple  $(\delta(G), z'(G), \Delta(G))$  for some graph  $G$ . We define a second parameter of a graph (one which is easily determined) and show that it is within 1 of the cochromatic index. We also show that this yields the cochromatic index of all bipartite graphs. (Received October 25, 1978.)

\*763-05-30 P. ERDOS, V. FABER, F. JONES, University of Colorado, Boulder, Colorado 80302. Projective  $(2n, n, \lambda, 1)$ -designs.

A projective  $(v, k, \lambda, t)$ -design is a set  $\mathcal{B}$  of  $k$ -element subsets (called blocks) of a  $v$ -element set  $V$  having the properties that (i) each  $t$ -element subset of  $V$  is a subset of  $\lambda$  blocks in  $\mathcal{B}$  and (ii) every two blocks in  $\mathcal{B}$  have a non-empty intersection. A  $(2n, n, \lambda, 1)$ -design is called a  $\lambda$ -tuple cover. This paper deals exclusively with projective  $(v, k, \lambda, t)$ -designs for  $k = n$ ,  $v = 2n$  and  $t = 1$ . Some selected theorems:

Theorem. A projective  $(2n, n, \lambda, 1)$ -design exists if and only if  $n \geq 3$  and either (a)  $n$  is a power of 2 and  $2 \leq \lambda \leq [\frac{1}{4}\binom{2n}{n}] - 1$ , or (b)  $n$  is not a power of 2 and  $2 \leq \lambda \leq \frac{1}{4}\binom{2n}{n} - 2$  or  $\lambda = \frac{1}{4}\binom{2n}{n}$ .

Theorem. Every projective  $(2n, n, \frac{1}{4}\binom{2n}{n}, 1)$ -design contains a triple cover and at least  $[\frac{1}{2(2n-1)} \frac{1}{4}\binom{2n}{n}]$  disjoint double covers. (Received October 25, 1978.)

## 06 ► Order, Lattices, Ordered Algebraic Structures

\*763-06-1 MAUREEN A. BARDWELL, St. Norbert College, De Pere, Wisconsin 54115. Right-Ordered Groups and Regular Ordered Permutation Groups.

It is well-known that the class of right-ordered groups corresponds with the class of regular ordered permutation groups, and that every o-primitive component of a regular lattice-ordered permutation group is order-isomorphic to a regular representation of a subgroup of  $\mathbb{R}$ . In this report, we exploit the correspondence mentioned to classify the possible o-primitive components of a regular ordered permutation group. These groups will be shown to differ widely from regular lattice-ordered permutation groups in that they may have non-archimedean, regular o-primitive components and that they may also have non-regular o-primitive components. Examples of regular ordered permutation groups exhibiting these phenomena will be given. (Received September 18, 1978.)

\*763-06-2 RUSSELL BELDING, IIT Research Institute, Annapolis, MD 21403. Structures Characterizing Partially Ordered Sets, and Their Automorphism Groups.

By finding invariant embeddings of a partially ordered set  $X$  into the semigroups it is shown that the semigroup of order ideals of  $X$ , where the semigroup operation is set union, and the semigroup and semiring of order preserving maps from  $X$  into the positive cone  $D^+$  of a partially ordered integral domain  $D$ , all characterize  $X$  to within poset isomorphism. The automorphism group of the semigroup of order ideals is isomorphic to the automorphism group of  $X$ . The same holds for the semiring of order preserving maps from  $X$  into the non-negative integers, and this semiring is the closed linear span of its idempotents. (Received October 12, 1978.)

\*763-06-3 M.E. ADAMS and J. SICHLER, University of Manitoba, Winnipeg, Manitoba, Canada. R3T 2N2 Frattini sublattices in varieties of lattices.

The Frattini sublattice of a lattice  $L$ , denoted  $\mathfrak{F}(L)$ , is the intersection of all maximal proper sublattices of  $L$ . Theorem: Let  $\mathcal{V}$  be a nontrivial variety of lattices. If the lattice  $L$  is a member of  $\mathcal{V}$ , then there exists a lattice  $L^+$  of  $\mathcal{V}$  such that  $L \cong \mathfrak{F}(L^+)$ . (Received October 16, 1978.)

\*763-06-4 A.A. Bishop, Western Illinois University, McComb, Ill 61455 and E.A. Schreiner, Western Michigan University, Kalamazoo, MI 49008.  $K(L)$ , the lattice of complete ideals of  $L$ .

Let  $L$  be a lattice with  $0$ . An ideal  $A$  of  $L$  is complete if  $X \subseteq A$  and  $\bigvee X$  exists implies  $\bigvee X \in A$ .  $K(L)$ , the set of all ideals of  $L$ , is a complete lattice and is a completion of  $L$ . We examine conditions which allow a description of the join in  $K(L)$  and use our results to investigate properties of  $K(L)$ . For an atomistic modular  $L$ ,  $K(L)$  is modular iff the join in  $K(L)$  of  $A$  and  $B$  is  $\{x \mid x = \bigvee S \text{ where } S \subseteq A \vee B, \text{ the } I(L) \text{ join}\}$ . (Received October 18, 1978.)

\*763-06-5 WAYNE B. POWELL, University of Kansas, Lawrence, Kansas 66044. Projectives in a variety of Lattice-ordered Modules.

The variety of  $f$ -modules consists precisely of those lattice-ordered modules which are subdirect products of totally ordered modules. A representation of the free  $f$ -module over an arbitrary set is given. This representation is used to show that over many lattice-ordered rings the free  $f$ -modules have nontrivial cardinal summands if and only if they are of rank 1. Thus, another approach must be used to determine projectives. An  $f$ -module  $N$  is said to be an  $l$ -retract of an  $f$ -module  $M$  if there exist  $l$ -homomorphisms  $g: N \rightarrow M, h: M \rightarrow N$  such that  $hg = id_N$ . Projective  $f$ -modules can be characterized as those  $f$ -modules that are  $l$ -retracts of some free  $f$ -module. This allows for the construction of many examples of projective  $f$ -modules using the representation given for the free  $f$ -modules.

(Received October 23, 1978.)

763-06-6 CARLOS A. INFANTOZZI, Inst. de Estudios Superiores, Atlántico 1514, Montevideo, Uruguay.  
Ordered fields: their completion by contiguous classes.

Every ordered field has a completion by contiguous classes. The archimedean property is not necessary. This completion can be nonisomorphic to  $(\mathbb{R}, +, \times)$ . All the fields complete by contiguous classes are complete by Cantor "Fundamentalfolgen" if the fundamental sequences are defined by means of the valuation derived from the order. The converse proposition is also true. But there are ordered fields which have a completion by contiguous classes and have not a completion by cuts. Both completions by contiguous classes and fundamental sequences are equivalent although the field does not admit a completion by cuts. The completion of an ordered field by  $\varphi$ -fundamental sequences relative to any valuation  $\varphi$  with range belonging to an ordered and complete by contiguous classes field, is an ordered field if this valuation is order-preserving (ample sense). Several consequences of these completions are also obtained, and the different concept "suites adjacentes" is moreover considered. (Received October 24, 1978.)

## 08 ► General Mathematical Systems

763-08-1 JAMES KEETON-WILLIAMS, Bowling Green State University, Bowling Green, Ohio 43402.  
Categorical structure theory.

The first section presents the definition of a categorical theory, and discusses associated methodological and philosophical problems. Section 2 contrasts the concept of a set preserving functor with that of a language reduct. Section 3 investigates the consequences of using single valued relations instead of mappings as the starting point for the development of a categorical theory. The resulting categorical theory is essentially Freyd's concept of a protcategory. The requisite structure theorem for the theory is based on Pultr's study of realizations of forgetful functors. The theory also affords an illustration of the extreme (though often reasonable) limitations that can result from studying only properties which are invariant under categorical equivalence. Section 4 contains a general structure theorem for the (E,M)-categories of Herrlich and Strecker. It also contains a general structure theorem for an extension of N. Bourbaki's theory of embeddings. The proofs take advantage of Platt's characterization of one-to-one and onto in algebraic categories. The categorical theories developed here are applied in the last section to give a concise formalization of a remark by S. A. Naimpally. The intuitive content of the theorem is that proximity spaces provide the simplest structuring of completely regular spaces that will support a convenient study of compacitification. (Received July 14, 1978.)

\*763-08-2 STEPHEN D. COMER, THE CITADEL, CHARLESTON, S.C. 29409.  
THE COUNTABLE CHAIN CONDITION IN QUASI-PRIMAL VARIETIES.

Suppose an algebra  $A$  contains a constant  $0$  and a binary operation among its fundamental operations. Let  $S_i(A)$  denote the class of all subdirectly irreducible factors of  $A$ .  $D \subseteq A$  is separable if for every distinct  $x, y \in D$  and  $h \in \text{Hom}(A, S_i(A))$  either  $h(x) = 0$  or  $h(y) = 0$ . The countable separability condition (c.s.c) holds in  $A$  if there is no uncountable separable set of nonzero elements in  $A$ . Under certain conditions c.s.c. is equivalent to the c.c.c. Theorem. Every free algebra in a quasi-primal variety satisfies c.s.c. As a corollary c.c.c holds for every free algebra in certain quasi-primal varieties. (Received October 17, 1978.)

763-08-3 WILLIAM P. WARDLAW, U. S. Naval Academy, Annapolis, Maryland 21402. Associativity in Finite Groupoids. Preliminary report.

A groupoid  $G$  (set with binary operation) is n-associative if the product of any  $n$  elements of  $G$  is independent of the way in which the factors are associated. A groupoid is finitely associative if it is  $n$ -associative for some positive integer  $n$ .

This study of associativity in finite groupoids is motivated by the fact that for  $n \geq 3$ ,  $n$ -associativity implies  $(n+1)$ -associativity. The result that if  $G^2 = G$  then  $n$ -associativity implies  $(n-1)$ -associativity is significantly generalized to prove the following

Theorem. If  $G$  is a finitely associative groupoid with  $n$  elements, then  $G$  is  $(2^{n-2}+1)$ -associative. (Received October 19, 1978.)

\*763-08-4 W. M. SANDERS, James Madison University, Harrisonburg, Virginia 22807  
Nondesarguesian finite projective planes derived from desarguesian planes.  
 Preliminary report.

A set of twenty-five permutation matrices which form the kernel of a canonical incidence matrix for a desarguesian plane of order twenty-five is obtained from a Galois field  $GF(5^2)$ . Incidence relations for a nondesarguesian plane are obtained by stacking desarguesian planes in a prescribed manner. From the incidence structure two binary operations are obtained. Under one operation the system yields an abelian group; under the other, the system is non-commutative, nonassociative, and at least one of the distributive laws fails. The procedure leads to the conjecture that for each odd prime  $p$ , a nondesarguesian plane of order  $n^2$  can be obtained from desarguesian planes of order  $p^2$ . (Received October 23, 1978.)

## 10 ► Number Theory

\*763-10-1 Bruce C. Berndt, University of Illinois, Urbana, Illinois, 61801. Chapter 14 of Ramanujan's second Notebook.

Chapter 14 of Ramanujan's second Notebook contains 87 formulas. A few of these formulas have been proved in print several times during the past few decades, but most have not. An attempt has been made to prove all 87 formulas, but one or two remain unproved. Many of the formulas have their origins in the theory of elliptic modular functions. Those formulas pertaining to modular functions will be discussed and their histories given. (Received August 3, 1978.)

763-10-2 Morris Newman, University of California, Santa Barbara, California 93106. The subgroup method in the theory of the elliptic modular functions.

The subgroup method is a powerful tool which may be used to derive identities and congruences for the coefficients of modular forms. The method will be described, and applications of the method to questions such as determining the number of representations of an integer by a positive definite diagonal quadratic form will be given. (Received August 15, 1978.)

\*763-10-3 D.M. BRESSOUD, The Pennsylvania State University, University Park, Pennsylvania 16802. Partition theory and  $q$ -orthogonal polynomials.

$q$ -series arise both as generalizations of orthogonal polynomials and as generating functions for partition functions. L.J. Rogers studied them as generalizations of the ultraspherical polynomials, and discovered identities which have implications in partition theory (e.g. the Rogers-Ramanujan identities). We shall use a purely partition theoretic argument to prove and extend to a multiset one of Rogers' identities. Our argument is representative of a technique which should shed light on the combinatorial nature of  $q$ -orthogonal polynomials. (Received September 11, 1978.)

\*763-10-4 DAVID A. JAMES, University of Michigan--Dearborn, Dearborn, Michigan 48128. Meromorphic factors of automorphy. Preliminary report.

A factor of automorphy  $v$  satisfies  $v(z, MN) = v(z, N) v(Nz, M)$ , for instance  $v(z, M) = (cz+d)^r$  where  $M$  is the  $2 \times 2$  matrix  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ . Previous classifications have been restricted either to nondiscontinuous groups  $\Gamma$  or to groups  $\Gamma$  with compact fundamental regions. THEOREM: If  $v$  is meromorphic on the complex plane and  $\Gamma$  is discontinuous then there is an entire nonvanishing  $h(z)$  such that  $v(z, M) = W(z, M) h(Mz)/h(z)$  for all  $M$  in  $\Gamma$ . Here  $W(z, M)$  depends only on the zeros and poles of  $v$ . Furthermore if  $v$  is rational in the sense of Petersson then (except for a few finite groups) there must be a rational function  $R(z)$  such that  $v(z, M) = u(M) (cz+d)^r R(Mz)/R(z)$ . (Received September 12, 1978.)

\*763-10-5 HARVEY COHN, Department of Mathematics, City College of New York, New York, N.Y., 10031. Complex multiplication on Fricke's torus-covering.

Let  $F = \langle A, B \rangle$ ,  $A, B \in SL_2(\mathbb{R})$ ,  $\text{trace } [A, B] = -2$ , so  $F$  is a free group represented by a Fricke group (of genus 1 in the upper half-plane). Let  $W \in F$  and let the abelianized  $W \rightarrow aA + bB$ , and let  $F_n = \{W \mid b \equiv 0 \pmod{n}\}$  so  $F_n$  also has a fundamental domain of genus 1 consisting of  $n$  copies of  $F$ . If  $F$  and  $F_n$  have the same modulus, we say a complex multiplication occurs. This agrees with the usual definition in the period structure of Fricke's torus-covering. Special cases are investigated where the fixed points of  $A$  and  $B$  generate Markoff forms.

(Received September 28, 1978.)

\*763-10-6 Anne L. Ludington, Hamilton College, Clinton, New York 13323. A Bound on Kaprekar Constants.

Let  $a$  be an  $r$ -digit  $g$ -adic integer with not all digits equal. Let  $a'$  be the integer formed by arranging these digits in descending order and let  $a''$  be the integer formed by arranging them in ascending order. Define  $T(a) = a' - a''$ . If repeated applications of  $T$  to any  $g$ -adic integer lead always to the same self-repeating integer, then that integer is called a Kaprekar Constant for the pair  $(g, r)$ . This paper proves that there are no Kaprekar Constants for a given  $g$  when  $r$  is sufficiently large. (Received September 22, 1978.)

763-10-7 L. ALAYNE PARSON, The Ohio State University, Columbus, Ohio 43210. A Class of Non-linear functional equations connected with modular functions. Preliminary report.

Motivated by the behavior of Klein's modular invariant  $j(u)$  which satisfies modular equations of order  $p$  for every prime  $p$ , Kurt Mahler considered solutions in formal Laurent series to certain non-linear functional equations and called such solutions  $S_p$ -series. Additional examples are given of  $S_p$ -series which are automorphic functions. The existence and uniqueness of  $S_n$ -series for  $n$  composite is discussed. (Received September 25, 1978.)

763-10-8 M. LAWRENCE GLASSER, Clarkson College of Technology, Potsdam, NY 13676. An Application of Number Theory to Crystal Physics.

The electrostatic contributions to the energies of ionic solids can be expressed as Epstein zeta-functions. The use of number theoretic methods for the evaluations of several such functions in two and three dimensions will be discussed. An application of these formulas to extend Selberg and Chowla's results dealing with the closed form evaluation of elliptic integrals will also be presented. (Received October 2, 1978.) (Author introduced by Dr. Stefan Burr).

\*763-10-9 RONALD L. GRAHAM, Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974. On a diophantine equation arising in graph theory

Recent work of Godsil, Schwenk and Watanabe has dealt with trees (i.e., connected acyclic graphs) having adjacency matrices with all eigenvalues integral. Many of these problems reduce to questions concerning solutions to certain types of diophantine equations. For example, associated with the trees having at most two vertices of degree exceeding two is the diophantine equation  $(x^2-1)(y^2-1) = z^2$ . In this talk we discuss this and related equations and classify their solution sets. In particular, the solutions to the displayed equation turn out to depend intimately on Chebyshev polynomials. (Received October 3, 1978.)

\*763-10-10 R. G. Jeroslow, Georgia Institute of Technology, Atlanta, GA 30332. Linear Diophantine Inequalities and Integer Programming.

We will survey the uses of a monoid basis theorem due to Hilbert, and results related to this theorem, in the algebraic approach to cutting-planes and relaxations of integer programs. These results on the solution form to linear diophantine inequalities, and related constraint systems, have proven valuable in the study of the relaxations of the algebraic approach. In these latter relaxations the basically boolean-logical aspects of commonly occurring pure integer programs are relaxed in favor of general integer variables. We will show that such relaxations can produce more information than branch-and-bound algorithms. From a similar perspective, the completeness theorems of Gomory and Choatal, and Blair, will be discussed and their generalizations will be cited. (Received October 4, 1978.)

\*763-10-11 KENNETH H. ROSEN, Ohio State University, Columbus, Ohio 43210. The Enumeration of Lattice Points in Certain Higher-Dimensional Tetrahedra and a Conjecture of Rademacher.

Let  $a_1, \dots, a_m$  be pairwise coprime positive integers. Define  $N_m(a_1, \dots, a_m)$  to be the number of lattice points in the  $m$ -dimensional tetrahedron described by the system of inequalities  $0 \leq x_i \leq a_i$ ,  $i = 1, 2, \dots, m$  and  $0 < \sum(x_i/a_i) < 1$ . In 1954, Rademacher proved that  $N_3(a_1, a_2, a_3) \equiv (a_1+1)(a_2+1)(a_3+1)/4 \pmod{2}$ . This result led him to conjecture in 1963 that for all positive integers  $m$  one has  $N_m(a_1, \dots, a_m) \equiv \frac{\prod_{i=1}^m (a_i+1)}{2^{m-1}} \pmod{2}$ . The author has shown that for  $m \geq 4$  this conjecture is not valid. Using a corrected version of a formula of Mordell for  $N_4$ , the author has proved that if  $q$  is an even positive integer not divisible by 3, then  $N_4(q-1, q, q+1, q+3)$  is odd if  $q \equiv 4$  or  $14 \pmod{24}$  and even otherwise. If Rademacher's conjecture were true  $N_4(q-1, q, q+1, q+3)$  would always be even. Hence there are infinitely many counterexamples to Rademacher's conjecture for  $m=4$ . Using these counterexamples it is easy to construct counterexamples for  $m>4$ . (Received October 4, 1978.)

763-10-12 DAVID W. MATULA, Southern Methodist University, Dallas, Texas 75275. Using Finite Precision Fractions For Computer Arithmetic. Preliminary report.

A finite precision rational number system denotes a system whose members are fractions where the numerators and denominators are subject to size limitations. Arithmetic is closed by rounding in the classical sense of best rational approximation. Number systems may be "fixed-slash" denoting that the numerators and denominators are each bounded by the constant  $n$  and thus are characterized by a Farey series, or "floating-slash" denoting that the sum of the number of bits necessary to represent the numerator and denominator in the packed machine word is no more than  $N$ . We extend the theory of Farey fractions to provide a foundation for both floating- and fixed-slash arithmetic. These systems provide much insight into the general structure of finite precision approximate arithmetic without the anomalies due to choice of base that confound analysis of "floating-point" systems. Time and storage efficiency of computer design of such a system is also contrasted with available floating-point computer architecture. (Received October 4, 1978.) (Author introduced by Dr. Stefan Burr).

763-10-13 A.O.L. Atkin, University of Illinois at Chicago Circle, Box 4348, 60680. Identical Formulae for Modular Forms on the Full Modular Group.

It is well known that some modular forms on the full modular group (with multipliers) have Fourier coefficients which are expressible as sums of complex divisors in the fields of the fourth and third roots of unity. Among

powers of Dedekind's eta function, the 2nd, 4th, and 6th, are trivially so expressible using Euler's and Jacobi's series; Klein exhibits the 8th power, and Rushforth, Winqvist, and the author, the 10th, 14th, and 26th powers.

We show here that there are six series of such formulae, each containing an infinite number of examples, covering four of the five nontrivial multiplier systems on the full group. The method of proof is classical and essentially due to Mordell. (Received October 4, 1978.)

\*763-10-14 JEFFREY D. VAALER, University of Texas, Austin, Texas 78712. Inequalities for the volume of certain convex bodies with applications to linear forms.

Let  $C_N = [-\frac{1}{2}, \frac{1}{2}]^N$  be the  $N$ -dimensional cube of volume one centered at the origin in  $\mathbb{R}^N$ . Let  $P_K$  be a  $K$ -dimensional subspace of  $\mathbb{R}^N$ . We prove that the  $K$ -dimensional Lebesgue measure of  $C_N \cap P_K$  is greater than or equal to one. A generalization is also proved in which  $C_N$  is replaced by a product of spheres of various dimensions. The proof uses techniques from probability. Suppose that  $L_j(x)$ ,  $j=1, 2, \dots, N$  are  $N$  linear forms in  $K$  variables,  $x = (x_1, \dots, x_K)$ , and  $\epsilon_j > 0$ ,  $j=1, 2, \dots, N$ . Minkowski's classical theorem on linear forms gives a sufficient condition for the existence of a nontrivial lattice point  $u$  such that  $|L_j(u)| \leq \epsilon_j$ ,  $j=1, 2, \dots, N$ , in terms of the  $K$ -dimensional measure of  $\{x \in \mathbb{R}^K: |L_j(x)| \leq \epsilon_j, j=1, 2, \dots, N\}$ . By applying our inequality we give a precise lower bound for the measure of this set which depends in a simple way on the  $N \times K$  matrix associated with the  $L_j$ 's and the  $\epsilon_j$ 's. We thus obtain a quantitative form of Minkowski's theorem which is new if  $1 \leq K < N$ . (Received October 6, 1978.)

\*763-10-15 Sidney Graham, California Institute of Technology, Pasadena, California 91125. On the greatest prime factor of the integers in an interval.

Let  $P(x)$  denote the greatest prime factor of the product of the integers in  $(x, x + x^{1/2}]$ . We show that  $P(x)$  exceeds  $x^{13/20}$  for sufficiently large  $x$ . The proof uses Vaughan's version of Vinogradov's method for estimating trigonometrical sums. (Received October 19, 1978.)

763-10-16 PAUL ERDŐS, Hungarian Academy of Sciences, Budapest, Hungary. Some number-theoretic problems in combinatorics and geometry. Preliminary report.

A number of problems will be discussed that at present have only partial solutions. (1) Denote by  $f(\epsilon, n)$  the largest number of points in a disk of radius  $n$  so that the distance between any two of them differs from an integer by more than  $\epsilon$ . Determine or estimate  $f(\epsilon, n)$  as well as possible. (2) Is there a one-to-one mapping  $f(x)$  between the integers  $1, \dots, n$  and  $m+1, \dots, m+n$  so that  $(x, f(x))=1$ ? (3) Estimate the minimum  $g(n)$  so that for any  $m$  there exists a one-to-one mapping  $f(x)$  of  $1, \dots, n$  into  $m, \dots, m+g(n)$  for which  $x | f(x)$ . (Received October 10, 1978.)

\*763-10-17 GERALD MYERSON, State University of New York at Buffalo, Buffalo, New York 14214. Number theory and numerical integration.

Fix an integer  $f \geq 2$ . For each prime  $p \equiv 1 \pmod{f}$  let  $\zeta_p$  be a primitive  $f$ -th root of unity  $\pmod{p}$ , and let  $\Lambda_p = \left\{ \left( \frac{a}{p}, \frac{a\zeta_p}{p}, \dots, \frac{a\zeta_p^{m-1}}{p} \right) : a=0, \dots, p-1 \right\}$ , where all coordinates are reduced  $\pmod{1}$  and  $m = \phi(f)$ , the Euler  $\phi$ -function. Then as  $p$  tends to infinity the discrepancy of  $\Lambda_p$  tends to zero. This has applications to multidimensional numerical integration and, in particular, integration of functions with singularities. (Received October 16, 1978.)

763-10-18 GEORGES G. GRINSTEIN, Auburn University at Montgomery, Montgomery, Alabama 36117. Functional Equations similar to those of Eisenstein Series. Preliminary report.

Let  $F_a(z) = \sum_{n=1}^{\infty} \sigma_{a-1}(n) e^{2\pi i n z}$  where  $\sigma_{a-1}(n) = \sum_{d|n} d^{a-1}$  with  $a \in \mathbb{C}$ . For  $a \in \mathbb{Z}$   $F_a(z)$  is well-known and has a functional equation. This paper studies the problem for  $a \in \mathbb{C}$ . (Received October 16, 1978.)



\*763-10-19 A. D. Pollington, Illinois State University, Normal, Illinois 61761. On generalized arithmetic and geometric progressions.

Let  $\alpha > 1$  and  $\beta$  be real numbers. We call the sequence  $[n\alpha + \beta]$ ,  $n=1,2,\dots$ , a generalized arithmetic progression. Erdos has asked that if we let  $(n_k)$  be a sequence of integers tending to infinity sufficiently fast is it true that the complement of  $(n_k)$  contains an infinite generalized arithmetic progression. Here  $[x]$  denotes the greatest integer  $\leq x$ . We shall show that given any sequence of integers  $(n_k)$  for which  $n_{k+1}/n_k \geq \delta > 3$ , for all  $k$ , we can always find a generalized arithmetic progression which does not meet the sequence  $(n_k)$ . (Received October 20, 1978.) (Author introduced by Professor L. C. Eggan).

763-10-20 CALVIN T. LONG, Washington State University, Pullman, Washington 99163, on leave at Clemson University, Clemson, South Carolina 29631. A Limited Arithmetic on Simple Continued Fractions, III.

If  $\xi$  is a quadratic surd, it has a simple continued fraction expansion that is eventually periodic and we write  $\xi = [a_0, a_1, \dots, \dot{a}_r, \dots, \dot{a}_{r+s}]$  where the notation is that of Hardy and Wright. The first two papers in this series primarily deal with properties of the simple continued fraction expansions of rational multiples of surds of the form  $\xi = [a, b]$ . Specialized to  $a = b = 1$ , we have  $\xi = (1 + \sqrt{5})/2$  and a typical result is that  $\xi \cdot F_n / L_n = [0, 1, \dot{2}, 1, \dots, 1, 3, 1, \dots, 1, \dot{4}]$  and  $\xi \cdot L_n / F_n = [3, \dot{1}, \dots, 1, 3, 1, \dots, 1, 2, \dot{4}]$  where  $F_n$  and  $L_n$  are the  $n$ th Fibonacci and Lucas numbers respectively and there are  $n-4$  ones in the first group and  $n-3$  ones in the second group for the first expansion and just the reverse in the second. In the present paper, we consider simple continued fraction expansions of powers of certain surds. In the special case where  $\xi = (1 + \sqrt{5})/2$ , a typical result is that  $\xi^{2n-1} = [L_{2n-1}, \dot{1}]$  and  $\xi^{2n} = [L_{2n-1}, \dot{1}, L_{2n-2}]$ . Results are also obtained for surds like  $(F_n + F_m \sqrt{5})/2$ ,  $(L_n + L_m \sqrt{5})/2$ , and so on. (Received October 20, 1978.)

763-10-21 M. D. CHOI, University of Toronto, Toronto, M5S1A1, Canada, T. Y. LAM, University of California, Berkeley, California 94720, and BRUCE REZNICK, University of California, Berkeley, California 94720. Non-negative Symmetric Quartic Forms.

It is well known that a real symmetric quadratic form  $aM_2 + bM_1^2 \left( M_r = \sum_{i=1}^n x_i^r \right)$  is non-negative (psd) if and only if  $a \geq 0$  and  $a+nb \geq 0$ . Necessary and sufficient conditions are found for a symmetric quartic form  $aM_4 + bM_1M_3 + cM_2^2 + dM_1^2M_2 + eM_1^4$  to be psd. Theorem: A symmetric quartic form is psd iff it is non-negative when evaluated at every  $n$ -tuple  $(x_1, \dots, x_n)$  with at most two distinct components. Consequences of this theorem, including explicit extremal forms, are discussed, as are circumstances under which a psd symmetric quartic is a sum of squares of quadratic forms. (Received October 20, 1978.)

763-10-22 PERSI DIACONIS, Bell Laboratories, Murray Hill, New Jersey 07974. The Analysis of Computer Algorithms and Probabilistic Number Theory.

Analysis of computer algorithms for the fast Fourier Transform involves the additive number theoretic function  $f(n) = \sum_{p|n} p$ . We derive the limiting distribution of this function and explain what the mathematical analysis says about the F.F.T. (Received October 23, 1978.)

\*763-10-23 Ferguson, H. R. P., Forcade, R. W., Brigham Young University, Provo, Utah 84602.  $\mathbb{Z}$ -linear dependence algorithms, Part I. Preliminary report.

A new approach to higher dimensional continued fraction algorithms is described. The problem of determining or approximating  $\mathbb{Z}$ -linear relations among a finite set of positive real numbers is related to the diminishing height of an associated matrix of intersection vertices. In particular, we show how the new theory improves the effectiveness of an old and elegantly simple algorithm due to Viggo Brun. (Received October 23, 1978.)

\*763-10-24 Ferguson, H. R. P., Forcade, R. W., Brigham Young University, Provo, Utah 84602.  $\mathbb{Z}$ -linear dependence algorithms, Part II. Preliminary report.

Applications of the theoretical developments of  $\mathbb{Z}$ -linear algorithms, Part I, include exclusion theorems for interesting real numbers of unknown arithmetic type (algebraic of some degree or transcendental). For a given polynomial with integral coefficients define the height of the polynomial to be the maximum of the absolute values of the coefficients of the polynomial. For example, we have proven that for the Euler-Mascheroni constant to satisfy a quadratic polynomial the height must exceed the integer 21033671776241859726874595624784950477866696020727598862581478765509485062309852945143. For Euler's constant to satisfy a cubic polynomial the height must exceed the integer 64337798485853810079882277883357885425. The height of a corresponding quartic must exceed the integer 10052913032425344. (Received October 23, 1978.)

\*763-10-25 Gordon D. Prichett, Hamilton College, Clinton, New York 13323. Terminating cycles for iterated difference values of five digit integers.

Choose  $a$  to be any  $g$ -adic integer with five digits, not all digits equal. Let  $a'$  be the integer formed by arranging these digits in descending order, and let  $a''$  be the integer formed by arranging these digits in ascending order. Define  $T(a) = a' - a''$ . Repeated applications of  $T$  commencing with  $a$  will always become periodic and terminate in a  $T$ -cycle. This paper completely characterizes all terminating  $T$ -cycles obtained from five digit  $g$ -adic integers. (Received October 23, 1978.)

763-10-26 Philip C. Tonne, Emory University, Atlanta, Georgia 30322. When is  $3^N$  congruent to 1 or 2 Modulo  $N$ ? Preliminary report.

Theorem. Suppose that  $p_0, p_1, \dots, p_m$  is an increasing sequence of primes,  $p_0 = 2, e_0, e_1, \dots, e_m$  is a sequence of positive integers, and  $n = 2^{e_0} \prod_{i=1}^m p_i^{e_i}$ . Then these statements are equivalent:

(1)  $3^n \equiv 1$  modulo  $n$ . (2) Either  $m = 0$  or else  $e_0 > 1$  and for each positive integer  $i$  in  $[1, m]$  there is an integer-sequence  $d_0, d_1, \dots, d_m$  such that  $d_0$  is in  $[2, e_0]$  and, if  $k$  is an integer in  $[1, m]$ ,  $d_k$  is in  $[0, e_k]$  such that if  $n' = 2^{e_0} \prod_{k=1}^{i-1} p_k^{d_k}$  then  $p_i$  divides  $3^{n'} - 1$ .

While we can demonstrate many classes of integers  $n$  such that  $3^n$  is not congruent 2 modulo  $n$  and can show that there is no "small" integer  $n$  exceeding 1 such that  $3^n \equiv 2$  modulo  $n$ , we present primes  $p, q$  and  $r$  such that  $3^{pqr}$  is congruent to 2 modulo  $pqr$ . (Received October 24, 1978.)

\*763-10-27 MELVYN B. NATHANSON, Southern Illinois University, Carbondale, Illinois 62901. Sumsets contained in infinite sets of integers.

If the positive integers are partitioned into a finite number of cells, then Hindman proved that there exists an infinite set  $B$  such that all finite, nonempty sums of distinct elements of  $B$  belong to one cell of the partition. Erdős conjectured that if  $A$  is a set of integers of positive asymptotic density, then there exist infinite sets  $B$  and  $C$  such that  $B + C \subseteq A$ . The conjecture is still not proved. This paper contains several results on sumsets contained in infinite sets of integers.

For example, if  $A$  is a set of integers of positive upper density, then for any  $n$  there exist sets  $B$  and  $F$  such that  $B$  has positive upper density,  $F$  has cardinality  $n$ , and  $B + F \subseteq A$ . (Received October 25, 1978.)

\*763-10-28 V. G. Tupitsyn, Southern Illinois University, Carbondale, Illinois 62901. Hamming's metric and T-orbits in a dynamical system.

Let  $R$  denote  $[0, 1)$  and let  $T_\theta^k = \{2^k\theta\}$ , where  $\theta \in R$ . Let  $\Delta_j^{(N)}(\theta) = (\epsilon_{j+1}(\theta), \dots, \epsilon_{j+N}(\theta))$  denote a finite subsequence of the binary series corresponding to  $\theta \in R$ . The relation  $\Delta_j^{(N)}(\theta) \subset \Delta_0^{(L)}(\theta)$  means that  $N$ -block occurs in  $L$ -block at the place  $j + 1$ . Identifying some  $N$ -blocks we get  $h \xrightarrow{\text{hom}} h^*$ , where  $h$  is the distance (Hamming's metric).

Theorem. Let  $L$  be a sufficiently large integer. Suppose that for any given  $\theta \in R$  and for some  $N$  we can find  $s_0 \geq 0$  and  $m_0 \geq 1$  with  $d(T_\theta^{s_0+im_0}, T_\theta^{s_0}) < \frac{1}{2^{2N+1}}$  where  $s_0 + (p-1)m_0 \leq L - 2N - 1$ ,  $i=0,1,\dots,p-1$ . Then for any given  $\theta \in R$  we can find  $s \geq 0$  and  $m \geq 1$  (where  $s + (p-1)m \leq L - N$ ) such that  $\Delta_{s+im}^{(N)}(\theta) \subset \Delta_0^{(L)}(\theta)$  is a system of  $P$   $h^*$ -identical  $N$ -blocks (i.e.,  $s + im$  is a monochromatic arithmetic progression. (Received October 25, 1978.) (Author introduced by Melvyn B. Nathanson).

763-10-29 G. V. CHUDNOVSKY, Columbia University, Department of Mathematics, New York, New York 10027. The bound for linear forms in elliptic logarithms for the complex multiplication case.

Let  $\wp(z)$  be a Weierstrass elliptic function with algebraic invariants  $g_2, g_3$  and complex multiplication by an order in the quadratic imaginary field  $\mathcal{K}$ . Let  $u_1, \dots, u_m$  be algebraic points of  $\wp(z)$ , linearly independent over  $\mathcal{K}$ . We put  $L = K(\wp(u_i), \wp'(u_i); i=1, \dots, m)$  and  $A_i = \max\{H(\wp(u_i)), H(\wp'(u_i))\}; i=1, \dots, m$ . For an algebraic  $\beta_i; i=1, \dots, m$  we found a Baker-Stark type of lower bound for  $\Lambda = \sum_{i=1}^m \beta_i u_i$ . Theorem. For fixed  $\beta_i; i=1, \dots, m$  and fixed  $\epsilon > 0$ ,  $|\Lambda| > \exp(-c_1 \times (\prod_{i=1}^m \log A_i)^{1+\epsilon})$ , where  $c_1 = c_1(\wp, \deg L, \beta_i, \epsilon) > 0$ . The analogical result is true for arbitrary  $\beta_i \in K$ . In the proof we use Schneider method with  $F(\bar{z}) = P(\bar{z}, \wp(\bar{z})), \bar{v} = (u_1, \dots, u_{m-1}), \bar{z} = (z_1, \dots, z_{m-1})$ . (Received October 24, 1978.)

763-10-30 GARY S. BLOOM, City College of New York, New York, New York 10031. Numbering graphs for fun and profit. Preliminary report.

For several years there has been interest in studying "numbered" graphs which have both a non-negative integer value  $\Psi(v)$  assigned to each vertex  $v$  and a non-negative integer value  $\Psi(uv) = |\Psi(u) - \Psi(v)|$  assigned to each edge  $uv$  with endpoints  $u$  and  $v$ . Imposition of additional constraints on both the numberings and the classes of graphs generates numerous number-theoretical questions as well as models with real-world applications ranging from astrophysics to x-ray crystallography. A sampling of numbered graph families, their applications, and open problems will be surveyed. (Received October 25, 1978.)

763-10-31 CARLOS J. MORENO, University of Illinois, Urbana, IL 61801 The Riemann-Siegel formula for automorphic L-functions. Preliminary report.

Riemann's Third proof of the functional equation for the zeta function depended on an integral formula of great arithmetical significance. Kronecker had used an analogous formula in one of his analytic proofs of the quadratic reciprocity law; already Riemann seems to have been aware of the possibility of using his formula to give an independent derivation of Jacobi's Theory of theta functions. The mystery that surrounds the structure of this formula and its connection with the analytic continuation of the constant term of the Eisenstein series deserves to be better understood. In this work we have undertaken the study of the asymptotic properties of the Riemann-Siegel formula associated to L-functions constructed from automorphic representations of  $GL(2)$ . We hope to relate this work with the Weil representation. The practical usefulness of the Riemann-Siegel formula for the study of zeros of L-functions on the critical line is well known in analytic number theory: for example in Levinson's proof of the " $\frac{1}{3}$  Theorem". (Received October 25, 1978.)

\*763-10-32 NEVILLE ROBBINS, University of San Francisco, San Francisco, CA 94117. On Fibonacci numbers which are powers.

Let the prime  $p \geq 5$ . Let  $m > 2$  be the least integer such that  $F_m = c^p$ .

Then (1)  $m$  is odd; (2) either (a)  $m \equiv \pm 1 \pmod{12}$  and  $c \equiv 1 \pmod{8}$  or (b)  $m \equiv \pm 5 \pmod{12}$  and  $c \equiv 5 \pmod{8}$ . (Received October 25, 1978.)

## 12 ► Algebraic Number Theory, Field Theory and Polynomials

\*763-12-1 JACOB T.B. BEARD, JR.\*<sup>\*</sup>, Emory University, Atlanta, GA 30322/University of Texas at Arlington and MICKIE SUE HARBIN, University of Texas at Arlington, Arlington, TX 76019. Toward non-splitting unitary perfect polynomials over each  $GF(p)$ .

In these Notices 24(1977), A-624 the authors conjectured that the cardinality  $NSUP(p^d)$  of the set of  $p^d$ -equivalence classes containing non-splitting unitary perfect polynomials over  $GF(p^d)$  is infinite for each prime  $p$  and each odd integer  $d > 1$ , a statement since confirmed in the cases  $p \leq 19$  or  $p = 2t+1$ ,  $(2,t) = 1$  [Notices 25(1978), A-351, November]. The current results establish the conjecture in the affirmative to the (overall) extent of this THEOREM.  $NSUP(p) \geq 1$  and  $NSUP(p^d) = \infty$  for each odd integer  $d > 1$  in the cases i)  $p < 97$ , or for odd  $p = 2^e t + 1$  with  $(2,t) = 1$ , ii)  $e \leq 2$ , iii)  $e \geq 3$  and each integer interval  $[\theta^t, \theta^{t'}]$  contains a square in  $GF^*(p) = \langle \theta \rangle = \{1, 2, \dots, p-1\}$  where  $\theta^t, \theta^{t'}$  are distinct odd powers of  $\theta$ . Excepting 2,17, all primes  $p < 97$  satisfy ii) or iii) and the proofs of these cases give explicit constructions of unitary perfect polynomials over  $GF(p)$  having precisely linear and quadratic prime factors. The Theorem and additional examples now establish  $NSUP(5) \geq 7$ ,  $NSUP(7) \geq 3$ ,  $NSUP(11) \geq 2$ , with  $NSUP(2) \geq 33$  and  $NSUP(3) \geq 16$  unchanged since [Notices 24(1977), A-624] to our knowledge. In conclusion, we conjecture that for fixed  $e \geq 3$  the condition of iii) above is satisfied for all sufficiently large admissible  $t$ . (Received September 14, 1978.)

763-12-2 R. P. KURSHAN and A. M. ODLYZKO, Bell Laboratories, 600 Mountain Ave., Murray Hill, N.J. 07974. Recursive Linear Digital Filters and the Values of Cyclotomic Polynomials at Roots of Unity.

A recursive linear digital filter which operates without roundoff errors is modeled by a periodic linear recursion with integer feedback coefficients whose characteristic polynomial optimally is cyclotomic. The frequency response of such a system is described in terms of its discrete Fourier series coefficients. The number of distinct amplitude levels in the spectrum of such a filter is of interest because in some applications it is desirable to have a spectrum with only one or very few high amplitudes, the remainder being low, while other applications require a uniform power distribution. The number of distinct amplitude levels for a filter of period  $m > 1$  is exactly  $\phi(m_0)/2$  when  $m_0$ , the largest square-free divisor of  $m$ , is greater than 2, and equals 1 when  $m_0 = 2$ . The proof of this result requires an investigation of the values of cyclotomic polynomials at roots of unity, and involves algebraic properties of cyclotomic polynomials as well as some analytic results about Dirichlet L-functions. (Received October 2, 1978.)

\*763-12-3 JAMES SOLDERITSCH, Villanova University, Villanova, Pennsylvania 19085. Class groups of quadratic number fields whose  $p$ -Sylow subgroups require three generators.

Several authors have considered the problem of finding sufficient conditions that a quadratic number field have  $p$ -rank  $\geq 2$ ; that is, the  $p$ -Sylow subgroup of the class group require at least two generators. Given the actual criteria, there remains the problem of producing discriminants for which the corresponding number fields satisfy the criteria. We review some of the work that has been done in this area. In particular, we are interested in  $p = 3, 5$  and  $7$  and those methods which produce discriminants small enough so that their class groups can be determined completely by computer. Unlike most of the recent work in the area, we have found a method of providing discriminants of  $p$  rank at least two for any odd  $p$ , many of which are small enough in the above sense. We have computationally verified the existence of cases 3-rank = 3 and 4 (these types have previously been produced by D. Shanks and others), 5-rank = 3 and most recently 7-rank = 3. We will discuss the computational and theoretical difficulty in extending our investigation to  $p > 7$  and in proving the existence of infinitely many examples of  $p$ -rank  $\geq 3$  for  $p$  other than three. (Received October 4, 1978.)

763-12-4 RAY MINES, New Mexico State University, Las Cruces, New Mexico 88003.  
Theory of Fields.

The theory of fields is developed using a definition based on the category of sets with equality and inequality relations. Among other results it is shown that the ability to factor polynomials over a field is equivalent to the ability to find roots of polynomials. (Received October 5, 1978.)

763-12-5 A. SEIDENBERG, University of California, Berkeley, California 94720. On the Lasker-Noether Decomposition Theorem.

Let  $R$  be a Noetherian integral domain and assume that one can construct (or is given) a Lasker-Noether decomposition for any given ideal of  $R$ . Then it is shown how to construct a Lasker-Noether decomposition for any given ideal of  $R[X]$ ,  $X$  an indeterminate. (One will have to make some necessary hypotheses on  $R$ , e.g., that one can (effectively) factor any polynomial in  $K[X] - 0$ , where  $K =$  quotient field of  $R$ , into irreducible factors.) In particular, one can construct a Lasker-Noether decomposition for any given ideal in a polynomial ring (in several variables) over the ring of integers (an assertion previously established - see a forthcoming issue of the American Journal). (Received October 6, 1978.)

763-12-6 Gary L. Mullen, The Pennsylvania State University, Sharon, Pennsylvania 16146. Equivalence of Sets of Functions over a Finite Field. Preliminary Report.

If  $k \geq 1$  let  $\Omega_1, \dots, \Omega_k$  be groups of permutations of  $K = GF(q)$ . If  $r \geq 1$  and  $f_i, g_i \in K[x_1, \dots, x_r]$  for  $i=1, \dots, k$  then the set  $(f_i)$  is left equivalent to  $(g_i)$  relative to  $\Omega_1, \dots, \Omega_k$  if there exist  $\phi_i \in \Omega_i$  such that  $\phi_i f_i = g_i$  for  $i=1, \dots, k$ . This generalizes work of S. Cavior [Acta Arith. 10(1964), 119-136 and J. für die Reine und Angew. Math. 225(1967), 191-202] and the author [Acta Arith. 29(1976), 353-358]. It is shown that many of the results in this general setting can be derived from the single function case.

For  $i=1, \dots, k$  let  $\Omega_i$  be a cyclic group of permutations of order  $n_i$  where the  $n_i$ 's are pairwise relatively prime. Let  $l(t_i)$  denote the number of invariant elements of the subgroup of  $\Omega_i$  of order  $t_i$ . If  $\lambda(\Omega_1, \dots, \Omega_k)$  denotes the number of classes induced by  $\Omega_1, \dots, \Omega_k$  then  $\lambda(\Omega_1, \dots, \Omega_k) = \prod_{i=1}^k \lambda(\Omega_i)$  where

$$\lambda(\Omega_i) = \sum_{t_i | n_i} t_i N(t_i) / n_i \qquad N(t_i) = \sum_{a | \frac{n_i}{t_i}} \mu(a) [l(at_i)]^{t_i}$$

and  $\mu(a)$  is the Mobius function. (Received October 16, 1978.)

\*763-12-7 RONALD P. INFANTE, Seton Hall University, South Orange, New Jersey 07079  
On the Inverse Problem in the Galois Theory of Difference Fields

Let  $F$  be a difference field of characteristic zero. Assume that the field of constants of  $F$ ,  $C$ , is algebraically closed. If  $G$  is a simple algebraic group defined over  $C$  then the following is true.

There is a difference field  $M$ , containing  $F$ , with  $M$  a strongly normal extension of  $F$  and  $\text{Gal}(M, F) \cong G$  provided there is an element of  $G$ ,  $\beta$ , which is rational over  $F$  satisfying  $\beta_1 \beta_2 \dots \beta_n \neq 1$  for all positive integers  $n$ .  $M = F \langle \alpha \rangle$  where  $\alpha$  is a generic element of  $G$ . (Received October 16, 1978.)

\*763-12-8 Nickolas Heerema and Tim Morrison, Florida State University, Tallahassee, FL 32306. A characterization of induced space of derivations in the p-adic Galois theory. Preliminary Report

Let  $K \supset K_0$  be p-adic fields with residue fields  $k \supset k_0$  with  $k/k_0$  finitely generated. This paper is concerned with the identification of those subspaces  $\bar{I}(K/K_0)$  of derivations on  $k$  which are induced by the integral derivations  $I(K/K_0)$  on  $K/K_0$ .

Given a distinguished subfield  $S$  of  $k$  over  $k_0$ ,  $S$  is said to be Jacobian if  $\overline{I}(K/K_0)|_S = \text{Der}_{k_0}(S, k)$ . It is proven that for any  $S$  there exists  $p$ -adic fields  $K \supset K_0$  such that  $S$  is Jacobian. J. Deveney has proved that in every case some  $S$  is Jacobian. It is shown that a given  $S$  is co-simple in  $k$  if and only if every distinguished subfield of  $k$  over  $k_0$  is co-simple in  $k$ . We call this the co-simple case. In the co-simple case it is shown that for a given pair  $K \supset K_0$  either  $S$  is Jacobian or  $\overline{I}(K/K_0)|_S$  is the kernel of a certain type of linear functional on the  $k$ -space  $\text{Der}_{k_0}(S, k)$  into  $k$  called a lifting form. Given any lifting form there exists  $K \supset K_0$  such that  $\overline{I}(K/K_0)|_S$  is the kernel of that lifting form. This provides a characterization of those subspaces of derivations on  $k$  over  $k_0$  of the form  $\overline{I}(K/K_0)$  except in the case when every distinguished subfield is Jacobian. An analysis of this case is also made. (Received October 23, 1978.)

\*763-12-9 JAMES G. HUARD, Southern Illinois University, Carbondale, Illinois 62901.  
The minimal index of a cyclic cubic field.

Let  $K$  be an algebraic number field and  $Z_K$  its ring of algebraic integers. For  $\theta$  in  $Z_K$  define the index of  $\theta$  to be the group index  $[Z_K : Z[\theta]]$ . We show that the minimal index of  $K$  is unbounded as  $K$  ranges over the set of cyclic cubic fields.

M. Hall (1937) has proved a similar result for pure cubic fields. (Received October 23, 1978.)

\*763-12-10 CHRIS ASH, Monash University, Clayton, Victoria, Australia 3168 and JOHN W. ROSENTHAL, Ithaca College, Ithaca, N.Y. 14850. On the lattice of algebraically closed subfields of an algebraically closed field of characteristic 0-II.

In these NOTICES (742-12-10 Jan. 1977) we announced there are ac (i.e. algebraically closed) subfields  $F, G$  of an ac field (of characteristic 0)  $K$  with  $\text{tr deg}(F \cap G) < \text{tr deg } F + \text{tr deg } G - \text{tr deg}(F + G)$  where  $\text{tr deg}$  is the transcendence degree over a base field  $k$  and  $F + G$  is the ac subfield of  $K$  generated by  $F \cup G$ .

We characterize (for  $\text{tr deg } F, \text{tr deg } G$  finite) when  $\text{tr deg}(F \cap G) = \text{tr deg } F + \text{tr deg } G - \text{tr deg}(F + G)$  as follows: Let  $x_1 \dots x_m$  be a (transcendence) basis of  $F$  (over  $k$ );  $x_{m+1} \dots x_n \in G$  with  $x_1 \dots x_n$  a basis of  $F + G$ ; and  $u_1 \dots u_\ell \in G$  with  $x_{m+1} \dots x_n, u_1 \dots u_\ell$  a basis of  $G$ . By rearranging  $x_1 \dots x_m$  we can assume  $u_1 \dots u_\ell, x_{\ell+1} \dots x_n$  are (algebraically) independent.

Let  $D = |\partial u_p / \partial x_q|$ ,  $p = 1 \dots \ell, q = 1 \dots \ell$ . For  $i = 1 \dots \ell, k = \ell + 1 \dots m$ , let  $U(i, k)$  be the determinant obtained by replacing in  $D$  derivatives with respect of  $x_i$  by derivatives with respect to  $x_k$ .

Theorem  $\text{tr deg}(F \cap G) = \ell$  iff  $U(i, k)/D \in F$  for  $i = 1 \dots \ell, k = \ell + 1 \dots m$ . In this case a basis of  $F \cap G$  can be obtained by specializing  $u_1 \dots u_\ell$ .

Using this we may prove results as:

Theorem Let  $x_1 \dots x_n$  be independent. Let  $u_1 \dots u_\ell \in \overline{k(x_0 \dots x_n)}$  be independent. Let  $F_i = \overline{k(x_0 \dots x_i, u_1 \dots u_\ell)}$ ,  $G = \overline{k(x_i, u_1 \dots u_\ell)}$ . If  $\text{tr deg}(F_i \cap G_i) = \ell$  for  $i = 1 \dots n$ , then  $\text{tr deg}(F_0 \cap G_0) = \ell$ . (Received October 23, 1978.)

\*763-12-11 DEBORAH TRIANTAPHYLLOU, University of California, Santa Cruz, CA 95064.  
Invariants of Finite Groups Acting Non-Linearly on Rational Function Fields.

$G$  denotes a finite group;  $F$  is a field;  $x$  and  $y$  are variables. We prove:

Theorem 1: Let  $G \subset \text{Aut}(F)$ , and let  $E(G, F) = \{\text{isomorphism classes of extensions } F(x)/F^G \text{ for some lifting } G \hookrightarrow \text{Aut}(F(x))\}$ . There is a natural bijection of sets:  $E(G, F) \leftrightarrow H^1(G, \text{PGL}_2(F))$ .

Theorem 2: Let  $G \subset \text{Aut}(F(x), F)$  (the automorphisms of  $F(x)$  which stabilize  $F$ ). If  $F$  is a  $C_1$  field, then  $F(x)/F^G$  is a pure (i.e. purely transcendental) extension.

Theorem 3: Let  $G \subset \text{Aut}(F(x), F)$ . If  $G$  has odd order, then  $F(x)/F^G$  is pure.

Theorem 4: Let  $G \subset \text{Aut}(F(x), F)$ , and let  $H$  be a 2-Sylow subgroup of  $G$ . Then  $F(x)/F^G$  is pure if and only if  $F(x)/F^H$  is pure.

Theorem 5: Let  $G \subset \text{Aut}_F(F(x, y), F(x))$ . If  $F$  is algebraically closed, then  $F(x, y)/F$  is pure. Furthermore, one of the two generators of  $F(x, y)/F$  may be chosen to be a rational function in  $x$ . (Received October 24, 1978.)

\*763-12-12 Andrew M. Baily, University of Michigan, Dearborn, Michigan 48128. On the density of discriminants of quartic fields.

For a group  $G$  let  $N_4(X, G)$  denote the number of non-conjugate quartic fields over  $Q$  whose normal closure has Galois group  $G$  and whose discriminant does not exceed  $X$  in absolute value. The possible groups are the symmetric group  $S_4$ , the alternating group  $A_4$ , the dihedral group  $D_4$ , the cyclic group  $C_4$ , and the Klein four group  $V_4$ . We show a)  $X \ll N_4(X, S_4) \ll X^{3/2} \log^4 X$  b)  $X^{1/2} \ll N_4(X, A_4) \ll X \log^4 X$  c)  $X \ll N_4(X, D_4) \ll X$  d)  $N_4(X, C_4) \sim a_C X^{1/2}$  e)  $N_4(X, V_4) \sim a_V X^{1/2} \log^2 X$  where  $a_C$  and  $a_V$  are constants. (Received October 25, 1978.)

\*763-12-13 Stanley J. Gurak, University of San Diego, San Diego, California 92110  
The Hasse norm principle in non-abelian extensions.

Recently several authors including myself have given criteria to determine when the Hasse norm principle holds in an abelian extension of a number field. In this paper we continue this investigation to find criteria for the Hasse norm principle to hold in non-abelian extensions. Our methods are, for the most part, idelic and make extensive use of the theories of the relative genus and relative central class field. Primarily we study how the validity of the Hasse norm principle in an arbitrary finite extension  $L/k$  is related to its validity in a subextension of  $L/k$ . We apply some of these results to give criteria for the Hasse norm principle to hold in non-normal extensions  $k(\sqrt[r_1]{\alpha_1}, \sqrt[r_2]{\alpha_2}, \dots, \sqrt[r_n]{\alpha_n})/k$  and in a given extension  $K/k$  of small degree ( $< 6$ ). (Received October 25, 1978.) (Author introduced by Roderic Deyo).

### 13 ► Commutative Rings and Algebras

763-13-1 MATTHEW MILLER, University of Illinois, Urbana, IL 61801. Self-duality of rank 2 reflexive modules.

Let  $R$  be a regular local ring and  $I$  an ideal of height 2. We show that  $\text{Hom}(-, R)$  induces an involution  $*$  on  $\text{Ext}(I, R)$ , which is actually multiplication by  $-1$  if  $I$  is unmixed and 2-generated at its associated primes. It follows that for every finitely generated rank 2 reflexive module  $M$  there is a skew symmetric self-duality isomorphism  $\varphi : M \rightarrow M^*$ . This isomorphism can be used to give an upper bound for  $k$ , where  $M$  is a  $k$ -th syzygy. The isomorphism  $\varphi$  is induced by an application of  $*$  to a short exact sequence  $R \rightarrow M \rightarrow I$  in which  $I$  has the properties mentioned above. Indeed, we can show that under much weaker conditions than regularity every finitely generated reflexive module  $M$  appears in an exact sequence  $R^d \rightarrow M \rightarrow P$  in which  $P$  is a height 2 analytically irreducible prime; this result should be regarded as an analogue of Bourbaki's theorem. (Received October 16, 1978.)

763-13-2 J. H. Kim, East Carolina University, Greenville, North Carolina 27834. R-Automorphisms of  $R[t][[X_1, \dots, X_n]]$ . Preliminary report.

Let  $R$  be a commutative ring with identity and  $R[t][[X_1, \dots, X_n]]$  the formal power series ring in  $n$  indeterminates  $X_1, \dots, X_n$  over  $R[t]$  where  $R[t]$  is the polynomial ring in an indeterminate  $t$  over  $R$ .

In this paper we consider  $R$ -automorphism of  $\phi$  of  $R[t][[X_1, \dots, X_n]]$  such that  $\phi(t)$  is not necessarily in  $R[t]$ . If  $g(t) \in R[t]$ ,  $\pi_i(g(t))$  will denote the coefficient of  $t^i$  in  $g(t)$ .

Theorem. Let  $\alpha = \sum_{j=0}^{\infty} \alpha_j t^j$ ,  $\beta_1 = \sum_{j=0}^{\infty} \beta_j^{(1)} t^j$ ,  $\dots$ ,  $\beta_n = \sum_{j=0}^{\infty} \beta_j^{(n)} t^j$  be homogeneous decompositions of elements of  $R[t][[X_1, \dots, X_n]]$  over  $R[t]$ . Then there exists an  $R$ -automorphism  $\phi$  of  $R[t][[X_1, \dots, X_n]]$  such that  $\phi(t) = \alpha$  and  $\phi(X_i) = \beta_i$  for each  $i$ , if and only if the following conditions are satisfied

- (i)  $(\beta_0^{(1)}, \dots, \beta_0^{(n)})$  is a nilpotent ideal of  $R[t]$ .
- (ii)  $R[t]\beta_1^{(1)} + \dots + R[t]\beta_1^{(n)} = R[t]X_1 + \dots + R[t]X_n$ .
- (iii)  $\pi(\alpha_0) \in R[\alpha][[\beta_1, \dots, \beta_n]]$  such that  $\pi_1(\alpha_0)$  is a unit of  $R$  and  $\pi_i(\alpha_0)$ , for  $i \geq 2$ , is nilpotent. (Received October 20, 1978.)

763-13-3 NICHOLAS S. FORD, The Pennsylvania State University, The Fayette Campus, Uniontown, Pa. 15401. Inertial Subalgebras of Algebras Possessing Finite Automorphism Groups.

Let  $R$  be a commutative ring with identity, and let  $A$  be a  $R$ -algebra of finite type with Jacobson radical  $N$  and center  $C$ . A  $R$ -inertial subalgebra of  $A$  is a  $R$ -separable subalgebra  $B$  with the property that  $B + N = A$ . Suppose  $A$  is separable over  $C$ , and that  $A$  possesses a finite group  $G$  of  $R$ -automorphisms whose restriction to  $C$  is faithful with fixed ring  $R$ . We find necessary and sufficient conditions for the existence of a  $R$ -inertial subalgebra of  $A$ , when  $R$  is an inertial subalgebra of  $C$ . In particular:

Theorem If the order of  $G$  is an unit in  $R$  and  $J_N = G$ , where  $J_N = \{g \mid g(x) - x \in N, x \in A\}$ , then  $A^G$  is a  $R$ -inertial subalgebra of  $A$  and, moreover,  $A \cong A^G \otimes_R C$ .

Using this result we extend a number of results by Ingraham in the commutative case to a non-commutative setting. (Received October 23, 1978.)

763-13-4 Nick Vaughan, North Texas State University, Denton, Texas 76203. An Integral domain with an almost division algorithm. Preliminary report.

Let  $F$  be a field,  $x$  an indeterminate, and  $D = \{f(x) \in F[x] \mid x \text{ coefficient of } f(x) \text{ is zero}\}$ . Theorem 1. If  $f(x) \in D$  and  $0 \neq g(x) \in D$ , then  $f(x) = q(x)g(x) + r(x)$ , with  $q(x), r(x) \in D$  and  $r(x) = 0$ , or  $\deg r(x) < \deg g(x)$  or  $\deg r(x) = 1 + \deg g(x)$ . Theorem 2. If  $N$  is an ideal of  $D$ , then  $N$  is generated by one or two elements. If we take  $D = \{f(x) \in F[x] \mid \text{the coefficients of } x, x^2, \dots, x^t \text{ are zero for some positive integer } t\}$ , then every ideal of  $D$  is generated by less than or equal to  $t$  elements. Moreover, there is again an almost division algorithm. (Received October 23, 1978.)

763-13-5 GARY A. HARRIS, Texas Tech University, Lubbock, Texas 79409. A Non Constant Rank Implicit Function Theorem. Preliminary report.

The following question is answered. Let  $\phi_1, \phi_2, \dots, \phi_n$  be convergent power series and suppose the generic rank of  $(\phi_1, \phi_2, \dots, \phi_n)$  is  $r < n$ . (The pointwise rank is NOT assumed to be constant.) Do there exist convergent power series  $G_1, G_2, \dots, G_{n-r}$  so that for each  $j = 1, 2, \dots, n-r$   $\phi_{r+j} = G_j(\phi_1, \dots, \phi_r)$ ? In other words, when do there exist  $n-r$  independent, non-singular analytic relations among the  $\phi_1, \phi_2, \dots, \phi_n$ ? (Received October 23, 1978.)

\*763-13-6 LUCY I. DECHENE, Fitchburg State College, Fitchburg, Massachusetts 01420. Adjacent extensions of rings. Preliminary report.

By a ring we mean a commutative ring with identity. A ring  $A$  is an adjacent extension of a ring  $R$  ( $R \subset A$  (adjac)) in case  $R \subset A$  and whenever  $S$  is any ring such that  $R \subset S \subseteq A$ , then  $S = A$ . If  $R$  is a quasi-local ring,  $R \subset A$  (adjac), and  $A$  has exactly two maximal ideals, then  $A$  is a 2-adjacent extension of  $R$ . We show how to construct an adjacent extension of an arbitrary ring and we discuss some equivalences of adjacent and 2-adjacent extensions. We also characterize the Catenary Chain Conjecture in terms of properties of 2-adjacent extensions. (Received October 23, 1978.)

\*763-13-7 DENNIS R. ESTES, University of Southern California, Los Angeles, California 90007. Determinants of Galois Automorphisms of Maximal Commutative Rings of  $2 \times 2$  Matrices

The determinants of solutions  $X$  of any of the  $2 \times 2$  matrix equations: (1)  $XAX^{-1} = A^t$ ,  $t$  denoting transpose; (2)  $A = XY$ ,  $X, Y$  symmetric; (3)  $X = AB - BA$ ; and (4)  $XAX^{-1} = A^*$ , the matrix of cofactors of  $A$  transposed, are characterized over a commutative ring  $R$ , via Galois cohomology and a generalized Latimer-MacDuffee correspondence between similarity classes and elements in the Picard group  $\text{Pic}(R[A])$ , as the negative of norms from the quadratic extension  $R[A]$  whenever certain elements in  $\text{Pic}(R[A])$  are trivial. Specializations at  $R = \mathbb{Z}$ , the ring of rational integers, appear in several articles by O. Taussky. (Received October 23, 1978.) (Author introduced by Professor G. Passty).



763-13-8 DAVID J. JOHN\*, Valdosta State College, Valdosta, Georgia 31601. MARY F. NEFF, Emory University, Atlanta, Georgia 30322. The word problem is solvable in  $N_0$ . Preliminary report.

Let  $\mathcal{N}_0$  be the variety of near-rings satisfying the extra identity  $0x = 0$ , and let  $N_0$  represent the free near ring in  $\mathcal{N}_0$ . Inductively,  $N_0$  can be constructed from a set of free generators as sums, products, and negatives of words. A set of reductions can be defined on the words of  $N_0$ , and a length function can be defined which is compatible with these reductions. Lemma 1 (Diamond): If  $w$  reduces to  $w_1$  and  $w_2$ , then there exists a  $w_3$  and reductions so that  $w_1$  and  $w_2$  reduce to  $w_3$ . Theorem 2: If  $C_w$  is the set of all words which can reduce to  $w$  or be reduced from  $w$ , then there exists a unique word in  $C_w$  of shortest length, i.e., the word problem is solvable in  $N_0$ . If  $N(S)$  represents the "free" distributively generated near ring, we prove, Theorem 3:  $N_0$  is isomorphically embedded in  $N(S)$  and Corollary 4:  $N_0$  is residually finite. (Received October 24, 1978.)

763-13-9 DENNIS R. ESTES and JACOB R. MATIJEVIC, University of Southern California, Los Angeles, California 90007. Local-Global Criteria for Outer Product Rings.

Let  $R$  be a commutative ring with unit.  $R$  is said to be an OP-ring if for each  $u \in \Lambda^n R^{n+1}$  there exist  $u_1, u_2, \dots, u_n \in R^{n+1}$  such that  $u = u_1 \wedge u_2 \wedge \dots \wedge u_n$ . Such rings are characterized by the following:

Theorem: Let  $R$  be reduced Noetherian ring. Then  $R$  is an OP-ring iff (1)  $\dim_{R/m} m/m^2 \leq 2$

for each maximal ideal  $m$  of  $R$  and (2) if  $P$  is a projective module of constant rank  $n$  such that  $\Lambda^n P \cong R$  then  $P$  is a free module.

As a consequence of the proof of the above theorem results can be obtained concerning the number of generators of ideals in rings with properties (1) and (2). (Received October 24, 1978.)

\*763-13-10 WILLIAM D. WEAKLEY, Northwestern University, Evanston, Illinois 60201. A structure theory for modules satisfying chain conditions over commutative rings. Preliminary report.

Fix a ring  $R$  and say that a simple  $R$ -module is 0-dcc-simple. Assume that  $\beta$ -dcc-simplicity of  $R$ -modules has been defined for all ordinals  $\beta < \alpha$ . An  $R$ -module  $N$  is said to be of dcc-dimension  $\beta$  iff there is a finite series of  $R$ -modules  $0 = N_0 \subset N_1 \subset \dots \subset N_k = N$  such that for each  $j$ ,  $1 \leq j < k$ ,  $N_j/N_{j-1}$  is  $\beta_j$ -simple and  $\sup \{\beta_j\} = \beta$ . We define an  $R$ -module  $M$  to be  $\alpha$ -dcc-simple iff for each proper submodule  $N$  of  $M$  there is an ordinal  $\beta_N, \beta_N < \alpha$ , such that  $N$  is of dcc-dimension  $\beta_N$ , but  $M$  does not have such a series. (For convenience the zero module is said to be  $-1$ -dcc-simple.)

Replacing "proper submodule" with "proper quotient" in the above gives the dual definitions of  $\beta$ -acc-dimension and  $\alpha$ -acc-simplicity. We have: Lemma Let  $R$  any ring,  $M$  a left  $R$ -module. TFAE: 1)  $M$  has the dcc (resp. acc) on left submodules; 2) For some ordinal  $\beta$ ,  $M$  is of dcc-dimension  $\beta$  (resp. acc-dimension  $\beta$ ). Some Jordan-Holder properties of the series decomposition are shown. Identifying finite ordinals with positive integers and using the Matlis duality, we have Theorem: Let  $R$  a commutative Noetherian ring,  $M$  an  $n$ -dcc-simple  $R$ -module. Then  $\text{End}_R(M)$  is a commutative local complete Noetherian domain of Krull dimension  $n$ . (Received October 24, 1978.)

763-13-11 Stuart S. Wang, Oakland University, Rochester, Michigan 48063. Extension of derivations. Preliminary report.

Let  $k$  be a commutative ring with identity. Let  $A \subseteq B$  be either (1) two polynomial rings over  $k$ ,  $A = k[Y_1, \dots, Y_n] \subseteq B = k[X_1, \dots, X_n]$ , or (2) two formal power series rings over  $k$ ,  $A = k[[Y_1, \dots, Y_n]] \subseteq B = k[[X_1, \dots, X_n]]$ , or (3) one polynomial ring over  $k$  and one formal power series ring over  $k$ ,  $A = k[X_1, \dots, X_n] \subseteq$

$k[[X_1, \dots, X_n]]$ . Then any  $k$ -derivation of  $A$  into itself can be uniquely extended

to  $B$  if and only if  $(\frac{\partial Y_i}{\partial X_j}) \in GL_n(B)$ . In case (1), where  $k[Y_1, \dots, Y_n] \subseteq$

$k[X_1, \dots, X_n]$ , an interpretation of the more relaxed condition that

$(\frac{\partial Y_i}{\partial X_j}) \in GL_n(k[[X_1, \dots, X_n]])$  but  $(\frac{\partial Y_i}{\partial X_j}) \notin GL_n(k[X_1, \dots, X_n])$  is also given. (Received

October 24, 1978.)

763-13-12 Roderic C. Deyo, University of California, San Diego, California 92093. Universal trace classes. Preliminary report.

Let  $R$  be a unital ring and  $\Sigma R$  the endomorphism category of finitely generated  $R$ -modules. Then along the lines of Stallings, we define a trace module  $T(R)$  and a morphism  $T: \Sigma R \rightarrow T(R)$ , which is universal for all maps of trace type. Theorem 1 If  $R$  is regular, then  $T(R) \cong R$ . Theorem 2 If  $R$  is artinian, then  $T(R) \cong R/J$ , where  $J$  is the Jacobson radical. A localization sequence  $0 \rightarrow \bigoplus_{p \text{ min prime}} T(R_p) \rightarrow T(R) \rightarrow R' \rightarrow 0$ , where  $R'$  is the integral closure, is given for  $R$  a commutative noetherian domain. This is used to show that  $T(R) \cong R'$  for  $\dim R = 1$ , with  $\text{char } R = 0$ . Using torsion in Pic  $R$ , we can also compute  $T(R)$  for  $\dim R = 2$ , when the torsion of Pic  $R$  is prime to the characteristic of  $R$ . If  $R$  is regular, we construct a bilinear form whose non-degeneracy implies the separability of a given ring extension of  $R$ . Generalizations to the abelian category of coherent sheaves over an  $R$ -scheme are given. An application of this theory to the calculation of Euler characteristics is also described. (Received October 25, 1978.)

763-13-13 FREDERICK CALL and THOMAS SHORES, University of Nebraska, Lincoln, Nebraska 68588. Torsion Theories with BSP.

Rings of this note are commutative with identity and modules are unital.

Hereditary torsion theories  $(\mathcal{T}, \mathcal{F})$  with the property that the torsion submodule of a module  $M$  is a direct summand of  $M$  if this submodule is bounded, i.e., killed by an ideal in the filter defining  $\mathcal{T}$ , are said to have BSP (the bounded splitting property). These torsion theories are a natural generalization of torsion modules over Dedekind rings. Such torsion theories are completely characterized in terms of ring-theoretic properties. The first step is a solution to the local problem. Next there follows a complete resolution of the case in which  $\mathcal{T}(R) = 0$ . Finally it is shown how to construct general torsion theories with BSP in terms of the case  $\mathcal{T}(R) = 0$ . (Received October 25, 1978.)

## 14 ► Algebraic Geometry

\*763-14-1 GUSTAVE A. FROYMSON, University of New Mexico, Albuquerque, New Mexico 87131. Sums of squares on planar Nash rings.

Let  $D$  be a connected semi-algebraic set in  $\mathbb{R}^3$ . Then  $f: D \rightarrow \mathbb{R}$  is a Nash function if  $f$  is real analytic and if  $f$  is algebraic, i.e., if  $\exists p(x, y, z)$ , a polynomial, with  $p(x, y, f(x, y)) \equiv 0$  on  $D$ . The set of Nash functions on  $D$  is a ring called the Nash ring  $N_D$ . Theorem: Let  $f \in N_D$ ,  $f \geq 0$  on  $D$ , then  $\exists f_1, f_2 \in N_D$  with  $f = f_1^2 + f_2^2$ . The analogous statement for polynomials is false. In fact, Motzkin gave an explicit example of a polynomial which is  $\geq 0$  on  $\mathbb{R}^2$  and cannot be expressed as the sum of squares of polynomials. This example homogenized gives an example of a Nash function on  $\mathbb{R}^3$  which cannot be written as a sum of squares of Nash functions. (Received October 6, 1978.)

Conjugacy classes of split subgroups.

Let  $G$  be a semisimple algebraic group defined over the real numbers. Let  $S$  be a maximal split torus and  $D$  the set of restricted roots of  $G$  on  $S$ . A subsystem  $B$  of  $D$  is a subset such that  $-B=B$  and such that if  $a, b \in B$  and if  $a+b \in D$  then  $a+b \in B$ . If  $B$  is a subsystem of  $D$ ,  $p_i(B) = \{b \mid b \in B \text{ and } b/2 \in B\}$ . Let  $H$  be a split semisimple subgroup of  $G$  with maximal rank.  $H$  is regular if the roots of  $H$ ,  $D_H$ , equal  $p_i(B)$  for some subsystem  $B$  of a set of restricted roots  $D$  of  $G$ .

Theorem. The  $G_R$ -conjugacy classes of regular split subgroups are indexed by the orbits of subsystems of maximal rank in  $D$ , the restricted roots of  $G$ , under the action of the Weyl group. (Received October 10, 1978.)

\*763-14-3 Robert Speiser, University of Minnesota, Minneapolis, Minn., 55455. Formal Meromorphic Functions on Rational Varieties, Preliminary Report.

A connected closed subscheme of  $\mathbb{P}^m \times \mathbb{P}^n$  is said to be positive if both projections onto the factors are positive dimensional.

THEOREM 1. Let  $Y \subset \mathbb{P}^m \times \mathbb{P}^n$  be a connected closed subscheme. Then  $Y$  is positive if and only if every formal meromorphic function along  $Y$  extends to  $\mathbb{P}^m \times \mathbb{P}^n$ , i.e.,  $G_3$  holds for  $Y \subset \mathbb{P}^m \times \mathbb{P}^n$ .

A connected closed subscheme of the Grassmannian  $G(m,n)$  of  $m$ -planes in  $\mathbb{P}^n$  is said to be positive if its intersections with subvarieties of the form  $G(m,n-1)$  (for hyperplanes  $\mathbb{P}^{n-1} \subset \mathbb{P}^n$ ) is nonzero modulo algebraic equivalence.

THEOREM 2. Let  $Y \subset G(m,n)$  be a positive closed subvariety. Then  $Y$  is connected, and every formal meromorphic function along  $Y$  extends to  $G(m,n)$ , i.e.,  $G_3$  holds for  $Y \subset G(m,n)$ .

There are Schubert cells on  $G(m,n)$  for which  $G_3$  fails. (Received October 20, 1978.) (Author introduced by Professor Joel Roberts).

\*763-14-4 Micheal J. Kent, Rutgers University, New Brunswick, NJ 08903 and Charles F. Schwartz, Texas Tech University, Lubbock, TX 79409. Specialization to Torsion of a Rational Point on an Elliptic Curve over  $\mathbb{Q}(t)$ .

It has been shown that the Mordell-Weil group (i.e., the group of  $\mathbb{C}(t)$ -rational solutions) of  $y^2 = x(x-1)(x+t^2+c)$  is generated by its torsion subgroup, together with a solution of infinite order:  $x = mt + b$ ,  $y = m(mt + b + t^2 + c)$ . In the present paper, the values of  $a$  for which the solution  $x = ma + b$ ,  $y = m(ma + b + a^2 + c)$  is of finite order in the group of  $\mathbb{Q}$ -rational solutions of  $y^2 = x(x-1)(x+a^2+c)$  are calculated. (Received October 23, 1978.)

\*763-14-5 BEN THOMAS, University of North Carolina, Chapel Hill, N. C. 27514. Singular points and Weierstrass points of algebraic plane curves.

Let  $V_0$  be the germ at 0 of an algebraic plane curve singularity with  $G^*$  action, ie a monomial curve. We extend  $V_0$  in the obvious way to get a compact curve  $V$  in  $\mathbb{P}^1 \times \mathbb{P}^1$ ; note that  $V_\infty$ , the germ of  $V$  at  $(\infty, \infty)$ , is a copy of  $V_0$ . If  $V$  is deformed,  $V_0$  is deformed "negatively" in the sense of Pinkham iff  $V_\infty$  is deformed "positively," ie equisingularly. If  $\bar{V} = V$  with  $V_\infty$  desingularized,  $\bar{V}$  has a Weierstrass point at  $\infty$  whose semigroup  $G$  equals the semigroup of the singularity  $V_0$ ; negative deformations of  $V_0$  correspond to deformations of  $\bar{V}$  which preserve  $G$ . In the case of negative deformations, this construction reproduces results of Pinkham in a more concrete way. We discuss applications of this construction to deformations of more general curve singularities. (Received October 24, 1978.)

\*763-14-6 J. R. Quine, The Florida State University, Tallahassee, Florida 32306. Plücker relations for  $p(e^{i\theta})$ .

Let  $p$  be a polynomial of degree  $n$ . In connection with the problem of determining whether  $p$  is univalent in the unit disc, we have previously shown that the closed

curve  $p(e^{i\theta})$  has at most  $(n-1)^2$  vertices, proving a conjecture of Titus. In this paper we derive the four Plücker relations for  $f$ , the algebraic completion of  $p(e^{i\theta})$ . Since Plücker's relations are classically for curves with simple singularities, we must examine carefully the contribution of the multiple singularities on the line at infinity. We discover that the first mentioned result is a consequence of one of the Plücker relations. We further show that the number of zeros of  $p'$  symmetric in  $|z| = 1$  is closely related to the Plücker characteristics of  $f$ . Finally, we show as a consequence of Plücker's relations that  $f$ , and therefore  $p(e^{i\theta})$ , has at most  $(n-1)(2n-3)$  double tangents. (Received October 25, 1978.)

\*763-14-7 Roy C. Smith, University of Georgia, Athens, Georgia, 30602. The degree of the Prym map in dimension five.

In 1974, Mumford wrote a paper on Prym varieties which stimulated renewed interest in an old idea of Schottky's for finding relations among the theta-nulls of abelian varieties which will characterize the Jacobian varieties of curves. The idea is to translate geometric relations on the theta-nulls of genus  $g+1$  curves to relations on the theta-nulls of  $g$ -dimensional abelian varieties via the Prym map and the "Schottky relations". A pertinent question in this procedure is the Torelli problem for the Prym map, still unsolved in general. We give the solution of the Torelli problem for the map from genus 6 curves to 5-dimensional Abelian varieties. That is, we show that the Prym map in that case is twenty-seven to one, and give evidence that the general fibre is parametrized by the lines of my cubic surface. (Received October 25, 1978.)

## 15 ► *Linear and Multilinear Algebra; Matrix Theory (finite and infinite)*

\*763-15-1 Stephen L. Campbell, North Carolina State University, Raleigh, North Carolina 27650. Continuity of the Drazin Inverse.

