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MULTIPLE REGRESSION TECHNIQUE FOR Pth DEGREE POLYNOMIALS WITH AND WITHOUT LINEAR CROSS PRODUCTS

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16. ABSTRACT

A multiple regression technique has been developed by which the nonlinear behavior of specified independent variables can be related to a given dependent variable. The polynomial expression can be of Pth degree and can incorporate N independent variables. Two cases are treated such that mathematical models can be studied both with and without linear cross products. The resulting surface fits can be used to summarize trends for a given phenomenon and provide a mathematical relationship for subsequent analysis.

To implement this technique, separate computer programs have been developed for the case without linear cross products and for the case incorporating such cross products which evaluate the various constants in the model regression equation. In addition, the significance of the estimated regression equation is considered and the standard deviation, the F statistic, the maximum absolute percent error, and the average of the absolute values of the percent of error evaluated.

The computer programs and their manner of utilization are described. Sample problems are included to illustrate the use and capability of the technique which show the output formats and typical plots comparing computer results to each set of input data.

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DEFINITION OF SYMBOLS

Symbol	Definition
A_0	model equation intercept coefficient
B_N	matrix quantity
B_P	model equation coefficients
b_0	defined quantity
b_{NP}	regression model coefficients
$c_{N-1,N}$	regression model cross product coefficients
F	F statistic
K_N	cross product model equation terms
k	cross product technique parameter pertaining to degrees of freedom
N	number of independent variables
n	number of data points
P	degree of equation
R	multiple correlation coefficient
S_{NN}	matrix quantity
S_{NY}	matrix quantity
S_{YY}	total sum of squares
S(Reg)	regression sum of squares
S(Res)	residual sum of squares
s	variance
Y	value of dependent variable

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
y	difference in observed and mean value of dependent variable
Z_N	value of Nth independent variable
z_{NP}	difference in observed and mean values to the Pth power for the Nth independent variable
$\lvert\epsilon\rvert_{AVG}$	average absolute percent error
$\sum_{i=1}$	$\sum\limits_{i=1}^{n}$, summation from the first to the nth term
Superscript	
c	calculated value
—	mean value
Subscript	
i	$i = 1, 2, \ldots n$
j	$j = 1, 2, \ldots N$

MULTIPLE REGRESSION TECHNIQUE FOR Pth DEGREE POLYNOMIALS WITH AND WITHOUT LINEAR CROSS PRODUCTS

I. INTRODUCTION

The investigation of physical processes and requirements for data analysis methods frequently requires the use of mathematical models which describe the processes. The model can be formulated such that certain variables interact according to physical theories associated with the particular process, or it may contain identified independent variables and unknown parameters. The relationship of the parameters identified in the model can be evaluated using the statistical tool commonly referred to as regression analysis. In principle, it should be possible to establish complex curves or surfaces for higher order multiple variable functions by regression techniques to summarize trends in data and to provide a means of predicting similar phenomena. Furthermore, such a technique might be used to establish unknown laws or relationships.

Most statistical textbooks treat the problem of linear multiple variable regression and of nonlinear regression of one independent variable. However, the study of many complex physical problems requires a method capable of determining nonlinear regression of multiple independent variables. By this means, an analytical representation of the observed data is provided which can be used in subsequent analysis.

The general procedure in regression analysis is to take partial derivatives of a specific model-dependent minimizing function. The set of equations obtained by setting these partial derivatives equal to zero is frequently referred to as the normal equations. If the normal equations are not transcendental in any of the unknown parameters, they can be solved by the usual algebraic methods. It is this situation which is of concern here.

Within this report a method is presented for evaluating multiple variable regression for Pth degree polynomials with and without linear cross products. Implementation of the technique, including error diagnostics has been accomplished on the UNIVAC 1108 computer. Program listings and illustrative example problems are given in the appendices.

The method developed is used in problems requiring the determination of mathematical relationships describing complex curves or surfaces from known physical data. Initial applications of this technique have concerned the external aerodynamics of aircraft and space vehicles and the internal aerodynamics of transonic wind tunnels and have produced excellent results. Applications in almost any other field of study should be expected to be equally as useful.

II. MULTIPLE REGRESSION TECHNIQUE WITHOUT CROSS PRODUCTS

A. Introduction

The derivation of a matrix solution for evaluating multiple variable regression for Pth degree polynomials without cross products is presented in Reference 1. In this section this solution is discussed for the general case where the polynomial expression can be of Pth degree with N independent variables. The mathematical procedures represent a rigorous least squares evolution of input data centered about the mean.

B. Regression Analysis Development

Assuming the observed dependent variable is to be estimated by the following model:

$$
\begin{aligned}
Y_i^c = {} & A_0 + (b_{11} Z_{1i} + b_{12} Z_{1i}^2 + \dots + b_{1P} Z_{1i}^P) \\
& + (b_{21} Z_{2i} + b_{22} Z_{2i}^2 + \dots + b_{2P} Z_{2i}^P) \\
& + \dots + (b_{N1} Z_{Ni} + b_{N2} Z_{Ni}^2 + \dots + b_{NP} Z_{Ni}^P)
\end{aligned}
\tag{1}
$$

where i = 1, 2, ..., nth set of data, and the intercept $A_0 = b_{10} + b_{20} + \dots + b_{N0}$.

For any given independent variable Z_N , the mean value is given by

$$
\bar{Z}_N = \frac{\sum Z_{Ni}}{n} \tag{2}
$$

If the input data are centered about the mean the model equation (1) becomes

$$
\begin{aligned}
Y_i^c = {} & b_0 + (b_{11} z_{11i} + b_{12} z_{12i} + \dots + b_{1P} z_{1Pi}) \\
& + (b_{21} z_{21i} + b_{22} z_{22i} + \dots + b_{2P} z_{2Pi})
\end{aligned}
\tag{3}
$$

$$+\ldots+(b_{N1}\,z_{N1i}+b_{N2}\,z_{N2i}+\ldots+b_{NP}\,z_{NPi}) \tag{3}$$

(Concluded)

Where, for convenience, the following quantities, which are indicated in equation (3), are defined by

$$\begin{aligned} b_0 = {} & (b_{10}+b_{11}\,\bar{Z}_1+b_{12}\,\bar{Z}_1^2+\ldots+b_{1P}\,\bar{Z}_1^P) \\ & +(b_{20}+b_{21}\,\bar{Z}_2+b_{22}\,\bar{Z}_2^2+\ldots+b_{2P}\,\bar{Z}_2^P) \\ & +\ldots+(b_{N0}+b_{N1}\,\bar{Z}_N+b_{N2}\,\bar{Z}_N^2+\ldots+b_{NP}\,\bar{Z}_N^P) \end{aligned} \tag{4}$$

$$\left.\begin{array}{l} \left.\begin{array}{l} z_{11i}=Z_{1i}-\bar{Z}_1 \\ z_{12i}=Z_{1i}^2-\bar{Z}_1^2 \\ \quad\vdots \\ z_{1Pi}=Z_{1i}^P-\bar{Z}_1^P \end{array}\right\} \\ \quad\vdots \\ \left.\begin{array}{l} z_{21i}=Z_{2i}-\bar{Z}_2 \\ z_{22i}=Z_{2i}^2-\bar{Z}_2^2 \\ \quad\vdots \\ z_{2Pi}=Z_{2i}^P-\bar{Z}_2^P \end{array}\right\} \\ \quad\vdots \\ \left.\begin{array}{l} z_{N1i}=Z_{Ni}-\bar{Z}_N \\ z_{N2i}=Z_{Ni}^2-\bar{Z}_N^2 \\ \quad\vdots \\ z_{NPi}=Z_{Ni}^P-\bar{Z}_N^P \end{array}\right\} \end{array}\right\} \tag{5}$$

The classical form of the least squares minimizing function is

$$M = \sum (Y_i - Y_i^c)^2 \qquad (6)$$

This result follows from the least squares principle that the best representation of the input data is that which makes the sum of the squares of the residuals a minimum. The condition which fulfills this requirement is that the partial derivatives of this function with respect to each of the unknowns be zero. Hence, the following normal equations for $i = 1, 2, \ldots n$ are written:

$$\left.\begin{array}{l} \frac{\partial M}{\partial b_0} = 0 \\[2ex] \frac{\partial M}{\partial b_{11}} = 0 \\[2ex] \frac{\partial M}{\partial_{12}} = 0 \\ \vdots \\ \frac{\partial M}{\partial b_{1P}} = 0 \end{array}\right\} \quad \left.\begin{array}{l} \frac{\partial M}{\partial b_{21}} = 0 \\[2ex] \frac{\partial M}{\partial b_{22}} = 0 \\ \vdots \\ \frac{\partial M}{\partial b_{2P}} = 0 \end{array}\right\} \quad \cdots \quad \left.\begin{array}{l} \frac{\partial M}{\partial b_{N1}} = 0 \\[2ex] \frac{\partial M}{\partial b_{N2}} = 0 \\ \vdots \\ \frac{\partial M}{\partial b_{NP}} = 0 \end{array}\right\} \qquad (7)$$

Through the use of the normal equation $\frac{\partial M}{\partial b_0} = 0$ it can be shown that

$$b_0 = \bar{Y} = \frac{\sum Y_i}{m} \qquad (8)$$

The following quantity is now defined for the difference in the observed value of Y_i and the mean value of the observed values:

$$y_i = Y_i - \bar{Y} \qquad (9)$$

Then, through the use of the remaining normal equations, the following matrix solution for the unknown constant of the independent variables is obtained.

$$\underset{\left[[NP\text{x}1]\right]}{\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix}} = \underset{[NP\text{x}NP]}{\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & & \vdots \\ S_{N1} & S_{N2} & & S_{NN} \end{bmatrix}^{-1}} \underset{[NP\text{x}1]}{\begin{bmatrix} S_{1y} \\ S_{2y} \\ \vdots \\ S_{ny} \end{bmatrix}} \tag{10}$$

where the following matrix quantities are given by:

$$\left.\begin{aligned} B_1 &= \begin{bmatrix} b_{11} \\ b_{12} \\ \vdots \\ b_{1P} \end{bmatrix} \\ B_2 &= \begin{bmatrix} b_{21} \\ b_{22} \\ \vdots \\ b_{2P} \end{bmatrix} \\ &\ \ \vdots \\ B_N &= \begin{bmatrix} b_{N1} \\ b_{N2} \\ \vdots \\ b_{NP} \end{bmatrix} \end{aligned}\right\} \tag{11}$$

$$
\left.
\begin{array}{l}
S_{1j} = \begin{bmatrix} \sum z_{11i}\, z_{j1i} & \sum z_{11i}\, z_{j2i} & \cdots & \sum z_{11i}\, z_{jPi} \\ \sum z_{12i}\, z_{j1i} & \sum z_{12i}\, z_{j2i} & \cdots & \sum z_{12i}\, z_{jPi} \\ \vdots & \vdots & & \vdots \\ \sum z_{1Pi}\, z_{j1i} & \sum z_{1Pi}\, z_{j2i} & \cdots & \sum z_{1Pi}\, z_{jPi} \end{bmatrix} \text{ for } j = 1, 2, \ldots, N \\[2em]
S_{2j} = \begin{bmatrix} \sum z_{21i}\, z_{j1i} & \sum z_{21i}\, z_{j2i} & \cdots & \sum z_{21i}\, z_{jPi} \\ \sum z_{22i}\, z_{j1i} & \sum z_{22i}\, z_{j2i} & \cdots & \sum z_{22i}\, z_{jPi} \\ \vdots & \vdots & & \vdots \\ \sum z_{2Pi}\, z_{j1i} & \sum z_{2Pi}\, z_{j2i} & \cdots & \sum z_{2Pi}\, z_{jPi} \end{bmatrix} \text{ for } j = 1, 2, \ldots, N \\[2em]
\quad \vdots \\[2em]
S_{Nj} = \begin{bmatrix} \sum z_{N1i}\, z_{j1i} & \sum z_{N1i}\, z_{j2i} & \cdots & \sum z_{N1i}\, z_{jPi} \\ \sum z_{N2i}\, z_{j1i} & \sum z_{N2i}\, z_{j2i} & \cdots & \sum z_{N2i}\, z_{jPi} \\ \vdots & \vdots & & \vdots \\ \sum z_{NPi}\, z_{j1i} & \sum z_{NPi}\, z_{j2i} & \cdots & \sum z_{NPi}\, z_{jPi} \end{bmatrix} \quad \text{ for } j = 1, 2, \ldots, N
\end{array}
\right\}
\tag{12}
$$

$$\left.\begin{array}{l}
S_{1y} = \begin{bmatrix} \sum z_{11i}\, y_i \\ \sum z_{12i}\, y_i \\ \vdots \\ \sum z_{1Pi}\, y_i \end{bmatrix} \\[2em]
S_{2y} = \begin{bmatrix} \sum z_{21i}\, y_i \\ \sum z_{22i}\, y_i \\ \vdots \\ \sum z_{2Pi}\, y_i \end{bmatrix} \\
\vdots \\
S_{Ny} = \begin{bmatrix} \sum z_{N1i}\, y_i \\ \sum z_{N2i}\, y_i \\ \vdots \\ \sum z_{NPi}\, y_i \end{bmatrix}
\end{array}\right\} \qquad (13)$$

Having determined the matrix solution indicated by equation (10), the intercept A_o of the fitted model expression may be determined as:

$$\begin{aligned} A_0 = \bar{Y} - (b_{11}\,\bar{Z}_1 + b_{12}\,\bar{Z}_1^{2} + \ldots + b_{1P}\,\bar{Z}_1^{P}) - (b_{21}\,\bar{Z}_2 + b_{22}\,\bar{Z}_2^{2} + \ldots \\ + b_2\,\bar{Z}_2^{P}) - \ldots - (b_{N1}\,\bar{Z}_N + b_{N2}\,\bar{Z}_N^{2} + \ldots + b_{NP}\bar{Z}_N^{P}) \end{aligned} \qquad (14)$$

C. Discussion

As noted by Graybill [2], there are an infinite number of solutions to the model relationship. However, only one solution must be found to have a useful result. Also note that the solution presented to the higher order multiple regression problem is a function of parameters such as the sum of the squares, cross products, and linear values similar to previously developed solutions for linear regression. In this case, one finds a more complex result with nested matrices, which becomes practical only in combination with digital computer techniques.

As discussed in Appendix A, a computer program has been developed to evaluate the unknown constants in the model equation with the solution specified by equation (10), as well as certain diagnostics reflecting the significance of the estimated regression equation which will be considered in Section IV.

It is required in the application of this technique that the S matrix be nonsingular and that the input data are reasonably well behaved. It is also required that $n \geqq NP$. That is, the number of data points n must be equal to or greater than the number of unknown parameters in the model equation.

Many sets of physical data have been studied using the previously mentioned computer program. Results have generally been excellent and it is felt that the technique can be highly useful in many fields of endeavor.

III. MULTIPLE REGRESSION TECHNIQUE WITH CROSS PRODUCTS

A. Introduction

The regression technique developed in Section II for Pth degree polynomials with N independent variables has proven to be extremely useful in correlating experimental wind tunnel data and aiding subsequent analysis. However, in certain applications the inclusion of cross products for the various independent variables can enhance the value of the technique. Such a case occurs when it is desired to optimize the dependent variable of the fitted mathematical expression with respect to the various independent variables. In such a case the determination of maximum and minimum values by solving for the critical points through the use of the necessary and sufficient conditions (that the first and second partial derivation of the response variable with respect to each of the independent variables be identically equal to zero) can only be determined when cross products are included in the model equation; otherwise the effect of all other independent variables is lost when the partial derivative with respect to a given independent variable is evaluated.

To deal with this and other similar problems, a method is developed in this section which treats all combinations of linear cross products for each of the N independent variables as well as the polynomial terms previously discussed in Section II. Higher order cross product terms could, in principle, be treated, although such a development is beyond the scope of the present work. The method for fitting polynomials with linear cross products is presented without formal derivation. However, the development is similar to that shown in Reference 1.

B. Regression Analysis Development with Linear Cross Products

Consider the following model:

$$
\begin{aligned}
Y_i^c = {} & A_0 + (b_{11} Z_{1i} + b_{12} Z_{1i}^2 + \ldots + b_{1P} Z_{1i}^P) \\
& + (b_{21} Z_{2i} + b_{22} Z_{2i}^2 + \ldots + b_{2P} Z_{2i}^P) \\
& + \ldots + (b_{N1} Z_{Ni} + b_{N2} Z_{Ni}^2 + \ldots + b_{NP} Z_{Ni}^P) \\
& + (C_{12} Z_{1i} Z_{2i} + c_{13} Z_{1i} Z_{3i} + \ldots + C_{1N} Z_{1i} Z_{Ni}) \\
& + (C_{23} Z_{21} Z_{3i} + C_{24} Z_{2i} Z_{4i} + \ldots + C_{2N} Z_{2i} Z_{Ni}) \\
& + \ldots + (C_{N-1,N} Z_{(N-1)i} Z_{Ni})
\end{aligned}
\qquad (15)
$$

where $i = 1, 2, \ldots,$ nth set of data, and the intercept $A_0 = b_{10} + b_{20} + \ldots + b_{n0}$. This model represents a polynomial expression of Pth degree with N independent variables and contains all combinations of linear cross products for each of the N independent variables.

If equal quantities are added and subtracted in equation (15), namely, those of the form $b_{NP}\bar{Z}_N^P$ and $C_{ji}\overline{Z_j Z_i}$, (where $j = 1, 2, \ldots, N-1$) it can be shown that the model equation is equivalent to:

$$Y_i^c = K_{1i} + B_{1i} + K_{2i} + B_{2i} + \ldots + K_{Ni} + B_{Ni} \tag{16}$$

where

$$\left.\begin{aligned}
K_{1i} &= b_{10} + b_{11}\bar{Z}_1 + b_{12}\bar{Z}_1^2 + \ldots + b_{1P}\bar{Z}_1^{\,P} + C_{12}\overline{Z_1Z_2} + C_{13}\overline{Z_1Z_3} + \ldots + C_{1N}\overline{Z_1Z_N}\\
K_{2i} &= b_{20} + b_{21}\bar{Z}_2 + b_{22}\bar{Z}_2^2 + \ldots + b_{2P}\bar{Z}_2^{\,P} + C_{23}\overline{Z_2Z_3} + C_{24}\overline{Z_2Z_4} + \ldots + C_{2N}\overline{Z_2Z_N}\\
&\vdots\\
K_{Ni} &= b_{N0} + b_{N1}\bar{Z}_N + b_{N2}\bar{Z}_N^2 + \ldots + b_{NP}\bar{Z}_N^{\,P} + C_{N-1,N}\overline{Z_{N-1}Z_N}
\end{aligned}\right\} \tag{17}$$

$$\left.\begin{aligned}
B_{1i} &= b_{11}\, z_{11i} + b_{12}\, z_{12i} + \ldots + b_{1P}\, z_{1Pi} + C_{12}\, \overset{+}{z}_{12i} + C_{13}\, \overset{+}{z}_{13i} + \ldots + C_{1N}\, \overset{+}{z}_{1Ni}\\
B_{2i} &= b_{21}\, z_{21i} + b_{22}\, z_{22i} + \ldots + b_{2P}\, z_{2Pi} + C_{23}\, \overset{+}{z}_{23i} + C_{24}\, \overset{+}{z}_{24i} + \ldots + C_{2N}\, \overset{+}{z}_{2Ni}\\
&\vdots\\
B_{Ni} &= b_{N1}\, z_{N1i} + b_{N2}\, z_{N2i} + \ldots + b_{NP}\, z_{NP} + C_{N-1,N}\, \overset{+}{z}_{N-1,N_i}
\end{aligned}\right\} \tag{18}$$

The jth mean value terms in equation (18) are given by

$$\bar{Z}_j = \sum \frac{Z_{ji}}{n} \tag{19}$$

$$\overline{Z_jZ_N} = \sum \frac{Z_{ji}\,Z_{Ni}}{n} \tag{20}$$

and for convenience z and z^+ terms in equation (18) have been defined as:

$$\left.\begin{array}{l}
\left.\begin{array}{l}
z_{11i} = Z_{1i} - \bar{Z}_1 \\
z_{12i} = Z_{1i}^2 - \bar{Z}_1^2 \\
\vdots \\
z_{1Pi} = Z_{1i}{}^P - Z_1{}^P
\end{array}\right\} \\
\\
\left.\begin{array}{l}
z_{21i} = Z_{2i} - \bar{Z}_2 \\
z_{22i} = Z_{2i}^2 - \bar{Z}_2^2 \\
\vdots \\
z_{2Pi} = Z_{2i}{}^P - Z_2{}^P
\end{array}\right\} \\
\\
\left.\begin{array}{l}
z_{N1i} = Z_{Ni} - Z_N \\
z_{N2i} = Z_{Ni}^2 - Z_N^2 \\
\vdots \\
z_{NPi} = Z_{Ni}{}^P - Z_N{}^P
\end{array}\right\}
\end{array}\right\} \tag{21}$$

$$\left.\begin{array}{l}
z_{12i}^{+} = Z_{1i}\, Z_{2i} - \overline{Z_1 Z_2} \\
z_{13i}^{+} = Z_{1i}\, Z_{3i} - \overline{Z_1 Z_3} \\
\vdots \\
z_{1Ni}^{+} = Z_{1i}\, Z_{Ni} - \overline{Z_1 Z_N}
\end{array}\right\} \tag{22}$$

$$\left.\begin{array}{l}\left.\begin{array}{l} z^{+}_{23i} = Z_{2i}\, Z_{3i} - \overline{Z_2 Z_3} \\ z^{+}_{24i} = Z_{2i}\, Z_{4i} - \overline{Z_2 Z_4} \\ \quad\vdots \\ z^{+}_{2Ni} = Z_{2i}\, Z_{Ni} - \overline{Z_2 Z_N} \end{array}\right\} \\ \qquad\qquad\qquad\vdots \\ \left. z^{+}_{N\text{-}1,N_i} = Z_{N\text{-}1,i}\, Z_{Ni} - \overline{Z_{N\text{-}1} Z_N} \right\} \end{array}\right\} \qquad \begin{array}{r}(22)\\ \text{(Concluded)}\end{array}$$

Regrouping equation (2) the following is obtained

$$Y_i^c = b_0 + B_{1i} + B_{2i} + \ldots B_{Ni} \qquad (23)$$

where

$$b_0 = K_{1i} + K_{2i} + \ldots + K_{Ni} \qquad (24)$$

Through the use of the normal equations resulting from the least squares minimizing function it is found that

$$b_0 = \bar{Y} = \frac{\sum y_i}{n} \qquad (25)$$

The difference in the observed value and the mean value of the observed values of the dependent variable is again denoted by:

$$y_i = Y_i - \bar{Y} \qquad (26)$$

Then the following matrix solution for the unknown constants of the model equation can be obtained through the use of the normal equations:

$$\underset{[M\mathrm{x}1]}{\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix}} = \underset{[M\mathrm{x}M]}{\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix}^{-1}} \underset{[M\mathrm{x}1]}{\begin{bmatrix} S_{1Y} \\ S_{2Y} \\ \vdots \\ S_{NY} \end{bmatrix}} \tag{27}$$

where

$$M = NP + N!/2(N-2)! \tag{28}$$

and the various matrix quantities are given by:

$$\left.\begin{aligned} B_1 &= \begin{bmatrix} b_{11} \\ \vdots \\ b_{1P} \\ \\ c_{12} \\ \vdots \\ c_{1N} \end{bmatrix} \\ B_2 &= \begin{bmatrix} b_{21} \\ \vdots \\ b_{2P} \\ \\ c_{23} \\ \vdots \\ c_{2N} \end{bmatrix} \end{aligned}\right\} \tag{29}$$

$$\left.B_N = \begin{bmatrix} b_{N1} \\ \vdots \\ b_{NP} \\ \\ c_{N-1,N} \end{bmatrix}\right\rbrace \qquad \begin{array}{r}(29)\\ \text{(Concluded)}\end{array}$$

$$\left.\begin{aligned}
S_{11} &= \begin{bmatrix}
\sum z_{11i} z_{11i} & \cdots & \sum z_{11i} z_{1Pi} & \sum z_{11i} z^{+}_{12i} & \cdots & \sum z_{11i} z^{+}_{1Ni} \\
\vdots & & & & & \\
\sum z_{1Pi} z_{11i} & \cdots & \sum z_{1Pi} z_{1Pi} & \sum z_{1Pi} z^{+}_{12i} & \cdots & \sum z_{1Pi} z^{+}_{1Ni} \\
\sum z^{+}_{12i} z^{+}_{11i} & \cdots & \sum z^{+}_{12i} z^{+}_{1Pi} & \sum z^{+}_{12i} z^{+}_{12i} & \cdots & \sum z^{+}_{12i} z^{+}_{1Ni} \\
\vdots & & & & & \\
\sum z^{+}_{1Ni} z^{+}_{11i} & \cdots & \sum z^{+}_{1Ni} z^{+}_{1Pi} & \sum z^{+}_{1Ni} z^{+}_{12i} & \cdots & \sum z^{+}_{1Ni} z^{+}_{1Ni}
\end{bmatrix} \\
S_{21} &= \begin{bmatrix}
\sum z_{21i} z_{11i} & \cdots & \sum z_{21i} z_{1Pi} & \sum z^{+}_{21i} z^{+}_{12i} & \cdots & \sum z_{21i} z^{+}_{1Ni} \\
\vdots & & & & & \\
\sum z_{2Pi} z_{11i} & \cdots & \sum z_{2Pi} z_{1Pi} & \sum z_{2Pi} z^{+}_{12i} & \cdots & \sum z_{2Pi} z^{+}_{1Ni} \\
\sum z^{+}_{23i} z^{+}_{11i} & \cdots & \sum z^{+}_{23i} z^{+}_{1Pi} & \sum z^{+}_{23i} z^{+}_{12i} & \cdots & \sum z^{+}_{23i} z^{+}_{1Ni} \\
\vdots & & & & & \\
\sum z^{+}_{2Ni} z^{+}_{11i} & \cdots & \sum z^{+}_{2Ni} z^{+}_{1Pi} & \sum z^{+}_{2Ni} z^{+}_{12i} & \cdots & \sum z^{+}_{2Ni} z^{+}_{1Ni}
\end{bmatrix} \\
&\vdots \\
S_{N1} &= \begin{bmatrix}
\sum z_{N1i} z_{11i} & \cdots & \sum z_{N1i} z_{1Pi} & \sum z_{N1i} z^{+}_{12i} & \cdots & \sum z_{N1i} z^{+}_{1Ni} \\
\vdots & & & & & \\
\sum z_{NPi} z_{11i} & \cdots & \sum z_{NPi} z_{1Pi} & \sum z_{NPi} z^{+}_{12i} & \cdots & \sum z_{NPi} z^{+}_{1Ni} \\
 & & & & & \\
\sum z^{+}_{N-1,Ni} z_{11i} & \cdots & \sum z^{+}_{N-1,Ni} z_{1Pi} & \sum z^{+}_{N-1,Ni} z^{+}_{12i} & \cdots & \sum z_{N-1,Ni} z^{+}_{1Ni}
\end{bmatrix}
\end{aligned}\right\rbrace \quad (30)$$

$$
\left.
\begin{array}{l}
S_{12} = \left[\begin{array}{cccccccc}
\sum z_{11i} z_{21i} & \cdots & \sum z_{11i} z_{2Pi} & & \sum z_{11i} z^{+}_{23i} & \cdots & \sum z_{11i} z^{+}_{2Ni} & \\
\vdots & & & & & & & \\
\sum z_{1Pi} z_{21i} & \cdots & \sum z_{1Pi} z_{2Pi} & & \sum z_{1Pi} z^{+}_{23i} & \cdots & \sum z_{1Pi} z^{+}_{2Ni} & \\
\vdots & & & & & & & \cdots \\
\sum z^{+}_{12i} z_{21i} & \cdots & \sum z^{+}_{12i} z_{1Pi} & & \sum z^{+}_{12i} z^{+}_{23i} & \cdots & \sum z^{+}_{12i} z^{+}_{2Ni} & \\
\vdots & & & & & & & \\
\sum z^{+}_{1Ni} z_{21i} & \cdots & \sum z^{+}_{1Ni} z_{2Pi} & & \sum z^{+}_{1Ni} z^{+}_{23i} & \cdots & \sum z^{+}_{1Ni} z^{+}_{2Ni} &
\end{array}\right] \\
\\
S_{22} = \left[\begin{array}{cccccccc}
\sum z_{21i} z_{21i} & \cdots & \sum z_{21i} z_{2Pi} & & \sum z_{21i} z^{+}_{23i} & \cdots & \sum z_{21i} z^{+}_{2Ni} & \\
\vdots & & & & & & & \\
\sum z_{2Pi} z_{21i} & \cdots & \sum z_{2Pi} z_{2Pi} & & \sum z_{2Pi} z^{+}_{23i} & \cdots & \sum z_{2Pi} z^{+}_{2Ni} & \\
\vdots & & & & & & & \cdots \\
\sum z^{+}_{23i} z_{21i} & \cdots & \sum z^{+}_{23i} z_{2Pi} & & \sum z^{+}_{23i} z^{+}_{23i} & \cdots & \sum z^{+}_{23i} z^{+}_{2Ni} & \\
\vdots & & & & & & & \\
\sum z^{+}_{2Ni} z_{21i} & \cdots & \sum z^{+}_{2Ni} z_{2Pi} & & \sum z^{+}_{2Ni} z^{+}_{23i} & \cdots & \sum z^{+}_{2Ni} z^{+}_{2Ni} &
\end{array}\right] \\
\vdots \\
S_{N2} = \left[\begin{array}{cccccccc}
\sum z_{N1i} z_{21i} & \cdots & \sum z_{N1i} z_{2Pi} & & \sum z_{N1i} z^{+}_{23i} & \cdots & \sum z_{N1i} z^{+}_{2Ni} & \\
\vdots & & & & & & & \\
\sum z_{NPi} z_{21i} & \cdots & \sum z_{NPi} z_{2Pi} & & \sum z_{NPi} z^{+}_{23i} & \cdots & \sum z_{NPi} z^{+}_{2Ni} & \cdots \\
 & & & & & & & \\
\sum z^{+}_{N-1,Ni} z_{21i} & \cdots & \sum z^{+}_{N-1,Ni} z_{2Pi} & & \sum z^{+}_{N-1,Ni} z^{+}_{23i} & \cdots & \sum z^{+}_{N-1,Ni} z^{+}_{2Ni} &
\end{array}\right]
\end{array}
\right\}
\quad \begin{array}{c} (30) \\ \text{(Continued)} \end{array}
$$

