## 2021 Preliminary Examination

## Secondary Four Express / Five Normal Academic

MATHEMATICS ..... 20 August 20214048/01

Name: $\qquad$ ( )

Class: $\qquad$

## READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid or tape.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142 , unless the question requires the answer in terms of $\pi$.

The number of marks is given in brackets [ ] at the end of each question or part question. The total number of marks for this paper is 80 .

| FOR MARKER'S USE |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Marks <br> Awarded | Max <br> Marks |  |
| Total |  | 80 |  |

## Mathematical Formulae

Compound interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
$$

Mensuration
Curved Surface area of a cone $=\pi r l$
Surface area of a sphere $=4 \pi r^{2}$

$$
\text { Volume of a sphere }=\frac{4}{3} \pi r^{3}
$$

Area of triangle $A B C=\frac{1}{2} a b \sin C$
Arc length $=r \theta$, where $\theta$ is in radians
Sector Area $=\frac{1}{2} r^{2} \theta$, where $\theta$ is in radians
Trigonometry

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{gathered}
$$

Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
\text { Standard deviation } & =\sqrt{\frac{\sum f x^{2}}{\sum f}-\left(\frac{\sum f x}{\sum f}\right)^{2}}
\end{aligned}
$$

1 Elly's height is 1.61 m and Adela's height is 1.73 m , measured correct to three significant figures.

Find the greatest possible difference in their heights in metres, correct to three significant figures.
$\qquad$

Sharp rise in number of new


Explain how the line graph above may be misleading.

Answer
$\qquad$
$\qquad$
$\qquad$

3 Simplify $\left(2 x^{2}\right)^{3} \div 4 \sqrt{x}$, giving your answer in the form of $a x^{n}$, where $a$ and $n$ are rational numbers.

> Answer

4 Given that $9172.05=\left(9 \times 10^{3}\right)+\left(1 \times 10^{2}\right)+(7 \times 10)+\left(2 \times 10^{a}\right)+\left(5 \times 10^{b}\right)$, write down the values of $a$ and of $b$.

$$
\text { Answer } \quad a=\ldots \ldots \ldots \ldots \ldots b=
$$

5 Write as a single fraction in its simplest form $\frac{3}{1-x^{2}}-\frac{2}{x+1}$.

$A B C D$ is a square and $A Q$ is perpendicular to $P R$.
$P Q R$ and $A D R$ are straight lines.
$D Q: D C=1: 2$.

Show that triangle $D Q R$ is congruent to triangle $C Q P$. Give a reason for each statement you make.

Answer

7 Use factorisation to solve the equation.

$$
2 h^{2}-11 h-21=0
$$

Answer $\quad h=$
or

8 A map is drawn to a scale of $1: n$. The actual distance between two points $X$ and $Y$ is 2.8 km . On the map, they are 4 cm apart.

Find the value of $n$.

9 Solve the inequalities $x \leq \frac{x+4}{3} \leq 2 x-1$.
$\qquad$

10 The acceleration, $a \mathrm{~m} / \mathrm{s}^{2}$, of a particle is inversely proportional to the square of its distance $x$ metres from a fixed point.
The distance of the particle is reduced to $0.5 x$.
Find the ratio of the acceleration to the original acceleration.
$\qquad$
$11 P Q S R$ is a parallelogram.
The coordinates of $P, Q$ and $R$ are $(0,2),(6,8)$ and $(10,2)$ respectively. Find area of the parallelogram $P Q S R$.


Answer ....................... units ${ }^{2}$

12 Sketch the graph of $y=(x+4)(10-x)$ on the axes below.
Indicate clearly the coordinates of the points where the graph crosses the axes and the maximum point on the curve.


13 In the diagram, $A$ is an obtuse angle such that $\sin A=\frac{5}{13}$.


Leaving your answer as a fraction, find the value of
(a) $\sin \left(180^{\circ}-A\right)$,
(b) $\cos A$.

14 The diagram shows a quadrilateral $P Q R S$.


On the diagram,
(a) construct the perpendicular bisector of $S R$ such that it meets $P Q$ at point $Z$.
Mark and label $Z$.
(b) measure and write down the size of angle $P Z S$.
$\qquad$

15 (a) Factorise completely $5 p r-2 p s-5 q r+2 q s$.

Answer
(b) Given that $p \neq q$, find the value of $\frac{r}{s}$ when $5 p r-2 p s-5 q r+2 q s=0$.