For  $A \in \mathbb{C}^{n \times n}$ , let  $A^D$  denote the Drazin inverse of  $A$ . It is well known that the operation of taking the Drazin inverse is not continuous as a function from  $\mathbb{C}^{n \times n}$  into  $\mathbb{C}^{n \times n}$ . We shall show that there is, in some sense, a continuity to the definition. That is, if a matrix  $\hat{A}$  comes close to satisfying the defining relations;  $XAX - X = 0$ ,  $XA - AX = 0$ , and  $A^{k+1}X - A^k = 0$ , then  $\hat{A}$  must be close to  $A^D$ . (Received October 12, 1978.)

\*763-15-2 JOHN D. FULTON, Clemson University, Clemson, South Carolina 29631. Generalized Inverses of Matrices Over Fields of Characteristic Two.

For  $F$  a field of characteristic two, the problem of determining which  $m \times n$  matrices of rank  $r$  have normalized generalized inverses and which have pseudoinverses is solved. For  $F_q$  a finite field of characteristic two, both the number of  $m \times n$  matrices of rank  $r$  over  $F$  which have normalized generalized inverses and the number of  $m \times n$  matrices of rank  $r$  over  $F_q$  which have pseudoinverses are determined. (Received October 20, 1978.)

\*763-15-3 Ray C. Shiflett, California State University, Fullerton, Fullerton, California 92634. A Birkhoff Theorem for Doubly Stochastic Matrices with Vector Entries. Preliminary report.

The famous Birkhoff Theorem states that the extreme  $n \times n$  doubly stochastic matrices with entries from the closed unit interval  $[0,1]$  are exactly the permutation matrices (i.e. those  $n \times n$  doubly stochastic matrices whose entries are extreme). A doubly stochastic matrix whose entries

come from a convex subset of the unit square  $[0,1] \times [0,1]$  is an  $n \times n$  matrix whose rows and columns sum to  $(1,1)$ . A characterization of the convex subsets of  $[0,1] \times [0,1]$  for which the Birkhoff Theorem holds is given. (Received October 20, 1978.)

763-15-4 Bruce H. Edwards, University of Florida, Gainesville, Florida 32611. Split Clifford Algebras. Preliminary Report.

Let  $V$  be a quadratic space over a field  $k$  of characteristic not two. The Clifford Algebra  $C(V)$  of  $V$  is assumed to be split; that is, a total matrix ring over  $k$ . The main involution and main antiautomorphisms of  $C(V)$  are studied in terms of this matrix representation. Similar results are derived for the case when the even Clifford Algebra  $C^+(V)$  is split. Applications to the structure of Spin, Pin and the orthogonal groups are given. (Received October 23, 1978.)

763-15-5 Helene Shapiro, California Institute of Technology, Pasadena, California 91125. The characteristic polynomial of a pencil generated by a pair of Hermitian matrices.

Let  $H_1$  and  $H_2$  be  $n \times n$  complex, Hermitian matrices. The pencil generated by  $H_1$  and  $H_2$  is  $\{xH_1 + yH_2 \mid x, y \in \mathbb{C}\}$ . The characteristic polynomial of this pencil is  $f(x, y, z) = \det(zI - xH_1 - yH_2)$ , a homogeneous polynomial of degree  $n$  in three variables. If  $f(x, y, z)$  factors into linear factors over  $\mathbb{C}$ , the pair  $H_1$  and  $H_2$  are said to have property L. Motzkin and Taussky (Trans. Amer. Math. Soc. 73 (1952) 108-114) showed that  $H_1$  and  $H_2$  have property L if and only if  $H_1H_2 = H_2H_1$ . If  $H_1$  and  $H_2$  can be simultaneously block diagonalized, then  $f(x, y, z)$  splits into factors corresponding to the blocks, but the converse does not hold in general. However, in some special cases, factors of  $f(x, y, z)$  indicate that  $H_1$  and  $H_2$  can be simultaneously block diagonalized.

Theorem. Suppose  $f(x, y, z) = (z - \alpha x - \beta y)^k g(x, y, z)$  where  $(z - \alpha x - \beta y)$  does not divide  $g(x, y, z)$  and that  $H_1 - \alpha I$  has rank  $n - k$ . Then if  $k > \frac{n}{3}$  the matrices  $H_1$  and  $H_2$  have a common eigenvector corresponding to the eigenvalue  $\alpha$  of  $H_1$  and  $\beta$  of  $H_2$ .

Theorem. If  $f(x, y, z) = [g(x, y, z)]^k$  where  $g(x, y, z)$  has degree 2, then  $H_1$  and  $H_2$  are simultaneously, unitarily similar to block diagonal matrices, each of which is a direct sum of  $k$  identical  $2 \times 2$  blocks. (Received October 23, 1978.)

\*763-15-6 John Jones, Jr., Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio 45433. Solutions of the Lyapunov Matrix Equation.

Theorem 1. If  $A(z)$ ,  $B(z)$ ,  $C(z)$  are  $n$  by  $n$  matrices of holomorphic functions of a single complex variable  $z$  belonging to a region  $\mathcal{R}$  and the pair of  $2n$  by  $2n$  matrices  $\begin{pmatrix} A(z) & C(z) \\ 0 & B(z) \end{pmatrix}$ ,  $\begin{pmatrix} A(z) & 0 \\ 0 & B(z) \end{pmatrix}$  are equivalent for  $z \in \mathcal{R}$ . Then the matrix equation  $A(z)X(z) - Y(z)B(z) = C(z)$  has a solution  $X(z)$ ,  $Y(z)$  a pair of  $n$  by  $n$  matrices of holomorphic functions for  $z \in \mathcal{R}$ . Theorem 2. Let  $A(z)$ ,  $B(z)$ ,  $C(z)$  by  $n$  by  $n$  matrices of holomorphic functions of  $z$  for  $z$  belonging to a closed bounded region  $\mathcal{R}$ . Let the characteristic equations of  $A(z)$ ,  $B(z)$  be  $|A(z) - \gamma(z)I| = 0$ ,  $|B(z) - \gamma(z)I| = 0$  where  $I$  is the identity matrix,  $I = (\delta_{ij})$ , where  $\delta_{ij}$  are the Kronecker symbols. Each characteristic root  $\gamma_i(z)$ ,  $i = 1, 2, \dots, m$  and their respective multiplicities  $h_i$  are such that  $\sum_{i=1}^m h_i = n$ . Each characteristic root  $\gamma_i(z)$  is a holomorphic function for  $z \in \mathcal{R}$ . If the pair of  $2n$  by  $2n$  matrices above are similar then the matrix equation  $A(z)X(z) - X(z)B(z) = C(z)$  has an  $n$  by  $n$  matrix solution  $X(z)$  of holomorphic functions for  $z \in \mathcal{R}$ . (Received October 23, 1978.)

\*763-15-7 R. E. Hartwig, North Carolina State University, Raleigh, North Carolina, 27650. Unit and unitary solutions to the matrix equation  $AX = B$ . Preliminary report.

It is shown that under quite weak conditions, the problem of characterizing all the unit and unitary solutions to the matrix equation  $AX = B$ , reduces to a study of idempotents and partial isometries. Closed form expressions are given for these sets of solutions, and the results are used to

generalize the orthogonal completion theorem of Hall and Ryser. In particular it is demonstrated that all partial isometries over a field with  $2 \neq 0$ , or over the integers, are unitary-regular; that is possess unitary inner inverses. (Received October 24, 1978.) (Author introduced by Professor John Franke).

763-15-8 PETER M. GIBSON, University of Alabama in Huntsville, Huntsville, Alabama 35807. Permanental polytopes of doubly stochastic matrices.

Let  $\Omega_n$  denote the polytope of all  $n \times n$  nonnegative doubly stochastic matrices. A subpolytope  $\Gamma$  of  $\Omega_n$  is said to be permanental if  $\text{per } A = \text{per } B$  for all  $A$  and  $B$  in  $\Gamma$ . Study [these Notices 25 (1978), A-652] of basic properties of permanental polytopes is continued. Permanental polytopes of  $\Omega_n$  of dimension  $(n-1)(n-2)/2$  are exhibited, and it is conjectured that no permanental polytope of  $\Omega_n$  of larger dimension exists. This conjecture is shown to hold for  $n < 4$ . (Received October 25, 1978.)

## 16 ► Associative Rings and Algebras

763-16-1 FRED RICHMAN, New Mexico State University, Las Cruces, New Mexico 88003. Finite dimensional algebras. Preliminary report.

A survey of constructive aspects of the theory of finite dimensional algebras over discrete fields.

Sample theorem: Every finite dimensional algebra over a discrete field  $k$  is either simple or has a nontrivial ideal, if and only if every polynomial over  $k$  is a product of irreducible polynomials.

Sample problem: What conditions on a field  $k$  imply that every finite dimensional algebra over  $k$  is either a division algebra or has a nonzero zero-divisor? (Received October 5, 1978.)

\*763-16-2 ROMAN W. WONG, Washington & Jefferson College, Washington, Pennsylvania 15301. Finitely generated ideals in free group algebras.

Let  $K$  be a skew field and  $G$  a free group. It is well-known that every left ideal of the free group algebra  $KG$  is free as  $KG$  module. Let  $I \subseteq KG$  be a left ideal generated by a finite set  $X$ . We show that  $X$  can be refined, in finitely many steps, to a free basis for  $I$ . This is an application of a technique developed in the author's paper Free Ideal Monoid Rings (J. of Alg., 53(1978), 21-35). (Received October 19, 1978.)

763-16-3 EARL J. TAFT, Institute for Advanced Study, Princeton, New Jersey 08540 and Rutgers University, New Brunswick, New Jersey 08903. Bialgebras with left antipodes. Preliminary report.

Let  $B$  be a bialgebra. A left antipode of  $B$  is a left inverse in  $\text{Hom}(B, B)$  of the identity mapping of  $B$  under convolution. A bialgebra possessing a left antipode is called a left Hopf algebra. Proposition. Let  $S$  and  $T$  be left antipodes of  $B$  which are bialgebra antiendomorphisms. Then  $S^2 = ST = TS = T^2$  (under composition of mappings). Using this idea, we are able to construct left Hopf algebras possessing  $n$  left antipodes which are bialgebra antiendomorphisms for an arbitrary prescribed cardinal number  $n$ . (Received October 18, 1978.)

\*763-16-4 RICHARD K. MOLNAR, Oakland University, Rochester, Michigan 48063. Hopf algebras and tensor products of semisimple representations. Preliminary report.

We say a Hopf algebra  $H$  (over a field  $k$ ) has the tensor product property if the tensor product of any two finite dimensional semisimple  $H$ -modules is semisimple as an  $H$ -module. We show that a finite dimensional Hopf algebra  $H$  has the tensor product property if and only if the Jacobson radical  $J(H)$  is a Hopf ideal of  $H$ . Applications to the modular representations of finite groups and restricted Lie algebras are given. Theorem 1. Let  $G$  be a finite group and  $k$  a field of

characteristic  $p \neq 0$ .  $G$  has the tensor product property over  $k$  if and only if  $G$  has a normal  $p$ -Sylow subgroup. Theorem 2. Let  $\mathfrak{L}$  be a restricted Lie algebra over a field  $k$  of characteristic  $p \neq 0$ .  $\mathfrak{L}$  has the tensor product property if and only if  $\mathfrak{L}$  is an extension of a toral Lie algebra by a  $p$ -unipotent Lie algebra. Related results for non finite dimensional Hopf algebras will be discussed. (Received October 19, 1978.)

763-16-5 GEORGE SZETO, Bradley University, Peoria, Illinois 61625. On a splitting theorem of Azumaya algebras. Preliminary report.

S. Parimala and P. Sridharan (J. Pure and Applied Alg. 9(1977), 181-193) defined a generalized quaternion algebra  $B[j]$  such that (1)  $B$  is a commutative ring with identity and with an automorphism group  $\{1, \sigma\}$  of order 2 invertible in  $B$ , and (2)  $B[j]$  is a free  $B$ -module with a basis  $\{1, j\}$  where  $j^2 = -1$  and  $jb = \sigma(b)j$  for all  $b$  in  $B$ . We shall show: Theorem. If  $B$  is Galois over the subring  $A$  with the Galois group  $\{1, \sigma\}$  then (1)  $B[j]$  is an Azumaya  $A$ -algebra, (2)  $A[j]$  is a splitting ring for  $B[j]$ , and (3)  $A[j]$  is Galois over  $A$ . More properties of  $B[j]$  are also given. (Received October 20, 1978.)

\*763-16-6 KIRBY C. SMITH and CARLTON J. MAXSON, Texas A&M University, College Station, Texas 77843. Simple near-ring centralizers of finite rings.

Let  $R$  be a finite ring with identity and let  $V$  be a finite unital  $R$ -module. The near-ring centralizer of  $R$  is  $C(R) = \{f: V \rightarrow V \mid f(av) = af(v) \text{ for all } a \in R, v \in V\}$ , the operations being function addition and composition. The structure of  $C(R)$  is investigated under various situations. In particular it is shown that if  $R$  is simple then  $C(R)$  is simple. Moreover we show that if  $C(R)$  is simple and  $R$  is not a field then  $C(R)$  must be a ring. (Received October 20, 1978.)

763-16-7 LLOYD CHAMBLESS, University of North Carolina, Chapel Hill, North Carolina 27514. N-Dimension and N-Critical Modules. Preliminary report.

The  $N$ -dimension of an  $R$ - (or  $R$ -S bi-) module  $M$  is defined as is Krull dimension, except ascending chains of  $R$ - (or  $R$ -S) submodules are used. Thus  $N_{(R, M_S)} = 0$  is the same as A.C.C. on  $R$ -S submodules of  $M$ . Notation:  $N_{(R, M)}$  (or  $N_{(R, M_S)}$ ), and  $K_{(R, M)}$  (or  $K_{(R, M_S)}$ ) for Krull dimension.  $K_P_{(R, M_S)}$  denotes  $P$ -relative Krull dimension, where  $P$  is a prime ideal of  $R$ .  $R^M$  is called  $\alpha$ - $N$ -critical if  $N_{(R, M)} = \alpha$  but  $N_{(R, W)} < \alpha$  for any proper  $R$ -submodule  $W$ .

1. Every artin  $R$ -module  $M$  has an " $N$ -critical composition series"  $0 = M_0 \subsetneq M_1 \subsetneq \dots \subsetneq M_n = M$  with  $M_{i+1}/M_i$   $\alpha_{i+1}$ - $N$ -critical,  $N_{(R, M)} = \alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n$ . (Uniqueness properties are given).

2. If  $(R, P)$  is commutative, local, noetherian, then  $\text{rank } P = K_{(P, R)} = N_{(R, E(R/P))}$ .

3. (Generalized Principal Ideal Theorem) If  $R$  is left noetherian and  $P$  a prime ideal minimal over  $x$ , where  $Rx$  is an ideal and  $E = E(R/P) = \bigcup_{n=0}^{\infty} \text{ann}_E x^n$ , then  $K_P_{(R, R)} \leq N_{(R, E_H)} \leq 1$ , where  $H = \text{End}_{(R, E)}$ . For  $R$  left fully bounded,  $\text{rank } P \leq K_{(R, R)} \leq N_{(R, E_H)} \leq 1$ .

4. If  $R$  is commutative noetherian, then  $N(M) \leq K(R)$  for any  $R$ -module  $M$ . (Received October 23, 1978.)

\*763-16-8 THOMAS R. SAVAGE, St. Olaf College, Northfield, Minnesota, 55057. Reduction of matrices over von Neumann regular rings. Preliminary report.

A ring  $R$  with identity is (von Neumann) regular if for every  $a$  there is an  $x$  such that  $a = axa$ , unit regular if  $x$  can always be chosen as a unit, and an elementary divisor ring if every square matrix over  $R$  is equivalent to a diagonal matrix. In [Arch. Math. 24 (1973), 133-141], M. Henriksen showed that every unit regular ring is an elementary divisor ring, and asked whether regular elementary divisor rings are necessarily unit regular.

We settle Henriksen's question in the negative with a theorem that yields the following example. If  $I$  is any proper ideal in the ring of all linear transformations on a vector space over a field  $F$ , then  $R = I + F$  is an elementary divisor ring. If  $I$  contains an element with infinite rank, then  $R$  is not unit regular. We also discuss other examples related to directly finite regular rings. (Received October 23, 1978.)

\*763-16-9 J. J. Malone, Worcester Polytechnic Institute, Worcester, MA 01609. A Non-Abelian 2-Group Whose Endomorphisms Generate a Ring, and Other Examples of E-Groups.

R. Faudree has given defining relations which, for each prime  $p$ , yield an E-group. These are the only published examples of E-groups. However, it has been shown that Faudree's relations do not yield a  $p$ -group for  $p=2$ . In the present paper the existence of a 2-group which is an E-group is demonstrated. In addition, for each odd prime  $p$ ,  $p$  E-groups, non-isomorphic to those of Faudree, are also given. The groups presented here were first noted by D. Jonah and M. Konvisser who proved that all automorphisms of these groups are central. Certain of the groups of Jonah and Konvisser are here shown to be E-groups by proving that the images of their (non-automorphism) endomorphisms must be in the center. (Received October 24, 1978.)

\*763-16-10 HORACE OLIVIER and HENRY HEATHERLY, University of Southwestern Louisiana, Lafayette, Louisiana 70504. H-monogenic near-rings, Preliminary report.

A near-ring  $(N, +, \cdot)$  with two-sided zero is said to be H-monogenic if it contains a subset  $H$  such that (1)  $N^2 \subseteq H$  and (2) if  $h_1, h_2$  are non-zero elements of  $H$ , then  $h_1 h_2 \neq 0$ . All near integral domains are H-monogenic; however, the class is wide enough to even include some distributive near-rings. An example of a commutative, H-monogenic near-ring is given and a general method of constructing H-monogenic near-rings is described which is related to Ferrero's construction of "fortemente monogeni" near-rings. It is shown that a finite, distributive, H-monogenic near-ring is the additive semidirect sum of its annihilator ideal and a subnear-ring which is a field. (Received October 24, 1978.)

\*763-16-11 WARREN NICHOLS, Florida State University, Tallahassee, Florida 32303. The Kostant structure theorems for  $K/k$  Hopf.

(1) An irreducible cocommutative  $K/k$  bialgebra (in characteristic zero) is isomorphic as a  $K/k$  bialgebra to the algebra of differential operators defined by its primitives. (2) A split pointed  $K/k$  Hopf algebra is the semidirect product  $K/k$  Hopf algebra of an irreducible  $K/k$  Hopf algebra and a group algebra. (Received October 25, 1978.)

\*763-16-12 L. O. Chung, Jiang Luh and A. Richoux, North Carolina State University, Raleigh, North Carolina 27650. Derivations and commutativity of rings.

The concept of a primary class of derivations of an associative ring is introduced. It is noted that the class of all inner derivations of a ring is always a primary class of derivations. Among the results proven for a ring  $R$  with center  $C$  having a primary class of derivations  $D$  are the following: Theorem 1. If for all  $x \in R$ ,  $\partial \in D$ , there exists a polynomial  $p_{x,\partial}(t)$  with integral coefficients such that  $\partial x = (\partial x)^2 p_{x,\partial}$ . Then  $R$  is commutative. Theorem 2. If for all  $x \in R$ ,  $\partial \in D$ , there exists a  $p = p(x,\partial) \in R$  such that  $\partial x - (\partial x)^2 p \in C$ . Then the nilpotent elements in  $R$  form an ideal  $N$  and  $R/N$  is a subdirect sum of division rings and commutative domains. Furthermore if for all  $x \in R$ ,  $\partial \in D$ ,  $p(x,\partial)$  is a polynomial in  $\partial x$  with integral coefficients, then  $\partial x \in C$  for all  $x \in R$ ,  $\partial \in D$  and  $R/N$  is commutative. (Received October 25, 1978.)

763-16-13 Gary F. Birkenmeier, Southeast Missouri State University, Cape Girardeau, Missouri 63701. Reduced and densely nil right ideals. Preliminary report.

$R$  denotes an associative ring with unity. A rt. ideal is reduced if it contains no nonzero nilpotent elements. A rt. ideal  $X$  is densely nil (DN) if either  $X = 0$ , or if  $X \neq 0$  then for every nonzero  $x \in X$  there is some  $r \in R$  such that  $xr \neq 0$  but  $(xr)^2 = 0$ . Some results are: (1) A rt. ideal of  $R$  is either reduced or it contains a nonzero DN rt. ideal of  $R$ . (2) In the set of rt. ideals of  $R$  the subset of DN rt. ideals and the subset of reduced rt. ideals are each closed under submodules and essential extensions. (3) Socle of  $R = A \oplus B$  where  $A$  is reduced and  $B$  is a DN ideal which contains all DN minimal rt. ideals. Also  $B = Y \oplus E$  where  $Y$  is a nil ideal which contains all nilpotent minimal rt. ideals,



and  $E$  is a direct sum of idempotent generated DN minimal rt. ideals. (4) If  $R = A \oplus B$  where  $B$  is the minimal direct summand containing the set of nilpotents (MDSN). Then: i)  $B$  contains an isomorphic copy of every DN rt. ideal; ii)  $B$  contains every DN ideal; iii) every direct summand of  $B$  contains a DN rt. ideal; iv)  $A$  contains an isomorphic copy of every idempotent generated reduced rt. ideal; v)  $A$  contains an isomorphic copy of every reduced rt. ideal iff  $B$  is DN. (Received October 24, 1978.)

763-16-14 MARSHA FINKEL JONES, Claremont Graduate School, Claremont, CA 91711. f-Projectivity and Flat Epimorphisms.

Let  $R$  be a ring with identity. A module is f-projective if for every finitely generated submodule the inclusion map factors through a free module.

Theorem: Let  $\varphi : R \rightarrow Q$  be a ring homomorphism and  $K = \text{Ker}(\varphi)$ . Then (1)  $\Rightarrow$  (2).

(1)  $Q_R$  is flat,  $\varphi$  is an epimorphism, and  $Q \cdot \ell_R(K) = Q$ , where  $\ell_R(K)$  is the left annihilator in  $R$  of  $K$ .

(2)  $Q_R$  is f-projective.

If  $(T, F)$  is a torsion theory for right  $R$ -modules with  $Q = \mathcal{L}_T(R_R)$ , then (2)  $\Rightarrow$  (1).

(In particular, if  $R_R \in F$ ,  $Q_R$  is a flat epimorphism precisely when  $Q_R$  is f-projective.)

Corollary: Let  $R$  be right Noetherian and  $(T, F)$  a torsion theory for right  $R$ -modules for which  $Q_R$  is flat, where  $Q = \mathcal{L}_T(R_R)$ . Then  $R \rightarrow Q$  is a ring epimorphism. (Received October 25, 1978.)

763-16-15 MICHAEL ARTIN, Department of Mathematics, Massachusetts Institute of Technology, Room 2-239, Cambridge, Massachusetts 02139. Some applications of algebraic geometry to ring theory.

- Algebraic geometry may be considered as the study of rings of polynomial functions on varieties in affine space. We will consider rings which are generated by matrices, whose entries are such polynomial functions. These rings arise in several contexts, for example in the study of matrix representations of arbitrary associative rings. They exhibit many of the complications of noncommutative rings, but are sufficiently near to commutative theory that methods of algebraic geometry can be applied to them. (Received October 25, 1978.)

## 17 $\blacktriangleright$ Nonassociative Rings and Algebras

763-17-1 Alex Jay Feingold, Drexel University, Philadelphia, Pennsylvania, 19104. Hyperbolic GCM Lie Algebras and Fibonacci Numbers. Preliminary Report.

Let  $g = g(A)$  be the Generalized Cartan Matrix (GCM) Lie algebra defined by matrix  $A$ . If  $A$  is a  $2 \times 2$  matrix,  $\begin{pmatrix} 2 & a \\ b & 2 \end{pmatrix}$ , such that  $ab > 4$  then  $g(A)$  is one of the hyperbolic GCM Lie algebras.

Investigations of the root systems of some of these algebras along lines suggested by J. Lepowsky (Rutgers University) have shown that the root spaces may be indexed in a fairly simple way by the Fibonacci numbers. The Weyl-Kac denominator formula for  $g$  can then be written down, except that the root multiplicities are unknowns. These unknowns may either be solved for by a recursive procedure, or else one can use the multiplicity formula recently discovered by R. V. Moody (University of Saskatchewan). (Received October 16, 1978.)

\*763-17-2 ANDREW SOBCZYK, Clemson University, Clemson, South Carolina 29631. Octonionic Linear Algebra. Preliminary Report.

Cayley numbers or octonions are a nonassociative alternative algebra, with seven basic imaginaries  $e_1, \dots, e_7$ . The unit 1 with  $e_1, e_2, e_4$  span a subalgebra of quaternions  $Q$ ; each octonion  $a = w + e_7 z = [w, z]$ , where  $w, z \in Q$ . Properties and a classification of left (right) and of (left and right) linear subspaces of, and of linear transformations in, the (left and right) space of octonion  $n$ -tuples  $(a^{(1)}, \dots, a^{(n)})$ , are derived, partly in terms of decompositions into quaternionic subspaces (associative scalars) corresponding to  $a = [w, z]: W, Z$ , with suitable structural relations between  $W$  and  $Z$ . Some applications in physics are indicated. (Received October 23, 1978.)

763-17-3 HARRY F. SMITH, Iowa State University, Ames, Iowa 50011. On nilpotency in certain varieties. Preliminary report.

A nonassociative algebra which satisfies the identities  $(wx,y,z) + (w,x,(y,z)) = w(x,y,z) + (w,y,z)x$  and  $(x,y,z) + (y,z,x) + (z,x,y) = 0$  is called a generalized  $(-1,1)$  algebra. For the variety of such algebras, with characteristic  $\neq 2$  or  $3$ , local nilpotence is a radical property; and each finitely-generated solvable algebra in this variety is nilpotent. As a consequence, finite-dimensional generalized  $(-1,1)$  nil algebras over fields with characteristic  $\neq 2$  or  $3$  are nilpotent. Nonassociative algebras which satisfy the identities  $(yz,x,x) = y(z,x,x) + (y,x,x)z$ ,  $(x,x,yz) = y(x,x,z) + (x,x,y)z$  and  $(x,x,x) = 0$  are called derivation alternator algebras. Two-dimensional anticommutative algebras belong to this variety, so that a finite-dimensional solvable derivation alternator algebra need not even be right nilpotent. (Received October 24, 1978.)

\*763-17-4 RALPH AMAYO, Southern Illinois University, Carbondale, Illinois 62901. Curtis Algebras.

A Lie algebra is said to be a Curtis algebra if its universal enveloping algebra is finitely generated as a module over its centre and the latter is a finitely generated polynomial ring over the field.

Theorem A: Curtis algebras are finite-dimensional. In prime characteristic they comprise all finite-dimensional Lie algebras and in characteristic zero they are the finite-dimensional abelian Lie algebras.

Theorem B: A Lie algebra is a Curtis algebra if and only if its abelian subalgebras are finite-dimensional and its universal envelope is finitely generated as a module over its centre. (Received October 25, 1978.)

763-17-5 ROBERT ROTH, Ohio State University, Columbus, Ohio 43210. The catalogue of Hall Triple Systems of small order.

A Hall Triple System is a Steiner Triple System in which every plane is affine. Such a design necessarily has  $3^n$  points (for  $n \geq 4$ ) and is said to have order  $n$ . Marshall Hall Jr. showed that his original H.T.S. was the unique H.T.S. of order 4. We give an independent proof of a result of L. Beneteau that there exists a unique H.T.S. of order 5 and we extend our methods to prove that there are exactly three non-isomorphic H.T.S.'s of order 6. Our method is to exploit the structure of the commutative exponent 3 Moufang loops, not groups, which correspond to H.T.S.'s. (Received October 25, 1978.)

## 18 ► Category Theory, Homological Algebra

\*763-18-1 MARGARET M. LASALLE, University of Southwestern Louisiana, Lafayette, Louisiana 70504. Construction of equivalent categories. Preliminary report.

A diffeomorphism on a manifold is a  $C^\infty$  bijective map such that it and its inverse is  $C^\infty$ . A case of a functor from the Category of Manifolds and Diffeomorphisms to the Category of Real Algebras and Algebraic Isomorphisms is shown. Further there exist  $\rho$ -equivariant diffeomorphisms sending invariant operators on  $M$  to invariant operators on  $N$ . (Received October 16, 1978.)

\*763-18-2 Paul Hill and Errin White\*, Auburn University, Auburn, Alabama 36830. Projective dimension of valuated vector spaces.

This paper is primarily concerned with characterizing for each positive integer  $n$  those valuated vector spaces having projective dimension  $n$ . Our characterization is based on the notion of separability. A subspace  $W$  of a space  $V$  with valuation  $|x|$  is  $\mathfrak{N}_n$ -separable in  $V$  if, for each  $x \in V$ , there exists a subset  $S$  of  $W$  having cardinality not exceeding  $\mathfrak{N}_n$  such that

$$\sup\{|x+w|: w \in W\} = \sup\{|x+s|: s \in S\}.$$

**Theorem:** Let  $V$  be a valuated vector space. Then  $\text{proj. dim.}(V) \leq n$  if and only if  $V$  is the union of a smooth ascending chain of  $\aleph_{n-1}$ -separable subspaces

$$0 = V_0 \subseteq V_1 \subseteq \dots \subseteq V_\alpha \subseteq \dots$$

such that  $\dim(V_{\alpha+1}/V_\alpha) \leq \aleph_n$ . Using the above characterization of  $n$ -projective spaces, we show the existence of valuated vector spaces having precisely projective dimension  $n$  for each positive integer  $n$ . We also prove that there exist valuated vector spaces having infinite projective dimension. (Received October 23, 1978.)

763-18-3 Paul Glenn, Union College, Schenectady, New York 12308. Cohomology groups and first-order theories. Preliminary report.

Recent work of Makkai, Reyes (SLN 611) establishes a reversible correspondence between certain kinds of useful first-order theories and "logical" categories, where the latter can be characterized by category-theoretic properties. "Logical" functors  $\underline{R} \rightarrow \underline{S}$  between such categories correspond bijectively to interpretations of the theory associated with  $\underline{R}$  in that of  $\underline{S}$ . Any logical category has a canonical extension to a Barr-exact category for which one may define cohomology groups (Notices AMS, 25/5/1978, 758-18-2). One thus has "cohomology of theories." Since a logical functor induces a homomorphism of cohomology groups, the existence of such comprises a necessary condition for there to be an interpretation of one theory in another. We will consider some examples of cohomologies of theories with the goal of relating comparisons of cohomologies to relative properties of the corresponding theories. (Received October 23, 1978.)

763-18-4 G. C. HEWITT, University of Montana, Missoula, MT 59812. On  $\aleph$ -Noetherian conditions.

There are many equivalent ways of saying a ring is right or left noetherian which have very natural generalizations to equivalent conditions for a ring to be right or left  $\aleph$ -noetherian. Included among these are such conditions as: the ascending chain condition, the maximal principle, that coproducts of injective modules are injective, that direct limits of injective modules are injective. Further, it is clear that there are equivalent generalizations to rings with several objects.

(Received October 24, 1978.)

## 20 ► Group Theory and Generalizations

763-20-1 DAVID I. ADU, Department of Mathematics, University of Lagos, Akoka-Yaba, Nigeria. Green's relations on the semigroup  $\mathbb{R}_X$ . Preliminary report.

In this paper we characterize Green's relations on the semigroup  $\mathbb{R}_X$  of all binary relations on a set  $X$ . To do this we introduce the notion of a right (left) skeleton of a binary relation which is a certain kind of subrelation. It turns out that any two right skeletons of the same relation are right equivalent where two relations  $\alpha$  and  $\beta$  are defined to be right equivalent if there is a bijection  $h$  from  $R(\alpha)$  (the range of  $\alpha$ ) onto  $R(\beta)$  such that  $\alpha^{-1}(y) = \beta^{-1}(h(y))$  for all  $y \in R(\alpha)$ . Green's relations are characterized using such concepts. For example, two relations are  $\mathcal{L}$ -equivalent if and only if their domains coincide and they contain right equivalent right skeletons. (Received September 27, 1978.)

\*763-20-2 LAUREL ROGERS, University of Colorado, Colorado Springs, Colorado 80907. Basic subgroups from a constructive viewpoint.

A number of classical theorems relating to basic subgroups are examined from a constructive point of view. Classically, every abelian  $p$ -group has a basic subgroup. However, there are groups for which a basic subgroup cannot be constructed; examples are

given. Necessary and sufficient conditions are given for a group (not necessarily discrete) to possess a countable basic subgroup, and a further equivalent condition for the existence of a basic subgroup in an Ulm group is found. If a group has a basic subgroup, it is shown that the Baer decomposition holds.

A classical result which does not hold constructively is that a bounded pure subgroup is a summand. However, if the group has a basic subgroup, and if the bounded pure subgroup can be extended to a basic subgroup, then the Baer decomposition can be used to guarantee a complementary summand. In particular, in such a group, every finite pure subgroup is a summand. (Received September 28, 1978.)

763-20-3 KENNETH D. MAGILL, JR., State University of New York at Buffalo, Buffalo, New York 14214. Automorphism groups of some semigroups are Schützenberger groups of others. Preliminary report.

Let  $S$  be a semigroup whose multiplication is denoted by juxtaposition. Each  $a \in S$  determines a new semigroup  $S_a$  where the product  $x*y$  of two elements  $x, y \in S_a$  is defined by  $x*y = xay$ .  $S$  is called the base semigroup and  $S_a$  is said to be a laminated semigroup of  $S$ . A few elementary facts about laminated semigroups in general are discussed and then laminated semigroups of full semigroups of continuous selfmaps are considered. It is shown that for certain spaces  $X$  and certain elements  $\alpha$  of  $S(X)$ ,  $\text{Aut } S(X)_\alpha$ , the automorphism group of the laminated semigroup  $S(X)_\alpha$  is isomorphic to  $\Gamma(\alpha)$  the Schützenberger group of the  $\mathbb{N}$ -class in  $S(X)$  which contains  $\alpha$ . This, together with some previously known results allows us to describe completely  $\text{Aut } S(\mathbb{R})_\alpha$  where  $\mathbb{R}$  denotes the reals and  $\alpha$  is a polynomial function of odd degree. It follows, among other things, that if  $\alpha$  is any polynomial of odd degree such that the roots of its derived polynomial are all real and distinct, then  $\text{Aut } S(\mathbb{R})_\alpha$  is isomorphic to the group of all homeomorphisms on  $\mathbb{R}$  which permute the points 1 and 2 if and only if there exist two linear polynomials  $L_1$  and  $L_2$  such that  $L_1 \circ \alpha \circ L_2$  is a Chebyshev polynomial of degree greater than one. (Received October 2, 1978.)

\*763-20-4 Karen Chase, University of Oklahoma, Norman, Oklahoma 73019. Maximal Groups in Semigroups of Binary Relations.

Let  $X$  be a finite set and  $R$  be an arbitrary but fixed binary relation on  $X$ . If  $A$  and  $B$  are relations on  $X$ , a product  $A*B$  is defined as follows:  $(a,b)$  is in  $A*B$  if there exist  $c$  and  $d$  with  $(a,c)$  in  $A$ ,  $(c,d)$  in  $R$  and  $(d,b)$  in  $B$ . Thus,  $A*B = A \circ R \circ B$  where  $\circ$  is the usual composition. This composition defines a sandwich semigroup of binary relations and is denoted  $B_X(R)$ . S.I. Montague and R.J. Plemmons have shown that given a finite group  $G$  there is some finite set  $X$  such that  $G$  is a maximal group in  $B_X$ , the usual semigroup of binary relations. The following theorem shows their result does not hold in  $B_X(R)$  for all choices of  $R$ .

THEOREM.

- (i) There is a class  $\Gamma$  of binary relations such that if  $R$  is in  $\Gamma$ , then all maximal groups in  $B_X(R)$  are trivial.
- (ii) There is a class  $\Lambda$  of binary relations such that if  $G$  is a finite group, then  $G$  is a maximal group in  $B_X(R)$  for some nonidentity relation  $R$  in  $\Lambda$  and some  $X$ .
- (iii) There is a class  $\Omega$  such that for any  $R$  in  $\Omega$  the only nontrivial groups in  $B_X(R)$  are either permutation groups or contain a nontrivial subgroup that is a permutation group. (Received October 10, 1978.) (Author introduced by Carlton J. Maxson).

\*763-20-5 David B. Surowski, Kansas State University, Manhattan, Kansas 66506. Permutation characters of finite groups of Lie type.

Let  $\underline{G}$  be a connected reductive linear algebraic group, and let  $G = \underline{G}_\sigma$  be the finite subgroup of fixed points, where  $\sigma$  is the generalized Frobenius endomorphism of  $G$ . Let  $x$  be a regular semi-simple element of  $G$ , and let  $w$  be a corresponding element of the Weyl group  $W$ . Let  $P$  be a parabolic subgroup of  $W$ . Let  $1_P^G, 1_P^W$  denote the induced permutation characters. Theorem. If  $G$  is untwisted we have  $1_P^G(x) = 1_P^W(w)$ . If  $G$  is twisted, then  $1_P^G(x)$  is given in terms of  $w, W_P$  and  $\sigma$ .

These results are obtainable via the Deligne-Lusztig theory (Ann. Math. 103 (1976)), but this seems like over-kill for the results above. Instead, the information concerning parabolic subgroups given by C. W. Curtis (J. Math. Soc. Japan 27 (1975)), together with the Schur-Zaussenhaus theorem suffice. (Received October 10, 1978.) (Author introduced by John E. Maxfield).

\*763-20-6 Deborah N. Gale, Montana State University, Bozeman, Montana 59717. Extensions of  $Z$  by  $Z(p^\infty)$ .

If  $G$  is an extension of  $Z$  by  $Z(p^\infty)$  then  $G$  is isomorphic to  $Q_{p^\infty}, Q_{p^\infty} \times Z(p^m)$  for some  $m \in Z^+$  or  $Z \times Z(p^\infty)$  where  $Z$  denotes the integers under addition,  $Z(p^\infty)$  is the quasi-cyclic group,  $Z(p^m)$  is the cyclic group of order  $p^m$  and  $Q_{p^\infty}$  is the set of all rationals whose denominator is a power of  $p$ . (Received October 13, 1978.)

\*763-20-7 Richard M. Davitt, University of Louisville, Louisville, Kentucky 40208. On the automorphism group of a finite  $p$ -group with a small central quotient.

Let  $G$  be a finite non-cyclic  $p$ -group of order greater than  $p^2$  such that the order of the central quotient is less than or equal to  $p^4$ . Then it is true that the order of  $G$  divides the order of the automorphism group of  $G$ . Reductions of the author and others are used to reduce the problem to the case where  $G$  is an irregular 2- or 3-group. It is established that in these remaining cases  $G$  is a metabelian group. Finally, specialized knowledge about the structure of groups of order  $3^4$  and  $2^4$  suffices to allow the construction of a  $p$ -subgroup of the automorphism group of  $G$  of large enough order to establish the result for the two exceptional primes. (Received October 12, 1978.)

\*763-20-8 LEO P. COMERFORD, JR., University of Wisconsin-Parkside, Kenosha, Wisconsin 53141 and CHARLES C. EDMUNDS, Mount Saint Vincent University, Halifax, Nova Scotia B3M 2J6, Canada. Quadratic equations over free products of groups.

An equation over a group  $G = \langle a_1, a_2, \dots; r_1, r_2, \dots \rangle$  is an expression  $W = 1$ , where  $W$  is a word on  $a_1, a_2, \dots$  (constants) and  $x_1, x_2, \dots$  (variables). An equation  $W = 1$  is called quadratic if each variable which occurs in  $W$  occurs exactly twice, with exponent  $+1$  or  $-1$ .

Theorem. Let  $G$  be the free product of the groups  $\{G_i: i \in I\}$ . There is an algorithm which determines whether or not quadratic equations over  $G$  have solutions if and only if there is a uniform algorithm for determining for all  $i \in I$  whether or not quadratic equations over  $G_i$  have solutions.

Corollary. If  $G$  is a free group, there is an algorithm which determines whether or not quadratic equations over  $G$  have solutions. (Received October 19, 1978.)

\*763-20-9 D.S. GAJENDRAGADKAR, University of Texas, El Paso, Texas 79968. A characteristic class of characters of finite  $\pi$ -separable groups.

Let  $G$  be a  $\pi$ -separable group. The set  $X_\pi(G)$  of all  $\chi$  in  $\text{Irr}(G)$  such that  $\chi(1)$  is a  $\pi$ -number and  $\chi$  is subnormally  $q$ -rational for every prime  $q$  not in  $\pi$  is considered. The interaction of  $X_\pi(G)$  with the normal structure of  $G$  is described. Theorem 1. The group  $G$  has a normal  $\pi$ -complement if and only if  $\sum_{\chi \in X_\pi(G)} \chi(1)^2 = |G|_\pi$ . Let  $Q(g_\pi)$  denote the cyclotomic field generated by a primitive  $g_\pi$ -th root of unity where  $g_\pi$  is the  $\pi$ -part of the exponent of  $G$ . Theorem 2. If  $G$  is  $\pi$ -solvable, then  $Q(g_\pi)$  is a splitting field for every  $\chi$  in  $X_\pi(G)$ . Theorem 3. Let  $\pi_1, \pi_2, \dots, \pi_n$  be pairwise disjoint sets. Suppose  $G$  is  $\pi_i$ -separable for each  $i$ . If  $\chi_i \in X_{\pi_i}(G)$ , then  $\prod_{i=1}^n \chi_i$  is irreducible. Furthermore if  $\chi_i' \in X_{\pi_i}(G)$  such that  $\prod_{i=1}^n \chi_i = \prod_{i=1}^n \chi_i'$ , then we have  $\chi_i = \chi_i'$  for every  $i$ . (Received October 19, 1978.)

\*763-20-10 C.C. Edwards, Indiana University-Purdue University at Fort Wayne, Fort Wayne, Ind. 46805. F-orthodox semigroups.

Let  $S$  be an orthodox semigroup and let  $E$  denote its band of idempotents. The relation  $\sigma$  defined on  $S$  by  $a \sigma b \iff eae = ebe$  for some  $e \in E$  is the minimum group congruence on  $S$  and the relation  $\leq$  defined on  $S$  by  $a \leq b \iff aS = bS$  and  $a = eb$  for some  $e \in E \cap R_a$  is a partial order on  $S$ .  $S$  is called F-orthodox if each  $\sigma$ -class of  $S$  contains a greatest element. Now let  $S$  be F-orthodox and let  $T$  denote its set of  $\sigma$ -class greatest elements.  $T$  under the binary operation  $*$ , where for  $u, v \in T$ ,  $u * v$  denotes the greatest element in  $(uv)\sigma$ , is the maximal group homomorphic image of  $S$ . It has been found that the structure of  $S$  is determined by  $E$ ,  $T$  and a set of endomorphisms of  $E$  satisfying certain conditions. This result extends, in an analogous manner, the McFadden-O'Carroll [F-inverse semigroups, Proc. London Math. Soc. (3) 22 (1971), 652-666] description of the F-inverse semigroup. Moreover, the property that  $a \wedge b$  exists  $\iff a \sigma b$  holds for elements of  $S \iff e \wedge f$  exists for each  $e, f \in E$ . In addition, if  $e \wedge f$  exists for each  $e, f \in E$ , then for  $a, b, x \in S$  such that  $a \sigma b$ ,  $x(a \wedge b) = xa \wedge xb$  [ $(a \wedge b) = ax \wedge bx$ ]  $\iff$  for each  $e, f, g \in E$ ,  $g(e \wedge f) = ge \wedge gf$  [ $(e \wedge f)g = eg \wedge fg$ ]. (Received October 19, 1978.)

\*763-20-11 N. J. A. SLOANE, Bell Laboratories, 600 Mountain Avenue, Murray Hill, N.J. 07974. Invariants of Finite Groups and Applications

This talk will describe some recent results on the polynomial invariants of finite groups, and applications to the construction of spherical designs - i.e., nice arrangements of points on the surface of a sphere in  $n$ -dimensional Euclidean space. (Received October 19, 1978.)

\*763-20-12 Larry J. Morley, Western Illinois U., Macomb, Illinois 61455 and Manley Perkel, Wright State U., Dayton, Ohio 45435. The nilpotency class of extensions of certain p-groups.

The nilpotency class of a semi-direct product  $AB$  of a  $p$ -group  $A$  of bounded exponent and a finite  $p$ -group  $B$  has been investigated by G.Baumslag, H.Liebeck, L.J.Morley, J.D.Meldrum and D.Shield, among others. In this paper, the least upper bound for the class is obtained in the case where  $B$  satisfies what we call the order representative property, i.e. when each generating coset of the factor groups determined by the lower central series of  $B$  contains a coset representative of the same order (in  $B$ ) as the order of the coset which it represents (e.g. groups of exponent  $p$ ; Sylow  $p$ -subgroups of  $GL(n, p^a)$ ). Suppose  $B$  is a finite  $p$ -group with this property and  $B = B_1 > \dots > B_L > 1$  the lower central series of  $B$ . Let  $r(i)$  be the rank of  $B_i/B_{i+1}$  and let  $p(i, j)$ ,  $1 \leq j \leq r(i)$ , be the descending prime power orders of the cyclic decomposition of  $B_i/B_{i+1}$ . Theorem: If  $A$  is an abelian group of exponent  $p^{n+1}$  and  $W=AB$ , then  $cl(W) \leq \sum_{i=1}^L \sum_{j=1}^{r(i)} i(p(i, j)-1) + nmp(m, 1)(p-1)p^{-1} + 1$ , where  $mp(m, 1) = \max_{1 \leq i \leq L} ip(i, 1)$ . Equality holds if  $W=AwrB$  (the standing wreath product). Corollary: If  $A$  is an abelian group of exponent  $p^{n+1}$ , and  $B$  is a finite group of exponent  $p$ , then  $cl(AB) \leq (p-1) \sum_{i=1}^L ir(i) + nL(p-1) + 1$ . Again equality holds for  $AwrB$ . (Received October 19, 1978.)

763-20-13 NALSEY B. TINBERG, Southern Illinois University, Carbondale, Illinois 62901. Some indecomposable modules of groups with split (B,N)-pairs. Preliminary report.

Let  $G = (G, B, N, R, U)$  be a finite group with a split  $(B, N)$ -pair of characteristic  $p$  ( $p$  prime) and  $k$  an algebraically closed field of the same characteristic. Fix  $J \subseteq R$ . For the parabolic subgroup  $G_J$  write  $Y_J \cong (k_U)^{G_J}$ ,  $E_J = \text{End}_{kG_J}(Y_J)$  where  $k_U$  is the trivial  $U$ -module  $k$ . We study the indecomposable components of  $Y_R$ . Our recent work on "unsaturated" split  $(B, N)$ -pairs generalizes the Curtis-Richen theory and is immediately applicable to  $G_J$ . We write  $Y_J = \Sigma^{\oplus} Y_J(S, \chi)$ , a decomposition into indecomposable  $kG_J$ -submodules. These components have simple head and socle and are in bijective correspondence with the set of simple  $E_J$ -modules  $\{\phi_J(S, \chi)\}$  (a fact first proved by Sawada in the split case). We find that  $Y_J(S, \chi)^G \cong \sum_{S=K \cap J}^{\oplus} Y_R(K, \chi)$  by examining the simple  $E_R$  and  $E_J$  modules. By considering the case  $S = J$  we deduce the dimension of each  $Y_R(K, \chi)$  using work by Solomon on Weyl groups. (Received October 20, 1978.)

\*763-20-14 WALTER S. SIZER, Southern Illinois University, Carbondale, Illinois 62901. Group Retractions Induced by a Total Order.

Let  $G$  be a group, and  $F(G)$  the semigroup of all finite subsets of  $G$ . A retraction is a semigroup homomorphism  $\sigma: F(G) \rightarrow G$  satisfying  $\sigma(\{g\}) = g$  for all  $g \in G$ . We have proved the

Theorem: Let  $G$  be a group. Let  $\sigma: F(G) \rightarrow G$  be a retraction satisfying  $\sigma(A) \in A$  for all  $A \in F(G)$ . Then  $\sigma$  is induced by a total order on  $G$ , with  $\sigma(A) = \max(A)$  for all  $A \in F(G)$ .

This answers a question raised by R. Byrd in 1976. (Received October 20, 1978.)

\*763-20-15 Barbara O'Brien, Texas Tech University, Lubbock, Texas 79409. On abelian groups which are pure-injective, as modules over their endomorphism rings.

We characterize those groups  $G$  which are pure-injective  $\text{End}_{\mathbb{Z}}(G, G)$ -modules, for certain subclasses of the class of abelian groups. (Received October 19, 1978.) (Author introduced by Dr. R. Bernard).

763-20-16 JOHN-TIEN HSIEH, University of Florida, Gainesville, Florida 32611. Fusion of p-elements in symmetric groups. Preliminary report.

In a finite group  $G$  whose order is divisible by a prime  $p$  the normalizers of  $p$ -subgroups are called local subgroups of  $G$ . The significance of local subgroups in the discussion concerning the fusion of  $p$ -elements, hence the relations between local and global properties of  $G$ , has been recognized in recent years. It is proved, as the main theorem of this paper, that in the Sylow  $p$ -subgroups of the symmetric groups of  $p$ -power degree, the normalizers of  $p$ -subgroups of only two different structures are needed to form a local fusion family. In fact, they are the normalizers of a big elementary abelian group and some small elementary abelian groups of order  $p^2$ .

(Received October 23, 1978.)

763-20-17 G. P. NIEBZWECKI and J. D. REID, Wesleyan University, Middletown, Connecticut 06457. Abelian groups projective over their endomorphism rings. Preliminary Report.

An  $E$ -ring is a ring that is isomorphic to the endomorphism ring of its underlying additive group via the regular representation (cf. P. Schultz, J. Austral. Math. Soc. 15 (1973), 60-69). We characterize the abelian groups that are finitely generated projective over their endomorphism rings in terms of  $E$ -rings as follows. Theorem 1: An abelian group  $G$  is cyclic projective over its endomorphism ring  $R$  iff the center,  $C$ , of  $R$  is an  $E$ -ring and  $C$  is isomorphic to a  $C$ -summand of  $G$ .

Theorem 2: For an  $E$ -ring  $C$  with identity  $e$  and for any  $C$ -module  $M$ ,  $C \otimes M$  is cyclic projective over its endomorphism ring iff  $\text{Hom}_{\mathbb{Z}}(C/\mathbb{Z}e, M) = 0$ . Theorem 3: An abelian group  $G$  is finitely generated projective over its endomorphism ring iff there exists an integer  $n$  such that  $G^n$  is cyclic projective over its endomorphism ring. Moreover all finitely generated projective groups arise in this way from cyclic projectives via an appropriate Morita equivalence. (Received

October 23, 1978.)

763-20-18 ANTHONY M. GAGLIONE, U. S. Naval Academy, Annapolis, Maryland 21402. A Generalization of a Theorem of P. Hall on Nilpotent Products. Preliminary report.

Here we state a generalization of a theorem of P. Hall. For all definitions see H. Waldinger and A. Gaglione, Can. J. Math., 1975, pp. 1185-1210.

The groups,  $G$ , we consider have the form  $G = \langle c_1, c_2, \dots, c_r; c_1^{\alpha_1}, c_2^{\alpha_2}, \dots, c_r^{\alpha_r} \rangle$  where the  $r$  exponents  $\alpha_i \geq 0$  so that at least one does not vanish.

Theorem: Let  $d_{\kappa_1} < a d_{\kappa_2} < a \dots < a d_{\kappa_{q_r(m)}}$  be the relation commutators of

pseudo-dimension  $\leq m$ . Let  $\omega = q_r(m)$  and  $\psi = q(m)$ . Consider the relation  $m$ -word

$\Pi = \prod_{i=1}^{\omega} d_{\kappa_i}^{\eta_i} \in G/G_{n+1}$  ( $m$  th nilpotent product).

(1) Then the basic commutator representation of  $\Pi$ ,  $\Pi = \prod_{i=1}^{\omega} c_i^{\varepsilon_i} \text{ mod } F_{m+1}$ , is such that each exponent  $\varepsilon_i = \sum_t \beta_{ti} \binom{\eta_1}{t} \binom{\eta_2}{2ti} \dots \binom{\eta_{\omega_i}}{s_{\omega_i} ti}$  where  $\beta_{ti}$  are integers and  $\binom{\eta_j}{jti}$  are binomial coefficients.

(2) Moreover,  $\varepsilon_i$  is independent of  $\eta_j$  if there exists a generator  $c_h (1 \leq h \leq r)$  such that  $c_h$  occurs in  $\langle d_{\kappa_j} \rangle$  but does not occur in  $\langle c_i \rangle$ . (Clearly,  $\varepsilon_i$  is a polynomial in  $\eta_1, \eta_2, \dots, \eta_r$ .  $\varepsilon_i$  is independent of  $\eta_j$  means  $\partial \varepsilon_i / \partial \eta_j = 0$ .) (Received October 23, 1978.)

\*763-20-19 ROBERT O. STANTON, St. John's University, Jamaica, New York 11439. S-groups.

The class of Abelian groups called  $S$ -groups, which has been defined by Warfield, is shown to be closed under direct summands. Groups of the form  $S \oplus W$ , where  $S$  is an  $S$ -group and  $W$  is a Warfield group (i.e. a direct summand of a simply presented group), are shown to be classified by cardinal invariants. (Received October 23, 1978.)

\*763-20-20 J. HIGGINS, Brigham Young University, Provo, Utah Topologies Induced in Free Semigroups. Preliminary Report.

One of the general solutions to the problem of embedding a semigroup  $S$  in a group  $G$  begins with the construction of the free semigroup  $E$  on any set  $M$  of generators of  $S$ .  $E$  is then extended in the natural way to the free group  $F$  and it can be shown that  $S$  is embeddable iff  $S$  embeds in  $F/\delta$  where  $\delta$  is the congruence of  $F$  generated by the relation  $\alpha^{-1} \beta \beta^{-1} \alpha$ , where  $\alpha, \beta$  are the natural homomorphisms of  $E$  onto  $S$  and into  $F$  respectively.

To obtain analogous results for topological semigroups it is necessary to find in  $E$  and  $F$  topologies naturally induced by that of  $S$ . This presentation considers a number of candidates (e. g.  $Q$  is open in  $E$  iff  $\beta(Q)$  is open in  $S$ ) and conditions on  $S$  such that  $E$  and  $F$  with the induced topologies are respectively a topological semigroup and topological group. (Received October 24, 1978.)

\*763-20-21 MARK R. HOPKINS, Drake University, Des Moines, Iowa 50311. On groups of Ree type.

Let  $G$  be a finite simple group in which the centralizer of every involution is isomorphic to that in  $G_2(q)$ , the group discovered by Ree [Amer. J. of Math, 83 (1961), 432-462]. We say  $G$  is of type  $R(q)$ . Thompson has shown [J. Algebra, 49 (1977), 162-166] that knowledge of two parameters determines the multiplication in  $G$ . We reduce the requirement to one parameter  $\mathfrak{F}$ . There is computer evidence that  $\mathfrak{F}$  can be determined by an identity of Thompson, although the calculations may not be feasible.

We catalogue the subgroups of  $G$  and apply induction to obtain this theorem.

THEOREM. Suppose  $G_2(q)$  is the only group of type  $R(q)$  whenever  $q = 3^n$  and  $n$  is a prime power. Then  $G_2(q)$  is the only group of type  $R(q)$  for all  $q$ . (Received October 24, 1978.)

\*763-20-22 KENNETH W. WESTON, University of Wisconsin-Parkside, Kenosha, Wisconsin 53140. On the elementary theory of a certain class of groups. Preliminary report.

Suppose  $K$  designates the class of groups defined by the following:  $G \in K$  if (1)  $G = N_1 \cdot N_2$ ; (2)  $N_i$  abelian ( $i = 1, 2$ ); (3)  $N_i$  is a direct product of  $N_1 \cap N_2$  and a subgroup  $A_G(i) \cong N_1 \cap N_2$  ( $i = 1, 2$ ). Designate by  $\alpha_G(i)$  ( $i = 1, 2$ ), the isomorphism between  $A_G(i)$  and  $N_1 \cap N_2$ . We define a statement over  $G \in K$  as quasi-elementary if it is couched in the language of first-order predicate calculus and uses only the operations of group multiplication and  $\alpha_G(i)$ . There is a syntactical inclusion of the elementary theory of nonassociative rings into the quasi-elementary theory of the groups of  $K$ . The author uses this to show that the equation problem for systems of commutator equations over a free nilpotent group of class 2 on 2 generators is unsolvable. The author also discusses the syntactical inclusion of the elementary theory of certain groups into the quasi-elementary theory of groups of  $K$ . (Received October 25, 1978.)



763-20-23 TEMPLE H. FAY, New Mexico State University, Las Cruces, New Mexico 88003. Torsion and Tensor Products for Valuated Groups.

The forgetful functor  $U$  from the category of valuated abelian groups to the category of abelian groups is a topological functor. Because of this, theorems usually used in categorical topology for lifting adjoints (both left and right) along  $U$  are applicable. In this manner a free valuated group over a valuated set functor is obtained which lifts the usual free abelian group over a set functor. A valuated tensor product functor which lifts the usual tensor product is defined and a valuated hom functor is lifted from the usual adjunction; that is, valuated groups form a symmetric closed monoidal category. The torsion product functor is described in a natural way using this setting. (Received October 25, 1978.)

763-20-24 BHAMA SRINIVASAN, Department of Mathematics, Clark University, Worcester, Massachusetts 01610. Representations of classical groups.

• At a meeting to commemorate the Fiftieth Anniversary of the Mathematical Association of America in 1965, Charles W. Curtis gave an invited address entitled "The classical groups as a source of algebraic problems" [American Mathematical Monthly 74(1967), 80-91]. We discuss the progress made since then in the solution to these problems, with special reference to the representation theory of finite classical groups. The interaction of this field with other areas in mathematics, for example, combinatorics, algebraic geometry, and number theory will also be discussed. (Received October 25, 1978.)

763-20-25 SIDNEY C. GARRISON and STEPHEN M. GAGOLA, JR., Texas A&M University, College Station, Texas 77843. Real characters and the Schur multiplier. Preliminary report.

The following result gives a character theoretic condition for the Schur multiplier of a finite group to have even order:

Theorem: Suppose  $x$  and  $y$  are two commuting elements of even order in a finite group  $G$ , with  $A = \langle x, y \rangle$  isomorphic to  $\langle x \rangle \times \langle y \rangle$ . Let  $\lambda_1, \lambda_2$  and  $\lambda_3$  denote the three distinct non-principal linear characters of  $A$  satisfying  $\lambda_1^2 = 1$ . Finally, let  $\chi$  be any faithful character of  $G$  afforded over the reals. If at least two of the inner products  $(\chi_A, \lambda_i)_A$  are odd, then  $H^2(G, \mathbb{C}^x)$  has even order (and in particular,  $G$  has a double cover.)

It should be noted that the character  $\chi$  is not assumed to be irreducible. (Received October 25, 1978.) (Author introduced by Kirby Smith).

\*763-20-26 STEPHEN C. KING, California State University, Los Angeles, California 90032. Small subgroups.

Following P. L. Sperry (Proc. Amer. Math. Soc. 24(1970), 148-153), we say that a subgroup  $H$  of a group  $G$  is small in  $G$  provided that whenever  $S \subseteq G$  and  $G = \langle S, H \rangle$ , then also  $G = \langle S \rangle$ . Every small subgroup of  $G$  is contained in  $\text{Fr}(G)$  (the Frattini subgroup of  $G$ ), and every subgroup of  $\text{Fr}(G)$  is small in  $G$  precisely in case  $\text{Fr}(G)$  is itself small in  $G$ ; but the Frattini subgroup of a nonidentity divisible abelian group is not small. For any set of words  $W$  and any group  $G$ , denote the associated verbal and marginal subgroups of  $G$  by  $W(G)$  and  $W^*(G)$ , and define the upper marginal series  $\{W_n^*(G) : n < \omega\}$  of  $G$  with respect to  $W$  in a manner analogous to the way the upper central series is defined. Theorem. For any group  $G$ , any set  $W$  of words, and any natural number  $n$ ,  $W_n^*(G) \cap W(G)$  is small in  $G$ . In particular, if  $W = \{[x, y]\}$ , we obtain that  $Z_n(G) \cap G'$  is small in  $G$  (where  $Z_n(G)$  is the  $n$ th term of the upper central series of  $G$ , and  $G'$  is the commutator subgroup of  $G$ ); this simultaneously generalizes the known facts that for any group  $G$ ,  $Z(G) \cap G' \cong \text{Fr}(G)$ ; and that the commutator subgroup of a nilpotent group is small. (Received October 25, 1978.)

\*763-20-27 ROGER H. HUNTER and FRED RICHMAN, New Mexico State University, Las Cruces, New Mexico 88003. Global Warfield Groups.

The theory of global Warfield groups is developed. The main theorems are the isomorphism theorem and the existence theorem, generalizations of Ulm's theorem and of Zippin's theorems, respectively. Additional results of interest are: (1) Each global Warfield group is the direct sum of a group of countable torsion-free rank and a simply presented group. (2) A direct sum of Warfield groups with torsion-free rank one can be expressed as the direct sum of a group with torsion-free rank one and a simply presented group. (Received October 25, 1978.)

## 22 ► Topological Groups, Lie Groups

763-22-1 GEORGE GRAHAM, University of Houston, Houston, Texas 77004. A Lie Algebra For Certain Local Differentiable Monoids.

$A$  is a regular set in  $R^n$  if  $A \subseteq R^n$  and  $A \subseteq \overline{A^0}$ . Differentiability of functions defined on regular sets is accomplished by an alteration of the definition of the Frechet derivative. Let  $S$  be a local monoid in  $R^n$  so that  $S$  is regular and the operation is differentiable in the sense alluded to above. Then  $S$  has cancellation near its identity. The local left-invariant vector fields of  $S$ , restricted to a suitable open set, form a Lie algebra of dimension  $n$ , under ordinary Lie bracket of vector fields. (Received October 16, 1978.)

\*763-22-2 R. K. OLIVER, 301 Roup Avenue, Pittsburgh, Pennsylvania 15232. On Bieberbach's analysis of discrete euclidean groups.

Let  $G$  be a subgroup of the group  $E_n$  of euclidean isometries of  $R^n$ . Let  $G^*$  denote the translation subgroup of  $G$ , let  $G_r$  denote the group generated by all  $(A, a)$  in  $G$  with  $\|1 - A\| < r$  ( $r > 0$ ,  $\|\cdot\|$  : operator norm on  $\text{End}(R^n)$ ,  $E_n$  identified with the semidirect product  $O_n \cdot R^n$ ), and let  $k_n(r)$  denote the maximum number of  $n \times n$  orthogonal matrices with mutual distances  $\geq r$  relative to the metric  $d(A, B) = \|A - B\|$ . We give an elementary, largely geometrical proof of the following results of Bieberbach: (1) If  $G$  is discrete, then  $G_{1/2}$  is abelian,  $G^* \triangleleft G_{1/2} \triangleleft G$ , and  $[G:G_{1/2}] \leq k_n(1/2)$ . (2)  $G$  is discrete if and only if  $G \subseteq O_{n-k} \times E_k$ , where  $G_k$  is discrete,  $G_k^*$  spans  $R^k$ , and the group  $\{u \in G_{n-k} : (u, 1) \in G\}$  is finite ( $G_{n-k}$  and  $G_k$  being respectively the  $O_{n-k}$  and  $E_k$  components of  $G$ ). (3) If  $G$  is crystallographic, then  $G^*$  spans  $R^n$  and  $[G:G^*] \leq k_n(1/2)$ ; if  $G$  is discrete and  $G^*$  spans  $R^n$ , then  $G$  is crystallographic. (Received October 19, 1978.)

\*763-22-3 AROLDO KAPLAN, University of Massachusetts, Amherst, Mass. 01003. A class of nilpotent Lie groups with analytically hypoelliptic sublaplacians.

We construct analytic fundamental solutions for the sublaplacians of a certain class of 2-step nilpotent Lie groups. Examples of these can be generated from the classical real division rings; such construction shows that the class of groups in question properly contains the known cases of analytic hypoellipticity for sublaplacians. (Received October 20, 1978.)

763-22-4 T. Christine Stevens, Mount Holyoke College, South Hadley, Massachusetts 01075. Weakening the topology of a Lie group. Preliminary report.

With any topological group  $(G, \mathcal{U})$  one can associate a locally arcwise-connected topological group  $(G, \mathcal{U}^*)$ , where  $\mathcal{U}^*$  is stronger than  $\mathcal{U}$ .  $(G, \mathcal{U})$  is a weakened Lie (WL) group if  $(G, \mathcal{U}^*)$  is a Lie group; all finite-dimensional metric groups are WL groups. We show that the WL groups with which a given connected Lie group  $(L, \mathcal{J})$  is associated are completely determined by a certain abelian subgroup  $H$  of  $L$  which is called decisive. If  $L$  has closed adjoint image, then  $H$  is the center  $Z(L)$  of  $L$ . Otherwise,  $Z(L)$  is a proper subgroup of  $H$  and  $H$  is, in the topology inherited from  $\mathcal{J}$ , the product of a vector group  $V$  and a group  $J$  that contains  $Z(L)$ .  $J/Z(L)$  is finite. (Received October 23, 1978.)

763-22-5 ERIC C. NUMMELA, St. Cloud State University, St. Cloud, MN 56301. Topological group rings. Preliminary report.

Let  $G$  be a group with identity  $e$ , let  $ZG$  be the (integral) group ring over  $G$ , and let  $IG$  be the augmentation ideal of  $ZG$ . Then  $G$  is injected into  $ZG$  by  $i(x) = x$  and into  $IG$  by  $j(x) = x - e$ . If  $G$  is a Hausdorff topological group, then it is well known that  $t_1$ , the finest group topology on  $ZG$  such that  $i$  is continuous, is Hausdorff. Let  $t_2$  be the finest such  $G$ -module topology on  $ZG$ . (Note that  $t_2$  is coarser than  $t_1$ .) Theorem 1. If  $G$  is compact, then  $t_1 = t_2$ ; in particular,  $t_2$  is Hausdorff. Theorem 2. Let  $t_3$  be the finest  $G$ -module topology on  $IG$  such that  $j$  is continuous. Then  $t_3$  is Hausdorff. Theorem 3. The topology induced on  $IG$  by  $t_2$  is  $t_3$ . Theorem 4.  $t_2$  is Hausdorff. Question. Is  $t_4$ , the finest ring topology on  $ZG$  such that  $i$  is continuous, Hausdorff? (Received October 23, 1978.)

A topological group is said to be locally invariant if there is a neighborhood base at the identity which is invariant under conjugation by elements of the group.

Lemma: If  $X$  is a non-discrete Tychonoff space then there is a continuous pseudo-metric  $\rho$  on  $X$  such that the metric space obtained by identifying points whose  $\rho$ -distance from each other is zero contains a convergent sequence. From this we obtain Theorem 1: the free topological group over a Tychonoff space is locally invariant if and only if it is discrete. Thus, the topology for the free group constructed by M. I. Graev in 1948 is almost never the free topology.

A related result is Theorem 2: the free topological group over a Tychonoff space is first countable (= metrizable) if and only if it is discrete. (Received October 25, 1978.)

## 26 ► Real Functions

\*763-26-1 MARK MANDELKERN, New Mexico State University, Las Cruces, New Mexico 88003.  
Located sets.

One often wishes to measure the distance  $\rho(x, F)$  between a point  $x$  and a set  $F$  on the real line. However, this is not always possible constructively. Because of this, Brouwer, in 1919, introduced the concept of located set, for which these distances always exist. Closed located sets are an unbounded generalization of compact sets; a set is compact if and only if it is closed, located, and bounded. Located sets are used extensively in constructive analysis.

The following gives a complete characterization of located sets on the line and provides an explicit procedure for their construction.

THEOREM. If  $F$  is a closed located set on the line, then there exists a sequence of disjoint open intervals  $I_n = (a_n, b_n)$  such that

- (1) there exists a sequence  $\{M_k\}$  of locating parameters such that  $n \leq M_k$  whenever  $I_n$  meets  $(-k, k)$  and  $\ell(I_n) > 1/k$ ,
- (2)  $F = \bigcap_n ((-\infty, a_n] \cup [b_n, +\infty))$ .

Conversely, whenever a sequence of disjoint open intervals satisfies (1), then the set  $F$  defined by (2) is closed and located. (Received October 5, 1978.) (Author introduced by Professor Ray Mines).

\*763-26-2 Charles H. Heiberg, U.S. Naval Academy, Annapolis, Maryland 21402.  
Extrema for functions of several variables.

The nature of a critical point  $a$  of a function  $f$  of two variables may be determined if the polynomial  $F$  consisting of the second order terms of the Taylor expansion  $T(f, a)$  of  $f(x) - f(a)$  about  $a$  is definite or indefinite. Since all non-zero terms  $b(x_1 - a_1)^{q_1} (x_2 - a_2)^{q_2}$  of  $T(f, a)$  satisfy  $q_1/2 + q_2/2 \geq 1$ , whereas all terms of  $F$  satisfy  $q_1/2 + q_2/2 = 1$ , the polynomial  $F$  is one possible outcome if one were to select a pair of positive numbers  $p_1, p_2$  such that  $q_1/p_1 + q_2/p_2 \geq 1$  for all non-zero terms of  $T(f, a)$  and then form the polynomial equal to the sum of those terms of  $T(f, a)$  for which  $q_1/p_1 + q_2/p_2 = 1$ . Call any such polynomial a critical polynomial of  $f$ . The main result of this paper implies that if any critical polynomial is positive (respectively negative) definite then  $f$  has a strict relative minimum (respectively maximum) at  $a$ . Moreover if  $f$  has a relative minimum (respectively maximum), then each critical polynomial of  $f$  is positive (respectively negative) semi-definite. (Received October 10, 1978.)

\*763-26-3 DAN MAULDIN, North Texas State University, Denton, Texas 76203. The set of continuous nowhere differentiable functions. Preliminary report.

Let  $C$  be the space of all continuous real-valued functions defined on  $[0,1]$  provided with the uniform norm. Let  $M = \{f \in C: f \text{ does not have a finite derivative anywhere}\}$ . It is shown that  $M$  forms a coanalytic non-Borel subset of  $C$ . (Received October 10, 1978.)

\*763-26-4 NICHOLAS PASSELL, New College, Sarasota, Florida 33580. Generalized Cantor Functions. Preliminary report.

A real function is a Generalized Cantor Function if it is monotone, continuous and has derivative zero almost everywhere. Theorem.  $f: [a,b] \rightarrow \mathbb{R}$  is a Generalized Cantor Function iff  $f$  is continuous and the arclength of the graph of  $f$  is  $|f(b)-f(a)| + |b-a|$ . This is established by a direct argument using the Vitali Covering Theorem, and an attractive construction for G.C.F.'s is given. (Received October 23, 1978.)

763-26-5 M. J. Frank, Illinois Institute of Technology, Chicago, Illinois 60616. Results on iteration of piecewise-monotonic functions. Preliminary report.

Let  $f$  be a continuous and piecewise-monotonic function mapping a closed real interval onto itself. Denote by  $\langle f \rangle$  the number of monotonic pieces of the graph of  $f$ . It is easy to show that the graph of the  $n$ -th iterate of  $f$  can consist of at most  $\langle f \rangle^n$  pieces; moreover, there are many functions for which this upper bound is achieved for all  $n$ .

The corresponding best-possible lower bound for  $\langle f^n \rangle$  is given by: (A)  $\langle f^n \rangle \geq \langle f \rangle$ ; but if the graph of  $f$  has no horizontal segments, this can be strengthened to: (B)  $\langle f^n \rangle \geq n \langle f \rangle - n + 1$ . Improvements can be obtained by imposing restrictions on the orbit structure of  $f$ . For instance, if  $f$  contains a 3-cycle (and thus cycles of all orders) and  $\langle f \rangle > 2$ , the above become:

$$(A') \quad \langle f^n \rangle \geq \langle f \rangle + F_{n+3} - 3, \quad (B') \quad \langle f^n \rangle \geq n \langle f \rangle + F_{n+3} - 2n - 1,$$

where  $F_k$  is the  $k$ -th Fibonacci number.

These and other similar inequalities, as well as examples of functions which achieve the various bounds, will be presented. (Received October 24, 1978.)

763-26-6 BENNETT SAWEY, Morningside College, Sioux City, Iowa 51106. Pseudo-Metric Spaces From the Power Set of A Metric Space. Preliminary Report.

In a metric space  $(M,d)$ , the metric difference  $D(A,B)$  between two subsets of  $M$  is the least upper bound of  $\{d(a,b) \mid a \in A\} \cup \{d(b,a) \mid b \in B\}$ . We write  $A \approx B$  iff this value is real. This relation is an equivalence relation on the power set of  $M$ , and each class in the partition induced is a pseudo-metric space with respect to metric difference. In the special case where  $M$  is a normed linear space, it follows that for any  $A \subset M$  the set of scalars  $\{k \mid kA \approx A\}$  is an Abelian group with respect to multiplication in the scalar field when  $k=0$  is excluded. If  $A \approx B$  then  $A$  and  $B$  are associated with the same group. A function  $f: M \rightarrow M'$  is continuous around a subset  $A$  of  $M$  iff  $\forall \epsilon > 0 \exists \delta > 0 \exists D(A,B) < \delta \Rightarrow D(f(A), f(B)) < \epsilon$ . From this definition one can prove that  $f$  is uniformly continuous iff  $f$  is continuous around every discrete subset of  $M$ . (Received October 25, 1978.)

## 28 ► Measure and Integration

763-28-1 SEKI A. CHOO, Manchester College, N. Manchester, Indiana 46962. Separability of a space of continuous vector-valued functions.

Let  $X$  be a completely regular Hausdorff space and  $E$  be a locally convex Hausdorff space. We define the strict topology  $\beta_0$  on  $C_p(X;E)$  to be the topology generated

by the semi-norm  $\| \cdot \|_{h,q}$ ,  $f \in C_b(X;E)$ ,  $\| f \|_{h,q} = \sup_{x \in X} q(h(x)f(x))$ , where  $q$  ranges over all continuous semi-norms on  $E$  and  $h$  varies through all real-valued functions on  $X$  vanishing at infinity. A topological space  $S$  is called submetrizable if it can be mapped by a one-to-one continuous function onto some metric space  $Z$ . If  $Z$  is separable, then we say that  $S$  is separably submetrizable. Theorem. For a separable space  $E$ ,  $(C_b(X;E), \beta_0)$  is separable if and only if  $X$  is separably submetrizable. Corollary. Let  $X$  be locally compact paracompact, and if  $(C_b(X;E), \beta_0)$  is separable, then  $X$  is metrizable. Some other topological properties of  $(C_b(X;E), \beta_0)$  are discussed. (Received October 12, 1978.)

\*763-28-2 Gerald W. Johnson and David L. Skoug, University of Nebraska-Lincoln, Lincoln, Nebraska 68588. Scale-Invariant Measurability in Wiener Space.

The fact that change of scale is a pathological transformation in Wiener space has long been known. For many problems, this pathology causes no special difficulties. However, it is sometimes necessary to consider functions of the form  $F(\lambda x)$  where  $\lambda$  varies over the positive reals and  $x$  varies over Wiener space. In this setting a variety of conceptual subtleties arise. We give a framework and several results which prove useful in dealing with these difficulties. Also we discuss several papers in the recent literature in the light of this framework. (Received October 23, 1978.)

763-28-3 CECILIA H. BROOK, Goucher College, Baltimore, Maryland 21204. Vector measures and projections in the universal measure space.

Given an algebra of sets, there is a universal measure on the algebra whose values lie in a complete locally convex space, the universal measure space (Graves, A.M.S. Memoir 195). With Arens' multiplication and a natural norm and involution, the universal measure space is a  $C^*$ -algebra. To each strongly countably additive vector measure (taking values in a complete locally convex space) is associated a projection (self-adjoint idempotent) in the universal measure space. Continuity and orthogonality of vector measures are described in terms of the projections. If the algebra is a  $\sigma$ -algebra, then continuity of measures is related to order-continuity and to the countable chain condition. (Received October 25, 1978.)

763-28-4 Karl O. Rehmer, Univ. of Missouri-Rolla, Rolla, Mo. 65401. A completion of a space of molecular measures. Preliminary report.

Let  $X$  be a  $T_2$ -uniform space and let  $(E, F)$  be paired real linear spaces where  $E$  is an LF-space with defining sequence  $\{E_n\}$ . Let  $U^c = \{f: X \rightarrow E \mid f \text{ is unif. conts. and } f[X] \text{ is precompact}\}$  and let  $U_n^c = \{f \in U^c \mid f[X] \subset E_n\}$ . The uniform convergence topology  $\mathcal{T}_n^c$  is placed on each  $U_n^c$  and  $U^c$  is given the inductive limit topology  $\mathcal{T}$  determined by the sequence  $\{(U_n^c, \mathcal{T}_n^c)\}$ . Let  $L$ , the space of molecular measures from  $X$  into  $F$ , be given the topology  $u^c$  of uniform convergence on the members of the family  $E^c$  of equicontinuous subsets of  $U^c$  whose members share a common precompact range. The completion of  $(L, u^c)$  is the space of all  $\mathcal{T}$ -continuous linear functionals  $m$  on  $U^c$  for which  $m_t$  is a uniform measure for all  $t \in E$ . (The functional  $m_t$  is defined by  $m_t(\phi) = m(\phi t)$  for all uniformly continuous  $\phi: X \rightarrow \mathbf{R}$ .) (Received October 25, 1978.)

763-28-5 Michael Keisler, Arkansas Tech University, Russellville, Arkansas 72801. Conditional extensions of quasi-measures. Preliminary report.

Z. Lipecki (Bull. Acad. Polon. Sci. Ser. Sci. Math. 22(1974), 19-27) applied a technique of Los-Marczewski to extend pairs of quasi-measures on an algebra of subsets to the power set and retain an absolute continuity relationship between the measures. Let  $G$  be a complete normed Abelian group and  $\Sigma$  an algebra of subsets of a set  $S$  and  $X(\Sigma)$  the collection of quasi-measures on  $\Sigma$  ( $2^S$ ) with values in  $G$ .

Theorem. There is  $g: X \rightarrow X' \subseteq \bar{X}$  and  $h: ba(S, \Sigma) \rightarrow Y \subseteq ba(S, 2^S)$  such that: (i)  $g$  and  $h$  are 1-1 and onto; (ii)  $u = g(u)|\Sigma$ , for  $u \in X$ , and  $v = h(v)|\Sigma$ , for  $v \in ba(S, \Sigma)$ ; (iii)  $|u| = |g(u)||\Sigma$ , for  $u \in X$ , and  $|v| = |h(v)||\Sigma$ , for  $v \in ba(S, \Sigma)$ , (hence  $h$  is an isometry w.r.t. the variation norm, and  $g$  is an isometry for  $bv(X)$ ); (iv) if  $|u| \in ba(S, \Sigma)$ , then  $|g(u)| = h(|u|)$ ; (v)  $g$  is additive and if  $k$  is a scalar and  $u \in X$  such that  $ku \in X$ , then  $g(ku) = kg(u)$  ( $g$  is linear if  $G$  is a vector space); (vi)  $h$  is linear; (vii) if  $u, v \in X$  such that  $|v| \in ba(S, \Sigma)$  and  $|u| \leq |v|$ , then  $|g(u)| \leq |g(v)|$ ; (viii) if  $u, v \in ba(S, \Sigma)$  such that  $|u| \leq |v|$ , then  $|h(u)| \leq |h(v)|$ ; (ix) if  $u$  is a sum of two valued measures in  $X$  ( $ba(S, \Sigma)$ ), then  $g(u)$  is ( $h(u)$  is); (x) if  $u \in X$  and  $v \in ba(S, \Sigma)^+$  such that  $u \ll v$  ( $u$  is absolutely continuous w.r.t.  $v$ ), then  $g(u) \ll h(v)$ . Examples are also given of various properties which cannot be retained by the Los-Marczewski type extension. (Received October 25, 1978.)

