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Group size and cooperation among strangers☆

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ABSTRACT

We study how group size affects cooperation in an infinitely repeated n-player Prisoner's Dilemma (PD) game. In each repetition of the game, groups of size $n \leq M$ are randomly and anonymously matched from a fixed population of size M to play the n-player PD stage game. We provide conditions for which the contagious strategy (Kandori, 1992) sustains a social norm of cooperation among all M players. Our main finding is that if agents are sufficiently patient, a social norm of society-wide cooperation becomes easier to sustain under the contagious strategy as n increases toward M. In an experiment where the population size M is fixed and conditions identified by our theoretical analysis hold, we find strong evidence that cooperation rates are higher with larger group sizes than with smaller group sizes in treatments where each subject interacts with $M - 1$ robot players who follow the contagious strategy. When the number of human subjects increases in the population, the cooperation rates decrease significantly, indicating that it is the strategic uncertainty among the human subjects that hinders cooperation.

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1. Introduction

What choice of group size maximizes (or minimizes) the possibility of achieving a social norm of cooperation in a finite population of self-interested strangers? This question would seem to be of some importance for the design of ad hoc committees, juries and teams. It is also of interest to experimentalists interested in understanding how the extent of pro-social behavior might depend on the matching group size of subject participants. In this paper we offer an answer to this question. Specifically, we consider a population of players of fixed size M. In every period, $t = 1, 2, \ldots, \infty$, players in this population are

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randomly and anonymously matched to form groups of size n and then play an n-person Prisoner's Dilemma game with all the members of their group. The total number of groups, M/n, is assumed to be an integer (i.e., M is a multiple of n).

The $n=2$ person version of this environment has been previously studied by Kandori (1992), who shows that a social norm of cooperation among anonymous, randomly matched players is sustainable under certain conditions on the game. Kandori further shows that a social norm of cooperation among strangers in the $n=2$ case becomes more difficult to sustain as M gets large and the possibility vanishes in the limit as $M \to \infty$. By contrast, in this paper we fix M and ask: for what value(s) of $n>2$ is a social norm of cooperation among strangers easiest to achieve? In other words, is there an optimal group size for maximizing the likelihood of cooperative outcomes?

Our answer is that under certain conditions – specifically if agents are sufficiently patient – a social norm of cooperation among strangers, which is sustained by universal play of a "contagious" trigger strategy, becomes steadily easier to achieve as n gets larger, and becomes easiest to achieve when $n=M$. That is, we find that cooperation can be easiest to sustain when the group size is as large as possible. This seemingly counterintuitive finding readily follows from the rational-choice logic of the contagious trigger strategy that is used to support cooperation among randomly matched, non-communicative and anonymous "strangers." Intuitively, if agents are sufficiently patient, then the costs of igniting a contagion toward mutual defection are greatest when the matching group size, n, equals the population size, M. On the other hand, once a defection has started in the community, the benefits to slowing down the contagious process are also minimized in this same case where $n=M$. Therefore, the players' incentives to follow the contagious strategy are easiest to satisfy when the group size is as large as possible.

Our main finding is consistent with Kandori's (1992) result. While we fix M and show that a social norm of cooperation is easier to achieve as n increases toward M, Kandori's result can be viewed as showing that for the case where n is fixed at 2, cooperation is easier to sustain for a smaller M. Still, our findings serve to generalize Kandori's (1992) extension of the folk theorem for repeated games with random, anonymous matchings to the multiple-player ($n>2$) Prisoner's Dilemma game. The n-player version of the Prisoner's Dilemma game is widely used to model a variety of *social* dilemmas including, e.g., the tragedy of the commons (Hardin, 1968). In addition, we show that our monotonicity result holds in an n-player binary public good game.

We also provide an empirical test of our main theoretical results by designing and implementing an experiment. In this experiment, we fix the population size, M, and the discount factor, δ, and study play of an indefinitely repeated game in which players from the population are randomly and anonymously matched in each repetition to play an n-player version of the Prisoner's Dilemma stage game. Within the population of size M, some fraction of players are robot players programmed to play according to the contagious strategy while the remaining fraction of players are human subjects and this ratio is public knowledge. In this setting we find strong evidence that, consistent with our theoretical predictions, cooperation rates are higher with larger matching groups, e.g., of size $n=6$ as compared with smaller matching groups, e.g., of size $n=2$ as subjects learn, with experience, the more immediate consequences of triggering an infectious wave of defection when the group size is larger. We show further how differences in cooperation rates between different group sizes vary in ways that reflect the predictions of our theory as we vary the payoff incentives of the game as well. Finally we show how our theoretical results find the strongest support when we eliminate strategic uncertainty by having subjects interact only with robot players.

Our paper contributes to the theoretical and experimental literature on sustaining cooperation among anonymous, randomly matched players. While this is an admittedly stark environment, it is an important benchmark case in both the theoretical and experimental literature and one that naturally characterizes many types of socio-economic interactions.[1] In addition to the original seminal paper by Kandori (1992), Ellison (1994) and Dal Bó (2007) provide further generalizations of how a social norm of cooperation may be sustained among anonymous, randomly matched players in 2-player Prisoner's Dilemma games. Xie and Lee (2012) extend Kandori's result to 2-player "trust" games under anonymous random matchings. Camera and Gioffre (2014) offers a tractable analysis of the contagious equilibria by characterizing a key statistic of contagious punishment processes and deriving closed-form expressions for continuation payoffs off the equilibrium path. Experimentally, Duffy and Ochs (2009) report on an experiment that examines play in an indefinitely repeated, two-player Prisoner's Dilemma game and find that a cooperative norm does not emerge in the treatments with anonymous random matching but does emerge under fixed pairings as players gain more experience. Camera and Casari (2009) examine cooperation under random matching by focusing on the role of private or public monitoring of the anonymous (or non-anonymous) players' choices. They find that such monitoring can lead to a significant increase in the frequency of cooperation relative to the case of no monitoring. Duffy et al. (2013) test the contagious equilibrium in the lab using trust games and find that information on past play significantly increases the level of trust and reciprocity under random matchings. Camera et al. (2012) report wide heterogeneity in strategies employed at the individual level in an experiment in which anonymous randomly matched subjects play the Prisoner's Dilemma game in sequences of indefinite duration. Compared with this previous literature, our paper is the first to theoretically and experimentally extend the analysis of the contagious equilibrium from a 2-player stage game to an n-player stage game. Our main theoretical finding, that a cooperative social norm is easier to sustain with a larger rather than a smaller group size, is new to the literature but finds support both in our own experiment

[1] There is also an experimental literature that studies cooperation in repeated Prisoner's Dilemma games of indefinite duration among fixed pairs of players (partners) e.g., Dal Bó (2005, 2007), Aoyagi and Fréchette (2009), Dal Bó and Fréchette (2011), Fudenberg et al. (2012). Engle-Warnick and Slonim (2006) examines a trust game of indefinite duration with fixed pairs.

and qualitatively in the previous literature as well, if one considers a large group size to be a partial substitute for public monitoring or fixed matching.[2]

There are several experimental papers that study the consequences of group size for contributions to a public good using the liner voluntary contribution mechanism (VCM). Isaac and Walker (1988) and Isaac et al. (1994) examine how groups of size 4, 10, 40 and 100 play a repeated public good game. One of their main findings is that, holding the marginal per capita return (MPCR) to the public good constant, an increase in the number of players, n, leads to no change or an increase (depending on the MPCR) in the mean percentage of each player's fixed and common endowment that is contributed toward the public good, and this effect is strongest with group sizes of 40 and 100 in comparison with group sizes of 4 and 10. Carpenter (2007) studies contributions under the VCM when players are randomly matched into groups of size 5 or 10, and he allows participants to monitor and punish each other following each repetition. In this setting, he also finds that contributions are higher with a larger group size. Xu et al. (2013) examine the effectiveness of an individual-punishment mechanism in larger groups of 40 participants compared with smaller groups of size four. They find that the individual punishment mechanism is effective when the MPCR is constant but not when the marginal group return (MGR) is held constant (in which case the MPCR is decreasing). Similarly, Nosenzo et al. (2015) find that it is more difficult to sustain cooperation in larger groups with high MPCR, however, with a low MPCR, considerations of the social benefits may dominate the negative effect of group size. Weimann et al. (2014) report that first-round contributions to a public good increases with the MPCR distance, which is defined as the difference between the actual MPCR and the minimal MPCR necessary to create a social dilemma for a given group size. They further demonstrate that small groups behave similar to large groups when the MPCR distance is controlled.

There are several important differences between our experimental results and linear VCM games (public good games). First, in our model and experimental design, we impose a payoff normalization under which players' payoffs are always equal to 1 when full cooperation is achieved, regardless of the group size. Equivalently, this corresponds to a public goods experiment where the MPCR decreases as the group size increases. Therefore, our monotonicity result and the evidence from our experiment show that cooperation increases with group size, even as the MPCR is decreasing, a result that we believe is new to this literature. Second, in our treatments where human subjects only interact with robot players, cooperation does not rely on any other-regarding preferences, which is an important explanatory variable in the experimental public goods literature. Third, the game we have implemented has several differences from the public good game: (1) the strategy space is continuous under the VCM and not binary as in the n-player Prisoner's Dilemma game that we study; (2) subjects in many of these public goods experiments (Carpenter (2007) being an exception) are in *fixed* matches of size n for all repetitions of the public good game whereas in our setup players are randomly and anonymously matched into groups of size n in each repetition of the game; and (3) perhaps most importantly, in all of these public good game experiments, the game is *finitely*-repeated so that in theory, positive public good provision ("cooperation") or efficiency is not possible according to standard backward induction arguments if players have the usual self-regarding preferences. By contrast, we study infinitely repeated, binary choice n-player prisoner's dilemma games where cooperation and efficiency are theoretically possible even if players are randomly and anonymously matched and have purely self-regarding preferences. We are not aware of any prior experimental study of indefinitely repeated n-player Prisoner's Dilemma games under anonymous random matching and our use of programmed robot players in this setting is also new to the literature.

The rest of the paper is organized as follows. Section 2 presents our model and Section 3 presents our main theoretical results on the consequences of group size for the sustainability of social norms of cooperation among anonymous and randomly matched strangers. Section 4 shows how our framework maps into the classic public good game of Isaac and Walker (1988). Section 5 describes the experimental design and Section 6 reports on the findings of an experiment testing our main theoretical results. Finally, Section 7 concludes with a brief summary and some suggestions for future research.