16 Written as a product of its prime factors,

$$
p=2^{1} \times 3^{x} \times 7^{y} \text { and } q=3 \times 7^{2} \times 11
$$

(a) Find the smallest value of $x$ and $y$ for which $p$ is a multiple of 21 .

$$
\text { Answer } \quad x=\ldots \ldots \ldots \ldots \ldots \cdot y=
$$

(b) Explain why $33 q$ is a perfect square.

Answer $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$


The diagram shows the speed-time graph of two cars, A and B.
Car A, starting from rest, accelerates uniformly for 2 seconds until it reaches a speed of $11 \mathrm{~m} / \mathrm{s}$.
It then continues to travel at constant speed.
2 seconds later, Car B starts from the same point as Car A.
(a) Find
(i) the acceleration of Car A when $t=1$,

$$
\text { Answer ........................... m/s }{ }^{2}
$$

(ii) the distance travelled by Car A for the first 2 seconds.

$$
\text { Answer ................................. } \mathrm{m}
$$

(b) Car B accelerates uniformly from rest.

It overtakes Car A when $t=9$ seconds.
Find $v$, the speed of Car B when it overtakes Car A.

$$
\text { Answer } \quad v=
$$

$18 \varepsilon=\{$ integers $x: 2 \leq x \leq 13\}$
$A=\{$ prime numbers $\}$
$B=\{$ multiples of 4$\}$
$C=\{$ factors of 12$\}$

List the elements in
(a) $B^{\prime}$,

## Answer

(b) $A \cap B^{\prime}$,

## Answer

(c) $(A \cup B)^{\prime}$,

## Answer

(d) $B \cap C$.
$\qquad$

19 The number of blue, white and black masks Julian has is in the ratio $3: 4: 5$. After exchanging 30 black masks for blue ones, the ratio becomes $9: 10: 11$.

Find the number of blue masks Julian has now.

20


The diagram shows two geometrically similar cup and saucer sets.
The diameter of the smaller saucer is 5 cm .
The diameter of the larger saucer is 6.5 cm .
A coffee shop sells the smaller cup of coffee at $\$ 1$ and the larger cup at $\$ 2$.

Calculate which is a better buy.
Explain your answer.

Answer $\qquad$
$\qquad$
$\qquad$
$\qquad$

21 The matrix below shows the results of three baseball teams in a series of competition.

$$
\mathbf{R}=\left(\begin{array}{ccc}
\text { Win } & \text { Draw } & \text { Lose } \\
\left(\begin{array}{ccc}
12 & 5 & 3 \\
3 & 8 & 7 \\
9 & 4 & 4
\end{array}\right) \begin{array}{c}
\text { Gratitude } \\
\text { Respect } \\
\text { Compassion }
\end{array}
\end{array}\right.
$$

(a) A win gains 3 points, a draw 1 point and a loss 0 point.

Represent this information with a $3 \times 1$ column matrix $\mathbf{P}$.

$$
\text { Answer } \quad \mathbf{P}=
$$

(b) Evaluate the matrix RP.
(c) Explain what your answer to (b) represents and state the name of the winning baseball team.

Answer $\qquad$
$\qquad$
$\qquad$

22 The diagram shows a sketch of the graph of $y=-10+2 x$. The line crosses the axes at $P$ and $Q$.

(a) Find the coordinates of $P$ and $Q$.

$$
\left.\begin{array}{r}
\text { Answer } \\
P(\ldots \ldots . . . . . . . . . . ., ~ . . . . . . . . . . . . . . . . .) ~
\end{array}\right) .
$$

(b) Calculate the length of the line joining $P$ to $Q$.


The diagram shows a goat tied to a pole at $A$. The length of the rope attached to the goat is 6 m .
$A$ is due west of the centre of the circle, $O$.
(a) Measure the bearing of the goat from $A$.
(b) The circle represents a grass patch of radius 4.5 m .

Find the probability that the goat is in the grass patch.

24 The cumulative frequency graph below shows the weight of 240 students in a school.


Use the graph to find
(a) the number of students with a weight greater than 55 kg ,

> Answer
(b) the interquartile range,

> Answer ................................. kg
(c) the median.

> Answer ................................ kg

It was discovered that the weighing machine used to measure the weight of the students was faulty.

The weight of each of the students was supposed to be 5 kg more than their recorded weights.
(d) Explain how the cumulative frequency curve of the corrected weights will differ from the given curve.

Answer
$\qquad$
$\qquad$
$\qquad$
$\qquad$

25 The first four terms of a sequence are 5, 9, 13 and 17.
(a) Write down the 8 th term of the sequence.

> Answer
(b) Find an expression, in terms of $n$, for the $n$th term of the sequence.

> Answer
(c) One term of the sequence is 205 .

Find the value of $n$ for this term.

