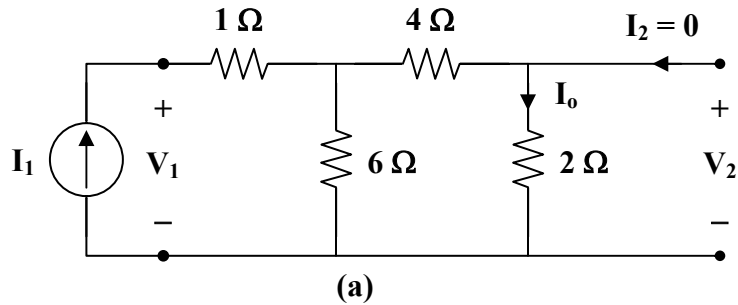


### Chapter 19, Solution 1.

To get  $z_{11}$  and  $z_{21}$ , consider the circuit in Fig. (a).

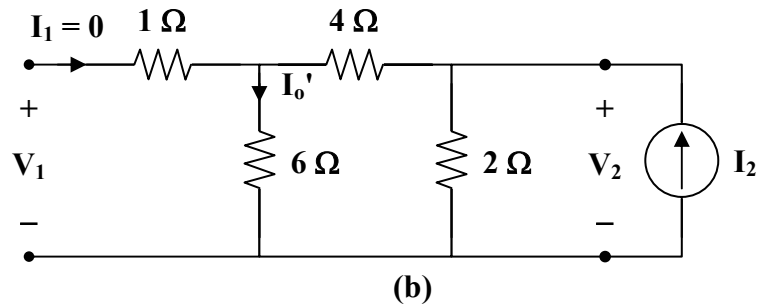


$$z_{11} = \frac{V_1}{I_1} = 1 + 6 \parallel (4 + 2) = 4 \Omega$$

$$I_o = \frac{1}{2} I_1, \quad V_2 = 2 I_o = I_1$$

$$z_{21} = \frac{V_2}{I_1} = 1 \Omega$$

To get  $z_{22}$  and  $z_{12}$ , consider the circuit in Fig. (b).



$$z_{22} = \frac{V_2}{I_2} = 2 \parallel (4 + 6) = 1.667 \Omega$$

$$I_o' = \frac{2}{2+10} I_2 = \frac{1}{6} I_2, \quad V_1 = 6 I_o' = I_2$$

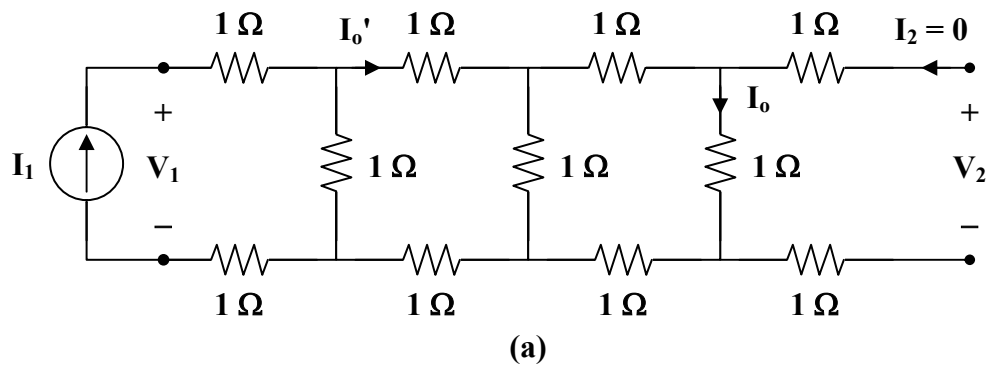
$$z_{12} = \frac{V_1}{I_2} = 1 \Omega$$

Hence,

$$[\mathbf{z}] = \underline{\underline{\begin{bmatrix} 4 & 1 \\ 1 & 1.667 \end{bmatrix} \Omega}}$$

**Chapter 19, Solution 2.**

Consider the circuit in Fig. (a) to get  $\mathbf{z}_{11}$  and  $\mathbf{z}_{21}$ .



$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 2 + 1 \parallel [2 + 1 \parallel (2 + 1)]$$

$$\mathbf{z}_{11} = 2 + 1 \parallel \left(2 + \frac{3}{4}\right) = 2 + \frac{(1)(11/4)}{1 + 11/4} = 2 + \frac{11}{15} = 2.733$$

$$\mathbf{I}_o = \frac{1}{1+3} \mathbf{I}_o' = \frac{1}{4} \mathbf{I}_o'$$

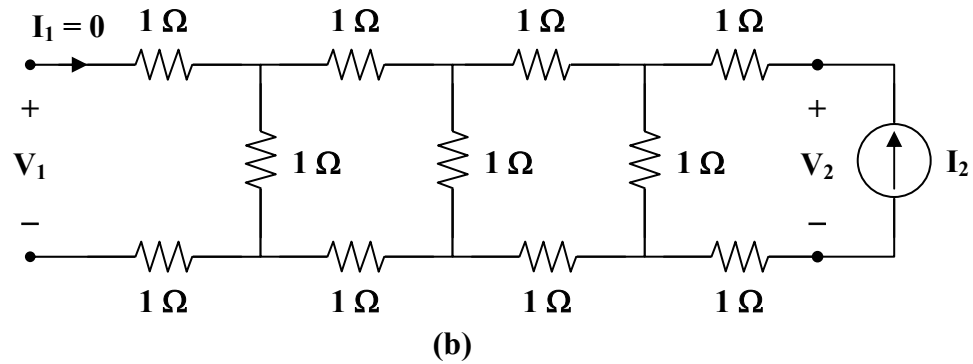
$$\mathbf{I}_o' = \frac{1}{1+11/4} \mathbf{I}_1 = \frac{4}{15} \mathbf{I}_1$$

$$\mathbf{I}_o = \frac{1}{4} \cdot \frac{4}{15} \mathbf{I}_1 = \frac{1}{15} \mathbf{I}_1$$

$$\mathbf{V}_2 = \mathbf{I}_o = \frac{1}{15} \mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{1}{15} = \mathbf{z}_{12} = 0.06667$$

To get  $z_{22}$ , consider the circuit in Fig. (b).



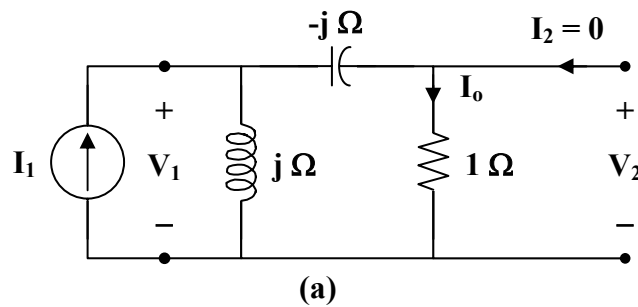
$$z_{22} = \frac{V_2}{I_2} = 2 + 1 \parallel (2 + 1 \parallel 3) = z_{11} = 2.733$$

Thus,

$$[z] = \begin{bmatrix} 2.733 & 0.06667 \\ 0.06667 & 2.733 \end{bmatrix} \Omega$$

### Chapter 19, Solution 3.

(a) To find  $z_{11}$  and  $z_{21}$ , consider the circuit in Fig. (a).



$$z_{11} = \frac{V_1}{I_1} = j \parallel (1 - j) = \frac{j(1-j)}{j+1-j} = 1 + j$$

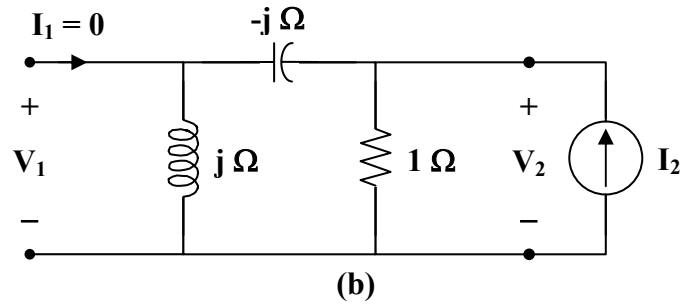
By current division,

$$I_o = \frac{j}{j+1-j} I_1 = j I_1$$

$$\mathbf{V}_2 = \mathbf{I}_o = j\mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = j$$

To get  $\mathbf{z}_{22}$  and  $\mathbf{z}_{12}$ , consider the circuit in Fig. (b).



$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = 1 \parallel (j - j) = 0$$

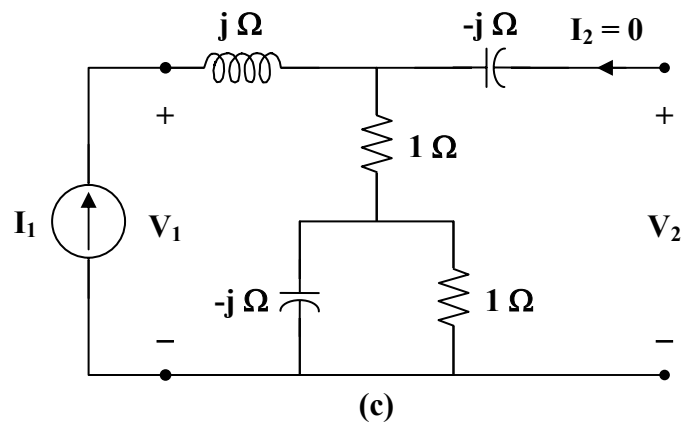
$$\mathbf{V}_1 = j\mathbf{I}_2$$

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = j$$

Thus,

$$\underline{\mathbf{z}} = \begin{bmatrix} 1+j & j \\ j & 0 \end{bmatrix} \Omega$$

(b) To find  $\mathbf{z}_{11}$  and  $\mathbf{z}_{21}$ , consider the circuit in Fig. (c).

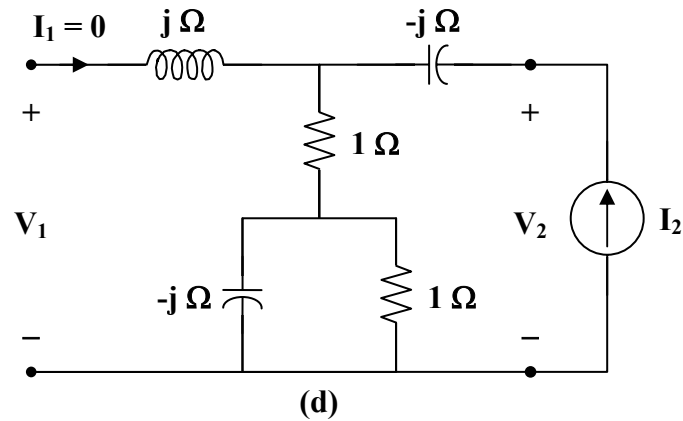


$$z_{11} = \frac{V_1}{I_1} = j + 1 + 1 \parallel (-j) = 1 + j + \frac{-j}{1-j} = 1.5 + j0.5$$

$$V_2 = (1.5 - j0.5)I_1$$

$$z_{21} = \frac{V_2}{I_1} = 1.5 - j0.5$$

To get  $z_{22}$  and  $z_{12}$ , consider the circuit in Fig. (d).



$$z_{22} = \frac{V_2}{I_2} = -j + 1 + 1 \parallel (-j) = 1.5 - j1.5$$

$$V_1 = (1.5 - j0.5)I_2$$

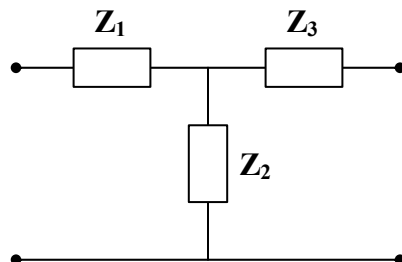
$$z_{12} = \frac{V_1}{I_2} = 1.5 - j0.5$$

Thus,

$$[z] = \begin{bmatrix} 1.5 + j0.5 & 1.5 - j0.5 \\ 1.5 - j0.5 & 1.5 - j1.5 \end{bmatrix} \Omega$$

#### Chapter 19, Solution 4.

Transform the  $\Pi$  network to a T network.



$$\mathbf{Z}_1 = \frac{(12)(j10)}{12 + j10 - j5} = \frac{j120}{12 + j5}$$

$$\mathbf{Z}_2 = \frac{-j60}{12 + j5}$$

$$\mathbf{Z}_3 = \frac{50}{12 + j5}$$

The z parameters are

$$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{Z}_2 = \frac{(-j60)(12 - j5)}{144 + 25} = -1.775 - j4.26$$

$$\mathbf{z}_{11} = \mathbf{Z}_1 + \mathbf{z}_{12} = \frac{(j120)(12 - j5)}{169} + \mathbf{z}_{12} = 1.775 + j4.26$$

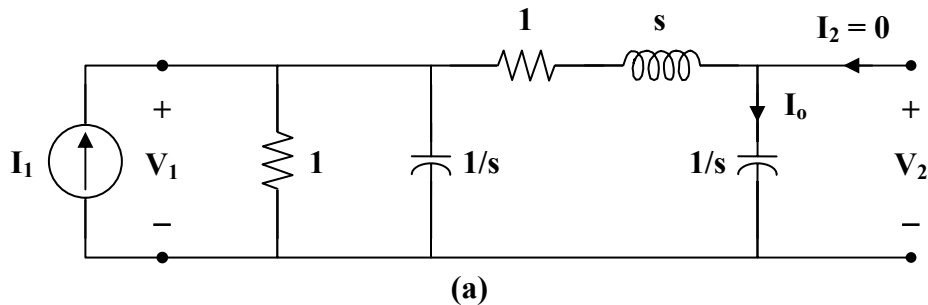
$$\mathbf{z}_{22} = \mathbf{Z}_3 + \mathbf{z}_{21} = \frac{(50)(12 - j5)}{169} + \mathbf{z}_{21} = 1.7758 - j5.739$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} 1.775 + j4.26 & -1.775 - j4.26 \\ -1.775 - j4.26 & 1.775 - j5.739 \end{bmatrix} \Omega$$

### Chapter 19, Solution 5.

Consider the circuit in Fig. (a).



$$\mathbf{z}_{11} = 1 \parallel \frac{1}{s} \parallel \left(1 + s + \frac{1}{s}\right) = \frac{1}{1 + \frac{1}{s}} \parallel \left(1 + s + \frac{1}{s}\right) = \frac{\left(\frac{1}{s+1}\right)\left(1 + s + \frac{1}{s}\right)}{\left(\frac{1}{s+1}\right) + 1 + s + \frac{1}{s}}$$

$$z_{11} = \frac{s^2 + s + 1}{s^3 + 2s^2 + 3s + 1}$$

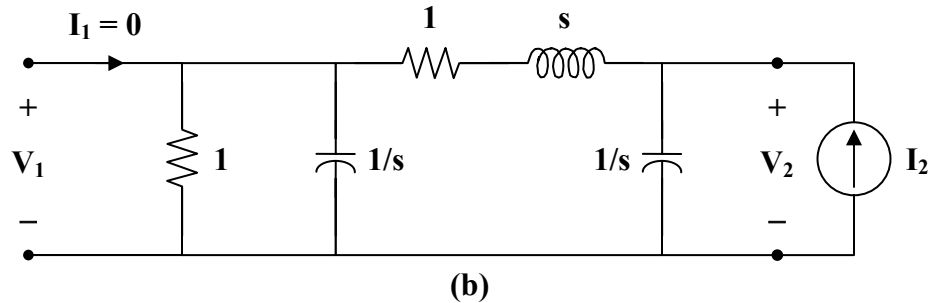
$$I_o = \frac{1 \parallel \frac{1}{s}}{1 \parallel \frac{1}{s} + 1 + s + \frac{1}{s}} I_1 = \frac{\frac{1}{s+1}}{\frac{1}{s+1} + 1 + s + \frac{1}{s}} I_1 = \frac{\frac{s}{s+1}}{\frac{s}{s+1} + s^2 + s + 1} I_1$$

$$I_o = \frac{s}{s^3 + 2s^2 + 3s + 1} I_1$$

$$V_2 = \frac{1}{s} I_o = \frac{I_1}{s^3 + 2s^2 + 3s + 1}$$

$$z_{21} = \frac{V_2}{I_1} = \frac{1}{s^3 + 2s^2 + 3s + 1}$$

Consider the circuit in Fig. (b).



$$z_{22} = \frac{V_2}{I_2} = \frac{1}{s} \parallel \left( 1 + s + 1 \parallel \frac{1}{s} \right) = \frac{1}{s} \parallel \left( 1 + s + \frac{1}{s+1} \right)$$

$$z_{22} = \frac{\left( \frac{1}{s} \right) \left( 1 + s + \frac{1}{s+1} \right)}{\frac{1}{s} + 1 + s + \frac{1}{s+1}} = \frac{1 + s + \frac{1}{s+1}}{1 + s + s^2 + \frac{s}{s+1}}$$

$$z_{22} = \frac{s^2 + 2s + 2}{s^3 + 2s^2 + 3s + 1}$$

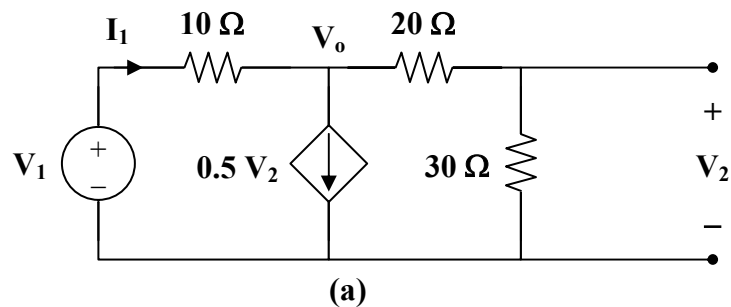
$$z_{12} = z_{21}$$

Hence,

$$[z] = \begin{bmatrix} \frac{s^2 + s + 1}{s^3 + 2s^2 + 3s + 1} & \frac{1}{s^3 + 2s^2 + 3s + 1} \\ \frac{1}{s^3 + 2s^2 + 3s + 1} & \frac{s^2 + 2s + 2}{s^3 + 2s^2 + 3s + 1} \end{bmatrix}$$

**Chapter 19, Solution 6.**

To find  $z_{11}$  and  $z_{21}$ , connect a voltage source  $V_1$  to the input and leave the output open as in Fig. (a).



$$\frac{V_1 - V_o}{10} = 0.5 V_2 + \frac{V_o}{50}, \quad \text{where } V_2 = \frac{30}{20 + 30} V_o = \frac{3}{5} V_o$$

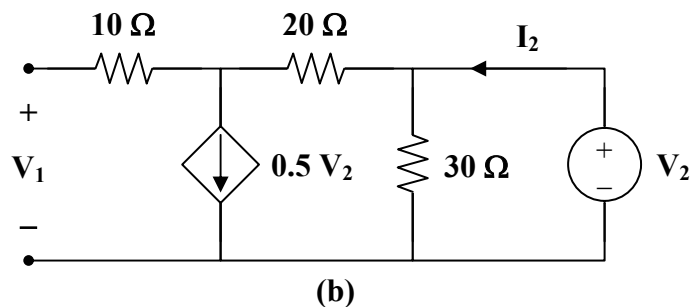
$$V_1 = V_o + 5 \left( \frac{3}{5} V_o \right) + \frac{V_o}{5} = 4.2 V_o$$

$$I_1 = \frac{V_1 - V_o}{10} = \frac{3.2}{10} V_o = 0.32 V_o$$

$$z_{11} = \frac{V_1}{I_1} = \frac{4.2 V_o}{0.32 V_o} = 13.125 \Omega$$

$$z_{21} = \frac{V_2}{I_1} = \frac{0.6 V_o}{0.32 V_o} = 1.875 \Omega$$

To obtain  $z_{22}$  and  $z_{12}$ , use the circuit in Fig. (b).





$$I_2 = 0.5V_2 + \frac{V_2}{30} = 0.5333V_2$$

$$z_{22} = \frac{V_2}{I_2} = \frac{1}{0.5333} = 1.875 \Omega$$

$$V_1 = V_2 - (20)(0.5V_2) = -9V_2$$

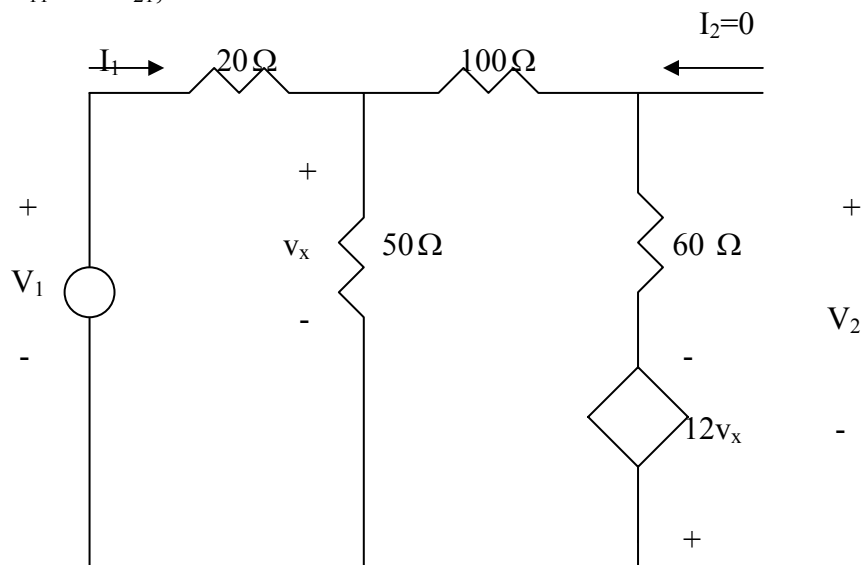
$$z_{12} = \frac{V_1}{I_2} = \frac{-9V_2}{0.5333V_2} = -16.875 \Omega$$

Thus,

$$[z] = \begin{bmatrix} 13.125 & -16.875 \\ 1.875 & 1.875 \end{bmatrix} \Omega$$

### Chapter 19, Solution 7.

To get  $z_{11}$  and  $z_{21}$ , we consider the circuit below.



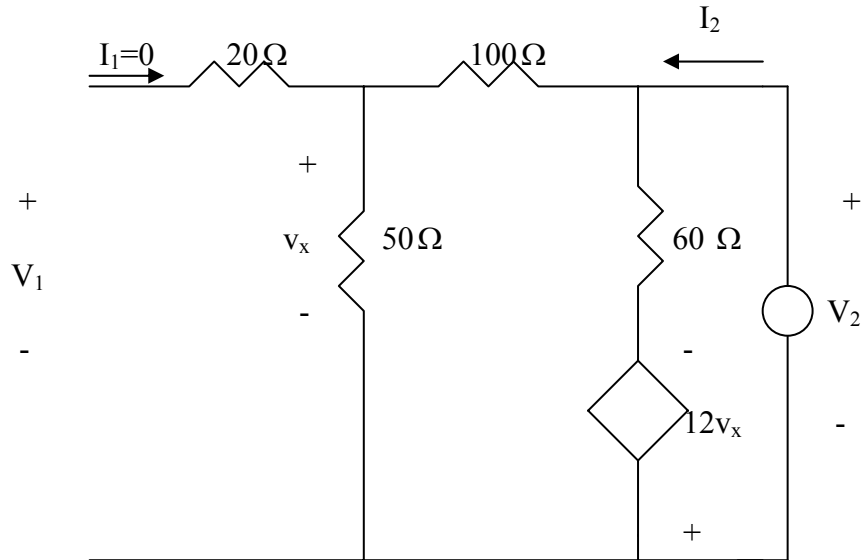
$$\frac{V_1 - V_x}{20} = \frac{V_x}{50} + \frac{V_x + 12V_x}{160} \longrightarrow V_x = \frac{40}{121}V_1$$

$$I_1 = \frac{V_1 - V_x}{20} = \frac{81}{121} \left( \frac{V_1}{20} \right) \longrightarrow z_{11} = \frac{V_1}{I_1} = 29.88$$

$$V_2 = 60\left(\frac{13V_x}{160}\right) - 12V_x = -\frac{57}{8}V_x = -\frac{57}{8}\left(\frac{40}{121}\right)V_1 = -\frac{57}{8}\left(\frac{40}{121}\right)\frac{20 \times 121}{81}I_1$$

$$= -70.37I_1 \longrightarrow z_{21} = \frac{V_2}{I_1} = -70.37$$

To get  $z_{12}$  and  $z_{22}$ , we consider the circuit below.



$$V_x = \frac{50}{100 + 50}V_2 = \frac{1}{3}V_2, \quad I_2 = \frac{V_2}{150} + \frac{V_2 + 12V_x}{60} = 0.09V_2$$

$$z_{22} = \frac{V_2}{I_2} = 1/0.09 = 11.11$$

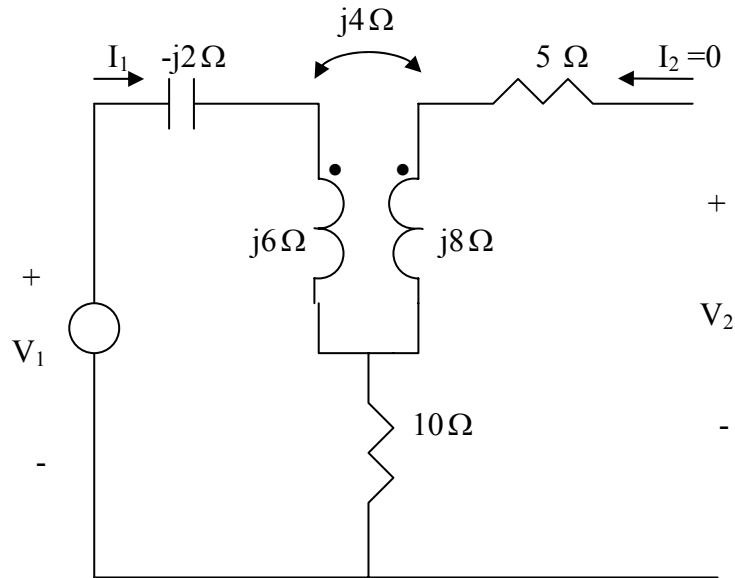
$$V_1 = V_x = \frac{1}{3}V_2 = \frac{11.11}{3}I_2 = 3.704I_2 \longrightarrow z_{12} = \frac{V_1}{I_2} = 3.704$$

Thus,

$$[z] = \begin{bmatrix} 29.88 & 3.704 \\ -70.37 & 11.11 \end{bmatrix} \Omega$$

**Chapter 19, Solution 8.**

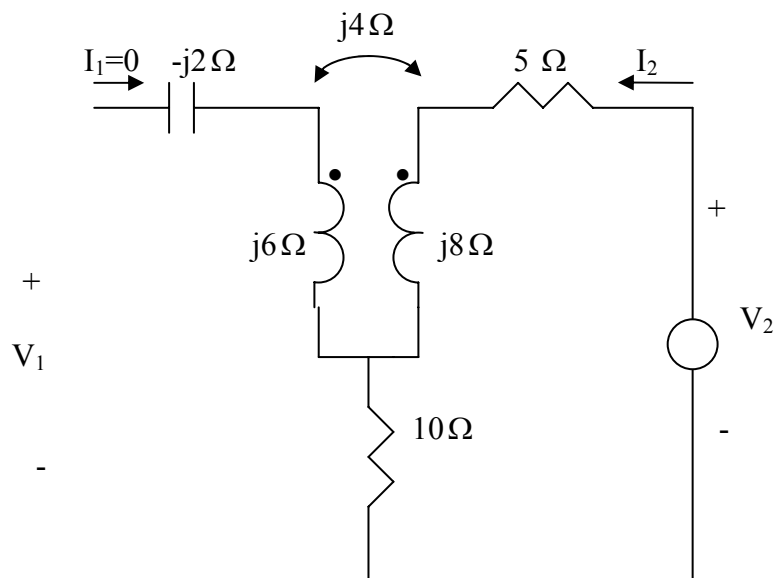
To get  $z_{11}$  and  $z_{21}$ , consider the circuit below.



$$V_1 = (10 - j2 + j6)I_1 \quad \longrightarrow \quad z_{11} = \frac{V_1}{I_1} = 10 + j4$$

$$V_2 = -10I_1 - j4I_1 \quad \longrightarrow \quad z_{21} = \frac{V_2}{I_1} = -(10 + j4)$$

To get  $z_{22}$  and  $z_{12}$ , consider the circuit below.



$$V_2 = (5 + 10 + j8)I_2 \quad \longrightarrow \quad z_{22} = \frac{V_2}{I_2} = 15 + j8$$

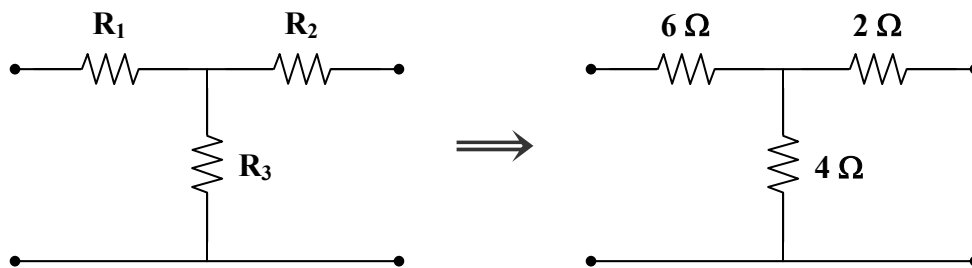
$$V_1 = -(10 + j4)I_2 \quad \longrightarrow \quad z_{12} = \frac{V_1}{I_2} = -(10 + j4)$$

Thus,

$$[z] = \begin{bmatrix} (10 + j4) & -(10 + j4) \\ -(10 + j4) & (15 + j8) \end{bmatrix} \Omega$$

### Chapter 19, Solution 9.

It is evident from Fig. 19.5 that **a T network is appropriate for realizing the z parameters.**



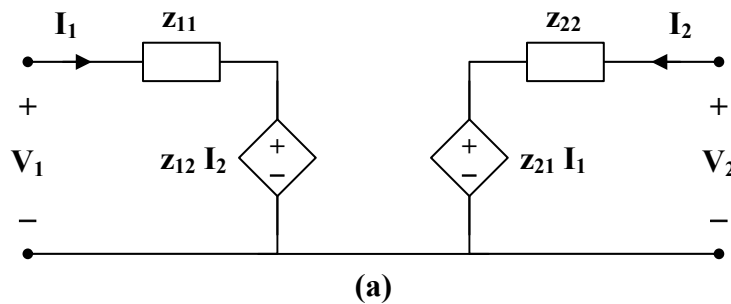
$$R_1 = z_{11} - z_{12} = 10 - 4 = \underline{6 \Omega}$$

$$R_2 = z_{22} - z_{12} = 6 - 4 = \underline{2 \Omega}$$

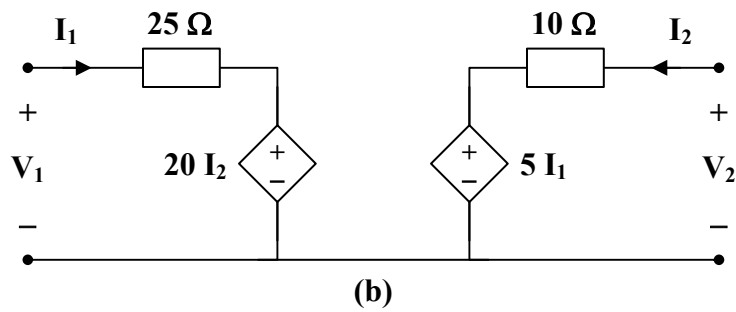
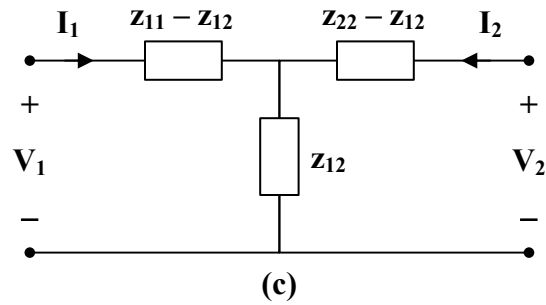
$$R_3 = z_{12} = z_{21} = \underline{4 \Omega}$$

### Chapter 19, Solution 10.

- (a) This is a non-reciprocal circuit so that **the two-port looks like the one shown in Figs. (a) and (b).**



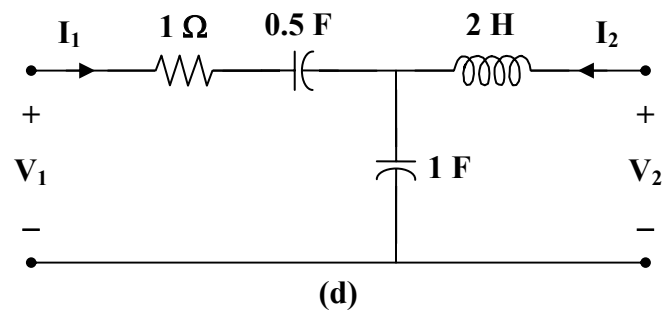
- (b) This is a reciprocal network and the two-port look like the one shown in Figs. (c) and (d).



$$z_{11} - z_{12} = 1 + \frac{2}{s} = 1 + \frac{1}{0.5s}$$

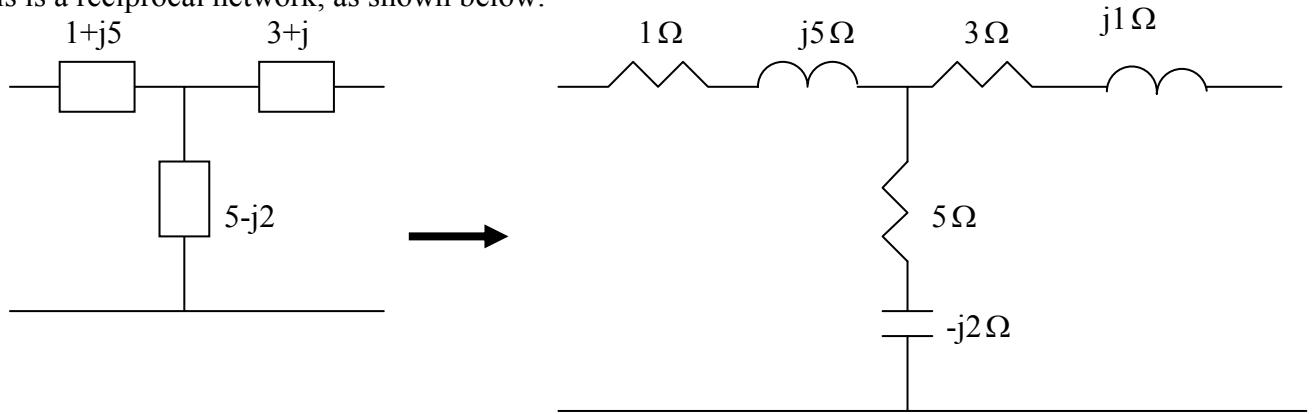
$$z_{22} - z_{12} = 2s$$

$$z_{12} = \frac{1}{s}$$



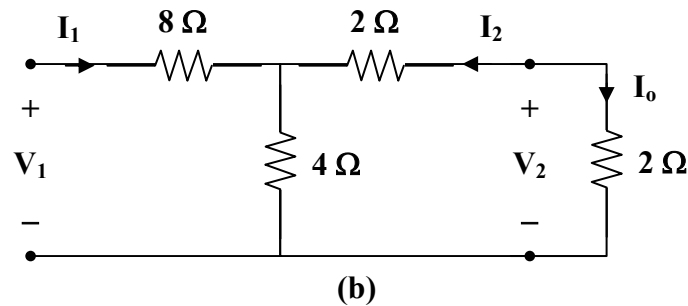
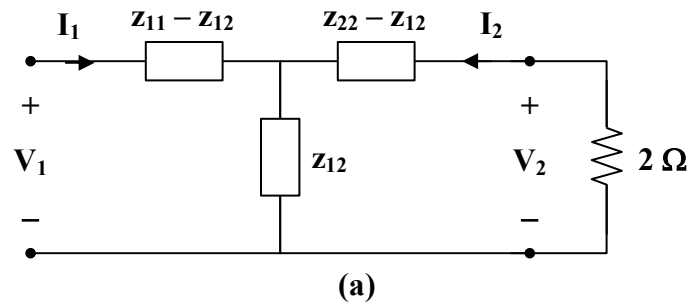
**Chapter 19, Solution 11.**

This is a reciprocal network, as shown below.



**Chapter 19, Solution 12.**

This is a reciprocal two-port so that it can be represented by the circuit in Figs. (a) and (b).



From Fig. (b),

$$V_1 = (8 + 4 \parallel 4)I_1 = 10I_1$$

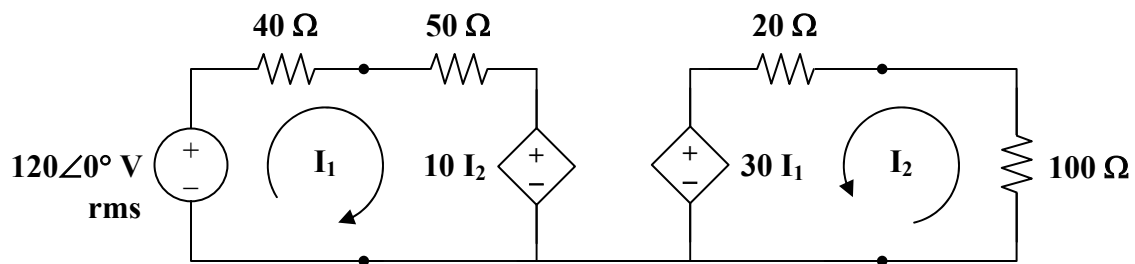
By current division,

$$\mathbf{I}_o = \frac{1}{2} \mathbf{I}_1, \quad \mathbf{V}_2 = 2 \mathbf{I}_o = \mathbf{I}_1$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\mathbf{I}_1}{10 \mathbf{I}_1} = \underline{\underline{0.1}}$$

### Chapter 19, Solution 13.

This is a reciprocal two-port so that the circuit can be represented by the circuit below.



We apply mesh analysis.