$$
\left.
\begin{array}{l}
S_{1N} = \begin{bmatrix}
\sum z_{11i} z_{N1i} \cdots & \sum z_{11i} z_{NPi} & \sum z_{11i} \overset{+}{z}_{N-1,Ni} \\
\vdots & & \\
\sum z_{1Pi} z_{N1i} \cdots & \sum z_{1Pi} z_{NPi} & \sum z_{1Pi} \overset{+}{z}_{N-1,Ni} \\
 & & \\
\sum \overset{+}{z}_{12i} z_{N1i} \cdots & \sum \overset{+}{z}_{12i} z_{NPi} & \sum \overset{+}{z}_{12i} \overset{+}{z}_{N-1,Ni} \\
\vdots & & \\
\sum \overset{+}{z}_{1Ni} z_{N1i} \cdots & \sum \overset{+}{z}_{1Ni} z_{NPi} & \sum \overset{+}{z}_{1Ni} \overset{+}{z}_{N-1,Ni}
\end{bmatrix} \\
\\
S_{2N} = \begin{bmatrix}
\sum z_{21} z_{N1i} \cdots & \sum z_{21i} z_{NPi} & \sum z_{21i} \overset{+}{z}_{N-1,Ni} \\
\vdots & & \\
\sum z_{2Pi} z_{N1i} \cdots & \sum z_{2Pi} z_{NPi} & \sum z_{2Pi} \overset{+}{z}_{N-1,Ni} \\
 & & \\
\sum \overset{+}{z}_{23i} z_{N1i} \cdots & \sum \overset{+}{z}_{23i} z_{NPi} & \sum \overset{+}{z}_{23i} \overset{+}{z}_{N-1,Ni} \\
\vdots & & \\
\sum \overset{+}{z}_{2Ni} z_{N1i} \cdots & \sum \overset{+}{z}_{2Ni} z_{NPi} & \sum \overset{+}{z}_{2Ni} \overset{+}{z}_{N-1,Ni}
\end{bmatrix} \\
\vdots \\
S_{NN} = \begin{bmatrix}
\sum z_{N1i} z_{N1i} \cdots & \sum z_{N1i} z_{NPi} & \sum z_{N1i} \overset{+}{z}_{N-1,Ni} \\
\vdots & & \\
\sum z_{NPi} z_{N1i} \cdots & \sum z_{NPi} z_{NPi} & \sum z_{NPi} \overset{+}{z}_{N-1,Ni} \\
 & & \\
\sum \overset{+}{z}_{N-1,Ni} z_{N1i} \cdots & \sum \overset{+}{z}_{N-1,Ni} z_{NPi} & \sum \overset{+}{z}_{N-1,Ni} \overset{+}{z}_{N-1,Ni}
\end{bmatrix}
\end{array}
\right\}
\qquad \begin{array}{r} (30) \\ \text{(Concluded)} \end{array}
$$

$$\left.\begin{aligned} S_{1y} &= \begin{bmatrix} \sum z_{11i}\, y_i \\ \vdots \\ \sum z_{1Pi}\, y_i \\ \\ \sum z_{12}^{+}\, y_i \\ \vdots \\ \sum z_{1Ni}^{+}\, y_i \end{bmatrix} \\ S_{2y} &= \begin{bmatrix} \sum z_{21i}\, y_i \\ \vdots \\ \sum z_{2Pi}\, y_i \\ \\ \sum z_{23i}^{+}\, y_i \\ \vdots \\ \sum z_{2Ni}^{+}\, y_i \end{bmatrix} \\ &\;\vdots \\ S_{Ny} &= \begin{bmatrix} \sum z_{N1i}\, y_i \\ \vdots \\ \sum z_{NPi}\, y_i \\ \\ \sum z_{N-1,Ni}^{+}\, y_i \end{bmatrix} \end{aligned}\right\} \tag{31}$$

Once the matrix solution given by equation (27) has been determined the intercept A_0 of the fitted model expression may be calculated as

$$
\begin{aligned}
A_0 = \bar{Y} &- (b_{11}\bar{Z}_1 + b_{12}\bar{Z}_1^2 + \ldots + b_{1P}\bar{Z}_1^{\,P} + c_{12}\overline{Z_1Z_2} + c_{13}\overline{Z_1Z_3} + \ldots + c_{1N}\overline{Z_1Z_N}) \\
&- (b_{21}\bar{Z}_2 + b_{22}\bar{Z}_2^2 + \ldots + b_{2P}\bar{Z}_2^{\,P} + c_{23}\overline{Z_2Z_3} + c_{24}\overline{Z_2Z_4} + \ldots + c_{2N}\overline{Z_2Z_n}) \\
&- \ldots - (b_{N1} + b_{N2}\bar{Z}_N^2 + \ldots + b_{NP}\bar{Z}_N^{\,P} + c_{N-1,N}\overline{Z_{N-1}Z_N})
\end{aligned}
\qquad (32)
$$

C. Comments

The solution developed for evaluating multiple variable regression for Pth degree polynomials with linear cross products is similar to that developed in Section II for the case without cross products. However, in this case, the nested matrices contain additional terms to account for the cross products and hence it is required that $n \geqq NP + \frac{N!}{2(N-2)!}$, which again indicates that the number of data points n must be equal to or greater than the number of unknown parameters in the model equation.

As discussed in Appendix B, a computer program has been developed to treat this case. It should be noted that while contrived problems have yielded excellent results, experience with several sets of physical data have generally yielded poorer results. This is evidently due to the larger matrices being manipulated and due to unknowns as to whether true linear cross coupling exists in the given physical process.

IV. SIGNIFICANCE OF THE ESTIMATED REGRESSION EQUATIONS

A. Introduction

To determine if the fitted regression equation obtained from the solution matrix is indeed a useful representation of the input data, it is desirable that certain diagnostics be evaluated. To this end the computer programs discussed in Appendix A and Appendix B determine the standard deviation of the observed data with respect to the fitted equation, the multiple correlation coefficient, and the F statistic as well as the average error and the maximum error of the observed data with respect to the fitted result.

Using these parameters, it is possible to assess the usefulness of the fitted expression for each given application.

B. Analysis of Variance

The significance of the estimated regression equation can be considered from the viewpoint of an analysis of variance as summarized in Table 1, where the total sum of squares is resolved into a component measuring the residual fitting error, and a component which measures the regression variation being tested.

TABLE 1. ANALYSIS OF VARIANCE (WITH AND WITHOUT CROSS PRODUCTS)

A. Analysis of Variance (without Cross Products)

Degrees of Freedom	Type Variation	Sum of Squares (SS)	Mean Square (MS)	F Value
n-1	Total	$S_{yy} = \sum (Y_i - Y)^2$		
NP-1	Residual	$S(RES) = \sum (Y_i - Y_i^c)^2$	$M(RES) = \frac{S(RES)}{NP-1}$	
n-NP	Regression	$S(REG) = \sum (Y_i^c - \bar{Y})^2$	$M(REG) = \frac{S(REG)}{n-NP}$	$\frac{M(REG)}{M(RES)}$

B. Analysis of Variance (with Cross Products)

Degrees of Freedom	Type Variation	Sum of Squares (SS)	Mean Square (MS)	F Value
	Total	$S_{yy} = \sum (Y_i - Y)^2$		
K-1	Residual	$S(RES) = \sum (Y_i - Y_i^c)^2$	$M(RES) = \frac{S(REG)}{k-1}$	
n-k	Regression	$S(REG) = \sum (Y_i^c - Y)^2$	$M(REG) = \frac{S(RES)}{n-k}$	$\frac{M(REG)}{M(RES)}$

where $MS = SS/\text{degrees of freedom}$

$S_{yy} = S(RES) + S(REG)$

$\bar{Y}$ = average of observed values

$k = NP + (N-1) + (N-2) + \ldots + 1$

C. Interpretation Diagnostics

As noted by Smille [3], the results of such an analysis of variance can be used to test the combined effect of all of the independent variables on the dependent variable. That is, the hypothesis that all of the population regression coefficients in the model regression equation are zero can be tested since the ratios of the regression mean square to the residual mean square are distributed in an F distribution as shown below:

$$F = \frac{S(REG)/\text{Regression degrees of freedom}}{S(RES)/\text{Residual degrees of freedom}} \tag{33}$$

where it is assumed that the observations are selected at random- from a normally distributed population with zero mean and constant variance, that $Z_1, Z_2, \ldots Z_N$ are independent variables following χ^2 distributions, and that only random errors are associated with the observations.

The F ratio calculated from equation (33) can be used to test the statistical significance of the regression equation under consideration by comparing it with the appropriate F_{TABLE} value at the desired probability level with the specified numerator and denominator degrees of freedom; that is, the following test of the null hypothesis may be performed:

$$\begin{aligned} &H_0 : b_{10} = b_{12} = \ldots b_{NP} = 0 \\ &\text{accept when} \quad F_c > F_{TABLE} \\ &\text{reject when} \quad F_c < F_{TABLE} \end{aligned} \tag{34}$$

D. Output Data

The desired coefficients of the model equation are outputted in E notation where A_0 is the computed intercept of the fitted polynomial expression and the B coefficients are printed out in ascending order of degree with the first P coefficients indicating b coefficients of the first independent variable, the second set of P coefficients indicating the b coefficient of the second independent variable, etc.

Also included in the output data are the number of independent variables N, the degree of the model equation P, and the number of input data points (sets) n. Further, the significance of the estimated regression equation is indicated by the standard deviation, the multiple correlation coefficient, the F ratio, the maximum percent error, and the average percent error.

The values of the dependent variable are also shown for the input data and for the computed values obtained from the fitted expression for each set of data input, as well as the residual difference in the input values and the computed values.

When called for, plotted results can also be obtained which compare the input and computed dependent variables as ordinates to the point interval along the abscissa (which is normally one of the physical independent variables, but which can be a unit indication of each data set in order of input to program). All computed points are connected by straight lines and are plotted with * symbols. The input points are plotted with + symbols and the points are not connected with lines.

Another useful parameter in testing the significance of the regression equation is the standard deviation which is estimated by:

$$s = \left[\frac{\sum (Y_i - Y_i^c)^2}{\text{Regression degrees of freedom} - 1}\right]^{1/2} = \left[\frac{S(RES)}{\text{Regression degrees of freedom} - 1}\right]^{1/2} \tag{35}$$

This result stems from the work of Junkin [4].

Earlier, the analysis of variance technique was used to test the combined effect of the independent variables on the dependent variable using the F statistic. A closely related statistic is the multiple correlation coefficient. R. Smille [3] defines this statistic as the simple correlation coefficient between the observed values of the dependent variable and those estimated by the multiple regression function as given by:

$$R = \frac{S(REG)}{S_{YY}}^{1/2} \tag{36}$$

If the observed and estimated values are completely unrelated, R will be zero and, if they are identical the multiple correlation coefficient will be unity. Values in between these limits represent different degrees of correlation or the closeness within which the regression equation describes the original data.

Also of interest is the average of the absolute values of the percent error of the dependent variable, considering each fitted observation:

$$|\epsilon|_{AVG} = \frac{1}{n} \sum \left| \frac{Y_i - Y_i^c}{Y_i} \right| \times 100$$

During this calculation the maximum absolute error condition can be determined for evaluation purposes.

V. SUMMARY

A powerful multiple regression technique for Pth degree polynomials with and without linear cross products has been developed with which the nonlinear behavior of identified independent variables can be related to a given dependent variable. The polynomial expression can be of the Pth degree and can incorporate N independent variables. The resulting surface fit can be used to summarize trends for a given phenomenon and the analytic results provide a mathematical basis for subsequent analysis.

It is required in the application of this technique that the S matrix be nonsingular and that the input data are reasonably well behaved. It is also required that the number of data points being fitted be equal to or greater than the number of unknown parameters in the model equation.

Two computer programs have been developed to implement this technique for the cases with and without linear cross products. These programs perform matrix operations in double precision and evaluate the various unknown constants in the model regression equation as well as the standard deviation, the multiple regression coefficient, the F statistic, the maximum absolute percent error, and the average of the absolute values of the percent error. Further included in these programs is the solution for the identity matrix to identify any problems in the original matrix inversion process and a means of obtaining machine plots, comparing the computer results to each set of input data.

Studies of many sets of physical data using the technique without cross products have generally yielded excellent results. However, much less experience has been obtained using the program with cross products, although test programs have shown very good correlation. It is, therefore, felt that the technique developed herein can be highly useful to many fields of endeavor.

APPENDIX A

MULTIPLE REGRESSION PROGRAM FOR CASE WITHOUT CROSS PRODUCTS

A. Input Data

The regression subroutine is called with the following statement:

```
CALL REGRES(Z,YO,LN,N,IP)
```

where:

Z is a two-dimensional array containing the independent variables used. The first subscript denotes the independent variable and the second subscript denotes the data set.

YO is an array containing the dependent variables which correspond to the independent variables.

LN is the number of data sets.

N is the number of independent variables used.

IP is the degree of the polynomial curve fit.

These arguments must be assigned a value in a driver program.

The regression subroutine incorporates several special routines: RDWT, IOWR, INVRT, and DMATML.

RDWT is a general FORTRAN I/O package which is capable of reading and/or writing on magnetic tape or drum in FORTRAN or non-FORTRAN format.

IOWR is an assembly language I/O routine called by RDWT.

INVRT is a double precision matrix inversion and simultaneous equation solver. The call to this routine is:

```
CALL INVRT(A,N,M,DETER)
```

where:

A is the input matrix for inversion or augmented matrix for simultaneous equations.

N is the order of the coefficient matrix.

M=0 for inversion only; otherwise, M is the number of constant vectors.

DETER is the determinant of the coefficient matrix.

DMATML is a double precision matrix multiplication routine with transpose options. The call to this routine is:

```
CALL DMATML(C,A,B,M,N,K)
```

where:

C is the product of matrix A times matrix B,

A is the first input matrix,

B is the second input matrix,

M is the number of rows of A,

N is the number of rows of B, and

K is the number of columns of B.

Certain variables have dimension changes that vary depending on the number of data points (LN), the number of independent variables (N), and the degree of the polynomial curve fit (IP). These will be dimensioned as follows:

DSN(NIP2), DSHAT(NIP,1), DB(NIP,1), Y(LN), RESID(LN), Z(N,LN), SHAT(NIP,NIP), ZBAR(N,IP), YO(LN), BZ(LN), B(N,IP), SVHAT(NIP,1), SH(NIP,NIP), XX(LN), PMAX(LN)

where:

LN,N,IP are as defined above and NIP is computed as: NIP = N*IP; NIP2 is NIP raised to the second power.

SMALLZ, SMALZ1, and SMALZ2 should be dimensioned greater than the number of data points and the dimension is set in a PARAMETER statement; for example:

```
PARAMETER IDIM = 100
```

B. Program

In order to run this program on the UNIVAC 1108, the deck is set up as follows:

```
@RUN,//T JBNAME,320590,VDARBYBIN406, 01, 100
@ASG,T 10,F/1/POS/2
@FOR,IS MAIN, MAIN
    (Source deck for inputting data)
@FOR,IS REGRES, REGRES
    (Source deck)
@FOR,IS RDWT, RDWT
    (Source deck)
@ASM,IS IOWR, IOWR
    (Source deck)
@FOR,IS INVRT, INVRT
    (source deck)
@FOR,IS DMATML, DMATML
    (Source deck)
@MAP,I AA, AA
    LIB SYS$*MSFC $.
@XQT AA
    (Input data)
@FIN
```

as illustrated in Figure 1.

A complete program listing for the case without cross products is shown in Table 2 and a flow chart indicating the operation of this computer program is indicated in Figure 2.

C. Output Data

The desired coefficients of the model equation are outputted in E notation where AO is the computed intercept of the fitted polynomial expression and the B coefficients are printed out in ascending order of degree (P) with the first P coefficients indicating the b coefficients of the first independent variable, the second set of P coefficients indicating the b coefficients of the second independent variable, etc.

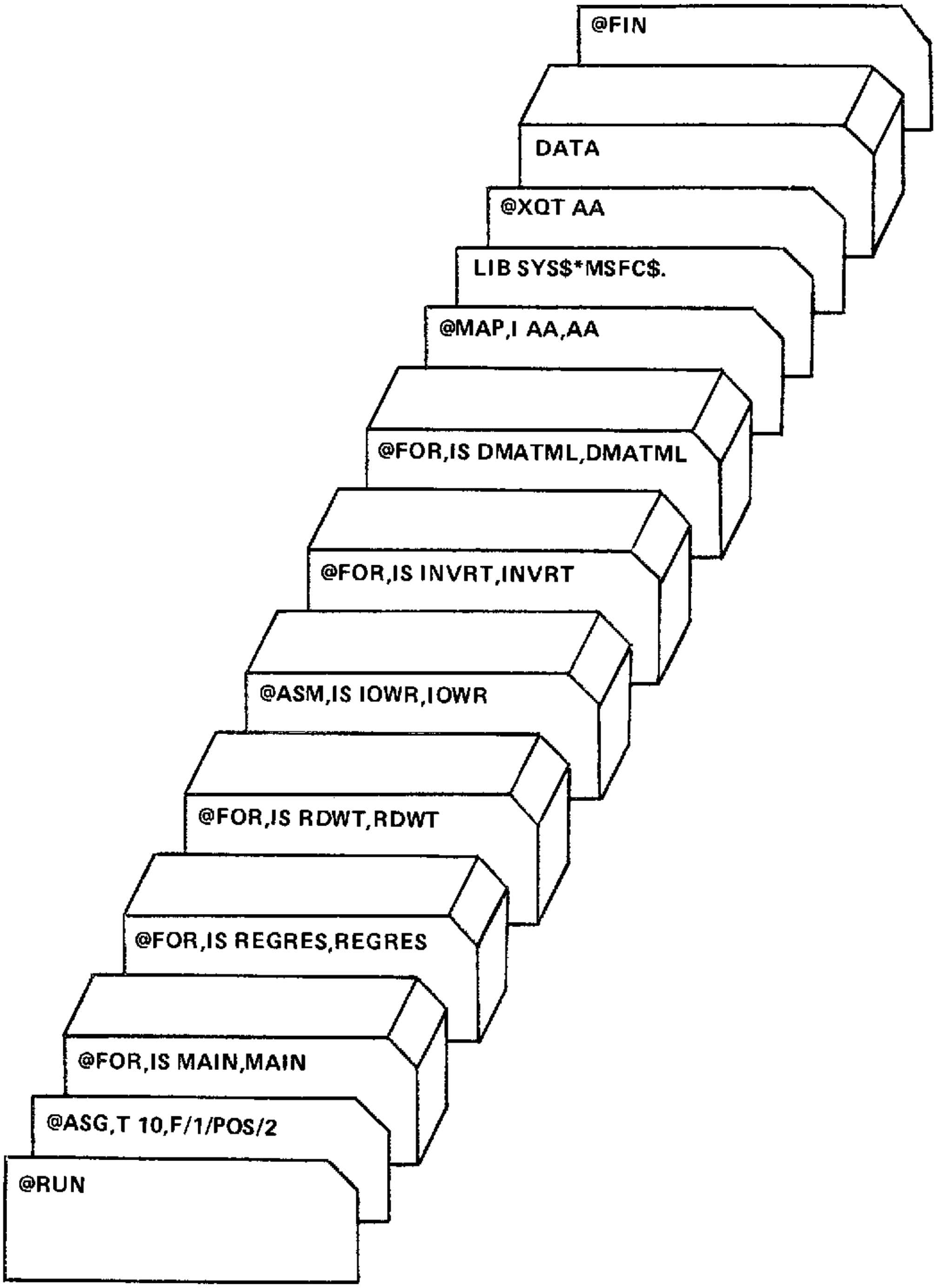


Figure 1. Program deck sequence.

TABLE 2. PROGRAM LISTING (WITHOUT CROSS PRODUCTS)

```
VV        VV  DDDDDDDDDD      AAAAAAAA    RRRRRRRRRRR   BBBBBBBBBBB   YY        YY
VV        VV  DDDDDDDDDDD    AAAAAAAAAA   RRRRRRRRRRRR  BBBBBBBBBBBB  YYY      YYY
 VV      VV   DD        DD  AA        AA  RR        RR  BB        BB   YYY    YYY
 VV      VV   DD        DD  AA        AA  RR        RR  BB        BB    YYY  YYY
  VV    VV    DD        DD  AA        AA  RR        RR  BB        BB     YYYYYY
  VV    VV    DD        DD  AAAAAAAAAAAA  RRRRRRRRRRRR  BBBBBBBBBBB       YYYY
   VV  VV     DD        DD  AAAAAAAAAAAA  RRRRRRRRRRR   BBBBBBBBBBB        YY
   VV  VV     DD        DD  AA        AA  RR    RR      BB        BB       YY
    VVVV      DD        DD  AA        AA  RR     RR     BB        BB       YY
    VVVV      DD        DD  AA        AA  RR      RR    BB        BB       YY
     VV       DDDDDDDDDDD   AA        AA  RR       RR   BBBBBBBBBBBB       YY
     VV       DDDDDDDDDD    AA        AA  RR        RR  BBBBBBBBBBB        YY

BBBBBBBBBBB      IIIIII     NN        NN       44  44      0000        666666
BBBBBBBBBBBB     IIIIII     NNN       NN       44  44    00000000    6666666666
BB        BB       II       NNNN      NN      44   44   000    000  666       66
BB        BB       II       NNNNN     NN     44    44  000      000 66
BB        BB       II       NN NNN    NN    44     44  00        00 66
BBBBBBBBBBB        II       NN  NNN   NN   44444444444 00        00 66 66666666
BBBBBBBBBBB        II       NN   NNN  NN   44444444444 00        00 66666666666
BB        BB       II       NN    NNN NN           44  00        00 66        66
BB        BB       II       NN     NNNNN           44  000      000 66        66
BB        BB       II       NN      NNNN           44   000    000  66        66
BBBBBBBBBBBB     IIIIII     NN       NNN           44    00000000    6666666666
BBBBBBBBBBB      IIIIII     NN        NN           44      0000        666666

    0000           11            11        999999     777777777777     333333
  00000000        111           111      9999999999   777777777777   3333333333
 000    000      1111          1111     99        99          777   333      333
000      000       11            11     99        99         777    33        33
00        00       11            11     99        99        777               33
00        00       11            11      99999999999       777              333
00        00       11            11       9999999 99      777               333
00        00       11            11               99     777                  33
000      000       11            11               99    777         33        33
 000    000        11            11     99        99   777          333      333
  00000000       111111        111111    9999999999   777            3333333333
    0000         111111        111111      999999     777              333333

    0000           44  44      0000        666666        222222     777777777777
  00000000         44  44    00000000    6666666666    2222222222   777777777777
 000    000       44   44   000    000  666       66  222      222          777
000      000     44    44  000      000 66            22      222          777
00        00    44     44  00        00 66                   222          777
00        00   44444444444 00        00 66 66666666         222          777
00        00   44444444444 00        00 66666666666        222          777
00        00           44  00        00 66        66      222          777
000      000           44  000      000 66        66     222          777
 000    000            44   000    000  66        66    222          777
  00000000             44    00000000    6666666666   222222222222  777
    0000               44      0000        666666     222222222222  777

@RUN,//Z WINNCP,322570,VDARBYBIN4D6,G1,1000
```

TABLE 2. (Continued)

```
 @MSG,N                   REMOVE AT CARD READER      01-18-73          CLOCK NO. 0093


 @ASG,T 13,F/1/POS/2

@FOR,IS MAIN,MAIN
RS0 009-01/19-00:56-(,0)-

  MAIN PROGRAM

  STORAGE USED: CODE(1) 000021; DATA(0) 000101; BLANK COMMON(2) 000000

  EXTERNAL REFERENCES (BLOCK, NAME)

   0003   REGRES
   0004   NINTR$
   0005   NWRNE$
   0006   NSTOP$

  STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

   0000 I 000052 IP       0000 I 000050 LN        0000 I 000051 N        0000   000055 NAM1       0000 R 000036 Y0
   0000 R 000000 Z


00101      1*          DIMENSION Z(3,10),Y0(10)
00103      2*          NAMELIST/NAM1/Z,Y0,LN,N,IP
00104      3*          READ(5,NAM1)
00107      4*          CALL REGRES(Z,Y0,LN,N,IP)
00110      5*          STOP
00111      6*          END
      END OF COMPILATION:        NO  DIAGNOSTICS.
```

TABLE 2. (Continued)

```
@FOR,IS REGRES,REGRES
HSO 029-01/19-00:59-(,0)

   SUBROUTINE REGRES     ENTRY POINT 001240

   STORAGE USED: CODE(1) 001310; DATA(0) 001662; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003     IDENT
    0004     IOARG$R
    0005     INVRT
    0006     DMATML
    0007     GUIK3L
    0010     CNDUU$
    0011     NRDU$
    0012     NIO2$
    0013     NEXP5$
    0014     SORT
    0015     NIOI$
    0016     NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0000     001501 1000F      0000     001470 10000F     0000     001541 10003F     0000     001552 10021F     0001     000075 137G
    0001     000076 142G       0001     000106 150G       0000     001531 1500F      0001     000112 155G       0001     000113 160G
    0001     000123 166G       0001     000127 173G       0001     000144 206G       0001     000146 211G       0001     000150 214G
    0000     001522 2222F      0001     000215 227G       0001     000217 232G       0001     000222 235G       0000     001476 24F
    0001     000264 250G       0000     001526 2500F      0000     001537 2502F      0001     000274 253G       0001     000320 261G
    0001     000330 264G       0001     000350 271G       0001     000422 314G       0001     000423 317G       0001     000451 330G
    0001     000470 340G       0001     000500 343G       0001     000532 350G       0001     000567 364G       0001     000570 367G
    0001     000610 377G       0001     000611 402G       0001     000633 414G       0001     000635 417G       0001     000637 422G
    0001     000667 431G       0001     000725 444G       0001     001004 460G       0001     001100 503G       0001     001101 505G
    0001     001175 505L       0001     001127 521G       0001     001145 536G       0001     001167 546G       0000 R   001331 ADARY
    0000 R   001466 AMAX       0000 R   001455 AO         0000 R   000644 B          0000 R   000632 BZ         0000 D   000264 DB
    0000 D   000306 DETER      0000 D   000242 DSHAT      0000 D   000000 DSN        0000 R   001371 FLDX       0000 R   001405 FLDY
    0000 R   001460 FRATIO     0000 I   001421 I          0000 I   001452 ICL        0000 I   001445 ICOL       0000 I   001443 IERR
    0000 I   001446 IK         0000     001611 INJP$      0000 I   001454 IR         0000 I   001444 IROW       0000 I   001442 ISECT
    0000 I   001440 IUNIT      0000 I   001451 I2         0000 I   001430 J          0000 I   001431 K          0000 I   001447 L
    0000 I   001461 LI         0000 I   001462 L1         0000 I   001463 L2         0000 I   001453 M          0000 I   001427 NIP
    0000 I   001450 NR         0000 I   001441 NSECT      0000 I   001422 NWDS       0000 R   001465 PERAVG     0000 R   001467 PERMAX
    0000 R   001357 PMAX       0000 R   000322 RESID      0000 R   001457 RHAT       0000 R   001434 RR         0000 R   001176 SH
    0000 R   000500 SHAT       0000 R   000334 SMALLZ     0000 R   000666 SMALZ1     0000 R   001032 SMALZ2     0000 R   001435 SREG
    0000 R   001437 SRES       0000 R   001456 STADEV     0000 R   001464 SUM        0000 R   000655 SVHAT      0000 R   001436 SYY
    0000 R   001423 XL         0000 R   001424 XR         0000 R   001317 XX         0000 R   000310 Y          0000 R   001425 YB
    0000 R   001432 YBARC      0000 R   001433 YBARO      0000 R   001426 YT         0000 R   000621 ZBAR


00101      1*          SUBROUTINE REGRES(Z,YO,LN,N,IP)
00101      2*     C      THIS SUBROUTINE IS DESIGNED TO DETERMINE SPECIFIED MODEL
```

