GASSER

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THERMAL ANALYSIS

FRACTIONAL WATT VUILLEUMIER CYROGENIC REFRIGERATOR PROGRAM ENGINEERING NOTEBOOK

74-9896-1

May 1974

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Prepared under Contract No. NAS 5-21715

for

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

GARRETT

AIRESEARCH MANUFACTURING COMPANY
OF CALIFORNIA

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FOREWORD

Under NASA Contract NAS 5-21715, the AiResearch Manufacturing Company, a Division of The Garrett Corporation, developed a $65^{0}K$ Vuilleumier (VM) cryogenic refrigerator for the NASA Goddard Space Flight Center (GSFC), Greenbelt, Maryland. During the program, thermal analysis and stress analysis notebooks were compiled for submittal to GSFC. This two-volume document contains (or references) all material compiled during the course of the thermal and stress analyses. In certain instances, copyrighted reference material was used during the analytical work and will not be reproduced in this document.

Volume 1, identified as AiResearch document 74-9896-1, presents the detailed thermal analysis that was conducted during the program.

Volume 2, identified as AiResearch document 74-9896-2 presents the detailed stress analysis that was conducted during the program on various component parts/assemblies of the VM refrigerator.

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SECTION 1

INTRODUCTION

SECTION 1

INTRODUCTION

The VM refrigerator thermal design is a lengthy iterative process. Initially, rough cut design calculations establish the design approach and basic sizing of the machine's elements. After the basic design is defined, effort must be concentrated on matching the thermodynamic design with that of the heat transfer devices (heat exchangers and regenerators). Typically, the heat transfer device configurations and volumes are adjusted to improve their heat transfer and pressure drop characteristics. These adjustments imply that changes be made to the active displaced volumes, compensating for the influence of the heat transfer devices on the thermodynamic processes of the working fluid. Then, once the active volumes are changed, the heat transfer devices require adjustment to account for the variations in flows, pressure levels, and heat loads. This iterative process is continued until the thermodynamic cycle parameters match the designs of the heat transfer devices. By examining several matched designs, a near-optimum refrigerator can be selected.

Many of the required iterations were performed during Task I, Preliminary Design (Reference 1). The final design configuration presented in the Task II report (Reference 2) involves only relatively minor modifications to the preliminary design machine. The modifications affect primarily four regions in the machine: (1) the cold end heat exchanger was redesigned to reduce weight and void volume, (2) the hot regenerator was lengthened to reduce losses, (3) the cold end insulation was changed to aluminized mylar to facilitate integration with the Honeywell DCA, and (4) the motor size was increased to provide sufficient break in power, thus eliminating the requirement for a break in magnetic coupling.

In this program, emphasis was not placed on complete optimization of the refrigerator with respect to thermal performance; long operational life and reliability were considered the most important. It is believed however, that the careful detail thermal design has resulted in a near-optimum thermal design.

The detail analyses leading to the final thermal design and performance prediction of the VM refrigerator are presented in this Volume 1 of the Engineering Notebook. The thermal design analyses are summarized, and engineering notes and calculations generated during the program are presented. The topics covered are:

- System Description
- Cycle Parameters and Performance
- Cold Regenerator Design
- Hot Regenerator Design

- Cold Heat Exchanger Design
- Hot Heat Exchanger Design
- Ambient Sump Heat Exchanger Design
- Cold-End Insulation
- Hot-End Insulation
- Flow Distributors
- Flow Passage Pressure Drop, Void Volume, and Flow Distribution
- Cold End Seal
- Hot End Seal
- Conduction Losses
- Sump Cooling Interface

The general method of analysis applied to both the hot and the cold regenerators, and the regenerator matrix characterization is presented in Appendix A of this report.

SECTION 2

SYSTEM DESCRIPTION

SECTION 2

SYSTEM DESCRIPTION

INTRODUCTION

The GSFC fractional watt Vuilleumier cryogenic refrigerator is a gas-cycle, reciprocating machine, intended to operate continuously for five years (two years minimum) at a design speed of 400 rpm. Continuous operation for such a long time period presents technical problems, namely long-life bearing and seal design. This technology has been developed, and is applied to a flight prototype machine for the first time in this design. As an aid in understanding the analyses described in subsequent sections of this report, a brief physical and functional description and summary of the VM refrigerator is given in this section. The layout drawing of this refrigerator is presented in Figure 2-1.

PHYSICAL DESCRIPTION

Although the primary energy input to the refrigerator is thermal energy, moving components (two displacers) are required so that the VM cycle will function properly. The two displacers (hot and cold) which make the refrigerator a reciprocating machine are driven by connecting rods attached to a crankshaft (Figure 2-1). Crankshaft throws are arranged 90-degrees apart so that the hot displacer is leading the cold displacer when the crankshaft is driven by an electric motor located at one end of the shaft. The displacers travel inside cylinders surrounded by packed-bed regenerators which in turn are enclosed in pressure shells joined to the crankshaft housing, making the entire assembly pressure tight.

The displacers are arranged in a horizontally opposed configuration, with the cold displacer positioned at an angle of 180 degrees from the hot displacer, thus simplifying the crankcase and sump heat exchanger mechanical design and minimizing the interaction between the hottest and coldest parts of the system.

As indicated earlier, the primary energy for driving the refrigerator is introduced at the hot end in the form of heat. The interface at the hot end of the machine is shown in Figure 2-1. For this design, an electric heater is used to supply the heat; future spacecraft systems may use radioisotope heat sources or a solar collector to conserve solar cell generated energy.

Proper operation of the VM cycle also depends on rejecting heat in the crankcase-sump region. The GSFC VM refrigerator rejects heat to water cooling coils that interface with the crankcase and sump heat exchanger. These water cooling coils replace the ammonia heat pipes and a radiator which would be used in a flight-type system.

The refrigeration heat load is absorbed at the cold end via the cold-end heat exchanger. For test purposes, the refrigeration load is generated by a small resistance-type heater bonded to the exterior surface of the cold-end heat exchanger. In a spacecraft system, the refrigerator load will be mounted directly to the cold end.

FOLDOUT FRAME 1

FOLDOUT FRAME 2

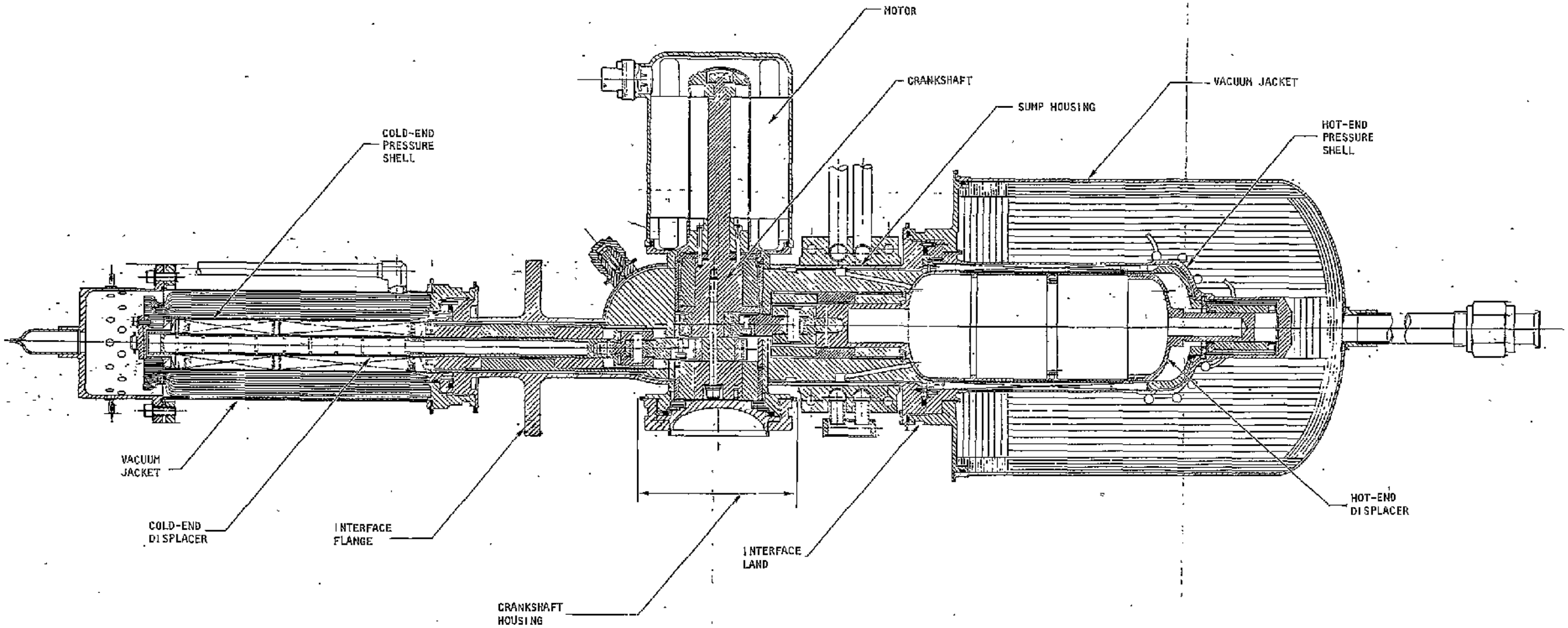

Figure 2-1. Fractional Watt Vuilleumier Cryogenic Refrigerator

DESIGN FEATURES

A number of design features incorporated in the VM refrigerator are intended to assure that the machine will meet the primary requirement of a minimum operating life of 2 years, with a 5-year operating life as a design goal. Design features of primary interest are: bearings; dynamic seals; absence of organic materials within the machine; effective heat transfer devices; and minimization of refrigeration loss caused by internal flow passages (Dead volume).

Bearings

Long-life bearings are a major factor in operational life of the VM refrigerator. After a lengthy test and evaluation program during the GSFC 5 watt refrigerator program, long-life bearing materials were selected--Boeing Compact 6-84-1 running against flame-sprayed tungsten carbide on hardened Inconel 718 or PH 13-8 Mo for the journal-type bearings; and Boeing Compact 6-84-1 running against flame-sprayed chrome carbide on the hot bearing shaft. Long test periods have shown that these bearings exhibit virtually no wear.

Dynamic Seals

Dynamic (non-contacting) seals are used in the hot and cold end of the machine. These seals performed well in tests conducted during the 5 watt refrigerator development program. Contacting surfaces of the journal bearings in the flow path from the dynamic seals function to backup operation of the seals.

Organic Materials

A primary design feature is the absence of any organic material within the machine. Organic materials were avoided due to their outgassing characteristics and the potential contamination of the working fluid during extended operating periods.

Effective Heat Transfer Devices

From a thermal design standpoint, a conscious effort was made to provide highly effective heat transfer devices. The design incorporates extended surface heat exchangers of the flow-through type for all internal heat exchangers. The cold regenerator makes use of monel shot as the matrix material. This material has superior thermo-physical properties for the specified operating temperature range. The hot regenerator matrix consists of stainless steel screens which provide a low pressure drop and high thermal performance.

Internal Flow Passages

The internal working fluid flow passages of the VM refrigerator contribute to refrigeration loss in two ways: pressure drop and void volume. Each of these factors decrease the net refrigeration. Optimization procedures have been developed that interrelate the two processes, thus assuring optimum designs without compromising the primary goal which is long life.

PERFORMANCE AND DESIGN SUMMARY

A summary of major performance and design parameters for the GSFC fractional watt Vuilleumier cryogenic refrigerator is given in Table 2-1.

TABLE 2-1

GSFC FLIGHT PROTOTYPE FRACTIONAL WATT VUILLEUMIER CRYOGENIC REFRIGERATOR - SUMMARY OF NOMINAL PERFORMANCE AND DESIGN VALUES

WORKING FLUID	HELIUM
TEMPERATURES	
Interfaces	
Cold End Interface	65°K (117°R)
Sump Interface	333.3K (600°R)
Hot End Interface	866°K (1560°R)
Gas	
Cold End	62.2°K (112°R)
Sump	344.4°K (620°R)
Hot End	853°K (1535°R)
PRESSURES	
Charge Gas Pressure at 297°K (535°R)	4.99X10⁶ N/m² (723 psia)
Maximum Cycle Pressure	6.895X10⁶ N/m² (1000 psia)
Minimum Cycle Pressure	6.088X10⁶ N/m² (883 psia)
THERMAL INPUT/OUTPUT	
Net Cold End Refrigeration	0.25 w
Cold End Insulation Loss	0.060 w
Total Hot End Input Power	80 w
Hot End Insulation Loss	6.1 w
Hot End Input Less Insulation Loss	73.9 w
Motor Input Power	10 w
Heat Rejection Rate	80.25 w
DRIVE MOTOR POWER INPUT	
Speed	400 rpm
Shaft Power	2.5 w
Electrical Input Power	10 w max.
COLD DISPLACER SPECIFICATIONS	
Length	13.97 cm (5.5 in)
Bore	1.017 cm (0.4 in)
Stroke	1.118 cm (0.44 in)
Displaced Volume	0.905 cm³ (0.0553 in³)

HOT DISPLACER SPECIFICATIONS	
Length	11.10 cm (4.37 in)
Bore	5.94 cm (2.336 in)
Stroke	1.219 cm (0.48 in)
Displaced Volume	30.85 cm³ (1.885 in3)
COLD REGENERATOR SPECIFICATIONS	
NOTE: Cold regenerator consists of 2 sections both packed with spheres of different diameters.	
Sump End	
Configuration	Annular
Matrix Material	Monel spheres, 0.0254 cm (0.010 in) dia.
Inside Diameter	1.077 cm (0.424 in)
Outside Diameter	2.60 cm (1.024 in)
Frontal Area	4.40 cm² (0.682 in²)
Length	7.62 cm (3 in)
Matrix Hydraulic Diameter	0.01087 cm (0.004272 in)
Matrix Surface/Volume Ratio	144.1 cm²/cm³ (366 in²/in³)
Void Fraction	0.39
Cold End	
Configuration	Annular
Matrix Material	Monel spheres. 0.01778 cm (0.007 in) dia.
Inside Diameter	1.077 cm (0.424 in)
Outside Diameter	2.60 cm (1.024 in)
Frontal Area	4.40 cm² (0.682 in²)
Length	5.08 cm (2 in)
Matrix Hydraulic Diameter	0.00759 cm (0.00299 in)
Matrix Surface/Volume Ratio	206 cm²/cm³ (523 in²/in³)
Void Fraction	0.39
HOT REGENERATOR SPECIFICATIONS	
Configuration	Annular
Matrix Material	Stainless Steel, 100 mesh
Inside Diameter	6.06 cm (2.386 in)

HOT REGENERATOR SPECIFICATIONS--Continued	
Outside Diameter	6.83 cm (2.686 in)
Frontal Area	7.42 cm² (1.15 in²)
Length	11.11 cm (4.375 in)
Matrix Hydraulic Diameter	0.0287 cm (0.01132 in)
Matrix Surface/Volume Ratio	100.7 cm²/cm³ (256 in²/in³)
Void Fraction	.725
VOID VOLUMES	
Cold End at 622°K (112°R)	0.573 cm³ (0.035 in³)
Sump at 344°K (620°R)	48.55 cm³ (2.963 in³)
Hot end at 853°K (1535°R)	5.67 cm³ (0.346 in³)
Cold Regenerator at 203°K (366°R)	21.88 cm³ (1.331 in³)
Hot Regenerator at 598°K (1077.5°R)	59.76 cm³ (3.647 in³)
HEAT TRANSFER CHARACTERISTICS	
Cold Heat Exchanger	
Maximum Pressure Drop	94.5 N/m² (0.0137 psi)
Conductance (ηhA)	0.404 w/°K (0.764 Btu/hr-°R)
Hot Heat Exchanger	
Maximum Pressure Drop	1.18X10³ N/m² (0.171 psi)
Conductance (ηhA)	6.51 w/°K (12.36 Btu/hr-°R)
Sump Heat Exchanger	
Maximum Pressure Drop	4.29X10² N/m² (0.0623 psi)
Conductance (ηhA)	27 w/°K (51.3 Btu/hr-°R)

SECTION 3

CYCLE PARAMETERS AND PERFORMANCE

SECTION 3

CYCLE PARAMETERS AND PERFORMANCE

INTRODUCTION

The final refrigerator design has a hot displaced volume of 30.85 cm^3 (1.885 $in.^3$) and a cold displaced volume of 0.905 cm^3 (0.0552 $in.^3$). The predicted refrigeration for the new machine is considerably greater than the 0.25 watt required. This capacity was designed into the machine to allow for performance degradation over a two year operating period.

PERFORMANCE AT NOMINAL DESIGN CONDITIONS

The output of the ideal VM cycle analysis computer program for the final design configuration of the refrigerator operating at nominal design conditions is presented in Figure 3-1. Table 3-1 presents the nomenclature for interpretation of the ideal cycle program output. The refrigerator internal volumes, pressure, and working fluid mass flow rates as a function of crankshaft position are shown in Figures 3-2, 3-3, and 3-4. The data for these plots were taken from Figure 3-1. The ideal cycle program is also capable of estimating the effects of internal pressure drops on the predicted refrigeration and heat input required. The cycle program output with the hot and cold end pressure drop effects included is shown in Figure 3-5. The pressure drops are calculated for all internal passages of the machine. The refrigeration and heat input requirements of Figure 3-5 were used in calculating the net performance of the fractional watt VM at the nominal design point. The gas temperatures in the cold, hot, and sump regions of the refrigerator were computed from the heat loads, interface metal temperatures, and performance characteristics of the respective heat exchangers. The regenerator gas temperatures input to the ideal cycle program are the numerical average of the temperatures at each end. Temperatures utilized in various regions of the machine are:

Cold end volume	62.2°K (112°R)
Cold regenerator	203.3°K (366°R)
Sump volume	344.4°K (620°R)
Hot regenerator	598.6°K (1077.5°R)
Hot end	852.8°K (1535°R)

The film temperature drop in the cold end heat exchanger is 0.62°K (1.12°R) and the wall temperature drop is an additional 0.37°K (0.67°R). The specified refrigeration temperature at the cold head (external surface of the cold end heat exchanger) is 65°K (117°R), thus a margin of 1.78°K (3.21°R) exists; that is, the actual surface temperature at the design conditions would be 64°K (115.21°R). This temperature at the cold end of the machine provides for 1.78°K (3.21°R) drop across the interface with the detector chip assembly before the 65°K (117°R) temperature level is reached. Since the calculated temperature

FRACTIONAL WATT VM AT NOMINAL DESIGN POINT

OPERATING PARAMETERS

COLD VOLUME TEMP.	=	112.00	R
SUMP VOLUME TEMP.	=	620.00	R
HOT VOLUME TEMP.	=	1535.00	R
COLD REGEN. TEMP.	=	366.00	R
HOT REGEN. TEMP.	=	1077.50	R
COLD DISPLACED VOL.	=	.05530	CU-IN
HOT DISPLACED VOL.	=	1.88500	CU-IN
COLD DEAD VOL.	=	.03500	CU-IN
SUMP DEAD VOL.	=	2.96320	CU-IN
HOT DEAD VOL.	=	.34600	CU-IN
COLD REGEN. VOL.	=	1.33060	CU-IN
HOT REGEN. VOL.	=	3.64760	CU-IN
GAS CONSTANT	=	4634.40	IN-LB/LBM-R
SPEED	=	400.00	RPM
CHARGE PRESSURE	=	722.81	PSIA
CHARGE TEMPERATURE	=	535.00	R
MASS OF FLUID	=	.0029	LBM
TOTAL VOLUME	=	10.26270	CU-IN

PRESSURE - MASS - FLOW PROFILE

ANGLE DEG	PRESS PSIA	VC CU-IN	VA CU-IN	VH CU-IN	MDOTC LB/SEC	MDOTA LB/SEC	MDOTH LB/SEC	MDOTRCA LB/SEC	MDOTRHA LB/SEC
20.	969.32	.0367	3.6370	1.6108	.00078	-.00956	.00551	.00244	.00712
40.	985.77	.0415	3.3488	1.8942	.00136	-.00846	.00462	.00262	.00584
60.	996.56	.0488	3.1310	2.1046	.00177	-.00613	.00304	.00244	.00369
80.	999.98	.0578	3.0100	2.2166	.00194	-.00286	.00099	.00190	.00096
100.	995.46	.0674	3.0003	2.2167	.00183	.00081	-.00120	.00110	-.00191
120.	983.75	.0765	3.1031	2.1049	.00146	.00428	-.00318	.00015	-.00444
140.	966.70	.0838	3.3060	1.8947	.00092	.00703	-.00464	-.00077	-.00626
160.	946.79	.0886	3.5846	1.6113	.00029	.00873	-.00543	-.00154	-.00719
180.	926.65	.0903	3.9051	1.2891	-.00034	.00928	-.00552	-.00208	-.00720
200.	908.66	.0886	4.2291	.9667	-.00089	.00874	-.00498	-.00236	-.00639
220.	894.65	.0838	4.5175	.6832	-.00132	.00730	-.00393	-.00237	-.00494
240.	885.94	.0765	4.7354	.4726	-.00160	.00516	-.00252	-.00213	-.00303
260.	883.24	.0675	4.8566	.3605	-.00171	.00255	-.00089	-.00169	-.00086
280.	886.78	.0579	4.8665	.3602	-.00165	-.00032	.00084	-.00107	.00140
300.	896.28	.0488	4.7638	.4718	-.00142	-.00322	.00251	-.00034	.00356
320.	910.92	.0415	4.5611	.6819	-.00103	-.00590	.00399	.00047	.00543
340.	929.33	.0367	4.2827	.9652	-.00050	-.00805	.00509	.00126	.00679
360.	949.58	.0350	3.9621	1.2874	.00013	-.00936	.00565	.00195	.00741

IDEAL REFRIGERATION AND HEAT INPUT

REFRIGERATION = 3.7372 WATTS
THERMAL HEAT = 25.6304 WATTS
MAX. PRESSURE = 1000.0000 PSIA AT ANGLE = 78.59 DEGREES

Figure 3-1. Ideal VM Cycle Computer Program Output for Nominal Design Conditions

TABLE 3-1

IDEAL VUILLEUMIER CYCLE ANALYSIS
NOMENCLATURE KEY

Symbol	Definition
PRESS	Cycle Pressure
ANGLE	Crankshaft angle referenced to cold displacer top dead center
VC	Cold displaced volume
VA	Ambient displaced volume
VH	Hot displaced volume
MDOTC	Flow rate into cold volume
MDOTA	Flow rate into ambient volume
MDOTH	Flow rate into hot volume
MDOTRCA	Flow rate into cold regenerator at the end toward the sump
MDOTRHA	Flow rate into hot regenerator at end toward the sump

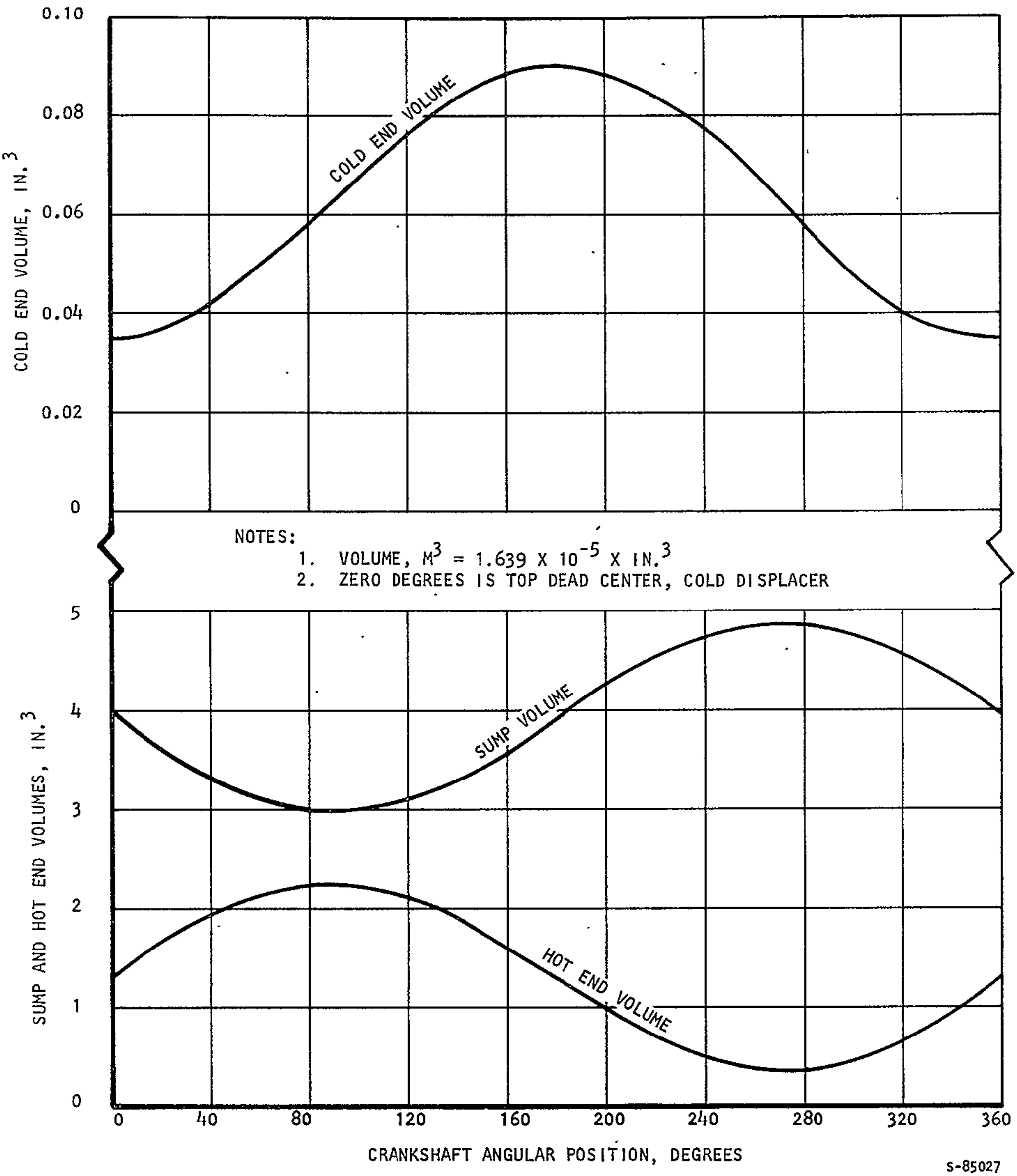

Figure 3-2. Internal Volumes of Fractional Watt VM Refrigerator

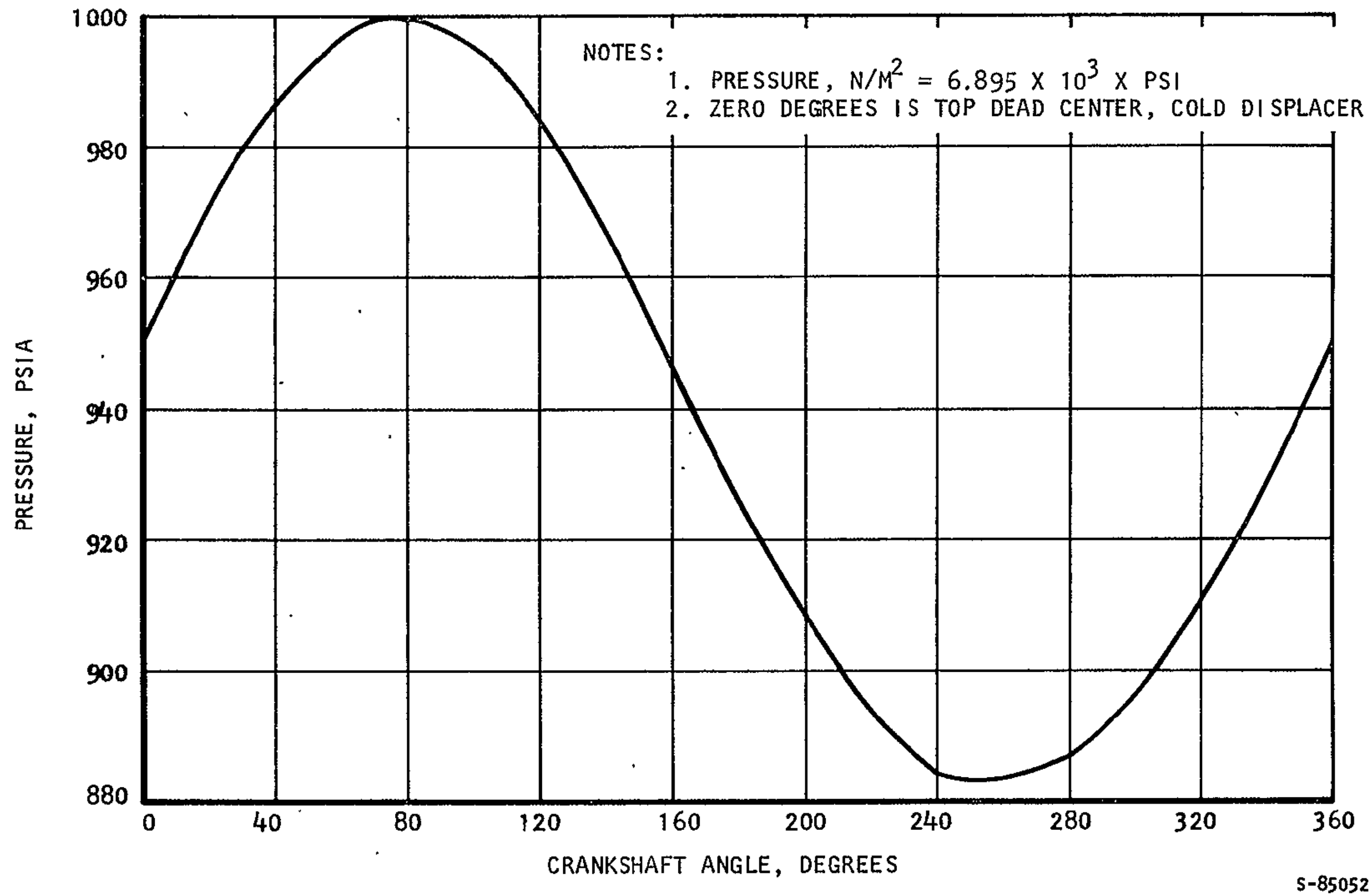

Figure 3-3. Fractional Watt Refrigerator Pressure Characteristic at Nominal Design Conditions

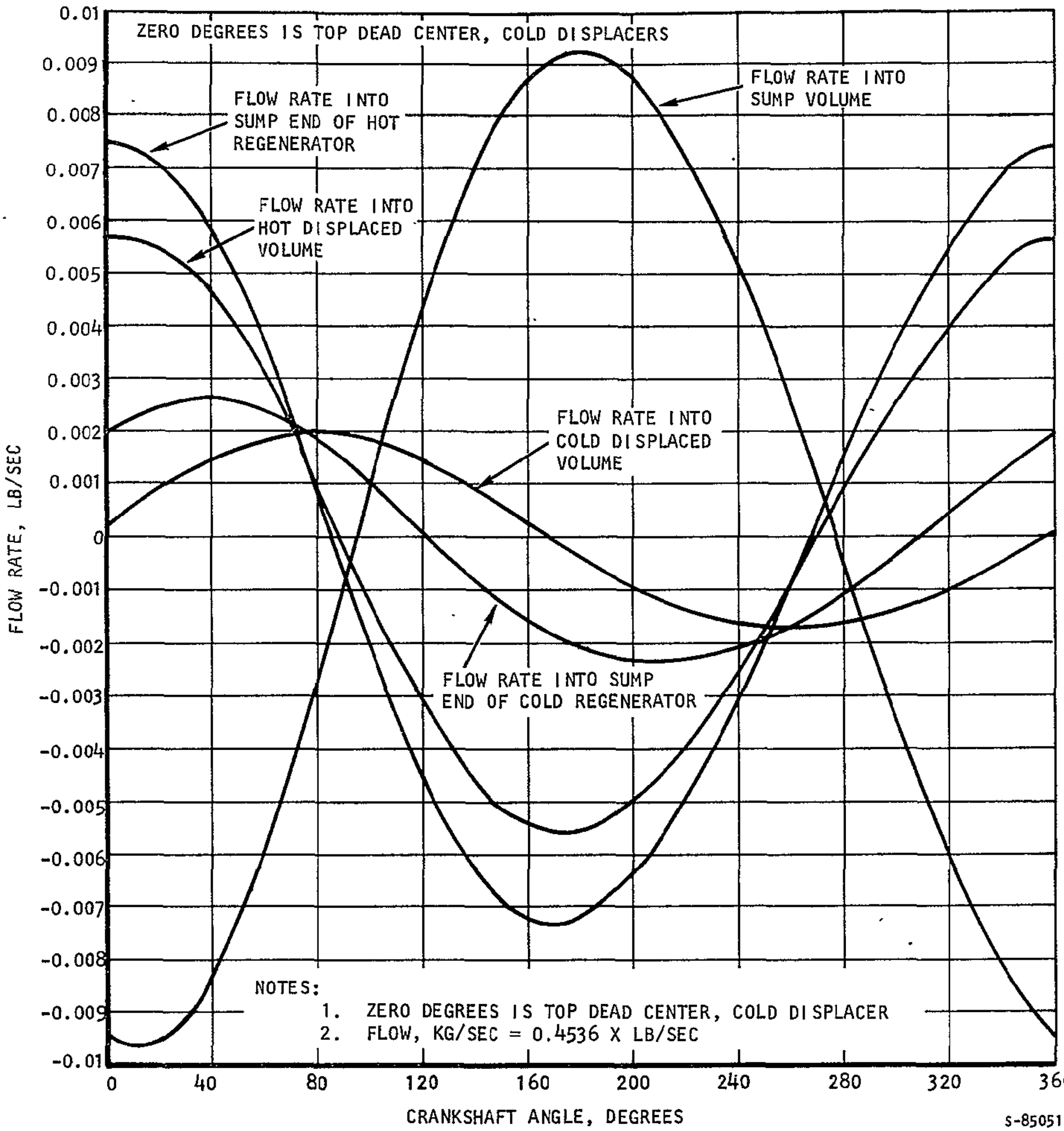

Figure 3-4. Fractional Watt Refrigerator Internal Flow Rates at Nominal Design Conditions

FRACTIONAL WATT VM AT NOMINAL DESIGN POINT WITH PRESSURE DROP

OPERATING PARAMETERS

```
COLD VOLUME TEMP.       =    112.00 R
SUMP VOLUME TEMP.       =    620.00 R
HOT VOLUME TEMP.        =   1535.00 R
COLD REGEN. TEMP.       =    366.00 R
HOT REGEN. TEMP.        =   1077.50 R
COLD DISPLACED VOL.     =    .05530 CU-IN
HOT DISPLACED VOL.      =   1.88500 CU-IN
COLD DEAD VOL.          =    .03500 CU-IN
SUMP DEAD VOL.          =   2.96320 CU-IN
HOT DEAD VOL.           =    .34600 CU-IN
COLD REGEN. VOL.        =   1.33060 CU-IN
HOT REGEN. VOL.         =   3.64760 CU-IN
GAS CONSTANT            =   4634.40 IN-LB/LBM-R
SPEED                   =    400.00 RPM

CHARGE PRESSURE         =    722.81 PSIA
CHARGE TEMPERATURE      =    535.00 R
MASS OF FLUID           =     .0029 LBM
TOTAL VOLUME            =  10.26270 CU-IN
```

PRESSURE - MASS - FLOW PROFILE

ANGLE DEG	PRESS PSIA	VC CU-IN	VA CU-IN	VH CU-IN	MDOTC LB/SEC	MDOTA LB/SEC	MDOTH LB/SEC	MDOTRCA LB/SEC	MDOTRHA LB/SEC
20.	969.32	.0367	3.6370	1.6108	.00078	-.00956	.00551	.00244	.00712
40.	985.77	.0415	3.3488	1.8942	.00136	-.00846	.00462	.00262	.00584
60.	996.56	.0488	3.1310	2.1046	.00177	-.00613	.00304	.00244	.00369
80.	999.98	.0578	3.0100	2.2166	.00194	-.00286	.00099	.00190	.00096
100.	995.46	.0674	3.0003	2.2167	.00183	.00081	-.00120	.00110	-.00191
120.	983.75	.0765	3.1031	2.1049	.00146	.00428	-.00318	.00015	-.00444
140.	966.70	.0838	3.3060	1.8947	.00092	.00703	-.00464	-.00077	-.00626
160.	946.79	.0886	3.5846	1.6113	.00029	.00873	-.00543	-.00154	-.00719
180.	926.65	.0903	3.9051	1.2891	-.00034	.00928	-.00552	-.00208	-.00720
200.	908.66	.0886	4.2291	.9667	-.00089	.00874	-.00498	-.00236	-.00639
220.	894.65	.0838	4.5175	.6832	-.00132	.00730	-.00393	-.00237	-.00494
240.	885.94	.0765	4.7354	.4726	-.00160	.00516	-.00252	-.00213	-.00303
260.	883.24	.0675	4.8566	.3605	-.00171	.00255	-.00089	-.00169	-.00086
280.	886.78	.0579	4.8665	.3602	-.00165	-.00032	.00084	-.00107	.00140
300.	896.28	.0488	4.7638	.4718	-.00142	-.00322	.00251	-.00034	.00356
320.	910.92	.0415	4.5611	.6819	-.00103	-.00590	.00399	.00047	.00543
340.	929.33	.0367	4.2827	.9652	-.00050	-.00805	.00509	.00126	.00679
360.	949.58	.0350	3.9621	1.2874	.00013	-.00936	.00565	.00195	.00741

THE CYCLE PRESSURE ABOVE IS IDEAL,THE P-V INTEGRALS HAVE BEEN MODIFIED FOR PRESSURE DROP
HOT MAXIMUM DP= 1.0000 PSI   COLD MAXIMUM DP= 2.8320 PSI

IDEAL REFRIGERATION AND HEAT INPUT

```
REFRIGERATION =     3.5994 WATTS
THERMAL HEAT  =    27.4648 WATTS
MAX. PRESSURE =  1000.0000 PSIA AT ANGLE =  78.59 DEGREES
```

Figure 3-5. Ideal VM Cycle Computer Program Output for Nominal Design Point With Effect of Pressure Drop

drop across this interface is 0.77°K (1.38°R), a slight margin in performance is provided. These figures are based on the specified net refrigeration load of 0.25 w.

The film temperature drop in the sump heat exchanger is approximately 2.96°K (5.33°R); this allows an additional 8.15°K (14.67°R) temperature drop across the sump pressure vessel wall and heat rejection interface clamp assembly for the specified 333°K (600°R) sump temperature. The calculated interface temperature drop is 3.71°K (6.67°R) or a total ΔT of 6.67°K (12°R), with water cooling coils. This provides a 4.44°K (8°R) design margin in the sump region of the VM.

The hot end heat exchanger has a film temperature drop of approximately 12.2°K (22°R) and a wall temperature drop of 0.78°K (1.4°R); the gas temperature of 852.8°K (1535°R) thus results in an outer wall temperature of 865.8°K (1558.4°R) at the nominal design conditions. The hot end heat exchanger receives its heat input directly by conduction from the heater brazed onto the hot end. Thus the heater operates at approximately 867°K (1560°R) while supplying 80 w of thermal power to the system.

The ideal refrigeration capacity of the system, considering pressure drops, is approximately 3.6 w, as given in Figure 3-5. Table 3-2 summarizes the thermal losses in the cold end of the machine that can be directly analyzed. Other factors contributing to the losses consist of mismatched temperature gradients along the displacer and cylinder walls and nonuniform flow distribution. Based on the experience from the GSFC 5 watt VM, and the detailed attention given to providing uniform flow distribution, no allowance has been made for these miscellaneous losses.

Table 3-2 includes the estimated losses for a machine that has been operating for two years. The indicated degradation (a loss of approximately 0.500 w of cooling) is caused by increased leakage of working fluid past the linear bearings that support the cold displacer. These bearings function as backup seals to the cold end labyrinth seals, greatly reducing cold end leakage and the associated thermal losses. The estimated degradation is believed very conservative; actual degradation is expected to be less since the worst cases of both bearing wear and pressure drop, which promote the leakage, were used in its calculation.

Subtracting the losses given in Table 3-2 from the ideal refrigeration yields net refrigeration capacities of 0.929 w and 0.429 w for the new machine and after two years of wear, respectively. These figures show a design margin of 0.179 watts at the end of two years of operation.

The ideal thermal power is given as 27.465 watts in Figure 3-5. The hot end losses are summarized in Table 3-3. The total thermal power required, 78.1 watts, is the sum of the ideal power, which includes pressure loss effects, and the losses. This input power provides a small design margin below the allowable thermal input power of 80 watts.

TABLE 3-2

COLD END LOSS SUMMARY

Refrigeration/Loss	Cooling/Loss (watts)
Ideal refrigeration	3.599
Regenerator loss ($\int MC_p \Delta T$)	0.872
Regenerator conduction loss	0.654
Regenerator inner wall conduction loss	0.200
Regenerator outer wall conduction loss	0.618
Displacer conduction loss	0.266
Insulation conduction loss	0.060
Ideal refrigeration minus losses (Net refrigeration, new machine)	0.929
Displacer leakage loss, 2 yr worn bearings	0.500
Net refrigeration, 2 yr old machine	0.429

TABLE 3-3

HOT-END POWER SUMMARY

	Heat Input, watts
Ideal power input	27.465
Regenerator Loss ($\int MC_p \Delta T$)	6.088
Regenerator conduction loss	1.69
Regenerator inner wall conduction loss	5.79
Regenerator outer wall conduction loss	14.07
Displacer conduction loss	16.9
Insulation conduction loss	6.1
Total hot end power required	78.103

GROWTH POTENTIAL OF BASIC DESIGN

The fractional watt VM meets all performance requirements when operated at the nominal design point. However, additional cooling capacity is available through operation at off-design conditions. The majority of the operating parameters of the VM are fixed by the design of the machine and the refrigeration and sump temperatures. The operating conditions that may be varied are the rotational speed, the hot-end temperature, and the cycle pressure. Increases in all three variables are possible with the present design. The relative merits of each are discussed below.

Rotational Speed

The selection of refrigerator speed was based on the extensive investigations performed in support of the GSFC 5 watt VM refrigerator. At very low speeds (200 rpm and below), the residence times of the gas in the heat transfer devices are such that near isothermal operation can be achieved. This would, however, require a refrigerator of relatively large size, and the other thermal losses, such as axial conduction, would eliminate any gains in thermal performance. If, on the other hand, the refrigerator was operated at a very high speed (above 600 rpm), the temperature response of the gas and heat exchanger devices would be such that the thermal performance of the refrigerator would be greatly decreased.

A speed of 400 rpm was selected as a good compromise. This speed also results in a very low bearing speed and low dynamic loading for the bearings to support. To further increase life, it may be possible to reduce the speed, with an accompanying reduction in capacity.

The effect of cycle speed on ideal refrigeration capacity is shown on Figure 3-6 for a wide range of operating conditions. The data were obtained by use of the ideal cycle computer program, and do not include the effects of pressure drop or other internal losses. Thus performance may be compared directly on an ideal basis.

Of the three parameters that may be varied in order to increase performance, only cycle speed has a direct effect on life. Therefore, although speed increases produce a very marked increase in ideal refrigeration, as shown in Figure 3-6, this method of increasing performance is the least desirable. In addition to increased dynamic loadings, the internal pressure drops and departure from isothermal operation both increase. The effects of increased pressure drops are to further increase bearing loading and also to increase the mechanical power input required. In addition, leakage and other internal losses are increased. Thus, the net gain in refrigeration will not increase as rapidly as the ideal refrigeration.

Hot End Temperature

The selection of the hot end temperature was based primarily on material considerations. With the use of electrical heating, the power source temperature does not play an important part in the selection of the hot end temper-

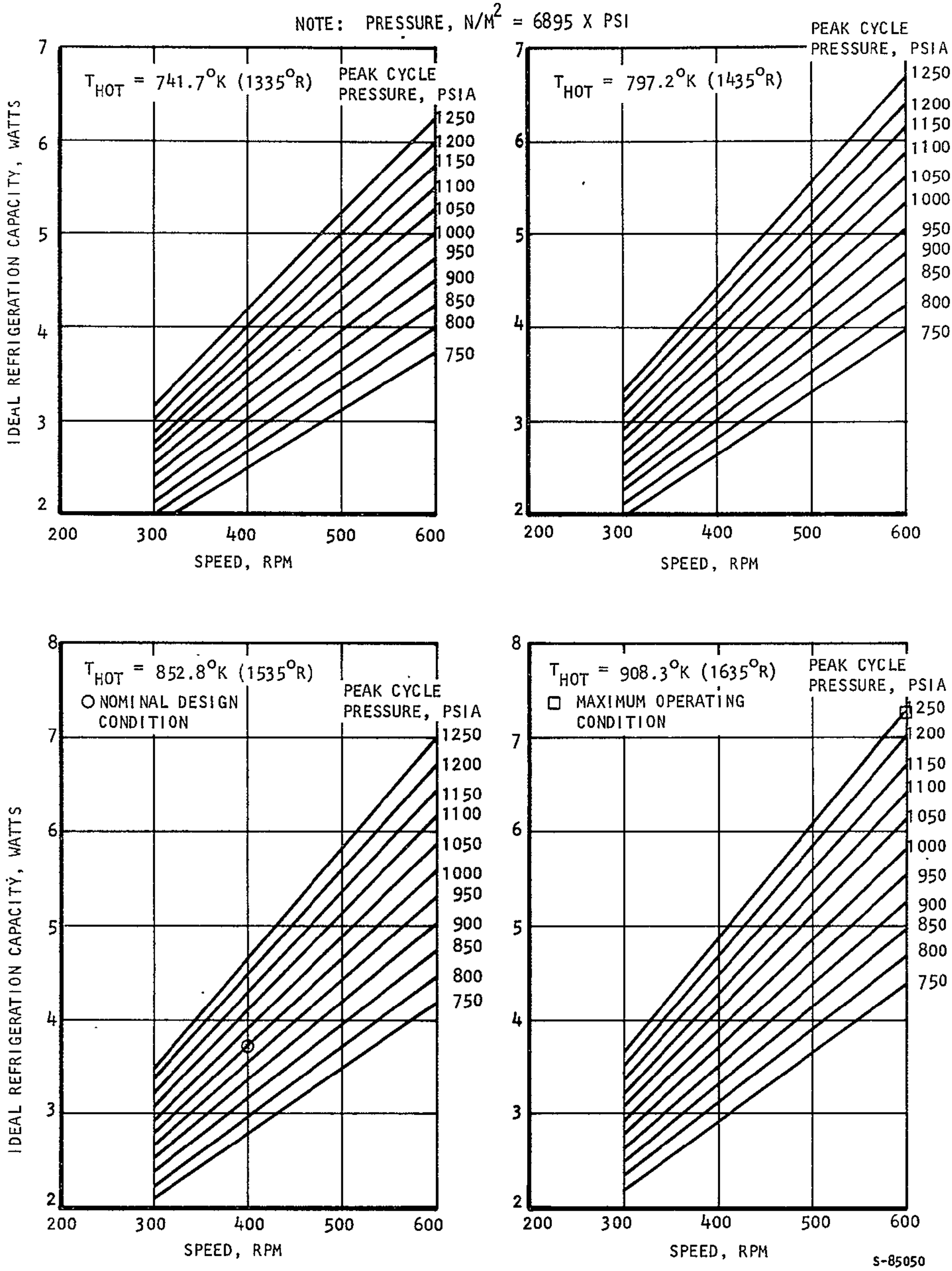

Figure 3-6. Effect of Rotational Speed on Ideal Performance of Fractional Watt VM.

ature. Heaters are available which are capable of operation at temperatures considerably above the maximum operating temperature of the hot end pressure containment dome. From a thermodynamic standpoint, the higher the hot end temperature, the higher the thermal efficiency. The selected temperature of 867°K (1560°R) was based on the strength characteristics of Inconel 718 and also on the bearing characteristics of Boeing Compact 6-84-1. Figure 3-7 shows the sudden drop in strength in Inconel 718 at temperatures above 992°K (1660°R). A 55.6°K (100°R) design margin of safety was allowed below this value. The hot end is designed structurally for pressure containment of 992°K (1660°R); thus the use of 867°K (1560°R) hot end temperature represents a margin of safety on performance, and not stress level.

The effect of hot-end temperature on ideal refrigeration is illustrated in Figure 3-8. The growth potential is readily apparent. The hot end temperature also affects the coefficient of performance (COP), or thermal efficiency, of the refrigerator. Figure 3-9 shows that COP increases with increasing temperature illustrating the desirability of operating at a higher hot-end temperature.

Thus, an increase of the hot-end temperature should be considered the prime candidate for increase in refrigeration capacity. A higher hot-end temperature not only increases total refrigeration, but the thermal efficiency is also increased. The internal losses will not increase as rapidly as with increase of rotational speed.

Pressure Influence on Cooling Capacity

The cycle pressure was selected in a somewhat qualitative manner. Thermodynamically, the higher the operating pressure, the higher the refrigeration capacity (or the smaller the size of the refrigerator). Originally, a maximum operating pressure of 10.33×10^6 N/m^2 (1500 psia) was selected in trying to minimize the effect of axial conduction losses for the cold end. For low pressures, the wall thickness of the cold end is determined by fabrication limitations. However, as the pressure increases, the wall thickness requirement remains constant until pressure level, rather than fabrication requirements becomes the design consideration.

The use of the original design pressure resulted in hot end pressure vessel thicknesses that gave excessive conduction losses. Therefore, in order to maintain the thermal power input below 80 watts, the maximum operating pressure was reduced to 8.62×10^6 N/m^2 (1250 psia).

By thermally designing the refrigerator to operate at 6.895×10^6 N/m^2 (1000 psia) and by structurally designing the refrigerator for 8.62×10^6 N/m^2 (1250 psia), the pressure of the working gas can later be increased by 25 percent, if necessary, to increase the capacity of the engine. This provides a margin of almost 25 percent in the thermal refrigeration capacity. The dependence of refrigeration on peak cycle pressure is shown in Figure 3-10 for a wide range of operating conditions.

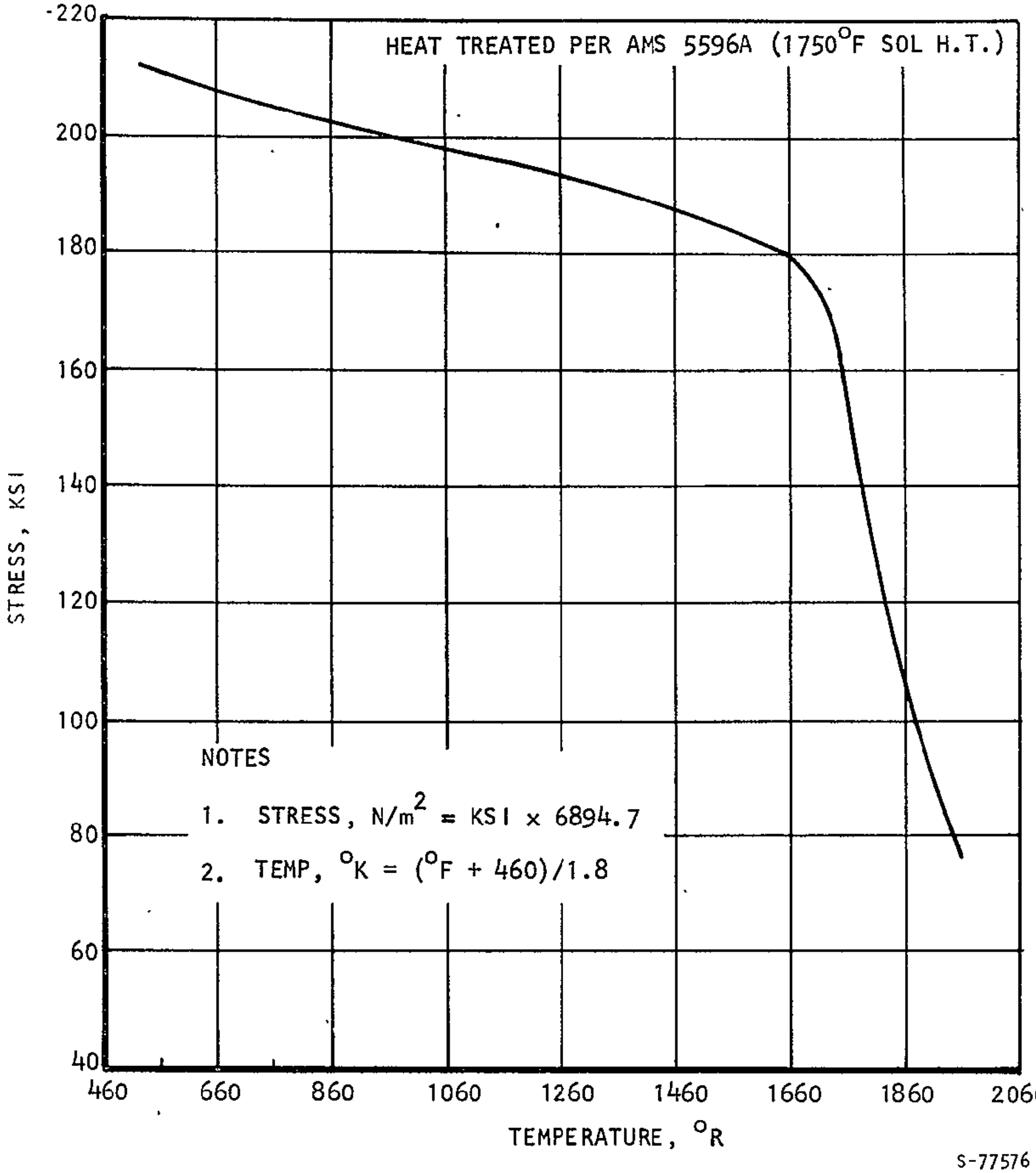

Figure 3-7. Ultimate Strength of Inconel 718

NOTES: 1. TEMP, $^oK = ^oR/1.8$

2. PRESSURE, N/M^2 = 6895 X PSI

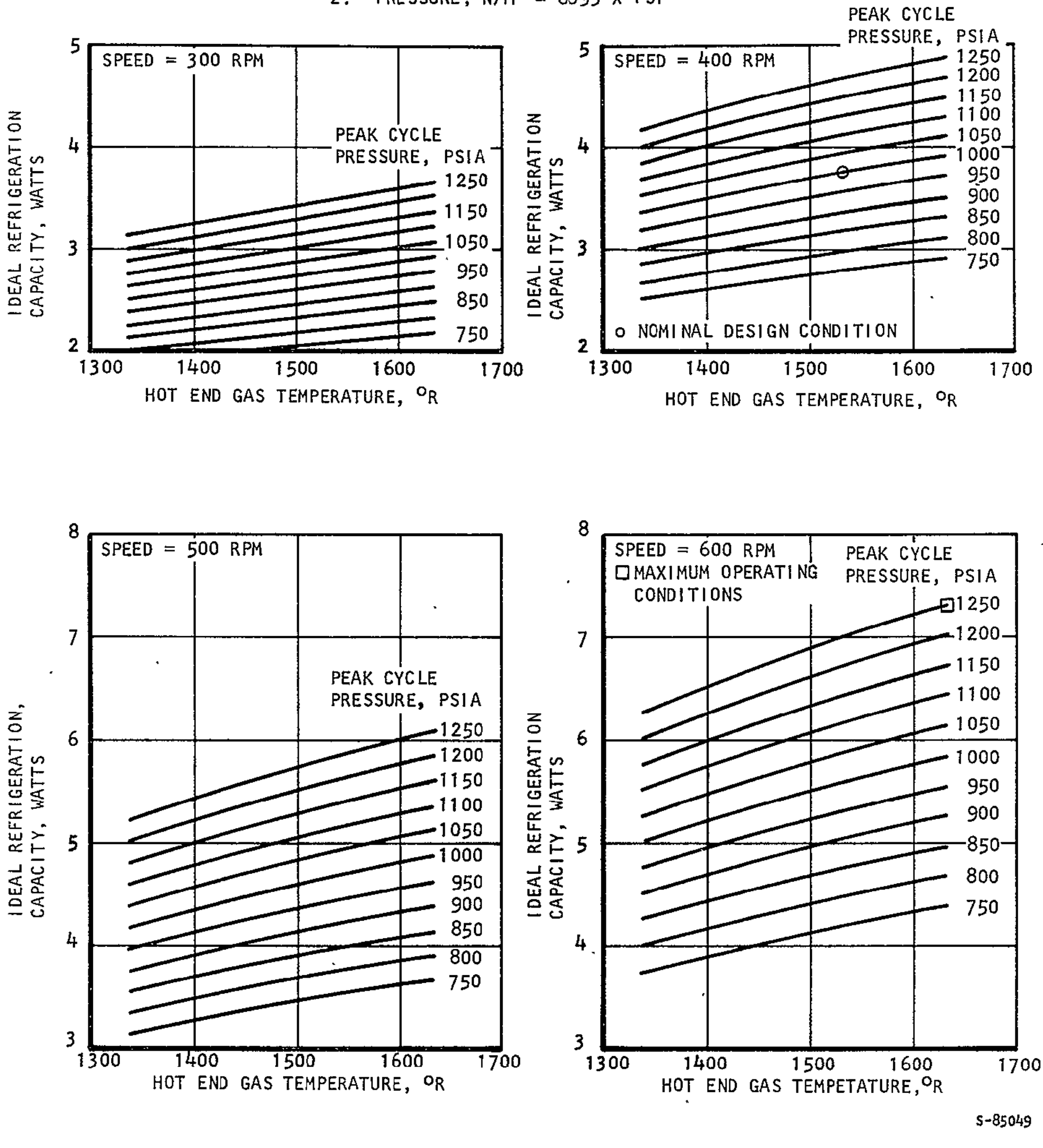

Figure 3-8. Effect of Hot End Temperature on Ideal Performance of Fractional Watt VM

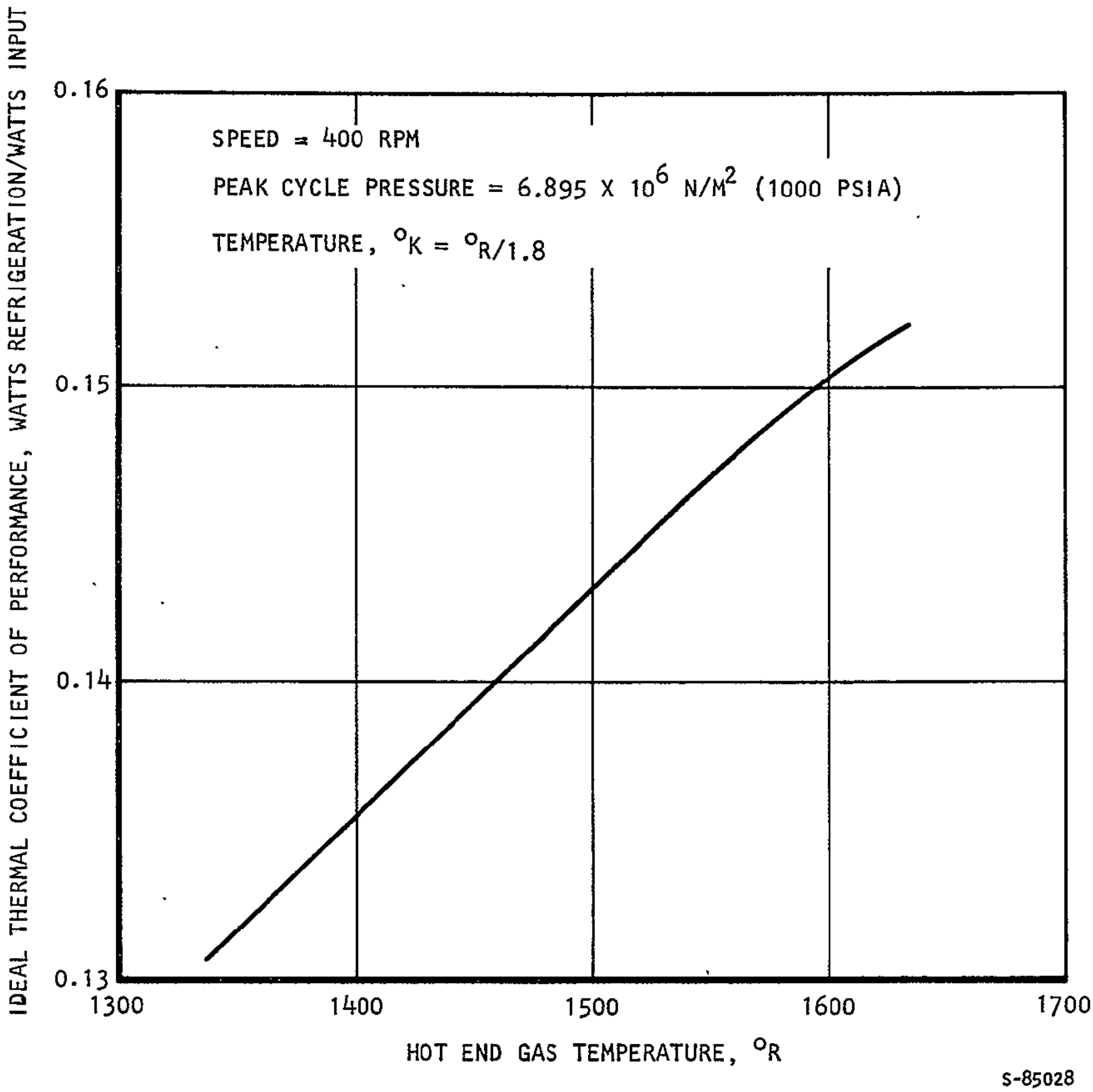

Figure 3-9. Effect of Hot End Temperature on Ideal Coefficient of Performance of Fractional Watt VM

Figure 3-10 indicates that a cycle pressure increase is an effective means of providing greater refrigeration capacity. In actual operation, the internal losses do increase with pressure increase, but not nearly as rapidly as with rotational speed increases. The COP, or efficiency, is not affected by pressure level. Since COP increases with hot-end temperature, as previously discussed, temperature level is still the preferred method of obtaining additional performance from the VM.

Figure 3-11 gives the peak cycle pressure as a function of charge pressure at 535°R ambient for several hot-end temperature operating conditions. This data is dependent on the cold-end and sump temperatures of the nominal design point, but is independent of rotational speed over the current range of interest. Data from Figure 3-11 should be used to determine the helium charging pressure for the refrigerator at ambient conditions. The maximum charge pressure allowable without exceeding the maximum working pressure during operation ranges from 897 to 918 psia, depending on the hot-end temperature at which the refrigerator is to be operated.

Summary of Improved Performance Parameters

Previous discussions concerning increased performance capability are summarized in Table 3-4.

IDEAL PERFORMANCE CAPABILITY

The ideal performance of the fractional watt VM at the extremes of all operating parameters is presented in Figure 3-12. Comparison of data from this figure with that of Figure 3-1 shows that the ideal refrigeration capacity may be nearly doubled. In view of the conservative assumptions made in estimating net refrigeration after two years of operation and the increased capacity available from varying the operating parameters, more than adequate design margin exists to overcome unforeseen contingencies. Figures 3-13 and 3-14 present the cyclic pressure and mass flow profiles when the VM is operated at design limits. These figures may be compared directly with 3-3 and 3-4, which present the same data at nominal operating conditions.