For mesh 1,

$$-120 + 90 \mathbf{I}_1 + 10 \mathbf{I}_2 = 0 \quad \longrightarrow \quad 12 = 9 \mathbf{I}_1 + \mathbf{I}_2 \quad (1)$$

For mesh 2,

$$30 \mathbf{I}_1 + 120 \mathbf{I}_2 = 0 \quad \longrightarrow \quad \mathbf{I}_1 = -4 \mathbf{I}_2 \quad (2)$$

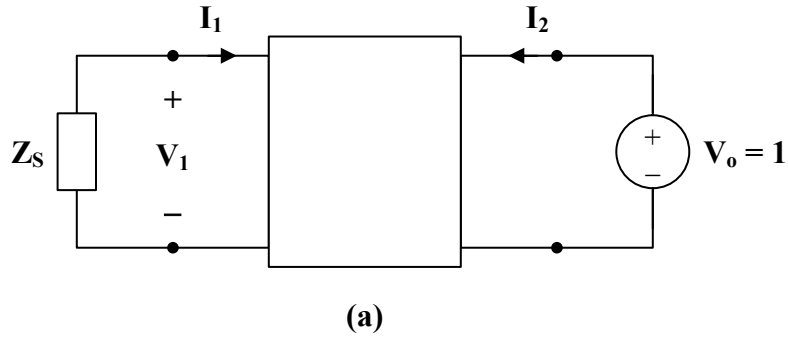
Substituting (2) into (1),

$$12 = -36 \mathbf{I}_2 + \mathbf{I}_2 = -35 \mathbf{I}_2 \quad \longrightarrow \quad \mathbf{I}_2 = \frac{-12}{35}$$

$$\mathbf{P} = \frac{1}{2} |\mathbf{I}_2|^2 \mathbf{R} = \frac{1}{2} \left( \frac{12}{35} \right)^2 (100) = \underline{\underline{5.877 \text{ W}}}$$

**Chapter 19, Solution 14.**

To find  $\mathbf{Z}_{Th}$ , consider the circuit in Fig. (a).



$$\mathbf{V}_1 = \mathbf{z}_{11} \mathbf{I}_1 + \mathbf{z}_{12} \mathbf{I}_2 \quad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21} \mathbf{I}_1 + \mathbf{z}_{22} \mathbf{I}_2 \quad (2)$$

But

$$\mathbf{V}_2 = 1, \quad \mathbf{V}_1 = -\mathbf{Z}_s \mathbf{I}_1$$

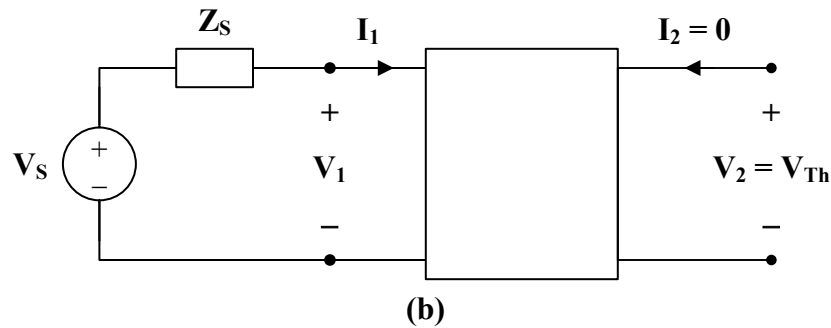
Hence,

$$0 = (\mathbf{z}_{11} + \mathbf{Z}_s) \mathbf{I}_1 + \mathbf{z}_{12} \mathbf{I}_2 \longrightarrow \mathbf{I}_1 = \frac{-\mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s} \mathbf{I}_2$$

$$1 = \left( \frac{-\mathbf{z}_{21} \mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s} + \mathbf{z}_{22} \right) \mathbf{I}_2$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{\mathbf{I}_2} = \underline{\underline{\mathbf{z}_{22} - \frac{\mathbf{z}_{21} \mathbf{z}_{12}}{\mathbf{z}_{11} + \mathbf{Z}_s}}}}$$

To find  $\mathbf{V}_{Th}$ , consider the circuit in Fig. (b).



$$\mathbf{I}_2 = 0,$$

$$\mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1 \mathbf{Z}_s$$



Substituting these into (1) and (2),

$$\mathbf{V}_s - \mathbf{I}_1 \mathbf{Z}_s = \mathbf{z}_{11} \mathbf{I}_1 \longrightarrow \mathbf{I}_1 = \frac{\mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_2 = \mathbf{z}_{21} \mathbf{I}_1 = \frac{\mathbf{z}_{21} \mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_2 = \underline{\underline{\frac{\mathbf{z}_{21} \mathbf{V}_s}{\mathbf{z}_{11} + \mathbf{Z}_s}}}$$

### Chapter 19, Solution 15.

(a) From Prob. 18.12,

$$Z_{\text{Th}} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_s} = 120 - \frac{80 \times 60}{40 + 10} = 24$$

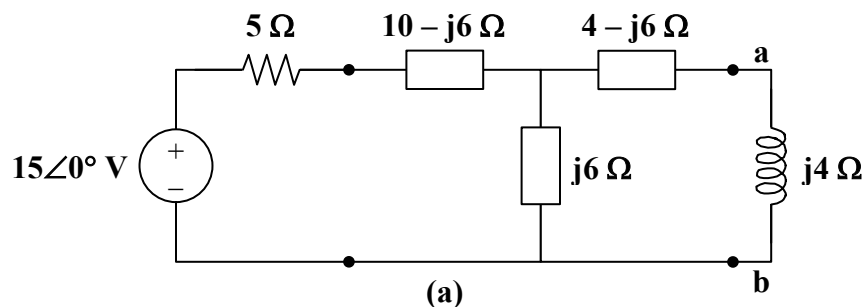
$$\underline{\underline{Z_L = Z_{\text{Th}} = 24 \Omega}}$$

$$(b) V_{\text{Th}} = \frac{z_{21}}{z_{11} + Z_s} V_s = \frac{80}{40 + 10} (120) = 192$$

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{8R_{\text{Th}}} = \frac{192^2}{8 \times 24} = \underline{\underline{192 \text{ W}}}$$

### Chapter 19, Solution 16.

As a reciprocal two-port, the given circuit can be represented as shown in Fig. (a).



At terminals a-b,

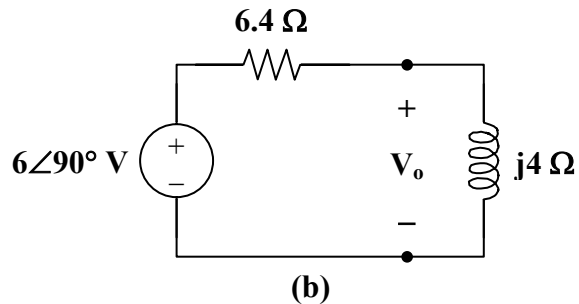
$$Z_{Th} = (4 - j6) + j6 \parallel (5 + 10 - j6)$$

$$Z_{Th} = 4 - j6 + \frac{j6(15 - j6)}{15} = 4 - j6 + 2.4 + j6$$

$$Z_{Th} = \underline{\underline{6.4 \Omega}}$$

$$V_{Th} = \frac{j6}{j6 + 5 + 10 - j6} (15 \angle 0^\circ) = j6 = \underline{\underline{6 \angle 90^\circ \text{ V}}}$$

The Thevenin equivalent circuit is shown in Fig. (b).



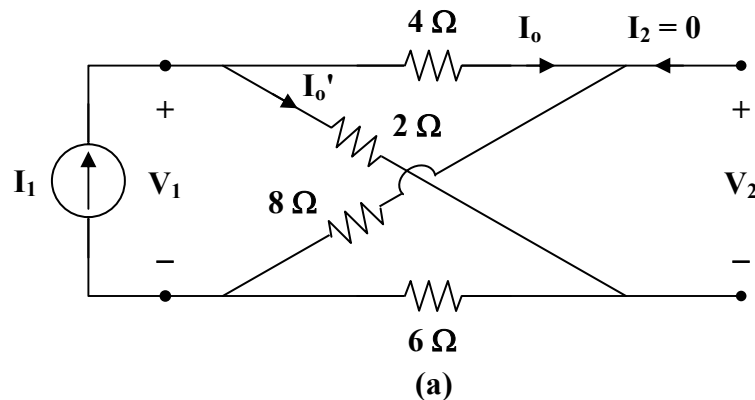
From this,

$$V_o = \frac{j4}{6.4 + j4} (j6) = 3.18 \angle 148^\circ$$

$$v_o(t) = \underline{\underline{3.18 \cos(2t + 148^\circ) \text{ V}}}$$

### Chapter 19, Solution 17.

To obtain  $z_{11}$  and  $z_{21}$ , consider the circuit in Fig. (a).



In this case, the 4- $\Omega$  and 8- $\Omega$  resistors are in series, since the same current,  $\mathbf{I}_o$ , passes through them. Similarly, the 2- $\Omega$  and 6- $\Omega$  resistors are in series, since the same current,  $\mathbf{I}_o'$ , passes through them.

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = (4 + 8) \parallel (2 + 6) = 12 \parallel 8 = \frac{(12)(8)}{20} = 4.8 \Omega$$

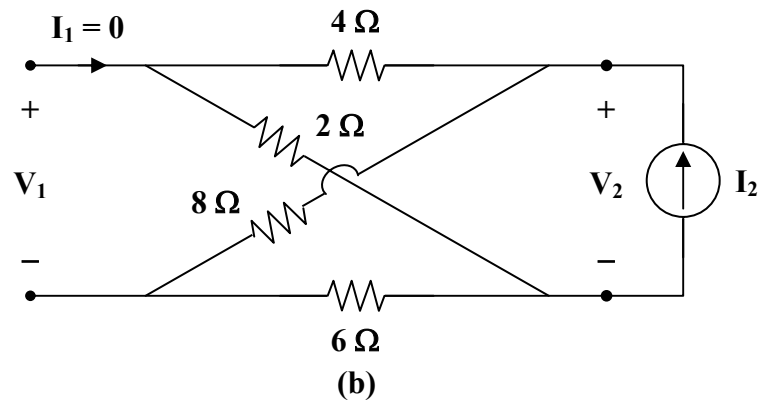
$$\mathbf{I}_o = \frac{8}{8+12} \mathbf{I}_1 = \frac{2}{5} \mathbf{I}_1 \quad \mathbf{I}_o' = \frac{3}{5} \mathbf{I}_1$$

But  $-\mathbf{V}_2 - 4\mathbf{I}_o + 2\mathbf{I}_o' = 0$

$$\mathbf{V}_2 = -4\mathbf{I}_o + 2\mathbf{I}_o' = \frac{-8}{5} \mathbf{I}_1 + \frac{6}{5} \mathbf{I}_1 = \frac{-2}{5} \mathbf{I}_1$$

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{-2}{5} = -0.4 \Omega$$

To get  $\mathbf{z}_{22}$  and  $\mathbf{z}_{12}$ , consider the circuit in Fig. (b).



$$\mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = (4 + 2) \parallel (8 + 6) = 6 \parallel 14 = \frac{(6)(14)}{20} = 4.2 \Omega$$

$$\mathbf{z}_{12} = \mathbf{z}_{21} = -0.4 \Omega$$

Thus,

$$\underline{\underline{[\mathbf{z}]} = \begin{bmatrix} 4.8 & -0.4 \\ -0.4 & 4.2 \end{bmatrix} \Omega}}$$

We may take advantage of Table 18.1 to get  $[\mathbf{y}]$  from  $[\mathbf{z}]$ .

$$\Delta_z = (4.8)(4.2) - (0.4)^2 = 20$$

$$y_{11} = \frac{z_{22}}{\Delta_z} = \frac{4.2}{20} = 0.21$$

$$y_{12} = \frac{-z_{12}}{\Delta_z} = \frac{0.4}{20} = 0.02$$

$$y_{21} = \frac{-z_{21}}{\Delta_z} = \frac{0.4}{20} = 0.02$$

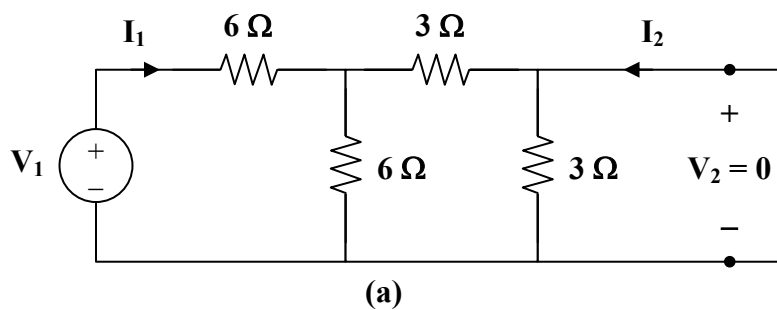
$$y_{22} = \frac{z_{11}}{\Delta_z} = \frac{4.8}{20} = 0.24$$

Thus,

$$[y] = \underline{\underline{\begin{bmatrix} 0.21 & 0.02 \\ 0.02 & 0.24 \end{bmatrix} \text{ S}}}$$

### Chapter 19, Solution 18.

To get  $y_{11}$  and  $y_{21}$ , consider the circuit in Fig.(a).



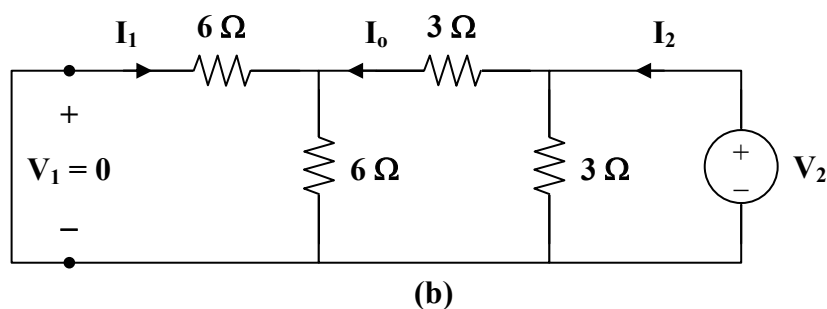
$$V_1 = (6 + 6 \parallel 3)I_1 = 8I_1$$

$$y_{11} = \frac{I_1}{V_1} = \frac{1}{8}$$

$$I_2 = \frac{-6}{6+3}I_1 = \frac{-2}{3} \frac{V_1}{8} = \frac{-V_1}{12}$$

$$y_{21} = \frac{I_2}{V_1} = \frac{-1}{12}$$

To get  $y_{22}$  and  $y_{12}$ , consider the circuit in Fig.(b).



$$y_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{3 \parallel (3+6 \parallel 6)} = \frac{1}{3 \parallel 6} = \frac{1}{2}$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_o}{2}, \quad \mathbf{I}_o = \frac{3}{3+6} \mathbf{I}_2 = \frac{1}{3} \mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_2}{6} = \left(\frac{-1}{6}\right) \left(\frac{1}{2} \mathbf{V}_2\right) = \frac{-\mathbf{V}_2}{12}$$

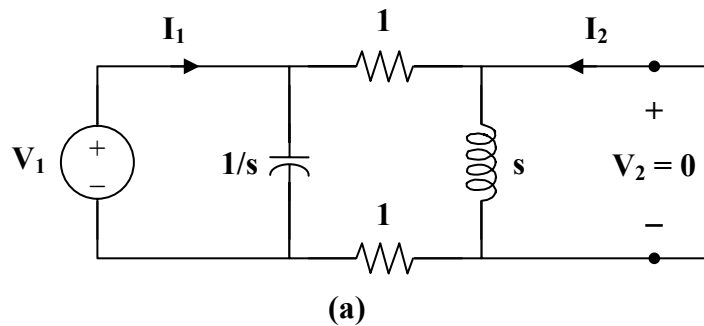
$$y_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{12} = y_{21}$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \frac{1}{8} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{1}{2} \end{bmatrix} \mathbf{S}$$

### Chapter 19, Solution 19.

Consider the circuit in Fig.(a) for calculating  $y_{11}$  and  $y_{21}$ .



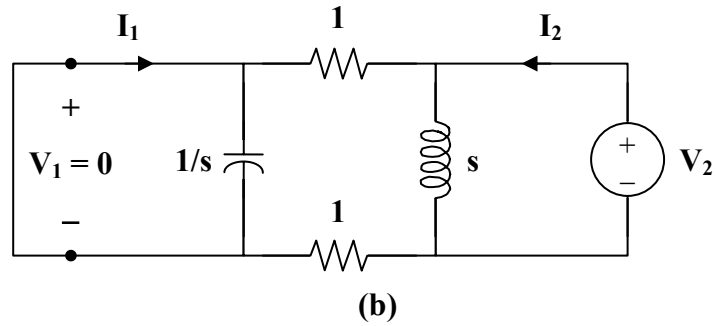
$$\mathbf{V}_1 = \left(\frac{1}{s} \parallel 2\right) \mathbf{I}_1 = \frac{2/s}{2 + (1/s)} \mathbf{I}_1 = \frac{2}{2s+1} \mathbf{I}_1$$

$$y_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{2s+1}{2} = s + 0.5$$

$$\mathbf{I}_2 = \frac{(-1/s)}{(1/s) + 2} \mathbf{I}_1 = \frac{-\mathbf{I}_1}{2s+1} = \frac{-\mathbf{V}_1}{2}$$

$$y_{21} = \frac{I_2}{V_1} = -0.5$$

To get  $y_{22}$  and  $y_{12}$ , refer to the circuit in Fig.(b).



$$V_2 = (s \parallel 2) I_2 = \frac{2s}{s+2} I_2$$

$$y_{22} = \frac{I_2}{V_2} = \frac{s+2}{2s} = 0.5 + \frac{1}{s}$$

$$I_1 = \frac{-s}{s+2} I_2 = \frac{-s}{s+2} \cdot \frac{s+2}{2s} V_2 = \frac{-V_2}{2}$$

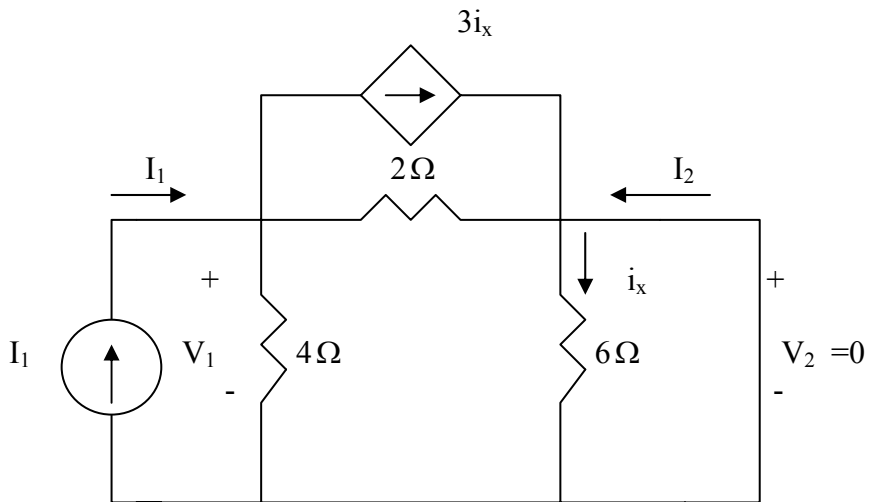
$$y_{12} = \frac{I_1}{V_2} = -0.5$$

Thus,

$$[y] = \underline{\underline{\begin{bmatrix} s+0.5 & -0.5 \\ -0.5 & 0.5+1/s \end{bmatrix} S}}$$

### Chapter 19, Solution 20.

To get  $y_{11}$  and  $y_{21}$ , consider the circuit below.

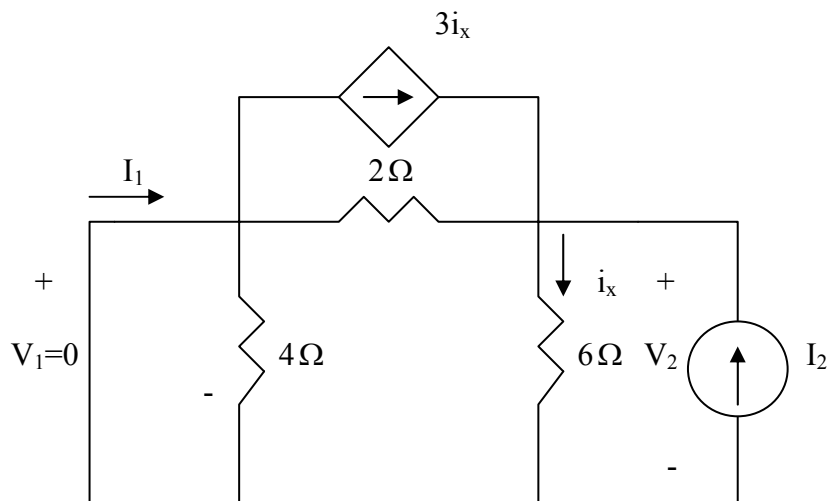


Since 6-ohm resistor is short-circuited,  $i_x = 0$

$$V_1 = I_1(4//2) = \frac{8}{6}I_1 \quad \longrightarrow \quad y_{11} = \frac{I_1}{V_1} = 0.75$$

$$I_2 = -\frac{4}{4+2}I_1 = -\frac{2}{3}\left(\frac{6}{8}V_1\right) = -\frac{1}{2}V_1 \quad \longrightarrow \quad y_{21} = \frac{I_2}{V_1} = -0.5$$

To get  $y_{22}$  and  $y_{12}$ , consider the circuit below.



$$i_x = \frac{V_2}{6}, \quad I_2 = i_x - 3i_x + \frac{V_2}{2} = \frac{V_2}{6} \quad \longrightarrow \quad y_{22} = \frac{I_2}{V_2} = \frac{1}{6} = 0.1667$$

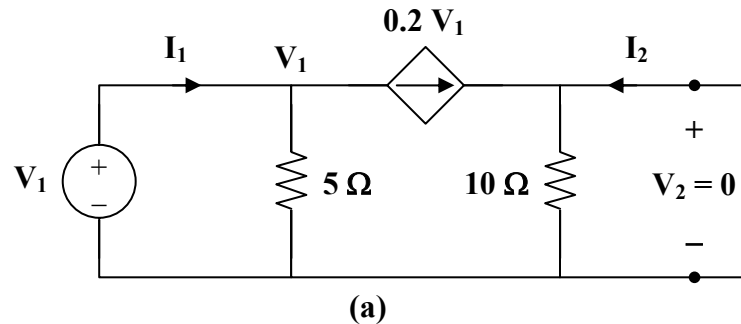
$$I_1 = 3i_x - \frac{V_2}{2} = 0 \quad \longrightarrow \quad y_{12} = \frac{I_1}{V_2} = 0$$

Thus,

$$[y] = \begin{bmatrix} 0.75 & 0 \\ -0.5 & 0.1667 \end{bmatrix} \text{ S}$$

### Chapter 19, Solution 21.

To get  $y_{11}$  and  $y_{21}$ , refer to Fig. (a).

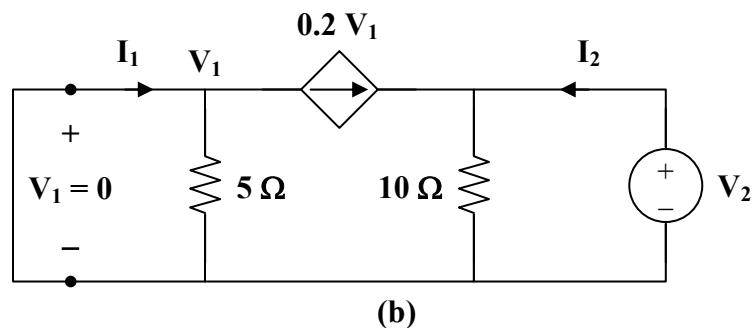


At node 1,

$$I_1 = \frac{V_1}{5} + 0.2V_1 = 0.4V_1 \quad \longrightarrow \quad y_{11} = \frac{I_1}{V_1} = 0.4$$

$$I_2 = -0.2V_1 \quad \longrightarrow \quad y_{21} = \frac{I_2}{V_1} = -0.2$$

To get  $y_{22}$  and  $y_{12}$ , refer to the circuit in Fig. (b).



Since  $V_1 = 0$ , the dependent current source can be replaced with an open circuit.



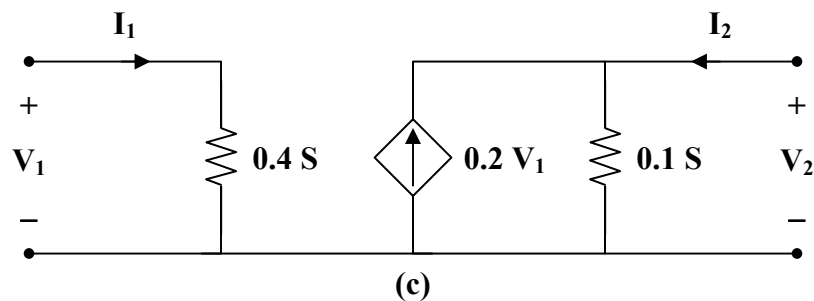
$$\mathbf{V}_2 = 10\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{10} = 0.1$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = 0$$

Thus,

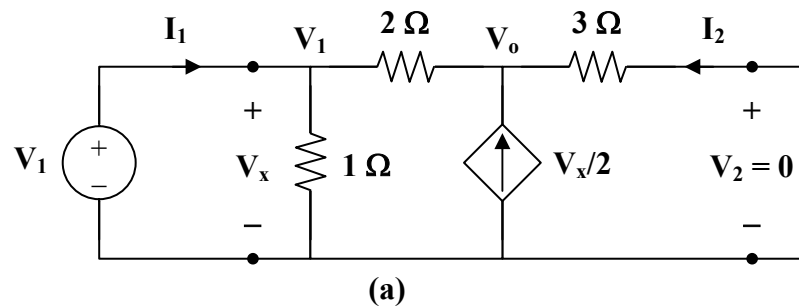
$$[\mathbf{y}] = \begin{bmatrix} 0.4 & 0 \\ -0.2 & 0.1 \end{bmatrix} \text{S}$$

Consequently, **the y parameter equivalent circuit is shown in Fig. (c).**



### Chapter 19, Solution 22.

(a) To get  $\mathbf{y}_{11}$  and  $\mathbf{y}_{21}$  refer to the circuit in Fig. (a).



At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{2} \longrightarrow \mathbf{I}_1 = 1.5\mathbf{V}_1 - 0.5\mathbf{V}_o \quad (1)$$

At node 2,

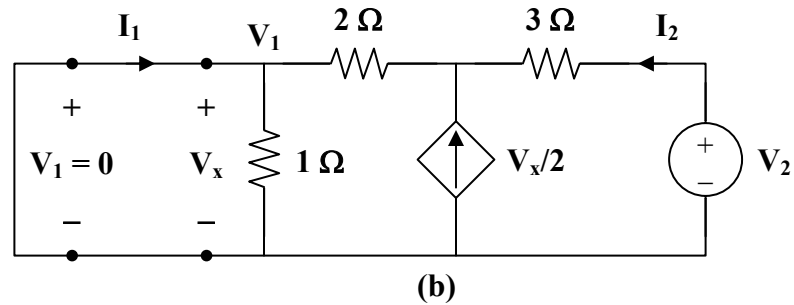
$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{2} + \frac{\mathbf{V}_1}{2} = \frac{\mathbf{V}_o}{3} \longrightarrow 1.2\mathbf{V}_1 = \mathbf{V}_o \quad (2)$$

Substituting (2) into (1) gives,

$$I_1 = 1.5V_1 - 0.6V_1 = 0.9V_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = 0.9$$

$$I_2 = \frac{-V_o}{3} = -0.4V_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = -0.4$$

To get  $y_{22}$  and  $y_{12}$  refer to the circuit in Fig. (b).



$V_x = V_1 = 0$  so that the dependent current source can be replaced by an open circuit.

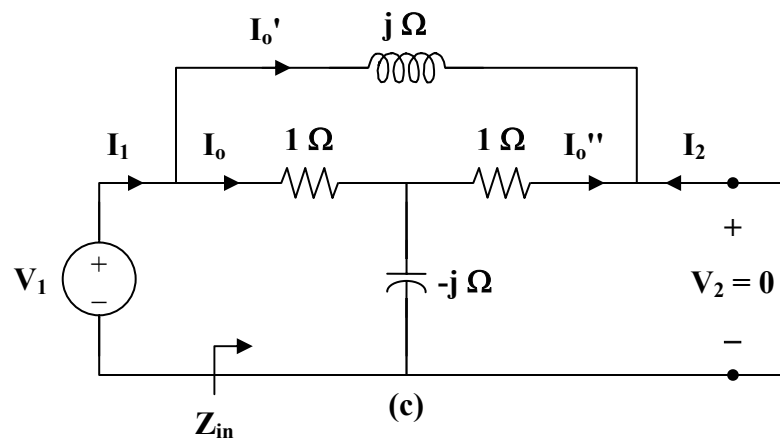
$$V_2 = (3 + 2 + 0)I_2 = 5I_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = \frac{1}{5} = 0.2$$

$$I_1 = -I_2 = -0.2V_2 \longrightarrow y_{12} = \frac{I_1}{V_2} = -0.2$$

Thus,

$$[y] = \begin{bmatrix} 0.9 & -0.2 \\ -0.4 & 0.2 \end{bmatrix} \text{ S}$$

(b) To get  $y_{11}$  and  $y_{21}$  refer to Fig. (c).



$$Z_{in} = j \parallel (1 + 1 \parallel -j) = j \parallel \left( 1 + \frac{-j}{1-j} \right) = j \parallel (1.5 - j0.5)$$

$$= \frac{j(1.5 - j0.5)}{1.5 + j0.5} = 0.6 + j0.8$$

$$\mathbf{V}_1 = \mathbf{Z}_{in} \mathbf{I}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{\mathbf{Z}_{in}} = \frac{1}{0.6 + j0.8} = 0.6 - j0.8$$

$$\mathbf{I}_o = \frac{j}{1.5 + j0.5} \mathbf{I}_1, \quad \mathbf{I}_o' = \frac{1.5 - j0.5}{1.5 + j0.5} \mathbf{I}_1$$

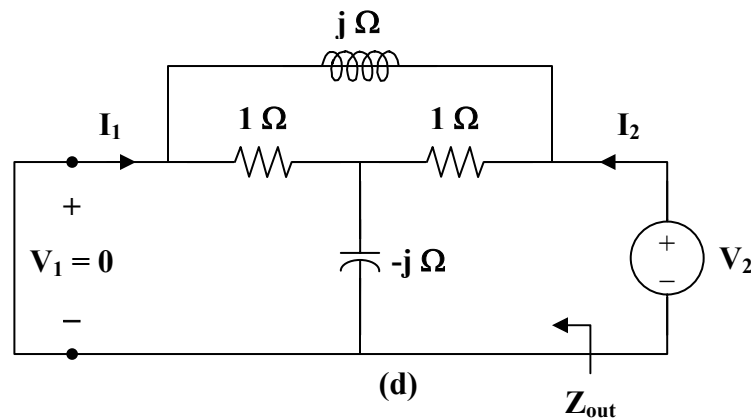
$$\mathbf{I}_o'' = \frac{-j}{1 - j} \mathbf{I}_o = \frac{\mathbf{I}_1}{(1 - j)(1.5 + j0.5)} = \frac{\mathbf{I}_1}{2 - j}$$

$$-\mathbf{I}_2 = \mathbf{I}_o' + \mathbf{I}_o'' = \frac{(1.5 - j0.5)^2}{2.5} \mathbf{I}_1 + \frac{2 + j}{5} \mathbf{I}_1 = (1.2 - j0.4) \mathbf{I}_1$$

$$-\mathbf{I}_2 = (1.2 - j0.4)(0.6 - j0.8) \mathbf{V}_1 = (0.4 - j1.2) \mathbf{V}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.4 + j1.2 = \mathbf{y}_{12}$$

To get  $\mathbf{y}_{22}$  refer to the circuit in Fig.(d).



$$\mathbf{Z}_{out} = j \parallel (1 + 1 \parallel -j) = 0.6 + j0.8$$

$$\mathbf{y}_{22} = \frac{1}{\mathbf{Z}_{out}} = 0.6 - j0.8$$

Thus,

$$\mathbf{[y]} = \begin{bmatrix} \mathbf{0.6 - j0.8} & \mathbf{-0.4 + j1.2} \\ \mathbf{-0.4 + j1.2} & \mathbf{0.6 - j0.8} \end{bmatrix} \mathbf{S}$$

**Chapter 19, Solution 23.**

(a)

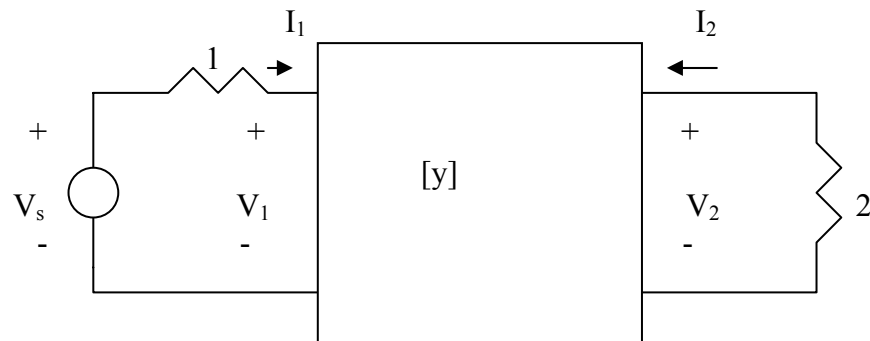
$$-y_{12} = 1 // \frac{1}{s} = \frac{1}{s+1} \quad \longrightarrow \quad y_{12} = -\frac{1}{s+1}$$

$$y_{11} + y_{12} = 1 \quad \longrightarrow \quad y_{11} = 1 - y_{12} = 1 + \frac{1}{s+1} = \frac{s+2}{s+1}$$

$$y_{22} + y_{12} = s \quad \longrightarrow \quad y_{22} = s - y_{12} = s + \frac{1}{s+1} = \frac{s^2 + s + 1}{s+1}$$

$$[y] = \begin{bmatrix} \frac{s+2}{s+1} & -\frac{1}{s+1} \\ -\frac{1}{s+1} & \frac{s^2 + s + 1}{s+1} \end{bmatrix}$$

(b) Consider the network below.



$$V_s = I_1 + V_1 \quad (1)$$

$$V_2 = -2I_2 \quad (2)$$

$$I_1 = y_{11}V_1 + y_{12}V_2 \quad (3)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \quad (4)$$

From (1) and (3)

$$V_s - V_1 = y_{11}V_1 + y_{12}V_2 \quad \longrightarrow \quad V_s = (1 + y_{11})V_1 + y_{12}V_2 \quad (5)$$

From (2) and (4),

$$-0.5V_2 = y_{21}V_1 + y_{22}V_2 \quad \longrightarrow \quad V_1 = -\frac{1}{y_{21}}(0.5 + y_{22})V_2 \quad (6)$$

Substituting (6) into (5),

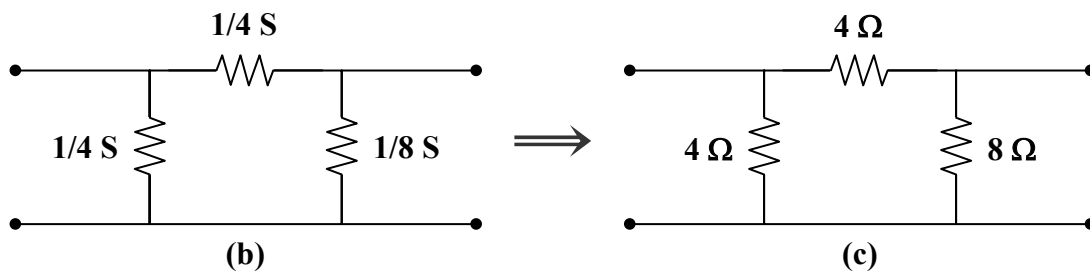
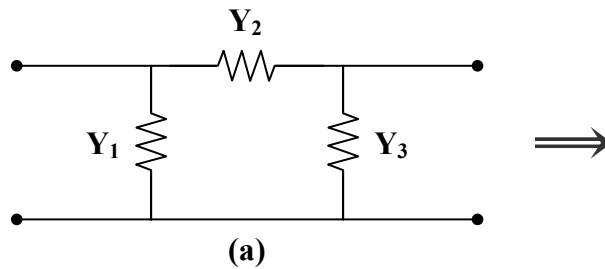
$$V_s = -\frac{(1 + y_{11})(0.5 + y_{22})}{y_{21}}V_2 + y_{12}V_2$$

$$= \frac{2}{s} \quad \longrightarrow \quad V_2 = \frac{2/s}{\left[ y_{12} - \frac{1}{y_{21}}(1 + y_{11})(0.5 + y_{22}) \right]}$$

$$V_2 = \frac{2/s}{-\frac{1}{s+1} + (s+1)\left(\frac{2s+3}{s+1}\right)\left(\frac{1}{2} + \frac{s^2+s+1}{s+1}\right)} = \frac{2(s+1)}{s(2s^3 + 6s^2 + 7.5s + 3.5)}$$

### Chapter 19, Solution 24.

Since this is a reciprocal network, **a  $\Pi$  network is appropriate, as shown below.**



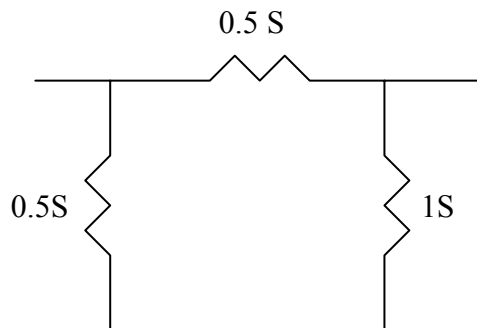
$$Y_1 = y_{11} + y_{12} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \text{ S}, \quad Z_1 = \underline{4 \Omega}$$

$$Y_2 = -y_{12} = \frac{1}{4} \text{ S}, \quad Z_2 = \underline{4 \Omega}$$

$$Y_3 = y_{22} + y_{21} = \frac{3}{8} - \frac{1}{4} = \frac{1}{8} \text{ S}, \quad Z_3 = \underline{8 \Omega}$$

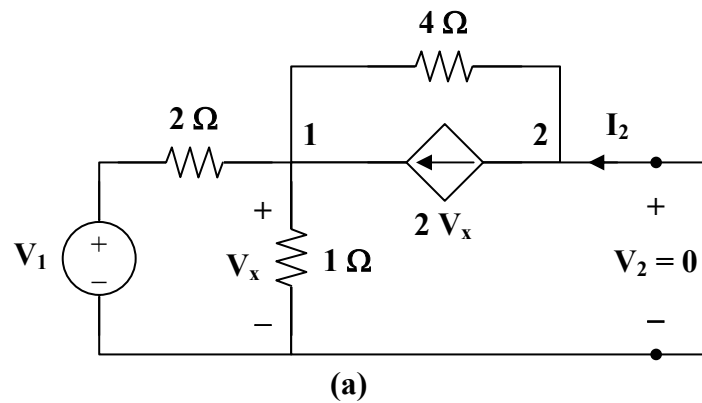
**Chapter 19, Solution 25.**

This is a reciprocal network and is shown below.



**Chapter 19, Solution 26.**

To get  $y_{11}$  and  $y_{21}$ , consider the circuit in Fig. (a).



At node 1,

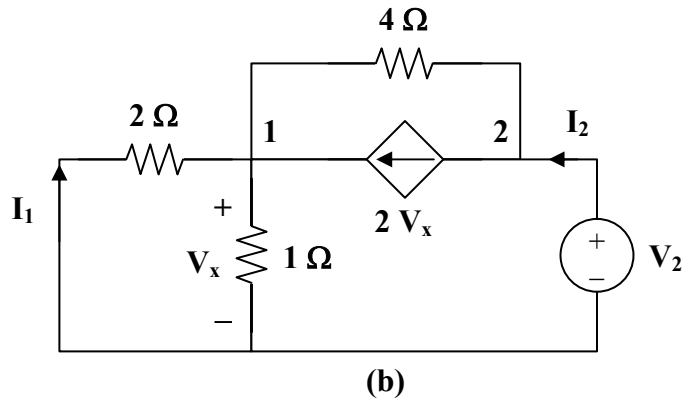
$$\frac{V_1 - V_x}{2} + 2V_x = \frac{V_x}{1} + \frac{V_x}{4} \longrightarrow 2V_1 = -V_x \quad (1)$$

But 
$$I_1 = \frac{V_1 - V_x}{2} = \frac{V_1 + 2V_1}{2} = 1.5V_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = 1.5$$

Also, 
$$I_2 + \frac{V_x}{4} = 2V_x \longrightarrow I_2 = 1.75V_x = -3.5V_1$$

$$y_{21} = \frac{I_2}{V_1} = -3.5$$

To get  $y_{22}$  and  $y_{12}$ , consider the circuit in Fig.(b).