2. The model

Consider a finite population of M players. Time is discrete, the horizon is infinite and all players have a common period discount factor, $\delta \in [0, 1]$. In each period, the M players are randomly and anonymously matched into m groups of size $n \leq M$, with all matchings being equally likely. We assume that M is a multiple, m, of n. The randomly matched group members then simultaneously and without communication play an n-player Prisoner's Dilemma game where each player chooses a strategy from the set $\{C, D\}$, with C representing cooperation and D representing defection. Let i denote the number of members of the group choosing to cooperate (i.e., the number of "cooperators") other than the representative player himself so that $0 \leq i \leq n-1$. Let C_i and D_i denote the payoffs to cooperation and defection, respectively, when there are i cooperators. An n-player Prisoner's Dilemma game is defined by the following three assumptions regarding these payoffs:

A1: $D_i > C_i$ for $0 \leq i \leq n-1$.
A2: $C_{i+1} > C_i$ and $D_{i+1} > D_i$ for $0 \leq i < n-1$.

[2] We note that when the group size, n, is set equal to the largest possible value, the population size M, then our model converges to one of perfect public monitoring and fixed matching. Thus for group sizes less than M, one can view larger group sizes as being closer approximations to perfect public monitoring and fixed matchings.

Table 1
The payoff matrix of the n-player Prisoner's Dilemma game.

Number of cooperators in the group	0	1	2	...	$n-1$
C	C_0	C_1	C_2	...	C_{n-1}
D	D_0	D_1	D_2	...	D_{n-1}

A3: $C_{n-1} > D_0$.

Assumption A1 says that defection is always a dominant strategy. Assumption A2 says that payoffs are increasing with the number of cooperators. Finally, assumption A3 says that if all participants adopt the dominant strategy, the outcome is sub-optimal relative to the mutual cooperation outcome. These conditions are standard in the literature on n-person Prisoner's Dilemma games (see, e.g., Okada, 1993, Assumption 2.1). We further suppose that the payoff matrix is symmetric for each player in the group and is as given in Table 1.

We next define the "contagious strategy" following Kandori (1992) and show that a social norm of cooperation can be sustained as a sequential equilibrium if all players adopt this strategy. Define a player as a "c-type" if in all previous repetitions of the game this player and all of the other $n-1$ group members with whom he has interacted in all prior periods have never chosen D, i.e., the outcome of the stage game played in every prior period has been cooperation, C, by every group member the player has encountered. Otherwise, the player is a "d-type" player. (Note that the presence of c-type players in any period does not preclude the presence of d-type players in the same period among the population (or "community") of players of size $M \geq n$). The "contagious strategy" can now be defined as follows: A player chooses C if he is c-type and chooses D if he is d-type.

We next provide a set of sufficient conditions that sustains the contagious strategy as a sequential equilibrium when the group size is n. We first introduce some notation. Let X_t denote the number of d-type players at time t. Define $A_n = (a_{ij}^n)$ to be an $M \times M$ transition probability matrix where $a_{ij}^n = Pr(X_{t+1} = j|X_t = i$ and all players follow the contagious strategy) given group size n. Define $B_n = (b_{ij}^n)$ as an $M \times M$ transition probability matrix where $b_{ij}^n = Pr(X_{t+1} = j|X_t = i$ and one d-type player deviates to playing C while all other players follow the contagious strategy) given group size n. Let $H_n = B_n - A_n$, which indicates how the diffusion of defection is delayed by the unilateral deviation of one of the d-type players. Define $Z_n = (\rho_0^n, \rho_1^n, \ldots, \rho_{n-1}^n)$, where $\rho_0^n, \rho_1^n, \ldots, \rho_{n-1}^n$ are $M \times 1$ vectors such that the ith element of ρ_j^n is the conditional probability that a d-type player meets j c-type players in the group when there are i d-type players in the community given that the group size is n (i.e., $Z_n = (z_{ij}^n)$ is an $M \times n$ matrix where $z_{ij}^n = Pr($a d-type player meets $j-1$ c-type players in his group in period $t|X_t = i)$ given a group size of n). Define e_i as a $1 \times M$ vector whose ith element is 1 and with zeros everywhere else. Finally, define column vectors $v_n = (D_0, D_1, \ldots, D_{n-1})^T$ and $u_n = (C_0, C_1, \ldots, C_{n-1})^T$, whose ith element is the payoff for a player from choosing D and C respectively, given that there are $i-1$ other players in the group who choose C.

Next we show that a one-shot deviation from the contagious strategy is unprofitable after any history. On the equilibrium path, a one-shot deviation is unprofitable if

$$\frac{C_{n-1}}{1-\delta} \geq \sum_{t=0}^{\infty} \delta^t e_1 A_n^t Z_n v_n. \tag{1}$$

The left hand side of (1) is the payoff from cooperating forever and the right-hand side of (1) is the payoff that the player earns if the player initiates a defection and defects forever afterward. Off the equilibrium path, following Kandori (1992), we identify a sufficient condition for a one-shot deviation to be unprofitable under any consistent beliefs. Suppose there are k d-type players, where $k = n, n+1, \ldots, M$.[3] Then a one-shot deviation off the equilibrium path is unprofitable if

$$\sum_{t=0}^{\infty} \delta^t e_k A_n^t Z_n v_n \geq e_k Z_n u_n + \delta \sum_{t=0}^{\infty} \delta^t e_k B_n A_n^t Z_n v_n. \tag{2}$$

The left hand side of (2) is the payoff that a d-type player earns from playing D forever when there are k d-type players including the player himself, while the right hand side of (2) is what a d-type player receives when he deviates from the contagious strategy, playing C today and then reverting back to playing D forever after. Inequalities (1) and (2) can be manipulated into equilibrium conditions 1 and 2 in the following lemma.

Lemma 1. *The contagious strategy constitutes a sequential equilibrium if the following two conditions are satisfied:*

$$\textit{Equilibrium Condition 1}: \quad C_{n-1} \geq (1-\delta) e_1 (I - \delta A_n)^{-1} Z_n v_n,$$

$$\textit{Equilibrium Condition 2}: \quad e_k Z_n (v_n - u_n) \geq \delta e_k H_n (I - \delta A_n)^{-1} Z_n v_n.$$

[3] Since the player under consideration is a d-type, there must be at least n d-type players in the community.

Table 2
The simpler payoff matrix for the n-player prisoner's dilemma game.

Number of cooperators in the group	0	1	2	...	$n-1$
C	0	β_n	$2\beta_n$	...	$(n-1)\beta_n$
D	α	$\alpha+\beta_n$	$\alpha+2\beta_n$	...	$\alpha+(n-1)\beta_n$

The intuition behind equilibrium conditions 1 and 2 is similar to that for the $n=2$ case studied by Kandori (1992). When a player is on the equilibrium path, he has no incentive to deviate from cooperation when δ is sufficiently large. When a player is off the equilibrium path, he has no incentive to deviate from continued play of the contagious strategy if the extra payoff from defection in the current period, $v_n - u_n$, is large enough. Using Lemma 1 we can prove the following theorem.

Theorem 1. *Under uniformly random matching, the contagious strategy described above constitutes a sequential equilibrium strategy for any finite population size, M, if δ, $C_{n-1} - D_o$, and all components of $v_n - u_n$ are sufficiently large.*

Proof. See Appendix A.

3. A monotonicity result

In this section we ask the following question: fixing the population size at M, which group size $n \le M$ maximizes the possibility of achieving a social norm of cooperation among strangers?[4] Although we can characterize the equilibrium conditions for the contagious strategy, we cannot derive closed-form solutions since the formulas for the elements of the transition matrices A and B become too complicated to derive for group sizes $n > 2$.[5] Therefore, in this section we switch to the use of numerical methods.[6]

Furthermore, for greater tractability, we focus on a simple symmetric specification for the payoff parameters that satisfy assumptions A1–A3. Specifically, we impose the following additional assumptions[7]:

A4: $C_0 = 0$.
A5: $D_i - C_i = \alpha$ for $0 \le i \le n-1$.
A6: $C_{i+1} - C_i = D_{i+1} - D_i = \beta_n$ for $0 \le i < n-1$.

Under these assumptions, the payoff matrix (Table 1) now takes on the specific form shown in Table 2. Finally, we note that under our parameterization it may be easier to achieve full cooperation with a larger group size since the payoff from full cooperation, $(n-1)\beta_n$, may grow with the group size, n. To properly correct for this dependency, we also normalize the payoff matrix in such a way that the payoff from full cooperation is fixed and constant by the following assumption:

A7: $(n-1)\beta_n = 1$ for any n.

Note that under this normalization, to satisfy assumption A3, we must have $\alpha < C_{n-1} = 1$ for all $n \ge 2$.

In order to examine the question raised above, we fix $M=12$ and examine changes in the two equilibrium conditions as the group size takes on the values $n = 2, 3, 4, 6, 12$. We find that as n increases toward M both the on-equilibrium and off-equilibrium conditions become easier to satisfy, which we refer as the *Monotonicity Result*.

3.1. Equilibrium condition 1

We first examine the effect of increases in the group size, n, on equilibrium condition 1. Although we are mainly interested in the case where payoffs are normalized to eliminate the dependency on n, for the moment we keep payoffs for equilibrium condition 1 in their original unnormalized form (i.e., $C_{n-1} = (n-1)\beta_n$), so that we can derive some intuition on the discounted summation of the probability of earning each payoff outcome.

[4] In Appendix B, we also ask how the answer to this question changes if instead of fixing M, we vary both M and n but in such a way that the number of groups, $m = M/n$, is held constant. We show there that we obtain similar results with respect to satisfaction of the equilibrium conditions needed for cooperation to be sustained as a social norm, if we vary n but hold the number of groups constant.
[5] Kandori (1989) provides transition matrix formulas for the $n=2$ case only.
[6] The Mathematica program used for our numerical results is available upon request.
[7] A slightly different normalization, for instance, $D_0 = 0$, $D_i = i\beta$, $C_i = D_i - \alpha$, $C_{n-1} = 1$, gives similar results.

Given a group of size n, we can write equilibrium condition 1 as:

$$\begin{aligned}(n-1)\beta_n &\geq p_0^n\alpha + p_1^n(\alpha+\beta_n) + \cdots + p_{n-1}^n(\alpha+(n-1)\beta_n)\\ &= \left(\sum_{j=0}^{n-1} p_j^n\right)\alpha + \sum_{j=1}^{n-1} j p_j^n \beta_n,\end{aligned}$$

where $p_j^n \equiv (1-\delta)e_1(I-\delta A_n)^{-1}\rho_j^n$ denotes the discounted summation of the probability of meeting j cooperators (c-types) in a group of size n once a player has initiated a defection. Then given that $\sum_{j=0}^{n-1} p_j^n = 1$ as shown in Lemma 2 in Appendix A and the normalization assumption A7, $(n-1)\beta_n = 1$, equilibrium condition 1 becomes

$$\alpha \leq p^n, \tag{3}$$

where

$$p^n \equiv 1 - \frac{\sum_{j=1}^{n-1} j p_j^n}{n-1}.$$

Eq. (3) says that the net payoff from defection (which is α) must be less than or equal to the net loss from initiating a defection (which is p^n).