\*763-28-6 WAYNE C. BELL, Murray State University, Murray, Kentucky 42071. Unbounded uniformly absolutely continuous sets of measures.

A uniformly absolutely continuous set of finitely additive measures can be decomposed into bounded and finite dimensional parts. (Received October 25, 1978.)

\*763-28-7 JACOB FELDMAN, University of California, Berkeley, Berkeley, California 94708. Time-change in flows.

• Given two measurable, aperiodic, finite measure-preserving, ergodic flows: when can they be made isomorphic by changing the time scale along the orbits of one of them? In the last three years, interesting and surprising partial answers have been found to this question. They shed light on various classical examples, as well as leading to new classes of examples. We describe these results, and also some still more recent developments relating to the  $n$ -dimensional case. (Received October 25, 1978.)

### 30 ► *Functions of a Complex Variable*

\*763-30-1 Norman PURZITSKY, York University, Downsview, Ontario, M3J 1P3. On Fricke Polygons, Signatures, and Canonical Forms for Infinitely Generated Fuchsian Groups.

From the classical theory of fuchsian groups one has that every finitely generated fuchsian group has a canonical or Fricke polygon (see Lehner p. 241), whereby one can read off the canonical presentation,  $a_1 b_1 a_1^{-1} b_1^{-1} \dots a_g b_g a_g^{-1} b_g^{-1} e_1 \dots e_r p_1 \dots p_\nu h_1 \dots h_\mu = 1$ . We obtain similar results for all but an identifiable class of infinitely generated groups and obtain Canonical forms for some groups. The groups from this exceptional class do not always have canonical polygons which are similar to the classical polygons. (Received August 2, 1978.) (Author introduced by Professor Marvin Knopp).

763-30-2 MARK SHEINGORN, University of Illinois, Urbana, Illinois 61801. Bound for the norm of  $\theta_q$ . Preliminary report.

Let  $q \geq 2$  be an integer. Let  $T$  be a finitely generated Fuchsian group acting on the upper half plane with a single parabolic cusp at  $\infty$ . (Assume the stabilizer of  $\infty$  ( $\Gamma_\infty$ ) is generated by  $z \rightarrow z+1$ .)  $\theta_q$ , the Poincaré operator, takes the Bers space  $A_q$  onto  $A_q(\Gamma)$ . We show that for  $q = 2$  the norm of this operator is  $\leq .9712$ . The proof employs a modification of Hecke's technique for estimating Fourier coefficients. Our estimates are crude at several points so this result is surely not best possible. Closer analysis may yield results for larger  $q$ . Also knowledge of the norm of either of the injections  $A_q(\Gamma) \xrightarrow{i} B_q(\Gamma)$  and  $A_q(\Gamma) \xrightarrow{j} A_q^2(\Gamma)$  would facilitate computations. Finally, the exact formula for the  $n$ -th Fourier coefficient of  $\theta_q(e^{2\pi i k z})$  may yield better insight. (Received September 5, 1978.)

\*763-30-3 Thomas A. Metzger, University of Pittsburgh, Pittsburgh, Pennsylvania 15260. The limit set of a Fuchsian group and the absolute convergence of Poincaré series. Preliminary report.

Let  $\Gamma$  be a Fuchsian group acting on the unit disk  $D$ . For  $\zeta$  in  $\mathbb{R}D$ , define  $S(\zeta) = \sum_{\gamma \in \Gamma} |\gamma'(\zeta)|$ , if  $\zeta$  is in  $L$ , the limit set of  $\Gamma$ , then  $S(\zeta)$  need not converge. The importance of the convergence of

$\mathbb{R}(\zeta)$  lies in the fact that there exists a measurable fundamental set for the action of  $\Gamma$  on  $L$  if and only if  $S(\zeta)$  converges a.e. on  $\partial D$ . Also, the convergence of  $S(\zeta)$  can be related to the Martin ideal boundary of the associated Riemann surface  $D/\Gamma$ . It is shown that the convergence of  $S(\zeta)$ , "very tangential" approach to  $\zeta$  by the set  $\Gamma_0 = \{\gamma_0: \gamma \text{ in } \Gamma\}$  and the exponent of convergence of the group  $\Gamma$ , i.e.,  $\delta(\Gamma) = \inf\{t: \sum_{\gamma \in \Gamma} |\gamma'(0)|^t < \infty\}$  are related in that any two imply the third. (Received September 22, 1978.)

763-30-4 JOSEPH LEHNER, University of Pittsburgh, Pittsburgh, PA 15260. Diophantine approximation on the Hecke groups. Preliminary report.

It is known that if  $G$  is an H-group with cusp set  $P$  and  $\alpha$  is real,  $\alpha \notin P$ , then there exists a largest constant  $h_G$  such that

$$|\alpha - a/c| < 1/h_G c^2, \quad V = (a \ b : c \ d)$$

for infinitely many  $V$  belonging to different cosets of  $G/G_\infty$  (Rankin, Canad. Jour. '57; Lehner, A.M.S. Surveys 8, 1964, p. 334). Here we study the "Hurwitz constant"  $h_G$  with special reference to the Hecke groups  $G(\lambda) = \langle (0 \ -1:1 \ 0), (1 \ \lambda:0 \ 1) \rangle$ ,  $\lambda = 2 \cos \pi/q$ ,  $q = 4, 5, \dots$ . (Received October 2, 1978.)

\*763-30-5 ELGIN H. JOHNSTON, Iowa State University, Ames, Iowa 50011. The boundary modulus of continuity of harmonic functions.

For  $0 < \alpha < 1$  and  $\epsilon > 0$ , let  $S(\alpha, \epsilon) = \{z: |\text{Arg } z| \leq \frac{\pi\alpha}{2}, 0 \leq \text{Re } z \leq \epsilon\}$ . If  $\zeta \in \mathbb{C}$  and  $\theta$  is real,  $S(\alpha, \epsilon, \zeta, \theta) = \zeta + e^{i\theta} S(\alpha, \epsilon)$ . A bounded, simply connected open domain  $G$  satisfies a (exterior) cone condition of order  $(\alpha, \epsilon)$  if for each  $\zeta \in \partial G$  there exists a real  $\theta = \theta(\zeta)$  such that  $S(\alpha, \epsilon, \zeta, \theta) \cap G = \emptyset$ .

THEOREM. Suppose  $G$  satisfies a cone condition of order  $(\alpha, \epsilon)$ . If  $u(z)$  is harmonic on  $G$ , continuous on  $\bar{G}$  and has boundary modulus of continuity  $\tilde{\omega}_u(\delta)$ , then the modulus of continuity  $\omega_u(\delta)$  of  $u$  on  $\bar{G}$  satisfies

$$\omega(\delta) \leq \tilde{\omega}(C\delta) + D\delta \int_{\delta}^{\frac{E}{\delta}} \frac{\tilde{\omega}(s)}{s^{\frac{1+\beta}{\beta}}} ds,$$

where  $\beta = 2-\alpha$  and  $C, D, E$  are positive constants depending on  $G$ . If  $\tilde{\omega}(\delta)$  is subadditive, then  $\omega(\delta) \leq C\delta^{\frac{1-\beta}{\beta}} \tilde{\omega}(\delta)$ .

This result can be shown to be sharp for special  $G$  and  $u(z)$  with  $\tilde{\omega}_u(\delta) = 0(\delta^\gamma)$  ( $0 < \gamma \leq 1$ ). (Received October 10, 1978.)

\*763-30-6 PETER J. NICHOLLS, Northern Illinois University, DeKalb, IL 60115. The orbital counting function for a Fuchsian group of infinite area.

For a Fuchsian group  $G$  acting in the unit disc and for  $r$  satisfying  $0 < r < 1$  we define  $n(r, 0)$  to be the cardinality of the set  $\{V \in G: |V(0)| < r\}$ . This function is important in the study of automorphic forms and several estimates are available. It has recently been shown by S. J. Patterson [Spectral theory and Fuchsian groups, Math. Proc. Camb. Philos. Soc. 81(1977), 59-75] that, for a group of infinite area,  $(1-r)n(r, 0) \rightarrow 0$  as  $r \rightarrow 1$ . We present a new proof of this result based on an ergodic property of the horocyclic flow for such a group. (Received October 10, 1978.)

\*763-30-7 H. SILVERMAN, College of Charleston, Charleston, South Carolina 29401 and D. N. TELAGE, University of Kentucky, Lexington, Kentucky 40506. Extreme points of subclasses of close-to-convex functions.

We consider functions  $f$ , analytic in the unit disk, for which there exists a convex function  $g$  satisfying either  $\text{Re}\{zf' | g\} > 0$  or  $\text{Re}\{(zf')' | g'\} > 0$ . Coefficient bounds, distortion and covering theorems, and the extreme points of the closed convex hulls are found for these and other subclasses of close-to-convex functions. All results are sharp. (Received October 13, 1978.)

\*763-30-8 Johnny E. Brown, University of Michigan, Ann Arbor, Michigan 48109. Geometric properties of a class of support points of univalent functions.

Let  $S$  denote the set of functions  $f(z)$  analytic and univalent in  $|z| < 1$ , normalized by  $f(0) = 0$  and  $f'(0) = 1$ . A function  $f$  is a support point of  $S$  if there exists a continuous linear functional  $L$ , nonconstant on  $S$ , for which  $f$  maximizes  $\operatorname{Re} L(g)$ ,  $g \in S$ . New examples of support points and extreme points are obtained by determining explicitly the support points of the point-evaluation functionals:  $L(g) = g(z_0)$ ,  $0 < |z_0| < 1$ . New geometric properties are found for the arcs omitted by these functions. Numerical calculations appear to indicate that the known bound  $\pi/4$  for the angle between radius and tangent vectors is actually best possible. (Received October 16, 1978.)

\*763-30-9 Ralph W. Wilkerson, Winthrop College, Rock Hill, South Carolina 29733. Linear maps of holomorphic functions on commutative algebras.

Let  $A$  be a commutative linear associative algebra of dimension  $n$  over the field of reals with basis  $\{e_i\}$ ,  $e_i e_j = \sum c_{ijk} e_k$ ; matrices  $R_i = (c_{isr})$ ,  $Q_i = (c_{rsi})$ ,  $r = \text{row index}$ ,  $s = \text{column index}$ ;  $M$  a  $n$ -dimensional manifold with boundary  $\partial M$ ;  $d\sigma$  a  $(n-1)$ -form (with coefficients in  $A$ );  $Z = x_1 e_1 + \dots + x_n e_n \in A$ ;  $f(Z) = \sum f_i(x_1, \dots, x_n) e_i \in A$ , with  $f_i$  real  $C^{(1)}$ -functions on a domain  $D \subset A$ . DEFS: Say  $f(Z)$  is regular in the sense of Scheffers ( $S$ -regular) (=classically differentiable) iff the  $f_i$  satisfy a system of first-order linear PDEs. Say  $f(Z)$  is regular in the sense of Fueter ( $F$ -regular) if  $\int_{\partial M} f(Z) d\sigma = 0$  for all  $M \subset D$ . THEOREM: Let  $f(Z)$  be  $S$ -regular. Then there exists a linear map  $A$  and component functions  $g_i$  with  $A [f_1 \dots f_n]^T = [g_1 \dots g_n]^T$  such that  $g(Z)$  is  $F$ -regular iff the space of matrices  $\{R_i Q_j\}$  has dimension less than  $n^2$ . As a corollary all algebras which satisfy the above property are classified, thus generalizing a result of H. H. Snyder (who established the existence of linear maps for functions defined on the real group algebra of the cyclic group of order  $n$ ). (Received October 19, 1978.)

763-30-10 Y. LEUNG, University of Hawaii, Honolulu, HI 96822. Integral means of derivatives of some univalent functions.

Let  $f(z)$  be analytic and univalent in the unit disk with normalization  $f(0) = 0$ ,  $f'(0) = 1$ . We say  $f$  is in  $B_\alpha$ , a subclass of Bazilevič functions, if it satisfies

$$z f'(z) = p(z) s(z)^\alpha f(z)^{1-\alpha},$$

where  $s(z)$  is a starlike function and  $p(z) = 1 + p_1 z + p_2 z^2 + \dots$  has positive real part. Let  $k(z) = z(1-z)^{-2}$ . We use the Baernstein  $*$ -functions to show, for  $f$  in  $B_\alpha$ ,  $0 < \alpha \leq 1$ ,

$$\int_{-\pi}^{\pi} |f'(re^{i\theta})|^p d\theta \leq \int_{-\pi}^{\pi} |k'(re^{i\theta})|^p d\theta,$$

for all  $p > 0$  and  $0 < r < 1$ . For fixed  $r$  in  $(0, 1)$ , equality holds only when  $f(z)$  is a rotation of  $k(z)$ . (Received October 20, 1978.)

\*763-30-11 EUGENE RODEMICH, Jet Propulsion Laboratory, Pasadena, California 91103. An inequality for univalent functions with real coefficients.

We consider the class of functions  $f(z) = z + a_2 z^2 + \dots$ , univalent in  $|z| < 1$ , with real coefficients  $\{a_n\}$ . Variational methods are used to show the following result:

Let  $q = |z - \bar{z}| / (1 - |z|^2)$ . Then

$$|\arg (1 - z^2) f'(z)| \leq \begin{cases} 2 \tan^{-1} q, & q \leq 1, \\ \frac{\pi}{2} + \log q, & q > 1. \end{cases}$$

(Received October 20, 1978.)



\*763-30-12 H. H. SNYDER, Southern Illinois University, Carbondale, Illinois 62901. An extended Cauchy integral formula on certain associative algebras.

Let  $Z = x + jy + j^2z$  be an element of the cyclic group algebra  $\mathbb{R}C_3$  over the reals,  $f(Z)$  holomorphic on a domain in  $\mathbb{R}C_3$  (cf. 720-30-18, Notices 22, no. 1). A natural extension of Cauchy's integral formula to  $\mathbb{R}C_3$  would take the form  $f(A) = \text{const.} \times \int_D f(Z)K(Z,A)d\sigma$  over the smooth boundary  $\partial D$  of a smooth compact and connected manifold  $D \subset \mathbb{R}^3$ , and  $K(Z,A)$  a holomorphic function, suitably singular at  $A$ . However, any such function  $K$  has line singularities which penetrate  $\partial D$  (since  $\mathbb{R}C_3$  is not a division algebra). We overcome this difficulty by embedding  $\mathbb{R}C_3$  in the direct product algebra  $\mathbb{R}C_3 \times \mathbb{Q}$  with the quaternions over  $\mathbb{R}$ . Let  $W = e_1x + e_2y + e_3z$  = position of quaternion of  $Z$ . Then if  $K(W,A) = (\|W-A\|(W-A))^{-1}$ , we prove that  $4\pi f(A) = \int_D f(Z)K(W,A)d\sigma - \int_D K(W,A)(e_1 + je_2 + j^2e_3)d\tau$ . Equating coefficients of the  $e_i$ 's, we obtain an integral formula involving only  $C_3$ -valued quantities. There seems no reason why this theorem may not be extended to arbitrary finite-dimensional non-division algebras with unity element. In place of quaternions, one would use an elliptic Clifford algebra which, while not a division algebra, always has a "division subspace." (Received October 23, 1978.)

\*763-30-13 Roger W. Barnard, Texas Tech University, Lubbock, Texas 79409 and Ted Suffridge, University of Kentucky, Lexington, Kentucky 40506. On Coefficient bounds on  $f$  in  $S$  when  $f'$  is univalent. Preliminary report.

Let  $S'$  be the subclass of the normalized univalent functions  $f$ ,  $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$  with  $f'$  univalent. In "Research Problems in Complex Analysis, 1977" J. Clunie posed the problem of determining the  $\max_{f \in S'} |a_n| = A_n$ . He noted that since  $z(1-z)^{-1} \in S'$ ,  $A_n \geq 1$ . The author has shown that the function  $F_a$  defined by  $F_a(z) = z + \sum_{n=2}^{\infty} \frac{2a(n-1)}{n} z^n$  is in  $S'$  for  $0 \leq a \leq 2(\pi-2)^{-1} = A \approx 1.75$ . Thus  $A_n \geq 2A(n-1)/n \geq A$  for all  $n \geq 2$ . Indeed from the nature of the function  $F_a$  the author wishes to make the following conjecture  $\max_{f \in S'} |a_n| = 2A(n-1)/n$ . The author has also obtained a number of distortion theorems for the class  $S'$  involving  $A_2 = \max |a_2|$  and has shown that  $1.75 \approx A \leq A_2 < 1.87$ . (Received October 23, 1978.)

\*763-30-14 John C. Plaster, Texas Tech University, Lubbock, Texas 79409. A Geometric Criterion for Determining Non-Bazilevichness. Preliminary report.

Let  $S$  be the standard class of normalized univalent functions on the unit disc, and  $B$  its subclass of normalized Bazilevic functions. We give a simple visual criterion for showing non-Bazilevichness. Using this technique we exhibit a class of non-Bazilevic polynomials, answering the problem posed by Doug Campbell as to the smallest degree of a non-Bazilevic polynomial. We also investigate the range of "a" for which the polynomial  $p(z) = z + az^2 + \frac{1}{3}z^3$  is Bazilevic. We further use these techniques to improve the upper bound for the radius of Bazilevichness in the class  $S$ . (Received October 23, 1978.) (Author introduced by Professor Roger W. Barnard).

\*763-30-15 WALTER PRANGER, DePaul University, Chicago, Illinois 60614. Bounded analytic functions. Preliminary report.

A class of Riemann surfaces is introduced which is "natural" for the study of bounded holomorphic functions. This class is characterized by four equivalent conditions, each of which involves some statement concerning holomorphic sections of unitary line bundles. (Received October 23, 1978.) (Author introduced by Professor Jerry Goldman).

\*763-30-16 CARL H. FITZGERALD, University of California, San Diego La Jolla, California 92093. Conditions for a function on an arc to have a bounded analytic extension.

Certain quadratic inequalities of Pick and Nevanlinna are reformulated to give conditions for boundary values. It is shown that analytic functions taking the unit disc into itself have boundary values which must satisfy these conditions. Conversely, any continuously differentiable complex valued function that satisfies these quadratic inequalities on an arc must have an analytic extension which takes the unit disc into itself. These observations are easily transformed to apply to functions that take the upper half plane into itself. The results generalize a theory of Loewner in which the boundary functions were required to be real valued. (Received October 24, 1978.)

\*763-30-17 Herb Silverman, College of Charleston, Charleston, South Carolina 29401 and Evelyn M. Silvia, University of California at Davis, Davis, California 95616. Convex Families of Starlike Functions. Preliminary report.

Special classes of starlike functions, defined in terms of auxiliary functions will be discussed. Coefficient bounds, distortion theorems, and extreme points are determined. (Received October 25, 1978.)

763-30-18 STEPHEN M. ZEMYAN, Georgia Institute of Technology, Atlanta, Georgia 30332. A minimal outer area problem in conformal mapping. Preliminary report.

For  $0 < p < 1$ , let  $S_p$  denote the class of meromorphic, univalent functions in the unit disk normalized so that  $f(0) = f'(0) - 1 = 0$  and  $f(p) = \infty$ . Let  $R_p = \{a : a = \text{Res}_{z=p} f(z), f \in S_p\}$ , and let  $S_p(a) = \{f \in S_p : \text{Res}_{z=p} f(z) = a\}$ . We show that there exists a dense and open subset  $N_p$  of  $R_p$  such that if  $a \in N_p$ , then  $\min_{f \in S_p(a)} \bar{A}(f) = 0$ , where  $\bar{A}(f)$  denotes the outer area of  $f$  (the area of the complement of the range of  $f$ ). (Received October 25, 1978.)

## 32 ► Several Complex Variables and Analytic Spaces

\*763-32-1 Ajaj A. Tarabay, Texas Tech University, Lubbock, Texas 79409. Local and global analysis on branched analytic covers.

We investigate conditions such that if  $\pi : X \rightarrow U$  is an analytic cover, where  $U$  is a domain of holomorphy in  $\mathbb{C}^k$ , then  $X$  is a closed subvariety of  $\mathbb{C}^n$  for some  $n$ . We prove the following two theorems:

Theorem 1  $X$  is an analytic subvariety iff  $\mathcal{O}(X)$  is a finitely generated  $\mathcal{O}(U)$ -module via  $\pi$ .

Theorem 2 Every one-dimensional analytic cover is an analytic subvariety. (Received October 2, 1978.)

763-32-2 JOHN E. FORNAESS, Princeton University, Fine Hall, Princeton, New Jersey 08540. Proper holomorphic mappings.

• Let  $\Omega_1$  and  $\Omega_2$  be two bounded open sets in  $\mathbb{C}^n$  with smooth ( $\mathcal{C}^\infty$ ) boundary and let  $F : \Omega_1 \rightarrow \Omega_2$  be an invertible holomorphic map. Conjecture.  $F$  has a smooth extension  $\bar{F} : \bar{\Omega}_1 \rightarrow \bar{\Omega}_2$ . We will discuss some positive results, under suitable extra conditions on  $\Omega_1$  and  $\Omega_2$ . (Received October 20, 1978.)

\*763-32-3 L.F. Heath, University of Texas at Arlington, Arlington, Texas 76019 and T.J. Suffridge, University of Kentucky, Lexington, Kentucky 40504. Holomorphic idempotents in  $\mathbb{C}^n$ .

Let  $D$  be the open unit ball in complex  $n$ -space with sup norm (i.e.  $D$  is a polydisc of radius one). We determine all holomorphic idempotents of  $D$ . That is, we determine all holomorphic mappings of  $D$  into the closure of  $D$  such that  $F \circ F = F$ . (Received October 23, 1978.)

763-32-4 JOSEPH A. BECKER and WILLIAM R. ZAME\*, State University of New York at Buffalo, Buffalo, New York, 14214. Primary ideals in rings of holomorphic germs.

Let  $X$  be a reduced complex-analytic space and  $K$  a compact subset of  $X$  which is holomorphically convex (e.g., the intersection of Stein open subsets of  $X$ ). We study the structure of primary ideals in  $\mathcal{O}(K)$ , the ring of germs on  $K$  of functions holomorphic near  $K$ . Our principal result is the following: THEOREM. In order that every ideal of  $\mathcal{O}(K)$  have an irredundant primary decomposition it is necessary and sufficient that for every variety  $V$  defined in a neighborhood of  $K$ ,  $V \cap K$  has only countably many connected components. (Received October 24, 1978.)

### 33 ► Special Functions

763-33-1 B. C. CARLSON, Ames Laboratory-DOE, Iowa State University, Ames, Iowa 50011. Integrals of the square root of a rational function. Preliminary report.

Published integral tables resort to enumeration of cases in treating integrals of the form

$$(*) \int_y^x (a+\alpha t)^{p/2} (b+\beta t)^{q/2} (c+\gamma t)^{r/2} (d+\delta t)^{s/2} dt,$$

where  $a, \alpha, \dots, d, \delta$  are real and  $p, q, r, s$  are integers (not all even). A glaring example is  $(p, q, r, s) = (1, -1, -1, -3)$ , for which 72 cases are listed without being exhaustive. Even elementary integrals like  $(-1, -1, -2, 0)$  require separation of circular and logarithmic cases. Enumeration of cases is unavoidable if the integrals are reduced to the usual standard functions (log, arctan, and Legendre's elliptic integrals). For given  $(p, q, r, s)$  all cases of (\*) can be unified by using a different choice of standard functions. Moreover, the functions do not need to be evaluated separately at the two limits of integration, and neither limit needs to be a branch point of the integrand. Algorithms are available for computing the standard functions numerically. (Received October 5, 1978.)

763-33-2 Allan J. Fryant, U. S. Naval Academy, Annapolis, Maryland 21402. Ultraspherical expansions and psuedoanalytic functions.

The theory of psuedo analytic functions and Bergman-Gilbert type integral operators are employed in the study of ultraspherical expansions, their conjugates, and the associated elliptic partial differential equations. The relation between these two approaches is examined, and analogs of results from the theory of analytic functions of a single complex variable are obtained. These include Schwartz's formula, Privaloff's theorem, results regarding the location of singularities, and uniform polynomial approximation. The manner in which analytic functions appear as limiting cases of psuedo analytic functions whose real parts are ultraspherical expansions will also be discussed. (Received October 18, 1978.)

\*763-33-3 DANIEL S. MOAK, University of Wisconsin, Madison, Wisconsin 53706. The q-gamma function. Preliminary report.

F. H. Jackson defined a q-analogue of the gamma function which extends the q-factorial  $(n!)_q = 1(1+q)(1+q^2)\dots(1+q+q^2+\dots+q^{n-1})$ . Askey obtained q-analogues of most of the classical facts about the gamma function for  $0 < q < 1, x > 0$ . In particular a logarithmically convex solution of  $f(1) = 1$  and  $f(x+1) = [(q^x-1)/(q-1)] f(x)$ , must be the q-gamma function, for  $0 < q < 1, x > 0$ . The uniqueness fails for  $q > 1$ ; a stronger condition than log convexity is needed. Two sufficient conditions are discussed. Askey studied the behavior of  $\Gamma_q$  as q changes for  $0 < q < 1, x > 0$ , and showed that as  $q \rightarrow \Gamma, \Gamma_q(x) \rightarrow \Gamma(x)$ . Similar results are obtained for  $q > 1$ , and for  $x < 0$ . In addition, we investigate the monotonic behavior of the relative extrema and their sizes as n or q changes, and discuss the limiting behavior of these points as  $x \rightarrow -\infty$ . (Received October 25, 1978.)

763-33-4 STEPHEN C. MILNE, Texas A&M University, College Station, Texas 77843. A multiple series transformation of the very well poised  ${}_{2k+4}\Psi_{2k+4}$ . Preliminary report.

A multiple series generalization of the q-analog of Whipple's theorem is derived for  ${}_{2k+4}\Psi_{2k+4}$  by applying recent analytical techniques of Askey and Ismail to Andrews' multiple series transformation of a well poised  ${}_{2k+4}\Phi_{2k+3}$ . The proof involves showing that two analytic functions agree infinitely often near a point that is an interior point of the set of analyticity. This result generalizes the  ${}_6\Psi_6$  summation formula (k=1) as well as a terminating form of the transformation of  ${}_8\Psi_8$  (k=2) given by M. Jackson. Moreover, this result has numerous applications to partition identities. (Received October 25, 1978.)

## 34 ► Ordinary Differential Equations

\*763-34-1 DON HINTON, University of Tennessee, Knoxville, TN 37916 and ROGER LEWIS, University of Alabama, Birmingham, AL 35294. Oscillation theory at a finite singularity.

Let  $L(y) = \sum_{i=0}^n (-1)^i (p_{n-i} y^{(i)})^{(i)}$  where the coefficients  $p_i$  are real continuous functions on  $(0,1]$  with  $p_0 > 0$ . The operator  $L$  is said to be oscillatory at 0 if for each  $\delta > 0$  there is a nontrivial solution  $y$  of  $L(y) = 0$  which has two zeros on  $(0,\delta)$  of multiplicity at least  $n$ . By application of the quadratic functional associated with  $L$ , conditions are given on the coefficients of  $L$  which are sufficient for  $L$  to be oscillatory at 0; other conditions are given which are necessary for  $L$  to be oscillatory at 0. As an application, criteria are given for a self-adjoint operator generated by  $L$  to have a spectrum which is discrete and bounded below. (Received September 11, 1978.)

763-34-2 Allan Edelson and Kurt Kreith, University of California, Davis, California 95616. Nonlinear relationships between oscillation and asymptotic behavior.

The fourth order differential equation  $(*) y^{(4)} = f(t,y)$  with  $\text{sgn } f = \text{sgn } y$  in  $[0,\infty) \times (0,\infty)$  is said to satisfy condition (H) if every eventually positive solution satisfies  $y'' > 0$  and  $y'y'' > 0$  for sufficiently large values of  $t$ . Generalizing upon a linear result of Ahmad [Pacific J. Math. 34 (1970), 289-299] conditions are established under which (H) is equivalent to a form of oscillatory behavior called condition (O). The implication  $(H) \Rightarrow (O)$  is extended to  $y^{(2n)} = f(t,y)$ . (Received September 18, 1978.)

763-34-3 SUNG J. LEE, Department of Mathematics and Statistics, University of Guelph, Guelph, Ontario, Canada. Operators with countably many generalized boundary conditions.

Abstract. Let  $H_1, H_2$  be Hilbert spaces. Let  $T: \mathcal{D}(T) \rightarrow H_2, B: \mathcal{D}(T) \rightarrow H_2$  be closed operators such that  $B$  is  $T$ -bounded where  $\mathcal{D}(T) \subset H_1$ . It is possible that  $\mathcal{D}(T)$  is not dense and  $B$  is of infinite dimensional range. We study a fixed linear operator  $L \subset T + B$ . In particular, we determine

1. Conditions under which  $L_1$  is closed.
2. The adjoint  $L^*$ .
3. Conditions under which a restriction of  $L$  is symmetric or self-adjoint.

We then apply this to the operator  $T$  generated by countably many ordinary linear differential expressions. The results are given in terms of Besselian-Hilbertian bases, and are generalization of the case when  $B$  is of finite dimensional range. These results will appear in the Indiana University Mathematics Journal. (Received September 28, 1978.)

763-34-4 J. K. Shaw, Virginia Polytechnic Institute and State University, Blacksburg, Va. 24061. Oscillation Properties of Singular Sturm-Liouville Operators. Preliminary report.

Let  $L$  be the Sturm-Liouville operator  $Ly' = -(Py)'' + Qy$ , defined on an interval  $[0, b^*)$  of regular points, but singular at  $b^*$ ,  $0 < b^* \leq \infty$ . Consider a singular boundary-value problem  $Ly = \lambda y, y(0) = B^*(y) = 0$ , where  $B^*$  is a boundary form associated with the singular endpoint  $b^*$ , with eigenvalues  $0 < \lambda_0 < \lambda_1 < \dots$  and eigenfunctions  $y_0(t), y_1(t), \dots$ . Denote by  $V(f)$  the number of sign changes of a function  $f(t)$  in  $(0, b^*)$ , and suppose  $V(y_k) = k, k = 0, 1, 2, \dots$ . A classical

result of E. Hille [Trans. Am. Math. Soc., 52 (1942), 463-497] asserts that for all  $f$  belonging to a certain class  $F$ , the conditions  $(L^k f)(0) = B^*(L^k f) = 0$  and  $V(L^k f) = N$ , for all  $k$ , imply  $f(t) = \sum_1^N C_k y_k(t)$ , with  $C_N \neq 0$ . A generalization of this result is obtained by means of a representation of the form  $f(t) = \sum_0^m [(L^k f)(0) P_{2k}(t) + B^*(L^k f) P_{2k+1}(t)] + \sum_1^N C_k y_k(t)$ ,  $C_N \neq 0$ . The conditions  $(L^k f)(0) = B^*(L^k f) = 0$  are replaced by summability conditions on the numbers  $(L^k f)(0)$  and  $B^*(L^k f)$ . (Received September 28, 1978.)

763-34-5 George H. Handelman and Joyce R. McLaughlin, Rensselaer Polytechnic Institute, Troy, New York, 12181. A Constructive Technique for solving a second order inverse eigenvalue problem. Preliminary Report.

Let  $\lambda_1^* < \lambda_2^* < \dots$  be the eigenvalues and  $\rho_1^*, \rho_2^*, \dots$ , be the corresponding normalization constants for a second order eigenvalue problem  $(r^* u')' + \lambda u^* = 0$ ,  $\sin \alpha^* r^*(0) u'(0) + \cos \alpha^* u(0) = 0$ ,  $\sin \beta^* r^*(1) u'(1) + \cos \beta^* u(1) = 0$ , where  $r^* \in C^1(0,1)$ ,  $p^* \in C(0,1)$ ,  $0 \leq \alpha^*, \beta^* < \pi$ . Let  $0 < \lambda_1 < \dots < \lambda_n < \lambda_{n+1}^*$  and  $\rho_1, \dots, \rho_n$  be the  $2n$  given positive numbers. A constructive technique will be displayed for finding  $r \in C^1[0,1]$ ,  $p \in C[0,1]$  and  $\alpha, \beta \in [0, \pi]$  such that  $\lambda_1, \dots, \lambda_n, \lambda_{n+1}^*, \lambda_{n+2}^*, \dots$  and  $\rho_1, \rho_2, \dots, \rho_n, \rho_{n+1}^*, \rho_{n+2}^*, \dots$  are eigenvalues and normalization constants for  $(ru')' + \lambda ru = 0$ ,  $\sin \alpha r(0) u'(0) + \cos \alpha u(0) = 0$ ,  $\sin \beta r(1) u'(1) + \cos \beta u(1) = 0$ . Solutions are not unique. Hypotheses for  $r, p, \alpha, \beta$  will be given in insure that solutions are unique. (Received September 28, 1978.)

763-34-6 T. A. BURTON, Southern Illinois University, Carbondale, Illinois 62901. Stability theory for Volterra equations.

In this paper we consider a Volterra equation (1)  $x'(t) = Ax(t) + \int_0^t C(t, s)x(s)ds + F(t)$  in which  $A$  is an  $n \times n$  constant matrix,  $C$  is  $n \times n$ , and  $x$  and  $F$  are column vectors. A Liapunov functional of the form

$$V(t, x(\cdot)) = \{[x^T B x]^{1/2} + \int_0^t [c_1 - c_2 \int_0^{t-s} \|C(u+s, s)\| du] |x(s)| ds + 1\} \exp - 2 \int_0^t |F(s)| ds$$

is constructed in which  $B$  is a positive definite matrix satisfying  $A^T B + BA = \pm I$ . Under various assumptions on  $A, C$ , and  $F$  we obtain boundedness, stability, and instability results. The functional is modified to cover equations of the form (2)  $x'(t) = g(t, x(t)) + \int_0^t p(t, s, x(s)) ds + F(t)$  as well as various perturbations. (Received October 2, 1978.)

\*763-34-7 KENT FOSTER, Emory University, Atlanta, Georgia 30322 and RONALD GRIMMER, Southern Illinois University, Carbondale, Illinois 62901. Nonoscillatory solutions of higher order delay equations. Preliminary report.

The existence and growth of nonoscillatory solutions of the equation  $x^{(n)} + f(t, x(\tau(t))) = 0$  is examined. Here  $f$  is continuous for  $t \geq 0$  and  $x$  real. Also  $f(t, x)$  is nondecreasing in  $x$  for fixed  $t$  and  $xf(t, x) > 0$  if  $x \neq 0$ . Criteria are derived which guarantee the existence of nonoscillatory solutions which satisfy  $x(t)x^{(i)}(t) > 0$  for  $i = 0, 1, \dots, \ell$  and  $(-1)^{n+i} x(t)x^{(i)}(t) < 0$  for  $i = \ell + 1, \dots, n$ . (Received October 3, 1978.)

\*763-34-8 JAMES R. WARD, Pan American University, Edinburg, Texas 78539. Existence of periodic solutions for nonlinearly perturbed conservative systems.

Consider the system (\*)  $x'' + \text{grad } G(x) = p(t, x)$ . Theorem: Suppose  $G \in C^2(\mathbb{R}^m, \mathbb{R})$  and  $p \in C(\mathbb{R} \times \mathbb{R}^m, \mathbb{R}^m)$  with  $p(t + 2\pi, x) = p(t, x)$ . Suppose  $p(t, x)$  is Lipschitz in  $x$  and  $|p(t, x)|/|x| \rightarrow 0$  as  $|x| \rightarrow \infty$ , uniformly in  $t$ . Suppose there exist real, constant, symmetric  $m \times m$  matrices  $A$  and  $B$  such

that if  $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_m$  and  $\mu_1 \leq \mu_2 \leq \dots \leq \mu_m$  are the eigenvalues of A and B, resp., then there are integers  $N_i \geq 0$  ( $i=1, \dots, m$ ) satisfying  $N_i^2 < \lambda_i \leq \mu_i < (N_i + 1)^2$ . Suppose there is a number  $r > 0$  such that for all  $a \in R^m$  with  $|a| \geq r$  the relation  $A \leq (\partial^2 G(a) / \partial x_i \partial x_j) \leq B$  is satisfied. Then (\*) has a  $2\pi$ -periodic solution. Other results for (\*) are also given. (Received October 4, 1978.)

\*763-34-9 GANGARAM S. LADDE, State University of New York College at Potsdam  
Potsdam, New York 13676. Stabilizing and Oscillizing Time-Delays.  
Preliminary report.

Sufficient conditions are given for the stability, oscillatory, and non-oscillatory behavior of first order retarded differential equations of the type:

$$(1) \quad y'(t) + q(t)y(t) + p(t)y(t-\tau) = 0,$$

where  $q$  and  $p$  are continuous functions defined on  $R$  into  $R$ ,  $p$  is either non-negative or non-positive for sufficiently large  $t$ , and  $\tau > 0$ . In particular, it is shown that the oscillations and stability are caused by retarded argument and they do not appear in the corresponding differential equations without time-delays. Furthermore, these results are also extended for more general functional differential equations. (Received October 6, 1978.)

\*763-34-10 Vadim Komkov, American Mathematical Society, 611 Church Street, Ann Arbor, Michigan  
48109. An embedding technique for a class of stability problems.

The stability of elastic systems is specifically considered in this paper, but the technique introduced here is quite general. The example of stability of the trivial solution of the buckling problem for a column is illustrating the principal idea. Two forms of energy are identified whose difference determines the stability of the trivial solution. Varying the so called constants occurring in the equations of state exhibits clearly the dependence of the energy forms on the values of these constants and permits to derive new forms of stability criteria. Upper and lower bounds are obtained on the first eigenvalue of the corresponding fourth order eigenvalue problem. (Received October 10, 1978.)

763-34-11 W. R. Utz, University of Missouri, Columbia, Missouri 65211. Properties of solutions of the differential equation  $xx'' - kx'^2 + f(x) = 0$ .

A continuation of work of L. Roth [Phil. Mag. 32 (1941), 155-164] and generalizations of this work are given. These results are then applied to a class of previously identified differential equations of polynomial class, particularly second order equations where there is an  $x'^2$  term. (Received October 10, 1978.)

\*763-34-12 ATHANASSIOS G. KARTSATOS, JORGE GONZALES TORO, University of South Florida, Tampa, Florida  
33620. Comparison theorems for equations with middle terms of order  $n-1$ .

Differential equations of the form: (\*)  $x^{(n)} + P_1(t, x, x', \dots, x^{(n-1)}) + H_1(t, x) = 0$ , are studied for  $n = \text{even}$ . It is first shown that the existence of a positive solution of (\*) implies the same fact for the equation: (\*\*)  $x^{(n)} + P_2(t)x^{(n-1)} + H_2(t, x) = 0$ , provided that the pairs  $P_1, P_2$  and  $H_1, H_2$  are properly related. These existence results are then used to show that the oscillation of (\*\*) implies that of (\*). Applications of these considerations are given for certain rather general second order equations which include as special cases generalized forms of the equation of Liénard and the equation of Van der Pol. Certain forced versions of the above results are also considered. The main difference between the above comparison results and the existing ones is that the comparison equation in the present work is, in general, a nonlinear equation which might satisfy weaker conditions ensuring its oscillation than those of a corresponding linear equation. (Received October 13, 1978.) (Author introduced by Professor M. N. Manougian).

Etgen and Pawlowski have recently given criteria for oscillation of matrix differential equations which involve the values of positive linear functionals on the matrices appearing in a matrix differential equation. We characterize these functionals and indicate relationships to eigenvalue criteria for oscillation. Our results are also useful for the detection of the oscillation of particular matrix differential equations. (Received October 16, 1978.) (Author introduced by James Ward).

We consider the eigenvalue problem

$$\begin{aligned} M[u] &= \eta^2 u^{(4)} - \{[\alpha(1-x)+1]u'\}' = \alpha \lambda u, \quad 0 < x < 1, \\ u(0) &= u'(0) = u''(1) = 0, \\ \eta^2 u'''(1) &= u'(1) - \lambda u(1), \end{aligned}$$

where  $u$  is the dimensionless time-independent displacement of a pendulum,  $\eta^2$  is the dimensionless stiffness,  $\alpha$  is the ratio of the wire mass and the bob mass, and  $\beta$  ( $\lambda = \beta^2$ ) is the dimensionless frequency. The method of integral equations is introduced to show that the eigenvalues of the above problem form a nonempty set which has no finite accumulation points. Lower bounds are established for the lowest frequency by the method of integral equations, which is directly applied to the above equation. Numerical results are obtained by utilizing a digital computer. We also obtain lower bounds for the lowest frequency by applying a modified minimum principle and the method of integral equations. Numerical results of the two methods are compared. Both theoretical and computational efficiencies for the method of integral equations are illustrated. (Received October 16, 1978.)

Consider the functional differential equation (\*)  $x'(t) = L(x_t) + g(t, x_t, \epsilon)$ , where  $L$  is linear,  $g$  is nonlinear, periodic in  $t$  and 'higher order' in the second variable. The existence of periodic solutions of (\*) for small values of the parameter  $\epsilon$  is considered in the resonance case. This problem is given an abstract formulation as a nonlinear equation in a Banach Space. The Lyapunov Schmidt method is used to derive a finite dimensional bifurcation equation to which methods of topological degree are applied. An example is given to illustrate the applicability of the method. (Received October 16, 1978.)

We give a qualitative analysis for the periodically forced relaxation oscillations described by the system

$$(1) \quad \epsilon \ddot{x} + \phi(x) \dot{x} + \epsilon x = b \sin t.$$

An equation of such a form was analyzed by Cartwright, Littlewood and Levinson. We give a geometrical explanation of their discoveries and apply our geometrical picture to obtain further results. Namely, we derive a complete analysis of the system (1) for "most" values of  $b$ . In particular, a complete description of the set of rotation numbers and of the "strange attractor" is given. The latter is described via the subshifts of finite type. Moreover, applying the recent results by Newhouse and Palis, we show that there exist uncountably many "exceptional" values of  $b$ , for each of which (1) has infinitely many stable periodic solutions. (Received October 16, 1978.)

\*763-34-17 Roger C. McCann, Mississippi State University, Mississippi State, Mississippi 39762. Asymptotically Stable Dynamical Systems Are Linear. Abstract.

If  $\pi$  is a dynamical system on a locally compact metric space  $X$  which has a globally asymptotically stable critical point, then  $\pi$  can be embedded into a dynamical system on  $\ell_2$  which is derived from a linear differential equation. If  $X$  is  $n$ -dimensional, then  $\ell_2$  may be replaced by  $\mathbb{R}^{2n}$ .  
(Received October 16, 1978.)

\*763-34-18 Roger T. Lewis, University of Alabama in Birmingham, Birmingham, Alabama 35294 and Lynne C. Wright, University of Alabama, University, Alabama 35486. A comparison theorem for even-order, vector-matrix differential equations.

A theorem, which relates the oscillation of the vector-matrix equation  $\sum_{k=0}^n (-1)^k (P_k(x)y^{(k)}(x))^{(k)} = 0$  to the oscillation of the associated scalar equation, will be presented. In the finite dimensional case, this theorem extends the recent second-order comparison theorem of Etgen and Lewis (Theorem 4.3, "Positive Functionals and Oscillation Criteria for Differential Systems", Optimal Control and Differential Equations, Academic Press, 1978, pp. 245-275). (Received October 16, 1978.)

\*763-34-19 ALLAN PETERSON, University of Nebraska, Lincoln, Nebraska 68588. Focal Green's functions for fourth order differential equations.

We are mainly concerned with the differential equation  $\rho_4(x)(\rho_3(x)(\rho_2(x)(\rho_1(x)(\rho_0(x)y)')')')')' = \lambda p(x)y$  where the coefficients are positive continuous functions on  $[a,b]$  and  $\lambda = \pm 1$ . The main result is that if this differential equation is disfocal on  $[a,b]$ , then the Green's functions for the two point focal type boundary value problems have constant sign on  $(a,b) \times (a,b)$ . The question of constancy of sign of various quasi partial derivatives of these Green's functions is also solved. (Received October 16, 1978.)

\*763-34-20 SAMUEL M. RANKIN III, West Virginia University, Morgantown, West Virginia 26506. A remark on cosine families.

Let  $C(t)$ ,  $t \in \mathbb{R}$ , be a strongly continuous cosine family and  $A$  its infinitesimal generator. Then the set  $E \stackrel{\text{def}}{=} \{x \in X: C(t)x \text{ is once continuously differentiable in } t \text{ on } \mathbb{R}\}$  of the Banach space  $X$  is contained in the domain of  $(-A)^\alpha$  for  $0 \leq \alpha < \frac{1}{2}$ . (Received October 18, 1978.)

\*763-34-21 Charles R. Powder, University of Dayton, Dayton, Ohio 45469. On the asymptotic behavior of a fundamental set of solutions.

We consider  $n^{\text{th}}$  order homogeneous linear ordinary differential equations whose coefficients have an asymptotic expansion as  $x \rightarrow \infty$  in terms of real powers of  $x$  and are analytic in sectors of the complex plane. We consider coefficients in a field of the type developed by Strodt (Trans. Amer. Math. Soc., 105 (1962), 229-250). By successive algebraic transformations, the asymptotic behavior of the solutions can be read from the equation. This generalizes the classical results when  $\infty$  is a singular point and the coefficients are analytic in neighborhoods of  $\infty$ . The strength of these results is that the coefficients need not be defined in a full neighborhood of  $\infty$ , and the asymptotic behavior can be read directly from the equation. (Received October 18, 1978)

763-34-22 F. V. Atkinson, University of Toronto, Toronto, Ont., M5S 1A1, Canada. Subgroups of the reals and asymptotic stability for second-order nonlinear oscillations.

Let  $f(t)$  be positive and nondecreasing on  $[0, \infty)$ , with  $f(t+0)/f(t-0)$  uniformly bounded. In a recent paper of the author (Proc. Roy. Soc. Edin.(A), 78,



299-314, (1978)), it was shown that one can associate with  $f$  an additive group  $S(f)$  of reals  $\lambda$  such that for some  $g \in L(0, \infty)$  the equation  $y'' + \lambda^2(r^2 + fg)y = 0$  has a solution not tending to zero as  $t \rightarrow \infty$ . The present paper derives a related, though slightly less precise, result for nonlinear equations such as  $y'' + f^2y^3 = 0$ , or similar equations with damping. The place of the parameter is now taken by the quantity  $\pi / \lim \{f(t)r(t)\}$ , where  $r(t)$  is the distance between consecutive zeros of  $y$  enclosing  $t$ , and  $y$  is a solution not tending to zero; this quantity must lie in a group which includes  $S(f)$ , but may still be a proper subset of the reals. The effect is to limit solutions not tending to zero to a certain set of possible limiting amplitudes. (Received October 18, 1978.)

\*763-34-23 James H. Lightbourne, III, Pan American University, Edinburg, Texas 78539. Semilinear functional differential equations via the variation of constant formula. Preliminary report.

Suppose  $X$  is a Banach space and the family  $\{A(t): t \geq 0\}$  generates a linear evolution system on  $X$ . Let  $r > 0$  and  $\mathcal{C} = \mathcal{C}([-r, 0]; X)$  denote the Banach space of continuous functions  $\phi: [-r, 0] \rightarrow X$  with  $\|\phi\| = \max \{ \|\phi(\theta)\| : -r \leq \theta \leq 0 \}$ . Suppose  $D \subset \mathcal{C}$  and  $F: [0, \infty) \times D \rightarrow X$ . We investigate the functional differential equation

$$* \quad x'(t) = A(t)x(t) + F(t, x_t), \quad t \geq 0$$

$$x_0 = \phi \in D$$

via the variation of constant formula satisfied by the map  $t \rightarrow x_t$ . We treat the questions of existence of solution to (\*), function space flow invariance for (\*), and existence of periodic solutions to (\*). Examples illustrating the applications of these results are given. (Received October 20, 1978.)

\*763-34-24 TAKAŠI KUSANO, Hiroshima University, Hiroshima, Japan. Nonlinear Oscillation of higher-order functional differential equations with deviating arguments.

The differential equation to be considered is

$$(*) \quad L_n x(t) + f(t, x(g_1(t)), \dots, x(g_m(t))) = 0,$$

where

$$L_n = \frac{1}{p_n(t)} \frac{d}{dt} \frac{1}{p_{n-1}(t)} \frac{d}{dt} \dots \frac{d}{dt} \frac{1}{p_1(t)} \frac{d}{dt} \cdot$$

and the following conditions are assumed to hold: (a)  $p_i \in C[[a, \infty), \mathbb{R}]$ ,  $p_i(t) > 0$ ,  $0 \leq i \leq n$ ,  $\int_a^\infty p_1(t) dt = \infty$ ,  $1 \leq i \leq n-1$ ; (b)  $g_j \in C[[a, \infty), \mathbb{R}]$ ,  $g_j(t) \rightarrow \infty$  as  $t \rightarrow \infty$ ,  $1 \leq j \leq m$ ; (c)  $f \in C[[a, \infty) \times \mathbb{R}^m, \mathbb{R}]$ ,  $u_1 f(t, u_1, \dots, u_m) > 0$  for  $u_1, u_j > 0$ ,  $1 \leq j \leq m$ . An attempt is made to obtain necessary and sufficient conditions in order that every solution of (\*) be oscillatory when  $n$  is even, and every solution be either oscillatory or tending monotonically to zero as  $t \rightarrow \infty$  when  $n$  is odd. (Received October 19, 1978.) (Author introduced by Professor John R. Graef).

763-34-25 HIROSHI ONOSE, Ibaraki University, Mito 310, Japan. Further results on the oscillation of functional differential equations. Preliminary report.

We consider the equation of the form (1)  $[r(t)y''(t)]' + f(y(g(t)), t) = 0$ , and use the notation

$$R_T(t) = \int_T^t \frac{(t-s)(s-T)}{r(s)} ds \text{ for some } T > 0. \text{ Theorem. Let equation (1) be either superlinear or sublinear.}$$

(i) A necessary and sufficient condition for (1) to have a solution  $y(t)$  such that  $\lim_{t \rightarrow \infty} y(t)/R_T(t) = a \neq 0$ , is that  $\int_a^\infty |f(cR_T(g(t)), t)| dt < \infty$  for some  $c \neq 0$ . (ii) A necessary and sufficient condition

for (1) to have a solution  $y(t)$  such that  $\lim_{t \rightarrow \infty} y(t) = b \neq 0$ , is that  $\int_{T_1}^{\infty} R_T(t) |f(c,t)| dt < \infty$  for some  $c \neq 0$ . By using this, we shall establish the oscillation theorems and shall mention to the related results. (Received October 18, 1978.)

763-34-26 MYRON K. GRAMMATIKOPOULOS, Department of Mathematics, University of Ioannina, Ioannina, Greece. Oscillation Theorems for Second Order Ordinary Differential Inequalities and Equations with Alternating Arguments. Preliminary report.

In this paper we study differential equations and inequalities of the following forms:

$$(*) \quad [r(t)x'(t)]' + p(t)f(x(t)) + H(t, x(t), x'(t)) = 0, \quad t \geq t_0$$

$$(**) \quad [r(t)x'(t)]' + p(t)f(x(t)) \leq 0, \quad t \geq t_0$$

where for the real-valued functions  $r$ ,  $p$ ,  $f$  and  $H$  the following assumptions are made:

- (i)  $r$  is continuous and positive on  $[t_0, \infty)$ ;
- (ii)  $p$  is locally summable on  $[t_0, \infty)$ ;
- (iii)  $f$  is increasing and continuously differentiable on  $\mathbb{R} - \{0\}$  and has the sign property  $yf(y) > 0$  for every  $y \in \mathbb{R} - \{0\}$
- (iv)  $H$  is continuous on the set  $E = [t_0, \infty) \times \mathbb{R}^2$  and such that for every  $(t, y, z) \in E$  with  $y \neq 0$   $yH(t, y, z) > 0$ .

Here we are interested in finding sufficient conditions in order that all solutions of the equation (\*) be oscillatory. The results obtained include previous ones of Kiguradze, Kamenev and others and also give some new oscillation criteria. (Received October 23, 1978.)

\*763-34-27 JOHN R. GRAEF and PAUL W. SPIKES, Mississippi State University, Mississippi State, Mississippi 39762. Nonoscillation and asymptotic properties of solutions of functional differential equations.

The nonoscillation and convergence to zero of oscillatory solutions of perturbed functional differential equations has recently received a good deal of attention. A brief discussion of the problem and some new results in this direction are presented. (Received October 23, 1978.)

\*763-34-28 GARY D. JONES, Murray State University, Murray, Kentucky 42071. Oscillation Properties of  $n^{\text{th}}$  Order Linear Differential Equations.

Oscillation properties of  $n^{\text{th}}$  order linear differential equations are discussed. In particular, an example is given showing that it is possible for

$$y^{(2n)} + py = 0$$

to have both oscillatory and nonoscillatory solutions where  $p$  is positive, which is contrary to a result of Nehari. (See Nehari, Green's function and disconjugacy. Archive for Rat. Mech. and Anal. 62, 53-76(1976).) (Received October 23, 1978.)

\*763-34-29 W. E. MAHFOUD, Murray State University, Murray, Kentucky 42071. Comparison theorems for delay differential equations.

We consider the delay equation  $x^{(n)}(t) + f(t, x(q(t))) = 0$  (1) where  $f$  is continuous,  $q$  is continuously differentiable and increasing, and  $xf(t, x) > 0$  if  $x \neq 0$ . We show that oscillation of (1) as well as boundedness of nonoscillatory solutions of (1) reduces to that of an ordinary differential equation and consequently any extension of such results (1) when known for  $x^{(n)} + f(t, x) = 0$  (2) becomes immediate. Illustrative examples are given and an improvement of some results is obtained. We also discuss the above results for a more general  $q(t)$  and conclude that for a bounded delay (1) and (2) have the same oscillatory behavior. (Received October 23, 1978.)

\*763-34-30 VASILIOS A. STAIKOS, Department of Mathematics, University of Ioannina, Ioannina, Greece.  
Differential Inequalities and Comparison Results in Oscillation Theory. Preliminary report.

An important problem in the Oscillation Theory is to give comparison results for the oscillatory and asymptotic behavior of solutions of differential equations of arbitrary order. Kartsatos [J. Math. Anal. Appl. 52(1975), pp. 1-9] was the first obtained some results on this subject for ordinary differential equations. Some results of the same nature but for differential equations with deviating arguments appeared later in the papers of Kartsatos and Onose [Bull. Austral. Math. Soc. 14(1976), pp. 343-347] and Onose [Bull. Austral. Math. Soc. 13(1975), pp. 13-19; Utilitas Math. 10(1976), pp. 185-191]. Here, we gather all results contained in the above mentioned papers, extend them in various directions, give some more ones of the same nature and we present the whole material as a distinct complete unit. The basic tool of the technique used is the introduction of differential inequalities assigned to the differential equations under consideration. (Received October 23, 1978).

\*763-34-31 CHARLES T. FULTON, Pennsylvania State U., Univ. Park, PA. 16802 and STEVEN PRUESS, University of New Mexico, Albuquerque, N.M. 87131. Numerical Methods for a Singular Eigenvalue Problem with Eigenparameter in the Boundary Conditions.

In this paper we consider the problem

$$(*) \begin{cases} (1) & \tau u = \frac{1}{k}[-(xu)'] = \lambda u, \quad x \in (0, b], \quad k(x) = k_1 x + k_2 x^2 + \dots \text{ near } x=0 \\ (2) & B(u) = 0, \quad B(\cdot) \text{ being the 'boundary value for } \tau \text{' which gives the Friedrichs' B.C. at } x=0 \\ (3) & -(\beta_1 u(b) - \beta_2 (xu)'(b)) = \lambda(\beta_1' u(b) - \beta_2' (xu)'(b)) \end{cases}$$

and show that the methods of Pruess, SIAM J. Numer. Anal. 10(1973), 55-68, and Numer. Math. 24(1975), 241-247, can be adapted to yield an algorithm for the computation of the eigenvalues and eigenfunctions, which is based on approximating the coefficient functions  $p(x)=x$  and  $k(x)$  by piecewise constant functions. The algorithm computes  $O(h^2)$ -approximations to the eigenvalues, but an a posteriori correction gives  $O(h^4)$ -approximations (supported by EROC data). As in the regular S.-L. problems studied by Pruess the algorithm is particularly well adapted to give good accuracy for the higher eigenvalues. The problem arises on separation of variables in the heat equation for a cylindrical rod which is immersed at time  $t=0$  in a finite mass of liquid. The mathematical theory has been given by J. Walter, Math. Z. 133(1973), 301-312, and Fulton, Proc. R.S.E. 77A(1977), 293-308, in the regular case, and by Fulton, "Singular Eigenvalue Problems with Eigenvalue Parameter contained in the boundary conditions," Proc. Roy. Soc. Edin., Ser. A (to appear; Abstract: A.M.S. Notices, Jan., 1978, p. A-97) in singular cases which include (\*). As usual, (\*) admits interpretation as the eigenvalue problem of a Self-Adjoint Operator in the direct sum Hilbert space,  $H = L_2(0, b) \oplus \mathcal{C}$  ( $\mathcal{C}$  = Complex numbers). (Received October 24, 1978).

\*763-34-32 RICHARD C. BROWN, University of Alabama, University, Alabama 35486.  
Adjoint and Extension Theory of Abstract Boundary Value Problems.

Let  $A: X \rightarrow Y$  be a densely defined closed operator where  $X$  and  $Y$  are Banach spaces. Let  $F$  be a locally convex t.v.s. and  $H: X \rightarrow F$  an operator such that  $D(H) \supset D(A)$  and  $D(H^*)$  is total over  $F$ . We compute  $A_H^*$  and  $A_H^*$  where  $A_H$  is the operator determined by  $A$  on  $N(H)$  and  $A_H(x) = (Ax, Hx)^t$ . We also characterise certain closed extensions of  $A_H$  and the adjoints of these extension. In particular application is made to the problem of determining self-adjoint extensions of symmetric operators restricted by boundary conditions in a Hilbert space. This material complements recent work by Coddington and Dijksma, and Lee. (Received October 23, 1978.)

763-34-33 LEIGH C. BECKER, Southern Illinois University, Carbondale, Illinois 62901. Some results for Volterra integrodifferential equations. Preliminary report.

The integrodifferential equations  $x'(t) = A(t)x(t) + \int_s^t C(t, u)x(u)du$  for  $0 \leq s \leq t < \infty$  are considered where  $x$  is an  $n$ -vector while  $A$  and  $C$  are  $n \times n$  matrices. Stability results for the case when  $A(t) \equiv A$ , a constant stable matrix, are obtained in terms of  $B$ , the symmetric positive definite matrix such that  $A^T B + BA = -I$ , via a Liapunov functional. Among other results, a variation of

parameters formula is obtained for  $x'(t) = A(t)x(t) + \int_0^t C(t,u)x(u)du + f(t)$  whereby a solution of the latter can be expressed in terms of solutions of the aforementioned family of equations.

(Received October 23, 1978.)

\*763-34-34 Bhagat Singh, University Center-Manitowoc County, Manitowoc, Wisconsin 54220  
Necessary and sufficient condition for eventual decay of oscillations in general functional equations with delays.

We study the equation

$$(A) \quad (r(t)y'(t))^{(n-1)} + a(t)h(y(g(t))) = f(t)$$

subject to some regularity assumptions on  $a(t), g(t), r(t)$  and  $f(t)$ ; and  $g(t) \leq t$ ,  $g(t) \rightarrow \infty$  and  $\int_0^\infty 1/r(t)dt = \infty$ ,  $r(t) > 0$ . Theorem 3\*2 In addition suppose  $\int_0^\infty t^{n-1}|f(t)|dt < \infty$  and

$\int_0^\infty t^{2n-2}|a(t)|dt < \infty$ , then all oscillatory solutions of (A) approach zero. This theorem is used to obtain the following main theorem.

Theorem (4\*1). Suppose  $a(t) > 0$ ,  $\int_0^\infty t^{2n-2}a(t)dt < \infty$  and  $\frac{f(t)}{t^{n-1}a(t)}$  approaches a limit. Then a necessary and sufficient condition for all oscillatory solutions of (A) to approach zero is

$$\lim_{t \rightarrow \infty} \frac{f(t)}{t^{n-1}a(t)} = 0. \quad (\text{Received October 23, 1978.})$$

763-34-35 JOHN W. HOOKER and C.E. LANGENHOP, Southern Illinois University, Carbondale, Ill. 62901. On regular systems of ordinary differential equations, I. Preliminary report.

In the system (1)  $A\dot{x} + Bx = f$  let  $A$  and  $B$  be  $n \times n$ ,  $x$  and  $f$  be  $n \times 1$  over the complex numbers  $C$ , and suppose  $A$  is singular. We say the system is regular if  $As + B$  is non-singular for some  $s \in C$ . In that case, for some integer  $\mu \geq 1$  and some  $n \times n$  matrices  $Q_k$  one has  $(A + zB)^{-1} = \sum_{k=-\mu}^\infty Q_k z^k$  for all  $z$  such that  $0 < |z| < \delta$ , some  $\delta > 0$ . If  $f$  is sufficiently differentiable, then each solution of (1) has the form

$$x(t) = [\exp(-Q_0 B t)] Q_0 A \xi + \int_0^t [\exp Q_0 B(\tau - t)] Q_0 f(\tau) d\tau + \sum_{k=0}^{\mu-1} (-Q_{-1} A)^k Q_{-1} f^{(k)}(t)$$

for some  $n \times 1$  vector  $\xi$ . (See Campbell, Meyer, and Rose, SIAM J. Appl. Math., 31 (1976), pp. 411-425, for an alternative representation involving Drazin inverses.) (Received October 23, 1978.)

763-34-36 C.E. LANGENHOP AND JOHN W. HOOKER, Southern Illinois University, Carbondale, Ill. 62901. On regular systems of ordinary differential equations, II. Preliminary report.

Consider the system (1)  $(\sum_{k=0}^r A_k D^k)x = 0$  ( $D=d/dt$ ) where  $x$  is  $n \times 1$  and the  $A_k$  are  $n \times n$  over the complex numbers  $C$  with  $A_r$  possibly singular. We assume  $\det \alpha(s) \neq 0$  where  $\alpha(s) = \sum_{k=0}^r A_k s^k$ ,  $s \in C$ . If  $\det \alpha(\lambda) = 0$  and  $v_1, \dots, v_\sigma$ ,  $\sigma \geq 1$ , are  $n \times 1$ ,  $v_1 \neq 0$  and  $\sum_{j=0}^r \frac{1}{j!} \alpha^{(j)}(\lambda) v_{p-j} = 0$ ,  $1 \leq p \leq \sigma$  with  $v_i = 0$  for  $i < 0$ , then  $v_1, \dots, v_\sigma$  is a singular sequence for  $\alpha(s)$  belonging to the singular value  $\lambda$ . For a singular value  $\lambda$  of multiplicity  $\mu$  there are singular sequences  $v_i(h)$ ,  $i = 1, \dots, \sigma(h)$ ,  $h = 1, \dots, \tau$  for some  $\tau \geq 1$  such that  $\sigma(1) + \dots + \sigma(\tau) = \mu$  and  $v_1(1), \dots, v_1(\tau)$  are linearly independent. The set of vector functions  $\sum_{j=0}^{i-1} \frac{1}{j!} t^j e^{\lambda t} v_{i-j}(h)$ ,  $i = 1, \dots, \sigma$ ,  $h = 1, \dots, \tau$  over all singular values of  $\alpha(s)$  then forms a basis for the solution space of (1). (Received October 23, 1978.)

\*763-34-37 CHAMPAK D. PANCHAL, University of North Florida, Jacksonville, Florida 32216. A differential equation for undamped forced nonlinear oscillations. Preliminary report.

For the boundary value problem  $u'' + u^3 = f$ ,  $u(0) = u(1) = 0$ , which arises in the study of undamped forced non-linear oscillations, it is known that a solution exists when  $f$  is a constant or  $f$  is very close

to zero in the norm of  $C[0,1]$ . The results of G. Morris (Proc. Camb. Phil. Soc, 51, 1955, 297-312 and 54, 1958, 426-438.) imply the existence of the solution for each  $f$  in  $C[0,1]$  satisfying  $f(0)=f(1)=0$ . We establish here the existence of solutions to the above problem for all  $f$  in  $C[0,1]$  satisfying  $\|f\| \leq 32\sqrt{6}/9$ . (Received October 23, 1978.)

\*763-34-38 A. S. VATSALA, The University of Texas at Arlington, Arlington, TX 76019 and R. L. VAUGHN, Texas Christian University, Fort Worth, TX 76129. Existence and comparison results for differential equations of Sobolev type. Preliminary report.

Local and global existence results for differential equations of Sobolev type are considered. After developing the theory of differential inequalities, the existence of maximal and minimal solution is discussed. These results are then used to derive comparison results. (Received October 23, 1978.)

\*763-34-39 ALFONSO CASTRO. Departamento de Matemáticas, CIEA del IPN, Apartado Postal 14-740, México 14, D.F., MEXICO. A two point BVP with jumping nonlinearities.

We study the problem (1)  $\{u''(t) + g(u(t)) = p(t) \quad t \in [0, \pi], u(0) = (\pi) = 0\}$ , where  $g$  and  $p$  are continuous functions. We prove that (1) has a solution if (i)  $\int_0^\pi p(t) \sin(t) dt \leq 0$ , (ii)  $g(u) = u$  for  $u \geq 0$ , (iii) there exists  $\alpha > 0$  such that  $g(u)/u \rightarrow 1 + \alpha$  as  $u \rightarrow -\infty$ , and, (1V)  $(g(u) - (\int_0^\pi g(s) ds)/u) \rightarrow 0$  as  $u \rightarrow -\infty$ .

This generalizes a result of L. Aguinaldo and K. Schmitt (Proc. A.M.S., Jan 1978) where a question of S. Fucik is answered. Our proof is entirely variational and provides a minmax characterization of the solution. (Received October 25, 1978.)

763-34-40 Kuo-Liang Chiou, Wayne State University, Detroit, Michigan 48202. Stability criteria for equation  $P(D)x(t) + f(t)q(D)x(t) = 0$ .

Consider the feedback control system (\*)  $P(D)x + f(t)q(D)x = 0$ ,  $t \geq 0$  where  $D = \frac{d}{dt}$ ,  $f(t)$  is a real continuous function,  $f(t+T) = f(t)$ ,  $T > 0$ ,  $P(s)$  and  $q(s)$  are polynomials in  $s$  with no common factors,  $P(s)$  is a monic polynomial, and the degree of  $p(s)$  is greater than the degree of  $q(s)$ . We have obtained several stability criteria via the theory of indefinite  $J$ -spaces and Hamiltonian systems. (Received October 25, 1978.)

## 35 ► Partial Differential Equations

763-35-1 M. A. AL-BASSAM, Kuwait University, Kuwait. On partial differential equations associated with Euler-Poisson-Darboux Equation.

Let the differential operator  $E_{\gamma, \delta, \lambda}^{\alpha, \beta} \equiv D_{xy} + ((\alpha + \gamma)/(x-y) + \delta/x) D_y - ((\beta + \gamma)/(x-y) + \lambda/y) D_x - [\gamma(\alpha + \beta + \gamma - 1)/(x-y)^2 + \lambda(\alpha + \gamma)/y(x-y) + \delta(\beta + \gamma)/x(x-y) + \lambda\delta/xy]$ , where  $\alpha, \beta, \gamma, \delta$  and  $\lambda$  are numbers. Mambriani [Ann. Scuola Norm. Sup. Pisa, 11(1942)] discussed the equations  $E_{\gamma, 0, 0}^{\alpha, \beta} u(x, y) = 0$ , and used the fractional derivative operator for solving  $E_{0, 0, 0}^{\alpha, \beta} u(x, y) = 0$ . In this work a study of the equations (A) :  $E_{\gamma, \delta, \lambda}^{\alpha, \beta} u(x, y) = 0$  has yielded the following results: (1) (A) are invariant under the substitution  $\begin{pmatrix} \alpha & \beta & \gamma & \delta & \lambda & x & y \\ \beta & \alpha & \gamma & -\lambda & -\delta & y & x \end{pmatrix}$ . (2) Differential operators identities have been obtained such as  $E_{0, \delta, 0}^{\alpha, \beta} (x-y)^\gamma y^{-\lambda} \equiv E_{\gamma, \delta, \lambda}^{\alpha, \beta}$ , and many others. (3) Properties of equivalence between these equations and their corresponding H-R transform equations have been established, by a similar approach used by the author in studying ordinary differential equations [J. Reine Angew. Math., 215(1964), 225(1967) and 288(1976)]. (4) By using the above results and applying the operational properties of the integro-differential operator of generalized order, solutions of (A) have been obtained. (Received June 30, 1978.)

A new theory based on the differential transform instead of integral transforms is proposed. The differential transform is related to a formal integral transform in which integrations have been replaced by differentiations by means of a formal application of Cauchy's integral formulas. Although the differential transform is free of integrals, nevertheless, it retains those properties to which integral transforms owe their usefulness. In particular, the property of a function to satisfy a linear partial differential equation (LPDE) and linear boundary conditions is invariant under the application of the differential transform. Hence, sometimes new solutions to LPDEs can be obtained by applying the differential transform to known solutions. An example is the new two-dimensional distortionless wave

$$\sqrt{r+x} \frac{1}{r} F(r-ct), \quad r^2 = x^2 + y^2,$$

which is not documented in the literature and was obtained by applying the differential transform to a known solution. This new two-dimensional distortionless wave fills the gap between the well known one- and three-dimensional distortionless waves  $F(x-ct)$  and  $F(R-ct)/R$ ,  $R^2 = x^2 + y^2 + z^2$ ,  $F$  being an arbitrary, twice differentiable function of its argument. (Received July 17, 1978.)

In this talk I shall describe an interpretation of Macdonald's  $\eta$ -function identities in terms of the heat equation. This interpretation is given by the following result.

**THEOREM** Let  $H_a(g, t)$  be the solution of the heat equation on the compact Lie group  $G$  with the initial data  $v(x) = \sum \chi_\lambda(a) \chi_\lambda(x)$  then  $H_a(g, t) = \prod_{n=1}^{\infty} \det(1 - \exp(2\pi i n t) \text{Ad}g)$ , where  $\chi_\lambda$  is the character with highest weight  $\lambda$  and  $a$  is a special point on  $G$  called "principal of type  $\rho$ ."

I shall then describe this result in more detail for the example of the group  $SU(2)$  and indicate how the theorem can be proved. (Received October 10, 1978.)

Two problems are considered for the even single-valued solutions of the equation

$$[\partial^2/\partial x^2 + \partial^2/\partial y^2 + (2\alpha+1)/x \partial/\partial x + (2\beta+1)/y \partial/\partial y] \phi = 0, \quad \alpha > \beta > -1/2.$$

(1) Let  $\phi$  be regular on  $\Sigma$ , the closed unit hypersphere about the origin in  $E^{2(\alpha+\beta+1)}$ . Classes  $R_{n,\nu}$  of multi-valued Newtonian potentials are constructed over which the Chebyshev norms

$$e_{n,\nu}(\phi) = \inf\{\sup\{|\phi(x,y) - N(x,y)| : (x,y) \in \Sigma\} : N \in R_{n,\nu}\}, n, \nu = 0, 1, 2, \dots$$

are defined. Properties of these norms characterize the singularities of the harmonic continuation of  $\phi$  to  $\text{comp}(\Sigma)$ .

(2) A Cauchy problem is considered where Chebyshev approximation to the data by Newtonian potentials defines a region in which  $\phi$  expands as a geometrically convergent series of Newtonian potentials. Information concerning the growth of an entire function  $\phi$  is calculated from the approximates. (Received October 16, 1978.)

We consider systems of reaction-diffusion equations on bounded domains with prescribed initial data, and homogeneous boundary conditions. We shall discuss some results on the stability of steady-state

solutions, and the construction of certain distinguished solutions. Our results are obtained via topological methods, using the Conley index of an isolated invariant set. Applications to ecological models with many species will be given. (Received October 23, 1978.)

763-35-6 DEAN A. PAYNE, Illinois State University, Normal, Ill. 61761. Bäcklund transformations in several variables. Preliminary report.

Bäcklund transformations have appeared in the recent literature on those nonlinear partial differential equations which admit special solutions called solitons. Until now these transformations have been limited to differential equations in two independent variables. This preliminary report describes a way to construct Bäcklund transformations for second order partial differential equations in any number of independent variables. The technique is based on a differential geometric approach which views a partial differential equation as a submanifold of an appropriate jet space. In this approach a Bäcklund transformation between equations in  $n$  independent variables is a  $3n$ -dimensional submanifold,  $M$ , (with certain regularity conditions) of  $J^1(\mathbb{R}^n, \mathbb{R}) \times J^1(\mathbb{R}^n, \mathbb{R})$  where  $J^1(\mathbb{R}^n, \mathbb{R})$  is the space of 1-jets of maps from  $\mathbb{R}^n$  to  $\mathbb{R}$ . The differential equations related by the transformation arise from integrability conditions for the two differential systems on  $M$  inherited from the two contact systems on the respective factors of  $J^1(\mathbb{R}^n, \mathbb{R}) \times J^1(\mathbb{R}^n, \mathbb{R})$ . (Received October 23, 1978.)

\*763-35-7 C. Y. CHAN and E. C. YOUNG, Florida State University, Tallahassee, Florida 32306. Comparison theorems for a coupled system of hyperbolic differential inequalities. Preliminary report.

Let  $R = D \times (0, T)$  where  $D$  is a bounded domain in  $E^n$ . It is shown that if  $(u, v)$  satisfies the coupled system of hyperbolic differential inequalities  $u_{tt} + (k/t)u_t - (a_{ij}u_{x_i}u_{x_j}) + bu - cv \geq 0$ ,  $v_{tt} + (k/t)v_t - (A_{ij}v_{x_i}v_{x_j}) + Bv + Cu \leq 0$  under appropriate conditions on the coefficients such that  $u > 0$  in  $R$ ,  $u = 0$  on  $\partial D \times [0, T]$ , and  $u(x, T) = 0$  for  $k > 0$ , then  $v$  must vanish at some point in  $R$ , provided  $v$  is positive somewhere in  $R$ . If in addition,  $u(x, 0) = 0$  for  $k \leq 0$ , then  $v$  must vanish in  $D \times [0, T)$ , provided  $v$  is positive somewhere in  $R$ . Similar results under more general boundary conditions are also given. (Received October 23, 1978.)

\*763-35-8 V. LAKSHMIKANTHAM and A. S. VATSALA, University of Texas at Arlington, Arlington, TX 76019 and S. LEE LA, SUNY at Geneseo, Geneseo, NY 14454. Comparison theorems for reaction-diffusion equations in abstract cones. Preliminary report.

One of the important and effective methods in the qualitative theory of differential equations is the comparison technique. This method has also proved to be a useful tool for the qualitative analysis of reaction-diffusion equations. Unfortunately, comparison theorems for systems demand certain monotonicity requirements on the reaction terms which are physically unreasonable. To circumvent this difficulty, one has two alternatives. This paper investigates these two situations. (Received October 23, 1978.)

763-35-9 R. K. Juberg, University of California, Irvine, California 92717. Stabilization for certain parabolic problems. Preliminary report.

A complete analysis is given of the stabilization problem for general mixed initial boundary value problems in a half space for certain parabolic partial differential equations of arbitrary order in one space variable. (Received October 24, 1978.)

\*763-35-10 David V. Chudnovsky, Columbia University, New York, N.Y. 10027. Infinite component non-linear Schrödinger and modified Korteweg-deVries equations. Preliminary report.

We investigate the following non-stationary equations in  $\varphi(x, t, k)$  and  $\psi(x, t, k)$ . The first system is  $i\varphi_t = \varphi_{xx} + \varphi \int_{-\infty}^{\infty} \varphi \psi dk$ ,  $-i\psi_t = \psi_{xx} + \psi \int_{-\infty}^{\infty} \varphi \psi dk$  (the generalized infinite component non-linear Schrödinger) and the second is  $\varphi_t = \varphi_{xxx} + \varphi \int_{-\infty}^{\infty} \varphi \psi dk +$

$\int_{-\infty}^{\infty} \psi_x \psi dx$ ,  $\psi_t = \psi_{xxxx} + \psi_x \int_{-\infty}^{\infty} \psi \psi dx + \psi \int_{-\infty}^{\infty} \psi_x \psi dx$  (the infinite component generalized non-linear modified KdV). The usual infinite component NLS and MKdV correspond to the

case  $\psi = \varphi^*$  and  $\psi = \varphi$ , respectively. Finite component NLS and MKdV arise when

$\varphi = \sum_{i=1}^n \varphi_i \delta(k-i)$ ,  $\psi = \sum_{i=1}^n \psi_i \delta(k-i)$ . We show that infinite component systems have infinitely many conservation laws and we show that the solutions of these systems are connected with the solution of inverse scattering problems for the Scrodinger operator. Particular non-meromorphic solutions corresponding to hyperelliptic curve were found. (Received October 24, 1978.)

\*763-35-11 R. KENT NAGLE, University of South Florida, Tampa, Florida 33620. Nonlinear elliptic boundary value problems at resonance. Preliminary report.

Consider the boundary value problem  $\Delta u + G(x, u, \dots, D^{2m-1}u) = f(x)$  for  $x$  in  $\Omega$ ,  $B(u) = 0$  on  $\partial\Omega$ . Let  $L$  be uniformly elliptic of order  $2m$  in a bounded domain  $\Omega$  and  $B(u) = 0$  be linear boundary conditions. Let  $G$  be a real-valued function of  $x, u$ , and the partial derivatives of  $u$  up to order  $2m-1$ . Assume the problem  $\Delta u = 0$  in  $\Omega$ ,  $B(u) = 0$  on  $\partial\Omega$  has nontrivial solutions. Conditions are given on  $f$  in terms of  $G$  which are sufficient for the existence of solutions. These conditions are compared with other Landesman - Lazer conditions. Of particular interest are nonlinearities involving derivatives and the corresponding Landesman - Lazer conditions. (Received October 25, 1978.)

\*763-35-12 Jagdish Chandra, U.S. Army Research Office, Research Triangle Park, NC 27709 and Paul Wm. Davis, Department of Mathematics, Worcester Polytechnic Institute, Worcester, MA 01609. Application of comparison principles to systems arising in combustion theory. Preliminary report.

An important question in the theory of combustion is determining the critical temperature to which a fuel-air mixture must be raised to initiate an explosion. For a prototype combustion problem, we illustrate the derivation of an appropriate comparison principle for the governing system of differential equations and its application to the estimation of the critical temperature. (Received October 25, 1978.)

763-35-13 HEINZ-OTTO KREISS, Department of Mathematics, California Institute of Technology, Pasadena, California 91125. Problems with different time scales and their numerical solution.