At node 2,

$$I_2 = 2V_x + \frac{V_2 - V_x}{4} \quad (2)$$

At node 1,

$$2V_x + \frac{V_2 - V_x}{4} = \frac{V_x}{2} + \frac{V_x}{1} = \frac{3}{2}V_x \longrightarrow V_2 = -V_x \quad (3)$$

Substituting (3) into (2) gives

$$I_2 = 2V_x - \frac{1}{2}V_x = 1.5V_x = -1.5V_2$$

$$y_{22} = \frac{I_2}{V_2} = -1.5$$

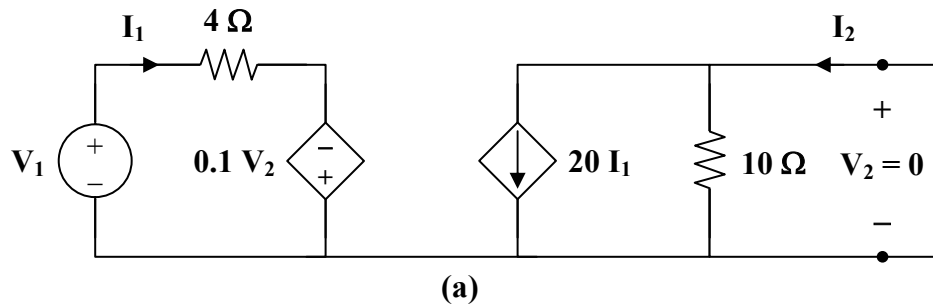
$$I_1 = \frac{-V_x}{2} = \frac{V_2}{2} \longrightarrow y_{12} = \frac{I_1}{V_2} = 0.5$$

Thus,

$$[y] = \underline{\underline{\begin{bmatrix} 1.5 & 0.5 \\ -3.5 & -1.5 \end{bmatrix} \text{S}}}$$

**Chapter 19, Solution 27.**

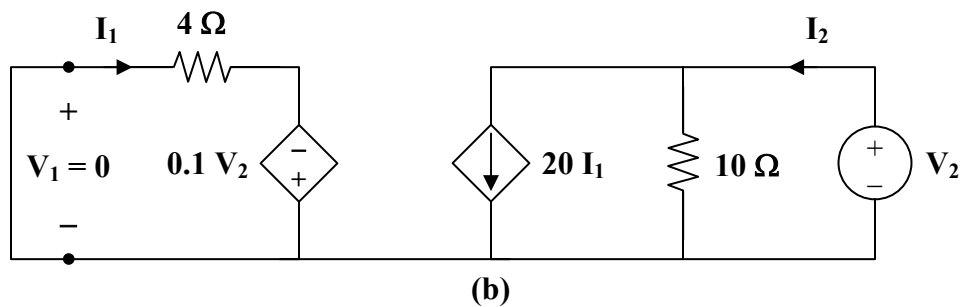
Consider the circuit in Fig. (a).



$$V_1 = 4I_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{I_1}{4I_1} = 0.25$$

$$I_2 = 20I_1 = 5V_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = 5$$

Consider the circuit in Fig. (b).



$$4I_1 = 0.1V_2 \longrightarrow y_{12} = \frac{I_1}{V_2} = \frac{0.1}{4} = 0.025$$

$$I_2 = 20I_1 + \frac{V_2}{10} = 0.5V_2 + 0.1V_2 = 0.6V_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = 0.6$$



Thus,

$$[y] = \underline{\underline{\begin{bmatrix} 0.25 & 0.025 \\ 5 & 0.6 \end{bmatrix} \text{S}}}$$

Alternatively, from the given circuit,

$$V_1 = 4I_1 - 0.1V_2$$

$$I_2 = 20I_1 + 0.1V_2$$

Comparing these with the equations for the h parameters show that

$$h_{11} = 4, \quad h_{12} = -0.1, \quad h_{21} = 20, \quad h_{22} = 0.1$$

Using Table 18.1,

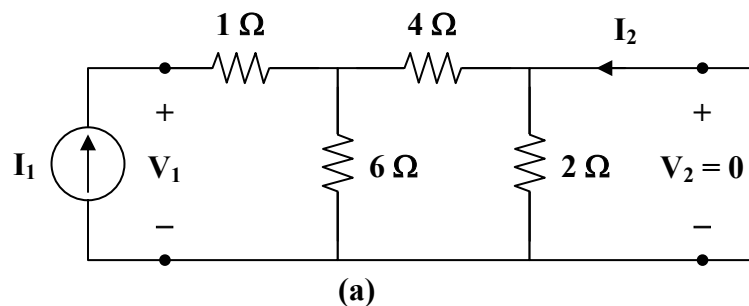
$$y_{11} = \frac{1}{h_{11}} = \frac{1}{4} = 0.25, \quad y_{12} = \frac{-h_{12}}{h_{11}} = \frac{0.1}{4} = 0.025$$

$$y_{21} = \frac{h_{21}}{h_{11}} = \frac{20}{4} = 5, \quad y_{22} = \frac{\Delta_h}{h_{11}} = \frac{0.4 + 2}{4} = 0.6$$

as above.

### Chapter 19, Solution 28.

We obtain  $y_{11}$  and  $y_{21}$  by considering the circuit in Fig.(a).



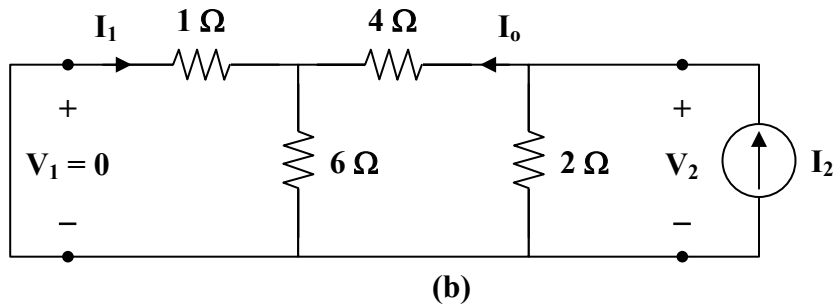
$$Z_{in} = 1 + 6 \parallel 4 = 3.4$$

$$y_{11} = \frac{I_1}{V_1} = \frac{1}{Z_{in}} = 0.2941$$

$$I_2 = \frac{-6}{10} I_1 = \left( \frac{-6}{10} \right) \left( \frac{V_1}{3.4} \right) = \frac{-6}{34} V_1$$

$$y_{21} = \frac{I_2}{V_1} = \frac{-6}{34} = -0.1765$$

To get  $y_{22}$  and  $y_{12}$ , consider the circuit in Fig. (b).



$$\frac{1}{y_{22}} = 2 \parallel (4 + 6 \parallel 1) = 2 \parallel \left(4 + \frac{6}{7}\right) = \frac{(2)(34/7)}{2 + (34/7)} = \frac{34}{24} = \frac{V_2}{I_2}$$

$$y_{22} = \frac{24}{34} = 0.7059$$

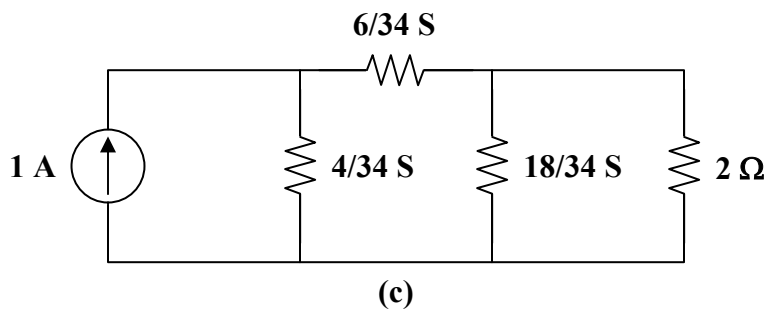
$$I_1 = \frac{-6}{7} I_0 \quad I_0 = \frac{2}{2 + (34/7)} I_2 = \frac{14}{48} I_2 = \frac{7}{34} V_2$$

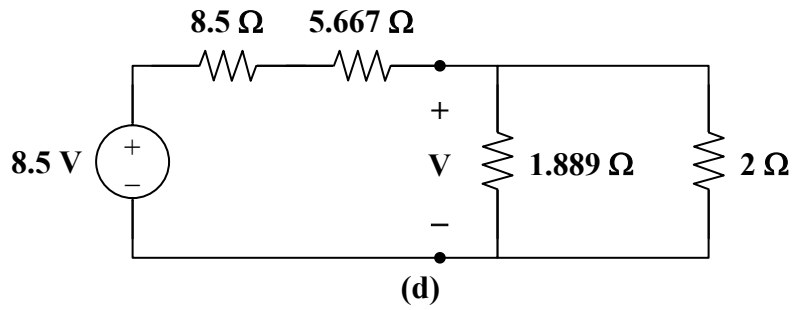
$$I_1 = \frac{-6}{34} V_2 \quad \longrightarrow \quad y_{12} = \frac{I_1}{V_2} = \frac{-6}{34} = -0.1765$$

Thus,

$$[y] = \begin{bmatrix} 0.2941 & -0.1765 \\ -0.1765 & 0.7059 \end{bmatrix} \text{S}$$

The equivalent circuit is shown in Fig. (c). After transforming the current source to a voltage source, we have the circuit in Fig. (d).



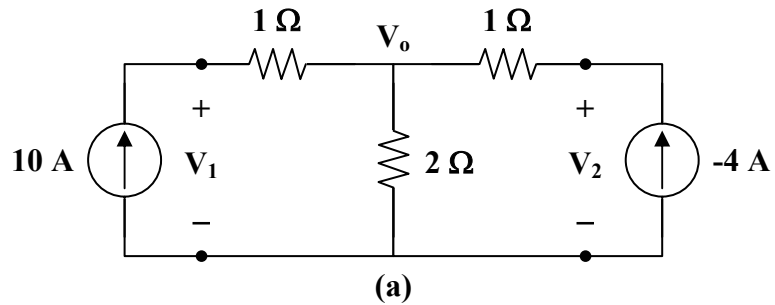


$$V = \frac{(2 \parallel 1.889)(8.5)}{2 \parallel 1.889 + 8.5 + 5.667} = \frac{(0.9714)(8.5)}{0.9714 + 14.167} = 0.5454$$

$$P = \frac{V^2}{R} = \frac{(0.5454)^2}{2} = \underline{\underline{0.1487 \text{ W}}}$$

**Chapter 19, Solution 29.**

- (a) Transforming the  $\Delta$  subnetwork to Y gives the circuit in Fig. (a).



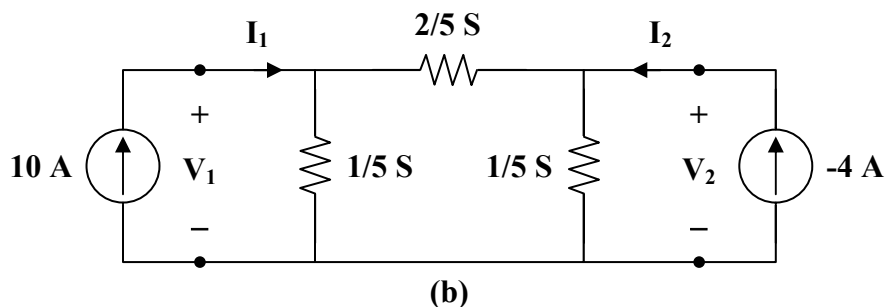
It is easy to get the z parameters

$$z_{12} = z_{21} = 2, \quad z_{11} = 1 + 2 = 3, \quad z_{22} = 3$$

$$\Delta_z = z_{11} z_{22} - z_{12} z_{21} = 9 - 4 = 5$$

$$y_{11} = \frac{z_{22}}{\Delta_z} = \frac{3}{5} = y_{22}, \quad y_{12} = y_{21} = \frac{-z_{12}}{\Delta_z} = \frac{-2}{5}$$

Thus, the equivalent circuit is as shown in Fig. (b).



$$\mathbf{I}_1 = 10 = \frac{3}{5}\mathbf{V}_1 - \frac{2}{5}\mathbf{V}_2 \longrightarrow 50 = 3\mathbf{V}_1 - 2\mathbf{V}_2 \quad (1)$$

$$\mathbf{I}_2 = -4 = \frac{-2}{5}\mathbf{V}_1 + \frac{3}{5}\mathbf{V}_2 \longrightarrow -20 = -2\mathbf{V}_1 + 3\mathbf{V}_2$$

$$10 = \mathbf{V}_1 - 1.5\mathbf{V}_2 \longrightarrow \mathbf{V}_1 = 10 + 1.5\mathbf{V}_2 \quad (2)$$

Substituting (2) into (1),

$$50 = 30 + 4.5\mathbf{V}_2 - 2\mathbf{V}_2 \longrightarrow \mathbf{V}_2 = \underline{\underline{8 \text{ V}}}$$

$$\mathbf{V}_1 = 10 + 1.5\mathbf{V}_2 = \underline{\underline{22 \text{ V}}}$$

- (b) For direct circuit analysis, consider the circuit in Fig. (a).

For the main non-reference node,

$$10 - 4 = \frac{\mathbf{V}_o}{2} \longrightarrow \mathbf{V}_o = 12$$

$$10 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_1 = 10 + \mathbf{V}_o = \underline{\underline{22 \text{ V}}}$$

$$-4 = \frac{\mathbf{V}_2 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_2 = \mathbf{V}_o - 4 = \underline{\underline{8 \text{ V}}}$$

### Chapter 19, Solution 30.

- (a) Convert to z parameters; then, convert to h parameters using Table 18.1.

$$\mathbf{z}_{11} = \mathbf{z}_{12} = \mathbf{z}_{21} = 60 \Omega, \quad \mathbf{z}_{22} = 100 \Omega$$

$$\Delta_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21} = 6000 - 3600 = 2400$$

$$\mathbf{h}_{11} = \frac{\Delta_z}{\mathbf{z}_{22}} = \frac{2400}{100} = 24, \quad \mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} = \frac{60}{100} = 0.6$$

$$\mathbf{h}_{21} = \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} = -0.6, \quad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}} = 0.01$$

Thus,

$$[\mathbf{h}] = \underline{\underline{\begin{bmatrix} 24 \Omega & 0.6 \\ -0.6 & 0.01 \text{ S} \end{bmatrix}}}$$

(b) Similarly,

$$\mathbf{z}_{11} = 30 \Omega$$

$$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{z}_{22} = 20 \Omega$$

$$\Delta_z = 600 - 400 = 200$$

$$\mathbf{h}_{11} = \frac{200}{20} = 10$$

$$\mathbf{h}_{12} = \frac{20}{20} = 1$$

$$\mathbf{h}_{21} = -1$$

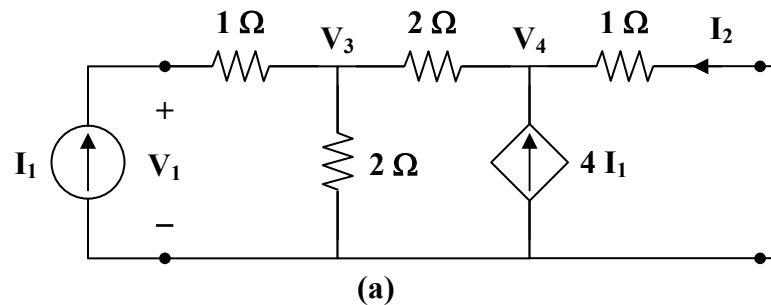
$$\mathbf{h}_{22} = \frac{1}{20} = 0.05$$

Thus,

$$[\mathbf{h}] = \underline{\underline{\begin{bmatrix} 10 \Omega & 1 \\ -1 & 0.05 \text{ S} \end{bmatrix}}}$$

### Chapter 19, Solution 31.

We get  $\mathbf{h}_{11}$  and  $\mathbf{h}_{21}$  by considering the circuit in Fig. (a).



At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_3}{2} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{2} \longrightarrow 2\mathbf{I}_1 = 2\mathbf{V}_3 - \mathbf{V}_4 \quad (1)$$

At node 2,

$$\frac{\mathbf{V}_3 - \mathbf{V}_4}{2} + 4\mathbf{I}_1 = \frac{\mathbf{V}_4}{1}$$

$$8\mathbf{I}_1 = -\mathbf{V}_3 + 3\mathbf{V}_4 \longrightarrow 16\mathbf{I}_1 = -2\mathbf{V}_3 + 6\mathbf{V}_4 \quad (2)$$

Adding (1) and (2),

$$18\mathbf{I}_1 = 5\mathbf{V}_4 \longrightarrow \mathbf{V}_4 = 3.6\mathbf{I}_1$$

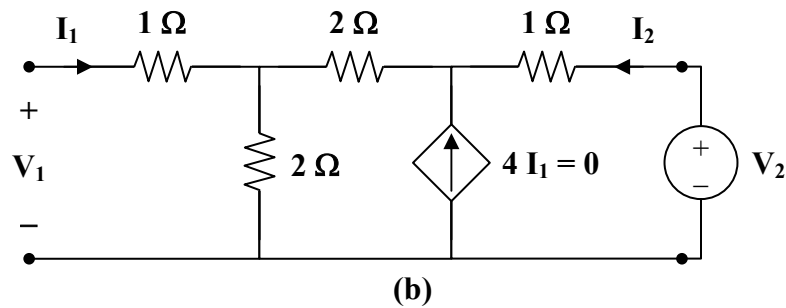
$$\mathbf{V}_3 = 3\mathbf{V}_4 - 8\mathbf{I}_1 = 2.8\mathbf{I}_1$$

$$\mathbf{V}_1 = \mathbf{V}_3 + \mathbf{I}_1 = 3.8\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 3.8 \Omega$$

$$\mathbf{I}_2 = \frac{-\mathbf{V}_4}{1} = -3.6\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = -3.6$$

To get  $\mathbf{h}_{22}$  and  $\mathbf{h}_{12}$ , refer to the circuit in Fig. (b). The dependent current source can be replaced by an open circuit since  $4\mathbf{I}_1 = 0$ .



$$\mathbf{V}_1 = \frac{2}{2+2+1}\mathbf{V}_2 = \frac{2}{5}\mathbf{V}_2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 0.4$$

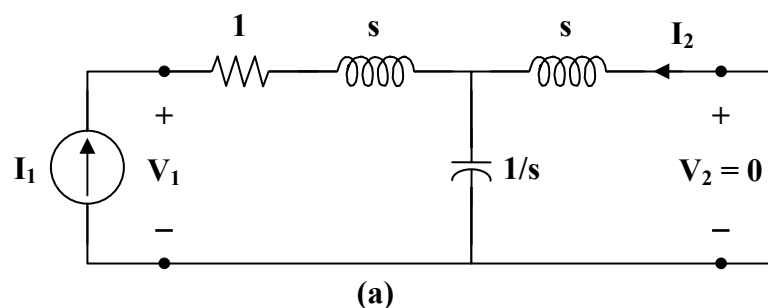
$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2+2+1} = \frac{\mathbf{V}_2}{5} \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{5} = 0.2 \text{ S}$$

Thus,

$$\underline{\underline{[\mathbf{h}] = \begin{bmatrix} 3.8 \Omega & 0.4 \\ -3.6 & 0.2 \text{ S} \end{bmatrix}}}$$

### Chapter 19, Solution 32.

(a) We obtain  $\mathbf{h}_{11}$  and  $\mathbf{h}_{21}$  by referring to the circuit in Fig. (a).



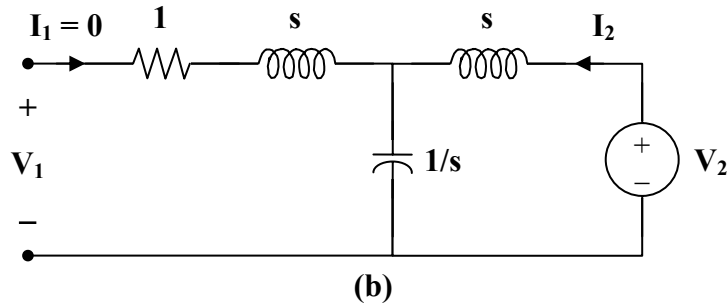
$$\mathbf{V}_1 = \left(1 + s + s \parallel \frac{1}{s}\right) \mathbf{I}_1 = \left(1 + s + \frac{s}{s^2 + 1}\right) \mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = s + 1 + \frac{s}{s^2 + 1}$$

By current division,

$$\mathbf{I}_2 = \frac{-1/s}{s + 1/s} \mathbf{I}_1 = \frac{-\mathbf{I}_1}{s + 1} \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-1}{s^2 + 1}$$

To get  $\mathbf{h}_{22}$  and  $\mathbf{h}_{12}$ , refer to Fig. (b).



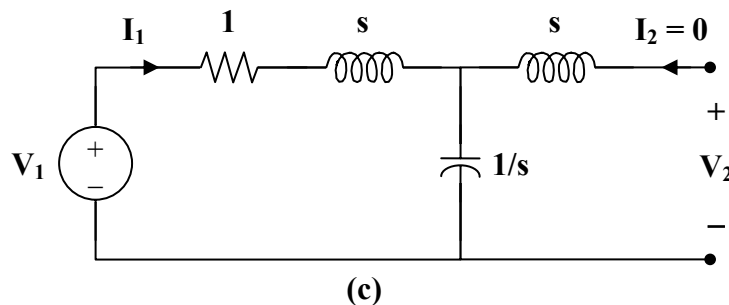
$$\mathbf{V}_1 = \frac{1/s}{s + 1/s} \mathbf{V}_2 = \frac{\mathbf{V}_2}{s^2 + 1} \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{s^2 + 1}$$

$$\mathbf{V}_2 = \left(s + \frac{1}{s}\right) \mathbf{I}_2 \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{s + 1/s} = \frac{s}{s^2 + 1}$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} s + 1 + \frac{s}{s^2 + 1} & \frac{1}{s^2 + 1} \\ \frac{-1}{s^2 + 1} & \frac{s}{s^2 + 1} \end{bmatrix}$$

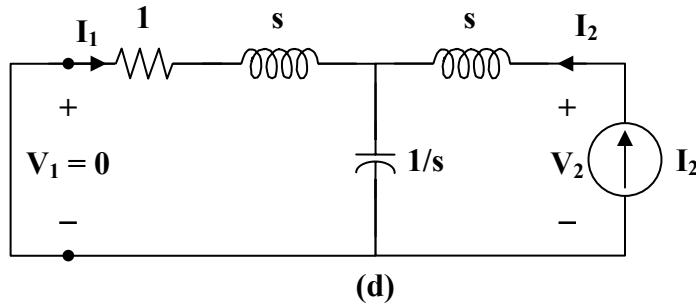
(b) To get  $\mathbf{g}_{11}$  and  $\mathbf{g}_{21}$ , refer to Fig. (c).



$$\mathbf{V}_1 = \left(1 + s + \frac{1}{s}\right) \mathbf{I}_1 \longrightarrow \mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{1 + s + 1/s} = \frac{s}{s^2 + s + 1}$$

$$\mathbf{V}_2 = \frac{1/s}{1 + s + 1/s} \mathbf{V}_1 = \frac{\mathbf{V}_1}{s^2 + s + 1} \longrightarrow \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{s^2 + s + 1}$$

To get  $\mathbf{g}_{22}$  and  $\mathbf{g}_{12}$ , refer to Fig. (d).



$$\mathbf{V}_2 = \left(s + \frac{1}{s} \parallel (s+1)\right) \mathbf{I}_2 = \left(s + \frac{(s+1)/s}{1 + s + 1/s}\right) \mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = s + \frac{s+1}{s^2 + s + 1}$$

$$\mathbf{I}_1 = \frac{-1/s}{1 + s + 1/s} \mathbf{I}_2 = \frac{-\mathbf{I}_2}{s^2 + s + 1} \longrightarrow \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-1}{s^2 + s + 1}$$

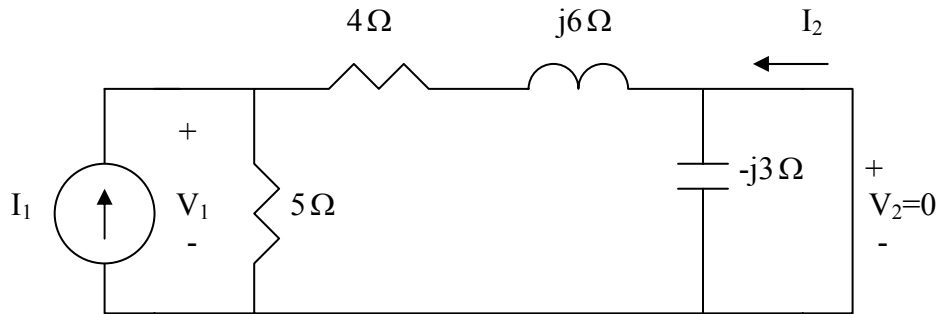
Thus,

$$[\mathbf{g}] = \begin{bmatrix} \frac{s}{s^2 + s + 1} & \frac{-1}{s^2 + s + 1} \\ \frac{1}{s^2 + s + 1} & s + \frac{s+1}{s^2 + s + 1} \end{bmatrix}$$

### Chapter 19, Solution 33.

To get  $h_{11}$  and  $h_{21}$ , consider the circuit below.

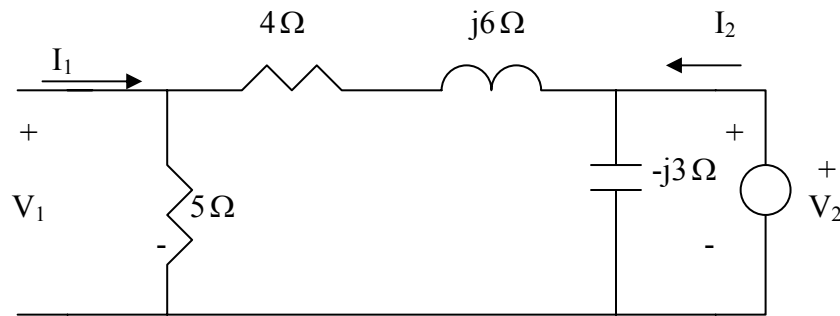




$$V_1 = 5 // (4 + j6) I_1 = \frac{5(4 + j6) I_1}{9 + j6} \quad h_{11} = \frac{V_1}{I_1} = 3.0769 + j1.2821$$

$$\text{Also, } I_2 = -\frac{5}{9 + j6} I_1 \quad \longrightarrow \quad h_{21} = \frac{I_2}{I_1} = -0.3846 + j0.2564$$

To get  $h_{22}$  and  $h_{12}$ , consider the circuit below.



$$V_1 = \frac{5}{9 + j6} V_2 \quad \longrightarrow \quad h_{12} = \frac{V_1}{V_2} = \frac{5}{9 + j6} = 0.3846 - j0.2564$$

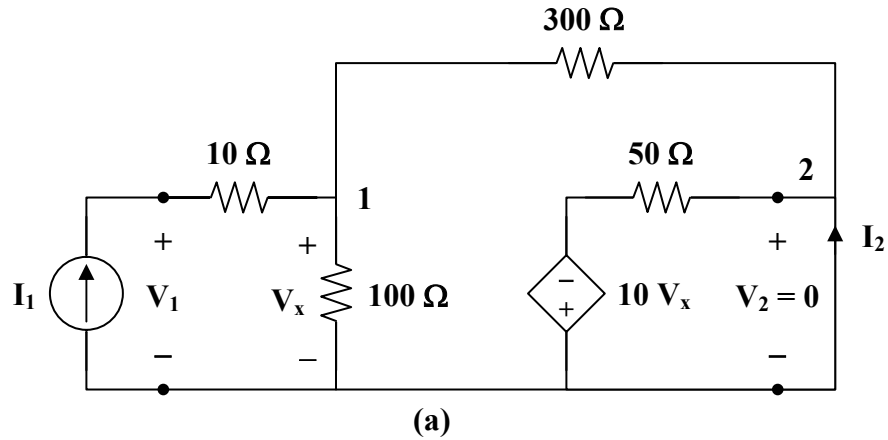
$$V_2 = -j3 // (9 + j6) I_2 \quad \longrightarrow \quad h_{22} = \frac{I_2}{V_2} = \frac{1}{-j3 // (9 + j6)} = \frac{9 + j3}{-j3(9 + j6)} = 0.0769 + j0.2821$$

Thus,

$$[h] = \begin{bmatrix} 3.0769 + j1.2821 & 0.3846 - j0.2564 \\ -0.3846 + j0.2564 & 0.0769 + j0.2821 \end{bmatrix}$$

**Chapter 19, Solution 34.**

Refer to Fig. (a) to get  $h_{11}$  and  $h_{21}$ .



At node 1,

$$I_1 = \frac{V_x}{100} + \frac{V_x - 0}{300} \longrightarrow 300I_1 = 4V_x \quad (1)$$

$$V_x = \frac{300}{4}I_1 = 75I_1$$

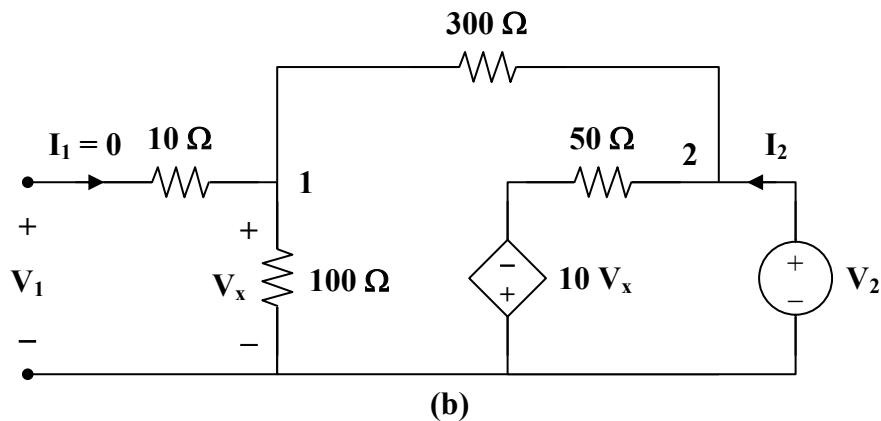
But  $V_1 = 10I_1 + V_x = 85I_1 \longrightarrow h_{11} = \frac{V_1}{I_1} = 85 \Omega$

At node 2,

$$I_2 = \frac{0 + 10V_x}{50} - \frac{V_x}{300} = \frac{V_x}{5} - \frac{V_x}{300} = \frac{75}{5}I_1 - \frac{75}{300}I_1 = 14.75I_1$$

$$h_{21} = \frac{I_2}{I_1} = 14.75$$

To get  $h_{22}$  and  $h_{12}$ , refer to Fig. (b).



At node 2,

$$I_2 = \frac{V_2}{400} + \frac{V_2 + 10V_x}{50} \longrightarrow 400I_2 = 9V_2 + 80V_x$$

But 
$$V_x = \frac{100}{400}V_2 = \frac{V_2}{4}$$

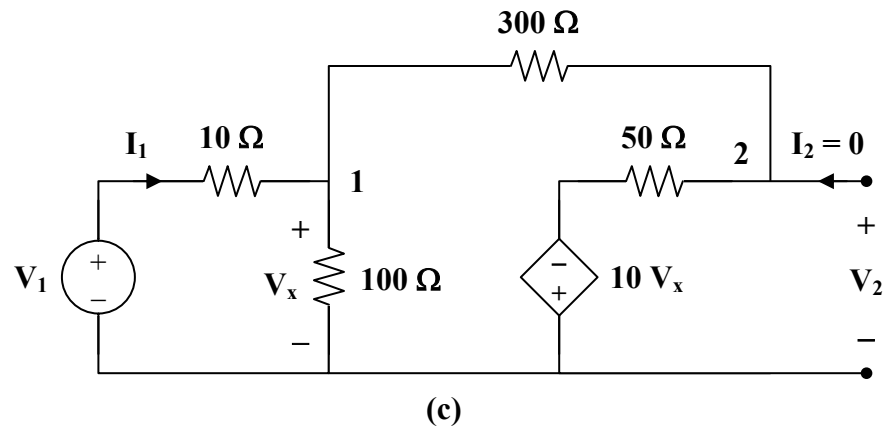
Hence, 
$$400I_2 = 9V_2 + 20V_2 = 29V_2$$

$$h_{22} = \frac{I_2}{V_2} = \frac{29}{400} = 0.0725 \text{ S}$$

$$V_1 = V_x = \frac{V_2}{4} \longrightarrow h_{12} = \frac{V_1}{V_2} = \frac{1}{4} = 0.25$$

$$[h] = \begin{bmatrix} 85 \Omega & 0.25 \\ 14.75 & 0.0725 \text{ S} \end{bmatrix}$$

To get  $g_{11}$  and  $g_{21}$ , refer to Fig. (c).



At node 1,

$$I_1 = \frac{V_x}{100} + \frac{V_x + 10V_x}{350} \longrightarrow 350I_1 = 14.5V_x \quad (2)$$

But 
$$I_1 = \frac{V_1 - V_x}{10} \longrightarrow 10I_1 = V_1 - V_x$$

or 
$$V_x = V_1 - 10I_1 \quad (3)$$

Substituting (3) into (2) gives

$$350\mathbf{I}_1 = 14.5\mathbf{V}_1 - 145\mathbf{I}_1 \longrightarrow 495\mathbf{I}_1 = 14.5\mathbf{V}_1$$

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{14.5}{495} = 0.02929 \text{ S}$$

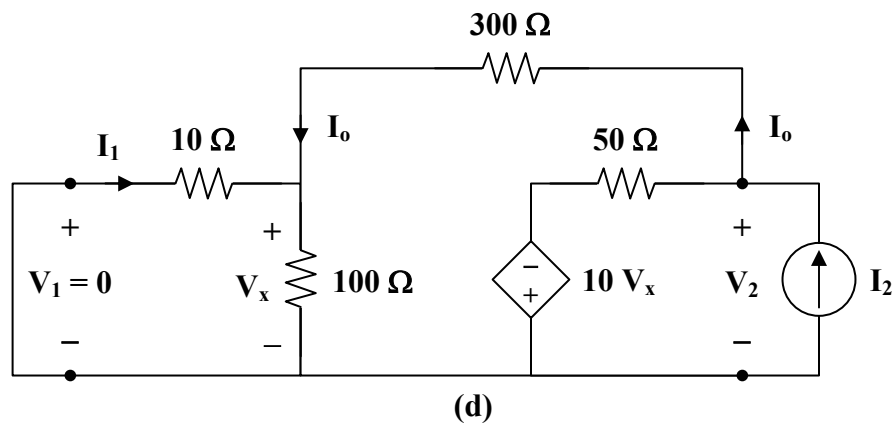
At node 2,

$$\mathbf{V}_2 = (50)\left(\frac{11}{350}\mathbf{V}_x\right) - 10\mathbf{V}_x = -8.4286\mathbf{V}_x$$

$$= -8.4286\mathbf{V}_1 + 84.286\mathbf{I}_1 = -8.4286\mathbf{V}_1 + (84.286)\left(\frac{14.5}{495}\right)\mathbf{V}_1$$

$$\mathbf{V}_2 = -5.96\mathbf{V}_1 \longrightarrow \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = -5.96$$

To get  $\mathbf{g}_{22}$  and  $\mathbf{g}_{12}$ , refer to Fig. (d).



$$10 \parallel 100 = 9.091$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2 + 10\mathbf{V}_x}{50} + \frac{\mathbf{V}_2}{300 + 9.091}$$

$$309.091\mathbf{I}_2 = 7.1818\mathbf{V}_2 + 61.818\mathbf{V}_x \quad (4)$$

But 
$$\mathbf{V}_x = \frac{9.091}{309.091}\mathbf{V}_2 = 0.02941\mathbf{V}_2 \quad (5)$$

Substituting (5) into (4) gives

$$309.091\mathbf{I}_2 = 9\mathbf{V}_2$$

$$g_{22} = \frac{V_2}{I_2} = 34.34 \Omega$$

$$I_o = \frac{V_2}{309.091} = \frac{34.34 I_2}{309.091}$$

$$I_1 = \frac{-100}{110} I_o = \frac{-34.34 I_2}{(1.1)(309.091)}$$

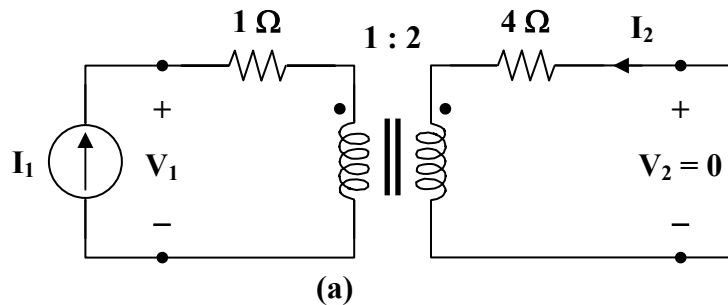
$$g_{12} = \frac{I_1}{I_2} = -0.101$$

Thus,

$$[g] = \underline{\underline{\begin{bmatrix} 0.02929 \text{ S} & -0.101 \\ -5.96 & 34.34 \Omega \end{bmatrix}}}$$

### Chapter 19, Solution 35.

To get  $h_{11}$  and  $h_{21}$  consider the circuit in Fig. (a).

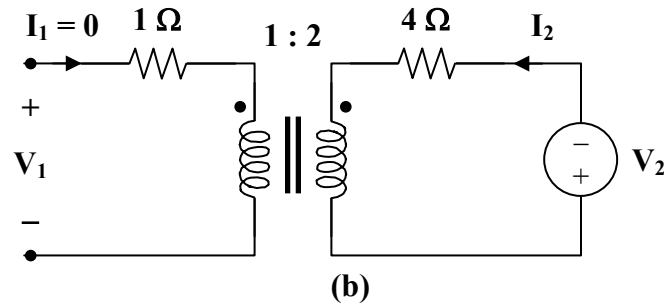


$$Z_R = \frac{4}{n^2} = \frac{4}{4} = 1$$

$$V_1 = (1+1)I_1 = 2I_1 \longrightarrow h_{11} = \frac{V_1}{I_1} = 2 \Omega$$

$$\frac{I_1}{I_2} = \frac{-N_2}{N_1} = -2 \longrightarrow h_{21} = \frac{I_2}{I_1} = \frac{-1}{2} = -0.5$$

To get  $h_{22}$  and  $h_{12}$ , refer to Fig. (b).



Since  $I_1 = 0$ ,  $I_2 = 0$ .

Hence,  $h_{22} = 0$ .

At the terminals of the transformer, we have  $V_1$  and  $V_2$  which are related as

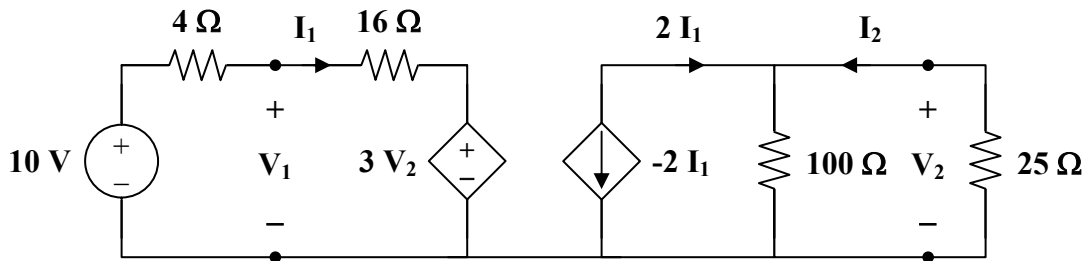
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n = 2 \longrightarrow h_{12} = \frac{V_1}{V_2} = \frac{1}{2} = 0.5$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} 2 \Omega & 0.5 \\ -0.5 & 0 \end{bmatrix}$$

### Chapter 19, Solution 36.

We replace the two-port by its equivalent circuit as shown below.



$$100 \parallel 25 = 20 \Omega$$

$$V_2 = (20)(2I_1) = 40I_1 \quad (1)$$

$$-10 + 20I_1 + 3V_2 = 0$$

$$10 = 20I_1 + (3)(40I_1) = 140I_1$$

$$\mathbf{I}_1 = \frac{1}{14}, \quad \mathbf{V}_2 = \frac{40}{14}$$

$$\mathbf{V}_1 = 16\mathbf{I}_1 + 3\mathbf{V}_2 = \frac{136}{14}$$

$$\mathbf{I}_2 = \left(\frac{100}{125}\right)(2\mathbf{I}_1) = \frac{-8}{70}$$

(a)  $\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{40}{136} = \underline{\underline{0.2941}}$

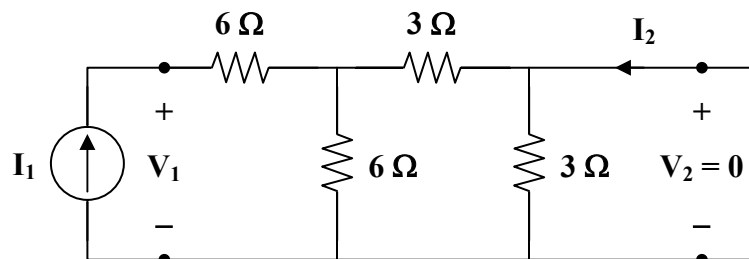
(b)  $\frac{\mathbf{I}_2}{\mathbf{I}_1} = \underline{\underline{-1.6}}$

(c)  $\frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{136} = \underline{\underline{7.353 \times 10^{-3} \text{ S}}}$

(d)  $\frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40}{1} = \underline{\underline{40 \Omega}}$

**Chapter 19, Solution 37.**

(a) We first obtain the h parameters. To get  $\mathbf{h}_{11}$  and  $\mathbf{h}_{21}$  refer to Fig. (a).



