(NASA-CR-3169)   A SIMPLIFIED COMPUTER   N79-28226
PROGRAM FOR THE PREDICTION OF THE LINEAR
STABILITY BEHAVIOR OF LIQUID PROPELLANT
COMBUSTORS (Colorado State Univ.)   59 p   Unclas
HC A04/MF A01   CSCL 21H H1/20   34097

NASA Contractor Report 3169

A Simplified Computer Program for the Prediction of the Linear Stability Behavior of Liquid Propellant Combustors

C. E. Mitchell and K. Eckert
Colorado State University
Fort Collins, Colorado

Prepared for
Lewis Research Center
under Grant NGR-06-002-095

National Aeronautics
and Space Administration

**Scientific and Technical
Information Branch**

1979

TABLE OF CONTENTS

NOMENCLATURE

Letters

a - speed of sound

AMF - aperture mean flow

BR - backing distance for slot absorber

F - quantity for integral governing equation

f - quantity for integral governing equation

G_N - modified Green's function, defined after Equation 15

i - $\sqrt{-1}$

J_m - Bessel Function of first kind of order m

K - acoustic impedance of slot absorber

L - nondimensional chamber length

ℓ - radial acoustic mode assumed

L_a - actual length of aperture for slot absorber

L_{eff} - effective length of aperture for slot absorber

M - mean flow Mach number

m - transverse acoustic mode assumed

$\dot{m}$ - mass generation rate

n - longitudinal acoustic mode assumed

n - interaction index

$\vec{n}$ - outward directed normal unit vector

P - pressure

r - radial dimension

r_c - radius of chamber

R_0 - resistance of slot absorber

S - surface of combustion chamber over which integration is to be carried out

T - temperature

t - time

u_θ - normal component of velocity oscillation in tangential direction

u_r - normal component of velocity oscillation in radial direction

u_t - transverse velocity

V - velocity or volume of combustion chamber

Wa - aperture width for slot absorber or length of acoustic liner

X_a - distance from injector face to beginning of slot absorber or acoustic liner

X_b - distance from injector face to end of slot absorber or acoustic liner

z - longitudinal dimension.

<u>Greek Letters</u>

β - acoustic admittance of a surface

γ - ratio of specific heats

ε - nondimensional wave amplitude

η - acoustic eigenvalue

θ - angle in radians

Λ - normalization factor defined after Equation 15

$\lambda_{\ell m}$ - root of Bessel Function of first kind, such that $J_m'(\lambda_{\ell m}) = 0$

μ - coefficient matrix

ρ - density

τ - sensitive time lag

Φ - velocity potential

ψ - normalization factor defined after Equation 15

$\Omega_{\ell mn}$ - acoustic eigenfunction

ω - complex frequency

Superscripts

$\rightarrow$ - vector quantities

$*$ - dimensional quantity

$'$ - derivative with respect to argument, or perturbation quantity

$-$ - mean or steady state quantity

Subscripts

I - injector

L - liner

N - nozzle

INTRODUCTION

The purpose of this report is to present an analytical technique and a computer program which can be used for the prediction of the linear stability behavior of liquid propellant combustors. The technique involved has been developed over the last few years at Colorado State University in the examination of several aspects of the instability problem. Basically, the approach employs a Green's function integral method in the iterative determination of combustor frequency, decay rate and spatial waveform.

This general approach has been applied to several different combustor models in the examination of different aspects of the linear instability problem. (Ref. 1-10.) This work was performed by several different people (mainly graduate students), and a wide variety of nomenclature and programming techniques has resulted. The details of the analytical approach have also varied from author to author though the general method remained the same. This more or less comprehensive compendium of programs and analyses as it exists in its several forms is cumbersome, somewhat redundant, and certainly hard to use as a designer's tool.

With this in mind it was decided to develop a simplified stability analysis and computer program which contained the most important features of the earlier work in a format that would be relatively easy to use. Consequently, the main goal of this effort has been the development of a computer program simple enough to be used effectively by a person without an exhaustive background in either advanced mathematics or stability theory.

In order to do this some compromises have had to be made as far as comprehensiveness and accuracy are concerned, and some aspects of the stability problem treated previously have not been included. For example, the effect of distributing combustion sources along the combustor axis (as opposed to having a concentrated combustion zone near the injector) on overall stability has been studied and analyzed using two different approaches (Ref. 8, 9). This effect is not included in the simplified model presented here, however. The justification for this is based on the fact that much greater complexity is introduced into both the analysis and the computer program when distributed sources of combustion are considered, while the qualitative stability behavior is very similar to that predicted for concentrated combustion. Moreover, the quantitative effect of distributing the combustion is stabilizing relative to the predictions for a concentrated combustion zone. Thus, the simplified model presented here will tend to give conservative estimates of combustor stability when the combustor being examined has its combustion zone well distributed (axially).

Other effects such as irrotationality, entropy variations, and droplet drag effects have also been ignored since their influence has been found to be small, stabilizing or both.

The body of the report is divided into three main sections. The first (called "Theory") presents the model and method of analysis. The second section (called "Computational Methods") presents the basics of the computational method and the user options available. The final section (called "Program MODULE") gives a user's manual, sample input and output and a flow chart. It is not necessary for a person wishing to use the computer program (MODULE), to follow the analytical details of the first section. It will be necessary, however, for him to understand the basics of the model and general method of approach as presented in that section so that appropriate input to the program may be made and correct interpretation of the output can result.

THEORY

Combustor Model

The motor configuration considered here is characterized by circular cylindrical geometry, a concentrated combustion zone located at the injector end of the combustor, a nozzle at the opposite end, and either an accustic liner or a slot absorber located in the cylindrical walls. A sketch of the combustor model is given in Figure 1. In the development of a linear stability model for a combustor of this type it is first necessary to represent the four main features of the configuration using appropriate mathematical models. The four aspects of the problem requiring such modeling are

1) The gasdynamic flow field
2) The combustion zone
3) The nozzle
4) The acoustic liner or absorber.

Each of these will be discussed separately before going on to a presentation of the global stability model and analytical technique.

1) The gasdynamic flow field

The flow field downstream of the concentrated combustion zone is taken to consist of a single component, single phase product gas which is non-conducting, inviscid and calorically perfect. The flow is assumed to be homentropic and irrotational. As long as the combustion zone is concentrated and pressure waves are of small amplitude, it has been shown that these approximations are not severely limiting and self-consistent (Ref. 11, 12). Before presenting the equations describing this flow field the relevant state and flow variables are non-dimensionalized as follows.

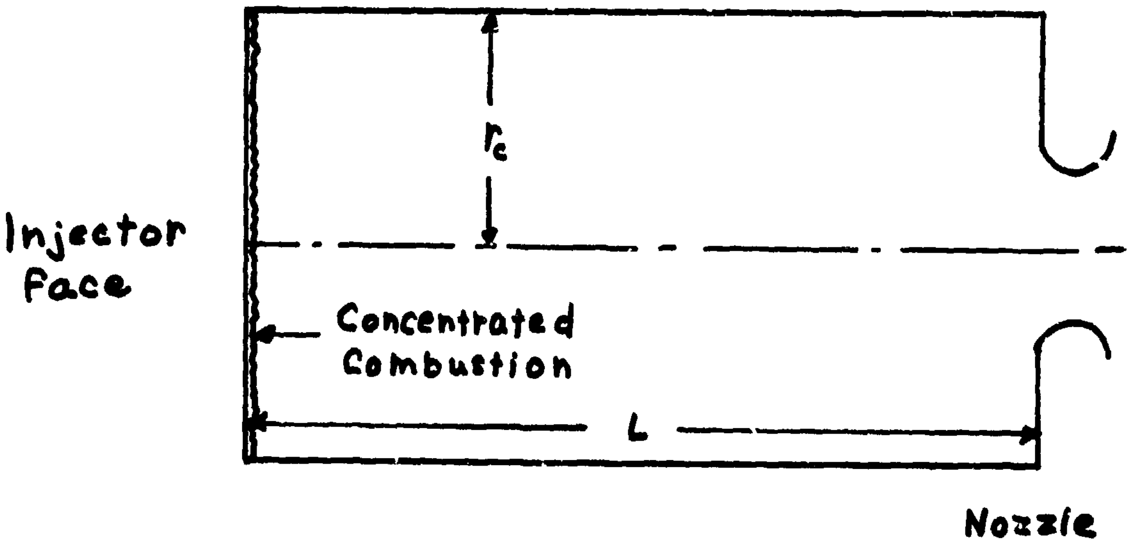

Figure 1. Combustion Chamber

$\rho = \rho^*/\bar{\rho}^*$, $T = T^*/\bar{T}^*$

$\vec{V} = \vec{V}^*/\bar{a}^*$, $P = P^*/\bar{P}^*$

The independent variables are nondimensionalized as follows.

$t = t^*(r_c^*/\bar{a}^*)$, $r = r^*/r_c^*$

$z = z^*/r_c^*$

where * denotes dimensional quantities and $\bar{}$ denotes mean chamber values. Using this nondimensional scheme the conservation equations become

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0 \qquad \text{CONTINUITY} \qquad (1)$$

$$\rho\frac{D\vec{V}}{Dt} + \frac{1}{\gamma}\vec{\nabla}P = 0 \qquad \text{MOMENTUM} \qquad (2)$$

$$P = \rho^\gamma \qquad \text{HOMENTROPIC} \qquad (3)$$

$$P = \rho T \qquad \text{STATE} \qquad (4)$$

$$\vec{V} = \vec{\nabla}\Phi \qquad \text{IRROTATIONALITY} \qquad (5)$$

Under the assumption of small amplitude oscillations the state and flow variables are represented as the sum of a mean (steady state) component and an oscillatory component, products of which are ignored as being higher order terms. Thus

$P = 1 + p'$ $\qquad \Phi = Mz + \phi$

$\rho = 1 + \rho$

$T = 1 + T'$ $\qquad \vec{V} = Me_z + \vec{\nabla}\phi$

where $\hat{e}_z$ is the unit vector in the axial direction and M is the mean flow Mach number.

After some manipulation the conservation equations can be reduced to a simple scalar partial differential equation

$$\nabla^2\phi - \frac{\partial^2\phi}{\partial t^2} = 2M\frac{\partial^2\phi}{\partial t\partial t} + M^2\frac{\partial^2\phi}{\partial z^2} \tag{6}$$

The equation relating the state variables to ϕ is

$$p' = -\gamma\left(\frac{\partial\phi}{\partial t} + M\frac{\partial\phi}{\partial z}\right) \tag{7}$$

Periodic oscillations in time are assumed so that

$$p' = p(r, \theta z)\, e^{i\omega t}$$
$$\phi = \phi(r, \theta, z)\, e^{i\omega t}$$

where $\omega = \omega_R + i\lambda$ is the complex frequency, ω_R the frequency, λ the decay rate. (If $\lambda > 0$ decay occurs.)

2) Combustion zone response model

It is assumed that all combustion occurs in a length small compared with the combustor's axial dimension. In the steady state mass is produced at the rate $\bar{\dot{m}} = M$ in the nondimensional system used here. No attempt to describe the details of the combustion process is made. Instead it is simply assumed that the combustion zone is sensitive to pressure oscillations and responds to these oscillations through a combustion zone admittance function β_I. Thus

$$\vec{\nabla}\phi \cdot \vec{n} \quad \beta_I\, p' \quad (z = 0) \tag{8}$$

β_I is taken to be a constant for the entire combustion zone, though p' is, of course, a function of r and z as well as time. In terms of the mass perturbation rate, $\dot{m}'$, the response condition is

$$\dot{m}' = (\frac{M}{\gamma} - \beta_I)\, p' \qquad (9)$$

β_I is, in general, complex so that all phasings between $\dot{m}'$ (or u') and p' are possible. Note that if the real part of β_I is greater than $\frac{M}{\gamma}$, the combustion zone provides a damping rather than driving effect.

It is also possible to relate β_I to the interaction index n, and time lag τ of the Crocco sensitive time lag model. The appropriate relationship is

$$\beta_I = M(\frac{1}{\gamma} - n\,(1 - e^{-i\omega\tau})\,)\ . \qquad (10)$$

Values of β_I (or n, and τ) must be supplied by the program user or calculated as output, given all other parameters. These options will be discussed later.

3) Nozzle model

Here again no attempt is made to investigate the details of the nozzle flow and, instead, a nozzle admittance function β_N is used.

$$\vec{\nabla}\phi \cdot \vec{n} = \beta_N\, p' \quad (z = L) \qquad (11)$$

Values of β_N are to be supplied by the user. Tables of admittance functions are given in Ref. (13), for example, for conical nozzles. In the absence of any knowledge of the nozzle response value it is suggested that the simple "short" nozzle value

$$\beta_N = M\left(\frac{\gamma-1}{2\gamma}\right)$$

be used.

4) Acoustic liner or slot absorber model.

Two possibilities are considered. The first is an acoustic liner of uniform average admittance, β_L , which is uniform in the azimuthal (θ) direction and extends along the cylindrical wall from $z = x_A$ to $z = x_B$. For this liner the appropriate boundary condition is

$$\vec{\nabla}\phi \cdot \vec{n} = \beta_L p' \tag{12}$$

No attempt is made to calculate β_L in either the analysis or computer program and therefore β_L must be supplied by the user.

The second absorber configuration considered is a circumferential slot machined into the cylindrical wall of the chamber and acting as a Helmholtz resonator. The geometry assumed is shown in Figure 2. All dimensions are nondimensional through division with the chamber radius.

The appropriate boundary condition at $r = 1$ (chamber wall) over the aperture width W_A $(=[x_B-x_A])$

$$\vec{\nabla}\phi \cdot \vec{n} = \beta_L p$$

or $$\vec{\nabla}\phi \cdot \vec{n} = \frac{1}{\gamma K} p$$

where $K = \frac{1}{\gamma\beta_L}$ is the impedance at the aperture entrance. K is used in this case to be consistent with existing treatments of Helmholtz resonators of this general type. β_L and K are, in general, complex with $K = R_0 + ik$, where R_0 is the resistance, k the reactance. Standard relationships for R_0 , resistance and L_{eff} taken from Reference (15) and (16) respectively, are given below.