NOTE: PRESSURE, N/M^2 = 6895 X PSI

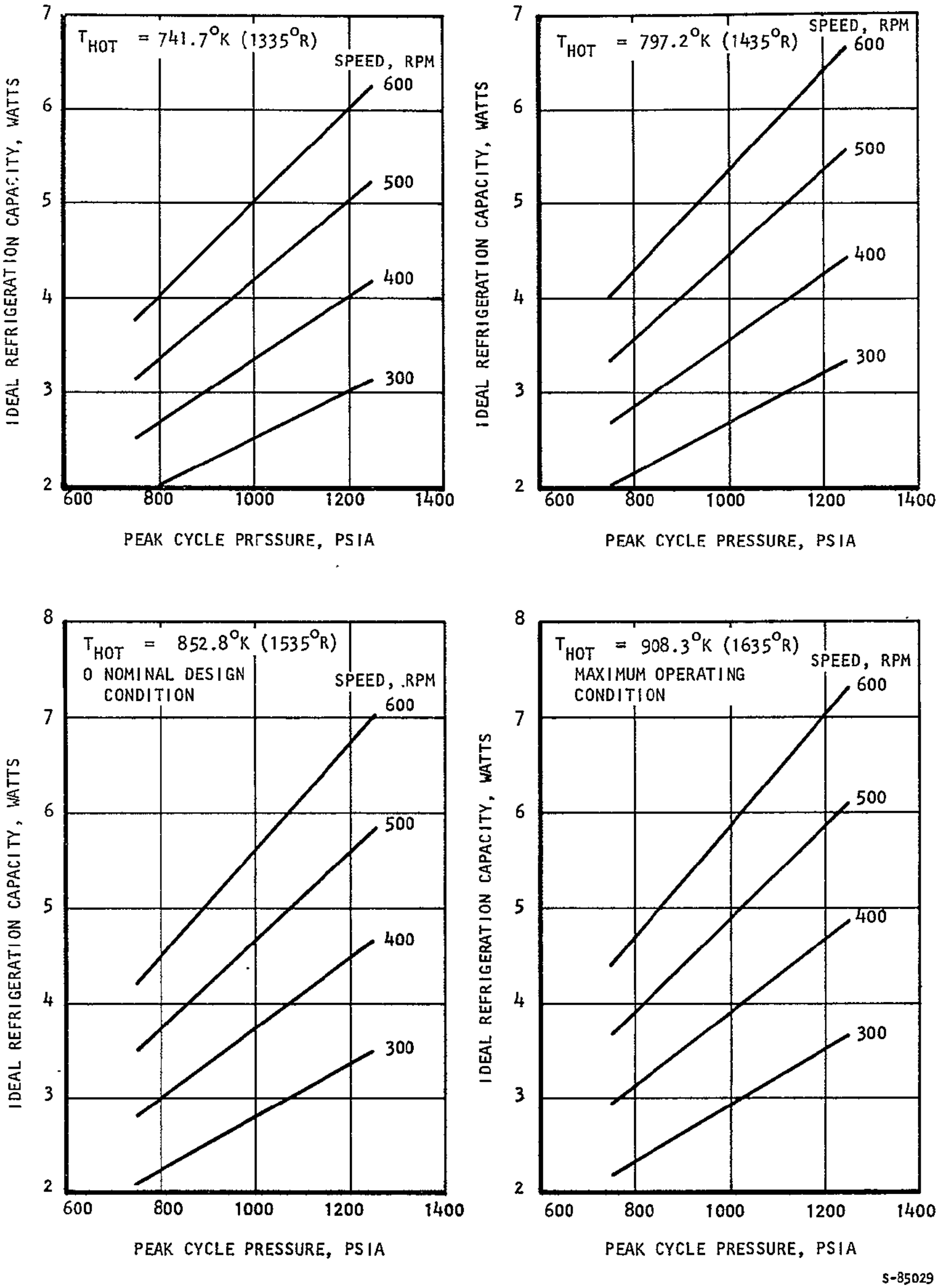

Figure 3-10. Effect of Peak Cycle Pressure on Ideal Performance of Fractional Watt VM

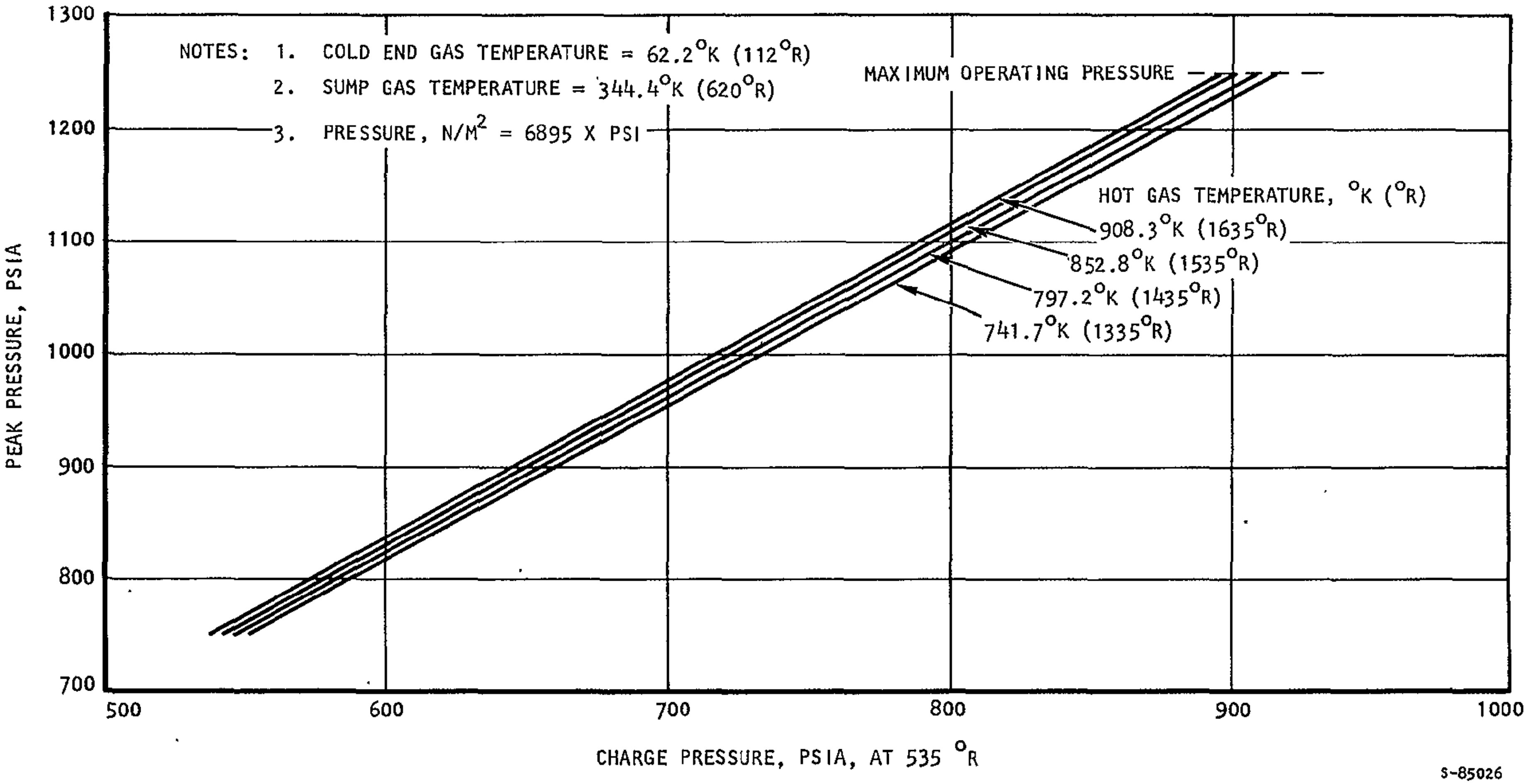

Figure 3-11. Peak Cycle Pressure as a Function of Charge Pressure

FRACTIONAL WATT VM AT MAXIMUM RANGE OF OPERATING CONDITIONS

OPERATING PARAMETERS

```
COLD VOLUME TEMP.      =   112.00 R
SUMP VOLUME TEMP.      =   620.00 R
HOT VOLUME TEMP.       =  1635.00 R
COLD REGEN. TEMP.      =   366.00 R
HOT REGEN. TEMP.       =  1077.50 R
COLD DISPLACED VOL.    =   .05530 CU-IN
HOT DISPLACED VOL.     =  1.88500 CU-IN
COLD DEAD VOL.         =   .03500 CU-IN
SUMP DEAD VOL.         =  2.96320 CU-IN
HOT DEAD VOL.          =   .34600 CU-IN
COLD REGEN. VOL.       =  1.33060 CU-IN
HOT REGEN. VOL.        =  3.64760 CU-IN
GAS CONSTANT           =  4634.40 IN-LB/LBM-R
SPEED                  =   600.00 RPM

CHARGE PRESSURE        =   897.62 PSIA
CHARGE TEMPERATURE     =   535.00 R
MASS OF FLUID          =    .0036 LBM
TOTAL VOLUME           = 10.26270 CU-IN
```

PRESSURE - MASS - FLOW PROFILE

ANGLE DEG	PRESS PSIA	VC CU-IN	VA CU-IN	VH CU-IN	MDOTC LB/SEC	MDOTA. LB/SEC	MDOTH. LB/SEC	MDOTRCA LB/SEC	MDOTRHA LB/SEC
20.	1209.31	.0367	3.6370	1.6108	.00147	-.01762	.00973	.00473	.01289
40.	1230.95	.0415	3.3488	1.8942	.00255	-.01564	.00817	.00505	.01059
60.	1245.27	.0488	3.1310	2.1046	.00332	-.01136	.00539	.00467	.00669
80.	1249.99	.0578	3.0100	2.2166	.00363	-.00532	.00175	.00360	.00172
100.	1244.31	.0674	3.0003	2.2167	.00342	.00147	-.00213	.00201	-.00349
120.	1229.17	.0765	3.1031	2.1049	.00272	.00790	-.00562	.00018	-.00808
140.	1207.01	.0838	3.3060	1.8947	.00169	.01297	-.00820	-.00160	-.01138
160.	1181.07	.0886	3.5846	1.6113	.00050	.01609	-.00958	-.00307	-.01303
180.	1154.83	.0903	3.9051	1.2891	-.00067	.01708	-.00971	-.00408	-.01300
200.	1131.34	.0886	4.2291	.9667	-.00170	.01608	-.00875	-.00456	-.01151
220.	1113.03	.0838	4.5175	.6832	-.00249	.01341	-.00690	-.00454	-.00888
240.	1101.56	.0765	4.7354	.4726	-.00299	.00949	-.00442	-.00405	-.00544
260.	1097.88	.0675	4.8566	.3605	-.00319	.00470	-.00155	-.00317	-.00153
280.	1102.29	.0579	4.8665	.3602	-.00307	-.00056	.00147	-.00197	.00253
300.	1114.43	.0488	4.7638	.4718	-.00264	-.00587	.00441	-.00055	.00642
320.	1133.29	.0415	4.5611	.6819	-.00190	-.01078	.00700	.00099	.00979
340.	1157.14	.0367	4.2827	.9652	-.00091	-.01476	.00895	.00251	.01225
360.	1183.49	.0350	3.9621	1.2874	.00025	-.01722	.00995	.00382	.01340

IDEAL REFRIGERATION AND HEAT INPUT

```
REFRIGERATION =       7.3140 WATTS
THERMAL HEAT  =      48.0927 WATTS
MAX. PRESSURE =    1250.0000 PSIA AT ANGLE =   79.04 DEGREES
```

Figure 3-12. Ideal VM Cycle Computer Program Output for Maximum Performance

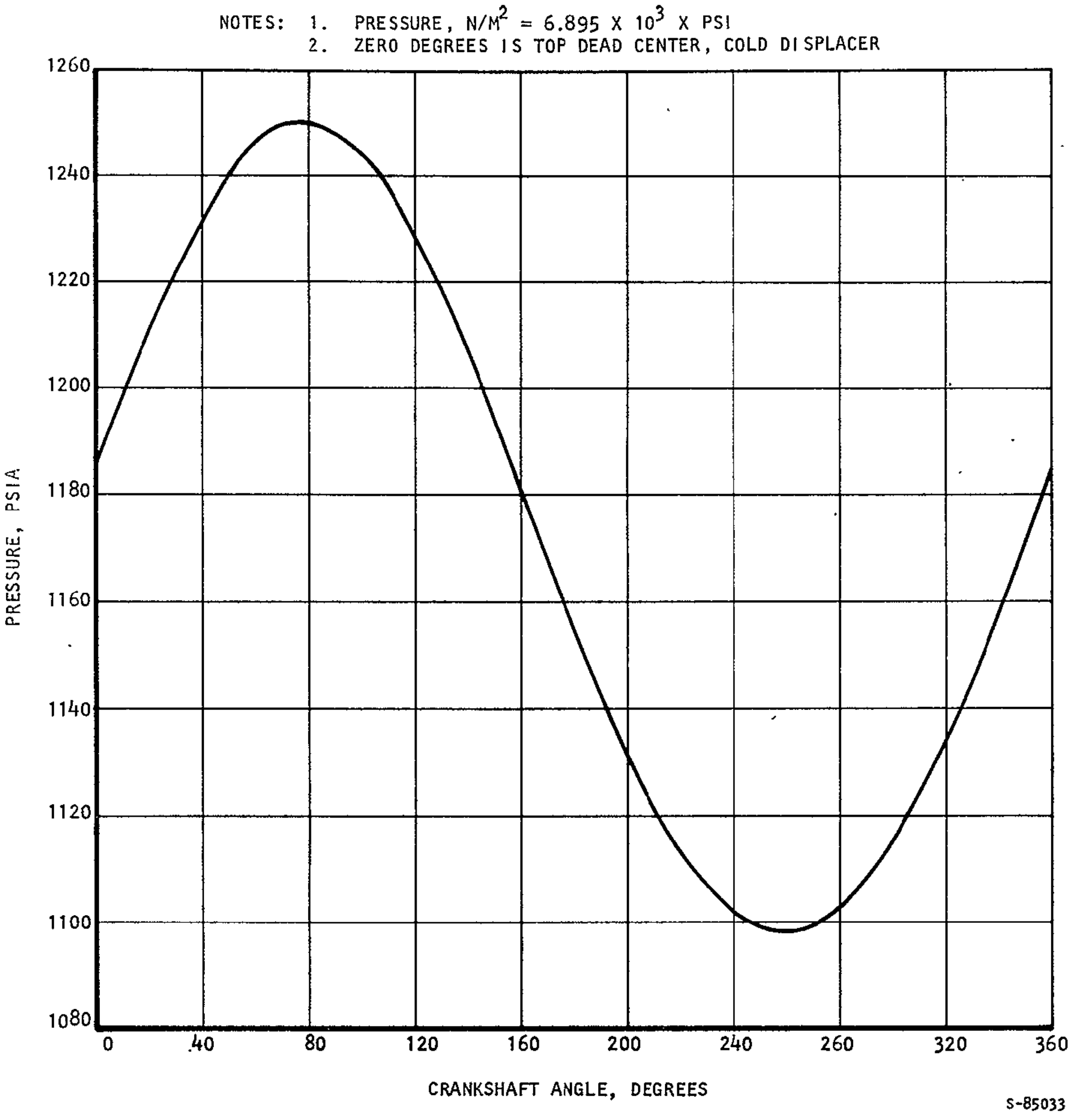

Figure 3-13. Fractional Watt Refrigerator Pressure Characteristic at Maximum Performance Conditions

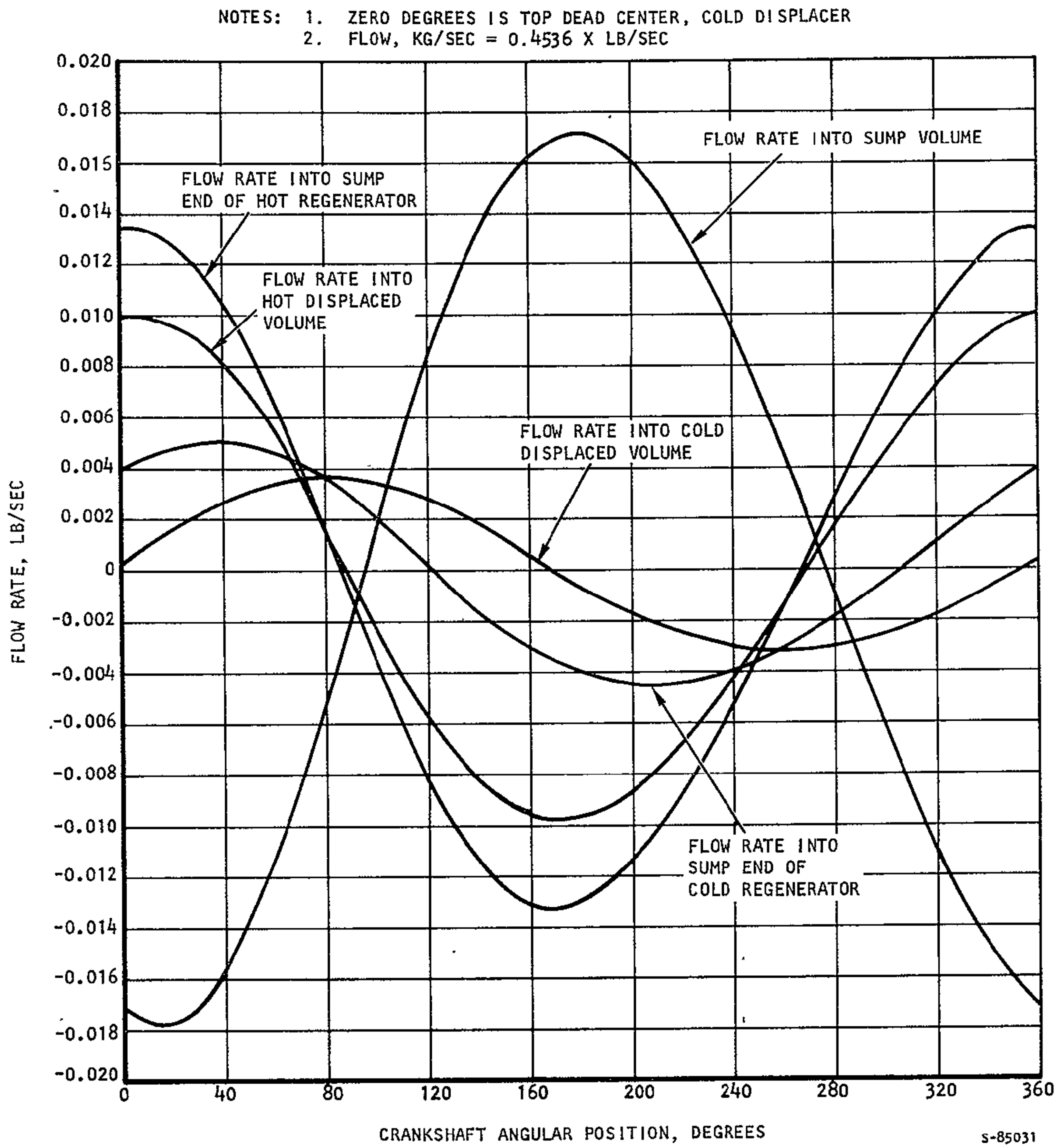

Figure 3-14. Fractional Watt Refrigerator Internal Flow Rates at Maximum Performance Conditions

TABLE 3-4

SUMMARY OF VM GROWTH POTENTIAL

Operating Parameter	Available Range	Comments
Hot End Temperature	Nominal operating 1100°F, maximum value 1200°F, set by material limitations	Most desirable method of obtaining increased performance since thermal efficiency also increases.
Peak Cycle Pressure	Nominal design 1000 psia, maximum structural limit 1250 psia	Good second choice for obtaining increased performance. Thermal efficiency only slightly decreased.
Machine Rotational Speed	Nominal design 400 rpm may be increased to 600 rpm	Least desirable method of obtaining increased refrigeration, since life is decreased.

SECTION 4

COLD REGENERATOR DESIGN

SECTION 4

COLD REGENERATOR DESIGN

INTRODUCTION

The cold regenerator is one of the most important components of the VM refrigerator, if not the most important. The regenerator cools gas as it flows from the sump region of the machine to the cold expansion volume. The gas expansion process further reduces the temperature, providing cooling at the cold temperature. After heat absorption from the refrigeration load, the gas is returned to the sump through the regenerator. This reverse flow process heats the gas by removal of energy stored in the regenerator matrix, and reestablishes the matrix temperature profile for cooling the gas during the next cycle. The periodic storage and removal of energy from the matrix allows the working fluid to pass from one essentially constant temperature portion of the machine to another. An ideal regenerator may thus be viewed as a temperature isolator.

The design requirements for an efficient regenerator demand that it must (1) absorb heat from the gas stream while at nearly the same temperature as the gas, (2) store this energy without significant temperature change in a given matrix locality, and (3) resupply the energy to the gas stream when the flow reverses direction, again while at a temperature very closely approaching that of the gas. The design requirements dictate that the regenerator packing (1) must have a very large heat capacity relative to that of the gas, (2) must have large values of heat transfer coefficient, heat transfer area, and thermal diffusivity, and (3) must be configured to limit axial conduction of heat from one end to the other. In addition, for the VM application, the void volume in the regenerator must be minimized. Obtaining these desirable regenerator characteristics from a specific packing generally leads to increased pressure drop as the thermal characteristics improve. Since pressure drop as well as void volume degrades the overall refrigerator performance, the basic tradeoff in the design of a regenerator is between heat transfer potential and the detrimental factors accompanying this potential.

METHOD OF ANALYSIS

The analysis of the regenerators for a VM refrigerator cannot be based on the classical effectiveness parameters that make use of end point temperatures. The system pressure fluctuates and a considerable amount of gas is stored in the regenerator void volume during various parts of the flow cycle. This periodic mass storage characteristic, coupled with the basic transient nature of regenerators, dictates that a finite difference technique be used in the analysis.

AiResearch has developed a computer program which utilizes finite difference techniques to analyze regenerators. The required program inputs are the regenerator physical characteristics, the matrix heat transfer and pressure drop characteristics, and the initial and boundary conditions. The computer program and the general characteristics of both spherical shot and screen regenerator packing materials are discussed in detail in Appendix A of this report.

The required boundary conditions are the time dependent pressures and flow rate profiles and the gas temperature at the sump end of the regenerator. The cold end gas temperature under reverse flow conditions is also required. These parameters are obtained from the design requirements of the VM, and the idealized cycle analysis computer program.

DESIGN CONFIGURATION AND PERFORMANCE

The regenerator design concept studies performed on the GSFC 5 watt VM program (Ref. 3) clearly indicated the advantages of the annular regenerator, as compared to those located inside the displacers or those located in parallel with the cold displacer. Therefore, the annular regenerator was the only type considered for the present machine.

The cold regenerator design was evolved along with the rest of the VM through numerous design iterations. However, certain initial ground rules were utilized which simplified the process. The cold displacer diameter was fixed early in the program, thus defining the inner diameter of the annulus. It was decided that, if at all possible, the packing would be composed entirely of spherical shot, in order to provide the maximum matrix heat capacity. The ground rule was also established that the annular height would be 30 shot diameters. This is necessary in order to prevent channeling of the gas along the regenerator wall.

The cold regenerator is identical to that presented in the fractional watt VM task I report (Ref. 1). Some modifications to the refrigerator have resulted in slight changes in flow rate and thus performance.

The cold regenerator final design selected for the fractional watt VM is of an annular configuration, packed with two sizes of spherical monel shot. Monel was chosen as the material because of its superior heat capacity in the temperature range of interest (see Appendix A). The total length is 0.127m (5 in.), and the annular height is 0.00762m (0.3 in.), 30 diameters of the largest shot, in order to prevent gas channeling along the walls. The first 0.0762m (3 in.) from the sump end are packed with 0.000354m (0.010 in.) diameter shot; the remaining .0508m (2 in.) are packed with 0.000178m (0.007 in.) diameter shot. The frontal area is $0.00044 m^2$ ($0.682 in.^2$).

The first section of the regenerator, 0.000254m (0.010 in.) dia shot, has a porosity of 39 percent, an area to volume ratio of $14,400 m^2/m^3$ ($366 in.^2/in.^3$), and a hydraulic diameter of 0.0001086 m (0.004274 in.). For the second section, 0.000178m (0.007 in.) dia shot, the porosity is 39 percent, the area to volume ratio is $20,600 m^2/m^3$ ($523 in.^2/in.^3$), and the hydraulic diameter is 0.0000759m (0.002988 in.).

The larger diameter shot is used in the warm end of the cold regenerator to reduce the pressure drop, but still maintain a large thermal capacity. The smaller diameter shot at the cold end yields the required high heat transfer coefficient.

Table 4-1 presents the detailed output from the regenerator analysis computer program for this preliminary design cold regenerator. The nodewise pressures and temperatures of the gas are listed as a function of the angular position of the crankshaft, or time. Angular displacement is referenced to the top dead center position of the cold displacer. The output parameters are matrix temperature, gas temperature, gas density, gas pressure, and gas flow rate. Positive gas flow rate denotes mass flow from the sump end of the regenerator toward the cold end. Node 0 represents the sump end of the regenerator, and Node 13 the outlet face at the cold end. Nodes 1 through 7 are the portion of the core packed with 0.000254m (0.010 in.) dia shot, and Nodes 8 through 12 represent the 0.000178m (0.007 in.) dia shot.

Figures 4-1 through 4-3 present plots of key parameters from the computer program printout. The matrix and gas temperatures at the cold end of the regenerator are plotted in Figure 4-1. The small difference between the two temperatures is indicative of excellent heat transfer, and the 0.833°K (1.5°R) temperature swing of the gas is indicative of adequate heat capacity.

Figure 4-2 indicates a maximum regenerator pressure drop of $1.24 \times 10^4 N/M^2$ (1.8 psi). This pressure drop, coupled with the low ΔP of the remainder of the components in the cold gas flow path, will limit leakage rates past the displacer to very low values. The regenerator represents the major portion of the cold gas pressure drop. However, attempts to decrease pressure drops would lead to losses in thermal performance which would impose a greater overall penalty on the VM. Figure 4-3 presents the gas flow rate at the cold end of the regenerator. This plot, along with the gas temperature, is used to determine the regenerator losses.

An ideal regenerator will always supply gas to the cold end of the VM at a constant temperature which is equal to that of the cold expansion volume temperature. In the actual case, this temperature varies as a result of the finite matrix heat capacity and heat transfer coefficient between the gas and matrix. This is evidenced by the temperature plots of Figure 4-1. The losses associated with this departure from ideal conditions are estimated by integrating the excess fluid energy supplied to the cold end over a complete cycle, or crankshaft revolution. The loss per cycle is expressed as:

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TABLE 4-1

PRELIMINARY DESIGN
COLD REGENERATOR
PERFORMANCE CHARACTERISTICS

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

$\theta = 0^{\circ}$

REGULAR PRINTOUTS DATE = 26 OCT 73 TIME = 18:37:14

TIME(SEC.) = 4.49983-01 4659 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19678+02	6.20000+02	5.52899-01	9.49540+02	1.94870-03
1	5.96877+02	5.97184+02	5.72920-01	9.49432+02	1.90223-03
2	5.56514+02	5.57098+02	6.13528-01	9.49241+02	1.80293-03
3	5.14273+02	5.14886+02	6.61621-01	9.49082+02	1.69610-03
4	4.71951+02	4.72578+02	7.19177-01	9.48952+02	1.58033-03
5	4.29636+02	4.30277+02	7.87578-01	9.48848+02	1.45400-03
6	3.87315+02	3.87971+02	8.67880-01	9.48767+02	1.31534-03
7	3.45002+02	3.45675+02	9.68598-01	9.48708+02	1.16140-03
8	3.02802+02	3.03200+02	1.09733+00	9.48662+02	9.88111-04
9	2.60400+02	2.60795+02	1.26675+00	9.48616+02	8.02971-04
10	2.18070+02	2.18487+02	1.49865+00	9.48589+02	5.86397-04
11	1.75895+02	1.76357+02	1.83554+00	9.48577+02	3.25353-04
12	1.35665+02	1.35893+02	2.34655+00	9.48574+02	-3.74597-07
13	1.12000+02	1.12000+02	2.81023+00	9.48575+02	-1.91324-04

$\theta = 30^{\circ}$

REGULAR PRINTOUTS DATE = 26 OCT 73 TIME = 18:37:31

TIME(SEC.) = 4.62468-01 4802 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19832+02	6.20000+02	5.68560-01	9.77374+02	2.55429-03
1	5.97080+02	5.97391+02	5.88903-01	9.77222+02	2.51987-03
2	5.56888+02	5.57471+02	6.30358-01	9.76947+02	2.44635-03
3	5.14657+02	5.15278+02	6.79631-01	9.76709+02	2.36727-03
4	4.72334+02	4.72972+02	7.38556-01	9.76506+02	2.28160-03
5	4.30015+02	4.30671+02	8.08634-01	9.76336+02	2.18814-03
6	3.87691+02	3.88366+02	8.90823-01	9.76196+02	2.08561-03
7	3.45377+02	3.46071+02	9.93888-01	9.76083+02	1.97181-03
8	3.03202+02	3.03615+02	1.12556+00	9.75985+02	1.84378-03
9	2.60792+02	2.61207+02	1.29870+00	9.75874+02	1.70707-03
10	2.18480+02	2.18913+02	1.53537+00	9.75794+02	1.54730-03
11	1.76337+02	1.76789+02	1.87872+00	9.75741+02	1.35497-03
12	1.36159+02	1.36612+02	2.39316+00	9.75710+02	1.11597-03
13	1.12248+02	1.12487+02	2.86883+00	9.75701+02	9.75860-04

NOTES:
1. TEMP, $^{\circ}K = {^{\circ}R}/1.8$
2. DENSITY, $kg/m^3 = 16.02 \times LB/FT^3$
3. PRESSURE, $N/m^2 = 6.895 \times 10^3 \times PSI$
4. FLOW, kg/sec = 0.4536 X LB/SEC

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TABLE 4-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

$\theta = 60^{o}$

REGULAR PRINTOUTS

DATE = 26 OCT 73 TIME = 18:37:48

TIME(SEC.) = 4.74999-01 4946 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19913+02	6.20000+02	5.78584-01	9.95219+02	2.44002-03
1	5.97274+02	5.97534+02	5.99140-01	9.95078+02	2.42339-03
2	5.57242+02	5.57724+02	6.41171-01	9.94820+02	2.38787-03
3	5.15028+02	5.15548+02	6.91232-01	9.94591+02	2.34966-03
4	4.72708+02	4.73246+02	7.51063-01	9.94391+02	2.30828-03
5	4.30391+02	4.30948+02	8.22247-01	9.94219+02	2.26314-03
6	3.88069+02	3.88647+02	9.05663-01	9.94073+02	2.21363-03
7	3.45760+02	3.46357+02	1.01025+00	9.93950+02	2.15870-03
8	3.03620+02	3.03979+02	1.14359+00	9.93839+02	2.09693-03
9	2.61216+02	2.61587+02	1.31897+00	9.93705+02	2.03103-03
10	2.18937+02	2.19330+02	1.55833+00	9.93603+02	1.95404-03
11	1.76852+02	1.77267+02	1.90475+00	9.93528+02	1.86150-03
12	1.36777+02	1.37195+02	2.42123+00	9.93476+02	1.74675-03
13	1.12679+02	1.12921+02	2.90413+00	9.93458+02	1.67949-03

$\theta = 90^{o}$

DATE = 26 OCT 73 TIME = 18:38:04

TIME(SEC.) = 4.87499-01 5080 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19954+02	6.20000+02	5.79939-01	9.97633+02	1.51513-03
1	5.97400+02	5.97540+02	6.00575-01	9.97559+02	1.52005-03
2	5.57468+02	5.57721+02	6.42787-01	9.97420+02	1.53057-03
3	5.15271+02	5.15552+02	6.93029-01	9.97294+02	1.54188-03
4	4.72958+02	4.73254+02	7.53065-01	9.97180+02	1.55413-03
5	4.30646+02	4.30960+02	8.24479-01	9.97078+02	1.56750-03
6	3.88331+02	3.88663+02	9.08144-01	9.96987+02	1.58216-03
7	3.46033+02	3.46386+02	1.01300+00	9.96909+02	1.59842-03
8	3.03924+02	3.04139+02	1.14626+00	9.96833+02	1.61670-03
9	2.61541+02	2.61775+02	1.32183+00	9.96737+02	1.63620-03
10	2.19301+02	2.19556+02	1.56122+00	9.96658+02	1.65896-03
11	1.77280+02	1.77560+02	1.90707+00	9.96596+02	1.68630-03
12	1.37315+02	1.37608+02	2.42057+00	9.96549+02	1.72016-03
13	1.13108+02	1.13297+02	2.90349+00	9.96531+02	1.74002-03

NOTES:
1. TEMP, $^{o}K = {}^{o}R/1.8$
2. DENSITY, $kg/m^3 = 16.02 \times LB/FT^3$
3. PRESSURE, $N/m^2 = 6.895 \times 10^3 \times PSI$
4. FLOW, kg/sec = 0.4536 X LB/SEC

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TABLE 4-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

θ = 120°

REGULAR PRINTOUTS

DATE = 26 OCT 73 TIME = 18:38:17

TIME(SEC.) = 4.99877-01 5186 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19971+02	6.20000+02	5.72234-01	9.83912+02	1.64234-04
1	5.97418+02	5.97339+02	5.92843-01	9.83909+02	1.89640-04
2	5.57497+02	5.57343+02	6.34820-01	9.83899+02	2.43915-04
3	5.15311+02	5.15179+02	6.84550-01	9.83887+02	3.02281-04
4	4.73006+02	4.72893+02	7.44012-01	9.83873+02	3.65508-04
5	4.30702+02	4.30610+02	8.14683-01	9.83858+02	4.34488-04
6	3.88397+02	3.88327+02	8.97533-01	9.83840+02	5.10157-04
7	3.46111+02	3.46065+02	1.00135+00	9.83822+02	5.94121-04
8	3.04021+02	3.04007+02	1.13266+00	9.83800+02	6.88471-04
9	2.61665+02	2.61680+02	1.30628+00	9.83767+02	7.89139-04
10	2.19457+02	2.19494+02	1.54303+00	9.83735+02	9.06681-04
11	1.77488+02	1.77554+02	1.88484+00	9.83706+02	1.04792-03
12	1.37608+02	1.37700+02	2.39140+00	9.83679+02	1.22281-03
13	1.13398+02	1.13493+02	2.86686+00	9.83667+02	1.32533-03

θ = 150°

DATE = 26 OCT 73 TIME = 18:38:28

TIME(SEC.) = 5.12465-01 5283 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19887+02	6.19631+02	5.57351-01	9.56885+02	-1.16677-03
1	5.97255+02	5.96773+02	5.77691-01	9.56938+02	-1.12013-03
2	5.57331+02	5.56832+02	6.18667-01	9.57025+02	-1.02048-03
3	5.15154+02	5.14647+02	6.67284-01	9.57100+02	-9.13283-04
4	4.72857+02	4.72342+02	7.25491-01	9.57154+02	-7.97113-04
5	4.30565+02	4.30041+02	7.94600-01	9.57193+02	-6.70354-04
6	3.88272+02	3.87736+02	8.75746-01	9.57219+02	-5.31244-04
7	3.45999+02	3.45442+02	9.77469-01	9.57234+02	-3.76793-04
8	3.03932+02	3.03615+02	1.10526+00	9.57242+02	-2.03289-04
9	2.61582+02	2.61507+02	1.27426+00	9.57246+02	-1.81477-05
10	2.19371+02	2.19072+02	1.50747+00	9.57244+02	1.98391-04
11	1.77430+02	1.77213+02	1.84224+00	9.57237+02	4.58811-04
12	1.37589+02	1.37444+02	2.33883+00	9.57226+02	7.81740-04
13	1.13485+02	1.13470+02	2.80060+00	9.57218+02	9.70977-04

NOTES:
1. TEMP, °K = °R/1.8
2. DENSITY, kg/m^3 = 16.02 X LB/FT^3
3. PRESSURE, N/m^2 = 6.895 X 10^3 X PSI
4. FLOW, kg/sec = 0.4536 X LB/SEC

TABLE 4-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

θ = 180°

REGULAR PRINTOUTS

DATE = 26 OCT 73 TIME = 18:38:44

TIME(SEC.) = 5.24999-01 5418 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19717+02	6.19412+02	5.40711-01	9.27047+02	-2.07995-03
1	5.96944+02	5.96383+02	5.60678-01	9.27168+02	-2.03529-03
2	5.57017+02	5.56430+02	6.00603-01	9.27382+02	-1.93982-03
3	5.14845+02	5.14247+02	6.47941-01	9.27562+02	-1.83711-03
4	4.72555+02	4.71946+02	7.04669-01	9.27711+02	-1.72579-03
5	4.30271+02	4.29653+02	7.71962-01	9.27831+02	-1.60430-03
6	3.87988+02	3.87360+02	8.51037-01	9.27925+02	-1.47090-03
7	3.45724+02	3.45089+02	9.50136-01	9.27997+02	-1.32276-03
8	3.03661+02	3.03303+02	1.07456+00	9.28054+02	-1.15625-03
9	2.61339+02	2.60976+02	1.24039+00	9.28113+02	-9.78362-04
10	2.19160+02	2.18792+02	1.46693+00	9.28149+02	-7.70313-04
11	1.77239+02	1.76871+02	1.79468+00	9.28168+02	-5.19657-04
12	1.37410+02	1.37221+02	2.27948+00	9.28176+02	-2.08452-04
13	1.13360+02	1.12000+02	2.75771+00	9.28176+02	-2.42536-05

θ = 210°

DATE = 26 OCT 73 TIME = 18:39:03

TIME(SEC.) = 5.37488-01 5568 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19520+02	6.19218+02	5.26538-01	9.01685+02	-2.38000-03
1	5.96589+02	5.96040+02	5.46164-01	9.01834+02	-2.34439-03
2	5.56654+02	5.56074+02	5.85160-01	9.02102+02	-2.26828-03
3	5.14485+02	5.13892+02	6.31375-01	9.02333+02	-2.18638-03
4	4.72196+02	4.71591+02	6.86813-01	9.02529+02	-2.09758-03
5	4.29917+02	4.29299+02	7.52533-01	9.02691+02	-2.00065-03
6	3.87639+02	3.87007+02	8.29830-01	9.02824+02	-1.89420-03
7	3.45379+02	3.44737+02	9.26715-01	9.02929+02	-1.77592-03
8	3.03305+02	3.02936+02	1.04841+00	9.03020+02	-1.64294-03
9	2.60982+02	2.60605+02	1.21076+00	9.03121+02	-1.50080-03
10	2.18794+02	2.18408+02	1.43278+00	9.03192+02	-1.33435-03
11	1.76864+02	1.76485+02	1.75440+00	9.03237+02	-1.13365-03
12	1.37179+02	1.36964+02	2.22914+00	9.03263+02	-8.84199-04
13	1.12375+02	1.12000+02	2.69312+00	9.03270+02	-7.36600-04

NOTES:
1. TEMP, °K = °R/1.8
2. DENSITY, kg/m^3 = 16.02 X LB/FT^3
3. PRESSURE, N/m^2 = 6.895 X 10^3 X PSI
4. FLOW, kg/sec = 0.4536 X LB/SEC

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TABLE 4-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

$\theta = 240^{\circ}$

REGULAR PRINTOUTS

DATE = 26 OCT 73 TIME = 18:39:21

TIME(SEC.) = 5.49999-01 5722 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19345+02	6.19090+02	5.17728-01	8.85941+02	-2.13003-03
1	5.96277+02	5.95821+02	5.37117-01	8.86072+02	-2.10850-03
2	5.56329+02	5.55844+02	5.75494-01	8.86309+02	-2.06245-03
3	5.14159+02	5.13661+02	6.20970-01	8.86517+02	-2.01291-03
4	4.71869+02	4.71358+02	6.75566-01	8.86696+02	-1.95919-03
5	4.29590+02	4.29066+02	7.40265-01	8.86848+02	-1.90053-03
6	3.87311+02	3.86772+02	8.16418-01	8.86975+02	-1.83609-03
7	3.45049+02	3.44496+02	9.11892-01	8.87080+02	-1.76447-03
8	3.02954+02	3.02631+02	1.03201+00	8.87173+02	-1.68392-03
9	2.60620+02	2.60287+02	1.19223+00	8.87281+02	-1.59776-03
10	2.18409+02	2.18063+02	1.41161+00	8.87361+02	-1.49682-03
11	1.76455+02	1.76112+02	1.72984+00	8.87418+02	-1.37496-03
12	1.36888+02	1.36681+02	2.19885+00	8.87454+02	-1.22341-03
13	1.12068+02	1.12000+02	2.65189+00	8.87466+02	-1.13382-03

$\theta = 270^{\circ}$

DATE = 26 OCT 73 TIME = 18:39:39

TIME(SEC.) = 5.62500-01 5868 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19234+02	6.19122+02	5.17205-01	8.85062+02	-1.39000-03
1	5.96085+02	5.95896+02	5.36501-01	8.85137+02	-1.40251-03
2	5.56124+02	5.55909+02	5.74781-01	8.85277+02	-1.42922-03
3	5.13949+02	5.13720+02	6.20154-01	8.85407+02	-1.45797-03
4	4.71653+02	4.71408+02	6.74638-01	8.85526+02	-1.48916-03
5	4.29369+02	4.29106+02	7.39229-01	8.85635+02	-1.52320-03
6	3.87084+02	3.86800+02	8.15268-01	8.85732+02	-1.56089-03
7	3.44813+02	3.44506+02	9.10636-01	8.85819+02	-1.60215-03
8	3.02690+02	3.02494+02	1.03105+00	8.85905+02	-1.64889-03
9	2.60337+02	2.60121+02	1.19134+00	8.86016+02	-1.69892-03
10	2.18091+02	2.17850+02	1.41105+00	8.86108+02	-1.75754-03
11	1.76100+02	1.75846+02	1.73013+00	8.86183+02	-1.82836-03
12	1.36599+02	1.36416+02	2.20046+00	8.86242+02	-1.91647-03
13	1.12010+02	1.12000+02	2.64874+00	8.86265+02	-1.96846-03

NOTES:
1. TEMP, $^{\circ}K = {^{\circ}R}/1.8$
2. DENSITY, $kg/m^3 = 16.02 \times LB/FT^3$
3. PRESSURE, $N/m^2 = 6.895 \times 10^3 \times PSI$
4. FLOW, kg/sec = 0.4536 X LB/SEC

TABLE 4-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

θ = 300°

REGULAR PRINTOUTS

DATE = 26 OCT 73 TIME = 18:39:53

TIME(SEC.) = 5.74918-01 5988 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19209+02	6.19262+02	5.23902-01	8.97090+02	-3.47365-04
1	5.96047+02	5.96160+02	5.43281-01	8.97102+02	-3.75651-04
2	5.56075+02	5.56166+02	5.81948-01	8.97127+02	-4.36110-04
3	5.13893+02	5.13966+02	6.27798-01	8.97154+02	-5.01172-04
4	4.71590+02	4.71643+02	6.82831-01	8.97184+02	-5.71722-04
5	4.29298+02	4.29329+02	7.48114-01	8.97215+02	-6.48738-04
6	3.87003+02	3.87009+02	8.24939-01	8.97247+02	-7.33331-04
7	3.44720+02	3.44697+02	9.21317-01	8.97280+02	-8.27335-04
8	3.02570+02	3.02530+02	1.04346+00	8.97317+02	-9.33152-04
9	2.60195+02	2.60132+02	1.20558+00	8.97370+02	-1.04632-03
10	2.17914+02	2.17823+02	1.42789+00	8.97419+02	-1.17895-03
11	1.75885+02	1.75769+02	1.75094+00	8.97465+02	-1.33913-03
12	1.36386+02	1.36271+02	2.22778+00	8.97505+02	-1.53848-03
13	1.12002+02	1.12000+02	2.67815+00	8.97522+02	-1.65590-03

θ = 330°

DATE = 26 OCT 73 TIME = 18:40:05

TIME(SEC.) = 5.87500-01 6082 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19417+02	6.20000+02	5.36360-01	9.20202+02	8.69995-04
1	5.96128+02	5.96359+02	5.56605-01	9.20166+02	8.29628-04
2	5.56205+02	5.56649+02	5.95824-01	9.20104+02	7.43381-04
3	5.14018+02	5.14483+02	6.42618-01	9.20056+02	6.50604-04
4	4.71706+02	4.72185+02	6.98714-01	9.20021+02	5.50019-04
5	4.29403+02	4.29900+02	7.65314-01	9.19998+02	4.40252-04
6	3.87095+02	3.87620+02	8.43547-01	9.19983+02	3.19736-04
7	3.44797+02	3.45391+02	9.41552-01	9.19975+02	1.85887-04
8	3.02625+02	3.03115+02	1.06636+00	9.19973+02	3.52489-05
9	2.60231+02	2.60461+02	1.23249+00	9.19974+02	-1.25904-04
10	2.17923+02	2.18078+02	1.45945+00	9.19979+02	-3.14651-04
11	1.75861+02	1.75957+02	1.78908+00	9.19989+02	-5.42442-04
12	1.36297+02	1.36290+02	2.27711+00	9.20003+02	-8.25820-04
13	1.12000+02	1.12000+02	2.73659+00	9.20011+02	-9.92599-04

NOTES:
1. TEMP, °K = °R/1.8
2. DENSITY, kg/m^3 = 16.02 X LB/FT^3
3. PRESSURE, N/m^2 = 6.895 X 10^3 X PSI
4. FLOW, kg/sec = 0.4536 X LB/SEC

TABLE 4-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

θ = 360°

REGULAR PRINTOUTS

DATE = 26 OCT 73 TIME = 18:40:20

TIME(SEC.) = 5.99983-01 6212 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.19672+02	6.20000+02	5.52899-01	9.49539+02	1.94868-03
1	5.96285+02	5.96597+02	5.73481-01	9.49431+02	1.90217-03
2	5.56492+02	5.57071+02	6.13557-01	9.49241+02	1.80285-03
3	5.14308+02	5.14920+02	6.61580-01	9.49082+02	1.69603-03
4	4.71992+02	4.72618+02	7.19116-01	9.48951+02	1.58027-03
5	4.29682+02	4.30323+02	7.87497-01	9.48848+02	1.45395-03
6	3.87367+02	3.88023+02	8.67768-01	9.48767+02	1.31532-03
7	3.45062+02	3.45734+02	9.68435-01	9.48708+02	1.16139-03
8	3.02901+02	3.03299+02	1.09700+00	9.48661+02	9.88155-04
9	2.60481+02	2.60876+02	1.26638+00	9.48616+02	8.03056-04
10	2.18156+02	2.18573+02	1.49810+00	9.48589+02	5.86544-04
11	1.76068+02	1.76530+02	1.83384+00	9.48577+02	3.25694-04
12	1.36358+02	1.36588+02	2.33464+00	9.48574+02	1.39544-06
13	1.12000+02	1.12000+02	2.81023+00	9.48574+02	-1.89553-04

NOTES:
1. TEMP, °K = °R/1.8
2. DENSITY, kg/m³ = 16.02 X LB/FT³
3. PRESSURE, N/m² = 6.895 X 10³ X PSI
4. FLOW, kg/sec = 0.4536 X LB/SEC

NOTES:
1. FRONTAL AREA = 0.00044 m^2 (0.68235 $IN.^2$)
2. L_1 = 0.0762 m (3 IN.) OF 0.000254 m (0.010 IN.) DIA MONEL SHOT
3. L_2 = 0.0508 m (2 IN.) OF 0.000178 m (0.007 IN.) DIA MONEL SHOT
4. SPEED = 400 RPM
5. MAXIMUM PRESSURE = $6.895 \times 10^6 N/m^2$ (1000 PSIA)
6. 0^o = TOP DEAD CENTER, COLD DISPLACER
7. TEMPERATURE, ^oK = $^oR/1.8$

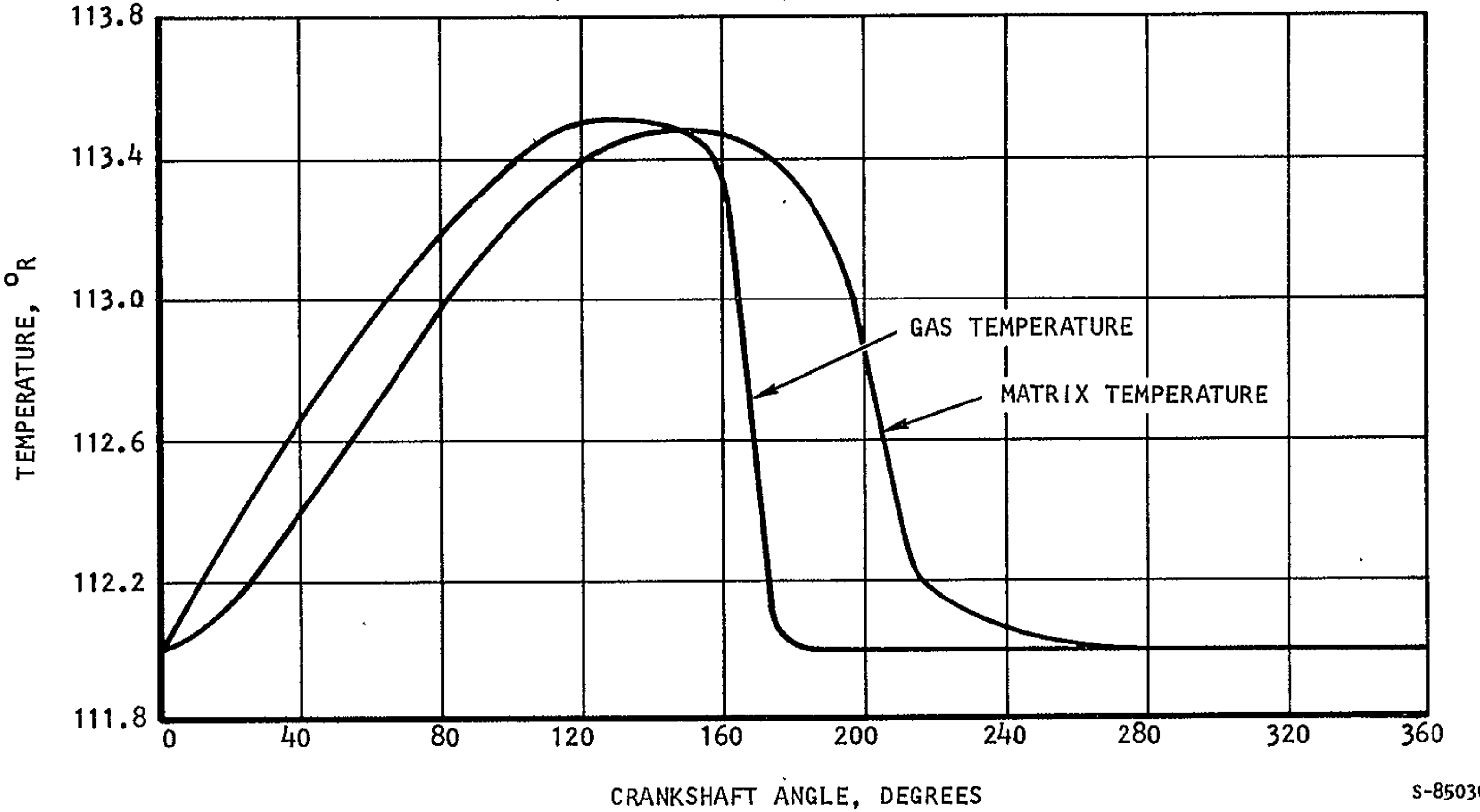

Figure 4-1. Temperature Variation at Cold End of Cold Regenerator

NOTES:
1. FRONTAL AREA = 0.00044 m^2 (0.68235 $IN.^2$)
2. L_1 = 0.0762 m (3 IN.) OF 0.000254 m (0.010 IN.) DIA MONEL SHOT
3. L_2 = 0.0508 m (2 IN.) OF 0.000178 m (0.007 IN.) DIA MONEL SHOT
4. SPEED = 400 RPM
5. MAXIMUM PRESSURE = 6.895 X 10^6 N/m^2 (1000 PSIA)
6. 0^o = TOP DEAD CENTER, COLD DISPLACER
7. PRESSURE DROP, N/m^2 = 6.895 X 10^3 X PSI

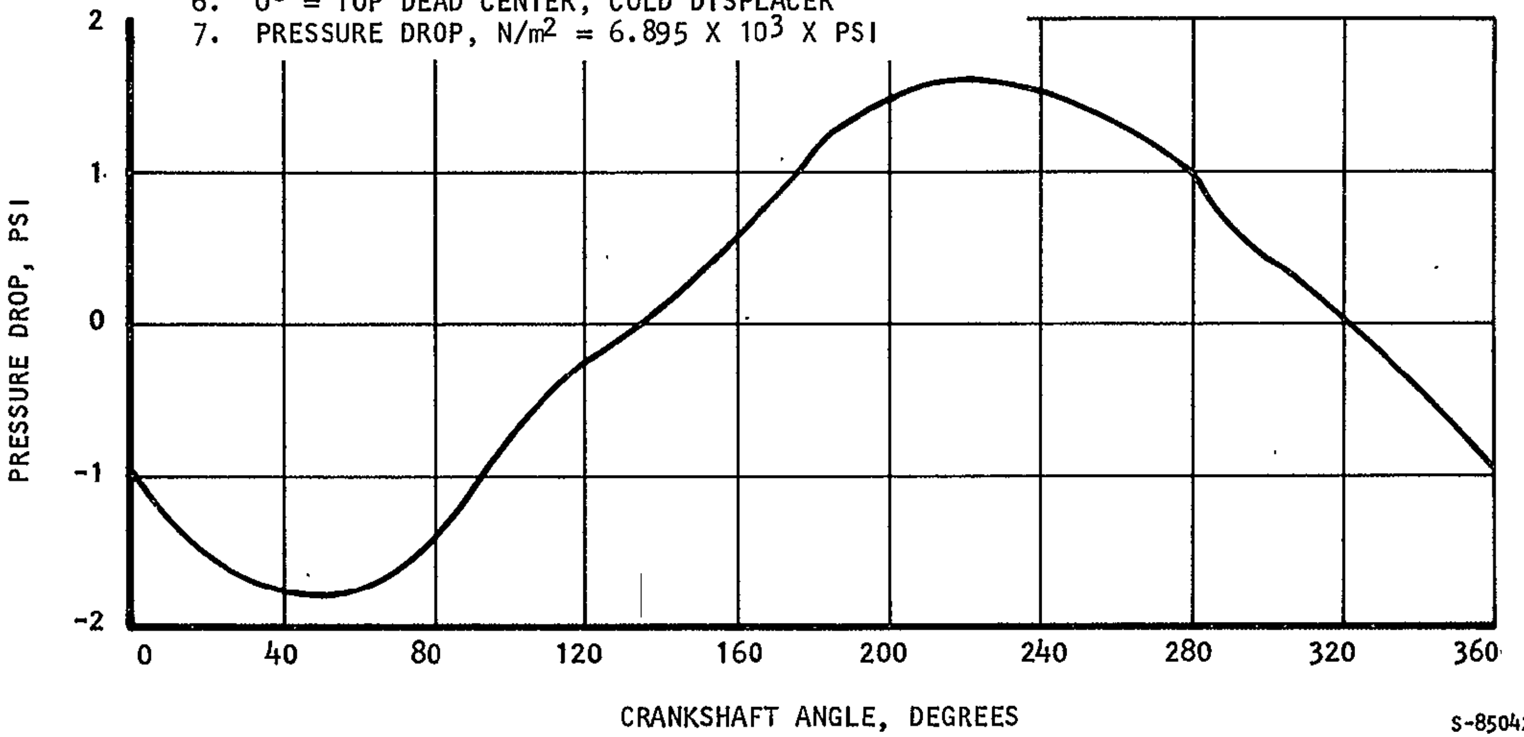

Figure 4-2. Pressure Drop of Cold Regenerator

NOTES:
1. FRONTAL AREA = 0.00044 m^2 (0.68235 $IN.^2$)
2. L_1 = 0.0762 m (3 IN.) OF 0.000254 m (0.010 IN.) DIA MONEL SHOT
3. L_2 = 0.0508 m (2 IN.) OF 0.000178 m (0.007 IN.) DIA MONEL SHOT
4. SPEED = 400 RPM
5. MAXIMUM PRESSURE = 6.895 X 10^6 N/m^2 (1000 PSIA)
6. 0^o = TOP DEAD CENTER, COLD DISPLACER
7. FLOWRATE, kg/sec = 0.4536 X LB/SEC

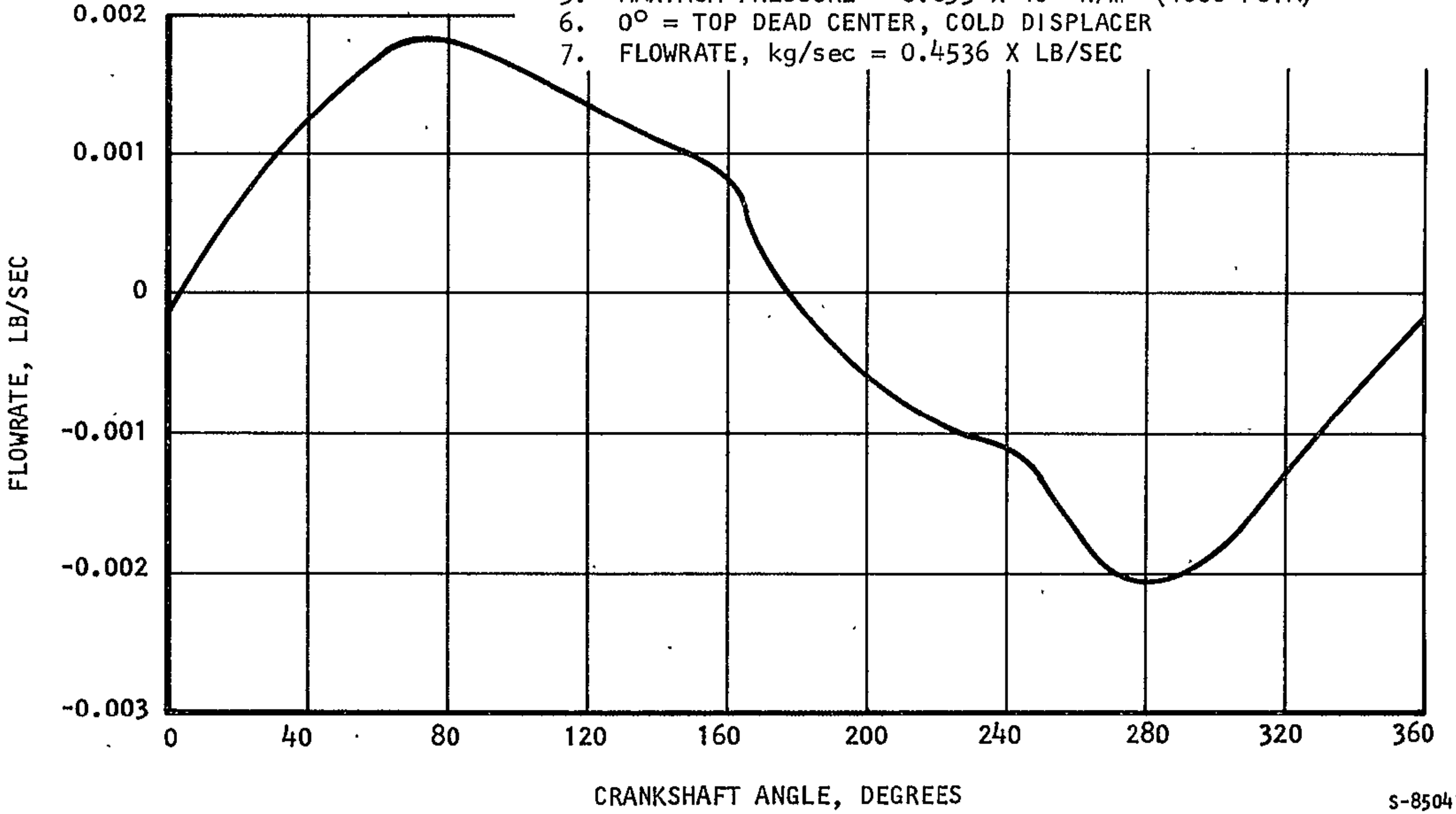

Figure 4-3. Cold End Flow Rate of Cold Regenerator

$$\dot{Q}_{loss} = \int_{0}^{2\pi} \dot{\omega}\, cp\, (T - T_{ref})\, d\theta \qquad (4\text{-}1)$$

Where

$\dot{Q}_{loss}$ = regenerator loss per revolution

$\dot{\omega}$ = fluid flow rate

cp = specific heat (function of temperature and pressure)

T = fluid temperature

T_{ref} = cold expansion volume temperature

Figures 4-1 and 4-3, along with the specific heat characteristics of helium, allow a stepwise evaluation of Equation 4-1. The integrated loss as a function of crank angle is presented in Figure 4-4. The total loss per cycle is 0.1309J (0.0001241 Btu). At a rotational speed of 400 rpm, this loss translates to a total of 0.872 watts.

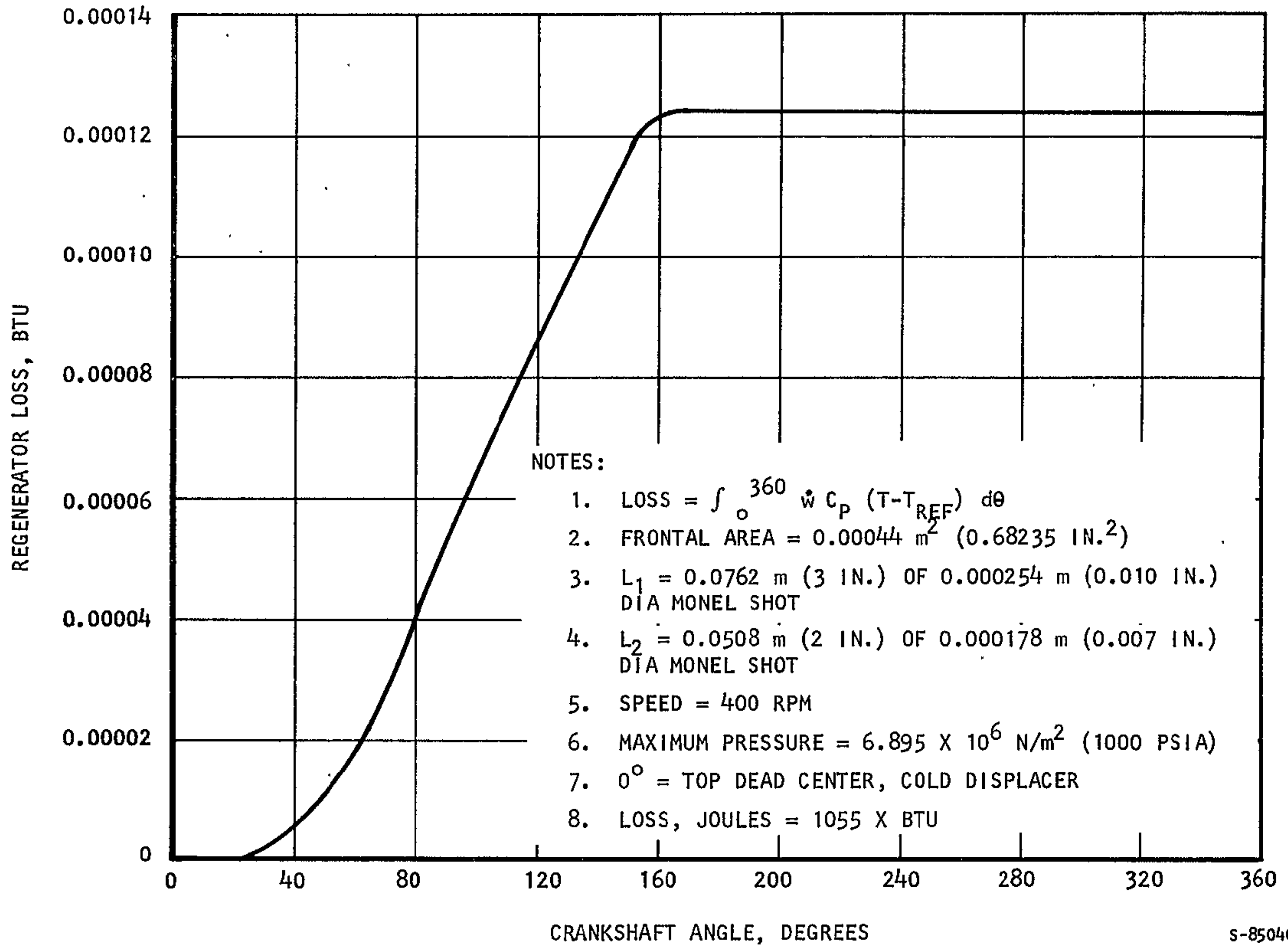

Figure 4-4. Preliminary Design Cold Regenerator Thermal Loss per Cycle

SECTION 5

HOT REGENERATOR DESIGN

SECTION 5

HOT REGENERATOR DESIGN

INTRODUCTION

The hot regenerator serves as a thermal isolator between the sump and the high temperature portions of the VM refrigerator. As the gas flows from the sump towards the hot end, it is heated to a temperature approaching the maximum in the cycle. At the time of flow reversal, an expansion process begins which allows heat to be added to the gas at essentially constant temperature. The expanding gas is cooled as it flows through the regenerator, thus storing energy in the regenerator matrix and reestablishing the temperature profile for the next half. Desirable design features are similar to those of the cold regenerator, with major emphasis placed on different items.

The hot displacer is considerably larger than the cold, and thus the pressure differential across the displacer strongly influences motor drive power. Thus regenerator pressure loss becomes an item of prime importance. Thermal efficiency assumes a less important role in the hot regenerator than in the cold. The thermal losses at the hot end are compensated for by additional heat input on a one to one basis, while at the cold end the thermal power input to offset the losses is magnified by the coefficient of performance of the refrigerator. Void volume at the higher temperature does not affect performance nearly as strongly as at colder temperatures, since the mass of gas stored in the high temperature region is smaller, due to the high temperature.

