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ANALYSIS (STEAP) PROGRAMS. VOLUME 2:
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SPACE TRAJECTORIES ERROR ANALYSIS (STEAP) PROGRAMS

Volume II - Programmers' Manual (Update)

December 1971

MARTIN MARIETTA CORPORATION DENVER DIVISION Denver, Colorado 80201

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SPACE TRAJECTORIES ERROR ANALYSIS PROGRAMS VERSION II

VOLUME II: PROGRAMMER'S MANUAL

March 1971 Space Navigation Technology Martin-Marietta Corporation Denver, Colorado

Volume II of Three Volumes

Final Report

Contract NAS 5-11795

Computer Program for Mission Analysis of Lunar and Interplanetary Missions

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Acknowledgements

The authors wish to acknowledge the invaluable support of several persons whose help was instrumental in the completion of this contract. Dr. Al Bradt, serving as Program Manager, efficiently solved the many administrative problems that arose during the course of the contract. Joanne Spofford did her usual excellent job of typing the voluminous documents. Deborah Bower was responsible for all the graphics work involved with the production of the flowcharts and mathematical analyses. A utility computer program developed by Jim Schnelker aided greatly in the program conversion from CDC single precision to IEM double precision. All of their work was greatly appreciated.

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FOREWORD

STEAP II is a series of three computer programs developed by the Martin Marietta Corporation for the mathematical analysis of interplanetary or lunar navigation and guidance. STEAP is an acronym for Space Trajectory Error Analysis Programs. The first series of programs under this name was developed under Contract NAS1-9745 for Langley Research Center and was documented in two volumes (STEAP Users' Manual, STEAP Analytical Manual) as NASA Contract Report 66818. Under contracts NAS5-11795 and NAS5-11873, the STEAP series was extensively modified and expanded for Goddard Space Flight Center. This second-generation series of programs is referred to as STEAP II.

STEAP II is composed of three independent yet related programs: NOMNAL, ERRAN, and SIMUL. All three programs require the integration on n-body trajectories for both interplanetary and lunar missions. The virtual-mass technique is the scheme used for this purpose in all three programs.

The first program named NOMNAL is responsible for the generation of n-body nominal trajectories (either lunar or interplanetary) performing a number of deterministic guidance events. These events include initial or injection targeting, midcourse retargeting, orbit insertion, and miniprobe targeting. A Variety of target parameters are available for the targeting events. The actual targeting is done iteratively either by a modified Newton-Raphson algorithm or by a steepest-descent/conjugate gradient scheme. Planar and non-planar strategies are available for the orbit insertion computation. All maneuvers may be executed either by a simple impulsive model or by a pulsing sequence model.

ERRAN, the second program of STEAP II, is used to conduct linear error analysis and generalized covariance analysis studies along specific targeted trajectories. The targeted trajectory may, however, be altered during flight by retargeting events (computed either by linear or nonlinear guidance) and by an orbit insertion event. Knowledge and control covariances are propagated along the trajectory through a series of measurements and guidance events in a totally integrated fashion. The knowledge covariance is processed through measurements using a Kalman-Schmidt or equivalent recursive weighted-least-squares filter with arbitrary solve-for/consider augmentation. Execution of guidance events may be modeled either by an impulsive approximation or by a pulsing sequence model. The resulting knowledge and control covariances can be analyzed by the program at various events to determine statistical data, including

probabilistic midcourse correction sizing and effectiveness, probability of impact, and biased aimpoint requirements. Probe release events are also available for studying missions employing multiprobe spacecraft.

The third and final program in the STEEP II series is the simulation program SIMUL. SIMUL is responsible for the testing of the mathematical models used in the navigation and guidance process. An "actual" dynamic model is used to propagate an "actual" trajectory. Noisy measurements from this "actual" trajectory are then sent to the estimation algorithm. Here the actual measurement, the statistics associated with that measurement, and an "assumed" dynamical model are blended together to generate the filter estimate of the trajectory state. This process is repeated continually through the measurement schedule. At guidance events, corrections are computed based on the estimate of the current state. These corrections are then corrupted by execution errors and added to the "actual" trajectory. The statistics and augmentation of the filter, the mismatches in the "actual" and "assumed" dynamics, and the execution errors and measurement biases may then be varied to determine the effects of these parameters on the navigation and guidance process. All guidance and probe release event options defined for ERRAN are also available in SIMUL.

The documentation for STEAP II consists of three volumes: the Analytic, Programmers' and User's Manuals. Each of these documents is self-contained.

The STEAP Analytic Manual consists of two major divisions. The first section provides a unified treatment of the mathematical analysis of the STEAP II programs. The general problem description, formulation, and solution are given in a tutorial manner. The second section of this report supplies the detailed analysis of those subroutines of STEAP II dealing with technical tasks.

The STEAP Programmers' Manual provides the reader with the information he needs to modify the programs. Both the overall structure of the programs as well as the computational flow and analysis of the individual subroutines is described in this manual.

The *Users' Manual* contains the information necessary to operate the programs. The input and output quantities of the programs are described in detail. Example cases are also given and discussed.

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1. INTRODUCTION

This Programmer's Manual is intended to supply the reader with sufficient information about the STEAP II programs to enable him to efficiently modify them. Both the overall structure of the programs and the computational flow of the individual subroutines are described in this manual.

. This section describes the contents of the Programmer's Manual. Following this discussion the nomenclature used throughout the report is presented.

The third section of this manual describes the four basic components of STEAP II: the n-body trajectory propagation package, the nominal trajectory generator NOMNAL, the error analysis program ERRAN, and the simulation program SIMUL. The general purpose and capability of each of the programs is briefly summarized.

Chapter 4 of this volume examines the STEAP II programs from a more detailed viewpoint. The operational structure of each of the main components is described at the subroutine level. The individual subroutines are defined and cross-referenced according to the three main programs of STEAP II.

Chapter 5 contains the definitions of the variables appearing in common blocks throughout the programs. The variables are first listed according to the common blocks to which they belong. The programs requiring each of these common blocks are also noted. Following this all the common variables are listed in alphabetical order with their common blocks referenced. Tables detailing the definitions of large, frequently referenced common arrays are also provided.

Chapter 6 comprises the bulk of this volume. Each of the subroutines is documented in detail in alphabetical order. The purpose
of the subroutine is supplied. Subroutines supported or required
by the subroutine are listed. Arguments and interval variables of
the subroutine are defined and usage of common variables is noted.
The mathematical analysis upon which the subroutine is based is then
discussed in full. Finally a flow chart of the computational flow of
the subroutine is provided.

2. NOMENCLATURE

A. Arabic symbols

Arabic sym	DOIR
Symbol	Definition
а	Semi-major axis of conic
В•Т	Impact plane parameter
B·R	Impact plane parameter
$\mathbf{c_{xx}_{s}}$	Correlation between position/velocity state and solve-for parameters
C _{xu}	Correlation between position/velocity state and dynamic consider parameters
c_{xv}	Correlation between position/velocity state and measure- ment consider parameters
c ×su	Correlation between solve-for parameters and dynamic consider parameters
c _{xs} v	Correlation between solve-for parameters and measurement consider parameters
e	Eccentricity of conic
E	Eccentric anomaly
f	True anomaly on conic
G	Observation matrix relating observables to dynamic consider parameter state
Н	Observation matrix relating observables to position/velocity state
i	Inclination of conic (reference body equatorial)
J	Measurement residual covariance matrix
K	Kalman gain constant for position/velocity state
L	Observation matrix relating observables to measurement consider parameter state Mean longitude
М	Observation matrix relating observables to solve-for parameter state Mean anomaly

n 1	Dimension of solve-for parameter state
n ₂	Dimension of dynamic consider parameter state
n ₃	Dimension of measurement consider parameter state
P	Semilatus rectum of conic Probability density function
P	Position/velocity covariance matrix
P	Unit vector to periapsis of conic
Ps	Solve-for parameter covariance matrix
Q	Dynamic noise covariance matrix
õ	Execution error matrix
Ŷ	Unit vector in plane of motion normal to P
r	Radius
r _{CA}	Radius of closest approach
r _{SI}	Radius of sphere of influence
R	Measurement noise covariance matrix
<u>R</u>	Actual noise covariance matrix
R	Unit vector normal to T in plane perpendicular to approach asymptote directed south ($R = S \times T$)
R _c	Target planet capture radius
S	Kalman gain constant for solve-for parameters
s j	Velocity correction covariance matrix
ŝ	Approach or departure asymptote
t CA	Time of closest approach to target body
t _{sI}	Time of intersection with sphere of influence of target body
Δt	Time interval
Ŷ	Unit vector lying in ecliptic plane normal to \hat{S} . $(\hat{T} = \frac{\hat{S} \times \hat{K}}{ \hat{S} \times \hat{K} } \text{ where } \hat{K} \text{ is unit normal to ecliptic plane.})$
uò	Dynamic consider parameter covariance matrix

v	Velocity
V	Measurement consider parameter covariance matrix
W j	Target parameter covariance matrix
พิ	Unit normal to orbital plane
Χ̈́	Actual position/velocity state
x	Targeted nominal position/velocity state
x .	Most recent nominal position/velocity state
B. Greek Symbol	.s
α	Auxiliary parameters
$\Gamma_{f j}$	Guidance matrix
Г	Flight path angle
δ	Declination of vector
Δv	Velocity increment
€	Measurement residual Errors in target paramèters
ŋ	Variation matrix relating position/velocity variations to target conditions
$\theta_{\mathbf{x}\mathbf{x}_{\mathbf{s}}}$	State transition matrix partition associated with solve-for parameters
$^{ heta}_{ ext{xu}}$	State transition matrix partition associated with dynamic consider parameters
. 0	Longitude or right ascension
Λj	Projection of target condition covariance matrix W into the impact plane ,
μ	Gravitational constant of body
$\overrightarrow{\mu}$	Biased aimpoint
u	Sampled measurement noise True anomaly

```
Correlation coefficient
       σ
                  Standard deviation
      Σ
                  Launch azimuth
      7
                   Target parameters
                   Targeting matrix
                   State transition matrix for position/velocity state
                   Latitude
       χ
                   Sensitivity matrix
                  Matrix relating guidance corrections to target condition
                     deviations
      \Omega
                  Longitude of ascending node
       \omega
                  Argument of periapsis
       \approx
                   Longitude of periapsis
C. Subscripts
       C
                  Control variable (Pg)
                  Closest approach (r_{CA})
       CA
       f
                  Final variable (t<sub>f</sub>)
                   Initial variable (t;)
       í
                   Index of current guidance event (P.)
       J
                   Index of current measurement (P_{l})
       k
                  Knowledge variable (P_v)
       K
       s
                  Solve-for parameter (x<sub>s</sub>)
        SI
                   Sphere of influence (tsT)
D. Superscripts
                  Augmented variable (\Phi^A)
       Α
                  Matrix transpose (\Phi^{T})
       T
                  Matrix inverse (\Phi^{-1})
       -1
```

Magnitude of gaussian approximation for midcourse correction

ρ

- Variable immediately before instant (P_{L}^{-} or v^{-})
- + Variable immediately after instant (P_k^+) or v^+)

E. Abbreviations

AU Astronomical unit

CA Closest approach to reference body

ERRAN Error analysis program

FTA Fixed time of arrival guidance policy

GHA Greenwich hour angle

J.D. Julian date (referenced either 0 yr or 1900 yr)

km Kilometers

M/C Midcourse correction

NOMNAL Nominal trajectory generation program

POI Probability of impact

Q-L Quasilinear filter event

S/C Spacecraft

SF/C Solve-for/consider

SIMUL Simulation program

SOI Sphere of influence

STM State transition matrix

STEAP Space Trajectories Error Analysis Programs

VM Virtual Mass

2VBP Two variable B-plane guidance policy

3VBP Three variable B-plane guidance policy

3. SUMMARY OF MODES

The Space Trajectory Error Analysis Programs (STEAP) consist of four subprograms or operational modes. The first mode, used as a subroutine by each of the other three programs, is the trajectory mode VMP by which an n-body trajectory (lunar or interplanetary) is propagated by the virtual mass technique. The second mode is the nominal trajectory generator or targeter (NOMNAL) by which a lunar or interplanetary trajectory meeting specified conditions is determined. The third mode is the error analysis program ERRAN in which the navigation and guidance characteristics of a nominal trajectory are analyzed by linearly propagating knowledge and control covariances along the trajectory. Finally the simulation mode SIMUL tests the mathematical models used in the navigation and guidance processes by modeling the tracking and correction of an "actual" trajectory. In this chapter a general description of each of these modes will be provided.

3.1 The Virtual Mass Propagator VMP

The dynamic model used by STEAP is supplied by the trajectory propagation package. The only external forces acting upon the spacecraft are assumed to be the gravitational forces of the celestial bodies considered in the integration. Both the spacecraft and the gravitational bodies are assumed to be point masses so neither spacecraft attitude nor planet asphericities are considered.

The celestial bodies to be in the integration are specified by the user and may include the sun, any of the nine planets, and the earth's moon. The motion of the planets about the sun and the moon about the earth are modeled by using mean ecliptic elements of date. If the user desires, each of the planets can be set in a fixed ellipse referenced to some epoch for speedier computation.

The coordinate system used in the integration is also specified by the user. The options available are either heliocentric ecliptic or barycentric ecliptic (nominally for lunar trajectories).

The actual scheme used in the propagation of the trajectory in the virtual mass or varicentric technique (see reference 15). No actual integration is performed by the trajectory mode; che key idea of the virtual mass technique is to build up an n-body trajectory by using a sequence of conic sections around a moving effective force center called the virtual mass. At each instantaneous moment along the trajectory, the combined effects of all the gravitational bodies can be viewed as resulting from a fictitious body of unique magnitude and position which is called the virtual mass. The computational pro-

cedure then assumes that over a small time interval, the motion of the spacecraft can be represented by a two-body conic section arc relative to this virtual mass. The complete trajectory is thus generated by a series of small arcs pieced together in steps while updating the position and magnitude of the effective force center. The main advantage of the virtual-mass technique is that the tedious numerical integration of the differential equations is avoided.

Another significant feature of the virtual-mass technique is its flexibility. By varying a simple parameter called the "accuracy level" related to the true anomaly increment of each step, trajectories ranging from a sequence of relatively few conic section arcs corresponding to a very approximate solution to those requiring a large number of arcs corresponding to highly accurate solutions may be generated.

3.2 The Nominal Trajectory Targeter NOMNAL

NOMNAL is responsible for the generation of a nominal trajectory for either lunar or interplanetary missions. The method or propagation in either case is the virtual-mass n-body integrator. The trajectory may be processed through a series of deterministic maneuvers including initial or injection targeting, subsequent retargeting, miniprobe targeting, and orbit insertion. A variety of target parameters are available for the targeting events. Both coplanar and nonplanar strategies are permitted in the orbit insertion maneuver.

If an initial state for the problem is known, this may be read in to start the trajectory. Otherwise NOMNAL generates its own zero iterate. In interplanetary missions this involves solving the Lambert time-of-flight equation for the massless planet trajectory that connects the desired initial and final positions in the specified time interval. Four options are available in describing these reference points.

Initial Point	Final Point
Launch Planet Launch Planet Specified Point Specified Point	Target planet Specified Point Target Planet Specified Point

If the initial point is referenced to the launch planet, a launch profile is consulted to generate a realistic set of injection conditions consistent with the heliocentric trajectory.

For lunar trajectories a slightly different procedure is used. The required data for the lunar zero iterate includes specification of the desired semimajor axis with respect to the moon, radius and time of closest approach to the moon, and inclination to the lunar equator. Then the zero iterate is generated by first targeting a patched conic trajectory and then a multiconic trajectory to the desired conditions.

A targeting event may be processed immediately after obtaining a zero iterate state or at any point along the nominal trajectory. At a targeting event the current velocity is refined to yield a trajectory satisfying target parameter constraints. The possible target parameter are:

1)	TPS	5)	B • T	9)	SMA (Lunar)	13)	DCP
2)	TSI	6)	B•R	10)	XF	14)	RAP
3)	TCS	7)	RCA	11)	YF	15)	TPR
4)	TCA	8)	INC	12)	ZF		

The targeting method to be used is specified by the user. Either a modified Newton-Raphson algorithm or a steepest descent/conjugate gradient technique may be used.

Orbit insertion events are also available in NOMNAL. At a specified time the spacecraft state relative to the target body is computed. The resulting conic trajectory relative to the target body is then compared with the desired orbit to determine the optimal time to make the insertion and the required correction. At the proper time the velocity correction is then implemented. Two strategies are permitted in the orbit insertion computation:

- Coplanar The desired semimajor axis, eccentricity, and periapsis shift of a coplanar orbit are specified;
- 2) Nonplanar The desired plane of the postinsertion state is specified along with nominal values of the orbit elements.

The targeted correction, orbit insertion correction, or an externally supplied correction may be executed if desired. Two models are available for this implementation—a simple impulsive addition or a more complex multiple pulse model.

NOMNAL is also capable of targeting a set of three miniprobes to three specified target sites. Since achieving impacts at three specified points on the planet surface constitutes a six-degree-of-freedom constraint while only four miniprobe release controls are available, any targeting process can, at most, achieve a minimum-miss solution. NOMNAL uses as its miss-index a weighted sum of the squares of the distances between the respective actual and desired B-plane asymptote pierce points. The weighting factors, which are supplied by the user, indicate the relative importance of securing nearby impacts at the respective target sites. NOMNAL computes its weighted least-squares solution by a hybrid pseudo-inverse and steepest-descent algorithm. The initial control iterate is constructed by approximately targeting the first miniprobe to one of the target sites using a single Newton-Raphson step.

Finally the program integrates and records all segments of the nominal trajectory between guidance events from injection at the launch planet until the appropriate termination condition input by the user. For a conglomerate vehicle NOMNAL records the separate branches of the trajectory belonging to the main probe and miniprobes as well as to the bus.

3.3 The Error Analysis/Generalized Covariance Analysis Program

The error analysis/generalized covariance analysis program
• ERRAN is a preflight mission analysis tool that is used to determine how selected error sources influence the orbit determination process for interplanetary or lunar missions.

In the error analysis mode, ERRAN provides three primary quantitative results: (1) knowledge covariance matrices, which provide a measure of how well the actual trajectory is known, (2) control covariance matrices, which when propagated forward to the target provide a measure of how well the nominal target conditions will be satisfied by the actual trajectory, and (3) statistical midcourse ΔVs , which provide a measure of the amount of fuel required for a successful mission.

In the generalized covariance analysis mode, ERRAN provides all of the above information plus corresponding "actual" statistical information. The three results discussed in the previous paragraph are all computed on the basis of statistical distributions assumed by the navigation filter to describe the significant error sources. In the generalized covariance analysis mode, "actual" knowledge covariances, control convariances, and statistical midcourse AVs are computed on the basis of statistical distributions that actually describe both error sources acknowledged by the navigation filter and the error sources ignored. The primary use of the generalized covariance analysis program is to study the sensitivity of filter performance to off-design conditions.

ERRAN allows for employing gain generators for user-specified linear recursive navigation filters. Two gain generators are currently available in ERRAN: (1) Kalman-Schmidt filter, and (2) equivalent recursive consider mode weighted-least-squares filter.

State transition matrices are required to propagate covariance matrices over an arbitrary interval of time. Three methods are available for computing the 6x6 position/velocity state transition matrix. The first two methods, which are analytical methods, are analytical patched conic and analytical virtual mass. The third method uses numerical differencing to compute the state transition matrix. To increase the accuracy of the analytical techniques over long intervals, a state transition matrix cascading option is also available. Augmented parameter state transition matrices are always computed using numerical differencing.

Up to 23 dynamic and measurement parameters may be solved-for or considered by the navigation filter. Parameters not acknowledged in design of the filter may be treated as ignore parameters when ERRAN is run in the generalized covariance analysis mode. The dynamic parameters include biases in the gravitational constants of the sun and the target planet and biases in the six orbital elements of the target planet. Measurement biases include biases in the locations of the three earth-based tracking stations, and biases in all measurements. Available measurement types are range, rangerate, star-planet angles, and apparent planet diameter measurements. Measurement noise for each measurement type is assumed to be constant.

The computational procedure in ERRAN is divided into basic cycle computations and event computations. Basic cycle computations are concerned with the propagation of covariances forward to a measurement time and processing the measurement. Events refer to a set of specialized computation, not directly concerned with measurement processing, that can be scheduled to occur at arbitrary times along the trajectory.

The four events available in ERRAN are eigenvector, prediction, guidance, and probe release. At an eigenvector event the position and velocity partitions of the knowledge covariance matrix are diagonalized to reveal geometric information about the size and orientation of the position and velocity_navigation uncertainties. Associated hyperellipsoids are also computed. At a prediction event the most recent covariance matrix is propagated forward to some critical trajectory time to determine predicted navigation uncertainties in the absence of further measurements.

The guidance event is the most complex event and yields much useful information for preflight mission analysis. Several types of guidance events are available in ERRAN. At a midcourse guidance event the user can choose from three midcourse guidance policies. The midcourse guidance event can also be constrainted to satisfy planetary quarantine requirements. At an orbital insertion guidance event the user can choose from two insertion policies. Options are also available for changing target conditions in midflight and retargeting the trajectory using nonlinear techniques, or for simply applying an externally supplied or precomputed AV at some arbitrary trajectory time. Two thrust models are available--impulse and impulse series. Execution error statistics are generated using an error model defined by a proportionality error, a resolution error, and two pointing angle errors. At a midcourse guidance event in ERRAN we also compute a statistical ΔV and the target condition covariance matrix both before and after the midcourse correction.

Probe release events provide the capability to study missions employing multiprobe spacecraft. The multiprobe spacecraft is modeled as (1) a primary vehicle, or bus, with thrusting capability, (2) a main probe, with no thrusting capability, and (3) three miniprobes located symmetrically on booms attached to the bus, with no thrusting capability, and released simultaneously with ΔVs provided by spinning the bus. Probe release events currently operate only in the error analysis mode of ERRAN. All measurement types and solve-for or consider parameters described previously are defined for all probes. Separate measurement schedules can be defined for the bus and the main probe. An additional measurement schedule can also be defined for all three miniprobes. Knowledge and control covariances are propagated for each probe in sequential fashion.

3.4 The Simulation Program SIMUL

The simulation program SIMUL is the most complex program in the STEAP set of programs. In SIMUL the validity of the navigation and guidance process is examined by simulating an actual mission. Spacecraft state estimates are generated in SIMUL, as well as knowledge covariance matrices. The results given by the error analysis program ERRAN become meaningful only when SIMUL shows that the estimated spacecraft trajectory converges, within reasonable bounds specified by the covariance matrix, to the simulated actual trajectory.

All state transition matrix, parameter augmentation, and measurement options described in section 3.3 are also available in SIMUL. As in ERRAN, the computational procedure in STMUL is divided into basic cycle computations. The SIMUL basic cycle is concerned with the generation of state estimates and an actual trajectory, together with all quantities generated in the ERRAN error analysis basic cycle. Eigenvector and prediction events in SIMUL involve all computations performed in the corresponding ERRAN events. In addition, the SIMUL prediction event propagates state estimates forward to the time to which we are predicting.

All options available in the ERRAN guidance event (see section 3.3) are also available in the SIMUL guidance event. The treatment of the midcourse guidance event, however, is different in several respects. First, since an estimated spacecraft state is generated in SIMUL, an actual midcourse ΔV can be computed rather than a statistical ΔV as in ERRAN. Also, all linear midcourse ΔV s computed in SIMUL can be recomputed using nonlinear techniques.

Finally, since an actual trajectory is generated in SIMUL, actual target errors after the midcourse correction are also computed.

Probe release events are also available in SIMUL. In addition to propagating knowledge and control covariance matrices for each probe, SIMUL also generates state estimates for each probe.

4. DESCRIPTION OF SUBROUTINES

4.1 Index of Subroutines

The subroutines making up the STEAP programs are listed according to category in Table 4.1 following. The programs are divided into three general classes: the subroutines making up the virtual mass propagation package used by the three basic programs, the additional subroutines required by NOMNAL and then the additional subroutines used in ERRAN and SIMUL. In Table 4.2 the subroutines are listed again by category with a brief summary of their purpose. Thus Table 4.2 can be used to track down the subroutine in which a specific task is performed. The individual subroutines are then documented in detail in alphabetical order in Chapter 6.

4.2 VMP Subroutine Hierarchy

The executive program for the virtual mass a-body trajectory propagator is named VMP. The reader should investigate the detailed analysis and flow chart of VMP in the individual subroutine documentation in Chapter 6. The summaries of the subroutines of VMP are given in the first part of Table 4.2. The subroutines are conveniently divided into four general classes:

Conic Subroutines based on-conic approximation	Conic	Subroutines	based	on-conic	approximation
------------------------------------------------	-------	-------------	-------	----------	---------------

Ephemeris Subroutines used to compute the positions and velocities of the gravitational bodies at

velocities of the gravitational bodies at different times along the trajectory

Propagation Subroutines used in the direct computation of

the trajectory of the spacecraft moving under the influence of all the gravitational bodies

Input/Output Subroutines processing either the input or

output from the virtual mass trajectory propagation

The calling hierarchy of the virtual mass programs is given in Figure 4.1. All subroutines within a given block are at an identical level relative to the calling hierarchy unless they are enclosed by parentheses. Subroutines within parentheses are called by the preceding subroutine. Otherwise calls to subroutines are indicated by arrows. Thus all subroutines within blocks connected directly to VMP are called directly from VMP.

4.3 NOMNAL Subroutine Hierarchy

The first of the three independent programs of STEAP is the nominal trajectory targeter NOMNAL. The main program controlling the processing of the program goes under the same name. is made to the complete documentation of NOMNAL in Chapter 6. subroutine hierarchy of NOMNAL is provided in Figure 4.2. BLOCK DATA loads the planetary constants used by many of the subroutines; it is therefore available to all subroutines of NOMNAL. PRELIM reads the input data and calls ZERIT for the computation of a zero iterate if necessary. ZERIT in turn calls HELIO or LUNA for the actual computation of the interplanetary or lunar zero iterate respectively. NOMNAL calls TRJTRY for the propagation of the nominal trajectory between guidance maneuvers. TRJTRY of course calls the VMP package described in Figure 4.1. NOMNAL calls GIDANS for the actual processing of any guidance event. GIDANS calls VMP to initialize arrays for the other events. If a targeting event requires a zero iterate computation, ZERIT is called. Subroutine TARGET controls the targeting events; INSERS controls the insertion decision computations. NOMNAL calls EXCUTE for the execution of either of these two types of events. GIDANS calls MPPROP to execute a main-probe propagation event (i.e., to provide a time history of the main-probe trajectory). Finally GIDANS calls TPRTRG to carry out a miniprobe targeting event (i.e., to obtain the minimum-miss release controls, record the impact data, and print histories of the minimum-miss miniprobe trajectories.

4.4 ERRAN and SIMUL Hierarchy

The calling hierarchy of the subroutines used in ERRAN and SIMUL is shown in Figures 4.3 and 4.4, respectively. The similar structure of ERRAN and SIMUL is apparent from these two figures. All subroutines can be classified under one or more of the following categories: input, output, basic cycle (measurement processing), or events.

The calling hierarchy of the subroutines is indicated by the level of the subroutine in Figures 4.3 and 4.4. A given subroutine calls all subroutines that are directly connected to the subroutine and are located on the next lower level. For the purposes of clarity, the lowest level subroutine on a given branch is enclosed in parentheses. BLOCK DATA is shown connected to the main hierarchy with a dashed line to indicate that the constants stored in BLOCK DATA are available to all subroutines.

The complete documentation of all subroutines used in ERRAN and SIMUL is given in Chapter 5 of this document.

Table 4.1 STEAP II Subroutines

I. Virtual-Mass Subroutines

Α.	Conic	B. Ep	hemeris	C.	Propagation	D.	Input/Output
	1. CAREL 2. ELCAR 3. 1MPACT 4. SOIPS	2. 3. 4. 5. 6.	TIME BLOCK DATA ORB EPHEM CENTER PECEQ EULMX SUBSOL		1. VMP 2. ESTMT 3. VECTOR 4. VMASS		1. TRAPAR 2. INPUTZ 3. PRINT 4. SPACE 5. NEWPGE

II. NOMNAL Subroutines

Α.	Executive	B. Zero Iterate C. Targeting	
	1. EXCUTE 2. GIDANS 3. MPPROP 4. NOMNAL 5. PRELIM 6. TRJTRY	1. BATCON 7. LUNTAR 1. DESENT 2. FLITE 8. MULCON 2. KTROL 3. HELIO 9. MULTAR 3. TARGET 4. LAUNCH 10. SERIE 4. TARMAX 5. LUNA 11. ZERIT 5. TAROPT 6. LUNCON	
D.	Insertion	E. Pulsing Arc F. Miniprobe Targeting	
	1. COPINS 2. INSERS 3. NONINS	1. (BATCON) 2. PERHEL 2. TPPROP 3. PREPUL 3. TRRTRG 4. PULSEX	
G.	Mathematical Functions and Operations	H. Conic I. Ephemeris	
	2. DINSIN 7. 3. JACOB 8. 4. MATIN 9.	SCAD 1. CAREL 6. HYPT 1. EPHEM SCAR 2. CONCAR 7. IMPACT 2. ORB THPSOM 3. DIMPCP 8. IMPCT 3. PECEQ USCALE 4. ELIPT 9. SPHIMP 4. SUBSOL UXV 5. HPOST 10. STIMP	

III. ERRAN and SIMUL Subroutines

A. Executive	C. Navigation	D. Event	E. Input/Output
A. Executive 1. ERRAN 2. SIMUL B. Dynamic Model 1. NTM 2. NTMS 3. PSIM 4. NDTM 5. PLND 6. MUND 7. PCTM 8. CONCZ 9. CASCAD	1. NAVM 2. GNAVM 3. GAIN1 4. GAIN2 5. SCHED 6. TRAKM 7. TRAKS 8. TARPRL 9. STAPRL 10. MEMO 11. MENOS 12. BIAS 13. RNUM 14. DYNO 15. DYNOS 16. GHA 17. JACOBI	1. SETEVN 2. SETEVS 3. PRED 4. PRESIM 5. BEPS 6. BATCON 7. ZRANS 8. ATANH 9. BPLANE 10. QUASI 11. GUIDM 12. GUISIM 13. GUID 14. GUIS 15. VARADA 16. VARSIM 17. PARTL	1. DATA 2. DATA1 3. GDATA 4. SKEDM 5. DATAS 6. DATA1S 7. CONURT 8. TRANS 9. CORREL 10. STMPR 11. SUB1 12. TITLE 13. GPRINT 14. MOMENT 15. PRINT3 16. PRNTS3 17. PRINT4
	14. DYNO 15. DYNOS 16. GHA 17. JACOBI 18. HYEIS 19. EIGHY	14. GUIS 15. VARADA 16. VARSIM	14. MOMENT 15. PRINT3 16. PRNTS3
		27. MINIQ 28. NTRY 29. GENGID 30. ATCEGV 31. GQCOMP	

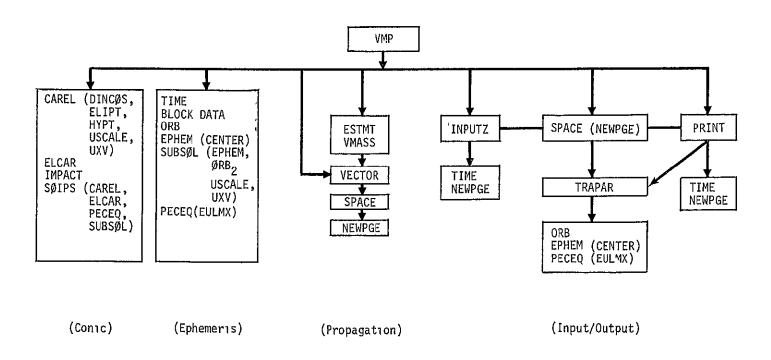


Figure 4.1 Subroutine Hierarchy of VMP

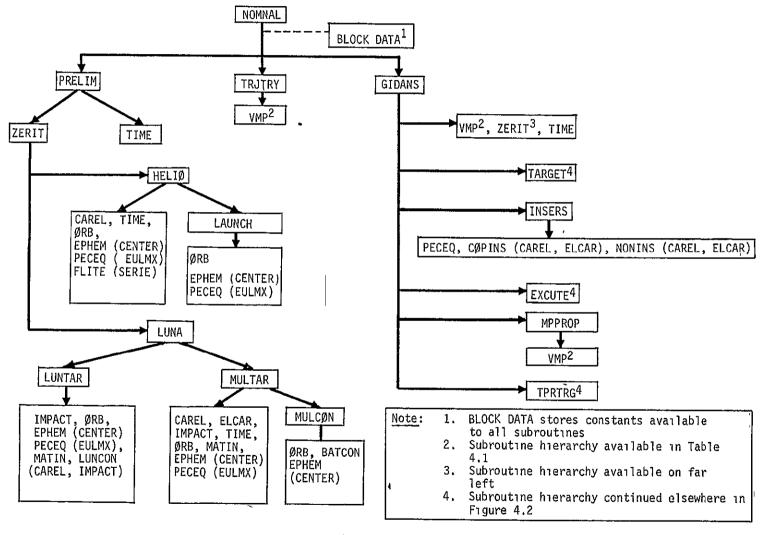


Figure 4.2a Subroutine Hierarchy of NOMNAL

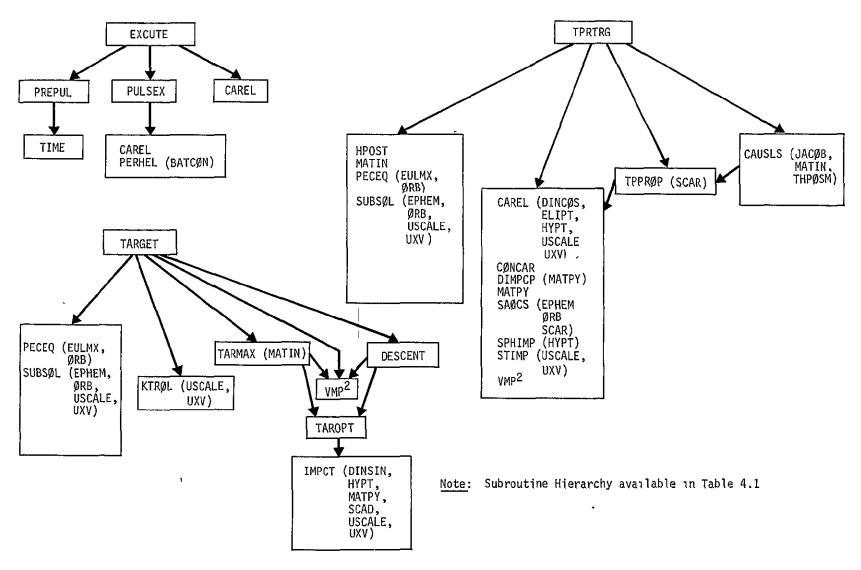


Figure 4.2b Subroutine Hierarchy of NOMNAL

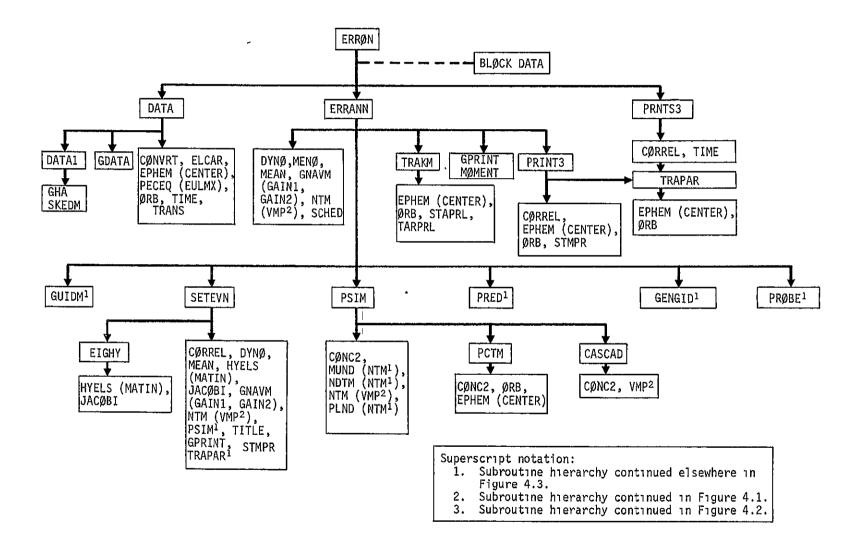


Figure 4.3a. Subroutine Hierarchy of ERRAN

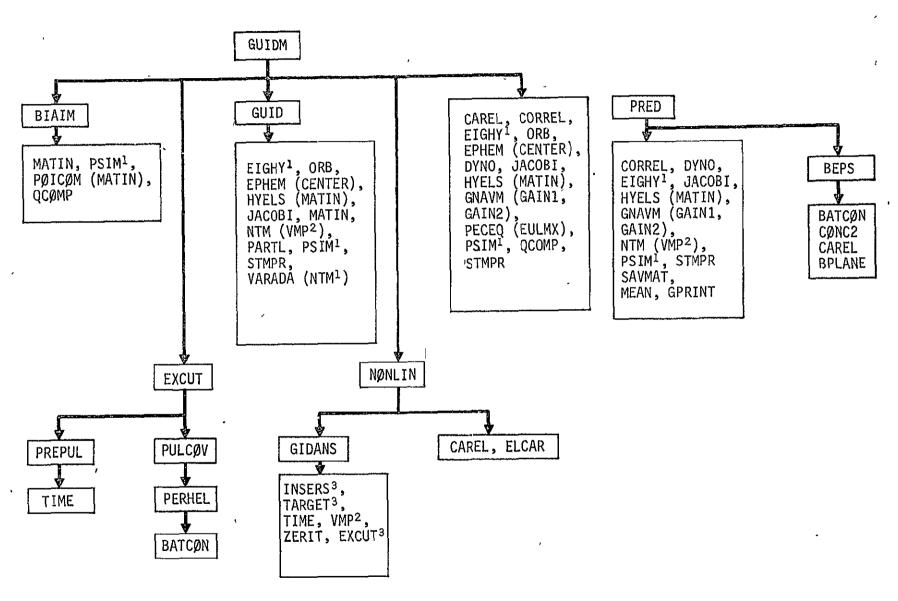


Figure 4.3b Subroutine Hierarchy of ERRAN

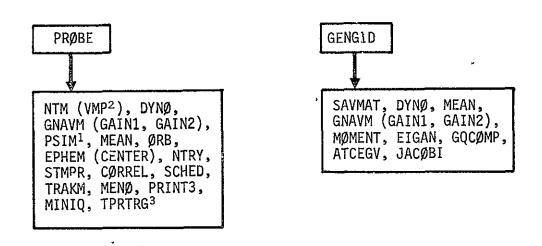


Figure 4.3c. Subroutine Hierarchy of ERRAN

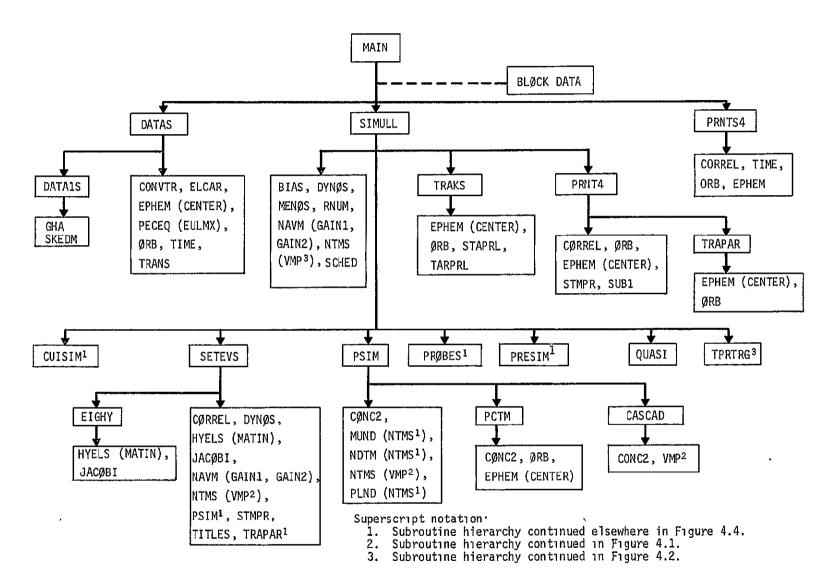


Figure 4.4a Subrouting Hierarchy of SIMUL

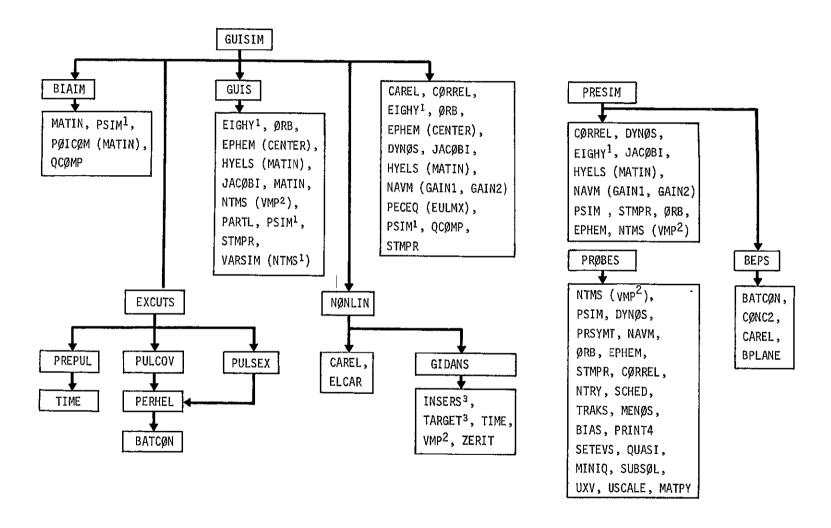


Figure 4.4b Subroutine Hierarchy of SIMUL (continued)

Table 4.2 STEAP II Subroutine Summaries

Subroutine

Function

I. Virtual-Mass Subroutines

1.	CAREL	Convert a Cartesian state to conic elements
2.	ELCAR	Convert conic elements to a Cartesian state
3.	IMPACT	Compute the impact-plane parameters
4.	SOIPS	Conically extrapolate from the nearest integration state to obtain impact data at the SOI and at the planet surface
Eph	emeris	

В.

1.	BLOCK DATA	Set the emphemeris constants of the gravitational bodies
2.	CENTER	Convert the states of bodies to

barycentric coordinates EPHEM

Compute the inertial state of a gravitational body at a given time

4. EULMX Compute the rotational transformation matrix from the Euler angles

ORB Compute the orbital elements of a gravitational body at a given time

6. PECEQ Compute the transformation matrix from ecliptic to equatorial coordinates

SUBSOL Compute the transformation matrix from ecliptic to subsolar coordinates

8. TIME Convert Julian dates epoch 1900 to calendar dates or vice versa

Subroutine Function

C. Propagation

1. ESTMT Determine final position and magnitude of the virtual mass on the current step

2. VECTOR Compute the final position of the spacecraft on the current step

3. VMASS Determine the virtual-mass data for the current step

4. VMP Direct the virtual-mass trajectory propagation

D. Input/Output

1. INPUTZ Convert the input data into a form on which VMP can operate

2. NEWPGE Print headings for each new page in VMP printout

3. PRINT PRINT periodic trajectory-status data

4. SPACE Space paper keeping tracking of paging

5. TRAPAR Compute and record navigation parameter data

II. NOMNAL Subroutines

A. Executive

1. EXCUTE Control the execution of a velocity-increment trajectory correction

2. GIDANS Control the computation of a velocity-increment trajectory correction

3. MPPROP Generate a time history of the main-probe trajectory

4. NOMNAL Control the generation of the nominal trajectory (main program) 5. PRELIM Perform preliminary data processing for NOMNAL TRJTRY 6. Propagate the nominal trajectory to the next guidance event Zero Iterate 1. BATCON Propagate a conic trajectory by means of the universal conic functions 2. FLITE Obtain the solution to Lambert's time-of-flight equation 3. HELIO Compute the heliocentric phase of the interplanetary zero iterate 4. LAUNCH Compute the launch phase of the interplanetary zero iterate 5. LUNA Control lunar zero-iterate generation 6. LUNCON Generate a patched conic lunar trajectory 7. LUNTAR Control the patched conic targeting 8. MULCON Generate the lunar multiconic trajectory 9. MULTAR Control the lunar multiconic targeting 10. SERIE Compute the universal conic functions used in FLITE 11. ZERIT Control the computation of the zero 1terate c. Targeting 1. DESENT Compute the interplanetary velocity targeting corrections using the descent scheme

Compute the heliocentric ecliptic 2. KTROL velocity corrections given the launchplanetocentric velocity controls Control the n-body targeting TARGET 3. Compute the Newton-Raphson targeting TARMAX matrix Set up the actual and auxiliary tar-TAROPT 5. get parameter arrays Insertion D. Compute the coplanar orbit insertion COPINS 1. maneuver Control the orbit insertion computa-2. INSERS tion Compute the nonplanar orbit insertion 3. NONINS E. Pulsing Arc Propagate a conic trajectory by means 1. (BATCON) of the universal conic functions Propagate a perturbed heliocentric 2. PERHEL conic Perform the preliminary data proces-3. PREPUL sing for a multiple-pulse trajectory correction Execute pulsing arc PULSEX Miniprobe Targeting Compute the sines and cosines of the 1. SAOCS spin-axis right ascension and declination given the spin-axis orientation mode Propagate the three miniprobe tra-2. TPPROP

jectories according to either a conic

or virtual-mass model

3. TPRTRG Control the muniprobe targeting procedure

G. Mathematical Functions and Operations

1.	DINCOS	•	Calcula	te	in c	degrees	the	inverse
			cosine	of	a re	eal numb	er	

- 2. DINSIN Calculate in degrees the inverse sine of a real number
- 3. JACOB Approximate by divided differences the Jacobian sensitivity matrix of a vector-valued function with respect to a vector variable
- 4. MATIN Invert a matrix of real-valued elements
- 5. MATPY Multiply two matrices of real-valued elements
- 6. SCAD Calculate both the sine and cosine of an angle given in degrees
- 7. SCAR Calculate both the sine and cosine of an angle given in radians
- 8. THPOSM Find the minimum of a function on a given interval by cubic interpolation
- 9. USCALE Scale the length of a three-vector to a specified real number
- 10. UXV Calculate the vector product of two three-vectors

H. Conic

- 1. CAREL Convert a Cartesian state to conic elements
- 2. CONCAR Convert a conic state in terms of r, θ , e, \underline{P} , \underline{Q} , and μ into a Cartesian state

3.	DIWPCP	Calculate the desired B-plane asymptote pierce-point coordinates given the right ascension and declination of a probe target site
4.	ELIPT	Calculate the time from periapsis on an elipse given the true anomaly
5.	HPOST	Calculate the radius and true anomaly on a hyperbola given the time from periapsis
6.	НҮРТ	Calculate the time from periapsis on a hyperbola given the true anomaly
7.	IMPACT	Compute the impact-plane parameters
8.	IMPCT	For auxiliary targeting compute actual and desired B-plane asymptote pierce points as well as actual target values
9.	SPHIMP	Calculate the true anomaly and time from periapsis at which a conic approach trajectory pierces a planetocentric sphere of a given radius
10.	STIMP	Calculate the B-plane asymptote pierce- point coordinates of a conic trajectory given a state upon it
Eph	emeris	
1.	ЕРНЕМ	Compute the inertial state of a gravitational body at a given time
2.	ORB	Compute the orbital elements of a gravitational body at a given time
3.	PECEQ	Compute the transformation matrix from ecliptic to equatorial coordinates
4.	SUBSOL	Compute the transformation matrix from ecliptic to subsolar coordinates

I,

III. ERRAN and SIMUL Subroutines

A.	Executive	
	1. ERRAN	Control error analysis program (main program)
	2. SIMUL	Control simulation program (main program)
в.	Dynamic Model	
	1. NTM	Control generation of trajectory data for ERRAN
	2. NTMS	Control generation of trajectory data for SIMUL
	3. PSIM *	Control computation of state transition matrix (STM)
1	4. NDTM	Compute unaugmented partition of STM by numerical differencing
	5. PLND	Compute STM partition associated with ephemeris biases
	6. MUND	Compute STM partition associated with gravitational constants
	7. FCTM	Compute unaugmented partition of STM by patched conic technique
	8. CONC2	Compute unaugmented partition of STM by virtual-mass technique
	9. CASCAD	' Compute unaugmented partition of STM by cascaded Darby matrizants
C.	Navigation	•
_	1. NAVM	Propagate covariance matrices between measurements and between events in SIMUL
-	2. GNAVM	Propagate assumed and actual covariance matrices between measurements and be-tween events in ERRAN

3.	GAIN1	Compute the Kalman GAIN matrices
4.	GAIN2	Compute the GAIN matrices for the equivalent recursive consider weighted-least-squares filter
5.	SCHED	Select next measurement time from measurement schedule
6.	TRAKM	Compute observation matrices
7.	TRAKS	Compute observation matrices and actual measurements
8.	TARPRL	Compute target planet position partials
9.	STAPRL	Compute station location position and velocity partials
10.	MENO	Compute assumed measurement noise covariance matrix
11.	MENOS	Compute assumed and actual measure- ment noise covariance matrices
12.	BIAS	Compute actual measurement bias
13.	RNUM	Generate random numbers
14.	DYNO	Compute dynamic noise covariance matrix
15.	DYNOS	Compute dynamic noise covariance matrix and actual dynamic noise
16.	GHA	Compute Greenwich hour angle
17.	JACOBI	Compute eigenvalues and eigenvectors of a matrix
18.	HYELS	Compute hyperellipsoids
19.	EIGHY	Control computation of eigenvalues, eigenvectors, and hyperellipsoids

20.	MEAN	Propagate and update means of actual state or parameter deviations and actual state of parameter estimation errors
. 21.	SAVMAT	Stores one vector in a second vector
D. Ever	ıt.	,
. 1.	SETEVN	Perform computations common to most events in ERRAN
~· 2.··	SETEVS	Perform computations common to most events in SIMUL
3.	PRED	Perform prediction event in ERRAN
4.	PRESIM	Perform prediction event in SIMUL
. 5 .	BEPS .	Compute B-Plane-Related covariances and state transition matrices
6.	BATCON	Compute trajectory data at time T given position and velocity at time 0
7.	ZRANS	Calculate transcendental functions used in the universal form of Kepler's equation
8.	ATANH	Find the angle Y whose TANH is X
9.	BPLANE	Compute B-plane parameters
10.	QUASI	Perform quasi-linear filtering event in SIMUL
11.	GUIDM	Perform guidance event in ERRAN
12.	GUISIM	Perform guidance event in SIMUL
13.	GUID	Compute guidance and variation matrices in ERRAN
-14.	GUIS	Compute guidance: and variation matrices in SIMUL
15.	VARADA	Compute 3VBP variation matrix in ERRAN

16.	VARSIM	Compute 3VBP variation matrix in SIMUL
17.	PARTL	Compute partials of B.T, B.R, wrt state
18.	BIAIM	Perform biased aimpoint guidance
19.	POLCOM	Compute probability of impact
20.	QCOMP	Compute execution error covariance matrix
21.	NONLIN	Control execution of nonlinear gui- dance events
22.	PULCOV	Propagate covariance matrix across a series of pulses
23.	EXCUT	Control execution of pulsing arc in ERRAN
24.	EXCUTS	Control execution of pulsing are in SIMUL
25.	PROBE	Control execution of probe release events in ERRAN
26.	PROBES	Control execution of probe release events in SIMUL
27.	MINIQ	Compute execution error covariance matrix for miniprobe release
28.	NTRY	Compute entry parameters, covariance, and communication angle
29.	GENGID	Generalized covariance technique applied to guidance processes
30.	ATCEGV	Compute eigenvalues and eigenvectors of actual target condition 2nd moment matrices
31.	GQCOMP	Compute actual execution error statistics

E. Input/Output

1.	DATA	Perform preliminary computations and read data in ERRAN
2.	DATAL	Continuation of DATA
3.	ÇDATA	Initialized generalized covariance quantities
4.	SKEDM	Set up bus, main probe, and miniprobe measurement schedules
5.	DATAS	Perform preliminary computations and read data in SIMUL
6.	DATAS 1	Continuation of DATAS
7.	CONVRT	Convert JPL injection conditions to Cartesian components
8.	TRANS	Compute coordinate transformations
9.	CORREL	Compute and print correlation matrix partitions and standard deviations
10.	STMPR	Print STM partitions
11,	SUB1	Compute position and velocity magnitudes
12.	TIŤLE	Print titles
13.	GPRINT	Print actual estimation error statistics
14.	MOMENT	Convert 2nd moment matrices to correlation matrices and print them
15.	PRINT3	Print basic cycle data in ERRAN
16.	PRNTS3	Print ERRAN summary
17.	PRINT4	Print basic cycle data in SIMUL
18.	PRNTS4	Print SIMUL summary

5. COMMON VARIABLE DEFINITIONS

THE BULK OF THE VARIABLES USED IN THE STEAP PROGRAMS ARE COMMON VARIABLES. THESE VARIABLES ARE DEFINED IN DETAIL IN THIS CHAPTER. THE FIRST SECTION LISTS THE COMMON BLOCKS IN ALPHABETICAL ORDER. THE PROGRAMS (NOMNAL, ERRAN, SIMUL) USING EACH COMMON BLOCK ARE NOTED. THE VARIABLES OF EACH COMMON BLOCK ARE DEFINED IN THE ORDER THAT THEY APPEAR IN THE COMMON BLOCK.

THE SECOND SECTION LISTS ALPHABETICALLY ALL VARIABLES APPEARING ANYWHERE IN COMMON. THE COMMON BLOCK TO WHICH THE VARIABLE BELONGS IS REFERENCED. THE DEFINITION OF THE VARIABLE IS THEN GIVEN.

THE THIRD SECTION SUPPLIES THE DEFINITIONS OF SEVERAL LARGE FREQUENTLY REFERENCED ARRAYS. THE ELEMENT APPEARING IN EACH COMPONENT OF EACH ARRAY IS NOTED.

5.1 COMMON VARIABLES BY BLOCKS

IN THIS SECTION COMMON BLOCKS APPEARING IN STEAP ARE LISTED IN ALPHABETICAL ORDER. VARIABLES WITHIN THESE BLOCKS ARE LISTED AND DEFINED IN THE ORDER THEY APPEAR IN THE PROGRAM.

/RATM / MODE FRRAN. SIMU

	BAIM / MODE ERRAN, SIMUL
	· · · · · · · · · · · · · · · · · · ·
ATRANS(6)	CLOSEST APPROACH STATE
TMPR (3)	MOST RECENT TARGET STATE
TNOMC (7)	NOMINAL CLOSEST APPROACH TARGET STATE, INCL. TIME
TNOMB(3)	NOMINAL B-PLANE TARGET STATE
PHI2 (3,3)	INVERSE OF VARIATION MATRIX PARTITION
VINF	HYPERBOLIC EXCESS VELOCITY
TINJ	INJECTION TIME
PROBI	ALLOWABLE PROBABILITY OF IMPACT
ADA(3,6)	VARIATION MATRIX
T3(10)	ARRAY OF GUIDANCE EVENT TIMES
IBAG	NOT USED
IPQ	NOT USED
IGUID(5,10)	ARRAY OF GUIDANCE EVENT CODES
II	GUIDANCE EVENT COUNTER

	** *** *** **** *** *** *** *** *** **
	/BLK / MODE: NOMNAL, ERRAN, SIMUL
Ŧ	TRAJECTORY TIME IN DAYS
PMASS(11)	GRAVITATIONAL CONSTANTS OF PLANETS IN A.U.**3/DAY**2
CN(80)	CONSTANTS USED TO CALCULATE THE ORBITAL ELEMENTS OF THE FIRST FIVE PLANETS (SEE LARGE ARRAY DEFINITIONS IN SECTION 5.3)
ST(50)	CONSTANTS USED TO CALCULATE THE ORBITAL ELEMENTS OF THE LAST FOUR PLANETS (SEE LARGE ARRAY DEFINITIONS IN SECTION 5.3)
EMN(15) ,	THE CONSTANTS USED TO CALCULATE THE ORBITAL ELEMENTS OF THE MOON (SEE LARGE ARRAY DEFINITIONS IN SECTION 5.3)
SMJR(18)	CONSTANTS USED TO CALCULATE THE SEMI-MAJOR AXES OF THE PLANETS
RADIUS(11)	THE RADIUS OF A GIVEN PLANET IN A.U.
RMASS(11)	THE RELATIVE GRAVITATIONAL CONSTANT OF A STATED PLANET WITH RESPECT TO THE SUN
ELMNT(80)	CONTAINS THE ORBITAL ELEMENTS OF THE PLANETS (SEE LARGE ARRAY DEFINITIONS IN SECTION 5.3)
SPHERE(11)	THE SPHERES OF INFLUENCE OF THE PLANETS IN A.U.
XP (6)	THE POSITION AND VELOCITY OF A PLANET IN INERTIAL ECLIPTIC COORDINATES
NO(11)	AN ARRAY OF PLANET CODES BEING USED TO GENERATE THE VIRTUAL MASS TRAJECTORY

	/CNTRIC/ MODE: NOMNAL, ERRAN, SIMUL
IHARY	REFERENCE COORDINATE SYSTEM CODE = 0 HELIOCENTRIC COORDINATES = 1 BARYCENTRIC COORDINATES
ICOORD	NON-FUNCTIONAL IN ERROR ANALYSIS MODE
INITAL	NON-FUNCTIONAL IN ERROR ANALYSIS MODE
	/COM / MODE: NOMNAL, ERRAN, SIMUL
V (16 , 7)	AN ARRAY WHICH STORES PERTINENT VECTORS USED IN THE CALCULATION OF THE VIRTUAL MASS TRAJECTORY (SEE LARGE ARRAY DEFNS IN SECT 5.3)
F (44,4)	CONTAINS THE POSITIONS AND VELOCITIES OF THE PLANETS AT A SPECIFIED TIME PLUS THE POSITIONS AND VELOCITIES OF THE SPACECRAFT RELATIVE TO . THE PLANETS (SEE LARGE ARRAY DEFNS IN SECT 5.3)
PI	THE VALUE OF THE MATHEMATICAL CONSTANT PI
RAD	THE NUMBER OF DEGREES PER RADIAN
ITRAT	IN INTERNAL CODE USED TO DETERMINE HOW MANY ITERATIONS HAVE BEEN ACCOMPLISHED IN THE VIRTUAL MASS PROCEDURE
KOUNT	A CODE WHICH SPECIFIES WHETHER PRINT-OUT IS TO OCCUR AFTER THIS TIME INCREMENT
INCMNT	NUMBER OF INCREMENTS USED
INCPR	SPECIFIES AFTER HOW MANY TIME INCREMENTS PRINT-OUT IS TO OCCUR
INC	DETERMINE WHETHER THE ABOVE OPTION IS TO BE USED
IPR	A CODE WHICH DETERMINES IF PRINT-OUT IS TO OCCUR AFTER A SPECIFIED NUMBER OF DAYS

IYUOBN	NUMBER OF BODIES CONSIDERED IN VIRTUAL MASS TRAJECTORY
NBODY	BASED ON ABOVE VALUEEQUAL TO 4*NBODYI-3
IPRT (4)	SPECIFIES PRINT OPTIONS (IN STEAP TRAJECTORY THIS OPTION IS OMITTED. WHEN PRINT-OUT OCCURS ALL SECTIONS ARE AUTOMATICALLY PRINTED)
KL	PROBLEM NUMBER (NOMNAL ONLY)
IPG	PAGE NUMBER (NOMNAL ONLY)
LINCT	LINE COUNT (NOMNAL' ONLY)
LINPGE	LINES PER PAGE (NOMNAL ONLY)

/CONST / MODE: EDDAN STRIP

70	MASI & MORES	ERRAN, SIMUL	

OMEG A	ROTATION RATE OF EARTH
EPS	OBLIQUITY OF EARTH
SAL(3)	ALTITUDES OF STATIONS
SLAT (3)	LATITUDES OF STATIONS
SLON(3)	LONGITUDES OF STATIONS
DNCN (3)	CONSTANTS FROM WHICH DYNAMIC NOISE IS COMPUTED
MNCN (12)	MEASUREMENT NOISE CONSTANTS
NST	NUMBER OF STATIONS TO BE USED (MAXIMUM 3)

	/CONST2/ MODE: ERRAN, SIMUL
UST (3)	DIRECTION COSINE ARRAYS OF THREE REFERENCE STARS
VST(3)	DIRECTION COSINE ARRAYS OF THREE REFERENCE STARS
WST(3)	DIRECTION COSINE ARRAYS OF THREE REFERENCE STARS
FOP	OFF-DIAGONAL ANNIHILATION VALUE FOR POSITION EIGENVALUES
FOV	OFF-DIAGONAL ANNIHILATION VALUE FOR VELOCITY EIGENVALUES
	/CONST3/ MODE: ERRAN, SIMUL
DELAXS	TARGET PLANET SEMI-MAJOR AXIS FACTOR USED IN NUMERICAL DIFFERENCING
DELECC	TARGET PLANET ECCENTRICITY FACTOR USED IN NUMERICAL DIFFERENCING
DELICL	TARGET PLANET INCLINATION FACTOR USED IN NUMERICAL DIFFERENCING
DELNOD	TARGET PLANET LONGITUDE OF THE ASCENDING NODE FACTOR USED IN NUMERICAL DIFFERENCING
DELW	TARGET PLANET ARGUMENT OF PERIAPSIS FACTOR USED IN NUMERICAL DIFFERENCING
DELMA	TARGET PLANET MEAN ANOMALY FACTOR USED IN NUMERICAL DIFFERENCING
DELMUS	SUN GRAVITATIONAL CONSTANT FACTOR USED IN NUMERICAL DIFFERENCING
DELMUP	TARGET PLANET GRAVITATIONAL CONSTANT FACTOR USED IN NUMERICAL DIFFERENCING

		DPNL	JM / MOD	E NO	MNAL						

	ZERO	THE	NUMBER	ZER 0	€03	TO	NINE	SIGNI	FICANT	FIGURES	
	ONE	THE	NUMBER	ONE	(1)	ro	NINE	SIGNI	FICANT	FIGURES	
	TWO	THE	NUMBER	OWT	(2)	TO	NINE	SIGNI	FICANT	FIGURES	
	THREE	THE	NUMBER	THREE	(3)	10	NINE	SIGNI	FICANT	FIGURES	
	FOUR	THE	NUMBER	FOUR	(4)	то	NINE	SIGNI	FICANT	FIGURES	
	FIVE	THE	NUMBER	FIVE	(5)	TO	NINE	SIGNI	FICANT	FIGURES	
•	EIGHT	THE	NUMBER	EIGHT	(8)	TO	NINE	SIGNI	FICANT	FIGURES	
	TEN	THE	NUMBER	TEN	(10)	TO	NINE	SIGNI	FICANT	FIGURES	
	NINETY		NUMBER URES	NINETY	((9)	3) 1	TO NI	NE SIG	NIFICA	NT	
I	HALF		NUMBER URES	ONE-HA	LF !	(1/2	2) TO	NINE	SIGNIF	ICANT	•

/DPNUM / MODE: ERRAN, SIMUL ZERO THE NUMBER ZERO (0) TO NINE SIGNIFICANT FIGURES ONE THE NUMBER ONE (1) TO NINE SIGNIFICANT FIGURES TWO THE NUMBER TWO (2) TO NINE SIGNIFICANT FIGURES HALF THE NUMBER ONE-HALF (1/2) TO NINE SIGNIFICANT FIGURES THREE THE NUMBER THREE (3) TO NINE SIGNIFICANT FIGURES EM1 THE NUMBER 1.E-1 TO NINE SIGNIFICANT FIGURES EM2 THE NUMBER 1.E-2 TO NINE SIGNIFICANT FIGURES EM3 THE NUMBER 1.E-3 TO NINE SIGNIFICANT FIGURES EM4 THE NUMBER 1.E-4 TO NINE SIGNIFICANT FIGURES EM5 THE NUMBER 1.E-5 TO NINE SIGNIFICANT FIGURES EM6 THE NUMBER 1.E-6 TO NINE SIGNIFICANT FIGURES EM7 THE NUMBER 1.E-7 TO NINE SIGNIFICANT FIGURES EM8 THE NUMBER 1.E-8 TO NINE SIGNIFICANT FIGURES EM9 THE NUMBER 1-E-9 TO NINE SIGNIFICANT FIGURES THE NUMBER 1.E-50 TO NINE SIGNIFICANT FIGURES EM50 TWOPI THE MATHEMATICAL CONSTANT 2.*PI

THE NUMBER 1.E-13 TO NINE SIGNIFICANT FIGURES

EM13

/EVENT / MODE:	ERRAN, SIMUL

TEV(50) TIMES OF EVENTS

TPT2(20) PREDICTION TIMES

SIGRES VARIANCE OF RESOLUTION ERROR

SIGPRO VARIANCE OF PROPORTIONALITY ERROR

SIGALP VARIANCE OF ERROR IN POINTING ANGLE 1

SIGBET VARIANCE OF ERROR IN POINTING ANGLE 2

NEV NUMBER OF EVENTS

IEVNT(50) CODES OF EVENTS

IHYP1 HYPERELLIPSOID CODE USED TO DETERMINE IF

K=1, K=3, OR BOTH

IEIG CODE USED TO DECIDE IF BOTH POSITION AND

VELOCITY EIGEN VECTORS ARE REQUESTED

NPE NUMBER OF PREDICTION EVENTS HAVING OCCURRED

NGE NUMBER OF GUIDANCE EVENTS HAVING OCCURRED

ICDQ3(10) ARRAY OF CODES WHICH DETERMINE WHICH

EXECUTION POLICIES ARE TO BE USED IN GUIDANCE

EVENTS

NEV1 TOTAL NUMBER OF EIGENVECTOR EVENTS

NEV2 TOTAL NUMBER OF PREDICTION EVENTS

NEV3 TOTAL NUMBER OF GUIDANCE EVENTS

NEV4 TOTAL NUMBER OF -COMCON- EVENTS

NQE QUASI-LINEAR FILTERING EVENTS HAVING OCCURRED

10PT7	ORBIT INSERTION VARIABLE. NON-FUNCTIONAL IN EXISTING PROGRAM
NEV8	ORBIT INSERTION VARIABLE. NON-FUNCTIONAL IN EXISTING PROGRAM
NEV9	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN EXISTING PROGRAM
NEV10	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN EXISTING PROGRAM
NEV11	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN EXISTING PROGRAM
	/EXE / MODE ERRAN, SIMUL
XXIN(6)	
YYINGP	STATE VECTOR TRANSFERRED TO EXCUT OR EXCUTS
DIPX	STATE VECTOR TRANSFERRED TO EXCUT OR EXCUTS JULIAN DATE TRANSFERRED TO EXCUT OR EXCUTS
DIPX	
DIPX DELPX(3)	JULIAN DATE TRANSFERRED TO EXCUT OR EXCUTS VELOCITY CORRETION TO BE MODELED AS AN IMPULSE
DIPX DELPX(3) QK(6,6)	JULIAN DATE TRANSFERRED TO EXCUT OR EXCUTS VELOCITY CORRETION TO BE MODELED AS AN IMPULSE SERIES
DIPX DELPX(3) QK(6,6)	JULIAN DATE TRANSFERRED TO EXCUT OR EXCUTS VELOCITY CORRETION TO BE MODELED AS AN IMPULSE SERIES EFFECTIVE EXECUTION COVARIANCE MATRIX

EXMEAN(4) ACTUAL IMPULSIVE EXECUTION ERROR MEANS

> = = +;	/GAINC / MODE: ERRAN
PMIN(6,6)	POSITION/VELOCITY COVARIANCE BEFORE MEASUREMENT USED TO COMPUTE WLS GAINS
PSMIN(12,12)	SOLVE-FOR PARAMETER COVARIANCE BEFORE MEASUREMENT USED TO COMPUTE WLS GAINS
CMIN(6,12)	CORRELATION BETWEEN POSITION/VELOCITY STATE AND SOLVE-FOR PARAMETERS BEFORE MEASUREMENT USED TO COMPUTE WLS GAINS
PPLU (6,6)	POSITION/VELOCITY COVARIANCE AFTER MEASUREMENT USED TO COMPUTE WLS GAINS
PSPLU(12,12)	SOLVE-FOR PARAMETER COVARIANCE AFTER MEASUREMENT USED TO COMPUTE WLS GAINS
CPLU(6,12)	CORRELATION BETWEEN POSITION/VELOCITY STATE AND SOLVE-FOR PARAMETERS AFTER MEASUREMENT USED TO COMPUTE WLS GAINS
RSAVE(6)	STATE AT TLAST
TLAST	TIME WHEN MEASUREMENT LAST PROCESSED
***	/GCA / MODE: ERRAN
XIG(24)	IGNORE PARAMETER LABELS
IAUGW(24)	IGNORE PARAMETER AUGMENTATION VECTOR
NDIM4	DIMENSION OF IGNORE PARAMETER STATE
IGEN	=0, PERFORM NO GENERALIZED COVARIANCE ANALYSIS =1, PERFORM GENERALIZED COVARIANCE ANALYSIS

	s co de de a serie to to to de a de de a de de a de de a de
4404445	/GENGD / MODE: ERRAN
EE (4)	VECTOR WITH THE FOLLOWING ELEMENTS: 1 - ACTUAL MEAN OF PROPORTIONALITY ERROR 2 - ACTUAL MEAN OF RESOLUTION ERROR 3 - ACTUAL MEAN OF POINTING ANGLE ALPHA ERROR 4 - ACTUAL MEAN OF POINTING ANGLE BETA ERROR
EEE(4)	VECTOR CONTAINING VARIANCES CORRESPONDING TO THE -EE- VECTOR MEANS
	/GENGD1/ MODE: ERRAN
GPG(6,6)	ACTUAL POSITION/VELOCITY CONTROL SECOND MOMENT MATRIX
GCXXSG(6,12)	ACTUAL CONTROL SECOND MOMENT MATRIX OF POSITION/ VELOCITY STATE AND SOLVE-FOR PARAMETERS
GCXUG(6,8)	ACTUAL: CONTROL SECOND MOMENT MATRIX OF POSITION/ VELOCITY STATE AND DYNAMIC CONSIDER PARAMETERS
GCXVG(6,15)	ACTUAL CONTROL SECOND MOMENT MATRIX OF POSITION/ VELOCITY STATE AND MEASUREMENT CONSIDER PARAMETERS
GCXWG(6,12)	ACTUAL CONTROL SECOND MOMENT MATRIX OF POSITION/ VELOCITY STATE AND IGNORE PARAMETERS
GPSG(12,12)	ACTUAL SOLVE-FOR PARAMETER CONTROL SECOND MOMENT MATRIX
GCXSUG(12,8)	ACTUAL CONTROL SECOND MOMENT MATRIX OF SOLVE-FOR PARAMETERS AND DYNAMIC CONSIDER PARAMETERS
GCXSVG(12,15)	ACTUAL CONTROL SECOND MOMENT MATRIX OF SOLVE-FOR PARAMETERS AND MEASUREMENT CONSIDER PARAMETERS
GCXSWG(12,12)	ACTUAL CONTROL SECOND MOMENT MATRIX OF SOLVE-FOR PARAMETERS AND IGNORE PARAMETERS

	/GENRL / MODE: ERRAN
	ACTUAL POSITION/VELOCITY SECOND MOMENT MATRIX
GCXXS(6,12)	ACTUAL SECOND MOMENT MATRIX OF POSITION/VELOCITY STATE AND SOLVE-FOR PARAMETERS
GCXU (6,8)	ACTUAL SECOND MOMENT MATRIX OF POSITION/VELOCITY STATE AND DYNAMIC CONSIDER PARAMETERS
GCXV(6,15)	ACTUAL SECOND MOMENT MATRIX OF POSITION/VELOCITY STATE AND MEASUREMENT CONSIDER PARAMETERS
GPS(12,12)	ACTUAL SOLVE-FOR PARAMETER SECOND MOMENT MATRIX
GCXSU(12,8)	ACTUAL SECOND MOMENT MATRIX OF SOLVE-FOR AND DYNAMIC CONSIDER PARAMETERS
GCXSV(12,15)	ACTUAL SECOND MOMENT MATRIX OF SOLVE-FOR AND MEASUREMENT CONSIDER PARAMETERS
GCXSW(12,12)	ACTUAL SECOND MOMENT MATRIX OF SOLVE-FOR AND IGNORE PARAMETERS
JPR(4,4)	ACTUAL MEASUREMENT RESIDUAL SECOND MOMENT MATRIX
TXW(6,12)	STATE TRANSITION MATRIX-PARTITION ASSOCIATED WITH IGNORE PARAMETERS
AN(4,12)	OBSERVATION MATRIX ASSOCIATED WITH IGNORE PARAMETER STATE
GCUV (8,15)	ACTUAL SECOND MOMENT MATRIX OF DYNAMIC CONSIDER AND MEASUREMENT CONSIDER PARAMETERS
GCUW (8,12)	ACTUAL SECOND MOMENT MATRIX OF DYNAMIC CONSIDER AND IGNORE PARAMETERS

GCVW (15,12)	ACTUAL SECOND MOMENT MATRIX OF MEASUREMENT CONSIDER AND IGNORE PARAMETERS
GU(8,8)	ACTUAL DYNAMIC CONSIDER PARAMETER SECOND MOMENT MATRIX
GV(15,15) .	ACTUAL MEASUREMENT CONSIDER PARAMETER SECOND MOMENT MATRIX
GW(12,12)	ACTUAL IGNORE PARAMETER SECOND MOMENT MATRIX
GDNCN(3)	CONSTANTS FROM WHICH ACTUAL DYNAMIC NOISE IS COMPUTED
GMNCN(12)	ACTUAL MEASUREMENT NOISE VARIANCES
EXI(6)	ACTUAL MEANS OF INITIAL POSITION/VELOCITY PARAMETER DEVIATIONS
EXSI (12)	ACTUAL MEANS OF INITIAL SOLVE-FOR PARAMETER DEVIATIONS
EU(8)	ACTUAL MEANS OF INITIAL DYNAMIC CONSIDER PARAMETER DEVIATIONS
EV(15) -	ACTUAL MEANS OF INITIAL MEASUREMENT CONSIDER PARAMETER DEVIATIONS
EW(12)	ACTUAL MEANS OF INITIAL IGNORE PARAMETER DEVIATIONS
QPR(6,6)	ACTUAL DYNAMIC NOISE SECOND MOMENT MATRIX
RPR(4,4)	ACTUAL MEASUREMENT NOISE SECOND MOMENT MATRIX

GCXH(6,12)	ACTUAL SECOND MOMENT MATRIX OF POSITION/VELOCITY STATE AND IGNORE PARAMETERS
EXT(6)	ACTUAL MEANS OF UPDATED ESTIMATION ERRORS FOR POSITION/VELOCITY STATE
EXST (12)	ACTUAL MEANS OF UPDATED ESTIMATION ERRORS FOR SOLVE-FOR PARAMETERS
EXTP (6)	ACTUAL MEANS OF PROPAGATED ESTIMATION ERRORS FOR POSITION/VELOCITY STATE
EXSTP(12)	ACTUAL MEANS OF PROPAGATED ESTIMATION ERRORS FOR SOLVE-FOR PARAMETERS
GCXWP(6,12)	ACTUAL SECOND MOMENT MATRIX OF POSITION/VELOCITY STATE AND IGNORE PARAMETERS PRIOR TO PROCESSING A MEASUREMENT
GCXSWP(12,12)	ACTUAL SECOND MOMENT MATRIX OF SOLVE-FOR AND IGNORE PARAMETERS PRIOR TO PROCESSING A MEASUREMENT
EMRES(4)	ACTUAL MEASUREMENT RESIDUAL MEAN
IGONF	ACTUAL DYNAMIC NOISE FLAG
IGMNF	ACTUAL MEASUREMENT NOISE-FLAG

	/GUI / MODE: ERRAN, SIMUL
PG (6,6)	POSITION/VELOCITY CONTROL COVARIANCE
CXXSG(6,12)	CONTROL CORRELATION BETWEEN POSITION/VELOCITY STATE AND SOLVE-FOR PARAMETERS
CXUG (6,8)	CONTROL CORRELATION BETWEEN POSITION/VELOCITY STATE AND DYNAMIC CONSIDER PARAMETERS
CXVG(6,15)	CONTROL CORRELATION BETWEEN POSITION/VELOCITY STATE AND MEASUREMENT CONSIDER PARAMETERS
PSG(12,12)	SOLVE-FOR PARAMETER CONTROL COVARIANCE
CXSUG(12,8)	CONTROL CORRELATION BETWEEN SOLVE-FOR PARAMETERS AND DYNAMIC CONSIDER PARAMETERS
CXSVG(12,15)	CONTROL CORRELATION BETWEEN SOLVE-FOR PARAMETERS AND MEASUREMENT CONSIDER PARAMETERS
XG(6)	POSITION/VELOCITY STATE AT MOST RECENT GUIDANCE EVENT
TG	TRAJECTORY TIME AT MOST RECENT GUIDANCE EVENT
EM(2,6)	VARIATION MATRIX RELATING POSITION/VELOCITY DEVIATIONS TO 8.T AND 8.R DEVIATIONS
	/GXXL / MODE: ERRAN

GUIDANCE EVENT POLICY LABEL ARRAY

GPL(34)

	/IMPTAR/ MODE: NOMNAL
DCP	ACTUAL TARGET VALUE OF PROBE SITE DECLINATION IN
	DEG RELATIVE TO PROBE-SPHERE FRAME (IT MAY DIFFER
,	FROM INPUT VALUE IN DTAR IF INPUT TARGET SITE
b •	CAN NOT BE ACHEIVED)
DIN	ACTUAL TARGET VALUE OF INCLINATION IN DEG AT
22	CLOSEST APPROACH (IT MAY DIFFER FROM INPUT VALUE
`	IN DTAR IF LATTER CAN NOT BE ACIEVED)
FCSS (3.3)	TRANSFORMATION MATRIX FROM PLANETOCENTRIC
2000 (0) 07	EGLIPTIC TO SUBSOLAR COORDINATES
RAP	ACTUAL TARGET VALUE OF PROBE SITE RIGHT-
	ASCENSION IN DEG RELATIVE TO PROBE-SPHERE FRAME
	(IT MAY DIFFER FROM INPUT VALUE IN DTAR IF INPUT TARGET SITE CAN NOT BE ACHIEVED)
•	' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' ' '

	/IMPTAR/ MODE ERRAN, SIMUL
•	
ANG	TARGET INCLINATION CONVERTED FROM INPUT FORMAT TO
	VALUE BETWEEN 0 AND 180 DEGREES AND SATISFYING
•,	APPROACH ASYMPTOTE CONSTRAINT
****	/JULY/ MODE: ERRANN
,	
A L DU A	ELEVATION ANGLE OF CAR FROM STATION
ALPNA	ELEVATION ANGLE OF S/C FROM STATION
BETA	AZIMUTH OF S/C FROM STATION

		. 	
_	/	LUNART/ MODE	
		TARGET VALUES GETING	OF SMA, B.T, AND B.R IN LUNAR TAR-
	PCON (3)	PERTURBATIONS	IN CONTROLS (ALPHA, DELTA, THETA)
	TTOL (3)	ALLOWABLE TOLE	ERANCES IN SMA, B.T, B.R
	BCON (3)	MAXIMUM STEP	SIZES OF CONTROLS
	RI(6)	GEOCENTRIC ST	ATE OF S/C AT LUNAR SOI
	RMQ(6)	GEOCENTRIC STA	ATE OF CENTER OF MOON AT TSI IN ORDINATES
	RSI(6)	SELENOCENTRIC	STATE OF S/C AT LUNAR SOI
	RME(6)	GEOCENTRIC STA	ATE OF CENTER OF MOON IN ECLIPTIC T TSI
	DECLIN	DECLINATION OF TO LUNAR EQUA	F APPROACH ASYMPTOTE WITH RESPECT
	OTAR (3)	DESTRED VALUES	S OF SMA, RCA, AND INC
	TCA	J.D. OF TIME	AT LUNAR CLOSEST APPROACH (DESIRED)
	RCA	RADIUS OF CLOS	SEST APPROACH TO MOON (DESIRED)
	SMA	SEMI-MAJOR AX	IS OF LUNAR HYPERBOLA (DESIRED)
	CAI	DESTRED CLOSES	ST APPROACH EQUATORIAL INCLINATION
	RPE	RADIUS OF EAR	TH PARKING ORBIT
	TSI	PROJECTED J.D	. AT SOI INTERSECTION
	EMU	GRAVITATIONAL	CONSTANT OF EARTH (KM3/SEC2)

TSPH RADIUS OF LUNAR SOI (KM)

EQLQ(3,3) TRANSFORMATION MATRIX FROM EARTH-EQUATORIAL TO

LUNAR EQUATORIAL COORDINATES

ITAG FLAG SPECIFYING STAGE OF TARGETING

=1 IN SMA TARGETING

=0 IN SMA, INC, RCA TARGETING

/MEAS / MODE: ERRAN, SIMUL

TMN(500) TIMES OF MEASUREMENTS

MCODE (500) ARRAY OF MEASUREMENT CODES

NMN TOTAL NUMBER OF MEASUREMENTS

MCNTR NUMBER OF MEASUREMENTS HAVING OCCURRED

/MISC / MODE: ERRAN, SIMUL

ACC ACCURACY FIGURE USED IN VIRTUAL MASS PROGRAM

FACP POSITION FACTOR USED IN NUMERICAL DIFFERENCING

FACV VELOCITY FACTOR USED IN NUMERICAL DIFFERENCING

BIA(12) MEASUREMENT BIASES

IDNF DYNAMIC NOISE FLAG

ICOOR STATE VECTOR CODE WHICH DETERMINES IN WHICH

COORDINATE SYSTEM THE VECTOR IS READ IN

ITR MODE FLAG

IMMF MEASUREMENT NOISE FLAG

ISP2 SPHERE OF INFLUENCE FLAG

/MNPR / MODE: ERRAN, SIMUL

ALFA RIGHT	ASCENSION	0F	SPIN	AXIS
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DELT DECLINATION OF SPIN AXIS

XPHI ROLL RELEASE ANGLE (IN RADIANS)

ABW MAGNITUDE OF SPIN AXIS

YYL BOOM LENGTH

XEE(5) PROBE RELEASE EXECUTION ERROR UNCERTAINTIES

QT(3,3) MINI-PROBE EXECUTION ERROR COVARIANCE MATRIX

ADV(3) ACTUAL MINI-PROBE RELEASE EXECUTION ERRORS

DW ACTUAL SPIN-RATE EXECUTION ERROR

DA ACTUAL SPIN-AXIS RIGHT ASCENSION EXECUTION ERROR

DL ACTUAL BOOM LENGTH ERROR

DD ACTUAL SPIN-AXIS DECLINATION EXECUTION ERROR

DP ACTUAL RELEASE ANGLE EXECUTION ERROR

/NAME / MODE: ERRAN, SIMUL

EVNM (11) EVENT NAME

MNNA ME (12,3) MEASUREMENT NA ME

CMPNM(30) COMPONENT NAME

	/OVER / MODE: SINUL
RF(6)	FINAL TARGETED NOMINAL STATE VECTOR
RF1(6)	FINAL MOST REGENT NOMINAL STATE VECTOR
	/OVER1 / MODE: SIMUL
RI(6)	INITIAL TARGETED NOMINAL STATE VECTOR
TEVN	TIME OF CURRENT EVENT
RI1(6)	INITIAL MOST RECENT NOMINAL STATE VECTOR
ICODE	EVENT CODE
NAFC	NON-FUNCTIONAL ADAPTIVE FILTER CODE
NR	NUMBER OF ROWS IN THE OBSERVATION MATRIX

	/OVERE / MODE: ERRAN
	FINAL TARGETED NOMINAL STATE
,	/OVERL / MODE ERRAN, SIMUL
	TIME INTERVAL BETWEEN ORBITAL INSERTION DECISION AND EXECUTION
	/OVERR / MODE ERRAN
RI(6)	STATE VECTOR AT EVENT TIME
TEVN	EVENT TIME
NOGEN	=1 CALL GENGID =0 DO NOT CALL GENGID
	/OVERX / MODE ERRAN, SIMUL
IX	NONLINEAR GUIDANCE CODE
JX	GUIDANCE EVENT COUNTER
XIN(6)	STATE VECTOR TRANSFERRED TO NONLIN
****	/OVERZ / MODE ERRAN, SIMUL
Ŗ F (6)	FINAL TARGETED STATE VECTOR
IGP	MIDCOURSE GUIDANCE POLICY CODE
GA (3,6)	GUIDANCE MATRIX

	/	PBLK / MODE ERRAN, SIMUL
	*	
	A(2,3)	FTA IMPACT PLANE TRANSFORMATION MATRIX
	XMUS (2)	NOMINAL IMPACT PLANE TARGET STATE
	EXEC (3, 3)	EXECUTION ERROR COVARIANCE MATRIX
	CR	CAPTURE RADIUS OF TARGET PLANET
	POI	PROBABILITY OF IMPACT
	XLAM(2,2)	PROJECTION OF TARGET CONDITION COVARIANCE MATRIX INTO THE IMPACT PLANE
	XLAMI(2,20	INVERSE OF XLAM(2,2)
	DVRB (3)	VELOCITY CORRECTION REQUIRED TO REMOVE AIMPOINT BIAS
	DVUP (3)	UPDATE VELOCITY CORRECTION
ł	PSTAR	NOMINAL PROBABILITY DENSITY FUNCTION EVALUATED AT TARGET PLANET CENTER
	DVN(3)	COMMANDED VELOCITY CORRECTION TRANSFERRED TO BIAIM
	DELV (3, 18)	ARRAY OF EXTERNALLY-SUPPLIED VELOCITY CHANGES
•	IIGP	MIDCOURSE GUIDANCE POLICY CODE
	IEND	=2 FOR 2 VARIABLE 8-PLANE =3 FOR 3 VARIABLE B-PLANE
	IBIAS	BIASED AIMPOINT GUIDANCE EVENT FLAG = 0 AIMPOINT NOT BIASED = 1 AIMPOINT BIASED
	IDENS	PROBABILITY DENSITY FUNCTION CODE. NON-FUNCTIONAL

/PRBE / MODE: ERRAN- SIMUL

PMN(12)	MEASUREMENT NOISE VARIANCES USED FOR MAIN PROBE
T6	MAIN PROBE RELEASE EVENT TIME
T7 _	MINI-PROBE RELEASE EVENT TIME
RPS	RADIUS OF PROBE SPHERE
SMN(12)	MEASUREMENT NOISE VARIANCES USED FOR MINI-PROBES
TIMPCT	APPROXIMATE TRAJECTORY TIME OF IMPACT FROM NOMNAL PROGRAM
TMN1 (100)	MAIN PROBE MEASUREMENT SCHEDULE TIMES
TMN2 (100)	MINI-PROBE MEASUREMENT SCHEDULE TIMES
IUTC	=1 TARGET CONTROLS DATA SUPPLIED BY USER =0 COMPUTE TARGET CONTROLS
NMNP (2)	NUMBER OF MEASUREMENTS TO BE PROCESSED (1) = MAIN PROBE (2) = MINI-PROBE
MCODE1(100)	ARRAY OF MAIN PROBE MEASUREMENT CODES
MCODE2(100)	ARRAY OF MINI-PROBE MEASUREMENT CODES
NENT1	NUMBER OF CARDS IN MEASUREMENT INPUT FOR MAIN PRB
NENT 2	NUMBER OF CARDS IN MEASUREMENT INPUT FOR MINI-PRB

MEASUREMENT COUNTER FOR PROBE RELEASE

MCNTRP

	/PROBD / MODE: NOMNAL
RPSP	RADIUS OF PROBE IMPACT SPHERE IN KM
IPCSP	FLAG INDICATING TYPE OF PLANETOCENTRIC COORDINATE SYSTEM FOR SPECIFYING PROBE IMPACT SITES =0 EQUATORIAL =1 SUBSOLAR ORBIT-PLANE
	/PRT / MODE NOMNAL
MONTH(12)	NAMES OF MONTHS
PLANET(11)	NAMES OF GRAVITATIONAL BODIES
	/PRT / MODE: ERRAN, SIMUL
PLANET(11)	NAMES OF PLANETS

/PULS / MODE NOMNAL

	/PULS / MODE NOMNAL
PULMAG	THRUST MAGNITUDE OF PULSING ENGINE
PULMAS	NOMINAL MASS OF SPACECRAFT DURING PULSING ARC
DUR	DURATION OF SINGLE PULSE
DfI	TIME INTERVAL (DAYS) BETWEEN SUCCESSIVE PULSES
DVI(3)	VELOCITY INCREMENT ADDED ON TYPICAL PULSE
DVF(3)	VELOCITY INCREMENT ADDED ON FINAL PULSE
PULT	TOTAL TIME INTERVAL OF PULSING ARC
RK(2,3)	POSITION VECTORS OF LAUNCH AND TARGET PLANETS AT IMPULSIVE TIME (MIDPOINT OF PULSING ARC)
VK(2,3)	VELOCITY VECTORS OF LAUNCH AND TARGET PLANETS AT IMPULSIVE TIME (MIDPOINT OF PULSING ARC)
FS (2,5)	F-SERIES COEFFICIENTS OF LAUNCH AND TARGET BODIES
GS (2,4)	G-SERIES COEFFICIENTS OF LAUNCH AND TARGET BODIES
GG(3)	GRAVITATIONAL CONSTANTS OF SUN, LAUNCH, AND TAR- GET BODIES
PSIGS	PULSING ARC ERROR MODEL RESOLUTION VARIANCE
PSIGK	PULSING ARC ERROR MODEL PROPORTION VARIANCE
PSIGA	PULSING ARC ERROR MODEL POINTING ANGLE A VARIANCE
PSIGB	PULSING ARC ERROR MODEL POINTING ANGLE B VARIANCE
NPUL	NUMBER OF PULSES IN PULSING ARC

	/SAVVAL/ MODE ERRAN, SIMUL
XBDT	ORIGINAL VALUE OF B.T IN NONLINEAR GUIDANCE
XBDR	ORIGINAL VALUE OF B.R IN NONLINEAR GUIDANCE
XOSI	ORIGINAL VALUE OF TSI IN NONLINEAR GUIDANCE
XRSI (3)	ORIGINAL VALUE OF RSI IN NONLINEAR GUIDANCE
XAZI (3)	ORIGINAL VALUE OF VSI IN NONLINEAR GUIDANCE
XRC(6)	ORIGINAL VALUE OF RC IN NONLINEAR GUIDANCE
XUC	ORIGINAL VALUE OF DC IN NONLINEAR GUIDANCE
	/SIMCNT/ MODE: SIMUL
· · · · · · · · · · · · · · · · · · ·	* * * * * * * * * * * * * * * * * * *
DMUSB '	BIAS IN GRAVITATIONAL CONSTANT OF SUN
DMUPB	BIAS IN GRAVITATIONAL CONSTANT OF TARGET PLANET
DAB	BIAS IN SEMI-MAJOR AXIS OF TARGET PLANET
DE8	BIAS IN ECCENTRICITY OF TARGET PLANET
DIB	BIAS IN INCLINATION OF TARGET PLANET
DNOB	BIAS IN LONGITUDE OF ASCENDING NODE
DWB	BIAS IN ARGUMENT OF PERIAPSIS
DMAB	BIAS IN MEAN ANOMALY
TTIM1	FIRST TIME USED FOR UNMODELLED ACCELERATION
TTIM2	SECOND TIME USED FOR UNMODELLED ACCELERATION

UNMAC(3,3)	UNMODELLED ACCELERATION
SLB(9)	BIASES IN STATION LOCATION CONSTANTS
AVARM(12)	VARIANCE OF ACTUAL MEASUREMENT NOISE
ARES (20)	ACTUAL RESOLUTION ERROR
APRO (20)	ACTUAL PROPORTIONALITY ERROR
AALP (20)	ACTUAL ERROR IN POINTING ANGLE 1
ABET (20)	ACTUAL ERROR IN POINTING ANGLE 2
IAHNF	ACTUAL MEASUREMENT NOISE FLAG

,	/SIM1 / MODE: SIMUL
X11(6)	INITIAL STATE VECTOR OF MOST RECENT NOMINAL TRAJECTORY
XF1(6)	FINAL STATE VECTOR OF MOST RECENT NOMINAL TRAJECTORY
ADEVX(6)	ACTUAL DEVIATION IN THE STATE VECTOR
ADEVXS(24)	ACTUAL DEVIATION IN SOLVE-FOR PARAMETERS
EDEVX(6)	ESTIMATED DEVIATION IN THE STATE VECTOR
EDEVXS(24)	ESTIMATED DEVIATION IN SOLVE-FOR PARAMETERS
W (6)	ACTUAL DYNAMIC NOISE
ZI (6)	INITIAL ACTUAL STATE VECTOR
ZF(6)	FINAL ACTUAL STATE VECTOR AFTER ADDING EFFECT OF UNMODELED ACCELERATION
ANOIS(4)	ACTUAL WHITE NOISE
RES(4)	RESIDUAL
EY(4)	ESTIMATED MEASUREMENT
AY (4)	ACTUAL MEASUREMENT
AR(4,4)	ACTUAL MEASUREMENT NOISE
ADEVX8(6)	ACTUAL DEVIATION IN STATE VECTOR AT BEGINNING OF TRAJECTORY
ADEVSB(24)	ACTUAL DEVIATION IN SOLVE-FOR PARAMETERS AT BEGINNING OF TRAJECTORY
AYMEY(4)	ACTUAL MEASUREMENT MINUS ESTIMATED MEASUREMENT
EDEVXM(6)	ESTIMATED DEVIATION IN THE STATE VECTOR (FOR ADAPTIVE FILTERING)
EDEVSM(24)	ESTIMATED DEVIATION IN SOLVE-FOR PARAMETERS (FOR ADAPTIVE FILTERING)

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	/SIM2 / MODE& SIMUL

NB1(11)	ARRAY OF PLANET CODES IN ACTUAL TRAJECTORY
ACC1	ACCURACY USED IN ACTUAL TRAJECTORY
NBOD1	NUMBER OF BODIES IN ACTUAL TRAJECTORY
	/SOIVMP/ MODE: NOMNAL, ERRAN, SIMUL
(
DCIMP	DECLINATION OF VEHICLE IN DEG RELATIVE TO
	PLANETOCENTRIC PROBE-SPHERE FRAME AT IMPACT
0	JULIAN DATE OF IMPACT WITH SPHERE OF INTEREST
J	ON OSCULATING PLANETOCENTRIC CONIC
DEPOC	JULIAN DATE EPOCH 1900 OF IMPACT WITH SPHERE OF
	INTEREST ON OSCULATING PLANETOCENTRIC CONIC
IP	INDEX IDENTIFYING IMPACTED PLANET
RAIMP	RIGHT ASCENSION OF VEHICLE IN DEG RELATIVE TO
. • . • • • • • • • • • • • • • • • • •	PLANETOCENTRIC PROBE-SPHERE FRAME AT IMPACT

	/STM / MODE: ERRAN, SIMUL
	,
P(6,6)	POSITION/VELOCITY COVARIANCE
CXXS (6,12)	CORRELATION BETWEEN POSITION/VELOCITY STATE AND SOLVE-FOR PARAMETERS
CXU(6,8)	CORRELATION BETWEEN POSITION/VELOCITY STATE AND DYNAMIC CONSIDER PARAMETERS
CXV(6,15)	CORRELATION BETWEEN POSITION/VELOCITY STATE AND MEASUREMENT CONSIDER PARAMETERS
PS(12,12)	SOLVE-FOR PARAMETER COVARIANCE
CXSU(12,8)	CORRELATION BETWEEN SOLVE-FOR PARAMETERS AND DYNAMIC CONSIDER PARAMETERS
CXSV(12,15)	CORRELATION BETWEEN SOLVE-FOR PARAMETERS AND MEASUREMENT CONSIDER PARAMETERS
U0(8,8)	DYNAMIC CONSIDER PARAMETER COVARIANCE MATRIX

VO(15,15) MEASUREMENT CONSIDER PARAMETER COVARIANCE MATRIX

NOTE IF THE ENTIRE COVARIANCE MATRIX WERE ASSEMBLED FROM THE GIVEN PARTITIONS THE RESULTANT MATRIX WOULD BE P(41,41) WITH THE SYMMETRIC STRUCTURE-

_	P(6,6)	CXXS(6,12),	CXU(6,8)	CXV(6,15)
P(41,41)=		PS(12,12)	CXSU(12,8)	CXSV(12,15)
F (41)41)-			UO (8,8)	CUV(8,15)
	•			VO (15,15)
PHI(6,6)	POS Į T	ION/VEL,OCITY S	TATE TRANSIT	ION MATRIX
TXXS (6, 12)		TRANSITION MA- FOR PARAMETER		ON ASSOCIATED WITH
TXU(6,8)		TRANSITION MA		ON ASSOCIATED WITH
Q(6,6)	DYNAM	IC NOISE COVAR	IANCE MATRIX	
R (4, 4)	MEASU	REMENT NOISE C	OVARIANCE MA	TRIX
AK(6,4)	KALMA State	N GAIN CONSTAN	T FOR POSITI	ON/VELOCITY
S(12,4)	KALMA Param	N GAIN CONSTAN Eters	T FOR SOLVE-	FOR
H(4,6)		VATION MATRIX ION/VELOCITY S		ERVABLES TO
AM(4,12)		VATION MATRIX I LVE-FOR PARAME		ERVABLES
G (4 ₇ '8)		VATION MATRIX		

AL (4,15)	OBSERVATION MATRIX RELATING OBSERVABLES TO MEASUREMENT CONSIDER PARAMETER STATE
HPHR (4,4)	NON-FUNCTIONAL IN PRESENT ERROR ANALYSIS PROGRAM
PP(6,6)	POSITION/VELOCITY COVARIANCE MATRIX PRIOR TO PROCESSING A MEASUREMENT
-	CORRELATION BETWEEN POSITION/VELOCITY STATE AND SOLVE-FOR PARAMETERS PRIOR TO PROCESSING A MEASUREMENT
CXUP (6,8)	CORRELATION BETWEEN POSITION/VELOCITY STATE AND DYNAMIC CONSIDER PARAMETERS PRIOR TO PROCESSING A MEASUREMENT
CXVP(6,15)	CORRELATION BETWEEN POSITION/VELOCITY STATE AND MEASUREMENT CONSIDER PARAMETERS PRIOR TO PROCESSING A MEASUREMENT
PSP(12,12)	SOLVE-FOR PARMETER COVARIANCE MATRIX PRIOR TO PROCESSING A MEASUREMENT
GXSUP(12,8)	CORRELATION BETWEEN SOLVE-FOR PARAMETERS AND DYNAMIC CONSIDER PARAMETERS PRIOR TO PROCESSING A MEASUREMENT
CXSVP(12,15)	CORRELATION BETWEEN SOLVE-FOR PARAMETERS AND MEASUREMENT CONSIDER PARAMETERS PRIOR TO PROCESSING A MEASUREMENT

 	/STMG / MODE: ERRAN
IGAIN	=1, USE GAIN1 SUBROUTINE =2, USE GAIN2 SUBROUTINE
	STVEC / MODE: ERRAN, SIMUL
XI(6)	INITIAL VEHICLE STATE VECTOR OF ORIGINAL NOMINAL
XF (6)	FINAL VEHICLE STATE VECTOR OF ORIGINAL NOMINAL
X8(6)	BEGINNING ORIGINAL NOMINAL VEHICLE STATE VECTOR
NDIM1	DIMENSION OF SOLVE-FOR PARAMETER STATE
NOIM2	DIMENSION OF DYNAMIC CONSIDER STATE
NDIM3	DIMENSION OF MEASUREMENT CONSIDER PARAMETER STATE
IAUGIN(24)	INPUT AUGMENTATION VECTOR OF ONE#S AND ZERO#S
IAUG (24)	AUGMENTATION VECTOR
IAUGDC(8)	DYNAMIC CONSIDER AUGMENTATION VECTOR
IAUGMC(15)	MEASUREMENT CONSIDER AUGMENTATION VECTOR

	/TAREAL/ MODE: NOMNAL, ERRAN, SIMUL
AC (5,10)	ACCURACY LEVELS (UP TO 5) USED IN EACH GUIDANCE EVENT
PHI(3,3)	TARGETING MATRIX
T LMG (10)	TIMES OF EACH GUIDANCE EVENT REFERENCED TO EPOCH- -INITIAL TIME, SOI TIME, OR CA TIME
TAR(6,10)	DESTRED VALUES OF TARGET PARAMETERS (UP TO 6 AVAILABLE) FOR EACH GUIDANCE EVENT
DAUX (3)	DESTRED AUXILIARY PARAMETER VALUES OF ITERATE
AAUX (3)	ACTUAL AUXILIARY VALUES OF ITERATE
DTAR (3)	DESIRED TARGET VALUES OF ITERATE
ATAR (3)	ACTUAL TARGET VALUES OF ITERATE
TOL(6,10)	ALLOWABLE TOLERANCES OF TARGET PARAMETERS FOR EACH GUIDANCE EVENT
TOLR (6)	NOT USED IN CURRENT TARGET VERSION
CTOL (6)	TOLERANCES FOR CURRENT EVENT
FAC(3)	SCALING FACTORS USED IN BAD STEP CHECK
TMPR	DAYS BETWEEN PRINTOUTS OF NOMINAL TRAJECTORY
PERV(10)	PERTURBATION SIZE FOR VELOCITY COMPONENTS IN CONSTRUCTING SENSITIVITY MATRICES IN TARGETING EVENTS
DINTG(10)	NOT USED IN CURRENT TARGET VERSION
DT (10)	JULIAN DATES OF TARGET TIMES
DELV (3,10)	EXTERNALLY SUPPLIED VELOCITY CORRECTION OR VELOCITY INCREMENT COMPUTED BY INSERTION DECISION

TRAJECTORY TIME (DAYS) REF. TO INJECTION

RIN(6) CURRENT STATE VECTOR AT I-TH EVENT

TIN JULIAN DATE AT INJECTION

D1 JULIAN DATE ASSOCIATED WITH RIN ARRAY

DG(10) JULIAN DATES OF EVENT TIMES

DELTAT NUMBER OF DAYS INTEGRATION IS TO CONTINUE IF

NO OTHER STOPPING CONDITION OCCURS

TMU GRAVITATIONAL CONSTANT OF TARGET PLANET

RRF(3) SPACECRAFT POSITION AT END OF INTEGRATION

DELTAY(3) CORRECTIONS TO BE ADDED TO VELOCITY COMPONENTS

FOR NEXT ITERATION

DVMAX(10) MAXIMUM ALLOWABLE CHANGE IN ANY VELOCITY

COMPONENT FOR EACH EVENT

ACKT TRAJECTORY INTEGRATION ACCURACY

EQECP(3,3) TRANSFORMATION FROM ECLIPTIC TO EQUATORIAL

SYSTEM FOR TARGET PLANE

TIMS INTERNAL CLOCK TIME AT START OF COMPUTER RUN

SPHFAC(10) REDUCTION FACTORS FOR TARGET PLANET SPHERE

OF INFLUENCE FOR EACH EVENT

RPS(10) RADIUS OF PROBE IMPACT SPHERE IN KM FOR VARIOUS

TYPES OF PROBE EVENTS

	/TARINT/ MODE: NOMNAL, ERRAN, SIMUL
	TOTAL NUMBER OF GUIDANCE EVENTS
NOOLD	TOTAL HOUSEN OF GOLDWIGH LANGE
KTIM(10)	EPOCH TO WHICH GUIDANCE EVENT TIMES ARE REFER-
)	ENCED
	=0 EVENT NOT PROCESSED =1 INITIAL TIME
~	=2 SOI TIME
	=3 CA TIME
	=4 CALENDAR DATE
KTYP(10)	TYPE OF GUIDANCE EVENT FOR EACH EVENT
	=-1 TERMINATION EVENT
	=1 TARGETING EVENT
	=2 RETARGETING EVENT
	=3 ORBIT INSERTION EVENT
KMXQ(10)	COMPUTE/EXECUTE MODES FOR EACH GUIDANCE EVENT
	=1 COMPUTE VELOCITY CORRECTION ONLY
	=2 EXECUTE VELOCITY CORRECTION ONLY
	#3 COMPUTE AND IMMEDIATELY EXECUTE CORRECTION #4 COMPUTE BUT EXECUTE CORRECTION LATER
	=4 COMPOSE BUT EXECUTE CURRECTION LATER
MDL(10)	EXECUTION MODELS FOR EACH GUIDANCE EVENT
	=1 IMPULSIVE '
	=2 PULSING ARC
NPAR (10)	NUMBER OF TARGET PARAMETERS IN EACH TARGETING
	EVENT
KTAR (6.10)	CODES OF TARGET PARAMETERS (UP TO 6) FOR EACH
K, AK (0, 10)	TARGETING EVENT OR ORBIT INSERTION OPTION FOR
	EACH INSERTION EVENT
KEYT AR (3)	KEY DEFINING DESIRED TARGET PARAMETERS FOR
RETTAREST	CURRENT EVENT
MAT(10)	TARGETING MATRIX COMPUTATION CODE FOR EACH TAR-
	GETING EVENT
	=1 COMPUTE TARGETING MATRIX ONLY AT FIRST LEVEL =2 COMPUTE TARGETING MATRIX AT EACH STEP
	-2 COMPUTE TARGETTHE MAIKIX AT EACH SIEP

BAD STEP FLAGS FOR EACH TARGETING EVENT IBAUS(10) #1 NEVER USE BAD STEP CHECK USE BAD STEP CHECK AT FINAL LEVEL ONLY **=** 2 =3 USE BAD STEP CHECK AT ALL LEVELS THE NUMBER OF TOTAL ITERATIONS ALLOWED AT THE NOIT(10) FIRST AND LAST LEVELS OF TARGETING EVENTS FOR EACH GUIDANCE EVENT THE NUMBER OF BAD STEPS ALLOWED DURING ANY TAR-MAXB(10) GETING EVENT NUMBER OF ACCURACY LEVELS FOR CURRENT EVENT LEVELS LEV CURRENT LEVEL IN CURRENT TARGETING EVENT ALLOWABLE NUMBER OF ITERATIONS FOR CURRENT EVENT NITS MAXIMUM NUMBER OF BAD ITERATIONS FOR CURRENT MAXBAD EVENT BAD STEP CHECK INDICATOR FOR CURRENT EVENT IBAST MATX MATRIX COMPUTATION CODE FOR CURRENT TARGETING EVENT (SEE DEFN OF MAT) ISTART STAGE OF INITIAL TARGETING =0 NO TARGETING STARTED =1 FIRST PHASE STARTED AND HAVE TARGETING MATRIX =2 SECOND PHASE STARTED AND HAVE MATRIX PHASE COUNTER FOR CURRENT TARGETING EVENT **IPHASE** NOPHAS NUMBER OF TARGETING PHASES FOR CURRENT EVENT FLAG TO CONTROL CONSTRUCTION OF TARGETING MATRIX ITARM =0 DO NOT COMPUTE TARGETING MATRIX =1 COMPUTE TARGETING MATRIX ON CURRENT ITERATION BAD STEP FLAG FOR CURRENT ACCURACY LEVEL IBAD

=1 DO NOT CHECK FOR BAD STEP

=2 CHECK FOR BAD STEP

ISTOP STOPPING CONDITION INDICATOR IN SUBROUTINE

TARGET

=1 STOP ON TIME

=2 STOP AT SPHERE OF INFLUENCE =3 STOP AT CLOSEST APPROACH

NOPAR NUMBER OF TARGET PARAMETERS FOR CURRENT EVENT

KWIT TERMINATION FLAG

=0 CONTINUE RUN =1 TERMINATE RUN

IPRE CASE FLAG

=0 FIRST CASE =1 STACKED CASE

NCPR NUMBER OF INTEGRATION INCREMENTS BETWEEN PRINT-

OUTS OF NOMINAL TRAJECTORY

IFINT (10) NOT USED IN THIS TRAJECTORY VERSION

KGYD(10) INDICES OF EVENTS TO BE PROCESSED

KSICA FLAG INDICATING STAGE OF NOMINAL TRAJECTORY

=1 SOI NOT YET INTERSECTED

=2 SOI INTERSECTED BUT NO CLOSEST APPROACH

=3 CLOSEST APPROACH ALREADY ENCOUNTERED

KUR INDEX OF CURRENT EVENT

KAXTAR(3) KEY DEFINING AUXILIARY PARAMETERS FOR CURRENT

EVENT

LVLS(10) NUMBER OF ACCURACY LEVELS TO BE USED ON EACH FAR-

GETING EVENT

NOSOI OUTER TARGETING FLAG

=0 NORMAL TARGETING =1 OUTER TARGETING

IPCS (10) FLAG SPECIFYING PLANETOCENTRIC COORDINATE SYSTEM

FOR VARIOUS TYPES OF PROBE EVENTS

= 0 EQUATORIAL

=1 SUBSOLAR ORBIT-PLANE

	/TAROIM/ MODE: NOMNAL
DBR	DESIRED VALUE OF AUXILIARY TARGET B.R IN KM TO ACHIEVE ACTUAL TARGET PAIRS OF EITHER INCLINATION AND RADIUS OF CLOSEST APPROACH OR RIGHT ASCENSION AND DECLINATION OF PROBE TARGET SITE
OBT	DESIRED VALUE OF AUXILIARY TARGET B.R IN KM TO ACHIEVE ACTUAL TARGET PAIRS OF EITHER INCLINATION AND RADIUS OF CLOSEST APPROACH OR RIGHT ASCENSION AND DECLINATION OF PROBE TARGET SITE
DOCP	ACTUAL TARGET VALUE OF PROBE SITE DECLINATION IN DEG RELATIVE TO PROBE-SPHERE FRAME
DINC	ACTUAL TARGET VALUE OF INCLINATION AT CLOSEST APPROACH IN DEG RELATIVE TO EQUATORIAL FRAME \
DRAP	ACTUAL TARGET VALUE OF PROBE SITE RIGHT ASCENSION IN DEG RELATIVE TO PROBE-SPHERE FRAME
DRCA	ACTUAL TARGET VALUE OF RADIUS AT CLOSEST APPROACH IN KM
IAUX	FLAG INDICATING TYPE OF AUXILIARY TARGETING =0 NO AUXILIARY TARGETING =1 AUXILIARY TARGETING WITH INCLINATION AND RADIUS AT CLOSEST APPROACH AS ACTUAL TARGETS =2 AUXILIARY TARGETING WITH RIGHT ASCENSION AND DECLINATION OF PROBE IMPACT SITE AS ACTUAL TARGETS
IINCRA	INDEX OF DESIRED INCLINATION OR PROBE SITE RIGHT ASCENSION IN DTAR ARRAY
IRCADC	INDEX OF DESIRED RADIUS OF CLOSEST APPROACH OR PROBE SITE DECLINATION IN DTAR ARRAY
ITARR	FLAG INDICATING OPERATING MODE OF TAROPT (SAME AS ITARO)
XATAR(3)	ARRAY OF ACTUAL TARGET VALUES (SAME AS ATAR)

	/TARVAR/ MODE ERRAN, SIMUL
,	## # # # # # # # # # # # # # # # # # #
XTAR (6, 10)	DESTRED TARGET VALUES
XTOL (6,10)	TOLERANCES ON TARGET PARAMETERS
XAC(5,10)	ACCURACY LEVELS EMPLOYED IN TARGETING
XPERV(10)	VELOCITY PERTURBATION USED TO COMPUTE TARGETING MATRIX
XOVMAX(10)	MAXIMUM ALLOWABLE VELOCITY CORRECTION
XFAC (10)	SPHERE OF INFLUENCE FACTORS
XDELV(3,10)	NCNLINEAR VELOCITY CORRECTION
TGT3(10)	DESIRED TARGET TIMES REFERENCED TO INITIAL TRAJECTORY TIME
LKTAR(6,10)	ARRAY DEFINING TARGET PARAMETERS
LKTP(10)	ARRAY OF TARGET PLANETS
LKLP(10)	ARRAY OF LAUNCH PLANETS
LNPAR(10)	NUMBER OF TARGET PARAMETERS DESIRED
LLVLS(10)	NUMBER OF INTEGRATION ACCURACY LEVLES USED

	#
	/TIM / MODE: ERRAN, SIMUL
	
DATEJ	JULIAN DATE OF INITIAL TRAJECTORY TIME (REFERENCED TO 1950)
	TRET EREMOLD TO 19507
TRTM1	INITIAL TRAJECTORY TIME
DELIM	TIME INCREMENT
FNTM	FINAL TRAJECTORY TIME
UNIVT	UNIVERSAL TIME
TOTUO	TO A SOT ON TIME AT DECEMBER OF THE POTTON
TRTMB	TRAJECTORY TIME AT BEGINNING OF TRAJECTORY
IRINB	
***************************************	/TMW2 / MODE: SIMUL
	/TMW2 / MODE: SIMUL
***************************************	/TMW2 / MODE: SIMUL
	/TMW2 / MODE: SIMUL
T1	/TMW2 / MODE: SIMUL EIGENVECTOR EVENT TIMES
T1 T2	/TMW2 / MODE: SIMUL EIGENVECTOR EVENT TIMES PREDICTION EVENT STARTING TIMES
T1 T2 T4	/TMW2 / MODE: SIMUL EIGENVECTOR EVENT TIMES PREDICTION EVENT STARTING TIMES CONIC COMPUTATION EVENT TIMES

	'/TRAJCD/ MODE: ERRAN, SIMUL
DTMAX	MAXIMUM TIME INCREMENT FOR WHICH ISTMC IS VALID
ACCND	ACCURACY USED IN NUMERICAL DIFFERENCING IF NDACC INDICATES
DTSUN	STATE TRANSITION INTEGRATION INTERVAL WHEN THE SUN IS CENTRAL BODY AND ISTM1=1
OTPL AN	STATE TRANSITION INTEGRATION INTERVAL WHEN TARGET PLANET IS CENTRAL BODY AND ISTM1=1
NTMC	NOMINAL TRAJECTORY CODE
ISTMC	STATE TRANSITION MATRIX CODE
ISTM1	ALTERNATE STATE TRANSITION MATRIX CODE
NDACC	NUMERICAL DIFFERENCING ACCURACY CODE

/TRIIAR / MODE: ERRAN, SIMUL

	OFFI THATTONS OF A MINT BRODE TAGETS
DC (P (3)	DECLINATIONS OF 3 MINI-PROBE TAGETS
RATP (3)	RIGHT ASCENSIONS OF 3 MINI-PROBE TARGETS
VTANGM	MINI-PROBE TANGENTIAL VELOCITY
DCSAF	FIXED SPIN AXIS DECLINATION AT RELEASE
RASAF	FIXED SPIN AXIS RIGHT ASCENSION AT RELEASE
Su	STEP SIZE UPPER BOUND IN THE CONTROL SPACE
ACTPP	VMP ACCURACY LEVEL FOR MINI-PROBE TARGETING
FACTR	FRACTION OF SOI INTEGRATED TO BEFORE CONIC PROPAGATION STARTS
XSAVE(6)	STATE VECTOR AT RELEASE
WFLS (3)	WEIGHTING FACTORS FOR TARGET SITES
UCNTRL(5)	TARGET CONTROLS
IPCSK	=1 SUBSOLAR COORDINATE SYSTEM =2 EQUATORIAL COORDINATE SYSTEM
ISAO	SPIN AXIS ORIENTATION FLAG
IPROPI	TRAJECTORY PROPAGATION CODE

	TRJ / MODE: ERRAN, SIMUL
RCA1 (6)	STATE AT CLOSEST APPROACH ON ORIGINAL NOMINAL
RCA2 (6)	STATE AT CLOSEST APPROACH ON MOST RECENT NOMINAL
RCA3 (6)	STATE AT CLOSEST APPROACH ON ACTUAL TRAJECTORY
	POSITION AT SPHERE OF INFLUENCE ON ORIGINAL NOMINAL
RS012(3)	POSITION AT SPHERE OF INFLUENCE ON MOST RECENT NOMINAL
	POSITION AT SPHERE OF INFLUENCE ON ACTUAL TRAJECTORY
	VELOCITY AT SPHERE OF INFLUENCE ON ORIGINAL NOMINAL
VS012(3)	VELOCITY AT SPHERE OF INFLUENCE ON MOST RECENT NOMINAL
A2013(3)	VELOCITY AT SPHERE OF INFLUENCE ON ACTUAL TRAJECTORY
TCA1	TIME AT CLOSEST APPROACH OF ORIGINAL NOMINAL
T C A 2	TIME AT CLOSEST APPROACH OF MOST RECENT NOMINAL
TCA3	TIME AT CLOSEST APPROACH OF ACTUAL TRAJECTORY
TS011	TIME AT SPHERE OF INFLUENCE OF ORIGINAL NOMINAL
TS012	TIME AT SPHERE OF INFLUENCE OF MOST RECENT NOMINAL
12013	TIME AT SPHERE OF INFLUENCE OF ACTUAL TRAJECTORY

	/TMTRIX/ MODE: NOMNAL, ERRAN, SIMUL
ICA3	CLOSEST APPROACH CODE FOR ACTUAL TRAJECTORY
ICA2	CLOSEST APPROACH CODE FOR MOST RECENT NOMINAL
ICA1	CLOSEST APPROACH CODE FOR ORIGINAL NOMINAL
15013	NOMINAL SPHERE OF INFLUENCE CODE FOR ACTUAL TRAJECTORY
IS0I2	SPHERE OF INFLUENCE CODE FOR MOST RECENT
ISOI1	SPHERE OF INFLUENCE CODE FOR ORIGINAL NOMINAL
BDRS 13	B DOT R ON ACTUAL TRAJECTORY
BDRS 12	B DOT R ON MOST RECENT NOMINAL
BORSI1	8 DOT R ON ORIGINAL NOMINAL
BOTS 13	B DOT I ON ACTUAL TRAJECTORY
BOTSIZ	B DOT T ON MOST RECENT NOMINAL
BDTS11	B DOT T ON ORIGINAL NOMINAL
BS13	B ON ACTUAL TRAJECTORY
BSI2	B ON MOST RECENT NOMINAL
BSI1	B ON ORIGINAL NOMINAL

CHI(3,3) SENSITIVITY MATRIX (TRANSFERRED FOR OUTPUT)

/TPTIN / MODE: ERRAN , SIMUL

GHUP GRAVITATIONAL CONSTANT OF TARGET PLANET

RTPS RADIUS OF PROBE SPHERE

T(3,3) COORDINATE SYSTEM TRANSFORMATION MATRIX

DJERN JULIAN DATE OF TARGETING EVENT

DTPRSC TIME FROM PERIAPSIS TO NOMINAL RELEASE TIME

RSCRPA(3) RELEASE POSITION OF S/C

VSCRPA(3) RELEASE VELOCITY OF S/C

RSCRPM MAGNITUDE OF RELEASE POSITION VECTOR

CSDCSA COS OF DECLINATION OF SPIN AXIS

SNDCSA SIN OF DECLINATION OF SPIN AXIS

CSRASA COS OF RIGHT ASCENSION OF SPIN AXIS

SNRASA SIN OF RIGHT ASCENSION OF SPIN AXIS

NNTP NUMBER OF THE TARGET PLANET

IMIN INDEX OF THE MINI-PROBE NEAREST THE S/C AT IMPACT

KKWIT =1 NO CONVERGENCE IN MINI-PROBE TARGETING

=0 CONVERGENCE IN MINI-PROBE TARGETING

	/TPTIN / MODE: NOMNAL
AATTP(3)	ARRAY OF ANGLES OF ATTACK IN DEG FOR MINIPROBES AT IMPACT
ACTPP	ACCURACY LEVEL OF VMP MINIPROBE PROPAGATION
CSDCSA	COSINE OF ECLIPTIC DECLINATION OF BUS SPIN AXIS AT RELEASE
CSRASA	COSINE OF ECLIPTIC RIGHT ASCENSION OF BUS SPIN AXIS AT RELEASE
DCTP (3)	ARRAY OF TARGET SITE DECLINATIONS OF MINIPROBES IN DEG RELATIVE TO PLANETOCENTRIC PROBE-SPHERE FRAME
DELTM	MAXIMUM VIRTUAL-MASS PROPAGATION INTERVAL FOR BUS AND MINIPROBES
DJEITP(3)	ARRAY OF JULIAN DATES OF IMPACT FOR MINIPROBES EPOCH 1900
DJERN	JULIAN DATE EPOCH 1900 OF MINIPROBE RELEASE
DTPRSC	TIME INTERVAL IN SEC ON BUS NEAR-PLANET OSCULATING CONIC FROM PERIAPSIS TO RELEASE STATE
FPATP(3)	ARRAY OF FLIGHT PATH ANGLES IN DEG FOR MINIPROBES AT IMPACT
GMUP	GRAVITATIONAL CONSTANT OF TARGET PLANET IN KM**3/SEC**2
IFIN2	FLAG INDICATING OPERATING MODE OF TPPROP =1 MISS-MINIMIZATION IS IN PROCESSOBTAIN PHI AS A FUNCTION OF UCNTRL =2 MISS-MINIMIZATION IS COMPLETEOBTAIN MINIPROBE IMPACT DATA FOR MINIMUM-MISS RELEASE CONTROLS
IMIN	INDEX OF MINIPROBE WHOSE IMPACT PLANE ASYMPTOTE PIERCE POINT IS NEAREST THAT OF BUS
IPROP	FLAG INDICATING MINIPROBE PROPAGATION MODE TO BE USED IN TPPROP (MAY DIFFER FROM THAT REQUESTED IN IPROPI) =1 CONIC =2 VIRTUAL-MASS
ISAO	FLAG INDICATING SPIN AXIS ORIENTATION MODE

- =1 BOTH SPIN AXIS DECLINATION AND RIGHT ASCENSION ARE FREE CONTROLS
- =2 SPIN AXIS IS COINCIDENT WITH OUS VELOCITY VECTOR AT RELEASE
- =3 SPIN AXIS IS NORMAL TO BUS/SUN LINE, PARALLEL TO ECLIPTIC PLANE, AND WITHIN 90 DEG OF BUS VELOCITY VECTOR
- =4 SPIN AXIS DECLINATION AND RIGHT ASCENSION ARE BOTH FIXED

RATP (3) ARRAY OF TARGET SITE RIGHT ASCENSIONS OF MINIPROBES IN DEG RELATIVE TO PLANETOCENTRIC PROBE-SPHERE FRAME HELIOCENTRIC ECLIPTIC POSITION VECTOR OF BUS IN RSCRHA(3) KM AT MINIPROBE RELEASE RSCRPA(3) PLANETOCENTRIC ECLIPTIC POSITION VECTOR IN KM ON BUS NEAR-PLANET OSCULATING CONIC AT EQUIVALENT RELEASE STATE MAGNITUDE OF PLANETOCENTRIC POSITION VECTOR OF RSCRPM BUS IN KM AT EQUIVALENT CONIC RELEASE STATE RTPS RADIUS OF MINIPROBE IMPACT SPHERE IN KM (SAME AS RADIUS OF PROBE IMPACT SPHERE) SNDCSA SINE OF ECLIPTIC DECLINATION OF BUS SPIN AXIS AT RELEASE SNRASA SINE OF ECLIPTIC RIGHT ASCENSION OF BUS SPIN AXIS AT RELEASE TRANSF TRANSFORMATION MATRIX FROM PLANETOCENTRIC ECLIPTIC TO PROBE-SPHERE COORDINATE FRAME VIMTP(3) ARRAY OF VELOCITY MAGNITUDES OF MINIPROBES AT IMPACT IN KM/SEC VSCRHA(3) HELIOCENTRIC ECLIPTIC VELOCITY VECTOR OF BUS IN KM/SEC AT MINIPROBE RELEASE VSCRPA(3) PLANETOCENTRIC ECLIPTIC VELOCITY VECTOR IN KM/SEC ON BUS NEAR-PLANET OSCULATING CONIC AT EQUIVALENT RELEASE STATE WFLS (3) ARRAY OF WEIGHTING FACTORS APPLIED TO IMPACT PLANE MISS DISTANCES AT RESPECTIVE MINIPROBE TARGET SITES IN LEAST-SQUARES MISS-MINIMIZATION PROCESS

/VM / MODE: NOMNAL, ERRAN, SIMUL

TM TIME UNITS PER DAY

ALNGTH

DELTP PRINT INCREMENTS (IN DAYS)

RC(6) STATE AT CLOSEST APPROACH

DC JULIAN DATE, EPOCH 1908, AT CLOSEST APPROACH

LENGTH UNITS PER A.U.

RSI(3) POSITION AT SPHERE OF INFLUENCE

VSI(3) VELOCITY AT SPHERE OF INFLUENCE

DSI JULIAN DATE, EPOCH 1900, AT SPHERE OF

INFLUENCE

RVS(6) POSITION OF VEHICLE RELATIVE TO VIRTUAL MASS

VMU GRAVITATIONAL CONSTANT OF VIRTUAL MASS

B B AT SPHERE OF INFLUENCE

BOT B DOT T

BOR BOOTR

DELTH INCREMENT IN TRUE ANOMALY USED 4

TIMINT TOTAL TIME USED

RE(6) POSITION AND VELOCITY OF EARTH

RTP(6) POSITION AND VELOCITY OF TARGET PLANET

CAINC INCLINATION AT CLOSEST APPROACH

RCA MAGNITUDE OF CLOSEST APPROACH POSITION VECTOR

TACA TRAJECTORY SEMIMAJOR AXIS WITH RESPECT TO TARGET

BODY AT CLOSEST APPROACH TO TARGET BODY

SSS(3) DIRECTION COSINE VECTOR OF SPACECRAFT SPIN AXIS

NLP CODE OF LAUNCH PLANET

NBOD NUMBER OF BODIES USED IN VIRTUAL MASS PROGRAM

NB(11) CODES OF PLANETS

NTP CODE OF TARGET PLANET

INPR PRINT INCREMENTS (IN INCREMENTS)