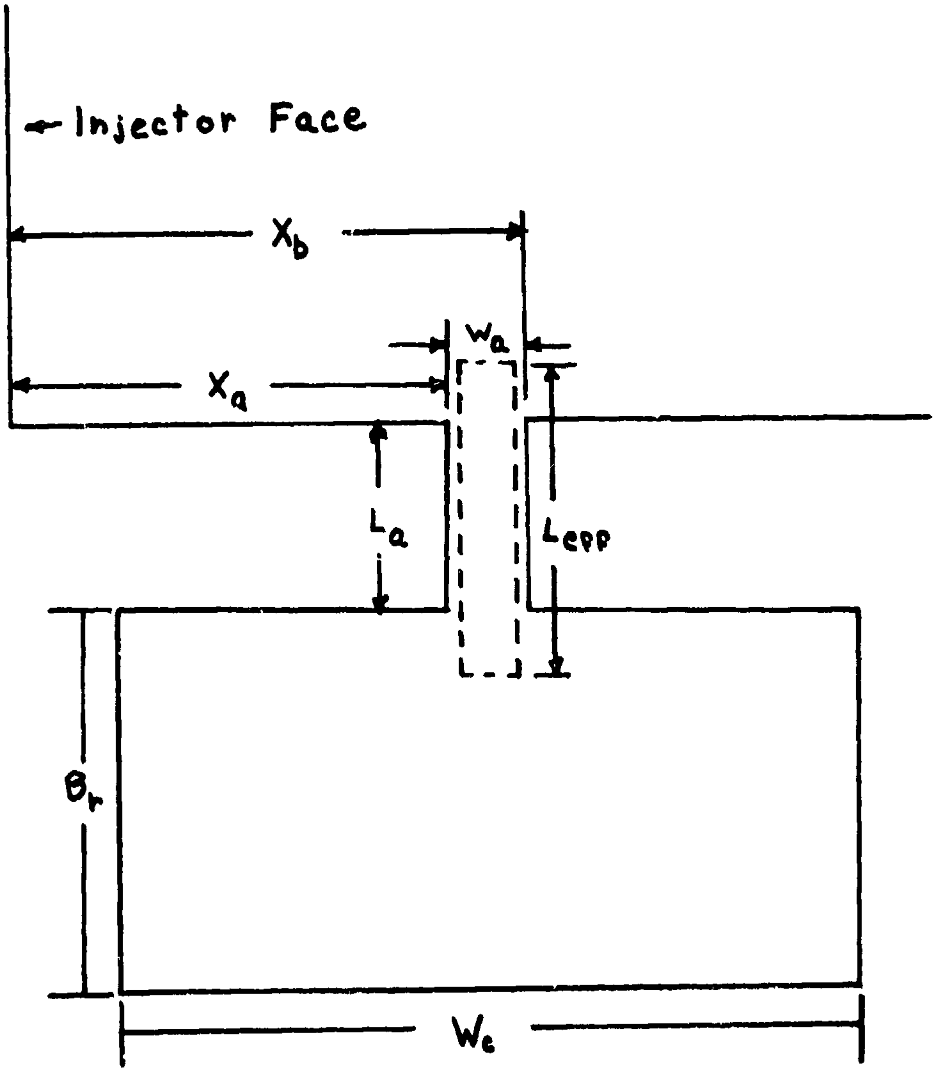

Figure 2. Slot Absorber Geometry

$$R_o = \left[.8\left(1.5(\mathrm{AMF})R_o + \frac{\varepsilon|p|}{\gamma\left(1 + \frac{k^2}{R_o{}^2}\right)^{\frac{1}{2}}}\right)\right]^{\frac{1}{2}} \qquad (15)$$

where ε is wave amplitude and p is the modulus of the pressure at $r = 1$.

$$L_{eff} = L_A + (0.375)(0.85)W_A\left[1 - 0.7\left(\frac{W_A}{W_C}\right)^{\frac{1}{2}}\right] \qquad (5)$$

An expression for the reactance k, comes directly from linear Helmholtz resonator theory and is given by

$$k = \omega_R\, \bar{\rho}_{ap}\, L_{eff} - \frac{W_A\, \bar{a}_a^2\, \bar{\rho}_a}{\omega_R\, W_C\, BR}$$

where $\bar{\rho}_{ap} = \left(\frac{\bar{\rho}^*_{ap}}{\bar{\rho}^*}\right)$, $\bar{a}_a^2 = \left(\frac{\bar{a}^*_a}{\bar{a}^*}\right)$, $\bar{\rho}_a = \left(\frac{\bar{\rho}^*_a}{\bar{\rho}^*}\right)$ are, respectively,

the nondimensional aperture density, absorber cavity mean sound speed, and absorber cavity density, and ω_R is the real part of the oscillation frequency. All these quantities, as well as the geometrical quantities in Fig. 2, AMF, and the assumed wave amplitude, ε, must be supplied by the user. The expression for R_o, Equation (4), is then solved iteratively for R_o as a function of frequency. Thus, an expression for $K(\omega)$ (or $\beta_L(\omega)$) is found numerically.

Method of Solution

The governing partial differential equation, Equation (6) along with the necessary boundary conditions (Equations (8) (10) (12) or (13)) are transformed to integral form using a Green's function, and the resulting integral equations are solved iteratively. Details of the transformation and solution method are presented in References (1, 2, 7). Only those relationships, definitions, and equations necessary for understanding and using the computer program which determines combustor stability will be presented here.

The transformed integral equations for $\phi(r, \theta, z)$ and ω are

$$\phi = \Omega_{\hat{\ell}\hat{m}\hat{n}} + \iiint_{V_0} G_N(\vec{r}/\vec{r}_0)\, F_1(\phi)\, dV_0 + \iint_{S_0} G_N(\vec{r}/\vec{r}_0)\, f_1(\phi)\, dS_0 \tag{14}$$

$$\omega^2 - \eta^2_{\hat{\ell}\hat{m}\hat{n}} = \iiint_V \Omega_{\hat{\ell}\hat{m}\hat{n}}\, F_1(\phi)\, dV + \iint_S \Omega_{\hat{\ell}\hat{m}\hat{n}}\, f_1(\phi)\, dS \tag{15}$$

where $F_1(\phi) = 2i\omega M \frac{\partial \phi}{\partial z} + M^2 \frac{\partial^2 \phi}{\partial z^2}$

$f_1 = -\beta p$

$\beta = \beta_N$ at nozzle $(z = L)$

$\beta = \beta_I$ at combustion zone $(z = 0)$

$\beta = \beta_L$ (or $\frac{1}{\gamma K}$) at liner (or absorber) $(r = 1)$

$\beta = 0$ on all other surfaces

$$G_N(\vec{r}/\vec{r}_0) = \sum_{\ell mn}\sum\sum \frac{\Omega_{\ell mn}(\vec{r})\ \Omega_{\ell mn}(\vec{r}_0)}{(\omega^2 - \eta^2_{\ell mn})}$$

$\ell \neq \hat{\ell}$. $m \neq \hat{m}$, $n \neq \hat{n}$, simultaneously

$$\Omega_{\ell mn} = \frac{J_m(\lambda_{\ell m} r) \cos \frac{n\pi z}{L} \cos m\theta}{\Lambda_{\ell mn}^{1/2}}$$

$$\Lambda_{\ell mn} = \iiint_V \left[J_m(\lambda_{\ell m} r) \cos \frac{n\pi z}{L} \cos m\theta\right]^2 dV$$

$\lambda_{\ell m}$ are the roots of $J'_m(\lambda_{\ell m}) = 0$

$$\eta^2_{\ell mn} = \lambda^2_{\ell m} + \left(\frac{n\pi}{L}\right)^2$$

$\Omega_{\ell mn}$ are the normalized eigenfunctions for a cylindrical chamber with no mean flow and non reactive walls. ℓ, m, n are the set of integers giving the radial, azimuthal, and axial character of the particular eigenfunction (or acoustic mode) in question. Thus, Ω_{110} represents a first transverse mode, Ω_{120} a second transverse mode, Ω_{200} a first radial mode, Ω_{001} a first axial mode, Ω_{111} a combined first transverse first axial mode, etc. The associated eigenvalues (acoustic frequencies) are

$$\eta^2_{\ell mn} = \lambda^2_{\ell m} + \left(\frac{n\pi}{L}\right)^2 .$$

The solution technique revolves around the assumption that the actual solution including mean flow and reactive walls has a character that is reasonably close to one of these acoustic modes. The particular acoustic mode most characteristic of the overall oscillation is called $\Omega_{\hat{\ell}\hat{m}\hat{n}}$, where $\hat{\ell}$, $\hat{m}$, $\hat{n}$ are the associated indices giving the radial, azimuthal, and axial character. The related eigenvalue (acoustic frequency) is $\eta_{\hat{\ell}\hat{m}\hat{n}}$. A discussion of the selection of $\Omega_{\hat{\ell}\hat{m}\hat{n}}$ in applications will be given later.

The equation for ϕ, Equation (14), implies that ϕ takes the following form

$$\phi = \sum_{\ell n m}\sum\sum \mu_{\ell mn} J_m(\lambda_{\ell m} r) \cos \frac{n\pi z}{L} \cos m\theta$$

where the coefficient matrix $\mu_{\ell mn}$ is determined by evaluation of the integrals on the right hand side of Equation (14).

Because of the symmetry in the θ direction which results from the assumptions concerning the boundary conditions, the series in m actually contains only one term, $\hat{m}$.

Thus, for the model used here ϕ may be written

$$\phi = \sum_{\ell n}\sum \mu_{\ell n} J_{\hat{m}}(\lambda_{\ell\hat{m}} r) \cos \frac{n\pi z}{L} \frac{\cos \hat{m}\theta}{\varepsilon_\theta} \qquad (16)$$

where $\varepsilon_\theta^2 = \int_0^{2\pi} (\cos \hat{m}\theta)^2 d\theta$. Exactly the same coefficient matrix would result if traveling waveforms were assumed. In this case

$$\Phi = \phi(r,z)e^{i(\omega t + \hat{m}\theta)}$$

$$\phi = \sum_{\ell}\sum \mu_{\ell n} J_{\hat{m}} (\lambda_{\ell\hat{m}} r) \cos \frac{n\pi z}{L} \qquad (17)$$

and $\mu_{\ell n}$ would be identical to the standing wave matrix.

The matrix $\mu_{\ell n}$ and the complex frequency ω are determined by an iterative process. The lowest order guess for ϕ (or $\mu_{\ell n}$) is used in the integral expressions of Equations (14) and (15) to compute improved values for the $\mu_{\ell n}$ and ω. The process continues until successive iterations are invariant to some degree of accuracy. A natural choice for the lowest order estimate for ϕ would be $\Omega_{\hat{\ell}\hat{m}\hat{n}}$; the lowest order frequency would then be $\eta_{\hat{\ell}\hat{m}\hat{n}}$. Though these initial guesses will work in general, experience has indicated that convergence can be slow and matrix sizes large, particularly when the mean flow Mach number is greater than about 0.3. Better convergence and a smaller matrix size are possible if the separation of variables solution for a combustor with mean flow but without an absorber is used. This solution was originally developed by Priem and Rice (Ref. (14)); in the modified form appropriate here it is discussed in References (1) and (2). The computer program presented later uses this form as the lowest order ϕ .

In addition to assuming a lowest order form for ϕ and ω and iterating, it is also possible to fix ω at some prescribed value (supplied by the user) and iterate to find the appropriate $\mu_{\ell n}$ and β_I (or n, and τ) from the same equations. The latter approach is used to solve for the combustion response necessary to sustain an oscillation of a given frequency and decay (growth) rate and known absorber and nozzle admittances. It would also be possible to set up the technique to solve iteratively for another parameter, such as nozzle admittance, for given combustion admittance; however, this has not been done in the program presented here.

COMPUTATIONAL METHODS

As discussed in the "Theory" section, Equations (14) and (15) are set up for iterative solution. Computer program MODULE is an algorithm for performing the necessary iterative computations on a digital computer. Several different choices are possible as far as input, output, and accuracy are concerned. These choices will be discussed in this section.

Matrix Sizing and Program Convergence

In the solution of Equations (14) and (15) two variables are always iterated. One of these is the perturbation velocity potential $\phi(r,\theta,z)$. The other is either the complex frequency ω $(\omega_R + i\lambda)$ or the complex combustion admittance, β_I (real (β_I) + i imag (β_I)).

The perturbation velocity potential is represented by a series expansion (Equation (16)).

$$\phi(r,\theta,z) = \left[\sum_{\ell n}\sum \mu_{\ell n} J_{\hat{m}} (\lambda_{\ell \hat{m}} r) \cos \frac{n\pi z}{L} \right] \frac{\cos \hat{m}\,\theta}{\varepsilon_\theta}$$

Thus, solution for the coefficient matrix $\mu_{\ell n}$ yields $\phi(r,\theta,z)$ and, in fact, it is this matrix which is the actual iterated variable in the solution algorithm. Formally, $\mu_{\ell n}$ is doubly infinite in ℓ and n. That is, $1 \le \ell < \infty$, $o \le n < \infty$. As a practical matter, however, limits on the largest values ℓ and n may take (in other words the dimensions of matrix $\mu_{\ell n}$) must be determined. It should be recalled here that the integers n are associated with axial dependence (through $\cos \frac{n\pi z}{L}$) while the integers ℓ are associated with radial dependence (through $J_{\hat{m}} (\lambda_{\ell \hat{m}} r)$).

Any choice for the maximum number of "ℓ terms" and "n terms" will limit accuracy. A compromise between program run time, storage requirements, and accuracy is desirable. Naturally, no one choice will be optimal for all combustor configurations. However, hundreds of runs with "typical" designs have indicated some rules of thumb to be used.

First of all, in none of the combustors investigated was any significant increase in accuracy obtained by keeping more than 50 terms in the axial direction or ten terms in the radial direction. That is, keeping 100 terms in the axial direction or 20 terms in the radial direction affected the values of the iterated variables only very slightly (<0.25%). Consequently, the program as written accepts a 10 x 50 matrix size for $\mu_{\ell n}$ as the maximum allowable. In the program variables this means LTS $\leq$ 10, NTS $\leq$ 50, where LTS and NTS are, respectively, the number of terms in the "ℓ" direction and the number of terms in the "n" direction.

The question as to the "best" values of LTS and NTS to use in a given combustor configuration is difficult to answer. Eckert (Ref. (14)) has studied optimal values for LTS and NTS for a "typical" configuration and suggests values of 3 for LTS and 16 for NTS. However, for a combustor with no absorber, a single term ($\hat{\ell}$) is necessary for description of the radial field and LTS = 1 in this case. On the other hand, if the Mach number is small, mean flow effects are less important and fewer terms in the axial direction (smaller NTS) would be needed. However, for configurations with large absorber effects or high Mach numbers (> 0.4) it is likely that "best" values for LTS and NTS could be greater than 3 and 16, respectively.

With this in mind it is suggested that the values LTS = 3 and NTS = 16 be used as a general rule. If strong absorber or high Mach number effects are present and may compromise accuracy, it is suggested that results with LTS = 9 and NTS = 50 be computed and compared with the smaller matrix results to estimate accuracy. Values of LTS and NTS larger than 3 and 16 could then be inserted until the desired accuracy relative to the 9 x 50 size was obtained. It should be noted that improvement in accuracy is monotonic with increasing NTS. The same is not true for LTS because of the alternating nature of the series involved and best results occur if LTS is an odd number (3, 5, 7, 9).