• Ordinary and partial differential equations which describe physical phenomena often leave solutions which vary on vastly different time scales. This leads to serious difficulties if one wants to solve these problems numerically. Various techniques and applications are discussed and the connection with singular perturbation theory is investigated. (Received October 25, 1978.)

### 39 ► *Finite Differences and Functional Equations*

763-39-1 William T. Patula, Southern Illinois University, Carbondale, IL 62901. Comparison theorems for second order linear difference equations. Preliminary report.

We consider second order linear difference equations of the form

(1)  $c_n x_{n+1} + c_{n-1} x_{n-1} = b_n x_n$ ,  $c_n > 0$ ,  $n \geq 0$ . Conditions for the existence of recessive and dominant solutions of (1) are discussed. Note that solutions  $u_n$  and  $v_n$  of (1) are called recessive and dominant, respectively, if  $u_n/v_n \rightarrow 0$ , as  $n \rightarrow \infty$ . Comparison results between recessive and dominant solutions of (1) and of (2) are established, under appropriate hypotheses, where (2) has the form  $r_n w_{n+1} + r_{n-1} w_{n-1} = d_n w_n$ ,  $r_n > 0$ . (Received October 2, 1978.)



763-39-2 Richard E. Rice, Union College, Schenectady, New York 12308. Fractional iterates of functions on restricted domains. Preliminary report.

Let  $f$  be a function on a set  $S$  with no acyclic orbits. For  $n \geq 0$  define  $f_n$  to be the restriction of  $f$  to the set  $D_n = \{x \in S : f^n(x) \text{ is an element of a cycle}\}$ , where  $f^n$  is the  $n^{\text{th}}$  iterate of  $f$ . A fractional iterate of  $f$  of order  $m > 1$  is any function  $g$  defined on the domain of  $f$  such that  $g^n = f$ . It is known that if  $f$  has a fractional iterate of order  $m$ , then for each  $n \geq 0$   $f_n$  has a fractional iterate of order  $m$ . We show that the converse is false. In particular, for fixed  $M > 0$  we construct a function  $f$  such that: (1)  $f_n$  has iterative roots of all orders; (2)  $f$  has an iterative root of order  $k$  if and only if  $k$  divides  $M$ . We also discuss various classes of functions for which the converse is true. (Received October 25, 1978.)

## 40 ► Sequences, Series, Summability

\*763-40-1 John Fridy, Kent State University, Kent, Ohio 44242. Tauberian theorems via block dominated matrices.

The principle result is a Tauberian theorem that can be applied to any real regular matrix summability method. The Tauberian condition is determined by the lengths of the blocks of consecutive terms that dominate the rows of the matrix  $A$ : for each  $n$ , let  $B_n$  be a block of consecutive indices such that  $\liminf_n \{ |\sum_{k \in B_n} a_{nk}| - \sum_{k \notin B_n} |a_{nk}| \} > 0$ , and let  $L_n$  denote the length of  $B_n$ ; then  $\max_{k \in B_n} |(\Delta x)_k| = o(L_n^{-1})$  is a Tauberian condition for  $A$  over the bounded sequences. If  $A$  is row-finite, then the Tauberian condition is valid without restriction to bounded sequences. These results and their variants are used to provide a unified method of proving some of the classical Tauberian theorems for the methods of Abel, Borel, and Euler-Knopp. The general technique also yields new theorems for Nörlund and Taylor matrices. (Received July 14, 1978.)

\*763-40-2 LOUISE A. RAPHAEL, Clark College, Atlanta, Georgia, 30314. Stieltjes summability and application.

A summability method for divergent integrals which is a generalization of the Stieltjes transform (i.e. the twice iterated Laplace transform) is defined. Properties of Stieltjes summability are proved and this method is compared with Cesaro (Reisz) and Abel summability methods for divergent integrals. As an application of Stieltjes summability consider an  $L_2[0, \infty)$  function  $f$  which is expressed in terms of the eigenfunctions associated with a singular Sturm Liouville system  $y''(x) - q(x)y(x) = -ax$ ,  $0 \leq x < \infty$ , and  $q$  is a bounded, continuous, real valued  $L_1[0, \infty)$  function. The spectrum of this system is mixed, i.e., discrete and continuous. The proof of the main theorem gives an algorithm for constructing a sequence of  $L_2[0, \infty)$  functions which yields a stable pointwise approximation to  $f$  whenever  $f$  is continuous. It is this property that H. Diamond, M. Kon and the author used to prove that the Sturm Liouville expansion of an  $L_2[0, \infty)$  function  $f$  is Stieltjes summable (pointwise) to  $f$  whenever  $f$  is continuous. (Received July 21, 1978.)

763-40-3 David F. Dawson, North Texas State University, Denton, Texas 76203. Summability factors for certain sequence spaces. Preliminary report.

Let  $m, C, c, ac$ , and  $qc$  denote, respectively, the bounded, Cesàro summable, convergent, absolutely convergent, and quasiconvex complex sequences, and let  $E = \{m, C, c, ac, qc\}$ . If  $u, v \in E$ , define  $(u, v)$  as follows:  $\{f_p\} \in (u, v)$  iff the sequence of partial sums of  $\sum_p a_p$  is in  $v$  whenever the sequence of partial sums of  $\sum_p a_p$  is in  $u$ . Hadamard characterized  $(c, c)$  in 1903 by showing that  $(c, c) = ac$ . It follows from a 1917 result of Kojima that  $(C, C) = qc$ . The purpose of this paper is to characterize the elements of  $E \times E$  not previously characterized. (Received September 14, 1978.)

\*763-40-4 David Borwein and F. P. JONES, The University of Western Ontario, London, Canada.  
Weighted means, generalised Hausdorff matrices and the Borel property.

The summability matrix  $A$  is said to have the Borel property if almost all sequences of zeros and ones are  $A$ -convergent to  $1/2$ .

A weighted mean matrix  $(M, d_n)$  is a triangular matrix  $A = \{a_{nk}\}$  with  $a_{nk} = d_k/D_n$ , where  $d_0 > 0$ ,  $d_n \geq 0$ ,  $\sum_{k=0}^n d_k = D_n$ . In 1954 J. D. Hill posed the question as to whether the conditions  $D_n \rightarrow \infty$  and  $d_n/D_n \rightarrow 0$  are sufficient for  $(M, d_n)$  to have the Borel property. We give an example to show that these conditions are in fact not sufficient for the Borel property to hold.

Hill showed that a regular Hausdorff matrix has the Borel property if and only if the terms on its diagonal tend to zero. Our example destroys the hope of any such simple criterion for a generalised Hausdorff matrix to have the Borel property. On the other hand we do give some sufficient conditions for a generalised Hausdorff matrix to have the Borel property. Hill also showed that a Hausdorff matrix is strongly regular if and only if it has the Borel property. We show that for weighted mean matrices and, a fortiori, for generalised Hausdorff matrices, there is no simple relation between having the Borel property and being strongly regular. (Received October 2, 1978.)

763-40-5 M. R. Parameswaran, University of Manitoba, Winnipeg, Canada R3T 2N2. The Modified Reduction Principle in Summability and some applications.

The concept of a Modified Reduction Principle in summability is introduced and is shown to establish the connection between Product theorems and Tauberian theorems. [For the original Reduction Principle, see Zeller-(Beekmann): Theorie der Limitierungsverfahren (Springer Verlag Berlin-Heidelberg-New York 1970), p. 87.] (Received October 2, 1978.) (Author introduced by Professor H.C. Finlayson).

763-40-6 Gordon G. Johnson, JSC and University of Houston, Houston, Texas 77004. Moments and Differential Equations.

Results from moment theory are used to construct solutions to linear systems of the form  $F' = PF + G$ . (Received October 2, 1978.)

763-40-7 DENNIS C. RUSSELL, York University, Toronto, Canada M3J 1P3, and AMNON JAKIMOVSKI, Tel-Aviv University, Tel-Aviv, Israel. Beta-duals of matrix fields.

We consider necessary and sufficient conditions for a sequence  $b$  to belong to the beta-dual of a BK sequence space  $F$  (that is, for  $b$  to be a convergence-factor sequence for  $F$ ), where  $F = E_A$ , the field of a matrix  $A$  with respect to a BK-space  $E$  with a Schauder basis. When  $A$  is a normal matrix whose inverse  $A^{-1}$  is positive only on its diagonal elements, the results can be extended to the case  $F = E_{AD^{-1}}$ , where  $D$  is a normal matrix whose inverse has only a fixed finite number of nonzero diagonals. We make use of these results to obtain inclusion theorems of the form  $E_A \subseteq E_B$ . A number of known results in summability theory can be deduced as special cases; for example, convergence factors for ordinary and absolute Cesàro and Nörlund means, and inclusion theorems for Riesz and generalized Cesàro means. (Received October 6, 1978.)

\*763-40-8 VIVIENNE M. MAYES, Baylor University, Waco, Texas 76703, and B. E. RHOADES, Indiana University, Bloomington, Indiana 47401. Some properties of the Leininger generalized Hausdorff matrix.

Generalizations of Hausdorff matrices have been defined by Hausdorff, Endl, Jakimovski, Harrell, and Leininger. It is shown that the Harrell and Leininger definitions are the same, and that the only matrices which are simultaneously Leininger and Jakimovski are those of Endl. Necessary and sufficient conditions are obtained for the Leininger matrix to be a Norlund or a weighted mean method. Sufficient conditions are established for the Leininger method to be conservative. (Received October 13, 1978.)

\*763-40-9 THOMAS A. KEAGY, Texas Eastern University, Tyler, Texas. Summability of matrix transforms of subsequences.

Conditions on the matrices T and A have been given (These Notices 25(1978) Abstract 78T-B85, A357) in order that for each sequence x with finite limit point k there exists a subsequence y of x such that each finite limit point of x is a T-limit point of Ay. In the current paper this result is utilized to obtain characterizations of several classes of sequences. The following is typical of these characterizations. Theorem. The sequence x converges if there exist a regular matrix T and a limit preserving by to c matrix A such that T(Ay) converges for each subsequence y of x. (Received October 16, 1978.)

763-40-10 VIVIENNE M. MAYES, Baylor University, Waco, Texas 76703. Classes of means which are common to Gronwall means. Preliminary report.

W.T. Scott and H.S. Wall Trans. Amer. Math. Soc. 51(1942)255-279 determined which Hausdorff means are also Gronwall means. In this paper necessary conditions are obtained for the Leininger and Jakimovski generalized Hausdorff means and The Norlund means to be Gronwall means. Also, necessary and sufficient conditions are obtained for the weighted means to be Gronwall means. (Received October 18, 1978.)

763-40-11 KARL-HEINZ INDLEKOFER, University of Paderborn, 4790 Paderborn, Fed. Rep. of Germany. On Euler- and Sonnenschein-matrices.

In 1949 J. Sonnenschein (Bull.Acad.Royale de Belgique 35 (1949), 594-601) defined a general class of sequence transformation matrices generated by a function  $u(z) = \sum_{k=0}^{\infty} u_k z^k$  with positive radius of convergence and satisfying  $u(1) = 1$ , the elements of the matrix  $a_{nk}$  being given by  $(u(z))^n = \sum_{k=0}^{\infty} a_{nk} z^k$  ( $n=0,1,\dots$ ). Euler-matrices are defined as transposed Sonnenschein-matrices. B. Bajanski (Acad.Serbe Sci.Publ.Inst.Math. 10 (1956), 131-152) gave general sufficient conditions for the permanence of the Sonnenschein-matrices. J. Clunie and P. Vermes (Acad.Roy.Belg.Cl.Sci. (5) 45 (1959), 930-954) showed, that the same conditions are also necessary for permanence. In this paper we give necessary and sufficient conditions for the permanence of Euler-matrices and prove sharp estimates of the rates of growth of the "norms"  $\sum_{n=0}^{\infty} |a_{nk}|$  and  $\sum_{k=0}^{\infty} |a_{nk}|$ . With these results we can prove a conjecture on Turán's equivalent power series. (Received October 19, 1978.)

763-40-12 JAMES DEFranza, Kent State University, Kent, Ohio 44240. Nonnegative  $\ell$ - $\ell$  Norlund Means. Preliminary report.

Let  $N_p$  be the Norlund mean defined by the transformation  $(N_p x)_n = P_n^{-1} \sum_k p_{n-k} x_k$ , where  $P_n$  is the  $n^{\text{th}}$  partial sum of p,  $p_n \geq 0$  for all n and  $p_0 > 0$ . This paper is a study of  $N_p$  as a mapping into  $\ell^1$  rather than c. In general, the matrix A is  $\ell$ - $\ell$  provided that  $\ell(A) \equiv A^{-1}[\ell^1] \supseteq \ell^1$ . Then  $N_p$  is  $\ell$ - $\ell$  iff  $p \in \ell^1$ . Results are obtained concerning the relative strength of these mappings. For example, an inclusion theorem is proved:  $\ell^1 \subseteq \ell(N_p) \subseteq \ell(N_q)$  iff  $b \in \ell^1$ , where  $b(x) = q(x)/p(x) = \sum b_n x^n$ ,  $p(x) = \sum p_n x^n$  and  $q(x) = \sum q_n x^n$ . THEOREM: The class of nonnegative  $\ell$ - $\ell$  Norlund means forms an ordered abelian semigroup with order relation given by  $\ell(N_p) \subsetneq \ell(N_q)$  and binary operation given by  $N_p \cdot N_q = N_g$ , where  $g_n = p_0 q_n + p_1 q_{n-1} + \dots + p_n q_0$ . (Received October 23, 1978.)

\*763-40-13 Martin Buntinas, Loyola University of Chicago, Chicago, Illinois 60626. Approximation by Abel means and Tauberian Theorems in BK-spaces.

Classically a Tauberian Theorem is a statement giving conditions under which some form of summability, such as Abel summability, of a series implies convergence of that series. Approximation by Abel means

in BK-spaces is considered instead of Abel summability and sectional convergence (AK) is considered instead of convergence. This results in approximation statements that can be defined on all BK-spaces but reduce to classical Tauberian Theorems when restricted to summability fields. It is shown that many of the Tauberian Theorems for the Abel summability method, such as Tauber's First and Second Theorem, the 0-Tauberian Theorem of Littlewood and the gap Tauberian Theorem of Hardy and Littlewood, can be so generalized. (Received October 24, 1978.)

\*763-40-14 John J. Sember, Simon Fraser University, Mathematics Department, Burnaby, B.C., Canada, V5A 1S6. The unrestricted sections of sequences.

If  $x$  is any sequence in a  $K$ -space, the set of unrestricted sections of  $x$  is

$$H(x) = \left\{ \sum_{k \in F} x_k \delta^k : F \in \Phi \right\}$$

where  $\Phi$  denotes the collection of all finite subsets of the positive integers. A survey will be given of recent results concerned with the topological properties of  $H(x)$ , with emphasis on the property of boundedness.

Interest in FK-spaces that are not solid, yet have the property that  $H(x)$  is bounded for every  $x$ , has led to the study of a class of sequence spaces which we call the  $\{N_\theta\}$  spaces, and the space  $|AC|$  of strongly almost convergent sequences. Some recent joint work with Freedman and Raphael in this area will also be discussed. (Received October 25, 1978.)

763-40-15 Bruce L. R. Shawyer, The University of Western Ontario, London, Ontario, Canada N6A 5B9. Maximal maximum modulus theorems for sequence spaces.

Let  $f(z) = \sum_0^\infty b_n z^n$  and  $x = \{x_k\}$  be a sequence of bounded variation. I have recently shown that

$$\|f(x)\|_{bv} \leq \sum_0^\infty |b_n|. \quad (1)$$

Here I characterise functions for which we obtain equality in (1). (Received October 25, 1978.) (Author introduced by Professor R.E. Rhoades).

\*763-40-16 Allen R. Freedman, Simon Fraser University, Burnaby, British Columbia, Canada V5A-1S6. Densities and summability.

A density is a function defined for all sets of positive integers and satisfying certain properties. The ordinary (lower) asymptotic density is one such density. Another is the uniform density defined by  $u(A) = \lim_{n \rightarrow \infty} \min_{m \geq 0} [(card(A \cap \{m+1, \dots, m+n\}))/n]$ . Again, a Banach limit restricted to (the characteristic sequences of) sets of integers is a density. We discuss several questions concerning the properties of densities in general and the application of densities to summability theory, in particular, to the characterization of certain strongly summable sequences, the convergence fields of certain matrices, and to the classification of Banach limits. (Received October 25, 1978.)

## 41 ► *Approximations and Expansions*

763-41-1 DOUGLAS S. BRIDGES, University College, Buckingham MK18 1EG, England.  
Constructive Aspects of Approximation Theory, Preliminary Report.

In this paper, we outline some recent work on best approximation theory within the framework of constructive mathematics. We discuss some general results, including a theorem asserting

the existence of best approximations under certain uniqueness conditions, and the application of this theorem to a constructive proof of existence of minimax polynomial approximations to elements of  $C[0,1]$ . This leads to further discussion of minimax polynomial approximations and the classical algorithms for their computation. The paper concludes with some remarks on constructive spline approximation, and suggestions for a systematic constructive investigation of the foundations of approximation theory, and those of numerical mathematics in general. (Received September 28, 1978.) (Introduced by Professor Fred Richman)

763-41-2 Cecelia H. Chu, DOE, Washington D. C. 20461. Some Examples of Integer Approximation.

Let  $E_n(f)$  be the best approximation to  $f(x)$  by polynomials, with real coefficients, of degree at most  $n$  and assume that  $f(x)$  and  $g(x)$  have derivatives up to order  $n+1$  on  $[-1,1]$ . We show that a well known theorem of S. N. Bernstein which states that  $E_n(f)$  less or equal to  $E_n(g)$  whenever  $f^{(n+1)}(x)$  less or equal to  $g^{(n+1)}(x)$  on  $[-1,1]$ , does not hold for integer approximation. (Received October 20, 1978.)

\*763-41-3 J. P. King, Lehigh University, Bethlehem, Pa. 18015. Probabilistic interpretation of some positive linear operators.

Let  $B_n(f)$  denote the  $n$ th Bernstein polynomial associated with the continuous function  $f$ . It is well-known that  $B_n(f)(x) = E(f(X_n))$ , where  $E$  denotes the expectation operator and  $X_n$  is the average number of successes in a sequence of  $n$  independent repetitions of an experiment with two outcomes. A natural way to generalize this procedure is to replace the probability of success  $x$  by a probability  $p_1(x)$  which varies from trial to trial. This method leads to operators  $L_n(f)(x) = \sum a_{nx}(x) f_n(k/n)$ , where the probabilities  $a_{nk}(x)$  are generated by the function  $g(z) = \prod_{i=1}^n p_i(x)z + 1 - p_i(x)$ . These operators, which were introduced by J.P. King [Can. J. Math. 18 (1969) 89-91] from the point of view of summability theory, are then natural probabilistic generalizations of the Bernstein polynomials.

A consideration of "waiting time" then leads to a probabilistic interpretation of generalized Baskakov operators and generalized Szasz operators. (Received October 23, 1978.)

763-41-4 Charles K. Chui and Lo-Yung Su, Texas A&M University, College Station, Texas, 77843. On best two-point local approximation. Preliminary report.

The study of best (one-point) local approximation has been shown to be equivalent to that of best (one-point) interpolation at least in  $L^2$  and  $L^\infty$ . In this paper, the problem of best two-point local approximation is introduced and studied in the general  $L^p$ -setting. Both positive and negative results will be discussed. (Received October 23, 1978.)

763-41-5 LYNN R. ZIEGLER, The Ohio State University, Columbus, Ohio 43210. Norm and Zero Asymptotics for extremal Polynomials.

Let  $\mu$  be a probability measure on  $[-1,1]$  with the property that  $0 < \underline{C} = \inf_{\mu(E)=1} C(E) \leq \overline{C} = C(S(\mu))$  where  $C(E)$  is the logarithmic capacity of  $E$  and  $S(\mu)$  is the support of  $\mu$ . Let  $P_n$  be a monic polynomial of degree  $n$  having minimal  $L_p(\mu)$  norm  $N_{n,p}$  amongst all such polynomials and let  $\nu_{n,p}$  be the probability measure having an atom of mass  $m/n$  at a zero of  $P_n$  of multiplicity  $m$ .

It is shown that  $\underline{C} \leq \lim_{n \rightarrow \infty} \frac{1}{n} N_{n,p} \leq \overline{\lim}_{n \rightarrow \infty} \frac{1}{n} N_{n,p} \leq \overline{C}$  and that  $\underline{C} \leq \lim_{p \rightarrow \infty} \lim_{n \rightarrow \infty} \frac{1}{n} N_{n,p} \leq \lim_{p \rightarrow \infty} \overline{\lim}_{n \rightarrow \infty} \frac{1}{n} N_{n,p} \leq \lim_{n \rightarrow \infty} \lim_{p \rightarrow \infty} \frac{1}{n} N_{n,p} = \overline{C}$  and that strict inequality and equality can hold in each case if  $S(\mu)$  is sufficiently nice.

Suppose that  $\nu$  is the weak limit of  $\{\nu_{n(k),p}\}$ . Then if  $p \geq 2$ ,  $\frac{1}{n(k),p} N_{n(k),p}$  converges to some number  $A$  and the logarithmic potential of  $\nu$  is bounded above by  $A$  and equals  $A$  on a carrier of  $\mu$  less a set of capacity 0. This can be used to show that  $\nu_{n,p} \rightarrow \bar{\mu}$  iff  $\frac{1}{n,p} N_{n,p} \rightarrow \overline{C}$  and  $\nu_{n,p} \rightarrow \underline{\mu}$  iff  $\frac{1}{n,p} N_{n,p} \rightarrow \underline{C}$  where  $\bar{\mu}$  is the equilibrium measure for  $S(\mu)$  and  $\underline{\mu}$  is the equilibrium measure for a carrier of capacity  $\underline{C}$ . (Received October 24, 1978.)

763-41-6 M. LACHANCE, E. B. SAFF, University of South Florida, Tampa, Florida 33620 and R. S. VARGA, Kent State University, Kent, Ohio 44242. Inequalities for polynomials with a prescribed zero.

For the collection of polynomials  $\pi_{s,m} := \{(z-1)^s \sum_{k=0}^m a_k z^k\}$  having a prescribed  $s$  order zero at  $z = 1$ , certain  $L_\infty$  extremal problems over the unit circle  $|z| = 1$  are reduced to real extremal problems in the interval  $[-1,1]$ . As a consequence we prove that, for each integer  $n \geq 1$ ,

$$\max_p |p(0)| = [\cos \frac{\pi}{2(n+1)}]^{n+1}$$

where  $p$  ranges over all polynomials of degree  $n$  with  $\max_{|z|=1} |p(z)| = 1$  and  $p(1) = 0$ . This solves a problem raised by G. Halasz (Bull. London Math. Soc., 4(1972), 354-366), and improves estimates obtained by Rahman and Schmeisser (Trans. Amer. Math. Soc., 216(1976), 91-103). In addition, growth estimates and convergence results are obtained for polynomials in  $\pi_{s,m}$  which are, in a limiting sense, best possible. (Received October 25, 1978.)

\*763-41-7 J. M. ASH and R. L. JONES, DePaul University, Chicago, IL 60614. Approximations of the second derivative. Preliminary report.

By a generalized second derivative quotient we mean

$$\Delta_{\underline{a}, \underline{b}} f(x, h) = h^{-2} \sum_{i=0}^{2+e} a_i f(x + b_i h)$$

where  $\sum a_i = \sum a_i b_i = 0$ ,  $\sum a_i b_i^2 = 2$ . (These conditions together with Taylor's theorem guarantee that  $f(x, h) \rightarrow f''(x)$  as  $h \rightarrow 0$  if  $f''(x)$  exists. Fix the integer  $e$  and say that  $(\underline{a}, \underline{b})$  is best if

$$(*) \quad \Delta_{(\underline{a}, \underline{b})} f(x, h) = f''(x) + O(h^m) \text{ for } f \in C^\infty$$

and no other choice of  $(\underline{a}, \underline{b})$  will produce a relation like (\*) with  $m$  replaced by a larger integer. Since  $(\underline{a}, \underline{b})$  has  $2(3+e)$  degrees of freedom, one might expect  $m$  to be  $2e+5$ . However  $m$  cannot exceed  $e+3$  for elementary reasons, which is surprising when  $e$  is large. When  $e=0$ ,  $m$  turns out to be 2. If  $f$  is considered to be analytic so that  $(\underline{a}, \underline{b})$  may be complex,  $m=3$ . Generalizations of this phenomena are also considered. (Received October 25, 1978.)

## 42 ► Fourier Analysis

763-42-1 J. -A. CHAO, University of Texas, Austin, Texas 78712. Hardy spaces of  $q$ -martingales. Preliminary report.

A  $q$ -adic martingale ( $q \geq 2$ ) is said to be in  $H^p$  if its maximal function is in  $L^p$ . Characterizations of  $H^p$   $q$ -martingales via square functions, atomic decomposition as well as conjugate transforms are given. Special attention is paid to the case  $p=1$ . Applications are also provided. (Received October 16, 1978.)

763-42-2 George Gasper, Northwestern University, Evanston, Illinois, 60201 and Walter Trebels, T. H. Darmstadt, D-6100 Darmstadt, West Germany. Multipliers for Fourier series and Jacobi series. Preliminary report.

Fix  $\alpha \geq \beta \geq -1/2$  and let  $R_n(x) = P_n^{(\alpha, \beta)}(x) / P_n^{(\alpha, \beta)}(1)$  where  $P_n^{(\alpha, \beta)}(x)$  is the Jacobi polynomial of degree  $n$ . Necessary conditions and sufficient conditions are given for a sequence  $\{m_n\}_{n=0}^{\infty}$  to be a multiplier for Jacobi series (and for Fourier series) by using the fact that if  $-1/2 < \gamma < \alpha + 2$ ,  $h_n = \|R_n\|_2^{-2}$ , and  $f(x) = \sum \hat{f}(n) h_n R_n(x)$  is a polynomial, then the weighted Parseval type relation

$$\int_{-1}^1 (1-x)^\gamma (f(x))^2 (1-x)^\alpha (1+x)^\beta dx \approx \sum (\Delta^\gamma \hat{f}(n))^2 h_n$$

holds where the fractional difference operator  $\Delta^\gamma$  is defined by

$$\Delta^\gamma m_n = \sum_{j=n}^{\infty} A_{j-n}^{-\gamma-1} m_j, \quad A_n^\gamma = \binom{n+\gamma}{n} = \frac{\Gamma(n+\gamma+1)}{\Gamma(n+1)\Gamma(\gamma+1)}. \quad (\text{Received October 17, 1978.})$$

763-42-3 W. B. JURKAT, Syracuse University, Syracuse, New York 13210 and G. SAMPSON, State University of New York at Buffalo, Buffalo, New York 14214. On weak restricted estimates and endperiod problems for convolutions with oscillating kernels (I). Preliminary report.

Throughout we consider  $K(t) = e^{i|t|^a} / |t|^b$ ,  $a > 0$ ,  $a \neq 1$ ,  $b < 1$  and  $t \in \mathbb{R}$ . Here we consider for fixed  $\lambda, \mu > 0$  the function  $B(\lambda, \mu; K) = B(\lambda, \mu) = \sup_{X_\lambda, X_\mu} \int X_\lambda(x) K * X_\mu(x) dx$  where the sup is

taken over all "characteristic" functions  $X_\lambda, X_\mu$  with complex signs (i.e.  $X_\mu$  is a measurable function for which  $|X_\mu| = 1$  on  $E$ ,  $|X_\mu| = 0$  off  $E$  and  $|E| \leq \mu$  ( $\mu > 0$ )). We have already shown sharp upper estimates for  $B(\lambda, \mu)$  (also see these notices Oct. 1978, pp. A-631, 760-B7). Here we improve our lower estimates and show that they match with our upper estimates. I.e. we are able to determine  $B(\lambda, \mu, K)$  up to constant factors for all  $\lambda$  and  $\mu$ . (Received October 20, 1978.)

\*763-42-4 P. ERDŐS, Hungarian Academy of Sciences, Budapest and B. SMITH, University of Kentucky, Lexington, KY 40506. Finite abelian group cohesion.

Let  $G$  be a finite abelian group. Let  $\text{card}(G) = n$ . For  $A, B \subset G$  let  $m(x, A, B) = \text{card}\{(a, b) : a+b = x\}$ . For  $E \subset G$  let  $E'$  denote its complement in  $G$ . Let

$$\text{coh } G = \min_{E \subset G} \max_{x \in G} |m(x, E, E) + m(x, E', E') - m(x, E, E') - m(x, E', E)|.$$

Theorem.  $\text{coh } G \geq n^{\frac{1}{2}}$ .

Theorem. Suppose  $G$  has no elements of order 2. Let  $\alpha > \frac{1}{2}$ . Then  $\text{coh } G < K \cdot n^\alpha$  ( $K$  depends only on  $\alpha$ .) (Received October 20, 1978.)

763-42-5 JOHN D. PESEK, Jr., University of Illinois, Urbana, Illinois 61801. Hardy spaces on  $\mathbb{R}^n$  defined by anisotropic geometries. Preliminary report.

$\mathbb{R}^n$  together with Lebesgue measure is made into a space of homogeneous type (Coifman and Weiss, Bull. Amer. Math. Soc. 83, 569-645) by means of a quasi-distance  $p(x-y)$ . To define  $p(x)$ , we postulate a sequence  $A_k$ ,  $k \in \mathbb{Z}$ , of linear maps on  $\mathbb{R}^n$  so that  $q|A_k x| \leq |A_{k+1} x| \leq r|A_k x|$  for some  $1 < q \leq r$  and let  $p(x) = 2^{-k}$  where  $|A_k x| \leq 1$  and  $|A_{k+1} x| > 1$  (analogous to Calderón and Torchinsky, Advances in Math. 16, 1-64). Hardy spaces can then be defined by atoms. To study questions relating to the Fourier transform we let  $B_k = A_{-k}^{-1T}$  and assume  $q|B_k x| \leq |B_{k+1} x| \leq r|B_k x|$  and define  $p^*(x) = 2^{-k}$  where  $|B_k x| \leq 1$  and  $|B_{k+1} x| > 1$  to get a dual structure. Our main result so far is an analogue of Paley's multiplier theorem. The function  $m(x)$  is a multiplier from  $H^1$  (relative to  $p(x)$ ) to  $L^2(\mathbb{R}^n)$  if  $\int_{p^*(x) < R} |m(x)|^2 dx \leq BR^2$  provided the  $A_k$  all commute. The commutativity

allows the choice of a preferred coordinate system with which we can overcome the lack of a dilation group. We believe that the situation for non-commutative  $A_k$  may be related to recent work on differentiation in lacunary directions of Strömberg, Cordoba and Fefferman, and Stein, Nagel and Wainger. (Received October 23, 1978.)

763-42-6 Jonathan Cohen, University of Georgia, Athens, Georgia 30602 and John Gosselin, University of Georgia, Athens 30602. A Multilinear singular integral in  $R^n$ .

We consider the maximal integral operator  $M(\bar{b}, f)(x) = \sup_{\epsilon > 0} \int_{|x-y| > \epsilon} H(x-y) \prod_{m_j} (b_j, x, y) f(y) dy$

where  $\bar{b} = (b_1, \dots, b_n)$ ,  $b_j^{(\alpha)} \in L^j(R^n)$  where  $\alpha \in Z^n$ ,  $|\alpha| = \alpha_1 + \dots + \alpha_n = m_j$  and

$H(x) = \Omega(x/|x|) |x|^{-n-M}$ ,  $M = m_1 + m_2 + \dots + m_n$ . Assume that  $\Omega$  satisfies one of the following sets of conditions for  $|x| = 1$ . (1)  $\Omega(-x) = (-1)^{M+1} \Omega(x)$  and  $\int_{|x|=1} |\Omega(x)| dx < \infty$

or (2)  $\Omega(-x) = (-1)^M \Omega(x)$ ,  $\int_{|x|=1} |\Omega(x)| \log^+ |\Omega(x)| dx < \infty$  and  $\int_{|x|=1} x^\alpha \Omega(x) dx = 0$  for

$|\alpha| = 1, 2, \dots, M$ . Then  $\|M(\bar{b}, f)\|_q \leq C \prod_{j=1}^n [ \sum_{|\alpha|=m_j} \|b_j^{(\alpha)}\|_{r_j} ] \|f\|_p$  where  $1 > \frac{1}{q} = \frac{1}{p} + \sum_{j=1}^n \frac{1}{r_j}$ ,

$1 < p < \infty$  and  $1 < r_j \leq \infty$ . The estimate follows from the one dimensional result via the method of rotations. (Received October 25, 1978.)

### 43 ► Abstract Harmonic Analysis

763-43-1 Sungwoo Suh, University of Missouri-Rolla, Rolla, Missouri 65401. Constructive proofs of the Rudin-Katznelson theorem and the Kahane theorem.

An elementary constructive proof (without using duality theory) of the following theorem in harmonic analysis is obtained using Herz's technique:

Theorem (Rudin-Katznelson). Let  $E$  be a compact subset of  $T$  with  $m(E) = 0$ . Then  $\overline{C_E^\infty(T)} = A(T)$ .

Consequently a constructive proof of Kahane theorem stated below also follows from our proof.

Theorem (Kahane). Let  $E$  be a compact subset of  $T$  with  $m(E) = 0$ . Then the restriction algebra of  $A(T)$  to  $E$  is generated by its idempotents. (Received October 13, 1978.)

\*763-43-2 G. Thomas Frey, St. Bonaventure University, St. Bonaventure, N.Y. 14778. Relatively Dense and Countably Dense Sets in Semi-groups. Preliminary report.

A subset  $H$  of a semi-group  $G$  is relatively (countably) dense in  $G$  if a finite (countable) number of left or right cosets of  $H$  form a cover of  $G$ . We establish 1) necessary and sufficient conditions for such subsets to be subgroups and normal subgroups, and 2) sequential and mean-theoretic sufficiency conditions for relative and countable density. (Received October 23, 1978.)

### 44 ► Integral Transforms, Operational Calculus

\*763-44-1 JOSEPH F. STOKES, Western Kentucky University, Bowling Green, Kentucky 42101. A New Glimpse at the Faltung Integral.

An algebraic classification of certain piecewise continuous functions by the use of the Faltung integral is discussed. Several theorems are stated and proved. (Received October 2, 1978.) (Author introduced by Dr. James B. Barksdale.)



763-44-2 Mangho Ahuja, Southeast Missouri State University, Cape Girardeau, Missouri 63701.  
Formally Real Rings of Distributions - II. Preliminary report.

Let  $C(\mathbb{R}_+^n)$  denote the set of real valued continuous functions over  $\mathbb{R}^n$  with support over the positive cone. In an earlier paper it was shown that  $C(\mathbb{R}_+^1)$  is formally real ring with addition + and convolution \* as operations. (A ring R is formally real iff  $a_i \in R$ ,  $\sum a_i^2 = 0$  implies  $a_i = 0$  for each i). In a 1953 paper Mikusinski and Nardzewski made use of a function from  $C(\mathbb{R}_+^n)$  to  $C(\mathbb{R}_+^1)$  to prove an extended version of Titchmarsh theorem. The same function is used here to prove that  $C(\mathbb{R}_+^n)$  is formally real. This gives us an easier method of showing that  $D(\mathbb{R}_+^n)$  (the set of all Schwartz distributions over  $\mathbb{R}^n$  with support over the positive cone and whose value is real for real-valued test functions) is formally real. (Received October 19, 1978.)

763-44-3 DAVID H. LUBBERS, Clemson University, Clemson, South Carolina 29631.  
Some theorems on the Hankel spectrum of a function. Preliminary report.

The Hankel spectrum of a function is derived from a Bochner-type theorem using the Hankel positive definiteness of the Hankel autocorrelation. The fact that the Hankel autocorrelation is the Hankel-Stieltjes transform of the Hankel spectrum is used to give several inversion theorems useful for calculating the Hankel spectrum of a function. Then the Hankel spectrums of the associated function and a convolution are given. The discussion concludes with a geometric interpretation and an example. (Received October 25, 1978.)

## 45 ► *Integral Equations*

763-45-1 C.L. Dolph, University of Michigan, Ann Arbor, Michigan 48104. Fredholm integral equations, scattering theory, and the singularity expansion method (SEM).

Since 1971 SEM has been extensively used in electromagnetic theory as is evident from the review article by C.E. Baum: "Emerging technology for transient and broad band analysis and synthesis of antennas and scatterers," Proc. I.E.E.E., 64, 1976, pp.1598-1616. The Fredholm integral equations and their complex singularities used in SEM will be discussed and SEM will be interpreted in terms of mathematical scattering theory. (Received August 31, 1978.)

\*763-45-2 C. CORDUNEANU, University of Tennessee, Knoxville, Tennessee 37916 and V. LAKSHMIKANTHAM\*, University of Texas at Arlington, Arlington, Texas 76019. Equations with unbounded delay: A survey.

Equations with unbounded delay (differential, integral, integro-differential, neutral) occur in several areas of investigation such as Control Theory, Population Dynamics, Nuclear Reactor Dynamics, Mechanics of Continua, and Systems Theory. During the past three decades, several basic results have been obtained which deal with both the linear and the nonlinear cases. Many basic problems still remain open. The paper contains a survey of results and open problems and emphasizes several methods of investigation. New results are included. (Received September 8, 1978.)

763-45-3 G. Milton Wing, Southern Methodist University, Dallas, Texas 75275  
An integral equation of transport theory and some generalizations.

The equation (\*)  $\phi(z) = S(z) + \gamma \int_a^b E_1(1-z-z')\phi(z')dz'$ ,  $E_1$  the exponential integral function, is a prototype equation in transport theory. Properties of

this equation provide understanding of many transport models. Information may sometimes be obtained from the linear Boltzmann equation from which (\*) is derived. Physical insight is often useful. An obvious generalization of (\*) replaces  $E_1(u)$  by  $K(u)$  where  $K$  has at least some of the properties of  $E_1$ . A pseudo-Boltzmann equation can sometimes be formulated. Many of the devices applicable to (\*) can be used to obtain information about the generalized equation. The current state of this type of analysis will be outlined, new results described, and open problems discussed. (Received September 11, 1978.)

\*763-45-4 Chaitan P. Gupta, Northern Illinois University, DeKalb, IL 60115. Some recent results on Fredholm and Hammerstein equations and applications.

Let  $X$  and  $Y$  be Banach spaces,  $L: D(L) \subset X \rightarrow Y$  a linear Fredholm operator,  $K: Y \rightarrow X$  a linear operator and  $N: X \rightarrow Y$  a nonlinear operator. A Fredholm equation is an equation of the form  $Lu + Nu = w$  and a Hammerstein equation is of the form  $v + KNv = w$ . In recent years great progress has been made in the study of these equations using the theory of operators of monotone type. It turns out that a Fredholm equation is related to a Hammerstein equation through the use of the Liapunov-Schmidt method. Some recent existence and uniqueness results for these equations in non-reflexive spaces are given. Conditions on the operators involved are natural and easy to verify in practice. These results are applied to certain nonlinear boundary value problems arising in partial and ordinary differential equations. (Received September 18, 1978.)

763-45-5 Albert T. Bharucha-Reid, Department of Mathematics, Wayne State University, Detroit, Michigan 48202 and Mark J. Christensen, School of Mathematics, Georgia Institute of Technology, Atlanta Georgia, 30332. Numerical Solutions of Random Fredholm Equations. Preliminary Report.

Algorithms for the solution of random Fredholm integral equations of the second kind based upon the program IESIMP of K. E. Atkinson (A survey of numerical methods for the solution of Fredholm integral equations of the second kind, SIAM Publications, Philadelphia, 1976) are presented. The case of random right hand side  $f$  as well as that of a random kernel  $K$  is discussed. Numerical results are presented showing the behavior of the averaged solutions, as well as the variance of the solutions. (Received October 4, 1978.)

\*763-45-6 ROBERT E. KALABA, University of Southern California, Los Angeles, California, 90007 and ELENA ZAGUSTIN, California State University, Long Beach, Long Beach, California, 90840. Cauchy System for Implicit Nonlinear Integral Equations.

In applications we often have to deal with an implicit nonlinear integral equation of the form

$$F[t, u(t, \lambda), \lambda \int_0^1 K(t, y) u(y, \lambda) dy] = 0, \quad 0 \leq t \leq 1, \quad 0 \leq \lambda \leq A;$$

furthermore we desire a parameter study in  $\lambda$ . It is shown that this problem, with light restrictions on the function  $F$ , can be reduced to an initial value problem in which  $\lambda$  plays the role of the time-like variable. The method is used to solve the nonlinear Ambarzumian's equation of radiative transfer and to find the bifurcation point at  $\lambda = 1$ . (Received October 6, 1978.)

\*763-45-7 John Locker and P. M. Prenter, Colorado State University, Fort Collins, CO. 80523. Regularization with differential operators.

The method of regularization is used to obtain least squares solutions of the linear equation  $Kx = y$ , where  $K$  is a bounded linear operator from one Hilbert space into another and the regularizing operator  $L$  is a closed densely defined linear operator. Existence, uniqueness, and convergence analyses are developed. An application is given to the special case when  $K$  is a first kind integral operator and  $L$  is an  $n$ th order differential operator in  $L^2[a, b]$ . Using weak least squares finite element procedures, spline approximate solutions are constructed and optimal  $L^2$  and  $L^\infty$  error estimates are derived. These methods apply to piecewise continuous kernels. (Received October 16, 1978.)

\*763-45-8 Dennis W. Brewer, University of Arkansas, Fayetteville, Arkansas 72701. The asymptotic stability of a nonlinear functional differential equation of infinite delay.

We consider the nonlinear functional differential equation of infinite delay

$$(1) \quad \begin{aligned} u'(t) + \alpha u(t) + Bu(t) &= F(u_t), \quad t \geq 0, \\ u(t) &= \phi(t), \quad t \leq 0, \end{aligned}$$

with an initial data space  $X$  of "fading memory type." Equation (1) is studied in the abstract setting of a Banach space  $E$ . The nonlinear functional  $F$  is a uniformly Lipschitz continuous mapping of  $X$  into  $E$ ,  $\alpha$  is a positive number, and  $B$  is  $m$ -accretive in  $E$ . Solutions of (1) are represented by a nonlinear contraction semigroup generated by a dissipative operator in  $X$ . The semigroup representation is used to prove the asymptotic stability of solutions of (1) under certain conditions on  $\alpha$ , the Lipschitz constant of  $F$ , and the norm in the initial data space  $X$ . (Received October 19, 1978.)

\*763-45-9 GEORGE C. HSIAO, University of Delaware, Newark, DE 19711 and WOLFGANG L. WENDLAND, T.H. Darmstadt, Germany. Finite-element Approximations of Integral Equations with Logarithmic Kernels.

A large class of boundary value problems in elasticity and hydrodynamics can be transformed to the integral equations of the first kind. This paper concerns, in particular, these integral equations with logarithmic kernels and their finite-element approximations. An efficient numerical procedure is developed, and results of numerical experiments are reported.

(Received October 23, 1978.)

763-45-10 THOMAS KAILATH, Stanford University, Stanford, California 94305. Exploiting Toeplitz-like structure for Fredholm integral equations.

It is known that Fredholm integral equations with Toeplitz (also called convolution or displacement or difference) kernels can be solved with an order of magnitude less computation than with an arbitrary kernel. However one might reasonably expect that certain kernels are less "nonToeplitz" than others and in this talk we shall quantify this notion. We shall show that we can associate with any kernel a so-called displacement rank  $\alpha$ ,  $\alpha$  a nonnegative integer, that provides a measure of how "close" the kernel is to a Toeplitz kernel. Several properties will be presented to justify this definition, including the fact that an equation with kernel of rank  $\alpha$  can be solved with about  $\alpha$  times as much effort as if the kernel were Toeplitz. Some relations between these results and certain integral equations of Wiener-Hopf type will also be noted. We might mention that a kernel  $K(t,s)$  has (first-order) displacement rank  $\alpha$  if  $\alpha$  is the smallest integer such that we can find some  $\{\phi_i(\cdot), \psi_i(\cdot)\}$  such that  $(\partial/\partial t + \partial/\partial s)K(t,s) = \sum_{i=1}^{\alpha} \phi_i(t)\psi_i(s)$ . (Received October 23, 1978.)

\*763-45-11 C. M. DAFERMOS, Brown University, and J. A. NOHEL, University of Wisconsin-Madison. A class of nonlinear hyperbolic Volterra equations.

We use energy methods to study the global existence, uniqueness, boundedness, and decay as  $t \rightarrow \infty$  of classical solutions for sufficiently "small" and smooth data of the Cauchy problem for the class of equations in the title. Such equations arise as mathematical models for either the motion of a nonlinear one-dimensional, unbounded viscoelastic body, or for nonlinear heat flow in an unbounded body of a material with "memory", under different natural assumptions concerning the "memory function" (kernel of the Volterra operator). In the absence of the memory term the underlying evolution process is a nonlinear wave equation which is known to have "shock" solutions, no matter how smooth or "small" one takes the data. The presence of the Volterra terms induces a dissipative mechanism which precludes the development of shocks under physically reasonable assumptions on the kernel. An earlier study of such problems uses Riemann invariants and is restricted to one space dimension. The energy method is simpler and yields more widely applicable results. (Received October 23, 1978.)

763-45-12 Elaine Hubbard, Kennesaw College, Marietta, Georgia 30061. Finite Dimensional Approximation and  $\beta$ -convergence for Solutions to Problems with Nonunique Minima. Preliminary report.

Let  $\phi$  be a weakly lower semicontinuous function defined on a Hilbert space,  $\bar{X}$ . Solutions of the minimization of  $\phi$  on  $\bar{X}$  are approximated by solving related minimization problems in finite dimensional subspaces of  $\bar{X}$ . Solutions of the minimization of  $\phi$  on  $\bar{X}$  are not necessarily unique. Convergence results are stated in terms of strong convergence when possible and in terms of the  $\beta$ -convergence of the solution sets otherwise. The results are applied to solving the Hammerstein equation,  $KFu=u$ . (Received October 23, 1978.)

763-45-13 Jamie J. Goode, Georgia Institute of Technology, Atlanta, Georgia 30332, and Elaine Hubbard, Kennesaw College, Marietta, Georgia 30061. The Frank-Wolfe Algorithm for Infinite Dimensional Problems with Nonunique Solutions. Preliminary report.

The Frank-Wolfe method is adapted to approximate the solution to the constrained minimization problem, minimize  $\phi(x)$ ,  $x \in S$ , where the dimension of the span of  $S$  is infinite and  $S$  is a weakly closed subset of a Hilbert space. The method is applied to the problem of finding the minimum norm element in a convex set as well as to solving the linear system,  $Ax=y$ ,  $x \in S$ . (Received October 23, 1978.)

\*763-45-14 M. ZUHAIR NASHED, University of Delaware, Newark, DE 19711 and MANSUK SONG, Daejeon, Korea. Finite element regularization of ill-posed integral equations: smooth and nonsmooth kernels.

Let  $K$  be a Fredholm integral operator whose kernel  $k(x,y)$  has a smoothing index  $q$  (e.g.  $(\partial^q/\partial x^q)k(x,y)=\delta(x-y)$  in the sense of distributions). Let  $w_0 = K^+g_0$  be the minimal norm solution to  $Kf=g_0$  for  $g_0 \in R(K)$ , and let  $u_{\alpha,h}^{\epsilon}$  be the element which minimizes  $\|Kf-g_{\epsilon}\|^2 + \alpha\|f\|_k^2$ ,  $\alpha > 0$ , over  $X_h^{t,k}$ , where  $X_h^{t,k}$  is a  $(t,k)$ -regular system with the inverse property (in the terminology of finite element approximations),  $\|f\|_k$  is the Sobolev norm of order  $k > 0$ , and  $\|g_{\epsilon}-g_0\| \leq \epsilon$ . Let  $h$ ,  $\alpha$  and  $\epsilon \rightarrow 0$  in such a way that  $h^{k+s}/\sqrt{\alpha}$  and  $\epsilon/\sqrt{\alpha}$  are bounded. Then  $\|u_{\alpha,h}^{\epsilon}-w_0\| \leq C\epsilon^{k/(k+s)}$ .

A perturbation-regularization method is developed to obtain error estimates for the convergence of finite element approximations for the case when the kernel is infinitely smooth, and a corresponding theorem is established. Generalizations are also given for the problem of approximation of the element of minimal  $\|Lx\|$  which minimizes  $\|Ax-y\|$ , where  $L$  is a bounded (or closed densely defined) linear operator with closed range,  $AN(L)$  and  $LN(A)$  both closed, and  $N(L) \cap N(A) = \{0\}$ . (Received October 23, 1978.)

763-45-15 GEORGE H. KNIGHTLY, University of Massachusetts, Amherst, Massachusetts 01002 and D. SATHER, University of Colorado, Boulder, Colorado 80309. Applications of group representations to nonlinear Hammerstein equations. Preliminary report.

A method is proposed to obtain multiple solutions of nonlinear Hammerstein equations of the form  $w + KF(w) = h$  where  $K: \mathcal{H} \rightarrow \mathcal{H}$  and  $F: \mathcal{H} \rightarrow \mathcal{H}$  are linear and nonlinear operators, respectively, on a real Hilbert space  $\mathcal{H}$ , and  $h$  is a given element of  $\mathcal{H}$ . If  $F$  is a continuous gradient operator the method is based in part upon the following result: Let  $g \rightarrow T_g$  be a unitary representation of a group  $G$  on a Hilbert space  $\mathcal{H}$ . If  $F$  is the gradient of a functional  $f: \mathcal{H} \rightarrow \mathbb{R}^1$  and if  $f$  is invariant under the representation  $g \rightarrow T_g$  in the sense that  $f(T_g w) = f(w)$  for all  $g \in G$  and all  $w \in \mathcal{H}$  then  $F$  is invariant under the representation  $g \rightarrow T_g$  in the sense that  $T_g F(w) = F(T_g w)$  for all  $g \in G$  and all  $w \in \mathcal{H}$ .

(Received October 23, 1978.) (Author introduced by Professor M. Z. Nashed).

763-45-16 YOSHIO HAYASHI, Nihon University, Tokyo, Japan, and National Research Council, Ottawa, Canada. The Fredholm integral equation of the first kind, in relation to the Dirichlet problem for an open boundary.

Let  $\Omega$  be a union of simple; smooth and bounded open arcs in a plane  $E$ , or of simple, smooth, simply or multiply connected, orientable and bounded open surfaces in a three-dimensional space  $E$ . Set  $\Omega(\rho) = \Omega - \{x: x \in \Omega, |x-y| < \rho \text{ for } \forall y \in \partial\Omega\}$ , where  $\rho > 0$  and  $\partial\Omega$  is the boundary of  $\Omega$ . Consider the integral equation (1)  $\Psi \tau \int_{\Omega} \psi(x,y) \tau(y) d\Omega(y) = g(x)$ , where  $g \in C(\Omega)$ ,  $\tau \in T(\Omega) = C(\Omega(\rho))_{A_1 L_1}(\Omega)$ , and  $\psi$  is the fundamental solution

of the Helmholtz equation in  $E$ . Then, the following theorems constitute the theory of Dirichlet problem for the two and three dimensional Helmholtz equations and for an open boundary  $\Omega$ , as well as that of the Fredholm integral equation of the first kind (1). Theorem 1. To solve the Dirichlet problem is equivalent to solve the equation (1) with  $g(x)$  obtained from the boundary data. Theorem 2.  $\Psi\tau = 0 \Leftrightarrow \tau = 0$ . Theorem 3.  $\Psi^{-1}$  is "continuous" in the following sense. Given  $\epsilon > 0$  and  $\rho > 0$ , there exists  $\delta > 0$  such that  $\|g\|_{C(\Omega)} < \delta$  implies  $\|\tau\|_{C(\Omega(\rho))} < \epsilon$ . Theorem 4. Let  $\{\phi_n\}$  be a complete system of functions in  $L_2(\Omega)$ , and set  $u_n = \Psi\phi_n$ . Then, the linear space generated by  $\{u_n\}$  is dense in  $C(\Omega)$ . Theorem 5. For any given function  $g \in C(\Omega)$ , there exists the (unique) solution  $\tau \in T(\Omega)$  of (1). (Received October 23, 1978.)

763-45-17 Theodore Laetsch, University of Arizona, Tucson, Arizona 85721. Minimax principles for nonlinear eigenvalue problems. Preliminary report.

Let the Hammerstein integral operator  $Au(x) = \int_{\Omega} K(x,y)f(u(y))dy$  be compact in some complete function space  $F$  with topological dual  $F^*$ , where  $K > 0$  on  $\Omega \times \Omega$  and  $f$  is strictly positive and increasing on  $[0, +\infty)$ . Let  $\Lambda$  be the set of positive numbers  $\lambda$  for which the equation  $u = \lambda Au$  has a positive solution, and let  $\lambda^* = \sup \Lambda$ . For arbitrary  $\phi \geq 0, u \geq 0$ , define the extended real-valued function  $L$  by  $L(\phi, u) = \phi(Au)/\phi(u)$  ( $= 0$  if  $\phi(Au) = \phi(u) = 0$ ). Then  $(\lambda^*)^{-1} = \inf_u \sup_{\phi} L(\phi, u)$ . If  $f$  is convex, then, under appropriate conditions,  $\lambda^* \in \Lambda$  if and only if  $(\lambda^*)^{-1}$  is a finite saddle value of  $L$ . The set of  $\lambda$  for which  $u = \lambda Au$  has more than one solution is related to the interval  $(\mu_{\infty}, \lambda^*)$ , where  $(\mu_{\infty})^{-1} = \sup_{\phi} \inf_u (\phi A)(0^+)(u)/\phi(u)$ , with  $(\phi A)(0^+)$  being the recession function of the convex functional  $(\phi A)$ . (Received October 23, 1978.)

\*763-45-18 Joel D. Pincus State University of New York, Stony Brook  
Symmetric Wiener-Hopf Equations

The author has shown that the structure of a pair of Unitary operators  $U, W$  with multiplicative commutator  $U^{-1}WUW^{-1} - 1$  of one dimensional range is completely determined (up to a simultaneous unitary equivalence of  $U$  and  $W$ ) by a certain "principal function"  $g(e^{i\alpha}, e^{i\beta})$  defined on the torus. (The existence of the principal function for the general trace class situation and associated transformation and index properties has also been obtained in joint work with Richard Carey.) A large class of symmetric operators can be discussed by means of this theory. The present talk will explain how the explicit solution theory of symmetric Wiener-Hopf equations with real, possibly unbounded, symbol  $k(x)$  satisfying  $\int_{-\infty}^{\infty} k(x)^2(1+x^2)^{-1}dx < \infty$  is obtained from the one dimensional theory. More than fifteen years ago the author derived the spectral diagonalization of bounded Wiener-Hopf and Toeplitz operators as a special case of his (still older) diagonalization of bounded self adjoint singular integral operators with Cauchy kernel. This was achieved by finding a somewhat ad hoc unitary equivalence. Now a full geometric picture exists. (Received October 23, 1978.)

763-45-19 W.E. Fitzgibbon, Department of Mathematics, University of Houston, Houston, Texas 77004,  
Representation and Approximation of Solutions to Semilinear Volterra Equations with Delay. Preliminary report.

We are concerned with the representation and approximation of semilinear Volterra equations which involve delay. We consider equations of this form  $x(\varphi)(t) = W(t, \tau)\varphi(0) + \int_0^t W(t,s)F(s, x_s(\varphi))ds$ .

Here  $X$  denotes a Banach space;  $C$  in the space of functions mapping an interval  $I = (\infty, 0)$  or  $I = [-r, 0]$  to  $X$  and  $F$  maps  $R \times C$  to  $X$ . The principal tool is the representation and approximation theory for nonlinear evolution equations due to Crandall and Pazy (Israel J. Math. 11 (1972), 57-94). Examples to partial differential equations involving delay are provided. (Received October 23, 1978.)

\*763-45-20 G. S. JORDAN, University of Tennessee, Knoxville, Tennessee 37916.  $L^1$  resolvents of Volterra integrodifferential equations.

Consider the Volterra integrodifferential system  $x'(t) = Ax(t) + \int_0^t B(t-s)x(s)ds$  where  $A$  and  $B(t)$  are  $n \times n$  matrices. If  $B(t) \in L^1(0, \infty)$ , then it is well known that the associated resolvent  $R(t) \in L^1(0, \infty)$  if and only if  $(*) \det[zI - A - B^*(z)] \neq 0$  ( $\text{Re } z \geq 0$ ), where  $I$  is the  $n \times n$  identity matrix and  $B^*(z)$  is the Laplace transform of  $B(t)$ . Applying condition  $(*)$  may be rather difficult. We extend some recent results of F. Brauer [J. Differential Equations 28(1978), 180-188] for the scalar case by giving easily checked conditions on the matrices  $A$  and  $B(t)$  which imply that  $(*)$  holds or that  $(*)$  does not hold. (Received October 23, 1978.)

\*763-45-21 V. LAKSHMIKANTHAM and A. S. VATSALA, University of Texas at Arlington, Arlington, TX 76019 and R. L. VAUGHN, Texas Christian University, Fort Worth, TX 76129. Existence and comparison results for a class of Volterra integral equations of Sobolev type. Preliminary report.

A new class of Volterra integral equations is considered special cases of which occur in imbedding method for solving Fredholm integral equations. After proving an existence result of Peono's type, the theory of integral inequalities is developed which is employed to show the existence of extremal solutions. A comparison theorem is then derived. The results considered here naturally include as special cases the corresponding theory of Volterra integral equations of usual type. (Received October 23, 1978.)

763-45-22 RALPH E. KLEINMAN, University of Delaware, Newark, Delaware 19711. Integral Equation Method for Scattering by Penetrable Objects with Corners.

An integral equation is derived for the problem of scattering of time harmonic scalar waves (in  $R^n$ ) by a penetrable bounded object with piecewise smooth boundary (in  $R^{n-1}$ ) where transition conditions (not necessarily continuous) are imposed. The equation is uniform in the sense that it holds for all points in  $R^n$  including corner points on the boundary (if any) and remains valid when the scattering object becomes rigid (impenetrable). It is shown that for restricted parameter ranges the equation may be solved by iteration. In the special case of continuous transition at the boundary and homogeneous scattering object, the method reduces to the well known Born approximation. (Received October 25, 1978.)

763-45-23 IGNACE I. KOLODNER, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Equations of Hammerstein type—A survey of recent results.

In this presentation we survey the application of monotonicity and isotonicity methods to these equations. (Received October 25, 1978.)

763-45-24 ROGER D. NUSSBAUM and MICHAEL MOCK, Rutgers University, New Brunswick, New Jersey 08903. A quadratic integral equation. Preliminary report.

This paper gives a reasonably complete treatment of the integral equation  $u(x) = f(x) + \lambda \int_x^1 u(y)u(y-x)dy$  for  $0 \leq x \leq 1$ . Here  $f(x)$  is a given continuous, real-valued function,  $\lambda$  is a real parameter and a continuous, real-valued solution  $u(x)$  is sought. The above equation arises in attempts to solve the Percus-Yevick equation from statistical mechanics. We determine for precisely what range of  $\lambda$  ( $\lambda$  positive or negative) a solution of the integral equation exists, how many solutions there are (with various other constraints), etc. For example, we have Theorem. If  $f(x)$  is positive and  $C^1$ , the integral equation has precisely two positive solutions for  $0 < \lambda < (2 \int_0^1 f(x)dx)^{-1} = \lambda_+$  and no real-valued solutions for  $\lambda > \lambda_+$ . This theorem was claimed by R. Ramalho for the case  $f(x) \equiv 1$ , but his argument uses a result of G. Pimbley for which the proof seems incorrect. (Received October 25, 1978.)

## 46 ► Functional Analysis

\*763-46-1 H.S. BEAR, University of Hawaii, Honolulu, Hawaii 96822. Approximate identities and pointwise convergence.

We give two additional conditions on an approximate identity, or positive kernel,  $\{K_\alpha\}$ , which insure that  $f * K_\alpha \rightarrow f$  a.e. if  $f \in L^1$  on the line or circle. Where the convolution defines a function on the disc or half-plane, as for positive kernels or heat kernels, then the theorem gives automatically the paths toward a boundary point along which pointwise convergence occurs. (Received September 15, 1978.)

763-46-2 WITHDRAWN

\*763-46-3 PATRICK FLINN, Ohio State University, Columbus, Ohio 43210. A characterization of M-ideals in  $B(\ell_p)$  for  $1 < p < \infty$ . Preliminary report.

A closed subspace  $M$  of a Banach space  $X$  is said to be an M-ideal of  $X$  if there is a subspace  $\hat{M}$  of  $X^*$  such that  $X^* = M^\perp \oplus \hat{M}$  and  $\|m + \hat{m}\| = \|m\| + \|\hat{m}\|$  for all  $m \in M$  and  $\hat{m} \in \hat{M}$ . For a Banach space  $X$ , let  $K(X)$  and  $B(X)$  denote the spaces of compact operators and bounded linear operators on  $X$ , respectively.

**THEOREM:** The only non-trivial M-ideal in  $B(\ell_p)$  for  $1 < p < \infty$  is  $K(\ell_p)$ .  
(Received October 16, 1978.)

\*763-46-4 S. Swaminathan, Dalhousie University, Halifax, Nova Scotia, B3H 4H8. Superreflexivity and Summability.

Let  $X$  be a Banach space. A property of Kakutani intermediate between uniform convexity and the property of Banach-Saks (BS) says that there exists a number  $\alpha$ ,  $0 < \alpha < 1$ , such that every sequence  $(x_n)$ ,  $\|x_n\| \leq 1$ , weakly convergent to 0, contains members  $x_{n_j}$  ( $j=1,2,\dots,k$ ) for which  $\|\sum_{j=1}^k x_{n_j}\| < \alpha$ . It is known that while BS strictly implies reflexivity, the latter can be characterised by a summability version of BS by considering a general summability method in lieu of the Cesaro means. It is shown how a similar summability version of Kakutani's property can characterise superreflexivity. (Received October 16, 1978.)

763-46-5 J.W. Kitchen, Duke University, Durham, NC 27706 and D.A. Robbins, Trinity College, Hartford, CT 06106. Sectional representation of Banach modules.

The present paper extends and generalizes results in an earlier paper by the authors (these Notices, 24 (1977), A115-116), regarding representation of a large class of Banach modules over commutative Banach algebras as spaces of sections of canonical bundles of Banach spaces. Throughout much of this paper, the assumptions that algebras be commutative or that modules be essential are dropped. The resulting general theory embraces work on sectional representation of  $C^*$ -algebras currently being investigated by K.H. Hofmann and M. Dupré, among others. (Received October 16, 1978.)

763-46-6 CHARLES A. AKEMANN, University of California, Santa Barbara, California 93106 and STEVE WRIGHT, Oakland University, Rochester, Michigan 48063. Compact and weakly compact derivations of C\*-algebras. Preliminary report.

Theorem 1. Let  $A$  be a C\*-algebra,  $\delta : A \rightarrow A$  a derivation. The following are equivalent:

- (a)  $\delta$  is compact.
- (b) There is an orthogonal sequence of minimal, finite-dimensional, central projections  $\{x_n\}$  of  $A$  and an element  $d \in A$  such that  $\delta = \text{ad } d$  and  $\sum_n x_n d$  converges uniformly to  $d$ .

Theorem 2. Let  $A$  be a C\*-algebra,  $\delta : A \rightarrow A$  a derivation. The following are equivalent:

- (a)  $\delta$  is weakly compact.
  - (b) There is a sequence  $\{I_n\}$  of orthogonal ideals of  $A$  such that  $I_n$  is an elementary C\*-algebra,  $n = 1, 2, \dots$ , and an element  $d$  in the restricted direct sum  $\hat{\bigoplus}_n I_n \subseteq A$  with  $\delta = \text{ad } d$ .
- (Received October 19, 1978.)

\*763-46-7 Alfred D. Andrew and William L. Green, Georgia Institute of Technology, Atlanta, Georgia 30332. On James' quasi-reflexive Banach space as a Banach algebra.

Let  $J$  be R. C. James' example of a non-reflexive Banach space isometric to its second dual. We show that  $J$  is a Banach algebra under an equivalent norm and pointwise multiplication. We identify  $J^{**}$  using largely Banach algebra techniques, and show that the multiplier algebra of  $J$  is  $J^{**}$ . Every algebra automorphism of  $J$  is bounded, and if  $\alpha$  is an automorphism with  $\|\alpha\| < \sqrt{2}$ , then  $\alpha$  is the identity map. (Received October 20, 1978.)

763-46-8 R. S. Doran, Texas Christian University, Fort Worth, Texas 76129. Essential subspaces. Preliminary report.

Let  $E$  be a normed linear space,  $M$  a closed linear subspace of  $E^*$ , and  $x \rightarrow \hat{x}$  the canonical embedding of  $E$  in  $E^{**}$ . If  $\|x\| = \|\hat{x}|M\|$  for each  $x$  in  $E$ , the subspace  $M$  is called essential. Let  $X$  be a locally compact Hausdorff space and  $M$  an essential subspace. A map  $\mu$  on  $X$  with values in  $E$  such that  $\phi \circ \mu$  is a regular Borel measure on  $X$  for each  $\phi$  in  $M$  is called an essential measure on  $X$ . Basic properties of essential subspaces and essential measures are studied. (Received October 20, 1978.)

\*763-46-9 N. J. Kalton, University of Missouri, Columbia, Missouri 65201, B. Turett, Oakland University, Rochester, Michigan 48063; and J. J. Uhl, Jr., University of Illinois, Urbana, Illinois 61801. Basically scattered measures.

Let  $X$  be a Banach space and  $\Sigma$  a  $\sigma$ -field of subsets of a set  $\Omega$ . A vector measure  $F: \Sigma \rightarrow X$  is basically scattered if  $F$  takes disjoint sequences of non- $F$ -null sets into basic sequences. For each non- $F$ -null set  $E$ , let  $\lambda(E)$  be the smallest number in  $[0, \infty]$  such that  $\|\sum_{n=1}^m \alpha_n F(E_n)\| \leq \lambda(E) \|\sum_{n=1}^{m+p} \alpha_n F(E_n)\|$  for all disjoint sequences  $(E_n)$  of non- $F$ -null subsets of  $E$ , for all sequences  $(\alpha_n)$  of real numbers, and for all positive integers  $m$  and  $p$ .  $F$  has a bounded basis constant on  $E$



if  $\lambda(E) < \infty$ . Theorem. Let  $F: \Sigma \rightarrow X$  be a non-atomic basically scattered vector measure. If  $A$  is a non- $F$ -null set, then there exists a non- $F$ -null subset  $B$  of  $A$  such that  $\lambda(B)$  is finite. In fact, there exist finitely many disjoint non- $F$ -null sets  $A_1, \dots, A_m$  such that  $\bigcup_{n=1}^m A_n = \Omega$  and  $\lambda(A_n)$  is finite for each  $n = 1, \dots, m$ . In spite of this theorem, a non-atomic basically scattered measure need not have a bounded basis constant on  $\Omega$ . Basically scattered measures are then used to characterize Banach spaces that contain a copy of  $L_p[0,1]$  ( $1 \leq p < \infty$ ). (Received October 20, 1978.)

\*763-46-10 Russell G. Bilyeu and Paul W. Lewis, North Texas State University, Denton, Texas 76203. Uniform Strong Additivity as A Lattice Property.

Suppose that  $X$  is a  $\sigma$ -complete,  $\sigma$ -order continuous Banach lattice and that  $x$  and  $y$  are non-negative members of  $X$ . Define  $P_x(y)$  to be  $\lim_n nx \wedge y$ ; if  $u$  is an arbitrary member of  $X$ , define  $P_x(u)$  to be  $P_x(u^+) - P_x(u^-)$ . Let  $\mathcal{O} = \{P_x : x \geq 0\}$ , and call two member  $P_x$  and  $P_y$  of  $\mathcal{O}$  disjoint if  $P_x P_y = 0$ . A subset  $K$  of  $X$  is said to be uniformly  $\mathcal{O}$ -exhaustive if  $\|P_{x_i}(u)\| \rightarrow 0$  uniformly for  $u \in K$  whenever  $(P_{x_i})$  is a disjoint sequence from  $\mathcal{O}$ . Theorem. Suppose that  $\Sigma$  is an algebra of sets and  $K \subseteq \text{ba}(\Sigma)$ . Then  $K$  is uniformly  $\mathcal{O}$ -exhaustive iff  $K$  is uniformly strongly additive. (Received October 20, 1978.)

\*763-46-11 C. J. Seifert, University of Kansas, Lawrence, Kansas, 66045. The Banach-Saks Property in Lebesgue-Bochner function spaces.

A Banach space  $X$  is said to have the Banach-Saks property if every bounded sequence in  $X$  admits a subsequence whose arithmetic means converge in norm. Let  $L^p(X)$  ( $p$  greater than 1 and finite) denote the Lebesgue-Bochner function space of  $X$ -valued functions on a finite measure space. We show that  $L^p(X)$  has the Banach-Saks property whenever  $X$  has it. (Received October 23, 1978.)

\*763-46-12 JOSEPH CREEKMORE, Kent State University, Kent, Ohio 44242. Type and cotype in Lorentz  $L_{pq}$  spaces. Preliminary report.

For  $1 < p < \infty$ ,  $1 \leq q \leq \infty$ , let  $L_{pq}$  be the Banach space of measurable functions  $f$  with  $\|f\|_{pq} = \left( \frac{q}{p} \int_0^\infty [t^{1/p} f^{**}(t)]^q \frac{dt}{t} \right)^{1/q}$  if  $q < \infty$  and  $\|f\|_{p,\infty} = \sup_{t>0} t^{1/p} f^{**}(t)$ . Here  $f^*$  is the nonincreasing rearrangement of  $f$  and  $f^{**}(t) = \frac{1}{t} \int_0^t f^*(s) ds$ . In case  $p = q$ ,  $L_{pq}$  reduces to the Lebesgue  $L_p$  space, and its type and cotype are well known. For  $q < p$ ,  $L_{pq}$  is of type  $\min(2, q)$  and if  $p \neq 2$ ,  $L_{pq}$  is of cotype  $\max(2, p)$ .  $L_{2,q}$  is of cotype  $2 + \epsilon$ , for all  $\epsilon > 0$ . When  $p < q < \infty$ ,  $L_{pq}$  is of cotype  $\max(2, q)$  and if  $p \neq 2$ ,  $L_{pq}$  is of type  $\min(2, p)$ .  $L_{2,q}$  is of type  $2 - \epsilon$ , for all  $\epsilon > 0$ . Finally,  $L_{p,\infty}$  is of no type  $r > 1$  and of no cotype  $s < \infty$ . (Received October 23, 1978.) (Author introduced by Professor Joseph Diestel).

\*763-46-13 M. Alpseymen, METU, Ankara, Turkey, M. S. Ramanujan, University of Michigan, Ann Arbor 48109 and T. Terzioglu, METU, Ankara, Turkey. Subspaces of some nuclear sequence spaces.

For locally convex spaces  $X$  and  $Y$ ,  $(X, Y) \in R$  if all continuous linear maps on  $X$  into  $Y$  are also compact. One method of showing that the infinite dimensional space  $X$  cannot be isomorphic to a subspace of  $Y$  is to show  $(X, Y) \notin R$ . For  $f$  a rapidly increasing or slowly increasing function and the corresponding Dragilov space  $L_f(b, \infty)$  we obtain a sufficient condition for  $(L_f(b, \infty), \lambda(A)) \in R$  to hold, where  $\lambda(A)$  is a Schwartz space and also obtain a partial converse to this. We show also that if  $X$  is a nuclear Frechet space with a basis which satisfies the  $(d_f)$ -condition (see the first author's University of Michigan thesis, 1978) and if  $L_f(b, \infty)$  is nuclear and stable then  $X$  has a complemented subspace isomorphic to a subspace of  $L_f(b, \infty)$  and therefore  $(X, L_f(b, \infty)) \in R$ . (Received October 23, 1978.)

\*763-46-14 C.D. ALIPRANTIS and O. BURKINSHAW, IUPUI, 1201 E. 38th Street, Indianapolis, Indiana 46205. Minimal topologies and  $L_p$ -spaces.

The paper deals with minimal topologies on Riesz spaces. A minimal topology is a Hausdorff locally solid topology that is coarser than any Hausdorff locally solid topology on the space. It is shown that every minimal topology satisfies the Lebesgue property, that an Archimedean Riesz space can admit a locally convex-solid topology that is minimal if and only if the space is discrete, that  $C[0,1]$  and  $L_\infty([0,1])$  do not admit a minimal topology, and that the topology of convergence in measure on  $L_p([0,1])$  ( $0 \leq p < \infty$ ) is a minimal topology. A similar result is shown for certain Orlicz spaces. (Received October 23, 1978.)

\*763-46-15 L. ASIMOW, Syracuse University, Syracuse, New York, 13210. Superharmonic Interpolation in Subspaces of  $C_\phi(X)$ .

Let  $E$  be a closed subset of the compact Hausdorff  $X$  and let  $A$  be a closed separating subspace of  $C_\phi(X)$ . Let  $\rho$  be a dominator (strictly positive, l.s.c.) defined on  $X \times T$ ,  $T$  the unit circle in  $\mathbb{C}$ . Conditions, formulated in terms of boundary measures, are discussed for approximate and exact solutions to the problem of finding  $\rho$ -dominated extensions in  $A$  of functions  $g \in (A|_E)^\perp$  satisfying  $\operatorname{re} tg(x) \leq \rho(x,t)$  on  $E \times T$ . Various interpolation theorems of Rudin-Carleson type for superharmonic dominators are incorporated into this framework. (Received October 23, 1978.)

\*763-46-16 VICTOR KAFTAL, Tel Aviv University, Israel, A weak convergence of vectors relative to a semifinite von Neumann algebra.

Let  $A$  be a semifinite von Neumann algebra operating on a separable Hilbert space  $H$ . We say that  $\xi_n \in H$  converges to  $\xi$  weakly relatively to  $A$  ( $\xi_n \xrightarrow{ARW} \xi$ ) if i)  $\xi_n$  is bounded ii)  $p\xi_n \xrightarrow{s} p\xi$  for every projection  $p$  finite relatively to  $A$ . We obtain that an operator is compact in  $A$  i.e. is a norm limit of operators with a finite range (relatively to  $A$ ) iff it maps ARW converging sequences into strongly converging ones (generalized Hilbert condition). We obtain also that if  $a \in A$  and there is a normalized  $\xi_n \xrightarrow{ARW} 0$  such that  $a\xi_n \xrightarrow{s} 0$  then  $a$  is a left-Fredholm operator relatively to  $A$  (generalized Wolf condition). (Received October 23, 1978.)

763-46-17 JOHN H. RILEY, JR., University of Connecticut, Storrs, Connecticut 06268. Subalgebras of  $H^\infty$  and the corona property. Preliminary report.

Let  $B$  be a closed subalgebra of  $H^\infty$  which contains  $A$ , the disk algebra. Let  $B_\lambda$  be the fibre algebra; the restriction of the Gelfand transform of  $B$  to  $M_\lambda(B)$ , the subset of the maximal ideal space of  $B$  which maps the function  $z$  to  $\lambda$  (the fibre of  $M(B)$  over  $\lambda$ ,  $|\lambda| = 1$ ). The  $w^*$ -density of  $\{|z| < 1\}$  is related to the distance of the polynomial convex hull of  $C(\lambda; F)$  to  $C(\lambda; F)$  where  $C(\lambda; F)$  is the cluster set of a finite set at  $\lambda$  and  $F$  runs through finite subsets of a set which generates  $B_\lambda$ .  $B$  has the corona property ( $\{|z| < 1\}$  is  $w^*$ -dense in  $M(B)$ ) iff the  $C(\lambda; F)$  at each point of  $\{|z| = 1\}$  become close to polynomially convex sets in a way that can be made precise. (Received October 23, 1978.)

763-46-18 STEVEN F. BELLENOT, The Florida State University, Tallahassee, Florida 32306. Basic sequences in non-Schwartz Fréchet spaces.

Obliquely normalized basic sequences are defined and used to characterize non-Schwartz Fréchet spaces. It follows that each non-Schwartz Fréchet space  $E$  has a non-Schwartz subspace with a basis and a quotient which is not Montel (which has a normalized basis if  $E$  is separable). Stronger results are given when more is known about  $E$ , for examples, if  $E$  is a subspace of

a Fréchet  $\ell_p$ -Kothe sequence space, then  $E$  has the Banach space  $\ell_p$  as a quotient and  $E$  has a subspace isomorphic to a non-Schwartz  $\ell_p$ -Kothe sequence space. Examples of Fréchet Montel spaces which are not subspaces of any Fréchet space with an unconditional basis are given. Non-standard analysis is used in some of the proofs and a new nonstandard characterization of Schwartz spaces is given. (Received October 23, 1978.)

763-46-19 AGGIE HO, Claremont Graduate School, Claremont, California 91711. Trees and extreme points in Banach spaces. Preliminary report.

It is known that a Banach space that contains a tree does not have the Radon-Nikodým property. It is shown that a Banach space that has certain types of "trees" does not have the Krein-Milman property. A separable, nondual Banach space is constructed in which there is a tree whose closed convex span has many extreme points. In fact, the closed convex span of these extreme points contains the tree itself. (Received October 24, 1978.)

\*763-46-20 William Tadd Franke, Emory University, Atlanta, Georgia 30322. Some Banach Spaces of Distributions.

Some norms are presented which produce a sequence of nested Banach spaces on subsets of distributions (in the sense of Laurent Schwartz). Several properties possessed by this collection of spaces, but not by  $\mathcal{D}'$ , are exhibited and an alternate order for these distributions is analyzed. (Received October 24, 1978.)

\*763-46-21 James Hagler\* and Francis Sullivan, The Catholic University of America, Washington, D. C. 20064. Smoothness and weak\* Sequential Compactness.

A Banach space  $E$  is said to have property  $(w)$  if every bounded sequence in  $E^*$  has a weak\* converging subsequence. Theorem: If  $E$  does not have  $(w)$  and  $F$  is a closed subspace of  $E$  which does have  $(w)$ , then  $E/F$  has no equivalent smooth norm. In particular, a smooth space has  $(w)$ .

This extends the result announced in these NOTICES (25) 1978, Abstract 78T-B179. (Received October 25, 1978.)

\*763-46-22 Francis Sullivan, The Catholic University of America, Washington, D.C. 20064. Nearly smooth norms for Banach spaces.

The norm for a Banach space  $E$  is smooth if for all  $\|x\| = 1$  and all  $y$

$$c(x, y) \equiv \limsup_{t \rightarrow 0^+} \frac{\|x+ty\| + \|x-ty\| - 2}{t} = 0$$

The norm is strongly rough if there is an  $\epsilon > 0$  so that for each  $\|x\| = 1$  there is a  $\|y\| \leq 1$  with  $c(x, y) > \epsilon$ .

It is known that if  $E$  has an equivalent smooth norm then  $E$  has no equivalent strongly rough norm. We prove a partial converse. Theorem. If  $E$  has a norm which is not strongly rough then every equivalent norm for  $E$  has the following property: For each  $\epsilon > 0$  there is a set of norm-1 vectors  $D_\epsilon$ , dense in the unit sphere of  $E$  such that for each  $x \in D_\epsilon$  and all  $y$ ,  $c(x, y) \leq \epsilon$  (Received October 25, 1978.)

763-46-23 W.A. Feldman and J.F. Porter\*, University of Arkansas, Fayetteville, Arkansas 72701. Representation spaces for Banach lattices.