IPROB PROBLEM NUMBER

ISPH SPHERE OF INFLUENCE CODE

=0 SPHERE OF INFLUENCE NOT INTERSECTED =1 SPHERE OF INFLUENCE ALREADY ENCOUNTERED

INCMT TOTAL INCREMENTS USED

IEPHEM EPHEMERIS CODE

ICL CLOSEST APPROACH CODE

=0 CLOSEST APPROACH NOT ENCOUNTERED =1 CLOSEST APPROACH ALREADY ENCOUNTERED

IPRINT PRINT CODE

=0 OUTPUT INITIAL AND FINAL DAFA

=1 DO NOT OUTPUT INITIAL AND FINAL DATA

ICL2 CLOSEST APPROACH TERMINATION CODE

=0 DO NOT STOP AT CLOSEST APPROACH

=1 STOP AT CLOSEST APPROACH

***	/XXXL / MODE: ERRAN, SIMUL
XSL(24)	SOLVE-FOR PARAMETER LABELS
XU(8)	DYNAMIC CONSIDER PARAMETER LABELS
XV (15)	MEASUREMENT CONSIDER PARAMETER LABELS
XLAB (6)	VEHICLE POSITION/ VELOCITY VECTOR COMPONENT NAMES
XNM(24)	AUGMENTATION PARAMETER LABELS
KPRINT	CORRELATION MATRIX PRINT CODE
~~~~	/ZERDAT/ MODE: NOMNAL
	**************************************
ZDAT (6)	ZERO ITERATE VECTOR
RP	PARKING ORBIT RADIUS
FI	INJECTION TRUE ANOMALY
PSI1	ANGLE OF FIRST BURN —
PS12	ANGLE OF SECOND BURN
TIM1	TIME INTERVAL OF FIRST BURN
TIM2	TIME INTERVAL OF SECOND BURN -
THELS	LONGITUDE OF LAUNCH SITE
PHILS	LATITUDE OF LAUNCH SITE
TI	NOT USED

TF NOT USED

RPRAT

THEDOT ROTATION RATE OF LAUNCH PLANET

PARKING ORBIT INVERSE RATE

SIGMAL NOMINAL LAUNCH AZIMUTH

IZERO ZERO ITERATION FLAG

=0 INITIAL STATE READ IN

=1 PLANET-TO-PLANET =2 PLANET-TO-POINT =3 POINT-TO-PLANET =4 POINT-TO-POINT =10 LUNAR TARGETING

KOAST PARKING ORBIT INDICATOR

> =-1 SHORT COAST =1 LONG COAST

TYPE OF MISSION FOR TARGETING LTARG

=0 INTERPLANETARY MISSION

=1 LUNAR MISSION

/ZOUT / MODE: NOMNAL

MAGNITUDE OF HYPERBOLIC EXCESS VELOCITY AT TARGET MAHA

BODY (VHP VECTOR)

DPA DECLINATION OF VHP

RAP RIGHT ASCENSION OF VHP

## 5.2 COMMON VARIABLES IN ALPHABETICAL ORDER

IN THIS SECTION ALL VARIABLES APPEARING IN COMMON ARE LISTED AND DEFINED IN ALPHABETICAL ORDER. THE SECOND FIELD SERVES TO IDENTIFY THE BLOCK IN WHICH THE VARIABLE APPEARS.

A(2,3)	-PBLK	FTA IMPACT PLANE TRANSFORMATION MATRIX
AALP(20)	SIMONT	ACTUAL ERROR IN POINTING ANGLE 1
AAUX(3)	TAREAL	ACTUAL AUXILIARY VALUES OF ITERATE
ABET (20)	SIMONT	ACTUAL ERROR IN POINTING ANGLE 2
AC(5,10)	TAREAL	ACCURACY LEVELS (UP TO 5) USED IN EACH GUIDANCE EVENT
ACC	MISC	ACCURACY FIGURE USED IN VIRTUAL MASS PRO- GRAM
ACC1	SIM2	ACCURACY USED IN ACTUAL TRAJECTORY
ACCND	TRAJCD	ACCURACY USED IN NUMERICAL DIFFERENCING IF NDACC INDICATES
ACKT	TAREAL	TRAJECTORY INTEGRATION ACCURACY
ADA(3,6)	BAIM	VARIATION MATRIX
ADEVS8(24)	SIM1	ACTUAL DEVIATION IN SOLVE-FOR PARAMETERS AT TRAJECTORY BEGINNING
ADEVX(6)	SIM1	ACTUAL DEVIATION IN THE STATE VECTOR
ADEVX8(6)	SIM1	ACTUAL DEVIATION IN STATE VECTOR AT BEGIN- NING OF TRAJECTORY
ADEVXS (24)	SIM1	ACTUAL DEVIATION IN SOLVE-FOR PARAMETERS
AINC?	EVENT	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN IN EXISTING PROGRAM
AK(6,4)	STM	KALMAN GAIN CONSTANT FOR POSITION/VELOCITY STATE

AL (4,15)	STM	OBSERVATION MATRIX RELATING OBSERVABLES TO MEASUREMENT CONSIDER PARAMETER STATE
ALNGTH	VM	LENGTH UNITS PER A.U.
AM (4 , 24)	STM	OBSERVATION MATRIX RELATING OBSERVABLES TO SOLVE-FOR PARAMETER STATE
ANODE7	EVENT	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN IN EXISTING PROGRAM
ANG	IMPTAR	TARGET INCLINATION CONVERTED FROM INPUT FORMAT TO VALUE BETWEEN 0 AND 180 DEGREES AND SATISFYING APPROACH ASYMPTOTE CONSTRAINT
ANOIS (4)	SIM1	ACTUAL WHITE NOISE
APRO (20)	SIMONT	ACTUAL PROPORTIONALITY ERROR
AR(4,4)	SIM1	ACTUAL MEASUREMENT NOISE
ARES (20)	SIMONT	ACTUAL RESOLUTION ERROR
ATAR(3)	TAREAL	ACTUAL TARGET VALUES OF ITERATE
ATRANS(6)	BAIM	CLOSEST APPROACH STATE
AVARM(12)	SIMONT	VARIANCE OF ACTUAL MEASUREMENT NOISE
AY (4)	SIM1	ACTUAL MEASUREMENT
AYMEY (4)	SIH1	ACTUAL MEASUREMENT MINUS ESTIMATED MEASUREMENT
В	VM	B AT SPHERE OF INFLUENCE
BCON(3)	LUNART	MAXIMUM STEP SIZES OF CONTROLS
BDR	VM	B DOT R
BOT	VM	B DOT T
BIA(12)	HISC	MEASUREMENT BIASES

BSI1	TRJ	B ON ORIGINAL NOMINAL
8212	TRJ	B ON MOST RECENT NOMINAL
BSI3	TRJ	B ON ACTUAL TRAJECTORY
BOTSI1	TRJ	B DOT T ON ORIGINAL NOMINAL
BDTSI2	TRJ	B DOT T ON MOST RECENT NOMINAL
BDTS13	TRJ	B DOT T ON ACTUAL TRAJECTORY
BDRSI1	TRJ	B DOT R ON ORIGINAL NOMINAL
BDRSI2	TRJ	B DOT R ON MOST RECENT NOMINAL
BDRS13	TRJ	B DOT R ON ACTUAL TRAJECTORY
CAI	LUNART	DESIRED CLOSEST APPROACH EQUATORIAL INCLINATION
CAINC	VM	INCLINATION AT CLOSEST APPROACH
CHI(3,3)	TMTRX	SENSITIVITY MATRIX (TRANSFERRED FOR OUTPUT)
CHPNM (30)	NA ME	COMPONENT NAME
CN(80)	BLK	CONSTANTS USED TO CALCULATE THE ORBITAL ELEMENTS OF THE FIRST FIVE PLANETS (SEE LARGE ARRAY DEFNS IN SECTION 5.3)
CN (80)	BLK	ELEMENTS OF THE FIRST FIVE PLANETS
		ELEMENTS OF THE FIRST FIVE PLANETS' (SEE LARGE ARRAY DEFNS IN SECTION 5.3)
CR .	PBLK	ELEMENTS OF THE FIRST FIVE PLANETS' (SEE LARGE ARRAY DEFNS IN SECTION 5.3)  CAPTURE RADIUS OF TARGET PLANET
CR (6)	PBLK TAREAL	ELEMENTS OF THE FIRST FIVE PLANETS' (SEE LARGE ARRAY DEFNS IN SECTION 5.3)  CAPTURE RADIUS OF TARGET PLANET  TOLERANCES FOR CURRENT EVENT  CORRELATION BETHEEN SOLVE-FOR, PARAMETERS
CR CTOL (6) CXSU(24,8)	PBLK TAREAL STM	ELEMENTS OF THE FIRST FIVE PLANETS' (SEE LARGE ARRAY DEFNS IN SECTION 5.3)  CAPTURE RADIUS OF TARGET PLANET  TOLERANCES FOR CURRENT EVENT  CORRELATION BETHEEN SOLVE-FOR PARAMETERS AND DYNAMIC CONSIDER PARAMETERS  CORRELATION BETHEEN SOLVE-FOR PARAMETERS AND DYNAMIC CONSIDER PARAMETERS AT

v		
CXSV (24,15)	STM	CORRELATION BETWEEN SOLVE-FOR PARAMETERS AND MEASUREMENT CONSIDER PARAMETERS
CXSV8(24,15)	STM	CORRELATION BETWEEN SOLVE-FOR PARAMETERS AND MEASUREMENT CONSIDER PARAMETERS AT INITIAL TIME
CXSVG (24,15)	GUI	CONTROL CORRELATION BETHEEN SOLVE-FOR PARAMETERS AND MEASUREMENT CONSIDER PARAMETERS
CXSVP(24,15)	STM	CORRELATION BETWEEN SOLVE-FOR PARAMETERS AND MEASUREMENT CONSIDER PARAMETERS PRIOR TO PROCESSING A MEASUREMENT
CXU(6,8)	STM	CORRELATION BETWEEN POSITION/VELOCITY STATE AND DYNAMIC CONSIDER PARAMETERS
CXUB (6,8)	STM	CORRELATION BETWEEN POSITION/VELOCITY STATE AND DYNAMIC CONSIDER PARAMETERS AT INITIAL TIME
CXUG (6,8)	GUI	CONTROL CORRELATION BETWEEN POSITION/ VELOCITY STATE AND DYNAMIC CONSIDER PARAMETERS
CXUP (6,8)	STM	CORRELATION BETWEEN POSITION/VELOCITY STATE AND DYNAMIC CONSIDER PARAMETERS PRIOR TO PROCESSING A MEASUREMENT
CXV(6,15)	STM	CORRELATION BETWEEN POSITION/VELOCITY STATE AND MEASUREMENT CONSIDER PARAMETERS
CXVB(6,15)	STM	CORRELATION BETWEEN POSITION/VELOCITY STATE AND MEASUREMENT CONSIDER PARMETERS AT INITIAL TIME
CXVG(6,15)	GUI	CONTROL CORRELATION BETWEEN POSITION/ VELOCITY STATE AND MEASUREMENT CONSIDER PARAMETERS
CXVP(6,15)	STM	CORRELATION BETWEEN POSITION/VELOCITY' STATE AND MEASUREMENT CONSIDER PARAMETERS PRIOR TO PROCESSING A MEASUREMENT

CXXS(6,24)	STM	CORRELATION BETWEEN POSITION/VELOCITY STATE AND SOLVE-FOR PARAMETERS
CXXSB(6,24)	STM	CORRELATION BETWEEN POSITION/VELOCITY STATE AND SOLVE-FOR PARAMETERS AT INITIAL TIME
CXXSG (6,24)	GUI	CONTROL CORRELATION BETWEEN POSITION/ VELOCITY STATE AND SOLVE-FOR PARAMETERS
CXXSP(6,24)	STM	CORRELATION BETWEEN POSITION/VELOCITY STATE AND SOLVE-FOR PARAMETERS PRIOR TO PROCESSING A MEASUREMENT
D1	TAREAL	JULIAN DATE ASSOCIATED WITH RIN ARRAY
DAB	SIMCNT	BIAS IN SEMI-MAJOR AXIS OF TARGET PLANET
DATEJ	TIME	JULIAN DATE OF INITIAL TRAJECTORY TIME (REFERENCED TO 1950)
DAUX(3)	TAREAL	DESIRED AUXILIARY PARAMETER VALUES OF ITERATE
DC	VM	JULIAN DATE, EPOCH 1900, AT CLOSEST APPROACH
DEB	SIMCNT	BIAS IN ECCENTRICITY OF TARGET PLANET
DECLIN	LUNART	DECLINATION OF APPROACH ASYMPTOTE WITH RESPECT TO LUNAR EQUATOR
DELAXS	CONST3	TARGET PLANET SEMI-MAJOR AXIS FACTOR USED IN NUMERICAL DIFFERENCING
DELECC	CONST3	TARGET PLANET ECCENTRICITY FACTOR USED IN NUMERICAL DIFFERENCING
DELICL	CONST3	TARGET PLANET INCLINATION FACTOR USED IN NUMERICAL DIFFERENCING
DELMA '	CONST3	TARGET PLANET MEAN ANOMALY FACTOR USED IN NUMERICAL DIFFERENCING
DELMUP	CONST3	TARGET PLANET GRAVITATIONAL CONSTANT FACTOR USED IN NUMERICAL DIFFERENCING
DELMUS	CONST3	SUN GRAVITATIONAL CONSTANT FACTOR USED IN NUMERICAL DIFFERENCING

DELNOD	CONST3	TARGET PLANET LONGITUDE OF THE ASCENDING NODE FACTOR USED IN NUMERICAL DIFFERENCING
DELPX(3)	EXĒ	VELOCITY CORRECTION TO BE MODELED AS AN IMPULSE SERIES
DELTAT	TAREAL	NUMBER OF DAYS INTEGRATION IS TO CONTINUE IF NO OTHER STOPPING CONDITION OCCURS
DELTAV(3)	TAREAL	CORRECTIONS TO BE ADDED TO VELOCITY COMPONENTS FOR NEXT ITERATION
DELTH	VM	INCREMENT IN TRUE ANOMALY USED
DELTM	TIME	TIME INCREMENT
DELTP	VM	PRINT INCREMENTS (IN DAYS)
DELV(3,10)	TAREAL	EXTERNALLY SUPPLIED VELOCITY CORRECTION OR VELOCITY INCREMENT COMPUTED BY INSERTION
DELV(3,10)	PBLK	ARRAY OF EXTERNALLY-SUPPLIED VELOCITY CHANGES
DELM	CONST3	TARGET PLANET ARGUMENT OF PERIAPSIS FACTOR USED IN NUMERICAL DIFFERENCING
DG(10)	TAREAL	JULIAN DATES OF EVENT TIMES
DIB	SIMCNT	BIAS IN INCLINATION OF TARGET PLANET
DINTG(10)	TAREAL	NOT USED IN CURRENT TARGET VERSION
DIPX	EXE	JULIAN DATE TRANSFERRED TO EXCUT OR EXCUTS
DMAB	SIMONT	BIAS IN MEAN ANOMALY
DMUPB	SIMONT	BIAS IN GRAVITATIONAL CONSTANT OF TARGET PLANET
DMUSB	SIMONT	BIAS IN GRAVITATIONAL CONSTANT OF SUN
DNCN(3)	CONST	CONSTANTS FROM WHICH DYNAMIC NOISE IS COMPUTED
DNOB	SIMONT	BIAS IN LONGITUDE OF ASCENDING NODE
DPA	ZOUT	DECLINATION OF VHP

DSI	VM	JULIAN DATE, EPOCH 1900, AT SPHERE OF INFLUENCE
07(10)	TAREAL	JULIAN DATES OF TARGET TIMES
DTAR(3)	TAREAL	DESIRED TARGET VALUES OF ITERATE
DTAR(3)	LUNART	TARGET VALUES OF SMA, B.T, AND B.R IN LUNAR TARGETING
DTI	PULS	TIME INTERVAL (DAYS) BETWEEN SUCCESSIVE PULSES
DTIME	OVERL	TIME INTERVAL BETWEEN ORBITAL INSERTION DECISION AND EXECUTION
DTMAX	TRAJCD	MAXIMUM TIME INCREMENT FOR WHICH ISTMC IS VALID
DTPLAN	TRAJCD	STATE TRANSITION INTEGRATION INTERVAL WHEN TARGET PLANET IS CENTRAL BODY AND ISTM1=1
DTSUN	TRAJCD	STATE TRANSITION INTEGRATION INTERVAL WHEN THE SUN IS CENTRAL BODY AND ISTM1=1
DUMMYQ(4)	EXE	ARRAY OF EXECUTION ERROR VARIANCES
DUR	PULS	DURATION OF SINGLE PULSE
DV8(3)	EVENT	ORBIT INSERTION VARIABLE. NON-FUNCTIONAL IN EXISTING PROGRAM
DVF(3)	PULS	VELOCITY INCREMENT ADDED ON FINAL PULSE
DVI(3)	PULS	VELOCITY INCREMENT ADDED ON TYPICAL PULSE
DVMAX(10)	TAREAL	MAXIMUM ALLOWABLE CHANGE IN ANY VELOCITY COMPONENT FOR EACH EVENT
DVN(3)	PBLK	COMMANDED VELOCITY CORRECTION TRANSFERRED TO BIAIM
DVRB(3)	PBLK	VELOCITY CORRECTION REQUIRED TO REMOVE AIMPOINT BIAS

DVUP(3)	PBLK	UPDATE VELOCITY CORRECTION
DMB	SIMONT	BIAS IN ARGUMENT OF PERIAPSIS
ECC7	EVENT	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN IN EXISTING PROGRAM
EDEVSM(24)	SIM1	ESTIMATED DEVIATION IN SOLVE-FOR PARAMETERS (FOR ADAPTIVE FILTERING)
EDEVX (6)	SIM1	ESTIMATED DEVIATION IN THE STATE VECTOR
EDEVXM(6)	SIM1	ESTIMATED DEVIATION IN THE STATE VECTOR (FOR ADAPTIVE FILTERING)
EBEVXS (24)	SIM1	ESTIMATED DEVIATION IN SOLVE-FOR PARAMETERS
EIGHT	DPNUM	THE NUMBER EIGHT (8) TO NINE SIGNIFICENT FIGURES. TARGETING MODE ONLY
ELMNT (80)	BLK	CONTAINS THE ORBITAL ELEMENTS OF THE PLANETS (SEE LARGE ARRAY, DEFNS IN SECTION 5.3)
EM(2,6)	GUI	VARIATION MATRIX RELATING POSITION /VELOCITY DEVIATIONS TO B.T AND B.R DEVIATIONS
EM1	DPNUM	THE NUMBER 1.E-1 TO NINE SIGNIFICANT FIGURES
EM2	DP NUM	THE NUMBER 1.E-2 TO NINE SIGNIFICANT FIGURES
EM3	DPNUM	THE NUMBER 1.E-3 TO NINE SIGNIFICANT FIGURES
EM4	DPNUM	THE NUMBER 1.E-4 TO NINE SIGNIFICANT FIGURES
EM5	DP NUM	THE NUMBER 1.E-5 TO NIME SIGNIFICANT FIGURES
EM6	DPNUM	THE NUMBER 1.E-6 TO NINE SIGNIFICANT FIGURES

EM7	DPNUM	THE NUMBER 1.E-7 TO NINE SIGNIFICANT FIGURES
EM8	DP NUM	THE NUMBER 1.E-8 TO NINE SIGNIFICANT FIGURES
EM9	DPNUM	THE NUMBER 1.E-9 TO NINE SIGNIFICANT FIGURES
EM13	DPNUM	THE NUMBER 1.E-13 TO NINE SIGNIFICANT FIGURES
E M 50	DP NUM	THE NUMBER 1.E-50 TO NINE SIGNIFICANT FIGURES
EMN(15)	8L K	THE CONSTANTS USED TO CALCULATE THE ORBIT- AL ELEMENTS OF THE MOON
EMU	LUNART	GRAVITATIONAL CONSTANT OF EARTH(KM3/SEC2)
EPS	CONST	OBLIQUITY OF EARTH
EQECP(3,3)	TAREAL	TRANSFORMATION FROM ECLIPTIC TO EQUATORIAL SYSTEM FOR TARGET PLANE
EQLQ(3,3)	LUNART	TRANSFORMATION MATRIX FROM EARTH- EQUATORIAL TO LUNAR EQUATORIAL COORDINTES
EVNM (11)	NAME	EVENT NAME
EXEC (3,3)	PBLK	EXECUTION ERROR COVARIANCE MATRIX
EY (4)	SIM1	ESTIMATED MEASUREMENT
F (44 ₉ 4)	COM	CONTAINS THE POSITIONS AND VELOCITIES OF THE PLANETS AT A SPECIFIED TIME PLUS THE POSITIONS AND VELOCITIES OF THE SPACE-CRAFT RELATIVE TO THE PLANETS
FACE33	TAREAL	SCALING FACTORS USED IN BAD STEP CHECK
FACP		
I NOF	MISC	POSITION FACTOR USED IN NUMERICAL DIFFER- ENCING
FACV	MISC	

FIVE	DPNUM	THE NUMBER FIVE (5) TO NINE SIGNIFICANT FIGURES. TARGETING MODE ONLY
FNTM	TIME	FINAL TRAJECTORY TIME
FOP	CONST2	OFF-DIAGONAL ANNIHILATION VALUE FOR POSITION EIGENVALUES
FOUR	DPNUM	THE NUMBER FOUR (4) TO NINE SIGNIFICANT FIGURES. TARGETING MODE ONLY
FOV	CONST2	OFF-DIAGONAL ANNIHILATION VALUE FOR VELOCITY EIGENVALUES
FS (2,5)	PULS	F-SERIES COEFFICIENTS OF LAUNCH AND TARGET BODIES
G(4,8)	STM	OBSERVATION MATRIX RELATING OBSERVABLES TO DYNAMIC CONSIDER PARAMETER STATE
GA(3,6)	OVERZ	GUIDANCE MATRIX
GG(3)	PULS	GRAVITIONAL CONSTANTS OF SUN, LAUNCH, AND TARGET BODIES
GS (2,4)	PULS	G-SERIES COEFFIEIENTS OF LAUNCH AND TARGET BODIES
H (4,6)	STM	OBSERVATION MATRIX RELATING OBSERVABLES TO POSITION/VELOCITY STATE
HALF	DP NUM	THE NUMBER ONE-HALF (1/2) TO NINE SIGNIF- ICANT FIGURES
НР7	EVENT	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN EXISTING PROGRAM
HPHR (4,4)	STM	NON-FUNCTIONAL IN PRESENT ERROR ANALYSIS PROGRAM
IAMNF	SIMCŅT	ACTUAL MEASUREMENT NOISE FLAG
IAUGIN(24)	STVEC	INPUT AUGMENTATION VECTOR OF ONE S AND ZERO-S
I AUG (24) -	STVEC	AUGMENTATION VECTOR
IAUGDC(8)	STVEC	DYNAMIC CONSIDER AUGMENTATION VECTOR

IAUGMC(15)	STVEC	MEASUREMENT CONSIDER AUGMENTATION VECTOR
IBAD	TARINT,	BAD STEP FLAG FOR CURRENT ACCURACY LEVEL =1, DO NOT CHECK FOR BAD STEP =2, CHECK FOR BAD STEP
IBADS(10)	TARINT	BAD STEP FLAGS FOR EACH TARGETING EVENT =1 NEVER USE BAD STEP CHEQUE =2 USE BAD STEP CHECK AT FINAL LEVEL ONLY =3 USE BAD STEP CHECK AT ALL LEVELS
IBAG	BAIM	NOT USED
IBARY	CNTRIC	REFERENCE COORDINATE SYSTEM CODE
IBAST	TARINT	BAD STEP CHECK INDICATOR FOR CURRENT EVENT
IBIAS	PBLK ,	BIASED AIMPOINT GUIDANCE EVENT FLAG = 0 AIMPOINT NOT BIASED =1 AIMPOINT BIASED
ICA1	TRJ	CLOSEST APPROACH CODE FOR ORIGINAL NOMINAL
ICA2	TRJ	CLOSEST APPROACH CODE FOR MOST RECENT NOMINAL
ICA3	TRJ	CLOSEST APPROACH CODE FOR ACTUAL TRAJECTORY
ICDQ3(20)	EVENT	ARRAY OF CODES WHICH DETERMINE WHICH EXECUTION POLICIES ARE TO BE USED IN GUIDANCE EVENTS
ICDT3(20)	EVENT	CODES WHICH DETERMINE WHICH GUIDANCE POLICIES ARE BEING USED
ICL	VM	CLOSEST APPROACH CODE =0 CLOSEST APPROACH NOT ENCOUNTERED =1 CLOSEST APPROACH ENCOUNTERED
ICFS	YH	CLOSEST APPROACH TERMINATION CODE =0 DO NOT STOP AT CLOSEST APPROACH =1 STOP AT CLOSEST APPROACH
ICODE	OVER1	MEASUREMENT CODE
ICOOR	MISC	CODE TO DETERMINE WHICH COORDINATE SYSTEM THE INITIAL STATE VECTOR IS INPUT

ICOORD	CNTRIC	NON-FUNCTIONAL IN ERROR ANALYSIS MODE
IDENS	PBLK	PROBABILITY DENSITY FUNCTION CODE. NON-FUNCT.ONAL
IDNF	MISC	DYNAMIC NOISE FLAG
IEIG	EVENT	CODE USED TO DECIDE IF BOTH POSITION AND VELOCITY EIGENVECTORS ARE REQUESTED
IEPHEM	VM	EPHENERIS CODE
IEVNT(50)	EVENT	CODES OF EVENTS
IFINT(10)	TARINT	NOT USED IN THIS TRAJECTORY VERSION
IGP	OVERZ	MIDCOURSE GIUDANCE POLICY CODE
IGUID(5,10)	BAIM	ARRAY OF GUIDANCE EVENT CODES
, IHYP1	EVENT	HYPERELLIPSOID CODE USED TO DETERMINE IF K=1, K=3, OR BOTH
II	BAIM	GUIDANCE EVENT COUNTER
IIGP	PBLK	MIDCOURSE GUIDANCE POLICY CODE
IIPOL	EVENT	CODE WHICH DETERMINES IF EITHER TWO -VARIBLE OR THREE-VARIABLE B-PLANE GUIDANCE POLICY HAS OCCURRED
IMNF	MISC	MEASUREMENT NOISE FLAG
INC	COM	DETERMINE WHETHER THE ABOVE OPTION IS TO BE USED
INCMNT	COM	NUMBER OF INCREMENTS USED
INCMT	٧M	TOTAL INCREMENTS USED
INCPR	COM	SPECIFIES AFTER HOW MANY TIME INCREMENTS PRINT, OUT IS TO OCCUR
INITIAL	CNTRIC	NON-FUNCTIONAL IN ERROR ANALYSIS MODE
INPR	VM	PRINT INCREMENTS (IN INCREMENTS)
INPX	EXE	IMPULSE SERIES CODE

IOPT7	EVENT	ORBIT INSERTION VARIABLE. NON-FUNCTIONAL IN EXISTING PROGRAMS
IPG	COM	PAGE NUMBER
IPHASE	TARINT	PHASE COUNTER FOR CURRENT TARGETING EVENT
IPOL	EVENT	CODE WHICH DETERMINES IF FIXED-TIME-OF- ARRIVAL GUIDANCE EVENT HAS OCCURED
IPQ	BAIM	NOT USED
IPR	COM	A CODE WHICH DETERMINES IF PRINT-OUT IS TO OCCUR AFTER A SPECIFIED NUMBER OF DAYS
IPRE	TARINT	CONTROLS INITIALIZATION OF PROGRAM CONSTANTS IN SUBROUTINE -PRELIM
IPRINT	VM	PRINT CODE =0 OUTPUT INITIAL AND FINAL DATA =1 DO NOT OUTPUT DATA
IPROB	VM	PROBLEM NUMBER
IPRT(4)	COM	SPECIFIES PRINT OPTIONS (NOT APPLICABLE TO STEAP TRAJECTORY)
ISOI1	TRJ	SPHERE OF INFLUENCE CODE FOR ORIGINAL NOMINAL
18012	TRJ	SPHERE OF INFLUENCE CODE FOR MOST RECENT NOMINAL
18013	TRJ	SPHERE OF INFLUENCE CODE FOR ACTUAL TRAJECTORY
ISP2	MISC	SPHERE OF INFLUENCE FLAG
ISPH	MV .	SPHERE OF INFLUENCE CODE =0 SPHERE OF INFLUENCE NOT INTERSECTED =1 SPHERE OF INFLUENCE INTERSECTED
ISTART	TARINT	STAGE OF INITIAL TARGETING  =0 NO TARGETING STARTED  =1 FIRST PHASE STARTED -HAVE TARG  MATRIX  =2 SECOND PHASE STARTED - HAVE TARG  MATRIX

ISTM1	TRAJCD	ALTERNATE STATE TRANSITION MATRIX CODE
ISTMC	TRAJCB	STATE TRANSITION MATRIX CODE
ISTOP	TARINT	STOPPING CONDITION INDICATOR IN SUBROUTINE TARGET =1, STOP ON TIME =2, STOP AT SPHERE-OF-INFLUENCE =3, STOP AT CLOSEST APPROACH
ITAG	LUNART	FLAG SPECIFYING STAGE OF TARGETING =1 IN SMA TARGETING =2 IN SMA, INC, RCA TARGETING
ITARM	TARINT .	FLAG TO CONTROL CONSTRUCTION OF TARGETING MATRICES  =0, DO NOT CALCULATE STATE TRANSITION  =1, CALCULATE STATE TRANSITION MATRIX  AFTER EACH ITERATION
ITR	MISC	MODE FLAG
ITRAT	COH	IN INTERNAL CODE USED TO DETERMINE HOW MANY ITERATIONS HAVE BEEN ACCOMPLISHED IN THE VIRTUAL MASS PROCEDURE
IX	OVERX	NONLINEAR GUIDANCE CODE
IZERO	ZERDAT	ZERO ITERATION FLAG  = 0 INITIAL STATE READ IN  = 1 PLANET TO PLANET  = 2 PLANET TO POINT  = 3 POINT TO PLANET  = 4 POINT TO POINT  = 10 LUNAR TARGETING
JX	OVERX	GUIDANCE EVENT COUNTER
KAXTAR(3)	TARINT	KEY DEFINING AUXILIARY PARAMETERS FOR CURRENT EVENT
KEYTAR(3)	TARINT	KEY DEFINING DESIRED TARGET PARAMETERS FOR CURRENT EVENT
KGYD(10)	TARINT	FLAG INDICATING GUIDANCE INFORMATION IN CORRESPONDING COLUMNS OF INPUT ARRAYS =1, INFORMATION =0, NO INFORMATION

KL	COH	PROBLEM NUMBER (NOMNAL ONLY)
KMXQ(10)	TARINT	COMPUTE/EXECUTE MODES FOR EACH EVENT =1 COMPUTE VELOCITY CORRECTION ONLY =2 EXECUTE VELOCITY CORRECTION ONLY =3 COMPUTE AND IMMEDIATELY EXECUTE CORRECTIONS =4 COMPUTE BUT EXECUTE CORRECTION LATER
KOAST	ZERDAT	=-1, SHORT COAST. =+1, LONG-COAST
KOUNT	COM	A CODE WHICH SPECIFIES WHETHER PRINT-OUT IS TO OCCUR AFTER THIS TIME INCREMENT
KPRINT	XXXL	CORRELATION MATRIX PRINT CODE
KSICA	TARINT	STOPPING CONDITION INDICATOR IN SUBROUTINE TRJTRY =1, STOP ON TIME =2, STOP AT SPHERE-OF-INFLUENCE =3, STOP AT CLOSEST APPROACH
KTAR (6,10)	TARINT	CODES OF TARGET PARAMETERS (UP TO 6) FOR EACH TARGETING EVENT OR ORBIT INSERTION OPTION FOR EACH INSERTION EVENT
KTIM(10)	TARINT	EPOCH TO WHICH GUIDANCE EVENT TIMES  ARE REFERENCED  OUT OF PROCESSED  INITIAL TIME  SOI TIME  CALENDAR DATE
KTYP (10)	TARINT	TYPE OF GUIDANCE EVENT FOR EACH EVENT =-1 TERMINATION EVENT =1 TARGETING EVENT =2 RETARGETING EVENT =3 ORBIT INSERTION EVENT
KMXQ (10)	TARINT	COMPUTE/EXECUTE MODES FOR EACH GUIDANCE EVENT =1 COMPUTE VELOCITY CORRECTION DNLY =2 EXECUTE VELOCITY CORRECTION ONLY =3 COMPUTE AND IMMEDIATELY EXECUTE CORRECTION =4 COMPUTE BUT EXECUTE CORRECTION LATER

KUR	TARINT	FLAG INDICATING WHICH EVENT IS THE CURRENT EVENT
KHIT	TARINT	TERMINATION INDICATOR =0, CONTINUE RUN =1, TERMINATE RUN
LEV	TARINT	CURRENT LEVEL IN CURRENT TARGETING EVENT
LEVELS	TARINT	NUMBER OF ACCURACY LEVELS FOR CURRENT EVENT
LINCT	COM	LINE COUNT (NOMNAL ONLY)
LINPGE	COM	LINES PER PAGE (NOMNAL ONLY)
LKLP(10)	TARVAR	ARRAY OF LAUNCH PLANETS
LKTAR (6,10)	TARVAR	ARRAY DEFINING TARGET PARAMETERS
LKTP(10)	TARVAR	ARRAY OF TARGET PLANETS
LLVES(10)	TARVAR	NUMBER OF INTEGRATION ACCURACY LEVELS USED
LNPAR(10)	TARVAR	NUMBER OF TARGET PARAMETERS DESIRED
LTARG	ZERDAT	= 0, INTERPLANETARY TARGETING. =1, LUNAR TARGETING
LVLS	TARINT	NUMBER OF ACCURACY LEVELS TO BE-USED ON EACH TARGETING EVENT
MAT(10)	TARINT	TARGETING MATRIX COMPUTATION CODE FOR EACH TARGETING EVENT =1 COMPUTE TARGETING MATRIX ONLY AT FIRST LEVEL =2 COMPUTE TARGETING MATRIX AT EACH STEP
МАТХ	TARINT	MATRIX COMPUTATION CODE FOR CURRENT TARGETING EVENT (SEE DEFN OF MAT)
MAXB(10)	TARINT	THE NUMBER OF BAD STEPS ALLOWED DURING ANY TARGETING EVENT
MAXBAD	TARINT	MAXIMUM NUMBER OF BAD ITERATIONS FOR CURRENT EVENT
MCNTR	MEAS	NUMBER OF MEASUREMENTS HAVING OCCURRED
MCODE (1000)	MEAS	ARRAY OF MEASUREMENT CODES

MDL(10)	TARINT	EXECUTION MODELS FOR EACH GUIDANCE EVENT =1 IMPULSIVE =2 PULSING ARC
MNCN (12)	CONST	MEASUREMENT NOISE CONSTANTS
HNNAME(12,3)	NAME	MEASUREMENT NAME
MONTH(12)	PRT	NAMES OF MONTHS
NAE	EVENT	ADAPTIVE FILTERING EVENTS HAVING OCCURRED.
NAF6 (20)	EVENT	ARRAY OF ADAPTIVE FILTERING EVENT CODES. NON-FUNCTIONAL IN EXISTING PROGRAM
NAFC	OVER1	ADAPTIVE FILTER FLAG
NB(11)	MA	CODES OF PLANETS
NB1(11)	SIM2	ARRAY OF PLANET CODES IN ACTUAL TRAJECTORY
NBOD.	VM	NUMBER OF BODIES USED IN VIRTUAL MASS PROGRAM
NBOD1	SIM2	NUMBER OF BODIES IN ACTUAL TRAJECTORY
NBODY	COM	EQUAL TO 4*NBODYI-3
NBODYI	COM	NUMBER OF BODIES CONSIDERED IN VIRTUAL MASS TRAJECTORY
NCPR	TARINT	NUMBER OF INTEGRATION INCREMENTS BETWEEN PRINT-OUTS OF NOMINAL TRAJECTORY
NDACC	TRAJCD	NUMERICAL DIFFERENCING ACCURACY CODE
NDIM1	STVEC	DIMENSION OF SOLVE-FOR PARAMETER STATE
NDIM2	STVEC	DIMENSION OF DYNAMIC CONSIDER STATE
NDIM3	STVEC	DIMENSION OF MEASUREMENT CONSIDER PARAMETER STATE
NEV	EVENT	NUMBER OF EVENTS
NEV1	EVENT	TOTAL NUMBER OF EIGENVECTOR EVENTS
NEVS	EVENT	TOTAL NUMBER OF PREDICTION EVENTS

NEV3	EVENT	TOTAL NUMBER OF GUIDANCE EVENTS
NEV4	EVENT	TOTAL NUMBER OF -COMCON- EVENTS
NEV5	EVENT	TOTAL NUMBER OF QUASI-LINEAR FILTERING EVENTS
NEV6	EVENT	TOTAL NUMBER OF ADAPTIVE FILTERING EVENTS. NON-FUNCTIONAL IN EXISTING PROGRAM
NEV7	EVENT	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN IN EXISTING PROGRAM
NEV8	EVENT	ORBIT INSERTION VARIABLE. NON-FUNCTIONAL IN EXISTING PROGRAMS
NEV9	EVENT	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN IN EXISTING PROGRAM
NEV10	EVENT	ORBIT INSERTION VARIABLES, NON-FUNCTIONAL IN IN EXISTING PROGRAM
NEV11	EVENT	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN IN EXISTING PROGRAM
NGE	EVENT	NUMBER OF GUIDANCE EVENTS HAVING OCCURRED
NINETY	DPNUM	THE NUMBER NINETY (90) TO NINE SIGNIFICANT FIGURES. TARGETING MODE ONLY—
NITS	TARINT	ALLOWABLE NUMBER OF ITERATIONS FOR CURRENT EVENT .
NLP	VH	CODE OF LAUNCH PLANET
NMN	MEAS	TOTAL NUMBER OF MEASUREMENTS
NO (11)	8L K	AN ARRAY OF PLANET CODES BEING USED TO GENERATE THE VIRTUAL MASS TRAJECTORY
NOGYD	TARINT	TOTAL NUMBER OF GUIDANCE EVENTS
NOIT (10)	TARINT	THE NUMBER OF TOTAL ITERATIONS ALLOWED AT THE FIRST AND LAST LEVELS OF TARGETING EVENTS FOR EACH GUIDANCE EVENT
NOPAR	TARINT	NUMBER OF TARGET PARAMETERS FOR CURRENT EVENT

NOPHAS	TARINT	NUMBER OF TARGETING PHASES FOR CURRENT EVENT
NOSOI	TARINT	OUTER TARGETING FLAG =0 NORMAL TARGETING =1 OUTER TARGETING
NPE	EVENT	NUMBER OF PREDICTION EVENTS HAVING OCCURRED
NPUL	PULS	NUMBER OF PULSES IN PULSING ARC
NQE	EVENT	QUASI-LINEAR FILTERING EVENTS HAVING OCCURRED
NR	OVER1	NUMBER OF ROWS IN THE OBSERVATION MATRIX
NST	CONST	NUMBER OF STATIONS TO BE USED (MAXIMUM 3)
NTMC	TRAJCD	NOMINAL TRAJECTORY CODE
NTP	VM	CODE OF TARGET PLANET
OMEGA	CONST	EARTH-S ROTATION RATE
ONE	DPNUM	THE NUMBER ONE (1) TO NINE SIGNIFICANT FIGURES
OTAR(3)	LUNART	DESIRED VALUES OF SMA,RCA, AND INC
P(6,6)	STM	POSITION/VELOCITY COVARIANCE
PB(6,6)	STM	POSITION/VELOCITY COVARIANCE AT INITIAL TIME
P7	EVENT	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN EXISTING PROGRAM
PCON(3)	LUNART	PERTURBATIONS IN CONTROLS (ALPHA, DELTA, THETA)
PERP7	EVENT	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN EXISTING PROGRAM.
PERV (10)	TAREAL	PERTURBATIONS SIZE FOR VELOCITY COMPONENTS IN CONSTRUCTING SENSITIVITY MATRICES IN TARGETING EVENTS
PG(6,6)	GUI	POSITION/VELOCITY CONTROL COVARIANCE

PHI(3,3)	TAREAL	TARGETING MATRIX. REQ-D ONLY IF ISTART=1,2
PHI(6,6)	STM	POSITION/VELOCITY STATE TRANSITION MATRIX
PHI2(3,3)	BAIM	INVERSE OF VARIATION MATRIX PARTITION
PHILS	ZERDAT	LATITUDE OF LAUNCH SITE
PI	СОМ	THE VALUE OF THE MATHEMATICAL CONSTANT PI
PLANET (11)	PRT	NAMES OF PLANETS
PMASS(11)	BLK	GRAVITATIONAL CONSTANTS OF PLANETS IN A.U.**3/DAY**2
POI	PBLK	PROBABILITY OF IMPACT
PP(6,6)	STM	POSITION/VELOCITY COVARIANCE MATRIX PRIOR TO PROCESSING A MEASUREMENT
PROBI	BAIM	ALLOWABLE PROBABILITY OF IMPACT
PS (24,24)	STM	SOLVE-FOR PARAMETER COVARIANCE
PSB(24,24)	STM	SOLVE-FOR PARAMETER COVARIANCE AT INITIAL TIME
PSG(24,24)	GUI	SOLVE-FOR PARAMETER CONTROL COVARIANCE
PSI1	ZERDAT	ANGLE OF FIRST BURN
PSI2	ZEROAT	ANGLE OF SECOND BURN
PSP(24,24)	STM	SOLVE-FOR PARAMETER COVARIANCE MATRIX PRIOR TO PROCESSING A MEASUREMENT
PSTAR	PBLK	NOMINAL PROBABILITY DENSITY FUNCTION EVALUATED AT TARGET PLANET CENTER
PULMAG	PULS	THRUST MAGNITUDE OF PULSING ENGIEN
PULMAS	PULS	NOMINAL MASS OF SPACECRAFT DURING PULSING ARC
PULT	PULS	TOTAL TIME INTERVAL OF PULSING ARC

Q(6,6)	STM	DYNAMIC NOISE COVARIANCE MATRIX
QK(6,6)	EXE	EFFECTIVE EXECUTION ERROR COVARIANCE MATRIX
R(4,4)	STM	MEASUREMENT NOISE COVARIANCE MATRIX
RAP	ZOUT	RIGHT ASCENSION OF VHP
RAD	COM	THE NUMBER OF DEGREES PER RADIAN
RADIUS (11)	BLK	THE RADIUS OF A GIVEN PLANET IN A.U.
RC(6)	VM	STATE AT CLOSEST APPROACH
RCA	VM	MAGNITUDE OF CLOSEST APPROACH POSITION VECTOR
RCA	LUNART	RADIUS OF CLOSEST APPROACH TO MOON (DESIRED)
RCA1(6)	TRJ	STATE AT CLOSEST APPROACH ON ORIGINAL NOMINAL
RCA2 (6)	TRJ	STATE AT CLOSEST APPROACH ON MOST RECENT NOMINAL
RCA3(6)	TRJ	STATE AT CLOSEST APPROACH ON ACTUAL: TRAJECTORY
RE (6)	VM	POSITION AND VELOCITY OF EARTH
RES(4)	SIM1	RESIDUAL
RF(6) ,	OVER	FINAL TARGETED NOMINAL STATE VECTOR
RF (6)	OVERZ	FINAL TARGETED STATE VECTOR
RF1(6)	OVER	FINAL MOST RECENT NOMINAL STATE VECTOR
RF1(6)	OVERZ	FINAL MOST RECENT NOMINAL STATE VECTOR
RI (6)	OVER1	INITIAL TARGETED NOMINAL STATE VECTOR
RI(6)	OVERR	STATE VECTOR AT EVENT TIME
RI(6)	LUNART	GEOCENTRIC STATE OF S/C AT LUNAR SOI
RI1	OVER1	INITIAL MOST RECENT NOMINAL STATE VECTOR

RIN(6)	TAREAL	CURRENT STATE VECTOR AT 1-TH EVENT
RK(2,3)	PULS	POSITION VECTORS OF LAUNCH AND TARGET PLANETS AT IMPULSIVE TIME (MIDPOINT OF PULSING ARC)
RMASS (11)	BLK	THE RELATIVE GRAVITATIONAL CONSTANT OF A STATED PLANET WITH RESPECT TO THE SUN
RME(6)	LUNART	GEOCENTRIC STATE OF CENTER OF MOON IN ECLIPTIC COORDINATES AT TSI
RMQ(6)	LUNART	GEOCENTRIC STATE OF CENTER OF MOON AT TSI IN EQUATORIAL COORDINATES
RP .	ZERDAT	PARKING ORBIT RADIUS
RPE	LUNART	RADIUS OF EARTH PARKING ORBIT
RPRAT	ZERDAT	PARKING ORBIT INVERSE RATE
RRF(3)	TAREAL	SPACECRAFT POSITION AT END OF INTEGRATION, USED IN BROKEN PLANE TARGETING
RSI(3)	VM	POSITION AT SPHERE OF INFLUENCE
ŖSI(6)	LUNART	SELENOCENTRIC STATE OF S/C AT LUNAR' SOI
RS0I1(3)	TRJ	POSITION AT SPHERE OF INFLUENCE ON ORIGINAL NOMINAL
RS012(3)	TRJ	POSITION AT SPHERE OF INFLUENCE ON MOST RECENT NOMINAL
RS013(3)	TRJ	POSITION AT SPHERE OF INFLUENCE ON ACTUAL TRAJECTORY
RTP(6)	VM	POSITION AND VELOCITY OF TARGET PLANET
RVS(6)	MA	POSITION OF VEHICLE RELATIVE TO VIRTUAL MASS
S (24,4)	STM	KALMAN GAIN CONSTANT FOR SOLVE-FOR PARAMETERS
SAL(3)	CONST	ALTITUDES OF STATIONS
SIGALP	EVENT	VARIANCE OF ERROR IN POINTING ANGLE 1

EVENT	VARIANCE OF ERROR IN POINTING ANGLE 2
ZERDAT	NOMINAL LAUNCH AZIMUTH
EVENT	VARIANCE OF PROPORTIONALITY ERROR
EVENT	VARIANCE OF RESOLUTION ERROR
CONST	LATITUDES OF STATIONS
SIMCNT	BIASES IN STATION LOCATION CONSTANTS
CONST	LONGITUDES OF STATIONS
LUNART	SIMI-MAJOR AXIS OF LUNAR HYPERBOLA (DESIRED)
BLK	CONSTANTS USED TO CALCULATE THE SEMI-MAJOR AXES OF THE PLANETS
BLK	THE SPHERES OF INFLUENCE OF THE PLANETS IN A.U.
	<b>,</b>
TAREAL	REDUCTION FACTORS FOR TARGET PLANET SPHERE OF INFLUENCE FOR EACH EVENT
TAREAL VM	· · · · · · · · · · · · · · · · · · ·
	OF INFLUENCE FOR EACH EVENT  DIRECTION COSINES VECTOR OF SPACEGRAFT SPIN AXIS
VM	OF INFLUENCE FOR EACH EVENT  DIRECTION COSINES VECTOR OF SPACECRAFT SPIN AXIS  CONSTANTS USED TO CALCULATE THE ORBITAL ELEMENTS OF THE LAST FOUR PLANETS
VM BL K	OF INFLUENCE FOR EACH EVENT  DIRECTION COSINES VECTOR OF SPACECRAFT SPIN AXIS  CONSTANTS USED TO CALCULATE THE ORBITAL ELEMENTS OF THE LAST FOUR PLANETS (SEE LARGE ARRAY DEFNS IN SECTION 5.3)
VM BLK BLK	OF INFLUENCE FOR EACH EVENT  DIRECTION COSINES VECTOR OF SPACECRAFT SPIN AXIS  CONSTANTS USED TO CALCULATE THE ORBITAL ELEMENTS OF THE LAST FOUR PLANETS (SEE LARGE ARRAY DEFNS IN SECTION 5.3)  TRAJECTORY TIME IN DAYS
VM BLK BLK TMW2	OF INFLUENCE FOR EACH EVENT  DIRECTION COSINES VECTOR OF SPACECRAFT SPIN AXIS  CONSTANTS USED TO CALCULATE THE ORBITAL ELEMENTS OF THE LAST FOUR PLANETS (SEE LARGE ARRAY DEFNS IN SECTION 5.3)  TRAJECTORY TIME IN DAYS  EIGENVECTOR EVENT TIMES
VM BLK BLK TMW2	OF INFLUENCE FOR EACH EVENT  DIRECTION COSINES VECTOR OF SPACECRAFT SPIN AXIS  CONSTANTS USED TO CALCULATE THE ORBITAL ELEMENTS OF THE LAST FOUR PLANETS (SEE LARGE ARRAY DEFNS IN SECTION 5.3)  TRAJECTORY TIME IN DAYS  EIGENVECTOR EVENT TIMES  PREDICTION EVENT STARTING TIMES
	EVENT EVENT CONST SIMENT CONST LUNART BLK

T6(20)	TMW2	NOT USED
T7	TMH2	NOT USED
TACA	VM	TRAJECTORY SEMIMAJOR AXIS WITH RESPECT TO TARGET BODY AT CLOSEST APPROACH TO TARGET BODY
TAR(6,10)	TAREAL	DESIRED VALUES OF TARGET PARAMETERS (UP TO 6 AVAILABLE) FOR EACH GUIDANCE EVENT
T AU7	EVENT	ORBIT INSERTION VARIABLES. NON-FUNCTIONAL IN IN EXISTING PROGRAM
TCA	LUNART	J.D. OF TIME AT LUNAR CLOSEST APPROACH (DESIRED)
TCA1	TRJ -	TIME AT CLOSEST APPROACH OF ORIGINAL NOMINAL
TCA2	TRJ	TIME AT CLOSEST APPROACH OF MOST RECENT NOMINAL
TCA3	TRJ	TIME AT CLOSEST APPROACH OF ACTUAL TRAJECTORY
TEN	DPNUM	THE NUMBER TEN (10) TO NINE SIGNIFICANT FIGURES. TARGETING MODE ONLY
TEV(50)	EVENT	TIMES OF EVENTS
TEVN	OVER1	TIME OF CURRENT EVENT
TEVN	OVERR	EVENT TIME
TF	ZERDAT	NOT USED
TG	GUI	TRAJECTORY TIME AT MOST RECENT GUIDANCE" EVENT
TGT3(10)	TARVAR	DESIRED TARGET TIMES REFERENCED TO INITIAL TRAJECTORY TIME
THEDOT	ZERDAT	ROTATION RATE OF LAUNCH PLANET
THELS	ZERDAT	LONGITUDE OF LAUNCH SITE

THREE	DPNUM	THE NUMBER THREE (3) TO NINE SIGNIFICANT FIGURES
, TI	ZERDAT	NOT USED
TIMG(10)	TAREAL	TIMES OF EACH GUIDANCE EVENT REFERENCED TO EPOCH-INITIAL TIME, SOI TIME, OR CA TIME
TIM1	ZERDAT	TIME OF FIRST BURN
TIŅ2	ZERDAT	TIME OF SECOND BURN
TIMINT	VM	TOTAL TIME USED
TIMS	TAREAL	INTERNAL CLOCK TIME AT START OF COMPUTER RUN
TIN	TAREAL	JULIAN DATE AT INJECTION
_UNIT,	BAIM	INJECTION TIME
, TM	VM	TIME UNITS PER DAY
_TMN(100,0)	MEAS	TIMES OF MEASUREMENTS
TMPR(3)	BAIM	HOST RECENT TARGET STATE
, TNOHB(3)	MIAB	NOMINAL B-PLANE, TARGET STATE
TNOHC (7)	BAIM	NOMINAL CLOSEST APPROACH TARGET STATE
, TMU	TAREAL	GRAVITATIONAL CONSTANT OF TARGET PLANET
, TMU	LUNART	GRAVITATIONAL CONSTANT OF MOON(KM3/SEC2)
TOL(6,10)	TAREAL	ALLOHABLE TOLERANCES OF TARGET PARAMETERS FOR EACH GUIDANCE EVENT
, TRTM	TAREAL	TRAJECTORY TIME (DAYS) REF. TO INJECTION
TOL (6,10)	TAREAL	TOLERANCES OF TARGET PARAMETERS FOR EACH TARGETING EVENT
TOLR (6)	TAREAL	NOT USED IN CURRENT TARGET VERSION

TPT2 (20)	EVENT	ARRAY OF TIMES TO WHICH A PREDICTION IS MADE
TRTM	TAREAL	TRAJECTORY TIME ON NOMINAL TRAJECTORY
TRTM1	TIME	INITIAL TRAJECTORY TIME
TRTMB	TIME	TRAJECTORY TIME AT BEGINNING OF TRAJECTORY
TSI	LUNART	PROJECTED J.D. AT SOI INERTSECTION
¥S0I1	TRJ	TIME AT SPHERE OF INFLUENCE OF ORIGINAL NOMINAL
TS0I2	TRJ	TÎME AT SPHERE OF INFLUENCE OF MOST RECENT NOMINAL
TS013	TRJ	TIME AT SPHERE OF INFLUENCE OF ACTUAL TRAJECTORY
TSPH	LUNART	RADIUS OF LUNAR SOI (KM)
TTIM1	SIMONT	FIRST TIME USED FOR UNMODELLED ACCELERATION
TTIMS	SIMONT	SECOND TIME USED FOR UNMODELLED ACCELERATION
TTOL (3)	LUNART	ALLOWABLE TOLERANCES IN SMA, B.T, B.R
TWO	DPNUM	THE NUMBER TWO (2) TO NINE SIGNIFICANT FIGURES
THOPI	DPNUM	THE MATHEMATICAL CONSTANT 2.*PI
TXU(6,8)	STM	STATE TRANSITION MATRIX PARTITION ASSOCIATED WITH DYNAMIC CONSIDER PARAMETERS
TXXS (6,24)	STH	STATE TRANSITION MATRIX PARTITION ASSOCIATED WITH SOLVE-FOR PARAMETERS
U0(8,8)	STM	DYNAMIC CONSIDER PARAMETER COVARIANCE MATRIX
TVINU	TIME	UNIVERSAL TIME
UNMAC (3,3)	SIMCNT	UNMODELLED ACCELERATION
UST(3)	CONST2	DIRECTION COSINES OF 3 REFERENCE STARS

V(16,7)	COM	AN ARRAY WHICH STORES PERTINANT VECTORS USED IN THE CALCULATION OF THE VIRTUAL MASS TRAJECTORY (SEE LARGE ARRAY DEFNS IN SECTION 5.3)
VO(15,15)	STM	MEASUREMENT CONSIDER PARAMETER COVARIANCE MATRIX
VHPM	ZOUT	MAGNITUDE OF HYPERBOLIC EXCESS VELOCITY AT TARGET BODY (VHP VECTOR)
VINF	BAIM	HYPERBOLIC EXCESS VOLOCITY
VK(2,3)	PULS	VELOCITY VECTORS OF LAUNCH AND TARGET PLANETS AT IMPULSIVE TIME (MIDPOINT OF PULSING ARC)
VMU	VM	GRAVITATIONAL CONSTANT OF VIRTUAL MASS
VSI(3).	VM	VELOCITY AT SPHERE OF INFLUENCE
VS0I1(3)	TRJ	VELOCITY AT SPHERE OF INFLUENCE ON ORIGINAL NOMINAL
VS0I2(3)	TRJ	VELOCITY AT SPHERE OF INFLUENCE ON MOST RECENT NOMINAL
VS013(3)	TRJ	VELOCITY AT SPHERE OF INFLUENCE ON ACTUAL TRAJECTORY
VST(3)	CONST2	DIRECTION COSINES OF 3 REFERENCE STARS
H (17)	SIM1	ACTUAL DYNAMIC NOISE
WST(3)	CONST2	DIRECTION COSINES OF 3 REFERENCE STARS
XAC(5,10)	TARVAR	ACCURACY LEVELS EMPLOYED IN TARGETING
XB(6)	STVEC	BEGINNING ORIGINAL NOMINAL VEHICLE STATE VECTOR
XBDT	SAVVAL	ORIGINAL VALUE OF B.T IN N-L GUIDANCE
XBDR	SAVVAL	ORIGINAL VALUE OF BOR IN N-L GUIDANCE
XDC	SAVVAL	ORIGINAL VALUE OF DC IN N-L GUIDANCE
XDELV(3,10)	TARVAR	NONLINEAR VELOCITY CORRECTION

XDSI	SAVVAL	ORIGINAL VALUE OF TSI IN N-L GUIDANCE
XDVMAX(10)	TARVAR	MAXIMUM ALLOWABLE VELOCITY CORRECTION
XF (6)	STVEC	FINAL VEHICLE STATE VECTOR OF ORIGINAL NOMINAL
XF1(6)	SIM1	FINAL STATE VECTOR OF MOST RECENT NOMINAL TRAJECTORY
XFAC(10)	TARVAR	SPHERE OF INFLUENCE FACTORS
XG(6)	GUI	STATE VECTOR AT TIME OF LAST GUIDANCE EVENT
XI (6)	STVEC	INITIAL VEHICLE STATE VECTOR OF ORIGINAL NOMINAL
XI1(6)	SIM1	INITIAL STATE VECTOR OF MOST RECENT NOMINAL
XIN(6)	OVERX	STATE VECTOR TRANSFERRED TO NONLIN
XLAB(6)	XXXL	VEHICLE POSITION/VELOCITY VECTOR COMPONENT NAMES
XL AM (2,2)	PBLK	PROJECTION OF TARGET CONDITION COVARIANCE MATRIX INTO THE IMPACT PLANE
XLAMI(2,2)	PBLK	INVERSE OF XLAM(2,2)
XMUS (2)	PBLK	NOMINAL IMPACT PLANE TARGET STATE
XNM(24)	XXXL	AUGUMENTATION PARAMETER LABELS
XP(6)	BLK	THE POSITION AND VELOCITY OF A PLANET IN HELIOCENTRIC ECLIPTIC COORDINATES
XPERV(10)	TARVAR	VELOCITY PERTURBATION USED TO COMPUTE TARGETING MATRIX
XRC(6)	SAVVAL	ORIGINAL VALUE OF RC IN N-L GUIDANCE
XRSI(3)	SAVVAL	ORIGINAL VALUE OF RSI IN N-L GUIDANCE

X\$L(24)	XXXL	SOLVE-FOR PARAMETER LABELS
XTAR (6,10)	TARVAR	DESIRED TARGET VALUES
XTOL (6,10)	TARVAR	TOLERANCES ON TARGET PARAMETERS
XU(8)	XXXL	DYNAMIC CONSIDER PARAMETER LABELS .
XV(15)	XXXF	MEASUREMENT CONSIDER PARAMETER LABELS
XVSI (3)	SAVVAL	ORIGINAL VALUE OF VSI IN N-L GUIDANCE
XXIN(6)	EXE	STATE VECTOR TRANSFERRED TO EXCUT OR EXCUTS
Z(17)	SIM1	ACTUAL STATE VECTOR
ZDAT (6)	ZERDAT	ZERO ITERATE VECTOR  IF IZERO=0 ZDAT(1-6) = INITIAL STATE  =2,3,4, ZDAT(1-3) = INITIAL  POSITION  ZDAT(4-6) = FINAL POSITION
ZERO	DPNUM	THE NUMBER ZERO (0) TO NINE SIGNIFICANT FIGURES
ZI(17)	SIM1	INITIAL ACTUAL STATE VECTOR
ZF(6)	SIM1	FINAL ACTUAL STATE VECTOR AFTER ADDING THE EFFECT OF UNMODELED ACCELERATION

### 5.3 Large Array Definitions

In this section large arrays appearing in COMMON will be displayed. The arrays depicted are frequently referenced in trajectory propagation subroutines in STEAP; hence the programmer studying such subroutines will find the following tables extremely useful.

Tables 5.1 to 5.5 describe arrays containing planetary ephemeris constants. The values actually stored in these arrays may be found in the documentation for BLOCK DATA. Tables 5.6 through 5.8 contain variables used in the virtual mass propagation procedure. Discussions of these variables may be found in VMP, EFHEM, ORB, and similar routines.

																- 7	
Constant	1.	Ω	$\widetilde{\omega}$	е	M	а	ω	E		<b>a</b> 0	8	1					
Mercury	1	2	3	4	5	6	٠7	8		1		2					
Venus	9	10	11	12		14	15	16		3		4					
Earth	17	18	19	20		:.22	23	24		5		6					
Mars	25	26	27	28	29	30	31	32		7		8					
Jupiter	33	34	35	36	37	38	39	40		9	1	l.O					
Saturn	41	42	43	44	45	46	47	48		11		.2					
Uranus	49	50	51	52	53	54	55	56		13		14					
Neptune	57	58	59	60	61	62	63	64		15		.6					
Pluto	65	66	67	68	69	70	71	72		17	1	.8					
Moon	73	74	75	76	77	78	79	80		,							
Table 5.1	ELM	INT A	rray		Coni	c El	emeį	nts		Tá	able	5,	. 2	SM	jr' <i>i</i>	Arra	ay
Constant	¹ 0	1 ₁ 1	2 ¹ 3	$\Omega_0$	$\Omega_1$	$\Omega_{3}$	$\widetilde{\omega}_0$	$\widetilde{\omega}_{\mathbf{i}}$	∞ ₂ ≈	e ₀	e ₁	e ₂	<b>e</b> 3	^M o	^M 1	^M 2	М3
Mercury	1	2	3 4	5	6	7 8	g	10	11 12	12	14	15	16	17	1 2	10	20
Venus	21	22 2	3 24	_	_	7 28			31 32				36	37	38	39	40
Earth									51 52	53	54	55	56	57		-	
Mars	61	62 6	3 64	65	66 6	7 68	69	70	71 72	73	74	75					-
Table 5.3	CN	Arra	у	In	ner P	lane	t Co	nsta	ants								
Constant	10	¹ 1	$\Omega_0$	$\Omega_{\mathrm{I}}$	$\widetilde{\omega}_0$	$\widetilde{\omega}_1$	е ₀	e ₁	M _O	^M 1							
Jupiter	1	2	3	4	5	6	7	8	9	10							
Saturn	11	12	13	14		16	17	18		20							
Uramıs	21	22	23	24	25	26	27	28	29	30							
Neptune	31	32	33	34	35	36	37	38	39	40							
Pluto	41	42	43	44	45	46	47	48	49	50							

Table 5.4 ST Array -- Outer Planet Constants

```
3 L<sub>0</sub> L<sub>1</sub> L<sub>2</sub> L<sub>3</sub> 1
Constant
                                                               2
                                                         1
                                1
                                      2
                                            3
                                                  0
                                                                           9 10
                                                                                        11 12 13 14 15
                                                               7
                                                                     8
                                                   5
                                                         6
                          1
                                2
                                       3
                                             4
       Moon
       Table 5.5 EMN Array -- Lunar Constants
             F-Array
                          z_1
 \mathbf{x}_{1}
             у<sub>1</sub>
                                     r,
                          ż<sub>1</sub>
             Ϋ́<sub>1</sub>
                                      v<sub>1</sub>
 ×ς1
             y<sub>S1</sub>
                          z<sub>S1</sub>
                                      r<sub>S1</sub>
                                                          Note:
                                                               Subscript i indicates component is
                    ż<sub>S1</sub>
             \dot{y}_{S1}
                                      v<sub>S1</sub>
                                                                    i-th body referenced to inertial
                                                                    coordinate system
 *2
              y<sub>2</sub> z<sub>2</sub>
                                       r<sub>2</sub>
                                                               Subscript si indicates component
                                                                    is spacecraft referenced to i-th
 \dot{x}_2
                                                                    body
             \dot{y}_2 \qquad \dot{z}_2
                                       v_2
              y<sub>S2</sub> z<sub>S2</sub>
  x<sub>S2</sub>
                                       r<sub>S2</sub>
              ý<sub>S2</sub>
```

Table 5.6 F-Array -- Ephemeris Data

TRG (1)	cos E	TRG (5)	cos i	$TRG(9)$ $\cos \omega$	TRG(13) $\cos(\dot{\omega} + f)$
TRG (2)	sin E	TRG (6)	sin i	TRG(10) $\sin\omega$	TRG(14) $sin(\omega + f)$
TRG (3)	cos f	TRG (7)	$\cos \mathcal{Q}$	TRG(11) $\cos \omega$	
TRG (4)	sin f	TRG (8)	$\sin arOmega$	TRG(12) sin ₩	

Table 5.7 TRG Array -- Trigonometric Functions

J				r <del></del> -	····		
1	1	2	3	4	5	6	7
1	(t _e ) _{dlm} ,t _B	(x _s ) , x _s	(y _a ) , y _a B	(z _s ) , ż _s dim	ω(deg/t), ω(rad/t) D		μ, -μ
2	t _a	¥ ₆	y _s	^z s _e	(t _F ) , t _F	(r _{is} ) ,r _{is} dim	(r _{2s} ) , r _{2s} f dim
3	(t _{ephe} ) , t _{ephB}	(x _{se} ), x _s B	(y _{se} ), y _{se}	(i ₈ ), i ₈ B	(Atp) , Atp	C ₂	
4	^t eph _e	× s _e	y _{ne}	z _e	<u>Δτ</u> r		ωD (velocity)
5	(µ _V ), µ _V B	(x _y ), x _y	(y _{ve} ), y _{vB}	(z _{Ve} ), z _{VB}	ωD ² (area rate)	ω ² D ² (velocity) ²	1 -μ
6	μ _A e	M _{x*} × _{ve}	M _y , y _{ve}	M _z , z _{ve}	ω ² D ³ (mass)	ω ³ D ³ (mass rate)	κ,Δτ
7	(µ, ) , µ, , µ, B	(x _{ve} ), x _{vB}	(y _{ve} ), y _{vB}	(z _{ve} ), z _{vB}	Δt _k	Δŧ	μ _γ average
8	μ̂ _{νε}	M _{x'} * _{Ye}	My, v	M _z , z _{ve}	(Δt) ²	й _ч аverage	B, _{"M}
9	r _{vs} B	×v8B	y vsB	² vs _B	x _{vsB} , (o _{vse} )	yvsB, (qnse h	z _{vsB} , (σ _{vse} )
10	r _{vse}	x _{vs} e	y _{vse}	z _{vs} e	x _{vavg} , x _{vavg}	y xavg xavg	z vavg zvavg
11	v _s B	× _{vs} B	y _{vs} B	z _{vs} B	x _{vsB} , n _x	y _{vsB} , ny	z _{vsB} , n _z
12		x _{vse} , e _x + x _{vse}	y _{vse} , e _y + $\frac{y_{vs_e}}{r_{vs_e}}$	z _{vse} , e _z z _{vse}	M _s , e _x + × _{vs_e} · · · · · · · · · · · · · · · · · ·	M _s , e _y + $\frac{y_{vs_o}}{r_{vs_e}}$	e _z + ^z vs _e r _{VB_Q}
13			. ^t p	Δt _{MA}	1e ²	(  1 e _e ²  )	k ²
14	e _e	e _{xe}	е  у _е	ė _{ze}	ee, e _{xe}	cos (t _e ), eye	sin (t _e ), e _{ze}
15	(k) _{dim}	(k _x )dim	(k _y ) dim	(k ₂ ) dim	ь _В . х _В	E _B	a - r _{vsB} , or r vs _B vs _B
16	k	k _x	^k y	k _z	k _{x'} k _{e'} b _{e'}	^k y, Ee	k _{z'} a - r _{vs'} or $\vec{r}_{vs_e}$ · $\vec{r}_{vs_B}$

Table 5.8 W-Array -- Virtual Mass Propagation Variables

#### 6. INDIVIDUAL SUBROUTINE DOCUMENTATION

This chapter contains the individual documentation for all the subroutines in the STEAP II series. The following information is given for each subroutine.

- 1. Purpose: The tasks performed by the subroutine.
- 2. Calling Sequence: The statement by which the subroutine is called.
- 3. Arguments: The arguments in the calling sequence, their definition, and identification as input, output, or both.
- 4. Subroutines Supported: A list of subroutines calling the subroutine being documented.
- 5. Subroutines Required: A list of subroutines called by the sub-routines being documented.
- 6. Local Symbols: The internal (non-common) variables used in the subroutine and their definitions.
- 7. Common Computed/Used: A list of variables appearing in common blocks which are both computed and used (see Chapter 3 for definitions).
- 8. Common Computed: A list of common variables which are set in the program.
- 9. Common Used: A list of common variables only used by the sub-routine.
- 10. Analysis: The detailed mathematical analysis on which the subroutine is based (if applicable).
- 11. Flowchart: A flowchart of the operation of the program (if required).

The reader is referred to Chapter 4 for an index of all subroutines of STEAP II (Tables 4.1 and 4.2) and for the calling hierarchies of the basic subprograms of STEAP II (Figures 4.1 to 4.4).

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ATANH-A

SUBROUTINE ATANH

PURPOSE: TO FIND THE ANGLE Y WHOSE TANH IS X

CALLING SEQUENCE: CALL ATANH(X,Y)

ARGUMENTS X I TANH(Y)

Y O ANGLE DESIRED

SUBROUTINES SUPPORTED: BATCON

LOCAL SYMBOLS: T1 INTERMEDIATE VARIABLE

T2 INTERMEDIATE VARIABLE

T3 INTERMEDIATE VARIABLE

SUBROUTINE ATCEGV

PURPOSE: TO COMPUTE EIGENVALUES AND EIGENVECTORS OF ACTUAL TARGET CONDITION 2ND MOMENT MATRIX

CALLING SEQUENCE: CALL ATCEGV(III, ATC, EDT, FOV)

ARGUMENTS: III I NUMBER OF ROWS IN ATC MATRIX

ATC I ACTUAL TARGET CONDITION MATRIX

EDT I ACTUAL TARGET STATE DEVIATION MEANS

FOV I FINAL OFF-DIAGONAL ANNIHILATION TERM FOR JACOBI

SUBROUTINES SUPPORTED: GENGID

SUBROUTINES REQUIRED: HYELS JACOBI

COMMON USED: IHYP1

LOCAL SYMBOLS: DUM1 OUTPUT MATRIX FOR JACOBI

EGVL EIGENVALUES

PEIG INTERMEDIATE ARRAY

ROW INTERMEDIATE VECTOR

S ATC COVARIANCE ARRAY(3,3)

SDUM ATC COVARIANCE ARRAY(2,2) FOR JACOBI

SSDUM ATC COVARIANCE ARRAY(2,2) FOR HYELS

SUBROUTINE BATCON

PURPOSE: TO FIND POSITION, VELOCITY, TRUE ANOMALY, RADIUS AND VELOCITY MAGNITUDE AT TIME T GIVEN POSITION AND VELOCITY AT TIME 0.

CALLING SEQUENCE: CALL BATCON(U,R,V,T,RT,VT)

ARGUMENTS: U I GRAVITATIONAL CONSTANT

R I POSITION VECTOR

V I VELOCITY VECTOR

T TIME AT WHICH POSITION AND VELOCITY IS DESIRED

RT O POSITION AT TIME T

VT O VELOCITY AT TIME T

SUBROUTINES SUPPORTED: BEPS

SUBROUTINES REQUIRED: ATANH ZRANS

LOCAL SYMBOLS: A SEMIMAJOR AXIS OF THE CONIC

ACC ACCURACY LEVEL__

AMOM INTERMEDIATE VECTOR

ANS INTERMEDIATE ARGUMENT

ARG INTERMEDIATE ARGUMENT

AO RECIPROCAL OF A

C COS OR COSH OF X

CFO COSINE TRUE ANOMALY

CF1 INTERMEDIATE VARIABLE

C1 INTERMEDIATE VARIABLE

E ECCENTRICITY

EPS INTERMEDIATE VARIABLE

EO INTERMEDIATE VARIABLE

E1 INTERMEDIATE VARIABLE

FINF INTERMEDIATE VARIABLE

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FT TRUE ANOMALY
FO INTERMEDIATE VARIABLE -
      INTERMEDIATE VARIABLE
F1
F2
       INTERMEDIATE VARIABLE
      INTERMEDIATE VARIABLE
HO
       INTERMEDIATE VARIABLE
H1
       COUNTER
K
       INTERMEDIATE VARIABLE
LM
Ρ
       SEMI-LATUS RECTUM OF THE CONIC
PI
       MATHEMATICAL CONSTANT
      INTERMEDIATE VARIABLE
RD
       INTERMEDIATE VARIABLE
RDT
      MAGNITUDE OF POSITION VECTOR AT TIME 0
RM
RMT MAGNITUDE OF POSITION VECTOR AT TIME T
RRDT INTERMEDIATE VARIABLE
      SIN OR SINH OF X
S
      SIN TRUE ANOMALY
SFO
SF1 INTERMEDIATE VARIABLE
T1 INTERMEDIATE VARIABLE
       INTERMEDIATE VARIABLE
T2
      INTERMEDIATE VARIABLE
13
UDTDX
       INTERMEDIATE VARIABLE
UTXN
       INTERMEDIATE VARIABLE
       MAGNITUDE OF VELOCITY VECTOR AT TIME O
VM
VMT
       MAGNITUDE OF VELOCITY VECTOR AT TIME T
       INTERMEDIATE VARIABLE
7.
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#### BATCON Analysis

BATCON is a conic propagator using the Battin universal variable formulation. A total derivation is too involved to be given here; rather the results of Battin's work will be given here.

Let the initial state of a point mass moving under the influence of a gravitational force  $\mu$  be given by  $\vec{r}_0$ ,  $\vec{v}_0$ . It is required to determine the state  $\vec{r}$ ,  $\vec{v}$  at a time T units later. It is useful to introduce the parameters

$$\sigma_{0} = \frac{\overrightarrow{r}_{0} \cdot \overrightarrow{v}_{0}}{\sqrt{\mu}}$$

$$\alpha = \frac{2}{r_{0}} - \frac{v_{0}^{2}}{\mu}$$
(1)

Battin's approach is to introduce a new independent variable x(t) in place of time by the relation

$$\frac{dx}{dt} = \frac{\sqrt{\mu}}{r(t)} \qquad x(0) = 0 \tag{2}$$

This parametrization greatly simplifies the conic propagation problem. For suppose that the value of x corresponding to -t-x T is given by X, i.e. x(T) = X. Then the final state is given by

$$\vec{r} = R_1(x) \vec{r}_0 + R_2(x) \vec{v}_0$$

$$\vec{v} = V_1(x) \vec{r}_0 + V_2(x) \vec{v}_0$$
(3)

where

$$R_{1}(X) = 1 - \frac{1}{r_{o}} U_{2}(X) \qquad R_{2}(X) = \sqrt{\frac{1}{\mu}} \left[ r_{o} U_{1}(X) + \sigma_{o} U_{2}(X) \right] \qquad (4)$$

$$V_{1}(X) = -\frac{\sqrt{\mu}}{r_{o}} U_{1}(X) \qquad V_{2}(X) = 1 - \frac{1}{r_{o}} U_{2}(X)$$

and where

The problem is thus reduced to the determination of X. X is generated iteratively by the recursive formulae

$$x_{n+1} = x_n - \frac{\sqrt{\mu} t_n - \sqrt{\mu} t}{r_n} = x_n - \Delta x$$
 (6)

where

$$\sqrt{\mu} \quad \mathbf{t}_{\mathbf{n}} = \mathbf{r}_{\mathbf{o}} \mathbf{U}_{1}(\mathbf{x}_{\mathbf{n}}^{L}) + \sigma \, \mathbf{U}_{2}(\mathbf{x}_{\mathbf{n}}^{L}) + \mathbf{U}_{3}(\mathbf{x}_{\mathbf{n}}^{L}) 
\mathbf{r}_{\mathbf{n}} = \mathbf{r}_{\mathbf{o}} \mathbf{U}_{\mathbf{o}}(\mathbf{x}_{\mathbf{n}}^{L}) + \sigma \, \mathbf{U}_{1}(\mathbf{x}_{\mathbf{n}}^{L}) + \mathbf{U}_{2}(\mathbf{x}_{\mathbf{n}}^{L})$$
(7)

To start the process the initial guess is set to

$$\dot{\mathbf{x}}_{0} = \frac{\sqrt{\mu} \, \mathbf{T}}{\mathbf{r}_{0}} \left\{ 1 - \frac{\sigma_{0}}{2\mathbf{r}_{0}^{2}} \sqrt{\mu} \, \mathbf{T} + \frac{1}{6\mathbf{r}_{0}^{4}} \left[ 3 \, \sigma_{0}^{2} - \mathbf{r}_{0} (1 - \alpha \, \mathbf{r}_{0}) \right] \mu \, \mathbf{T}^{2} \right\}$$
(8)

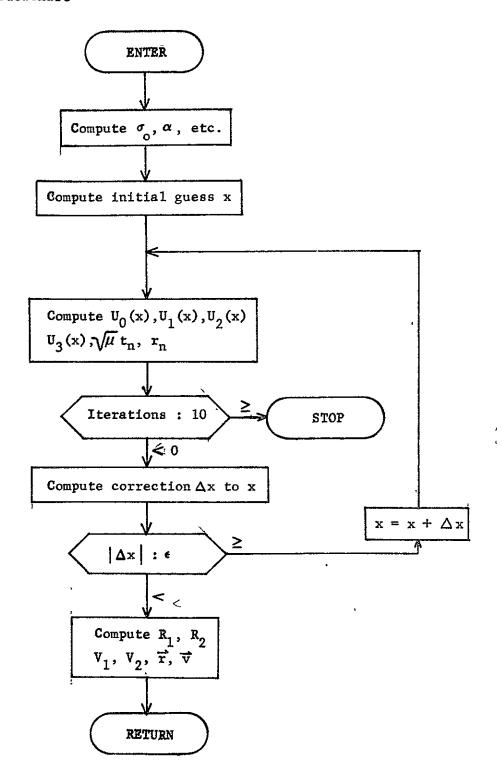
The program sets  $X = x_n$  when the correction  $\Delta x$  is less than  $10^{-8}$ . It terminates if the number of iterations exceeds 10.

# References:

Battin, R.H., Astronautical Guidance, McGraw-Hill Book Co., New York, 1964.

Battin, R. H. and Fraser, D.C., Space Guidance and Navigation, AIAA Professional Study Series, 1970.

## BATCON Flowchart



SUBROUTINE BEPS

PURPOSE TO COMPUTE POSITION/VELOCITY COVARIANCE AT THE B-PLANE AND TO COMPUTE THE STATE TRANSITION MATRIX RELATING CHANGE IN POSITION/VELOCITY TO B-PLANE PARAMETER CHANGES

CALLING SEQUENCE: CALL BEPS (GMU, R, V, GAMMA, BPS)

ARGUMENTS: GMU I GRAVITATIONAL CONSTANT OF TARGET PLANET

R I POSITION VECTOR AT TIME OF EVENT

V I VELOCITY VECTOR AT TIME OF EVENT

GAMMA O STATE TRANSITION MATRIX RELATING STATE TO

B-PLANE PARAMETERS

BPS O POSITION/VELOGITY COVARIANCE AT B-PLANE

SUBROUTINES SUPPORTED: PRED

SUBROUTINES REQUIRED: CAREL BATCOX CONC2 BPLANE

LOCAL SYMBOLS: A SEMIMAJOR AXIS OF THE CONIC

BPV B-PLANE VARIABLES VECTOR

CF INTERMEDIATE VARIABLE

COSFX INTERMEDIATE VARIABLE

DELT TIME DIFFERENCES BETWEEN SOI AND PERIAPSIS

E ECCENTRICITY OF THE CONIC

FX INTERMEDIATE VARIABLE

HSINCF INTERMEDIATE VARIABLE

PP DUMMY ARRAY FOR CAREL CALL

QQ DUMMY ARRAY FOR CAREL CALL

WW DUMMY ARRAY FOR CAREL CALL

PSI STATE TRANSITION MATRIX

PVALUE POSITION/VELOCITY PERTURBATION VECTOR

RN DUMMY VECTOR FOR BPLANE CALL

SN DUMMY VECTOR FOR BPLANE CALL

TN DUMMY VECTOR FOR BPLANE CALL

RR INTERMEDIATE VECTOR

VV INTERMEDIATE VECTOR

RS POSITION AT THE B-PLANE

VS VELOCITY AT THE B-PLANE

RSI SPHERE OF INFLUENCE RADIUS IN KM.

SINFX INTERMEDIATE VARIABLE

SUM INTERMEDIATE VARIABLE

TA INSTANTANEOUS TRUE ANAMOLY OF THE CONIC

TANCF INTERMEDIATE VARIABLE

TFP TIME FROM PERIAPSIS

TPSI INTERMEDIATE VARIABLE

W ARGUMENT OF PERIAPSIS OF THE CONIC

XI INCLINATION OF THE CONIC TO REF. FRAME

XN LONGITUDE OF ASCENDING NODE OF THE CONIC

COMMON USED: ALNGTH NTP SPHERE

SUBROUTINE BLAIM

PURPOSET TO PERFORM BIASED AIMPOINT GUIDANCE.

CALLING SEQUENCE: CALL BIAIM(RI, TEVN)

ARGUMENTS RI I NOMINAL SPACECRAFT STATE AT TIME OF BIASED AIMPOINT GUIDANCE EVENT

TEVN I TIME OF BLASED AIMPOINT GUIDANCE EVENT

SUBROUTINES SUPPORTED: GUISIN GUIDN

SUBROUTINES REQUIRED: MATIN POICOM PSIM QCOMP

LOCAL SYMBOLS ADA1 VARIATION MATRIX AT TIME T(J+1)

BB RIGHT HALF PARTITION OF ADA1 MATRIX

CSQ CONSTANT DEFINING CONSTRAINT ELLIPSE

C1 A COEFFICIENT IN THE NECESSARY CONDITION

C2 A COEFFIEIENT IN THE NECESSARY CONDITION

C3 A COEFFICIENT IN THE NECESSARY CONDITION

C4 A COEFFICIENT IN THE NECESSARY CONDITION

C5 A COEFFICIENT IN THE NECESSARY CONDITION

C SQUARE ROOT OF CSQ

DELMU AIMPOINT BIAS IN IMPACT PLANE

DENOM INTERMEDIATE VARIABLE

DET DETERMINANT OF PROJECTED TARGET CONDITION COVARIANCE MATRIX

DVBIAS BIAS VELOCITY CORRECTION

DATT TOTAL VELOCITY CORRECTION IF BIAS IS REMOVED

DVT TOTAL VELOCITY CORRECTION IF BIAS IS APPLIED

DVUPP UPDATE VELOCITY ITERATE

D1 PARTIAL DERIVATIVE USED IN NEWTON ITERATION JECHNIQUE

D2 PARTIAL DERIVATIVE USED IN NEWTON ITERATION TECHNIQUE

D3 PARTIAL DERIVATIVE USED IN NEWTON ITERATION TECHNIQUE

D4 PARTIAL DERIVATIVE USED IN NEWTON ITERATION TECHNIQUE

EE MATRIX DEFINING FUNCTION TO BE MINIMIZED

IKNT COUNTER ON NEWTON ITERATION LOOP

IS INDEX OF NEXT GUIDANCE EVENT

ITRN COUNTER ON OUTER ITERATION LOOP

NDIM1S STORAGE FOR NDIM1

NDIM2S STORAGE FOR NDIM2

PHII INVERSE OF STATE TRANSITION MATRIX

PHI1 INTERMEDIATE ARRAY

PSIJ1 GUIDANCE MATRIX PSI AT T(J+1)

PSIJ GUIDANCE MATRIX PSI AT EVENT TIME T(J)

. QUOT INTERMEDIATE VARIABLE

RF DUMMY VECTOR

SAVET STORAGE FOR TRTM1

SUM1 INTERMEDIATE VARIABLE

SUM INTERMEDIATE VARIABLE

TWOE CONSTANT DEFINING CONSTRAINT ELLIPSE

VCA SPACECRAFT CLOSEST APPROACH VELOCITY RELATIVE TO TARGET PLANET

XK4 INTERMEDIATE VARIABLE

XMU MOST RECENT IMPACT PLANE AIMPOINT

XM1 AIMPOINT ITERATE

XM2 AIMPOINT ITERATE

XM STORAGE FOR MOST RECENT AIMPOINT ITERATE

XN1	NEGATIVE OF CONSTRAINT EQUATION EVALUATED AT MOST RECENT AIMPOINT ITERATE
XN2	NEGATIVE OF NECESSARY CONDITION EVALUATED AT MOST RECENT AIMPOINT ITERATE
AA	INTERMEDIATE VARIABLE
ZH	AIMPOINT INCREMENT FOR MOST RECENT ITERATION
ZK	AIMPOINT INCREMENT FOR MOST RECENT ITERATION
COMMON COMPUTED/USED:	A CR DVN DVRB DVUP EXEC IBIAS IIGP PHI2 RCA TMPR TRTM1 XMUS
COMMON COMPUTEDS	DELTM
COMMON USEDS	ADA ALNGTH ATRANS DUMMYQ EM3 IDENS IEND IGUID II ISTMC ITR NTP ONE PHI PMASS POI PROBI RADIUS TINJ TM TNOMB TNOMC TWO T3 VINF XLAMI XLAM ZERO

### BIAIM Analysis

Subroutine BIAIM performs biased aimpoint guidance computations. If planetary quarantine constraints are in effect at injection or at a midcourse correction, and if the nominal aimpoint does not satisfy these constraints, subroutine BIAIM will compute a biased aimpoint and the required bias velocity correction such that the constraints are satisfied and some performance functional is minimized.

Aimpoint biasing is performed in the impact plane and as such permits only two degrees of freedom in the selection of the biased aimpoint. The general aimpoint in the impact plane will be denoted by the 2-dimensional vector  $\vec{\mu}_j$ , where the j-subscript indicates that the biased aimpoint guidance event is occurring at time t_j. Three midcourse guidance policies are available in STEAP, and it will be necessary to relate  $\vec{\mu}_j$  to the specific aimpoint for each of these three policies. These relationships are summarized below:

(a) Two-variable B-plane (2VBP):

$$\vec{\mu}_{j} = \begin{bmatrix} B \cdot T \\ B \cdot R \end{bmatrix} \tag{1}$$

(b) Three-wariable B-plane (3VBP):

$$\vec{\mu}_{j} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} B \cdot T \\ B \cdot R \\ t_{SI} \end{bmatrix}$$
 (2)

(c) Fixed-time-of-arrival (FTA):

$$\vec{\mu}_{j} = A \vec{r}_{CA} \tag{3}$$

where  $\vec{r}_{CA}$  is the nominal closest approach position of the spacecraft relative to the target planet. Coordinate transformation A projects the 3-dimensional vector  $\vec{r}_{CA}$  (referred to ecliptic coordinates) into an equivalent FTA impact plane which is defined to be the plane containing  $\vec{r}_{CA}$  and perpendicular to the spacecraft closest approach velocity  $\vec{v}_{CA}$  relative to the target planet. If the ecliptic coordinates of  $\vec{r}_{CA}$  and  $\vec{v}_{CA}$  are denoted by  $r_x$ ,  $r_y$ ,  $r_z$  and  $v_x$ ,  $v_y$ ,  $v_z$ , respectively, then the transformation A is given by

$$A = \begin{bmatrix} \frac{r_x}{r_{CA}} & \frac{r_y}{r_{CA}} & \frac{r_z}{r_{CA}} \\ \frac{r_y v_z - r_z v_y}{r_{CA} v_{CA}} & \frac{r_z v_x - r_x v_z}{r_{CA} v_{CA}} & \frac{r_x v_y - r_y v_x}{r_{CA} v_{CA}} \end{bmatrix}$$

$$(4)$$

Spacecraft state variations at  $t_j$  are related to aimpoint variations (target condition variations) by the variation matrix  $\eta_j$ , which is always available prior to calling BIAIM. Thus, the statistical state dispersions about the nominal following the guidance correction at  $t_j$  and represented by the control covariance  $P_{c_j}^+$ , can be related to the dispersions about the nominal aimpoint represented by  $W_j^+$  according to the equation

$$W_{j}^{+} = \eta_{j} P_{c_{j}}^{+} \eta_{j}^{T}$$
 (5)

The control covariance  $P_{c_{j}}^{+}$  is computed from

$$P_{c_{j}}^{+} = P_{k_{j}}^{-} + \begin{bmatrix} 0 & i & 0 \\ 0 & i & \tilde{Q}_{j} \end{bmatrix}$$
 (6)

where  $P_{k}$  is the knowledge covariance prior to the guidance event and  $\vec{Q}_{\,i}$  is the execution error covariance.

Transformations employed in equations (1) through (3) can also be employed to project  $W_j^+$  into the impact plane. The resulting projection is denoted by the covariance  $\Lambda_j$ , and is obtained from  $W_j^+$  according to the following equations:

(a) 
$$2VBP$$
:  $\Lambda_{i} = W_{j}^{+}$  (7)

(b) 3VBP: 
$$\Lambda_{j} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w_{j}^{\dagger} & 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$
 (8)

(c) FTA: 
$$\Lambda_{j} = A W_{j}^{+} A^{T}$$
 (9)

With covariance  $\Lambda_j$  available, it is now possible to compute the probability of impact PØI. Assuming the probability density function associated with  $\Lambda_j$  is gaussian and nearly constant over the target planet capture area permits us to compute PØI using the equation

$$P\emptyset I = \pi R_c^2 p \tag{10}$$

where R is the target planet capture radius and p represents the gaussian density function evaluated at the target planet center and is given by

$$p = \frac{1}{2\pi \left| \Lambda_{j} \right|^{\frac{1}{2}}} \exp \left[ -\frac{1}{2} \vec{\boldsymbol{\mu}} *^{T} \Lambda_{j}^{-1} \vec{\boldsymbol{\mu}} * \right]$$
 (11)

The nominal impact plane simpoint is denoted by  $\vec{\mu}^*$ . Subroutine BIAIM calls subroutine PØICØM to perform the computations involved in equations (7) through (11).

Capture radius R is simply the physical radius R of the target planet if the FTA guidance policy is employed, while for the two B-plane policies the capture radius is given by

$$R_{c} = R_{p} \sqrt{1 + \frac{2 \mu_{p}}{V_{\infty}^{2} R_{p}}}$$
 (12)

where  $\mu_{p}$  is the target planet gravitational constant and  $V_{co}$  is the hyperbolic excess velocity.

If the probability of impact PØI does not exceed the permissible impact probability  $P_{\underline{I}}$ , and if the nominal aimpoint has not been previously biased, we simply return to subroutine GUIDM (or GUISIM). If the nominal aimpoint has been previously biased, a velocity correction  $\Delta \overline{V}_{RB}$  required

to remove that bias is computed prior to returning. But if PØI exceeds  $P_{1}$ , an aimpoint bias  $\delta \vec{\mu}_{j}$  and the associated bias velocity correction  $\Delta \vec{v}_{B_{j}}$ 

must be computed. Before describing the details of the biasing technique it is necessary to define the relationship between  $\Delta \vec{V}$  and  $\delta \vec{\mu}$  for linear midcourse guidance policies.

Linear impulsive guidance policies have form

$$\Delta \vec{v}_{j} = \Gamma_{j} \delta \hat{\vec{x}}_{j} \tag{13}$$

where  $\Gamma_j$  is the guidance matrix and  $\delta \vec{X}_j$  is the spacecraft state deviation from the targeted nominal trajectory. (These guidance policies are discussed in more detail in the subroutine GUIS analysis section.) Such guidance policies can be readily generalized to account for changes in the target conditions from their nominal values. This generalized version of equation (13) has form

$$\Delta \vec{v}_{j} = \Gamma_{j} \delta \vec{x}_{j} + \Psi_{j} \delta \vec{\mu}_{j}$$
 (14)

where  $\psi_j$  can also be referred to as a guidance matrix. For the purposes of the BIAIM analysis, we shall assume that  $\delta \vec{\mu}_j$  in equation (14) is always an aimpoint change in the impact plane. Thus,  $\psi_j$  will be a 3x2 guidance matrix. The derivation of the  $\psi_j$  matrix is quite similar to the derivation of the  $\Gamma_j$  matrix and will not be presented here. If we partition the previously discussed variation matrix  $\eta_j$  as follows:

$$\eta_{i} = \left[ \eta_{1} \mid \eta_{2} \right] \tag{15}$$

then the  $\psi$  matrices for the three midcourse guidance policies are given by the following equations:

(a) 2VBP : 
$$\Psi_{j} = \eta_{2}^{T} (\eta_{2} \eta_{2}^{T})^{-1}$$
 (16)

(b) 
$$3VBP : \Psi_{j} = \eta^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$
 (17)

(c) FTA: 
$$\Psi = \eta^{-1} A^{T}$$
 (18)

If an aimpoint bias were to be removed at time t, the required velocity correction would be given by

$$\Delta \vec{v}_{RB_{j}} = -\Psi_{j} \delta \vec{\mu}_{j}$$
 (19)

If an aimpoint bias were to be imparted at time  $t_j$ , the bias velocity correction would be given by

$$\Delta \vec{v}_{B_{j}} = \Psi_{j} \delta \vec{\mu}_{j}$$
 (20)

If an aimpoint bias  $\delta \hat{\mu}_{j}$  had been previously imparted, and if a new aimpoint bias  $\delta \hat{\mu}_{j}$  is to be imparted, then the total bias velocity correction would be given by

$$\Delta \hat{\nabla}_{B_{i}} = \Psi_{j} \left[ \delta \vec{\mu}_{j}^{(2)} - \delta \vec{\mu}_{j}^{(1)} \right]$$
 (21)

The general statement of the biased aimpoint guidance problem is as follows: Find an aimpoint  $\hat{\mu}_j$  in the impact plane which satisfies the impact probability constraint

$$\mathbf{P} \emptyset \mathbf{I} \notin \mathbf{P}_{\mathsf{T}} \tag{22}$$

and minimizes a performance functional having form

$$J = (\vec{\mu}_{1} - \vec{\mu}^{*})^{T} \tilde{A} (\vec{\mu}_{1} - \vec{\mu}^{*})$$
 (23)

where  $\vec{\mu}$  's the nominal aimpoint and  $\vec{A}$  is a constant symmetric matrix that will be defined subsequently.

The solution of this problem is detailed in the section on biased aimpoint guidance in the analytical manual. Only the results will be presented here. The assumption of constant probability density over the target planet capture area permits us to rewrite constraint equation (22) as

$$\lambda_1 \mu_1^2 + 2\lambda_3 \mu_1 \mu_2 + \lambda_2 \mu_2^2 = c^2$$
 (24)

where

$$c^{2} = 2 \ln \left[ \frac{R_{c}^{2}}{2 |\mathcal{A}|^{\frac{1}{2}} P_{I}} \right]$$
 (25)

and 
$$\vec{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$$
 and  $\Lambda^{-1} = \begin{bmatrix} \lambda_1 & \lambda_3 \\ \lambda_3 & \lambda_2 \end{bmatrix}$ .

The inequality has been replaced by an equality since the solution can be shown to lie on the constraint boundary, which, from inspection of equation (24) is an ellipse centered at the target planet.

If t_j is the time of the final midcourse correction, matrix  $\tilde{A}$  will be chosen as a 2x2 identity matrix. The minimization of J is then equivalent to minimization of the miss distance  $|\tilde{\mu}_j - \tilde{\mu}^*|$ . If t_j is not the final midcourse correction time,  $\tilde{A}$  will be defined as follows:

$$\tilde{A} = \overset{T}{\psi} \qquad \psi \qquad (26)$$

Here  $\psi_{j+1}$  denotes the aimpoint guidance matrix for the next midcourse correction occurring at time  $t_{j+1}$ . In this case the minimization of J is equivalent to the minimization of  $\left\|\Delta V_{RB_{j+1}}\right\|$ , i.e., the velocity required to remove bias  $\delta \tilde{\mu}_{j}$  at time  $t_{j+1}$  will by minimized. The computation of  $\psi_{j+1}$  is based on the variation matrix  $\eta_{j+1}$ , just as  $\psi_{j}$  was based on  $\eta_{j}$ . However,  $\eta_{j+1}$  can be computed more efficiently by using the relationship

$$\eta = \eta \Phi^{-1}$$

$$j+1 \quad j \quad j+1, \quad j$$
(27)

where  $\Phi_{j+1,j}$  is the state transition matrix over  $[t_j, t_{j+1}]$ .

If we define

$$\overset{\circ}{A} = \begin{bmatrix} a_1 & a_3 \\ a_3 & a_2 \end{bmatrix}$$

then the necessary condition for a minimum is given by

$$(a_{1} \lambda_{3} - a_{3} \lambda_{1}) \mu_{1}^{2} + (a_{3} \lambda_{2} - a_{2} \lambda_{3}) \mu_{2}^{2} + (a_{1} \lambda_{2} - a_{2} \lambda_{1}) \mu_{1} \mu_{2}$$

$$+ (-a_{1} \lambda_{3} \mu_{1}^{*} - a_{3} \lambda_{3} \mu_{2}^{*} + a_{3} \lambda_{1} \mu_{1}^{*} + a_{2} \lambda_{1} \mu_{2}^{*}) \mu_{1} +$$

$$(-a_{1} \lambda_{2} \mu_{1}^{*} - a_{3} \lambda_{2} \mu_{2}^{*} + a_{3} \lambda_{3} \mu_{1}^{*} + a_{2} \lambda_{3} \mu_{2}^{*}) \mu_{2} = 0$$

$$(28)$$

Thus, our problem is reduced to finding  $\mu_1$  and  $\mu_2$  which satisfy equations (24) and (28). Since the analytical solution of these equations proved intractable, a standard Newton iteration technique is employed in BIAIM which quickly converges to solutions for  $\mu_1$  and  $\mu_2$ . The iteration process is started with an initial guess defined as the intersection of the extended  $\vec{\mu}^*$  vector and the constraint boundary defined by equation (24). This initial guess is given by

$$\mu_{1}^{\circ} = \left(\begin{array}{c} \frac{\mu_{1}^{*}}{\mu_{2}^{*}} \right) \mu_{2}^{\circ}$$

$$\mu_{2}^{\circ} = \operatorname{sgn}(\mu_{2}^{*}) \frac{c}{\sqrt{\lambda_{1} \left(\frac{\mu_{1}^{*}}{\mu_{2}^{*}}\right)^{2} + 2\lambda_{3} \left(\frac{\mu_{1}^{*}}{\mu_{2}^{*}}\right) + \lambda_{2}}}$$
(29)

where c is defined by equation (25).

In addition to the previously described iteration process, subroutine BIATM also employs an outer iteration loop which accounts for the dependence of  $\vec{Q}_j$  (equation (6)) on  $\delta \vec{\mu}_j$ . The execution error covariance  $\vec{Q}_j$  is a function of the total velocity correction at  $t_j$ , but the total velocity correction, in particular  $\Delta \vec{\nabla}_B$ , depends on  $\delta \vec{\mu}_j$ . This coupling is resolved by recomputing  $\vec{Q}_j$  at the end of the previously described biasing technique and repeating the biasing cycle until the error function

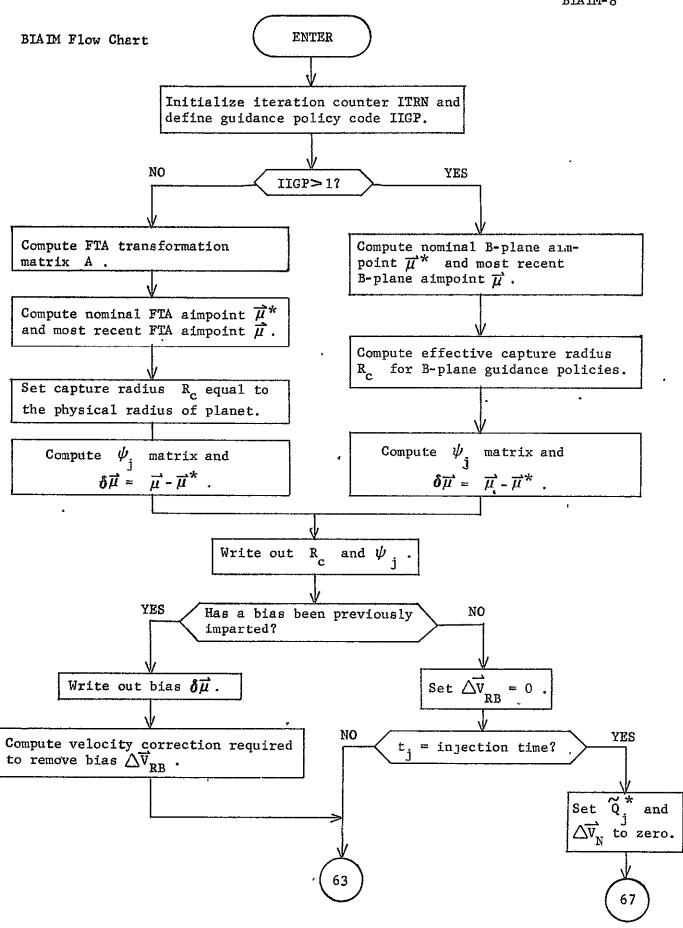
is satisfied. This outer iteration process is not performed, however, if  $t_j$  = injection time since at injection equation (6) is replaced by the equation

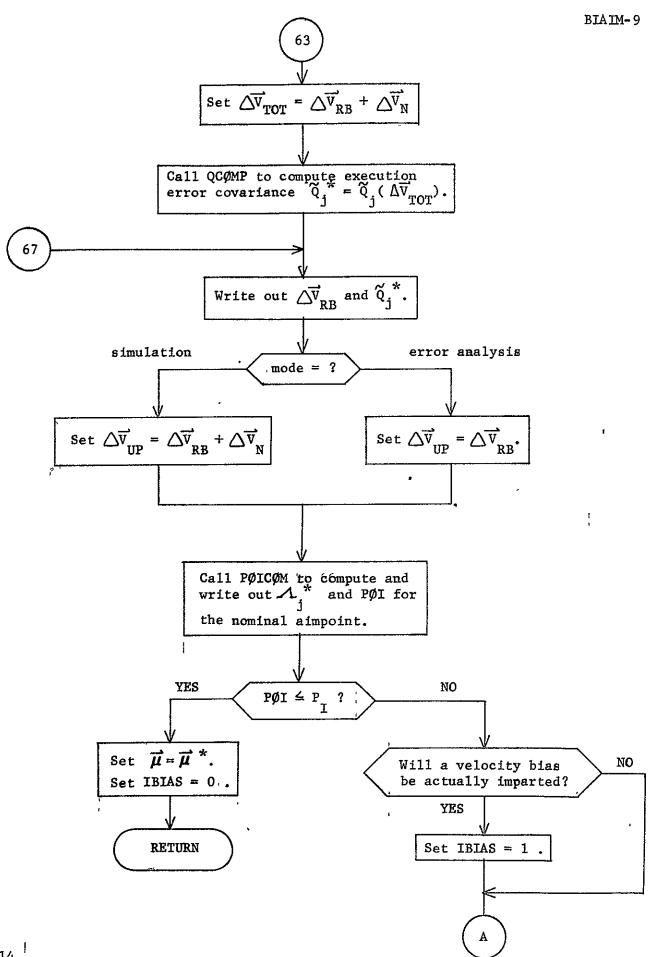
$$P_{c_{j}} = P_{k_{j}}$$

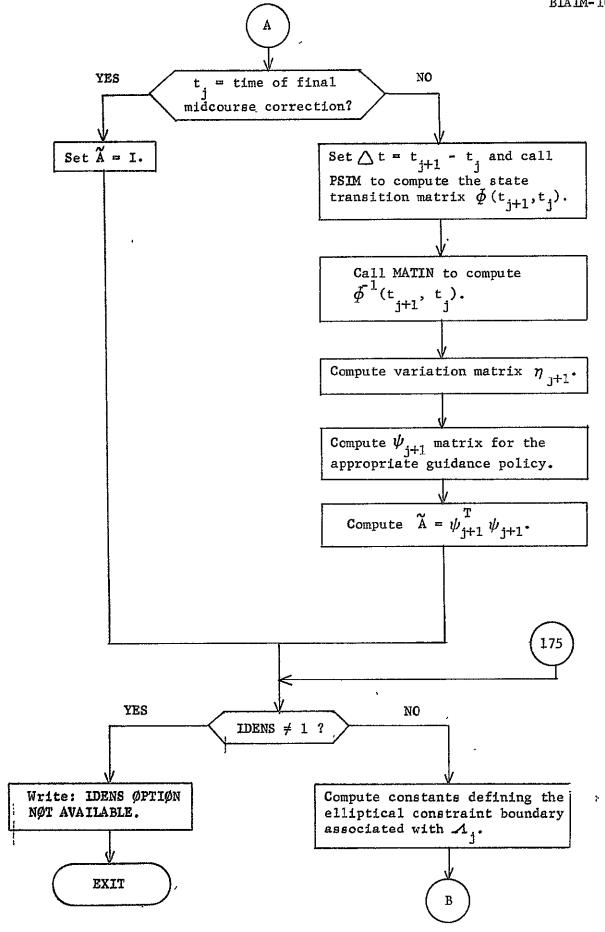
and  $\tilde{Q}$  is always zero.

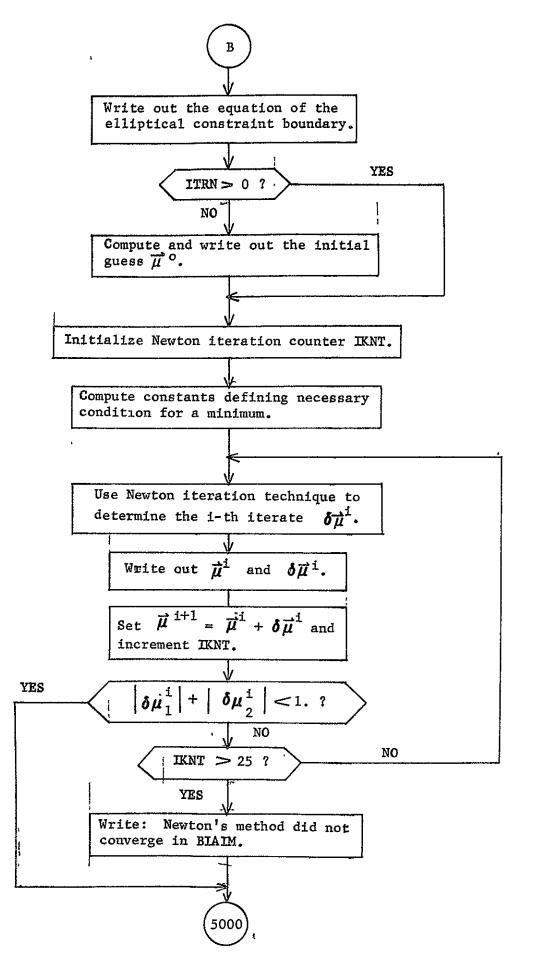
Reference: Mitchell, R. T., and Wong, S. K.: Preliminary Flight Path Analysis Orbit Determination and Maneuver Strategy Mariner Mars 1969. Project Document 138, Jet Propulsion Laboratory, 1968.

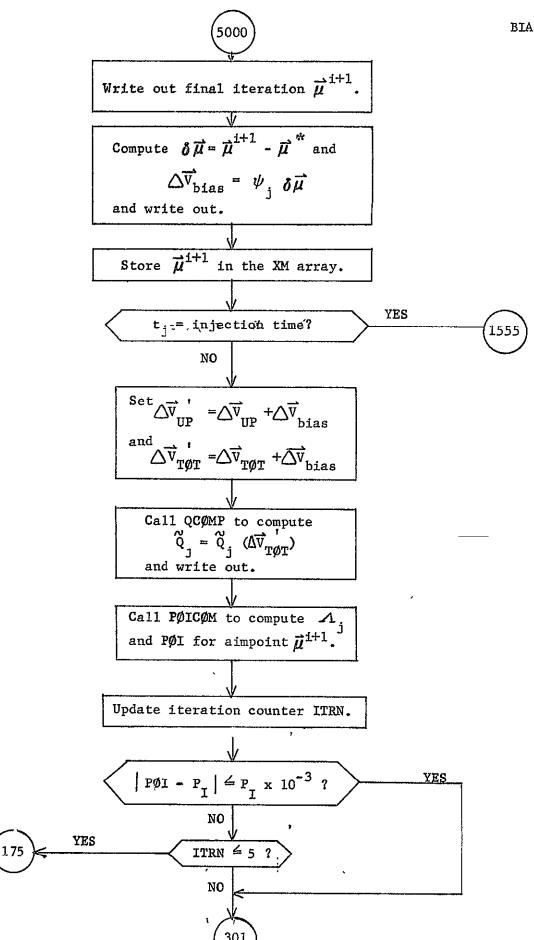
113

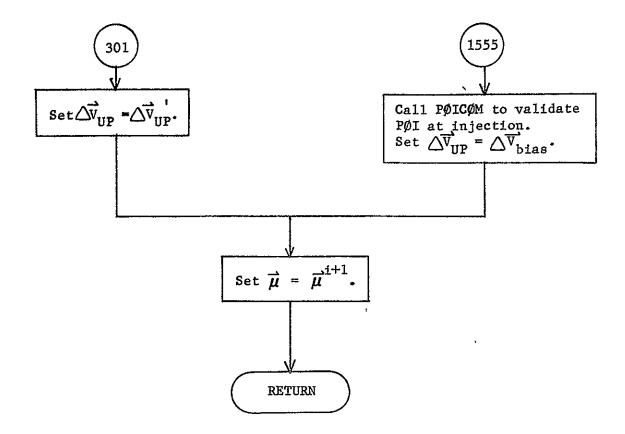












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SUBROUTINE BIAS

PURPOSE: COMPUTE THE ACTUAL MEASUREMENT BIAS IN THE SIMULATION PROGRAM

RETURN THE ACTUAL MEASUREMENT BIAS TO BE USED IN THE SIMULATION MODE.

CALLING SEQUENCE: CALL BIAS (MCODE, BVAL)

ARGUMENT: BVAL O THE ACTUAL BIAS TO BE USED IN THE MEASUREMENT

NCODE I NEASUREMENT TYPE CODE

SUBROUTINES SUPPORTED & SIMULL

COMMON USED: BIA

BIAS Analysis

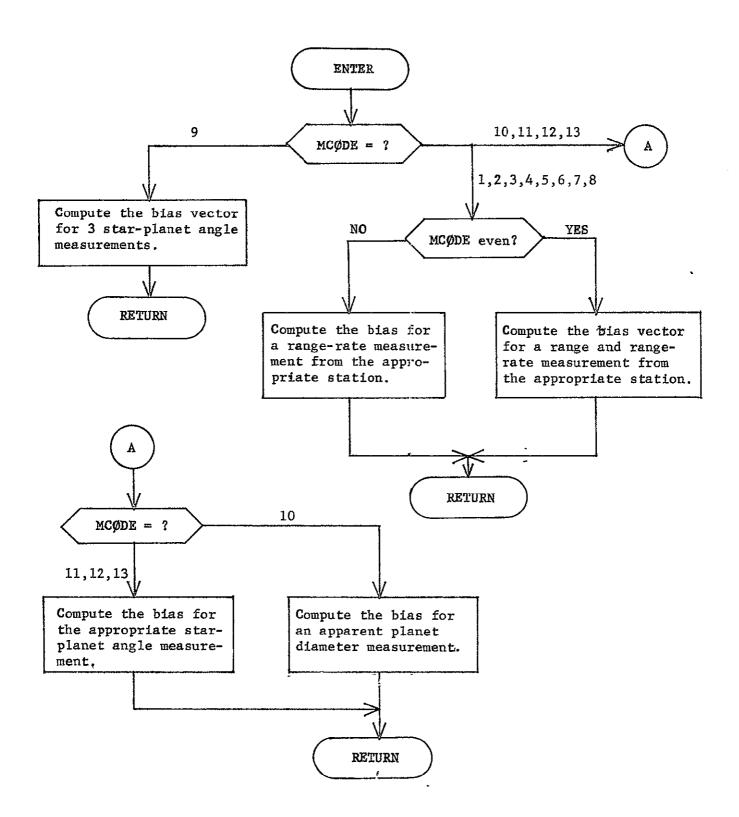
The actual measurement  $Y_k^a$  at time  $t_k$  is given by

$$Y_k^a = \underline{Y}_k + b_k + \nu_k$$

where  $\underline{Y}_k$  is the ideal measurement, which would be made in the absence of instrumentation errors,  $b_k$  is the actual measurement bias and  $\nu_k$  represents the actual measurement noise.

The function of subroutine BIAS is to compute the measurement bias  $\,b_k^{}\,$   $\,$  for the appropriate measurement type. The constant biases for all measurement devices are stored in the vector BIA. Subroutine BIAS selects the appropriate elements from this vector to construct the actual measurement bias.

## BIAS Flow Chart



BLOCK DATA

PURPOSE: TO LOAD CONSTANTS INTO COMMON LOCATIONS USED IN VARIOUS OTHER PARTS OF THE PROGRAM.

CALLING SEQUENCE: NONE

ARGUMENTS NONE

SUBROUTINES SUPPORTED HALF THE SUBROUTINES USE THE CONSTANTS

STORED BY THIS SUBROUTINE

SUBROUTINES REQUIRED NONE

COMMON LOADED CN1 CN ELMNT EMM EVNM

RADIUS RAD RMASS SMJR SPHERE F MNNAME PI PLANET PMASS

ST

## BLKDAT Analysis

Subroutine BIKDAT is responsible for setting up constants used in computing ephemeris data for the gravitating bodies.

The arrays set up by BLKDAT and their definitions are as follows:

Array	Definition
CN(80)	Constants defining mean elements for inner planets
ST(50)	Constants defining mean elements for outer planets
SMJR(18)	Constants defining semi-major axes for planets and
	moon
EMN(15)	Constants defining lunar elements
PMASS (11)	Gravitational constants of sun, planets, and moon
RMASS(11)	Mass of bodies relative to sun
RADIUS (11)	Surface radii of sun, planets, and moon
SPHERE (11)	Sphere of influence radii of sun, planets, and moon
MONTH (12)	Names of months for output purposes
PLANET(11)	Names of planets for output purposes

The definitions of the CN, ST, SMJR, and EMN arrays are provided in Tables 2 through 5 on the following page. The actual constants stored in those arrays are the ephemeris data listed on the next pages following.

The constants stored in the other arrays are given below.

Body	PMASS (AU ³ /day ² )	rmass*	RADIUS (AU)	SPHERE (AU)
Sun	2.959122083(-4)	1.0	4.66582(-3)	NA
Mercury	4.850 <b>(-</b> 11 <b>)</b>	1.639(-7)	1.617(-5)	7.46 <b>(-</b> 4)
Venus	7.243(~10)	2.448(-6)	4.044(-5)	4.12(-3)
Earth	8.88757(-10)	3.003(-6)	4.263(-5)	6.18(-3)
Mars	9.5497905(-11)	3.236(-7)	2.279(-5)	3.78(-3)
Jupiter	2.8252 ( <b>-</b> 7)	9.547(-4)	4.7727( <del>-</del> 4)	.3216
Saturn	8.454(~8)	2.857(-4)	4.0374(-4)	.3246
Uranus	1.290(-8)	4.359(-5)	1.5761(-4)	.346
Neptune	1.5(-8)	5.069( <b>-</b> 5)	1.4906(-4)	.5805
Pluto	7.4(-10)	2.501(-6)	4.679(-5)	.2366
Moon	1.0921748(-11)	3.696(-8)	1.161(-5)	3.71394(-4)

^{*} Truncated from program values

# Array Definitions

Constant	i.	Ω	$\widetilde{\omega}$	е	M	a	w	E	^a 0	a 1
Mercury	1	2	3	4	5	6	7	8	1	2
Venus	9	10	11	12	13	14	15	16	3	4
Earth	17	18	19	20	21	22	23	24	5	6
Mars	25	26	27	28	29	30	31	32	7	8
Jupiter	33	34	35	36	37	38	39	40	9	10
Saturn	41	42	43	44	45	46	47	48	11	12
Uranus	49	50	51	52	53	54	55	56	13	14
Neptune	57	58	59	60	61	62	63	64	15	16
Pluto	65	66	67	68	69	70	71	72	17	18
Moon	73	74	75	76	77	78	79	80		

Table 1. ELMNT Array -- Conic Elements

Table 2. SMJR Array

Constant	i ₀	i ₁	¹ 2	í ₃	$\Omega_{0}$	$\Omega_{1}$	$\Omega_2$	$\Omega_3$	$\widetilde{\omega}_{0}$	$\widetilde{\omega}_1$	$\widetilde{\omega}_{_{\! 2}}$	$\widetilde{\omega}_{3}$	е ₀	е ₁	е ₂	е ₃	М ₀	^M 1	^M 2	<b>м</b> ₃
Mercury	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Venus							27													
Earth	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Mars	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80

Table 3. CN Array -- Inner Planet Constants

Constant -	¹ 0	iì	$\Omega_0$	$\mathcal{Q}_1$	$\ddot{\omega}_{_{0}}$	$\widetilde{\omega}_{_{1}}$	е ₀	<b>e</b> 1	Mo	M ₁
Jupiter	1	2	3	4	5	6	7	8	۰ 9	10
Saturn	11	12	13	14	15	16	17	18	19	20
Uranus	21	22	23	24	25	26	27	28	29	30
Neptune	31	32	33	34	35	36	37	38	39	40
Pluto	41	42	43	44	45	46	47	48	49	50

Table 4. ST Array -- Outer Planet Constants

Constant	$\Omega_0$	$\mathcal{\Omega}_1$	$\Omega_2$	$\Omega_3$	$\omega_{0}$	$\widetilde{\omega}_1$	$\widetilde{\omega}_2$	$\widetilde{\omega}_3$	$\mathbf{r}^0$	L ₁	L ₂	L ₃	i	е	а
Moon	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Table 5. EMN Array -- Lunar Constants

## Planetary and Lunar Ephemerides

# Mean Elements of Mercury

$$i = 0.1222233228 + 3.24776685 \times 10^{-5} T - 3.199770295 \times 10^{-7} T^{2}$$

$$\Omega = 0.8228518595 + 2.068578774 \times 10^{-2} \text{ T} + 3.034933644 \times 10^{-6} \text{ T}^2$$

$$\widetilde{\omega} = 1.3246996178 + 2.714840259 \times 10^{-2} \text{ T} + 5.143873156 \times 10^{-6} \text{ T}^2$$

$$e = 0.20561421 + 0.00002046 T - 0.000000030 T^2$$

$$M = 1.785111955 + 7.142471000 \times 10^{-2} d + 8.72664626 \times 10^{-9} D^{2}$$

$$a = 0.3870986 \text{ A.U.} = 57,909,370 \text{ km}$$

## Mean Elements of Venus

$$i = 0.0592300268 + 1.7555510339 \times 10^{-5} T - 1.696847884 \times 10^{-8} T^{2}$$

$$\Omega = 1.3226043500 + 1.570534527 \times 10^{-2} \text{ T} + 7.155849933 \times 10^{-6} \text{ T}^2$$

$$\widetilde{\omega} = 2.2717874591 + 2.457486613 \times 10^{-2} \text{ T} + 1.704120089 \times 10^{-5} \text{ T}^2$$

$$e = 0.00682069 - 0.00004774 T + 0.000000091 T^2$$

$$M = 3.710626172 + 2.796244623 \times 10^{-2} d + 1.682497399 \times 10^{-6} p^2$$

$$a = 0.7233316 \text{ A.U.} = 108,209,322 \text{ km}$$

# Mean Elements of Earth

$$i = 0$$

$$\Omega = 0$$

$$\widetilde{\omega} = 1.7666368138 + 3.000526417 \times 10^{-2} \text{ T} + 7.902463002 \times 10^{-6} \text{ T}^2 + 5.817764173 \times 10^{-8} \text{ T}^3$$

$$e = 0.01675104 - 0.00004180 T - 0.000000126 T^2$$

$$M = 6.256583781 + 1.720196977 \times 10^{-2} d - 1.954768762 \times 10^{-7} p^{2}$$
$$- 1.22173047 \times 10^{-9} p^{3}$$

$$a = 1.0000003 \text{ A.U.} = 149.598.530 \text{ km}$$

## Mean Elements of Mars

$$i = 0.0322944089 - 1.178097245 \times 10^{-5} T + 2.201054112 \times 10^{-7} T^{2}$$

$$\Omega = 0.8514840375 + 1.345634309 \times 10^{-2} \text{ T} - 2.424068406 \times 10^{-8} \text{ T}^2$$
  
- 9.308422677 × 10⁻⁸ T³

$$\widetilde{\omega} = 5.8332085089 + 3.212729365 \times 10^{-2} \text{ T} + 2.266503959 \times 10^{-6} \text{ T}^2$$

$$-2.084698829 \times 10^{-8} \text{ T}^3$$

$$e = 0.09331290 + 0.000092064 T - 0.000000077 T^2$$

$$M = 5.576840523 + 9.145887726 \times 10^{-3} d + 2.365444735 \times 10^{-7} p^2 + 4.363323130 \times 10^{-10} p^3$$

$$a = 1.5236915 \text{ A.U.} = 227,941,963 \text{ km}$$

# Mean Elements of Jupiter

$$1 = 0.0228410270 - 9.696273622 \times 10^{-5} T$$

$$\Omega = 1.7355180770 + 1.764479392 \times 10^{-2} \text{ T}$$

$$\widetilde{\omega} = 0.2218561704 + 2.812302353 \times 10^{-2} \text{ T}$$

$$e = 0.0483376 + 0.00016302 T$$

$$M = 3.93135411 + 1.450191928 \times 10^{-3} d$$

## Mean Element of Saturn

$$i = 0.0435037861 = 7.757018898 \times 10^{-8} T$$

$$\Omega = 1.9684445802 + 1.523977870 \times 10^{-2} \text{ T}$$

$$\widetilde{\omega} = 1.5897996653 + 3.419861162 \times 10^{-2} \text{ T}$$

$$e = 0.0558900 - 0.00034705 T$$

$$M = 3.0426210430 + 5.837120844 \times 10^{-4} d$$

$$a = 9.538843 \text{ A.U.} = 1.426.996.160 \text{ km}$$

## Mean Elements of Uranus

$$i = 0.0134865470 + 0.696273622 \times 10^{-6} T$$

$$\Omega = 1.2826407705 + 8.912087493 \times 10^{-3} \text{ T}$$

$$\widetilde{\omega} = 2.9502426085 + 2.834608631 \times 10^{-2} \text{ T}$$