Answer $n=$
(d) Explain why 50 is not part of the sequence.

Answer $\qquad$
$\qquad$
$\qquad$
$\qquad$

## 2021 Preliminary Examination

## Secondary Four Express/Five Normal Academic

MATHEMATICS
4048/02

Name: $\qquad$ ( )

Class: $\qquad$

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| :---: | :---: | :---: | :---: |
|  | Marks <br> Awarded | Max <br> Marks |  |
| Total |  | 100 |  |

This question paper consists of $\underline{\mathbf{3}}$ printed pages including the cover page.

Compound interest

$$
\text { Total amount }=P\left(1+\frac{r}{100}\right)^{n}
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Mensuration

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\begin{gathered}
\text { Curved Surface area of a cone }=\pi r l \\
\text { Surface area of a sphere }=4 \pi r^{2} \\
\text { Volume of a sphere }=\frac{4}{3} \pi r^{3} \\
\text { Volume of a cone }=\frac{1}{3} \pi r^{2} h
\end{gathered}
$$

$$
\text { Area of triangle } A B C=\frac{1}{2} a b \sin C
$$

Arc length $=r \theta$, where $\theta$ is in radians
Sector Area $=\frac{1}{2} r^{2} \theta$, where $\theta$ is in radians
Trigonometry

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\begin{aligned}
& \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A
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Statistics

$$
\begin{aligned}
\text { Mean } & =\frac{\sum f x}{\sum f} \\
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\end{aligned}
$$

1 (a) Simplify $\frac{16 a^{4} b^{2}}{5} \div \frac{8 a^{3}}{25 a b^{3}}$.
(b) (i) Express $-4 x+x^{2}-6$ in the form $a+(x+b)^{2}$, where $a$ and $b$ are integers.
(ii) Write down the coordinates of the minimum point of the graph of $y=-4 x+x^{2}-6$.
$\qquad$ , $\square$
(c) $\quad l=\frac{1}{2}\left(m^{2}-n\right)$
(i) Evaluate $l$ when $m=4$ and $n=-5$.

$$
\text { Answer } l=
$$

(ii) Express $n$ in terms of $l$ and $m$.

> Answer
$\qquad$
(d) Solve $\frac{x+1}{2 x+3}+\frac{3 x}{4 x^{2}-9}=2$.

2 (a) A children indoor playground can accommodate 160 people in a session. Ticket price for an accompanying adult and a child is $\$ 9$ and $\$ 32.50$ respectively.
(i) On a particular weekend afternoon, the playground is $60 \%$ full. $75 \%$ of the patrons are children.

Calculate the total amount collected from the sales of tickets.

> Answer \$
(ii) On that same particular weekend evening, $\$ 3900$ was collected from the sales of tickets for children.

Calculate the percentage increase in the number of children who patronized the playground on the weekend evening compared to the afternoon.
(b) Arielle plans to invest $\$ 25000$ over a period of 2 years.

Plan $A$ offers simple interest of $6.2 \%$ per annum.
Plan $B$ offers $6 \%$ per annum interest compounded quarterly.
Determine which plan offers a better return for her.
Justify your answer.
Answer

3 (a) Students in a class were asked how many siblings they have. The results are shown in the table.

| Number of <br> sibling | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> students | 5 | 18 | 10 | $x$ | 2 |

(i) The modal number of siblings is 1 .

Calculate the greatest possible number of students in the class.

> Answer
(ii) The median number of siblings is 2 .

Calculate the smallest possible value of $x$.

Answer $x=$
(iii) The mean number of siblings is 1.525 .

Calculate the value of $x$.
(b) A dart board comprises of 3 colours as shown below.

## [Turn Over



A pouch contains six 10 cents coins, three 20 cents coins and one 50 cents coin.

Tim throws a dart first then picks a coin from the pouch.
(i) Draw a tree diagram to show the probabilities of the possible outcomes.

Answer
(ii) Calculate the probability that the dart hits the red region and a 20 cent coin is picked.

Answer
(iii) State one assumption made.

Answer $\qquad$
$\qquad$
$\qquad$

4 (a) A regular $m$-sided polygon and a regular $n$-sided polygon are joined together.
Part of the polygons are shown below.
$\angle B C D: \angle D C G=5: 3$


Mike made some calculations and claimed that $m=6$ and $n=10$.
Determine if Mike's claim is accurate.
Justify your answer.
Answer
(b)


Triangle $A B C$ is an equilateral triangle.
$A C=C D$.
$\angle A D F=20^{\circ}$
$A D$ bisects $\angle C A F$.
(i) Show that $A F$ is parallel to $C D$. State your reasons.