METHOD OF ANALYSIS

The hot regenerator is analyzed in the same manner as the cold. A description of the analytical methods used in the computer program, along with the physical characteristics of candidate packings, is presented in Appendix A of this report.

DESIGN CONFIGURATION AND PERFORMANCE

The hot regenerator is one of the components of the VM which has been changed from the configuration presented in Task I report (Reference 1). Very soon after initiation of detail design, it became apparent that the hot end power requirement of the preliminary design machine was too high. In an effort to reduce the power input, the maximum operating pressure was reduced from 10.34×10^6 to 8.619×10^6 N/M^2 (1500 to 1250 psia), as discussed earlier. This resulted in thinner pressure vessel walls and reduced conduction losses. However, it was decided that a further reduction in hot end input power was desirable, and the hot regenerator and displacer were thus lengthened. The regenerator length was increased from 0.1016 to 0.1112 in. (4 to 4.375 in.).

The hot regenerator is annular in shape with an annulus height of 0.00381m (0.150 in.). The total length is 0.1112m (4.375 in.), which is packed entirely with 100 mesh stainless steel screen. The screen has a porosity of 72.5 percent, an area to volume ratio of 10,080m^2/m^3 (257 $in.^2/in.^3$) and a hydraulic diameter of 0.000288m (0.01132 in.). The frontal area of the regenerator is 0.000741m^2 (1.15 $in.^2$).

The screen packing was selected instead of sperical shot, or a combination of the two, in order to minimize pressure drop. The high operating temperature of the hot regenerator minimizes the effect of the increased void volume of screen packing as compared to spheres. Pressure drop through the screen matrix is also predicted more accurately than for the spheres. This is caused by the inherent uniformity of the screen material rather than the analytical methods used. Slight variations in shot size or departure from a perfectly spherical shape may cause deviations in pressure drop. This is an important factor in the hot regenerator packing selection because of the strong influence of pressure drop on drive motor power. Finally, the selection of stainless steel was based on the superior heat sink capacity in the operating temperature range of interest (see Appendix A).

Table 5-1 presents the detailed output from the regenerator analysis computer program for the preliminary design hot regenerator. The output parameters listed in the table are metal temperature, gas temperature, density, pressure, and flow rate. These parameters are printed as functions of time (angular crankshaft displacement) and of position in the regenerator matrix. The angular position is referenced to top dead center of the cold displacer. Node 0 represents the gas inlet face at the sump end, and Node 11 the outlet at the hot end. Nodes 1 through 10 are internal to the matrix. Positive flow rate indicates flow from the sump toward the hot end.

Figures 5-1 through 5-3 present plots of key parameters from the data of Table 3-6. The hot gas and matrix temperatures are plotted as functions of crankshaft position in Figure 5-1. The small difference between the two temperatures is indicative of adequate heat transfer. The matrix temperature swing of 1.355°K (2.44°R) shows that the matrix heat capacity is sufficient.

Figure 5-2 indicates a maximum pressure drop of 5590 N/m^2 (0.811 psi). This value of pressure drop is higher than that of the preliminary design unit due to the increased length. The pressure drop is still acceptable from a motor power standpoint.

At any time gas is introduced into the hot end of the machine at a temperature less than the operating level, additional heat must be added to make up for the deficiency. Thus compensation must be made for the loss represented by the regenerator inefficiency. This loss may be evaluated by use of Figures 5-1 and 5-3, which presents the gas flow rate at the hot end of the regenerator. Equation 4-1 is used to evaluate the regenerator loss, as was done for the cold regenerator. The cumulative losses are shown as a function of crank angle in Figure 5-4. The total loss per crankshaft revolution is 0.914J (0.0008661 Btu), which translates to a total loss of 5.67 watts at a speed of 400 rpm.

TABLE 5-1

PRELIMINARY DESIGN
HOT REGENERATOR
PERFORMANCE CHARACTERISTICS

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

REGULAR PRINTOUTS

$\theta = 0^{\circ}$

DATE = 26 OCT 73 TIME = 11:44:26

TIME(SEC.) = 4.49959-01 3611 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.21125+02	6.20000+02	5.54267-01	9.49480+02	7.40777-03
1	6.72842+02	6.71118+02	5.13751-01	9.49459+02	7.27362-03
2	7.57571+02	7.55042+02	4.57892-01	9.49412+02	7.03381-03
3	8.48135+02	8.45494+02	4.09655-01	9.49359+02	6.81871-03
4	9.39648+02	9.37109+02	3.70144-01	9.49301+02	6.62394-03
5	1.03132+03	1.02890+03	3.35575-01	9.49237+02	6.44701-03
6	1.12300+03	1.12068+03	3.01013-01	9.49164+02	6.28793-03
7	1.21467+03	1.21242+03	2.66469-01	9.49083+02	6.14668-03
8	1.30629+03	1.30407+03	2.31959-01	9.48989+02	6.02327-03
9	1.39734+03	1.39516+03	1.97666-01	9.48879+02	5.91757-03
10	1.48369+03	1.48165+03	1.65105-01	9.48746+02	5.82869-03
11	1.53366+03	1.53241+03	1.46001-01	9.48669+02	5.78918-03

$\theta = 30^{\circ}$

DATE = 26 OCT 73 TIME = 11:44:36

TIME(SEC.) = 4.62405-01 3741 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.20647+02	6.20000+02	5.70101-01	9.77423+02	6.55584-03
1	6.71911+02	6.70221+02	5.29188-01	9.77405+02	6.45611-03
2	7.56214+02	7.53653+02	4.71897-01	9.77367+02	6.27775-03
3	8.46735+02	8.44042+02	4.22153-01	9.77323+02	6.11778-03
4	9.38316+02	9.35709+02	3.81375-01	9.77275+02	5.97296-03
5	1.03006+03	1.02756+03	3.45806-01	9.77220+02	5.84140-03
6	1.12180+03	1.11939+03	3.10248-01	9.77158+02	5.72311-03
7	1.21350+03	1.21116+03	2.74714-01	9.77088+02	5.61806-03
8	1.30514+03	1.30285+03	2.39217-01	9.77007+02	5.52626-03
9	1.39621+03	1.39396+03	2.03942-01	9.76912+02	5.44761-03
10	1.48263+03	1.48053+03	1.70429-01	9.76797+02	5.38146-03
11	1.53301+03	1.53173+03	1.50613-01	9.76729+02	5.35208-03

NOTES:
1. TEMP, $^{\circ}K = {}^{\circ}R/1.8$
2. DENSITY, $kg/m^3 = 16.02 \times LB/FT^3$
3. PRESSURE, $N/m^2 = 6.895 \times 10^3 \times PSI$
4. FLOW, $kg/sec = 0.4536 \times LB/SEC$

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TABLE 5-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

REGULAR PRINTOUTS

θ = 60°

DATE = 26 OCT 73 TIME = 11:44:45

TIME(SEC.) = 4.74932-01 3849 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
=0	6.20407+02	6.20000+02	5.80152-01	9.95161+02	3.70871-03
1	6.71208+02	6.69834+02	5.38862-01	9.95153+02	3.66041-03
2	7.55161+02	7.53047+02	4.80649-01	9.95136+02	3.57398-03
3	8.45642+02	8.43421+02	4.29977-01	9.95115+02	3.49648-03
4	9.37269+02	9.35116+02	3.88426-01	9.95092+02	3.42632-03
5	1.02906+03	1.02700+03	3.52232-01	9.95066+02	3.36258-03
6	1.12084+03	1.11885+03	3.16048-01	9.95037+02	3.30526-03
7	1.21257+03	1.21064+03	2.79891-01	9.95004+02	3.25437-03
8	1.30422+03	1.30234+03	2.43773-01	9.94965+02	3.20988-03
9	1.39531+03	1.39347+03	2.07878-01	9.94919+02	3.17177-03
10	1.48179+03	1.48007+03	1.73768-01	9.94863+02	3.13971-03
11	1.53250+03	1.53145+03	1.53531-01	9.94831+02	3.12548-03

θ = 90°

DATE = 26 OCT 73 TIME = 11:44:50

TIME(SEC.) = 4.86999-01 3907 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
=0	6.20323+02	6.19982+02	5.81641-01	9.97761+02	-3.11768-04
1	6.70942+02	6.69995+02	5.40105-01	9.97764+02	-2.97470-04
2	7.54751+02	7.53210+02	4.81791-01	9.97773+02	-2.71893-04
3	8.45214+02	8.43708+02	4.30976-01	9.97812+02	-2.48957-04
4	9.36852+02	9.35320+02	3.89404-01	9.97858+02	-2.28194-04
5	1.02866+03	1.02714+03	3.53150-01	9.97875+02	-2.09332-04
6	1.12045+03	1.11902+03	3.16873-01	9.97884+02	-1.92371-04
7	1.21219+03	1.21084+03	2.80616-01	9.97890+02	-1.77310-04
8	1.30384+03	1.30258+03	2.44388-01	9.97895+02	-1.64146-04
9	1.39494+03	1.39377+03	2.08377-01	9.97899+02	-1.52869-04
10	1.48144+03	1.48040+03	1.74169-01	9.97904+02	-1.43386-04
11	1.53229+03	1.53175+03	1.53891-01	9.97906+02	-1.39176-04

NOTES:
1. TEMP, $^{o}K = {^{o}R}/1.8$
2. DENSITY, kg/m^3 = 16.02 X LB/FT^3
3. PRESSURE, N/m^2 = 6.895 X 10^3 X PSI
4. FLOW, kg/sec = 0.4536 X LB/SEC

TABLE 5-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

REGULAR PRINTOUTS

$\theta = 120^{\circ}$

DATE = 26 OCT 73 TIME = 11144155

TIME(SEC.) = 4.99961-01

3971 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.20582+02	6.21972+02	5.72099-01	9.83801+02	-4.42872-03
1	6.71292+02	6.73344+02	5.30005-01	9.83811+02	-4.35542-03
2	7.55103+02	7.57254+02	4.72715-01	9.83834+02	-4.22430-03
3	8.45542+02	8.47615+02	4.23184-01	9.83860+02	-4.10663-03
4	9.37154+02	9.39135+02	3.82571-01	9.83889+02	-4.00005-03
5	1.02894+03	1.03085+03	3.46855-01	9.83922+02	-3.90325-03
6	1.12072+03	1.12258+03	3.11132-01	9.83958+02	-3.81622-03
7	1.21246+03	1.21428+03	2.75417-01	9.84000+02	-3.73898-03
8	1.30411+03	1.30591+03	2.39731-01	9.84047+02	-3.67150-03
9	1.39519+03	1.39686+03	2.04306-01	9.84104+02	-3.61373-03
10	1.48162+03	1.48268+03	1.70879-01	9.84171+02	-3.56510-03
11	1.53291+03	1.53500+03	1.50499-01	9.84211+02	-3.54358-03

$\theta = 150^{\circ}$

DATE = 26 OCT 73 TIME = 11145105

TIME(SEC.) = 5.12446-01

4082 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.21312+02	6.22956+02	5.56130-01	9.56932+02	-6.79392-03
1	6.72323+02	6.74674+02	5.14854-01	9.56950+02	-6.65998-03
2	7.56154+02	7.58603+02	4.59310-01	9.56992+02	-6.42037-03
3	8.46526+02	8.48868+02	4.11302-01	9.57037+02	-6.20527-03
4	9.38070+02	9.40293+02	3.71912-01	9.57088+02	-6.01038-03
5	1.02981+03	1.03194+03	3.37168-01	9.57149+02	-5.83336-03
6	1.12155+03	1.12362+03	3.02407-01	9.57208+02	-5.67424-03
7	1.21327+03	1.21530+03	2.67644-01	9.57279+02	-5.53302-03
8	1.30492+03	1.30692+03	2.32906-01	9.57360+02	-5.40969-03
9	1.39594+03	1.39779+03	1.98446-01	9.57455+02	-5.30410-03
10	1.48210+03	1.48329+03	1.66020-01	9.57569+02	-5.21520-03
11	1.53366+03	1.53500+03	1.46407-01	9.57634+02	-5.17580-03

NOTES:
1. TEMP, $^{\circ}K = ^{\circ}R/1.8$
2. DENSITY, kg/m^3 = 16.02 X LB/FT^3
3. PRESSURE, N/m^2 = 6.895 X 10^3 X PSI
4. FLOW, kg/sec = 0.4536 X LB/SEC

TABLE 5-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

REGULAR PRINTOUTS

$\theta = 180^{\circ}$

DATE = 26 OCT 73 TIME = 11:45:16

TIME(SEC.) = 5.24909-01

4216 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.22275+02	6.24092+02	5.38416-01	9.27047+02	-7.20002-03
1	6.73689+02	6.76240+02	4.97999-01	9.27068+02	-7.07242-03
2	7.57549+02	7.60202+02	4.44384-01	9.27115+02	-6.84405-03
3	8.47841+02	8.50390+02	3.98049-01	9.27166+02	-6.63898-03
4	9.39305+02	9.41736+02	3.60006-01	9.27224+02	-6.45314-03
5	1.03098+03	1.03332+03	3.26331-01	9.27288+02	-6.28434-03
6	1.12269+03	1.12496+03	2.92634-01	9.27360+02	-6.13264-03
7	1.21439+03	1.21642+03	2.58927-01	9.27441+02	-5.99804-03
8	1.30601+03	1.30820+03	2.25244-01	9.27535+02	-5.88050-03
9	1.39695+03	1.39896+03	1.91864-01	9.27644+02	-5.77988-03
10	1.48274+03	1.48401+03	1.60582-01	9.27776+02	-5.69512-03
11	1.53419+03	1.53500+03	1.41820-01	9.27853+02	-5.65749-03

$\theta = 210^{\circ}$

DATE = 26 OCT 73 TIME = 11:45:27

TIME(SEC.) = 5.37406-01

4347 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.23185+02	6.24816+02	5.23593-01	9.01762+02	-5.71441-03
1	6.74981+02	6.77305+02	4.83988-01	9.01778+02	-5.61310-03
2	7.58869+02	7.61280+02	4.31945-01	9.01812+02	-5.43175-03
3	8.49091+02	8.51406+02	3.86973-01	9.01851+02	-5.26886-03
4	9.40483+02	9.42690+02	3.50031-01	9.01895+02	-5.12119-03
5	1.03211+03	1.03423+03	3.17292-01	9.01943+02	-4.98711-03
6	1.12377+03	1.12583+03	2.84448-01	9.01998+02	-4.86661-03
7	1.21545+03	1.21747+03	2.51630-01	9.02060+02	-4.75968-03
8	1.30706+03	1.30903+03	2.18836-01	9.02131+02	-4.66633-03
9	1.39791+03	1.39971+03	1.86358-01	9.02214+02	-4.58643-03
10	1.48334+03	1.48447+03	1.56000-01	9.02314+02	-4.51909-03
11	1.53450+03	1.53500+03	1.37896-01	9.02373+02	-4.48916-03

NOTES:
1. TEMP, $^{\circ}K = ^{\circ}R/1.8$
2. DENSITY, $kg/m^3 = 16.02 \times LB/FT^3$
3. PRESSURE, $N/m^2 = 6.895 \times 10^3 \times PSI$
4. FLOW, kg/sec = 0.4536 X LB/SEC

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TABLE 5-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

REGULAR PRINTOUTS

$\theta = 240^\circ$ DATE = 26 OCT 73 TIME = 11:45:36

TIME(SEC.) = 5.49910-01 4456 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.23829+02	6.25048+02	5.14572-01	8.86040+02	-3.05139-03
1	6.75898+02	6.77602+02	4.75566-01	8.86047+02	-2.99022-03
2	7.59806+02	7.61561+02	4.24438-01	8.86062+02	-2.88072-03
3	8.49981+02	8.51650+02	3.80255-01	8.86079+02	-2.78236-03
4	9.41326+02	9.42902+02	3.43954-01	8.86098+02	-2.69319-03
5	1.03291+03	1.03442+03	3.11726-01	8.86120+02	-2.61221-03
6	1.12456+03	1.12601+03	2.79470-01	8.86143+02	-2.53944-03
7	1.21622+03	1.21764+03	2.47198-01	8.86170+02	-2.47487-03
8	1.30781+03	1.30920+03	2.14981-01	8.86201+02	-2.41850-03
9	1.39859+03	1.39986+03	1.83021-01	8.86238+02	-2.37025-03
10	1.48376+03	1.48456+03	1.53189-01	8.86282+02	-2.32959-03
11	1.53467+03	1.53500+03	1.35422-01	8.86308+02	-2.31151-03

$\theta = 270^\circ$ DATE = 26 OCT 73 TIME = 11:45:41

TIME(SEC.) = 5.62446-01 4516 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.24066+02	6.20000+02	5.17745-01	8.85028+02	2.55301-04
1	6.76267+02	6.77822+02	4.74871-01	8.85027+02	2.19839-04
2	7.60187+02	7.62209+02	4.23591-01	8.85024+02	1.56361-04
3	8.50350+02	8.52556+02	3.79414-01	8.85020+02	9.93424-05
4	9.41684+02	9.43848+02	3.43211-01	8.85019+02	4.76542-05
5	1.03326+03	1.03538+03	3.11006-01	8.85019+02	7.24931-07
6	1.12490+03	1.12696+03	2.78788-01	8.85019+02	-4.14455-05
7	1.21656+03	1.21852+03	2.46571-01	8.85019+02	-7.88587-05
8	1.30815+03	1.30997+03	2.14396-01	8.85019+02	-1.11920-04
9	1.39890+03	1.40053+03	1.82534-01	8.85020+02	-1.39479-04
10	1.48394+03	1.48477+03	1.52897-01	8.85022+02	-1.63053-04
11	1.53473+03	1.53500+03	1.35224-01	8.85023+02	-1.73536-04

NOTES:
1. TEMP, $^\circ K = ^\circ R/1.8$
2. DENSITY, $kg/m^3 = 16.02 \times LB/FT^3$
3. PRESSURE, $N/m^2 = 6.895 \times 10^3 \times PSI$
4. FLOW, kg/sec = 0.4536 X LB/SEC

TABLE 5-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

REGULAR PRINTOUTS

$\theta = 300^{\circ}$

DATE = 26 OCT 73 TIME = 11:45:46

TIME(SEC.) = 5.74971-01 4576 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.23149+02	6.20000+02	5.24624-01	8.97167+02	3.55252-03
1	6.76084+02	6.74862+02	4.83415-01	8.97159+02	3.47177-03
2	7.59974+02	7.58428+02	4.31383-01	8.97140+02	3.32723-03
3	8.50133+02	8.48521+02	3.86269-01	8.97120+02	3.19745-03
4	9.41485+02	9.39952+02	3.49184-01	8.97097+02	3.07987-03
5	1.03308+03	1.03164+03	3.16493-01	8.97071+02	2.97309-03
6	1.12473+03	1.12336+03	2.83793-01	8.97043+02	2.87709-03
7	1.21640+03	1.21507+03	2.51096-01	8.97012+02	2.79190-03
8	1.30798+03	1.30668+03	2.18437-01	8.96976+02	2.71750-03
9	1.39874+03	1.39745+03	1.86079-01	8.96934+02	2.65378-03
10	1.48379+03	1.48259+03	1.55726-01	8.96883+02	2.60008-03
11	1.53462+03	1.53384+03	1.37461-01	8.96853+02	2.57625-03

$\theta = 330^{\circ}$

DATE = 26 OCT 73 TIME = 11:45:55

TIME(SEC.) = 5.87430-01 4684 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
-0	6.22018+02	6.20000+02	5.37594-01	9.20056+02	6.14852-03
1	6.75435+02	6.73771+02	4.96227-01	9.20039+02	6.03284-03
2	7.59138+02	7.56899+02	4.42979-01	9.20002+02	5.82570-03
3	8.49274+02	8.46918+02	3.96603-01	9.19960+02	5.63977-03
4	9.40673+02	9.38401+02	3.58444-01	9.19914+02	5.47134-03
5	1.03232+03	1.03016+03	3.24921-01	9.19862+02	5.31839-03
6	1.12401+03	1.12193+03	2.91395-01	9.19803+02	5.18083-03
7	1.21570+03	1.21369+03	2.57875-01	9.19738+02	5.05875-03
8	1.30730+03	1.30533+03	2.24401-01	9.19668+02	4.95210-03
9	1.39807+03	1.39613+03	1.91235-01	9.19574+02	4.86074-03
10	1.48317+03	1.48138+03	1.60099-01	9.19468+02	4.78375-03
11	1.53422+03	1.53307+03	1.41224-01	9.19406+02	4.74959-03

NOTES:
1. TEMP, $^{\circ}K = {}^{\circ}R/1.8$
2. DENSITY, $kg/m^3 = 16.02 \times LB/FT^3$
3. PRESSURE, $N/m^2 = 6.895 \times 10^3 \times PSI$
4. FLOW, kg/sec = 0.4536 X LB/SEC

TABLE 5-1 (Continued)

NODE NO.	MATRIX TEMPERATURE	GAS TEMPERATURE	GAS DENSITY	GAS PRESSURE	GAS FLOW RATE

REGULAR PRINTOUTS

$\theta = 360^{\circ}$

DATE = 26 OCT 73 TIME = 11146105

TIME(SEC.) = 5.99959-01

4814 = COUNT (NO. OF CALCULATIONS)

N	TM(N) DEG.R	TG(N) DEG.R	RG(N) LBM/CF	PG(N) PSIA	WG(N) LBM/SEC
=0	6.21173+02	6.20000+02	5.54268-01	9.49482+02	7.40781-03
1	6.74505+02	6.72712+02	5.12488-01	9.49460+02	7.27399-03
2	7.57897+02	7.55420+02	4.57666-01	9.49413+02	7.03432-03
3	8.47988+02	8.45368+02	4.09714-01	9.49361+02	6.81921-03
4	9.39450+02	9.36913+02	3.70218-01	9.49303+02	6.62439-03
5	1.03117+03	1.02875+03	3.35634-01	9.49238+02	6.44742-03
6	1.12291+03	1.12059+03	3.01049-01	9.49166+02	6.28833-03
7	1.21464+03	1.21238+03	2.66484-01	9.49084+02	6.14709-03
8	1.30626+03	1.30404+03	2.31970-01	9.48990+02	6.02367-03
9	1.39704+03	1.39487+03	1.97774-01	9.48880+02	5.91792-03
10	1.48222+03	1.48022+03	1.65644-01	9.48748+02	5.82876-03
11	1.53361+03	1.53232+03	1.46036-01	9.48671+02	5.78925-03

NOTES:
1. TEMP, $^{\circ}K = ^{\circ}R/1.8$
2. DENSITY, $kg/m^3 = 16.02 \times LB/FT^3$
3. PRESSURE, $N/m^2 = 6.895 \times 10^3 \times PSI$
4. FLOW, kg/sec = 0.4536 X LB/SEC

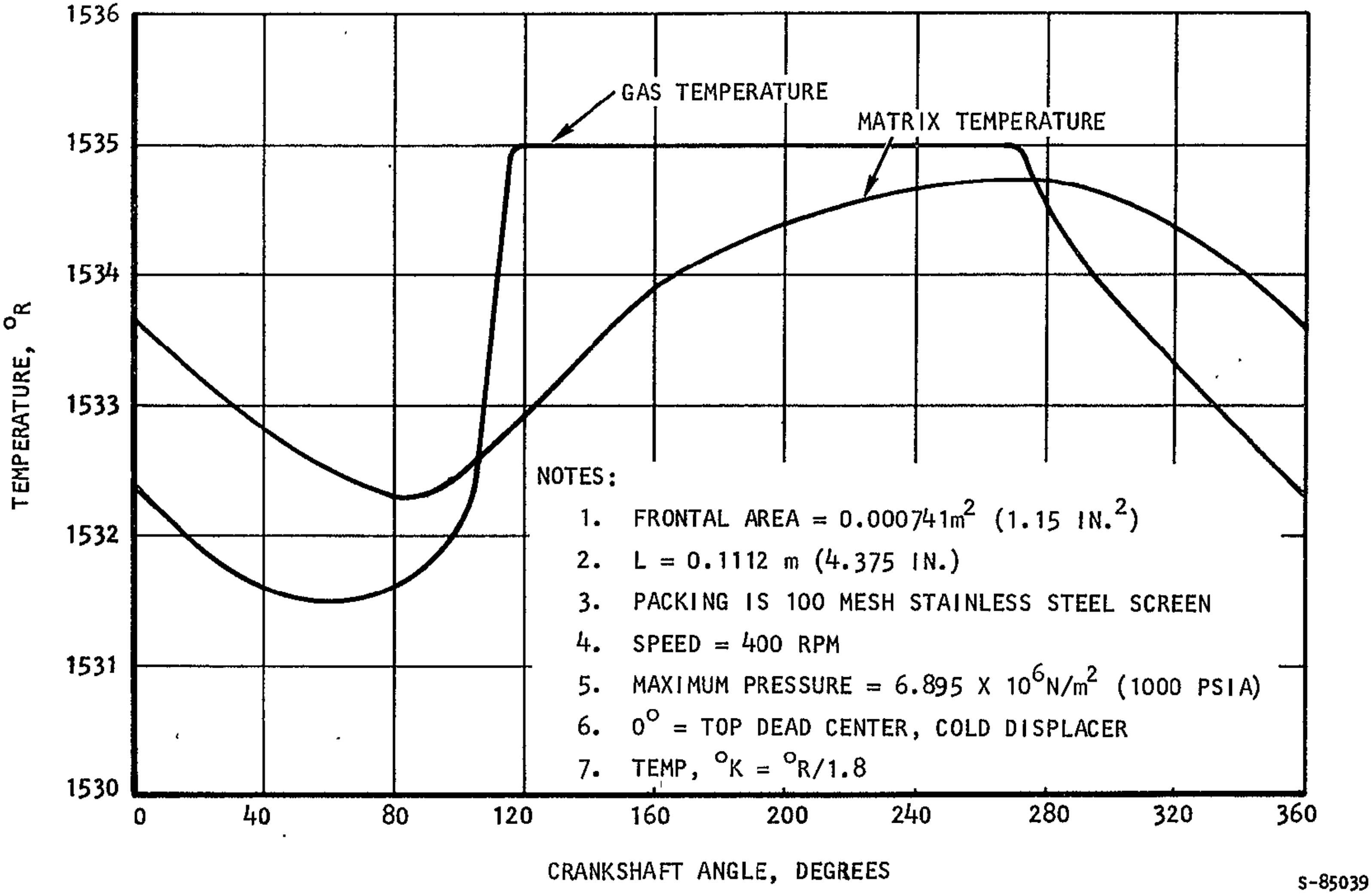


Figure 5-1. Temperature Variation at Hot End of Hot Regenerator

NOTES:

1. FRONTAL AREA = 0.000741 m^2 (1.15 $IN.^2$)
2. L = 0.1112m (4.375 in.)
3. PACKING IS 100 MESH STAINLESS STEEL SCREEN
4. SPEED = 400 RPM
5. MAXIMUM PRESSURE = 6.895 X 10^6 N/m^2 (1000 PSIA)
6. 0^o = TOP DEAD CENTER, COLD DISPLACER
7. PRESSURE DROP, N/m^2 = 6.895 X 10^3 X PSI

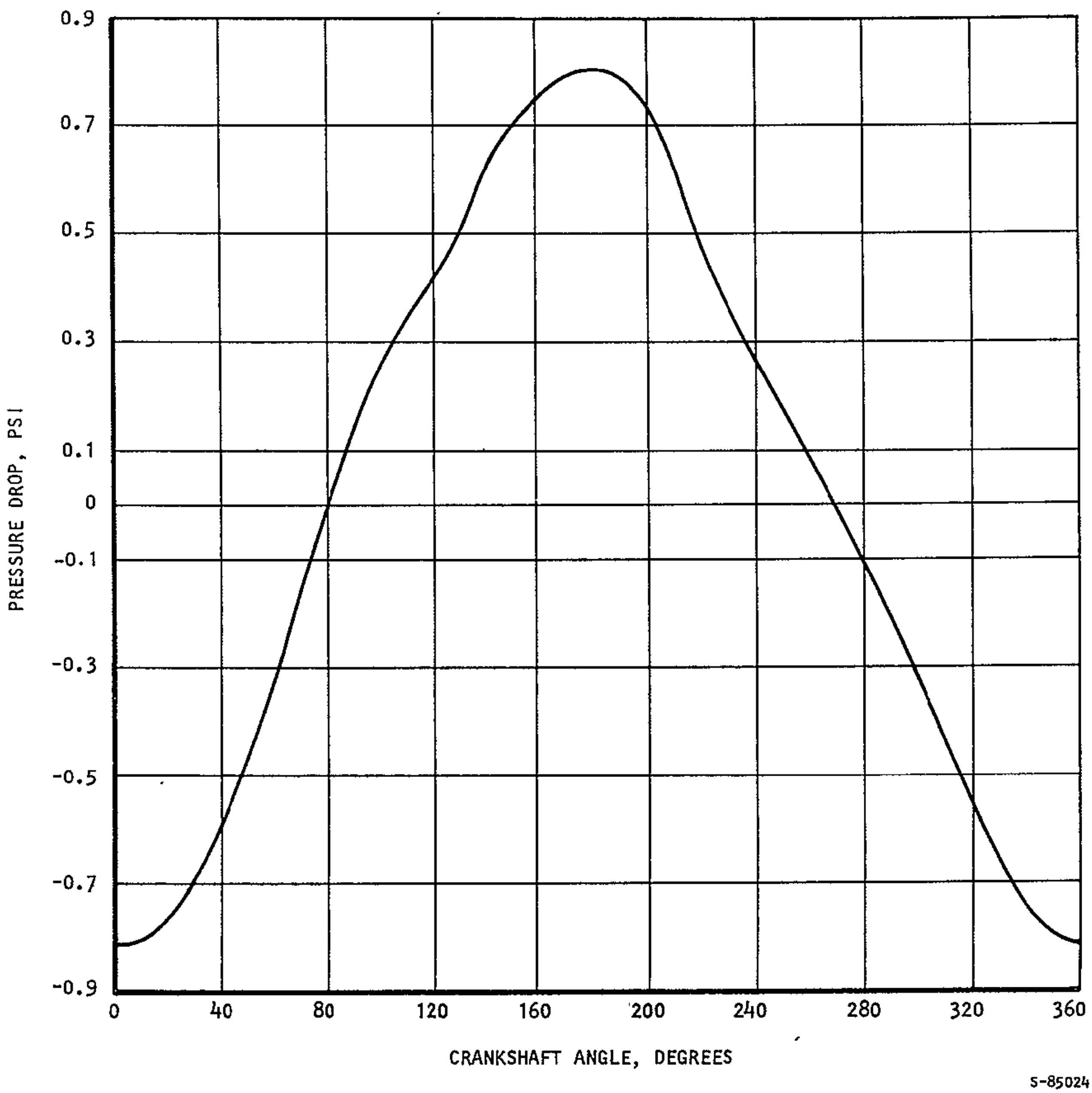


Figure 5-2. Pressure Drop of Hot Regenerator

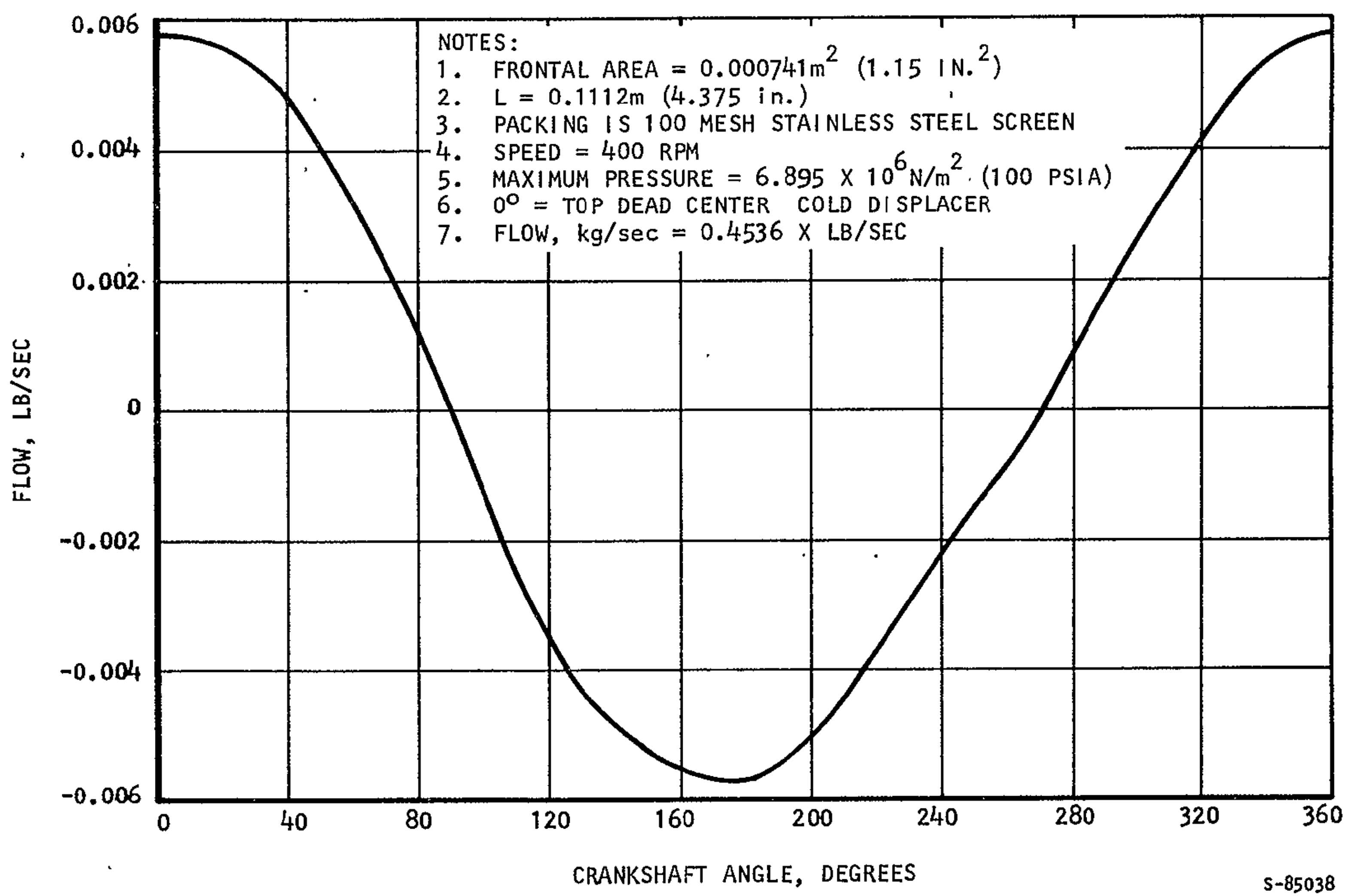


Figure 5-3. Hot End Flowrate of Hot Regenerator

NOTES:
1. $LOSS = \int_{o}^{360} WC_P(T-T_{REF})d\theta$
2. FRONTAL AREA = $0.000741 m^2$ (1.15 IN.)
3. L = 0.1112m (4.375 in.)
4. PACKING IS 100 MESH STAINLESS STEEL SCREEN
5. SPEED = 400 RPM
6. MAXIMUM PRESSURE = $6.895 \times 10^6 N/m^2$ (1000 PSIA)
7. 0^o = TOP DEAD CENTER, COLD DISPLACER
8. LOSS, JOULES = 1055 X BTU

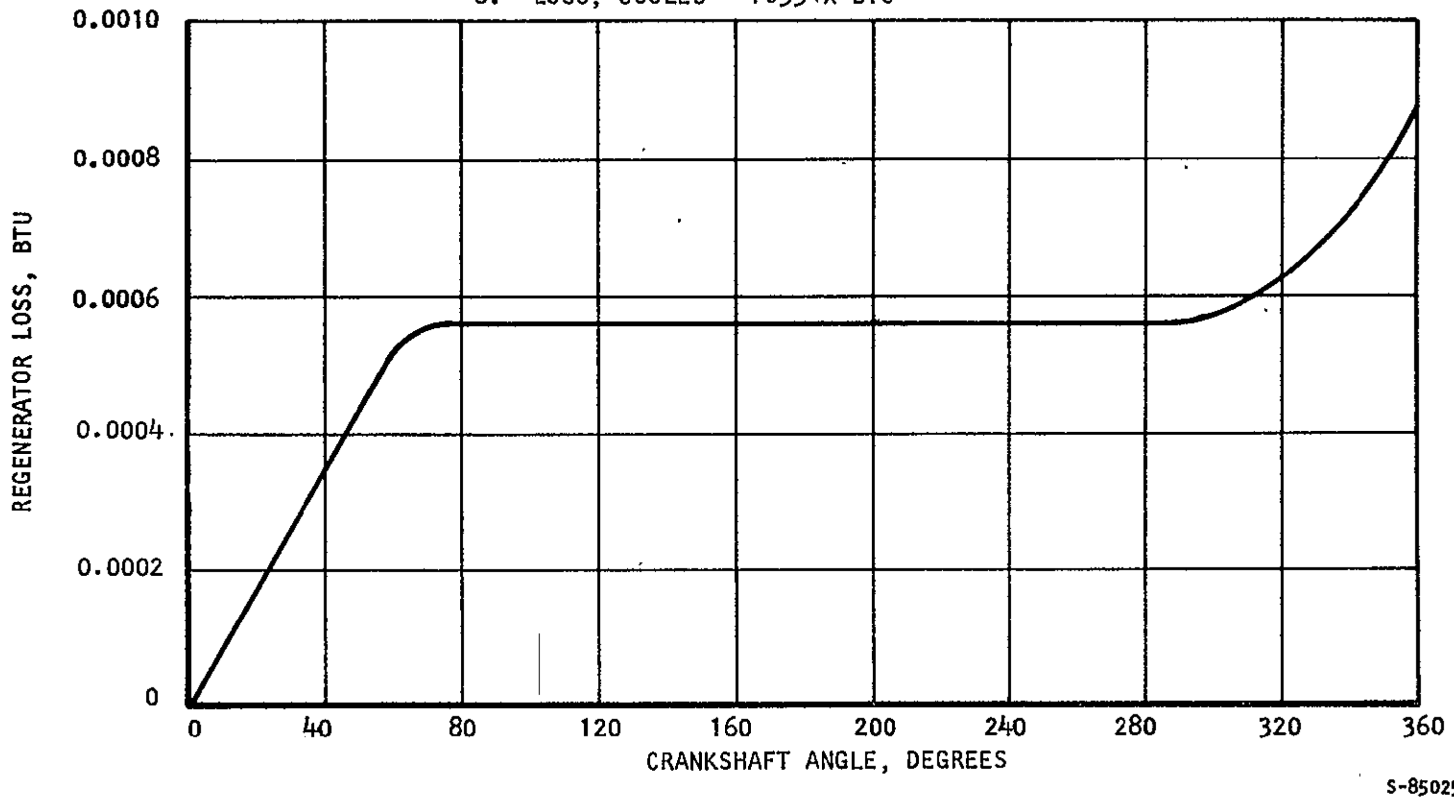


Figure 5-4. Hot Regenerator Thermal Loss per Cycle

SECTION 6

COLD END HEAT EXCHANGER

SECTION 6

COLD END HEAT EXCHANGER DESIGN

INTRODUCTION

The cold end heat exchanger transfers the refrigeration heat load from the cold finger of the VM refrigerator to the working fluid. The primary design criteria are:

Minimum temperature difference between working fluid and refrigeration load--This is an extremely important consideration. Since the refrigeration temperature is fixed, the larger the temperature difference, the lower the working fluid temperature the refrigerator must produce with inherent decreased thermodynamic efficiencies.

Low working fluid pressure drop--The heat exchanger must provide high thermal performance and still not impose excessive working fluid pressure drop on the system. The heat exchanger pressure drop subtracts directly from the cold expansion volume pressure variations, which produce the refrigeration.

Low void or internal volume--Void volume at low temperatures significantly decreases the refrigeration capacity of VM refrigerators. An evaluation of this effect was presented in the Task I report (reference 3-1) and is discussed under flow distributors elsewhere in this report.

Flow distribution--Uniform flow within each element of the heat exchanger must occur. Potential problems here are twofold: nonuniform flow leads to (1) reduced heat exchanger conductance, and (2) fluid elements at different temperatures. Subsequent mixing of these elements results in entropy increase and reduces the thermodynamic efficiency of the refrigerator.

Heat exchanger interfaces--The cold end heat exchanger interfaces with the cold regenerator, the cold expansion volume, and the refrigeration heat load. These interface requirements control the heat exchanger configuration to some extent. The annular flow passages of the heat exchanger interface with the cold regenerator through a perforated plate flow distributor and with the cold expansion volume through ports in the displaced volume dome. The interface with the refrigeration heat load is provided by means of an axial clamp and a high thermal conductivity (copper plating) heat path to the heat exchanger outer wall.

DESIGN CONFIGURATION AND PERFORMANCE

The cold-end heat exchanger has been changed in physical configuration from that of the preliminary design unit. However, the flow and heat transfer areas have been retained, and the performance is thus identical. The change involved reduction of the cold finger outer diameter, thus placing the heat exchanger at the inner diameter of the cold regenerator, rather than the outer. The change was made primarily to reduce weight at the cold end; an additional benefit realized was reduction of the cold end void volume. A summary of cold end void volume is presented in Table 6-1.

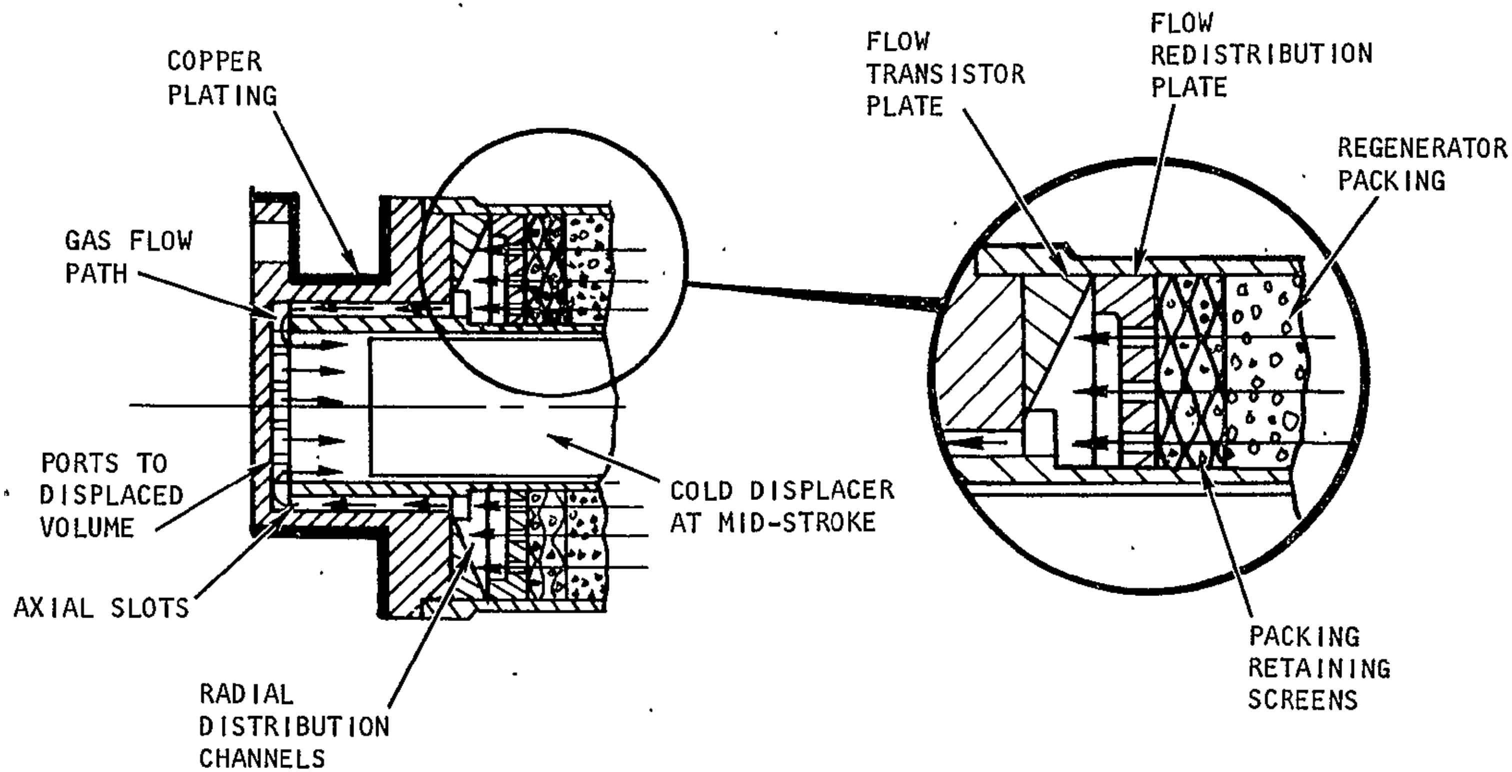
The cold heat exchanger is shown in Figure 6-1. Twelve axial slots are arranged in an annular configuration around the regenerator inner wall. This configuration, with the reduction in outer wall diameter at the cold end of the regenerator, eliminated the use of a relatively heavy filler block for the heat exchanger, and thus reduced the cantilevered weight and the void volume of the cold end.

The cyclic flow enters (and exits) the heat exchanger from the cold expansion volume via ports at the outer diameter of the displacer bore. The working fluid traverses the axial slots and enters the radial distribution channels. The flow from the channels is distributed uniformly over the cold face of the regenerator by the perforated plate flow distributor.

TABLE 6-1

COLD END VOID VOLUME BREAKDOWN

Location of Volume	Volume, m^3 (in.^{3})
Displacer clearance at maximum stroke	2.06×10^{-8} (0.001257)
Heat exchanger	3.059×10^{-7} (0.018665)
Transition-heat exchanger to flow distributor	1.461×10^{-7} (0.008917)
Flow distributor	7.907×10^{-8} (0.004825)
Total	5.516×10^{-7} (0.033664)
To account for tolerance buildup, use 5.735×10^{-7} (0.035 in.^{3})	



S-86223

Figure 6-1. Schematic of Cold End Heat Exchanger

The twelve flow channels are 0.00953m (0.375 in.) long, and are rectangular in shape. The cross section of each channel is 0.001017 (0.04) x 0.002034m (0.08 in.), with the shorter side in the radial direction. This provides a minimum radial height and a maximum prime surface for heat transfer.

The average flow during the cyclic flow process is used for heat transfer performance evaluation. The total heat transfer conductance (ηhA) of the cold heat exchanger is 0.403 watts/^{o}K (0.224 watts/^{o}R). At a heat load of 0.25 watts, the corresponding film temperature drop from the metal heat exchanger wall to the working fluid is 0.621^{o}K (1.12^{o}R). The pressure drop, evaluated at the maximum flow rate of the cycle, is 94.5 N/m^2 (0.0137 psi).

The refrigeration heat load is mounted on a removable copper end cap. This end cap is clamped to the cold end of the machine with spring loaded axial bolts, and indium foil is employed at the interface. The outer diameter of the cold end of the refrigerator is copper plated, in order to allow uniform heat distribution over the outer surface of the heat exchanger. Thus a minimum temperature drop from the refrigeration load to the heat exchanger wall is assured.

COLD HEAT EXCHANGER ANALYSIS

The analysis shown on the following pages was performed for an early configuration of the cold end heat exchanger. The diameter of the cold end was subsequently reduced, and the flow slots for the heat exchanger located at the inner diameter of the cold regenerator. The number and size of the flow slots remained the same, and thus the analysis applies. The pressure drop of the reconfigured design is expected to be slightly lower than calculated here, and void volume is reduced significantly.

7/18/72
NWJM

Cold Heat Exchanger

Ref: 1. F.B. Tarlton Sketch, Rev A, 6/26/72
2. L145804, 7/13/72 No Change

Maximum Flow = .00193 lb/sec = 6.95 lb/hr

Average Flow = .00116 lb/sec = 4.18 lb/hr

1. Heat Transfer

Since a radial clamp will be used for the load, consider only the axial slots in the heat transfer calculation. The maximum cycle pressure is 1000 psia and minimum is 885. Therefore evaluate physical properties for heat transfer calculations at $P_{av} = 942.5$ psia.

$P_r = 0.69$

$\mu = 0.01848$ lbm/ft-hr

$K = 0.03434$ BTU/ft-hr-°F

$C_p = 1.291$ BTU/lbm-°R

Free flow Area 12 slots, .04 x .08 in.

$$A_c = 12 \times .04 \times .08/144 = 0.0002665 \text{ ft}^2$$

Hydraulic Diameter

$$D_h = 4A_c/wp = 4 \times .04 \times .08/2(.04+.08) = .0533 \text{ in} = .00444 \text{ ft.}$$

Mass Velocity

$$G = \frac{\dot{w}}{A_c} = \frac{4.18}{0.0002665} = 15{,}680 \ \mathrm{lb/ft^2\text{-}hr}$$

Reynolds Number

$$Re = \frac{\rho D_h V}{\mu} \qquad \text{however, } \rho V = \frac{\dot{w}}{A} = G$$

Therefore

$$Re = \frac{G D_h}{\mu} = \frac{15{,}680 \ \mathrm{lb_m} \quad \mathrm{ft\text{-}hr} \quad \times 0.00444 \ \mathrm{ft}}{\mathrm{ft^2\text{-}hr} \quad 0.01848 \ \mathrm{lb_m}} = 3765$$

Slot Length = 0.375, $L/D_h = .375/.0533 = 7.04$

from Fig. 9-21, McAdams, $\frac{h_L}{c_p G\left[1+(D/L)^{.7}\right]} = .0054$ (Based on interpolation between $L/D = 5$ and $L/D = 11$)

The value of h_L obtained from this correlation is the mean value between $x=0$ and $x=L$.

Thus $h_L = .0054 \ c_p \ G\left[1+(D/L)^{.7}\right] = .0054 \times 1.291 \times 15{,}680\left[1+\frac{1}{(7.04)^{.7}}\right]$

$$= 109.3 \times 1.255 = 137.2 \ \mathrm{BTU/ft^2\text{-}hr\text{-}{}^\circ F}$$

The above value of h is for circular channels. From pg. 248, McAdams, the maximum deviation between circular tubes and rectangular channels with aspect ratios as high as 7.9 (ours is 2.0) is minus 24 percent.

Therefore, To be conservative, use $h_L = h_L/1.24$

$$\therefore h_L = 137.2/1.24 = 110.8 \text{ BTU/ft}^2\text{-hr-}^\circ\text{F}$$

The prime heat Transfer area is That located on the outer edge of each slot.

$$A_{prime} = .08 \times 12 \times .375/144 = .0025 \text{ ft}^2$$

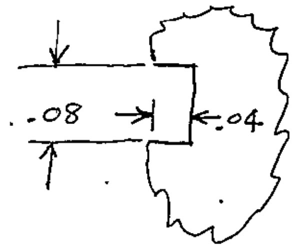
$$hA_{prime} = 110.8 \times .0025 = .2766 \text{ BTU/hr-}^\circ\text{F} = 0.0819 \text{ Watts/}^\circ\text{F}$$

The secondary surface performance is degraded by conduction loss both Through The metal and across The helium gap between The outer wall and The insert. First obtain The resistance of The helium gap. The maximum radial clearance is .0002 in., and The diameter at The interface is 1.024 in.

$$\text{Area} = \pi DL - 12 \times .08 \times .375 = .375(1.024\pi - .96) = 0.847 \text{ in}^2$$
$$= 0.00589 \text{ ft}^2$$

$$\frac{KA}{L} = \frac{0.03434 \text{ BTU} \times .00589 \text{ ft}^2 \times 12 \text{ in}}{\text{ft-hr-}^\circ\text{F} \times .0002 \text{ in ft}} = 12.11 \text{ BTU/hr-}^\circ\text{F}$$

now obtain fin efficiency



$$\eta_f = \frac{\mathrm{Tanh}\, m\ell_e}{m\ell_e}$$

where ℓ_e = effective fin length

$m = \sqrt{\frac{h_c}{k\delta}}$ for a single sided fin

K = Thermal conductivity of metal
= 3.1 BTU/FT-hr-°F for Inco 718 at 112°R

δ = fin Thickness

$\ell_e = .04 + .5 \times .08 = .08$ in.

The slots are spaced 30° apart on a 1.024 in. Dia
Thus the chord length between them is:

$$L = 1.024\pi \times \frac{30}{360} - .08 = .268 - .08 = .188 \text{ in.}$$

use 1/3 of this length as fin Thickness, $\delta = .0626$ in.

$$m\ell_e = \frac{.08 \cancel{in}\, ft}{12 \cancel{in}} \sqrt{\frac{110.8\ BTU\ \cancel{ft\text{-}hr\text{-}°F} \times 12\ \cancel{in}}{ft^2\text{-}hr\text{-}°F \cdot 3.1\ BTU \times .0626\ \cancel{in}\ ft}} = .00667\,\cancel{ft}\sqrt{\frac{6850}{ft^2}}$$

$$= 0.552$$

$$\eta_f = .91$$

secondary surface area

$$A_s = 12 \times .375(.08+.04+.04) = .72\ in^2 = .005\ ft^2$$

$$\eta h A = .91 \times 110.8 \times .005 = .505\ BTU/hr\text{-}°F$$

combining $\eta h A$ and conduction loss

$$\eta h A' = \frac{1}{\frac{1}{\eta h A} + \frac{1}{kA/L}} = \frac{1}{\frac{1}{.505} + \frac{1}{12.11}} = \frac{1}{1.98 + .0826} = \frac{1}{2.0626}$$

$$= 0.485\ BTU/hr\text{-}°F$$

$$= .142\ watts/°F$$

$$\text{Total } \eta h A = hA\text{prime} + \eta h A' = .0819 + .142 = .2239\ watts/°F$$

For a heat load of 0.25 watts,

$$\Delta T_{film} = \frac{0.25\ watts}{0.2239\ watts/°F} = 1.118\ °R$$

This is a reasonable number since the cold junction temperature is 117 °R and the cold end gas temperature is 112 °R. This leaves 3.9°R allowable metal conduction losses.

2. Pressure Drop

Use maximum cold flow, 0.00193 lb/sec. The pressure drop is expected to be primarily shock loss, with friction contributing only a small portion. The maximum flow occurs at maximum pressure, 1000 psia. Therefore use density at this pressure.

$\rho = 2.911 \ lb/ft^3$

Calculate velocity head first

$$H_v = \frac{\rho V^2}{2g_c} \qquad \text{from continuity, } V^2 = \left(\frac{\dot{w}}{A\rho}\right)^2 = \frac{G^2}{\rho^2}$$

and $$H_v = \frac{G^2}{2g_c\rho}$$

$$G = \frac{\dot{w}}{A_c} = \frac{.00193}{0.0002665} = 7.24 \ lb_m/ft^2\text{-}sec$$

$$H_v = \frac{7.24^2 \ \cancel{lb_m^2} \ \ lb_f\text{-}\cancel{sec^2} \ \cancel{ft^3} \ \cancel{ft^2}}{\cancel{ft^4}\text{-}\cancel{sec^2} \ \ 2 \times 32.2 \ \cancel{lb_m}\text{-}\cancel{ft} \times 2.911 \ \cancel{lb_m} \times 144 in^2} = .00194 \ lb_f/in^2$$

The Reynolds number may be ratioed from that calculated for Heat Transfer

$$Re = \frac{.00193}{.00116} \times 3765 = 6270$$

$4f = .04 @ \epsilon/D = 0.004$

$4f\ L/D_h = .04 \times 7.04 = 0.282$

The flow passage from the heat exchanger slots to the cold displaced volume has been designed with a constant flow cross sectional area equal to that of the slots. However, the hydraulic diameter is different, which will affect the friction factor and pressure drop. The hydraulic diameter varies because of variation in wetted perimeter as the fluid progresses radially inward. The variation in wetted perimeter is a direct function of diameter, and my thus be ~~averaged~~ evaluated at an average diameter.

$$D_{av} = \frac{1.024 + .4}{2} = 0.712\ in.$$

$$D_h = \frac{4 \times .04 \times .08 \times 12}{2\pi \times .712} = .0342\ in = .00286\ ft.$$

$$Re = \frac{.00286}{.00444} 6270 = 4040$$

$4f = 0.044$

$4f\,L/D_h = .044 \times .38/.0342 = 0.489$

As expected, the frictional loss will be low. In order to be conservative, we will consider all turns as 90° square duct mitered elbows, which have a loss coefficient of 1.6 H.v. There are three turns, one at the inlet to the slots, one at the end of the slots, and one into the cold displaced volume. In addition, allow 1 H_v loss for sudden expansion at exit, and 0.5 H_v loss for sudden contraction at inlet. The loss coefficients are summarized below.

	Loss Coefficient
Slot friction	0.282
Radial friction	0.489
inlet contraction	0.5
outlet expansion	1.0
3 Turns at 1.6 each	4.8
Total	7.071

thus maximum $\Delta P = 7.071\ H_v = 7.071 \times .00194 = 0.01372$ psi

This is an acceptable value

3. Dead Volume

a. Slot Volume = $A_c \times L = .0384 \times .375 = 0.0144\ in^3$

b. The radial flow path to the displacer has equal flow area and $L = .38\ in.$ $V = .0384 \times .38 = 0.0146\ in^3$

c. There is .125 inches of slot length at entrance from ~~displacer~~ regenerator which is not effective for heat transfer, but must be included in the volume.
$V = .0384 \times .125 = 0.0048\ in^3$

d. The transition slots from the regenerator ar triangular in shape. The base is .125 in and the height is $.3 - 0.04 = 0.26\ in.$ slot width = .04 in.

$$V = 0.5 \times .125 \times .26 \times .04 \times 12 = 0.0078\ in^3$$

Thus Total volume is:

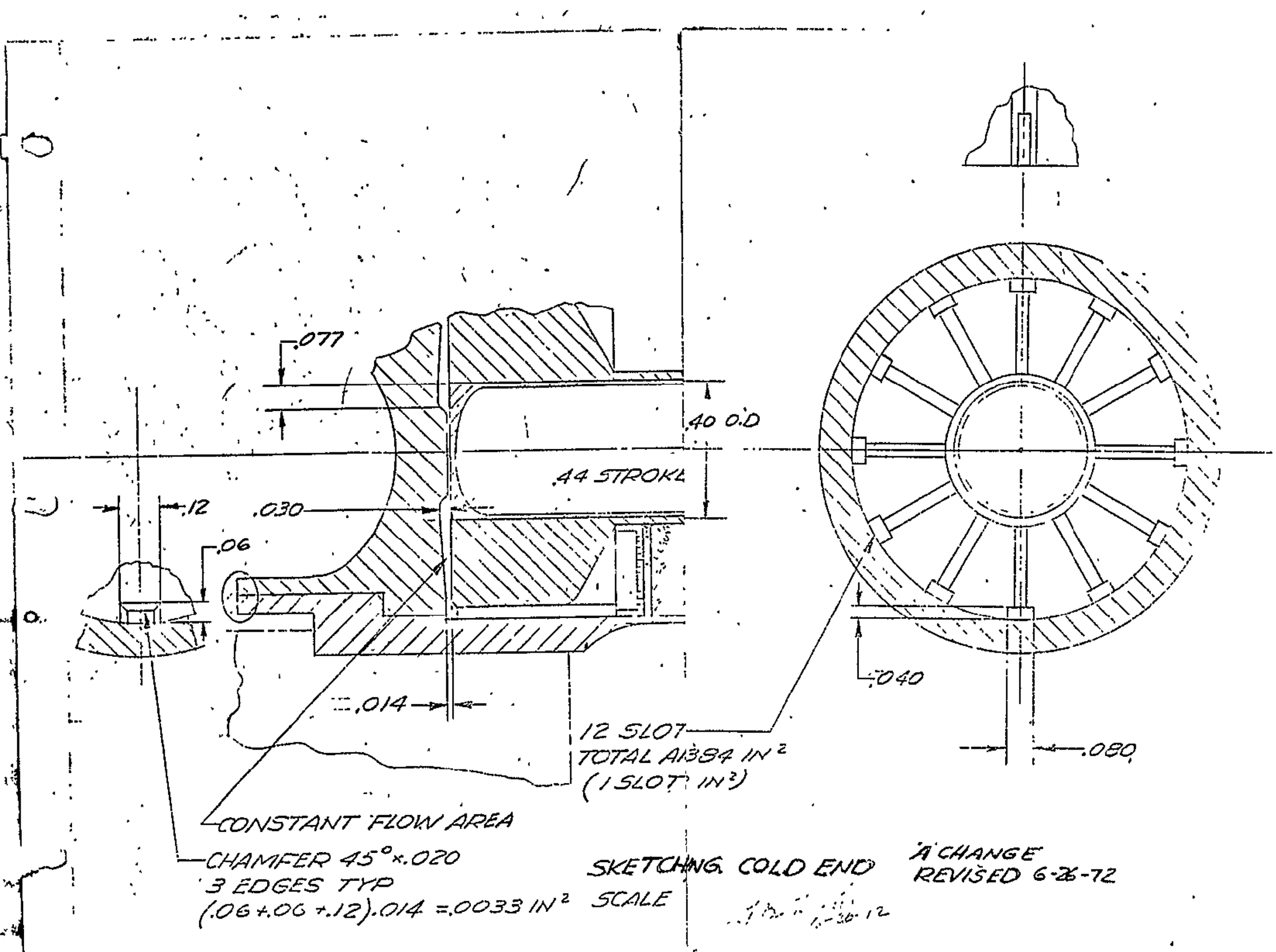
Slots	0.0144
radial	0.0146
Slot extension	0.0048
Transition Slots	0.0078
	$0.0416\ in^3$

The volume of the flow distributor and the displacer clearance must be added to this. They are calculated elsewhere.

4. Additional Pressure Drop Considerations

The chamfer at the transition from the slots to the radial flow path has been added to prevent local velocity increases. As the flow emerges from the slots, the flow area is the axial gap of .014 in multiplied by the sum of three sides of the slot. The chamfer is .020, so the sides at this point are $.08+2 \times .02 = .12$ and $.04+.02 = .06$.

Thus $A_c = (.12+.06+.06).014 = 0.00336\ in^2$.
The slot A_c is $0.0032\ in^2$, so this will not cause local increases in velocity or pressure drop.


.077
.40 O.D
.44 STROKE
.12
.030
.06
.014
.040
.080
12 SLOT
TOTAL A1384 IN²
(1 SLOT IN²)
CONSTANT FLOW AREA
CHAMFER 45° × .020
3 EDGES TYP
(.06 + .06 + .12) .014 = .0033 IN²
SKETCHING COLD END
SCALE
'A' CHANGE
REVISED 6-26-72

SECTION 7

HOT END HEAT EXCHANGER DESIGN

SECTION 7

HOT END HEAT EXCHANGER DESIGN

INTRODUCTION

The hot-end heat exchanger transfers thermal energy to the working fluid of the VM refrigerator, thus providing the power input necessary to drive the system. Critical design criteria are similar to those of the cold heat exchanger, with emphasis on various parameters changed.

Low working fluid pressure drop--As with the hot regenerator, low pressure drop is of prime importance in the hot end heat exchanger. This parameter directly affects drive motor power, and thus must be minimized. In addition, pressure drop reduces the cold end pressure fluctuations, and thus refrigeration capacity. The high fluid temperature, and thus low density, in the hot end heat exchanger requires that careful attention be paid to pressure drop. High fluid velocities occur for flow rates and cross-sectional areas similar to lower temperature portions of the machine.

Low Film Temperature Drop--The maximum temperature of the heat exchanger wall is set by structural considerations. Thus a low film temperature drop allows a close working fluid approach to this maximum temperature. This in turn increases thermodynamic efficiency, which is a direct function of hot end gas temperature.

Flow distribution--Non-uniform flow in the hot end heat exchanger must be avoided for the same reasons given for the cold end heat exchanger.

Low void or internal volume--The hot end of the VM refrigerator is the least sensitive portion of the machine to void volume. This dependency, caused by the high operating temperature, is discussed in the task I report (Reference 3-1).

Heat exchanger interfaces--The hot heat exchanger provides working fluid transitions with both the hot displaced volume and the hot regenerator. The electrical heater, which provides the energy input to the system is bonded directly to the outer surface of the heat exchanger--pressure dome. The heat input is transferred directly to the working fluid from the heat exchanger walls.

DESIGN CONFIGURATION AND PERFORMANCE

The hot end heat exchanger was patterned after that used on the GSFC 5 watt VM, with modifications that simplify the required manufacturing processes. The design configuration is shown in Figure 7-1.