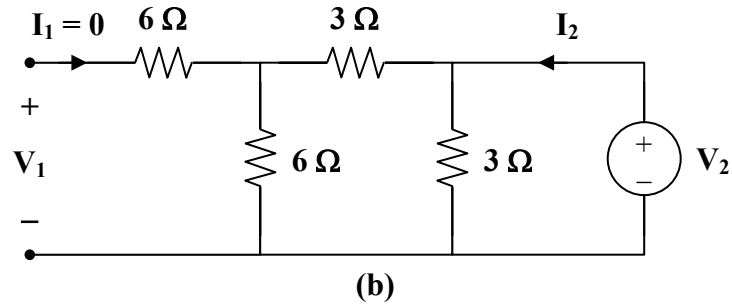
(a)

$$3 \parallel 6 = 2$$

$$\mathbf{V}_1 = (6 + 2)\mathbf{I}_1 = 8\mathbf{I}_1 \quad \longrightarrow \quad \mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 8 \Omega$$

$$I_2 = \frac{-6}{3+6} I_1 = \frac{-2}{3} I_1 \longrightarrow h_{21} = \frac{I_2}{I_1} = \frac{-2}{3}$$

To get  $h_{22}$  and  $h_{12}$ , refer to the circuit in Fig. (b).



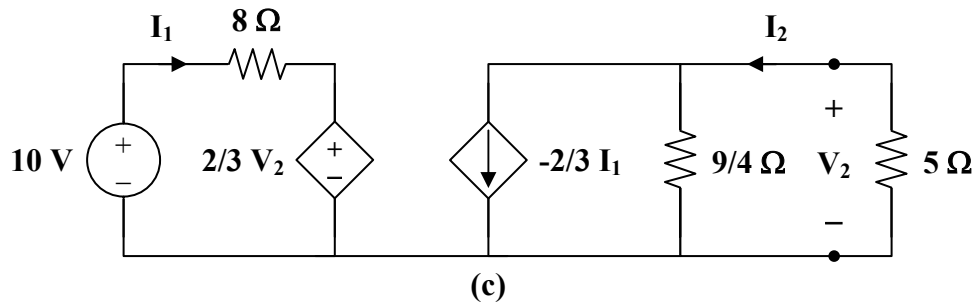
$$3 \parallel 9 = \frac{9}{4}$$

$$V_2 = \frac{9}{4} I_2 \longrightarrow h_{22} = \frac{I_2}{V_2} = \frac{4}{9}$$

$$V_1 = \frac{6}{6+3} V_2 = \frac{2}{3} V_2 \longrightarrow h_{12} = \frac{V_1}{V_2} = \frac{2}{3}$$

$$[h] = \begin{bmatrix} 8 \Omega & \frac{2}{3} \\ -\frac{2}{3} & \frac{4}{9} \text{ S} \end{bmatrix}$$

The equivalent circuit of the given circuit is shown in Fig. (c).



$$8I_1 + \frac{2}{3}V_2 = 10 \quad (1)$$



$$V_2 = \frac{2}{3} I_1 \left( 5 \parallel \frac{9}{4} \right) = \frac{2}{3} I_1 \left( \frac{45}{29} \right) = \frac{30}{29} I_1$$

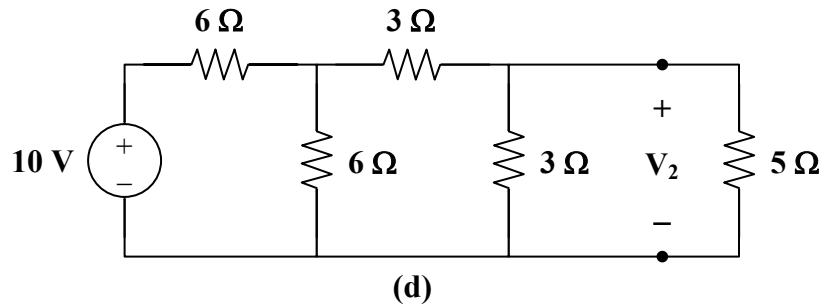
$$I_1 = \frac{29}{30} V_2 \quad (2)$$

Substituting (2) into (1),

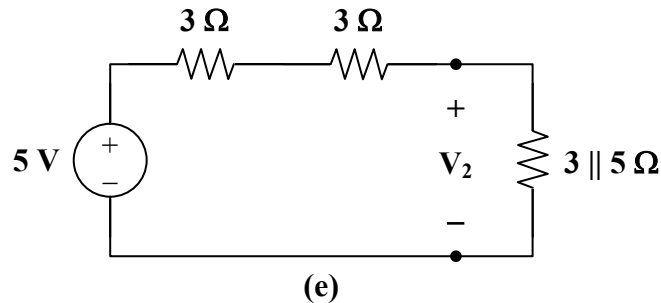
$$(8) \left( \frac{29}{30} \right) V_2 + \frac{2}{3} V_2 = 10$$

$$V_2 = \frac{300}{252} = \underline{\underline{1.19 \text{ V}}}$$

(b) By direct analysis, refer to Fig.(d).



Transform the 10-V voltage source to a  $\frac{10}{6}$ -A current source. Since  $6 \parallel 6 = 3 \Omega$ , we combine the two 6- $\Omega$  resistors in parallel and transform the current source back to  $\frac{10}{6} \times 3 = 5 \text{ V}$  voltage source shown in Fig. (e).

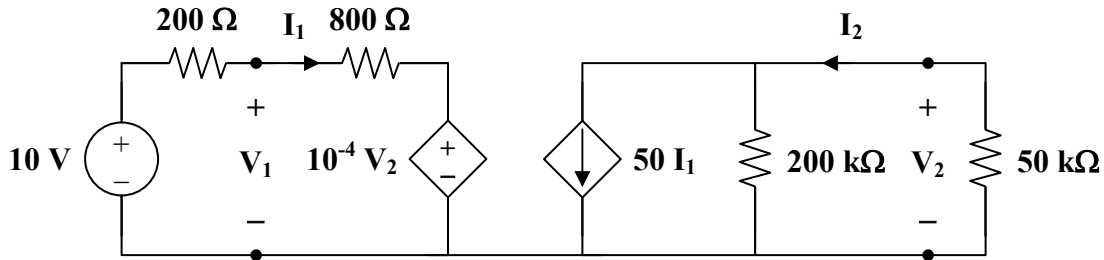


$$3 \parallel 5 = \frac{(3)(5)}{8} = \frac{15}{8}$$

$$V_2 = \frac{15/8}{6+15/8}(5) = \frac{75}{63} = \underline{\underline{1.19 \text{ V}}}$$

**Chapter 19, Solution 38.**

We replace the two-port by its equivalent circuit as shown below.



$$Z_{\text{in}} = \frac{V_s}{I_1}, \quad 200 \parallel 50 = 40 \text{ k}\Omega$$

$$V_2 = -50 I_1 (40 \times 10^3) = (-2 \times 10^6) I_1$$

For the left loop,

$$\frac{V_s - 10^{-4} V_2}{1000} = I_1$$

$$V_s - 10^{-4} (-2 \times 10^6 I_1) = 1000 I_1$$

$$V_s = 1000 I_1 - 200 I_1 = 800 I_1$$

$$Z_{\text{in}} = \frac{V_s}{I_1} = \underline{\underline{800 \Omega}}$$

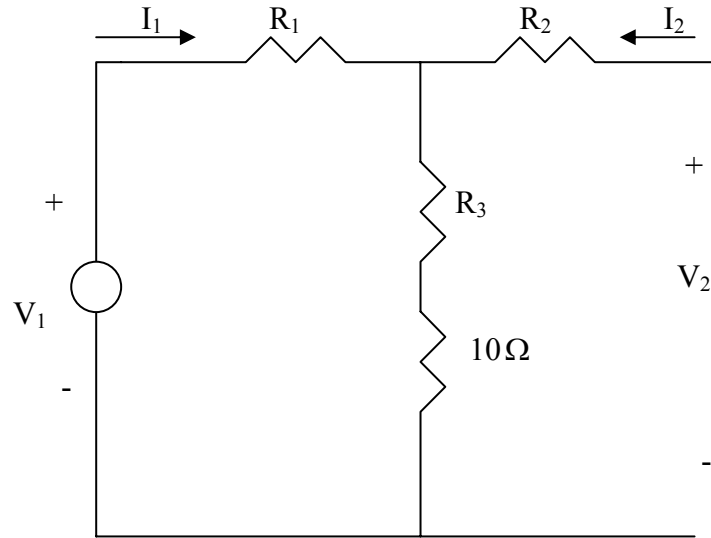
Alternatively,

$$Z_{\text{in}} = Z_s + h_{11} - \frac{h_{12} h_{21} Z_L}{1 + h_{22} Z_L}$$

$$Z_{\text{in}} = 200 + 800 - \frac{(10^{-4})(50)(50 \times 10^3)}{1 + (0.5 \times 10^{-5})(50 \times 10^3)} = \underline{\underline{800 \Omega}}$$

**Chapter 19, Solution 39.**

To get  $g_{11}$  and  $g_{21}$ , consider the circuit below which is partly obtained by converting the delta to wye subnetwork.

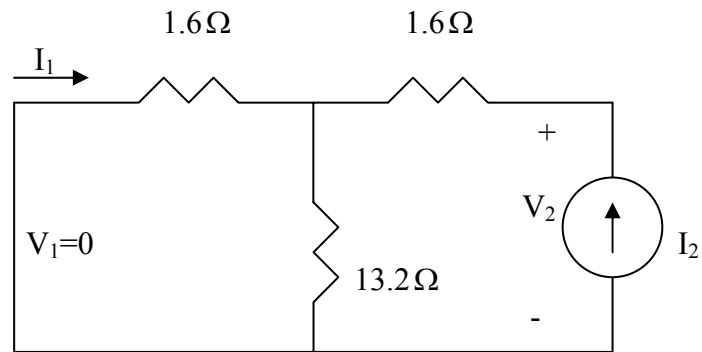


$$R_1 = \frac{4 \times 8}{8 + 8 + 4} = 1.6 = R_2, \quad R_3 = \frac{8 \times 8}{20} = 3.2$$

$$V_2 = \frac{13.2}{13.2 + 1.6} V_1 = 0.8919 V_1 \quad \longrightarrow \quad g_{21} = \frac{V_2}{V_1} = 0.8919$$

$$V_1 = I_1(1.6 + 3.2 + 10) = 14.8 I_1 \quad \longrightarrow \quad g_{11} = \frac{I_1}{V_1} = \frac{1}{14.8} = 0.06757$$

To get  $g_{22}$  and  $g_{12}$ , consider the circuit below.



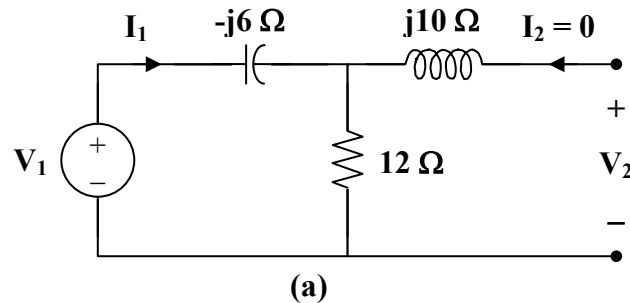
$$I_1 = -\frac{13.2}{13.2+1.6}I_2 = -0.8919I_2 \quad \longrightarrow \quad g_{12} = \frac{I_1}{I_2} = -0.8919$$

$$V_2 = I_2(1.6 + 13.2 // 1.6) = 3.027I_2 \quad \longrightarrow \quad g_{22} = \frac{V_2}{I_2} = 3.027$$

$$[g] = \begin{bmatrix} 0.06757 & -0.8919 \\ 0.8919 & 3.027 \end{bmatrix}$$

### Chapter 19, Solution 40.

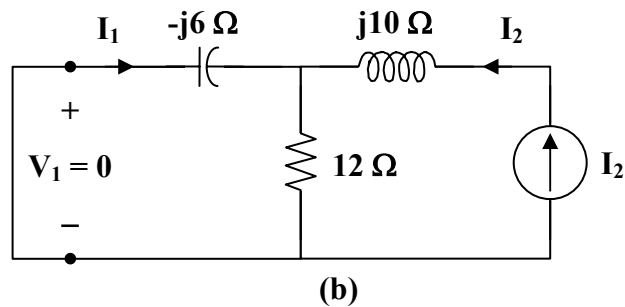
To get  $g_{11}$  and  $g_{21}$ , consider the circuit in Fig. (a).



$$V_1 = (12 - j6)I_1 \quad \longrightarrow \quad g_{11} = \frac{I_1}{V_1} = \frac{1}{12 - j6} = 0.0667 + j0.0333 \text{ S}$$

$$g_{21} = \frac{V_2}{V_1} = \frac{12I_1}{(12 - j6)I_1} = \frac{2}{2 - j} = 0.8 + j0.4$$

To get  $g_{12}$  and  $g_{22}$ , consider the circuit in Fig. (b).



$$\mathbf{I}_1 = \frac{-12}{12 - j6} \mathbf{I}_2 \longrightarrow \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-12}{12 - j6} = -\mathbf{g}_{21} = -0.8 - j0.4$$

$$\mathbf{V}_2 = (j10 + 12 \parallel -j6) \mathbf{I}_2$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = j10 + \frac{(12)(-j6)}{12 - j6} = 2.4 + j5.2 \Omega$$

$$[\mathbf{g}] = \underline{\underline{\begin{bmatrix} 0.0667 + j0.0333 \text{ S} & -0.8 - j0.4 \\ 0.8 + j0.4 & 2.4 + j5.2 \Omega \end{bmatrix}}}$$

### Chapter 19, Solution 41.

For the g parameters

$$\mathbf{I}_1 = \mathbf{g}_{11} \mathbf{V}_1 + \mathbf{g}_{12} \mathbf{I}_2 \quad (1)$$

$$\mathbf{V}_2 = \mathbf{g}_{21} \mathbf{V}_1 + \mathbf{g}_{22} \mathbf{I}_2 \quad (2)$$

But  $\mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1 \mathbf{Z}_s$  and

$$\mathbf{V}_2 = -\mathbf{I}_2 \mathbf{Z}_L = \mathbf{g}_{21} \mathbf{V}_1 + \mathbf{g}_{22} \mathbf{I}_2$$

$$0 = \mathbf{g}_{21} \mathbf{V}_1 + (\mathbf{g}_{22} + \mathbf{Z}_L) \mathbf{I}_2$$

or 
$$\mathbf{V}_1 = \frac{-(\mathbf{g}_{22} + \mathbf{Z}_L)}{\mathbf{g}_{21}} \mathbf{I}_2$$

Substituting this into (1),

$$\mathbf{I}_1 = \frac{(\mathbf{g}_{22} \mathbf{g}_{11} + \mathbf{Z}_L \mathbf{g}_{11} - \mathbf{g}_{21} \mathbf{g}_{12})}{-\mathbf{g}_{21}} \mathbf{I}_2$$

or 
$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \underline{\underline{\frac{-\mathbf{g}_{21}}{\mathbf{g}_{11} \mathbf{Z}_L + \Delta_g}}}}$$

Also, 
$$\begin{aligned} \mathbf{V}_2 &= \mathbf{g}_{21} (\mathbf{V}_s - \mathbf{I}_1 \mathbf{Z}_s) + \mathbf{g}_{22} \mathbf{I}_2 \\ &= \mathbf{g}_{21} \mathbf{V}_s - \mathbf{g}_{21} \mathbf{Z}_s \mathbf{I}_1 + \mathbf{g}_{22} \mathbf{I}_2 \\ &= \mathbf{g}_{21} \mathbf{V}_s + \mathbf{Z}_s (\mathbf{g}_{11} \mathbf{Z}_L + \Delta_g) \mathbf{I}_2 + \mathbf{g}_{22} \mathbf{I}_2 \end{aligned}$$

But 
$$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{\mathbf{Z}_L}$$

$$\mathbf{V}_2 = \mathbf{g}_{21} \mathbf{V}_s - [\mathbf{g}_{11} \mathbf{Z}_s \mathbf{Z}_L + \Delta_g \mathbf{Z}_s + \mathbf{g}_{22}] \left[ \frac{\mathbf{V}_2}{\mathbf{Z}_L} \right]$$

$$\frac{\mathbf{V}_2 [\mathbf{Z}_L + \mathbf{g}_{11} \mathbf{Z}_s \mathbf{Z}_L + \Delta_g \mathbf{Z}_s + \mathbf{g}_{22}]}{\mathbf{Z}_L} = \mathbf{g}_{21} \mathbf{V}_s$$

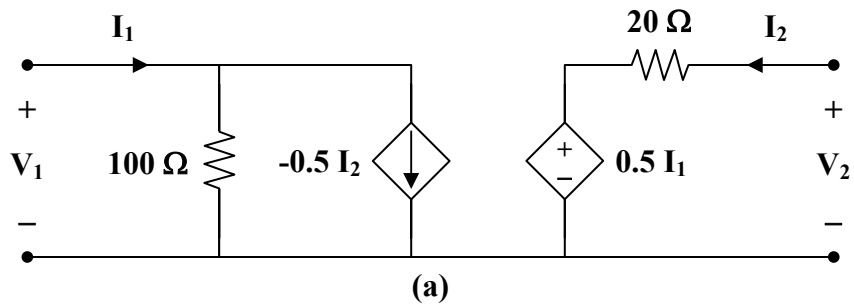
$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21} \mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11} \mathbf{Z}_s \mathbf{Z}_L + \Delta_g \mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21} \mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11} \mathbf{Z}_s \mathbf{Z}_L + \mathbf{g}_{11} \mathbf{g}_{22} \mathbf{Z}_s - \mathbf{g}_{21} \mathbf{g}_{12} \mathbf{Z}_s + \mathbf{g}_{22}}$$

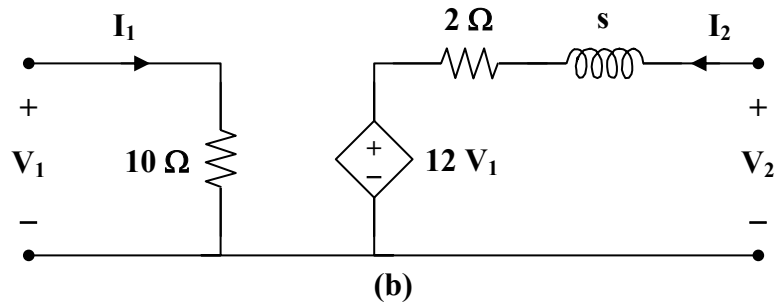
$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21} \mathbf{Z}_L}{\underline{\underline{(\mathbf{1} + \mathbf{g}_{11} \mathbf{Z}_s)(\mathbf{g}_{22} + \mathbf{Z}_L) - \mathbf{g}_{12} \mathbf{g}_{21} \mathbf{Z}_s}}}$$

**Chapter 19, Solution 42.**

(a) The network is shown in Fig. (a).

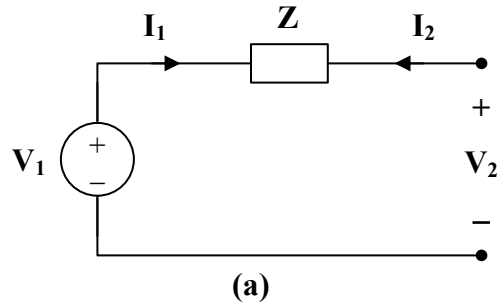


(b) The network is shown in Fig. (b).



**Chapter 19, Solution 43.**

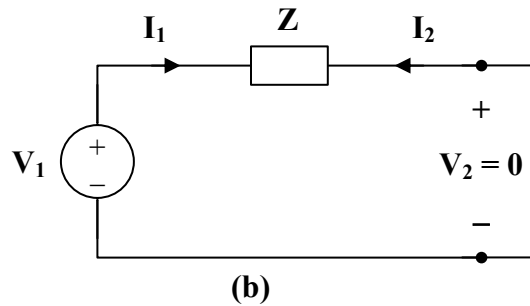
(a) To find **A** and **C**, consider the network in Fig. (a).



$$V_1 = V_2 \longrightarrow A = \frac{V_1}{V_2} = 1$$

$$I_1 = 0 \longrightarrow C = \frac{I_1}{V_2} = 0$$

To get **B** and **D**, consider the circuit in Fig. (b).



$$V_1 = ZI_1, \quad I_2 = -I_1$$

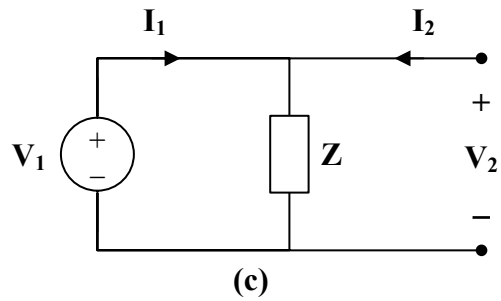
$$B = \frac{-V_1}{I_2} = \frac{-ZI_1}{-I_1} = Z$$

$$D = \frac{-I_1}{I_2} = 1$$

Hence,

$$\underline{[T]} = \underline{\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}}$$

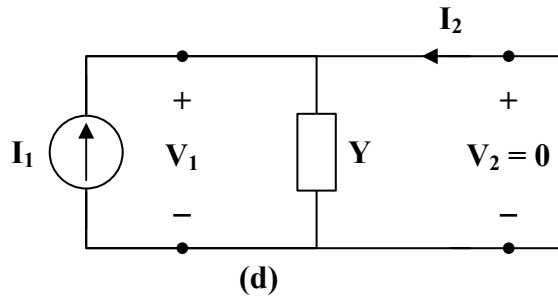
(b) To find **A** and **C**, consider the circuit in Fig. (c).



$$V_1 = V_2 \longrightarrow A = \frac{V_1}{V_2} = 1$$

$$V_1 = ZI_1 = V_2 \longrightarrow C = \frac{I_1}{V_2} = \frac{1}{Z} = Y$$

To get **B** and **D**, refer to the circuit in Fig.(d).



$$V_1 = V_2 = 0 \qquad I_2 = -I_1$$

$$B = \frac{-V_1}{I_2} = 0, \qquad D = \frac{-I_1}{I_2} = 1$$

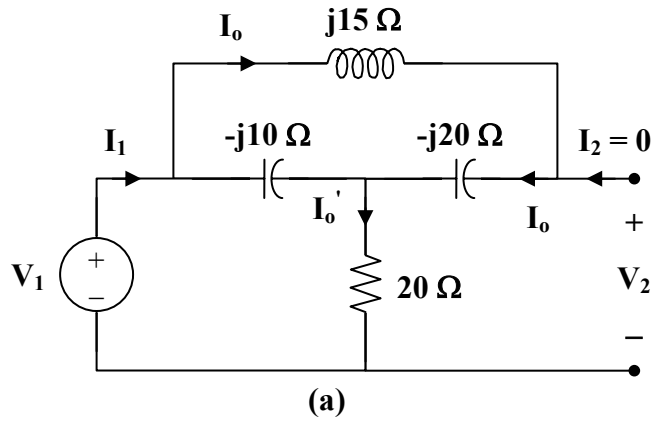
Thus,

$$\underline{[T]} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

### Chapter 19, Solution 44.

To determine **A** and **C**, consider the circuit in Fig.(a).





$$V_1 = [20 + (-j10) \parallel (j15 - j20)] I_1$$

$$V_1 = \left[ 20 + \frac{(-j10)(-j5)}{-j15} \right] I_1 = \left[ 20 - j\frac{10}{3} \right] I_1$$

$$I_o' = I_1$$

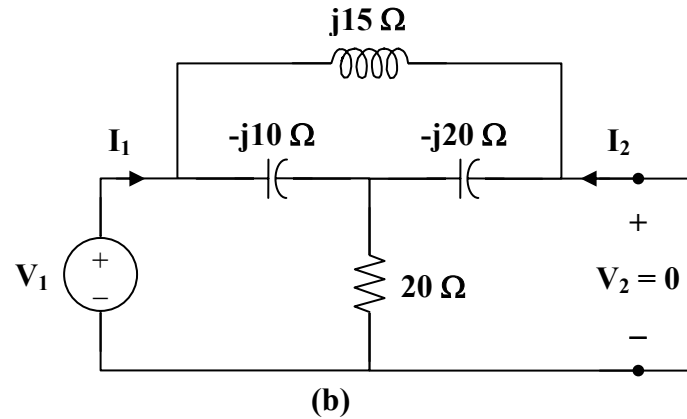
$$I_o = \left( \frac{-j10}{-j10 - j5} \right) I_1 = \left( \frac{2}{3} \right) I_1$$

$$V_2 = (-j20)I_o + 20I_o' = -j\frac{40}{3}I_1 + 20I_1 = \left( 20 - j\frac{40}{3} \right) I_1$$

$$A = \frac{V_1}{V_2} = \frac{(20 - j10/3)I_1}{\left( 20 - j\frac{40}{3} \right) I_1} = 0.7692 + j0.3461$$

$$C = \frac{I_1}{V_2} = \frac{1}{20 - j\frac{40}{3}} = 0.03461 + j0.023$$

To find **B** and **D**, consider the circuit in Fig. (b).

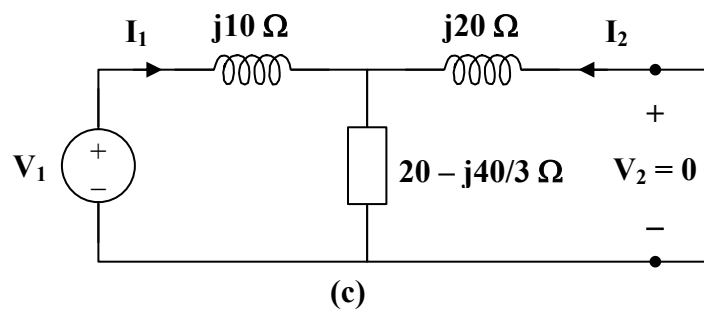


We may transform the  $\Delta$  subnetwork to a T as shown in Fig. (c).

$$\mathbf{Z}_1 = \frac{(j15)(-j10)}{j15 - j10 - j20} = j10$$

$$\mathbf{Z}_2 = \frac{(-j10)(-j20)}{-j15} = -j\frac{40}{3}$$

$$\mathbf{Z}_3 = \frac{(j15)(-j20)}{-j15} = j20$$



$$-\mathbf{I}_2 = \frac{20 - j40/3}{20 - j40/3 + j20} \mathbf{I}_1 = \frac{3 - j2}{3 + j} \mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{3 + j}{3 - j2} = 0.5385 + j0.6923$$

$$\mathbf{V}_1 = \left[ j10 + \frac{(j20)(20 - j40/3)}{20 - j40/3 + j20} \right] \mathbf{I}_1$$

$$\mathbf{V}_1 = [j10 + 2(9 + j7)]\mathbf{I}_1 = j\mathbf{I}_1(24 - j18)$$

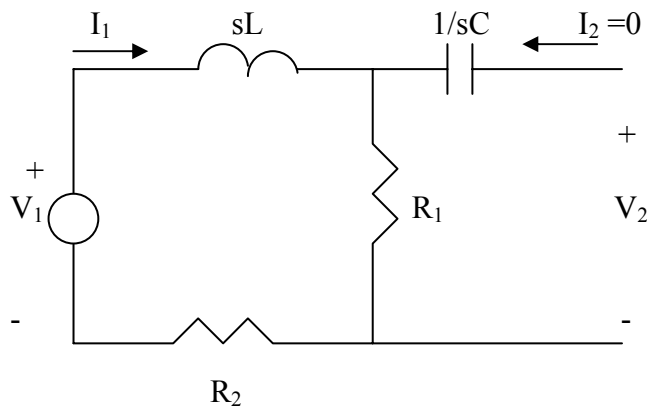
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{-j\mathbf{I}_1(24 - j18)}{\frac{-(3 - j2)}{3 + j}\mathbf{I}_1} = \frac{6}{13}(-15 + j55)$$

$$\mathbf{B} = -6.923 + j25.385 \Omega$$

$$\underline{\underline{[\mathbf{T}] = \begin{bmatrix} \mathbf{0.7692 + j\mathbf{0.3461} & -\mathbf{6.923 + j\mathbf{25.385} \Omega} \\ \mathbf{0.03461 + j\mathbf{0.023} S} & \mathbf{0.5385 + j\mathbf{0.6923} \end{bmatrix}}}$$

### Chapter 19, Solution 45.

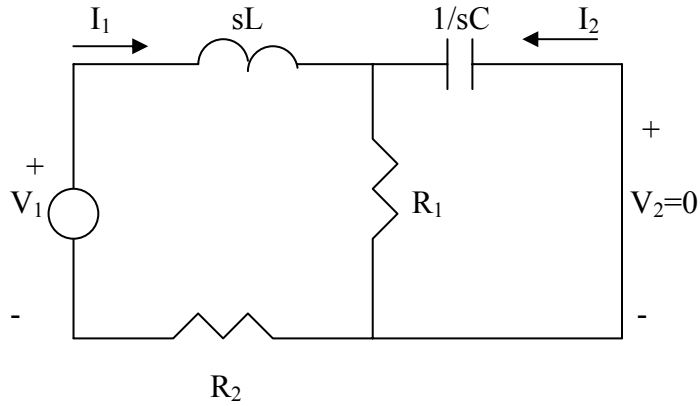
To obtain A and C, consider the circuit below.



$$V_2 = \frac{R_1}{R_1 + R_2 + sL} V_1 \quad \longrightarrow \quad A = \frac{V_1}{V_2} = \underline{\underline{\frac{R_1 + R_2 + sL}{R_1}}}$$

$$V_2 = I_1 R_1 \quad \longrightarrow \quad C = \underline{\underline{\frac{I_1}{V_2} = \frac{1}{R_1}}}$$

To obtain B and D, consider the circuit below.



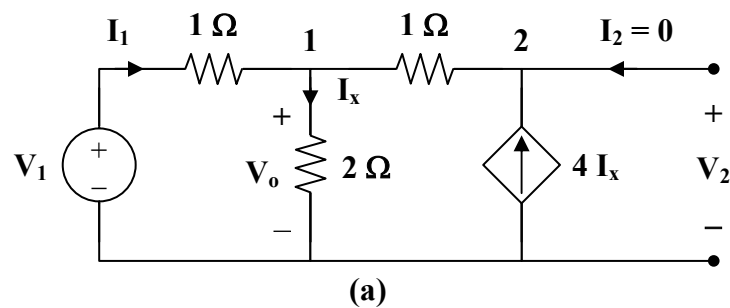
$$I_2 = -\frac{R_1}{R_1 + \frac{1}{sC}} I_1 = -\frac{sR_1C}{1 + sR_1C} I_1 \quad \longrightarrow \quad \underline{D = -\frac{I_1}{I_2} = \frac{1 + sR_1C}{sR_1C}}$$

$$V_1 = \left( R_2 + sL + \frac{\frac{R_1}{sC}}{R_1 + \frac{1}{sC}} \right) I_1 = -\frac{[(1 + sR_1C)(R_2 + sL) + R_1](1 + sR_1C)}{1 + sR_1C} \frac{I_1}{sR_1C} I_2$$

$$\underline{B = -\frac{V_1}{I_2} = \frac{1}{sR_1C} [R_1 + (1 + sR_1C)(R_2 + sL)]}$$

**Chapter 19, Solution 46.**

To get **A** and **C**, refer to the circuit in Fig.(a).



At node 1,

$$I_1 = \frac{V_o}{2} + \frac{V_o - V_2}{1} \quad \longrightarrow \quad 2I_1 = 3V_o - 2V_2 \quad (1)$$

At node 2,

$$\frac{V_o - V_2}{1} = 4I_x = \frac{4V_o}{2} = 2V_o \longrightarrow V_o = -V_2 \quad (2)$$

From (1) and (2),

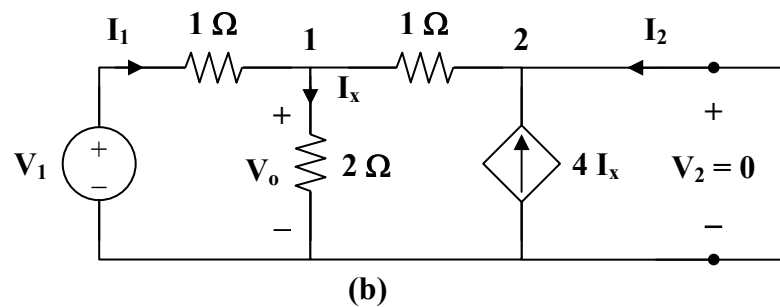
$$2I_1 = -5V_2 \longrightarrow C = \frac{I_1}{V_2} = \frac{-5}{2} = -2.5 \text{ S}$$

But 
$$I_1 = \frac{V_1 - V_o}{1} = V_1 + V_2$$

$$-2.5V_2 = V_1 + V_2 \longrightarrow V_1 = -3.5V_2$$

$$A = \frac{V_1}{V_2} = -3.5$$

To get **B** and **D**, consider the circuit in Fig. (b).



At node 1,

$$I_1 = \frac{V_o}{2} + \frac{V_o}{1} \longrightarrow 2I_1 = 3V_o \quad (3)$$

At node 2,

$$I_2 + \frac{V_o}{1} + 4I_x = 0$$

$$-I_2 = V_o + 2V_o = 0 \longrightarrow I_2 = -3V_o \quad (4)$$

Adding (3) and (4),

$$2I_1 + I_2 = 0 \longrightarrow I_1 = -0.5I_2 \quad (5)$$

$$D = \frac{-I_1}{I_2} = 0.5$$

But 
$$\mathbf{I}_1 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{1} \longrightarrow \mathbf{V}_1 = \mathbf{I}_1 + \mathbf{V}_o \quad (6)$$

Substituting (5) and (4) into (6),

$$\mathbf{V}_1 = \frac{-1}{2}\mathbf{I}_2 + \frac{-1}{3}\mathbf{I}_2 = \frac{-5}{6}\mathbf{I}_2$$

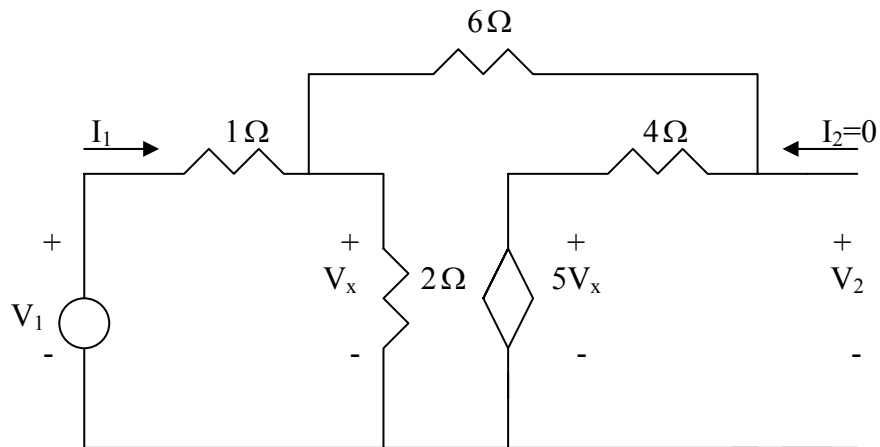
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{5}{6} = 0.8333 \Omega$$

Thus,

$$\mathbf{[T]} = \underline{\underline{\begin{bmatrix} -3.5 & 0.8333 \Omega \\ -2.5 \text{ S} & -0.5 \end{bmatrix}}}$$

### Chapter 19, Solution 47.

To get A and C, consider the circuit below.



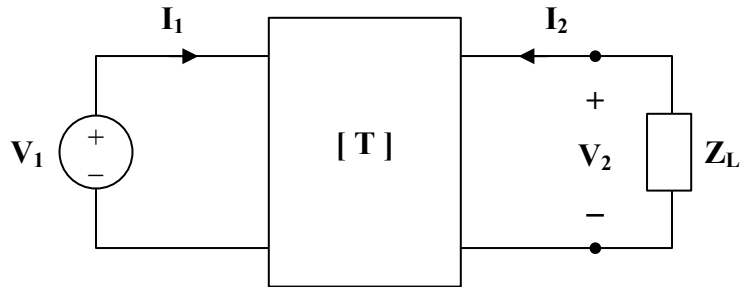
$$\frac{V_1 - V_x}{1} = \frac{V_x}{2} + \frac{V_x - 5V_x}{10} \longrightarrow V_1 = 1.1V_x$$

$$V_2 = 4(-0.4V_x) + 5V_x = 3.4V_x \longrightarrow A = \frac{V_1}{V_2} = 1.1/3.4 = 0.3235$$

$$I_1 = \frac{V_1 - V_x}{1} = 1.1V_x - V_x = 0.1V_x \longrightarrow C = \frac{I_1}{V_2} = 0.1/3.4 = 0.02941$$

**Chapter 19, Solution 48.**

(a) Refer to the circuit below.



$$V_1 = 4V_2 - 30I_2 \quad (1)$$

$$I_1 = 0.1V_2 - I_2 \quad (2)$$

When the output terminals are shorted,  $V_2 = 0$ .

So, (1) and (2) become

$$V_1 = -30I_2 \quad \text{and} \quad I_1 = -I_2$$

Hence,

$$Z_{in} = \frac{V_1}{I_1} = \underline{\underline{30 \Omega}}$$

(b) When the output terminals are open-circuited,  $I_2 = 0$ .

So, (1) and (2) become

$$\begin{aligned} V_1 &= 4V_2 \\ I_1 &= 0.1V_2 \quad \text{or} \quad V_2 = 10I_1 \\ V_1 &= 40I_1 \end{aligned}$$

$$Z_{in} = \frac{V_1}{I_1} = \underline{\underline{40 \Omega}}$$

(c) When the output port is terminated by a  $10\text{-}\Omega$  load,  $V_2 = -10I_2$ .