Answer

Calculate
(ii) reflex $\angle B A F$,
(iii) $\angle F D E$.

5 (a) Complete the table of values for $y=2 x^{3}-x^{2}-10 x$.

| $x$ | -2 | -1.5 | -1 | -0.5 | 0 | 1 | 1.5 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 0 | 6 | 7 | 4.5 | 0 | -9 | -10.5 |  |

(b) On the grid opposite, draw the graph of $y=2 x^{3}-x^{2}-10 x$ for $-2 \leq x \leq 2$.
(c) The equation $2 x^{3}-x^{2}-5 x=-8$ only has one solution.
(i) Explain how this can be seen from your graph.

Answer
(ii) Use your graph to find the solution of the equation $2 x^{3}-x^{2}-5 x=-8$.

> Answer
(d) Use your graph to find the $x$-coordinate of the maximum point.

Canberra Secondary School 2021 Prelim EM P2


6 The diagram represents a flat plot of land $P Q R S$.
$P Q=40 \mathrm{~m}, P R=75 \mathrm{~m}, Q R=90 \mathrm{~m}, S R=60 \mathrm{~m}$ and $\angle P R S=70^{\circ}$. [Turn Over
$S$ is due north of $R$ and $Q$ is due east of $R$.

(a) Find
(i) the area of the land $P Q R S$,

> Answer
$\qquad$
$\mathrm{m}^{2}$ [2]
(ii) the length of $P S$,
(iii) the angle $P S R$,

Answer -
(iv) the shortest distance from $S$ to $P R$.

Answer m
(b) $T$ is the top of a vertical tower at $S$. The angle of elevation of $T$ from $R$ is $10^{\circ}$.
(i) Find the height of the tower.

Answer
(ii) Hence, find the greatest angle of elevation of $T$ viewed from $P R$.

Answer。

7 (a) A circle with centre $O$ has a tangent $X Y$ at $D$. $\angle O D C=70^{\circ}, \angle B C D=120^{\circ}$ and $\angle O D A=15^{\circ}$.


Find
(i) $\angle C D Y$,

> Answer
(ii) $\angle D A B$,

## Answer

$\qquad$
(iii) $\angle O B A$.
(b) The diagram shows a sector $O A B C$ of a circle, centre $O$ and an isosceles triangle $O C D$.
$O C=O D=10 \mathrm{~cm}$ and $\angle C O D=1.2$ radians .


Find
(i) the area of the shaded segment $A B C$,

Answer $\mathrm{cm}^{2}$
(ii) the length $C D$.

8 Old MacDonald has a farm with a rock wall around it. The figure shows a portion of his farm where he plans to build two enclosures.


Legend: $\times \times \times \times \times \times \times \times \times$ Fence
The total length of the fence is 180 m .
(a) Show that $y=90-\frac{3}{2} x$.

Answer
(b) $A$ is the total areas of the 2 enclosures.

Form a quadratic equation for $A$ in terms of $x$.
Answer
(c) If $A=2100 \mathrm{~m}^{2}$, solve for $x$.

Leave your answer in 2 decimal places.

Answer $x=$ or
(d) Using your answers in part (c), find the coordinates of the turning point of the quadratic equation of area, $A$.

(e) Explain if the answer found in part (d) represents a maximum or minimum area.

Answer
$\qquad$
$\qquad$

9 An industrial mixer is made up of an open cylinder connected to a cone. The height of the cylinder and the cone is 1.2 m and 0.5 m respectively. The diameter of the mixer is 1.5 m .

(a) Find the capacity of the mixer in litres,
(b) Water is poured into the mixer.

Find the height of the water level, in centimetres, given that the volume is
(i) 500 litres,

$$
\text { Answer } \quad \mathrm{cm} \text { [3] }
$$

(ii) 150 litres.

Answer
cm
(c) The outside of the mixer is to be painted bright yellow.

1 can of paint can cover $4.5 \mathrm{~m}^{2}$.
How many cans are needed to paint 6 such mixers?

10 John is planning a trip from Singapore to Paris. His flight will stop over in Delhi before continuing its journey to Paris.

Answer the following questions using the information from the map below.


Map is drawn to scale.
(a) What is the bearing of Paris from Delhi?

Answer $\qquad$ [1]
(b) The distance from Singapore to Delhi is 4150 km .

What is the distance from Delhi to Paris?