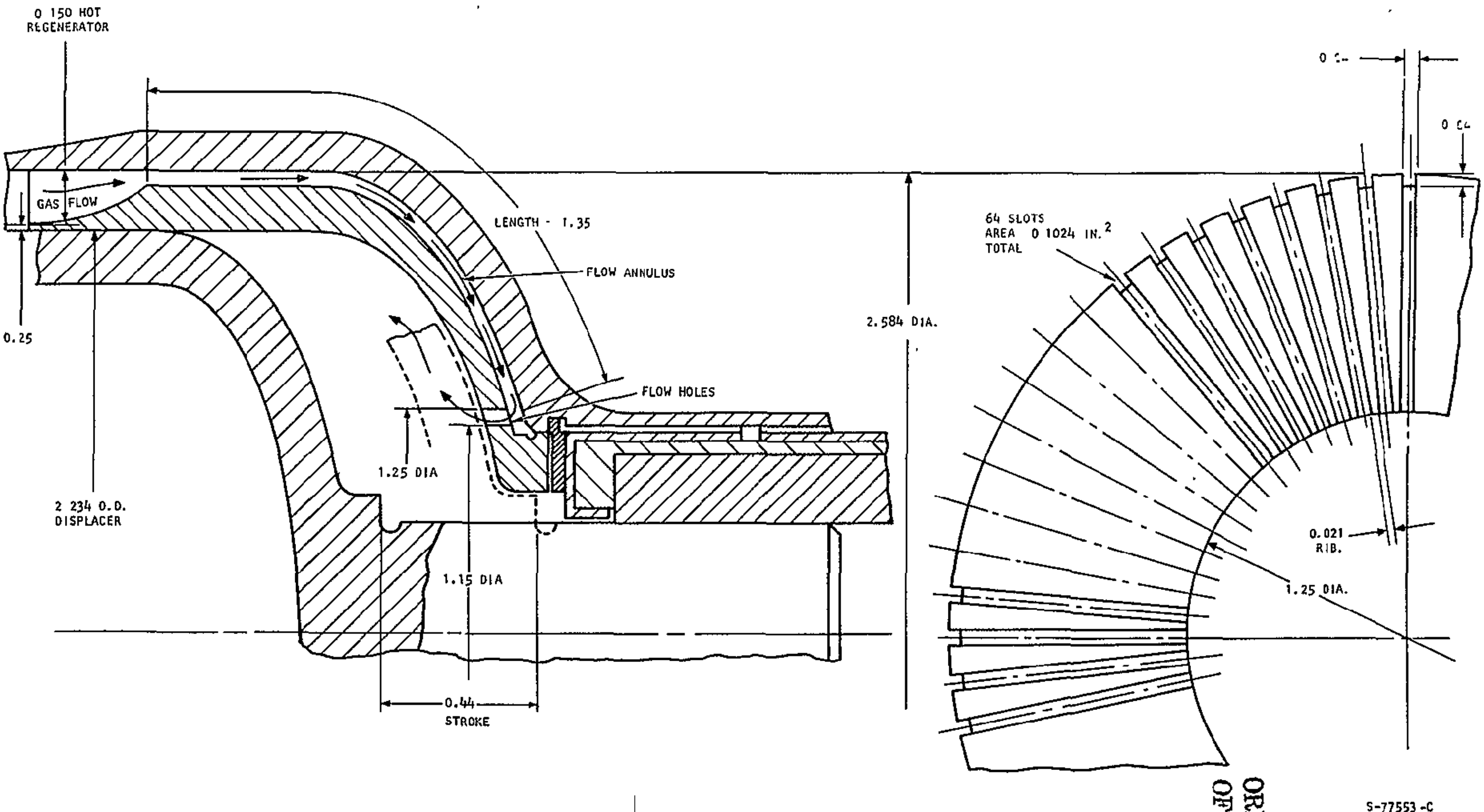


Figure 7-1. Schematic of Hot End Heat Exchanger

The working fluid enters and exits the heat exchanger at the hot displaced volume interface by a series of holes bored through the dome at a circle diameter corresponding to the hot end bearing support ring. The flow enters a total of 64 square passages contained between the heat exchanger inner wall and the outer pressure dome. These passages direct the flow initially in a radially outward direction. The passages follow the pressure dome contour, until the flow emerges in an axial direction into transition slots that form the interface with the hot regenerator.

The flow passages extend the full length of the heat exchanger, with no changes in cross-sectional area or number of slots. This configuration provides adequate heat transfer and a very low pressure drop, which are the desired characteristics of the heat exchanger. The constant cross-sectional area slots are simpler to produce in manufacture than intersecting ribs and variable flow cross-sectional area.

Heat transfer to the working fluid is accomplished at both primary and secondary heat transfer surfaces. The primary surface consists of the portions of the outer pressure dome which form the boundaries of the slots. Heat is transferred directly to the working fluid from this surface. The secondary heat transfer surface is composed of the sides and inner walls of the flow passages. Heat is conducted from the other surface to these secondary surfaces, and then to the working fluid. The secondary surface is approximately 93 percent as effective as the primary surface.

The heat transfer performance calculation is based on the average value of working fluid flow rate during the cycle, which is considered a conservative estimate. The flow passage length is less than 40 hydraulic diameters, which is generally too short for fully developed laminar flow to occur. However, under the further conservative assumption that fully developed laminar flow does occur, the total ηhA of the hot heat exchanger is 6.51 watts/^{o}K (3.62 watts/^{o}R). This translates to a film temperature drop of 12.28^{o}K (22.1^{o}R) at a total heat load of 80 watts.

The pressure drop of the hot heat exchanger is calculated at the maximum working fluid flow rate during the cycle, and thus represents worst case conditions. The pressure drop under these conditions is 1179N/m^2 (0.171 psi), which is considered an acceptable value.

Hot Heat Exchanger

The Heat exchanger consists of 64 slots, 0.04 x 0.04 x 1.35 in long, following the contour of the hot dome. The interface with the hot regenerator is formed by slots cut into the dome at a 45° angle. The interface with the hot displaced volume is formed by 32 holes which have a flow area greater than that of the slots.

1. Heat Transfer

Physical properties

$C_p = 1.24$ BTU/lb·°F, $\mu = .0943$ lb/ft-hr, $K = .1743$ BTU/ft-hr°F,

$P_r = .67$, $P_r^{2/3} = .776$, $\alpha = P_r^{2/3}/C_p = 0.618$, $\rho = .23$ lb/ft.^{3}

Flow rate = 14.28 lb/hr.

Free flow area $= .04 \times .04 \times 64/144 = .000711$ ft^2

Total Area $= 4 \times .04 \times 64 \times 1.35/144 = .096$ ft^2

Prime Area $= .25 \times$ Total Area $= .25 \times .096 = .024$ ft^2

Secondary Area $= .75 \times$ Total Area $= .75 \times .096 = .072$ ft^2

Hydraulic Dia. $= .04$ in. $= .00333$ ft.

Mass Velocity, $G = \dot{w}/A = 14.28/.000711 = 20{,}084$ lb/ft^2-hr.

Reynolds Number $= GD_h/\mu = \dfrac{20{,}084 \text{ lb} \times .00333 \text{ ft}}{\text{ft}^2\text{-hr}} \dfrac{\text{ft-hr}}{.0943 \text{ lb}} = 709$

This is laminar flow, look at L/D_h

$$L/D_h = 1.35/.04 = 33.75$$

Therefore, use fig 7-2 of Kays and London, square passages, and $L/D_h = 40$.

Colburn Modulus, $j = .0069$

$$h = j\,G/\alpha = \frac{.0069 \times 20.084\ \text{lb}}{\text{ft}^2\text{-hr}}\ \frac{\text{BTU}}{.0618\ \text{lb-}°\text{F}} = 2.24\ \text{BTU}/\text{ft}^2\text{-hr-}°\text{F}$$

For Prime Area,

$$\eta h A = 1 \times 2.24 \times .024 = 5.38\ \text{BTU}/\text{hr-}°\text{F}$$

For the secondary area, the fin efficiency must be calculated, and also the effect of clearance between the pressure dome and the inner piece that forms the heat exchanger.

The clearance is .001 in. maximum, and the conduction area is calculated below.

At the minimum diameter, the ribs are .020 in wide. We will calculate the rib width at the maximum diameter, and use the average of maximum and minimum. This is a very close approximation and will also be used for fin thickness.

The maximum diameter of the ribs is 2.584 in

Circumference $= \pi D = \pi \times 2.584 = 8.12$ in.

maximum rib width $= \dfrac{8.12 - 64 \times .04}{64} = .087$ in

Average width $= \dfrac{.087 + .020}{2} = .0535$ in.

Conduction area $= .0535 \times 1.35 \times 64 / 144 = .0321\ ft^2$

$$KA/L = \frac{1743\ BTU \times .0321\ ft^2 \times 12\ in}{ft\text{-}hr\text{-}°F \times .001\ in.\ ft} = 67.14\ BTU/hr\text{-}°F$$

now we need the fin effectiveness

$$\eta_f = \frac{\tanh m l_e}{m l_e}, \quad m l_e = l_e \sqrt{\frac{2h}{k\delta}}$$

$l_e = .04 + .04/2 = .06$ in $= .06/12 = .005$ ft.

$\delta =$ average rib width $= .0535$ in $= .004458$ ft.

$K_{metal} = 11$ BTU/ft-hr-°F

$$\therefore\ m l_e = .005\ ft \sqrt{\frac{2 \times 224\ BTU\ ft\text{-}hr\text{-}°F}{ft^2\text{-}hr\text{-}°F\ 11\ BTU\ .004458\ ft}} = .0478$$

$\eta_f = 0.93$

thus Secondary $\eta_f h A = 0.93 \times 224 \dfrac{BTU}{ft^2\text{-}hr\text{-}°F} \times 0.072\ ft^2 = 15$ BTU/hr-°F

Total Secondary Surface Conductance

$$\eta h A_{sec} = \frac{1}{\frac{1}{\eta_f h A} + \frac{1}{KA/L}} = \frac{1}{\frac{1}{15} + \frac{1}{67.14}} = 12.26 \ BTU/hr\text{-}°F$$

Total Hx Conductance $= \eta h A_{prime} + \eta h A_{sec} = 5.38 + 12.26 = 17.64$ BTU/hr-°F

The heat load is 80 watts X 3.41 = 273 BTU/hr,

and The fluid To metal ΔT is Thus $\frac{273}{17.64} = 15.48$ °F

As a measure of conservatism, look at what happens if fully developed laminar flow were To occur. From Kays and London, Table 6-1, fully developed laminar flow in a square passage with constant Wall Temperature, Nusselt Number = 2.89

$$Nu = \frac{h D_h}{K}, \quad \text{Thus } h = \frac{Nu\ K}{D_h} = \frac{2.89 \times 0.1713 BTU}{ft\text{-}hr\text{-}°F \times .0033 ft} = 148.67 \frac{BTU}{ft^2 hr\text{-}°F}$$

The prime Area conductance = 148.67 X .024 = 3.568 BTU/hr-°F

$$m l_e = .478 \sqrt{\frac{148.67}{224}} = .389, \quad \eta_f = .95$$

$$\eta_f h A = .95 \times 148.67 \times .072 = 10.17 \ BTU/hr\text{-}°F$$

Total Secondary Surface Conductance with fully developed laminar flow

$$\eta h A_{sec} = \frac{1}{\frac{1}{10.17} + \frac{1}{67.14}} = 8.83$$

Total Hx Conductance = 3.568 + 8.83 = 12.398 BTU/hr-°F

and the fluid to metal ΔT is thus $\frac{273}{12.398} = 22.02$°F

This is a conservative number, as we saw on the previous page. However, it is the worst case that could ever occur, and we will quote it.

2 Pressure Drop

The maximum flow is used for pressure Drop calculations, 20.4 lb/hr.

Mass Velocity, $G = \dot{w}/A = 20.4\ lb/hr / .000711\ ft^2 = 28{,}692\ lb/ft^2hr$

in seconds, $28{,}692/3600 = 7.97\ lb/ft^2\text{-}sec$.

Velocity head $= \rho v^2/2g$, but $v^2 = \left(\frac{\dot{w}}{A\rho}\right)^2 = \frac{G^2}{A^2}$,

so $VH = \frac{G^2}{2g\rho}$

$$VH = \frac{7.97^2\ lb_m^2\ lb_f\text{-}sec^2\ ft^3\ ft^2}{ft^4\ sec^2 \times 2 \times 32.2\ lb_m\text{-}ft\ .23\ lb_m\ 144\ in^2} = .0297\ psi$$

(8

Now we must calculate the shock and frictional losses for flow in both directions. Since the flow passage is curved, there will be an increase in the friction factor. Since the flow is laminar, the Prandtl correlation applies (Schlicting, pg 570)

$$f/f_{straight} = 0.37\, De^{.36}$$

where De = Dean Number $= \frac{Re}{2}\left(r_T/r_c\right)^{1/2}$

Reynolds Number may be ratioed from Heat Transfer,

$Re = \frac{20.4}{14.28} \times 709 = 1013$, minimum radius of curvature = .375 in.

$$\text{Dean No.} = \frac{1013}{2}\left(.01/.375\right)^{1/2} = \cancel{765}\ 82.7$$

$$f/f_{straight} = .37\left(\cancel{765}\,82.7\right)^{.36} = 1.81$$

we will use a factor of 1.25, since the sharp curve only occurs in 15 percent of the total flow length.

$$4f = 64/R_e = 64/1013 = .0632 \times 1.25 = .07897$$

$$4f\, L/D_h = .07897 \times 1.35/.04 = 2.67$$

The Frictional losses are the same for flow in either direction. Now look at shock losses for flow into the hot displaced volume.

1 Entrance loss, Treat as a sudden Contraction with a sharp corner (this is conservative), Loss Coeff = .5

2. Friction, $4fL/D_h$ = 2.67

3. Exit Turning loss, Treat as a 90° miter bend in a square Duct, Loss Coeff = 1.6

4. Exit Expansion loss into infinite area, Coeff = 1.0

$\Sigma K = 5.77$

Pressure Drop = $\Sigma K \times \cancel{V}H = 5.77 \times .0297 \simeq 0.1713$ psi

For Flow Out of The Hot Displaced Volume

1. Entrance loss, $K = 0.5$

2. Turning loss, $K = 1.6$

3. Friction loss $K = 2.67$

4. Exit loss, coeff $= \left(1 - \frac{A_1}{A_2}\right)^2 = \left(1 - \frac{.04}{.3}\right)^2 = .75$

$\Sigma K = 5.52$

$\Delta P = 5.52 \times .0297 = 0.1639$ psi.

Thus flow into The hot displaced Volume produces The maximum pressure drop.

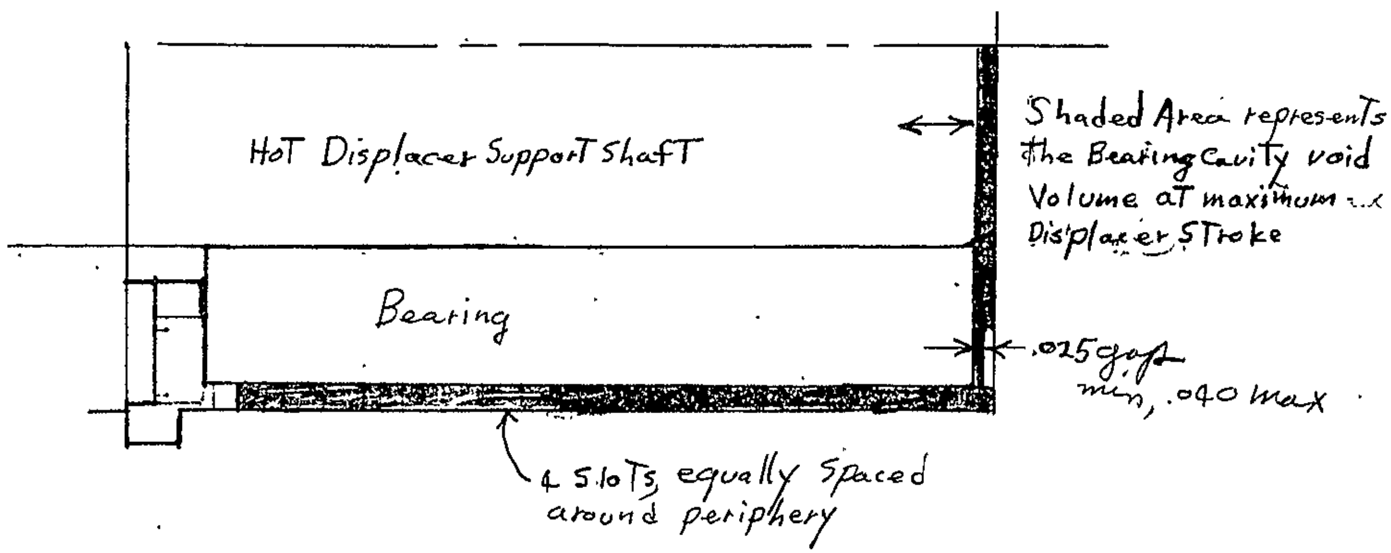
Hot End Bearing Cavity

The hot end bearing has a volume at the far outboard end which oscillates with displacer motion. As this volume varies it must alternatly be filled and emptied with working fluid, helium. The amount of fluid stored in the passages leading to and from the bearing cavity volume also varies due to the fluctuating pressure, and must also be accounted for. The maximum flow rate is determined in the same manner as the crankcase region (Section 12 of this report), following the method of analysis developed for the GSFC 5 watt VM (Reference 4). The flow passages consist of 4 slots milled into the outer diameter of the bearing slot, and the clearance between the end of the bearing and the bearing housing. The total volume of these passages is 0.095 in^3 (see next page).

The swept volume is calculated as the area of the bearing bore times the stroke

$$V_S = \frac{\pi}{4} \times 0.635^2 \times .48 = 0.152 \text{ in.}^3$$

Schematic of Hot End Bearing Flow Passages



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The Total volume may Then be expressed as:

$$V_T = 0.095 + 0.076(1+\sin\theta) = 0.171 + 0.076\sin\theta$$

and $\frac{dV_T}{d\theta} = 0.076 \cos\theta$

The flow rate may be obtained (Ref. 4) from The perfect gas equation as:

$$\dot{W} = \frac{\pi N}{30 Z R T}\left[P\frac{dV}{d\theta} + V\frac{dP}{d\theta}\right]$$

Substituting The appropriate constants and The above expressions for volume and $dV_T/d\theta$, The flow rate is expressed as:

$$\dot{W} = 5.89\times10^{-6}\left[0.076 P\cos\theta + (0.171 + 0.076\sin\theta)\frac{dP}{d\theta}\right],\ \text{lb/sec.}$$

The values of P and $dP/d\theta$ are Taken from the sump region calculations (section 12 of This report) which were in Turn evaluated from The output of The ideal cycle program.

The stepwise evaluation of flowrate as a function of crankshaft angle is presented on The next page.

④

$$\omega = \frac{400\pi}{30\times1\times386\times12\times.535}\left[.076P\cos\theta+(.171+.076\sin\theta)\frac{dP}{d\theta}\right]$$

$$= 5.89\times10^{-6}\left[.076P\cos\theta + (.171+.076\sin\theta)\frac{dP}{d\theta}\right]$$ we can get P and dP/dθ from sump calcs

θ	P	dP/dθ	.076 P cos θ	.171+.076 sin θ	( ) dP/dθ	Σ	ω
10	964.02	51.36	72.15	.1842	9.4605	81.61	.0004807
30	980.36	42.25	64.52	.209	8.83	73.35	.0004320
50	992.48	27.16	48.48	.2292	6.225	54.70	.0003222
70	998.58	7.82	25.96	.2424	1.89	27.85	.0001640
90	997.72	-12.78	0	.247	-3.16	-3.16	-.00001861
110	990.02	-31.34	-25.73	.2424	-7.60	-33.33	-.0001963
130	976.68	-45.06	-47.71	.2292	-10.33	-58.04	-.0003418
150	958.69	-52.31	-63.16	.209	-10.93	-74.09	-.0004364
170	941.35	-52.77	-70.45	.1842	-9.72	-80.17	-.0004722
190	923.93	-47.01	-69.15	.1578	-7.42	-76.57	-.0004510
210	909.38	-36.38	-59.85	.133	-4.84	-64.69	-.0003810
230	899.13	-22.32	-43.92	.1128	-2.52	-46.44	-.0002735
250	894.14	-6.30	-23.24	.09958	-.627	-23.87	-.0001406
270	891.25	10.25	0	.095	.974	.974	.00000574
290	901.14	25.93	23.42	.09958	2.58	26.0	.000153
310	912.53	39.33	44.58	.1128	4.44	49.02	.0002887
330	927.73	48.90	61.06	.133	6.51	67.57	.0003980
350	945.76	53.26	70.78	.1578	8.41	79.19	.0004664

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The flowrates evaluated on the previous page are plotted on the next page, and the maximum value of flow rate is obtained as 0.00048 lb/sec.

The flow rate per slot is one fourth of this, 0.00012 lb/sec.

Using minimum dimensions for the slot, the area is:

$$A_c = 0.038 \times \cancel{0.1}\,0.1/144 = 0.0000264\ ft.^2$$

$$\text{Mass Velocity, } G = \dot{w}/A = 0.00012/0.0000264 = 4.54\ lb/ft^2\text{-sec.}$$

$$\text{The density, } \rho = 0.234\ lb/ft.^3$$

$$\text{Velocity head} = \frac{G^2}{2g\rho} = \frac{4.54^2\ lb_m^2}{ft^4\text{-}sec^2}\ \frac{lb_f\text{-}sec^2\ ft^3\ ft^2}{2 \times 32.2\ lb_m\text{-}ft. \times 0.234\ lb_m\ 144\ in^2}$$

$$= 0.0095\ psi$$

$$\text{The Slot Hydraulic Diameter} = \frac{4A}{wp} = \frac{4 \times .0038}{2 \times .1 + 2 \times .038} = 0.055\ in.$$

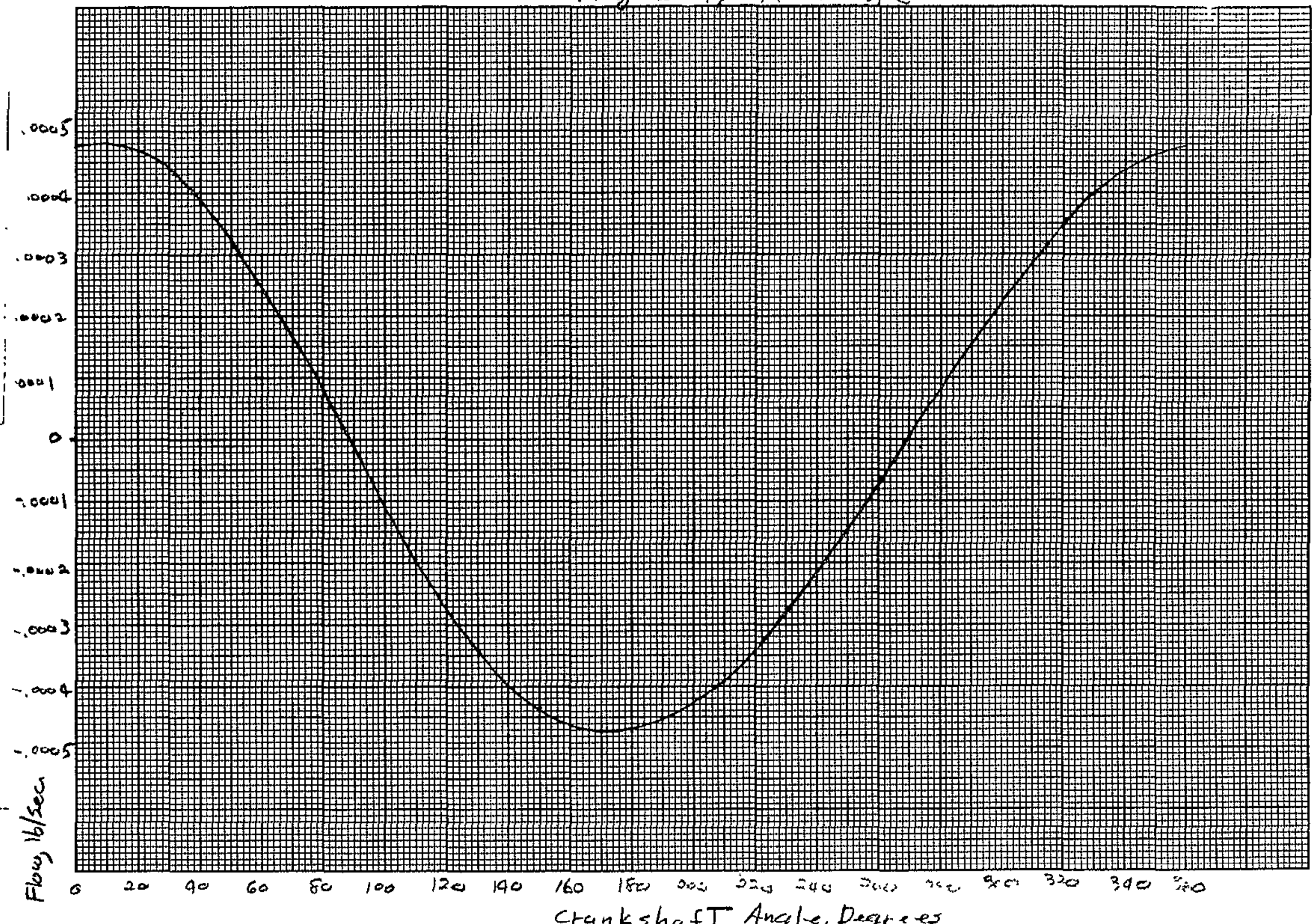
$$D_h = .055/12 = .00459\ ft$$

$$\text{viscosity} = 0.0943\ lb/ft\text{-}hr$$

$$\text{Reynolds Number} = \frac{G D_h}{\mu} = \frac{4.54\ lb \times .00459\ ft}{ft^2\text{-}sec\ \ 0.0943\ lb}\ \frac{ft\text{-}hr \times 3600\ sec}{hr}$$

$$= 795$$

Hot Bearing Cavity flow Rate



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This is laminar flow, and the friction factor is $f = \frac{16}{Re} = \frac{16}{795} = 0.0201$

$$4fL/D_h = 4 \times .0201 \times 1.115 \text{ in.}/0.055 \text{ in} = 1.63$$

we will assume 3 more velocity heads lost in shock, so the total pressure loss is 3+1.63 = 4.63 velocity heads.

$$\Delta P = 4.63 \times 0.0095 \text{ psi} = 0.044 \text{ psi}$$

This value of pressure drop is of the same order of magnitude as that across the somp ports to the back side of the hot displacer and also those to the crankcase region. Thus large pressure imbalances will not occur over various portions of the hot displacer area, and the hot end bearing is considered acceptable as designed.

SECTION 8

AMBIENT SUMP HEAT EXCHANGER

SECTION 8

AMBIENT SUMP HEAT EXCHANGER

INTRODUCTION

The sump heat exchanger functions to transfer heat from the working fluid of the VM refrigerator for rejection from the system. The design criteria for this heat exchanger are similar to those of the cold- and hot-end heat exchangers with changed emphasis on the various items. The preliminary design criteria consist of:

- Low Working Fluid Pressure Drop--The heat exchanger must provide good thermal performance and yet not lead to an excessive pressure drop of the working fluid. As with the cold and hot end heat exchangers; the pressure drop subtracts from the pressure-volume variations in the cold expansion volume thereby reducing the refrigeration capacity. In addition, the pressure drop across the sump heat exchanger can have a significant affect on the drive motor power requirements unless this pressure drop is minimized.

- Low Void or Internal Volume--Void volumes reduce the refrigeration capacity of VM refrigerators by decreasing the pressure variations or pressure ratio; minimization of the heat exchanger internal void volume is therefore important. The relative importance of void volume in this heat exchanger as compared to other regions of the machine is discussed in the Task I report (Reference 3-1).

- Minimization of the Film Temperature Drop--The thermodynamic efficiency of the refrigerator increases as the temperature of the gas in the sump decreases. Minimization of the sump heat exchanger film temperature drop allows maximum performance for the fixed heat rejection and heat sink temperature.

- Flow Distribution--Uniform flow within the heat exchanger is important for two reasons: (1) non-uniform flow leads to reduced conductance of the heat exchanger and (2) non-uniform flow leads to fluid elements at different temperatures; subsequent mixing of these elements results in an increase in entropy and reduced thermodynamic efficiency of the refrigerator.

- Heat Exchanger Interfaces--The sump heat exchanger must interface with both the hot and cold regenerators, fluid passages into the sump volume, and a cooling collar or clamp which provides the heat sink.

DESIGN CONFIGURATION

The configuration of the ambient sump heat exchanger is shown in Figure 8-1. This configuration is a refinement of the design evolved for the GSFC 5 watt VM and is unchanged from the preliminary design configuration.

The annular shaped heat exchanger is divided into two sections as shown, with both sections being identical in configuration except for length. The heat transfer surface of each section is formed by brazing an offset copper plate fin to the inside surface of the cylindrical section of the sump pressure vessel wall. The cylindrical sump filler block fits inside the plate fin thereby forming an annular passage forcing flow through the finned surface. At the right hand side (Figure 8-1), flow of the working fluid enters and exits Section 1 of the heat exchanger as it flows to and from the hot regenerator during the cyclic flow process. The average flow rate in this section of the heat exchanger is approximately 3 times that in Section 2; this accounts for the greater length (larger heat transfer surface) required for this section. At the other end of Section 1 of the heat exchanger, the flow enters and exits from a flow distribution passage cut into the sump filler block. This distribution passage or slot is supplied working fluid via ports that connect to the active cycle volumes in the crank case and behind the hot displacer as shown. The distribution slot is sized to provide uniform flow across the face of the heat exchanger.

On the left hand side of Figure 8-1, flow enters and exits Section 2 of the heat exchanger as it flows to and from the cold regenerator. This section of the heat exchanger is pneumatically connected to the cold regenerator via the sump filler block (not shown in Figure 8-1) and the cold-end linear bearing support. The right hand end of this section of the heat exchanger interfaces and shares the central flow distribution slot with the other section of the heat exchanger.

The path for heat transfer from both sections of the heat exchanger is from the gas to the plate fin surface, from the finned surface through the pressure vessel wall and on into the aluminum sump cooling collar. Indium foil is placed between the sump pressure vessel and the cooling collar; this foil is maintained under a 100 psi interface pressure to ensure good thermal contact. Heat is finally rejected from the system to water cooling coils brazed into channels cut in the cooling collar. This collar is designed to allow interfacing of the refrigerator with water cooling coils or ammonia heat pipes interchangeably.

HEAT EXCHANGER CHARACTERIZATION

The rate of heat transfer for each section of sump heat exchanger can be expressed as:

$$\dot{Q} = h(Ap + \eta_f A_f)\,\overline{\Delta T} \qquad (8\text{-}1)$$

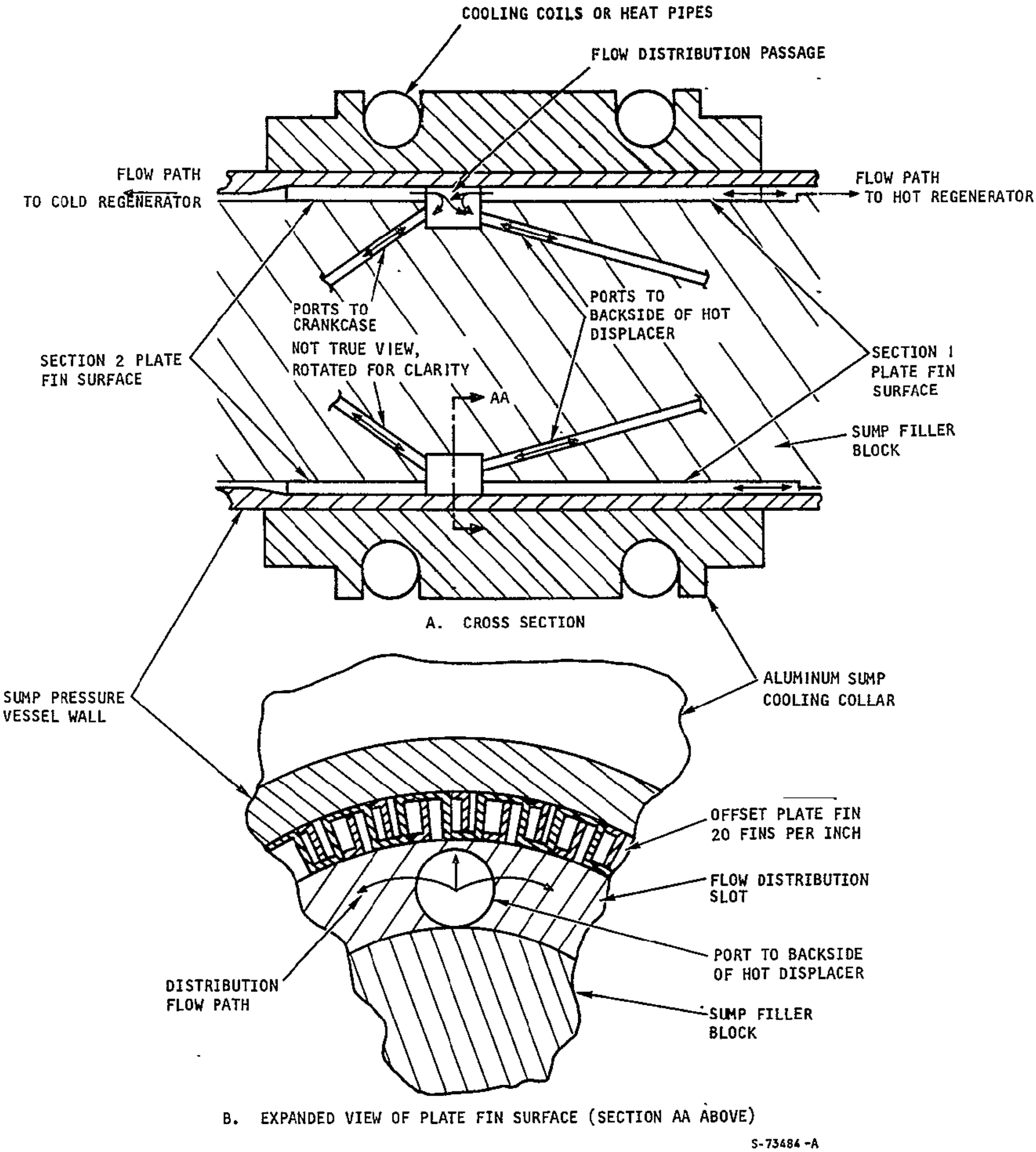


Figure 8-1. Sump Heat Exchanger Configuration

where h = the average heat transfer coefficient

Ap = basic area of the plate

η_f = fin effectiveness

A_f = fin area

$\overline{\Delta T}$ = average temperature difference between the working fluid and the heat transfer surface

Referring to Figure 8-2 the following relations can be derived:

Plate area

$$Ap = (1 - N\delta)WL \tag{8-2}$$

where N = fins per inch

W = plate width

L = plate length

Fin area

$$A_f = \left\{2N(b - \delta) + \frac{N}{2}\left(\frac{1}{N} - \delta\right)\right\} WL \tag{8-3}$$

Note: This neglects the fin area exposed between fin and the sump filler block--a conservative approach.

Fin effectiveness

$$\eta_f = \frac{\text{Tanh } (ML_e)}{ML_e} \tag{8-4}$$

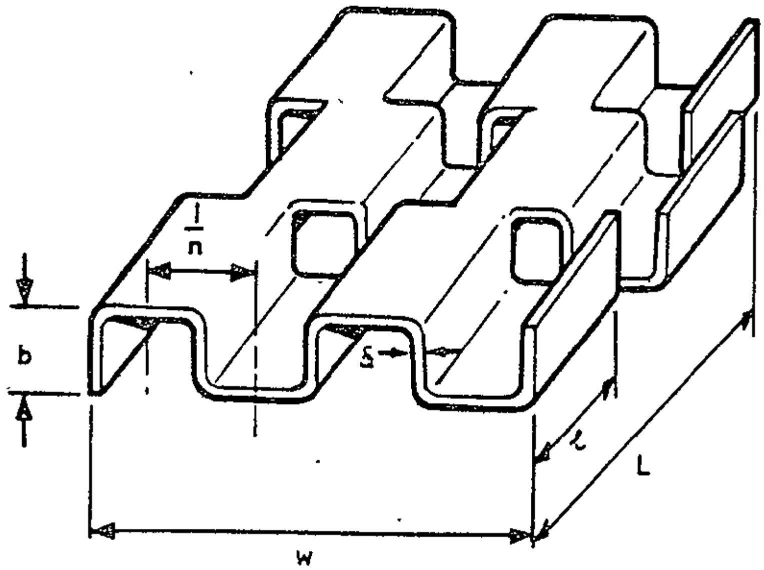
where $$M = \sqrt{\frac{2h}{k\delta}} \tag{8-5}$$

k = fin material thermal conductivity and the fin length L_e is given by

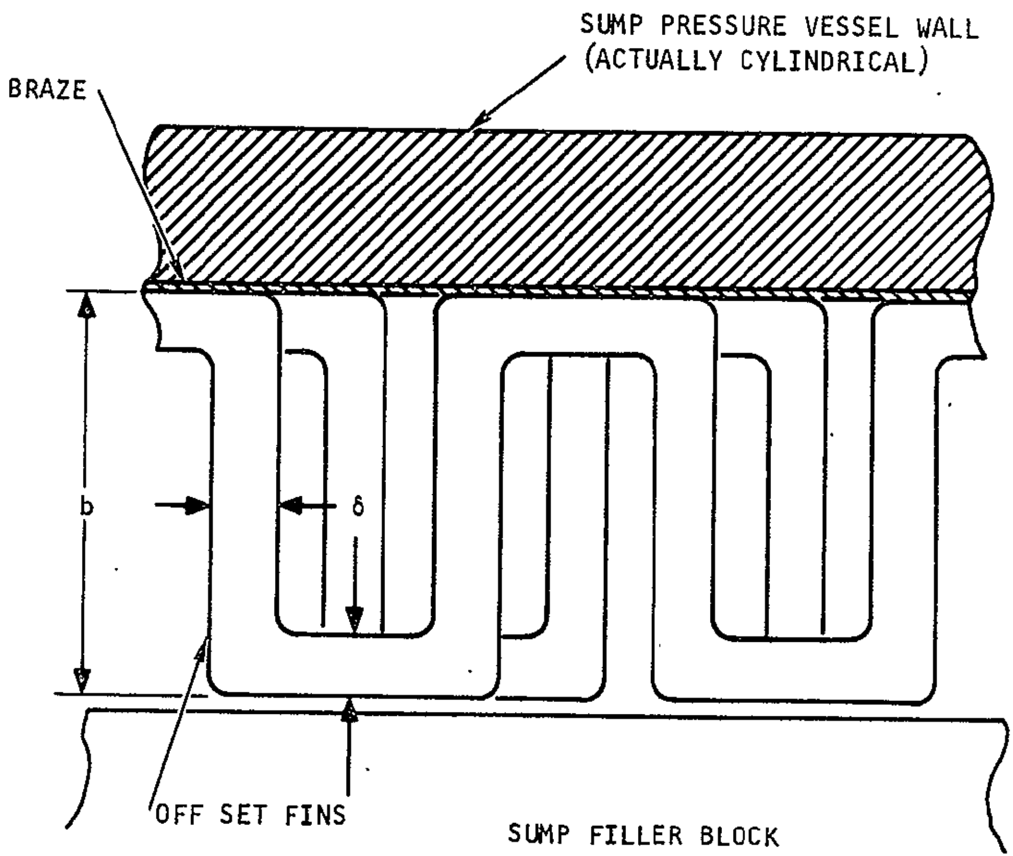
$$L_e = b + \frac{1}{2}\left(\frac{1}{N} - \delta\right) \tag{8-6}$$

Flow cross sectional area

$$A_f = \left\{b - \delta(N(b - \delta) + 1)\right\} W \tag{8-7}$$



a. NOMENCLATURE FOR OFFSET FIN S-41922



b. INSTALLED PLATE-FIN SURFACE S-73483

Figure 8-2. Rectangular Offset Plate-Fin VM Refrigerator Sump Heat Exchanger

Hydraulic diameter

$$D_H = \frac{2(b - \delta)\left(\frac{1}{N} - \delta\right)}{(b - \delta) + \left(\frac{1}{N} - \delta\right)} \qquad (8\text{-}8)$$

Performance characteristics unique to a given plate fin surface are generally presented as plots of Colburn's j factor and Fanning's friction factor f as functions of Reynolds Number. Colburn's j factor is defined by:

$$j = \frac{h}{C_p G} Pr^{2/3} \qquad (8\text{-}9)$$

where C_p = gas heat capacity

Pr = Prandtl number

$G = \frac{\dot{\omega}}{A_f}$

$\dot{\omega}$ = flow rate

Reynolds number is defined as

$$Re = \frac{D_H G}{\mu} \qquad (8\text{-}10)$$

Figure 8-3 gives the Colburn j factor for the fin used in the GSFC VM refrigerator sump heat exchanger. This surface has 787 fins/m (20 fins/in.), an 0.00254 m (0.1 in.) offset length, fin length of 0.000762 m (0.030 in.) and a fin thickness of 0.0001016 m (0.004 in.). Figure 8-4 gives the friction factor for this surface. The pressure drop is then computed by use of

$$\Delta P = \frac{4fL}{D_H}\left(\frac{V^2}{2g_c}\right)\rho \qquad (8\text{-}11)$$

where V = gas velocity

ρ = gas density

g_c = gravitational constant

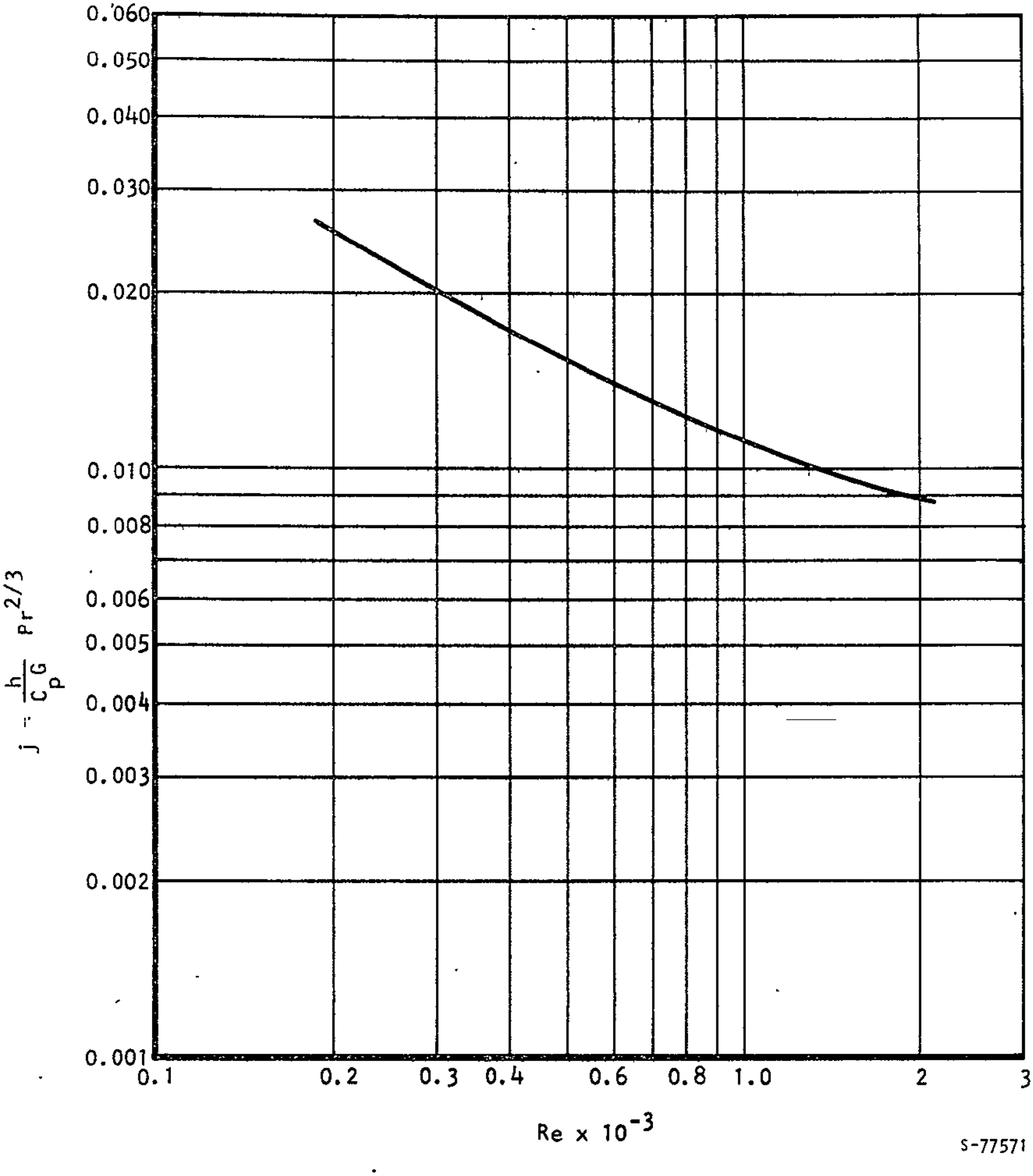


Figure 8-3. Colburn j Factor vs Reynolds Number for Sump Heat Exchanger Heat Transfer Surface

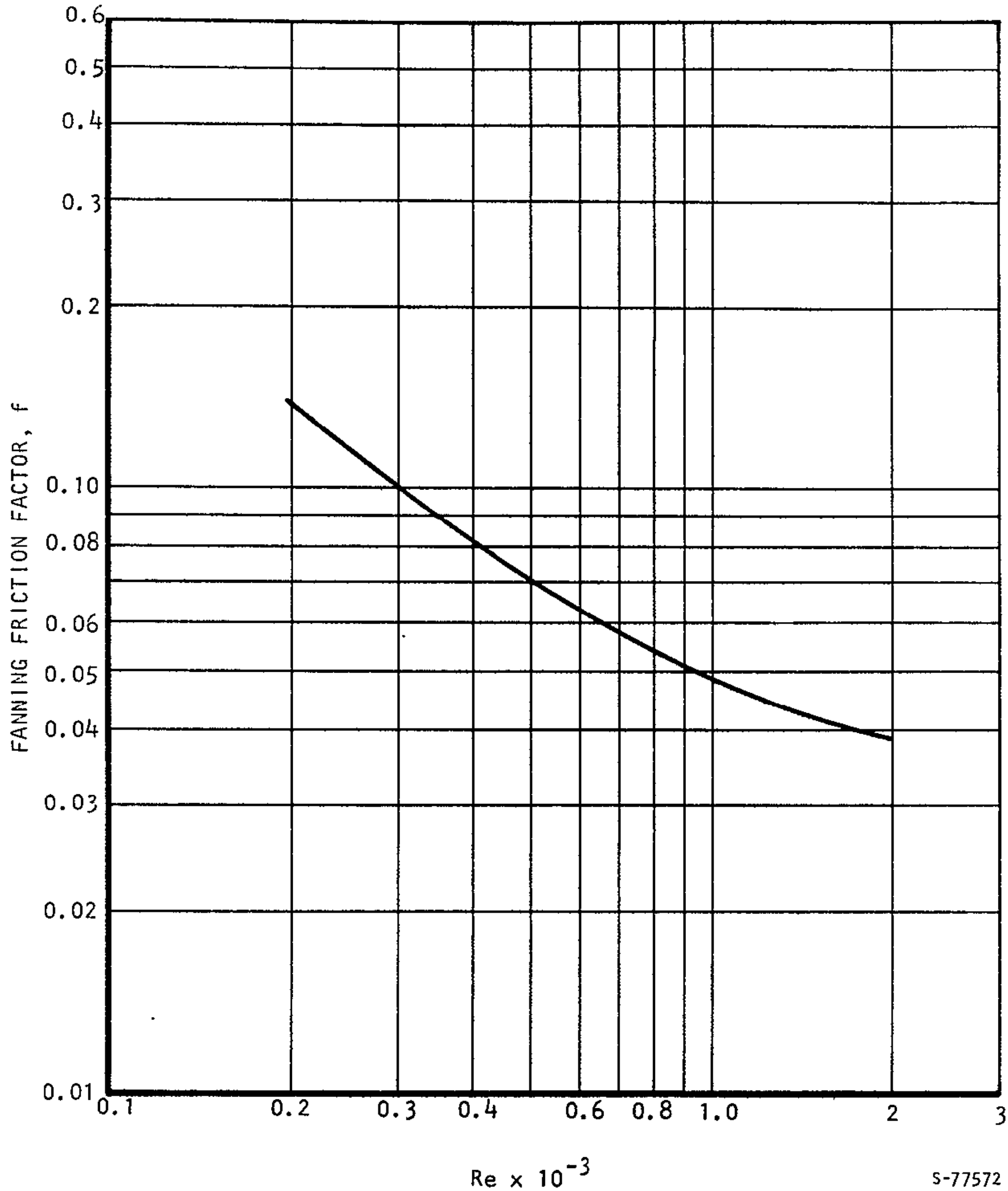


Figure 8-4. Fanning Friction Factor vs Reynolds Number for Sump Heat Exchanger Heat Transfer Surface

PERFORMANCE CHARACTERISTICS

The sump heat exchanger performance characteristics are summarized in Table 8-1. The heat transfer performance is based on the average flow in each section of the heat exchanger. For the pressure drop, maximum flows were assumed. The lengths of the two sections of the heat exchanger have been chosen to produce a heat transfer conductance ratio in proportion to the ratio of flows in the two sections. The film temperature drop at a heat load of 80 watts is 2.96°K (5.33°R).

TABLE 8-1

SUMP HEAT EXCHANGER DESIGN AND PERFORMANCE SUMMARY

Parameter	Section 1	Section 2
Total Area, m^2 (ft^2)	0.01475 (0.1588)	0.00794 (0.0855)
Maximum Flow, kg/sec (lb/sec)	0.00329 (0.00724)	0.001237 (0.00272)
Average Flow, kg/sec (lb/sec)	0.00208 (0.00458)	0.000764 (0.00168)
Fin Effectiveness	0.837	0.884
Fluid Temperature, °K (°R)	344 (620)	344 (620)
Heat Transfer Coefficient, watts/m^2-°K (Btu/ft^2 °R hr)	1489 (262)	991 (174.2)
Conductance (ηhA), watts/°K (watts/°R)	19.71 (10.95)	7.30 (4.06)
Pressure Drop, N/m^2 (psi)	429 (0.0623)	52.7 (0.00765)
	Total for Sections 1 and 2	
Conductance (ηha), watts/°K (watts/°R)	27.01 (15.01)	

Sump Heat Exchanger

8/1/72

OD of fin is 2.34 in.

if we keep .030 hi fin, ID = 2.34 − .06 = 2.28 in.

$D_{mean} = 2.34 - .03 = 2.31$ in

$A_{Fr} = 2.31\pi \times .03/144 = .001512\ ft^2$

for 20 fpi, .030 hi, .004 thick

$A_c/A_{Fr} = \dfrac{(b-t)(1-nt)}{b} = \dfrac{(.03-.004)(1-20\times.004)}{.03} = \dfrac{.026\times.92}{.03} = .797$

$r_h = \dfrac{(b-t)(1-nt)}{2(1+nb-2nt)} = \dfrac{.02395}{2(1+.6-.16)} = \dfrac{.02395}{2\times1.44} = .0083\text{ in} = .000692\ ft$

$A_T/L_W = 2n(\cancel{y-t}) + 1.5n(\frac{1}{n}-t) = 40(.026) + 30\times.046 = 1.042 + 1.38 = 2.422$

$A_F/A_T = \dfrac{2n(y-t) + .5n(\frac{1}{n}-t)}{A_T/L_W} = \dfrac{1.042+.46}{2.422} = \dfrac{1.502}{2.422} = .62$

$l_e = b + .5(\frac{1}{n}-t) = .03 + .5\times.046 = .053\text{ in} = .00442\ ft$

in hot portion, max flow = .00724 lb/sec, average = .00458

in cold portion, max flow = .00272 lb/sec, average = .00168 lb/sec

$A_c = .797\times.001512 = .001208\ ft^2$

calculate heat transfer first

mass velocity, G

hot

$$G = \frac{.00458 \times 3600}{.001208} = 13{,}700 \text{ lb/ft}^2\text{-hr}$$

cold

$$G = \frac{.00168 \times 3600}{.001208} = 5010$$

$$Re = \frac{G D_h}{\mu} = \frac{4 G r_h}{\mu}$$

physical properties

$\mu = .0521$ lb/ft-hr

$\rho = .5808$ lb$_m$/ft^3 @ 1000 psi

$C_p = 1.243$ BTU/lb$_m$-°R

$P_r = 0.67$

$\alpha = P_r^{2/3}/C_p = 0.624$

hoT

$$Re = \frac{4 \times 13{,}700 \times .000692}{.0521} = 728$$

cold

$$Re = \frac{4 \times 5010 \times .000692}{.0521} = 266$$

for this fin

hot $j = .0128$

cold $j = .0217$

$h = j\,G/\alpha$

hot $h = .0128 \times 13{,}700/.624 = 262\ BTU/ft^2\text{-}hr\text{-}°F$

cold $h = .0217 \times 5010/.624 = 174.2\ BTU/ft^2\text{-}hr\text{-}°F$

now calculate fin efficiency

$\eta_f = \dfrac{\tanh m l_e}{m l_e} \qquad m = \sqrt{\dfrac{2h}{kt}}$

hot $m l_e = .00442\sqrt{\dfrac{2 \times 262 \times 12}{50 \times .004}} = .00442\sqrt{31{,}400} = .782$

cold $m l_e = .00442\sqrt{\dfrac{2 \times 174.2 \times 12}{50 \times .004}} = .00442\sqrt{20{,}900} = .639$

hot $\eta_f = .837$ cold $\eta_f = .884$

overall surface efficiency

$\eta_o = 1 - \dfrac{A_F}{A_T}(1 - \eta_f)$

hot $\eta_o = 1 - .62(1 - .837) = .899$; cold $\eta_o = 1 - .62(1 - .884) = .928$

total area

width of passage $= w = \pi D_{mean} = 2.31\pi = 7.26$ in.

hot flow length is 1.2 inches.

thus $A_T = 7.26 \times 1.2 \times 2.422 = 21.1 \text{ in}^2 = 0.1467 \text{ ft}^2$

cold section flow length is 0.8 in,

$$A_T = \frac{0.8}{1.2} \, 0.1467 = 0.0977 \text{ ft}^2$$

hot $\eta_o hA = .899 \times 262 \times .1467 = 34.5$ BTU/hr-°F

cold $\eta_o hA = .928 \times 174.2 \times .0977 = 15.82$ BTU/hr-°F

$$\eta h A_{TOT} = 50.32 \text{ BTU/hr-°F}$$

$Q = 80$ watts $= 273$ BTU/hr

$$\Delta T = \frac{Q}{\eta h A} = \frac{273}{50.32} = 5.43\ °F$$

this is an acceptable value

now check and see if the proper amount of surface has been allocated to the hot and cold ends. In order to provide an equal film temperature drop, the ηhA in each section should be proportional to the percentage of flow in that section. This desired split will not be obtained exactly, since the fin length is determined by integral offsets; thus the fin length is incremented by 1/10 of an inch at a time.

in the hot section, $\frac{\eta hA}{(\eta_o hA)_{Total}} = \frac{34.5}{50.32} = 0.686$

$$\frac{\text{hot average flow}}{\text{total average flow}} = \frac{.00458}{.00458+.00168} = \frac{.00458}{.00626} = 0.73$$

if we keep the total flow length of the heat exchanger constant at 2 inches (set by mechanical design), we will add one offset (0.1 in) to the hot flow length and subtract a like amount from the cold.

thus the new hot $\eta hA = \frac{1.3}{1.2} \times 34.5 = 37.4$ BTU/hr-°F

cold $\eta hA = \frac{0.7}{0.8} 15.82 = 13.83$ BTU/hr-°F new total $\eta_o hA = 51.23$

$$\frac{(\eta_o hA)_{hot}}{(\eta_o hA)_{Total}} = \frac{37.4}{\cancel{15.8}\ 51.23} = 0.729$$

this gives a better match to the flows.

film ΔT is now $\frac{273}{51.23} = 5.33\ °F$

now look at pressure drop.

the hot end maximum flow occurs at a pressure of 952 psia. thus $\rho = .554\ lb_m/ft^3$

first obtain velocity head

$$H_v = \frac{G^2}{2g_c\rho} \quad , \quad G_{hot} = \frac{.00724}{.001208} = 6.00\ lb_m/ft^2\text{-}sec$$

$$H_v = \frac{6.0^2\ \cancel{lb_m^2}\text{-}lb_f\text{-}\cancel{sec^2}\text{-}\cancel{ft^3}\ \cdot\ \cancel{ft^2}}{\cancel{ft^4}\ \cancel{sec^2} \times 2 \times 32.2\ \cancel{lb_m}\text{-}\cancel{ft} \times .554\ \cancel{lb_m} \times 144\ in^2} = .00702\ lb_f/in^2$$

Reynolds number

$$Re = \frac{6 \times 3600 \times 4 \times .000692}{.0521} = 1148$$

$f = .047$

$$\frac{fL}{r_h} = \frac{.047 \times 1.3}{.0083} = 7.36$$

in addition to friction loss, we will lose an additional 1.5 velocity heads in shock losses at the entrance and exit.

thus total pressure loss $= \left(\frac{fL}{r_h} + 1.5\right) h_v$

$$\Delta P = (7.36 + 1.5)\ .00702 = 8.86 \times .00702 = .0623\ psi$$

cold end ΔP

maximum flow $= .00272$ at 993 psi, $\rho = .577$ lb/ft^3

$$G = \frac{.00272}{.001208} = 2.25\ \text{lb/ft}^2\text{-sec}$$

$$H_v = \frac{2.25^2}{2 \times 32.2 \times .577 \times 144} = .000946\ \text{lb/in.}^2$$

$$R_e = \frac{2.25 \times 3600 \times 4 \times .000692}{.0521} = 431$$

$$f = .078$$

$$\frac{fL}{r_h} = \frac{.078 \times .7}{.0083} = 6.58$$

$$\frac{fL}{d_h} + 1.5 = 6.58 + 1.5 = 8.08$$

$$\Delta P = 8.08\ H_V = 8.08 \times .000946 = .00765\ psi$$

SECTION 9

COLD END INSULATION

SECTION 9

COLD END INSULATION

INTRODUCTION

The cold end insulation functions to limit the heat transferred from the ambient atmosphere to the cryogenic cold end of the VM refrigerator. The insulation system analysis is somewhat unique as compared to the more common problem of insulating a constant temperature cryogenic heat sink from the constant temperature ambient heat source. The cold finger of the VM is composed of two regions: (1) a constant temperature portion comprised of the cold end heat exchanger and the refrigeration load mounting surface and (2) the cold regenerator outer wall, which has a linear temperature gradient imposed on it. This linear gradient varies from the cold end temperature to the sump temperature, which is above ambient temperature.

In discussions between NASA/GSFC, AiResearch, and Honeywell Radiation Center (HRC) personnel, further characteristics of the cold end insulation scheme were agreed upon. Since the VM refrigerator is a flight prototype machine, it is desirable to be able to evacuate the cold end vacuum enclosure and seal it off without the requirement for further pumping at a later date. This requirement thus dictated not only a leak tight enclosure, but also the use of materials which do not outgas with time under hard vacuum. Thus the use of the cryogenic superinsulations (aluminized mylar) was originally precluded, along with any other organic shield supported by fiberglass pads.

A detailed analysis of the cold end heat leak with discrete metallic radiation shields was performed during the preliminary design effort (Reference 1). An acceptable design was evolved which utilized two rhodium plated aluminum radiation shields. However, later studies showed that the shields would have to be shortened in order to install the HRC provided lead ring matrix. A major percentage of the cold end was thus exposed to direct radiation from the ambient temperature outer cover. The resultant heat leak would be high; thus the use of aluminized mylar insulation was agreed upon by NASA, AiResearch, and HRC. This change represents one of the major differences between the preliminary design refrigerator and the final VM design.

DESCRIPTION AND PERFORMANCE

The entire outer diameter of the cold end of the VM is wrapped with NRC-2 aluminized mylar superinsulation. The material will be wrapped at approximately 60 layers per inch, and will fill the evacuated annulus. The method of installation and the material utilized are virtually identical to that of the GSFC 5 watt VM.

The heat leak was calculated with the conservative assumption that the entire cold finger of the machine, including the regenerator, is at the cold end temperature. With this assumption, the calculated heat leak is 60 milliwatts, which represents a 13 milliwatt increase over that predicted for the discrete radiation shield approach of the preliminary design unit.

Cold End Insulation

With the use of multilayer insulation in the cold end, the determination of the heat leak becomes a conduction calculation. The heat leak with two radiation shields was determined by use of the thermal analyzer computer program for the Task I effort, and is documented in the Task I final report, Reference 1. Since the insulation scheme has been changed to aluminized mylar for interfacing reasons, the discrete radiation shield analysis is not repeated here.

The heat leak to the circular end of the cold finger is accounted for in the Honeywell contributed heat load. Therefore the insulation calculations reduce to conduction ~~betwe~~ through a thick wall cylinder. The thermal resistance of the metallic walls is neglected.

The worst case assumption, the entire cold finger including the regenerator at the cold end temperature of 112°R, will be made, as was done for the 5 watt VM. Between the temperature limits of 112°R and 530°R, AiResearch Tests indicate that the product of thermal conductivity and temperature difference, KΔT, for NRC-2 is 0.051 BTU/ft-hr.

The heat leak is then expressed as

$$Q = \frac{2\pi L \, K\Delta T}{\ln r_o/r_i}$$

where L = Length of Cold Finger
r_o = radius of outer vacuum jacket
r_i = radius of pressure shell
$K\Delta T$ = 0.051 BTU/ft-hr for NRC-2 and the temperature limits of interest

$r_o = 1.135$ in., $r_i = 0.54$ in., $L = 5.75$ in. $= 0.479$ ft.

$$Q = \frac{2\pi \times 0.479\,\cancel{ft.} \times 0.051\ BTU}{\ln 1.135/0.54 \quad hr\text{-}\cancel{ft.}} = 0.2067\ BTU/hr.$$

$= 0.0606$ watts

we will use 60 milliwatts

SECTION 10

HOT END INSULATION

SECTION 10

HOT END INSULATION

INTRODUCTION

The hot end insulation limits the heat transferred from the heater, the hot end pressure dome, and the hot regenerator outer wall to the ambient atmosphere surrounding the VM refrigerator. It is essential to limit this hot end loss to a minimum practical value, as heat loss in this manner is a direct loss of input power.

DESCRIPTION AND PERFORMANCE

A composite insulation system utilizing fiberglass and Min-K has been utilized, similar to that incorporated into the GSFC 5 watt VM. This type of insulation system is attractive for the fractional watt VM, since the heater is bonded directly to the hot end dome. The heater thus operates at essentially the hot end temperature, as compared to that on the 5 watt machine which transfers its energy by radiation. The insulation system has been designed so that the temperature at the interface between the Min-K and the fiberglass is 742°K (1335°R).