```
e = 0.0470463 + 0.00027204 T
```

$$M = 1.2843599198 + 2.046548840 \times 10^{-4} d$$

a = (19.182281 - 0.00057008 T) A.U. = (2,869,640,310 - 85271 T) km

# Mean Elements of Neptune

$$1 = 0.0310537707 - 1.599885148 \times 10^{-4} T$$

$$\Omega = 2.2810642235 + 1.923032859 \times 10^{-2} \text{ T}$$

$$\widetilde{\omega} = 0.7638202701 + 1.532704516 \times 10^{-2} \text{ T}$$

$$e = 0.00852849 + 0.00007701 T$$

$$M = 0.7204851506 + 1.033089473 \times 10^{-4} d$$

a = (30.057053 + 0.001210166 T) A.U. = (4,496,490,000 + 181039 T) km

#### Mean Elements of Pluto

i = 0.2996706970859694

 $\Omega = 1.1914337550102258$ 

 $\widetilde{\omega} = 3.909919302791948$ 

e = 0.2488033053623924

$$M = 3.993890007 + 0.6962635708298997 \times 10^{-4}$$

a = 39.37364135300176 A.U. = 5,890,213,786,146,730 km

#### Mean Elements of Moon

$$1 = 5.1453964^{\circ}$$

$$\Omega = 259.183275^{\circ} - 0.0529539222d + 0.002078 \text{ }T^2 + 0.000002 \text{ }T^3$$

$$\widetilde{\omega} = 334.329556^{\circ} + 0.1114040803d - 0.010325 T^{2} - 0.000012 T^{3}$$

$$L = 270.434164^{\circ} + 13.1763965268d - 0.001133 T^{2} + 0.0000019 T^{3}$$

 $a \approx .00256954448 \text{ A.U.}$ 

e = 0.054900489

- Note 1: The above elements are referred to the mean equinox and ecliptic of date except for Pluto.
- Note 2: The elements for pluto are oscillating values for epoch 1960 September 23.0 E.T. = J.D. 2437200.5
- Note 3: The time interval from the epoch is denoted by T when measured in Julian centuries of 36,525 ephemeris days, by D = 3.6525 T when measured in units of 10,000 ephemeris days, and by d = 10,000D = 36,525 T when measured in ephemeris days. Times are measured with respect to the epoch 1900 January 0.5 E.T. = J.D. 2415020.0.
- Note 4: Angular relations are expressed in radians for planets and degrees for moon.

References: (1) Space Research Conic Program, Phase III, J.P.L., May 1969 (Planetary constants)

⁽²⁾ The American Ephemeris and Nautical Almanac - 1965, U.S. Government Printing Office, Washington, p. 493 (Lunar constants)

SUBROUTINE BPLANE

PURPOSE: TO COMPUTE B-PLANE PARAMETERS

CALLING SEQUENCE: CALL BPLANE(GMX,R,V,BUT,BDR,TF,SDR,SDT,C3,INDX,RN,SN,TN)

ARGUMENTS: GMX I GRAVITATIONAL CONSTANT OF TARGET PLANET

R I POSITION VECTOR

V I VELOCITY VECTOR

BOT O B DOT T

BDR O B DOT R

TF 0 TIME FROM PERIAPSIS

SOR O S DOT R

SDT O S DOT T

C3 O PLANET DEPARTURE ENERGY

INDX I =1 FILL RN, SN, TN VECTORS, =2 DO NCT FILL

RN OR VECTOR

SN O S VECTOR

TN O T VECTOR

SUBROUTINES SUPPORTED: BEPS

LOCAL SYMBOLS: A SEMIMAJOR AXIS OF THE CONIC

AB INTERMEDIATE VARIABLE

AUXF INTERMEDIATE VARIABLE

8 INTERMEDIATE VARIABLE

BV B VECTOR

CTA COSINE TRUE ANOMALY

C1 ANGULAR MOMENTUM CONSTANT

E ECCENTRICITY

NINETY 90.

ONE 1.

P SEMI-LATUS RECTUM

PI MATHEMATICAL CONSTANT

PV P VECTOR

QV Q VECTOR

RAD DEGREES PER RADIAN

RO INTERMEDIATE VARIABLE

RM INTERMEDIATE VARIABLE

RRD INTERMEDIATE VARIABLE

RV R VECTOR

SINHF INTERMEDIATE VARIABLE

STA SINE TRUE ANOMALY

SV S VECTOR

TA INTERMEDIATE VARIABLE

TANG INTERMEDIATE VARIABLE

TCA INTERMEDIATE VARIABLE

TV T VECTOR

TWO 2.

VX INTERMEDIATE VARIABLE

WV INTERMEDIATE VARIABLE

Z INTERMEDIATE VARIABLE

ZERO 0.

SUBROUTINE CAREL

PURPOSES TRANSFORM CARTESIAN COORDINATES TO CONIC ELEMENTS

CALLING SEQUENCE: OALL CAREL(GM.R.V.TFP.A.E.W.XI,XN,TA,PP.QQ,WM)

ARGUMENT: GM I GRAVITATIONAL CONSTANT OF THE CENTRAL BODY

R(3) I POSITION VECTOR RELATIVE TO CENTRAL BODY

V(3) I VELOCITY VECTOR RELATIVE TO CENTRAL BODY

TFP O TIME OF FLIGHT FROM PERIAPSIS ON THE CONIC

A O SEMI-MAJOR AXIS OF THE CONIC

E O ECCENTRICITY OF THE CONIC

W O ARGUMENT OF PERIAPSIS OF THE CONIC

XI O INCLINATION OF THE CONIC TO THE REFERENCE FRAME

XN O LONGITUDE OF THE ASC(NDING NODE OF THE CONIC

TA O INSTANTANEOUS TRUE ANOMALY OF THE CONIC

PP(3) O UNIT VECTOR TOWARD PERIAPSIS ON CONIC

QQ(3) O UNIT VECTOR NORMAL TO PP IN ORBITAL PLANE

WH(3) O UNIT VECTOR NORMAL TO ORBITAL PLANE

SUBROUTINES SUPPORTED: TAROPT LUNCON MULTAR EXCUTE COPINS NONINS CPROP VMP GUISIM NONLIN PULSEX GUIDM

SUBROUTINES REQUIRED 8 NONE

LOCAL SYMBOLS: AUXF ECCENTRIC ANOMALY (HYPERBOLIC CASE)

AVA MEAN ANOMALY (ELLIPTIC CASE)

COSEA COSINE OF THE ECCENTRIC ANOMALY (ELLIPTIC CASE)

CTA COSINE OF THE TRUE ANOMALY

C MAGNITUDE OF THE ANGULAR MOMENTUM

DIV INTERMEDIATE VARIABLE IN CALCULATION OF ECCENTRIC ANOMALY

ECCENTRIC ANOMALY (ELLIPTIC CASE) EΑ SEMI-LATUS RECTUM OF THE CONIC DEGREES TO RADIANS CONVERSION CONSTANT RAD TIME DERIVATIVE OF RADIUS RD MAGNITUDE OF CARTESIAN POSITION VECTOR RM SINE OF THE ECCENTRIC ANOMALY (ELLIPTIC SINEA CASE SINHF HYPERBOLIC SINE OF AUXF STA SINE OF THE TRUE ANOMALY TANG INTERMEDIATE VARIABLE USED TO CALCULATE SINHE MAGNITUDE OF THE CARTESIAN VELOCITY VECTOR ٧M INTERMEDIATE VECTOR USED TO CALCULATE Z PP, QQ VECTORS

### CAREL Analysis

CAREL converts the cartesian state (position and velocity) of a massless point referenced to a gravitational body to the equivalent conic elements about that body.

Let the cartesian state be denoted  $\overline{r}$ ,  $\overline{v}$  and let the gravitational constant of the central body be  $\mu$ .

The angular momentum constant c is

$$c = |\vec{r} \times \vec{v}| \tag{1}$$

The unit normal W to the orbital plane is

$$\hat{W} = \vec{r} \times \vec{v}$$
 (2)

The semilatus rectum p is

$$p = \frac{c^2}{\mu} \tag{3}$$

The semi-major axis a is

$$a = \frac{r}{2 - \frac{rv^2}{\mu}}$$
 (4)

Thus a > 0 for elliptical motion, a < 0 for hyperbolic motion. The eccentricity e is

$$e = \sqrt{1 - \frac{p}{a}} \qquad . \tag{5}$$

Thus e < 1 for elliptical motion, e > 1 for hyperbolic motion. The inclination of the orbit i is computed from

$$\cos i = \hat{W}_z \tag{6}$$

The longitude of the ascending node  $\Omega$  is defined by

$$\tan \Omega = \frac{\widehat{w}_x}{-\widehat{w}_y}$$
 (7)

The true anomaly f at the given state is computed from

$$\cos f = \frac{p - r}{e r} \qquad \sin f = \frac{c \dot{r}}{\mu e} \tag{8}$$

Now define an auxiliary vector 2 by

$$\hat{Z} = \frac{r}{c} \vec{v} - \frac{\dot{r}}{c} \vec{r}$$
 (9)

Then  $\ \widehat{P}$  , the unit vector to periapsis, and  $\ \widehat{Q}$  , the in-plane normal to  $\ \widehat{P}$  , are defined by

$$\hat{P} = \hat{r} \cos f - \hat{z} \sin f \qquad (10)$$

$$\hat{Q} = \hat{r} \sin f + \hat{z} \cos f \tag{11}$$

where  $\hat{r} = \frac{\vec{r}}{r}$ . The argument of periapsis  $\omega$  is then computed from

$$\tan \omega = \frac{\hat{p}}{\hat{Q}_z}$$
 (12)

The conic time from periapsis t is computed from different formulae depending upon the sign of the semi-major axis. For a>0 (elliptical motion)

$$t_{D} = \sqrt{\frac{a^3}{\mu}}$$
 (E - e sin E)

$$\cos E = \frac{e + \cos f}{1 + e \cos f} \qquad \sin E = \frac{\sqrt{1 - e^2 \sin f}}{1 + e \cos f} \tag{13}$$

For  $a \le 0$  (hyperbolic motion) the time from periapsis is

$$t_p = \sqrt{\frac{3}{\mu}}$$
 (e sinh H - H)

$$\tanh \frac{H}{2} = \sqrt{\frac{e-1}{e+1}} \tan \frac{f}{2}$$
 (14)

Reference: Battin, R. H., Astronautical Guidance, McGraw-Hill Book Co., New York, 1964.

SUBROUTINE CASCAD

PURPOSE: TO COMPUTE THE STATE TRANSITION MATRIX DEFINING STATE PERTURBATIONS OVER AN ARBITRARY TIME INTERVAL BY CASCADING DANBY MATRIZANTS OVER SEGMENTS OF THE INTERVAL USING EITHER PATCHED CONIC OR VIRTUAL MASS THO BODY FORMULAE.

CALLING SEQUENCES CALL CASCAD(RI, STMAT)

ARGUMENTS RI I POSITION AND VELOCITY OF VEHICLE AT BEGINNING OF TIME INTERVAL

STMAT O STATE TRANSITION MATRIX OVER DESIRED INTERVAL

SUBROUTINES SUPPORTED: PSIM

SUBROUTINES REQUIRED: CONC2 VMP

LOCAL SYMBOLS: DELTAT TIME INTERVAL USED IN A SINGLE ITERATE

DELT TIME INTERVAL OF CURRENT PROPAGATION

D1 INITIAL TIME OF ITERATE

IFLAG FLAG TO DETERMINE WHETHER ITERATION IS

IOS FLAG INDICATING HELIOCENTRIC OR PLANETOCENTRIC PHASE

ISP3 FLAG USED AS TRAJECTORY INTEGRATION SPHERE OF INFLUENCE STOPPING CODE

PHI CUMULATIVE STATE TRANSITION MATRIX OVER INTERVAL ( TO, TK)

PSI CUMULATIVE STATE TRANSITION MATRIX OVER INTERVAL ( TO, TK+1)

PTP STATE OF TARGET RELATIVE TO INERTIAL COORDINATE AT TIME TK

RAV STATE OF SPACECRAFT RELATIVE TO DOMINANT BODY FOR MATRIZANT

RHO STATE TRANSITION MATRIX OVER INTERVAL (TK,TK+1)

RS INERTIAL SPACECRAFT STATE AT TK

RSF INERTIAL SPACECRAFT STATE AT TK+1

R1 SPACECRAFT STATE RELATIVE TO VIRTUAL MASS

AT TK

R2 SPACECRAFT STATE RELATIVE TO VIRTUAL MASS

AT TK+1

SUM INTERMEDIATE VARIABLE

TIME CUMULATIVE TRAJECTORY TIME FROM INITIAL

TIME TO TK+1

XHU VIRTUAL MASS MAGNITUDE AT TK

YMU VIRTUAL MASS MAGNITUDE AT TK+1

COMMON COMPUTED: ICL

COMMON USEDS ACC ALNGTH DATEJ DELTM DTPLAN

DTSUM ISTHI NTP PMASS RTP RVS TM TRTH1 VMU V CASCAD Analysis

CASCAD approximates the state transition matrix  $\phi_{f,o}$  defining state pergurbations over an arbitrary interval  $\begin{bmatrix} t_o, t_f \end{bmatrix}$  by recursively computing state transition matrices over intervals  $\begin{bmatrix} t_o, t_1 \end{bmatrix}$ ,  $\begin{bmatrix} t_o, t_2 \end{bmatrix}$ ,...,  $\begin{bmatrix} t_o, t_f \end{bmatrix}$ .

The recursive formula for the k+1 iteration based on the k*th iteration is given by

$$\phi_{k+1,o} = \psi_{k+1,k} \phi_{k,o}$$
 (1)

where  $\Psi_{k+1,k}$  is the state transition matrix for the k+1-st interval  $\begin{bmatrix} t_k, & t_{k+1} \end{bmatrix}$ :

The time interval  $\Delta t_{k+1} = t_{k+1} - t_k$  is determined by the position vector  $\overrightarrow{r}_k$  of the spacecraft relative to the target planet along the nominal n-body trajectory at the time  $t_k$ . Then if  $R_{SOI}$  denotes the radius of the sphere of influence of the target planet the time interval is defined by

where  $\Delta t_{
m planet}$  and  $\Delta t_{
m sun}$  are input parameters. For the last interval a partial step may be required so that  $\Delta t_{
m n}=t_{
m n-1}$  .

The  $\psi_{k+1,k}$  matrix may be computed by either of two models. In the patch conic model the position and velocity vectors  $\overrightarrow{R}_k$ ,  $\overrightarrow{V}_k$  of the spacecraft relative to the dominant body (the sun if  $\Delta t_{k+1} = \Delta t_{sun}$  or  $\Delta t_{sol}$ , the target planet if  $\Delta t_{k+1} = \Delta t_{planet}$ ) at the time  $t_k$  is used to define a

conic with respect to the dominant body and the Danby matrizant over the given interval defines  $\Psi_{k+1,k}$  (CØNC2) .

In the virtual mass model the position and velocity vectors  $\overrightarrow{R}_k$ ,  $\overrightarrow{V}_k$  are computed relative to the virtual mass and the gravitational constant used is that of the virtual mass magnitude at the time  $t_k$ . The Danby matrizant corresponding to this conic then is used to compute  $\psi_{k+1,k}$  (CONC2).

The recursive process continues until the state transition matrix over the entire interval  $\begin{bmatrix} t_o, t_f \end{bmatrix}$  is determined.

Reference: Danby, J.M.A., "The Matrizant of Keplerian Motion," AIAA Journal, vol 2, no 1, January, 1964.

(1)

SUBROUTINE CENTER

PURPOSES TO CONVERT THE POSITION AND VELOCITY VECTORS OF THE GRAVITATING BODIES FROM REFERENCE BODY ECLIPTIC TO BARYCENTRIC ECLIPTIC AND STORE THEM IN THE F ARRAY.

CALLING SEQUENCE: CALL CENTER

SUBROUTINES SUPPORTED8 EPHEM

SUBROUTINES REQUIRED: NONE

LOCAL SYMBOLS BARYC POSITION AND VELOCITY OF CENTER OF MASS RELATIVE TO EARTH. (AU. AU/DAY)

F ARRAY OF PLANET EPHEMERIS DATA IN AU, AU/DAY UNITS. DATA INDICATED BY THE FOLLOWING INDICES

4*I-2,J VELOCITY OF I-TH PLANET RELATIVE TO THE SUN (INPUT) AND RELATIVE TO THE BARYCENTER (OUTPUT)

4*I-3,J POSITION OF I-TH PLANET RELATIVE TO THE SUN (INPUT) AND RELATIVE TO THE BARYCENTER (OUTPUT)

4*IM-2,J VELOCITY OF MOON RELATIVE TO EARTH (INPUT) AND RELATIVE TO BARYCENTER OUTPUT

4*IM-3,J POSITION OF MOON RELATIVE TO EARTH (INPUT) AND RELATIVE TO BARYCENTER (OUTPUT)

GEOP POSITION AND VELOCITY OF BODIES RELATIVE TO THE EARTH. (AU, AU/DAY)

INDEX USED TO EXTRACT EARTH EPHENERIS DATA RELATIVE TO SUN FROM F-ARRAY

IX INDEX OF IJ-TH GRAVITATIONAL BODY

SUM OF GRAVITATIONAL CONSTANTS
(AU##3/DAY##2)

SUN POSITION AND VELOCITY OF SUN RELATIVE TO EARTH (AU. AU/DAY)

COMMON COMPUTED/USED: F INITAL V

COMMON USED8 NBODYI NO PMASS ZERO

#### CENTER Analysis

Let the state vector of position and velocity of the gravitating bodies (excluding the moon) in heliocentric ecliptic coordinates be denoted  $\rho_{\rm i}$ ,  $\omega_{\rm i}$  at some reference time. Let the index of the earth be i_E. Then the coordinates of all bodies (excluding the moon) relative to the earth is

$$\overrightarrow{r}_{i} = \overrightarrow{\gamma}_{i} - \overrightarrow{\rho}_{i_{E}} \qquad i=1,n, i\neq i_{M}$$

$$\overrightarrow{v}_{i} = \overrightarrow{\omega}_{i} - \overrightarrow{\omega}_{i_{E}} \qquad i=1,n, i\neq i_{M}$$
(1)

Let the position and velocity of the moon relative to the earth be denoted  $\vec{r}_{i_M}$ ,  $\vec{v}_{i_M}$ .

Define the radius vector to the center of mass (in earth ecliptic coordinates) by

$$R_{CM} = \frac{1}{M} \sum_{i=1}^{n} \mu_i \overrightarrow{r}_i \qquad M = \sum_{i=1}^{n} \mu_i \qquad (2)$$

Its velocity relative to the earth may then be found by differentiation.

$$\overrightarrow{\mathbf{v}}_{\mathbf{CM}} = \frac{1}{\mathbf{M}} \sum_{i=1}^{n} \mu_i \overrightarrow{\mathbf{v}}_i$$
 (3)

The coordinates of all gravitating bodies relative to the center of mass may then be computed

$$\vec{R}_{i} = \vec{r}_{i} - \vec{R}_{CM}$$

$$\vec{\nabla}_{i} = \vec{\nabla}_{i} - \vec{\nabla}_{CM}$$
(4)

# SUBROUTINE CONCAR

PURPOSE: TO CONVERT A CONIC STATE IN TERMS OF R, THETA, E, P, PA, QA AND GMU INTO A CARTESIAN STATE

_			
ARGUMENT &	CSTA	I	COSINE OF TRUE ANAMOLY AT CURRENT STATE
	£	I	ECCENTRICITY OF CONIC
	GMU	I	GRAVITATIONAL CONSTANT OF PLANET IN KM**3/SEC**2
	PA	1	UNIT VECTOR IN DIRECTION OF PERIAPSIS
	p	1	SEMI-LATUS RECTUM OF CONIC IN KM
	QA	I	UNIT VECTOR IN ORBIT PLANE IN DIRECTION 90 DEG ADVANCED FROM PA
	RV	0	POSITION VECTOR OF CURRENT STATE IN KM
	R	I	MAGNITUDE OF POSITION VECTOR AT CURRENT STATE IN KM
	SNTA	Î	SINE OF TRUE ANAMOLY AT CURRENT STATE
SUBROUTINE	VV S SUPPORT	0 ED:	VELOCITY VECTOR-OF CURRENT STATE IN KM/SEC TPPROP TPRTRG
LOCAL SYMB	OLS: HOP		ANGULAR MOMENTUM DIVIDED BY SEMI-LATUS

RECTUM IN KM/SEC

POSPA PROJECTION OF POSITION VECTOR ON PA IN KM

POSQA PROJECTION OF POSITION VECTOR ON QA IN KM

VELPA PROJECTION OF VELOCITY ON PA IN KM/SEC

VELQA PROJECTION OF VELOCITY ON QA IN KHISEC

SUBROUTINE CONC2

PURPOSE® COMPUTE STATE TRANSITION MATRIX USING ANALYTICAL PATCHED CONIC OR ANALYTICAL VIRTUAL MASS TECHNIQUES

CALLING SEQUENCES CALL CONC2(R, V, DELT, GMX, PSIEC)

ARGUMENTS DELT I TIME INCREMENT OVER WHICH THE STATE TRANSITION MATRIX IS BEING COMPUTED

GMX I GRAVITATIONAL CONSTANT OF GOVERNING BODY

PSIEC O STATE TRANSITION MATRIX

R I POSITION OF THE VEHICLE RELATIVE TO THE GOVERNING BODY

V I VELOCITY OF THE VEHICLE RELATIVE TO THE GOVERNING BODY

SUBROUTINES SUPPORTED: PSIM CASCAD PCTM

LOCAL SYMBOLS 8 A SEMI-MAJOR AXIS

A1 INTERMEDIATE VARIABLE

AZ INTERMEDIATE VARIABLE

A3 INTERMEDIATE VARIABLE

AM2 INTERMEDIATE VARIABLE

C1 MAGNITUDE OF RXV

CSE COSINE OF ECCENTRIC ANOMALY

CTA COSINE OF TRUE ANOMALY

CTA2 COSINE OF TRUE ANOMALY ON ELLIPSE

DOXO INTERMEDIATE VARIABLE

DDYO INTERMEDIATE VARIABLE

DXO INTERMEDIATE VARIABLE

DYO INTERMEDIATE VARIABLE

E ECCENTRICITY

EA ECCENTRIC ANOMALY

FMI1 INTERMEDIATE VECTOR

FMI INTERMEDIATE VECTOR

F INTERMEDIATE VARIABLE

N INTERMEDIATE VARIABLE

OPEC INTERMEDIATE VECTOR

ORB INTERMEDIATE VARIABLE

P SEMI-LATUS RECTUM

PI HATHEMATICAL CONSTANT

PSIOP INTERHEDIATE STATE TRANSITION MATRIX

PV INTERMEDIATE VECTOR

Q INTERMEDIATE VECTOR

RO R DOT V DIVIDED BY MAGNITUDE OF R

RM MAGNITUDE OF R

RRD R DOT V

RTHO INTERMEDIATE VARIABLE

RZ INTERMEDIATE VARIABLE

R3 INTERMEDIATE VARIABLE

SNE SINE OF ECCENTRIC ANOMALY

SNF SINE OF F

STA SINE OF TRUE ANOMALY

STA2 SINE OF TRUE ANOMALY ON ELLIPSE

TIM1 INTERMEDIATE TIME

TIM2 INTERMEDIATE TIME

VM MAGNITUDE OF V

WV RXV

XO INTERMEDIATE VARIABLE

YO INTERMEDIATE VARIABLE

	Z	INTER	MEDIATE '	VECTOR		
•				*	•	
COMMON U	SED®	EM8 Two	HALF ZERO	ONE	THREE	TWOPI
			~ ·- ' \ U			

CONC2 Analysis

CONC2 is responsible for the computation of a state transition matrix about a conic trajectory using the Danby matrizant analytic formula.

Danby has shown (see Reference 2) that the state transition matrix (or matrizant) has a particularly simple form if written in the orbital plane coordinate system. The state transition matrix  $\phi$  defined by

$$\delta x_f = \Phi(t_f, t_o) \delta X_o$$
 (1)

where  $\delta x_f$ ,  $\delta x_o$  refer to perturbations about a conic trajectory at time  $t_f$ ,  $t_o$  respectively may be written in the orbital plane system

$$\cdot \cdot \overline{\Phi}(t_f, t_o) = M(t_f) M^{-1}(t_o)$$
 (2)

where M(t), M¹(t) may be computed from the following formulae

$$\mathbf{M}^{-1} = \mathbf{A} \mathbf{J} \mathbf{M}^{\mathrm{T}} \mathbf{J}^{\mathrm{T}} \tag{4}$$

where  $X,Y,\dot{X},\dot{Y},\ddot{X},\ddot{Y}$  are evaluated at the time the is the angular momentum constant  $\tau$  is the time interval from the to some epoch (periapsis)

and 
$$A = \text{diag } (a/\mu, a/\mu h, 1/h, a/\mu, a/\mu h, 1/h)$$
 (5)

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{6}$$

Thus to use the Danby formulation one must determine the transformation from the reference frame to the orbital plane coordinates, compute the values of the quantities  $X,Y,X,\mathring{Y},\mathring{X},\mathring{Y}$  and h and  $\tau$  at the times t, and then use the above equations.

Let the initial state of the conic be denoted  $\overline{r}$ ,  $\overline{v}$ , the gravitational force  $\hat{\mu}$ , and the time interval  $\triangle t$ . Then the unit vectors  $\widehat{P}$  in the direction of periapsis,  $\widehat{W}$  in the direction of the angular momentum vector, and  $\widehat{Q} = \widehat{W} \times \widehat{P}$  defining the orbital plane coordinate system may be computed by the following conic equations

$$h = |\vec{r} \times \vec{v}|$$
 (7)

$$\widehat{\mathbf{W}} = \frac{\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{v}}}{h} \tag{8}$$

$$\dot{r} = \frac{\vec{r} \cdot \vec{v}}{v} \tag{9}$$

$$p = \frac{h^2}{\mu} \tag{10}$$

$$a = \frac{r}{2 - rv^2/\mu} \tag{11}$$

$$e = \sqrt{1 - P/a} \tag{12}$$

$$\cos f = \frac{p - r}{er} \qquad \sin f = \frac{\dot{r} h}{\mu e} \tag{13}$$

$$\vec{z} = \frac{\mathbf{r}}{\mathbf{h}} \vec{\mathbf{v}} - \frac{\dot{\mathbf{r}}}{\mathbf{h}} \vec{\mathbf{r}} \tag{14}$$

$$\hat{\mathbf{r}} = \cos \mathbf{f} \, \frac{\vec{\mathbf{r}}}{\mathbf{r}} - \sin \mathbf{f} \, \vec{\mathbf{z}} \tag{15}$$

$$\widehat{Q} = \sin f \frac{\widehat{\mathbf{r}}}{\mathbf{r}} + \cos f \widehat{\mathbf{z}}$$
 (16)

$$\dot{f} = \frac{c}{r^2} \tag{17}$$

The transformation matrix from the original  $\vec{r}$ ,  $\vec{v}$  system to the orbital plane system may then be written

$$T = \left[ \hat{P} \mid \hat{Q} \mid \hat{W} \right]$$
 (18)

Let the true anomaly at the pertinent time  $(t_0 \text{ or } t_f)$  be denoted f. Then the quantities required in (3) are written

$$X = r \cos f$$

$$Y = r \sin f$$

$$\dot{X} = \dot{r} \cos f - r\dot{f} \sin f \quad \dot{Y} = \dot{r} \sin f + r\dot{f} \cos f$$

$$\ddot{X} = -\frac{\mu X}{3}$$

$$\ddot{Y} = -\frac{\mu Y}{3}$$

$$(19)$$

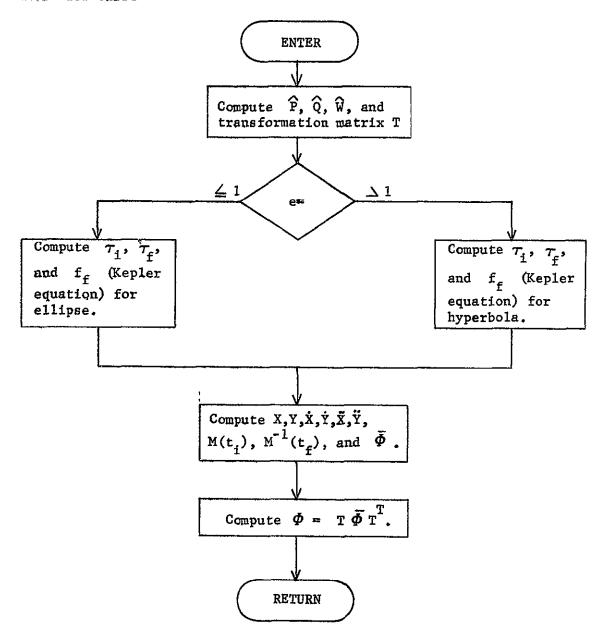
Having computed the state transition matrix  $\vec{\Phi}$  corresponding to the orbital plane system by equations (2), (3), (4), it is an easy task to convert it to the normal reference system

$$\phi = T \overline{\phi} T^{T}$$
 (20)

References: Battin, R. H., Astronautical Guidance, McGraw-Hill Book Co., New York, 1964.

Danby, J.M.A., Matrizant of Keplerian Motion, AIAA J., vol. 3, no. 4, April, 1965.

## CONC2 Flow Chart



SUBROUTINE CONVRT.

PURPOSE: TO COMPUTE THE GEOCENTRIC EQUATORIAL COORDINATES OF THE VEHICLE.

CALLING SEQUENCE: CALL CONVRT(R, PHI, THETA, VEL, GAMMA, SIGMA, X, Y, Z, VX, VY, VZ)

ARGUMENTS GAMMA I PATH ANGLE

R I GEOCENTRIC RADIUS

PHI I DECLINATION

THETA I RIGHT ASCENSION

VEL I VELOCITY

SIGHA I AZIMUTH

X O X COMPONENT OF POSITION IN GEOCENTRIC EQUITORIAL COORDINATES

Y O Y COMPONENT OF POSITION IN GEOCENTRIC EQUITORIAL COORDINATES

Z O Z COMPONENT OF POSITION IN GEOCENTRIC EQUITORIAL COORDINATES

VX O X COMPONENT OF VELOCITY IN GEOCENTRIC EQUITORIAL COORDINATES

VY O Y COMPONENT OF VELOCITY IN GEOCENTRIC EQUITORIAL COORDINATES

VZ O Z COMPONENT OF VELOCITY IN GEOCENTRIC EQUITORIAL COORDINATES

## SUBROUTINES SUPPORTED: DATA ' DATAS

LOCAL SYMBOLS: B1 INTERMEDIATE VARIABLE

82 INTERMEDIATE VARIABLE

BJ INTERMEDIATE VARIABLE

CG COSINE OF PATH ANGLE

CP COSINE OF DECLEMATION

CT COSINE OF RIGHT ASCENSION

SG SINE OF PATH ANGLE

SP SINE OF DECLINATION

ST SINE OF RIGHT ASCENSION

## CONVRT Analysis

Geocentric equatorial position and velocity components are related to geocentric radius, declination, right ascension, velocity magnitude, flight path angle, and azimuth through the following equations:

 $x = r \cos \phi \cos \theta$ 

 $y = r \cos \phi \sin \theta$ 

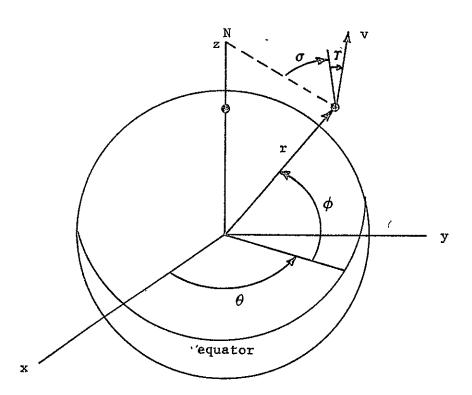
 $z = r \sin \phi$ 

 $\dot{x} = v \left( \sin \tau \cos \phi \cos \theta - \cos \tau \sin \sigma \sin \theta - \cos \tau \cos \sigma \sin \phi \cos \theta \right)$ 

 $\dot{y} = v \left( \sin \tau \cos \phi \sin \theta + \cos \tau \sin \sigma \cos \theta - \cos \tau \cos \sigma \sin \phi \sin \theta \right)$ 

 $\dot{z} = v \left( \sin \beta \sin \phi + \cos \tau \cos \sigma \cos \phi \right)$ 

The definitions of pertinent quantities are apparent in the following figure.



SUBROUTINE COPINS

PURPOSE: TO DETERMINE THE IMPULSIVE CORRECTION AND TIME REQUIRED TO INSERT FROM AN APPROACH HYPERBOLA INTO A COPLANAR ELIPTICAL ORBIT.

CALLING SEQUENCE: CALL COPINS(GM,R,V,DA,DE,DELW,TEX,DELV,IEX)

ARGUMENTS: SM I GRAVITATIONAL CONSTANT

R(3) I POSITION VECTOR AT DECISION

V(3) I VELOCITY VECTOR AT DECISION

DE I DESIRED SEMIMAJOR AXIS

DE I QESIRED ECCENTRICITY

DELW I DESIRED PERIAPSIS SHIFT

TEX O TIME FROM DECISION TO EXECUTION (SECONDS)

DELV(3) O INSERTION VELOCITY CORRECTION

IEX O EXECUTION CODE

=0 EVENT IS EXECUTABLE

=1 NO EXECUTABLE SOLUTION FOUND

SUBROUTINES SUPPORTED: INSERS

SUBROUTINES REQUIRED: CAREL ELCAR

LOCAL SYMBOLS: AA COEFFICIENT DEFINING TANGENTIAL SOLUTION FOR A

AH HYPERBOLIC SEMIMAJOR AXIS

ARC THE CONSTANT 180

A1 CANIDATE SOLUTION FOR SEMIMAJOR AXIS

AZ CANIDATE SOLUTION FOR SEMIMAJOR AXIS

A TARGET SEMIMAJOR AXIS

BB COEFFICIENT DEFINING TANGENTIAL SOLUTION FOR A

B TANGENTIAL SOLUTION CONSTANT

CC COEFFICIENT DEFINING TANGENTIAL SOLUTION FOR A

COE 1/E

COSTH COS(THETA)

COSW COS(W)

C TANGENTIAL SOLUTION CONSTANT

DELVM MAGNITUDE OF FINAL CORRECTION

DISC DISCRIMINANT OF SOLUTION FOR THETA

DISK DISCRIMINANT OF TANGENTIAL SOLUTION

FOR A

DRA DESIRED APOAPSIS RADIUS

DRP DESIRED PERIAPSIS RADIUS

DAM MAGNITUDE OF VELOCITY CORRECTION FOR

CANDIDATE SOLUTION

DV YELOCITY CORRECTION OF CANDIDATE SOLUTION

D TANGENTIAL SOLUTION CONSTANT

EH HYPERBOLIC ECCENTRICITY

ERRNAX SCALAR ERROR ASSOCIATED WITH IMPOSSIBLE

SOLUTION

ERR SCALAR ERRORS OF CANDIDATE SOLUTIONS

ER RADIUS ON ELLIPSE AT INSERTION

ETA TRUE ANOMALY ON ELLIPSE AT INSERTION

E ECCENTRICITY OF ELLIPSE

HI INCLINATION OF HYPERBOLA

HN ASCENDING NODE OF HYPERBOLA

HRP HYPERBOLIC PERIAPSIS RADIUS

HR RADIUS OF HYPERBOLA AT INSERTION

IOPT TYPE OF SOLUTION

= 0 ORBITS INTERSECT

=1 MUST MODIFY ORBIT TO QBTAIN SOLUTION

ISOL INDEX OF SOLUTION

MIN INDEX OF HINIMUM LOSS FUNCTION SOLUTION

NSOLS NUMBER OF SOLUTIONS

PH HYPERBOLIC SEMILATUS RECTUM

PI THE MATHEMATICAL CONSTANT PI

PP UNIT VECTOR TOWARD PERIAPSIS

P ELLIPTICAL SEMILATUS RECTUM

QQ UNIT VECTOR IN ORBIT PLANE NORMAL TO PP

RAD QEGREE TO RADIAN TRANSFORMATION

RA APOAPSIS RADIUS

RD RADIUS TO DECISION STATE

REMG HAGNITUDE OF RADIUS ON ELLIPSE AFTER

INSERTION

RE POSITION VECTOR ON ELLIPSE AFTER

INSERTION

RH POSITION ON HYPERBOLA BEFORE INSERTION

RMAG MAGNITUDE OF RADIUS ON HYPERBOLA-BEFORE

INSERTION

RP PERIAPSIS RADIUS

SGN PARAMETER IN TANGENTIAL SOLUTION

SINW SIN(W)

STA TRUE ANOMALY ON HYPERBOLA AT DECISION

SYGN POSITIVE OR NEGATIVE SIGN IN QUADRATIC

S INTERMEDIATE VARIABLE

TFPE TIME FROM PERIAPSIS ON ELLIPSE

TFPH HYPERBOLIC TIME FROM PERIAPSIS AT INSERT

THA TRUE ANOMALY OF INSERTION ON HYPERBOLA

TINDX TIME FROM DECISION TO EXECUTION

TIMD TIME FROM PERIAPSIS AT DECISION

VD	SPEED AT DECISION
AEMG	SPEED ON ELLIPSE AFTER INSERTION
VE	VELOCITY VECTOR ON ELLIPSE AFTER INSERTION
AH	VELOCITY VECTOR ON HYPERALOA BEFORE INSERTION
VMAG	SPEED ON HYPERBOLA BEFORE INSERTION
	HYPERBOLIC ARGUMENT OF PERIAPSIS
MM	UNIT NORMAL TO ORBITAL PLANE
ЯX	ARGUNENT OF PERIAPSIS OF ELLIPSE
W	PERIAPSIS SHIFT
X	CONSTANT IN EQUATION DEFINING HYPERBOLIC TRUE ANOMALY AT INSERTION
Υ	CONSTANT IN EQUATION DEFINING HYPERBOLIC TRUE ANOMALY AT INSERTION
Z	CONSTANT IN EQUATION DEFINING HYPERBOLIC TRUE ANOMALY AT INSERTION

## COPINS Analysis:

COPINS determines the impulsive correction and time required to insert from an approach hyperbola into a coplanar elliptical orbit. The approach hyperbola is specified by a planetocentric state  $\vec{r}$ ,  $\vec{v}$  at a decision time  $t_d$ . The desired elliptical orbit is prescribed by input parameters a, e,  $\triangle \omega$  where a and e are the semi-major axis and eccentricity of the desired ellipse and  $\triangle \omega$  is the angle (measured counter clockwise) from the hyperbolic periapsis to the periapsis of the desired orbit. The situation is illustrated in Figure 1.

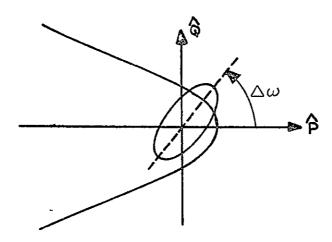


Figure 1. Approach Hyperbola and Desired Orbit

The planetocentric ecliptic state  $\vec{r}$ ,  $\vec{v}$  at the time of decision  $t_d$  is first converted to Keplerian elements.  $(a_H, e_H, i_H, \mu, \Omega_H, t_{Hd})$  via subroutine CAREL where  $t_{Hd}$  is the time from periapsis (negative on the approach ray). The angle  $f_{\infty}$  between the hyperbolic periapsis and the approach asymptote  $\hat{S}$  is computed from

$$\cos f_{\infty} = \frac{1}{e} \qquad 0 < f_{\infty} < 90^{\circ}$$
 (1)

Thus the angle  $\omega$  between the hyperbolic periapsis and the desired elliptical periapsis is given by

$$\omega = \Delta \omega \tag{2}$$

The hyperbola and ellipse may therefore be described in the PQ plane by standard conic formula, specifically,

$$r_{H} = \frac{P_{H}}{1 + e_{H} \cos \theta}$$

$$r_{E} = \frac{P_{E}}{1 + e_{E} \cos (\theta - \omega)}$$
(3)

where  $\theta$  is measured counter-clockwise from  $\hat{P}$  and  $p_H, p_E$  are the semilatus rectum of the hyperbola and ellipse respectively. Obviously if an angle of intersection  $\theta^*$  is known, the states on both conics  $(\hat{r},\overset{\vee}{v_H})$  and  $(\hat{r},\overset{\vee}{v_E})$  may be computed from conic formulae and the desired impulsive correction is given by

$$\overrightarrow{\Delta v} = \overrightarrow{v_H}^* - \overrightarrow{v_H}^* \tag{4}$$

Likewise the time from periapsis to the intersection point t* may be computed using hyperbolic formula and therefore the time from decision to execution is given by

$$\Delta t = t^* - t_d \tag{5}$$

Thus the coplanar insertion problem reduces to the determination of the optimal angle  $\theta^*$  for the impulsive maneuver.

From (3) the values of  $\theta$  for which  $r_H = r_E$  are given by

$$\cos \theta = \frac{-xy \pm z \sqrt{D}}{y^2 + z^2} \tag{6}$$

where

$$x = p_{H} - p_{E}$$

$$y = p_{H} e_{E} \cos \omega - p_{E} e_{H}$$

$$z = p_{H} e_{E} \sin \omega$$

$$D = y^{2} + z^{2} - x^{2}$$
(7)

If the discriminant  $D \ge 0$  there are at most two real non-extraneous solutions  $\theta_1, \theta_2$  such that  $r_E(\theta) = r_H(\theta)$ . Note that the angle  $\theta$  may not lie in the region inside the approach and departure asymptotes. If there are two solutions, both  $\Delta v$ 's are computed by (4) and the minimum  $\Delta v$  transfer is selected.

If D < 0, the applied hyperbola and the desired orbit do not intersect and there is no impulsive transfer between the two conics. In such a case the desired elements  $a_{E}$  and  $e_{E}$  are modified to determine the "best" tangential solution possible. Three different modifications are tested:

- (1) Vary  $r_a$  while holding  $r_p$  at the desired value. (2) Vary  $r_p$  while holding  $r_a$  at the desired value. (3) Vary  $a_E$  while holding  $e_E$  at the desired value.

The three modification schemes are illustrated in Figure 2 where the original nonintersecting orbit is shown by the broken lines.

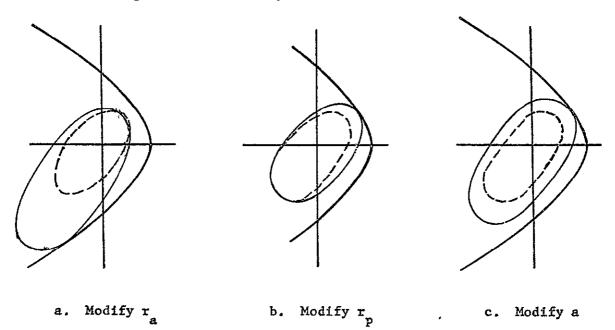


Figure 2. Candidate Orbit Modifications'

It is desired to modify the "a" and the "e" of the desired orbit to achieve the tangential configurations. From (6) it is obvious that a necessary condition for a tangential solution is given by D=0. Using (7) D may be written

$$D = p_{H}^{2} (e_{E}^{2} - 1) + p_{E}^{2} b + 2p_{H}p_{E} - cp_{E}e_{E}$$
where  $b = e_{H}^{2} - 1$ 

$$c = 2p_{H}e_{H} \cos \omega$$
 (8)

where it is observed the approach hyperbola is fixed and it is desired not to vary the  $\,\omega\,$  of the desired ellipse so that subsequent apsidal rotations are avoided.

Modification Option 1: Rewriting (8a) in terms of a and r leads to

$$a^{2}D = (4 r_{p}^{2} b + 4 r_{p} p_{H} - 2 r_{p} c) a^{2}$$

$$+ (-2 p_{H}^{2} r_{p} - 4 r_{p}^{3} b - 2 p_{H} r_{p}^{2} + 3 r_{p}^{2} b) a$$

$$+ (p_{H}^{2} r_{p}^{2} + r_{p}^{4} b - c r_{p}^{3})$$
(9)

Now if D is set equal to 0,  $r_p$  held at its desired value, and the resulting quadratic solved for "a", the solution will correspond to the tangential solution which holds  $r_p$  constant. If  $a \le 0$  or imaginary, the solution is disregarded. The modified eccentricity is of course defined by

$$e = 1 - \frac{r_p}{a} \tag{10}$$

Modification Option 2: Rewriting (8a) in terms of a and  $r_{s}$  leads to

$$a^{2}D = (4 r_{a}^{2} b + 4 r_{a} p_{H} + 2 r_{a} c)a^{2}$$

$$+ (-2 p_{H}^{2} r - 4 r_{a}^{3} b - 2 p_{H} r_{a}^{2} - 3 r_{a}^{2} c)a$$

$$+ (p_{H}^{2} r_{a}^{2} + r_{a}^{4} b + c r_{a}^{3})$$
(11)

For computational purposes the similarity between (9) and (11) may be exploited. Again setting D=0 and holding  $r_a$  at its desired value, the value of "a" may be determined which specifies the tangential solution holding  $r_a$  constant. Having determined a realistic value of "a", the corresponding eccentricity is given by

$$e = \frac{r_a}{a} - 1 \tag{12}$$

Modification Option 3: Rewriting (8a) in terms of a and  $e_{R}$  leads to

$$D = (d^{2}b)a^{2} + (2p_{H}d - cd e_{E})a - dp_{H}^{2}$$

$$d = (1 - e_{E}^{2})$$
(13)

Setting D = 0 and solving for "a" while holding  $e_E$  at its desired value then defines the option 3 solution.

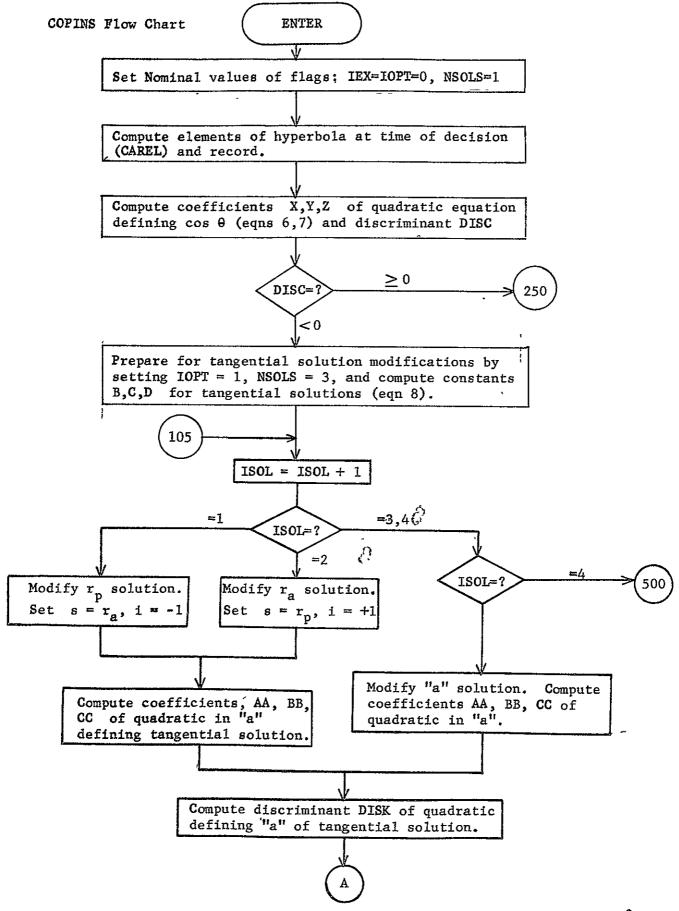
To determine the "best" modified orbit from the three candidate options a rather arbitrary scheme is used. A scalar error is assigned to each option according to a weighting factor and the difference between the desired and achieved values of the periapsis and apoapsis radii:

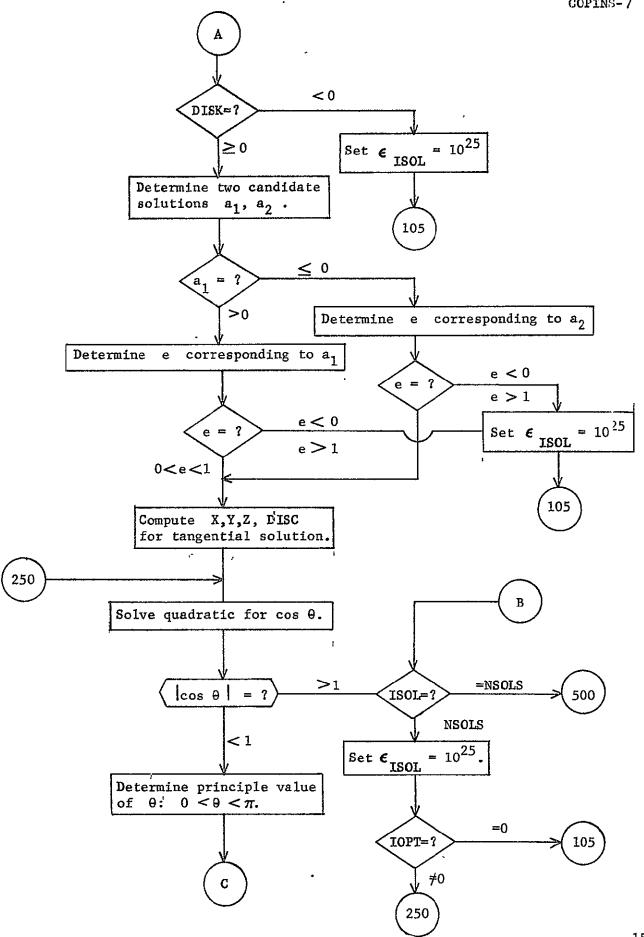
$$E_i = W_i (|\Delta r_a| + |\Delta r_p|)$$

where the scalar factor  $W_1$  is set to 1,2,3 respectively for the three options. Thus the preferred strategy is the one which requires a correction only at apoapsis while the least desired scheme requires subsequent corrections both at periapsis and apoapsis.

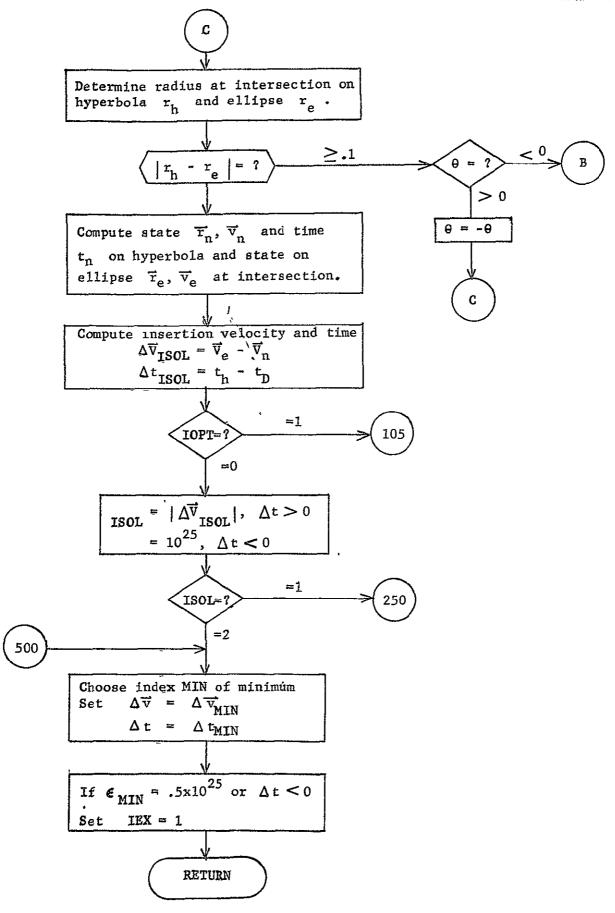
Maving determined orbital elements that necessarily lead to a tangential solution, (6) may now be used to compute the angle of intersection  $\theta$ .

Q





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SUBROUTINE CORREL

COMMON USED 8

PURPOSE & CONVERT COVARIANCE MATRIX PARTITIONS TO CORRELATION MATRIX PARTITIONS AND STANDARD DEVIATIONS AND WRITE THEM OUT

CALLING SEQUENCE: CALL CORRELEPP.CXXSP.PSP.CXUP.UD.CXVP.VA.CXSUP

CALLING SEQ	JENCE8 CAL	L CORREL(PP,CXXSP,PSP,CXUP,U0,CXVP,V0,CXSUP,CXSVP)
ARGUMENT 8	PP I	POSITION/VELOCITY COVARIANCE MATRIX
	CXXSP I	CORRELATION BETWEEN SOLVE-FOR PARAMETERS AND POSITION/VELOCITY STATE
	PSP I	SOLVE-FOR PARAMETER COVARIANCE MATRIX
	CXUP I	CORRELATION BETWEEN POSITION/VELOCITY STATE AND DYNAMIC CONSIDER PARAMETERS
	UO I	DYNAMIC CONSIDER PARAMETER COVARIANCE MATRIX
	CXVP I	CORRELATION BETWEEN POSITION/VELOCITY STATE AND MEASUREMENT CONSIDER PARAMETERS
	vo I	MEASUREHENT CONSIDER PARAMETER COVARIANCE MATRIX
	CXSUP I	CORRELATION BETWEEN SOLVE-FOR PARAMETERS AND DYNAMIC CONSIDER PARAMETERS
,	CXSVP I	CORRELATION BETHEEN SOLVE-FOR PARAMETERS AND MEASUREMENT CONSIDER PARAMETERS
SUBROUTINES	SUPPORTED	PRINT4 SETEVS GUISIM PRESIM PRNTS4 PRINT3 SETEVN GUIDM PRED PRNTS3
LOCAL SYMBO	LS8 DUM	INVERSE OF SQUARE ROOT OF DIAGONAL ELEMENTS IN DYNAMIC AND MEASUREMENT CONSIDER COVARIANCE PARTITIONS
	IEND	COUNTER INDICATING TOTAL NUMBER OF AUGMENTED STATE VARIABLES
	ŖOW	INTERMEDIATE COMPUTATION AND OUTPUT VECTOR
	SQP	INVERSE OF THE SQUARE ROOT OF DIAGONAL ELEMENTS IN VEHICLE AND SOLVE-FOR COVARIANCE PARTITIONS
	ZZ	STANDARD DEVIATION

KPRINT NDIM1 NDIM2

ND IM3

ONE

SUBROUTINE DATA

PURPOSE: TO READ INPUT DATA, TRANSLATE THIS DATA INTO PROPER INTERNAL VALUES, ASSIGN VALUES TO UNSPECIFIED NAMELIST VARIABLES, SET NECESSARY INITIAL VALUES, COMPUTE DIMENSIONS OF STATE TRANSITION, OBSERVATION, AND COVARIANCE MATRIX PARTITIONS, ORDER MEASURMENT AND EVENT SCHEDULES, AND PRINT OUT INITIAL CONDITIONS IN THE ERRAN PROGRAM.

CALLING SEQUENCE & CALL DATA

ARGUMENT & NONE

SUBROUTINES SUPPORTED: ERRON

SUBROUTINES REQUIRED: CONVRT EPHEM GHA ORB PECEQ
TIME TRANS DATA1 GOATA ELCAR

LOCAL SYMBOLS: AI INCLINATION

AMIN INTERMEDIATE VARIABLE

ANODE LONGITUDE OF ASCENDING NODE

A SEMIMAJOR AXIS

DGTR DEGREE TO RADIAN CONVERSION

DUM1 INTERMEDIATE STORAGE ARRAY

DUM INTERMEDIATE STORAGE ARRAY

D INTERMEDIATE JULIAN DATE

DATE ARRAY CONTAINING FINAL JULIAN DATE

EARTH CALENDAR DATE AT WHICH EARTHS ORBITAL ELEMENTS WILL BE CALCULATED

E ECCENTRICITY

FNDT DATE OF FINAL TIME

GAMMA PATH ANGLE

GM GRAVITATIONAL CONSTANT OF CENTRAL BODY

IDAY CALENDAR DAY OF FINAL TIME

IHR CALENDAR HOUR OF FINAL TIME

IMIN CALENDAR MINUTES OF FINAL TIME

IMO CALENDAR MONTH OF FINAL TIME

IPMN FLAG FOR MAIN PROBE MEASUREMENT NOISE

ISMN FLAG FOR MINI-PROBE MEASUREMENT NOISE

IYR CALENDAR YEAR OF FINAL TIME

JUPITER CALENDAR D'ATE AT WHICH ORBITAL ELEMENTS OF JUPITER WILL BE CALCULATED

LDAY CALENDAR DAY OF INITIAL TIME

LHR CALENDAR HOURS OF INITIAL TIME

LMIN CALENDAR MINUTES OF INITIAL TIME

LMO CALENDAR MONTH OF INITEAL TIME

LYR CALENDAR YEAR OF INITIAL TIME

MARS CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF MARS WILL BE CALCULATED

MERCURY CALENDAR ¿DATE AT WHICH ORBITAL ELEMENTS OF MERCURY WILL BE CALCULATED

MOON CALENDAR, DATE AT WHICH ORBITAL ELEMENTS OF EARTHS MOON WILL BE CALCULATED

NENT NUMBER OF ENTRIES IN MEASUREMENT SCHEDULE

NEPTUNE CALENDAR DATE AT WHICH GRBITAL ELEMENTS OF NEPTUNE WILL BE CALCULATED

OME ARGUMENT OF PERIAPSIS

PHIT DECLINATION

PLUTO CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF PLUTO WILL BE CALUCLATED

PRD INITIAL SEMI-LATUS RECTUM OF SPACECRAFT ORBIT

RDS GEOCENTRIC RADIUS OF VEHICLE

RD MAGNITUDE OF POSITION VECTOR

' SATURN CALENDAR' DATE AT WHICH ORBITAL ELEMENTS OF SATURN WILL BE CALCULATED

SEC INTERMEDIATE CALENDAR SECONDS SECI CALENDAR SECONDS AT FINAL TIME SECL CALENDAR SECONDS AT INITIAL TIME SIGMA AZIMUTH TA TRUE ANOMALY OF INSTANTANEOUS POSITION AND VELOCITY THETA RIGHT ASCENSION CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF URANUS URANUS WILL BE CALCULATED VEL INJECTION VELOCITY RELATIVE TO EARTH VENUS CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF - VENUS WILL BE CALCULATED ٧L MAGNITUDE OF VELOCITY VECTOR VUNIT INTERMEDIATE VELOCITY CONVERSION FACTOR COMMON COMPUTED/USED # ACCND ACC **ALNGTH** CXSU CXSV CXU **DELAXS** CXV CXXS DELECC DELMUS DELICL DELMA DELMUP DELNOD DELTP DELW DYMAX DTPLAN DTSUN EM1 EM4 EM5 EM6 EM7 EM8 EPS EP50 FACP FACV **FNTM** FOP FOV IAUGIN IBARY **ICOOR** ICDQ3 ICOORD IONF IEIG **IEPHEM IEVNT** IHYP1 IMNF INPR **IPRINT IPRT** ISP2 ISTMC ISTM1 **KPRINT** MNCN NBOD NDACC NDIM1 NDIM2 NDI M3 NEV10 NEV11 NEV1 NEV2 NEV3 NEV4 NEV5 NEV6 NEV7 NEV 9 NEV NMN NO NST NTMC ONE PS RAD SAL SIGALP SIGBET SIGPRO SIGRES SLAT SLON TEV SSS TMN TM TRTM1 TWO T UST UO VST

VO

**ACTPP** 

IBIAS

IUTC

NDIM4

**PSIGS** 

PMN

IPCSK .

WST

DELV

INPR

ISAO

NENT1

PROBI

**PUL MAG** 

IDENS

XΙ

FACTR

IGAIN

**IPGCK** 

NENT2

PSIGA

PULMAS

LKTP

ΧP

IAUGW

IGEN

IPQ

LKLP

NEV4

**PSIGB** 

RAD

**ZERO** 

IBAG

IGUID

IPROPI

NBODYI

NMNP

SMN

**PSIGK** 

•	80	TINJ	VTANGM	XDELV	XFAG
COMMON COMPUTEDS	BDRSI1 BDTSI3 CXSUG CXVB EM13 HALF ICA2 INCMT ISOI3 NAE NGE PSG RSOI2 TG TSOI2 VSOI3 XU	BDRSI2 BSI1 CXSVB CXVG EM2 IAUGDC ICA3 INITAL ISPH NBODYI OMEGA RCA1 RSOI3 THREE TSOI3 XB	BDRSI3 BSI2 CXSVG CXSVB EM3 IAUGMC ICL2 ITR NEV8 PB RCA2 ICA1 TIMINT TWOPI XF	BDTSI1 BSI3 CXUB CXXSG EM9 IAUG ICL ISOI1 MCNTR NGE PG RCA3 TCA2 TRTMB VSOI1 XG	BDTSI2 CXSUB CXUG DELTM EM ICA1 ISOI2 MCODE NPE PSB RSOI1 TCA3 TSOI1 VSOI2 XSL
COMMON USED:	AINC7 ELMNT IPROB PERP7 TAU7	ANDOE7 EVNM MNNAME PI TPT2	DATEJ F NB PLANET XLAB	DNCN G NLP PMASS XNM	ECC7 HP7 NTP P7

PROGRAH

DATAS

PURP OSE 8

TO READ INPUT DATA, TRANSLATE THIS DATA INTO PROPER INTERNAL VALUES. ASSIGN VALUES TO UNSPECIFIED NAMELIST VARIABLES, SET NECESSARY INITIAL VALUES, COMPUTE DIMENSIONS OF STATE TRANSITION, OBSERVATION, AND COVARIANCE MATRIX PARTITIONS, ORDER MEASURMENT AND EVENT SCHEDULES. AND PRINT OUT INITIAL CONDITIONS IN THE SIMUL PROGRAM.

CALLING SEQUENCE: CALL DATAS

SUBROUTINES SUPPORTED: MAIN

SUBROUTINES REQUIRED: CONVRT DATAIS ELCAR EPHEM

PECEQ

088

TRANS

LOCAL SYMBOLS

AΙ

INITIAL INCLINATION OF SPACECRAFT ORBIT

ANODE

INITIAL LONGITUDE OF ASCENDING NODE OF

SPACECRAFT ORBIT

TIME

A

INITIAL SEMI-MAJOR AXIS OF SPACECRAFT

ORBIT

DATE

ARRAY CONTAINING FINAL JULIAN DATE

DUM1

PLANETO-CENTRIC ECLIPTIC SPACECRAFT STATE

DUM

COORDINATE TRANSFORMATION FROM PLANETO-CENTRIC EQUATORIAL TO PLANETO-CENTRIC

**ECLIPTIC COORDINATES** 

D

JULIAN DATE AT LAUNCH

EARTH

CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF

EARTH WILL BE CALCULATED

Ε

INITIAL ECCENTRICITY OF SPACECRAFT ORBIT

FNDT

FINAL JULIAN DATE

GAMNA

INJECTION PATH ANGLE

GM

GRAVITATIONAL CONSTANT OF TARGET PLANET

IDAY

DAY OF FINAL COMPUTATION

IHR

HOUR OF FINAL COMPUTATION

IMIN

MINUTE OF FINAL COMPUTATION

IMO .

MONTH OF FINAL COMPUTATION

IYR YEAR OF FINAL COMPUTATION

JUPITER CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF JUPITER WILL BE CALCULATED

LDAY LAUNCH DAY

LHR LAUNCH HOUR

LMIN LAUNCH MINUTE

LMO LAUNCH MONTH

LYR LAUNCH YEAR

MARS CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF MARS WILL BE CALCULATED

MERCURY CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF MERCURY WILL BE CALCULATED

MOON GALENDAR DATE AT WHICH ORBITAL ELEMENTS OF MOON WILL BE CALCULATED

NENT NUMBR OF ENTRIES IN MEASUREMENT SCHEDULE

NEPTUNE CALENDAR DATE AT WHICH ORBITAL ELEMETHS OF NEPTUNE WILL BE CALCULATED

ONE INITIAL ARGUMENT OF PERIAPSIS OF SPACE-CRAFT ORBIT

PHIT INJECTION DECLINATION

PLUTO CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF PLUTO WILL BE CALCULATED

PRD INITIAL SEMI-LATUS RECTUM OF SPACECRAFT ORBIT

RDS EARTH-CENTERED INJECTION RADIUS

RD DUMMY VARIABLE

SATURN CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF SATURN WILL BE CALCULATED

SECI SECOND OF FINAL COMPUTATION

SECL LAUNCH SECOND

SEC SECOND OF CALENDAR DATE AT WHICH ORBITAL

1

ACC

ACC1

ELEMENTS OF A PLANET WILL BE CALCULATED

SIGMA INJECTION AZIMUTH

TA INITIAL SPACECRAFT TRUE ANOMALY

THETA INJECTION RIGHT ASCENSION

URANUS CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF

URANUS WILL BE CALCULATED

VEL INJECTION VELOCITY RELATIVE TO EARTH

VENUS CALENDAR DATE AT WHICH ORBITAL ELEMENTS OF

VENUS WILL BE CALCULATED .

VL DUMMY VARIABLE

AALP

VUNIT VELOCITY CONVERSION FACTOR

ABET

	COMMON	COMPUT	ED/I	USEDS
--	--------	--------	------	-------

ADEVX	ALNGTH	APRO	ARES	BIA
CXSU	CXSA	GXU	CXA	CXXS
DAB	DEB	DELAXS	DELECC	DELICL
DELMA	DELMUP	DELMUS	DELNOD	DELTP
DELM	DIB	DMAB	DMUPB	DMUSB
DNOB	DTMAX	DTPLAN	DTSUN	DWB
EM1	EM4	EM5	EM6	EM7
EHO	EPS	EP50	FACP	FACV
fnth	FOP	FOV .	IAMNF	IAUGIN
IBARY	ICDT3	ICOORD	ICOOR	IDNF
IEIG	iephen	IHYP1	innf,	INPR
EOPT7	IPRINT	IPRT	ISP2	ISTMC
istm1	KPRINT	MNCN	NB OD 1	NBOD
NB1	NDACC	NDIN1	SMIDN	NDIM3
NEV10	NEV11	NEV1	NEVS	NEV3
nev4	NEV5	NEV6	NE V7	NEV9
MO	NST	NTMC	ONE	PS
P	RAD	SAL	SIGALP	SIGBET
SIGPRO	SIGRES	SLAT	SLB	SLON
SSS	TM	trtm1	TTIM1	SMITT
TWO	T	UNMAC	UST	UO
VST	V O	<b>HST</b>	XI	ΧP
ZERO				•

ACCND

COMMON COMPUTED:

ADEVXS	BORSII	BDRSIZ	BDRSI3	BOTSI1
SIZTOB	BOTSI3	BSI1	BSIS	BSI3
DELTN	EM13	EM2	EM3	EM9
MALF	IAUGDC	IAUGHC	IAUG	ICA1
ICA2	IGAZ	ICDQ3	ICTS	ICL
INCHT	INITAL	ISOI1	ISOIS	ISOI3
isph	ITR	NBODYI	NEAB	RCA1
	Ŷ			

	RCA2 TCA1 TIMINT TWOPI XSL	RCA3 TCA2 TRTMB VSOI1 XU	RSOI1 TCA3 TSOI1 VSOI2 XV	RSOI2 TEV TSOI2 VSOI3 ZI	RSOI3 THREE TSOI3 XI1
COMMON USED:	AINC7 ECC7 NAF6 PI TPT2 T5	ANODE7 ELMNT NB PLANET T1 T6	AVARM F NLP PHASS T2 T7	DATEJ HP7 NTP P7 T3 XNM	BNCN IPROB PERP7 TAU7 T4

SUBROUTINE DATA1

PURPOSE: TO CONTINUE THE INITIALIZATION PROCESS DESCRIBED UNDER DATA.

CALLING SEQUENCE: CALL DATA1 (NENT)

ARGUMENT: NENT I NUMBER OF CARDS IN THE MEASUREMENT SCHEDULE

SUBROUTINES REQUIRED: GHA SKEDM

SUBROUTINES SUPPORTED: DATA,

LOCAL SYMBOLS:	AMIN	INTERMEDIATE VARIABLE
	AP	INTERMEDIATE TIME ARRAY
	DGTR	DEGREE TO RADIAN CONVERSION
	ICNT	COUNTER ON MEASUREMENT SCHEDULE CARDS
	IGO	INTERNAL FLAG
	IROW	INTERMEDIATE ROW INDEX
	MEAS	MEASUREMENT CODES
	NOUT	DIMENSION OF AUGUMENTED COVARIANCE MATRIX

COMMON	COMPUTED/USED:	IEVNT TEV T4	NEV TMN T5	NMN T1 T6	SLAT T2 T7	SLON T3 T	
COMMON	COMPUTED:	CXSUB CXUG EM MCODE OMEGA TG	CXSUG CXV8 EPS NAE PB XB	CXSVB CXVG IIPOL NGE PG XF	CXSVG CXXSB IPOL NPE PSB XG	CXUB CXXSG MCNTR NGE PSG	
COMMON	USED:	CXSU DATEJ EVNM IDNF MNCN NEV1 NEV6 P U0 MCODE1 TMN1	CXS V DNCN FACP IGUID MNNAME NEV 2 NEV 7 RAD VO MCODE2 TMN2	CXU EM7 FACV IHYP1 NDIM1 NEV3 NST SAL XI NENT1 T6	CXV EM8 FNTM IMNF NDIM2 NEV 4 ONE TPT 2 ZERO NENT2 T7	CXXS EP50 ICDQ3 ISTMC NDIM3 NEV5 PS TRTM1	171

SUBROUTINE DATA1S

PURPOSE: TO CONTINUE THE INITIALIZATION PROCESS DESCRIBED UNDER DATAS.

CALLING SEQUENCE: CALL DATA1S(NENT)

ARGUMENTS MENT I NUMBER OF CARDS IN THE HEASUREMENT SCHEDULE

SUBROUTINES SUPPORTED: DATAS

SUBROUTINES REQUIRED: GHA

LOCAL SYMBOLS & AMIN INTERMEDIATE VARIABLE

AP INTERMEDIATE TIME ARRAY

ICNT COUNTER ON MEASUREMENT SCHEDULE CARDS

IROW INTERMEDIATE ROW INDEX

MEAS MEASUREMENT CODES

NOUT DIMENSION OF AUGUMENTED COVARIANCE MATRIX

PARAM ARRAY OF AUGHENTED BIASES

SCHED ARRAY OF TIMES IN MEASUREMENT SCHEDULE

	<b>;</b>					
COMMON	COMPUTED/USED:	ADEVXS SLB T2 T7	IEVNT SLON T3 T	NEV TEV T4	NMN TMN TS	SLAT T1 T6
COMMON	COMPUTED:	ADEVSB CXSVG CXXSB EPS NAE PB XB	ADEVXB CXVB CXXSG IIPOL NGE PG XF	CXSUB CXUG EDEVXS IPOL NPE PSB XG,	CXSUG CXVB EDEVX HCNTR NQE PSG XI1	CXSVB CXVG EM MCQDE OMEGA TG ZI
COMMON	USED:	ACC1 CXSV DATEJ DMUSB EMS' FMTM IDNF MNNAME NDIM3	ADEVX CXU DEB DNCN EP50 IAMMF IHYP1 NBOD1 NEV1	AVARM CXV DIB DNOB EVNM IAUGIN IMNF NB1 NEV2	BIA CXXS DMAB DW3 FACP ICDQ3 ISTMC NDIM1 NEV3	CXSU DAB DMUPB EM7 FACV ICDT3 MNCN NDIM2 NEV4

NE V5	NEVO	NE V7	NST	ONE
<b>PLANET</b>	PS	P	RAD	SAL
TPT2	TRYM1	TTIM1	TTIM2	UNMAC
un	V O	AUTIM	YT	7F80

SUBROUTINE DESENT

PURPOSES TO COMPUTE A CORRECTION TO AN INITIAL VELOCITY BY THE STEEPEST DESCENT OR CONJUGATE GRADIENT TECHNIQUES FOR USE BY TARGET.

CALLING SEQUENCES CALL DESENT (ERC, IT, KREK, GMP, PP)

ARGUMENTS 8 ERC I SCALAR ERROR OF CURRENT ITERATE

IT I/O ITERATION COUNTER

KREK I STEEPEST DESENT RECTIFICATION NUMBER

GMP I/O PREVIOUS GRADIENT MAGNITUDE (INPUT)
CURRENT GRADIENT MAGNITUDE (OUTPUT)

PP(3) I/O PREVIOUS GRADIENT (INPUT)

CURRENT GRADIENT (OUTPUT)

SUBROUTINES SUPPORTED: TARGET

SUBROUTINES REQUIRED: TAROPT VMP

LOCAL SYMBOLS: ACK CURRENT ACCURACY LEVEL

AER ABSOLUTE ERRORS OF TARGET PARAMETERS

AUXN VALUES OF AUXILIARY PARAMETERS OF CURRENT

ITERATE

DD DIRECTIONAL DERIATIVE

DELVM MAGNITUDE OF PREDICTED CORRECTION

DEVIATION OF NOMINALLY-PREDICTED AUXILIARY PARAMETERS FROM CURRENT ITERATE VALUES

DUMMY VARIABLES

DUM DUMMY VARIABLES

DVEE VELOCITY PERTURBATIONS

DVH MAXIMUM ALLOWABLE VELOCITY_INCREMENT

ERB SCALAR ERROR OF NOMINALLY-PREDICTED STEP

GC CURRENT GRADIENT

GHC MAGNITUDE OF GC

HB NOMINALLY PREDICTED STEP MAGNITUDE

		нн	G	ORRECTIO	n magni	TUDE AFT	ER CONST	raints (	
		нѕ	C	ORRECTIO	N MAGNI	TUDE AFT	ER PARAB	OLIG FI	T
		IEN			TO 1 IF RBED TRA	TOLERAN Jectory	CES ACCE	PTABLE	
		ISI	=		STOP A	DITION F T SOI	LAG		
PC PERR PM			0	DIRECTION OF CORRECTION					
			RR P	PERTURBED ERRORS					
			M	MAGNITUDE OF UNNORMALIZED DIRECTION VECTOR					
		QC	U	INIT VECT	OR IN D	IRECTION	OF GRAD	IENT	
		RSI	F F	INAL STA	TE OF I	NTEGRATI	ON ,		
	,					1			
	CONMON	COMPUTED/US	ED: D	ELTAV 1	SPH	RIN	TEN		
	соймои	COMPUTEDS	1	CL2	CCL (	INCMT	RRF		
	Сомнон	USED8	0 1	ELTAT (	TAR ESTOP	DVMAX Kur	D1 . Lev	DAUX FAC LVLS ZERO	

#### DESENT Analysis

DESENT computes a correction to an initial velocity by the steepest descent or conjugate gradient techniques for use by TARGET.

The technique used is determined by the value of METHOD. DESENT takes n steps in the conjugate gradient directions before rectifying by making a steepest descent step where n = METHOD - 1. Thus if METHOD = 1, all steps are taken in the steepest descent direction.

Let the current iterate initial state be denoted r, v. Let the scalar error of the auxiliary parameters corresponding to this state be denoted  $\epsilon$ . Let the perturbation size for the sensitivities be dv.

The current gradient  $\vec{g}_c$  is computed by numerical differencing. For the k-th component of  $\vec{g}_c$  the corresponding component of velocity is perturbed by dv

$$\vec{v}_p = \vec{v} + dv \left[ \delta_{1K}, \delta_{2K}, \delta_{3K} \right]^T$$
 (1)

The initial state ( $\vec{r}$ ,  $\vec{v}_p$ ) is then propagated to the final stopping conditions. Let the auxiliary parameters of that trajectory be denoted  $\vec{\alpha}_p$ . The error associated with the perturbed state is then

$$\epsilon_{p} = \overrightarrow{W} \cdot (\overrightarrow{\alpha}_{p} - \overrightarrow{\alpha}^{*})$$
 (2)

where  $\overline{W}$  represents the weighting factors and  $\overline{\alpha}^*$  are the desired target conditions. The k-th component of the current gradient is then

$$g_{c_{K}} = \frac{\dot{\epsilon_{p}} - \dot{\epsilon}}{dv}$$
 (3)

The corrected gradient is given by

$$\vec{P}_c = \vec{g}_c$$
 steepest descent step
$$= \frac{|g_c|^2}{|g_p|^2} \vec{p}_p + \vec{g}_c \quad \text{conjugate gradient step} \quad (4)$$

where the subscript c refers to a current parameter, p refers to a previous-step parameter.

The unit vector in the direction of the next step is then given by

$$\vec{q}_{c} = -\frac{\vec{p}_{c}}{P_{c}} \tag{5}$$

The directional derivative of the scalar error in the the direction  $\vec{q}_c$  is

$$d = \vec{g} \cdot \vec{q}$$
 (6)

The nominal step size  $\overline{\,h\,}$  is computed from a linear approximation to null the error

$$\overline{h} = \frac{\epsilon}{-d} \tag{7}$$

The initial state corrected by this nominal correction is then propagated to the final stopping conditions and the resulting error  $\overline{\epsilon}$  computed. The three conditions

$$y(o) = \epsilon$$
  
 $y(h)_{,} = \overline{\epsilon}$   
 $y'(o) = d$  (8)

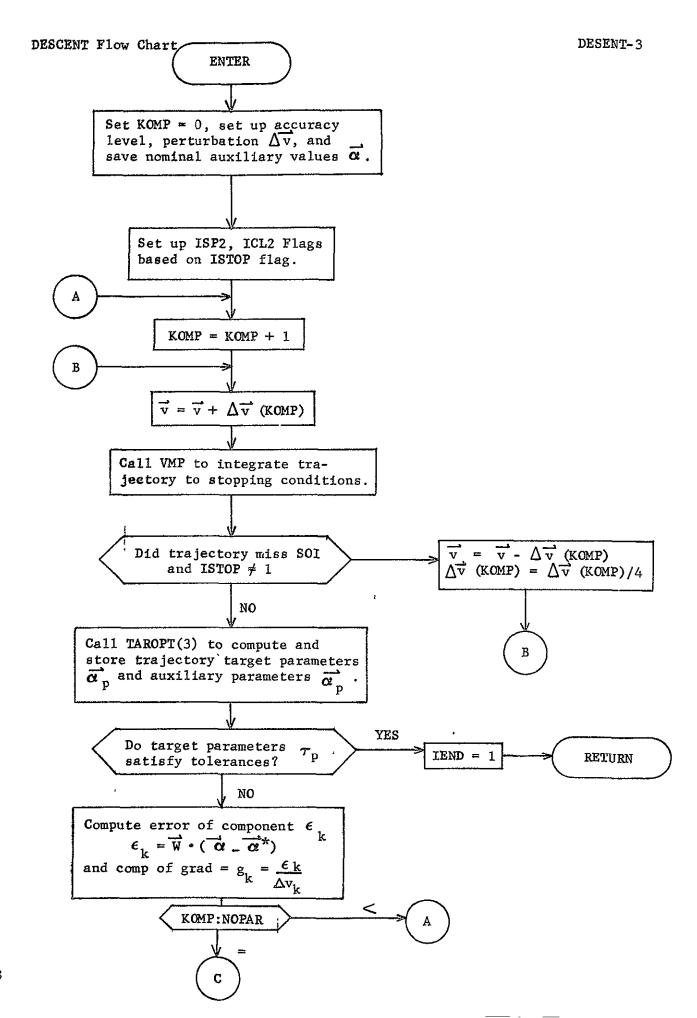
may now be applied to the formula of a parabola  $y-\epsilon^*=a(x-h^*)^2$  to predict the optimal step size  $h^*$  yielding the minimum error  $\epsilon^*$ 

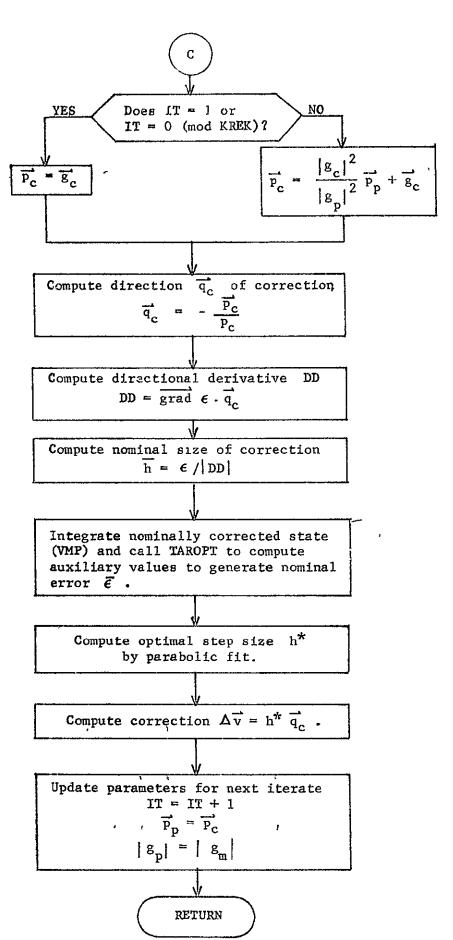
$$h^* = \frac{dh^2}{2(dh + \epsilon - \overline{\epsilon})}$$
 (9)

The correction for the current is then given by

$$\Delta v = h^* \vec{q}_g$$
 (10)

Reference: Myers, G. E., "Properties of the Conjugate Gradient and Davidon Methods", AAS Paper 68-081. Presented at 1968 AAS/AIAA Astrodynamics Specialist Conference, Jackson, Wyoming.





# SUBROUTINE DIMPCP

PURPOSE: TO CALCULATE THE DESIRED IMPACT PLANE ASYMPTOTE PIERCE POINT COORDINATES GIVEN THE RIGHT ASCENSION AND DECLINATION OF A PROBE TARGET SITE

ARGUMENT:	A	I	SEMI-MAJOR AXIS OF APPROACH HYPERBOLA IN KM
	84	0	MAGNITUDE OF B VECTOR IN KM
	OBR	0	DESIRED R.R IN KM
	DBT	0	DESIRED B.T IN KM
	DCP	I	DESIRED DECLINATION IN PROBE-SPHERE COORDINATES OF IMPACT SITE IN DEG
	RAP	I	DESIRED RIGHT ASCENSION IN PROBE-SPHERE COORDINATES OF IMPACT SITE IN DEG
	RPR	I	RADIUS OF THE PROBE SPHERE IN KM
	RV	I	PLANETOCENTRIC VECTOR IN DIRECTION OF CROSS PRODUCT OF SV BY ECLIPTIC POLE VECTOR
	SV	I	PLANETOCENTRIC VECTOR IN DIRECTION OF TRAJECTORY ASYMPTOTE
	TV	I	CROSS PRODUCT OF SV BY RV
	Ť	I	TRANSFORMATION MATRIX FROM ECLIPTIC TO PROBE-SPHERE COORDINATES

## SUBROUTINES SUPPORTED: TPPROP TPRTRG

### SUBROUTINES SUPPORTED: LOCAL SYMBOLS: BV

PLANETOCENTRIC UNIT VECTOR IN DIRECTION OF B VECTOR

CSDCP COSINE OF DECLINATION OF DESIRED IMPACT SITE

CSDIF COSINE OF THE ARC LENGTH BY WHICH DESIRED IMPACT SITE IS CLOSER TO SV THAN IS PERIAPSIS

CSPHI COSINE OF ANGLE BETWEEN SV AND VECTOR TO DESIRED IMPACT SITE

CSRAP COSINE OF RIGHT ASCENSION OF DESIRED IMPACT SITE

CSTHTS COSINE OF TRUE ANAMOLY OF TRAJECTORY ASYMPTOTE

C1 COEFFICIENT USED IN CALCULATING REPOSITIONED IMPACT SITE

C2 COEFFICIENT USED IN CALCULATING REPOSITIONED IMPACT SITE

DCPSAV SAVED VALUE OF IMPACT SITE DECLINATION IN DEG

E ECCENTRICITY OF CONIC

FOUR CONSTANT 4.

HALF CONSTANT .5

ONE CONSTANT 1.

RAPSAV SAVED VALUE OF IMPACT SITE RIGHT ASCENSION IN DEG

RPVRV PLANETOCENTRIC UNIT VECTOR TO DESIRED IMPACT SITE IN ECLIPTIC COORDINATES

RPV PLANETOCENTRIC UNIT VECTOR TO DESIRED IMPACT SITE IN PROBE-SPHERE COORDINATES

RTD CONVERSION FACTOR FROM RADIANS TO DEGREES

SNDCP SINE OF DECLINATION OF DESIRED IMPACT SITE

SNPHI SINE OF ANGLE BETWEEN SV AND VECTOR TO DESIRED IMPACT SITE

SNRAP SINE OF RIGHT ASCENSION OF DESIRED IMPACT SITE

SNTHTS SINE OF TRUE ANAMOLY OF TRAJECTORY ASYMPTOTE

# DIMPCP Analysis

Subroutine DIMPCP converts the actual probe target parameters of declination  $\delta$  and right ascension  $\alpha$  of the trajectory impact point on the planet into the auxiliary target parameters of equivalent B·T and B·R. To do so it assumes the direction of the hyperbolic excess velocity and the energy of the trajectory are known so the S, T, and R* in the ecliptic frame and the semimajor axis, a, of the approach hyperbola are available as inputs. To complete the specification of the probe impact point, the subroutine also requires the radius, r, of the planet at impact as well as the transformation, L, from the inertial ecliptic frame to the coordinate system to which the right ascension and delcination are referenced.

Derivation of the necessary equations is relatively straightforward once the appropriate variables are defined. Let  $\underline{\rho}$  be a planet-centered unit vector in the direction of the impact point. Then, in the inertial ecliptic system,

$$\underline{\rho} = L^{T} \begin{pmatrix} \cos \alpha \cos \delta \\ \sin \alpha \cos \delta \\ \sin \delta \end{pmatrix} . \tag{1}$$

Define  $\phi$  to be the unique angle on the closed interval from 0 to  $\pi$  between  $\underline{\rho}$  and  $\underline{S}$  (see Fig. 1). Finally denote the true anomalies of  $\underline{\rho}$  and  $\underline{S}$  by  $\overline{\theta}$  and  $\theta_{\underline{S}}$ , respectively.

First DIMPCP determines whether the desired impact point is indeed targetable. It is apparent from Figure 1 that

$$|\theta| = \phi - \theta_{g} \tag{2}$$

It is further obvious from the figure that the approach hyperbola will intersect the planet surface at true anomalies of both  $+\theta$  and  $-\theta$ . Obviously only the negative true anomaly impact points are physically realizable since the trajectory stops at the first intersection with the planet. Hence DIMPCP requires the

$$\theta_{S} \leq \phi$$
 . (3)

^{*}For definitions of these vectors see the analysis section of the subroutine STIMP.

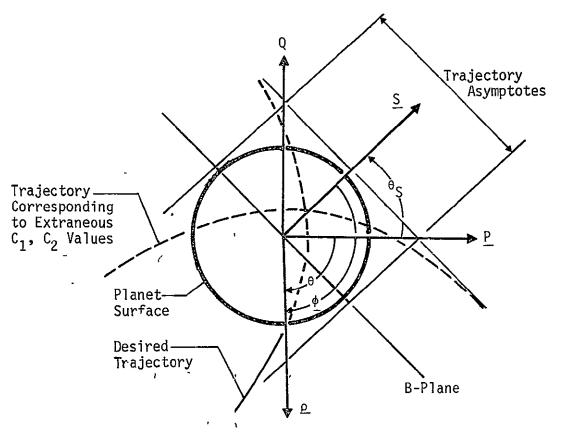


Figure 1 Geometry of Probe Impact

In other words, there is a circular region on the planet surface of radius  $\theta_S$  about the outgoing pierce point of the  $\underline{S}$  vector inside of which no probes can be targeted. If  $\rho$  falls in this untargetable region, DIMPCP repositions the desired impact point direction to  $\underline{\rho}$ , the nearest acceptable direction in the plane determined by  $\underline{S}$  and  $\underline{\rho}$ . Analytically this is done by expressing  $\underline{\rho}$  as a linear combination of  $\underline{\rho}$  and  $\underline{S}$ ; that is

$$\underline{\rho}' = d_1 \underline{\rho} + d_2 \underline{S} \qquad . \tag{4}$$

Then the constraints

$$\left|\left|\underline{\rho}'\right|\right| = 1 \tag{5}$$

and

$$\underline{\rho} \cdot \underline{\rho}' = \cos (\theta_{S} - \phi)$$
 (6)

are applied. These result in the following pair of simultaneous equations for  $\mathbf{d}_3$  and  $\mathbf{d}_2$  :

$$1 = d_1^2 + 2 d_1 d_2 \cos \phi + d_2^2$$
 (7)

$$\cos (\theta_S - \phi) = d_1 + d_2 \cos \phi . \qquad (8)$$

Solving (8) for  $d_1$  in terms to  $d_2$  and substituting into (7) produces the quadratic

$$1 - \cos^2(\theta_S - \phi) = d_2^2(1 - \cos^2\phi)$$
 . (9)

Assuming  $|\cos \phi| \neq 1$  leads then to the conclusion that

$$d_2 = \pm \sqrt{\frac{1 - \cos^2(\theta_S - \phi)}{1 - \cos^2\phi}}$$
 (10)

Figure 2 geometrically interprets the two roots of equation (9) given by (10).

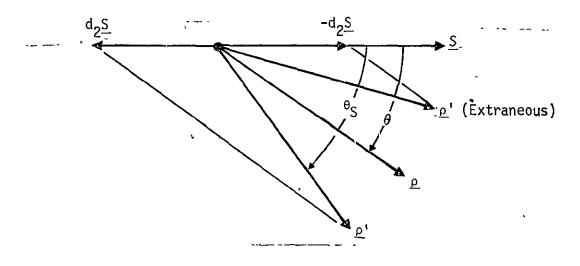


Figure 2 Geometrical Interpretation of the Two  $d_2$ -Roots

Clearly the  $d_2$  value corresponding to the positive radical is extraneous since it produces a  $\underline{\rho}$  nearer  $\underline{S}$  than  $\underline{\rho}$ . Hence

$$d_2 = -\sqrt{\frac{1 - \cos^2(\theta_S - \phi)}{1 - \cos^2\phi}}$$
 (11)

$$d_{1} = \cos (\theta_{S} - \phi) + \cos \phi \sqrt{\frac{1 - \cos^{2} (\theta_{S} - \phi)}{1 - \cos^{2} \phi}} . \tag{12}$$

The exceptional case that  $\cos \phi = -1$  cannot occur since then  $\phi = \pi$  and hence  $\theta_S < \phi$ . However,  $\cos \phi$  can equal 1. In this case  $\underline{\rho}$  and  $\underline{S}$  are coincident so  $\underline{\rho}$ ' cannot be taken as a linear combination of the two. Further, no particular point on the boundary of the circular untargetable region recommends itself. Hence DIMPCP arbitrarily puts  $\underline{\rho}$ ' in the S-T plane as

$$\underline{\rho'} = \underline{S} \cos \theta_{S} + \underline{T} \sin \theta_{S} . \qquad (13)$$

On repositioning  $\underline{\rho}$ , DIMPCP prints out the right ascention  $\alpha'$  and declination  $\delta'$  of  $\underline{\rho}'$  making use of the formulae

$$\alpha^{\dagger} = \tan^{-1} \left( \rho_{2}^{\dagger} / \rho_{1}^{\dagger} \right) \tag{14}$$

$$\delta^{\dagger} = \sin^{-1} (\rho_3^{\dagger}) \qquad . \tag{15}$$

Having repositioned  $\underline{\rho}$ , if necessary, DIMPCP calculates the magnitude of the desired  $\underline{B}$ . It can readily be shown that

$$\cos \theta_{S} = \frac{1}{e} \tag{16}$$

$$\sin \theta_{\rm S} = \frac{\sqrt{\rm e^2 - 1}}{\rm e} \tag{17}$$

$$B = |a| \sqrt{e^2 - 1} . {18}$$

Recall the polar equation of the near-planet conic trajectory, namely

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta}$$
 (19)

Substituting equation (2) into (19) and rearranging gives

$$a(1 - e^2) = r \left[1 + e \left(\cos \phi \cos \theta_S + \sin \phi \sin \theta_S\right)\right] . \quad (20)$$

Using equations (16) and (17) in (2) yields

$$a(1 - e^2) = r (1 + \cos \phi + \sqrt{e^2 - 1 \sin \phi})$$
 (21)

Eliminating the eccentricity from equation (19) by means of (18) produces a quadratic in B, that is,

$$-B^2/a = r (1 + \cos \phi) - rB/a$$
 (22)

Applying the quadratic formula to (20) gives

$$B = \frac{[r \sin \phi \pm \sqrt{r^2 \sin^2 \phi - 4 \arctan (1 + \cos \phi)}]}{2}$$
 (23)

Since  $1 + \cos \phi \ge 0$  for all  $\phi$  and a < 0 for hyperbolic approach. trajectories, the root corresponding to the negative radical in (21) produces a negative magnitude of  $\underline{B}$  and hence must be extraneous. Thus

$$B = \frac{[r \sin \phi + \sqrt{r^2 \sin^2 \phi - 4 \arctan (1 + \cos \phi)}]}{2}$$
 (24)

One can further conclude from the radicand of (23) that a solution for B will exist if, and only if,

4 ar 
$$(1 + \cos \theta) \le r^2 (1 - \cos^2 \phi)$$
, (25)

or equivalently if  $\cos \phi \neq -1$ 

$$\cos \phi \le 1 - \frac{4a}{r} \quad , \tag{26}$$

Since a is negative for a hyperbolic approach, this last inequalities always true. Further if  $\cos \phi = -1$ , B = 0. Hence equation (22) always has the unique nonnegative solution given by (24).

Next DIMPCP computes the direction of  $\underline{B}$ . Since the desired  $\underline{B}$  must lie in the plane determined by  $\underline{S}$  and  $\underline{\rho}$ , there must exist real numbers  $C_1$  and  $C_2$  so that

$$\underline{B}/B = C_1 \underline{S} + C_2 \underline{\rho} . \qquad (27)$$

Applying the constraints that  $\left| \left| \underline{B} \middle| B \right| \right| = 1$  and  $\underline{B} \cdot \underline{S} = 0$  respectively

$$C_1^2 + 2 C_1 C_2 \cos \phi + C_2^2 = 1$$
 (23)

$$C_1 + C_2 \cos \phi = 0$$
 . (29)

Solving these two equations simultaneously for a and b gives

$$C_2 = \pm 1/\sin \phi \tag{30}$$

$$C_1 = \mp \cot \phi$$
 (31)

The negative  $C_2$  root and the corresponding positive  $C_1$  root are extraneous since they place  $\underline{B}$  on the side of  $\underline{S}$  opposite to  $\underline{\rho}$  as shown in Figure 1. Substituting the correct pair of roots from (30) and (31) into (27) gives the direction of the desired  $\underline{B}$  as

$$\underline{B}/B = (\underline{\rho} - \underline{S} \cos \phi) / \sin \phi . \qquad (32)$$

Clearly in the exceptional case that  $\sin \phi = 0$ , the trajectory passes through the center of the planet coinciding with its asymptote so that  $\underline{B} = \underline{0}$ .

Finally DIMPCP calculates the desired  $B \cdot T$  and  $B \cdot R$  coordinates now that B is known:

$$B \cdot T = B_1 T_1 + B_2 T_2 \tag{33}$$

$$B \cdot R = B_1 R_1 + B_2 R_2 + B_3 R_3$$
 (34)

FUNCTION DINCOS

PURPOSE: TO CALCULATE THE INVERSE COSINE IN DEGREES OF GIVEN REAL ARGUMENT SETTING RESULT TO 0. (180.) IF ARGUMENT IS GREATER THAN 1. (LESS THAN -1.)

ARGUHENT: X I REAL NUMBER WHOSE INVERSE COSINE IS TO BE EVALUATED

SUBROUTINES SUPPORTED + CAREL TPPROP

LOCAL SYMBOLS: DINCOS INVERSE COSINE IN DEGREES OF ARGUMENT X

ONE CONSTANT 1.

RTD CONVERSION FACTOR FROM RADIANS TO DEGREES

FUNCTION DINSING

PURPOSE® TO CALCULATE THE INVERSE SINE IN DEGREES OF GIVEN REAL ARGUMENT SETTING RESULT TO 90. (-90.) IF ARGUMENT IS GREATER THAN 1. (LESS THAN -1.)

ARGUMENT: X REAL NUMBER WHOSE INVERSE SINE IS TO BE EVALUATED

SUBROUTINES SUPPORTED: DIMPCP IMPCT TPPROP TPRTRG

LOCAL SYMBOLS: DINSIN INVERSE SINE IN DEGREES OF ARGUMENT X

ONE CONSTANT 1.

RTD CONVERSION FACTOR FROM RADIANS TO DEGREES

SUBROUTINE DYNO

PUPPOSE: COMPUTE ASSUMED AND ACTUAL DYNAMIC NOISE COVARIANCE MATRIX IN THE ERROR ANALYSIS PROGRAM

CALLING SEQUENCE: CALL DYNO(ICODE)

ARGUMENT & ICODE I =0 ASSUMED =1 ACTUAL

SUBROUTINES SUPPORTED: ERRANN SETEVN GUIDM PRED

LOCAL SYMBOLS: D2 SQUARE OF (DELTM*TM)

COMMON COMPUTED: Q QPR

COMMON USED: DELTM DNCN IDNF TM
IGDNF GDNCN

#### DYNØ Analysis

Subroutine DYNØ evaluates the assumed dynamic covariance matrix Q over the time interval  $t=t_{k+1}-t_k$  if ICØDE=0. If ICØDE=1 the actual dynamic noise covariance matrix Q' is evaluated over the same interval. In either case the dynamic noise covariance matrix is assumed to have the form

Q = diag (
$$\frac{1}{4}$$
 K₁  $\Delta t^4$ ,  $\frac{1}{4}$  K₂  $\Delta t^4$ ,  $\frac{1}{4}$  K₃  $\Delta t^4$ , K₁  $\Delta t^2$ , K₂  $\Delta t^2$ , K₃  $\Delta t^2$ )

where dynamic noise constants  $K_1$ ,  $K_2$ , and  $K_3$  have units of  $\mathrm{km}^2/\mathrm{s}^4$ . To compute the actual dynamic noise covariance matrix Q', we simply replace  $K_1$ ,  $K_2$ , and  $K_3$  with the actual dynamic noise constants  $K_1'$ ,  $K_2'$ , and  $K_3'$ , respectively.

SUBROUTINE DYNOS

PURPOSE® COMPUTE DYNAMIC NOISE COVARIANCE MATRIX AND THE ACTUAL DYNAMIC NOISE (UNMODELED ACCELERATION) IN THE SIMULATION PROGRAM

CALLING SEQUENCE: CALL DYNOS(ICODE)

ARGUMENTS ICODE I INTERNAL CODE TO DETERMINE IF THE DYNAMIC NOISE MATRIX IS COMPUTED OR IF THE ACTUAL DYNAMIC NOISE IS CALCULATED

SUBROUTINES SUPPORTED & SIMULL SETEVS GUISIM PRESIM

LOCAL SYMBOLS: DT INTERNAL TIME INCREMENT

D2 SQUARE OF (DELTM*TM)

IC INTERNAL CODE ON DT CALCULATION

T1 CURRENT TIME

T2 CURRENT TIME + DELTA TIME

COMMON COMPUTED/USED: H

COMMON COMPUTEDS Q

COMMON USED: DELTH DNCN HALF IBNF TM
TRTM1 TTIM1 TTIM2 UNHAC ZERO

### DYNØS Analysis

Subroutine DYNØS performs two functions. It's first function is identical to that of subroutine DYNØ, namely, to evaluate the dynamic noise covariance matrix Q over the time interval  $\triangle$  t  $\neq$  t_{k+1} - t_k.

The second function of subroutine DYN is to compute the actual dynamic noise  $\overline{\omega}_{k+1}$ , which represents the integrated effect of unmodelled accelerations acting on the spacecraft over the time interval  $\triangle t$ . Actual dynamic noise  $\overline{\omega}_{k+1}$  is used elsewhere in the program to compute the actual state deviations of the spacecraft from the most recent nominal trajectory.

If we define 
$$\overrightarrow{\omega}_{k+1} = \begin{bmatrix} \overrightarrow{\omega}_{r_{k+1}}, & \overrightarrow{\omega}_{v_{k+1}} \end{bmatrix}^T$$
, where

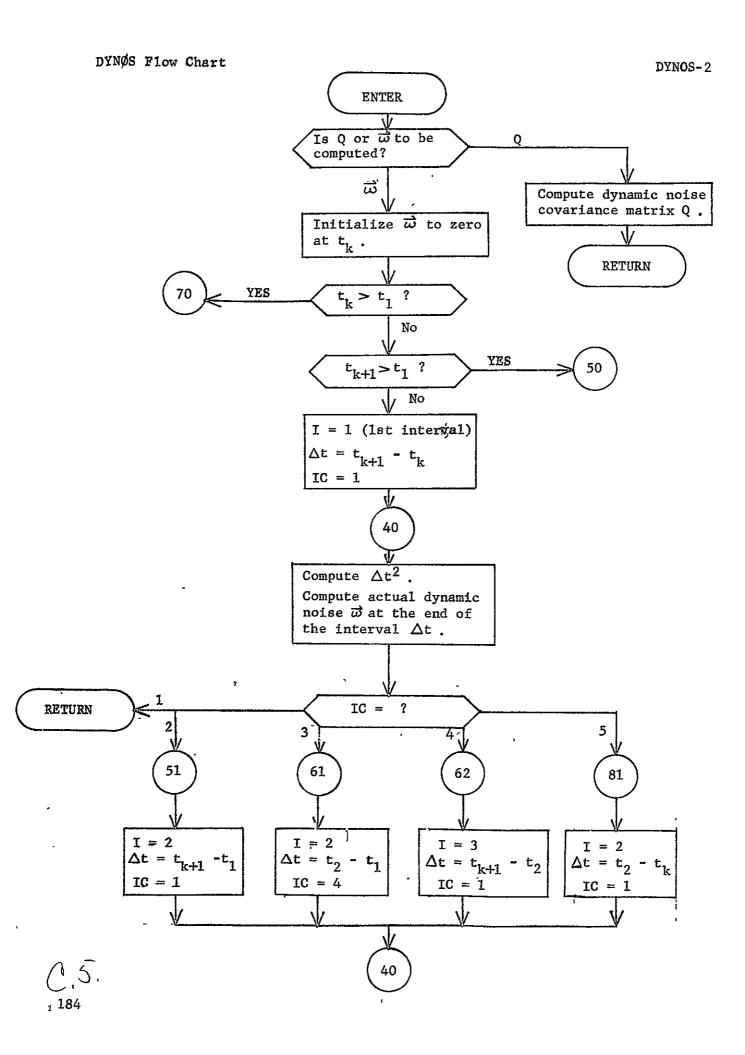
$$\overrightarrow{\omega}_{\mathbf{r_{k+l}}}$$
 and  $\overrightarrow{\omega}_{\mathbf{v_{k+l}}}$  denote the contributions of unmodelled

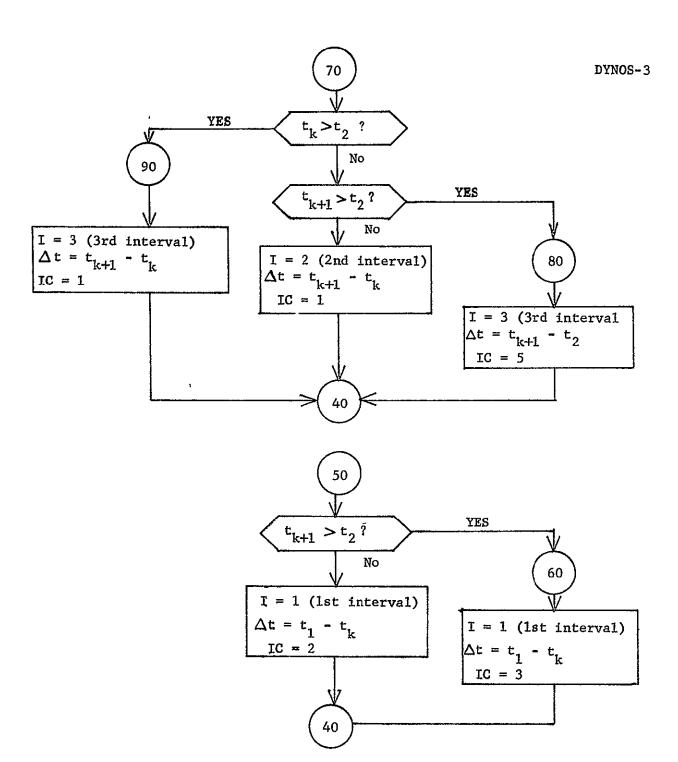
accelerations to spacecraft position and velocity, respectively, and if we assume constant unmodelled acceleration a, then

$$\overrightarrow{\omega}_{\mathbf{r}_{k+1}} = \frac{\overrightarrow{a}}{2} (t_{k+1} - t_k)^2 + \overrightarrow{\omega}_{\mathbf{v}_k} (t_{k+1} - t_k) + \overrightarrow{\omega}_{\mathbf{r}_k}$$

$$\overrightarrow{\omega}_{v_{k+1}} = \overrightarrow{a}(t_{k+1} - t_k) + \overrightarrow{\omega}_{v_k}$$

The program permits the entire trajectory to be divided into three arbitrary consecutive intervals, over each of which a different constant unmodelled acceleration a can be specified. These intervals are represented by  $(t_0,\,t_1),\,(t_1,\,t_2),\,$  and  $(t_2,\,t_f),\,$  where  $t_0$  is the initial trajectory time and  $t_f$  is the final trajectory time. If  $t_k$  and  $t_{k+1}$  occur in different intervals, then the above equations must be evaluated piece-wise over  $\left(t_k,\,t_{k+1}\right)$ .





SUBROUTINE EIGHY

PURPOSE: TO CONTROL THE COMPUTATION OF EIGENVALUES, EIGENVECTORS, AND HYPERELLIPSOIDS.

CALLING SEQUENCE: CALL EIGHY (VEIG, FOX, HARG, IFMT)

ARGUMENTS YEIG I MATRIX TO BE DIAGONALIZED

FOX I FINAL OFF-DIAGONAL ANNIHILATION VALUE

HARG I MATRIX FOR WHICH THE HYPERELLIPSOID IS TO

BE COMPUTED

IFMT I FORMAT FLAG

=1, PRINT POSITION EIGENVALUE TITLE

=2, PRINT VELOCITY EIGENVALUE TITLE

=3, PRINT EIGENVALUE TITLE

SUBROUTINES SUPPORTED: SETEVS GUISIM GUISS PRESIM GUIDM

SUBROUTINES REQUIRED: HYELS JACOBI

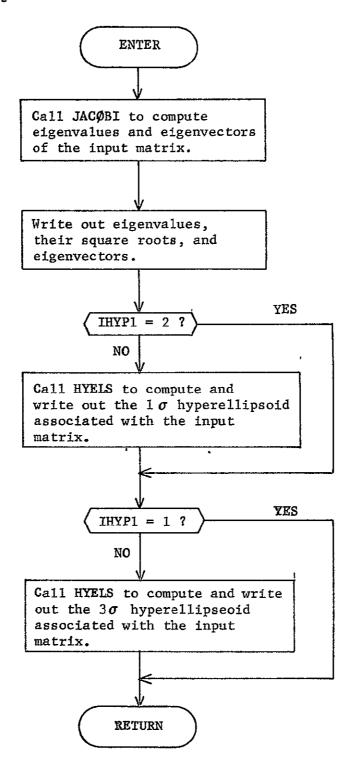
LOCAL SYMBOLS: EGYCT EIGENVECTOR MATRIX

EGVL EIGENVALUE MATRIX

OUT SQUARE ROOTS OF EIGENVALUES

COMMON USED: IHYP1

# EIGHY Flow Chart



SUBROUTINE ELCAR

PURPOSE: TRANSFORMATION OF CONIC ELEMENTS TO CARTESIAN COORDINATES

CALLING SEQUENCES CALL ELCAR(GM, A, E, H, XI, XN, TA, R, RH, V, VM, TFP)

ARGUMENTS GM I GRAVITATIONAL CONSTANT OF CENTRAL BODY

A I SEMIMAJOR AXIS

E I ECCENTRICITY

W I ARGUMENT OF PERIAPSIS

XI I INCLINATION IN REGERENCE SYSTEM

XN I LONGITUDE OF ASCENDING NODE

TA I TRUE ANOMALY

R(3) 0 POSITION VECTOR IN REFERENCE SYSTEM

RH 0 POSITION MAGNITUDE

V(3) 0 VELOCITY VECTOR IN REFERENCE SYSTEM

VM 0 VELOCITY MAGNITUDE

TFP 0 TIME FROM PERIAPSIS

SUBROUTINES SUPPORTED 8 DATAS VMP NONLIN COPINS NONINS DATA HELIO MULTAR CPROP

SUBROUTINES REQUIRED 8 NONE

LOCAL SYMBOLS B AUXF ECCENTRIC ANOMALY (HYPERBOLIC CASE)

AVA MEAN ANOMALY (ELLIPTIC CASE)

CI COSINE OF INCLINATION

CK VELOCITY FACTOR USED TO CALCULATE FINAL

VELOCITY VECTOR

CN COSINE OF LONGITUDE OF ASCENDING NODE .

COSEA COSINE OF ECCENTRIC ANOMALY (ELLIPTIC

CASE

CT COSINE OF TRUE ANOMALY

CW COSINE OF SUM OF ARGUMENT OF PERIAPSIS AND TRUE ANOMALY/ COSINE OF ARGUMENT OF PERIAPSIS

DIV THE SUM 1.+E*(CQS(TA/RAD)). USED AS A DIVISOR IN SUBSEQUENT EQUATIONS TO CALCULATE TFP

EA ECCENTRIC ANOMALY (ELLIPTIC CASE)

P SEMI-LATUS RECTUM

RAD DEGREES TO RADIANS CONVERSION FACTOR

SINEA SINE OF ECCENTRIC ANOMALY (ELLIPTIC CASE)

SINHF HYPERBOLIC SINE OF AUXF

SI SINE OF INCLINATION

SN SINE OF LONGITUDE OF ASCENDING NODE

ST SINE OF TRUE ANOMALY

SM SINE OF THE SUM OF ARGUMENT OF PERIAPSIS AND TRUE ANOHALY/ SINE OF ARGUMENT OF PERIAPSIS

TANG INTERMEDIATE VARIABLE USED TO CALCULATE SINHF

#### ELCAR Analysis

ELCAR transforms the standard conic elements of a massless point referenced to a gravitational body to cartesian position and velocity components with respect to that body.

Let the gravitational constant of the body be denoted  $\mu$  and the given conic elements (a, e, i,  $\omega$ , Q, f). The semilatus rectum p is

$$p = a(1 - e^2)$$
 (1)

Then the magnitude of the radius vector is given by

$$r = \frac{p}{1 + e \cos f} \tag{2}$$

The unit vector in the direction of the position vector is

$$u_{X}^{5} = \cos (\omega + f) \cos \Omega - \cos i \sin (\omega + f) \sin \Omega$$

$$u_{Y} = \cos (\omega + f) \sin \Omega + \cos i \sin (\omega + f) \cos \Omega$$

$$u_{Z} = \sin (\omega + f) \sin i$$
(3)

The position vector  $\vec{r}$  is therefore

$$\vec{r} = r \hat{u} \tag{4}$$

The velocity vector  $\overrightarrow{v}$  is given by

$$v_{x} = \sqrt{\frac{\mu}{p}} \left[ (e + \cos f) (-\sin \omega \cos \Omega - \cos i \sin \Omega \cos \omega) - \sin f (\cos \omega \cos \Omega - \cos i \sin \Omega \sin \omega) \right]$$

$$v_{y} = \sqrt{\frac{\mu}{p}} \left[ (e + \cos f) (-\sin \omega \sin \Omega + \cos i \cos \Omega \cos \omega) - \sin f (\cos \omega \sin \Omega + \cos i \cos \Omega \sin \omega) \right]$$

$$v_{z} = \sqrt{\frac{\mu}{p}} \left[ (e + \cos f) \sin i \cos \omega - \sin f \sin i \sin \omega \right]$$
 (5)

The conic time from periapsis t is computed from different formulae depending upon the sign of the semi-major axis. For a>0 (elliptical motion)

$$t_p = \sqrt{\frac{3}{\mu}}$$
 (E - e sin E)

$$\cos E = \frac{e + \cos f}{1 + e \cos f} \qquad \sin E = \frac{\sqrt{1 - e^2} \sin f}{1 + e \cos f} \qquad (6)$$

For a < 0 (hyperbolic motion) the time from periapsis is

$$t_{p} = \sqrt{\frac{a^{3}}{\mu}} \text{ (e sinh H - H)}$$

$$\tanh \frac{H}{2} = \sqrt{\frac{e-1}{e+1}} \tan \frac{f}{2} \tag{7}$$

SUBROUTINE ELIPT

PURPOSE: TO CALCULATE TIME FROM PERIAPSIS ON AN ELLIPSE GIVEN TRUE ANAMOLY

ARGUMENT: A I SEMI-MAJOR AXIS OF ELLIPSE IN KM

E I ECCENTRICITY OF ELLIPSE

ORBE I RECIPROCAL OF MEAN ORBITAL RATE IN

SEC/RAD

R I RADIUS IN KM

SNTA I SINE OF TRUE ANAMOLY

TIME O TIME FROM PERIAPSIS IN SEC

SUBROUTINES SUPPORTED: CAREL

LOCAL SYMBOLS: CSE COSINE OF ECCENTRIC ANAMOLY

SNE SINE OF ECCENTRIC ANAMOLY

TWOPI CONSTANT 2.*PI

#### SUBROUTINE EPHEM

PURPOSE: TO COMPUTE THE CARTESIAN STATE OF DESIRED BODIES AT SPECIFIED TIMES ACCORDING TO TWO OPTIONS:

- (1) ECLIPTIC COORDINATES OF ONE BODY RELATIVE TO ITS REFERENCE BODY (SUN FOR PLANETS, EARTH FOR MOON)
- (2) ECLIPTIC COORDINATES OF ALL GRAVITATIONAL BODIES RELATIVE TO THE INERTIAL COORDINATE SYSTEM (EITHER HELIOCENTRIC).

### CALLING SEQUENCES CALL EPHEM(IC,D,N)

ARGUMENT: D I JULIAN DATE OF REFERENCE TIME (REFERENCED 1950)

IC I FLAG SET EQUAL TO 1 FOR OPTION 1 AND TO 0 FOR OPTION 2

N I NUMBER OF GRAVITATIONAL BODIES TO BE COMPUTED

SUBROUTINES SUPPORTED: HELIO LAUNCH LUNTAR MULCON **HULTAR** VMP DATAS PCTM TRAPAR EXCUTE PSIM **GUISS** TRAKS GUISIM PRINT4 PRNTS4 DATA PRINT3 TRAKM GUIDM GUID

SUBROUTINES REQUIRED CENTER

LOCAL SYMBOLS A SEMI-MAJOR AXIS OF LUNAR CONIC

DD ONE TEN-THOUSANDTH TIMES THE INPUT ARGUMENT D FOR COMPUTATIONS IN FN1, FN2

E ECCENTRICITY OF LUNAR CONIC

ECAM ECCENTRIC ANGMALY USED TO SOLVE KEPLER EQUATION

ECC ECCENTRICITY USED TO SOLVE KEPLER EQUATION

EM MEAN ANOMALY OF LUNAR CONIC

E2 E SQUARED

E3 E CUSED

FCTR VELOCITY DIVIDED BY RADIUS

FN1 STATEMENT FUNCTION DEFINING A THIRO ORDER POLYMOMINAL. USED IN COMPUTATION OF MEAN ANOMALY OF INNER PLANETS AND OF MOON

TIONS OF THE OUTER PLANETS INDEX FOR LOGIC CONTROL I IJKL INCREMENT COUNTER IN SOLUTION OF KEPLER EQUATION IN INDEX, ROW OF F OF LUNAR COORDINATES INDEX, ROW OF F OF COORDINATES OF THE IND I-TH PLANET ITEMP INTERMEDIATE VARIABLE USED TO NORMALIZE CONIC ANGLES ITEST INTERNAL CODE WHICH DETERMINES IF COORDINATES OF EARTH ARE BEING CALCULATED IN ORDER TO COMPUTE THOSE OF MOON ITEST2 INTERNAL CODE WHICH DETERMINES IF COORDINATES OF EARTH HAVE BEEN COMPUTED PRIOR TO COMPUTING THOSE OF THE MOON K INDEX USED IN CALCULATION OF HEAN ANOMALY P SEMI-LATUS RECTUM PI2 TWO TIMES THE MATHEMATICAL CONSTANT PI HELIOCENTRIC RADIUS OF PLANET R ARRAY OF TRIGONOMETRIC FUNCTIONS OF TRG SPECIFIED ANGLES VEL VELOCITY OF PLANET WΧ X-COMPONENT OF INTERMEDIATE VECTOR, W WY Y-COMPONENT OF INTERMEDIATE VECTOR, W WZ Z-COMPONENT OF INTERMEDIATE VECTOR, W COMMON COMPUTED/USED: ELMNT F T XΡ COMMON USED® CN EMN IBARY NBODYI NO ONE PMASS ST TWOPI TWO ZERO

STATEMENT FUNCTION DEFINING A FIRST ORDER POLYNOMINAL. USED IN MEAN ANOMALY COMPUTA-

FN2

#### EPHEM Analysis

EPHEM first determines the current value for the mean anomaly of the pertinent body. The mean anomaly M is computed from

$$M = M_0 + M_1 t + M_2 t^2 + M_2 t^3$$
 for inner planets  

$$M = M_0 + M_1 t$$
 for outer planets  

$$M = L_0 + L_1 t + L_2 t^2 + L_3 t^3 - \widetilde{\omega}(t)$$
 for the moon

Kepler's equation M = E - e sin E is then solved iteratively to determine the eccentric anomaly E. The subsequent computations are basic conic manipulations:

$$p = a(1 - e^{2})$$

$$r = a(1 - e \cos E)$$

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$\cos f = \frac{p - r}{er}$$

$$\sin f = \sqrt{1 - \cos^{2}} f \operatorname{sgn}(\sin E)$$

$$\cos \gamma = \sqrt{\frac{\mu p}{rv}}$$

$$\sin \gamma = \sqrt{1 - \cos^{2}} \operatorname{sgn}(\sin E)$$

$$\omega = \widetilde{\omega} - Q$$

The cartesian position and velocity relative to the reference body are then

$$\overline{r} = r_x \stackrel{\wedge}{i} + r_y \stackrel{\wedge}{j} + r_z \stackrel{\wedge}{k}$$

$$r_x = r \cos(\omega + f) \cos \Omega - r \sin(\omega + f) \sin \Omega \cos i$$

$$r_y = r \cos(\omega + f) \sin \Omega + r \sin(\omega + f) \cos \Omega \cos i$$

$$r_z = r \sin(\omega + f) \sin i$$

$$\overline{v} = \frac{v}{r} \left[ (\stackrel{\wedge}{w} \times \overline{r}_r) \cos \gamma + \overline{r} \sin \gamma \right]$$

where  $\hat{\mathbf{w}} = (\sin i \sin \Omega)^{\hat{1}} - (\sin i \cos \Omega)^{\hat{1}} + (\cos i)^{\hat{1}}$ 

1 9/

When option 1 is used, the reference body for all the planets is the sun while the reference body for the moon is the earth.

When option 2 is used with heliocentric inertial coordinates, the cartesian state of the earth is added to the cartesian state of the moon to convert the state of the moon to heliocentric coordinates before storing that state in the F-array.

When option 2 is used with barycentric inertial coordinates, sub-routine CENTER is called to convert all elements to barycentric coordinates before storing in the F-array.

PROGRAM ERRANN

PURPOSE: TO CONTROL THE COMPUTATIONAL FLOW THROUGH THE BASIC CYCLE (MEASUREMENT PROCESSING) AND ALL EVENTS IN THE ERPOR ANALYSIS MODE.

SUBROUTINES SUPPORTED: ERRON

SUBROUTINES REQUIRED: SCHED NTM PSIM DYNO TRAKM MENO GNAVM PRINT3 SETEVN GUIDM MEAN GPRINT

LOCAL SYMBOLS: AY DUMMY VARIABLE

ICL2S TEMPORARY STORAGE FOR ICL2

ICODE EVENT CODE

IPRN MEASUREMENT COUNTER FOR PRINTING

ISP2S TEMPORARY STORAGE FOR ISP2

NEVENT EVENT COUNTER

SPHERS TEMPORARY STORAGE FOR SPHERE (NTP)

TRTM2 TIME OF THE MEASUREMENT

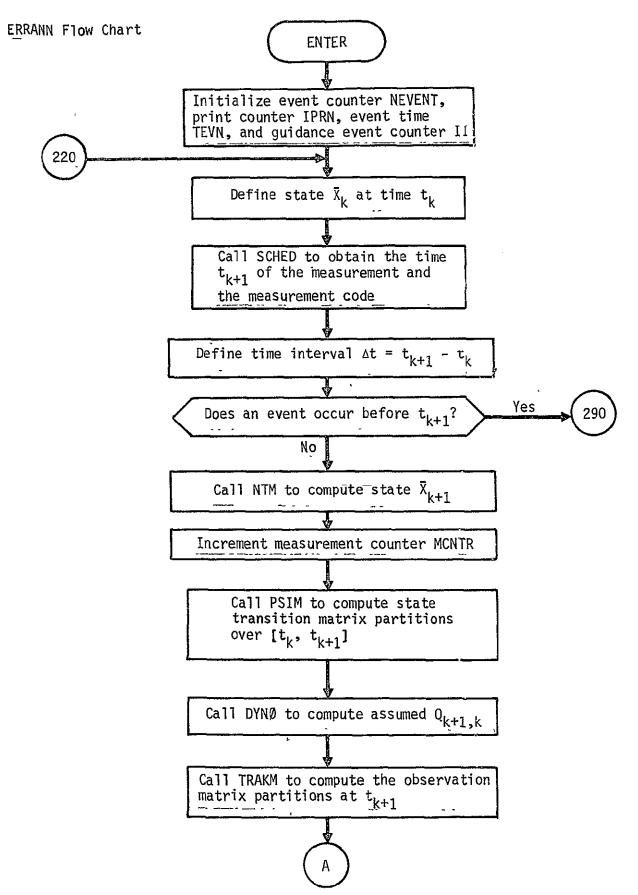
COMMON COMPUTED/USED: ICODE MCNTR RI TEVN TRTM1 XF ΧI COMMON COMPUTED: TLAST DELTM SPHERE ICL 2 ISP2 XPHI ALFA DELT ABW COMMON USED: **FNTM** IEVNT **IPRINT** ISTMC NEV NMN NR NTMC RF TEV UCNTRL IUTC KKWIT NOGEN T7

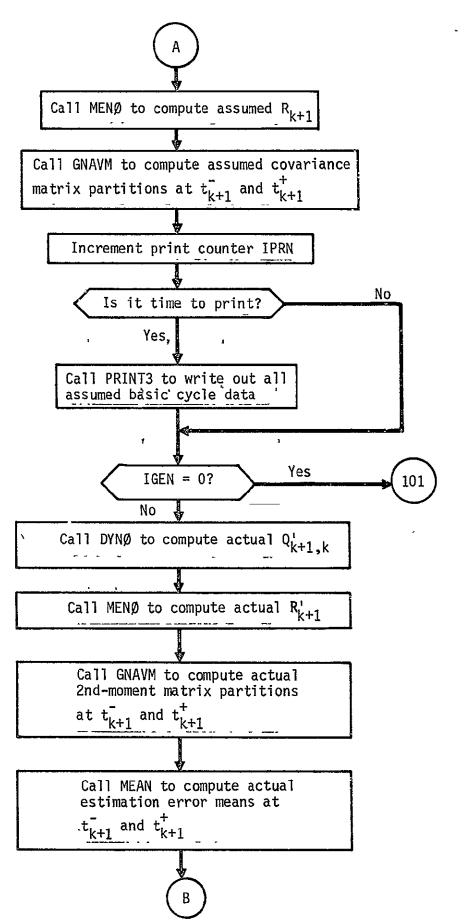
## ERRANN Analysis

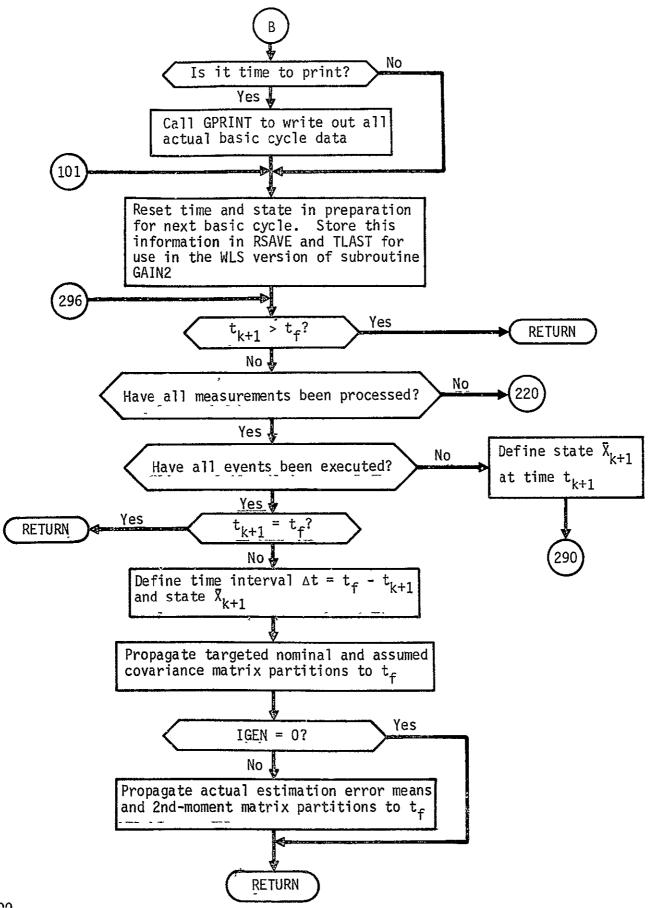
Subroutine ERRANN controls the computational flow through the basic cycle (measurement processing) and all events in the error analysis/generalized covariance analysis program.

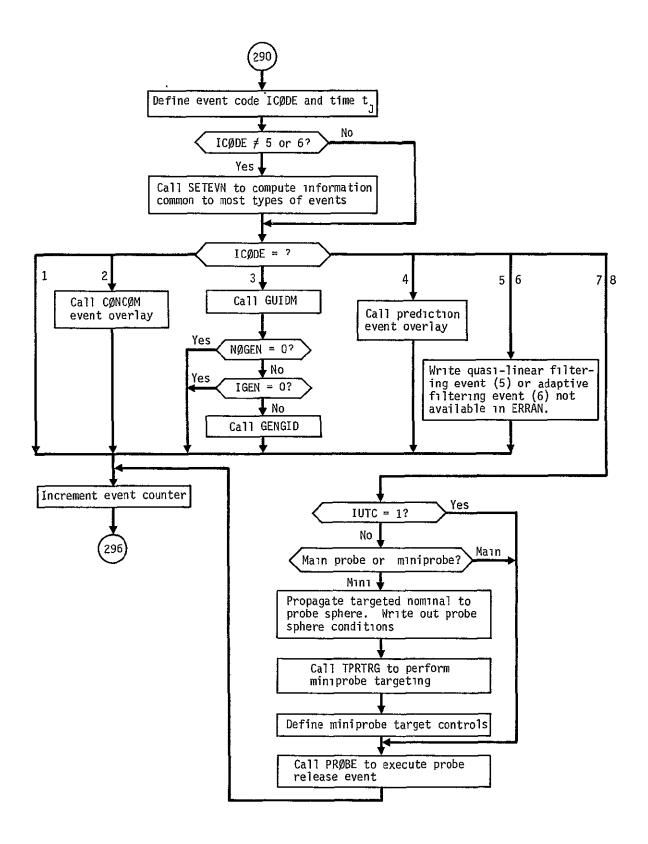
In the basic cycle the first task of ERRANN is to control the generation of the targeted nominal spacecraft state  $\overline{X}_{k+1}$  at time  $t_{k+1}$ , given the state  $\overline{X}_k$  at time  $t_k$ . Then calling PSIM, DYNØ, TRAKM, and MENØ, successively, ERRANN controls the computation of all matrix information required by subroutine GNAVM to compute the actual and assumed knowledge covariance matrix partitions at time  $t_{k+1}^+$  immediately following the measurement.

At an event, ERRANN simply calls the proper event subroutine or overlay where all required computations are performed. Subroutine ERRANN also controls miniprobe targeting in the error analysis program.









PROGRAM ERRON

PURPOSE: TO CONTROL THE ERROR ANALYSIS OVERLAY SCHEHE

SUBROUTINES SUPPORTED: NONE

SUBROUTINES REQUIRED: DATA ERRAN PRNTS3

LOCAL SYMBOLS: IRUNX TOTAL NUMBER OF DATA CASES

IRUN DATA CASE COUNTER

SUBROUTINE ESTAT

PURPOSE: TO UPDATE THE FINAL VALUES OF THE PRECEDING COMPUTATION INTERVAL WHICH SERVE AS INITIAL VALUES FOR THE NEW STEP, TO DETERMINE THE DESIRED SIZE OF THE NEXT TIME INCREMENT ON THE BASIS OF TRUE ANOMALY OR REQUESTED PRINTTINE, AND TO ESTIMATE THE FINAL POSITION AND MAGNITUDE OF THE VIRTUAL MASS.

CALLING SEQUENCE: CALL ESTMT(D1, DELTM, TRTM)

ARGUMENTS D1 I JULIAN DATE, EPOCH 1900, OF THE INITUAL TRAJECTORY TIME

DELTM I TIME INTERVAL OVER WHICH THE TRAJECTORY WILL BE PROPAGATED (DAYS)

TRTM I INITIAL TRAJECTORY TIME (DAYS) REFERENCED TO INJECTION

SUBROUTINES SUPPORTED: VMP

SUBROUTINES REQUIRED: NONE

COMMON COMPUTED/USED: INCMNT V

COMMON COMPUTED: ITRAT KOUNT

#### ESTMT Analysis

The initial values of the state variables are first set equal to the values at the end of the previous interval. The nominal time interval to be used during the current step is computed from

$$\Delta t_{k} = \frac{c_{2} r_{VS_{B}}}{v_{VS_{B}}}$$
 (1)

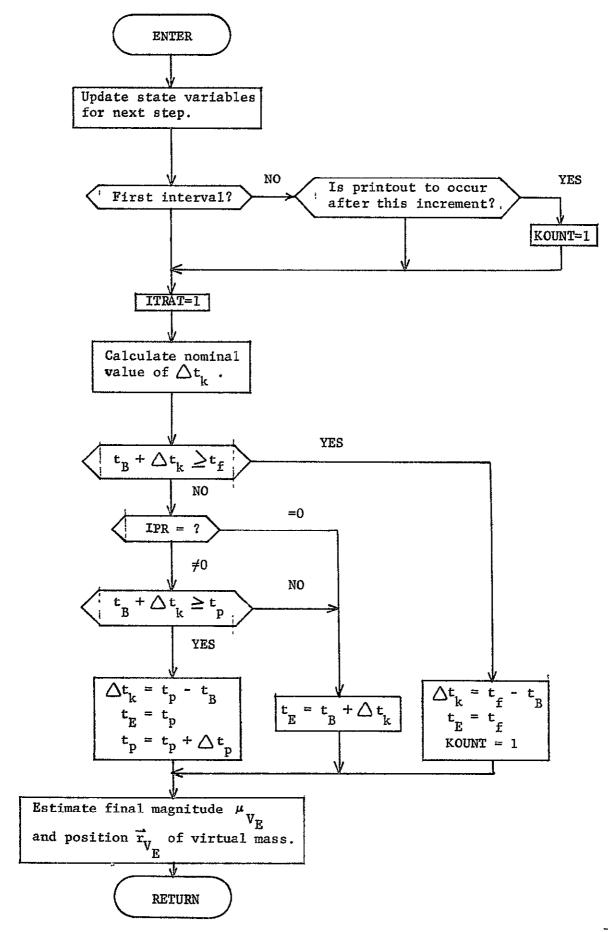
where  $\epsilon_2$  is the constant input true anomaly increment relative to the virtual mass trajectory.

The time interval to the final time  $t_f$  or to the next time printout  $t_p$  is computed and the current time interval  $\Delta t$  is adjusted if necessary.

Finally the virtual mass final position and magnitude are estimated by the expansions

$$\mu_{V_{E}} = \mu_{V_{B}} + \dot{\mu}_{V_{B}} \triangle t + \ddot{\mu}_{V} \triangle t^{2}$$

$$\vec{r}_{V_{E}} = \vec{r}_{V_{B}} + \dot{\vec{r}}_{V_{B}} \triangle t + \ddot{\vec{r}}_{V_{av}} \triangle t^{2}$$
(2)



SUBROUTINE EULMX

PURPOSE: TO COMPUTE THE MATRIX REQUIRED TO DEFINE TRANSFORMATIONS FROM ONE COORDINATE SYSTEM TO ANOTHER.

CALLING SEQUENCE: CALL EULMX (ALP:NN, BET, MM, GAM, LL, P)

ARGUMENT8 ALP I FIRST ROTATION ANGLE (RADIANS)

NN I FIRST AXIS OF ROTATION

BET I SECOND ROTATION ANGLE (RADIANS)

MM I SECOND AXIS OF ROTATION

GAM I THIRD ROTATION ANGLE (RADIANS)

LL I THIRD AXIS OF ROTATION

P(3,3) 0 TRANSFORMATION MATRIX

SUBROUTINES SUPPORTED & PECEQ

SUBROUTINES REQUIRED: NONE

LOCAL SYMBOLS: A INTERMEDIATE ROTATION MATRIX

ALPHA TEMPORARY LOCATION FOR EACH OF THE ROTATION ANGLES: ALP, BET, AND GAM

D INTERMEDIATE PRODUCT MATRIX

F TRANSFORMATION MATRIX FOR ANGLE ALP

G TRANSFORMATION MATRIX FOR ANGLE BET

H TRANSFORMATION MATRIX FOR ANGLE GAM

N COUNTER SHOWING NUMBER OF COORDINATE AXES FOR WHICH CALCULATIONS REMAIN

NAXIS TEMPORARY LOCATION FOR EACH OF THE AXES

OF ROTATION: NN, MM, AND LL

COMMON USED: ONE ZERO

SUBROUTENE EXCUT

PURPOSE CONTROL EXECUTION OF A VELOCITY CORRECTION MODELED AS AN IMPULSE SERIES IN THE ERROR ANALYSIS PROGRAM

CALLING SEQUENCES CALL EXCUT

SUBROUTINES SUPPORTED: GUIDM

SUBROUTINES REQUIRED: PREPUL PULCOV PULSEX

COMMON COMPUTED/USED: XXIN

COMMON COMPUTEDS OK

COMMON USED: DELPX, DIPX, TM, INPX

SUBROUTINE EXCUTE

PURPOSE: TO CONTROL THE ACTUAL EXECUTION OF THE VELOCITY INCREMENT DELTAY.

CALLING SEQUENCE: CALL EXCUTE

SUBROUTINES SUPPORTED: GIDANS

SUBROUTINES REQUIRED: PREPUL PULSEX CAREL PECEQ

LOCAL SYMBOLS: A SEMIMAJOR AXIS OF DOMINANT BODY CONIC

DVM HAGNITUDE OF VELOCITY INCREMENT

E ECCENTRICITY OF DOMINANT BODY CONIC

INDEX CODE OF BODY BEING TESTED FOR DOMINANT

BODY

IND INDEX OR CODE OF DOMINANT BODY

ISUN SUN VALUE OF IND

I INDEX

JX INDEX OF S/C-REL-TO-BODY ROW OF F-ARRAY

MODEL EXECUTION HODEL (1=IMPULSIVE, 2=PULSE ARC)

PP UNIT VECTOR TO PERIAPSIS IN ORBITAL PLANE

QQ UNIT VECTOR NORMAL TO PP IN ORBITAL PLANE

RN POSITION AND VELOCITY OF S/C AT END OF

EXECUTION BY PULSING ARC

RSI POSITION VECTOR OF S/C RELATIVE TO DOMINANT BODY AT EXECUTION TIME

1

RTB RADIUS MAGNITUDE TO BODY BEING TESTED FOR

DOMINANT BODY

TA TRUE ANOMALY ON DOMINANT BODY CONIC

TFP TIME FROM PERIAPSIS ON DOMINANT BODY CONIC

VSI VELOCITY VECTOR OF S/C RELATIVE TO DOMINANT BODY AT EXECUTION TIME

WH UNIT NORMAL TO ORBITAL PLANE

W ARGUMENT OF PERIAPSIS OF DOMINANT BODY

CONIC

XI INCLINATION OF DOMINANT BODY CONIC

XMU GRAVITIONAL CONSTANT OF DOMINANT BODY

XN LONGITUDE OF ASCENDING NODE OF DOMINANT

BODY

COMMON COMPUTED/USED: DELTAV D1 RIN TRTM

COMMON COMPUTED: KTIM

COMMON USED: ALNGTH DELV F KUR MDL NBOD NB PHASS PULT SPHERE

TM THO V

#### EXCUTE Analysis

EXCUTE is the executive subroutine controlling the actual execution of the velocity increment  $\Delta v$ . The  $\Delta v$  is computed by TARGET or INSERS or read in by the user.

Before executing the correction EXCUTE computes peripheral information of interest to the user. It first determines the dominant body acting on the spacecraft. If the spacecraft is in the moon's SOI (with respect to the earth), the moon is the dominant body. If not in the moon's SOI but in any of the planets' SOI (with respect to the sun) that planet is the dominant body. Otherwise the sun is the dominant body.

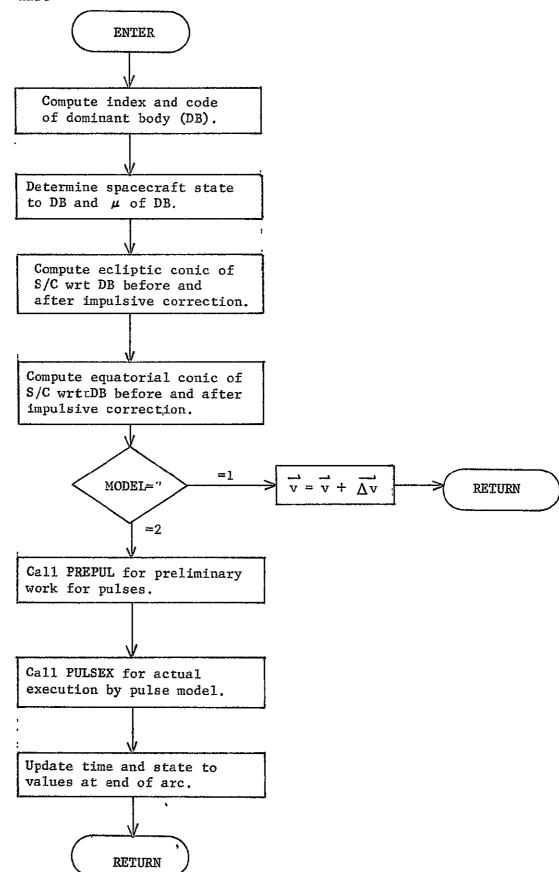
Having determined the dominant body EXCUTE computes the state of the space-craft relative to that body. It then computes the conic elements of the trajectory both before and after an impulsive addition of the  $\overline{\Delta v}$  in ecliptic coordinates.

If the dominant body is not the sun, it makes the same computations in equatorial coordinates.

EXCUTE then operates on the current value MODEL of the array MDL. If MODEL = 1, the impulsive model of execution is commanded. The  $\Delta v$  is therefore added to the current inertial ecliptic velocity before peturning to GIDANS.

If MODEL = 2, the pulsing arc model of execution is required. PREPUL is called to perform the preliminary work needed for the pulsing arc. PULSEX then actually propagates the trajectory through the series of pulses. At the completion of the arc EXCUTE updates the time and inertial ecliptic state (both position and velocity) of the nominal trajectory to the state determined by PULSEX.

#### EXCUTE Flow Chart



SUBROUTINE EXCUTS

PURPOSE CONTROL EXECUTION OF A VELOCITY CORRECTION MODELED AS AN IMPULSE SERIES IN THE SIMULATION PROGRAM

CALLING SEQUENCE: CALL EXCUTS

SUBROUTINES SUPPORTED: GUISIM

SUBROUTINES REQUIRED: PREPUL, PULCOV, PULSEX

LOCAL SYMBOLS RN EFFECTIVE SPACECRAFT STATE AFTER A

VELOCITY CORRECTION MODELED AS AN IMPULSE

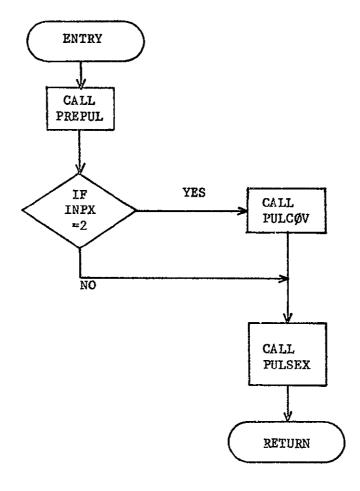
SERIES

-COMMON COMPUTED/USED: XXIN

COMMON COMPUTED: QK

COMMON USED8 DELPX, DIPX, TM, INPX

## EXCUTS Flow Chart



FUNCTION F1

PURPOSE: TO CALCULATE SQUARE OF LENGTH OF CONSTRAINT VECTOR AFTER TAKING STEP OF GIVEN LENTH AND IN GIVEN DIRECTION FROM CURRENT CONTROL VECTOR

ARGUMENT: ALPHA I STEP LENGTH IN SEARCH DIRECTION FPHI FUNCTION RELATING CONSTRAINT VECTOR TO I CONTROL VECTOR DIMENSION OF CONSTRAINT VECTOR I M I CONTROL VECTOR N CURRENT CONSTRAINT VECTOR PHI I Ρ Ι UNIT VECTOR IN SEARCH DIRECTION I CURRENT CONTROL VECTOR X

SUBROUTINES SUPPORTED'S GAUSES

SUBROUTINES REQUIRED: FPHI

LOCAL SYMBOLS: F1 SQUARE OF LENGTH OF CONSTRAINT VECTOR AFTER TAKING PRESCRIBED STEP

XALPHA VECTOR OBTAINED FROM ORIGINAL CONTROL VECTOR BY TAKING STEP OF LENGTH ALPHA IN DIRECTION P

### SUBROUTINE FLITE

PURPOSE: TO SOLVE THE TIME OF FLIGHT EQUATION (LAMBERT-S THEOREM) USING BATTIN-S UNIVERSAL EQUATION FORMULATION.

CALLING SEQUENCE: CALL FLITE(R1, R2, THETA, GM, TF, A, E, K)

ARGUMENTS R1 I INITIAL RADIUS

R2 I FINAL RADIUS

THETA I CENTRAL ANGLE

GM 'I GRAVITIONAL CONSTANT

TF I TIME OF FLIGHT

A O SEMIMAJOR AXIS

E O ECCENTRICITY

K 0 ERROR CODE =0 NO ERROR =1 ERROR

SUBROUTINES SUPPORTED: HELIO

SUBROUTINES REQUIRED: NONE

LOCAL SYMBOLS: AMISS ERROR IN ITERATE

BIGNO CONSTANT = E+25

B1 CONSTANT = S 3/2

CHECK ERROR IN ITERATE

CX BATTIN C-FUNCTION OF X

CY BATTIN C-FUNCTION OF Y

C CHORD LENGTH

DEM INTERMEDIATE VARIABLE

P SEHILATUS RECTUM

ROOT INTERMEDIATE VARIABLE

SLOP VALUE OF DERIATIVE OF T(X)

SX BATTIN S-FUNCTION OF X

SY BATTIN S-FUNCTION OF Y

S1 SEMIPERIMETER

S =INTERMEDIATE VARIABLE (=1-C/S1)

TIME FLIGHT TIME CORRESPONDING TO ITERATE X

T NORMALIZED TIME OF FLIGHT

U FLAG SET TO 1 IF X LESS THAN PI 2,-1 ELSE

VB1 INTERMEDIATE VARIABLE

V FLAG SET TO 1 FOR TYPE I,-1 FOR TYPE II

X1 STARTING VALUE FOR X

X VARIABLE INTRODUCED TO REPLACE A

Y INTERMEDIATE VARIABLE AS FUNCTION OF X

FLITE Analysis

FLITE solves the time of flight equation (Lambert's theorem) using Battin's universal equation formulation. Stated functionally Lambert's theorem states that the time of flight  $t_{\rm f}$  is a function

$$t_f = t_f (r_1 + r_2, c, a)$$
 (1)

solely of the sum  $r_1+r_2$  of the distances of the initial and final points of the trajectory from the central body, the length c of the chord joining these points, and the length of the semimajor axis a of the trajectory. Usually the time of flight is known and it is desired to solve for the semimajor axis. The standard formulation involves different equations for the elliptic, parabolic, and hyperbolic cases, all of which then iterate on a to determine the solution.

In Battin's approach the semimajor axis a is replaced by a new variable x. By further introducing two new transcendental functions S(x) and C(x), the special cases of the flight-time equation are combined into one single, better behaved formula. The functions S(x) and C(x) are defined by

$$S(x) = \frac{\sqrt{x} - \sin \sqrt{x}}{3} \qquad C(x) = \frac{1 - \cos \sqrt{x}}{x} \qquad x > 0$$

$$= \frac{\sinh \sqrt{-x} - \sqrt{-x}}{\sqrt{x}} \qquad = \frac{\cosh \sqrt{-x} - 1}{-x} \qquad x < 0$$

$$= \frac{1}{6} \qquad = \frac{1}{2} \qquad x = 0 \qquad (2)$$

A parameter Q is introduced as

$$Q = \frac{s = c}{s}$$
where
$$c = (r_1^2 + r_2^2 - 2r_1 \cos \theta)^{\frac{1}{2}}$$

$$s = \frac{1}{2}(r_1 + r_2 + c)$$
(3)

The universal flight-time formula is

$$T = \frac{S(x)}{c^{3/2}(x)} = \frac{1}{c^{3/2}} \frac{S(y)}{c^{3/2}(y)}$$

$$yC(y) = Q \times C(x) \tag{4}$$

 $T = \sqrt{\frac{\mu}{3}} t_f$ . The choice of the upper or lower sign is

made according to whether the transfer angle  $\theta$  is less or greater than 180° respectively.

The development of equations (4) is too long and complex to be given here. It may be obtained from the first reference listed below. The following steps of that reference are noted:

- (1) the two body problem on pp. 15,16

- (2) the "vis viva" equation and Kepler's equation on pp. 50,51
  (3) Lambert's theorem proved from Kepler's equation on p. 71
  (4) the basic flight-time formula and detailed analysis on pp. 72-78
- (5) The universal formulation on pp. 80,81.

Instead of using the equations (4) the authors of refence 2 (listed below) determined y as a function of x as

$$y = 4 \arcsin^{2} \sqrt{\frac{xsC(x)}{2}} \qquad x \ge 0$$

$$= -4 \ln^{2} \left\{ \sqrt{\frac{xsC(x)}{2}} + \sqrt{\frac{xsC(x)}{2}} + 1 \right\}^{\frac{1}{2}} \qquad x < 0 \qquad (5)$$

Therefore a single variable iteration is possible. Newton's method is used to solve (4a) given T and Q as

$$x = x - \frac{T(x_n) - T}{T'(x_n)}$$
 (6)

where 
$$T(x) = \frac{S(x)}{c^{3/2}(x)} + Q^{3/2} \frac{S(y)}{c^{3/2}(y)}$$
 (7).

$$T'(x) = \frac{1 + k \left[ -\frac{3}{2} - 1.5 \sqrt{2 - yC(y)} \right]}{2x \sqrt{C(x)}}$$
 (8)

$$k = sgn (\pi^2 - x) \sqrt{\frac{2-xC(x)}{2-yC(y)}},$$
 (9)

As  $|2-yC(y)| \longrightarrow 0$ ,  $k \longrightarrow 1$ . Therefore if  $|2-yC(y)| < 10^{-4}$  k is set to 1. Also T'(x) breaks down as  $x \longrightarrow 0$ . Therefore the approximation is used:

$$T'(x) = \frac{1 \mp q^{3/2}}{2\pi^2} \qquad |x| < 10^{-6} \qquad (10)$$

The starting value for x is given by  $x = x_1 - \Delta x(T,Q)$  where

$$x_1 = 82.1678 + 352.8045 T$$
- (123954.8504  $T^2 + 43904.0083 T + 13423.6819$ )

$$\Delta x(T,Q) = \mp \left(\frac{2.36}{T^2 + .15} + \frac{3.1}{T + .1}\right) \quad (0.3 \ Q^2 + 0.7 \ Q) \tag{11}$$

To insure that the routine will not fail for large or small values of T certain restrictions on T are built into the program. The nominal value of T is forced to be no larger than 950,000 and no smaller than  $10^{-6}$ . This forces the corresponding limits for x of -823.0473  $\leq$  x  $\leq$  39.14553.

Finally convergence is achieved when  $T(x_n) - T < \frac{T}{100000}$ .

Having solved for semimajor axis a , the semilatus rectum p is given by

$$p = \frac{1}{2} \left\{ \frac{r_1 r_2 \sin \theta}{c} \sqrt{\frac{1}{s-c} - \frac{1}{2a}} \pm sgn(t_m - t) \sqrt{\frac{1}{s} - \frac{1}{2a}} \right\}^2$$
 (12)

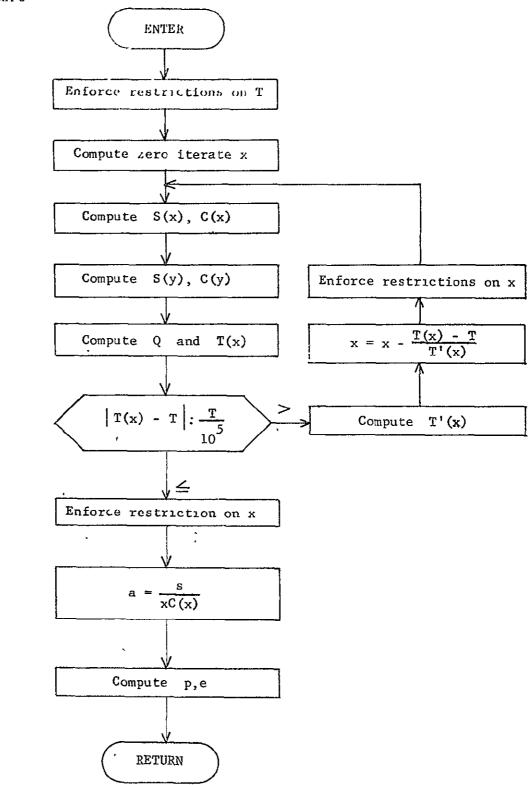
Then the eccentricity e is given by

$$e = 1 - \frac{p}{a}$$
 (13)

References:

- (1) Battin, R. H., Astronomical Guidance, McGraw Hill Book Co., Inc., New York, 1964.