The table below shows the Greenwich Meridian Time (GMT) for several locations in the world.

| Location | GMT |
| :--- | :---: |
| Brisbane, Australia | $+10: 00$ |
| Delhi, India | $+05: 30$ |
| Greenwich, England | $00: 00$ |
| Los Angeles, United States | $-07: 00$ |
| Paris, France | $+02: 00$ |
| Singapore, Singapore | $+08: 00$ |
| Toronto, Canada | $-04: 00$ |

Note: Brisbane is 10 hours ahead of Greenwich and Los Angeles in 7 hours behind Greenwich.

John departed from Singapore at $11: 15 \mathrm{pm}$ on a Saturday.
His flight will stop over at Delhi for 2 hours before continuing to Paris. The average speed of an airplane is $850 \mathrm{~km} / \mathrm{h}$.
(c) On what day and at what time will John arrive in Paris?

Answer

## Answers

| 1 | $1.734-1.605=0.129 \mathrm{~m}$ OR $1.7349-1.6050=0.130 \mathrm{~m}$ |
| :---: | :---: |
| 2 | The title of the line graph is biased as it does not allow reader to make judgement. <br> OR <br> The vertical axis does not start from 0 , which exaggerated the differences. (Can accept without the reasoning) |
| 3 | $2 x^{5 \frac{1}{2}}$ |
| 4 | $a=0, \quad b=-2$ |
| 5 | $=\frac{1+2 x}{1-x^{2}} \text { or } \frac{1+2 x}{(1+x)(1-x)}$ |
| 7 | $h=-\frac{3}{2} \quad \text { or } \quad h=7$ |
| 8 | $n=70000$ |
| 9 | $1 \frac{2}{5} \leq x \leq 2$ |
| 10 | $a_{\text {now }}: a_{\text {original }}=4: 1$ |
| 11 | 60 square units |
| 13a | $\sin \left(180^{\circ}-A\right)=\frac{5}{13}$ |
| 13b | $\cos A=-\frac{12}{13}$ OR shown adjacent side is 12 units ( 1 m ) |
| 14b | $\angle P Z S=33^{\circ}$ (accept angle values of $\pm 1^{\circ}$ ) |
| 15a | $(p-q)(5 r-2 s)$ |
| 15b | $\frac{r}{s}=\frac{2}{5}$ |
| 16a | Smallest $x=1$, Smallest $y=1$ |
| 16b | Since the indices of the prime factors are multiples of $2,33 q$ is a perfect square. |
| 17ai | $5.5 \mathrm{~m} / \mathrm{s}^{2}$ |
| 17aii | 11 m |
| 17b | $v=25.1 \quad \text { or } 25 \frac{1}{7} \quad \text { or } \quad \frac{176}{7}$ |
| 18a | $\{2,3,5,6,7,9,10,11,13\}$ |
| 18b | $\{2,3,5,7,11,13\}$ |
| 18c | $\{6,9,10\}$ |
| 18d | $\{4,12\}$ |
| 19 | There are 180 blue masks. |
| 20 | The larger cup is a better buy since it could have been more costly. |


| $\mathbf{2 1 a}$ | $\mathbf{P}=\left(\begin{array}{l}3 \\ 1 \\ 0\end{array}\right)$ |
| :--- | :--- |
| $\mathbf{2 1 b}$ | $\mathbf{R P}=\left(\begin{array}{l}41 \\ 17 \\ 31\end{array}\right)$ |
| $\mathbf{2 1 c}$ | Team Gratitude scores 41 points, Team Respect scores 17 points and <br> Team Compassion scores 31 points. <br> Team Gratitude is the winner. |
| $\mathbf{2 2 a}$ | $P(0,-10) \& Q(5,0)$ |
| $\mathbf{2 2 b}$ | $\sqrt{(5)^{2}+(10)^{2}}=11.2$ units |
| $\mathbf{2 3 a}$ | $067^{\circ}$ |
| $\mathbf{2 3 b}$ | Probability $=0.624$ ( to 3 sf) or 0.351 ( to 3 sf) |
| $\mathbf{2 4 a}$ | 2 to 4 students |
| $\mathbf{2 4 b}$ | interquartile range $=6$ kg |
| $\mathbf{2 4 c}$ | median $=45.5$ kg |
| $\mathbf{2 4 d}$ | The cumulative frequency curve will shift to the right by 5 kg. |
| $\mathbf{2 5 a}$ | 33 |
| $\mathbf{2 5 b}$ | $4 n+1$ |
| $\mathbf{2 5 c}$ | 51 |
| $\mathbf{2 5 d}$ | Since n is not an integer, 50 is not a term of the sequence. |