For a Banach lattice  $V$ , H.H. Schaefer defined a locally compact space  $X$  to be a representation space for  $V$  if the space  $C_\infty(X)$  of functions continuous on  $X$  with compact support can be identified with a dense ideal in  $V$ . A characterization is provided for the existence of a representation space, extending results reported in these Notices (25(1978): A-121). (Received October 25, 1978.)

763-46-24 ROBERT E. HUFF, The Pennsylvania State University, University Park, Pennsylvania 16802  
Spaces which are nearly uniformly convex. Preliminary report.

For  $\delta < 1$ , a  $\delta$ -slice of the unit ball  $B$  of a Banach space  $X$  is a set of the form  $\{x \in B : x^*(x) \geq \delta\}$  for some  $\|x^*\| = 1$ .  $X$  is uniformly convex (UC) iff for every  $\varepsilon > 0$  there exists  $\delta < 1$  such that every  $\delta$ -slice has diameter  $< \varepsilon$ . Call  $X$  nearly uniformly convex (NUC) if for every  $\varepsilon > 0$  there exists  $\delta < 1$  such that every  $\delta$ -slice has a finite  $\varepsilon$ -net. Then (NUC)-spaces are reflexive, there exist (NUC)-spaces which cannot be renormed to be (UC), and there exist non-(NUC) reflexive spaces. The author does not yet know if every reflexive space can be renormed to be (NUC). (Received October 25, 1978.)

763-46-25 J. F. COLOMBEAU, B. PERROT, Université de Bordeaux-I, France, and T. A. W. DWYER, III, Northern Illinois University, De Kalb, IL 60115. Convolution equations in non-metric and non-dual-metric domains. Preliminary report.

Existence theorems for convolution equations in spaces of analytic functions on infinite-dimensional domains  $E$  have until now been restricted to the case when  $E$  is either metrizable or the dual of a metrizable space: cfr., e.g. Ph. Boland, Proc. on Infinite-dimensional holomorphy, Springer Lecture Notes in Math. No. 364 (1974), p.143, Theorem 2.2, or T. A. W. Dwyer, III, Rendiconti di Matematica 10, No.2-3 (1977-78), p.286, Theorem 2.4.2 and p.291, Theorem 2.6.2. This excludes cases such as  $E = D$  or  $D'$  (distributions). In this paper we show how, given an equation  $g(d)\psi = \varphi$  with  $\varphi$  analytic on  $E$  and  $g$  on  $E'$  (symbol of a convolution operator), one can construct a Fréchet space  $\tilde{E}$  (depending on  $\varphi$  and  $g$ ) into which  $E$  is continuously mapped, to solve the equation for  $\psi$  over  $\tilde{E}$ , and then restricting  $\psi$  to  $E$  to get a solution to the equation over  $E$ . Previous existence theorems then extend to domains in  $D$  or  $D'$ . (Received October 25, 1978.)

\*763-46-26 ANTON R. SCHEP, California Institute of Technology, Pasadena, California 91125. Generalized Carleman operators.

Let  $L_p = L_p(Y, \nu)$  be a normed function space with order continuous norm  $\rho$ . Theorem. For a linear operator  $T$  from  $L_p$  into  $M(X, \mu)$  the following are equivalent: 1)  $T$  is an integral operator with kernel  $T_x(y) = T(x, y) \in L'_p(Y, \nu)$  (= 1<sup>st</sup> associate space). 2) There exists  $g \in M(X, \mu)$  such that  $|Tf| \leq g \cdot \rho(f)$  for all  $f \in L_p$ . 3) If  $f_n \in L_p$  and  $\rho(f_n) \rightarrow 0$ , then  $Tf_n(x) \rightarrow 0$  a.e. As an application we get an order theoretic characterization of integral operators of finite double norm. (Received October 25, 1978.)

763-46-27 PETER MORRIS, The Pennsylvania State University, University Park, PA 16802. A non-uniquely minimal Fourier projection.

Let  $\Gamma$  be a finite symmetric set of integers. Denote by  $\Pi$  the closed subspace of  $C_{2\pi}$  (the real  $2\pi$ -periodic continuous functions) spanned by  $\{e^{int} : n \in \Gamma\}$ . Let  $P$  be the Fourier projection onto  $\Pi$ . The well-known Berman formula shows that  $P$  has least norm among all projections of  $C_{2\pi}$  onto  $\Pi$ . S. Fisher, D. Wulbert and myself have shown (in a paper to appear) that  $P$  is often unique in this respect. However, if  $\Gamma = \{0, \pm 2, \pm 3, \pm 4\}$ ,  $P$  is not unique. (Received October 25, 1978.)

## 47 ► Operator Theory

763-47-1 Scott W. Brown, University of California, Santa Barbara, Ca. 93106. Banach algebras that are dual spaces. Preliminary report.

Let  $\mathcal{B}$  be a Banach algebra which is isometrically isomorphic to the dual of a Banach space  $C$ . Define for any  $A \in \mathcal{A}$  the operators  $R_A$  and  $L_A$  by  $R_A(B) = BA$  and  $L_A(B) = AB$  for all  $B \in \mathcal{A}$ . Assume that  $L_A, R_A: (\mathcal{B}, \text{weak}^*) \rightarrow (\mathcal{B}, \text{weak}^*)$  are continuous for all  $A \in \mathcal{A}$ . These operators induce, in natural fashion, operators on  $C$ . These operators on  $C$  are discussed. (Received August 21, 1978.)

\*763-47-2 Ronald E. Bruck, University of Southern California, Los Angeles 90007. Asymptotic behavior of certain second-order equations involving monotone operators.

Let  $A$  be a maximal monotone operator on a real Hilbert space  $H$  and consider the equation (\*):  $d^2u/dt^2 \in Au + f(t)$ , where  $f \in L^2_{loc}(0, \infty; H)$ . A strong solution of (\*) on  $[0, T]$  is a function  $u$  in  $W^{2,2}(0, T; H)$  satisfying (\*) a.e.; a weak solution on  $[0, T]$  is a  $W^{1,2}$  limit of strong solutions. A strong (weak) solution on  $[0, \infty)$  is one whose restriction to each  $[0, T]$  is a strong (weak) solution. THEOREM. If  $u$  is a bounded strong solution of (\*) on  $[0, \infty)$  and  $f$  has period  $p$ , then there exists a  $p$ -periodic weak solution  $v$  such that  $u(t) - v(t) \rightarrow 0$  as  $t \rightarrow \infty$ . It is not known whether  $v$  is actually a strong solution. (Received October 23, 1978.)

\*763-47-3 Robert Olin\* and James Thomson, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Algebras of Subnormal Operators.

Let  $S$  be a subnormal operator on a separable Hilbert space  $\mathcal{H}$  with minimal normal extension  $N$  on  $\mathcal{K}$ . Let  $\mathcal{Q}(S)$  denote the  $\sigma$ -weakly closed algebra generated by  $S$  and the identity.  $\mathcal{W}(S)$  denotes the weak closure of  $\mathcal{Q}(S)$ .

Theorem. The map  $i(T) = T$  for  $T \in \mathcal{Q}(S)$  is a homeomorphism from  $(\mathcal{Q}(S), \sigma\text{-weak})$  onto  $(\mathcal{W}(S), \text{weak})$ . The proof of this relies on a suitable "factorization" formula for any linear functional  $L \in (\mathcal{Q}(S), \sigma\text{-weak})^*$ . This in turn is accomplished by a modification of S. Brown's remarkable technique (Some Invariant Subspaces for Subnormal Operators). (Received September 5, 1978.)

\*763-47-4 SANDY GRABINER, Pomona College, Claremont, California 91711. Operator Ranges and Invariant Subspaces.

Suppose that  $A$  is an algebra of bounded operators on the Banach space  $X$ , and that  $C$  and  $D$  are non-zero compact operators on  $X$ . If  $A$  is the range of a bounded operator from some Banach space to the algebra of operators on  $X$ , and if  $CA \subseteq AD$ , we prove that both  $A$  and its commutant have invariant subspaces (the analogous result when  $AC \subseteq DA$  was proved by Fong, Nordgren, et al). We prove a result on ranges of multilinear operators which allows us to prove similar theorems about joint invariant subspaces of finite collections of operator range algebras. Using results on invariant subspaces of operator range algebras, together with results about formal power series which we obtained earlier, we prove that if  $T$  is a quasi-nilpotent operator and  $g$  is an analytic function for which either  $TC = Cg(T)$  or  $CT = g(T)C$ , then  $T$  has a hyperinvariant subspace, provided that  $|g'(0)| < 1$  or that  $\{\|T^n\|\}$  satisfies a reasonable growth condition. (Received September 18, 1978.)

\*763-47-5 William L. Paschke, University of Kansas, Lawrence, Kansas 66045, and Norberto Salinas, University of Kansas, Lawrence, Kansas 66045.  $C^*$ -algebras associated with free products of groups.

We show that the  $C^*$ -algebra of the left regular representation of the free product of two non-trivial groups, not both of order 2, is simple and has a unique tracial state. In the case of the free product of cyclic groups, we investigate weak versus strong triviviality for extensions of this  $C^*$ -algebra. One consequence of our extension - theoretic results is that the algebras of  $n \times n$  matrices ( $n = 1, 2, \dots$ ) over the  $C^*$ -algebra of the left regular representation of the free product of two cyclic groups are pairwise non-isomorphic. (Received September 18, 1978.)

\*763-47-6 GEORGE H. PIMBLEY, National Science Foundation, Washington, D. C. 20550 and Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545. Nonoccurrence of secondary bifurcation for certain nonlinear operators in Banach space. Preliminary report.

This talk concerns the eigenvalue problem  $F(x) = \lambda x$ ,  $x \in B$ , where  $B$  is a separable Banach space with a normal reproducing minihedral cone  $K$ , and  $F(x)$  is an odd compact Frechet differentiable superlinear (sublinear) operator mapping  $B$  into itself. The pencil of Frechet derivatives  $F'(x)$  is considered on a Hilbert space  $H$  containing  $B$  setwise, and is assumed to have the property that the  $i$ th eigenelement  $h_i^{(x)}$  corresponding to the eigenvalue  $\mu_i^{(x)}$  is related to the corresponding eigenelement of the adjoint operator  $F'(x)^*$  by a positive definite

symmetric transformation  $P$ , independently of  $x \in B$ . With this condition on  $F'(x)$ , the theory is applicable to, but not limited to, problems of Hammerstein type with positive definite kernel. Within these strictures, a form of coercive monotone nonlinear operator,  $F(x)$ , is defined for which a given continuous branch  $x_\lambda^{(i)}$  of eigenelements, with primary bifurcation point at  $\mu_i^{(0)}$ , does not undergo secondary bifurcations. (Received September 21, 1978.)

763-47-7 PAUL R. HALMOS, Indiana University, Bloomington, Indiana 47401. Closure and continuity, weak and strong.

There are several famous closure and continuity theorems in operator theory for the weak and for the norm topologies. (E.g., the weak closure of the set of unitary operators on  $H$  is the unit ball of  $B(H)$ , and the norm closure of the set of reducible operators on  $H$  is  $B(H)$ ; norm is weakly lower semicontinuous, and spectrum is norm upper semicontinuous.) For the strong topology less is known, and the most impressive known fact (the strong closure of the set of normal operators is the set of subnormal operators) has not yet found any applications. The purpose of this note is to study some strong closures (e.g., of the set of co-isometries and of the set of hyponormal operators), answer some strong continuity questions (e.g., is the mapping  $\text{Lat}$  from  $B(H)$  to lattices of subspaces upper semicontinuous?), and indicate some applications (e.g., to the determination of some invariant subspace lattices). Some curious lemmas turn out to be useful. Sample: if a sequence of vectors in  $H$  converges weakly to 0, then some subsequence is arbitrarily near to an orthonormal sequence, and, similarly, if a sequence of projections of finite (but not necessarily bounded) rank in  $H$  converges weakly to 0, then some subsequence is arbitrarily near to an orthogonal sequence of projections. (Received October 3, 1978.)

\*763-47-8 JOEL ANDERSON, Pennsylvania State University, University Park, PA 16802. Pathology in the Calkin Algebra.

One of the main stumbling blocks in the study of the Calkin algebra is the fact that it is non-separable. The theme of this report is that sometimes this difficulty can be overcome by using the continuum hypothesis (CH) in conjunction with separable results. For example, CH and Voiculescu's theorem can be used to show that each state on the Calkin algebra restricts to a homomorphism on a maximal abelian self-adjoint subalgebra of the Calkin algebra. Other results and open questions will also be discussed. (Received October 4, 1978.)

\*763-47-9 ERIC A. NORDGREN, University of New Hampshire, Durham, New Hampshire 03824. Lomonosov's theorem: extensions and limitations.

Some recent extensions of Lomonosov's invariant subspace theorem will be reported. Also an example will be presented of a weighted shift operator whose commutant contains no nonscalar operator that commutes with a compact operator. Thus there exist operators not covered by Lomonosov's theorem. The set of such operators is in fact strongly dense. (Received October 5, 1978.)

763-47-10 John B. Conway, Indiana University, Bloomington, In. 47401. A result on quasi-similar subnormal operators. Preliminary report.

For  $k = 1, 2$ , let  $S_k$  be a subnormal operator on a separable Hilbert space  $H_k$ , and let  $A_k$  be the weak\* (i.e., ultraweakly) closed algebra generated by  $S_k$ . THEOREM If  $S_1$  and  $S_2$  are quasi-similar via the quasi-invertible operators  $X: H_1 \rightarrow H_2$  and  $Y: H_2 \rightarrow H_1$ , then there is an isometric algebra isomorphism  $\rho: A_1 \rightarrow A_2$  such that: (a)  $\rho$  is a weak\* homeomorphism; (b)  $\rho(S_1) = S_2$ ; (c) for every  $A_1$  in  $A_1$ ,  $XA_1 = \rho(A_1)X$  and  $A_1Y = Y\rho(A_1)$ . It is already known that quasi-similar hyponormal operators have equal spectra (Clary, P.A.M.S., 53(1975)); the preceding theorem can be interpreted as a refinement of this fact for subnormal operators. In fact, if  $\mu_k$  is a scalar-valued spectral measure for the minimal normal extension of  $S_k$ , then  $\mu_1$  and  $\mu_2$  must have the same "Sarason hull" when  $S_1$  and  $S_2$  are quasi-similar. This has several applications. (Received October 6, 1978.)

\*763-47-11 WILLIAM MARGULIES, California State University, Long Beach, Long Beach, California 90840. Numerical Range. Preliminary report.

Let  $A: H \rightarrow H$  be a bounded operation on a Hilbert space. Let  $N(A) = \{(Ax, x) \mid |x| = 1\}$ .

Def.  $z \in N(A)$  is a corner point if any ellipse or line segment containing  $z$  in the interior intersects the complement of  $N(A)$ . It is shown that if  $z$  is a corner point and  $z = (Ax, x)$  then  $x$  is an eigenvector of  $A$ . (Received October 10, 1978.)

\*763-47-12 DONALD D. ROGERS, U. S. Naval Academy, Annapolis, Maryland, 21402. On Fuglede's Theorem and Operator Topologies.

For each normal operator  $N$  and neighborhood  $E$  of 0 in the strong or weak operator topology, a neighborhood  $D$  of 0 in the same topology is constructed such that  $\|B\| \leq 1$  and  $NB - BN$  in  $D$  imply  $N^*B - BN^*$  in  $E$ . A sequence  $\{N_k\}$  of unitarily equivalent normal operators and an operator  $B$  are constructed such that  $N_k B - B N_k^*$  converges to 0 in the strong operator topology, and  $N_k^* B - B N_k^*$  converges in the weak operator topology to a non-zero operator. (Received October 16, 1978.)

\*763-47-13 W. N. ANDERSON, Jr, West Virginia University, Morgantown, WV 26506, T.D. MORLEY\*, University of Illinois, Urbana, IL 61801, and G. E. TRAPP, West Virginia University. The geometric mean and the Asplund average of positive operators.

Given two positive operators  $A$  and  $B$ , the Asplund average of  $A$  and  $B$  is given by the limit of two different monotone sequences of positive operators. These sequences are formed by successively computing arithmetic and harmonic means. Explicit estimates of the differences in consecutive terms of the sequences may be given in terms of the arithmetic and harmonic differences. The Asplund average generalizes the concept of the geometric mean of positive operators, and reduces to the classical square root definition whenever the operators commute. (Received October 16, 1978.)

763-47-14 CHARLES A. BERGER and LEWIS A. COBURN, Yeshiva University, New York, New York 10033. A fully explicit index theory for the  $C^*$ -algebra generated by two Toeplitz isometries.

Suppose that  $\varphi$  and  $\psi$  are two inner functions in  $H^\infty(\mathbb{T})$ . Let  $T_\varphi$  and  $T_\psi$  be the associated Toeplitz operators acting on  $H^2(\mathbb{T})$ , and let  $\mathcal{A}$  be the  $C^*$ -algebra they generate. Berger, Coburn, and Lebow established necessary and sufficient conditions for an element of  $\mathcal{A}$  to be a Fredholm operator when  $T_\varphi^* T_\psi - T_\psi^* T_\varphi$  is compact. An explicit computation of the index of such an operator is now provided. (Received October 16, 1978.)

\*763-47-15 WILLIAM E. KAUFMAN, University of Houston, Houston, Texas 77004. Semiclosed operators in Hilbert space.

In a Hilbert space  $H$ , an operator  $C$  is semiclosed provided that there is a bounded operator  $B$ , from  $H$  onto the domain of  $C$ , such that  $CB$  is bounded. We prove that the family  $SC(H)$  of all such operators is the smallest which contains all closed operators in  $H$ , all sums and products of its members, and all inverses of its one-to-one members. In fact, each member of  $SC(H)$  is (1) the sum of two closed one-to-one operators in  $H$  with the same domain and closed ranges, (2) the product of a bounded operator on  $H$  with a closed positive operator in  $H$ , (3) the strong limit on its domain of a sequence of bounded operators on  $H$ . Thus  $SC(H)$  is the algebraic and topological closure of the set of all closed operators in  $H$ , and the latter may be characterized as the set of all  $C$  in  $SC(H)$  such that, if  $B$  is a bounded operator from  $H$  onto the domain of  $C$  and  $A$  denotes  $CB$ , then  $\text{range}(A^*) + \text{range}(B^*)$  is a closed subspace of  $H$ . In view of these facts, it is perhaps surprising that, if  $T$  is any linear operator in  $H$  with domain having countable algebraic dimension then  $T$  has an extension belonging to  $SC(H)$ . (Received October 16, 1978.)

763-47-16 HEINZ W. ENGL, Kepler-Universität, A-4045 Linz, Austria, and University of Delaware, and M. ZUHAIR NASHED, Dept. of Mathematical Sciences, University of Delaware, Newark, DE 19711. Measurability of outer inverses of linear random operators.

M. Z. Nashed and H. Salehi proved that under mild conditions the Moore-Penrose inverse of a linear random operator is again a random operator (SIAM J. Appl. Math. 25(1973)). Here we consider the question of measurability of outer inverses of linear random operators and prove:

Theorem: Let  $X, Y$  be real separable Banach spaces,  $(W, S, m)$  a  $\sigma$ -finite measure space,  $L: W \times X \rightarrow Y$  a bounded linear random operator. Then there is a countable set of bounded linear random operators  $S_n: W \times X \rightarrow X$  such that for all  $w \in W$  and  $n \in \mathbb{N}$ ,  $S_n(w)L(w)S_n(w) = S_n(w)$ . Furthermore, for  $m$ -almost all  $w_0 \in W$  the following holds: If  $T \in L(Y, X)$  is an arbitrary outer inverse of  $L(w_0)$ , then there exists a sequence  $n_1, n_2, n_3, \dots$  of natural numbers such that for all  $y \in Y$ ,  $(S_{n_i}(w_0)y)$  converges to  $Ty$ .

The need for measurable outer inverses stems from applications to Newton's method for random nonlinear operator equations and to inverse mapping theorems for random operators. (Received October 18, 1978.)

\*763-47-17 G. D. ALLEN and J. D. WARD, Texas A & M University, College Station, Texas 77843, and D. A. LEGG, IUPUI-Fort Wayne, Fort Wayne, Indiana 46805. Hermitian liftings in Orlicz sequence spaces.

Let  $X$  be a Banach space,  $B(X)$  the space of bounded linear operators on  $X$ , and  $C(X)$  the space of compact operators on  $X$ . The quotient space  $B(X)/C(X)$  is called the Calkin algebra. An operator  $T \in B(X)$  is Hermitian if its numerical range is real, and essentially Hermitian if the coset  $T + C(X)$  has real numerical range in  $B(X)/C(X)$ .

Theorem: Let  $X$  be a reflexive Orlicz sequence space such that 2 is not in the associated interval. Then every essentially Hermitian operator is a compact perturbation of an operator which has a real diagonal representation with respect to the natural basis in  $X$ . (Received October 20, 1978.)

\*763-47-18 GREGORY B. PASSTY, University of Southern California, Los Angeles, California 90007. Ergodic Convergence to a Zero of the Sum of Monotone Operators. Preliminary report.

In the real Hilbert space  $H$ , let  $\{A^i: 1 \leq i \leq N\}$  be a set of nonlinear maximal monotone operators with maximal monotone sum. Consider the multiple resolvent scheme given by:  $x_0 = x \in H$ ;  $x_n = \prod_{i=1}^N (I + \lambda_n A^i)^{-1} x_{n-1}$  for  $n \geq 1$ , where  $\{\lambda_n\}$  is a sequence of positive reals. For each  $n$ , let  $z_n = \frac{\sum_{i=1}^n \lambda_i x_i}{\sum_{i=1}^n \lambda_i}$ , the weighted average of the iterates. Theorem: If  $\{\lambda_n\} \in \ell^2 \setminus \ell^1$ , then either (i)  $\|z_n\| \rightarrow +\infty$ , in which case 0  $\notin$  the range of  $\sum_{i=1}^N A^i$ ; or (ii)  $\{z_n\}$  converges weakly to a zero of  $\sum_{i=1}^N A^i$ . Weak ergodic convergence is also obtained for iterations consisting of combinations of forward and backward steps, including those allowing permutations of  $\{A^i\}$ . Strong convergence of  $\{x_n\}$  is proved for suitably conditioned operators. (Received October 20, 1978.)

\*763-47-19 Robert C. Sine, University of Rhode Island, Kingston, Rhode Island 02881. Fixed points for certain nonlinear contraction semigroups.

Fixed points for nonexpansive semigroups with precompact orbits on certain kinds of domains are obtained. Typical results: Theorem 1. Every nonexpansive self map of the ball of  $L_\infty$  has a fixed point. Theorem 2. Every nonexpansive semigroup of self maps of the ball of  $C_0$  with at least one precompact orbit has a fixed point. Theorem 3. Every nonexpansive semigroup of self maps of the ball of  $C(X)$  with every orbit precompact has a fixed point. (Received October 20, 1978.)

\*763-47-20 MARK A. KON, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139.

Asymptotic Probability Distributions of Observables in Canonical Ensembles.

In quantum statistical mechanics, there is associated with any hamiltonian operator  $H$  the density operator  $\frac{e^{-\beta H}}{\text{tr } e^{-\beta H}}$ , with  $\beta > 0$ , which exists if  $e^{-\beta H}$  is trace class.  $e^{-\beta H}$  is never trace class if  $H$  has a continuous component in its spectrum, which motivates a procedure discussed here for the "approximation" of a self-adjoint operator with a continuous spectrum by a net  $\{H_\epsilon\}_{\epsilon > 0}$  of discrete self-adjoint operators whose spectra approach that of  $H$  as  $\epsilon \downarrow 0$ . The hamiltonian  $H$  which is approximated using this technique has the form  $H = d\Gamma(A)$ , which is that of the many particle hamiltonian governing a system with an indefinite number of particles (as obtained, for instance, through non-interacting quantum field theory) each of which evolves



under the action of the positive single particle energy operator  $A$ .  $H$  may describe either fermions or bosons, with different mappings  $d\Gamma$  acting on the same single particle operator  $A$  for the two cases. The notion of a *distribution* of a self-adjoint operator is introduced, and the  $\epsilon$ -discrete approximation  $A_\epsilon$  of  $A$  is defined with respect to a distribution  $\mu$  of  $A$ . We then make the definition  $H_\epsilon = d\Gamma(A_\epsilon)$ . The density operator  $\frac{e^{-\beta H_\epsilon}}{\text{tr } e^{-\beta H_\epsilon}}$  yields particle number and energy probability distributions whose asymptotic forms (which depend on the distribution  $\mu$ ) are derived as  $\epsilon \downarrow 0$ . Within the class of distributions  $\mu$  of self-adjoint operators under consideration, the normalizations (to mean 0 and variance 1) of these asymptotic probability distributions have either the normal or the extreme value distribution. (Received October 25, 1978.)

\*763-47-21 J. J. BUONI, Youngstown State University, Youngstown, Ohio and BHUSHAN L. WADHWA, Cleveland State University, Cleveland, Ohio. Joint Browder Spectrum. Preliminary report.

Let  $T_1$  and  $T_2$  be two commuting bounded operators on a Banach Space  $Y$ . Following the natural ideas of Waelbroeck, (J. Math. Pures Appl. 1954), we define the joint Browder spectrum of  $T_1$  and  $T_2$ , denoted by  $\sigma_b(T_1, T_2) = \{(\lambda_1, \lambda_2) : p(\lambda_1, \lambda_2) \in \sigma_b(p(T_1, T_2)) \text{ for all polynomials } p: C^2 \rightarrow C\}$  where  $\sigma_b(T)$  denotes the usual definition of Browder spectrum. The joint Fredholm spectrum  $\sigma_\phi(T_1, T_2)$  and joint spectrum  $\sigma(T_1, T_2)$  are also defined in this algebra independent manner. Our main result is then: Theorem  $\sigma_b(T_1, T_2) = \sigma_\phi(T_1, T_2) \cup \{\text{accumulation points of } \sigma(T_1, T_2)\}$ . (Received October 23, 1978.)

\*763-47-22 George L. Cain, Jr., Georgia Institute of Technology, Atlanta, Georgia 30332. Fixed points and stability of condensing multifunctions on random normed spaces.

A fixed point theorem for probabilistically condensing set valued operators on complete random normed spaces is obtained and applied to stability problems for a related class of set valued operators on such spaces. These results extend to set valued operators fixed point results of G. Bocsan [Sem. de Teoria Funct. si Mat. Apl. - A. Spatii metriche prob., N. 13, 1974], and for Banach spaces, certain stability results of Cain and Nashed [Pac. J. Math. 39(1971), 581-592]. (Received October 23, 1978.)

\*763-47-23 BRIAN W. MCENNIS, The Ohio State University, Marion, Ohio 43302. The distortion coefficient of an operator with bounded characteristic function.

An operator  $T$  on a Hilbert space  $\mathcal{H}$  is similar to a contraction if  $\|STS^{-1}\| \leq 1$  for some invertible operator  $S$  on  $\mathcal{H}$ . For such an operator  $T$ , we define the distortion coefficient  $M(T)$  by  $M(T) = \inf \{\|S\| \|S^{-1}\| : \|STS^{-1}\| \leq 1\}$ .

Ch. Davis and C. Foias [Acta Sci. Math., 32(1971), 127-139] have shown that for  $T$  to be similar to a contraction, it is sufficient that the characteristic function  $\theta_T$  be bounded:  $\sup \{\|\theta_T(\lambda)\| : |\lambda| < 1\} = C < \infty$ . In this paper, we obtain an upper bound for the distortion coefficient  $M(T)$  for an operator with bounded characteristic function. The estimate of  $M(T)$  is given as an explicit function of the bound  $C$  of  $\theta_T$ . This solves a problem originally posed to the author by John Holbrook.

(Received October 23, 1978.)

\*763-47-24 JEFFREY R. BUTZ, University of Oklahoma, Norman, OK 73019. Hankel Operators with Hilbert Space Range.

The functional calculus of Sz.-Nagy and Foias for completely non-unitary contractions is used to define a class of operators from the Hardy space  $H^2$  to an arbitrary (separable complex) Hilbert space  $X$ . These operators have as special cases of them both the classical Toeplitz operators and the classical Hankel operators. Properties of this class of operators are derived which extend previous results (such as Nehari's Theorem) known for the classical case. (Received October 23, 1978.)

\*763-47-25 James V. Herod, Georgia Institute of Technology, Atlanta, Georgia 30332 and R. W. McKelvey, University of Montana, Missoula, Montana 59801. A Hille-Yosida Theory for Evolutions.

This presentation will be concerned with providing generators for a class of evolution operators defined as follows:  $\{X_p, \|\cdot\|_p\}_{p=0}^\infty$  is a chain of Banach spaces such that  $X_{p+1}$  is dense in  $X_p$  and  $\|x\|_{p+1} \geq \|x\|_p$  for each  $x$  in  $X_{p+1}$ ,  $p = 0, 1, 2, \dots$ . The evolution class consists of operator valued functions  $U$  having these properties: (a)  $U(r,s)U(s,t) = U(r,t)$  for  $r \geq s \geq t$ , (b) there is a continuous, multiplicative function  $\mu$  such that if  $a \geq b$  then  $\|[U(a,b)-1]x\|_p \leq [\mu(a,b)-1]\|x\|_{p+1}$ ,  $x \in X_{p+1}$ , and (c)  $U$  is  $m$ -contractive on the chain  $\{X_p, \|\cdot\|_p\}_{p=0}^\infty$  in the sense that if  $a \geq b$  then  $U(a,b)$  has domain  $X_0$ ,  $U(a,b)X_p \subset X_p$ , and  $\|U(a,b)x\|_p \leq \|x\|_p$  for  $x$  in  $X_p$ ,  $p = 0, 1, 2, \dots$ . If  $T$  is a  $C_0$ , contractive semigroup on a Banach space and  $U(s,t) = T(s-t)$ ,  $s \geq t$ , then there is a chain of Banach spaces such that  $U$  is in the evolution class described above.

(Received October 24, 1978.)

\*763-47-26 Michael A. Freedman, Georgia Institute of Technology, Atlanta, Georgia 30332. Integral Equations and Evolution Operations. Preliminary report.

The generators  $A$  for the class of evolution operators defined in the above abstract by Herod and McKelvey satisfy (a)  $A(r,s)x + A(s,t)x = A(r,t)x$  for  $r \geq s \geq t$  and  $x \in X_1$ , (b) there is a continuous, additive function  $\alpha$  such that if  $a \geq b$  then  $\|A(a,b)x\|_p \leq \alpha(a,b)\|x\|_{p+1}$  for  $x \in X_{p+1}$  and (c)  $A$  is  $m$ -dissipative on the chain  $\{X_p, \|\cdot\|_p\}_{p=0}^\infty$  in the sense that if  $a \geq b$ ,  $(1-A(a,b))(X_{p+1}) \supseteq X_{p+1}$  and  $\|(1-A(a,b))x\|_p \geq \|x\|_p$  for  $x \in X_p$ ,  $p = 0, 1, 2, \dots$ . The analysis of the present paper allows for  $\alpha$  and  $\mu$  to be discontinuous. In fact, if property (d)  $\|(1+A(a,a^+))x\|_p \leq \|x\|_p$  and  $\|(1+A(a^-,a))x\|_p \leq \|x\|_p$  for  $x \in X_p$ ,  $\forall a, p = 1, 2, \dots$  is added, we demonstrate a one-to-one pairing from this class  $A$  to elements in the multiplicative class  $M$  defined by Herod and McKelvey amended to allow discontinuities in  $\mu$ . If  $A \in A$  we solve the integral equations

$$F(a,b)x = x + (L) \int_a^b F(a,\cdot)Ax \quad \text{and} \quad F(a,b)x = x + (R) \int_a^b AF(\cdot,b)x \quad \text{for } x \in X_1,$$

by looking into class  $M$ . (Received October 24, 1978.)

763-47-27 D.J. DOWNING, Oakland University, Rochester, Michigan 48063. Some Coincidence Theorems in Metric and Banach Spaces. Preliminary report.

Let  $A$  be an arbitrary set,  $(M,d)$  a metric space, and  $f:A \rightarrow M$  a mapping. Suppose  $T:A \rightarrow M$  is a mapping with  $T(A)$  complete. The following is an immediate consequence of Caristi's fixed point theorem. Theorem. Suppose  $f(A) \subset T(A)$  and, in addition, suppose that there exists a l.s.c.  $\phi:T(A) \rightarrow \mathbb{R}^+$  (the positive reals) such that  $d(T(x),f(x)) \leq \phi(T(x)) - \phi(f(x))$  for all  $x \in A$ . Then there exists  $\tilde{x} \in A$  with  $T(\tilde{x}) = f(\tilde{x})$ . A result of K. Goebel and a result of Furi, Martelli, and Vignoli are obtained as corollaries. A number of other coincidence theorems are also proven.

(Received October 24, 1978.)

\*763-47-28 LAWRENCE R. WILLIAMS, The University of Texas, Austin, Texas 78712. Equality of essential spectra of certain quasisimilar seminormal operators

Let  $A$  and  $B$  be quasisimilar seminormal (bounded, linear) operators on a separable, infinite dimensional, complex Hilbert space. Theorem 1. If  $A$  and  $B$  are biquasitriangular, then  $A$  and  $B$  have equal essential spectra. Theorem 2. If one of the quasiaffinities that implement the quasisimilarity of  $A$  and  $B$  is compact, then  $A$  and  $B$  have equal essential spectra.

Theorem 3. If  $A$  and  $B$  are both weighted unilateral shifts or both weighted bilateral shifts, then  $A$  and  $B$  have equal essential spectra. Theorem 4. If  $A$  is cohyponormal and  $B$  is a non normal hyponormal weighted (unilateral or bilateral) shift, then there exists no nonzero operator  $X$  such that  $XA = BX$ . (Received October 24, 1978.)

763-47-29 KAREN SINGKOFER. University of South Florida, Tampa, Florida 33620. Solvability of non-linear equations using K-monotonicity. Preliminary report.

Let  $H$  and  $H_1$  be real Hilbert space with norms  $\|\cdot\|$  and  $\|\cdot\|_1$ . Consider the equation  $Lx + N(x) = f$ ,  $x, f \in H$  where  $L$  is an unbounded linear operator and  $N$  is a nonlinear operator with  $D(L) \subset H_1 \subset D(N) \subset H$ . Let  $K$  be a linear operator and assume  $L$  and  $N$  satisfy  $(Lx, Kx) \geq a\|x\|_1^2 - b\|x\|^2$  and  $(N(x) - N(y), K(x-y)) \geq -c\|x-y\|_1^2 - d\|x-y\|^2$  where  $a, b, c, d$  are nonnegative constants. Existence Theorems are given under various conditions on  $K$  and  $N$ . When  $K$  is not invertible, alternative methods are used. Applications are given to boundary value problems for ordinary differential equations. The conditions imposed on  $N$  allow for nonlinearities involving derivatives. (Received October 25, 1978.) (Author introduced by Professor R. Kent Nagle).

763-47-30 THAKYIN HU, Department of Mathematics, Tamkang College of Arts and Sciences, Tamsui, Taipei, Taiwan, Republic of China. An extension of Markov-Kakutani theorem.

Abstract. Mappings defined on a star-shaped domain of a topological linear space are investigated in this paper. Some interesting results are derived. Our main result is a fixed point theorem for a commutative family of continuous affine mappings defined on a non-convex domain which extends the classical Markov-Kakutani Theorem. (Received October 24, 1978.) (Author introduced by Professor Frank T. Birtel).

\*763-47-31 JULIEN HENNEFELD, Department of Mathematics, Brooklyn College, Brooklyn, New York 11210. Smooth compact operators.

It is well known that for  $T$  a norm one compact operator on a real Hilbert space,  $T$  is smooth  $\Leftrightarrow \|Tx_1\| = \|Tx_2\| = \|T\|$  for some  $\|x_1\| = \|x_2\| = 1$  implies  $x_1 = \pm x_2$ . We show that this same characterization for smooth, compact operators holds for a large class of Banach spaces including  $\ell_p$ ,  $L_p[0, 1]$ , and  $d(a, p)$  where  $1 < p < \infty$ . (Received October 25, 1978.)

763-47-32 RICHARD J. FLEMING AND JAMES E. JAMISON, Memphis State University, Memphis, Tennessee 38152. Hermitian Operators on  $C(X, E)$ . Preliminary report.

A bounded operator  $T$  on a complex Banach space is norm hermitian if  $[Tx, x]$  is real for every element  $x$  of the space where  $[ \ , \ ]$  is a semi-inner product for which  $[x, x] = \|x\|^2$ . Let  $E$  be a complex Banach space,  $X$  a compact Hausdorff space, and  $C(X, E)$  the Banach space of continuous functions on  $X$  with values in  $E$ . Theorem. An operator  $T$  on  $C(X, E)$  is norm hermitian if and only if for each  $x \in X$  there is a norm hermitian operator  $H(x)$  on  $E$  such that  $Tf(x) = H(x)f(x)$  for all  $f \in C(X, E)$ . This extends a well-known result for the case where  $E$  is the field of complex numbers. (Received October 25, 1978.)

763-47-33 RICHARD J. FLEMING AND JAMES E. JAMISON, Memphis State University, Memphis, Tennessee 38152. The Isometries of  $s_p$ . Preliminary Report.

Let  $\| (x_k) \|_p^p = \sum |x_k|^p + \sum |x_{k+1} - x_k|^p$  where  $p > 1$  and  $p \neq 2$ . Let  $s_p$  be the linear space of all complex sequences for which  $\| (x_k) \|_p^p < \infty$ . Theorem. If  $v$  is a surjective isometry of  $s_p$  then there is a complex number  $\lambda$  with  $|\lambda| = 1$  such that  $v = \lambda I$  or  $v(x_k) = \lambda(x_2 - x_1, x_2, x_3, x_4, \dots)$ . (Received October 25, 1978.)

763-47-34 JAMES E. ROBINSON, LeMoyné-Owen College, Memphis, Tennessee 38126.  
A Comparison of Shifts on Banach Spaces. Preliminary Report.

R. Crownover defined a shift operator  $T$  on a Banach Space  $X$  to be characterized by the properties:

- (1)  $T$  is injective and has closed range.
- (2)  $T$  has corank 1.
- (3) If  $x$  is in  $X$ , and  $x$  is non-zero, then  $x$  is not infinitely divisible by  $T$ .

R. Gellar defined a weighted basis Shift  $T$  on a Banach Space  $X$  with a Schauder basis  $(x_i)$  as an operator that maps  $x_i$  onto  $b_i x_{i+1}$  where  $b_i$  is non-zero for each scalar  $b_i$ .

This paper determines that Gellar shifts can be strongly approximated by Crownover shifts on  $l_p$ , where  $p$  is greater than or equal to 1. Results on the interrelationships of a variety of shift operators are also shown. (Received October 25, 1978.) (Author introduced by Professor Richard Fleming).

\*763-47-35 R. Kannan, University of Texas at Arlington, Arlington, Texas 76019. Approximate solutions of nonlinear operator equations.

We consider the question of approximation of solutions of nonlinear operator equations of the type  $Eu = Nu$ , where  $E$  is a linear differential operator with a finite dimensional nontrivial kernel and  $N$  is a nonlinear operator over a real Hilbert space. Results from the theory of approximate solutions of nonlinear Hammerstein equations are utilized. (Received October 25, 1978.)

\*763-47-36 RICHARD W. CAREY, University of Kentucky, Lexington, Kentucky, 40506. Principal functions and real valued Cauchy transforms. Preliminary Report.

We shall discuss some connections between the principal function for almost commuting operators and the theory of weakly differentiable functions.

Let  $A$  be an operator on Hilbert space with  $[A^*, A] = A^*A - AA^*$  in trace class. For smooth functions  $\phi$  the mapping  $\phi \mapsto \text{trace}[\phi(A), A]$  defines a distribution  $\mathcal{J}$  whose support is a subset of the essential spectrum of  $A$ . The Cauchy transform of  $\mathcal{J}$  is a real valued function  $g$ , the so-called principal function of  $A$ . When  $\mathcal{J}$  is continuous in the sup norm and thus is represented by integration (by an annihilating measure) then  $g$  is weakly differentiable, i.e. satisfies the Gauss-Green theorem. This happens for example if the spectral multiplicity functions of the absolutely continuous parts of  $\text{Re}(A)$  and  $\text{Im}(A)$  are summable or if  $A$  is subnormal and the projection back has a trace class commutator with the normal extension; in this case  $H^p(\text{gdxdy})$  has bounded point evaluations ( $1 \leq p < \infty$ ), which in some cases leads to bounded evaluations for  $A$ . This result prompts the search for annihilating measures whose Cauchy transform is real valued. (Received October 25, 1978.) (Author introduced by Professor Douglas N. Clark).

## 49 ► *Calculus of Variations and Optimal Control*

763-49-1 I.M. Anderson and T. Duchamp\*, Department of Mathematics, University of Utah, Salt Lake City, Utah 84112.  
On the existence of global variational principals.

The inverse problem of the calculus of variations is to determine when a given system of differential equations can be identified with the Euler-Lagrange equations associated to a variational problem. The local version of the inverse problem has been solved (independently) many times. In this paper we solve the global inverse problem: for differential equations on a fiber bundle  $P$  over an  $m$ -dimensional manifold we show that the global obstructions to the solution of the inverse problem lie in the deRham cohomology group  $H_{DR}^{m+1}(P)$ . (Received September 5, 1978.)

\*763-49-2 KAREN K. UHLENBECK, University of Illinois at Chicago Circle, Chicago, Illinois 60680. Zermelo's navigation problem and null geodesics.

The following navigation problem was posed and solved by Zermelo. A wind distribution on  $R^2$  is given by a time-dependent vector field  $W(x, t)$  and a ship moves with constant speed  $k$  relative to the air. How should the ship be steered to reach a goal in the shortest time? By introducing an internal parameter  $\lambda$ , Carathéodory converts this to a standard Mayer problem in space-time. We recognize the constraint on  $s: [0, 1] \rightarrow M \times R$  as  $|s'(\lambda)| = 0$  in the Lorentz metric  $(ds)^2 = (k^2 - W^2)(dt)^2 - 2kdt(W \cdot dx) - (dx)^2$ . The solution is easily seen from Fermat's principle. A path of a light-ray minimizes travel-time between points in space. Analysis of light-cones in Lorentz geometry gives insight to a whole class of problems in the calculus of variations. (Received October 4, 1978.)

\*763-49-3 Loren D. Meeker, Univ<sup>th</sup> of New Hampshire, Durham, NH, 03824. Local time-optimal feedback control of n-order two-input systems.

This paper is concerned with the time-optimal control of the autonomous system  $\dot{x} = Ax + Eu$ ,  $x \in R^n$ ,  $u \in R^r$ . Let  $K(T, k)$  be the points of  $R^n$  controllable to the origin in time  $T$  by extremal control functions having less than  $k$  switches and  $K(T) = \bigcup K(T, k)$ . The system is minimally controllable if  $K(T, n)$  is a neighborhood of the origin. THEOREM: Let  $r = 2$  and  $B = [b^1, b^2]$ . Then the normal system  $\dot{x} = Ax + Bu$  is minimally controllable if, and only if, the  $n+1$  determinants  $d(j) = b_\lambda^1 A b_\lambda^1 \dots A^{n-1-j} b_\lambda^1 b_\lambda^2 A b_\lambda^2 \dots A^{j-1} b_\lambda^2$ ,  $j = 0, 1, 2, \dots, n$ , are not zero. (Received October 25, 1978.)

763-49-4 Vernon L. Bakke, University of Arkansas, Fayetteville, Arkansas 72701. Optimal Fields for Problems with Delays. Preliminary report.

The theory of optimal fields is developed for optimal control problems in which the states are solutions of integral equations with variable delays. The results obtained include a maximum principle and the Hamilton-Jacobi equations. The Hilbert integral is used to derive necessary conditions which are considered "natural" from the standpoint of variational theory in the sense that the special character of the state equations are reflected in the Hamiltonian function. In this case the results agree with those obtained by variational methods. (Received October 25, 1978.)  
(Author introduced by Professor James F. Porter).

## 50 ► Geometry

763-50-1 ROBERT B. GARDNER, University of North Carolina, Chapel Hill, North Carolina 27514. Applications of the Derived Flag of Pfaffian Systems. Preliminary report.

Let  $M$  be an  $m$ -manifold and  $\Gamma(M)$  the module of 1-forms over the  $C^\infty$ -functions. A Pfaffian system  $I$  is a submodule of  $\Gamma(M)$ . Let  $\pi: T(M) \rightarrow \Gamma(M)/I$  be the canonical projection and let  $\delta = (\Lambda^2 \pi) \circ d$  where  $d$  is the exterior derivative and  $\Lambda^2 \pi$  is the second exterior power of  $\pi$ , then  $\ker \delta = I^{(1)}$  defines a submodule of  $I$  and inductively defines a flag structure  $I^{(N)} \subset \dots \subset I^{(1)} \subset I$  known as the derived flag. The basic fact, exploited heavily by E. Cartan, is that given a  $n$ -manifold  $N$ , a Pfaffian system  $J$  on  $N$  and a map  $f: M \rightarrow N$  satisfying  $f^* J \subset I$  then  $f^* J^{(k)} \subset I^{(k)}$ . These ideas will be applied to prove Backlund's theorem that  $r^{\text{th}}$  order contact transformations are prolongations of 1st order contact transformations and to survey a program of E. Cartan concerning Pfaffian systems whose solutions depend on  $n$ -arbitrary functions and their derivatives up to order  $r$ . (Received September 1, 1978.)

\*763-50-2 K. Bolling Farmer and Mark P. Hale, Jr., University of Florida, Gainesville, Florida 32611. Geometries containing Dual Affine planes. Preliminary report.

If the vector space  $V$  of dimension  $n$  over the Galois field  $GF(q)$  is endowed with a nonzero alternative bilinear form  $f$ , a finite geometry can be defined

on V. The points and lines of the symplectic geometry (relative to f) are the 1-dimensional subspaces of V which are not in the radical of f and the 2-dimensional subspaces on which f is not identically zero. A planar symplectic geometry can be thought of as a projective plane with one point (radical) and the lines through it (isotropic 2-spaces) removed; hence, this is a dual affine plane. We prove:

Geometries in which all planes are dual affine must be symplectic geometries.

(Received October 19, 1978.)

\*763-50-3 Elwyn H. Davis, Pittsburg State University, Pittsburg, Kansas 66762. Translation Planes of Order 25 with Non-trivial X-OY Perspectivities.

A computer generated list of 1-spread sets sufficient to yield all translation planes of order 25 with non-trivial X-OY perspectivities is given. An identification of the known planes is given.

The list contains some new planes. (Received October 20, 1978.)

\*763-50-4 ALAN H. SCHOEN, Southern Illinois University, Carbondale, Illinois 62901. Loxodromic polyhedra with plane quadrangular faces.

Let n be any positive rational number p/q. Inscribe in a sphere of radius  $\rho$  a cyclic chain of p congruent rectangles R, each with length:width ratio r ( $0 < r \leq 1$ ). Label the vertices of each R in cyclic order: a, b, c, d. Latitudes  $\theta_a$  and  $\theta_c = 0$ ; the azimuths  $\phi_a$  and  $\phi_c$  differ by  $2\pi/n$ . The vertex a of each R coincides with the vertex c of an adjacent R. Construct the family  $\{M_i\}$  of p congruent loxodromes L and the conjugate family  $\{N_i\}$  of p congruent L's, each family having  $C_p$  symmetry, so that edge ab of the i<sup>th</sup> R is a chord of  $M_i$ , and edge bc of the i<sup>th</sup> R is a chord of  $N_i$ . Inscribe chords in each L between every pair of points at which L intersects an L of the conjugate family. The set of all such chords defines the graph  $G(n, r, \rho)$ , which has a countable infinity of quadrangular faces. Assign consecutive integer subscripts k to consecutive intersection points  $q_k$  on each L;  $k=0$  for  $\theta_k = 0$  and is monotone increasing with  $\theta_k$ . Let  $\alpha_k, \beta_k, \gamma_k, \delta_k$ , in cyclic order, be the edges of a face  $f_k$  with two opposite vertices at  $\theta_k$ ;  $\alpha_k$  is the longer, and  $\beta_k$  the shorter, of the two edges whose common vertex has smallest  $|\theta_k|$  in  $f_k$ . Let  $\{g_k\}$  be the Pell sequence  $\dots, -2, 1, 0, 1, 2, 5, \dots$  and  $\{h_k\}$  its Lucasian analog  $\dots, 3, -1, 1, 1, 3, 7, \dots$

Theorem 1: The faces of  $G(n, r, \rho)$  are plane.

Theorem 2:  $|\alpha_k| |\delta_k| = |\beta_k| |\gamma_k|$  ( $k = 0, \pm 1, \pm 2, \dots$ ).

Theorem 3: In  $G(4, 1, \sqrt{2})$ , the (xyz)-coordinates of consecutive  $q_k$ 's on a given L are  $(1/h_k)(1, 1, 2g_k)$  (k even) and  $(1/g_k)(1, 0, h_k)$  (k odd) (or the images of these terms under the rotation and reflection isometries of the group  $D_4$ ). (Received October 23, 1978.) (Author introduced by Professor Melvyn B. Nathanson).

763-50-5 MARK P. HALE JR., University of Florida, Gainesville, Florida 32611. Synthetic problems in spaces with mixed planes. Preliminary report.

Partial geometries can be made from projective spaces in various algebraic ways, e.g. remove all points and lines which correspond to isotropic subspaces in some bilinear form. The planes in such a geometry are projective with some data deleted, e.g. one point, one line, a vee, all lines through one point, an oval, etc. In trying to determine these geometries from their plane assortment, one encounters the following "synthetic geometry" problem: given a geometry containing a certain mixture of planes, can it be embedded in a projective space in a reasonable way? Progress on this problem will be described. (Received October 25, 1978.)

\*763-50-6 L. M. BATTEN, University of Winnipeg, 515 Portage Avenue, Winnipeg, Manitoba, Canada R3B 2E9. Embeddability of linear spaces with  $p = f(n)$  points and line range a subset of  $\{n-1, n, n+1\}$ .

Two papers are involved here. One treats  $p \leq (n+1)^2$  and line range  $\{n, n+1\}$ , and the other  $p \leq n^2$  and line range  $\{n-1, n+1\}$ . In both cases it is shown that for n larger than some fixed

positive integer, the space is embeddable in a projective plane of order  $n - 1$ ,  $n$ ,  $n + 1$  or  $n + 2$  depending on  $p$ . For some small values of  $n$ , we get block designs about which, in some cases, not much is known. For the second type of line range, we apply a theorem due to Bruck on nets.

(Received October 25, 1978.)

## 53 ► Differential Geometry

763-53-1 WILLIAM M. BOOTHBY and PHILIP B. ZWART, Washington University, St. Louis, Missouri 63130. On compact, homogeneous, symplectic and contact manifolds.

In this paper results contained in the dissertation of one of the authors (Zwart, Ph.D. Thesis, Washington University (1965) and Abstract 693-D11, these Notices (v19 No.2) February 1972) are given improved proofs and extended. In brief the structure of connected, compact, homogeneous symplectic spaces  $G/K$  are first studied. [It is important to observe that no assumptions are made concerning  $\pi_1(G/K)$  nor  $H^1(G/K)$ .] It is shown that the semi-simple part of  $G$  is compact and that  $G/K$  decomposes into the Cartesian product of compact, homogeneous symplectic spaces corresponding to the semi-simple and solvable parts of  $G$ . The latter of these is then analyzed in some detail. The results are applied to the analysis of compact, homogeneous contact manifolds, again without assumptions on the fundamental group. The proofs use special properties of nilpotent groups and the orbits of their representations, and the Borel-Selberg density theorems.

(Received August 4, 1978.)

\*763-53-2 DAVID L. JOHNSON, Texas A&M University, College Station Texas 77843. Curvature and Euler Characteristic For Six-Dimensional Kähler Manifolds.

Abstract: Let  $M$  be a compact, six real-dimensional Kähler manifold. It is shown that, for this class of manifolds, a well-known conjecture of H. Hopf is true. If the sectional curvature of  $M$  is nonnegative (resp., nonpositive), the Euler characteristic of  $M$  is nonnegative (resp., nonpositive). The methods employed are entirely pointwise; that is, the Gauss-Bonnet integrand is shown to be of the proper sign at each point via algebraic methods. This is not true in the real case, as has been shown by R. Geroch. (Received August 17, 1978.)

\*763-53-3 TAN VO VAN, University of Massachusetts, Boston, Massachusetts 02125. On the kahlerity of strongly pseudoconvex manifolds.

Definition: A non compact  $\mathbb{C}$ -analytic manifold  $X$  is called strongly pseudoconvex if

- i) there exists  $\varphi \in \mathcal{C}_R^{\infty}(X)$  such that  $\{x \mid \varphi(x) < c\} \Subset X$  for  $\forall c \in \mathbb{R}$
- ii)  $\varphi$  is strongly plurisubharmonic at any point  $x \in X \setminus K$  for some compact subset  $K \subset X$ .

In the special case where  $K = \emptyset$ , one can prove that the strongly pseudoconvex manifold  $X$  is actually stein; in particular  $X$  is then kahlerian. However, one has

Example A: We'll construct a 3-dimensional strongly pseudoconvex manifold  $X$ , carrying a compact 1 dimensional cycle  $T$  which is homologous to zero. Hence  $X$  is not kahlerian.

Dimensionwise, such an example is sharp since we'll prove

Theorem B: Any 2-dimensional strongly pseudoconvex manifold is kahlerian.

Furthermore with the Example A in mind, a deeper investigation will show us that, in higher dimension i.e. for  $\dim X \geq 3$ , the kahlerity of strongly pseudoconvex manifolds is completely characterized by some intrinsic analytic structure in the neighborhood of the compact subset  $K$  in  $X$ . (Received September 20, 1978.) (Author introduced by Professor R. Seeley).

\*763-53-4 THOMAS E. CECIL, Holy Cross College, Worcester, MA 01610 and PATRICK J. RYAN, Indiana University at South Bend, South Bend, IN 46615. Tight and taut immersions into hyperbolic space.

Let  $f: M \rightarrow H^m$  be an immersion into real hyperbolic space. For  $p$  in  $H^m$ ,  $x$  in  $M$ , define  $L_p(x)$  to be the square of the distance from  $p$  to  $f(x)$  in  $H^m$ . For a hyperplane  $\pi$  in  $H^m$ , define  $L_\pi(x)$  to be the distance in  $H^m$  from  $f(x)$  to  $\pi$ . As is done for sub-

manifolds of Euclidean space,  $f$  is called tight, respectively taut, if every non-degenerate function  $L_f$ , respectively  $L_p$ , has the minimum number of critical points. We prove initial results about both conditions, among which are some which differ from Euclidean analogues. In particular, if  $f: M \rightarrow H^m$  is tight and  $i: H^m \rightarrow H^{m+k}$  is a totally geodesic embedding, then  $i \circ f$  need not be tight. In fact, there are tight embeddings  $f: S^n \rightarrow H^{n+1}$  with this property. We find that the class of tight immersions of  $S^n$  into  $H^{n+1}$  is composed of those Euclidean convex hypersurfaces in the disk  $D^{n+1}$  which remain Euclidean convex under all isometries of the Poincaré metric on  $D^{n+1}$ . Hence, this class lies strictly between the class of Euclidean spheres and the class of Euclidean convex hypersurfaces in  $D^{n+1}$ . (Received September 21, 1978.)

763-53-5 Kinetsu Abe, University of Connecticut, Storrs, Connecticut 06268. Some structures on the boundary of an analytic set. Preliminary report.

Let  $M$  be a  $C^\infty$ -manifold of dimension  $2n+1$ . Assume that  $M$  is a boundary of a domain  $D$  in an analytic variety  $V$  of complex dimension  $n+1$ . Furthermore, assume that there is at most one singular point of  $V$  in  $D$ .

It is shown that there are contact and (strongly) pseudo convex structures on  $M$ . They are intimately related to the analytic structure of  $D$ . The interrelations between these structures are discussed. Also discussed is the impact of the existence of a singular point in  $D$  on these structures and vice versa. (Received September 22, 1978.)

763-53-6 ROBERT L. BRYANT, University of North Carolina, Chapel Hill, North Carolina 27514. Pfaffian Derived Types with no local invariants.

Given a Pfaffian system  $I$  on a manifold  $M$ , there are several invariants which can be attached to  $I$  by local constructions. We say that  $I$  is non-singular of dimension  $p$  if for each  $x \in M$ , the vector space  $L_x \subseteq T_x^*M$  is of dimension  $p$ . We let  $E^*(M)$  be the exterior algebra of forms on  $M$  and let  $(I)$  be the ideal generated by  $I$  in  $E^*(M)$ . Then  $I' = \{\omega \in I \mid d\omega = 0 \text{ mod } (I)\}$  is called the first derived system of  $I$ . The Engel half-rank of  $I$  is defined to be the smallest integer  $q$  such that  $(d\omega)^{q+1} = 0 \text{ mod } (I)$  for all  $\omega \in I$ . The Cartan system  $C(I)$  is defined (in the case that  $I$  and  $I'$  are non-singular) to be the smallest completely integrable system containing  $I$  such that one can write down a system of generators for  $I$  using only the variables of  $C(I)$ . One has the known inequality in the case that  $C(I)$  is non-singular

$$\dim C(I) - \dim I \leq q(p+1).$$

For further details concerning these invariants and others one can associate with  $I$ , consult R. Gardner, Invariants of Pfaffian systems, Trans. A.M.S. 126 (1967), pp. 514-533.

We show that, when  $p \neq 2$  and equality obtains in the above inequality,  $I$  is locally equivalent to the contact system of  $J^1(R^q, R^p)$ . The case  $p = 1$  is the classical Darboux theorem stating that the only local invariant of a 1-form is its rank. In the case  $p = 2$ , the theorem is false, as shown by Cartan in his Five Variables paper (1910). (Received September 28, 1978.)

763-53-7 Lee Whitt, Texas A&M University, College Station, Texas, 77843. Isometric homotopies and codimension two isometric immersions of spheres into Euclidean space.

Given an isometric immersion of a Riemannian manifold  $M$  into Euclidean space, a method is given for constructing  $C^\infty$  homotopies through isometric immersions (isometric homotopies). Namely, by varying the second fundamental form and the normal connection so that the Gauss and Codazzi-Mainardi equations hold, we obtain an isometric homotopy. This is used to show that any two isometric immersions of the usual  $n$ -sphere  $S^n$  into  $R^{n+2}$  are related by an isometric homotopy. Furthermore, it is shown that the space of isometric immersions  $S^n \rightarrow R^{n+2}$  has the special orthogonal group  $SO(n+2)$  as a strong deformation retract ( $C^\infty$  topology). Similar results are obtained for isometric immersions between Euclidean spaces with flat normal connections. (Received September 28, 1978.)

\*763-53-8 J-H ESCHENBURG, University of Münster, Germany and J. J. O'Sullivan, The Pennsylvania State University, University Park, PA 16802. Lagrange Tensors and Ricci Curvature.

Let  $c$  be a geodesic on an  $(n+1)$ -dimensional manifold, let  $c^\perp$  be its normal bundle and let  $\varepsilon(c)$  be the trivial  $n$ -dimensional vector bundle over  $c$ . A Jacobi tensor field  $Y$  along  $c$  is a section



of the bundle  $\text{Hom}(\varepsilon(c), c^\perp)$  which satisfies the equation.

$$\nabla_c \nabla_c Y + R_c \circ Y = 0$$

where  $\nabla$  is the connection induced on  $\text{Hom}(\varepsilon(c), c^\perp)$  by the Riemannian connection on  $M$  and  $R_c \in \Gamma \text{Hom}(c^\perp, c^\perp)$  is the restriction of the Riemannian curvature tensor of  $M$ . Lagrange tensor fields are precisely those Jacobi tensor fields which arise as the variation tensor fields of  $n$ -Parameter normal geodesic variations of  $c$ , i.e. variations of  $c$  through geodesics which are initially perpendicular to some  $n$ -dimensional submanifold of  $M$ . Such variations are called congruences in the book of Hawking-Ellis.

We develop the main properties of Lagrange tensor fields and use them to study relationships between the Ricci tensor of  $M$  and, among other things; volume; conjugate points; focal points of hypersurfaces of  $M$ ; mean curvature of hypersurfaces of  $M$ . (Received October 2, 1978.)

763-53-9 ROBERT C. REILLY, University of California, Irvine, California 92717.  
Geometric applications of the solvability of certain boundary-value problems.

Differential geometers have always been among the primary users of the existence theorems which analysts have produced. Normally they use such analytic existence theorems to derive corresponding geometric existence theorems. In contrast, I shall show how some important geometric uniqueness theorems (for example: Aleksandrov's Theorem on embedded hypersurfaces of constant mean curvature; Fiala's version of the isoperimetric inequality) follow from the solvability of certain boundary-value problems. (Received October 2, 1978.) (Author introduced by Professor Richard Millman).

\*763-53-10 Frank Flaherty, Oregon State University, Corvallis, OR 97331  
Maximal Hypersurfaces in Lorentzian manifolds.

A spacelike hypersurface (codimension 1) in a Lorentzian manifold is called a maximal surface iff it extremizes the hypervolume functional. Though superficially analogous to minimal submanifolds in Riemannian geometry, maximal surfaces offer some striking contrasts. We shall discuss these contrasts, along with existence and uniqueness theorems and applications of maximal surfaces to general relativity. (Received October 2, 1978.)

763-53-11 Martin A. Magid, Wellesley College, Wellesley, MA 02181. Some Problems Involving Indefinite Riemannian Submanifolds. Preliminary report.

I will talk about L. Graves' classification of isometric immersions  $\mathbb{L}^n \rightarrow \mathbb{L}^{n+1}$ , when  $\mathbb{L}^k$  is  $k$ -dimensional Lorentz space and my work classifying certain codimension 2 isometric immersions between indefinite flat spaces with parallel second fundamental forms. (Received October 2, 1978.)

\*763-53-12 SU-SHING CHEN, Georgia Institute of Technology, Atlanta, GA 30332 and PATRICK EBERLEIN University of North Carolina, Chapel Hill, NC 27514. Isometry Groups.

$H$  denotes a complete, simply connected Riemannian manifold of nonpositive sectional curvature with isometry group  $I(H)$  and points at infinity  $H(\infty)$ . A point  $x \in H(\infty)$  is flat if every geodesic belonging to  $x$  bounds an imbedded flat half plane. A subgroup  $D \subseteq I(H)$  satisfies the duality condition if for every geodesic  $\gamma$  of  $H$  there exists a sequence  $\{\varphi_n\} \subseteq D$  such that  $\varphi_n p \rightarrow \gamma(\infty)$  and  $\varphi_n^{-1} p \rightarrow \gamma(-\infty)$  for any point  $p$  in  $H$ . We investigate groups  $D$  that satisfy the duality condition. For example: 1) If  $D$  contains a normal abelian subgroup  $A \neq 1$ , then for each  $\varphi \in A$  the function  $p \rightarrow d(p, \varphi p)$  is constant. If  $D$  contains a solvable subgroup of finite index, then  $H$  is flat. 2) If  $A \subseteq H(\infty)$  is a closed set invariant under  $D$ , then  $\partial A$  consists of flat points. If all sectional curvatures at one point  $p$  in  $H$  are negative, then

$H(\infty)$  has no flat points and  $D$  acts minimally in  $H(\infty)$ ; moreover, if  $M = H|D$  is a smooth manifold, then the geodesic flow has a dense orbit in  $T_1 M$ . 3) If  $H$  is homogeneous and  $I(H)$  satisfies the duality condition, then  $H$  is the Riemannian product of a Euclidean space and a symmetric space of non-compact type. (Received October 3, 1978.)

\*763-53-13 DOMINIC S.P. LEUNG, U.S. Naval Academy, Annapolis, MD 21402. On the local existence and deformations of submanifolds with constant mean curvature in a Riemannian manifold.

A submanifold  $N$  of a Riemannian manifold  $M$  is said to be with constant mean curvature if the mean curvature vector field of  $N$  has constant length. Using the Cartan-Kähler existence theorem, the following fact is proved: "Locally any  $(n-1)$ -dimensional analytic submanifold  $S$  of an analytic Riemannian manifold  $M$  whose dimension is greater than  $n$  can be extended to an  $n$ -dimensional submanifold  $N$  of  $M$  with constant mean curvature  $C$ ,  $C \neq 0$  being an arbitrary constant. Furthermore, the normalized mean curvature vector of  $N$  can be preassigned in an arbitrary manner along  $S$ . The equations of variations of the differential system used to construct such a submanifold  $N$  give rise to a system of differential equations on the sections of certain vector bundle over  $N$ , whose solutions are called generalized Jacobi fields. The following fact is also proved: Under the assumption of real analyticity, locally every generalized Jacobi field defined on a submanifold of constant mean curvature is induced by a one-parameter family of such submanifolds. (Received October 5, 1978.)

763-53-14 H. GUGGENHEIMER, Polytechnic Institute of New York, 333 Jay Street, Brooklyn, New York 11201. Differential invariants that withstand catastrophes. Preliminary report.

The methods of A. Tresse lead to the theorem: Two subvarieties  $V_1, V_2$  of an homogeneous space  $G/H$  are images of one another in a transformation of  $G$  if (a)  $v_i : V \rightarrow G/H, v_i(V) = V_i (i = 1, 2)$  are differentiable maps for which no  $r$ -jet vanishes identically for sufficiently large  $r$  and (b) the differential invariants of  $j_{x^1}^r v_1, j_{x^2}^r v_2$  coincide for all  $x \in V$ . Applications are given to the differential geometry of curves and surfaces in 3-space with algebraic singularities of several kinds, and euclidean and affine invariants that characterize different kinds of singularities. (Received October 6, 1978.)

\*763-53-15 RICHARD L. BISHOP, University of Illinois, Urbana, Illinois 61801. Cylindricity of infinitesimal deformation of hyperplanes.

The following infinitesimal analogue of the cylindricity theorem for complete flat hypersurfaces has been discovered. Theorem. If  $M$  is a hyperplane in a Euclidean space, then a 2nd-order infinitesimal isometric deformation has as its 1st-order part a vector field  $V$  which is cylindrical, up to an infinitesimal rigid deformation. That is,  $M$  has a Euclidean product structure  $M = N \times E$ , where  $N$  is a line, such that on each  $\{n\} \times E$  the vector field  $V$  is parallel in the ambient space.

On the other hand, for every nonplanar complete flat hypersurface we can construct a 2nd-order infinitesimal deformation for which the 1st-order part is not cylindrical. (Received October 11, 1978.)

763-53-16 RALPH ALEXANDER and STEPHANIE ALEXANDER, University of Illinois, Urbana, Illinois 61801. Geodesics in Riemannian manifolds-with-boundary.

The global theory of geodesics in manifolds-with-boundary promises to be an interesting one. However, since at present there does not seem to exist even a local theory, some fundamental local questions must be answered first. For example, we show that a shortest path  $\alpha$  in a Riemannian  $C^3$ -manifold-with- $C^1$ -boundary  $M$  is  $C^1$  (clearly no higher differentiability class

could be hoped for, regardless of the class of  $M$ ), and that  $\alpha$  has an osculating plane normal to  $\partial M$  whenever  $\alpha$  touches  $\partial M$ . The first statement has been announced independently by F.-E. Wolter. Our method involves the construction of convex subsets of  $M$ . The method also gives information about the question of local uniqueness of shortest paths from a given point. (Received October 16, 1978.)

\*763-53-17 Stanley M. Zoltek, Wright State University, Dayton, OH 45435. A Normal Form for a Special Class of 5-Dimensional Manifolds.

In the case of a 4-dimensional oriented Einstein manifold, Singer and Thorpe found a canonical form for the curvature tensor and used it to show that the curvature function is completely determined by its critical point behavior. We extend these results to 5-dimensional oriented manifolds whose curvature tensor, pointwise, commutes with an element of  $\Lambda^4$ . (Received October 19, 1978.)

763-53-18 STEVEN G. HARRIS, University of Chicago, Chicago, Illinois 60637. Geodesic triangles in a Lorentz manifold. Preliminary report.

A Rauch-type comparison theorem is established for Jacobi fields along time-like geodesics in Lorentz manifolds (non-degenerate pseudo-Riemannian, signature  $+...+-$ ). Using this, a Toponogov-type result is obtained for triangles formed of time-like geodesic segments: Under suitable hypotheses, an upper bound on the sectional curvature of time-like planes in a Lorentz manifold yields a lower bound for the length of the third side of such a triangle, in terms of the lengths of the first two sides and the angle between them (comparison is made with a constant-curvature Lorentz manifold). (Received October 23, 1978.)

763-53-19 JOHN R. HERRING AND JOHN J. O'SULLIVAN, The Pennsylvania State University, University Park, Pennsylvania 16802. The Jacobi Equation on a Lie Group.

Let  $G$  be a simply connected  $n$ -dimensional Lie group with a left invariant Riemannian metric. Let  $\mathfrak{g}$  be its Lie algebra and  $x \in \mathfrak{g}$  be such that  $c(t) = \text{Exp}(tx)$ , the one-parameter group defined by  $x$ , is a geodesic. We study the behaviour of solutions to the Jacobi equation along such geodesics. If  $x$  is in the center of  $\mathfrak{g}$ , there are conjugate points along  $c(t)$  unless  $G$  is isometric to  $\mathbb{R} \times \bar{G}$ , where  $\mathbb{R}$  is the one-parameter subgroup  $\text{Exp}(tx)$  and  $\bar{G}$  is a  $n-1$  dimensional Lie subgroup of  $G$ . Using the fact that the derived algebra  $\mathfrak{g}' = [\mathfrak{g}, \mathfrak{g}]$  of a solvable Lie algebra is nilpotent, and so has nontrivial center, this leads to a method for finding conjugate points on a large class of solvable Lie groups with left invariant metrics. If  $x \in \mathfrak{g}$  is perpendicular to  $\mathfrak{g}'$ , then the Jacobi equation along  $c$  has a particularly simple form with the type of solutions depending solely upon the eigenvalues of  $\text{ad}_x$ . In a solvable Lie group where  $\mathfrak{g}'$  is codimension one, and  $z$  is perpendicular to  $\mathfrak{g}'$ , then the  $\mathfrak{g}'$  contains an  $x$  as above if and only if  $\text{ad}_z$  is not definite. We study these geodesics in some detail. (Received October 23, 1978.)

\*763-53-20 DETLEF GROMOLL, SUNY at Stony Brook, Stony Brook, N.Y. 11794 and KARSTEN GROVE, University of Copenhagen, Denmark. Rigidity of positively curved manifolds with large diameter.

Let  $M^n$  be a complete Riemannian manifold with sectional curvature  $K \geq 1$  and diameter  $d \geq \frac{\pi}{2}$ . By the work of Berger, Grove, and Shiohama,  $M$  is homeomorphic to the sphere  $S^n$  if  $d > \frac{\pi}{2}$  (General Sphere Theorem). If  $d = \frac{\pi}{2}$  and  $M$  simply connected, one expects that  $M$  is either homeomorphic to  $S^n$ , or isometric to a symmetric space of rank 1. We prove this rigidity conjecture in low dimensions (so far  $n \leq 7$ ), and in general reduce the non-simply connected case to the simply connected case. (Received October 24, 1978.)

\*763-53-21 HOWARD JACOBOWITZ, Rutgers University, Camden, New Jersey 08102. Surfaces in real projective space.

The absolute invariants under projective transformations of negatively curved surfaces are derived using a normal form introduced by Wilczynski (Trans. A.M.S., 1908). A new derivation of the projective normal is also discussed.

(Received October 25, 1978.)

763-53-22 EDWARD D. DELOFF, Rutgers University, New Brunswick, New Jersey 08903. Naturally Reductive Homogeneous Riemannian Manifolds with Nonpositive Curvature. Preliminary report.

Let  $M$  be a simply connected, homogeneous Riemannian manifold with nonpositive curvature. We show that if  $M$  has a naturally reductive metric, then  $M$  is symmetric. We do this by first proving the following (using the theory of Azencott and Wilson):

Thm: Let  $M$  be a simply connected, homogeneous Riemannian manifold with nonpositive curvature, which has no Euclidean factors. If there exists a connected Lie group of isometries  $G$  such that its Lie algebra  $\mathfrak{g} = [\mathfrak{g}, \mathfrak{g}]$ , then  $M$  is symmetric.

We then show that when  $M$  has a naturally reductive metric, such a group of isometries must exist.

(Received October 25, 1978.)

763-53-23 TERENCE GAFFNEY AND MARIA RUAS, Brown University, Providence, Rhode Island 02912. Singularities of mappings and orthogonal projections of spaces. Preliminary report.

Given a surface  $M^2$  generically immersed in  $\mathbb{R}^3$  by a map  $i$ , the orthogonal projections of  $i(M^2)$  onto planes or lines form 2 stable mappings,  $\pi_p : M^2 \times S^2 \rightarrow TS^2$  and  $\pi_L : S^2 \times M^2 \rightarrow NS^2$ . Applying the theory of singularities of stable maps, many local and global results can be proven. Some examples:

Theorem 1 (Banchoff, Gaffney, McCrory). A point  $m$  of  $M^2$  is a cusp of the Gauss map of  $i(M^2)$  if and only if for every neighborhood  $U$  of  $m$ , there is a plane in  $\mathbb{R}^3$  tangent to  $i(U)$  at two points.

Theorem 2. Suppose  $M$  is compact, oriented, generically immersed in  $\mathbb{R}^3$ . Let  $C$  be a curve in the region of negative curvature, then the number of tangencies to  $C$  of a given asymptotic line field is equal mod 2 to the number of points of  $C$  which are inflection points of integral curves of the line field. (Received October 25, 1978.)

## 54 ► General Topology

\*763-54-1 JERRY E. VAUGHAN, University of North Carolina at Greensboro, North Carolina 27412. A countably compact, first countable, non-normal  $T_2$ -space.

Spaces having the properties listed in the title have been constructed using extra assumptions such as  $\diamond$ , CH, and MA + not-CH. We construct such a space within ZFC which is in addition locally compact, zero dimensional,  $\omega$ -bounded and has cardinality  $\aleph_1$ . (Received September 22, 1978.)

\*763-54-2 GORDON BERG, Motorola Inc., Mesa, Arizona, 85202. Constructive dimension theory.

Constructive definitions of covering dimension and inductive dimension for compact metric spaces are given and the equivalence of the two is shown. Finite sum and product theorems are demonstrated. A new characterization of dimension is introduced and shown to be equivalent to the existence of stable points of mappings into  $I^n$ . (Received October 2, 1978.)

(Author introduced by Professor Ray Mines).

\*763-54-3 ROBERT A. HERRMANN, Math. Dept., U.S. Naval Academy, Annapolis, MD.21402  
A nonstandard approach to pseudotopological compactifications.

In this long paper, we use the q-monad theory of Puritz to establish numerous standard results. Some examples are as follows: THM.1 The collection  $H'$  of all Hausdorff convergence space compactifications of  $(X, q)$  contains a projective max. iff  $W^+$  is nuclear iff  $H'$  contains a projective min. THM. 2. The set  $W^+$  is nuclear iff  $(\hat{X}, \hat{q})$  is the projective max. in the class of all pseudotopological Hausdorff compactifications of  $X$ . THM. 3. If  $X$  is a pretopological space, then  $(X^+, q^+)$  is pretopological iff  $W^+$  is nuclear iff  $(\hat{X}, \hat{q})$  is a projective max. in  $H$  iff  $X^+$  is a projective min. in  $N$ . Applications are given to one-point H-closures, in general; the uniqueness of one-point H-closures and one-point near-compactifications. For example, for the special pretopological space  $(X, \theta)$ , the  $\theta$ -convergence space, we have that  $W^+ = Nuc T$ , where  $T$  generates Obreanu's one point continuous projective max. quasi-H-closure of  $X$ . (Received October 4, 1978.)

763-54-4 WILLIAM H. JULIAN, New Mexico State University, Las Cruces, New Mexico 88003.  
The Jordan Curve Theorem.

Given a Jordan curve and a point whose distance to the curve is positive, we give a finite procedure to decide whether the point is inside or outside the curve. Given two points on the same side of the curve, we construct a polygonal path joining the two points and bounded away from the curve. Finally we construct a point inside the curve. (Received October 6, 1978.)

\*763-54-5 August Lau, North Texas State University, Denton, Texas 76203. Certain Local Homeomorphisms of Continua are Homeomorphisms, I

The original intention of the paper is just to answer the question: Is every local homeomorphism on a nonseparating plane continuum a homeomorphism? However, a much more general theorem was proved: if  $X$  is a simple limit, then every local homeomorphism on  $X$  is a homeomorphism if and only if no prime order group can act freely on  $X$ . This answers the original question in the affirmative. (Received October 10, 1978.)

\*763-54-6 SCOTT W. WILLIAMS, Ohio University and State University of New York at Buffalo, Buffalo, New York 14214. A note on co-absolutes of Stone-Cech remainders. Preliminary report.

The unique extremally disconnected space  $E(X)$  which maps onto  $X$  by a perfect irreducible function is called the absolute of  $X$ . Regular spaces are known as co-absolutes whenever  $E(X)$  and  $E(Y)$  are homeomorphic. Denote the Stone-Cech remainder of non-compact space  $X$  by  $X^*$ . Theorem 1: If  $X$  is a free union of locally compact  $\sigma$ -compact spaces of  $\pi$ -weight at most  $2^\omega$ , then  $X^*$  is co-absolute with a linearly ordered space having a dense set of P-points. Theorem 2: If  $X$  is a locally compact metric space without isolated points and density at most  $2^\omega$ , then  $X^*$  and  $\mathbb{R}^*$  are co-absolute. Theorem 3: CH implies that  $X$  and  $N^*$  are co-absolute whenever  $X$  is compact of  $\pi$ -weight at most  $2^\omega$ , has no isolated points, and zero sets of  $X$  are regular-closed. Theorem 4: There are models  $\mathfrak{M}$  of ZFC in which  $\mathfrak{M} \models 2^\omega > \omega_1$  and  $\mathfrak{M} \models X^*$  and  $N^*$  co-absolute whenever  $X$  is paracompact, locally compact, real compact of  $\pi$ -weight at most  $2^\omega$ . Problem: Is  $\mathbb{R}^*$  always co-absolute with  $N^*$ ? (Received October 12, 1978.)

\*763-54-7 R. F. Dickman, Jr., Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. A proof of a conjecture of A. H. Stone.

In this paper we use the techniques of analytic topology to establish a conjecture of A. H. Stone: A perfectly normal, locally connected, connected space is multicoherent if and only if there exists four non-empty, closed and connected subsets  $A_0, A_1, A_2, A_3$  of  $X$  such

3  
 that  $\bigcup_{i=0}^3 A_i = X$  and the nerve of  $\{A_0, A_1, A_2, A_3\}$  forms a closed 4-gon, i.e.  $A_i$  meets  $A_{i+1}$  and  $A_{i-1}$  and no others (the suffices being taken mod 3). (Received October 12, 1978.)

763-54-8 JOHN B. FOULKES, University of Delaware, Newark, Delaware 19711.  
Convexity and the  $J_\epsilon$ -function. Preliminary report.