TABLE 2. (Continued)

```
00101     3*   C       PARAMETERS USING REGRESSION TECHNIQUES
00103     4*           PARAMETER IDIM=100
00104     5*           DOUBLE PRECISION DSN(81),DSHAT(9,1),DD(9,1)
00105     6*           DOUBLE PRECISION DETER
00106     7*           DIMENSION Y(10),RESID(10),Z(3,10),SMALLZ(IDIM),SHAT(9,9),
00106     8*          *ZBAR(3,3),YO(15),BZ(10),B(3,3),SVHAT(9,1),
00106     9*          *SMALZ1(IDIM),SMALZ2(IDIM)
00107    10*           DIMENSION SM(9,9),XX(10)
00110    11*           DIMENSION ADARY(22)
00111    12*           DIMENSION PMAX(10)
00112    13*           DIMENSION FLOX(12),FLOY(12)
00113    14*           DATA FLOY/72H                              Y VALUES
00113    15*          *                    /
00115    16*           DATA FLOX/72H                  ,   POINT INTERVALS
00115    17*          *                    /
00117    18*           DATA (ADARY(I),I=1,3)/6HONE   ,6H  COPY,6HFLO  /
00121    19*           CALL IDENT(Y,ADARY)
00122    20*           NWDS=IDIM
00123    21*           XL=0.0
00124    22*           XR=11.0
00125    23*           YB=4.0
00126    24*           YT=12.0
00127    25*           WRITE(6,10000) N,IP,LN
00134    26*     10000 FORMAT(1H1,2HN=I4,3X,2HP=I4,3X,3HLN=I4)
00135    27*           NIP=N*IP
00136    28*           DO 1234 I=1,N
00141    29*           DO 1234 J=1,IP
00144    30*      1234 ZBAR(I,J)=0.0
00147    31*           DO 1235 K=1,IDIM
00152    32*      1235 SMALLZ(K)=0.0
00154    33*           DO 1236 I=1,NIP
00157    34*           DO 1236 J=1,NIP
00162    35*      1236 SHAT(I,J)=0.0
00165    36*           DO 1237 I=1,NIP
00170    37*      1237 SVHAT(I,1)=0.0
00172    38*           DO 1238 I=1,LN
00175    39*      1238 BZ(I)=0.0
00177    40*           YBARC=0.0
00200    41*           YBARO=0.0
00201    42*           RR=0.0
00202    43*           SREG=0.0
00203    44*           SYY=0.0
00204    45*           SRES=0.0
00204    46*   C       DETERMINE THE SMALL Z VALUES FOR THE ASSUMED MODEL FORM
00204    47*   C
00205    48*           DO 225 I=1,N
00210    49*           DO 225 J=1,IP
00213    50*           DO 210 K=1,LN
00216    51*       210 ZBAR(I,J)=ZBAR(I,J)+Z(I,K)**J
00220    52*       225 ZBAR(I,J)=ZBAR(I,J)/LN
00223    53*           IUNIT=10
00224    54*           NSECT=(NWDS*27)/28
00225    55*           ISECT=1
00226    56*           DO 250 I=1,N
00231    57*           DO 250 J=1,IP
00234    58*           DO 245 K=1,LN
```

TABLE 2. (Continued)

```
00237     59*         245 SMALLZ(K)=Z(I,K)**J-ZBAR(I,J)
00241     60*             CALL IORDWR(1,IUNIT,ISECT,SMALLZ,NWDS,IERR)
00242     61*         250 ISECT=ISECT+NSECT
00242     62*     C
00242     63*     C       COMPUTE SHAT MATRICES FOR USE IN THE LEAST SQUARES SOLUTION
00242     64*     C         OF FINDING THE B'S.
00242     65*     C
00242     66*     C
00245     67*             IROW=0
00246     68*             ICOL=1
00247     69*             DO 450 IK=1,N
00252     70*             DO 400 L=1,IP
00255     71*             ISECT=(IK-1)*IP*NSECT+(L-1)*NSECT+1
00256     72*             CALL IORDWR(2,IUNIT,ISECT,SMALZ1,NWDS,IERR)
00257     73*             IROW=IROW+1
00260     74*             DO 350 J=1,N
00263     75*             DO 350 K=1,IP
00266     76*             ISECT=(J-1)*IP*NSECT+(K-1)*NSECT+1
00267     77*             CALL IORDWR(2,IUNIT,ISECT,SMALZ2,NWDS,IERR)
00270     78*             DO 310 I=1,LN
00273     79*         300 SHAT(IROW,ICOL)=SHAT(IROW,ICOL)+SMALZ1(I)*SMALZ2(I)
00274     80*             SH(IROW,ICOL)=SHAT(IROW,ICOL)
00275     81*         310 CONTINUE
00277     82*             IF(ICOL.EQ.N*IP) ICOL=0
00301     83*         350 ICOL=ICOL+1
00304     84*         400 CONTINUE
00306     85*         450 CONTINUE
00310     86*             NR=IROW
00311     87*             I2=1
00312     88*              ICL=N*IP
00313     89*             DO 1322 I=1,ICL
00316     90*             DO 1322 J=1,IROW
00321     91*             DSH(I2)=SH(J,I)
00322     92*        1322 I2=I2+1
00325     93*             M=0
00326     94*             CALL INVRT(DSH,NR,M,DETER)
00327     95*             DO 650 I=1,LN
00332     96*         650 YBARO=YBARO+YO(I)
00334     97*             YBARO=YBARO/LN
00335     98*             NIP=N*IP+1
00336     99*             IROW=1
00337    100*             DO 710 I=1,N
00342    101*             DO 710 J=1,IP
00345    102*             ISECT=(I-1)*IP*NSECT+(J-1)*NSECT+1
00346    103*             CALL IORDWR(2,IUNIT,ISECT,SMALZ2,NWDS,IERR)
00347    104*             DO 700 K=1,LN
00352    105*             SVHAT(IROW,1)=SVHAT(IROW,1)+SMALZ2(K)*(YO(K)-YBARO)
00353    106*         700 DSHAT(IROW,1)=SVHAT(IROW,1)
00355    107*         710 IROW=IROW+1
00355    108*     C       COMPUTE B'S FOR THE ASSUMED MODEL.
00355    109*     C
00360    110*             IR=IROW-1
00361    111*             CALL DMATML(DB,DSH,DSHAT,IR,IR,1)
00362    112*             I2=1
00363    113*             DO 1331 I=1,N
00366    114*             DO 1331 J=1,IP
```

TABLE 2. (Continued)

```
00371    115*              B(I,J)=DB(I2,I)
00372    116*      1331   I2=I2+1
00375    117*              A0=YBAR0
00376    118*              DO 23 I=1,N
00401    119*              DO 23 J=1,IP
00404    120*         23   A0=A0-B(I,J)*ZBAR(I,J)
00407    121*              WRITE(6,24) A0
00412    122*         24   FORMAT(1H0,3HA0=,E16.9)
00413    123*              DO 750 I=1,LN
00416    124*              DO 750 J=1,N
00421    125*              DO 750 K=1,IP
00424    126*        750   BZ(I)=BZ(I)+B(J,K)*Z(J,I)**K
00424    127*     C
00424    128*     C        COMPUTE THE Y POINTS
00424    129*     C
00430    130*              DO 775 I=1,LN
00433    131*              Y(I)=A0+BZ(I)
00434    132*              YBARC=YBARC+Y(I)
00435    133*              RESID(I)=YO(I)-Y(I)
00436    134*              RR=RR+RESID(I)**2
00437    135*        775   CONTINUE
00441    136*              YBARC=YBARC/LN
00441    137*     C
00441    138*     C        COMPUTE THE STANDARD DEVIATION.
00441    139*     C
00442    140*              STADEV=SQRT(RR/(LN-N*IP))
00442    141*     C
00442    142*     C        COMPUTE THE MULTIPLE CORRELATION COEFFICIENT,RHAT,AND
00442    143*     C        THE F RATIO,FRATIO.
00442    144*     C
00443    145*              DO 800 I=1,LN
00446    146*              SREG=SREG+(Y(I)-YBARC)**2
00447    147*              SYY=SYY+(YO(I)-YBARO)**2
00450    148*        800   SRES=SRES+RESID(I)**2
00452    149*              RHAT=SQRT(SREG/SYY)
00453    150*              IF(RHAT.GT.1.0) RHAT=2.0-RHAT
00455    151*              FRATIO=(SREG/(N*IP-1))/(SRES/(LN-N*IP))
00456    152*              XX(1)=1.
00457    153*              DO 83 LI=2,LN
00462    154*         83   XX(LI)=XX(LI-1)+1.
00464    155*              L1=-1
00465    156*              L2=0
00466    157*              CALL QUIK3L(L1,XL,XR,YB,YT,40,FLDX,FLDY,-LN,XX,Y)
00467    158*              CALL QUIK3L(L2,XL,XR,YB,YT,34,FLDX,FLDY,LN,XX,YO)
00470    159*              WRITE(6,1000) STADEV,RHAT,FRATIO
00475    160*       1000   FORMAT(1H0,19HSTANDARD DEVIATION=E16.9,4X,33HMULTIPLE CORRELATION
00475    161*             *COEFFICIENT=E16.9,4X,8HF RATIO=E16.9)
00476    162*              WRITE(6,2222)
00500    163*       2222   FORMAT(1H0,'B COEFFICIENTS')
00501    164*              WRITE(6,2500) ((B(I,J),J=1,IP),I=1,N)
00512    165*       2500   FORMAT(1H0,5E21.9/)
00513    166*              WRITE(6,1500)
00515    167*              WRITE(6,10003)
00517    168*       1500   FORMAT(1H0,16X,2HYO,16X,2HYC,15X,5HYO-YC)
00520    169*              DO 2000 I=1,LN
00523    170*              WRITE(6,2502) YO(I),Y(I),RESID(I)
```

TABLE 2. (Continued)

```
00530    171.      2532 FORMAT(1H0,3E21.9)
00531    172.      2500 CONTINUE
00533    173.     10003 FORMAT(1H0,12X,8HOBSERVED,12X,8HCOMPUTED,12X,8HRESIDUAL)
00534    174.           SUM=0.
00535    175.           DO 501 L=1,LN
00540    176.           PMAX(L)=ABS((YO(L)-Y(L))/Y(L))
00541    177.       501 SUM=SUM+PMAX(L)
00543    178.           PERAVG=(SUM/LN)*100.
00544    179.           AMAX=PMAX(1)
00545    180.           DO 505 L=2,LN
00550    181.           IF(AMAX-PMAX(L)) 506,505,505
00553    182.       506 AMAX=PMAX(L)
00554    183.       505 CONTINUE
00556    184.           PERMAX=AMAX*100.
00557    185.           WRITE(6,10021) PERMAX,PERAVG
00563    186.     10021 FORMAT(1H0,22HMAXIMUM PERCENT ERROR=E16.9,6X,22HAVERAGE PERCENT ER
00563    187.          *ROR=E16.9)
00564    188.           CALL ENDJOB
00565    189.           RETURN
00566    190.           END

     END OF COMPILATION:        NO  DIAGNOSTICS.
```

TABLE 2. (Continued)

```
@FOR,IS RDWT,RDWT
HSO 009-01/19-01:00-(,0)

   SUBROUTINE RDWT       ENTRY POINT 000270
              REDTPR     ENTRY POINT 000273
              WRITER     ENTRY POINT 000320
              CLOSE      ENTRY POINT 000342
              OPEN       ENTRY POINT 000351
              IORDWR     ENTRY POINT 000360

   STORAGE USED: CODE(1) 000402; DATA(0) 000127; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   IOWR
    0004   NERR2$
    0005   NWDU$
    0006   NIO1$
    0007   NIO2$
    0010   NSTOP$
    0011   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001     000071 1GL        0000     000070 100F       0001     000116 20L        0001     000140 21L        0001     000242 215G
    0001     000144 22L        0001     000211 24L        0001     000227 25L        0001     000234 26L        0001     000033 6L
    0001     000035 7L         0001     000050 8L         0000 I   000061 I          0000 I   000065 IBLANK     0000 I   000000 ICWD
    0000 I   000003 IFLD       0000     000111 INJP$      0000 I   000066 IOP        0000 I   000046 IPACKT     0000 I   000067 ITWD
    0000 I   000062 I1         0000 I   000063 I2         0000 I   000064 I3         0000 I   000056 KNTWD      0000 I   000040 KODE


00100      1*    C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
00100      2*    C       GENERAL FORTRAN I/O PACKAGE          EXEC VIII            APRIL,1970
00100      3*    C       READ/WRITE MAG TAPE OR DRUM IN FORTRAN OR NON-FORTRAN FORMAT.
00100      4*    C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
00101      5*            SUBROUTINE RDWT
00103      6*            DIMENSION ARRAY( 1),ICWD(3),IFLD(29),KODE(6),IPACKT(8),KNTWD(3)
00104      7*            DATA (IFLD(I),I=1,29)/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,2H10,
00104      8*           12H11,2H12,2H13,2H14,2H15,2H16,2H17,2H18,2H19,2H20,2H21,2H22,2H23,
00104      9*           22H24,2H25,2H26,2H27,2H28,2H29/
00106     10*            DATA (KODE(I),I=1,6)/000430000000,000200000000,000150000000,
00106     11*           1                          000100000000,000110000000,000400000000/
00110     12*            DATA I1/262144/,I2/1073741824/,I3/0010001/
00114     13*            DATA IBLANK/6H      /
00114     14*      C*****
00116     15*            ENTRY REDTPR(IUNIT,MODE,IERR,NW,NWDS,ARRAY)
00120     16*            IOP=MODE                   @ =1,READ FTN FORMAT. =2,READ NON-FTN.
00120     17*      C*****
00121     18*            GO TO 10
00122     19*            ENTRY WRITER(IUNIT,MODE,IERR,NWDS,ARRAY)
```

TABLE 2. (Continued)

```
00124     20*          IOP=MODE+2              @ =3,WRITE FTN FORMAT. =4,WRITE NON-FTN.
00125     21*          KNTWD(1)=NWDS*I1+I3     @ BUILD FORTRAN RECORD-CONTROL WORD.
00126     22*          KNTWD(2)=IBLANK         @ DUMMY CHECKSUM ERROR WORD.
00127     23*          KNTWD(3)=KNTWD(1)
00130     24*          GO TO 10
00130     25*    C*****
00131     26*          ENTRY CLOSE(IUNIT,JOP)
00133     27*          ITWO=1
00134     28*          GO TO (6,6,7,25),JOP    @ JOP=4, NO ACTION,RETURN.
00135     29*        6 ITWO=2                  @ JOP=1,2, WRITE EOF AND REWIND IUNIT.
00136     30*        7 IOP=5                   @ =5, WRITE END-OF-FILE.
00137     31*          GO TO 10
00137     32*    C*****
00140     33*          ENTRY OPEN(IUNIT,JOP)
00142     34*          IF(JOP.GT.1) RETURN     @ JOP=2,3, NO ACTION,RETURN.
00144     35*        8 IOP=6                   @ =6, REWIND IUNIT.
00145     36*          GO TO 10
00145     37*    C*****
00146     38*          ENTRY IORDWR(ICODE,IUNIT,IADDRS,ARRAY,NWDS,IERR)
00150     39*          IF(ICODE.EQ.1) IOP=4    @ =4, SEQUENTIAL WRITE.
00152     40*          IF(ICODE.EQ.2) IOP=2    @ =2, SEQUENTIAL READ.
00154     41*          IPACKT(6)=IADDRS        @ BEGINNING DRUM-SECTOR ADDRESS.
00154     42*    C*****
00155     43*       10 CONTINUE
00156     44*          IPACKT(1)=IFLD(IUNIT)              @ IUNIT IN FIELD-DATA.
00157     45*          IPACKT(2)=IBLANK
00160     46*          IPACKT(3)=0
00161     47*       11 IPACKT(4)=KODE(IOP)                @ OPERATION CODE TO IOWS.
00162     48*          GO TO (20,21,20,21,22,22),IOP
00163     49*       20 ICWD(1)=   1*I1+LOC(KNTWD(1))      @  FIRST CONTROL ACCESS WORD.
00164  *DIAGNOSTIC* THE VARIABLE ARRAY IS DIMENSIONED.
00164     50*          ICWD(2)=NWDS*I1+LOC(ARRAY)         @ SECOND CONTROL ACCESS WORD.
00165     51*          ICWD(3)=   2*I1+LOC(KNTWD(2))      @  THIRD CONTROL ACCESS WORD.
00166     52*          IPACKT(5)= 3*I1+LOC(ICWD(1))       @ NMBR AND LOC OF 1ST C/A WORD.
00167     53*          GO TO 22
00170  *DIAGNOSTIC* THE VARIABLE ARRAY IS DIMENSIONED.
00170     54*       21 IPACKT(5)=NWDS*I1+LOC(ARRAY)       @ NMBR AND LOC OF DATA WORDS.
00171     55*       22 CALL IOWR(IPACKT)                  @ ROUTINE TO ISSUE EXEC REQUEST.
00172     56*          IF(IOP.EQ.6) RETURN                @ RETURN AFTER REWINDING IUNIT.
00174     57*          IF(IOP.EQ.5) GO TO (25,8),ITWO     @ ITWO=1,RETURN, =2,REWIND IUNIT.
00176     58*          IERR=IPACKT(4)/I2                  @ ERROR STATUS IN S1 PORTION.
00177     59*          IF(IERR.EQ.0) GO TO 24             @ NORMAL OPERATION,NO ERROR.
00201     60*          IF(IERR.GE.3) GO TO 26             @ TERMINATING ERROR.
00203     61*          IERR=2                             @ END-OF-FILE OR END-OF-TAPE.
00204     62*          RETURN
00205     63*       24 NW=KNTWD(1)/I1                     @ NW FROM FTN CONTROL WORD.
00206     64*          IF(MODE.EQ.1) GO TO 25
00210     65*          NW=IPACKT(4)-IPACKT(4)/I1*I1       @ NW FROM IOWS.
00211     66*       25 IERR=1
00212     67*          RETURN
00213     68*       26 WRITE(6,100) (IPACKT(I),I=1,6)
00221     69*      100 FORMAT(///50H*****EXECUTION TERMINATED IN 'RDWT' I/O ROUTINE. //
00221     70*         114H I/O PACKET IS 6O16 )
00222     71*          STOP
00223     72*          END
```

TABLE 2. (Continued)

```
 END OF COMPILATION:          2 DIAGNOSTICS.

 @ASM,IS IOWR,IOWR
 ASM11F-01/19-01:00-(,0)

      1.                                              $(1)    AXRS
      2.       01    000000  27 00 14 13 0 000000   IOWR*    L        A0,0,X11
      3. U           000001  72 11 00 00 0 000000            ER       IOWS
      4.             000002  74 04 00 13 0 000002            J        2,X11
      5.                                                     END
                      IOWS
  END ASM ERRORS : NONE
```

TABLE 2. (Continued)

```
@FOR,IS INVRT,INVRT
HSO-009-01/19-01:00-(,0)

   SUBROUTINE INVRT      ENTRY POINT 000370

   STORAGE USED: CODE(1) 000416; DATA(0) 000334; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000016 111G       0001   000144 115L       0001   000027 117G       0001   000034 123G       0001   000042 131G
    0001   000114 162G       0001   000260 181L       0001   000166 201G       0001   000206 207G       0001   000230 220G
    0001   000266 234G       0001   000336 235L       0001   000312 245G       0001   000346 250L       0001   000071 75L
    0001   000073 76L        0001   000110 90L        0000 D 000002 AMAX       0000 I 000273 I          0000 I 000276 IC
    0000 I 000274 IND        0000 I 000100 INDEX      0000 I 000300 IND2       0000   000306 INJP$      0000 I 000004 IPIV
    0000 I 000275 IR         0000 I 000273 J          0000 I 000272 K          0000 I 000277 L          0000 I 000271 NN
    0000 D 000000 SIGN


00101      1*           SUBROUTINE INVRT(A,N,M,DETER)
00103      2*           PARAMETER IDIM=60
00103      3*      C    MATRIX INVERSION AND SIMULTANEOUS EQUATIONS SOLVER
00103      4*      C    A=INPUT MATRIX FOR INVERISON OR AUGUMENTED MATRIX FOR SIME. EQS.  IG500030
00103      5*      C    N=ORDER OF COEFFICIENT MATRIX                                    IG500040
00103      6*      C    M=0 FOR INVERISON ONLY
00103      7*      C    M=NUMBER OF CONSTANT VECTORS
00103      8*      C    DETER=DETERMINANT OF COEFFICIENT MATRIX
00104      9*           DOUBLE PRECISION A(1),DETER,SIGN,AMAX
00105     10*           DIMENSION IPIV(IDIM),INDEX(IDIM,2)
00106     11*           DETER=1.0D0
00107     12*           SIGN=1.0D0
00110     13*           DO 20 J=1,N
00113     14*        20 IPIV(J)=0
00115     15*           NN=N+M
00116     16*           DO 182 K=1,N
00121     17*           AMAX=0.0D0
00122     18*        40 DO 76 I=1,N
00125     19*           IF (IPIV(I)-1)50,76,50
00130     20*        50 DO 75 J=1,N
00133     21*           IF (IPIV(J)-1)55,75,250
00136     22*        55 IND=(J-1)*N+I
00137     23*           IF(AMAX-DABS(A(IND))) 60,75,75
00142     24*        60 IR=I
00143     25*           IC=J
00144     26*           AMAX=DABS(A(IND))
00145     27*        75 CONTINUE
00147     28*        76 CONTINUE
```

TABLE 2. (Continued)

```
00151      29*           IPIV(IC)=IPIV(IC)+1
00152      30*           IF (IR-IC)90,115,90
00155   *DIAGNOSTIC*     CONTROL CAN NEVER REACH THE NEXT STATEMENT
00155      31*           IF (IR-IC)90,115,90
00160      32*        90 SIGN=-SIGN
00161      33*           DO 110 L=1,NN
00164      34*           IND=(L-1)*N+IR
00165      35*           IND2=(L-1)*N+IC
00166      36*           AMAX=A(IND)
00167      37*           A(IND)=A(IND2)
00170      38*       110 A(IND2)=AMAX
00172      39*       115 INDEX(K,1)=IR
00173      40*           INDEX(K,2)=IC
00174      41*           IND=(IC-1)*N+IC
00175      42*           AMAX=A(IND)
00176      43*           DETER=DETER*AMAX
00177      44*       140 A(IND)=1.0D0
00200      45*           DO 150 L=1,NN
00203      46*           IND=(L-1)*N+IC
00204      47*       150 A(IND)=A(IND)/AMAX
00206      48*           DO 181 L=1,N
00211      49*           IF (L-IC)165,181,165
00214      50*       165 IND=(IC-1)*N+L
00215      51*           AMAX=A(IND)
00216      52*           A(IND)=0.0D0
00217      53*           DO 180 I=1,NN
00222      54*           IND=(I-1)*N+L
00223      55*           IND2=(I-1)*N+IC
00224      56*           A(IND)=A(IND)-A(IND2)*AMAX
00225      57*       180 CONTINUE
00227      58*       181 CONTINUE
00231      59*       182 CONTINUE
00233      60*           DO 235 I=1,N
00236      61*           L=N+1-I
00237      62*           IR=INDEX(L,1)
00240      63*           IC=INDEX(L,2)
00241      64*           IF (IR-IC)210,235,210
00244      65*       210 DO 230 K=1,N
00247      66*           IND=(IR-1)*N+K
00250      67*           IND2=(IC-1)*N+K
00251      68*           AMAX=A(IND)
00252      69*           A(IND)=A(IND2)
00253      70*       230 A(IND2)=AMAX
00255      71*       235 CONTINUE
00257      72*           DETER=SIGN*DETER
00260      73*           RETURN
00261      74*       250 M=-1
00262      75*       255 RETURN
00263      76*            END

       END OF COMPILATION:          1 DIAGNOSTICS.
```

TABLE 2. (Continued)

```
@FOR,IS DMATML,DMATML
H50-009-01/19-01:00-(,0)

  SUBROUTINE DMATML     ENTRY POINT 000140

  STORAGE USED: CODE(1) 000176; DATA(0) 000053; BLANK COMMON(2) 000000

  EXTERNAL REFERENCES (BLOCK, NAME)

   0003   NERR3$

  STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

   0001   000031 1L        0001   000061 130G      0001   000067 135G      0001   000102 143G      0001   000035 2L
   0001   000046 3L        0001   000052 4L        0000 D 000000 CD        0000 I 000005 IA1       0000 I 000006 IA2
   0000 I 000002 IA3       0000 I 000007 IB1       0000 I 000010 IB2       0000 I 000013 IB3       0000 I 000003 IM
   0000 I 000004 IN        0000   000022 INJP$     0000 I 000015 LA        0000 I 000016 LB        0000 I 000012 LC
   0000 I 000011 LM        0000 I 000017 LN        0000 I 000014 LP


00101      1*          SUBROUTINE DMATML(C,A,B,M,N,K)
00101      2*    C*    ABSTRACT
00101      3*    C     GENERAL MATRIX MULTIPLICATION ROUTINE  WITH TRANSPOSE OPTIONS
00101      4*    C      WHERE, M IS THE NUMBER OF ROWS OF (A)
00101      5*    C             N IS THE NUMBER OF ROWS OF (B)
00101      6*    C             K IS THE NUMBER OF COLUMNS OF (B) OR (B)T
00101      7*    C     TRANSPOSE OPTIONS ARE CONTROLLED BY THE SIGNS OF M AND N.
00101      8*    C     THE FOLLOWING PRODUCTS MAY BE OBTAINED
00101      9*    C      (C)=(A)(B)       M AND N POSITIVE
00101     10*    C      (C)=(A)T(B)      M NEGATIVE FOR (A)T
00101     11*    C      (C)=(A)(B)T      N NEGATIVE FOR (B)T
00101     12*    C      (C)=(A)T(B)T     M AND N NEGATIVE
00101     13*    C      WHERE T INDICATES TRANSPOSE
00101     14*    C      IF M IS NEGATIVE, M IS THE NUMBER OF ROWS OF (A)T
00101     15*    C      IF N IS NEGATIVE, N IS THE NUMBER OF ROWS OF (B)T
00101     16*    C
00101     17*    C*    OUTPUT ARGUMENT * C
00103     18*          DIMENSION C(1)
00103     19*    C
00103     20*    C*    INPUT ARGUMENTS * A,B,M,N,K
00104     21*          DIMENSION A(1),B(1)
00104     22*    C
00104     23*    C**************************************************************
00105     24*          DOUBLE PRECISION CD,C,A,B
00106     25*          IA3=1
00107     26*          IM=IABS(M)
00110     27*          IN=IABS(N)
00111     28*          IF(M .LT. 0)GO TO 1
00113     29*          IA1=IM
00114     30*          IA2=1
```

TABLE 2. (Continued)

```
00115     31*           GO TO 2
00116     32*     1     IA1 = 1
00117     33*           IA2=IN
00120     34*     2     IF(N .LT. 0)GO TO 3
00122     35*           IB1=1
00123     36*           IB2=IN
00124     37*           GO TO 4
00125     38*     3     IB1=K
00126     39*           IB2=1
00127     40*     4     DO 7 LM=1,IM
00132     41*           LC=LM
00133     42*           IB3=1
00134     43*           DO 6 LP=1,K
00137     44*           CD = 0.D00
00140     45*           LA=IA3
00141     46*           LB=IB3
00142     47*           DO 5 LN=1,IN
00145     48*           CD = CD + A(LA)*B(LB)
00146     49*           LA=LA+IA1
00147     50*     5     LB=LB+IB1
00151     51*           C(LC) = CD
00152     52*           LC = LC + IM
00153     53*     6     IB3=IB3+IB2
00155     54*     7     IA3=IA3+IA2
00157     55*           RETURN
00160     56*           END

     END OF COMPILATION:          NO  DIAGNOSTICS.
```

TABLE 2. (Continued)

```
@MAP,IS AA,AA
MAP 17R2-01/19-01:30-(,0)

   1.        LIB SYS$*MSFC$.

ADDRESS LIMITS    001000 033207        040000 054642
STARTING ADDRESS  033167
WORDS DECIMAL       13448 IBANK        6563 DBANK

             SEGMENT MAIN           001000 033207      040000 054642

NSWTC$/FOR                  1   001000 001021
NRBLK$/MSFCS5               1   001022 001110      0   040000 040001
NEXPA$/MSFCS7               1   001111 001303      2   040002 040053
ALOG$/FOR51                 1   001304 001421      2   040054 040114
CSIG6V/SC4020               1   001422 001710      0   040115 040153
                                                   2   BLANK$COMMON
CERMRK/SC4020               1   001711 001740      0   040154 040167
                                                   2   BLANK$COMMON
CLABLV/SC4020               1   001741 003003      0   040170 040276
                                                   2   BLANK$COMMON
NRWND$/FOR50                1   003004 003063      2   040277 040310
NAEF$/MSFCS5                1   003064 003320      2   040311 040331
NFTCH$/FOR57                1   003321 003620      2   040332 040367
CYMODV/SC4020               1   003621 003646      0   040370 040376
                                                   2   BLANK$COMMON
CACCHY/SC4020               1   003647 003670      0   040377 040407
                                                   2   BLANK$COMMON
CXMODV/SC4020               1   003671 003716      0   040410 040416
                                                   2   BLANK$COMMON
CONCAT/MSFC                 1   003717 004070      0   040417 040440
SETINT/SC4020               1   004071 004123      0   040441 040446
                                                   2   BLANK$COMMON
CHOLLV/SC4020               1   004124 004206      0   040447 040463
                                                   2   BLANK$COMMON
CHONLN/SC4020               1   004207 004614      0   040464 040535
                                                   2   BLANK$COMMON
CLINRV/SC4020               1   004615 005470      0   040536 040651
                            3   GGG                2   BLANK$COMMON
CYSCLV/SC4020               1   005471 005673      0   040652 040700
                            3   GGG                2   BLANK$COMMON
CXSCLV/SC4020               1   005674 006076      0   040701 040727
                            3   GGG                2   BLANK$COMMON
CERNEV/SC4020               1   006077 006253      0   040730 040742
```