Proposition 1. *If p^n is increasing in n, then Condition (3) (equilibrium condition 1) is monotonically less restrictive as the group size n increases.*

3.2. Equilibrium condition 2

We next examine the effects of increases in the group size, n, on equilibrium condition 2. Given our payoff specification that $D_i - C_i = \alpha$ for $i = 0, 1, \ldots, n-1$, the left hand side of equilibrium condition 2, representing the extra payoff from defection, is equal to α. The right hand side of equilibrium condition 2, the payoff to a d-type player from slowing down the contagious process, achieves its highest value when the number of d-type players are at a minimum, i.e., when $k = n$. Thus it is sufficient to compare equilibrium condition 2 at $k = n$ for different group sizes, $n = 2, 3, 4, 6, 12$. Similar to equilibrium condition 1, we first present equilibrium condition 2 with the original payoff parameters and then we impose our normalization later.

Given a group of size n, we can write equilibrium condition 2 as:

$$\begin{aligned}\alpha &\geq q_0^n\alpha + q_1^n(\alpha+\beta_n) + \cdots + q_{n-1}^n(\alpha+(n-1)\beta_n)\\ &= \left(\sum_{j=0}^{n-1} q_j^n\right)\alpha + \sum_{j=1}^{n-1} j q_j^n \beta_n,\end{aligned}$$

where $q_j^n \equiv \delta e_n H_n (I-\delta A_n)^{-1}\rho_j^n$ denotes the change in the discounted summation of the probability of meeting j c-type players in the group when the d-type player reverts back to playing cooperation instead of defection given that the group size is n and there are $k = n$ d-type players in the population. Given that $\sum_{j=0}^{n-1} q_j^n = 0$ as shown in Lemma 2 and the normalization assumption A7, $(n-1)\beta_n = 1$, equilibrium condition 2 becomes

$$\alpha \geq q^n, \tag{4}$$

where

$$q^n \equiv \frac{\sum_{j=1}^{n-1} j q_j^n}{n-1}.$$

Eq. (4) says that the net payoff from continuing a defection (which is α) must be greater than or equal to the net benefit from slowing down a contagious defection (which is q^n).

Proposition 2. *If q^n is decreasing in n, then Condition (4) (equilibrium condition 2) is monotonically less restrictive as the group size n increases.*

3.3. Numerical findings for different values of δ

Propositions 1 and 2 require that p^n is increasing and q^n is decreasing in n so that the sufficient equilibrium conditions for the contagious strategy to sustain a social norm of cooperation among strangers becomes monotonically less restrictive

Table 3
Numerical results on p^n and q^n for different n and δ ($M = 12$).

δ	0.01	0.1	0.3	0.5	0.7	0.9	0.99
p^n for given n and δ							
$n=2$	0.000925	0.011028	0.051716	0.141773	0.331384	0.704922	0.966359
$n=3$	0.001859	0.022562	0.100818	0.239648	0.460745	0.788192	0.977226
$n=4$	0.002785	0.033155	0.137519	0.298782	0.524404	0.821914	0.981270
$n=6$	0.004600	0.050899	0.185380	0.363489	0.585281	0.850813	0.984588
$n=12$	0.010000	0.100000	0.300000	0.500000	0.700000	0.900000	0.990000
q^n for given n and δ							
$n=2$	0.000839	0.009584	0.039064	0.088720	0.168391	0.290770	0.363919
$n=3$	0.001195	0.012416	0.040398	0.072725	0.109571	0.151112	0.171385
$n=4$	0.000926	0.009293	0.028096	0.047192	0.066581	0.086265	0.095219
$n=6$	0.000059	0.000590	0.001771	0.002952	0.004132	0.005313	0.005844
$n=12$	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

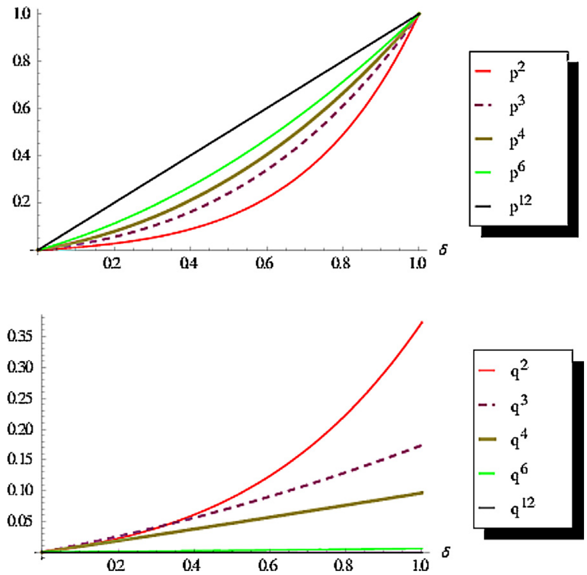

Fig. 1. p^n and q^n as functions of δ ($M = 12$).

as the group size, n, increases. We next ask whether these conditions hold by computing p^n and q^n for different group sizes, n, and for different discount factors, δ, all under a fixed $M = 12$.[8]

Our numerical exercises on p^n and q^n illustrate some interesting results. Given any group size n, Table 3 reveals that p^n and q^n increase with δ. Therefore, if a player cares more about the future, then the extra loss from initiating a contagious wave of defection and the extra benefit from slowing down a contagion both become larger.

Next we ask: given a fixed δ, how do p^n and q^n change with increases in the group size n? First, in all cases, the contagious equilibrium always exists, i.e., the numerical value in each cell for p^n is always larger than the value in the corresponding cell for q^n. Thus by choosing any α between q^n and p^n both equilibrium conditions 1 and 2 hold. Furthermore, we find that p^n is monotonically increasing in n given any δ, and q^n is monotonically decreasing in n if δ is sufficiently large enough (greater than 0.5). Given the numerical results in Table 3, we conjecture that there exists a threshold value for the discount factor, $\bar{\delta}$ such that, for any $\delta > \bar{\delta}$, p^n is monotonically increasing in n and q^n is monotonically decreasing in n. This observation is verified in Fig. 1.

Intuitively, with a larger group size, an initial defection spreads to more "innocent" (c-type) players. Furthermore, via the random re-matching each period, defection spreads to the entire population of M players much faster since there are fewer groups given the fixed population size, M, and a larger group size, n. These two effects together imply that the contagious process is faster with a larger group size n and thus the payoff from starting a defection is reduced (i.e., the net loss p^n from starting a defection is increased), making the condition on the equilibrium path easier to satisfy. Off the equilibrium path, condition 2 becoming less restrictive with a larger group size is also due to the faster contagious process associated with a larger group size. When the speed of contagion is faster, the effectiveness of a single d-type player slowing down

[8] Notice that p^n and q^n are both functions of δ, n and M.

Table 4
The payoff matrix for the n-player public goods game.

Number of contributors in the group	0	1	2	...	$n-1$
C (invest in the public account)	μ_n	$2\mu_n$	$3\mu_n$	...	$n\mu_n$
D (invest in the private account)	$\gamma+\mu_n$	$\gamma+2\mu_n$	$\gamma+3\mu_n$	...	$\gamma+n\mu_n$

the contagious process becomes smaller. Thus, the d-type player has less of an incentive to deviate from the contagious strategy off the equilibrium path by reverting back to playing cooperation again (i.e., the net benefit from slowing down the contagion q^n monotonically decreases).

Summarizing, our main finding is that, for a fixed population, M, and a sufficiently high δ, the conditions under which the contagious strategy sustains play of the cooperative strategy in a n-player Prisoner's Dilemma game by all anonymously and randomly matched players in each period are monotonically more easily satisfied as the group size, n increases toward M.

4. An application to a public goods game

In this section we show that with a slightly different normalization of the payoff matrix, the n-person Prisoner's Dilemma game can be re-interpreted as a public goods game so that our previous monotonicity result continues to hold in this public goods game version of the stage game.

As before, we assume a population of M players, who are anonymously and randomly assigned to groups of size n in each period to play an n-player public goods game. Here we study a binary choice version of the classic public good game (Isaac and Walker, 1988) where each player is endowed with a single token and must decide whether or not to invest that token in his own privately held account or in a public account. Each token invested in the public account yields a payoff of μ_n for each group member. A token invested in the private account yields an additional payoff of γ, but only to the player associated with that private account. Table 4 represents the payoff matrix for the player from choosing to invest in the public account (C) or in the private account (D) given the number of other contributors to the public account in the group of size n. The standard public good game setup has $\mu_n > 0$ and $\gamma > 0$, so that non-contribution to the public good is always a dominant strategy in the one-shot, n-player game, and further that $\gamma + \mu_n < n\mu_n$, which implies that the social optimum is achieved when all n players contribute to the public good. Notice that these restrictions also satisfy assumptions A1–A3, as defined in Section 2 for an n-player Prisoner's Dilemma game.

When this public goods game serves as the stage game played by a population of M players, who are randomly divided up into groups of size n in every period, the sufficient conditions to sustain the contagious equilibrium are very similar to those shown before. On the equilibrium path we must have:

$$n\mu \geq p_0^n(\gamma + \mu_n) + p_1^n(\gamma + 2\mu_n) + \cdots + p_{n-1}^n(\gamma + n\mu_n),$$

while off the equilibrium path we require that:

$$\gamma \geq q_0^n(\gamma + \mu_n) + q_1^n(\gamma + 2\mu_n) + \cdots + q_{n-1}^n(\gamma + n\mu_n).$$

Define

$$\tilde{p^n} \equiv \frac{n-1}{n}p^n = \frac{n-1}{n} - \frac{\sum_{j=1}^{n-1} jp_j^n}{n}$$

and

$$\tilde{q^n} \equiv \frac{n-1}{n}q^n = \frac{\sum_{j=1}^{n-1} jq_j^n}{n}.$$

Then with the normalization that $\mu_n = 1/n$, equilibrium condition 1 becomes

$$\gamma \leq \tilde{p^n}$$

and equilibrium condition 2 becomes

$$\gamma \geq \tilde{q^n}.$$

Based on the previous numerical results (Table 3), it is easy to show that the monotonicity pattern still holds for the public goods game when δ is sufficiently large, with the threshold value for $\bar{\delta}$ slightly increased.

5. Experimental design

In the next two sections we report on an experiment that tests the monotonicity results of Propositions 1 and 2. For this purpose we return to the simplified parameterization of the model used in Section 3. In our experimental design, we always

Table 5
The payoff matrices for the 2-player and 6-player Prisoner's Dilemma game.