Let  $X$  be a metric space, and let  $F(X)$  be the hyperspace of nonempty closed subsets of  $X$  with the Vietoris topology. For a fixed  $\epsilon > 0$ , we define the set-valued map  $G_\epsilon: F(X) \rightarrow F(X)$  by  $G_\epsilon(A) = \{x: d(x,A) \leq \epsilon\}$ , and define the set-valued map  $J_\epsilon: F(X) \rightarrow F(X)$  to be  $G_\epsilon \circ G_\epsilon$ . Investigations are made as to what implies and what is implied by the condition:  $J_\epsilon(A) = A$  for  $A \in F(\mathbb{R}^2)$ . The following theorem is established: Let  $A \in F(\mathbb{R}^2)$  such that  $\text{Int}[\text{CH}(A)] \neq \emptyset$ , where  $\text{CH}(A)$  denotes the convex hull of  $A$ . Then  $\text{CH}(A) = \bigcup_{\epsilon > 0} J_\epsilon(A)$ , and so  $A$  is convex if and only if  $J_\epsilon(A) = A$  for each  $\epsilon > 0$ . A similar formula for  $\text{CH}(A)$  is established which is free of the interior condition. These formulas give representations for convex hulls strictly in terms of a set-valued mapping, and can be used to represent a "generalized" convex hull in spaces where convexity in the usual sense is not meaningful. (Received October 16, 1978.)

\*763-54-9 D.J. Lutzer, Texas Tech University, Lubbock, Tx. 79409 and R.A. McCoy, V.P.I. & S.U., Blacksburg, Va. 24061. Second Category Function Spaces

For a completely regular space  $X$ , let  $C(X)$  be the collection of all continuous, real-valued functions on  $X$ , equipped with the topology of pointwise convergence. In this note we investigate the Baire category theory of  $C(X)$ , proving, for example, (1) if  $X$  contains an infinite pseudocompact subspace, then  $C(X)$  is first category in itself; (2) if  $X$  is pseudonormal and if every countable subset of  $X$  is closed, then  $C(X)$  is of second category in itself (equivalently, for homogeneous spaces, is a Baire space); (3) if  $\omega \subset X \subset \beta\omega$  and if  $X - \omega$  is a countable, discrete-in-itself space, then  $C(X)$  is of second category in itself. The techniques used to obtain (3) involve certain infinite games with ultrafilters on  $\omega$ . (Received October 16, 1978.)

\*763-54-10 Ludvik Janos, Mathematical Reviews, Ann Arbor, Michigan 48109.  
A linearization of semiflows in the Hilbert space  $l_2$ . Embeddability of semiflows into flows.

By a semiflow  $(R^+, X, \alpha)$  we understand a continuous semigroup action  $\alpha: R^+ \times X \rightarrow X$  of the addition semigroup of nonnegative reals  $R^+ = [0, \infty)$  on a topological space  $X$ . We say that a semiflow  $(R^+, X, \alpha)$  can be embedded in a flow  $(R, Y, \beta)$  on a topological space  $Y$  if there exists a topological embedding  $i: X \rightarrow Y$  such that the group action  $\beta: R \times Y \rightarrow Y$  restricted to  $R^+ \times i(X)$  coincides with the action of  $\alpha$ , i.e.,  $i \alpha(t, x) = \beta(t, ix)$  for all  $t \geq 0$  and  $x \in X$ . Given  $c > 0$  we denote by  $(R, l_2, \alpha_c)$  the natural flow on the Hilbert space  $l_2$  with the similarity constant  $c$ , i.e., for every  $t \in R$  and  $x \in l_2$  we set  $\alpha_c(t, x) = c^t x$ . Theorem. If the semiflow  $(R^+, X, \alpha)$  is such that  $X$  is compact metric, the function  $\alpha(t, \cdot): x \rightarrow \alpha(t, x)$  is one to one for every  $t \in R^+$  and the family  $\{\alpha(t, \cdot): t \in R^+\}$  is evenly continuous, then for every  $c \in (0, 1)$  the semiflow  $(R^+, X, \alpha)$  can be embedded into the product  $(R, l_2, \alpha_c) \times (R, l_2, \beta)$  of flows on  $l_2$  where  $\beta(t, \cdot)$  is a unitary operator for every  $t \in R$ . (Received October 16, 1978.)

\*763-54-11 RICHARD H. WARREN, University of Nebraska at Omaha, Omaha, NE 68182  
Topological spaces with a unique compatible uniformity.

Four characterizations of Tychonoff (i.e. completely regular and Hausdorff) spaces with a unique compatible uniformity are extended to completely regular spaces. The method is to formulate a uniform identification space and demonstrate a 1-1 correspondence between the family of compatible uniformities on a completely regular space and the family of compatible uniformities on its  $T_0$ -identification space. (Received October 20, 1978.)

\*763-54-12 SOO BONG CHAE, University of South Florida, New College, Sarasota, Florida 33580. Remote points.

A remote point of  $\beta X$  is a point which does not belong to the closure of any discrete subset of  $X$ . If  $X$  is a metric space with no isolated points, then the existence of a remote point of  $\beta X$  is equivalent to the existence of a free  $z$ -ultrafilter on  $X$ , no member of which is nowhere dense. A topological space  $X$  is called a G-space if for any  $n$  in  $\mathbb{N}$  there is a collection  $C$  of closed sets with the  $n$ -intersection property such that each dense open set contains a set in  $C$ . Without using the CH we obtain the following result: Theorem. Let  $\mathcal{B}$  be a ring of closed sets in  $X$ . If  $X$  is a nonpseudocompact G-space, then  $X$  admits at least  $2^{\mathfrak{c}}$  free  $\mathcal{B}$ -ultrafilters on  $X$ , no member of which is nowhere dense. If  $X$  is also normal, then  $\mathcal{B}$ -ultrafilters in the theorem can be replaced by  $z$ -ultrafilters. The class of G-spaces include metric spaces and some  $\sigma$ -spaces. (Received October 23, 1978.)

763-54-13 C. E. AULL, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. C-Embedding in Functionally Normal and Related Spaces.

Definitions. Two sets  $A$  and  $B$  are functionally separated (ordered functionally separated) [ $\sigma$ -functionally separated] if there is a continuous function  $f$  [a denumerable family  $\{f_n\}$  of continuous functions] to the real line such that  $f(A) \cap f(B) = \emptyset$  ( $f(A) < f(B)$ ), [ $\times_n f_n(A) \cap \times_n f_n(B) = \emptyset$ ]. A space  $X$  is functionally normal, FN, (ordered functionally normal, OFN) [ $\sigma$ -functionally normal,  $\sigma$ -FN] if every pair of disjoint closed sets are functionally separated (ordered functionally separated) [ $\sigma$ -functionally separated]. A space is cofunctionally normal, CFN, if every pair of functionally separated closed sets are completely separated. The latter spaces include P-spaces and countably compact spaces and are special cases of Z-spaces. The Tychonoff plank is OFN and the Niemyzki plane is  $\sigma$ -FN. Theorems. A pseudocompact  $T_1$  FN-space is C-embedded in every  $T_1$  FN-space it is embedded in as a closed set. A  $T_4$  space (a  $\sigma$ -FN Tychonoff space) [a CFN Tychonoff space] is C-embedded in every OFN-space ( $\sigma$ -FN Tychonoff space) [CFN Tychonoff space] it is embedded in as a closed set iff it is countably compact (pseudocompact) [almost Lindelöf]. (Received October 23, 1978.)

\*763-54-14 James E. Joseph, Howard University, Washington, D. C. 20059. Some Remarks on  $\theta$ -Rigidity.

The notion  $\theta$ -rigidity was introduced in the paper, " $\theta$ -perfect and  $\theta$ -absolutely closed functions" (Illinois J. Math. 21 (1977), 42-60). It is established that a  $\theta$ -closed subset of an  $H(i)$  topological space is  $\theta$ -rigid. This leads to improvements of a number of known results on subsets of  $H$ -closed spaces and some new results. The new results include a characterization of those Hausdorff spaces in which the Fomin  $H$ -closed extension operator commutes with the projective cover (absolute) operator and new characterizations of locally  $H$ -closed Hausdorff spaces. A product theorem for  $\theta$ -rigid subsets is also produced. (Received October 23, 1978.)

763-54-15 BRIAN M. SCOTT, Cleveland State University, Cleveland, Ohio 44115.  
Spaces in which points have bilinearly ordered bases. Preliminary report.

Given partial orders  $\langle P, \leq \rangle$ ,  $\langle Q, \leq \rangle$ , we order  $P \times Q$  by  $\langle p, q \rangle \leq \langle p', q' \rangle$  iff  $p \leq p'$  and  $q \leq q'$ . A collection,  $\mathcal{B}$ , of sets is bilinearly ordered (by reverse inclusion) iff  $\langle \mathcal{B}, \supseteq \rangle$  is order-isomorphic to  $P \times Q$  for some linear orders  $P, Q$ . We investigate such local cardinal functions as character, pseudo-character, and tightness at points having bilinearly ordered local bases. (Received October 23, 1978.)

763-54-16 WAYNE LEWIS, Texas Tech University, Lubbock, Texas 79409. Periodic Homeomorphisms on Chainable Continua.

We prove that for every prime  $n$  there exists a homeomorphism of period  $n$  on the pseudo-arc and a homeomorphism of period  $2n$  on the wedge of two pseudo-arcs. Periodic homeomorphisms on other chainable continua are also discussed. (Received October 23, 1978.)

\*763-54-17 J.M. WORRELL, Jr. and H.H. WICKE, Ohio University, Athens, Ohio 45701. A covering property which implies isocompactness I.

The authors define a general covering property  $P$  for topological spaces. The property  $P$  is implied by weak  $\delta\theta$ -refinability. Theorem. If a space is countably compact and has property  $P$  then it is compact. This result has as a corollary the isocompactness of weakly  $\delta\theta$ -refinable spaces [see the authors' papers Can. J. Math. 17(1965)820-830 and Proc. Amer. Math. Soc. 55(1976) 427-431]. Other applications of the property  $P$  are made to topics such as closed-completeness and realcompactness. (Received October 23, 1978.)

\*763-54-18 GEORGE M. REED, Institute for Medicine and Mathematics, Ohio University, Athens, Ohio 45701. Q-sets and Shelah's Principle.

An uncountable space which is not  $\mathcal{C}$ -discrete but in which each subset is an  $F_{\mathcal{C}}$ -set is said to be a Q-set. Definition. (SP\*) There exists a collection  $S$  of almost disjoint, infinite sequences of  $\omega_1$  such that (1) any collection of subsequences which contains a final part from each sequence in  $S$  is not pairwise disjoint and (2) for any coloring of  $S$  by two colors, there exists a coloring of  $\omega_1$  agreeing with each of the sequences in  $S$  on a final part. Note that (SP\*) is a direct consequence of Shelah's Principle, and hence it both follows from (MA +  $\neg$ CH) and is consistent with (GCH). Recall that the author has previously observed that, under (V = L), normal, first countable Q-sets do not exist [Notices AMS 22(1975), A216 and Fund. Math., to appear]. In this paper, the author establishes the following result. Theorem. (SP\*) is true if and only if there exists a metric Q-set of weight  $\omega_1$ . Corollary. It is consistent with (GCH) that there exists a metric Q-set. (Received October 23, 1978.)

\*763-54-19 M. Solveig Espelie, Howard University, Washington, D. C. 20059 and James E. Joseph, Howard University, Washington, D. C., 20059. Properties of  $\theta$ -Closure.

Let  $X$  be a topological space and let  $A \subset X$ . The  $\theta$ -closure of  $A$  ( $cl_{\theta}(A)$ ) is  $\{x \in X: \text{each closed neighborhood of } x \text{ contains points of } A\}$  and  $A$  is  $\theta$ -closed if  $A = cl_{\theta}(A)$ .  $A$  is  $\theta$ -rigid (see R.F. Dickman Jr. and J.R. Porter,  $\theta$ -perfect and  $\theta$ -absolutely closed functions, Illinois J. Math. 21 (1977), 42-60) if for each cover,  $\Omega$ , of  $A$  by open subsets of  $X$  there is a finite  $\Omega^* \subset \Omega$  such that the family of closures of elements of  $\Omega^*$  covers  $A$ . It is shown that if  $A$  is  $\theta$ -rigid, then  $cl_{\theta}(A) = \bigcup_{x \in A} cl_{\theta}(x)$ , and some implications of this result are drawn. It is also shown that the following statements are equivalent: 1)  $X$  is compact, 2) For each upper-semicontinuous (u.s.c.) multifunction,  $\lambda$ , on  $X$ , the multifunction,  $\mu$ , on  $X$  defined by  $\mu(x) = cl_{\theta}(\lambda(x))$  assumes a maximal value under set inclusion, 3) Each u.s.c. multifunction,  $\lambda$ , on  $X$  with  $\lambda(x)$   $\theta$ -closed for each  $x$  assumes a maximal value under set inclusion, 4) For each u.s.c. multifunction,  $\lambda$ , from  $X$  to a regular space, the multifunction,  $\mu$ , defined by  $\mu(x) = cl(\lambda(x))$  assumes a maximal value under set inclusion. (Received October 23, 1978.)

\*763-54-20 DOUGLAS E. MILLER, University of Illinois at Chicago Circle, Chicago, Illinois 60680. Borel Selectors for Separated Quotients.

Theorem A. Let  $G$  be a Polish topological group which acts on a Polish space  $Y$  via a Borel measurable function of class  $\gamma \geq 0$ . Suppose  $A \subset Y$  is an invariant Borel set of ambiguous class  $\alpha \geq 0$  such that the quotient Borel space  $A/G$  is countably separated by projections of sets of ambiguous class  $\alpha$ . Then there is a  $\gamma + \alpha$ -Borel selector function for  $A$ .



**Theorem B.** Let  $Y$  be a Polish space and let  $E$  be an equivalence on  $Y$  whose equivalence classes are  $G_\delta$  sets. Suppose that the  $E$ -saturation of each basic open set is of ambiguous Borel class  $\alpha \geq 0$ . Let  $\beta = \sup\{\alpha + \mu : \mu < \alpha\}$ . Then there is a  $\beta$ -Borel selector function for  $E$  on  $Y$ .

A key lemma for each Theorem is an even more general version of the "General Theorem on Selectors" of Kuratowski and Ryll-Nardzewski. (Received October 24, 1978.)

\*763-54-21 Peter Nyikos, Department of Mathematics, Ohio University, Athens, Ohio 45701 and Auburn University, Auburn, Alabama 36830. Countable small rank and cardinal invariants II.

A collection of subsets of a set  $X$  is called fixed if it has nonempty intersection, and irreducible if each member has an element not contained in any of the others. A collection of subsets of a set  $X$  is of subinfinite [resp. countable] small rank if every fixed irreducible subcollection is finite [resp. countable]. In the first paper with this title [pp. 344-347 in: General Topology and its Relations to Modern Analysis and Algebra IV, Society of Czechoslovak Mathematicians and Physicists, Prague] the author showed that a separable space is of countable spread if, and only if, it has a base of countable small rank. Here it is shown that every separable regular space with a base of subinfinite small rank is hereditarily Lindelöf ("No  $S$ -space has a base of subinfinite small rank, but all have bases of countable small rank") and that a separable regular space is Lindelöf if every open cover has an open refinement of subinfinite small rank. (Received October 25, 1978.)

\*763-54-22 Gary Gruenhage, Auburn University, Auburn, Alabama 36830. On the product of closed images of metric spaces.

Assuming the continuum hypothesis, Y. Tanaka has given necessary and sufficient conditions for the product of two closed images of metric spaces to be a  $k$ -space. We show that Tanaka's theorem is independent of the usual axioms of set theory. If  $f$  and  $g$  are functions from  $\omega$  to  $\omega$ , define  $f \leq g$  if and only if  $f(n) \leq g(n)$  for all but finitely many  $n \in \omega$ . We show that Tanaka's theorem is equivalent to the set-theoretic axiom "not  $BF(\omega_2)$ ", i.e., to the existence of an unbounded collection  $\mathcal{F} \subset \omega^\omega$  with  $\text{card}(\mathcal{F}) = \omega_1$ . This axiom is weaker than CH, and contradicts  $MA + \text{not } CH$ . (Received October 25, 1978.)

\*763-54-23 Allan C. Cochran, University of Arkansas, Fayetteville, Arkansas 72701. A topology series for a convergence space. Preliminary report.

The object of this note is the construction of an ordinal sequence of topologies, associated with a given convergence space, which culminates in the topological modification of the given space. The length of this "T-series" gives a measure of how "topological" a given convergence space is. The main concept used in the construction is that of "interior hull" [defined in A characterization of singly generated convergence spaces, Math. Japonica 22 (1977), 347-355]. The construction is somewhat dual to the regularity series of a convergence space as given by Kent and Richardson [Bull. Australian Math. Soc. 13 (1975), 21-44]. The present note gives some general results of the type "if the convergence space has property  $P$  then the length of the series is less than  $n$ ". Application is made to completion - compactification theory. (Received October 25, 1978.)

\*763-54-24 Robert Warren Button, Ithaca College, Ithaca, New York 14850. The Maximal  $\mathcal{D}$ -compactification.

Recently we introduced a new definition for filter  $\mathcal{D}$ -convergence and used it to characterize  $\mathcal{D}$ -compactness as follows:

Definition. Let  $(X, \mathcal{C})$  be a topological space, let  $\mathcal{D}$  be a non-principal ultrafilter on an index set  $I$ , let  $x \in X$ , and let  $\mathcal{F}$  be a filter on  $X$  with a base  $\{F_i : i \in I\}$  such that (i) for each  $i \in I$ ,  $\{j : F_j \subset F_i\} \in \mathcal{D}$ , and (ii) for each neighborhood  $U$  of  $x$ ,  $\{i : F_i \cap U \neq \emptyset\} \in \mathcal{D}$ . Then we shall say that  $\mathcal{F} \mathcal{D}$ -converges to  $x$ . A

filter base satisfying condition (i) will be called a companion base, and  $\mathfrak{F}$  and  $\mathfrak{D}$  will be said to be companion filters if  $\mathfrak{F}$  has a companion base.

Theorem. For any space  $(X, \mathfrak{J})$  and non-principal ultrafilter  $\mathfrak{D}$  on an index set  $I$ , the following conditions are equivalent: (i)  $(X, \mathfrak{J})$  is  $\mathfrak{D}$ -compact, (ii) each companion filter  $\mathfrak{F}$  on  $X$   $\mathfrak{D}$ -converges, and (iii) each companion base of closed subsets of  $X$  is fixed.

We now use filter  $\mathfrak{D}$ -convergence to describe an analogue to  $\beta X$ , the maximal  $\mathfrak{D}$ -compactification. (Received October 25, 1978.)

763-54-25 J. L. SOLOMON, Mississippi State University, Mississippi State, Mississippi 39762. Structure of the set of subsequential limit points.

Let  $(X, d)$  be a metric space and  $f$  a continuous self-map. For  $x$  in  $X$ , we denote the set of subsequential limit points under  $f$  by  $L(x)$ . Several results concerning the structure of the set of subsequential limit points under  $f$  together with an example in which the derived set,  $L'(x)$ , of  $L(x)$  is uncountable and  $f(L'(x)) \neq L'(x)$  are presented. (Received October 25, 1978.)

## 55 ► Algebraic Topology

\*763-55-1 JOHN MCCLEARY, Bates College, Lewiston, Maine 04240. Realization of algebraic homomorphisms by continuous maps.

Let  $M$  and  $M'$  be unstable modules over the mod  $p$  Steenrod algebra such that there are spaces  $Y$  and  $Y'$  with  $H^*(Y; Z_p) = U(M)$  and  $H^*(Y'; Z_p) = U(M')$  where  $U(\ )$  is the free-graded-commutative-associative-unstable algebra functor introduced by Steenrod.

Suppose  $g: M' \rightarrow M$  is a morphism of unstable modules. We develop an obstruction theory which decides when  $g$  can be realized by a map  $G: Y_{(p)} \rightarrow Y'_{(p)}$  that is,

$g = H^*(G; Z_p)|_{M'}$ . We apply this obstruction theory to obtain  $p$ -equivalences of

certain  $H$ -spaces with products of simpler spaces which reflect the cohomology structure of the  $H$ -space. (Received September 22, 1978.)

\*763-55-2 ALAN D. UNELL, Northwestern University, Evanston, Illinois 60201. A non-embedding result for certain exotic spheres. Preliminary report.

Theorem. For each integer  $j \geq 4$  there exist exotic  $n$ -spheres,  $\Sigma^n$ ,  $n = 2^j$ , such that  $\Sigma^n$  does not smoothly embed in Euclidean  $n + \frac{n}{2} - 3$  space.

When  $j \geq 4$  and  $n = 2^j + 1$  or  $n = 2^j + 2$  we obtain the following

Corollary. There exist exotic  $n$ -spheres,  $\Sigma^n$ , such that  $\Sigma^n$  does not smoothly embed in Euclidean  $n + 2^{j-1} - 3$  space. (Received October 18, 1978.)

763-55-3 C. A. McGibbon, University of Pennsylvania, Philadelphia, PA 19104 Stable Properties of Rank 1 Loop Spaces. Preliminary report.

Let  $Q(X) = \lim_{\rightarrow} \Omega^n \Sigma^n X$ . A theorem of Graeme Segal [Quart. J. Math. Oxford(2), 24(1973)] states that  $Q(BS^1) \simeq BU \times F$ , where  $F$  is a space with finite homotopy groups. We prove an analogous result for the other rank-1 loop spaces.

Theorem 1. Let  $p$  be an odd prime and let  $B$  be a loop structure on  $S_{(p)}^{2n-1}$  (i.e.,  $\Omega B \simeq S_{(p)}^{2n-1}$ ). Then  $Q(B) \simeq X \times Y$  where  $X$  is a retract of  $BU_{(p)}$  and  $Y$  has finite homotopy groups. Moreover, the inclusion  $B \hookrightarrow Q(B)$  factors through  $X$ .

Theorem 2. Any two loop structures on  $S_{(p)}^{2n-1}$  have the same stable homotopy type.

Corollary 3. The infinite suspension  $E^\infty: \pi_* B \rightarrow \pi_*^S B$  is trivial on  $p$ -torsion.

(Received October 19, 1978.)

In Kosniowski's paper, Generators of the  $Z/p$  bordism ring, Math. Z. 149 (1976), 121-130, he determined the generators of the unitary  $Z_p$  bordism ring  $U_*(Z_p)$  for any odd prime  $p$ . However, the result he obtained is not true for  $p \geq 5$ . The purpose of this paper is to give the complete generating set of the unitary bordism ring  $U_*(Z_5)$  and the oriented bordism ring  $O_*(Z_5)$ . (Received October 20, 1978.) (Author introduced by James R. Weaver).

\*763-55-5 SAMUEL MASIH, Albany State College, Albany, Georgia 31705. Fixed point index of weighted maps. Preliminary report.

An ordered triple  $(X, f, U)$  is said to be admissible if  $X$  is a compact connected triangulable orientable and differentiable  $n$ -manifold,  $f: X \rightarrow X$  is a weighted map (G. Darbo, Teoria dell'omologia in una categoria di mappe plurivalenti ponderate, Questi Rendiconti, 28(1958), 188-220) and  $U$  is an open subset of  $X$  which has no fixed point on the boundary  $\partial U$  of  $U$ . We define a function  $I$  from the set of all admissible triples to the set of rational numbers and show that this defines an index for weighted maps. Let  $I(X, f, U)$  denote the number associated with the admissible triple  $(X, f, U)$  defined by the function  $I$ . Theorem. (i) Let  $(X, f, U)$  be admissible. If  $f$  has no fixed point in  $U$ , then  $I(X, f, U) = 0$ . (ii)  $I(X, f, X) = L(f)$ , where  $L(f)$  is the Lefschets number of the weighted map  $f$ . (iii) If  $(X, f, U)$  and  $(X, g, U)$  are admissible such that  $f|_U = g|_U$ , then  $I(X, f, U) = I(X, g, U)$ . (iv) If  $(X, f, U)$  and  $(X, g, U)$  are admissible such that  $f$  is  $\sigma$ -homotopic to  $g$  by a  $\sigma$ -homotopy  $h$  which has no fixed point in  $\partial U \times I$ , then  $I(X, f, U) = I(X, g, U)$ . (v) If  $f: X \rightarrow Y$  is a single valued map and  $g: Y \rightarrow Z$  is a weighted map such that  $(Y, fg, U)$  is admissible, then  $(X, gf, f^{-1}(U))$  is admissible and  $I(Y, fg, U) = I(X, gf, f^{-1}(U))$ . Using the above theorem we define a fixed point index of the weighted maps of compact absolute neighborhood retracts. (Received October 23, 1978.)

\*763-55-6 SUSHIL JAJODIA, University of Oklahoma, Norman, OK 73019. Classification problem for Lens spaces for one-relator groups with torsion.

Let  $\mathbb{E}$  be a one-relator group with presentation  $R = (x_1, \dots, x_n : R^p)$  where  $R$  is not a proper power and  $p \geq 2$ . Then given any integer  $q$ , relatively prime to  $p$ , we can construct the Lens spaces  $L(p, q)$  for  $\mathbb{E}$  obtained from the cellular model  $C(R)$  of the presentation  $R$  by attaching a 3-cell via the attaching map  $R^q - 1$ , which generates the ideal  $Z\mathbb{E}(R - 1) \approx \Pi_2(C(R))$ . We prove:

Theorem 1. If the single relator  $R^p$  is a power of a primitive element  $R$  in the free group  $F$  generated by  $x_1, \dots, x_n$ , then two Lens spaces  $L(p, q)$  and  $L(p, r)$  for  $\mathbb{E}$  have the same homotopy type if and only if  $qr$  or  $-qr$  is a quadratic residue mod  $p$ . We are able to solve the classification problem for the Lens spaces for the remaining one-relator groups modulo the conjecture: If  $R$  is not primitive in the free group  $F$ , then each automorphism  $\Phi \in \text{Aut } \mathbb{E}$  is induced by an automorphism of the free group  $F$ .

Theorem 2. If the above conjecture holds for  $\mathbb{E}$ , then two lens spaces  $L(p, q)$  and  $L(p, r)$  for  $\mathbb{E}$  have the same homotopy type if and only if  $q \equiv \pm r \pmod{p}$ . We give several examples of one-relator groups for which the conjecture is true. (Received October 24, 1978.)

\*763-55-7 JULIUS L. SHANESON, Department of Mathematics, Hill Center, Rutgers University, New Brunswick, New Jersey 08903. Manifolds and submanifolds.

- This talk will survey developments and open problems in the theory of manifolds and submanifolds. The general question is: when can one manifold be embedded in another, how can such embeddings be classified, and how locally nice can they be made. A typical situation of interest is the case of a hypersurface in a complex algebraic variety. Homotopy theory, algebraic K-theory, local piecewise differential geometry and characteristic classes all coalesce into the existence and classification of embedded or immersed manifolds of codimension two (= complex codim 1) and the structure of the singularities (= points where the immersion is not locally smoothable).

A few results (from the joint work with S. Cappell on this area) giving the flavor of the subject. (1) Let  $M$  be a closed 4-manifold of even Euler characteristic. Then  $M$  has a piecewise smooth (P. L. or P. D.) immersion in Euclidean space  $\mathbb{R}^6$ . (2)  $M$  as in (1), but a spin manifold. Then  $M$  has a P. D. embedding in  $\mathbb{R}^6$ . (3)  $M^n$  closed manifold with the stable tangent bundle fibre homotopically trivial. Then  $M^n$  has a P. D. immersion in  $\mathbb{R}^{n+2}$ . (4) For  $n$  odd, concordance classes of P. D. immersions of the sphere  $S^n$  in  $\mathbb{R}^{n+2}$  are in 1-1 correspondence with the stable stem  $\lim_{k \rightarrow \infty} \pi_{n+k}(S^k)$ . Each of (1), (2), (3) comes with many examples where the singular set can never be trivial. E.g. in (3), if  $M$  has Pontrjagin classes, their duals must be cycles in the singular set. Problem. Give a combinatorial formula for these cycles. Problem connected with (4). Interpret standard operations (Toda bracket, etc.) in  $\pi_n^S$  geometrically. (Received October 25, 1978.)

763-55-8 MICHAEL R. COLVIN, Louisiana State University, Baton Rouge, Louisiana 70803. Multi-valued mappings of closed and bounded subsets of a normed linear space—A mapping degree. Preliminary report.

Extending the concept of degree initiated by D. G. Bourgin, we construct a degree for set-valued mappings defined on closed and bounded subsets of a normed linear space into itself. Using this degree we obtain fixed point theorems. (Received October 25, 1978.)

## 57 ► *Manifolds and Cell Complexes*

\*763-57-1 John R. Harper, University of Rochester, Rochester, New York 14627 and S.J. Lomonaco, Jr., State University of New York, Albany, New York 12222. The homotopy groups of knots III. The k-invariant of knots. Preliminary report.

A method is given for computing the first  $k$ -invariant of smooth  $(4,2)$ -knots from their diagram. Various results and examples are given. The geometric significance of these results is discussed. (Received September 13, 1978.)

763-57-2 BRADD CLARK, University of Southwestern Louisiana, Lafayette, Louisiana 70504. The genus of 3-manifolds obtained by surgery on cable links. Preliminary report.

Let  $C$  be a cube-with- $K$ -knotted hole. One can find a sequence of disjoint 3-cells  $c_1, \dots, c_k$  such that (1)  $c_i \subset C$  for  $1 \leq i \leq k$  (2)  $c_i \cap \partial C = D_{i,1} \cup D_{i,2}$ , a pair of disjoint 2-cells and (3)  $C - \cup c_i$  is a cube-with-handles. We associate with the knot  $K$  the integer  $k$ , the smallest number of "tunnels" needed to transform  $C$  into a cube-with-handles. Let  $g(M)$  stand for the genus of the closed 3-manifold  $M$ . Theorem. If  $M$  is a manifold obtained by surgery on a cable-link-about  $K$  with  $n$  components, then  $g(M) \leq n + k + 1$ . (Received October 2, 1978.)

763-57-3 Richard Randell, University of Michigan, Ann Arbor, Michigan 48109. Some topology of cyclic multiple planes.

The cyclic multiple planes  $z^n = f(x,y)$ , branched over the curve  $C = \{f(x,y) = 0\}$  in the projective plane  $P^2$ , were studied by O. Zariski many years ago. We will apply recent results in the computation of  $\pi_1(P^2 - C)$  to obtain refinements and generalizations of some of Zariski's theorems.

(Received October 20, 1978.)

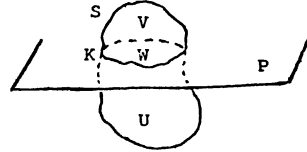
\*763-57-4 L.R. Hitt, University of South Alabama, Mobile, Alabama 36688. Characterizations of ribbon  $n$ -knots.

An  $n$ -knot,  $K = (S^{n+2}, fS^n)$ , is an oriented codimension-two sphere pair.  $K$  is defined to be a *ribbon  $n$ -knot* iff  $fS^n$  bounds an immersed  $(n+1)$ -disk,  $\bar{F}(D^{n+1}) \subset S^{n+2}$ , the singularities being of "ribbon type"—i.e., proper  $n$ -disks intersecting interior  $n$ -disks (see Abstract 755-G3 for a formal definition). Several characterizations of ribbon 1- and 2-knots are known. We show that these, as well as some new, characterizations generalize naturally to ribbon  $n$ -knots.

Theorem: For each  $n \geq 1$  the following statements are equivalent: (1)  $K = (S^{n+2}, fS^n)$  is a ribbon  $n$ -knot; (2)  $fS^n$  is a fusion of a trivial  $n$ -link in  $S^{n+2}$ ; (3)  $fS^n$  has a Seifert manifold of the form  $D^{n+1} \cup \{h_1^n | 1 \leq i \leq k\} \cup \{h_2^j | 1 \leq i \leq k\}$  where  $D^{n+1} \cup \{h_1^n | 1 \leq i \leq k\}$  is contained in an equatorial  $S^{n+1} \subset S^{n+2}$  ( $h_2^j$  denotes a  $j$ -handle); (4)  $fS^n$  bounds a semi-unknotted  $(n+1)$ -manifold in  $S^{n+2}$ ; (5)  $fS^n \times 0$  bounds a ribbon  $(n+1)$ -disk in  $S^{n+2} \times I$ ; (6)  $K$  is cobordant to the unknot via a cobordism built up with only 1- and 2- handles in special cancelling pairs from the unknotted end; (7)  $K$  bounds a disk pair obtained from the unknotted disk pair by attaching 1- and 2- handles in special cancelling pairs. (Received October 23, 1978.)

\*763-57-5 GARY RICHTER, Southwestern University, Georgetown, Texas 78626. Spanning arcs of patched disks.

Suppose a topological 2-sphere  $S$  in  $E^3$  intersects a horizontal plane  $P$  in a continuum  $K$  such that  $S - K$  has only two components,  $U$  lying below  $P$  and  $V$  lying above  $P$ , with  $K$  the boundary in  $S$  of each component.  $W$  is the bounded component of  $P - K$ . The point set



$S' = (S - V) \cup W$  is a patched 2-sphere.  $S'$  may not be a topological 2-sphere and may even contain a simple closed curve which fails to separate it. If  $D$  is a disk in  $S$  with  $V \subseteq \text{Int } D$ , then  $D' = (D - V) \cup W$  is a patched disk whose "boundary" is  $\text{Bd } D$ .  $D'$  may contain a spanning arc which fails to separate it. However, the following can be proved:

Theorem. No patched disk contains disjoint spanning arcs  $\alpha$  and  $\beta$  such that the end points of  $\alpha$  separate the end points of  $\beta$  from each other in the boundary of the patched disk. (Received October 23, 1978.)

\*763-57-6 JOAN S. BIRMAN, Columbia University, New York, N.Y. 10027. Fibered knots and branched coverings of  $S^3$ . Preliminary report.

We generalize a construction due to Goldsmith ["Symmetric fibered links", Annals of Math. Studies #84 (1975), pp. 3-24]. Theorem: Let  $\tilde{\alpha}$  be a fibered knot in a homology sphere  $M$ . Then there is a 3-fold irregular covering  $p: M \rightarrow S^3$  branched over a link  $\beta$ , and a representation of  $\beta$  as a closed braid with braid axis  $\alpha$  such that  $\tilde{\alpha} = p^{-1}(\alpha)$  and such that each fiber  $\tilde{A}$  is the pre-image  $p^{-1}(A)$  of a fiber  $A$  in the standard fibration of  $S^3 - \alpha$ . The monodromy  $\beta_* = p\beta_*p^{-1}$  where  $\beta_*$  is the autohomeomorphism induced by  $\beta$  on the punctured disc  $(A - \beta \cap A)$ .

Conjecture: If  $M$  is  $S^3$ , then  $\beta$  can be taken to be the unlink of 2 components.

(Received October 23, 1978.)

763-57-7 E. H. ANDERSON, Mississippi State University, Mississippi State, Mississippi 39762. Uncomplicated group presentations for simply-connected three-manifolds. Preliminary report.

Suppose Poincare's Conjecture for three-manifolds is false. Then, there is a compact, connected, simply-connected three-manifold  $M$  which is not a three-sphere and a positive integer  $N$  such that the fundamental group of  $M$  has a presentation  $\{a_1, a_2, \dots, a_N \mid R_1, R_2, \dots, R_N\}$  where

- (i) each relation contains 2 or 3 generators all of which are distinct,
- (ii) each generator appears 2 or 3 times,
- (iii) no two generators appear together in more than one relation, and
- (iv) the arcs and disks in the Heegard diagram of the presentation form a connected set and, equivalently, their complement is a collection of open disks. (Received October 23, 1978.)

\*763-57-8 John D. Blanton, St. John Fisher College, Rochester, New York 14618.  
On the Morse characteristic of fibre bundles. Preliminary report.

The Morse characteristic of a differentiable manifold is the minimum number of critical points which any differentiable function on the manifold can have. Let the closed manifold  $M$  be the total space of a fibre bundle with base space  $B$  and fibre  $F$ . If there are functions  $f$  with  $m$  critical points and  $g$  with  $n$  critical points on  $F$  and  $B$  respectively, then a vector field is defined locally on  $M$  by the gradients of  $f$  and  $g$ . Using partitions of unity the vector field is defined globally and then used to define a differentiable function on  $M$  with  $mn$  critical points. We conclude that the Morse characteristic of the total space is less than or equal to the product of the Morse characteristic of the base space and the fibre. (Received October 23, 1978.)

763-57-9 FRANK QUINN, Virginia Polytechnic Institute and State University, Blacksburg, Va. 24061.  
Resolutions of homology manifolds.

Every ANR homology manifold of dimension  $\geq 5$  is shown to have a resolution (ie. is a cell-like image of a topological manifold.) Combined with a result of R.D. Edwards this yields the characterization of manifolds: a space is a manifold of dimension  $\geq 5$  if it is an ANR homology manifold, and has the disjoint 2-disc property (two singular 2-discs can be approximated by ones with disjoint images).

(Received October 25, 1978.)

## 58 ► *Global Analysis, Analysis on Manifolds*

763-58-1 WILLIAM J. FITZPATRICK, University of Southern California, Los Angeles, California 90007.  
Convergent solutions of singular Pfaffian systems. Preliminary report.

Consider a Pfaffian system of the form

$$x^{p+1}y^{q+1}dz = y^{q+1}f(x,y,z)dx + x^{p+1}g(x,y,z)dy,$$

where  $p$  and  $q$  are nonnegative integers,  $x$  and  $y$  are in  $\mathbb{C}$ ,  $z$  is in  $\mathbb{C}^m$  and  $f$  and  $g$  are functions from  $\mathbb{C}^2 \times \mathbb{C}^m$  into  $\mathbb{C}^m$ . Using the theory of asymptotic solutions, R. Gerard and Y. Sibuya (C.R. Acad. Sc. Paris, 284 (1977), 57-60) have obtained several results concerning the existence of holomorphic solutions of such systems in a neighborhood of the origin. In this report, it is shown that such results can also be obtained by employing certain functional analysis techniques. A number of existence theorems for related systems are also obtained. (Received October 23, 1978.)

## 60 ► *Probability Theory and Stochastic Processes*

\*763-60-1 KENNETH ALEXANDER, University of Washington, Seattle, Washington 98195 and VICTOR GOODMAN, Indiana University and The University of Wisconsin, Madison, Wisconsin 53706.  
Quadratic convergence and almost sure convergence of sums of independent random variables.

Let  $K_\infty$  denote the set of sequences  $\underline{\alpha} = \{\alpha_i\}$  such that  $\alpha_i$  is a positive integer and  $\alpha_i \neq 1$  for finitely many  $i = 1, \dots$ .  $K_\infty$  is partially ordered by  $\underline{\alpha} \leq \underline{\beta}$  iff  $\alpha_i \leq \beta_i$  for all  $i$ . There exists a family of independent mean zero Gaussian random variables  $\{X_\alpha\}$ ,  $\alpha \in K_\infty$ , such that the partial sums  $\sum_{\alpha \leq \underline{\beta}} X_\alpha$  converge in  $L^2(\Omega)$  and diverge almost surely. This answers a question

raised by Gabriel, Ann. Probability 5 779-785, who asked if the structure of the index set affects the relation between quadratic and almost sure convergence. The example is a consequence of the following lemma for the  $N$ -parameter Wiener process,  $W^N(\underline{t})$ . Lemma.  $\sup_{0 \leq \underline{t} \leq \underline{1}} W^N(\underline{t})$  converges in probability

to  $+\infty$  as  $N \rightarrow +\infty$ . The following sufficient condition for a.s. convergence is not a necessary condition. However, we conjecture that it is sharp. Theorem. In order that the partial sums of a family  $\{X_\alpha\}$ ,  $\alpha \in K_\infty$ , of independent random variables converge, it is sufficient that the family  $\{X'_\alpha\}$  obtained by truncating about medians satisfies

$$\sum_{\alpha \in K_\infty} P[X'_\alpha \neq X_\alpha] < \infty, \lim_{\rightarrow} \sum E[X'_\alpha] \text{ exists, } \sum_{n=1}^{\infty} \frac{\dim\{A_{n+1} - A_n\}}{\sum_{\beta \in A_{n+1} - A_n} \sigma^2(X'_\beta)} < \infty$$

for some generating sequence  $\{\alpha_n\}$ . Here  $A_n \equiv \{\beta : \beta \leq \alpha_n\}$ . (Received September 18, 1978.)

763-60-2 Mark J. Christensen, School of Mathematics, Georgia Institute of Technology, Atlanta, Georgia 30332. A Theorem on the Average Roots of Random Algebraic Polynomials. Preliminary report.

It has been shown (Šparo & Šur, Vestnik Moskov, Univ. Ser. I Mat. Meh., 1962) that the roots of the random algebraic polynomial equation;

$$a_n z^n + a_{n-1} z^{n-1} + \dots + a_1 z^1 + a_0 = 0, \quad (1)$$

where the coefficients are i.i.d. random variables, "cluster" (in the sense of random measures) around the unit circle as  $n$  goes to infinity. Strukov and Timan (Sibirskii Matematicheskii Zhurnal, 1977) have given estimates for the difference;

$$| E( f( \vec{a} ) ) - f( E( \vec{a} ) ) | \quad (E = \text{expected value})$$

for the case of  $f$  continuous on  $C^n$  using Bernstein polynomials. We give stronger estimates for this quantity for the case of the roots of (1) using techniques from several complex variables, as well as the theory of Semi Amarts. (Received October 2, 1978.) (Author introduced by Professor A. T. Bharucha-Reid).

763-60-3 Y. K. CHAN, Energy Technology Applications Division, Boeing Computer Services Company, 565 Andover Park West, M/S 9C-01, Tukwila, Washington 98188. Constructive probability theory and related topics. Preliminary report.

This is a review of results in constructive probability theory. The basic tools and theorems have been recast in the constructive framework with no major difficulty: independence, characteristic functions, laws of large number, and central limit theorems. Results and some open problems in the following areas are discussed: renewal theorems, construction of Markov processes, martingale theorems, and classical potential theory. (Received October 5, 1978.) (Author introduced by Professor Ray Mines).

763-60-4 JOHN A. MORRISON, Bell Laboratories, Murray Hill, New Jersey 07974. Distribution of waiting times for two discrete-time queues in tandem. Preliminary report.

A discrete-time queueing problem involving two queues of unlimited size in tandem, with unit service times, is considered. The joint process of the numbers of (correlated) external arrivals to the two queues at unit times forms a sequence of i.i.d. vector random variables. In addition, departures from the first queue arrive instantaneously at the second queue. At both queues all arrivals at a given time are served ahead of subsequent arrivals. Arrivals to the second queue from the first are either served ahead of, or behind, simultaneous external arrivals, or the order of service is random. With the help of a previously derived formula for the generating function of the joint equilibrium distribution of the two queue lengths, the joint equilibrium distribution of waiting times in the two queues for arrivals to the first queue, and the equilibrium distribution of waiting times for external arrivals to the second queue, are derived in the three cases. (Received October 10, 1978.)

\*763-60-5 FARAG A. ATTIA, Mathematics Department, Kuwait University, Kuwait, Arabian Gulf. On the probability distribution of level crossings by a stochastic process. Preliminary report.

In this paper some results concerning the probability distributions of both the number of crossings of a level by a stochastic process and the interval between them are obtained using a method similar to that used by Longuet-Higgins (1962). The results obtained are then compared with some well known results. (Received October 10, 1978.)

763-60-6 HILL, THEODORE P., Georgia Institute of Technology, Atlanta, GA 30332. "On Reaching a Goal Quickly in a Finite State Gambling Problem."

In every gambling problem with finite state space  $F$  and goal  $g \in F$ , there exists a stationary family of strategies which, uniformly in  $F$ , not only (nearly) maximizes the probability of reaching  $g$ , but also (nearly) minimizes the expected time to  $g$ .

A finite state gambling house is a pair  $(F, \Gamma)$  in which  $F$  is a finite set and  $\Gamma$  a function which associates with each  $f$  in  $F$  a non-empty collection  $\Gamma(f)$  of transition probabilities on  $F$ . A strategy is a rule which selects a transition probability  $\gamma_n$  at time  $n$  as a function of the history  $(f_1, f_2, \dots, f_n)$  of the system ( $\gamma_n$  must be in  $\Gamma(f_n)$ ). A strategy  $\sigma$  is stationary if  $\sigma(f_1, \dots, f_n) = \sigma(f_1', \dots, f_m')$  whenever  $f_n = f_m'$ . (Notation follows that of Dubins and Savage.)

Suppose the objective is to reach a goal  $g \in F$ . Let  $V(\sigma)$  be the probability of reaching the goal using strategy  $\sigma$ , and let  $V(f) = \sup \{V(\sigma) : \sigma \text{ is available at } f\}$ . Let  $W(\sigma)$  be the expected time to the goal using  $\sigma$ , and let  $W(f) = \inf \{W(\sigma) : \sigma \text{ is available at } f\}$ .

Theorem. Let  $(F, \Gamma)$  be a finite state gambling problem with goal. Then for each  $\epsilon > 0$  there exists a stationary strategy  $\sigma$  in  $\Gamma$  satisfying (i) and (ii) for all  $f \in F$ .

$$(i) \quad V(\sigma[f]) \geq V(f) - \epsilon$$

$$(ii) \quad W(\sigma[f]) \leq W(f) + \epsilon$$

(Received October 13, 1978.)

\*763-60-7 Debra D. Qualls, University of Arkansas, Fayetteville, Arkansas 72701. A Potential Theory for Geometrically Ergodic Infinite Interacting Particle Systems.

The theory of infinite interacting particle systems doesn't fit neatly into any of the existing class of potential theories. In the case of geometric ergodicity, we present adaptations of recurrent and transient denumerable Markov chain potential theoretic ideas in terms of cylinder sets. This is done by means of the dual, or reversed time, process and the Radon-Nikodym derivative. The notions of equilibrium potential and capacity are included. (Received October 16, 1978.)

763-60-8 P. Z. Daffer, Louisiana Tech University, Ruston, LA, 71272. On Compact Convex Subsets of  $D[0,1]$ .

It is proved that a subset  $K$  of  $D[0,1]$  which is convex and conditionally compact with respect to the Skorokhod topology is conditionally compact with respect to the uniform topology on  $D[0,1]$ . Consequences of this result are indicated for limit laws for generalized random variables in  $D[0,1]$  which use tightness of measures as a hypothesis. A characterization of the convex, conditionally compact subsets of  $D[0,1]$  is given in terms of the modulus of continuity and finitely many jump points. (Received October 16, 1978.)

\*763-60-9 ED WAYMIRE, University of Mississippi, Oxford, Mississippi 38677. Zero-Range Interaction at Bose-Einstein Speeds under a Positive Recurrent Single Particle Law.

The equilibrium states and the time asymptotic behavior of Spitzer's zero-range interaction system are studied in the case of a positive-recurrent irreducible single particle law and an attractive speed function. It is shown that within the natural phase space of finitely many particles per cell, "explosions" occur which lead to infinite occupancy by certain individual cells. These cells are then identified in terms of the parameters of the model. (Received October 19, 1978.)

\*763-60-10 CHULL PARK and FREDERICK J. SCHUURMANN, Miami University, Oxford, Ohio 45056. Evaluations of absorption probabilities for Wiener process on large intervals.

Let  $\{W(t), 0 \leq t < \infty\}$  be the standard Wiener process. The computation schemes developed in the past do not work effectively for the absorption probabilities of the type  $P\{\sup_{0 \leq t \leq T} W(t) - f(t) \geq 0\}$  when either  $T$  is large or  $f(0) > 0$  is small. It is known that a large class of useful Gaussian processes can be transformed into Wiener processes. When we do this, a process on a relatively small interval sometimes transforms into the Wiener process on a huge interval. Thus at times it is inevitable to evaluate the probabilities on large intervals. The paper gives an efficient and accurate algorithm to compute such probabilities. (Received October 23, 1978.)



763-60-11 LAIF SWANSON, Texas A&M University, College Station 77840. An automorphism all of whose n-fold Cartesian products are loosely Bernoulli. Preliminary Report.

Loosely Bernoulli automorphisms (see Feldman, Israel Journal 24, No. 1) have been shown to be exactly those which are Kakutani equivalent to Bernoulli shifts or group rotations. Being loosely Bernoulli, unlike being Bernoulli, is not a property which carries to Cartesian products; T may be loosely Bernoulli without  $T \times T$  being loosely Bernoulli.

We use cutting and stacking to construct an automorphism T, similar to an automorphism constructed by Katok, with  $T \times T \times \dots \times T$  (n-fold Cartesian product) loosely Bernoulli for every n.

(Received October 20, 1978.)

763-60-12 T. C. SUN, Department of Mathematics, Wayne State University, Detroit, Michigan 48202. A finite element method for random differential equations with random coefficients.

A finite element method is derived for solving equations of the following type  $-(p(x)u'(x, \omega))' + (q(x) + r(x)\lambda(\omega))^2 u(x, \omega) = f(x, \omega)$ ,  $0 \leq x \leq l$  with boundary conditions  $u(0, \omega) = u(l, \omega) = 0$  where (i)  $0 < p \in C^1[0, l]$ , (ii)  $q, r \in C[0, l]$ , (iii)  $f(x, \omega) = B'(x, \omega)$  is the white noise, (iv)  $\lambda(\omega)$  is a standard Gaussian random variable and is independent of  $B(x, \omega)$  all  $0 \leq x \leq l$ . We can think of  $(q(x) + r(x)\lambda(\omega))^2$  as a random perturbation of the deterministic  $q(x)^2$ . The error of the finite element solution is estimated and possible extensions are discussed.

(Received October 23, 1978.)

763-60-13 WITHDRAWN

\*763-60-14 HUI-HSIUNG KUO, Dept. Math. Louisiana State Univ. Baton Rouge, La. 70803 Central Limit Theorem for  $\mathfrak{F}'$ -valued Random Variables.

Let  $\{X_n\}$  be a sequence of independent, identically distributed  $\mathfrak{F}'$ -valued random variables, where  $\mathfrak{F}'$  is a countably Hilbert nuclear space. Assume that the common distribution  $\mu$  on  $\mathfrak{F}'$  has mean 0 and that  $\Gamma(\phi, \psi) = \int_{\mathfrak{F}'} (x, \phi)(x, \psi) \mu(dx)$  is continuous. Then it is shown that the distribution of  $(X_1 + \dots + X_n) / \sqrt{n}$  converges weakly to a Gaussian measure on  $\mathfrak{F}'$ . An application to the tempered distributions-valued random variables is also given. (Received October 25, 1978.)

\*763-60-15 Thomas A. O'Connor, University of Louisville, Louisville, Kentucky 40208. Infinitely Divisible Distributions Similar to Class L Distributions.

Infinitely divisible characteristic functions belong to the class L if and only if the Lévy spectral function M satisfies  $\text{sgn } x |x| dM(x)$  does not increase on  $(-\infty, 0)$  and on  $(0, +\infty)$ . Another NASC for the characteristic function  $\phi$  to belong to the class L is that it be a solution of the functional equation  $\phi(u) = \phi(ru) \phi_r(u)$ ,  $0 < r < 1$ , and  $\phi_r$  is a characteristic function. The problem under concern here is to determine which infinitely divisible characteristic functions have Lévy spectral functions satisfying  $\text{sgn } x |x|^\alpha dM(x)$  do not increase on  $(-\infty, 0)$  and on  $(0, +\infty)$  for  $0 \leq \alpha < 1$  and identify the functional equation these characteristic functions must obey. Also the role these characteristic functions play as limit laws will be examined. (Received October 25, 1978.)

\*763-60-16 CHI-SHANG SOONG, Villanova University, Villanova, Pa. 19085. A Central limit theorem for exchangeable sequences in Banach Spaces. Preliminary Report.

Let  $\{x_n\}$  be an exchangeable sequence of random elements in a type 2 separable Banach space E.

We assume that the expectation  $EX_1$  exists (in the sense of Bochner or Pettis) and  $E\|X_1\|^2 < \infty$ .

The following form of central limit theorem is proved:

$$\frac{1}{\sqrt{n}} \sum_{j=1}^n (x_j - E(x_j|\lambda)) \xrightarrow{d} \int_E x w_\lambda(dx)$$

where  $\lambda$  is a random measure generated by the exchangeable sequence  $\{x_n\}$ ,  $E(x_j|\lambda)$  is a conditional expectation in certain sense,  $\xrightarrow{d}$  denotes convergence in distribution,  $w_\lambda$  is a randomized white noise. (Received October 25, 1978.) (Author introduced by Professor J. Galambos).

763-60-17 GRANT A. RITTER, University of Florida, Gainesville, Florida, 32611. Sums of Independent Random Variables Conditioned to Stay Bounded. Preliminary Report.

Following Iglehart let  $S_n = \sum_{i=1}^n \xi_i$  be the sum of i.i.d. random variables with  $E\xi_i = 0$ ,  $E\xi_i^2 = \sigma^2$ , and  $E|\xi_i|^3 < \infty$ . For hitting times of the form  $T_n(x) = \min(k>0: S_k \notin [0, xn^{1/2}])$  we obtain a limit law for  $(S_n/\sigma n^{1/2} | T_n(x) > n)$  and prove the invariance principle for  $(S_{[tn]}/\sigma n^{1/2} | T_n(x) > n)$ . (Received October 25, 1978.)

## 62 ► Statistics

\*763-62-1 PERRY HAALAND, University of Miami, PO Box 249085, Coral Gables, FL 33124. Characterization of Certain Partitioning Methods with Application to Statistical Monitoring Systems.

The end result of a cluster analysis may be viewed as a partition of the observations into subgroups which correspond to the clusters. It is possible to evaluate the significance of clustering if the occupancy numbers of the partition follow a multinomial distribution. Under a null hypothesis of no significant clustering, the occupancy numbers follow a multinomial distribution and the partitioning method satisfies certain invariance restrictions if and only if there is a fixed a priori classification of clusters and each observation is assigned to a cluster independently of the other observations. These principles are applied to the problem of significance testing in a statistical monitoring system. (Received October 10, 1978.)

\*763-62-2 JANOS GALAMBOS, Temple University, Philadelphia, Pennsylvania 19122. A statistical test for extreme value distributions.

The paper is a continuation of the investigation concerning the following test which was first announced by the author at the Fourth Australian Statistical Conference in Canberra (July, 1978). If, in an extreme value model, an approximation by independent and identically distributed random variables is possible (see Sections 3.6-3.13 in the author's book The Asymptotic Theory of Extreme Order Statistics, Wiley, New York, 1978, for such possibilities), then the asymptotic distribution is a classical one. In order to avoid a decision on a population distribution, we want to use observations to decide that which of the classical limiting distributions applies to the model investigated. Assume that the unknown population distribution  $F(x)$  satisfies  $F(x) < 1$  for all  $x$ . Then the possible limiting distributions of properly normalized maxima are the location and scale families of  $\exp(-e^{-x})$  and  $\exp(-x^{-h})$ . By a transformation of the observations, a test free of the mentioned parameters, is developed. (Received October 16, 1978.)

763-62-3 Gary Richardson, East Carolina University, Greenville, North Carolina 27834. Application of convergence structures to statistics. Preliminary report.

Let  $T$  denote the sequential convergence space of all test functions on a subspace of  $n$ -dimensional Euclidean space and let  $C(Y)$  denote the set of all continuous real-valued functions on  $Y$ . It is shown that  $T$  can be embedded as a closed subspace of  $C_c(Y)$  for some sequential convergence space  $Y$  and sequential convergence structure  $c$  for  $C(Y)$ . Then an Ascoli-type theorem for sequential convergence spaces gives an alternate proof to the well-known result that  $T$  is sequentially compact. (Received October 23, 1978.)

763-62-4 J. H. B. KEMPERMAN, University of Rochester, Rochester, New York 14627. On the norm of an unbiased estimator.

Let  $\{P_\theta\}$  be a collection of probability measures on a measurable space  $X$ . The set of estimators (measurable functions)  $T: X \rightarrow \mathbb{R}$  with  $E_\theta T = g(\theta)$  for all  $\theta$  will be denoted as  $H_g$ . Theorem 1 gives an explicit formula for  $\inf \{\|T\|_L : T \in H_g\}$ . Here  $L$  denotes a Banach function space relative to a  $\sigma$ -finite measure  $Q$

on  $X$ . It is assumed that  $L$  is the dual of a Banach function space  $K$  which contains all elements  $dP_\theta/dQ$ . This generalizes a 1949 result of E. Barankin, Theorem 2. Suppose that the  $P_\theta$  are dominated by a single  $\sigma$ -finite measure. Let  $\phi$  be a fixed Orlicz function (such as  $\phi(t) = |t|^s$  with  $s > 1$ ). Then the following are equivalent. (i) Each class  $H_g$  which contains a bounded estimator also contains an estimator  $T_0$  such that  $E_\theta \phi(T_0) \leq E_\theta \phi(T)$  for all  $T \in H_g$  and all  $\theta$ . (ii) The (sufficient)  $\sigma$ -field  $\mathcal{G}$  generated by the densities  $dP_\theta/dP_\theta$ , is boundedly complete. That is, there exists no nontrivial  $\mathcal{G}$ -measurable bounded estimator  $T$  with  $E_\theta T = 0$  for all  $\theta$ . This generalizes a 1957 result of R. R. Bahadur with  $\phi(t) = t^2$ . One can generalize Theorem 2 to situations where the goodness of an estimator  $T_0$  is measured relative to a collection  $\{Q_\sigma\}$  distinct from  $\{P_\theta\}$ , at least when the latter is finite. (Received October 24, 1978.)

\*763-62-5 CHRIS P. TSOKOS, University of South Florida, Tampa, Florida 33620 and JOHN C. TURNER, United States Naval Academy, Annapolis, Maryland 21402. A Stochastic Algorithm for the Design of Multi-Spectral Sensors. Preliminary report.

The present study is concerned with developing a stochastic algorithm to handle the problem of determining the optimal choice of a subset of  $K$  filters from  $N$  possible ones in order to discriminate among  $M$  possible sources. Various objective functions can be used in determining the best filter combination. The misclassification probability can be calculated fairly quickly by a Monte-Carlo scheme. The other objective functions can be evaluated directly from the means and covariance matrices of the filter outputs. The step up and step down algorithms represent significant cost savings over exhaustive search. (Received October 25, 1978.)

## 65 ► Numerical Analysis

763-65-1 L.J. LARDY, Syracuse University, Syracuse, N.Y. 13210. An analysis of a numerical method for Hammerstein equations. Preliminary report.

Consider the Hammerstein equation  $\phi(s) - \int_0^1 k(s,t)F(t,\phi(t)) dt = f(s)$  where the given functions  $k(s,t)$ ,  $F(t,u)$ , and  $f(s)$  are continuous. We present and analyze a numerical method for calculating an approximate solution of such an equation. The method involves the construction of certain projection operators and uses them in conjunction with Nyström's approximate equation. The objective is to obtain an approximate solution which is nearly as accurate as that produced by Nyström's method but that does not require the solution of so large a nonlinear system. The analysis is based on convergence properties of the projections. (Received September 5, 1978.)

763-65-2 ALVIN BAYLISS, ICASE, Hampton, Virginia 23665 and ELI TURKEL, Courant Institute, New York 10012. Boundary conditions for the dynamic solution of acoustic equations in an unbounded region.

The numerical solution for acoustic waves in the mean flow of a jet is computed from the time dependent equations rather than assuming an  $e^{i\omega t}$  time dependence as is conventionally done. This approach permits the solution of many more interesting physical problems, for example moving sources in a jet. With this approach one solves a linear hyperbolic system in an unbounded domain. Artificial boundaries must be introduced in order to get a finite domain of computation. A fundamental difficulty is in developing boundary conditions for the artificial boundaries which in addition to being well posed permit the computational domain to be sufficiently constricted so that solutions can be obtained efficiently. Conditions are required which simulate outgoing waves at outflow boundaries and which simulate the inflow of the jet at inflow boundaries. Such boundary conditions are developed for this problem and their validity is considered theoretically and numerically. (Received September 11, 1978.) (Author introduced by Dr. Robert J. Thompson).

\*763-65-3 John Gregory, S.I.U., Carbondale, IL 62901 and Ralph Wilkerson, Winthrop College, Rock Hill, S.C. 29733. The Double Eigenvalue Problem. Preliminary report.

A new abstract theory and numerical algorithm for computing eigenvalues of second order differential equations of the form  $(p(t)x'(t))' + q(t)x(t) + \lambda r(t)x(t) + \mu s(t)x(t) = 0$ ,  $x(a) = x(b) = 0$ ,

$p(t) > 0$  has been developed. The numerical algorithm is very fast, accurate, and easy to implement. The theoretical and numerical ideas follow from the first author's approximation theory of quadratic forms on Hilbert Spaces. By setting  $\mu = 0$  we obtain a "generalization" of classical Raleigh-Ritz theory of eigenvalues.

The numerical algorithm is derived as follows: The above differential equation is converted to the equivalent two parameter quadratic form which is in turn approximating by a finite dimensional quadratic form using Spline "hat" functions. The "Euler-Lagrange" equation of this finite dimensional quadratic form is constructed and is the numerical solution to a two parameter differential equation. A simple search procedure on  $\lambda$  and  $\mu$  yields the desired solution. Efficiency is obtained since the "coefficient" matrices are computed only once. (Received September 14, 1978.) (Author introduced by Professor Ronald B. Kirk).

\*763-65-4 Riaz A. Usmani, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2. Spline Solutions of a Boundary Value Problem.

Methods of order two and four are developed and analysed for the smooth continuous approximation of the solution of the two point boundary value problem

$$\begin{cases} y^{(4)}(x) + p(x) y(x) = q(x), & a < x < b, \\ y(a) - A_1 = y(b) - A_2 = y'(a) - B_1 = y'(b) - B_2 = 0, \end{cases}$$

via quintic and sextic spline functions respectively. The boundary value problem of this type usually arises in plate deflection theory. It is also noted that this differential system has a unique solution provided

$$\inf_x p(x) = -\eta > -500.564/(b-a)^4.$$

In three typical numerical examples, the results are briefly summarized to demonstrate the practical usefulness of our methods. (Received October 2, 1978.)

763-65-5 IAN H. SLOAN, University of Maryland, College Park, Md. 20742, and University of New South Wales, Australia. Fredholm equations of the second kind by product integration via polynomial interpolation. Preliminary report.

Consider the second-kind Fredholm equation  $y(t) = f(t) + \int_{-1}^1 h(t,s)m(t,s)y(s)ds$ , where  $y$  and  $f$  are continuous,  $h$  is weakly singular but of a standard type, for example  $h(t,s) = |t-s|^\alpha$ ,  $\alpha > -1$ , and  $m$  is continuous (and preferably smoother). In Atkinson's product-integration method,  $m(t,s)y(s)$  is approximated by piecewise-linear or piecewise-quadratic interpolation with respect to  $s$ , after which the integration with respect to  $s$  is performed exactly. Here we replace piecewise-polynomial interpolation by polynomial interpolation at the classical Chebyshev points. Despite the well known difficulties introduced by polynomial interpolation, we show, using arguments based on the mean convergence of interpolating polynomials, that a completely satisfactory convergence theory exists. Numerical examples suggest that the method may have a useful role in practice. (Received October 16, 1978.) (Author introduced by Professor M. Z. Nashed).

763-65-6 Gilbert N. Lewis, Michigan Technological University, Houghton, Michigan, 49931. Predicting resonance in turning point problems by numerical methods. Preliminary report.

Olver's sufficiency condition for resonance in the solution of the singularly perturbed turning point problem

$\epsilon y'' + f(x,\epsilon) y' + g(x,\epsilon)y = 0$ ,  $y(a) = A$ ,  $y(b) = B$ , where  $f(x,\epsilon) = 0$  for some  $x \in [a,b]$ , and  $\epsilon > 0$  is "small", is not applicable in certain common examples. For instance,  $\epsilon y'' - (a+\epsilon)x y' + (b+\epsilon cx + dx^2)y = 0$  and  $\epsilon y'' - x(1+x^2)y' + ky = 0$  are two examples of equations to which the condition does not apply. In the first example, certain combinations of the different constants will definitely yield resonance, while others will not. In the second, it is not known whether any value of the constant,  $k$  (except 0), will yield resonance. Some numerical results which predict whether or not resonance will occur will be presented. (Received October 19, 1978.)

\*763-65-7 Luis Kramarz, Emory University, Atlanta, Georgia 30322. A two-mesh collocation method for nonlinear two-point boundary value problems. Preliminary report.

The collocation solution of a nonlinear two-point boundary value problem using piecewise polynomials on a fine mesh is obtained with an iterative scheme, in which each step consists of solving two linear problems by collocation. The first problem is on the fine mesh but requires no Jacobian evaluations or inversions; the second is on an auxiliary fixed coarser mesh. The scheme can be particularly useful if Jacobians are not reasonably easy to obtain and/or if storage becomes critical. (Received October 23, 1978.)

763-65-8 RICHARD C. MACCAMY, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213 and SAMUEL P. MARIN, General Motors Research Center, Warren, Michigan 48090. Variational Procedures for a Class of Diffraction Problems. Preliminary Report.

A numerical procedure for the approximate solution of some two dimensional diffraction problems is developed. The basic idea is to reduce problems involving scattering of waves by obstacles in an infinite region to variational problems over a finite region. This is accomplished by the introduction of non-local boundary conditions for the finite region. Effective numerical procedures have been developed and convergence results established. Numerical experiments confirm the theoretical convergence rates. (Received October 23, 1978.)

\*763-65-9 F. H. Mathis, Auburn University, Auburn, Alabama 36830, and G. W. Reddien, Vanderbilt University, Nashville, Tennessee 37235. Convergence of the Ritz-Treffitz Method for Optimal Control Problems.

We consider a Ritz-Treffitz procedure to approximate solutions of the optimal control problem: minimize the cost function  $J(u) = \frac{1}{2} \int_a^b [x(t)^T M x(t) + u(t)^T N u(t)] dt$ , where  $x$  solves the linear differential equation,  $\dot{x}(t) = Ax(t) + Bu(t)$ ,  $x(a) = x_0$ . We take our approximations in spaces of piecewise polynomials and establish optimal-order  $L^2$  error estimates for both the control and cost functions. (Received October 24, 1978.)

\*763-65-10 LEROY J. DERR\*, CURTIS L. OUTLAW, and DIRAN SARAFYAN, University of New Orleans, New Orleans, Louisiana 70122. Derivation of Algebraic Equations Associated with Runge-Kutta Formulas Relative to Systems of Ordinary Differential Equations.

Consider the differential equation  $dy/dx = f(x,y)$  subject to the initial condition  $y(x_0) = y_0$ . In order to obtain Runge-Kutta formulas for its approximate solution, first a system of algebraic equations is derived. The solutions of this system then provides the constants which determines the Runge-Kutta formulas. In 1956, A. Huta, (see Acta Facultatis Rerum Naturalium Universitatis Comenianae, Tom. 1, Fasc. IV-VI, pp. 201-224), facilitated the derivation of the algebraic system. However Huta's method is valid only for a single equation. It is the aim of this paper to extend Huta's method to the vector case, that is, to systems of ordinary differential equations. (Received October 25, 1978.)

763-65-11 CHARLES S. PESKIN, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012. The heart valve problem of cardiac fluid dynamics and its numerical solution.

• The heart valve leaflets are thin, flexible, elastic membranes with essentially zero mass. The leaflets move at the local fluid velocity and, at the same time, exert forces on the fluid that alter the fluid motion. In particular, the closed valve prevents backflow and the open valve forms vortices that participate in valve closure. Accordingly, the heart valve problem involves an immersed boundary that interacts with the fluid. The motion of this boundary is not known in advance. In this talk, we shall discuss the mathematical formulation of this problem and its numerical solution. Computational results will be presented in the form of a computer-generated movie. Some related problems of cardiac physiology will be briefly discussed. In the formulation of the heart valve problem, the boundary forces play a critical role. These can be expressed as a distribution with support along the immersed

boundary. The boundary forces are determined from the boundary configuration, which has the equation of motion that the boundary points move at the local fluid velocity. In this formulation the boundary appears as a place in the fluid where extra forces are applied. The numerical solution of the heart valve problem involves a fixed grid for the fluid, a moving collection of points for the boundary, and a numerical representation of the  $\delta$ -function that is used to connect the boundary with the fluid. Specific rules for the computation of the boundary forces in the case of natural valves, artificial valves, and heart muscle have been developed. To achieve numerical stability, the boundary forces are not calculated explicitly. Instead, a certain minimization problem is solved at each time step. The computational results (in the form of a computer-generated movie) illustrate how the method can be used in the design of artificial valves. The flow patterns and motions of the mitral valve in a contractile cardiac chamber are computed and displayed. The film involves the natural mitral valve (normal and pathological behavior) and also several examples of artificial valves. (Received October 25, 1978.)

## 68 ► Computer Science

\*763-68-1 VLADIMIR LIFSCHITZ, Brigham Young University, Provo, Utah 84602, and LEON PESOTCHINSKY, University of California, Santa Barbara, California 93106. The Analysis of a Partition Algorithm

We analyze a backtrack algorithm which generates, for any given integers  $x_1, \dots, x_N, M$ , all sets  $\Delta \subset \{1, \dots, N\}$  such that  $M = \sum_{i \in \Delta} x_i$ . For  $x_i$  taken at random in an initial segment of the positive integers, the computation time is shown to be  $O(e^{\gamma N})$ ,  $\gamma < 1$ , with a probability close to 1 for large  $N$ , provided  $M$  is small in comparison with  $x_1 + \dots + x_N$ . This bound is essentially smaller than the computing time of the trivial algorithm based on the testing of all  $2^N$  sets  $\Delta$ . The result shows how the probabilistic approach to the analysis of algorithms can be used to describe theoretically the relative efficiency of backtrack algorithms, well-known from computational practice. (Received October 23, 1978.)

\*763-68-2 JOHN SADOWSKY, U.S. Bureau of the Census, Washington, D.C. 20233. Addresses in Computer Data Files Which Are Ordered as Lattices. Preliminary report

In several applications at the U.S. Bureau of the Census, it is desirable to structure large data sets with the ordering of a lattice. Unfortunately, the computation of certain lattice theoretic structures, mainly the join (l.u.b.) and meet (g.l.b.) would require a great amount of costly computer I/O. This problem has motivated a search for an addressing method in which the addresses of the meet and join could be computed arithmetically from the addresses of the defining elements.

In developing an addressing algorithm, the necessary and sufficient conditions for a lattice to be a sublattice of the power set lattice were rediscovered. This addressing algorithm is presented and problems with the size of the addresses are discussed. A "paging" problem is posed, which attempts to minimize the size of the addresses. (Received October 23, 1978.)

## 76 ► Fluid Mechanics

\*763-76-1 RICHARD P. KENDALL, Exxon Production Research Co., Box 2189, Houston, TX 77001 and JIM DOUGLAS, JR., University of Chicago, Chicago, IL 60637, MARY F. WHEELER and BRUCE L. DARLOW, Rice University, Houston, TX 77001. Self-Adaptive Galerkin Methods for One Dimensional Miscible and Immiscible Displacements. Preliminary Report.

Several Galerkin methods employing self-adaptive mesh modification are discussed for approximating the solutions of miscible and immiscible displacement problems in a single space variable. In the case of miscible displacement the transport term is taken so as to dominate the diffusion term, and in the immiscible case capillarity is ignored so that the solution is discontinuous. Thus, in both cases the solution varies very rapidly in a small, moving portion of the domain.

The Galerkin methods for the immiscible problem are based on the weak formulation given by

$$(s_t, v) - (f(s), v) + (s, v) \Big|_0^1 + \epsilon (g(s, s_x) s_x, v) - \epsilon g(s, s_x) s_x v \Big|_0^1 = 0$$

for test functions  $v(x)$ , where the terms involving  $\epsilon g$  represent an artificial diffusivity that is

required to enforce the entropy condition that selects the physical solution out of the infinite set of solutions of the conservation law  $s_x + f(s)_x = 0$ . Two conceptually different Galerkin methods have been studied. The first is based on the use of continuous, piecewise-polynomial functions for trial and test spaces. The second method employs discontinuous, piecewise-polynomial spaces, and it is necessary to modify the equation giving the weak formulation by adding terms that penalize the jumps in the solution and the jumps in the space derivative of the solution across the nodes. In both methods a method of characteristics is used to approximate the location of the front and thereby to allow the self-adaptive modification of the mesh.

The same concepts are applied to the miscible problem. (Received September 11, 1978.)

\*763-76-2 LOKENATH DEBNATH, Math Department, East Carolina University, Greenville, North Carolina 27834 U.S.A. and UMA BASU, Applied Math Department, University of Calcutta, Calcutta, India. Capillary-gravity waves against a vertical cliff.

The initial value problem of capillary-gravity waves generated by an oscillatory pressure distribution of given frequency  $\omega$  against a vertical cliff is investigated. The problem is treated with the aid of generalized functions (distributions). The ultimate steady-state solutions for the capillary-gravity waves is explicitly determined with physical significance. A comparison of the present solution with that in water unbounded in both horizontal directions reveals a very interesting conclusion about the waves produced by a vertical cliff. The steady-state solution consists of two progressive waves, and the wave propagating towards the vertical cliff is reflected back on reaching it. The reflection is physically realistic because the wave moving toward the cliff carries some energy with it, and there is no mechanism to absorb the incoming wave energy. The wave must be reflected back. Several limiting cases of interest are discussed. (Received September 28, 1978.)

## 81 ► Quantum Mechanics

763-81-1 SHIGEMITSU KADEKAWA, Indiana University, Bloomington, Indiana 47401. Asymptotic behavior of solutions for the nonlinear Schrödinger equation. Preliminary report.

The time decay of solutions for the nonlinear Schrödinger equation  $i u_t - \Delta u + g|u|^{p-1}u = 0$  with  $u(x,0) = u_0(x)$ ,  $p > 1$  is shown to satisfy  $\|u(t)\|_{p+1} \leq \text{const}(1+t)^{-\frac{d(p-1)}{2(p+1)}}$  by combining Ginibre-Velo's pseudo-conformal invariance and a version of Gronwall's inequality. It is also shown that, for the  $p > 2$  case, this decay is suitable for the scattering in  $L^2(\mathbb{R}^3)$  norm. (Received October 25, 1978.) (Author introduced by R. Glassey).

## 83 ► Relativity

\*763-83-1 John K. Beem\* and Paul E. Ehrlich, University of Missouri, Columbia, Missouri 65211. Lorentzian distance and Lorentzian geometry.

Recent results using the Lorentzian distance function to construct maximal geodesics will be described. In globally hyperbolic space-times, the existence of maximal nonspacelike rays, the timelike diameter and the nonspacelike cut locus will be considered. Furthermore, sufficient conditions for a space-time to be causally disconnected will be given. (Received September 8, 1978.)

763-83-2 PHILLIP E. PARKER, Syracuse University, Syracuse, New York 13210. Some new directions in relativity.

This will be a survey talk providing a coherent theoretical framework for certain recent work. The methods discussed include Hironaka's resolution of singularities, distributional (compared with differential) geometry, and some drawn from spectral geometry. In addition, the work of Michor on manifolds of smooth maps will be briefly described. (Received September 13, 1978.)

\*763-83-3 MURRAY CANTOR, The University of Texas, Austin, Texas 78712. Recent progress in the existence of asymptotically flat spacetimes.

One systematic approach for constructing solutions to the Einstein field equations is found in the initial value formulation. Using the Lichnerowicz-York conformal methods and weighted function

spaces, there has been substantial progress in showing the existence of asymptotically flat solutions for open spacetimes.

This talk will be expository in nature and will summarize recent results and discuss some open questions. (Received September 15, 1978.)

\*763-83-4 JUDITH M. ARMS, University of Utah, Salt Lake City, Utah 84112. Coupled gravitational and gauge fields.

Let  $S$  be a spacetime with compact Cauchy surface  $M$ , and  $g$  the (Lorentz) metric on  $S$ . The gauge field is represented as a principal fiber bundle over  $S$  with connection  $\omega$ . Consider the set of solutions to the constraint equations on  $M$  for the gravitational and gauge fields. The implicit function theorem may be used to show that this set is a manifold at most points. The obstruction to applying the implicit function theorem is the set of infinitesimal symmetries of the fields: vector fields on the bundle whose flows preserve  $\omega$  and the pullback of  $g$  to the bundle. This identification is achieved by relating vector fields on the bundle to the Lagrange multipliers in the variational principle which gives rise to the field equations. When the implicit function theorem fails, these multipliers give the components of a vector field restricted to  $M$  which can be extended to a symmetry on the entire bundle. Conversely, a vector field which preserves  $\omega$  and  $g$  must be invariant under the group action and thus gives rise to values for the Lagrange multipliers which obstruct the application of the theorem. (Received October 2, 1978.)

\*763-83-5 FRANK J. TIPLER, University of California, Berkeley, California 94720. On the non-existence of time periodic and almost time periodic spacetimes.

I show that physically realistic closed universes cannot be either time periodic or almost time periodic. A spacetime  $(M,g)$  is said to be time periodic if there exist disjoint Cauchy surfaces  $S_1, S_2$  such that the initial data on  $S_1$  is isometric to the initial data on  $S_2$ . Theorem. If  $(M,g)$  contains compact Cauchy surfaces and satisfies both the generic and the timelike convergence conditions, then  $(M,g)$  is not time periodic. Furthermore, if in addition the matter fields  $\Psi$  and their first derivatives  $\Psi'$  obey the restrictions on page 254 of Hawking & Ellis, then for any neighborhood  $U$  of any Cauchy surface  $S_1$ , there exists a number  $\epsilon > 0$  such that  $\|(h, \chi, \Psi, \Psi') - (h_1, \chi_1, \Psi_1, \Psi'_1)\|_{5+a} > \epsilon$  for the initial data on any Cauchy surface  $S$  with  $U \cap S$  empty, where  $\| \cdot \|_{5+a}$  is the Sobolev norm (cf. page 234 of Hawking & Ellis). (Received October 2, 1978.) (Author introduced by Professor Paul Ehrlich).

763-83-6 A. H. TAU $\bar{U}$ , University of California, Berkeley, CA 94720. Accelerating Masses in Robertson-Walker Space-times

In this paper it is shown that if a metric tensor  $\hat{g}$  is of the form  $\hat{g} = g + 2H \ell \otimes \ell$  where  $g$  is a Lorentzian metric tensor,  $H$  is a scalar function and  $\ell$  is a null, geodesic, shear free vector field in the space-time with metric  $g$ , then  $\ell$  is a null, geodesic, shear free vector field in the space-time with metric  $\hat{g}$ . Further the Einstein tensor computed from  $\hat{g}$  when written in mixed form, that is with one contravariant and one covariant index is a linear function of  $H$ . These results are used to generalize the notion of a Kerr-Schild space-time. In particular they provide a generalization of W. Kinnersley's (1) description of the gravitational field of an accelerating mass in Minkowski space where  $g$  is the metric of a flat space-time. A description of the gravitational field of an accelerating point mass in a Robertson-Walker space-time is obtained by setting  $g$  to be the metric tensor of such a space-time. Since this space-time is conformally flat, Minkowski space still plays an important role in the discussion and the values obtained for  $H$  and  $\ell$  are simply related to those given by Kinnersley. When  $H$  and  $\ell$  are taken to be related to those corresponding to the Kerr metric, Vaidya's (2) results describing the Kerr metric in a cosmological background are obtained. (1) W. Kinnersley: Physical Review 186, 1335-1336 (1969)  
(2) P. C. Vaidya: Pramāna, 8, 512-517 (1977). (Received October 5, 1978.)