Canberra Secondary School 2021 Prelim EM P2 24

Answers

| 1a | $10 a^{2} b^{5}$ |
| :---: | :--- |
| bi | $(x-2)^{2}-10$ |
| ii | $(2,-10)$ |
| ci | 10.5 |
| ii | $n=m^{2}-2 l$ |
| d | $x=1.76 \quad$ or $\quad x=-1.42$ |
| 2ai | $\$ 2556$ |
| ii | $66 \frac{2}{3} \%$ |
| b | Plan B offers better returns with a higher interest. |
| 3ai | $x=17$ |
|  | $5+18+10+17+2=52$ |
| ii | 12 |
| iii | $x=5$ |
| bii | $\frac{1}{10}$ |
| iii | The dart will always hit one of the coloured region. |
| 4a | Mike's claim is accurate. |
| ii | $240^{\circ}$ |
| iii | $130^{\circ}$ |
| 5a | -8 |
| ci | Plot $y=-5 x-8$ |
|  | Since the line intersects the curve only at one point, there is only one solution for |
| ii | $2 x^{3}-x^{2}-5 x=-8$. |
| d | -1.9 |
| 6 ai | 3270 |
| ii | 78.4 m |
| iii | $\angle P S R=64.0^{\circ}$ |
| v | 56.4 m |
| bi | 10.6 m |
| ii | $10.6^{\circ}$ |
| 7 ai | $20^{\circ}$ |
| ii | $60^{\circ}$ |
| iii | $45^{\circ}$ |
| bi | $50.5 \mathrm{~cm}^{2}$ |
| ii | 11.3 cm |
| 8 a | $y=90-\frac{3}{2} x$ |
| b | $A=180 x-3 x^{2}$ |
| c | $x=44.14 \mathrm{~m} \quad$ or $x=15.86 \mathrm{~m}$ |


| 8 d | $x$-coordinate of turning point $=30$ <br> $y$-coordinate of turning point $=180(30)-3(30)^{2}=2700 \mathrm{~m}^{2}$ |
| :---: | :--- |
| e | Area is maximum, <br> Because the coefficient of $x^{2}$ is negative. |
| 9 a | 2415 or 2420 litres |
| bi | 61.6 cm |
| ii | 39.9 cm |
| c | 11 cans |
| 10 a | Bearing $=360-70=290^{\circ}$ |
| b | 8540 km |
| c | Time difference between Singapore and Paris $=8-2=6$ hours <br> Singapore 6 hours ahead of Paris <br> Total distance $=4150+8540=12,690$ km <br> Total time including stop over $=\frac{12690}{850}+2=16.929$ |
|  | Adjustment for time difference $=16$ hrs 56 mins 56 mins -6 hrs $=10$ hrs 56 mins <br> $11: 15$ pm $\rightarrow 12: 00$ am Saturday $(45$ minutes $)$ |
|  | $12: 00$ am $\rightarrow 10: 00$ am Sunday $(10$ hours $)$ <br> $10: 00$ am $\rightarrow 10: 11$ am Sunday $(11$ minutes $)$ <br> Reach Paris at $10: 11$ am Sunday |

## Canberra Secondary School <br> 4E/5NA O Level Mathematics Paper 1 Preliminary Examination 2021 Marking Scheme

| Question | Marking Scheme | Marks |
| :---: | :---: | :---: |
| 1 | $\begin{aligned} & \text { Greatest possible difference } \\ & \begin{array}{lll} =1.734-1.605 & \text { OR } & =1.7349-1.6050 \\ =0.129 \mathrm{~m} & & =0.130 \mathrm{~m} \end{array} \end{aligned}$ | B1 |
| 2 | The title of the line graph is biased as it does not allow reader to make judgement. <br> The vertical axis does not start from 0 , which exaggerated the differences. (Can accept without the reasoning) | B1 <br> (either one) |
| 3 | $\begin{aligned} & \left(2 x^{2}\right)^{3} \div 4 \sqrt{x} \\ & =8 x^{6} \div 4 x^{\frac{1}{2}} \\ & =2 x^{5 \frac{1}{2}} \end{aligned}$ | M1 <br> A1 |
| 4 | $\begin{aligned} & 9172.05=9 \times 10^{3}+1 \times 10^{2}+7 \times 10+2 \times 10^{0}+5 \times 10^{-2} \\ & a=0, \quad b=-2 \end{aligned}$ | B1, B1 |
| 5 | $\begin{aligned} & \frac{3}{1-x^{2}}-\frac{2}{x+1} \\ & =\frac{3}{(1-x)(1+x)}-\frac{2}{x+1} \\ & =\frac{3-2(1-x)}{1-x^{2}} \\ & =\frac{3-2+2 x}{1-x^{2}} \\ & =\frac{1+2 x}{1-x^{2}} \text { or } \frac{1+2 x}{(1+x)(1-x)} \end{aligned}$ | M1 <br> A1 |
| $6$ | $\angle D Q R=\angle C Q P$ (vertically opposite angles) <br> $D Q=C Q$ (given that ratio $D Q: D C=1: 2$ ) <br> $\angle Q D R=180^{\circ}-90^{\circ}$ (adjacent angles on straight line) $=90^{\circ}$ <br> $\angle Q D R=\angle Q C P=90^{\circ} \quad$ (property of a square) <br> $\therefore \triangle D Q R$ is congruent to $\triangle C Q P$. (ASA) (shown) | M1 A1 |
| 7 | $\begin{aligned} & 2 h^{2}-11 h-21=0 \\ & (2 h+3)(h-7)=0 \\ & 2 h+3=0 \quad \text { or } \quad h-7=0 \\ & h=-\frac{3}{2} \quad \text { or } \quad h=7 \end{aligned}$ | M1 <br> A1 |