TABLE 2. (Continued)

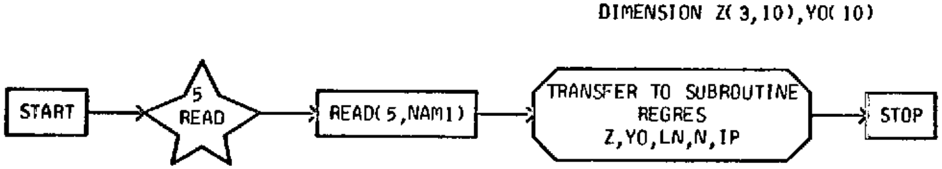
```
                                                2   BLANK$COMMON
CERRLN/SC4020            1   006254 006363     0   040743 040760
                                                2   BLANK$COMMON
CSETCV/SC4020            1   006364 006424     0   040761 040771
                                                2   BLANK$COMMON
CSETMV/SC4020            1   006425 006503     0   040772 041006
                                                2   BLANK$COMMON
CXAXIS/SC4020            1   006504 006707     0   041007 041042
                                                2   BLANK$COMMON
VCHARV/SC4020            1   006710 007153     0   041043 041062
RITE2V/SC4020            1   007154 007404     0   041063 041111
BPLOTK/SC4020            1   007405 007465     0   041112 041471
                                                2   BLANK$COMMON
CFRAM/SC4020             1   007466 007705     0   041472 041556
                         3   GGG               2   BLANK$COMMON
CCAMRA/SC4020            1   007706 007757     0   041557 041570
                         3   GGG               2   BLANK$COMMON
TABLIV/SC4020                                  0   041571 042111
NBDCV$/FOR57             1   007760 010113     2   042112 042151
NFTY$/FOR                1   010114 010136
NCLOS$/HSFC57            1   010137 010305     2   042152 042177
NBBLK$/HSFC57            1   010306 010474     0   042200 042201
NBSBL$/FOR               1   010475 010532
NUPOA$/FOR               1   010533 010566
NBFOD$/FOR                                     2   042202 044403
NCNVT$/FOR57             1   010567 011021     2   044404 044473
NINIT$/HSFC55            1   011022 011253     2   044474 044515
CPLOTV/SC4020            1   011254 011436     0   044516 044554
                         3   GGG               2   BLANK$COMMON
CLINEV/SC4020            1   011437 012022     0   044555 044633
                                                2   BLANK$COMMON
YSCLVI/SC4020            1   012023 012141     0   044634 044644
                         3   GGG               2   BLANK$COMMON
XSCLVI/SC4020            1   012142 012257     0   044645 044655
                         3   GGG               2   BLANK$COMMON
CAPLOT/SC4020            1   012260 012533     0   044656 044731
                         3   GGG               2   BLANK$COMMON
CAPRNV/SC4020            1   012534 012616     0   044732 044750
                                                2   BLANK$COMMON
CPRNTV/SC4020            1   012617 013210     0   044751 045005
                         3   GGG               2   BLANK$COMMON
CGRDIV/SC4020            1   013211 014115     0   045006 045121
                         3   GGG               2   BLANK$COMMON
CDXDYV/SC4020            1   014116 014710     0   045122 045206
                                                2   BLANK$COMMON
CBRITV/SC4020            1   014711 015012     0   045207 045222
                         3   GGG               2   BLANK$COMMON
CMARGN/SC4020            1   015013 015071     0   045223 045264
                                                2   BLANK$COMMON
CNBLNK/SC4020            1   015072 015141     0   045265 045300
                                                2   BLANK$COMMON
BMOV/HSFC                1   015142 015222     0   045301 045311
                                                2   BLANK$COMMON
TRACE                    1   015223 015346     0   045312 045317
                                                2   045320 045402
SPCFRM/HSFC                                    0   045403 045456
```

TABLE 2. (Concluded)

```
CTOBCD/MSFC                 1    015347 015645        0    045457 045630
BOXLTR/CSC                  1    015646 016500        0    045631 045647
CIDENT/SC4020               1    016501 017653        0    045650 046033
                            3    GGG                  2    BLANK$COMMON
NOTIM$/MSFC55               1    017654 020204        2    046034 046044
NOUT$/MSFC57                1    020205 021166        2    046045 046076
NFMT$/MSFC57                1    021167 022074        2    046077 046115
NIOER$/MSFC57               1    022075 022254        2    046116 046240
NFCHK$/MSFC57               1    022255 023073        2    046241 046415
                                                      4    046416 046467
NTAB$/MSFC55                                          2    046470 046556
NLINP$/MSFC57               1    023074 024640        2    046557 046770
SQRT$/FOR55                 1    024641 024700        2    046771 047002
NEXP5$/FOR57                1    024701 024764        2    047003 047012
GGG (COMMON BLOCK)                                         047013 047146
CQUIKL/SC4020               1    024765 025402        0    047147 047231
                            3    GGG                  2    BLANK$COMMON
IDENT/SC4020                1    025403 026645        0    047232 051027
H$MONITOR/MSFC55            1    026646 027755        2    051030 051614
NIER$/FOR52                 1    027756 030040        2    051615 051744
NOBUF$/FOR51                1    030041 030100
ERU$/MSFC55
NERR$/FOR57                 1    030101 030435        2    051745 052121
BLANK$COMMON (COMMON BLOCK)
DMATML                      1    030436 030633        0    052122 052174
                                                      2    BLANK$COMMON
INVRT                       1    030634 031251        0    052175 052530
                                                      2    BLANK$COMMON
IOAR                        1    031252 031254
RDWT                        1    031255 031656        0    052531 052657
                                                      2    BLANK$COMMON
REGRES                      1    031657 033166        0    052660 054541
                                                      2    BLANK$COMMON
MAIN                        1    033167 033207        0    054542 054642
                                                      2    BLANK$COMMON


SYS$*RLIB$. LEVEL MS7-5
END OF COLLECTION - TIME 3.518 SECONDS

@XQT AA
```



```
                         D I M E N S I O N E D   V A R I A B L E S

SYMBOL     STORAGES  SYMBOL     STORAGES  SYMBOL     STORAGES  SYMBOL     STORAGES  SYMBOL     STORAGES

 Y          10        RESID      10        Z          3,10      SMALLZ     IDIM      SHAT       9,9
 ZBAR       3,3       YO         10        BZ         10        B          3,3       SVHAT      9,1
 SMALZ1     IDIM      SMALZ2     IDIM      SH         9,9       XX         10        ADARY      22
 PMAX       10        FLDX       12        FLDY       12
```

Figure 2. Operation of the computer program (noncross product).

SUBROUTINE REGRES(Z,YO,LN,N,IP)

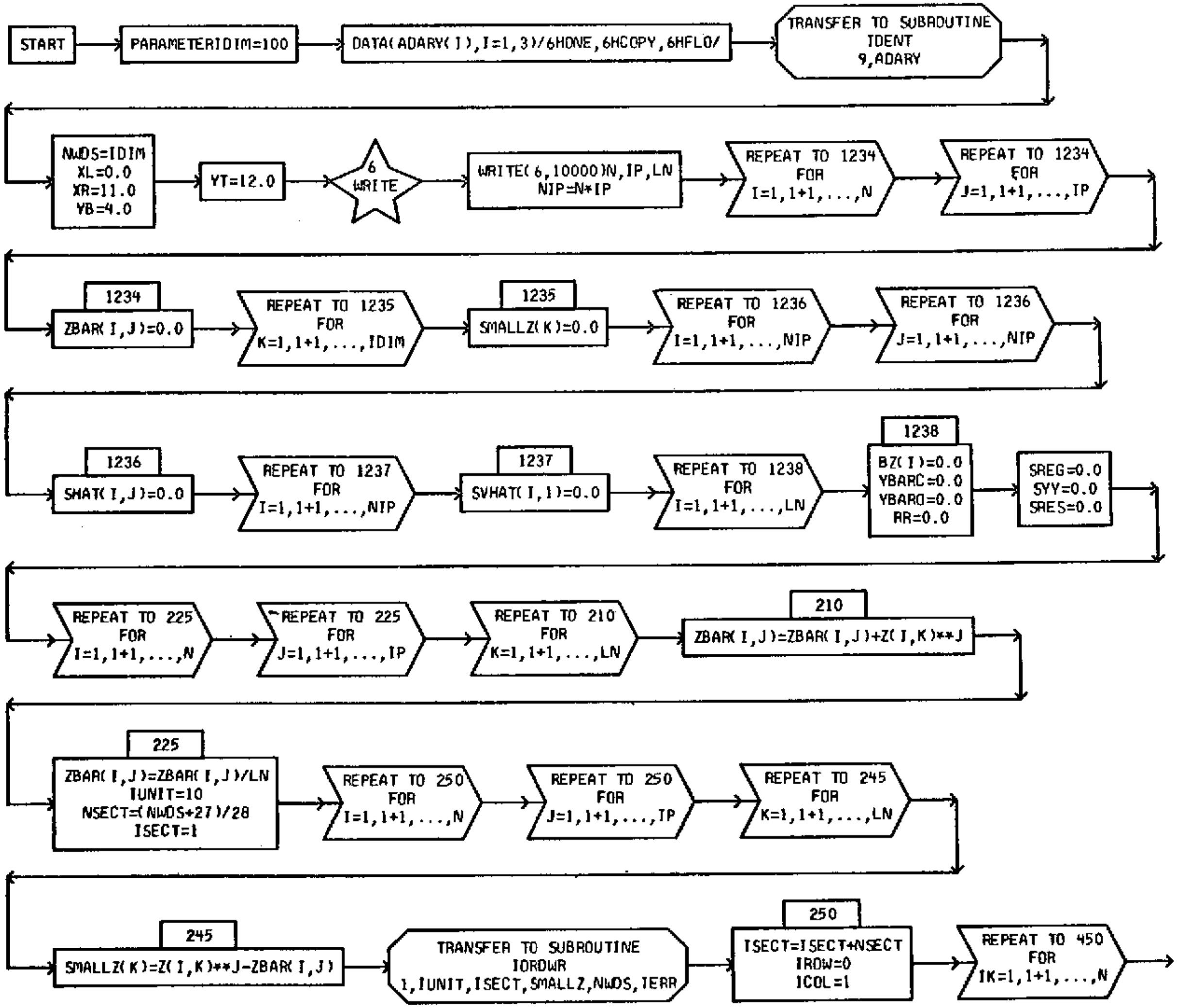


Figure 2. (Continued)

SUBROUTINE REGRES(Z,Y0,LN,N,IP)

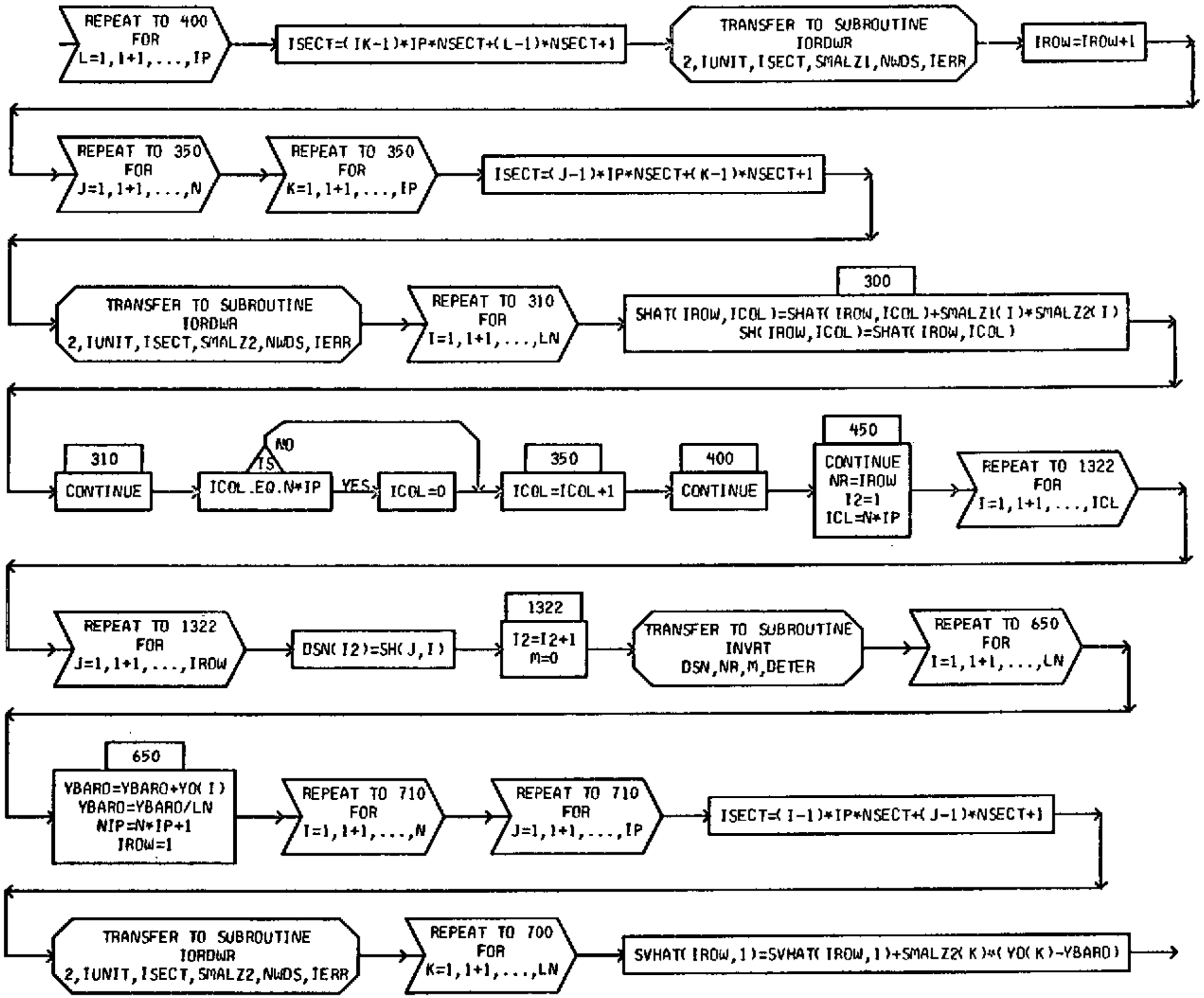


Figure 2. (Continued)

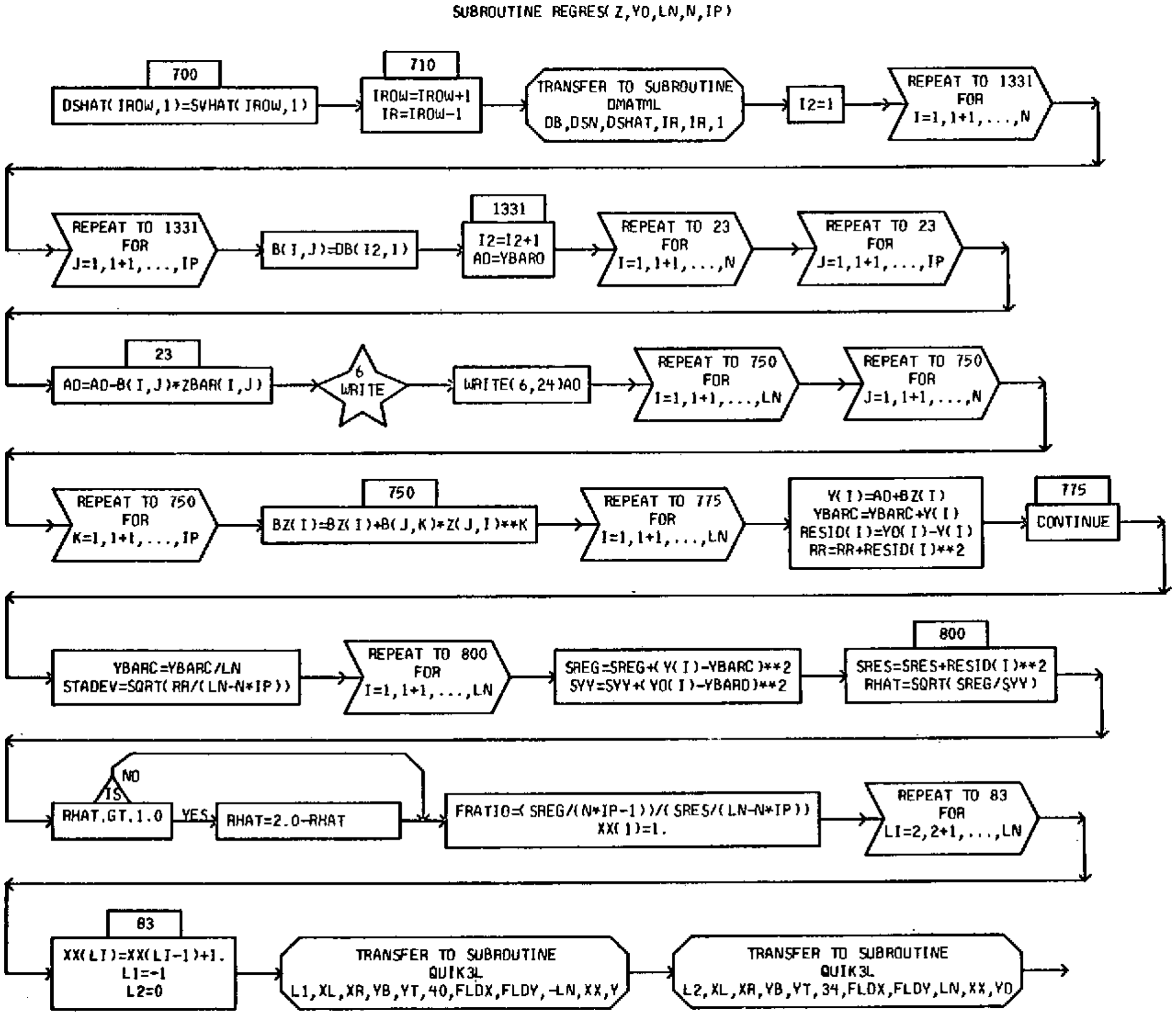


Figure 2. (Continued)

SUBROUTINE REGRES(Z,YO,LN,N,IP)

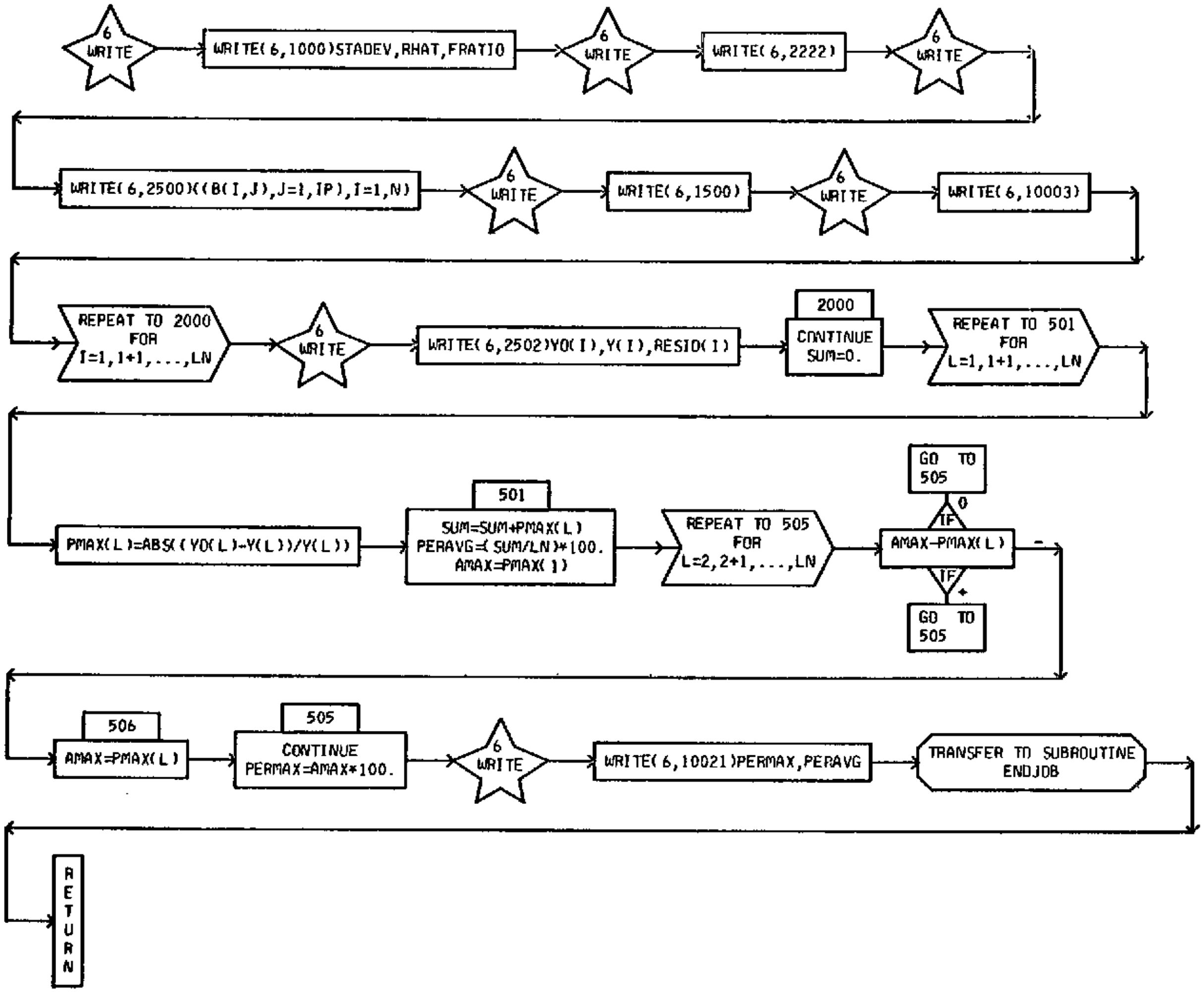


Figure 2. (Continued)

```
                         GENERAL FORTRAN I/O PACKAGE       EXEC VIII        APRIL,1970
                          READ/WRITE MAG TAPE OR DRUM IN FORTRAN OR NON-FORTRAN FORMAT.


                                   D I M E N S I O N E D   V A R I A B L E S

SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES

 ARRAY     1           ICWD      3           IFLD      29          KODE      6           IPACKT    8
 NTWD      3
```

Figure 2. (Continued)

SUBROUTINE RDWT

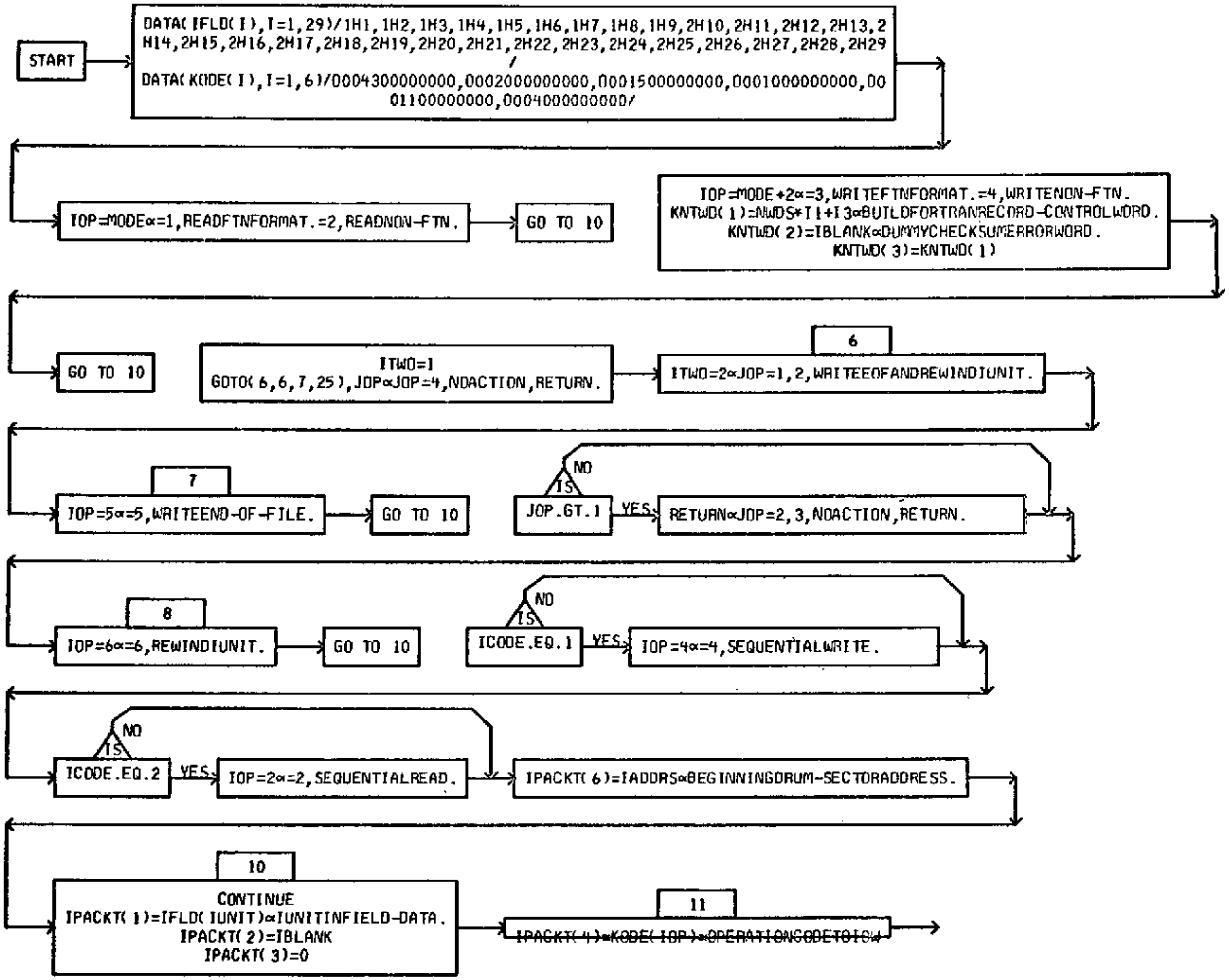


Figure 2. (Continued)

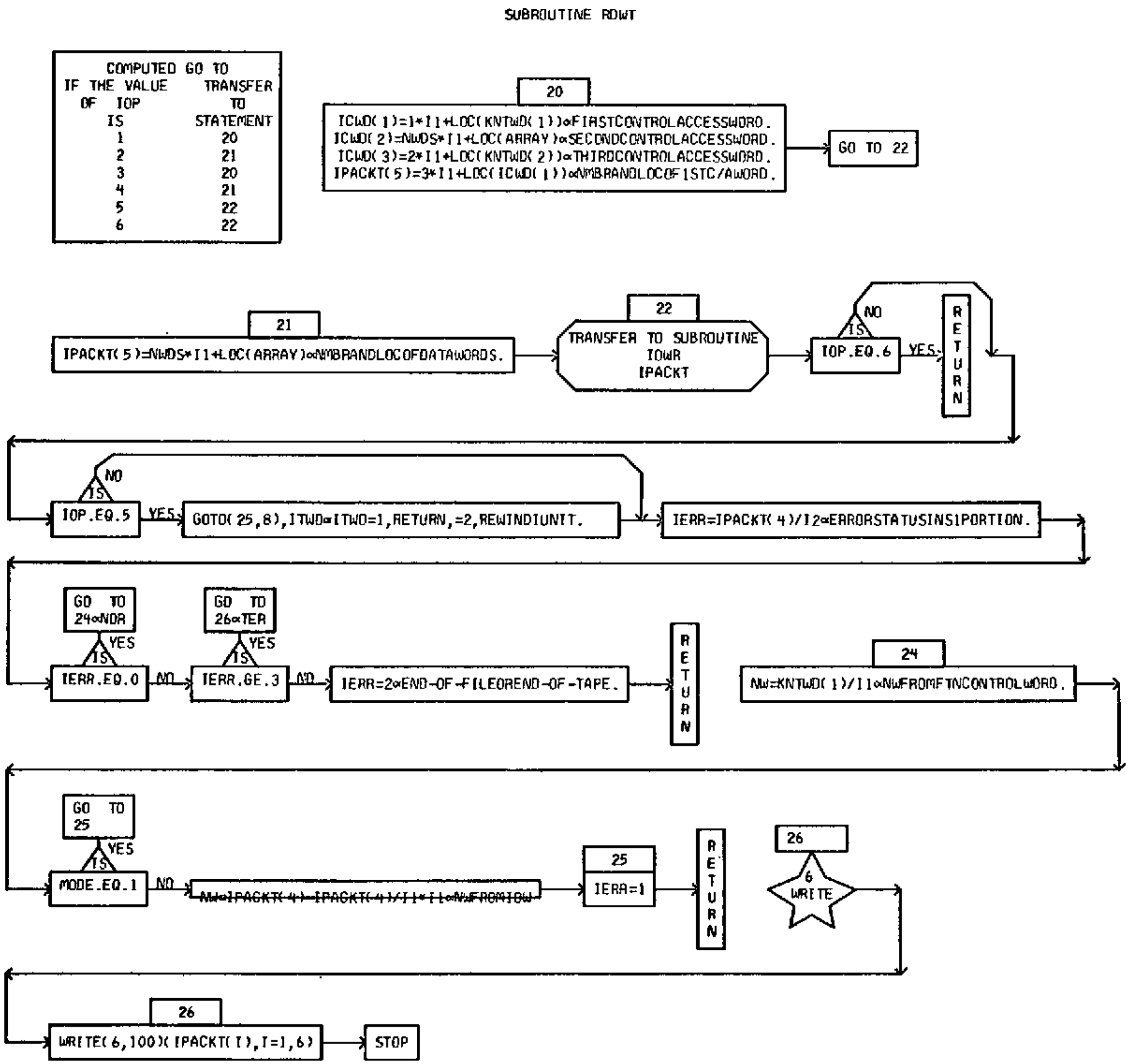


Figure 2. (Continued)

```
                                   D I M E N S I O N E D   V A R I A B L E S

SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES   SYMBOL     STORAGES

 IPIV        IDIM      INDEX       IDIM,2
```