Payoff matrix for $n = 2$

The other player's choice	D	C
C	0	1
D	α	$\alpha + 1$

Payoff matrix for $n = 6$

Number of others playing C	0	1	2	3	4	5
C	0	0.2	0.4	0.6	0.8	1
D	α	$\alpha + 0.2$	$\alpha + 0.4$	$\alpha + 0.6$	$\alpha + 0.8$	$\alpha + 1$

Table 6
Summary of treatments.

Treatment	Session	Group size	α	Players	On-equm	Off-equm	Cooperation?
1H11R_A	1	$n=6$	$\alpha=0.5$	1 HS 11 robots	$\alpha \le 0.65$	$\alpha \ge 0.004$	Yes
1H11R_B	2	$n=2$	$\alpha=0.5$	1 HS 11 robots	$\alpha \le 0.40$	$\alpha \ge 0.19$	No
1H11R_C	3	$n=2$	$\alpha=0.3$	1 HS 11 robots	$\alpha \le 0.40$	$\alpha \ge 0.19$	Yes
1H11R_D	4	$n=3$	$\alpha=0.6$	1 HS 11 robots	$\alpha \le 0.53$	$\alpha \ge 0.12$	No
1H11R_E	5	$n=6$	$\alpha=0.6$	1 HS 11 robots	$\alpha \le 0.65$	$\alpha \ge 0.004$	Yes
6H6R_A	6, 7	$n=6$	$\alpha=0.5$	6 HS 6 robots	$\alpha \le 0.65$	$\alpha \ge 0.004$	Yes
2H10R_A	8	$n=6$	$\alpha=0.5$	2 HS 10 robots	$\alpha \le 0.65$	$\alpha \ge 0.004$	Yes

consider communities of size $M = 12$ and a discount factor $\delta = 0.75$. Our main two treatment variables are the group size, n and the value of α. With the normalized payoff in Table 2, $\beta_n = \frac{1}{n-1}$ for different groups of size n. For instance, the stage game payoff for the 2-person PD and the 6-person PD are shown in Table 5.

A third treatment variable in our experimental design is the number of human subjects in each 12-player community. We start with the treatments in which each 12-player community consists of just one human subject who interacts with 11 other "robot" players as opposed to allowing 12 human subjects to interact with one another. We employ this design in order to avoid the coordination problem of strategy selection among 12 human players and thereby remove strategic uncertainty. As with other folk-theorem type results, the contagious equilibrium is not the unique equilibrium of the infinitely repeated n-player PD game that we implement in our experiment. There exist many other non-cooperative equilibria including the one where all players choose to defect in every round of the supergame. Empirically, when players face both the selection of their own strategy and the uncertainty of strategy selection by other players, the outcome of play can be far from that predicted by the contagious equilibrium.[9] Furthermore, we would expect that this problem of strategic uncertainty is naturally more severe as the group size n gets larger. For these reasons, we chose to first eliminate the strategic uncertainty dimension from our experimental design by having our players interact with robot players programmed to play according to the contagious strategy so as to provide a cleaner test of our monotonicity results.[10] While our baseline design has just 1 human subject and 11 robots per community of size $M = 12$, in other treatments, we increase the number of human subjects in each community to 2 and 6 while decreasing the number of robots to 10 and 6, respectively, i.e., holding M fixed at 12.

Using the three treatment variables discussed above, i.e., the group size n, the value of α and the number of human subjects in each community, we summarize all of our experimental treatments in Table 6. The treatments are denoted using labels of the form "aHbR_X," where a is the number of human subjects and b is the number of robots in each community of 12 players. $X \in \{A, B, C, D, E\}$ represents different pairs of values for the group size, n, and payoff parameter, α, as indicated in Table 6. This same table also provides the on-equilibrium and off-equilibrium conditions on α, given the group size chosen for each treatment. Using these conditions, and the treatment value for α, Table 6 also indicates (under the heading "Cooperation?") whether or not cooperation by all players can be sustained as a sequential equilibrium of the indefinitely repeated game where players are randomly and anonymously matched in groups of size n in each round. We conducted one session each for the treatments involving 1H11R or 2H10R and two sessions for the treatment involving 6H6R. Note that each session gives multiple independent observations – Table 7 below provides further details.

The value of α in each treatment is varied based on several considerations. First, we wanted a parameterization that could sustain the contagious strategy as an equilibrium in a community of a fixed population size *under a larger group size but not under a smaller group size* so as to test our main monotonicity result. Second, we chose to focus on the on-equilibrium-path condition rather than the off-equilibrium-path condition; if α was instead chosen in such a way that the on-equilibrium-path condition (but not the off-equilibrium-path condition) was always satisfied for both group size treatments, e.g., a choice of $\alpha = 0.1$, then we might observe that subjects seldom chose to defect (with the consequence that they were seldom actually off the equilibrium path) under either group size, making it difficult to detect any treatment effect. Therefore, we chose to

[9] See for example, Duffy et al. (2013) and Duffy and Ochs (2009).
[10] We note that this type of experimental design involving robot players has not previously been implemented to test the contagious equilibrium prediction.

Table 7
Description of experimental sessions.

Treatment	Session	No. of subjects	No. of obs.	No. of supergames	No. of rounds	Average earnings
1H11R_A	1	10	10	18	75	USD 22.59
1H11R_B	2	12	12	20	76	USD 15.33
1H11R_C	3	12	12	19	75	USD 13.11
1H11R_D	4	10	10	22	77	USD 20.21
1H11R_E	5	7	7	18	77	USD 15.31
6H6R_A	6	12	2	19	81	USD 11.49
6H6R_A	7	12	2	15	83	USD 14.63
2H10R_A	8	18	9	15	65	USD 18.09
Average	N/A	12	N/A	18	76	USD 16.35

set $\alpha = 0.5$ to test the group size effect between $n = 6$ and $n = 2$ (Sessions 1 and 2). By the same principle, we chose $\alpha = 0.6$ to test the group size effect between $n = 3$ and $n = 6$ (Sessions 4 and 5). In addition, in Session 3 we set $\alpha = 0.3$ for a group size $n = 2$ so that cooperation via the contagious equilibrium is supported in this case, as opposed to $\alpha = 0.5$ for group size $n = 2$ as in Session 2. In sessions 1–5, there is just one human subject per community of size 12. By contrast, in sessions 6, 7, and 8, the number of human subjects in the 12-player community is varied but we always set $\alpha = 0.5$ and $n = 6$ as in Session 1 so that the contagious equilibrium can be sustained.

Given our choice of α in each treatment and a fixed $\delta = 0.75$ and $M = 12$, we are able to test the following hypotheses using our experimental data.

Hypotheses 1–3 concern aggregate treatment effects on cooperation.

Hypothesis 1. The overall cooperation rate is higher with a larger group size than with a smaller group size given the same value of α in the 1H11R treatments (Session 1 vs. 2, Session 5 vs. 4).

Hypothesis 2. The overall cooperation rate is higher when the equilibrium condition is satisfied with a smaller α than when the equilibrium condition is not satisfied with a larger α given the same group size in the 1H11R treatments (Session 3 vs. 2).

Hypothesis 3. The overall cooperation rate is lower with an increase in the ratio of human subjects to robots (playing the contagious strategy) in a community given the same group size, n and value of α (Sessions 1 vs. 6 and 7 vs. 8).

Hypotheses 4–7 concern the individual behavior of the human subjects:

Hypothesis 4. The frequency with which subjects are on the equilibrium path is larger when the on-equilibrium-path condition is satisfied than when the condition is not satisfied.

Hypothesis 5. The cooperation rate when subjects are c-types (on-the-equilibrium-path) is higher with a larger group size than with a smaller group size given the same value of α.

Hypothesis 6. The cooperation rate when subjects are d-types (off-the-equilibrium-path) is not different between a larger group size and a smaller group size.

Hypothesis 7. The cooperation rate when subjects are c-types is higher when the number of human subjects in the community decreases, given the same group size n and value of α.

In our 1H11R treatments, we explicitly told our subjects that in each round of a supergame (or "sequence" as it was referred to in the experiment) they would be randomly matched with $n - 1$ other robot players (out of a total population of 11 robot players) and not with any other human subjects. Since $n < M$, subjects were told that there would also be robot–robot group interactions that they would not be a part of given the uniform random matching that we used. Subjects were further instructed that the robots in each community played according to the rules of the contagious strategy. Specifically subjects were told:

"The robots are programmed to make their choices according to the following rules:

- choose X in the first round of each new sequence;
- if, during the current sequence, any of a robot's group members, including you or any other robot players have chosen Y in any prior round of that sequence, then the robot will switch to choosing Y in all remaining rounds of the sequence;
- otherwise, the robot will continue to choose X."

Here X refers to the cooperative action C, while Y refers to the defect action D.[11] Thus subjects had complete knowledge of the strategies to be played by their opponents. We did not provide subjects with any further information, such as the

[11] We used the neutral labels X and Y rather than (Cooperate and Defect) in our experimental implementation of the n-player game.

number of periods it might take for them to meet a defecting player once they (the human subject) had initiated a defection, as this calculation was one that we wanted subjects to make on their own.

In our 2H10R and 6H6R treatments, everything remained the same except that subjects knew that there would be 2(6) human subjects and 10(6) robot players in their 12-player community for the duration of the session. Subjects remained in the same community with the same 1 other or the same 5 other human subjects in all supergames (sequences) of a session. Further, subjects were told that there would be no spillovers from human subjects interacting in different communities of size $M = 12$. Therefore, as in the 1H11R treatments, each community of size 12 in the 2H10R and 6H6R treatments constitutes an independent observation.

One may be concerned that explicitly telling subjects the contagious strategy used by robots will induce a kind of experimenter demand effect by which the human subjects will also follow this same contagious strategy. However, if that were the case, then our hypotheses that the group size n matters would not find any support since subjects in all of our treatments were told the same information about the contagious strategy played by the robot players. On the other hand, if we observe a higher cooperation rate under a group size of $n = 6$ than under a group size of $n = 2$ or $n = 3$, then it implies that subjects rationally choose to follow the contagious strategy more frequently when the equilibrium conditions were satisfied.

To implement an infinite-horizon n-player PD game in the laboratory, we use the standard random termination methodology (Roth and Murnighan, 1978) in which subjects participate in supergames that consist of an indefinite number of rounds, where the probability of continuation from one round to the next is a known constant equal to the discount factor, $\delta \in (0, 1)$. With our choice of $\delta = 0.75$, the expected duration of a supergame is 4 rounds.