Once the dimensions of the $\mu_{\ell n}$ matrix are determined it is next necessary to decide upon an acceptable convergence condition for the iteration process. The second iterated variable (either ω or β_I depending upon the application) is used to do this. Successive values of the iterated variable are compared. When the difference between the two values is less than some value, adequate convergence is assumed. Since both ω and β_I are complex numbers, it is necessary that both the real and imaginary parts converge in the sense just mentioned. In this program, however, it is convenient instead to deal with ω (or β_I) in complex polar notation, and require that successive values of the modulus and phase angle converge. This is because the phase angle is frequently near zero and can cause problems in the definition of convergence for the imaginary part of the iterated variable. In program MODULE convergence is assumed when the percent change in the modulus of the iterated variable is less than the value ERROR and, at the same time, the absolute value of the change in the phase angle is also less than ERROR. ERROR can take values between 10^{-5} and 1.0.

In most cases convergence to within 0.1% or less is rapid, usually occurring in ten iterations or less. However, for some choices of parameters and for some program options it can be much slower or not occur at all. For this reason a maximum desired number of iterations must be specified. This is done through program variable IDMAX which can take any integer value. If convergence does not occur in the number of iterations specified by IDMAX, the iterative loop terminates, and program values at the last iteration are output.

Program Options

In addition to choosing either ω or β_I as the iterated variable, choices are possible as far as the form of the combustion response model and the acoustic absorber. Taken together this results in six distinct ways of running the program. These possibilities are labelled options and are described sequentially below. For all of the options it is necessary that certain design or program variables be specified by the user. These parameters are γ (ratio of specific heats), M (mean flow Mach number), L (chamber length to radius ratio), β_N (complex nozzle admittance), ERROR (maximum error allowable in determining convergence), LTS (number of terms in radial direction kept), and NTS (number of terms kept in the axial direction).

Option 1

This option is designed to compute frequency and decay rate (complex frequency) for known combustion zone admittance and known acoustic absorber (or liner) length and admittance. The iterated variable is the complex frequency. Required to be input to the program are β_I , β_L , X_A and X_B . X_A and X_B are the nondimensional distances to the start and end of the acoustic absorber, respectively. Output are ω_R and λ, $\mu_{\ell n}$, the input parameters, and n and τ , the interaction index and time lag corresponding to the given β_I and the converged value for ω .

Option 2

Option 2 is similar to Option 1 except that the combustion response is described by n and τ instead of β_I. In this case n and τ are input and β_I is output. Other input and output parameters are the same as for Option 1.

Option 3

In this option the acoustic absorber is of the slot design type described earlier. The combustion response is described by β_I, and ω is the iterated variable. Required input variables are β_I, β_R (absorber backing distance), W_c (absorber cavity width), L_a (absorber aperture length), X_A, X_B, $\bar{a}_a$, (ratio of sound speed in the cavity to sound speed in the main chamber), $\bar{\rho}_a$ (nondimensional aperture gas density), AMF (aperture mean flow), and ε (wave amplitude of the oscillation). Output variables are ω, n and τ, $\mu_{\ell n}$ and β_L, the equivalent absorber admittance for the given geometry.

Option 4

This option is the same as Option 3 except that n and τ are input and β_I is output.

Option 5

The last two options use β_I as the iterated variable. They are most useful in generating stability maps in terms of n and τ (Option 5) or real (β_I) and imag (β_I), (Option 6). Examples of such stability maps are presented in References 1, 2, 3 and 14. Frequency is used as parameter along these curves.

Option 5 is designed to compute β_I for a given complex frequency and a slot absorber. All the slot absorber parameters necessary for Option 3 must be supplied here as well, in addition to the complex frequency. Output includes $\mu_{\ell n}$, β_I , n and τ , and β_L , the equivalent liner admittance.

<u>Option 6</u>

This option also uses β_I as the iterated variable. In this option, however, the absorber is characterized by an admittance, β_L , and a length $W_a = X_B - X_A$. For this option ω , β_L , X_A and X_B must be supplied and output will give β_I , $\mu_{\ell n}$, and n and τ .

A summary of the principal input and output variables for the six options is given in Table 1 below.

TABLE 1

OPTION	ONE	TWO	THREE	FOUR	OUTPUT
1	real(β_I)	imag(β_I)	real(β_L)	imag(β_L)	ω , n & τ
2	n	τ	real(β_L)	imag(β_L)	ω , β_I
3	real(β_I)	imag(β_I)	BR	AMF	ω , n & τ, β_L
4	n	τ	BR	AMF	ω , β_I , β_L
5	real(ω)	imag(ω)	BR	AMF	β_I , n & τ , β_L
6	real(ω)	imag(ω)	real(β_L)	imag(β_L)	β_I , n & τ

For convenience four main input variables are called ONE, TWO, THREE and FOUR both in the program and in the table. These variables represent different quantities in the different options. For example, in Option 1 variable TWO represents the imaginary part of β_I , whereas in Option 2 it represents τ , the time lag.

Choice of Fundamental Acoustic Mode

As was mentioned in the "Theory" section, the success of the iteration process revolves around the assumption that the oscillation with active walls and mean flow is similar to one of the normal acoustic modes (no flow, hard walls) of the combustion chamber. The most useful variable for determining the suitability of an acoustic mode choice is the real part of the complex frequency. As a general rule, when the frequency of oscillation in the combustor is within 10% of a particular acoustic mode frequency, convergence will usually occur if that acoustic mode is used for $\Omega_{\hat{\ell}\hat{m}\hat{n}}$ in the iterative process. Since the imaginary part of the frequency can be as large as the deviation of the frequency from its acoustic value, nondimensional decay (or growth) rates as large as 0.20 (of the order of 1000 sec^{-1} for typical $\bar{a}^*$ and R^*) can occur for these conditions.

For lower acoustic modes there is considerable separation in frequencies. At higher frequencies a given frequency may be close to two (or more) acoustic modes. In this latter case convergence problems can occur and it may be necessary to test all of the possible acoustic modes sequentially. Experience with the program must be the guide in these cases.

For many (if not most) applications the acoustic mode choice is clear. For example, suppose that in a given combustor of diameter 2 ft, length 2 ft and average sound speed, $\bar{a}^*$, of 3000 ft/sec, an oscillation of frequency 5700 sec^{-1} were observed. The real part of the nondimensional frequency would be $\omega_R = (5700)/3000 = 1.90$. This value is within 10% of 1.841, the acoustic frequency of the first transverse mode. $(J_1(\lambda_{11}r)$, $\ell = 1$, $m = 1$, $n = 0$, $\eta_{\ell mn} = \lambda_{11})$ Hence, when investigating this oscillation using the iterative model, $\Omega_{\hat{\ell}\hat{m}\hat{n}} = \Omega_{110}$ (i.e., $\hat{\ell} = 1$, $\hat{m} = 1$, $\hat{n} = 0$). Indeed, the choice of mode would be the same for frequencies between about 4970 sec^{-1} and 6075 sec^{-1}. On the other hand, if the observed frequency were 6900 sec^{-1} the oscillation would be closer in frequency to the combined first transverse, first longitudinal mode frequency of 7260 sec^{-1} ($J_1(\lambda_{11}r)$ $\cos\frac{\pi z}{L}$, $\ell = 1$, $m = 1$, $n = 1$, $\eta^2_{\ell mn} = \left(\lambda^2_{11} + \frac{\pi^2}{L^2}\right)$) and Ω_{111} ($\hat{\ell} = 1$, $\hat{m} = 1$, $\hat{n} = 1$) should be used. When the frequency of oscillation is "in between" two acoustic frequencies and is within 10% of neither, it is usually best to pick the higher mode. For the example given, if $\omega_R =$ 6300 sec^{-1} it would be within 10% of neither the pure first transverse frequency nor the combined first transverse, first longitudinal frequency. The best choice in this case would be Ω_{111} rather than Ω_{110}.

The choice of acoustic mode is input to the program through the choice of the three integers, $\hat{\ell}$ (radial), $\hat{m}$ (azimuthal), $\hat{n}$ (axial). In the program these are called LHAT, MHAT, and NHAT, respectively.

REFERENCES

1. Mitchell, C. E., W. R. Espander, and M. R. Baer, "Stability of Combustors with Partial Length Acoustic Liners," NASA CR-120889, 1972.

2. Mitchell, C. E., W. R. Espander, and M. R. Baer, "Determination of Decay Coefficients for Combustors with Acoustic Absorbers," NASA CR-120836, 1972.

3. Baer, M. R. and C. E. Mitchell, "A Theoretical Evaluation of Rigid Baffles in the Suppression of Combustion Instability," NASA CR-134986, 1975.

4. Espander, W. R., "Partial Length Liners in Rocket Motors," M.S. Thesis, Dept. of Mechanical Engineering, Colorado State University, 1971.

5. Baer, M. R., "Combustion Instability in Rocket Motors with Distributed Combustion and Acoustic Liners," M.S. Thesis, Dept. of Mechanical Engineering, Colorado State University, 1973.

6. Baer, M. R., "A Theoretical Evaluation of Rigid Baffles in Suppression of Combustion Instability," Ph.D. Thesis, Dept. of Mechanical Engineering, Colorado State University, 1975.

7. Mitchell, C. E., "Stability of Combustors with Partial Length Acoustic Liners," Combustion Science and Technology, Vol. 6, pp. 61-70, 1972.

8. Baer, M. R. and C. E. Mitchell, "Stability of Partially Lined Combustors with Distributed Combustion," AIAA Journal, Vol. 12, No. 4, pp. 475-480, 1974.

9. Mitchell, C. E., "Stability Predictions for Combustors with Acoustic Absorbers and Continuous Combustion Distributions," AIAA Journal, Vol. XIII, No. 8, pp. 1107-1109, 1975.

10. Mitchell, C. E. and Y. Jotiban, "Velocity Dependent Combustion Stability," presented at the 13th JANNAF Combustion Meeting, Monterey, Calif., 1976.

11. Mitchell, C. E., "The Effect of Entropy Waves on High Frequency Pressure Oscillations in Liquid Rocket Motors," Combustion Science and Technology, Vol. 1, pp. 269-274, 1970.

12. Crocco, L. and W. A. Sirignano, Behaviour of Supercritical Nozzles under Three-Dimensional Oscillatory Conditions, AGARDograph No. 117, Butterworths, 1967.

13. Eckert, K. W., "Computer Programs for Linear and Nonlinear Analysis of Rocket Engine Stability," M.S. Thesis, Dept. of Mechanical Engineering, Colorado State University, 1978.

14. Harrje, D. T. ed., F. H. Reardon, assoc. ed., Liquid Propellant Rocket Combustion Instability, NASA SP-194, 1972.

15. Phillips, B., Hannum, N. P., and L. M. Russell, "On the Design of Acoustic Liners for Rocket Engines: Helmholtz Resonators Evaluated with a Rocket Combustor," NASA TN D-5171, April 1969.

Program MODULE

In this section a general description of the program will be given first. Next, discussions of input and output formats will be given. Finally, a sample run will be presented and discussed, and a complete program listing will be given.

General Description of Program

The structure of the programs is as follows. (See Figure 3 for a flow diagram and Table 2 for a listing of program nomenclature.) After the non-default type variables have been declared, and the matrices and arrays have been dimensioned, the values for constants (such as PI [π]) are stored. Next, values for the two program variables K and IDCR are stated. The values for the iterated variable and the percent error in the modulus, and the absolute change in angle, will be printed out the first, last and every K^{th} iteration. IDCR is an arbitrary number, after which a percent error in the modulus of over 50% will indicate that the problem is not converging. The values for the two constants K and IDCR (ID critical), may need to be changed, but it was felt they would not be changed often enough to warrant including them with the other input data.

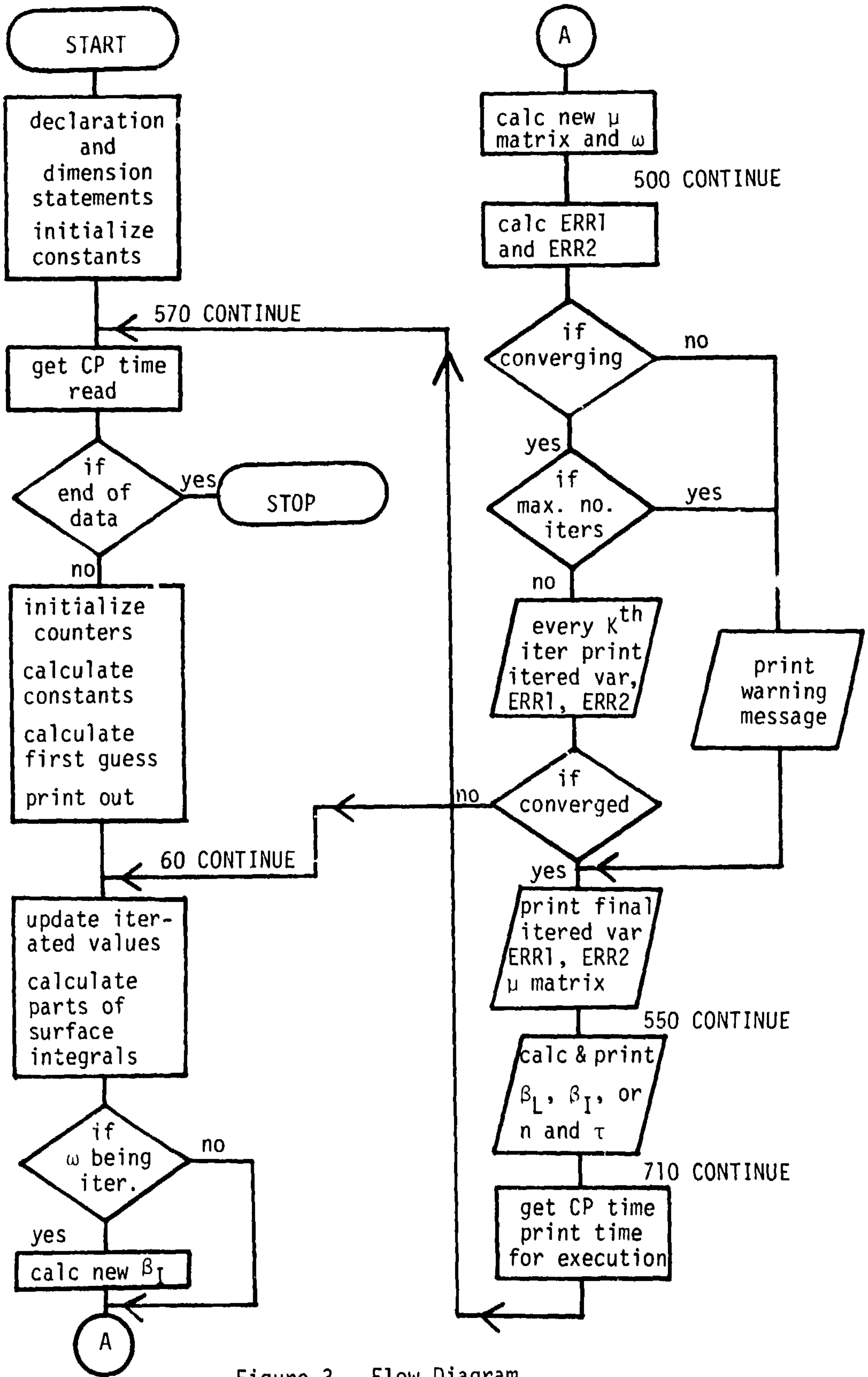

Figure 3. Flow Diagram

TABLE 2

Computer Program Nomenclature

Input Variables

AMF - aperture mean flow

ACAV - A_C nondimensional sound speed in slot absorber cavity

BETAI - β_I, acoustic admittance of injector

BETAL - β_L, acoustic admittance of liner

BETAN - β_N, acoustic admittance of nozzle

BR - backing distance

EPSIL - ε, amplitude of wave oscillation (only used in calculation of absorber resistance)