So, (1) and (2) become

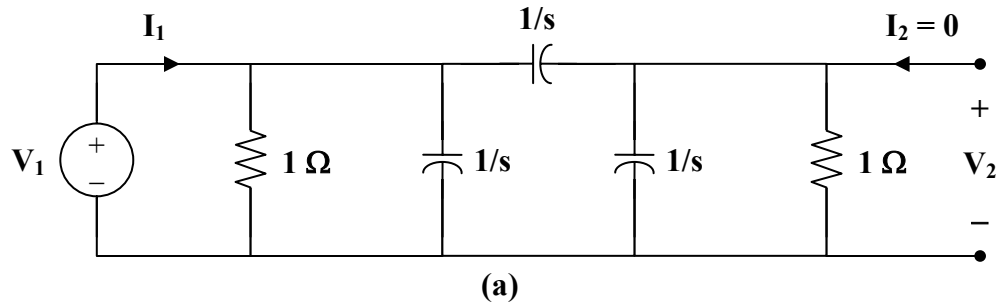
$$\begin{aligned} V_1 &= -40I_2 - 30I_2 = -70I_2 \\ I_1 &= -I_2 - I_2 = -2I_2 \\ V_1 &= 35I_1 \end{aligned}$$

$$Z_{in} = \frac{V_1}{I_1} = \underline{\underline{35 \Omega}}$$

Alternatively, we may use  $\mathbf{Z}_{in} = \frac{\mathbf{A}\mathbf{Z}_L + \mathbf{B}}{\mathbf{C}\mathbf{Z}_L + \mathbf{D}}$

**Chapter 19, Solution 49.**

To get **A** and **C**, refer to the circuit in Fig.(a).



$$1 \parallel \frac{1}{s} = \frac{1/s}{1 + 1/s} = \frac{1}{s+1}$$

$$\mathbf{V}_2 = \frac{1 \parallel 1/s}{1/s + 1 \parallel 1/s} \mathbf{V}_1$$

$$\mathbf{A} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\frac{1}{s+1}}{\frac{1}{s} + \frac{1}{s+1}} = \frac{s}{2s+1}$$

$$\mathbf{V}_1 = \mathbf{I}_1 \left( \frac{1}{s+1} \right) \parallel \left( \frac{1}{s} + \frac{1}{s+1} \right) = \mathbf{I}_1 \left( \frac{1}{s+1} \right) \parallel \left( \frac{2s+1}{s(s+1)} \right)$$

$$\frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\left( \frac{1}{s+1} \right) \cdot \left( \frac{2s+1}{s(s+1)} \right)}{\frac{1}{s+1} + \frac{2s+1}{s(s+1)}} = \frac{2s+1}{(s+1)(3s+1)}$$

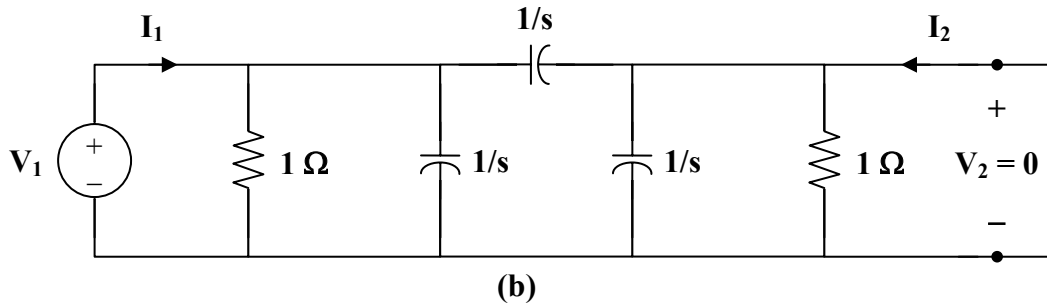
But  $\mathbf{V}_1 = \mathbf{V}_2 \cdot \frac{2s+1}{s}$



Hence, 
$$\frac{V_2}{I_1} \cdot \frac{2s+1}{s} = \frac{2s+1}{(s+1)(3s+1)}$$

$$C = \frac{V_2}{I_1} = \frac{(s+1)(3s+1)}{s}$$

To get **B** and **D**, consider the circuit in Fig. (b).



$$V_1 = I_1 \left( 1 \parallel \frac{1}{s} \parallel \frac{1}{s} \right) = I_1 \left( 1 \parallel \frac{1}{2s} \right) = \frac{I_1}{2s+1}$$

$$I_2 = \frac{\frac{-1}{s+1} I_1}{\frac{1}{s+1} + \frac{1}{s}} = \frac{-s}{2s+1} I_1$$

$$D = \frac{-I_1}{I_2} = \frac{2s+1}{s} = 2 + \frac{1}{s}$$

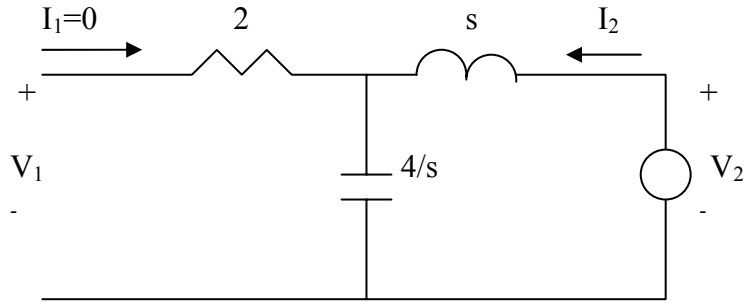
$$V_1 = \left( \frac{1}{2s+1} \right) \left( \frac{2s+1}{-s} \right) I_2 = \frac{I_2}{-s} \longrightarrow \mathbf{B} = \frac{-V_1}{I_2} = \frac{1}{s}$$

Thus,

$$[\mathbf{T}] = \begin{bmatrix} \frac{2}{2s+1} & \frac{1}{s} \\ \frac{(s+1)(3s+1)}{s} & 2 + \frac{1}{s} \end{bmatrix}$$

### Chapter 19, Solution 50.

To get a and c, consider the circuit below.

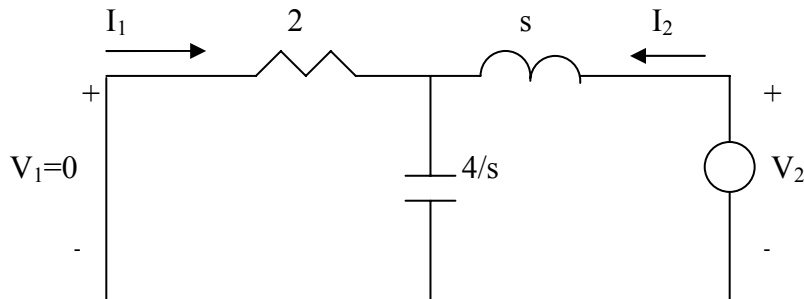


$$V_1 = \frac{4/s}{s + 4/s} V_2 = \frac{4}{s^2 + 4} V_2 \quad \longrightarrow \quad a = \frac{V_2}{V_1} = 1 + 0.25s^2$$

$$V_2 = (s + 4/s)I_2 \quad \text{or}$$

$$I_2 = \frac{V_2}{s + 4/s} = \frac{(1 + 0.25s^2)V_1}{s + 4/s} \quad \longrightarrow \quad c = \frac{I_2}{V_1} = \frac{s + 0.25s^3}{s^2 + 4}$$

To get b and d, consider the circuit below.



$$I_1 = \frac{-4/s}{2 + 4/s} I_2 = -\frac{2I_2}{s + 2} \quad \longrightarrow \quad d = -\frac{I_2}{I_1} = 1 + 0.5s$$

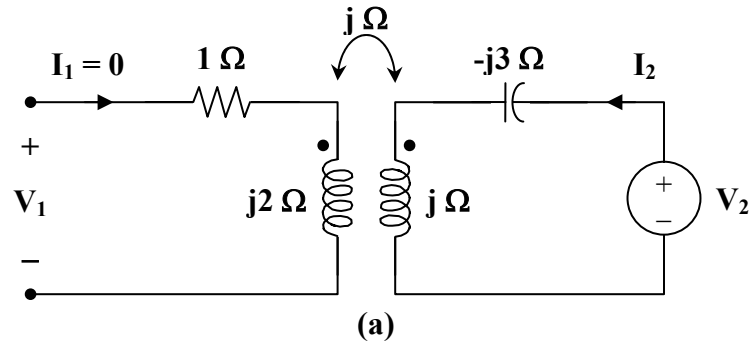
$$V_2 = (s + 2 // \frac{4}{s})I_2 = \frac{(s^2 + 2s + 4)}{s + 2} I_2$$

$$= -\frac{(s^2 + 2s + 4)(s + 2)}{s + 2} I_1 \quad \longrightarrow \quad b = -\frac{V_2}{I_1} = 0.5s^2 + s + 2$$

$$[t] = \begin{bmatrix} 0.25s^2 + 1 & 0.5s^2 + s + 2 \\ \frac{0.25s^2 + s}{s^2 + 4} & 0.5s + 1 \end{bmatrix}$$

Chapter 19, Solution 51.

To get **a** and **c**, consider the circuit in Fig. (a).



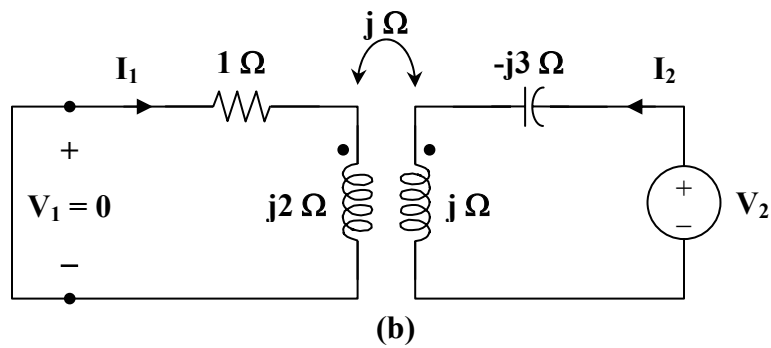
$$V_2 = I_2 (j - j3) = -j2 I_2$$

$$V_1 = -j I_2$$

$$a = \frac{V_2}{V_1} = \frac{-j2 I_2}{-j I_2} = 2$$

$$c = \frac{I_2}{V_1} = \frac{1}{-j} = j$$

To get **b** and **d**, consider the circuit in Fig. (b).



For mesh 1,

$$0 = (1 + j2) I_1 - j I_2$$

or

$$\frac{I_2}{I_1} = \frac{1 + j2}{j} = 2 - j$$

$$\mathbf{d} = \frac{-\mathbf{I}_2}{\mathbf{I}_1} = -2 + j$$

For mesh 2,

$$\mathbf{V}_2 = \mathbf{I}_2 (j - j3) - j\mathbf{I}_1$$

$$\mathbf{V}_2 = \mathbf{I}_1 (2 - j)(-j2) - j\mathbf{I}_1 = (-2 - j5)\mathbf{I}_1$$

$$\mathbf{b} = \frac{-\mathbf{V}_2}{\mathbf{I}_1} = 2 + j5$$

Thus,

$$[\mathbf{t}] = \underline{\underline{\begin{bmatrix} 2 & 2 + j5 \\ j & -2 + j \end{bmatrix}}}$$

### Chapter 19, Solution 52.

It is easy to find the z parameters and then transform these to h parameters and T parameters.

$$[\mathbf{z}] = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix}$$

$$\begin{aligned} \Delta_z &= (R_1 + R_2)(R_2 + R_3) - R_2^2 \\ &= R_1R_2 + R_2R_3 + R_3R_1 \end{aligned}$$

$$(a) \quad [\mathbf{h}] = \begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{z_{22}}{-z_{21}} & \frac{1}{z_{22}} \end{bmatrix} = \begin{bmatrix} \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2 + R_3} & \frac{R_2}{R_2 + R_3} \\ \frac{-R_2}{R_2 + R_3} & \frac{1}{R_2 + R_3} \end{bmatrix}$$

Thus,

$$\underline{\underline{h_{11} = R_1 + \frac{R_2R_3}{R_2 + R_3}}}, \quad \underline{\underline{h_{12} = \frac{R_2}{R_2 + R_3} = -h_{21}}}, \quad \underline{\underline{h_{22} = \frac{1}{R_2 + R_3}}}$$

as required.

$$(b) \quad [\mathbf{T}] = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix} = \begin{bmatrix} \frac{R_1 + R_2}{R_2} & \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \\ \frac{1}{R_2} & \frac{R_2}{R_2 + R_3} \end{bmatrix}$$

Hence,

$$\underline{\underline{\mathbf{A} = 1 + \frac{\mathbf{R}_1}{\mathbf{R}_2}}}, \quad \underline{\underline{\mathbf{B} = \mathbf{R}_3 + \frac{\mathbf{R}_1}{\mathbf{R}_2}(\mathbf{R}_2 + \mathbf{R}_3)}}, \quad \underline{\underline{\mathbf{C} = \frac{1}{\mathbf{R}_2}}}, \quad \underline{\underline{\mathbf{D} = 1 + \frac{\mathbf{R}_3}{\mathbf{R}_2}}}$$

as required.

### Chapter 19, Solution 53.

For the z parameters,

$$\mathbf{V}_1 = \mathbf{z}_{11} \mathbf{I}_1 + \mathbf{z}_{12} \mathbf{I}_2 \quad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{12} \mathbf{I}_1 + \mathbf{z}_{22} \mathbf{I}_2 \quad (2)$$

For ABCD parameters,

$$\mathbf{V}_1 = \mathbf{A} \mathbf{V}_2 - \mathbf{B} \mathbf{I}_2 \quad (3)$$

$$\mathbf{I}_1 = \mathbf{C} \mathbf{V}_2 - \mathbf{D} \mathbf{I}_2 \quad (4)$$

From (4),

$$\mathbf{V}_2 = \frac{\mathbf{I}_1}{\mathbf{C}} + \frac{\mathbf{D}}{\mathbf{C}} \mathbf{I}_2 \quad (5)$$

Comparing (2) and (5),

$$\mathbf{z}_{21} = \frac{1}{\mathbf{C}}, \quad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}}$$

Substituting (5) into (3),

$$\begin{aligned} \mathbf{V}_1 &= \frac{\mathbf{A}}{\mathbf{C}} \mathbf{I}_1 + \left( \frac{\mathbf{AD}}{\mathbf{C}} - \mathbf{B} \right) \mathbf{I}_2 \\ &= \frac{\mathbf{A}}{\mathbf{C}} \mathbf{I}_1 + \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}} \mathbf{I}_2 \end{aligned} \quad (6)$$

Comparing (6) and (1),

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}}, \quad \mathbf{z}_{12} = \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}} = \frac{\Delta_T}{\mathbf{C}}$$

Thus,

$$\underline{\underline{[\mathbf{Z}] = \begin{bmatrix} \frac{\mathbf{A}}{\mathbf{C}} & \frac{\Delta_T}{\mathbf{C}} \\ \frac{1}{\mathbf{C}} & \frac{\mathbf{D}}{\mathbf{C}} \end{bmatrix}}}$$

### Chapter 19, Solution 54.

For the y parameters

$$\mathbf{I}_1 = \mathbf{y}_{11} \mathbf{V}_1 + \mathbf{y}_{12} \mathbf{V}_2 \quad (1)$$

$$\mathbf{I}_2 = \mathbf{y}_{21} \mathbf{V}_1 + \mathbf{y}_{22} \mathbf{V}_2 \quad (2)$$

From (2),

$$\mathbf{V}_1 = \frac{\mathbf{I}_2}{\mathbf{y}_{21}} - \frac{\mathbf{y}_{22}}{\mathbf{y}_{21}} \mathbf{V}_2$$

or

$$\mathbf{V}_1 = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{21}} \mathbf{V}_2 + \frac{1}{\mathbf{y}_{21}} \mathbf{I}_2 \quad (3)$$

Substituting (3) into (1) gives

$$\mathbf{I}_1 = \frac{-\mathbf{y}_{11} \mathbf{y}_{22}}{\mathbf{y}_{21}} \mathbf{V}_2 + \mathbf{y}_{12} \mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}} \mathbf{I}_2$$

or

$$\mathbf{I}_1 = \frac{-\Delta_y}{\mathbf{y}_{21}} \mathbf{V}_2 + \frac{\mathbf{y}_{11}}{\mathbf{y}_{21}} \mathbf{I}_2 \quad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{V}_1 = \mathbf{A} \mathbf{V}_2 - \mathbf{B} \mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C} \mathbf{V}_2 - \mathbf{D} \mathbf{I}_2$$

clearly shows that

$$\underline{\mathbf{A} = \frac{-\mathbf{y}_{22}}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{B} = \frac{-1}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{C} = \frac{-\Delta_y}{\mathbf{y}_{21}}}, \quad \underline{\mathbf{D} = \frac{-\mathbf{y}_{11}}{\mathbf{y}_{21}}}$$

as required.

### Chapter 19, Solution 55.

For the z parameters

$$\mathbf{V}_1 = \mathbf{z}_{11} \mathbf{I}_1 + \mathbf{z}_{12} \mathbf{I}_2 \quad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21} \mathbf{I}_1 + \mathbf{z}_{22} \mathbf{I}_2 \quad (2)$$

From (1),

$$\mathbf{I}_1 = \frac{1}{\mathbf{z}_{11}} \mathbf{V}_1 - \frac{\mathbf{z}_{12}}{\mathbf{z}_{11}} \mathbf{I}_2 \quad (3)$$

Substituting (3) into (2) gives

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}} \mathbf{V}_1 + \left( \mathbf{z}_{22} - \frac{\mathbf{z}_{21} \mathbf{z}_{12}}{\mathbf{z}_{11}} \right) \mathbf{I}_2$$

or 
$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}} \mathbf{V}_1 + \frac{\Delta_z}{\mathbf{z}_{11}} \mathbf{I}_2 \quad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{I}_1 = \mathbf{g}_{11} \mathbf{V}_1 + \mathbf{g}_{12} \mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{g}_{21} \mathbf{V}_1 + \mathbf{g}_{22} \mathbf{I}_2$$

indicates that

$$\underline{\underline{\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}}}, \quad \underline{\underline{\mathbf{g}_{12} = \frac{-\mathbf{z}_{12}}{\mathbf{z}_{11}}}}, \quad \underline{\underline{\mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}}}, \quad \underline{\underline{\mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}}}$$

as required.

### Chapter 19, Solution 56.

(a)  $\Delta_y = (2 + j)(3 - j) + j4 = 7 + j5$

$$[\mathbf{z}] = \begin{bmatrix} y_{22}/\Delta_y & -y_{12}/\Delta_y \\ -y_{21}/\Delta_y & y_{11}/\Delta_y \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0.2162 - j0.2973 & -0.2703 - j0.3784 \\ 0.0946 - j0.0676 & 0.2568 - j0.0405 \end{bmatrix} \Omega}}$$

(b)  $[\mathbf{h}] = \begin{bmatrix} 1/y_{11} & -y_{12}/y_{11} \\ y_{21}/y_{11} & \Delta_y/y_{11} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0.4 - j0.2 & -0.8 - j1.6 \\ -0.4 + j0.2 & 3.8 + j0.6 \end{bmatrix} \Omega}}$

(c)  $[\mathbf{t}] = \begin{bmatrix} -y_{11}/y_{12} & -1/y_{12} \\ -\Delta_y/y_{12} & -y_{22}/y_{12} \end{bmatrix} = \underline{\underline{\begin{bmatrix} -0.25 + j0.5 & j0.25 \\ -1.25 + j1.75 & 0.25 + j0.75 \end{bmatrix} \Omega}}$

### Chapter 19, Solution 57.

$$\Delta_T = (3)(7) - (20)(1) = 1$$

$$[\mathbf{z}] = \begin{bmatrix} \frac{\mathbf{A}}{\mathbf{C}} & \frac{\Delta_T}{\mathbf{C}} \\ \frac{1}{\mathbf{C}} & \frac{\mathbf{D}}{\mathbf{C}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 3 & 1 \\ 1 & 7 \end{bmatrix} \Omega}}$$

$$[\mathbf{y}] = \begin{bmatrix} \frac{\mathbf{D}}{\mathbf{B}} & \frac{-\Delta_T}{\mathbf{B}} \\ -1 & \frac{\mathbf{A}}{\mathbf{B}} \\ \frac{\mathbf{B}}{\mathbf{B}} & \frac{\mathbf{B}}{\mathbf{B}} \end{bmatrix} = \begin{bmatrix} \frac{7}{20} & \frac{-1}{20} \\ -1 & \frac{3}{20} \\ \frac{20}{20} & \frac{20}{20} \end{bmatrix} \mathbf{S}$$

$$[\mathbf{h}] = \begin{bmatrix} \frac{\mathbf{B}}{\mathbf{D}} & \frac{\Delta_T}{\mathbf{D}} \\ -1 & \frac{\mathbf{C}}{\mathbf{D}} \\ \frac{\mathbf{D}}{\mathbf{D}} & \frac{\mathbf{D}}{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} \frac{20}{7} \Omega & \frac{1}{7} \\ -1 & \frac{1}{7} \mathbf{S} \\ \frac{7}{7} & \frac{1}{7} \mathbf{S} \end{bmatrix}$$

$$[\mathbf{g}] = \begin{bmatrix} \frac{\mathbf{C}}{\mathbf{A}} & \frac{-\Delta_T}{\mathbf{A}} \\ \frac{1}{\mathbf{A}} & \frac{\mathbf{B}}{\mathbf{A}} \\ \frac{\mathbf{A}}{\mathbf{A}} & \frac{\mathbf{A}}{\mathbf{A}} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \mathbf{S} & \frac{-1}{3} \\ \frac{1}{3} & \frac{20}{3} \Omega \\ \frac{3}{3} & \frac{3}{3} \Omega \end{bmatrix}$$

$$[\mathbf{t}] = \begin{bmatrix} \frac{\mathbf{D}}{\Delta_T} & \frac{\mathbf{B}}{\Delta_T} \\ \frac{\Delta_T}{\mathbf{C}} & \frac{\Delta_T}{\mathbf{A}} \\ \frac{\Delta_T}{\Delta_T} & \frac{\Delta_T}{\Delta_T} \end{bmatrix} = \begin{bmatrix} \frac{7}{1} & \frac{20 \Omega}{3} \\ 1 \mathbf{S} & 3 \end{bmatrix}$$

**Chapter 19, Solution 58.**

The given set of equations is for the h parameters.

$$[\mathbf{h}] = \begin{bmatrix} 1 \Omega & 2 \\ -2 & 0.4 \mathbf{S} \end{bmatrix} \quad \Delta_h = (1)(0.4) - (2)(-2) = 4.4$$

$$(a) \quad [\mathbf{y}] = \begin{bmatrix} \frac{1}{\mathbf{h}_{11}} & \frac{-\mathbf{h}_{12}}{\mathbf{h}_{11}} \\ \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} & \frac{\Delta_h}{\mathbf{h}_{11}} \\ \frac{\mathbf{h}_{11}}{\mathbf{h}_{11}} & \frac{\mathbf{h}_{11}}{\mathbf{h}_{11}} \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 4.4 \end{bmatrix} \mathbf{S}$$

$$(b) \quad [\mathbf{T}] = \begin{bmatrix} \frac{-\Delta_h}{\mathbf{h}_{21}} & \frac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \frac{\mathbf{h}_{21}}{-\mathbf{h}_{22}} & \frac{-1}{\mathbf{h}_{21}} \\ \frac{\mathbf{h}_{21}}{\mathbf{h}_{21}} & \frac{\mathbf{h}_{21}}{\mathbf{h}_{21}} \end{bmatrix} = \begin{bmatrix} 2.2 & 0.5 \Omega \\ 0.2 \mathbf{S} & 0.5 \end{bmatrix}$$



**Chapter 19, Solution 59.**

$$\Delta_g = (0.06)(2) - (-0.4)(2) = 0.12 + 0.08 = 0.2$$

$$(a) \quad [z] = \begin{bmatrix} \frac{1}{g_{11}} & \frac{-g_{12}}{g_{11}} \\ \frac{g_{21}}{g_{11}} & \frac{\Delta_g}{g_{11}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 16.667 & 6.667 \\ 3.333 & 3.333 \end{bmatrix} \Omega}}$$

$$(b) \quad [y] = \begin{bmatrix} \frac{\Delta_g}{g_{22}} & \frac{g_{12}}{g_{22}} \\ \frac{-g_{21}}{g_{22}} & \frac{1}{g_{22}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0.1 & -0.2 \\ -0.1 & 0.5 \end{bmatrix} \text{S}}}$$

$$(c) \quad [h] = \begin{bmatrix} \frac{g_{22}}{\Delta_g} & \frac{-g_{12}}{\Delta_g} \\ \frac{-g_{21}}{\Delta_g} & \frac{g_{11}}{\Delta_g} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 10 \Omega & 2 \\ -1 & 0.3 \text{ S} \end{bmatrix}}}$$

$$(d) \quad [T] = \begin{bmatrix} \frac{1}{g_{21}} & \frac{g_{22}}{g_{21}} \\ \frac{g_{11}}{g_{21}} & \frac{\Delta_g}{g_{21}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 5 & 10 \Omega \\ 0.3 \text{ S} & 1 \end{bmatrix}}}$$

**Chapter 19, Solution 60.**

$$\Delta_y = y_{11} y_{22} - y_{12} y_{21} = 0.3 - 0.02 = 0.28$$

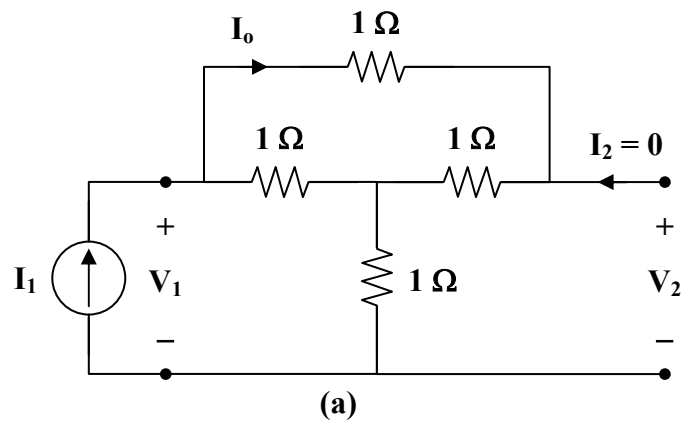
$$(a) \quad [z] = \begin{bmatrix} \frac{y_{22}}{\Delta_y} & \frac{-y_{12}}{\Delta_y} \\ \frac{-y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1.786 & 0.7143 \\ 0.3571 & 2.143 \end{bmatrix} \Omega}}$$

$$(b) \quad [h] = \begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1.667 \Omega & 0.3333 \\ -0.1667 & 0.4667 \text{ S} \end{bmatrix}}}$$

$$(c) \quad [t] = \begin{bmatrix} \frac{-y_{11}}{y_{12}} & \frac{-1}{y_{12}} \\ \frac{-\Delta_y}{y_{12}} & \frac{-y_{22}}{y_{12}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 3 & 5 \Omega \\ 1.4 \text{ S} & 2.5 \end{bmatrix}}}$$

**Chapter 19, Solution 61.**

(a) To obtain  $z_{11}$  and  $z_{21}$ , consider the circuit in Fig. (a).



$$V_1 = I_1 [1 + 1 \parallel (1 + 1)] = I_1 \left( 1 + \frac{2}{3} \right) = \frac{5}{3} I_1$$

$$z_{11} = \frac{V_1}{I_1} = \frac{5}{3}$$

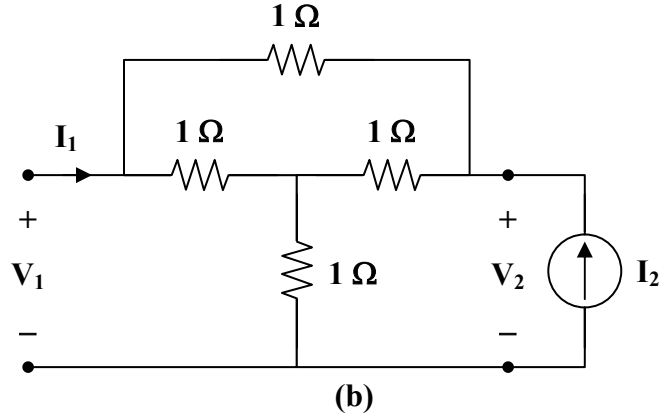
$$I_0 = \frac{1}{1+2} I_1 = \frac{1}{3} I_1$$

$$-V_2 + I_0 + I_1 = 0$$

$$V_2 = \frac{1}{3} I_1 + I_1 = \frac{4}{3} I_1$$

$$z_{21} = \frac{V_2}{I_1} = \frac{4}{3}$$

To obtain  $z_{22}$  and  $z_{12}$ , consider the circuit in Fig. (b).



Due to symmetry, this is similar to the circuit in Fig. (a).

$$z_{22} = z_{11} = \frac{5}{3}, \quad z_{21} = z_{12} = \frac{4}{3}$$

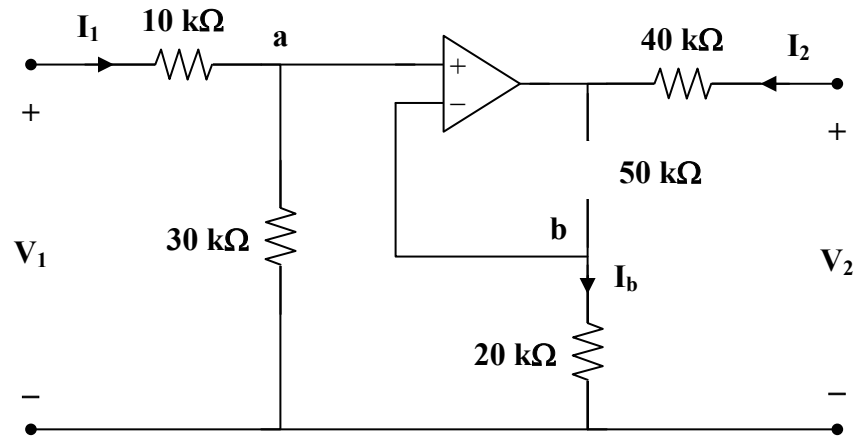
$$[z] = \begin{bmatrix} \frac{5}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{5}{3} \end{bmatrix} \Omega$$

$$(b) \quad [h] = \begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{z_{22}}{-z_{21}} & \frac{1}{z_{22}} \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \Omega & \frac{4}{5} \\ -4 & \frac{3}{5} S \end{bmatrix}$$

$$(c) \quad [T] = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & \frac{3}{4} \Omega \\ \frac{3}{4} S & \frac{5}{4} \end{bmatrix}$$

### Chapter 19, Solution 62.

Consider the circuit shown below.



Since no current enters the input terminals of the op amp,

$$V_1 = (10 + 30) \times 10^3 I_1 \quad (1)$$

But 
$$V_a = V_b = \frac{30}{40} V_1 = \frac{3}{4} V_1$$

$$I_b = \frac{V_b}{20 \times 10^3} = \frac{3}{80 \times 10^3} V_1$$

which is the same current that flows through the 50-kΩ resistor.

Thus, 
$$V_2 = 40 \times 10^3 I_2 + (50 + 20) \times 10^3 I_b$$

$$V_2 = 40 \times 10^3 I_2 + 70 \times 10^3 \cdot \frac{3}{80 \times 10^3} V_1$$

$$V_2 = \frac{21}{8} V_1 + 40 \times 10^3 I_2$$

$$V_2 = 105 \times 10^3 I_1 + 40 \times 10^3 I_2 \quad (2)$$

From (1) and (2),

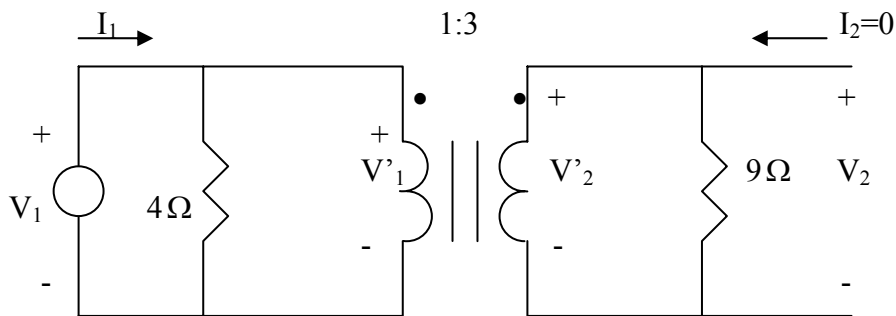
$$[z] = \underline{\underline{\begin{bmatrix} 40 & 0 \\ 105 & 40 \end{bmatrix} \text{ k}\Omega}}$$

$$\Delta_z = z_{11} z_{22} - z_{12} z_{21} = 16 \times 10^8$$

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ 1 & \frac{z_{22}}{z_{21}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0.381 & 15.24 \text{ k}\Omega \\ 9.52 \text{ }\mu\text{S} & 0.381 \end{bmatrix}}}$$

### Chapter 19, Solution 63.

To get  $z_{11}$  and  $z_{21}$ , consider the circuit below.

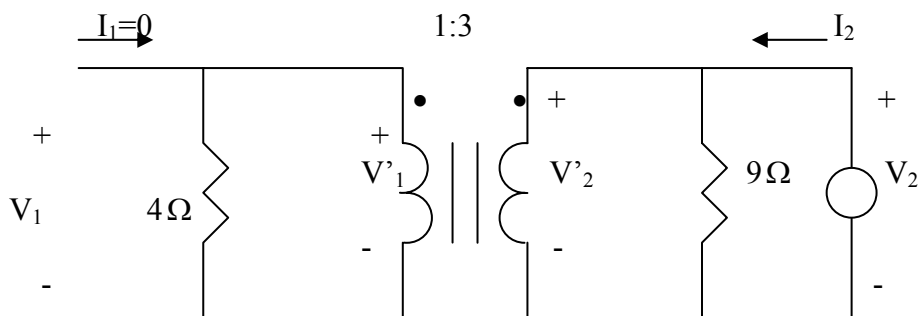


$$Z_R = \frac{9}{n^2} = 1, \quad n = 3$$

$$V_1 = (4 // Z_R)I_1 = \frac{4}{5}I_1 \quad \longrightarrow \quad z_{11} = \frac{V_1}{I_1} = 0.8$$

$$V_2 = V_2' = nV_1' = nV_1 = 3\left(\frac{4}{5}\right)I_1 \quad \longrightarrow \quad z_{21} = \frac{V_2}{I_1} = 2.4$$

To get  $z_{21}$  and  $z_{22}$ , consider the circuit below.



$$Z_R' = n^2(4) = 36, \quad n = 3$$

$$V_2 = (9 // Z_{R'}) I_2 = \frac{9 \times 36}{45} I_2 \longrightarrow z_{22} = \frac{V_2}{I_2} = 7.2$$

$$V_1 = \frac{V_2}{n} = \frac{V_2}{3} = 2.4 I_2 \longrightarrow z_{21} = \frac{V_1}{I_2} = 2.4$$

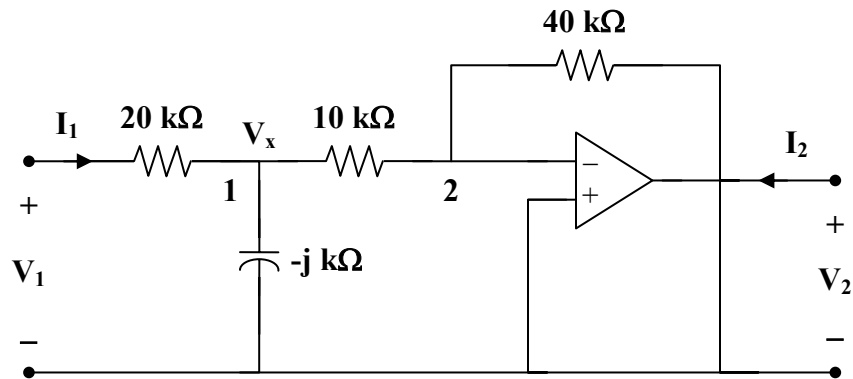
Thus,

$$[z] = \begin{bmatrix} 0.8 & 2.4 \\ 2.4 & 7.2 \end{bmatrix} \Omega$$

### Chapter 19, Solution 64.

$$1 \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{-j}{(10^3)(10^{-6})} = -j \text{ k}\Omega$$

Consider the op amp circuit below.



At node 1,

$$\frac{V_1 - V_x}{20} = \frac{V_x}{-j} + \frac{V_x - 0}{10}$$

$$V_1 = (3 + j20) V_x \quad (1)$$

At node 2,

$$\frac{V_x - 0}{10} = \frac{0 - V_2}{40} \longrightarrow V_x = \frac{-1}{4} V_2 \quad (2)$$

But 
$$I_1 = \frac{V_1 - V_x}{20 \times 10^3} \quad (3)$$

Substituting (2) into (3) gives

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 + 0.25\mathbf{V}_2}{20 \times 10^3} = 50 \times 10^{-6} \mathbf{V}_1 + 12.5 \times 10^{-6} \mathbf{V}_2 \quad (4)$$

Substituting (2) into (1) yields

$$\mathbf{V}_1 = \frac{-1}{4}(3 + j20)\mathbf{V}_2$$

or 
$$0 = \mathbf{V}_1 + (0.75 + j5)\mathbf{V}_2 \quad (5)$$

Comparing (4) and (5) with the following equations

$$\begin{aligned} \mathbf{I}_1 &= \mathbf{y}_{11} \mathbf{V}_1 + \mathbf{y}_{12} \mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{y}_{21} \mathbf{V}_1 + \mathbf{y}_{22} \mathbf{V}_2 \end{aligned}$$

indicates that  $\mathbf{I}_2 = 0$  and that

$$[\mathbf{y}] = \underline{\underline{\begin{bmatrix} 50 \times 10^{-6} & 12.5 \times 10^{-6} \\ 1 & 0.75 + j5 \end{bmatrix} \text{ S}}}$$

$$\Delta_y = (77.5 + j25) \times 10^{-6} - 12.5 \times 10^{-6} = (65 + j250) \times 10^{-6}$$

$$[\mathbf{h}] = \begin{bmatrix} \frac{1}{\mathbf{y}_{11}} & \frac{-\mathbf{y}_{12}}{\mathbf{y}_{11}} \\ \frac{\mathbf{y}_{21}}{\mathbf{y}_{11}} & \frac{\Delta_y}{\mathbf{y}_{11}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2 \times 10^4 \Omega & -0.25 \\ 2 \times 10^4 & 1.3 + j5 \text{ S} \end{bmatrix}}}$$

### Chapter 19, Solution 65.

The network consists of two two-ports in series. It is better to work with z parameters and then convert to y parameters.