- (2) Lesh, H. F/, and Travis, C., FLIGHT: a Subroutine to Solve the Flight Time Problem, JPL Space Programs Summary 37-53, Vol. II.

FLITE Flow Chart



SUBROUTINE GAIN1

PURPOSE: TO COMPUTE THE KALMAN GAIN MATRICES

CALLING SEQUENCE: CALL GAIN1 (NR, AJ, AKW, SW, IEND)

ARGUMENTS: NR I NUMBER OF ROWS IN THE OBSERVATION MATRIX

AJ I MEASUREMENT RESIDUAL COVARIANCE AND ITS

INVERSE

AKW I INTERMEDIATE ARRAY

SW I INTERMEDIATE ARRAY

IEND I NR-1

SUBROUTINES SUPPORTED: GNAVM

SUBROUTINES REQUIRED: MATIN

LOCAL SYMBOLS: DUM INTERMEDIATE VECTOR

XJ INTERMEDIATE ARRAY

SUM INTERMEDIATE VARIABLE

COMMON COMPUTED: AK S

COMMON USED: ONE HALF ZERO

## GAIN1 Analysis

Subroutine GAIN1 computes the Kalman-Schmidt filter gain matrices  ${\rm K}_{k+1}$  and  ${\rm S}_{k+1}$  that are used in subroutines GNAVM and NAVM to update estimation error covariance matrices after a measurement has been processed.

The measurement residual covariance matrix  $J_{k+1}$  and the auxiliary matrices  $A_{k+1}$  and  $B_{k+1}$  are assumed to be available (from GNAVM or NAVM) when GAIN1 is called. Subroutine GAIN1 then evaluates the following equations to determine the filter gain matrices:

$$K_{k+1} = A_{k+1}^{:} J_{k+1}^{-1}$$
 (1)

$$S_{k+1} = B_{k+1} J_{k+1}^{-1}$$
 (2)

SUBROUTINE GAIN2

PURPOSE: TO COMPUTE THE GAIN MATRICES FOR THE EQUIVALENT RECURSIVE WEIGHTED LEAST SQUARES CONSIDER FILTER

CALLING SEQUENCE: CALL GAINZ (NR)

ARGUMENT: NR I NUMBER OF POWS IN OBSERVATION MATRIX

SUBROUTINES SUPPORTED: NAVM GNAVM

SUBROUTINES REQUIRED: PSIM DYNO MATIN

LOCAL SYMBOLS: AJ MEASUREMENT RESIDUAL COVARIANCE MATRIX AND ITS INVERSE

AKW INTERMEDIATE VARIABLE

DELTMS INTERMEDIATE STORAGE FOR DELTM

DUM INTERMEDIATE VECTOR

IEND NR-1

IFLAG =0 IF STATE TRANSITION MATRICES USED ARE IDENTICAL TO STM FOR THE MEASUREMENT =1 IF STM ARE RECALCULATED TO CORRESPOND TO THE DT BETWEEN THE LAST MEASUREMENT AND THE PRESENT MEASUREMENT

'NDIM2'S TEMPORARY STORAGE FOR NDIM2

NDIM3S TEMPORARY STORAGE FOR NDIM3

PHISV TEMPORARY STORAGE FOR PHI

PSAVE INTERMEDIATE ARRAY

SUM INTERMEDIATE VARIABLE

SW INTERMEDIATE ARRAY

TRTS INTERMEDIATE STORAGE FOR TRTM1

TXUSV INTERMEDIATE STORAGE FOR TXU ARRAY

TXXSSV INTERMEDIATE STORAGE FOR TXXS ARRAY

COMMON COMPUTED/USED: AΚ CMIN CPLU DELTM PHI PMIN PPLU **PSMIN PSPLU** TXXS COMMON USED: AM CXXSG II MCNTR Н NOIM1 NDIM2 NDIM3 ONE PG

PSG RF RSAVE S TEVN TLAST TRTM1 TXU ZERO

## GAIN2 Analysis

Subroutine GAIN2 computes filter gain matrices  $K_{k+1}$  and  $K_{k+1}$  for an equivalent recursive weighted-least-squares (WLS) consider filter. The equations required to compute  $K_{k+1}$  and  $K_{k+1}$  are identical to those used to compute the Kalman filter gains, but with all consider parameter covariances removed.

Subroutine GAIN2 propagates and updates (at a measurement) a set of covariance matrix partitions that are completely independent of those processed in subroutines NAVM or GNAVM for the sole purpose of generating filter gain matrices  $K_{k+1}$  and  $S_{k+1}$ .

The propagation and update equations employed in GAIN2, which are a subset of the NAVM and GNAVM propagation and update equations, are summarized below. For definitions of all matrices, see either the subroutine NAVM or GNAVM analysis section.

Propagation equations:

$$P_{k+1}^{-} = \left( \Phi P_{k}^{+} + \theta_{xx_{s}} C_{xx_{s_{k}}}^{+^{T}} \right) \Phi^{+} + C_{xx_{s_{k+1}}}^{-} \theta_{xx_{s}}^{T} + Q_{k}$$
 (1)

$$C_{xx_{s_{k+1}}}^{-} = \Phi C_{xx_{s_k}}^{+} + \theta_{xx_{s}} P_{s_k}^{+}$$
(2)

$$p_{s_{k+1}}^- = p_{s_k}^+$$
 (3)

Gain equations:

$$A_{k+1} = P_{k+1}^{-} H_{k+1}^{T} + C_{xx}^{-} M_{k+1}^{T}$$

$$(4)$$

$$B_{k+1} = P_{s_{k+1}}^{-} M_{k+1}^{T} + C_{xx_{s_{k+1}}}^{-} H_{k+1}^{T}$$
(5)

$$J_{k+1} = H_{k+1} A_{k+1} + M_{k+1} B_{k+1} + R_{k+1}$$
 (6)

GAIN2-2

$$K_{k+1} = A_{k+1} J_{k+1}^{-1}$$
 (7)

$$S_{k+1} = B_{k+1} J_{k+1}^{-1}$$
 (8)

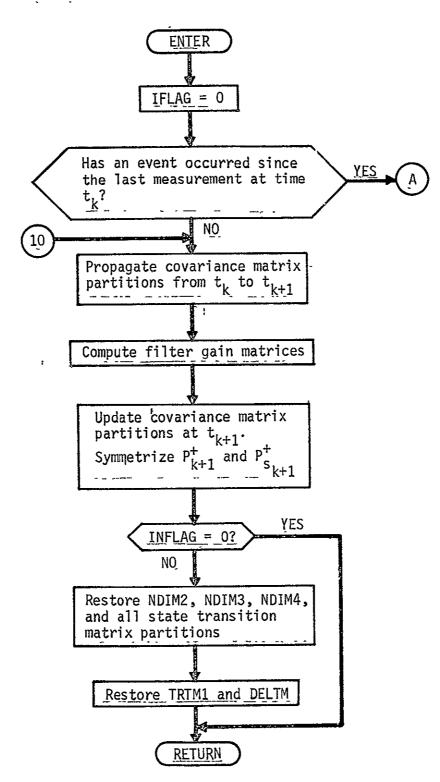
Update equations:

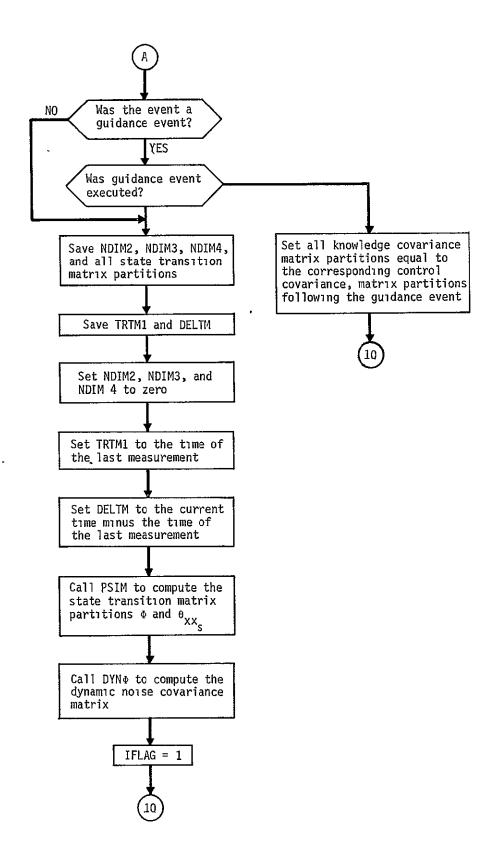
$$P_{k+1}^{+} = P_{k+1}^{-} - K_{k+1} A_{k+1}^{T}$$
(9)

$$C_{xx_{s_{k+1}}}^{+} = C_{xx_{s_{k+1}}}^{-} - K_{k+1} B_{k+1}^{T}$$
 (10)

$$P_{s_{k+1}}^{+} = P_{s_{k+1}}^{-} - S_{k+1} B_{k+1}^{T}$$
(11)

# GAIN2 Flow Chart





#### SUBROUTINE GAUSLS

PURPOSE: TO FIND CONTROL VECTOR OF DIMENSION N WHICH MINIMIZES SQUARE OF LENGTH OF CONSTRAINT VECTOR OF DIMENSION M

- ARGUMENT: C1 I WEIGHTING FACTOR. APPLIED TO CHANGE IN LENGTH OF CONTROL VECTOR IN CONVERGENCE CRITERION
  - C2 I WEIGHTING FACTOR APPLIED TO CHANGE IN MAGNITUDE OF MISS INDEX IN CONVERGENCE CRITERION
  - DELTA I COMMON PERTURBATION SIZE APPLIED TO ALL CONTROL COMPONENTS IN APPROXIMATING JACOBIAN SENSITIVITY MATRIX BY DIVIDED DIFFERENCES
  - EPS I UPPER BOUND ON WEIGHTED SUM OF CHANGES IN CONTROL VECTOR AND IN MISS INDEX FOR CONVERGENCE
  - FPHI I FUNCTION RELATING CONSTRAINT VECTOR TO CONTROL VECTOR
  - ICONV1 O FLAG INDICATING WHETHER OR NOT CONVERGENCE OCCURRED =1 CONVERGENCE := 2 NO CONVERGENCE
  - ITLIM I UPPER BOUND ON NUMBER OF PERMISSIBLE ITERATIONS BEFORE ALGORITHM IS STOPPED
  - M I NUMBER OF CONSTRAINTS(DIMENSION OF CONSTRAINT VECTOR)
  - N I NUMBER OF CONTROLS(DIMENSION OF CONTROL VECTOR)
  - PHI I CONSTRAINT VECTOR
  - I MAXIMUM PERMISSIBLE LENGTH OF PSEUDO-INVERSE CONTROL STEP (LARGER STEPS ARE REPLACED BY STEEPEST DESCENT CORRECTION)
  - X I/O CONTROL VECTOR
  - YM O FINAL MINIMUM VALUE OF MISS INDEX

SUBROUTINES SUPPORTED: TPRTRG

SUBROUTINES REQUIRED: FPHI JACOB MATIN THPOSM

LOCAL SYMBOLS: ALPAH PRODUCT OF ALPHA AND AMBOA

ALPHA FACTOR DETERMINING LOCATION OF INTERMEDIATE FUNCTION EVALUATION ON INTERVAL FROM 0. TO AMBDA FOR CUBIC INTERPOLATION

AMBDA LENGTH OF INTERVAL USED IN CUBIC INTER-POLATION TO FIND MINIMUM IN SEARCH DIRECTION

DXMS MAGNITUDE SQUARED OF CONTROL CORRECTION

DXM MAGNITUDE OF CONTROL CORRECTION

GRADYM MAGNITUDE OF GRADIENT OF MISS INDEX

GRADY GRADIENT OF MISS INDEX

HALF CONSTANT 0.5

ILOC INDEX USED IN TRANSFORMING A SQUARE MATRIX INTO A COLUMN VECTOR

ILS FLAG INDICATING WHETHER PREVIOUS STEP WAS PSEUDO-INVERSE OR STEEPEST DESCENT CONTROL CORRECTION

=1 PSEUDO-INVERSE

=2 STEEPEST DESCENT

ITM1 CURRENT ITERATION NUMBER LESS ONE

1TP1 CURRENT ITERATION NUMBER PLUS ONE

IT CURRENT ITERATION NUMBER

IYC FLAG INDICATING WHETHER OR NOT MISS INDEX WAS CALCULATED FROM LAST CONTROL CORRECTION

=1 MISS INDEX CALCULATED

=2 MISS INDEX NOT CALCULATED

PROJM PSEUDO-INVERSE MATRIX

PSDIJ INVERSE OF PRODUCT OF JACOBIAN MATRIX BY
JACOBIAN MATRIX ITSELF

PSDJ PRODUCT OF TRANSPOSE OF JACOBIAN MATRIX BY JACOBIAN MATRIX ITSELF

RECORD ARRAY CONTAINING CONTROL VECTORS, MISS INDICES, AND GRADIENTS OF MISS INDICES FOR ALL ITERATIONS

RJCBM JACOBIAN SENSITIVITY MATRIX OF CONSTRAINT VECTOR WITH RESPECT TO CONTROL VECTOR

TEMPV COLUMN VECTOR REPRESENTATION OF SQUARE MATRIX PSDJ

TEMPW COLUMN VECTOR REPRESENTATION OF SQUARE MATRIX PSDIJ

TWO CONSTANT 2.0

WINCS WEIGHTED SUM OF CHANGE IN LENGTH OF CONTROL VECTOR AND MAGNITUDE MISS INDEX USED IN CONVERGENCE CRITERION

XMIN LENGTH OF STEP WHICH MINIMIZES MISS INDEX IN SEARCH DIRECTION

YALAM VALUE OF MISS INDEX AFTER STEP OF LENGTH ALPHA TIMES LAMBDA IN SEARCH DIRECTION

YLAM VALUE OF MISS INDEX AFTER STEP OF LAMBDA IN SEARCH DIRECTION

YP VALUE OF MISS INDEX ON PREVIOUS ITERATE

Y CURRENT VALUE OF MISS INDEX

ZERO CONSTANT 0.

GAUSLS Analysis

GAUSLS is a hybrid algorithm to obtain a least-squares solution to the system

$$\phi(x) = 0 \tag{1}$$

where  $\underline{x}$  is an n-dimensional control vector,  $\underline{\phi}$  is an m-dimensional constraint vector, and  $\underline{m} \geq n$ . Current array dimensions in GAUSLS require that  $n \leq 5$  and  $\underline{m} \leq 10$ . By least-squares we mean that the square of the standard Euclidean norm of  $\phi$ , namely

$$\left| \left| \underline{\phi} \right| \right|^2 = \underline{\phi}^{\mathrm{T}} \underline{\phi} \quad , \tag{2}$$

is minimized. The principal algorithm used is the well-known pseudo-inverse scheme originally due to Gauss. When, however, the dependence of  $\phi$  on  $\underline{x}$  deviates substantially from the approximate linearity tactily assumed by the Gauss method, a best-step steepest descent algorithm is invoked. Either of two indications of nonlinearity can cause GAUSLS to transfer from the normal pseudo-inverse mode to best-step steepest descent technique: (1) the Gauss control correction is larger in norm than an input upper bound ,  $s_0$ , or (2) the Gauss step actually increases the miss index,  $||\phi||^2$ , over the previous iterate.

The Gauss procedure can readily be derived since it is simply the exact one-step solution to equation (1) when  $\phi$  depends on  $\chi$  linearly. Let J represent the Jacobian or sensitivity matrix of  $\phi$  with respect to  $\chi$ ; that is

$$J_{ij} = \frac{\partial \phi_{i}}{\partial x_{j}} \qquad j=1,...,n \qquad . \tag{3}$$

$$m \ge n$$

Next let y denote the least-squares miss index; that is

$$y = ||\phi||^2 \tag{4}$$

Then the gradient of the miss-index is simply

$$\nabla y = 2 J^{T} \phi \tag{5}$$

Now a necessary condition for the miss-index to be minimized after a control correction of  $\Delta x$  is

$$\underline{\nabla}y \ (\underline{x} + \Delta\underline{x}) = \underline{0} \qquad . \tag{6}$$

Substituting equation (5) into (6) gives

$$2 J^{T} (\underline{x} + \Delta \underline{x}) \cdot \underline{\phi} (\underline{x} + \underline{\Delta} \underline{x}) = \underline{0} . \tag{7}$$

Assuming J either constant or approximately so and using the first two terms of the Taylor's series for  $\phi$  yields the approximation

$$2 JT [\phi(x) + J\Delta x] = 0 .$$
 (8)

Solving for the control correction then yields the pseudo-inverse control correction

$$\Delta \mathbf{x} = -(\mathbf{J}^{\mathrm{T}}\mathbf{J})^{-1} \ \mathbf{J}^{\mathrm{T}} \underline{\phi}(\underline{\mathbf{x}}) \qquad . \tag{9}$$

Clearly equation (9) is exact if  $\dot{\phi}$  is a linear function of  $\underline{x}$  so that the Taylor series of  $\dot{\phi}$  ( $\underline{x} + \Delta \underline{x}$ ) has only two terms and J is independent of  $\underline{x}$ . Since one can reasonably expect that if the dependence of  $\dot{\phi}$  on  $\underline{x}$  is approximately linear, formula (9) can be applied iteratively to yield a convergent sequence of control vectors converging to the least-squares solution and one arrives at the Gauss algorithm, namely,

$$\Delta \underline{\mathbf{x}}_{k} = (\mathbf{J}^{T}\mathbf{J})^{-1} \mathbf{J}^{T}\underline{\phi}(\underline{\mathbf{x}}_{k})$$
 (10)

$$k = 0.1.2...$$

$$\underline{\mathbf{x}}_{k+1} = \underline{\mathbf{x}}_k + \Delta \underline{\mathbf{x}}_k \tag{11}$$

where  $\underline{x}_0$  is an initial control estimate suggested by other sources. GAUSLS requires as an input parameter this zero-iterate control estimate, together with the corresponding constraints  $\phi(\underline{x}_0)$ .

Since equation (6) only guarantees an extremum of the miss index (i.e., a minimum, a maximum, or an inflection point), no more can be said for the Gauss algorithm. It must be assumed that the initial estimate of  $\underline{x}_0$  is sufficiently near a local minimum and that y is well enough behaved that the algorithm indeed leads to that minimum. It is interesting to note that in the case that m=n, equations (10) and (11) reduce to the familiar Newton-Raphson scheme for solving nonlinear systems of equations.

The logic behind the steepest-descent mode is less elegant but more straightforward than the Gauss procedure. First, the gradient of the miss index is computed via equation (5). Next a search is conducted in the negative gradient direction until the miss index is observed to begin increasing. Let a denote the step length in the search direction where y is first observed to increase. Then the subroutine THPØSM is called to find a minimum of y on the step length interval from 0 to a by cubic interpolation. Let  $\lambda_m$  denote the step length value corresponding to the minimum returned by THPØSM. Then the control correction for the kth iterate is taken to be

$$\Delta_{\underline{\mathbf{x}}_{\mathbf{k}}} = -\lambda_{\underline{\mathbf{m}}} \, \underline{\nabla} \mathbf{y} / ||\underline{\nabla} \mathbf{y}|| \qquad . \tag{12}$$

The convergence of this scheme is only asymptotic with no acceleration as the minimum-miss controls are approached. Nevertheless, the steepest descent algorithm seems to be the best available for extremely nonlinear miss indices since it involves no linear extrapolation and since it searches in the only direction in which improvement is guaranteed. Its poor terminal convergence is no handicap in the hybrid GAUSLS routine because once the iteration; sequence falls inside a suitably linear region about the miss-index minimum, the rapidly convergent Gauss scheme takes over.

GAUSLS calculates the Jacobian matrix J by numerical differencing through a call to the subroutine JAC $\emptyset$ B. Hence the user is required to supply a perturbation size  $\delta$  to GAUSLS for us by JAC $\emptyset$ B in approximating J by the forward-divided difference

$$J_{ij} \approx \frac{\left[\phi_{i} \left(x_{j} + \delta\right) - \phi_{i} \left(x_{j}\right)\right]}{\delta} \tag{13}$$

The user could conceivably use an analytical Jacobian matrix by replacing the call to JACOB by formulae for the appropriate partial derivatives.

The convergence criterion in either mode of GAUSLS is the same. Adequate convergence is assumed when a weighted sum of the length of the change in the control vector and the magnitude of the change in the miss index fall below a preassigned value; i.e.,

$$C_1 |\Delta x| + C_2 |\Delta y| < \varepsilon$$
 (14)

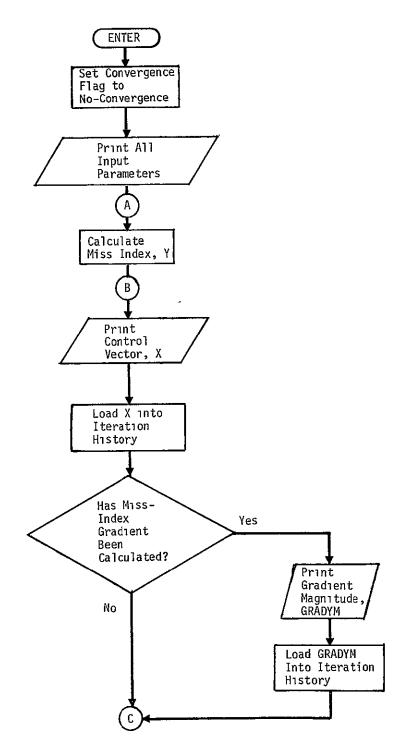
The user must supply  $C_1$ ,  $C_2$ , and  $\varepsilon$  as input parameters to GAUSLS. To expedite convergence, the user should scale the components of  $\underline{x}$  so they are, as far as possible, all of the same order of magnitude, say in the range from 0.1 to 10. This scaling makes meaningful the use of a single perturbation size  $\delta$  for all components of  $\underline{x}$  in approximating J and avoids numerical problems in matrix inversion and search direction calculation. Further he must supply as an input parameter the maximum number of iterations k he will allow before terminating the algorithm.

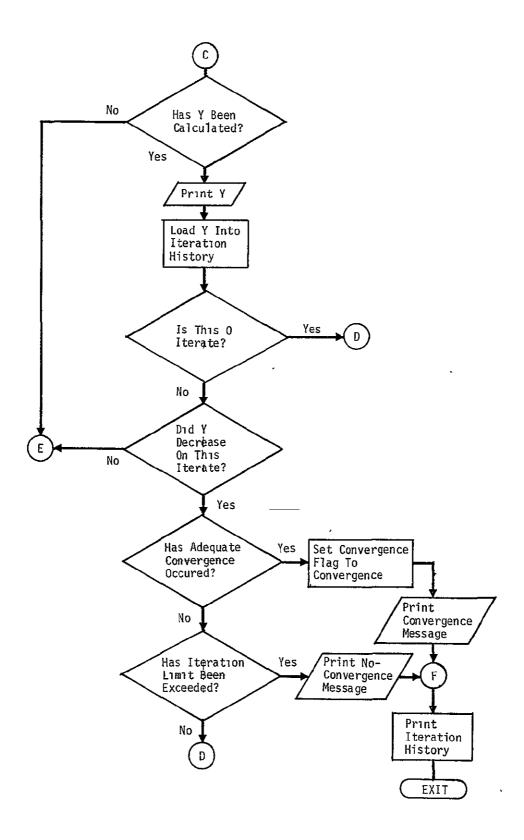
GAUSLS supplies enough output to adequately monitor either mode of the iterative least-squares process. Initially under the heading "Gauss Least-Squares Routine," it prints out all of the input parameters. These include n, m,  $\delta$ ,  $C_1$ ,  $C_2$ ,  $\epsilon$ ,  $s_0$ , and  $k_{max}$ . the user-supplied initial-control estimate  $\underline{x}_0$ , together with the corresponding miss index  $y(\underline{x}_0)$ , are printed out under the heading "Gauss Iteration Point." Then the printout relative to the general kth iterate begins. All data concerning the Jacobian matrix J are printed from the subroutine JACOB under the heading "Jacobian Matrix Routine." Each iterate, of course, starts with a Jacobian matrix computation even if it eventually ends in a steepestdescent step. All of the control vectors and corresponding constraint vectors that go into the approximation of the Jacobian matrix are printed under the heading "Nominal and Perturbed Function Values." The divided-difference approximation to J is then printed under the heading "Jacobian Matrix." Next GAUSLS prints out the Gauss pseudo inverse matrix,  $(J^TJ)^{-1}$   $J^T$ , under the heading "Projection Matrix." Finally the next Gauss centrol vector iterate, the corresponding miss index, and the gradient magnitude of the previous iterate are printed out under the heading "Gauss Iteration Point." If the length of the control correction  $\Delta x$  exceeds  $s_0$ , however, the miss index is neither calculated nor printed.

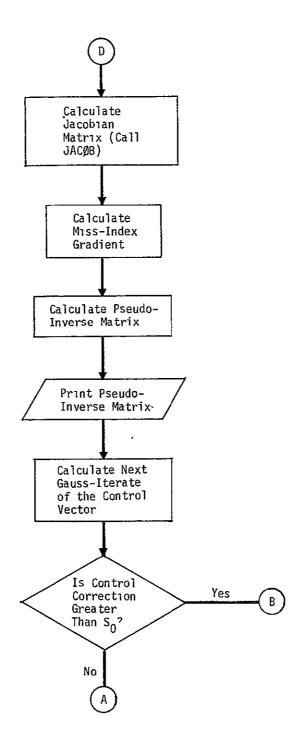
If the Gauss iterate is such that a steepest-descent step is required, GAUSLS prints out all of the pertinent data. Under the heading "Cubic Interpolation Routine" are printed all trial step lengths and corresponding miss induces used in bracketing the minimum, the input parameters to the routine THPØSM, and the minimum miss step length and index are returned by THPØSM. If the miss index decreases monotonically in the search direction, a message to that effect is printed out and execution of the program is stopped. Finally the steepest descent control iterate and the corresponding miss index is printed out under the heading "Best-Step Steepest-Descent Iteration Point." The iteration printout then is repeated with each successive iterate. When convergence finally occurs, the message "Adequate convergence occurred on previous step" is printed after the last iterate and the convergence flag, ICONV1 is set to 1. If, on the other hand, convergence fails to occur in k iterations, the message "Convergence did not oc-

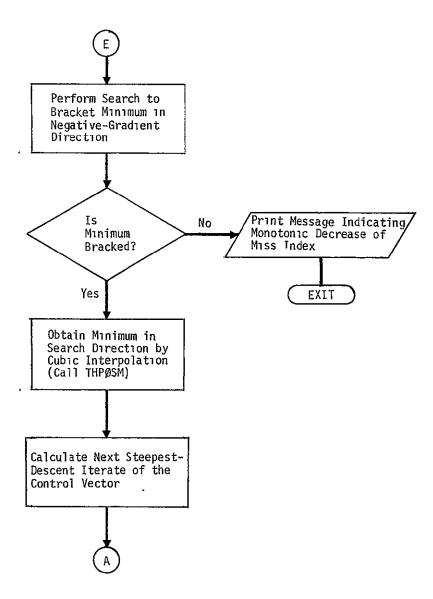
cur" is supplied after the last iteration point and ICØNV1 is set to 2. After either of these two stopping conditions is reached, a summary of the iteration points is printed under the heading "Iteration History." This summary contains the control vector, the miss index, and the gradient to the miss index at each of the iterates in consecutive order.

# GAUSLS Flow Chart









SUBROUTINE, GDATA

PURPOSE: TO INITIALIZE GENERALIZED COVARIANCE QUANTITIES

CALLING SEQUENCE: CALL GDATA

SUBROUTINES SUPPORTED: DATA

COMMON COMPUTED/USED:	EU EVS EXT GCXSUG GCXU GCXWG GP GV VARK	EV EW GCUV GCXSV GCXUG GCXXS GPG GN VARS	EVA EXI GCUW GCXSVG GCXV GCXXSG GPS IDNF	EVB EXSI GCVW GCXSW GCXVG GDNCN GPSG VARA	EVK EXST GCXSU GCXSWG GCXW GMNCN GU VARB
COMMON USED:	CXSU DNCN NDIM3 SIGBET V0	CXSV IDNF NDIM4 SIGPRO ZERO	CXU MNCN P SIGRES	CXV NDIM1 PS TG	CXXS NDIM2 SIGALP U0

SUBROUTINE GENGIO

PURPOSE 8 TO GENERATE THE ENSEMBLE STATISTICS OF THE ACTUAL COMMANUED VELOCITY CORRECTION, THE ACTUAL EXECUTION ERROR AND THE ACTUAL TARGET MISS

CALLING SEQUENCE: CALL GENGID

SUBROUTINES SUPPORTED: ERRANN

SUBROUTINES REQUIRED: SAVMAT DYNO GNAVM MEAN MOMENT EIGHY GQCOMP ATCEGV JACOBI

LOCAL SYMBOLS: AMAX INTERMEDIATE VARIABLE

ATC ACTUAL TARGET CONDITION 2ND MOMENT MATRIX

B INTERMEDIATE VARIABLE

BBBB BLANK LABEL ARRAY

C INTERMEDIATE VARIABLE

DELTM TIME DIFFERENCE

EBOVB MAGNITUDE OF ACTUAL STATISTICAL DELTA-V

EDVN MEAN OF ACTUAL COMMANDED VELOCITY CORRECTION

EGM MAGNITUDE OF EIGENVECTOR CORRESPONDING TO MAXIMUM EIGENVALUE

EGVCT EIGENVECTOR ARRAY

EGVL EIGENVALUE VECTOR

ELAB LABEL

EXTS STORAGE FOR EXI

EXSIS STORAGE FOR EXSI

EXTS STORAGE FOR EXT

EXV ACTUAL STATISTICAL DELTA-V

GAP ACTUAL VELOCITY CORRECTION 2ND MOMENT MATRIX

GPSAVE STORAGE FOR GP

GSAVE STORAGE FOR ACTUAL CONTROL 2ND MOMENT

MATRICES

ΕV

GA

GV

QPR

XIG

GCXV

NDIM1

EW

GW

RPR

XSL

GCXSU

GCXW.

NDIM2

GSAVE2 STORAGE FOR ACTUAL CONTROL 2ND MOMENT MATRICES TIME OF ACTUAL GUIDANCE EVENT GTG =1 BEFORE GUIDANCE EVENT IFLAG =2 AFTER GUIDANCE EVENT INDEX DEPENDING ON GUIDANCE EVENT TYPE III INDEX OF MAXIMUM EIGENVALUE MAP INTERMEDIATE ARRAY PEIG ACTUAL EXECUTION ERROR 2ND MOMENT MATRIX Q INTERMEDIATE VECTOR ROW S INTERMEDIATE ARRAY SUM INTERMEDIATE VARIABLE ACTUAL COMMANDED VELOCITY CORRECTION U INTERMEDIATE VECTOR **VEIG** ZL AB LABEL Z۷ ACTUAL EXECUTION ERROR MEANS ZZ INTERMEDIATE VARIABLE COMMON COMPUTED/USED: D UMMY Q EXI EXMEAN EXSI EXT GCXSVG GCXUG GCXVG GCXSUG GCXSWG GCXWG GCXXSG GP GPG GPSG XLAB EEE ΕU ADA DVUP EE

FOP

GPS

TG

X۷

GCXSW

IGUID

NDIM4

FOV

GU

II

PΙ

TINJ

GCXU

EXST

GCXSV

GCXXS

NDIM3

TEVN

ΧU

IGP

COMMON USED#

## GENGID Analysis

Subroutine GENGID controls the execution of generalized guidance events. Generalized guidance has been extended to all guidance options defined for subroutine GUIDM except for biased aimpoint guidance and impulse series thrusting.

Unlike GUIDM, which computes target dispersions and fuel budgets based on filter-generated statistics, subroutine GENGID computes target dispersions and fuel budgets based on actual statistics. In other words, the generalized covariance technique as applied to the guidance process is programmed in GENGID. The required equations are summarized below.

Before the guidance event at time  $t_j$  can be executed, it is necessary to propagate the actual control mean and control 2nd-moment matrix partitions forward to  $t_j$  from the previous guidance event at time  $t_{j-1}$ . The control mean propagates according to

$$\overline{x_{j}} = \Phi \overline{x_{j-1}} + \theta \overline{x_{s}} \overline{x_{s}} + \theta \overline{u_{o}} + \theta \overline{w_{o}}$$
(1)

where  $\Phi$ ,  $\theta$ ,  $\theta$ , and  $\theta$  are state transition matrix partitions over the interval  $\begin{bmatrix} t_{j-1}, t_{j} \end{bmatrix}$ , and x,  $x_{s}$ , u, and w denote actual position/velocity and solve-for, dynamic-consider, and ignore parameter deviation means. The notation () indicates actual values as opposed to the unprimed assumed values, while () and () indicate values immediately before and after the execution of the guidance event, respectively. The actual control position/velocity 2nd-moment matrix is defined by

$$P_{c_{j}} = E \left[ x_{j}^{\prime} x_{j}^{T} \right]. \tag{2}$$

The remaining control 2nd-moment matrix partitions are defined similarly. The propagation equations appearing in subroutine GNAVM are used to propagate the control 2nd-moment matrix partitions over the interval  $\begin{bmatrix} t \\ j-1 \end{bmatrix}$ .

The actual target state deviation  $\delta r_j$  is related to the actual state deviation  $x_i$  at time  $t_j$  according to

$$\delta i = n_j x_j$$
 (3)

where  $\eta_j$  is the variation matrix for the appropriate midcourse guidance policy. The mean of  $\delta\tau_i^*$  is given by

$$E \left[\delta \tau_{j}\right] = \eta_{j} E \left[x_{j}\right]. \tag{4}$$

The statistical target dispersions are represented by the actual target condition 2nd-moment matrix  $W_i$ , which is defined as

$$W_{j} = E \left[ \delta \tau_{j} \delta \tau_{j}^{T} \right]. \tag{5}$$

Substitution of equation (3) into equation (5) yields

$$W_{j} = \eta_{j} P_{c_{i}} \eta_{j}^{T}. \qquad (6)$$

Equations (4) and (6) are evaluated immediately before and after the guidance correction to determine how much the target errors have actually been reduced by the velocity correction at t;.

The actual commanded velocity correction 2nd-moment matrix is defined by

$$S_{j} = E \left[ \Delta \hat{V}_{j} \Delta \hat{V}_{j}^{T} \right]$$
 (7)

where the actual commanded velocity correction is given by

$$\Delta \hat{\mathbf{V}}_{\mathbf{j}}' = \Gamma_{\mathbf{j}} \hat{\mathbf{x}}_{\mathbf{j}}' = \Gamma_{\mathbf{j}} \left( \mathbf{x}_{\mathbf{j}}' + \hat{\mathbf{x}}_{\mathbf{j}}' \right). \tag{8}$$

The guidance matrix  $\Gamma_j$  corresponds to the appropriate linear midcourse guidance policy. The equation used to evaluate  $S_j$  is given by

$$S_{j}' = \Gamma_{j} \left( P_{c_{j}}' - P_{k_{j}}' \right) \Gamma_{j}^{T}$$
(9)

where all  $E\left[x, x, T\right]$  terms have been neglected in the derivation of equation (9).

The mean of the actual commanded velocity correction is obtained by applying the expectation operator to equation (8):

$$E\left[\Delta \hat{\mathbf{v}}_{\mathbf{j}}^{\prime}\right] = \Gamma_{\mathbf{j}} \left\{ E\left[\mathbf{x}_{\mathbf{j}}^{\prime}\right] + E\left[\hat{\mathbf{x}}_{\mathbf{j}}^{\prime}\right] \right\}. \tag{10}$$

Since this equation gives no useful information for fuel-sizing studies, the Hoffman-Young formula will be used to evaluate  $\begin{bmatrix} \Delta \hat{V}_j \end{bmatrix}$ 

$$\mathbb{E}\left[\left|\Delta\hat{V}_{j}^{\prime}\right|\right] = \sqrt{\frac{2A}{\pi}}\left(1 + \frac{B(\pi - 2)}{A^{2}\sqrt{5.4}}\right) \tag{11}$$

where

A = trace 
$$S_j$$
  
B =  $\lambda_1^2 \lambda_2^2 + \lambda_1^2 \lambda_3^2 + \lambda_2^2 \lambda_3^2$ ,

and  $\lambda_1'$ ,  $\lambda_2'$ , and  $\lambda_3'$  are the eigenvalues of the 2nd-moment matrix  $S_1'$ .

The actual effective or statistical  $\Delta V$  is defined as

"E 
$$\left[\hat{\Delta V_{j}}\right]$$
" = E  $\left[\hat{\Delta V_{j}}\right]$   $\cdot \alpha_{j}$  (12)

where  $\alpha_j^c$  denotes a unit vector in the most likely direction of the velocity correction. The most likely direction is assumed to be aligned with the eigenvector associated with the maximum eigenvalue of  $S_i^c$ .

With "E  $\left[\Delta\hat{V}_{j}\right]$ " available, the actual execution error statistics can be computed (by calling subroutine GQCØMP). These are the actual execution error mean E  $\left[\delta\Delta V_{j}\right]$  and 2nd-moment maxtrix  $\hat{Q}_{j}^{\prime}$  defined as

$$\tilde{Q}_{j}' = E \left[ \delta \Delta V_{j}' \delta \Delta V_{j}^{T} \right]. \tag{13}$$

It remains to summarize the equations which are used to update all actual control and knowledge means and 2nd-moment matrix partitions immediately following the execution of a guidance event. The actual estimation error means and 2nd-moment matrix partitions are updated using the following equations:

$$E\begin{bmatrix} v_{j}^{\prime} + \\ v_{j}^{\prime} \end{bmatrix} = E\begin{bmatrix} v_{j}^{\prime} - \\ v_{j}^{\prime} \end{bmatrix} - \Lambda \cdot E \begin{bmatrix} \delta \Delta V_{j}^{\prime} \end{bmatrix}$$
 (14)

$$E\begin{bmatrix} x_{s_{j}}^{+} \\ x_{s_{j}} \end{bmatrix} = E\begin{bmatrix} x_{s_{j}}^{-} \\ x_{s_{j}} \end{bmatrix}$$
 (15)

$$P_{k_{j}}^{+} = P_{k_{j}}^{-} + A \hat{Q}_{j}^{'} A^{T} - A \cdot E \left[\delta \Delta V_{j}^{'}\right] \cdot E \begin{bmatrix} \hat{x}_{j}^{-} \end{bmatrix} - E \begin{bmatrix} \hat{x}_{j}^{-} \end{bmatrix}$$

$$\cdot \ \mathbb{E} \left[ \phi \Delta \mathbf{V}_{\mathbf{j}}^{\mathsf{T}} \right] \cdot \mathbf{A}^{\mathsf{T}}$$
 (16)

$$P_{s_{k_{j}}}^{+} = P_{s_{k_{j}}}^{-}$$

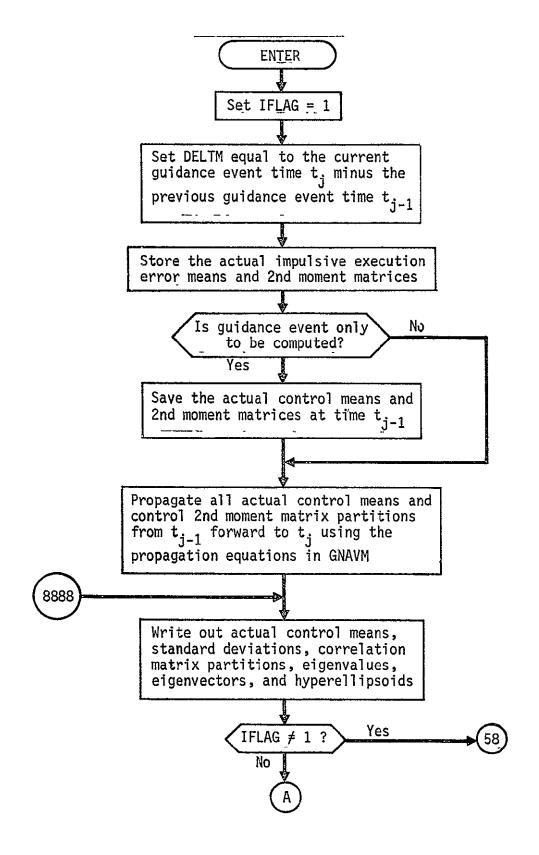
$$(17)$$

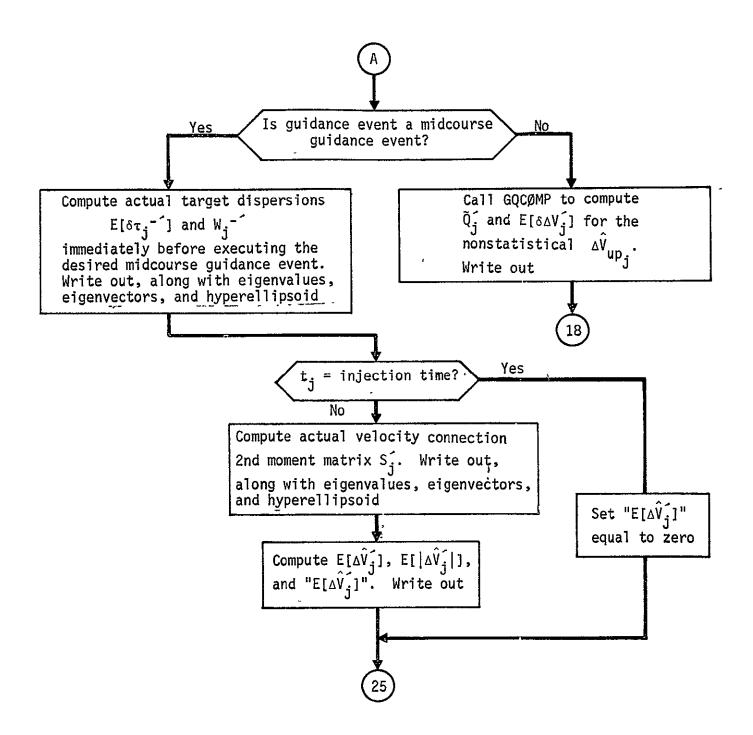
where  $A = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$ . The actual deviation means are updated using the following equations:

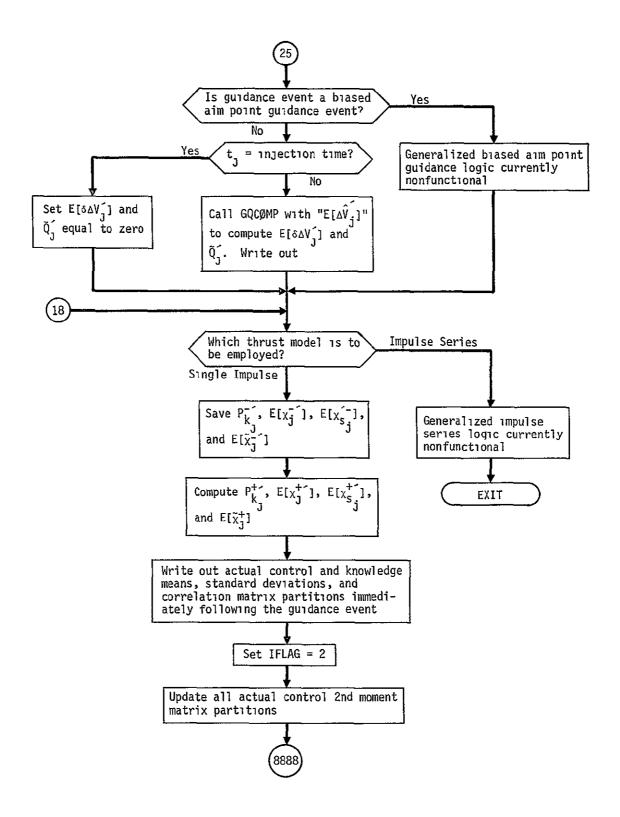
$$E\begin{bmatrix} x_{j}^{+} \end{bmatrix} = -E\begin{bmatrix} x_{j}^{+} \\ x_{j}^{-} \end{bmatrix}$$
 (18)

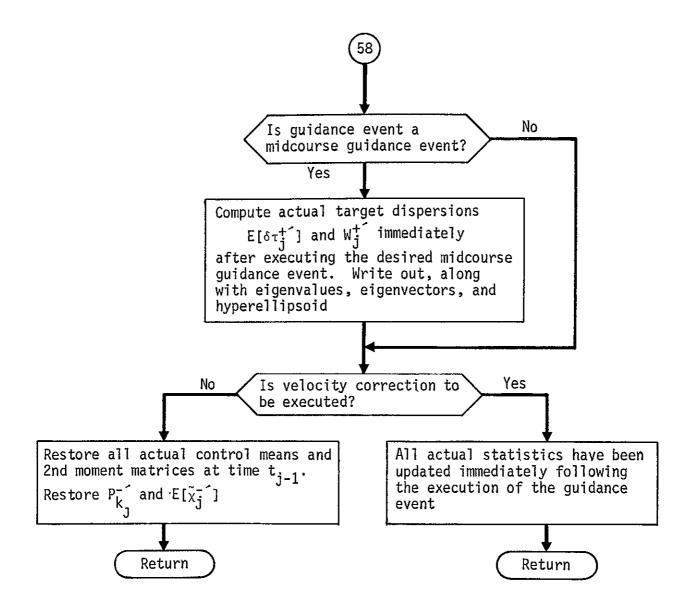
$$E\left[x_{s_{j}}^{+}\right] = -E\left[x_{s_{j}}^{+}\right]$$
(19)

The entire set of actual control 2nd-moment matrix partitions is updated by equating them to the corresponding actual knowledge 2nd-moment matrix partitions at  $t_i^+$ .









SUBROUTINE GHA

PURPOSE: TO COMPUTE THE GREENWICH HOUR ANGLE AND THE UNIVERSAL TIME (IN DAYS) WHICH IS USED IN THE TRACKING MODULE TO ORIENT THE TRACKING STATIONS ON A SPERICAL ROTATING EARTH.

CALLING SEQUENCE: CALL GHA

ARGUMENTS: NONE

SUBROUTINES SUPPORTED: DATA1S DATA1

LOCAL SYMBOLS: D NUMBER OF DAYS IN TSTAR

EQMEG EARTH ROTATION RATE

GH GREENWICH HOUR ANGLE

IO INTERMEDIATE VARIABLE

REFJD JULIAN DATE OF JAN. 0, 1950

TFRAC FRACTION OF DAY IN TSTAR

TSTAR JULIAN DATE, EPOCH JAN. 0, 1950, OF

INITIAL TRAJECTORY TIME

COMMON COMPUTED: UNIVI

COMMON USED8 DATEJ EM13

GHA Analysis

Subroutine GHA computes the Greenwich hour angle in degrees and days at some epoch T* referenced to 1950 January  $1^d0^h$ . Epoch T* is computed from

$$T* = J.D._0 + 2415020.0 - J.D._{REF}$$

where

J.D.  $_{0}$  = Julian date at launch time t referenced to 1900 January  $0^{d}12^{h}$ .

J.D. Reference Julian date 2433282.5

= 1950 January 1^d0^h referenced to January 0^d12^h of the year 4713 B.C.

and 2415020.0 = 1900 January  $0^{d}12^{h}$  referenced to January  $0^{d}12^{h}$  of the year 4713 B.C.

Then T* is the Julian date at launch time t referenced to 1950 January  $1^{d_0h}$  .

The Greenwich hour angle corresponding to T* is given by '_____

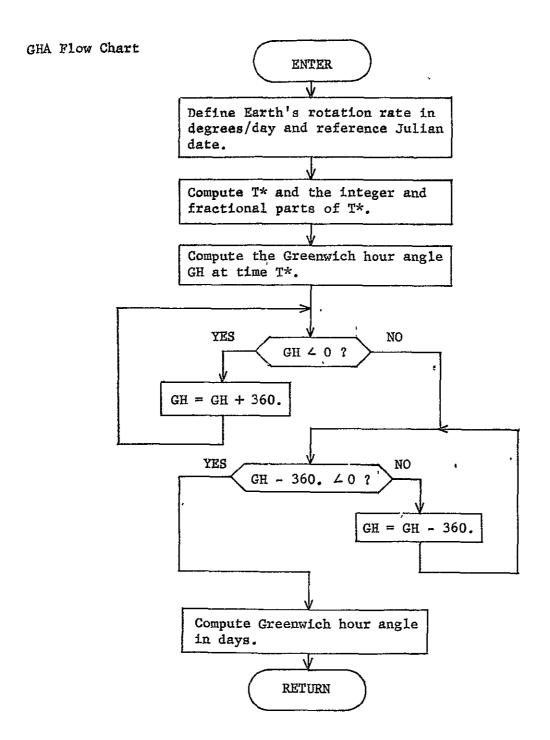
GHA(T*) =  $100.0755426 + 0.985647346d + 2.9015 \times 10^{-13} d^2 + \omega t$ 

where  $0 \leq GHA(T*) < 360^{\circ}$ 

and d = integer part of T*, t = fractional part of T*,

and  $\omega = \text{Earth's rotation rate is degrees/day}$ .

The Greenwich hour angle in days is given by  $\frac{GHA}{\omega}$ .



SUBROUTINE GIDANS

PURPOSES EXECUTIVE ROUTINE FOR COMPUTATION OF REQUIRED GUIDANCE EVENT

CALLING SEQUENCES CALL GIDANS

SUBROUTINES SUPPORTED: NOMNAL NOMLIN

SUBROUTINES REQUIRED: EXCUTE ZERIT TARGET INSERS TIME VMP

LOCAL SYMBOLS: DTIME DELTA TIME (DAYS) BETWEEN ORBIT INSERTION COMPUTATION AND EXECUTION

IZER VECTOR OF CODES USED FOR RETARGETING
IZER(KUR)=0, DO NOT RECOMPUTE ZERO ITERATE

\$0, RECOMPUTE ZERO ITERATE

I INDEX

KTYPE VALUE OF KTYP(KUR) INDICATING TYPE OF EVENT

=1, ORIGINAL TARGETING

=2, RETARGETING

=3, ORBIT INSERTION

MODEL DOES NOT APPEAR IN CURRENT VERSION SEE EXECUTE

COMMON COMPUTED/USEDS KMXQ KHIT TIMG COMMON COMPUTEDS DELV KTIM ZDAT

COMMON USED: DELTAV KTYP KUR MOL RIN

#### GIDANS Analysis

GIDANS is an executive routine responsible for processing a guidance manuever for the computation of the velocity increment  $\Delta v$  to the execution of the correction.

Before entry to GIDANS, TRJTRY has computed the index of the current event (KUR) and has integrated the nominal trajectory to the time of the event. GIDANS now evaluates the KUR component of two integer arrays — KTYP and KMXQ. The values of these flags determine the operation of GIDANS. The flag KTYP specifies the type of guidance event to be performed, while KMXQ prescribes the compute/execute mode to be used according to

- KTYP = -1 Termination event
  - 1 Targeting event
  - 2 Retargeting event
  - 3 Orbit insertion
  - 4 Main probe propagation
  - 5 Miniprobe targeting
- KMXQ = 1 Compute  $\Delta \hat{y}$  only
  - 2 Execute Δv only
  - 3 Compute and execute  $\Delta y$
  - 4 Compute but execute Δv-later

GIDANS first checks for a termination event. If the current index prescribes such an event, the flag KWIT is set to 1 and a return is made to the main program  $N\emptyset MNAL$ .

In prepration for a normal guidance event, GIDANS calls VMP with the current spacecraft heliocentric state and a time increment of zero to restore the F and V arrays providing the current geometry of spacecraft and planets. If the current event is an execute-only mode, the transfer is made to the execution section of GIDANS for the addition of the preset velocity increment.

Otherwise GIDANS interrogates KTYP for the type of maneuver to be computed. For a targeting event, subroutine TARGET is called directly for the computation of the  $\Delta v$  necessary to satisfy input target conditions. After calling TARGET, the F and V arrays are restored as indicated above.

A retargeting event is defined as a targeting event that requires computation of a new zero iterate. Thus a retargeting event is an event in which the current nominal state when integrated forward would miss the target conditions badly. Such an event would be the broken-plane correction. For this event TRJTRY stores the current position (and possibly the target position) in the ZDAT array. It then calls ZERIT for the computation of the massless-planet's initial velocity consistent with the target conditions. It then operates identically to the targeting event.

The third guidance maneuver is the insertion event. GIDANS calls INSERS for the computation of the velocity increment  $\Delta v$  and the time interval  $\Delta t$  before it is to be executed.

The main-probe propagation event involves storing the current spacecraft state, propagating the main probe to an appropriate stopping condition while printing a time history, and restoring the original state in preparation for the next event. It is carried out in a single call to the subroutine MPPRØP. Upon return to GIDANS, the F and V arrays are restored as indicated above.

The miniprobe targeting event, although somewhat complicated, is completely executed by the single subroutine TPRTRG. The current bus state is first stored. Next the miniprobe release controls are calculated to apply at the current time to target three miniprobes respectively to three target sites characterized by imput values of declination and right ascension. Using the minimum-miss release controls, each miniprobe is then propagated from release to a stopping condition while a time history is concurrently printed. Finally, the original hus state is restored. On returning to GIDANS, the F and V arrays are restored as usual.

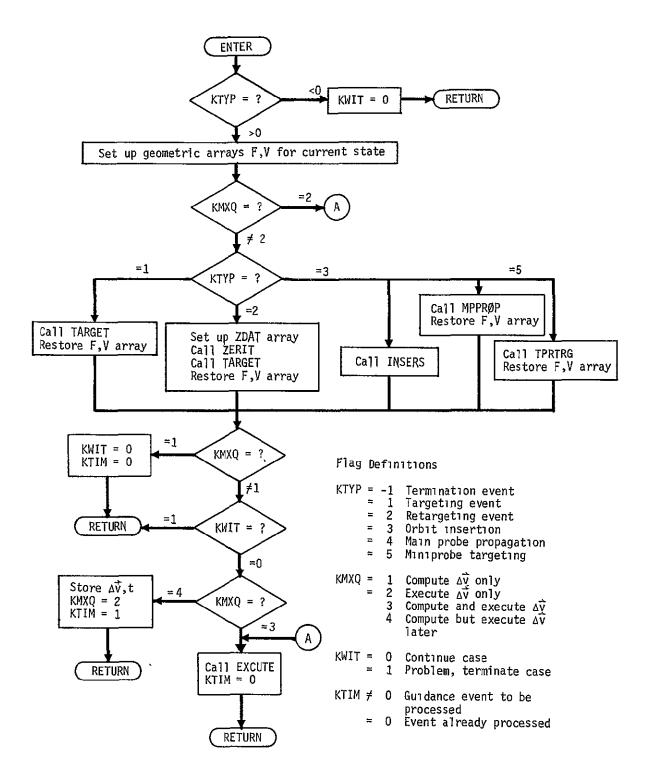
The three subroutines TARGET, INSERS, and TPRTRG signal trouble to GIDANS via the flag KWIT. If problems are encountered in their execution, e.g., failure to converge in TARGET or TPRTRG or the impossibility of insertion in INSERS, KWIT is set to 1. Otherwise KWIT = 0. On return to NØMNAL, if KWIT = 1 the current case is terminated while if KWIT = 0 it is continued.

If the current event is a compute-only mode, TRJTRY now sets KWIT = 0 (so that the program will continue regardless of whether the correction computations were successful) and returns to NØMNAL. However if the current event failed (KWIT = 1) and was to be executed (KMXQ  $\neq$  1) GIDANS consideres this a fatal error for the current case and returns with KWIT = 1.

If the compute/execute mode is compute-execute later (KMXQ = 4) as is the insertion event, GIDANS now sets up for the subsequent execute-only event. The  $\Delta v$  computed is stored in the DELV array, the time of the execution is computed  $\begin{pmatrix} t \\ ex \end{pmatrix} + \Delta t$  and stored in the TIMG array, and the KMXQ flag is set to a 2 (execute-only). The return is then made to NØMINAL.

For an event to be executed at the current time (KMXQ = 2,3), GIDANS now calls EXCUTE for the completion of that task.

It should be noted that for all events that are completed at this time, the KUR components of the KTIM array are set equal to 0 so they are no longer considered in determining the next event in TRJTRY. Only in the case of KMXQ = 4 is the KTIM flag nonzero on exit from GIDANS.



#### SUBROUTINE GNAVM

PURPOSE: TO PROPAGATE ASSUMED COVARIANCE MATRIX PARTITIONS P,
CXXS,CXU,CXV,PS,CXSU,CXSV,OR ACTUAL SECOND MOMENT MATRIX
PARTITIONS GP,GCXXS,GCXU,GCXV,GCXW,GPS,GCXSU,GCXSV,
GCXSW FROM THE TIME OF THE LAST MEASUREMENT OR EVENT TO
THE PRESENT TIME AND TO UPDATE THESE MATRIX PARTITIONS
IF A MEASUREMENT IS TO BE PROCESSED

CALLING SEQUENCE: CALL GNAVM(NR, IFLAG1, ICODE, U0, V0, GCXW, GCXSW, P, CXXS, CXU, CXV, PS, CXSU, CXSV, Q, R)

ARGUMENTS:	NR	I	NUMBER OF ROWS IN THE OBSERVATION MATRIX
	IFLAG1	I	=1 FOR ASSUMED COVARIANCE PROCESSING =2 FOR ACTUAL SECOND MOMENT PROCESSING
	ICODE	I	=0 FOR UPDATE =1 FOR PROPAGATION
	UO	Ι	ACTUAL OR ASSUMED DYNAMIC CONSIDER PARAMETER 2ND MOMENT MATRIX
	<b>V</b> 0	I	ACTUAL OR ASSUMED MEASUREMENT CONSIDER PARAMETER 2ND MOMENT MATRIX
	C CXM	I	ACTUAL POSITION-VELOCITY STATE / IGNORE PARAMETER 2ND MOMENT MATRIX
	GCXSW	Ι	ACTUAL SOLVE-FOR PARAMETER / IGNORE PARAMETER 2ND MOMENT MATRIX
	Р	I	ACTUAL OR ASSUMED POSITION-VELOCITY 2ND MOMENT MATRIX 2ND MOMENT MATRIX
	CXXS	I	ASSUMED OR ACTUAL POSITION-VELOCITY STATE / SOLVE-FOR PARAMETER 2ND MOMENT MATRIX
	CXU	I	ASSUMED OR ACTUAL POSITION-VELOCITY STATE / DYNAMIC CONSIDER PARAMETER 2ND MOMENT MATRIX
	CXA	I	ASSUMED OR ACTUAL POSITION-VELOCITY STATE / MEASUREMENT CONSIDER PARAMETER 2ND MOMENT MATRIX
	PS	I	ASSUMED OR ACTUAL SOLVE-FOR PARAMETER COVARIANCE OR 2ND MOMENT MATRIX
	CXSU	I	ASSUMED OR ACTUAL SOLVE-FOR PARAMETER

MATRIX

/ DYNAMIC CONSIDER PARAMETER 2ND MOMENT

	CXSV	I	ASSUMED OR ACTUAL SOLVE-FOR PARAMETER / MEASUREMENT CONSIDER PARAMETER 2ND MOMENT MATRIX				
	Q	I	ASSUMED MATRIX	OR ACTUA	L DYNAMI	C NOISE	2ND MOMENT
	R	I	ASSUMED 2ND MOME	OR ACTUA NT MATRI		EMENT NO	ISE
SUBROUTINES	SUPPORTED	<b>3</b> E	ERRANN S	ETEVN G	UIDM PR	ED GENG	ID PROBE
SUBROUTINES REQUIRED: GAIN1 GAIN2							
LOCAL SYMBOL	S: AKW		INTERMED	IATE ARR	RAY		
	อร		INTERMEDIATE ARRAY				
	ES		INTERMEDIATE ARRAY				
	FS		INTERMEDIATE ARRAY				
	IEND		NR-1				
	NOIM4	S	NDIM4 VALUE STORAGE				
	N1		NDIM1-1				
	SUM		INTERMEDIATE VARIABLE				
	SW		INTERME	DIATE AR	RAY		
COMMON COMP	UTED/USED	1	AK CXSVP GCUV HPHR JPR	AL CXUP GCUW IGAIN ZERO	AM CXVP GCVH GH PP	AN CXXSP GCXSWP H PSP	CXSUP G GCXWP HALF S
COMMON USED	<b>1</b>		NDIM1 TXU	NDIM2 TX#	NDIM3 TXXS	NDIM4	PHI

# GNAVM Analysis

Subroutine GNAVM propagates and updates (at a measurement) both assumed (or filter) covariance matrix partitions and actual 2nd moment matrix partitions. The equations programmed in GNAVM are independent of the filter algorithm employed to generate gain matrices.

The covariance and 2nd moment matrix partitions manipulated by GNAVM are defined as follows:

$$P = E[\tilde{x} \ \tilde{x}^{T}] \qquad P_{g} = E[\tilde{x}_{g} \ \tilde{x}_{g}^{T}]$$

$$C_{xx_{g}} = E[\tilde{x} \ \tilde{x}^{T}] \qquad C_{x_{g}u} = E[\tilde{x}_{g} \ \tilde{u}^{T}]$$

$$C_{xu} = E[\tilde{x} \ \tilde{u}^{T}] \qquad C_{x_{g}v} = E[\tilde{x}_{g} \ \tilde{v}^{T}] \qquad (1)$$

$$C_{xv} = E[\tilde{x} \ \tilde{v}^{T}] \qquad C_{x_{g}w} = E[\tilde{x}_{g} \ \tilde{v}^{T}] \qquad (2)$$

$$C_{xv} = E[\tilde{x} \ \tilde{v}^{T}] \qquad C_{x_{g}w} = E[\tilde{x}_{g} \ \tilde{v}^{T}] \qquad (3)$$

The following matrix partitions are used in GNAVM, but are not changed in GNAVM:

$$C_{uv} = E[\tilde{u} \ \tilde{v}^{T}]$$

$$C_{uw} = E[\tilde{u} \ \tilde{w}^{T}]$$

$$C_{vw} = E[\tilde{v} \ \tilde{w}^{T}]$$

$$V = E[\tilde{u} \ \tilde{u}^{T}]$$

$$V = E[\tilde{v} \ \tilde{v}^{T}]$$

$$W = E[\tilde{v} \ \tilde{w}^{T}]$$

In these definitions  $\tilde{\mathbf{x}}$ ,  $\tilde{\mathbf{x}}$ ,  $\tilde{\mathbf{u}}$ ,  $\tilde{\mathbf{v}}$ , and  $\tilde{\mathbf{w}}$  represent, respectively, the estimation errors in position/velocity state, solve-for parameters, dynamic consider parameters, measurement consider parameters, and ignore parameters. Ignore parameters, of course, are not defined when assumed (or filter) covariance matrix partitions are being propagated or updated. Furthermore, the assumed  $\mathbf{C}_{\mathbf{u}\mathbf{v}}$  has been set to zero.

The equations used to propagate covariances or 2nd moment matrices from time  $t_k$  to  $t_{k+1}$  are summarized:

$$P_{k+1}^{-} = \left( \Phi P_{k}^{+} + \theta_{xx_{s}} C_{xx_{s_{k}}}^{+T} + \theta_{xu} C_{xu_{k}}^{+T} + \theta_{xw} C_{xw_{k}}^{+T} \right) \Phi^{T}$$

$$+ c_{xx_{s_{k+1}}}^{-} \theta_{xx_{s}}^{T} + c_{xu_{k+1}}^{-} \theta_{xu_{k+1}}^{T} + c_{xw_{k+1}}^{-} \theta_{xw}^{T} + Q_{k+1}$$
 (3)

$$C_{xx_{s_{k+1}}}^{-} = \Phi C_{xx_{s_{k}}}^{+} + \theta_{xx_{s}} P_{s_{k}}^{+} + \theta_{xu} C_{x_{s}u}^{+T} + \theta_{xw} C_{x_{s}w}^{+T}$$
(4)

$$C_{xu_{k+1}}^{-} = \Phi C_{xu_{k}}^{+} + \theta_{xx_{s}} C_{s_{t}}^{+} + \theta_{xu} U_{o} + \theta_{xw} C_{uw_{o}}^{T}$$
(5)

$$C_{xv_{k+1}}^{-} = \Phi C_{xv_{k}}^{+} + \theta_{xx_{s}} C_{x_{s}v_{k}}^{+} + \theta_{xu} C_{uv_{o}}^{-} + \theta_{xw} C_{vw_{o}}^{T}$$
(6)

$$C_{xw_{k+1}}^{-} = \Phi C_{xw_{k}}^{+} + \theta C_{xx_{s}}^{+} + \theta C_{xu_{s}}^{+} + \theta C_{xu_{s}}^{-} + \theta W_{s}^{-}$$
(7)

$$P_{s_{k+1}}^- = P_{s_k}^+ \tag{8}$$

$$\begin{array}{ccc}
c_{x_S u} & = c^{\dagger} \\
k+1 & s u
\end{array}$$
(9)

$$C_{\mathbf{x}_{\mathbf{S}}\mathbf{v}}^{-} = C_{\mathbf{x}_{\mathbf{S}}\mathbf{v}}^{+} \tag{10}$$

$$C_{x_s^W} = C_{x_s^W}^+ \qquad (11)$$

In these equations () indicates immediately prior to processing a measurement; () , immediately after. The state transition matrices over the interval [t_k, t_{k+1}] are indicated by  $\Phi$ ,  $\theta_{xx}$ ,  $\theta_{xu}$ , and  $\theta_{xw}$ . The dynamic noise covariance or 2nd moment matrix is denoted by  $Q_{k+1}$ .

Before covariance (or 2nd moment) matrix partitions can be updated at a measurement, the measurement residual covariance (or 2nd moment) matrix, defined by

$$J_{k+1} = E \left[ \varepsilon_{k+1} \quad \varepsilon_{k+1}^{T} \right]$$
 (12)

must be computed. The required equations are summarized

$$J_{k+1} = HA_{k+1} + MB_{k+1} + GD_{k+1} + LE_{k+1} + NF_{k+1} + R_{k+1}$$
 (13)

$$A_{k+1} = P_{k+1}^{-} H^{T} + C_{xx}^{-} M^{T} + C_{xu}^{-} G^{T} + C_{xv}^{-} L^{T} + C_{xw}^{-} N^{T}$$
(14)

$$B_{k+1} = P_{s_{k+1}}^{-} M^{T} + C_{xx_{s_{k+1}}}^{-T} H^{T} + C_{x_{s_{k+1}}}^{-} G^{T} + C_{x_{s_{k+1}}}^{-} I^{T} + C_{x_{s_{k+1}}}^{-} N^{T}$$
(15)

$$D_{k+1} = C_{xu_{k+1}}^{-T} H^{T} + C_{x_{su_{k+1}}}^{-T} M^{T} + U_{o}G^{T} + C_{uw_{o}}^{-} N^{T} + C_{uv_{o}}^{-} L^{T}$$
 (16)

$$E_{k+1} = C_{xv_{k+1}}^{-T} H^{T} + C_{x_{s}v_{k+1}}^{-T} M^{T} + C_{vw_{o}}^{-} N^{T} + V_{o}L^{T} + C_{uv_{o}}^{-T} G^{T}$$
(17)

1----

$$F_{k+1} = W_0 N^T + C_{xw_{k+1}}^{-T} H^T + C_{xw_{k+1}}^{-T} M^T + C_{vw_0}^{-T} L^T + C_{uw_0}^{-T} G^T .$$
 (18)

In these equations H, M, G, L, and N represent observation matrix partitions, and  $R_{k+1}$  represents the measurement noise covariance (or 2nd moment) matrix.

Gain matrices  $K_{k+1}$  and  $K_{k+1}$  are also required before covariance (or 2nd moment) matrix partitions can be updated. These are not computed in GNAVM but are obtained by calling either subroutine GAIN1 or GAIN2, depending on which recursive estimation algorithm is desired.

With  $J_{k+1}$ ,  $K_{k+1}$ , and  $S_{k+1}$  available, the following equations are used in the updating process:

$$P_{k+1}^{+} = P_{k+1}^{-} - K_{k+1} A^{T} - AK_{k+1}^{T} + K_{k+1} J_{k+1} K_{k+1}^{T}$$
 (19)

$$c_{xx_{8k+1}}^{+} = c_{xx_{8k+1}}^{-} - K_{k+1} B^{T} - AS_{k+1}^{T} + K_{k+1} J_{k+1} S_{k+1}^{T}$$
 (20)

$$C_{xu_{k+1}}^{\dagger} = C_{xu_{k+1}}^{-} - K_{k+1} D^{T}$$
 (21)

$$C_{xv_{k+1}}^+ = C_{xv_{k+1}}^- - K_{k+1} D^T$$
 (22)

$$C_{xw_{k+1}}^+ = C_{xw_{k+1}}^- - K_{k+1} F^T$$
 (23)

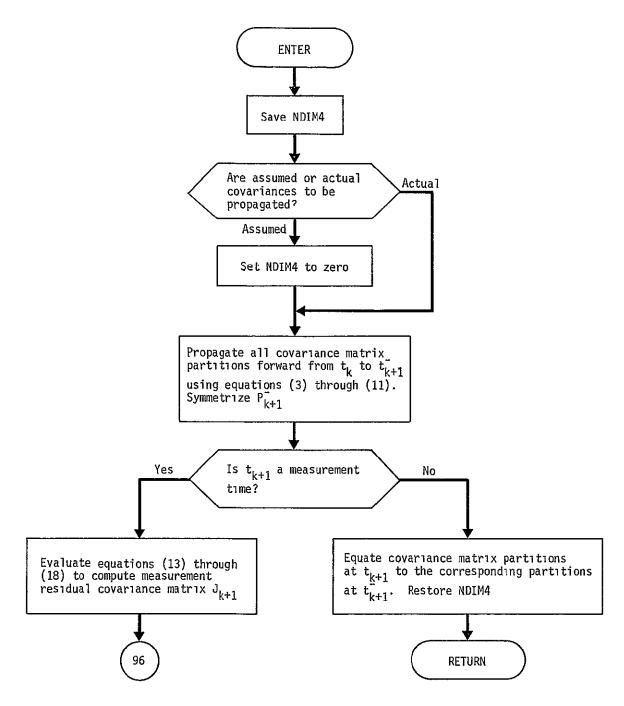
$$P_{s_{k+1}}^{+} = P_{s_{k+1}}^{-} - S_{k+1} B^{T} - BS_{k+1}^{T} + S_{k+1} J_{k+1} S_{k+1}^{T}$$
 (24)

$$C_{x_{g_{k+1}}}^{+} = C_{x_{g_{k+1}}}^{-} - S_{k+1} D^{T}$$
 (25)

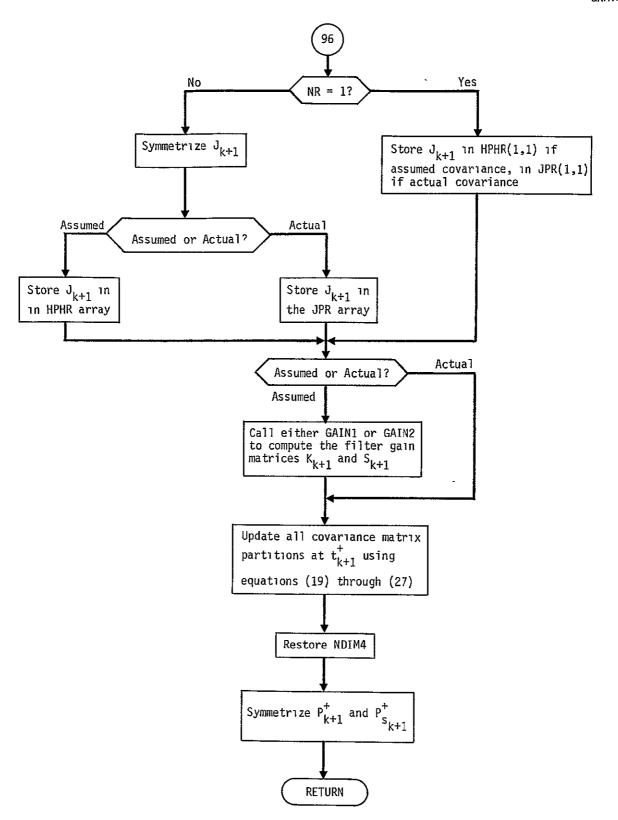
$$c_{x_{g}v_{k+1}}^{+} = c_{x_{g}v_{k+1}}^{-} - s_{k+1} E^{T}$$
 (26)

$$c_{x_{s_{k+1}}}^{+} = c_{x_{s_{k+1}}}^{-} - s_{k+1}^{T}$$
 (27)

It should be noted that propagation equations (3) through (11) are also used to propagate both assumed control covariance and actual 2nd moment matrix partitions over the time interval separating two successive guidance events. The update equations, of course, are not used in this situation.



GNAVM Flow Chart



SUBROUTINE GPRINT

PURPOSE: TO PRINT ACTUAL ESTIMATION~ERROR STATISTICS

CALLING SEQUENCE: CALL GPRINT(IFLAG, TIMM)

ARGUMENTS: IFLAG I =3 PRINT ACTUAL STATISTICS AT A

GUIDANCE EVENT

=10 PRINT ACTUAL ESTIMATION ERROR STATISTICS

=2 PRINT ACTUAL ESTIMATION ERROR STATISTICS AT A PREDICTION EVENT

TIMM I TIME TO BE PRINTED

SUBROUTINES SUPPORTED: PRED ERRANN SETEVN

SUBROUTINES REQUIRED: MOMENT

LOCAL SYMBOLS: A HOLLERITH WORD -AFTER-

8 HOLLERITH WORD -BEFORE-

DUM INTERMEDIATE VECTOR

EXSTSV TEMPORARY STORAGE FOR EXST

EXTSV TEMPORARY STORAGE FOR EXT -

ROW INTERMEDIATE VECTOR

ZZ INTERMEDIATE VARIABLE

COMMON USED:

CXSUP	CXSVP	CXUP	CXVP	CXXSP
EMRES	ΕU	E۷	EW	EXI
EXSI	EXST	EXSTP	EXTP	GCXSU
GCXSV	GCXSW	GCXSWP	GCXU	GC XV
GCXW	GCXWP	GCXXS	GP	GPS
GU	G۷	GW	JPR	NDIM1
NDIM2	NDIM3	NDIM4	NR	PР
PSP	RPR	TRTM2	XIG	XL AB
XSL	ΧU	ΧV		

SUBROUTINE GQCOMP

PURPOSE: TO COMPUTE ACTUAL EXECUTION ERROR STATISTICS

CALLING SEQUENCE: CALL GQCOMP(VV, EE, EEE, EV, Q)

ARGUMENTS: VV I ACTUAL COMMANDED VELOCITY CORRECTION

EE I MEANS OF ACTUAL EXECUTION ERRORS

FEE I 2ND MOMENTS OF ACTUAL EXECUTION ERRORS

EV O EXPECTED VALUE OF ACTUAL EXECUTION ERROR

Q O ACTUAL EXECUTION ERROR 2ND MOMENT MATRIX

SUBROUTINES SUPPORTED: GENGID

LOCAL SYMBOLS: FACTR INTERMEDIATE VARIABLE

RHOP MAGNITUDE OF VV VECTOR

RHOP2 RHOP**2

V1 V(1)**2

V2 V(2)**2

V3 V(3) **2

V4 V1*V2*V3

XI INTERMEDIATE VARIABLE

XMUP INTERMEDIATE VARIABLE

ZETA INTERMEDIATE VARIABLE

## GQCØMP Analysis

Subroutine GQCØMP computes the actual execution error mean and 2nd moment matrix for use in the generalized covariance analysis of a guidance event. The actual execution error  $\delta\Delta V^{\dagger}$  is assumed to have the form

$$\delta \Delta V_{j}^{i} = k' \Delta \hat{V}_{j}^{i} + s' \frac{\Delta \hat{V}_{j}^{i}}{|\Delta \hat{V}_{j}^{i}|} + \delta \Delta V_{pointing}^{i}$$
 (1)

where k' denotes the actual proportionality error; s', the actual resolution error;  $\delta \Delta V'_{pointing}$ , the actual pointing error; and  $\Delta \hat{V}'_{4}$ , the actual commanded velocity correction.

The means of the three ecliptic components of  $\delta\Delta V_{j}^{\prime}$  are given as:

$$E[\delta\Delta V_{x}^{\dagger}] = \left(\vec{k}^{\dagger} + \frac{\vec{s}^{\dagger}}{\rho^{\dagger}}\right) \Delta \hat{V}_{x}^{\dagger} + \frac{\rho^{\dagger}\Delta \hat{V}_{y}^{\dagger} \ \delta\alpha^{\dagger} + \Delta \hat{V}_{x}^{\dagger} \ \Delta \hat{V}_{z}^{\dagger} \ \delta\beta^{\dagger}}{\mu^{\dagger}}$$
(2)

$$\mathbb{E}\left[\delta\Delta \mathbf{V}_{\mathbf{y}}^{\dagger}\right] = \left(\tilde{\mathbf{k}}^{\dagger} + \frac{\bar{\mathbf{s}}^{\dagger}}{\rho^{\dagger}}\right) \Delta\hat{\mathbf{V}}_{\mathbf{y}}^{\dagger} + \frac{\Delta\hat{\mathbf{V}}_{\mathbf{y}}^{\dagger} \Delta\hat{\mathbf{V}}_{\mathbf{z}}^{\dagger} \overline{\delta\beta}^{\dagger} - \rho^{\dagger}\Delta\hat{\mathbf{V}}_{\mathbf{x}}^{\dagger} \overline{\delta\alpha}^{\dagger}}{\mu^{\dagger}}$$
(3)

$$E[\delta \Delta V_{z}^{\dagger}] = \left(\overline{k}^{\dagger} + \frac{\overline{s}^{\dagger}}{\rho^{\dagger}}\right) \Delta \hat{V}_{z}^{\dagger} - \mu^{\dagger} \overline{\delta \beta}^{\dagger}$$
(4)

where  $\rho' = |\Delta \hat{V}'|$ ,  $\mu' = [\Delta \hat{V}_x^{'2} + \Delta \hat{V}_y^{'2}]^{\frac{1}{2}}$ , and  $\delta \alpha'$  and  $\delta \beta'$  are the actual pointing angle errors, and both E() and () indicate mean values.

The actual execution error 2nd moment matrix is defined by

$$\tilde{Q}_{j}^{\dagger} = E \left[ \delta \Delta V_{j}^{\dagger} \delta \Delta V_{j}^{\dagger}^{T} \right] \qquad (5)$$

the elements  $Q_{ik}^{!}$  of matrix  $Q_{j}^{!}$  are given as:

$$\tilde{Q}_{11}^{\prime} = \xi^{\prime} \Delta \hat{V}_{\mathbf{x}}^{\prime 2} + \frac{1}{\mu^{\prime 2}} \left( \rho^{\prime 2} \Delta \hat{V}_{\mathbf{y}}^{\prime 2} \overline{\delta \alpha^{\prime} \delta \alpha^{\prime}} + \Delta \hat{V}_{\mathbf{x}}^{\prime 2} \Delta \hat{V}_{\mathbf{z}}^{\prime 2} \overline{\delta \beta^{\prime} \delta \beta^{\prime}} + . \right)$$

$$2\rho^{\dagger} \Delta \hat{\mathbf{v}}_{\mathbf{x}}^{\dagger} \Delta \hat{\mathbf{v}}_{\mathbf{y}}^{\dagger} \Delta \hat{\mathbf{v}}_{\mathbf{z}}^{\dagger} \overline{\delta \alpha}^{\dagger} \overline{\delta \beta}^{\dagger} + \frac{2\Delta \hat{\mathbf{v}}_{\mathbf{x}}^{\dagger}}{\mu^{\dagger}} \zeta^{\dagger} \left( \underline{\rho} \Delta \hat{\mathbf{v}}_{\mathbf{y}}^{\dagger} \overline{\delta \alpha}^{\dagger} + \Delta \hat{\mathbf{v}}_{\mathbf{x}}^{\dagger} \Delta \hat{\mathbf{v}}_{\mathbf{z}}^{\dagger} \overline{\delta \beta}^{\dagger} \right)$$
(5)

$$\begin{split} \tilde{Q}_{22}^{i} &= \, \xi^{\, i} \, \Delta \hat{V}_{y}^{i\, 2} \, + \, \frac{1}{\mu^{\, i}\, Z} \, \left( \Delta \hat{V}_{y}^{i\, 2} \, \Delta \hat{V}_{z}^{i\, 2} \, \overline{\delta \beta^{\, i} \, \delta \beta^{\, i}} \, + \, \rho^{\, i\, 2} \, \Delta \hat{V}_{x}^{i\, 2} \, \overline{\delta \alpha^{\, i} \, \delta \alpha^{\, i}} \, - \\ & 2 \rho^{\, i} \, \Delta \hat{V}_{x}^{i} \, \Delta \hat{V}_{y}^{i} \, \Delta \hat{V}_{z}^{i} \, \overline{\delta \alpha^{\, i} \, \delta \beta^{\, i}} \right) \, + \, \frac{2 \Delta \hat{V}_{y}^{i}}{\mu^{\, i}} \, \zeta^{\, i} \, \left( \Delta \hat{V}_{y}^{i} \, \Delta \hat{V}_{z}^{i} \, \overline{\delta \beta^{\, i}} \, - \, \rho^{\, i} \, \Delta \hat{V}_{x}^{i} \, \overline{\delta \alpha^{\, i}} \right) \quad (6) \\ \tilde{Q}_{33}^{i} &= \, \xi^{\, i} \, \Delta \hat{V}_{z}^{i} \, 2 \, + \, \mu^{\, i\, 2} \, \overline{\delta \beta^{\, i} \, 6 \beta^{\, i}} \, - \, 2 \Delta \hat{V}_{z}^{i} \, \mu^{\, i} \, \zeta^{\, i} \, \overline{\delta \beta^{\, i}} \, \qquad (7) \\ \tilde{Q}_{12}^{i} &= \, \tilde{Q}_{21}^{i} \, = \, \xi^{\, i} \, \Delta \hat{V}_{x}^{i} \, \Delta \hat{V}_{y}^{i} \, + \, \frac{\xi^{\, i}}{\mu^{\, i}} \, \left[ 2 \Delta \hat{V}_{x}^{i} \, \Delta \hat{V}_{y}^{i} \, \Delta \hat{V}_{z}^{i} \, \overline{\delta \beta^{\, i}} \, - \, \rho^{\, i} \left( \Delta \hat{V}_{x}^{i\, 2} \, - \, \Delta \hat{V}_{y}^{i\, 2} \right) \, \overline{\delta \alpha^{\, i}} \, \right] \, + \\ & \frac{1}{\mu^{\, i}\, 2} \, \left[ - \, \rho^{\, i\, 2} \, \Delta \hat{V}_{x}^{i} \, \Delta \hat{V}_{y}^{i} \, \overline{\delta \alpha^{\, i} \, \delta \alpha^{\, i}} \, + \, \rho^{\, i} \, \Delta \hat{V}_{z}^{i} \, \left( \Delta \hat{V}_{y}^{i\, 2} \, - \, \Delta \hat{V}_{x}^{i\, 2} \right) \, \overline{\delta \alpha^{\, i}} \, \overline{\delta \beta^{\, i}} \, + \\ & \Delta \hat{V}_{x}^{i} \, \Delta \hat{V}_{y}^{i} \, \Delta \hat{V}_{z}^{i} \, \overline{\delta \beta^{\, i}} \, \overline{\delta \beta^{\, i}} \, \right] \, \qquad \qquad (8) \\ \tilde{Q}_{13}^{i} &= \, \tilde{Q}_{31}^{i} \, = \, \xi^{\, i} \, \Delta \hat{V}_{x}^{i} \, \Delta \hat{V}_{z}^{i} \, + \, \xi^{\, i} \, \left[ \frac{\Delta \hat{V}_{z}^{i}}{\mu^{\, i}} \, \left( \rho^{\, i} \Delta \hat{V}_{y}^{i} \, \overline{\delta \alpha^{\, i}} \, + \, \Delta \hat{V}_{x}^{i} \, \Delta \hat{V}_{z}^{i} \, \overline{\delta \beta^{\, i}} \right) - \, \mu^{\, i} \, \Delta \hat{V}_{x}^{i} \, \overline{\delta \beta^{\, i}} \, \right] \\ & - \, \rho^{\, i} \Delta \hat{V}_{y}^{i} \, \overline{\delta \alpha^{\, i}} \, \overline{\delta \beta^{\, i}} \, - \, \Delta \hat{V}_{x}^{i} \, \Delta \hat{V}_{z}^{i} \, \overline{\delta \beta^{\, i}} \, \overline{\delta \beta^{\, i}} \, - \, \rho^{\, i} \Delta \hat{V}_{x}^{i} \, \overline{\delta \alpha^{\, i}} \, \overline{\delta \beta^{\, i}} \, - \, \Delta \hat{V}_{x}^{i} \, \Delta \hat{V}_{z}^{i} \, \overline{\delta \beta^{\, i}} \, \overline{\delta \beta^{\, i}} \, - \, \rho^{\, i} \Delta \hat{V}_{x}^{i} \, \overline{\delta \alpha^{\, i}} \, \right] \\ & - \, \rho^{\, i} \Delta \hat{V}_{y}^{i} \, \overline{\delta \alpha^{\, i}} \, \overline{\delta \beta^{\, i}} \, - \, \Delta \hat{V}_{x}^{i} \, \Delta \hat{V}_{z}^{i} \, \overline{\delta \beta^{\, i}} \, \overline{\delta \beta^{\, i}} \, - \, \rho^{\, i} \Delta \hat{V}_{x}^{i} \, \overline{\delta \beta^{\, i}} \, - \, \rho^{\, i} \Delta \hat{V}_{x}^{i} \, \overline{\delta \beta^{\, i}} \, \right] \\ & - \, \rho^{\, i} \Delta \hat{V}_{y}^{i} \, \overline{\delta \alpha^{\,$$

where

$$\xi^{\dagger} = \overline{\mathbf{k}^{\dagger} \ \mathbf{k}^{\dagger}} + \frac{2}{\rho^{\dagger}} \overline{\mathbf{k}^{\dagger}} \overline{\mathbf{s}^{\dagger}} + \frac{\overline{\mathbf{s}^{\dagger}} \mathbf{s}^{\dagger}}{\rho^{\dagger 2}}$$
 (11)

and

$$\zeta^{\dagger} = \overline{k^{\dagger}} + \frac{\overline{s^{\dagger}}}{\rho^{\dagger}} \qquad (12)$$

SUBROUTINE GUID

PURPOSE COMPUTE GUIDANCE MATRIX, VARIATION MATRIX, AND TARGET COMDITION COVARIANCE MATRIX AT A MIDCOURSE GUIDANCE EVENT IN THE ERROR ANALYSIS PROGRAM

SUBROUTINES SUPPORTED: GUIDM

CALLING SEQUENCE: CALL GUID

SUBROUTINES REQUIRED: EPHEN HYELS JACOBI MATIN NTM
ORB PARTL PSIM STMPR VARADA

LOCAL SYMBOLS A TWO-VARIABLE B-PLANE GUIDANCE SUB-MATRIX A

BB TWO-VARIABLE B-PLANE GUIDANCE SUB-MATRIX B

BDRS B DOT R

BDR1 VALUE OF B DOT R RETURNED FROM PARTL (NOT

USED)

BDTS B DOT T

BDT1 VALUE OF B DOT T RETURNED FROM PARTL (NOT

USED)

BS MAGNITUDE OF B VECTOR

B1 VALUE OF B RETURNED FROM PARTL (NOT USED)

D INTERMEDIATE JULIAN DATE

DUM1 ARRAY OF EIGENVECTORS

EGVCT ARRAY OF EIGENVECTORS

EGVL ARRAY OF EIGENVALUES

ICS INTERNEDIATE STORAGE FOR ICL2

ICLS INTERMEDIATE STORAGE FOR ICL

INCHTS INTERMEDIATE STORAGE FOR INCHT

IPR INTERMEDIATE STORAGE FOR IPRINT

ISP INTERMEDIATE STORAGE FOR ISP2

PBR PARTIAL OF B DOT R WITH RESPECT TO STATE

VECTOR

PBT PARTIAL OF B DOT T WITH RESPECT TO STATE

# VECTOR

PHI1	INTERMEDIATE ARRAY					
PHI2	INTERNEDIATE ARRAY					
PHI3	INTERMEDIATE ARRAY					
RI	NOMINAL SPACECRAFT STATE AT GUIDANCE EVENT					
ROW	TARGET CONDITION CORRELATION MATRIX					
RTPS	INERTIAL SPACECRAFT STATE AT SPHERE OF INFLUENCE					
SQP	TARGET CONDITION STANDARD DEVIATIONS					
TCA	TRAJECTORY TIME AT CLOSEST APPROACH					
TSI	TRAJECTORY TIME AT SPHERE OF INFLUENCE					
XCA	INERTIAL SPECECRAFT STATE AT CLOSEST APPROACH					
XSIP	SPACECRAFT POSITION RELATIVE TO TARGET PLANET AT SPHERE OF INFLUENCE					
XSIV	SPACECRAFT VELOCITY RELATIVE TO TARGET PLANET AT SPHERE OF INFLUENCE					
COMMON COMPUTED/USED:	ICL2 IPRINT ISPH ISP2 NO XP					
COMMON COMPUTED:	DELTH EM TRTM1 TSOI1					
COMMON USEDS	ALNGTH BDR BDT B DATEJ DC DSI FNTM FOV F IBARY ICL IHYP1 ISTMC NBOD NB NTMC NTP ONE PHI P RC RSI TM VSI ZERO					

## GUID Analysis

Subroutine GUID is used in the error analysis mode to compute the same quantities which subroutine GUIS computes in the simulation mode. Subroutine GUID differs from GUIS in that instead of calling NTMS and VARSIM as does GUIS, subroutine GUID calls NTM and VARADA. In addition, the state transition and variation matrices computed in GUID are referenced to the targeted nominal since the most recent nominal is not defined for the error analysis mode. These differences entail only minor logic differences in the flow chart for GUID, and for this reason no GUID flow chart is presented. See subroutine GUIS analysis and flow chart for further details.

SUBROUTINE GUIDM

PURPOSE CONTROL EXECUTION OF A GUIDANCE EVENT IN THE ERROR ANALYSIS PROGRAM

CALLING SEQUENCE: CALL GUIDM

SUBROUTINES SUPPORTED: ERRANN

SUBROUTINES REQUIRED: CORREL DYNO GUID HYELS JACOBI

NAVM PSIM STMPR

LOCAL SYMBOLS: ADA VARIATION MATRIX

AMAX INTERMEDIATE VARIABLE USED TO FIND MAXIMUM

EIGENVALUE OF VELOCITY CORRECTION

COVARIANCE MATRIX (S MATRIX)

CXSU1 STORAGE FOR CXSU KNOWLEDGE COVARIANCE

CXSV1 STORAGE FOR CXSV KNOWLEDGE COVARIANCE

CXU1 STORAGE FOR CXU KNOWLEDGE COVARIANCE

CXV1 STORAGE FOR CXV KNOWLEDGE COVARIANCE

CXXS1 STORAGE FOR CXXS KNOWLEDGE COVARIANCE

DUM1 INTERMEDIATE VARIABLE

DUM VECTOR SUM OF UPDATE AND STATISTICAL

**VELOCITY CORRECTIONS** 

EGM MAXIMUM EIGENVALUE OF S MATRIX

EGYCT ARRAY OF EIGENVECTORS

EGVL ARRAY OF EIGENVALUES

EXEC EXECUTION ERROR COVARIANCE MATRIX

EXV EXPECTED VALUE OF VELOCITY CORRECTION

GA GUIDANCE MATRIX

GAP INTERHEDIATE ARRAY EQUAL TO GA TIMES P

ICODE INTERNAL CONTROL FLAG

ICODE2 INTERNAL CONTROL FLAG

IGP MIDCOURSE GUIDANCE POLICY CODE

IQP EXECUTION ERROR CODE ISPHC TEMPORARY STORAGE FOR ISPH MAP INDEX OF MAXIMUM EIGENVALUE OF S OUT SPACECRAFT VELOCITY RELATIVE TO TARGET PLANET IN PLANETO-CENTRIC EQUATORIAL COORDINATES PS1 STORAGE FOR PS KNOWLEDGE COVARIANCE P1 STORAGE FOR P KNOWLEDGE COVARIANCE RF NOMINAL TRAJECTORY STATE AT GUIDANCE EVENT MAGNITUDE OF STATISTICAL DELTA-V RHO ROW INTERMEDIATE VECTOR . . SDV STANDARD DEVIATION OF MAGNITUDE OF STATISTICAL DELTA-Y SQP INTERMEDIATE VECTOR TRS TRACE OF S MATRIX U INTERMEDIATE VARIABLE VEIG MATRIX TO BE DIAGONALIZED Z INTERMEDIATE ARRAY COMMON COMPUTED/USED: CXSUG CXSU CXSVG CXSV CXUG CXU CXVG CXV CXXSG CXXS ISPH NGE PG PSG PS TG XG COMMON COMPUTEDS DELTM TRTM1 XI COMMON USEDS FOP FOV ICDQ3 ICDT3 IEIG IHYP1 ISTMC NDIM1 SMIGN NDIM3 ONE Q SIGALP SIGBET SIGPRO SIGRES TWO UO VO XF ZERO

#### GUIDM Analysis

Subroutine GUIDM is the executive guidance subroutine in the error analysis program. In addition to controlling the computational flow for all types of guidance events, GUIDM also performs many of the required guidance computations itself.

Before considering each type of guidance event, the treatment of a general guidance event will be discussed. Let  $t_j$  be the time at which the guidance event occurs. Before any guidance event can be executed the targeted nominal state  $\overline{X_j}$ , knowledge covariance  $P_{\overline{X_j}}$ , and control covariance  $P_{\overline{X_j}}$  must all be available, where ( ) indicates values immediately before the event. The first two quantities are available prior to entering GUIDM. However, GUIDM controls the propagation of the control covariance over the interval  $\begin{bmatrix} t_{j-1}, t_j \end{bmatrix}$ , where  $t_{j-1}$  denotes the time of the previous guidance event.

The next step in the treatment of a general guidance event is concerned with the computation of the commanded velocity correction and the execution error covariance. In the error analysis program a non-statistical velocity correction is computed whenever the nominal target conditions are changed; otherwise, only a statistical velocity correction can be computed. The commanded velocity correction  $\Delta \hat{V}_j$  is then used to compute the execution error covariance matrix  $\hat{Q}_j$ . A summary of the execution error model and the equations used to compute  $\hat{Q}_j$  can be found in the subroutine  $QCOMP^{\Lambda^*}$  analysis section.

The last step is concerned with the updating of required quantities prior to returning to the basic cycle. An assumption underlying the modeled guidance process is that the targeted nominal is always updated by the commanded velocity correction. In the error analysis program only the non-statistical component is used to perform the state update and is indicated by the variable  $\Delta V_{\rm UP}$ . Thus, the targeted nominal state immediately following the guidance event is given by

$$\overline{X}_{j}^{+} = \overline{X}_{j}^{-} + \begin{bmatrix} 0 \\ \Delta V_{UP_{j}} \end{bmatrix}$$
.

The knowledge covariance is updated using the equation

$$P_{K_{j}}^{+} = P_{K_{j}}^{-} + \left[ \frac{0}{0} - \frac{1}{1} - \frac{0}{\widetilde{Q}_{j}} \right]$$

if an impulsive thrust model is assumed. If the thrust is modeled as a series of impulses, then an effective execution error covariance  $\widetilde{Q}_{eff}$  is computed and the knowledge covariance is updated using the equation

$$P_{K_j}^+ = P_{K_j}^- + \tilde{Q}_{eff}$$
.

In either case the control covariance is updated simply by setting

$$P_{c_{j}}^{+} = P_{K_{j}}^{+}$$
.

This equation is a direct consequence of the assumption that the targeted nominal state is always updated at a guidance event.

A "compute only" option is available in GUIDM in which all of the ( ) the quantities will still be computed and printed. However, the state and all covariances are then reset to their former ( ) values prior to returning to the basic cycle.

Each specific type of guidance event involves the computation of other quantities not discussed above. These will be covered in the following discussion of specific guidance events.

Midcourse and Biased Aimpoint Guidance

Linear midcourse guidance policies have form

$$\Delta \hat{\mathbf{v}}_{\mathbf{N}_{\mathbf{j}}} = \mathbf{r}_{\mathbf{j}} \delta \hat{\mathbf{x}}_{\mathbf{j}}$$

where the subscript N indicates that this is the velocity correction required to mull out deviations from the nominal target state. This notation is required to differentiate between this type of velocity correction and velocity corrections required to achieve an altered target

state. Linear midcourse guidance policies are discussed in more detail in the subroutine GUIS analysis section.

Subroutine GUIDM calls GUID to compute the guidance matrix,  $\Gamma_j$ , and the target condition covariance immediately prior to the guidance event, W , and then uses  $\Gamma_j$  to compute the velocity correction covariance S , which is defined as

$$\mathbf{s}_{j} = \mathbf{E} \left[ \Delta \hat{\mathbf{v}}_{\mathbf{N}_{j}} \Delta \hat{\mathbf{v}}_{\mathbf{N}_{j}}^{\mathbf{T}} \right],$$

and is given by the equation

$$s_j = \Gamma_j (P_{c_j} - P_{K_j}) \Gamma_j^T$$
.

This equation assumes that an optimal estimation algorithm is employed in the navigation process, since the derivation of this equation requires the orthogonality of the estimate and the estimation error.

In the error analysis program  $\Delta V_N$  is never available since no estimates  $\delta X_j$  are ever generated. Only the ensemble statistics of  $\delta X_j$  are available which means only a statistical or effective velocity correction "E  $\left[\Delta \hat{V}_N\right]$ " can be computed. In the STEAP error analysis program this effective velocity correction is assumed to have form

"E 
$$\left[\Delta \hat{V}_{K_{j}}\right]$$
" =  $\rho_{j} \frac{\alpha_{j}}{|\alpha_{j}|}$ .

The magnitude  $\rho$ , is given by the Hoffman-Young approximation

$$\rho_{j} = \sqrt{\frac{2A}{\pi}} \left( 1 + \frac{B(\pi - 2)}{A^2 \sqrt{5.4}} \right)$$

where

A = trace 
$$S_j = \lambda_1 + \lambda_2 + \lambda_3$$
,  

$$B = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3$$
,

and  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$  are the eigenvalues of  $S_j$ . The direction of the effective velocity correction is assumed to coincide with the eigenvector corresponding to the maximum eigenvalue of  $S_j$ . This eigenvector is denoted by  $\alpha_j$ . An alternate model assumes the direction coincides with the vector  $(\lambda_1, \lambda_2, \lambda_3)$ .

If planetary quarantive constraints must be satisfied at a midcourse correction, GUIDM calls BIAIM to compute the new aimpoint  $\mu_j$  and the (non-statistical) bias velocity correction  $\Delta V_B$ . All computations in BIAIM are based on linear guidance theory. However, an option is available in GUIDM to recompute  $\Delta V_B$ , but not  $\mu_j$ , using nonlinear techniques. This option is recommended if a biased aimpoint guidance event occurs at  $t_j$  = injection time. It should also be noted that  $\widetilde{Q}_j$  is set to zero if  $t_j$  = injection time since it is assumed that the injection covariance does not change for small changes in injection velocity.

After the updated control covariance  $P_{c}^{\dagger}$  has been computed, the target condition covariance matrix  $W_{j}^{\dagger}$  following the guidance correction is computed using the equation

$$(\mathbf{W}) = \eta_{\mathbf{j}} \mathbf{P}_{\mathbf{c}_{\mathbf{j}}}^{+} \eta_{\mathbf{j}}^{\mathbf{T}}$$

where variation matrix  $\eta_i$  has been previously computed in subroutine GUID.

#### 2. Re-targeting

In the error analysis (and simulation) program a re-targeting event is defined to be the computation of a velocity correction  $\Delta \widehat{V}_{RT}$  required to achieve a new set of target conditions using nonlinear techniques. Since the original targeted nominal will be used as the zero-th iterate in the re-targeting process, the new target conditions must be close enough to the original nominal target condition to ensure a covergent process.

It should be noted that after a re-targeting event the new target conditions are henceforth treated as the nominal target conditions.

## 3. Orbital insertion

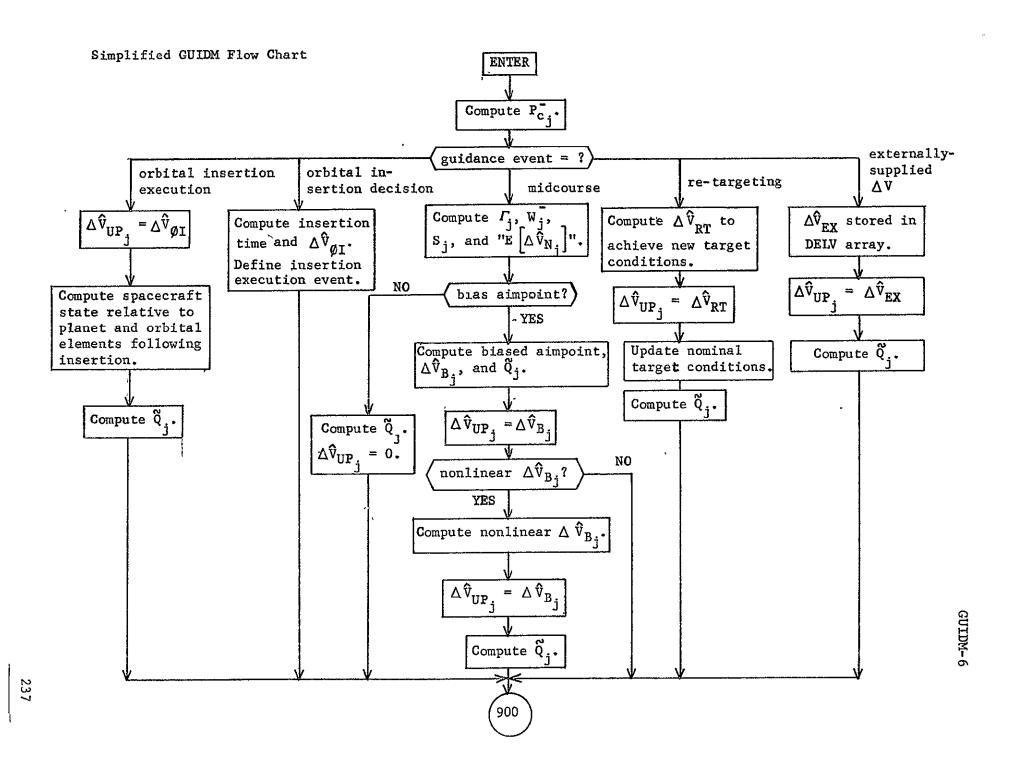
An orbital insertion event is divided into a decision event and an execution event. At a decision event the orbital insertion velocity correction  $\Delta V_{\rho I}$  and the time interval  $\Delta t$  separating decision and execution are computed based on the targeted nominal state at  $t_j$ . The relevant equations can be found in the subroutine CPPINS analysis section for coplanar orbital insertion; in NPPINS, for non-planar orbital insertion. Before returning to the basic cycle, GUIDM schedules the orbital insertion execution event to occur at  $t_j$  +  $\Delta t$  and re-orders the necessary event arrays accordingly.

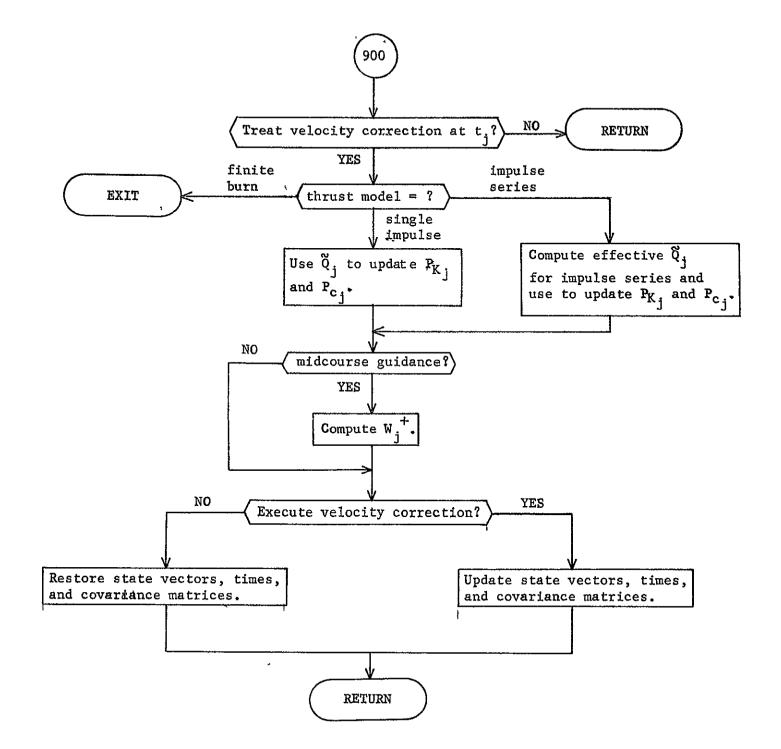
At an orbital insertion execution event the targeted nominal state is updated using the previously computed  $\Delta \hat{V}_{pl}$ . In addition, the planeto-centric equatorial components of  $\Delta \hat{V}_{pl}$  and the nominal spacecraft cartesian and orbital element state following the insertion maneuver are computed.

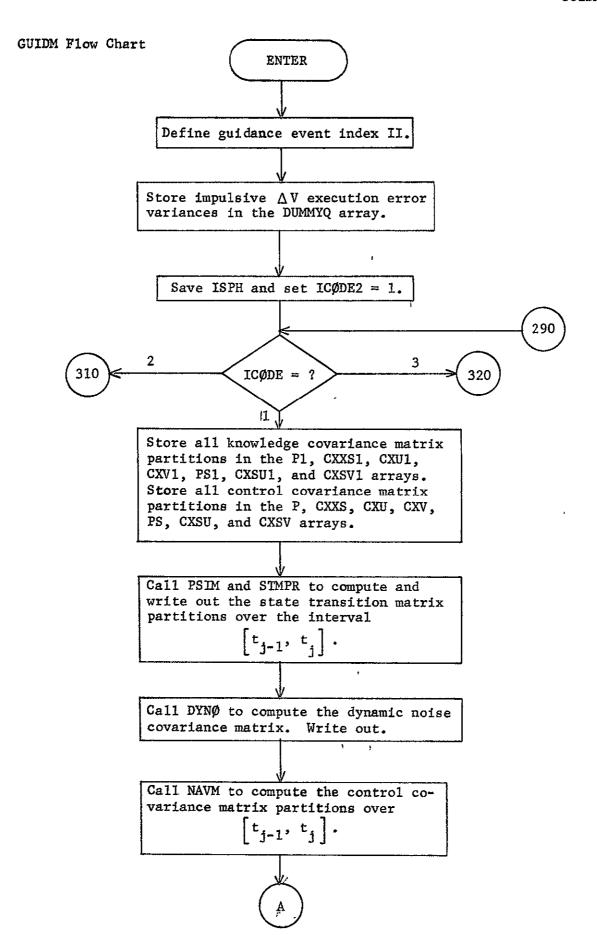
### 4. Externally-supplied velocity correction

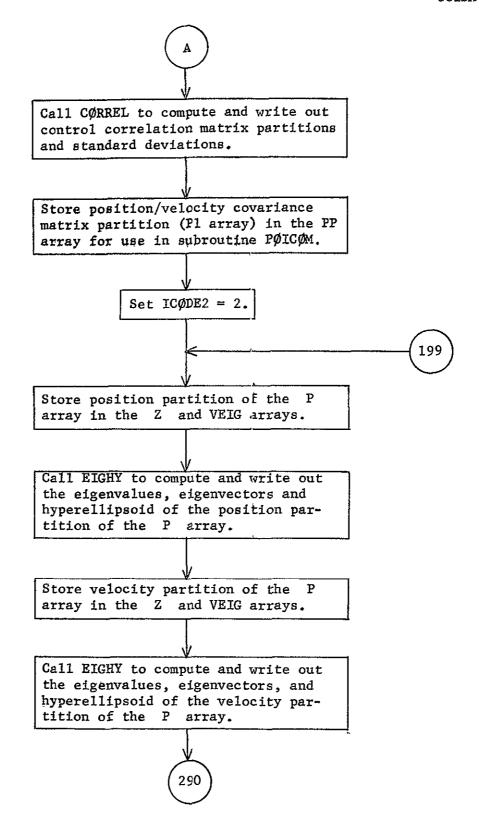
At this type of guidance event the targeted nominal state is simply updated using the externally-supplied velocity correction  $\Delta \hat{V}_{EX}$ .

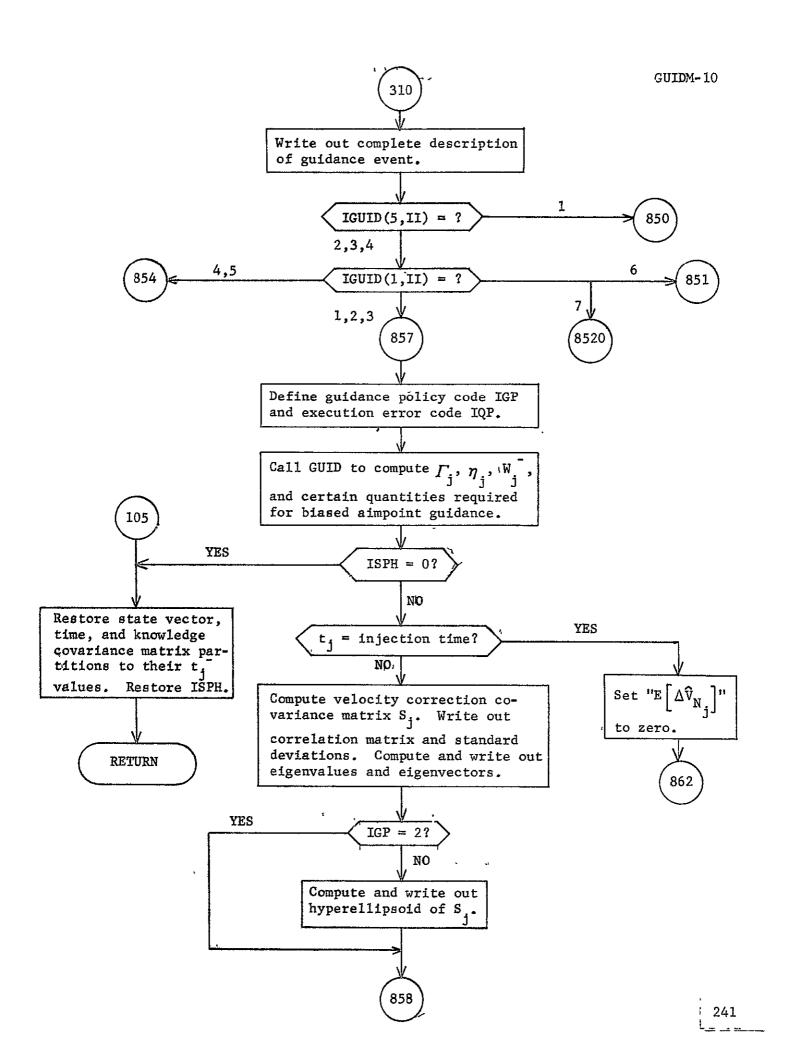
Because of the complexity of the GUIDM flow chart, a simplified flow chart depicting the main elements of the GUIDM structure precedes the complete GUIDM flow chart.

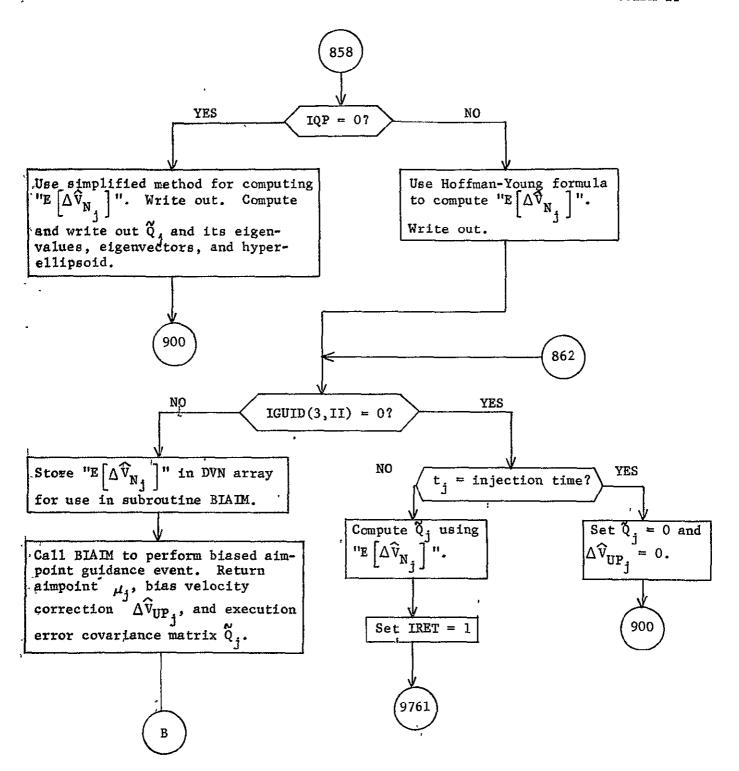


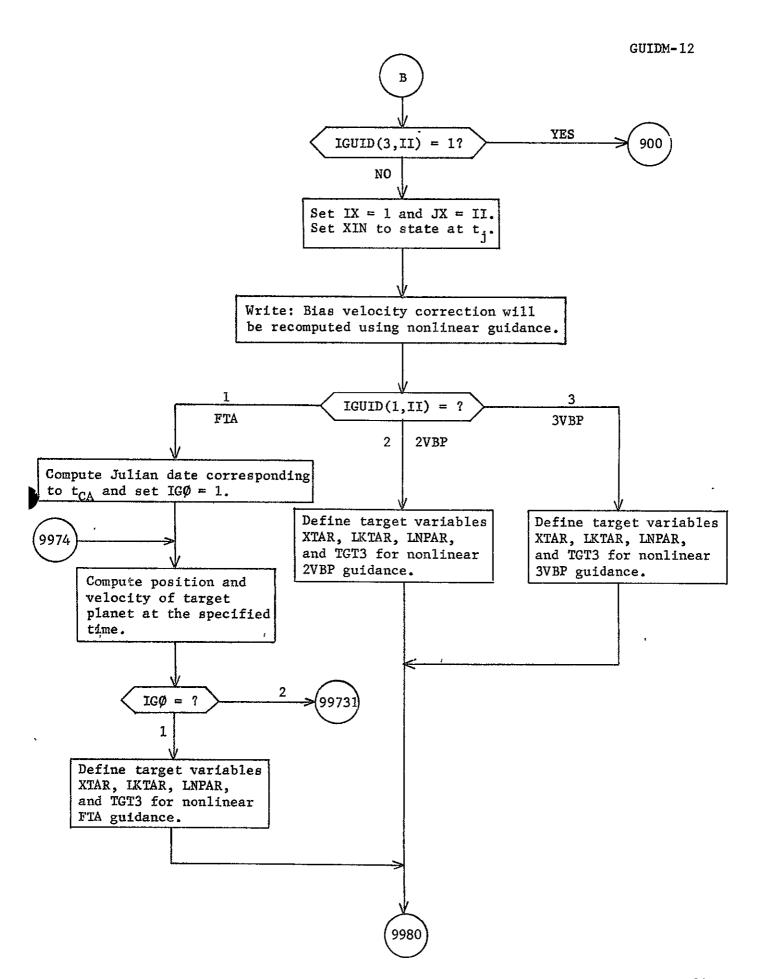


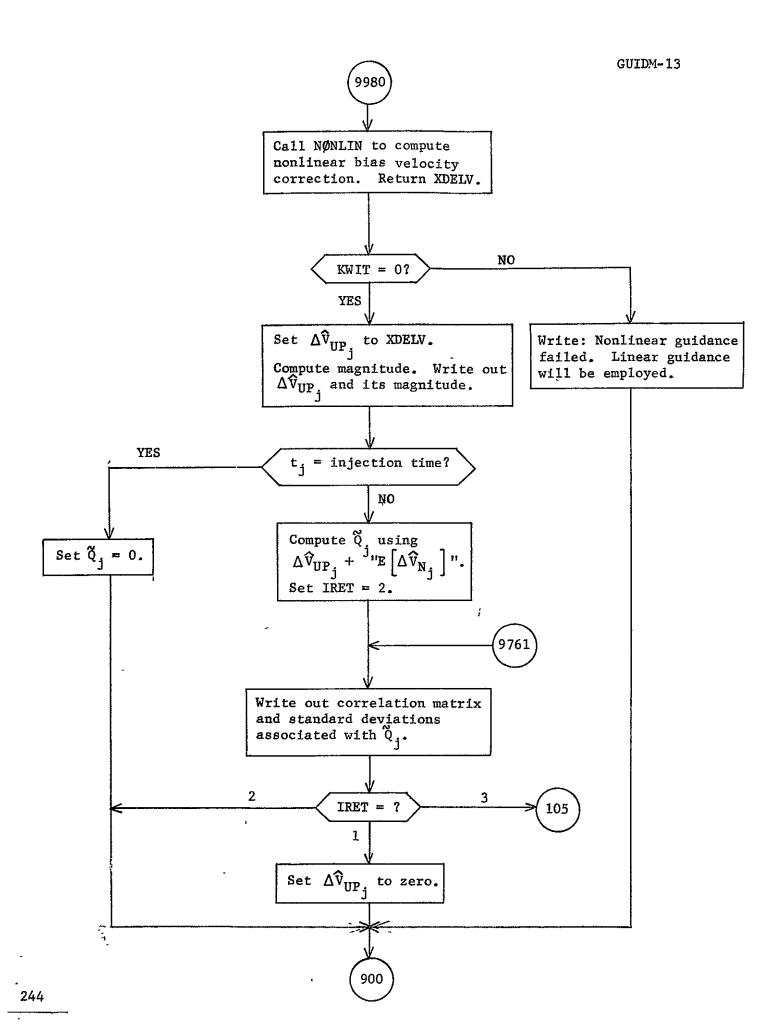


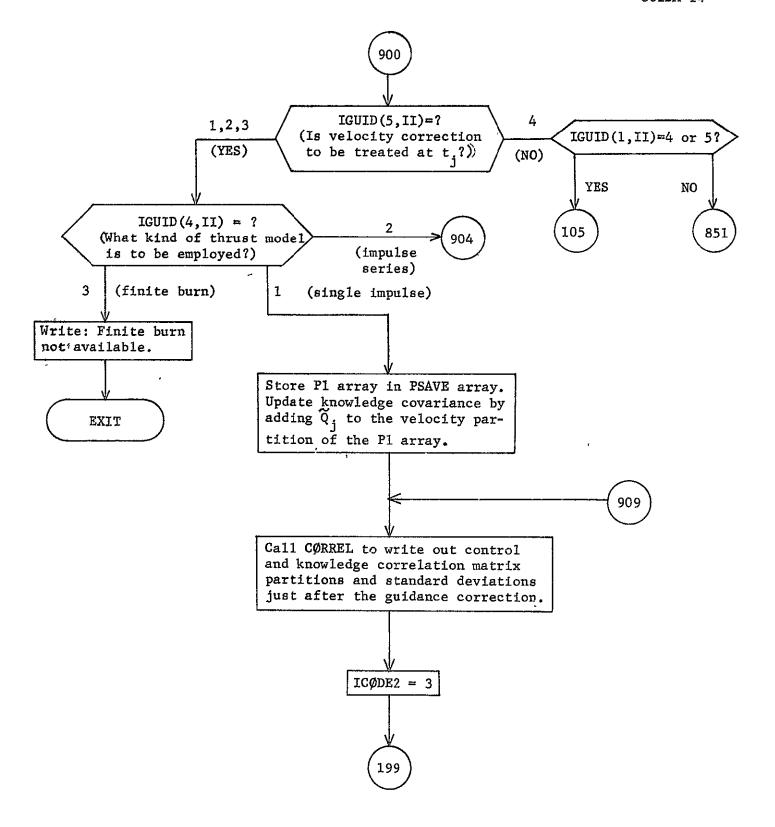


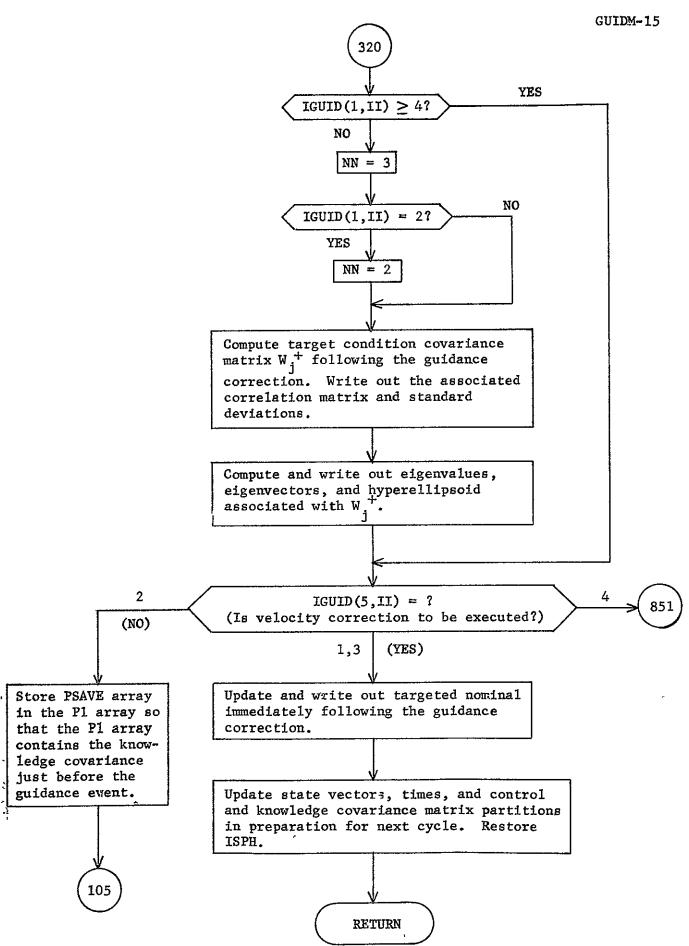


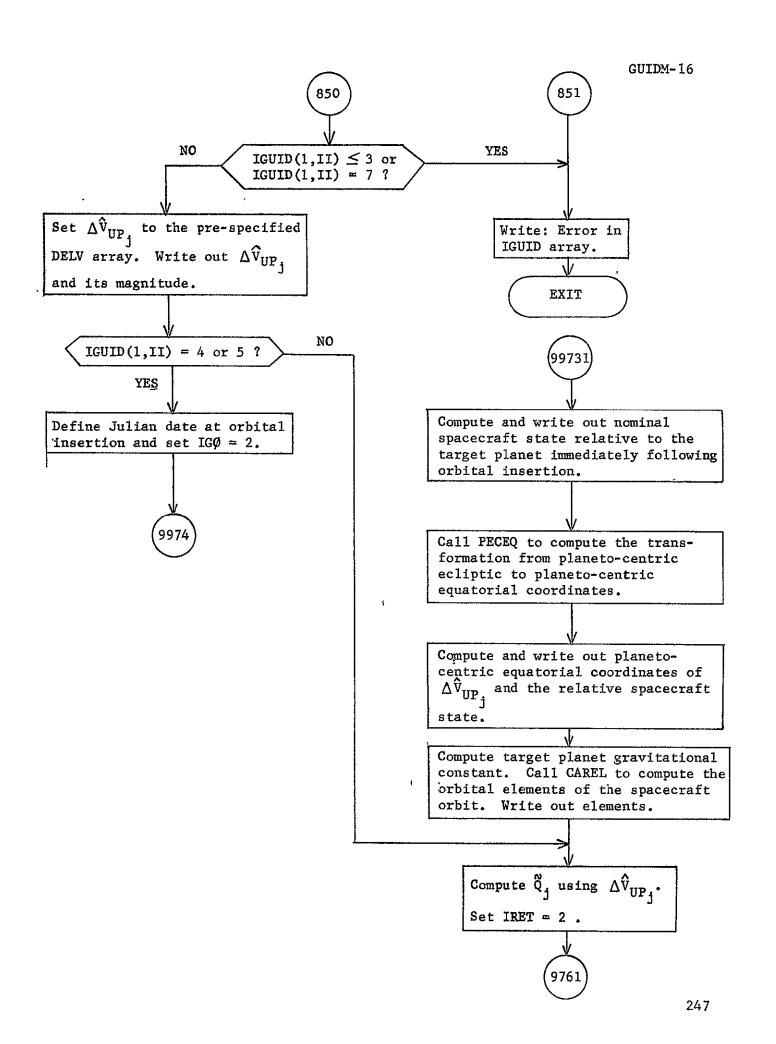


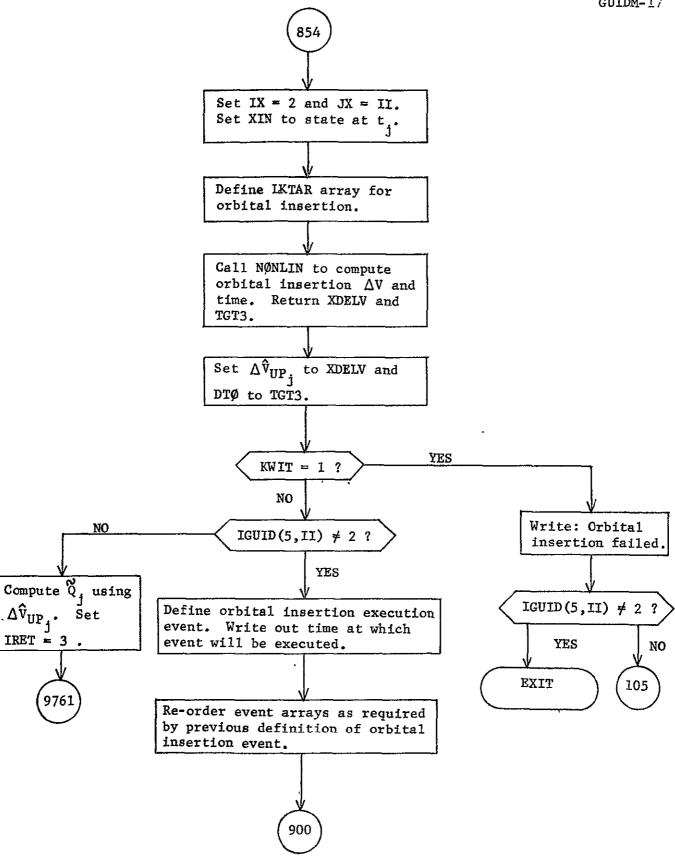


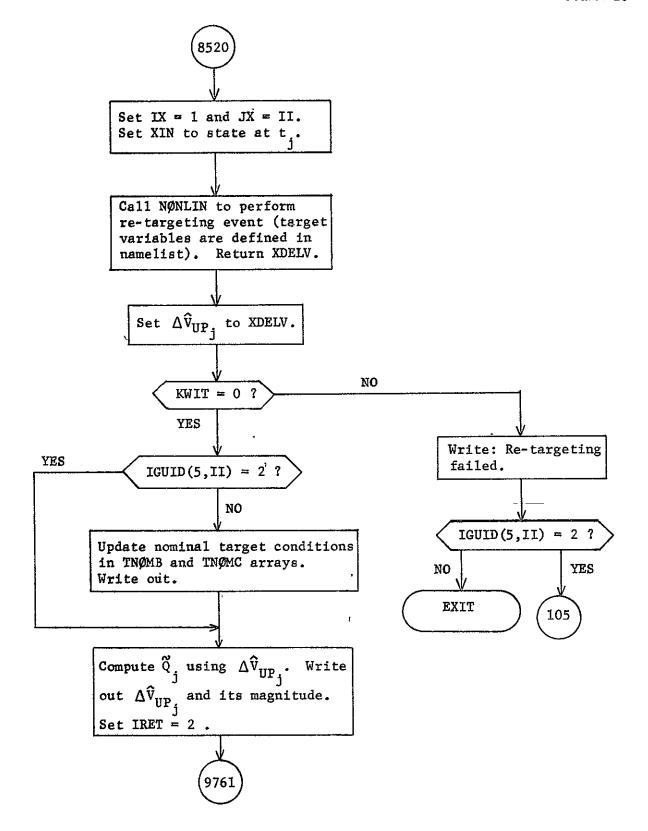


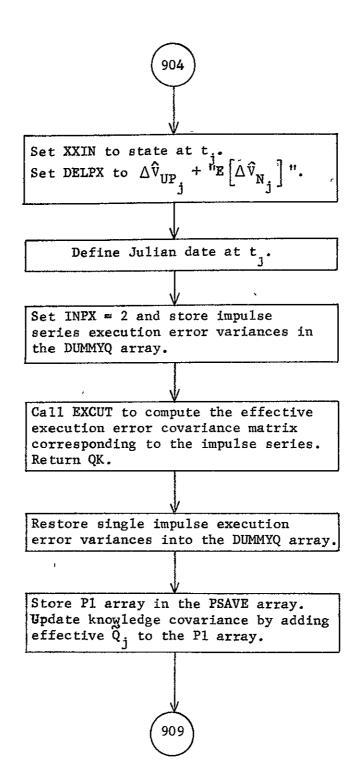












PROGRAM GUIS

PURPOSE COMPUTE GUIDANCE MATRIX, VARIATION MATRIX, AND TARGET

CONDITION COVARIANCE MATRIX AT A MIDCOURSE GUIDANCE

EVENT IN THE SIMULATION PROGRAM

SUBROUTINES SUPPORTED & GUISIM

SUBROUTINGS REQUIRED: EPHEM HYELS JACOBI HATIN NTHS

ORB PARTL PSIN STMPR VARSIM

LOCAL SYMBOLS A TWO-VARIABLE B-PLANE GUIDANCE SUB-MATRIX A

BB TWO-VARIABLE B-PLANE GUIDANCE SUB-MATRIX B

BDR1 VALUE OF B DOT R RETURNED FROM PARTL (NOT

USED)

BDT1 VALUE OF B DOT T RETURNED FROM PARTL (NOT

USED)

81 MAGNITUDE OF 8 VECTOR RETURNED FROM PARTL

(NOT USED)

DUM1 ARRAY OF EIGENVECTORS

DUM INTERMEDIATE ARRAY

EGYCT ARRAY OF EIGENVECTORS

EGVL ARRAY OF EIGENVALUES

ICLS INTERMEDIATE STORAGE FOR ICL

ICS INTERMEDIATE STORAGE FOR ICL2

IPR INTERMEDIATE STORAGE FOR IPRINT

ISPS INTERMEDIATE STORAGE FOR ISP2

PBR' PARTIAL OF B DOT R WITH RESPECT TO STATE

VECTOR

PBT PARTIAL OF B DOT 1 WITH RESPECT TO STATE

VECTOR

PHI1 INTERMEDIATE ARRAY

PHI2 INTERMEDIATE ARRAY

PHI3 INTERMEDIATE ARRAY

RI1	MOST RECENT NOMINAL SPACECRAFT STATE AT GUIDANCE EVENT						
ŔI	TARGETED NOMINAL SPACECRAFT STATE AT GUIDANCE EVENT						
RMCA	SPACECRAFT DISTANCE FROM TARGET PLANET AT CLOSEST APPROACH						
RMSI	SPACECRAFT DISTANCE FROM TARGET PLANET A SPHERE OF INFLUENCE						
ROW	TARGET CONDITION CORRELATION MATRIX						
RTPS	INERTIAL SPACECRAFT STATE AT SPHERE OF INFLUENCE						
SQP	TARGET CONDITION STANDARD DEVIATIONS						
TCA	TRAJECTORY TIME AT CLOSEST APPROACH						
TSI	TRAJECTORY TIME AT SPHERE OF INFLUENCE						
VMCA	MAGNITUDE OF VELOCITY OF VEHICLE RELATIVE TO TARGET PLANET AT CLOSEST APPROACH						
VMSI	MAGNITUDE OF VELOCITY OF VEHICLE RELATIVE TO TARGET PLANET AT SPHERE OF INFLUENCE SPACECRAFT VELOCITY RELATIVE TO TARGET PLANET AT SPHERE OF INFLUENCE						
XCA							
COMMON COMPUTED/USED8 \	EM ICL2 IPRINT ISPH ISP2 NO XP						
COMMON COMPUTED:	DELTM TRTM1						
COMMON USED:	ALNGTH BDR BDT B DATEJ DC DSI FNTM FOV F IBARY IHYP1 ISOI1 ISTMC NBOD NB NGE NQE NTMC NTP ONE PHI P RC RSI TM VSI ZERO						

GUIS Analysis

Subroutine GUIS is called at a midcourse guidance event at t in the simulation mode to compute three primary quantities for the selected midcourse guidance policy. These three quantities are the variation matrix  $\eta_j$ , the target condition covariance matrix prior to the velocity correction  $W_j$ , and the guidance matrix  $\Gamma_j$ . Three midcourse guidance policies are available: fixed-time-of-arrival (FTA), two-variable B-plane (2VBP), and three-variable B-plane (3VBP). All are linear impulsive guidance policies having form

$$\triangle \hat{v}_{j} = \Gamma_{j} \delta \hat{x}_{j}$$

where  $\triangle \hat{v}_j$  is the commanded velocity correction, and  $\delta \hat{x}_j$  is the estimate of the spacecraft position/velocity deviation from the targeted nominal. The relevant equations for each guidance policy will be summarized below.

The variation matrix  $\eta_j$  for FTA guidance relates deviations in spacecraft state at  $t_j$  to position deviations at time of closest approach  $t_{CA}$ , and is given by

$$\eta_{i} = \left[ \varphi_{1} \mid \varphi_{2} \right]$$

where  $\begin{bmatrix} \emptyset_1 & \emptyset_2 \end{bmatrix}$  is the upper half of the state transition matrix  $\Phi(t_{CA}, t_j)$ . The guidance matrix for FTA guidance is given by

$$\Gamma_{1} = \left[-\varphi_{2}^{-1} \varphi_{1} \right] - 1$$

The variation matrix for 3VBP guidance relates deviations in spacecraft state at t to deviations in B-T, B-R, and  $t_{\rm SI}$ , where  $t_{\rm SI}$  is the time at which the sphere of influence is pierced. Unlike the variation matrix for FTA guidance, which can be computed analytically or by numerical differencing, the 3VBP variation matrix must always be computed using numerical differencing since no good analytical formulas are available which relate deviations in spacecraft state at t to deviations in  $t_{\rm SI}$ . If the

variation matrix is written as

$$\eta_{j}^{\lambda} = \left[ \eta_{1} \mid \eta_{2} \right]$$

then the guidance matrix for 3VBP guidance is given by

$$\Gamma_{\mathbf{J}}^{,} = \left[ -\eta_{2}^{-1} n_{\mathbf{J}} \right] - \mathbf{I}$$

The variation matrix for 2VBP guidance relates deviations in spacecraft state at t to deviations in B.T and B.R and is given by  $\hat{J}$ 

$$\eta_{j} = M \Phi (t_{SI}, t_{j})$$

where M is an analytically computed matrix relating B·T and B·R deviations to spacecraft state deviations at  $t_{SI}$ , and  $\Phi(t_{SI},t_j)$  is the state transition matrix over  $\left[t_j,t_{SI}\right]$ . If  $\eta_i$  is written as

$$\eta_{i} = [A \mid B]$$

then the guidance matrix for 2VBP guidance is given by

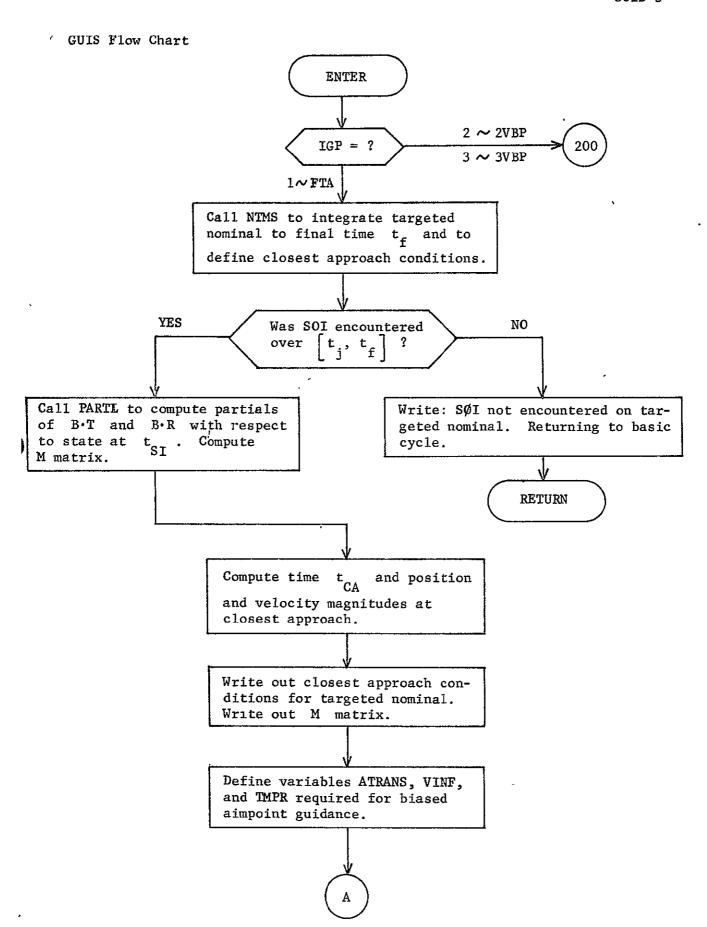
$$\Gamma = \left[ -B^{T} (BB^{T})^{-1} A \right] -B^{T} (BB^{T})^{-1} B$$

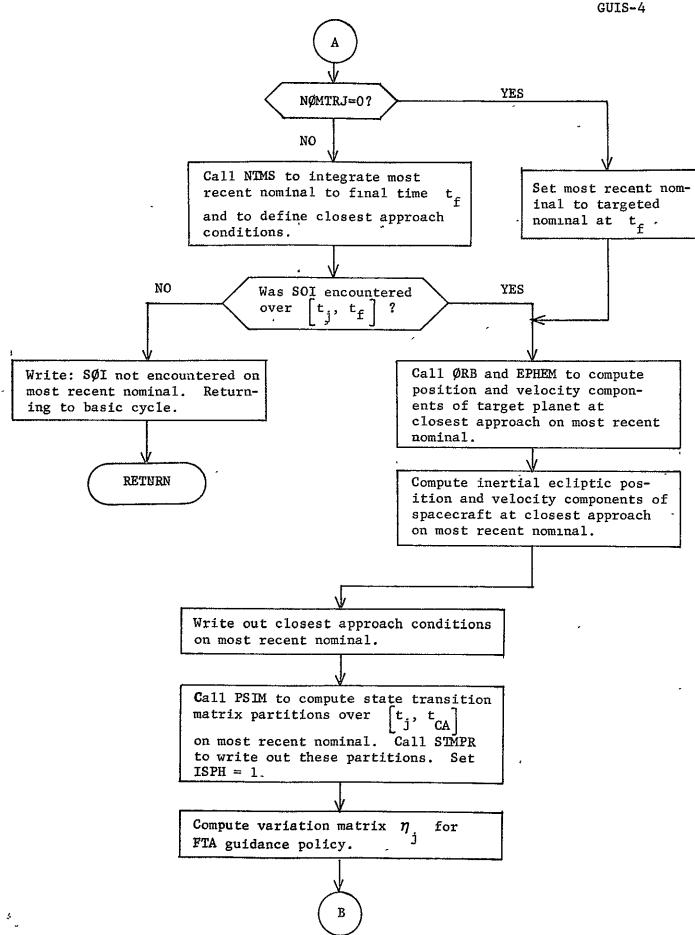
All state transition matrices and, hence, all variation matrices used by the above three guidance policies are referenced to the most recent nominal trajectory for improved numerical accuracy.

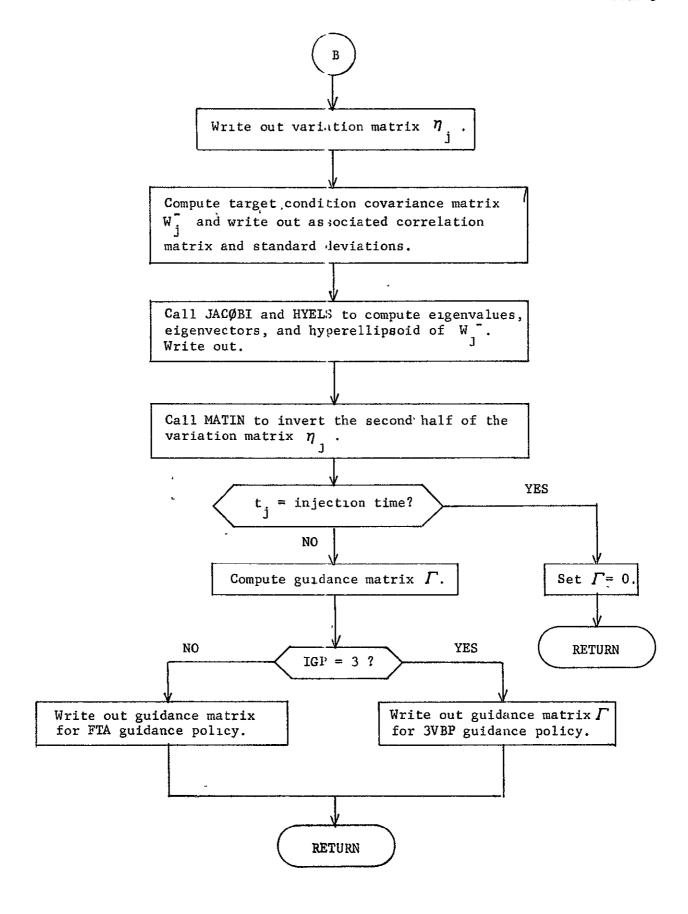
Once the variation matrix  $\eta_j$  is available for any of the above guidance policies, the target condition covariance matrix can be computed using

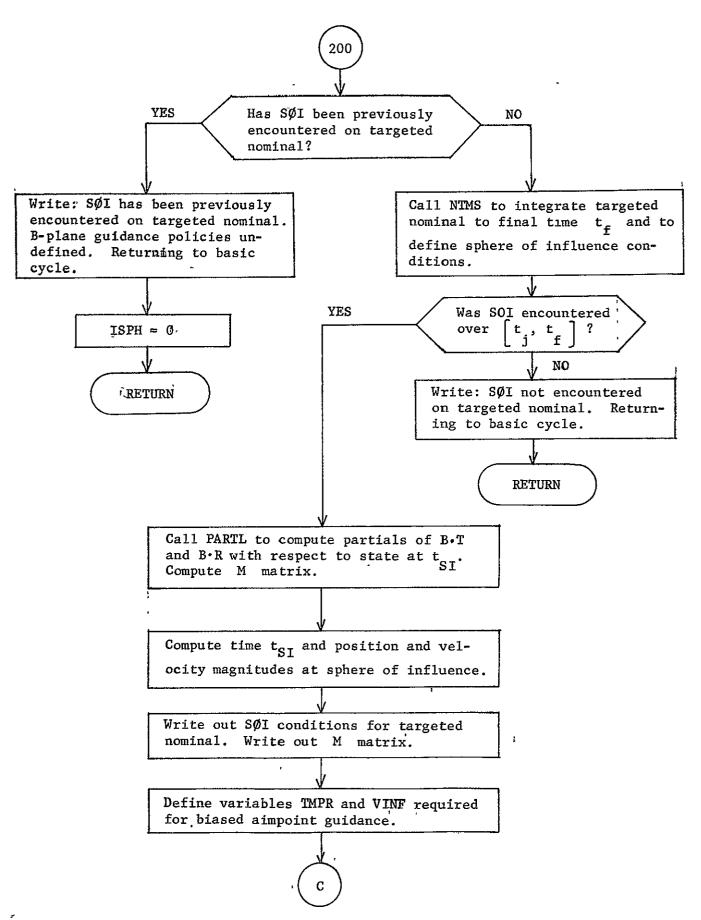
$$\mathbf{w}_{\mathbf{j}} = \eta_{\mathbf{j}} \mathbf{P}_{\mathbf{c}_{\mathbf{j}}} \mathbf{\eta}_{\mathbf{j}}^{\mathbf{T}}$$

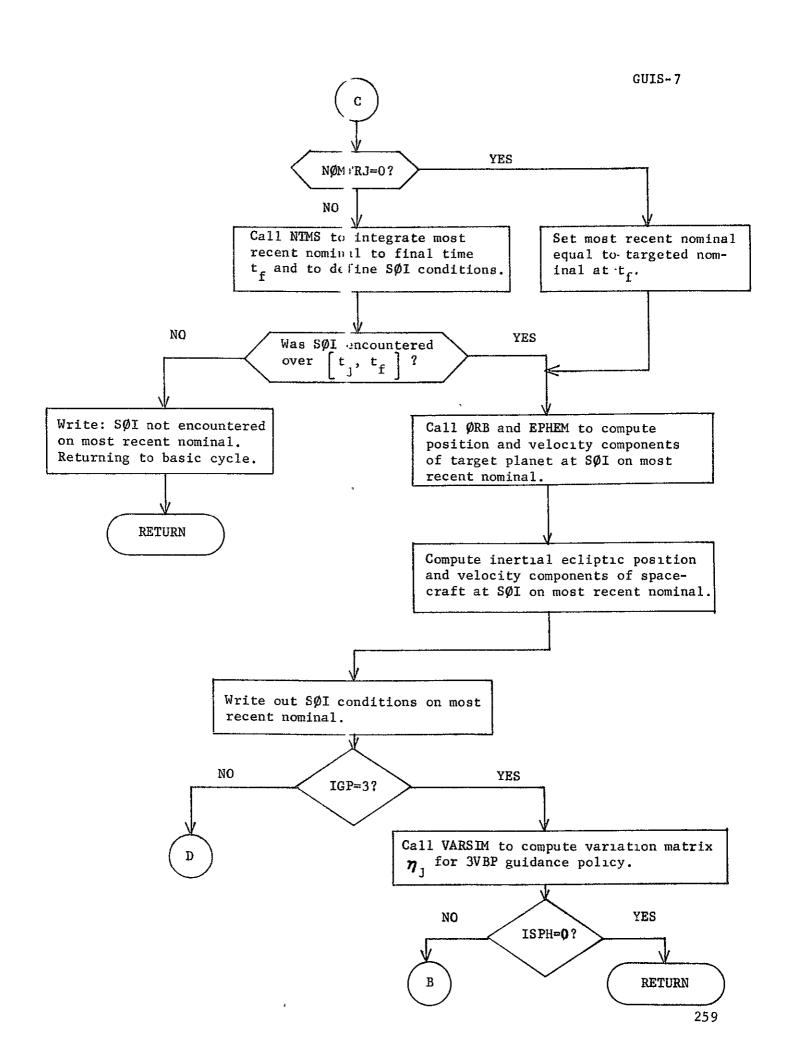
where  $P_{c}$  is the control covariance matrix immediately prior to the guidance event.

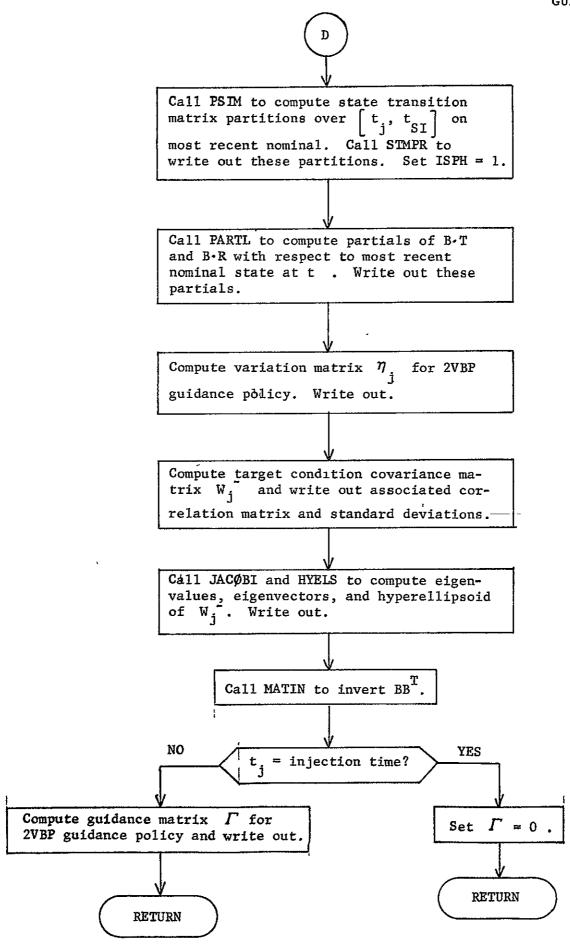












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PROGRAM GUISIM

PURPOSE CONTROL EXECUTION OF GUIDANCE EVENT IN THE SIMULATION PROGRAM

SUBROUTINES SUPPORTED: SIMULL

SUBROUTINES REQUIRED: CORREL DYNOS GUIS HYELS JACOBI

NAVM PSIM STMPR

LOCAL SYMBOLS8 ADA VARIATION MATRIX

AK1 ACTUAL RESOLUTION ERROR

AL1 ACTUAL ERROR IN POINTING ANGLE ALPHA

BT1 ACTUAL ERROR IN POINTING ANGLE BETA

CXSU1 STORAGE FOR CXSU KNOWLEDGE COVARIANCE

CXSV1 STORAGE FOR CXSV KNOWLEDGE COVARIANCE

CXU1 STORAGE FOR CXU KNOWLEDGE COVARIANCE

CXV1 STORAGE FOR CXV KNOWLEDGE COVARIANCE

CXXS1 STORAGE FOR CXXS KNOWLEDGE COVARIANCE

DELX ESTIMATED STATE DEVIATION FROM TARGETED

NOMINAL TRAJECTORY

DUM1 INTERMEDIATE VARIABLE

DUM2 ARRAY OF EIGENVECTORS

DVCH MAGNITUDE OF COMMANDED MIDCOURSE VELOCITY

CORRECTION

DVC COMMANDED MIDCOURSE VELOCITY CORRECTION

DVE ERROR IN MIDCOURSE VELOCITY CORRECTION

DUE TO NAVIGATION UNCERTAINTY

DV PERFECT MIDCOURSE VELOCITY CORRECTION

DX ACTUAL STATE DEVIATION FROM TARGETED

NOMINAL TRAJECTORY

EGVCT ARRAY OF EIGENVECTORS

EGVL ARRAY OF EIGENVALUES

EXEC EXECUTION ERROR COVARIANCE MATRIX

	ΨΛI1	IINGIISIO	J_ U. U.		••		
	GAP	INTERME	DIATE AR	RAY EQUA	L TO GA	TIMES P	
	GA	GUIDANCE MATRIX					
	ICODE2	INTERNAL CONTROL FLAG					
	IGP	MIDCOURSE GUIDANCE POLICY CODE					
	OUT	SPACECRAFT VELOCITY RELATIVE TO TARGET PLANET IN PLANETO-CENTRIC EQUATORIAL COORDINATES					
	PS1	STORAGE FOR PS KNOWLEDGE COVARIANCE					
	P1	STORAGE FOR P KNOWLEDGE COVARIANCE					
	RF1 MOST RECENT NOMINAL SPACECRAFT STATE GUIDANCE EVENT						
	RAFT STA	TATE AT					
	ROW	INTERMEDIATE VECTOR					
	SQP	INTERMEDIATE VECTOR					
	S1	ACTUAL PROPORTIONALITY ERROR					
	VEIG	MATRIX TO BE DIAGONALIZED INTERMEDIATE ARRAY					
	Z						
COMMON COMPUTED	/USED 8	ADEVX CXUG CXXS PSG XF1	CXU	CXSU CXVG ICODE P	CXV	CXSV CXXSG PG TG	
COMMON COMPUTED:		DELTM	TRTM1	XII	XI		
COMMON USED8		AALP EDEVXS IHYP1 NDIM3 SIGRES XF	ABET FOP ISPH Q TEVN XSL	ADEVXS FOV ISTMC SIGALP UO ZERO	APRO ICDT3 NDIM1 SIGBET VO	ARES IEIG NDIM2 SIGPRO W	

EXM

MAGNITUDE OF UPDATE VELOCITY CORRECTION

## GUISIM Analysis

Subroutine GUISIM is the executive guidance subroutine in the simulation program. In addition to controlling the computational flow for all types of guidance events, (UISIM also performs many of the required guidance computations itself.

Before considering each type of guidance event, the treatment of a general guidance event will be discussed. Let  $t_j$  be the time at which the guidance event occurs. Before any guidance event can be executed the targeted nominal state  $\overline{X_j}$ , most recent nominal state  $\overline{X_j}$ , estimated state deviation  $\delta \overline{X_j}$  from most recent nominal, actual state deviation  $\delta \overline{X_j}$  from most recent nominal, knowledge covariance  $P_{K_j}$ , and control covariance  $P_{C_j}$  must all be available, where () indicates values immediately before the event. Only the control covariance is not available prior to entering GUISIM. The propagation of the control covariance over the interval  $[t_{j-1}, t_j]$ , where  $t_{j-1}$  denotes the time of the previous guidance event, is performed within GUISIM.

The next step in the treatment of a general guidance event is concerned with the computation of the commanded velocity correction, execution error covariance, actual execution error, and actual velocity correction. In the simulation program a non-statistical commanded velocity correction can always be computed. This commanded velocity correction  $\Delta V_{j}$  is used to compute the execution error povariance matrix  $\widetilde{Q}_{j}$  and the actual execution error  $\delta \Delta V_{j}$ . A summary of the execution error model and the equations used to compute  $\widetilde{Q}_{j}$  and  $\delta \Delta V_{j}$  can be found in the subroutine QCPMP analysis section. The actual velocity correction is then computed using the equation

$$\Delta V_{j} = \Delta \hat{V}_{j} + \delta \Delta V_{j}$$

The last step is concerned with the updating of required quantities prior to returning to the basic cycle. An assumption underlying the modeled guidance process is that the targeted nominal is always updated by the commanded velocity correction. In the simulation program the update velocity correction  $\Delta \hat{V}_{UP}$  is always identical to the commanded velocity correction  $\Delta \hat{V}_{i}$ . This is in contrast to the error analysis program where  $\Delta \hat{V}_{UP}$  is equated with the non-statistical component of  $\Delta \hat{V}_{i}$ . The

most recent and targeted nominal states immediately following the guidance event are updated using the equations

$$\widetilde{\mathbf{x}}_{\mathbf{j}}^{+} = \widetilde{\mathbf{x}}_{\mathbf{j}}^{-} + \delta \widetilde{\widetilde{\mathbf{x}}}_{\mathbf{j}}^{-} + \left[ -\frac{0}{\delta \Delta \widetilde{\mathbf{v}}_{\mathrm{UP}_{\mathbf{j}}}} \right]$$

$$\overline{x}_1^+ = \widetilde{x}_1^+$$

The actual and estimated state deviations from the most recent nominal are given by

$$\delta \widetilde{\mathbf{x}}_{\mathbf{j}}^{+} = \delta \widetilde{\mathbf{x}}_{\mathbf{j}}^{-} - \delta \widetilde{\mathbf{x}}_{\mathbf{j}}^{-} + \begin{bmatrix} 0 \\ -\delta \widetilde{\Delta} \mathbf{v}_{\mathbf{j}}^{-} \end{bmatrix}$$

$$\delta \hat{x}_{1}^{+} = 0$$

The previous 4 equations assume an impulsive thrust model. If, instead, the thrust is modeled as an impulse series, then an effective estimated state  $\hat{X}_{eff}$  and an effective actual state  $\hat{X}_{eff}$  are computed.

The equations used to compute these effective states are summarized in the subroutine PULSEX analysis section. The previous update equations are then replaced by the following equations

$$\tilde{X}_{1}^{+} = \hat{X}_{eff}$$

$$\overline{x}_1^+ = \widetilde{x}_1^+$$

$$\delta \widetilde{x}_{j}^{+} = x_{eff} - \widehat{x}_{eff}$$

$$\delta X_{4}^{+} = 0$$

The knowledge covariance is updated using the equation

$$P_{K_{j}}^{+} = P_{K_{j}}^{-} + \left[ \frac{0}{0} + \frac{1}{0} - \frac{0}{0} \right]$$

if an impulsive thrust model is assumed. If the thrust is modeled as a series of impulses, then an effective execution error covariance  $\widetilde{Q}_{eff}$  is computed and the knowledge covariance is updated using the equation

$$P_{K}^{+} = P_{K}^{-} + \tilde{Q}_{eff}$$

In either case the control covariance is updated simply by setting

$$P_{\mathbf{c}_{\mathbf{j}}}^{\dagger} = P_{\mathbf{K}_{\mathbf{j}}}^{\dagger}$$

This equation is a direct consequence of the assumption that the targeted nominal is always updated at a guidance event.

A "compute only" option is available in GUISIM in which all of the () quantities will still be computed and printed. However, states, deviations, and covariances are then reset to their former () values prior to returning to the basic cycle.

Each specific type of guidance event involves the computation of other quantities not discussed above. These will be covered in the following discussion of specific guidance events.

1. Midcourse and biased aimpoint guidance.

Linear midcourse guidance policies have form

$$\Delta \hat{v}_{N_{i}} = \Gamma_{j} \delta \hat{x}_{j}$$

where the subscript N indicates that this is the velocity correction required to null out deviations from the nominal target state. This notation is required to differentiate between this type of velocity correction and velocity corrections required to achieve an altered target state. Linear midcourse guidance policies are discussed in more detail in the subroutine GUIS analysis section.

Subroutine GUISIM calls GUIS to compute the guidance matrix,  $\Gamma_j$ , and the target condition covariance immediately prior to the guidance event,  $W_j$ , and then uses  $\Gamma_j$  to compute the velocity correction covariance  $S_j$ , which is defined as

$$\mathbf{s}_{1} = \mathbf{E} \left[ \Delta \hat{\mathbf{v}}_{\mathbf{N}_{1}} \Delta \hat{\mathbf{v}}_{\mathbf{N}_{1}}^{\mathbf{T}} \right],$$

and is given by the equation

$$\mathbf{s}_{\mathbf{j}} = \Gamma_{\mathbf{j}} (P_{\mathbf{c}_{\mathbf{j}}}^{-} - P_{\mathbf{K}_{\mathbf{j}}}^{-}) \Gamma_{\mathbf{j}}^{T}$$

This equation assumes that an optimal estimation algorithm is employed in the navigation process, since the derivation of this equation requires the orthogonality of the estimate and the estimation error.

Since state estimates  $\delta \hat{X}_j$  are generated in the simulation program, an actual  $\Delta \hat{V}_N$  can always be computed. This is in contrast to the error analysis program where only a statistical or effective  $\Delta \hat{V}_N$  can be computed. The perfect velocity correction  $\Delta \hat{V}_j$ , defined as the velocity correction required to null out actual deviations from the nominal target state, is also computed for midcourse guidance events. Assuming linear guidance theory, the perfect velocity correction is given by

$$\Delta V_{i} = \Gamma_{i} \delta X_{i}$$

where  $\delta X_j$  is the actual deviation from the targeted nominal. An option is also available in GUISIM for re-computing  $\Delta \hat{V}_{N_j}$  using nonlinear techniques. However, it should be noted that the nonlinear two-variable B-plane guidance policy, unlike the corresponding linear policy, constrains the z-component of  $\Delta \hat{V}_{N_j}$  to be zero.

If planetary quarantine constraints must be satisfied at a midcourse correction, GUISIM calls BIAIM to compute the new aimpoint  $\mu_j$  and the bias velocity correction  $\Delta \hat{V}_B$ . All computations in BIAIM are based on linear guidance theory. However, an option is available in GUISIM to recompute the total velocity correction  $\Delta \hat{V}_B + \Delta \hat{V}_A$ , but not  $\mu_j$ , using nonlinear techniques. This option is recommended if a biased aimpoint guidance event occurs at  $t_j$  injection time. It should also be noted that  $\hat{Q}_j$  is set to zero if  $t_j$  injection time since it is assumed that the injection

covariance does not change for small changes in injection velocity.

After the updated control covariance  $P_c^+$  has been computed, the target condition covariance matrix  $W_j^+$  following the guidance correction is computed using the equation

$$W_{j}^{+} = \eta_{j} P_{c_{j}}^{+} \eta_{j}^{T}$$

where variation matrix  $\eta_j$  has been previously computed in subroutine GUIS.

## 2. Re-targeting.

In the simulation (and error analysis) program a re-targeting event is defined to be the computation of a velocity correction  $\triangle V_{RT}$  required to achieve a new set of target conditions using nonlinear techniques. Since the state estimate  $\widetilde{X}_j^- + \delta \widetilde{\widehat{X}}_j^-$  is used as the zero-th iterate in the re-targeting process, the new target conditions must be close enough to the original nominal target conditions to ensure a convergent process.

It should be noted that after a re-targeting event the new target conditions are henceforth treated as the nominal target conditions.

## 3. Orbital insertion,

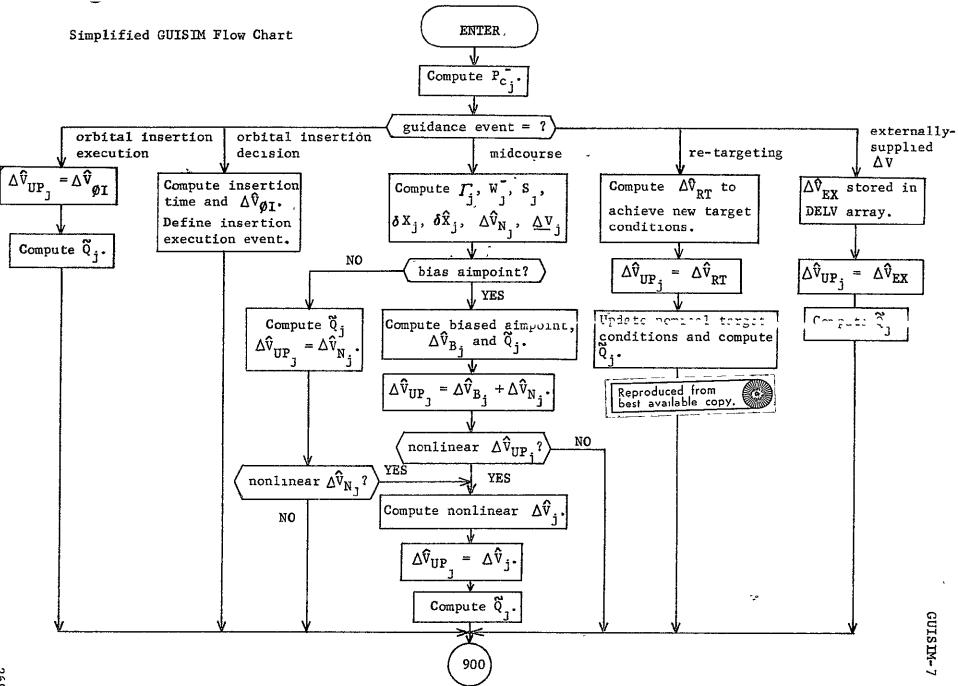
An orbital insertion event is divided into a decision event and an execution event. At a decision event the orbital insertion velocity correction  $\Delta \hat{V}_{\emptyset I}$  and the time interval  $\Delta t$  separating decision and execution are computed based on the state estimate  $\widetilde{X} + \delta \widetilde{\widehat{X}}$ . The relevant equations can be found in the subroutine COPINS analysis section for coplanar orbital insertion; in NOPINS, for non-planar orbital insertion. Before returning to the basic cycle, GUISIM schedules the orbital insertion execution event to occur at  $t_1 + \Delta t$  and re-orders the necessary event arrays accordingly.

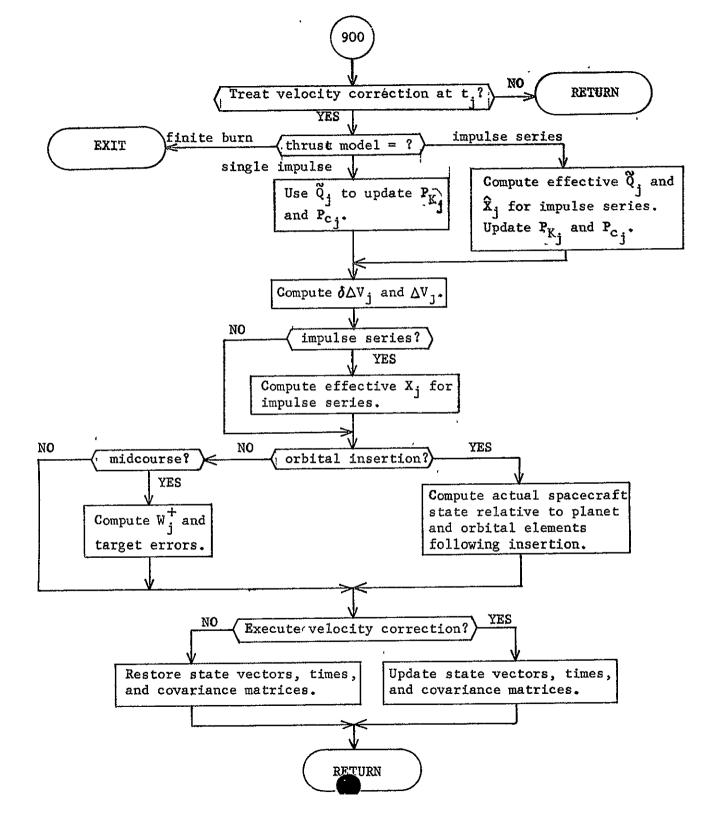
At an orbital insertion execution event the previously computed  $\Delta V_{0l}$  is used to update the targeted nominal state. In addition, the planeto-centric equatorial components of  $\Delta V_{0l}$  and the actual spacecraft cartesian and orbital element states following the insertion maneuver are computed.

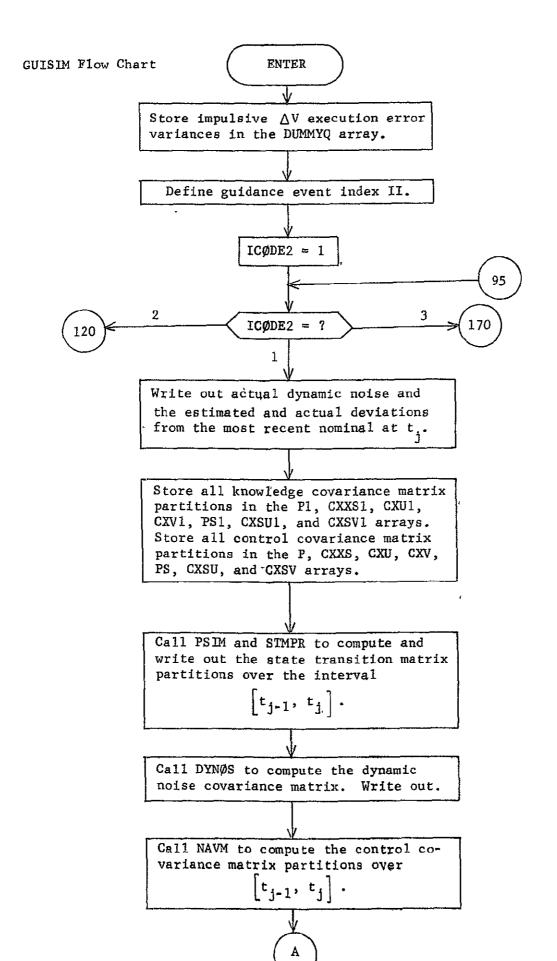
4. Externally-supplied velocity correction.

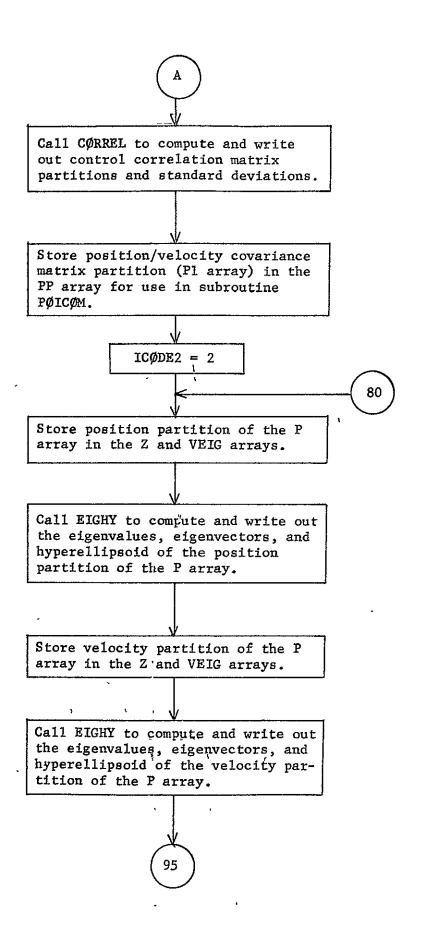
At this type of guidance event the state estimate  $\widetilde{X}_j^- + \widetilde{\widehat{X}}_j^-$  is simply updated using the externally-supplied velocity correction  $\triangle \widehat{V}_{EX}$ .

Because of the complexity of the GUISIM flow chart, a simplified flow chart depicting the main elements of the GUISIM structure precedes the complete GUISIM flow chart.



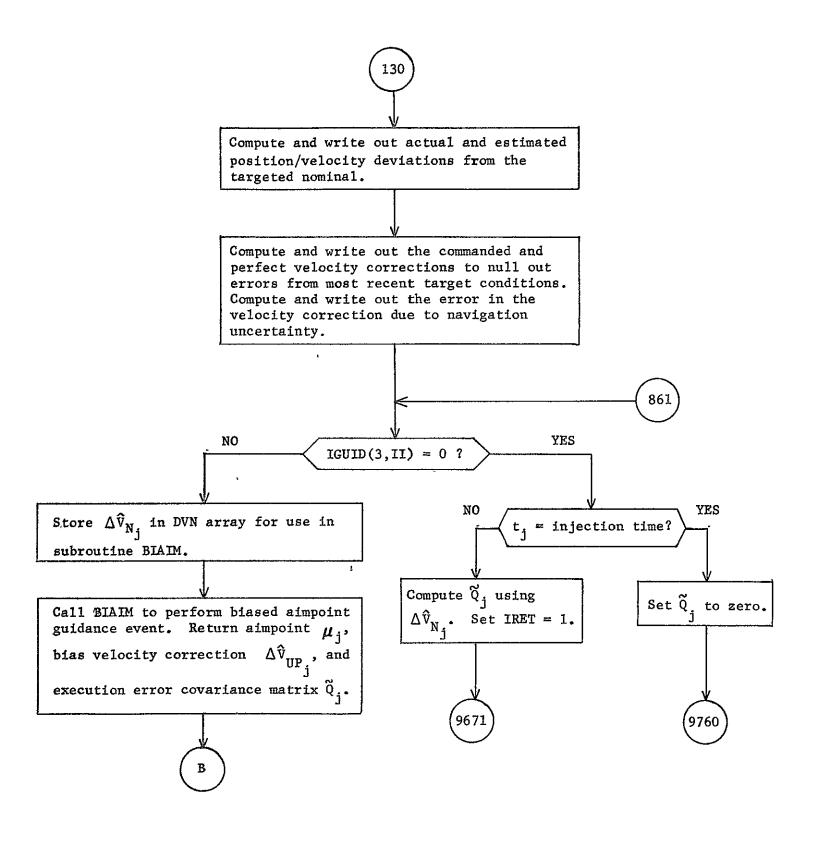




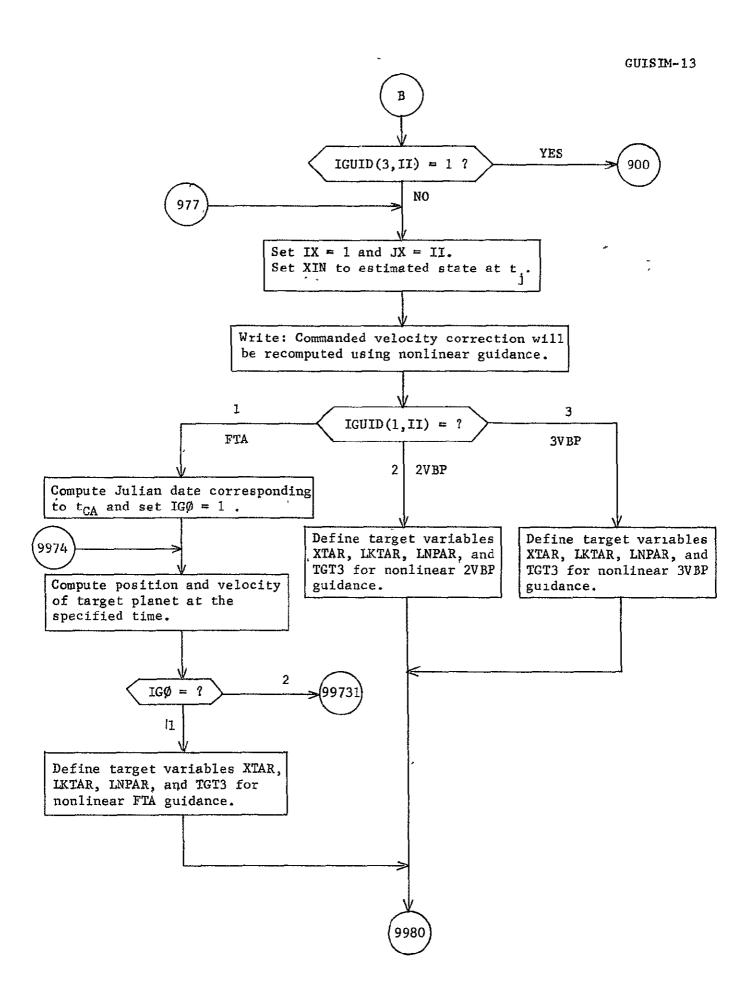


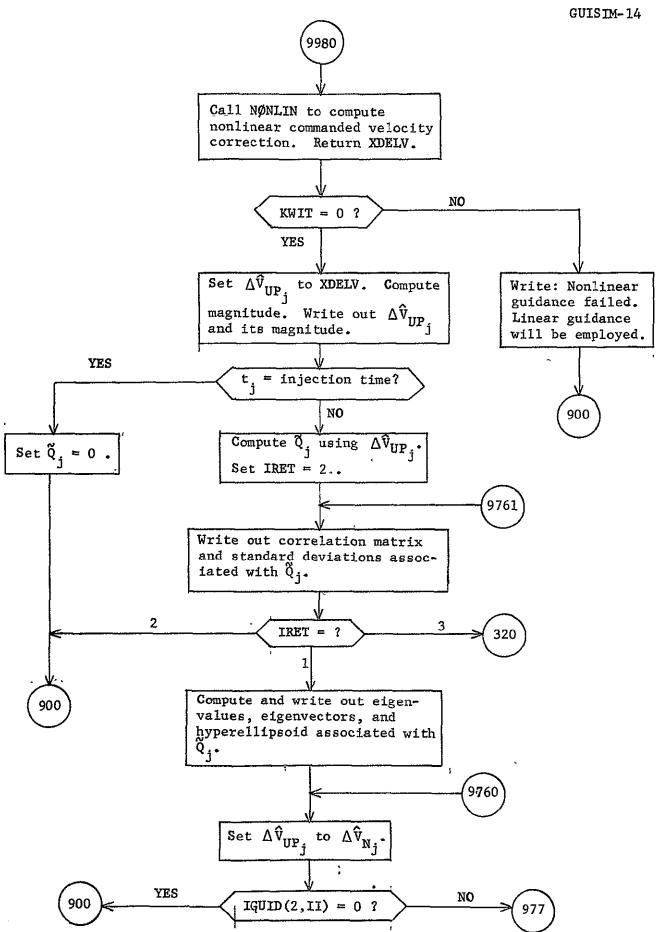
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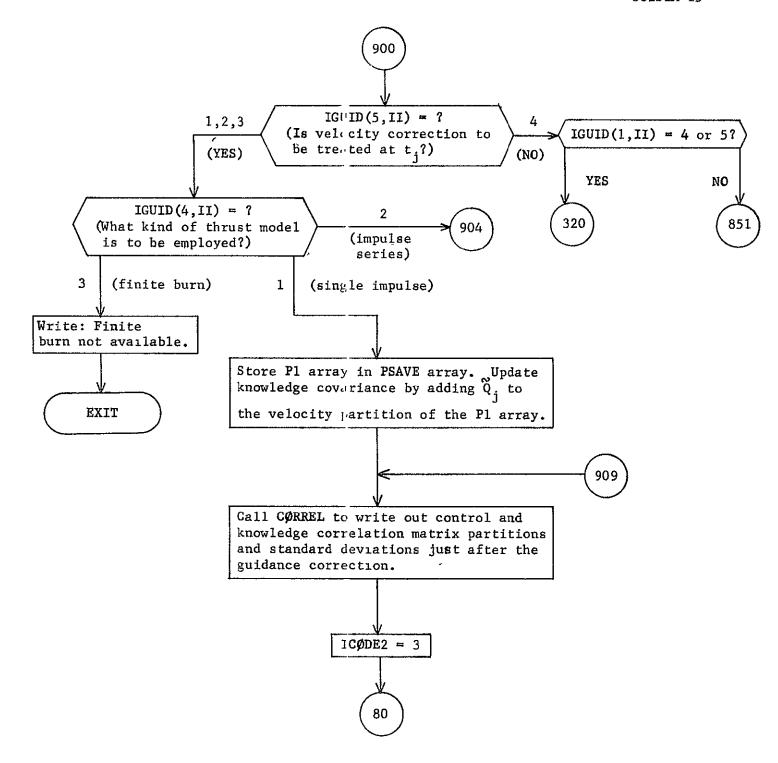
values.



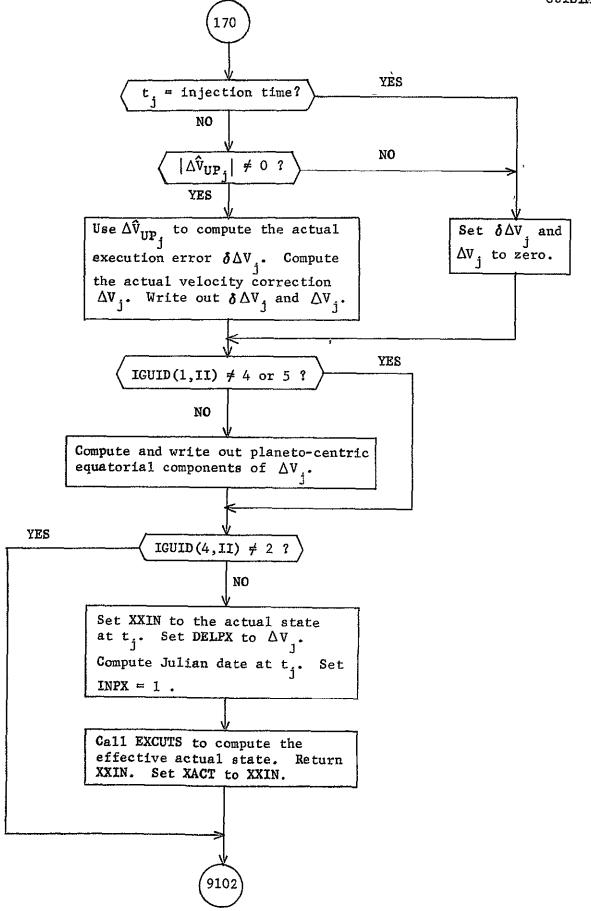
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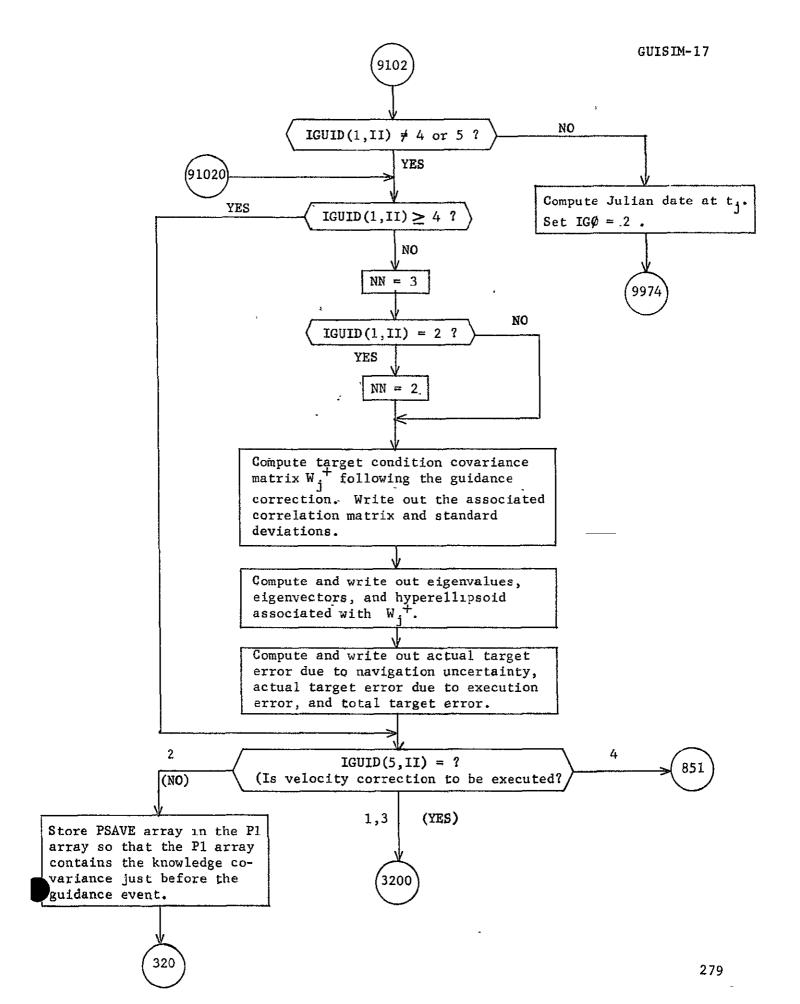


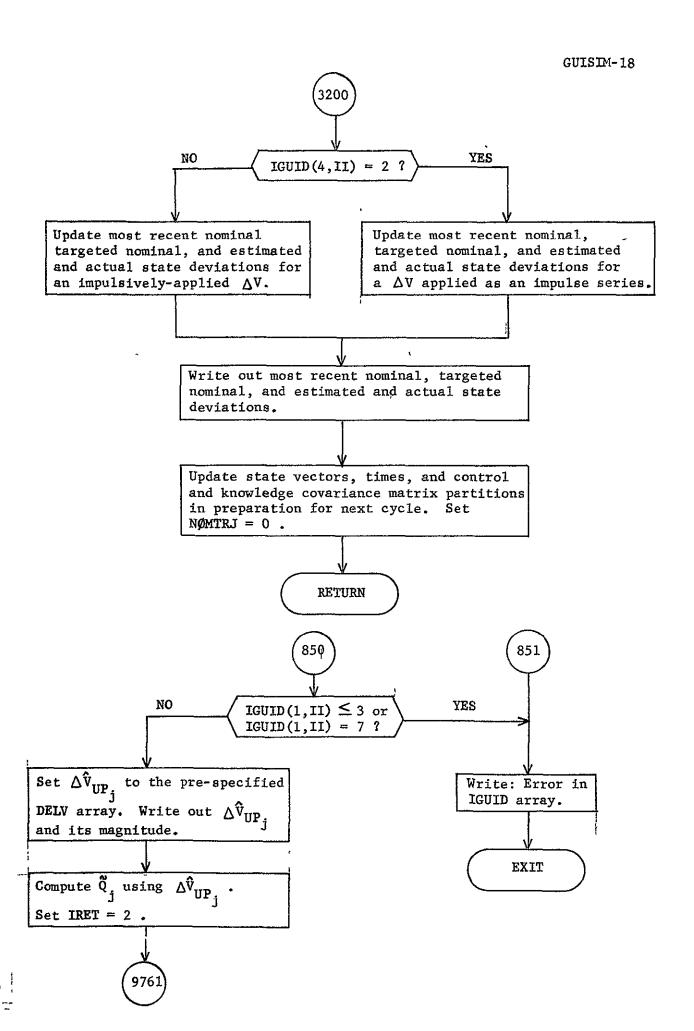


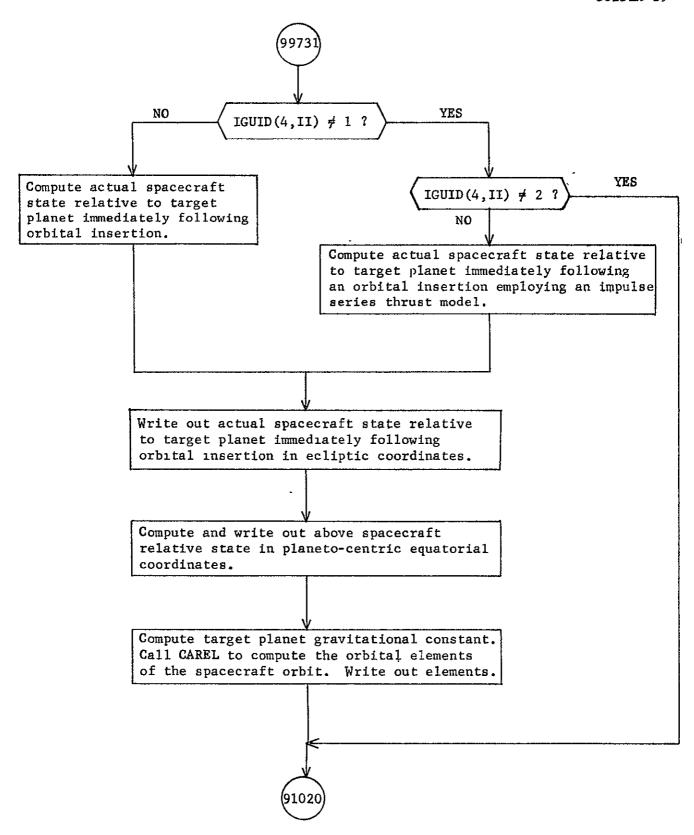


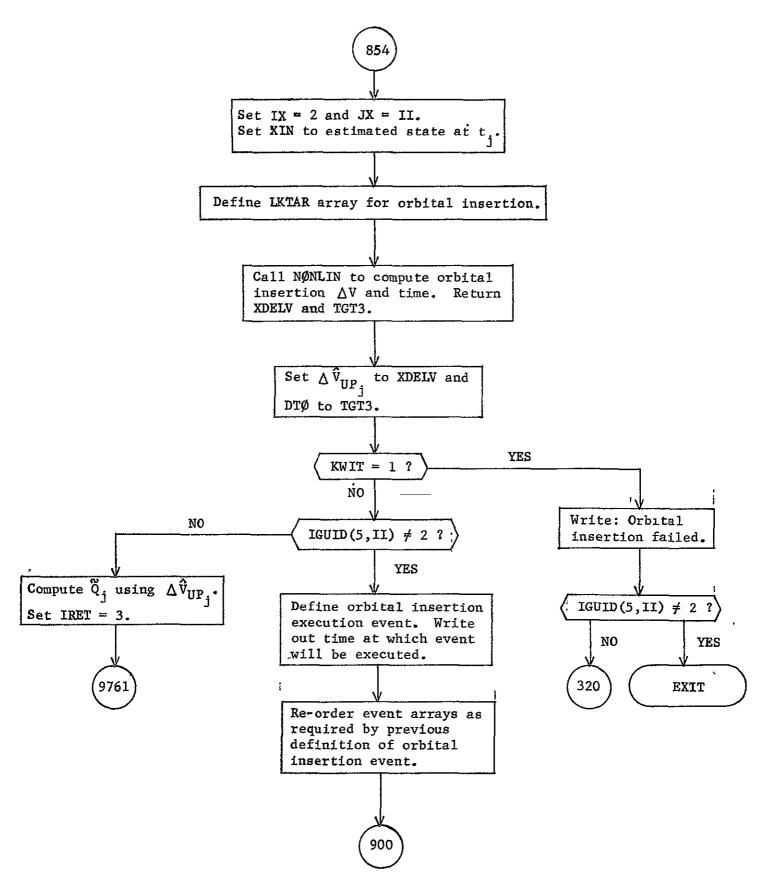


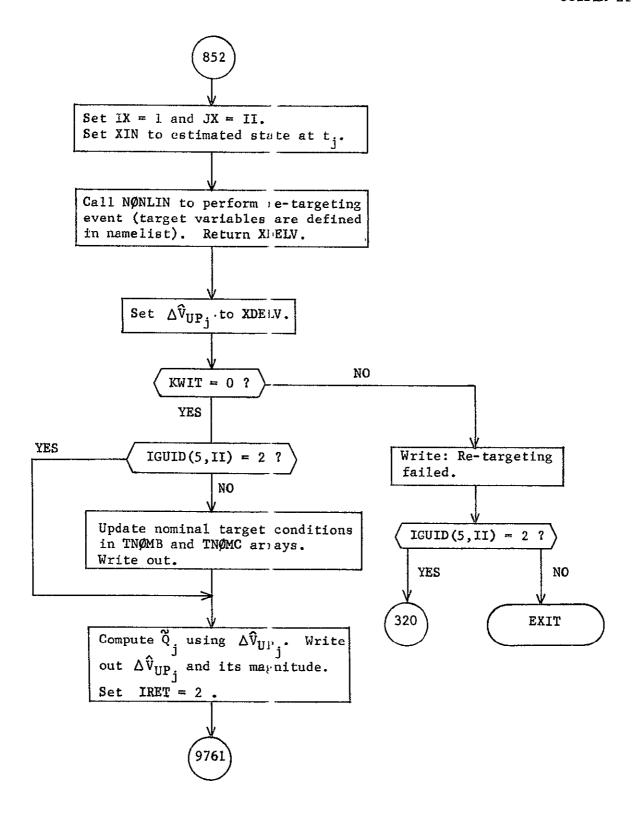


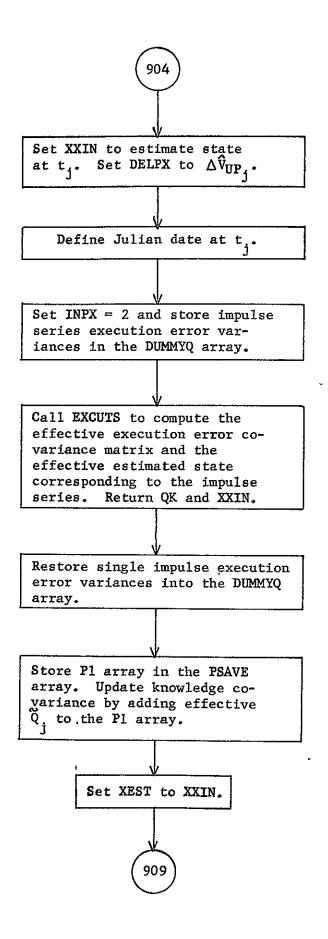












SUBROUTINE HELIO

PURPOSES TO COMPUTE THE ZERO ITERATE INJECTION STATE FOR INTERPLANETARY TARGETING

CALLING SEQUENCE: CALL HELIO

SUBROUTINES SUPPORTED: ZERIT

SUBROUTINES REQUIRED: LAUNCH FLITE ELCAR EPHEM ORB PECEQ TIME

LOCAL SYMBOLS: AHEL SEMI-MAJOR AXIS OF THE HELIOCENTRIC CONIC

ARGP ARGUMENT OF PERIAPSIS OF THE HELIOCENTRIC CONIC IN RADIANS

ASCND LONGITUDE OF THE ASCENDING NODE OF THE HELIOCENTRIC CONIC IN RADIANS

ATP SEMI-MAJOR AXÍS OF TARGET PLANETOÇENTRIC CONIC

AZF AZIMUTH AT DF ON THE HELIOCENTRIC CONIC IN DEGREES

AZI AZIMUTH AT DI ON THE HELIOCENTRIÇ ÇONÎC IN DEGREES

B2 SQUARE OF THE B VECTOR MAGNITUDE OF THE TARGET PLANETOCENTRIC CONIC

CBDR DESIRED B.R MAGNITUDE AT DF OF THE TARGET PLANETOCENTRIC CONIC

CBDT DESIRED B.T MAGNITUDE AT DF OF THE TARGET PLANETOCENTRIC CENTER

COSASN CONSINE OF ASCND

COSB INTERMEDIATE VARIABLE FOR AZI, AZF EQUATION

COSFS COSINE OF FS

COSF' COSINE OF TAI

COSPSI COSINE OF PSI

COSTHE COSINE OF THETAI

CRCA DESIRED RCA MAGNITUDE AT DF OF THE TARGET PLANETOCENTRIC CONIC

TIME OF FLIGHT (SECS) OF HELICOENTRIC DELT CONIC FINAL JULIAN DATE OF HELIOCENTRIC CONIC DF DI INITIAL JULIAN DATE OF HELIOCENTRIC CONIC DSICA DELTA TIME (DAYS) FROM SPHERE-OF-INFLUENCE TO CLOSEST APPROACH OF THE TARGET PLANET OCENTRIC CONIC ECCENTRICITY OF THE HELIOCENTRIC CONIC EHEL EQEC TRANSFORMATION MATRIX FROM ECLIPTIC TO TARGET PLANET EQUATORIAL AT DF ECCENTRICITY OF THE TARGET PLANETOCENTRIC ETP CONIC FF INTERMEDIATE VARIABLE FOR COMPUTATION OF DSICA FS TRUE ANOMALY OF THE TARGET PLANETOCENTRIC CONIC IDAT CALENDER DATE CORRESPONDING TO DF DUMMY ARGUMENT FOR CALL TO SUBROUTINE IDUM FLITE ITIM INDICATES COMPUTATION OF HELIOCENTRIC STATES =0, COMPUTE INITIAL AND FINAL STATES =1, COMPUTE FINAL STATE ONLY I INDEX INDEX CALENDER DATE CORRESPONDING TO DI LDAT OAPO APOAPSIS RADIUS OF THE HELIOCENTRIC CONIC OASN ASCND CONVERTED TO DEGREES OECC OUTPUT ECCENTRICITY OF THE HELIOCENTRIC CONIC OGAF FLIGHT PATH ANGLE AT DF OF THE HELIO-CENTRIC CONIC FLIGHT PATH ANGLE AT DI OF THE HELIO-OGAI CENTRIC CONIC

OHCA	CENTRAL ANGLE OF THE HELIOCENTRIC CONIC
OINC	INCLINATION OF THE HELIOCENTRIC CONIC
OLAF	LATITUDE AT DF OF HELIOCENTRIC CONIC
OLAI	LATITUDE AT DI OF HELIOCENTRIC CONIC
OLOF	L'ONGITUDE AT DF OF HELIOCENTRIC CONIC
OLOI	LONGITUDE AT DI OF HELIOCENTRIC CONIC
OPER	ARGUMENT OF PERIAPSIS OF THE HELIOCENTRIC CONIC IN DEGREES
ORCA	PERIAPSIS RADIUS OF HELIOCENTRIC CONIC
ORF	MAGNITUDE OF HELIOCENTRIC POSITION AT DF IN OUTPUT UNITS
ORI	MAGNITUDE OF HELIOCENTRIC POSITION AT DI IN OUTPUT UNITS
OSMA	SEMI-MAJOR AXIS OF THE HELIOCENTRIC CONIC IN OUTPUT UNITS
OTAF	TRUE ANOMALY AT DF OF HELIOCENTRIC CONIC IN DEGREES
OTAI	TRUE ANOMALY AT DI OF HELIOCENTRIC CONIC IN DEGREES