ERROR - acceptable % error in magnitude and absolute change in radians between two successive iterations

FOUR - OPTION = 1,2 or 6, imag(BETAL)
OPTION = 3,4 or 5, AMF

GAMMA - γ, ratio of specific heats

IDMAX - maximum number of iterations

IN - n, interaction index

K - iterated variable is output every K^{th} iteration

LENGTH - L, nondimensional chamber length

LHAT - $\hat{\ell}$, radial acoustic mode, integer

LTS - number of terms in radial direction

MACH - M, Mach number

MHAT - $\hat{m}$, transverse acoustic mode, integer

NHAT - $\hat{n}$, longitudinal acoustic mode, integer

NTS - number of terms in radial direction

ONE - OPTION = 2 or 3, real(BETAI)
OPTION = 2 or 4, IN
OPTION = 5 or 6, real(OMEGA)

OPTION - integer value between 1 and 6, sets which way the problem will be calculated

ROAP - $\bar{\rho}_a$, density in slot absorber aperture

ROCAV - $\bar{\rho}_c$, density in slot absorber cavity

TAU - τ , sensitive time lag

THREE - OPTION = 1, 2 or 6, real(BETAL)
OPTION = 3, 4 or 5, BR

TWO - OPTION = 1 or 3, imag(BETAI)
OPTION = 2 or 4, TAU
OPTION = 5 or 6, imag(OMEGA)

WCAV - nondimensional slot absorber cavity width

XA - distance from injector to beginning of acoustic liner

XB - distance from injector to end of acoustic liner.

Program Variables

A1 - ϕ evaluated at injector

A2 - ϕ evaluated at nozzle

BES - $\dfrac{J_m^2(\lambda_{\ell m})(\lambda_{\ell m}^2 - m^2)}{2\lambda_{\ell m}^2} = \int_0^1 J_m^2(\lambda_{\ell m} r) r dr$

BETAIN - new BETAI

CIOM - $i\omega$

DZP1L - $\dfrac{\partial\phi}{\partial z}$ evaluated at liner midpoint

ETA - $\eta_{\ell n}^2$

ETA1 - $\eta_{\hat{\ell}\hat{n}}^2$

ID - iteration counter

LAMDA2 - $\lambda_{\ell m}^2$ where, $J_m'(\lambda_{\ell m}) = 0$

LEFF - ℓ_{eff}

MU - μ matrix

MUX - old AMU

NORM - $\Lambda_{\ell n}^{1/2}$

NORM1 - $\Lambda_{\hat{\ell}\hat{n}}^{1/2}$

NPIL2 - $(\frac{\hat{n}\pi}{L})^2$

OMEGAN - new OMEGA

PIL - $\frac{\pi}{L}$

PIXL - $\frac{\pi XL}{L}$

PSI - Ψ

PIL - ϕ evaluated at liner midpoint

SINJ - $\iint dS_{INJ}$

SLIN - $\iint dS_{LIN}$

SNOZ - $\iint dS_{NOZ}$

VOL - $\iiint dV$

WA - W_a, width of liner aperture

WWO - $\frac{OMEGA}{ETA1}$

XL - distance from injector to midpoint of liner.

Output Variables (not defined above)

KI - Imaginary part of absorber impedance (reactance)

RO - R_0 real part of absorber impedance (resistance)

A point right after this (570 CONTINUE) is where the program returns to begin execution of a given set of data. The central processor (CP) time is stored at the beginning of execution of a set of data. This time is used to calculate the execution time for the set of data. Next, the data is read in and the counter ID is initialized. The constants for the set of data, such as $(\frac{n\pi}{L})$, are then calculated. The first guess for the iterated variable and the μ matrix (MU) are calculated, using the separation of variables solution. The input data and first guess are printed out. The setup is now complete, and each iteration returns to a point just below this (60 CONTINUE).

For each iteration the iterated variable is first updated, then variables that are functions of the iterated variable are updated. The μ matrix is stored in an extra matrix (MUX). The portion of the program from here to 500 CONTINUE, is designed to evaluate Equations (14) and (15), which give a new μ matrix and iterated variable, respectively. The next section, down to 550 CONTINUE, does the following: Calculates the percent error in the modulus (ERR1), and the absolute error in the angle (ERR2). Then checks to see if the problem appears to be converging (ID greater than IDCR and ERR1 greater than 50%), or has gone the maximum number of iterations. If neither of the above has happened, the iterated variable, ERR1 and ERR2 are printed out, if first or K^{th} iteration. Then checks to see if the problem has converged (ERR1 and ERR2 are both less than ERROR). If the problem has not converged, the program returns to 60 CONTINUE. If the problem is not converging, has gone the maximum number of iterations, or has converged, the final values for the iterated variable, ERR1, ERR2, and the final μ matrix, are printed out. If the problem does not converge, a message is printed out.

The next segment of the program, down to 710 CONTINUE, calculates and prints the other information that is to be output. If a slot absorber is used, an equivalent BETAL is calculated. If IN (n) and TAU (τ) are used, the equivalent BETAI is calculated; otherwise, the corresponding IN and TAU are calculated for the final OMEGA and BETAI. The CP time at the end of the program is stored. Running time is calculated and printed out. The calculations for this set of data are then complete and the program returns to 570 CONTINUE to begin calculations for the next set of data. If no more data is found, the program jumps to 6000 CONTINUE and stops without an error message.

The program is capable of handling up to 10 terms in the radial direction (LTS), 50 terms in the longitudinal direction (NTS), and a transverse mode as high as 4 (MHAT). This should be satisfactory for most cases. However, if the number of terms in the radial or transverse directions must be increased, the Bessel values and Bessel roots for higher modes must be added to subroutines BESVL and BESRT, respectively. Also, all relevant dimension statements must be increased. To increase the number of terms in the longitudinal direction, only the dimensions of MU and MUX must be increased. If MHAT and NHAT are zero (0), LHAT cannot be one (1). This is a trivial case.

The best compromise between good accuracy and fast running time (as discussed previously) occurs with 15-20 terms in the longitudinal direction (NTS), and 3 or 5 terms in the radial direction (LTS). LTS should always be odd, because the series has alternating signs in the radial direction. If the Mach number is greater than .40, more terms should be kept in the longitudinal direction.

Commonly, for liners covering less than one-third of the chamber walls, evaluating the integral over the surface of the liner, by evaluating at the midpoint and multiplying by the width, gives a good approximation to the integral with a big saving in running time. The program is set up to run this way. However, if it is desired to carry the integration out, replace the SLIN card with the CALL LINER card and include SUBROUTINE LINER. (See program listing after 40 CONTINUE.)

The final μ matrix, which is printed out, should always be checked to be sure that the term corresponding to the acoustic mode assumed is the largest term in the matrix. If this is not the case, the wrong primary acoustic mode has been assumed, and the answer is not a characteristic of the primary mode assumed.

Program Input

The input necessary and the formats for typing the data cards are listed in Table 3. Before the first data card can be typed it must be decided from what is known about the engine (or desired from the calculation) what value OPTION must take. It will be helpful to refer to the "Computational Methods" section in making this determination. Once the value of OPTION is fixed, Table 1 is used to determine which values the variables ONE, TWO, THREE and FOUR must take. The first data card can then be typed.

The second data card contains the model information which must be supplied regardless of option. All inputs are real numbers. The third card contains program variable information, including convergence and matrix size limitation information. All variables on card three are of the integer type. The fourth data card needs to be included <u>only</u> when

TABLE 3

The first card:

COLUMNS	VARIABLE	TYPE
1-20	ONE	Real number
21-40	TWO	Real number
41-60	THREE	Real number
61-80	FOUR	Real number

The second card:

COLUMNS	VARIABLE	TYPE
1-10	real(BETAN)	Real number
11-20	imag(BETAN)	Real number
21-30	GAMMA	Real number
31-40	MACH	Real number
41-50	LENGTH	Real number
51-60	XA	Real number
61-70	XB	Real number
71-80	ERROR	Real number

The third card:

COLUMNS	VARIABLE	TYPE
1-10	LHAT	Integer
11-20	MHAT	Integer
21-30	NHAT	Integer
31-40	LTS	Integer
41-50	NTS	Integer
51-60	IDMAX	Integer
61-70	OPTION	Integer

The fourth card:

COLUMNS	VARIABLE	TYPE
1-10	EPSIL	Real
11-20	ROAP	Real
21-30	ACAV	Real
31-40	ROCAV	Real
41-50	WCAV	Real
51-60	LA	Real

a slot type absorber is present in the combustor. All variables appearing on this card are real.

When setting up the program the correspondence between text and program variables given in Table 2 will be useful. Also, it should be remembered that all variables in the program and text are nondimensional.

Program Output

The primary outputs of MODULE are the matrix $\mu_{\ell n}$ and the iterated variable, either ω or β_I . Values for these quantities are computed at every step and ω (or β_I) is written out for every K^{th} iteration. K can be changed by the user by replacing a single card. In the program as presented here $K = 5$. The first and last iterations of $\mu_{\ell n}$ are also printed out. The first iteration is a solution with no liner effect, consequently all terms in $\mu_{\ell n}$ except $\mu_{\hat{\ell}\hat{n}}$ are null entries.

In addition, all model design variables are printed and labelled according to the program names of Table 2. The example to be discussed next demonstrates the typical form the output takes.

Sample Run

The combustor used for this sample run has a ratio of length to radius (LENGTH) of 2.0, a mean flow Mach number (MACH) of .3, and a ratio of specific heats (GAMMA) of 1.2. From the Mach number and ratio of specific heats, a nozzle response (BETAN) was calculated using the equation for a short nozzle given in the theory section of this report. The value for BETAN is .025 + 0.0i. There is an acoustic liner, of known admittance (BETAL), of .075 + 0.0i in place covering 10% of the cylindrical surface of the chamber, beginning one-tenth of the chamber length downstream of the injector face. Since space dimensions are nondimensionalized by dividing by the chamber radius, and the nondimensional chamber length is 2.0, this gives an Xa of .2 and an Xb of .4. The interaction index IN (n) and sensitive time lag TAU (τ) are known to be equal to .30097 and 2.219497, respectively, for the injector response. With the information known about the injector and liner responses and looking at Table 1, it is determined that OPTION must equal 2, and ONE is IN, TWO is TAU, THREE is real (BETAL), and FOUR is imag (BETAL). Table 1 shows that (ω) OMEGA, the nondimensional frequency and decay rate, and the effective BETAI for the injector will be calculated. The first transverse acoustic mode is chosen as the primary mode. This corresponds to LHAT = 1, MHAT = 1, and NHAT = 0. A good compromise between running time and accuracy was desired, so by referring to the discussion in this report, the number of terms chosen in the radial direction (LTS) is equal to 3, and the number of terms chosen in the longitudinal direction (NTS) is equal to 16. A high precision is desired, so ERROR is chosen as .01. The maximum number of iterations allowed for this set of data (IDMAX) will be 50. Since an acoustic liner is used, no fourth data card is needed. The input values are then typed up on three data cards, as described in Table 3. The cards as punched and submitted appear in Figure 4.

The output from the sample run is shown in Figure 5. The program first prints out the value of every variable that is input on the data cards. This is to allow for double checking, to be sure all the input data is correct, and also so there is a complete description of the rocket engine that was simulated. Next, the fundamental frequency for the primary mode assumed, and the first guess of the iterated variable, and the μ

matrix are printed. (These are the separation of variables values.) If OPTION is 5 or 6, the first guess of BETAI is printed under the injector response description. Next, the value of the iterated variable is printed for the first, every K^{th}, and last iteration. In the sample run $K = 5$, so the first, fifth and sixth (last) iteration values are printed, along with the errors in the modulus and phase angle of the iterated variable (ω in this case). The last $\mu_{\ell n}$ matrix (iteration 6) is then printed out. It can be seen that the term corresponding to $\ell = \hat{\ell} = 1$ and $n = \hat{n} = 0$ is, indeed, the largest. In fact, in this case it is an order of magnitude larger than any other matrix element. The other output information is then printed. In this case, the BETAI calculated from the input IN and TAU and the final OMEGA. The last thing printed out for each set of data is the beginning time TBG, ending time TEND, and execution time TEX, for this set of data.

Program Listing

A complete program listing is presented at the end of this report. Comment cards are used liberally and much of the program is self-explanatory. The computer program MODULE conforms to Fortran IV ANSI standards.