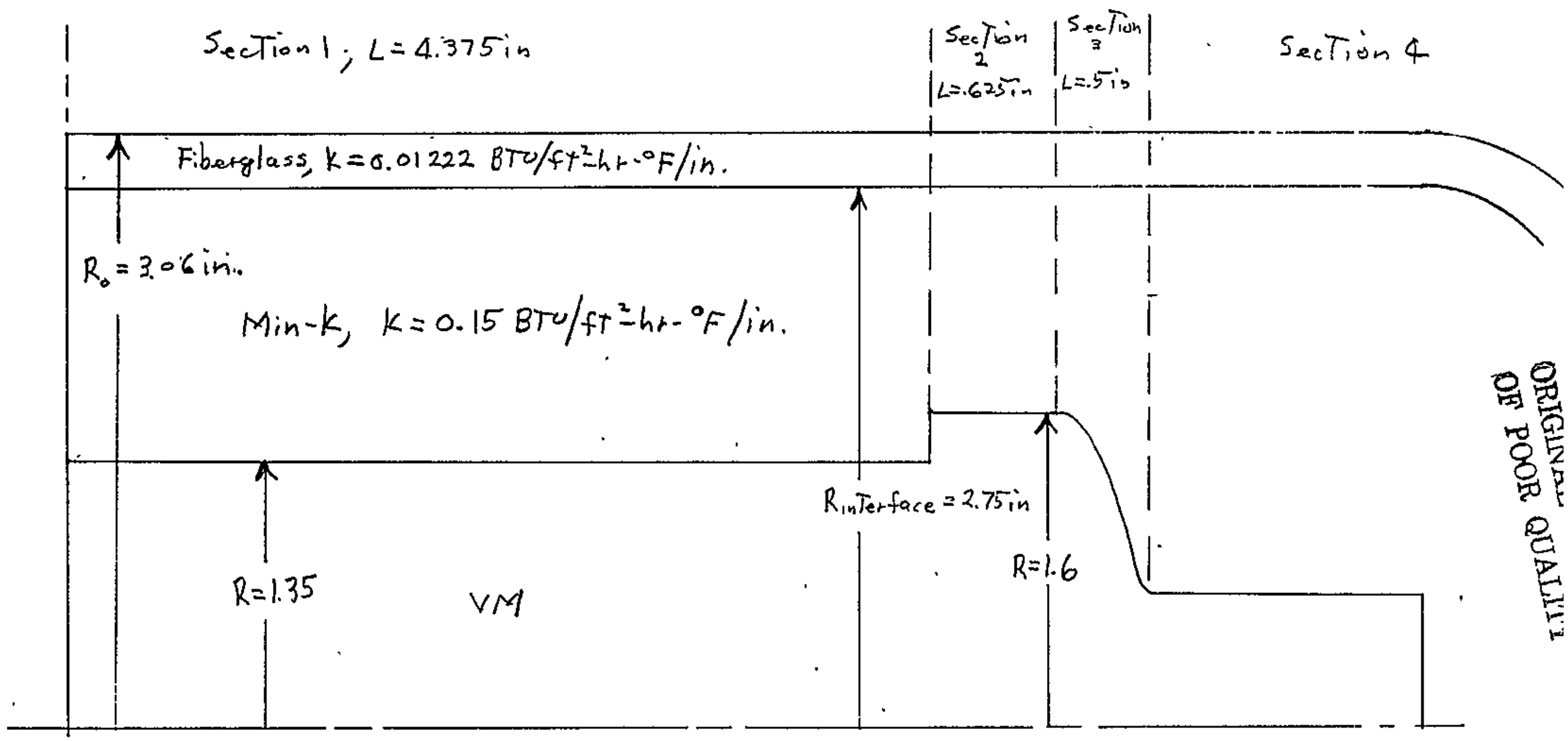
This system is unchanged from the preliminary design configuration with the exception of length, and the calculated heat leak to ambient is 6.1 watts. This is compatible with the overall power input requirements for the fractional watt VM. The insulation system is shown on the outline drawing (Figure 2-1).

Hot End Insulation

The hot End Insulation is a composite of Evacuated Fiberglass and High Temperature Min-K insulation, as with The 6SFC 5 watt VM. The major restriction to heat flow is The fiberglass, with The primary function of The Min-K being The limitation of The fiberglass Temperature to an acceptable level. The Temperature at The interface between The Two materials should be below 900°F.

The insulation heat loss is calculated by considering The hot end in four separate sections, making simplifying and conservative assumptions where necessary, and Then calculating The conduction heat loss of composite Thick wall geometric shapes. A sketch of The model considered is shown on The next Page.

The 5/16" Thick fiberglass is installed at The outer periphery of The insulation system, and sufficient Min-K is installed to cover all contours of the VM and interface with The Fiberglass.



Model For Hot End Heat Leak Analysis

(2)

A. Heat Leak

3.

Section 1.

This is a composite thick wall cylinder and conduction heat flow is defined as

$$Q = U A_o \Delta T$$

$$\text{where } U = \frac{1}{\dfrac{r_o \ln r_i/r}{k_{min\text{-}k}} + \dfrac{r_o \ln r_o/r_i}{k_{fiberglass}}}$$

$$U = \frac{1}{\dfrac{3.06 \text{ in } \ln \frac{2.75}{1.75} \text{ ft}^2\text{-hr-}^\circ\text{F}}{.15 \text{ BTU-in}} + \dfrac{3.06 \text{ in. } \ln \frac{3.06}{2.75} \text{ ft}^2\text{-hr-}^\circ\text{F}}{.01222 \text{ BTU-in}}}$$

$$U = 0.02423 \text{ BTU/ft}^2\text{-hr-}^\circ\text{F}$$

$$A_o = 2\pi r_o L = 2\pi \times 3.06 \times 4.375/144 = 0.584 \text{ ft}^2$$

The temperature gradient along the regenerator is nearly linear, and thus the average temperature may be used. The outer surface will be assumed as 100°F, to allow for natural convection to the surrounding atmosphere.

Thus $T_{hot} = \frac{1100+140}{2} = 620°F$

And $\Delta T = 620-100 = 520°F$

Thus Section 1 $Q = \frac{0.02423\ BTU}{ft^2\text{-}hr\text{-}°F} \times 0.584\ ft^2 \times 520°F$

$= 7.358\ BTU/hr$

$= 2.157$ watts

Section 2

In this portion of the hot end, the radius of the VM is taken as 1.6 in, which is representative of the outer diameter of the coiled heater.

As above, $Q = U A_o \Delta T$

here, $T_{hot} = 1100°F$ and $\Delta T = 1000°F$

$A_o = 2\pi \times 3.06 \times 0.625/144 = 0.0834\ ft^2$

$$U = \frac{1}{\frac{3.06 \ln 2.75/1.6}{.15} + \frac{3.06 \ln 3.06/2.75}{.01222}} = \frac{1}{11.048 + 26.747}$$

$= .02646\ BTU/ft^2\text{-}hr\text{-}°F$

Thus Section 2 $Q = .02646 \times .0834 \times 1000 = 2.207$ BTU/hr.

$= .6468$ watts

Section 3

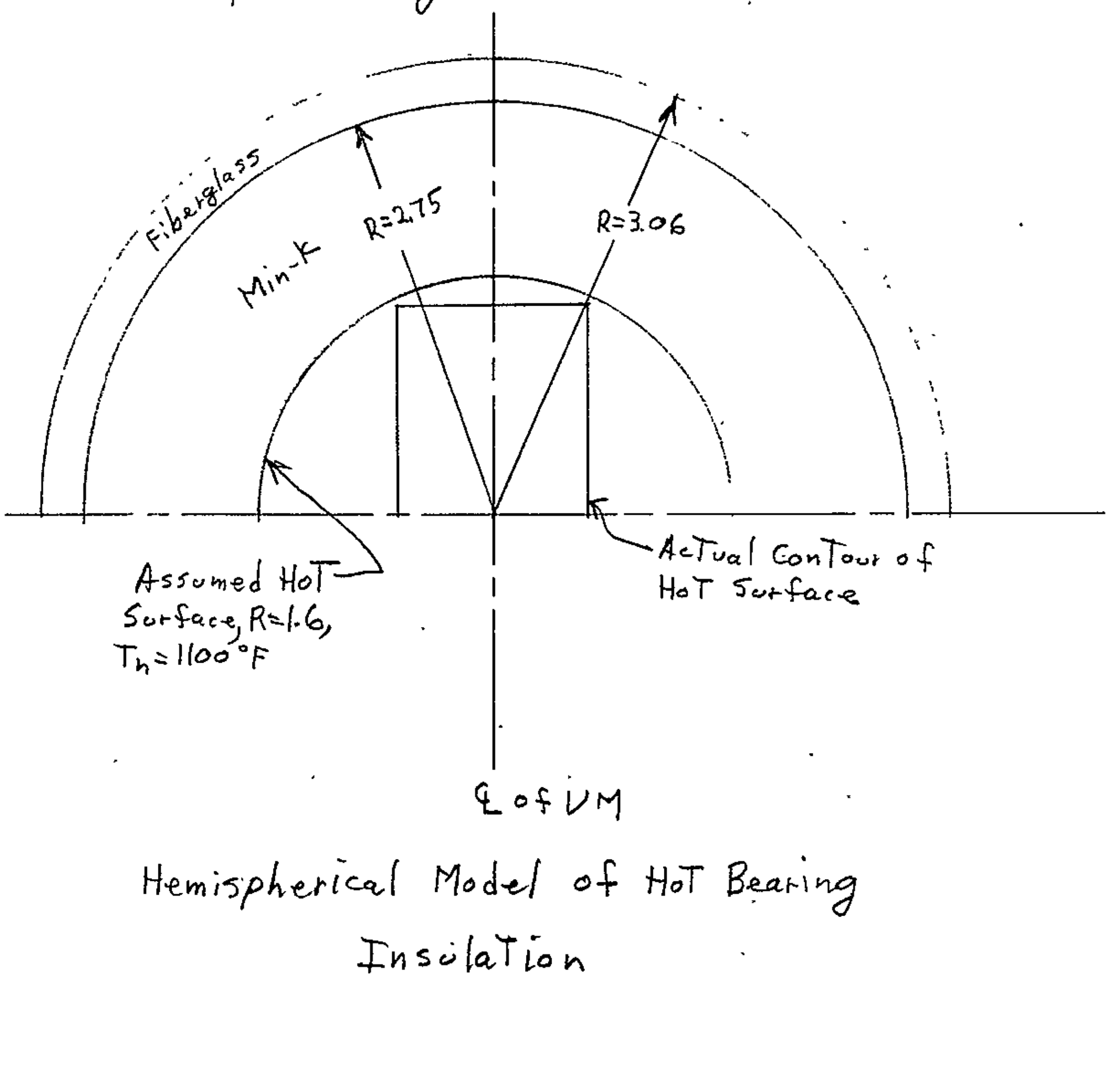
For this portion of the hot end, rather than perform a complicated two dimensional analysis, the assumption will be made that the entire surface is a continuation of section 2, and may be treated as a cylinder. This is a conservative assumption, since the actual amount of min-k between the hot portion of the machine and the fiberglass is greater in all areas than implied by using the cylindrical calculation.

Under this assumption, the heat flow through section 3 may be ratioed directly from that of section 2, by the relative lengths of the sections.

$$Q_3 = \frac{L_3}{L_2} Q_2 = \frac{0.5}{0.625} 0.6468 = 0.5174 \text{ watts}$$

Section 4

This section will also be Treated as a simple geometrical shape. We will assume a hollow hemisphere, as shown below, which is a continuation of the cylindrical geometry of the preceeding sections



Hemispherical Model of Hot Bearing Insolation

For The hollow composite hemisphere, the series resistance concept will be used:

$$Q = \frac{\Delta T}{\Sigma R}$$

where The Resistance of each section is defined as $\frac{L}{K A_m}$

per McAdams, Eqn 2-10a, The proper mean area for spherical shapes is $\sqrt{A_1 A_2}$

Thus for the portion of the hemisphere filled with Min-k,

$$R_{min\text{-}k} = \frac{L}{K_{min\text{-}k}\sqrt{A_1 A_2}}$$

Area of hemispherical Surface $= \frac{\pi}{2} D^2$

$A_1 = \frac{\pi}{2}(2\times 1.6)^2/144 = 0.1117\ ft.^2$

$A_2 = \frac{\pi}{2}(2\times 2.75)^2/144 = 0.330\ ft.^2$

$L = 2.75 - 1.6 = 1.15\ in.$

$$R = \frac{1.15\ in.\ ft^2\text{-}hr\text{-}^\circ F}{.15\ BTU\text{-}in\sqrt{.33\times .1117\ ft^4}} = 39.932\ \frac{hr\text{-}^\circ F}{BTU}$$

For The Fiberglass Section

$A_1 = 0.33$ ft^2 from preceeding section

$A_2 = \frac{\pi}{2}(2 \times 3.06)^2/144 = 0.4086$ ft.^{2}

$L = 3.06 - 2.75 = 0.31$ in.

$$R = \frac{0.31}{.01222\sqrt{.33 \times .4086}} = 69.085 \frac{hr\text{-}°F}{BTU}$$

Now $\Sigma R = 39.932 + 69.085 = 109.017$ hr-°F/BTU

$$Q = \frac{\Delta T}{\Sigma R} = \frac{1000°F \ BTU}{109.017 \ hr\text{-}°F} = 9.173 \ BTU/hr$$

$$= 2.689 \text{ watts}$$

The Total heat loss for the insulation System is summarized below.

Section	Heat loss, Watts
1	2.157
2	0.647
3	0.517
4	2.689
Total	6.01 we will quote 6.1

B. Interface Temperature

In section 2, the min-K thickness is a minimum, and this will define interface temperature. The maximum hot end temperature for the VM is 1200°F, and we will use this for determination of interface temperature.

From Section 2 heat leak calculations,

$$U = \frac{1}{R_{min\text{-}k} + R_{fiberglass}} = \frac{1}{11.048 + 26.747}$$

Thus $R_{min\text{-}k} = 11.048$ and $R_{Total} = 11.048 + 26.744 = 37.795$

The Min-k represents $\frac{11.048}{37.795} = 0.2923$ of the Total Resistance. Thus 29.23% of the Total Temperature drop occurs across the Min-k. Total $\Delta T = 1200 - 100 \simeq 1100°F$

$$\Delta T_{min\text{-}k} = .2923 \times 1100 = 321.5°F$$

(10)

The interface temperature between the Min-K and the fiberglass is thus

$T_I = 1200 - 321.5 = 878.5\,^\circ F$. This is an acceptable value, and we will call it $875^\circ F = 1335\,^\circ R$

SECTION 11

FLOW DISTRIBUTORS

SECTION 11

FLOW DISTRIBUTORS

INTRODUCTION

The prime importance of obtaining uniform flow distribution through all heat transfer devices, both heat exchangers and regenerators, has been repeatedly stressed throughout this report. Test results from the AiResearch IR&D VM refrigerator revealed non-uniform flow in the cold end of the refrigerator. The configuration of the cold end of the AiResearch VM refrigerator (cold end heat exchanger, displacer, and cold regenerator) is very similar to that of the GSFC VM refrigerator. Initial tests on the AiResearch refrigerator indicated an unbalance in flow in the cold end heat exchanger and low temperature end of the cold regenerator. To overcome this problem, a flow distributor was designed and installed in this refrigerator. As a result of the successful testing of this flow distributor, the same basic design was incorporated into the GSFC 5 watt VM refrigerator. Similar devices are employed in the fractional watt machine, one at each end of the cold regenerator.

The flow distributors have been optimized for minimum loss of refrigeration by a method developed during the preliminary design effort. The optimization procedure will be summarized, and the application to the actual devices is then described.

OPTIMIZATION PROCEDURES

A flow distributor optimization procedure was developed utilizing the tradeoff factors determined during the Task I effort. Factors of prime importance affecting flow distributor design are the ratio of axial to circumferential pressure drop, the total pressure drop, and the void volume contribution to the machine. These factors are interrelated in such a manner that the combined effect on refrigeration is determined. This expression is then differentiated with respect to the axial dimension and set equal to zero. In this manner flow distributor dimensions are determined such that minimum loss of refrigeration occurs.

In general, the dimensions of a flow distributor, such as that at the end of a regenerator, are fixed by the geometry of the VM, with the exception of the axial length. A sketch of a typical flow distributor (Figure 11-1) illustrates this.

Thus the length, X, may be varied in order to obtain optimum performance. When the expression for overall refrigeration loss is differentiated, set equal to zero, and solved for length, Equation 11-1 results. The reader is referred to Reference 1 for details of the derivation.

$$X = \left[\frac{2C_1 k_1}{C_2 k_2}\right]^{1/3} \qquad (11\text{-}1)$$

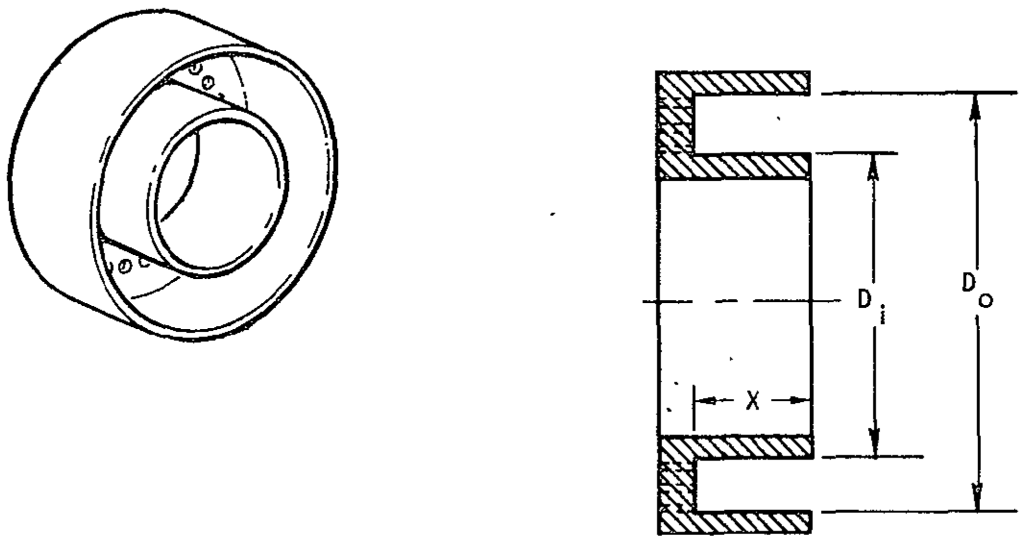


Figure 11-1. Typical Flow Distributor

S-73895

where

C_1 = pressure drop trade factor, watts refrigeration lost/psi

C_2 = void volume trade factor, watts refrigeration lost/in.^{3}

$$K_1 = \frac{K_c W^2 (N+1)}{2 g_c \rho \; (D_o - D_i)^2} \text{ , } lb_f$$

K_c = the total loss coefficient in the circumferential flow direction
W = appropriate mass flow rate
N = ratio of axial to circumferential pressure drop
g_c = gravitational constant
ρ = gas density
D_o, D_i = dimensions defined by Figure 11-1

$$K_2 = \frac{\pi}{4} (D_o{}^2 - D_i{}^2) \text{ , } in^2$$

The pressure drop and void volume trade factors, C_1 and C_2, were developed during the Task I effort by use of the VM ideal cycle computer program. Pressure drop subtracts directly from the pressure variations in the cold end, and thus the net refrigeration. Since volume variation must occur with the pressure variations in order to produce refrigeration, the void or non-cyclic volume also detracts from refrigeration. The relative magnitude of these effects is of prime importance in trading void volume against pressure drop in any given component. Table 11-1 presents the tradeoff factors utilized in optimization of the various components of the VM. These parameters are specific to the operating conditions and physical characteristics of the fractional watt VM.

As shown in Table 11-1, the void volume trade factors are a function of temperature. This variation is plotted on Figure 11-2. This dependence occurs because the mass of gas contained in the volume is dependent on temperature. Since the working fluid, helium, is not a perfect gas at cryogenic temperatures, the product of compressibility and temperature is more representative of the mass in a given volume. Figure 11-2 also presents the tradeoff factors of Table 11-1 as a function of ZT. A linear relationship is evident. Thus the tradeoff values could be reduced to a single constant for this particular machine, which would then be modified by a temperature-compressibility factor. The use of a plot such as Figure 11-2 is considered more simple when trade factors at various temperatures are desired.

TABLE 11-1

TRADEOFF PARAMETERS FOR FRACTIONAL WATT VM REFRIGERATOR

Void Volume		
Location of Void Volume	Temperature of Void Volume, ^oK (^oR)	Tradeoff Factor, $\Delta Q_c/\Delta V$, watts/m^3 (watts/cu in.)
Cold end	62.3 (112)	1.143×10^5 (1.875)
Cold regenerator	204 (366)	0.380×10^5 (0.623)
Sump	344 (620)	0.232×10^5 (0.380)
Hot regenerator	559 (1077.5)	0.134×10^5 (0.2194)
Hot end	853 (1535)	0.935×10^5 (0.1531)
Pressure Drop Between Sump and Cold End		
Refrigeration Loss = 6.88×10^{-6} watts/N/M^2 (0.04738 watts/psi)		

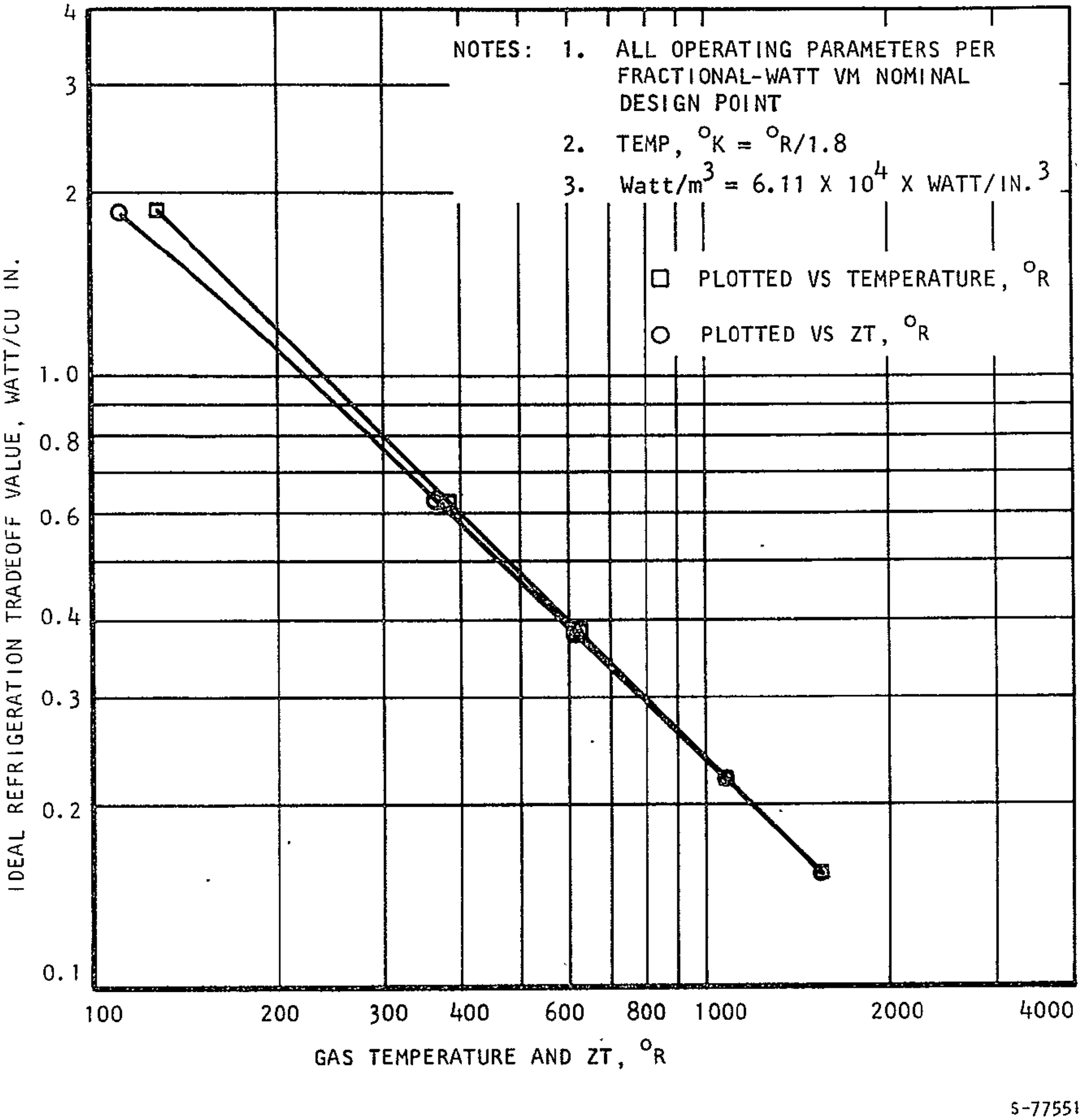


Figure 11-2. Refrigeration Tradeoff Values as a Function of Void Volume Temperature

COLD END FLOW DISTRIBUTOR

The optimization procedure described above was applied to the cold end flow distributor.

The cold regenerator fixes the inner and outer diameter of the annulus into which the distributor is installed at 0.01078 m (0.424 in.) and 0.026 m (1.024 in.) respectively. Thus, allowing 0.000508 m (0.020 in.) thick walls on the distributor, D_i = 0.01178 m (0.464 in.) and D_o = 0.0250 m (0.984 in.) (See Figure 11-1 for nomenclature). From the preceeding section, K_2 = 0.000379 m^2 (0.587 $in.^2$).

The axial to circumferential pressure drop ratio (N) is chosen as 15, and the total loss coefficient in the circumferential direction (K_C) is taken as 5.0. The maximum gas flow rate at the cold end of the cold regenerator is reduced by a factor of 12, since there are 12 flow slots in the heat exchanger. Substitution of these values yields K_1 = 0.00000276 kgf (0.00000608 lbf). From the preceding section, the void volume trade factor at the cold end of the machine (C_2) is 1.143×10^5 watts/m^3 (1.875 watts/$in.^3$) and the pressure drop tradeoff factor (C_1) is 6.88×10^{-6} watts/N/m^2 (0.04738 watts/psi). Substitution of these values into Equation 11-1 yields

$$X = \left[\frac{2\ C_1\ K_1}{C_2\ K_2}\right]^{1/3} = \left[\frac{2 \times 0.04738 \text{ watts} \times 0.00000608 \text{ lbf}}{\text{lbf/in.}^2\ 0.587 \text{ in.}^2 \times 1.875 \text{ watts/in.}^3}\right]^{1/3}$$

$$= 0.000205 \text{ m } (0.00806 \text{ in.})$$

Thus the optimum annular flow length for the cold flow distributor is slightly greater than 0.000203 m (0.008 in.). The total pressure drop for the flow distributor is obtained as 645 N/m^2 (0.0936 psi) and the refrigeration loss due to this pressure drop is 4.43 milliwatts. The associated void volume in $7.69 \times 10^{-8} m^3$ (0.00470 $in.^3$) and the refrigeration loss is 8.81 mw. Thus the total refrigeration loss associated with the optimized flow distributor is approximately 13.2 mw.

The circumferential pressure drop is obtained by dividing the total pressure drop by (N + 1). Thus ΔP_c = 40.3 N/m^2 (0.00585 psi) and the axial holes must be sized to yield a pressure drop of 605 N/m^2 (0.0877 psi). If eighty axial holes are chosen as a reasonable number from manufacturing considerations, and a loss coefficient of 1.5 velocity heads is assumed, the resulting hole size is 0.000269 m (0.0106 in.) diameter. A nominal value of 0.000254 m (0.010 in.) was used to ensure against tolerances that would make the pressure drops too low and thereby allow maldistribution.

COLD REGENERATOR FLOW DISTRIBUTOR

A flow distributor is also utilized at the sump end of the cold regenerator, and performs the same function as the one at the cold end. The sump end distributor acts as a further safeguard against maldistribution, since a distribution slot is employed in the bearing housing. Thus by using two devices, each capable of providing uniform flow, no maldistribution will occur.

The optimization procedure described previously was also applied to the cold regenerator flow distributor. The void volume trade factor at the sump temperature was utilized. Since the detrimental effects of void volume at the sump temperature are not as great as at the cold end, the axial dimension that results from the analysis is greater than that at the cold end. Thus the minimum loss of refrigeration occurs with a larger void volume and lower pressure drop.

The axial dimension, X, for this flow distributor is 0.000457 m (0.018 in.). The axial hole pattern with 80 holes is retained, and the hole diameter is 0.000584 m (0.023 in.). The total (axial plus circumferential) pressure drop is 300.6 N/m^2 (0.0436 psi), and the corresponding refrigeration loss is 2.06 mw. The dead volume is $1.78 \times 10^{-7} m^3$ (0.01087 $in.^3$) which causes a refrigeration loss of 4.13 mw. The total refrigeration loss of the optimized flow distributor at the sump end of the cold regenerator is approximately 6.2 mw.

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Flow Distributors

We would like to provide a flow distributor at each end of the cold regenerator, in order to assure maximum performance. The reasons for providing as near perfect flow distribution as possible have been reiterated many times, and need not be repeated here. The distributor is configured as an annular ring, with the only variable being the axial length, X. The optimum value of X, which minimizes loss of refrigeration caused by void volume and pressure loss, has been shown (Fractional Watt Task I Report) to be:

$$X_{opt} = \left(\frac{2C_1 K_1}{C_2 K_2} \right)^{1/3}$$

$$K_1 = (N+1)\, K_c \frac{\dot{w}^2}{(D_o - D_i)^2} \frac{1}{2g_c\rho}$$

At the cold End

N = Desired ratio of axial to circumferential pressure drop (we will use 15).

K_c = Loss coefficient in the circumferential direction (choose 5 for conservatism).

$\dot{w}$ = Mass flow rate. There are 12 flow slots in the cold Heat Exchanger, so

The appropriate mass flow rate is 1/12 of the total; i.e. 1/12 of the total flow must be distributed around 1/12 of the circumference.

D_o = outer diameter of annular flow passage in flow distributor = 0.9847 in.

D_i = Inner diameter of annular flow passage in flow distributor = 0.4694 in.

g_c = gravitational constant

ρ = Density of fluid

$$\text{Thus } K_1 = (15+1)5 \frac{(.000162)^2 \cancel{lb_m}^2}{(.9847-.4694)^2 in^2 sec^2} \left(\frac{lb_f\text{-}\cancel{sec}^2\ \cancel{ft}^3\ 144\,in^2}{2 \times 32.2\ \cancel{lb_m}\text{-}\cancel{ft} \times 2.911\ \cancel{lb_m},\ ft^2} \right)$$

$$= 6.08 \times 10^{-6}\ lb_f$$

$$K_2 = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}(.9847^2 - .4694^2) = 0.587\ in^2$$

The tradeoff factors were defined by the parametric studies performed with the ideal cycle computer program. At the cold end of the machine, the void volume trade factor is:

$$C_2 = 1.875 \text{ watts/in}^3$$

and the pressure drop tradeoff factor is:

$$C_1 = 0.04738 \text{ watts/lb}_f\text{/in.}^2$$

The optimum value of X is thus:

$$X_{opt} = \left(\frac{2 \times .04738 \text{ watts-in}^2 \times 6.08 \times 10^{-6} \text{ lb}_f \text{ in}}{1.875 \text{ watts lb}_f \times .587 \text{ in}^2} \text{ in}^3 \right)^{1/3} = 0.008059 \text{ in}$$

The pressure drop associated with the flow distributor is K_1/X^2

$$\Delta P = \frac{6.08 \times 10^{-6} \text{ lb}_f}{(.00806 \text{ in})^2} = .0936 \text{ psi}$$

The circumferential pressure drop is obtained by dividing the total pressure drop by N+1, thus

$$\Delta P_{cir} = .0936/16 = .00585 \text{ psi}$$

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The axial pressure drop that must be provided by the holes is .0936-.00585 = .08775 psi

The refrigeration loss associated with the pressure loss in the cold-end flow distributor is $C_1 \Delta P = .04738 \frac{\text{watts}}{\text{psi}} \times .0936 \text{ psi} = .00443 \text{ watts}$

The void volume is $K_2 X = .587 \text{in}^2 \times .008 \text{in} = .0047 \text{in}^3$ and the refrigeration loss due to void volume is $C_2 V = 1.875 \frac{\text{watts}}{\text{in}^3} \times .0047 \text{in}^3 = .00881 \text{ watts}$

Thus total loss of refrigeration is the sum of the two losses

$$Q_{lost} = .00443 + .00881 = .01324 \text{ watts} = 13.24 \text{ milliwatts}$$

Now we need to size the axial holes to provide the required pressure drop. Eighty holes appears to be a reasonable number from a layout, using an involute pattern. 1.5 velocity heads will be lost.

Thus velocity head, $q_1 = \Delta P/1.5 = .0877/1.5 = .0585 \text{psi}$

Velocity head is defined as $q = \left(\frac{\dot{w}}{A}\right)^2 / 2g\rho$

Thus $$A_{req'd} = \left[\frac{\dot{w}^2}{2q\, g_c \rho}\right]^{1/2}$$

$$A = \left[\frac{.001943^2 \, lb_m^2 \, in^2 \, \cancel{lb_f\text{-}sec^2} \, ft^3 \quad ft^2}{sec^2 \; .0585 \, lb_f \times 2 \times 32.2 \, lb_m\text{-}ft \times 2.911 \, lb_m \; 144 \, in^2}\right]^{1/2} = 4.889 \times 10^{-5} \, ft^2$$

$$A = \frac{4.889 \times 10^{-5} \, ft^2 \times 144 \, in^2}{80 \text{ holes } ft^2} = 8.801 \times 10^{-5} \, in^2/hole.$$

$$Dia = \left[\frac{4}{\pi} A\right]^{1/2} = \left[\frac{4}{\pi} 8.801 \times 10^{-5}\right]^{1/2} = 0.0106 \text{ in.}$$

We will use 0.01 Dia. holes.

(i)

We will now apply The same procedure To The flow distributor at The warm end of The cold regenerator.

K_2 is The same as at The cold end, $0.587\ in^2$

The flow rate and density are different and K_1 must Therefore be reevaluated. Here, there are 24 slots leading To The distributor, so divide flow by 24.

$$K_1 = (15+1)5\ \frac{(.0001133)^2\ \cancel{lb_m}^2}{(.9847-.4694)^2\ \cancel{in^2}\ sec^2}\left(\frac{lb_f\ sec^2\ ft^3\ 144\ in^2}{2 \times 32.2\ \cancel{lb_m}\ ft \times 0.577\ lb_m\ ft^2}\right)$$

$$= 1.499 \times 10^{-5}\ lb_f$$

The pressure drop Trade factor is identical at This end, $C_1 = .04738$ watts/psi.

but C_2 is different in This Temperature zone

$$C_2 = 0.38\ watts/in.^3$$

$$\text{Thus } X_{opt} = \left[\frac{2 \times .04738\ watts - in^2\ in^3 \times 1.499 \times 10^{-5}\ lb_f}{0.38\ watts\ \ lb_f \times 0.587\ in^2}\right]^{1/3}$$

$$= 0.0185\ in.$$

$\Delta P = K_1/X^2 = \frac{1.499 \times 10^{-5} lb_f}{.0185^2 in^2} = 0.0438 psi$

circumferential $\Delta P = .0438/16 = 0.002737 psi$

and axial $\Delta P = .0438 - .002737 = 0.04106$

Refrigeration lost from pressure drop $= C_1 \Delta P = .0438 \times .04738$

$\doteq .00207$ watts

Void Volume $= K_2 X = .0185 \times .587 = .01086 \; in.^3$

and Refrigeration lost from Void Volume $= C_2 V = .38 \times .01086$

$= .00413$ watts

Thus Total refrigeration lost $= .00207 + .00413 = .0062$ watts

$= 6.2$ milliwatts

now we need to size the axial holes. A pattern of 80 holes will be retained, and the loss coefficient is 1.5

Thus Velocity head $= \Delta P_{axial}/1.5 = .04106/1.5 = .02737$ psi.

$$A_{req'd} = \left[\frac{\dot{w}^2}{2q\,g_c\,\rho}\right]^{1/2} = \left[\frac{.00272^2\ \cancel{lb_m}^2\ \cancel{in^2}\ \cancel{lb_f}\text{-}\cancel{sec^2}\ ft^3\ \cancel{ft}}{\cancel{sec^2}\ 2\times .02737\ \cancel{lb_f}\times 32.2\ \cancel{lb_m}\text{-}\cancel{ft}\times .577\ \cancel{lb_m}\times 144\ \cancel{in^2}}\right]^{1/2}$$

$$= 2.248\ ft^2 \times 10^{-4}$$

$$A/hole = \frac{2.248\times 10^{-4}\ ft^2 \times 144\ in^2}{80\ holes\quad ft^2} = 4.0464\ in^2/hole \times 10^{-4}$$

$$Dia = \left[\frac{4}{\pi}\ 4.0464\times 10^{-4}\ in^2\right]^{1/2} = 0.0227\ in. \qquad \text{use } .023$$

SECTION 12

FLOW PASSAGE PRESSURE DROP, VOID VOLUME, AND FLOW DISTRIBUTION

SECTION 12

FLOW PASSAGE PRESSURE DROP, VOID VOLUME, AND FLOW DISTRIBUTION

INTRODUCTION

In the sump region of the refrigerator, three porting systems are provided which pneumatically connect (1) the cold regenerator to the sump heat exchanger, (2) the hot regenerator to the sump heat exchanger, and (3) the sump heat exchanger, and (3) the sump heat exchanger to the active sump volumes. Pressure drop and void volumes are important considerations in each of these systems of passages, with different emphasis on each in the various areas.

METHOD OF ANALYSIS

Cold Regenerator to Sump Heat Exchanger

This system of passages consists of axial slots in the outer diameter of the cold-end bearing support and a controlled gap between the sump filler block and the pressure vessel housing. The bearing housing, shown schematically in Figure 12-1, includes a flow distribution slot in order to assure uniform flow to the cold regenerator. The sump filler block-pressure vessel arrangement is shown schematically in Figure 12-2. A uniform gap is provided between the filler block and the housing in the cylindrical portion. In the hemispherical portion, the gap varies inversely with the diameter such that a constant flow area is provided.

The bearing support slots were analyzed using a graphical optimization method. Some basic assumptions were first made concerning the relative pressure drops of the axial slots and the distribution slot. It was then possible to calculate the total dead volume and pressure drop for a given axial slot width. Making use of the pressure drop and void volume tradeoff factors, the refrigeration loss is calculated as a function of slot width. When the results are plotted over a range of slot width, a minimum loss of refrigeration is obtained. Thus, although an exact analytical expression is not obtained, the bearing support flow slots and distribution slot are optimized with respect to minimum refrigeration loss.

A similar procedure is utilized for the sump filler block. The pressure drop and void volume are calculated over a range of gaps between the filler block and housing. Using the appropriate trade factors, a total loss of refrigeration is calculated for each gap. When the results are plotted, a minimum refrigeration loss is clearly evident.

Hot Regenerator to Sump Heat Exchanger

The approach taken here was similar to that utilized for the GSFC 5 watt VM refrigerator (Reference 4), and differs from the cold end analysis. Pressure drop is of prime importance in any of the passages that connect the hot and

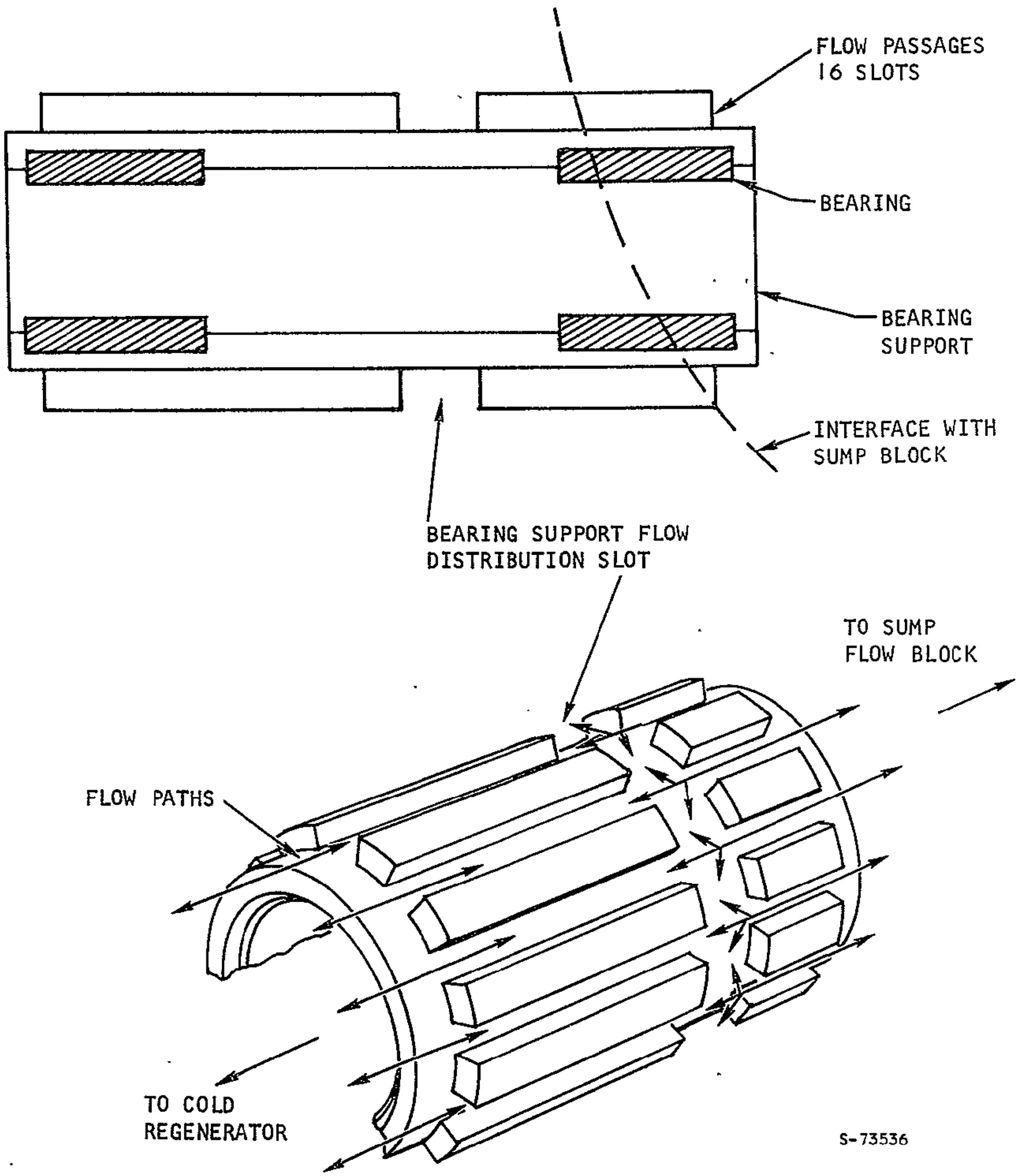

Figure 12-1. Bearing Support Flow Passages

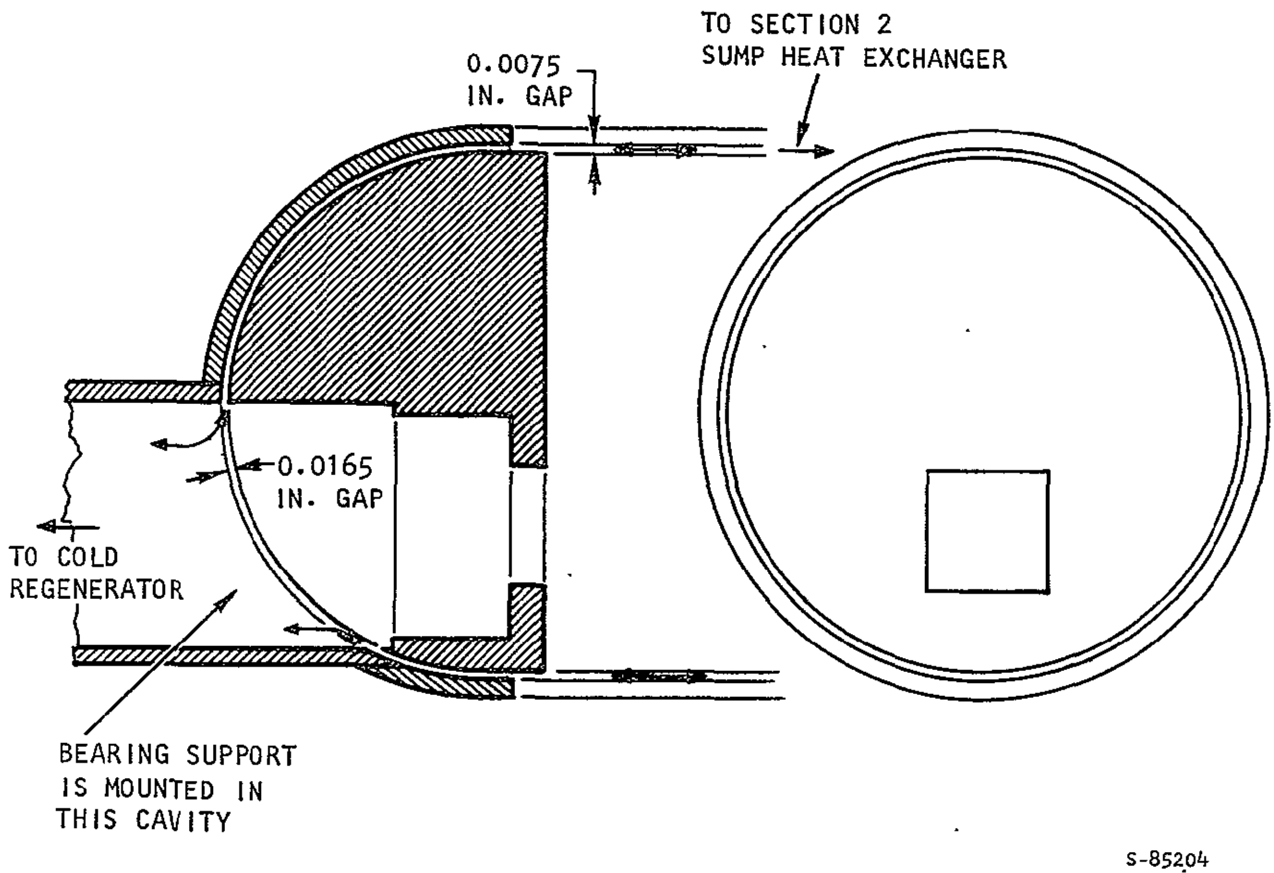

Figure 12-2. Flow Path Around Sump Filler Block

ambient ends of the hot displacer because of the large effect on motor power. Thus, in the region between the hot regenerator and sump heat exchanger, a very low pressure drop was provided, consistent with flow distribution and void volume considerations.

Sump Heat Exchanger to Active Sump Volumes

The flow passages in this region are shown schematically on Figure 8-1. Pressure drop and flow distribution in the sump heat exchanger are the items of primary importance in this region. Void volume is of secondary importance, since the primary concern is the effect of pressure drop on motor power.

The distribution slot between two sections of the sump heat exchanger was sized using the analysis developed for the GSFC 5 watt refrigerator. Since the pressure drop of the sump heat exchanger is a fixed value, the distribution slot is sized to provide a pressure drop of approximately one tenth that of the sump heat exchanger.

The ports from the distribution slot to the ambient end of the hot displacer are also sized to provide a very low pressure drop. Again, motor power is of primary importance.

The ports from the sump heat exchanger flow distribution slot to the crankcase region are sized to provide a pressure drop equivalent to that of the hot displacer ports. This approach was taken to provide equal pressures on the wrist pin retainer and sump end of the hot displacer. Thus, the overall pressure drop across the hot displacer also acts over equal areas. The clearance between the cold displacer wrist pin housing and the bearing retainer was sized using the graphical optimization procedure, because pressure drop across the cold displacer does not drastically affect motor power.

ANALYSIS AND DESIGN CONFIGURATION

Cold Regenerator to Sump Heat Exchanger

The analysis in this region consists of two parts, the ambient bearing support for the cold displacer and the sump filler block. Each will be discussed separately.

The physical dimensions of the refrigerator dictate certain characteristics of the bearing support. The outer diameter is fixed, as is the maximum practical slot depth. The number of slots chosen is 16; a practical value due to manufacturing considerations. The assumptions concerning the distribution slot are the same as those of the GSFC 5 watt VM; i.e., one sixth of the flow must be distributed around one half of the circumference with a pressure drop equal to 1/20 of the axial value.

The first step in the analysis is to calculate distribution slot pressure drop over a range of slot widths. The pressure drop was calculated using the assumptions listed above, and the results are presented in Figure 12-3. This data is used in the following manner; for each axial slot width investigated, the

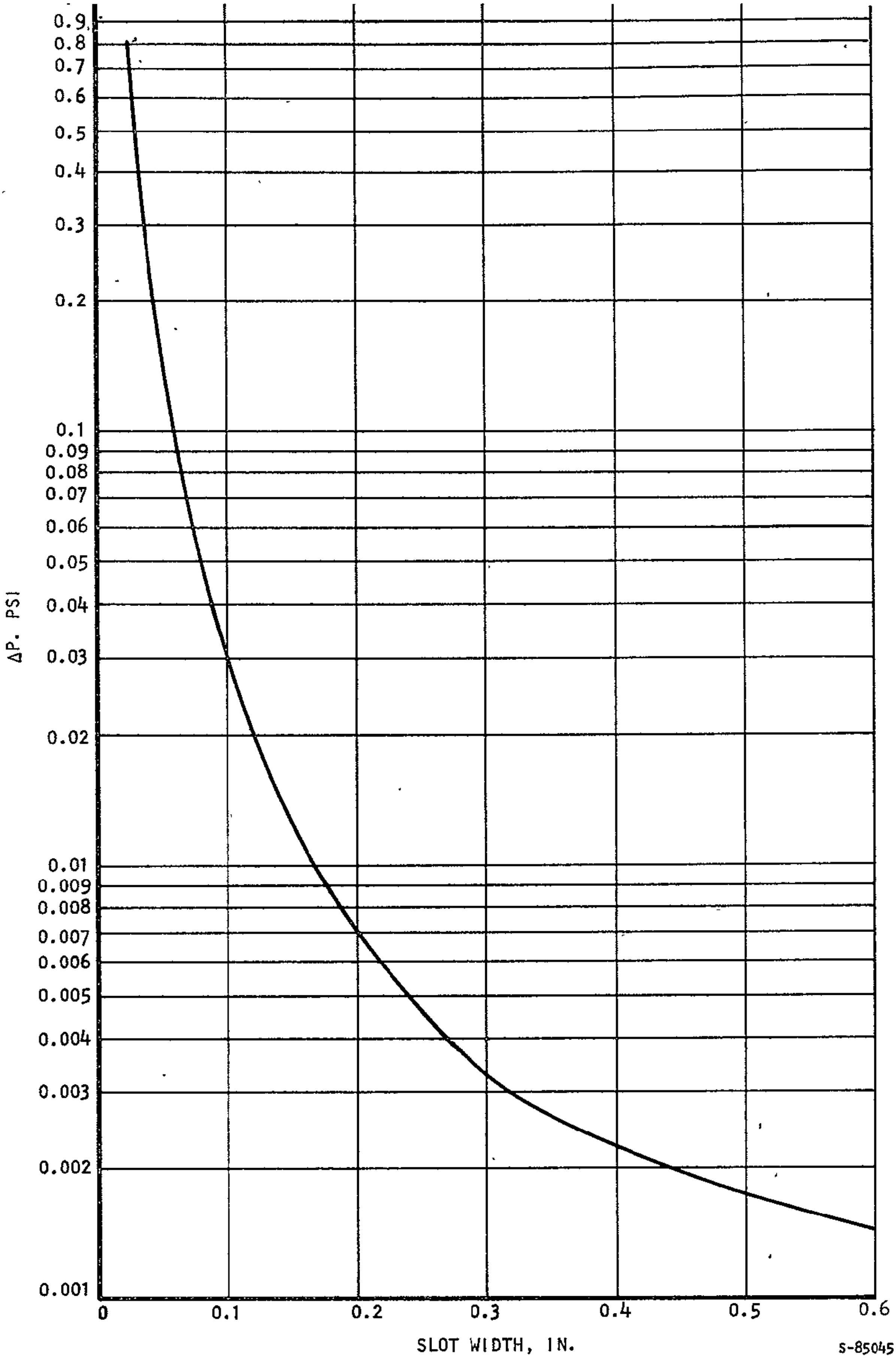

Figure 12-3. Variation of ΔP as a Function of Slot Width in Cold Bearing Support

pressure drop is calculated. This value is then divided by 20 to yield the required pressure drop of the distribution slot. Figure 12-3 is then utilized to determine the distribution slot width required. The total volume and pressure drop of the combined axial and distribution slots are then calculated and converted to refrigeration loss by use of the tradeoff factors presented previously in this report.

The resultant total refrigeration lost is then available as a function of axial slot width. A plot of these data, Figure 12-4, yields the slot width which provides optimum performance. As shown in Figure 12-4, the optimum slot width is 0.000762m (0.030 in.). The distribution slot width is 0.003175m (0.125 in.); the corresponding combined pressure drop is 2708 N/m^2 (0.39 psi) and the total dead volume is $1.81 \times 10^{-6} m^3$ (0.1106 $in.^3$).

The sump filler block to pressure vessel housing was analyzed in a manner similar to that described above. Flow around the filler block is divided into two regions; (1) the cylindrical portion that extends from the sump heat exchanger past the crankshaft and (2) the hemispherical region that extends from the cylindrical portion to the interface with the bearing housing. The flow area was held constant over both regions. This results in a constant gap in region 1 and a gap that varies linearly in region 2. The linear variation is such that the cross-sectional area at the smallest diameter (intersection with the bearing housing) and at the largest diameter (sump diameter) are equal.

The pressure drops and void volumes for a range of cylindrical gaps were calculated, and converted to the corresponding total loss in refrigeration. The results are presented in Figure 12-5. The refrigeration loss is plotted as a function of the annular gap in the cylindrical portion of the VM sump. Each of these gaps has a unique associated tapering gap in the hemispherical region. The minimum refrigeration loss occurs at a gap of 1.905×10^{-4}m (0.0075 in.). This cylindrical gap, and the corresponding taper in the hemispherical region have been incorporated in the design of the fractional watt VM.

The flow passages in this portion of the VM sump and the flow distributors described previously have been optimized to yield minimum refrigeration loss. The resulting pressure drops in all components have resulted in an overall pressure drop from the sump to the cold end which are slightly higher than originally allocated in the preliminary design. The overall pressure drop was used in the ideal cycle analysis, and in the dynamic analysis. The motor power requirement is within acceptable limits when the increased cold end pressure drop is considered. Thus the one parameter not considered in the optimization procedure, the effect of cold end pressure drop on motor power, is not adversely affected.

Hot Regenerator to Sump Heat Exchanger

As discussed previously, the primary concern in this region of the VM sump is pressure drop. The components that will be considered are: the hot regenerator retainer; the mounting flange of the sump filler block; and the

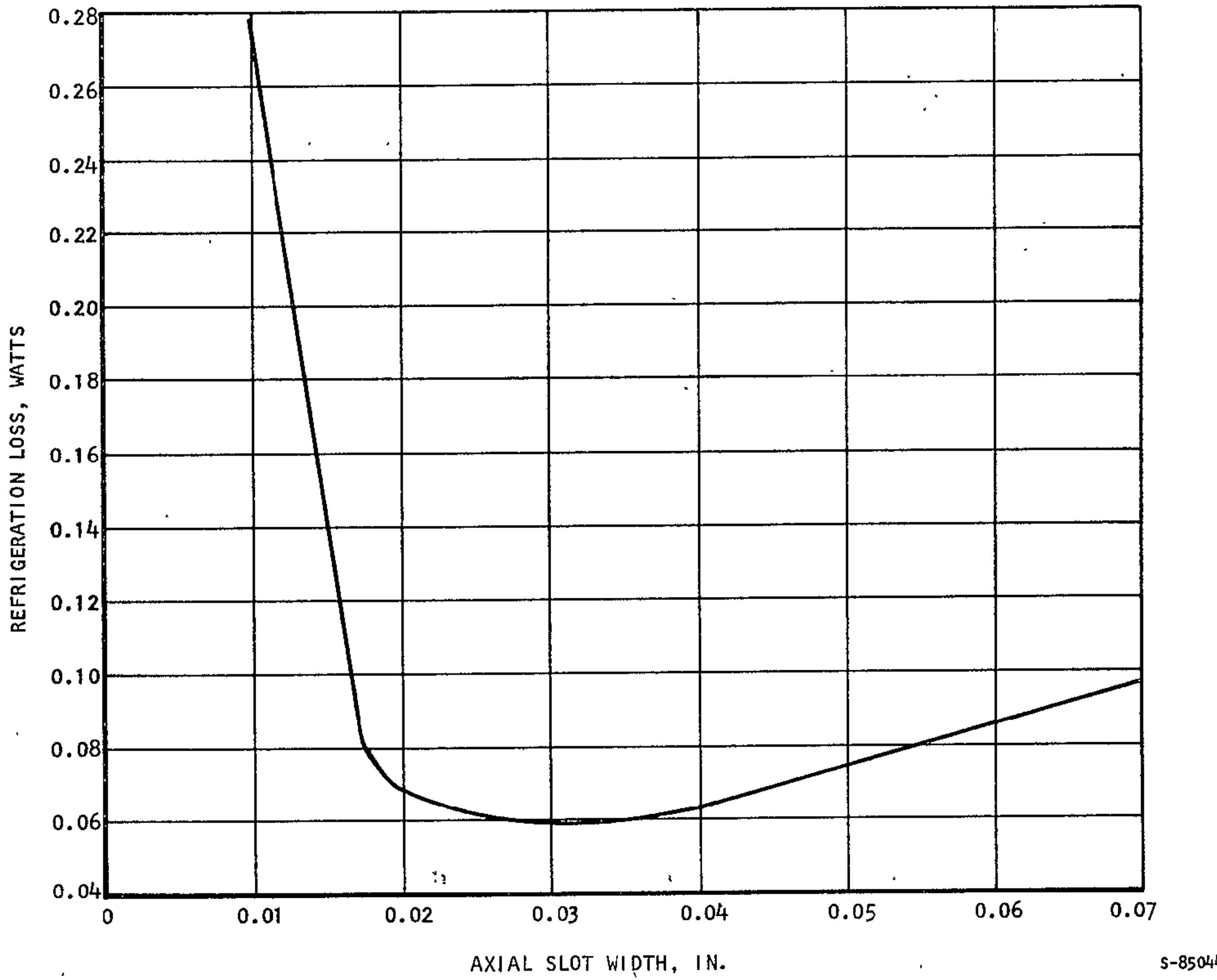

Figure 12-4. Bearing Support Flow Passage Width vs Refrigeration Loss

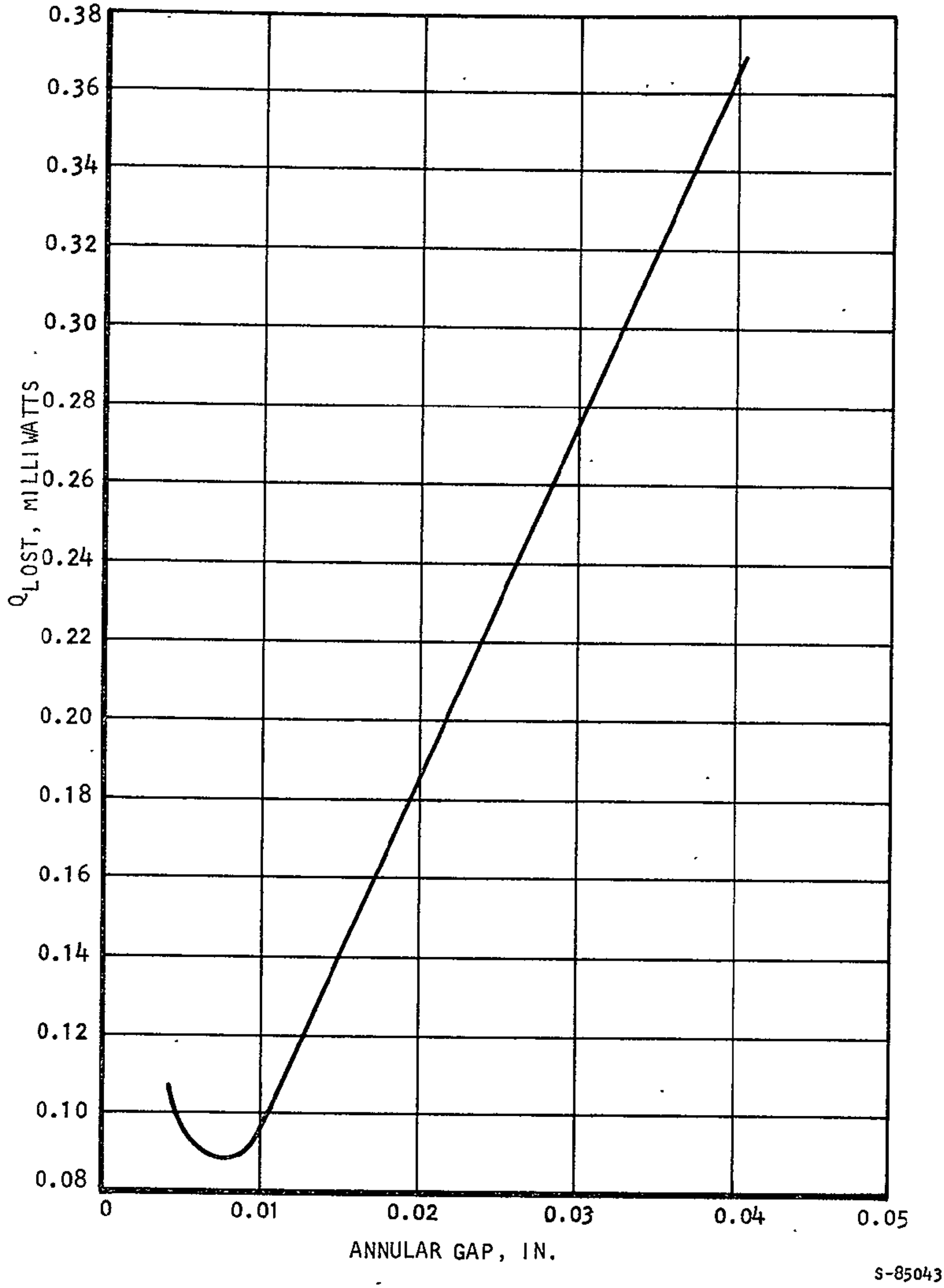

Figure 12-5. Refrigeration Loss as a Function of Angular Gap of Sump Filler Block

transition slot between the mounting flange and the sump heat exchanger. The flow passages in the first two components consist of a series of holes in a circular pattern. The interconnecting passage is an annular slot, whose flow area may be easily varied by changing the filler block diameter. The arrangement of these components is shown on the layout drawing, Figure 2-1.

The hot regenerator retainer is provided with forty eight holes, 0.00159m (0.0625 in.) in diameter. The pressure drop through the holes at maximum flow is 38.39 N/m^2 (0.00557 psi), which is considered an acceptable value. The sump filler block mounting flange is provided with an identical set of holes. Thus the pressure drop is also identical to that of the hot regenerator retainer.

The radial dimension of the slot between the filler block mounting flange and the sump heat exchanger was set to provide a total flow area equal to that of the axial holes in the other two components. Since the pressure drop is caused primarily by shock loss (expansion and contraction losses), the pressure drop is nearly identical to that of the axial holes.

The pressure drops of these three components are a very small contribution to the overall value between the hot and ambient ends of the hot displacer. Thus the designs are considered adequate.

Sump Heat Exchanger to Active Sump Volumes

The flow passages in this area are composed of four separate elements: (1) the sump heat exchanger flow distribution slot, (2) ports to the back side of the hot displacer, (3) ports to the crankcase region of the machine and (4) an annular passage interconnecting the two ends of the cold wrist pin housing. These porting system elements will be discussed separately below.

1. Sump Heat Exchanger Distribution Slot

The sump heat exchanger distribution slot analysis was performed in the same manner as the procedure developed for the GSFC 5 watt VM (Reference 4). The model is shown in Figure 12-6, and is predicted on the assumptions of uniform distribution of the total flow between the sump ports and zero circumferential flow at the midpoint between ports. As the flow progresses from a given sump port, the flow rate in the slot decreases due to flow into the sump heat exchanger. Thus a turbulent and a laminar region may exist. The total pressure drop is therefore composed of a turbulent and a laminar contribution. The final expression for circumferential pressure drop is given by equation 12-1.