Figure 2. (Continued)

7

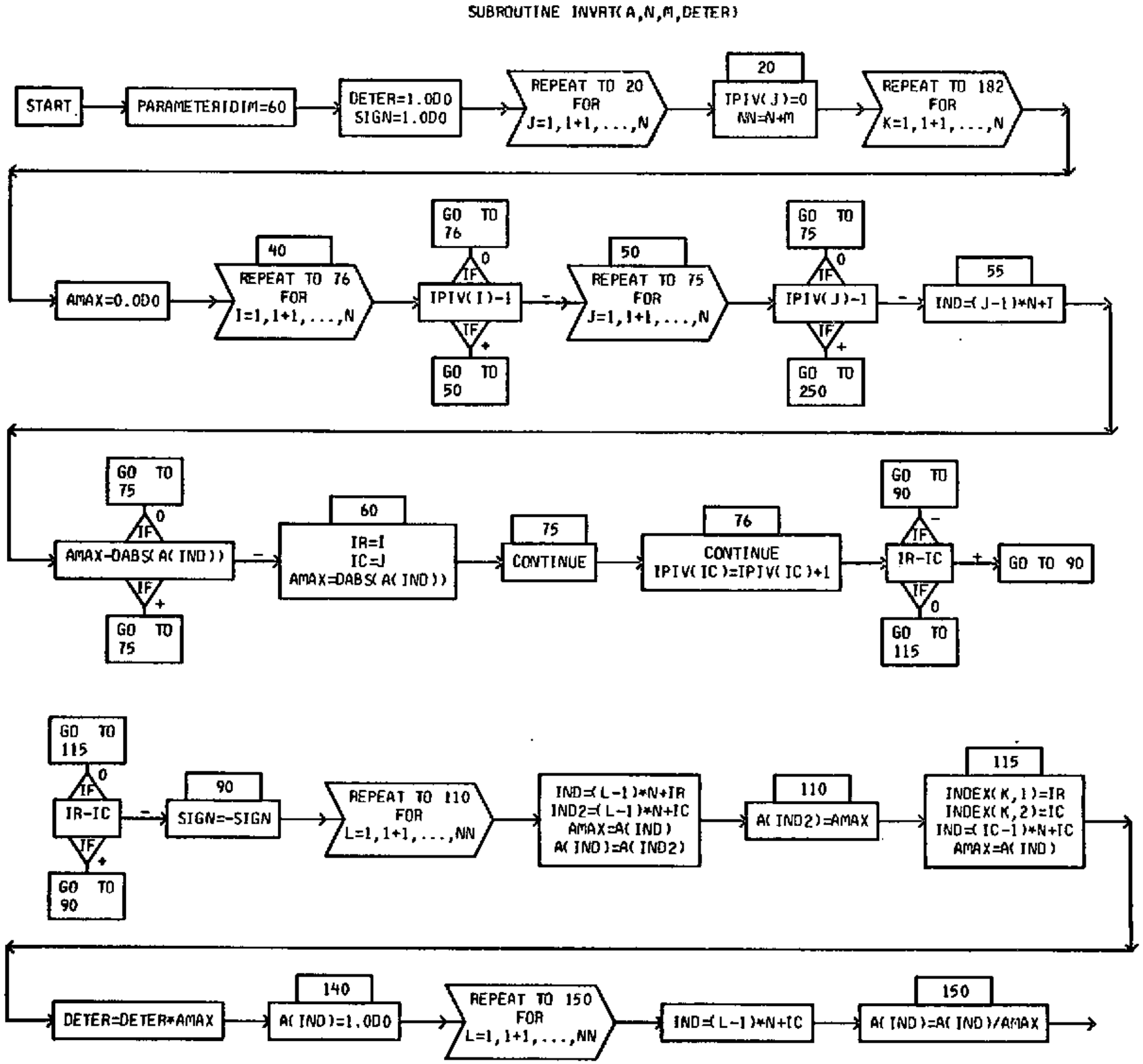


Figure 2. (Continued)

SUBROUTINE INVRT(A,N,M,DETER)

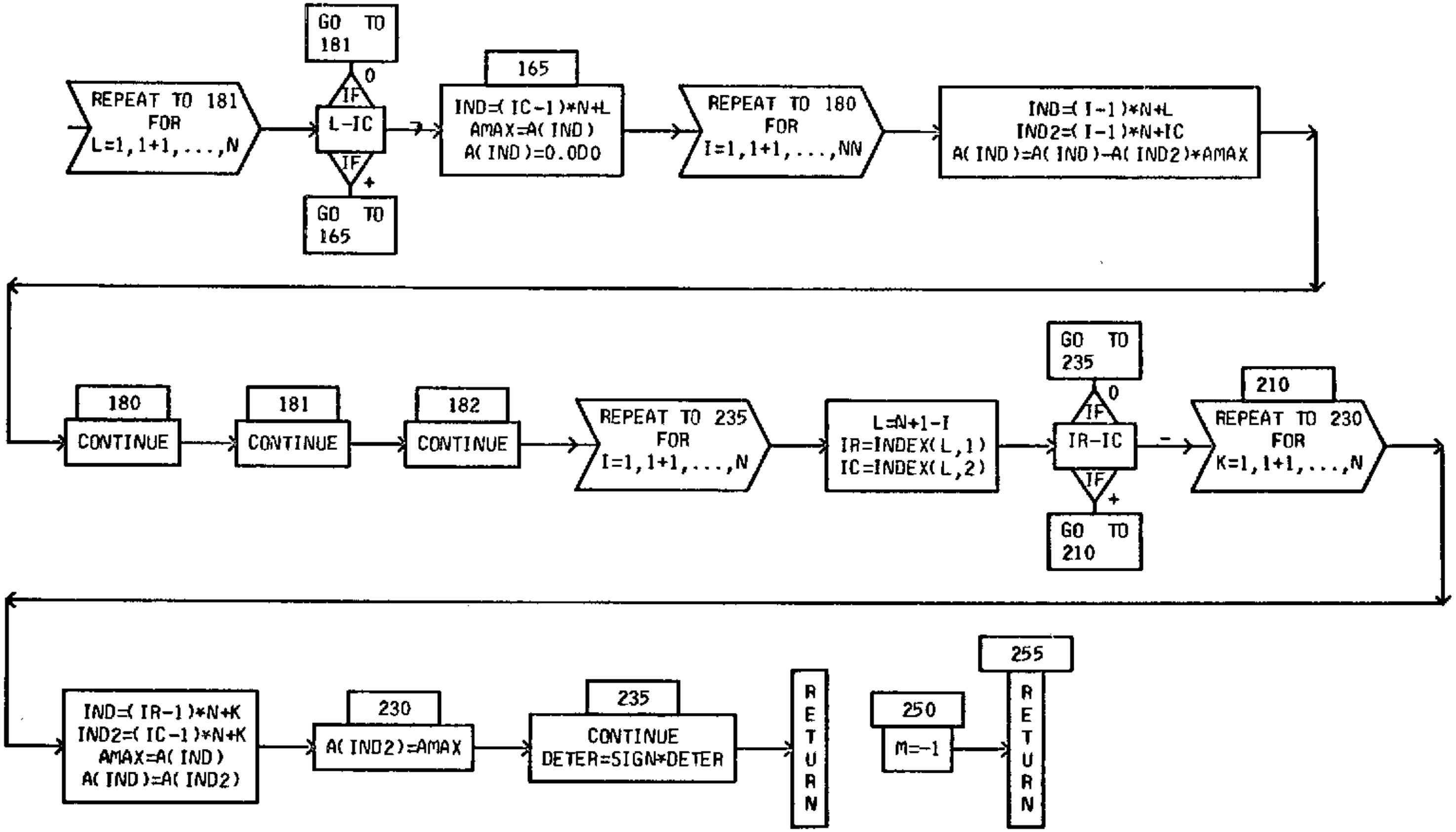


Figure 2. (Continued)

DIMENSIONED VARIABLES

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
C	1	A	1	B	1				

SUBROUTINE DMATML(C,A,B,M,N,K)

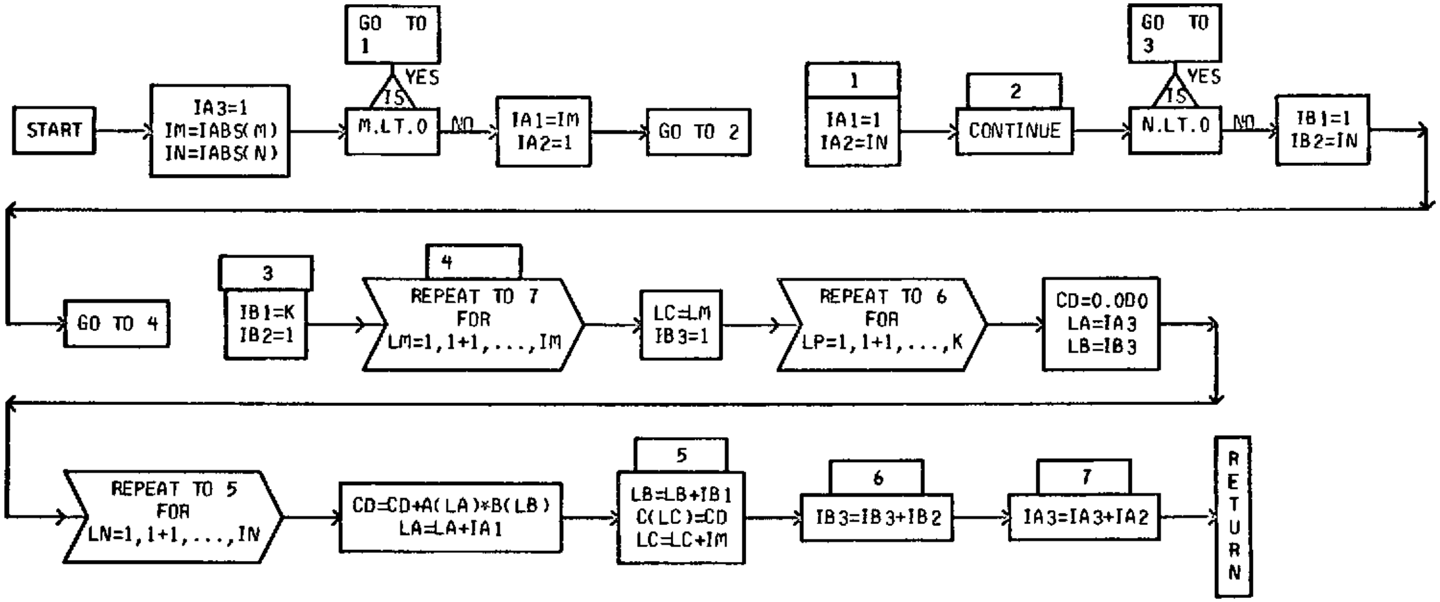


Figure 2. (Concluded)

Also included in the output are the number of independent variables N, the degree of the model equation P, and the number of input data points (sets) n denoted in the output as LN. Further, the significance of the estimated regression equation is indicated by the standard deviation, the multiple correlation coefficient, the F ratio, the maximum percent error, and the average percent error.

The values of the dependent variable are also shown for the input data and for the computed values obtained from the fitted expression for each set of data input, as well as for the residual difference in the input values and the computed values.

Plotted results are also obtained which compare the input and the computed dependent variables as ordinates to the point interval along the abscissa (which is normally one of the physical independent variables, but which can be a unit indication of each data set in order of input to the program). The computed points are connected by straight lines and are plotted with * symbols. The input points are plotted with + symbols and these points are not connected with lines.

D. Illustrative Problem

For the purpose of illustrating the use of the program and its capabilities the following arbitrary equation was selected for use:

$$\begin{aligned} Y = {} & 9 - 6X_1 + 7X_1^2 + 5X_1^3 \\ & - 4X_2 + 1X_2^2 + 7X_2^3 \\ & - 6X_3 + 7X_3^2 - 6X_3^3 \end{aligned} \qquad (37)$$

From equation (37) the following set of input data (Table 3) was developed containing exact dependent variable values (to three decimal places) for arbitrary values of the three independent variables.

TABLE 3. INPUT DATA, DEPENDENT VARIABLE VALUES

Data Point Number	Y	X_1	X_2	X_3
1	5.719	0.5	0.4	0.4
2	7.111	0.6	0.3	0.2
3	5.216	0.2	0.6	0.6
4	5.592	0.2	0.6	0.5
5	10.915	0.9	0.0	0.5
6	7.569	0.7	0.0	0.6
7	7.450	0.3	0.9	0.7
8	9.618	0.9	0.3	0.6
9	5.019	0.5	0.6	0.7
10	8.021	0.3	0.8	0.2

The input for this sample problem was read in through namelist NAM1 as follows:

```
Col. 2
     $NAM1
          Z = 0.5, 0.4, ..., 0.2,
          YO = 5.719, 7.111, ..., 8.021,
          LN = 10,
          N = 3,
          IP = 3,
     $
```

The computed results are shown in Table 4 for this example problem and the plotted results are shown in Figure 3.

The plot graphs the point intervals on the X axis against the exact and computed dependent variables. The exact dependent variables are plotted with (+) and the points are not connected with lines. The computed dependent variables are plotted with (*) and the points are connected by straight line segments.

Inspection of the digital and plotted results reveals that the computed regression relation yields an almost exact representation of the input data for this arbitrary case. Experience with many sets of physical data have also shown excellent results.

TABLE 4. OUTPUT DATA LISTING FOR ILLUSTRATIVE PROBLEM (CASE WITHOUT CROSS PRODUCTS)

```
N=    3    P=    3    LN=   10

A0=    .899989665+01

STANDARD DEVIATION=    .149373568-04    MULTIPLE CORRELATION COEFFICIENT=    .999999970+00    F RATIO=    .190450291+11

B COEFFICIENTS

        -.599952072+01         .699866408+01         .500090784+01         -.400001669+01         .999836586+00

         .700009459+01        -.599887073+01         .699684674+01         -.599740386+01

                   YO                    YC                  YO-YC

              OBSERVED              COMPUTED              RESIDUAL

         .571899998+01         .571900237+01        -.238418579-05
         .711100000+01         .711100280+01        -.280141830-05
         .521600002+01         .521600103+01        -.101327896-05
         .559200001+01         .559199802+01         .119209290-05
         .109150000+02         .109150084+02        -.846385956-05
         .756900001+01         .756899226+01         .774860382-05
         .744999999+01         .745000219+01        -.220537186-05
         .961800003+01         .961799359+01         .643730164-05
         .501899999+01         .501900327+01        -.327826546-05
         .802100003+01         .802099574+01         .429153442-05

MAXIMUM PERCENT ERROR=    .102372490-03        AVERAGE PERCENT ERROR=    .517097560-04
```

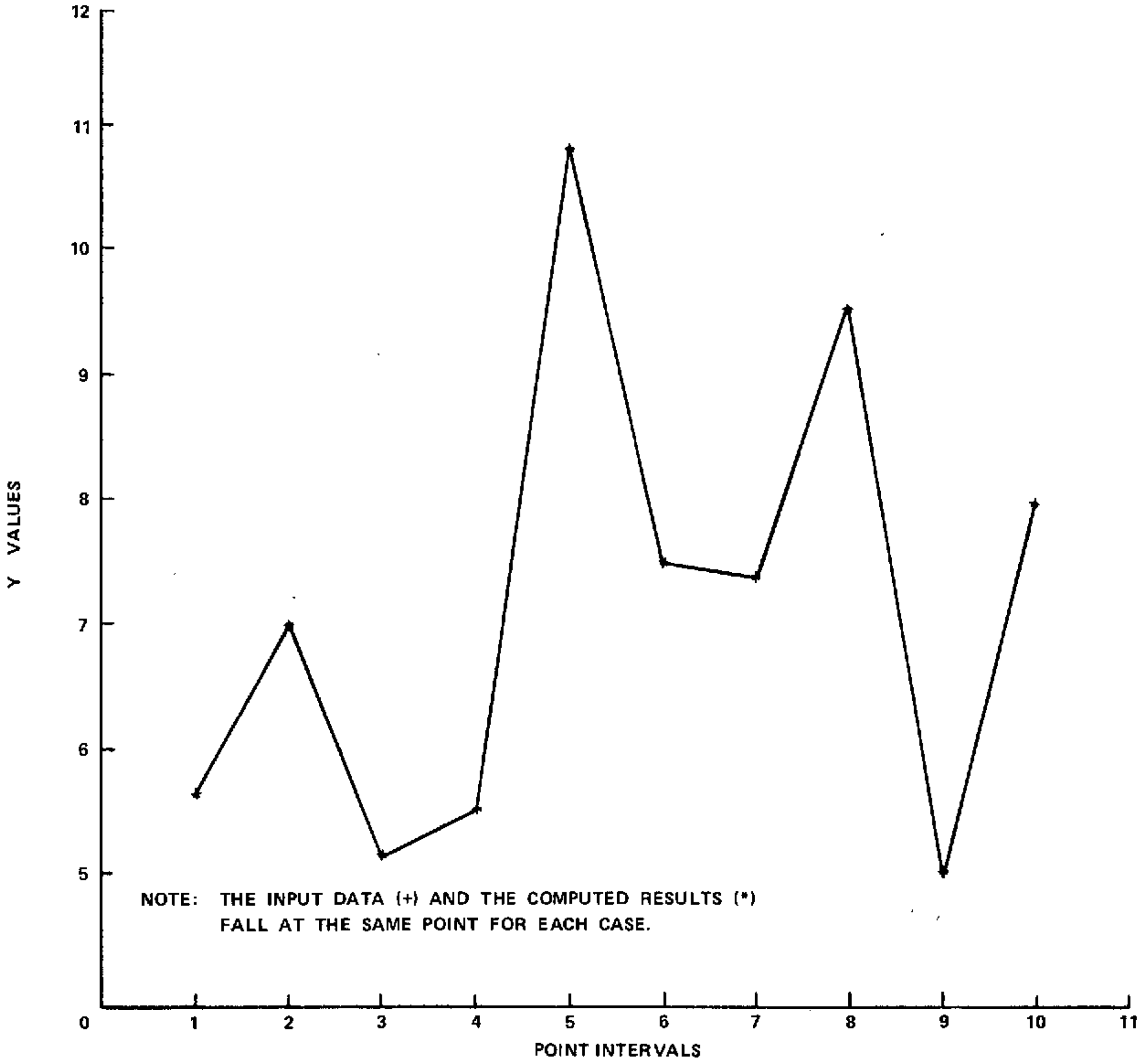


Figure 3. Comparison of input and computed values for illustrative problem without cross products.

APPENDIX B

MULTIPLE REGRESSION PROGRAM FOR CASE WITH CROSS PRODUCTS

A. Input Data

The regression subroutine is called with the following statement:

CALL REGRES(Z, YO, LN, N, IP)

where:

Z is a two-dimensional array containing the independent variables used. The first subscript denotes the independent variable and the second subscript denotes the data set.

YO is an array containing the dependent variables which correspond to the independent variables.

LN is the number of data sets.

N is the number of independent variables used.

IP is the degree of the polynomial curve fit.

These arguments must be assigned a value in a driver program.

The regression subroutine incorporates several special routines: RDWT, IOWR, INVRT, and DMATML.

RDWT is a general FORTRAN I/O package which is capable of reading and/or writing on magnetic tape or drum in FORTRAN or non-FORTRAN format.

IOWR is an assembly language I/O routine used by RDWT.

INVRT is a double precision matrix inversion and simultaneous equations solver. The call to this routine is:

CALL INVRT(A,N,M,DETER)

where:

A is the input matrix for inversion or augmented matrix for simultaneous equations.

N is the order of the coefficient matrix.

M=0 for inversion only; otherwise, M is the number of constant vectors.

DETER is the determinant of the coefficient matrix.

DMATML is a double precision matrix multiplication routine with transpose options. The call to this routine is:

CALL DMATML(C,A,B,M,N,K)

where:

C is the product of matrix A times matrix B.

A is the first input matrix,

B is the second input matrix,

M is the number of rows of A,

N is the number of rows of B, and

K is the number of columns of B.

Certain variables have dimension changes that vary depending on the number of data points (LN), the number of independent variables (N), and the degree of the polynomial curve fit (IP). These will be dimensioned as follows:

DSN(NCL2), DSVHAT(NCL,1), DB(NCL,1), X(LN), Z(N,LN), ZBAR(N,IP), ZZBAR(NN,IP), SHAT(NCL,NCL), YO(LN), SVHAT(NCL,1), YC(LN), BZ(LN), RESID(LN), B(N,IP), A(NN,IP)

where:

LN,N,IP are as defined above and
NCL is computed as follows:

NCL = N*IP + N!/[2*(N-2)!]

NCL2 is NCL raised to the second power.
NN is N minus 1.

SMALLZ, SMALZ1, and SMALZ2 should be dimensioned greater than the number of data points and the dimension is set in a PARAMETER statement; for example,

```
PARAMETER IDIM = 100
```

B. Program

In order to run this program on the UNIVAC 1108, the deck is set up as follows:

```
@RUN,//T JBNAME,320590,UDARBYBIN406,01.100
@ASG,T 10,F/1/POS/2
@FOR,IS MAIN, MAIN
    (Source deck for inputting data)
@FOR,IS REGRES, REGRES
    (Source deck)
@FOR,IS RDWT, RDWT
    (Source deck)
@ASM,IS IOWR, IOWR
    (Source deck)
@FOR,IS INVRT, INVRT
    (Source deck)
@FOR,IS DMATML,DMATML
    (Source deck)
@MAP,I AA, AA
    LIB SYS$*MSFC$.
@XQT AA
    (Input data)
@FIN
```

as illustrated in Figure 1.

A complete program listing for the case with linear cross products is shown in Table 5 and a flow chart indicating the operation of this computer program is indicated in Figure 4.

C. Output Data

The desired coefficients of the model equation are outputted in E notation where BHAT is the computed intercept A_O of the fitted polynomial expression and the B coefficients are printed out in ascending order of degree (P) with the first P coefficients indicating the b coefficients of the first independent variable, the second set of P coefficients indicating the b coefficients of the second independent variable, etc. The A coefficient designation represents the C coefficients of the various cross products in the following order:

$C_{12}, C_{13}, \ldots, C_{1N}$

$C_{23}, C_{24}, \ldots, C_{2N}$

$C_{N-1,N}$

Also included in the output are the number of independent variables N, the degree of the model equation P, and the number of input data points (sets) n denoted in the output as LN. Further, the significance of the estimated regression equation is indicated by the standard deviation, the multiple correlation coefficient, the F ratio, the maximum percent error, and the average percent error.

The values of the dependent variable are also shown for the input data and for the computed values obtained from the fitted expression for each set of data input, as well as for the residual difference in the input values and the computed values.

Plotted results are also obtained which compare the input and the computed dependent variables as ordinates to the point interval along the abscissa (which is normally one of the physical independent variables, but which can be a unit indication of each data set in order of input to the program). The computed points are connected by straight lines and are plotted with * symbols. The input points are plotted with + symbols and the points are not connected with lines.

D. Illustrative Problem

For the purposes of illustrating the use of the cross product program and its capabilities, the following arbitrary equation was selected for use:

$$\begin{aligned} Y = {} & 9 - 6X_1 + 7X_1^2 + 5X_1^3 \\ & - 4X_2 + 1X_2^2 + 7X_2^3 \\ & - 6X_3 + 7X_3^2 - 6X_3^3 \\ & + 5X_1X_2 + 5X_1X_3 - 6X_2X_3 \end{aligned} \tag{38}$$

TABLE 5. PROGRAM LISTING (WITH CROSS PRODUCTS)

```
VV          VV     DDDDDDDDDD          AAAAAAAA        RRRRRRRRRRR       BBBBBBBBBB        YY          YY
VV          VV     DDDDDDDDDDD        AAAAAAAAAA       RRRRRRRRRRRR      BBBBBBBBBBB       YYY        YYY
 VV        VV      DD        DD      AA        AA      RR        RR      BB        BB       YYY      YYY
 VV        VV      DD        DD      AA        AA      RR        RR      BB        BB        YYY    YYY
  VV      VV       DD        DD      AA        AA      RR        RR      BB        BB         YYYYYY
  VV      VV       DD        DD      AAAAAAAAAAAA      RRRRRRRRRRRR      BBBBBBBBBB            YYYY
   VV    VV        DD        DD      AAAAAAAAAAAA      RRRRRRRRRRR       BBBBBBBBBB             YY
   VV    VV        DD        DD      AA        AA      RR    RR          BB        BB           YY
    VVVV           DD        DD      AA        AA      RR     RR         BB        BB           YY
    VVVV           DD        DD      AA        AA      RR      RR        BB        BB           YY
     VV            DDDDDDDDDDD       AA        AA      RR       RR       BBBBBBBBBBB            YY
     VV            DDDDDDDDDD        AA        AA      RR        RR      BBBBBBBBBB             YY

BBBBBBBBBB            IIIIII         NN        NN          44    44         OOOO              666666
BBBBBBBBBBB           IIIIII         NNN       NN          44    44       OOOOOOOO          6666666666
BB        BB            II           NNNN      NN         44     44      OOO    OOO        666       66
BB        BB            II           NNNNN     NN        44      44      OOO    OOO        66
BB        BB            II           NN NNN    NN       44       44      OO      OO        66
BBBBBBBBBB              II           NN  NNN   NN      44444444444       OO      OO        66 6666666
BBBBBBBBBB              II           NN   NNN  NN      44444444444       OO      OO        66666666666
BB        BB            II           NN    NNN NN               44       OO      OO        66        66
BB        BB            II           NN     NNNNN               44       OOO    OOO        66        66
BB        BB            II           NN      NNNN               44        OOO  OOO         66        66
BBBBBBBBBBB           IIIIII         NN       NNN               44        OOOOOOOO          6666666666
BBBBBBBBBB            IIIIII         NN        NN               44          OOOO              666666

   OOOO                 11               11             666666        777777777777         333333
 OOOOOOOO              111              111           6666666666      777777777777       3333333333
OOO    OOO            1111             1111          666       66              777      333      333
OOO    OOO              11               11          66                       777       33        33
OO      OO              11               11          66                      777                  33
OO      OO              11               11          66 6666666             777                 333
OO      OO              11               11          66666666666           777                  333
OO      OO              11               11          66        66         777                     33
OOO    OOO              11               11          66        66        777          33          33
 OOO  OOO               11               11          66        66       777           333        333
 OOOOOOOO             111111           111111         6666666666       777             3333333333
   OOOO               111111           111111           666666        777                333333

   OOOO               222222            OOOO         555555555555          44    44      333333
 OOOOOOOO           2222222222        OOOOOOOO       555555555555         44    44     3333333333
OOO    OOO         222       222     OOO    OOO      55                  44    44      333      333
OOO    OOO         22        222     OOO    OOO      55                 44     44      33        33
OO      OO                  222      OO      OO      55                44      44                33
OO      OO                222        OO      OO      5555555555       444444444444             333
OO      OO              222          OO      OO      55555555555      444444444444             333
OO      OO            222            OO      OO                55             44                 33
OOO    OOO           222             OOO    OOO                55             44       33        33
 OOO  OOO            222              OOO  OOO       55        55             44       333      333
 OOOOOOOO            222222222222     OOOOOOOO        5555555555              44        3333333333
   OOOO              222222222222       OOOO           55555555               44          333333

$RUN,//1 WIN-CF,320570,VDARBYBIN406,03,1000
```

TABLE 5. (Continued)

```
    @MSG,N              REMOVE AT CARD READER        01-16-73      CLOCK NO.  9353


    @ASG,T 10,F/1/POS/2

@FOR,IS MAIN,MAIN
FOR 009-01/16-13:51 (,0)

   MAIN PROGRAM

   STORAGE USED: CODE(1) 000021; DATA(0) 000125; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003    REGRES
    0004    NINTR$
    0005    NRNLS
    0006    NSTOP$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0000 I 000076 IP       0000 I 000074 LN       0000 I 000075 N        0000   000101 NAM1      0000 R 000055 YO
    0000 R 000000 Z


00101      1*          DIMENSION Z(3,15),YO(15)
00103      2*          NAMELIST/NAM1/Z,YO,LN,N,IP
00104      3*          READ(5,NAM1)
00107      4*          CALL REGRES(Z,YO,LN,N,IP)
00110      5*          STOP
00111      6*          END
       END OF COMPILATION:        NO  DIAGNOSTICS.
```