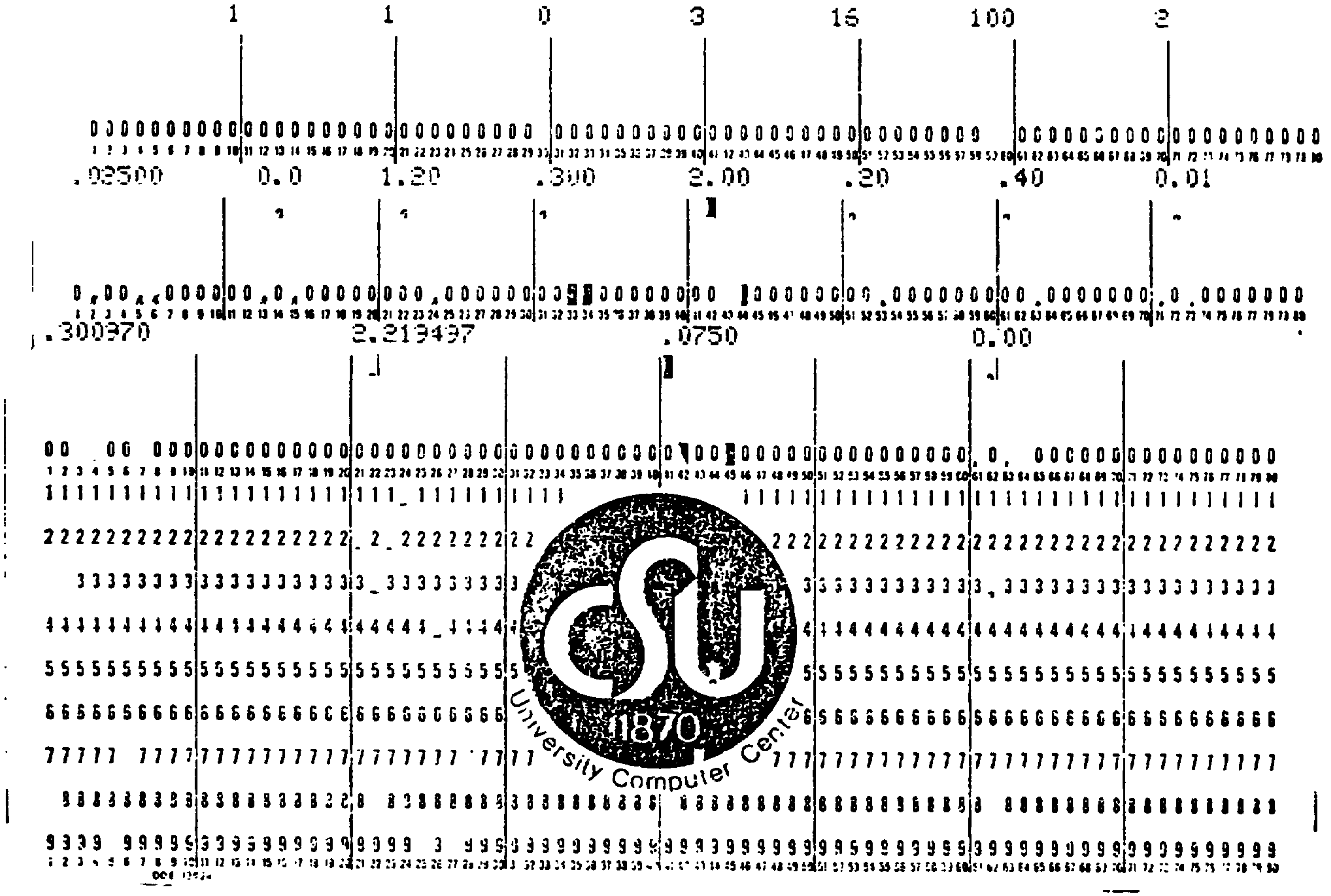

Figure 4. Data Cards for Sample Run

```
THE FOLLOWING CALCULATIONS ARE MADE FOR A COMBUSTOR WITH THE FOLLOWING CONFIGURATION

                         THE LENGTH TO RADIUS =  2.000000                          MEAN FLOW MACH NO. =  .300000
                         LHAT = 1      MHAT = 1      NHAT = 0                      OPTION =  2
                         GAMMA =  1.200000

THE ABSORBER HAS THE FOLLOWING GEOMETRY

                         THE LINER STARTS AT   .200000 AND ENDS AT   .400000      APERTURE WIDTH =   .200000

                         BETAL =   .075000    0.000000

THE INJECTOR RESPONSE IS DESCRIBED BY

                         FOR THE CROCCO SENSITIVE TIME LAG THEORY                  N =   .300970    TAU =  2.21949

                         INITIAL BETAI =   .100195     .067914

THE NOZZLE RESPONSE IS DESCRIBED BY

                         BETAN =   .025000    0.000000

MISCELLANEOUS INFORMATION FOR THIS PROBLEM

                         NUMBER OF TERMS    RADIAL   3    LONGITUDINAL 16          ITMAX =  100
                         ERROR =   .010000
                         THE FUNDAMENTAL FREQUENCY FOR THIS MODE IS  1.841184   0.0
                         THE FIRST GUESS OF OMEGA IS 1.798879     .000124


   THE FIRST GUESS FOR THE MU MATRIX IS AS FOLLOWS
     N                   L=1                          L=2                          L=3
     0      2.04682      0.            0.             0.            0.             0.
     1      .250455     -.114844       0.             0.            0.             0.
     2      .373181E-01 -.757680E-01   0.             0.            0.             0.
     3      .150049E-01 -.206982E-01   0.             0.            0.             0.
     4      .832489E-02 -.173684E-01   0.             0.            0.             0.
     5      .518523E-02 -.754186E-02   0.             0.            0.             0.
     6      .362518E-02 -.760205E-02   0.             0.            0.             0.
     7      .261627E-02 -.398588E-02   0.             0.            0.             0.
     8      .202474E-02 -.425354E-02   0.             0.            0.             0.
     9      .157549E-02 -.235504E-02   0.             0.            0.             0.
    10      .129159E-02 -.271562E-02   0.             0.            0.             0.
    11      .105224E-02 -.157797E-02   0.             0.            0.             0.
    12      .895343E-03 -.188335E-02   0.             0.            0.             0.
    13      .752381E-03 -.113039E-02   0.             0.            0.             0.
    14      .657049E-03 -.138253E-02   0.             0.            0.             0.
    15      .564553E-03 -.849324E-03   0.             0.            0.             0.
```

Figure 5. Output for Sample Run

```
ITER        REAL OMEGA       IMAG OMEGA         ERRORS        MODULUS            ANGLE

  1         1.79984          .162375E-01                      1.007752           .010132
  5         1.78849          .177707E-01                       .034144           .006235
  6         1.78837          .176785E-01                       .008533           .000051


 THE FINAL MU MATRIX IS AS FOLLOWS
   N                     L=1                                 L=2                                    L=3

   0     2.04682       0.             .177943E-03   .533416E-02  -.154135E-03  -.236943E-02
   1     .253107      -.205382        .568428E-03   .874843E-02  -.315714E-03  -.408957E-02
   2     .283958E-01  -.955142E-01    .751661E-03   .465732E-02  -.293297E-03  -.246717E-02
   3     .112628E-01  -.230215E-01    .614310E-03   .940368E-03  -.221466E-03  -.570694E-03
   4     .706775E-02  -.164754E-01    .305628E-03  -.135034E-02  -.447724E-04   .970463E-03
   5     .449316E-02  -.514159E-02    .450557E-04  -.232432E-02   .297045E-04   .146020E-02
   6     .355532E-02  -.550611E-02   -.117743E-03  -.239017E-02   .124761E-03   .208678E-02
   7     .252901E-02  -.212943E-02   -.179382E-03  -.193983E-02   .163614E-03   .191175E-02
   8     .210348E-02  -.331027E-02   -.171988E-03  -.125494E-02   .159652E-03   .124569E-02
   9     .152422E-02  -.194670E-02   -.125628E-03  -.572083E-03   .115600E-03   .569155E-03
  10     .129658E-02  -.290075E-02   -.706609E-04   .113653E-04   .689273E-04  -.640822E-05
  11     .941207E-03  -.203506E-02   -.156440E-04   .416124E-03   .156615E-04  -.443549E-03
  12     .826920E-03  -.255602E-02    .223523E-04   .624884E-03  -.225883E-04  -.682451E-03
  13     .606747E-03  -.176608E-02    .455299E-04   .654050E-03  -.430918E-04  -.726772E-03
  14     .568850E-03  -.194487E-02    .507790E-04   .547060E-03  -.522592E-04  -.615505E-03
  15     .409482E-03  -.121884E-02    .479618E-04   .355904E-03  -.490861E-04  -.404735E-03


 THE FINAL BETAI IS  .096181      .069174


 TBG=  14.419 TEND=  15.541 TEX=   1.122
```