763-83-7 JERROLD E. MARSDEN, University of California, Berkeley, Berkeley, California 94720.

Supergravity for mathematicians. Preliminary report.

We shall report on recent clarifications of just what classical (i.e. nonquantum) supergravity is, following Kostant, Sternberg, Zumino and especially Singer. Spacetime is replaced by a graded algebra built on the Dirac spin bundle over spacetime. We shall also report on Grisar's attempt to prove positivity of mass (recently proved by Schoen and Yau) by using classical supergravity to express the gravitational mass as a sum of squares. (Received October 13, 1978.)

763-83-8 ABRAHAM H. TAUB, University of California, Berkeley, California 94720. Space-times with distribution valued curvature tensors.

• This paper considers problems in general relativity for which the sources of gravitational fields are described by distribution valued stress-energy tensors. In view of the Einstein field equations, this means that we should deal with space-times whose Ricci tensors are distribution valued. Since the conformal tensor (the Weyl tensor) is related to the Ricci tensor by means of the Bianchi identities one must expect that the entire Riemann-Christoffel curvature tensor should be distribution valued. Such a curvature tensor will arise if one deals with a space-time in which in an admissible coordinate system the metric tensor is continuous but has discontinuous first and second derivatives where the discontinuities occur across submanifolds of three, two or one dimension or even at a single event. The theory of a gravitational shock wave is one in which there is a hypersurface  $\Sigma$  across which discontinuities in the first and second derivatives of the metric tensor occur. These are described by two second order tensors  $b_{\mu\nu}$  and  $\hat{b}_{\mu\nu}$  defined on  $\Sigma$ . These quantities may be shown to satisfy algebraic equations involving the vector  $l_{\mu}$ , the normal to  $\Sigma$  and differential equations describing their propagation in the direction of the normal to  $\Sigma$ . Thus one is able to characterize the "singular" hypersurface  $\Sigma$  and describe its development in time. In this case one considers space-time to include a singular region and one studies the behavior of this region. One can treat other space-times with distribution valued curvature tensors in a similar manner and thus discuss the behavior of shock waves, shells of matter, the history of line sources, singular world-lines and singular events. We shall be mainly concerned with space-times whose curvature tensors contain Dirac delta functions with supports on submanifolds or even isolated events. When one recalls that the curvature tensor is linear in the second derivatives of the metric tensor and quadratic in the first derivatives, one sees that for a space-time in which the first derivative of the metric tensor has a finite jump across a submanifold, then its curvature tensor will contain a Dirac delta function with support on the submanifold. The jump in the first derivative is describable by a Heaviside function which will enter the curvature tensor quadratically. Fortunately the product of such distributions is quite tractable. (Received October 23, 1978.)

\*763-83-9 J. G. MILLER, Texas A&M University, College Station, Texas 77843. Pseudospherically symmetric tachyon fluids.

It has been suggested that there exists a pseudospherically tachyon analog of the spherically symmetric Schwarzschild solution in general relativity. It is shown that no such solution exists with zero pressure at the boundaries of the fluid. (Received October 23, 1978.)

\*763-83-10 DAVID LERNER, Department of Mathematics, University of Kansas, Lawrence, Kansas 66045. Self-dual solutions to Einstein's equations.

We discuss a general methods for constructing self-dual solutions to Einstein's equations due to Penrose and give several illustrative examples. (Received October 25, 1978.) (Author introduced by Professor Paul Ehrlich).

## 90 ► Economics, Operations Research, Programming, Games

\*763-90-1 ALEXANDER J. FEDEROWICZ, Westinghouse R&D Center, Pittsburgh, Pennsylvania 15235. Asymptotic solutions to engineering design problems.

A question of practical interest in the design of many electro-mechanical devices (transformers, motors, generators, etc.) is whether there exists an economy of scale as the device rating increases (i.e. Cost = K Rating $^{\alpha}$ ,  $\alpha < 1$ ). If a strong economy of scale exists then economics support the

design and construction of increasingly large and presumably centralized units. Conversely if  $\alpha > 1$  then smaller individual units are less costly per unit of capacity.

This question was posed to the author in conjunction with the design of transformers. By using the duality theory of Geometric Programming it was possible to show that the asymptotic cost behavior can be found by solving a well defined min-max linear programme. For most such problems closed form analytic solutions can be found. (Received September 18, 1978.) (Author introduced by Dr. Robert J. Thompson).

\*763-90-2 BERNIE L. HULME, Sandia Laboratories, Albuquerque, New Mexico 87185. Graph-Theoretic Path Problems Arising in Facility Safeguards Studies. Preliminary report.

One of the subproblems arising in the evaluation of safeguards effectiveness at a nuclear fuel-cycle facility is the determination of good routes for a thief or saboteur within the facility. The problem can be solved by finding shortest paths in an appropriately weighted digraph. A facility digraph has nodes representing the locations of targets and possible penetration points of barriers, such as walls, fences, locked doors, etc., and arcs representing physical paths from one node to another. The arc weights are quantities to be minimized by an adversary, typically delay time and/or detecting probability. The talk will focus on: (a) the various types of adversaries and their associated path problems; (b) possible objective functions for an adversary and how these affect the solution process; and (c) basic pathfinding algorithms that may be employed and how they compare in efficiency when applied to sparse digraphs. (Received September 18, 1978.) (Author introduced by Dr. Robert J. Thompson).

763-90-3 WILLIAM H. RUCKLE, Clemson University, Clemson, South Carolina 29631. On the constructability of solutions to two person games.

Let  $B$  consist of all subsets of  $[0,1]$  with at most  $k$  points and let  $R$  consist of all subsets of  $[0,1]$  with total length at least  $a < 1$ . Consider the following two games  $G$  and  $H$  in which the opponents BLUE and RED choose  $B$  from  $B$  and  $R$  from  $R$  respectively. In the game  $G$  the payoff to RED is 1 if  $B \cap R$  is empty and 0 otherwise. In the game  $H$  the payoff to BLUE is 1 if  $B \cap R$  is empty and 0 otherwise. A reasonable solution can be found to  $G$  but not to  $H$  unless  $k = 1$ . Use of the Axiom of Choice leads to paradoxical solutions to  $H$ . We define the concept of constructable solution and prove  $H$  has no constructable solution. (Received September 21, 1978.)

\*763-90-4 T.E.S. RAGHAVAN, University of Illinois at Chicago, Chicago, Illinois 60680. Completely mixed games and M. matrices.

**Any non-singular  $M$ . matrix is a completely mixed matrix game with positive value. This property is used to give game - theoretic proofs of several well-known characterizations of such matrices. The same methods also yield many theorems on  $S_0$  - irreducible matrices.** (Received September 22, 1978.) (Author introduced by Professor William Lucas).

\*763-90-5 T. PARTHSARATHY, University of Kansas, Lawrence, Kansas 66045. Markov games. Preliminary report.

The concept of (finite) Markov games or stochastic games were first introduced by Shapley in 1953. Around the same time Gillette also considered Markov games in extensive form in his Ph.D. dissertation. Since then scores of papers on Markov games have appeared in the literature. Two types of pay-offs namely discounted and undiscounted (or limiting average pay-off) have been considered. Results in the discounted case are complete whereas the results in the undiscounted case are far from complete. In this report we briefly review the current status of the problem under consideration. (Received September 25, 1978.) (Author introduced by Professor William Lucas).

\*763-90-6 PHILIP D. STRAFFIN JR., Department of Mathematics, Beloit College, Beloit, Wisconsin 53511 and JAMES P. HEANEY, Department of Environmental Engineering, University of Florida, Gainesville, Florida 32611. Game Theory and the Tennessee Valley Authority.

Problems of fair allocation of joint costs were considered by the TVA in the 1930's, in relation to apportioning costs of dam systems among participatory uses. Methods proposed to solve this problem foreshadowed various game theory solution concepts including the core, a special case of the nucleolus, and the imputation which minimizes the maximum propensity to disrupt. A method equivalent to the latter, but with a different rationale, is now in standard use among water resource engineers. (Received September 27, 1978.)

763-90-7 JAMES CASE, Federal Trade Commission, Washington, D.C. 20580. A game of N buyers in comparative shopping. Preliminary report.

When essentially identical items are available in a particular market at widely varying prices, as is true of eye-glasses or insurance policies, for instance, prospective buyers must engage in comparative shopping. Two questions thus present themselves, namely

- (i) what is the form of the optimal shopping policy, and
- (ii) what distribution of prices should be observed in the market.

To answer both requires the solution of a game between the N buyers and on sellers which is both dynamic and (at least partially) cooperative. Only partial answers for the two questions are presently available. (Received September 27, 1978.)

\*763-90-8 GUILLERMO OWEN, Instituto SER, Calle 19A, 4-19E, Bogota, Colombia and NANCY JOHNSON, Jones School of Administration, Rice University, Houston, TX 77001. A game-theoretic approach to a problem of cost allocation.

We consider the problem of allocating the cost of a common facility among several (n) activities. Three game-theoretic concepts are considered: the core, the nucleolus, and the value. It is shown that the nucleolus can be easily computed by an algorithm previously developed for (n, n-1) games, i.e., games in which any coalition with n-2 or fewer players is totally defeated. (Received September 27, 1978.) (Author introduced by Professor William Lucas).

763-90-9 Edward Azoff, University of Georgia, Athens, GA. 30602 and Charles Bird, General Motors Research Laboratories, Warren, MI. 48090. On the formation of coalitions in infinite player games.

This paper investigates the problem of determining when an infinite player game can be partitioned into countably many stable coalitions. An example is given of a superadditive scalar measure game for which this is not possible and several positive results are presented. The final section of the paper takes up the problem of when a process of successive coalition formation can eventually lead to division of all available resources. A theorem and a counterexample are presented for countable player games and the positive result is applied to the vector measure games considered earlier in the paper. The resulting solution, called the core-stem solution, is shown to exist for all continuous vector measure games which are differentiable at the origin and superadditive. (Received September 28, 1978.)

763-90-10 PRAKASH P. SHENOY and P. L. YU, University of Kansas, Lawrence, Kansas 66045. A reframing of two person non-zero-sum games. Preliminary report.

More realistic situations that can be modelled as a game involve partial cooperation and partial conflict. Even in situations where the players are not allowed to communicate directly with each other (e.g. due to antitrust laws in industry), the players can signal to each other by their choice patterns on previous plays in repeated plays of a game. For such situations, a one stage two person game model is not adequate. Here, we shall formulate this game in such a way that a player's strategy is a function of the observed strategy used by the opponent on previous plays of the game. We emphasize the dynamic aspects of these situations by using some concepts from stochastic processes and study some stability concepts to describe the optimal behavior of the players. (Received October 3, 1978.) (Authors introduced by Professor William Lucas).

\*763-90-11 Edward W. Packel, Lake Forest College, Lake Forest, Illinois 60045. A Stochastic Solution Concept for n-Person Games.

Let  $X$  be a set of outcomes among which a set of  $N$  players, each having a preference relation on  $X$ , must choose. Let  $v: 2^N \rightarrow 2^A$  be a game in generalized characteristic function form, where  $v(C)$  denotes the set of outcomes that a coalition  $C$  can guarantee its members regardless of actions by players outside of  $C$ . By letting  $c(x,y)$  denote the number of minimal coalitions via which  $y$  is directly accessible from  $x$ , a Markov chain model of outcome selection is developed. By establishing convergence results for both finite and spatial ( $X \subseteq R^m$ ) outcome cases, a probability distribution on  $X$  is obtained and referred to as the stochastic solution of the game. After presenting some initial results on the dependence of the stochastic solution on the initial outcome distribution, it is shown under reasonable assumptions that a (strong) core, if it exists, must occur with probability one. Generally, the results obtained have natural interpretations and proofs using the language and theory of Markov chains. Finally, some examples and previous experimental results are considered in terms of the model. Stochastic solution values obtained and their agreement with available experimental values appear to be very encouraging. (Received October 4, 1978.) (Author introduced by Ivan Filippenko).

\*763-90-12 HARVEY R. DIAMOND, West Virginia University, Morgantown, WV 26506. Asymptotic Equilibria in a Class of Symmetric N-Person Games on  $[0, T]$ .

We consider the asymptotic calculation, for large  $N$ , of the symmetric equilibrium in the following game:  $N$  players each independently choose a  $t \in [0, T]$ . Let  $t_1 < t_2 < \dots < t_N$  be the choices. Define  $t_0 = 0$ . Then the player who chose  $t_i$  receives the payoff  $G(t_i - t_{i-1})$  where  $G$  is monotone increasing, differentiable, and  $G(0) = 0$ . The equilibrium probability distribution  $F(s) = \Pr(t \leq s)$  then satisfies the following: For some  $a > 0$ ,  $F(a) = 0$ ;  $F(T) = 1$  and

$$G(a) = [1 - F(t+a)]^{N-1} [G(t+a) - G(t)] + \int_0^t [1 - F(t+a) + F(t+a-s)]^{N-1} dG(s) \quad 0 \leq t \leq T-a$$

The equilibrium payoff is  $G(a)$ .

Uniform asymptotic approximations are developed for the cases  $G(t) = t$  and  $G(t) \sim \sum_0^n b_n t^{r+n}$ ,  $r \geq 0$ , with  $T = 1$ ; and in general to first order for  $T = N$ . The solutions have the properties that  $a = 0(T/N)$ ;  $F(t+a)$  exhibits boundary layer behavior for  $t = 0(T/N)$ ; outside the boundary layer  $F$  is uniform to first order. The approximations are obtained by developing inner and outer expansions for  $F$ , performing an asymptotic matching, and applying  $F(T) = 1$ .

Some pseudo-applications are included. (Received October 4, 1978.)

\*763-90-13 ROBERT JAMES WEBER, Yale University, New Haven, Connecticut 06520. Recent results on the valuation of games.

Much attention has been given to methods for measuring the "value" of playing a particular role in an  $n$ -person game. Some recent work has involved the study of semivalues (symmetric positive linear operators on a space of games, which leave additive games fixed). We will discuss both axiomatic and noncooperative approaches to valuation, along with applications to games with a large number of players. (Received October 5, 1978.)

763-90-14 D. MARC KILGOUR, Wilfrid Laurier University, Waterloo, Ontario, Canada, N2L 3C5. Equilibrium Points of Infinite Stationary Truels.

Stationary Truels are a class of truels including both sequential and random truels. A stationary truel is described by its (time-independent) marksmanship functions, which incorporate not only the players' abilities but also the firing rules of the game. The equilibrium point structure of Infinite Stationary Truels is studied in the case when the marksmanship functions are multilinear and satisfy certain other restrictions, when the players' duel values are preassigned parameters, and when the players are restricted to stationary strategies. The equilibrium point structure of Infinite Stationary Truels is shown to resemble that of Infinite Sequential Truels. (Received October 5, 1978.)

\*763-90-15 GABRIEL J. TURBAY, Drake University, Des Moines, Iowa 50311. A Duality Analysis of Balanced Sets.

Balanced collections are studied in the context of two different types of duality relations. The first type of duality which has been used in a different context to develop a theory of solutions for  $n$ -person cooperative games, is used here to construct prespecified power structures. The second type of analysis reveals properties that relate balanced sets and their complements. (Received October 6, 1978.) (Author introduced by Professor William Lucas).

763-90-16 JIM A. FLANIGAN, University of California, Los Angeles, California 90024. Selective and shortened selective sums of loopy partizan games. Preliminary report.

In Chapter 14 of *On Numbers and Games*, J. H. Conway provides a simple theory for selective and shortened selective sums of impartial games whose graphs are trees. In *Partizan and Impartial Combinatorial Games* (an unpublished paper, 1976) R. K. Guy offers theories for selective and shortened selective sums of partizan (unimpartial) games whose graphs are trees. We provide a theory for selective and shortened selective sums of partizan games whose graphs may contain repetitive cycles of moves, or loops. Plays which last indefinitely are considered draws. Our analysis of a selective sum is accomplished by considering its "offside" and its "onside". In each case, a certain type of stopping value and suspense number are employed. To compute the Left and Right offside and onside stopping values of a position, we locate its four Dedekind sections as indicated in a paper of S-Y. R. Li, Sums of Zuchswang Games [Journal of Combinatorial Theory (A) 21, 52-67 (1967)]. The theory for shortened selective sums utilizes a certain type of remoteness number. (Received October 23, 1978.)

763-90-17 Bruce R. Ebanks, Department of Mathematics, Texas Tech University, Lubbock, Texas 79409. A Condition for a Utility Function to be Separable.

Beckmann and Funke (Z. Operations Research 22 (1978), 1-11) have proposed a household utility function parametrized by "taste" or "attractiveness" variables. It is of the form  $u = u(a, x)$ , where  $x = (x_1, x_2, \dots, x_n)$  is a vector of product quantities and  $a = (a_1, a_2, \dots, a_n)$  a vector of product "attractions." In practice, however, the only utility functions considered are the separable ones,  $u = \sum_{i=1}^n \phi(a_i, x_i)$ . In an effort to find conditions under which a utility function will be separable, we characterize utility functions which satisfy a property analogous to the branching property in information theory. When these functions are also symmetric under permutations of the pairs  $(a_i, x_i)$ , they are separable. (Received October 23, 1978.)

## 92 ► *Biology and Behavioral Sciences*

\*763-92-1 S. P. Diaz and M. Humi, Worcester Polytechnic Institute, Worcester, Massachusetts 01609. Principles and Quantitative Measurements of Visual Illusions.

Hoffman's principle postulates the interaction of two (visual) Lie generators as the cause that gives rise to a visual illusion. From a physiological point of view this implies interference between two specialized visual centers in the brain. The lecture will examine this possible explanation of the Zöllner and other illusions from this point of view. (Received September 21, 1978.)

763-92-2 JOHN PAUL BOYD, University of California, Irvine, California 92717. Theory and data as adjoints in a topos.

It is argued that fuzzy sets can be modeled as a topos, which is the type of category proposed as a generalization of the category of sets and functions. In these categories of fuzzy sets, one can define deductive systems where all the theorems of intuitionistic mathematics are true. This is illustrated for the case of several simple theorems in measurement and clustering using the concept of data and theory as adjoint pairs. (Received September 21, 1978.) (Author introduced by Professor William C. Hoffman).

763-92-3 THADDEUS M. COWAN, Kansas State University, Manhattan, Kansas 66506. Topological Analysis of Impossible Figures.

In the study of impossible torus objects the two problems most extensively investigated concern: (1) the algorithm for identifying (and generating) impossible objects and (2) the conceptual cues responsible for the puzzling effect these figures have on the perceiver. The first of these problems is a formal one and two approaches, one using graph theory (Huffman) the other braid theory (Cowan) will be described and compared. The second problem has been attacked by Cowan and Pringle and

independently by Draper. These two papers represented independent discoveries of the fact that net levels of depth experienced with a single scan around a torus is directly related to judgments of impossibility. This is not the only cue, however. Cowan and Pringle describe a set of figures that look impossible even though the measure of net depth is zero, and Draper discusses the importance of "axis systems" (essentially axes with different orientations). It will be shown how the formal braid analysis of these figures can be used to describe and supplement this property of different axis systems. (Received September 21, 1978.) (Author introduced by Professor William C. Hoffman).

763-92-4 WILLIAM C. HOFFMAN, Oakland University, Rochester, Michigan 48063. The Simplicial Category as a Model for Cognitive Information Processing.

The visual field of view constitutes a manifold  $M$  over which form recognition and the action of the psychological constancies (size, shape, motion-invariant perception, etc.) are defined. The result, at the cortical level, is an equivariant fibration, the conformal group and its prolongations comprising the Lie group of symmetries acting. We postulate that a functorial map exists between the associated category of fibre bundles and the simplicial category that describes cognitive information processing. Higher faculties then come about as functorial maps to the category of simplicial objects. Some of the relations of this model to Piaget-Bruner developmental psychology have previously been explored. Here its connection with information-processing psychology is developed and possible neuropsychological correlates suggested. (Received September 21, 1978.)

\*763-92-5 Manfred Kochen, University of Michigan, Ann Arbor, Michigan 48104. Induction in Adaptive Learning.

Advances in knowledge, both in an individual and in a community, depend on what is known, how it is represented and organized and the dynamics for going beyond what is known. For the mathematical community, the problem is to explain how it continues to advance mathematics when the estimated 200,000 theorems per year that are contributed will be beyond anyone's grasp in their entirety well enough to judge what is important. At an individual level, the problem is to explain how the human mind has invented such concepts as "infinity" or "torus" and discovered "laws" such as Gödel's incompleteness theorem or Maxwell's equations.

We present a mathematical model for the formation of certain kinds of concepts, conjectures and ways of organizing (e.g., mathematical) knowledge into quality-oriented maps by which inquiring systems can steer themselves so as to adapt increasingly well to their environment. We analyze the model with the partial help of a computer, to clarify what induction requires beyond the abstract machinery of evaluating functionals in  $\Sigma_i$ ,  $i=0,1,2,3$  of the Kleene Arithmetic Hierarchy. The model may connect ideas of the brain of an inquiring system described by the continuous mathematics of Lie groups and the brain as a logical symbol processor. (Received September 21, 1978.)

763-92-6 HAROLD R. LINDMAN, Indiana University, Bloomington, Indiana 47401. A non-Euclidean model of visual illusions. Preliminary report.

Geometric visual illusions demonstrate that the metric of visual space is affected by the figures in it. A model analogous to the general theory of relativity therefore has potential for explaining at least some aspects of these illusions. A model is proposed in which the metric at any point is determined by the combined effects of all figures in the visual field. The effect of any small portion of the figure depends on the angle and distance from the point. Preliminary data fits to the Muller-Lyer illusion appear promising. (Received September 21, 1978.) (Author introduced by Professor William C. Hoffman).

763-92-7 ANATOL RAPPOPORT, University of Toronto, Toronto, Canada M5S 1A1. Separating and Assessing Motivation Pressures in 2 x 2 Experimental Games.

In deciding between the two strategies in a 2 x 2 non-cooperative game played once, a player may use any of several optimization criteria, e.g. dominance, maximization of expected payoff, maximin, Pareto optimality, and the like. Of special psychological interest are 2 x 2 games in

which a player is indifferent between the two strategies with reference to his own payoffs. Thus, motivations generated by considerations of the co-player's payoffs can be "separated out" in these games. Problems in constructing games which separate out components of motivation are considered. Some results on games of this sort are reported. (Received September 21, 1978.)

763-92-8 DAVID A. SMITH, Duke University, Durham, N. C. 27706. Differential models for perception of optical illusions.

The background of differential models of visual perception will be discussed, as will the details of Hoffman's Lie-theoretic model (SIAM Rev. 13, 1971, 169-184) and the author's modification of it (J. Math. Psych. 17, 1978, 64-85). The implications for these models of recent work by Day and Dickinson (British J. Psych. 67, 1976, 537-552), Diaz and Humi ("Principles and quantitative measurements of visual illusions", preprint, 1978), and others will be noted. The presentation will be illustrated with both classical and new illusions. (Received September 21, 1978.)

763-92-9 DONALD G. SAARI, Northwestern University, Evanston, Illinois 60201. A dynamical model for cognitive development. Preliminary report.

The canonical story for cognitive development involves a real world-model world adaptive interaction. In order to investigate the consequences of this story, it is modelled by means of a partially controlled dynamical system where equilibria positions are represented by an attractor while the adjustment procedure is given by the movement toward the attractor. The qualitative dynamical conclusions resulting from this model are then compared with empirical results from short term memory experiments and from the Piagetian theory. (Received September 21, 1978.)

763-92-10 A. S. BLAVAIS, formerly Institute of Higher Nervous Activity, Moscow, USSR. A Model of Color Vision of Trichromatic Mammals Based on a Group-Theoretic Approach.

The physiological and psychological data of trichromatic color vision suggest the possibility of description in terms of a series expansion of the visual image in vector spherical harmonics. After conversion in the primate retina the trichromatic visual image may be expressed in terms of a 3-component vector function  $G(\theta, \vartheta) = (F_G(\theta, \vartheta), F_R(\theta, \vartheta), F_B(\theta, \vartheta))$ , where  $\theta$  and  $\vartheta$  are spherical coordinates on the retina. The series sought may then be expressed as  $G(\theta, \vartheta) = \sum_{LJM} A_{JM}^L Y_{JM}^L(\theta, \vartheta)$ , which constitutes a generalization of an earlier achromatic model. The expansion coefficients  $A_{JM}^L$  are readily interpreted physiologically as three successive nerve impulse trains. The vector spherical harmonics constitute local sensitivity functions for the receptive fields. The three invariants contain both the color and spatial frequency information from the visual image. (Received September 27, 1978.) (Author introduced by Professor William C. Hoffman).

763-92-11 WILLIAM F. LUCAS, Cornell University, Ithaca, New York, 14853. On what others receive in a fair division. Preliminary report.

Several fair division schemes have been proposed for dividing a set of divisible or indivisible objects among several participants which guarantee, under reasonable assumptions, that each one will receive what he himself considers a fair share. In practice, however, one's personal judgement about equity is often upset after the division because of what was awarded to others. Methods for making fair divisions which include one's evaluation for what others might obtain will be presented. (Received October 2, 1978.)

763-92-12 PAUL FIFE, The University of Arizona, Tucson, Arizona 85721. Modeling with reaction-diffusion systems.

Multi-species populations whose individuals migrate in some random fashion and can be created and annihilated by interaction are often modeled by reaction-diffusion systems. A close look is taken at the question of when such models are appropriate, and an attempt is made to formulate minimal sets of assumptions under which that is the case. Approaches via continuous distributions and via colony models are outlined. (Received October 2, 1978.)

763-92-13 DONALD G. ARONSON, University of Minnesota, School of Mathematics, Minneapolis, Minnesota 55455. Density dependent diffusion.

I will discuss nonlinear diffusion processes in which the diffusivity depends on the local density of the diffusing substance. In particular, I will focus my attention on processes in which the diffusivity vanishes with the densities. The classical example is gas flow in a porous medium, but there are also biological examples. Of the latter, I will consider, in some detail, the Gurtin-MacCamy population model and a class of predator-prey systems in which the dispersal of each species is governed by the local density of the other. (Received October 2, 1978.)

763-92-14 WILLIAM H. BATCHELDER, University of California, Irvine, California 92717. Some problem foci in mathematical psychology. Preliminary report.

This paper discusses several problem foci of current interest to mathematical psychology. The emphasis is on posing general problems and describing the sorts of mathematical formalisms that have been brought to bear on them. The problem foci that will be discussed are representing similarity data, probabilistic choice theory, and learning theory. The distinction between problems concerning data reduction and problems of theory formation will be emphasized. (Received October 4, 1978.) (Author introduced by Professor William C. Hoffman).

763-92-15 PETER C. DODWELL, Queen's University at Kingston, Ontario, Canada. K7L 3N6. Mapping the visual field on to the brain. Preliminary report.

The properties of visual space (the space we perceive) differ from those of physical space. One task for psychologists is to describe and, if possible, explain the differences. One of the most fruitful approaches is to ask whether specific geometries of visual space (or space-time) will fit the available psychophysical data on spatial orientation, the constancies, optico-geometrical illusions, apparent movement, etc.

Several lines of evidence suggest that conformal mapping is a general characteristic of visual processing. Examples: the mapping from physical to "Cyclopean" space (Lunenburg), from retina to thalamus to visual cortex (Schwartz); the process of adaptation to optically induced distortions of the normal visual field (Dodwell). It appears that "stabilization" of visual world under quite general transformations induced by, e.g., observer locomotion, can be treated in the same way (Hoffman & Dodwell).

Conformal mapping implies strong local processing constraints, together with the possibility of great diversity of transformations in going from one domain to another, just the characteristics of the visual system. What the mechanisms of transformation may be remains to be determined. (Received October 6, 1978.) (Author introduced by Professor William C. Hoffman).

\*763-92-16 N. D. KAZARINOFF, State University of New York at Buffalo, Buffalo, New York 14214 and PAULINE VAN DEN DRIESSCHE, University of Victoria, B. C., Canada V8W 2Y2. Control of Oscillations in Haematopoiesis.

Results of a mathematical analysis of models of haematopoietic systems introduced by Mackey and Glass are given. The models include a constant time lag, and it is shown that this lag has a critical value above which oscillations in blood cell concentration occur. To reduce the likelihood of disease associated with such oscillations, physiologists should seek to learn how to increase this critical value of the lag. (Received October 13, 1978.)



\*763-92-17 Anthony Leung, University of Cincinnati, Cincinnati, Ohio 45221. Conditions for Global Stability Concerning a Prey-Predator Model with Delay Effects. Preliminary report.

An integrodifferential system concerning prey-predator interaction is studied. By means of appropriately constructed Lyapunov functions, sufficient conditions concerning the lag effects are derived for global stability of the equilibrium. The effects of various forms of continuously distributed delays are examined. (Received October 16, 1978.)

763-92-18 THOMAS G. HALLAM, University of Tennessee, Knoxville, Tennessee 37916. Asymptotic Behavior of Lotka-Volterra Systems.

The asymptotic behavior of Lotka-Volterra models of food webs is discussed from a persistence-extinction perspective. Food chain behavior and structural effects of an interacting species such as a competitor, a mutualist, or predator on a competitive subsystem are emphasized. (Received October 16, 1978.)

763-92-19 LOUIS N. HOWARD, Mass. Inst. of Tech., Cambridge, Mass. 02139. Some oscillatory reaction-diffusion wave structures in finite domains.

A class of solutions of certain reaction diffusion equations which are periodic in time and heterogeneous in space will be described. They may be thought of as consisting of several patches of wave trains, joined by transition zones of one or the other of two qualitatively different types, and resemble certain phenomena which occur in real oscillatory chemical reactions. Methods of showing the existence of such solutions and of calculating them by singular perturbation and numerical methods will be presented. (Received October 19, 1978.)

763-92-20 R. Shonkwiler and Maynard Thompson, Georgia Institute of Technology, Atlanta, Ga. 30332 and Indiana University, Bloomington, Ind. 47401. A Model for Common Source Epidemics.

A stochastic model is developed for common source epidemics-one in which the infectious agent originates outside the target population and infected individuals can not directly spread the disease. Examples of this are Malaria, Giardiasis, Histoplasmosis and Toxoplasmosis. The model is quite general and results in a Volterra type integral equation. An application is made to an outbreak of Toxoplasmosis. (Received October 23, 1978.)

763-92-21 ARTHUR T. WINFREE, University of Purdue, West Lafayette IN 47907. A 2-dimensional continuum of circadian oscillators.

As ascomycete fungus (like bread mold) typically comprises a 2-dimensional network of filaments through which metabolic substances are continually transported both by diffusion and by grosser mechanical modes of translocation. Many such fungi harbor "biological clocks" of one sort or another which rhythmically bias the fate of newly created "cells" toward conspicuous spore production or away from it. Because the fungus grows only at its edges, this rhythmical bias conspires with rectilinear growth to create rhythmic patterns in space. The topological variants of such patterns reveal the modes of organization of metabolic oscillations in space and in time. These modes must inevitably represent solutions of some generalized reaction/diffusion equation. I have recently acquired a new mutation of Neurospora which lends itself to experimental manipulation of these patterns using patterned exposures to blue light. As yet I have no interesting experimental results, but I trust that will change before January 1979. (Received October 23, 1978.) (Author introduced by Professor Murray Gerstenhaber).

763-92-22 LEE A. SEGEL, Department of Applied Mathematics, Weizmann Institute, Rehovot, Israel and Department of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, New York 12181. On leave 1978-79, academic year in Division of Applied Science, Harvard University, Cambridge, Massachusetts 02138. Examples of spatio-temporal patterns in biophysics, cell biology, and ecology.

In this survey talk, examples will be given on travelling waves in a layer of bound enzyme and in a population of chemotactic organisms (including a brief report of recent work by Odell on "preytaxis"). Theories on the development of patterns in leaves will be mentioned, with emphasis on the work of Meinhardt. The final section of the talk will deal with the generation of patterns in ecology. In the mathematical models considered, the "diffusion" terms are often replaced by more general expressions for flux or mixing. (Received October 16, 1978.)

763-92-23 K.-H. Chen, Department of Genetics, Stanford University Medical School, Stanford, CA 94305 (also, SIAM Institute of Mathematics and Society). The Diffusion Approximation to Models of Cultural Evolution. Preliminary report.

A diffusion theory approximation widely used in population genetics is applied to a generalized cultural transmission model originally due to Feldman and Cavalli-Sforza. This model of a dichotomous trait includes vertical transmission from parent to child, horizontal transmission from age peers to offspring, and oblique transmission from non-parents in the parental generation to progeny. The latter modes of transmission comprise a linear form due to approximation of mean average of a series of contacts and a quadratic form representing the effect of epidemic transfer. In a closed population with natural selection, assortative and random matings, and with binomial sampling, conditions for the existence of the probability of eventual phenotypic fixation, starting from  $u_0$ ,  $F(u_0)$ , are determined. Then, under further natural constraints, monotone properties of  $F(u_0)$  as a function of the parameters are identified. Finally, the average number of generations until phenotypic fixation and/or loss are computed. (Received October 25, 1978.)

763-92-24 JAMES A. PENNLINE, Virginia Commonwealth University, Richmond, Virginia 23284. An integral formulation of a reaction-diffusion equation. Preliminary report.

A model of the processes of convection and glucose reabsorption in a renal tubule

$$c'' = (v(x)c(x))' + Bc/(c+K), \quad c(0) = 1, \quad c'(1) = 0,$$

is converted to an integral equation. Sufficient conditions for existence and uniqueness of a solution are discussed as well as some difficulties involved in attempting a numerical solution. (Received October 25, 1978.)

## 93 ► *Systems, Control*

\*763-93-1 DAVID A. LEE, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio 45433. System Identification: Examples of Regularizable Ill-Posed Problems.

Rather frequently, applications lead to uniquely solvable problems whose solutions do not depend continuously on the data, so that the problems are not well-posed in Hadamard's sense. From the conventional point of view it is pointless to attempt to infer solutions of these ill-posed problems from experimental data, since experimental data are always corrupted by observational errors. However, many of the ill-posed problems in applications are regularizable in the sense of Tikhonov, which means, roughly, that there exist suitably bounded operators which generate approximate solutions of the problems. Rational error estimates and even rational error bounds can be given for approximate solutions of regularizable ill-posed problems generated from noisy data. An important class of regularizable ill-posed problems comes from the field of system identification, in which one wishes to infer useful descriptions of a system from observations of its responses to known excitations. We present a number of examples of system identification problems which lead to regularizable ill-posed problems, and discuss in detail the treatment of a case in which one wishes to infer the spatial distribution of dopant in a doped semiconductor, from observations of X-rays induced by bombarding the semiconductor with proton beams of varying energies. (Received August 15, 1978.) (Author introduced by Dr. Robert J. Thompson).

Integral equations of the type obtained in least-squares estimation of stationary stochastic processes are considered and some fast algorithms for the solution of these are derived. (Received October 4, 1978.) (Author introduced by Professor M. Z. Nashed).

## 94 ► Information and Communication, Circuits, Automata

\*763-94-1 Gustavus J. Simmons, Sandia Laboratories, Albuquerque, New Mexico 87185. Some number theoretic questions arising in asymmetric encryption techniques.

All cryptosystems currently in use are symmetric in the sense that either the same information (key) is held in secret by both communicants or else that the secret information held by either is simply derivable from the other. There has been enormous interest in the past two years in a proposed class of asymmetric encryption techniques in which the information held by the transmitter and receiver is not only different - but that it is also "computationally complex" to derive either from the other.

Two of the asymmetric encryption schemes which have been proposed are most naturally represented as J-rings, i.e., for every plain text message M there is an exponent  $\alpha_m$  for which  $M^{\alpha_m} = M$  in the ring. It is easy to show that a finite ring is a J-ring if and only if it is the direct sum of fields, and in the schemes to be considered the cryptosystem is "broken" if this representation can be deduced. The cryptosecurity of a particular ring is therefore dependent on the "line spectra" of the messages - and for the MIT scheme also dependent on the spectra of a derived ring for the exponents  $\alpha_m$ . The investigation of these spectra is a difficult number theoretic question: for example, the validity of Schinzel's Conjecture H would imply for the MIT scheme the existence of arbitrarily insecure J-rings whose message spectrum was at the same time asymptotically best possible. (Received September 15, 1978.) (Author introduced by Professor Stefan A. Burr).

\*763-94-2 Lilian T. Sheng & Pushpa N. Rathie, State University of Campinas, Campinas, Brazil. On the measurable solutions of a non-additive functional equation.

Theorem: Functions  $f, g$  &  $h: [0,1] \rightarrow \mathbb{R}$  are Lebesgue measurable and satisfy the functional equation

$$\sum_{j=1}^m f(x_i, y_j) = \sum_{i=1}^n g(x_i) + \sum_{j=1}^m h(y_j) + \lambda \sum_{i=1}^n \sum_{j=1}^m g(x_i) h(y_j).$$

for  $n, m=2, 3$  and for all  $x_i, y_j \in [0,1]$  ( $i=1, \dots, n; j=1, \dots, m$ ) with  $\sum_{i=1}^n x_i = \sum_{j=1}^m y_j = 1$ , and  $\lambda > 0$  is a constant, iff there exist non-zero constants  $a, b$  such that

$$f(x) = \frac{x}{\lambda}(ax^{\beta-1}-1), \quad g(x) = \frac{x}{\lambda}(ax^{\beta-1}-1), \quad h(x) = \frac{x}{\lambda}(bx^{\beta-1}-1), \quad \beta > 0; \quad \text{or}$$

$$f(x) = g(x) = -\frac{x}{\lambda}, \quad h(x) \text{ arbitrary}; \quad \text{or} \quad f(x) = h(x) = -\frac{x}{\lambda}, \quad g(x) \text{ arbitrary}.$$

This has application in characterizing non-additive entropy of order  $\beta$ . The result is extended to the two variable case in a similar manner, and which has application in characterizing some other generalized information measures of type  $(\alpha, \beta)$ , directed divergence of order  $\alpha$  ( $\alpha=1$ ), and inaccuracy of type  $1+\beta$ . (Received September 28, 1978.)

\*763-94-3 CHARLES P. DOWNEY, University of Nebraska at Omaha, Omaha, Nebraska 68182  
Gaussian Channel Codes from Abelian Groups.

The optimal initial vector problem for Gaussian channel codes is shown to be equivalent to one of three linear programming problems in case the generating group is Abelian. (Received October 20, 1978.)

\*763-94-4 JOHN KARLOF, University of Nebraska at Omaha, Omaha, Nebraska 68182. Optimal (M,3) Group Codes for the Gaussian Channel

The structure of finite groups of real, orthogonal, three by three matrices is used to find all optimal (M,3) nonplanar group codes for the Gaussian channel. (Received October 20, 1978.)

\*763-94-5 RICHARD L. THOMPSON, Bhaktivedanta Institute, 2820 Ponderosa Circle, Atlanta, Ga. 30359. Information and Random Automata.

In 1968 Kolmogorof suggested that the length of the shortest algorithm computing a binary number,  $X$ , could be taken as a measure,  $L(X)$ , of the information content of  $X$ . In 1974 G. Chaitin showed that for large enough  $n$ , the statement,  $L(X) \geq n$ , cannot be proven for any  $X$ . In this talk we show how knowledge of  $L(X)$  enables one to make predictions about the behavior of random automata. We also propose a method of estimating  $L(X)$ . This involves two main theorems. (1) If  $M(X) \geq 0$  and  $\sum_X M(X) \leq T$ , then  $M(X) \leq 2^{c+\log(T)+m-L(X)}$ , where  $c$  is a constant, and  $m$  is the length of the shortest algorithm computing the function,  $M$ . (2) We can specify a function,  $F(n, k, w, X)$ , with the following property. Let  $X$  be a binary number of  $1+nm$  digits. Let  $X_0=1$  and let  $X_k$  be the number consisting of the leftmost  $1+nk$  digits of  $X$ . Then there are numbers,  $K_k > 1$  and  $1 \leq w_k < 2^{K_k}$ , so that  $X_k = F(n, K_k, w_k, X_{k-1})$  for  $k=1, \dots, m$ , and  $\sum_{k=1}^m (K_k - 1) \leq L(X) + 63$ . (Received October 24, 1978.)

\*763-94-6 Pl. Kannappan, Univ. of Waterloo, Ontario, Canada. On some symmetric recursive inset measures of randomized system of events.

In the probabilistic theory of information the entropies and other measures of information are supposed to depend only upon the probabilities of the events. On the other hand, in the non-probabilistic theory of information, these measures do not depend upon the probabilities at all, only directly upon the events themselves. Aczel has proposed a mixed theory of information, where measures of information depend both upon the events and their probabilities.

The general forms of i) symmetric and  $\beta$ -recursive inset entropies and ii) symmetric, recursive and regular inset deviations of randomized systems of events are determined in the framework of the mixed theory of information, which include the Shannon entropy, the entropies of degree  $\beta$  ( $\neq 1$ ), the inaccuracy and the directed divergence of the purely probabilistic information theory as special cases. A sample theorem: Theorem. The sequence  $K_n$  (called inset deviation) is recursive, 3-symmetric and measurable iff there is a  $g: B$  (ring of sets)  $\rightarrow R$  (reals) such that  $K_n(X|P|Q) = g(\bigcup_{i=1}^n x_i) - \sum_{i=1}^n p_i g(x_i) - a \sum_{i=1}^n p_i \log p_i + b \sum_{i=1}^n p_i \log q_i$ ; where  $a$  and  $b$  are arbitrary constants. (Received October 25, 1978.)

## 97 ► *Mathematical Education, Secondary*

\*763-97-1 Robert P. Hostetler, The Behrend College of The Pennsylvania State University, Erie Pennsylvania 16563. The Old Math, the New Math, and the Aftermath.

This paper discusses some of the strengths and weaknesses of the old math prior to the sixties and of the new math whose era has come to a close. The major thrust relates to the present "aftermath" - a period of remediation, frustration, and uncertainty. Educators are searching for new direction and yet few are willing to boldly step out with new programs. The paper discusses reasons for the remedial math boom at the college level and expresses reservations about the back-to-basics movement, about mastery learning, and about programmed and individualized instructional programs.

A new direction is called for, to turn the aftermath into a productive mathematics program; a direction characterized by a marriage of method and application made efficient by skill development. One prerequisite for such a program is a return of the teacher to his or her proper place at the center of the learning process. The paper concludes with some suggestions for teaching students to be problem solvers. (Received September 20, 1978.) (Author introduced by Professor Roland E. Larson).

## 98 ► *Mathematical Education, Collegiate*

\*763-98-1 ROBERT C. ESLINGER, Hendrix College, Conway, Arkansas 72032. A program of undergraduate mathematical research.

In 1970 the Department of Mathematics at Hendrix College instituted a pilot project for early identification of potentially research-oriented undergraduate students. Since that time this

project has evolved into an active undergraduate research program which enhances the preparation of students with a variety of post-graduate objectives. Each year students ranging from the sophomore to senior level investigate questions or topics which often are not encountered in the undergraduate curriculum and which generate independent (sometimes original) discovery by the students. Their subsequent papers are presented at regional and national meetings and in colloquia at other institutions, and frequently they are published in undergraduate and professional journals. This paper discusses the history and development of the program, outlines its present features, and examines its contribution to the mathematics curriculum.

(Received October 23, 1978.)

763-98-2 JANICE T. ASTIN and JULIA P. KENNEDY, Georgia State University, Atlanta, Georgia 30303. Considerations in beginning a mathematics assistance center. Preliminary report.

Georgia State, in the heart of Atlanta, is a commuting university whose student population have diverse mathematics backgrounds. The Department of Mathematics decided to initiate a center to supplement the classroom instruction for mathematics courses offered in the first two years of college. Decisions had to be made in many areas: type of assistance to be offered; location and evaluation of available audio-visual aids; type of facilities; administration, including budget, staffing and cataloging. The presentation will emphasize sources to tap for material and a compilation of software that is presently available for the first two years of college mathematics.

(Received October 25, 1978.)

763-98-3 WITHDRAWN

## 99 ► *None of the above*

\*763-99-1 Saul Stahl, University of Kansas, Lawrence, Kansas 66045. Permutation-partition Pairs. Preliminary report.

Let  $T$  be a fixed set of finite cardinality  $n$ . For any permutation  $P$  of  $T$ , let  $||P||$  denote the number of cycles (orbits) in the disjoint cycle decomposition of  $P$ . A  $PP(n,k)$  pair  $(P, \Pi)$  consists of a permutation  $P$  of  $T$  and a partition  $\Pi = \{\Pi_i\}_{i=1}^k$  of the same set  $T$ . The maximum orbiticity  $\mu(P, \Pi)$  of such a pair equals  $\max\{||PQ||\}$  where  $Q$  varies over all the permutations whose orbits consist of the members of  $\Pi$ . The maximum orbiticity of all  $PP(n,k)$  pairs is evaluated for  $k = 1, 2, 3$ . These results are strong enough to prove virtually all the known theoretical results of topological graph theory as well as some new ones. This work extends that published by the author under the title of "A Counting Theorem for Topological Graph Theory". (Received October 23, 1978.)

## ERRATUM—Volume 25

ALISON C. BRIGHT, On the integer  $(2^{p-1}-1)/p$ . Abstract 78T-A198, October 1978, Page A-580.

Line 3, for  $(2^{p-1}-1)/p = -(\sum_{k=1}^n (k)^{-1} \pmod{p})/3$ , read  $(2^{p-1}-1)/p = -(\sum_{k=1}^n (k)^{-1})/3 \pmod{p}$ .

## AMS Policy on Recruitment Advertising

No listing for a position will be printed if it expresses or implies any preference, limitation, specification, or discrimination based on color, age, sex, race, religion, or national origin in contravention of any federal or state statute prohibiting discrimination in employment on the basis of these criteria. The publisher reserves the right to edit any listing prior to publication which does not conform with federal or state law on discrimination in employment.

All employers in the United States are required to abide by the requirements of Title VII of the Civil Rights Act of 1964, announcing a national policy of equal employment opportunity in private employment, without discrimination because of race, color, religion, sex or national origin. All U.S. listings are accepted with the understanding that the employer complies with federal requirements. Advertisers not subject to U.S. laws are required to sign a statement that they do not discriminate in employment on grounds of age, race, color, religion, sex, or national origin. *Applicants should be aware that institutions outside the U. S. may not be legally bound to conform to these or similar requirements and the AMS cannot assume responsibility for enforcing compliance. Applicants are advised to inform themselves of conditions that may exist at the institutions to which they apply.*

In particular, readers should note that the Equal Employment Opportunity Act (42 U. S. C., § 2000e et seq.), which prohibits discrimination in employment on the basis of sex, race, religion, or national origin, contains (in §2000e-1) an exception from the provisions of the Act for any religious corporation, association, educational institution, or society with respect to employment of individuals of a particular religion to perform work connected with the carrying on by such corporation, association, educational institution, or society of its activities.

The Age Discrimination in Employment Act of 1967 (29 U.S.C., § 621 et seq.), makes it unlawful for an employer to discriminate against any individual between the ages of 40 and 65 because of age. Thus it is legal to seek as an employee someone who is "over 30," but not one "over 50"; neither is it legal to express a preference for someone who is "young," or is a "recent graduate," since the latter tend (on statistical grounds) to be young.

*This policy, which has been in force for some time with respect to display advertisements in the NOTICES as well as listings in EMPLOYMENT INFORMATION IN THE MATHEMATICAL SCIENCES, will also apply to classified advertisements beginning with the February issue of the NOTICES.*

## CLASSIFIED ADVERTISEMENTS

**SUGGESTED USES** for classified advertising are books or lecture notes for sale, books being sought, positions available, summer or semester exchange or rental of houses, mathematical typing services and special announcements of meetings. The rate is \$3.00 per line. The same ad run in seven consecutive issues is \$2.50 per line. Ads will be typed in the AMS office and will be typed solid. If centering and spacing of lines is requested, the charge will be per line with the same rate for open space as for solid type.

**TO CALCULATE** the length of an ad assume that one line will accommodate 52 characters and spaces.

**DEADLINES** for the next few issues are: for February, *February 9*; for April, *March 9*; for June, *May 4*.

**SEND AD AND CHECK TO:** Advertising Department, AMS, P. O. Box 6248, Providence, Rhode Island 02940. Individuals are requested to pay in advance, institutions are not required to do so.

**CONTRIBUTIONS REQUESTED** (Publication-Readied) for "Open Questions", contributed by—or attributed to—renowned scientists. (See Springer-Verlag "Mathematical Intelligencer".)

Dr. Henney  
George Washington University  
Washington, D. C. 20052

### FOR SALE

1979 MATHEMATICAL CALENDAR! Colorful tessellations, biographies, history of symbols, humor. \$5 postpd. (for N. C. add 18¢ tx). ROME PRESS, Dept. N, Box 31451, Raleigh, N. C. 27612. 16 x 20 print of Gauss, \$1.50.

MR 1-36, Pac. J. 1-27, Annals 36-88, J. Alg. 1-51, Duke J. 1-43, P. A. M. S. 1-19. Mostly bound. D. D. Miller, Lucina 5913, Shell Point, Ft. Myers, Florida 33908.

Topological Silver Jewelry, Moebius Band Rings \$20, with gemstone cabochons \$30. Pins and pendants also available. Talisman, 5505 S. Kimbark, Chicago, Illinois 60637.

### POSITIONS AVAILABLE

#### WESTERN ILLINOIS UNIVERSITY

Three tenure-track positions (one in Computer Science, another in Operations Research, Mathematical Programming or Optimization Theory) and one or more temporary positions are available. The Computer Science position calls for experience in some area of theoretical computer science and an interest in expanding and developing what is now a small undergraduate program. The Operations Research position calls for an interest in the computer implementation of solutions to real-world problems. Starting date: August 1979. Rank and salary open. Resumes, transcripts, three letters of recommendation, and offprints and preprints if possible to Professor J. Burlak, Mathematics Department, Western Illinois University, Macomb, IL 61455, by 1 March 1979. Equal opportunity/affirmative action employer.

Assistant Professor, Fall 1979, Ph. D. required. Candidates should have research interests in algebraic geometry or a closely related area such as commutative algebra or several complex variables. Responsibilities will include research and teaching. Tenure-track position. Applications and three letters of recommendation to: Gustavo Efrogmson, Chairman Search Committee, Department of Mathematics and Statistics, UNIVERSITY OF NEW MEXICO, Albuquerque, NM 87131. AA/EOE.

#### SOMERVILLE COLLEGE, OXFORD

Applications are invited for a Fellowship and Tutorship in Pure Mathematics restricted by the College Statutes made under the Universities of Oxford and Cambridge Act 1923 to women. The successful candidate will be eligible for appointment by the Faculty Board to a part-time University Lecturership associated with the Fellowship. Further particulars obtainable from The Principal, Somerville College, Oxford, England. Closing date for applications 28 February 1979.

#### TECHNICAL UNIVERSITY OF CRETE

The Administrative Committee of the Technical University of Crete is interested in compiling information concerning scientists who may be interested in taking a professional or research position in one of the Schools or Research Institutes of the University. It therefore invites all who may be interested in such a position, to send to the Main Office of the University (Zaimi 2, Athens TT 148, GREECE, Attention: Professor P. Theoharis) a Curriculum Vitae containing full name and address, scientific accomplishments and areas of scientific interests. In this way the Administrative Committee will be able to keep those interested informed about the vacancies.

It is expected that at the beginning of the next calendar year there will be 15 new professorial positions in the first three Schools and two Research Institutes of the University. The creation of more positions for the remaining Schools and Institutes will take place gradually within a couple of years.

For more detailed information please send your inquiries to the address given above.

---

Department of Mathematics  
Southern Illinois University at Carbondale

Qualified persons are invited to apply for a position as Assistant Professor in Number Theory at Southern Illinois University, Carbondale. Continuing (tenure-track) position starting August 16, 1979. Applicants are expected to hold a Ph. D. by August 15, 1979 and have a strong record of research and publications. All fields of number theory will be considered. Closing date: February 15, 1979. Applications and inquiries should be sent to: Number Theory Position, c/o Alphonse Baartmans, Chairman, Department of Mathematics, Southern Illinois University, Carbondale, Illinois 62901. Southern Illinois University is an equal opportunity/affirmative action employer.

Qualified persons are invited to apply for a position as an Assistant Professor in Partial Differential Equations at Southern Illinois University, Carbondale. Continuing (tenure-track) position starting August 15, 1979. Ph. D. preferred. Individuals who expect to have completed their Ph. D. prior to August 15, 1979 will be considered. All candidates must have serious research interests. Closing date is February 15, 1979. Interested persons should write to: Partial Differential Equations Position, c/o Alphonse Baartmans, Chairman, Department of Mathematics, Southern Illinois University, Carbondale, Illinois 62901. Southern Illinois University is an equal opportunity/affirmative action employer.

The Mathematics Department of Southern Illinois University, Carbondale, is seeking a junior-level numerical analyst to fill a tenure-track assistant professorship. Ph. D. preferred. Individuals who expect to complete their Ph. D. prior to August 15, 1979 will be considered. Candidates will be evaluated on their potential for superior teaching at both undergraduate and graduate levels, as well as on their interest and ability to develop the research credentials expected of a tenured faculty member in a Ph. D.-granting department. The salary will be competitive. Applications and inquiries should be directed to: Numerical Analysis Position, c/o Alphonse Baartmans, Chairman, Department of Mathematics, Southern Illinois University, Carbondale, Illinois 62901. SIU is an equal opportunity/affirmative action employer. Closing date is February 15, 1979.

Mathematics position in Operations Research anticipated at Southern Illinois University-Carbondale.

---

SPECIAL YEAR IN LOGIC, 1979-1980. The Department of Mathematics of the University of Connecticut will support several candidates with strong research records in Recursion Theory or Model Theory for either one or two semesters. Preference will be given to Recursion Theorists for the Fall semester positions, and to Model Theorists for the Spring semester positions. Send curriculum vitae and references by February 1 to: Joseph Landin, Head, Department of Mathematics, the UNIVERSITY OF CONNECTICUT, Storrs, CT 06268. An Equal Opportunity Employer.

---

The UNIVERSITY OF CONNECTICUT is seeking applicants for a senior position in its Mathematics Department to begin September 1979. Preference will be given to applied mathematicians. Candidates should have already done outstanding research and demonstrated leadership in their fields. They should have ability to interact with mathematicians and with specialists in applied areas, and should have active interests in undergraduate and graduate teaching as well as in departmental affairs. Curriculum vitae should be sent by February 1, 1979, to Joseph Landin, U-9, University of Connecticut, Storrs, CT 06268. An Equal Opportunity Employer.

---

The UNIVERSITY OF MISSISSIPPI is seeking applicants for tenure-track positions at the Assistant Professor or Associate Professor rank, beginning in the 1979 Fall Semester. Applicants in any area may apply. Resume and three letters of recommendation should be sent by February 1, 1979, to Dr. Eldon L. Miller, Chairman, Department of Mathematics, University of Mississippi, University, MS 38677. Equal Opportunity Employer, M/F/H.

---

UNIVERSITY OF WISCONSIN-MADISON  
Van Vleck Assistant Professorships in Mathematics

We invite applications from outstanding mathematicians who are recipients of a doctorate. We are looking for people who interact well with members of our department, who care about teaching, and who can contribute to our research and instructional programs.

The regular teaching load is two courses per semester, with at least one in your specialty every other year. There is a high probability of additional income through research or teaching during summers between consecutive years of appointment.

The salary will be dependent on experience and will be at least \$16,000 per academic year.

All positions are for specified two- or three-year terms.

Application blanks may be obtained by writing Professor Joshua Chover, Chairman, Department of Mathematics, 213 Van Vleck Hall, University of Wisconsin, Madison, WI 53706. University of Wisconsin-Madison is an equal opportunity employer.

---

The Mathematics Department of the University of Wisconsin-Madison solicits applications for a tenure-track assistant professorship (or possibly a beginning associate professorship) commencing in Fall 1979. We are interested in a person of established excellence in any area of mathematics. In particular, the Department has identified needs in the areas of Lie Algebras and Applied Mathematics. An applicant should send a vita, a list of publications and selected reprints/preprints, by February 1, 1979, to Professor Joshua Chover, Chairman, Department of Mathematics, 213 Van Vleck Hall, University of Wisconsin, Madison, WI 53706. Three letters of recommendation concerning the applicant's research and teaching should also be sent as soon as possible. The University of Wisconsin-Madison is an Equal Opportunity Employer.

---

CHAIRPERSON  
DEPARTMENT OF MATHEMATICS

TEXAS TECH UNIVERSITY, a state supported institution with an enrollment of 22,000 invites applications and nominations for the position of chairperson of the Department of Mathematics. The department has 50 faculty members and offers programs at the undergraduate and graduate levels including the Ph. D. Appointment effective no later than Fall 1979. Candidate must have Ph. D. or equivalent, classroom experience and a record of research and publication. Administrative experience desirable. Send complete resume and names of three references to:

Donald R. Haragan, Chairperson  
Search Committee for Chairperson of Mathematics  
Texas Tech University  
P. O. Box 4320  
Lubbock, Texas 79409

Closing date for application is February 15, 1979.

TEXAS TECH UNIVERSITY IS AN AFFIRMATIVE ACTION  
EQUAL EMPLOYMENT OPPORTUNITY EMPLOYER

---

ASSISTANT PROFESSOR - College of Charleston. One or possibly two tenure-track positions. Available 8-20-79. Teaching in an undergraduate mathematics and computer science program (12 hrs/wk). A sincere concern for and commitment to teaching undergraduates, a Ph. D. in mathematics, and continuing interest in research is required. A strong background in applications of mathematics, statistics, or computer science is preferred. All applicants with outstanding teaching and research credentials will be seriously considered. Minimum salary: \$14,000. Three letters of recommendation and a resume should be sent to W. Hugh Haynsworth, Chairman, Department of Mathematics, COLLEGE OF CHARLESTON, Charleston, South Carolina 29401. Equal Opportunity/Affirmative Action Employer.

---

DEPARTMENT OF MATHEMATICS  
COLORADO STATE UNIVERSITY

Applications are invited for two positions for 1979-1980. Salary and rank open.

**TENURE-TRACK:** Applicants with potential for excellence in research and teaching are sought from all subdisciplines—especially all aspects of optimization.

**VISITING PROFESSOR:** Applicants with noteworthy achievements in research are sought from all subdisciplines for a one- or two-semester appointment.

Send resume and three letters of reference to:

R. E. Gaines, Head  
Department of Mathematics  
Colorado State University  
Fort Collins, Colorado 80523

DEADLINE for receiving applications MARCH 1, 1979

Colorado State University is an EEO/Title IX employer. Equal Opportunity Office: 314 Student Services Building.

---

Distinguished Professorships  
University of Maryland, College Park

One position in Algebra and Number Theory. Of particular interest is a number theorist or an algebraist or an algebraic geometer with collateral interests in number theory.

One position in Functional Analysis. Of particular interest is a person who works in harmonic analysis or topological dynamics or in the interactions of functional analysis with other fields.

The University of Maryland is an Equal Opportunity/Affirmative Action Employer.

Application and inquiries should be sent by January 31, 1979 to W. E. Kirwan, Chairman, Department of Mathematics, UNIVERSITY OF MARYLAND, College Park, Maryland 20742.

---

The Mathematics Department at University of Maryland, Baltimore County has a tenure-track position for an Assistant Professor. Candidates should have Ph.D., the potential for research collaboration with some department faculty and a commitment to teaching. Postdoctoral research experience is preferred. Major current fields of research interest are Applied Mathematics, Complex Analysis, Computer Science, Control Systems Science, Numerical Analysis, Operations Research, Statistics and Probability. The department offers the B.S. degree in Mathematics, M.S. and Ph.D. degrees in Applied Mathematics. Preference will be given to applicants in areas of Computer Science and Operations Research. Applicants should send a curriculum vitae, a graduate school transcript, thesis abstract, reprints, and three letters of recommendation to: Y. M. Lynn, Chairman, Department of Mathematics, UNIVERSITY OF MARYLAND BALTIMORE COUNTY, Baltimore, Maryland 21228. Affirmative Action Employer.

---

Chairperson and Professor, Computer Science Department. The University of Maryland at College Park invites applications and nominations for the position of professor and chairperson of the Computer Science Department. Candidates for the position should have a well-established reputation in research and teaching and ability to lead in further developing the programs of a rapidly growing Department. The Department has twenty-seven professorial-rank positions and offers undergraduate and graduate (M.S. and Ph.D.) degrees. The prospective appointment date is July 1 or August 15, 1979; applications received by February 15, 1979, will receive full consideration. Inquiries, nominations, and applications should be addressed to Dr. Robert O. Harger, Chairman of the Search Committee, Electrical Engineering Department, University of Maryland, College Park, Maryland 20742. The UNIVERSITY OF MARYLAND is an equal opportunity, affirmative action employer.

---

---

COMPUTER SCIENCE,  
COMPUTER ENGINEERING POSITIONS

TWO ASSISTANT PROFESSORS, (1 Mathematics; 1 EE), Tenure-Track, in new undergraduate programs at the UNIVERSITY OF THE PACIFIC in Stockton, California (95211), available September 1, 1979. Teaching and professional development supporting the complementary programs emphasized. Salary negotiable. Contact by February 15, 1979.

Computer Science: Ph. D. in C.S. or allied field required. Preference: Data Base Systems. Send resume to William Topp, Mathematics Department.

Computer Engineering: Ph. D. in Computer Engineering or allied field required. Preference: Computer Architecture/Distributed Processing. Program includes required CO-OP. Send resume to I. Dale Dunmire, School of Engineering.

EO/AA Employer.

---

NORTHEASTERN UNIVERSITY  
Department of Mathematics

Applications are invited for a position of Instructor or Assistant Professor.

**Responsibilities:** To teach two courses per quarter, hold three 65-minute conference hours per week and to contribute in an appropriate area of mathematical activity to the ongoing programs of the Department.

**Qualifications:** Ph.D. in Mathematics preferred. Candidates with evidence of strong mathematical talent will be considered first. The current needs of the Department lie in the areas of applied mathematics, computer science and statistics.

Please send resumes to:

Professor M. Gilmore (504UR)  
Chairman, Mathematics Department  
Northeastern University  
Boston, Massachusetts 02115  
Telephone (617) 437-2450

---

NORTHEASTERN UNIVERSITY  
Department of Mathematics

Applications are invited for a position of Instructor or Assistant Professor, for the academic year 1979-1980 only.

**Responsibilities:** To teach three courses per quarter, hold three 65-minute conference hours, and participate in the department programs.

**Qualifications:** Ph.D. in Mathematics is preferred. Candidates with evidence of excellency in teaching will be considered first.

Please send resume to:

Professor M. Gilmore (504UR)  
Chairman, Mathematics Department  
Northeastern University  
Boston, Massachusetts 02115  
Telephone (617) 437-2450

---

THE DEPARTMENT OF MATHEMATICAL SCIENCES  
CLEMSON UNIVERSITY

announces a nine-month, tenure-track faculty opening at the Assistant Professor or Associate Professor level. Qualifications include a Ph. D. degree in applied or pure mathematics, mathematical computing, operations research, or statistics and the desire to interact with a mathematical sciences faculty. It is desirable that applicants have research interests in the applications of the mathematical sciences to the solutions of real world problems. Starting date is August 15, 1979. Application deadline is February 15, 1979. Apply with vitae and three letters of reference to John D. Fulton, Head, Mathematical Sciences Department, Clemson University, Clemson, SC 29631. Clemson University is an EEO/AAE.

---



---

ASSOCIATE HEAD  
Department of Mathematical Sciences  
Clemson University  
Clemson, S. C. 29631

The Department seeks an Associate Head to administer some matters of the instructional programs and the research efforts. The position is a twelve-month tenure-track position at the Associate Professor or Professor level. Qualifications include a Ph. D. in the mathematical sciences, college teaching experience, and leadership ability. Administrative experience is desirable, but not required. The Department with approximately fifty faculty members offers B. A., B. S., M. S., and Ph. D. degrees which integrate core mathematics, mathematical computing, operations research, and statistics. Starting date is July 1, 1979. The application deadline is February 15, 1979. Apply with vitae and three letters of reference to SEARCH COMMITTEE CHAIRPERSON, Mathematical Sciences Department, Clemson University, Clemson, SC 29631. Clemson University is an equal employment opportunity/affirmative action employer.

---

THE DEPARTMENT OF MATHEMATICAL SCIENCES  
CLEMSON UNIVERSITY

announces two tenure-track nine-month faculty openings at the Associate Professor or Professor levels. Qualifications include a Ph. D. in the mathematical sciences with research interests in applied analysis or in mathematical computing and the desire to interact with a faculty in core mathematics, mathematical computing, operations research, or statistics. Starting date is August 15, 1979. Application deadline is February 15, 1979. Apply with vitae and three letters of reference to John D. Fulton, Head, Mathematical Sciences Department, Clemson University, Clemson, SC 29631. Clemson University is an EEO/AEE.

---

HUNTER COLLEGE of the City University of New York announces five tenure-track positions in computer science, statistics, developmental mathematics skills, and applied mathematics beginning in September 1979. Rank and salary are open. The applicant should have a Ph. D. in the appropriate field and a serious commitment to both research and teaching. Address inquiry and curriculum vitae to Professor Thomas Jambois, Chairman, Department of Mathematical Sciences, Hunter College, CUNY, 695 Park Avenue, New York, NY 10021. An Equal Opportunity/Affirmative Action Employer.

---

---

UNIVERSITY OF CALIFORNIA  
SANTA BARBARA

The Department of Computer Science is seeking qualified applicants for tenure-track positions beginning September 1979. A Ph. D. is required, some experience in teaching is preferred, and both promise and significant achievement are expected in an area of mainstream Computer Science research. The positions will entail teaching at both the undergraduate and graduate levels, developing extramurally-funded research programs and directing doctoral research. Academic rank and salary dependent on qualifications. Applications at any level, assistant professor and above, will be considered. Send résumé along with the names of at least four references to Professor E. G. Coffman, Jr., Department of Computer Science, University of California, Santa Barbara, CA 93106, by 15 February 1979. An Equal Opportunity/Affirmative Action Employer.

---

UNIVERSITY OF CALIFORNIA  
SANTA BARBARA

The Department of Computer Science expects to have several temporary positions beginning Fall 1979. A Ph. D. is required and the positions will entail teaching and research in mainstream areas of computer science. Appointments will be made in the Visiting Professor series, the Acting Professor series, the Lecturer series, or the Associate series at a rank and salary depending upon qualifications, and will be one-year appointments. Send résumé along with the names of at least three references to Professor E. G. Coffman, Jr., Department of Computer Science, University of California, Santa Barbara, CA 93106, by February 15, 1979. An Equal Opportunity/Affirmative Action Employer.

---

UNIVERSITY OF SOUTH CAROLINA  
Department of Mathematics  
Computer Science, and Statistics  
Columbia, SC 29208

Expected tenure-track openings in computer science, mathematics, applied mathematics, and statistics. Ph. D. degree with commitment to excellence in teaching and strong potential for research are required. Each applicant should send a vita and arrange for four letters of recommendation to be sent to R. M. Stephenson, Jr., Chairman. USC is an affirmative action/equal opportunity employer.

---

## CBMS REGIONAL CONFERENCE SERIES IN MATHEMATICS

### *POLYNOMIAL RINGS AND AFFINE SPACES*

by Masayoshi Nagata

This volume contains expository lectures from the Conference Board of the Mathematical Sciences Regional Conference held at Northern Illinois University on July 25-29, 1977.

In addition to an Introduction and References, the Table of Contents is:

*Elementary properties of polynomial rings*  
*Affine spaces and projective spaces*  
*Ruled surfaces and rational surfaces*

*Automorphism groups of polynomial rings*  
*Cremona groups*  
*Group actions on affine rings*  
*Complete reducibility of rational representations*  
*Rings of invariants*  
*Some remarks on orbits*

Number 37

40 pages

List price \$5.60; individual price \$4.20

ISBN 0-8218-1687-X; LC 78-8264

Publication date: July 31, 1978

To order, please specify CBMS/37

Prepayment is required for all American Mathematical Society publications.  
Send for the book(s) above to: AMS, P. O. Box 1571, Annex Station, Providence, RI 02901

# COMPREHENSIVE

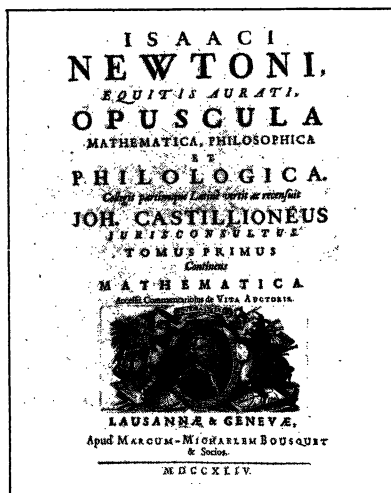
## New Fourth Edition **CALCULUS AND ANALYTIC GEOMETRY**

by **John A. Tierney**, Director, Cushing  
Hospital, Framingham, Mass.

The new edition of this widely adopted text is ideal for the 3 or 4 term calculus sequence for science, math and engineering students of average mathematical sophistication.

Throughout, a wide variety of applied examples and problems stress the relevance of analytic geometry and calculus to such fields as physics, mechanics, chemistry, biology, economics, statistics, and engineering. Over 900 new review problems now follow each chapter, plus many sections are rewritten for greater clarity. The notation  $\log x$  for the natural logarithm now appears as  $1n - x$  to facilitate problem solving using hand calculators. A Self-Test Supplement, a Student Solutions Manual, and an Instructor's Manual accompany the text.

1979 7½ × 9¼ Est. 800 pp.



## **Also coming this Spring** \_\_\_\_\_

**Developing Skills in Algebra: A Lecture Worktext, Third Edition**  
— Volumes I and II or Combined Third Edition  
by *Louis Nanney* and *John Cable*, both of Miami Dade Community College, 1979.

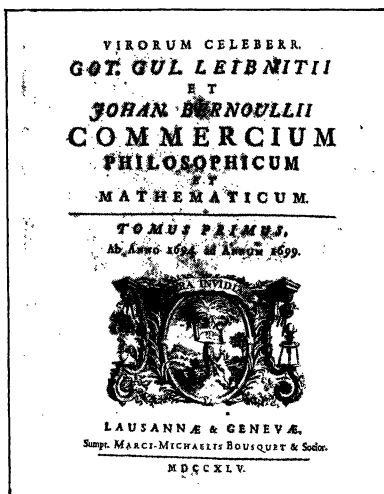
**Trigonometry: A Skills Approach**  
by *Louis Nanney* and *John Cable*, both of Miami Dade Community College, 1979

**Practical Finite Mathematics**  
by *Gareth Williams*, Stetson University, 1979

# ALLYN AND

College Division, Department 893

# AND ACCESSIBLE



## New! DIFFERENTIAL EQUATIONS

by **John A. Tierney**, Director, Cushing Hospital, Framingham, Mass.

This new text is specifically geared for a one term course for students of Math, Science and Engineering and related disciplines. A working knowledge of precalculus and elementary calculus topics is required.

Tierney maintains a consistent level of rigor without becoming overly theoretical. However, he does provide ample material and references for a more ambitious course of study. The text contains material for hand calculator usage, plus computer programs in BASIC for the Euler and Runge-Kutta methods. A wide variety of applications, precise language, large lists of graded exercises, thorough explanations and interwoven historical references promote readability and student appeal. A detailed Solutions Manual accompanies the text.

1979 6 $\frac{3}{8}$   $\times$  9 $\frac{1}{4}$  Est. 384 pp.

---

### **Mathematics: An Informal Approach**

by *Albert B. Bennett*, University of New Hampshire; and  
*Leonard T. Nelson*, Portland State University, 1979

### **Mathematics: An Activity Approach**

by *Albert B. Bennett*, University of New Hampshire; and  
*Leonard T. Nelson*, Portland State University, 1979

### **Introduction to Vector Analysis, Fourth Edition**

by *Harry F. Davis*, The University of Waterloo; and *Arthur David Snider*, The University of South Florida, 1979

# BACON, INC.

470 Atlantic Avenue, Boston, MA 02210

# WE'VE GOT YOUR NUMBER (at Booths 44 & 45)

## **Discrete and Switching Functions**

**Marc Davio, Jean-Pierre Deschamps and André Thayse**

A unified treatment of the algebraic theory of discrete functions, this new text is addressed to the logician, the systems designer, and to the advanced student or scientist. The subject matter is divided into two principal parts: the general representation theory and the study of functional properties.

ISBN 07-015509-7 1978 752 pp. \$67.00

## **Modern Methods in Partial Differential Equations: An Introduction**

**Martin Schechter**

This book introduces the modern techniques and methods used in the new theories of differential equations. Problems of greater generality than those considered in the classical theory texts are discussed and background material is kept to a minimum.

ISBN 07-055193-6 1977 264 pp. \$28.50

## **Advanced Mathematical Methods for Scientists and Engineers**

**Carl M. Bender and Steven A. Orszag**

This outstanding new book presents and explains mathematical methods for obtaining approximate analytical solutions to differential and difference equations that cannot be solved exactly. It develops the insights and methods that are most useful for attacking a new problem and provides computer plots that compare exact and approximate answers.

ISBN 07-004452-X 1978 736 pp. \$28.50

## **Modern Methods in Topological**

**Vector Spaces**  
**Albert Wilansky**

Written by an internationally recognized specialist, this book is more detailed and wider in its coverage than existing books in the field. Many refined proofs and treatments of standard theorems and topics have been developed and among those included are a short proof of a Mahowald's characterization of barrelled spaces and a unified treatment of the class of Hellinger Toeplitz theorems.

ISBN 07-070180-6 1978 304 pp. \$34.50

## **The Four-Color Problem: Assaults and Conquest**

**Thomas L. Saaty and Paul C. Kainen**

The only modern comprehensive treatise on the solution of the famous 125-year-old mathematical four-color problem. The authors give a self-contained description of the innovative methods used to settle this problem, including the relevant combinatorial theory needed to understand the proof.

The book focuses on the full mathematical development of the conjecture and places special attention on the heuristic, probabilistic and computational themes involved in the Appel-Haken solution.

ISBN 07-054382-8 1977 232 pp. \$22.00

## **Inference and Linear Models**

**D. A. S. Fraser**

Here is a well-organized presentation of inference methods and their applications to a wide range of problems that have some linear component in the model. The text combines inference results from the literature of the field along with new research results available for the first time. A wide spectrum of numerical and computer analyses of representative problems is discussed, with a particularly detailed treatment of the non-normal analysis of location-scale and regression problems.

ISBN 07-021910-9 1979 304 pp. (tent.) \$32.00 (tent.)

## **Finite Element Analysis in Fluid Dynamics**

**T. J. Chung**

This, the first monograph on finite elements to deal exclusively with fluid dynamics problems, discusses the basic concepts of the finite element method before summarizing fluid mechanics equations. Applications for both compressible and incompressible flows are discussed, and step-by-step procedures are demonstrated for an ideal flow, both in two-dimensional and axisymmetric geometrics.

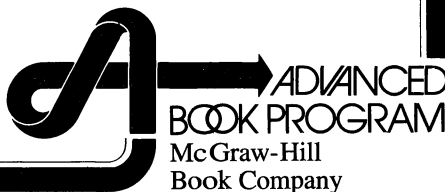
ISBN 07-010830-7 1978 416 pp. \$38.00



*Prices subject to change.*

## **ORDER YOUR COPIES NOW!!**

Write to: O. Graham, 27th fl., Rm 179-N,  
McGraw-Hill Book Company, 1221 Ave. of the Americas,  
New York, N.Y. 10020. Or contact your McGraw-Hill  
representative.



# New Titles & New Editions...

## **Intermediate Algebra for College Students, Second Edition**

Louis Leithold, *University of Southern California*  
1979/528 pages (approx.)/ISBN 0-02-369640-0

Here is a text that combines sound mathematical content with a student-oriented approach to provide a valuable learning tool for all students of intermediate algebra. The full range of standard topics receives comprehensive and detailed treatment.

Changes in this second edition include a clearer and more concise writing style. Many new problems have been added, especially single step exercises which help students build basic skills and confidence. A simplification and reordering of the material in the first five chapters has provided an easier and less rigorous review of sets, real numbers, polynomials, rational numbers, and inequalities. An *Instructor's Manual* is available gratis.

## **Finite Mathematics with Applications, Third Edition**

A. W. Goodman and J. S. Ratti,  
*University of South Florida*  
1979/640 pages/ISBN: 0-02-344760-5

Once again the authors of this leading textbook have joined forces to bring you the most up-to-date material available for the finite math course. The clarity of style and abundance of interesting examples and exercises that marked the success of the earlier editions are retained in this third edition of *Finite Mathematics with Applications*.

New to the third edition is a chapter on statistics, expanded material on linear programming, and many more chapter review problems. In this edition, answers to odd-numbered problems *only* are provided within the text itself. A separate *Instructor's Manual* for even-numbered problems is available, gratis, to the instructor.

## **Elementary Technical Mathematics with Calculus**

James. F. Connelly and Robert A. Fratangelo,  
*Monroe Community College*  
1979/904 pages (approx.)/ISBN 0-02-324440-2

Here is the ideal text for teaching two semesters of mathematics to technology students at all types of colleges, universities, and technical institutes. Comprehensive and readable, this book gives students ample opportunity to practice the application of mathematical principles to technical problems, and incorporates those aspects of calculus most useful to the technician.

The key features of *Elementary Technical Mathe-*

*matics With Calculus* include more than 900 solved examples and 3000 end-of-chapter review questions and exercises, both graded in their degree of difficulty. Three chapters of basic algebra review begin the book, with two chapters covering differentiation and its applications and three chapters on integration also included.

An *Instructor's Manual* containing solutions for all even numbered exercises is available, gratis.

## **TRIGONOMETRY: An Analytic Approach** Irving Drooyan, Walter Hadel, and Charles C. Carico, *Los Angeles Pierce College* 1979/416 pages (approx.)/ISBN: 0-02-330240-2

This third edition continues to emphasize the periodic nature of the circular and trigonometric functions and the relationships between these functions. Over 1800 exercises and more than 340 explanatory examples guide your students to success in the course. Topics of special interest to students planning to continue with the study of math are stressed.

This new edition has been arranged so that instructors can accommodate their classroom procedures to students who use a calculator and to those who do not. All exercise sets have been revised to ensure that they are graded. Answers, including graphs, to odd-numbered exercises are provided in the text. A student's *Solutions Supplement* contains detailed solutions to even-numbered problems. A complimentary *Test Bank* is available for the instructor.

## **Algebra, Second Edition** Saunders MacLane, *University of Chicago*, and Garrett Birkhoff, *Harvard University* 1979/592 pages/ISBN 0-02-374310-7

This important text has been revised to keep abreast of mathematical developments and to clarify and systematize the presentation of modern algebra to undergraduate and graduate students. It continues the theme of the first edition in combining algebraic manipulations and the abstract axiomatic approach with the striking general ideas which have recently developed in the conceptual organization of mathematics.

This second edition has been completely rewritten, with a new chapter on Galois Theory, a simplified explanation of proofs, and new coverage of power series.