For  $N_a$ , 
$$[\mathbf{z}_a] = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$$

For  $N_b$ , 
$$[\mathbf{z}_b] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b] = \begin{bmatrix} 6 & 3 \\ 3 & 3 \end{bmatrix}$$

$$\Delta_z = 18 - 9 = 9$$

$$[\mathbf{y}] = \begin{bmatrix} \frac{\mathbf{z}_{22}}{\Delta_z} & \frac{-\mathbf{z}_{12}}{\Delta_z} \\ \frac{-\mathbf{z}_{21}}{\Delta_z} & \frac{\mathbf{z}_{11}}{\Delta_z} \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} \mathbf{S}}}$$

### Chapter 19, Solution 66.

Since we have two two-ports in series, it is better to convert the given y parameters to z parameters.

$$\Delta_y = \mathbf{y}_{11} \mathbf{y}_{22} - \mathbf{y}_{12} \mathbf{y}_{21} = (2 \times 10^{-3})(10 \times 10^{-3}) - 0 = 20 \times 10^{-6}$$

$$[\mathbf{z}_a] = \begin{bmatrix} \frac{\mathbf{y}_{22}}{\Delta_y} & \frac{-\mathbf{y}_{12}}{\Delta_y} \\ \frac{-\mathbf{y}_{21}}{\Delta_y} & \frac{\mathbf{y}_{11}}{\Delta_y} \end{bmatrix} = \begin{bmatrix} 500 \Omega & 0 \\ 0 & 100 \Omega \end{bmatrix}$$

$$[\mathbf{z}] = \begin{bmatrix} 500 & 0 \\ 0 & 100 \end{bmatrix} + \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix} = \begin{bmatrix} 600 & 100 \\ 100 & 200 \end{bmatrix}$$

i.e.  $\mathbf{V}_1 = \mathbf{z}_{11} \mathbf{I}_1 + \mathbf{z}_{12} \mathbf{I}_2$   
 $\mathbf{V}_2 = \mathbf{z}_{21} \mathbf{I}_1 + \mathbf{z}_{22} \mathbf{I}_2$

or  $\mathbf{V}_1 = 600 \mathbf{I}_1 + 100 \mathbf{I}_2$  (1)

$$\mathbf{V}_2 = 100 \mathbf{I}_1 + 200 \mathbf{I}_2$$
 (2)

But, at the input port,

$$\mathbf{V}_s = \mathbf{V}_1 + 60 \mathbf{I}_1$$
 (3)

and at the output port,

$$\mathbf{V}_2 = \mathbf{V}_o = -300 \mathbf{I}_2$$
 (4)

From (2) and (4),

$$100 \mathbf{I}_1 + 200 \mathbf{I}_2 = -300 \mathbf{I}_2$$

$$\mathbf{I}_1 = -5 \mathbf{I}_2$$
 (5)

Substituting (1) and (5) into (3),

$$\mathbf{V}_s = 600 \mathbf{I}_1 + 100 \mathbf{I}_2 + 60 \mathbf{I}_1$$



$$\begin{aligned}
 &= (660)(-5)\mathbf{I}_2 + 100\mathbf{I}_2 \\
 &= -3200\mathbf{I}_2
 \end{aligned} \tag{6}$$

From (4) and (6),

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{-300\mathbf{I}_2}{-3200\mathbf{I}_2} = \underline{\underline{\mathbf{0.09375}}}$$

### Chapter 19, Solution 67.

The y parameters for the upper network is

$$[\mathbf{y}] = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, \quad \Delta_y = 4 - 1 = 3$$

$$[\mathbf{z}_a] = \begin{bmatrix} \frac{\mathbf{y}_{22}}{\Delta_y} & \frac{-\mathbf{y}_{12}}{\Delta_y} \\ \frac{-\mathbf{y}_{21}}{\Delta_y} & \frac{\mathbf{y}_{11}}{\Delta_y} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$$[\mathbf{z}_b] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b] = \begin{bmatrix} 5/3 & 4/3 \\ 4/3 & 5/3 \end{bmatrix}$$

$$\Delta_z = \frac{25}{9} - \frac{16}{9} = 1$$

$$[\mathbf{T}] = \begin{bmatrix} \frac{\mathbf{z}_{11}}{\Delta_z} & \frac{\Delta_z}{\mathbf{z}_{21}} \\ \frac{1}{\mathbf{z}_{21}} & \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1.25 & 0.75 \Omega \\ 0.75 \text{ S} & 1.25 \end{bmatrix}}}$$

### Chapter 19, Solution 68.

For the upper network  $N_a$ ,  $[\mathbf{y}_a] = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$

and for the lower network  $N_b$ ,  $[\mathbf{y}_b] = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

For the overall network,

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}$$

$$\Delta_y = 36 - 9 = 27$$

$$[\mathbf{h}] = \begin{bmatrix} \frac{1}{\Delta_y} & \frac{-\mathbf{y}_{12}}{\Delta_y} \\ \frac{\mathbf{y}_{11}}{\Delta_y} & \frac{\mathbf{y}_{11}}{\Delta_y} \\ \frac{\mathbf{y}_{21}}{\Delta_y} & \frac{\Delta_y}{\Delta_y} \\ \frac{\mathbf{y}_{11}}{\Delta_y} & \frac{\mathbf{y}_{11}}{\Delta_y} \end{bmatrix} = \begin{bmatrix} \frac{1}{27} & \frac{-1}{9} \\ \frac{6}{27} & \frac{-3}{27} \\ \frac{-3}{27} & \frac{6}{27} \\ \frac{6}{27} & \frac{-3}{27} \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{-1}{9} \\ \frac{2}{9} & \frac{-1}{9} \\ \frac{-1}{9} & \frac{2}{9} \\ \frac{2}{9} & \frac{-1}{9} \end{bmatrix} \mathbf{S}$$

### Chapter 19, Solution 69.

We first determine the  $y$  parameters for the upper network  $N_a$ .

To get  $\mathbf{y}_{11}$  and  $\mathbf{y}_{21}$ , consider the circuit in Fig. (a).

$$n = \frac{1}{2}, \quad \mathbf{Z}_R = \frac{1/s}{n^2} = \frac{4}{s}$$

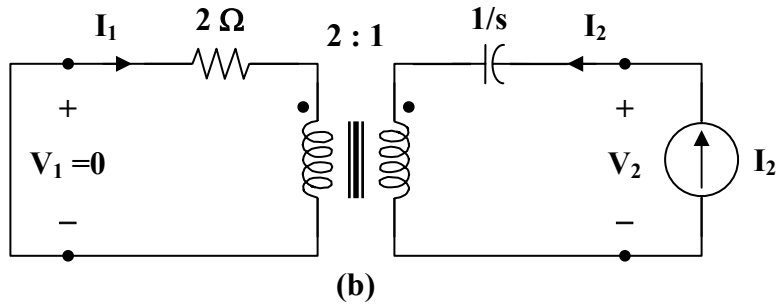
$$\mathbf{V}_1 = (2 + \mathbf{Z}_R) \mathbf{I}_1 = \left(2 + \frac{4}{s}\right) \mathbf{I}_1 = \left(\frac{2s+4}{s}\right) \mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{s}{2(s+2)}$$

$$\mathbf{I}_2 = \frac{-\mathbf{I}_1}{n} = -2\mathbf{I}_1 = \frac{-s\mathbf{V}_1}{s+2}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-s}{s+2}$$

To get  $\mathbf{y}_{22}$  and  $\mathbf{y}_{12}$ , consider the circuit in Fig. (b).



$$\mathbf{Z}_R' = (n^2)(2) = \left(\frac{1}{4}\right)(2) = \frac{1}{2}$$

$$\mathbf{V}_2 = \left(\frac{1}{s} + \mathbf{Z}_R'\right)\mathbf{I}_2 = \left(\frac{1}{s} + \frac{1}{2}\right)\mathbf{I}_2 = \left(\frac{s+2}{2s}\right)\mathbf{I}_2$$

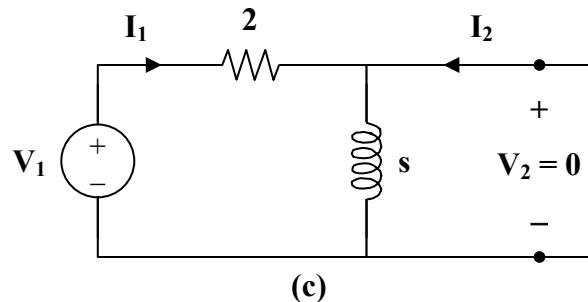
$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{2s}{s+2}$$

$$\mathbf{I}_1 = -n\mathbf{I}_2 = \left(\frac{-1}{2}\right)\left(\frac{2s}{s+2}\right)\mathbf{V}_2 = \left(\frac{-s}{s+2}\right)\mathbf{V}_2$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-s}{s+2}$$

$$[\mathbf{y}_a] = \begin{bmatrix} \frac{s}{2(s+2)} & \frac{-s}{s+2} \\ \frac{-s}{s+2} & \frac{2s}{s+2} \end{bmatrix}$$

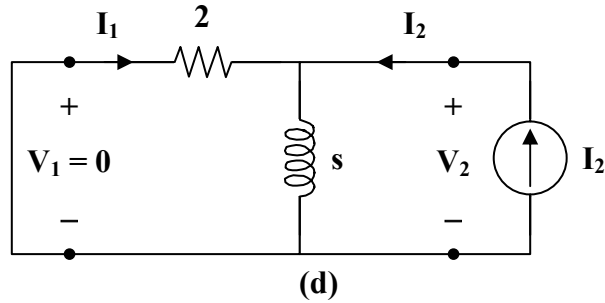
For the lower network  $N_b$ , we obtain  $\mathbf{y}_{11}$  and  $\mathbf{y}_{21}$  by referring to the network in Fig. (c).



$$\mathbf{V}_1 = 2\mathbf{I}_1 \longrightarrow \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{2}$$

$$\mathbf{I}_2 = -\mathbf{I}_1 = \frac{-\mathbf{V}_1}{2} \longrightarrow \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{2}$$

To get  $\mathbf{y}_{22}$  and  $\mathbf{y}_{12}$ , refer to the circuit in Fig. (d).



$$\mathbf{V}_2 = (s \parallel 2)\mathbf{I}_2 = \frac{2s}{s+2}\mathbf{I}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{s+2}{2s}$$

$$\mathbf{I}_1 = -\mathbf{I}_2 \cdot \frac{-s}{s+2} = \left(\frac{-s}{s+2}\right)\left(\frac{s+2}{2s}\right)\mathbf{V}_2 = \frac{-\mathbf{V}_2}{2}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{2}$$

$$[\mathbf{y}_b] = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & (s+2)/2s \end{bmatrix}$$

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \underline{\underline{\begin{bmatrix} \frac{s+1}{s+2} & \frac{-(3s+2)}{2(s+2)} \\ \frac{-(3s+2)}{2(s+2)} & \frac{5s^2+4s+4}{2s(s+2)} \end{bmatrix}}}$$

### Chapter 19, Solution 70.

We may obtain the  $g$  parameters from the given  $z$  parameters.

$$[\mathbf{z}_a] = \begin{bmatrix} 25 & 20 \\ 5 & 10 \end{bmatrix}, \quad \Delta_{z_a} = 250 - 100 = 150$$

$$[\mathbf{z}_b] = \begin{bmatrix} 50 & 25 \\ 25 & 30 \end{bmatrix}, \quad \Delta_{z_b} = 1500 - 625 = 875$$

$$[\mathbf{g}] = \begin{bmatrix} \frac{1}{Z_{11}} & \frac{-Z_{12}}{Z_{11}} \\ \frac{Z_{21}}{Z_{11}} & \frac{\Delta_z}{Z_{11}} \end{bmatrix}$$

$$[\mathbf{g}_a] = \begin{bmatrix} 0.04 & -0.8 \\ 0.2 & 6 \end{bmatrix}, \quad [\mathbf{g}_b] = \begin{bmatrix} 0.02 & -0.5 \\ 0.5 & 17.5 \end{bmatrix}$$

$$[\mathbf{g}] = [\mathbf{g}_a] + [\mathbf{g}_b] = \underline{\underline{\begin{bmatrix} 0.06 \text{ S} & -1.3 \\ 0.7 & 23.5 \Omega \end{bmatrix}}}$$

### Chapter 19, Solution 71.

This is a parallel-series connection of two two-ports. We need to add their  $g$  parameters together and obtain  $z$  parameters from there.

For the transformer,

$$V_1 = \frac{1}{2}V_2, \quad I_1 = -2I_2$$

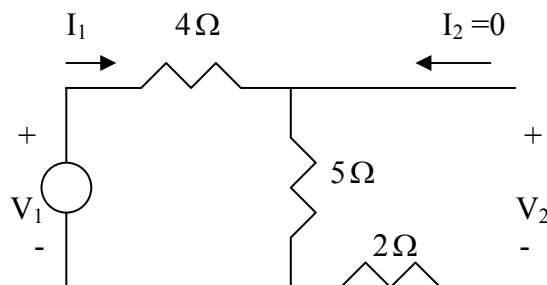
Comparing this with

$$V_1 = AV_2 - BI_2, \quad I_1 = CV_2 - DI_2$$

shows that

$$[T_{b1}] = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$$

To get  $A$  and  $C$  for  $T_{b2}$ , consider the circuit below.

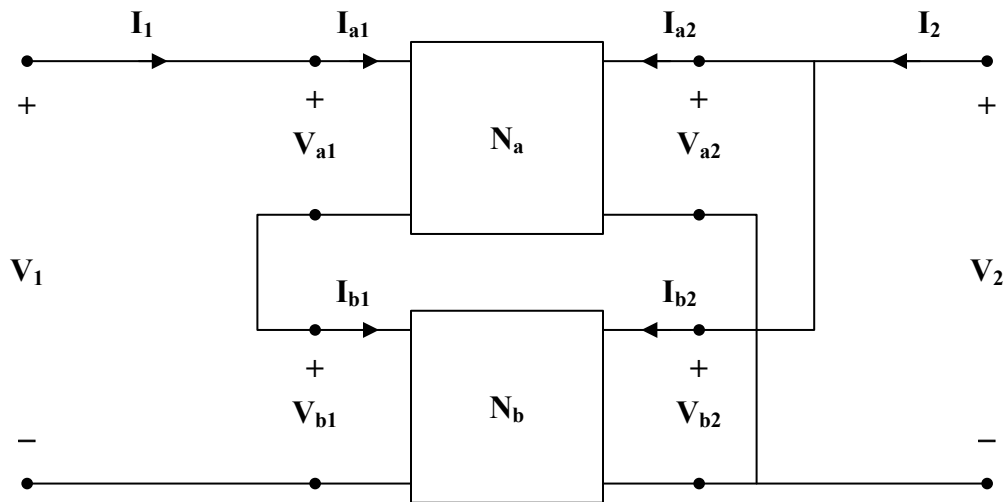


$$V_1 = 9I_1, \quad V_2 = 5I_1$$

$$A = \frac{V_1}{V_2} = 9/5 = 1.8, \quad C = \frac{I_1}{V_2} = 1/5 = 0.2$$

**Chapter 19, Solution 72.**

Consider the network shown below.



$$V_{a1} = 25I_{a1} + 4V_{a2} \quad (1)$$

$$I_{a2} = -4I_{a1} + V_{a2} \quad (2)$$

$$V_{b1} = 16I_{b1} + V_{b2} \quad (3)$$

$$I_{b2} = -I_{b1} + 0.5V_{b2} \quad (4)$$

$$V_1 = V_{a1} + V_{b1}$$

$$V_2 = V_{a2} = V_{b2}$$

$$I_2 = I_{a2} + I_{b2}$$

$$I_1 = I_{a1}$$

Now, rewrite (1) to (4) in terms of  $I_1$  and  $V_2$

$$V_{a1} = 25I_1 + 4V_2 \quad (5)$$

$$I_{a2} = -4I_1 + V_2 \quad (6)$$

$$V_{b1} = 16I_{b1} + V_2 \quad (7)$$

$$I_{b2} = -I_{b1} + 0.5V_2 \quad (8)$$

Adding (5) and (7),

$$\mathbf{V}_1 = 25\mathbf{I}_1 + 16\mathbf{I}_{b1} + 5\mathbf{V}_2 \quad (9)$$

Adding (6) and (8),

$$\mathbf{I}_2 = -4\mathbf{I}_1 - \mathbf{I}_{b1} + 1.5\mathbf{V}_2 \quad (10)$$

$$\mathbf{I}_{b1} = \mathbf{I}_{a1} = \mathbf{I}_1 \quad (11)$$

Because the two networks  $N_a$  and  $N_b$  are independent,

$$\mathbf{I}_2 = -5\mathbf{I}_1 + 1.5\mathbf{V}_2$$

or 
$$\mathbf{V}_2 = 3.333\mathbf{I}_1 + 0.6667\mathbf{I}_2 \quad (12)$$

Substituting (11) and (12) into (9),

$$\mathbf{V}_1 = 41\mathbf{I}_1 + \frac{25}{1.5}\mathbf{I}_1 + \frac{5}{1.5}\mathbf{I}_2$$

$$\mathbf{V}_1 = 57.67\mathbf{I}_1 + 3.333\mathbf{I}_2 \quad (13)$$

Comparing (12) and (13) with the following equations

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2$$

indicates that

$$[\mathbf{z}] = \underline{\underline{\begin{bmatrix} 57.67 & 3.333 \\ 3.333 & 0.6667 \end{bmatrix} \Omega}}$$

Alternatively,

$$[\mathbf{h}_a] = \begin{bmatrix} 25 & 4 \\ -4 & 1 \end{bmatrix}, \quad [\mathbf{h}_b] = \begin{bmatrix} 16 & 1 \\ -1 & 0.5 \end{bmatrix}$$

$$[\mathbf{h}] = [\mathbf{h}_a] + [\mathbf{h}_b] = \begin{bmatrix} 41 & 5 \\ -5 & 1.5 \end{bmatrix} \quad \Delta_h = 61.5 + 25 = 86.5$$

$$[\mathbf{z}] = \begin{bmatrix} \frac{\Delta_h}{\mathbf{h}_{22}} & \frac{\mathbf{h}_{12}}{\mathbf{h}_{22}} \\ \frac{\mathbf{h}_{22}}{-\mathbf{h}_{21}} & \frac{1}{\mathbf{h}_{22}} \\ \frac{\mathbf{h}_{22}}{\mathbf{h}_{22}} & \frac{\mathbf{h}_{22}}{\mathbf{h}_{22}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 57.67 & 3.333 \\ 3.333 & 0.6667 \end{bmatrix} \Omega}}$$

as obtained previously.

**Chapter 19, Solution 73.**

From Example 18.14 and the cascade two-ports,

$$[\mathbf{T}_a] = [\mathbf{T}_b] = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b] = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 7 & 12 \Omega \\ 4 \text{ S} & 7 \end{bmatrix}}}$$

When the output is short-circuited,  $V_2 = 0$  and by definition

$$V_1 = -\mathbf{B}\mathbf{I}_2, \quad \mathbf{I}_1 = -\mathbf{D}\mathbf{I}_2$$

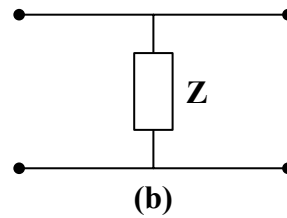
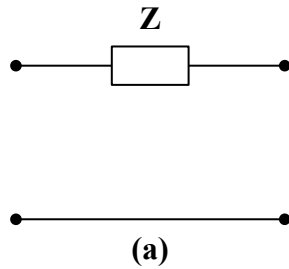
Hence,

$$\mathbf{Z}_m = \frac{V_1}{\mathbf{I}_1} = \frac{\mathbf{B}}{\mathbf{D}} = \underline{\underline{\frac{12}{7} \Omega}}$$

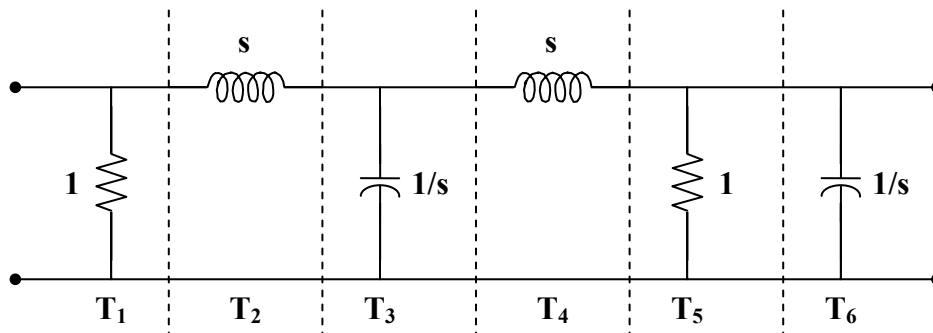
**Chapter 19, Solution 74.**

From Prob. 18.35, the transmission parameters for the circuit in Figs. (a) and (b) are

$$[\mathbf{T}_a] = \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix}, \quad [\mathbf{T}_b] = \begin{bmatrix} 1 & 0 \\ 1/\mathbf{Z} & 1 \end{bmatrix}$$



We partition the given circuit into six subcircuits similar to those in Figs. (a) and (b) as shown in Fig. (c) and obtain  $[\mathbf{T}]$  for each.





$$[\mathbf{T}_1] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad [\mathbf{T}_2] = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}, \quad [\mathbf{T}_3] = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

$$[\mathbf{T}_4] = [\mathbf{T}_2], \quad [\mathbf{T}_5] = [\mathbf{T}_1], \quad [\mathbf{T}_6] = [\mathbf{T}_3]$$

$$\begin{aligned} [\mathbf{T}] &= [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4][\mathbf{T}_5][\mathbf{T}_6] = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4] \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix} \\ &= [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4] \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix} = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3] \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix} \\ &= [\mathbf{T}_1][\mathbf{T}_2] \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix} \begin{bmatrix} s^2 + s + 1 & s \\ s+1 & 1 \end{bmatrix} \\ &= [\mathbf{T}_1] \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s^2 + s + 1 & s \\ s^3 + s^2 + 2s + 1 & s^2 + 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s^4 + s^3 + 3s^2 + 2s + 1 & s^3 + 2s \\ s^3 + s^2 + 2s + 1 & s^2 + 1 \end{bmatrix} \\ [\mathbf{T}] &= \underline{\underline{\begin{bmatrix} s^4 + s^3 + 3s^2 + 2s + 1 & s^3 + 2s \\ s^4 + 2s^3 + 4s^2 + 4s + 2 & s^3 + s^2 + 2s + 1 \end{bmatrix}}} \end{aligned}$$

Note that  $\mathbf{AB} - \mathbf{CD} = 1$  as expected.

### Chapter 19, Solution 75.

(a) We convert  $[z_a]$  and  $[z_b]$  to T-parameters. For  $N_a$ ,  $\Delta_z = 40 - 24 = 16$ .

$$[\mathbf{T}_a] = \begin{bmatrix} z_{11}/z_{21} & \Delta_z/z_{21} \\ 1/z_{21} & z_{22}/z_{21} \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0.25 & 1.25 \end{bmatrix}$$

For  $N_b$ ,  $\Delta_y = 80 + 8 = 88$ .

$$[\mathbf{T}_b] = \begin{bmatrix} -y_{22}/y_{21} & -1/y_{21} \\ -\Delta_y/y_{21} & -y_{11}/y_{21} \end{bmatrix} = \begin{bmatrix} -5 & -0.5 \\ -44 & -4 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} -186 & -17 \\ -56.25 & -5.125 \end{bmatrix}$$

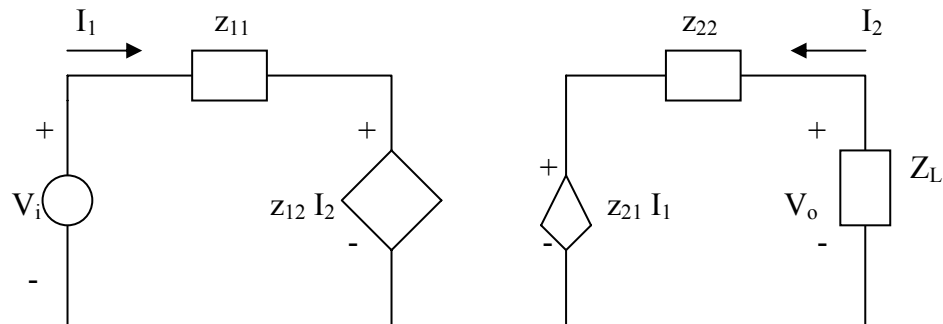
We convert this to y-parameters.  $\Delta_T = AD - BC = -3$ .

$$[y] = \begin{bmatrix} D/B & -\Delta_T/B \\ -1/B & A/B \end{bmatrix} = \begin{bmatrix} 0.3015 & -0.1765 \\ 0.0588 & 10.94 \end{bmatrix}$$

(b) The equivalent z-parameters are

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 3.3067 & 0.0533 \\ -0.0178 & 0.0911 \end{bmatrix}$$

Consider the equivalent circuit below.



$$V_i = z_{11}I_1 + z_{12}I_2 \quad (1)$$

$$V_o = z_{21}I_1 + z_{22}I_2 \quad (2)$$

$$\text{But } V_o = -I_2 Z_L \quad \longrightarrow \quad I_2 = -V_o / Z_L \quad (3)$$

From (2) and (3),

$$V_o = z_{21}I_1 - z_{22} \frac{V_o}{Z_L} \quad \longrightarrow \quad I_1 = V_o \left( \frac{1}{z_{21}} + \frac{z_{22}}{Z_L z_{21}} \right) \quad (4)$$

Substituting (3) and (4) into (1) gives

$$\frac{V_i}{V_o} = \left( \frac{z_{11}}{z_{21}} + \frac{z_{11}z_{22}}{z_{21}Z_L} \right) - \frac{z_{12}}{Z_L} = -194.3 \quad \longrightarrow \quad \frac{V_o}{V_i} = \underline{\underline{-0.0051}}$$

**Chapter 19, Solution 76.**

To get  $z_{11}$  and  $z_{21}$ , we open circuit the output port and let  $I_1 = 1\text{A}$  so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \quad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 3.849, \quad z_{21} = V_2 = 1.122$$

Similarly, to get  $z_{22}$  and  $z_{12}$ , we open circuit the input port and let  $I_2 = 1\text{A}$  so that

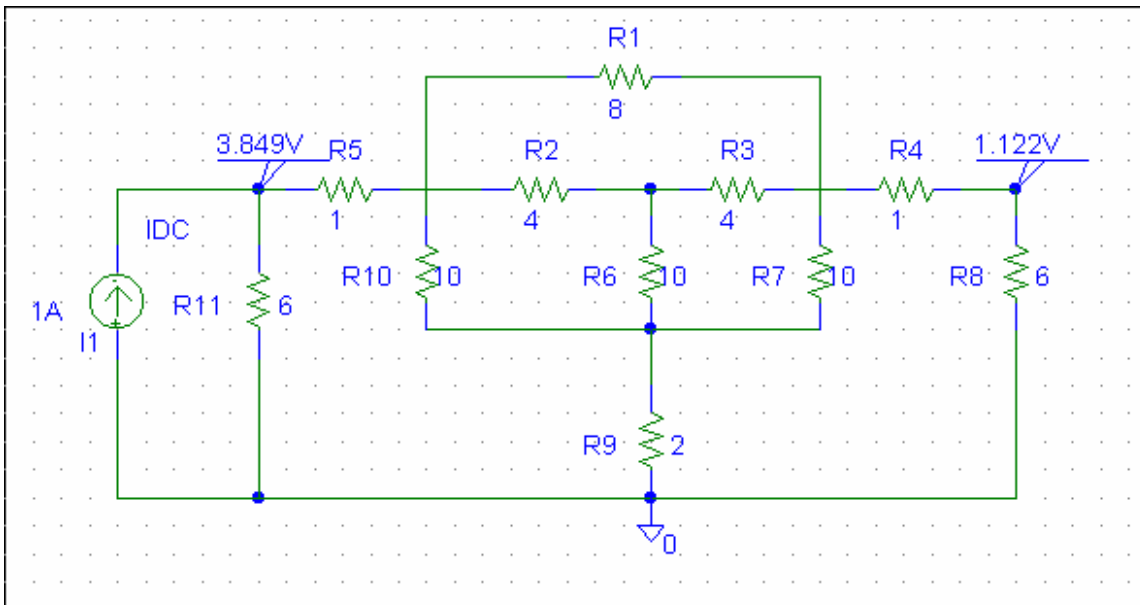
$$z_{12} = \frac{V_1}{I_2} = V_1, \quad z_{22} = \frac{V_2}{I_2} = V_2$$

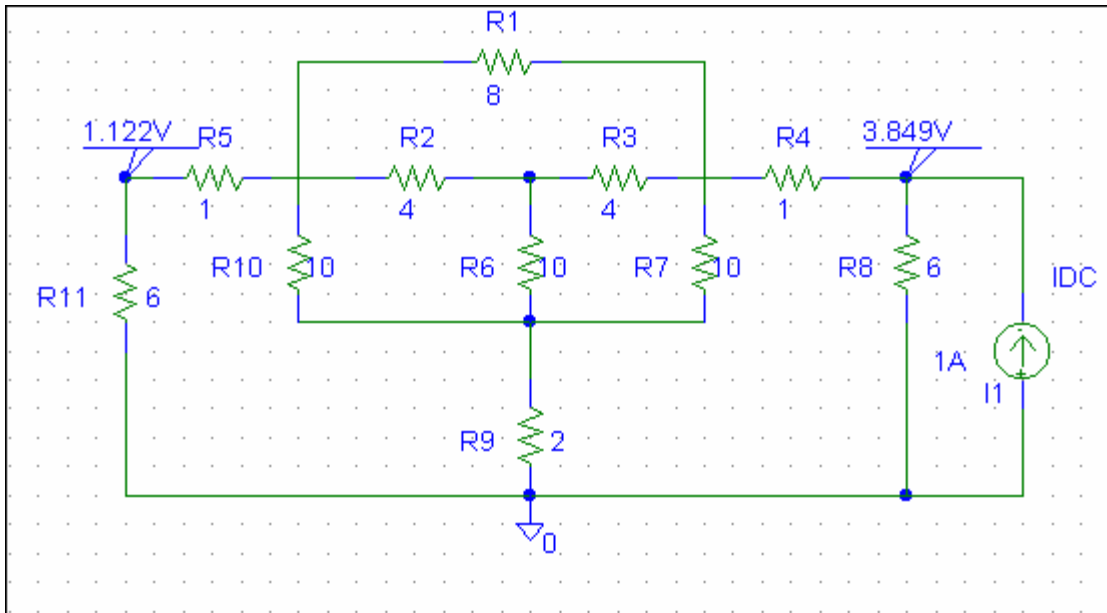
The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 1.122, \quad z_{22} = V_2 = 3.849$$

Thus,

$$[z] = \begin{bmatrix} 3.949 & 1.122 \\ 1.122 & 3.849 \end{bmatrix} \Omega$$





**Chapter 19, Solution 77.**

We follow Example 19.15 except that this is an AC circuit.

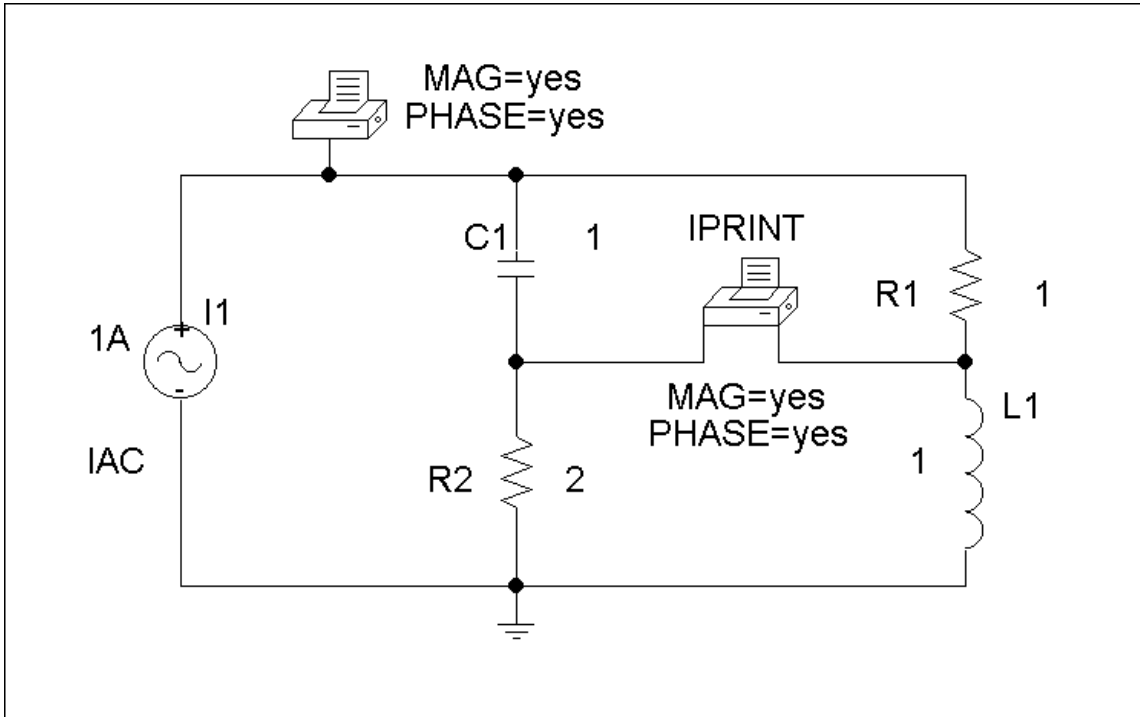
(a) We set  $V_2 = 0$  and  $I_1 = 1$  A. The schematic is shown below. In the AC Sweep Box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E-01	3.163 E-01	-1.616 E+02
FREQ	VM(\$N_0001)	VP(\$N_0001)
1.592 E-01	9.488 E-01	-1.616 E+02

From this we obtain

$$h_{11} = V_1/I_1 = 0.9488 \angle -161.6^\circ$$

$$h_{21} = I_2/I_1 = 0.3163 \angle -161.6^\circ.$$



(b) In this case, we set  $I_1 = 0$  and  $V_2 = 1V$ . The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

FREQ	VM(\$N_0001)	VP(\$N_0001)
1.592 E-01	3.163 E-01	1.842 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E-01	9.488 E-01	-1.616 E+02

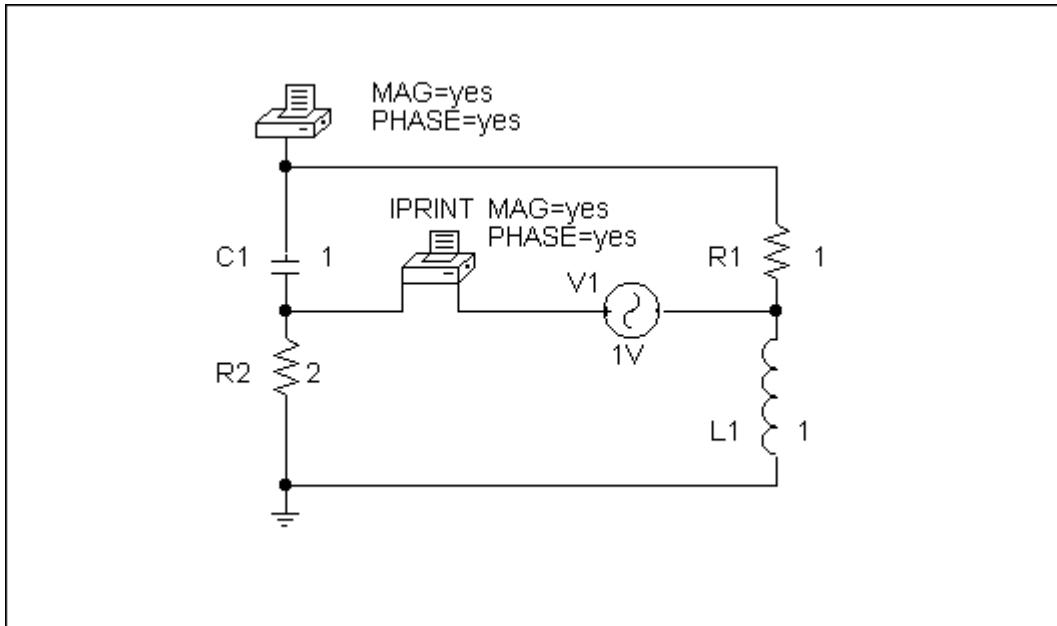
From this,

$$h_{12} = V_1/1 = 0.3163 \angle 18.42^\circ$$

$$h_{21} = I_2/1 = 0.9488 \angle -161.6^\circ.$$

Thus,

$$[h] = \underline{\underline{\begin{bmatrix} 0.9488 \angle -161.6^\circ & 0.3163 \angle 18.42^\circ \\ 0.3163 \angle -161.6^\circ & 0.9488 \angle -161.6^\circ \end{bmatrix}}}$$



### Chapter 19, Solution 78

For  $h_{11}$  and  $h_{21}$ , short-circuit the output port and let  $I_1 = 1\text{A}$ .  $f = \omega / 2\pi = 0.6366$ . The schematic is shown below. When it is saved and run, the output file contains the following:

```

FREQ          IM(V_PRINT1) IP(V_PRINT1)
  6.366E-01    1.202E+00  1.463E+02
FREQ          VM($N_0003) VP($N_0003)
  6.366E-01    3.771E+00 -1.350E+02

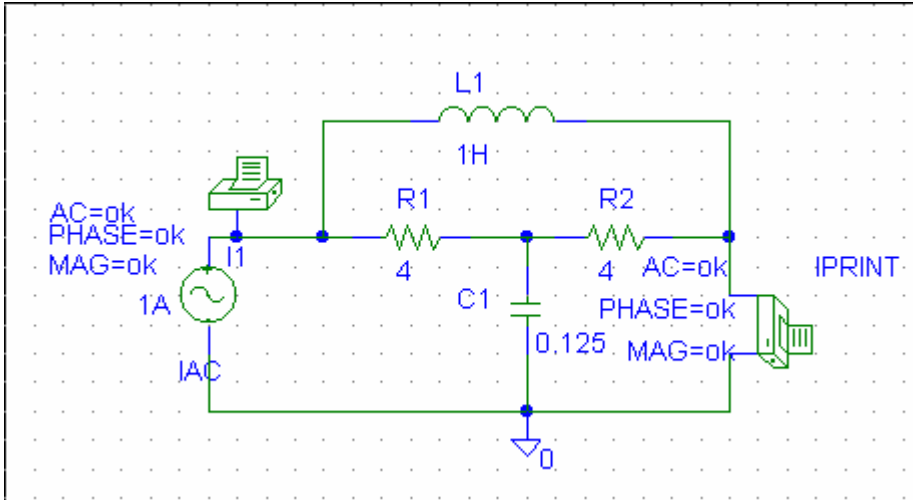
```

From the output file, we obtain

$$I_2 = 1.202 \angle 146.3^\circ, \quad V_1 = 3.771 \angle -135^\circ$$

so that

$$h_{11} = \frac{V_1}{I_1} = 3.771 \angle -135^\circ, \quad h_{21} = \frac{I_2}{I_1} = 1.202 \angle 146.3^\circ$$



For  $h_{12}$  and  $h_{22}$ , open-circuit the input port and let  $V_2 = 1V$ . The schematic is shown below. When it is saved and run, the output file includes:

```

FREQ          VM($N_0003) VP($N_0003)
6.366E-01    1.202E+00 -3.369E+01
FREQ          IM(V_PRINT1) IP(V_PRINT1)
6.366E-01    3.727E-01 -1.534E+02

```

From the output file, we obtain

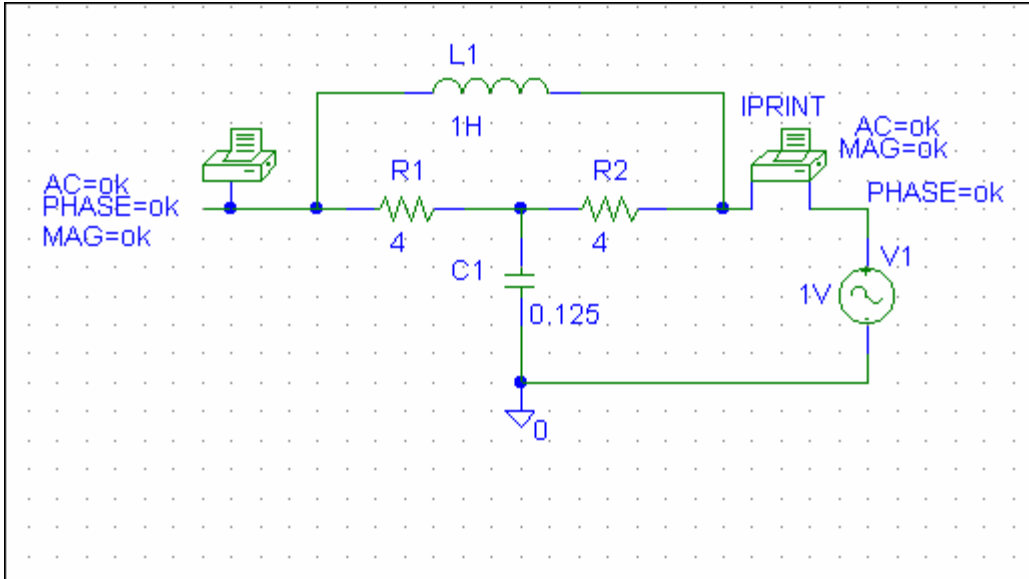
$$I_2 = 0.3727 \angle -153.4^\circ, \quad V_1 = 1.202 \angle -33.69^\circ$$

so that

$$h_{12} = \frac{V_1}{1} = 1.202 \angle -33.69^\circ, \quad h_{22} = \frac{I_2}{1} = 0.3727 \angle -153.4^\circ$$

Thus,

$$[h] = \begin{bmatrix} 3.771 \angle -135^\circ & 1.202 \angle -33.69^\circ \\ 1.202 \angle 146.3^\circ & 0.3727 \angle -153.4^\circ \end{bmatrix}$$



### Chapter 19, Solution 79

We follow Example 19.16.