ORIGINAL PAGE IS
OF POOR QUALITY

Figure 5. (Continued)

```
      PROGRAM MODULE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C**********************************************************************
C*    COMPUTER PROGRAM MODULE
C*    WRITTEN AT COLORADO STATE UNIVERSITY
C*    DEPARTMENT OF MECHANICAL ENGINEERING
C*    FORT COLLINS, COLORADO 80523
C*    SPONSORED BY NASA LEWIS RESEARCH CENTER
C*    GRANT NGR 06 - 002 - 095
C*    DIRECTED BY RICHARD J. PRIEM
C*    THIS PROGRAM IS DOCUMENTED IN THE MASTERS THESIS OF KURTIS W. ECKERT
C*    1978, COLORADO STATE UNIVERSITY, AND CONFORMS TO ALL FORTRAN IV ANSI
C*    STANDARDS.
C*
C*          IT IS WRITTEN TO GIVE A LINEAR ANALYSIS OF HIGH FREQUENCY
C*    COMBUSTION STABILITY IN LIQUID PROPELLANT ROCKET ENGINES.  THE
C*    PHYSICAL MODEL USED IS A RIGHT CIRCULAR CYLINDER.  THE COMBUSTION
C*    IS MODELED AS EITHER AN ARBITRARY ACOUSTIC ADMITTANCE (BETAI), OR
C*    BY THE CROCCO SENSITIVE TIME LAG THEORY (IN AND TAU), THE NOZZLE
C*    IS MODELED AS AN ARBITRARY ACOUSTIC ADMITTANCE (BETAN), THE LINER
C*    IS MODELED AS EITHER AN ARBITRARY ACOUSTIC ADMITANCE (BETAL) OVER
C*    SOME PORTION OF THE CYLINDRICAL WALL OF THE CHAMBER, OR AS A SLOT
C*    ABSORBER.
C*
C*          THIS PROGRAM HAS BEEN WRITTEN TO SOLVE FOR  THE NONDIMENSIONAL
C*    COMPLEX FREQUENCY IF ALL THE BOUNDARY RESPONSES ARE GIVEN, OR SOLVE
C*    FOR THE INJECTOR RESPONSE IF THE COMPLEX FREQUENCY AND LINER AND
C*    NOZZLE RESPONSES ARE GIVEN.  THIS LEADS TO A TOTAL OF 6 OPTIONS
C*    FOR RUNNING THE COMPUTER PROGRAM.
C*
C*          FOLLOWING IS A TABLE TO USE IN DETERMINING WHICH VALUE OF OP-
C*    TION TO USE.  THE TABLE SHOWS WHAT INFORMATION MUST BE GIVEN AND
C*    WHAT WILL BE CALCULATED FOR EACH OPTION.  R() MEANS THE REAL PART OF
C*    THE COMPLEX VALUE INSIDE THE PARENTHESES, AND I() MEANS THE IMAGIN-
C*    ARY PART OF THE COMPLEX VALUE INSIDE THE PARENTHESES.
C*
C*
C*    TABLE OF OPTIONS
C*
C*                   INPUT VARIABLES                       OUTPUT VARIABLES
C*    OPTION    ONE          TWO        THREE       FOUR
C*
C*       1      R(BETAI)   I(BETAI)   R(BETAL)   I(BETAL) OMEGA, IN & TAU
C*       2         IN         TAU     R(BETAL)   I(BETAL) OMEGA, BETAI
C*       3      R(BETAI)   I(BETAI)      BR         AMF   OMEGA, BETAL, IN & TAU
C*       4         IN         TAU        BR         AMF   OMEGA, BETAI, BETAL
C*       5      R(OMEGA)   I(OMEGA)      BR         AMF   BETAI, BETAL, IN & TAU
C*       6      R(OMEGA)   I(OMEGA)   R(BETAL)   I(BETAL) BETAI, IN & TAU
C*
C*
C*          FOLLOWING IS A TABLE LISTING THE INPUT VARIABLES THAT ARE PUT
C*    ON EACH DATA CARD.  AFTER THE VARIABLE NAME THE TYPE OF VARIABLE IS
C*    SHOWN. THEN A BRIEF DESCRIPTION OF THE VARIABLE IS GIVEN, THEN AT
C*    THE END OF THE LINE ARE THE COLUMNS OF THE DATA CARD WHICH THE VALUE
C*    FOR THIS VARIABLE MUST BE TYPED IN.  ALWAYS BE SURE TO RIGHT JUSTIFY
C*    INTEGER VALUES. THE FOURTH DATA CARD IS USED ONLY WHEN OPTION EQUALS
C*    3, 4 OR 5.
C*
C*
C*    LIST OF INPUT VARIABLES
C*
C*    FIRST CARD
C*     ONE      REAL        SEE TABLE ABOVE                               1-20
C*     TWO      REAL        SEE TABLE ABOVE                              21-40
C*     THREE    REAL        SEE TABLE ABOVE                              41-60
C*     FOUR     REAL        SEE TABLE ABOVE                              61-80
C*
C*    SECOND CARD
C*     BETAN    COMPLEX     ACOUSTIC ADMITTANCE OF NOZZLE         1-10 & 11-20
C*                          MODULUS LESS THAN .5
C*                          IF NO KNOWN VALUE USE SHORT NOZZLE
C*     GAMMA    REAL        RATIO OF SPECIFIC HEATS                      21-30
```

```
C*    MACH     REAL      MEAN FLOW MACH NUMBER  0 < MACH < .5            31-40
C*    LENGTH   REAL      LENGTH OF CHAMBER/RADIUS OF CHAMBER             41-50
C*    XA       REAL      DISTANCE FROM INJECTOR FACE TO START            51-60
C*                       OF LINER/RADIUS OF CHAMBER
C*                       0 < XA < XB
C*    XB       REAL      DISTANCE FROM INJECTOR FACE TO END              61-70
C*                       OF LINER/RADIUS OF CHAMBER
C*                       XA < XB < LENGTH
C*    ERROR    REAL      MAXIMUM ALLOWABLE % ERROR IN MODULUS            71-80
C*                       OR ABSOLUTE DIFFERANCE IN RADIANS OF
C*                       ANGLE TO DETERMINE CONVERGENCE OF
C*                       ITERATED VARIABLE   10E-5 < ERROR < 1
C*
C*   THIRD CARD
C*    LHAT      INTEGER  ASSUMED MODE IN RADIAL DIRECTION                 1-10
C*                       TYPICALLY 1 OR 2
C*    MHAT      INTEGER  ASSUMED MODE IN TRANSVERSE DIRECTION            11-30
C*                       TYPICALLY 0, 1 OR 2
C*    NHAT      INTEGER  ASSUMED MODE IN LONGITUDINAL DIRECTION          21-30
C*                       TYPICALLY 0, 1 OR 2
C*    LTS       INTEGER  NUMBER OF TERMS KEPT IN RADIAL DIRECTION        31-40
C*                       ODD < 10   TYPICALLY 3 OR 5
C*    NTS       INTEGER  NUMBER OF TERMS KEPT IN LONG. DIRECTION         41-50
C*                       NTS < 50 TYPICALLY 15 < NTS < 20
C*    IDMAX     INTEGER  MAXIMUM NUMBER OF ITERATIONS ALLOWED            51-60
C*                       TYPICALLY 100
C*    OPTION    INTEGER  SEE TABLE ABOVE                                 61-70
C*                       1, 2, 3, 4, 5, OR 6
C*
C*   FOURTH CARD
C*    EPSIL    REAL      WAVE AMPLITUDE                                   1-10
C*    ROAP     REAL      APERTURE DENSITY RATIO                          11-20
C*    ACAV     REAL      CAVITY SOUND SPEED                              21-30
C*    ROCAV    REAL      CAVITY DENSITY RATIO                            31-40
C*    WCAV     REAL      CAVITY WIDTH                                    41-50
C*    LA       REAL      APERTURE LENGTH                                 51-60
C*
C*************************************************************************
C
      REAL IN1,IN0,IN,NPIL2,LENGTH,LEFF,LA,LAMDA,LAMDA2,NORM,MACH,MACH2
     1,NORM1
      INTEGER OPTION,CHECK
C
      COMPLEX BETAN,BETAL,BETAI,MU,MUX,OMEGA,OMEGAN,CIOM,CERROR,CI
     1,CZERO,F1,HLINER,PIL,DZPIL,PRES,A1,A2,SUM1,SUM2,
     2AV,BASEW,BETAIN,WWO
      COMPLEX CTERM,B1,B2,B1SQ,B2SQ,A,EXP1,EXP2,TERM1,TERM2,VOL,SINJ,
     1SNOZ,SINJB,SLIN,PSI
C
      DIMENSION MU(50,10),MUX(50,10),A1(10),A2(10)
C
C     INITIALIZE CONSTANTS
C
      PI=3.14159265359
      CI=CMPLX(0.,1.)
      CZERO = CMPLX(0.,0.)
      IDCR= 20
      K=5
C
C     READ IN DATA
C
  570 CONTINUE
      CALL SECOND (TBGN)
      READ (5,100) ONE,TWO,THREE,FOUR,BETAN,GAMMA,MACH,LENGTH,XA,XB,
     1ERROR,LHAT,MHAT,NHAT,LTS,NTS,IDMAX,OPTION
      IF(EOF(5)) 6000,580,6000
  580 CONTINUE
      IF (OPTION.GE.3.AND.OPTION.LE.5) READ (5,101) EPSIL ,ROAP,
     1 ACAV,ROCAV,WCAV,LA
C
C     INITIALIZE VARIABLES
C
```

```
      ANG1= 0.0
      ID=0
      IF (OPTION.EQ.1.OR.OPTION.EQ.3) BETAIN= CMPLX(ONE,TWO)
      IN= ONE
      TAU= TWO
      BETAL= CMPLX(THREE,FOUR)
      BR= THREE
      AMF= FOUR
C
C     CALCULATE CONSTANTS
C
      PIL= PI/LENGTH
      NPIL2= (FLOAT(NHAT)*PIL)**2
      WA = XB -XA
      XL = XA + WA/2.0
      PIXL=PI*XL/LENGTH
      LAMDA2=BESRT(MHAT,LHAT)**2
      IF (OPTION.LE.2.OR.OPTION.EQ.6) GO TO 30
      LEFF = LA + 0.375*0.85*WA*( 1. - 0.7 *SQRT( WA/WCAV))
   30 CONTINUE
      ETA1= SQRT(LAMDA2 + NPIL2)
C
C     CALCULATE FIRST GUESS FOR ITERATED VARIABLES
C
      OMEGAN= .98*CMPLX(ETA1,0.0)
      IF (OPTION.GE.5) OMEGAN= CMPLX(ONE,TWO)
      WWO=OMEGAN/ETA1
      IF (OPTION.EQ.2.OR.OPTION.EQ.4) BETAIN=MACH*(1./GAMMA - IN*(1. -C
     1EXP(-CI*OMEGAN*TAU)))
C
C     CHECK VALUES OF INPUT VARIABLES
C
      TERM= WA*REAL(BETAL)
      CHECK= 0
      IF (MACH.LE.0.0.OR.MACH.GE.1.0) CHECK= 1
      IF (MACH.GT.0.5) WRITE (6,900) MACH
      IF (GAMMA.LT.1.0.OR.GAMMA.GT.1.67) CHECK= 1
      IF (GAMMA.LT.1.1) WRITE (6,920) GAMMA
      IF (LENGTH.LE.0.0) CHECK= 1
      IF (LENGTH.GE.3.0) WRITE (6,902) LENGTH
      IF (LENGTH.LE.1.0) WRITE (6,922) LENGTH
      IF (REAL(BETAN).LT.0.0) CHECK= 1
      IF (REAL(BETAN).GT.0.3) WRITE (6,906) BETAN
      IF (OPTION.GT.2.AND.OPTION.LT.6) GO TO 44
      IF (TERM.LT.0.0) CHECK= 1
      IF (TERM.GT.0.3) WRITE (6,904) TERM
   44 CONTINUE
      IF (OPTION.NE.2.AND.OPTION.NE.4) GO TO 46
      IF (IN.LT.0.0.OR.TAU.LT.0.0) CHECK= 1
      IF (IN.GT.3.0) WRITE (6,912) IN
      IF (TAU.GT.4.0) WRITE (6,914) TAU
   46 CONTINUE
      IF (OPTION.NE.1.AND.OPTION.NE.3) GO TO 47
      IF (CABS(BETAIN).GT.2.0) WRITE (6,916) BETAIN
   47 CONTINUE
      IF (XA.LT.0.0.OR.XA.GT.XB.OR.XB.GT.LENGTH) CHECK= 1
      IF (WA.GT.0.5) WRITE (6,930) WA
      IF (LHAT.LE.7.AND.MHAT.LE.4.AND.NHAT.LE.10) GO TO 48
      CHECK= 1
      WRITE (6,980) LHAT,MHAT,NHAT
   48 CONTINUE
      IF (NTS.LT.NHAT + 10) WRITE (6,940) NTS,NHAT
      IF (LTS.LT.LHAT + 2) WRITE (6,942) LTS,LHAT
      IF (REAL(WWO).GT.1.1) WRITE (6,950)
      IF (OPTION.LT.3.OR.OPTION.EQ.6) GO TO 49
      IF (EPSIL.LT.0.0.OR.BR.LE.0.0.OR.AMF.LT.0.0) CHECK= 1
      IF (BR.GT.0.2) WRITE (6,908) BR
      IF (AMF.GT.0.2) WRITE (6,910) AMF
   49 CONTINUE
      IF (CHECK.EQ.1) WRITE (6,960) MACH,GAMMA,LENGTH,BETAL,BETAN,XA,XB,
     1IN,TAU
      IF (CHECK.EQ.1) GO TO 710
```

```
C
C     INITIALIZE FIRST GUESS OF MU MATIX
C
      DO  50 N1=1,NTS
      DO  50 L=1,LTS
      MU(N1,L)= CZERO
   50 CONTINUE
C
C     THE TILDA SOLUTION
C
      BV= BESVL(MHAT,LHAT)
      EP= 1.
      IF (NHAT.EQ.0) EP= 2.
      BES1= BV*BV*(LAMDA2 - FLOAT(MHAT)**2)/2./LAMDA2
      NORM1= SQRT(BES1*EP*LENGTH/2.)
      MACH2= MACH*MACH
      DO 75 N= 1,100
      CTERM= MACH2*OMEGAN*OMEGAN+(MACH2-1.)*(LAMDA2-OMEGAN*OMEGAN)
      CTERM= CSQRT(CTERM)
      B1= (MACH*OMEGAN+CTERM)/(1.-MACH2)
      B2= (MACH*OMEGAN-CTERM)/(1.-MACH2)
      B1SQ=B1*B1
      B2SQ=B2*B2
      A=-(CEXP(CI*LENGTH*(B1-B2))*(B1+BETAN*GAMMA*(OMEGAN+MACH*B1))/
     1(B2+BETAN*GAMMA*(OMEGAN+MACH*B2)))
      EXP1=CEXP(CI*LENGTH*B1)
      EXP2=CEXP(CI*LENGTH*B2)
      TERM1= B1*(EXP1*(-1.)**NHAT-1.)/(B1SQ-NPIL2)
      TERM2= A*B2*(EXP2*(-1.)**NHAT-1.)/(B2SQ-NPIL2)
      VOL= MACH*(MACH*(B1SQ*TERM1+B2SQ*TERM2)+2.*OMEGAN*(B1*TERM1+
     1B2*TERM2))
      SINJ=GAMMA*(OMEGAN*(1.+A)+MACH*(B1+A*B2))
      SINJB=BETAIN*SINJ
      SNOZ= BETAN*GAMMA*(-1.)**NHAT*(OMEGAN*(EXP1+A*EXP2)+MACH*(B1*
     1EXP1 + A*B2*EXP2))
      IF (OPTION.GE.5) GO TO 70
      OMEGA= -(VOL + SINJB + SNOZ)/(TERM1 + TERM2) + ETA1**2
      OMEGA= CSQRT(OMEGA)
      IF(OPTION.EQ.2. OR.OPTION.EQ.4) BETAIN= MACH*(1./GAMMA-IN*(1.-C
     1EXP(-CI*OMEGAN*TAU)))
      CERROR= OMEGA - OMEGAN
      OMEGAN= OMEGA
      IF ( ABS(REAL(CERROR)).GT.0.0001) GO TO 75