To enable subjects to gain some experience with the play of an indefinitely repeated game, we had them participate in multiple supergames in a session. As noted above, subjects were informed that at the start of each and every new supergame (sequence) all of the robot players in their community of size 12 would start out each new supergame as c-types playing the cooperative strategy (X) and that robot players would only change to playing the defect strategy (Y) if they became d-types during that supergame. This feature allows subjects to treat each new supergame as a fresh start rather than viewing the entire session as one long indefinitely repeated game, with the possibility of switching to a different strategy at the start of each new supergame. We did not fix the number of supergames played in advance. Instead, during the experiment, we allowed subjects to play for at least one hour, and the supergame in progress beyond one hour was determined to be the last supergame (we did not inform subjects of our stopping rule); when that final supergame was completed, the session was declared to be over. Following the completion of the experiment, three sequences were randomly selected from all played and subjects were paid their total earnings from those three sequences in addition to a $7 show-up fee.[12] All sessions were completed within the two hour time-horizon for which we recruited subjects; a typical session required about 30 min for the reading of instructions followed by 1 h of play of multiple supergames.

The experiment was conducted at the Experimental Social Science Laboratory (ESSL) of the University of California, Irvine using undergraduate students with no prior experience with our experimental design. Instructions were read aloud and then subjects completed a brief comprehension quiz. The instructions used in the $n = 6$ treatment are provided in Appendix C; instructions for other treatments are similar.

6. Experimental findings

Table 7 reports some details about all of our experimental sessions. As this table reveals, on average, each session involved 18 supergames and 76 rounds. In total, 93 subjects participated in our experiment, with average earnings of USD $16.35. In the 1H11R treatments, since each subject interacted with an independent group of 11 other robots all playing according to the contagious strategy, each subject's behavior amounts to a single, independent observation. Thus, the number of subjects we have for each of sessions 1–5 corresponds to the number of independent observations. For treatment 6H6R_A, there are 6 human subjects in each community and 12 subjects in each session, so each session produces two independent observations, and as we have two sessions of 6H6R_A, we have a total of 4 independent observations of this treatment. For treatment 2H10R_A, there are 2 human subjects in each community; since we recruited 18 human subject for this treatment, we thus have 9 independent observations of this treatment.

In the remainder of this section, we will begin with an aggregate data analysis, followed by analysis of data at the individual level. For the aggregate results, we will first provide a summary of findings on the group size effect, the effect of α, and the effect of the number of human subjects within a community. After that we will present results from Mann-Whitney tests that compare the cooperation rates in the different treatments all together in Table 9.

We first examine results from the treatments with one human subject in each community (1H11R). Fig. 2 shows the average cooperation rate per round over time for treatments 1H11R_A (B, D, E). The blue line (with diamonds) in each panel is the average cooperation rate of the human subjects only, while the red line (with squares) in each panel is the average cooperation rate by the entire 12-member community as a whole, consisting of 1 human subject and 11 robots. The start of each new supergame is indicated by a vertical line.

[12] We chose to pay for three randomly sequences, as opposed to just one, so as to avoid the possibility that a "short" (e.g., 1-round) supergame was chosen for payment under the pay-one-sequence-only protocol.

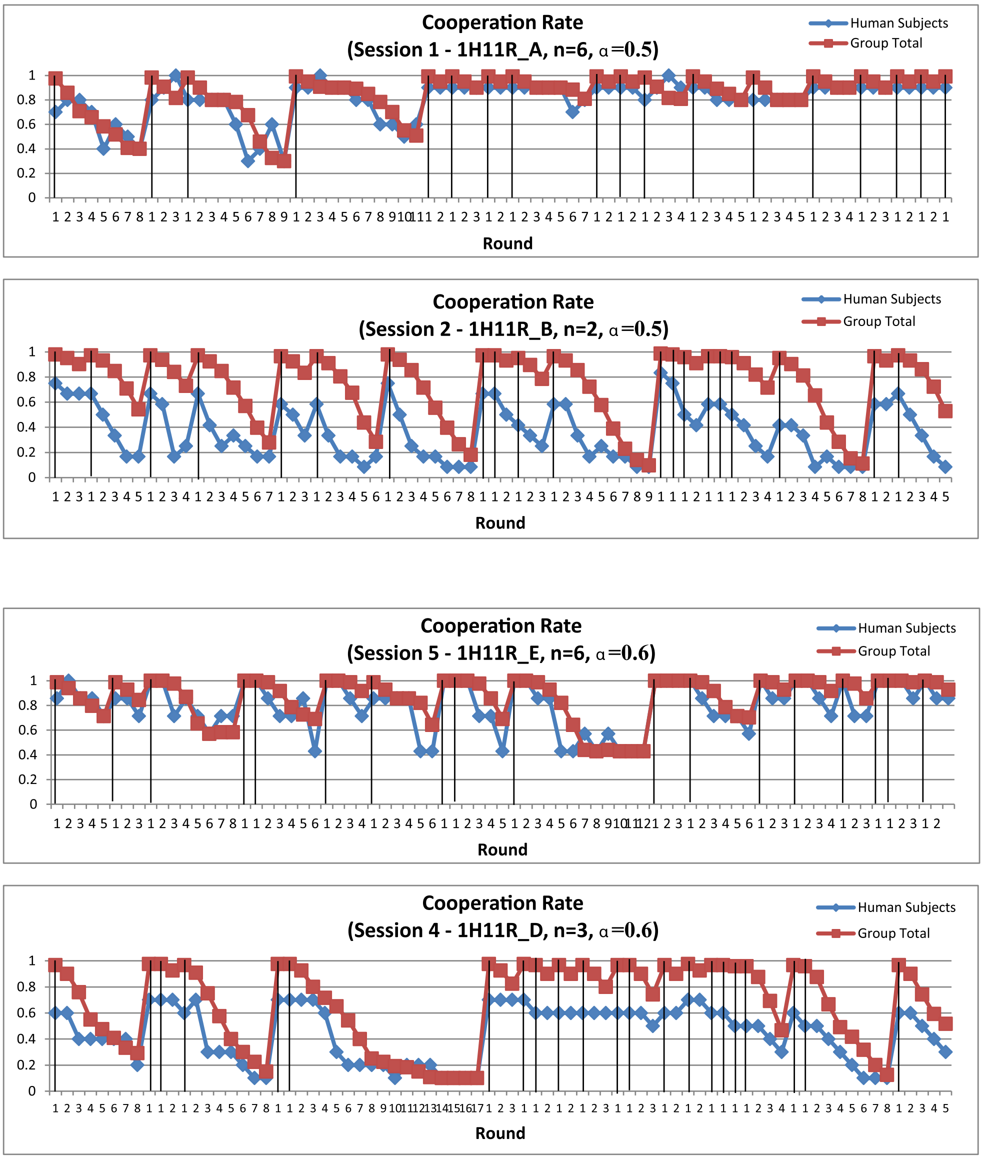

Fig. 2. Cooperation rates by human subjects and communities over time (treatment 1H11R_A,B,D,E).

The two top panels of Fig. 2 show the average cooperation rates for the group size of $n=6$ and $n=2$ respectively with $\alpha=0.5$. For Session 2 (the $n=2$ group size), we observe a decline in the cooperation rate over time by the human subjects in almost all supergames lasting more than 2 rounds, which indicates that more human subjects began to switch from cooperation to defection from round 2 if they did not choose to defect from the beginning of the supergame. Consistently, the cooperation rate at the community level shows a similar pattern but remains above the cooperation rate by the human subjects alone as it takes some time for the contagious strategy, as played by the robot players, to spread throughout the population of size 12. Across all supergames of the session with $n=2$, there is no obvious learning effect or convergence.

By contrast, the cooperation rates in Session 1 under a group size of $n=6$ exhibit a very different pattern over time. Indeed, consistent with our theory, the overall cooperation rate is higher in Session 1 ($n=6$) than in Session 2 ($n=2$). Although there is also a decline in cooperation over the course of each sugergames at the beginning of Session 1, the cooperation rate eventually becomes high, at around 90%, following the fourth supergame of this session (approximately after the first one-third of the session has been completed) and remains high for the remaining supergames of that session. This finding indicates that, given the payoff parameters we have chosen and the strategy followed by the robots, most subjects learn over time that it is in their best interest to follow the contagious strategy when n is large ($n=6$), relative to the case where n is small ($n=2$). A comparison of the cooperation rates between groups of size $n=2$ and $n=6$ indicates that the human subjects responded to the payoff incentives of the game. They choose to start defecting more frequently when the contagious effect of a single

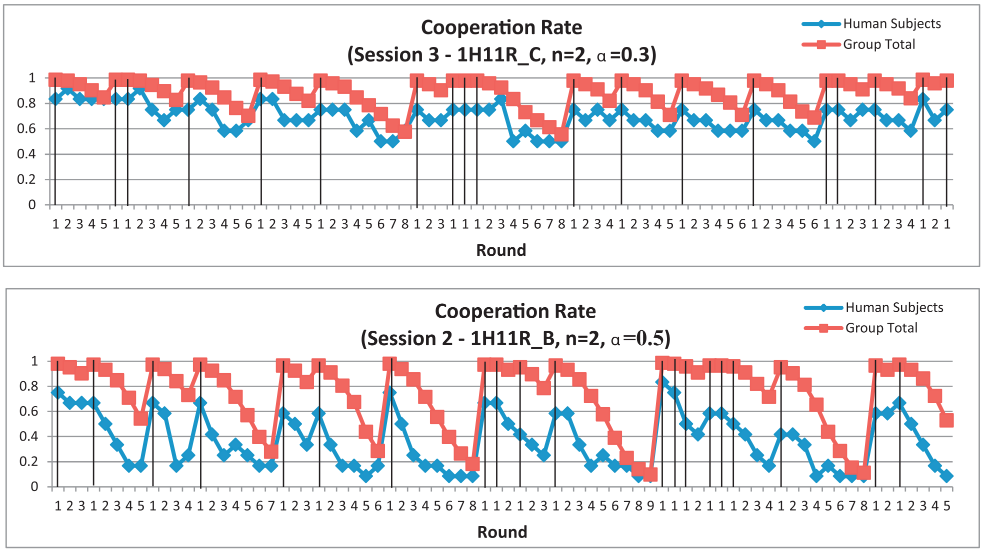

Fig. 3. Cooperation rates by human subjects and communities over time (treatments 1H11R_B,C).

defection is much slower in Session 2 ($n = 2$) and this tendency to defect was not diminished by experience. By contrast, when the contagious effect of a defection is more immediate, as in Session 1 ($n = 6$), subjects learned to avoid triggering a wave of defection.

The two bottom panels of Fig. 2 show the average cooperation rates for the group of size $n = 6$ and $n = 3$ respectively with the choice of $\alpha = 0.6$. This treatment comparison provides a robustness check as to whether the group size effect continues to hold with a different parameterization for n and α. Indeed, comparing the different group sizes $n = 6$ and $n = 3$, we continue to find that, consistent with our theory, cooperation rates are higher with the larger group of size $n = 6$ than under the smaller group of size $n = 3$. We summarize the group size effect in Finding 1, which supports Hypothesis 1. Detailed statistical support can be found in Table 9.