OVF	MAGNITUDE OF HELIOCENTRIC VELOCITY AT DF IN KILOMETERS
OVI	MAGNITUDE OF HELIOCENTRIC VELOCITY AT DI IN KILOMETERS
OVPF	VELOCITY OF TARGET PLANET AT DF
OVPI	VELOCITY OF TARGET PLANET AT DI
PHEL	SEMI-LAUS RECTUM OF HELIOCENTRIC CONIC
PLINC	INCLINATION (IN RADIANS) OF HELIOCENTRIC CONIC
PSI	CENTRAL ANGLE (IN RADIANS) OF HELIOCENTRIC CONIC
PTP	SEMI-LATUS RECTUM OF TARGET PLANETOCENTRIC CONIC AT DF. USED TO CALCULATE DSICA

RF MAGNITUDE OF TARGET PLANET POSITION AT DF

RI MAGNITUDE OF TARGET PLANET POSITION AT DI

RTM MAGNITUDE OF RT VECTOR

RT HELIOCENTRIC POSITION VECTOR OF THE FINAL CONIC CORRESPONDING TO OTAF

RZM MAGNITUDE OF THE RZ VECTOR

RZ HELIOCENTRIC POSITION VECTOR OF THE FINAL CONIC CORRESPONDING TO OTAL

SGN INTERNAL SIGN VARIABLE USED TO DEFINE THE TRANSFER PLANE ORIENTATION

SINASH SINE OF ASCHD

SINF SIN OF TAI

SINHF HYPERBOLIC SINE OF THE AUXILIARY VARIABLE F USED TO CALCULATE DSICA

SINPSI SINE OF PSI

SI SECONDS IN CALENDER DATE IDAT

SL SECONDS IN CALENDER DATE LDAT

SUNMU GRAVITATIONAL CONSTANT OF SUN'IN KM##3/SEC##2

TAF OTAF IN RADIANS

TAI OTAI IN RADIANS.

TANF TANGENT OF THE AUXILIARY VARIABLE F USED TO CALCULATE DSICA

TERM INTERMEDIATE VARIABLE USED TO CALCULATE CRCA

TEST INTERMEDIATE VARIABLE USED TO CALCULATE AZIMUTHS AND PATH ANGLES

TFP DUMMY VARIABLE USED TO CALL ELCAR

THETAI INTERMEDIATE ANGLE USED TO DEFINE ARGP

TSPH SPHERE-OF-INFLUENCE OF TARGET PLANET IN KILOMETERS

VÉ	VELOCITY OF THE TARGET PLANET AT DF
· VHAT	INTERMEDIATE VECTOR USED TO DEFINE AZI, AZF
VHP	HYPERBOLIC EXCESS VELOCITY OF THE TARGET PLANETOCENTRIC CONIC AT OF
VHPM	MAGNITUDE OF THE VHP VECTOR USED TO CALCULATE DSICA
VI	VELOCITY OF LAUNCH PLANET AT DI
VMAG	INTERMEDIATE VARIABLE USED TO DEFINE AZI, AZF
VTM -	MAGNITUDE OF VT VECTOR
vT	HELIOCENTRIC VELOCITY VECTOR OF THE FINAL CONIC CORRESPONDING TO OTAF
V ZM	MAGNITUDE OF THE VZ VECTOR
٧Z	HELIOCENTRIC VELOCITY VECTOR OF THE FINAL CONIC CORRESPONDING TO OTAL
TAHW	UNIT VECTOR NORMAL TO THE TRANSFER PLANE
WMAG	MAGNITUDE OF THE NON-UNITIZED WHAT VECTOR
XF	POSITION OF THE TARGET PLANET AT DF
XI	POSITION OF THE LAUNCH PLANET AT DI
COMMON COMPUTED/USED:	TMU VHPM
COMMON COMPUTEDS	DPA NO RAP RIN TIN
COMMON USED8	ALNGTH DG DT IZERO KTAR KUR NLP NTP ONE PI PMASS RAD SPHERE TAR TM TWO XP ZDAT ZERO

HELIO Analysis

HELIO computes the zero iterate initial state for interplanetary trajectories. The initial and final states are determined either by an arbitrary position vector or by the location of a specified planet at a prescribed time according to

IZERO = 1 planet to planet

2 planet to arbitrary final point

3 arbitrary initial point to planet

4 arbitrary initial point to final point

The final time used in locating a planet must correspond to the closest approach (CA) to the planet. Therefore if the target time is read in as a sphere of influence (SOI) time, a modification is required. The heliocentric conic is computed (as described below) using the to time to determine the final position. The approach asymptote  $\overrightarrow{V}$  corresponding to that trajectory is used with the desired  $r_{CA}$  to compute the time from SOI to CA. If  $r_{CA}$  is not a target variable then the target values of B-T and B-R are used to estimate the  $r_{CA}$ 

$$r_{CA} = -\frac{\mu}{V_{HP}} + \frac{1}{2} \sqrt{\left(\frac{2\mu}{V_{HP}^2}\right)^2 + 4B^2}$$
 (1)

Then the approximate approach hyperbola is given by

$$a_{h} = \frac{\mu r_{SI}}{2\mu - V_{HP}^{2} r_{SI}}$$

$$e_{h} = 1 - \frac{r_{CA}}{a}$$

$$p_{h} = a_{n}(1 - e_{h}^{2})$$
(2)

and the hyperbolic time to go from SOI to CA is given by

$$\Delta t_{SICA} = \frac{\mu}{V_{HP}} \quad (e \sinh F - F)$$
 (3)

where

$$\tanh \frac{F}{2} = \sqrt{\frac{\frac{e}{n}-1}{\frac{e}{h}+1}} \quad \tanh \frac{f}{2}$$

$$\cos f = \frac{1}{e} \left( \frac{P_h}{r_{SI}} - 1 \right) \tag{4}$$

The final time is then given by  $t_f = t_{SI} + \Delta t_{SICA}$ .

The initial and final positions  $\overrightarrow{r}$  and  $\overrightarrow{r}$  of the heliocentric conic are either input or computed from the positions of planets determined by ORB and EPHEM. The unit normal to the heliocentric orbit plane is

$$\hat{\mathbf{W}} = \frac{\hat{\mathbf{r}}_{1} \times \hat{\mathbf{r}}_{f}}{\left|\hat{\mathbf{r}}_{1} \times \hat{\mathbf{r}}_{f}\right|}$$
(5)

The inclination to that plane is

$$\cos i = V_{z} \tag{6}$$

The ascending node of the plane is given by

$$\tan \Omega = \frac{\widehat{W}_{x}}{\widehat{W}_{y}}$$
 (7)

The central angle of transfer is defined by

$$\cos \Psi = \frac{\vec{r}_1 \cdot \vec{r}_f}{r_i r_f} \tag{8}$$

The semi-major axis a and eccentricity e of the heliocentric conic are computed from Lambert's theorem in subroutine FLITE. The true anomaly  $\mathbf{f_i}$  at the initial and final points are computed from

$$p = a (1 - e^{2})$$

$$\cos f_{i} = \frac{p - r_{i}}{e r_{i}} \quad \sin f_{i} = \frac{\cos f_{i} \cos \Psi - \frac{p - r_{f}}{e r_{f}}}{\sin \Psi}$$
 (8)

$$f_f = f_i + \Psi$$

Finally, the argument of periapsis  $\omega$  is computed from

$$\cos(\omega + f_{i}) = \frac{\frac{1}{r} \cdot \hat{V}}{r_{i}}$$
 (10)

where  $\widehat{U} = (\cos \Omega, \sin \Omega, 0)$ .

Therefore the initial or final states ( $\vec{r}_i$ ,  $\vec{v}_i$ ) or ( $\vec{r}_f$ ,  $\vec{v}_f$ ) may now be computed by ELCAR. Let ( $\vec{r}$ ,  $\vec{v}$ ) denote either state and let ( $\vec{r}$ ,  $\vec{v}$ ) denote the state of the relevant planet. The departure (or approach) asymptote is then given by

$$\overrightarrow{v}_{HP} = \overrightarrow{v}_{f} - \overrightarrow{v}_{p} \qquad \overrightarrow{v}_{HE} = \overrightarrow{v}_{i} - \overrightarrow{v}_{p} \qquad (11)$$

The latitude and longitude of the position vector are

$$\sin \theta = \frac{\dot{r}}{r}$$
  $\tan \theta = \frac{r_y}{r_x}$  (12)

The path angle F may be computed from

$$\cos \Gamma = \frac{V_{\mu p}}{r v} \tag{13}$$

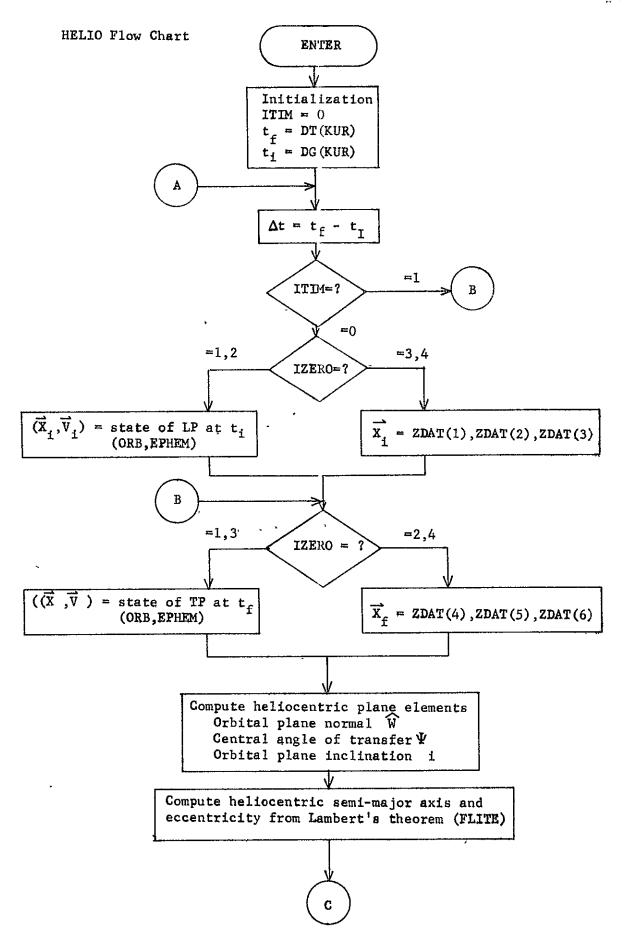
The azimuth of the relevant state is

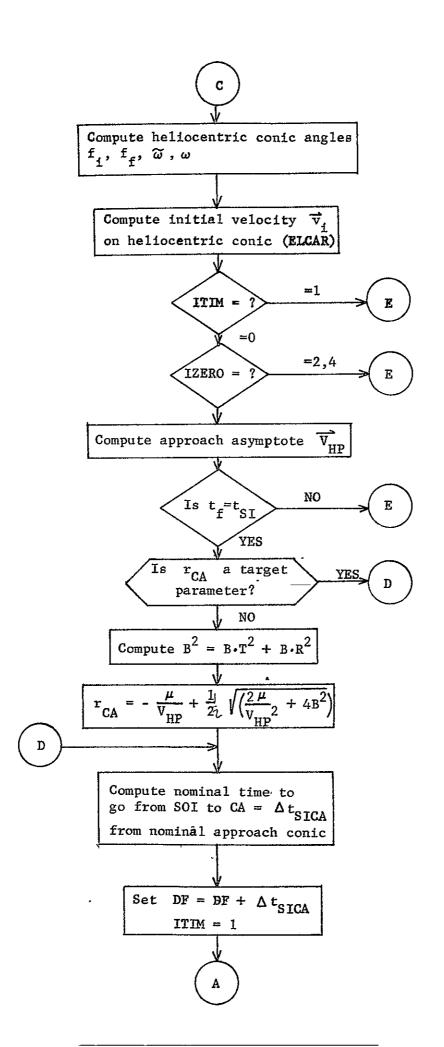
$$\sin \Sigma = \frac{(\vec{r} \times \vec{v}) \cdot \hat{U}}{|\vec{r} \times \vec{v}|}$$
(14)

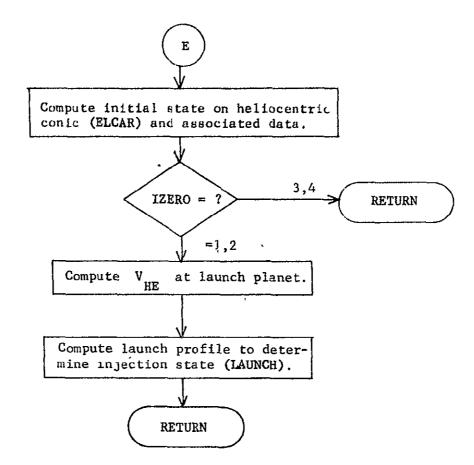
$$\cos \Sigma = \frac{\overrightarrow{V} \cdot \widehat{U}}{V \cos \Gamma}$$
 (15)

If the initial state is referenced to a planet, subroutine LAUNCH is called to convert the departure asymptote and launch profile into an injection radius, velocity, and time. Otherwise the initial state is returned as the initial state on the heliocentric conic.

Reference: Space Research Conic Program, Phase III, May 1, 1969, Jet Propulsion Laboratory, Pasadena, California.







SUBROUTINE HPOST

PURPOSE: TO CALCULATE RADIUS AND TRUE ANAHOLY ON A HYPERBOLA GIVEN TIME FROM PERIAPSIS

ARGUMENT 4	A	I	SEMI-MAJOR AXIS OF HYPERBOLA IN KM
	CTA	0	COSINE OF TRUE ANAMOLY
	OTPS	I	TIME INTERVAL FROM PERIAPSIS TO DESIRED STATE IN SEC
	ε	I	ECCENTRICITY OF HYPERBOLA
	GMU	I	GRAVITATIONAL CONSTANT OF PLANET IN KM**3/SEC**2
	P	I	SEMI-LATUS RECTUM OF HYPERBOLA IN KM

R O RADIUS IN KM

STA O SINE OF TRUE ANAMOLY

SUBROUTINES SUPPORTED: TPRTRG

LOCAL SYMBOLS: ABSA ABSOLUTE VALUE OF SEMI-MAJOR AXIS IN KM

ABSOLUTE VALUE OF TIME FROM PERIAPSIS TO DESIRED STATE IN SEC

AM1 ITERATED VALUE OF MEAN ANAMOLY
CORRESPONDING TO CURRENT NEWTON ITERATE
OF HYPERBOLIC ANAMOLY IN RAD

AM MEAN ANAMOLY IN RAD

CSF HYPERBOLIC COSINE OF HYPERBOLIC ANAMOLY

DLE ITERATION INCREMENT TO HYPERBOLIC ANAMOLY.

DTS CONVERSION FACTOR FROM DAYS TO SECONDS

F HYPERBOLIC ANAMOLY IN RAD

ONE CONSTANT 1.

P1 CONSTANT PI

RMEAN MEAN ORBITAL RATE IN RADISEC

SNF HYPERBOLIC SINE OF HYPERBOLIC ANAMOLY TM8 CONSTANT 1.0E-08

TWO CONSTANT 2.

VHE MAGNITUDE OF HYPERBOLIC EXCESS VELOCITY IN

KM/SEC

EXPONENTIAL FUNCTION OF HYPERBOLIC ANAMOLY

, -

## SUBROUTINE HYELS

PURPOSE: TO COMPUTE AND PRINT THE THO-DIMENSIONAL OR THREE-DIMENSIONAL HYPERELLIPSOID OF A SPECIFIED MATRIX.

CALLING SEQUENCE: CALL HYELS(KS,P,N)

ARGUMENT: KS I SIGMA LEVEL OF THE HYPERELLIPSOID

P I MATRIX FOR WHICH THE HYPERELLIPSOID IS TO BE COMPUTED

I DIMENSION LIMITS OF THE SQUARE MATRIX P

SUBROUTINES SUPPORTED: EIGHY GUISIN GUISS SETEVN GUIDN

GUID PRED

SUBROUTINES REQUIRED: MATIN

LOCAL SYMBOLS: K2 SQUARE OF SIGNA LEVEL

PI INVERSE OF MATRIX P

P12 TWICE THE VALUE OF (1,2) ELEMENT OF PI

P13 TWICE THE VALUE OF (1,3) ELEMENT OF PI

P23 THICE THE VALUE OF (2,3) ELEMENT OF PI

V TEMPORARY STORAGE VECTOR FOR ARRAY P

COMMON USED: THO

HYELS Analysis

Subroutine HYELS computes and writes out hyperellipsoids associated with a 2 or 3 dimensional covariance matrix P.

If P is agained to be the covariance matrix of an n-dimensional random variable x having a gaussian distribution with mean zero, then the probability density function is given by

$$p = \frac{1}{(2\pi)^{n/2}} \left[ \frac{1}{2} \left[ \frac{1}{2} \right] \left[ \frac{1}{2}$$

Re-writing this equation as

$$\dot{x}^{T} P^{-1} \dot{x} = 2 \ln \left[ \frac{1}{(2\pi)^{n/2} p |P|^{1/2}} \right] = k^{2}$$

shows that the surface of constant probability density p is an m-dimensional ellipsoid, where m is the rank of P. The constant k can be shown to correspond to the sigma level of the ellipsoid.

For n = 3, the above equation has form

$$ax^{2} + by^{2} + cz^{2} + d xy + e xz + f yz = k^{2}$$
where
 $a = a_{11}$ 
 $b = a_{22}$ 
 $c = a_{33}$ 
 $d = 2a_{12}$ 
 $e = 2a_{13}$ 

and the a_{ij} are the elements of P⁻¹.

Subroutine HYELS uses this equation to compute a 3-dimensional hyperellipsoid, and sets the appropriate constants to zero to compute a 2-dimensional hyperellipsoid.

Reference: H. Sorenson. "Kalman Filtering", Advances in Control Systems, Vol. 3, C. T. Leades (Ed.). New York: Academic Press, 1966, p. 219.

SUBROUTINE HYPT

PURPOSE: TO CALCULATE TIME FROM PERIAPSIS ON A HYPERBOLA GIVEN TRUE ANAMOLY

ARGUMENT: CSTA I COSINE OF TRUE ANAMOLY

EH I ECCENTRICITY OF HYPERBOLA

ORBH I RECIPROCAL OF MEAN ORBITAL RATE IN SEC/RAD

SNTA I SINE OF TRUE ANAMOLY

T O TIME FROM PERIAPSIS IN SEC

SUBROUTINES SUPPORTED: CAREL IMPCT SPHIMP

LOCAL SYMBOLS: HA HYPERBOLIC ANAMOLY IN RAD

HSNHA HYPERBOLIC SINE OF HYPERBOLIC ANAMOLY

ONE CONSTANT 1.

SUBROUTINE IMPACT

PURPOSE: TO COMPUTE THE ACTUAL IMPACT PLANE PARAMETERS BOT AND BDR CORRESPONDING TO ANY POINT ON AN INCOMING HYPERBOLA. IT HAS THE OPTION TO CONVERT TARGET VALUES OF INCLINATION XIN AND RADIUS OF CLOSEST RCA INTO EQUIVALENT TARGET VALUES OF DBT AND DBR.

CALLING SEQUENCE: CALL IMPACT(R, V, GMX, T, BDT, BDR, XIN, RCA, DBT, DBR, TCA, KOPT)

'ARGUMENTS R(3) I POSITION VECTOR TO CENTRAL BODY AT EPOCH

V(3) I VELOCITY VECTOR TO CENTRAL BODY AT EPOCH

GMX I GRAVITATIONAL CONSTANT OF CENTRAL BODY

T(3,3) I TRANSFORMATION MATRIX FROM REFERENCE TO INCLINATION SYSTEM

BDT O VALUE OF ACTUAL B.T EVALUATED AT EPOCH

BDR O VALUE OF ACTUAL B.R EVALUATED AT EPOCH

XIN I DESIRED INCLINATION (DEG) (OPTIONAL)

RCA I DESIRED RADIUS OF CLOSEST APPROACH (OPTION)

DBT O TARGET VALUE OF B.T BASED ON XIN, RCA

DBR O TARGET VALUE OF B.R BASED ON XIN, RCA

TCA O TIME FROM PERIAPSIS ON CONIC

KOPT I TARGET VALUE COMPUTATION FLAG =0 DO NOT COMPUTE TARGET VALUES =1 COMPUTE TARGET VALUES OF 8.T, 8.R

COMPUTE TARGET VALUES OF B.T, B.R (MUST READ IN OPTIONAL INPUT)

SUBROUTINES SUPPORTED: TAROPT LUNCON LUNTAR MULTAR VMP

SUBROUTINES REQUIRED: NONE

LOCAL SYMBOLS: AB INTERMEDIATE VARIABLE FOR CALCULATION OF RV, SV, TV SYSTEM

AIN TARGET INCLINATION IN RADIANS. AFTER NORMALIZATION

ANG OUTPUT VARIABLE WHEN DECLINATION CONSTRAINT IS VIOLATED

AUXF ECCENTRIC ANOMALY (HYPERBOLIC CASE) A SEMI-MAJOR AXIS OF R-V CONIC BMAG MAGNITUDE OF DESIRED B VECTOR BV ACTUAL/DESIRED B VECTOR MAGNITUDE OF ACTUAL B VECTOR COSINE OF DECL CDECL CDELW COSINE OF DELW CTA COSINE OF TA CM COSINE OF W MAGNITUDE OF VECTOR NORMAL TO ORBITAL C1 PLANE IN INERTIAL SYSTEM DB DESIRED MAGNITUDE OF DESIRED B VECTOR DECLINATION OF APPROACH ASYMPTOTE IN DECL INCLINATION SYSTEM LONGITUDE OF ASCENDING NODE IN INCLINATION DELW SYSTEM Ε ECCENTRICITY OF THE R-V CONIC II INCLINATION SIGN INDICATOR. =1. INCLINATION IS POSITIVE =-1, INCLINATION IS NEGATIVE INDICATOR FOR DIRECTION OF MOTION OF THE IM TRAJECTORY =1, MOTION IS POSIGRADE =-1, MOTION IS RETROGRADE PI MATHEMATICAL CONSTANT 3.141592653589793 INTERMEDIATE VECTOR USED TO CALCULATE P۷ DESIRED B VECTOR P SEMI-LATUS RECTUM INTERMEDIATE VECTOR USED TO CALCULATE QV ACTUAL B VECTOR RAD DEGREES TO 'RADIANS CONVERSION CONSTANT

RD TIME DERIVATIVE OF RM'

RM HAGNITUDE OF THE POSITION VECTOR R

RRD DOT PRODUCT OF R AND V VECTORS

RV' VECTOR USED TO CALCULATE ACTUAL AND DESIRED B DOT R

SDECL SINE OF DECL

SDELW SINE OF DELW

SINHF HYPERBOLIC SINE OF AUXF

ļ

STA SINE OF TA

SV VECTOR USED TO CONSTRUCT RY, TV VECTORS.
PARALLEL TO THE APPROACH ASYMPTOTE

SW SINE OF W

SX VARIABLE USED TO DETERMINE SIGNS OF DBT,
DBR

TANG INTERMEDIATE VARIABLE FOR CALCULATION OF AUXF

TA TRUE ANOMALY FOR CALCULATION OF AUXF

THS INTERMEDIATE ANGLE FOR CALCULATION OF W

TV VECTOR USED TO CALCULATE ACTUAL AND DESIRED B DOT T

VINH VELOCITY AT INFINITY

VX MAGNITUDE OF THE VELOCITY VECTOR V

WMAG , MAGNITUDE OF VECTOR NORMAL TO ORBITAL PLANE IN INCLINATION SYSTEM

WV VECTOR NORMAL TO ORBITAL PLANE IN INCLINATION AND INERTIAL SYSTEMS

W ARGUMENT OF PERIAPSIS

Z APPROACH ASYMPTOTE IN INCLINATION SYSTEM

COMMON USED: NINETY, ONE TWO, ZERO

IMPACT Analysis

The impact parameters  $B \cdot T$  and  $B \cdot R$  form a convenient set of variables for the description of the approach geometry for lunar and interplanetary missions. Let a reference carte ian coordinate system XYZ (ecliptic in STEAP) be established at the center of the target body. Let  $\overline{V}_{\infty}$  denote the hyperbolic excess velocity of the spacecraft in the XYZ system. An auxiliary coordinate system R-S T may be constructed relative to the  $\overline{V}_{\infty}$  by the definitions

$$\hat{S} = \overrightarrow{V}_{\infty} / \overrightarrow{\nabla}_{\infty} \qquad \hat{T} = \frac{\hat{S} \times \hat{K}}{|\hat{S} \times \hat{K}|} \qquad \hat{R} = \hat{S} \times \hat{T} \qquad (1)$$

Therefore S is in the direction of the approach asymptote. I lies along the intersection of the impact p ane (the plane normal to S and passing through the center of the planet) and the reference plane (XY-plane), and R completes the right hand system. The B vector lies in the impact plane and is directed to the incoming asymptote. Then B.T and B.R have the usual vector definitions.

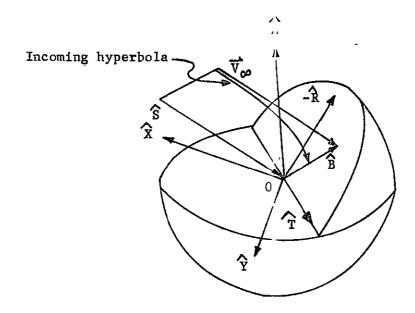


Figure 1. Impact Plane Parameters

In the optional part of the subroutine, the target impact parameter B* associated with  $\hat{S}$  and a target inclination i (relative to target planet equator) and radius of closest approach  $r_{CA}$  is computed. However given an approach asymptote  $\hat{S}$  there are generally four trajectories with the same values of i and  $r_{CA}$ . Two of these trajectories are retrograde and

two are posigrade. For each type of motion there are two distinct planes that have the same inclination and include the  $\hat{S}$  vector. These are distinguished by the direction of motion when the approach asymptote is crossed, i.e., whether the motion is from north to south (northern approach) or from south to north (southern approach). Let  $0 \le \alpha \le 90^\circ$ . Then setting the target inclinations to the following values determines the trajectory which will be specified:

í	Trajectory		
α	posigrade with northern approach		
-α	posigrade with southern approach		
180+α	retrograde with northern approach		
180-α	retrograde with southern approach		

The possible trajectories are illustrated in Figure 2.

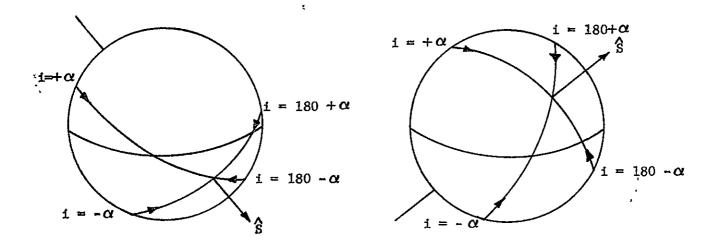


Figure 2. Possible Trajectories with Same Inclination

The detailed computations for the basic part of the program are straightforward. Using the standard conic abbreviations,

$$c = |\overrightarrow{r} \times \overrightarrow{v}| \tag{2}$$

$$\hat{\mathbf{v}} = \frac{\vec{\mathbf{r}} \times \vec{\mathbf{v}}}{c} \tag{3}$$

$$p = \frac{c^2}{u} \tag{4}$$

$$a = \frac{r}{2-rv^2/\mu} \tag{5}$$

$$e^2 = 1 - \frac{p}{a} \tag{6}$$

$$b = \sqrt{p \mid a \mid} \tag{7}$$

$$\cos f = \frac{p-r}{er} \tag{8}$$

$$\sin f = \frac{\dot{r} c}{e u} \tag{9}$$

$$\hat{Z} = \frac{r}{c} \quad v - \frac{\dot{r}}{c} \vec{r} \tag{10}$$

$$\hat{P} = \frac{\vec{r}}{r} \cos f - \vec{z} \sin f \qquad (11)$$

$$\hat{Q} = \frac{\vec{r}}{r} \sin f + \hat{Z} \cos f$$
 (12)

$$\hat{S} = -\frac{a}{\sqrt{a^2 + b^2}} \hat{P} + \frac{b}{\sqrt{a^2 + b^2}} \hat{Q}$$
 (13)

$$\hat{\mathbf{R}} = \hat{\mathbf{S}} \times \hat{\mathbf{T}} \tag{15}$$

$$\vec{B} = \frac{b^2}{\sqrt{a^2 + b^2}} \hat{P} + \frac{ab}{\sqrt{a^2 + b^2}} \hat{Q}$$
 (16)

$$B \cdot T = \overrightarrow{B} \cdot \overrightarrow{T} \tag{17}$$

$$B \cdot R = \overrightarrow{B} \cdot \overrightarrow{R} \tag{18}$$

The computations for the optional part of the program which converts the i and  $r_{CA}$  into an equivalent  $\overrightarrow{B^*}$  proceed as follows. The approach asymptote is first converted into target planet equatorial coordinates and its right ascension and declination computed

$$\hat{S}_{q} = \phi_{ECEQ} \hat{S}$$

$$\theta_{S} = \tan^{-1} \frac{(S_{q})_{y}}{(S_{q})_{x}}$$

$$\delta_{S} = \sin^{-1} (S_{q})_{z}$$
(19)

The angle  $\triangle 0$  between the ascending node of the trajectory and the right ascension of the approach asymptote is from Napiers rule

$$\sin \theta = \frac{\tan \delta_{S}}{\tan i} \tag{20}$$

after assuring that  $|i| \ge |\delta_S|$ . The ascending node of the trajectory is then computed recalling the definitions of the angle i

$$Q = \theta_{S} + \Delta \theta \quad (+\pi) \tag{21}$$

Thus the unit vector to the ascending node is given by

$$\hat{R}_{A} = (\cos \Omega, \sin \Omega, 0)$$
 (22)

The normal to the orbital plane (in target planet equatorial coordinates) is

$$\widehat{W}_{q} = \frac{\widehat{S}_{q} \times \widehat{R}_{A}}{|\widehat{S}_{q} \times \widehat{R}_{A}|}$$
(23)

This is now converted to the ecliptic coordinate system

$$\hat{\mathbf{W}}_{\mathbf{C}} = \emptyset_{\mathbf{ECEQ}} \hat{\mathbf{W}}_{\mathbf{q}}$$
 (24)

The unit vector in the desired B* direction is

$$\hat{\mathbf{B}}^* = \frac{\hat{\mathbf{S}} \times \hat{\mathbf{W}}_{\mathbf{C}}}{\left|\hat{\mathbf{S}} \times \hat{\mathbf{W}}_{\mathbf{C}}\right|} \tag{25}$$

The magnitude of the B* vector is given by

$$B* = r_{CA} \sqrt{1 + \frac{2 \mu}{r_{CA} \hat{V}_{\infty}^2}}$$
 (26)

Then the target impact parameter is  $\overrightarrow{B*} = \overrightarrow{B*} = \overrightarrow{B*}$ . The target values are then given by their obvious definitions

$$B \cdot T^* = \overrightarrow{B^*} \cdot \overrightarrow{T}$$

$$B \cdot R^* = \overrightarrow{B^*} \cdot \widehat{R}$$
(27)

Finally the hyperbolic time from  $(\vec{r}, \vec{v})$  to periapsis is computed from the conic formula

$$\tanh \frac{F}{2} = \sqrt{\frac{e-1}{e+1}} \tan \frac{f}{2}$$

$$t = \sqrt{\frac{-a^3}{u}} \quad (e \sinh F - F)$$
(28)

Reference: Kizner, W., A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories, Ballistic Missiles and Space Technology vol III, Pergamon Press, New York, 1961.

SUBROUTINE IMPCT

PURPOSE: TO COMPUTE FOR AUXILIARY TARGETING ACTUAL AND DESIRED B-PLANE ASYMPTOTE PIERCE POINTS AS WELL AS ACTUAL TARGET VALUES

ARGUMENT: KOPT I FLAG INDICATING SUBROUTINE OPERATING MODE

=1 CALCULATE ACTUAL B-PLANE ASYMPTOTE

PIERCE POINT COORDINATES (B.T AND B.R)

ONLY

- =2 CALCULATE ACTUAL INCLINATION, RADIUS, AND TIME AT CLOSEST APPROACH AS WELL AS ACTUAL B.T AND B.R
- =3 CALCULATE ACTUAL RIGHT ASCENSION, DECLINATION, AND TIME AT IMPACT AS WELL AS ACTUAL B.T AND B.R

R I PLANETOCENTRIC POSITION VECTOR ON WHICH OSCULATING CONIC IS TO BE BASED IN KM

V I PLANETOCENTRIC VELOCITY VECTOR ON WHICH OSCULATING CONIC IS TO BASED IN KM/SEC

SUBROUTINES SUPPORTED: TAROPT

SUBROUTINES REQUIRED: HYPT MATPY SCAD USCALE UXV

LOCAL SYMBOLS: AB ECCENTRICITY TIMES MAGNITUDE OF SEMI-MAJOR AXIS IN KM

AIN ACUTE INCLINATION REFERENCE ANGLE IN RAD

A SEMI-MAJOR AXIS OF OSCULATING CONIC IN KM

BMAG MAGNITUDE OF B VECTOR IN KM USED IN CALCULATION OF DESIRED B.T AND B.R FROM INCLINATION AND RADIUS OF CLOSEST APPROACH

BM MAGNITUDE OF B VECTOR IN KM USED IN CALCULATION OF DESIRED B.Y AND B.R FROM PROBE TARGET SITE

BV B VECTOR ITSELF IN KM OR UNIT VECTOR IN DIRECTION OF B VECTOR

CDECL COSINE OF DECLINATION OF TRAJECTORY
ASYMPTOTE RELATIVE TO INCLINATION SYSTEM

COELW COSINE OF DIFFERENCE BETWEEN RIGHT
ASCENSION OF ASCENDING MODE AND RIGHT
ASCENSION OF TRAJECTORY ASYMPTOTE RELATIVE
TO INCLINATION SYSTEM

CSDCP COSINE OF DECLINATION OF PROBE IMPACT SITE RELATIVE TO PROBE-SPHERE FRAME CSDIF COSINE OF ARC LENGTH BY WHICH DESIRED IMPACT SITE IS CLOSER TO TRAJECTORY ASYMPTOTE THAN IS PERIAPSIS COSINE OF THE ANGLE BETWEEN TRAJECTORY CSPHI ASYMPTOTE AND VECTOR TO DESIRED IMPACT SITE CSRAP COSINE OF RIGHT ASCENSION OF PROBE IMPACT SITE RELATIVE TO PROBE-SPHERE FRAME **CSTHTS** COSINE OF TRUE ANAMOLY OF TRAJECTORY ASYMPTOTE COSINE OF TRUE ANAMOLY OF GIVEN STATE CTA CTPS COSINE OF TRUE ANAMOLY AT PROBE SPHERE COSINE OF RIGHT ASCENSION OF ASCENDING CW NODE IN INCLINATION SYSTEM C1 COEFFICIENT USED IN CALCULATING REPOSITIONED PROBE IMPACT SITE COEFFICIENT USED IN CALCULATING C2 REPOSITIONED PROBE IMPACY SITE 08 MAGNITUDE OF DESIRED B VECTOR CALCULATED DB. FROM DESIRED INCLINATION AND RADIUS AT CLOSEST APPROACH DECLINATION OF TRAJECTORY ASYMPTOTE IN DECL RAD RELATIVE TO INCLINATION SYSTEM DIFFERENCE IN RAD BETWEEN RIGHT ASCENSION DELW OF ASCENDING NODE AND RIGHT ASCENSION OF TRAJECTORY ASYMPTOTE RELATIVE TO INCLINATION SYSTEM DTR CONVERSION FACTOR FROM DEGREES TO RADIANS DTS CONVERSION FACTOR FROM DAYS TO SECONDS Ε ECCENTRICITY OF OSCULATING CONIC HREV 180. DEG

II INCLINATION SIGN INDICATOR INCLINATION IS POSITIVE =1 INCLINATION IS NEGATIVE IM INDICATOR FOR DIRECTION OF MOTION OF TRAJECTORY MOTION IS POSIGRADE **=1** MOTION IS RETROGRADE =2 RECIPROCAL OF MEAN ORBITAL RATE IN ORBH SEC/RAD PI MATHEMATICAL CONSTANT PI DIRECTION OF PERIAPSIS ON OSCULATING CONIC PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN PV þ SEMI-LATUS RECTUM OF OSCULATING CONIC IN KM PLANETOGENTRIC ECLIPTIC UNIT VECTOR IN QV ORBIT PLANE ADVANCED 90 DEG FROM PV TIME RATE OF CHANGE OF OSCULATING CONIC RD RADIUS IN KM/SEC RM MAGNITUDE OF PLANETOCENTRIC OSCULATING CONIC POSITION VECTOR IN KM RPR RADIUS OF PROBE SPHERE IN KM RPVRV PLANETOCENTRIC ECLIPTIC UNIT VECTOR TO IMPACT SITE RPV PLANETOCENTRIC PROBE-SPHERE UNIT VECTOR TO IMPACT SITE RRO PRODUCT OF OSCULATING CONIC RADIUS BY ITS TIME RATE OF CHANGE IN KM**2/SEC RTO CONVERSION FACTOR FROM RADIANS TO DEGREES PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN RV DIRECTION OF CROSS PRODUCT OF OSCULATING CONIC ASYMPTOTE BY ECLIPTIC POLE VECTOR SDECL SINE OF DECLINATION OF OSCULATING CONIC ASYMPTOTE RELATIVE TO INCLINATION SYSTEM SINE OF DIFFERENCE BETWEEN RIGHT SDELW ASCENSION OF ASCENDING NODE AND RIGHT ASCENSION OF ASYMPTOTE FOR OSCULATING CONIC RELATIVE TO INCLINATION SYSTEM

SNDCP SINE OF DECLINATION PROBE IMPACT SITE RELATIVE TO PROBE-SPHERE FRAME SINE OF ANGLE BETWEEN OSCULATING CONIC SNPHI ASYMPTOTE AND VECTOR TO DESIRED IMPACT SITE SNRAP SINE OF RIGHT ASCENSION OF PROBE IMPACT SITE RELATIVE TO PROBE-SPHERE FRAME SINE OF TRUE ANAMOLY OF OSCULATING CONIC SNTHTS **ASYMPTOTE** STA SINE OF TRUE ANAMOLY ON OSCULATING CONIC AT GIVEN STATE STPS SINE OF TRUE ANAMOLY ON OSCULATING CONIC AT PROBE SPHERE SV PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN DIRECTION OF OSCULATING CONIC ASYMPTOTE SW SINE OF RIGHT ASCENSION OF ASCENDING NODE OF OSCULATING CONIC IN INCLINATION SYSTEM TEST QUANTITY USED TO DETERMINE PROPER POLARITY OF WV THS RIGHT ASCENSION IN DEG OF OSCULATING CONIC ASYMPTOTE REALTIVE TO INCLINATION SYSTEM **TPSCA** TIME INTERVAL IN SEC FROM PROBE SPHERE TO CLOSEST APPROACH ON OSCULATING CONIC TSICA TIME INTERVAL IN SEC FROM SOI TO CLOSEST APPROACH TV CROSS PRODUCT OF SV BY RV VINF MAGNITUDE OF HYPERBOLIC EXCESS VELOCITY IN KM/SFC VS SQUARE OF MAGNITUDE OF PLANETOCENTRIC VELOCITY IN KM**2/SEC**2 WEQ PLANETOCENTRIC EQUATORIAL UNIT VECTOR IN DIRECTION OF OSCULATING CONIC ANGULAR MOMENTUM WMAG UN-NORMALIZED MAGNITUE OF WV

WV		ON OF OS	ECLIPTIC CULATING		
W			OF ASCE		DE OF CLINATION
Z	DIRECTI OR PLAN ORBIT F	ON OF OS	CULATING RIC ECLIP DEG ADVA	CONIC A	VECTOR IN SYMPTOTE VECTOR IN M GIVEN
COMMON COMPUTED/USED:	8	ОСР	DIN	RAP	XATAR
COMMON COMPUTED:	AAUX	BDR	80T	DBR	DBT
COMMON USED:	CAINC DRCA FOUR IRCADC NOPAR ZERO	DC DSI GMX ITARR ONE	DDCP DTAR HALF KEYTAR RGA	DINC ECSS IINCRA KUR RPS	ORAP EQECP IPHASE NINETY TWO

## IMPCT Analysis

The subroutine IMPCT is responsible for computing all of the target parameter data associated with auxiliary targeting. Three basic types of target information are required given a planetocentric ecliptic state. First, what are the actual B-plane pierce point coordinates,  $B_{\rm A}{}^{}\cdot T$  and  $B_{\rm A}{}^{}\cdot R$ ? Second, what are the values of the actual target parameters? These may be triples of inclination, radius and time at closest approach or right ascension, declination, and time at impact. Third, what are the B-plane pierce point coordinates,  $B_{\rm D}{}^{}\cdot T$  and  $B_{\rm D}{}^{}\cdot R$  on the current trajectory required to achieve the desired values of the corresponding actual target parameters. In addition to supplying all of this information, IMPCT places it in the appropriate locations for sorting by the processing routine TARØPT.

Whenever IMPCT is called, it first calculates the actual B-plane pierce point coordinates for the current state. In the process it also calculates other useful information about the osculating conic, including the parameters a, e,  $\theta$ ,  $\underline{W}$ ,  $\underline{S}$ ,  $\underline{T}$  and  $\underline{R}$ . For the equations giving these quantities see the subroutine STIMP analysis. If option flag KPT is 1, only this information is desired and a return to the calling program is executed.

The values of the actual target parameters are calculated if K $\emptyset$ PT is not 1. If the targets are inclination, radius, and time at closest approach, K $\emptyset$ PT must be 2. If TARGET is not in the second phase of a two-phase targeting case, the target parameters, i,  $r_{CA}$ , and  $t_{CA}$  are obtained by conic extrapolation from the current state (SOI):

$$r_{CA} = a (1 - e).$$
 (1)

Let D denote the transformation from planetocentric ecliptic coordinates to the planetocentric equatorial frame:

$$i = \cos^{-1} \left[ \left( D \underline{W} \right)_3 \right] \tag{2}$$

If TARGET is in a second phase, the virtual mass program will have integrated the trajectory all the way to closest approach rather than stopping at the SOI. Hence refined values of all three target parameters are available from VMP.

Suppose, on the other hand, that the targets are right ascension  $\alpha$ , declination  $\delta$ , and time  $t_{\tau}$  at impact. Then KØPT must be 3.

Again if TARGET is not involved in a second phase, these target parameters are calculated by extrapolating conically from the SOI. Let  $\underline{r}_{1}$  and  $\underline{\rho}_{1}$  denote the position vectors of the vehicle at im-

pact in the planetocentric ecliptic and probe-sphere frames, respectively. Let C denote the transformation from the former frame to the latter. Obviously

$$\underline{\rho}_{\mathrm{I}} = \underline{\mathrm{Cr}}_{\mathrm{I}}. \tag{4}$$

The right ascension and declination at impact are then

$$\alpha = \tan^{-1} \left[ \left( \rho_{I} \right)_{2} / \left( \rho_{I} \right)_{1} \right], \qquad (5)$$

and

$$\delta = \sin^{-1} \left[ \left( \rho_{I} \right)_{3} / \rho_{I} \right]. \tag{6}$$

Let  $\Delta t_{SC}$  and  $\Delta t_{IC}$  be the times from the sphere of influence and from the probe sphere to closest approach, respectively. Let  $\theta_{I}$  denote the true anomaly on the osculating conic at the probe sphere:

$$\cos \theta_{T} = \left( p / r_{T} - 1/a \right) / e \tag{7}$$

where

$$p = a (1 - e^2)$$
 (8)

is the semilatus rectum of the conic. Since impact occurs before peripsis,

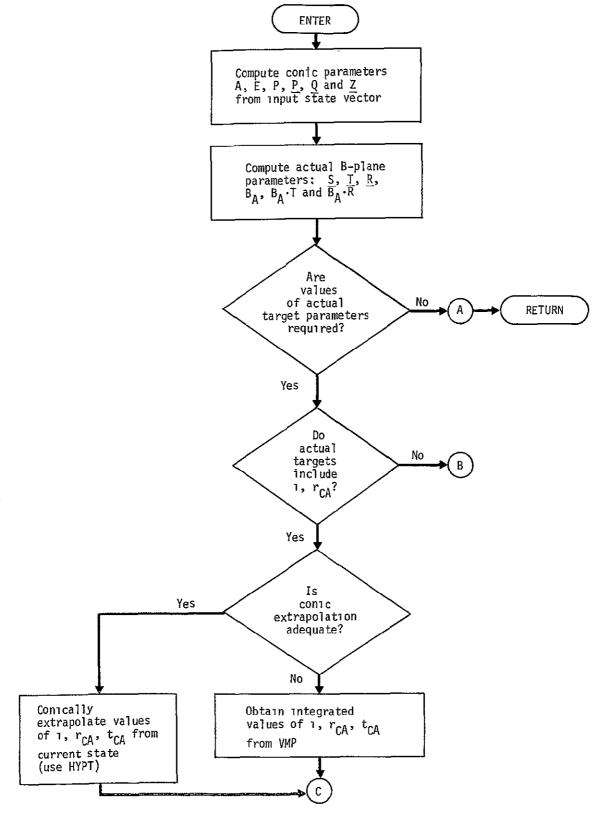
$$\sin \theta_{T} = -\sqrt{1 - \cos^{2} \theta_{T}}.$$
 (9)

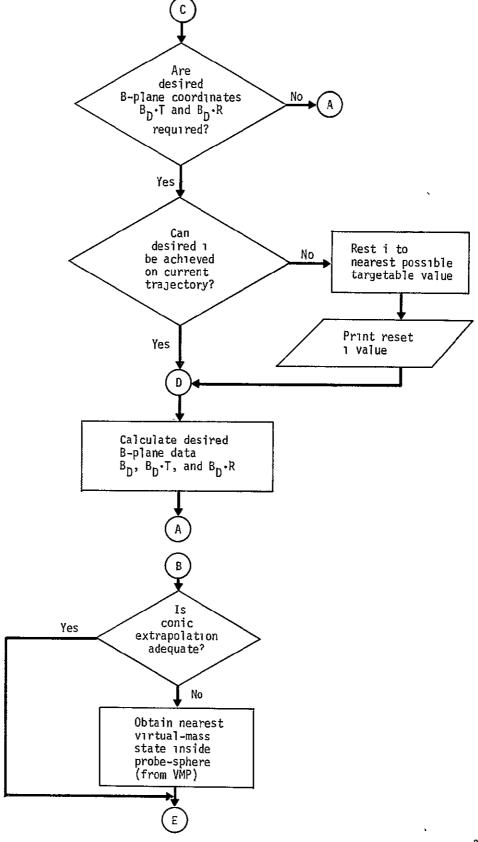
With the true anomalies on the conic at the current state and at the probe sphere available, IMPCT cals HYPT to determine  $\Delta t_{SC}$  and  $\Delta t_{IC}$ . If, as above,  $t_{SOI}$  denotes the time from the sphere of influence to periapsis,

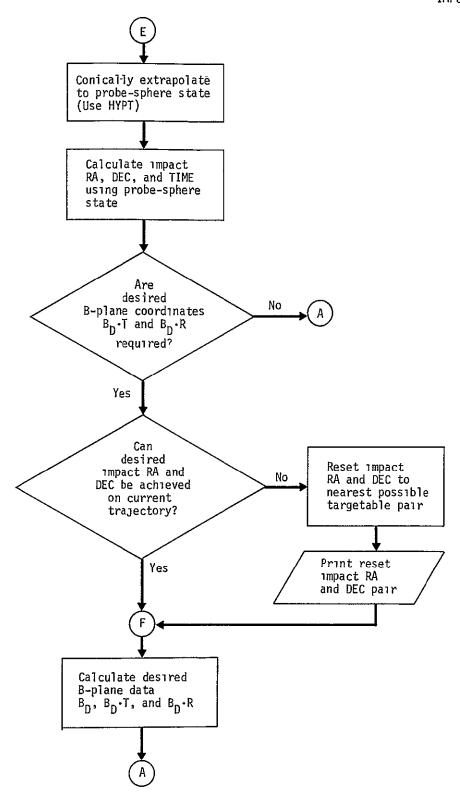
$$t_{\rm I} = t_{\rm SOI} + (\Delta t_{\rm SC} - \Delta t_{\rm IC}). \tag{10}$$

Finally, if TARGET is involved in the second phase of targeting, the virtual-mass algorithm integrates the trajectory all the way to the first integration increment inside the probe sphere. Hence the preceding conic extrapolation formulae can be used to obtain accurate impact target parameters by replacing the state at the SOI with the first state inside the probe sphere.

IMPCT calculates the desired B-plane pierce point coordinates  $^{B}D^{\cdot T}$  and  $^{B}D^{\cdot R}$  for either the i and  $^{C}A$  or  $\alpha$  and  $\beta$  target options if the flag ITARR is 2 indicating that a new control iteration is being made. The equations and logical flow of this calculation for the former target option is given in the subroutine IMPACT while tose for the latter are presented in DIMPCP.







SUBROUTINE INPUTZ

PURPOSE: TO CONVERT THE INPUT INFORMATION FOR THE VIRTUAL MASS PROGRAM INTO VARIABLES COMPATIBLE WITH THE REST OF THE VIRTUAL MASS SUBROUTINES

CALLING SEQUENCE: CALL INPUTZ(RS, NTP, IPRINT)

ARGUMENTS RS(6) I INERTIAL STATE OF S/C AT INITIAL TIME

NTP I CODE OF TARGET BODY

IPRINT I INITIAL INFORMATION PRINT FLAG =0 PRINT INITIAL DATA =1 00 NOT PRINT INITIAL DATA

SUBROUTINES SUPPORTED: VMP

SUBROUTINES REQUIRED: TIME SPACE

LOCAL SYMBOLS: D INTERMEDIATE VARIABLE FOR PRINTOUT PURPOSES

D2 JULIAN DATE OF FINAL TRAJECTORY TIME

IDAY DAY OF CALENDAR DATE OF FINAL TRAJECTORY TIME

IHR HOUR OF CALENDAR DATE OF FINAL TRAJECTORY TIME

IMIN MINUTE OF CALENDAR DATE OF FINAL TRAJECTORY TIME

INO MONTH OF CALENDAR DATE OF FINAL TRAJECTORY

INERR NOT USED

IP CODE OF I-TH PLANET FOR STORAGE OF PMASS ARRAY

IYR YEAR OF CALENDAR DATE OF FINAL TRAJECTORY TIME

LDAY DAY OF CALENDAR DATE OF INITIAL TIME

LHR HOUR OF CALENDAR DATE OF INITIAL TIME

LMIN MINUTE OF CALENDAR DATE OF INITIAL TIME

LMO MONTH OF CALENDAR DATE OF INITIAL TIME

LYR	YEAR OF CALENDAR DATE OF INITIAL TIME
SECI	SECOND OF CALENDAR DATE OF FINAL TIME
SECL	SECOND OF CALENDAR DATE OF INITIAL TIME
TP	INTERMEDIATE VARIABLE FOR CALCULATION OF COMPUTING INTERVAL
COMMON COMPUTED/USED:	V
COMMON COMPUTED:	F INC IPR ITRAT KOUNT NBODY

COMMON USED: NBODYI NO PMASS ZERO

SUBROUTINE INSERS

**PURPOSE**: TO CONTROL THE PROCESSING OF AN ORBITAL INSERTION EVENT.

CALLING SEQUENCE: CALL INSERS(DTIME)

TIME INTERVAL FROM DECISION TO EXECUTION **ARGUMENT** 0 DTIME

(DAYS)

SUBROUTINES SUPPORTED: GIDANS

SUBROUTINES REQUIRED: COPINS NONINS PECEG

LOCAL SYMBOLS: DA DESIRED SEMIMAJOR AXIS

> DE DESIRED ECCENTRICITY

DI DESIRED INCLINATION

DN DESIRED LONGITUDE OF ASCENDING NODE

DHTP DESIRED ARGUMENT OF PERIAPSIS SHIFT OR

DESIRED ARGUMENT OF PERIAPSIS

ECEQI ECLIPTIC TO EQUATORIAL TRANSFORMATION

GH GRAVITIONAL CONSTANT OF TARGET BODY

UNEXECUTABLE EVENT CODE IEX

=0 EVENT IS EXECUTABLE

=1 NO EXECUTABLE SOLUTION FOUND

IOPT INSERTION STRATEGY OPTION

=1 COPLANAR INSERTION

=2 NONPLANER INSERTION

RSP SPACECRAFT POSITION IN ECLIPTIC COORDS

SPACECRAFT POSITION IN EQUATORIAL COORDS RSQ

TEX TIME INTERVAL TO EXECUTION (SECONDS)

VSP SPACECRAFT VELOCITY IN ECLIPTIC COORDS

VSQ SPACECRAFT VELOCITY IN EQUATORIAL COORDS

'CONNON CONPUTED/USED: DELTAV

COMMON COMPUTED: DELV KTIM KHIT

COMMON USEDS ALNGTH D1 KMXQ KTAR

KUR NBOD NB NTP **PMASS**  INSERS Analysis

INSERS controls the processing of an orbital insertion event. The sub-routine COPINS and NONINS perform the actual computations for the co-planar and non-planar options respectively.

INSERS first records the specific parameter values for the current orbit insertion event.

It then computes the current state  $(\vec{r}, \vec{v})$  of the spacecraft in target-planet centered ecliptic coordinates. Subroutine PECEQ is called to compute the transformation matrix  $\Phi_{\text{ECEQ}}$  from ecliptic to equatorial coordinates. The planet centered equatorial coordinates are then

$$\vec{r}_q = \Phi_{ECEQ} \vec{r}$$

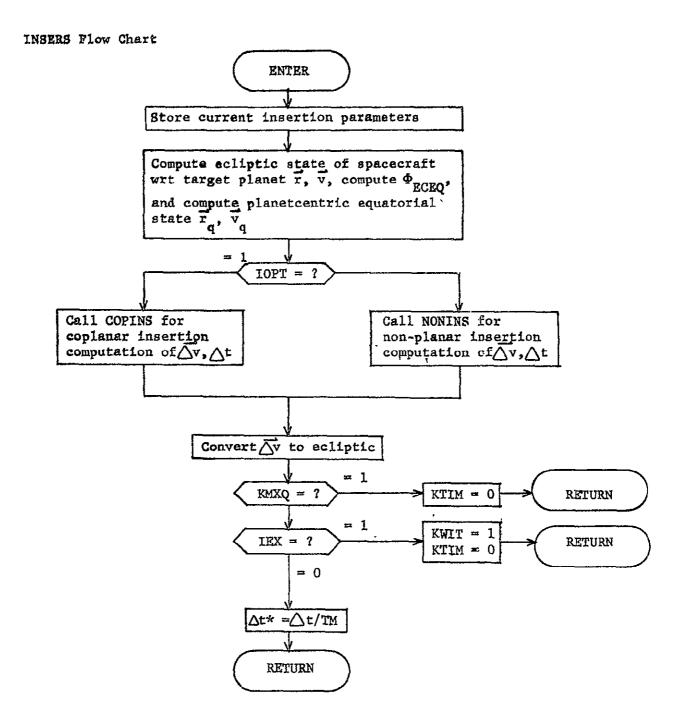
$$\vec{v}_{q} = \Phi_{ECEQ} \vec{v}$$

This state is then sent to COPINS or NONINS for the computation of the insertion velocity  $\triangle v$  and the time interval t between the current time and the time at which the insertion should take place (based on conic propagation about the target body). The correction  $\triangle v$  is then converted to ecliptic coordinates

$$\overrightarrow{\triangle} \mathbf{v} = \mathbf{\Phi}^{\mathrm{T}} \overrightarrow{\triangle} \mathbf{v}_{\mathbf{q}}$$

If the event is a compute-only mode, the return is made to GIDANS.

If the event is to be executed the flag IEX (set by COPINS or NONINS to indicate success or failure) is then interrogated. If IEX = 1, no acceptable insertion event was found and so the executive flag KWIT is set to 1 before returning. If IEX = 0 an acceptable insertion was determined and so it is set up.



SUBROUTINE JACOB

PURPOSE: TO APPROXIMATE BY DIVIDED DIFFERENCES THE JACOBIAN SENSITIVITY MATRIX OF A VECTOR-VALUED FUNCTION WITH RESPECT TO A VECTOR ARGUMENT

ARGUMENT *	DELTA	I	COMMON PERTURBATION SIZE USED FOR ALL COMPONENTS OF INDEPENDENT VECTOR
	FPHI	I	FUNCTION RELATING DEPENDENT VECTOR TO INDEPENDENT VECTOR
	М	I	DIMENSION OF DEPENDENT VECTOR
	N	I	DIMENSION OF INDEPENDENT VECTOR
	PHI	I	DEPENDENT VECTOR
	RJCBM	0	JACOBIAN SENSITIVITY MATRIX OF PHI WITH RESPECT TO X
	X	I	INDEPENDENT VECTOR

SUBROUTINES SUPPORTED: GAUSES

SUBROUTINES REQUIRED: FPHI

LOCAL SYMBOLS: IZERO

PHÍN PERTURBED VALUES OF DEPENDENT VECTOR

XSAVE ORIGINAL VALUE OF COMPONENT OF INDEPENDENT VECTOR CURRENTLY BEING PERTURBED

SUBROUTINE JACOBI

PURPOSE: TRANSFORMATION OF A REAL SYMMETRIC MATRIX TO DIAGONAL FORM BY A SUCCESSION OF PLANE ROTATIONS TO ANNIHILATE THE OFF-DIAGONAL ELEMENTS AND SUBSEQUENT COMPUTATION OF THE EIGENVALUES AND EIGENVECTORS OF THAT MATRIX

CALLING SEQUENCE: CALL JACOBI (A, H2, V, N, FOD)

ARGUMENT: A I MATRIX TO BE DIAGONALIZED (WILL BE DESTROYED)

W2 O VECTOR OF EIGENVALUES (LENGTH N)

V O MATRIX OF EIGENVECTORS (N BY N DIMENSION)

N I DIMENSION OF SQUARE MATRIX A

FOD I FINAL OFF-DIAGONAL ANNIHILATION VALUE

SUBROUTINES SUPPORTED: EIGHY GUISIH GUISS PRESIM SETEVN GUIDH GUID PRED

LOCAL SYMBOLS: AIIP INTERMEDIATE VARIABLE

AIPIP INTERMEDIATE VARIABLE-A(IPIP)

AIPJP INTERMEDIATE VARIABLE-A(IPJP)

AJPJP INTERMEDIATE VARIABLE-A(JPJP)

CS INTERMEDIATE VARIABLE

DEL DIFFERENCE IN ELEMENTS OF A

IREDO COUNTER

KR DIMENSION OF A

KRP1 KR + 1

NM1 N - 1

RAD INTERMEDIATE VARIABLE

SN INTERMEDIATE VARIABLE

TN INTERMEDIATE VARIABLE

11 LARGEST OFF-DIAGONAL ELEMENT

VIIP INTERMEDIATE VARIABLE

## JACOBI Analysis

The Jacobi method subjects a real, symmetric matrix A to a sequence of transformations based on a rotation matrix:

$$O_{K} = \begin{bmatrix} \cos \phi_{K} & -\sin \phi_{K} \\ \sin \phi_{K} & \cos \phi_{K} \end{bmatrix}$$

where all other elements of the rotation matrix are identical with the unit matrix. After n multiplications A is transformed into:

$$A' = 0_N^{-1} \dots 0_1^{-1} A 0_1 \dots 0_N$$

If  $\emptyset_K$  is chosen at each step to make a pair of off-diagonal elements zero, then A' will approach diagonal form with the eigenvalues on the diagonal. The columns of  $0_1$   $0_2$ ... $0_N$  correspond to the eigenvectors of A.

The angle of rotation  $\emptyset$  is chosen in the following way. If the four entries of 0 are in (i,1), (i,j), (j,i) and (j,j) then the corresponding elements of  $0_1^{-1}$  A  $0_1$  are

$$b_{ij} = a_{i1} \cos^2 \emptyset + 2a_{ij} \sin \emptyset \cos \emptyset + a_{jj} \sin^2 \emptyset$$

$$b_{ij} = b_{j1} = (a_{jj} - a_{ii}) \sin \emptyset \cos \emptyset + a_{ij} (\cos^2 \emptyset - \sin^2 \emptyset)$$

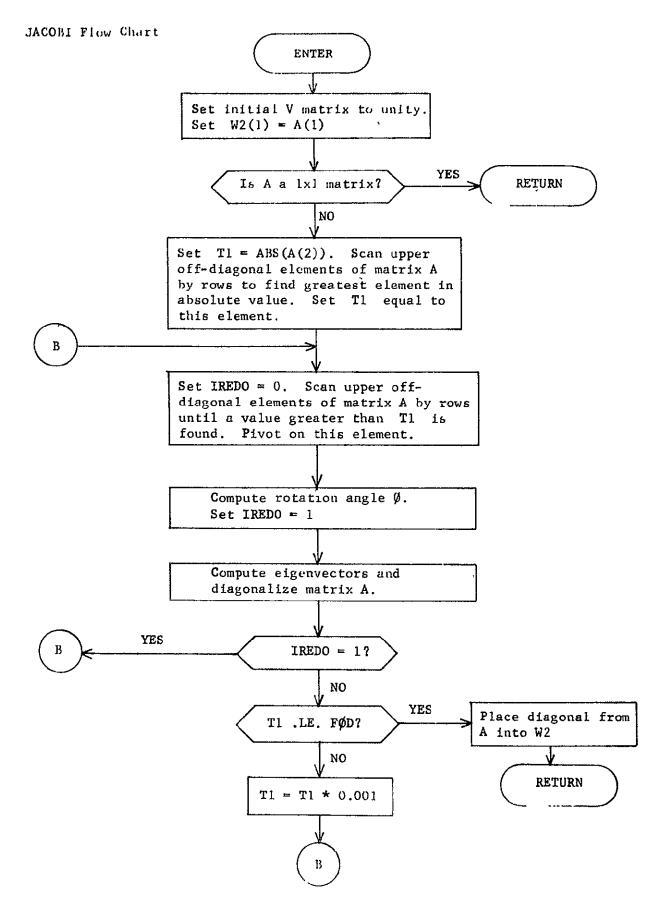
$$b_{jj} = a_{i1} \sin^2 \emptyset - 2a_{ij} \sin \emptyset \cos \emptyset + a_{jj} \cos^2 \emptyset$$

If  $\emptyset$  is chosen so that  $\tan 2\emptyset = 2a_{1j}/(a_{1j} - a_{jj})$  then

$$b_{ij} = b_{ji} = 0$$

Each multiplication creates a new pair of zeros but will introduce a non-zero contribution to positions zeroed out on previous steps. However, successive matrices of the form  $0_2^{-1}$   $0_1^{-1}$  A  $0_1$   $0_2$  will approach the required diagonal form.

Reference: Scheid, Frances: Theory and Problems of Numerical Analysis, McGraw-Hill Book Company, Inc., New York, 1968.



## SUBROUTINE KTROL

PURPOSE:	TO COMPUTE HELIOCENTRIC ECLIPTIC VELOCITY CORRECTIONS
	GIVEN LAUNCH-PLANETOCENTRIC VELOCITY CONTROLS

	GIVEN	LAUNCH-	PLANETOCENTRIC VELOCITY CONTROLS
AUGUMENT :	CON	I	VECTOR OF VELOCITY CONTROLS IN KM/SEC AND RAD
	עמ	0	CORRECTION IN HELIOCENTRIC VELOCITY IN KM/SEC PRODUCED BY VELOCITY CONTROLS
	IOPT	I	FLAG INDICATING WHICH CONTROLS ARE TO BE PERTURBED
	X	į	LAUNCH-PLANETOCENTRIC ECLIPTIC STATE VECTOR AT MANEUVER POINT IN KM AND KM/SEC
SUBROUTINE	ES SUPI	PORTED	TARMAX TARGET
SUBROUTINE	ES REQU	JIRED:	USCALE UXV
LOCAL SYME	30LS:	CSC2	COSINE OF IN-PLANE ROTATION ANGLE
ŗ	•	CSC3	COSINE OF OUT-OF-PLANE ROTATION ANGLE
		ONE	CONSTANT 1.
		RTD	CONVERSION FACTOR FROM RADIANS TO DEGREES
		SNC2	SINE OF IN-PLANE ROTATION ANGLE
		SNC3	SINE OF OUT-OF-PLANE ROTATION ANGLE
		U	CROSS PROBUCT OF W BY V
		V	LAUNCH-PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN DIRECTION OF VELOCITY AT MANEUVER POINT
		MA	MAGNITUDE OF VELOCITY VECTOR AT MANEUVER

VM MAGNITUDE OF VELOCITY VECTOR AT MANEUVER
POINT

W LAUNCH-PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN DIRECTION OF ANGULAR MOMENTUM AT MANEUVER POINT

WM MAGNITUDE OF CROSS PRODUCT OF LAUNCH-PLANETOCENTRIC ECLIPTIC POSITION VECTOR AND UNIT VELOCITY VECTOR AT MANEUVER POINT

## KTRØL Analysis

KTRØL calculates the targeting velocity increment  $\Delta v$  given the targeting state vector  $\underline{X} = (\underline{r}/\underline{v})^T$  of the spacecraft relative to the launch planet and the launch-planetocentric velocity controls  $c_1$ ,  $c_2$ , and  $c_3$ . This computation is required in two distinct situations. The first is in calculating the sensitivity matrix of the auxiliary parameters to the velocity controls by successively perturbing each control while holding the remaining two constant. The second is in applying the control correction indicated by the Newton-Raphson algorithm to arrive at the next iterate to the postimpulse targeting state.

In either case the three unit vectors  $\underline{V}$ ,  $\underline{U}$ ; and  $\underline{W}$  that serve to define the local, spherical, velocity-control coordinate system are first computed

$$\underline{\mathbf{v}} = \frac{\mathbf{v}}{\mathbf{v}} \tag{1}$$

$$\underline{W} = \frac{\underline{r} \times \underline{V}}{||\underline{r} \times \underline{V}||} \tag{2}$$

$$\underline{\mathbf{u}} = \underline{\mathbf{w}} \times \underline{\mathbf{v}} . \tag{3}$$

 $\underline{V}$  specifies the direction of zero latitude and zero longitude in the control frame while the W axis determines the +z or polar direction. Then  $\mathbf{c}_2$  and  $\mathbf{c}_3$  are, respectively, the latitude and longitude of the posttargeting velocity while  $\mathbf{c}_1$  is the increase in length of that velocity. Figure 1 defines the controls pictorially when the earth is the launch planet. The velocity increments required in either of the two situations mentioned above can readily be calculated in terms of the vector  $\underline{V}$ ,  $\underline{U}$ , and  $\underline{W}$ .

First consider the calculation of the increment  $\Delta \underline{v}_{\underline{i}}$  produced by perturbing the ith control an amount  $c_{\underline{i}}$  while fixing the other controls at zero as required in the sensitivity approximation. KTRØL performs this conputation when IOPT = i. Reasoning from Figure 1 it follows immediately that

$$\underline{\Delta \mathbf{v}_1} = \mathbf{c}_1 \underline{\mathbf{v}} \tag{4}$$

$$\underline{\Delta v_2} = \|\underline{v}\| \left(\cos c_2 - 1\right) V + \sin c_2 \underline{U}$$
 (5)

$$\underline{\Delta v_3} = \|\underline{v}\| \left(\cos c_3 - 1\right) \underline{v} + \sin c_3 \underline{w}. \tag{6}$$

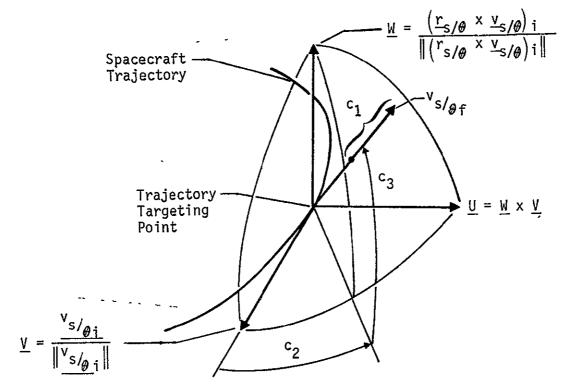


Figure 1 Pictorial Definition of Launch-Planetocentric Targeting Controls

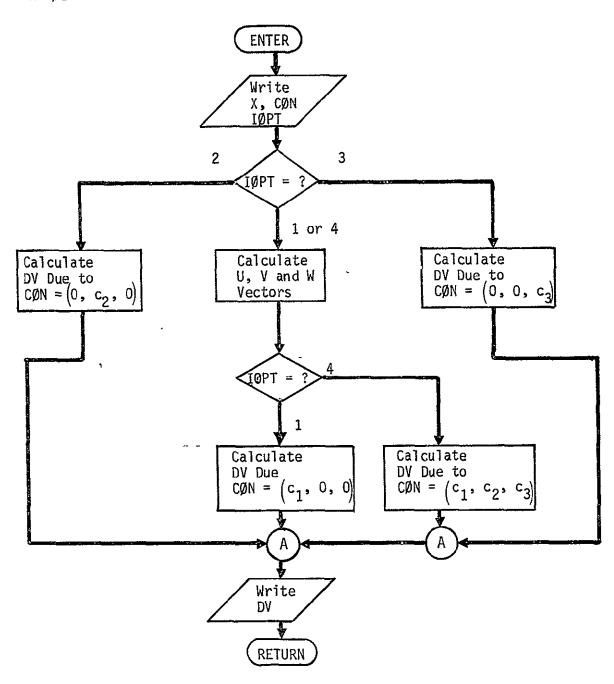
Next consider the calculation of the increment  $\Delta v$  produced by perburbing all three controls simultaneously as required in the Newton-Raphson control correction. KTRØL performs this computation when IØPT = 4. Reasoning again from Figure 1

$$\Delta \mathbf{v} = \left[ \left( \left\| \underline{\mathbf{v}} \right\| + \mathbf{c}_{1} \right) \cos \mathbf{c}_{2} \cos \mathbf{c}_{3} - \left\| \underline{\mathbf{v}} \right\| \right] \underline{\mathbf{v}}$$

$$+ \left( \left\| \underline{\mathbf{v}} \right\| + \mathbf{c}_{1} \right) \sin \mathbf{c}_{2} \cos \mathbf{c}_{3} \underline{\mathbf{u}} + \left( \left\| \underline{\mathbf{v}} \right\| + \mathbf{c}_{1} \right) \sin \mathbf{c}_{3} \underline{\mathbf{w}}. \tag{7}$$

Note that equation (7) degenerates to equations (4), (5), and (6) when the appropriate controls are set to zero.

KTRØL Flow Chart



SUBROUTINE LAUNCH

PURPOSE: TO COMPUTE THE INJECTION TIME, POSITION, AND VELOCITY FROM THE DEPARTURE ASYMPTOTE AND THE LAUNCH PROFILE

ARGUMENT: DI JULIAN DATE AT INJECTION (OUTPUT)

R7 POSITION VECTOR AT INJECTION (OUTPUT)

VZ VELOCITY VECTOR AT INJECTION (OUTPUT)

SUBROUTINES SUPPORTED: HELIO

SUBROUTINES REQUIRED: FPHEM ORB PECEQ

LOCALS SYMBOLS: ANGLE INTERMEDIATE ANGLE USED TO DEFINE TL

AZI PLANETOCENTRIC AZUMUTH AT INJECTION (DEG)

BHAT UNIT VECTOR NORMAL TO SHAT AND WHAT USED TO DEFINE THE P-Q ELEMENTS OF THE DEPARTURE HYPERBOLA

BMAG MAGNITUDE OF THE NON-UNITIZED BHAT VECTOR

COSFL COSINE OF FL

COSFS COSINE OF FS

COSGAM COSINE OF GAMMAI

COSPHI COSINE OF FI

COSSIG CONSINE OF SIGMAL

COSHL CONSINE OF HL

VIS VIVA ENERGY ON THE DEPARTURE HYPERBOLA

DD INTERMEDIATE VARIABLE USED TO CALCULATE GREENWICH HOUR ANGLE

DLA PLANETOCENTRIC EQUATORIAL DECLINATION OF THE DEPARTURE ASYMPTOTE

EQEC TRANSFORMATION MATRIX FROM ECLIPTIC TO LAUNCH PLANET EQUATORIAL

FL TRUE ANOMALY OF LAUNCH SITE POSITION VECTOR

FS TRUE ANOMALY OF DEPARTURE ASYMPTOTE

GAMMAI FLIGHT PATH ANGLE AT INJECTION

GH GREENWICH HOUR ANGLE

GMLP GRAVITATIONAL CONSTANT OF THE LAUNCH

PLANET IN KM**3/SEC**2

HE ECCENTRICITY OF THE DEPARTURE HYPERBOLA

ID INTERMEDIATE VARIABLE USED TO COMPUTE

GREENWICH HOUR ANGLE

IHR HOUR OF INJECTION

IMN MINUTE OF INJECTION

I INDEX

J INDEX

LHR HOUR OF LAUNCH

LMN MINUTE OF LAUNCH

PHAT UNIT VECTOR POINTING TOWARD PERIAPSIS OF

THE HYPERBOLA

PHII LATITUDE OF INJECTION

PSIB THE ANGLE FROM LAUNCH TO INJECTION

QHAT UNIT VECTOR NORMAL TO PHAT POINTING IN THE

DIRECTION OF MOTION

RAI RIGHT ASCENSION AT INJECTION

RAL RIGHT ASCENSION OF DEPARTURE ASYMPTOTE

REFJD JULIAN DATE FOR 1950

RIMAG MAGNITUDE OF THE SPACECRAFT POSITION AT

INJECTION

RI SPACECRAFT POSITION AT INJECTION

RLHAT LAUNCH SITE POSITION UNIT VECTOR

SECI SECOND OF INJECTION

SECL SECOND OF LAUNCH

SHAT UNIT SPACECRAFT VELOCITY VECTOR IN EQUATORIAL SYSTEM AT INJECTION

SINFL SINE OF FL

SINFS SINE OF FS

SINGAM SINE OF GAMMAI

SINPHI SINE OF FI

SINSIG SINE OF SIGMAL

SINWL SINE OF WL

SLR SIMI-LATUS RECTUM OF THE DEPARTURE HYPERBOLA

THE BETHEEN LAUNCH AND INJECTION IN SECONDS

TC LENGTH OF PARKING ORBIT COAST IN SECONDS

TEST INTERMEDIATE VARIABLE TO TEST FOR VIOLATION OF AZIMUTH CONSTRAINT

TFRAC INTERMEDIATE VARIABLE USED TO CALCULATE GREENWICH HOUR ANGLE

THETAI LONGITUDE AT INJECTION

TH INTERMEDIATE VARIABLE USED TO CALCULATE CLOCK TIMES OF LAUNCH AND INJECTION

TI INJECTION TIME IN DAYS REFERENCED TO MIDNIGHT OF THE LAUNCH DAY

TL LAUNCH TIME IN DAYS REFERENCED TO MIDNIGHT OF THE LAUNCH DAY

TMN INTERMEDIATE VARIABLE USED TO CALCULATE CLOCK TIMES OF LAUNCH AND INJECTION

TSTAR INTERMEDIATE VARIABLE USED TO COMPUTE GREENWICH HOUR ANGLE

TWOFOR CONSTANT VALUE, EQUAL TO 24.

VHL MAGNITUDE OF VZ, THE INPUT VECTOR OF THE DEPARTURE ASYMPTOTE

VIMAG MAGNITUDE OF SPACECRAFT VELOCITY AT INJECTION

WHAT UNIT VECTOR NORMAL TO THE LAUNCH PLANE IN

EQUATORIAL SYSTEM

WL RIGHT ASCENSION OF THE LAUNCH SITE

WMAG MAGNITUDE OF THE NON-UNITIZED WHAT VECTOR

XTIM INTERMEDIATE VARIABLE USED TO COMPUTE

CLOCK TIMES OF LAUNCH AND INJECTION

COMMON COMPUTED/USED: SIGMAL

COMMON COMPUTED 8 NO

COMMON USED: ALNGTH DPA FI FOUR KOAST

**PMASS** NINETY NLP ONE PHILS PSI1 PSI2 RAD RAP RPRAT THEDOT RP THELS TIM1 TIM2 VHPM TM XΡ TWO **ZERO** 

### LAUNCH Analysis

LAUNCH computes the injection time, position and velocity from the departure velocity  $\vec{v}_{HE}$  (computed in HELIO) and the launch profile parameters input by the user.

The rotation matrix  $\Phi_{\mbox{ECEQ}}$  defining the transformation from ecliptic to equatorial coordinates is first computed (PECEQ). The departure velocity is then normalized and converted into ecliptic coordinates to yield the departure asymptote  $\mbox{$\hat{S}$}$  .

$$\hat{S} = \Phi_{\text{ECEQ}} \frac{\vec{v}_{\text{HE}}}{v_{\text{HE}}} \tag{1}$$

Auxiliary information associated with  $\hat{S}$  is then computed. The energy  $C_3$ , the declination  $\Phi_S$  and the right ascension  $\theta_S$  of the departure asymptote, and the eccentricity of the departure hyperbola are given by

$$C_3 = v_{HE}^2$$

$$\sin \Phi_S = S_z$$

$$\tan \theta_S = \frac{S_y}{S_x}$$

$$e = 1 + \frac{r_p C_3}{\mu}$$
(2)

where r is the desired parking orbit radius and  $\mu$  is the gravitational constant of the launch planet.

The unit normal  $\widehat{W}$  to the launch plane in equatorial coordinates is then computed.  $\widehat{W}$  is defined by

$$W_{z} = \cos \Phi_{L} \sin \Sigma_{L}$$

$$W_{y} = \frac{-W_{z} S_{y} S_{z} + k S_{x} \left[1 - (S_{z}^{2} + W_{z}^{2})\right]^{\frac{1}{2}}}{S_{x}^{2} + S_{y}^{2}}$$

$$W_{z} = \frac{-(W_{y} S_{y} + W_{z} S_{z})}{S_{z}}$$
(3)

where  $\Phi_L$  is the launch site latitude,  $\Sigma_L$  is the launch azimuth, and  $k=\pm 1$  or -1 for the long or short coast time models respectively. The second equation defines an implicit constraint on  $\Sigma_T$ .

$$\sin^2 \Sigma_{L} \leq \frac{\cos^2 \Phi}{\cos^2 \Phi} \tag{4}$$

The right ascension at launch  $\Theta$  may now be defined by

$$\cos \Theta_{L} = \frac{\frac{W_{x} \sin \Phi_{L} \sin \Sigma_{L} + W_{y} \cos \Sigma_{L}}{W_{z}^{2} - 1}$$

$$\sin \Theta_{L} = \frac{W_{y} \sin \Phi_{L} \sin \Sigma_{L} - W_{x} \cos \Sigma_{L}}{W_{z}^{2} - 1}$$
 (5)

and the unit vector toward the launch position is then

$$R_{L} = (\cos \Phi_{L} \cos \Theta_{L}, \cos \Phi_{L} \sin \Theta_{L}, \sin \Phi_{L})$$
 (6)

The complementary unit vectors  $\widehat{P}$ ,  $\widehat{Q}$  defining the orientation of the hyperbola within the launch plane are now introduced. Let

$$\hat{\mathbf{B}} = \hat{\mathbf{S}} \times \hat{\mathbf{W}} \tag{7}$$

The true anomaly of the departure asymptote is  $\cos f = -\frac{1}{e}$ . Then  $\widehat{P}$  and  $\widehat{Q}$  are given as

$$\hat{P} = \hat{S} \cos f_{g} + \hat{B} \sin f_{g}$$

$$\hat{Q} = \hat{S} \sin f_{g} + \hat{B} \cos f_{g}$$
(8)

The true anomaly of the launch site  $f_T$  may now be given

$$\cos f_{L} = \widehat{R}_{L} \cdot \widehat{P}$$

$$\sin f_{L} = \widehat{R}_{L} \cdot \widehat{Q}$$
(9)

The angle  $\Psi_{R}^{}$  between launch and injection is

$$\Psi_{\rm B} = 2\pi - f_{\rm L} + f_{\rm T} \tag{10}$$

where  $f_{\tilde{I}}$  is the desired true anomaly at injection read in as input. The coast time  $t_{\tilde{c}}$  may now be computed from

$$t_{c} = \left[\Psi_{B} - (\Psi_{1} + \Psi_{2})\right] k_{\widetilde{\Phi}}$$
 (11)

where  $\Psi$  and  $\Psi$  are the angles of the first and second burns and k. is the inverse of the parking orbit coast rate, all of which are read in as input.

The time between launch and injection is therefore

$$t_B = t_1 + t_2 + t_c$$
 (12)

where  $t_1$  and  $t_2$  are the input time durations of the first and second burns.

The unit vector to injection is

$$\widehat{R}_{I} = \widehat{P} \cos f_{I} + \widehat{Q} \sin f_{I}$$
(13)

The semi-latus rectum p is

$$p = \frac{\mu(e^2 - 1)}{C_3}$$
 (14)

The radius magnitude to injection is

$$R_{I} = \frac{p}{1 + e \cos f_{T}} \tag{15}$$

The injection speed is

$$v_{I} = \sqrt{c_{3} + \frac{2\mu}{R_{I}}}$$
 (16)

The path angle at injection is

$$\cos \Gamma_{I} = \frac{\sqrt{\mu_{P}}}{R_{I} V_{I}}$$
 (17)

The injection latitude is

$$\sin \Phi_{I} = \widehat{R}_{I_{Z}}$$
 (18)

The injection right ascension is

$$tan \Theta = \frac{R_{I}}{R_{I_{X}}}$$
 (19)

The injection longitude is

$$\theta_{I} = \theta_{L} + \Theta_{I} - \Theta_{L} - \omega t_{B}$$
 (20)

where  $\theta$  is the longitude of the launch site and  $\omega$  is the rotation rate L of the launch planet, both being read in as input.

The injection azimuth is

$$\cos \Sigma_{I} = \frac{S_{z} - \cos (f_{s} - f_{I}) \sin \Phi_{I}}{\sin (f_{s} - f_{I}) \cos \Phi_{I}}$$
(21)

The launch time on the day of launch is

$$t_{L} = \frac{(\Theta_{L} - \Theta_{L} - GHA) \mod 2\pi}{\omega}$$
 (22)

where GHA is the Greenwich hour angle at 0^h UT of the launch date

$$GHA = 100.07554260 + 0.9856473460 T_d + 2.9015 \times 10^{1-3} T_d^3$$
 (23)

where T_d = days past 0^h January 1, 1950.

The injection radius vector is now computed from

$$\overrightarrow{R}_{I} = R_{I} \widehat{R}_{I}$$

$$\overrightarrow{V}_{I} = \frac{V_{I}}{R_{I}} \left[ (\widehat{W} \times \overline{R}_{I}) \cos \Gamma_{I} + \overline{R}_{I} \sin \Gamma_{I} \right]$$
(24)

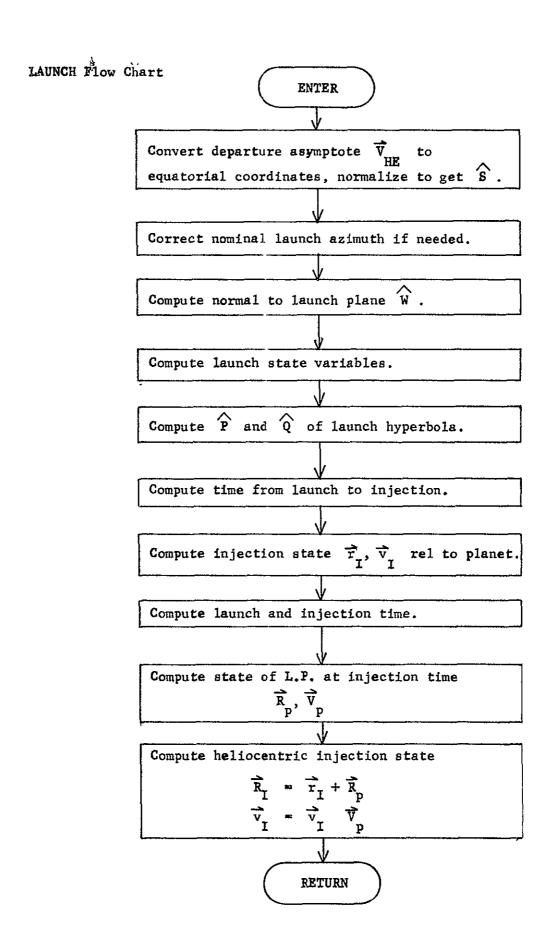
The injection time is

$$T_{I} = T_{O} + t_{L} + t_{B}$$
 (25)

where T is the Julian date of the launch calendar date.

The injection position and velocity are now rotated into the ecliptic plane. The position and velocity of the launch planet at the time  $T_{\rm I}$  are computed and added to the injection state to get the heliocentric injection state.

Reference: Space Research Conic Program, Phase III, May 1, 1969, Jet Propulsion Laboratory, Pasadena, California.



SUBROUTINE LUNA

PURPOSE: TO CONTROL THE GENERATION OF THE ZERO ITERATE FOR LUNAR TARGETING

CALLING SEQUENCE: CALL LUNA

SUBROUTINES SUPPORTED: ZERIT

SUBROUTINES REQUIRED: LUNTAR MULTAR

LOCAL SYMBOLS: I INDEX

OSPH ORIGINAL SPHERE OF INFLUENCE OF TARGET

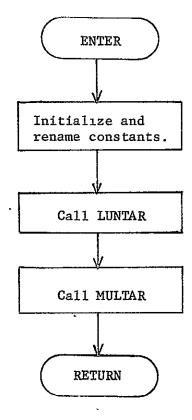
PLANET IN A.U.

COMMON COMPUTED/USED : OTAR SPHERE ICOORD PCON COMMON COMPUTED: BCON CAI IBARY RCA RPE SMA TCA **TSPH** TTOL ALNGTH FOUR NTP COMMON USEDS DT KUR ONE RP SPHFAC TEN ZDAT

## LUNA Analysis

LUNA is the controlling subroutine for lunar zero iterate targeting. It first serves an interface role in which it initializes constants and renames variables for the other lunar targeting routines. It then calls LUNTAR for the targeting of the lunar patched conic. When that is completed it calls MULTAR for the targeting of the multi conic trajectory, It then returns control to PRELIM.

#### LUNA Flow Chart



SUBROUTINE LUNCON

TO COMPUTE THE ACTUAL VALUES OF THE TARGET PARAMETERS PURPOSE: (A, BDT, BDR) FOR A LUNAR PATCHED CONIC TRAJECTORY DETERMINED BY CONTROL VALUES OF ALPHA, DELTA, AND THETA.

CALLING SEQUENCE: CALL LUNCON(ALPHAI, DELTAI, THETAI, AM, BDT, BDR, SIGMAL.ITR)

ARGUMENTS: ALPHAI I ANGLE DEFINING PERIGEE OF TRANSFER CONIC (RAD)

> DELTAI I DELINATION OF LSI POINT (RAD)

THETAI I RIGHT ASCENSION OF LSI POINT (RAD)

AM 0 SEMIMAJOR AXIS OF LUNAR CONIC

IMPACT PARAMETER OF LUNAR CONIC BDT 0

BDR 0 IMPACT PARAMETER OF LUNAR CONIC

SIGMAL I/O NOMINAL LAUNCH AZIMUTH OR THAT REQUIRED

0 OUTPUT ITERATION COUNTER ITR

SUBROUTINES SUPPORTED: LUNTAR

SUBROUTINES REQUIRED: CAREL IMPACT

ALPHAI IN DEGREES LOCAL SYMBOLS: ALPHA

> - AOUT TEMPORARY LOCATION FOR AM

CC ANGULAR MOMENTUM OF THE EARTH CENTERED

TRANSFER CONIC

COSINE OF DELTAI CDEL

CECC ECCENTRICITY OF THE EARTH CENTERED

TRANSFER CONIC

COSDEC COSINE OF DECLIN

COSPL COSINE OF PHIL

INTERMEDIATE VARIABLE TO TEST FOR COSPS

VIOLATIN OF SIGNAL CONSTAINT

COSSIG COSINE OF SIGNAL

CP SEMI-LATUS RECTUM OF EARTH CENTERED

TRANSFER CONIC

CSMA SEMI-MAJOR AXIS OF EARTH CENTERED TRANSFER

CONIC

CT COSINE OF THETAI

DELTA DELTAI IN DEGREES

EM ECCENTRICITY OF LUNAR CONIC

GAMMI INTERMEDIATE ANGLE USED TO COMPUTE EARTH

CENTERED TRANSFER CONIC

I INDEX

PHIL LATITUDE OF LAUNCH SITE

POS SPACECRAFT POSITION AND VELOCITY AT LSI

POINT IN MOON-CENTERED EARTH EQUATORIAL

COORDINATES

PPM DUMMY VARIABLE FOR CALL CAREL

QQM DUMMY VARIABLE FOR CALL TO CAREL

RAD RADIANS TO DEGREES CONVERSION FACTOR

RMAG MAGNITUDE OF THE RI VECTOR

ROUT VELOCITY AT LSI IN GEOCENTRIC EQUATORIAL

SYSTEM

RPM RADIUS OF PERIAPSIS OF LUNAR CONIC

SDEL SINE OF DELTAI

SHAT UNIT VECTOR POINTING FROM THE EARTH TO THE

POINT DEFINED BY DELTAI, THETAI

SIGM SIGMAL IN DEGREES

SINDEC SINE OF DECLIN

SIMPS INTERMEDIATE VARIABLE USED TO TEST FOR

VIOLATION OF SIGNAL CONTRAINT

SINSIG SINE OF SIGNAL

SX SINE OF THETAI

TAM TRUE ANOMALY OF THE LUNAR CONIC

CORRESPONDING TO THE RSI VECTOR

TFLP TIME OF FLIGHT FROM PERIAPSIS CORRESPOND-

ING TO THE RSI VECTOR

THETAI IN DEGREES THETA

SPACECRAFT VELOCITY MAGNITUDE USED TO VMAG

CALCULATE DECLIN

UNIT VECTOR NORMAL TO THE EARTH-PHASE WHAT

WMAG ANGULAR MOMENTUM CONSTANT

WH ARGUMENT OF PERIAPSIS OF THE LUNAR CONIC

DUMNY VARIABLE FOR CALL TO CAREL MWW

SAME AS SHAT VECTOR XHAT

XIM INCLINATION OF THE CONIC

XNM LONGITUDE OF THE ASCENDING NODE OF THE

LUNAR CONIC

CROSS PRODUCT OF THE WHAT AND XHAT VECTORS YHAT

COMMON COMPUTED/USED 8 RI RSI

COMMON COMPUTEDS DECLIN

COMMON USEDS EMU EQLQ KOAST NINETY ONE SIGMA TMU

PHILS RMQ RPE TSPH THO ZERO

#### LUNCON Analysis

The point of intersection of the Earth-centered conic with the lunar sphere of influence (LSI) is determined by the angles and  $\delta$ . Relative to the moon in Earth-equatorial coordinates that point is

$$\vec{r}_{SI} = \begin{bmatrix} R_{SI} \cos \delta \cos \theta \\ R_{SI} \cos \delta \sin \theta \\ R_{SI} \sin \delta \end{bmatrix}$$
 (1)

where  $R_{\rm SI}$  is the radius of the LSI. Relative to the earth that point is

$$\overline{R}_{G} = \overline{R}_{M} + \overline{r}_{SI}$$
 (2)

where  $\overline{R_M}$  is the radius vector to the center of the moon at the time of LSI intersection  $t_{SI}$  in earth equatorial coordinates.

There are at most two planes which contain  $R_q$  and satisfy the launch latitude  $\emptyset$  and azimuth  $\Sigma$  constraints. Let  $\widehat{\mathbb{W}}$  denote the unit mormal to either of these planes. Now let  $R_L$ ,  $\theta_L$ ,  $\theta_L$ , denote the unit vector, longitude, and latitude of the launch site. Construct a local horizon coordinate system at the launch site as indicated in Figure 1.

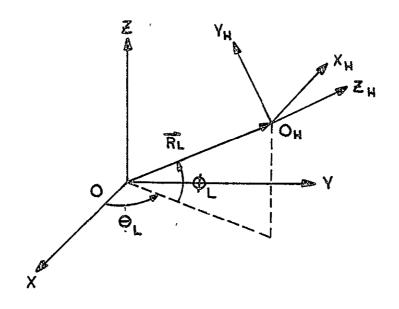


Figure 1. Local Horizon Coordinate System

Here  $\hat{Z}_h = \frac{\hat{R}_L}{R_L}$ ,  $\hat{Y}_L$  is normal to  $\hat{Z}_h$  in the  $\hat{Z}$ -0- $\hat{R}_L$  plane, and  $\hat{X}_h = \hat{Y}_h \times \hat{Z}_h$ .

In the local horizon system, the position and velocity are very simply represented

$$\overline{R}_{h} = R \left[0, 0, \overline{1}\right]^{T}$$

$$\overline{V}_{h} = V \left[\cos \delta \sin \Sigma, \cos \delta \cos \Sigma, \sin \overline{\delta}\right]^{T}$$
(3)

where  $\Sigma$  is the launch azimuth and  $\delta$  is the declination wrt the local horizontal. Thus

$$\widehat{W}_{h} = \frac{\overline{R}_{h} \times \overline{V}_{h}}{|\overline{R}_{h} \times \overline{V}_{h}|} = \begin{bmatrix} -\cos \Sigma \\ \sin \Sigma \\ 0 \end{bmatrix}$$
(4)

The transformation matrix converting a vector in the local horizon system to the equatorial system is

$$T = \begin{bmatrix} -\sin \theta_{L} & -\sin \theta_{L} \cos \theta_{L} & \cos \theta_{L} \cos \theta_{L} \\ \cos \theta_{L} & -\sin \theta_{L} \cos \theta_{L} & \cos \theta_{L} \sin \theta_{L} \end{bmatrix}$$

$$0 \qquad \cos \theta_{L} \qquad \sin \theta_{L} \qquad (5)$$

Therefore since  $\overline{W_q} = T \overline{W_h}$ , the z-component of  $\overline{W}$  in the equatorial coordinate system is

$$\widehat{W}_{Z} = \cos \emptyset_{L} \sin \Sigma$$
 (6)

Since  $\widehat{\mathbb{W}}$  is a unit normal it must satisfy both  $\widehat{\mathbb{W}} \cdot \widehat{\mathbb{W}} = 1$  and  $\widehat{\mathbb{W}} \cdot \widehat{\mathbb{S}} = 0$  where  $\widehat{\mathbb{S}} = \frac{\widehat{\mathbb{R}}_q}{\widehat{\mathbb{R}}_q}$ . Solving for the two remaining components of  $\widehat{\mathbb{W}}$ ,

$$\widehat{\mathbf{w}}_{\mathbf{y}} = \frac{-\widehat{\mathbf{w}}_{\mathbf{z}} \, \widehat{\mathbf{s}}_{\mathbf{y}} \, \widehat{\mathbf{s}}_{\mathbf{z}} \pm \widehat{\mathbf{s}}_{\mathbf{x}} \, \sqrt{1 - (\widehat{\mathbf{s}}_{\mathbf{z}}^2 + \widehat{\mathbf{w}}_{\mathbf{z}}^2)}}{\widehat{\mathbf{s}}_{\mathbf{x}}^2 + \widehat{\mathbf{s}}_{\mathbf{y}}^2}$$
(7)

$$\hat{\mathbf{w}}_{x} = -\frac{(\hat{\mathbf{w}}_{y} \hat{\mathbf{s}}_{y} + \hat{\mathbf{w}}_{z} \hat{\mathbf{s}}_{z})}{\hat{\mathbf{s}}_{y}}$$
(8)

To eliminate the ambiguity of sign in (7) the short-coast plane corresponding to the negative sign is used. Note that (7) also imposes a constraint on the launch azimuth

$$\sin^2 \Sigma \leqslant \frac{1 - \hat{S}^2}{\cos^2 \emptyset} \tag{9}$$

Now choose  $\widehat{\mathbf{U}} = \widehat{\mathbf{W}} \times \widehat{\mathbf{S}}$  to complete a right hand system  $(\widehat{\mathbf{S}}, \widehat{\mathbf{U}}, \widehat{\mathbf{W}})$ . Then the position at LSI relative to the earth is  $(\mathbf{R}_{\mathbf{I}}, 0, 0)$ . Now let  $\alpha$  determine the perigee point in the orbital plane  $(\widehat{\mathbf{W}} = 0)$  measured counterclockwise from the  $-\widehat{\mathbf{S}}$  axis. Then the perigee point is  $(-\mathbf{r}_{\mathbf{p}} \cos \alpha, -\mathbf{r}_{\mathbf{p}} \sin \alpha, 0)$  where  $\mathbf{r}_{\mathbf{p}}$  is the parking orbit radius (input). Therefore the true anomaly of the earth centered conic at the LSI is given by

$$f_{SI} = 180 - \alpha \tag{10}$$

The two equations  $R_I = \frac{a(1-e^2)}{1+e\cos f_{SI}}$  and  $r_p = a(1-e)$  may be

solved simultaneously for the semi-major axis a and eccentricity e of the unique earth centered conic

$$e_{g} = \frac{R_{I} - r_{p}}{r_{p} - R_{I} \cos f_{SI}}$$
 (11)

$$a_{g} = \frac{r_{p}}{1-e_{g}} \tag{12}$$

Thus the velocity of the earth centered conic at the LSI is in the  $(\hat{S}, \hat{U}, \hat{W})$  system

$$\overline{V}_{o} = \sqrt{a(1-e^{2})} e \sin f_{SI}$$

$$\mu a(1-e^{2}) / R_{I}$$
0 (13)

Transforming to the earth equatorial coordinate system

$$\overline{V}_{q} = \begin{bmatrix} S_{x} & U_{x} & W_{x} \\ S_{y} & U_{y} & W_{y} \end{bmatrix}$$

$$S_{z} \quad U_{z} \quad W_{z}$$

$$S_{z} \quad U_{z} \quad W_{z}$$

$$(14)$$

Now if  $(\overline{R_{MQ}}, \overline{V_{MQ}})$  are the position and velocity of the moon at  $t_{SI}$  Earth-centered coordinates and  $(\overline{R_Q}, \overline{V_Q})$  are the position and velocity of the spacecraft at  $t_{SI}$  then the state of the spacecraft with respect to the moon at  $t_{SI}$  is in earth equatorial coordinates

$$\overrightarrow{\mathbf{r}_{SI}} = \overrightarrow{R_{Q}} - \overrightarrow{R_{MQ}}$$

$$\overrightarrow{\mathbf{v}_{SI}} = \overrightarrow{V_{Q}} - \overrightarrow{V_{MQ}}$$
(15)

Using the transformation matrix  $\emptyset_{\text{EQIQ}}$  defining transformations from earth equatorial to lunar equatorial the state in the IQ system is

$$\overline{\mathbf{r}_{sq}} = \emptyset_{EQLQ} \overline{\mathbf{r}_{SI}}$$

$$\overline{\mathbf{v}_{sq}} = \emptyset_{EQLQ} \overline{\mathbf{v}_{SI}}$$
(16)

The impact plane parameters B·T and B·R, and the inclination ignary now be computed by calling subroutines ACTB and CAREL.  $^{i}\chi$ 

SUBROUTINE LUNTAR

PURPOSE® TO GENERATE A PATCHED CONIC TRAJECTORY FOR LUNAR MISSIONS CONSISTENT WITH TARGET PARAMETERS AT THE MOON OF (ACA, RCA, ICA, TCA) AND LAUNCH PARAMETERS (PHIL, THETAL, SIGNAL).

CALLING SEQUENCE: CALL LUNTAR

SUBROUTINES SUPPORTED: LUNA

SUBROUTINES REQUIRED: LUNCON EPHEM IMPACT MATIN ORB

LOCAL SYMBOLS 8 AA SEMI-MAJOR AXIS OF THE LUNAR CONIC FOR THE NOMINAL TRAJECTORY

ALNGTH SAME AS AU

ALPHAI REFINED ANGLE (RADIANS) DEFINING POSITION OF PERIGEE ON THE TRANSFER CONIC (NOMINALLY SET TO FIVE DEGREES)

ALPI PERTURBED VALUE OF ALPHAI USED TO SOLVE FOR RCA, ICA, ACA

AUDAY CONVERTS KM/SEC TO AU/DAY

AUS SAME AS AU

AU CONVERTS KILOMETERS (KM) TO ASTRONOMICAL UNITS (AU)

BDR B QOT R FOR THE NOMINAL TRAJECTORY

BDT B DOT T FOR THE NOMINAL TRAJECTORY

BINC OBTAINABLE INCLINATION USED TO CALCULATE DESIRED B DOT T, B DOT R

DELI PERTURBED VALUE OF DELTAI USED TO SOLVE FOR RCA, ICA, ACA

DELTAI REFINED ANGLE (RADIANS) DEFINING DECLINATION OF THE LSI POINT (NOMINALLY SET TO DELTAO)

DELTAO DECLINATION OF THE MOONS POSITION AT TIME TSI

DELT TIME FROM TSI TO TCA IN SECONDS

DEL REFINING VALUES FOR ALPHAI, DELTAI, THETAI

DENOM INTERMEDIATE VARIABLE USED TO LIMIT THE DEL VALUES FOR EACH ITERATION

ECC DESIRED ECCENTRICITY OF THE LUNAR CONIC

ECEQ TRANSFORMATION MATRIX FROM ECLIPTIC TO EARTH EQUATORIAL

ECLQ TRANSFORMATION MATRIX FROM ECLIPTIC TO LUNAR EQUATORIAL

ERR VECTOR OF DIFFERENCES BETWEEN DESIRED AND NOMINAL VALUES OF B DOT T, B DOT R, ACA

ITAR LOGIC CONTROLLING INDICATOR
=1 IMPROVE ACA ONLY
=2 IMPROVE RCA, ICA, ACA

ITER ITERATION COUNTER FOR NOMINAL TRAJECTORIES

IT ITERATION COUNTER FOR PERTURBED TRAJECT-ORIES

I INDEX

J INDEX

K INDEX

ONEMAT UNIT DUMMY MATRIX FOR CALL TO IMPACT

PAI DUMMY VARIABLE FOR CALL TO LUNCON WHEN ITAR=1

PARP DUMMY VARIABLE FOR CALL TO LUNCON WHEN ITAR=1

PARTA INTERMEDIATE VARIABLE USED TO REFINE ACA WHEN ITAR=1

PARTH INTERMEDIATE VARIABLE USED TO COMPUTE DELT

PARTX INTERMEDIATE VARIABLE USED TO COMPUTE DELT

PARTY INTERMEDIATE VARIABLE USED TO COMPUTE DELT

PARTZ INTERMEDIATE VARIABLE USED TO COMPUTE DELT

PHI MATRIX RELATING PERTURBATIONS IN ALPHAI, DELTAI, AND THETAI TO CHANGES IN 8 DOT T, 8 DOT R, AND ACA

PSI TARGETING MATRIX RELATING PERTURBATIONS IN 8 DOT T, 8 DOT R, AND ACA TO CHANGES IN ALPHAI, DELTAI, AND THETAI

PTAR PERTURBED BALUES OF AA, BDT, BDR, USED TO CALCULATE PHI

RAD CONVERTS DEGREES TO RADIANS

RMAG HAGNITUDE OF THE RMQ VECTOR

SIGMAL LAUNCH AZIMUTH SET IN LUNCON (NOMINALLY 90 DEGREES)

TAR NOMINAL VALUES OF AA, BDT AND BDR USED TO CALCULATE PHI

THEI PERTURBED VALUES OF THETAI USED TO SOLVE FOR RCA, ICA, AND ACA

THETAI REFINED ANGLE (RADIANS) DEFINING RIGHT ASCENSION OF THE LSI POINT (NOMINALLY SET TO THETAO)

THETAO RIGHT ASCENSION OF THE MOONS POSITION AT TIME TSI

TM CONSTANT VALUE OF SECONDS PER DAY

TSICA DUMMY ARGUMENT FOR CALL TO IMPACT

COMMON COMPUTED/USED:	OT AR TMU	EQLQ TSI	ITAG	RMQ	RSI
COMMON COMPUTED:	EMU	ИО	RME		
COMMON USEDS	BCON PCON TSPH	DECLIN PHASS TTOL	FIVE RCA THO	ONE Sma Xp	OTAR TCA Zero

#### LUNTAR Analysis

LUNTAR generates a patched conic trajectory arriving at closest approach to ghe Moon at a specified time  $t_{CA}$  and meeting prescribed target values at that point as well as standard launch quantities. The target parameters are

t CA	Julian date of required closest approach (CA) referenced 1900
^r ca	Radius of CA
i _{CA}	Inclination (relative to lunar equator) at $\mathtt{CA}^1$
^a CA	Semi-major axis at CA

The launch parameters

The eccentricity of the moon-centered hyperbola may be computed

$$e_{CA} = 1 - \frac{r_{CA}}{a_{CA}} \tag{1}$$

where a_{CA}<0. The hyperbolic time At to go from R_{SI} (radius of lunar sphere of influence (LSI)) to periapsis may be computed from

$$\Delta t = f(\mu_M, a_{CA}, e_{CA}, R_{SI})$$
 (2)

where  $\mu_{ ext{M}}$  is the lunar gravitational constant. The time at which the probe should intersect the LSI is then

$$t_{SI} = t_{CA} - \Delta t \tag{3}$$

The inclination must be specified according to the format described in IMPACT. For  $0 \le i < 90^{\circ}$  the inclinations  $\pm i$  prescribe posigrade orbits while  $180^{\circ}$   $\pm i$  define retrograde orbits. The positive signs denote approaches from the north, the negative signs designate southern approaches.

The position  $\overline{R}_{ME}$  and velocity  $\overline{V}_{ME}$  of the moon at  $t_{SI}$  relative to the earth in earth ecliptic (EC) coordinates are computed by calling ORB and EPHEM. Transformation matrices  $\emptyset_{ECEQ}$  and  $\emptyset_{EQIQ}$  defining transformations from EC to EQ (earth equatorial) and EQ to IQ (lunar equatorial) respectively are then computed by PECEQ. The position and velocity of the moon in the EQ system are

$$\vec{R}_{MQ} = \vec{\emptyset}_{ECEQ} \vec{R}_{ME}$$

$$\vec{V}_{MQ} = \vec{\emptyset}_{ECEQ} \vec{V}_{ME}$$
(4)

Call the point of intersection of the vector  $\mathbf{R}_{MQ}$  with the LSI the bullseye point. Then in moon-centered Earth-equatorial coordinates the vector to the bullseye point is given by

$$\vec{r}_{B} = -\left(\frac{\vec{R}_{MQ}}{R_{MQ}}\right) R_{SI}$$
 (5)

From this vector one can calculate a set of angular coordinates ( $\delta_0$ ,  $\theta_0$ ) of the bullseye point. Any other point on the LSI is determined by giving general coordinates ( $\delta$ ,  $\theta$ ) = ( $\delta_0 + \Delta \delta$ ,  $\theta_0 + \Delta \theta$ ).

Now let such a set of coordinates by given. They determine a vector  $R_I$  from earth to the LSI (in the EQ-system). The vector  $R_I$  along with the launch parameters  $\theta_L$ ,  $\theta_L$ ,  $E_L$  then determines the plane of the Earth-LSI transfer (see LUNCON). Now let  $\alpha$  be measured counter-clockwise in that plane from  $-R_I$ . The parameter  $\alpha$  specifies the location of the perigee point of the transfer conic, thus the vector to perigee is fixed as r where the perigee magnitude r is fixed as input. The vectors r and r then determine a unique conic for the Earth-LSI phase (see LUNCON). Let the state at the LSI on that conic (relative to Earth-equatorial coordinates) be denoted by r . The state relative to the moon may then be computed as

$$\vec{\hat{T}}_{I} = \vec{R} - \vec{R} 
\vec{V}_{I} = \vec{V}_{I} - \vec{V}_{MO}$$
(6)

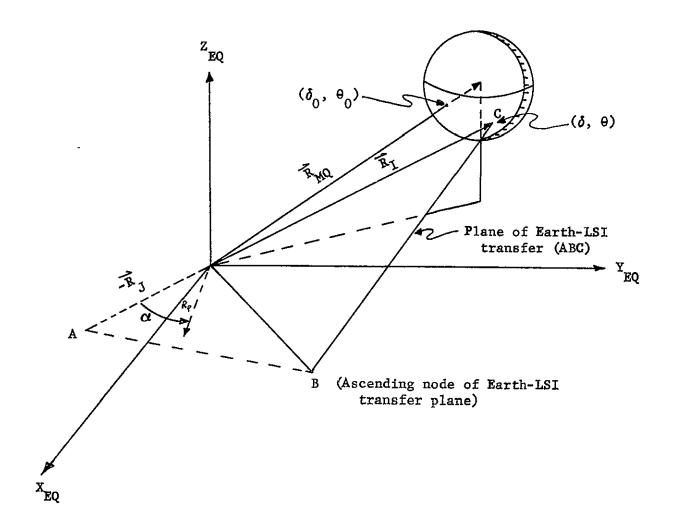


Figure 1. Lunar Patched Conic Targeting

Thus the elements relative to the moon may be computed from standard conic formula. The three angles ( $\delta$ ,  $\theta$ ,  $\alpha$ ) form a set of independent controls to be varied to meet the three constraints ( $r_{CA}$ ,  $i_{CA}$ ,  $a_{CA}$ ). The controls are depicted in Figure 1.

LUNTAR uses the standard Newton-Raphson algorithm to refine the controls to meet the constraints. This targeting is done in two stages. In the first stage the controls  $\delta$  and  $\theta$  are held fixed at the bullseye point ( $\delta_0$ ,  $\theta_0$ ) while  $\alpha$  is varied until the semi-major axis target  $\alpha_{\rm CA}$  is met. Then all three controls are varied to satisfy the three target constraints. The preliminary targeting of  $\alpha_{\rm CA}$  is essential to the success of the procedure. Once the in initial targeting is completed, the semi major axis of future

iterations in the second stage will not vary much from the target value a_{CA}. For such iterates the excess hyperbolic velocity at the moon will be generally constant. This permits the substitution of the auxiliary impact plane parameters B·T and B·R for the less linear parameters of r_{CA} and i_{CA} (see IMPACT). In LUNTAR the impact plane parameters are referenced to the LQ system.

The procedure may now be described in detail. Suppose that in the first stage of targeting the current value of  $\alpha$  is  $\alpha_K$ . Using the controls  $(\alpha_K, \delta_0, \theta_0)$  the resulting semi-major axis is found to be  $\mathbf{a}_K$  (LUNCON). A perturbed value for the first control is then used  $(\alpha_K + \Delta \alpha, \delta_0, \theta_0)$  producing a perturbed value of semi-major axis  $(\mathbf{a}_K + \Delta \mathbf{a})$ . The (k+1)st value of  $\alpha$  is then given by the standard numerical differencing approximation

$$\alpha_{K+1} = \alpha_K + \frac{\Delta \alpha}{\Delta a} (a_{CA} - a_K)$$
 (7)

The second stage of the targeting of the lunar patched conic uses the vector analogue of the above procedure. The current iterate ( $\alpha_K$ ,  $\delta_K$ ,  $\theta_K$ ) is input to LUNCON to obtain the current target values ( $\alpha_K$ ,  $\alpha_K$ ,  $\alpha_K$ ). The target values B.T and B.R are determined from subroutine IMPACT and the errors of the kth iterate are computed ( $\alpha_K$ ,  $\alpha_K$ ). If all three errors are within tolerances, the procedure is terminated. Otherwise the sensitivity matrix  $\beta$  is computed by numerical differencing as in the first stage

$$\emptyset = \begin{bmatrix} \frac{\Delta a_{\alpha}}{\Delta \alpha} & \frac{\Delta a_{\delta}}{\Delta \delta} & \frac{\Delta a_{\theta}}{\Delta \theta} \\ \frac{\Delta B \cdot T}{\Delta \alpha} & \frac{\Delta B \cdot T_{\delta}}{\Delta \delta} & \frac{\Delta B \cdot T_{\theta}}{\Delta \theta} \\ \frac{\Delta B \cdot R_{\alpha}}{\Delta \alpha} & \frac{\Delta B \cdot R_{\delta}}{\Delta \delta} & \frac{\Delta B \cdot R_{\theta}}{\Delta \theta} \end{bmatrix}$$
(8)

The inverse of  $\emptyset$  is the targeting matrix. The k+l iterate is then defined to be

$$\begin{bmatrix} \alpha \\ \delta \\ \theta \end{bmatrix}_{K+1} = \begin{bmatrix} \alpha \\ \delta \\ \theta \end{bmatrix}_{K} + \emptyset^{-1} \begin{bmatrix} a_{CA} - a_{K} \\ B \cdot T - B \cdot T_{K} \\ B \cdot R - B \cdot R_{K} \end{bmatrix}$$
(9)

This procedure is repeated until convergence is achieved.

PROGRAM MAIN

PURPOSE: TO CONTROL THE SIMULATION OVERLAY, SCHEME

SUBROUTINES SUPPORTED & NONE

SUBROUTINES REQUIRED: DATAS SIMUL PRNTS4

LOCAL SYMBOLS: IRUNX TOTAL NUMBER OF DATA CASES

IRUN DATA CASE COUNTER

SUBROUTINE MATIN

PURPOSE TO COMPUTE THE INVERSE OF A MATRIX.

CALLING SEQUENCE: CALL MATIN (A,R,N)

ARGUMENTS A (N.N) I MATRIX TO BE INVERTED

R(N,N) O RESULTANT INVERSE OF MATRIX A

N I DIMENSION OF A AND R

SUBROUTINES SUPPORTED: HYELS NAVM BIAIM POICOM GUISS TARMAX GUID LUNTAR MULTAR GAIN1

GAIN2 GAUSLS TPRTRG

SUBROUTINES REQUIRED: NONE

LOCAL SYMBOLS: AL A(LL) + S (INTERMEDIATE VARIABLE)

ALBAR INTERMEDIATE VARIABLE

B INTERMEDIATE VECTOR

DETR INTERMEDIATE VECTOR

G INTERMEDIATE VECTOR

IX INTERMEDIATE VECTOR

KR DIMENSION OF A

MIXI INTERMEDIATE VARIABLE

MIXJ INTERMEDIATE VARIABLE

MIXL INTERMEDIATE VARIABLE

S INTERMEDIATE VARIABLE

X INTERMEDIATE VARIABLE

XOFF INTERMEDIATE VARIABLE

COMMON USED: EM7 EM9. ONE ZERO

# SUBROUTINE MATPY

PURPOSE 1 T6	MULTIPLY	TWO	MATRICES OF REAL-VALUED ELEMENTS
ARGUMENT:	A	I	PRE-FACTOR MATRIX
,	8	I	POST-FACTOR MATRIX
	c	0	PRODUCT MATRIX
	L	I	NUMBER OF ROWS OF A
	М	I	NUMBER OF COLUMNS OF A (=NUMBER OF ROWS OF B)
	N	I	NUMBER OF COLUMNS OF B
SUBROUTINE	S SUPPORT	EDS	DIMPCP IMPCT TPPROP TPRTRG
LOCAL SYMB	ols: II		INDEX USED IN ADDRESSING ELEMENTS OF C TREATED AS COLUMN VECTOR
	JJ		INDEX USED IN ADDRESSING ELEMENTS OF A TREATED AS COLUMN VECTOR
	KK		INDEX USED IN ADDRESSING ELEMENTS OF B TREATED AS COLUMN VECTOR

SUBROUTINE MEAN

PURPOSE: TO PROPAGATE AND UPDATE MEANS OF ACTUAL STATE AND PARAMETER DEVIATIONS AND ACTUAL STATE AND PARAMETER ESTIMATION ERRORS

CALLING SEQUENCE: CALL MEAN(EXTP, EXSTP, IFLAG, IFLAG1, NR)

ARGUMENTS: EXTP I STATE DEVIATIONS OR ESTIMATION ERRORS

EXSTP I SOLVE-FOR PARAMETER DEVIATIONS OR ESTIMATION ERRORS

IFLAG I =1 FOR UPDATE

=2 FOR PROPAGATION

IFLAG1 I =1 FOR DEVIATION MEANS

=2 FOR ESTIMATION ERROR MEANS

NR I NUMBER OF ROWS IN THE OBSERVATION MATRIX

SUBROUTINES SUPPORTED: ERRANN SETEVN PROBE GENGID PRED

LOCAL SYMBOLS: IGO INTERNALLY SET FLAG

SUM INTERMEDIATE STORAGE

ZERO VALUE 0.0

COMMON COMPUTED/USED: DUME EU EV EW EXIP EXSIP

COMMON USED: AK AL AM AN G
H NDIM1 NDIM2 NDIM3 NDIM4
PHI S TXU TXW TXXS

#### MEAN Analysis

Subroutine MEAN propagates and updates actual estimation error means over the time interval  $[t_k,\ t_{k+1}]$  separating two successive measurements or events. The equations programmed in MEAN are independent of the filter algorithm employed to generate gain matrices. Gain matrices are assumed to have been computed during a prior call to subroutine GNAVM. The propagation equations programmed in MEAN are also used to propagate actual deviation means over the time interval separating two successive guidance events. The update equations, of course, are not used in this situation.

The actualestimation errors for position/velocity state, solve-for parameters, dynamic consider parameters, measurement consider parameters, and ignore parameters are defined, respectively, by the following:

$$\tilde{x}_{k+1} = \hat{x}_{k+1} - x_{k+1}$$
 (1)

$$\tilde{x}_{s_{k+1}} = \hat{x}_{s_{k+1}} - x_{s_{k+1}}$$
 (2)

$$\tilde{u}_{k+1} = u_{k+1} - u_{k+1} = -u_0$$
 (3)

$$\tilde{v}_{k+1} = \hat{v}_{k+1} - v_{k+1} \approx -v_{o}$$
 (4)

$$\tilde{w}_{k+1} = \hat{w}_{k+1} - w_{k+1} = -w_0$$
 (5)

where ( $^{\circ}$ ) indicates estimated values, and x, x, u, v, and w are the actual deviations from nominal.

Only the means of  $\tilde{x}$  and  $\tilde{x}$  are paopagated and updated since the means of y, x, and w are constant. The propagation equations are summarized

$$\mathbb{E}\left[\tilde{\mathbf{x}}_{k+1}^{-}\right] = \Phi \cdot \mathbb{E}\left[\tilde{\mathbf{x}}_{k}^{+}\right] + \theta_{\mathbf{x}\mathbf{x}_{s}} \cdot \mathbb{E}\left[\tilde{\mathbf{x}}_{s_{k}}^{+}\right] - \theta_{\mathbf{x}\mathbf{u}} \cdot \bar{\mathbf{u}}_{o} - \theta_{\mathbf{x}\mathbf{w}} \cdot \bar{\mathbf{w}}_{o} \quad (6)$$

$$\mathbb{E}\left[\tilde{\mathbf{x}}_{s_{k+1}}^{-}\right] = \mathbb{E}\left[\tilde{\mathbf{x}}_{s_{k}}^{+}\right] \tag{7}$$

where  $\Phi$ ,  $\theta_{xx_s}$ ,  $\theta_{xu}$ , and  $\theta_{xw}$  are state transition matrices over  $[t_k, t_{k+1}]$ .

Before the means of x and  $x_g$  can be updated at a measurement, the mean of the measurement residual k+1 must first be computed using

$$E\left[\varepsilon_{k+1}\right] = -H \cdot E\left[\tilde{x}_{k+1}\right] - M \cdot E\left[\tilde{x}_{s_{k+1}}\right] + G\bar{u}_{o} + L\bar{v}_{o} + N\bar{w}_{o}$$
 (8)

where H, M, G, L, and N are observation matrix partitions.

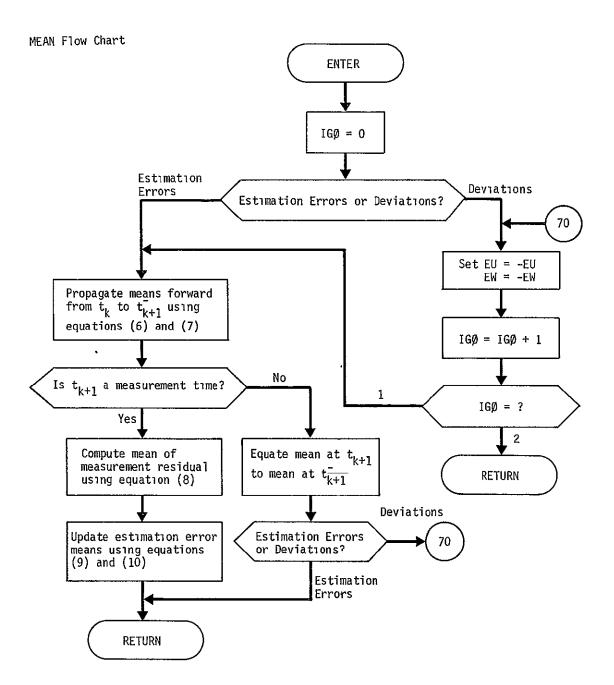
The update equations are summarized as:

$$\mathbb{E}\left[\tilde{\mathbf{x}}_{k+1}^{+}\right] = \mathbb{E}\left[\tilde{\mathbf{x}}_{k+1}^{-}\right] + \mathbb{K}_{k+1} \cdot \mathbb{E}\left[\varepsilon_{k+1}\right] \tag{9}$$

$$\mathbb{E}\left[\tilde{\mathbf{x}}_{k+1}^{+}\right] = \mathbb{E}\left[\tilde{\mathbf{x}}_{s_{k+1}}^{-}\right] + S_{k+1} \cdot \mathbb{E}\left[\varepsilon_{k+1}\right]$$
 (10)

where  $K_{k+1}$  and  $S_{k+1}$  are the filter gain matrices.

To propagate actual deviation means requires that x and  $x_s$  be replaced by  $\tilde{x}$  and  $\tilde{x}_s$ , respectively, in equations (6) and (7), and that the minus signs in equations (6) be replaced with plus signs.



SUBROUTINE MENO

PURPOSE: COMPUTE ASSUMED AND ACTUAL MEASUREMENT NOISE COVARIANCE MATRICES IN THE ERROR ANALYSIS PROGRAM

CALLING SEQUENCE? CALL MENO(MMCODE, ICODE)

ARGUMENT: I CODE I INTERNAL CODE USED TO DISTINGUISH BETWEEN THE TWO ALTERNATIVES LISTED ABOVE

MMCODE I MEASUREMENT MODEL CODE

SUBROUTINES SUPPORTED: ERRANN

COMMON COMPUTED: R RPR

COMMON USED: IMMF MNCN IGMMF GMNCN

MENØ Analysis

The linearized observation equation employed by the navigation process is given by

$$\delta Y_k = H_k^A \delta X_k^A + \eta_k$$

where  $\delta Y_k$  is the measurement deviation from the nominal measurement,  $H_k^A$  is the augmented observation matrix,  $\delta X_k^A$  is the augmented state deviation from the nominal augmented state, and  $\eta_k$  is the assumed measurement noise.

The function of subroutine MENØ is to compute the assumed measurement noise covariance matrix

$$R_{k} = E \left[ n_{k} \quad n_{k}^{T} \right]$$

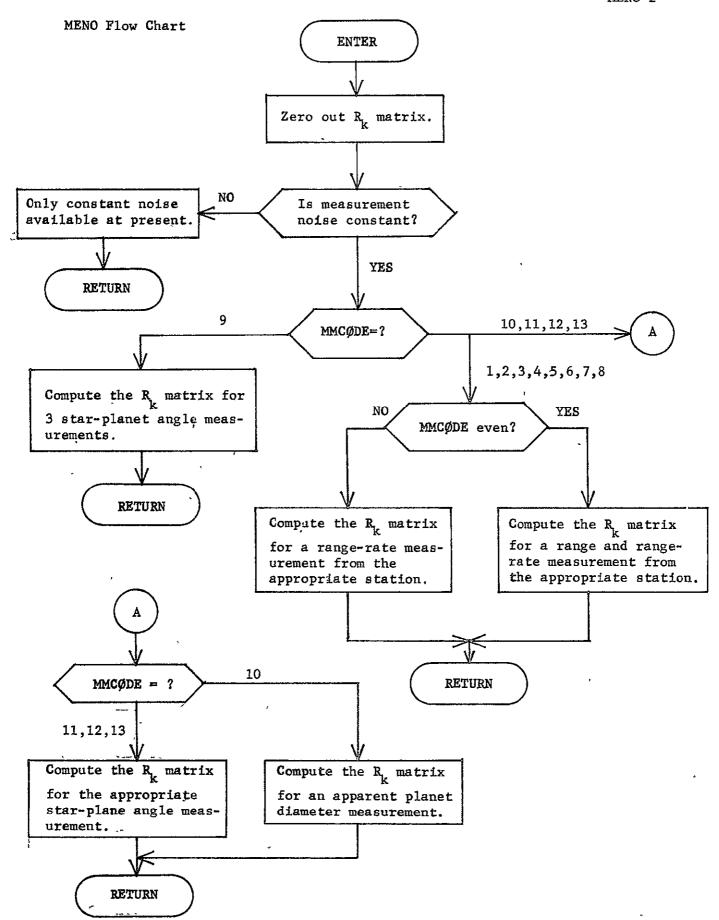
if ICØDE = 0. The constant measurement noise variances associated with all available measurement types are stored in the vector MNCN. Subroutine MENØ selects the appropriate element from this vector to construct  $\boldsymbol{R}_L$ .

If ICODE # 0 the actual measurement noise covariance matrix

$$R_k' = E \left[ n_k' n_k'^T \right]$$

where  $\eta_k^{\, \text{t}}$  is the actual measurement noise, is computed instead. In this case subroutine MENØ selects the appropriate actual measurement noise variances from the vector GMNCN to construct  $R_k^{\, \text{t}}$ .

The accompanying flow chart indicates the computational flow for computing  $\mathbf{R}_k$  . An identical procedure is used to compute  $\mathbf{R}_k^{\,\prime}$  .



SUBROUTINE MENOS

PURPOSE: COMPUTE ASSUMED AND ACTUAL MEASUREMENT NOISE COVARIANCE MATRICES IN THE SIMULATION PROGRAM

CALLING SEQUENCE: CALL MENOS (MMCODE, ICODE)

ARGUMENT: ICODE I INTERNAL CODE USED TO DISTINGUISH BETWEEN, THE TWO ALTERNATIVES LISTED ABOVE

MMCODE I MEASUREMENT MODEL CODE

SUBROUTINES SUPPORTED: SIMULL

COMMON COMPUTED/USED: R

COMMON COMPUTED: AR

COMMON USED: AVARM IAMNF IMMF MNCN ZERO

MENØS Analysis

The linearized observation equation employed by the navigation process is given by

$$\delta Y_k = H_k^A \delta X_k^A + \eta_k$$

where  $\delta Y_k$  is the measurement deviation from the nominal measurement,  $H_k^A$  is the augmented observation matrix,  $\delta X_k^A$  is the augmented state deviation from the nominal augmented state, and  $\eta_k$  is the assumed measurement noise.

The actual measurement  $Y_k^a$  is given by

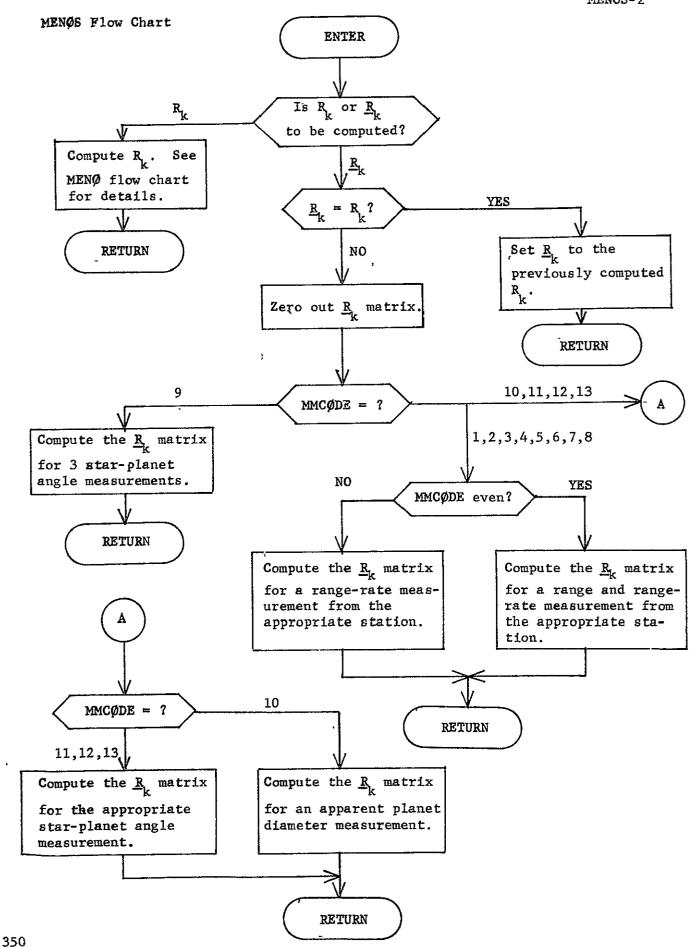
$$Y_k^a = \underline{Y}_k + b_k + \nu_k$$

where  $\underline{Y}_k$  is the ideal measurement, which would be made in the absence of instrumentation errors,  $b_k$  is the actual measurement bias, and  $\nu_k$  represents the actual measurement noise.

Subroutine MENOS performs two functions. It's first function, which is identical to that of subroutine MENO, is to compute the measurement noise covariance matrix  $R_k$  which describes the statistics of noise  $\eta_k$ . The constant variances for the assumed measurement noises associated with all available measurement devices are stored in the vector MNCN. Subroutine MENOS selects the appropriate elements from this vector to construct the measurement noise covariance matrix  $R_k$ .

The second function of MENOS is to compute the measurement noise covariance matrix  $\frac{R}{k}$  which describes the statistics of the actual noise  $\nu_k$ . The constant variances for the actual measurement noises associated with all available measurement devices are stored in the vector AVARM. Subroutine AVARM selects the appropriate elements from this vector to construct the measurement noise covariance matrix  $\frac{R}{k}$ .

C.9.



SUBROUTINE MINIQ

PURPOSE: TO COMPUTE EXECUTION ERROR COVARIANCE MATRIX FOR MINI-PROBE RELEASE

CALLING SEQUENCE: CALL MINIQ (ISACT)

ARGUMENTS: ISACT I ISACT=0 IF OPERATING IN ERRAN =1 IF OPERATING IN SIMUL

SUBROUTINES SUPPORTED PROBE

LOCAL SYMBOLS: ZERO 0.0

CAPA MINI-PROBE ROLL RELEASE ANGLE

CCAPA COS(CAPA)

DTR RADIAN VALUE OF ONE DEGREE

DVDA PARTIAL OF V WRT ALFA

DVDD PARTIAL OF, V WRT DELT

DVDL PARTIAL OF V HRT BOON LENGTH

DVDP PARTIAL OF V WRT PHI

DVDW PARTIAL OF V WRT OMEGA

FACTR 2*PI/3 RADIANS

SCAPA SIN(CAPA)

U INTERMEDIATE VECTOR

V INTERMEDIATE VECTOR

XX INTERMEDIATE VECTOR

ZZ INTERMEDIATE VECTOR

COMMON COMPUTED/USED8 ADV QT

COMMON USED:

ABW ALFA DA DD DELT

DL DP DW EE XPHI

YYL

# MINIQ Analysis

Subroutine MINIQ computes the execution error covariance matrix and the actual execution error associated with the spin-release of a miniprobe. The actual execution error is computed only when MINIQ is used in the simulation program SIMUL.

The velocity increment imparted to the ith probe at release is given by

$$\Delta \vec{V}^{1} = \vec{\omega} \times \vec{\ell}^{1} \tag{1}$$

where  $\vec{\omega}$  is the spin vector and  $\vec{k}^1$  denotes the position of the ith probe relative to the primary vehicle. Referred to the  $\hat{\mathbf{u}}$   $\hat{\mathbf{v}}$   $\hat{\mathbf{h}}$  cocordinate system, which is defined in subroutine TPRTRG, equation (1) becomes

$$\Delta \vec{V}^{i} = \ell \omega \cos \left\{ \left[ \phi + (i-1) \frac{2\pi}{3} \right] \hat{u} + \sin \left[ \phi + (i-1) \frac{2\pi}{3} \right] \hat{v} \right\}$$
 (2)

where  $\left|\phi+(i-1)\frac{2\pi}{3}\right|$  is the roll release angle of the ith probe, and  $\hat{u}$  and  $\hat{v}$  are unit vectors.

Define  $\vec{p}=(\omega, \ell, \alpha, \delta, \phi)$  as the release execution parameter vector, where  $\omega$  is the spin rate magnitude,  $\ell$  is the boom length,  $\alpha$  is the right ascension of the spin axis,  $\delta$  is the declination of the spin axis, and  $\phi$  is the roll release angle. Then the release execution error can be written as

$$\delta \Delta \vec{\nabla}^{\vec{1}} = \frac{\partial \Delta \vec{\nabla}^{\vec{1}}}{\partial \vec{p}} \quad \delta \vec{p} \tag{3}$$

where  $\delta \vec{p}$  represents the error in the release parameter vector. The jth component of  $\delta \Delta \vec{V}^{\hat{I}}$  is given by

$$\delta \Delta V_{j}^{i} = \sum_{m=1}^{5} \frac{\partial \Delta V_{j}^{i}}{\partial P_{m}} \delta P_{m}$$
 (4)

The execution error covariance matrix is defined as

$$\tilde{Q}^{1} = E \left[ \delta \Delta \vec{V}^{1} \cdot \delta \Delta \vec{V}^{1} \right]$$
 (5)

and the element  $\tilde{\tilde{Q}}_{jk}^i$  of matrix  $\tilde{\tilde{Q}}^i$  is given by

$$\tilde{Q}_{jk}^{i} = E \left[ \sum_{m=1}^{5} \frac{\partial \Delta V_{j}^{i}}{\partial p_{m}} \delta p_{m} \cdot \sum_{n=1}^{5} \frac{\partial \Delta V_{k}^{i}}{\partial p_{n}} \delta p_{n} \right].$$
 (6)

Assuming  $E\left[\delta_{p_m} \delta_{p_n}\right] = 0$  for  $m \neq n$ , equation (6) reduces to

$$\tilde{Q}_{jk}^{i} = \sum_{m=1}^{5} \frac{\partial \Delta V_{j}^{i}}{\partial p_{m}} \cdot \frac{\partial \Delta V_{k}^{i}}{\partial p_{n}} \cdot \mathbb{E} \left[ \delta p_{m}^{2} \right] . \tag{7}$$

The partial derivatives required for evaluation of equations (4) and (7) are summarized as:

$$\frac{\partial \Delta \vec{\nabla}^{i}}{\partial \omega} = \ell \left\{ \cos \left[ \phi + (i-1) \frac{2\pi}{3} \right] \hat{\mathbf{u}} + \sin \left[ \phi + (i-1) \frac{2\pi}{3} \right] \hat{\mathbf{v}} \right\}$$
 (8)

$$\frac{\partial \Delta \vec{\nabla}^{1}}{\partial \ell} = \omega \left\{ \cos \left[ \phi + (i-1) \frac{2\pi}{3} \right] \hat{\mathbf{u}} + \sin \left[ \phi + (i-1) \frac{2\pi}{3} \right] \hat{\mathbf{v}} \right\}$$
 (9)

$$\frac{\partial \Delta \hat{\vec{V}}^{\hat{1}}}{\partial \alpha} = \&\omega \left\{ \cos \left[ \phi + (i-1) \frac{2\pi}{3} \right] \frac{\partial \hat{u}}{\partial \alpha} + \sin \left[ \phi + (i-1) \frac{2\pi}{3} \right] \frac{\partial \hat{v}}{\partial \alpha} \right\}$$
(10)

$$\frac{\partial \Delta \hat{\nabla}^{\hat{\mathbf{i}}}}{\partial \delta} = \ell \omega \left\{ \cos \left[ \phi + (\mathbf{i} - 1) \frac{2\pi}{3} \right] \frac{\partial \hat{\mathbf{u}}}{\partial \delta} + \sin \left[ \phi + (\mathbf{i} - 1) \frac{2\pi}{3} \right] \frac{\partial \hat{\mathbf{v}}}{\partial \delta} \right\}$$
 (11)

$$\frac{\partial \Delta \vec{V}^{i}}{\partial \phi} = \hbar \omega \left\{ -\sin \left[ \phi + (i-1) \frac{2\pi}{3} \right] \hat{\mathbf{u}} + \cos \left[ \phi + (i-1) \frac{2\pi}{3} \right] \hat{\mathbf{v}} \right\}$$
 (12)

where

$$\hat{\mathbf{u}} = (\sin \alpha, -\cos \alpha, 0)$$
 (13)

$$\hat{\mathbf{v}} = (\sin \delta \cos \alpha, \sin \delta \sin \alpha, -\cos \delta)$$
 (14).

$$\frac{\partial \hat{\mathbf{u}}}{\partial \alpha} = (\cos \alpha, \sin \alpha, 0) \tag{15}$$

$$\frac{\partial \hat{\mathbf{v}}}{\partial \alpha} = (-\sin \delta \sin \alpha, \sin \delta \cos \alpha, 0) \tag{16}$$

$$\frac{\partial \hat{\mathbf{u}}}{\partial \delta} = (0, 0, 0) \tag{17}$$

$$\frac{\partial \hat{\mathbf{v}}}{\partial \delta} = (\cos \delta, \cos \alpha, \cos \delta \sin, \alpha, \sin \delta) \tag{18}$$

when referred to the ecliptic coordinate system.

SUBROUTINE MOMENT

PURPOSE: TO CONVERT AN ARBITRARY NON-SQUARE 2ND MOMENT MATRIX TO THE ASSOCIATED CORRELATION MATRIX PARTITION AND PRINT IT. ALSO COMPUTE AND PRINT EIGENVALUES, EIGENVECTORS, AND HYPERELLIPSOIDS

CALLING SEQUENCE: CALL MOMENT(N1,N2,EXYT,EX,EY,CORW,CORW1,ABL,I1, I2,IFLAG,IF2)

ARGUMENTS: N1 I NUMBER OF ROWS IN 2ND MOMENT MATRIX

N2 I NUMBER OF COLS IN 2ND MOMENT MATRIX

EXYT I N1 BY N2 2ND MOMENT MATRIX OF X AND Y

EX I N1 VECTOR MEAN OF X

EY I N2 VECTOR MEAN OF Y

CORW I 2ND MOMENT MATRIX CORRESPONDING TO VECTOR X OF DIMENSION N1

CORW1 I 2ND MOMENT MATRIX CORRESPONDING TO VECTOR
Y OF DIMENSION N2

ABL I VECTOR OF ROW LABELS CORRESPONDING TO CORW1

I1 I ROW INDEX MAXIMUM

I COL INDEX MAXIMUM

IFLAG I =0 DO NOT COMPUTE EIGENVECTORS, ETC.

IF2 I =0 DO NOT COMPUTE STD. DEV.

SUBROUTINES SUPPORTED: GPRINT GENGID

SUBROUTINES REQUIRED: EIGHY

LOCAL SYMBOLS: OUT INTERMEDIATE ARRAY

PEIG INTERMEDIATE VECTOR

ROW INTERMEDIATE VECTOR

SQP INTERMEDIATE VECTOR

SQP1 INTERMEDIATE VECTOR

VEIG INTERMEDIATE VECTOR

Z7 INTERMEDIATE VARIABLE

COMMON USED: FOP FOV

MØMENT Analysis

Subroutine MØMENT transforms an arbitrary 2nd moment matrix  $E[xy^T]$  into a correlation matrix and, if x = y, into a vector of standard deviations. The transformation consists of two steps:

1) Transform E[xy^T] into the covariance matrix

cov 
$$(x,y) = E[xy^T] - E[x] \cdot E[y^T];$$

2) Transform cov (x,y) into the correlation matrix having correlation coefficients

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_{i} \sigma_{i}} \qquad i \neq j$$

where

$$\sigma_{ij} = E[x_i y_j]$$

$$\sigma_i = E[x_1^2]^{\frac{1}{2}}$$

$$\sigma_j = E[y_j^2]^{\frac{1}{2}}$$

Subroutine MØMENT writes out the correlation matrix and, if they exist, the standard deviations. Subroutine MØMENT can also compute and write out the eigenvalues, eigenvectors, and hyperellipsoid of cov(x,y) if x = y.