      IF ( ABS(AIMAG(CERROR)).GT.0.0001) GO TO 75
      GO TO 71
   70 CONTINUE
      BETAIN= ((ETA1**2- OMEGAN**2)*(TERM1 +TERM2) - VOL - SNOZ)/SINJ
      GO TO 71
   75 CONTINUE
   71 CONTINUE
      PSI= (-2.*CI/LENGTH)*(TERM1 + TERM2)
      IF (NHAT.EQ.0) PSI= PSI/2.
      MU(NHAT+1,LHAT)= CMPLX(1./NORM1,0.0)
      DO 55 N1=1,NTS
      N= N1-1
      RN= FLOAT(N)
      IF (N.EQ.NHAT) GO TO 55
      TERM= FLOAT((-1)**N)
      NPIL2= (RN*PIL)**2
      EP= 1.
      IF (N.EQ.0) EP= 2.
      NORM= BES1*EP*LENGTH/2.
      ETA= LAMDA2 + NPIL2
      TERM1= (EXP1*TERM - 1.)/(B1SQ - NPIL2)
      TERM2= (EXP2*TERM - 1.)/(B2SQ - NPIL2)
      VOL= MACH2*(B1SQ*B1*TERM1 + A*B2SQ*B2*TERM2) + 2.*MACH*OMEGAN*(B
     11SQ*TERM1 + A*B2SQ*TERM2)
      SNOZ= BETAN*GAMMA*TERM*(OMEGAN*(EXP1 +A*EXP2) + MACH*(B1*EXP1+
     1 A*B2*EXP2))
      SINJB= BETAIN*GAMMA*(OMEGAN*(1. +A) + MACH*(B1 + A*B2))
      MU(N1,LHAT)= BES1*CI*(VOL+ SNOZ +SINJB)/(OMEGAN**2 -ETA)/NORM1
     1/NORM/PSI
```

```
   55 CONTINUE
C
C     PRINT OUT PROBLEM DESCRIPTION AND INITIAL VALUES
C
      WRITE (6,1)
      WRITE (6,200)
      WRITE (6,210) LENGTH,MACH,LHAT,MHAT,NHAT,OPTION,GAMMA
      WRITE (6,220)
      WRITE (6,230) XA,XB,WA
      IF (OPTION.GE.3.AND.OPTION.LE.5) GO TO 20
      WRITE (6,235) BETAL
      GO TO 22
   20 CONTINUE
      WRITE (6,240) BR,AMF,LA,LEFF,EPSIL,WCAV,ROCAV,ROAP,ACAV
   22 CONTINUE
      WRITE (6,250)
      IF (OPTION.GE.5) GO TO 10
      IF (OPTION.EQ.2.OR.OPTION.EQ.4) GO TO 12
      WRITE (6,260) BETAIN
      GO TO 14
   12 CONTINUE
      WRITE (6,270) IN,TAU,BETAIN
      GO TO 14
   10 CONTINUE
      WRITE (6,280) OMEGAN,WWO
      WRITE (6,285) BETAIN
   14 CONTINUE
      WRITE (6,290)
      WRITE (6,310) BETAN
      WRITE (6,320) LTS,NTS,IDMAX,ERROR
      WRITE (6,300) ETA1
      WRITE (6,305) OMEGAN
      WRITE (6,105)
      DO 35 N=1,NTS
      NM1= N - 1
      WRITE (6,104) NM1,(MU(N,L),L=1,LTS)
   35 CONTINUE
      WRITE (6,2)
      IF (OPTION.LE.4) WRITE (6,350)
      IF (OPTION.GE.5) WRITE (6,360)
C
C     BEGIN ITERATION
C
   60 CONTINUE
C
C     UPDATE ITERATED VARIABLES
C
      IF (OPTION.EQ.2.OR.OPTION.EQ.4) BETAIN=MACH*(1./GAMMA - IN*(1. -C
     1EXP(-CI*OMEGAN*TAU)))
      BETAI= BETAIN
      OMEGA=OMEGAN
      CIOM=CI*OMEGA
      WR= REAL(OMEGA)
C
C     STORE NEW MU MATRIX IN EXTRA MATRIX
C
      DO 800 N1=1,NTS
      DO 800 L=1,LTS
      MUX(N1,L)=MU(N1,L)
  800 CONTINUE
C
C     CALCULATE PHI AT INJECTOR, NOZZLE,MIDPOINT OF LINER
C
      P1L= CZERO
      DZP1L= CZERO
      DO 130 L=1,LTS
      SUM1= CZERO
      SUM2= CZERO
      BV= BESVL(MHAT,L)
C
C
```

```
      DO 120 N1=1,NTS
      N= N1 - 1
      RN= FLOAT(N)
      AV=MU(N1,L)
      SUM1= SUM1+AV
      SUM2= SUM2 + AV*FLOAT((-1)**N)
      PIL=PIL+BV*AV*COS(RN*PIXL)
      DZPIL= DZPIL + BV*AV*RN*PIL*SIN(RN*PIXL)
  120 CONTINUE
C
      A1(L)=SUM1
      A2(L)=SUM2
  130 CONTINUE
C
C     CALCULATE BLINER FOR MIDPOINT OF LINER
C
      PRES= GAMMA*(CIOM*PIL - MACH*DZPIL)
      BLINER = WA*PRES*BETAL
C
C     CALCULATE BLINER FROM LINER GEOMETRY
C
      IF (OPTION.LE.2.OR.OPTION.EQ.6) GO TO 3000
      AK1 = LEFF*WR*ROAP-WA*ACAV*ROCAV/WR/WCAV/BR *ACAV
      PRE1 = CABS(PRES)
      RO=1.
      RO1 = RO
C
      DO 133 I=1,100
      F = SQRT( 1.0 + (AK1/RO)**2 )
      BASE= EPSIL*PRE1/F/GAMMA
      RO = SQRT( 0.8*( 1.5*AMF*RO + BASE ) )
      RO2 = ABS( (RO1-RO)/RO1 )
      IF(RO2.LT.1.0E-04) GO TO 134
      RO1= RO
  133 CONTINUE
C
  134 CONTINUE
      BLINER = WA*PRES/GAMMA/(RO+CI*AK1)
 3000 CONTINUE
      IF (OPTION.LE.4) GO TO 3001
C
C     CALCULATE NEW BETAT
C
      VOL= CZERO
      DO 45 N1=2,NTS
      NM1= N1-1
      IF (NM1.EQ.NHAT) GO TO 45
      K1= NM1 + NHAT
      K2= NM1 - NHAT
      L1=(-1)**K1-1
      L2=(-1)**K2-1
      LSUM=L1+L2
      IF(LSUM.EQ.0) GO TO 45
      C1= FLOAT(L1)/FLOAT(K1)
      C2= FLOAT(L2)/FLOAT(K2)
      SC= (C1 + C2)*FLOAT(NM1)
      VOL= VOL + MUX(N1,LHAT)*SC
   45 CONTINUE
      VOL= VOL*MACH*CIOM*BES1
      VOL= VOL - MUX(NHAT+1,LHAT)*BES1*(MACH*FLOAT(NHAT)*PIL)**2*LENGTH
     1/2.
      VOL= VOL/NORM1
      SNOZ= BETAN*GAMMA*CIOM*BES1*A2(LHAT)*FLOAT((-1)**NHAT)/NORM1
      SLIN= BLINER*BESVL(MHAT,LHAT)*COS(FLOAT(NHAT)*PIXL)/NORM1
      SINJ= GAMMA*CIOM*BES1*A1(LHAT)/NORM1
      BETAIN= (OMEGA**2 -ETA1**2 -VOL -SNOZ -SLIN)/SINJ
 3001 CONTINUE
C
C     START DO LOOP FOR L SUMMATION
C
      DO 500 L=1,LTS
```

```
C
      LAMDA= BESRT(MHAT,L)**2
      BV2= BESVL(MHAT,L)**2
C
C     START DO LOOP FOR N SUMMATION
C
      DO 500 NX=1,NTS
C
C     INITIALIZE AND CALCULATE CONSTANTS FOR THIS SUMMATION
C
      N= NX - 1
      VOL= CZERO
      ETA= LAMDA + (FLOAT(N)*PIL)**2
      BES= BV2*(LAMDA - FLOAT(MHAT)**2)*.5/LAMDA
      EP= 0.5
      IF (N.EQ.0) EP= 1.
      NORM= BES*EP*LENGTH
      NORM= SQRT(NORM)
      BASEW=OMEGA*OMEGA-ETA
C
C     CALCULATE PROPAGATION TERMS
C
      DO 40 N1= 2,NTS
      NM1= N1 - 1
      IF (NM1.EQ.N) GO TO 40
      K1= NM1 + N
      K2= NM1 - N
      L1=(-1)**K1-1
      L2=(-1)**K2-1
      LSUM=L1+L2
      IF(LSUM.EQ.0) GO TO 40
      C1= FLOAT(L1)/FLOAT(K1)
      C2= FLOAT(L2)/FLOAT(K2)
      SC= (C1 + C2)*FLOAT(NM1)
      VOL= VOL + MUX(N1,L)*SC
   40 CONTINUE
      VOL= VOL*MACH*CIOM*BES
      VOL= VOL - MUX(NX,L)*BES*(MACH*FLOAT(N)*PIL)**2*LENGTH/2.
      VOL= VOL/NORM
C
C     CALCULATE NOZZLE INTEGRAL
C
      SNOZ= BETAN*GAMMA*CIOM*BES*A2(L)*FLOAT((-1)**N)/NORM
C
C     CALCULATE LINER INTEGRAL
C*    TO CARRY OUT THE INTEGRATION OVER THE LINER USE THE FOLLOWING TWO
C*    CARDS, AND PUT A "C" IN THE FIRST COLUMN OF THE CARD FOR APPROX-
C*    IMATING THE INTEGRAL, HENCE MAKING IT A COMMENT CARD.
C
C     CALL LINER(XA,XB,PIL,L,N,MHAT,MUX,CIOM,BLINER,LTS,NTS,CZERO)
C     SLIN= GAMMA*BESVL(MHAT,L)*BETAL*BLINER/NORM
C
C*    TO APPROXIMATE THE LINER INTEGRAL BY EVALUATING AT THE MIDPOINT
C*    AND MULTIPLYING BY THE WIDTH, USE THE FOLLOWING CARD, AND PUT A
C*    "C" IN THE FIRST COLUMNS OF THE TWO PRECEDING CARDS FOR CARRYING
C*    OUT THE INTEGRATION, HENCE MAKING THEM COMMENT CARDS. "SUBROUTINE
C*    LINER" NEED NOT BE COMPILED IF THE INTEGRAL IS TO BE APPROXIMATED.
C
      SLIN= BLINER*BESVL(MHAT,L)*COS(FLOAT(N)*PIXL)/NORM
C
C     CALCULATE INJECTOR INTEGRAL
C
      SINJ= GAMMA*CIOM*BES*A1(L)/NORM
      SINJB= SINJ*BETAIN
C
C     CALCULATE NEW MU TERM
C
      F1= VOL + SINJB + SLIN + SNOZ
      IF (N.EQ.NHAT.AND.L.EQ.LHAT) GO TO 430
      MU(NX,L)= F1/NORM/BASEW
      GO TO 500
  430 CONTINUE
```

```
C
C     FOR PRIMARY MODE CALCULATE MU TERM AND NEW OMEGA
C
      MU(NX,L)= CMPLX(1./NORM,0.0)
      IF (OPTION.LE.4) OMEGAN= CSQRT(F1 + ETA)
  500 CONTINUE
      ID= ID + 1
      IK= IFIX(FLOAT(ID)/FLOAT(K))
      RK= FLOAT(ID)/FLOAT(K)
      RK= RK - FLOAT(IK)
      IF (OPTION.LE.4) GO TO 4000
C
C     CHECK FOR CONVERGENCE ON BETAI
C
      CERROR= BETAIN - BETAI
      ERR1= CABS(CERROR)/CABS(BETAIN)*100.
      ANG2= ATAN(AIMAG(BETAIN)/REAL(BETAIN))
      ERR2= ABS(ANG2 - ANG1)
      ANG1= ANG2
      IF (ID.GE.IDCR.AND.ERR1.GE.50.) GO TO 540
      IF (ID.GE.IDMAX) GO TO 540
      IF (ID.EQ.1.OR.RK.LE.0.0001) WRITE (6,666) ID,BETAIN,ERR1,ERR2
      IF (ID.LT.5.OR.ERR1.GT.ERROR.OR.ERR2.GT.ERROR) GO TO 60
      GO TO 545
  540 CONTINUE
      WRITE (6,365)
  545 CONTINUE
      WRITE (6,666) ID,BETAIN,ERR1,ERR2
      GO TO 550
C
C     CHECK FOR CONVERGENCE ON OMEGA
C
 4000 CONTINUE
      CERROR = OMEGAN - OMEGA
      ANG2= PI/2.
      ERR1= 0.0
      IF (CABS(OMEGAN).EQ.0.0) GO TO 43
      ERR1= CABS(CERROR)/CABS(OMEGAN)*100.
      IF (REAL(OMEGAN).EQ.0.0) GO TO 43
      ANG2= ATAN(AIMAG(OMEGAN)/REAL(OMEGAN))
   43 CONTINUE
      ERR2= ABS(ANG2-ANG1)
      ANG1= ANG2
      IF (ID.GE.IDCR.AND.ERR1.GE.50.) GO TO 547
      IF (ID.GE.IDMAX) GO TO 547
      IF (ID.EQ.1.OR.RK.LE.0.0001) WRITE (6,666) ID,OMEGAN,ERR1,ERR2
      IF (ID.LT.5.OR.ERR1.GT.ERROR.OR.ERR2.GT.ERROR) GO TO 60
      GO TO 546
  547 CONTINUE
      WRITE (6,365)
  546 CONTINUE
      WRITE (6,666) ID,OMEGAN,ERR1,ERR2

  550 CONTINUE
      IF (ID.GE.IDMAX) WRITE(6,390)
C
C     PRINT OUT FINAL MU MATRIX
C
      WRITE (6,2)
      WRITE(6,110)
      DO 106 N1=1,NTS
      N= N1 - 1
      WRITE (6,104) N,(MU(N1,L),L=1,LTS)
  106 CONTINUE
      WRITE(6,2)
C
C     CALCULATE AND PRINT EQUIVALENT BETAL IF CALCULATED FROM GEOMETRY
C
      IF (OPTION.GE.3.AND.OPTION.LE.5) WRITE (6,501) RO,AK1
      IF (OPTION.GE.3.AND.OPTION.LE.5) BETAL= BLINER/WA/PRES
      IF (OPTION.GE.3.AND.OPTION.LE.5) WRITE (6,370) BETAL
```

```
C
C     CALCULATE AND PRINT EITHER N, TAU OR BETAI WHICHEVER IS APPROPRIATE
C
      IF (OPTION.NE.1.AND.OPTION.NE.3) WRITE (6,380) BETAIN
      IF (OPTION.EQ.2.OR.OPTION.EQ.4) GO TO 710
      WR= REAL (OMEGAN)
      WI= AIMAG(OMEGAN)
      BIR= REAL(BETAIN)
      BII= AIMAG(BETAIN)
      IN0= -((BIR - MACH/GAMMA)**2+BII**2)/(2.*MACH*(BIR-MACH/GAMMA))
      DO 11 N=1,3
      RN= FLOAT(N) - 1.
      TAU0= ABS(ASIN(-BII/MACH/IN0) + RN*PI)/WR
      BETAIN= MACH*(1./GAMMA - IN0*(1. - CEXP(-CI*OMEGAN*TAU0)))
      ERR1= ABS((BIR - REAL(BETAIN))/BIR)
      ERR2= ABS((BII - AIMAG(BETAIN))/BII)
      IF (ERR1.LE.0.30.AND.ERR2.LE.0.30) GO TO 19
      RN= -RN
      TAU0= ABS(ASIN(-BII/MACH/IN0) + RN*PI)/WR
      BETAIN= MACH*(1./GAMMA - IN0*(1. - CEXP(-CI*OMEGAN*TAU0)))
      ERR1= ABS((BIR - REAL(BETAIN))/BIR)
      ERR2= ABS((BII - AIMAG(BETAIN))/BII)
      IF (ERR1.LE.0.30.AND.ERR2.LE.0.30) GO TO 19
   11 CONTINUE
      WRITE (6,3)
      GO TO 710
   19 CONTINUE
      WRITE (6,386) RN,IN0,TAU0,BETAIN
      IF( WI.EQ.0.0) GO TO 710
      RN= -3.0
   18 CONTINUE
      RN= RN + 1.0
      IF (RN.GE.3.5) GO TO 17
      IN1= IN0
      TAU1= TAU0
      DO 15 I=1,200
      TERM= BIR - MACH/GAMMA + MACH*IN1
      TAU= ABS(ATAN(-BII/TERM) + RN*PI)/WR
      IN= TERM/(MACH*EXP(WI*TAU)*COS(WR*TAU))
      IF (ABS(TAU -TAU1).LT.1.E-5.AND.ABS(IN - IN1).LT.1.E-5.AND.I.GE.5)
     1 GO TO 16
      IF (I.GE.5.AND.IN.LT.0.0) GO TO 16
      IF (I.GE.5.AND.TAU.LT.0.0) GO TO 16
      IN1= IN
      TAU1= TAU
   15 CONTINUE
   16 CONTINUE
      IF (IN.LT.0.00.OR.TAU.LT.0.00) GO TO 18
      BETAIN= MACH*(1./GAMMA - IN*(1. - CEXP(-CI*OMEGAN*TAU)))
      ERR1= ABS((BIR - REAL(BETAIN))/BIR)
      ERR2= ABS((BII - AIMAG(BETAIN))/BII)
      IF (ERR1.GE.0.15.OR.ERR2.GE.0.15) GO TO 18
      WRITE (6,385) RN,IN,TAU,BETAIN
      GO TO 710
   17 CONTINUE
      SWT= SIN(WR*TAU0)
      CWT= COS(WR*TAU0)
      EWT= EXP(WI*TAU0)
      EC1= EWT*CWT - 1.
      WTW= WR/TAN(WR*TAU0) + WI
      TAU1= (BIR/MACH/EC1 + BII/MACH/EWT/SWT - 1./GAMMA/EC1)/(IN0*
     1 (EWT*( WI*CWT - WR*SWT)/EC1 - WTW))
      IN1= -BII/MACH/EWT/SWT - IN0 - IN0*TAU1*WTW
      TAU= TAU0 + TAU1
      IN= IN0 + IN1