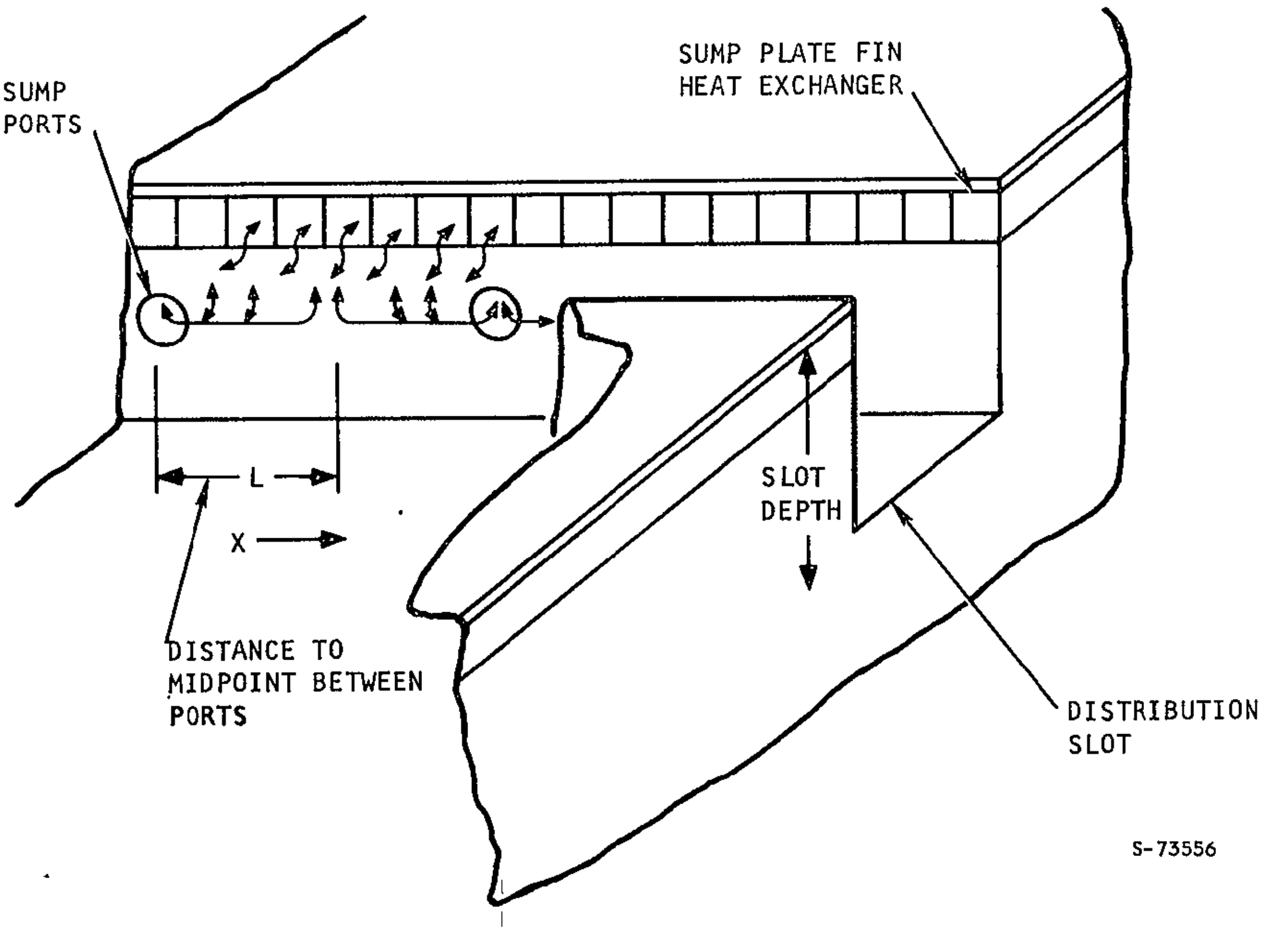

Figure 12-6. Flow Distribution Slot Model Schematic

$$\Delta P_{cir} = \left[\frac{0.042}{g_c \rho} (2\mu)^{1/4} \left(\frac{\dot{W}_o}{A_F}\right)^{1.75} \frac{L}{\left(D_H\right)^{1.25}}\right]\left[\left(1 - \frac{X_i}{L}\right)^{2.75} - 1\right]$$

$$+ \left(1 - \frac{X_i}{L}\right)^2 \frac{-K\mu \dot{W}_o L}{2g_c D_H^2 \rho A_F} \qquad (12\text{-}1)$$

where:

ΔP_{cir} = Pressure drop in the circumferential direction

g_c = Gravitational constant

ρ = Fluid density

μ = Fluid viscosity

$\dot{W}_o$ = Flow rate from one sump port

A_F = Slot flow area in circumferential direction

D_H = Slot hydraulic diameter

L = Distance to midpoint between ports (Figure 3-36)

X_i = Distance from port where transition to laminar flow occurs

K = A constant between 13 and 24, depending on slot geometry

The development of Equation 12-1 is given in Reference 4.

Manufacturing considerations set the distribution slot width at 0.00508m (0.2 in). The variable dimension is the slot depth. The total effective depth is the sum of the fin height and slot dimension; thus the calculations are based on slot depth plus 7.62×10^{-4}m (0.03 in.). The pressure drop in the circumferential direction was calculated for a series of slot depths. The pressure drop of section 2 of the sump heat exchanger is 9.47 times that of a slot with a depth of 0.00127m (0.05 in.). This is considered adequate to distribute the flow uniformly over the heat exchanger inlet face. When compared to section 1 of the sump heat exchanger, the ratio of pressure drops is 77 to 1. The void volume of the slot is $1.93 \times 10^{-6}m^3$ (0.118 $in.^3$), which is considered an acceptable value.

2. Ports to Ambient End of Hot Displacer

The design procedure for these ports is a straightforward procedure; however, the selection of the final design involves engineering judgement concerning the allowable pressure drop (effect on motor power). A very low pressure drop involves the introduction of relatively large void volume. The number and size of ports selected yield a pressure drop and void volume consistent with the overall design goals of the VM.

The number of ports was selected somewhat arbitrarily as six. This was the number of ports utilized in the GSFC 5 watt VM, and results in reasonable flow lengths between ports in the sump heat exchanger distribution slot. The port diameter selected was 0.004191m (0.165 in.). The void volume is $3.67 \times 10^{-6}m^3$ (0.224 $in.^3$), and the pressure drop is 301.3N/M^2 (0.0437 psi). These values are consistent with the overall design goals, and are considered acceptable.

3. Ports to Crankcase Region

In considering the ports to the crankcase region, the use of maximum sump flow for port design is overly conservative, since the major portion of the flow enters the ambient end of the hot displacer. Only a small fraction of the gas flows to and from the crankcase region. Thus the first step is establishing the sump flow rate. The method follows that of the GSFC 5 watt VM, Reference 4. The flow rate is expressed as the differentiated form of the perfect gas equation with respect to time.

$$\dot{W} = \frac{dM}{d\tau} = \left(\frac{d\theta}{d\tau}\right)\left(\frac{dM}{d\theta}\right) = \left(\frac{d\theta}{d\tau}\right)\left(\frac{P}{ZRT} \quad \frac{dVs}{d\theta} + \frac{Vs}{ZRT} \quad \frac{dP}{d\theta}\right) \tag{12-2}$$

where

$\dot{W}$ = Flow rate

θ = Crank angle

τ = Time

$\frac{d\theta}{d\tau} = \frac{2\pi\,(rpm)}{60}$ in radians/sec

P = Pressure

Z = Compressibility of working fluid

R = Gas constant of working fluid

T = Temperature of sump region (constant)

Vs = Crankcase volume, function of angular position

$\frac{dVs}{d\theta}$ = Rate of change of crankcase volume

$\frac{dP}{d\theta}$ = Rate of change of cycle pressure

The crankcase volume and thus its derivative may be expressed in terms of the geometry of the machine. For the fractional watt VM, the appropriate expressions are:

$$V_s = 1.6436 - .1564 \sin\,(\theta - 10.17^\circ) \tag{12-3}$$

$$\frac{dV_s}{d\theta} = -.1564 \cos\,(\theta - 10.17^\circ) \tag{12-4}$$

Substitution of equations 12-3, 12-4, and the appropriate constants into equation 12-2 yields the expression for flow rate into the crankcase region of the fractional watt VM.

$$W = 1.455 \times 10^{-5} \left\{\left[-0.1564P \cos(\theta - 10.17^\circ)\right] + \left[1.644 - 0.1564 \sin(\theta - 10.17^\circ)\right] \frac{dP}{d\theta}\right\} \tag{12-5}$$

Equation 12-5 is evaluated over a complete crankshaft revolution utilizing the pressure and pressure derivative evaluated from the ideal cycle program printout (Figure 3-1). The results are presented in Figure 12-7. The maximum absolute value of flowrate, 0.000514 kg/sec (0.00113 lb/sec) is used to size the ports to the crankcase region.

The port arrangement chosen consists of four holes, 0.001905 m (0.075 in.) in diameter, extending from the sump heat exchanger distribution slot to the inboard end of the hot displacer bearing assembly. A relief groove provided at that point allows the flow to enter the crankcase region. The void volume of the porting system is 1.33×10^{-6} M^3 (0.0812 in.^{3}) and the pressure drop is 256 N/M^2 (0.0372 psi). This pressure drop value is nearly equivalent to that of the ports from the heat exchanger slot to the ambient end of the hot displacer. Thus the pressure in all portions of the sump will be nearly equal.

Cold Wrist Pin Housing

As the cold displacer moves in a reciprocating motion, the working fluid must be free to flow to and from the cavity at the outboard end of the wrist pin housing. The required flow passage is provided by a controlled annular gap between the wrist pin retainer and the cold-end bearing housing. The flow rate to this volume is first determined in a manner similar to the crankcase flow. The procedure of evaluating the differentiated perfect gas equation is followed. Upon substitution of the constants and expressions for volume and volume derivative, the following equation for flowrate is obtained.

$$W = 1.455 \times 10^{-5} \left[0.007845P \sin\theta + (0.08596 - 0.07845 \cos\theta) \frac{dP}{d\theta}\right] \qquad (12\text{-}6)$$

Equation 12-6 plotted for a full crankshaft revolution is shown on Figure 12-8 The maximum absolute value of flow rate obtained from this figure is 5.18×10^{-4} kg/sec (0.00114 lb/sec), which is used in the design calculations for the required clearance between the bearing housing and wrist pin retainer.

Having determined the flowrate to be utilized for designs, the graphical optimization procedure is used in calculating the required flow passage dimensions. The pressure drop and void volume were calculated for a range of annular gaps between the bearing housing and wrist pin retainer. Utilizing the appropriate trade factors, a total refrigeration loss was determined for each gap size. The results are presented in Figure 12-9, which indicates an optimum flow passage at a gap of 2.54×10^{-4} (0.01 in.). The pressure drop of this passage is 368N/M^2 (0.0534 psi) and the void volume is $3.376 \times 10^{-7} M^3$ (0.0206 in.^{3}). The optimum design indicated by figure 12-9 has been incorporated in the design of the fractional watt VM.

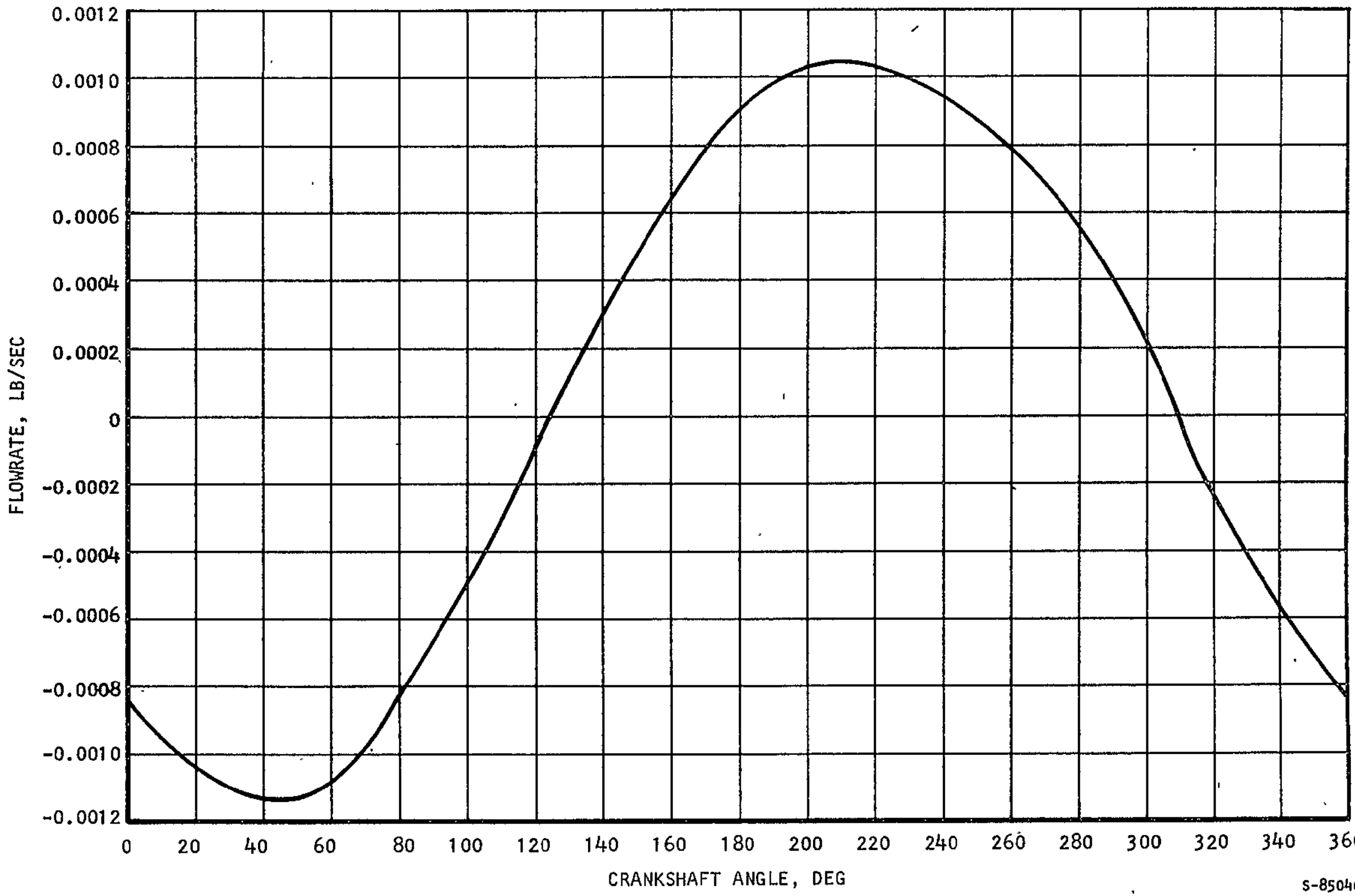

Figure 12-7. Crankcase Region Flowrate versus Crankshaft Angle

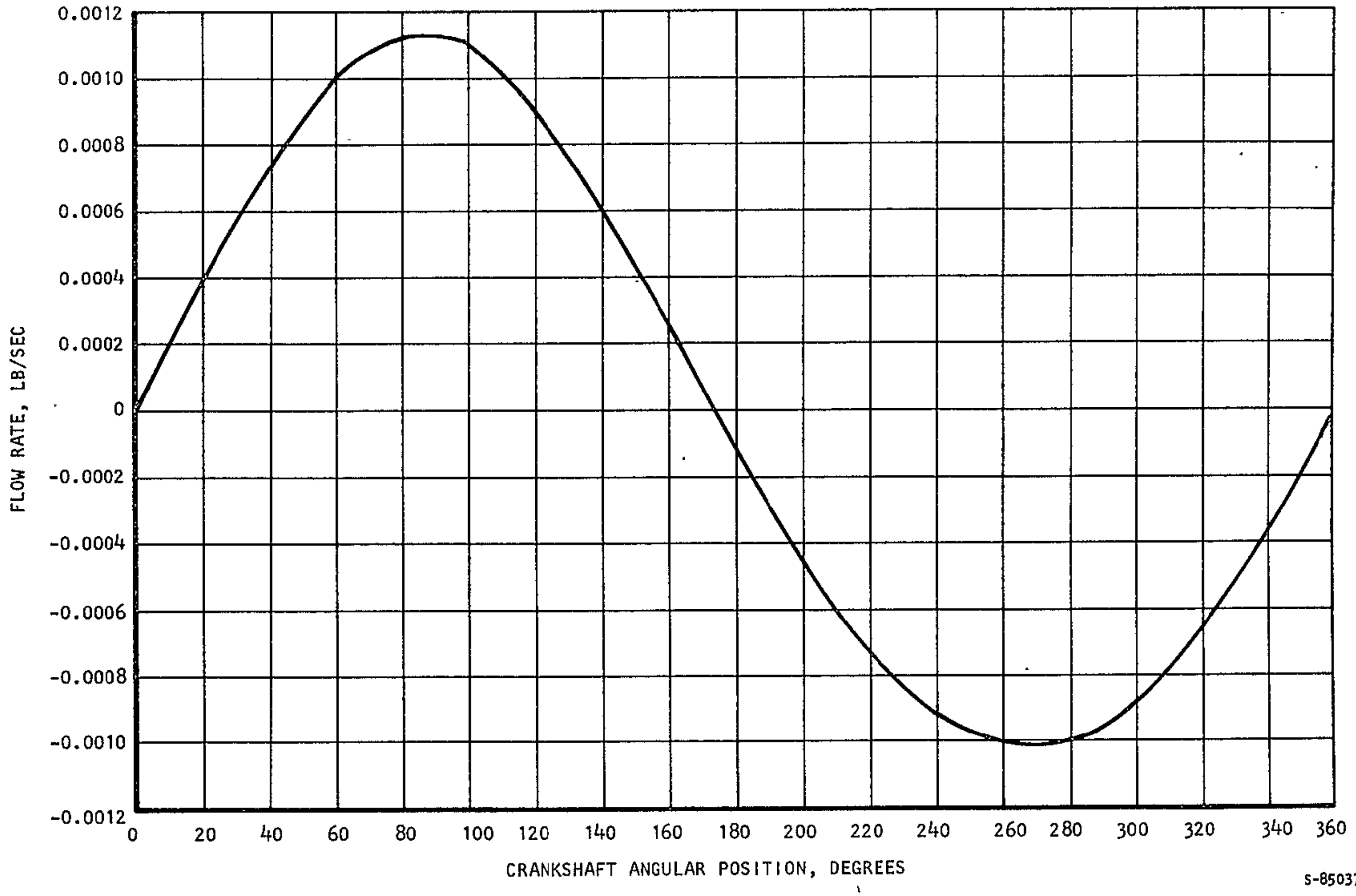

Figure 12-8. Sump End Flow Rate, Cold Displacer

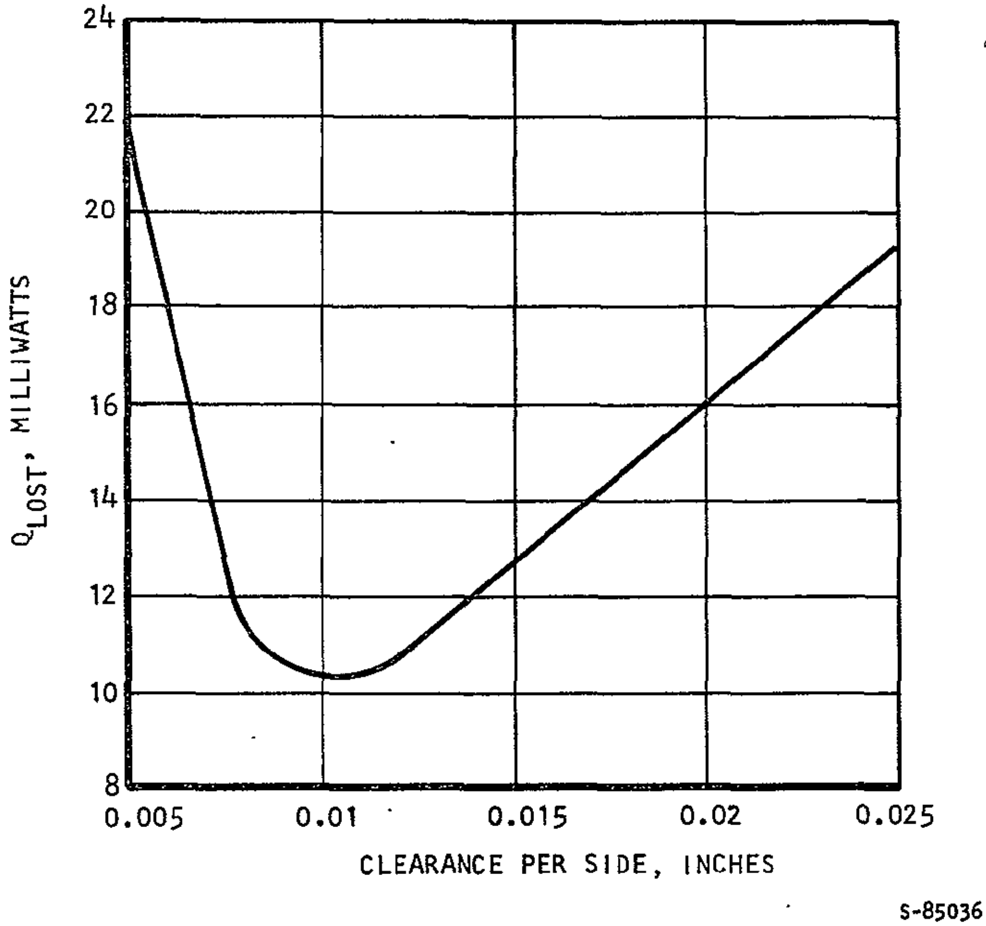

Figure 12-9. Optimization of Gap Between Cold Wrist Pin Housing and Bearing Housing Bore

Cold Bearing Support

Flow Slots and Distribution Slot

The axial slots in the outer diameter of the cold end bearing support connect the cold regenerator to the sump region. The helium that flows to and from the cold end must pass through these slots in order to reach the sump heat exchanger and reject the refrigeration heat load to ambient. We are very interested in providing uniform flow to the cold regenerator, so that it will function properly. Since maldistributed flow may emerge from the sump region of the VM, we will provide a means of redistributing the flow equally between all axial slots. This function is performed by a circumferential slot with a low pressure drop compared to the axial slots. The circumferential distribution slot acts as a backup device to the flow distributor which is provided at the entrance to the regenerator.

The distribution slot will be sized using the same basic assumptions as the 5 watt VM; i.e. 1/4 of the total flow must be distributed around 1/2 of the circumference of the bearing support. The allowable pressure drop to accomplish this will be 1/20 of the pressure drop of the axial slots.

We can optimize the flow passages with respect to refrigeration by use of the tradeoff parameters. Each axial slot area will have a unique pressure drop and void volume. By virtue of the pressure drop being fixed, there is also a unique distribution slot size required. Thus each axial slot size will define a pressure drop and void volume which may then be converted to a refrigeration loss by use of the tradeoff factors. If this is done for a range of slot sizes, a minimum refrigeration loss should be evident.

The geometry and manufacturing considerations of the bearing support indicate that 16 axial slots 0.05 in. deep comprise a reasonable configuration. The average length of the slots is 3.9 inches.

The first step is to determine the pressure drop of the circumferential distribution slot as a function of width. This will in turn allow rapid determination of slot width required for a given axial pressure drop.

The appropriate flow rate is Total flow/6

$$\dot{w} = .00272\ lb/sec/6 = .000453\ lb/sec$$

$$\mu = .00001445\ lb/ft\text{-}sec.$$

$$L = .5\pi D = .5 \times \pi \times 1.0 = 1.57,\quad 4L = 6.28$$

$$Re = GD_h/\mu = \cancel{GD_h/4.55}\ \frac{G\,lb\ D_h\,in\ \cancel{ft\text{-}sec}\text{-}ft}{\cancel{ft^2\text{-}sec}\ \ \mu\,lb\ 12\,in}$$

$$= \frac{GD_h}{12 \times .00001445} = .577 \times 10^4\, GD_h$$

with G in $lb/ft^2\text{-}sec$
D_h in inches

$f = 16/Re$ in laminar flow, $f = .084/Re^{1/4}$ in turbulent flow

$$\Delta P = 4fL/D_h\ VH$$

$$VH = G^2/2g\rho$$

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Set up Table

Slot Width, in	A, ft²	G lb/ft²-Sec.	VH, Psi	D_h in.	Re	f	$4fL/D_h$	ΔP, Psi
.025	.0000868	52.19	0.508	0.0333	10,060	.0084	1584	.805
.05	.0001736	26.09	0.127	0.05	7540	.00902	1156	.1467
.1	.0003472	13.05	0.0318	0.0667	5020	.00998	.9396	.0299
.2	.0006944	6.524	0.00794	0.08	3000	.0113	.887	.00704
.3	.0001042	4.347	0.00353	0.0857	2150	.0123	901	.00318
.4	.0001389	3.261	0.001983	0.0889	1665	.016*	1130	.00224
.6	.0002083	2.175	0.000883	0.0923	1158	.024*	1163	.00144

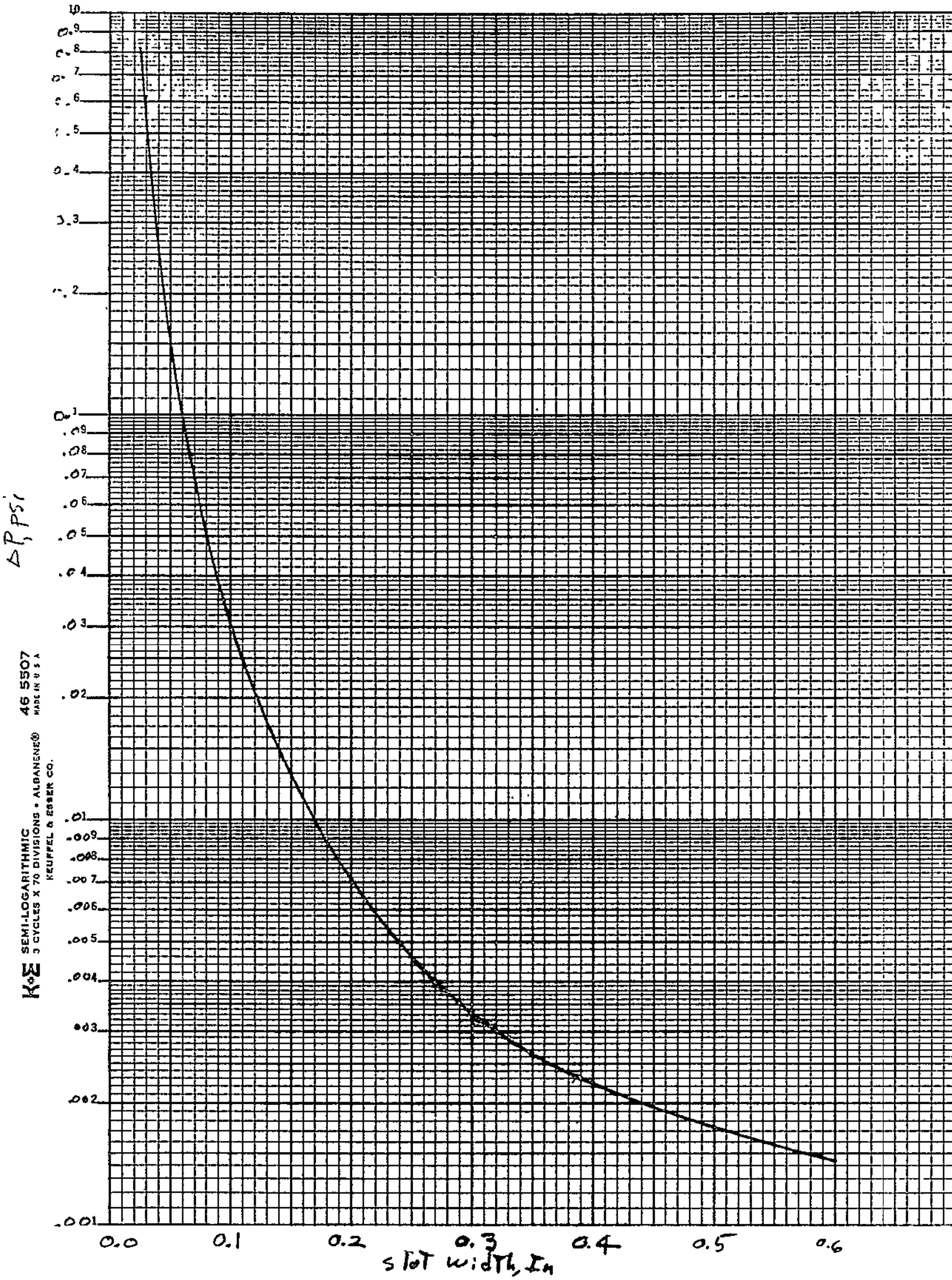
* These laminar friction factors are from Kays and London for flow in rectangular channels. They are higher than the laminar f in round tubes, and are thus used to insure conservatism.

The above pressure drops are plotted as a function of slot width on the next page. They will be used for rapid determination of required slot width.

Now we will do the axial slots.

In addition to frictional pressure drop, we will consider 3 velocity heads lost due to expansions and contractions. This loss considers 1½ velocity heads lost at the distribution slot and 1½ at

Pressure Drop of Circumferential Flow Distribution slot (5)



the entrance and exit. We will set up a table as was done for the circumferential slot.

Slot width in.	A, ft^2	G, $lb/ft^2 sec$	VH, psi	D_h, in	Re	f	$4fL/D_h+3$
.01	$.556 \times 10^{-4}$	48.9	.446	.01667	4700	.01015	12.5
.02	1.112×10^{-4}	24.45	.1118	.0286	4040	.0105	8.72
.03	1.668×10^{-4}	16.3	.0496	.0375	3520	.01092	7.55
.04	2.22×10^{-4}	12.25	.028	.0444	3140	.01123	6.75
.05	2.78×10^{-4}	9.78	.01782	.050	2820	.0115	6.59
.06	3.335×10^{-4}	8.16	.01243	.0545	2570	.0118	6.38
.07	3.89×10^{-4}	6.99	.00912	.05833	2375	.01203	6.22

We are now in a position to calculate the pressure loss and void volume of each axial slot and its associated circumferential distribution slot. These will then be converted to refrigeration loss. The appropriate trade factors are:

Pressure Drop Trade factor: 0.04738 watts/psi

Void Volume Trade factor: 0.380 watts/in.^{3}

The total length of the axial slots is 4.1 inches. The width of the appropriate circumferential slot will be deducted from the total length in calculating void volume.

(7)

Axial slot width, in	ΔP axial, psi	ΔPaxial/20 psi	Distribution* slot width, in.	Axial slot Volume, in^3	Distribution slot Volume, in^3
.01	5.57	0.278	0.04	0.0324	0.00635
.02	0.975	0.0487	0.081	0.0644	0.01285
.03	0.374	0.0187	0.125	0.0955	0.01983
.04	0.1947	0.00973	0.172	0.1258	0.02729
.05	0.1173	0.00587	0.220	0.155	0.0349
.06	0.0794	0.00397	0.270	0.184	0.0428
.07	0.0567	0.00284	0.333	0.2115	0.0528

* From the curve on Page 5

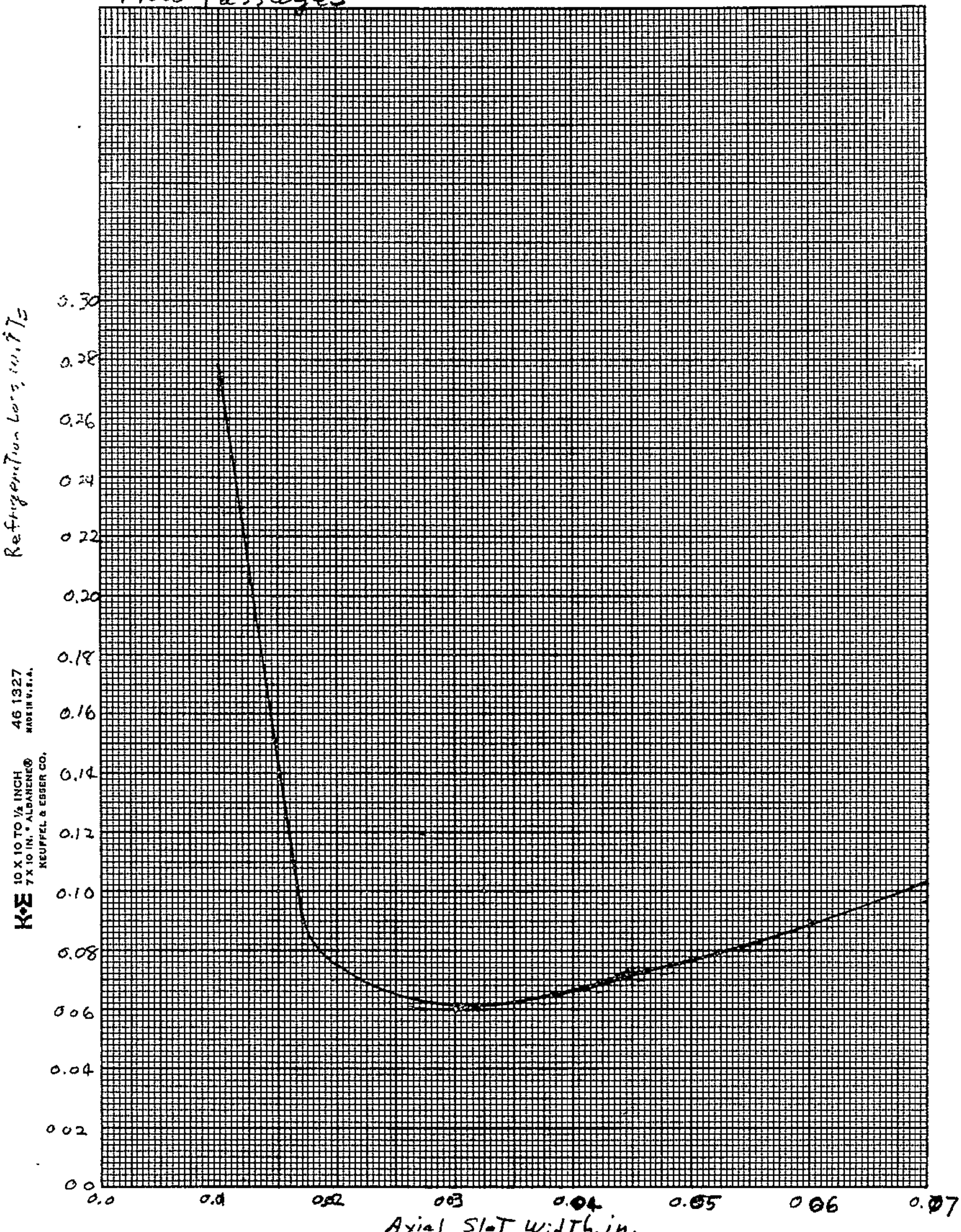
Axial slot width, in	Total Volume, in^3	Void Volume** Refrigeration Loss, watts	Pressure Drop*** Refrigeration Loss, watts	Total Refrigeration Loss, watts
.01	.0387	0.01742	0.264	0.2787
.02	.0772	0.0293	0.0462	0.0755
.03	.1153	0.0438	0.0177	0.0615
.04	.1531	0.0582	0.0092	0.0674
.05	.1899	0.0722	0.0056	0.0777
.06	.2268	0.0862	0.0038	0.0900
.07	.2643	0.1004	0.0027	0.1031

** 0.38 X Void Volume
*** 0.04738 X Pressure Drop

The total refrigeration loss as a function of axial slot width is plotted on page 8.

Refrigeration Loss of Cold End bearing Support Flow Passages

(8)



(c-1)

The curve on page 8 indicates a minimum refrigeration loss at an axial slot width of 0.030 inches. The associated circumferential distribution slot is 0.125 inches wide. These values will be incorporated in the final design of the VM.

Sump Filler Block

Flow Passage

Flow passages must be provided between the sump heat exchanger and the cold end bearing support flow passages. The most direct path is around the outer surface of the sump filler block, as was done on the GSFC 5 watt VM. However, the individual flow passages that were chem milled into the 5 watt sump filler block presented manufacturing problems, and a slightly different approach will be used. The sump filler block is installed and supported in such a manner that a gap may be provided between it and the pressure vessel inner wall. The gap may be controlled very accuratly with relatively simple dimensional control.

The flow passage may be divided into two regions, a straight annular gap that extends from section 2 of the sump heat exchanger past the crankshaft assembly, and the hemispherical transition to the cold displacer bearing support.

Even Though The gas at different circumferential locations experiences different flow lengths, we will calculate pressure drop based on The longest flow length. Some maldistribution will occur. However, for flow Toward the cold regenerator, we have Two flow distribution devices To remove The maldistribution. For flow Towards The sump heat exchanger, any maldistribution will be caused by The hemispherical portion of the flow passage In The straight annular section, the resistance To circumferential flow is essentially The same as in The axial direction. Therefore the flow will Tend To redistribute itself pretty well by The Time it reaches The sump heat exchanger.

The straight annular gap will be maintained at a constant ~~length~~ value over its entire length. In The hemispherical section, The gap will be increased linearly as a function of distance Travelled around The hemisphere, so That a constant flow area is maintained.

Since each gap size will imply a unique value of pressure drop and void volume, we may calculate the refrigeration loss associated with each size. If we do this for a range of gap size, a minimum refrigeration loss will be found.

First, look at the straight annular gap. The diameter of the sump in this ~~region~~ region is 2.34 in., and the flow length is 2.75 in.

Flow area $= \pi D C$ where C = clearance per side

$= 2.34 \pi C / 144$ FT.^{2}

$D_h = 2C$

$G = \dot{w}/A$

$R_e = G D_h / \mu \qquad \mu = 1.445 \times 10^{-5}$ lbm/FT-sec.

The Reynolds number will be a constant for all values of clearance, since the change in free flow area is exactly offset by the change in Hydraulic Diameter.

$\Delta P = 4 f L / D_h \; VH$ where $VH = G^2 / 2 g_c \rho$ and $f = 24 / R_e$

now set up Table

gap, c in	A, FT^2	G lb/FT^2-sec	VH, psi	Re	f	$4fL/D_h$	ΔP, psi	Volume, in^3
.005	.000255	10.66	.0212	615	.039	42.9	.909	.05015
.0075	.000383	8.0	.01192	↓	↓	28.6	.341	.0751
.010	.00051	5.33	.0053			21.45	.1138	.1003
.015	.000765	3.56	.00236			14.3	.0338	.1504
.020	.00102	2.66	.001325			10.72	.0142	.2006
.040	.00204	1.33	.000331			5.36	.001775	.4012

Now look at the hemispherical portion of the block. At the intersection of the cold displacer bearing support, the diameter is 1.06. Thus the gap at the tip should be 2.34/1.06 = 2.207 times that in the straight annular portion in order to provide equal areas. The average gap will be used for hydraulic diameter calculation. The maximum flow length is .25πD = .25π×2.34 = 1.84 in. now set up a table similar to that above.

Hemispherical Portion of Sump Block

Axial Gap, c, in	Average Gap, in.	Area, ft^2	G lb/ft^2-sec	VH, psi	$4fL/D_h$	ΔP, psi	Volume, in^3
.005	.008	Same	Same	Same	11.22	.238	.0687
.0075	.012	as	as	as	7.55	.09	~~[illegible]~~ .1032
.010	.016	for	for	for	5.66	.03	~~[illegible]~~ .1375
.015	.024	axial	axial	axial	3.74	.00882	.206
.020	.032	slot	slot	slot	2.805	.00372	.275
.040	.064				1.402	.000464	.55

Note: Re = Constant = 985, f = .0244

Now we are in a position to add up the pressure drops and volumes, and convert to refrigeration loss. The appropriate trade factors are:

Pressure Drop Trade Factor = 0.04738 watts/psi

Void Volume Trade Factor = 0.380 watts/in^3

Axial Gap, c, in.	Total Volume, in^3	Total ΔP, psi	Refrig. loss by ΔP, watts	Refrig. loss by Vol, watts	Total Refrig. loss, watts
.005	.1189	1.147	.0543	.0451	.0994
.0075	.1783	0.431	.0204	.0678	.0882
.010	.2378	0.1438	.00674	.0904	.0914
.015	.3564	.04262	.00202	.1355	.1375
.020	.4756	.01792	.00085	.1808	.1816
.040	.9512	.002239	.000106	.362	.3621

Refrigeration loss is plotted against axial gap on the next page.

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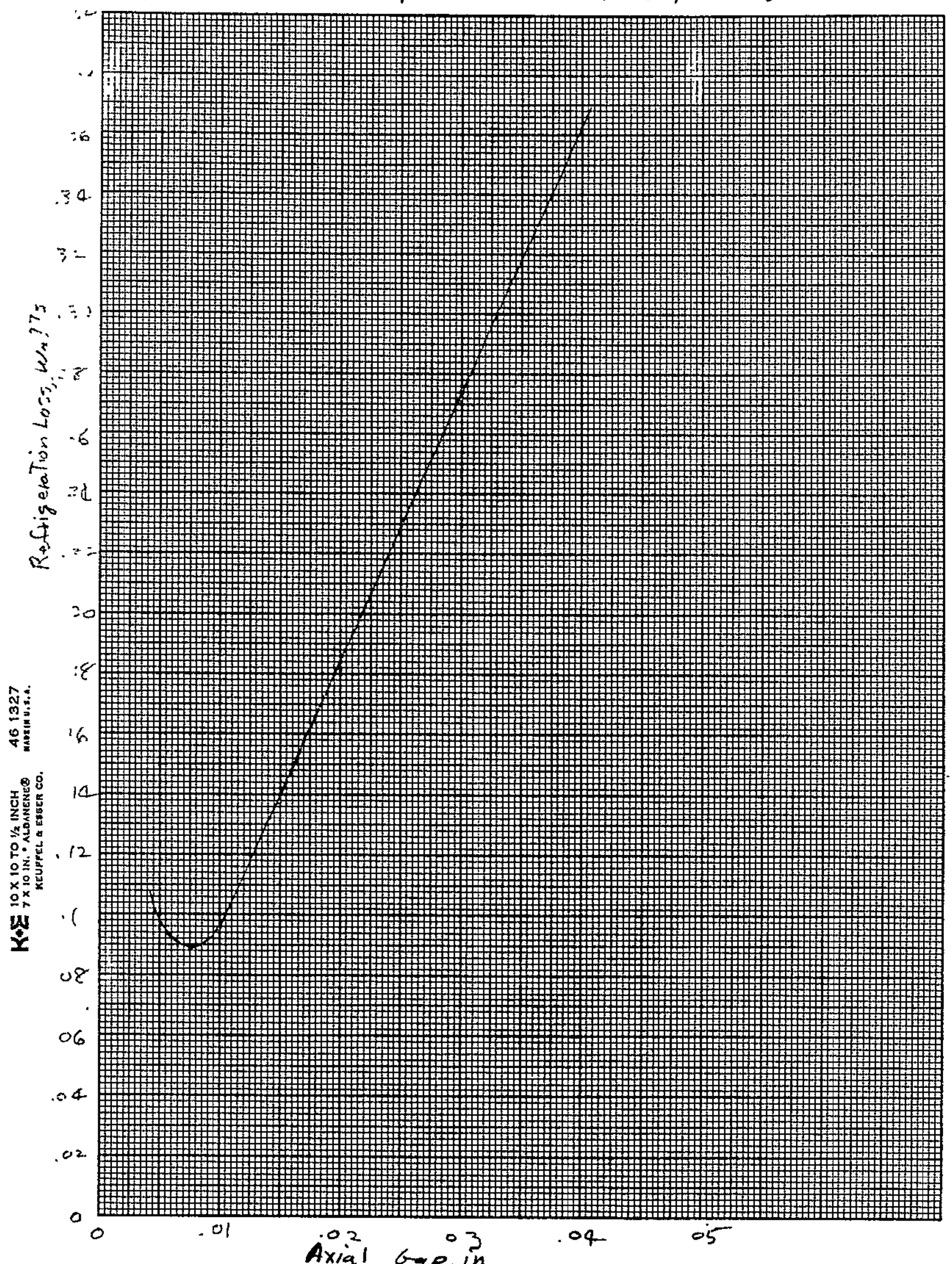
Optimization of sump filler block flow passage

(6)



The minimum refrigeration loss occurs at an axial gap of 0.0075 in. The corresponding gap at the tip of the hemisphere is 0.0165 inches The pressure loss is 0.431 psi, and the void volume is 0.1783 in.^{3}

These optimum gap dimensions will be incorporated in the VM.

Hot Regenerator To Sump Heat Exchanger

The Hot Regenerator retainer has 48 holes in it, as does the mounting flange of the sump filler block. These holes are presently 0.0625 in. diameter. We will check and see if the pressure drop is acceptably low. The flow rate is .00723 lb/sec, and the appropriate density is 0.5808 lb/ft^3. The pressure drop is primarily shock, and we will assume 1.5 velocity heads lost at each plate.

$$A = 48\ \pi/4\ 0.0625^2/144 = .001023\ ft^2$$

$$G = w/A = .00723\ lb/sec\ .001023\ ft^2 = 7.07\ lb/ft^2\text{-}sec$$

$$\text{velocity head} = G^2/2g_c\rho = \frac{7.07^2\ \cancel{lbm^2}\ lb_f\text{-}\cancel{sec^2}\ ft^3\ \cancel{ft^2}}{\cancel{ft^4}\text{-}\cancel{sec^2} \times 2 \times 32.2\ \cancel{lbm}\ ft \times .5808\ \cancel{lbm} \times 144\ in^2}$$

$$= 0.00928\ psi$$

$$\Delta P = 3 \times .00928 = 0.02784\ psi$$

In addition to the holes, there is a

circumferential slot between the heat exchanger entrance and the end of the sump filler block. We will adjust the width of the slot to match that of the holes.

$$A_{req'd} = .001023\ ft^2 = \pi D G$$

$$C = \frac{.001023\ ft^2 \times 144\ in^2}{2.34\ in \times \pi\ \ ft^2} = .02004\ in.$$

use .020 inch wide gap.

$$Re = \frac{G D_h}{\mu} = \frac{7.07\ lb \times .04\ in \cdot ft}{ft^2\text{-}sec} \frac{ft\text{-}sec}{12\ in \times 1.447 \times 10^{-5}\ lb} = 1628$$

$$f \cong .01,\quad 4fL/D_h = 4 \times .01 \times .25/.04 = .25$$

Thus $\Delta P_{friction} = .25 \times .00928 = 0.00232$ psi

and $\Delta P_{Total} = 0.03016$

This appears to be an acceptable value, and we will maintain the hot regenerator retainer and sump block as is.

Sump Heat Exchanger
Flow Distribution Slot

The purpose of this slot is to accept the flow from the discrete ports leading to the ambient end of both displacers and distribute this flow uniformly into both sections of the sump heat exchanger. The general ground rule will be to provide a circumferential pressure drop in the slot of about 1/10 of that in the sump heat exchanger. The distribution slot pressure drop analysis developed for the 5 watt VM will be utilized, where the flow leaving the slot and entering the heat exchanger as a function of circumferential distance is accounted for. The pressure drop of section 2 of the sump heat exchanger is 0.00765 psi, which is the lowest of the two heat exchanger sections. We must therefore provide a slot ΔP of approximatly 7.65×10^{-4} psi.

As flow emerges from an ambient active volume port and flows circumferentially in the heat exchanger distribution slot, both turbulent and laminar flow regeimes may occur. Therefore the pressure drop analysis is broken into two parts. In the turbulent region, the pressure drop is expressed as:

$$\Delta P = -\frac{0.01815}{g_c \rho} \mu^{.25} \left(\frac{\dot{W}_o}{A_F}\right)^{1.75} \frac{L}{D_h^{1.25}} \left(1 - \left(1 - \frac{X_i}{L}\right)^{2.75}\right)$$

This expression is valid for Reynolds number equal to or greater than 2100, where Reynolds No. is defined as:

$$Re = \frac{\dot{W}_o D_h}{2 A_F \mu} \left(1 - \frac{X_i}{L}\right)$$

In the laminar region, i.e. Re less than 2100, the pressure drop is defined as:

$$\Delta P = -\frac{K \mu \dot{W}_o L}{2 g_c D_h^2 \rho A_F} \left(1 - \frac{X_i}{L}\right)^2$$

The maximum slot width is set by the geometry of the sump region as 0.2 in, so we will look at various depths. We will define slot depth as the depth of cut into the sump block. The actual passage height, h, is 0.03 inches greater, which is the fin height of the sump heat exchanger.

The flow rate that will be used is the maximum value of both sections of the heat exchanger combined, 0.00996. We will actually use 0.01 lb/sec. Since there are six ports from the back side of the hot displacer, the proper w_o is $w_{TOT}/6 = 0.001667$ lb/sec. The total flow length is one-half of the port spacing $= 2.34\pi/12 = 0.612$ in. $= 0.051$ ft. The first step is to set up the various constants in the pressure drop equations.

Physical Properties

$$\rho = 0.577 \text{ lb/ft}^3$$

$$\mu = 1.445 \times 10^{-5} \text{ lb/ft-sec.}$$

Turbulent Region

$$\Delta P = \frac{0.01815\ lb_f\text{-}sec^{.25}\ ft^3\ ft^2\ (1.445\times10^{-5})^{.25}\ lb_m^{.25}\ (.001667)^{1.75}\ lb_m^{1.75}\ .057\ ft}{32.2\ lb_m\text{-}ft \times 0.577\ lb_m\ 144\ in^2\ ft^{.25}\ sec^{.25}\ sec^{1.75}}$$

$$\times \frac{1}{(A_F)^{1.75}\ ft^{3.5}\ (D_h)^{1.25}\ ft^{1.25}}\left(1-\left(1-\frac{x_i}{L}\right)^{2.75}\right)$$

$$= \frac{2.934\times10^{-13}}{(A_F)^{1.75}(D_h)^{1.25}}\left(1-\left(1-\frac{x_i}{L}\right)^{2.75}\right)$$

ΔP_{Turb} in psi with A_F in ft^2 and D_h in ft.

look at slot ~~heights~~ depths of 0.1, 0.2, and 0.3 in. First get areas and D_h and check to see if Turbulent flow exists in all slots.

d, in.	h, in.	A_F, ft^2	$A_F^{1.75}$	D_h, ft.	$D_h^{1.25}$
.1	.13	.0001805	2.812×10^{-7}	.01313	.004443
.2	.23	.0003194	7.633×10^{-7}	.01783	.006514
.3	.33	.0004583	1.436×10^{-6}	.02076	.007881

If the flow is Turbulent in the largest slot ($d=.3$) at $x=0$, all slots will have a Turbulent flow region.

AT $x=0$, The expression for Reynolds Number reduces To:

$$Re = \frac{\dot{w}_o D_h}{2 A_F \mu} = \frac{.001667\ lbm \times .02076\ ft.\ ft\text{-}sec}{2\ \ sec \times 4.583\times10^{-4}\ ft^2 \times 1.445\times10^{-5}\ lbm}$$

$$= 2613$$

Thus all slots will have a Turbulent region

Now we need The Transition flow length, x_i, where $Re = 2100$, and we can Then calculate the Turbulent portion of the pressure Drop.

$$Re = 2100 = \frac{\dot{w}_o D_h}{2 A_f \mu}\left(1 - \frac{x_i}{L}\right)$$

$$1 - \frac{x_i}{L} = \frac{2 \times 2100 \times 1.445\times10^{-5} A_f}{.001667\ D_h} = 36.41 \frac{A_F}{D_h}$$

$$x_i = L\left(1 - 36.41 \frac{A_F}{D_h}\right) = 0.051\left(1 - 36.41\, A_F/D_h\right)$$

d, in.	x_i, ft.	$(1-x_i/L)$	$(1-x_i/L)^{2.75}$	$(1-(1-x_i/L)^{2.75})$
.1	.02547	.5005	.1491	.8509
.2	.01774	.6522	.3087	.6913
.3	.01001	.8037	.5484	.4516

we now have all of the terms in the turbulent pressure drop equation, i.e $A_F^{1.75}$, $D_h^{1.25}$, the constant 2.934×10^{-13}, and $(1-(1-X_i/L)^{2.75})$, and the pressure drop may be calculated.

d, in.	ΔP turbulent, psi
.1	1.998×10^{-4}
.2	4.079×10^{-5}
.3	1.171×10^{-5}

In the laminar region

$$\Delta P = \frac{16 \times 1.445 \times 10^{-5}\ lb_m \times .001667\ lb_m\ lb_f\text{-}sec^2\ ft^3\ ft^2 \times .051\ ft}{ft\text{-}sec.\ sec \times 2 \times 32.2\ lb_m\text{-}ft \times 0.577\ lb_m\ 144\ in^2}\left(\frac{\left(1-\frac{X_i}{L}\right)^2}{D_h^2 A_f}\right)$$

$$= 3.6734 \times 10^{-12}\ \frac{(1-X_i/L)^2}{D_h^2 A_f}$$

ΔP in psi with D_h and A_f in units of ft.

d, in.	$(1-X_i/L)^2$	D_h^2	ΔP_{lam}, psi
.1	.2505	1.724×10^{-4}	2.957×10^{-5}
.2	.4254	3.179×10^{-4}	1.539×10^{-5}
.3	.6460	4.310×10^{-4}	1.202×10^{-5}

now add laminar and turbulent

d, in	ΔP total, psi
.1	2.292×10^{-4}
.2	5.617×10^{-5}
.3	2.374×10^{-5}

①

The smallest slot still has a very low ΔP, lets compare it to the heat exchanger

$$\Delta P_{Hx}/\Delta P_{slot} = 7.65\times10^{-3}/2.292\times10^{-4} = 33.38$$

This is a much higher ratio than we need, and would introduce unnecessary void volume. lets try a slot depth of 0.05 in, Total height = 0.08 in.

$$A_F = .08\times0.2/144 = 1.111\times10^{-4}\ ft.^2,\quad A_f^{1.75} = 1.2025\times10^{-7}$$

$$D_h = \frac{4A_F}{w_p} = \frac{4\times1.111\times10^{-4}}{2(.2+.08)/12} = 9.524\times10^{-3}\ ft.$$

$$D_h^{1.25} = 2.9752\times10^{-3}$$

$$1 - X_i/L = 36.41\ A_F/D_h = 36.41\times1.111\times10^{-4}/9.524\times10^{-3}$$

$$= .4248$$

$$1-(1-X_i/L)^{2.75} = 1-(.4248)^{2.75} = 1-.09494 = .90506$$

$$\Delta P_{Turb} = \frac{2.934\times10^{-13}\times.90506}{1.2025\times10^{-7}\times2.9752\times10^{-3}} = 7.422\times10^{-4}\ psi$$

for laminar flow, the only term we don't have is $(1 - x_i/L)^2$

$$.4248^2 = .18045$$

$$\Delta P_{lam} = \frac{3.6734 \times 10^{-4} \times .18045}{(9.524 \times 10^{-3})^2 \times 1.111 \times 10^{-4}} = 6.578 \times 10^{-5}\ psi$$

$$\Delta P_{Tot} = 7.422 \times 10^{-4} + 6.578 \times 10^{-5} = 8.0798 \times 10^{-4}\ psi$$

$\Delta P_{Hx}/\Delta P_{slot} = 7.65 \times 10^{-3}/8.0798 \times 10^{-4} = 9.47$ This is fine,

now compare with the pressure drop of section 1 of the heat exchanger.

Section 1 $\Delta P = .0623\ psi$

ratio $= .0623/8.0798 \times 10^{-4} = 77.1$

This ratio is of course much higher than necessary, but we must accept it in order to provide good flow distribution in Section 2.

The void volume of this slot is:

$$V = .2 \times .08 \times 2.34\pi = .1176 \text{ in.}^3$$

The slot depth of 0.05 in. will be incorporated in the final design of the VM.

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Ports To Ambient End of Hot Displacer

The primary concern in sizing these ports is to provide a low pressure drop, which minimizes motor input power required. Six ports are provided between the sump heat exchanger distribution slot and the ambient end of the hot displacer. For conservatism, we will consider one velocity head lost at the port exit (complete expansion) and one half velocity head lost due to the inlet contraction. Thus total losses are friction plus 1.5 velocity heads. The flow is the same as in the heat exchanger distribution slot, 0.001667 lb/sec. We will examine a series of port sizes. The flow length is 1.75 inches

Port Dia, in	A, ft.²	G, lb/ft²-sec	Velocity head, psi	Re	f	4fL/D	4fL/D+1.5	ΔP, psi
.05	1.364×10^{-5}	122.26	2.792 ⇐ Too high					
.09	4.418×10^{-5}	37.73	.266 ⇐ Too high					
.125	8.522×10^{-5}	19.56	.0715	14,100	.00771	.432	1.932	.138
.15	1.227×10^{-4}	13.58	.0345	11,747	.00807	.376	1.876	.0647
.165	1.485×10^{-4}	11.23	.0235	9714	.00846	.359	1.859	.0437

The original calculations for these ports, which were sized at 0.125 in. diameter, contained a mistake. The incorrect calculations indicated a pressure drop of about 0.04 psi. Thus we will change the port diameter to 0.165 inches.

The void volume of the new port size is:

$$V = \frac{\pi}{4}(.165)^2 \times 6 \times 1.75 = .2245 \text{ in.}^3$$

The old volume was 0.129 in^3

The difference is $.2245 - .129 = .0955 \text{ in}^3$

Lets see how this will affect refrigeration. The void volume trade factor in this region is 0.38 watts/in.^{3}

Thus refrigeration loss = 0.0955 X .38 = .036 watts. We can stand this loss, and the motor power will not be affected.

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Ports To Crankcase Region

The volume in the crankcase region of the VM varies with crankshaft rotation. Thus this region must be connected to the other active volumes of the machine, or excessive compression work will occur, drastically increasing motor power. The first step in sizing the ports from the sump heat exchanger distribution slot to the crankcase is to determine the maximum flowrate. The volume and pressure both vary as a function of crankshaft position. The flow rate is thus expressed as the perfect gas equation differentiated with respect to time. The derivation of this equation is presented in reference 4. The result is presented below.

$$\dot{w} = \frac{dM}{d\tau} = \left(\frac{d\theta}{d\tau}\right)\left(\frac{dM}{d\theta}\right) = \frac{d\theta}{d\tau}\left(\frac{P}{ZRT}\frac{dV_s}{d\theta} + \frac{V_s}{ZRT}\frac{dP}{d\theta}\right)$$

The crankcase volume is expressed in terms of the constant or void volume plus the reciprocating volume as a function of crankcase position.

The void, or non varying volume is 1462 in.^{3} The calculation of this value is presented on page 3, and the crankcase layout from which the calculations were made is shown on page 4.

The total volume in the crankcase is:

$$V_s = V_{SD} + \frac{V_H}{2} + \frac{V_c}{2} - \frac{V_c}{2}\left[\frac{\sin\left(\theta - \tan^{-1}\frac{V_c}{V_H}\right)}{\sin\left(\tan^{-1}\frac{V_c}{V_H}\right)}\right]$$

V_H = Volume swept by hot displacer wrist pin housing

$$V_H = \frac{\pi}{4}\,.905^2 \times .48 = 0.308 \text{ in.}^3$$

V_C = Volume swept by cold displacer wrist pin housing

$$V_C = \frac{\pi}{4}\,.4^2 \times .44 = 0.0553 \text{ in.}^3$$

Thus

$$V_s = 1.462 + \frac{.308}{2} + \frac{.0553}{2} - \frac{.0553}{2}\left[\frac{\sin\left(\theta - \tan^{-1}\frac{.0553}{.308}\right)}{\sin\left(\tan^{-1}\frac{.0553}{.308}\right)}\right]$$

$$V_s = 1.6436 - .1564 \sin(\theta - 10.17°)$$

Form 2437

AIRESEARCH MFG. CO.

DATE 3-9-73
PREPARED BY [signature]
CHECKED BY

CALC. NO.
MODEL
PART NO.

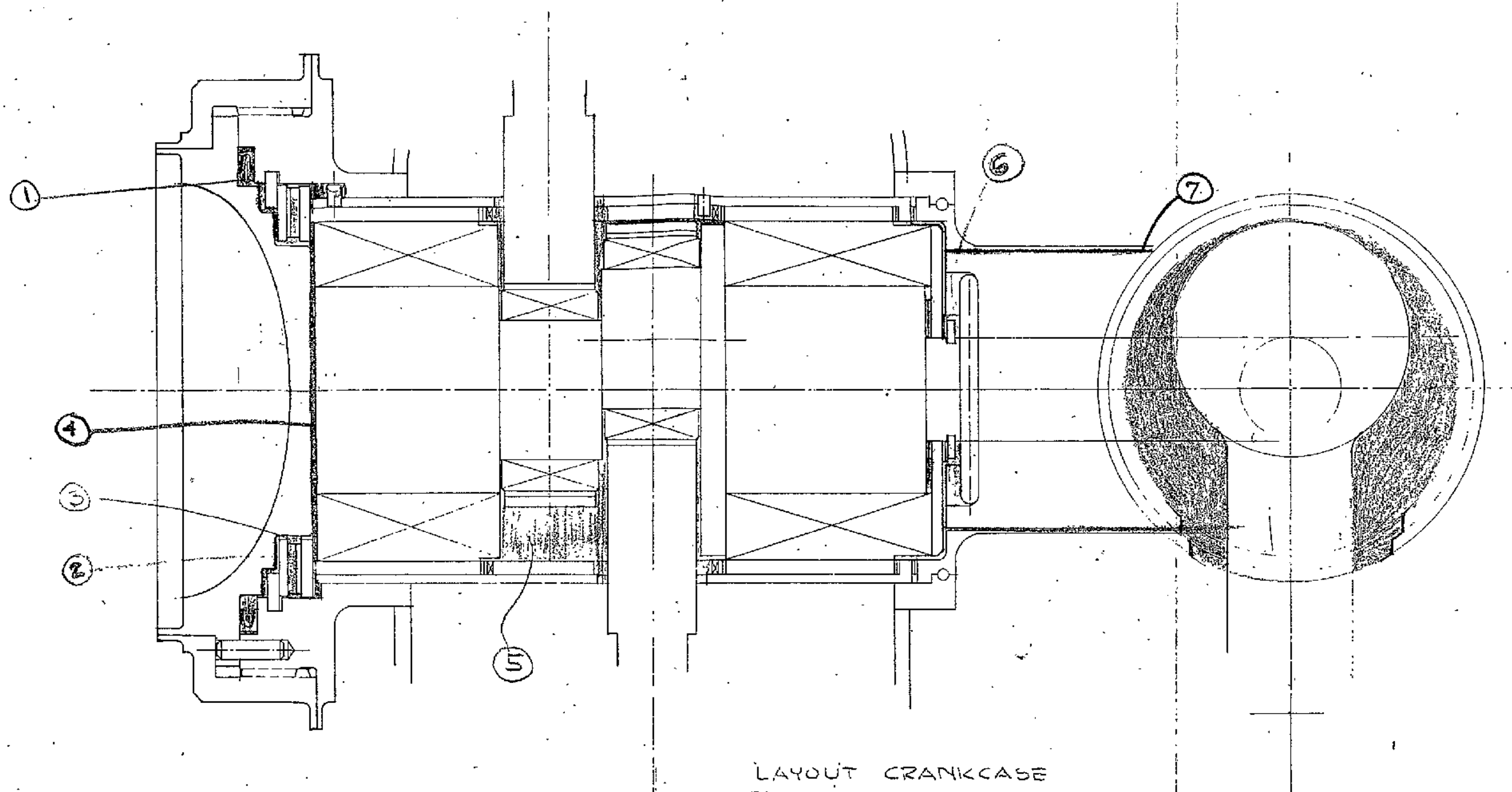
VOLUME CRANKCASE VUILLEMIER ½ WATT

1	$\frac{\pi}{4}(2.25^2 - 1.95^2).075$	=	.0742
2	$\frac{\pi}{4}\{(1.95^2 - 1.4^2).020 + (1.75^2 - 1.7^2).062\}$	=	.0373
3	$\frac{\pi}{4}\{(1.43^2 - 1.41^2).080 + (1.98^2 - 1.4^2).060 - (1.95^2 - 1.48^2).014\}$	=	.0782
4	$\frac{\pi}{4}\, 2^2\, .014$	=	.0440
5	$2[\frac{\pi}{4}\{(1.63^2 \times .5) - (1.15^2 \times .48)\} - (.6 \times .32) + (.11 \times 2)]$	=	1.1460
6	$\frac{\pi}{4}\{(1^2 - .5^2).022 + (1.6^2 - 1.55^2).2 + (1.6^2 - .5^2).015 + (.7^2 - .5^2).075 + 2(.080 \times .06 \times .4)$	=	.0820
			1.4617

REPORT
PAGE OF

FOLDOUT FRAME 1

FOLDOUT FRAME 2



LAYOUT CRANKCASE
TWICE SCALE

3-5-73

(5)

We also need the derivative of volume with respect to crankcase position (θ).

$$\frac{dV_s}{d\theta} = \frac{d}{d\theta}\left[1.6436 - .1564\sin(\theta - 10.17^\circ)\right]$$

$$= -.1564\cos(\theta - 10.17^\circ)$$

The time rate of change of θ is constant.

$$\frac{d\theta}{d\tau} = \frac{2\pi \times 400\ \text{rpm}}{60\ \text{sec/min}} = 41.867\ \text{rad/sec}.$$

now substitute into the expression for flowrate

$$\dot{w} = 41.867\frac{1}{ZRT}\left(P\frac{dV_s}{d\theta} + V_s\frac{dP}{d\theta}\right) = \frac{41.867\ \text{rad}\ \text{lb}_m\ {}^\circ\text{R}\ \text{ft}}{\text{sec}\ 1 \times 386.2\ \text{ft}\ \text{lb}_f \times 620^\circ\text{R} \times 12\ \text{in}}\left(\frac{\text{lb}_f\ \text{in}^3}{\text{in}^2\ \text{rad}}\right)$$

$$= 1.455 \times 10^{-5}\left(P\frac{dV_s}{d\theta} + V_s\frac{dP}{d\theta}\right)\ \text{lbm/sec}$$

now substitute expressions for V_s and $dV_s/d\theta$

$$\dot{w} = 1.455 \times 10^{-5} \left\{ \left[-.1564 P \cos(\Theta - 10.17^\circ) \right] + \left[1.6436 - .1564 \sin(\Theta - 10.17^\circ) \right] \frac{dP}{d\Theta} \right\}$$

This expression must be evaluated utilizing the pressure and rate of change of pressure from the ideal cycle program printout, figure 3-1. The detailed evaluation is presented on pages (7) and (8), and the results are plotted on page (9).