SUBPOUTINF MPPROP

PURPOSE: TO GENERATE A TIME HISTORY OF MAIN PROBE TRAJECTORY

SUBPOUTINES SUPPORTED: GIDANS

SUBROUTINES REQUIRED: VMP

LOCAL SYMBOLS: DELTM MAXIMUM DURATION OF PROPAGATION IN DAYS

DELTPS SAVED VALUE OF VMP VARIABLE DELTP SPECIFYING TIME INTERVAL IN DAYS BETWEEN SUCCESSIVE TRAJECTORY STATUS PRINTOUTS

ICLS SAVED VALUE OF VMP CONDITION CODE ICL INDICATING WHETHER OR NOT CLOSEST APPROACH HAS BEEN REACHED

ICL2S SAVED VALUE OF VMP TERMINATION CODE ICL2
SPECIFYING WHETHER OR NOT TRAJECTORY IS TO
BE STOPPED AT CLOSEST APPROACH

IEPS SAVED VALUE OF VMP FLAG IEPHEM SPECIFYING WHETHER OR NOT ORB IS TO BE CALLED BEFORE CALLING EPHEM

INPRS SAVED VALUE OF VMP VARIABLE INPR SPECIFYING NUMBER OF INTEGRATION STEPS BETWEEN SUCCESSIVE TRAJECTORY STATUS PRINTOUTS

IPRNTS SAVED VALUE OF VMP FLAG IPRINT
SPECIFYING WHEHTER OR NOT TRAJECTORY
INFORMATION IS TO BE PRINTED

ISPHS SAVED VALUE OF VMP CONDITION CODE ISPH SPECIFYING WHETHER OR NOT TRAJECTORY IS TO BE STOPPED AT SOI

TRTMS SAVED VALUE OF VMP VARIABLE TRTM INDICATING ELAPSED TIME IN DAYS FROM START OF TRAJECTORY

XSF FINAL HELIOCENTRIC ECLIPTIC STATE VECTOR OF MAIN PROBE AFTER PROPAGATION IN KM AND KM/SEC

COMMON COMPUTED/USED: DELTP FIVE ICL ICL 2 **IEPHEM** IPGS IPRINT ISPH KUR NINETY TRTM COMMON COMPUTED* DELTP ICL ICLZ IEPHEM INCMT IPCSP IPRINT ISPH RPSP TRTM

### MPPROP Analysis

MPPRØP serves the single purpose of providing a time history of the targeted main probe trajectory from release to the appropriate stopping condition. The process of providing such a time history is classified as a separate type of guidance event in the GIDANS execution logic. Although the main probe propagation event is currently only applied in the NØMNAL program to propagate the main probe once it begins to deviate from the bus trajectory due to some guidance maneuver on the latter, it could be used to treat any branched trajectory.

On being called by GIDANS, MPPROP first prints out the title "Main Probe Propagation Event" followed by the heading "Main Probe Approach Trajectory." Next it stores the current spacecraft heliocentric ecliptic state and the VMP trajectory condition and instrument flags so that they can be returned to after the event. Then the VMP flags are set for propagating the main probe. Codes are used to stop the trajectory after 90 days of propagation or at closest approach but not at the sphere of influence. The VMP stopping condition of impacting the planet is slightly modified. Rather than using NØMNAL's own value of target-planet radius, MPPROP transmits to VMP the radius of the probe sphere input by the user and applied throughout probe targeting. The print routine is activated and the print increments are set at 5-day and 100-integration steps. Next VMP is called to propagate and print the trajectory until a stopping condition is reached. Finally, the original spacecraft state and the VMP flags are restored before returning to GIDANS.

SUBROUTINE MULCON

TO PROPAGATE A SET OF CARTESIAN COORDINATES ALONG A **PURPOSE**: LUNAR MULTI-CONIC TRAJECTORY OVER A SPECIFIED TIME INTERVAL.

CALLING SEQUENCE: CALL MULCON(SEI,TLI,TF,DT,SMF)

ARGUMENTS: SEI(6) T INITIAL SPACECRAFT GEOCENTRIC STATE

> INITIAL INJECTION TIME (JD EPOCH 1950) TLI I

TIME INTERVAL OF PROPAGATION TF

I STEP SIZE USED IN MULTICONIC PROPAGATION DT

SHF (6) 0 FINAL SPACECRAFT SELENOCENTRIC STATE

SUBROUTINES SUPPORTED: MULTAR

SUBROUTINES REQUIRED: CPROP EPHEN ORB

LOCAL SYMBOLS: ALNGTH CONVERTS KILOMETERS TO ASTRONOMICAL UNITS

> A PERTURBING ACCELERATION VECTOR OF THE MOON

OVER THE ITERATION INTERVAL

COSINE OF TRUE ANOMALY ON SELONOCENTRIC COSF CONIC AT' END OF ITERATION INTERVAL

DF FINAL TIME USED IN ITERATION INTERVAL

DI INITIAL TIME USED IN ITERATION INTERVAL

EMM MAGNITUDE OF FIRST THREE ELEMENTS OF EM

EM GEOCENTRIC ECLIPTIC STATE OF MOON

STOPPING CONDITION INDICATOR IDONE

=0 PROPAGATION CONTINUES

=1 STOPPING CONDITION REACHED

I INDEX

TIM JULIAN DATE OF FINAL TIME ON THE ITERATION

INTERVAL

TM CONVERTS SECONDS TO DAYS

SPACECRAFT VELOCITY VECTOR WITH RESPECT TO H EARTH AND/OR MOON BEFORE AND AFTER LUNAR PERTURBATIONS AT DI AND DE

XM	MAGNITUDE OF THE X VECTOR	
x	GEOCENTRIC POSITION OF SPACECRAFT AT DI AND GEOCENTRIC POSITION OF MOON AT DF	
Y	GEOCENTRIC VELOCITY OF SPACECRAFT AT DI AND GEOCENTRIC VELOCITY OF MOON AT DF	
Z	SPACECRAFT POSITION VECTOR WITH RESPECT EARTH AND/OR MOON AT DI AND DF BEFORE AN AFTER LUNAR PERTURBATIONS	
COMMON COMPUTED:	NO	
COMMON USED:	EMU THU THO XP ZERO	

MULCON Analysis

The equations of motion of a spacecraft traveling under the influence of the earth and moon may be written

$$\frac{\vec{r}_E}{\vec{r}_E} = -\frac{\mu_E \vec{r}_E}{r_E^3} - \frac{\mu_M \vec{r}_M}{r_M^3} - \frac{\mu_M \vec{R}_{EM}}{r_{EM}^3}$$
(1)

where  $\vec{r}_{\rm E}$ ,  $\vec{r}_{\rm M}$ ,  $\vec{R}_{\rm EM}$  are the position vectors of the spacecraft-to-earth, the spacecraft-to-moon, and the moon-to-earth respectively and  $\mu_{\rm E}$ ,  $\mu_{\rm M}$  are the gravitational constants of the earth and moon respectively.

The multi-conic approximation of the solution to (1) proceeds as follows. Let  $\overline{r}_{E,k}$ ,  $\overline{v}_{E,k}$  be the geocentric state at some time  $t_k$ . This state is propagated by conic formulae to obtain an estimate of the geocentric state at time  $t_{k+1} = t_k + \Delta t$  given by  $\overline{r}_{E,k+1}$ ,  $\overline{v}_{E,k+1}$ .

To account for the third term perturbations, the state of the moon relative to the earth at the two timepoints is computed, denoted by  $(R_{EM,k}, V_{EM,k})$  and  $(R_{EM,k+1}, V_{EM,k+1})$ . The average value of this acceleration is then determined from

$$\overrightarrow{A} = -\frac{\mu_{M}}{2} \left[ \frac{\overrightarrow{R}_{EM,k}}{\overrightarrow{R}_{EM,k}} + \frac{\overrightarrow{R}_{EM,k+1}}{\overrightarrow{R}_{EM,k+1}} \right]$$
(2)

The corrected geocentric state is then given by

$$\vec{r}_{E,k+1} = \vec{r}_{E,k+1} + \frac{1}{2} \vec{A} (\Delta t)^{2}$$

$$\vec{v}_{E,k+1} = \vec{v}_{E,k+1} + \vec{A} \Delta t$$
(3)

The effect of the direct lunar perturbations is then added. The state of the spacecraft relative to the moon is first computed

$$\vec{r}_{M,k+1} = \vec{r}_{E,k+1} - \vec{R}_{EM,k+1}$$

$$\vec{v}_{M,k+1} = \vec{v}_{E,k+1} - \vec{v}_{EM,k+1}$$
(4)

This state is then propagated linearly backwards in time over the time interval  $\Delta t$  to obtain

$$\overrightarrow{r}_{M,k} = \overrightarrow{r}_{M,k+1} - \overrightarrow{v}_{M,k+1} \Delta t$$

$$\overrightarrow{v}_{M,k} = \overrightarrow{v}_{M,k+1}$$
(5)

This state is now propagated forward in a selenocentric conic to obtain a final state relative to the moon  $(\vec{r}_{M,k+1}, \vec{v}_{M,k+1})$ . The geocentric state of the spacecraft at time  $t_{k+1}$  after considering all terms of (1) is then given by

$$\overrightarrow{r}_{E,k+1} = \overrightarrow{r}_{M,k+1} + \overrightarrow{R}_{EM,k+1}$$

$$\overrightarrow{v}_{E,k+1} = \overrightarrow{v}_{M,k+1} + \overrightarrow{V}_{EM,k+1}$$
(6)

This completes one cycle of the multi-conic propagation.

The multi-conic propagation proceeds until an input final time is reached or until the selenocentric conic passes through pericynthion.

Reference: Byrnes, D. V. and Hooper, H. L., Multi-Conic: A Fast and Accurate Method of Computing Space Flight Trajectories, AAS/AIAA Astrodynamics Conference, Santa Barbara, Cal., 1970, AIAA Paper 70-1062.

SUBROUTINE MULTAR

.PURPOSE: TO CALCULATE THE TRANSLUNAR INJECTION CONDITIONS FROM TARGETED PATCHED-CONIC CONDITIONS AND CALLS VMP TO PERFORM THE NOMINAL TRAJECTORY NEEDED BY ITERAT.

CALLING SEQUENCE: CALL MULTAR

SUBROUTINES SUPPORTED: LUNA

SUBROUTINES REQUIRED: HULCON CAREL ELCAR EPHEM IMPACT
MATIN ORB PECEG TIME

LOCAL SYMBOLS: AE SEMI-MAJOR AXIS OF THE EARTH-ECLIPTIC,
TARGETED PATCHED-CONIC TRAJECTORY

ATARN NOMINAL VALUES OF THE TARGET VARIABLES

ATAR DESIRED VALUES OF THE TARGET VARIABLES

ATOL TOLERANCES OF TARGET VARIABLES

BCOR MAXIMUM STEPS ALLOWED IN ITERATIVE CORRECTION OF CONTROL VARIABLES

BJ ZERO TRUE ANOMALY USED TO DEFINE PERIGEE OF THE TARGETED PATCHED-CONIC TRAJECTORY

BSTEP MULTI-CONIC STEP SIZE (HOURS)

CHI SENSITIVITY MATRIX RELATING PERTURBATIONS IN CONTROL VARIABLES TO CHANGES IN TARGET VARIABLES

DELP VALUE USED TO PERTURB TLI FOR CONSTRUCTION OF CHI

DELTH NOMINAL TIME FOR PROPAGATION

DELT INTEGRATION TIME TO BE USED, AND TIME ACTUALLY USED, IN THE MULTI-CONIC PROPAGATION

DELV VALUE USED TO PERTURB VELOCITY COMPONENTS
OF RT FOR CONSTRUCTION OF CHI

DV CORRECTION ACTUALLY ADDED TO CONTROL VARIABLES

ECEQP TRANSFORMATION MATRIX FROM EARTH ECLIPTIC TO LUNAR EQUATORIAL

EECEQ TRANSFORMATION MATRIX FROM EARTH ECLIPTIC

TO EARTH EQUATORIAL

EE ECCENTRICITY OF THE EARTH-ECLIPTIC, TARGETED PATCHED-CONIC TRAJECTORY

ERR ITERATE ERRORS IN TARGET CONDITIONS

FAC INTERMEDIATE VARIABLE USED TO CHECK FOR MAXIMUM STEP

FMAG INTERMEDIATE VARIABLE USED TO COMPUTE PHIA, PHIB, PHIC

HYT HYPERBOLIC TIME TO LUNAR PERIAPSIS (DAYS)

IDATE CALENDAR DATE OF INJECTION

IST INDICATOR FOR CONTROL VARIABLE PERTURBATION

IT ITERATIONS COUNTER

I INDEX

JERTH INDEX OF EARTH IN F-ARRAY

JMOON INDEX OF HOON IN F-ARRAY

J . INDEX

K INDEX

MITS MAXIMUM NUMBER OF ITERATIONS ALLOWED

NOEX SAME AS JERTH

PERMN HINIMUM PERTURBATION OF CONTROL VARIABLES FOR CONSTRUCTION OF CHI

PERMX MAXIMUM PERTURBATION OF CONTROL VARIABLES FOR CONSTRUCTION OF CHI

PERT PERTURBATION VALUES USED TO CONSTRUCT CHI

PHIA TRANSFORMATION MATRIX FROM RTH TO EC SYSTEM AT TLI

PHIB TRANSFORMATION MATRIX FROM RTW TO EC SYSTEM AT PERTURBED TLI

PHIC PRODUCT OF PHIB AND TRANSPOSE OF PHIA

PPE DUMMY VARIABLE FOR CALL TO CAREL

PSI TARGET MATRIX (INVERSE OF CHI) RELATING PERTURBATIONS IN TARGET VARIABLES TO CHANGES IN CONTROL VARIABLES

PTAR PERTURBED TARGET VALUES

PV PREDICTED CORRECTIONS TO CONTROL VARIABLES

QQE DUMMY VARIABLE FOR CALL TO CAREL

REMAG DUMMY VARIABLE FOR CALL TO ELCAR

REPET MINIMUM ALLOWABLE INJECTION TIME DIFFERENCE IN KTH AND K+2 ITERATIONS TO AVOID REPETITION-TRAP CORRECTION

RM MAGNITUDE OF THE SCH POSITION VECTOR

RSE INJECTION STATE IN EARTH EQUATORIAL SYSTEM AT TLI

RS. ROTATED INJECTION STATE FOR TIME DIFFERENTIAL

RT INJECTION STATE USED IN PERTURBED MULIT-CONIC PROPAGATIONS

SCM FINAL STATE IN LUNAR ECLIPTIC SYSTEM ON THE MULTI-CONIC

SEC SECONDS OF CALENDAR DATE OF TLI

STEP MULTI-CONIC STEP SIZE (SECONDS)

STLI ORIGINAL VALUE OF TLI, RESTORED FOR SUCESSIVE ITERATIONS

THE TRUE ANOMALY OF EARTH-ECLIPTIC TARGETED PATCHED-CONIC TRAJECTORY

TBR DUMMY VARIABLE FOR CALL TO IMPACT

TBT DUMMY VARIABLE FOR CALL, TO IMPACT

TFP TIME OF FLIGHT FROM PERIGEE OF THE EARTH-ECLIPTIC, TARGETED PATCHED-CONIC TRAJECTORY

TIMM1 INJECTION DATE ON K-1 ITERATION

TIMMS INJECTION DATE ON K-2 ITERATION

TMU

TLI INJECTION JULIAN DATE

TTP DUMMY VARIABLE FOR CALL TO ELCAR

VEMAG DUMMY VARIABLE FOR CALL TO ELCAR

VX MAGNITUDE OF THE SCH VELOCITY VECTOR

WE ARGUMENT OF PERIAPSIS OF THE EARTH-ECLIPTIC, TARGETED PATCHED-CONIC

TRAJECTORY

HHE DUNNY ARGUMENT FOR CALL TO CAREL

XIE INCLINATION OF THE EARTH-ECLIPTIC, TARGETED PATCHED-CONIC TRAJECTORY

XNE LONGITUDE OF ASCENDING NODE OF THE EARTH-ECLIPTIC, TARGETED PATCHED-CONIC

TRAJECTORY

COMMON COMPUTED/USED: NO RI

COMMON COMPUTED: ICOORD RIN TIN

COMMON USED: ALNGTH CAI ENU F HALF KUR NBOD NB ONE RCC

SMA TAR TCA TEN TM TSI THO ZERO

ï

#### MULTAR Analysis

Let the earth equatorial state of the probe at the LSI as computed from the patched conic targeting be denoted  $\vec{r}_{LS}$ ,  $\vec{v}_{LS}$ . Subroutine CAREL is called to compute the conic elements and conic time from perigee  $\Delta t$  based on the geocentric conic. The time of injection is then computed as

$$t_{TLI} = t_{SI} - \Delta t \tag{1}$$

The position and velocity of the probe at  $t_{TLI}$  is given by the state along the conic at perigee (true anomaly of zero) and determined by ELCAR to be  $\overrightarrow{r}_{TLI}$ ,  $\overrightarrow{v}_{TLI}$ . If  $\phi_{ECEQ}$  is the transformation matrix from the EC (earth ecliptic) to the EQ (earth equatorial) system, then the patched conic injection state in EC coordinates is

$$\overrightarrow{r}_{I} = \phi_{\text{ECEQ}}^{T} \overrightarrow{r}_{\text{TLI}}$$

$$\overrightarrow{v}_{I} = \phi_{\text{ECEO}}^{T} \overrightarrow{v}_{\text{TLI}}$$
(2)

Since the earth is revolving about the E-M barycenter in time, the EC injection state must be rotated if an earlier or later injection time is to be used. The necessary rotation matrix may be easily computed through the introduction of the R-T-W coordinate system. Let the state of the earth at some time  $t_k$  in BC (barycentric ecliptic) coordinates be denoted  $\overrightarrow{R}_k$ ,  $\overrightarrow{V}_k$ . Construct the  $\widehat{R}^k$ - $\widehat{T}$ - $\widehat{W}$  system at that point as

$$\hat{R}_{k} = \frac{\hat{R}_{k}}{R_{k}} \qquad \hat{W}_{k} = \frac{\hat{R}_{k} \times \hat{V}_{k}}{|\hat{R}_{k} \times \hat{V}_{k}|} \qquad \hat{T}_{k} = \hat{W}_{k} \times \hat{R}_{k} \qquad (3)$$

The transformation matrix from the  $\hat{R}_k - \hat{T}_k - \hat{W}_k$  system to the ecliptic system to the ecliptic system is then given by

$$\boldsymbol{\Phi}_{k} = \begin{bmatrix} \hat{R}_{k} & \hat{T}_{k} & \hat{W}_{k} \end{bmatrix} \tag{4}$$

At a time  $t_{k+1}$  the state of the earth in BC coordinates is given by  $\overrightarrow{R}_{k+1}$ ,  $\overrightarrow{V}_{k+1}$  and the transformation from the  $\widehat{R}_{k+1}$  -  $\widehat{T}_{k+1}$  -  $\widehat{W}_{k+1}$  system to ecliptic coordinates is given by  $\Phi_{k+1}$  in accordance with (4). Injection

states at times  $t_k$  and  $t_{k+1}$  will be called "equivalent" if they are identical when expressed in the pertinent  $\hat{R}$ - $\hat{T}$ - $\hat{W}$  system. Therefore if ( $\vec{r}_k$ ,  $\vec{v}_k$ ) is the injection state in EC coordinates at time  $t_k$ , the equivalent state in EC coordinates at  $t_{k+1}$  is given by

$$\begin{bmatrix} \vec{r}_{k+1} \\ \vec{v}_{k+1} \end{bmatrix} = \begin{bmatrix} \Psi_{k+1,k} & 0 \\ 0 & \Psi_{k+1,k} \end{bmatrix} \begin{bmatrix} \vec{r}_{k} \\ \vec{v}_{k} \end{bmatrix}$$
(5)

where the rotation matrix  $\Psi$  is defined by

$$\Psi_{k+1,k} = \Phi_{k+1} \Phi_k^T$$
 (6)

The targeting algorithm used by MULTAR may now be described. Let the injection state in EC coordinates on the k=th iteration be denoted  $(t_k, \overrightarrow{r_k}, \overrightarrow{v_k})$ . This state is propagated forward using the multi-conic propagator MULCØN to determine a final state  $\overrightarrow{r_M}$ ,  $\overrightarrow{v_M}$  to the moon in ecliptic coordinates. IMPACT is then called to compute the B·T_k, B·R_k and t_{CA}, k actually achieved on the trajectory and the target values of B*·T_k, B*·R_k required to satisfy the  $\overrightarrow{i_{CA}}$  and  $\overrightarrow{i_{CA}}$  constraints. The semi-major axis  $a_k$  of the k-th iterate is computed from the conic formula

$$a = r_{M} \left( 2 - \frac{r_{M} v_{M}}{\mu_{M}} \right) + 1$$
 (7)

Errors in the four target conditions

$$\Delta \tau = \begin{bmatrix} \Delta a \\ \Delta B \cdot T \\ \Delta B \cdot R \\ \Delta t_{CA} \end{bmatrix} = \begin{bmatrix} a - a^* \\ B \cdot T_k - B^* \cdot T_k \\ B \cdot R_k B^* \cdot R_k \\ t_{CA,k} - t_{CA} \end{bmatrix}$$
(8)

if the error in each parameter is less than the allowable tolerance, the process stops.

If convergence has not been achieved a Newton-Raphson iteration is entered. The four controls are  $\overline{v_k}_x$ ,  $\overline{v_k}_y$ ,  $\overline{v_k}_z$ , and  $t_k$ . For the velocity components

a perturbation  $\Delta v$  is added to the pertinent component while the rest of the injection state is held constant before propagating with the multi-conic. For the time perturbation, the rotation matrix  $\Psi_{\Delta}$  corresponding to the perturbed time  $t_k + \Delta t$  (6) is first computed. The injection state used

in the perturbed propagation for time is then  $\begin{bmatrix} t_k + \Delta t, \psi_{\!\!A} \, \overrightarrow{r}_k, \psi_{\!\!A} \, \overrightarrow{v}_k \end{bmatrix}$ . A sensitivity matrix is computed using the results of the numerical differencing:

$$X = \begin{bmatrix} \frac{\Delta a_{x}}{\Delta v_{x}} & \frac{\Delta a_{y}}{\Delta v_{y}} & \dots \\ \frac{\Delta B T_{x}}{\Delta v_{x}} & \vdots & \vdots \\ \frac{\Delta B R_{x}}{\Delta v_{x}} & \vdots & \vdots \\ \frac{\Delta t_{CA_{x}}}{\Delta v_{x}} & \frac{\Delta t_{CA_{t}}}{\Delta t} \end{bmatrix}$$

$$(9)$$

where in the term  $\frac{\Delta \alpha_{\beta}}{\Delta \beta}$ ,  $\Delta \alpha_{\beta}$  is the change in the  $\alpha$  target parameter produced by the variation of the  $\beta$  control component and  $\Delta \beta$  is the change in the  $\beta$  control component. The k+l iterate controls are then given by

$$\Delta C = \begin{bmatrix} \delta v_x \\ \delta v_y \\ \delta v_z \\ \delta t \end{bmatrix} = X^{-1} \Delta \tau$$
 (10)

The k+1 injection state is then computed by first determining the injection state after rotation due to the change in injection time and then adding the injection velocity corrections

$$\begin{array}{rcl}
t_{k+1} &=& t_k + \delta t \\
\overrightarrow{r}_{k+1} &=& \Psi_{\delta} \ \overrightarrow{r}_k \\
\overrightarrow{v}_{k+1} &=& \Psi_{\delta} \ \overrightarrow{v}_k + \delta \overrightarrow{v}
\end{array} \tag{11}$$

The iteration process is repeated until tolerable errors are met. The converged injection state is then integrated in the virtual mass trajectory.

SUBROUTINE MUND

PURPOSE: TO COMPUTE THE AUGMENTED PORTION OF THE STATE TRANSITION MATRIX WHEN THE GRAVITATIONAL CONSTANT OF THE SUN OR OF THE TARGET PLANET HAS BEEN AUGMENTED TO THE BASIC STATE VECTOR.

CALLING SEQUENCE: CALL MUND(RI, RF, POSS)

ARGUMENT: RF I POSITION AND VELOCITY OF THE VEHICLE AT THE END OF THE TIME INTERVAL

RI I POSITION AND VELOCITY OF THE VEHICLE AT THE BEGINNING OF THE TIME INTERVAL

POSS I DISTANCE OF THE VEHICLE FROM THE TARGET PLANET AT THE INITIAL TIME

SUBROUTINES SUPPORTED: PSIM

SUBROUTINES REQUIRED: NTM

LOCAL SYMBOLS: IC COUNTER FOR VARIABLES AUGMENTED TO STATE

VECTOR

RPER ALTERED POSITION AND VELOCITY OF VEHICLE

AT FINAL TIME

SAVE TEMPORARY STORAGE LOCATION FOR GRAVITA-

TIONAL CONSTANTS OF SUN AND TARGET PLANET

COMMON COMPUTED/USED: IPRINT PMASS

COMMON COMPUTED: TXU TXXS TXW

COMMON USED: ALNGTH DELMUP DELMUS IAUGDO IAUGIN

IAUG NTMC NTP SPHERE TM

IAUGW

## MUND Analysis

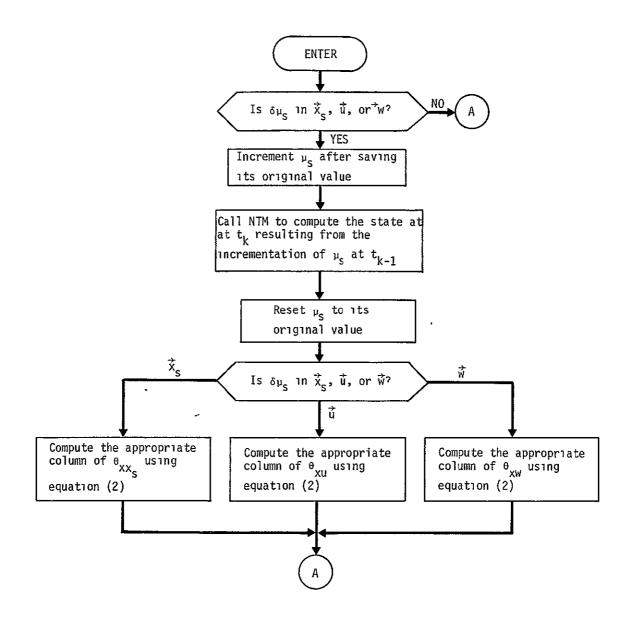
The nonlinear equations of motion of the spacecraft can be written symbolically as

$$\dot{\mathbf{x}} = \dot{\mathbf{f}}(\dot{\mathbf{x}}, \dot{\boldsymbol{\mu}}, t) \tag{1}$$

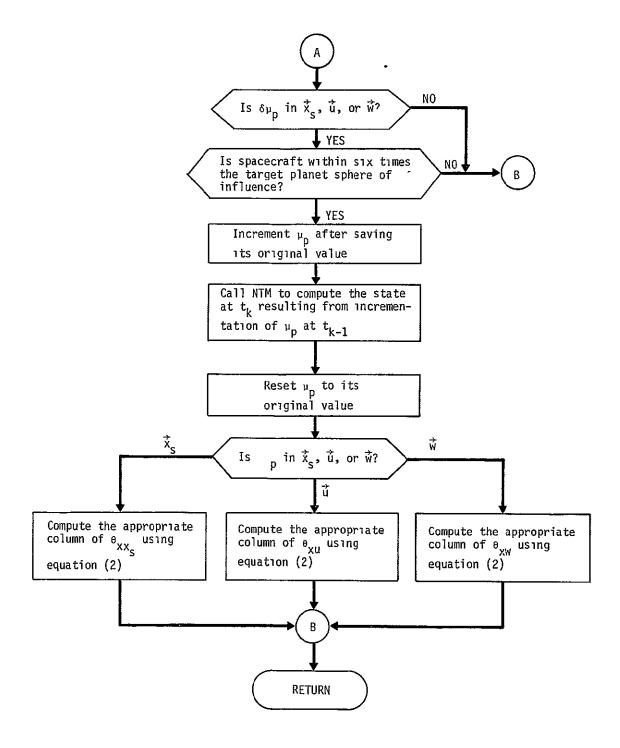
where  $\vec{x}$  is the spacecraft position/velocity state and  $\vec{\mu}$  is a vector composed of the gravitational constants of the sun and the target planet.

Suppose we wish to use numerical differencing to compute those columns of  $\theta_{xx}$ ,  $\theta_{xu}$ , and  $\theta_{xw}$  associated with gravitational constant blases included in the augmented state vector over the time interval  $[t_{k-1},\ t_k]$ . Let  $\dot{\theta}_j$   $(t_k,\ t_{k-1})$  represent the column asasociated with the j-th gravitational constant bias. We assume we have available the nominal states  $\dot{x}^*(t_{k-1})$  and  $\dot{x}^*(t_k)$ , which, of course, were obtained by numerically solving equation (1) using nominal  $\dot{\mu}$ . To obtain  $\dot{\theta}_j(t_k,\ t_{k-1})$  we increment the j-th gravitational constant bias by the pertinent numerical differencing factor  $\Delta\mu_j$  and numerically integrated equation (1) over the interval  $[t_{k-1},\ t_k]$  to obtain the new spacecraft state  $\dot{x}_j(t_k)$ , where the j-subscript on the spacecraft state indicates that it was obtained by incrementing the j-th gravitational constant bias. Then

$$\vec{\theta}_{j} (t_{k}, t_{k-1}) = \frac{\vec{x}_{j}(t_{k}) - \vec{x}^{*}(t_{k})}{\Delta \mu_{j}} . \qquad (2)$$



MUND Flow Chart



SUBROUTINE NAVM

PURPOSE: TO PROPAGATE COVARIANCE MATRIX PARTITIONS P, CXXS, CXU, CXV, PS-, CX-SU, CXSV FROM THE TIME OF THE LAST MEASURE-MENT OR EVENT TO THE PRESENT TIME USING A CONSIDER RE-CURSIVE ALGORITHM.

CALLING SEQUENCE: CALL NAVM(NR, ICODE)

ARGUMENT: ICODE I INTERNAL CODE WHICH DETERMINES IF A MEASUREMENT IS BEING PROCESSED

NR I NUMBER OF ROWS IN THE OBSERVATION MATRIX

SUBROUTINES SUPPORTED: SIMULL SETEVS GUISIM PRESIM PROBES

SUBROUTINES REQUIRED: MATIN GAIN1 GAIN2

LOCAL SYMBOLS AJ MEASUREMENT RESIDUAL COVARIANCE MATRIX AND

ITS INVERSE

AKW INTERMEDIATE ARRAY

DUM INTERMEDIATE VECTOR

PSAVE INTERMEDIATE ARRAY

SUM INTERMEDIATE VARIABLE

SW INTERMEDIATE ARRAY

COMMON COMPUTED/USEO: AK CXSUP CXSU CXSVP CXSV CXSVP CXSV CXXSP

CXXS PP PSP PS P

S

COMMON USED: AL AM G H NCIM1

NDIM2 NDIM3 ONE PHI Q
R TXU TXXS U0 V0

ZERO

NAVM Analysis

The augmented deviation state vector is defined as

$$\vec{x}^{A} = \begin{bmatrix} \vec{x}, \vec{x}_{g}, \vec{v}, \vec{v} \end{bmatrix}^{T}$$

where

x = position and velocity state (dimension 6)

 $\overline{x}_s$  = solve-for parameter state (dimension  $n_1$ )

 $\vec{u}$  = dynamic consider parameter state (dimension  $n_2$ )

 $\vec{v}$  = measurement consider parameter state (dimension  $n_3$ )

The linearized equations of motion have form

$$\dot{\vec{x}} = F_1 \vec{x} + F_2 \vec{x}_s + F_3 \vec{u}$$

$$\frac{\mathbf{x}}{\mathbf{x}} = 0$$

$$\frac{1}{\nabla} = 0$$

and solution

$$\vec{x}_{k+1} = \phi(k+1,k)\vec{x}_k + \theta_{xx_g}(k+1,k)\vec{x}_{s_k} + \theta_{xu}(k+1,k)\vec{u}_k + \vec{q}_k$$

$$\vec{\nabla}_{k+1} = \vec{\nabla}_{k}$$

where dynamic noise  $\overline{q}_k$  has been added to the solution of  $\overline{x}_{k+1}$ . This solution can be written in augmented form

$$\vec{x}_{k+1}^A = \phi^A (k+1,k) \vec{x}_k^A + \vec{q}_k^A$$

where the augmented state transition matrix  $\phi^{A}$  (k+1,k) is defined as

$$\Phi^{A}(k+1,k) = \begin{bmatrix} \Phi & \theta_{xx_8} & \theta_{xu} & 0 \\ 0 & I_{n_1x & n_1} & 0 & 0 \\ 0 & 0 & I_{n_2x & n_2} & 0 \\ 0 & 0 & 0 & I_{n_3x & n_3} \end{bmatrix}$$

Henceforth state transition matrix partitions will be written without stating the associated interval of time, which will always be assumed to be k, k+1 .

The measurement deviation vector y (dimension m) is related to the augmented deviation state vector through the equation

$$\vec{y}_k = H_k^A \vec{x}_k^A + \vec{\eta}_k$$

where the augmented observation matrix is defined as

$$H_k^A = \begin{bmatrix} H_k & M_k & G_k & L_k \end{bmatrix}$$

 $H_k^A = \begin{bmatrix} H_k & M_k & G_k & L_k \end{bmatrix}$  and  $\overline{\eta}_k$  is measurement noise. The augmented state covariance matrix  $P_k^A$  can be written in terms of its partitions as

Propagation and update equations for the partitions appearing in the previous equation will be written below. Equations need not be written for the consider parameter covariances U and V since these do not change with time. Also, C will be set to zero because of the assumption that no cross-correlation exists between dynamic- and measurement-consider parameters. In the equations below, Q and R represent the covariances of the dynamic and measurement noises, respectively, defined previously. A minus superscript on covariance partitions indicates the covariance partition immediately prior to processing a measurement; a plus superscript, immediately after processing a measurement. If ICODE indicates that a measurement is not to be processed, the update equations are bypassed. To improve numerical accuracy and avoid nonpositive definite covariance matrices, P, P, P⁺, and P⁺, are always symmetrized after their computation.

The propagation equation are given by

$$P_{k+1}^{-} = \left( \phi P_{k}^{+} + \theta_{xx_{s}} C_{xx_{s_{k}}}^{+T} + \theta_{xu} C_{xu_{k}}^{+T} \right) \phi^{T} + C_{xx_{s_{k+1}}}^{-} = \left( \phi P_{k}^{+} + \theta_{xx_{s}} C_{xx_{s_{k}}}^{+T} + \theta_{xu} C_{xu_{k}}^{+T} \right) \phi^{T} + C_{xx_{s_{k+1}}}^{-} = \left( \phi C_{xx_{s_{k}}}^{+} + \theta_{xx_{s}} C_{x_{s_{k}}}^{+} + \theta_{xu} C_{x_{s}}^{+T} \right) \phi^{T} + C_{xx_{s_{k+1}}}^{-} = \left( \phi C_{xx_{s_{k}}}^{+} + \theta_{xx_{s}} C_{x_{s_{k}}}^{+} + \theta_{xu} C_{x_{s}}^{+T} \right) \phi^{T} + C_{xx_{s_{k+1}}}^{-} = \left( \phi C_{xx_{s_{k}}}^{+} + \theta_{xx_{s_{k}}} C_{x_{s_{k}}}^{+} + \theta_{xu} C_{x_{s}}^{+} \right) \phi^{T} + C_{xx_{s_{k}}}^{-} + C_{xx_$$

The measurement residual covariance matrix is given by

$$J_{k+1} = H_{k+1} A_{k+1} + M_{k+1} B_{k+1} + G_{k+1} D_{k+1} + L_{k+1} E_{k+1} + R_{k+1}$$

where

$$\begin{split} & A_{k+1} = P_{k+1}^{-} H_{k+1}^{T} + C_{xx}^{-} H_{k+1}^{T} + C_{xx}^{-} H_{k+1}^{T} + C_{xu}^{-} H_{k+1}^{T} + C_{xv}^{-} H_{k+1}^{T} \\ & B_{k+1} = P_{s_{k+1}}^{-} H_{k+1}^{T} + C_{xx}^{-} H_{k+1}^{T} + C_{xs}^{-} H_{k+1}^{T} + C_{xs}^{-} H_{k+1}^{T} + C_{xs}^{-} H_{k+1}^{T} \\ & D_{k+1} = C_{xu}^{-T} H_{k+1}^{T} + C_{xs}^{-T} H_{k+1}^{T} + C_{xs}^{-T} H_{k+1}^{T} + U_{o} G_{k+1}^{T} \\ & E_{k+1} = C_{xv_{k+1}}^{-T} H_{k+1}^{T} + C_{xs}^{-T} H_{k+1}^{T} + C_{xs}^{-T} H_{k+1}^{T} + V_{o} L_{k+1}^{T}. \end{split}$$

The covariance matrix partitions immediately after processing a measurement are given by the following update equations:

$$P_{k+1}^{+} = P_{k+1}^{-} - K_{k+1} A_{k+1}^{T} - A_{k+1} K_{k+1}^{T} + K_{k+1} J_{k+1} K_{k+1}^{T}$$

$$C_{xx}^{+} = C_{xx}^{-} - K_{k+1} B_{k+1}^{T} - A_{k+1} S_{k+1}^{T} + K_{k+1} J_{k+1} S_{k+1}^{T}$$

$$P_{s_{k+1}}^{+} = P_{s_{k+1}}^{-} - S_{k+1} B_{k+1}^{T} - B_{k+1} S_{k+1}^{T} + S_{k+1} J_{k+1} S_{k+1}^{T}$$

$$C_{xu_{k+1}}^{+} = C_{xu_{k+1}}^{-} - K_{k+1} D_{k+1}^{T}$$

$$C_{x_{s_{k+1}}}^{+} = C_{x_{s_{k+1}}}^{-} - S_{k+1} D_{k+1}^{T}$$

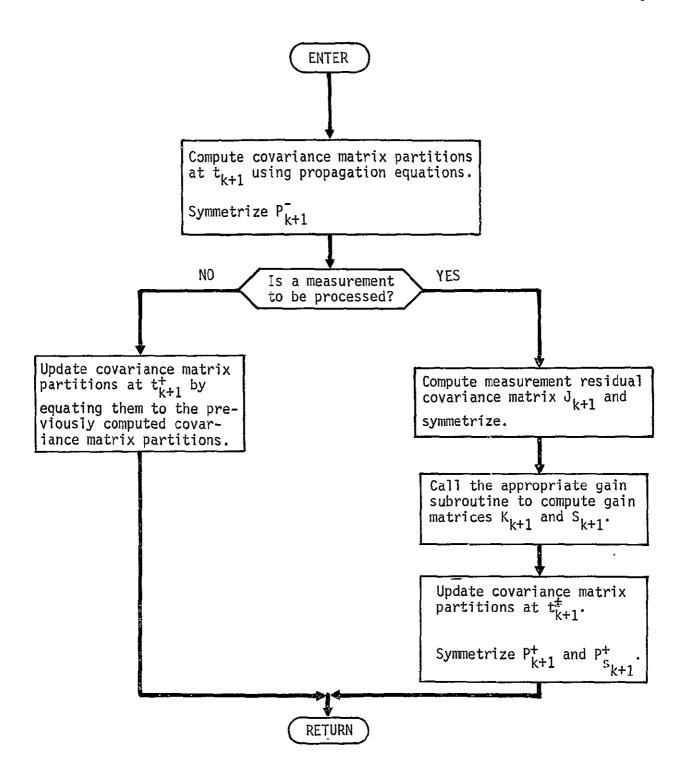
$$C_{xv_{k+1}}^{+} = C_{xv_{k+1}}^{-} - K_{k+1} D_{k+1}^{T}$$

$$C_{xv_{k+1}}^{+} = C_{xv_{k+1}}^{-} - K_{k+1} E_{k+1}^{T}$$

$$C_{x_{s_{k+1}}}^{+} = C_{x_{s_{k+1}}}^{-} - K_{k+1} E_{k+1}^{T}$$

$$C_{x_{s_{k+1}}}^{+} = C_{x_{s_{k+1}}}^{-} - K_{k+1} E_{k+1}^{T}$$

where gain matrices  $\mathbf{K}_{k+1}$  and  $\mathbf{S}_{k+1}$  are computed in the appropriate gain subroutine--GAIN1, if Kalman-Schmidt, GAIN2, if WLS.



SUBROUTINE NOTH -

PURPOSE: TO COMPUTE THE UNAUGMENTED PORTION OF THE STATE TRANS-ITION MATRIX USING THE NUMERICAL DIFFERENCE TECHNIQUE.

CALLING SEQUENCE: CALL NOTH (RI, RF)

ARGUMENT: RF I POSITION AND VELOCITY OF THE VEHICLE AT THE END OF THE 'TIME INTERVAL

RI I POSITION AND VELOCITY OF THE VEHICLE AT THE BEGINNING OF THE TIME INTERVAL

SUBROUTINES SUPPORTED: PSIM

SUBROUTINES REQUIRED 8 NTM

LOCAL SYMBOLS: F1 TEMPORARY STORAGE FOR FACP

F2 TEMPORARY STORAGE FOR FACY

IPR INTERMEDIATE STORAGE FOR IPRINT

RP POSITION AND VELOCITY OF VEHICLE AT

INITIAL TIME

SAVE TEMPORARY STORAGE FOR ACC

T ALTERED POSITION AND VELOCITY OF VEHICLE

AT INITIAL TIME

U ALTERED POSITION AND VELOCITY OF VEHICLE -

AT FINAL TIME

COMMON COMPUTED/USED: ACC FACP FACV IPRINT

COMMON COMPUTED: PHI

COMMON USED: ACCND DELTM NDACC ONE ZERO

NDTM Analysis

The nonlinear equations of motion of the spacecraft can be written symbol-lically as

$$\frac{1}{x} = \vec{f}(\vec{x}, t) \tag{1}$$

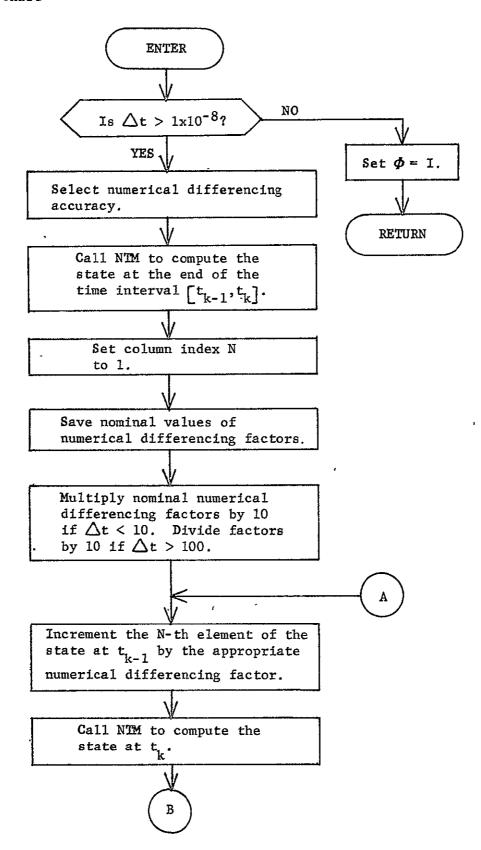
where  $\vec{x}$  is the spacecraft position/velocity state.

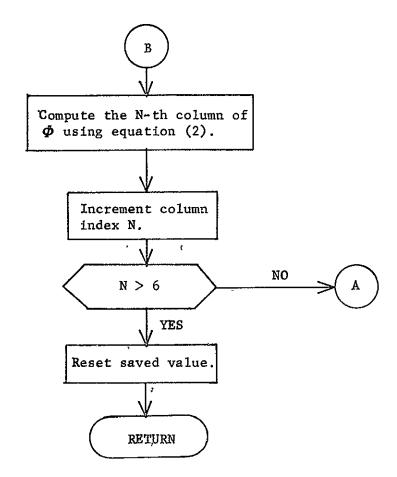
Suppose we wish to use numerical differencing to compute the state transition matrix  $\phi(t_k, t_{k-1})$ . Let  $\vec{\phi}(t_k, t_{k-1})$  represent the j-th column of  $\phi_j(t_k, t_{k-1})$ . We assume we have available the nominal states  $\vec{x}^*(t_{k-1})$  and  $\vec{x}^*(t_k)$ . To obtain  $\vec{\phi}_j(t_k, t_{k-1})$  we increment the j-th element of  $\vec{x}^*(t_{k-1})$  by the numerical differencing factor  $\triangle x_j$  and numerically integrate equation (1) over the time interval  $\begin{bmatrix} t_{k-1}, t_k \end{bmatrix}$  to obtain the new spacecraft state  $\vec{x}_j(t_k)$ . The j-subscript indicates  $\vec{x}_j(t_k)$  was obtained by incrementing the j-th element of  $\vec{x}^*(t_{k-1})$ . Then

$$\vec{\phi}_{j}(t_{k}, t_{k-1}) = \frac{\vec{x}_{j}(t_{k}) - \vec{x}^{*}(t_{k})}{\Delta x_{j}}$$

$$j = 1, 2, \dots, 6$$
(2)

### NDTM Flow Chart





SUBROUTINE NEWPGE

PURPOSE: PRINTS APPROPRIATE HEADING AT THE TOP OF EACH PAGE WHEN PRINTOUT OF TRAJECTORY INFORMATION IS DESIRED

CALLING SEQUENCE CALL NEWPGE

SUBROUTINES SUPPORTED INPUTZ PRINT SPACE

SUBROUTINES REQUIRED: NONE

COMMON COMPUTED/USED: IPG

COMMON COMPUTED: LINCT

COMMON USED: KL

PROGRAM NOMNAL

PURPOSES TO CONTROL THE ENTIRE GENERATION OF A NOMINAL TRAJECTORY FROM INJECTION TARGETING THROUGH MIDCOURSE CORRECTIONS AND ORBIT INSERTION.

CALLING SEQUENCE: NONE (MAIN PROGRAM)

SUBROUTINES SUPPORTED : NONE (MAIN PROGRAM) .

SUBROUTINES REQUIRED: GIDANS PRELIM TRJTRY

COMMON COMPUTED: IPRE

COMMON USED: KWIT

## NOMNAL Analysis

NOMNAL is the executive program controlling the entire generation of a nominal trajectory from injection targeting through midcourse corrections and orbit insertion.

NOMNAL begins by calling PRELIM for the preliminary work including initialization of variables, reading of the input data, and computation of zero iterate values of initial time, position, and velocity if required.

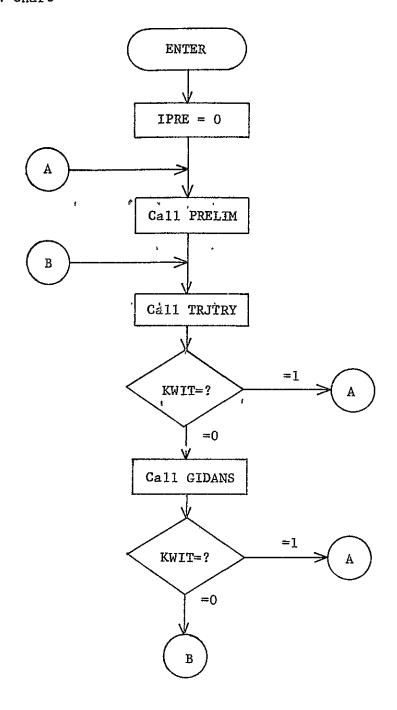
NOMNAL then calls TRJTRY. TRJTRY first determines the time of the next guidance event. It then integrates and records the nominal trajectory to that time. TRJTRY then returns control to NOMNAL.

NOMNAL next calls GIDANS. GIDANS.processes the computation and execution of the current guidance event. NOMNAL then reenters its basic cycle by calling TRJTRY to propagate the corrected trajectory to its next guidance event.

Two flags are used by NOMNAL. The flag IPRE is initialized at zero in NOMNAL. During the processing of the first data case PRELIM sets it to unity. PRELIM uses IPRE to determine whether to preset constants to internally stored values or leave them at their previous values before reading the next data case.

The second flag KWIT determines whether the current case should be continued or terminated according to the flag value zero or unity respectively. Termination is indicated when a fatal error occurs during trajectory propagation or guidance event computation or when the desired end time is reached.

# NOMNAL Flow Chart



SUBROUTINE NONINS

PURPOSE: TO DETERMINE THE TIME AND CORRECTION VECTOR FOR AN INSERTION FROM AN APPROACH HYPERBOLA INTO A SPECIFIED PLANE AND AS NEAR AS POSSIBLE TO A PRESCRIBED CLOSED ORBIT.

CALLING SEQUENCE: CALL NONINS(GM, X, Z, DA, DE, DWTP, DI, DN, TMEX, VEL, TFX)

ARGUMENT:	GM	I	GRAVITATIONAL CONSTANT
	x (3)	I	POSITION VECTOR AT DECISION
	Z(3)	I	VELOCITY VECTOR AT DECISION
	DA	I	DESIRED SEMIMAJOR AXIS
	DE	I	DESIRED ECCENTRICITY
	DHTP	I	DESIRED ARGUMENT OF PRRIAPSIS
	DNTP	I	DESIRED ARGUMENT OF PERIAPSIS
	DI	I	DESIRED INCLINATION
	DN	I	DESIRED LONGITUDE OF ASCENDING NODE
	THEX	0	TIME FROM DECISION TO EXECUTION (SECONDS)
	VEL(3)	0	INSERTION VELOCITY VECTOR
	IEX	0	EXECUTION CODE =0 EXECUTABLE SOLUTION DETERMINED =1 NO EXECUTABLE SOLUTION FOUND

SUBROUTINES SUPPORTED: INSERS

SUBROUTINES REQUIRED: CAREL ELCAR

LOCAL SYMBOLS AH HYPERBOLIC SEMIMAJOR AXIS.

ANG TRUE ANOMALY OF HYPERBOLIC ASYMPTOTE

ARC2 360.

ARC 180.

SEMIMAJOR AXIS OF MODIFIED ELLIPSE

CEI COSINE OF DI

CEN COSINE OF DN

CHI COSINE OF HI

CHN COSINE OF HN

CTAE COSINE OF ETA

CTASY COSINE OF TASY

CUTXE COSINE OF HIXE

CHTXH COSINE OF WIXH

DELV VELOCITY CORRECTIONS OF CANDIDATE SOLUTION

DRA DESIRED APOAPSIS RADIUS

DRP DESIRED PERIAPSIS RADIUS

DTA DUMMY VARIABLE FOR OUTPUT

DVM MAGNITUDES OF CANDIDATE CORRECTIONS

DV MAGNITUDES OF CANDIDATE CORRECTIONS

EH HYPERBOLIC ECCENTRICITY

ERRMAX SCALAR ERROR ASSIGNED TO IMPOSSIBLE

SOLUTION

ERR ARRAY OF SCALAR ERRORS OF SOLUTIONS

ETAX TRUE ANOMALIES AT INTERSECTION POINTS ON

ELLIPSE

E ECCENTRICITY OF MODIFIED ELLIPSE

HI HYPERBOLIC INCLINATION

HN HYPERBOLIC LONGITUDE OF ASCENDING NODE

HRP HYPERBOLIC PERIAPSIS RADIUS

HTAX TRUE ANOMALIES AT INTERSECTION POINTS ON

HYPERBOLA

HTA CANDIDATE HYPERBOLIC TRUE ANOMALY AT

INTERSECTION

MININ INDEX OF OPTIMAL INTERSECTION POINT

MIN INDEX OF OPTIMAL SOLUTION (POINT AND MOD)

NCPOS FLAG INDICATING WHETHER ANGLE BETWEEN NODE AND INTERSECTION IS GREATER OR LESS THAN 180

NSOLS NUMBER OF SOLUTIONS

NT1 INDEX OF FIRST SOLUTION

NT2 INDEX OF LAST SOLUTION

PH HYPERBOLIC SEMILATUS RECTUM

PI THE MATHEMATICAL CONSTANT PI

PP UNIT VECTOR TOWARD PERIAPSIS

QQ UNIT VECTOR IN ORBITAL PLANE NORMAL TO PP

RAD DEGREE TO RADIAN FACTOR

RA APOAPSIS RADIUS

RHYP HYPERBOLIC CANDIDATE RADII TO INTERSECTION

RMAG RADIUS TO INTERSECTION POINT

RM RADIUS AT DECISION

RP PERIAPSIS RADIUS

RX RADIUS TO INTERSECTION POINT ON ELLIPSE

R1 RADIUS VECTOR TO HYPERBOLA AT INTERSECTION

R RADIUS VECTOR TO ELLIPSE AT INTERSECTION

SEI SINE OF DI

SEN SINE OF DN

SGNZ SIGN OF DECLINATION OF INTERSECTION POINT

SHI SINE OF HI

SHN SINE OF HN

STA TRUE ANOMALY AT DECISION

TAE ARRAY OF ELLIPTIC TRUE ANOMALIES

TASY TRUE ANOMALY OF ASYMPTOTE

TAKE TRUE ANOMALY AT INTERSECTION POINT ON

#### ELLIPSE

TAXH	TRUE ANOMALY	AT	INTERSECTION	POINT	ON
	HYPERBOLA				

TA TRUE ANOMALY

TOEC TIME FROM PERIAPSIS AT DECISION

TEXC TIME FROM PERIAPSIS AT EXECUTION ON HYPER-BOLA

TEX ARRAY OF CANDIDATE TIMES FROM DECISION TO EXECUTION

TIF TIME FROM PERIAPSIS AT DECISION ON ELLIPSE

VM SPEED AT DECISION

VI VELOCITY VECTOR ON HYPERBOLA AT EXECUTION

V VELOCITY VECTOR ON ELLIPSE AT EXECUTION

WE ARGUMENT OF PERIAPSIS ON ELLIPSE

WH ARGUMENT OF PERIAPSIS ON HYPERBOLA

WTXE ANGLE BETWEEN ASCENDING NODE AND INTER-SECTION POINT ON ELLIPSE

WTXH ANGLE BETWEEN ASCENDING NODE AND INTER-SECTION POINT ON HYPERBOLA

WW UNIT NORMAL TO ORBITAL PLANE

W ARGUMENT OF PERIAPSIS

XINT X COMPONENT OF INTERSECTION POINT

YINT Y COMPONENT OF INTERSECTION POINT

ZINT Z COMPONENT OF INTERSECTION POINT

### NONINS Analysis

NONINS determines the time and correction vector for an impulsive insertion from an approach hyperbola into a specified plane and as near as possible to a prescribed closed orbit. The approach hyperbola is specified by giving the planetocentric equatorial state  $\overline{\mathbf{r}}, \overline{\mathbf{v}}$  at the time of decision  $\mathbf{t}_{\mathbf{d}}$ . The final orbit is defined by giving its desired orbital elements  $(\mathbf{a}_{\mathbf{E}}, \mathbf{e}_{\mathbf{E}}, \mathbf{i}_{\mathbf{E}}, \boldsymbol{\omega}_{\mathbf{E}}, \Omega_{\mathbf{E}})$  again in planetocentric equatorial coordinates.

Subroutine CAREL is first called to convert the hyperbolic state at decision  $\overrightarrow{r}$ ,  $\overrightarrow{v}$  into Keplerian conic elements ( $a_H$ ,  $e_H$ ,  $i_H$ ,  $\omega_H$ ,  $\Omega_H$ ,  $t_{Hd}$ ) where  $t_{Hd}$  is the time from periapsis at decision (negative on the approach ray).

The points of intersection of the approach orbital plane and the desired orbital plane are then determined. The elements defining the two planes are therefore given by  $i_H$ ,  $f_H$  and  $i_E$ ,  $f_E$ . Let  $f_H$  denote the unit vector toward the ascending node of an orbit and  $f_H$  denote the in-plane normal to  $f_H$  in the direction of motion. Then

$$\hat{A} = (\cos \Omega, \sin L, 0)$$
 (1)

$$\hat{B} = (-\sin\Omega \cos i, \cos\Omega \cos i, \sin i)$$
 (2)

Hence the normal to the orbital plane  $\hat{C}$  is given by  $\hat{C} = \hat{A} \times \hat{B}$  or

$$\hat{C} = (\sin \Omega \sin i, -\cos \Omega \sin i, \cos i)$$
 (3)

The direction of the line of intersection of the two planes is therefore determined by  $\hat{X} = \hat{C}_{\mu} \times \hat{C}_{\mu}$  or

$$\overrightarrow{X} = (\cos i_{H} \sin i_{E} \cos \Omega_{E} - \sin i_{H} \cos i_{E} \cos \underline{\mathcal{L}}_{H},$$

$$\cos i_{H} \sin i_{E} \sin \Omega_{E} - \sin i_{H} \cos i_{E} \sin \underline{\Omega}_{H},$$

$$\sin i_{H} \sin i_{E} (\cos \Omega_{H} \sin \Omega_{E} - \sin \underline{\mathcal{L}}_{H} \cos \underline{\mathcal{L}}_{E})) \quad (4)$$

Then the unit vector along the line of intersection toward the northern hemisphere is given by

$$\hat{X} = \operatorname{sgn} \vec{X}_3 \cdot \overline{\vec{X}}$$
 (5)

Therefore the true anomaly f_{HX} along the hyperbola at the northern intersection point is given by

$$cos (\omega_{H} + f_{HX}) = \hat{X} \cdot \hat{A}_{H} \qquad (6)$$

The true anomaly on the hyperbola at the southern point is therefore  $f_{\mu\nu} + 180^{\circ}$ . Note that there exists a region of true anomalies lying between the incoming and outgoing asymptotes for which the hyperbola is not defined. Similar equations define the true anomaly on the ellipse at the two points of intersection. Note that this implies that the modified ellipse will have the same  $\omega$  as the desired ellipse.

For the intersection true anomaly  $f_{\mbox{HX}}$  the radius magnitude on the hyperbola may be determined

$$r_{I} = \frac{a_{H}(1 - e_{H}^{2})}{1 + e_{H} \cos f_{H}}$$
 (7)

To permit an impulsive insertion,  $a_E$  and  $e_E$  must be modified to satisfy

$$r_{I} = \frac{a_{E}(1 - e_{E}^{2})}{1 + e_{E} \cos f_{E}}$$
 (8)

There are three candidate modifications examined to determine a "best" one: (1) Vary r while holding r constant

- (1) Vary  $r_a$  while holding  $r_p$  constant (2) Vary  $r_p$  while holding  $r_a$  constant
- (3) Vary a while holding e constant

"Best" is defined below in terms of a weighted scalar function of the changes in  $r_{\rm a}$  and  $r_{\rm p}$  .

Rewriting (8) in terms of  $r_a$  and  $r_p$  (using  $a = \frac{r_a + r_p}{2}$ ,  $e = \frac{r_a - r_p}{r_a r_p}$ ) yields the useful relation

$$r_a(1 + \cos f_E) + r_p(1 - \cos f_E) = \frac{2r_a r_p}{r_T}$$
 (9)

Equation (9) may be solved for r as

$$r_{a} = \frac{r_{I}r_{p}(1 - \cos f_{E})}{2 r - r (1 + \cos f)}$$
(10)

This yields the  $r_a$  which defines the modified orbit holding  $r_p$  at its desired value. The semi-major axis and eccentricity are then computed from  $a=\frac{r_a+r_p}{2}$ ,  $e=\frac{r_a-r_p}{r_a+r_p}$ .

Similarly (9) may be solved for  $r_p$  as

$$r_{p} = \frac{r_{I}r_{a}(1 + \cos f)}{2 r_{a} - r_{I}(1 - \cos f)}$$
(11)

Finally (8) may be solved trivially for the a required to produce intersection for the desired eccentricity.

$$a_{E} = \frac{r_{I}(1 + e_{E} \cos f_{E})}{(1 - e_{E}^{2})}$$
 (12)

An error is assigned to each of the candidate solutions as

$$E_{i} = W_{i} \left[ \left| \Delta r_{a} \right| + \left| \Delta r_{p} \right| \right]$$

where  $\Delta r_a$ ,  $\Delta r_p$  are the errors between the desired and modified values of  $r_a$  and  $r_p$ . The weighting factor  $W_i$  is assigned rather arbitrarily. Currently the weighting factor is  $W_i = w_{1i} w_{2i}$  where

 $w_{1i} = 1$  if the true anomaly is on the incoming ray 2 if the true anomaly is on the outgoing ray

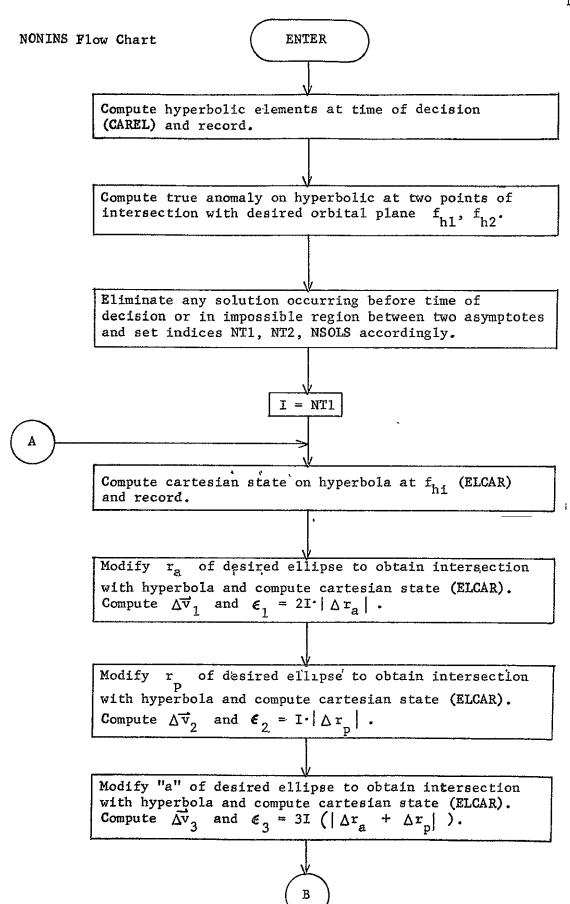
w_{2i} = 1 if option 1 2 if option 2 3 if option 3

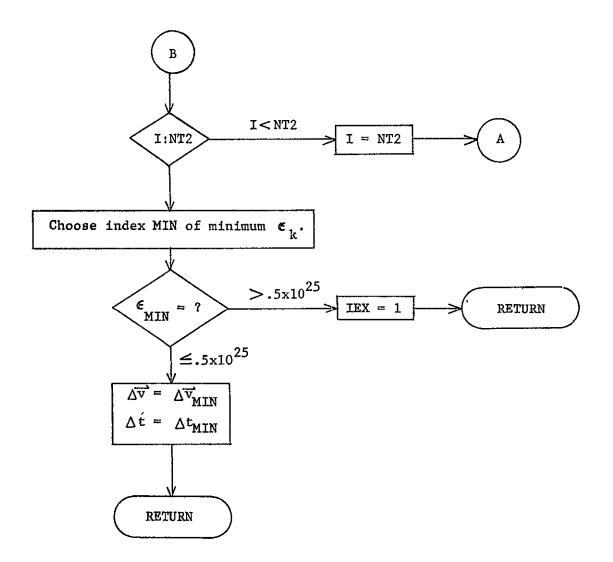
Thus a solution on the incoming asymptote is preferred over one on the outgoing asymptote and one subsequent trim is preferred over two subsequent trims.

Having determined the elements of an intersecting orbit the insertion parameters are easily computed. The velocity on the hyperbola at the intersection point may be computed from ELCAR as  $\overrightarrow{v}_H$ . The velocity on the ellipse following the insertion is computed by calling ELCAR with the modified elliptical element to get  $\overrightarrow{v}_E$ . The impulsive  $\overrightarrow{\Delta v}$  is then given by

$$\overrightarrow{\Delta v} = \overrightarrow{v}_{E} - \overrightarrow{v}_{H}$$

The time interval from the decision to the execution is given by the hyperbolic time from the initial point to the relevant intersection point.





SUBROUTINE NONLIN

PURPOSE: TO CONTROL EXECUTION OF NON-LINEAR GUIDANCE EVENTS.

CALLING SEQUENCES CALL NONLIN

SUBROUTINES SUPPORTED & GUISIM GUIDM

SUBROUTINES REQUIRED & CAREL ELCAR GIDANS

LOCAL SYMBOLS: AA ARGUMENT FOR SUBROUTINE CAREL

DI JULIAN DATE OF EVENT

EE ARGUMENT FOR SUBROUTINE CAREL

ISNPR SAVE INPR VALUE

ISPRNT SAVE IPRINT VALUE

KEY INTERMEDIATE VARIABLE IN SETTING UP TARGET

ARRAY

KICL2 SAVE ICL2 VALUE

KICL SAVE ICL VALUE

KISPH SAVE ISPH VALUE

KISP2 SAVE ISP2 VALUE

ODELY SAVE ORIGINAL DELTP VALUE

OSPH SAVE ORIGINAL SPHERE OF INFLUENCE OF

TARGET PLANET

PP ARGUMENT RETURNED FROM CAREL

QQ ARGUMENT RETURNED FROM CAREL

RMAG ARGUMENT FOR SUBROUTINE ELCAR

TAA TRUE ANOMALY

TFFP TIME OF FLIGHT FROM PERIAPSIS

TFP TIME FROM PERIAPSIS

TRTIME TRAJECTORY TIME OF THE GUIDANCE EVENT

VHAG ARGUMENT FOR SUBROUTINE ELCAR

WW ARGUMENT FOR SUBROUTINE CAREL

	XXI	ARGUMENT	FOR SUB	ROUTINE	CAREL	
	XXN	ARGUMENT	FOR SUB	ROUTINE	CAREL	
	XYZTAA	ZERO TRU ELCAR	IE ANOMAL	Y ARGUME	NT FOR S	UBROUTINE
COMMON	COMPUTED/USED:	DELTP ICL2 ISP2 KTP NOIT TGT3 XDC	DELV ICL KLP KWIT NTP TIN XDELV	DSI INPR KMXQ MAT RSI TMU XRC	DT IPRINT KTAR MAXB SPHERE TOL ZDAT	IBADS ISPH KTIM HDL TAR VSI
COMMON	COMPUTED:	ACKT DG KTYP NPAR TIMG	AC DVMAX KUR PERV TRTH	BDR D1 LVLS RC	BDT ISTART NLP RIN	DC IZERO NOGYD SPHFAC
COMMON	USED:	ACX JX LNPAR XBDR XIN XVSI	ALNGTH LKLP PMASS XBDT XPERV ZERO	DATEJ LKTAR TM XDSI XRSI	DELTAV LKTP T3 XDVMAX XTAR	IX LLVLS XAC XFAC XTOL

## NØNLIN Analysis

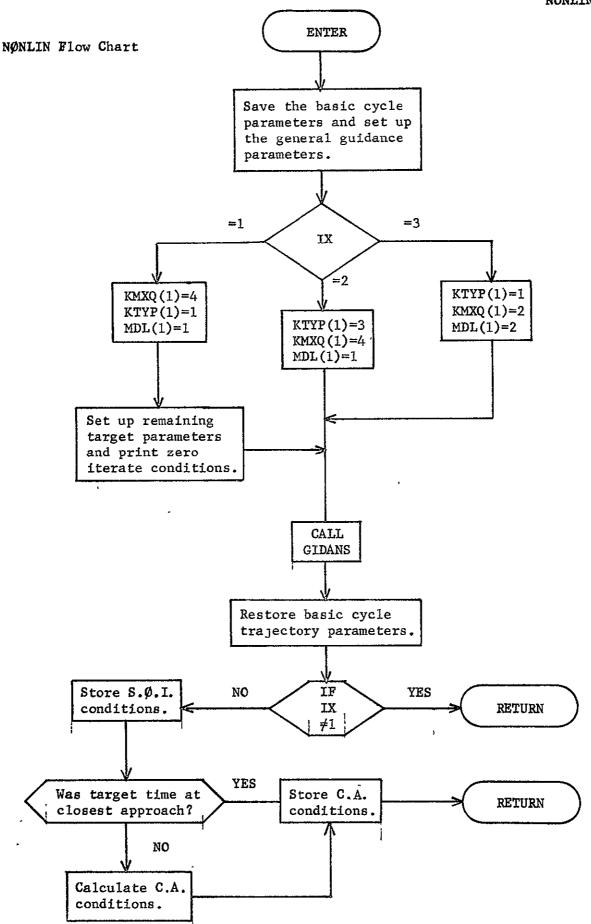
NØNLIN is the interface subroutine between the non-linear guidance subroutines of NØMNAL and subroutine GUIDM of ERRAN and subroutine GUISIM of SIMUL. NØNLIN selects the necessary data from the ERRAN and SIMUL common blocks and stores into the common blocks of NØMNAL the information needed to compute and/or execute the  $\Delta \overline{V}$  required in order to meet specified target conditions.

The most important task performed by NØNLIN is the selection of the desired guidance scheme. The variable IX is tested and control is transformed according to the following:

- IX = 1, retargeting to specified target parameters
  - ='2, orbit insertion to specified orbit
  - = 3,  $\Delta \overline{V}$  execution by a series of specified pulses

For each type of event, NØNLIN then sets up values controlling the type of guidance event (KTYP), implementation code (KMXQ), and execution model code (MDL). For retargeting only, NØNLIN stores the remaining values needed for  $\Delta V$  calculation and prints the zero iterate conditions.

NØMNAL calls GIDANS to perform the guidance event and restores parameters necessary for the basic cycles of ERRAN and SIMUL. For retargeting only, NØMNAL then stores the conditions at sphere of influence and closest approach of the target planet which were calculated by subroutine TARGET.



SUBROUTINE NTM

PURPOSE: CONTROL COMPUTATION OF NOMINAL TRAJECTORY IN THE ERROR ANALYSIS PROGRAM

CALLING SEQUENCES CALL NTM(RI, RF, NTMC, ICODE)

ARGUMENT: ICODE I INTERNAL CODE THAT DETERMINES WHICH TRAJECTORY IS BEING RUN AND WHAT INFORMATION IS DESIRED

NTMC I NOMINAL TRAJECTORY MODULE CODE THAT
DETERMINES WHICH TYPE OF TRAJECTORY PROGRAM
IS TO BE USED (NOTE ONLY THE VIRTUAL MASS
TECHNIQUE IS SUPPLIED WITH THIS PROGRAM.
HOWEVER, WITH LITTLE EFFORT ANY TRAJECTORY
PROGRAM MAY BE ADDED AS AN EXTRA OPTION.)

RF 0 POSITION AND VELOCITY OF THE VEHICLE AT THE END OF THE TIME INTERVAL

RI I POSITION AND VELOCITY OF THE VEHICLE AT THE BEGINNING OF THE TIME INTERVAL

SUBROUTINES SUPPORTED: ERRANN MUND NOTH PLND PSIM SETEVN GUID VARADA PRED

SUBROUTINES REQUIRED: VMP

LOCAL SYMBOLS'S D1 JULIAN DATE, EPOCH JAN.O, 1900, OF INITIAL TRAJECTORY TIME

RMP DISTANCE OF VEHICLE FROM TARGET PLANET AT SPHERE OF INFLUENCE OR CLOSEST APPROACH

RC

RSI

TRTN1

YMM MAGNITUDE, OF THE VELOCITY VECTOR

COMMON COMPUTED/USED: BDRSI1 BDRSI2 BDRSI3 **BDTSI1** BDTS12 BDTS13 BSI1 BSI2 BS I3 ICA1 ICA2 ICA3 ICL ISOI1 ISOI2 ISOI3 ISPH RCA1 RCA2 RCA3 RSOI1 RSOI2 RS013 TCA1 TCA2 TCA3 TSOI1 TSOI2 TS0I3 VSOI1 VSOI2 VS0I3 COMMON USEDS ACC BDR BOT DATEJ DC DELTM DSI IPROB ISP2

NQE

ITR

# NTM Analysis

Subroutine NTM is used to generate the (most recent) targeted nominal trajectory in the error analysis mode. Subroutine NTM is equivalent to a subroutine NTMS from which all loops associated with ICODE = -3, -2, 2, 3 have been removed. For this reason no further analysis and no flow chart will be presented for subroutine NTM. Refer to subroutine NTMS.

SUBROUTINE NTHS

PURPOSE: CONTROL COMPUTATION OF TARGETED NOMINAL, MOST RECENT NOMINAL, AND ACTUAL TRAJECTORIES IN THE SIMULATION PROGRAM

CALLING SEQUENCES CALL NTMS (RI, RF, NTMC, ICODE)

ARGUMENT8 ICODE I INTERNAL CODE THAT DETERMINES WHICH TRAJECTORY IS BEING RUN AND WHAT INFORMATION IS DESIRED

NTMC I NOMINAL TRAJECTORY MODULE CODE THAT

DETERMINES WHICH TYPE OF TRAJECTORY PROGRAM

IS TO BE USED (NOTE ONLY THE VIRTUAL MASS

TECHNIQUE IS SUPPLIED WITH THIS PROGRAM.

HOWEVER, WITH LITTLE EFFORT ANY TRAJECTORY

PROGRAM MAY BE ADDED AS AN EXTRA OPTION.)

RF O POSITION AND VELOCITY OF THE VEHICLE AT THE END OF THE TIME INTERVAL

RI I POSITION AND VELOCITY OF THE VEHICLE AT THE BEGINNING OF THE TIME INTERVAL

SUBROUTINES SUPPORTED: SIMULL MUND NOTM PLND PSIM SETEVS GUISS VARSIM PRESIM

SUBROUTINES REQUIRED: VMP

LOCAL SYMBOLS: ACCS INTERMEDIATE STORAGE FOR ACCURACY

D1 JULIAN DATE, EPOCH JAN., 1900, OF INITIAL TRAJECTORY TIME

K1 INDEX FOR SEMIMAJOR AXIS ELEMENT

K2 INDEX FOR ECCENTRICITY ELEMENT

K3 INDEX FOR INCLINATION ELEMENT

K4 INDEX FOR ASCENDING NODE ELEMENT

KS INDEX FOR PERIAPSIS ELEMENT

K6 INDEX FOR MEAN ANOMALY. ELEMENT

NBODS INTERNEDIATE STORAGE FOR NBOD

NBS INTERMEDIATE STORAGE FOR NB ARRAY

RMP DISTANCE OF VEHICLE FROM TARGET PLANET

:	SAVE10	INTERMED	IATE STO	RAGE		
	SAVE1	INTERMED	TATE STO	RAGE		
	SAVE2	INTERMED	TATE STO	RAGE		
	SAVE3	INTERMED	IATE STO	RAGE		
	SAVE4	INTERMED	TATE STO	RAGE		
	SAVE5	INTERMED	IATE STO	RAGE		
	SAVE6	INTERME	DIATE STO	RAGE		
	SAVE7	INTERMED	DIATE STO	RAGE		
	SAVE8	INTERMED	TATE STO	RAGE		
	VHM	MAGNITUE	E OF THE	VELOCI1	TY VECTOR	<b>ર</b>
CONHON COMPUTED/	USED:	NBOD RCA3	BDTSI3 EMN ISOI1	PMASS	BDRSI3 BSI2 ICA2 ISOI3 RCA1 RSOI3 TCA3 VSOI2	RCA2
COMMON USED:		ACC1 DAB DIB DSI NBOD1 RC	ALNGTH DATEJ DMAB DW8 NB1 RSI	BOR DC DMUPB IPROB NGE TM	BDT DEB DMUSB ISP2 NQE TRTM1	B DELTH DNOB ITR NTP VSI

### NTMS Analysis

Subroutine NTMS is used to generate any of the three trajectories required in the simulation mode -- the (most recent) targeted nominal trajectory, the most recent nominal trajectory, and the actual trajectory.

The input variable ICODE is used to distinguish between these trajectories. It is unimportant to the virtual mass technique which trajectory is being computed. However, it is important to keep them separated so that the proper codes are set that check for approaching the sphere of influence of the target planet and reaching closest approach. It is also important to keep separate the conditions at which these occur for each trajectory. The following list describes ICODE completely.

- ICODE = 2, NIMS performs the same operations as described above for the
   most recent nominal trajectory.
- ICODE = 1, NTMS again checks for sphere of influence and closest approach as above for the targeted nominal trajectory.
- ICODE = 0, the only important information in this situation is the state
   vector at the end of the time interval. Therefore, NTMS does
   not check to see if closest approach or sphere of influence is
   encountered. This might occur in numerical differencing, for
   example.
- ICODE = -2, the same comments may be made as if ICODE = -1, except this
  is on the most recent nominal trajectory.
- ICODE = -3, again, this value of ICODE is treated the same as is ICODE =
   -1, for the actual trajectory.

Physical constants, planetary ephemerides, and other information relating to the dynamic model are the same for the targeted and most recent nominal trajectories. This is not true for the actual trajectory. There may be biases in the target planet ephemerides and the gravitational constants of the Sun and target planet. The numerical accuracy and the number of celestial bodies employed in the generation of the actual trajectory may also differs

Ephemeris biases are specified as biases in orbital elements a, e, i,  $\Omega$ ,  $\omega$ , and M... However, within the program are stored the ephemeris constants of a, e, i,  $\Omega$ ,  $\widetilde{\omega}$ , and M for the planets and a, e, i,  $\Omega$ ,  $\widetilde{\omega}$ , and L for the moon, where

$$\widetilde{\omega} = \omega + \Omega$$

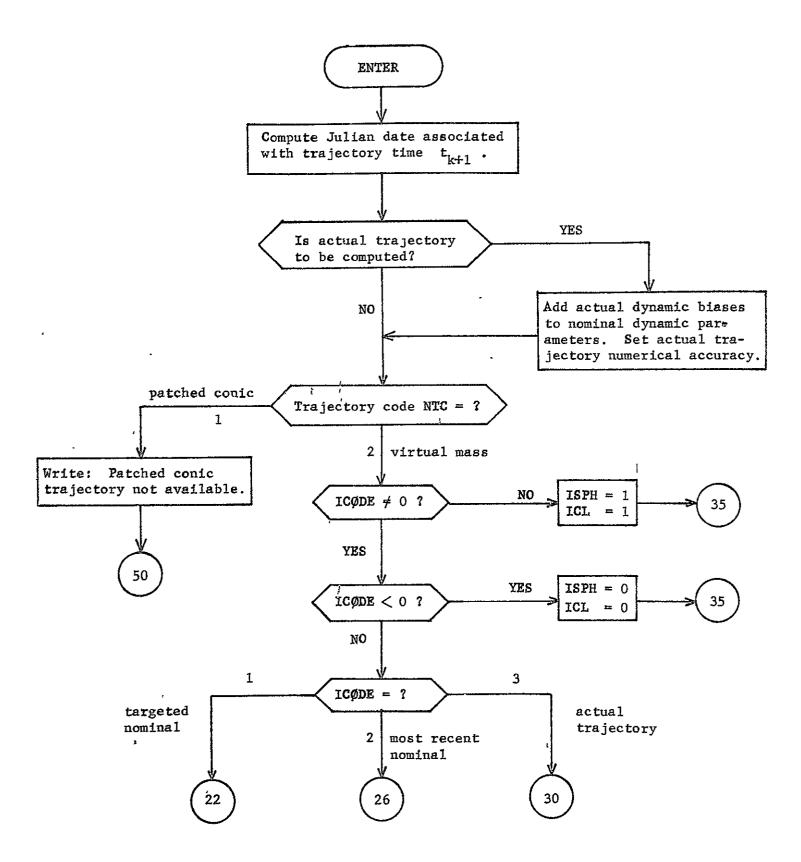
and

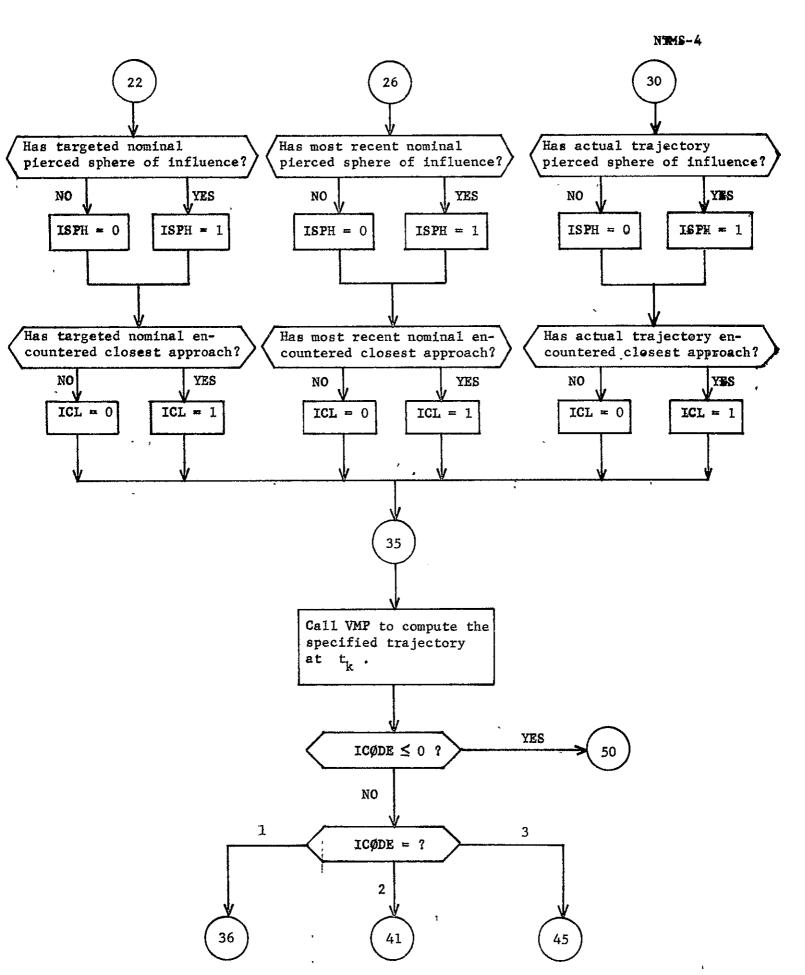
$$L = M + \omega + \Omega.$$

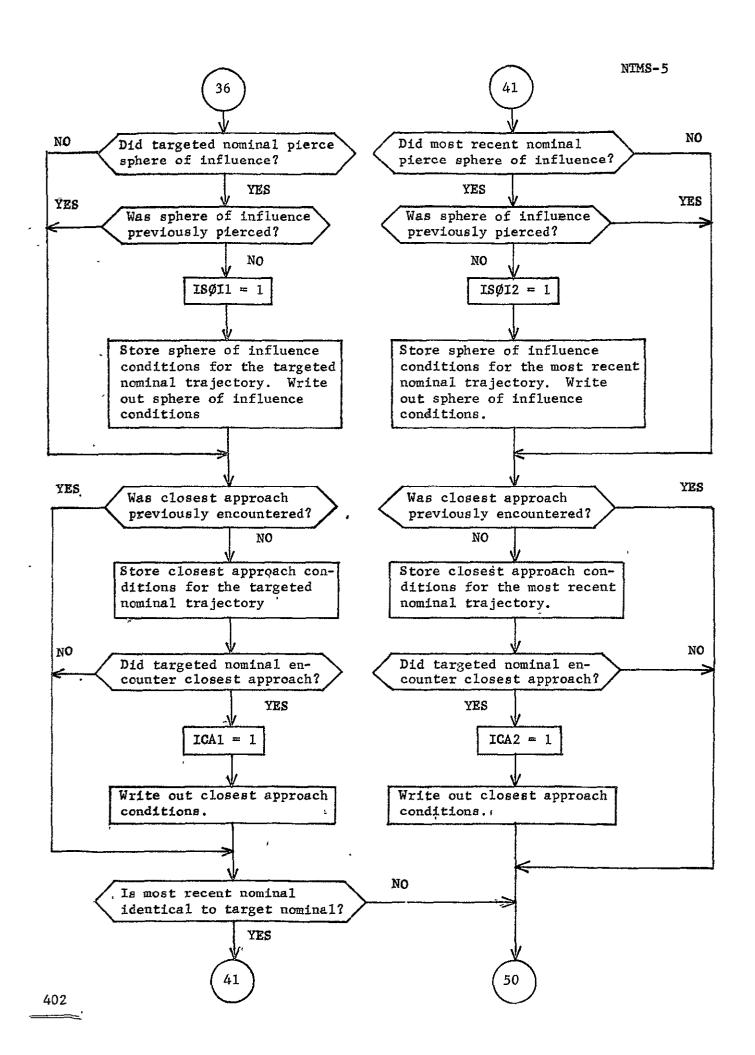
Incrementation of  $\widetilde{\omega}$  and L requires addition of biases in  $\Omega$ ,  $\omega$ , and M as indicated by the above equations.

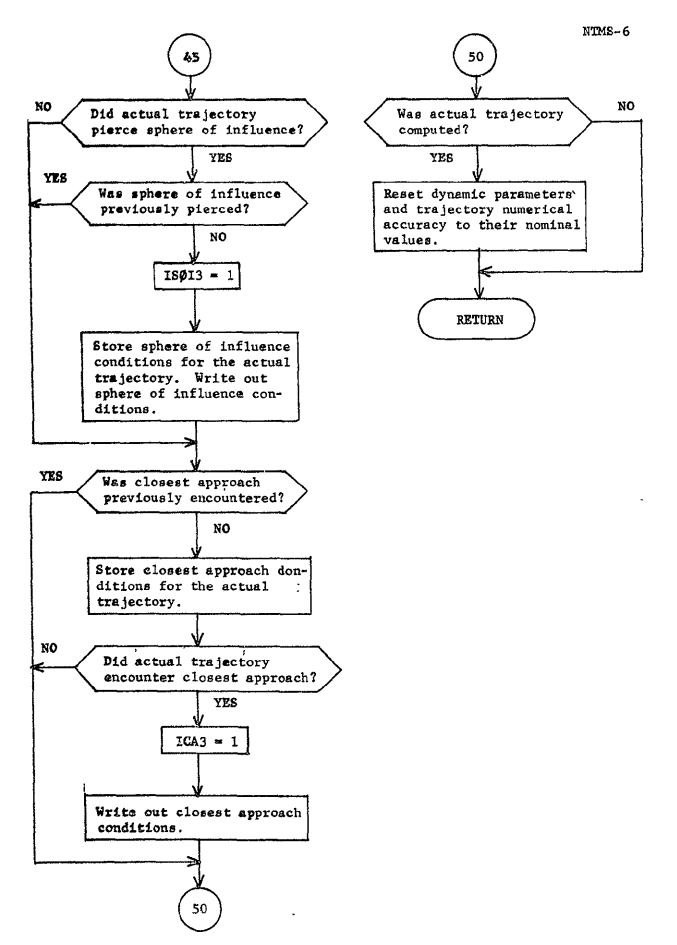


# NTMS Flow Chart









SUBROUTINE NTRY

PURPOSE TO COMPUTE ENTRY PARAMETERS, ENTRY COVARIANCE AND COMMUNICATION ANGLE

CALLING SEQUENCE & CALL NTRY (TE, XPS, P, MODE)

ARGUMENTS: TE I ENTRY TIME (TRAJECTORY TIME)

'XPS I PLANETOCENTRIC ECLIPTIC STATE AT PROBE

SPHERE

P I STATE COVARIANCE MATRIX

MODE I NOT FUNCTIONAL CURRENTLY

SUBROUTINES SUPPORTED: PROBE PROBES

SUBROUTINES REQUIRED: ORB EPHEM TIME SUBSOL MATPY

LOCAL SYMBOLS: A TRANSFORMATION FROM STEAP CARTESIAN STATE

VARIABLES TO LTR ENTRY PARAMETER STATE

VARIABLES

B INTERMEDIATE VARIABLE

H VEHICLE ALTITUDE

T TRANSFORMATION FROM PLANETOCENTRIC TO SUB-

SOLAR COORDINATES

V VEHICLE VELOCITY RELATIVE TO PLANET AT

PROBE SPHERE

AA INTERMEDIATE VARIABLE

ER UNIT VECTOR ALIGNED WITH SUBSOLAR RADIUS

**VECTOR** 

EN VECTOR NORMAL TO ENTRY PLANE

HI ENTRY ORBIT ANGULAR MOMENTUM/UNIT MASS

MO MONTH

TZ JULIAN DATE CORRESPONDING TO TE

UP INTERMEDIATE VARIABLE

IHR HOUR

IYR YEAR

PPP COMMUNICATION ANGLE IN DEGREES

SEC SECONDS

SUM INTERMEDIATE VARIABLE

XIS INCLINATION OF ENTRY PLANE TO SUBSOLAR PLANE

XRE POSITION OF EARTH AT TE

XVE VELOCITY OF EARTH AT TE

XRP POSITION OF TARGET PLANET AT TE

XVP VELOCITY OF TARGET PLANET AT TE

COSP COS(COMMUNICATION ANGLE)

FAC1 INTERMEDIATE VARIABLE

FAC2 INTERMEDIATE VARIABLE

FAC3 INTERMEDIATE VARIABLE

GAMA FLIGHT PATH ANGLE

IDAY DAY

IMIN MINUTE

OMEGS REFERENCE LONGITUDE OF ASCENDING NODE OF ENTRY PLANE RELATIVE TO SUBSOLAR COORDINATE SYSTEM

PHIPS ANGLE BETWEEN ASCENDING NODE AND THE PHI REFERENCE LINE

PHIX PHI REFERENCE (SET EQUAL TO ZERO)

PLTR COVARIANCE MATRIX FOR LTR

PSAV INTERMEDIATE ARRAY

RDOT INTERMEDIATE VARIABLE

RSIS SUBSOLAR POSITION COORDINATES OF SPACE CRAFT AT THE PROBE SPHERE

SUM1 INTERMEDIATE VARIABLE

SUM2 INTERMEDIATE VARIABLE

11

SUM3 INTERMEDIATE VARIABLE

SUM4 INTERMEDIATE VARIABLE

SUMS INTERMEDIATE VARIABLE

SUM6 INTERMEDIATE VARIABLE

VSIS SUBSOLAR VELOCITY COORDINATES OF SPACE

CRAFT AT THE PROBE SPHERE

88 INTERMEDIATE VARIABLE

COMMON COMPUTED/USED: NO XP

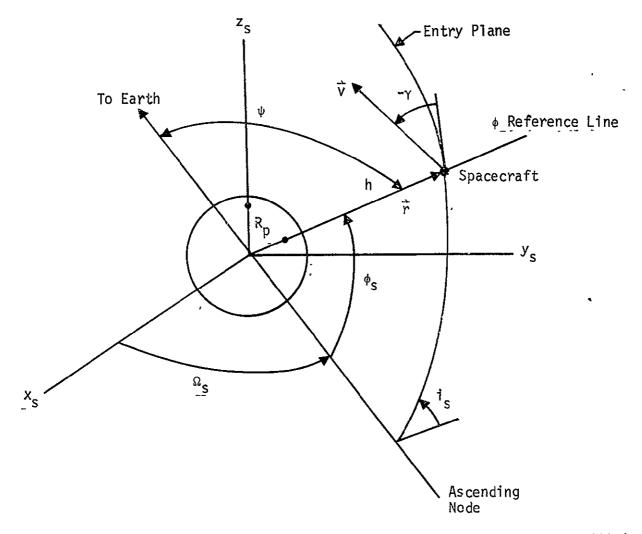
COMMON USED: DATEJ DSI NTP PI RAD

RPS TRIMB

#### NTRY Analysis

Subroutine NTRY transforms the heliocentric ecliptic spacecraft state and covariance matrix to entry parameter coordinates. This information is useful in defining initial data for the *Lander Trajectory Reconstruction (LTR)* program. Subroutine NTRY also computes the communication angle at entry.

The entry parameter state is defined by altitude h, velocity v relative to the planet, flightpath angle  $\gamma$ , longitude of the ascending node  $\Omega_{\rm S}$ , inclination is of the entry plane, and the angle  $\phi_{\rm S}$  between the ascending node and the  $\phi$  reference line. These latter angles are all defined relative to the subsolar orbital-plane coordinate system x y z , which is defined in subroutine SUBSØL. All entry parameters are shown in the following figure, as is the communication angle  $\psi$ .



The transformation of the heliocentric ecliptic spacecraft state to the entry parameter state requires first that the target planet heliocentric ecliptic state be subtracted to obtain the relative spacecraft position  $\hat{r}$  and velocity  $\hat{v}$ . The equations for transforming  $\hat{r} = \begin{pmatrix} r_x, r_y, r_z \end{pmatrix}$  and  $\hat{v} = \begin{pmatrix} v_x, v_y, v_z \end{pmatrix}$  to  $\begin{pmatrix} h, v, \gamma, \phi_s, 1_s, \Omega_s \end{pmatrix}$  are summarized below.

Define  $e_r = \frac{\dot{r}}{r}$  and  $\dot{e}_n = \frac{\dot{r} \times \dot{v}}{|\dot{r} \times \dot{v}|}$ , where  $e_r$  is a unit vector aligned with  $\dot{r}$  and  $\dot{e}_n$  is a unit vector normal to the entry plane. Let  $\dot{e}_z$  denote a unit vector aligned with the  $z_s$ -axis. The Cartesian subsolar orbital-plane components of these three unit vectors will be denoted as follows:

$$\dot{e}_{r} = \begin{pmatrix} e_{r_{x}}, e_{r_{y}}, e_{r_{z}} \end{pmatrix}$$

$$\dot{e}_{n} = \begin{pmatrix} e_{n_{x}}, e_{n_{y}}, e_{n_{z}} \end{pmatrix}$$

$$\dot{e}_{z} = (0, 0, 1).$$

Altitude h and velocity v are readily obtained:

$$h = |\dot{r}| - R_{p} \tag{1}$$

$$v = |\vec{v}| \tag{2}$$

where  $\boldsymbol{R}_{p}$  is the target planet radius. Flightpath angle  $\gamma$  is computed from

$$\gamma = \sin^{-1}\left(\frac{\dot{\mathbf{r}}}{\mathbf{v}}\right) \tag{3}$$

where

$$\dot{\mathbf{r}} = \dot{\mathbf{v}} \cdot \dot{\mathbf{e}}_{\mathbf{r}}$$
.

Longitude of the ascending node  $\Omega_s$  is given by

$$\Omega_{g} = \tan^{-1} \left( \frac{e_{n}}{-e_{n}} \right), \tag{4}$$

while inclination i is obtained from

$$i_s = \cos^{-1} \left( \begin{pmatrix} e_n \\ e_n \end{pmatrix} \right). \tag{5}$$

The angle  $\varphi_{\mathbf{S}}$  is given by

$$\phi_{S} = \tan^{-1} \left( \frac{\sin \phi_{S}}{\cos \phi_{S}} \right) \tag{6}$$

where

$$\sin \phi_{s} = \frac{e_{r}}{\sin i_{s}}.$$

and

$$\cos \phi_{s} = -\frac{\langle (e_{n_{y}} e_{r_{x}} + e_{n_{x}} e_{r_{y}}) \rangle}{\left[e_{n_{y}}^{2} + e_{n_{x}}^{2}\right]^{\frac{1}{2}}}.$$

If  $i_s = 0$  or 180 degrees, the following equation for  $\phi_s$  is used instead:

$$\phi_{s} = \tan^{-1} \left( \frac{e_{r_{y}}}{e_{r_{z}}} \right) - \Omega_{s}. \tag{7}$$

The desired entry parameter covariance matrix is defined by

$$P = E \left[ \begin{array}{c} x & x \\ \end{array} \right]$$
 (8)

where  $\mathbf{x} = (\delta \mathbf{h}, \delta \mathbf{v}, \delta \gamma, \delta \phi_s)$ . Given the covariance matrix

$$P' = E \left[ \dot{x}, \dot{x}, T \right] \tag{9}$$

where  $x' = (\delta r_x, \delta r_y, \delta r_z, \delta v_x, \delta v_y, \delta v_z)$ , the desired covariance matrix can be obtained from

$$P = A P' A^{T}$$
 (10)

where transformation matrix A is defined by

$$\dot{\vec{x}} = \vec{A} \dot{\vec{x}}. \tag{11}$$

The elements  $a_{i,j}$  of the 4x6 matrix A are found by computing the differentials of equations (1), (2), (3), and (6). The results of this process are summarized as:

$$a_{11} = \frac{r_{x}}{r}, \ a_{12} = \frac{r_{y}}{r}, \ a_{13} = \frac{r_{z}}{r}, \ a_{14} = a_{15} = a_{16} = 0$$

$$a_{21} = a_{22} = a_{23} = 0, \ a_{24} = \frac{v_{x}}{v}, \ a_{25} = \frac{v_{y}}{v}, \ a_{26} = \frac{v_{z}}{v}$$

$$a_{31} = \frac{1}{h} \left[ v_{x} - \frac{r_{x}}{r^{2}} (\vec{r} \cdot \vec{v}) \right], \ a_{32} = \frac{1}{h} \left[ v_{y} - \frac{r_{y}}{r^{2}} (\vec{r} \cdot \vec{v}) \right]$$

$$a_{33} = \frac{1}{h} \left[ v_{z} - \frac{r_{z}}{r^{2}} (\vec{r} \cdot \vec{v}) \right], \ a_{34} = \frac{1}{h} \left[ r_{x} - \frac{v_{x}}{v^{2}} (\vec{r} \cdot \vec{v}) \right]$$

$$a_{35} = \frac{1}{h} \left[ r_{y} - \frac{v_{y}}{v^{2}} (\vec{r} \cdot \vec{v}) \right], \ a_{36} = \frac{1}{h} \left[ r_{z} - \frac{v_{z}}{v^{2}} \vec{r} \cdot \vec{v} \right]$$

$$a_{41} = -\frac{r_{x} \tan \phi_{s}}{r_{z} r^{2}}, \ a_{42} = -\frac{r_{y} \tan \phi_{s}}{r_{z} r^{2}}, \ a_{43} = \frac{\tan \phi_{s}}{r_{z}} \left( 1 - \frac{r_{z}}{r^{2}} \right)$$

$$a_{44} = a_{45} = a_{46} = 0$$

where  $h' = |\vec{r} \times \vec{v}|$  and is the orbit angular momentum/unit mass.

Communication angle  $\psi$  is computed from

$$\cos \psi = \frac{\vec{r} \cdot (\vec{r}_e - \vec{r}_p)}{|\vec{r}| \cdot |\vec{r}_e - \vec{r}_p|}$$
 (12)

where

r = spacecraft position relative to planet

 $r_e$  = Earth position relative to sun

r = planet position relative to sun.

ORB SUBROUTINE

TO COMPUTE THE ORBITAL ELEMENTS -- INCLINATION, PURPOSE ! LONGITUDE OF ASCENDING NODE, LONGITUDE OF PERIHELION, ECCENTRICITY, AND LENGTH OF SEMIMAJOR AXIS -- FOR A SPECIFIED PLANET AT A GIVEN TIME.

CALLING SEQUENCE: CALL ORB(IP,D)

JULIAN DATE, EPOCH 1900, OF THE TIME AT D ARGUMENT * WHICH THE ELEMENTS ARE TO BE CALCULATED

> I CODE NUMBER OF PLANET IP

> > =1 SUN

> > > =2 MERCURY

=3 VENUS

EARTH =4

=5 MARS

=6 JUPITER

=7 SATURN

=8 URANUS

=10 PLUTO

=11 MOON

PRINT3 PRINT4 SUBROUTINES SUPPORTED: DATA DATAS PCTM TRAPAR VMP TRAKM TRAKS PSIM GUISS PRNTS3 GUISIM GUIDM GUID MULTAR HELIO LAUNCH LUNTAR MULCON SACCS SUBSOL: PROBE **PROBES** PECEQ PRNTS4 TRAPAR .

SUBROUTINES REQUIRED: NONE

STATEMENT FUNCTION DEFINING A THIRD ORDER LOCAL SYMBOLS: FN1

POLYNOMIAL

STATEMENT FUNCTION DEFINING A FIRST ORDER FN2

POLYNOMIAL

INTERMEDIATE VARIABLE ITEMP

TWICE THE MATHEMATICAL CONSTANT PI PI2

COMMON COMPUTED/USED: ELMNT T

CN EMN SMJR ST TWOPI COMMON USED:

## ORB Analysis

ORB determines the mean orbital elements for any gravitational body at a specified time.

The elements used are semi-major axis a , eccentricity e , inclination i , longitude of the ascending node  $\Omega$  , and longitude of periapsis  $\widetilde{\omega}$ . These elements are referenced to heliocentric ecliptic for the planets or geocentric ecliptic for the moon.

The mean elements are computed from time expansions as follows. Let  $\alpha$  be any of the elements. Then the value of  $\alpha$  at any time t is given by

$$\alpha(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3$$

where the constants  $\alpha_k$  are stored by BIKDAT. These constants are stored into the arrays CN, ST, and EMN for inner planets, outer planets, and the moon respectively. The definitions of these arrays and the values stored are provided in the analysis of the previous subroutine BIKDAT. The element value as computed from the above equation is then returned in the EIMNT array according to the gravitational body code k as

ELMNT(8k-15) = i	k = 1	Sun	= 1	Saturn
$ELMNT(8k-14) = \Omega$	2	Mercury	8	Uranus
$ELMNT(8k-13) = \widetilde{\omega}$	.3	Venus	. 9	Neptune
ELMNT(8k-12) = e	4	Earth		Pluto
ELMNT(8k-10) = a	5	Mars	11	Moon
$EIMNT(8k-9) = \omega$	6	Jupiter		

SUBROUTINE PARTL

PURPOSE COMPUTE PARTIALS OF B DOT T AND B DOT R WITH RESPECT TO SPACECRAFT POSITION AND VELOCITY

CALLING SEQUENCE: CALL PARTL(R, V, B, BDT, BDR, PBT, PBR)

ARGUMENT: B O IMPACT PLANE PARAMETER

BDR O B DOT R

BOT O B DOT T

PBR O PARTIAL OF B DOT R WITH RESPECT TO R AND V

PBT O PARTIAL OF B DOT T WITH RESPECT TO R AND V

R I POSITION OF VEHICLE RELATIVE TO PLANET

V I VELOCITY OF VEHICLE RELATIVE TO PLANET

SUBROUTINES SUPPORTED: GUISS GUID

LOCAL SYMBOLS: H3 INTERMEDIATE VARIABLE

RU INTERMEDIATE VARIABLE

S MAGNITUDE OF VELOCITY

U INTERMEDIATE VARIABLE

U2 SQUARE OF U

UZPVZ INTERMEDIATE VARIABLE

UV INTERMEDIATE VARIABLE

UV3 CUBE OF UV

V2 SQUARE OF MAGNITUDE OF VELOCITY

COMMON USED8 ZERO

PARTL Analysis

PARTL is responsible for the computation of the partials of B.T and B.R with respect to the cartesian components of position and velocity.

Let the state of the spacecraft with respect to the target body at intersection with its sphere of influence be denoted

$$\vec{r} = \begin{bmatrix} x, y, z \end{bmatrix}^T \qquad r = \sqrt{x^2 + y^2 + z^2}$$

$$\vec{v} = \begin{bmatrix} \dot{x}, \dot{y}, \dot{z} \end{bmatrix}^T \qquad v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$
(1)

$$\vec{v} = [\dot{x}, \dot{y}, \dot{z}]^T \qquad v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$
 (2)

Introduce the approach asymptote S and approximate it by the direction of ₹.

$$\hat{S} = \frac{\vec{v}}{v} \tag{3}$$

The B-plane is the plane normal to X containing the center of the target body. Any vector  $\vec{\beta}$  within the B-plane must satisfy therefore

$$\widehat{S} \cdot \overrightarrow{\beta} = 0 \tag{4}$$

The impact parameter vector B is determined by the intersection of the B-plane and the incoming asymptote. The incoming asymptote is given parametrically by

$$\cdot \vec{\sigma} = \vec{r} + \vec{v} t \tag{5}$$

The time at which the asymptote intersects the B-plane may be determined by applying the B-plane condition (4)

$$\hat{\mathbf{s}} \cdot \vec{\mathbf{r}} + \hat{\mathbf{s}} \cdot \vec{\mathbf{v}} \mathbf{t} = 0$$

$$t = -\frac{\vec{r} \cdot \vec{v}}{v^2} \tag{6}$$

Therefore the B-vector is given by

$$\vec{B} = \vec{r} - \frac{\vec{l} \cdot \vec{v}}{2} \vec{v}$$

$$\vec{B} = \left[ x - \alpha \dot{x}, y - \alpha \dot{y}, z - \alpha \dot{z} \right]^{T}$$
 (7)

where

$$\alpha = \frac{\vec{\mathbf{r}} \cdot \vec{\mathbf{v}}}{\mathbf{v}^2} = \frac{\mathbf{x} \cdot \mathbf{x} + \mathbf{y} \cdot \mathbf{y} + \mathbf{z} \cdot \mathbf{z}}{\mathbf{x}^2 + \mathbf{v}^2 + \mathbf{z}^2}$$

Now assuming that the T axis is to lie in the x-y reference plane and the B-plane, it is defined as

$$\widehat{T} = \frac{\widehat{S} \times \widehat{K}}{|\widehat{S} \times \widehat{K}|}$$

$$\widehat{T} = \begin{bmatrix} \dot{y} & -\dot{x} & 0 \end{bmatrix}$$
(8)

where  $u^2 = \dot{x}^2 + \dot{y}^2$ . The  $\hat{R}$  axis is defined by

$$\hat{R} = \hat{S} \times \hat{T}$$

$$\widehat{R} = \frac{1}{uv} \left[ \dot{x}\dot{z}, \dot{y}\dot{z}, -u^2 \right]^{T}$$
(9)

Now combining (7), (8), (9) B.T and B.R may be computed in terms of the state components

$$B \cdot T = \frac{1}{u} (x\dot{y} - \dot{x}y)$$

$$B \cdot R = \frac{1}{uv} \left[ (x\dot{x} + y\dot{y})\dot{z} - u^2 z \right]$$
(10)

where  $u^2 = \dot{x}^2 + \dot{y}^2$ ,  $v^2 = u^2 + \dot{z}^2$ .

The partials may now be computed from differentiation of the above equations.

$$\frac{\partial B \cdot T}{\partial x} = \frac{\dot{y}}{u}$$

$$\frac{\partial B \cdot R}{\partial x} = \frac{\dot{x}\dot{z}}{uv}$$

$$\frac{\partial B \cdot R}{\partial y} = -\frac{\dot{x}}{u}$$

$$\frac{\partial B \cdot R}{\partial y} = \frac{\dot{y}\dot{z}}{uv}$$

$$\frac{\partial B \cdot R}{\partial z} = 0$$

$$\frac{\partial B \cdot R}{\partial z} = -\frac{u}{v}$$

$$\frac{\partial B \cdot R}{\partial x} = -\frac{\dot{y}}{u^3} (x\dot{x} + y\dot{y}) \qquad \frac{\partial B \cdot R}{\partial \dot{x}} = \frac{\dot{z}}{u^3 v^3} \left[ u^2 (v^2 x - \dot{x}z\dot{z}) - \dot{x}(u^2 + v^2) (x\dot{x} + y\dot{y}) \right]$$

$$\frac{\partial B \cdot T}{\partial y} = \frac{\dot{x}}{u^3} (x\dot{x} + y\dot{y}) \qquad \frac{\partial B \cdot R}{\partial \dot{y}} = \frac{\dot{z}}{u^3 v^3} \left[ u^2 (v^2 y - \dot{y}z\dot{z}) - \dot{y}(u^2 + v^2) (x\dot{x} + y\dot{y}) \right]$$

$$\frac{\partial B \cdot T}{\partial z} = 0 \qquad \frac{\partial B \cdot R}{\partial z} = \frac{u}{v^3} (x\dot{x} + y\dot{y} + z\dot{z})$$

SUBROUTINE PCTM

PURPOSE: CONTROL COMPUTATION OF STATE TRANSITION MATRIX USING THE ANALYTICAL PATCHED CONIC TECHNIQUE

CALLING SEQUENCE: CALL PCTM(RI)

ARGUMENT: RI I POSITION AND VELOCITY OF THE VEHICLE AT THE BEGINNING OF THE TIME INTERVAL

SUBROUTINES SUPPORTED: PSIM

SUBROUTINES REQUIRED: CONC2 EPHEH' ORB

LOCAL SYMBOLS: D JULIAN DATE, EPOCH JAN.0, 1900, OF INITIAL

TIME

DELT LENGTH OF TIME INCREMENT IN PROPER UNITS

DUM TEMPORARY STORAGE FOR STATE TRANSITION

MATRIX

GHS GRAVITATIONAL CONSTANT OF GOVERNING BODY

IP CODE OF PLANET

RM DISTANCE FROM SPECIFIED PLANET

RS POSITION OF VEHICLE RELATIVE TO SPECIFIED

PLANET

VS VELOCITY OF VEHICLE RELATIVE TO SPECIFIED

**PLANET** 

COMMON COMPUTED/USED: XP

COMMON COMPUTED: NO PHI ...

COMMON USED: ALNGTH DATEJ DELTM F IBARY

NBOD NB PMASS SPHERE TM

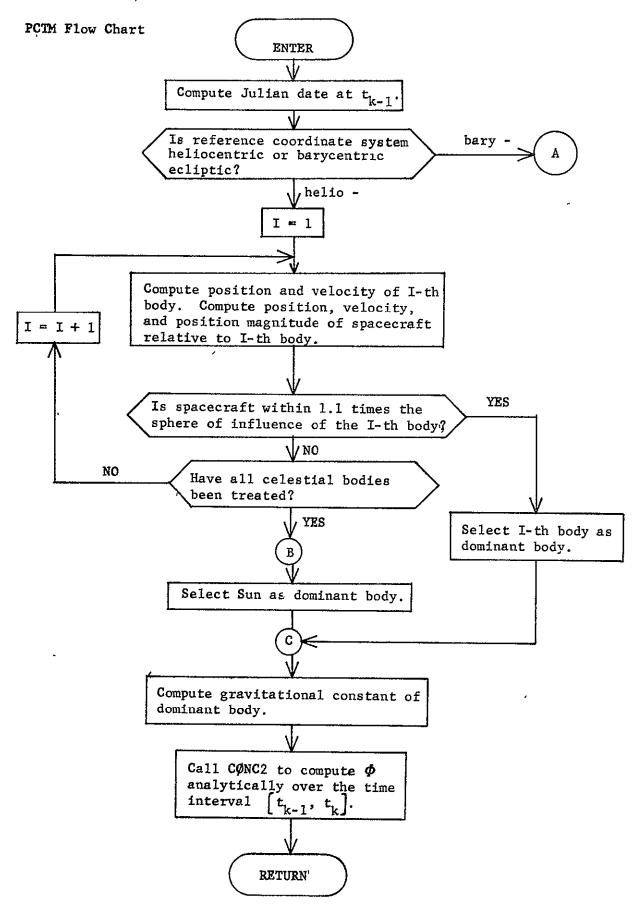
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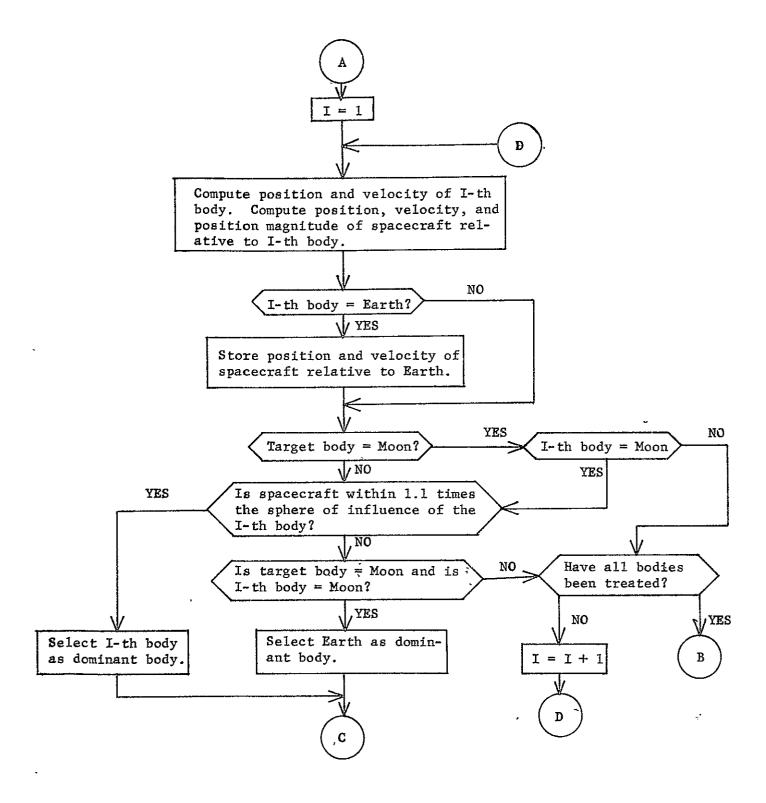
### PCTM Analysis

Subroutine PCTM does not actually compute the state transition matrix  $\phi(t_k, t_{k-1})$  itself; this is accomplished by calling CONC2 from within PCTM. The primary function of PCTM is to determine the dominant body at time  $t_{k-1}$  to be used in the computation of  $\phi(t_k, t_{k-1})$  by means of the analytical patched conic technique.

On interplanetary trajectories we compute the distance separating the spacecraft from each of the celestial bodies included in the analysis. If the distance between the spacecraft and the i-th body is less than or equal to 1.1 times the sphere of influence of the i-th body, the i-th body is selected as the dominant body. Otherwise, the Sun is selected as the dominant body.

On lunar trajectories we compute the distance separating the spacecraft from the Moon. If this distance is less than or equal to 1.1 times the sphere of influence of the Moon, the moon is selected as the dominant body. If not, the Earth is selected as the dominant body.





SUBROUTINE PECEQ

PURPOSE: TO COMPUTE THE MATRIX DEFINING THE TRANSFORMATION FROM PLANET CENTERED ECLIPTIC COORDINATES TO PLANET CENTERED EQUATORIAL COORDINATES AS A FUNCTION OF THE PARTICULAR PLANET AND TIME.

CALLING SEQUENCE: CALL PECEQ (NP,D, ECEQ)

ARGUMENT NP I CODE OF PLANET

I JULIAN DATE, EPOCH 1980, OF REFERENCE TIME

ECEQ(3,3) O COORDINATE TRANSFORMATION MATRIX FROM PLANETOCENTRIC ECLIPTIC TO PLANETOCENTRIC EQUATORIAL COCRDINATES

SUBROUTINES SUPPORTED: TARGET HELIO LAUNCH LUNTAR MULTAR
INSERS TRAPAR VMP DATAS GUISIM
DATA GUIDM EXCUTE SOIPS TPRTRG

SUBROUTINES REQUIRED: EULMX ORB

LOCAL SYMBOLS: AGCAC COORDINATE TRANSFORMATION MATRIX FROM ORBITAL PLANE TO EQUATORIAL COORDINATES FOR MOON

AHCGC COORDINATE TRANSFORMATION MATRIX FROM GEOCENTRIC ECLIPTIC TO GEOCENTRIC EQUATORIAL COORDINATES FOR EARTH - FROM ECLIPTIC TO ORBITAL PLANE COORDINATES FOR MOON

CSDECL COSINE OF DECL

CSEOBL COSINE OF EOBL

CSINM COSINE OF INM

CSNDM COSINE OF NODEM

CSRASC COSINE OF RASC

DECL DECLINATION OF TARGET PLANET POLE

DGTR CONVERSION FACTOR FROM DEGREES TO RADIANS

ED JULIAN DATE , EPOCH 4713 B.C.

EOBL OBLIQUITY OF ECLIPTIC

INM INDEX 413

NODEM INDEX

NORM UNIT VECTOR NORMAL TO TARGET PLANET

ORBITAL PLANE

PBAR CROSS PRODUCT OF POLE AND NORM

PMAG MAGNITUDE OF PBAR

POLE UNIT VECTOR ALIGNED WITH TARGET PLANET

POLAR AXIS

POLMAG MAGNITUDE OF POLE

QBARP CROSS PRODUCT OF POLE AND PBAR

QMAG MAGNITUDE OF QBARP

RASC RIGHT ASCENSION OF TARGET PLANET POLE

SNDECL SINE OF DECL

SNEOBL SINE OF EOBL

SNINM SINE OF INCLINATION INM

SNNDM SINE OF NODE NDM

SNRASC SINE OF RASC

TPRIM BESSELIAN DATE

XI INTERMEDIATE VALUE

XIQ INTERMEDIATE VALUE

XL INTERMEDIATE VALUE

XLQ INTERMEDIATE VALUE

COMMON USED: EMN ONE T ZERO

## PECEQ Analysis

Subroutine PECEQ computes the coordinate transformation maxtrix A from planetocentric ecliptic to planetocentric equatorial coordinates for an arbitrary planet.

The derivation of A for a planet other than the earth or moon will be summarized. Matrix A is defined by

$$A = \begin{bmatrix} \hat{X} & \hat{Y} & \hat{Z} \end{bmatrix}^T \tag{1}$$

where  $\hat{X}$ ,  $\hat{Y}$ , and  $\hat{Z}$  are unit vectors aligned with the planetocentric equatorial coordinate axes and referenced to the planetocentric ecliptic coordinate system. Unit vector  $\hat{Z}$  is aligned with the planet pole. Unit vector  $\hat{X}$  lies along the intersection of the of the planet equatorial and orbital planes and points at the planet vernal equinox. Unit vector  $\hat{Y}$  completes the orthogonal triad and is given by

$$\hat{Y} = \hat{Z} \times \hat{X}. \tag{2}$$

It remains to obtain expressions for  $\hat{X}$  and  $\hat{Z}$ . Let  $\hat{N}$  denote the unit vector normal to the planet orbital plane, and let  $\hat{P}$  denote the unit vector aligned with the planet pole. Then

$$\hat{Z} = \hat{P} \qquad (3)$$

and

$$X = \frac{\hat{P} \times \hat{N}}{|\hat{P} \times \hat{N}|}.$$
 (4)

The unit vector  $\hat{N}$ , referred to the ecliptic coordinate system, is given by

$$\hat{N} = \begin{bmatrix} \sin i \sin \Omega \\ -\sin i \cos \Omega \\ \cos i \end{bmatrix}$$
 (5)

where i and  $\Omega$  are the inclination and longitude of the ascending node, respectively, of the planet orbital plane. The unit vector  $\hat{P}$ , referred to the ecliptic system is given by

$$\stackrel{\cdot}{P} = \begin{bmatrix}
\cos \alpha \cos \delta \\
\cos \varepsilon \sin \alpha \cos \delta + \sin \varepsilon \sin \delta \\
-\sin \varepsilon \sin \alpha \cos \delta + \cos \varepsilon \sin \delta
\end{bmatrix}$$
(6)

where  $\alpha$  and  $\delta$  are the right ascension and declination, respectively, of the planet pole relative to the geocentric equatorial coordinate system, and  $\epsilon$  is the obliquity of the ecliptic. Expressions for  $\alpha$  and  $\delta$  for each planet were obtained from JPL TR 32-1306, Constants and Related Information for Astrodynamic Calculations, 1968, by Melbourne, et al.

For the earth and the moon, the transformation matrix A is written as the produce of two transformation matrices

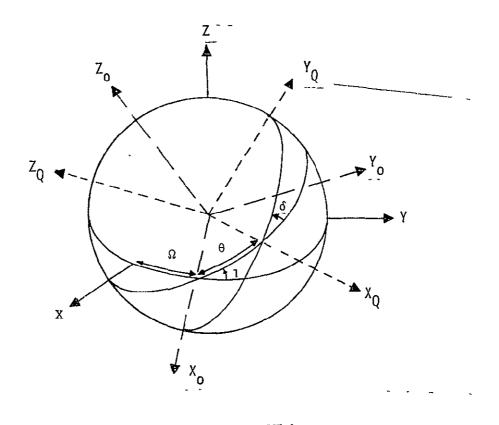
$$A = A_2 A_1. \tag{7}$$

For the earth  $\mathbf{A}_2$  is the identity matrix and  $\mathbf{A}_1$  is given by

$$A_{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon & -\sin \varepsilon \\ 0 & \sin \varepsilon & \cos \varepsilon \end{bmatrix}. \tag{8}$$

The following figure defines the transformations  $A_1$  and  $A_2$ , using the definitions given.

XYZ Ecliptic coordinate axes
X_QY_QZ_Q Orbital plane coordinate axes
½_QY_QZ_Q Moon's equatorial coordinate axes
i Inclination of moon's orbital plane to ecliptic plane
Ω Right ascension of moon's orbital plane to ecliptic plane
δ Inclination of moon's equatorial to orbital plane
β Right ascension of moon's equatorial to orbital plane
θ Right ascension of moon's equatorial to orbital plane

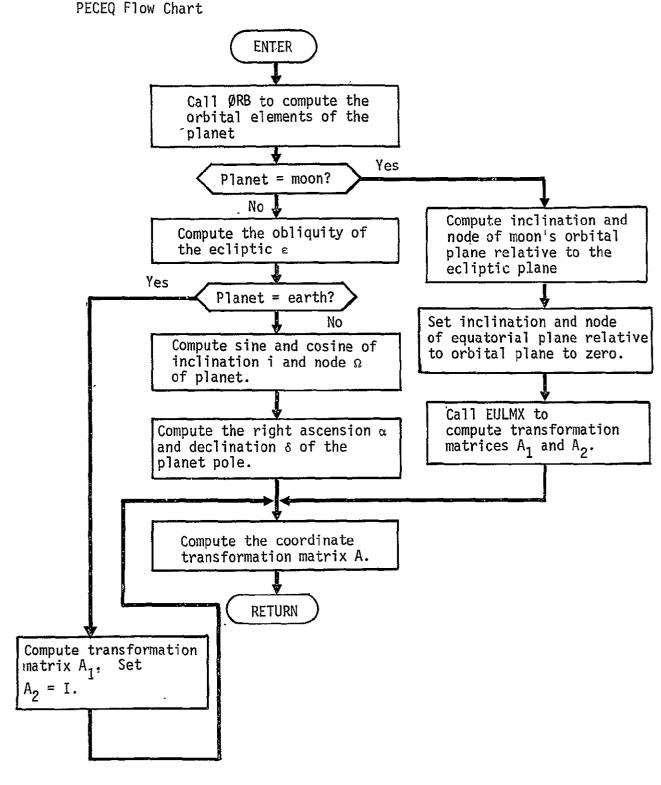


The transformation  $A_1$  from ecliptic to orbital plane coordinates is performed by rotating about the z-axis through an angle  $\Omega$  and then about the resulting x-axis through an angle i. Symbolically,

$$A_1 = (\Omega \text{ about 3, i about 1)}.$$
 (9)

The transformation  $A_2$  from orbital plane to equatorial coordinates can be written similarly as

$$A_2 = (\theta \text{ about } -3, \delta \text{ about } -1).$$
 (10)



414-4

SUBROUTINE PERHEL

PURPOSE: TO PROPAGATE A HELIOCENTRIC TRAJECTORY CONSIDERING THE PERTUBATIONS PRODUCED BY BOTH THE LAUNCH AND TARGET BODIES.

CALLING SEQUENCE: CALL PERHEL(GM, HSI, HLTI, HLTF, QELT, HSF)

ARGUMENT8 GM(3) I GRAVITATIONAL CONSTANTS OF SUN, LAUNCH AND TARGET PLANETS

HSI(6) I HELIOCENTRIC ECLIPTIC SPACECRAFT STATE (INITIAL)

HLTI(2,3) I INITIAL HELIOCENTRIC STATES OF LAUNCH AND TARGET BODIES

HLTF(2,3) I FINAL HELIOCENTRIC STATES OF LAUNCH AND TARGET BODIES

DELT I TIME INTERVAL OF PROPAGATION

HSF(6) O HELIOCENTRIC ECLIPTIC SPACECRAFT STATE (FINAL)

SUBROUTINES SUPPORTED: PULCOV PULSEX

SUBROUTINES REQUIRED: BATCON

LOCAL SYMBOLS COM INTERMEDIATE VARIABLE

DELR RF-RI

PER PERTURBATION IN FINAL STATE

PSF SPACECRAFT POSITION RELATIVE TO PLANET (FINAL)

PSI SPACECRAFT POSITION RELATIVE TO PLANET (INITIAL)

RAV , AVERAGE OF RI AND RF

RA INTERMEDIATE VARIABLE

RF MAGNITUDE OF PSF

RH INTERMEDIATE VARIABLE

RI MAGNITUDE OF PSI

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PERHEL Analysis

PERHEL is responsible for propagating a heliocentric trajectory considering the perturbations produced by both the launch and target bodies. The equations of motion of a body moving under the influence of the sun while perturbed by a smaller mass are

$$\frac{\ddot{r}}{\ddot{r}} = -\frac{\mu_{o}\vec{r}}{r^{3}} - \frac{\mu(\vec{r} - \vec{r}_{m})}{\left|\vec{r} - \vec{r}_{m}\right|^{3}} - \frac{\mu\vec{r}_{m}}{r_{m}^{3}}$$
(1)

where  $\vec{r}$  is the vector radius from the sun to the spacecraft  $\vec{r}$  is the vector radius from the sun to the perturbative mass  $\mu_0$ ,  $\mu$  are the gravitational constants of the sun and mass respectively.

Assuming that the indirect term is small, attention may be directed to the first two terms only. Suppose that  $(\vec{r}_0(t), \vec{v}_0(t))$  satisfy

$$\dot{\vec{r}}_{o} = \vec{v}_{o}$$

$$\dot{\vec{v}}_{o} = \mu_{o} \dot{\vec{r}}_{o}$$
(2)

Then  $(r_o(t), v_o(t))$  are given by the familar equations of conic motion.

A first order corrected solution necessary to account for the direct term force must then satisfy

$$\dot{\vec{r}} = \dot{\vec{r}}_{o} + \dot{\vec{\delta}}\vec{r} = \vec{v}$$

$$\dot{\vec{v}} = \dot{\vec{v}} + \dot{\vec{\delta}}\vec{v} = -\frac{\mu_{o}\vec{r}_{o}}{r_{o}^{3}} - \mu \frac{(\vec{r}_{o} - \vec{r}_{m})}{|\vec{r}_{o} - \vec{r}_{m}|^{3}}$$
(3)

Applying the conditions (2) leads to the equations defining the corrections

$$\frac{\dot{\delta}}{\dot{\delta}} = \frac{\dot{\delta}}{\dot{v}}$$

$$\dot{\delta} = -\mu \frac{\dot{R}}{R^3}$$
(4)

where  $R = r_0(t) - r_m(t)$  is the position vector of the spacecraft with respect to the perturbing mass.

One further assumption enables one to solve in closed form the perturbations produced by the third mass. Generally  $\overline{R}$  (t) and R (t) are nearly linear functions of time. Therefore suppose that the initial and final values of these variables are known to be  $\overline{R}_1$ ,  $\overline{R}_2$ ,  $\overline{R}_1$ ,  $\overline{R}_2$  over the interval  $\Delta t$ .

Introduce the definitions

$$\overline{\Delta R} = \overline{R_2} - \overline{R_1}$$

$$\Delta R = R_2 - R_1 \quad (\text{not } |\overline{\Delta R}|)$$

$$\langle R \rangle = \frac{1}{2} (R_1 + R_2)$$

$$\overline{\Delta R} = \frac{\overline{R_2}}{R_2} - \frac{\overline{R_1}}{R_1}$$
(5)

Then the equation defining the velocity perturbation would be

$$\frac{\hat{\delta} \mathbf{v}}{\hat{\delta} \mathbf{v}} = -\mu \frac{\hat{\mathbf{a}} + \hat{\mathbf{b}} \mathbf{t}}{(\mathbf{c} + \mathbf{d} \mathbf{t})^3} \qquad \hat{\mathbf{a}} = \hat{\mathbf{R}}_1 \qquad \mathbf{c} = \hat{\mathbf{R}}_1$$

$$\hat{\mathbf{b}} = \frac{\hat{\Delta} \hat{\mathbf{R}}}{\hat{\Delta} \mathbf{t}} \qquad \mathbf{d} = \frac{\hat{\Delta} \mathbf{R}}{\hat{\Delta} \mathbf{t}}$$
(6)

It is more convenient however to transform from time t to position magnitude  $\rho$  as the independent variable. This may be done since the position magnitude is assumed to be linear in time with  $\dot{\rho} = \frac{\Delta R}{\Delta t}$ .

According to the assumptions, the position vector  $\overline{R}$  is a linear function of  $\rho$  also

$$\hat{R} = \hat{A} + \hat{B} \rho \tag{7}$$

Since  $\vec{R}$  ( $\rho_1$ ) =  $\vec{R}_1$  and  $\vec{R}$  ( $\rho_2$ ) =  $\vec{R}_2$ , the constants are

$$\overrightarrow{A} = \overrightarrow{R}_1 - \frac{\overrightarrow{\Delta R}}{\overrightarrow{\Delta R}} \quad R_1 = -\frac{R_1 R_2}{\overrightarrow{\Delta R}} \quad \overrightarrow{\Delta R}$$

$$\overrightarrow{B} = \frac{\overrightarrow{\Delta R}}{\overrightarrow{\Delta R}} \tag{8}$$

In terms of  $\rho$  the equations defining the perturbations may be written (with primes indicating differentiation with respect to  $\rho$  )

$$\frac{\delta \mathbf{r'}}{\delta \mathbf{v'}} = \frac{\Delta \mathbf{t}}{\Delta \mathbf{R}} \frac{\delta \mathbf{v}}{\delta \mathbf{v}}$$

$$\frac{\delta \mathbf{v'}}{\delta \mathbf{v'}} = -\frac{\mu \Delta \mathbf{t}}{\Delta \mathbf{R}} \frac{\mathbf{A} + \mathbf{B} \rho}{\rho^3}$$
(9)

These equations are easily integrated to determine the perturbations caused as the spacecraft moves from  $\overline{R}_1$  to  $\overline{R}_2$  relative to the perturbative body:

$$\overline{\delta v} = -\frac{\mu \Delta t}{\Delta R} \int_{R_1}^{\rho} \frac{\overrightarrow{\Delta} + \overrightarrow{B} \rho}{\rho^3} d\rho$$

$$= \frac{\mu \Delta t}{\Delta R} \left[ \frac{\overrightarrow{A}}{2} \left( \frac{1}{\rho^2} - \frac{1}{R_1^2} \right) + \overrightarrow{B} \left( \frac{1}{\rho} - \frac{1}{R_1} \right) \right]$$

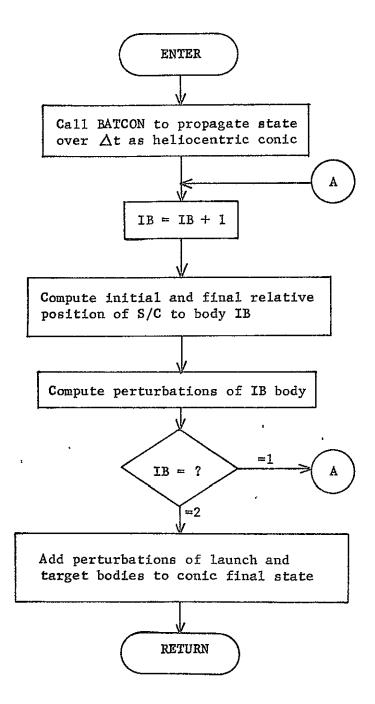
$$= \frac{\mu \Delta t}{R_1 R_2 \Delta R} \left[ \langle R \rangle \stackrel{\wedge}{\Delta R} - \stackrel{\rightarrow}{\Delta R} \right], \quad \rho = R_2 \qquad (10)$$

$$\overline{\delta r} = \frac{\mu \Delta t^2}{\Delta R^2} \int_{R_1}^{R_2} \left[ \frac{\overrightarrow{A}}{2} \left( \frac{1}{\rho^2} - \frac{1}{R_1^2} \right) + \overrightarrow{B} \left( \frac{1}{\rho} - \frac{1}{R_1} \right) \right] d\rho$$

$$= \frac{\mu \Delta t^2}{\Delta R} \left[ \frac{1}{2} \stackrel{\wedge}{R_1} + \stackrel{\rightarrow}{\Delta R} \right] \left( \ln \left( \frac{R_2}{R_1} \right) - \frac{\Delta R}{R_1} \right) \right] d\rho$$
(11)

PERHEL calls BATCON for the generation of the uncorrected heliocentric conic, computes the initial and final positions of the spacecraft relative to each of the launch and target planets, and computes the perturbations based on equations (10) and (11) above.

## PERHEL Flow Chart



SUBROUTINE PLND

PURPOSE: TO COMPUTE COLUMNS OF THE STATE TRANSITION MATRIX
PARTITIONS TXXS, TXW AND TXU ASSOCIATED WITH TARGET PLANET
PLANET EPHEMERIS BIASES INCLUDED IN THE AUGMENTED STATE
VECTOR BY A NUMERICAL DIFFERENCING TECHNIQUE.

CALLING SEQUENCE: CALL PLND(RI, RF)

ARGUMENT: RF I POSITION AND VELOCITY OF THE VEHICLE AT THE END OF THE TIME INTERVAL

RI I POSITION AND VELOCITY OF THE VEHICLE AT THE BEGINNING OF THE TIME INTERVAL

SUBROUTINES SUPPORTED: PSIM

SUBROUTINES REQUIRED: NTM

LOCAL SYMBOLS: DEL TEMPORARY STORAGE FOR TARGET PLANET SEMI-MAJOR AXIS FACTOR USED IN NUMERICAL DIFFERENCING

IC COUNTER FOR VARIABLES AUGMENTED TO STATE VECTOR

IEMN VECTOR OF INDICES FOR ORBITAL ELEMENTS OF THE 'MOON

IEND FLAG FOR VARIABLES AUGMENTED TO STATE VECTOR

NEW VECTOR OF INDICES FOR ORBITAL ELEMENTS OF INNER AND OUTER PLANETS

RPER ALTERED FINAL POSITION AND VELOCITY OF VEHICLE

SAVE1 TEMPORARY STORAGE FOR CONSTANTS OF AUGUMENTED ELEMENTS OF TARGET PLANET

SAVE2 SAME COMMENTS AS SAVE1

SAVE3 SAME COMMENTS AS SAVE1

COMMON COMPÚTED/USED: CN EMN IPRINT SMJR ST

ì

COMMON COMPUTED: TXU TXXS TXW

COMMON USED: ALNGTH DELAXS DELX IAUGDC IAUGW IAUG NTMC NTP

421

;

### PLND Analysis

The nonlinear equations of motion of the spacecraft can be written symbolically as

$$\dot{x} = \dot{f}(\dot{x}, \dot{e}(t), t) \tag{1}$$

where  $\dot{x}$  is the spacecraft position/velocity state and  $\dot{e}(t)$  is a vector composed of the six orbital elements a, e, i,  $\Omega$ ,  $\omega$ , and M of the target planet. The motion of the spacecraft is, of course, dependent on the positions of other celestial bodies, but this dependency need not be explicitly stated for the purposes of this analysis.

Suppose we wish to use numerical differencing to compute those columns of  $\theta_{xx}$ ,  $\theta_{xu}$ , and  $\theta_{xw}$  associated with target planet ephemeris biases included in the augmented state vector over the time interval  $[t_{k-1}, t_k]$ . Let  $\dot{\theta}_j(t_k, t_{k-1})$  represent the column associated with the j-th ephermeris bias. We assume we have available the nominal states  $\dot{x}^*(t_{k-1})$  and  $\dot{x}^*(t_k)$ , which, of course, were obtained by numerically solving equation (1) using nominal  $\dot{e}(t)$ . To obtain  $\dot{\theta}_j(t_k, t_{k-1})$ , we increment the j-th orbital element by the pertinent numerical differencing factor  $\Delta e_j$  and numerically integrate equation (1) over the interval  $[t_{k-1}, t_k]$  to obtain the new spacecraft state  $\dot{x}_j(t_k)$ , where the j-subscript on the spacecraft state indicates that it was obtained by incrementing the j-th orbital element. Then

$$\dot{\theta}_{j}(t_{k}, t_{k-1}) = \frac{\dot{x}_{j}(t_{k}) - \dot{x}^{*}(t_{k})}{\Delta e_{i}}$$
 (2)

Ephemeris biases are defined as biases of the basic set of orbital elements a, e, i,  $\Omega$ ,  $\omega$ , and M. However the ephemeris constants of a, e, i,  $\Omega$ ,  $\widetilde{\omega}$ , and M for the planets and a, e, i,  $\Omega$ ,  $\widetilde{\omega}$ , and L for the moon are stored in the program. Thus to increment certain of the basic elements, we must increment certain combinations of the stored ephemeris constants.

The elements  $\omega$  and M are related to the longitude of perihelion  $\tilde{\omega}$  and the mean longitude L as follows:

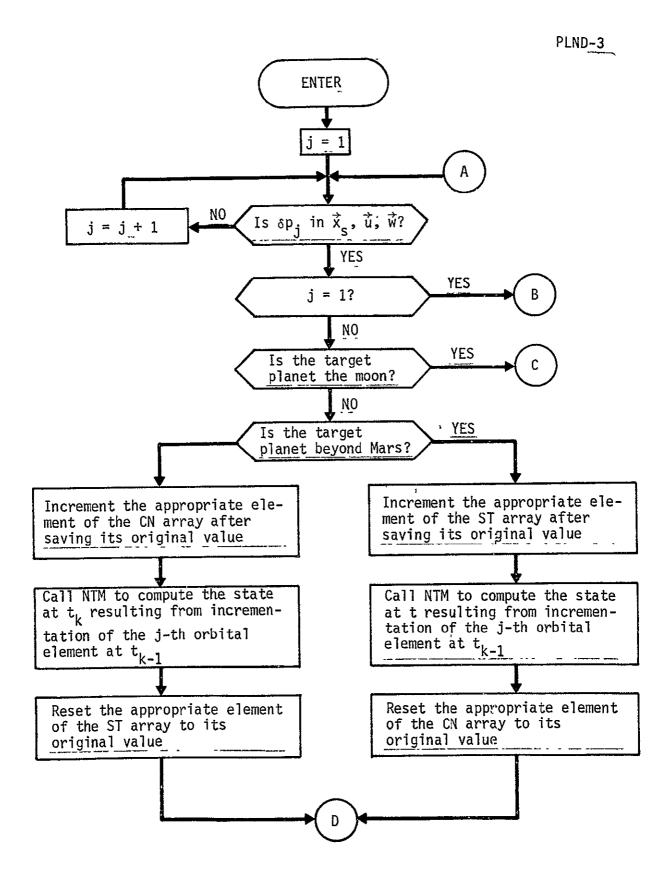
$$\omega = \tilde{\omega} - \Omega$$

$$M = L - \tilde{\omega}$$
.

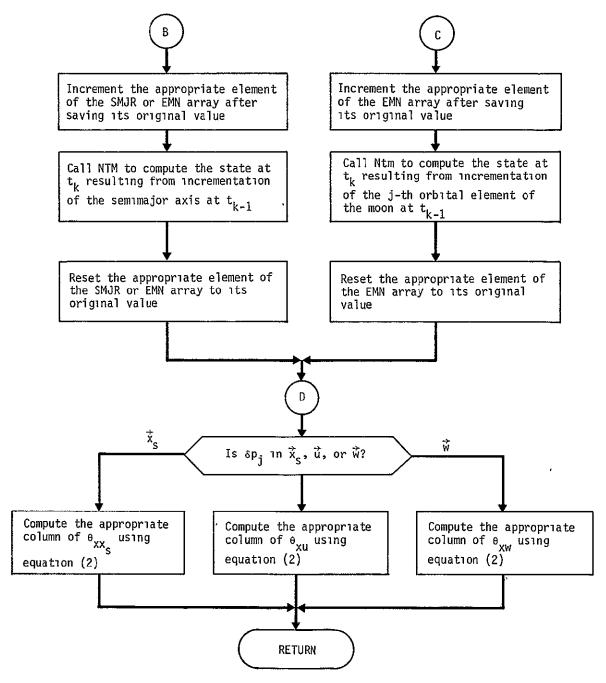
Thus, to increment  $\Omega$  by  $\Delta\Omega$  without changing the other five basic elements requires that we also increment  $\tilde{\omega}$  by  $\Delta\Omega$  for the case of a planet, and both  $\tilde{\omega}$  and L by  $\Delta\Omega$  for the case of the moon. To increment  $\omega$  by  $\Delta\omega$  we simply increment  $\tilde{\omega}$  by  $\Delta\omega$  for a planet, while for the moon we must increment both  $\tilde{\omega}$  and L by  $\Delta\omega$ . To increment M by  $\Delta M$  for the moon we simply increment L by  $\Delta M$ .

In the PLND flow chart we employ the following definition:

$$P_{j} = \begin{cases} a & j = 1 \\ e & 2 \\ i & 3 \\ \Omega & 4 \\ \omega & 5 \\ M & 6 \end{cases}.$$



PLND Flow Chart



IIGP

SUBROUTINE POICOM

PURPOSE COMPUTE PROBABILITY OF IMPACT

CALLING SEQUENCE: CALL POICOM(XXXX,DET)

ARGUMENT: XXXX I AIMPOINT IN THE IMPACT PLANE VECTOR

DET I DETERMINANT OF LAMBDA MATRIX

SUBROUTINES SUPPORTED: BIAIM

SUBROUTINES REQUIRED: MATIN

LOCAL SYMBOLS: PMQM P+ MQM TRANSPOSE

SAVE INTERMEDIATE VARIABLE

SUM INTERMEDIATE VARIABLE

W ADA* PMQM* ADA TRANSPOSE

COMMON COMPUTED/USED: IEND POI PSTAR XLAM

COMMON USED: ADA A CR EXEC

ONE PI PP TWO XLAMI

ZERO

## POICOM Analysis

Subroutine POICOM computes the target condition covariance  $W_{j}^{\dagger}$  after a guidance correction, the projection of  $W_{j}^{\dagger}$  into the impact plane, and the probability of impact of the spacecraft with the target planet.

The target condition covariance matrix  $W_{j}^{+}$  is defined as

$$W_{j}^{+} = \eta_{j} (P_{k_{j}}^{-} + M\widetilde{Q}_{j}M^{T}) \eta_{j}^{T}$$

where  $\eta_j$  is the variation matrix for the appropriate guidance policy,  $P_k^j$  is the knowledge covariance prior to the guidance correction,  $\tilde{Q}_j$  is the execution error covariance, and M is defined as the following 6 x 3 matrix:

$$M = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$$

Before the probability of impact can be computed, it is necessary to compute the projection  $\bigwedge_j$  of  $W_j$  into the impact plane. The covariance  $\bigwedge_j$  is computed as follows for each of the three available midcourse guidance policies.

a. Fixed-time-of-arrival:

$$\Lambda_{j} = A W_{j}^{+} A^{T}$$

where transformation A is defined in the subroutine BIAIM analysis.

b. Two-variable B-plane:

c. Three-variable B-plane:

$$\Lambda_{j} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} W_{j}^{+} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Assuming the probability density function associated with  $\bigwedge_j$  is Gaussian and nearly constant over the target planet capture area permits us to compute the probability of impact using the equation

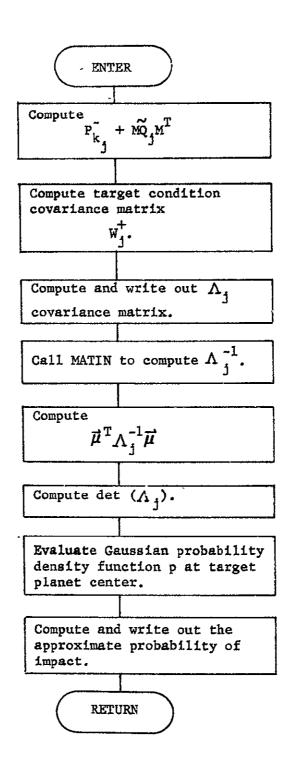
POI = 
$$\pi R_c^2 p$$

where R is the target planet capture radius and p is the Gaussian probability density function evaluated at the target planet center and given by

$$p = \frac{1}{2\pi|\Lambda_{j}|^{1/2}} \exp\left[-\frac{1}{2}\vec{\mu}^{T} \Lambda_{j}^{-1} \vec{\mu}\right]$$

where  $\overrightarrow{\mu}$  is the aimpoint in the impact plane.

## POICOM Flow Chart



PROGRAM PRED

PURPOSE CONTROL EXECUTION OF A PREDICTION EVENT IN THE ERROR ANALYSIS PROGRAM

SUBROUTINES SUPPORTED: ERRANN

SUBROUTINES REQUIRED: CORREL DYNO HYELS JACOBI GNAVM
NTM PSIM STMPR SAVMAT EIGHY
MEAN GPRINT ORB EPHEM BEPS

LOCAL SYMBOLS: BLAB LABEL

BPS B-PLANE PARAMETER COVARIANCE

CXSU1 STORAGE FOR CXSU COVARIANCE ARRAY

CXSV1 STORAGE FOR CXSV COVARIANCE ARRAY

CXU1 STORAGE FOR CXU COVARIANCE ARRAY

CXV1 STORAGE FOR CXV COVARIANCE ARRAY

CXXS1 STORAGE FOR CXXS COVARIANCE ARRAY

D JULIAN DATE PREDICTED TO

DUMM INTERMEDIATE VARIABLE

DUM2 ARRAY OF EIGENVECTORS

DUM3 ARRAY OF EIGENVALUES

DUM B DOT T AND B DOT R COVARIANCE MATRIX

EGVCT ARRAY OF EIGENVECTORS

EGVL ARRAY OF EIGENVALUES,

EXSTS TEMPORARY STORAGE FOR EXST

EXTS TEMPORARY STORAGE FOR EXT

EXTIJ INTERMEDIATE STORAGE

ICODE INTERNAL CONTROL FLAG

IGO INTERNAL FLAG

IPR STORAGE FOR IPRINT

OUT ARRAY OF STANDARD DEVIATIONS AND CORRELATION COEFFICIENTS

		PEIG	MATRIX WHOSE HYPERELLIPSOID IS TO BE COMPUTED				O BE
		PSAVE	TEMPORAR	Y STORAG	E FOR P		
		PSUBB	B-PLANE	PARAMETE	R UNCERT	AINTIES	•
		PS1	STORAGE	FOR PS C	OVARIANC	E ARRAY	
		P1	STORAGE	FOR P C	OVARIANC	E ARRAY	
		RF	NOMINAL	SPACECRA	FT STATE	AT TIME	TPT
		RFMAG	VELOCITY	MAGNITU	DE		
		ROW	INTERMED	DIATE VAR	IABLE		
		RPC	PLANETO	ENTRIC S	STATE		
		RPCSV	TEMPORARY STORAGE FOR RPC				
		SQP	INTERMED	DIATE VAR	RIABLE		
		SUM THETA	INTERMED ANGLE	DIATE VAF	RIABLE		
		TPT	TIME TO	WHICH PE	REDICTION	IS TO E	BE MADE
		TRANSG			N MATRIX TY TO B-F		CHANGE IN
		VEIG	,INTERME	DIATE VEC	TOR		
COMMON	COMPUTED	/USED#		CXSV NPE GCXW	C XU PS	C XV	CXXS
СОММОИ	COMPUTED	:	DELTM	TRTM1	xı		
COMMON	USĘD:		EM ISTMC NTMC GP, U0	FOP NDIM1 ONE EXT VO	FOV NDIM2 Q EXST XF	IEIG NOIM3 TPT2	IHYP1 NGE TSOI1

## PRED Analysis

Subroutine PRED executes a prediction event in the error analysis/generalized covariance analysis program. Subroutine PRED differs from subroutine PRESIM in three respects. First, the propagated knowledge covariance matrix partitions are based on the (most recent) targeted nominal, rather than on the most recent nominal as in PRESIM. Second, estimated position/velocity deviations are not propagated in PRED since estimates are processed only in the simulation program and not in the error analysis program. And third, subroutine PRED treats both assumed and actual knowledge covariance matrix partitions, whereas subroutine PRESIM treats only assumed knowledge covariance matrix partitions. Subroutine PRED uses the propagation equations in subroutine GNAVM to propagate both assumed and actual covariances.

A flow chart for PRED is not presented here because of its similarity to the PRESIM flow chart (see PRESIM for further details).