Finding 1. Cooperation rates are higher under larger groups than under smaller groups given the same α.

Not only did our subjects respond to the incentives induced by having different group sizes, but also they responded to the incentives associated with changes in α when holding the group size constant. Fig. 3 shows a comparison between Session 3 ($\alpha = 0.3$, $n = 2$) and Session 2 ($\alpha = 0.5$, $n = 2$). With a larger α, players have a larger temptation to initiate a defection, thus making it more difficult to sustain cooperation within the community. Fig. 3 and Finding 2 confirm Hypothesis 2. In the top panel of Fig. 3 where $\alpha = 0.3$, the cooperation rate is sustained at a level above 50%. By contrast, as seen in the bottom panel of the same figure where $\alpha = 0.5$, the cooperation rate drops below 20% if the supergame lasts for 4 or more rounds. Results from Mann–Whitney tests regarding Finding 2 are presented in Table 9.

Finding 2. Cooperation rates are higher under a smaller α than under a larger α given the same group size, n.

We next compare the 1H11R treatment with the 2H10R and 6H6R treatments, restricting attention to the case where $n = 6$ and $\alpha = 0.5$, so that the contagious equilibrium is supported by the same set of parameters in the three treatments and we can examine whether or not the ratio of humans to robots in each community of size 12 will affect the possibility that the contagious equilibrium is selected.[13] The comparison is made using Fig. 4 and statistical support is included in Table 9. The first panel of this figure shows cooperation rates for the 1H11R_A treatment, the second panel for the 2H10R_A treatment and the last two panels show cooperation rates for the 6H6R_A treatment (2 sessions). As the ratio of human subjects to robots in a community increases, we find that the average cooperation rate monotonically decreases. Recall that for the 1H11R_A treatment, the cooperation rate converges to about 90% after subjects gained some experience in the first four supergames. By contrast, in the 2H10R_A treatment, the cooperation rate is closer to 50% throughout the session.[14] Finally, in the 6H6R_A treatment, the cooperation rate almost always drops to zero after a supergame lasts for 2 or 3 rounds. Based on the results of the 6H6R treatment, we did not think it necessary to run treatments where the community consisted entirely of 12 human subjects and 0 robots, as we expect that cooperation rates would likely have been close to 0 in these treatments as well.

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[13] We did not study the case of a group size of $n = 2$ across these three treatments involving different ratios of humans to robots, since when $n = 2$ and $\alpha = 0.5$, our theory predicts that a social norm of mutual cooperation is not sustainable using the contagious strategy.

[14] Note that for the 2H10R treatment, the aggregate cooperation rate masks some heterogeneity at the community level. Focusing on behavior in the last three sequences of the 2H10R_A treatment, we found that 3 out of 9 human groups (each consisting of 2 human subjects) always chose to cooperate, another 3 out of 9 human groups always chose to defect, and the remaining 3 groups cooperated at average levels of 10%, 20%, and 70%, respectively.

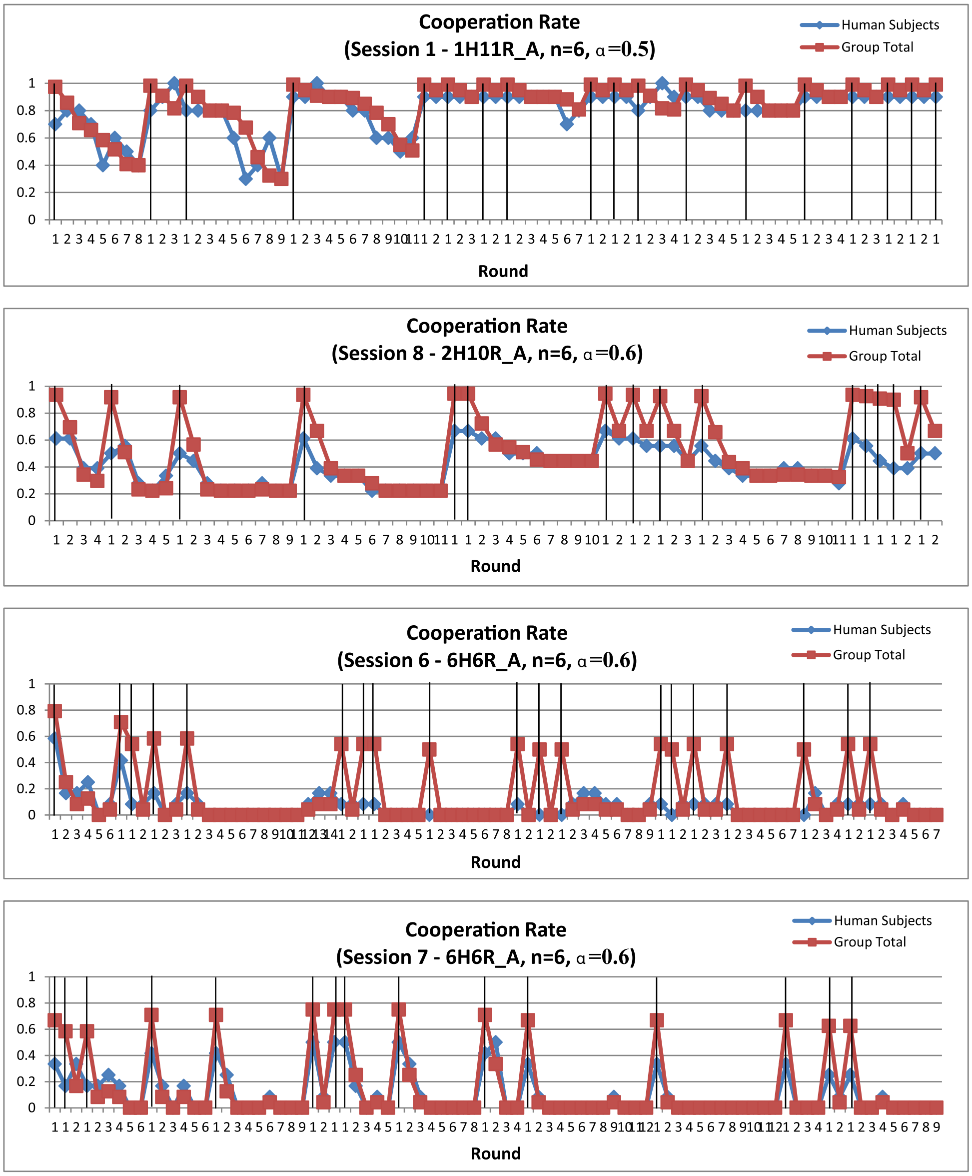

Fig. 4. Cooperation rates by human subjects and communities over time (treatments 1H11R_A, 2H10R_A, 6H6R_A).

Finding 3. Cooperation rates monotonically decrease as the number of human subjects in a community increases, given the same group size n and value of α.

Tables 8 and 9 provide further detailed evidence that support Findings 1–3. Table 8 reports cooperation rates calculated based on human subjects' choices as well as on community-wide action choices (humans plus robot players) over all rounds of all supergames. We further calculated the cooperation rates over the first and second halves of each session. Table 9 reports p-values from two-tailed Mann–Whitney tests on the cooperation rates between treatments. The first two rows test Hypothesis 1 (the group size effect) for $n = 6$ vs. $n = 2$ given $\alpha = 0.5$ (treatment 1H11R_A vs. 1H11R_B), and $n = 6$ vs. $n = 3$ given $\alpha = 0.6$ (treatment 1H11R_D vs. 1H11R_E). The third and fourth rows test Hypothesis 2 (the effect of α) for $\alpha = 0.5$ vs. $\alpha = 0.3$ given $n = 2$ (treatment 1H11R_B vs. 1H11R_C) and $\alpha = 0.5$ vs. $\alpha = 0.6$ given $n = 6$ (treatment 1H11R_A vs. 1H11R_E), where we only expect a significant difference in the case of $n = 2$. The last three rows test Hypothesis 3 (the number of human subjects in a community) given $n = 6$ and $\alpha = 0.5$.

The p-values shown in Table 9 are all consistent with a careful examination of the cooperation rates shown in Figs. 2–4. We find the most significant group size effect between the $n = 2$ and $n = 6$ treatments. On the other hand, strategic uncertainty (about the play of other human subjects) also plays an important role in de-stabilizing cooperation in the environment we

Table 8
Cooperation rates for each session.

Treatment	Session	Human subjects			Communities		
		Whole session	1st half	2nd half	Whole session	1st half	2nd half
1H11R_A	1	0.800	0.732	0.870	0.846	0.776	0.918
1H11R_B	2	0.364	0.373	0.355	0.743	0.754	0.733
1H11R_C	3	0.692	0.732	0.651	0.883	0.898	0.869
1H11R_D	4	0.444	0.379	0.508	0.667	0.534	0.797
1H11R_E	5	0.800	0.812	0.788	0.865	0.875	0.855
6H6R_A	6	0.062	0.079	0.047	0.148	0.159	0.142
6H6R_A	7	0.104	0.152	0.058	0.147	0.204	0.090
2H10R_A	8	0.418	0.368	0.466	0.519	0.441	0.594

Table 9
p-values from Mann–Whitney tests on cooperation rates.

Treatment	No. of obs.	Human subjects			Communities		
		Whole session	1st half	2nd half	Whole session	1st half	2nd half
1H11R_A vs. 1H11R_B	10 vs. 12	0.006	0.008	0.003	0.015	0.391	0.005
1H11R_D vs. 1H11R_E	10 vs. 7	0.039	0.009	0.128	0.044	0.005	0.403
1H11R_B vs. 1H11R_C	12 vs. 12	0.067	0.018	0.097	0.045	0.017	0.067
1H11R_A vs. 1H11R_E	10 vs. 7	0.767	0.488	0.290	1.000	0.348	0.291
1H11R_A vs. 6H6R_A	10 vs. 24 (10 vs. 4)	0.000	0.000	0.000	0.005	0.005	0.003
1H11R_A vs. 2H10R_A	10 vs. 18 (10 vs. 9)	0.034	0.034	0.029	0.049	0.059	0.034
2H10R_A vs. 6H6R_A	18 vs. 24 (9 vs. 4)	0.001	0.001	0.003	0.005	0.020	0.005

study here where players are randomly and anonymously matched in each repetition of the stage game. Although nearly full cooperation can be sustained in the second half of our sessions where $n = 6$ and each human subject interacts only with 11 robot players programmed to play according to the contagious strategy, the cooperation rate drops to close zero when n remains equal to 6, but the community consists of 6 human and 6 robot players, even though the equilibrium conditions for sustenance of a social norm of cooperation continue to be satisfied. In order to further ensure that these treatment effects are not driven by subjects' reactions to the instructions used in different treatments, we compared the subjects' choice in the very first period of the sessions and did not found any significant difference across treatments.