TABLE 5. (Continued)

```
@FOR,IS REGRES,REGRES
MSG 009-01/16-13:51 (.0)

    SUBROUTINE REGRES     ENTRY POINT 001617

    STORAGE USED: CODE(1) 001665; DATA(0) 002140; BLANK COMMON(2) 000000

    EXTERNAL REFERENCES (BLOCK, NAME)

     0003    IDENT
     0004    IORDER
     0005    INVRT
     0006    DMATML
     0007    NOIK3L
     0010    LNUJ$B
     0011    NWDU$
     0012    NIO2$
     0013    NEXP6$
     0014    SQRT
     0015    NIO1$
     0016    NSTOP$
     0017    NERR3$

    STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

     0000     001744 10000F     0000     001752 10006F     0000     001755 10009F     0000     001776 10010F     0000     002002 10011F
     0000     002005 10012F     0000     002011 10014F     0000     002013 10026F     0000     002024 10027F     0000     002027 10033F
     0001     000115 142G       0001     000116 145G       0001     000126 153G       0001     000127 156G       0001     000137 164G
     0001     000143 171G       0001     000144 174G       0001     000154 202G       0001     000160 207G       0001     000200 225G
     0001     000202 230G       0001     000204 233G       0001     000240 244G       0001     000267 250G       0001     000270 253G
     0001     000327 264G       0001     000331 271G       0001     000334 274G       0001     000375 306G       0001     000422 312G
     0001     000424 315G       0001     000462 327G       0001     000505 340G       0001     000542 351G       0001     000556 357G
     0001     000575 363G       0001     000623 376G       0001     000640 405G       0001     000654 411G       0001     000676 422G
     0001     000677 425G       0001     000742 442G       0001     000743 445G       0001     000764 456G       0001     000777 462G
     0001     001015 472G       0001     001016 475G       0001     001033 504G       0001     001235 505L       0001     001046 510G
     0001     001316 513L       0001     001066 522G       0001     001070 525G       0001     001072 530G       0001     001121 537G
     0001     001126 543G       0001     001147 547G       0001     001167 556G       0001     001205 567G       0001     001227 577G
     0001     001246 611G       0001     001260 621G       0001     001333 634G       0001     001422 660G       0001     001423 662G
     0001     001444 675G       0001     001457 701G       0001     001501 716G       0001     001521 734G       0000 R 001653 A
     0000 R 000522 AYARY        0000 R 001731 AMAX         0000 R 001642 B            0000 R 001726 BHAT         0000 R 001604 BZ
     0000 D 000472 DF           0000 D 000000 DETER        0000 D 000002 DSN          0000 D 000442 DSVHAT       0000 R 000567 FLDX
     0000 R 000603 FLDY         0000 R 001737 FRATIO       0000 I 001662 I            0000 I 001720 ICOL         0000 I 001711 IERR
     0000   002066 INJP$        0000 I 001717 IROW         0000 I 001740 IS           0000 I 001710 ISECT        0000 I 001721 ISECT1
     0000 I 001706 IUNIT        0000 I 001661 IZERO        0000 I 001722 I2           0000 I 001724 I21          0000 I 001672 J
     0000 I 001704 K            0000 I 001734 KHAT         0000 I 001712 KK           0000 I 001705 L            0000 I 001743 LI
     0000 I 001741 L1           0000 I 001742 L2           0000 I 001725 M            0000 I 001716 MCOL         0000 I 001713 NFACT
     0000 I 001714 NFACT2       0000 I 001733 NI           0000 I 001671 NIP          0000 I 001670 NN           0000 I 001723 NR
     0000 I 001707 NSECT        0000 I 001663 NADS         0000 I 001715 NZ           0000 R 001730 PERAVG       0000 R 001732 PERMAX
     0000 R 000550 RMAX         0000 R 001701 R            0000 R 001623 RESID        0000 R 001736 RHAT         0000 R 001674 RR
     0000 R 001331 SHAT         0000 R 000655 SMALLZ       0000 R 001021 SMALZ1       0000 R 001165 SMALZ2       0000 R 001675 SREG
     0000 R 001676 SRES         0000 R 001703 SS           0000 R 001735 STADEV       0000 R 001727 SUM          0000 R 001551 SVHAT
     0000 R 001700 SYY          0000 R 001677 VAR          0000 R 000617 X            0000 R 001664 XL           0000 R 001665 XR
```

TABLE 5. (Continued)

```
 0000 R 001666 YB          0000 R 001673 YBAR        0000 R 001565 YC          0000 R 001667 YT          0000 R 001702 YY
 0000 R 000636 ZBAR        0000 R 000647 ZZBAR


00101      1*        SUBROUTINE REGRES(Z,YO,LN,N,IP)
00103      2*        PARAMETER IDIM=100
00104      3*        DOUBLE PRECISION DETER
00105      4*        DOUBLE PRECISION DSN(144),DSVHAT(12,1),DB(12,1)
00106      5*        DIMENSION ADARY(22)
00107      6*        DIMENSION PMAX(15)
00110      7*        DIMENSION FLDX(12),FLDY(12)
00111      8*        DIMENSION X(15)
00112      9*        DIMENSION Z(3,15),ZBAR(3,3),ZZBAR(2,3),SMALLZ(IDIM),
00112     10*       *SMALZ1(IDIM),SMALZ2(IDIM),SHAT(12,12),YG(15),SVHAT(12,1),
00112     11*       *YC(15),BZ(15),RESID(15),B(3,3),A(2,3)
00113     12*        DATA IZERO/0/
00115     13*        DATA FLDY/72H                              Y VALUES
00115     14*       *                   /
00117     15*        DATA FLDX/72H                                POINT INTERVALS
00117     16*       *                   /
00121     17*        DATA (ADARY(I),I=1,3)/6HONE    ,6H  COPY,6HFLO    /
00123     18*        CALL IDENT(9,ADARY)
00124     19*        NROS=IDIM
00125     20*        XL=0.
00126     21*        XR=16.
00127     22*        YB=1.
00130     23*        YT=17.
00131     24*        WRITE(6,10000) N,IP,LN
00136     25*  10000 FORMAT(1H1,2HN=I4,3X,2HP=I4,3X,3HLN=I4)
00136     26*  C     INITIALIZE VARIABLES
00137     27*        NN=N-1
00140     28*        NIP=N*IP
00141     29*        DO 15 I=1,N
00144     30*        DO 15 J=1,IP
00147     31*     15 ZBAR(I,J)=0.0
00152     32*        DO 16 I=1,NN
00155     33*        DO 16 J=1,IP
00160     34*     16 ZZBAR(I,J)=0.0
00163     35*        DO 7 I=1,IDIM
00166     36*      7 SMALLZ(I)=0.0
00170     37*        DO 8 I=1,NIP
00173     38*        DO 8 J=1,NIP
00176     39*      8 SHAT(I,J)=0.0
00201     40*        DO 9 I=1,NIP
00204     41*      9 SVHAT(I,1)=0.0
00206     42*        DO 5 I=1,LN
00211     43*      5 BZ(I)=0.0
00213     44*        YBAR=0.0
00215     45*        RR=0.0
00215     46*        SRLG=0.0
00216     47*        SRES=0.0
00217     48*        VAR=0.0
00220     49*        SYY=0.0
00221     50*        S=0.0
```

TABLE 5. (Continued)

```
00222              YY=0.0
00223              SS=0.0
00223    C         COMPUTE ZBAR
00224              DO 125 I=1,N
00227              DO 125 J=1,IP
00232              DO 100 K=1,LN
00235          100 ZBAR(I,J)= ZBAR(I,J)+ Z(I,K)**J
00237          125 ZBAR(I,J)= ZBAR(I,J)/LN
00237    C         COMPUTE ZZBAR
00242              K=1
00243              DO 150 I=1,NN
00246              K=K+1
00247              DO 150 J=K,N
00252              DO 130 L=1,LN
00255          130 ZZBAR(I,J)=ZZBAR(I,J)+Z(I,L)*Z(J,L)
00257          150 ZZBAR(I,J)=ZZBAR(I,J)/LN
00257    C         COMPUTE SMALLZ'S
00262              IUNIT=10
00263              NSECT=(NWDS+27)/28
00264              ISECT = 1
00265              DO 200 I=1,N
00270              DO 200 J=1,IP
00273              DO 175 K=1,LN
00276          175 SMALLZ(K)=Z(I,K)**J-ZBAR(I,J)
00300              CALL IORDWR(1,IUNIT,ISECT,SMALLZ,NWDS,IERR)
00301          200 ISECT=ISECT+NSECT
00304              KK=1
00305              DO 230 I=1,NN
00310              KK=KK+1
00311              DO 230 J=KK,N
00314              DO 225 K=1,LN
00317          225 SMALLZ(K)=Z(I,K)*Z(J,K)-ZZBAR(I,J)
00321              CALL IORDWR(1,IUNIT,ISECT,SMALLZ,NWDS,IERR)
00322          230 ISECT=ISECT+NSECT
00322    C         COMPUTE NUMBER OF COLUMNS
00325              NFACT=1
00326              DO 250 I=1,N
00331          250 NFACT=NFACT*I
00333              NFACT2=1
00334              N2=N-2
00335              IF(N2.EQ.0) N2=1
00337              DO 275 I=1,N2
00342          275 NFACT2=NFACT2*I
00344              NCOL=N*IP+(NFACT/(2*NFACT2))
00344    C         COMPUTE ELEMENTS OF THE SHAT MATRIX
00345              ISECT=1
00346              NSECT=(NWDS + 27)/28
00347              IROW=1
00350              DO 310 J=1,NCOL
00353              CALL IORDWR(2,IUNIT,ISECT,SMALZ1,NWDS,IERR)
00354              ICOL=1
00355              ISECT1=1
00356              DO 305 K=1,NCOL
00361              CALL IORDWR(2,IUNIT,ISECT1,SMALZ2,NWDS,IERR)
00362              DO 300 I=1,LN
00365          300 SHAT(IROW,ICOL)=SHAT(IROW,ICOL)+SMALZ1(I)*SMALZ2(I)
```

TABLE 5. (Continued)

```
00367          ICOL=ICOL+1
00370      305 ISECT1=ISECT1+NSECT
00372          IROW=IROW+1
00373      310 ISECT=ISECT+NSECT
00373    C     COMPUTE SVHAT
00375          DO 325 I=1,LN
00400      325 YBAR=YBAR+YO(I)
00402          YBAR=YBAR/LN
00403          ISECT=1
00405          DO 350 K=1,NCOL
00407          CALL IGRDAR(2,IUNIT,ISECT,SMALZI,NWDS,IERR)
00410          DO 330 I=1,LN
00413          SVHAT(K,1)=SVHAT(K,1)+SMALZI(I)*(YO(I)-YBAR)
00414      330 DSVHAT(K,1)=SVHAT(K,1)
00416      350 ISECT=ISECT+NSECT
00420          I2=1
00421          DO 375 I=1,NCOL
00424          DO 375 J=1,NCOL
00427          DSN(I2)=SHAT(I,J)
00430      375 I2=I2+1
00433          NR=NCOL
00434          I21=I2-1
00434    C     FIND SHAT MATRIX INVERSE
00435          M=0
00436          CALL INVRT(DSN,NR,M,DETER)
00436    C     COMPUTE B AND A COEFFICIENTS FOR THE ASSUMED MODEL
00437          CALL DMATML(DB,DSN,DSVHAT,NR,NR,1)
00440          I2=1
00441          DO 400 I=1,N
00444          DO 400 J=1,IP
00447          B(I,J)=DB(I2,1)
00450      400 I2=I2+1
00453          I21=I2
00454          K=1
00455          DO 410 I=1,NN
00460          K=K+1
00461          DO 410 J=K,N
00464          A(I,J)=DB(I21,1)
00465      410 I21=I21+1
00470          BHAT=YBAR
00471          DO 420 I=1,N
00474          DO 420 J=1,IP
00477      420 BHAT=BHAT-B(I,J)*ZBAR(I,J)
00502          K=1
00503          DO 425 I=1,NN
00506          K=K+1
00507          DO 425 J=K,N
00512      425 BHAT=BHAT-A(I,J)*ZZBAR(I,J)
00515          WRITE(6,10006) BHAT
00520    10006 FORMAT(1H0,5HBHAT=E16.9)
00521          DO 450 I=1,LN
00524          DO 450 J=1,N
00527          DO 450 K=1,IP
00532      450 BZ(I)=BZ(I)+B(J,K)*Z(J,I)**K
00536          DO 475 I=1,LN
00541          KK=1
```

TABLE 5. (Continued)

```
00542          DO 475 J=1,NN
00545          KK=KK+1
00546          DO 475 L=KK,N
00551      475 BZ(I)=BZ(I)+A(J,L)*Z(J,I)*Z(L,I)
00551     C    COMPUTE THE Y POINTS
00555          DO 500 I=1,LN
00560          YC(I)=BHAT+BZ(I)
00561          RESID(I)=YO(I)-YC(I)
00562          RR=RR+RESID(I)**2
00563      500 CONTINUE
00565          SUM=0.
00566          DO 501 L=1,LN
00571          PMAX(L)=ABS((YO(L)-YC(L))/YC(L))
00572      501 SUM=SUM+PMAX(L)
00574          PERAVG=(SUM/LN)*100.
00575          AMAX=PMAX(1)
00576          DO 505 L=2,LN
00601          IF(AMAX-PMAX(L)) 506,505,505
00604      506 AMAX=PMAX(L)
00605      505 CONTINUE
00607          PERMAX=AMAX*100.
00610          DO 507 L=1,LN
00613          R=R+ABS(RESID(L))
00614          YY=YY+YO(L)
00615      507 CONTINUE
00615     C    COMPUTE STANDARD DEVIATION
00617          N2=0
00620          DO 510 I=1,NN
00623          NI=N-I
00624      510 N2=N2+NI
00626.         KHAT=N*IP+N2
00627          IF((LN-KHAT-1).LE.0) GO TO 513
00631          STADEV=SQRT(RR/(LN-KHAT-1))
00632      513 CONTINUE
00632     C    COMPUTE THE MULTIPLE CORRELATION COEFFICIENT, AND FRATIO
00633          DO 525 I=1,LN
00636          SREG=SREG+(YC(I)-YBAR)**2
00637          SYY=SYY+(YO(I)-YBAR)**2
00640          SS=SS+(YC(I)-YBAR)*(YO(I)-YC(I))
00641      525 SRES=SRES+RESID(I)**2
00643          RHAT=SREG/SYY
00644          FRATIO=(SREG/(KHAT-1))/(SRES/(LN-KHAT))
00645          WRITE(6,10009) STADEV,RHAT,FRATIO
00652    10009 FORMAT(1H0,19HSTANDARD DEVIATION=E16.9,4X,33HMULTIPLE CORRELATION
00652         *COEFFICIENT=E16.9,4X,8HF RATIO=E16.9)
00653          WRITE(6,10010)
00655    10010 FORMAT(1H0,14HB COEFFICIENTS)
00656          WRITE(6,10011) ((B(I,J),J=1,IP),I=1,N)
00667    10011 FORMAT(1H ,5E21.9/)
00670          WRITE(6,10012)
00672    10012 FORMAT(1H0,14HA COEFFICIENTS)
00673          K=1
00674          DO 530 I=1,NN
00677          K=K+1
00700          DO 530 J=K,N
00703          WRITE(6,10014) A(I,J)
```

TABLE 5. (Continued)

```
00706    219*     10014 FORMAT(1H ,E21.9)
00707    220*       530 CONTINUE
00712    221*           WRITE(6,10026)
00714    222*     10026 FORMAT(1H0,12X,8HOBSERVED,12X,8HCOMPUTED,12X,8HRESIDUAL)
00715    223*           DO 1233 IS=1,LN
00720    224*           WRITE(6,10027) YO(IS),YC(IS),RESID(IS)
00725    225*      1233 CONTINUE
00727    226*     10027 FORMAT(1H ,3E21.9/)
00730    227*           L1=-1
00731    228*           L2=0
00732    229*           X(1)=1.
00733    230*           DO 649 LI=2,LN
00736    231*       649 X(LI)=X(LI-1)+1.
00740    232*           CALL QUIK3L(L1,XL,XR,YB,YT,40,FLDX,FLDY,-LN,X YC)
00741    233*           CALL QUIK3L(L2,XL,XR,YB,YT,34,FLDX,FLDY,LN,X,YO)
00742    234*           WRITE(6,10033) PERMAX,PERAVG
00746    235*     10033 FORMAT(1H0,22HMAXIMUM PERCENT ERROR=E16.9,6X,22HAVERAGE PERCENT ER
00746    236*          *ROR=E16.9)
00747    237*      1234 CONTINUE
00750    238*           CALL ENDJOB
00751    239*           STOP
00752    240*           END

        END OF COMPILATION:          NO DIAGNOSTICS.
```

TABLE 5. (Continued)

```
@FOR,IS RDWT,RDWT
HSO 009-01/16-13:51 (,0)

   SUBROUTINE RDWT       ENTRY POINT 000270
              REDTPR     ENTRY POINT 000273
              WRITER     ENTRY POINT 000320
              CLOSE      ENTRY POINT 000342
              OPEN       ENTRY POINT 000351
              IORDWR     ENTRY POINT 000360

   STORAGE USED: CODE(1) 000404; DATA(0) 000127; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   IOFR
    0004   NERR2$
    0005   NWDU$
    0006   NIO1$
    0007   NIO2$
    0010   NSTOP$
    0011   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000071 10L       0000   000070 100F      0001   000116 20L       0001   000140 21L       0001   000242 215G
    0001   000144 22L       0001   000211 24L       0001   000227 25L       0001   000234 26L       0001   000033 6L
    0001   000235 7L        0001   000250 9L        0000 I 000061 I         0000 I 000065 IBLANK    0000 I 000000 ICWD
    0000 I 000003 IFLD      0000   000111 INJP$     0000 I 000066 IOP       0000 I 000046 IPACKT    0000 I 000067 ITWO
    0000 I 000062 I1        0000 I 000063 I2        0000 I 000064 I3        0000 I 000056 KNTWD     0000 I 000040 KODE


00100      1*     C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
00100      2*     C      GENERAL FORTRAN I/O PACKAGE         EXEC VIII           APRIL,1970
00100      3*     C      READ/WRITE MAG TAPE OR DRUM IN FORTRAN OR NON-FORTRAN FORMAT.
00100      4*     C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
00101      5*            SUBROUTINE RDWT
00103      6*            DIMENSION ARRAY(1),ICWD(3),IFLD(29),KODE(6),IPACKT(8),KNTWD(3)
00104      7*            DATA (IFLD(I),I=1,29)/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,2H10,
00104      8*           12H11,2H12,2H13,2H14,2H15,2H16,2H17,2H18,2H19,2H20,2H21,2H22,2H23,
00104      9*           22H24,2H25,2H26,2H27,2H28,2H29/
00106     10*            DATA (KODE(I),I=1,6)/000430000000,000200000000,000150000000,
00106     11*           1                        000100000000,000110000000,000400000000/
00110     12*            DATA I1/262144/,I2/1073741824/,I3/00100001/
00114     13*            DATA IBLANK/6H
00114     14*     C*****
00116     15*            ENTRY REDTPR(IUNIT,MODE,IERR,NW,NADS,ARRAY)
00120     16*            IOP=MODE                  $ =1,READ FTN FORMAT, =2,READ NON-FTN.
00120     17*     C*****
00121     18*            GO TO 10
00122     19*            ENTRY WRITER(IUNIT,MODE,IERR,NADS,ARRAY)
```

TABLE 5. (Continued)

```
00124    20*          IOP=MODE+2               @ =3,WRITE FTN FORMAT. =4,WRITE NON-FTN.
00125    21*          KNTWD(1)=NWDS*I1+I3      @ BUILD FORTRAN RECORD-CONTROL WORD.
00125    22*          KNTWD(2)=IBLANK          @ DUMMY CHECKSUM ERROR WORD.
00127    23*          KNTWD(3)=KNTWD(1)
00130    24*          GO TO 10
00130    25*    C*****
00131    26*          ENTRY CLOSE(IUNIT,JOP)
00133    27*          ITWO=1
00134    28*          GO TO (6,6,7,25),JOP     @ JOP=4, NO ACTION,RETURN.
00135    29*        6 ITWO=2                   @ JOP=1,2, WRITE EOF AND REWIND IUNIT.
00136    30*        7 IOP=5                    @ =5, WRITE END-OF-FILE.
00137    31*          GO TO 10
00137    32*    C*****
00140    33*          ENTRY OPEN(IUNIT,JOP)
00142    34*          IF(JOP.GT.1) RETURN      @ JOP=2,3, NO ACTION,RETURN.
00144    35*        8 IOP=6                    @ =6, REWIND IUNIT.
00145    36*          GO TO 10
00145    37*    C*****
00146    38*          ENTRY IORDWR(ICODE,IUNIT,IADDRS,ARRAY,NWDS,IERR)
00150    39*          IF(ICODE.EQ.1) IOP=4     @ =4, SEQUENTIAL WRITE.
00152    40*          IF(ICODE.EQ.2) IOP=2     @ =2, SEQUENTIAL READ.
00154    41*          IPACKT(6)=IADDRS         @ BEGINNING DRUM-SECTOR ADDRESS.
00154    42*    C*****
00155    43*       10 CONTINUE
00155    44*          IPACKT(1)=IFLD(IUNIT)              @ IUNIT IN FIELD-DATA.
00157    45*          IPACKT(2)=IBLANK
00160    46*          IPACKT(3)=0
00161    47*       11 IPACKT(4)=KODE(IOP)                @ OPERATION CODE TO IOWS.
00162    48*          GO TO (20,21,20,21,22,22),IOP
00163    49*       20 ICWD(1)=    1*I1+LOC(KNTWD(1))     @ FIRST CONTROL ACCESS WORD.
00164  *DIAGNOSTIC*  THE VARIABLE ARRAY IS DIMENSIONED.
00164    50*          ICWD(2)=NWDS*I1+LOC(ARRAY)         @ SECOND CONTROL ACCESS WORD.
00165    51*          ICWD(3)=    2*I1+LOC(KNTWD(2))     @  THIRD CONTROL ACCESS WORD.
00166    52*          IPACKT(5)= 3*I1+LOC(ICWD(1))       @ NMBR AND LOC OF 1ST C/A WORD.
00167    53*          GO TO 22
00170  *DIAGNOSTIC*  THE VARIABLE ARRAY IS DIMENSIONED.
00170    54*       21 IPACKT(5)=NWDS*I1+LOC(ARRAY)       @ NMBR AND LOC OF DATA WORDS.
00171    55*       22 CALL IOWR(IPACKT)                  @ ROUTINE TO ISSUE EXEC REQUEST.
00172    56*          IF(IOP.EQ.6) RETURN                @ RETURN AFTER REWINDING IUNIT.
00174    57*          IF(IOP.EQ.5) GO TO (25,8),ITWO     @ ITWO=1,RETURN, =2,REWIND IUNIT.
00176    58*          IERR=IPACKT(4)/I2                  @ ERROR STATUS IN S1 PORTION.
00177    59*          IF(IERR.EQ.0) GO TO 24             @ NORMAL OPERATION,NO ERROR.
00201    60*          IF(IERR.GE.3) GO TO 26             @ TERMINATING ERROR.
00203    61*          IERR=2                             @ END-OF-FILE OR END-OF-TAPE.
00204    62*          RETURN
00205    63*       24 NW=KNTWD(1)/I1                     @ NW FROM FTN CONTROL WORD.
00206    64*          IF(MODE.EQ.1) GO TO 25
00210    65*          NW=IPACKT(4)-IPACKT(4)/I1*I1       @ NW FROM IOWS.
00211    66*       25 IERR=1
00212    67*          RETURN
00213    68*       26 WRITE(6,100) (IPACKT(I),I=1,6)
00221    69*      100 FORMAT(///50H*****.EXECUTION TERMINATED IN 'ROWT' I/O ROUTINE. //
00221    70*         114H I/O PACKET IS 6O16 )
03222    71*          STOP
00223    72*          END
```

TABLE 5. (Continued)

```
END OF COMPILATION:            2 DIAGNOSTICS.


@ASM,IS IOWR,IOWR
ASM11F 01/16-13:51-(,0)

     1.                                                         $(1)   AXRS
     2.        01    000000  27 00 14 13 0 000000  IOWR*  L        A0,0,X11
     3. U             000001  72 11 00 00 0 000000         ER       IOW$
     4.               000002  74 04 00 13 0 000002         J        2,X11
     5.                                                    END
                        IOW$
 END ASM ERRORS : NONE
```

TABLE 5. (Continued)

```
@FOR,IS INVRT,INVRT
RSO 009-01/16-13:51 (,0)

   SUBROUTINE INVRT     ENTRY POINT 000364

   STORAGE USED: CODE(1) 000412; DATA(0) 000202; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001     000016 111G      0001     000140 115L      0001     000027 117G      0001     000034 123G      0001     000042 131G
    0001     000110 157G      0001     000162 176G      0001     000254 181L      0001     000202 204G      0001     000224 215G
    0001     000262 231G      0001     000332 235L      0001     000306 242G      0001     000342 250L      0001     000071 75L
    0001     000073 76L       0000 D 000002 AMAX      0000 I 000141 I         0000 I 000144 IC        0000 I 000142 IND
    0000 I 000042 INDEX       0000 I 000146 IND2      0000   000154 INJP$     0000 I 000004 IPIV      0000 I 000143 IR
    0000 I 000136 J           0000 I 000140 K         0000 I 000145 L         0000 I 000137 NN        0000 D 000000 SIGN


00101      1*             SUBROUTINE INVRT(A,N,M,DETER)
00103      2*             PARAMETER IDIM=30
00103      3*       C     MATRIX INVERSION AND SIMULTANEOUS EQUATIONS SOLVER
00103      4*       C     A=INPUT MATRIX FOR INVERISON OR AUGUMENTED MATRIX FOR SIME. EQS.  IG500030
00103      5*       C     N=ORDER OF COEFFICIENT MATRIX                                      IG500040
00103      6*       C     M=0 FOR INVERISON ONLY
00103      7*       C     M=NUMBER OF CONSTANT VECTORS
00103      8*       C     DETER=DETERMINANT OF COEFFICIENT MATRIX
00104      9*             DOUBLE PRECISION A(1),DETER,SIGN,AMAX
00105     10*             DIMENSION IPIV(IDIM),INDEX(IDIM,2)
00106     11*             DETER=1.0D0
00107     12*             SIGN=1.0D0
00110     13*             DO 20 J=1,N
00113     14*          20 IPIV(J)=0
00115     15*             NN=N+M
00115     16*             DO 182 K=1,N
00121     17*             AMAX=0.0D0
00122     18*          40 DO 76 I=1,N
00125     19*             IF (IPIV(I)-1)50,76,50
00130     20*          50 DO 75 J=1,N
00133     21*             IF (IPIV(J)-1)55,75,250
00136     22*          55 IND=(J-1)*N+I
00137     23*             IF(AMAX-DABS(A(IND))) 60,75,75
00142     24*          60 IR=I
00143     25*             IC=J
00144     26*             AMAX=DABS(A(IND))
00145     27*          75 CONTINUE
00147     28*          76 CONTINUE
00151     29*             IPIV(IC)=IPIV(IC)+1
```

TABLE 5. (Continued)

```
00152     30*           IF (IR-IC)90,115,90
00155     31*        90 SIGN=-SIGN
00156     32*           DO 110 L=1,NN
00161     33*           IND=(L-1)*N+IR
00162     34*           IND2=(L-1)*N+IC
00163     35*           AMAX=A(IND)
00164     36*           A(IND)=A(IND2)
00165     37*       110 A(IND2)=AMAX
00167     38*       115 INDEX(K,1)=IR
00170     39*           INDEX(K,2)=IC
00171     40*           IND=(IC-1)*N+IC
00172     41*           AMAX=A(IND)
00173     42*           DETER=DETER*AMAX
00174     43*       140 A(IND)=1.0D0
00175     44*           DO 150 L=1,NN
00200     45*           IND=(L-1)*N+IC
00201     46*       150 A(IND)=A(IND)/AMAX
00203     47*           DO 181 L=1,N
00206     48*           IF (L-IC)165,181,165
00211     49*       165 IND=(IC-1)*N+L
00212     50*           AMAX=A(IND)
00213     51*           A(IND)=0.0D0
00214     52*           DO 180 I=1,NN
00217     53*           IND=(I-1)*N+L
00220     54*           IND2=(I-1)*N+IC
00221     55*           A(IND)=A(IND)-A(IND2)*AMAX
00222     56*       180 CONTINUE
00224     57*       181 CONTINUE
00226     58*       182 CONTINUE
00230     59*           DO 235 I=1,N
00233     60*           L=N+1-I
00234     61*           IR=INDEX(L,1)
00235     62*           IC=INDEX(L,2)
00236     63*           IF (IR-IC)210,235,210
00241     64*       210 DO 230 K=1,N
00244     65*           IND=(IR-1)*N+K
00245     66*           IND2=(IC-1)*N+K
00246     67*           AMAX=A(IND)
00247     68*           A(IND)=A(IND2)
00250     69*       230 A(IND2)=AMAX
00252     70*       235 CONTINUE
00254     71*           DETER=SIGN*DETER
00255     72*           RETURN
00256     73*       250 M=-1
00257     74*       255 RETURN
00260     75*           END

        END OF COMPILATION:          NO  DIAGNOSTICS.
```