      BETAIN= MACH*(1./GAMMA - IN*(1. - CEXP(-CI*OMEGAN*TAU)))
      WRITE (6,5) IN,TAU,BETAIN
C
C     CALCULATE RUNNING TIME FOR THIS SET OF DATA
C
  710 CONTINUE
      CALL SECOND (TEND)
```

```
      TEX= TEND - TBGN
      WRITE (6,80) TBGN,TEND,TEX
C
C     RETURN FOR NEXT SET OF DATA
C
      GO TO 570
 6000 CONTINUE
      STOP
C
C     FORMAT STATEMENTS
C
    1 FORMAT (1H1,///)
    2 FORMAT(///)
    3 FORMAT (48H NO AND TAU0 WOULD NOT CONVERGE FOR THIS PROBLEM,//)
    4 FORMAT (66H   N AND TAU DID NOT CONVERGE SO A LINEARIZATION SOLUTI
     1ON IS GIVEN,//)
    5 FORMAT (61H N AND TAU DID NOT CONVERGE SO A LINEARIZED SOLUTION I
     1 GIVEN,/,5X,29HTHE CALCULATED N AND TAU ARE ,F9.6,5H AND ,F9.6,/,
     25X,18HGIVING A BETAI OF ,F9.6,3X,F9.6,///)
   80 FORMAT (7H   TBG=,F8.3,6H TEND=,F8.3,5H TEX=,F8.3,///)
  100 FORMAT (4F20.10,/,8F10.8,/,7I10)
  101 FORMAT(8F10.8 )
  104 FORMAT(2X,I3,3X,8G13.6,/,(8X,8G13.6,/))
  105 FORMAT (49H  THE FIRST GUESS FOR THE MU MATRIX IS AS FOLLOWS,//5X,
     11HN,16X,3HL=1,24X,3HL=2,24X,3HL=3,/)
  110 FORMAT (35H  THE FINAL MU MATRIX IS AS FOLLOWS,//5X,1HN,16X,3HL=1,
     124X,3HL=2,24X,3HL=3,//)
  200 FORMAT(85H THE FOLLOWING CALCULATIONS ARE MADE FOR A COMBUSTOR WIT
     1H THE FOLLOWING CONFIGURATION,///)
  210 FORMAT(20X,23HTHE LENGTH TO RADIUS = ,F9.6,24X,20HMEAN FLOW MACH N
     10. =,F9.6//,20X,7HLHAT = ,I1,5X,7HMHAT = ,I1,5X,7HNHAT = ,I1,22X,
     29HOPTION = ,I2,//,20X,8HGAMMA = ,F9.6,///)
  220 FORMAT(40H THE ABSORBER HAS THE FOLLOWING GEOMETRY ,///)
  230 FORMAT (20X,20HTHE LINER STARTS AT ,F9.6,13H AND ENDS AT ,F9.6,5X,
     117HAPERTURE WIDTH = ,F9.6,//)
  235 FORMAT (20X,8HBETAL = ,F9.6,3X,F9.6,///)
  240 FORMAT (20X,23HTHE BACKING DISTANCE = ,F9.6,24X,25HTHE APERTURE ME
     1AN FLOW = ,F9.6,//,20X,18HAPERTURE LENGTH = ,F9.6,28X,19HEFFECTIVE
     2 LENGTH = ,F9.6,//,20X,10HEPSILON = ,F9.6,36X,14HCAVITY WIDTH =,
     3 F9.6,//,20X,16HCAVITY DENSITY = ,F9.6,30X,18HAPERTURE DENSITY =
     4 ,F9.6,//,20X,20HCAVITY SOUND SPEED = ,F9.6,///)
  250 FORMAT (39H THE INJECTOR RESPONSE IS DESCRIBED BY ,///)
  260 FORMAT (20X,8HBETAI = ,F9.6,3X,F9.6,///)
  270 FORMAT (20X,40HFOR THE CROCCO SENSITIVE TIME LAG THEORY,16X,4HN =
     1,F9.6,4X,6HTAU = ,F9.6,//
     2                         ,20X,16HINITIAL BETAI = ,F9.6,3X,F9.6,///)
  280 FORMAT (20X,80HFOR THIS OPTION FREQUENCY IS INPUT AND BETAI IS CAL
     1CULATED.  INPUT FREQUENCY IS ,F9.6,3X,F9.6,//,20X,6HWWO = ,F9.6,3X
     2,F9.6,/)
  285 FORMAT (20X,27HTHE FIRST GUESS OF BETAI IS,F9.6,3X,F9.6,///)
  290 FORMAT (37H THE NOZZLE RESPONSE IS DESCRIBED BY ,///)
  300 FORMAT (20X,43HTHE FUNDAMENTAL FREQUENCY FOR THIS MODE IS ,F9.6,3X
     1,3H0.0,/)
  305 FORMAT (20X,27HTHE FIRST GUESS OF OMEGA IS,F9.6,3X,F9.6,///)
  310 FORMAT (20X,8HBETAN = ,F9.6,3X,F9.6,///)
  320 FORMAT (43H MISCELLANEOUS INFORMATION FOR THIS PROBLEM,///,20X,26H
     1NUMBER OF TERMS    RADIAL ,I2,4X,13HLONGITUDINAL ,I2,9X,8HIDMAX =
     2,I4,//,20X,8HERROR = ,F9.6,/)
  350 FORMAT (78H ITER       REAL OMEGA      IMAG OMEGA       ERRORS
     1 MODULUS          ANGLE,//)
  360 FORMAT (78H ITER       REAL BETAI      IMAG BETAI       ERRORS
     1 MODULUS          ANGLE,//)
  365 FORMAT (1X,51H***THIS PROBLEM DOES NOT APPEAR TO BE CONVERGING***)
  370 FORMAT (27H   THE EQUIVALENT BETAL IS ,F9.6,3X,F9.6,///)
  380 FORMAT (21H   THE FINAL BETAI IS,F9.6,3X,F9.6,///)
  385 FORMAT (28H N AND TAU CONVERGED FOR RN=,F9.6,/,
     1     5X,29HTHE CALCULATED N AND TAU ARE ,F9.6,5H AND ,F9.6,16H
     2RESPECTIVELY,/,5X,18HGIVING A BETAI OF ,F9.6,3X,F9.6,///)
  386 FORMAT (30H NO AND TAU0 CONVERGED FOR RN=,F9.6,/,
     1   5X,31HTHE CALCULATED NO AND TAU0 ARE ,F9.6,5H AND ,F9.6,16H
     2RESPECTIVELY,/,5X,18HGIVING A BETAI OF ,F9.6,3X,F9.6,///)
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
390 FORMAT (/,44H*****  THIS PROBLEM HAS NOT CONVERGED  *****,///)
501  FORMAT (26H   THE SLOT IMPEDANCES ARE,
   1                                         10X,3HRO=,G13.6,5X,3HKI=,G13.6,
   2 ///)
666 FORMAT (2X,I3,5X,G13.6,2X,G13.6,15X,F10.6,5X,F10.6)
900 FORMAT (5X,37HYOUR VALUE FOR MACH IS EXTREMELY HIGH,G10.4,//)
902 FORMAT (5X,39HYOUR VALUE FOR LENGTH IS EXTREMELY HIGH,G10.4,//)
904 FORMAT(5X,42HYOUR PRODUCT OF WA*BETAL IS EXTREMELY HIGH,2G10.4,//)
906 FORMAT (5X,38HYOUR VALUE FOR BETAN IS EXTREMELY HIGH,2G10.4,//)
908 FORMAT (5X,35HYOUR VALUE FOR BR IS EXTREMELY HIGH,G10.4,//)
910 FORMAT (5X,36HYOUR VALUE FOR AMF IS EXTREMELY HIGH,G10.4,//)
912 FORMAT (5X,35HYOUR VALUE FOR IN IS EXTREMELY HIGH,G10.4,//)
914 FORMAT (5X,36HYOUR VALUE FOR TAU IS EXTREMELY HIGH,G10.4,//)
916 FORMAT (5X,55HTHE MAGNITUDE OF YOUR VALUE FOR BETAI IS EXTREMELY H
   1IGH,2G10.4,//)
920 FORMAT (5X,37HYOUR VALUE FOR GAMMA IS EXTREMELY LOW,G10.4,//)
922 FORMAT (5X,38HYOUR VALUE FOR LENGTH IS EXTREMELY LOW,G10.4,//)
924 FORMAT (5X,34HYOUR VALUE FOR BR IS EXTREMELY LOW,G10.4,//)
926 FORMAT (5X,35HYOUR VALUE FOR AMF IS EXTREMELY LOW,G10.4,//)
930 FORMAT (5X,109HWITH THIS APERTURE THE INTEGRATION OVER THE LINER S
   1HOULD BE CARRIED OUT  SEE COMMENT CARDS ABOVE 500 CONTINUE,G10.4,
   2//)
940 FORMAT (5X,89HYOU HAVE NOT KEPT ENOUGH TERMS IN THE LONGITUDINAL D
   1IRECTION FOR THE MODE YOU HAVE CHOSEN,/,6HNTS = ,I2,5X,7HNHAT = ,
   2I2,//)
942 FORMAT (5X,84HYOU HAVE NOT KEPT ENOUGH TERMS IN THE RADIAL DIRECTI
   1ON FOR THE MODE YOU HAVE CHOSEN,/,5X,6HLTS = ,I2,5X,7HLHAT = ,I2,
   2//)
950 FORMAT (5X,61HTHIS FREQUENCY IS EXTREMELY HIGH FOR THE MODE YOU HA
   1VE CHOSEN,)
960 FORMAT (5X,98HONE OF THE VALUES LISTED BELOW IS NOT PHYSICALLY MEA
   1NINGFUL  THIS SET OF DATA WILL NOT BE EXECUTED,/,5X,7HMACH = ,G10.
   24,5X,8HGAMMA = ,G10.4,5X,9HLENGTH = ,G10.4,5X,8HBETAL = ,2G10.4,/,
   35X,8HBETAN = ,2G10.4,5X,5HXA = ,G10.4,5X,5HXB = ,G10.4,5X,5HIN = ,
   4G10.4,5X,6HTAU = ,G10.4,////)
980 FORMAT (5X,70HTHE ACOUSTIC MODE YOU SELECTED IS TOO HIGH FOR THIS
   1PROGRAM TO HANDLE,/,5X,7HLHAT = ,I2,5X,7HMHAT = ,I2,5X,7HNHAT = ,
   2I2)
    END
```

```
      FUNCTION BESRT(M,L)
C*    THIS FUNCTION SUBROUTINE IS A TABLE OF THE ROOTS OF THE BESSEL
C*    FUNCTION OF THE FIRST KIND.
      DIMENSION A(4,10)
      DIMENSION B(1,10)
      IF (M.EQ.0) GO TO 10
      A(1 ,1) = 1.84118378
      A(1 ,2) = 5.33144277
      A(1 ,3) = 8.53631637
      A(1 ,4) = 11.7060049
      A(1 ,5) = 14.86358863
      A(1 ,6) = 18.01552786
      A(1 ,7) = 21.16436986
      A(1 ,8) = 24.31132666
      A(1 ,9) = 27.45705057
      A(1,10) = 30.60192297
      A(2 ,1) = 3.05423693
      A(2 ,2) = 6.70613319
      A(2 ,3) = 9.96946782
      A(2 ,4) = 13.17037086
      A(2 ,5) = 16.34752232
      A(2 ,6) = 19.51291278
      A(2 ,7) = 22.67158177
      A(2 ,8) = 25.82603714
      A(2 ,9) = 28.97767277
      A(2,10) = 32.12732702
      A(3 ,1) = 4.20118894
      A(3 ,2) = 8.01523660
      A(3 ,3) = 11.34592431
      A(3 ,4) = 14.58584829
      A(3 ,5) = 17.78874787
      A(3 ,6) = 20.97247694
      A(3 ,7) = 24.14489743
      A(3 ,8) = 27.31005793
      A(3 ,9) = 30.47026881
      A(3,10) = 33.62694918
      A(4 ,1) = 5.31755313
      A(4 ,2) = 9.28239629
      A(4 ,3) = 12.68190844
      A(4 ,4) = 15.96410704
      A(4 ,5) = 19.19602880
      A(4 ,6) = 22.40103227
      A(4 ,7) = 25.58975968
      A(4 ,8) = 28.76783622
      A(4 ,9) = 31.93853934
      A(4,10) = 35.10391668
      BESRT=A(M,L)
      RETURN
   10 CONTINUE
      B(1 ,1) = 0.00000000
      B(1 ,2) = 3.83170597
      B(1 ,3) = 7.01558667
      B(1 ,4) = 10.17346814
      B(1 ,5) = 13.32369194
      B(1 ,6) = 16.47063005
      B(1 ,7) = 19.61585851
      B(1 ,8) = 22.76008438
      B(1 ,9) = 25.90367209
      B(1,10) = 29.04682853
      BESRT= B(1,L)
      RETURN
      END
```

```
      FUNCTION BESVL(M,L)
C*    THIS FUNCTION SUBROUTINE IS A TABLE OF THE VALUES OF THE BESSEL
C*    FUNCTION OF THE FIRST KIND.
      DIMENSION A(4,10)
      DIMENSION C(1,10)
      IF (M.EQ.0) GO TO 10
      A(1 ,1) =  0.58186522
      A(1 ,2) = -0.34612620
      A(1 ,3) =  0.27329994
      A(1 ,4) = -0.23330442
      A(1 ,5) =  0.20701265
      A(1 ,6) = -0.18801749
      A(1 ,7) =  0.17345905
      A(1 ,8) = -0.16183821
      A(1 ,9) =  0.15228207
      A(1,10) = -0.14424290
      A(2 ,1) =  0.48649868
      A(2 ,2) = -0.31353045
      A(2 ,3) =  0.25474416
      A(2 ,4) = -0.22088158
      A(2 ,5) =  0.19793743
      A(2 ,6) = -0.18101000
      A(2 ,7) =  0.16783553
      A(2 ,8) = -0.15719517
      A(2 ,9) =  0.14836378
      A(2,10) = -0.14087833
      A(3 ,1) =  0.4343942763
      A(3 ,2) = -0.2911584413
      A(3 ,3) =  0.240738175
      A(3 ,4) = -0.210965204
      A(3 ,5) =  0.190419022
      A(3 ,6) = -0.175048405
      A(3 ,7) =  0.162954965
      A(3 ,8) = -0.153102409
      A(3 ,9) =  0.144866574
      A(3,10) = -0.137844513
      A(4 ,1) =  0.3996514545
      A(4 ,2) = -0.2743809949
      A(4 ,3) =  0.229590468
      A(4 ,4) = -0.202763849
      A(4 ,5) =  0.184029896
      A(4 ,6) = -0.169878516
      A(4 ,7) =  0.158655372
      A(4 ,8) = -0.149451156
      A(4 ,9) =  0.141714307
      A(4,10) = -0.135086328
      BESVL=A(M,L)
      RETURN
   10 CONTINUE
      C(1 ,1) =  1.00000000
      C(1 ,2) = -0.4027588095
      C(1 ,3) =  0.301128303
      C(1 ,4) = -0.249704877
      C(1 ,5) =  0.218359407
      C(1 ,6) = -0.19645371
      C(1 ,7) =  0.180063375
      C(1 ,8) = -0.167184600
      C(1 ,9) =  0.156724985
      C(1,10) = -0.148011108
      BESVL= C(1,L)
      RETURN
      END
```

```
      SUBROUTINE LINER(XA,XB,PIL,L,N,MHAT,MUX,CIOM,MACH,BLINER,LTS,
     1NTS,CZERO)
C*    THIS SUBROUTINE IS USED TO CARRY OUT THE INTEGRATION OVER THE
C*    LINER
      REAL NMNP,NPNP,MACH
      COMPLEX MUX,CIOM,BLINER,SUM1,SUM2,CZERO
      DIMENSION MUX(50,10)
      BLINER= CZERO
      DO 47 LP=1,LTS
      SUM1= CZERO
      SUM2= CZERO
      DO 45 N1=1,NTS
      NP= N1 - 1
      RN= FLOAT(NP)
      RNPIL= RN*PIL
      IF (N.EQ.NP) GO TO 42
      NMNP= FLOAT(NP - N)*PIL
      NPNP= FLOAT(NP + N)*PIL
      SUM1= SUM1 + MUX(N1,LP)*(SIN(NMNP*XB)/2./NMNP + SIN(NPNP*XB)/2./
     1NPNP - SIN(NMNP*XA)/2./NMNP - SIN(NPNP*XA)/2./NPNP)
      SUM2= SUM2 + MUX(N1,LP)*RNPIL*(COS(NMNP*XB)/2./NMNP + COS(NPNP*
     1 XB)/2./NPNP - COS(NMNP*XA)/2./NMNP - COS(NPNP*XA)/2./NPNP)
      GO TO 45
   42 CONTINUE
      IF (NP.EQ.0) GO TO 44
      SUM1= SUM1 + MUX(N1,LP)*((XB - XA)/2. + (SIN(2.*RNPIL*XB) -
     1  SIN(2.*RNPIL*XA))/(4.*RNPIL))
      SUM2= SUM2 - MUX(N1,LP)*RNPIL*(SIN(RNPIL*XB)**2 - SIN(RNPIL*XA)*
     1*2)/(2.*RNPIL)
      GO TO 45
   44 CONTINUE
      SUM1= MUX(N1,LP)*(XB - XA)
   45 CONTINUE
      BLINER= BLINER - BESVL(MHAT,LP)*(CIOM*SUM1 + MACH*SUM2)
   47 CONTINUE
      RETURN
      END
```