Importantly, our results indicate that subjects understand how to evaluate payoffs in the indefinitely repeated games we induce despite the relatively complicated environment, and they correctly respond to the incentives provided. Our findings from varying the ratio of humans to robots suggest that it is *strategic uncertainty* about the play of others and not bounded rationality with regard to payoff calculations that may explain the failure of human subjects to achieve the efficient equilibrium in indefinitely repeated games where Kandori's contagious equilibrium construction can be used to support a social norm of cooperative play by all players.

We next ask whether there is a significant learning effect when the contagious equilibrium condition is or is not satisfied. In our experiment, we find such a learning effect for some but not for all of our treatments/sessions. Table 10 reports *p*-values for the two-tailed Wilcoxon matched-pair signed ranks tests between the first and the second halves of each session. Learning is most significant in treatment 1H11R_A ($n = 6$, session 1), in which subjects learn to cooperate with the 11 robot players who are playing according to the contagious strategy, and in treatment 6H6R_A ($n = 6$, sessions 6 and 7), where cooperation rates converge to zero as half of the community members are human subjects. In sessions 3 and 5, the cooperation rates decrease significantly from the 1st half of sessions to the 2nd half of sessions, but the magnitude of this decrease is small.

We next analyze individual strategic behavior. Specifically, we ask to what extent the subjects in each treatment behaved according to the contagious strategy that is needed to support a social norm of cooperation among strangers. Table 11 shows

Table 10
p-value of Wilcoxon matched-pair signed ranks tests on cooperation rates (1st vs. 2nd half of the session).

Treatment	Session	Cooperation?	Human subjects	Communities	Pattern over time
1H11R_A	1	Yes	0.081	0.018	Increasing
1H11R_B	2	No	0.692	0.346	Flat
1H11R_C	3	Yes	0.010	0.054	Decreasing
1H11R_D	4	No	0.185	0.006	Increasing
1H11R_E	5	Yes	0.050	0.050	Decreasing
6H6R_A	6, 7	Yes	0.001	0.144	Decreasing
2H10R_A	8	Yes	0.614	0.056	Increasing

Table 11
Strategy analysis of human subjects.

Treatment	Session	Frequency on equm path	Cooperate rate on equm path	Defect rate off equm path	Frequency using contagious strategy
1H11R_A	1	82.27%	86.49%	74.48%	91.60%
1H11R_B	2	50.99%	56.50%	94.46%	79.50%
1H11R_C	3	76.11%	74.35%	93.96%	89.00%
1H11R_D	4	57.40%	60.35%	97.48%	84.42%
1H11R_E	5	84.42%	90.43%	85.76%	91.47%
6H6R_A	6	23.66%	11.31%	95.41%	75.51%
6H6R_A	7	19.08%	37.58%	96.40%	85.54%
2H10R_A	8	50.68%	61.67%	96.53%	87.69%

Table 12
p-value of Mann–Whitney tests on individual strategy.

Treatment	Frequency on equm path	Cooperate rate on equm path	Defect rate off equm path	Frequency using contagious strategy
1H11R_A vs. 1H11R_B	0.008	0.008	0.118	0.015
	(10 vs. 12)	(10 vs. 12)	(8 vs. 11)	(10 vs. 12)
1H11R_D vs. 1H11R_E	0.043	0.127	0.017	0.301
	(10 vs. 7)	(10 vs. 7)	(9 vs. 4)	(10 vs. 7)
1H11R_B vs. 1H11R_C	0.090	0.086	0.479	0.061
	(12 vs. 12)	(12 vs. 12)	(11 vs. 6)	(12 vs. 12)
1H11R_A vs. 1H11R_E	0.843	0.693	1.000	1.000
	(10 vs. 7)	(10 vs. 7)	(8 vs. 4)	(10 vs. 7)
1H11R_A vs. 6H6R_A	0.005	0.024	0.494	0.047
	(10 vs. 4)	(10 vs. 4)	(8 vs. 4)	(10 vs. 4)
1H11R_A vs. 2H10R_A	0.085	0.163	0.381	0.251
	(10 vs. 9)	(10 vs. 9)	(8 vs. 7)	(10 vs. 9)
2H10R_A vs. 6H6R_A	0.009	0.031	0.706	0.216
	(9 vs. 4)	(9 vs. 4)	(7 vs. 4)	(9 vs. 4)

the average of individual frequencies with which subjects in each treatment are on the equilibrium path, the average of individual frequencies with which subjects chose to cooperate when on the equilibrium path and the average of individual frequencies of choosing to defect when off the equilibrium path.[15] Finally, for each human subject, we also calculated the frequency with which that subject played the contagious strategy, which is given by:

$$\begin{aligned}&\text{freq.offollowingthecontagiousstrategy}\\&\quad=\text{freq.ontheequilibriumpath}\times\text{cooperationrateontheequilibriumpath}+\text{freq.offtheequilibriumpath}\\&\qquad\times\text{defectionrateofftheequilibriumpath.}\end{aligned}$$

In Table 12 we report the *p*-values from two-tailed Mann Whitney tests comparing all of these frequencies across various treatments. The tests are conducted by using the average of the individual frequencies of human subjects within each community, which constitutes an independent observation since human subjects in different communities did not have any interactions with one another by our experimental design.

Most of the results shown in Tables 11 and 12 are consistent with Hypotheses 4–7 regarding individual strategic behavior. For the treatments with 1H11R, the average frequency with which subjects are found to be on the equilibrium path is always significantly larger in sessions where the on-equilibrium-path condition is satisfied than in those treatments where this same condition is not satisfied. On the other hand, an increase in the number of human subjects in a community, as in treatments 2H10R_A and 6H6R_A, significantly lowers the frequency of on-equilibrium-path play of the contagious strategy. The average frequency of on-equilibrium-path play moves from about 80% to 50% and further declines to 20% when comparing across the 1H11R_A, 2H10R_A and 6H6R_A treatments, respectively ($p<0.1$ or $p<0.01$).

Conditional on being on the equilibrium path (column 3 in Table 12), the difference in cooperation rates between the $n=6$ and $n=2$ treatments is significant (session 1 vs. 2, $p<0.01$) but there is no significant difference between the $n=6$ and $n=3$ treatments (session 4 vs. 5, $p=0.127$). The cooperation rates on the equilibrium path are not significantly different between 1H11R_A and 2H10R_A treatments ($p=0.163$), but are significantly different between 1H11R_A and 6H6R_A treatments and between 2H10R_A and 6H6R_A treatments ($p<0.05$). Conditional on being off the equilibrium path (column 4 in Table 12), as

[15] Recall that a player is defined to be on the equilibrium path in the first round of a supergame or when the player has never experienced a defection by his group members or himself in the past rounds of a supergame. Otherwise, a player is off the equilibrium path.

Hypothesis 6 states, the cooperation rates are not significantly different between treatments except for treatments 1H11R_D and 1H11R_E, which may be due to the small number of subjects off the equilibrium path in treatment 1H11R_E. Finally, the frequency with which subjects followed the contagious strategy (column 5 in Table 12) is significantly larger when the on-equilibrium-path condition is satisfied and for a 1H11R community. For the 2H10R_A treatment, the frequency that subjects followed the contagious strategy is in between those in found for the 1H11R_A and 6H6R_A treatments, although it is not significantly different. However, this frequency is significantly different between 1H11R_A and 6H6R_A treatments ($p < 0.05$).

Finding 4. With a larger group size $n = 6$ and with a 1H11R community, subjects are more often on the equilibrium path, choose cooperation more frequently when on the equilibrium path, and more often follow the contagious strategy, as compared with the corresponding 1H11R treatment with a smaller group size $n = 2$ and the treatment with $n = 6$ and 6H6R.

Summarizing our experimental results, we find that the behavior of the human subjects is consistent with our theoretical predictions on the impact of group size for cooperative play. Given the same payoff parameter α, cooperation rates increase when the group size increases. Under a small group size, the cooperation rate declines over time and this pattern repeats itself across supergames even as subjects gain repeated experience with the environment. By contrast, under a larger group size, subjects learn to stick with cooperation after experiencing the much quicker consequences of triggering a contagious wave of defection in their community.

When there is more than one human subject in each community, cooperation rates decrease as the ratio of human subjects to robots increases. Comparing the 1H11R treatment with the 6H6R treatment with the same group size $n = 6$ and α that always supports the contagious equilibrium, the cooperation rate by the human subjects drops from 80% in 1H11R to less than 10% in 6H6R, suggesting that it is greater strategic uncertainty that works to destroy cooperation in communities with randomly and anonymously matched players.

7. Conclusions

We have examined the effect of group size, n, on the equilibrium conditions needed to sustain cooperation via the contagious strategy as a sequential equilibrium in repeated play of an n-player Prisoner's Dilemma game, given a finite population of players of size $M \geq n$ and random and anonymous matching of players in each repetition of the game. We find that, if agents are sufficiently patient, the equilibrium conditions, both on the equilibrium path and off the equilibrium path, become less restrictive, and thus more easily satisfied as the group size n increases toward M. This result arises from the faster speed with which a contagious wave of defections can occur as the group size becomes larger. Our findings expand upon Kandori's (1992) idea that a social norm of cooperative behavior among randomly matched strangers can be policed by community-wide enforcement. Specifically, we show that community–wide enforcement becomes easier to sustain as the speed with which information travels becomes faster, which is here proxied by increases in the matching group size, n.

Other interpretations of n are possible. Consider, for example, two towns with similar populations. The traffic rules are commonly known to all but there is no official police force that regulates drivers to follow these traffic rules in either town. It's always more efficient if everyone follows the traffic rules than if no one does. Suppose the only difference between the two towns is in the structure of the roads. One town has several main streets with a few crosses. The other town has many small streets with a lot of crosses. The first town's road structure corresponds to our large group structure whereas the second town's road structure corresponds to our small group structure. Disobedience of traffic laws has a more immediate impact in the first town than in the second town, and so, by the logic of our theory, one might expect greater adherence to traffic laws in the first town than in the second. As another example, centralized communication or monitoring mechanisms (credit bureaus) might also perform the same role played by larger group sizes in easing the conditions under which a social norm of cooperation is sustained in a large population of players. After all, as n increases to M, we effectively achieve increasingly perfect monitoring of the actions chosen by all players.