SUBROUTINE PRELIM

PURPOSE TO PERFORM THE PRELIMINARY WORK ASSOCIATED WITH THE NOMNAL PROGRAM INCLUDING THE READING OF THE INPUT DATA, INITIALIZATION OF CONSTANTS, AND THE COMPUTATION OF A ZERO ITERATE IF REQUIRED

CALLING SEQUENCES -CALL PRELIM

SUBROUTINES SUPPORTED: NOMNAL

SUBROUTINES REQUIRED:

LOCAL SYMBOLS* DF JULIAN DATE CORRESPONDING TO KALF ARRAY

DI JULIAN DATE CORRESPONDING TO KALI ARRAY

TIME

CPWMS

GS ARRAY OF VALUES OF SECONDS CORRESPONDING TO KALG ARRAY

ZERIT

I INDEX

J INDEX

KALF CALENDAR DATE OF FINAL TRAJECTORY TIME

KALG ARRAY OF CALENDAR DATES OF GUIDANCE EVENTS

KALI CALENDAR DATE OF INITIAL TRAJECTORY TIME

KALT ARRAY OF CALENDAR DATES OF TARGET TIMES

KEY LOCAL VARIABLE USED TO COMPLETE INFORMATION IN THE ARRAY

SF SECONDS OF FINAL TRAJECTORY TIME

SI SECONDS OF INITIAL TRAJECTORY TIME

TS SECONDS OF TARGET TIMES CORRESPONDING TO KALT ARRAY

COMMON COMBUTED/USED:

AC	ALNGTH	DG	D1	FI
IBADS	IBARY	ICOORD	ÎFINT	IPRE
ISTART	IZERO	KGYD	KMXQ	KOAST
KTIM	KTYP	LTARG	LVLS	MAT
MAXB	MDL	NBOD	NB	NCPR
NOGYD	NCIT	NPAR	ONE	PERV
PHILS	PSI1	PSI2	RIN	RPRAT
RP	SIGMAL	SPHFAC	SSS	TAR
THEDOT	THELS	TIMG	TIM1	TIM2
TIN	TMPR	TM	ZDAT	<del>-</del>

COMMON COMPUTED 8	DINTG IEPHEM NINETY TRIM	EIGHT IPRINT RAD TWO	FIVE KSICA TEN ZERO	FOUR KUR THREE	HALF NBODYI TMU
COMMON USEDS	ACKT	DELV	DT	DVMAX	IBAST
	KTAR	LEVELS	MAXBAD	NITS	NLP
	NTP	PHI	PMASS	TIMS	TOL

#### PRELIM Analysis

PRELIM is responsible for the preliminary work required by NOMNAL including the initialization of variables, the reading of input, and the computation of zero iterate values for initial time, position, and velocity if necessary.

On the first call to PRELIM, PRELIM presets constants to be used on the entire series of runs. These constants include the double precision numbers and the launch profile parameters. On subsequent calls these variables are not reset.

PRELIM then presets constants for individual runs. These constants presently include most of the guidance event parameters. The user may easily change the two sets of constants for his particular needs.

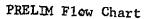
PRELIM then accepts the input data. It reads data in the NAMELIST format.

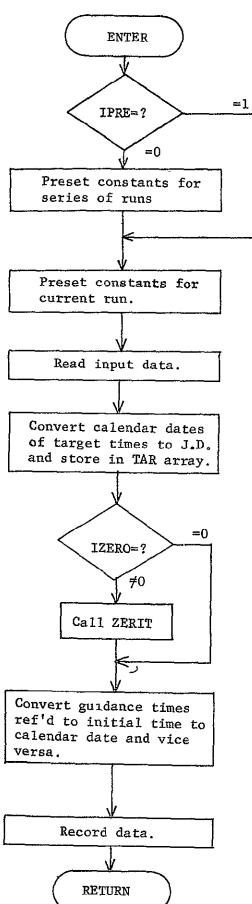
Target times must be read in as calendar dates. PRELIM next converts these to Julian date referenced 1900 and stores the converted values in the TAR array.

If the flag IZERO is nonzero, ZERIT is called for the computation of the zero iterate values of initial time, position, and velocity. ZERIT in turn calls HELIO for interplanetary trajectories and LUNA for lunar trajectories.

PRELIM then converts guidance event times referenced to initial time to calendar data and converts times read in as calendar dates to times referenced to the initial time. When the latter is done, it sets-KTIM to acknowledge that conversion.

Finally PRELIM records all pertinent data.





SUBROUTINE PREPUL

PURPOSE: TO PERFORM THE PRELIMINARY COMPUTATIONS REQUIRED FOR THE PULSING ARC MODEL.

CALLING SEQUENCE: CALL PREPUL(RIN, DELTAY, D1)

ARGUMENTS* RIN(6) I INERTIAL STATE OF SPACECRAFT AT NOMINAL TIME OF CORRECTION

DELTAV(3) I TOTAL VELOCITY INCREMENT TO BE ADDED

D1 I JULIAN DATE OF NOMINAL TIME OF CORRECTION

SUBROUTINES SUPPORTED: EXCUTE EXCUTS

SUBROUTINES REQUIRED: TIME

LOCAL SYMBOLS: A SEMIMAJOR AXIS

C INTERMEDIATE VARIABLE IN F AND G SERIES

DB JULIAN DATE AT BEGINNING OF PULSING ARC

DELVM MAGNITUDE OF TOTAL IMPULSIVE CORRECTION

DE JULIAN DATE AT END OF PULSING ARC

DVFM MAGNITUDE OF FINAL PULSE OF—SEQUENCE

DVIM MAGNITUDE OF TYPICAL PULSE OF SEQUENCE

D INTERMEDIATE VARIABLE IN F AND G SERIES

G GRAVITATIONAL CONSTANT OF BODY UNDER CONSIDERATION

ID CALENDAR DATE OF CRITICAL TIMES FOR OUTPUT

MAXP MAXIMUM NUMBER OF PULSES ALLOWED

NDX ARRAY OF CODES OF LAUNCH AND TARGET BODIES

NX INDEX OF GIVEN PLANET COORDINATES IN F-ARRAY

RD TIME DERIVATIVE OF RADIUS MAGNITUDE OF PLANET

RR MAGNITUDE OF RADIUS

SD SECONDS OF CRITICAL TIMES FOR OUTPUT

	vv	SPEED 0	F PLANET			
COMMON	COMPUTED/USED:	B GS	DVF NPUL	DV.I Pult	FS RK	GG VK
COHMON	USED#	ALNGTH F NTP TH	DTI NBOD PMASS THO	DUR NB PULMAG V	FIVE NINETY PULHAS	FOUR NLP THREE

#### PREPUL Analysis

PREPUL is responsible for performing the preliminary computations required for the pulsing arc model.

PREPUL first determines the nominal pulsing arc. Let the following definitions be made:

T magnitude of pulsing engine thrust

m . nominal mass of spacecraft

Δt duration of single pulse

Δt, time interval between pulses

 $\overline{\Delta v}$  total velocity increment to be added

The velocity increment imparted by a single pulse is

$$\Delta v_{i} = \frac{T \Delta t}{m} \tag{1}$$

The number of pulses required is then

$$N_{p} = \left[\frac{\Delta v}{\Delta v_{i}}\right] + 1 \tag{2}$$

where  $[\cdot]$  denotes the greatest integer function. The magnitude of the final pulse must be set to

$$\Delta v_f = \Delta v - (N_p - 1) \Delta v_f$$
 (3)

The vector nominal pulse and final pulse are therefore given by

$$\overline{\Delta v}_{i} = \Delta v_{i} \frac{\overline{\Delta v}}{\Delta v}$$

$$\overline{\Delta v}_{f} = \Delta v_{f} \frac{\overline{\Delta v}}{\Delta v}$$
(4)

The duration of the pulsing arc is then given by

$$\Delta T = (N_p - 1) \Delta t_i$$
 (5)

Later computations require time histories of the position vectors of the launch and target bodies. An efficient means of obtaining this involves the f and g series. Given the state  $\vec{r}_0$ ,  $\vec{v}_0$  of body moving in a conic section about a central body of gravitational constant  $\mu$ , the position vector as a function of t measured from the initial time is given by

$$\vec{r}(t) = f(t) \vec{r}_0 + g(t) \vec{v}_0$$
 (6)

where

$$f(t) = \sum_{k=0}^{n} f_k t^k$$
  $g(t) = \sum_{k=1}^{n} g_k t^k$  (7)

The constants fk, gare computed in PREPUL as

$$f_{0} = 1$$

$$f_{1} = 0$$

$$f_{2} = \frac{-\mu}{2r_{0}^{3}}$$

$$f_{3} = \frac{\mu\dot{r}_{0}}{2r_{0}^{4}}$$

$$f_{4} = \frac{\mu^{2}}{24r_{0}^{6}} \quad (4 - 15\frac{r_{0}\dot{r}_{0}^{2}}{\mu} - 3\frac{r_{0}}{a})$$

$$f_{5} = \frac{-\mu^{2}\dot{r}_{0}}{8r_{0}^{7}} \quad (4 - \frac{7r_{0}\dot{r}_{0}^{2}}{\mu} - 3\frac{r_{0}}{a})$$

$$f_{6} = \frac{\mu^{3}}{720r_{0}^{9}} \left[ -70 + 114\frac{r_{0}}{a} + 840\frac{r_{0}\dot{r}_{0}^{2}}{\mu} - 630\frac{r_{0}^{2}\dot{r}_{0}^{2}}{\mu a} - 450\left(\frac{r_{0}\dot{r}_{0}^{2}}{\mu}\right)^{2} - 45\frac{r_{0}^{2}}{a^{2}} \right]$$

$$g_{1} = 1$$

$$g_{2} = 0$$

$$g_{3} = \frac{1}{3}f_{2}$$

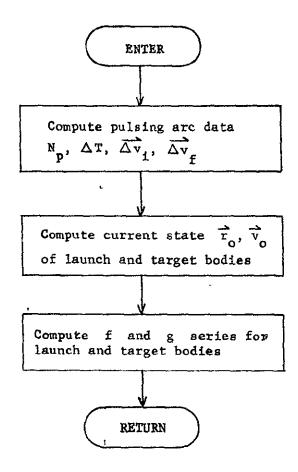
$$g_{4} = \frac{1}{2}f_{3}$$

$$g_{5} = \frac{3}{5}f_{4} - \frac{1}{15}f_{2}^{2}$$

$$g_{6} = \frac{2}{3}f_{5} - \frac{1}{6}f_{2}f_{3}$$

Reference: Baker, R. M. L. and Makemson, M. W., An Introduction to Astrodynamics, Academic Press, New York, 1967.

### FREPUL Flow Chart



PROGRAM PRESIM

PURPOSE CONTROL EXECUTION OF A PREDICTION EVENT IN THE SIMULATION PROGRAM

SUBROUTINES SUPPORTED: SIMULL

SUBROUTINES REQUIPED: CORREL DYNOS HYELS JACOBI NAVM NTMS PSIM STMPR EIGHY ORB

EPHEM BEPS

LOCAL SYMBOLS: BLAB LABEL

BPS B-PLANE PARAMETER COVARIANCE

CXSU1 STORAGE FOR CXSU COVARIANCE ARRAY

CXSV1 STORAGE FOR CXSV COVARIANCE ARRAY

CXU1 STORAGE FOR CXU COVARIANCE ARRAY

CXV1 STORAGE FOR CXV GOVARIANCE ARRAY

CXXS1 STORAGE FOR CXXS COVARIANCE ARRAY

D JULIAN DATE PREDICTED TO

DM B DOT T AND B DOT R COVARIANCE MATRIX

DM2 ARRAY OF EIGENVECTORS

DM3 ARRAY OF EIGENVALUES

DUMM INTERMEDIATE VARIABLE

EGVCT ARRAY OF EIGENVECTORS

EGVL ARRAY OF EIGENVALUES

GMU GRAVITATIONAL CONSTANT OF TARGET PLANET

IPR STORAGE FOR IPRINT

OUT ARRAY OF STANDARD DEVIATIONS AND CORRELATION COEFFICIENTS

PEIG MATRIX WHOSE HYPERELLIPSOID IS TO BE COMPUTED

PSUBB B-PLANE PARAMETER UNCERTAINTIES

PS1 STORAGE FOR PS COVARIANCE ARRAY

Pi	STORAGE FOR P COVARIANCE ARRAY
RFMAG	VELOCITY MAGNITUDE
RF1	MOST RECENT NOMINAL SPACECRAFT STATE AT TIME TPT2
ROW	INTERMEDIATE VARIABLE
RPC	PLANETOCENTRIC STATE
SQP	INTERMEDIATE VARIABLE
SUM	INTERMEDIATE VARIABLE
THETA	ANGLE
TPT	TIME TO WHICH PREDICTION IS TO BE MADE
TRANSG	STATE TRANSITION MATRIX RELATING CHANGE IN POSITION/VELOCITY TO 8-PLANE PARAMETERS
VEIG	MATRIX TO BE DIAGONALIZED
COMMON COMPUTED/USED!	CXSU CXSV CXU CXV CXXS ICODE IPRINT NPE PS P RI1
COMMON COMPUTED:	DELTM RI TRTM1 XI1 XI
COMMON USED:	ADEVXS ADEVX EDEVXS EDEVX EM FOP FOV IEIG IHYP1 ISTMC NDIM1 NDIM2 NDIM3 NGE NTMC ONE PHI Q TEVN TPT2 TSOI1 TXXS U0 V0 W XF1 XF XSL ZERO

### PRESIM Analysis

Subroutine PRESIM executes a prediction event in the simulation program SIMUL. At a prediction event, the knowledge covariance partitions, and the estimated position/velocity deviations from the most recent nominal trajectory are propagated forward to  $t_{\rm p}$ ,

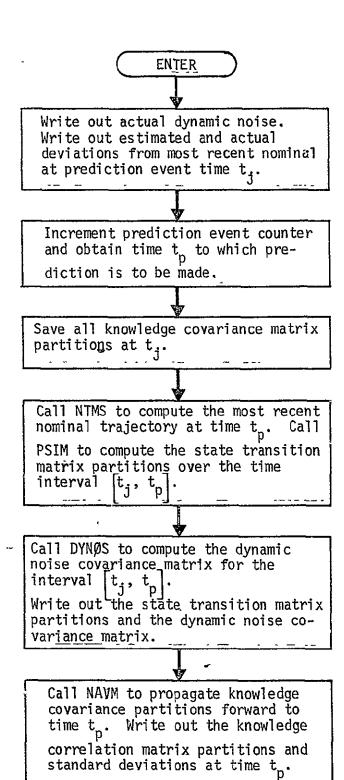
the time to which the prediction is to be made. The knowledge covariance partitions are propagated using the prediction equations found in the NAVM Analysis section. The estimate is propagated using the equation

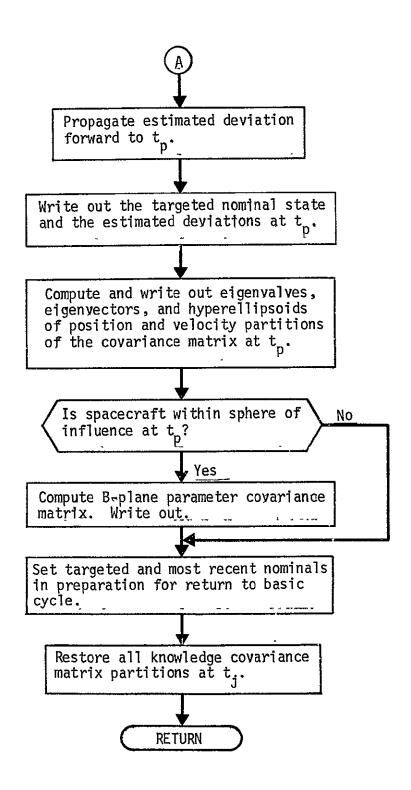
$$\delta \tilde{\hat{x}}_{p} = \Phi (t_{p}, t) \delta \tilde{\hat{x}}_{j} + \theta_{xx_{s}} (t_{p}, t_{j}) \delta \tilde{\hat{x}}_{s_{j}}$$

where  $\Phi$  and  $\theta_{\mbox{\tiny KK}}$  are the state transition matrix partitions over the time interval [t_j, t_p].

The position and velocity partitions of the propagated knowledge covariance matrix are diagonalized at time t and the eigenvalues, eigenvectors, and hyperellipsoids are computed.

If t occurs within the target planet sphere of influence, the Cartesian position/velocity covariance matrix is transformed to a B-plane parameter covariance matrix. The B-plane parameters are B·T, B·R, time-of-flight, S·R, S·T, and  $C_3$ .





SUBROUTINE PRINT

PURPOSE: TO PRINT THE VIRTUAL MASS INFORMATION SPECIFIED.

CALLING SEQUENCE: CALL PRINT

SUBROUTINES SUPPORTED: VMP

SUBROUTINES REQUIRED: TIME TRAPAR NEWPGE SPACE

LOCAL SYMBOLS: D INTERMEDIATE VARIABLE USED FOR PRINTOUT

IDAY DAY OF CALENDAR DATE OF CURRENT TIME

IHR HOUR OF CALENDAR DATE OF CURRENT TIME

INCMNT CURRENT TOTAL INCREMENTS FOR PRINTOUT

IP CODE OF I-TH PLANET FOR PRINTOUT PURPOSES

IYR YEAR OF CALENDAR DATE OF CURRENT TIME

MIN MINUTES OF CALENDAR DATE OF CURRENT TIME

MO MONTH OF CALENDAR DATE OF CURRENT TIME

RP RADIUS OF I-TH PLANET RELATIVE TO INERTIAL

FRAME

RS RADIUS OF VEHICLE RELATIVE TO INERTIAL

FRAME

RV RADIUS OF VIRTUAL MASS RELATIVE TO

INERTIAL FRAME

SEC SECONDS OF CALENDAR DATE OF CURRENT TIME

TMP POSITION AND VELOCITY OF VIRTUAL MASS

RELATIVE TO PLANETS

VMR MAGNITUDE OF VELOCITY OF VEHICLE RELATIVE

TO VIRTUAL MASS

VP MAGNITUDE OF VELOCITY OF I-TH PLANET FOR

PRINTOUT PURPOSES

VS MAGNITUDE OF VELOCITY OF VEHICLE RELATIVE

TO INERTIAL FRAME

VSP MAGNITUDE OF VELOCITY OF VEHICLE RELATIVE

TO I-TH PLANET FOR PRINTOUT PURPOSES

VV MAGNITUDE OF VELOCITY OF VIRTUAL MASS

0.11

RELATIVE TO INERTIAL FRAME

COMMON COMPUTED/USED8 F V

COMMON USED: INCMNT IPRT NBODYI NBODY NO

PLANET ZERO

SUBROUTINE PRINTS

PURPOSE: TO PRINT THE PERTINENT INFORMATION AT THE END OF EACH MEASUREMENT.

CALLING SEQUENCE: CALL PRINT3

SUBROUTINES SUPPORTED # ERRANN PROBE

SUBROUTINES REQUIRED: CORREL EPHEM ORB STMPR TRAPAR

LOCAL SYMBOLS: D INTERMEDIATE DATE

IA STATION NUMBER

D3 JULIAN DATE OF INITIAL TIME

D4 JULIAN DATE OF FINAL TIME

IDAY CALENDAR DAY OF FINAL TIME

IHR CALENDAR HOUR OF FINAL TIME

IMIN CALENDAR MINUTE OF FINAL TIME

IMO CALENDAR MONTH OF FINAL TIME

ITEMP INTERMEDIATE VARIABLE

IYR CALENDAR YEAR OF FINAL TIME

LDAY CALENDAR DAY OF INITIAL TIME

LHR CALENDAR HOUR OF INITIAL TIME

LMIN CALENDAR MINUTES OF INITIAL TIME

LMO CALENDAR MONTH OF INITIAL TIME

LYR CALENDAR YEAR OF INITIAL TIME

M NUMBER OF MEASUREMENT

RME GEOCENTRIC RADIUS OF VEHICLE

RMP DISTANCE OF VEHICLE FROM TARGET PLANET

SECI CALENDAR SECONDS OF FINAL TIME

SECL CALENDAR SECONDS OF INITIAL TIME

TRTM2 TRAJECTORY TIME AT END OF INTERVAL

VME	MAGNITUE TO EARTH		LOCITY	OF VEHICL	E RELATIVE TO
V MP		DE OF VE		OF VEHICL	E RELATIVE TO
COMMON COMPUTED/USED:	NO	RE	RTP	XP	
COMMON USED:	AK CXSU CXVP DELTM IPROB NDIM1 PSP S XF XV AN	ALNGTH CXSVP CXV F IPRT NDIM2 PS TM XI ALPHA	AL CXSV CXXSP G MCNTR NDIM3 P TRTM1 XLAB BETA	AM CXUP CXXS H N80D NTP Q U0 XSL NDIM4	CXSUP CXU DATEJ IBARY NB PP R V0 XU XIG

SUBROUTINE PRINT4

PURPOSE: THIS SUBROUTINE PRINTS RELEVANT DATA AT THE END OF EACH MEASUREMENT IN THE SIMULATION MODE

CALLING SEQUENCE: CALL PRINT4 (MMCODE, NR)

ARGUMENT: MMCODE I MEASUREMENT CODE

NR I NUMBER OF ROWS IN THE OBSERVATION MATRIX

SUBROUTINES SUPPORTED: SIMULL

SUBROUTINES REQUIRED: CORREL EPHEN ORB STMPR SUB1

TRAPAR

LOCAL SYMBOLS: ADON ACTUAL STATE DEVIATION FROM TARGETED

NOMINAL TRAJECTORY

AODI ACTUAL ORBIT ESTIMATION ERROR

D INTERMEDIATE DATE

EDON ESTIMATED STATE DEVIATION FROM TARGETED

NOMINAL TRAJECTORY

IA STATION NUMBER

IB STAR PLANET ANGLE NUMBER

M MEASURMENT NUMBER

ROH ARRAY OF CORRELATION COEFFICIENTS

SQP VECTOR OF STANDARD DEVIATIONS

TRIMZ TRAJECTORY TIME AT END OF INTERVAL

XE1 POSITION AND VELOCITY OF EARTH AT TRIM1

XE2 POSITION AND VELOCITY OF EARTH AT TRIM2

XP1 POSITION AND VELOCITY OF TARGET PLANET AT

TRTM1

XP2 POSITION AND VELOCITY OF TARGET PLANET AT

TRTM2

COMMON COMPUTED/USED: NO

COMMON USED: ADEVXS ADEVX AK ALNGTH AL

AN ANOIS AR AY CHSUP

CXSU	CXSVP	CXSV	CXUP	CXU
CXVP	CXV	CXXSP	CXXS	DATEJ
DELTM	EDEVXS	EDEVX	EY	F
G	HPHR	H	IBARY	IPROB
IPRT	MCNTR	NBOD	NB	NDIM1
SMIGN	NDIM3	NTP	pр	PSP
PS	P	Q	RES	R
S	TM	TRTM1	ប០	VO
W	XF1	XF	XI1	XI
XLAB	ΧP	XSL	ΧU	XA
ZF	ΖĮ			

PROGRAM PRNTS3

PURPOSE: TO PRINT A SUMMARY OF THE ERROR ANALYSIS MODE

SUBROUTINES SUPPORTED: ERRON

SUBROUTINES REQUIRED: CORREL TIME

LOCAL SYMBOLS: D8 HOLLERITH LABEL INITIAL

D9 HOLLERITH LABEL FINAL

D1 JULIAN DATE, EPOCH JAN. 0, 1900, OF INITIAL TIME

D2 JULIAN DATE, EPOCH JAN. 0,1900, OF FINAL TIME

D3 JULIAN DATE OF INITIAL TIME

D4 JULIAN DATE OF FINAL TIME

F FUNCTION= SQUARE ROOT OF SUM OF 3 SQUARES

IDAY CALENDAR DAY OF FINAL TIME

IHR CALENDAR HOUR OF FINAL TIME

IMIN CALENDAR MINUTES OF FINAL TIME

IMO CALENDAR MONTH OF FINAL TIME

IYR CALENDAR YEAR OF FINAL TIME

LOAY CALENDAR DAY OF INITIAL TIME

LHR CALENDAR HOUR OF INITIAL TIME

LMIN CALENDAR MINUTES OF INITIAL TIME

LMO CALENDAR MONTH OF INITIAL TIME

LYR CALENDAR YEAR OF INITIAL TIME

RI POSITION AND VELOCITY OF VEHICLE AT INITIAL TIME

RMF HELIOCENTRIC RADIUS OF VEHICLE AT FINAL TIME

RMI HELIOCENTRIC RADIUS OF VEHICLE AT INITIAL TIME

CALENDAR SECONDS OF FINAL TIME SECI

CALENDAR SECONDS OF INITIAL TIME SECL

TRTM2 TRAJECTORY TIME AT END OF TRAJECTORY

POSITION AND VELOCITY OF VEHICLE RELATIVE ٧E

TO EARTH AT FINAL TIME

MAGNITUDE OF VELOCITY OF VEHICLE AT FINAL VMF

TIME

VMI MAGNITUDE OF VELOCITY OF VEHICLE AT

INITIAL TIME

POSITION AND VELOCITY OF VEHICLE RELATIVE TO ٧T

TARGET PLANET AT FINAL TIME

DC COMMON COMPUTED/USED: DSI

COMMON COMPUTED:

TRTM1

COMMON USED:

ACCND	ACC	ALNGTH	BDRSI1	BOTSI1
BSI1	В	CXSU	CXSV	C XU
CXA	CXXS	DATEJ	DEL TH	DELX
DNCN	DTMAX	FACP	FACV	FNTM
IAUGIN	IDNF	IEPHEM	IMNF	IPROB
ISPH	ISTMC	ISTM1	MNCN	MNNAME
NDACC	NDIM1	NDIM2	NDI M3	NEV1
NEVS	NEV3	NEV	NMN	NST
NTMC	NTP	₽B	PLANET	P SB
PS	P	RCA1	RE	RSOI1
RTP	SAL	SIGALP	SIGBET	SIGPRO
SIGRES	SLAT	SLON	TCA 1	TM
TRTMB	TS0I1	UST	υo	VSOI1
VST	VO	WST	XB	X NM

PROGRAM PRNTS4

PURPOSES TO PRINT OUT A SUMMARY OF THE SIMULATION HODE

SUBROUTINES SUPPORTED: SIMULL

SUBROUTINES REQUIRED: CORREL EPHEN ORB TIME

NOMINAL

LOCAL SYMBOLS: ADON ACTUAL STATE DEVIATION FROM TARGETED

NOMINAL TRAJECTORY AT FINAL TIME

AODI ACTUAL ORBIT ESTIMATION ERROR AT FINAL

TIME

BLANK BLANK HOLLERITH CHARACTER

D1 JULIAN DATE, EPOCH JAN.0,1900, OF INITIAL

TIME

D2 JULIAN DATE, EPOCH JAN.0,1900, OF FINAL

TIME

D3 JULIAN DATE OF INITIAL TIME

D4 JULIAN DATE OF FINAL TIME

EDON ESTIMATED STATE DEVIATION FROM TARGETED

NOHINAL TRAJECTORY

IDAY CALENDAR DAY OF FINAL TIME

IHR CALENDAR HOUR OF FINAL TIME

ININ CALENDAR MINUTES OF FINAL TIME

IMO CALENDAR MONTH OF FINAL TIME

IVR CALENDAR YEAR OF FINAL TIME

LDAY CALENDAR DAY OF INITIAL TIME

LHR CALENDAR HOURS OF INITIAL TIME

LMIN CALENDAR MINUTES OF INITIAL TIME

LMO CALENDAR MONTH OF INITIAL TIME

LYR CALENDAR YEAR OF INITIAL TIME

RE1 POSITION AND VELOCITY OF VEHICLE RELATIVE TO

TO EARTH ON TARGETED NOMINAL

RE2 POSITION AND VELOCITY OF VEHICLE RELATIVE TO TO EARTH ON MOST RECENT NOMINAL RE3 POSITION AND VELOCITY OF VEHICLE RELATIVE TO TO EARTH ON ACTUAL TRAJECTORY RME1 GEOCENTRIC RADIUS OF VEHICLE ON TARGETED NOMINAL AT FINAL TIME GEOCENTRIC RADIUS OF VEHICLE ON HOST RME2 RECENT NOMINAL AT FINAL TIME RME3 GEOCENTRIC RADIUS OF VEHICLE ON ACTUAL TRAJECTORY AT FINAL TIME RME , GEOCENTRIC RADIUS OF VEHICLE AT INITIAL TIME RMP1 DISTANCE OF VEHICLE FROM TARGET PLANET ON TARGETED NOMINAL AT FINAL TIME DISTANCE OF VEHICLE FROM TARGET PLANET ON RMP2 MOST RECENT NOMINAL AT FINAL TIME RMP3 DISTANCE OF VEHICLE FROM TARGET PLANET ON ACTUAL TRAJECTORY AT FINAL TIME RMP DISTANCE OF VEHICLE FROM TARGET-PLANET AT INITIAL TIME RMS1 HELIOCENTRIC RADIUS OF VEHICLE AT FINAL TIME ON TARGETED NOMINAL RMS2 HELIOCENTRIC RADIUS OF VEHICLE AT FINAL TIME ON MOST RECENT NOMINAL HELIOCENTRIC RADIUS OF VEHICLE AT FINAL RMS3 TIME ON ACTUAL TRAJECTORY RMS HELIOCENTRIC RADIUS OF VEHICLE AT INITIAL TIME STATE OF VEHICLE RELATIVE TO TARGET PLANET RP1 AT FINAL TIME ON TARGETED NOMINAL RP2 STATE OF VEHICLE RELATIVE TO TARGET PLANET AT FINAL TIME ON MOST RECENT NOMINAL RP3 STATE OF VEHICLE RELATIVE TO TARGET PLANET AT FINAL TIME ON ACTUAL TRAJECTORY SECI CALENDAR SECONDS AT FINAL TIME

SECL	CALENDAR SECONDS AT INITIAL TIME
VHE	MAGNITUDE OF VELOCITY OF VEHICLE RELATIVE TO EARTH AT INITIAL TIME
VME1	HAGNITUDE OF VELOCITY OF VEHICLE RELATIVE TO EARTH ON TARGETED NOMINAL AT FINAL TIME
VME2	MAGNITUDE OF VELOCITY OF VEHICLE RELATIVE TO EARTH ON HOST RECENT NOMINAL AT FINAL TIME
VME3	HAGNITUDE OF VELOCITY OF VEHICLE RELATIVE TO EARTH ON ACTUAL TRAJECTORY AT FINAL TIME
VMP	MAGNITUDE OF VELOCITY OF VEHICLE RELATIVE TO TARGET PLANET AT INITIAL TIME
VMP1	MAGNITUDE OF VELOCITY OF VEHICLE RELATIVE TO TARGET PLANET ON TARGETED NOMINAL AT FINAL TIME
VMP2	MAGNITUDE OF VELOCITY OF VEHICLE RELATIVE TO TARGET PLANET ON MOST RECENT NOMINAL AT FINAL TIME
VMP3	MAGNITUDE OF VELOCITY OF VEHICLE RELATIVE TO TARGET PLANET ON ACTUAL TRAJECTORY AT FINAL TIME
VMS	MAGNITUDE OF VELOCITY OF VEHICLE AT INITIAL TIME
VMS1	MAGNITUDE OF VELOCITY OF VEHICLE AT FINAL TIME ON TARGETED NOMINAL
VMS2	MAGNITUDE OF VELOCITY OF VEHICLE AT FINAL TIME ON MOST RECENT NOMINAL TRAJECTORY
VMS3	MAGNITUDE OF VELOCITY OF VEHICLE AT FINAL TIME ON ACTUAL TRAJECTORY
COMMON COMPUTED/USED:	NO RE RTP XP ZI
COMMON USED8	AALP ABET ACCND ACC1 ACC ADEVXB ADEVXS ADEVX ALNGTH APRO ARES AVARM BDRSI1 BDRSI2 BDRSI3 BDTSI1 BDTSI2 BDTSI3 BSI1 BSI2

BS13	8	CXSUB	CXSU	CXSVB
CXSV	CXUB	CXU	CXAB	CXV
CXXSB	CXXS	DAB	DATEJ	DEB
DELMUP	DELMUS	DELX	DIB	DMAB
DMUPB	DMUSB	DNCN	DNOB	DTMAX
DMB	EDEVXS	EDEVX	FACP	FACV
FNTM	F	H	IAMNF	IAUGIN
IBARY	IDNF	IMNF	IPROB	ISOI1
ISOI2	ISOI3	ISTMC	ISTM1	MNCN
MNNAME	NBOD1	NBOD	NB1	ИВ
NDACC	NDIM1	NDIM2	NDIM3	NEV1
NEVS	NEV3	NEV5	NEA	nmn
NTMC	NTP	PB	PLANET	PSB
PS	P	RCA1	RCA2	RCA3
RS011	RSOI2	RS0I3	SAL	SIGALP
SIGBET	SIGPRO	SIGRES	SLAT	SLON
TCAL	TCA2	TCA3	TM	TRTMB
TSOI1	TSOI2	TS0I3	TTIM1	TTIM2
UNMAC	UST	ŲŪ	VS0I1	VSOI2
A2013	VST	V O	HST	XB
XF1	XF	XLAB	XNM	ZF

SUBROUTINE PROBE

PURPOSE! TO CONTROL THE EXECUTION OF ALL PROBE RELEASE EVENTS IN ERRAN

CALLING SEQUENCE: CALL PROBE

SUBROUTINES SUPPORTED: ERRANN

SUBROUTINES REQUIRED: NTM PSIM DYNO SNAVM ME AN ORB EPHEM STMPR NTRY CORREL SCHED TRAKM MENO PRINT3 TPRTRG MINIO

LOCAL SYMBOLS: AI MINI-PROBE ROLL RELEASE ANGLE

AY DUMMY ARGUMENT FOR TRAKM

COSA COS(ALFA)

COSAI COS(AI)

COSD COS(DELT)

ICL2S TEMPORARY STORAGE FOR ICL2

IPRN PRINT COUNTER FOR MEASUREMENT PROCESSING

ISP2S TEMPORARY STORAGE FOR ISP2

MAMI =1 IF MAIN PROBE BEING PROCESSED =2 IF MINI-PROBE BEING PROCESSED

MAMIP =MAMI + 1 (INPUT TO SCHED, POINTS TO PROPER SCHEDULE OF MEASUREMENTS)

MCNTRS TEMPORARY STORAGE FOR MCNTR

NMNS TEMPORARY STORAGE FOR NMN

NMP NUMBER OF MINI-PROBE BEING PROCESSED

PI MATHEMATICAL CONSTANT

RF STATE VECTOR AT END OF TIME INTERVAL

SINA SIN(ALFA)

SINAI SIN(AI)

SIND SIN(DELT)

SMNCH TEMOPRARY STORAGE FOR MNCH ARRAY

### SPHERS TEMOPRARY STORAGE FOR SPHERE (NTP)

TE	TIME	ΑТ	THE	PROBE	SPHERE
, ,				111700	

TEVNS TEMPORARY STORAGE FOR TEVN

VT TANGENTIAL VELOCITY OF MINI-PROBE

XFP HELIOCENTRIC STATE AT PROBE SPHERE

XPS PLANETOCENTRIC STATE AT PROBE SPHERE

XSAVE HELIOCENTRIC STATE AT BEGINNING OF PROBE

# COMMON COMPUTED/USED: C

CXSU	CXSUG	CXSV	CXSVG	CXU
CXUG	CXA	CXVG	CXXS	CXXSG
DELTM	EXI	EXSI	EXST	EXT
GCXSW	GCXM	GCXUG	GC XV G	GC XW
GCXWG	GCXXSG	GU	GV	GPG
GPSG	ICLZ	MCNTR	MMCODE	NMN
01/	NR	P	₽G	PMN
P\$	PSG	Q	QPR	R
RI	RPR	RSI	TIMPCT	TRTM1
TRTM2	ΧG	XI		

## COMMON USED:

ABW	ALFA	ALNGTH	DATEJ	DELT
IPRINT	ISPH	·ISP2	ISTMC	IUTC
MNCN	NDIM1	NDÌM2	ND IM 3	NTMC
NTP	QT	RPS	RTP	SMN
SPHERE	TEVN	TG	TM	TRTMB
T6	T7	បត្	V O	

### PRØBE Analysis

are given by

Subroutine PRØBE controls the execution of both main probe and miniprobe release events. When a probe release event occurs, PRØBE saves all states, covariance matrices, etc relating to the bus and initializes all states, covariance matrices, etc for the probe under consideration. The probe state at release is then propagated forward to entry, along with the probe control covariance matrix partitions to obtain the probe state and control dispersions at entry. Next, the probe is tracked from release to entry and probe knowledge covariance matrix partitions are propagated and updated accordingly.

Let t be the time of probe release and let X denote the nominal bus state at release. Denote all main probe quantities with a superscript zero, and all miniprobe quantities with a superscript i (for the ith miniprobe where i = 1, 2, 3). Then, following release, the probe states are given by

$$\overline{X}_{j}^{0} = \overline{X}_{j} \tag{1}$$

$$\overline{X}_{j}^{i} = \overline{X}_{j} + \left[ \frac{0}{\Delta V_{j}^{(i)}} \right]$$
 (2)

where  $\Delta V_j^{(i)}$  is the velocity increment imparted to the ith miniprobe by the spin release at t_j. The velocity increment  $\Delta V_j^{(i)}$  and the miniprobe release controls are used in subroutine MINIQ to compute the execution error covariance matrix  $\tilde{Q}_j^{(i)}$  associated with the spin release of the ith miniprobe. Denote the bus position/velocity knowledge and control covariance matrices at release by  $P_k$  and  $P_c$ , respectively. Then, immediately following release, the probe position/velocity knowledge and control covariance matrices

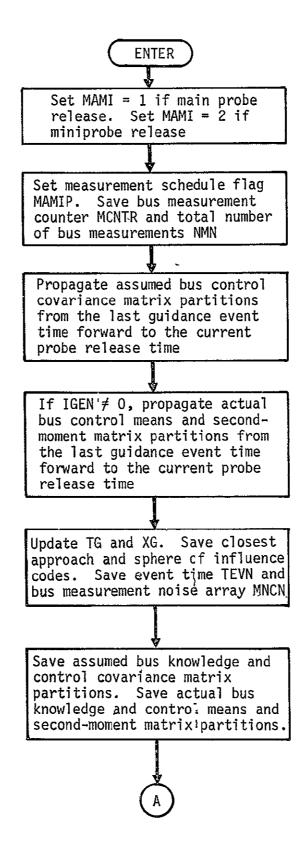
$$P_{k_{j}}^{0} = P_{k_{j}}$$
(3)

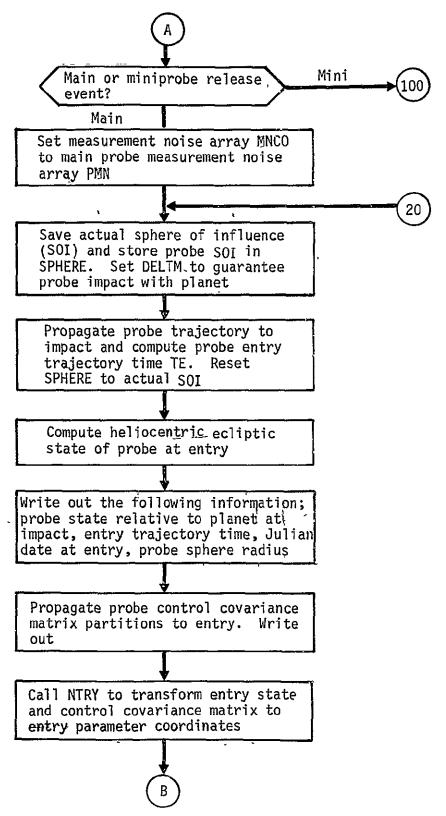
$$P_{c_{j}}^{0} = P_{c_{j}}; \qquad (4)$$

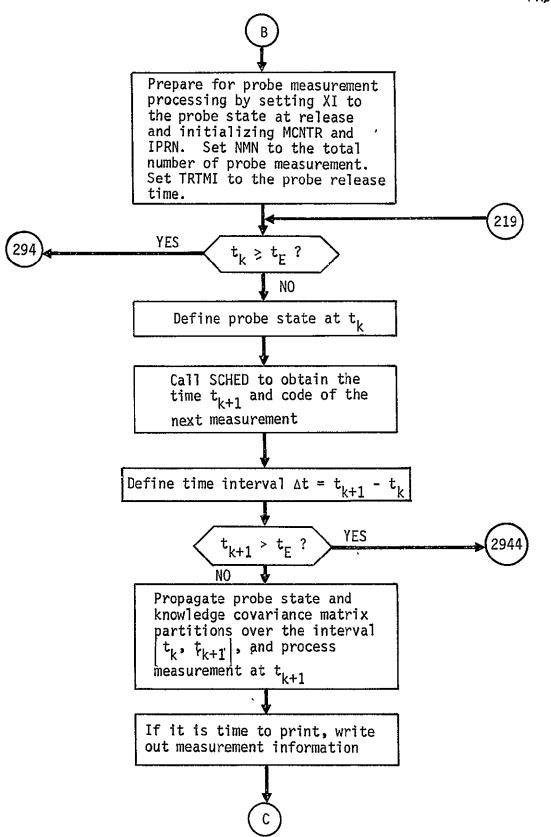
$$P_{k_{j}}^{(i)} = P_{k_{j}} + \begin{bmatrix} 0 & 0 \\ 0 & \tilde{Q}_{j}^{(i)} \end{bmatrix}$$
 (5)

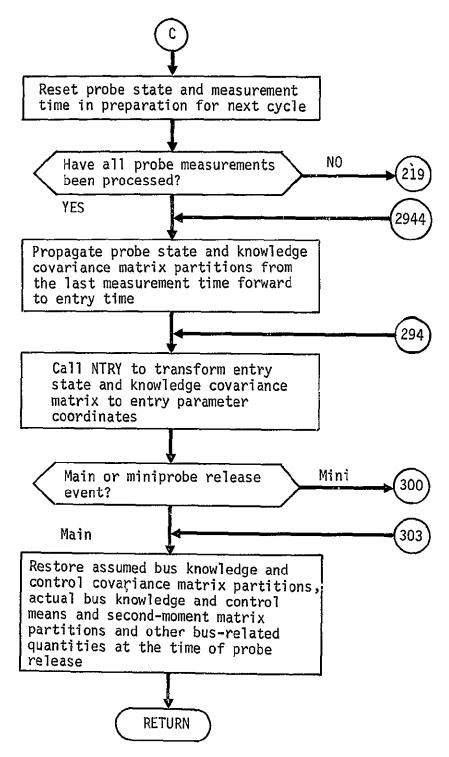
$$P_{c_{j}}^{(i)} = P_{k_{j}}^{(i)}$$
 (6)

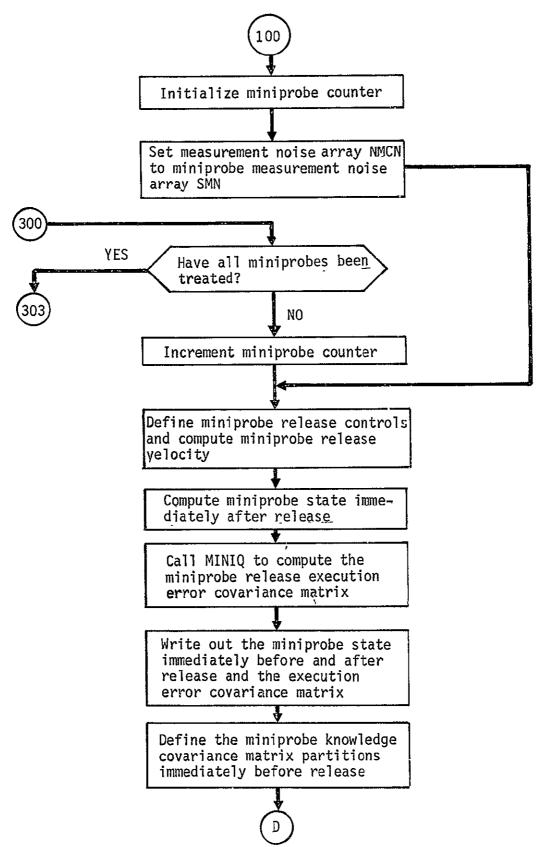
The above probe control matrices, along with all related partitions, are propagated forward to entry time  $\mathbf{t}_E$  using the propagation equations appearing in subroutine GNAVM to obtain the probe entry dispersions. Beginning with the above knowledge covariance matrix of the probe under consideration and all related partitions, the probe knowledge covariance matrix partitions are propagated and updated as each probe measurement is processed using the update equations appearing in subroutine GNAVM.

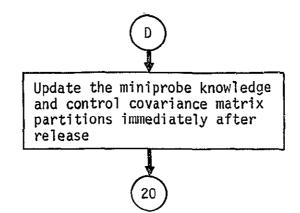












SUBROUTINE PROBES

PURPOSE: TO CONTROL THE EXECUTION OF ALL PROBE RELEASE EVENTS IN SIMUL

CALLING SEQUENCE: CALL PROBES

SUBROUTINES SUPPORTED: SIMULL

SUBROUTINES REQUIRED: NTMS PSIM DYNOS NA VM MEAN ORB EPHEM STMPR CORREL NTRY **TPRTRG** SCHED TRAKS MENOS PRINT4 MINIQ

LOCAL SYMBOLS: AI MINI-PROBE ROLL RELEASE ANGLE

BVAL BIAS VALUE VECTOR

COSA COS(ALFA)

COSAI COS(AI)

COSD COS(DELT)

DISUNS TEMPORARY STORAGE FOR DISUN

DUM INTERMEDIATE VECTOR

DUMM INTERMEDIATE VARIABLE

ICL2S TEMPORARY STORAGE FOR ICL2

IPRN PRINT COUNTER FOR MEASUREMENT PROCESSING

IQL QUASI-LINEAR EVENT COUNTER

ISP2S TEMPORARY STORAGE FOR ISP2 /

MAMI =1 IF MAIN PROBE BEING PROCESSED =2 IF MINI-PROBE BEING PROCESSED

MAMIP = MAMI + 1 (INPUT TO SCHED, POINTS TO PROPER SCHEDULE OF MEASUREMENTS)

MCNTRS TEMPORARY STORAGE FOR MCNTR

MMCODE MEASUREMENT CODE

NEVENT EVENT COUNTER

NMNS TEMPORARY STORAGE FOR NMN

NMP NUMBER OF MINI-PROBE BEING PROCESSED

		PI	MATHEMAT	ICAL CON	STANT		
		PVS	PLANET OC	ENTRIC S	TATE AT	PROBE SP	HERE
		SADVX	TEMPORAR	Y STORAG	E FOR AD	EVX	
		SEDVX	TEMPORAR	Y STORAG	E FOR ED	EVX	
		SAVRM	TEMPORAR	Y STORAG	E FOR AV	ARM	
		SINA	SIN(ALFA	.)			
		SINAI	SIN(AI)				
		SIND	SIN(DELT	·)			
		SMNCN	TEMOPRAR	Y STORAG	E FOR MI	CN ARRAY	
		SPHERS	TEMOPRAR	Y STORAG	SE FOR SE	PHERE (NTF	<b>)</b>
		SXF1	TEMPORAR	Y STORAG	E FOR XF	⁻ 1	
		SZF	TEMPORAR	RY STORAG	SE FOR ZE	<del>.</del>	
		TE	TIME AT	THE PROP	SE SPHERE	<b>:</b>	
		TEVNS	TEMPORAR	Y STORAG	E FOR TE	EVN	
		TRTM2	TIME OF	NEXT ME	SUREMENT	OR EVEN	IT
		VT	TANGENTI	AL VELO	CITY OF N	INI-PROE	3E
		XFP	HELIOCEN	ITRIC STA	ATE AT PE	ROBE SPHE	RE
		XPS	PLANETO	ENTRIC S	STATE AT	PROBE SP	HERE
		XSAVE	HELIOCEN EVENT	ITRIC ST	ATE AT 88	EGINNING	OF PROBE
COMMON	COMPUTEDA	/USED#	CXSU CXUG DELTM	CXSUG CXV ICL2	CXSV CXVG MCNTR	CXXS CXXS	CXU CXXSG NMN
			NO PS	NR PSG	P Q	PG R	PMN
			RI		RSI	TIMPCT	TRTM1
			TRTM2	XG	XI		
COMMON	USED:		ABW	ALFA	ALNGTH	DATEJ	DELT
			IPRINT MNCN	ISPH NDIM1	ISP2 NDIM2	ISTMC NDIM3	IUTC NTMC
58-12			· · · · · · · · · · · · · · · · · · ·	1740 41 (4	1104116	1157 441 0	

OLDX TEMPORARY STORAGE FOR XF

NTP	QT	RPS	RTP	SMN
SPHERE	TEVN	TG	TM	TRTMB
TA	T 7	110	VÕ	

### PRØBS Analysis

Subroutine PRØBS controls the execution of both main probe and miniprobe release events in the simulation program SIMUL. When a probe release event occurs, PRØBS saves all nominal and actual states, deviation estimates, covariance matrices, etc relating to the bus and initializes all nominal and actual states, deviation estimates, covariance matrices, etc for the probe under consideration. The probe state at release is the propagated forward to entry, along with the probe control covariance matrix partitions, to obtain the probe state and control dispersions at entry. Next the probe is tracked from release to entry and probe knowledge covariance matrix partitions and deviation estimates are propagated and updated accordingly.

Let t denote the time of probe release. Denote the bus targeted nominal and most recent nominal states at release by  $\tilde{X}_j$  and  $\tilde{X}_j$ , respectively. Denote actual and estimated deviations of the bus state from the most recent nominal by  $\delta \tilde{X}_j$  and  $\delta \tilde{X}_j$ , respectively. All main probe quantities will be denoted with a superscript zero and all miniprobe quantities with a superscript i (for the ith miniprobe, where i = 1,2,3). Then following release, the probe nominal states are given by

$$\bar{\mathbf{x}}_{\mathbf{j}}^{0} = \bar{\mathbf{x}}_{\mathbf{j}} \tag{1}$$

$$\tilde{\mathbf{x}}_{\mathbf{j}}^{0} = \tilde{\mathbf{x}}_{\mathbf{j}} \tag{2}$$

$$\tilde{\mathbf{x}}_{j}^{i} = \tilde{\mathbf{x}}_{j} + \delta \tilde{\hat{\mathbf{x}}}_{j} + \left[ -\frac{\mathbf{0}}{\Delta \mathbf{v}_{i}^{i}} \right]$$
 (3)

$$\tilde{X}_{j}^{i} = \tilde{X}_{j}^{i} \tag{4}$$

where  $\Delta V_{j}^{1}$  is the velocity increment imparted to the ith miniprobe by the spin-release at  $t_{j}$ . The velocity increment  $\Delta V_{j}^{1}$  and the miniprobe release controls are used in subroutine MINIQ to compute the execution error covariance matrix  $\tilde{Q}_{j}^{1}$  and the actual execution  $\delta \Delta V_{i}^{1}$ .

The actual and estimated deviations of the probe states from the most recent nominals are given by

$$\delta \tilde{\hat{\mathbf{x}}}_{\mathbf{j}}^{0} = \delta \tilde{\hat{\mathbf{x}}}_{\mathbf{j}} \tag{5}$$

$$\delta \tilde{x}_{j}^{0} = \delta \tilde{x}_{j} \tag{6}$$

$$\delta \tilde{\hat{X}}_{j}^{i} = 0 \tag{7}$$

$$\delta \tilde{x}_{j}^{i} = \delta \tilde{x}_{j} - \delta \tilde{\hat{x}}_{j} + \left[ -\frac{0}{\delta \Delta v_{i}^{i}} \right] . \tag{8}$$

Denote the bus position/velocity knowledge and control covariance matrices at release by P  $_{K}$  and P  $_{c}$  , respectively. Then, immediately  $\dot{j}$ 

following release, the probe position/velocity knowledge and control covariance matrices are given by

$$P_{K_{j}}^{0} = P_{K_{j}}$$
(9)

$$P_{c_{i}}^{0} = P_{c_{i}} \tag{10}$$

$$P_{K_{j}}^{i} = P_{K_{j}} + \begin{bmatrix} 0 & \frac{1}{\tilde{Q}_{i}^{i}} \\ 0 & \tilde{Q}_{i}^{i} \end{bmatrix}$$
 (11)

$$P_{c_{j}}^{i} = P_{K_{j}}^{i} . \qquad (12)$$

The above probe control covariance matrices, along with all related partitions, are propagated forward to entry time  $t_{\rm E}$  using the

propagation equations appearing in subroutine NAVM to obtain the probe entry dispersions. Beginning with the above knowledge covariance matrix of the probe under consideration and all related partitions, the probe knowledge covariance matrix partitions are

propagated and updated as each probe measurement is processed using the equations appearing in subroutine NAVM. The probe estimates are propagated and updated using the equations appearing in subroutine SIMUL.

A flow chart for subroutine PRØBS is not presented since it would be quite similar to the flow chart for subroutine PRØBE (see subroutine PRØBE for details).

SUBROUTINE PSIM

PURPOSE: TO COMPUTE THE STATE TRANSITION MATRIX PARTITIONS PHI, TXXS, TXW, AND TXU OVER AN ARBITRARY INTERVAL OF TIME (TK .TK+1).

CALLING SEQUENCE: CALL PSIM(RI, RF, ISC)

ARGUMENT: ISC I CODE SPECIFYING WHICH TECHNIQUE IS TO BE
USED TO COMPUTE THE STATE TRANSITION MATRIX
PARTITION PHI

RF I POSITION AND VELOCITY OF THE VEHICLE AT THE END OF THE TIME INTERVAL

RI I POSITION AND VELOCITY OF THE VEHICLE AT THE BEGINNING OF THE TIME INTERVAL

SUBROUTINES SUPPORTED: SIMULL SETEVS BIAIM GUISIM GUISS
PRESIM ERRANN SETEVN GUIDM GUID
PRED PROBE PROBES

SUBROUTINES REQUIRED: CASCAD CONC2 EPHEM MUND NDTM
ORB PCTM PLND

LOCAL SYMBOLS: D INTERMEDIATE JULIAN DATE

DELT TIME INTERVAL IN CORRECT UNITS

DUM TEMPORARY STORAGE FOR STATE TRANSITION MATRIX

IANS VARIABLE USED IN EXAMINING IAUG , IAUGIN

ICLC TEMPORARY STORAGE FOR ICL

ISPHC TEMPORARY STORAGE FOR ISPH

IPX TEMPORARY STORAGE FOR IPRINT

POSS DISTANCE OF THE VEHICLE FROM THE TARGET PLANET AT INITIAL TIME

RP STATE VECTOR

RS POSITION OF VEHICLE RELATIVE TO GOVERNING BODY AT INITIAL TIME

SAVE TEMPORARY STORAGE FOR ACC

THSP CONSTANT EQUAL TO SIX TIMES THE SPHERE OF INFLUENCE OF TARGET PLANET

VFC		ID VELOCITY OF		RELATIVE TO
vs	VELOCITY OF BODY AT INI	VEHICLE REL	ATIVE TO	GOVERNING
COMMON COMPUTED/USED:	NO XP			
COMMON COMPUTED:	PHI TXU	TXXS	TXW	
COMMON USED:	ALNGTH DAT IAUGDC IAU NB NDI SPHERE TM	JG IBARY	DTMAX ISTM1 NTP VMU	F NEOD RVS ZERO

## **PSIM Analysis**

Subroutine PSIM controls the computation of each partition appearing in the augmented state transition matrix

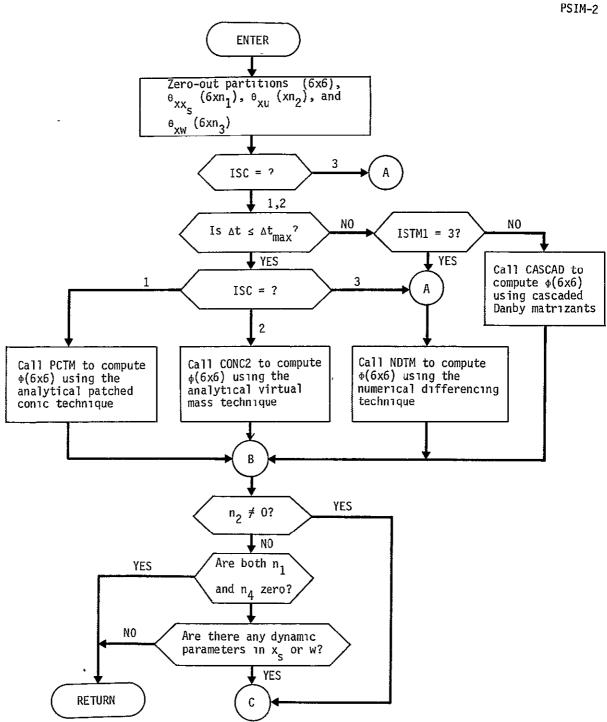
$$\Phi^{A}(k+1,k) = \begin{bmatrix} \Phi(k+1,k) & \theta_{XX}(k+1,k) & \theta_{XU}(k+1,k) & 0 & \theta_{XW}(k+1,k) \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$$

The first part of the subroutine deals solely with the computation of  $\Phi(k+1,k)$  by one of the three techniques — analytical patched conic, analytical virtual mass, or numerical differencing. If an analytical technique is selected for computing  $\Phi(k+1,k)$  over an interval of time greater than the maximum time interval for which the analytical technique is considered valid, we compute  $\Phi(k+1,k)$  using numerical differencing or by cascading Danby matrizants.

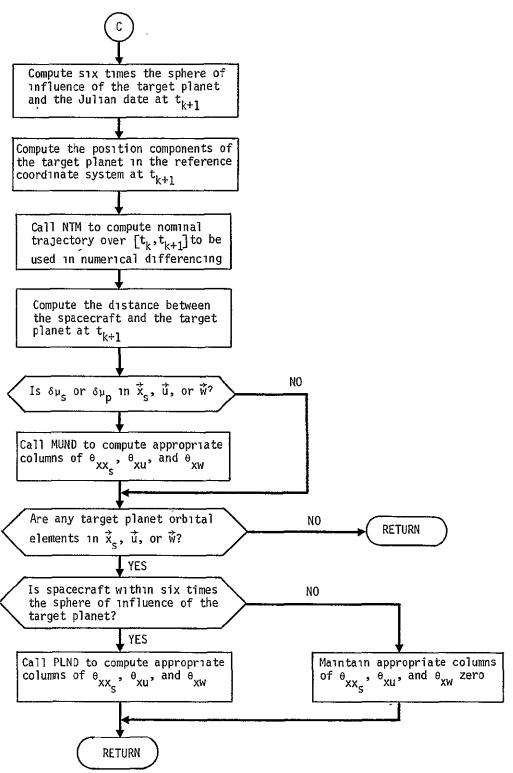
The remaining partitions,  $\theta_{xx}$ ,  $\theta_{xu}$ , and  $\theta_{xw}$ , are always computed

by numerical differencing. Columns in these partitions associated with target planet gravitational constant or orbital elements are computed only if the spacecraft is within six times the sphere of influence of the target planet at  $t_{k+1}$ . Otherwise, these columns are set to zero.





PSIM Flow Chart



SUBROUTINE PULCOV

PURPOSE COMPUTE EFFECTIVE EXECUTION ERROR COVARIANCE MATRIX FOR A VELOCITY CORRECTION MODELED AS AN IMPULSE SERIES

CALLING SEQUENCE: CALL PULCOV(RIN, DELTAV, TH, QK)

ARGUMENTS: RIN(6) I INERTIAL STATE OF SPACECRAFT AT NOMINAL

TIME OF CORRECTION

DELTAV(3) I TOTAL VELOCITY INCREMENT TO BE ADDED

TM I TIME UNITS PER DAY

QK(6,6) O DEVIATION MATRIX RESULTING FROM EXECUTION

ERRORS

SUBROUTINES SUPPORTED: EXCUTS

SUBROUTINES REQUIRED: PERHEL QCOMP

LOCAL SYMBOLS: DELR PERTURBATION IN POSITION

DELV PERTURBATION IN VELOCITY

DVFM HAGNITUDE OF FINAL PULSE

DVIN MAGNITUDE OF TYPICAL PULSE

FSER F-SERIES CONSTANT FOR PLANET

GSER G-SERIES CONSTANT FOR PLANET

HLTF STATES OF LAUNCH AND TARGET BODIES AT END

OF PROPAGATION INTERVAL

PERT CURRENT PERTURBATION

PHI STATE TRANSITION MATRIX OVER TYPICAL

INTERVAL

QQ DEVIATION MATRIX DURING PROPAGATION

THROUGH PULSES

Q TYPICAL VELOCITY EXECUTION ERROR

COVARIANCE

RF NOMINAL INERTIAL STATE OF SPACECRAFT AT

END OF TYPICAL INTERVAL

RPF PERTURBED INERTIAL STATE OF SPACECRAFT AT

END OF TYPICAL INTERVAL

R INERTIAL STATE OF SPACECRAFT AT BEGINNING

TIME	INTERVAL	BETWEEN	PULSES	
71**2	2			
71**	3			
T1 ##!	la.			

T5 T1**5

**T1** 

**T2** 

**T3** 

**T**4

T6 T1**6

COMMON USED:	DTI GS	DVF NPUL	DVI ONE	FS PSIGA	GG PSIGB
	PSIGK	PSIGS	RK	THO	VK
	ZERO				

OF TYPICAL INTERVAL

#### PULCØV Analysis

PULCOV processes the control covariance through the pulsing arc to determine a measure of the probabilistic deviation of the corrected trajectory from the desired trajectory resulting from execution errors.

The pulsing arc itself is computed in PREPUL. It consists of N_p-1 pulses  $\overline{\Delta v}_i$  and a final pulse  $\overline{\Delta v}_f$  satisfying

$$(N_{p}-1) \ \overrightarrow{\Delta v}_{1} + \overrightarrow{\Delta v}_{f} = \overrightarrow{\Delta v}$$
 (1)

where  $\Delta v$  is the equivalent single impulse. The pulses are separated by a time interval  $\Delta t_i$ . The duration of the entire sequence of pulses is given by  $\Delta T = (N_p-1) \Delta t_i$ .

PULCØV must compute the execution error matrices Q,  $Q_f$  corresponding to the nominal pulse  $\Delta v_i$  and the final pulse  $\Delta v_f$  respectively. The error model for the engine is defined by the input specifications

 $\sigma_{k}^{2}$  = proportionality error

 $\sigma_{k}^{2} = \text{resolution error}$ 

 $\sigma_{\alpha}^{2}$  = first pointing error

 $\sigma_{\beta}^2$  = second pointing error

The execution error matrix measuring the probabilistic deviation of the actual velocity increment from the desired velocity increment is computed by QCOMP.

The exact equations defining the propagation of the covariance matrix are recursive in nature. If  $P_k^+$  is the control covariance immediately after the  $k^{th}$  pulse, the covariance will propagate to the time of the next pulse  $t_{k+1}$  by the formula

$$\mathbf{P}_{k+1}^{-} = \Phi_{k+1,k} \quad \mathbf{P}_{k}^{+} \quad \Phi_{k+1,k}^{T}$$
(2)

where  $\Phi_{k+1,k}$  is the 6x6 state transition matrix relating perturbations at t . Adding the pulse at t expands the covariance by

$$P_{k+1}^{+} = P_{k+1}^{-} + \begin{bmatrix} 0 & 0 & 0 \\ - & - & - \\ 0 & 0 & Q \end{bmatrix}$$
 (3)

where  $\,Q\,$  is set equal either to the nominal or final form of  $\,Q\,$  .

To start the process the control covariance following the first pulse is given by

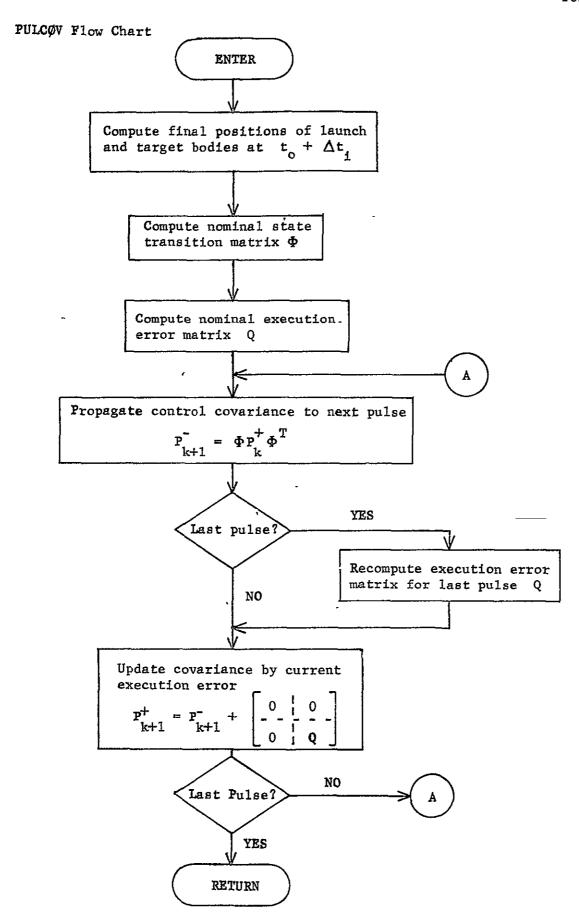
$$P_{1}^{+} = \begin{bmatrix} 0 & 0 & 0 \\ - & 0 & 0 \\ 0 & Q \end{bmatrix}$$
 (4)

For efficiency one simplification is made in the process. Instead of recomputing the state transition matrix over each interval, the value of that matrix is held constant at the value corresponding to the "average interval". To explain this, let the state of the spacecraft at the time  $t_0$  of the impulsive  $\Delta v$  computation be denoted  $\vec{r}_0$ ,  $\vec{v}_0$ . Then the "average interval" is defined to be the perturbed heliocentric trajectory (PERHEL) resulting from the propagation of the state  $(\vec{r}_0, \vec{v}_0 + \frac{1}{2} \Delta \vec{v})$  over the interval  $(t_0, t_0 + \Delta t_1)$ .

The constant state transition matrix  $\Phi$  is computed by numerical differencing. The initial state  $(\overrightarrow{r}_0, \overrightarrow{v}_0 + \frac{1}{2} \overrightarrow{\Delta v})$  is first propagated over the  $\Delta t_1$  time interval (using PERHEL) resulting in the state  $(\overrightarrow{r}, \overrightarrow{v})$ . Then the x-component of initial position is perturbed by  $\Delta x$ , leading to a final state of  $(\overrightarrow{r}_{pf}, \overrightarrow{v}_p)$  upon propagation. The first column of the matrix is then computed by

$$\Phi_{1} = \begin{bmatrix} \overrightarrow{r}_{pf} - \overrightarrow{r}_{f} \\ \Delta x \end{bmatrix}^{T} \qquad (5)$$

The other columns of  $\Phi$  are computed by similar computations using the remaining components of position and velocity  $(y, z, \dot{x}, \dot{y}, \dot{z})$ .



SUBROUTINE PULSEX

PURPOSE: TO CONTROL EXECUTION OF THE PULSING ARC MODEL.

CALLING SEQUENCE: CALL PULSEX(RIN, DELTAY, RE, TM, IRE)

ARGUMENTS 8 RIN(6) I INERTIAL STATE OF SPACECRAFT AT NOMINAL TIME OF CORRECTION

DELTAV(3) I TOTAL VELOCITY INCREMENT TO BE ADDED

RE(6) O FINAL INERTIAL STATE OF SPACECRAFT (IRE)

TM I TIME UNITS PER DAY

IRE I FLAG DETERMINING FINAL STATE

=0 RETURN FINAL STATE AT END OF PULSE ARC

=1 RETURN FINAL STATE AT ARC MIDPOINT

SUBROUTINES SUPPORTED: EXCUTE EXCUTS EXCUT

SUBROUTINES REQUIRED: CAREL PERHEL

LOCAL SYMBOLS: A SEMIMAJOR AXIS

TA

DTS TIME INTERVAL IN TIME UNITS

DT DUMMY VARIABLE FOR OUTPUT

E ECCENTRICITY

FSER F-SERIES CONSTANT

GSER G-SERIES CONSTANT

HLTF STATES OF LAUNCH AND TARGET BODIES AT END OF PROPAGATION INTERVAL

HLTI STATES OF LAUNCH AND TARGET BODIES AT BEGINNING OF PROPAGATION INTERVAL

IPUL PULSE COUNTER

PP UNIT VECTOR TO PERIAPSIS

QQ UNIT VECTOR IN ORBITAL PLANE NORMAL TO PP

RB INERTIAL STATE OF SPACECRAFT AT BEGINNING OF PROPAGATION INTERVAL

TFP TIME FROM PERIAPSIS

TRUE ANOMALY

	1 K	TIME PROM STAKE OF PULSING ARC
	TS	INTERMEDIATE VARIABLE
	T1	TIME INTERVAL FROM MIDPOINT OF ARC
	T2	T1**2
	13	71**3
	T4	T1**4
	<b>T</b> 5	T1**5
	<b>T</b> 6	T1**6
	W	ARGUMENT OF PERIAPSIS
	HH	UNIT NORMAL TO ORBITAL PLANE
	хI	INCLINATION
	XN	LONGITUDE OF ASCENDING NODE
COMMON USED:		DTI DVF DVI FS GG GS NPUL ONE PULT RK TWO VK ZERO

PULSEX Analysis

PULSEX is responsible for the actual execution of the pulsing arc. Experiments have shown that adding an impulsive  $\Delta v$  at time to may be approximated quite closely by centering an equivalent sequence of smaller impulses about the nominal time to .

This equivalent sequence of thrusts is computed by PREPUL. It consists of N - 1 pulses  $\overline{\Delta v}$  and a final pulse  $\overline{\Delta v}_f$  satisfying

$$(N_p - 1) \overline{\Delta v}_i + \overline{\Delta v}_f = \overline{\Delta v}$$
 (1)

The pulses are separated by a time interval  $\Delta t$ . The duration of the entire sequence of pulses is given by  $\Delta T = (N_p^i - 1) \Delta t_i$ .

For efficiency the perturbed heliocentric conic propagator PERHEL is used to propagate the trajectory between pulses. PERHEL requires the positions of the launch and target bodies at the beginning and end of each propagation interval. PREPUL stores the position and velocity of the launch and target bodies at the reference time  $t_0: (\vec{r}_{L0}, \vec{v}_{L0})$  and  $(\vec{r}_{T0}, \vec{v}_{T0})$  and stores the constants of the f and g series for those states  $(f_{Lk}, g_{Lk}, f_{Tk}, g_{Tk}, k=1,6)$ . The position of the launch body at some time t relative to the reference time t is then given by

$$\vec{r}_L(t) = f_L(t) \vec{r}_{L0} + g_L(t) \vec{v}_{L0}$$
 (2)

where 
$$f_{L}(t) = \sum_{k=0}^{6} f_{Lk} t^{k}$$

$$g_{L}(t) = \sum_{k=1}^{6} g_{Lk} t^{k}$$
(3)

with similar equations holding for the target body.

The procedure of PULSEX is straightforward. The positions of the launch and target bodies are computed at the time the pulsing arc should begin:  $t_B = t_o - \Delta T/2.$  PERHEL is then called to propagate the spacecraft from  $t_o$  backwards to  $t_B$ . The actual pulsing arc cycle is now entered. The nominal velocity increment  $\Delta v_i$  is added to the current velocity impulsively

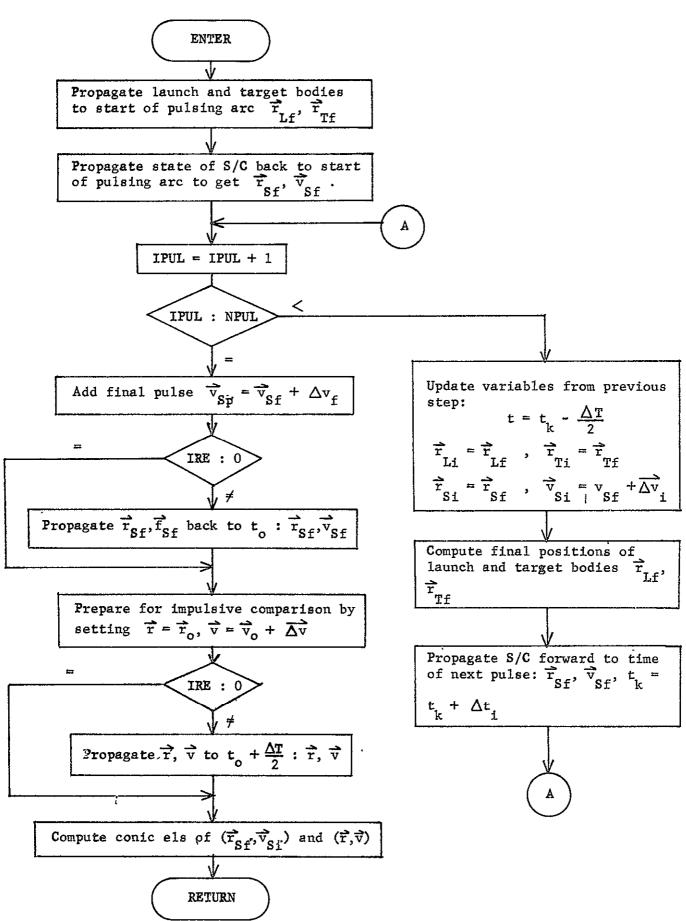
$$\vec{v} = \vec{v} + \vec{\Delta v}_{1} \tag{4}$$

and the resulting state ( $\vec{r}$ ,  $\vec{v}$ ) is propagated forward over the time interval  $\Delta t_i$  by PERHEL. Another pulse is added and the process repeated until N_p - 1 pulses have been added. Finally a pulse of  $\Delta v_f$  is added.

Two options are now permitted. If IRE = 0, the final state is not altered (NOMNAL). If IRE = 1, the final state is propagated backwards back to t for use in ERRAN and SIMUL.

Finally CAREL is called to compute the conic elements of the final state. For comparison purposes, the impulsive  $\Delta v$  is added to the state at t_o, propagated to the final time t_E = t_o +  $\Delta T/2$  by PERHEL, and those elements computed.

PULSEX Flow Chart PULSEX-3



SUBROUTINE QCOMP

PURPOSE: TO COMPUTE THE EXECUTION ERROR COVARIANCE MATRIX FOR A VELOCITY CORRECTION.

CALLING SEQUENCE: CALL QCOMP(V,EM,Q)

ARGUMENTS V I VELOCITY CORRECTION

Q O EXECUTION ERROR MATRIX

EN I ERROR MODEL (SIGRES, SIGPRO, SIGALP, SIGBET)

SUBROUTINES SUPPORTEDS BIAIM GUISIM PULCOV GUIDM

LOCAL SYMBOLS: AU SIGALP/U2

BRK SIGPRO+ SIGRES/R2

BU SIGBET/U2

R2 U2+Z2

U2 X2+Y2

X2 V(1) SQUARED

Y2 V(2) SQUARED

Z2 V(3) SQUARED

QCOMP Analysis

Subroutine QCØMP computes the execution error covariance matrix  $\widetilde{Q}_j$  for a velocity correction  $\overrightarrow{\triangle V} = (\triangle V_x, \triangle V_y, \triangle V_z)$  occurring at time  $t_j$ . If the execution error is assumed to have form

$$\delta \triangle V = k \triangle V + s \frac{\overrightarrow{\triangle} V}{\triangle V} + \delta \overrightarrow{\triangle} V$$
 pointing

where k is the proportionality error and s is the resolution error, then the elements of the  $\widetilde{Q}_1$  matrix are given by

$$\widetilde{Q}_{11} = \Delta V_{x}^{2} \left[ \sigma_{k}^{2} + \frac{\sigma_{s}^{2}}{\rho^{2}} \right] + \frac{\Delta V_{y}^{2} \rho^{2} \sigma_{\delta \alpha}^{2}}{\mu^{2}} + \frac{\Delta V_{x}^{2} \Delta V_{z}^{2} \sigma_{\delta \beta}^{2}}{\mu^{2}}$$

$$\widetilde{Q}_{12} = \widetilde{Q}_{21} = \Delta V_{x} \Delta V_{y} \left[ \sigma_{k}^{2} + \frac{\sigma_{s}^{2}}{\rho^{2}} - \frac{\rho^{2} \sigma_{\delta \alpha}^{2}}{\mu^{2}} + \frac{\Delta V_{z}^{2} \sigma_{\delta \beta}^{2}}{\mu^{2}} \right]$$

$$\widetilde{Q}_{13} = \widetilde{Q}_{31} = \Delta V_{x} \Delta V_{z} \left[ \sigma_{k}^{2} + \frac{\sigma_{s}^{2}}{\rho^{2}} - \sigma_{\delta \beta}^{2} \right]$$

$$\widetilde{Q}_{22} = \Delta V_{y}^{2} \left[ \sigma_{k}^{2} + \frac{\sigma_{s}^{2}}{\rho^{2}} \right] + \frac{\Delta V_{x}^{2} \rho^{2} \sigma_{\delta \alpha}^{2}}{\mu^{2}} + \frac{\Delta V_{y}^{2} \Delta V_{z}^{2} \sigma_{\delta \beta}^{2}}{\mu^{2}}$$

$$\widetilde{Q}_{23} = \widetilde{Q}_{32} = \Delta V_{y} \Delta V_{z} \left[ \sigma_{k}^{2} + \frac{\sigma_{s}^{2}}{\rho^{2}} - \sigma_{\delta \beta}^{2} \right]$$

$$\widetilde{Q}_{33} = \Delta V_{z}^{2} \left[ \sigma_{k}^{2} + \frac{\sigma_{s}^{2}}{\rho^{2}} \right] + \mu^{2} \sigma_{\delta \beta}^{2}$$

where  $\mu^2 = \Delta v_x^2 + \Delta v_y^2$ ,  $\rho^2 = \mu^2 + \Delta v_z^2$ , and  $\sigma_s^2$ ,  $\sigma_k^2$ ,  $\sigma_{\alpha}^2$ , and  $\sigma_{\beta\alpha}^2$  are the variances associated with the resolution, proportionality, and two pointing errors, respectively.

PROGRAM QUASI

PURPOSES PERFORM QUASI-LINEAR FILTERING EVENT IN THE SIMULATION

PROGRAM

CALLING SEQUENCE: CALL QUASI

SUBROUTINES SUPPORTED: SIMULL

LOCAL SYMBOLS &

COMMON COMPUTED/USED: ADEVX EDEVX NGE XF1

COMMON COMPUTEDS TRIM1 XI1 XI

COMMON USED8 ADEVXS EDEVXS NDIM1 TEVN W

XF XSL ZERO

#### QUASI Analysis

At a quasi-linear filtering event the most recent nominal trajectory is updated by using the most recent state deviation estimate. If  $\widetilde{X}_j$  is the most recent nominal position/velocity state immediately preceding the event at time  $t_j$ , and if  $\delta \widetilde{X}_j$  is the position/velocity deviation estimate, then immediately following the quasi-linear filtering event, the most recent nominal position/velocity state is given by

$$\tilde{x}_{j}^{+} = \tilde{x}_{j}^{-} + \delta \tilde{\hat{x}}_{j}^{-}$$

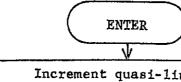
The estimated and actual deviations from the most recent nominal trajectory must also be updated:

$$\delta \widetilde{X}_{j}^{+} = 0$$

$$\delta \widetilde{X}_{j}^{+} = \delta \widetilde{X}_{j}^{-} - \delta \widetilde{X}_{j}^{-}$$

A quasi-linear filtering event in no way alters the knowledge and control uncertainties at time  $t_j$ . Thus knowledge covariance  $P_k$  and control covariance  $P_c$  remain constant across a quasi-linear filtering event. Furthermore, since no velocity correction is performed, the (most recent) targeted nominal  $\overline{X}_j$  is unchanged. Neither is the solve-for parameter state updated at a quasi-linear filtering event.

# QUASI Flow Chart



Increment quasi-linear filtering event counter.

Write out actual dynamic noise and estimated and actual deviations from most recent nominal at time t_i immediately

preceding the event.

Update and write out most recent nominal position/velocity state.

Update and write out estimated and actual state deviations at time  $t_j^+$  immediately following the event.

Reset state vectors and trajectory time in preparation for next cycle. Set NØMTRJ = 1 since targeted nominal and most recent nominal no longer coincide.

RETURN

FUNCTION RNUM

PURPOSE: TO RETURN RANDOM NUMBERS ON A NORMAL DISTRIBUTION WITH MEAN ZERO AND STANDARD DEVIATION SIGMA.

CALLING SEQUENCES Z=RNUM(SIGNA)

ARGUMENTS SIGMA I STANDARD DEVIATION

SUBROUTINES SUPPORTED: SIMULL

LOCAL SYMBOLS: A SUM OF THELVE RANDOM NUMBERS BETWEEN ZERO

AND ONE

NX CONTROLLING INTEGER

N INTERMEDIATE INTEGER

Q INTERMEDIATE VARIABLE

RNUM RANDOM NUMBER FROM NORMAL DISTRIBUTION

WITH MEAN ZERO AND STANDARD DEVIATION

SIGMA

RR INTERMEDIATE VARIABLE

SS INTERMEDIATE VARIABLE

WW INTERMEDIATE VARIABLE

WI INTERMEDIATE VARIABLE

YY INTERMEDIATE VARIABLE

Y1 INTERMEDIATE VARIABLE

ZZ INTERMEDIATE VARIABLE

Z1 INTERMEDIATE VARIABLE

# RNUM Analysis

Function subprogram RNUM supplies random numbers on a normal distribution with near zero and standard deviation  $\sigma$  .

Twelve random numbers  $X_i$  between 0 and 1 are computed, which are then used to compute the returned random number RNUM using the following equation:

RNUM = 
$$\left[ \sum_{i=1}^{12} x_i - 6 \right] \cdot \sigma$$

SUBROUTINE SAOCS

PURPOSE TO COMPUTE SINES AND COSINES OF SPIN-AXIS RIGHT

ASCENSION AND DECLINATION GIVEN SPIN-AXIS ORIENTATION

MODE

ARGUMENT: CSDCSA O COSINE OF ECLIPTIC DECLINATION OF SPIN

AXIS

CSRASA O COSINE OF ECLIPTIC RIGHT ASCENSION OF

SPIN AXIS

DJR I JULIAN DATE EPOCH1900 OF MINIPROBE RELEASE

ISÃO I SPIN ÁXIS ORIENTÁTION HODE INDICATOR

NP I NUMBER OF TARGET PLANET

SNDCSA O SINE OF ECLIPTIC DECLINATION OF SPIN AXIS

SNRASA O SINE OF ECLIPTIC RIGHT ASCENSION OF SPIN

AXIS

UCNTRL I MINIPROBE RELEASE CONTROL VECTOR

VSCRP I PLANETOCENTRIC ECLIPTIC VELOCITY VECTOR OF

BUS AT MINIPROBE RELEASE IN KM/SEC

SUBROUTINES SUPPORTED: TPPROP TPRTRG

SUBROUTINES REQUIRED: EPHEM ORB SCAR

LOCAL SYMBOLS: HDOTY DOT PRODUCT OF PLANETOCENTRIC BUS VELOCITY

WITH CROSS PRODUCT OF PLANETOCENTRIC

VECTOR TO SUN

ONE CONSTANT 1.0

TEMP1 MAGNITUDE OF CROSS PRODUCT OF VECTOR TO

SUN BY ECLIPTIC POLE VECTOR .

VSCRPM MAGNITUDE OF PLANETOCENTRIC VELOCITY OF

BUS AT MINIPROBE RELEASE

ZERO CONSTANT 0.

COMMON COMPUTEDS NO

COMMON USED: XP

SUBROUTINE SAVMAT

PURPOSE: TO STORE N VALUES OF VECTOR A IN VECTOR B

CALLING SEQUENCE: CALL SAVMAT(A,8,N)

ARGUMENTS: A I VECTOR TO BE STORED

B O VECTOR IN WHICH A IS STORED

N I NUMBER OF ELEMENTS TO BE TRANSFERRED

SUBROUTINES SUPPORTED: PRED GENGIO

SUBROUTINE SCAD

PURPOSE & TO CALCULATE BOTH SINE AND COSINE OF AN ANGLE IN DEGREES

ARGUMENT: CSA O COSINE OF ANGLE

DA I ANGLE IN DEGREES

SNA O SINE OF ANGLE

SUBROUTINES SUPPORTED: DIMPCP IMPCT

LOCAL SYMBOLS: DTR CONVERSION FACTOR FROM DEGREES TO RADIANS

ONE CONSTANT ONE

QREV 90. DEGREES

RA ANGLE IN RADIANS

REV 360. DEGREES

THOREV 270. DEGREES

SUBROUTINE SCAR

PURPOSE: TO CALCULATE BOTH SINE AND COSINE OF AN ANGLE GIVEN IN RADIANS

ARGUMENT: CSA O COSINE OF ANGLE

RA I ANGLE IN RADIANS

SNA O SINE OF ANGLE

SUBROUTINES SUPPORTED: SAOCS TPPROP

LOCAL SYMBOLS: HALFPI CONSTANT PI/2

ONE CONSTANT 1.

THHFPI CONSTANT 3.*PI/2.

TWOPI CONSTANT 2.*PI

SUBROUTINE SCHED

PURPOSE: TO DETERMINE WHAT TYPE OF MEASUREMENT IS TO BE TAKEN NEXT AND AT WHAT TIME IT WILL OCCUR.

CALLING SEQUENCE: CALL SCHED(T1,T2,MMCODE, IPOINT)

ARGUMENT: MMCODE O MEASUREMENT MODEL CODE

T1 I PRESENT TRAJECTORY TIME

T2 O TRAJECTORY TIME AT WHICH THE NEXT MEASUREMENT OCCURS

IPOINT I =1 FOR BUS

=2 FOR MAIN PROBE =3 FOR MINI-PROBE

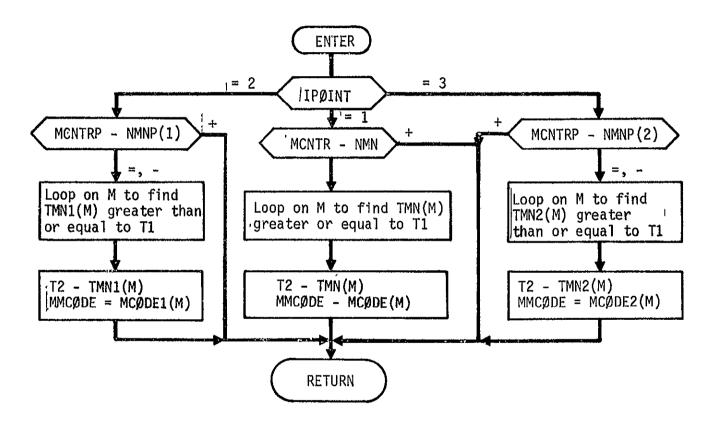
SUBROUTINES SUPPORTED: SIMULL ERRANN

LOCAL SYMBOLS: M INDEX

COMMON USED 8 MCNTR MCODE NMN TMN

MCNTRP NMNP TMN1 MCODE1 TMN2

MCODE 2



SUBROUTINE SERIE

PURPOSE: TO COMPUTE THE TRANSCENDENTAL FUNCTIONS USED IN FLITE.

CALLING SEQUENCE: CALL SERIE(X,SX,CX)

ARGUMENTS 8 X I INDEPENDENT VARIABLE

SX O BATTIN S-FUNCTION OF X

CX O BATTIN C-FUNCTION OF X

SUBROUTINES SUPPORTED: FLITE

SUBROUTINES REQUIRED: NONE

LOCAL SYMBOLS: COSH STATEMENT FUNCTION FOR HYPERBOLIC COSINE

SINH STATEMENT FUNCTION FOR HYPERBOLIC SINE

E SQRT OF ABS VALUE OF X

### SERIE Analysis

SERIE computes the transcendental functions S(x) and C(x) used in the FLITE program in the solution of Lambert's theorem.

The functions S(x) and C(x) are defined by

$$S(x) = \frac{\sqrt{x - \sin \sqrt{x}}}{3} \qquad x > 0$$

$$= \frac{\sinh \sqrt{-x - \sqrt{-x}}}{\sqrt{-x}} \qquad x < 0$$

$$= \frac{1}{6} \qquad x = 0 \qquad (1)$$

$$C(x) = \frac{1 - \cos \sqrt{x}}{x} \qquad x > 0$$

$$= \frac{\cosh \sqrt{-x - 1}}{-x} \qquad x < 0$$

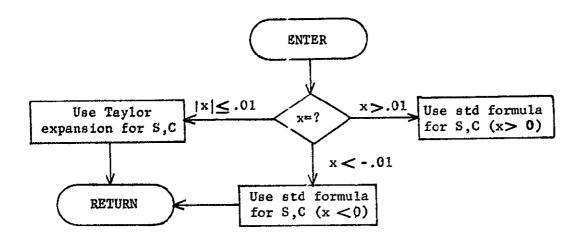
$$= \frac{1}{2} \qquad x = 0 \qquad (2)$$

For small values of | x | the Taylor series expansions are used

$$S(x) = \frac{1}{3!} - \frac{x}{4!} + \frac{x^2}{5!} + \dots$$

$$C(x) = \frac{1}{2!} - \frac{x}{3!} + \frac{x^2}{4!} + \dots$$
(3)

## SERIE Flow Chart



SUBROUTINE SETEVN

PURPOSE PERFORM ALL COMPUTATIONS COMMON TO MOST EVENTS IN THE ERROR ANALYSIS PROGRAM

CALLING SEQUENCE: CALL SETEVN(RI, TEVN, NCODE)

ARGUMENT: NCODE I EVENT CODE

RI I TARGETED NOMINAL SPACECRAFT STATE AT PREVIOUS MEASUREMENT OR EVENT TIME

TEVN I EVENT TIME

SUBROUTINES SUPPORTED: ERRANN

SUBROUTINES	REQUIRED:	CORREL	DYNO	HYELS	JACOBI	GNAVM
					TITLE	TRAPAR
		GPRINT	MEAN	EIGHY		

LOCAL SYMBOLS BLANK DUMMY CALLING ARGUMENT

EGVCT ARRAY OF EIGENVECTORS CORRESPONDING TO THE COLUMNS OF A GIVEN MATRIX

EGVL ARRAY OF EIGENVALUES RELATED TO THE EIGENVECTORS CONTAINED IN EGVCT

EXTIJ INTERMEDIATE VARIABLE

ICODE INTERNAL CONTROL FLAG

OUT SQUARE ROOTS OF EIGENVALUES

PEIG MATRIX FOR WHICH HYPERELLIPSOID IS TO BE

COMPUTED

RF NOMINAL SPACECRAFT STATE AT EVENT TIME

VEIG MATRIX TO BE DIAGONALIZED

COMMON COMPUTED/USED: TRTM1 XF

COMMON COMPUTED: DELTM XI

COMMON USED: CXSU CXSV CXU CXV CXXS FOV IEIG IHYP1 IPRT FOP NTMC PS þ ISTMC Q

XLAB UD Vũ G۷ GCXW GCXSW IGEN GU GP CCXXS GCXU GCXV GPS GCXSU GCXSV QPR RPR EXT

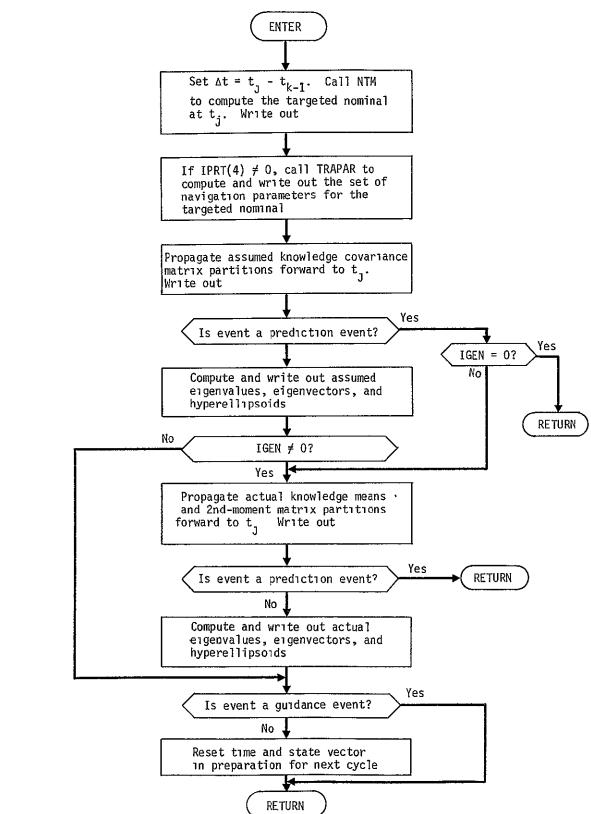
EXST

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## SETEVN Analysis

Before executing any event in the error analysis/generalized covariance analysis program subroutine SETEVN is called to perform a series of computations that are common to all events. Subroutine SETEVN computes the targeted nominal trajectory at t and propagates the assumed and actual knowledge covariance partitions at  $t_{k-1}$  the time of the previous event or measurement — forward to time t using the propagation equations found in subroutine GNAVM. The actual estimation error means are also propagated forward to t using the propagation equations found in subroutine MEAN.

For any event other than a prediction event, subroutine SETEVN also computes eigenvalues, eigenvectors, and hyperellipsoids of the position and velocity partitions of the assumed and actual knowledge covariance at  $t_i$ .



SUBROUTINE SETEVS

PURPOSE PERFORM ALL COMPUTATIONS COMMON TO MOST EVENTS IN THE SIMULATION PROGRAM

CALLING SEQUENCE: CALL SETEVS(RI, TEVN, RI1, NCODE)

ARGUMENTS NCODE I EVENT CODE

RI I TARGETED NOMINAL SPACECRAFT STATE AT PREVIOUS MEASUREMENT OR EVENT TIME

RI1 I MOST RECENT NOMINAL SPACECRAFT STATE AT PREVIOUS MEASUREMENT OR EVENT TIME

TEVN I EVENT TIME

SUBROUTINES SUPPORTED: SIMULL

SUBROUTINES REQUIRED: CORREL DYNOS HYELS JACOBI NAVM
NTHS PSIM STMPR TITLES TRAPAR

LOCAL SYMBOLS DUM INTERMEDIATE VECTOR

EGVT ARRAY OF EIGENVECTORS

EGVL ARRAY OF EIGENVALUES

ICODE INTERNAL CONTROL FLAG

OUT SQUARE ROOTS OF EIGENVALUES

PEIG MATRIX WHOSE HYPERELLIPSOID IS TO BE COMPUTED

RF1 MOST RECENT NOMINAL SPACECRAFT STATE AT EVENT TIME

RF TARGETED NOMINAL SPACECRAFT STATE AT EVENT TIME

VEIG MATRIX TO BE DIAGONALIZED

COMMON COMPUTED/USED: ADEVX EDEVX TRTH1 XF1 XF XI1 ZF ZI

COMMON COMPUTED: DELTM XI

COMMON USED® ADEVXS CXSU CXSV CXU CXV CXXS EDEVXS FOP FOV IEIG

IHYP1 IPRT ISTMC NDIM1 NGE

NQE NTMC PHI PS P Q TXXS U0 V0 W XLAB

### SETEVS Analysis

Prior to executing any event in the simulation mode, subroutine SETEVS is called to perform a series of computations which are common to all events. After computing the targeted nominal and most recent nominal states at the time of the event  $t_{\mbox{\scriptsize j}}$ , knowledge covariance partitions are propagated forward to time  $t_{\mbox{\scriptsize j}}$  from time  $t_{\mbox{\scriptsize k-1}}$  of the previous event or measurement using the prediction equations found in the NAVM Analysis section. The actual trajectory state at  $t_{\mbox{\scriptsize j}}$  is computed using

$$X_i = Z_j + \omega_i$$

where Z is the actual trajectory state assuming no unmodeled acceleration has been acting on the spacecraft, and  $\omega_{\rm j}$  is the contribution of the actual unmodeled acceleration to the actual trajectory state at t . The actual and predicted position/velocity deviations from the most recent nominal at t is are given by

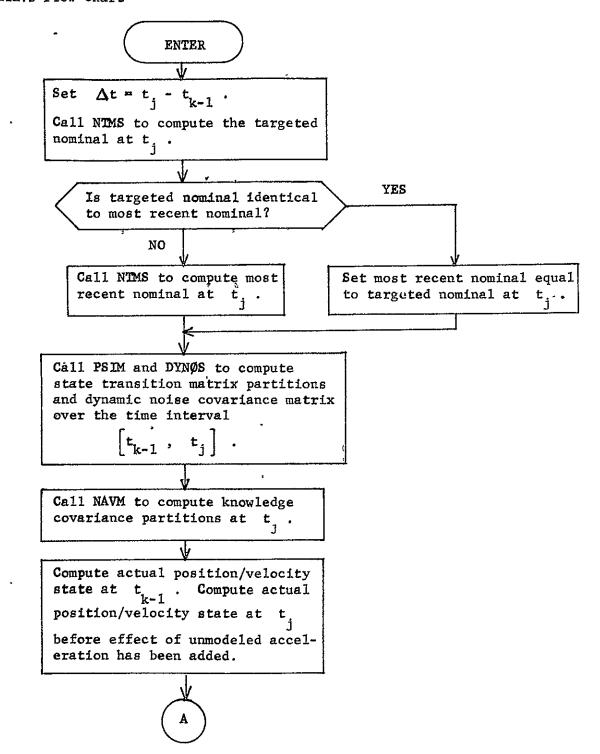
$$\delta \widetilde{X}_{j} = X_{j} - \widetilde{X}_{j}$$

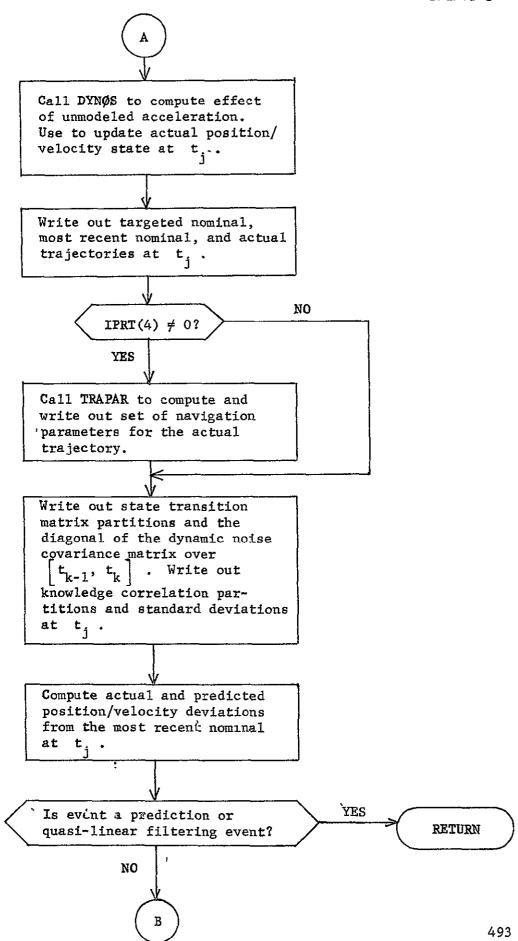
$$\delta \widetilde{X}_{j} = \Phi(t_{j}, t_{k-1}) \delta \widetilde{\hat{X}}_{k-1} + \theta_{XX_{s}} (t_{j}, t_{k-1}) \delta \widetilde{\hat{X}}_{s_{j}},$$

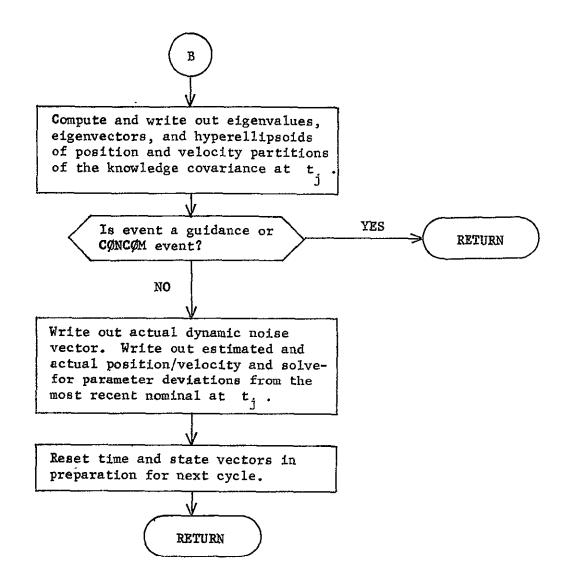
respectively, where  $\Phi$  and  $\theta_{xx_8}$  are the state transition matrix partitions over  $\left[\ t_{k-1},\ t_j\ \right]$  .

For any event other than prediction and quasi-linear filtering events, subroutines SETEVS also computes eigenvalues, eigenvectors, and hyperellipsoids of the position and velocity partitions of the knowledge covariance at  $t_i$ .

#### SETEVS Flow Chart







PROGRAM SIMUL

PURPOSE: TO CONTROL THE COMPUTATIONAL FLOW THROUGH THE BASIC

CYCLE (MEASUREMENT PROCESSING) AND ALL EVENTS IN THE

SIMULATION PROGRAM

SUBROUTINES SUPPORTED: MAIN

SUBROUTINES REQUIRED: BIAS DYNOS NENOS NAVM NTMS

TRAKS PRINT4 PSIM SCHED SETEVS

LOCAL SYMBOLS BYAL ACTUAL MEASUREMENT BIAS VECTOR

DUMM INTERMEDIATE VARIABLE

DUM INTERMEDIATE VECTOR

IPRN MEASUREMENT PRINT TIME COUNTER

MMCODE MEASUREMENT CODE

NEVENT EVENT COUNTER

RNUM RANDOM MEASUREMENT NOISE

TRTM2 TIME OF THE MEASUREMENT

COMMON COMPUTED/USED: ADEVX ANGIS AY EDEVXS EDEVX EY ICODE MONTR NAFC RES RF1 TRTM1 RI1 RI TEVN XF1 XF XII XI ZF

ZI

COMMON COMPUTED: AYMEY DELTH EDEVSM. EDEVXM

COMMON USEDS AR AK FNTM AM М **IEVNT IPRINT** NAF6 ISTMC NAE NGE NDIM1 NEA NMN NGE

NR NIMC PHI RF TEV TXXS W ZERO

S

SIMUL Analysis

The primary function of subroutine SIMUL is to control the computational flow through the basic cycle (measurement processing) and all events in the simulation mode. Subroutine SIMUL also performs some computations in the basic cycle. All event-related analysis is presented in the event subroutines themselves and will not be treated below.

In the basic cycle the first task of SIMUL is to control the generation of targeted nominal and most recent nominal spacecraft states,  $\overline{X}_{k+1}$  and  $\overline{X}_{k+1}$ , respectively, at time t, given states  $\overline{X}_k$  and  $\overline{X}_k$  at time t,  $\overline{X}_k$ . Then, calling PSIM, DYNØS, TRAKS, and MENØS, successively, SIMUL controls the computation of all matrix information required by subroutine NAVM in order to compute the covariance matrix partitions at time t, immediately following the measurement.

After computing the actual state  $X_k$  at time  $t_k$  from

$$X_{k} = X_{k} + \delta X_{k}$$
 (1)

where  $\tilde{\mathfrak{d}}_k^X$  is the actual spacecraft state deviation from the most recent nominal, SIMUL controls the generation of the actual state  $Z_{k+1}$  at time  $t_k$  before the effect of unmodeled acceleration has been added. Then, having called DYNØS to compute the effect of unmodeled acceleration  $\omega_{k+1}$ , SIMUL computes the actual state and actual state deviation at time  $t_{k+1}$ :

$$X_{k+1} = Z_{k+1} + \omega_{k+1}$$
 (2)

$$\delta \tilde{x}_{k+1} = x_{k+1} - \tilde{x}_{k+1} \tag{3}$$

With both the most recent nominal and actual spacecraft states available at  $t_{k+1}$ , SIMUL calls TRAKS twice in succession to compute the ideal measurements  $\tilde{Y}_{k+1}$  and  $\tilde{Y}_{k+1}$ , respectively, which would be made at each of these trajectory states. Calling MENØS, RNUM, and BIAS to compute the actual measurement noise and bias corrupting the ideal measurement associated with the actual state, SIMUL computes the actual measurement at time  $t_{k+1}$  using

$$y_{k+1}^a = \underline{y}_{k+1} + b_{k+1} + \nu_{k+1}$$
 (4)

where  $\mathbf{b}_{\mathbf{k}+1}$  and  $\boldsymbol{\nu}_{\mathbf{k}+1}$  represent the actual measurement bias and noise, respectively.

All information required for computing both predicted and filtered state deviations from the most recent nominal at  $t_{k+1}$  is now available. With  $otation{\mbox{\boldmath $\phi$}}$  and  $\theta_{xx_s}$  denoting state transition matrix partitions over the time interval  $\left[t_k, t_{k+1}\right]$ , SIMUL computes the predicted spacecraft state deviations and solve-for parameter deviations at  $t_{k+1}$  using

$$\delta \tilde{x}_{k+1} = \Phi \delta \tilde{x}_{k}^{+} + \theta_{xx_{s}} \delta \tilde{x}_{s_{t}}^{+}$$
 (5)

$$\delta \tilde{X}_{s_{k+1}} = \delta \tilde{X}_{s_{k}} + \tag{6}$$

Prior to computing filtered deviations, SIMUL computes the measurement residual from

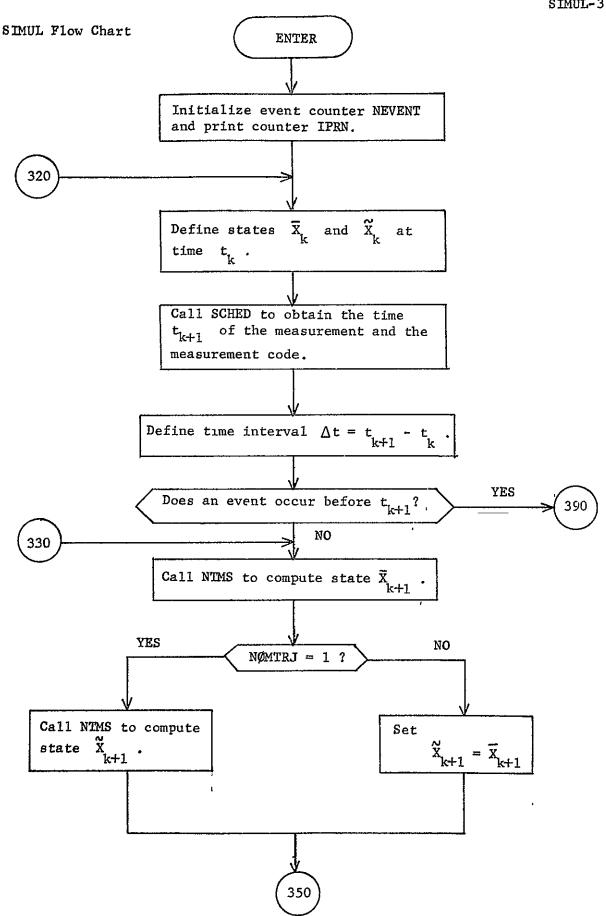
$$\epsilon_{k+1} = (Y_{k+1}^{a} - \tilde{Y}_{k+1}^{a}) - H_{k+1} \delta \hat{\tilde{X}}_{k+1}^{-} - M_{k+1} \delta \hat{\tilde{X}}_{s_{k+1}}^{-}$$
 (7)

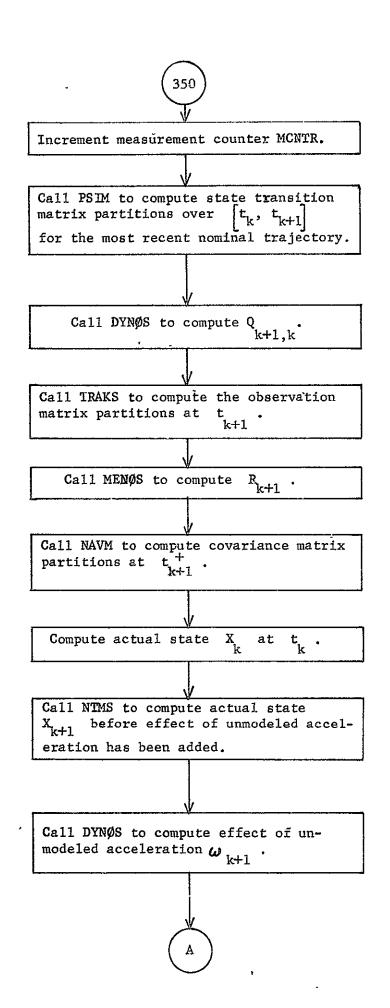
where H and M are observation matrix partitions. Filtered k+1 k+1 spacecraft state deviations and solve-for parameter deviations are then computed from

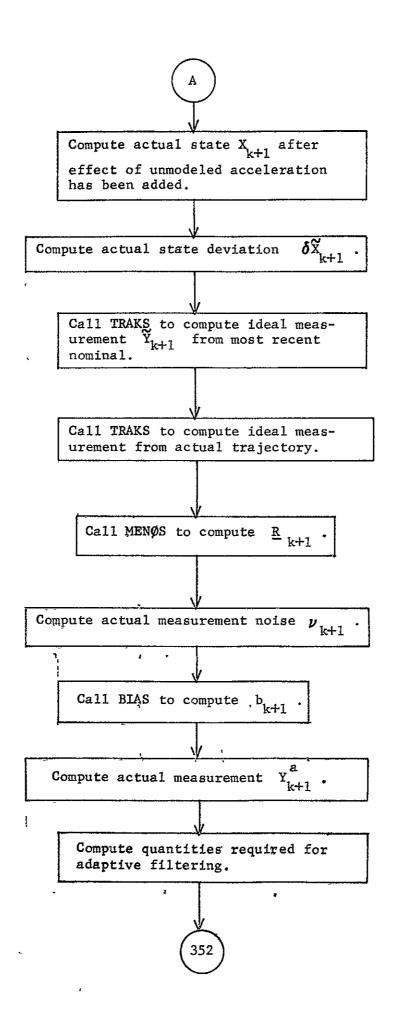
$$\delta \hat{\tilde{x}}_{k+1}^{+} = \delta \hat{\tilde{x}}_{k+1}^{-} + K_{k+1} \epsilon_{k+1}$$
(8)

$$\delta \hat{\hat{\mathbf{x}}}_{\mathbf{s}_{k+1}}^{+} = \delta \hat{\mathbf{x}}_{\mathbf{s}_{k+1}}^{-} + \mathbf{s}_{k+1} \boldsymbol{\epsilon}_{k+1}$$
(9)

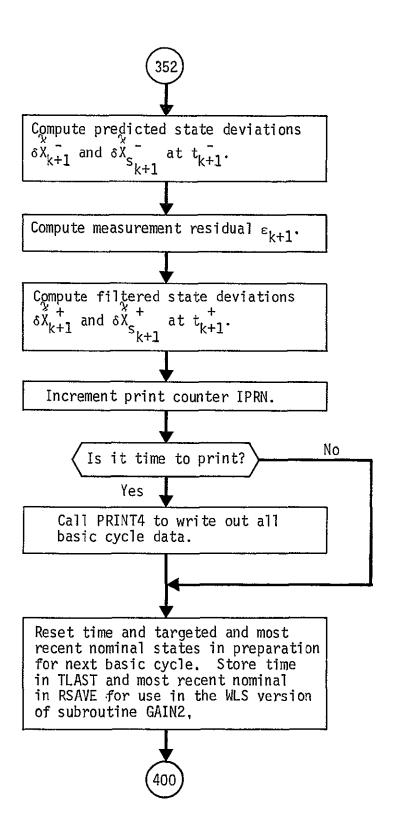
where  $K_{k+1}$  and  $S_{k+1}$  are the filter gain constants.

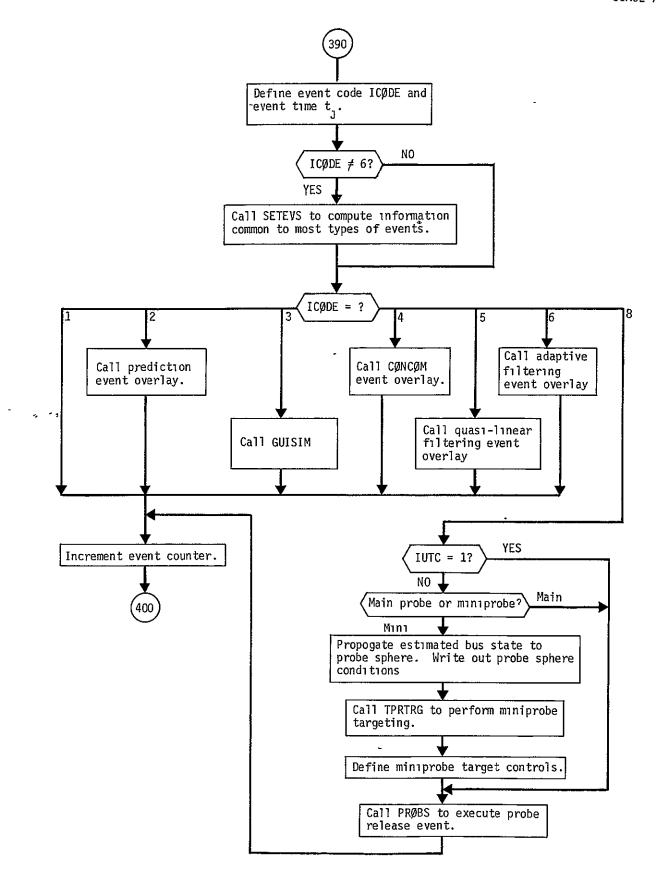


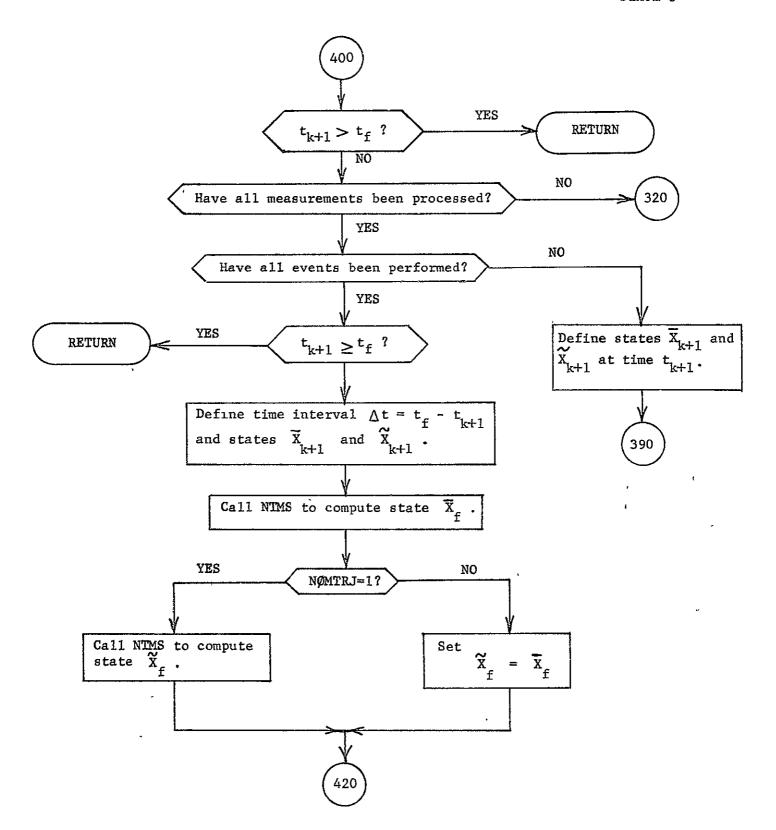


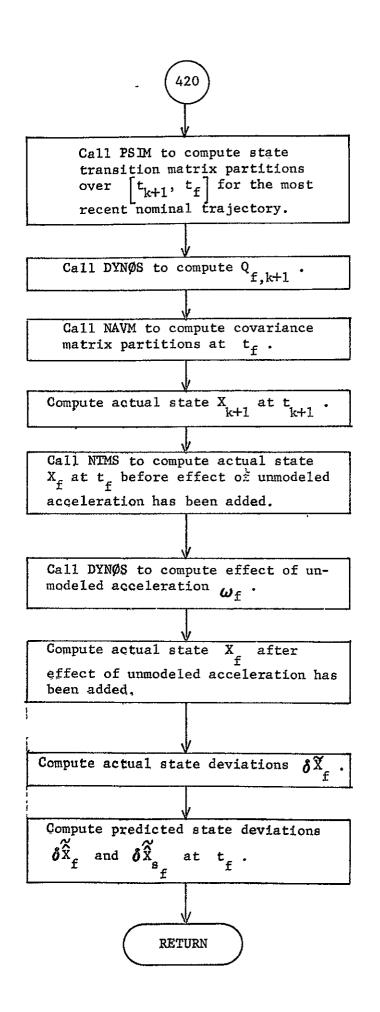


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SUBROUTINE SKEDM

PURPOSE: TO SET UP THE MEASUREMENT SCHEDULE USED INTERNALLY

CALLING SEQUENCE: CALL SKEDM(NENT, TMN, MCODE, NMN, FNTM, INUM, IFLG)

ARGUMENTS: NENT I NUMBER OF MEASUREMENT CARDS

TMN O VECTOR OF TIMES OF MEASUREMENTS

MCODE O VECTOR OF HEASUREMENT CODES CORRESPONDING

TO ENTRIES IN TMN

NMN O MAXIMUM NUMBER OF ENTRIES IN THN VECTOR

FNTM I FINAL TRAJECTORY TIME

INUM I DIMENSION OF THN AND MCODE VECTORS

IFLG I INDEX FOR PRINTING TITLE

SUBROUTINES SUPPORTED: DATA1 DATA1S

LOCAL SYMBOLS: AMIN INTERMEDIATE TIME VARIABLE

AP INTERMEDIATE TIME ARRAY

EP50 10. TO 50TH POWER

ICNT COUNTER

IROW ROW INDEX

MEAS MEASUREMENT CODE ARRAY

SCHED' MEASUREMENT TIME ARRAY

XLAB LABEL

SUBROUTINE SOIPS

PURPOSE: TO CONICALLY EXTRAPOLATE FROM NEAREST INTEGRATION STATE TO OBTAIN IMPACT DATA AT SOI AND AT PLANET SURFACE

ARGUMENT: ISOIPS I FLAG INDICATION OPERATING MODE OF SUBROUTINE

= 1 EXTRAPOLATE FOR SOI IMPACT DATA

= 2 EXTRAPOLATE FOR PLANET IMPACT DATA

RSPHM O MAGNITUDE OF PLANETOCENTRIC POSITION VECTOR OF TRAJECTORY AT SPHERE OF INTEREST IN KM PLANETOCENTRIC ECLIPTIC

RSPH O PLANETOCENTRIC ECLIPTIC POSITION VECTOR OF TRAJECTORY AT SPHERE OF INTEREST IN KM

TMU I GRAVITATIONAL CONSTANT OF TARGET PLANET IN KM**3/SEC**2

VSPHM O MAGNITUDE OF PLANETOCANTRIC VELOCITY OF TRAJECTORY AT SPHERE OF INTEREST IN KM/SEC

VSPH O PLANETOCENTRIC ECLIPTIC VELOCITY VECTOR OF TRAJECTORY AT SPHERE OF INTEREST IN KM/SEC

SUBROUTINES SUPPORTED: VMP

SUBROUTINES REQUIRED: CAREL ELCAR PECEQ SUBSOL

LOCAL SYMBOLS: AS SEMIMAJOR AXIS OF PLANETOCENTRIC OSCULATING CONIC IN KM

CSTAS COSINE OF TRUE ANOAMOLY OF PLANETOCENTRIC OSCULATING CONIC AT SPHERE OF INTEREST

DJEPOC JULIAN DATE EPOCH 1900 WHEN TRAJECTORY PIERCES SPHERE OF INTEREST

DTNIS TIME INTRRVAL IN SECONDS ALONG PLANETOCENTRIC OSCULATING CONIC FROM NEAREST INTEGRATION STATE INSIDE SPHERE OF INTEREST TO SPHERE ITSELF

DTPNI TIME INTERVAL IN SECONDS ALONG PLANETOCENTRIC OSCULATING CONIC FROM PERAPSIS TO NEAREST INTEGRATION STEP INSIDE SPHERE OF INTEREST

OTPS TIME INTERVAL IN SECONDS ALONG PLANETOCENTRIC OSCULATING CONIC FROM PERIAPSIS TO SPHERE OF INTEREST

DTS CONVERSION FACTOR FROM DAYS TO SECONDS

ES ECCENTRICITY OF PLANETOCENTRIC OSCULATING CONIC

PAS PLANETOCENTRIC ECLIPTIC UNIT VECTOR POINTING IN DIRECTION OF PERIAPSIS OF PLANETOCENTRIC / OSCULATING CONIC

PS SEMILATUS RECTUM OF PLANETOCENTRIC OSCULATING CONIC IN KM

QAS PLANETOCENTRIC ECLIPTIC UNIT VECTOR LYING IN ORBITAL PLANE OF PLANETOCENTRIC OSCULATING CONIC 9 DEGREES ADVANCED FROM PA

RS RADIUS OF SPHERE OF INTEREST IN KM

RTD CONVERSION FACTOR FROM RADIANS TO DEGREES

SNTAS SINE OF TRUE ANAMOLY OF PLANETOCENTRIC OSCULATING CONIC AT SPHERE OF INTEREST

TAS TRUE ANAMOLY OF PLANETOCENTRIC OSCULATING CONIC AT NEAREST INTEGRATION STATE INSIDE SPHERE OF INTEREST

TRANSF TRANSFORMATION MATRIX FROM ECLIPTIC TO PROBE-SPHERE COORDINATES

WAS PLANETOCENTRIC ECLIPTIC UNIT VECTOR POINTING IN DIRECTION OF ANGULAR MOMENTUM OF PLANETOCENTRIC OSCULATION CONIC

WS ECLIPTIC ARGUMENT OF PERIAPSIS IN DEGREES OF PLANETOCENTRIC OSCULATING CONIC

XIS ECLIPTIC INCLINATION IN DEGREES OF PLANETOCENTRIC OSCULATING CONIC

XNS ECLIPTIC RIGHT ASCENSION OF ASCENDING NODE IN DEGREES OF PLANETOCENTRIC OSCULATING CONIC

COMMON COMPUTED/USED: DEPOC D

COMMON COMPUTED: DCIMP RAIMP

COMMON USED: ALNGTH IPCSP IP NTP ONE RADIUS RPSP SPHERE

SUBROUTINE SPHIMP

PURPOSE: TO CALCULATE TRUE ANAMOLY AND TIME FROM PERIAPSIS AT WHICH CONI APPROACH TRAJECTORY PIERCES PLANETOCENTRIC SPHERE OF GIVEN RADIUS

ARGUMENT: A I SEMIMAJOR AXIS OF CONIC IN KM

CSTAI O COSINE OF TRUE ANAMOLY AT SPHERE OF INTEREST

DTPI O TIME INTERVAL IN SECONDS ON CONIC FROM PERIAPSIS T SPHERE OF INTEREST

E I 'ECCENTRICITY OF CONIC

GMUP I GRAVITATIONAL CONSTANT OF PLANET IN KM***3/SEC**2

RS I RADIUS OF SPHERE OF INTEREST IN KM

SNTAI O SINE OF TRUE ANAMOLY AT SPHERE OF INTEREST

SUBROUTINES SUPPORTED: TPPROP TPRTRG |

SUBROUTINES REQUIRED: HYPT

LOCAL SYMBOLS: ONE CONSTANT 1.

ORBH RECIPROCAL OF MEAN ORBITAL RATE IN SEC/RADIANS

P SEMILATUS RECTUM OF CONIC IN KM

SUBROUTINE SPACE

PURPOSE® COUNTS THE NUMBER OF LINES BEING PRINTED TO DETERMINE WHEN TO SKIP TO THE NEXT PAGE WITH A NEW HEADING

CALLING SEQUENCE CALL SPACE(LINES)

ARGUMENT LINES I NUMBER OF LINES THAT WILL BE WRITTEN IN THE NEXT OUTPUT STATEMENT

SUBROUTINES SUPPORTED: INPUTZ PRINT VECTOR VMP

SUBROUTINES REQUIRED: NEWPGE

COMMON COMPUTED/USED8 LINCT

COMMON USED: LINPGE

SUBROUTINE STAPRL

PURPOSE: TO COMPUTE THE PARTIAL DERIVATIVES OF STATION LOCATION ERRORS.

CALLING SEQUENCE: CALL STAPRL(AL, ALON, ALAT, PAT2, VEC, PA)

ARGUMENTS AL I ALTITUDE OF THE STATION

ALAT I LATITUDE OF THE STATION

ALON I LONGITUDE OF THE STATION

PA O PARTIAL OF STATION POSITION AND VELOCITY WITH RESPECT TO ALTITUDE, LATITUDE AND LONGITUDE

PAT2 I LONGITUDE + OMEGA* (CURRENT TIME-LAUNCH TIME)

VEC UNUSED

SUBROUTINES SUPPORTED & TRAKE TRAKE

LOCAL SYMBOLS: G1 SINE OF LATITUDE

G2 COSINE OF LATITUDE

G3 SINE (PHI + OMEGA (T-UNIVT))

G4 COSINE(PHI +OMEGA(T-UNIVT))

WHERE PHI =LONGITUDE

OMEGA=EARTH ROTATION RATE

T =TIME

UNIVT=UNIVERSAL TIME ..

G5 SINE OF OBLIQUITY OF EARTH

G6 COSINE OF OBLIQUITY OF EARTH

OMEG OMEGA IN PROPER UNITS

COMMON USED8 EPS OMEGA TM

## STAPRL Analysis

The ecliptic components of the position and velocity of a tracking station relative to the Earth are related to station location parameters R,  $\theta$ , and  $\phi$  through the following set of equations:

$$X_s = R \cos \theta \cos G$$
 $Y_s = R \cos \theta \cos \epsilon \sin G + R \sin \theta \sin \epsilon$ 
 $Z_s = -R \cos \theta \sin \epsilon \sin G + R \sin \theta \cos \epsilon$ 
 $\dot{X}_s = -\omega R \cos \theta \sin G$ 
 $\dot{Y}_s = \omega R \cos \theta \cos \epsilon \cos G$ 
 $\dot{Z}_s = -\omega R \cos \theta \sin \epsilon \cos G$ 

where  $G = \phi + \omega (t - T)$ , and T is the universal time at some epoch (usually launch time).

Subroutine STAPRL computes the negative of the partials of the previous quantities with respect to the station location parameters. R,  $\theta$ , and  $\phi$ . These partials are summarized below:

$$-\frac{\partial X_{g}}{\partial R} = -\cos \theta \cos G$$

$$-\frac{\partial X_{g}}{\partial \theta} = R \sin \theta \cos G$$

$$-\frac{\partial X_{g}}{\partial \phi} = R \cos \theta \sin G$$

$$-\frac{\partial Y_{g}}{\partial R} = -\left[\sin \epsilon \sin \theta + \cos \epsilon \cos \theta \sin G\right]$$

$$-\frac{\partial Y_{g}}{\partial \theta} = R \cos \epsilon \sin \theta \sin G - R \sin \epsilon \cos \theta$$

$$-\frac{\partial Y_{g}}{\partial \phi} = -R \cos \epsilon \cos \theta \cos G$$

$$-\frac{\partial Z_{g}}{\partial R} = \sin \epsilon \cos \theta \sin G - \cos \epsilon \sin \theta$$

$$-\frac{\partial Z_B}{\partial \theta} = -\left[R \sin \epsilon \sin \theta \sin G + R \cos \epsilon \cos \theta\right]$$

$$-\frac{\partial Z_B}{\partial \phi} = R \sin \epsilon \cos \theta \cos G$$

$$-\frac{\partial \dot{X}_B}{\partial R} = \omega \cos \theta \sin G$$

$$-\frac{\partial \dot{X}_B}{\partial \theta} = -\omega R \sin \theta \sin G$$

$$-\frac{\partial \dot{X}_B}{\partial \phi} = \omega R \cos \theta \cos G$$

$$-\frac{\partial \dot{X}_B}{\partial \theta} = -\omega \cos \theta \cos G$$

$$-\frac{\partial \dot{X}_B}{\partial \theta} = \omega R \cos \theta \cos G$$

$$-\frac{\partial \dot{X}_B}{\partial \theta} = \omega R \cos \epsilon \sin \theta \cos G$$

$$-\frac{\partial \dot{X}_B}{\partial \theta} = \omega R \cos \epsilon \cos \theta \sin G$$

$$-\frac{\partial \dot{X}_B}{\partial \phi} = \omega R \cos \epsilon \cos \theta \sin G$$

$$-\frac{\partial \dot{X}_B}{\partial \phi} = \omega R \cos \epsilon \cos \theta \cos G$$

$$-\frac{\partial \dot{X}_B}{\partial \phi} = \omega R \cos \epsilon \cos \theta \cos G$$

$$-\frac{\partial \dot{X}_B}{\partial \phi} = \omega R \sin \epsilon \cos \theta \cos G$$

 $-\frac{\partial \dot{z}_{s}}{\partial \phi} = -\omega R \sin \epsilon \cos \theta \sin G$ 

SUBROUTINE	STI	MP	
PURPOSE :			-PLANE ASYMPTOTE PIERCE-POINT COORDINATES LANE OF A HYPERBOLA, GIVEN THE STATE
ARGUMENT :	A	1	SEMI-MAJOR AXIS OF HYPERBOLA IN KM
	BDR	0	PROJECTION OF BV ON RV (B.R) IN KM
	80 T	0	PROJECTION OF BV ON TV (B.T) IN KM
	В	0	MAGNITUDE OF BV IN -KM
	GMX	I	GRAVITATIONAL CONSTANT OF PLANET
	RV	0	PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN DIRECTION OF CROSS PRODUCT OF HYPERBOLA ASYMPTOTE BY ECLIPTIC POLE VECTOR
	R	I	PLANETOCENTRIC ECLIPTIC POSITION VECTOR OF GIVEN HYPERBOLA STATE IN KM
	SV	0	PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN DIRECTION OF HYPERBOLA ASYMPTOTE
	TV	0	CROSS PRODUCT OF SV AND RV
	٧		PLANETOCENTRIC ECLIPTIC VELOCITY VECTOR OF GIVEN HYPERBOLA STATE IN KM/SEC
SUBROUTINES SUPPORTED:		PORTED:	TPPROP TPRTRG
SUBROUTINES REQUIRED:			USCALE UXV
LOCAL SYME	OLS :	AB	PRODUCT OF HYPERBOLA ECCENTRICITY AND MAGNITUDE OF SEMI-MAJOR AXIS IN KM
		BV	PLANETOCENTRIC ECLIPTIC VECTOR IN KM TO HYPERBOLA ASYMPTOTE PIERCE POINT IN IMPACT PLANE
		CTA	COSINE OF TRUE ANOMALY OF GIVEN STATE
		C1	MAGNITUDE OF HYPERBOLA ANGULAR MOMENTUM
		DUM	MAGNITUDE OF CROSS PRODUCT OF SV AND ECLIPTIC POLE VECTOR
		E	ECCENTRICITY OF HYPERBOLA
		ONE	CONSTANT = 1.
		PV	PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN

	DIRECTION OF HYPERBOLA PERIAPSIS
p	SEMI-LATUS RECTUM OF HYPERBOLA IN KM
ōΛ	PLANETOCENTRIC ECLIPTIC UNIT VECTOR LYING IN HYPERBOLA PLANE AND ADVANCED 90 DEGREES FROM PA
RD	TIME RATE OF CHANGE OF HYPERBOLA RADIUS AT GIVEN STATE
RM	MAGNITUDE OF PLANETOCENTRIC POSITION
RRD	PRODUCT OF HYPERBOLA RADIUS AND ITS TIME RATE OF CHANGE OF GIVEN STATE
STA	SINE OF TRUE ANOMALY
TWO	CONSTANT = 2.
٧S	SQUARED MAGNITUDE OF PLANETOCENTRIC VELOCITY OF GIVEN STATE
MV	PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN DIRECTION OF HYPERBOLA ANGULAR MOMENTUM
ZERO	CONSTANT = 0.
Z	PLANETOCENTRIC ECLIPTIC UNIT VECTOR LYING IN HYPERBOLA PLANE AND 90 DEGREES ADVANCED FROM POSITION VECTOR

STIMP Analysis

Subroutine STIMP converts a planetocentric ecliptic state vector  $(\underline{r}/\underline{v})^T$  to the more readily targetable impact plane coordinates B·T and B·R. These coordinates are preferred as target variables for two basic reasons: (1) they generally exhibit reasonably linear dependence on the targeting controls, and (2) in probe targeting they obviate the need for defining a pseudoimpact point when the probe misses the planet in an early interation.

The impact coordinates are defined in terms of the direction of the trajectory hyperbolic excess velocity,  $\underline{v}$ , and the north ecliptic pole vector,  $\underline{K}$ . Let

$$\underline{S} = \underline{v}_{\infty} v_{\infty} \tag{1}$$

$$\underline{\mathbf{T}} = \frac{\mathbf{S} \times \mathbf{K}}{\|\mathbf{S} \times \mathbf{K}\|} \tag{2}$$

$$\underline{\mathbf{R}} = \underline{\mathbf{S}} \times \underline{\mathbf{T}} \tag{3}$$

where all the vectors are assumed to originate at the planet center. Thus  $\underline{T}$ ,  $\underline{R}$  and  $\underline{S}$  form a right-handed Cartesian frame with  $\underline{S}$  pointing in the direction of the hyperbolic excess velocity and  $\underline{T}$  lying in the ecliptic plane. The plane containing the vector vector  $\underline{T}$  and  $\underline{R}$  is known as the impact plane.  $\underline{B}$  is defined as that unique vector from the planet center to the point where the trajectory asymptote pierces the impact plane.  $\underline{B}$   $\cdot \underline{T}$  and  $\underline{B}$   $\cdot \underline{R}$  are then simply the components of  $\underline{B}$  along the  $\underline{T}$  and  $\underline{R}$  axes, respectively. Figure 1 illustrates all of these terms for the case of a single probe trajectory.

In addition to calculating B·T and B.R, STIMP also makes available other approach trajectory parameters useful in the STEAP auxiliary targeting scheme. These are  $\underline{S}$ ,  $\underline{T}$ .  $\underline{R}$ , and  $\underline{B}$  in the inertial ecliptic frame as well as the approach hyperbola semimajor axis, a.

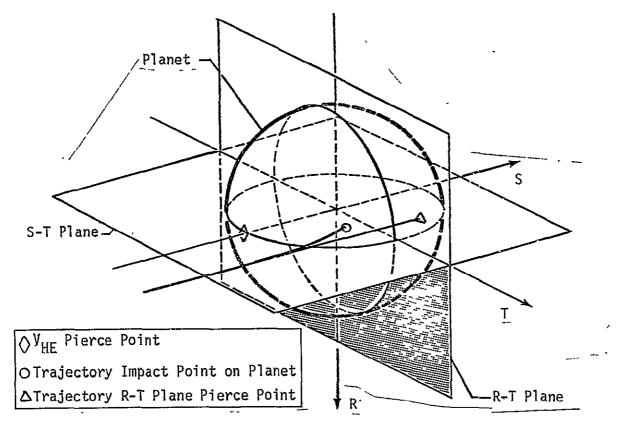


Figure 1 Single-Probe Auxiliary Targeting Illustration

The formulae used in calculating these elements are well known results derived in most engineering treatments of two-body motion (see, for example, Ref 2). Hence they will simply be listed here in the order they are used. The standard conic symbolism is used throughout:

$$\underline{\mathbf{h}} = \underline{\mathbf{r}} \times \underline{\mathbf{v}} \tag{4}$$

$$\underline{W} = \underline{h}/h \tag{5}$$

$$\dot{\mathbf{r}} = \underline{\mathbf{r}} \cdot \underline{\mathbf{v}} / \mathbf{r} \tag{6}$$

$$p = h^2/\mu \tag{7}$$

$$a = r/(2 - rv^2/\mu)$$
 (8)

$$e = \sqrt{1 - p/a} \tag{9}$$

$$\cos \theta = (p - r)/er \tag{10}$$

$$\sin \theta = \dot{r}h/e\mu$$
 (11)

$$B = \sqrt{p|a|}$$
 (12)

$$\underline{Z} = (\underline{r}\underline{v} - \dot{r}\underline{r})/h \tag{13}$$

$$\underline{P} = \frac{r}{r} \cos \theta - \underline{Z} \sin \theta \tag{14}$$

$$\underline{Q} = \frac{\mathbf{r}}{\mathbf{r}} \sin \theta + \underline{\mathbf{z}} \cos \theta \tag{15}$$

$$\underline{S} = \underline{P}/e + \frac{Q}{e}\sqrt{e^2 - 1} \tag{16}$$

$$\underline{B} = p\underline{P}/e + \frac{aQ}{e}\sqrt{e^2 - 1} \tag{17}$$

$$\underline{B} = pP/e + \frac{aQ}{e} \sqrt{e^2 - 1}$$

$$\underline{T} = \left(s_2^2 - s_1^2, 0\right)^T / \sqrt{s_1^2 + s_2^2}$$
(17)

$$\underline{\mathbf{R}} = \left( -\mathbf{S}_{3} \mathbf{T}_{2}, \ \mathbf{S}_{3} \mathbf{T}_{1}, \ \mathbf{S}_{1} \mathbf{T}_{2} - \mathbf{S}_{2} \mathbf{T}_{1} \right)^{T}$$
 (19)

$$B \cdot T = B_1 T_1 + B_2 T_2 \tag{20}$$

$$B \cdot R = B_1 R_1 + B_2 R_2 + B_3 R_3 \tag{21}$$

SUBROUTINE STMPR

PURPOSET TO PRINT OUT THE TRANSPOSES OF THE STATE TRANSITION MATRIX PARTITIONS PHI, TXXS, TXH, AND TXU OVER AN ARBITRARY INTERVAL OF TIME.

CALLING SEQUENCE: CALL STMPR(TRTM1, TRTM2)

ARGUMENT: TRTM1 I TIME AT BEGINNING OF INTERVAL OVER WHICH STATE TRANSITION MATRIX PARTITIONS HAVE BEEN COMPUTED

TRTM2 I TIME AT END OF INTERVAL OVER WHICH STATE TRANSITION MATRIX PARTITIONS HAVE BEEN COMPUTED

SUBROUTINES SUPPORTED: PRINT4 SETEVS GUISIM GUISS PRESIM PRINT3 SETEVN GUIDM GUID PRED PROBES

COMMON USED:

NDIM1 NDIM2 · PHI TXU TXXS

XLAB XSL XU

NDIM4 TXH

# SUBROUTINE SUB1

PURPOSES TO COMPUTE POSITION AND VELOCITY MAGNITUDES.

CALLING SEQUENCE: CALL SUB1(X,XE,XP)

ARGUMENT: X I INERTIAL POSITION/VELOCITY OF THE VEHICLE

XE I EARTH-S POSITION/VELOCITY

XP I POSITION/VELOCITY OF THE TARGET PLANET

SUBROUTINES SUPPORTED: PRINT4

LOCAL	SYMBOLS:	RX	MAGNITUDE	OF	INERTIAL	POSITION	4 VECTO	R
		RY	MAGNITUDE	OF	GEOCENTR	IC POSIT	ION VEC	TOR
		RZ	MAGNITUDE VECTOR	OF	PLANETOCE	ENTRIC P	OSITION	1
		٧x	MAGNITUDE	OF	INERTIA	r AEFOCI	TY VECT	OR
		VY	MAGNITUDE	OF	GEOCENTR:	IC VELOC	ITY VEC	TOR
		VZ	MAGNITUDE VECTOR	OF	PLANETOCI	ENTRIC V	ELOCITY	,
	Y	GEOCENTRIO VEHICLE	C P	OSITION/VI	ELOCITY	OF THE		
		2	PLANETOCE VEHICLE	NTR	IC POSITI	ONTAEFOC	ITY OF	THE

SUBROUTINE SUBSOL

PURPOSE'S TO COMPUTE TRANSFORMATION MATRIX FROM ECLIPTIC

TO SUBSOLAR ORBITAL COORDINATES

ARGUMENT: 0 I JULIAN DATE EPOCH 1900

EQSS O TRANSFORMATION FROM PLANETOCENTRIC

ECLIPTIC TO SUBSOLAR ORBITAL

NP I INDEX OF PLANET

SUBROUTINES SUPPORTED: SOIPS TARGET TPRTRG

SUBROUTINES REQUIRED: EPHEM ORB USCALE UXV

LOCAL SYMBOLS: C2 MAGNITUDE OF PLANETOCENTRIC VECTOR TO SUN

EXS PLANETOCENTRIC ECLIPTIC VECTOR TO SUN

EYS CROSS PRODUCT OF EZS AND EXS

EZS PLANETOCENTRIC ECLIPTIC UNIT VECTOR

IN DIRECTION OF ANGULAR MOMENTUM OF PLANET

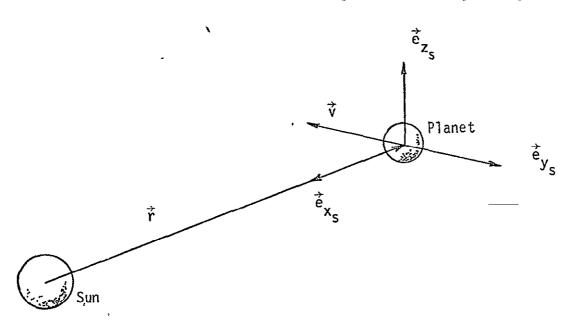
ONE CONSTANT = 1.

COMMON COMPUTED: NO

COMMON USED: XP

# SUBSØL Analysis

Subroutine SUBSØL computes the transformation from planetocentric ecliptic coordinates to subsolar planet orbital plane coordinates for an arbitrary planet. The subsolar planet orbital plane coordinate system is defined as the planetocentric system whose x-axis points directly at the sun, whose z-axis is normal to the planet's orbital plane, and whose y-axis is normal to the xz-plane and lies in the planet's orbital plane. In the figure below  $\dot{r}$  and  $\dot{v}$  denote the position and velocity vectors, respectively, of the planet relative to the sun. Unit vectors  $\dot{e}$ ,  $\dot{e}$ , and  $\dot{e}$  are aligned with the axes of the subsolar planet orbital plane system.



These unit vectors are defined as

$$\dot{e}_{x_s} = -\frac{\dot{r}}{r}$$

$$\dot{e}_{x_s} = \dot{e}_{x_s} \times \dot{e}_{x_s}$$

$$\dot{e}_{z_s} = \frac{\dot{r} \times \dot{v}}{|\dot{r} \times \dot{v}|}$$

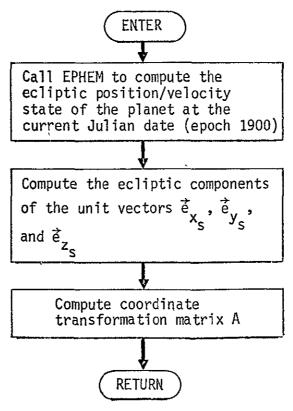
If these unit vectors are referred to the ecliptic coordinate system, the coordinate transformation A from planetocentric ecliptic to subsolar planet orbital plane coordinates is given by

$$A = \begin{bmatrix} \overset{\rightarrow}{e^{T}} \\ x_{s} \\ - & - & - \\ \overset{\rightarrow}{e^{T}} \\ y_{s} \\ - & - & - \\ \overset{\rightarrow}{e^{T}} \\ z_{s} \end{bmatrix}$$

Thus

$$\dot{\mathbf{x}}_{\text{subsolar}} = A \dot{\dot{\mathbf{x}}}_{\text{ecliptic}}$$

SUBSØL Flow Chart



SUBROUTINE TARGET

PURPOSES TO PERFORM EXECUTIVE FUNCTIONS OF THE TARGETING MODE AS CALLING REQUIRED SUBROUTINES TO READ THE INPUT DATA, COMPUTING THE ZERO ITERATE IF NECESSARY AND PERFORMING THE ACTUAL TARGETING THROUGH THE PROGRESSIVE STAGES USED BY STEAP.

CALLING SEQUENCES CALL TARGET

SUBROUTINES SUPPORTED: GIDANS

SUBROUTINES REQUIRED: TAROPT TARMAK DESENT PECEQ VMP

LOCAL SYMBOLS: ABV INTERMEDIATE VARIABLE USED TO LIMIT EACH

DELTAY COMPONENT CHANGE

ACC VECTOR OF ACCURACY LEVELS FOR THE CURRENT

TARGETING EVENT

ACK ACTUAL ACCURACY USED BY SUBROUTINE VMP

AER ABSOLUTE VALUES OF DIFFERENCES BETWEEN

DESIRED AND NOMINAL END CONDITIONS

CERROR CURRENT SUM OF WEIGHTED DIFFERENCES OF DESIRED AUXILIARY AND NOMINAL AUXILIARY

END CONDITIONS

DEV DIFFERENCES (ERRORS) BETWEEN AUXILIARY END

CONDITIONS (DESIRED AND NOMINAL)

ISP2 INDICATOR USED BY SUBROUTINE VMP

=1 STOP AT SPHERE-OF-INFLUENCE

=0 DO NO STOP AT SPHERE-OF-INFLUENCE

ITBAD BAQ STEP COUNTER

ITER ITERATION COUNTER

ITOL CONVERGENCE INDICATOR

=1 CASE CONVERGED

=0 CASE DID NOT CONVERGE

IT INDICATOR USED TO LOCATE DESIRED TIME

VALUE FOR OUTER TARGETING

I INDEX

J INDEX

LOWHI INDICATOR USED TO CALCULATE THE PHASE 2

# TARGETING MATRIX

NOMORE	INDICATOR USED TO LIMIT OUTER TARGETING =0 OUTER TARGETING HAS NOT BEEN PERFORMED =1 OUTER TARGETING HAS ALREADY BEEN PERFORMED
ОЅРН	ORIGINAL SPHER OF INFLUENCE OF THE TARGET PLANET
PERROR	PREVIOUS VALUE OF CERROR
REDUC	INTERMEDIATE VARIABLE USED IN BAD STEP REDUCTION
RIS	LOCAL VECTOR USED TO SAVE AND RESTORE THE RIN VECTOR
RR	INTERMEDIATE VARIABLE FOR QUIER TARGETING
RSF	FINAL SPACECRAFT STATE RETURNED BY VMP
SSOI	INTERMEDIATE VARIABLE FOR QUIER TARGETING
STOL	INTERMEDIATE VARIABLE FOR OUTER TARGETING
, TMDF,	INTERMEDIATE VARIABLE FOR OUTER TARGETING
TVH	PHASE 1 TARGETED VELOCITY AT HIGHEST ACCURACY
TVL	PHASE 1 TARGETED VELOCITY AT LOSEST ACCURACY
AA	INTERMEDIATE VARIABLE FOR QUIER TARGETING
XTIME .	CURRENT DT TIME USED TO CALCULATE EQECP FOR TARGET PLANET
COMMON CÓMPUTED/USED:	CTOL DAUX DELTAV DTAR IBAD IBAST IPHASE ISPH ISTART ISTOP ITARH KEYTAR LEVELS LEV MATX MAXBAD NITS NOPAR NOPHAS NOSOI PHI RIN SPHERE
COMMON COMPUTEDS	DELTP DELV ICL2 ICL INCMT INPR IPRINT KAXTAR KHIT RRF

CO	MMAN	USEDS

AAUX	AC	ALNGTH	ATAR	DC .
DELTAT	DT	DVMAX	01	EQECP
FAC	IBADS	KTAR	KUR	LVLS
MAT	MAXB	NOIT	NPAR	NTP
ONE	RC	SPHFAC	TAR	TM
TOI	TOTM	TWO	7FR0	~

## TARGET Analysis

TARGET is responsible for the control of any targeting (nonlinear guidance) event. The targeting is done either by the Newton-Raphson technique or by a steepest descent-conjugate gradient algorithm, the method being specified by the user. In either case numerical differencing is used to compute the required sensitivities.

#### I. Preliminaries

The current inertial state of the spacecraft upon entering TARGET is first saved (RIS=RIN) along with the original SOI radius (OSPH=SPHERE) since both variables may be changed during the course of the targeting. Before exiting from TARGET these values are restored.

The index of the current event KUR has been computed by TRJTRY. This enables the specific targeting parameters for the current event to be set:

Parameter	Definition
METHOD	Triggers Newton Raphson (=0) or Steepest Descent (\noting) technique
MATX	Determines whether Newton-Raphson matrix is computed always (=2) or only at low level (=1)
IRAST	Determines whether bad step checks are made never (=1), high level only (=2) or always (=3)
LEVELS	Number of integration accuracy levels to be used
NOPAR	Number of target parameters to be used
ACC	Actual accuracy levels used in targeting

The following flags are then initialized to zero

Flag	Definition
ITDS LOWHI	Counter for steepest descent iterations Flag indicating whether first phase complete (=1) or not (=0)
NOMORE	Flag indicating whether outer targeting has been done (=1) or not (=0)

The target time is computed and using that time the transformation matrix  $\phi_{\text{ECEQ}}$  from ecliptic to target planet equatorial coordinates is calculated (PECEQ).

### II. Phase Preparations

TARGET performs the targeting in one phase unless targeting to TCA (time of closest approach). In that case the trajectory is targeted in two phases: the first phase targets to the target planet SOI (sphere of influence), the second phase to the closest approach conditions. IPHASE is the phase counter, NOPHAS is the number of phases needed.

If all the phases have been completed, the program prepares to exit. If the last iterate satisfied the target tolerances ITOL will have been set to a 1. If it did not, ITOL will be zero and this requires that KWIT be set to 1 to terminate the program upon return to the basic cycle.

If the last phase has not yet been completed TAROPT is now called with an argument 1 to compute the following phase parameters:

Parameter	Definition				
KEYTAR(3)	Vector of codes of target parameters				
KAXTAR(3)	Vector of codes of auxiliary parameters				
DTAR(3)	Vector of desired values of target parameters				
DAUX(3)	Vector of desired values of auxiliary parameters				
FAC(3)	Weighting factors for loss function for auxiliary parameters				
ISTOP	Flag indicating integration stopping conditions with ISTOP = 1,2,3 indicating fixed final time, SOI, or CA encounter				

The target parameters are the parameters actually desired, the auxiliary parameters are the parameters used to do the targeting. The target and auxiliary parameters are identical except when  $i_{CA}$  and  $i_{CA}$  are targets. In that case the corresponding auxiliary parameters are B·T and B·R which are much more linear variables. The codes of the target and auxiliary parameters are as follows:

Code 1 3 4 5 6 1 8 10 11 12 TRF* TSI TCS TCA Parameter BDTBDR RCA INC SMA XRF YRF ZRF

* not currently available

### III. Level Preparations

Within any phase TARGET operates through a series of integration accuracy levels prescribed by the user. After completing each level TARGET checks to see if the maximum number of levels LEVELS has been exceeded. If it has the program cycles to the beginning of the "phase loop" to go to the next phase. If the current level LEV is less than LEVELS the following computations are made.

The flag ITARM controls whether the previous targeting matrix is to be used (=1) or whether the matrix is to be recomputed (=2) during the current level. ITARM is set according to the current values of MATX, ISTART, and LEV.

The flag IBAD controls the bad step logic. If IBAD=1 no bad step check will be made during the current level; if IBAD=2 the bad step check will be in effect. TARGET sets IBAD according to the values of IBAST and LEV.

The flags ITOL, ITER, ITEAD are set to 0 to begin the iterations. The allowable iterations NITS and bad iterations MAXBAD are also set at this time.

#### IV. Iterate Calculations

Within each level the program makes one or more iterations. After each iteration the program updates the iteration counter ITER. If the maximum number of iterations for this level NITS has been exceeded, the program sets KWIT to 1 and prepares for the return from TARGET. Otherwise TARGET computes the target and auxiliary values corresponding to the current iterate values of state (position and velocity) RIN.

The integration parameters are first set. VMP is then called to propagate the initial state to the final stopping conditions. Checks are made to insure that the target planet SOI was intersected if the stopping conditions were SOI or CA. If it was not intersected and this is the first iteration, the "outer targeting" phase is entered (see below). If "outer targeting" has already been performed, the bad-step check is entered to reduce the previous correction by REDUC.

Otherwise TAROPT is called with the argument 2 to compute the desired and actual target (DTAR, ATAR) and auxiliary (DAUX, AAUX) parameter values. The absolute error in target values AER and the error in auxiliary values DEV are then computed.

If the current iterate is the first integration at the low level during the second phase of targeting (LOWHI=1) TARMAX is now called to compute the phase 2 targeting matrix. Then the state RIN is reset to the targeted velocity at the high level TVH to prepare for the second phase targeting. The program then returns to the level *loop.

Otherwise the program now checks the actual target variables to determine whether they satisfy the input tolerances or not.

### V. Tolerances Satisfied

If the tolerances are satisfied, the program first checks to see if the current targeting phase is outer targeting. If it is TARGET restores the original target parameters and initiates the normal targeting (see Outer Targeting below).

If the current targeting is already normal targeting, TARGET sets ITOL=1 to indicate the satisfaction of the tolerances. If the problem is a 2-phase and the current level is the highest level in phase 1 targeting, the targeted high level velocity TVH=RIN is saved, LOWHI is set to 1 and the targeted low level velocity is recalled RIN=TVL for the construct of the phase 2 targeting matrix. Then the level loop is reentered.