(a) We set  $I_1 = 1$  A and open-circuit the output-port so that  $I_2 = 0$ . The schematic is shown below with two VPRINTs to measure  $V_1$  and  $V_2$ . In the AC Sweep box, we enter Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

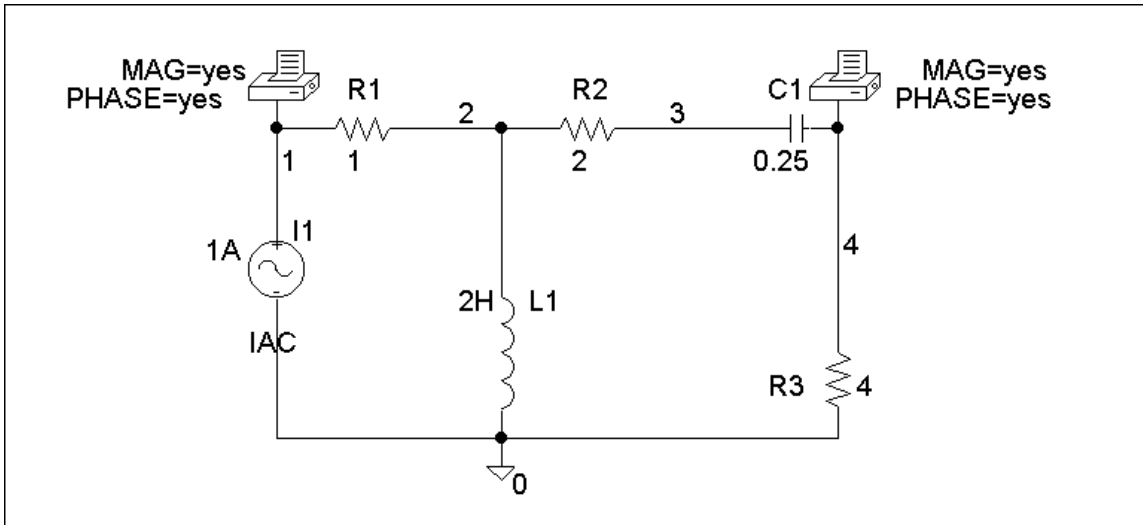
FREQ	VM(1)	VP(1)
3.183 E-01	4.669 E+00	-1.367 E+02
FREQ	VM(4)	VP(4)
3.183 E-01	2.530 E+00	-1.084 E+02

From this,

$$z_{11} = V_1/I_1 = 4.669\angle-136.7^\circ/1 = 4.669\angle-136.7^\circ$$

$$z_{21} = V_2/I_1 = 2.53\angle-108.4^\circ/1 = 2.53\angle-108.4^\circ.$$





(b) In this case, we let  $I_2 = 1$  A and open-circuit the input port. The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

FREQ	VM(1)	VP(1)
3.183 E-01	2.530 E+00	-1.084 E+02

FREQ	VM(2)	VP(2)
3.183 E-01	1.789 E+00	-1.534 E+02

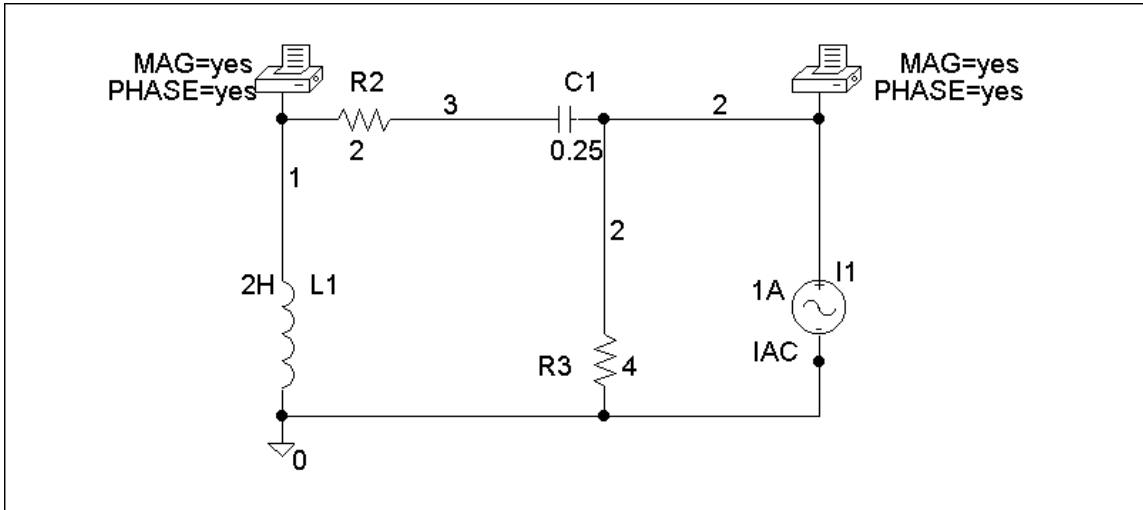
From this,

$$z_{12} = V_1/I_2 = 2.53 \angle -108.4^\circ / 1 = 2.53 \angle -108.4^\circ$$

$$z_{22} = V_2/I_2 = 1.789 \angle -153.4^\circ / 1 = 1.789 \angle -153.4^\circ.$$

Thus,

$$[z] = \begin{bmatrix} 4.669 \angle -136.7^\circ & 2.53 \angle -108.4^\circ \\ 2.53 \angle -108.4^\circ & 1.789 \angle -153.4^\circ \end{bmatrix}$$



### Chapter 19, Solution 80

To get  $z_{11}$  and  $z_{21}$ , we open circuit the output port and let  $I_1 = 1\text{A}$  so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \quad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 29.88, \quad z_{21} = V_2 = -70.37$$

Similarly, to get  $z_{22}$  and  $z_{12}$ , we open circuit the input port and let  $I_2 = 1\text{A}$  so that

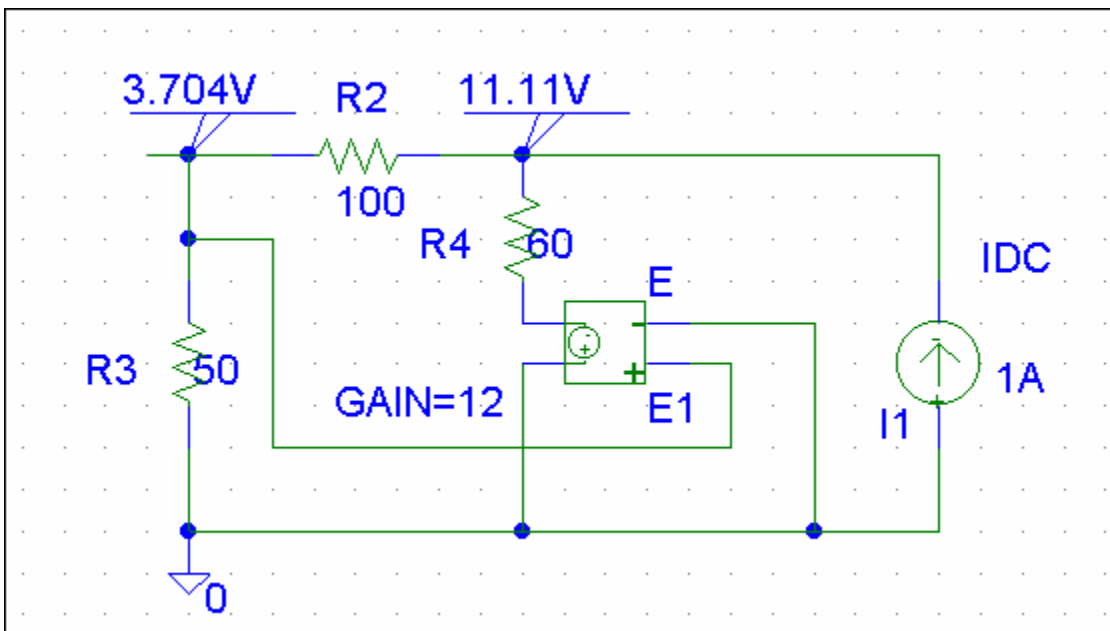
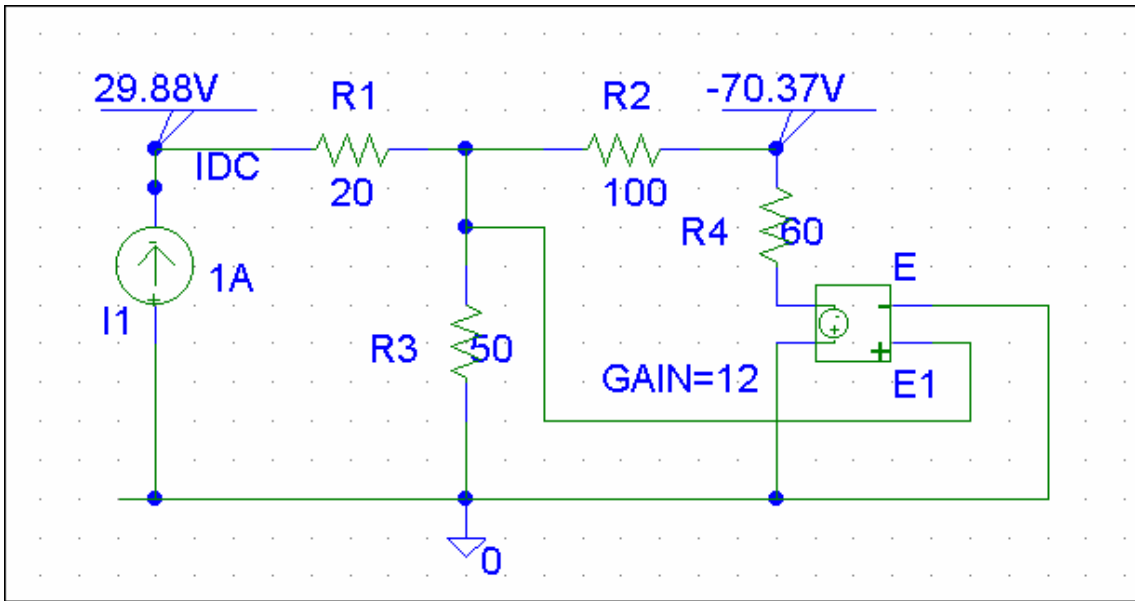
$$z_{12} = \frac{V_1}{I_2} = V_1, \quad z_{22} = \frac{V_2}{I_2} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 3.704, \quad z_{22} = V_2 = 11.11$$

Thus,

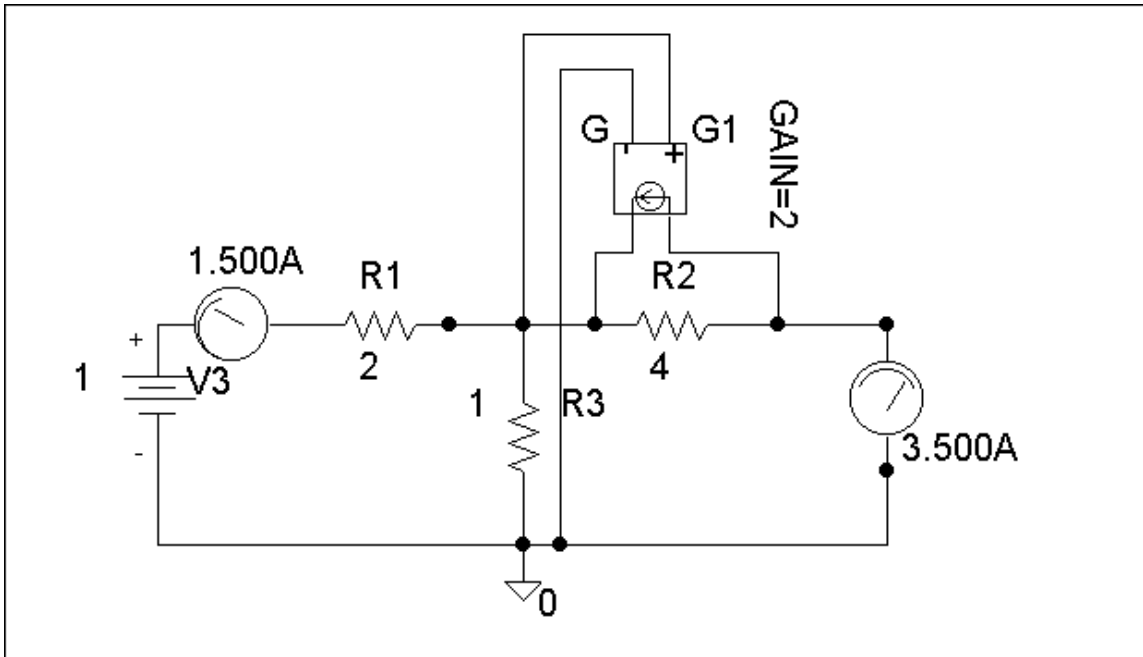
$$[z] = \begin{bmatrix} 29.88 & 3.704 \\ -70.37 & 11.11 \end{bmatrix} \Omega$$



### Chapter 19, Solution 81

(a) We set  $V_1 = 1$  and short circuit the output port. The schematic is shown below. After simulation we obtain

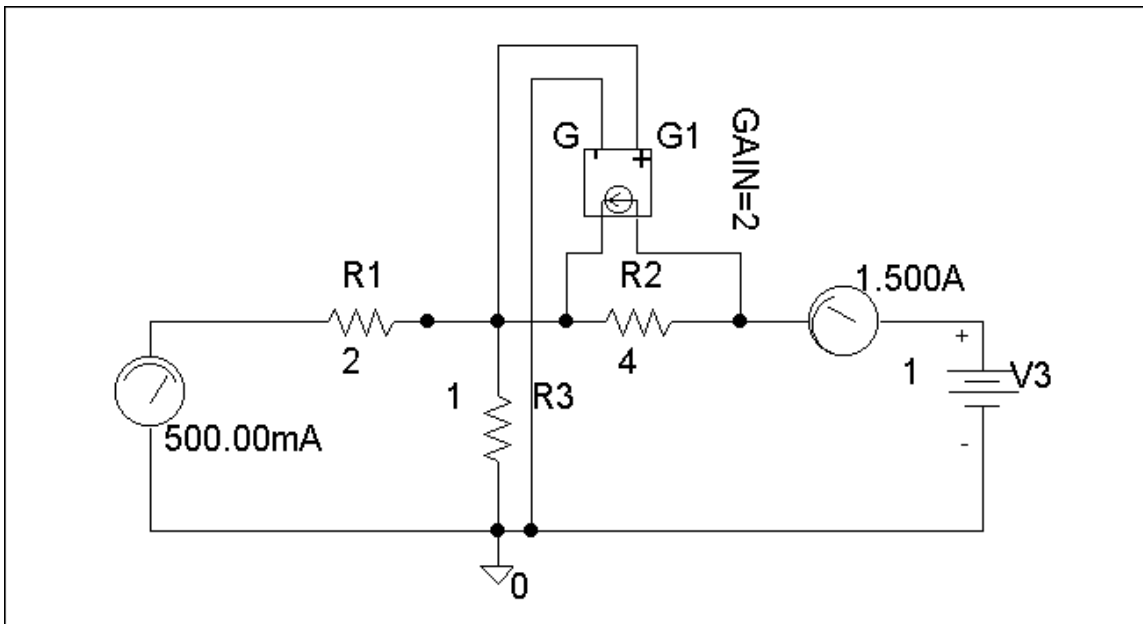
$$y_{11} = I_1 = 1.5, \quad y_{21} = I_2 = 3.5$$



(b) We set  $V_2 = 1$  and short-circuit the input port. The schematic is shown below. Upon simulating the circuit, we obtain

$$y_{12} = I_1 = -0.5, y_{22} = I_2 = 1.5$$

$$[Y] = \begin{bmatrix} 1.5 & -0.5 \\ 3.5 & 1.5 \end{bmatrix}$$

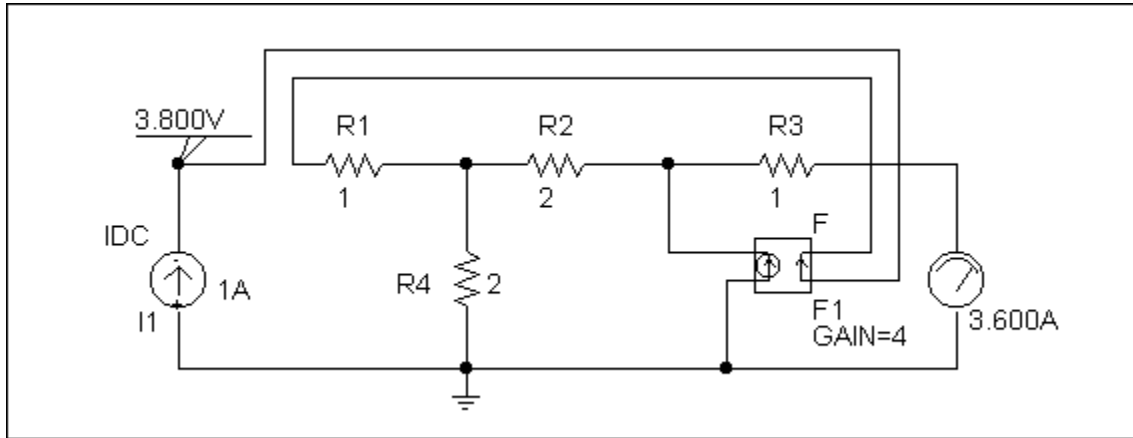


## Chapter 19, Solution 82

We follow Example 19.15.

(a) Set  $V_2 = 0$  and  $I_1 = 1\text{A}$ . The schematic is shown below. After simulation, we obtain

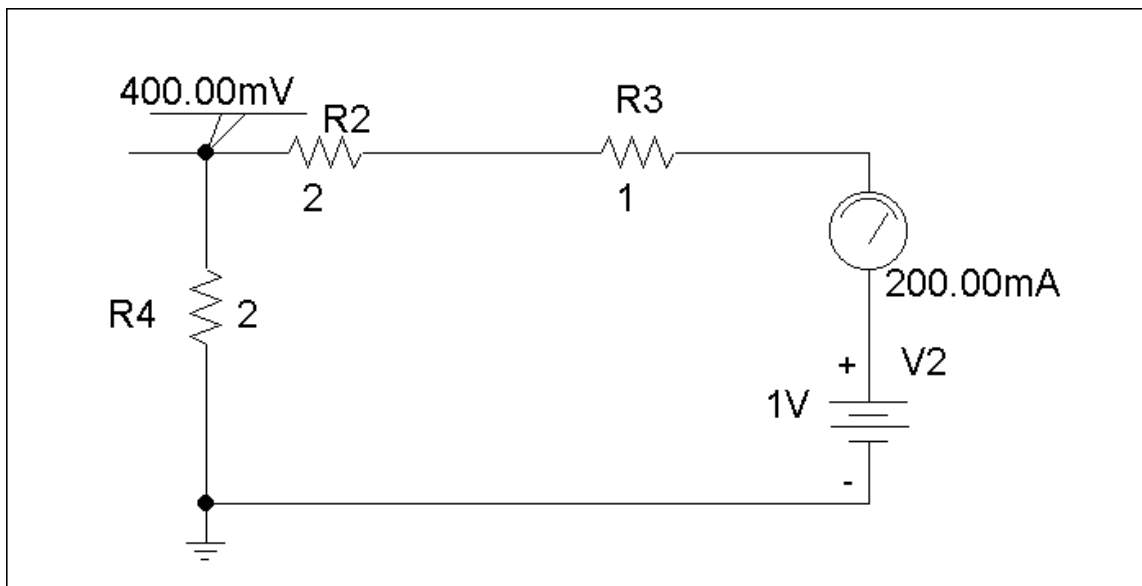
$$h_{11} = V_1/1 = 3.8, \quad h_{21} = I_2/1 = 3.6$$



(b) Set  $V_1 = 1\text{V}$  and  $I_1 = 0$ . The schematic is shown below. After simulation, we obtain

$$h_{12} = V_1/1 = 0.4, \quad h_{22} = I_2/1 = 0.25$$

Hence, 
$$[h] = \begin{bmatrix} 3.8 & 0.4 \\ 3.6 & 0.25 \end{bmatrix}$$



### Chapter 19, Solution 83

To get A and C, we open-circuit the output and let  $I_1 = 1\text{A}$ . The schematic is shown below. When the circuit is saved and simulated, we obtain  $V_1 = 11$  and  $V_2 = 34$ .

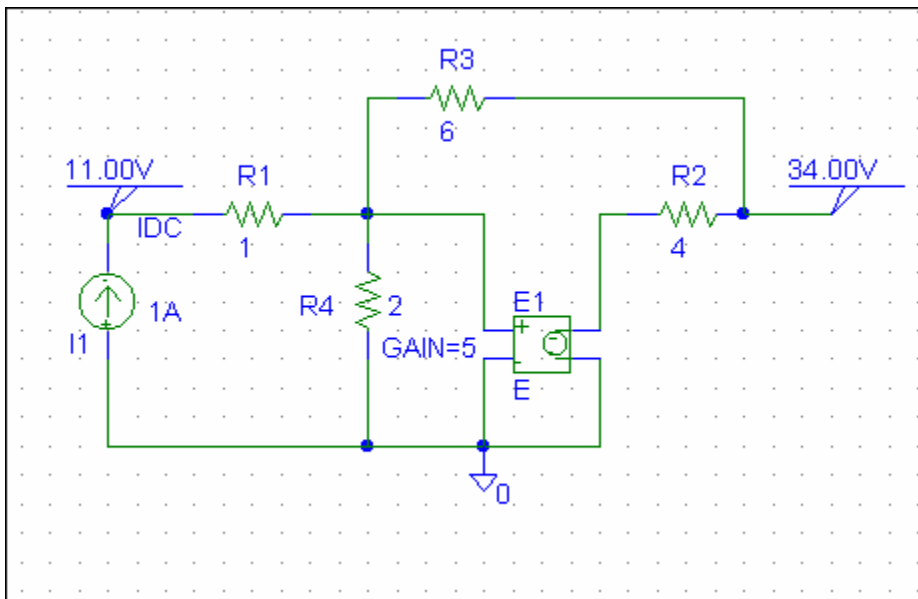
$$A = \frac{V_1}{V_2} = 0.3235, \quad C = \frac{I_1}{V_2} = \frac{1}{34} = 0.02941$$

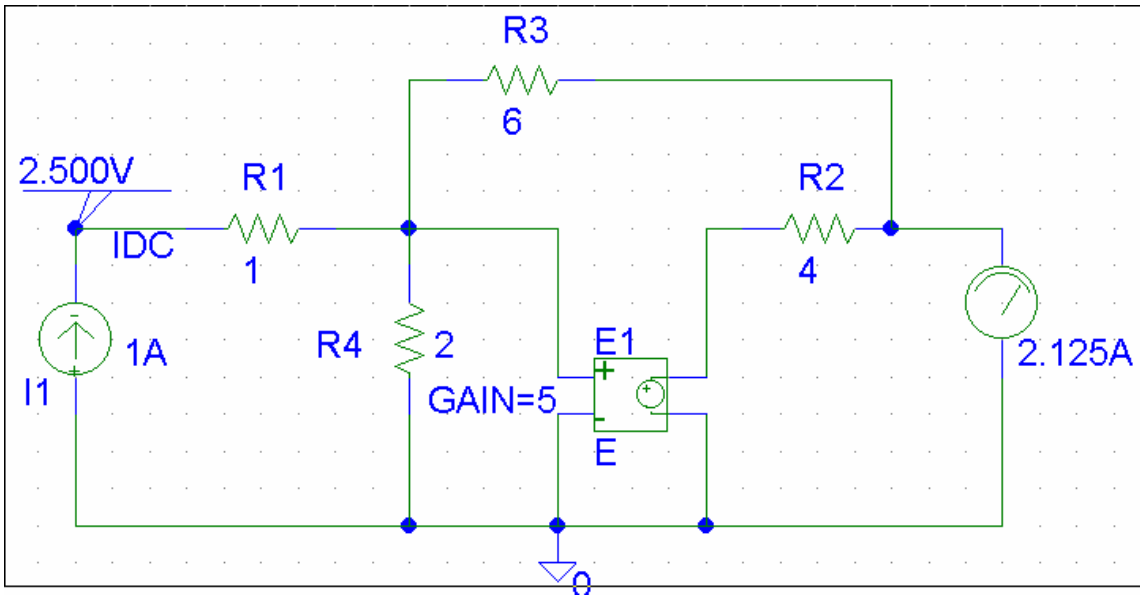
Similarly, to get B and D, we open-circuit the output and let  $I_1 = 1\text{A}$ . The schematic is shown below. When the circuit is saved and simulated, we obtain  $V_1 = 2.5$  and  $I_2 = -2.125$ .

$$B = -\frac{V_1}{I_2} = \frac{2.5}{2.125} = 1.1765, \quad D = -\frac{I_1}{I_2} = \frac{1}{2.125} = 0.4706$$

Thus,

$$[T] = \begin{bmatrix} 0.3235 & 1.1765 \\ 0.02941 & 0.4706 \end{bmatrix}$$



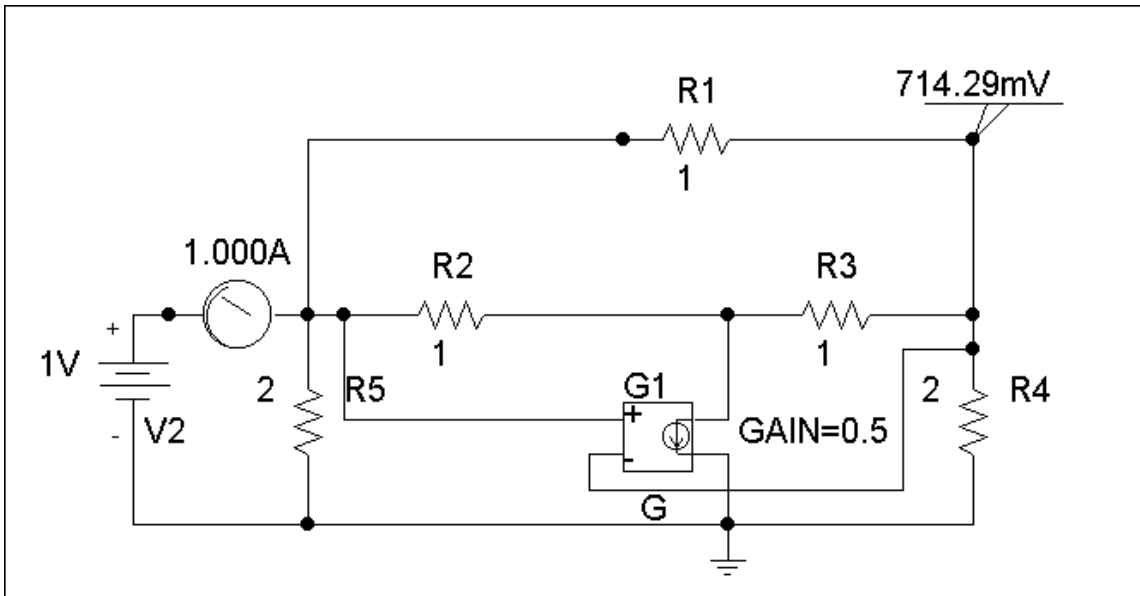


**Chapter 19, Solution 84**

- (a) Since  $A = \left. \frac{V_1}{V_2} \right|_{I_2=0}$  and  $C = \left. \frac{I_1}{V_2} \right|_{I_2=0}$ , we open-circuit the output port and let  $V_1 = 1$  V. The schematic is as shown below. After simulation, we obtain

$$A = 1/V_2 = 1/0.7143 = 1.4$$

$$C = I_2/V_2 = 1.0/0.7143 = 1.4$$



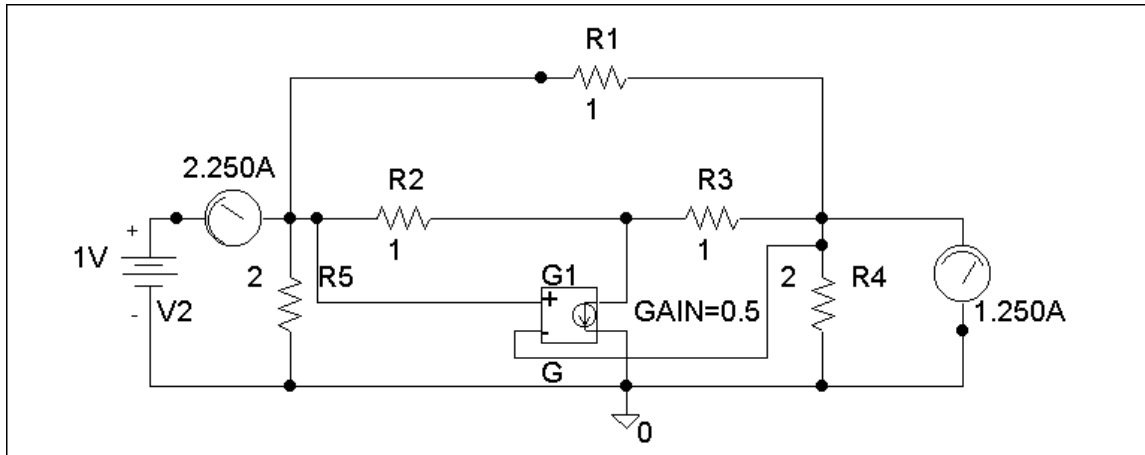
(b) To get B and D, we short-circuit the output port and let  $V_1 = 1$ . The schematic is shown below. After simulating the circuit, we obtain

$$B = -V_1/I_2 = -1/1.25 = -0.8$$

$$D = -I_1/I_2 = -2.25/1.25 = -1.8$$

Thus

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1.4 & -0.8 \\ 1.4 & -1.8 \end{bmatrix}}}$$



### Chapter 19, Solution 85

(a) Since  $A = \left. \frac{V_1}{V_2} \right|_{I_2=0}$  and  $C = \left. \frac{I_1}{V_2} \right|_{I_2=0}$ , we let  $V_1 = 1$  V and open-circuit the output port. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

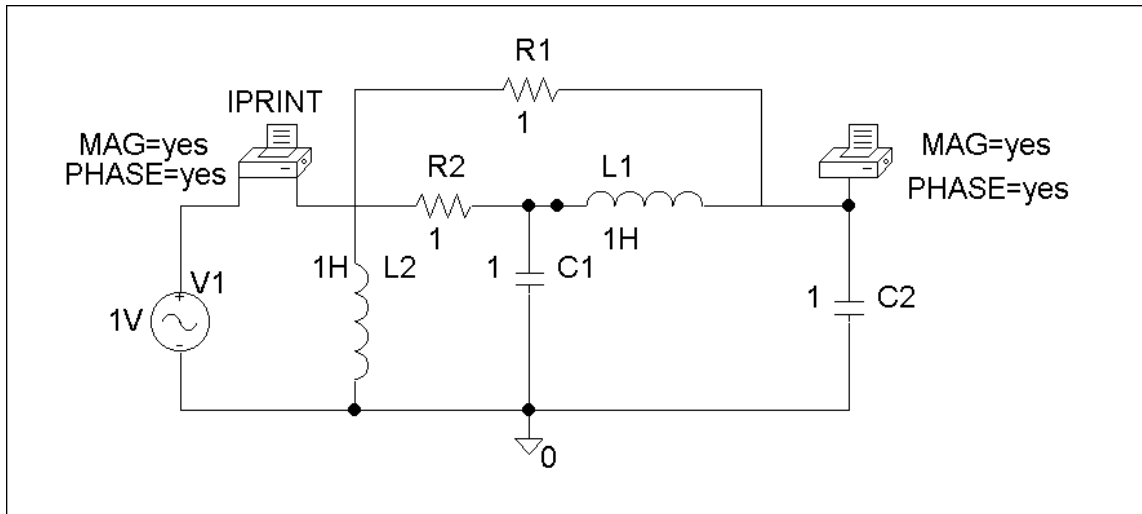
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E-01	6.325 E-01	1.843 E+01
FREQ	VM(\$N_0002)	VP(\$N_0002)
1.592 E-01	6.325 E-01	-7.159 E+01

From this, we obtain

$$A = \frac{1}{V_2} = \frac{1}{0.6325 \angle -71.59^\circ} = 1.581 \angle 71.59^\circ$$



$$C = \frac{I_1}{V_2} = \frac{0.6325 \angle 18.43^\circ}{0.6325 \angle -71.59^\circ} = 1 \angle 90^\circ = j$$



(b) Similarly, since  $B = \left. \frac{V_1}{I_2} \right|_{V_2=0}$  and  $D = -\left. \frac{I_1}{I_2} \right|_{V_2=0}$ , we let  $V_1 = 1$  V and short-circuit the output port. The schematic is shown below. Again, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. After simulation, we get an output file which includes the following results:

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E-01	5.661 E-04	8.997 E+01

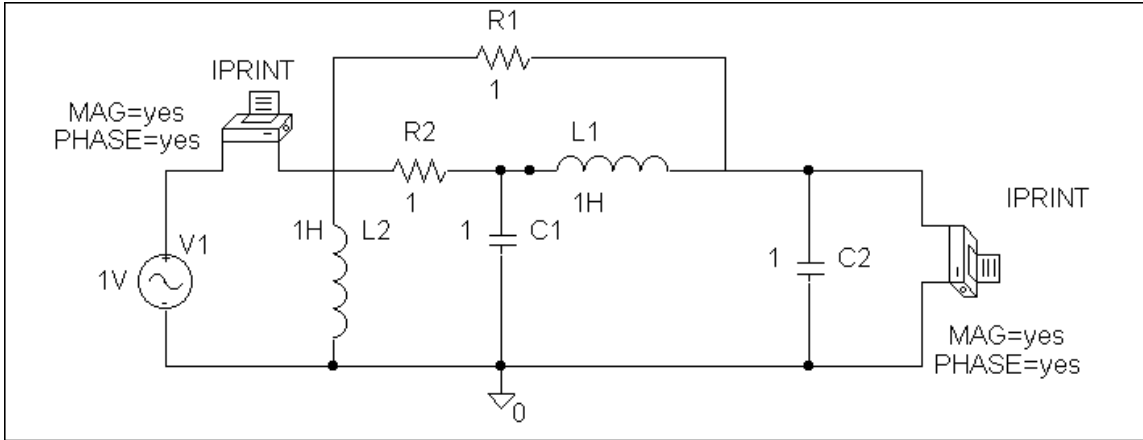
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E-01	9.997 E-01	-9.003 E+01

From this,

$$B = -\frac{1}{I_2} = -\frac{1}{0.9997 \angle -90^\circ} = -1 \angle 90^\circ = -j$$

$$D = -\frac{I_1}{I_2} = -\frac{5.661 \times 10^{-4} \angle 89.97^\circ}{0.9997 \angle -90^\circ} = 5.661 \times 10^{-4}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.581 \angle 71.59^\circ & -j \\ j & 5.661 \times 10^{-4} \end{bmatrix}$$



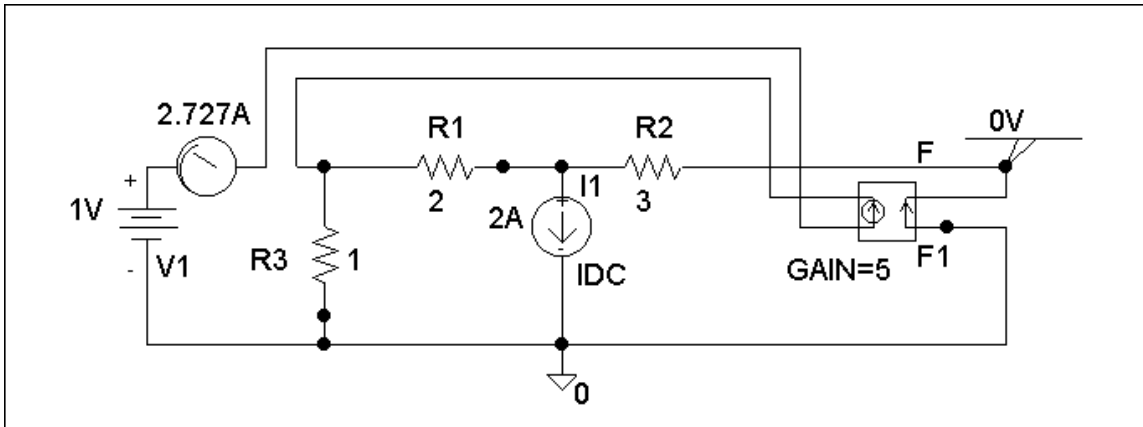
**Chapter 19, Solution 86**

(a) By definition,  $g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0}$ ,  $g_{21} = \left. \frac{V_1}{V_2} \right|_{I_2=0}$ .