We have also empirically evaluated our theory by designing and reporting on an experiment exploring some of the comparative statics implications of the theory. Consistent with the theory, we found that subjects *were* able to achieve higher rates of cooperation when randomly and anonymously matched into larger rather than smaller groups of size n to play a indefinitely repeated n-player Prisoner's Dilemma game. In many of our experimental treatments, we removed strategic uncertainty by having human participants play only against robot players who were known to always play according to the contagious strategy. By contrast, Camera et al. (2012) found large heterogeneity in strategies adopted by participants in a 2-player Prisoner's Dilemma game with random and anonymous matching. When there is heterogeneity in strategies or a belief that strategies are heterogeneous, the incentives for agents to play according to the contagious strategy may be greatly weakened or become non-existent. We have found evidence for this phenomenon as well, as we observed declines in cooperation rates as we increased the ratio of human subjects to robots in the population of players from which groups were randomly formed.

A next step in this literature might be to examine whether the fraction of robot players playing according to the contagious strategy could be very gradually reduced, e.g., from $n - 1$ robots on down to 0, and gradually replaced with human subjects, so as to keep the total population size, M, fixed. Alternatively, one could first have 12 human subjects play for a time in our 1H11R treatment condition and subsequently have those same 12 subjects interact with one another (no robots -12H0R) playing the same indefinitely repeated n-player PD game. It would be of interest to know whether the human subjects, who would be free to choose any strategy, could learn to coordinate on the cooperative outcome in such settings. We leave these projects to future research.

Appendix A.

We first show that the equilibrium conditions in Lemma 1 are equivalent to the equilibrium conditions provided by Kandori (1992) when the group size $n=2$. Translating our notation to that used by Kandori (1992), $C_{n-1}=1$, $u_n=(-l, 1)^T$, $v_n=(0, 1+g)^T$, $Z_n=(i_M-\rho\rho)$ $(\rho=\frac{1}{M-1}(M-1, M-2, \ldots, 1, 0)^T$, in which the ith element of ρ is the conditional probability that a d-type player meets a c-type when there are i d-types, and i_M is a $1\times M$ vector with all elements equal to 1), $A_n=A$, $B_n=B$, $H_n=H$. Thus condition 1, Eq. (1) can be written as:

$$\begin{aligned}1 &\geq (1-\delta)e_1(I-\delta A)^{-1}(i_M-\rho\,\rho)\begin{pmatrix}0\\1+g\end{pmatrix}\\ &=(1-\delta)e_1(I-\delta A)^{-1}\rho(1+g),\end{aligned}$$

which is the same as equilibrium condition 1 in Kandori (1992). Condition 2, Eq. (2) can be written as

$$e_k(i_M-\rho\,\rho)\begin{pmatrix}l\\g\end{pmatrix}\geq \delta e_k H(I-\delta A)^{-1}(i_M-\rho\,\rho)\begin{pmatrix}0\\1+g\end{pmatrix},$$

i.e.,

$$(\frac{M-k}{M-1})g+(\frac{k-1}{M-1})l\geq \delta e_k H(I-\delta A)^{-1}\rho(1+g),$$

which is the same as equilibrium condition 2 in Kandori (1992).

Lemma 2. *Define* $p_j^n\equiv(1-\delta)e_1(I-\delta A_n)^{-1}\rho_j^n$ *and* $q_j^n\equiv\delta e_n H_n(I-\delta A_n)^{-1}\rho_j^n$ $(j=0, \ldots, n-1)$*, then* $\sum_{j=0}^{n-1}p_j^n=1$ *and* $\sum_{j=0}^{n-1}q_j^n=0$.

Proof. By definition, p_j^n denotes the discounted summation of the probability of meeting j c-type players in the group once a defection has started when the group size is n, and q_j^n denotes the change in the discounted summation of the probability of meeting j c-type players in the group when the d-type player reverts back to playing cooperation instead of defection given that the group size is n and there are $k=n$ d-type players. Notice that by definition the summation of the elements in each row of matrix Z_n, A_n, and B_n is always equal to 1. Denote i_k as a $1\times k$ vector with all elements equal to 1. Thus $Z_n i_n=i_M$, $A_n^t i_M=i_M$ and $B_n^t i_M=i_M$ for any group size n and $t=0, 1, \ldots, \infty$. Therefore we have

$$\sum_{j=0}^{n-1}p_j^n=(1-\delta)e_1(I-\delta A_n)^{-1}Z_n i_n=(1-\delta)e_1(I-\delta A_n)^{-1}i_M=(1-\delta)\sum_{t=0}^{\infty}\delta^t e_1 A_n^t i_M=1,$$

$$\sum_{j=0}^{n-1}q_j^n=e_n H_n(I-\delta A_n)^{-1}Z_n i_n=e_n H_n(I-\delta A_n)^{-1}i_M=\sum_{t=0}^{\infty}\delta^t e_n(B_n-A_n)A_n^t i_M=0.$$

□

Proof. [Theorem 1]

We first show that $\lim_{\delta\to1}(I-\delta A_n)^{-1}\rho_j^n<\infty$ for $j=1, \ldots, n-1$. (Therefore, $\lim_{\delta\to1}p_j^n=0$ for $j=1, \ldots, n-1$ and $\lim_{\delta\to1}p_0^n=1$.) The proof is similar as in Kandori's (1992) proof for Theorem 1. Since $X_t=M$ is the absorbing state and the Mth element of ρ_j^n is zero for $j=1, \ldots, n-1$,

$$(I-\delta A_n)^{-1}\rho_j^n=\sum_{t=0}^{\infty}\delta^t A_n^t\rho_j^n=\sum_{t=0}^{\infty}\delta^t\tilde{A}_n^t\rho_j^n=(I-\delta\tilde{A}_n)^{-1}\rho_j^n,\quad \text{for} j=1,\ldots,n-1$$

where $\tilde{A}_n$ is a matrix obtained by replacing the last column of A_n by zeros. Given this, we have only to show the existence of $(I-\tilde{A}_n)^{-1}$. Since the number of d-types never declines, $\tilde{A}_n$ is upper-triangular and so is $(I-\tilde{A}_n)$. The determinant of an upper-triangular matrix is the products of its diagonal elements, which are all strictly positive for $(I-\tilde{A}_n)$. Therefore, $\lim_{\delta\to1}p_j^n=\lim_{\delta\to1}(1-\delta)e_1(I-\delta A_n)^{-1}\rho_j^n\to0$ and $q_j^n=\delta e_n H_n(I-\delta A_n)^{-1}\rho_j^n$ is finite for $j=1, \ldots, n-1$.

Now the r.h.s. of equilibrium condition 1, $(1-\delta)e_1(I-\delta A_n)^{-1}Z_n v_n = p_o^n D_o + \sum_{j=1}^{n-1} p_j^n D_j \leq D_o + \sum_{j=1}^{n-1} p_j^n D_j$, where the inequality comes from $\sum_{j=0}^{n-1} p_j^n = 1$ and $p_o^n \leq 1$. Therefore, equilibrium condition 1 is satisfied if $C_{n-1} - D_o \geq \sum_{j=1}^{n-1} p_j^n D_j$, which is satisfied when $C_{n-1} - D_o$ and δ are sufficiently large.

Similarly, the r.h.s. of equilibrium condition 2, $\delta e_n H_n(I-\delta A_n)^{-1}Z_n v_n = q_o^n D_o + \sum_{j=1}^{n-1} q_j^n D_j$, is finite because $\sum_{j=0}^{n-1} q_j^n = 0$ and so $q_o^n = -\sum_{j=1}^{n-1} q_j^n$. Therefore, equilibrium condition 2 is satisfied when $v_n - u_n$ is sufficiently large.□

Appendix B. Fixing the number of groups, *m* = *M*/*n*

In this appendix we examine the case where M and n are varied in such a way that the number of groups $m = M/n$ is held constant. In particular, we compare the equilibrium conditions in three cases where $m = 3$: (1) $M = 12$ and $n = 4$; (2) $M = 9$ and $n = 3$; and (3) $M = 6$ and $n = 2$. Our aim here is to understand whether variations in the group size n continue to matter for satisfaction of the equilibrium conditions needed for cooperation to be sustained as a social norm, when the number of groups is held constant. The following numerical results are obtained holding fixed $\delta = 0.9$.

Equilibrium condition 1

First we consider equilibrium condition 1 for each of the three cases where $M/n = 3$:

$M = 6$ and $n = 2$: $\beta_n \geq 0.773647\alpha + 0.226353(\alpha + \beta_n)$; $M = 9$ and $n = 3$: $2\beta_n \geq 0.774597\alpha + 0.0696314(\alpha + \beta_n) + 0.155771(\alpha + 2\beta_n)$; $M = 12$ and $n = 4$: $3\beta_n \geq 0.783465\alpha + 0.030344(\alpha + \beta_n) + 0.0546601(\alpha + 2\beta_n) + 0.131531(\alpha + 3\beta_n)$.

Imposing the normalization condition $\beta_n = (n-1)^{-1}$, these conditions can be simplified as follows:

$M = 6$ and $n = 2$: $\alpha \leq 0.773647$ for $\beta_n = 1$; $M = 9$ and $n = 3$: $\alpha \leq 0.809415$ for $\beta_n = 1/2$; $M = 12$ and $n = 4$: $\alpha \leq 0.821913$ for $\beta_n = 1/3$.

We observe that when the number of groups $m = M/n$ is fixed (at 3) the results are very similar to those in the case where the population size M is fixed: equilibrium condition 1 is observed to become less restrictive as the group size n becomes larger. Intuitively, again, this is driven by the faster contagious process with a larger group size. The extent of the tendency for cooperation to become more easily sustainable as n increases is smaller than in the case where M is fixed, since in the latter case the contagious process becomes faster not only due to a larger group size but also due to there being a smaller number of groups as n increases.

Equilibrium condition 2

Finally, we consider equilibrium condition 2 for each of the three cases where $M/n = 3$:

$M = 6$ and $k = n = 2$: $\alpha \geq -0.270439\alpha + 0.270439(\alpha + \beta_n)$; $M = 9$ and $k = n = 3$: $\alpha \geq -0.219582\alpha + 0.15486(\alpha + \beta_n) + 0.064722(\alpha + 2\beta_n)$; $M = 12$ and $k = n = 4$: $\alpha \geq -0.187568\alpha + 0.126324(\alpha + \beta_n) + 0.0512613(\alpha + 2\beta_n) + 0.00998238(\alpha + 3\beta_n)$.

If we further impose our payoff normalization, then we have:

$M = 6$ and $k = n = 2$: $\alpha \geq 0.270439$ for $\beta_n = 1$; $M = 9$ and $k = n = 3$: $\alpha \geq 0.142152$ for $\beta_n = 1/2$; $M = 12$ and $k = n = 4$: $\alpha \geq 0.086265$ for $\beta_n = 1/3$.

As with equilibrium condition 1, the results for equilibrium condition 2 under a fixed ratio for M/n are similar to those found under a fixed M. Equilibrium condition 2 becomes less restrictive with increases in the group size, n.

Appendix C. Supplementary Data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.jebo.2016.02.007.

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