TABLE 5. (Continued)

```
@FOR,IS DMATML,DMATML
HSO 009-01/16-13:51 (,0)

   SUBROUTINE DMATML     ENTRY POINT 000160

   STORAGE USED: CODE(1) 000176; DATA(0) 000053; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003    NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001    000031 1L          0001    000061 130G         0001   000067 135G         0001    000102 143G         0001    000035 2L
    0001    000046 3L          0001    000052 4L           0000 D 000000 CB           0000 I 000005 IA1          0000 I 000006 IA2
    0000 I 000002 IA3          0000 I 000007 IB1           0000 I 000010 IB2          0000 I 000013 IB3          0000 I 000003 IM
    0000 I 000004 IN           0000    000022 INJP$        0000 I 000015 LA           0000 I 000016 LB           0000 I 000012 LC
    0000 I 000011 LM           0000 I 000017 LN            0000 I 000014 LP


00101      1*           SUBROUTINE DMATML(C,A,B,M,N,K)
00101      2*     C*    ABSTRACT
00101      3*     C     GENERAL MATRIX MULTIPLICATION ROUTINE  WITH TRANSPOSE OPTIONS
00101      4*     C      WHERE, M IS THE NUMBER OF ROWS OF (A)
00101      5*     C             N IS THE NUMBER OF ROWS OF (B)
00101      6*     C             K IS THE NUMBER OF COLUMNS OF (B) OR (B)T
00101      7*     C     TRANSPOSE OPTIONS ARE CONTROLLED BY THE SIGNS OF M AND N.
00101      8*     C     THE FOLLOWING PRODUCTS MAY BE OBTAINED
00101      9*     C      (C)=(A)(B)       M AND N POSITIVE
00101     10*     C      (C)=(A)T(B)      M NEGATIVE FOR (A)T
00101     11*     C      (C)=(A)(B)T      N NEGATIVE FOR (B)T
00101     12*     C      (C)=(A)T(B)T     M AND N NEGATIVE
00101     13*     C      WHERE T INDICATES TRANSPOSE
00101     14*     C      IF M IS NEGATIVE, M IS THE NUMBER OF ROWS OF (A)T
00101     15*     C      IF N IS NEGATIVE, N IS THE NUMBER OF ROWS OF (B)T
00101     16*     C
00101     17*     C*    OUTPUT ARGUMENT * C
00103     18*           DIMENSION C(1)
00103     19*     C
00103     20*     C*    INPUT ARGUMENTS * A,B,M,N,K
00104     21*           DIMENSION A(1),B(1)
00104     22*     C
00104     23*     C**********************************************************************
00105     24*           DOUBLE PRECISION CB,C,A,B
00106     25*           IA3=1
00107     26*           IM=IABS(M)
00110     27*           IN=IABS(N)
00111     28*           IF(M .LT. 0)GO TO 1
00113     29*           IA1=IM
00114     30*           IA2=1
```

TABLE 5. (Continued)

```
00115     31*              GO TO 2
00116     32*      1       IA1 = 1
00117     33*              IA2=IN
00120     34*      2       IF(N .LT. 0)GO TO 3
00122     35*              IB1=1
00123     36*              IB2=IN
00124     37*              GO TO 4
00125     38*      3       IB1=K
00126     39*              IB2=1
00127     40*      4       DO 7 LM=1,IM
00132     41*              LC=LM
00133     42*              IB3=1
00134     43*              DO 6 LP=1,K
00137     44*              CD = 0.0D0
00140     45*              LA=IA3
00141     46*              LB=IB3
00142     47*              DO 5 LN=1,IN
00145     48*              CD = CD + A(LA)*B(LB)
00146     49*              LA=LA+IA1
00147     50*      5       LB=LB+IB1
00151     51*              C(LC) = CD
00152     52*              LC = LC + IM
00153     53*      6       IB3=IB3+IB2
00155     54*      7       IA3=IA3+IA2
00157     55*              RETURN
00160     56*              END
          END OF COMPILATION:          NO  DIAGNOSTICS.
```

TABLE 5. (Continued)

```
@MAP,I AA,AA
MAP 17M2-01/16-13:51 -(,0)

    1.         LIB SYS$*MSFCS.



ADDRESS LIMITS     001000 033560      040000 055012
STARTING ADDRESS   033540
WORDS DECIMAL       13681 IBANK         6667 DBANK


              SEGMENT MAIN            001000 033560      040000 055012

NSWTC$/FOR                1   001000 001021
NRBLK$/MSFC55             1   001022 001110       0     040000 040001
NEXP6$/MSFC57             1   001111 001303       2     040002 040053
ALOG$/FOR51               1   001304 001421       2     040054 040114
CSIG6V/SC4020             1   001422 001710       0     040115 040153
                                                  2     BLANK$COMMON
CSNFRK/SC4020             1   001711 001740       0     040154 040167
                                                  2     BLANK$COMMON
CLASLV/SC4020             1   001741 003003       0     040170 040276
                                                  2     BLANK$COMMON
NRAND$/FOR50              1   003004 003063       2     040277 040310
NVEF$/MSFC55              1   003064 003320       2     040311 040331
NSTCH$/FOR57              1   003321 003620       2     040332 040367
CYMCDV/SC4020             1   003621 003646       0     040370 040376
                                                  2     BLANK$COMMON
CACCBY/SC4020             1   003647 003670       0     040377 040407
                                                  2     BLANK$COMMON
CAMCDV/SC4020             1   003671 003716       0     040410 040416
                                                  2     BLANK$COMMON
CONCAT/MSFC               1   003717 004070       0     040417 040440
SETIMT/SC4020             1   004071 004123       0     040441 040446
                                                  2     BLANK$COMMON
CHOLLV/SC4020             1   004124 004206       0     040447 040463
                                                  2     BLANK$COMMON
CHONLN/SC4020             1   004207 004614       0     040464 040535
                                                  2     BLANK$COMMON
CLINRV/SC4020             1   004615 005470       0     040536 040651
                          3   GGG                 2     BLANK$COMMON
CYSCLV/SC4020             1   005471 005673       0     040652 040700
                          3   GGG                 2     BLANK$COMMON
CASCLV/SC4020             1   005674 006076       0     040701 040727
                          3   GGG                 2     BLANK$COMMON
CERNLV/SC4020             1   006077 006253       0     040730 040742
```

TABLE 5. (Continued)

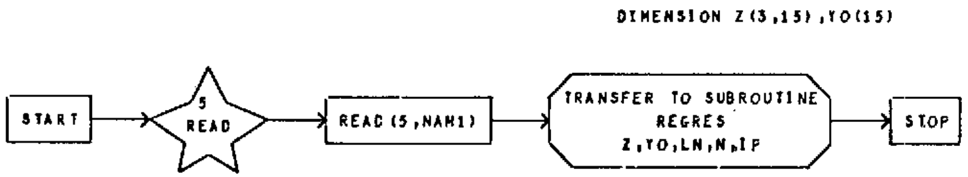
```
                                                   2    BLANK$COMMON
CERRLN/SC4020        1    006254 006363            0    040743 040760
                                                   2    BLANK$COMMON
CSETCV/SC4020        1    006364 006424            0    040761 040771
                                                   2    BLANK$COMMON
CSETMV/SC4020        1    006425 006503            0    040772 041006
                                                   2    BLANK$COMMON
CXAXIS/SC4020        1    006504 006707            0    041007 041042
                                                   2    BLANK$COMMON
VCHARV/SC4020        1    006710 007153            0    041043 041062
RITE2V/SC4020        1    007154 007404            0    041063 041111
BPLOTK/SC4020        1    007405 007465            0    041112 041471
                                                   2    BLANK$COMMON
CFRAM/SC4020         1    007466 007705            0    041472 041556
                     3    GGG                      2    BLANK$COMMON
CCAMRA/SC4020        1    007706 007757            0    041557 041570
                     3    GGG                      2    BLANK$COMMON
TABLEV/SC4020                                      0    041571 042111
NBDCV$/FOR57         1    007760 010113            2    042112 042151
NFTV$/FOR            1    010114 010136
NCLOS$/MSFC57        1    010137 010305            2    042152 042177
NMBLK$/MSFC57        1    010306 010474            0    042200 042201
NOSOL$/FOR           1    010475 010532
NRPOA$/FOR           1    010533 010566
NBFUS$/FOR                                         2    042202 044403
NCNVT$/FOR57         1    010567 011021            2    044404 044473
MINIM$/MSFC55        1    011022 011253            2    044474 044515
CPLOTV/SC4020        1    011254 011436            0    044516 044554
                     3    GGG                      2    BLANK$COMMON
CLINEV/SC4020        1    011437 012022            0    044555 044633
                                                   2    BLANK$COMMON
YSCLVI/SC4020        1    012023 012141            0    044634 044644
                     3    GGG                      2    BLANK$COMMON
XSCLVI/SC4020        1    012142 012257            0    044645 044655
                     3    GGG                      2    BLANK$COMMON
CAPLOT/SC4020        1    012260 012533            0    044656 044731
                     3    GGG                      2    BLANK$COMMON
CAPRNV/SC4020        1    012534 012616            0    044732 044750
                                                   2    BLANK$COMMON
CPRNTV/SC4020        1    012617 013210            0    044751 045005
                     3    GGG                      2    BLANK$COMMON
CGRDIV/SC4020        1    013211 014115            0    045006 045121
                     3    GGG                      2    BLANK$COMMON
COXOYV/SC4020        1    014116 014710            0    045122 045206
                                                   2    BLANK$COMMON
CBRITV/SC4020        1    014711 015012            0    045207 045222
                     3    GGG                      2    BLANK$COMMON
CMARGN/SC4020        1    015013 015071            0    045223 045264
                                                   2    BLANK$COMMON
CNBLNK/SC4020        1    015072 015141            0    045265 045300
                                                   2    BLANK$COMMON
BNOV/MSFC            1    015142 015222            0    045301 045311
                                                   2    BLANK$COMMON
TRACE                1    015223 015346            0    045312 045317
                                                   2    045320 045402
SPCFRM/MSFC                                        0    045403 045456
```

TABLE 5. (Concluded)

```
CTOBCD/MSFC                     1   015347 015645       0    045457 045630
BOXLTR/CSC                      1   015646 016500       0    045631 045647
CIDENT/SC4020                   1   016501 017653       0    045650 046033
                                3   GGG                 2    BLANK$COMMON
NOTIN$/MSFC55                   1   017654 020204       2    046034 046044
NOUT$/MSFC57                    1   020205 021166       2    046045 046076
NFMT$/MSFC57                    1   021167 022074       2    046077 046115
NIOER$/MSFC57                   1   022075 022254       2    046116 046240
NFCHK$/MSFC57                   1   022255 023073       2    046241 046415
                                                        4    046416 046467
NTAB$/MSFC55                                            2    046470 046556
NLIOP$/MSFC57                   1   023074 024640       2    046557 046770
SQRT$/FOR55                     1   024641 024700       2    046771 047002
NEXP$$/FOR57                    1   024701 024764       2    047003 047012
GGG (COMMON BLOCK)                                           047013 047146
CQUIKL/SC4020                   1   024765 025402       0    047147 047231
                                3   GGG                 2    BLANK$COMMON
IDEAT/SC4020                    1   025403 026645       0    047232 051027
MSMONITOR/MSFC55                1   026646 027755       2    051030 051614
NTEN$/FOR52                     1   027756 030040       2    051615 051744
NUBUF$/FOR51                    1   030041 030100
ERO$/MSFC55
NERR$/FOR57                     1   030101 030435       2    051745 052121
BLANK$COMMON (COMMON BLOCK)
DMATML                          1   030436 030633       0    052122 052174
                                                        2    BLANK$COMMON
INVRT                           1   030634 031245       0    052175 052376
                                                        2    BLANK$COMMON
IOUR                            1   031246 031250
ROOT                            1   031251 031652       0    052377 052525
                                                        2    BLANK$COMMON
REGRES                          1   031653 033537       0    052526 054665
                                                        2    BLANK$COMMON
MAIN                            1   033540 033560       0    054666 055012
                                                        2    BLANK$COMMON

SYS$*RLIB$. LEVEL MS7-Q
END OF COLLECTION - TIME 3.887 SECONDS

@XQT AA
```



```
                                   D I M E N S I O N E D   V A R I A B L E S

SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES    SYMBOL     STORAGES

 ADARY      22          PMAX       15          FLDX       12          FLDY       12          X          15
 Z          3,15        ZBAR       3,3         ZZBAR      2,3         SMALLZ     IDIM        SMALZ1     IDIM
 SMALZ2     IDIM        SHAT       12,12       YO         15          SVHAT      12,1        YC         15
 BZ         15          RESID      15          B          3,3         A          2,3
```

Figure 4. Operation of the computer program (cross product).

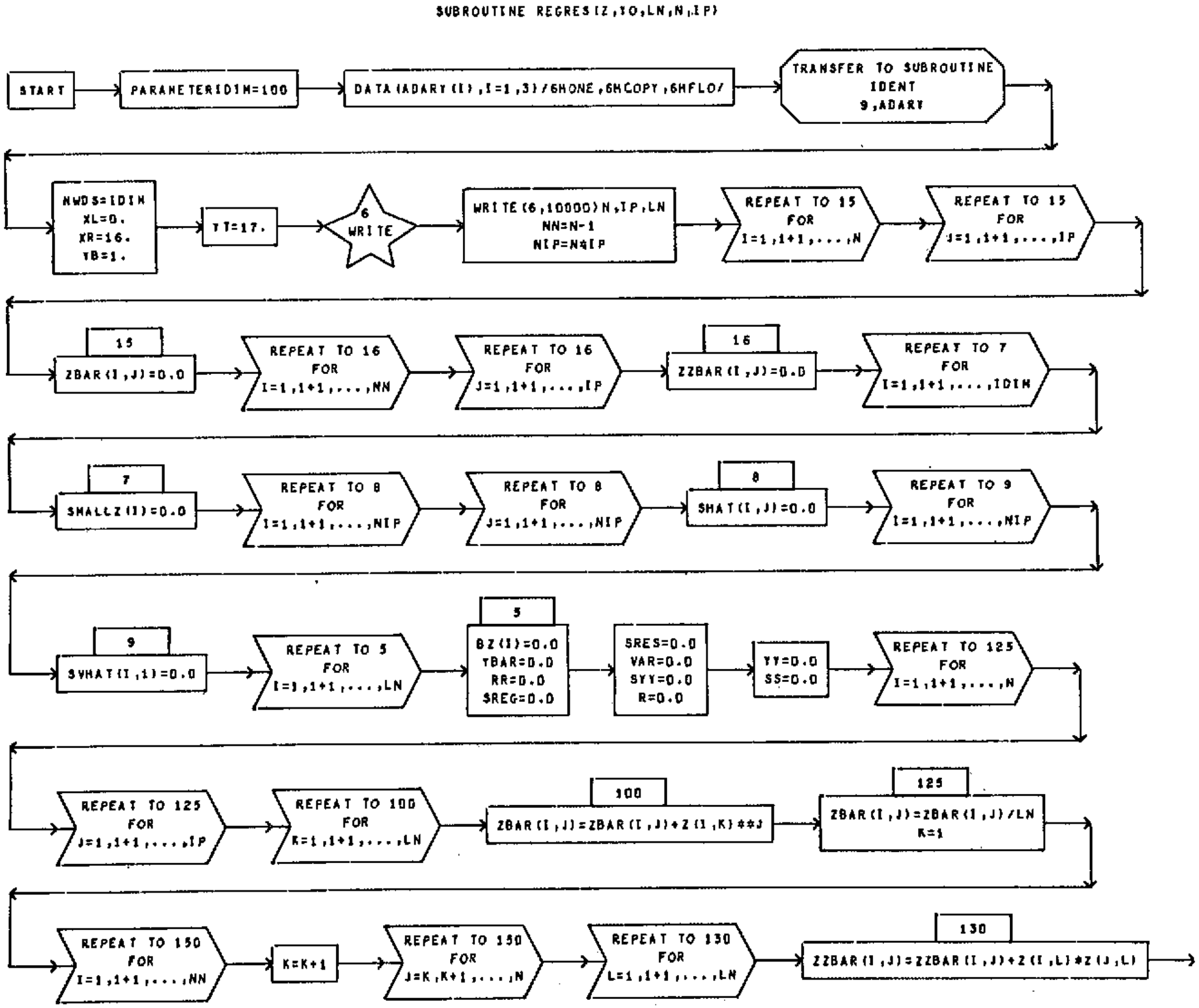


Figure 4. (Continued)

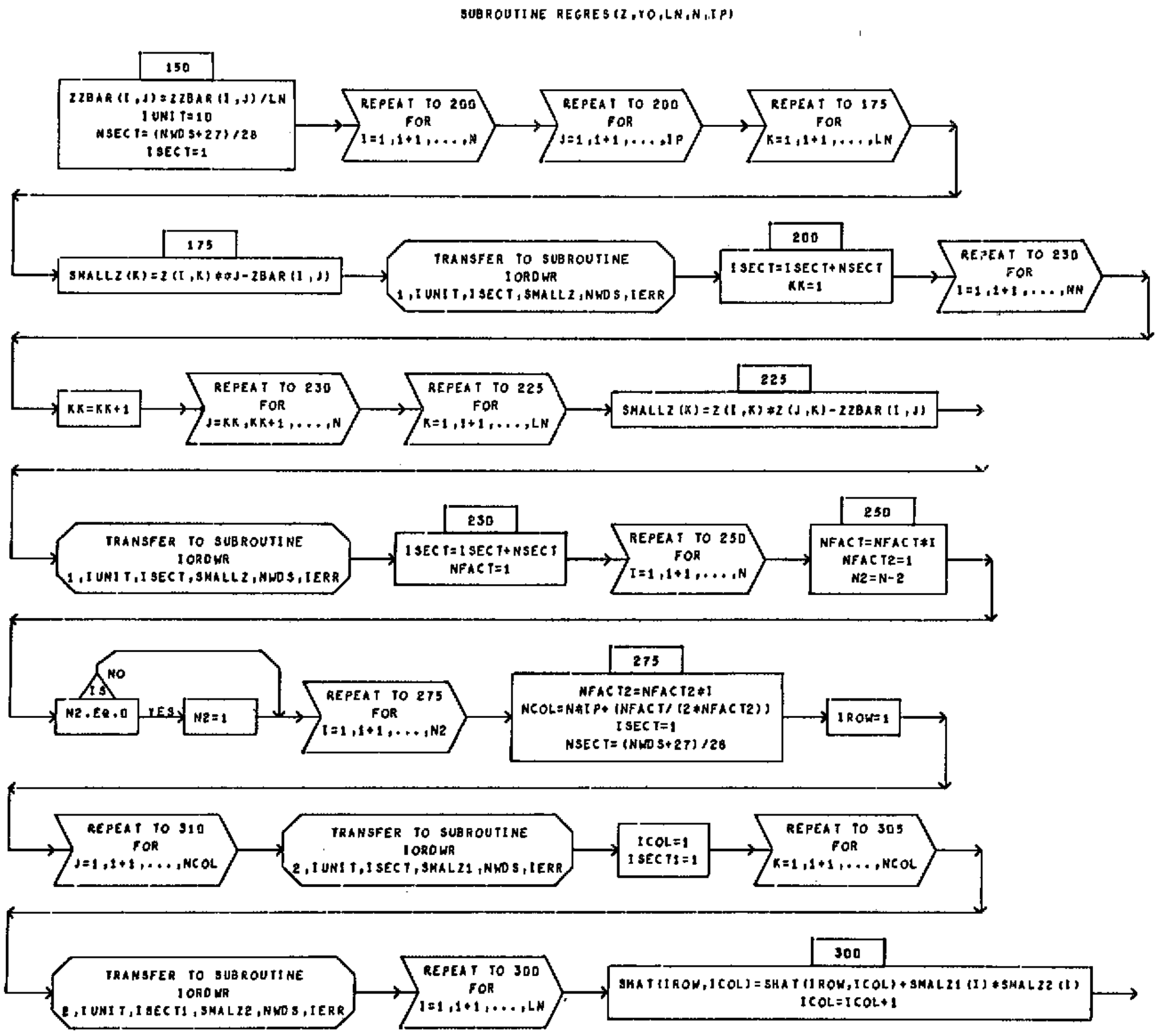


Figure 4. (Continued)

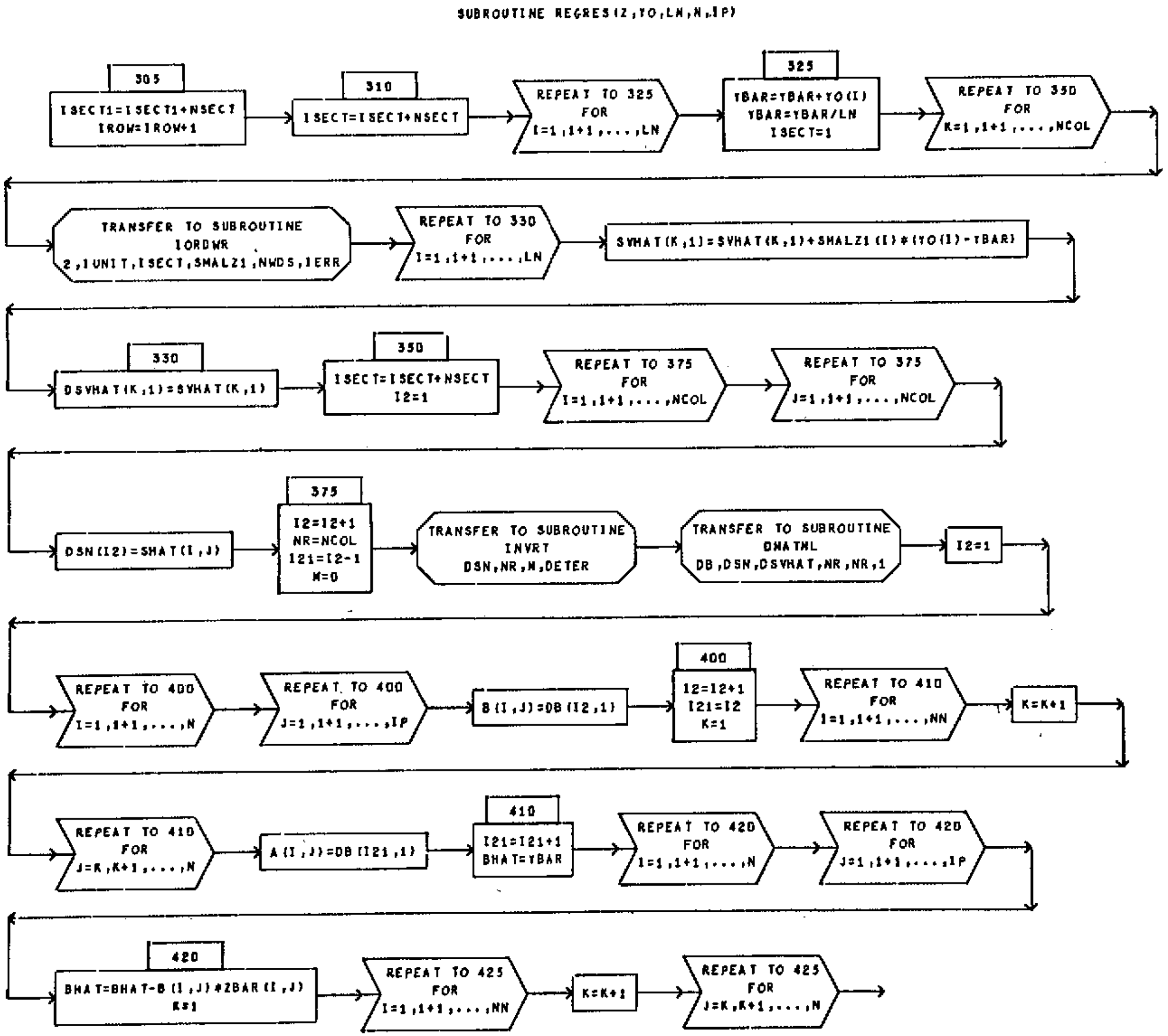


Figure 4. (Continued)

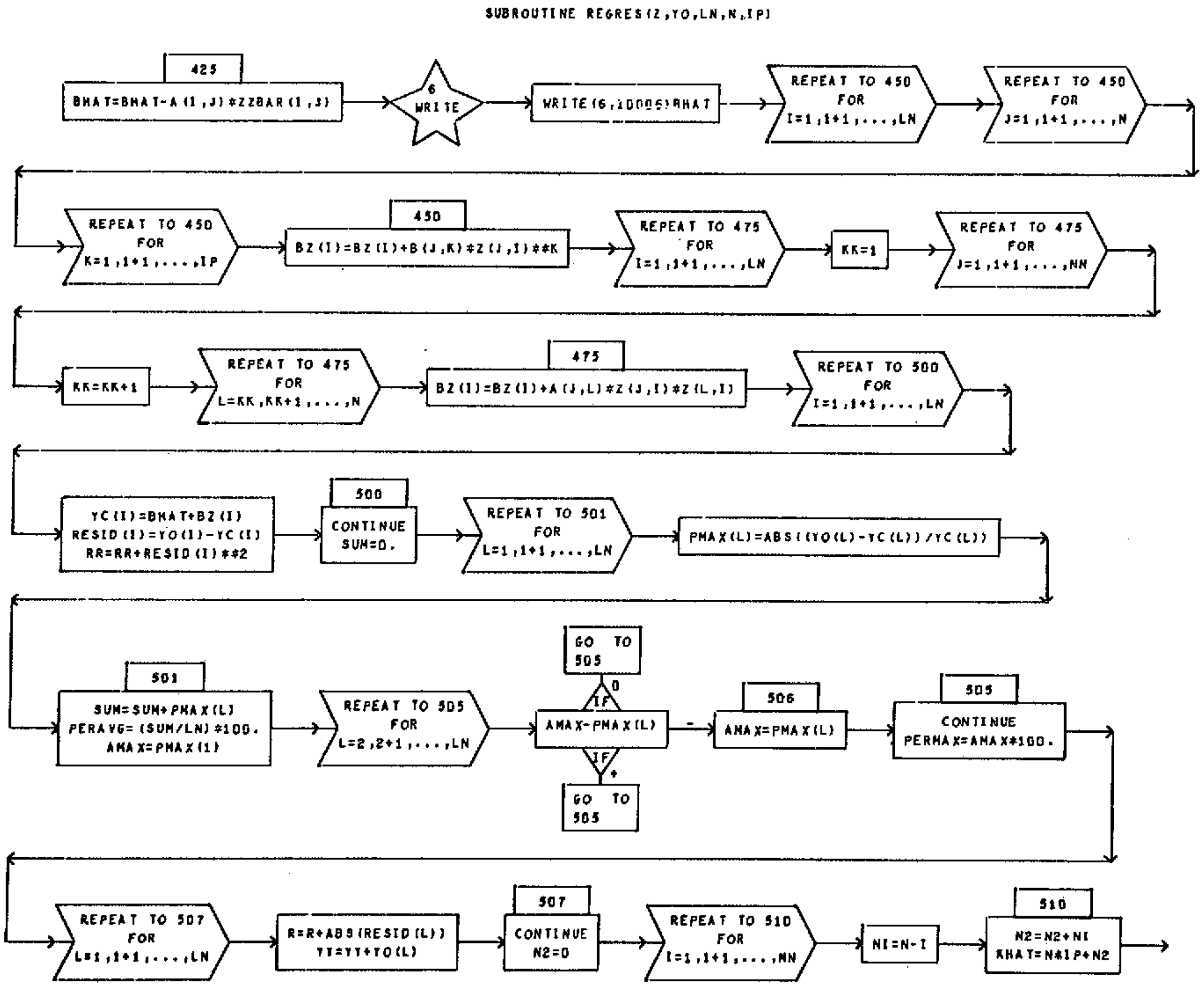


Figure 4. (Continued)