**MACMILLAN**  
PUBLISHING  
CO., INC.

866 Third Avenue.  
New York, N.Y. 10022

# Visit Our Booth at the AMS Meeting

## PRINCIPLES AND PROCEDURES OF NUMERICAL ANALYSIS

**Ferenc Szidarovszky and Sidney Yakowitz**

Develops the principles and procedures of numerical analysis, using modern methods which have proven reliable and enjoy attractive convergence properties, including spline interpolation, conjugate gradient methods, and quasilinearization for two-point differential equations. approx. 325 pp., illus., 1978, \$24.50

## DIFFERENTIAL EQUATIONS WITH SMALL PARAMETERS AND RELAXATION OSCILLATIONS

**E. F. Mishchenko and B. Kh. Rozov**

Translated from Russian by

**F. M. C. Goodspeed**

Describes a method for determining asymptotic solutions of autonomous systems of ordinary differential equations where small parameters multiply certain derivatives, enabling the reader to obtain various characteristics of the solutions. approx. 250 pp., illus., 1979, \$29.50

## APPLIED MATHEMATICS An Intellectual Orientation

**Francis J. Murray**

This unusually practical text provides a framework for applying mathematics to actual problems in technological systems and scientific research, elucidating many of the practical demands made upon the "pure" mathematician and suggesting appropriate responses. 239 pp., illus., 1978, \$29.50

## INTEGRAL TRANSFORMS IN SCIENCE AND ENGINEERING

**Kurt Bernardo Wolf**

Provides a systematic, detailed examination of the theoretical and applied aspects of finite transforms, function vector spaces, Fourier and related integral transforms, canonical transformations in quantum mechanics, hyperdifferential operator realizations for the transforms, and similarity groups for a class of differential equations. approx. 475 pp., illus., 1978, \$32.50

## SINGULAR OPTIMAL CONTROLS

**R. Gabasov and F. M. Kirillova**

Translated from Russian by **N. H. Choksy**

Examines special aspects of the theory of optimal processes in relation to singular controls and sliding states, including new methods for analyzing optimal controls using the Pontryagin extremals as a basis for investigation. approx. 260 pp., illus., 1978, \$29.50

## MATHEMATICAL LOGIC

### An Introduction to Model Theory

**A. H. Lightstone**

Edited by **H. B. Enderton**

Develops a form for the classical predicate calculus that provides a foundation for nonstandard analysis and axiomatic set theory. 352 pp., 1978, \$22.50

## INTRODUCTION TO THE LAPLACE TRANSFORM

**Peter K. F. Kuhfittig**

Combines theory and practical application to provide a thorough and lucid introduction to the Laplace transform for one-semester undergraduate courses. 216 pp., illus., 1978, \$19.50

## PRINCIPLES OF OPTIMAL CONTROL THEORY

**R. V. Gamkrelidze**

Translated from Russian by **Karol Makowski**

Translation edited by **Leonard D. Berkovitz**

Intended for graduate and post-graduate students specializing in control theory, this book presents the principles of general control theory as well as proofs of the maximum principle and basic theorems of optimal control theory. 188 pp., 1978, \$24.50

The above titles are the most recent volumes in the *Mathematical Concepts and Methods in Science and Engineering* series, edited by Angelo Miele.

Prices slightly higher outside the U.S.



**PLENUM PUBLISHING CORPORATION**  
227 West 17th Street, New York, N.Y. 10011

# Check Holt's Math



NEW FOR 1979

## Mathematics for Business and Social Sciences

A. W. Goodman and J. S. Ratti,  
*University of South Florida*

An intuitive, practical introduction to the basics of finite mathematics (matrices, linear programming, probability and statistics) and calculus.

ISBN 0-03-022161-7  
1979, 576 pages, cloth

## Elementary Algebra Second Edition

Robert T. Stephens, *Moorpark College*

To its intuitive, non-technical introduction to algebra, this work/text's Second Edition adds mastery tests and topics such as exponents and roots.

ISBN 0-03-046371-8  
1979, 512 pages, paper



ONE YEAR YOUNG

## Trigonometry A Modern Approach

Jack Ceder,  
*University of California, Santa Barbara*

ISBN 0-03-020901-3  
1978, 230 pages, cloth

## Essentials of Business Mathematics, Second Edition

W. Alton Parish and  
William L. Kindsfather,  
*Tarrant County Junior College, Fort Worth*  
ISBN 0-03-021171-9  
1978, 376 pages, paper.



SOME LASTING FAVORITES

## Modern Elementary Algebra for College Students, Third Edition

Vivian Shaw Groza and  
Susanne M. Shelley,  
*Sacramento City College*

ISBN 0-03-018846-6  
1977, 320 pages, cloth

## Modern Intermediate Algebra for College Students, Second Edition

Vivian Shaw Groza and  
Susanne M. Shelley,  
*Sacramento City College*

ISBN 0-03-007461-4  
1974, 543 pages, cloth

## An Introduction to the History of Mathematics, Fourth Edition

Howard Eves, *University of Maine*

ISBN 0-03-089539-1  
1976, 608 pages, cloth

For complimentary copies, send course title and approximate enrollment to: James S. Ryder



HOLT, RINEHART  
AND WINSTON  
383 MADISON AVENUE  
NEW YORK, NY 10017

# Two Best Sellers!

**UNDERSTANDING  
COLLEGE  
ALGEBRA**

**UNDERSTANDING  
ALGEBRA AND  
TRIGONOMETRY**

available Feb. 1979

**Gene R. Sellers,**  
Sacramento City College

**Comprehension** is the keynote of these new Sellers texts designed for your college algebra and precalculus courses. A blend of an intuitive style with a traditional approach makes them especially promising.

- clear, low-powered writing style which promotes student interest
- numerous exercises for drill and application
- chapter introductions and summaries
- end-of-chapter reviews with self-checking problems
- informal marginal notes for students
- calculator application
- Student Study Guide, Instructor's Manual, and Test Bank

**WRITE CHRIS COLE, DEPT. AMS, FOR EXAMINATION COPY**

**CHARLES E. MERRILL PUBLISHING CO.**

A BELL & HOWELL COMPANY  
1300 Alum Creek Drive • Columbus, Ohio 43216

 **BELL & HOWELL**

Information systems. For work, education and entertainment.



# New... from Wiley-Interscience

## **THEORETICAL NUMERICAL ANALYSIS** **An Introduction to Advanced** **Techniques** Peter Linz

Provides an understanding of mathematical foundations that fosters an efficient, well-constructed algorithm whose behavior is predictable, and whose results can be judged by more than mere plausibility. Emphasizes content of theorems rather than proof. 224 pp. (1-04561-6) Dec. 1978 \$17.95

## **TOPOLOGICAL UNIFORM** **STRUCTURES** Warren Page

A unifying theme of topologies compatible with algebraic structures. Studies mathematics as a structured, coherent, and harmonious whole. Includes details of uniform spaces; topological groups, vector spaces, and algebras; and abstract harmonic analysis. approx. 416 pp. (1-02231-4)  
Dec. 1978 \$34.95 (tent.)

## **FINITE DYNAMIC PROGRAMMING** **An Approach to Standardised Markov** **Decision Processes** D. J. White

Examines the dynamic programming approach to standardized finite state, finite action bounded reward Markov decision processes, over finite and infinite time horizons. approx. 256 pp. (1-99629-7)  
Dec. 1978 \$29.95

## **STATISTICAL METHODS FOR** **ENGINEERS AND SCIENTISTS** **A Students' Course Book** A. C. Bajpai, I. M. Calus, & J. A. Fairley

Provides greater detail of the statistical methods required in science and engineering than most conventional texts. Emphasizes applications. Illustrates the material with many practical examples. 444 pp. (1-99644-0)  
1978 \$14.50 Paper

## **THE ASYMPTOTIC THEORY OF** **EXTREME ORDER STATISTICS** Janos Galambos

Discusses the stochastic regularity in extreme behavior, presenting the asymptotic theory of extremes as the number of components making up the extremes increases indefinitely. 352 pp. (1-02148-2) 1978 \$24.95

## **LECTURES IN SEMIGROUPS** Mario Petrich

A systematic exposition of the most important topics in semigroup theory, including bands, matrix and normal band decompositions, and lattices of subsemigroups. Prerequisite topics are summarized and are dealt with at considerable depth. 168 pp. (1-99514-2) 1978 \$17.95

## **A COURSE OF APPLIED FUNCTIONAL** **ANALYSIS** Arthur Wouk

Bridges the gap created by traditional programs in this area. Examines approximate and iterative methods in functional spaces and applies them to the solution of functional equations and the minimization of functionals. Covers both linear and nonlinear cases, using contraction mappings, projection methods, steepest descent, Newton's methods, convexity, separation theorems, convex duality, Kuhn-Tucker theory, and monotonicity. approx. 576 pp. (1-96238-4)  
Feb. 1979 \$24.95 (tent.)

## **AN INTRODUCTION TO** **MATHEMATICAL MODELING** Edward A. Bender

A practical learning approach for developing mathematical models. Provides over 100 models from all fields of science, engineering, and operations research. 256 pp. (1-02951-3) 1978 \$16.95  
**Solutions Manual**  
32 pp. (1-03407-X) 1978 \$3.50

## **MATHEMATICS FOR OPERATIONS** **RESEARCH** W. H. Marlow

Shows how the different parts of mathematics—algebra, geometry, and analysis—work together for optimizations and linear systems. Focuses on those parts of mathematics (except probability and statistics) that are used frequently in operations research, engineering, systems sciences, statistics, economics, and the many other fields where optimization and linear systems figure prominently. 483 pp. (1-57233-0) 1978 \$19.95

## **PRINCIPLES OF ALGEBRAIC** **GEOMETRY** Phillip Griffiths & Joseph Harris

A comprehensive, self-contained treatment, presenting main, general results of the theory. Establishes a geometric intuition and a working facility with specific geometric practices. Emphasizes applications to the study of interesting examples and to the development of computational tools. Coverage ranges from analytic to geometric. 813 pp. (1-32792-1) 1978 \$42.00

Order any of these books for free 30-day examination. Write to Nat Bodian, Dept. 092 A 3468.



### **WILEY-INTERSCIENCE**

a division of John Wiley & Sons, Inc.  
605 Third Avenue  
New York, N.Y. 10016

In Canada: 22 Worcester Road, Rexdale, Ontario  
Prices subject to change without notice.  
092 A 3468-51

## DOs\*

Plenty of exercises — *correctly*  
answered  
Clear explanations  
Graded exercises  
Intuitive treatments with a  
reasonable balance of rigor  
Realistic and varied  
applications  
High readability  
Calculator-related content  
and exercises when  
appropriate

\*D.C. Heath and Company National Survey of College Calculus Instructors

## Calculus with Analytic Geometry

by

Roland E. **Larson** and Robert P. **Hostetler**  
The Pennsylvania State University, The Behrend College

January 1979

Casebound

1008 pages

## DOES!

*Student Solutions Manual*

*Instructor's Manual*

### Just Published!

#### **Arithmetic by Example**

William J. **Hemmer**

1979 *Paperbound* 608 pages

This complete, highly flexible program in basic arithmetic can be used in a wide variety of teaching formats from traditional lecture to self-paced. Self-contained lessons can be taught out of sequence or omitted according to class needs. An informal, highly personalized writing style and abundant learning aids lend special appeal for students. *Student Solutions Manual. Instructor's Manual. Instructor's Resource Manual with Tests. Audio Tapes.*

### Just Published!

#### **Mathematics: Models and Applications**

Lawrence C. **Eggen** and Charles **Vanden Eynden**, Illinois State University

1979 *Casebound* 464 pages

Written for students who will probably go no further in mathematics, the text focuses on major concepts, practical topics, and abundant applications — all of which are clearly useful in daily life. *Instructor's Manual.*



### Just Published!

#### **Arithmetic for College Students, Third Edition**

D. Franklin Wright, Cerritos College

1979 Casebound 307 pages

Written expressly to help students avoid common pitfalls, the text achieves a compromise between traditional and self-paced approaches and includes newer techniques without forcing students to learn a large new vocabulary. Self-quizzes, summaries, other study aids abound. *Instructor's Manual, Student Workbook.*

#### **College Algebra and Trigonometry, Second Edition**

William L. Hart, University of Minnesota

Bert K. Waits, Ohio State University  
1978 Casebound 411 pages

*Answers to Odd-Numbered Problems, Index — 70 pages*

A complete foundation in college algebra and an efficient modern treatment of trigonometry, including only a moderate amount of numerical trigonometry. *Instructor's Manual.*

#### **College Algebra, Sixth Edition**

William L. Hart, University of Minnesota

Bert K. Waits, Ohio State University  
1978 Casebound 322 pages

*Appendix, Tables, Answers to Odd-Numbered Problems, Index — 56 pages*

Stresses associated analytic geometry and a logical ordering of topics. Numerous examples with solutions give students frequent models for their own work. *Instructor's Manual.*

#### **Basic Mathematics: A Program for Semi-Independent Study**

John D. Baley, Martin Holstege, and Gail M. Hughes, Cerritos College

1978 Paperbound 370 pages

Practical applications (computing sales commissions, interest on savings and investments, etc.) are stressed in this text/workbook offering a complete course in basic mathematics skills. *Test Program. Audio Tapes.*

#### **Algebra by Example:**

An Elementary Course

Herbert I. Gross, Bunker Hill Community College

1978 Paperbound 434 pages

Presents beginning algebra in a relaxed, conversational style students can easily grasp. *Student Solutions Manual. Instructor's Resource Manual with Tests. Audio Tapes.*

#### **Understandable Statistics:**

Concepts and Methods

Charles Henry Brase, Regis College (Colorado) and Corrinne Pellillo Brase, Arapahoe Community College (Colorado)

1978 Paperbound 392 pages

A firm grounding in statistics with abundant problems and practical applications. *Test Program and Key Steps to Solutions for Even-Numbered Problems.*

#### **Elementary Probability and Statistical Reasoning**

Howard E. Reinhardt and Don O. Loftsgaarden, University of Montana, Missoula

1977 Casebound 417 pages

Probability theory is tied to statistical applications by introducing students early to simulation, elementary modeling, and tests of fit. *Instructor's Manual.*



**HEATH**

For details or sample copies, call us toll free: 800-225-1388.  
In Massachusetts, call collect: 617-862-6650, ext. 1344.

**D.C. Heath and Company**

Home Office: 125 Spring Street, Lexington, Massachusetts 02173

Sales Offices: Atlanta, Ga. 30318 / Rockville, Md. 20852 / Park Ridge, Ill.

60068 / San Antonio, Texas 78217 / Novato, Calif. 94947 / Toronto,

Ontario M5H 1S9

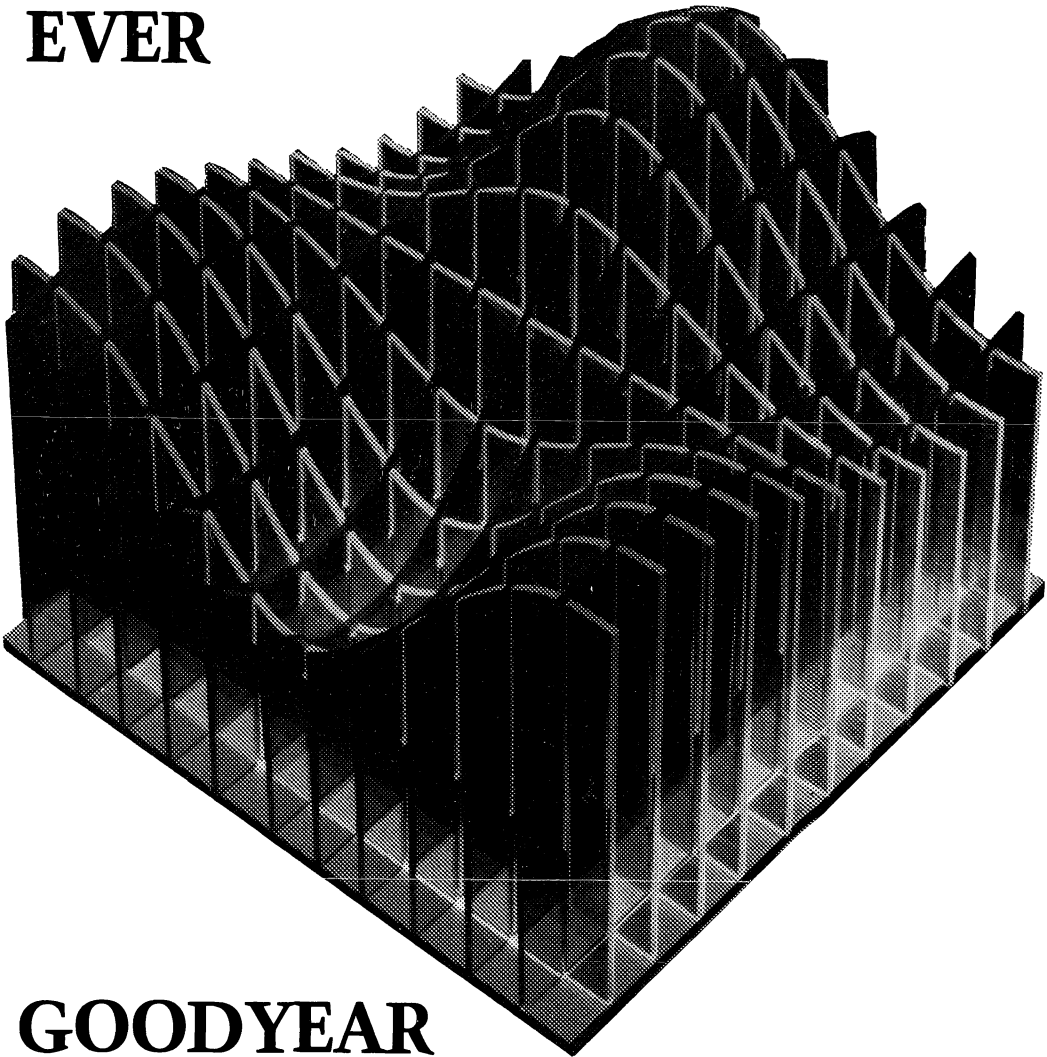
A Raytheon Company

# **SHENK 2ND**

---

---

**SEVENTY-FIVE THOUSAND  
USERS CLOSER TO BECOMING  
THE MOST SUCCESSFUL  
CALCULUS TEXT,  
EVER**



**GOODYEAR**

**Goodyear  
does  
it  
again      and  
again      and  
again**

---

Kertz for  
liberal arts  
math.

Khazanie for  
elementary  
statistics.

Gilligan & Nenno  
for finite math.

Two years ago  
you had never  
heard of  
Shenk, either.

Booth 23-24

# At Booth 6, you'll find new solutions to your math problems.

Stop by in Biloxi and talk to our Harper & Row representatives about new ways to reach – and teach more math to – more students.

## **CALCULUS WITH ANALYTIC GEOMETRY**

DANIEL J. FLEMING & JAMES J. KAPUT

Extensive class-testing, an intuitive, geometric approach backed by mathematical precision, and extraordinarily well-developed pedagogy have led us to call this new text "The Careful Calculus." Emphasizes physical science and engineering applications, includes examples from business, economics, social and life sciences. More than 7000 graded exercises. Approx. 750 pp. \$18.95 (tentative).

## **TOPICS IN CONTEMPORARY MATHEMATICS**

Second Edition

JACK BRITTON & IGNACIO BELLO

A new edition of a proven text, stressing the application of basic mathematical concepts to consumer problems. New material on the metric system, computers, and the use of statistics, plus an additional chapter on basic algebra, make it especially suitable for liberal arts and elementary education students. Approx. 575 pp. \$15.95 (tentative).

## **INTRODUCTORY ALGEBRA** **A Dual Approach**

RONALD HOLLOWAY & VICTOR STROZAK

The unique dual approach provides self-study and extra help as needed. Each chapter has two sections: the first emphasizes concepts and is to be used in class; the second provides self-help and allows for review as individual difficulties arise. Includes pre- and post-chapter tests. Approx. 500 pp. Paperbound. \$11.95 (tentative).

Prices subject to change without notice. Prices quoted by Harper & Row are suggested list prices only and in no way reflect the prices at which these books may be sold by suppliers other than Harper & Row.



**Harper & Row**

1700 Montgomery • San Francisco • California 94111

# 16mm math films

## New Releases

### **DRAGON FOLD—And Other Ways to Fill Space**

A new animated film by Bruce and Katharine Cornwell which introduces the logic and beauty of space filling curves in college-level geometry.

### **ZOOMS ON SELF SIMILAR FIGURES and THE SIERPINSKI CURVE FILLS SPACE**

New releases from the Topology Films Project supported by the National Science Foundation by project director Nelson L. Max.

## Earlier Releases

### **THE TOPOLOGY SERIES**

Four 16mm color films with computer animation by the Educational Development Center for the Topology Films Project with support from the National Science Foundation. Nelson L. Max, Topology Films Project Director.

### **THE GEOMETRY SERIES**

Twelve animated films developed by the College Geometry Project at the University of Minnesota and supported by the National Science Foundation.

### **THE TRIANGLE SERIES**

Four animated color films by Bruce and Katharine Cornwell.

*To receive more information, complete and return the coupon below.*

To: International Film Bureau Inc.  
332 South Michigan Avenue  
Chicago, Illinois 60604

Name \_\_\_\_\_

Position \_\_\_\_\_

School \_\_\_\_\_

Address \_\_\_\_\_

City, State, Zip \_\_\_\_\_

# The P-H Math List for '79.

## THE LANGUAGE OF MATHEMATICS

Nelda Roueche and Barbara Washburn Mink—both of Austin Community College  
576 pp. (est.) Cloth \$12.95

## MODERN ELEMENTARY STATISTICS, 5/e

John E. Freund—Arizona State University  
480 pp. (est.) Cloth \$14.95

## ORDINARY DIFFERENTIAL EQUATIONS:

**A Computational Approach**  
Charles E. Roberts, Jr.—Indiana State University  
416 pp. (est.) Cloth \$16.50

## ARITHMETIC AND ALGEBRA, 2/e

Daniel D. Benice—Montgomery College  
350 pp. (est.) Paper \$10.95

## MATHEMATICS: Contemporary Topics and Applications

Howard A. Silver—Chicago State University  
608 pp. (est.) Cloth \$14.95

## FUNDAMENTALS OF MATHEMATICS: For Business, Social, and Life Sciences

William J. Adams—Pace University  
672 pp. (est.) Cloth \$17.95

## MULTIVARIABLE MATHEMATICS: Linear Algebra, Calculus, Differential Equations, 2/e

Richard E. Williamson—Dartmouth College  
Hale F. Trotter—Princeton University  
576 pp. (est.) Cloth \$19.95

## ALGEBRA AND TRIGONOMETRY: A Precalculus Approach

Max A. Sobel—Montclair State College; Norbert Lerner—S. U. N. Y.  
576 pp. (est.) Cloth \$14.95

## INTERMEDIATE ALGEBRA

Francis J. Mueller  
480 pp. (est.) Paper \$13.95

## BASIC ALGEBRA

Marilyn Frankenstein—University of Massachusetts  
480 pp. (est.) Paper \$12.95

## ALGEBRA: An Incremental Approach, Volumes I and II

John H. Saxon, Jr.—Oscar Rose Junior College  
Vol. I—464 pp. (est.) Paper \$12.95  
Vol. II—496 pp. (est.) Paper \$12.95

## ELEMENTARY LINEAR ALGEBRA WITH APPLICATIONS

Francis G. Florey—University of Wisconsin  
400 pp. (est.) Cloth \$13.95

## LINEAR ALGEBRA

Stephen H. Friedberg, Arnold J. Insel, and Lawrence E. Spence—all of Illinois State University  
512 pp. (est.) Cloth \$16.95

## MATHEMATICS FOR BIOLOGICAL SCIENCES

Jagdish C. Arya and Robin W. Lardner—both of Simon Fraser University, B. C.  
704 pp. (est.) Cloth \$18.95

Prices subject to change without notice.

For further information, or to order or reserve examination copies, please write: Robert Jordan, Dept. J-436, Prentice-Hall, Inc., Englewood Cliffs, NJ 07632.

# Prentice-Hall

# New

ANNOUNCING A NEW JOURNAL

## JOURNAL OF OPERATOR THEORY

Editors

CONSTANTIN APOSTOL  
INCREST

R. G. DOUGLAS  
SUNY at Stony Brook

CARL M. PEARCY  
University of Michigan

DAN VOICULESCU  
INCREST

ZOIA CEAUȘESCU, *Managing Editor*  
INCREST

The Journal is published by the Mathematics Department of the National Institute for Scientific and Technical Creation in Bucharest. Two volumes are published each year, and each volume consists of two issues.

The Journal endeavors to publish significant research articles in all areas of operator theory, which is one of the fastest growing

and most exciting areas of mathematical research today and, in addition, furnishes important techniques to almost all of the applied sciences.

Manuscripts for publication should be submitted to one of the four editors and may be written in English, French, German or Russian.

### TABLE OF CONTENTS

VOLUME I, NUMBER 1, 1979

1. J. A. Ball, *A lifting theorem for operator models of finite rank on multiply connected domains.*
2. G. D. Allen and J. D. Ward, *Hermitian liftings in  $B(l_p)$ .*
3. B. Simon, *Maximal and minimal Schrödinger forms.*
4. J. Barria and D. A. Herrero, *Closure of similarity orbits of nilpotent operators. I. Finite rank operators.*
5. L. Pitt, *A compactness condition for linear operators on function spaces.*
6. M. Pimsner, S. Popa and D. Voiculescu, *Homogeneous  $C^*$ -extensions of  $C(X) \otimes K(H)$ . Part I.*
7. J. D. Morgan, III, *Schrödinger operators whose potentials have separated singularities.*
8. S. Brown, *Connections between an operator and a compact operator that yield hyperinvariant subspaces.*
9. T. Fall, W. Arveson and P. Muhly, *Perturbations of nest algebras.*

Published Quarterly. Price \$44 per year for 2 volumes of 2 issues each.

Subscriptions to the Journal of Operator Theory will be filled by the American Mathematical Society. The first issue will be available in early 1979, and the first two issues will be given free of charge to subscribers. Subscription orders must be prepaid (Volume 1 is free and Volume 2 is \$22) and should be sent to:

American Mathematical Society  
P.O. Box 1571, Annex Station  
Providence, Rhode Island USA 02901

NAME (please print) \_\_\_\_\_

STREET \_\_\_\_\_ CITY \_\_\_\_\_

STATE or COUNTRY \_\_\_\_\_ ZIP CODE \_\_\_\_\_



## **CONTINUOUS CROSSED PRODUCTS AND TYPE III VON NEUMAN ALGEBRAS**

ALFONS VAN DAELE

*London Mathematical Society Lecture Note Series 31* \$7.50

## **UNIFORM ALGEBRAS AND JENSEN MEASURES**

T. W. GAMELIN

*London Mathematical Society Lecture Note Series 32* \$13.50

## **SPHERICAL MODELS**

MAGNUS J. WENNINGER

t Hardcover \$17.95 Paper \$7.95

## **VECTORS IN THREE DIMENSIONAL SPACE**

J. S. R. CHISHOLM

Hardcover \$33.50 Paper \$9.95

## **SPECIAL RELATIVITY**

*The Foundation of Macroscopic Physics*

W. G. DIXON \$34.50

## **LINEAR ALGEBRA AND GEOMETRY**

DAVID M. BLOOM

Hardcover \$34.50 Paper \$18.95

## **LOGIC FOR MATHEMATICIANS**

A. G. HAMILTON

Hardcover \$31.00 Paper \$9.50

## **THE UNPROVABILITY OF CONSISTENCY**

*An Essay in Modal Logic*

GEORGE BOOLOS t \$16.50

(t = tentative)

*Cambridge Books are on exhibit at the Annual Meeting/Booth #27*

**Cambridge University Press**

32 East 57th Street, New York, N.Y. 10022

## **DEGREE THEORY**

N. G. LLOYD

*Cambridge Tracts in Mathematics 73* \$21.00

## **AN INTRODUCTION TO CATEGORIES, HOMOLOGICAL ALGEBRA AND SHEAF COHOMOLOGY**

JAN R. STROOKER \$28.50

## **GEOMETRIC SYMMETRY**

E. H. LOCKWOOD and R. H. MACMILLAN \$24.50

## **A COURSE ON GROUP THEORY**

J. S. ROSE

Hardcover \$29.50 Paper \$9.95

## **MULTIPLE-CONCLUSION LOGIC**

T. J. SMILEY and D. J. SHOESMITH \$35.00

*Now in Paperback . . .*

## **PRINCIPLES OF STATISTICAL TECHNIQUES**

*Second Edition*

P. G. MOORE \$9.95

## **A FIRST COURSE IN MATHEMATICAL ANALYSIS**

J. C. BURKILL \$8.95

## **MATHEMATICAL MODELS FOR THE GROWTH OF HUMAN POPULATIONS**

J. H. POLLARD \$7.50

*Journal . . .*

## **MATHEMATICAL PROCEEDINGS OF THE CAMBRIDGE PHILOSOPHICAL SOCIETY**

J. C. BURKILL, Editor

# North Holland is the

---

---

## INTRODUCTION TO NUMERICAL METHODS AND OPTIMIZATION TECHNIQUES

Richard W. Daniels, Bell Telephone Laboratory

This new text combines numerical methods and optimization techniques in one introductory course. It shows undergraduates how to apply a computer to many different types of problems.

Rather than overwhelming the reader with many techniques, the author focuses on one or two to solve each problem. He covers the solution of linear and non-linear equations, interpolation, differentiation and integration. Then, he shows how to use these methods in optimization techniques, including: Simplex, steepest-descent, Fletcher-Powell, least squares and least p-th.

End-of-chapter problems check the reader's knowledge of subtle points, illustrate program applications, and present the computer as a labor saving device as well as an ally in future endeavors. All computer programs used in the book are written in a version of time-sharing FORTRAN similar to FORTRAN IV, and were run on a Control Data Cyber 70 system.

0-444-00263-4 1978 308 pp. \$19.95

An instructor's manual is also available from the publisher.

## TOPICS IN TOPOLOGY

Arlo W. Schurle, University of North Carolina at Charlotte

Here's a versatile introduction to topology for undergraduate or beginning graduate students. It presents introductory topological concepts in two parts, allowing the reader to concentrate on either metric space or general space results.

The book covers basic topics in the foundations of topology, with a special chapter on examples and pathologies. This chapter examines Cantor Sets, Peano Spaces, embeddings, continuous non-differentiable functions and differential equations. Additional chapters explore winding numbers, combinatorial topology and the fundamental group. An ample supply of challenging exercises accompanies each section.

0-444-00285-5 March 1979 216 pp. \$19.95

## JOURNAL OF INTEGRAL EQUATIONS

Editors:

A.T. Bharucha-Reid  
Dept. of Mathematics  
Wayne State University  
Detroit, Michigan 48202

M.Z. Nashed  
Dept. of Mathematics  
University of Delaware  
Newark, Delaware 19711

JIE will publish high-quality research papers in the area of integral equations and their applications. JIE will be particularly devoted to: the deterministic and probabilistic theories of integral equations; numerical analysis and approximation methods for integral equations; and applications of integral equations in the sciences and technology.

Write to the publisher for a free sample copy.

Subscription Information: Volume 1 (4 issues): March 1979.

\$33.00 Individual \$65 Institutional.

---

---

# ELSEVIER NORTH HOLLAND<sup>INC</sup>

52 VANDERBILT AVENUE NEW YORK, NEW YORK 10017

Prices subject to change without notice.

# name to remember..

---

---

Announcing a new series in

## **PROBABILITY AND APPLIED MATHEMATICS**

**A.T. Bharucha-Reid**, Wayne State University, **Series Editor**

This series of texts and monographs is devoted to the role of modern probability theory (particularly the theory of stochastic processes) in the general field of applied mathematics. The books will be devoted to probabilistic analysis and its applications as well as to computational methods for stochastic processes and their applications.

## **STOCHASTIC METHODS IN QUANTUM MECHANICS**

**S. Gudder**, University of Denver

This graduate level book shows how stochastic methods can be used to obtain a deeper understanding of the foundations of quantum mechanics. Particular consideration is given to quantum logic, operational and algebraic approaches and the study of hidden variables, as well as the Feynman integral, quantum stochastic processes, random fields, the quantum moment problem and quantum field theory.

In preparation for March 1979 0-444-00299-5 227 pp.

## **AN INTRODUCTION TO STOCHASTIC PROCESSES**

**D. Kannan**, University of Georgia, Athens

A text intended for mathematicians who want a rigorous introduction to stochastic processes. The book covers all major areas, with emphasis on discrete Markov chains, martingales and diffusion. Analytical in nature, this modern approach gives a systemized and orderly presentation of a vast amount of material.

In preparation for March 1979 0-444-00301-0 296 pp.

## **APPROXIMATE SOLUTION OF RANDOM EQUATIONS**

**A.T. Bharucha-Reid**, Wayne State University, **Editor**

This is the first book in any language devoted to the approximate solution of random equations. The eleven articles, by leading research workers in the field, utilize results from numerical functional analysis and the theory of stochastic processes, and employs computers for numerical studies.

In preparation for 1979

---

To order books, or for more information, contact the publisher. To request examination copies, please send full course information on school letterhead to James van Maanen, **Elsevier North Holland, Inc., 52 Vanderbilt Ave., New York, N.Y. 10017.**

---

---

---

**ELSEVIER NORTH HOLLAND**<sup>INC</sup>  
52 VANDERBILT AVENUE NEW YORK, NEW YORK 10017

# *An Information Treasury*

---

## STUDIES IN MATHEMATICS AND ITS APPLICATIONS:

*editors:* J. L. LIONS, Paris; G. PAPANICOLAOU, New York; R. T. ROCKAFELLAR, New York.

Vol. 5: **Asymptotic Analysis for Periodic Structures**

by A. BENSOUSSAN, J. L. LIONS  
and G. PAPANICOLAOU

1978 xxiv + 698 pages.  
Price: US \$49.00/Dfl. 110.00  
ISBN 0-444-85172-0

Vol. 3: **Differential Forms in Mathematical Physics**

by C. VON WESTENHOLZ

1978 xvi + 500 pages.  
Price: US \$66.75/Dfl. 150.00  
ISBN 0-7204-0537-8

Vol. 4: **The Finite Element Method for Elliptic Problems**

by P. G. CIARLET

1978 xviii + 534 pages.  
Price: US \$57.75/Dfl. 130.00  
ISBN 0-444-85028-7

Vol. 1: **Convex Analysis and Variational Problems**

by I. EKELAND and R. TEMAM

1975 x + 402 pages.  
Price: US \$37.75/Dfl. 85.00  
ISBN 0-7204-2838-6

## NORTH-HOLLAND SERIES IN APPLIED MATHEMATICS AND MECHANICS:

*editors:* W. T. KOITER, Delft; H. A. LAUWERIER, Amsterdam.

Vol. 25: **Three-Dimensional Problems of Elasticity and Thermoelasticity**

by V. D. KUPRADZE, T. G. GEGELIYA, M. O. BASHELEISHVILI  
and T. V. BURCHULADZE

1978 960 pages.  
Price: US \$144.50/Dfl. 325.00  
ISBN 0-444-85148-8

Vol. 23: **Unidirectional Wave Motion**

by H. LEVINE

1978 xvi + 494 pages.  
Price: US \$71.00/Dfl. 160.00  
ISBN 0-444-85043-0

Vol. 24: **Theoretical Kinematics**

by O. BOTTEMA and B. ROTH

1979 xii + 546 pages.  
Price: US \$80.00/Dfl. 180.00  
ISBN 0-444-85124-0

Vol. 22: **The Theory of Elastic Waves and Waveguides**

by J. MIKLOWITZ

1978 xviii + 602 pages, 180 figures.  
Price: US \$71.00/Dfl. 160.00  
ISBN 0-7204-0551-3

## NORTH-HOLLAND MATHEMATICAL LIBRARY:

*editors:* M. ARTIN, Cambridge, Mass; H. BRASS, New York; J. EELLS, Coventry; W. FEIT, New Haven; P. FREYD, Philadelphia; F. W. GEHRING, Michigan; H. HALBERSTAM, Nottingham; L. V. HÖRMANDER, Lund; M. KAC, New York; J. KEMPERMAN, New York; H. A. LAUWERIER, Amsterdam; W. A. J. LUXEMBURG, Pasadena; F. P. PETERSON and I. M. SINGER, Cambridge, Mass; A. C. ZAAANEN, Leiden.

Vol. 17: **Some Classes of Singular Equations**

by S. PRÖSSDORF

1978 365 pages.  
Price: US \$53.25/Dfl. 120.00  
ISBN 0-7204-0501-7

Vol. 16: **The Theory of Error-Correcting Codes**

by F. J. MACWILLIAMS  
and N. J. A. SLOANE

*Hardbound Student Edition*  
1978 xx + 762 pages.  
Price: US \$39.50/Dfl. 95.00  
ISBN 0-444-85193-3

# New Mathematics Titles with Lasting Value from NORTH-HOLLAND

---

## NORTH-HOLLAND MATHEMATICS STUDIES

(NOTAS DE MATEMATICA):

*editor:* L. NACHBIN, Rio de Janeiro.

Vol. 31: **Differential Equations  
and Applications**

by **W. ECKHAUS** and **E. M. DE JAGER**

1978 x + 264 pages.

Price: US \$31.00/Dfl. 70.00

ISBN 0-444-85185-2

Vol. 29: **Probability and  
Potentials**

by **C. DELLACHERIE** and  
**P.-A. MEYER**

1978 300 pages.

Price: US \$28.75/Dfl. 65.00

ISBN 0-7204-0701-X

## ANNALS OF DISCRETE MATHEMATICS:

*editor:* P. L. HAMMER, Waterloo, Can.

Vol. 3: **Advances in Graph  
Theory**

edited by **BELA BOLLABÁS**

1978 viii + 295 pages.

Price: US \$49.00/Dfl. 110.00

ISBN 0-444-85075-9

Vol. 2: **Algorithmic Aspects  
of Combinatorics**

edited by **B. ALSPACH**, **P. HELL**  
and **D. J. MILLER**

1978 x + 222 pages.

Price: US \$42.25/Dfl. 95.00

ISBN 0-444-85145-3

## STUDIES IN LOGIC AND THE FOUNDATIONS OF MATHEMATICS:

*editors:* J. BARWISE and H. J. KEISLER, Madison;  
P. SUPPES, Stanford; A. S. TROELSTRA, Utrecht; D. KAPLAN, Los Angeles.

Vol. 92: **Classification Theory  
and the Number of  
Non-Isomorphic Models**

by **S. SHELAH**

1978 xvi + 536 pages.

Price: US \$62.25/Dfl. 140.00

ISBN 0-7204-0757-5

Vol. 90: **Handbook of  
Mathematical Logic**

edited by **JON BARWISE**

1978 1st reprint ix + 1165 pages.

Price: US \$84.50/Dfl. 190.00

ISBN 0-7204-2288-X

## MATHEMATICAL PROGRAMMING STUDIES:

*editor-in-chief:* M. L. BALINSKI, New Haven.

Vol. 9: **Mathematical  
Programming in Use**

edited by **M. L. BALINSKI** and  
**C. LEMARECHAL**

1978 vii + 200 pages.

Price: US \$24.00/Dfl. 54.00

ISBN 0-444-85195-X

Vol. 8: **Polyhedral  
Combinatorics**

edited by **M. L. BALINSKI** and  
**A. J. HOFFMAN**

1978 190 pages.

Price: US \$22.00/Dfl. 49.50

ISBN 0-444-85196-8

0659NH

---

## NORTH-HOLLAND PUBLISHING COMPANY

52 Vanderbilt Avenue, New York, N. Y. 10017,  
P. O. Box 211, 1000AE Amsterdam, The Netherlands

---

Please visit our booths, nrs. 33 and 34, at the AMS Annual Meeting in Biloxi.



**Norton**

**Just Published – the Second Edition of**

# **CALCULUS**

by **Leonard Gillman**, *University of Texas, Austin*  
and **Robert H. McDowell**, *Washington University*

Detailed critiques from users of the first edition have helped the authors refine the text and make it suitable for an even wider audience.

928 pages

©1978

(9051)

\$19.95 cloth

**And a new text with a “real-world” approach**

# **FINITE MATHEMATICS**

by **Steven C. Althoen**, *University of Michigan—Flint*  
and **Robert J. Bumcrot**, *Hofstra University*

A practical, class-tested introduction to probability, basic statistics, linear programming, and other important mathematical tools used to solve problems and make decisions in modern business and industry.

582 pages

©1978

(9046)

\$14.95 cloth

For complimentary examination copies, visit the Norton booth at the annual meetings in Biloxi.

**Norton**



**W. W. NORTON & COMPANY, INC.**

500 Fifth Avenue, New York, N. Y. 10036

**Last spring,  
we claimed we had  
“a better brand  
of calculus.”**

**This fall,  
you’re helping us  
prove it.**

**Booth 20**



**HARCOURT  
BRACE  
JOVANOVIICH,  
INC.**

New York • San Diego  
Chicago • San Francisco • Atlanta

## **CALCULUS**

**With Analytic Geometry**

**ROBERT ELLIS** and **DENNY GULICK**,  
both of the University of Maryland  
at College Park

Just published in March of 1978, this introductory textbook for the three-semester course has already been recognized as one of the clearest and best presentations of calculus ever published. Especially designed and written for students in the sciences, engineering and mathematics, *Calculus with Analytic Geometry* provides complete coverage of functions, limits, differentiation and integration in one and several variables, analytic geometry, vectors, and series.

993 pages

Accompanied by a comprehensive two-part *Solutions Manual*

**Part I: Chapters 1 through 10**

Paperbound. 337 pages

**Part II: Chapters 11 through 15**

Paperbound. 218 pages

# The Proof is in the Press

## Princeton's new and forthcoming books

### INFINITE LOOP SPACES

**J. Frank Adams**

*Annals of Mathematics Studies, 90*  
Cloth, \$14.00 Paperback, \$5.50

### SEMINAR ON SINGULARITIES OF SOLUTIONS OF LINEAR PARTIAL DIFFERENTIAL EQUATIONS

**Edited by Lars Hörmander**

*Annals of Mathematics Studies, 91*  
Cloth, \$18.50 Paperback, \$7.50  
(Spring 1979)

### THE CLASSIFYING SPACES FOR SURGERY AND COBORDISM OF MANIFOLDS

**Ib Madsen and R. James Milgram**

*Annals of Mathematics Studies, 92*  
Cloth, \$20.00 Paperback, \$8.50  
(May 1979)

### SEMINAR ON MICRO-LOCAL ANALYSIS

**Victor Guillemin, Masaki Kashiwara,  
and Takahiro Kawai**

*Annals of Mathematics Studies, 93*  
Cloth, \$17.00 Paperback, \$6.50  
(Summer 1979)

### THE MOTION OF A SURFACE BY ITS MEAN CURVATURE

**Kenneth A. Brakke**

*Mathematical Notes, 20*  
\$9.00

### NOTES ON CRYSTALLINE COHOMOLOGY

**Pierre Berthelot and Arthur Ogus**

*Mathematical Notes, 21*  
\$7.50

### ON UNIFORMIZATION OF COMPLEX MANIFOLDS

The Role of Connections

**Robert C. Gunning**

*Mathematical Notes, 22*  
\$5.50

### WORKS ON THE FOUNDATIONS OF STATISTICAL PHYSICS

**Nikolai Sergeevitch Krylov**

Translated by A. B. Migdal, Ya. G. Sinai

Introduction by A. B. Migdal and V. A. Fock

Afterword by Ya. G. Sinai

*Princeton Series in Physics*  
Cloth, \$19.50 Paperback, \$7.50  
(Summer 1979)

### CONVEXITY IN THE THEORY OF LATTICE GASES

**Robert B. Israel**

With an Introduction by Arthur W. Wightman

*Princeton Series in Physics*  
Cloth, \$16.00 Paperback, \$6.95  
(February 1979)

### TWO APPLICATIONS OF LOGIC TO MATHEMATICS

**Gaisi Takeuti**

*Publications of the  
Mathematical Society of Japan, 13*  
\$17.50

*Introducing a New International Monograph  
Series in Astrophysics for Researchers and  
Students, edited by J. P. Ostriker*

### THEORY OF ROTATING STARS

**Jean-Louis Tassoul**

*Princeton Series in Astrophysics, 1*  
Cloth, \$40.00 Paperback, \$15.00  
(January 1979)

### AEPINUS'S ESSAY ON THE THEORY OF ELECTRICITY AND MAGNETISM

Introductory Monograph and Notes

by R. W. Home

Translation by R. J. Conner

\$35.00 (April 1979)

### ELIE CARTAN AND ALBERT EINSTEIN, LETTERS ON ABSOLUTE PARALLELISM

Edited by Robert Debever

\$16.50  
(Spring 1979)

### MATHEMATICAL STATISTICAL MECHANICS

**Colin J. Thompson**

Cloth, \$15.00 Paperback, \$5.95  
(Summer 1979)

Come see our books at Booth 22  
and take advantage of  
our 20% convention discount.

# Princeton University Press

Princeton, New Jersey 08540

THE MATHEMATICS DEPARTMENT  
of the  
OHIO STATE UNIVERSITY

invites applications for several anticipated positions on its regional campuses. These campuses located at Lima, Mansfield, Marion and Newark provide instruction in mathematics at the Freshman and Sophomore levels.

Appointments would be at the rank of Assistant Professor and would require the Ph.D. degree. Since these are "tenure track" positions, continuing research as well as excellent teaching is expected of those appointed.

The Ohio State University is an equal opportunity employer.

Please send inquiries and resumes, and have letters of recommendation sent to:

JOHN RINER, Vice Chairman  
Department of Mathematics  
The Ohio State University  
231 W. 18th Ave.  
Columbus, Ohio 43210

STATE UNIVERSITY OF NEW YORK AT ALBANY

Applications are invited for the position of

**PROFESSOR AND CHAIRPERSON**  
Department of Mathematics and Statistics

The applicant should be an internationally recognized scholar with outstanding research accomplishments and broad professional interests. The chairperson will be expected to provide the Department with leadership, to represent it effectively and to promote the continued strengthening of its programs.

The Department currently has 38 regular faculty members, with research interests in the following broad areas: *Algebra, Algebraic Geometry, Analysis (Real and Complex), Differential Equations, Harmonic Analysis, Probability, Statistics and Topology*. Courses and degrees are offered at all levels. Preference will be given to candidates with research interests compatible with those of the Department.

Position available September 1, 1979.

Send applications with vita and names of four references to:

**PROFESSOR BORIS KORENBLUM**  
Chairman, Search Committee  
Department of Mathematics & Statistics  
SUNY at Albany  
Albany, New York 12222

The State University of New York is an Equal Opportunity/Affirmative Action Employer. Applications from minority persons, women and handicapped persons are particularly welcome.

**CHAIRMAN**  
**DEPARTMENT OF MATHEMATICS AND STATISTICS**

STEPHEN F. AUSTIN STATE UNIVERSITY

Applications invited for position of Chairman of Department of Mathematics and Statistics. Stephen F. Austin is a state supported, multidisciplinary university enrolling approximately 10,500 students. Located in Nacogdoches, Texas, a town of approximately 30,000, Stephen F. Austin offers a wide range of bachelor's and master's degrees and one doctorate.

Department has faculty of twenty-one, twelve doctorate, it offers B. S. and M. S. degrees in pure mathematics, mathematics teaching, and statistics. Chairman retiring after extended period of service.

Qualifications include:

1. Earned doctorate
2. Proven leadership and administrative capability
3. Demonstrated teaching excellence, research accomplishments, and other qualifications for appointment at a senior rank.

Salary: Competitive, 12-month appointment.

Duties begin in Fall Semester of 1979.

Letter of application, resume, three letters of recommendation, and placement credentials postmarked by February 15, 1979.

DR. R. G. DEAN — Chairman of Search Committee  
Department of Mathematics & Statistics  
Stephen F. Austin State University  
Nacogdoches, TX 75962

SFASU is an Affirmative Action, Equal Opportunity Employer.



*"A fine modern treatment of differential and integral calculus."*

—W. Richard Stark, University of Texas, Austin

## **Applied Calculus for Business and Economics**

With an Introduction to Matrices

**Gerald Beer**

California State University, Los Angeles

Gerald Beer's new text offers an important alternative to the short calculus texts and cumbersome mathematical analysis texts instructors must currently choose between. It focuses on those aspects of calculus needed by business and economics students, plus matrices and determinants, sigma notation, and significant elements of the mathematics of finance. It also features an extensive algebra review. Beer de-emphasizes mathematical formalism, stressing instead business and economic terminology and realistic applications.

Cloth approx. 512 pages March 1978

A Solutions Manual is also available.

## **College Algebra with Applications**

**Sabah Al-Hadad**

**C. H. Scott**

Both at California Polytechnic State University

Within a format carefully designed to facilitate learning, Profs. Al-Hadad and Scott cover all topics essential in a pre-calculus algebra course. A distinctive feature of the text is its introduction of a unique structured analytical technique for solving word problems. Unusually extensive exercise sets, graded to allow for flexible use, include a wide variety of *applied* exercises. COLLEGE ALGEBRA WITH APPLICATIONS offers more applications than any competing text.

Cloth approx. 576 pages March 1978

An Instructor's Manual is also available.

*Three new applied work-texts for career-oriented students:*

## **Mathematics for Business Occupations**

Paper approx. 592 pages February 1978

## **Mathematics for Technical Occupations**

Paper approx. 544 pages March 1978

## **Mathematics for Health Occupations**

Paper approx. 528 pages February 1978

by

**Dennis Bila**

Washtenaw Community College

**Ralph Bottorff**

Washtenaw Community College

**Paul Merritt**

Highland Park Community College

**Donald Ross**

Washtenaw Community College

Each of the texts in this eagerly-awaited new series covers the basic mathematical skills students will need in their chosen occupational field. The skills are taught using the vocabulary of that field, and reinforced with real-world applications. The semi-programmed format (developed and extensively class-tested by the authors) allows students with different math backgrounds and abilities to work at their own pace.

A separate Instructional Kit is available for each volume.

**For further information, or to order examination copies, please write to Sara Black, Dept. 1078.**



**WINTHROP PUBLISHERS, INC.**

17 Dunster St., Cambridge, Mass. 02138

**UNIVERSITY OF PETROLEUM AND MINERALS  
DHAHRAN, SAUDI ARABIA**

The University of Petroleum and Minerals, Dhahran, Saudi Arabia, will have graduate faculty positions open for the Academic Year 1979–80 for teaching / research in the areas of *Chemical Engineering, Civil Engineering, Electrical Engineering, Mechanical Engineering, and Mathematics*. The responsibilities include:

- 1) Six contact hours teaching load per semester
- 2) Research in individual's area of specialization
- 3) Advising of Senior projects and MS thesis

Applicants should hold PhD degree and should have at least five years experience in research/graduate education.

Minimum regular contract for two years, renewable. Competitive salaries and allowances, air conditioned and furnished housing provided. Free air transportation to and from Dhahran each two year tour. Attractive educational assistance grants for school-age dependent children. All earned income without Saudi taxes. Ten months duty each year with two months vacation paid and possibility of participation in University's ongoing Summer program with good additional compensation.

Apply with complete resume on *academic and professional background, list of references, a complete list of publications with clear indication of those papers published in refereed professional magazines/journals, research details, and with copies of transcripts/degrees/testimonials, including personal data such as addresses, telephone numbers* to:

**UNIVERSITY OF PETROLEUM & MINERALS  
c/o Saudi Arabian Educational Mission  
2223 West Loop South, Suite 400  
Houston, Texas 77027**

# Addison-Wesley Advanced Book Program

## UNITARY GROUP REPRESENTATIONS

in Physics, Probability, and Number Theory  
*MATHEMATICS LECTURE NOTE SERIES*

George W. Mackey, *Harvard University*

Here are Professor Mackey's well known Oxford lectures finally in book form. The theory of unitary group representations in Hilbert space is a mathematical tool of considerable power and scope. It includes Fourier analysis, spectral theory, and the representation theory of finite groups as special cases and may be regarded as a unification of these. The purpose of this book is to present the main outlines of this theory and then give a rather detailed account of its extensive applications to probability, quantum physics, and number theory.

**Contents:** Introduction. Representations of Finite Commutative Groups. Characters and the Decomposition of Representations of Finite Groups which are not Necessarily Commutative. Preliminaries Concerning Infinite Groups and Measure Spaces. Compact Groups and the Peter-Weyl Theorem. Locally Compact Commutative Groups. Direct Integrals of Unitary Representations in the General Case. Primary Representations and the von Neumann-Murray Theory of Factors. Hilbert G. Bundles, Systems of Imprimitivity and Induced Representations. The Unitary Representations of Semi-Direct Products. Unitary Representations of General Groups Extensions. Simple Groups. The Irreducible Unitary Representations of Compact Simple Groups. The Irreducible Unitary Representations of Noncompact Simple Groups. Ergodic Theory. Applications to Probability Theory. The Foundations of Quantum Mechanics. The Quantum Mechanics of One Particle Systems. Particle Interactions. Bound States and the Structure of Matter. Collision Theory and the S Operator. Radiation, Special Relativity, and Field Theory. Temperature, Heat, and Thermodynamics. Statistical Mechanics. Fourier Analysis and Analytic Functions. Number Theory. Diophantine Equations and Fourier Analysis. Semi-direct Products, Algebraic Number Fields and Diophantine Equations. Quadratic Forms and Automorphic Forms. Automorphic Forms and Group Representations. The Weil-Siegel Formula. Index.

1978, 416 pp., illus.  
Hardbound 36702 \$31.50  
Paperbound 36703 \$19.50

SECOND EDITION,  
Revised, Enlarged, and Reset

## FOUNDATIONS of MECHANICS

Ralph Abraham, *University of California, Santa Cruz*  
Jerrold E. Marsden, *University of California, Berkeley* with the assistance of  
Tudor Ratiu and Richard Cushman

"We have updated the material on symmetry groups and qualitative theory, added new sections on the rigid body, topology and mechanics, and quantization, and other topics, and have made numerous corrections and additions. In fact some of the results in this edition are new." —From authors' Preface

**Contents:** PRELIMINARIES: Differential Theory. Calculus on Manifolds. ANALYTICAL DYNAMICS: Hamiltonian and Lagrangian Systems. Hamiltonian Systems with Symmetry. Hamilton-Jacobi Theory and Mathematical Physics. AN OUTLINE OF QUALITATIVE DYNAMICS: Topological Dynamics. Differentiable Dynamics. Hamiltonian Dynamics. CELESTIAL MECHANICS: The Two-Body Problem. The Three-Body Problem. Appendix. Bibliography. Glossary of Symbols. Index. Numerous references. **Portrait gallery.**

1978, 832 pp., illus.  
Hardbound 30102 \$39.50 (prepublication price  
in effect until Jan. 31, 1979 — thereafter \$44.50)

Prices are quoted in U.S. dollars.  
Outside U.S.A. prices may vary somewhat  
from those listed, reflecting distribution cost  
and currency fluctuations.  
Prices are subject to change without notice.



**Addison-Wesley • Benjamin/Cummings**  
Advanced Book Program, Reading, Massachusetts, 01867, U.S.A.

P.O. Box 363, Crow's Nest N.S.W. 2065, Australia  
West End House, 11 Hills Place, London W1R 2LR, U.K.  
De Lairesestr. 90, Amsterdam 1071, The Netherlands  
36 Prince Andrew Pl., Don Mills, Ontario M3C 2T8, Canada

DEPARTMENT OF MATHEMATICS  
UNIVERSITY OF HOUSTON  
(Central Campus)

is seeking applicants for new faculty positions available at the beginning of the academic year 1979–1980:

1. a senior position (Professor) in Applied Mathematics,
2. a junior position (Assistant Professor, tenure-track) in all areas of Mathematics.

Strong recommendations will be required. Salaries open to negotiation. Please send applications with vitae, resumes, references, and lists of publications to:

**Professor A. Lelek**  
Chairman, Recruiting Committee  
Department of Mathematics  
University of Houston  
Houston, TX 77004

The University of Houston is *an equal opportunity employer*.

**TOPICS IN OPERATOR THEORY**

Edited by Carl Pearcy

Mathematical Surveys, Number 13

1. *Invariant Subspaces* by Donald Sarason
2. *Weighted Shift Operators and Analytic Function Theory* by Allen L. Shields
3. *A Version of Multiplicity Theory* by Arlen Brown
4. *Canonical Models* by R. G. Douglas
5. *A Survey of the Lomonosov Technique in the Theory of Invariant Subspaces* by Carl Pearcy and Allen L. Shields

235 pages, 1974, list price \$28.00; institutional member \$21.00; individual member \$14.00

Orders must be prepaid.  
Specify code SURV/13 when ordering

AMERICAN MATHEMATICAL SOCIETY  
P.O. BOX 1571, ANNEX STATION  
PROVIDENCE, RHODE ISLAND 02901

**THE DEPARTMENT OF MATHEMATICS**  
TEXAS A & M UNIVERSITY

announces openings beginning in the academic year, 1979–80. Significant research accomplishments or exceptional research promise as well as evidence of good teaching ability are expected of the successful applicants. Some preference will be shown to applied mathematicians. Salary and rank will depend on qualifications and experience. Send applications and resumes to:

**PROFESSOR C. J. MAXSON**  
Department of Mathematics  
Texas A & M University  
College Station, Texas 77843

*Equal Employment Opportunity Through Affirmative Action.*

**MATH SCI PRESS**

53 Jordan Road, Brookline, MA 02146  
617-738-0307

**Hilbert's papers on Invariant Theory.**  
Translated by M. Ackerman,  
comments by R. Hermann. \$35.  
(Lie Groups: History, Frontiers & Applications, vol. 8).

**Yang-Mills, Kaluza-Klein, and the Einstein Program,**  
by R. Hermann. \$20.  
(Interdisciplinary Mathematics, vol. 19).

**25% cash discount to individuals.**  
**27 titles in print—write for information.**

# NEW ADDITIONS FOR 1979 FROM THE LEADER IN MATHEMATICS

***Calculus and Analytic Geometry, Fifth Edition*** by George B. Thomas, Jr., Massachusetts Institute of Technology and Ross L. Finney, University of Illinois-Urbana/Champaign

An effective revision of one of the most widely-used calculus texts. Features additional applications to a variety of disciplines, a more concise treatment of topics, several organizational changes, and a general redesign. The authors have now made it easier for students to work with limits throughout the text. Student Supplement, Self-Study Guide, *Pocket Calculator Supplement for Calculus*, and Solutions Manual accompany the text.

07540 COMPLETE HARDBOUND PART I 07541 PART II 07542

***Pocket Calculator Supplement for Calculus*** by J. Barkley Rosser and Carl de Boor, both of University of Wisconsin-Madison

Organized into chapters by calculator topics, this supplement provides exercises for the scientific hand-held calculator.

06502 PAPER

***Calculus of Several Variables, Second Edition*** by Serge Lang, Yale University  
Meticulously revised and reorganized text emphasizing applications. Linear topics are introduced only as needed. Special attention is focused on 2- and 3-dimensions throughout. Contains additional worked-out examples with detailed solutions.

04299 HARDBOUND

***Analytic Geometry, Fifth Edition*** by Gordon Fuller, Texas Tech University  
Strengthened and expanded edition presenting new material on transcendental functions and the intersection of lines. Features expanded exercises, amplified solutions, and many applied problems.

02414 HARDBOUND

***Finite Mathematics with Calculus for the Management, Life, and Social Sciences***  
by William R. Derrick and Judith L. Derrick, University of Montana  
A balanced treatment of concepts and methods of finite mathematics and intuitive calculus. Demonstrates the applications of quantitative methods in each field.

Includes relevant, applicable supplementary materials within the text.

01400 HARDBOUND

If you would like to be considered for a complimentary copy, write to Alfred Walters, Information Services, Addison-Wesley. Please include title of the course, enrollment, and author of the text now in use.

WE WILL NOT BE OUTNUMBERED



Science & Mathematics Division  
ADDISON-WESLEY PUBLISHING COMPANY  
Reading, Mass. 01867

# MATHEMATICS, MISSISSIPPI

Come on by the MIT Press booth at the 1979 AMS meeting in Biloxi

*The monumental EDM—  
mathematics from A  
through Z*

## **Encyclopedic Dictionary of Mathematics**

*by the Mathematical Society  
of Japan*

*edited by Shōkichi Iyanaga  
and Yukiyoji Kawada*

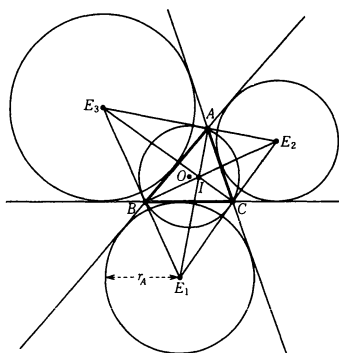
translated by the Mathema-  
tical Society of Japan with the  
cooperation of the American  
Mathematical Society  
translation reviewed by

Kenneth O. May  
two volumes 7 X 11¼ inches  
each

1750 pages

436 articles—appendices—  
27,000 index entries

First published in late 1977,  
the *Encyclopedia Dictionary  
of Mathematics* is already  
established as the definitive  
reference to the whole of  
mathematics, and is now in  
its third printing.  
\$125.00



*New volumes in the series  
Mathematicians of Our  
Time, Gian-Carlo Rota,  
general editor:*

## **Percy Alexander Mac-Mahon: Collected Papers**

Volume I: Combinatorics  
*edited by George E. Andrews*  
\$75.00

## **Oscar Zariski: Collected Papers**

Volume III: Topology of Curves  
and Surfaces, and Special  
Topics in the Theory of  
Algebraic Varieties  
*edited by M. Artin and B.  
Mazur*  
\$35.00

## **Mark Kac: Selected Papers**

Probability, Number Theory,  
and Statistical Physics  
*edited by K. Baclawski and  
M. D. Donsker*  
\$35.00 (available February  
1979)

*An important new text,  
available in early 1979:*

## **Infinitesimal Calculus**

*by James M. Henle and  
Eugene M. Kleinberg*

The authors of this radically  
new text writes that "In the ear-  
ly 1960s the mathematician  
Abraham Robsinon pio-  
neered a body of work known  
as Nonstandard Analysis  
which makes precise and  
mathematically rigorous the  
intuitively pleasing concept of  
infinitesimal. . . .

"A most natural place for  
Robinson's insight is as a next

(and probably final) point in  
the evolution of the teaching of  
calculus. We can now develop  
calculus using infinitesimals  
and enjoy all of their sim-  
plicity and intuitive power, yet  
at the same time work in a  
mathematically precise and  
rigorous atmosphere. . . .

"The only prerequisite  
assumed for this book is a  
good foundation in high  
school mathematics."

The text includes exercises  
and historical notes on the  
development of various con-  
cepts.  
\$12.50

*Two classroom-proved  
texts recently published  
in paperback editions:*

## **An Introduction to Number Theory**

*by Harold M. Stark*  
\$8.95, paperback

## **Ordinary Differential Equations**

*by V. I. Arnold*  
*Translated from the Russian by  
Richard A. Silverman*  
\$8.95, paperback

## **The MIT Press**

**MASSACHUSETTS**  
Institute of Technology  
Cambridge,  
**MASSACHUSETTS**  
02142

# NEW

International Journal  
in Mathematics

## Integral Equations and Operator Theory

### Editor:

I. Gohberg, Tel-Aviv  
University, Ramat-Aviv, Israel

### Editorial Office:

Department of Mathematics  
Tel-Aviv University  
Ramat-Aviv Israel

### Editorial Board:

K. Clancey, Athens - L. Coburn,  
New York R. Douglas, Stony  
Brook - H. Dym, Rehovot - A. Dynin,  
Princeton - P. Fillmore, Halifax -  
P. Fuhrmann, Beer Sheva -  
S. Goldberg, College Park - B.  
Gramsch, Mainz - W. Helton, La  
Jolla - D. Herrero, Caracas -  
M. Kaashoek, Amsterdam -  
T. Kailath, Stanford - S. Kuroda,  
Tokyo - P. Lancaster, Calgary -  
L. Lerer, Haifa - E. Meister, Darm-  
stadt - J. Pincus, Stony Brook -  
M. Rosenblum, Charlottesville -  
D. Sarason, Berkeley - H. Widom,  
Santa Cruz.

### Honorary and Advisory

#### Editorial Board:

P. Halmos, Santa Barbara - E. Hille,  
La Jolla - T. Kato, Berkeley -  
R. Phillips, Stanford. B. Sz.-Nagy,  
Szeged.

### Subscription Information:

1978. Volume 1 (4 issues, 600 pages  
per year) sFr. 96.-; about US \$ 48.-

Please ask for a specimen copy  
(free of charge)

Please order through your  
bookseller, news-agent,  
or directly from the publisher  
Birkhäuser Verlag, Subscription  
Department, P.O. Box 34,  
CH-4010 Basel/Switzerland

### Contents

#### Volume 1 (1978) No. 1

*Bart H., Gohberg I., Kaashoek M. A.* Operator polynomials as inverses of characteristic functions  
*Den Boer B.* Linearization of operator functions on arbitrary open sets  
*Grabmüller H.* Asymptotic behaviour of solutions of linear integro-differential equations  
*Herrero D. A.* On multicyclic operators  
*Kato T.* Remarks on Schrödinger operators with vector potentials  
*Mitiagin B.* Linearization of holomorphic operator functions I.  
Short Communications:  
*Anderson B. D. O. / Kailath T.* Fast algorithms for the integral equations of the inverse scattering problem

The first and second issues have already appeared

#### Volume 1 (1978) No. 2

*Costabel M.* A singular integral operator related to the one-sided Hilbert transformation  
*Dym H.* Trace formulas for pair operators  
*Gohberg I., Lerer L., Rodman L.* On canonical factorization of operator polynomials, spectral divisors and Toeplitz matrices  
*Hille E.* On spectral Weyl-Titchmarsh problem  
*Mitiagin B.* Linearization of holomorphic operator-functions II  
*Sunder V. S.* Two characterization theorems for integral operators  
Short Communications:  
*Dym H., Kravitsky N.* On the inverse spectral problem for the string equation  
*Gohberg I., Kaashoek M. A.* Unsolved problems in matrix and operator theory, I. Partial multiplicities and additive perturbations.

## Birkhäuser Verlag Basel and Stuttgart

## HISTORY OF ANALYSIS

Edited by *R. J. Stanton* and *R. O. Wells, Jr.*  
Rice University Studies, Vol. 64, Nos. 2 and 3,  
1978, approx. 228 p.

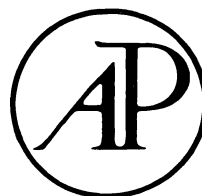
This volume contains the proceedings of a conference on the history of analysis held 12–13 March 1977 at Rice University. The papers reflect the diverse ways to approach the history of mathematics: the role of mathematics in societies in which it has existed, the origins of mathematical concepts, the detailed process of discovery in mathematics, and a retrospective survey exposing common themes in mathematics and science.

### CONTENTS

- Mathematics and Society — A Historical View  
by *Felix E. Browder*
- The Emergence of Analysis in the Renaissance  
and After  
by *Salomon Bochner*
- The Creation of the Theory of Group Characters  
by *Thomas Hawkins*
- Harmonic Analysis as the Exploitation of Symmetry — A Historical Survey  
by *George W. Mackey*

Order from:                      Price: \$11.50 including  
Rice Campus Store            handling (Texas resi-  
Rice University                dents add 55¢ sales  
Houston, TX 77001            tax)

# THE WINNER'S CIRCLE



## THE FULL SPECTRUM OF QUALITY TEXTBOOKS IN MATHEMATICS

### ACADEMIC PRESS, INC.

*A Subsidiary of  
Harcourt Brace Jovanovich, Publishers*  
111 FIFTH AVENUE, NEW YORK, N.Y. 10003

## FUNCTION THEORY ON THE UNIT CIRCLE

BY DONALD SARASON

These notes (138 pages), based on a series of 10 lectures given by the author at a conference at Virginia Polytechnic Institute and State University, are an introduction to certain recent developments in harmonic analysis. This includes some basic background material on the conjugation operator and the  $H^p$  spaces, and a discussion of topics of more recent origin such as functions of bounded mean oscillation and of vanishing mean oscillation, Douglas algebras, boundedness of the conjugation operator in spaces with weights, Hankel and Toeplitz operators and related approximation problems. The notes are intended to be accessible to graduate students with a basic knowledge of real, complex and functional analysis.

To order send the completed form below with check or money order for \$6 per copy (\$8 outside U. S. A. and Canada) to:

VIRGINIA POLYTECHNIC INSTITUTE and STATE UNIVERSITY  
ATTN: SHARON IRVIN  
DEPARTMENT OF MATHEMATICS  
BLACKSBURG, VIRGINIA 24061

NAME \_\_\_\_\_

No. of copies  
requested \_\_\_\_\_

ADDRESS \_\_\_\_\_

Total remittance  
enclosed \_\_\_\_\_



**NEW**

**Mathematics Revealed**

ELIZABETH BERMAN

*University of Missouri—Kansas City*

Demystify math for even your most math-anxious students with this solid introduction to arithmetic, geometry, and the rudiments of algebra. Practical consumer-oriented problems and examples bring math down to earth for the beginning student.

1979 about 525 pages paper

**Elementary Algebra**

CHARLES P. McKEAGUE

*Cuesta College*

Lots of word problems; a comfortable, conversational style; and abundant exercises, graded in difficulty are the winning combination that helps your students overcome their fear of mathematics.

1978 312 pages

**NEW**

**Intermediate Algebra**

CHARLES P. McKEAGUE

*Cuesta College*

In this new text, McKeague writes with the same 'straight & simple' style that made ELEMENTARY ALGEBRA so popular with students and instructors alike. His supportive step-by-step style gives students the confidence they need to master the fundamentals of intermediate algebra.

1979 about 450 pages

**Applied Finite Mathematics with Calculus**

HOWARD ANTON and BERNARD KOLMAN  
*both, Drexel University*

Now, the number one applied finite math text is available with an outstanding five chapter introduction to calculus.

1978 673 pages

**Applied Finite Mathematics**

**Second Edition**

HOWARD ANTON and BERNARD KOLMAN  
*both, Drexel University*

The most widely used textbook in the field, this revision offers a new chapter on *Mathematics of Finance*, a revised chapter on computers—using BASIC, and a significantly rewritten chapter on probability.

1978 487 pages

**NEW**

*To accompany both Anton/Kolman texts:*

**Algebra Review**

CHARLES DENLINGER and  
ELAINE JACOBSON

Perfect for in-class use or self-study, this booklet reviews the fundamentals of high school algebra.

1978 about 120 pages

**Mathematics:**

**Ideas and Applications**

DANIEL D. BENICE

*Montgomery College*

A captivating tour of mathematics, its history and development, its role and applications in our world. Perfect for the liberal arts student.

1978 430 pages

**Calculus**

STANLEY I. GROSSMAN

*University of Montana*

Retained by more than 90% of its original adopters, Grossman's CALCULUS offers 983 fully worked examples, nearly 6000 problems, a vast array of exciting applications, and an intuitive approach.

1977 1005 pages

**Calculus with Analytic Geometry**

HARLEY FLANDERS and JUSTIN PRICE

Two leading educators share their vast experience and insights into the field in this refreshing introduction to calculus. Now available in a larger, more open format.

1978 951 pages

**NEW**

**Boundary Value Problems**

**Second Edition**

DAVID L. POWERS

*Clarkson College*

Remember what the *American Math Monthly* said of the first edition? "It was easy and fun for students to learn, and at the same time perceive how the material presented all nicely tied together . . . Powers' presentations on the derivation of each equation, the boundary conditions and regions which allow the method of separation of variables to work, and the formulation of problems in terms of dimensionless units are excellent."

With 57% more problems, twice as many diagrams, and a new introductory review chapter, the new edition is a sure success.

1979 about 360 pages



For your complimentary examination copy of any of these titles, write to:

**College Department/A-2**

**ACADEMIC PRESS, INC.**

*A Subsidiary of  
Harcourt Brace Jovanovich, Publishers*  
111 FIFTH AVENUE, NEW YORK, N.Y. 10003  
*Please state course title, enrollment,  
and present text.*

# ROOM RESERVATION FORM

NEW YORK CITY MEETING  
THE BILTMORE HOTEL  
APRIL 19-20, 1979

The Biltmore Hotel in New York City is the official headquarters hotel. Requests for housing accommodations should be sent directly to the Room Reservations Office, The Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York 10017. Be sure to specify the names of persons for whom the reservation is being made. Bring the confirmation slip with you as proof of your reservation. Unless otherwise requested, the hotel will hold reservations until 6:00 p.m. on the day of your arrival. Checkout time is 1:00 p.m.

In order to obtain the special rates listed below, please copy or tear out this form and send it to the hotel for your room reservation. THE DEADLINE FOR RESERVATIONS IS APRIL 4, 1979.

## RATES

Single	\$36	Single occupancy
Twin/Double	\$42	Double occupancy

NOTE: The above rates are subject to the current New York City sales tax of eight percent, and \$1 per night occupancy tax.

-----

MAIL RESERVATION TO: Room Reservations Office  
The Biltmore Hotel  
Madison Avenue at 43rd Street  
New York, New York 10017

Please enter my reservation for the April 19-20, 1979 meeting of the AMERICAN MATHEMATICAL SOCIETY.

_____	Single room(s)	@ \$36
_____	Twin room(s)	@ \$42
_____	Double room(s)	@ \$42

Name (please print) \_\_\_\_\_

Address \_\_\_\_\_

\_\_\_\_\_

I will share a room with \_\_\_\_\_  
(provide name, if other than spouse)

I/We will arrive on \_\_\_\_\_ at \_\_\_\_\_ a.m./p.m.  
and depart on \_\_\_\_\_.

# Differential Geometry, Lie Groups, and Symmetric Spaces

By SIGURDUR HELGASON

A Volume in the PURE AND APPLIED MATHEMATICS Series

CONTENTS: (Chapter Heads): Elementary Differential Geometry. Lie Groups and Lie Algebras. Structure of Semisimple Lie Algebras. Symmetric Spaces. Decomposition of Symmetric Spaces. Symmetric Spaces of the Noncompact Type. Symmetric Spaces of

the Compact Type. Hermitian Symmetric Spaces. Structure of Semisimple Lie Groups. The Classification of Simple Lie Algebras and of Symmetric Spaces.

1978, about 628 pp., \$27.00

ISBN: 0-12-338460-5

## Stochastic Analysis

Edited by AVNER FRIEDMAN and MARK PINSKY

This collection of nineteen articles dealing with stochastic differential equations and their applications is based on the International Conference on Stochastic Analysis held in April 1978 at Northwestern University. The conference brought together leading workers in stochastic differential equations and related areas. Specific areas covered include foundations, stochastic optimization, stochastic differential geometry,

limit theorems, and martingale methods. Most papers report on work done in the past few years, and tend to be expository. Providing a good perspective on the present stage of this new interdisciplinary field, *Stochastic Analysis* will interest mathematicians as well as electrical and industrial engineers.

1978, 336 pp., \$18.00 ISBN: 0-12-268380-3

## Continuation Methods

*Proceedings of a Symposium at the University of Linz, Austria October 3-4, 1977*

Edited by HANSJÖRG WACKER

CONTENTS: *Hj. Wacker*, A Summary on Imbedding Methods. *J. C. Alexander*, The Topological Theory of an Imbedding Method. *F. J. Drexler*, A Homotopy Method for the Calculation of All Zeroes of Zero-Dimensional Polynomial Ideals. *J. Hackl*, Solution of Optimization Problems With Nonlinear Restrictions via Continuation Methods. *G. Heindl*, Some Remarks on the Convergence of Descent Methods. *H. Jeggle*, Existence and Discrete Approximation of Bifurcation

Points. *M. Prüfer*, Calculating Global Bifurcation. *J. W. Schmidt*, Selected Contributions to Imbedding Methods for Finite Dimensional Problems. *Hj. Wacker et al.*, Optimal Stepsize Control for the Globalized Newton Method. *W. L. Wendland*, On the Imbedding Method for Semilinear First Order Elliptic Systems and Related Finite Element Methods.

1978, 352 pp., \$19.50 ISBN: 0-12-729250-0

## Nonlinear Evolution Equations

Edited by MICHAEL G. CRANDALL

*Proceedings of a Symposium, Conducted by the Mathematics Research Center*

*The University of Wisconsin Madison October 17-19, 1977*

CONTENTS: *R. J. DiPerna*, Entropy and the Uniqueness of Solutions to Hyperbolic Conservation Laws. *A. J. Chorin*, Computational Aspects of Glimm's Method. *T. Nishida*, The Initial Value Problem of the Boltzmann Equation and Its Fluid Dynamical Limit at the Level of Compressible Euler Equation. *J. L. Lions*, On Some Problems Connected With Navier Stokes Equations. *W. A. Strauss*, Everywhere Defined Wave Operators. *C. M. Dafermos*, Asymptotic Behavior of Solutions of Evolution Equations. *P. C. Fife*, Results and Open Questions in the Asymptotic The-

ory of Reaction-Diffusion Equations. *H. Brezis*, Asymptotic Behavior of Some Evolution Systems. *T. Kato*, Trotter's Product Formula for Some Nonlinear Semigroups. *L. C. Evans*, Application of Nonlinear Semigroup Theory to Certain Partial Differential Equations. *J. M. Ball*, Finite Time Blow-Up in Nonlinear Problems. *P. D. Lax*, A Hamiltonian Approach to the KDV and Other Equations. *P. H. Rabinowitz*, A Variational Method for Finding Periodic Solutions of Differential Equations.

1978, 272 pp., \$11.00 ISBN: 0-12-195250-9

Send payment with order and save postage and handling charge.

Prices are subject to change without notice.

**ACADEMIC PRESS, INC.**

A Subsidiary of Harcourt Brace Jovanovich, Publishers

111 FIFTH AVENUE, NEW YORK, N.Y. 10003

24-28 OVAL ROAD, LONDON NW1 7DX

Now available in North America from  
Springer-Verlag, New York

## PUBLICATIONS MATHÉMATIQUES

An International Journal of the  
**Institut Des Hautes Études Scientifiques**

Editor: Jean Dieudonné

Advisory committee: Jacques Tits, Pierre Deligne,  
Dennis Sullivan, René Thom.

Published by Presses Universitaires de France, Paris

Starting in 1979 the Publications Mathématiques will appear as a regular journal. Approximately 400 pages will be published each year in two hardcover issues.

### Recent issues included:

- P. Deligne, *La conjecture de Weil, I.*
- D. Sullivan, *A counterexample to the periodic orbit conjecture.*
- B. Mazur, *Modular curves and the Eisenstein ideal.*
- R. Connelly, *A counterexample to the rigidity conjecture for polyhedra.*
- D. Ruelle, *Integral representation of measures associated with a foliation.*

### Subscription Information

Issues 49-50 1979:

Institutional rate: \$48.50

Personal rate: \$32.50

Subscribers for 1979 may take advantage of the following special offer:

Issues 48-50

Institutional rate: \$69.00

Personal rate: \$46.00

Individual issues are available separately.

Subscriptions are entered with prepayment only. Please add \$1.00 per issue for postage and handling. Prices subject to change without notice.

Send your order or request to:

**Springer-Verlag New York Inc.**  
175 Fifth Avenue  
New York, NY 10010

TIME VALUE

AMERICAN MATHEMATICAL SOCIETY

P. O. Box 6248

Providence, Rhode Island 02940

Return Postage Guaranteed