From the plot on page (9), we will use a maximum absolute value of flowrate of 0.00113 lb/sec to size the ports to the crankcase region. We can drill 4 ports from the sump heat exchanger distribution slot to the end of the ambient bearing housing. A relief in the housing at that point will allow access to the crankcase region. We will try 0.075 inch diameter ports first. The flow length is 1.5 inches, and we will assume 1.5 velocity heads lost by expansion and contraction losses.

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7

θ	$\bar{\theta}$	P, Psia	$\bar{P}$	ΔP	$\Delta P/\Delta\theta$	$(\theta-10.17)$	sin( )	cos( )
20		972.99						
	30		980.36	14.75	42.25	19.83	.3392	.9402
40		987.74						
	50		992.48	9.48	27.16	39.83	.6405	.7679
60		997.22						
	70		998.58	2.73	7.82	59.83	.8645	.5026
80		999.95						
	90		997.72	-4.46	-12.78	79.83	.9843	.1766
100		995.49						
	110		990.02	-10.94	-31.34	99.83	.9853	-.1707
120		984.55						
	130		976.68	-15.73	-45.06	119.83	.8675	-.4974
140		968.82						
	150		959.69	-18.26	-52.31	139.83	.6450	-.7641
160		950.56						
	170		941.35	-18.42	-52.77	159.83	.3448	-.9386
180		932.14						
	190		923.93	-16.41	-47.01	179.83	.002967	-.9999
200		915.73						
	210		909.38	-12.7	-36.38	199.83	-.3392	-.9407
220		903.03						
	230		899.13	-7.79	-22.32	219.83	-.6405	-.7679
240		895.24						
	250		894.14	-2.2	-6.30	239.83	-.8645	-.5026
260		893.04						
	270		891.25	3.58	10.25	259.83	-.7843	-.1766
280		896.62						
	290		901.14	9.05	25.93	279.83	-.9853	.1702
300		905.67						
	310		912.53	13.73	39.33	299.83	-.8675	.4974
320		919.40						
	330		927.93	17.07	48.90	319.83	-.6450	.7641
340		936.47						
	350		945.76	18.59	53.26	339.83	-.3448	.9387
360		955.06						
	10		964.025	17.93	51.36	-.17	-.002967	.9999

$$\Delta\theta = \left(\frac{180}{\pi}\right)^{-1} \times 20 = .347 \text{ rad}$$

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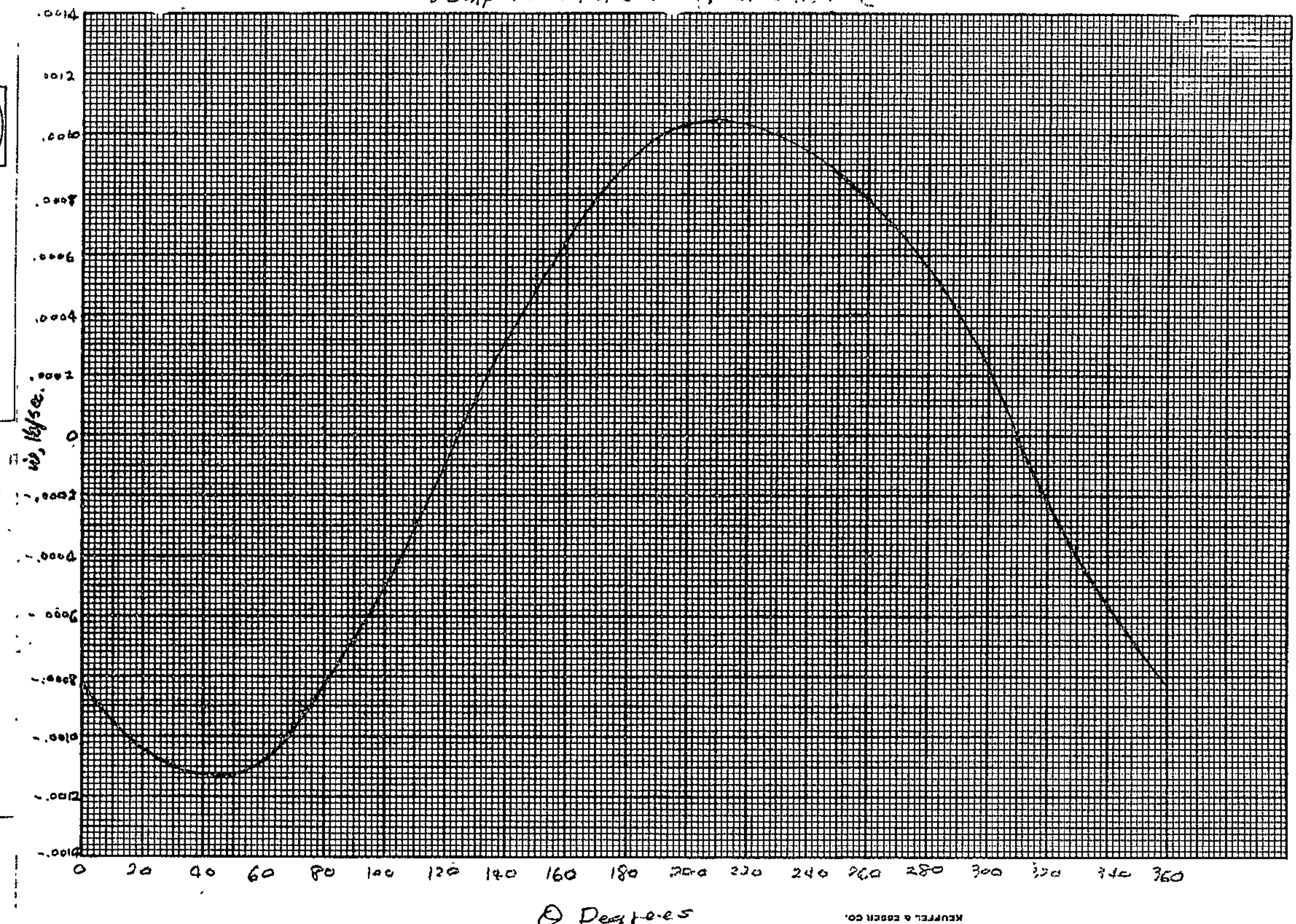
(8)

θ	(2) $-.1564\,P\cos(\theta-10.17)$	(1) $\left\{\left[1.6436-.1564\sin(\theta-10.17)\right]\frac{dP}{d\theta}\right\}$	(1)+(2)	$\dot{w}$, lb/sec
30	-142.2	67.3	-74.9	-.001090
50	-119.3	41.9	-77.4	-.001128
70	-78.45	11.8	-66.65	-.000970
90	-27.75	-19.02	-46.77	-.000680
110	26.44	-46.6	-20.26	-.000295
130	76.0	-67.9	8.1	.000118
150	114.8	-80.7	34.1	.000496
170	138.4	-83.9	54.5	.000793
190	144.6	-77.3	67.3	.000978
210	133.9	-61.6	72.3	.001052
230	108.2	-38.9	69.3	.001008
250	70.3	-11.2	59.1	.00086
270	24.63	18.4	43.03	.000626
290	-24.0	46.6	22.6	.000329
310	-71.0	69.9	-1.1	-.000016
330	-110.8	85.4	-25.4	-.00037
350	-139.0	90.6	-48.4	-.000705
10	-150.8	86.1	-64.7	-.000941

Sump Flowrate : Crank ...



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$$\text{Area} = \frac{\pi}{4 \times 144} .075^2 = 3.07 \times 10^{-5} \text{ ft}^2$$

$$\text{Flow rate/port} = \frac{.00113 \text{ lb/sec}}{4} = 2.825 \times 10^{-4} \text{ lb/sec}$$

$$G = \frac{2.825 \times 10^{-4} \text{ lb}}{3.07 \times 10^{-5} \text{ ft}^2 \text{ sec}} = 9.21 \text{ lb/ft}^2\text{-sec}$$

$$\text{Velocity head} = G^2/2g\rho = 9.21^2/2 \times 32.2 \times .577 \times 144 = .0158 \text{ psi}$$

$$Re = \frac{G D_H}{\mu} = \frac{9.21 \times .075}{1.445 \times 10^{-5} \times 12} = 3983$$

$$f = .084/Re^{1/4} = .0106$$

$$4fL/D = 4 \times .0106 \times 1.5/.075 = .846$$

$$\Delta P = (1.5 + .846)\ .0158 = 0.0371 \text{ psi}$$

This is of the same order of magnitude as the pressure drop of the ports to the backside of the hot displacer. Thus the effect on motor power will be minimal. In addition, the pressure forces acting on the ambient end of the hot displacer and the wrist pin housing will be approximatly equal. Thus no pressure imbalances will exist.

The void volume of these ports is:

$$V = 4\frac{\pi}{4}(.075)^2 \times 1.5 = .0265 \text{ in}^3$$

In addition, the ambient bearing will have a relief .030 in axial length to allow helium access to the crankcase region. The outer diameter of this relief is 1.525 in, and the volume of a disc. This size will be utilized.

$$V = \frac{\pi}{4} 1.525^2 \times .03 \doteq .0548 \text{ in}^3$$

This design is acceptable, and will be incorporated in the VM.

Cold Wrist Pin
Housing

Since the cold displacer and its wrist pin housing are of different diameters, helium must flow past the housing as the displacer oscillates. No special porting is necessary as the flow path is provided by a gap between the wrist pin housing and the bearing housing. We must first determine the proper maximum flow rate, and will then determine the optimum gap using the tradeoff factors. The flow rate is determined in a similar manner to that used for the crankcase. First an expression for volume as a function of angle is developed.

$$V = V_{min} + \frac{V_\Delta}{2} - \frac{V_\Delta}{2}\cos\theta$$

V_{min} is the volume when the cold displacer is at top dead center, and V_Δ is the swept volume. The displacer ~~od~~ diameter is 0.4 in, and the wrist pin housing is 0.8 in. OD.

The maximum axial clearance between the outboard end of the wrist pin housing and bearing housing is 0.020 in, and the stroke is 0.44 in.

$$\text{Thus } V_{min} = \frac{\pi}{4}(.8^2 - .4^2)\,.02 = .007508 \text{ in}^3$$

$$\text{and } V_{\Delta} = .007508 \times \frac{.44}{.02} = .1569 \text{ in}^3$$

$$\text{Thus } V = .007508 + \frac{.1569}{2} - \frac{.1569}{2}\cos\theta$$

$$= .08596 - 0.07845 \cos\theta$$

differentiating

$$dV/d\theta = .07845 \sin\theta$$

$$\dot{w} = \frac{d\theta}{d\tau}\,\frac{1}{ZRT}\left[P\frac{dV}{d\theta} + V\cdot\frac{dP}{d\theta}\right]$$

$\frac{d\theta}{d\tau}$ and $\frac{1}{ZRT}$ are identical to the corresponding terms for the crankcase region.

Substituting the appropriate Terms,

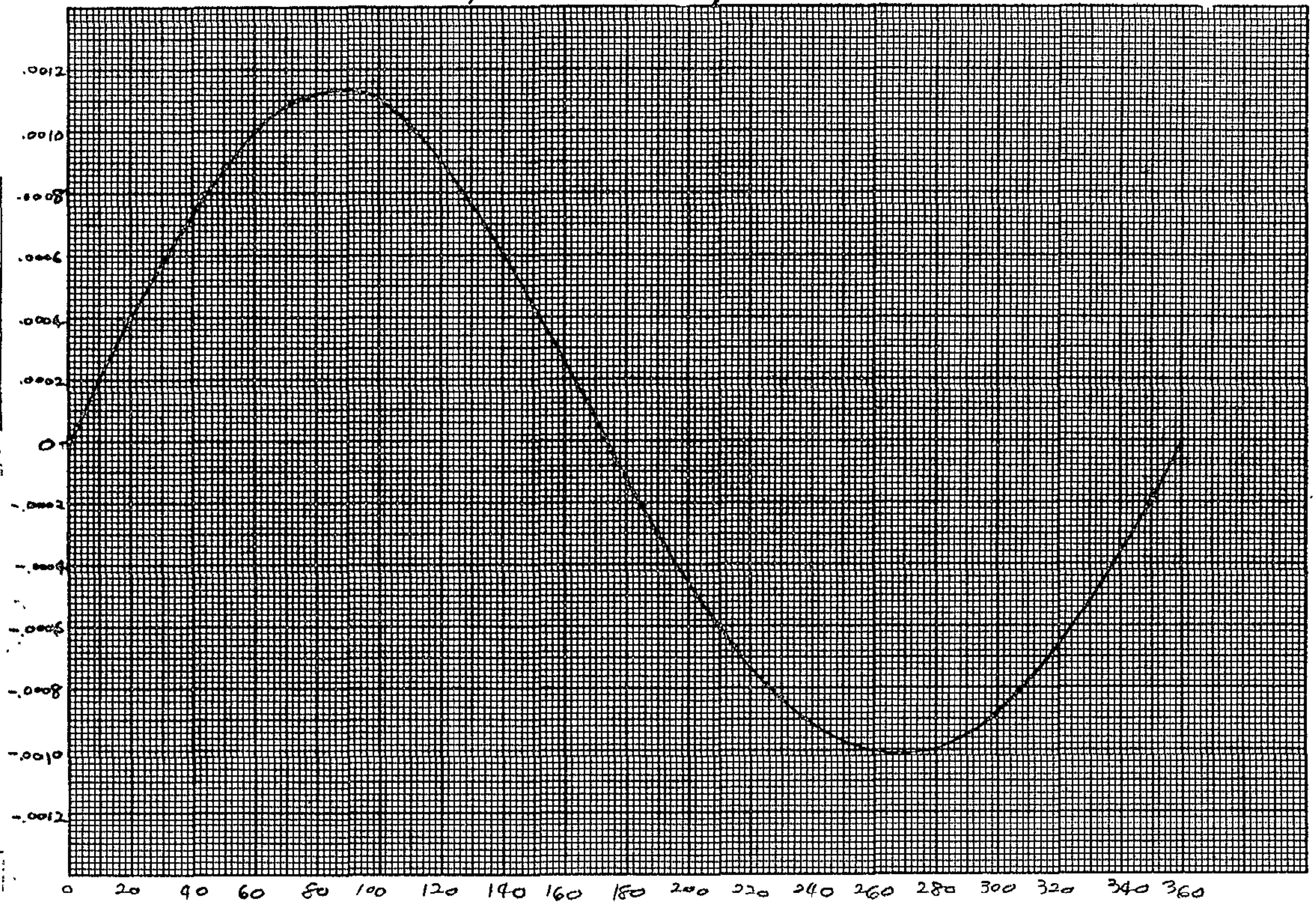
$$\dot{w} = 1.455 \times 10^{-5}\left[.07845\, P \sin\theta + (.08596 - .07845 \cos\theta)\frac{dP}{d\theta}\right]$$

The pressure and dP/dθ were evaluated for the crankcase region and also apply here. The stepwise evaluation of this equation is presented below.

θ	P	$\sin\theta$	$.078448 P \sin\theta$	$.078448 \cos\theta$	$dP/d\theta$	$\frac{dP}{d\theta}(\ )$	$\dot{w}$
10	964.02	.1736	13.13	.07726	51.36	.4426	.0001975
30	980.36	.5	38.45	.06794	42.25	.7611	.0005705
50	992.48	.766	59.64	.05042	27.16	.9652	.0008817
70	998.58	.9397	73.61	.02683	7.82	.4624	.001078
90	997.72	1	78.27	0	-12.78	0	.001139
110	990.02	.9397	72.98	-.02683	-31.34	-3.535	.001010
130	976.68	.766	58.69	-.05042	-45.06	-6.145	.0007645
150	959.69	.5	37.64	-.06794	-52.31	-8.050	.0004305
170	941.35	.1736	12.82	-.07726	-52.77	-8.613	.00006121
190	923.93	-.1736	-12.59	-.07726	-47.01	-7.673	-.0002948
210	909.38	-.5	-35.67	-.06794	-36.38	-5.599	-.0006005
230	899.13	-.766	-54.03	-.05042	-22.32	-3.044	-.0008304
250	894.14	-.9397	-65.91	-.02683	-6.30	-.7105	-.0009693
270	891.25	-1	-69.92	0	10.25	0	-.001017
290	901.14	-.9397	-66.43	.02683	25.93	1.533	-.0009462
310	912.53	-.766	-54.84	.05042	37.33	1.3976	-.0007776
330	927.73	-.5	-36.39	.06794	48.90	.8810	-.0005166
350	945.76	-.1736	-12.87	.07726	53.26	.4631	-.0001805

We will plot these results to determine the maximum flow.

cold displacer, sump end flow rate



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4

The maximum flowrate is 0.00114 lb/sec. Look at various clearances and calculate pressure drop and void volume. Convert these to refrigeration loss, and plot versus clearance to obtain a minimum. The length of flow path is 0.82 in

C, in.	Ac, ft²	G, lb/ft²-sec	Velocity Head, psi	Re	f, 24/Re	4fL/DH.5	ΔP
.005	8.73×10^{-5}	13.06 ~~6.53~~	.0318	753	.0318	11.92	.38
.01	1.747×10^{-4}	6.53	.00795	753	↓	6.72	.0534
.015	2.618×10^{-4}	4.9	.00448	753		4.98	.0223
.02	3.49×10^{-4}	3.265	.00199	753		4.11	.00819
.04	6.98×10^{-4}	1.632	.000497	753		2.81	.0014

Volume = L x Ac x 144

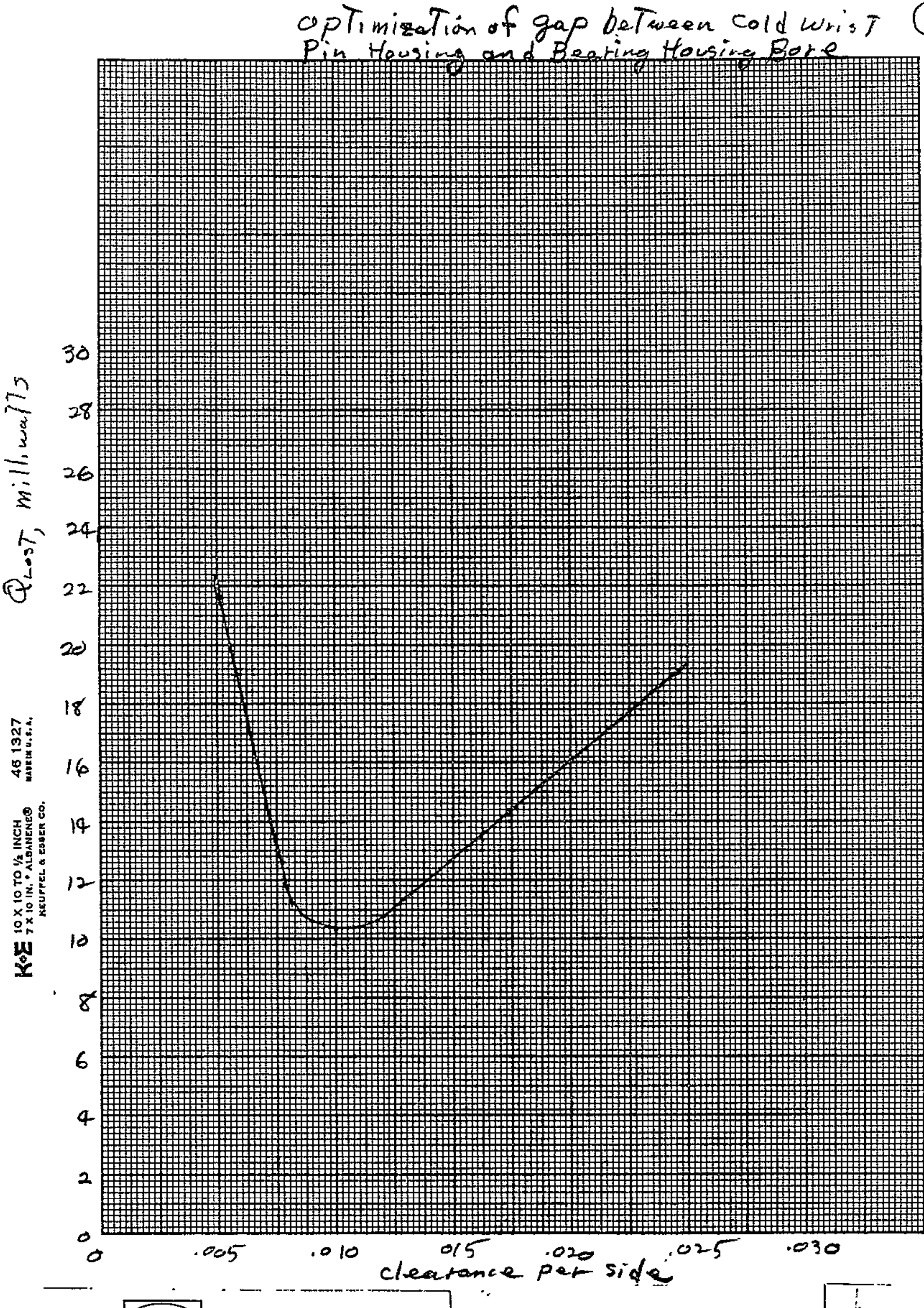
Void Volume Trade Factor = 0.38 watts/~~psi~~ in.³

Pressure Drop Trade Factor = 0.04738 watts/psi

C, in	Void Vol, in.³	Q_{lost}, Vol, watts	Q_{lost}, ΔP, watts	Q_{lost}, Total, watts
.005	.0103	.00392	.018	.02192
.01	.0206	.00784	.00254	.01038
.015	.0309	.01173	.00106	.01279
.02	.0412	.01568	.00039	.01607
.04	.0825	.03136	.000067	.03143

Plot These on next Page

(6)


opTimization of gap between cold wrisT Pin Housing and Bearing Housing Bore
QLosT, milliwaTTs
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
0
.005
.010
.015
.020
.025
.030
clearance per side
K&E 10 X 10 TO ½ INCH 7 X 10 IN. ALBANENE® KEUFFEL & ESSER CO.
46 1327 MADE IN U.S.A.

(5)

The optimum gap is .010 inches clearance per side. This value will be incorporated into the VM design.

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Sump Void Volume

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The Sump region void volume is summarized here

Bearing Support (Cold Displacer)	.1106
Filler Block Flow Passages	.1783
Tapered Slot at Hx Entrance	.0174
Sump Heat Exchanger	.348
Sump Heat Exchanger Distribution Slot	.1176
Hot Displacer Ports	.2245
Hot Regenerator Retainer	.081
Crankcase	1.462
Crankcase Ports	.0813
Flow Distributor and Associated Hardware	.1024
Maximum Additional Crankcase Volume (Fully Extended Hot Displacer)	.336
Total Volume, in^3.	3.0591

SECTION 13

DESIGN OF COLD END SEAL

SECTION 13

DESIGN OF COLD END SEAL

The cold end seals function to control the rate of leakage which bypasses the cold regenerator. This function is critical in the refrigerator design since leakage past the seals can result in a significant loss in thermal performance. Leakage past the seals bypasses the regenerator by flowing through the annular space between the cold displacer and cold cylinder walls. At low leakage rates, the displacer and cylinder walls effectively regenerate the leakage fluid temperatures and the resulting thermal losses are small. As leakage rates increase, the walls can no longer function as an effective regenerator, and significant losses in overall thermal performance occur.

DESIGN CONFIGURATION

The basic configuration of the cold end sealing system is shown in Figure 13-1. Seals incorporated in the cold end sealing system, from the cold end toward the sump end of the cold displacer, are as follows:

(a) A 0.0254 m (1.0 in.) long, close-fit annular seal with a clearance between the inner wall of the regenerator and the seal of 0.0000635 m (0.0025 in.) maximum.

(b) A 5-groove labyrinth seal with a tip clearance of 0.0000635 m (0.0025 in.), a groove spacing of 0.00127 m (0.05 in.), and a nominal tip width of 0.000127 m (0.005 in.)

(c) The first linear bearing, which acts as an annular seal with a clearance of 0.00001017 m (0.0004 in.) and a length of 0.0254 m (1.0 in.).

(d) The bearing support member, which acts as an annular seal with a clearance of 0.000089 m (0.0035 in.) and a length of 0.0429 m (1.69 in.).

(e) The second linear bearing which is identical to the first.

The arrangement of the machine is such that these sealing elements are in series.

As with the GSFC 5 watt VM the linear bearings and the bearing support are used as part of the sealing system. The mechanical arrangement of the machine allows the use of these components as seals without penalty. The only disadvantage in their use is that the leakage rate is dependent on bearing clearance and increases as the bearing wears. However, the loading and rate of wear of these bearings is low, and will provide over two years of operation before bearing wear affects the performance (even in the worst case analysis).

METHOD OF ANALYSIS

The correlation for analyzing the leakage past both labyrinth and annular seals were developed and modified to match the seal test data in Ref. 5. The appropriate equations are:

for Labyrinth seal

$$\Delta P_i = \frac{\dot{\omega}_L{}^2 T_o R \eta}{C^2 A^2 \quad P_o K_L{}^2 \zeta \; 2g_c} \tag{13-1}$$

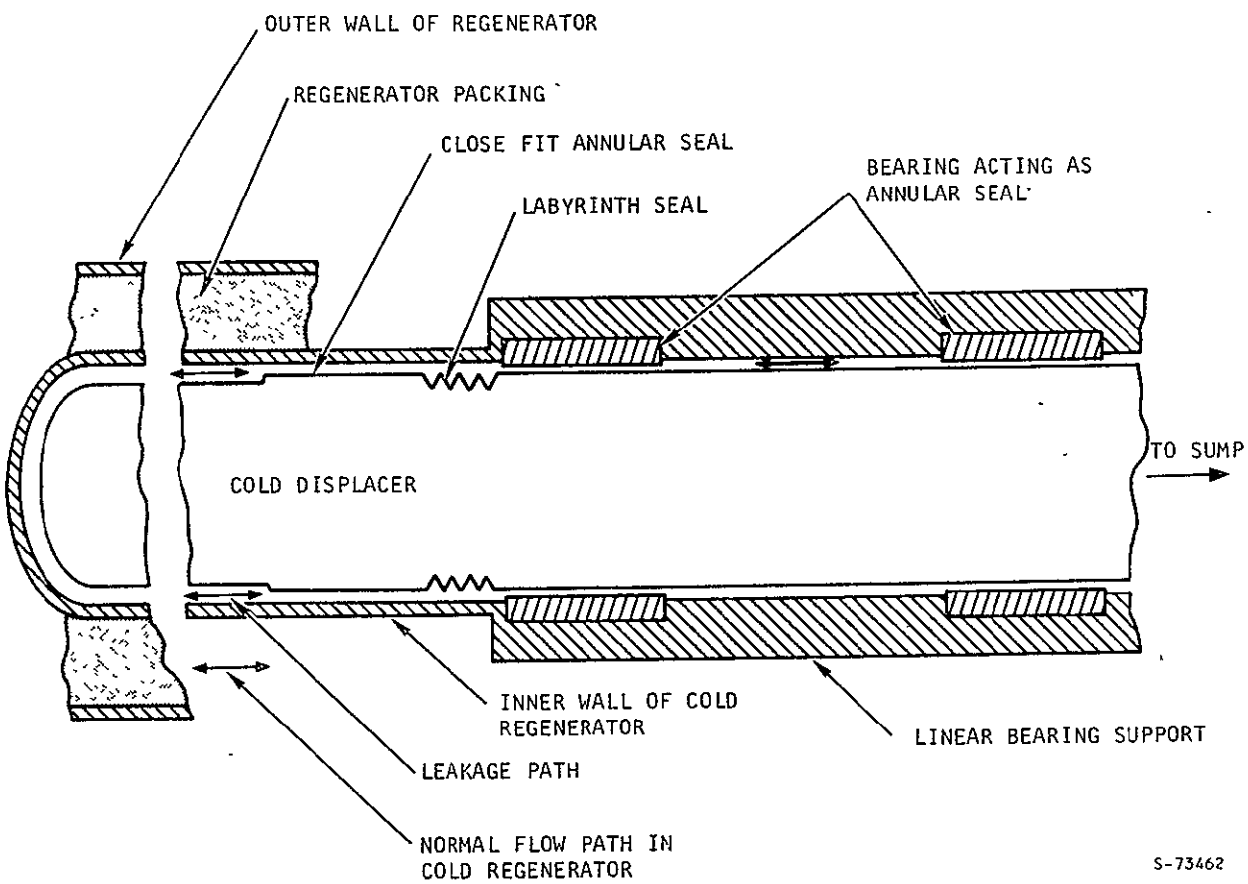

Figure 13-1. Cold Displacer Sealing Design Configuration Schematic

for Annular Seal

$$\Delta P_i = \frac{\dot{\omega}_A L}{K_A \rho} \left[\frac{48\mu}{D_H^{\;2} g_c A} \right] \tag{13-2}$$

Then noting that:

$$\Delta P_T = \sum_{0}^{N} \Delta P_i = \sum_{0}^{N} f_i(\dot{\omega}_i) \tag{13-3}$$

Where

P_T = the total pressure drop across the series of sealing elements

i and N = the identity and number of elements respectively

then further noting that

$$\dot{\omega}_T = \dot{\omega}_L = \dot{\omega}_A = \dot{\omega}_i \tag{13-4}$$

that is, the flow past each element is the same, the pressure drop can be computed as a function of leakage rate from Equation (13-3).

PERFORMANCE CHARACTERISTICS

Leakage Rate

The configuration of the cold end of the refrigerator is such that seal elements can be added in series (or subtracted) as desired by providing flow passages into the active cycle volume at different locations. Figure 13-2 gives the leakage rate as a function of pressure drop for the combination of all seal elements in series. The design combines all possible seal elements to minimize the losses due to leakage.

Figure 13-2 presents leakage rate data for seal systems with both new bearings and bearings after 2 years of wear. As the bearings wear, the clearance of the annular seal (the seal which the bearings form with the cold displacer) increases and hence the leakage rate increases. The bearing wear rate used in the analysis is based on data for the wear rate experienced in the linear bearings of the AiResearch IR&D VM refrigerator after 5000 hours of operation (Reference 6). The wear rate experienced during the test has been doubled for conservatism.

The dashed vertical line in Figure 13-2 corresponds to the maximum limit of pressure drop across the cold end seal. During the preliminary design phase, it was planned to utilize a special screening process in order to assure uniform size spherical slot in the cold regenerator. This would insure that the pressure drop was as predicted for the uniformly packed bed of spheres. The special screening process has since been eliminated, as it is felt to be unnecessary. The resulting minor variations in regenerator packing size may

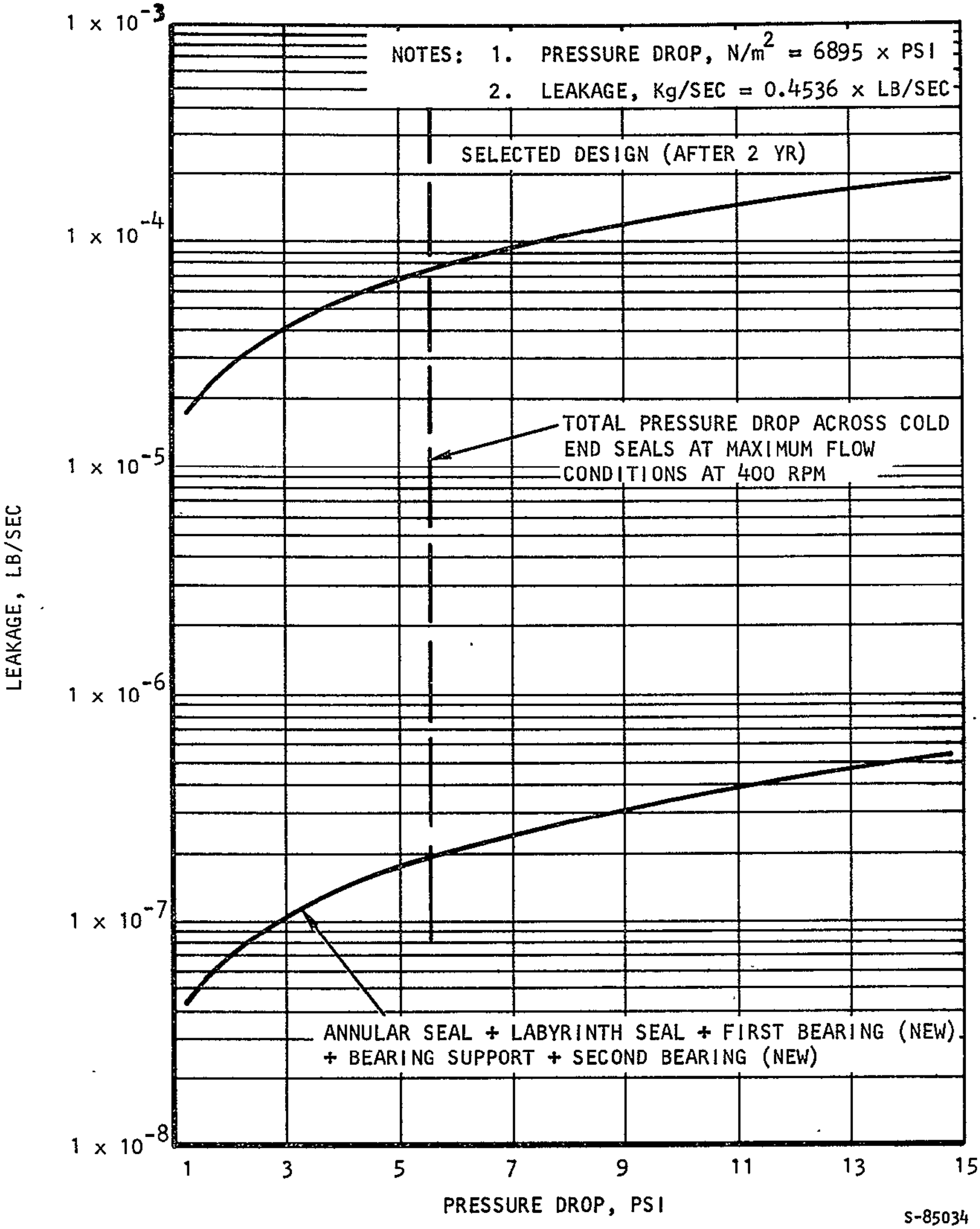

Figure 13-2. Cold-End Seal Leakage--Pressure Drop Characteristics

lead to pressure drop increases. Therefore, as a further measure of conservatism the vertical line shown on Figure 13-2 includes a regenerator pressure drop 2.5 times greater than that calculated.

Thermal Losses

The cold end leakage is important, in that it influences refrigerator thermal performance. A good approximation of the thermal losses associated with the cold end leakage is provided by the simple model developed below.

Figure 13-3 depicts the basic elements of the thermal loss model employed. Taking an element of length (dX) along the annular flow passage between displacer and inner regenerator wall, the following differential equation can be written:

$$\frac{dT_f}{dX} + \frac{hA_c}{\dot{\omega}C_p} T_f = \frac{hA_c}{\dot{\omega}C_p} T_\omega \qquad (13\text{-}5)$$

Where

T_f = temperature of the leakage gas at any location X

T_ω = temperature of the regenerator and displacer walls at any location X

h = local heat transfer coefficient between leakage gas and the surrounding walls

A_c = heat transfer area per unit length along the leakage path

$\dot{\omega}$ = rate of leakage

C_p = heat capacity of leakage gas

Then if a linear temperature distribution is assumed along the displacer and regenerator walls and ratio of heat transfer coefficient to gas heat capacity is taken as a constant, Equation 13-5 can be written as:

$$\frac{dT_f}{dX} + \alpha T_f = \alpha (T_a - T_o) \frac{X}{X_e} + \alpha T_o \qquad (13\text{-}6)$$

Where the new terms are:

$$\alpha = \frac{hA_c}{\dot{\omega}C_p}$$

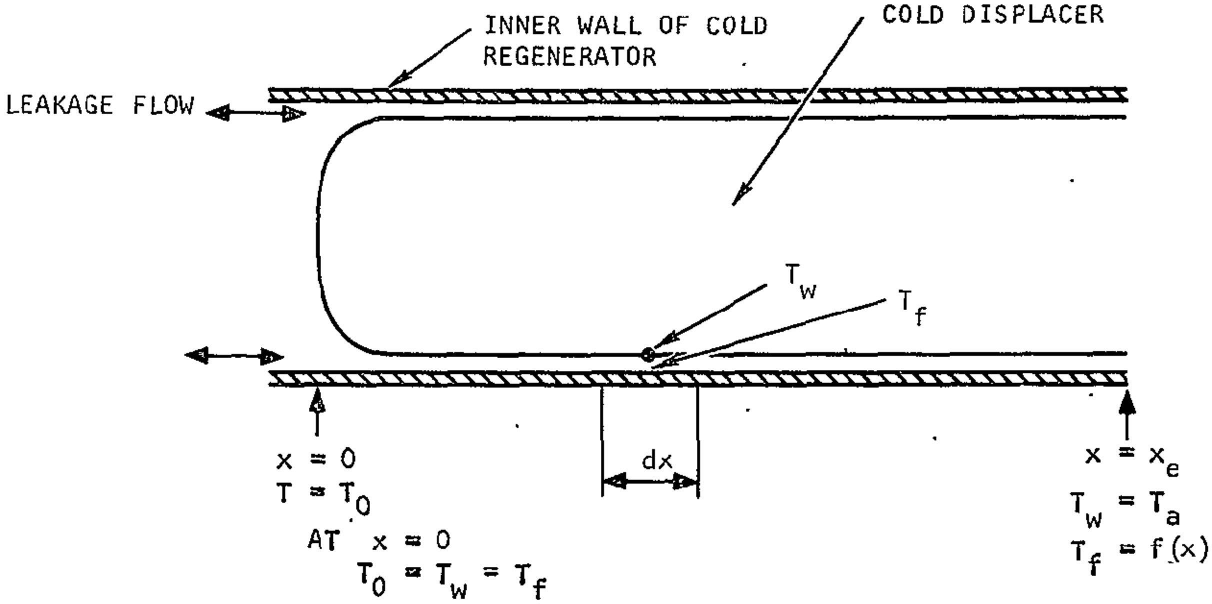

Figure 13-3. Leakage Thermal Loss Model

T_a = wall temperature at the sump end of the displacer

T_o = refrigeration temperature or wall temperature at X = o

X_e = length of the displacer

Solving Equation 13-6 with the boundary conditions of $T_f = T_o$ at X = o yields:

$$T_f = \frac{T_a - T_o}{X_e} \left\{ X - \frac{1}{\alpha} + \frac{1}{\alpha} e^{-\alpha X} \right\} + T_o \qquad (13\text{-}7)$$

If the leakage flow were completely regenerated by the walls, the temperature of the fluid would be that of the wall at the end of the displacer; that is:

$$T_f \longrightarrow T_a \text{ @ } X = X_e$$

or if flow (leakage) is considered in the reverse direction

$$T_f \longrightarrow T_o \text{ @ } X = 0$$

In setting up the relation for T_f, this latter condition was used as a boundary condition assuming flow in the positive X direction (see Figure 13-3). Relationships for flow in the reverse direction are similar but are not developed here.

In the actual case, due to the cyclic operation of the VM refrigerator, the leakage flow does reverse direction. The leakage losses can be estimated, however, by considering flow in one direction with an appropriate time span. With this consideration the thermal losses per cycle due to leakage can be expressed as:

$$Q_L = \oint |\dot{\omega}| \ c_p \left\{ (T_a - T_o) \frac{1}{\alpha X_e} \left(1 - e^{-\alpha X_e}\right) \right\} d\tau \qquad (13\text{-}8)$$

Where

$\oint$ implies integration around the cycle and

τ = time

Assuming a constant leakage rate, the leakage thermal losses can be expressed as:

$$\dot{Q}_L = \dot{\omega} C_p \left\{ (T_a - T_o) \frac{1}{\alpha X_e} \left(1 - e^{-\alpha X_e}\right) \right\} \qquad (13\text{-}9)$$

Figure 13-4 gives the losses (Equation 13-9) as a function of leakage rate. The thermal losses at leakage rates corresponding to the design limit levels of pressure drop and associated cold end leakage rates (see Figure 13-2) are shown in Figure 13-4. It is noted these losses are estimated for the machine after two years of wear; thermal losses due to leakage are negligible for a new machine. In fact, the leakage thermal losses for the machine after 2 years of wear are expected to be considerably lower than indicated in Figure 13-4 for two reasons. First, the bearing wear rate used in estimating the leakage is considered to yield a conservative result as previously mentioned. Second, constant leakage rates at their maximum levels were used in estimating the losses by use of Equation 13-9. In the actual case, the leakage rate is a periodic function with an average value on the order of 70 percent of the maximum. Thus not only are the losses lower due to the decreased leakage flow, but the regenerator and displacer walls more effectively regenerate the leakage gas, further reducing the losses below the maximum values indicated in Figure 13-4.

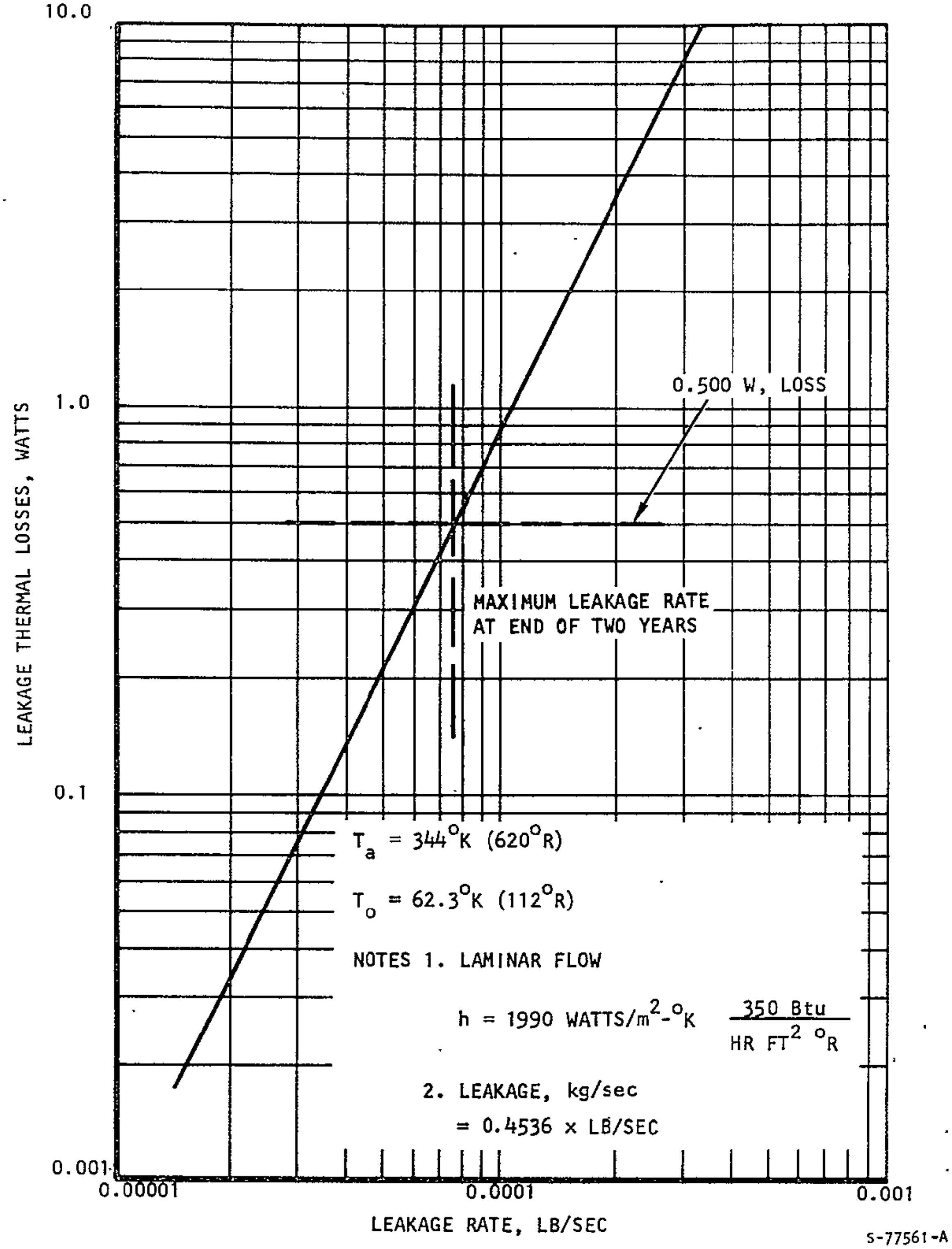

Figure 13-4. Cold End Thermal Losses Due to Leakage as a Function of Leakage Rate

Cold End Leakage

The proceedure of determining the effect on refrigeration of leakage past the cold displacer sealing system is broken up into two steps. First, the leakage flow is determined, and then the effect of bypassing that amount of flow around the regenerator is calculated.

I Leakage

The sealing system consists of four individual seals placed in series with each other. The pressure drop vs flow characteristic of each is calculated by methods developed for the 5 watt VM, and then combined for the entire system. The four sealing system components are (1) a close fit annular seal, (2) a labyrinth seal, (3) the bearing support, and (4) the two linear bearings. The leakage characteristics of the bearings are calculated for two conditions: (a) new bearings and (b) bearings worn an amount equivalent to

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Two years of operation.

1. Annular seal

from The 5 watt seal Testing,

$$\dot{w} = \frac{\rho \Delta P}{L}\left[\frac{D_h^2 g_c A_c}{48\mu}\right] \quad \text{in laminar flow}$$

The clearance per side is 0.0025 in and The length is 1. inch $D_h = 2C = .005$ in.

Physical Properties:

$A_c = \pi D C = .4 \times \pi \times .0025$
$= .00314 \text{ in}^2$

$\mu = .0521$ lb/ft-hr

$\rho = 0.5808$ lb/ft.^{3}

Thus we will solve for $\dot{w}$ as a function of ΔP

$$\dot{w} = \frac{.5808 \text{ lb}_m \; (\Delta P \text{ lb}_f)}{\text{ft}^3 \; 1 \text{ in} \; (\text{in}^2)}\left[\frac{(.005)^2 \text{in}^2 \times 32.2 \text{ lb}_m\text{-ft} \times .00314 \text{ in}^2 \text{ ft-hr} \; 3600 \text{ sec} \; \text{ft}}{48 \times .0521 \text{ lb}_m \; \text{lb}_f\text{-sec}^2 \; \text{hr} \; 12 \text{ in}}\right]$$

$$= 1.761 \times 10^{-4} \, \Delta P$$ with ΔP in psi and $\dot{w}$ in lb/sec

now calculate flow for a range of ΔP, and Then check To be sure That laminar flow occurs.

ΔP, psi	$\dot{w}$, lb/sec
0.1	1.761×10^{-5}
1.0	1.761×10^{-4}
2.0	3.52×10^{-4}
3.0	5.28×10^{-4}
5.0	8.81×10^{-4}
10.0	1.761×10^{-3}
0.2	3.52×10^{-5}
0.5	8.81×10^{-5}

$$Re_{max} = \frac{G D_h}{\mu} = \frac{\dot{w}_{max} D_h}{A_c \, \mu} = \frac{1.761 \times 10^{-3} \text{ lbm}}{\text{sec } .00314 \text{ in}^2} \; \frac{.005 \text{ in} \; \text{ft-hr} \; 3600 \text{ sec} \; 12 \text{ in}}{.0521 \text{ lbm} \quad \text{hr} \quad \text{ft}}$$

$= 2325$ turbulent (or Transition)

calculate maximum flow where the flow is laminar, ie at $Re = 2100$

$$\dot{w} = \frac{Re \, A_c \, \mu}{D_h} = \frac{2100 \times .00314 \text{ in}^2 \times .0521 \text{ lbm}}{.005 \text{ in} \cdot \frac{12 \text{ in}}{\text{ft}} \; \frac{3600 \text{ sec}}{\text{hr}} \; \text{ft-hr}} = 1.59 \times 10^{-3} \text{ lb/sec.}$$

The pressure drop corresponding to this flow is $\Delta P = \frac{\dot{w}}{1.761 \times 10^{-4}} = \frac{1.59 \times 10^{-3}}{1.761 \times 10^{-4}} = 9.029 \text{ psi.}$

A pressure drop this high across only one component of the sealing system will not occur, and we may proceed with the analysis without exceeding the limits of the correlations.

2. Bearing Support

This is essentially another annular seal, with a clearance of .0035 inches and a flow length of 1.5 inches. Thus the flow-pressure drop relationship may be ratioed from the one developed above.

$A_c = .4\pi \times .0035 = .0044 \text{ in}^2, D_h = .007 \text{ in}$

$$\dot{w} = 1.761 \times 10^{-4} \Delta P \underbrace{\left(\frac{1.0}{1.5}\right)}_{L} \underbrace{\left(\frac{.007}{.005}\right)^2}_{D_h} \underbrace{\left(\frac{.0044}{.00314}\right)}_{A_c} = 3.22 \times 10^{-4} \Delta P$$

now calculate ΔP at same flows determined in (1.) above.

$\dot{w}$, lb/sec	ΔP, psi
1.761×10^{-5}	.0547
3.52×10^{-5}	.1093
8.81×10^{-5}	.274
1.761×10^{-4}	.547
3.52×10^{-4}	1.093
5.28×10^{-4}	1.64
8.81×10^{-4}	2.74

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3. Labyrinth Seal

From the 5 watt Thermal Notebook, the labyrinth seal flow rate - pressure drop characteristic may be reduced to

$$\dot{w} = 0.399\, A_c \left[\frac{P_o \Delta P}{T_o N}\right]^{1/2}$$

The Labyrinth seal contains 5 elements, thus N = 5. The clearance is the same as the close fit annular seal, Thus $A_c = .00314\ in.^2$ P_o is 1000 psia, and $T_o = 620°R$.

Thus

$$\dot{w} = 0.399 \times .00314 \left[\frac{1000}{5 \times 620}\right]^{1/2} \Delta P^{1/2} = .000712\, \Delta P^{1/2}$$

and

$$\Delta P = \left(\frac{\dot{w}}{.000712}\right)^2 = \frac{\dot{w}^2}{5.069 \times 10^{-7}}$$

$\dot{w}$, lb/sec	ΔP, psi
1.761×10^{-5}	6.118×10^{-4}
3.52×10^{-5}	2.444×10^{-3}
8.81×10^{-5}	.0153
1.761×10^{-4}	.0612
3.52×10^{-4}	.2444
5.28×10^{-4}	.550
8.81×10^{-4}	1.531

4. Linear bearings

A. New Bearings

This is again a close fit annular seal, and we may ratio the length, flow area, and hydraulic diameter. The combined length is 2 inches, clearance is .0004 in., and D_h = .0008 in.

$$A_c = .4\pi \times .0004 = 5.03 \times 10^{-4} \text{ in}^2$$

$$\dot{w} = 1.761 \times 10^{-4}\left(\frac{1}{2}\right)\left(\frac{.0008}{.005}\right)^2\left(\frac{.000503}{.00314}\right)\Delta P = 3.61 \times 10^{-7} \Delta P$$

ΔP, psi	$\dot{w}$, lb/sec
1.0	3.61×10^{-7}
5.0	1.81×10^{-6}
10.0	3.61×10^{-6}
15.0	5.42×10^{-6}

These flows are so low compared to those through the other components at comparable pressure drops that the bearings will be the controlling seal in a new machine. Thus the above numbers will be used as is for the new machine.

B. Bearings worn Two years.

The distance Traveled by The displacer in Two years is $D = 2 \times .44 \times 400 \times 1.05 \times 10^6 = 3.7 \times 10^8$ in.

We now need a wear rate. The IR+D VM exhibited a linear bearing wear rate of 5×10^{-6} mg/m-in^2 after 5000 hours of operation. we will use a rate of 10×10^{-6} mg/m-in^2 for conservatism.

Converting units, $\Delta M = \frac{10^{-5} \text{ mg}}{\text{m-in}^2} \frac{\text{m}}{37.37 \text{ in}} \frac{2.2 \text{ lb}}{\text{kg}} \frac{10^{-6} \text{ kg}}{\text{mg}} = .5588 \times 10^{-12} \frac{\text{lb}}{\text{in}^2\text{-in}}$

The change is clearance may be expressed as $\Delta C = \frac{\Delta M\, L}{\rho} = \frac{.5588 \times 10^{-12} \text{ lb} \times 3.7 \times 10^8 \text{ in in}^3}{\text{in}^2\text{-in} \quad .21 \text{ lb}} = .0009845$ in.

we will use 0.001 inches wear, thus The clearance at The end of Two years is .0014 inches. $A_c = .4\pi \times .0014 = .001759$ in^2 and $D_h = .0028$ in.

again, from annular seal

$$\dot{w} = 1.761\times10^{-4}\left(\frac{1}{2}\right)\left(\frac{.0028}{.005}\right)^2\left(\frac{.001759}{.00314}\right)\Delta P = 1.547\times10^{-5}\Delta P$$

$\dot{w}$, lb/sec	ΔP, psi
1.761×10^{-5}	1.139
3.52×10^{-5}	2.277
8.81×10^{-5}	5.699
1.761×10^{-4}	11.39

It looks like we will cover the ΔP range of interest with these values of flow. Therefore add the ΔP of the components of the sealing system.

5. Total ΔP of sealing system.

Flow, lb/sec	ΔP, psi: Annular Seal	Bearing Support	Labyrinth Seal	2yr Worn Bearings	Total ΔP
1.761×10^{-5}	0.1	.0547	6.112×10^{-4}	1.139	1.294
3.52×10^{-5}	0.2	.1093	2.444×10^{-3}	2.277	2.588
8.81×10^{-5}	0.5	.274	.0153	5.699	6.488
1.761×10^{-4}	1.0	.547	.0612	11.39	12.998

This flow-pressure drop characteristic of the sealing system will be used to determine seal losses.

II Refrigeration loss due to leakage

Again from the 5 watt analysis, we obtain the expression for the helium ΔT after passing along the displacer. The amount of heat carried to the cold end of the displacer by this leakage gas is a direct loss of refrigeration.

The ΔT between the helium and the cold end of the displacer is:

$$\Delta T = \left(T_{sump} - T_{cold}\right)\left[\frac{-1}{X_L}\left(X_L - \frac{1}{\alpha} + \frac{1}{\alpha} e^{-\alpha X_L}\right)\right]$$

where $\alpha = \frac{hA_c}{w c_p}$ and $X_L = 5 \text{ in} = .417 \text{ ft}$.

A Nusselt Number of 8.23 will be used,

$$h = \frac{Nu\,k}{D_h} = \frac{8.23 \times .0708\ \text{BTU}\ 12\ \text{in}}{\text{ft-hr-}^\circ\text{F} \times .02\ \text{in}\ \text{ft}} = 350\ \text{BTU/ft}^2\text{-hr-}^\circ\text{F}$$

The only term that varies with flow rate is α, so lets set up the constants in the equation and look at a range of flows.

$$\Delta T = (620-112)\left[1-\frac{1}{.417}\left(.417-\frac{1}{\alpha}+\frac{1}{\alpha}e^{-.417\alpha}\right)\right]$$

$$\alpha = \frac{350\ \text{BTU}\ \times .23\ \text{FT}^2\ \text{lb}_m\cdot{}^\circ\text{F}\quad \text{sec}\quad \text{hr}}{\text{FT}^2\text{-hr}\cdot{}^\circ\text{F}\quad \text{FT}\times 1.246\ \text{BTU}\quad \dot{w}\ \text{lb}_m\ 3600\ \text{sec}} = \frac{\cancel{.0179}\ .01794}{\dot{w}}\ \frac{1}{\text{FT}}$$

$\dot{w}$	α	αX_L	$e^{-\alpha X_L}$	ΔT, °R	$\dot{Q}_L$, $\dot{w}c_p\Delta T$, watts
10^{-5}	1794	748	0	~~1628~~ .679	~~.0314~~ .0087
10^{-4}	179	74.8	≈ 0	~~16.28~~ 6.805	~~3.159~~ .874
10^{-3}	17.94	7.48	5.6×10^{-4}	~~162.7~~ 67.9	~~219.7~~ 89.2
2×10^{-5}	897	374	0	1.35	.0357
5×10^{-5}	359	149	≈ 0	3.39	.223
2×10^{-4}	89.7	37.4	≈ 0	13.58	3.57
5×10^{-4}	35.9	14.9	3.15×10^{-7}	33.93	22.29

we will plot these, over a much more limited range than calculated above, and then determine refrigeration loss with two year worn bearings.

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III Total Refrigeration loss.

The regenerator ΔP is 1.8 psi. We will multiply this by 2.5 to account for the possibility of deformed spheres.

$$\Delta P_{regen} = 1.8 \times 2.5 = 4.5 \text{ psi}$$
$$\text{other cold end } \Delta P = \underline{1.03 \text{ psi}}$$
$$\text{Total } \Delta P = 5.53 \text{ psi}$$

from plot of flow vs ΔP, $\dot{w} = 7.6 \times 10^{-5}$

and from plot of Q_{LOST} vs flow,

$$Q_{LOST} = 0.5 \text{ watts}$$

SECTION 14

DESIGN OF HOT END SEAL

SECTION 14

DESIGN OF HOT END SEAL

The hot-end seal functions to control the leakage rate of the working fluid which bypasses the hot regenerator. Leakage bypasses the regenerator by flowing through the annular space between the hot displacer and the inner wall of the regenerator. Excessive leakage results in a loss in thermal performance, therefore hot end seal design is an important consideration.

At low leakage rates, the hot displacer and cylinder walls effectively regenerate the leakage fluid temperatures and the resulting thermal losses are small. As the leakage rates increase, the walls can no longer function as an effective regenerator; thus significant losses in overall thermal performance result.

DESIGN CONFIGURATION

The hot end seal is unchanged from the preliminary design configuration (Figure 14-1) and consists of the following two elements, from the hot end toward the sump end: (1) a 5-groove labyrinth seal with a tip clearance of 0.0000635 m (0.0025 in.), a groove spacing of 0.00127 m (0.05 in.), and a nominal tip width of 0.000127 m (0.005 in.) and (2) a 0.0254 m (1.0 in.) long close fit annular seal with clearance between the inner wall of the regenerator and the seal of 0.0000635 m (0.0025 in.) maximum.

These sealing elements are similar to those used in the cold-end seal except for the diameter. One major difference is the use of linear bearings as part of the sealing system for the cold end. The bearings are not used in the hot-end seal and thus the performance of this seal is independent of wear (or operational time).

It should be noted that the seal is located in the highest temperature region possible within the machine. The leakage rate is an inverse function of the temperature, therefore, this location minimizes the leakage rate for a given seal configuration. The seal design selected provides very low losses as discussed in the following paragraphs.

Method of Analysis

The method of analysis is identical to that used for the cold end seals.

Performance Characteristics

Figure 14-2 gives the hot-end seal leakage rate as a function of the pressure drop across the seal. The design limit pressure drop and the associated leakage rate shown in Figure 14-2 corresponds to the preliminary design goal maximum pressure drop across the seal. This pressure drop includes the

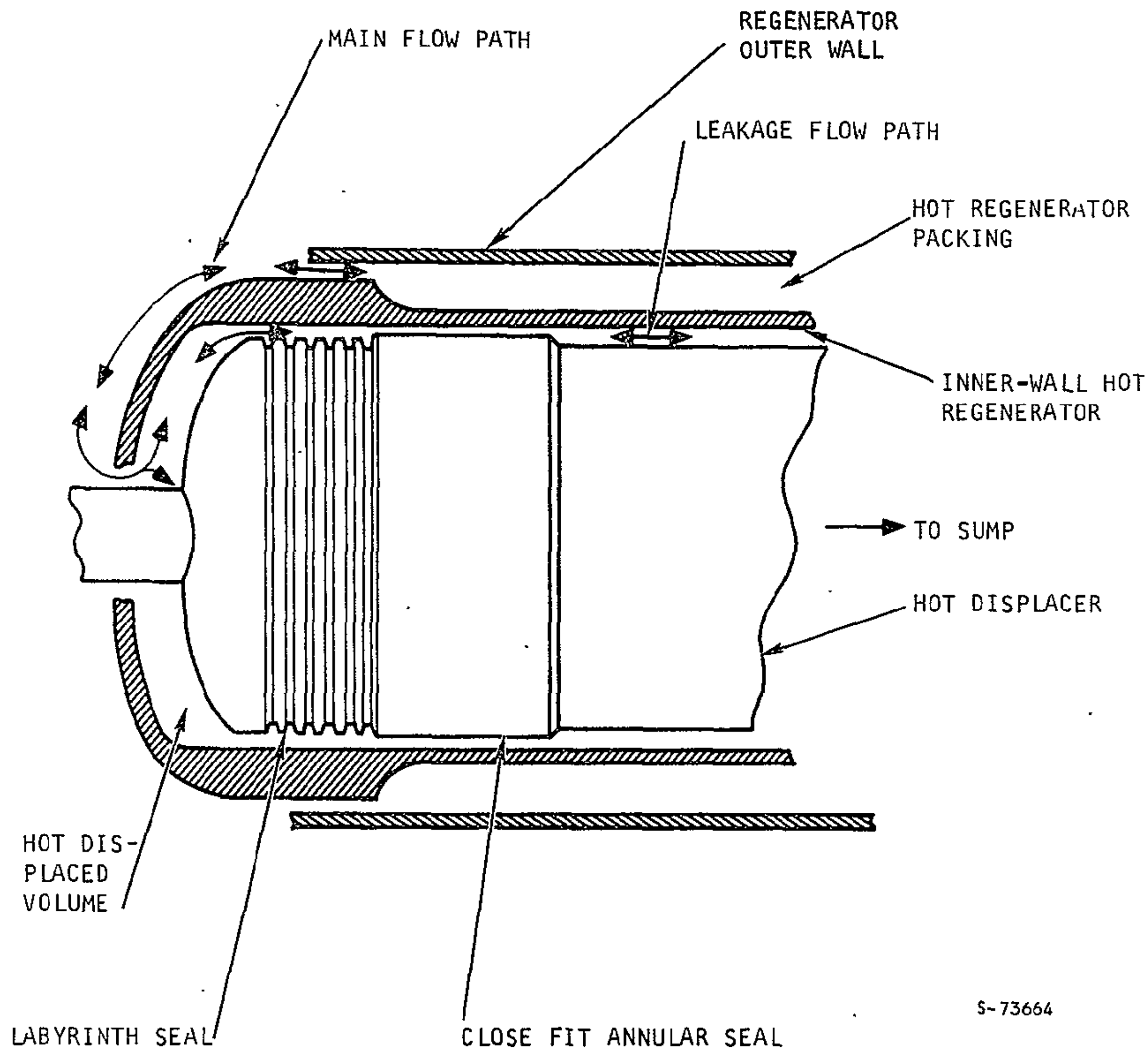

Figure 14-1. Hot-End Seal Configuration

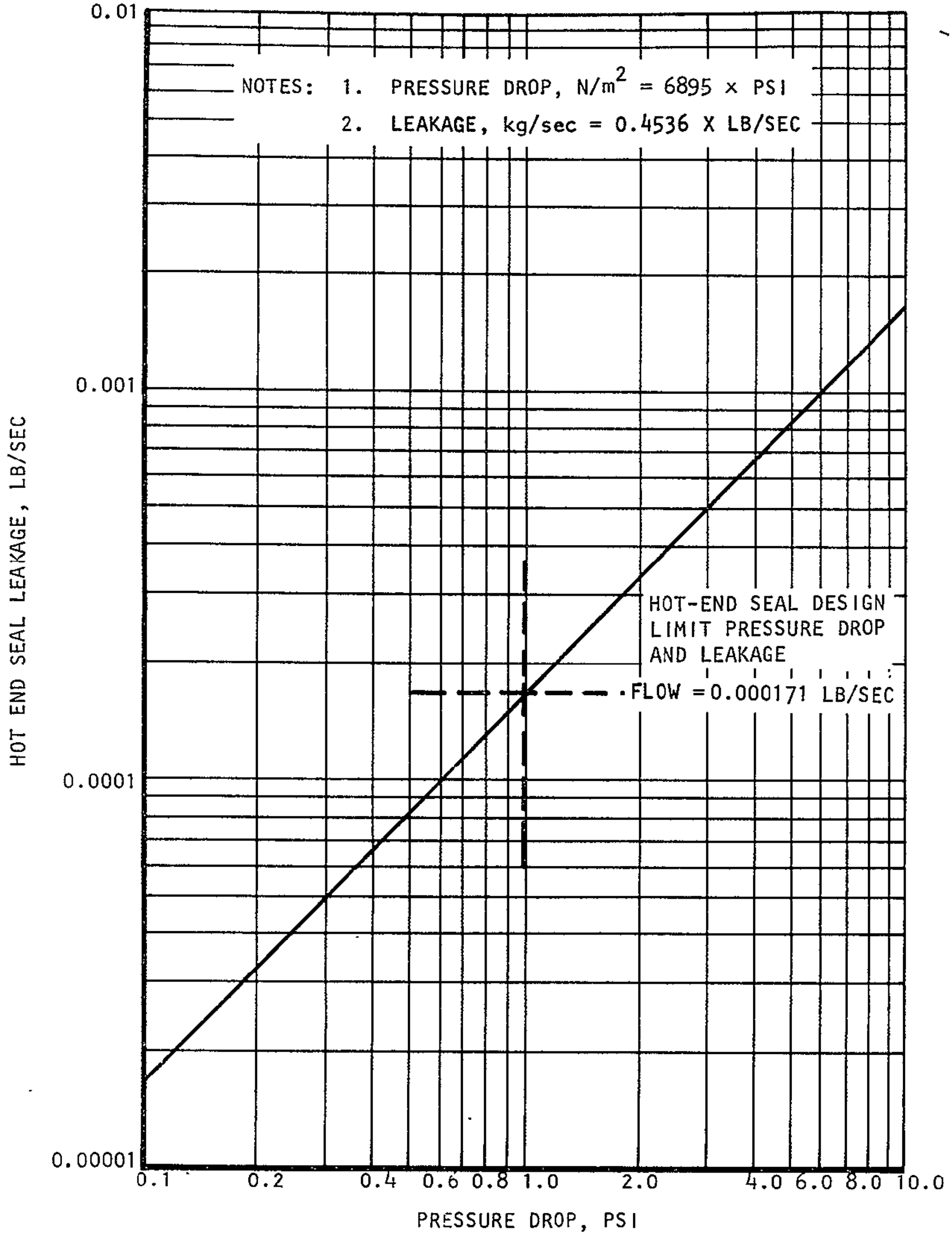

Figure 14-2. Hot-End Seal Leakage Rate vs Pressure Drop

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maximum pressure drops across: (1) sump ports to backside of hot displacer, (2) Section 1 of the sump heat exchanger, (3) interface between sump heat exchanger and hot regenerator, (4) hot regenerator and (5) the hot-end heat exchanger. Due to the use of maximum pressure drops--pressure drops corresponding to the maximum flow rates at a rotational speed of 400 rpm--the indicated design leakage rate is conservative.