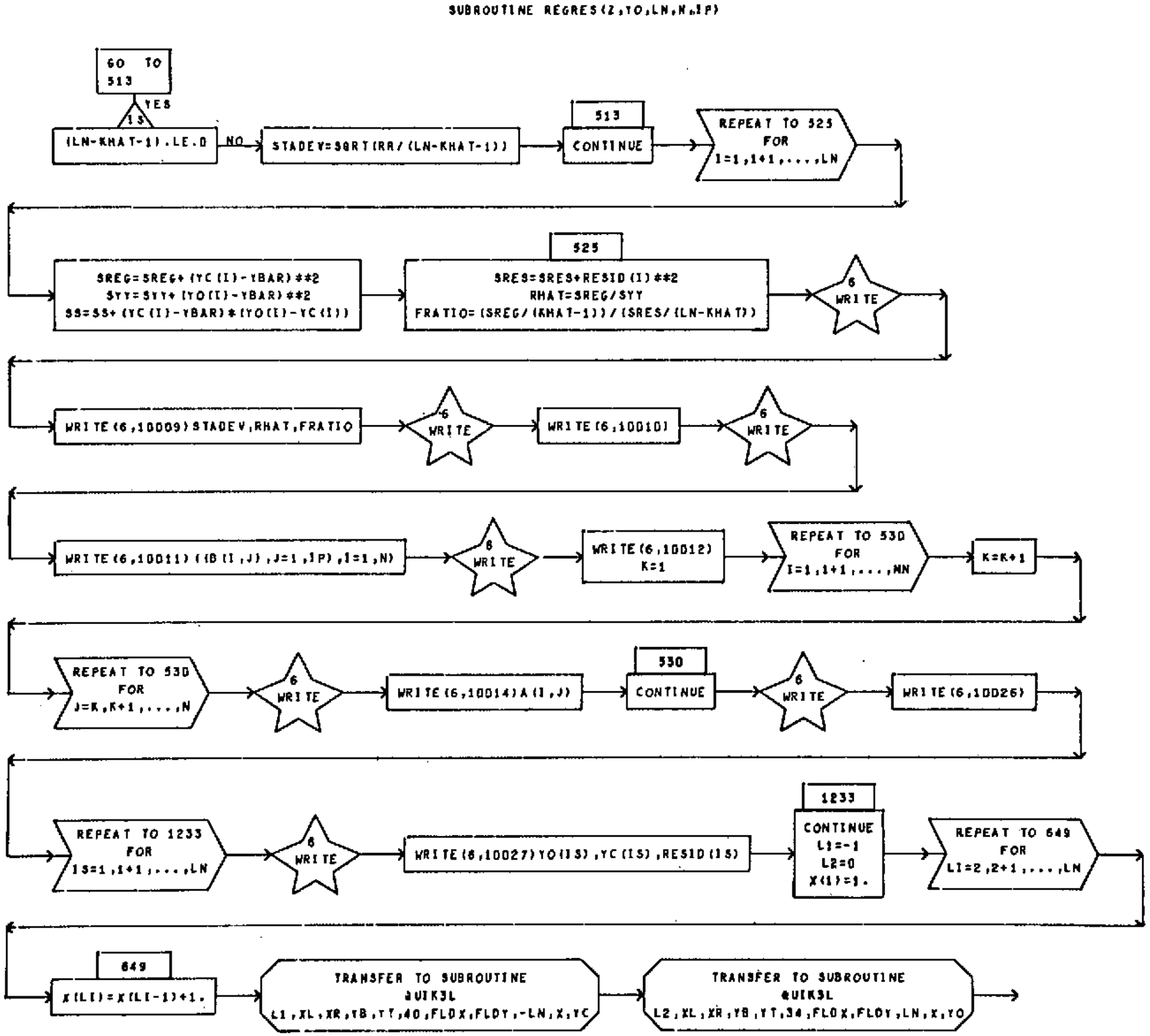
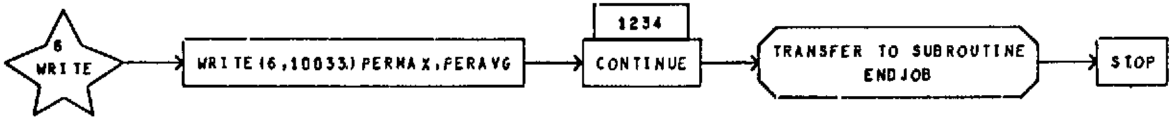


Figure 4. (Continued)

SUBROUTINE REGRES(Z,YO,LN,N,IP)



```
GENERAL FORTRAN I/O PACKAGE        ,EXEC VIII          ,APRIL,1970

READ/WRITE MAG TAPE OR DRUM IN FORTRAN OR NON-FORTRAN FORMAT.



                        D I M E N S I O N E D   V A R I A B L E S

SYMBOL       STORAGES'    SYMBOL       STORAGES     SYMBOL       STORAGES     SYMBOL       STORAGES     SYMBOL       STORAGES

 ARRAY          1          ICWD           3           IFLD          29           KODE          6          .IPACKT        6

 NTWD           3
```

Figure 4. (Continued)

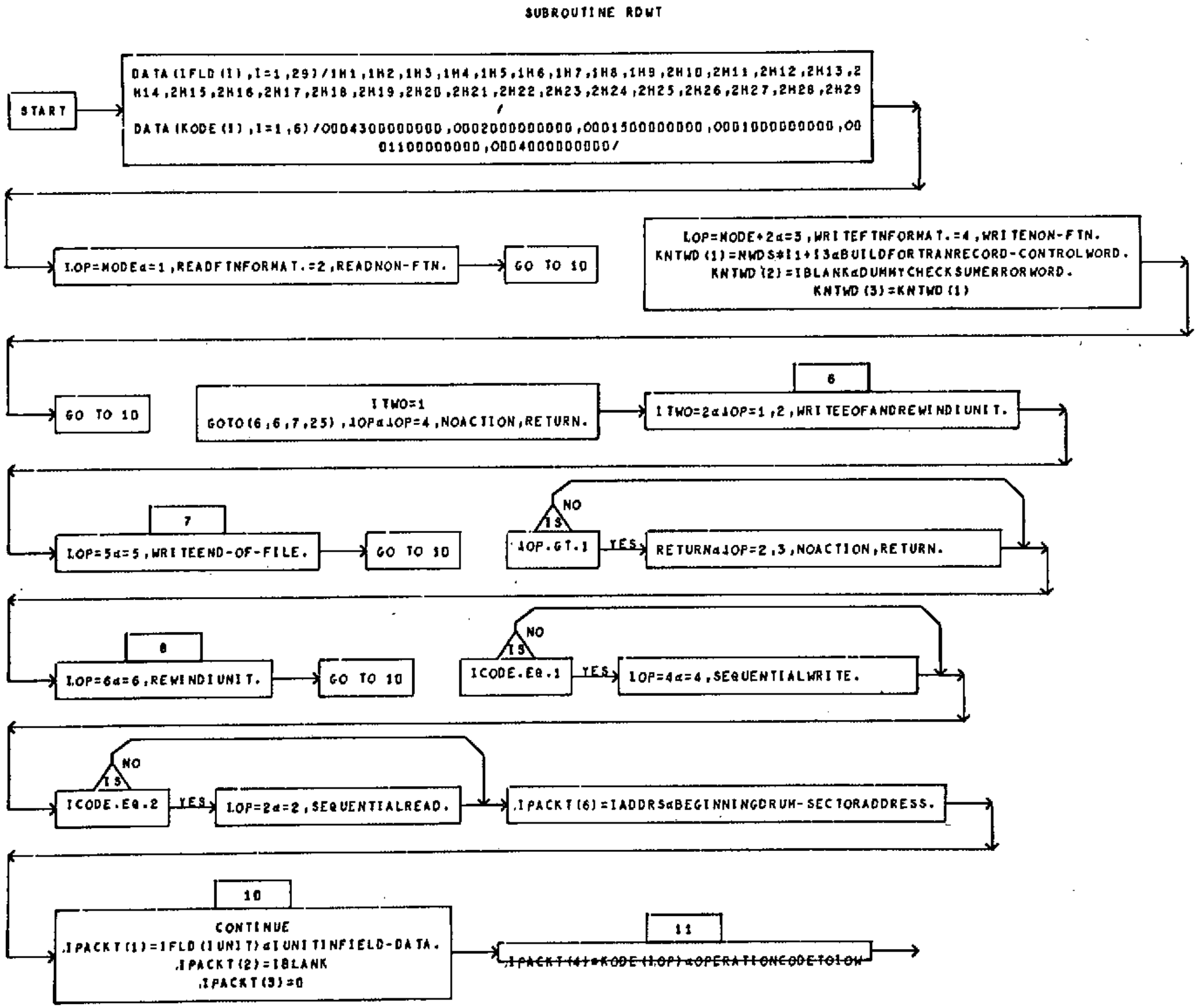


Figure 4. (Continued)

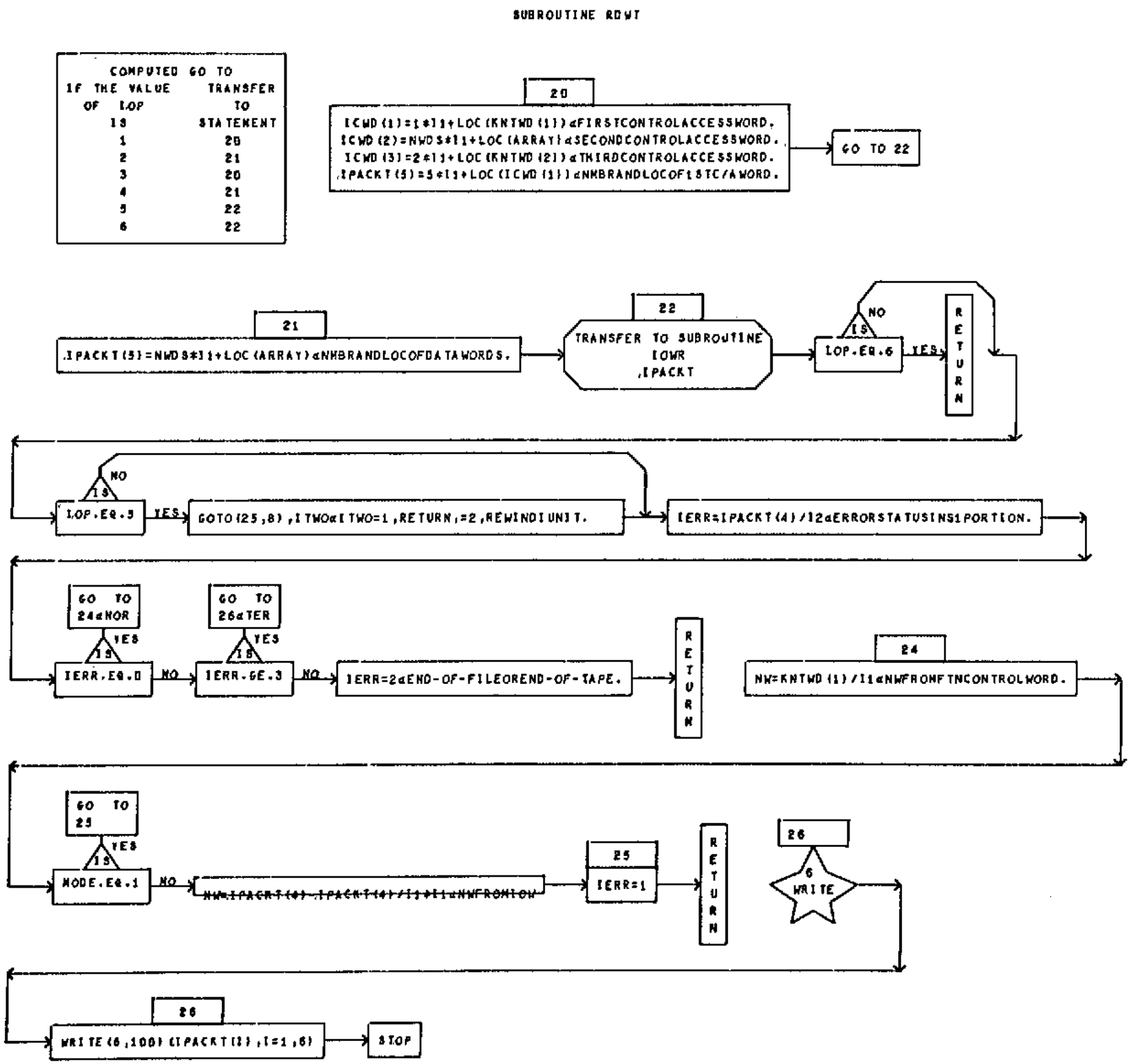


Figure 4. (Continued)

```
                             D I M E N S I O N E D   V A R I A B L E S

SYMBOL       STORAGES  SYMBOL       STORAGES  SYMBOL       STORAGES  SYMBOL       STORAGES  SYMBOL       STORAGES

IPIV         IDIM      INDEX        IDIM,2
```

Figure 4. (Continued)

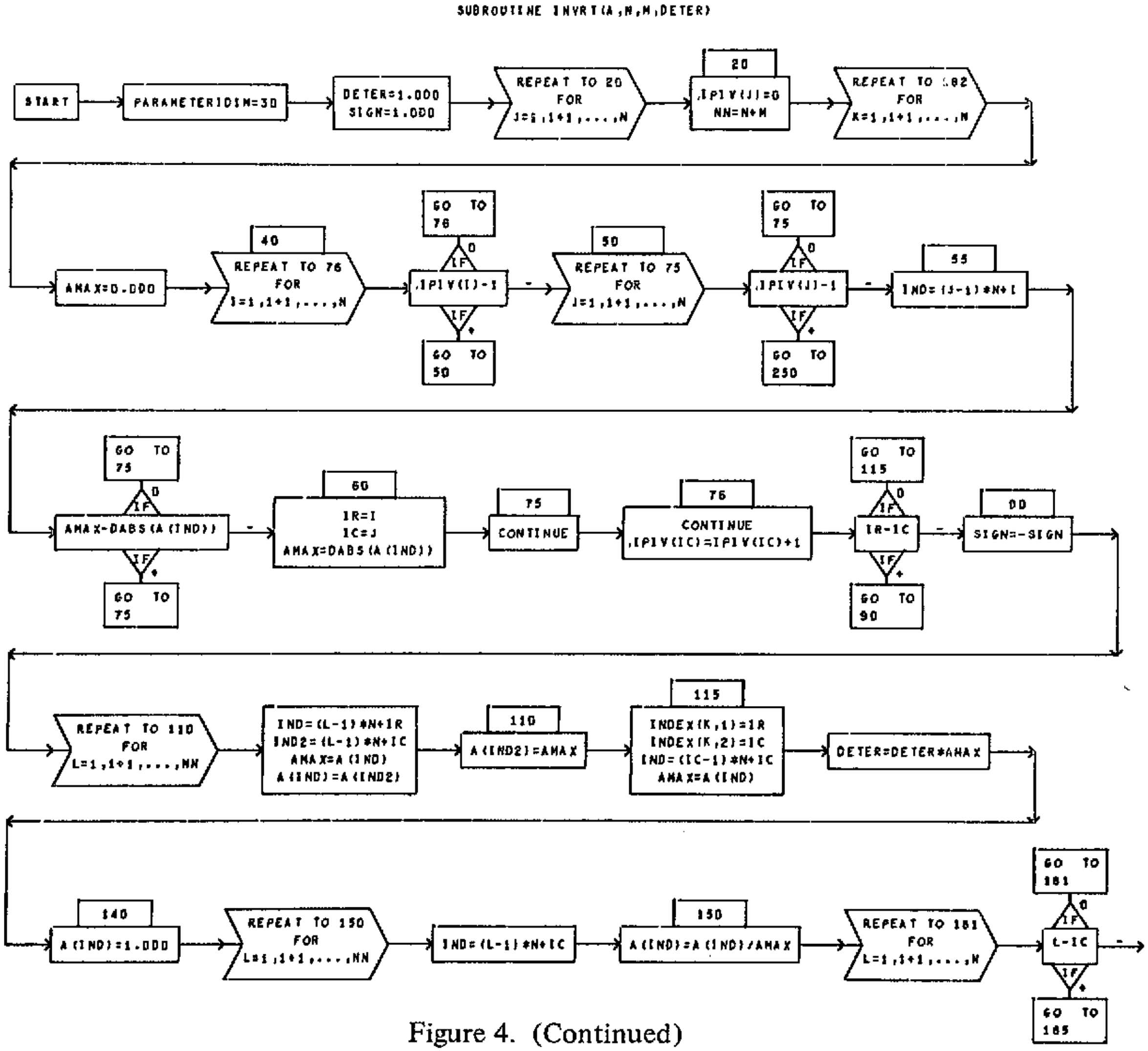


Figure 4. (Continued)

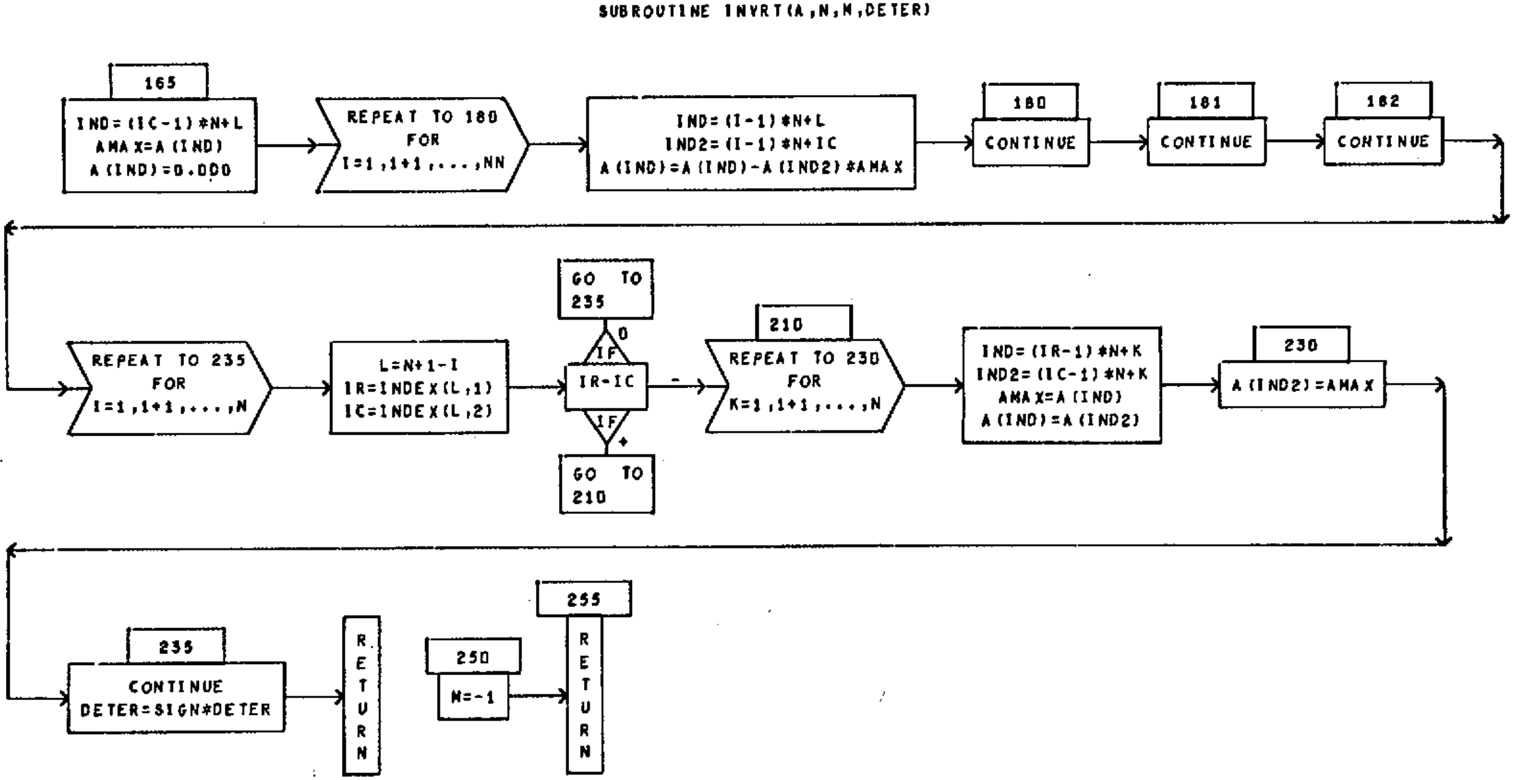


Figure 4. (Continued)

DIMENSIONED VARIABLES

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
C	1	A	1	B	1				

SUBROUTINE DMATML(C,A,B,M,N,K)

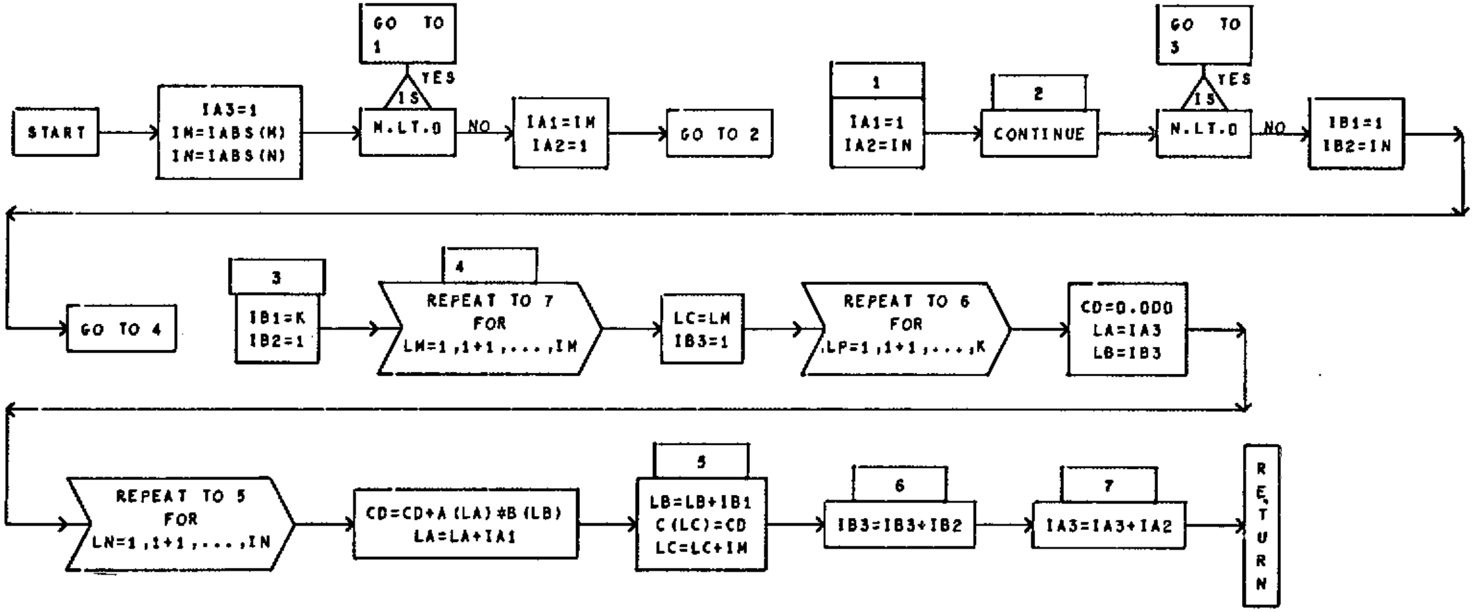


Figure 4. (Concluded)

From equation (38) the following set of input data (Table 6) was developed, containing exact dependent variable values (to three decimal places) for arbitrary values of the three independent variables. In as far as was possible the problem selected in Appendix A was duplicated for comparison purposes.

TABLE 6. INPUT DATA, DEPENDENT VARIABLE VALUES

Data Point Number	Y	X_1	X_2	X_3
1	6.759	0.5	0.4	0.4
2	8.251	0.6	0.3	0.2
3	4.256	0.2	0.6	0.6
4	4.892	0.2	0.6	0.5
5	13.165	0.9	0.0	0.5
6	9.669	0.7	0.0	0.6
7	6.070	0.3	0.9	0.7
8	12.588	0.9	0.3	0.6
9	5.749	0.5	0.6	0.7
10	8.561	0.3	0.8	0.2
11	11.333	0.2	0.9	0.0
12	10.589	0.8	0.5	0.6
13	16.117	0.9	0.8	0.3
14	3.525	0.1	0.4	0.7
15	2.333	0.0	0.5	0.8

The input for this sample program was read in through namelist NAM1 as follows:

```
Col. 2
$NAM1

    Z = 0.5, 0.4, ..., 0.8,
    YO = 6.759, 8.251, ..., 2.333,
    LN = 15,
    N = 3,
    IP = 3,
$
```

The computed results are shown in Table 7 for this example problem and the plotted results are shown in Figure 5.

TABLE 7. OUTPUT DATA LISTING FOR ILLUSTRATIVE PROBLEM (CASE WITH CROSS PRODUCTS)

```
N=   3   P=   3   LN=  15

BHAT=   .900033772+01

STANDARD DEVIATION=   .945426711-05    MULTIPLE CORRELATION COEFFICIENT=   .100000000+01    F RATIO=   .335245672+12

B COEFFICIENTS
       -.600040036+01       .699976504+01       .500027460+01      -.400052220+01       .100033142+01

        .699985909+01      -.600018770+01       .699929118+01      -.599732355+01

A COEFFICIENTS
        .500028425+01
        .500044143+01
       -.599971730+01
           OBSERVED             COMPUTED             RESIDUAL
        .675900000+01        .675899339+01        .661611557-05

        .825100005+01        .825100231+01       -.226477650-05

        .425599998+01        .425599325+01        .673532486-05

        .489200002+01        .489205279+01       -.268220901-05

        .131650000+02        .131649994+02        .596046448-06

        .966900063+01        .966900027+01       -.238418579-06

        .606999999+01        .607000053+01       -.536441803-06

        .125880001+02        .125880098+02       -.715255737-06

        .574900001+01        .574900067+01       -.655651093-06

        .856099999+01        .856100559+01       -.560283661-05

        .113329999+02        .113329972+02        .274181366-05

        .105890000+02        .105890003+02       -.357627869-06

        .161170001+02        .161169991+02        .953674316-06

        .352500001+01        .352500546+01       -.545382500-05

        .233300000+01        .233299756+01        .244379044-05

MAXIMUM PERCENT ERROR=   .158255063-03     AVERAGE PERCENT ERROR=   .486492650-04
```

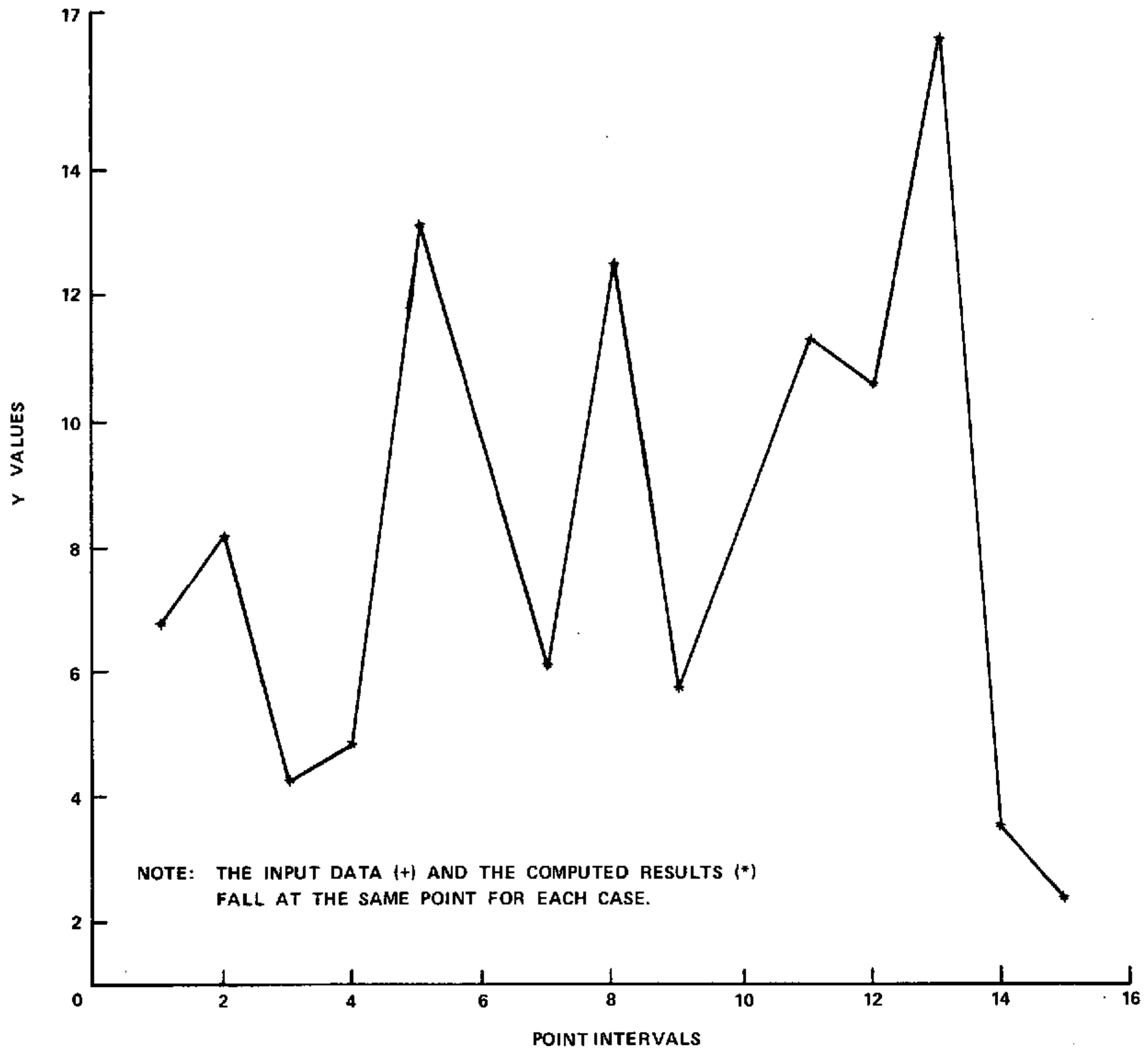


Figure 5. Comparison of input and computed values for illustrative problem with cross products.

The plot graphs the point intervals on the X axis against the exact and computed dependent variables. The exact dependent variables are plotted with (+) and the points are not connected by line segments. The computed dependent variables are plotted with (*) and the points are connected by straight line segments.

Inspection of the digital and plotted results reveals that the computed regression relation yields an almost exact representation of the input data for this arbitrary case. However, experience with several sets of physical data have generally yielded poorer results evidently due to the larger matricies being manipulated and due to unknowns as to whether true linear cross coupling exists in the physical process.

REFERENCES

1. Davis, J.W.: Optimization of Wave Cancellation in Variable Porosity Transonic Wind Tunnel Flows. NASA TN D-7432, 1973.

2. Graybill, F.A.: An Introduction to Linear Statistical Models. Volume I, McGraw-Hill, New York, 1961.

3. Smille, K.W.: An Introduction to Regression and Correlation. The Ryerson Press, New York, 1966.

4. Junkin, B.G.: Regression Analysis Procedures for the Evaluation of Tracking System Measurement Errors. NASA TN D-4826, 1968.

NASA-Langley, 1973 —— 19