We let  $V_1 = 1 \text{ V}$  and open-circuit the output port. The schematic is shown below. After simulation, we obtain

$$g_{11} = I_1 = 2.7$$

$$g_{21} = V_2 = 0.0$$



(b) Similarly,

$$g_{12} = \left. \frac{I_1}{I_2} \right|_{V_1=0}$$

$$g_{22} = \left. \frac{V_2}{I_2} \right|_{V_1=0}$$

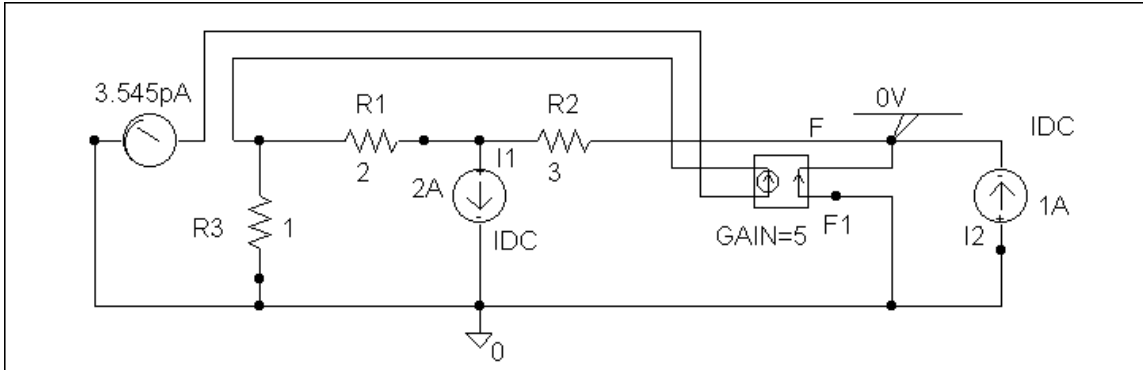
We let  $I_2 = 1 \text{ A}$  and short-circuit the input port. The schematic is shown below. After simulation,

$$g_{12} = I_1 = 0$$

$$g_{22} = V_2 = 0$$

Thus

$$[g] = \begin{bmatrix} 2.727S & 0 \\ 0 & 0 \end{bmatrix}$$



### Chapter 19, Solution 87

(a) Since  $a = \left. \frac{V_2}{V_1} \right|_{I_1=0}$  and  $c = \left. \frac{I_2}{V_1} \right|_{I_1=0}$ ,

we open-circuit the input port and let  $V_2 = 1$  V. The schematic is shown below. In the AC Sweep box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

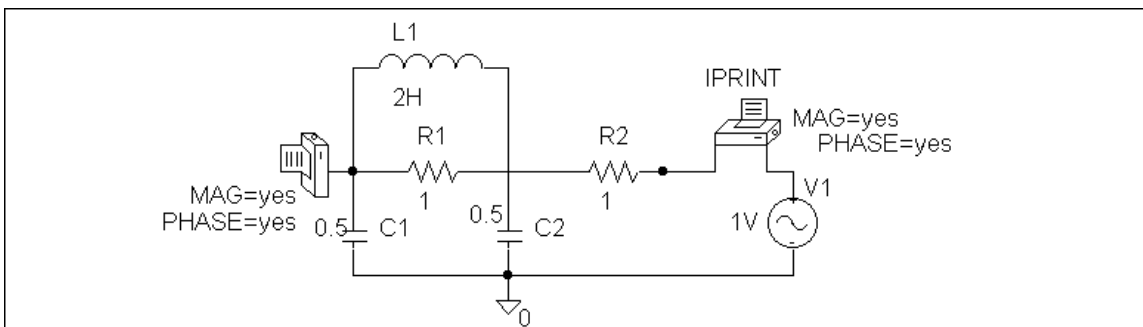
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E-01	5.000 E-01	1.800 E+02

FREQ	VM(\$N_0001)	VP(\$N_0001)
1.592 E-01	5.664 E-04	8.997 E+01

From this,

$$a = \frac{1}{5.664 \times 10^{-4} \angle 89.97^\circ} = 1765 \angle -89.97^\circ$$

$$c = \frac{0.5 \angle 180^\circ}{5.664 \times 10^{-4} \angle 89.97^\circ} = -882.28 \angle -89.97^\circ$$



(b) Similarly,

$$b = -\frac{V_2}{I_1} \Big|_{V_1=0} \quad \text{and} \quad d = -\frac{I_2}{I_1} \Big|_{V_1=0}$$

We short-circuit the input port and let  $V_2 = 1$  V. The schematic is shown below. After simulation, we obtain an output file which includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E-01	5.000 E-01	1.800 E+02

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E-01	5.664 E-04	-9.010 E+01

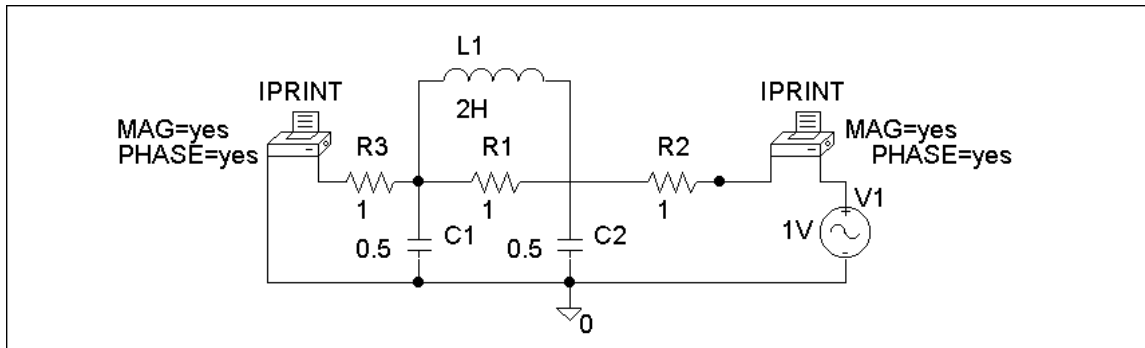
From this, we get

$$b = -\frac{1}{5.664 \times 10^{-4} \angle -90.1^\circ} = -j1765$$

$$d = -\frac{0.5 \angle 180^\circ}{5.664 \times 10^{-4} \angle -90.1^\circ} = j888.28$$

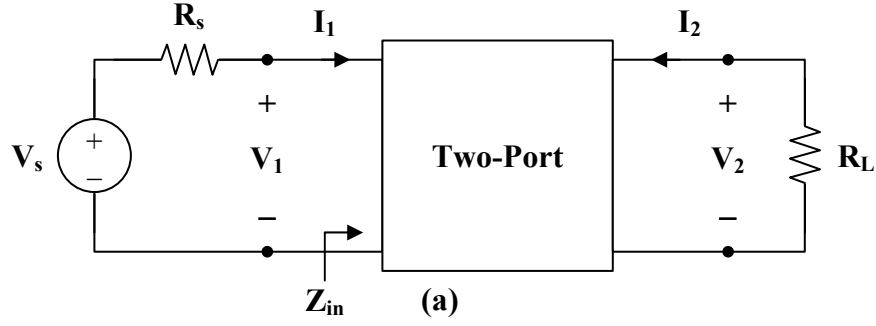
Thus

$$[t] = \begin{bmatrix} -j1765 & -j1765 \\ j888.2 & j888.2 \end{bmatrix}$$



## Chapter 19, Solution 88

To get  $Z_{in}$ , consider the network in Fig. (a).



$$\mathbf{I}_1 = \mathbf{y}_{11} \mathbf{V}_1 + \mathbf{y}_{12} \mathbf{V}_2 \quad (1)$$

$$\mathbf{I}_2 = \mathbf{y}_{21} \mathbf{V}_1 + \mathbf{y}_{22} \mathbf{V}_2 \quad (2)$$

But 
$$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{\mathbf{R}_L} = \mathbf{y}_{21} \mathbf{V}_1 + \mathbf{y}_{22} \mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{-\mathbf{y}_{21} \mathbf{V}_1}{\mathbf{y}_{22} + 1/\mathbf{R}_L} \quad (3)$$

Substituting (3) into (1) yields

$$\mathbf{I}_1 = \mathbf{y}_{11} \mathbf{V}_1 + \mathbf{y}_{12} \cdot \left( \frac{-\mathbf{y}_{21} \mathbf{V}_1}{\mathbf{y}_{22} + 1/\mathbf{R}_L} \right), \quad \mathbf{Y}_L = \frac{1}{\mathbf{R}_L}$$

$$\mathbf{I}_1 = \left( \frac{\Delta_y + \mathbf{y}_{11} \mathbf{Y}_L}{\mathbf{y}_{22} + \mathbf{Y}_L} \right) \mathbf{V}_1, \quad \Delta_y = \mathbf{y}_{11} \mathbf{y}_{22} - \mathbf{y}_{12} \mathbf{y}_{21}$$

or 
$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{y}_{22} + \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11} \mathbf{Y}_L}$$

$$\mathbf{A}_i = \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{\mathbf{y}_{21} \mathbf{V}_1 + \mathbf{y}_{22} \mathbf{V}_2}{\mathbf{I}_1} = \mathbf{y}_{21} \mathbf{Z}_{in} + \left( \frac{\mathbf{y}_{22}}{\mathbf{I}_1} \right) \left( \frac{-\mathbf{y}_{21} \mathbf{V}_1}{\mathbf{y}_{22} + \mathbf{Y}_L} \right)$$

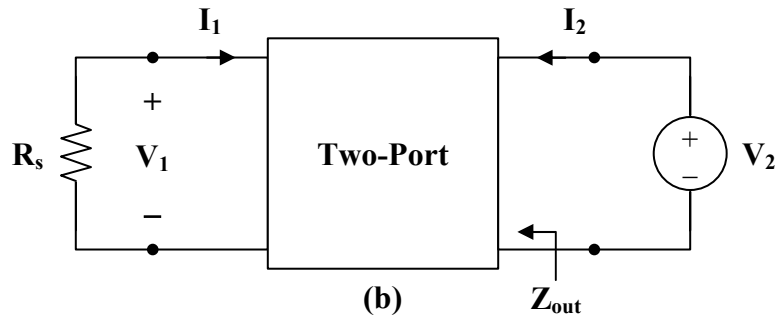
$$= \mathbf{y}_{21} \mathbf{Z}_{in} - \frac{\mathbf{y}_{22} \mathbf{y}_{21} \mathbf{Z}_{in}}{\mathbf{y}_{22} + \mathbf{Y}_L} = \left( \frac{\mathbf{y}_{22} + \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11} \mathbf{Y}_L} \right) \left( \mathbf{y}_{21} - \frac{\mathbf{y}_{22} \mathbf{y}_{21}}{\mathbf{y}_{22} + \mathbf{Y}_L} \right)$$

$$\mathbf{A}_i = \frac{\mathbf{y}_{21} \mathbf{Y}_L}{\Delta_y + \mathbf{y}_{11} \mathbf{Y}_L}$$

From (3),

$$\mathbf{A}_v = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{-\mathbf{y}_{21}}{\mathbf{y}_{22} + \mathbf{Y}_L}$$

To get  $Z_{out}$ , consider the circuit in Fig. (b).



$$Z_{out} = \frac{V_2}{I_2} = \frac{V_2}{y_{21} V_1 + y_{22} V_2} \quad (4)$$

But  $V_1 = -R_s I_1$

Substituting this into (1) yields

$$\begin{aligned} I_1 &= -y_{11} R_s I_1 + y_{12} V_2 \\ (1 + y_{11} R_s) I_1 &= y_{12} V_2 \end{aligned}$$

$$I_1 = \frac{y_{12} V_2}{1 + y_{11} R_s} = \frac{-V_1}{R_s}$$

or  $\frac{V_1}{V_2} = \frac{-y_{12} R_s}{1 + y_{11} R_s}$

Substituting this into (4) gives

$$\begin{aligned} Z_{out} &= \frac{1}{y_{22} - \frac{y_{12} y_{21} R_s}{1 + y_{11} R_s}} \\ &= \frac{1 + y_{11} R_s}{y_{22} + y_{11} y_{22} R_s - y_{21} y_{22} R_s} \\ Z_{out} &= \frac{y_{11} + Y_s}{\Delta_y + y_{22} Y_s} \end{aligned}$$

### Chapter 19, Solution 89

$$A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + (2640 \times 16 \times 10^{-6} - 2.6 \times 10^{-4} \times 72) \cdot 10^5}$$

$$A_v = \frac{-72 \cdot 10^5}{2640 + 1824} = \underline{\underline{-1613}}$$

$$\text{dc gain} = 20 \log |A_v| = 20 \log(1613) = \underline{\underline{64.15}}$$

### Chapter 19, Solution 90

$$(a) \quad Z_{in} = h_{ie} - \frac{h_{re} h_{fe} R_L}{1 + h_{oe} R_L}$$

$$1500 = 2000 - \frac{10^{-4} \times 120 R_L}{1 + 20 \times 10^{-6} R_L}$$

$$500 = \frac{12 \times 10^{-3}}{1 + 2 \times 10^{-5} R_L}$$

$$500 + 10^{-2} R_L = 12 \times 10^{-3} R_L$$

$$500 \times 10^2 = 0.2 R_L$$

$$R_L = \underline{\underline{250 \text{ k}\Omega}}$$

$$(b) \quad A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-120 \times 250 \times 10^3}{2000 + (2000 \times 20 \times 10^{-6} - 120 \times 10^{-4}) \times 250 \times 10^3}$$

$$A_v = \frac{-30 \times 10^6}{2 \times 10^3 + 7 \times 10^3} = \underline{\underline{-3333}}$$

$$A_i = \frac{h_{fe}}{1 + h_{oe} R_L} = \frac{120}{1 + 20 \times 10^{-6} \times 250 \times 10^3} = \underline{\underline{20}}$$

$$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie}) h_{oe} - h_{re} h_{fe}} = \frac{600 + 2000}{(600 + 2000) \times 20 \times 10^{-6} - 10^{-4} \times 120}$$

$$Z_{out} = \frac{2600}{40} \text{ k}\Omega = \underline{\underline{65 \text{ k}\Omega}}$$

$$(c) \quad A_v = \frac{V_c}{V_b} = \frac{V_c}{V_s} \longrightarrow V_c = A_v V_s = -3333 \times 4 \times 10^{-3} = \underline{\underline{-13.33 \text{ V}}}$$

### Chapter 19, Solution 91

$$R_s = 1.2 \text{ k}\Omega, \quad R_L = 4 \text{ k}\Omega$$

$$(a) \quad A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-80 \times 4 \times 10^3}{1200 + (1200 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80) \times 4 \times 10^3}$$

$$A_v = \frac{-32000}{1248} = \underline{\underline{-25.64}}$$

$$(b) \quad A_i = \frac{h_{fe}}{1 + h_{oe} R_L} = \frac{80}{1 + 20 \times 10^{-6} \times 4 \times 10^3} = \underline{\underline{74.074}}$$

$$(c) \quad Z_{in} = h_{ie} - h_{re} A_i$$

$$Z_{in} = 1200 - 1.5 \times 10^{-4} \times 74.074 \cong \underline{\underline{1.2 \text{ k}\Omega}}$$

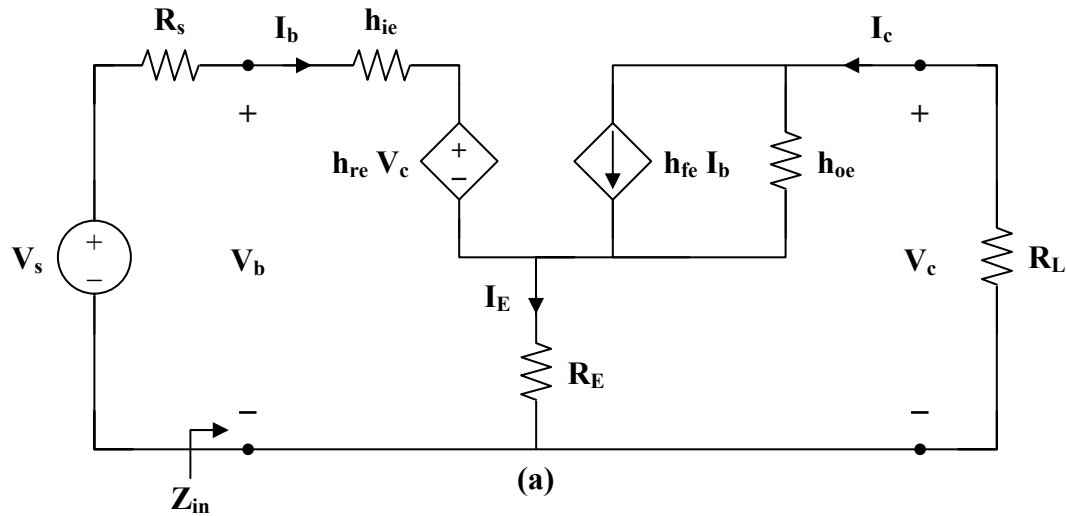
$$(d) \quad Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie}) h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{1200 + 1200}{2400 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80} = \frac{2400}{0.0468} = \underline{\underline{51.282 \text{ k}\Omega}}$$



## Chapter 19, Solution 92

Due to the resistor  $R_E = 240 \Omega$ , we cannot use the formulas in section 18.9.1. We will need to derive our own. Consider the circuit in Fig. (a).



$$I_E = I_b + I_c \quad (1)$$

$$V_b = h_{ie} I_b + h_{re} V_c + (I_b + I_c) R_E \quad (2)$$

$$I_c = h_{fe} I_b + \frac{V_c}{R_E + 1/h_{oe}} \quad (3)$$

But  $V_c = -I_c R_L \quad (4)$

Substituting (4) into (3),

$$I_c = h_{fe} I_b - \frac{R_L}{R_E + 1/h_{oe}} I_c$$

or  $A_i = \frac{I_c}{I_b} = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L)} \quad (5)$

$$A_i = \frac{100(1 + 240 \times 30 \times 10^{-6})}{1 + 30 \times 10^{-6}(4,000 + 240)}$$

$$A_i = \underline{\underline{79.18}}$$

From (3) and (5),

$$\mathbf{I}_c = \frac{h_{fe}(1+R_E)h_{oe}}{1+h_{oe}(R_L+R_E)} \mathbf{I}_b = h_{fe} \mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + 1/h_{oe}} \quad (6)$$

Substituting (4) and (6) into (2),

$$\mathbf{V}_b = (h_{ie} + R_E) \mathbf{I}_b + h_{re} \mathbf{V}_c + \mathbf{I}_c R_E$$

$$\mathbf{V}_b = \frac{\mathbf{V}_c (h_{ie} + R_E)}{\left( R_E + \frac{1}{h_{oe}} \right) \left[ \frac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)} - h_{fe} \right]} + h_{re} \mathbf{V}_c - \frac{\mathbf{V}_c R_E}{R_L}$$

$$\frac{1}{A_v} = \frac{\mathbf{V}_b}{\mathbf{V}_c} = \frac{(h_{ie} + R_E)}{\left( R_E + \frac{1}{h_{oe}} \right) \left[ \frac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)} - h_{fe} \right]} + h_{re} - \frac{R_E}{R_L} \quad (7)$$

$$\frac{1}{A_v} = \frac{(4000 + 240)}{\left( 240 + \frac{1}{30 \times 10^{-6}} \right) \left[ \frac{100(1 + 240 \times 30 \times 10^{-6})}{1 + 30 \times 10^{-6} \times 4240} - 100 \right]} + 10^{-4} - \frac{240}{4000}$$

$$\frac{1}{A_v} = -6.06 \times 10^{-3} + 10^{-4} - 0.06 = -0.066$$

$$A_v = \underline{\underline{-15.15}}$$

From (5),

$$\mathbf{I}_c = \frac{h_{fe}}{1+h_{oe} R_L} \mathbf{I}_b$$

We substitute this with (4) into (2) to get

$$\mathbf{V}_b = (h_{ie} + R_E) \mathbf{I}_b + (R_E - h_{re} R_L) \mathbf{I}_c$$

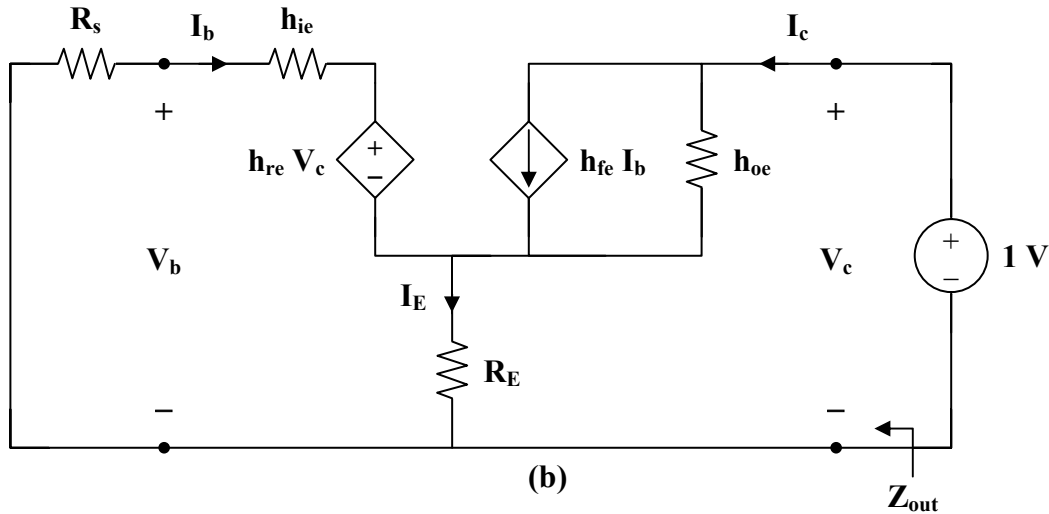
$$\mathbf{V}_b = (h_{ie} + R_E) \mathbf{I}_b + (R_E - h_{re} R_L) \left( \frac{h_{fe}(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)} \mathbf{I}_b \right)$$

$$Z_{in} = \frac{\mathbf{V}_b}{\mathbf{I}_b} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re} R_L)(1+R_E h_{oe})}{1+h_{oe}(R_L+R_E)} \quad (8)$$

$$Z_{in} = 4000 + 240 + \frac{(100)(240 \times 10^{-4} \times 4 \times 10^3)(1 + 240 \times 30 \times 10^{-6})}{1 + 30 \times 10^{-6} \times 4240}$$

$$Z_{in} = \underline{\underline{12.818 \text{ k}\Omega}}$$

To obtain  $Z_{out}$ , which is the same as the Thevenin impedance at the output, we introduce a 1-V source as shown in Fig. (b).



From the input loop,

$$I_b (R_s + h_{ie}) + h_{re} V_c + R_E (I_b + I_c) = 0$$

But

$$V_c = 1$$

So,

$$I_b (R_s + h_{ie} + R_E) + h_{re} + R_E I_c = 0 \quad (9)$$

From the output loop,

$$I_c = \frac{V_c}{R_E + \frac{1}{h_{oe}}} + h_{fe} I_b = \frac{h_{oe}}{R_E h_{oe} + 1} + h_{fe} I_b$$

or

$$I_b = \frac{I_c}{h_{fe}} - \frac{h_{oe}}{1 + R_E h_{oe}} \quad (10)$$

Substituting (10) into (9) gives

$$(R_s + R_E + h_{ie}) \left( \frac{I_c}{h_{fe}} \right) + h_{re} + R_E I_c - \frac{(R_s + R_E + h_{ie}) \left( \frac{h_{oe}}{h_{fe}} \right)}{1 + R_E h_{oe}} = 0$$

$$\frac{R_s + R_E + h_{ie}}{h_{fe}} I_c + R_E I_c = \frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}} \left( \frac{h_{oe}}{h_{fe}} \right) - h_{re}$$

$$\mathbf{I}_c = \frac{(h_{oe}/h_{fe}) \left[ \frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}} \right] - h_{re}}{R_E + (R_s + R_E + h_{ie})/h_{fe}}$$

$$Z_{out} = \frac{1}{\mathbf{I}_c} = \frac{R_E h_{fe} + R_s + R_E + h_{ie}}{\left[ \frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}} \right] h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{240 \times 100 + (1200 + 240 + 4000)}{\left[ \frac{1200 + 240 + 4000}{1 + 240 \times 30 \times 10^{-6}} \right] \times 30 \times 10^{-6} - 10^{-4} \times 100}$$

$$Z_{out} = \frac{24000 + 5440}{0.152} = \mathbf{193.7 \text{ k}\Omega}$$

### Chapter 19, Solution 93

We apply the same formulas derived in the previous problem.

$$\frac{1}{A_v} = \frac{(h_{ie} + R_E)}{\left( R_E + \frac{1}{h_{oe}} \right) \left[ \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe} \right]} + h_{re} - \frac{R_E}{R_L}$$

$$\frac{1}{A_v} = \frac{(2000 + 200)}{(200 + 10^5) \left[ \frac{150(1 + 0.002)}{1 + 0.04} - 150 \right]} + 2.5 \times 10^{-4} - \frac{200}{3800}$$

$$\frac{1}{A_v} = -0.004 + 2.5 \times 10^{-4} - 0.05263 = -0.05638$$

$$A_v = \mathbf{-17.74}$$

$$A_i = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} = \frac{150(1 + 200 \times 10^{-5})}{1 + 10^{-5} \times (200 + 3800)} = \mathbf{144.5}$$

$$Z_{in} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re} R_L)(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)}$$

$$Z_{in} = 2000 + 200 + \frac{(150)(200 - 2.5 \times 10^{-4} \times 3.8 \times 10^3)(1.002)}{1.04}$$

$$Z_{in} = 2200 + 28966$$

$$Z_{in} = \underline{\underline{31.17 \text{ k}\Omega}}$$

$$Z_{out} = \frac{R_E h_{fe} + R_s + R_E + h_{ie}}{\left[ \frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}} \right] h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{200 \times 150 + 1000 + 200 + 2000}{\left[ \frac{3200 \times 10^{-5}}{1.002} \right] - 2.5 \times 10^{-4} \times 150} = \frac{33200}{-0.0055}$$

$$Z_{out} = \underline{\underline{-6.148 \text{ M}\Omega}}$$

### Chapter 19, Solution 94

We first obtain the **ABCD** parameters.

Given  $[\mathbf{h}] = \begin{bmatrix} 200 & 0 \\ 100 & 10^{-6} \end{bmatrix}$ ,  $\Delta_h = \mathbf{h}_{11} \mathbf{h}_{22} - \mathbf{h}_{12} \mathbf{h}_{21} = 2 \times 10^{-4}$

$$[\mathbf{T}] = \begin{bmatrix} \frac{\Delta_h}{\mathbf{h}_{21}} & \frac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \frac{\mathbf{h}_{21}}{-\mathbf{h}_{22}} & \frac{\mathbf{h}_{21}}{-1} \\ \mathbf{h}_{21} & \mathbf{h}_{21} \end{bmatrix} = \begin{bmatrix} -2 \times 10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}$$

The overall **ABCD** parameters for the amplifier are

$$[\mathbf{T}] = \begin{bmatrix} -2 \times 10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix} \begin{bmatrix} -2 \times 10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix} \cong \begin{bmatrix} 2 \times 10^{-8} & 2 \times 10^{-2} \\ 10^{-10} & 10^{-4} \end{bmatrix}$$

$$\Delta_T = 2 \times 10^{-12} - 2 \times 10^{-12} = 0$$

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{B} & \frac{\Delta_T}{\mathbf{D}} \\ \mathbf{D} & \mathbf{D} \\ -1 & \mathbf{C} \\ \mathbf{D} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} 200 & 0 \\ -10^{-4} & 10^{-6} \end{bmatrix}$$

Thus,  $h_{ie} = 200$ ,  $h_{re} = 0$ ,  $h_{fe} = -10^{-4}$ ,  $h_{oe} = 10^{-6}$

$$A_v = \frac{(10^4)(4 \times 10^3)}{200 + (2 \times 10^{-4} - 0) \times 4 \times 10^3} = \underline{\underline{2 \times 10^5}}$$

$$Z_{in} = h_{ie} - \frac{h_{re} h_{fe} R_L}{1 + h_{oe} R_L} = 200 - 0 = \underline{\underline{200 \Omega}}$$

### Chapter 19, Solution 95

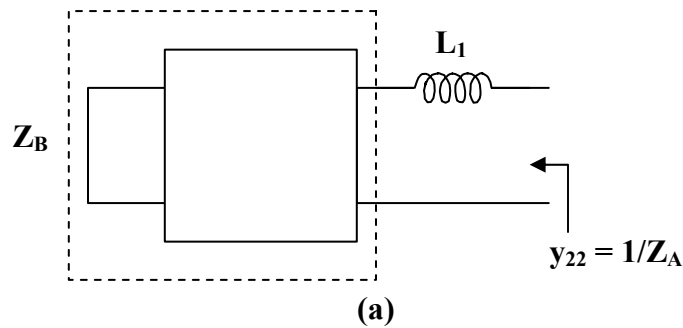
$$\text{Let } Z_A = \frac{1}{y_{22}} = \frac{s^4 + 10s^2 + 8}{s^3 + 5s}$$

Using long division,

$$Z_A = s + \frac{5s^2 + 8}{s^3 + 5s} = sL_1 + Z_B$$

i.e.  $L_1 = 1 \text{ H}$  and  $Z_B = \frac{5s^2 + 8}{s^3 + 5s}$

as shown in Fig (a).



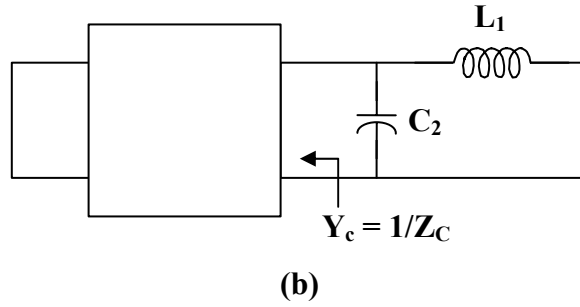
$$Y_B = \frac{1}{Z_B} = \frac{s^3 + 5s}{5s^2 + 8}$$

Using long division,

$$Y_B = 0.2s + \frac{3.4s}{5s^2 + 8} = sC_2 + Y_C$$

where  $C_2 = 0.2 \text{ F}$  and  $Y_C = \frac{3.4s}{5s^2 + 8}$

as shown in Fig. (b).

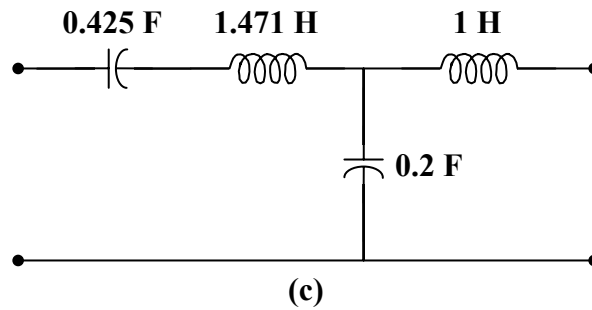


$$Z_C = \frac{1}{Y_C} = \frac{5s^2 + 8}{3.4s} = \frac{5s}{3.4} + \frac{8}{3.4s} = sL_3 + \frac{1}{sC_4}$$

i.e. an inductor in series with a capacitor

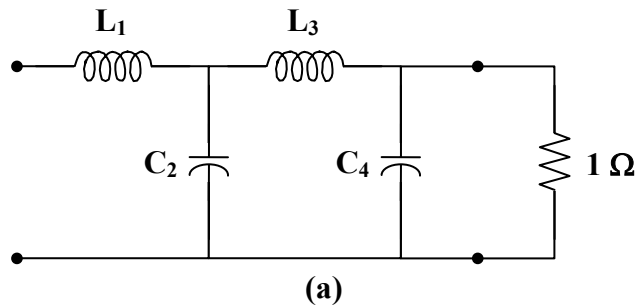
$$L_3 = \frac{5}{3.4} = 1.471 \text{ H} \quad \text{and} \quad C_4 = \frac{3.4}{8} = 0.425 \text{ F}$$

Thus, the LC network is shown in Fig. (c).



### Chapter 19, Solution 96

This is a fourth order network which can be realized with the network shown in Fig. (a).



$$\Delta(s) = (s^4 + 3.414s^2 + 1) + (2.613s^3 + 2.613s)$$

$$H(s) = \frac{1}{1 + \frac{2.613s^3 + 2.613s}{s^4 + 3.414s^2 + 1}}$$

which indicates that

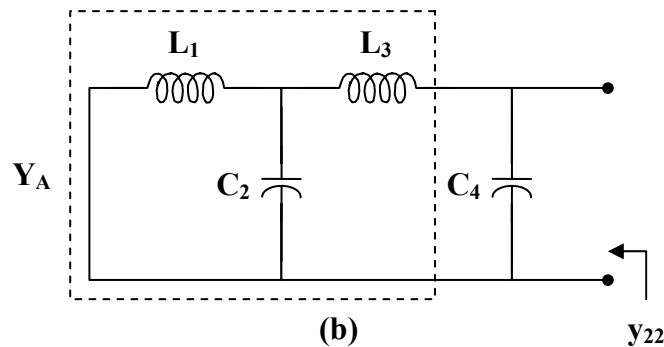
$$y_{21} = \frac{-1}{2.613s^3 + 2.613s}$$

$$y_{22} = \frac{s^4 + 3.414s^2 + 1}{2.613s^3 + 2.613s}$$

We seek to realize  $y_{22}$ . By long division,

$$y_{22} = 0.383s + \frac{2.414s^2 + 1}{2.613s^3 + 2.613s} = sC_4 + Y_A$$

i.e.  $C_4 = 0.383 \text{ F}$  and  $Y_A = \frac{2.414s^2 + 1}{2.613s^3 + 2.613s}$   
as shown in Fig. (b).



$$Z_A = \frac{1}{Y_A} = \frac{2.613s^3 + 2.613s}{2.414s^2 + 1}$$

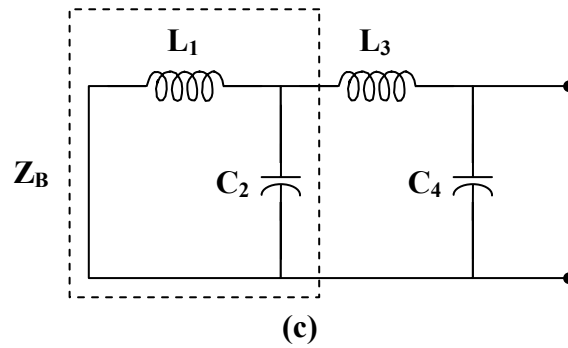
By long division,

$$Z_A = 1.082s + \frac{1.531s}{2.414s^2 + 1} = sL_3 + Z_B$$

i.e.  $L_3 = 1.082 \text{ H}$  and  $Z_B = \frac{1.531s}{2.414s^2 + 1}$



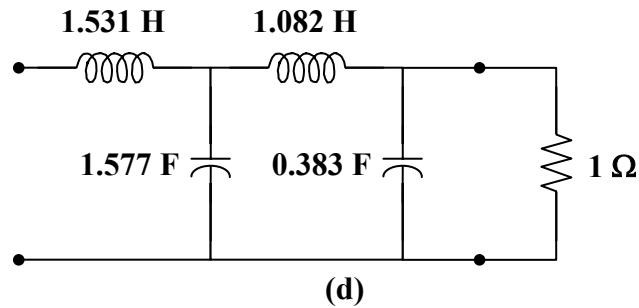
as shown in Fig.(c).



$$Y_B = \frac{1}{Z_B} = 1.577s + \frac{1}{1.531s} = sC_2 + \frac{1}{sL_1}$$

i.e.  $C_2 = 1.577 \text{ F}$  and  $L_1 = 1.531 \text{ H}$

Thus, **the network is shown in Fig. (d).**



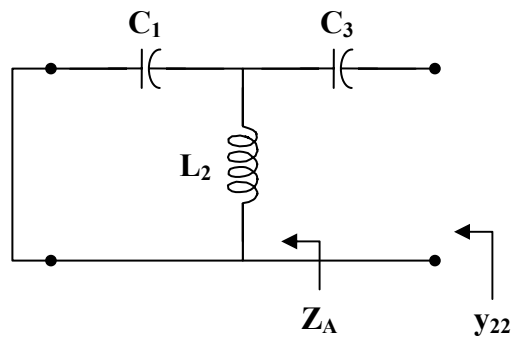
### Chapter 19, Solution 97

$$H(s) = \frac{s^3}{(s^3 + 12s) + (6s^2 + 24)} = \frac{s^3}{s^3 + 12s + 6s^2 + 24} = \frac{s^3}{1 + \frac{6s^2 + 24}{s^3 + 12s}}$$

Hence,

$$y_{22} = \frac{6s^2 + 24}{s^3 + 12s} = \frac{1}{sC_3} + Z_A \quad (1)$$

where  $Z_A$  is shown in the figure below.



We now obtain  $C_3$  and  $Z_A$  using partial fraction expansion.

$$\text{Let } \frac{6s^2 + 24}{s(s^2 + 12)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 12}$$

$$6s^2 + 24 = A(s^2 + 12) + Bs^2 + Cs$$

Equating coefficients :

$$s^0: \quad 24 = 12A \quad \longrightarrow \quad A = 2$$

$$s^1: \quad 0 = C$$

$$s^2: \quad 6 = A + B \quad \longrightarrow \quad B = 4$$

Thus,

$$\frac{6s^2 + 24}{s(s^2 + 12)} = \frac{2}{s} + \frac{4s}{s^2 + 12} \quad (2)$$

Comparing (1) and (2),

$$C_3 = \frac{1}{A} = \frac{1}{2} \text{ F}$$

$$\frac{1}{Z_A} = \frac{s^2 + 12}{4s} = \frac{1}{4}s + \frac{3}{s} \quad (3)$$

$$\text{But } \frac{1}{Z_A} = sC_1 + \frac{1}{sL_2} \quad (4)$$

Comparing (3) and (4),

$$C_1 = \frac{1}{4} \text{ F} \quad \text{and} \quad L_2 = \frac{1}{3} \text{ H}$$

Therefore,

$$C_1 = \underline{\underline{0.25 \text{ F}}}, \quad L_2 = \underline{\underline{0.3333 \text{ H}}}, \quad C_3 = \underline{\underline{0.5 \text{ F}}}$$

### Chapter 19, Solution 98

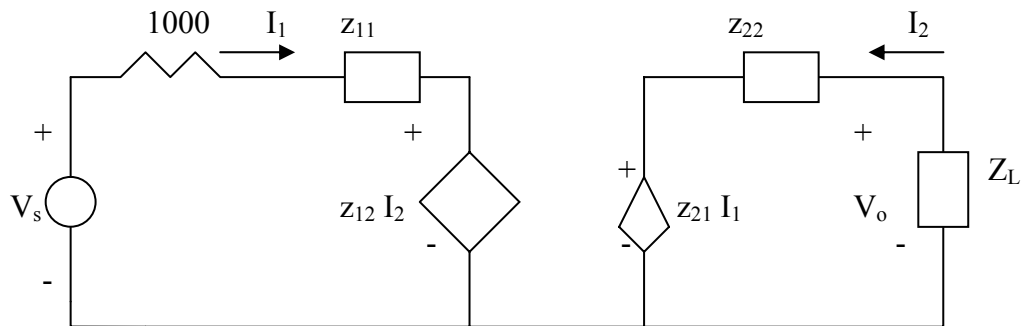
$$\Delta_h = 1 - 0.8 = 0.2$$

$$[T_a] = [T_b] = \begin{bmatrix} -\Delta_h/h_{21} & -h_{11}/h_{21} \\ -h_{22}/h_{21} & -1/h_{21} \end{bmatrix} = \begin{bmatrix} -0.001 & -10 \\ -2.5 \times 10^{-6} & -0.005 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} 2.6 \times 10^{-5} & 0.06 \\ 1.5 \times 10^{-8} & 5 \times 10^{-5} \end{bmatrix}$$

We now convert this to z-parameters

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 1.733 \times 10^3 & 0.0267 \\ 6.667 \times 10^7 & 3.33 \times 10^3 \end{bmatrix}$$



$$V_s = (1000 + z_{11})I_1 + z_{12}I_2 \quad (1)$$

$$V_o = z_{22}I_2 + z_{21}I_1 \quad (2)$$

$$\text{But } V_o = -I_2 Z_L \quad \longrightarrow \quad I_2 = -V_o / Z_L \quad (3)$$

Substituting (3) into (2) gives

$$I_1 = V_o \left( \frac{1}{z_{21}} + \frac{z_{22}}{z_{21} Z_L} \right) \quad (4)$$

We substitute (3) and (4) into (1)

$$\begin{aligned}
 V_s &= (1000 + z_{11}) \left( \frac{1}{z_{11}} + \frac{z_{22}}{z_{21}Z_L} \right) V_o - \frac{z_{12}}{Z_L} V_o \\
 &= 7.653 \times 10^{-4} - 2.136 \times 10^{-5} = \underline{744 \mu V}
 \end{aligned}$$

### Chapter 19, Solution 99

$$Z_{ab} = Z_1 + Z_3 = Z_c \parallel (Z_b + Z_a)$$

$$Z_1 + Z_3 = \frac{Z_c(Z_a + Z_b)}{Z_a + Z_b + Z_c} \quad (1)$$

$$Z_{cd} = Z_2 + Z_3 = Z_a \parallel (Z_b + Z_c)$$

$$Z_2 + Z_3 = \frac{Z_a(Z_b + Z_c)}{Z_a + Z_b + Z_c} \quad (2)$$

$$Z_{ac} = Z_1 + Z_2 = Z_b \parallel (Z_a + Z_c)$$

$$Z_1 + Z_2 = \frac{Z_b(Z_a + Z_c)}{Z_a + Z_b + Z_c} \quad (3)$$

Subtracting (2) from (1),

$$Z_1 - Z_2 = \frac{Z_b(Z_c - Z_a)}{Z_a + Z_b + Z_c} \quad (4)$$

Adding (3) and (4),

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c} \quad (5)$$

Subtracting (5) from (3),

$$Z_2 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} \quad (6)$$

Subtracting (5) from (1),

$$Z_3 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c} \quad (7)$$

Using (5) to (7)

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a\mathbf{Z}_b\mathbf{Z}_c(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)}{(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)^2}$$

$$\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1 = \frac{\mathbf{Z}_a\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \quad (8)$$

Dividing (8) by each of (5), (6), and (7),

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

as required. Note that the formulas above are not exactly the same as those in Chapter 9 because the locations of  $\mathbf{Z}_b$  and  $\mathbf{Z}_c$  are interchanged in Fig. 18.122.