| 8 | $\begin{aligned} & 4 \mathrm{~cm}: 2.8 \mathrm{~km} \\ & 4: 280000 \\ & 1: 70000 \\ & \therefore n=70000 \end{aligned}$ | M1 <br> A1 |
| :---: | :---: | :---: |
| 9 | $\begin{aligned} & x \leq \frac{x+4}{3} \leq 2 x-1 \\ & 3 x \leq x+4 \leq 3(2 x-1) \\ & 3 x \leq x+4 \leq 6 x-3 \\ & 3 x \leq x+4 \text { and } x+4 \leq 6 x-3 \\ & 2 x \leq 4 \quad \text { and }-5 x \leq-7 \\ & x \leq 2 \quad \text { and } x \geq 1 \frac{2}{5} \\ & \therefore 1 \frac{2}{5} \leq x \leq 2 \end{aligned}$ | M1 <br> M1 <br> A1 |
| 10 | $a_{\text {original }}=\frac{k}{x^{2}}$, where $k$ is a const $\begin{aligned} & a_{\text {now }}=\frac{k}{(0.5 x)^{2}} \\ & a_{\text {now }}=\frac{k}{0.25 x^{2}} \\ & \frac{a_{\text {now }}}{a_{\text {original }}} \frac{k}{0.25 x^{2}} \div \frac{k}{x^{2}} \\ & \frac{a_{\text {now }}}{a_{\text {original }}} \frac{1}{0.25} \\ & \frac{a_{\text {now }}}{a_{\text {original }}} \frac{4}{1} \\ & \therefore a_{\text {now }}: a_{\text {original }}=4: 1 \end{aligned}$ | M1 <br> M1 <br> A1 |
| 11 | $\begin{aligned} & \text { Base of figure }=10 \text { units } \\ & \text { Vertical height }=6 \text { units } \\ & \begin{aligned} \text { Area } & =10 \times 6 \\ & =60 \text { square units } \end{aligned} \end{aligned}$ | M1 <br> M1 <br> Al |



| 14b | $\angle P Z S=33^{\circ}$ <br> (accept angle values of $\pm 1^{\circ}$ ) | B1 |
| :---: | :---: | :---: |
| 15a | $\begin{aligned} & 5 p r-2 p s-5 q r+2 q s \\ & =p(5 r-2 s)-q(5 r-2 s) \\ & =(p-q)(5 r-2 s) \end{aligned}$ | $\begin{aligned} & \mathrm{M} 1 \\ & \mathrm{~A} 1 \end{aligned}$ |
| 15b | $\begin{aligned} & 5 p r-2 p s-5 q r+2 q s=0 \\ & (p-q)(5 r-2 s)=0 \\ & p-q=0 \quad \text { or } \quad 5 r-2 s=0 \\ & p=q \quad \text { or } \quad \frac{r}{s}=\frac{2}{5} \\ & \text { (reject) } \end{aligned}$ | M1 <br> A1 |
| 16a | Smallest $x=1$ <br> Smallest $y=1$ | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \end{aligned}$ |
| 16 b | $\begin{aligned} & 33 q \\ & =3 \times 11 \times 3 \times 7^{2} \times 11 \\ & =3^{2} \times 7^{2} \times 11^{2} \end{aligned}$ <br> Since the indices of the prime factors are multiples of $2,33 q$ is a perfect square. | M1 $\mathrm{Al}$ |
| 17ai | $5.5 \mathrm{~m} / \mathrm{s}^{2}$ | B1 |
| 17aii | 11 m | B1 |
| 17b | $\begin{aligned} & \frac{1}{2} \times 7 \times v=\frac{1}{2} \times 2 \times 11+7 \times 11 \\ & 3.5 v=88 \\ & v=25.1 \quad \text { or } 25 \frac{1}{7} \quad \text { or } \quad \frac{176}{7} \end{aligned}$ | M1 <br> A1 |
| 18a | $\{2,3,5,6,7,9,10,11,13\}$ | B1 |
| 18b | $\{2,3,5,7,11,13\}$ | B1 |
| 18c | $\{6,9,10\}$ | B1 |
| 18d | \{4, 12 \} | B1 |
| 19 | $\begin{aligned} & 3: 4: 5 \rightarrow 9: 10: 11 \\ & 7.5: 10: 12.5 \\ & 1.5 \text { units } \rightarrow 30 \text { masks } \\ & 1 \text { unit } \rightarrow 20 \text { masks } \\ & 9 \text { units } \rightarrow 180 \text { masks } \end{aligned}$ <br> There are 180 blue masks. | M1 <br> M1 <br> M1 <br> A1 |
| 20 | $\begin{aligned} & \left(\frac{6.5}{5}\right)^{3}=\frac{x}{\$ 1} \\ & x=\$ 2.197 \end{aligned}$ <br> The larger cup is a better buy since it could have been more costly. | $\begin{gathered} \text { M1 } \\ \text { M1 } \\ \text { M1, A1 } \end{gathered}$ |