Comparing Figure 14-2 with Figure 13-2 (Figure 13-2 gives the cold-end leakage rate vs pressure drop) a major difference is noted. The hot-end seal is completely non-contacting and is not subject to wear. The hot-end leakage is therefore independent of operational life.

Figure 14-3 gives the thermal loss for leakage past the hot-end seal as a function of leakage rate. The method of calculating this thermal loss, which takes into account the regenerative effect of the displacer and cylinder walls, is identical to that used for the cold-end seal thermal losses. The loss indicated for the design limit conditions is not significant compared to the 80 watts maximum power input to the hot-end required to operate the system.

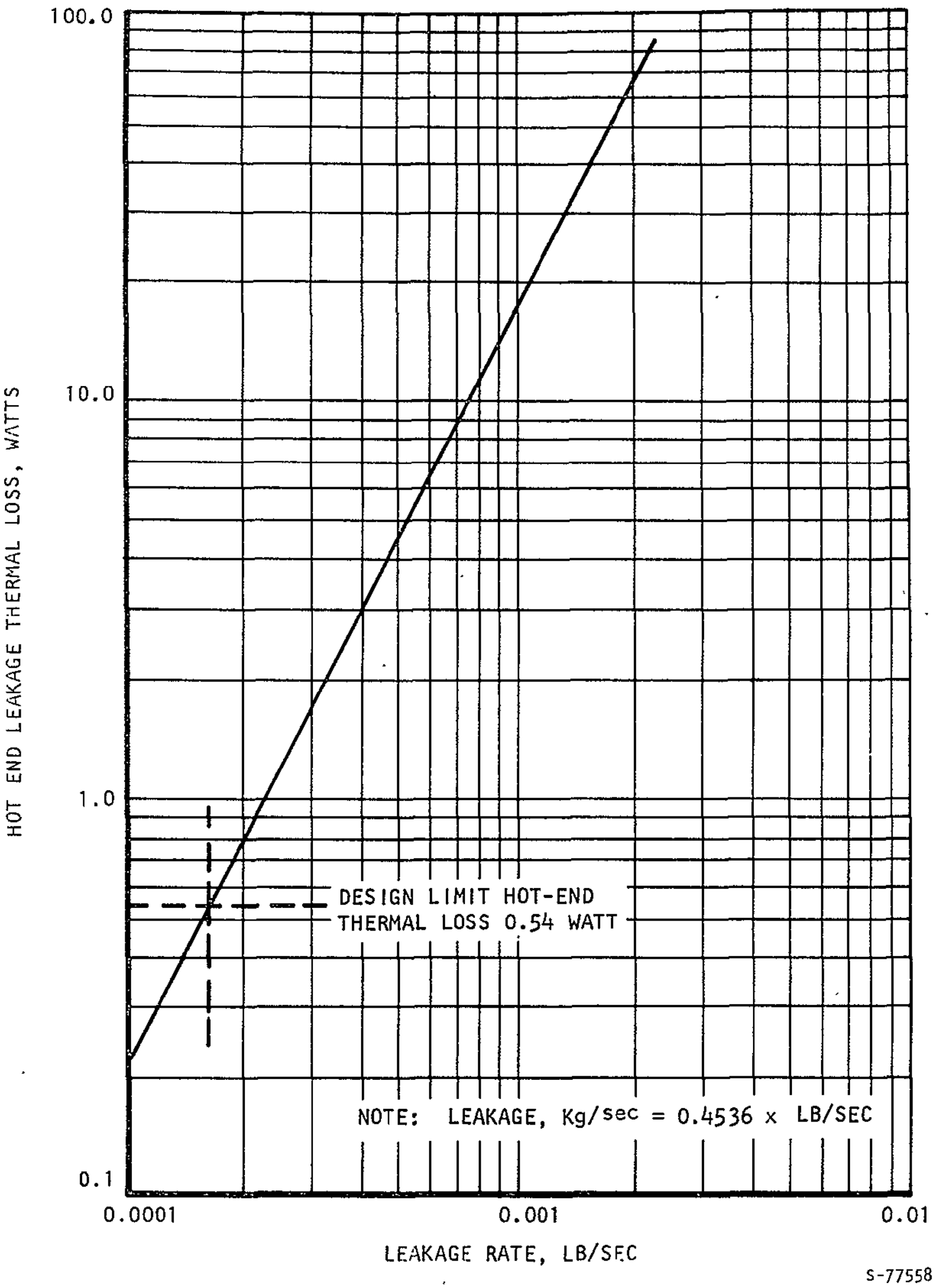


Figure 14-3. Hot-End Thermal Loss vs Leakage Rate

Hot End Seal

This analysis is similar To The Cold End. The leakage rate is calculated for each component of The sealing system, and The loss Then determined

I Leakage

1. Annular Seal

$$\dot{w} = \frac{\rho \Delta P}{L} \frac{D_h^2 g_c A_c}{48 \mu}$$

clearance = .0025, $A_c = 2.336\pi \times .0025 = .0183 \text{ in}^2$

$D_h = .005$

$\mu = 3.75 \times 10^{-5}$ lb/sec-fT

$\rho = .267$ lb/fT^3

$$\dot{w} = \frac{.267 \text{ lb}_m (\Delta P) \text{ lb}_f}{\text{ft}^3 \; 1 \text{ in} \; \text{in}^2} \left[\frac{.005^2 \text{ in}^2 \times 32.2 \text{ lb}_m\text{-ft} \; .0183 \text{ in}^2 \; \text{ft-sec ft}}{48 \; \text{lb}_f\text{-sec}^2 \; 3.75 \times 10^{-5} \text{ lb}_m \; 12 \text{ in}} \right]$$

$= 1.821 \times 10^{-4} \Delta P$, $\dot{w}$ in lb/sec and ΔP in psi

Calculate $\dot{w}$ over a range of ΔP

ΔP, psi	$\dot{w}$, lb/sec.
.05	9.11×10^{-6}
.1	1.821×10^{-5}
.5	9.11×10^{-5}
1	1.821×10^{-4}
5.	9.11×10^{-4}
10.	1.821×10^{-3}

2. For the remainder of the displacers (except Labyrinth) the clearance is 0.010 inches and length is 2.95 in.

$A_c = .0732 \text{ in}^2$

$$\dot{w} = 1.821 \times 10^{-4} \left(\frac{1}{2.95}\right)\left(\frac{.02}{.005}\right)^2 \left(\frac{.0732}{.0183}\right) \Delta P = 3.66 \times 10^{-3} \Delta P$$

$\dot{w}$, lb/sec	ΔP, psi
9.11×10^{-6}	.0025
1.821×10^{-5}	.00498
9.11×10^{-5}	.025
1.821×10^{-4}	.0498
9.11×10^{-4}	0.25
1.821×10^{-3}	0.498

3. Labyrinth Seal

$$\dot{W} = .399 A_c \sqrt{\frac{P_o \Delta P}{T_o N}} = .399 \times .0183 \sqrt{\frac{1000 \Delta P}{6 \times 1395}}$$

$$= .00276 \Delta P^{1/2}$$

Thus $\Delta P = \dfrac{\dot{w}^2}{.761 \times 10^{-5}}$

$\dot{w}$	ΔP
9.11×10^{-6}	1.09×10^{-5}
1.821×10^{-5}	4.36×10^{-5}
9.11×10^{-5}	1.09×10^{-3}
1.821×10^{-4}	4.36×10^{-3}
9.11×10^{-4}	0.109
1.821×10^{-3}	0.436

4. Add up The Total ΔP

$\dot{w}$ lb/sec	ΔP, psi Annular	remainder of Displacer	Labyrinth	Total ΔP, Psi
9.11×10^{-6}	.05	.0025	1.09×10^{-5}	.0525
1.821×10^{-5}	.1	.00498	4.36×10^{-5}	.1050
9.11×10^{-5}	.5	.025	1.09×10^{-3}	.5261
1.821×10^{-4}	1.	.0498	4.36×10^{-3}	1.0544
9.11×10^{-4}	5.	.25	.109	5.659
1.821×10^{-3}	10.	.498	.436	10.936

II Losses

The equation for losses may be reduced to (Ref 4):

$$Q_L = \frac{w\,c_p}{\alpha X_L}(T_h - T_a)\left(1 - e^{-\alpha X_L}\right)$$

$X_L = .354\,\text{FT}$

$$\alpha = \frac{hA}{\dot{w} c_p}$$

$$A = 2\pi \times 2.336/12 = 1.221\ \text{ft}^2/\text{ft}$$

At $Nu = 8.23$, $h = 8.23 \times .135 \times 12/.02 = 666\ \text{BTU/ft}^2\text{-hr-}^\circ\text{R}$

$$c_p = 1.24$$

$$\text{Thus } \alpha = \frac{666\ \frac{\text{BTU}}{\text{ft}^2\text{-hr-}^\circ\text{R}} \times 1.221\ \text{sec}\ \text{hr}\ \text{ft}^2\ \text{lb-}^\circ\text{R}}{1.24\ \dot{w}\ \text{lb}\ 3600\ \text{sec}\ \text{ft}\ \text{BTU}} = \frac{.18216}{\dot{w}}\ \frac{1}{\text{ft}}$$

$\dot{w}$	α	αX_L	$e^{-\alpha X_L}$	Q_{LOST}, WATTS
9.11×10^{-6}	19996	7078	0	.0015
1.821×10^{-5}	10 000	3541	0	.0061
9.11×10^{-5}	1999	707.8	0	.154
1.821×10^{-4}	1000	354	0	.615
9.11×10^{-4}	200	70.78	~ 0	15.39
1.821×10^{-3}	100	35.4	4.1×10^{-16}	61.52

III Total Losses

The Plot of Pressure Drop versus flow yields a flow rate of 1.71×10^{-4} lb/sec leakage flow at the hot end pressure drop of 1.0 psi.

This flow rate corresponds to a hot end loss of 0.54 watts, which is perfectly acceptable.

SECTION 15

CONDUCTION LOSSES

SECTION 15

CONDUCTION LOSSES

The VM refrigerator contains three distinct temperature zones: the hot end, the sump region, and the cold end. Any material which connects regions of different temperature is thus exposed to a temperature gradient, and will conduct heat from the higher temperature region to the colder region. Heat conducted from the hot end to the sump is rejected to the cooling water without contributing to the cycle pressure variations. These conduction losses must be minimized in order to maintain the thermal power input within acceptable limits. Heat conducted from the sump region to the cold end represents additional refrigeration load, which must be returned to the sump region by the working fluid. The conduction losses are included in the calculation of net performance, presented in Tables 3-2 and 3-3. The losses are summarized here for convenient reference.

METHOD OF ANALYSIS

The method of determining conduction losses is straight forward; details are given in Volume I of the Engineering Notebook, "Thermal Analysis". The only conduction calculation requiring special information is the one associated with the packing (matrix) of the regenerators. Here the properties of the packed beds were taken from Reference 7.

CONDUCTION LOSSES SUMMARY

The hot-end and cold-end conduction losses are summarized in Table 15-1.

TABLE 15-1

HOT-END AND COLD-END CONDUCTION LOSSES

Element	Hot-End Losses, Watts	Cold-End Losses, Watts
Displacer		
Walls	15.90	0.200
Packing*	1.00	0.066
Subtotal	16.90	0.266
Regenerator		
Walls	19.86	0.818
Matrix	1.69	0.654
Subtotal	21.55	1.472
Dewar	--	0.060
Total	38.45	1.793

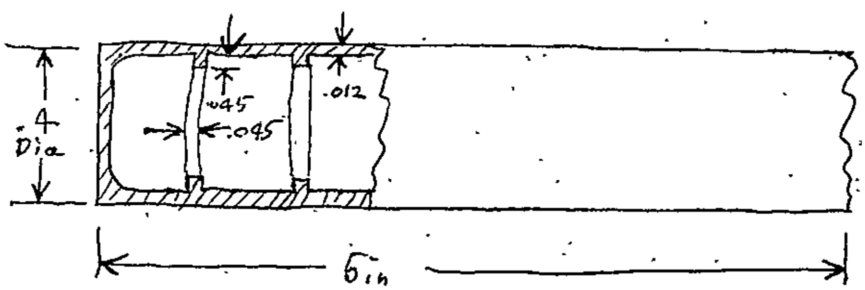
*Each displacer contains a low conductivity packing to eliminate convective heat transfer due to gas contained within the sealed displacers.

Conduction Losses

The conduction losses must be calculated for all paths of heat flow between the sump and the hot or cold ends of the machine. Conduction to the cold end represents additional refrigeration load, whereas conduction from the hot end to the sump region represents heat that is not used by the cycle.

I Cold End

A. Displacer



There are 9 internal ribs, Total length = 9 x .045/12 = .0338 ft.

Inner Diameter = .31 in.

$$A = \frac{\pi}{4}(.4^2 - .31^2)/144 = .00035\ ft.^2$$

AT 366°R (Average Temperature) $k = 5.2\ BTU/ft^2\text{-}hr\text{-}°F$

Thus for ribs,

$$\frac{KA}{L} = \frac{5.2 \times .00035}{.0338} = 0.0538 \text{ BTU/hr-°F}$$

For the remainder of the Displacer,

$$L = (5 - .405)/12 = 0.383 \text{ ft.}$$

$$A = (.4 - .012)\pi \times .012/144 = .0001016 \text{ ft}^2$$

$$\frac{KA}{L} = \frac{5.2 \times .0001016}{.383} = .00138 \text{ BTU/hr-°F}$$

The Total Resistance is

$$R_T = \frac{1}{\frac{1}{.00138} + \frac{1}{.0538}} = .001343 \text{ BTU/hr-°F}$$

$$Q = R_T \Delta T = .001343(620 - 112) = 0.683 \text{ BTU/hr}$$

$$= 0.200 \text{ watts}$$

For the Packing inside the Displacer, $K = .02$ BTU/ft-hr-°F

$$A = \frac{\pi}{4}(.4 - .024)^2/144 = .00925 \text{ ft}^2$$

$$L = 5/12 = .417 \text{ ft}$$

$$KA/L = \frac{.02 \times .00925}{.417} = .000443 \text{ BTU/hr-°F}$$

$$Q = .000443 \times 508 = .225 \text{ BTU/hr} = 0.066 \text{ watts}$$

(3)

B Regenerator Walls

1. inner wall

$L = 5'' = .417$ FT.

Average Diameter $= .412$ in, wall Thickness is $.012$

$A = .412\pi \times .012/144 = .000108 \text{ FT}^2$

$Q = \dfrac{5.2 \times .000108}{.417} 508 = .684$ BTU/hr $= .200$ watts

2. Outer wall

The Taper at the ambient end makes the effective length 4.75 in. The maximum wall Thickness is .014 and the mean diameter is 1.036 in.

$A = 1.036\pi \times .014/144 = .000316 \text{ FT}^2$

$L = 4.75/12 = .396$ FT.

$Q = \dfrac{5.2 \times .000316}{.396} 508 = 2.11$ BTU/hr $= 0.618$ watts

C. Regenerator Packing

The effective thermal conductivity of a bed of packed spheres filled with a gas is obtained from McAdams (Ref 7), figure 11-7. The thermal conductivity of the spheres is 4.1 BTU/FT-hr-°F, and for the helium is 0.069 BTU/FT-hr-°F.

Thus $k_{sphere}/k_{gas} = 4.1/.069 = 59.4$.

From figure 11-7, McAdams, at a porosity of 39%, $k_{bed}/k_{gas} = 5.9$

Thus $k_{bed} = 5.9 \times .069 = 0.407$ BTU/ft-hr-°F.

$A = \cancel{.0566}\ .00472$ FT.^{2}, $L = 5$ in $= .417$ FT.

$$Q = \frac{0.407 \times .00472}{.417}\, 508 \times .293 = 0.654 \text{ watts}$$

II Hot End

A Displacer

The displacer on the hot end has four distinct regions in the outer wall for conduction. These are (1) the labyrinth seal, (2) the straight section of wall, (3) the stiffening rib, and (4) the joint of the two pieces.

(1) labyrinth seal

Wall thickness = .090 in, L = .45 in

$K = 10.25$ BTU/ft-hr-°F

$A = 2.289\pi \times .09/144 = .0045 \text{ ft}^2$

$L = .45/12 = .0375$ ft.

$$KA/L = \frac{10.25 \times .0045}{.0375} = 1.232 \text{ BTU/hr-°F}$$

(2) straight wall

Wall thickness = .040 in, Effective length at this thickness = 3.79 in. = .3158 ft.

ratio area from above, $A = \frac{.04}{.09} .0045 = .002 \text{ ft}^2$

$$\frac{KA}{L} = \frac{10.25 \times .002}{.3158} = .0649 \text{ BTU/hr-°F}$$

(3) stiffening rib width = .14 in, L = .08 in.

$$A = \frac{.14}{.04} .002 = .007\ ft^2$$

$$L = .08/12 = .006667$$

$$KA/L = \frac{10.25 \times .007}{.006667} = 10.76$$

(4) joint

The joint between the two halves of the displacer has two parallel conduction paths

The first has a thickness of .045 in and length of .23 in

$$A = \frac{.045}{.04} \times .002 = .00225$$

$$L = .23/12 = .01916$$

$$\frac{KA}{L} = \frac{10.25 \times .00225}{.01916} = 1.203$$

Second path has a length of .46 in. and same area

$$\therefore KA/L = \frac{.23}{.46} 1.203 = .602$$

$$\Sigma\ KA/L = 1.203 + .602 = 1.805$$

5. Calculate Conduction in metallic portion of Displacer.

$$\text{Total Resistance} = \frac{1}{\frac{1}{1.272} + \frac{1}{.0649} + \frac{1}{10.76} + \frac{1}{1.805}}$$

$$= .0593 \text{ BTU/hr-}^\circ\text{F}$$

$$\Delta T = 1535 - 620 = 915\ ^\circ R$$

$$Q = .0593 \times 915 = 54.26 \text{ BTU/hr}$$

$$= 15.9 \text{ watts}$$

6. Packing in Displacer

ID = 2.143 in

$A = \frac{\pi}{4} 2.143^2/144 = .025 \text{ ft}^2$

Use effective Length = 3.4 in = .283 ft

K = .042

$$Q = \frac{.042 \times .025}{.283} 915 = 3.39 \text{ BTU/hr}$$

$$= 1.0 \text{ watts}$$

B Regenerator walls

1. Inner Wall

Wall Thickness = .015 in, D_{av} = 2.344 in

$A = 2.344\pi \times .015/144 = .000769\ ft^2$

$L = 4.375\ in = .3646\ ft.$

$k = 10.25$

$$Q = \frac{10.25 \times .000769}{.3646} 915 = 19.78\ BTU/hr$$

$$= 5.79\ watts$$

2. Outer wall

This is composed of four sections of different thickness

(a) Taper at sump end.

Thickness Tapers from .0292 To .0575

average Thickness = (.0292 + .0575)/2 = .0434 in.

D_{av} = 2.73 in.

$A = 2.73\pi \times .0434/144 = .00259\ ft^2$

$L = .25\ in = .0208\ ft.$

$KA/L = 10.25 \times .00259/.0208 = 1.276\ BTU/hr\cdot°F$

(b) Straight Section

Thickness = .0292 in., L = 1.75 in. = .146 fT.

$$A = \frac{.0292}{.0434}\,.00259 = .001742\ ft^2$$

$$KA/L = \frac{10.25 \times .001742}{.146} = .1222\ BTU/hr\cdot°F$$

(c) Tapered Section

Wall Thickness goes from .0292 To .0325 in.

$T_{av} = (.0292 + .0325)/2 = .0307$ in

$$A = \frac{.0307}{.0292}\,.001742 = .001831$$

L = 2.11 in = .1758 fT.

$$\frac{KA}{L} = \frac{10.25 \times .001831}{.1758} = .1067\ BTU/hr\cdot°F$$

(d). Sharp Taper aT hoT End.

L = .25 in = .0208 fT.

Thickness goes from .0325 To .065 in.

$T_{av} = (.0325 + .065)/2 = .0487$ in.

$$A = \frac{.0487}{.0307}\,.001831 = .002904\ ft^2$$

$$\frac{K.A}{L} = \frac{10.25 \times .002904}{.0208} = 1.431 \ BTU/hr\text{-}^\circ F$$

(e) Total Conduction of outer wall

$$\text{Total Resistance} = \frac{1}{\frac{1}{1.276} + \frac{1}{.1222} + \frac{1}{.1067} + \frac{1}{1.431}}$$

$$= .0525 \ BTU/hr\text{-}^\circ F$$

$$Q = .0525 \times 915 = 48.06 \ BTU/hr$$

$$= 14.08 \text{ watts}$$

c. Regenerator Packing

The McAdams (Ref. 7) correlation for packed beds is used here to obtain the effective thermal conductivity of the regenerator. As in the 5 watt VM, one half of the value for 50% porosity will be used for the 72.5% porous regenerator

(11)

The Stainless Steel Thermal conductivity is 10.69 BTU/ft-hr-°F, and That of helium is .132.

$K_{ss}/K_{He} = 10.69/.132 = 81$

from fig. 11-7, McAdams at 50% porosity,

$K_{bed}/K_{He} = 4$

$\therefore K_{bed} = 4 \times .5 \times .132 = .264$ BTU/ft-hr-°F

$L = 4.375$ in $= .3646$ ft.

$A = 1.15$ in^2 $= .007986$ ft.^{2}

$$Q = \frac{.264 \times .007986}{.3646} 915 = 5.566 \text{ BTU/hr}$$

$= 1.63$ watts

SECTION 16

SUMP COOLING INTERFACE

SECTION 16

SUMP COOLING INTERFACE

To properly function, VM refrigerators must reject heat from the sump or crankcase region. The amount of heat that must be rejected is equal to the sum of: the hot end input power; the refrigeration load; all losses; and the drive motor input power. In the GSFC fractional-watt VM refrigerator, the heat rejection rate is 80 watts.

During Task I of the program, an ambient heat pipe assembly was designed for the purpose of rejecting heat from the refrigerator to a simulated space radiator. The heat pipe design was carried only to the conceptual stage, in order to provide compatibility with either water cooling coils or heat pipes. The reader is referred to Reference 1 for details of the heat pipe sump cooling assembly

DESIGN CONFIGURATION

The sump cooling assembly consists of an aluminum block which is mounted around the sump region of the VM refrigerator. Water cooling coils are mounted in grooves in the block, as shown in Figure 16-1. Indium foil is utilized at the interface between the block and the VM refrigerator in order to minimize thermal resistance.

The aluminum block design, depicted in Figure 16-1 employs an interface clamp design which is adaptable to use with either a water cooling system or an ambient heat pipe assembly. The cooling coil is soft soldered to the aluminum block, as shown in Figure 16-1. The soft solder provides good thermal contact between the cooling coil and the aluminum and, also, allows heat to be distributed over most of the coil circumference.

The two halves are mounted on the engine sump with indium foil 0.0000762 m (0.003-in.) thick at the interface and four bolts located in slots as shown in Figure 16-1. This scheme provides good thermal contact between the block and the sump while providing detachability of either the heat pipe assembly or the cooling water assembly.

The bolts use compression washers under each bolt head and each nut to achieve an interface pressure in the range of 3.477×10^5 (50) to 6.895×10^5 N/m^2 (100 psi) (between the engine sump and aluminum block). The fluctuations in the engine sump pressure will result in an increase or decrease of the deflection experienced by the washers while maintaining the interface pressure in the desired range.

An analysis of the water cooling coils was performed similar to that for the 5 watt VM. The analysis indicated that the cooling water must pass through the four sections of the water cooling coil in series. The water flow was assumed at 0.00315 m^3/sec (50 gal/hr). The maximum temperature drop from the sump gas to the cooling water with this configuration is approximately 6.67°K (12°R), which is less than the 11.1°K (20°R) allowed in the design.

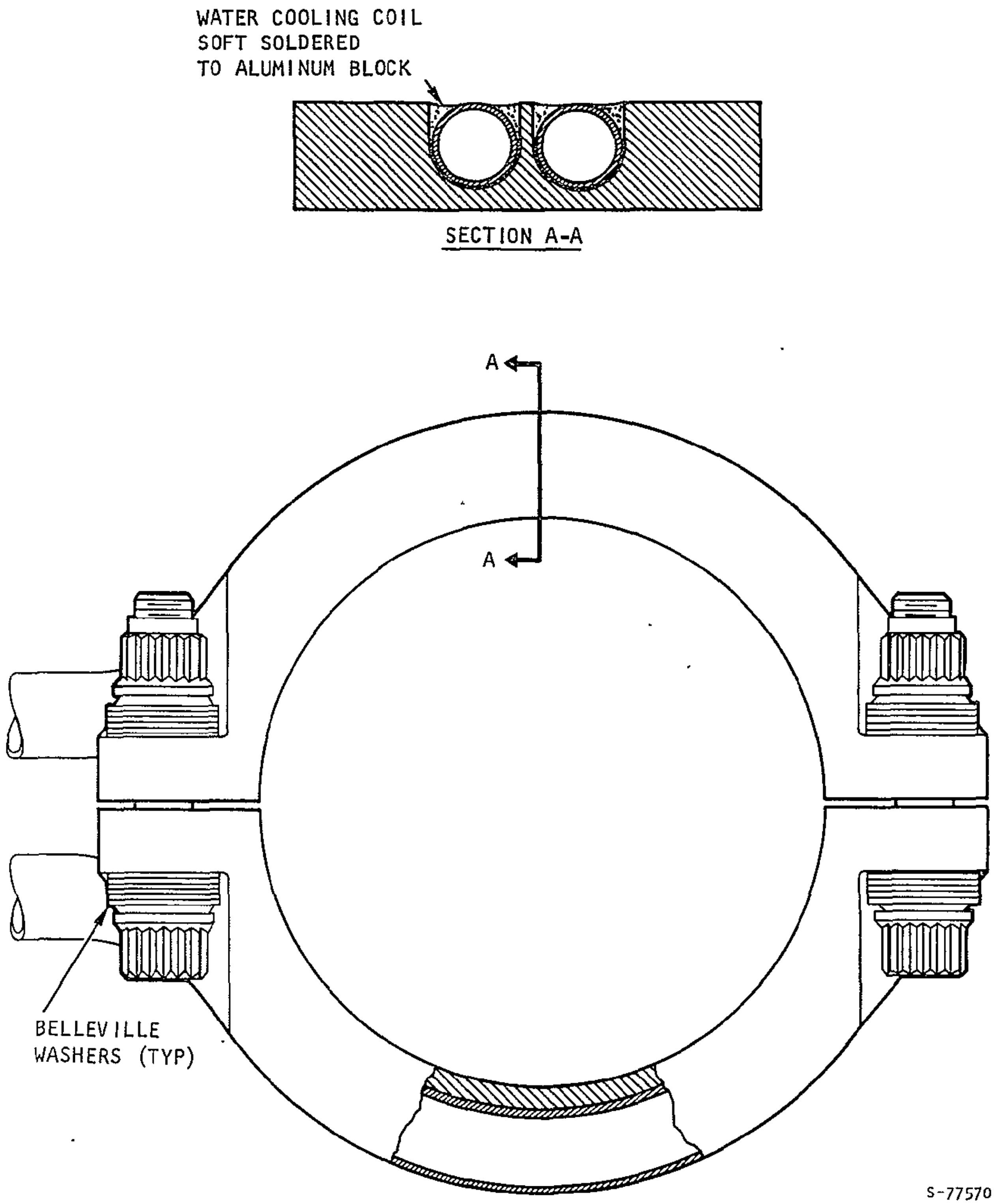


Figure 16-1. Aluminum Block Design Details

DESIGN OF WATER COOLING COILS AND SUMP INTERFACE

The analysis presented on the following pages was performed when it was planned to use copper for the sump cooling collar. Subsequently, the material was changed to aluminum in order to save weight. The material change affects the conduction temperature drops and fin efficiency calculations presented here. However, it has been determined that the overall temperature drop from the sump gas to the water in the cooling coils is within acceptable limits with the use of aluminum.

Design of Water

Cooling Coils, Sump Interface

The sump water cooling coils must be compatible with the sump interface clamp defined for the heat pipe interface. Thus the coil will be ~~composed of~~ 0.375 OD tubing and will make two passes around each half of the sump interface clamp. The first thing will be to define the heat transfer area in the coil.

Each half of the collar is 180°. However, the copper tubing does not contact the collar for this entire arc length. The angle of contact for the coils on the 5 watt machine is 147.4°. Use 145° of arc until further definition of the clamp for the fractional watt machine is available. The sump diameter is 2.5 in. Thus use 2.5+0.375 in = 2.875 as mean diameter of the coil.

Also assume that the tubing wall thickness is 0.032 in, as for the 5 watt machine. The prime area is 1/2 of the tubing circumference, and the fin area is the other half.

$$A_{prime} = A_{fin} = .5\pi D_i 4 S$$

$$S = \frac{145}{360}\pi D_c = \frac{145}{360}\pi\, 2.875 = 3.635 \text{ in.}$$

$$D_i = 0.375 - 2\times.032 = 0.311 \text{ in.}$$

$$A_{prime} = 4\times.5\pi\times.311\times3.635/144 = 0.0493 \text{ ft}^2$$

now we must evaluate the heat transfer coefficient. assume the water flow is 50 gal/hr. at 140°F. Based on the 5 w. VM, use a single flow path.

$$\mu = 1.2 \text{ lb/ft-hr},\ k = 0.38 \text{ BTU/ft-hr-°F},\ Pr = 3.2$$

$$\dot{w} = 50\times8.34 = 417 \text{ lb/hr}$$

$$D_h = D_i = .311 \text{ in} = \cancel{0.0216}\ 0.0259 \text{ ft.}$$

$$A_c = \frac{\pi}{4}D_i^2 = \frac{\pi}{4}.311^2 = 0.076 \text{ in}^2 = 0.000527 \text{ ft}^2$$

$$G = \dot{w}/A = 417/.000527 = 791{,}000 \text{ lb/ft}^2\text{-hr}$$

$$Re = GD_h/\mu = 791000\times\cancel{.0216}\,.0259/1.2 = \cancel{14\ldots}\ 17{,}100$$

$$h = 0.023 \frac{K}{D} Re^{.8} Pr^{.4} = 0.023 \frac{0.38\ BTU}{ft\text{-}hr\text{-}^\circ F \times .0259 ft} (17{,}100)^{.8} (3.2)^{.4}$$

$$= 0.337\ (2420)(1.592) = 1300\ BTU/ft^2\text{-}hr\text{-}^\circ F$$

now we need fin efficiency

$$\eta_f = \frac{\tanh m l_e}{m l_e}$$

l_e = fin length $= .25 \times \pi D_i = .25 \pi \times .311 = 0.244\ in = .02036\ ft.$

$m = \sqrt{\frac{h}{K \delta}}$ for single side fin

$$K_{cu} = 225\ BTU/ft\text{-}hr\text{-}^\circ F$$

$$\delta = .032\ in = .002665\ ft$$

$$m = \sqrt{\frac{1300\ BTU\ ft\text{-}hr\text{-}^\circ F}{ft^2\text{-}hr\text{-}^\circ F\ 225\ BTU \times .002665\ ft}} = \sqrt{\frac{2170}{ft^2}} = \frac{46.6}{ft}$$

$$m l_e = .02036\ ft \times 46.6/ft = .948$$

$$\eta_f = \tanh .948 / .948 = .76$$

$$\eta h A = h(A_p + \eta_f A_f) = \frac{1300\ BTU}{ft^2\text{-}hr\text{-}^\circ F}(.0493\ ft^2 + .76 \times .0493\ ft^2)$$

$$= 112.8\ BTU/hr\text{-}^\circ F$$

for extra conservation, assume Q = 100 watts

$Q = 100 \times 3.413 = 341.3\ BTU/hr.$

$$\Delta T = \frac{Q}{\eta h A} = \frac{341.3\ BTU\ hr\text{-}^\circ F}{hr\ 112.8\ BTU} = 3.02\,^\circ F$$

with 100 watts, calculate the water ΔT

$$\Delta T_{H_2O} = \frac{341.3\ BTU\ \cancel{lb}\text{-}^\circ F\ \cancel{hr}}{\cancel{hr} \times 1\ BTU \times 417\ \cancel{lb}} = 0.817\ ^\circ F$$

thus at the outlet the water will be at approximatly 141 °F. The tube wall will be ~~operating~~ at 141 + 3 = 144 °F = 604 °R from the interface calculations for the heat pipe, the copper block ΔT = 0.5 °F, the solder joint ΔT = 0.6 °F, and the indium foil ΔT = 0.1 °F.

Thus the sump wall temperature = 604 + .5 + .6 + .1 = 605.2 °F.
The sump heat exchanger ΔT is ~ 6 °R.
Thus the helium in the sump is 611.7 °R. This is acceptable, since we are using a

sump temperature of 620 °R in the design of the refrigerator.

~~At~~ Now make a rough check on pressure drop. assume the total tubing length is 3 times that in contact with the sump interface block. $L = 3 \times 45 = 12 \times 3.635/12 = 3.635$ ft.

at $Re = 17{,}100$ $4f = .027$ for smooth tubes.

$$\Delta P = 4f\, L/D\; G^2/2g\rho = \frac{.027 \times 3.635\ \text{ft}}{.0259\ \text{ft}} \; \frac{791000^2\ \text{lb}_m^2\ \text{lb}_f\text{-sec}^2\ \text{hr}^2\ \text{ft}^3\ \text{ft}^2}{144\ \text{in}^2\ \text{ft}^4\ \text{hr}^2\ 2 \times 32.2\ \text{lb}_m\text{-ft}\ 3600^2\ \text{sec}^2\ 62.4\ \text{lb}_m}$$

$$= 0.316\ \text{lb}_f/\text{in}^2$$

if ~~it~~ we assume that turning losses, etc, are 10 times this, $\Delta P = 11 \times .316 = 3.48$ psi

this is perfectly acceptable since water supply pressures are seldom below ~ 40 psi.

APPENDIX A

REGENERATOR ANALYSIS

APPENDIX A

REGENERATOR ANALYSIS

General

The regenerator is one of the most important components of the Vuilleumier cycle refrigerator. It is a heat storage device which is used at both the hot and cold ends of the refrigerator. At the cold end, it is used to cool the gas as it flows from the sump region to the cold expansion volume of the machine. The expansion process of the gas reduces the gas temperature further in order to provide cooling at the cold temperature. After absorbing heat from the refrigeration load, the gas is then exhausted from the expansion volume through the regenerator where it regains energy previously stored, to the sump region. This reestablishes the temperature profile in the cold regenerator for the incoming gas for the next cycle.

The process in the hot regenerator is essentially the same as that at the cold end. The gas flowing from the sump region to the hot end is heated in the regenerator. As the flow reverses, an expansion process begins, which tends to cool the gas. This allows heat to be added to the gas at essentially constant temperature from the hot end heater. The gas is exhausted from the expansion volume through the regenerator where it transfers energy to the regenerator matrix and is cooled. This reestablishes the temperature profile in the hot regenerator for the incoming gas for the next cycle.

The design requirements for a good regenerator are that the regenerator packing be of a material with a very large heat capacity and that the packing have a large heat transfer coefficient and a large heat transfer area. The basic tradeoff for the design of a regenerator is between irreversible pressure drop and heat transfer potential with a minimum void volume.

The analysis of the regenerators for a Vuilleumier refrigerator cannot be based on the classical effectiveness parameters which make use of inlet and outlet temperatures. The system pressure fluctuates and a considerable amount of gas is stored in the regenerator void volume during the flow period. This results in the mass flow of gas into the regenerator not being equal to the mass flow of gas out of the regenerator at all points in time.

Computer Program

In order to analyze the regenerators properly, a computer program making use of finite difference techniques has been developed by AiResearch. The regenerator is broken into axial nodes; these separate nodes are used for both the gas and the matrix. The following basic equations were used to develop the difference equations which the computer program solves.

1. Gas Nodes

The continuity equation can be written as

$$\frac{\partial \rho}{\partial \tau} + \frac{\partial(\rho u)}{\partial x} = 0 \qquad \text{(A-1)}$$

where ρ = gas density

τ = time

u = gas velocity

x = coordinate

Setting

$$G = \rho u$$

$$-\frac{\partial G}{\partial x} = \frac{\partial \rho}{\partial \tau}$$

Now writing the difference form

$$\left(\frac{G_n - G_{n+1}}{\Delta x_n}\right) = \left(\frac{\rho_n' - \rho}{\Delta \tau}\right) \quad \text{(A-2)}$$

therefore

$$G_n = G_{n+1} + \Delta x_n \left(\frac{\rho_n' - \rho}{\Delta \tau}\right) \quad \text{(A-3)}$$

The momentum equation for the gas is

$$\rho \frac{\partial u}{\partial \tau} + \rho u \frac{\partial u}{\partial x} + \frac{4f}{2D_h} \left(\rho u^2\right) + \frac{\partial P}{\partial x} g_c = 0 \quad \text{(A-4)}$$

where f = Fanning friction factor

D_h = characteristic length

P = static pressure

g_c = 32.2 lbm-ft/lbf-sec^2

Taking the steady-state form and substituting G,

$$g_c \frac{\partial P}{\partial x} = \frac{\partial (G^2/\rho)}{\partial x} + \left(\frac{4f}{2D_h}\right) \frac{G|G|}{\rho} \quad \text{(A-5)}$$

Now taking the difference form of this equation,

$$g_c \frac{(P_n - P_{n+1})}{(\Delta x_n)} = \frac{(G_{n+1}{}^2/\rho_{n+1}) - (G_n{}^2/\rho_n)}{(\Delta x_n)} + \frac{1}{2}\left[\left(\frac{4f_{n+1}}{2D_h}\right) \frac{G_{n+1}|G_{n+1}|}{\rho_{n+1}} + \left(\frac{4f_n}{2D_h}\right) \frac{G_n|G_n|}{\rho_n}\right] \quad \text{(A-6)}$$

Now solving for P_n

$$P_n = P_{n+1} + \frac{1}{g_c}\left(\frac{G_{n+1}}{\rho_{n+1}}\right)\left[G_{n+1} + \left| G_{n+1} \right| \left(\frac{\Delta x_n f_{n+1}}{D_h}\right)\right]$$
$$+ \frac{1}{g_c}\left(\frac{G_n}{\rho_n}\right)\left[-G_n + \left| G_n \right| \left(\frac{\Delta x_n f_n}{D_h}\right)\right] \qquad \text{(A-7)}$$

The energy equation is

$$\frac{\partial(c_v \rho T)}{\partial \tau} + \frac{\partial(c_p \rho u T)}{\partial x} = \left(\frac{UA_\ell}{V_\ell}\right)(t - T) \qquad \text{(A-8)}$$

where c_v = specific heat at constant volume

c_p = specific heat at constant pressure

T = gas temperature

U = overall heat transfer coefficient

A_ℓ = heat transfer surface area per unit length

V_ℓ = void volume in regenerator per unit length

t = matrix temperature

This equation can be reduced to

$$\bar{c}_v \bar{\rho} \frac{\partial T}{\partial \tau} + \frac{\partial(c_p\ GT)}{\partial x} = \left(\frac{UA_\ell}{V_\ell}\right)(t - T) \qquad \text{(A-9)}$$

The overall heat transfer coefficient u is defined as

$$u = \frac{1}{\left(\frac{1}{h_c}\right) + \left(\frac{\Delta}{k}\right)} \qquad \text{(A-10)}$$

where $h_c = \left(\frac{jG\ c_p}{Pr^{2/3}}\right)$

j = Colburn j-factor

P_r = Prandtl number

Δ = equivalent conduction length

Taking the difference form and solving for future gas temperature

$$T_n' = T_n + \left[\frac{(\Delta\tau_g)(G_n c_{pn} T_n - G_{n+1} c_{pn+1} T_{n+1})}{\Delta x_n \bar{c}_{vn} \bar{\rho}_n}\right] \tag{A-11}$$

$$+ (\Delta\tau_g)\left(\frac{UA_\ell}{\bar{c}_{vn}\bar{\rho}_n V_\ell}\right)(t_n - T_n)$$

The stability criterion for this difference equation is

$$\Delta\tau_g \le \left[\frac{1.0}{\left(\frac{UA_\ell}{\bar{c}_{vn}\bar{\rho}_n V_\ell}\right) - \left(\frac{G_n c_{pn}}{\bar{c}_{vn}\bar{\rho}_n \Delta x_n}\right)}\right] \tag{A-12}$$

In the case that the time increment $\Delta\tau_g$ required by the stability criterion for the gas is very small compared to that of the matrix, steady-state solutions may be selected. In this case, the steady-state energy equation is

$$\frac{\partial(c_p GT)}{\partial x} = \left(\frac{UA_\ell}{V_\ell}\right)(t - T) \tag{A-13}$$

Writing the difference form

$$\frac{(G_{n+1}\, c_{pn+1}\, T_{n+1} - G_n c_{pn} T_n)}{\Delta x_n} = \left(\frac{UA_\ell}{V_\ell}\right)(t_n - T_n) \tag{A-14}$$

Solving for gas temperature

$$T_n = \frac{\left[\left(\frac{UA_\ell}{V_\ell}\right) tn - \left(\frac{G_n c_{pn}}{\Delta x_n}\right) T_{n+1}\right]}{\left[\left(\frac{UA_\ell}{V_\ell}\right) - \left(\frac{G_n\, c_{pn}}{\Delta x_n}\right)\right]} \tag{A-15}$$

2. Matrix Nodes

The energy equation for the matrix is

$$\frac{\partial t}{\partial\tau} = \left(\frac{UA_\ell}{C_m M_\ell}\right)(T - t) + \frac{1}{C_m M_\ell}\frac{\partial}{\partial x}\left[k_m A_x \frac{\partial t}{\partial x}\right] \tag{A-16}$$

where t = matrix temperature

C_m = matrix specific heat

M_ℓ = matrix mass per unit length

k_m = matrix conductivity

A_x = matrix effective heat transfer area for conduction

Writing the difference equation and solving for future matrix temperature,

$$t_n' = t_n + \Delta\tau_m \frac{UA_\ell}{C_m M_\ell} (T_n - t_n) + \left(\frac{\Delta\tau_m}{C_m M_\ell \Delta x_n}\right)\left[\frac{(t_{n-1} - t_n)}{R_{n-1,n}} + \frac{(t_{n+1} - t_n)}{R_{n+1,n}}\right] \quad \text{(A-17)}$$

where $R_{m,n} = \left[\left(\frac{\Delta x_{m/2}}{k_m A_{xm}}\right) + \left(\frac{\Delta x_{n/2}}{k_n A_{xn}}\right) + \frac{1}{(H\ A)_i}\right]$

where $(H\ A)_i$ = interface or boundary heat transfer characteristic.

The stability criterion for the matrix time increment is

$$\Delta\tau_m \leq \left[\frac{1.0}{\left(\frac{UA_\ell}{C_m M_\ell}\right) + \left[\frac{1}{C_m M_\ell \Delta x_n}\right]\left[\left(\frac{1}{R_{n-1,n}}\right) + \left(\frac{1}{R_{n+1,n}}\right)\right]}\right] \quad \text{(A-18)}$$

3. Discussion

The computer program is capable of analyzing both transient performance and cyclically steady performance. The inputs required by this computer program are the physical characteristics of the regenerator, heat transfer and friction loss characteristics, initial conditions, and boundary conditions. The physical characteristics are reflected in the matrix areas, volume, length, heat capacity, thermal conductivity, and mass. The heat transfer and friction loss characteristics are read into the computer program in the form of Colburn j-factor and Fanning friction factor as a function of Reynolds number. The initial conditions must be fully described in terms of pressure and temperatures of the gas and the matrix. The boundary conditions that are required are the time dependent characteristics of pressure, mass flow rate, and gas temperature at one end of the regenerator, and the return gas temperature at the other end (basically defining the thermodynamic process at the other end).

Regenerator Characterization

In the analyses of the regenerators, friction coefficients and the Colburn j-factor have been taken from the data presented by Kays and London. The data for randomly stacked screens are shown in Figure A-1 and the data for regenerators packed with spherical shots are shown in Figure A-2. Standard screen mesh data, shown in Table A-1, was used to characterize the screen matrix. Figures A-3 and A-4 show the specific heats of Monel, stainless steel, and Inconel as a function of temperature.

1. Area to Volume Ratio

For screens, the area to volume ratio can be derived to be

$$\beta = \frac{4(1 - \epsilon)}{d} \tag{A-19}$$

where

β = area to volume ratio

ϵ = porosity

d = wire diameter

For spheres, the area to volume ratio is

$$\beta = \frac{6(1 - \epsilon)}{d} \tag{A-20}$$

where

d = sphere diamter

2. Hydraulic Diameter

Another regenerator characteristic which is required is the hydraulic diameter. Defining the hydraulic radius as being equal to the flow volume divided by the total surface area, the hydraulic radius of a screen stack is found to be

$$r_H = \frac{\epsilon d}{4 (1-\epsilon)} \tag{A-21}$$

For a sphere, the hydraulic radius is equal to

$$r_H = \frac{\epsilon d}{6 (1-\epsilon)} \tag{A-22}$$

The hydraulic diameter is equal to 4 times the hydraulic radius.

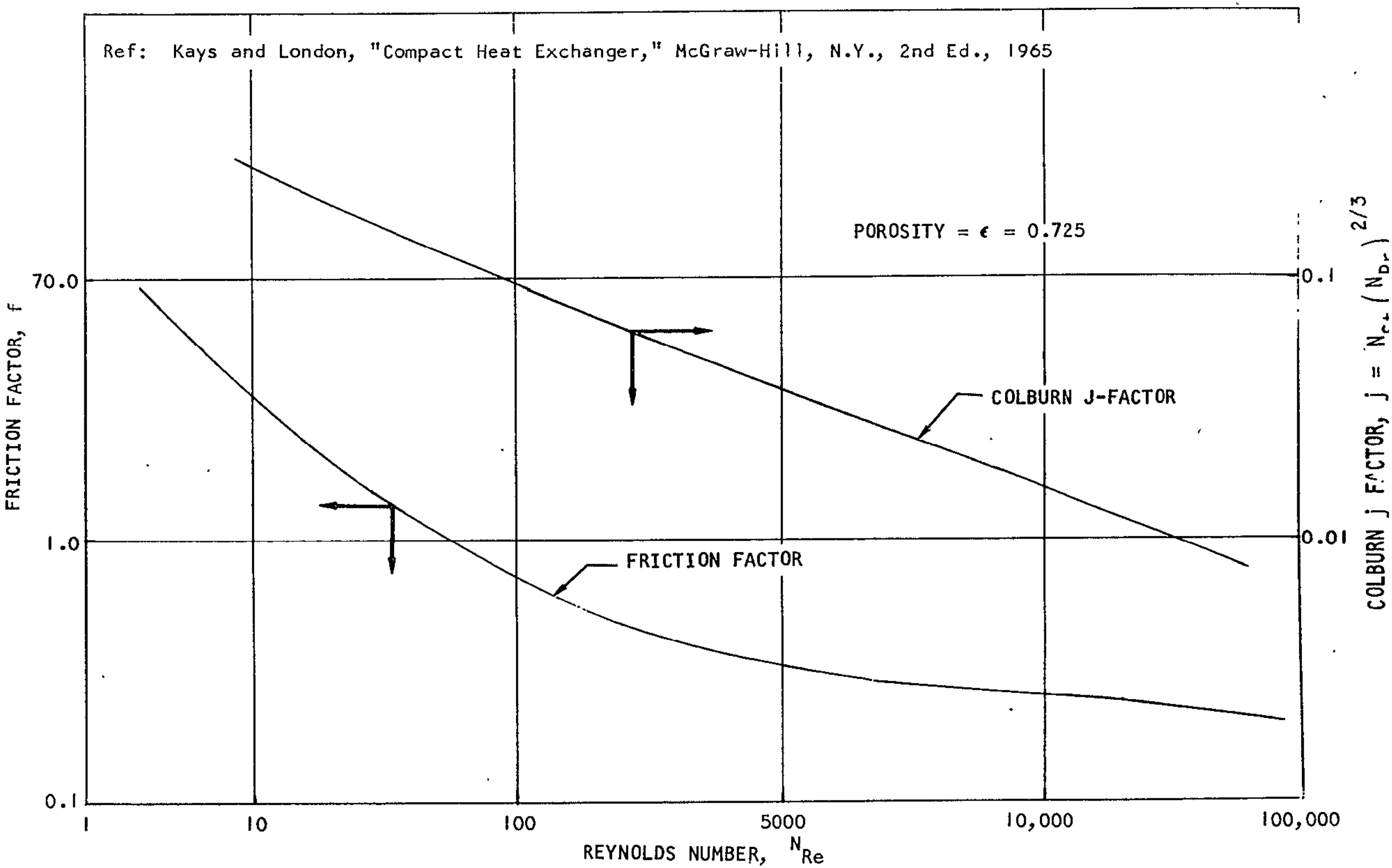


Figure A-1. Heat Transfer and Pressure Drop Data for Stacked Screens

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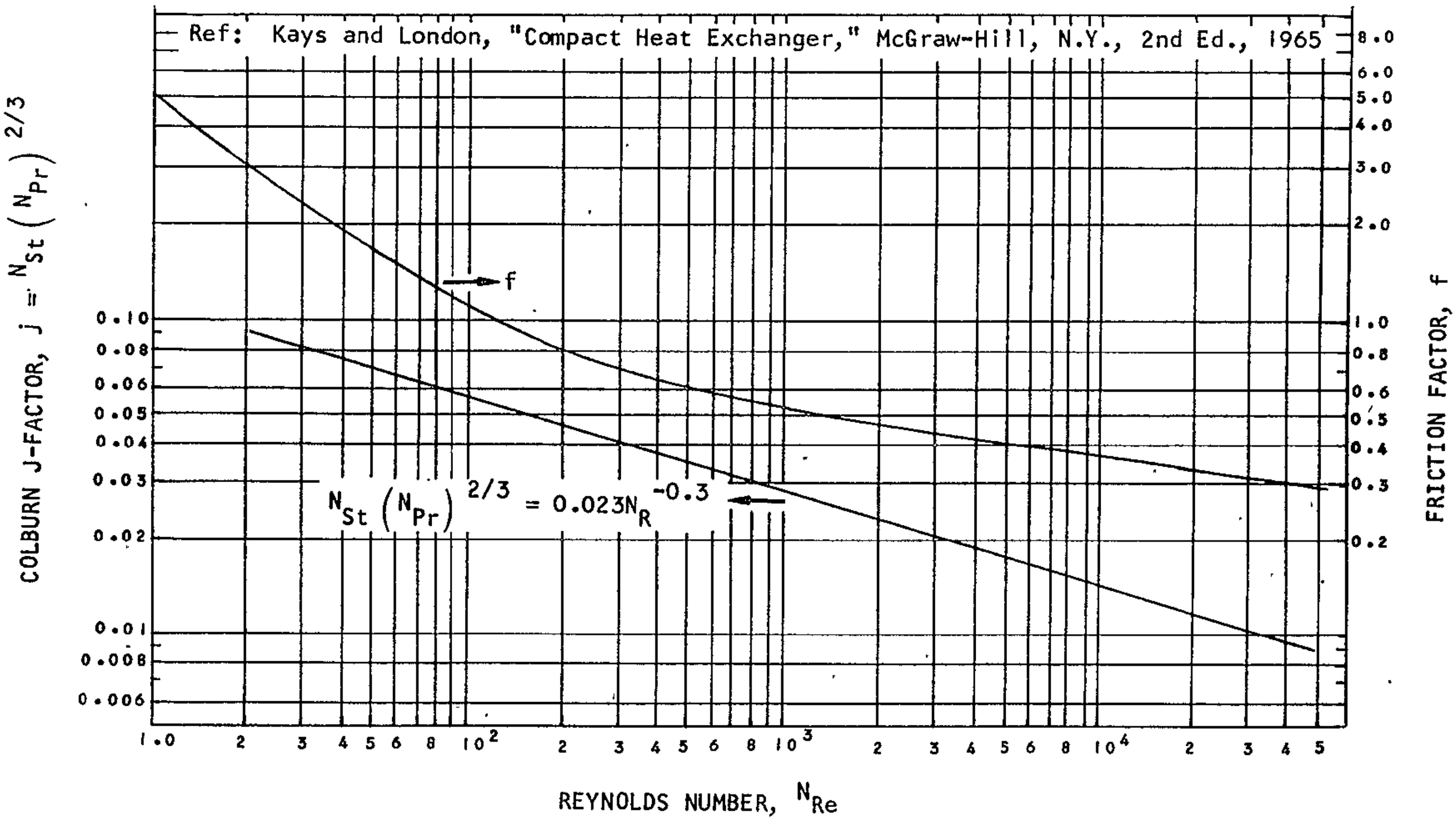

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Figure A-2. Gas Flow through an Infinite Randomly Stacked Sphere Matrix

TABLE A-1

U.S. SIEVE SERIES AND
TYLER EQUIVALENTS

A.S.T.M. - E-11-61

Sieve Designation		Sieve Opening		Nominal Wire Diameter		
Standard	Alternate, No.	mm	Inches, approx. equiv.	mm	Inches, approx. equiv.	Tyler Equiv. Designation, mesh
1.41 mm*	14	1.41	0.0555	0.725	0.0285	12
1.19 mm	16	1.19	0.0469	0.650	0.0256	14
1.00 mm*	18	1.00	0.0394	0.580	0.0228	16
841 micron	20	0.841	0.0331	0.510	0.0201	20
707 micron*	25	0.707	0.0278	0.450	0.0177	24
595 micron	30	0.595	0.0234	0.390	0.0154	28
500 micron*	35	0.500	0.0197	0.340	0.0134	32
420 micron	40	0.420	0.0165	0.290	0.0113	35
354 micron*	45	0.354	0.0129	0.247	0.0097	42
297 micron	50	0.297	0.0117	0.215	0.0085	48
250 micron*	60	0.250	0.0098	0.180	0.0071	60
210 micron	70	0.210	0.0083	0.152	0.0060	65
177 micron*	80	0.177	0.0070	0.131	0.0052	80
149 micron	100	0.149	0.0059	0.110	0.0043	100
125 micron*	120	0.125	0.0049	0.091	0.0036	115
105 micron	140	0.105	0.0041	0.076	0.0030	150
88 micron*	170	0.088	0.0035	0.064	0.0025	170
74 micron	200	0.074	0.0029	0.053	0.0021	200
63 micron*	230	0.063	0.0024	0.044	0.0017	250
53 micron	270	0.053	0.0021	0.037	0.0015	270
44 micron*	325	0.044	0.0017	0.030	0.0012	325
37 micron	400	0.037	0.0015	0.025	0.0010	400

*These sieves correspond to those proposed as an international (ISO) standard. It is recommended that wherever possible these sieves be included in all sieve analysis data or reports intended for international publication.

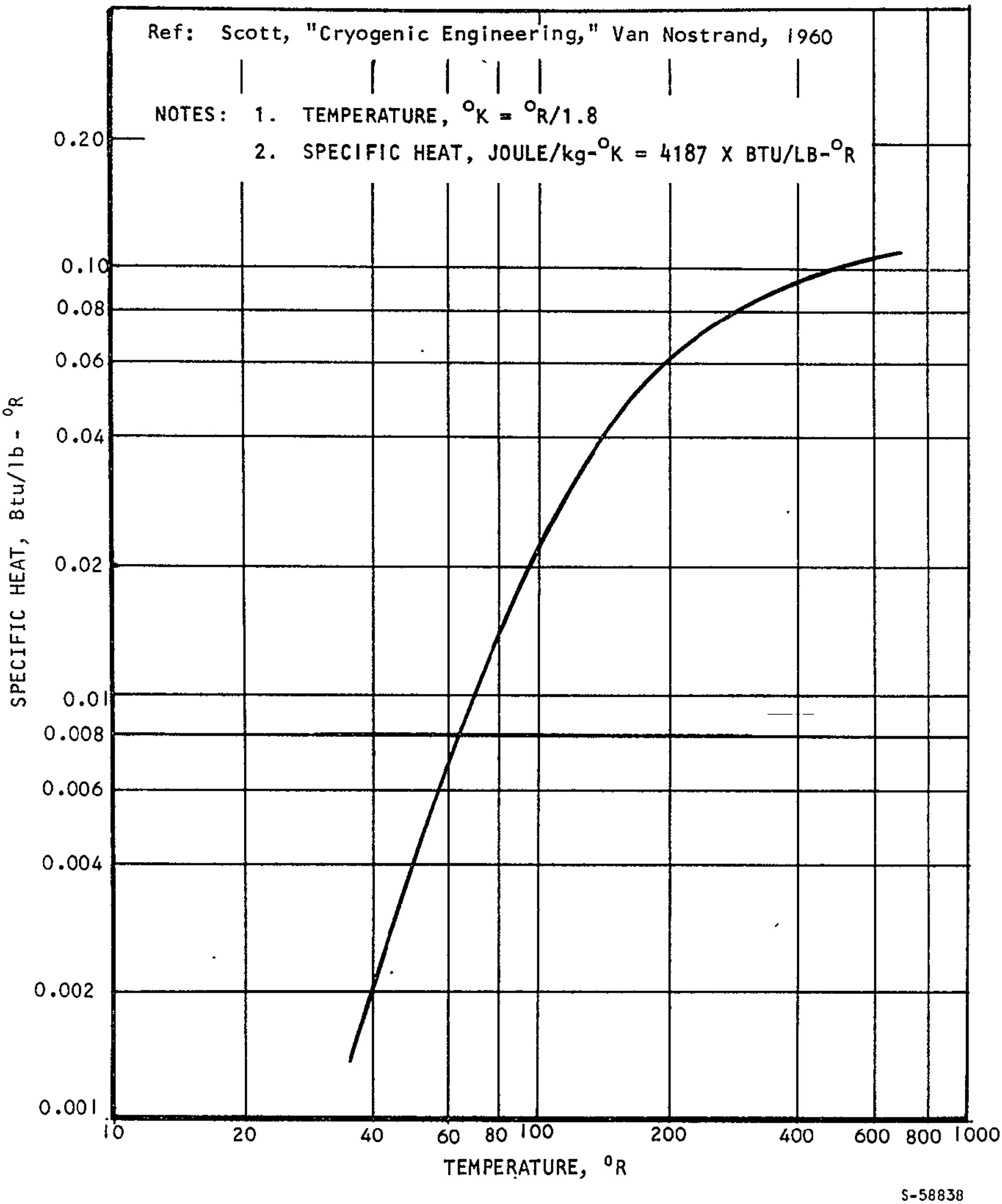

Figure A-3. Monel Specific Heat

Ref: AiResearch Material Design Data
347 STAINLESS STEEL: ASTM STP NO. 227
INCONEL 100: G.E. HANDBOOK, SEC. 2

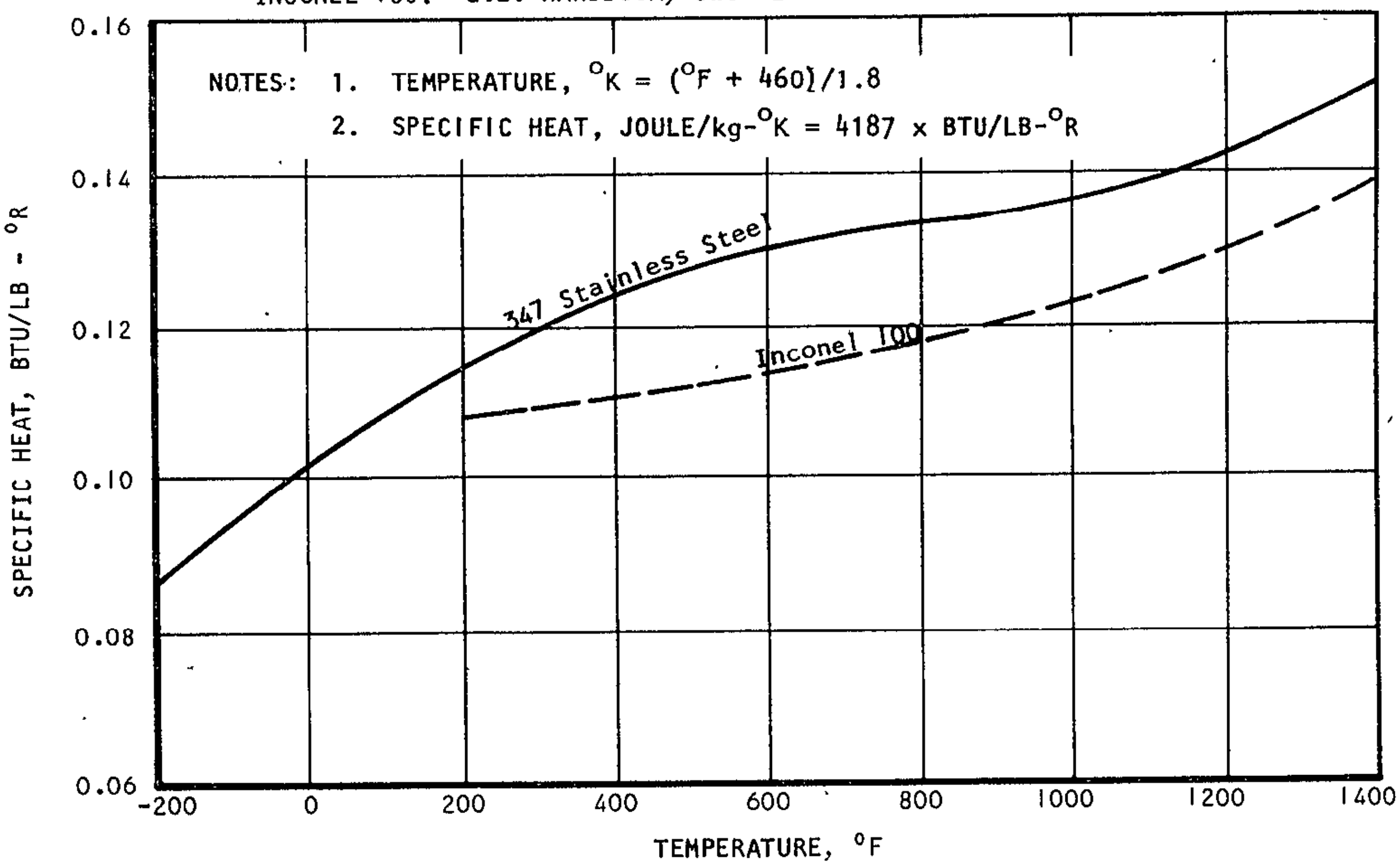

Figure A-4. Stainless Steel and Inconel Specific Heats