\begin{tabular}{|c|c|c|}
\hline 21a \& $\mathbf{P}=\left(\begin{array}{l}3 \\ 1 \\ 0\end{array}\right)$ \& B1 <br>
\hline 21b \& $$
\begin{aligned}
\mathbf{R} \mathbf{P} & =\left(\begin{array}{ccc}
12 & 5 & 3 \\
3 & 8 & 7 \\
9 & 4 & 4
\end{array}\right)\left(\begin{array}{l}
3 \\
1 \\
0
\end{array}\right) \\
& =\left(\begin{array}{l}
41 \\
17 \\
31
\end{array}\right)
\end{aligned}
$$ \& M1

A1 <br>

\hline 21e \& | Team Gratitude scores 41 points, Team Respect scores 17 points and Team Compassion scores 31 points. |
| :--- |
| Team Gratitude is the winner. | \& B1 <br>

\hline 22a \& $$
\begin{aligned}
& P(0,-10) \\
& Q(5,0)
\end{aligned}
$$ \& B1

B1 <br>

\hline 22b \& $$
\begin{aligned}
& \sqrt{(5)^{2}+(10)^{2}} \\
& =11.2 \text { units }
\end{aligned}
$$ \& M1

A1 <br>
\hline 23a \& $067^{\circ}$ \& B1 <br>
\hline 23b \&  \& M1

M1 <br>
\hline
\end{tabular}

|  | $\begin{aligned} & \begin{aligned} \text { Area of sector } & =\frac{48.2 \times 2}{360} \times \pi(6)^{2} \\ & =30.2784 \mathrm{~m}^{2} \end{aligned} \\ & \begin{aligned} \text { Area of segment } \mathrm{B} & =\frac{83.6}{360} \times \pi(4.5)^{2}-\frac{1}{2} \times(4.5)^{2} \sin 83.6^{\circ} \\ & =4.7114 \mathrm{~m}^{2} \end{aligned} \\ & \begin{aligned} & \text { Probability }= \frac{30.2784+2(4.7114)}{(4.5)^{2} \pi} \text { or } \begin{aligned} & \frac{30.2784+2(4.7114)}{(6)^{2} \pi} \\ & =0.624(\text { to } 3 \mathrm{sf}) \end{aligned} \\ &=0.351(\text { to } 3 \mathrm{sf} \end{aligned} \end{aligned}$ | A1 |
| :---: | :---: | :---: |
| 24a | 2 or 4 students | B1 |
| 24b | $\begin{aligned} \text { interquartile range } & =48.5-42.5 \\ & =6 \mathrm{~kg} \end{aligned}$ | $\begin{aligned} & \mathrm{M} 1 \\ & \mathrm{~A} 1 \end{aligned}$ |
| 24 c | median $=45.5 \mathrm{~kg}$ | B1 |
| 24d | The cumulative frequency curve will shift to the right by 5 kg . | B2 |
| 25a | 33 le | B1 |
| 25b | $4 n+1$ | B2 |
| 25c | 51 | B1 |
| 25d | $\begin{aligned} & 4 n+1=50 \\ & 4 n=49 \\ & n=49 \div 4 \end{aligned}$ | M1 |
|  | Since n is not an integer, 50 is not a term of the sequence. | A1 |