# VI. Bad Step Reduction

If the target parameter values of any iterate are not within the acceptable tolerances TARGET now assigns a scalar error  $\epsilon$  to the iterate using the weighting factors  $\vec{W}$ 

$$\epsilon = \vec{w} \cdot \Delta \vec{\tau}$$

If the bad-step check is to be made on this iterate the current error  $\epsilon$  is compared to the previous error  $\epsilon$ . If  $\epsilon \searrow \epsilon$  and the maximum

number of bad steps has not been exceeded, the previous correction  $\Delta \vec{v}$  is reduced by REDUC (usually 1/4). The initial state RIN is adjusted by this and the iterate loop is reentered. If the maximum number of bad steps has been made, KWIT is set to 1 and the preparations for return are made.

#### VII. Generation of Next Iterate

The correction  $\Delta v$  to any iterate may be computed from either of two techniques selected by the flag METHOD. If METHOD  $\neq 0$ , subroutine DESCENT is called for the computation of the  $\Delta v$  by a steepest descent algorithm. The numerical value of METHOD determines the number n of conjugate gradient steps between each straight gradient step where n = METHOD-1. Thus if METHOD=1, every step is in the direction of the gradient. But if METHOD=5, four steps are taken following the conjugate gradient direction before rectification by the gradient direction.

If METHOD=0, the Newton-Raphson correction is used. If ITARM=0, TARMAX is called for the computation of the targeting matrix  $\phi$  by numerical differencing. If any of the integrations made in constructing that matrix satisfy the tolerances in  $\tau$ , the flag IEND is set to 1 before returning to TARGET. Thus a check must be made on IEND. If ITARM=1 the previous targeting matrix is used. The correction is then given by

$$\vec{\Delta v} = \phi \cdot \vec{\Delta \alpha}$$

where  $\Delta \hat{\alpha}$  are the deviations in the iterate auxiliary values. The  $\Delta \hat{v}$  is checked to insure that the maximum step size DVMAX is not violated: if it is, the  $\Delta \hat{v}$  is reduced proportionately to satisfy it. The next iterate is then set to

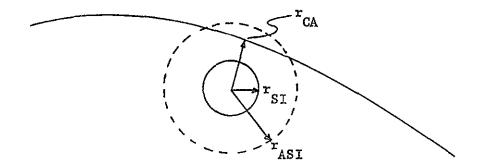
$$(\overrightarrow{r}, \overrightarrow{v}) = (\overrightarrow{r}, \overrightarrow{v} + \overrightarrow{\Delta v})$$

and the return is made to the iterate loop.

### VIII. Outer Targeting

Occasionally the zero iterate initial state leads to a trajectory missing the target body SOI. Since all target options except one (targeting to a specified position, i.e., KTAR = 10,11,12) require the trajectory to intersect the target body SOI steps must be taken to correct this.

Let the initial state propagated forward lead to a trajectory with a closest approach to the target body of  $r_{CA}$  with  $r_{CA} > r_{SI}$  where  $r_{SI}$  is the radius of the SOI.



Until the initial trajectory intersects the SOI the usual targeting can not be done. Therefore an "artificial" SOI is introduced having a radius of

$$r_{ASI} = 1.2 \times r_{CA}$$

The initial trajectory obviously intersects the artificial SOI and hence may be targeted to conditions on the ASOI. If the target conditions are established as  $B \cdot T_A = B \cdot R_A = 0$ , when this artificial targeting is completed,

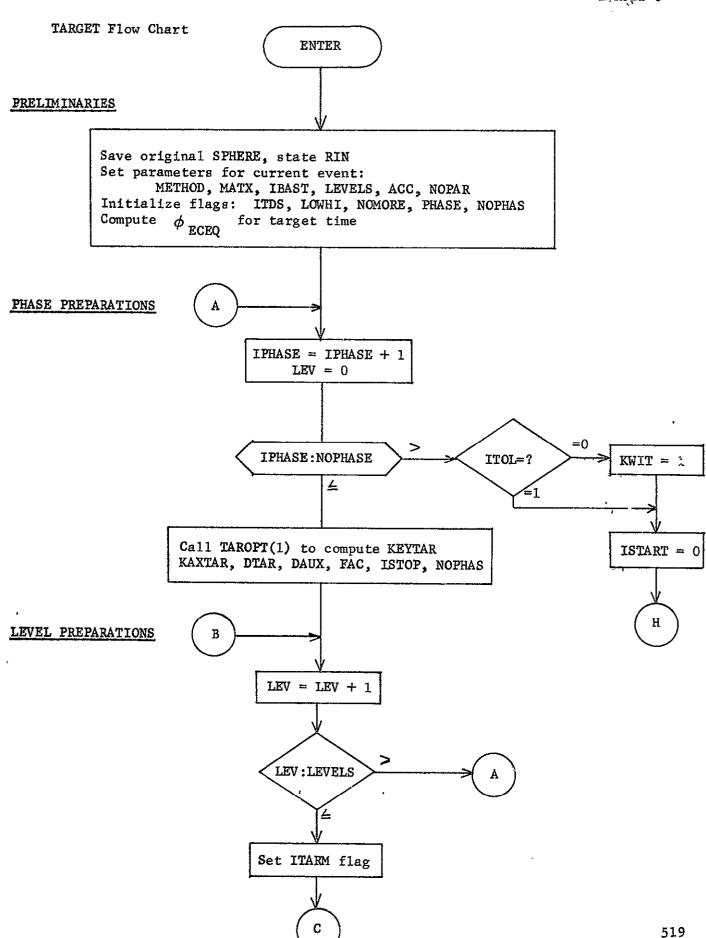
the refined trajectory will be headed straight for the target body when it hits the ASOI. Thus the refined trajectory should automatically hit the normal SOI when propagated past the ASOI. To insure that the time of intersection with the normal SOI is consistent with the target time, an artificial target time is also used. Let the speed of the spacecraft with respect to the target body at  $r_{\rm CA}$  be  $v_{\rm CA}$ . Make the approximation that

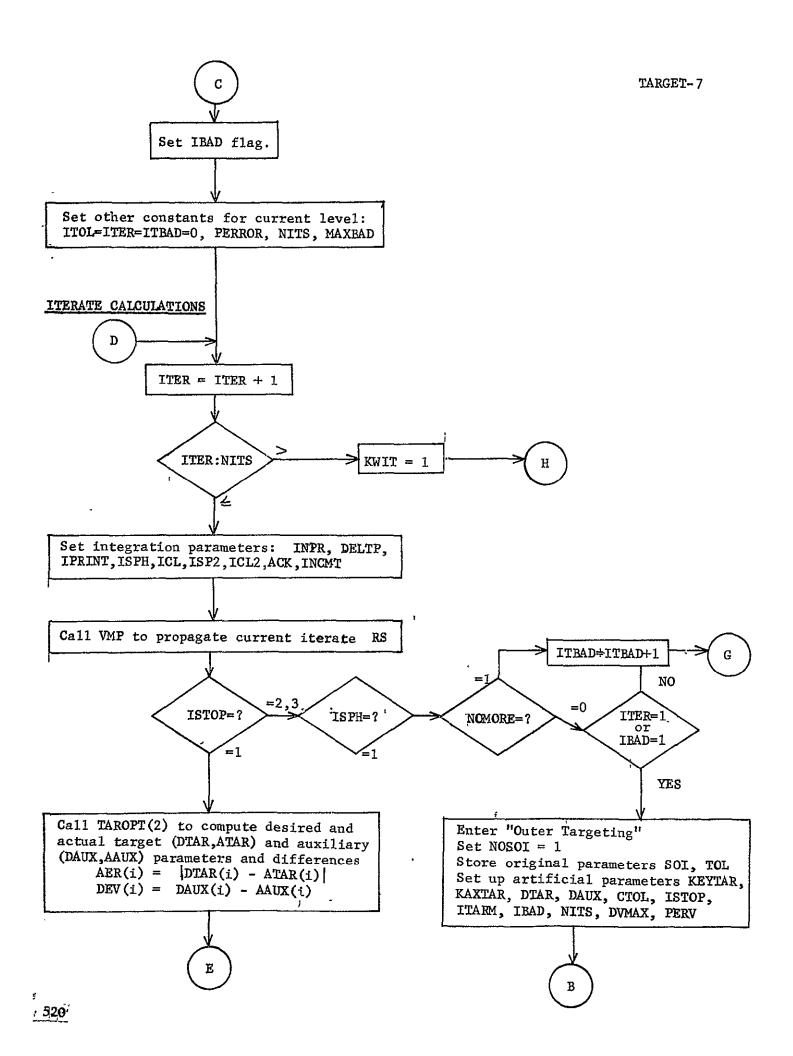
this speed will be roughly the same for the refined trajectory. Then the time that the spacecraft should intersect the ASOI is

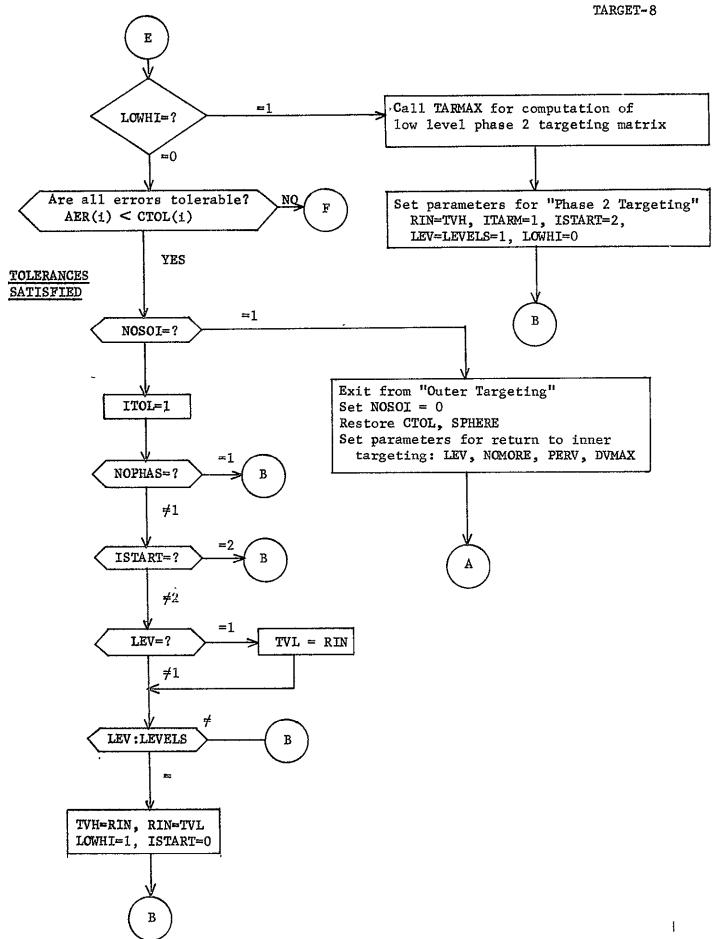
$$t_{ASI} = t_{CA} - \frac{r_{ASI}}{v_{CA}}$$
or
$$t_{ASI} = t_{SI} - \frac{r_{ASI} - r_{SI}}{v_{CA}}$$

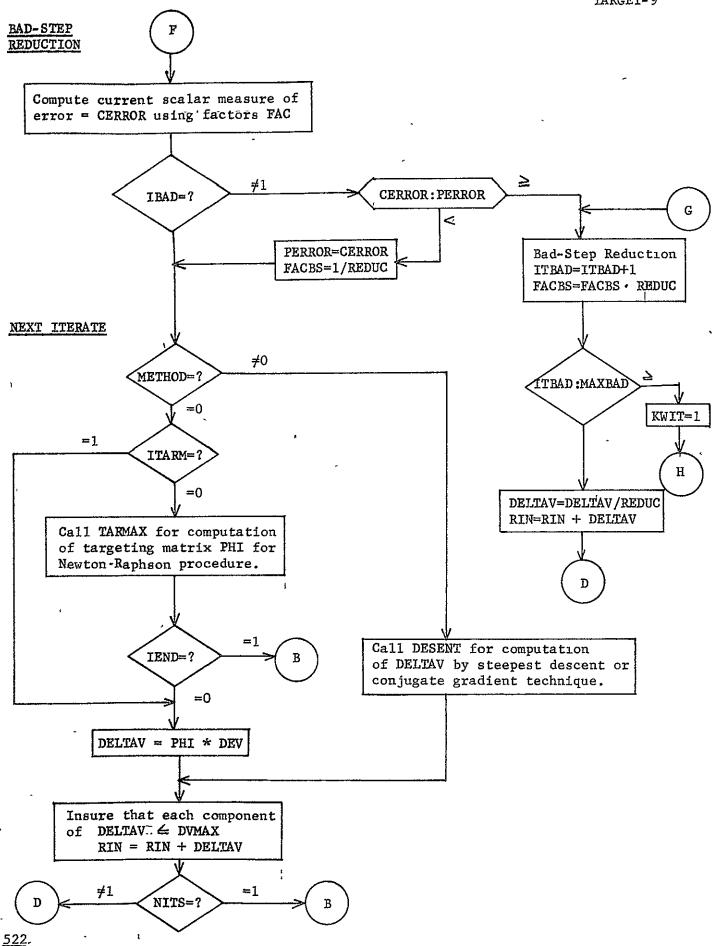
where the first formula should be used if the target time is t or t and the second formula is used for  $t_{ST}$ .

Thus when a trajectory is found which misses the normal SOI, the closest approach state  $r_{CA}$ ,  $v_{CA}$  is recorded. The normal SOI radius is stored, and the artificial SOI radius given above is used in its place. Target parameters of B·T_A, B·R_A, and t_{ASI} are then set up as the targets. When targeting of this artificial problem is complete, the resulting trajectory will intersect the normal SOI and the original problem may be solved.

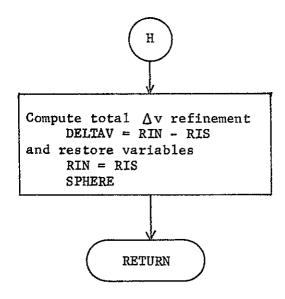








# PREPARATIONS FOR RETURN



SUBROUTINE TARMAX

PURPOSE: TO CALCULATE A TARGET MATRIX FROM NOMINAL INJECTION CONDITIONS, AND A PERTURBATION FACTOR BDELV FOR A GIVEN

ACCURACY LEVEL.

CALLING SEQUENCE: CALL TARMAX

SUBROUTINES SUPPORTED & TARGET

SUBROUTINES REQUIRED: MATIN TAROPT VMP

LOCAL SYMBOLS & ACK ACCURACY USED TO GENERATE THE TARGET

MATRIX

AER DIFFERENCES BETWEEN DESIRED AND ACTUAL END

CONDITIONS

AUXN NOMINAL AUXILIARY END CONDITIONS

CHI STATE TRANSITION MATRIX RELATING

PERTURBATIONS IN THE RIN VECTOR TO CHANGES

IN AUXN

DVEE VECTOR OF VELOCITY COMPONENT PERTURBATIONS

ISP2 INDICATOR USED BY SUBROUTINE VMP

=0 DO NOT STOP AT SHPERE OF INFLUENCE

=1 STOP AT SPHERE OF INFLUENCE

I INDEX

J INDEX

KOMP INDEX

PSI TARGET MATRIX FOR 2 X 2 EASE, STORED INTO

PHI

RSF FINAL SPACECRAFT STATE RETURNED BY VMP

COMMON COMPUTED/USED: ISPH PHI RIN TRTM

COMMON COMPUTEDS ICLS ICL INCHT

COMMON USEDS AAUX AC ATAR CTOL DAUX

DELTAT DELTAV DTAR D1 ISTOP KUR LEV LVLS NOPAR PERV

6 44

ZERO

## TARMAX Analysis

TARMAX computes the targeting matrix used by TARGET for Newton-Raphson refinements. The targeting matrix is computed by numerical differencing.

Let the current iterate initial state be denoted  $\overline{r}$ ,  $\overline{v}$ . Let the auxiliary parameters corresponding to this state be  $\overline{\alpha}$ . Let the perturbation size for the sensitivities be  $\bigwedge v$ .

The k-th column of the sensitivity matrix is computed as follows. Perturb the k-th component of velocity by  $\Delta v$ :

$$\vec{\mathbf{v}}_{p} = \vec{\mathbf{v}} + \Delta \mathbf{v} \left[ \boldsymbol{\delta}_{1K}, \ \boldsymbol{\delta}_{2K}, \ \boldsymbol{\delta}_{3K} \right]^{T}$$
 (1)

Propagate the perturbed initial state ( $\vec{r}$ ,  $\vec{v}_p$ ) to the final stopping conditions. Let the auxiliary parameters of that trajectory be denoted  $\alpha_p$ . The k-th column of the sensitivity matrix x is then given by

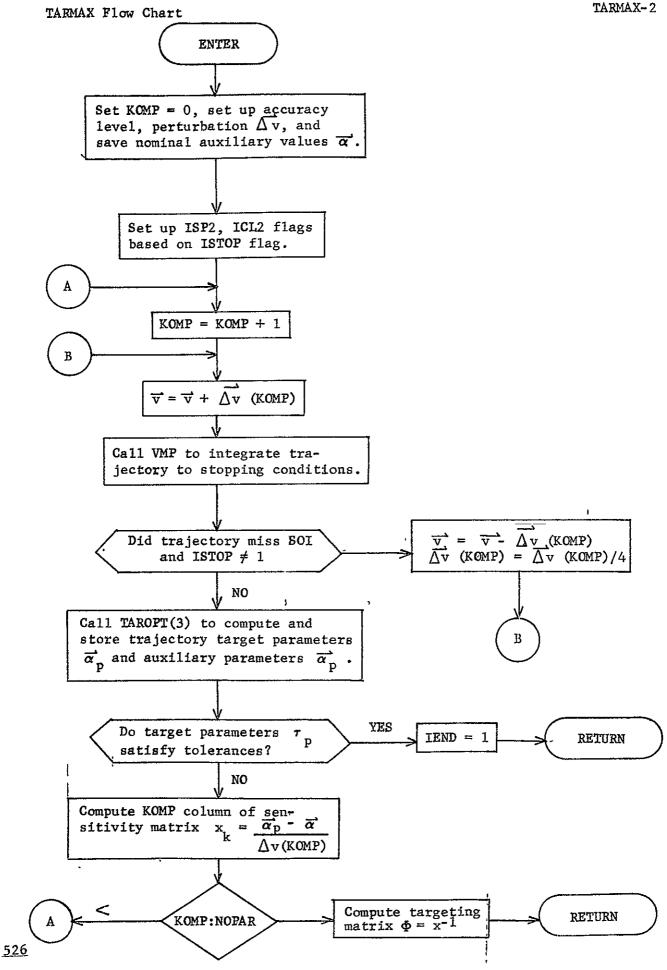
$$x_{k} = \frac{\vec{\alpha}_{p} - \vec{\alpha}}{\Delta v}$$
 (2)

Having computed all the columns of x, the targeting matrix is then given by the inverse of x:

$$\Phi = x^{-1} \tag{3}$$

The targeting matrix then has the property that to obtain a change  $\Delta \alpha$  in the nominal auxiliary parameters, the velocity should be changed by the amount

$$\overrightarrow{\Delta v} = \Phi \cdot \overrightarrow{\Delta \sigma} \tag{4}$$



SUBROUTINE TAROPT

PURPOSE: TO COMPUTE THE DESIRED AND ACHIEVED TARGET PARAMETER VALUES FOR ALL THE TARGETING SUBROUTINES.

CALLING SEQUENCE: CALL TAROPT(ITARO)

ARGUMENTS ITARO I OPTION FLAG

=1 SET UP TARGETING PARAMETERS FOR TARGET

KEYS

=2 COMPUTE ACTUAL VALUES OF PARAMETERS

=3 COMPUTE ACTUAL AND DESIRED VALUES OF

PARAMETERS

SUBROUTINES SUPPORTED: TARGET TARMAX DESENT

SUBROUTINES REQUIRED: CAREL CPWMS IMPACT

LOCAL SYMBOLS: ACK CURRENT ACCURACY BEING USED

A SEMI-MAJOR AXIS OF THE TARGET

PLANETOCENTRIC CONIC

CPT TOTAL COMPUTER TIME USED (SECS)

DBDR DESIRED VALUE OF 8 DOT R

DBDT DESRIED VALUE OF 8 DOT T

DINC DESIRED VALUE OF INCLINATION

DRCA DESIRED VALUE OF RCA

E ECCENTRICITY OF THE TARGET PLANETOCENTRIC

CONIC

IAUX INDICATOR FOR AUXILIARY END CONDITIONS

=0 TARGET TO ACTUAL END CONDITIONS

=1 TARGET TO AUXILIARY END CONDITIONS

IINC LOCATES DESIRED INCLINATION IN THE DTAR

ARRAY

IRCA LOCATES DESIRED RCA IN THE DTAR ARRAY

I INDEX

KEY LOCAL VARIABLE USED TO COMPLETE

INFORMATION IN THE KAXTAR AND KEYTAR ARRAY

PP DUMMY VARIABLE FOR CALL TO CAREL

QQ DUMMY VARIABLE FOR CALL TO CAREL

	RM du		UDE OF SP AJOR AXIS		USED TO	COMPUTE
	TA	DUMMY	ARGUMENT	FOR CALL	TO CARE	<b>L</b>
	TOBR	DUMMY	ARGUEMENT	FOR CALL	TO IMP	ACT
	TBDT	DUMMY	ARGUEMENT	FOR CALL	TQ IMP	ACT
	TFP		F FLIGHT			N THE
	TIMC	INTERM	EDIATE VA	RIABLE TO	COMPUT	E CPT
	TSICA	DUMMY	VARIABLE	FOR CALL	TO IMPA	CT
	٧X		EDIATE VA Major axis			ALCULATE
	нн	DUMMY	VARIABLE	FOR CALL	TO CARE	L
	Ħ	DUMMY	VARIABLE	FOR CALL	TO CARE	L
	XI	DUMMY	VARIABLE :	FOR CALL	TO CARE	L
	XN	DUMMY	VARIABLE	FOR CALL	TO CARE	<b>_</b>
COMMON COMPUTED/	USED:	AAUX Istop RCA	ATAR KAXTAR	DAUX KEYTAR	DELTAT NOPAR	DTAR NOPHAS
COMMON COMPUTEDS		CTOL	FAC			
COMMON USED8		AC DG ICL2 LEV RIN TMU	BDR DSI INCHT NOSOI RRF TM	IPHASE.	CAINC EQECP KTAR ONE TAR TWO	DC IBAD KUR RC TIMS VSI

## TAROPT Analysis

TAR $\emptyset$ PT is responsible for computing the desired and achieved target parameter values for all the targeting subroutines. To add any new target parameters, TAR $\emptyset$ PT is the only subroutine that must be modified.

The key variables used by TAROPT and their definitions are:

Variable - Definition;

KTAR(6,10) - Codes of target parameters of all targeting event

TAR(6,10) - Desired values of target parameters of all targeting events;

KEYTAR(3) - Codes of target parameters of current event;

ATAR(3) - Actual values of target parameters of current iterate;

KAXTAR(3) - Codes of auxiliary parameters of current iterate;

DAUX(3) - Desired values of auxiliary parameters on current iterate;

AAUX(3) - Actual values of auxiliary parameters of current iterate.

The available target parameters and their codes and definitions are tabulated.

Code	Parameter	Definition
1	^t ps	Time at probe sphere impact (n-body integration to sphere of influence (SOI), conic propagation to probe-sphere)
2	t _{SI}	Time at SOI of target body (n-body integration to SOI)
3	^t cs	Time at CA (n-body integration to SOI, conic propagation to CA)
4	t _{CA}	Time at CA (n-body integration to CA)
5	В∙Т	Impact parameter B•T
6	B•R	Impact parameter B•R
7 '	i	Inclination to target planet equator
8	r _{CA}	Radius of closest approach to target body
9	^a SI	Semimajor axis of conic with respect to target body
10	x _f	X-component of final state (inertial ecliptic system)
11	y _f	Y-component of final state
12	z _f	Z-component of final state
13	δ _I	Declination of probe target point in planeto- centric probe-sphere coordinate system speci- fied by IPCS
14	αΙ	Right ascension of probe target point in planetocentric probe-sphere coordinate system specified by IPCS
15	t _{PR}	Time at probe-sphere impact (n-body integration to probe sphere)

The term target parameter refers to a variable with a final value that conforms to a desired value. The term auxiliary parameter refers to a variable used to compute the progressive corrections. The target parameters and auxiliary parameters are identical unless the target parameters include either of the pairs i and  $r_{CA}$  or  $\alpha_{T}$  and  $\delta_{T}$ . In these cases the more linear parameters B·T and B·R are used as auxiliary targets in place of the actual ones. The desired values of the asymptote pierce-point coordinates,  $B_{D}$ T and  $B_{D}$ R, as well as the actual values,  $B_{A}$ T and  $B_{A}$ R, are computed by IMPCT for each successive iterate of the trajectory, based on the desired values of i and  $r_{CA}$  or  $\alpha_{T}$  and  $\delta_{T}$ .

TAR $\emptyset$ PT is called under three different options, which are distinguished by an argument ITAR $\emptyset$ . The three different options will be discussed in order.

TARØPT(1) is called by TARGET at the beginning of each phase to set up the proper variables for the targeting. The arrays KEYTAR, KAXTAR, DTAR, and DAUX are set to the current event values of KTAR and TAR. If  $t_{CA}$  or  $t_{PR}$  is a target parameter, the number of phases NØPHAS is set to 2. Then for the first phase of a two-phase problem, the time target variable  $t_{CA}$  or  $t_{PR}$  is replaced by  $t_{CS}$  or  $t_{PS}$ , respectively, in the KEYTAR and KAXTAR arrays. Thus in the initial high-speed phase of a high-precision double-phase targeting problem, integration is stopped at the SOI and the trajectory is extrapolated to the target conditions as in a single-phase case. If 1 and  $r_{CA}$  or  $\alpha_{I}$  and  $\delta_{I}$  are included in the adtual target parameters, the corresponding indices of the KAXTAR array arè set for the auxiliary targets B·T and B·R. TARØPT then sets up the integration parameters.

There are three propagation stop modes. The first terminates the trajectory after a maximum prepagation time interval,  $\Delta t$ , set equal to the nominal difference between the target time  $t_T$  and the current guidance event time  $t_C$ :

$$\Delta t = t_{T} - t_{G} \qquad (1)$$

For this stopping condition, ISTØP is set to 1. The second termination mode stops propagation at the SOI (or at impact if the SOI radius is temporarily set to that of the probe sphere). This ISTØP = 2 mode is used when the target time is  $t_{SI}$ ,  $t_{CS}$ ,  $t_{PS}$ , or  $t_{PR}$ .

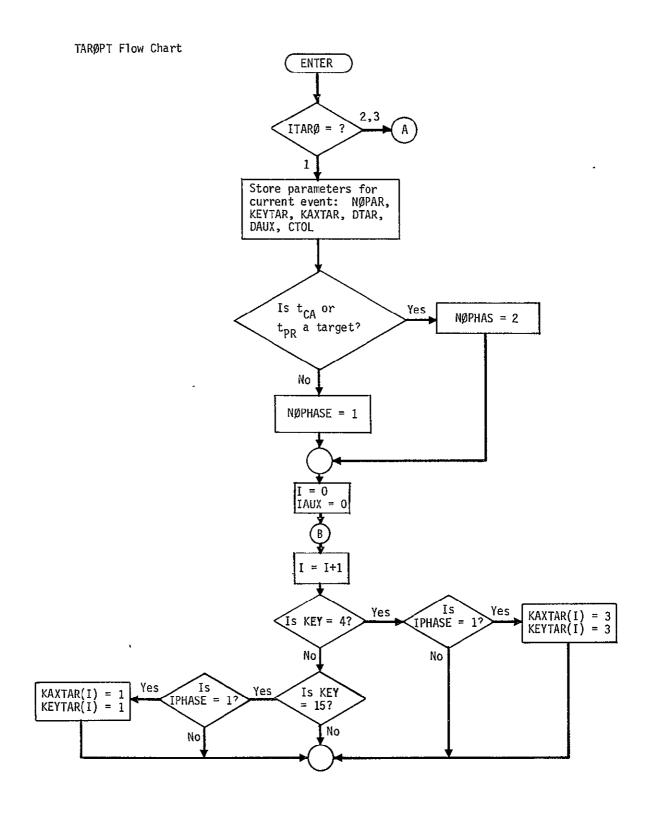
The final ISTØP = 3 mode terminates the propagation at closest approach. It is used only when the target time is  $t_{CA}$ . In either of the last two modes, the maximum propagation time interval is set to 1.1 times the value assigned for the first mode except when one of the target parameters is  $a_{SI}$ , the semimajor axis of the planetocentric conic at the target time. In this case  $\Delta t$  is set to the same value as in the first stopping mode. The augmented propagation interval guarantees that the trajectory will be propagated long enough to reach the desired targeting termination.

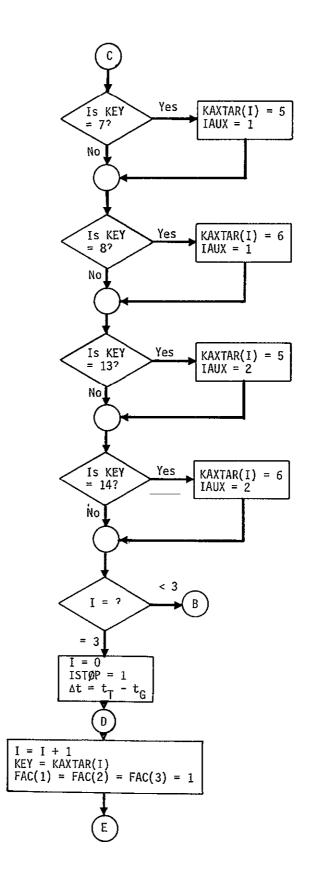
Finally the weighting factors FAC(3) used in computing the scalar loss function are set. Since the loss function is calculated solely from the auxiliary targets, which all have units of either length or time, only the relative weight of time to length need be input. Thus the length factors are set to unity and the time factor to the input value in WGHTM.

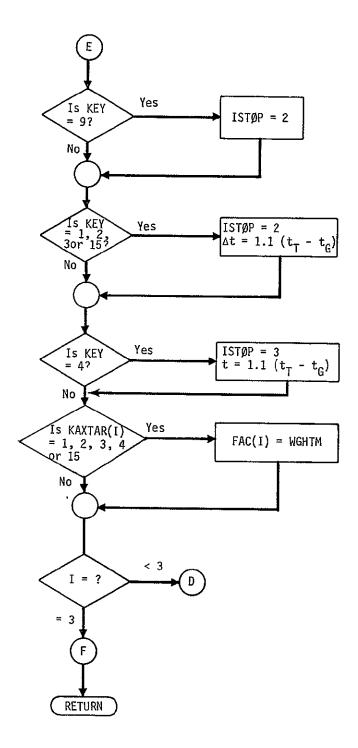
TARØPT(2) is called by TARGET after integrating each iterate to the final stopping conditions. Here TARØPT performs mainly a book-keeping role. It must fill the proper cells of the ATAR, AAUX, and DAUX arrays with values generally computed by the virtual-mass routines. If auxiliary targeting is required, both the actual and desired values of the B-plane coordinates are computed by calling IMPCT.

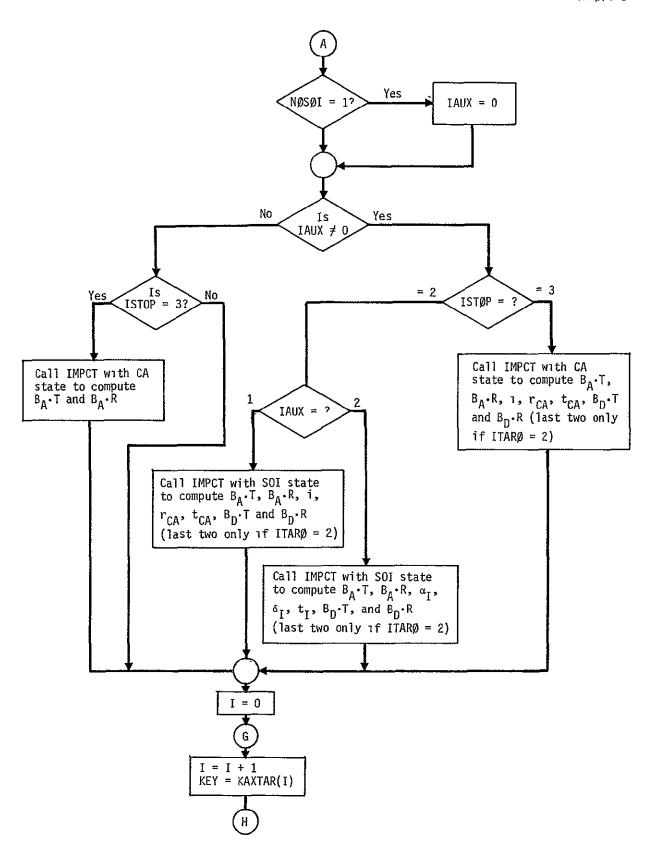
Since TARØPT(3) is called by TARMAX and DESENT after integrating each perturbed trajectory to compute the perturbed values of the auxiliary parameters, the desired values of DAUX need not be computed at this time. If auxiliary targeting is in process, the actual values of the B-plane coordinates,  $\rm B_A \cdot T$  and  $\rm B_A \cdot R$ , are computed by a call to IMPCT. Once again this task is simply a bookkeeping job to store the trajectory data correctly in the ATAR and AAUX cells. TARMAX and DESENT may then operate easily on these arrays to compute the targeting matrix or gradient directions.

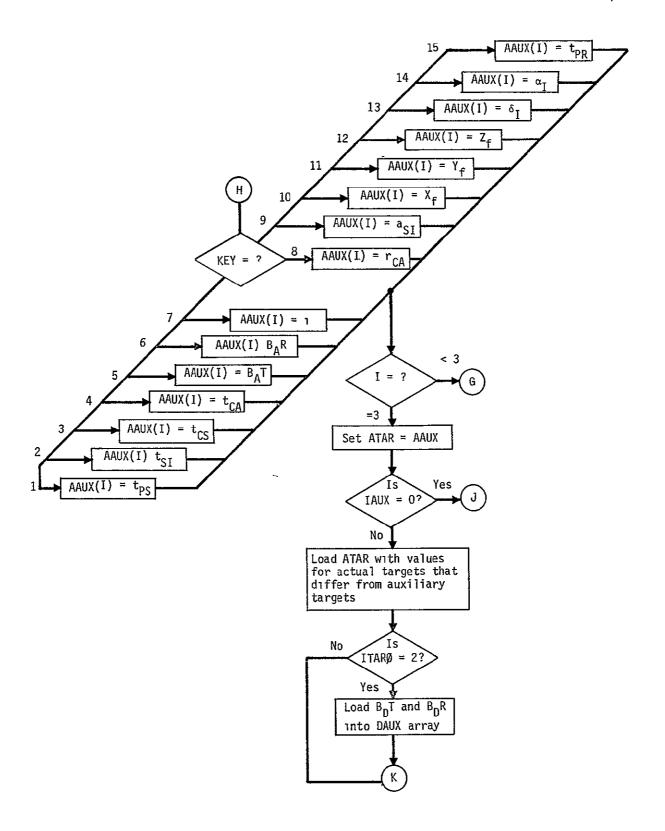
In both calls  $TAR \emptyset PT(2)$  and  $TAR \emptyset PT(3)$ , the trajectory data are printed out before exiting from  $TAR \emptyset PT$ .

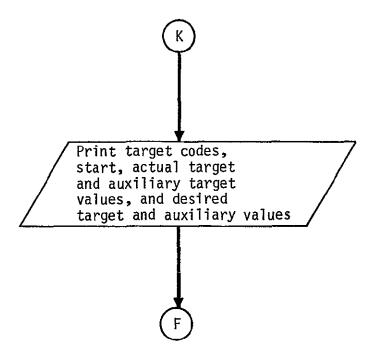












SUBROUTINE TARPRL

PURPOSE: TO COMPUTE THE PARTIAL DERIVATIVES OF THE POSITION COMPONENTS OF A PLANET WITH RESPECT TO EACH OF ITS ORBITAL ELEMENTS.

CALLING SEQUENCE: CALL TARPRL(ICODE, PAR)

ARGUMENT & I CODE I CODE DEFINING ORBITAL ELEMENT OF INTEREST

PAR O VECTOR OF 3 POSITION PARTIALS WITH RESPECT
TO THE ORBITAL ELEMENT OF INTEREST

SUBROUTINES SUPPORTED: TRAKS TRAKM

LOCAL SYMBOLS: CBO COSINE OF LONGITUDE OF ASCENDING NODE

CI COSINE OF ANGLE OF INCLINATION

CLO COSINE OF ARGUMENT OF PERIAPSIS

COSNU COSINE OF TRUE ANOMALY

COSONU COSINE OF THE SUM OF THE ANGLES OF TRUE ANOMALY PLUS THE ARGUMENT OF PERIAPSIS

DNUDE PLANET DISTANCE TIMES THE PARTIAL OF TRUE ANOMALY WITH RESPECT TO ECCENTRICITY

DNUDM PARTIAL OF TRUE ANOMALY WITH RESPECT TO MEAN ANOMALY

DPAR 1./R*(*R/*E + (*R/*NU)*(*NU/*E))

WHERE R= PLANET DISTANCE

NU=TRUE ANOMALY

E= ECCENTRICITY

#= PARTIAL OF

DRDNU PARTIAL R WITH RESPECT TO NU

E2 SQUARE OF ECCENTRICITY

IND INDEX USED IN ARRAY STORING ORBITAL ELEMENTS OF PLANETS

PCOMP SEMI-MAJOR AXIS TIMES THE TERM (1-E*E)
WHERE E=ECCENTRICITY

R PLANET DISTANCE

SBO SINE OF LONGITUDE OF THE ASCENDING MODE

SI SINE OF INCLINATION

SINNU SINE OF TRUE ANOMALY

SINE OF THE SUM OF THE ANGLES OF TRUE ANOMALY PLUS THE ARGUMENT OF PERIAPSIS SINONU

SLO SINE OF THE ARGUMENT OF PERIAPSIS

XXX SCRATCH CELL

COMMON USED: ALNGTH ELMNT NTP ONE XP

ZERO

## TARPRL Analysis

The position components of a planet are related to its orbital elements a, e, i,  $\Omega$ ,  $\omega$ , and M through the following set of equations:

$$x = r \left[ \cos \Omega \cos(\omega + \nu) - \sin \Omega \sin(\omega + \nu) \cos i \right]$$

$$y = r \left[ \sin \Omega \cos(\omega + \nu) + \cos \Omega \sin(\omega + \nu) \cos i \right]$$
(2)

$$y = r \left[ \sin \Omega \cos(\omega + v) + \cos \Omega \sin(\omega + v) \cos i \right]$$
 (2)

$$z = r \sin(\omega + \nu) \sin i$$
 (3)

$$\mathbf{r} = \underline{\mathbf{a}(1 - \mathbf{e}^2)} \tag{4}$$

$$\tan \frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}} \quad \tan \frac{E}{2} \tag{5}$$

$$M = E - e \sin E \tag{6}$$

We can write equations (1), (2), (3), and (4) symbolically as

$$x_i = f_i(a, e, i, \Omega, \omega, \nu)$$

and equations (5) and (6) as

$$\nu = \nu \text{ (e, M)}$$
.

Then the partials of x with respect to a, e, i,  $\Omega$ ,  $\omega$ , and M can be evaluated as follows: evaluated as follows:

$$\frac{\partial x_1}{\partial a} = \frac{\partial f_1}{\partial a} \tag{7}$$

$$\frac{\partial x_{i}}{\partial e} = \left(\frac{\partial f_{i}}{\partial e}\right)_{\nu} + \frac{\partial f_{i}}{\partial \nu} \cdot \frac{\partial \nu}{\partial e}$$
 (8)

$$\frac{\partial x_i}{\partial t} = \frac{\partial f_i}{\partial t} \tag{9}$$

$$\frac{\partial x_1}{\partial \Omega} = \frac{\partial f_1}{\partial \Omega} \tag{10}$$

$$\frac{\partial x_i}{\partial \omega} = \frac{\gamma f_i}{\partial \omega} \tag{11}$$

$$\frac{\partial x_i}{\partial M} = \frac{\partial f_i}{\partial \nu} \cdot \frac{\partial \nu}{\partial M}$$
 (12)

Only  $\frac{\partial \nu}{\partial e}$  and  $\frac{\partial \nu}{\partial M}$  require further consideration before equations (7) through (11) can be used to obtain expressions for the 18 desired partial derivatives.

We obtain  $\frac{\partial \nu}{\partial M}$  by first differentiating equation (5) with respect to E and equation (6) with respect to M to obtain

$$\frac{\partial \nu}{\partial E} = \frac{a}{r} \sqrt{1 - e^2}$$

and

$$\frac{\partial E}{\partial M} = \frac{a}{r}$$
.

Then

$$\frac{\partial \nu}{\partial M} = \frac{\partial \nu}{\partial E} \cdot \frac{\partial E}{\partial M} = \left(\frac{a}{r}\right)^2 \sqrt{1 - e^2} . \qquad (13)$$

We obtain  $\frac{\partial v}{\partial e}$  by first differentiating equation (6) with respect to e to obtain

$$\frac{\partial M}{\partial e} = -\frac{\sqrt{1 - e^2} \sin \nu}{1 + e \cos \nu} .$$

This result is then combined with equation (13) to yield

$$\frac{\partial \nu}{\partial e} = \frac{\partial \nu}{\partial M} \quad \frac{\partial M}{\partial e} = -\left(\frac{a}{r}\right)^2 \frac{(1-e^2)\sin\nu}{1+e\cos\nu} \tag{14}$$

The evaluation of the desired partials can now proceed. The results are summarized below.

a. Partials with respect to a.

$$\frac{\partial x}{\partial a} = \frac{x}{a}$$

$$\frac{\partial z}{\partial s} = \frac{z}{s}$$

b. Partials with respect to e.

$$\frac{\partial \dot{x}}{\partial e} = \frac{xq}{r} + r \frac{\partial \nu}{\partial e} \left[ -\cos\Omega\sin(\omega + \nu) - \sin\Omega\cos(\omega + \nu) \cos i \right]$$

$$\frac{\partial y}{\partial e} = \frac{yq}{r} + r \frac{\partial \nu}{\partial e} \left[ -\sin \Omega \sin(\omega + \nu) + \cos \Omega \cos(\omega + \nu) \cos i \right]$$

$$\frac{\partial z}{\partial e} = \frac{zq}{r} + r \frac{\partial \nu}{\partial e} \cos(\omega + \nu) \sin i$$

where 
$$q = \frac{r}{ae(1-e^2)} \left[r - a - ae^2(1 + \sin^2 \nu)\right]$$

c. Partials with respect to i.

$$\frac{\partial x}{\partial i} = r \sin \Omega \sin(\omega + \nu) \sin i$$

$$\frac{\partial y}{\partial i} = -r \cos \Omega \sin(\omega + \nu) \sin i$$

$$\frac{\partial z}{\partial i} = r \sin(\omega + \nu) \cos i$$

d. Partials with respect to  $\, \Omega \,$  .

$$\frac{\partial x}{\partial Q} = -y$$

$$\frac{\partial y}{\partial \Omega} = x$$

$$\frac{\partial z}{\partial \theta} = 0$$

e. Partials with respect to  $\omega$ .

$$\frac{\partial x}{\partial \omega} = r \left[ -\cos \Omega \sin(\omega + \nu) - \sin \Omega \cos(\omega + \nu) \cos i \right]$$

$$\frac{\partial y}{\partial \omega} = r \left[ -\sin \Omega \sin(\omega + \nu) + \cos \Omega \cos(\omega + \nu) \cos i \right]$$

$$\frac{\partial z}{\partial \omega} = r \cos(\omega + \nu) \sin i$$

f. Partials with respect to M.

$$\frac{\partial x}{\partial M} = \frac{xs}{r} + r \frac{\partial v}{\partial M} \left[ -\cos \Omega \sin(\omega + v) - \sin \Omega \cos(\omega + v) \cos i \right]$$

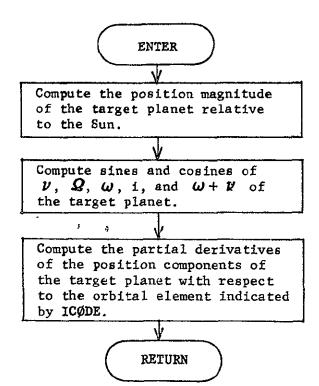
$$\frac{\partial y}{\partial M} = \frac{ys}{r} + r \frac{\partial v}{\partial M} \left[ -\sin \Omega \sin(\omega + v) + \cos \Omega \cos(\omega + v) \cos i \right]$$

$$\frac{\partial z}{\partial M} = \frac{zs}{r} + r \frac{\partial v}{\partial M} \cos(\omega + v) \sin i$$

where 
$$s = \frac{ae \sin \nu}{\sqrt{1 - e^2}}$$

Reference: Battin, R. H.: Astronautical Guidance, McGraw-Hill Book Company, Inc., New York, 1964.

## TARPRL Flow Chart



SUBROUTINE THPOSM

PURPOSE: TO FIND MINIMUM OF FUNCTION ON A GIVEN INTERVAL BY CUBIC INTERPOLATION

ARGUMENT: ALPHA I FRACTION OF INTERVAL FROM 0. TO AMEDA AT WHICH THIRD FUNCTION VALUE IS FOUND

AMBDA I UPPER END OF SEARCH INTERVAL AT WHICH FUNCTION VALUE IS FOUND (LOWER END IS 0.)

FALAM I VALUE OF FUNCTION AT ALPHA TIMES AMBDA

FLAM I VALUE OF FUNCTION AT AMBDA

FOP I DERIVATIVE FUNCTION VALUE AT ABSCISSA OF 0.

FO I FUNCTION VALUE AT ABSCISSA VALUE OF 0.

XMIN O ABSCISSA VALUE AT MINIMUM OF FITTED CUBIC POLYNOMIAL

# SUBROUTINES SUPPORTED: GAUSLS

A1 COEFFICIENT OF QUADRATIC TERM IN FITTED CUBIC POLYNOMIAL

DISC DISCRIMINANT OF QUADRATIC EQUATION FORMED BY SETTING DERIVATIVE OF CUBIC POLYNOMIAL TO ZERO

THRAO THREE TIMES COEFFICIENT OF CUBIC TERM IN FITTED CUBIC POLYNOMIAL

## THPØSM Analysis

THPØSM locates by cubic interpolation the minimum of a scalar function f of one variable on the closed interval from 0 to  $\lambda$  (positive), assuming f has a unique point on that interval where its derivative vanishes and that this extreme value is indeed a minimum. The algorithm fits a cubic polynomial through function values at 0,  $\alpha\lambda$ , and  $\lambda$  as well as through the function's slope at 0. Here  $\alpha\lambda$  should be on the open interval from 0 to  $\lambda$  and preferably near the middle. The abscissa of the minimum is then approximately by the abscissa of the corresponding minimum of the cubic.

The coefficients of the approximating cubic can readily be derived once and for all and cast into a form facilitating speedy execution. This approach proves much more economical of machine time than solving for them each pass with a linear system routine as is frequently done in polynomial fitting.

Denote the approximating cubic polynomial by

$$c(X) = a_0 X^3 + a_1 X^2 + a_2 X + a_3.$$
 (1)

Then clearly

$$a_3 = f(0) \tag{2}$$

and

$$a_2 = f'(0)$$
 (3)

One also has that

$$f(\lambda) = a_0 \lambda^3 + a_1 \lambda^2 + f'(0) \lambda + f(0)$$
 (4)

and

$$f(\alpha\lambda) = a_0 \alpha^3 \lambda^3 + a_1 \alpha^2 \lambda^2 + f'(0) \alpha \lambda + f(0).$$
 (5)

Solving these last two equations for  $a_0$  and  $a_1$  yields

$$a_0 = \frac{1}{\lambda^3 \alpha^2} \left[ \lambda f'(0) \alpha + f(0) (1+\alpha) + \frac{\alpha^2 f(\lambda) - f(\alpha \lambda)}{1 - \alpha} \right]$$
 (6)

and

$$a_1 = \frac{1}{\lambda^2 \alpha^2} \left[ \frac{f(\alpha \lambda) - \alpha^3 f(\lambda)}{1 - \alpha} - \lambda \alpha (1 + \alpha) f'(0) - f(0) (1 + \alpha + \alpha^2) \right]. \quad (7)$$

For an extreme value,  $X_e$ , one knows that  $c'(X_e) = 0$ , that is,

$$3a_0^{X_e^2} + 2a_1^{X_e} + a_2 = 0.$$
 (8)

Hence

$$X_{e_{1,2}} = \frac{-a_1 \pm \sqrt{a_1^2 - 3a_0^a}}{3a_0}$$
.

The question remains as to which of these two extrema is a minimum. It is shown in elementary calculus that an extremum is a minimum if, and only if, the second derivative is positive there.

$$c''(X) = 6a_0 X + 2a_1$$
.

Hence

$$c''\left(x_{e_{1,2}}\right) = 6a_0 \left[\frac{-a_1 \pm \sqrt{a_1^2 - 3a_0^2}}{3a_0}\right] + 2a_1$$

$$= \pm 2\sqrt{a_1^2 - 3a_0^2}.$$

Thus the cubic has its minimum at

$$x_{\min} = \frac{-a_1 + \sqrt{a_1^2 - 3a_0^2}}{3a_0}.$$

The preceding formula for the minimum will obviously be inadequate when  $a_0 = 0$  as is the case when minimizing a quadratic.

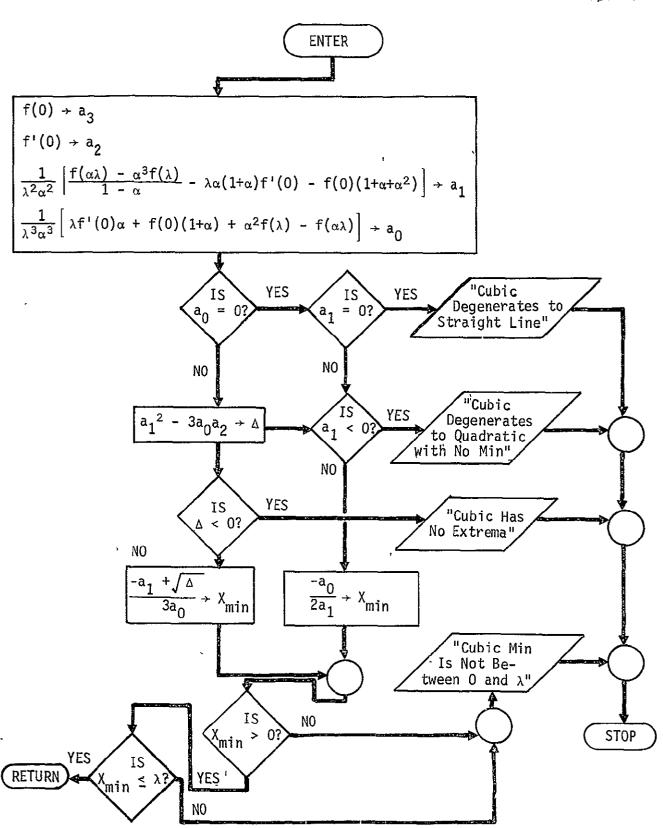
However it can be shown that

$$\lim_{a_0 \to 0} x_{\min} = -\frac{a_2}{2a_1}$$

and hence when  $a_0 = 0$  we take

$$X_{\min} = -\frac{a_2}{2a_1}$$

THP $\emptyset$ SM provides diagnostic printouts and stops execution of the calling program in the three conceivable cases of difficulty: (1) the cubic degenerates to a straight line, (2) the cubic degenerates to a quadratic with no minimum, (3) the cubic is nondegenerate but has no extrema, or (4) the minimum of the cubic does not fall between 0 and  $\lambda$ .



SUBROUTINE TIME

PURPOSE: TO COMPUTE THE JULIAN DATE, EPOCH 1900, FROM THE CALENDAR DATE OR TO COMPUTE THE CALENDAR DATE FROM THE JULIAN DATE.

CALLING SEQUENCE: CALL TIME(DAY, IYR, MO, IDAY, IHR, MIN, SEC, ICODE)

ARGUMENT * DAY I/O JULIAN DATE, EPOCH 1900

IYR O/I CALENDAR YEAR

MO O/I CALENDAR MONTH

IDAY O/I CALENDAR DAY

IHR O/I HOUR OF THE DAY

MIN O/I MINUTE OF HOUR

SEC O/I FRACTIONAL SECONDS

ICODE I OPERATIONAL MODE

- = 1, INDICATES THE JULIAN DATE IS INPUT, CALENDAR DATE IS OUTPUT
- = 0; INDICATES THE CALENDAR DATE IS INPUT,
  JULIAN DATE IS OUTPUT

SUBROUTINES SUPPORTED: DATAS INPUTZ PRINT VMP GIDANS PREPUL PRNTS4 DATA PRNTS3 PRELIM GIDANS HELIO MULTAR

SUBROUTINES REQUIRED: NONE

LOCAL SYMBOLS: IA NUMBER OF CENTURIES

IB YEARS IN PRESENT CENTURY

IP NUMBER OF MONTH (BASED ON MARCH AS NUMBER ZERO)

IQ NUMBER OF YEARS

IR NUMBER OF CENTURIES DIVIDED BY 4

IS NUMBER OF YEARS SINCE LAST 400 YEAR SECTION BEGAN

IT NUMBER OF LEAP YEARS IN PRESENT CENTURY

IU NUMEER OF YEARS SINCE LAST LEAP YEAR

IV NUMBER OF DAYS IN LAST YEAR

# PRECEDING PAGE BLANK NOT FILMED

IX	INTERMEDIATE INTEGER
J	INTERMEDIATE INTEGER
JD	NUMBER OF DAYS IN JULIAN DATE
Ρ	JULIAN DATE
R	FRACTIONAL PORTION OF DAY IN JULIAN DATE

SUBROUTINE TITLE

PURPOSES TO PRINT TITLES FOR ERRAN.

CALLING SEQUENCES CALL TITLE(LINES, TEVN, ICODE)

ARGUMENT: LINES NOT USED

TEVN I EVENT TIME

ICODE I EVENT CODE

SUBROUTINES SUPPORTED: SETEVN

LOCAL SYMBOLS: TPT TIME PREDICTING TO

COMMON USED: IPROB NPE TPT2

SUBROUTINE TITLES

PURPOSES TO PRINT TITLES FOR SIMUL.

CALLING SEQUENCE: CALL TITLES (TEVN, ICODE)

ARGUMENT: TEVN I EVENT TIME

ICODE I EVENT CODE

SUBROUTINES SUPPORTED: SETEVS

LOCAL SYMBOLS: TPT TIME PREDICTING TO

COMMON USED: IPROB NPE TPT2

SUBROUTINE TPPROP

PURPOSE: PROPAGATE THREE MINIPROBE TRAJECTORIES ACCORDING TO EITHER CONIC OR VIRTUAL - MASS MODEL

ARGUMENT: PSI I CONSTRAINT VECTOR

UCNTRL O CONTROL VECTOR

SUBROUTINES SUPPORTED: TPRTRG

SUBROUTINES REQUIRED: CAREL CONCAR DIMPCP, MATPY SAOCS
SCAR SPHIME STIME VMP

LOCAL SYMBOLS: ANPTP SEMI-MAJOR AXIS OF I-TH PROBE CONIC

ATP INTERMEDIATE VARIABLE USED TO FIND UCNIRL

AUTKM CONVERSION FACTOR FROM AU TO KM

BM MAGNITUDE OF B-VECTOR OF I-TH CONIC

CSRRA COSINE OF ROLL RELEASE ANGLE OF I-TH PROBE

CSTITP COSINE AT IMPACT OF TRUE ANOMALY

CSTP03 INTERMEDIATE VARIABLE

DBR INTERMEDIATE VARIABLE

DBT INTERMEDIATE VARIABLE

DJEDUM JULIAN BATE EPOCH 1900

DTPITP TIME FROM PERIAPSIS TO IMPACT OF I-TH CONIC

DTPRTP TIME FROM PERIAPSIS TO EXTRAPOLATION STATE

DTR CONVERSION FACTOR FROM DEGREES TO RADIANS

DTS CONVERSION FACTOR FROM DAYS TO SECONDS

EMS1 MISS DISTANCE SQUARED FROM SECOND PROBE TO FIRST TARGET SITE TO WHICH PROBE ONE IS NOT TARGETED

EMS2 MISS DISTANCE SQUARED FROM SECOND PROBE TO SECOND TARGET SITE TO WHICH PROBE ONE IS NOT TARGETED

ENPTP ECCENTRICITY OF I-TH PROBE CONIC

FACTR INTERMEDIATE VARIABLE

ICL2S SAVED VALUE OF VMP FLAG ICL2

INMIN1 INDEX OF FIRST SITE TO WHICH PROBE ONE IS NOT TARGETED

INMIN2 INDEX OF SECOND SITE TO WHICH PROBE ONE IS NOT TARGETED

ISITE INDEX OF DESIRED I-TH PROBE TARGET SITE

ISP2 CALL ARGUMENT TO VMP

I INDEX

NTPI INDEX FOR F AND V ARRAY USAGE

ONE CONSTANT = 1.

PNPTP SEMI-LATUS RECTUM OF I-TH PROBE CONIC

PPNPTP UNIT ECLIPTIC PERIAPSIS VECTOR OF CONIC

QQNPTP UNIT VECTOR NORMAL TO PPNPTP IN ORBIT PLANE

RTD CONVERTS RADIANS TO DEGREES

RTPIP UNIT ECLIPTIC VECTOR TO I-TH PROBE SITE

RTPISS PROBE-SPHERE UNIT VECTOR TO 1-TH PROBE SITE

RV INTERMEDIATE VECTOR

SNRRA SINE OF ROLL RELEASE ANGLE OF I-TH PROBE

SNTITP SINE OF TRUE ANOMALY OF CONIC AT IMPACT

SNTPO3 INTERMEDIATE VARIABLE

SPHERS S.O.I. RADIUS CF TARGET PLANET

SV UNIT VECTOR NORMAL TO RV

TM2 CONSTANT = 1.E-2

TP2 CONSTANT = 1.E+2

TRIMS TIME OF PROBE RELEASE IN DAYS RELATIVE TO START OF VMF TRAJECTORY

TV CROSS PRODUCT OF SV AND RV

TWO CONSTANT = 2.

VTANG TANG. VELOCITY OF ALL PROBES AT RELEASE

VTPIP ECLIPTIC VELOCITY VECTOR OF I-TH PROBE

VTPRPA ECLIPTIC VELOCITY VECTOR OF I-TH PROBE AT RELEASE OR AT BEGINNING OF THE CONIC EXTRAPOLATION

WWNPTP CROSS PRODUCT OF PPNPTP AND QQNPTP

XSF FINAL HELIOCENTRIC ECLIPTIC STATE OF I-TH PROBE RETURNED BY VMP IN KM, KM/SEC

XSI INITIAL HELIQCENTRIC ECLIPTIC STATE OF I-TH PROBE RELEASE PROPAGATED BY VMP

ZERO CONSTANT = 0.

COMMON COMPUTED/USED:	ICL2 VSCRPA	RSCRPA	SPHERE	TRTM	VIMTP
COMMON COMPUTED:	AATTP FPATP TM	ADCTP ICL	ALNGTH INCMT	ARATP IPRINT	DJEITP ISPH
COMMON USEO:	ACTPP CSRASA F ISAO RSCRHA TRANSF	BDR DCTP GMUP NBOD RSI VSCRHA	BOT DELTM IFIN2 NB RTPS VSI	B DJERN IMIN NTP SNDCSA V	CSDCS A DSI IPROP RATP SNRASA WFLS

#### TPPRØP Analysis

The subroutine TPPRØP has the sole responsibility for propagating miniprobes. It is called on to do so in two basic applications. The first is in calculating the miniprobe targeting constraint  $\psi$  given the release control  $\underline{u}$  as repeatedly required in the missminimization process (see analysis of TPRTRG). The second is in describing the minimum-miss miniprobe approach trajectories once the optimal release controls have been found. This includes generating impact data for both conic and virtual-mass propagation as well as time histories of the latter. The logic of TPPRØP is considerably complicated by the requirement that it be able to propagate the approach trajectories according to either a high-speed conic model or an accurate virtual-mass n-body integrator.

First consider the problem of calculating  $\psi$  given  $\underline{u}$ . For this application of TPPRØP, the miss-minization status flag, IFIN2, is set to 1. In either propagation mode the velocity,  $\underline{v}_{iR}$ , of the ith miniprobe just after release must be computed first of all. Let  $v_{BR}$  be the velocity of the bus at release and  $\phi_i$  be the release roll angle of the ith probe. Let  $\underline{U}$ ,  $\underline{V}$  and  $\underline{H}$  be the probe release reference vectors defined in the analysis section of TPRTRG. By  $\phi_i$  we mean the angle the release velocity increment of the ith miniprobe makes with the U direction. It should not be confused with the angle the ith probe arm makes with the U direction, which is  $\pi/2$  radians smaller. Next define  $v_T$  to be the common tangential velocity of the miniprobes at release. Then the velocity of the ith miniprobe at release is

$$\underline{\underline{v}}_{iR} = v_T \left[ \cos \phi_i \underline{\underline{U}} + \sin \phi_i \underline{\underline{V}} \right] + \underline{\underline{v}}_{BR} \qquad i = 1, 2, 3. \quad (1)$$

Let  $\alpha_H$  and  $\delta_H$  denote the right ascension and declination, respectively, of the spin axis at release. Then by expressing  $\underline{U}$  and  $\underline{V}$  in terms of these angles, equation (1) reduces to the following in the planetocentric ecliptic system:

$$\underline{v}_{iR} = v_{T} \begin{bmatrix} \cos \phi_{i} \begin{pmatrix} \sin \alpha_{H} \\ -\cos \alpha_{H} \end{pmatrix} + \sin \phi_{i} & \begin{pmatrix} \cos \alpha_{H} \sin \delta_{H} \\ \sin \alpha_{H} \sin \delta_{H} \\ -\cos \delta_{H} \end{bmatrix} \\ + \underline{v}_{BR} \qquad i = 1, 2, 3 \quad (2)$$

Simplifying equation (2) yields the computational form of the inth miniprobe welvoity increment:

$$\underline{v}_{iR} = v_{T} \begin{pmatrix} \sin \alpha_{H} \cos \phi_{i} + \cos \alpha_{H} \sin \delta_{H} \sin \phi_{i} \\ -\cos \alpha_{H} \cos \phi_{i} + \sin \alpha_{H} \sin \delta_{H} \sin \phi_{i} \\ -\cos \delta_{H} \sin \phi_{i} \end{pmatrix} + \underline{v}_{BR}$$
(3)
$$i = 1, 2, 3.$$

The sines and cosines of  $\alpha_H$  and  $\delta_H$  necessary in equation (3) are all calculated in a single call to the subroutine SAØCS given the spin-axis orientation flag, ISAØ.

At this point the algorithms for calculating the constraint  $\psi$ diverge for the two miniprobe propagation models. In the conic propagation mode, signaled by the flag IPROP set to 1, vBR in equation (3) represents the velocity of the equivalent conic release state of the bus (see TPRTRG analysis). Hence, to determine the actual B-plane pierce point coordinates  $\begin{pmatrix} B & & T \\ 1 & & 1 \end{pmatrix}_A$ and  $(B_i \cdot R_i)_A$  as well as the parameters  $\underline{S}$ ,  $\underline{T}$ ,  $\underline{R}$  and a for the ith miniprobe, TPPROP simply applies the subroutine STIMP to the state  $\left(r_{BR}^{T} \mid v_{1R}^{T}\right)^{T}$  where  $\underline{r}_{BR}$  denotes the equivalent conic position of the bus (and hence also of the ith probe) at release. Determining the desired B-plane pierce point coordinates,  $(B_i \cdot T_i)$ and  $(B_i \cdot R_i)_n$ , of the ith probe is complicated by the fact that the miniprobes must be correctly paired with the impact sites. The first miniprobe is targeted to the miniprobe site whose pierce point at the time of the calculation of the initial control estimate in TPRTRG was nearest the bus pierce point. Hence  $\begin{pmatrix} B_1 & T_1 \end{pmatrix}_D$  and  $\begin{pmatrix} B_1 & R_1 \end{pmatrix}_D$  are readily calculated by the appropriate call to DIMPCP, with the right ascension and declination of the target site used in the initial control estimate and the S and a of the current miniprobe 1 trajectory. Next TPRTRG computes two sets of desired B-plane coordinate pairs,  $(B_2 \cdot T_2)_D$ and  $(B_2 \cdot R_2)_p$ , by applying DIMPCP successively to each of the remaining pairs of miniprobe target sites using in both cases the

 $\underline{\mathbf{S}}$  and a of the current miniprobe 2 trajectory. With these two sets of desired pierce point coordinate pairs relative to miniprobe 2 now available for comparison, TPPR $\phi$ P selects the set whose Euclidean distance from the pierce point of miniprobe 2 is the smaller. Finally,  $\begin{pmatrix} \mathbf{B}_3 & \mathbf{T}_3 \end{pmatrix}_D$  and  $\begin{pmatrix} \mathbf{B}_3 & \mathbf{R}_3 \end{pmatrix}_D$  are calculated and  $\begin{pmatrix} \mathbf{B}_3 & \mathbf{R}_3 \end{pmatrix}_D$  are calculated and  $\begin{pmatrix} \mathbf{B}_3 & \mathbf{R}_3 \end{pmatrix}_D$ 

lated by calling DIMPCP with right ascension and declination of the remaining miniprobe target site and the <u>S</u> and a of the current miniprobe 3 trajectory. At this point an approximately inherent in the application of the subroutine DIMPCP to the miniprobes must be noted. The time-varying transformation from planetocentric-ecliptic coordinates to the probe-impact frame required by DIMPCP is held fixed at the time of the bus impact. This approximation is more than adequate for engineering purposes. All of the required B-plane data for the three miniprobes having been assembled, TPPRØP can now calculate the ith and (i+3)rd components of the constraint vector:

$$\psi_{\underline{i}} = C_{\underline{i}} \left[ \left( B_{\underline{i}} \cdot T_{\underline{i}} \right)_{D} - \left( B_{\underline{i}} \cdot T_{\underline{i}} \right)_{\underline{A}} \right]$$

$$\psi_{\underline{i+3}} = C_{\underline{i}} \left[ \left( B_{\underline{i}} \cdot R_{\underline{i}} \right)_{D} - \left( B_{\underline{i}} \cdot T_{\underline{i}} \right)_{\underline{A}} \right]$$

$$i = 1, 2, 3, \quad (4)$$

Here the C_i's are weighting factors indicating the relative importance of achieving nearby impacts at the respective miniprobe target sites. Finally, the release roll angle for the next miniprobe can be found from that of the current one by applying the addition formulas for the sine and cosine:

$$\sin \phi_{i+1} = \left(\sqrt{3} \sin \phi_i + \cos \phi_i\right) / 2 \tag{5}$$

$$\cos \phi_{i+1} = \left(\cos \phi_i - \sqrt{3} \sin \phi_i\right) / 2. \tag{6}$$

The iterative process is started by noting that  $\phi_1$  is simply  $\phi$  , the first component of the control vector.

For virtual-mass propagation indicated by IPR $\emptyset$ P=2, the general structure of the  $\underline{\psi}$  computation remains the same but the interpretation of the contituent state vectors changes. The  $\underline{v}_{BR}$  in equation (3) is taken as the velocity of the actual virtual-mass release state. Then  $\underline{VMP}$  is called to integrate the virtual mass state  $\begin{pmatrix} r & 1 & v_{IR} \\ r_{BR} & v_{IR} \end{pmatrix}^T$  of the ith miniprobe just after

release to a pseudosphere whose radius is one-tenth that of the actual Laplacian sphere of influence. From this distance inward, conic extrapolation of the current state suffices for engineering calculations. For the actual B-plane pierce-point coordinates,  $(B_i \cdot T_i)_A \text{ and } (B_i \cdot R_i)_A , \text{ as well as the quantities } \underline{S}, \underline{T}, \underline{R} \text{ and }$ 

for the ith miniprobe, TPPRØP again simply makes a single call to STIMP but this time with the virtual-mass state at the pseudo-sphere rather than the equavalent conic state at release. The remainder of the virtual-vass constraint vector computation including the computation of the desired miniprobe B-plane pierce points via the subroutine DIMPCP proceeds exactly as for the conic model,

Next consider the provisions in TPRTRG for describing the minimum-miss miniprobe approach trajectories. Throughout this application, the miss-minimization status flag, IFIN2, must be set to 2. First in this area TPPRØP must supply the impact data for the miniprobes using the conic propagation model on the conic minimum-miss release controls. This is done by setting the progation mode flag, IPRØP to 1. Then with IFIN2=2, TPPRØP uses the state

 $\left(\frac{T}{T_{BR}}\right)^T$  constructed from the bus equivalent conic release state, the conic minimum-miss release controls, and equation (3) to generate an osculating conic by a call to CAREL. The sine and cosine of the true anomaly and the time at impact are determined by a call to SPHIMP. Then the Cartesian planetocentric ecliptic state is evaluated by a call to CONCAR. From these data the right ascension and declination of the impact site, as well as the time, speed, and flightpath angle at impact for the ith miniprobe can be calculated from the formulas used in computing the same quantities for the bus. These are described in the TPRTRG analysis. The angle of attack,  $\alpha_1$ , for the ith miniprobe

however requires separate treatment. It is assumed that the longitudinal body axies of each miniprobe remains parallel until impact to the spacecraft spin axis at realse. Thus if  $\mathbf{v}_{iI}$  represents the velocity of the ith miniprobe at impact,

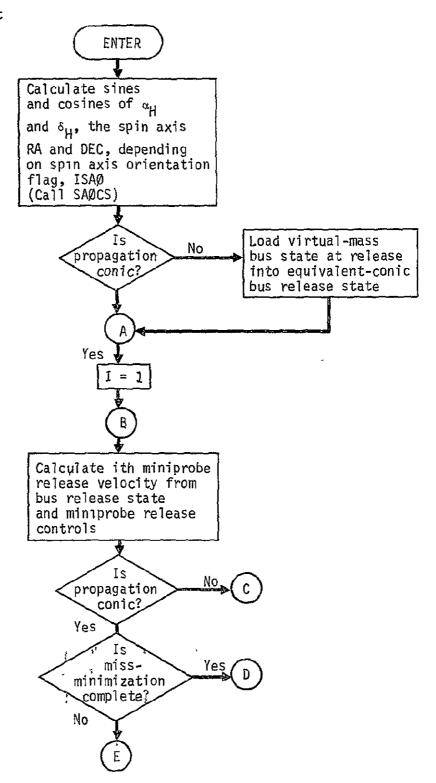
$$\alpha_{i} = \cos^{-1} \left( \underline{H} \cdot \underline{v}_{iI} / ||v_{iI}|| \right) \qquad i = 1, 2.3. \tag{7}$$

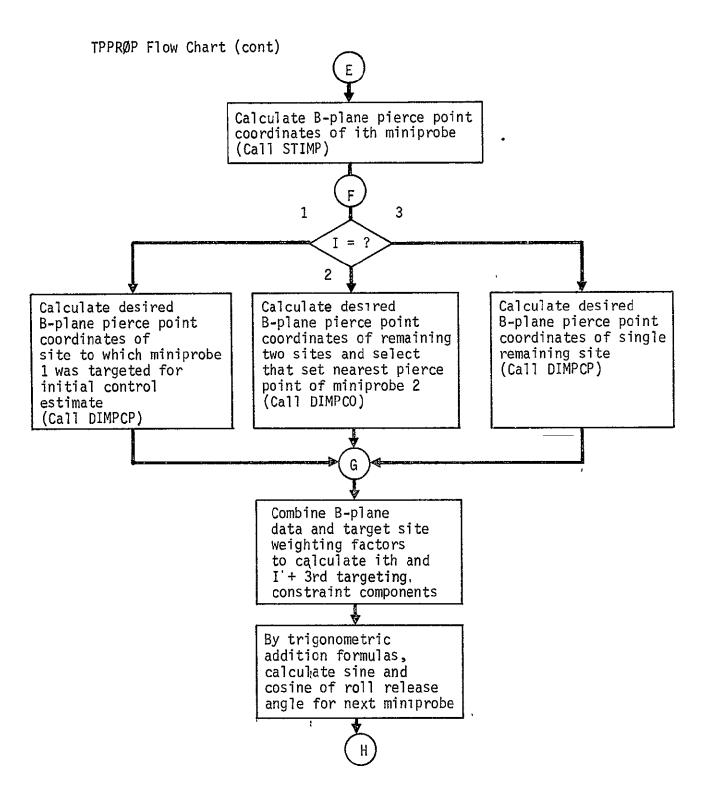
Equation (5) can be expressed in terms of the angles  $\alpha_H^{}$  and  $\delta_H^{}$  as follows:

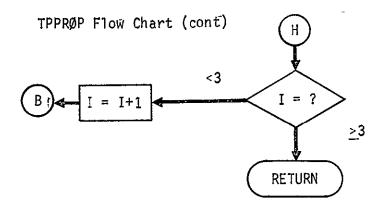
$$\alpha_{i} = \cos^{-1} \left\{ \left( \cos \delta_{H} \left[ \left( v_{iI} \right)_{1} \cos \alpha_{H} + \left( v_{iI} \right)_{2} \sin \alpha_{H} \right] + \left( v_{iI} \right)_{3} \sin \delta_{H} \right) \middle/ \left\| v_{iI} \right\| \right\} \qquad i = 1, 2.3.$$
 (8)

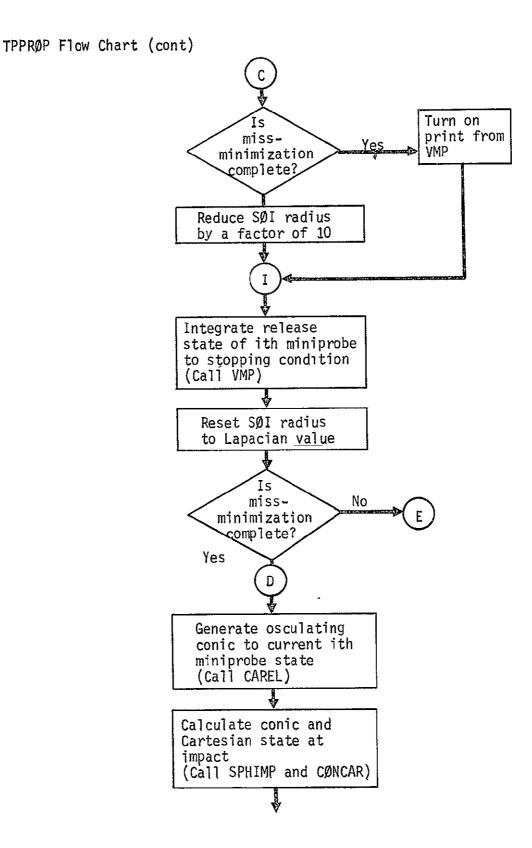
The second trajectory-descriptive function of TPPR $\emptyset$ P is calculating the virtual-mass miniprobe approach-trajectory time histories and impact data. This is always done for the conic minimum-miss release controls and also for the virtual-mass controls whenever these are calculated. The propagation flag, IPR $\emptyset$ P is simply set to 2 and the heading "Miniprobe I Minimum-Miss Approach Trajectory" is printed. Then with IFIN2=2, the state

### TPPRØP Flow Chart

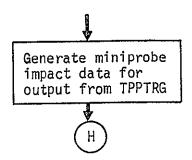








# TPPRØP Flow Chart (concl)



SUBROUTINE TPRTRG '

PURPOSE # TO CONTROL MINIPROBE TARGETING PROCEDURE

SUBROUTINES SUPPORTED: GIDANS

SUBROUTINES REQUIRED: CAREL CONCAR DIMPCP GAUSLS HPOST MATIN MATPY PECEQ SADCS SPHIMP

STIMP SUBSOL TPPROP VMP

LOCAL SYMBOLS: ANPT SEMI-MAJOR AXIS OF BUS NEAR-PLANET OSCULATING CONIC IN KM

AP SEMI-MAJOR AXIS OF PERTURBED BUS NEAR-PLANET OSCULATING CONIC IN KM

ASC SAME AS ANPT

BDRP PROJECTION ON RVP OF PLANETOCENTRIC

ECLIPTIC VECTOR TO IMPACT PLANE ASYMPTOTE
PIERCE POINT OF PERTURBED NEAR-PLANET
OSCULATING CONIC IN KM

BDRSC PROJECTION OF RVSC OF PLANETOCENTRIC
ECLIPTIC VECTOR TO IMPACT PLANE ASYMPTOTE
PIERCE POINT OF BUS NEAR-PLANET OSCULATING
CONIC IN KM

BDTP PROJECTION ON TVP OF PLANETOCENTRIC ECLIPTIC VECTOR TO IMPACT PLANE ASYMPTOTE PIERCE POINT OF PERTURBED BUS NEAR-PLANET OSCULATING CONIC IN KM

BDTSC PROJECTION ON TVSC OF PLANETOCENTRIC
ECLIPTIC VECTOR TO IMPACT PLANE ASYMPTOTE
PIERCE POINT OF BUS NEAR-PLANET OSCULATING
CONIC IN KM

BP MAGNITUDE OF PLANETOCENTRIC VECTOR TO IMPACT PLANE ASYMPTOTE FIERCE POINT OF PERTURBED BUS NEAR-PLANET OSCULATING CONIC IN KM

BSC MAGNITUDE OF PLANETOCENTRIC VECTOR TO IMPACT PLANE ASYMPTOTE PIERCE POINT OF BUS NEAR-PLANET OSCULATING CONIC IN KM

CSTISC COSINE OF TRUE ANAMOLY ON BUS NEAR-PLANET OSCULATING CONIC AT IMPACT

CSTRSC COSINE OF TRUE ANAMOLY ON BUS NEAR-PLANET OSCULATING CONIC AT EQUIVALENT RELEASE

#### STATE

- DBRTP ARRAY OF DESIRED RVSC-AXIS IMPACT PLANE COORDINATES FOR THREE MINIPROBES TO ACHIEVE RESPECTIVE THREE MINIPROBE TARGET SITES
- DBTTP ARRAY OF DESIRED TVSC-AXIS IMPACT PLANE COORDINATES FOR THREE MINIPROBES TO ACHIEVE RESPECTIVE THREE MINIPROBE TARGET SITES
- DCSAF ECLIPTIC DECLINATION OF SPIN AXIS IN DEG FOR FIXED ORIENTATION SPIN AXIS MODE
- DCSC DECLINATION IN DEG OF BUS IMPACT SITE RELATIVE TO PLANETOCENTRIC PROBE-SPHERE FRAME
- DDCS MINIMUM-MISS ECLIPTIC DECLINATION OF SPIN AXIS IN DEG
- DELTPS SAVED VALUE OF TIME DURATION BETWEEN VMP PRINTOUTS
- DELTUV COMMON VELOCITY INCREMENT LENGTH IN KM/SEC USED IN U AND V DIRECTIONS IN APPROXIMATING JACOBIAN MATRIX SENSM BY DIVIDED DIFFERENCES
- DJECA JULIAN DATE EPOCH 1900 ON BUS NEAR-PLANET OSCULATING, CONIC AT CLOSEST APPROACH
- OJEISC JULIAN DATE EPOCH 1900 ON BUS NEAR-PLANET OSCULATING CONIC AT IMPACT
- DJENPS JULIAN DATE EPOCH 1900 ON BUS NEAR-PLANET OSCULATING CONIC AT EXTRAPOLATION STATE
- DJEPOC JULIAN DATE ON JANUARY .5 1900 E.T.
- DRAS MINIMUM-MISS ECLIPTIC RIGHT ASCENSION OF SPIN AXIS IN DEG
- DRRA MINIMUM-MISS RELEASE ROLL ANGLE OF MINIPROBE 1 IN DEG
- DTPISC TIME INTERVAL IN SEC FROM PERIAPSIS TO IMPACT ON BUS NEAR-PLANET OSCULATING CONIC
- DTPNPS TIME INTERVAL IN SEC FROM PERIPSIS TO EXTRAPOLATION STATE ON BUS NEAR-PLANET OSCULATING CONIC

CONVERSION FACTOR FROM DEGREES TO RADIANS

DTS CONVERSION FACTOR FROM DAYS TO SECONDS

EMSMN SQUARE OF DISTANCE BETWEEN ASYMPTOTE PIERCE POINTS IN IMPACT PLANE OF BUS AND NEAREST MINIPROBE IN KM**2

EMS SQUARE OF DISTANCE BETWEEN ASYMPTOTE PIERCE POINTS IN IMPACT PLANE OF BUS AND ITH MINIPROBE IN KM**2

ENPT ECCENTRICITY OF BUS NEAR-PLANET OSCULATING CONIC .

EPSLS UPPER BOUND ON WEIGHTED SUM OF CHANGE IN LENGTH OF CONTROL VECTOR AND CHANGE IN MAGNITUDE OF MISS INDEX USED IN CONVERGENCE CRITERION FOR LEAST-SQUARES ROUTINE

FPASC FLIGHT PATH ANGLE OF BUS IN DEG AT IMPACT ON NEAR-PLANET OSCULATING CONIC

ICLS SAVED VALUE OF VMP FLAG ICL INDICATING WHETHER OR NOT TRAJECTORY HAS REACHED CLOSEST APPROACH

ICL2S SAVED VALUE OF VMP FLAG ICL2 INDICATING WHETHER OR NOT TRAJECTORY IS TO BE STOPPED AT CLOSE'ST APPROACH

ICONV1 CONVERGENCE INDICATOR FOR LEAST-SQUARES ROUTINE

=1 CONVERGENCE

=2 NO CONVERGENCE

IEPHEMS SAVED VALUE OF VMP FLAG IEPHEM INDICATING WHETHER OR NOT ORB IS TO SE CALLED BEFORE CALLING EPHEM

INPRS SAVED VALUE OF VMP VARIABLE INPR
INDICATING NUMBER OF INTEGRATION STEPS
BETWEEN PRINTOUTS

JPCSK FLAG INDICATING THE PLANETGCENTRIC PROBESPHERE COORDINATE SYSTEM OFTION
=1 EQUATORIAL
=2 SUBSOLAR ORBIT PLANE

IPRNTS SAVED VALUE OF VMP FLAG IPRINT INDICATING WHETHER OR NOT PRINTOUT, IS TO OCCUR

IPROPI FLAG INDICATING WHETHER CONIC OR VIRTUALMASS MINIPROBE PROPAGATION IS DESIRED

=1 CONIC

=2 VIRTUAL-MASS

ISAOP MODIFIED SPIN AXIS ORIENTATION MODE FLAG
USED IN ORIENTING SPIN AXIS FOR GENERATING
INITIAL RELEASE CONTROL ESTIMATE

ISPHS SAVED VALUE OF VMP FLAG ISPH INDICATING WHETHER OR NOT SPHERE OF INFLUENCE HAS BEEN PIERCED

NCNTRL NUMBER OF RELEASE CONTROLS

PNPT SEMI-LATUS RECTUM OF BUS NEAR-PLANET OSCULATING CONIC

PPNPT PLANETUCENTRIC ECLIPTIC UNIT VECTOR IN DIRECTION OF PERIAPSIS OF BUS OSCULATING NEAR-PLANET CONIC

PSI IMPACT PLANE CONSTRAINT VECTOR

QQNPT PLANETOCENTRIC ECLIPTIC UNIT VECTOR LYING IN PLANE OF NEAR-PLANET OSCULATING CONIC 90 DEG ADVANCED FROM PPNPT

RASAF ECLIPTIC RIGHT ASCENSION OF SPIN AXIS FOR FIXED ORIENTATION SPIN AXIS MODE

RASC RIGHT ASCENSION OF BUS IMPACT SITE IN DEG RELATIVE TO PLANETOCENTRIC PROBE-SPHERE FRAME

RCM MAGNITUDE OF PLANETOCENTRIC POSITION VECTOR OF BUS AT IMPACT 'IN KM

RPVEC PLANETOCENTRIC ECLIPTIC UNIT VECTOR TO BUS IMPACT SITE

RPVSS PLANETOCENTRIC SUBSOLAR UNIT VECTOR TO BUS IMPACT POINT

RSCNPS PLANETOCENTRIC ECLIPTIC POSITION VECTOR OF BUS EXTRAPOLATION STATE IN KM

RTD CONVERSION FACTOR FROM RADIANS TO DEGREES

RVP PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN DIRECTION OF CROSS PRODUCT OF ASYMPTOTE OF NEAR-PLANET OSCULATING CONIC FOR

PERTURBED BUS BY ECLIPTIC POLE VECTOR PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN RVSC DIRECTION OF CROSS PRODUCT OF ASYMPTOTE OF NEAR-PANET OSCULATING CONIC FOR BUS BY ECLIPTIC POLE VECTOR SENSM JACOBIAN SENSITIVITY MATRIX OF BUS NEAR-PLANET OSCULATING CONIC ASYMPTOTE PIERCE POINT COORDINATES TO VELOCITY INCREMENTS AT RELEASE IN THE U AND V DIRECTIONS SINE OF TRUE ANAMOLY ON BUS NEAR-PLANET SNTISC OSCULATING CONIC AT IMPACT SINE OF TRUE ANAMOLY ON BUS NEAR-PLANET SNTRSC OSCULATING CONIC AT EQUIVALENT RELEASE STATE SPHERS SAVED VALUE OF RADIUS OF SOI FOR TARGET PLANET IN ASTRONOMICAL UNITS SVP PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN DIRECTION OF ASYMPTOTE OF NEAR-PLANET OSCULATING CONIC OF PERTURBED BUS PLANETO CENTRIC ECLIPTIC · UNIT VECTOR IN SVSC DIRECTION OF ASYMPTOTE OF NEAR-PLANET OSCULATING CONIC OF BUS SO MAXIMUM PERMISSIBLE LENGTH OF PSEUDO-INVERSE CONTROL STEP TARGM NEWTON-RAPHSON MATRIX FOR TARGETING BUS TO NEAREST MINIPROBE TARGET SITE TM3 CONSTANT 1.0E-03 TM4 CONSTANT 1.0E-04 TM5 CONSTANT 1.0E-05 TP20 CONSTANT 1. DE+20 CONSTANT 1.0E+02 TP2

TP4

TVP

TVSC

CONSTANT 1.8E+04

CROSS PRODUCT OF SVP BY RVP

CROSS PRODUCT OF SVSC BY RVSC

UCNTRL RELEASE CONTROL VECTOR

UTPR APPROXIMATE VELOCITY INCREMENT IN U DIRECTION AT RELEASE IN KM/SEC NECESSARY TO TARGET BUS TO NEAREST MINIPROBE TARGET SITE

VCM MAGNITUDE OF PLANETOCENTRIC VELOCITY OF BUS AT IMPACT IN KM/SEC

VSCNPS PLANETUCENTRIC ECLIPTIC VELOCITY VECTOR OF BUS EXTRAPOLATION STATE IN KM/SEC

VSCRPM MAGNITUDE OF VELOCITY OF BUS AT EQUIVALENT CONIC RELEASE STATE IN KM/SEC

VSCRPP PLANETOCENTRIC ECLIPTIC VELOCITY VECTOR AT EQUIVALENT CONIC RELEASE STATE IN KM/SEC USED IN EVALUATING SENSM

VTANG MINIMUM-MISS TANGENTIAL VELOCITY OF MINIPROBES AT RELEASE IN KM/SEC

VTPR APPROXIMATE VELOCITY INCREMENT IN V
DIRECTION AT RELEASE IN KM/SEC NECESSARY
TO TARGET BUS TO NEAREST MINIPROBE TARGET
SITE

WWNPT PLANETOCENTRIC ECLIPTIC UNIT VECTOR IN DIRECTION OF ANGULAR MOMENTUM FOR NEAR-PLANET BUS OSCULATING CONIC

XSF FINAL HELIOCENTRIC ECLIPTIC BUS STATE
IN KM AND KM/SEC RETURNED BY VMP AS THE
FIRST STATE INSIDE PROBE SPHERE

XSI INITIAL HELIOCENTRIC ECLIPTIC BUS STATE IN KM AND KM/SEC PROPAGATED TO IMPACT BY VMP

YM FINAL MINIMUM VALUE OF MISS INDEX

COMMON COMPUTED/USED: ACTPP DCTP DELTM DELTP DJERN ' DTPRSC GMUP ICL2 ICL **IEPHEM** IMIN INPR IPROP IPRINT ISAO ISPH RATP RSCRHA RTPS SPHERE VSCRHA COMMON COMPUTED: IPCSP IFIN2 KHIT RPSP WFLS COMMON USED: AATTP ACKT ADCTP ALNGTH ARATP CSDCSA CSRASA DJEITP DG DSI

FIVE	FPAIP	IPCS	KTAR	KUR
NINETY	NTP	ONE	PMASS	RIN
RPS	RSCRPA	RSCRPM	RSI	SNDCSA
SNRASA	TAR	TM	TOL	TRANSF
TWO ZERO	VIMTP	VSCRPA	VSI	WGHTM

## TPRTRG Analysis

TPRTRG is the executive routine directing multiprobe targeting. To do so it must accomplish four basic tasks: (1) process miniprobe targeting input data, (2) generate initial estimates for the release controls, (3) apply the least-squares routine, GAUSLS, to minimize the miniprobe miss index, and (4) use the miniprobe propagating routine, TPPROP, to generate minimum-miss approach trajectories and impact data. Each of these objectives, as well as the routine's printout, is discussed in the following paragraphs.

The processing of the miniprobe targeting input involves three major aspects. First an equivalence must be set up between the multipurpose targeting variables of NØMINAL and the mnemonic symbolism of TPRTRG. Second, a conic miniprobe release state must be determined that is equivalent to the virtual-mass release state for use in the high-speed conic miniprobe propagation model. By equivalent we mean that although one state falls on a conic and the other on a virtual-mass trajectory, both states occur at the same absolute time and the two trajectories are tangent at impact. To generate the equivalent conic release state, the subroutine VMP is called to propagate the bus state from release to impact. Before doing this, however, the VMP trajectory condition flags are stored for subsequent restoration after exiting TPRTRG. The virtual-mass impact state is then fit with an osculating conic by a call to CAREL. The osculating conic is then propagated back to the actual release time via calls to the subroutines HPOST and CONCAR. Using this conic release state, a set of minimum-miss controls for the conic model can now readily be determined. The third phase of processing miniprobe input data, namely setting up the several targeting options, is next begun. First the coordinate transformation matrix, C, from the planetocentric inertial ecliptic coordinates to the Cartesian frame in which the miniprobe impact sites are specified is generated by a call to either the subroutine SUBSOL or PECEQ, depending on the state of the coordinatesystem flag IPCSK. The spin axis orientation mode desired is stored in the flag ISAØ. It is used by TPRTRG in devising an initial control estimate and by TPPRØP in all miniprobe propagations. Finally, the desired miniprobe propagation mode is stored in the flag IPROPI. This variable is used solely by TPRTRG in deciding on which propagation mode to request from TPPRØP via the common flag IPRØP.

TPRTRG next deals with the problem of generating an initial estimate of the control vector. First consider the problem of estimating the conic-model minimum-miss controls. Initial estimates for two of the controls (the spin axis orientation angles) depend of course on the spin axis orientation mode. In three of the four possible modes, the inertial-ecliptic declination and right ascension of the spin axis are fixed rather than free controls. " Hence no initial estimates for them need be provided. In the remaining mode, however, both of these controls are free, and the initial estimates provided for them are simply those that bring the spin axis into coincidence with the spacecraft velocity vector at release. This orientation was chosen for the initial estimate since it produces the widest distribution of miniprobe entry sites for a given combination of the remaining two release controls. Regardless of the orientation mode, the spin axis pointing direction is specified throughout miniprobe targeting by the sines and cosines of its ecliptic declination and right ascension. These trigonometric functions are always calculated by the subroutine SAØCS from the appropriate components of the control vector and the spin axis orientation flag. Since the spin axis pointing direction is necessary in initially estimating the other two controls, TPRTRG must call SAØCS to obtain the above trigonometric characterization.

Initial estimates for the roll release angle, \$\phi\$, of the first probe and the tangential velocity at release,  $v_m$ , can then be generated by merely targeting the first miniprobe to the miniprobe target site nearest the impact point for the bus on the trajectory existing at release. First, the B-plane pierce point of the bus is calculated by calling the subroutine STIMP with the bus impact state. Then the desired B-plane pierce point corresponding to each of the three miniprobe target sites is computed through a call to DIMPCP. It must be noted that this calculation is only approximate since it assumes all of the miniprobes have the same S and impact time as the bus. Nonetheless, the accuracy is more than sufficient for engineering purposes. Next the pierce points of the desired miniprobe impact sites are compared to find the one nearest that of the bus. The release velocity increment perpendicular to the bus spin axis that would target the first miniprobe is then approximated by a single Newton-Raphson step. Let B_R•T, B_R•R, B₁•T, and B₁•R denote the B-plane pierce point components corresponding to the bus and the desired miniprobe impact sites, respectively. Define a constraint vector as

$$\psi = \begin{pmatrix} B_1 \cdot T + B_B \cdot T \\ B_1 \cdot R - B_B \cdot R \end{pmatrix} \tag{1}$$

Let  $\underline{H}$  denote a unit vector in the direction of the spin axis of the spacecraft. Using  $\underline{H}$ , define  $\underline{U}$  and  $\underline{V}$  as

$$\underline{\mathbf{U}} = \underline{\mathbf{H}} \times \underline{\mathbf{Z}}_{ec} / || \underline{\mathbf{H}} \times \underline{\mathbf{Z}}_{ec} ||$$
 (2)

$$V = H \times U \tag{3}$$

where  $\underline{Z}_{ec}$  is the inertial ecliptic pole vector. Then a convenient probe-release Cartesian frame is defined by the triple  $\underline{U}$ - $\underline{V}$ - $\underline{H}$ . Let  $\Delta v_{\underline{U}}$  and  $\Delta u_{\underline{V}}$  denote the components of the release velocity increment in the  $\underline{U}$  and  $\underline{V}$  directions, respectively.

Then the control vector is given by

$$\underline{\mathbf{X}} = \begin{pmatrix} \Delta \mathbf{v}_{\mathbf{U}} \\ \Delta \mathbf{v}_{\mathbf{V}} \end{pmatrix}. \tag{4}$$

Let J denote the Jacobian matrix of  $\underline{\psi}$  with respect to  $\underline{X}$ ; i.e.,

$$Jij = \frac{\partial \psi_i}{\partial Xj} \qquad i = 1,2$$

$$j = 1,2.$$
(5)

One Newton-Raphson step then approximates the targeting control vector as

$$\underline{\mathbf{X}} = \mathbf{J}^{-1} \ \underline{\boldsymbol{\psi}}. \tag{6}$$

TPRTRG computes the Jacobian matrix by divided differencing. Let  $v_{BR}$  represent the equivalent conic planetocentric velocity of the bus at release. Let  $\delta_{\underline{U}}\underline{v}_{BR}$  and  $\delta_{\underline{V}}\underline{u}_{BR}$  denote the perturbations in  $\underline{v}_{BR}$  caused by a velocity increment of magnitude  $\delta v$  in the  $\underline{U}$  and  $\underline{V}$  directions, respectively. Then clearly

$$\delta_{\underline{U}}\underline{V}_{BR} = \delta v\underline{U} \tag{7}$$

$$\delta_{V - BR} = \delta_{V \underline{V}}. \tag{8}$$

Let  $\alpha_H$  and  $\delta_H$  be the right ascension and declination of the spin axis, respectively. By expressing the definitions of  $\underline{U}$  and  $\underline{V}$  in terms of these angles, equations (7) and (8) can be expressed in the planetocentic ecliptic frame as

$$\delta_{U} \underline{v}_{BR} = \delta v \left( \sin \alpha, -\cos \alpha, 0 \right)^{T}$$
 (9)

$$\delta_{V \to BR} = \delta_{V} (\cos \alpha \sin \delta, \sin \alpha \sin \delta, -\cos \delta)^{T}.$$
 (10)

Let  $\underline{r}_{BR}$  be the equivalent conic planetocentric position of the spacecraft at release. Then by applying the subroutine STIMP consecutively to the states  $\left(\underline{r}_{BR}, \underline{v}_{BR} + \delta_{U}v_{BR}\right)^{T}$  and  $\left(\underline{r}_{BR}, \underline{v}_{BR} + \delta_{U}v_{BR}\right)^{T}$ , the perturbed state vectors  $\delta_{U}\psi$  and  $\delta_{V}\psi$  can be generated. TPRTRG then approximates J as  $\left(\delta_{U}\psi \mid \delta_{V}\psi\right)/\delta v$ . Having approximated  $\Delta v_{U}$  and  $\Delta v_{V}$ , the roll release angle of miniprobe 1 and the tangential velocity at release are readily calculated from the formulae

$$v_{T} = \sqrt{\Delta v_{U}^{2} + \Delta v_{V}^{2}}$$
 (11)

$$\phi = \tan^{-1} \left( \Delta v_{V} / \Delta v_{U} \right). \tag{12}$$

It must be noted that  $\phi$  represents the angle the velocity increment of the first miniprobe makes with the U axis. It should not be confused with the angle between the first probe arm and the U direction, which is  $\pi/2$  radians less than  $\phi$ .

Consider next the initial control estimate for virtual-mass minimum-miss controls. TPRTRG simply uses the minimum-miss conic control for this estimate. Hence, irrespective of the desired miniprobe propagation mode indicated by IPRØPI, the conic controls are first found. Then if only the conic controls are desired, a return is made to the calling program; otherwise the least-squares algorithm is repeated with virtual-mass rather than conic miniprobe propagation.

The third basic task of TPRTRG, namely using the subroutine GAUSLS to calculate the minimum-miss index controls, requires some explanation. First the four miniprobe release controls must be

identified. They are simply  $\phi$  ,  $v_T^{}$  ,  $\delta_H^{}$  , and  $\alpha_H^{}$  (see Fig. 1). Thus if u denotes the control vector, then

$$\underline{\mathbf{u}} = \begin{pmatrix} \phi \\ \mathbf{v}_{\mathbf{T}} \\ \delta_{\mathbf{H}} \\ \alpha_{\mathbf{H}} \end{pmatrix}. \tag{13}$$

By measuring the angles  $\phi$ ,  $\alpha_{\rm H}$ , and  $\delta_{\rm H}$  in radians and the velocity  $v_{\rm T}$  in decameters/s all of the components of  $\underline{u}$  fall in the range from 0.1 to 10 as required by the subroutine GAUSLS when Jacobian matrices are generated by a uniform control perturbation of  $10^{-5}$ .

Let  $(B_i \cdot T_i)_A$  and  $(B_i \cdot R_i)_A$  denote the actual B-plane pierce point coordinates of the ith miniprobe when the release control  $\underline{u}$  is applied. Let  $(B_i \cdot T_i)_D$  and  $(B_i \cdot R_i)_D$  represent the de-

sired B-plane pierce point coordinates of the ith miniprobe based on the actual  $\underline{S}$  and the energy of that miniprobe. The six components of the constraint vector are then given as

$$\psi_{i} = C_{i} \left[ \begin{pmatrix} B_{i} \cdot T_{i} \end{pmatrix}_{A} - \begin{pmatrix} B_{i} \cdot T_{i} \end{pmatrix}_{D} \right] \qquad \qquad (14)$$

$$\psi_{i+3} = C_{i} \left[ \begin{pmatrix} B_{i} \cdot R_{i} \end{pmatrix}_{A} - \begin{pmatrix} B_{i} \cdot R_{i} \end{pmatrix}_{A} \right] \qquad (15)$$

Here the  $C_1$ 's are weighting factors input by the user to indicate the relative importance of achieving nearby impacts at the various miniprobe target sites. The subroutine TPPRØP is always used to calculate  $\underline{\psi}$  given  $\underline{u}$  for whichever miniprobe propagation mode is specified by the flag IRPØP. Thus GAUSLS can be called to carry out the entire least-squares process of minimizing the miss-index  $y = \underline{\psi}^T\underline{\psi}$  for either propagation mode once the initial control estimate  $\underline{u}$  and the corresponding constraint  $\underline{\psi}$  ( $\underline{u}$ 0) have been

estimate uo and the corresponding constraint  $\psi$  (uo) have been calculated. If the least-squares routine should fail to converge, the universal NØMNAL trouble flag, KWITT, is set to 1 to cause execution to terminate on return to the main program, and a return is made to the calling program, GIDANS.

The final responsibility of TPRTRG is to calculate the n-body miniprobe approach trajectory time histories and impact data for the bus and the miniprobes using the minimum-miss release controls corresponding to both the conic and virtual-mass propagation modes. The impact data for the bus are calculated in TPRTRG itself. Let  $\underline{r}_{BI}$  and  $\underline{\rho}_{BI}$  denote the impact position vectors in the planetocentric ecliptic and probe-impact coordinate frames. The  $\underline{\rho}_{BI}$  is calculated from  $\underline{r}_{BI}$ , which is available from the virtual-mass propagation, as

$$\underline{\rho}_{\mathrm{RT}} = C\underline{\mathbf{r}}_{\mathrm{RT}}.\tag{16}$$

The right ascension,  $\alpha_B$ , and declination,  $\delta_B$ , of the bus impact site relative to the probe-impact frame are then readily calculated as

$$\alpha_{\rm B} = \tan^{-1} \left( {^{\rho}_{\rm BI}} \right)_2 / \left( {^{\rho}_{\rm BI}} \right)_1 \tag{17}$$

$$\delta_{B} = \sin^{-1} \left( \rho_{BI} \right). \tag{18}$$

The flightpath angle, 
$$\gamma_{BI} = \tan^{-1} \left( e r_{I} \sin \theta_{I} / p \right)$$
 (19)

where  $r_{\tilde{I}}$  is the radius of the impact sphere, e is the eccentricity, p is the semilatus rectum and  $\theta_{\tilde{I}}$  is the true anomaly at impact. All of these conic dements refer to osculating hyperbola at 1m-pact. The magnitude of the bus impact velocity is calculated as

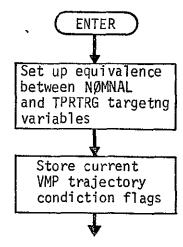
$$v_{BI} = \sqrt{u(2/r_I - 1/a)}$$
 (20)

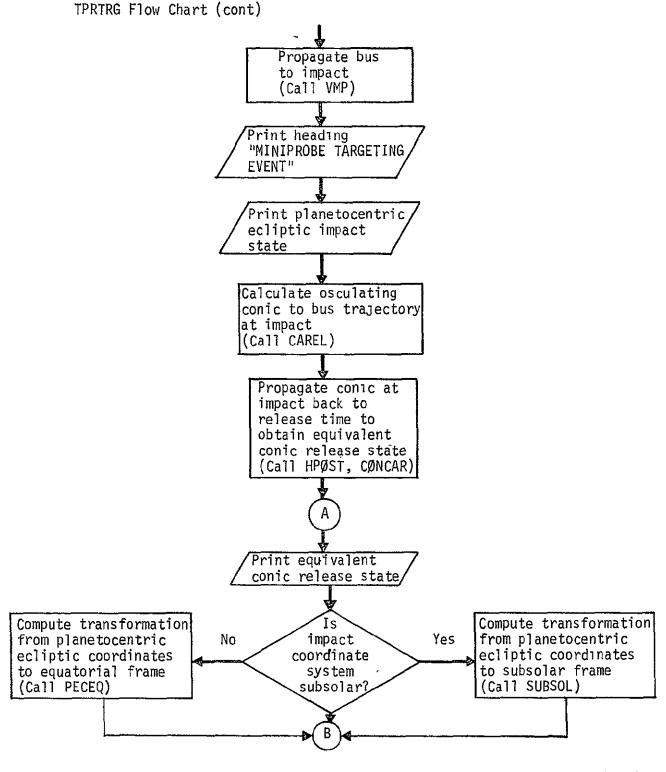
where a is the semimajor axis of the osculating conic, and u is the gravitational constant of the planet.

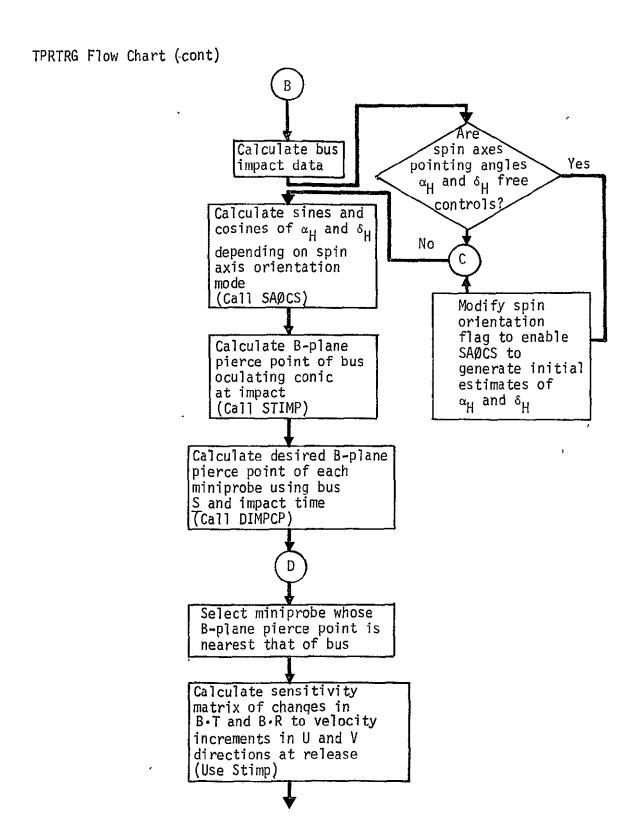
For the miniprobes, both the impact data and the virtual-mass approach trajectory time histories are generated by a call to TPPRØP with the least-squares status flag IFIN2 set to indicate that the miss-minimizing procedure is complete. When the least-squares algorithm is performed using conic miniprobe propagation, impact data are computed for both the conic and virtual-mass models.

The printout from TPRTRG is designed to meet two objectives: (1) to completely describe the minimum-miss miniprobe approach trajectories, and (2) to reveal any errors in the minimum-miss release controls caused by improper use of the program. To identify the type of nolinear guidance event, the heading "Miniprobe Targeting Event" is printed first of all. Next pertient data at release are printed. These consist of the planetocentric bus impact state and the equivalent conic release state as previously described. Then the printout from the miss-minimizing algorithm GAUSLS is provided in its entirety. After the minimum-miss release controls are found, they are printed out with a phrase indicating whether they correspond to the conic or virtual-mass model. If conic miniprobe propagation was used for the miss minimization, the conic-model probe impact data are printed. These include right ascension and declination of the impact point, together with time, velocity, and flightpath angle at impact for each of the miniprobes as well as the bus (numbered as probe number 0 in the printout). The bus data provided here are actually based on the initial virtual-mass propagation from the release state. The miniprobe data also contain the angles of attach, assuming the minprobe longitudinal body axes reamin parallel to the spacecraft spin axis at release. Next the time histories of the miniprobe minimum-miss approach trajectories are printed in succession from the subroutine VMP with print intervals of 5 days and 100 integration steps. Finally, the virtualmass model probe impact information is printed. It is-identical in content to the conic impact data except that the information is now based on virtual-mass propagation from the release state.

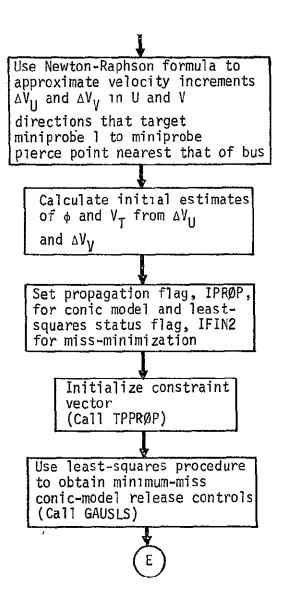
#### TPRTRG Flow Chart

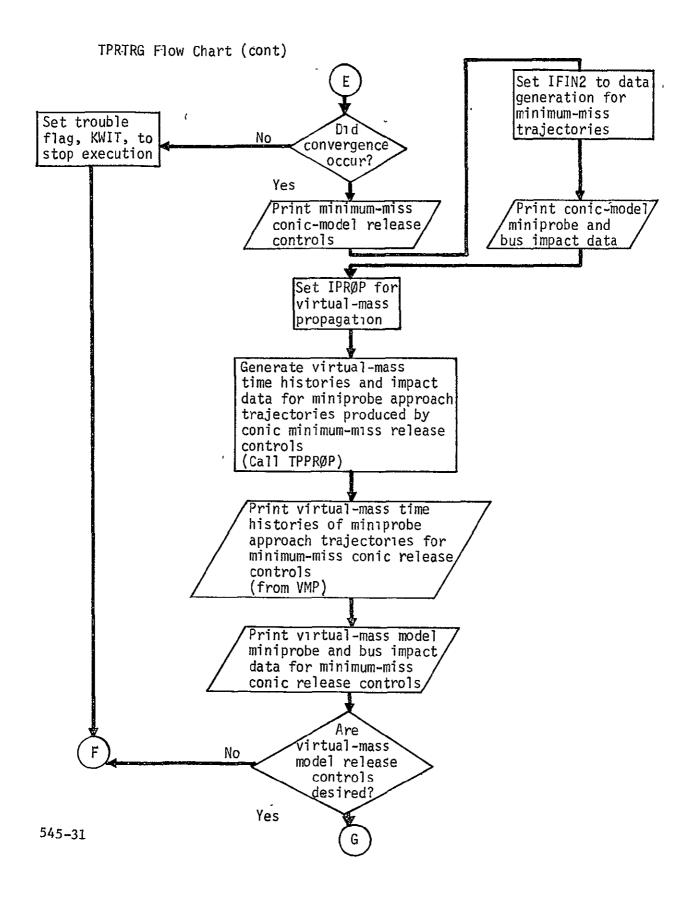


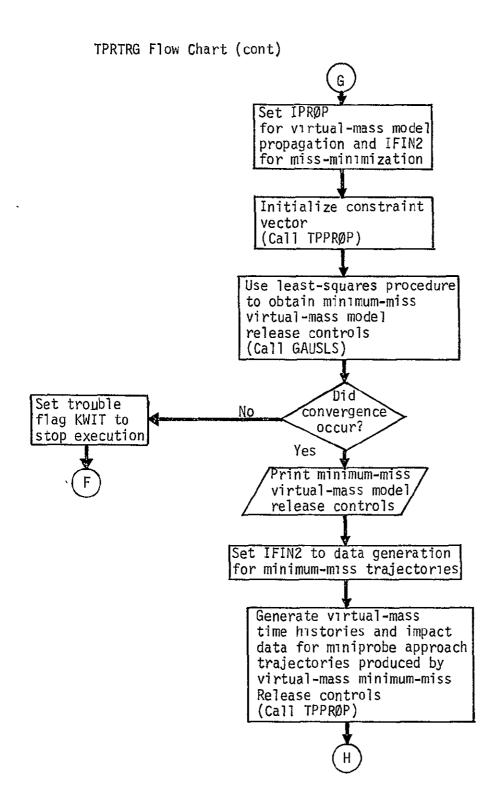




# TPRTRG Flow Chart (cont)







# Print virtual mass time histories of miniprobe approach trajectories for minimum-miss virtual-mass release controls (From VMP) Print virtual-mass model miniprobe and bus impact data for minimummiss virtual-mass release controls Restore previous VMP trajectory condition flags

## Key to Symbols

- right ascension of spin axis - declination of spin axis - roll release angle of first miniprobe Y_Ti - tangential release velocity of ith miniprobe <u>H</u> - spacecraft spin-axis unit vector

 $\frac{\underline{U}}{\underline{Y}} = \frac{\underline{H} \times \underline{Z}_{ec}}{\underline{H} \times \underline{Z}_{ec}}$   $= \frac{\underline{H} \times \underline{U}}{\underline{X}_{ec}}, \quad \underline{Y}_{ec}, \quad \underline{Z}_{ec}$   $= \frac{\underline{H} \times \underline{U}}{\underline{I}_{ec}}$   $= \frac{\underline{H} \times \underline$ 

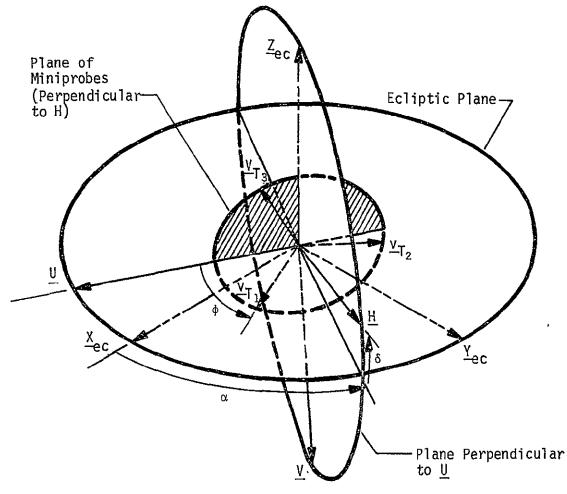


Figure 1 Miniprobe Release Geometry

SUBROUTINE TRAKM

PURPOSE: THE OBSERVATIONS AND OBSERVATION MATRIX FOR A GIVEN TYPE OF MEASUREMENT IS COMPUTED BY THIS ROUTINE

CALLING SEQUENCE: CALL TRAKM (HECV, ITRK, NR, IOBS, VECTOR)

ARGUMENT: HEGY I POSITION AND VELOCITY OF SPACECRAFT AT TIME OF MEASUREMENT

IOBS I CODE WHICH SPECIFIES IF MEASUREMENT OR OBSERVATION MATRIX IS TO BE COMPUTED

ITRK I CODE WHICH SPECIFIES MEASUREMENT TYPE (CALLED MMCODE ELSEWHERE IN PROGRAM)

NR O NUMBER OF ROWS IN THE OBSERVATION MATRIX

VECTOR O ACTUAL MEASUREMENT

SUBROUTINES SUPPORTED: ERRANN PROBE

SUBROUTINES REQUIRED: EPHEM ORB STAPRL TARPRL

LOCAL SYMBOLS: AD1 INTERMEDIATE VARIABLE

AD2 INTERMEDIATE VARIABLE

AD3 INTERMEDIATE VARIABLE

A INTERMEDIATE VARIABLE

AL ALTITUDE

ALAT LATITUDE

ALON LONGITUDE

ALRAD INTERMEDIATE VARIABLE

A1 PARTIAL OF RANGE WITH RESPECT TO X

A2 PARTIAL OF RANGE WITH RESPECT TO Y

A3 PARTIAL OF RANGE WITH RESPECT TO Z

B1 PARTIAL OF RANGE-RATE WITH RESPECT TO X

B2 PARTIAL OF RANGE-RATE WITH RESPECT TO Y

B3 PARTIAL OF RANGE-RATE WITH RESPECT TO Z

CE COSINE OF OBLIQUITY OF EARTH

COAL COSINE OF STAR-PLANET ANGLE COSA INTERMEDIATE VARIABLE COSAZ COSINE AZIMUTH COSRA COSINE RIGHT ASSENSION CP COSINE OF LONGITUDE + CONSTANT PARTIALS OF STAR-PLANET ANGLE WITH RESPECT DADP TO VEHICLE POSITION AND VELOCITY DBDP PARTIALS OF APPARENT PLANET DIAMETER WITH RESPECT TO VEHICLE POSITION AND VELOCITY DD INTERMEDIATE VARIABLE INTERNEDIATE VARIABLE DENOM D INTERMEDIATE TIME RANGE-RATE PARTIAL WITH RESPECT TO STATION ĒΚ LOCATION ERRORS GEOCENTRIC EQUATORIAL COORDINATES OF **GECS** STATION GELS GEOCENTRIC ECLIPTIC COORDINATES OF STATION HE CE COORDINATES OF EARTH HE CP COORDINATES OF TARGET PLANET TRACKING STATION LOCATION SELECTION CODE IΑ CODE CORRESPONDING TO TRACKING STATION ICO LOCATION ERRORS IC COLUMN NUMBER IN OBSERVATION MATRIX PARTITION WHERE EK IS TO BE STORED TEND VARIABLE INDEX VALUE IR STAR-PLANET ANGLE INDEX INCREMENT VALUE NΑ STAR-PLANET ANGLE INDEX LOWER LIMIT =1 FOR 3 STAR-PLANET ANGLES =ITRK-10 FOR SINGLE STAR-PLANET ANGLES STAR-PLANET ANGLE INDEX UPPER LIMIT NC

=3 FOR 3 STAR-PLANET ANGLES

=ITRK-10 FOR SINGLE STAR-PLANET ANGLES

PAR PARTIALS RETURNED FROM SUBROUTINE TARPEL

PAT1 INTERMEDIATE VARIABLE

PAT2 INTERMEDIATE VARIABLE

PA PARTIALS

P2, INTERMEDIATE VARIABLE

RA RIGHT ASCENSION

RADNIP RADIUS OF TARGET PLANET

RAS INTERMEDIATE VARIABLE

RHOP INTERMEDIATE VECTOR

RRATE RANGE-RATE

R1 RANGE

SE

R2 SQUARE OF RANGE

SA PARTIALS OF STAR-PLANET ANGLES WITH RESPECT TO VEHICLE POSITION

SINE OF OBLIQUITY OF EARTH

SIAL SINE OF STAR-PLANET ANGLE

SINAZ SINE AZIMUTH

SIND INTERMEDIATE VARIABLE

SINRA SINE RIGHT ASCENSION

SP SINE OF LONGITUDE + CONSTANT

SUM INTERMEDIATE VARIABLE

SUM1 INTERMEDIATE VARIABLE

VEC INTERMEDIATE VECTOR

ZZ1 INTERMEDIATE VARIABLE

ZZ2 INTERMEDIATE VARIABLE

COMMON COMPUTED/USED: AAL AM H NO T

		ΧP	AN			
COMMON	COMPUTED:	G				
COMMON	USED:	ALNGTH F IBARY ONE TM VST	DATEJ IAUGDC NBOD RADIUS TRIM1 WST	DELTM IAUGIN NB SAL TWO ZERO	EM3 IAUGMC NTP SLAT UNIVT	EPS IAUG OMEGA SLON UST

549

TRAKM Analysis

The linearized observation equation can be written as

$$y = Hx + Mx + Gu + Lv + Nw$$

where y is the observable, x is the spacecraft state, and  $x_s$ , u, v, and w are solve-for, dynamic consider, measurement consider, and ignore parameter vectors, respectively. The function of subroutine TRAKM is to compute the observation matrix partitions H, M, G, L, and N, which indicate the sensitivity of the observable y to changes in x,  $x_s$ , u, v, and w, respectively, in the error analysis/generalized covariance analysis program. The matrix N is computed only for a generalized covariance analysis.

Except for computation of the ignore parameter observation matrix partition N, TRAKM is equivalent to subroutine TRAKS, which is used in the simulation program. See subroutine TRAKS for further analytical details and a flow chart.

SUBROUTINE TRAKS

PURPOSE: TO COMPUTE ALL OBSERVATION MATRIX PARTITIONS FOR THE MEASUREMENT TYPE AND TO COMPUTE THE MEASUREMENT ITSELF.

CALLING SEQUENCE: CALL TRAKS (HECV, ITRK, NR, 108S, VECTOR)

ARGUMENT: HECV I POSITION AND VELOCITY OF SPACECRAFT AT TIME OF MEASUREMENT

I CODE WHICH SPECIFIES IF MEASUREMENT OR OBSERVATION MATRIX IS TO BE COMPUTED

ITRK I CODE WHICH SPECIFIES MEASUREMENT TYPE (CALLED MMCODE ELSEWHERE IN PROGRAM)

NR O NUMBER OF ROWS IN THE OBSERVATION MATRIX

VECTOR O ACTUAL MEASUREMENT

SUBROUTINES SUPPORTED: SIMULL

SUBROUTINES REQUIRED: EPHEM ORB STAPRL TARPRL

LOCAL SYMBOLS: AD1 INTERMEDIATE VARIABLE

AD2 INTERMEDIATE VARIABLE

AD3 INTERMEDIATE VARIABLE

A INTERMEDIATE VARIABLE

ALAT LATITUDE

ALON LONGITUDE

AL ALTITUDE

ALRAD INTERMEDIATE VARIABLE

A1 PARTIAL OF RANGE WITH RESPECT TO X

A2 PARTIAL OF RANGE WITH RESPECT TO Y

A3 PARTIAL OF RANGE WITH RESPECT TO Z

B1 PART'IAL OF RANGE-RATE WITH RESPECT TO X

BZ PARTIAL OF RANGE-RATE WITH RESPECT TO Y

B3 PARTIAL OF RANGE-RATE WITH RESPECT TO Z

CE COSINE OF OBLIQUITY OF EARTH

COAL COSINE OF STAR-PLANET ANGLE

COSA INTERMEDIATE VARIABLE

COSAZ COSINE AZIMUTH

COSRA COSINE RIGHT ASCENSION

CP COSINE OF LONGITUDE + CONSTANT

DADP PARTIALS OF STAR-PLANET ANGLE WITH RESPECT TO VEHICLE POSITION AND VELOCITY

DBDP PARTIALS OF APPARENT PLANET DIAMETER WITH RESPECT TO VEHICLE POSITION AND VELOCITY

DD INTERMEDIATE VARIABLE

DENOM INTERMEDIATE VARIABLE

D INTERMEDIATE TIME

EK RANGE-RATE PARTIAL WITH RESPECT TO STATION LOCATION ERRORS

GECS GEOCENTRIC EQUATORIAL COORDINATES OF STATION

GELS GEOCENTRIC ECLIPTIC COORDINATES OF STATION

HECE COORDINATES OF EARTH

HECP COORDINATES OF TARGET PLANET

IA TRACKING STATION LOCATION SELECTION CODE

ICD CODE CORRESPONDING TO TRACKING STATION LOCATION ERRORS

IC COLUMN NUMBER IN OBSERVATION MATRIX PARTITION WHERE EK IS TO BE STORED

IEND VARIABLE INDEX VALUE

IR STAR-PLANET ANGLE INDEX INCREMENT VALUE

NA STAR-PLANET ANGLE INDEX LOWER LIMIT

=1 FOR 3 STAR-PLANET ANGLES

=ITRK-10 FOR SINGLE STAR-PLANET ANGLES

NC STAR-PLANET ANGLE INDEX UPPER LIMIT =3 FOR 3 STAR-PLANET ANGLES =ITRK-10 FOR SINGLE STAR-PLANET ANGLES

PAR PARTIALS RETURNED FROM SUBROUTINE TARPEL

PAT1 INTERMEDIATE VARIABLE

PAT2 INTERMEDIATE VARIABLE

PA PARTIALS

P2 INTERMEDIATE VARIABLE

RA RIGHT ASCENSION

RADNTP RADIUS OF TARGET PLANET

RAS INTERMEDIATE VARIABLE

RRATE RANGE-RATE

R1 RANGE

R2 SQUARE OF RANGE

SA PARTIALS OF STAR-PLANET ANGLES WITH

RESPECT TO VEHICLE POSITION

SE SINE OF OBLIQUITY OF EARTH

SIAL SINE OF STAR-PLANET ANGLE

SINAZ SINE AZIMUTH

SIND INTERMEDIATE VARIABLE

SINRA SINE RIGHT ASCENSION

SP SINE OF LONGITUDE + CONSTANT

SUM INTERMEDIATE VARIABLE

SUM1 INTERMEDIATE VARIABLE

VEC INTERMEDIATE VECTOR

ZZ1 INTERMEDIATE VARIABLE

ZZ2 INTERMEDIATE VARIABLE

COMMON COMPUTED/USED: AAL AM H NO XP

COMMON	COMPUTED:	G				
COMMON	USED:	ALNGTH F IBARY ONE SLON UNIVT	DATEJ IAUGDC NBOD RADIUS TM UST	DELTM IAUGIN NB SAL TRIMB VST	EM3 IAUGMC NTP SLAT TRTM1 WST	EPS IAUG OMEGA SLB TWO ZERO

TRAKS Analysis

Subroutine TRAKS performs two functions in the simulation mode. The first function, which corresponds to IØBS = 0, is to compute all observation matrix partitions for the measurement type indicated by ITRK. The second function, which corresponds to  $IØBS \neq 0$ , is to compute the measurement itself. If IØBS = 1, TRAKS computes the measurement corresponding to the most recent nominal spacecraft state. If IØBS = 2, TRAKS computes the measurement corresponding to the actual spacecraft state, and, if the measurement is a range or range-rate measurement, to the actual tracking station locations. The number of rows, NR, in the measurement and the observation matrix partitions is also computed.

A general measurement has form

$$\frac{\rightarrow}{Y} = \frac{\rightarrow}{Y} (\frac{\rightarrow}{X}, \frac{\rightarrow}{p}, t)$$

where  $\overline{X}$  is the spacecraft position/velocity state at time t and  $\overline{p}$  is a vector of parameters. This equation can be linearized about nominal  $\overline{X}$  and  $\overline{p}$  to obtain

$$\delta \overrightarrow{Y} = \left(\frac{\partial \overrightarrow{Y}}{\partial \overrightarrow{X}}\right)^* \delta \overrightarrow{X} + \left(\frac{\partial \overrightarrow{Y}}{\partial \overrightarrow{P}}\right)^* \delta \overrightarrow{P}$$

where ( )* indicates matrices are evaluated at the nominal condition. This perturbation equation can be rewritten as

$$\delta \vec{Y} = H \delta \vec{X} + M \delta \vec{x}_s + G \delta \vec{u} + L \delta \vec{v}$$

where  $H = \left(\frac{\partial \overrightarrow{Y}}{\partial \overrightarrow{X}}\right)^*$ , and  $\left(\frac{\partial \overrightarrow{Y}}{\partial \overrightarrow{p}}\right)^*$  is distributed among the M, G, and L

partitions to correspond to the partition of the parameter vector  $\delta \vec{p}$  into solve-for parameters  $\delta \vec{x}$ , dynamic consider parameters  $\delta \vec{u}$ , and measurement consider parameters  $\delta \vec{v}$ .

In the remainder of this section the measurement equation and all partial derivatives required to construct the H, M, G, and L observation matrix partitions will be summarized for each measurement type.

### A. Range measurement P .

A range measurement has form

$$\rho = \rho (\bar{x}, R, \theta, \emptyset, t)$$

where R,  $\theta$ , and  $\emptyset$  are the radius, latitude, and longitude of the relevant tracking station.

More explicitly,

$$\rho = \left[ (x - x_E - x_S)^2 + (y - y_E - y_S)^2 + (z - z_E - z_S)^2 \right]^{\frac{\pi}{2}}$$

where X, Y, Z = inertial position components of spacecraft  $X_E$ ,  $Y_E$ ,  $Z_E$  = inertial position components of Earth  $X_S$ ,  $Y_S$ ,  $Z_S$  = station position components relative to Earth.

 $X_S$ ,  $Y_S$ , and  $Z_S$  are related to R,  $\theta$ , and  $\emptyset$  as follows:

$$X_{S} = R \cos \theta \cos G$$

$$Y_S = R \cos \theta \cos \epsilon \sin G + R \sin \theta \sin \epsilon$$

$$Z_S = -R \cos \theta \sin \epsilon \sin G + R \sin \theta \cos \epsilon$$

where  $\epsilon$  is the obliquity of the Earth, and

$$G = \emptyset + GHA$$

where GHA is the Greenwich hour angle at time t.

Partials of ho with respect to spacecraft state are given by

$$\frac{\partial \rho}{\partial x} = \frac{1}{\rho} (x - x_E - x_S) \qquad \frac{\partial \rho}{\partial \dot{x}} = 0$$

$$\frac{\partial \rho}{\partial y} = \frac{1}{\rho} (y - y_E - y_S) \qquad \frac{\partial \rho}{\partial \dot{y}} = 0$$

$$\frac{\partial \rho}{\partial z} = \frac{1}{\rho} (z - z_E - z_S) \qquad \frac{\partial \rho}{\partial \dot{z}} = 0$$

Partials of  $\rho$  with respect to R,  $\theta$ , and  $\emptyset$  are given by

$$\frac{\partial \rho}{\partial R} = \frac{\partial \rho}{\partial X_{S}} \cdot \frac{\partial X_{S}}{\partial R} + \frac{\partial \rho}{\partial Y_{S}} \cdot \frac{\partial Y_{S}}{\partial R} + \frac{\partial \rho}{\partial Z_{S}} \cdot \frac{\partial Z_{S}}{\partial R}$$

$$\frac{\partial \rho}{\partial \theta} = \frac{\partial \rho}{\partial X_{S}} \cdot \frac{\partial X_{S}}{\partial \theta} + \frac{\partial \rho}{\partial Y_{S}} \cdot \frac{\partial Y_{S}}{\partial \theta} + \frac{\partial \rho}{\partial Z_{S}} \cdot \frac{\partial Z_{S}}{\partial \theta}$$

$$\frac{\partial \rho}{\partial \theta} = \frac{\partial \rho}{\partial X_{S}} \cdot \frac{\partial X_{S}}{\partial \theta} + \frac{\partial \rho}{\partial Y_{S}} \cdot \frac{\partial Y_{S}}{\partial \theta} + \frac{\partial \rho}{\partial Z_{S}} \cdot \frac{\partial Z_{S}}{\partial \theta}$$

$$\frac{\partial \rho}{\partial \theta} = \frac{\partial \rho}{\partial X_{S}} \cdot \frac{\partial X_{S}}{\partial \theta} + \frac{\partial \rho}{\partial Y_{S}} \cdot \frac{\partial Y_{S}}{\partial \theta} + \frac{\partial \rho}{\partial Z_{S}} \cdot \frac{\partial Z_{S}}{\partial \theta}$$

where

$$\frac{\partial \rho}{\partial x_{S}} = -\frac{\partial \rho}{\partial x} , \quad \frac{\partial \rho}{\partial y_{S}} = -\frac{\partial \rho}{\partial y} , \quad \frac{\partial \rho}{\partial z_{S}} = -\frac{\partial \rho}{\partial z}$$

and the negatives of the partials of  $X_S$ ,  $Y_S$ , and  $Z_S$  with respect to R,  $\theta$ , and  $\emptyset$  are summarized in the subroutine STAPRL analysis.

# B. Range-rate measurement p.

A range-rate measurement has form

$$\dot{\rho} = \dot{\rho} (\vec{X}, R, \theta, \emptyset, t)$$

where all arguments have been defined previously. More explicitly,

$$\dot{\rho} = \frac{\rho_{x} \dot{\rho}_{x} + \rho_{y} \dot{\rho}_{y} + \rho_{z} \dot{\rho}_{z}}{\rho}$$

where

$$\rho_{x} = x - x_{E} - x_{S} \qquad \dot{\rho}_{x} = \dot{x} - \dot{x}_{E} - \ddot{x}_{S} 
\rho_{y} = y - y_{E} - y_{S} \qquad \dot{\rho}_{y} = \dot{y} - \dot{y}_{E} - \dot{y}_{S} 
\rho_{z} = z - z_{E} - z_{S} \qquad \dot{\rho}_{z} = \dot{z} - \dot{z}_{E} - \dot{z}_{S}$$

 $\dot{x}_{S}$ ,  $\dot{y}_{S}$ , and  $\dot{z}_{S}$  are related to R,  $\theta$ , and  $\emptyset$  as follows:

$$\begin{array}{lll}
\overset{\bullet}{X}_{S} &=& -\omega R \cos \theta \sin G \\
\overset{\bullet}{X}_{S} &=& \omega R \cos \theta \cos \epsilon \cos G \\
\overset{\bullet}{Z}_{S} &=& -\omega R \cos \theta \sin \epsilon \cos G
\end{array}$$

where  $\omega$  is the rotational rate of the Earth.

Partials of  $\stackrel{\circ}{oldsymbol{
ho}}$  with respect to spacecraft state are given by

$$\frac{\partial \dot{\rho}}{\partial x} = \frac{\dot{\rho}_{x}}{\rho} - \frac{\rho_{x}\dot{\rho}}{\rho^{2}} \qquad \frac{\partial \dot{\rho}}{\partial x} = \frac{\rho_{x}}{\rho}$$

$$\frac{\partial \dot{\rho}}{\partial x} = \frac{\dot{\rho}_{y}}{\rho} - \frac{\rho_{y}\dot{\rho}}{\rho^{2}} \qquad \frac{\partial \dot{\rho}}{\partial \dot{x}} = \frac{\rho_{y}}{\rho}$$

$$\frac{\partial \dot{\rho}}{\partial z} = \frac{\dot{\rho}_{z}}{\rho} - \frac{\rho_{z}\dot{\rho}}{\rho^{2}} \qquad \frac{\partial \dot{\rho}}{\partial \dot{z}} = \frac{\rho_{z}}{\rho}$$

The partial of  $\dot{oldsymbol{
ho}}$  with respect to R is given by

$$\frac{\partial \dot{\rho}}{\partial R} = \frac{\partial \dot{\rho}}{\partial X_{S}} \cdot \frac{\partial X_{S}}{\partial R} + \frac{\partial \dot{\rho}}{\partial Y_{S}} \cdot \frac{\partial Y_{S}}{\partial R} + \frac{\partial \dot{\rho}}{\partial Z_{S}} \cdot \frac{\partial Z_{S}}{\partial R} +$$

where

$$\frac{\partial \dot{\rho}}{\partial x_{S}} = -\frac{\partial \dot{\rho}}{\partial x} , \text{ etc.}$$

and 
$$\frac{\partial \dot{\rho}}{\partial \dot{x}_S} = -\frac{\partial \dot{\rho}}{\partial \dot{x}}$$
, etc.

The negatives of the partials of  $X_S$ ,  $Y_S$ ,  $Z_S$ ,  $X_S$ ,  $Y_S$ , and  $Z_S$  with respect to R,  $\theta$ , and  $\emptyset$  are summarized in the subroutine STAPRL analysis. Partials of  $\hat{\rho}$  with respect to  $\theta$  and  $\emptyset$  are treated similarly.

# C. Star-planet angle measurement & .

A star-planet angle measurement has form &

$$\alpha = \alpha$$
 ( $\vec{X}$ , a, e, i,  $\Omega$ ,  $\omega$ , M)

where a, e, i, arOmega ,  $\omega$  , and M are the standard set of target planet orbital elements.

If we define  $\vec{\rho} = (\rho_x, \rho_y, \rho_z)$  to be the position of the target planet relative to the spacecraft and (u, v, w) to be the direction cosines of the relevant star, then

$$\alpha = \cos^{-1} \left[ \frac{1}{\rho} (u \rho_{x} + v \rho_{y} + w \rho_{z}) \right]$$

where

$$\rho_{x} = X_{p} - X, \quad \rho_{y} = Y_{p} - Y, \quad \rho_{z} = Z_{p} - Z,$$

and  $(X_p, Y_p, Z_p)$  represent the position coordinates of the target planet.

Partials of  $\alpha$  with respect to spacecraft state are given by

$$\frac{\partial \alpha}{\partial x} = \frac{1}{\sin \alpha} \left( \frac{u}{\rho} - \frac{\rho_x \cos \alpha}{\rho^2} \right) \qquad \frac{\partial \alpha}{\partial \dot{x}} = 0$$

$$\frac{\partial \alpha}{\partial y} = \frac{1}{\sin \alpha} \left( \frac{v}{\rho} - \frac{\rho_y \cos \alpha}{\rho^2} \right) \qquad \frac{\partial \alpha}{\partial \dot{y}} = 0$$

$$\frac{\partial \alpha}{\partial z} = \frac{1}{\sin \alpha} \left( \frac{w}{\rho} - \frac{\rho_z \cos \alpha}{\rho^2} \right) \qquad \frac{\partial \alpha}{\partial \dot{z}} = 0$$

where

$$\sin \alpha = + \left[1 - \cos^2 \alpha\right]^{\frac{1}{2}}.$$

The partial of  $\alpha$  with respect to target planet semi-major axis is given by

$$\frac{\partial \alpha}{\partial a} = \frac{\partial \alpha}{\partial x_p} \cdot \frac{\partial x_p}{\partial a} + \frac{\partial \alpha}{\partial y_p} \cdot \frac{\partial y_p}{\partial a} + \frac{\partial \alpha}{\partial z_p} \cdot \frac{\partial z_p}{\partial a}$$

where 
$$\frac{\partial \alpha}{\partial x_p} = -\frac{\partial \alpha}{\partial x}$$
,  $\frac{\partial \alpha}{\partial y_p} = -\frac{\partial \alpha}{\partial y}$ ,  $\frac{\partial \alpha}{\partial z_p} = -\frac{\partial \alpha}{\partial z}$ ,

and partials of  $\mathbf{X}_p$ ,  $\mathbf{Y}_p$ , and  $\mathbf{Z}_p$  with respect to semi-major axis are summarized in the subroutine TARPRL analysis. Partials of  $\mathbf{X}_p$  with respect to  $\mathbf{X}_p$ ,  $\mathbf{Y}_p$ , and  $\mathbf{Z}_p$  do not appear in the above expression since they are all zero. Partials of  $\mathbf{X}_p$  with respect to the remaining target planet orbital elements are treated similarly.

# D. Apparent planet diameter measurement $\beta$ .

An apparent planet diameter measurement has form

$$\beta \approx \beta(\vec{X}, a, e, i, \Omega, \omega, M)$$

where all arguments have been defined previously.

Defining  $\vec{\rho} = (\rho_x, \rho_y, \rho_z)$  to be the position of the target planet relative to the spacecraft and  $R_p$  to be the radius of the target planet, the apparent planet diameter can then be written as

$$\beta = 2 \sin^{-1}\left(\frac{R_p}{\rho}\right)$$

Partials of  $oldsymbol{eta}$  with respect to spacecraft state are given by

$$\frac{\partial \beta}{\partial x} = \frac{2 R_{p} \rho_{x}}{\rho^{2} \left[\rho^{2} - R_{p}^{2}\right]^{\frac{1}{2}}} \qquad \frac{\partial \beta}{\partial \dot{x}} = 0$$

$$\frac{\partial \beta}{\partial Y} = \frac{2 R_p \rho_y}{\rho^2 \left[\rho^2 - R_p^2\right]^{\frac{1}{2}}} \qquad \frac{\partial \beta}{\partial Y} = 0$$

$$\frac{\partial \beta}{\partial z} = \frac{2 R_{p} \rho_{z}}{\rho^{2} \left[\rho^{2} - R_{p}^{2}\right]^{\frac{1}{2}}} \qquad \frac{\partial \beta}{\partial \dot{z}} = 0$$

The partial of  $\,eta\,$  with respect to target planet semi-major axis is given by

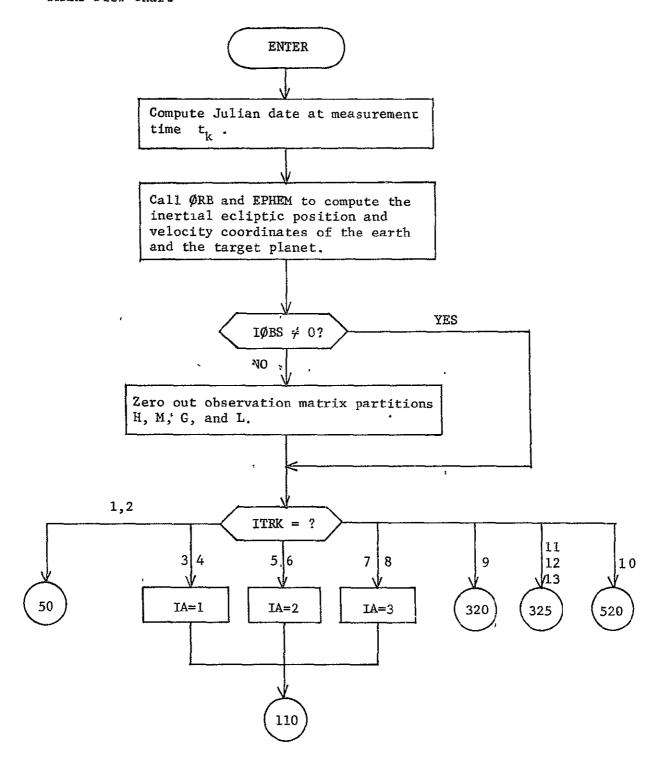
$$\frac{\partial \beta}{\partial a} = \frac{\partial \beta}{\partial x_{p}} \cdot \frac{\partial x_{p}}{\partial a} + \frac{\partial \beta}{\partial y_{p}} \cdot \frac{\partial y_{p}}{\partial a} + \frac{\partial \beta}{\partial z_{p}} \cdot \frac{\partial z_{p}}{\partial a}$$

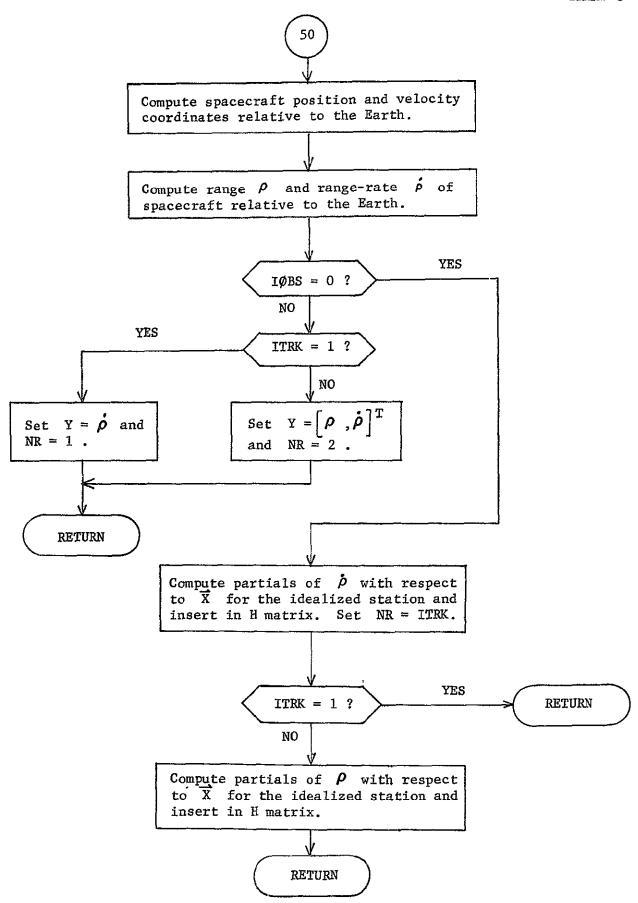
where 
$$\frac{\partial \beta}{\partial x_p} = -\frac{\partial \beta}{\partial x}$$
,  $\frac{\partial \beta}{\partial y_p} = -\frac{\partial \beta}{\partial y}$ ,  $\frac{\partial \beta}{\partial z_p} = -\frac{\partial \beta}{\partial z}$ ,

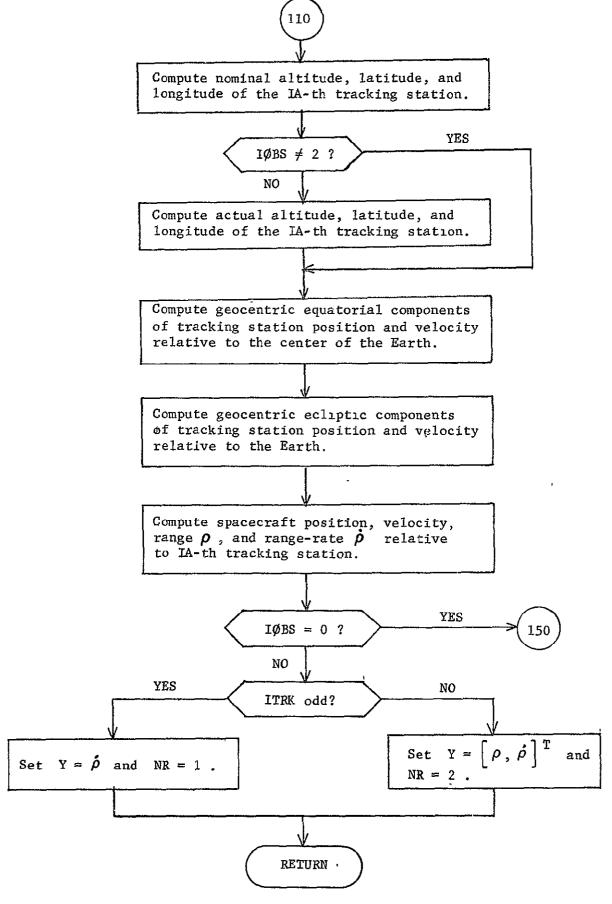
and partials of  $X_p$ ,  $Y_p$ , and  $Z_p$  with respect to semi-major axis are summarized in the subroutine TARPRL analysis.

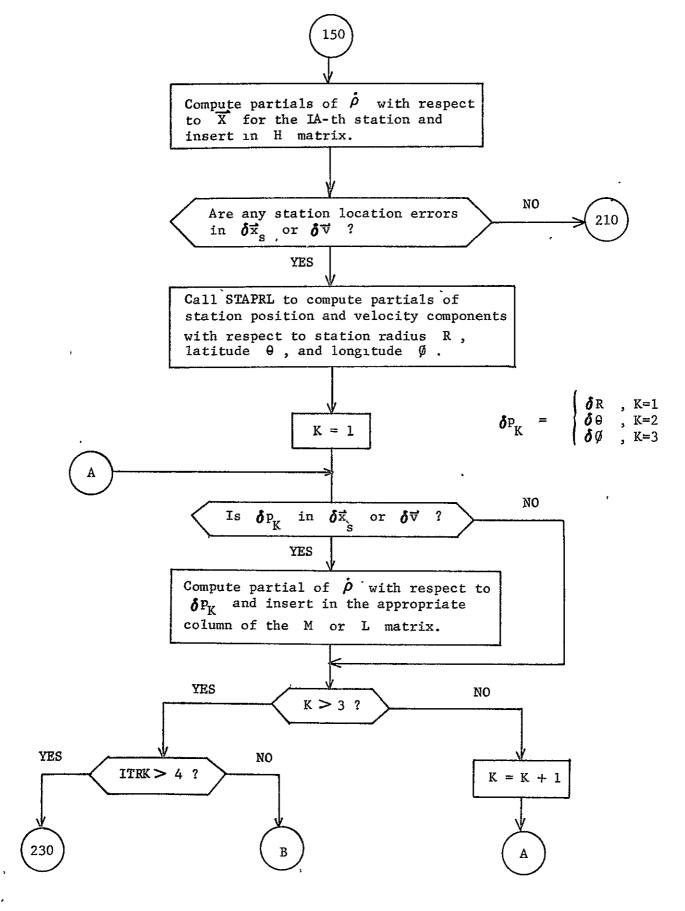
Partials of  $\boldsymbol{\beta}$  with respect to  $\dot{X}_p$ ,  $\dot{Y}_p$ , and  $\dot{Z}_p$  do not appear in the above expression since they are all zero. Partials of  $\boldsymbol{\beta}$  with respect to the remaining target planet orbital elements are treated similarly.

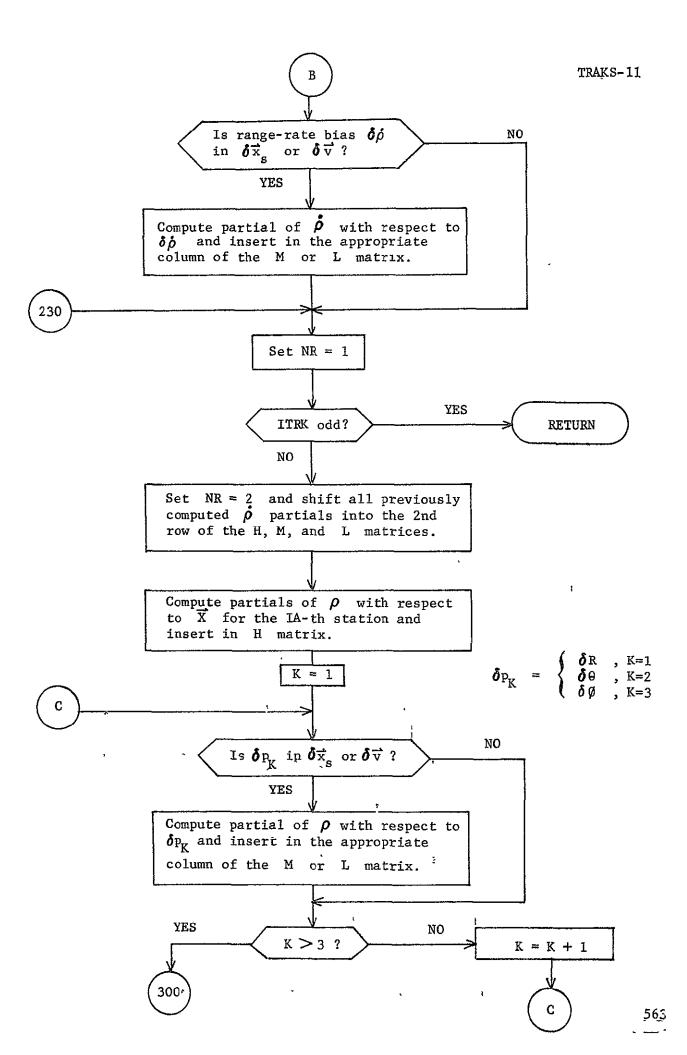
## TRAKS Flow Chart

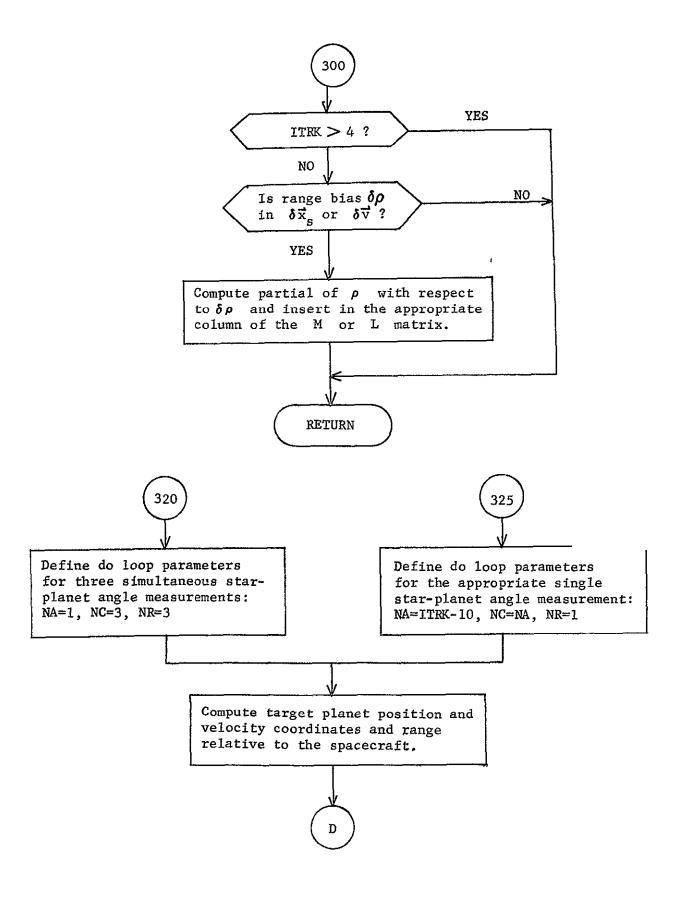


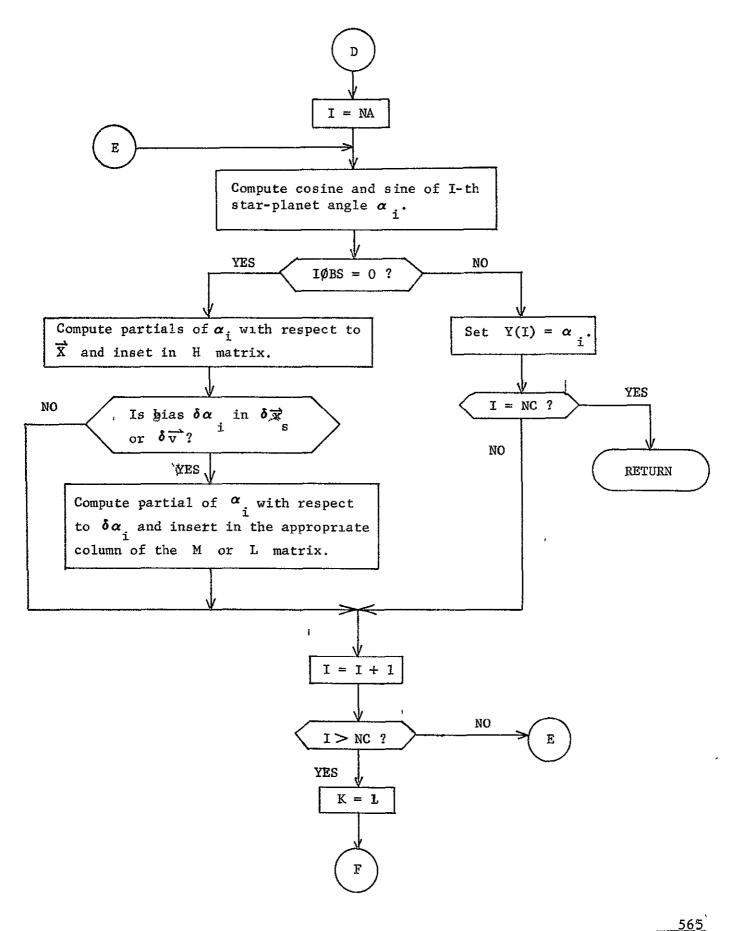


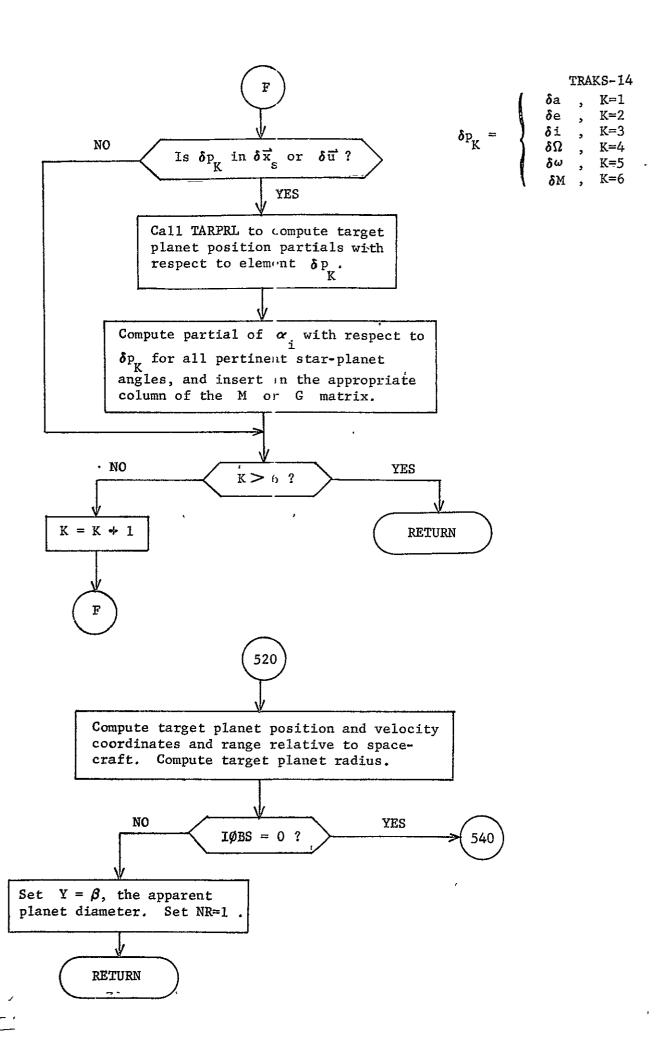


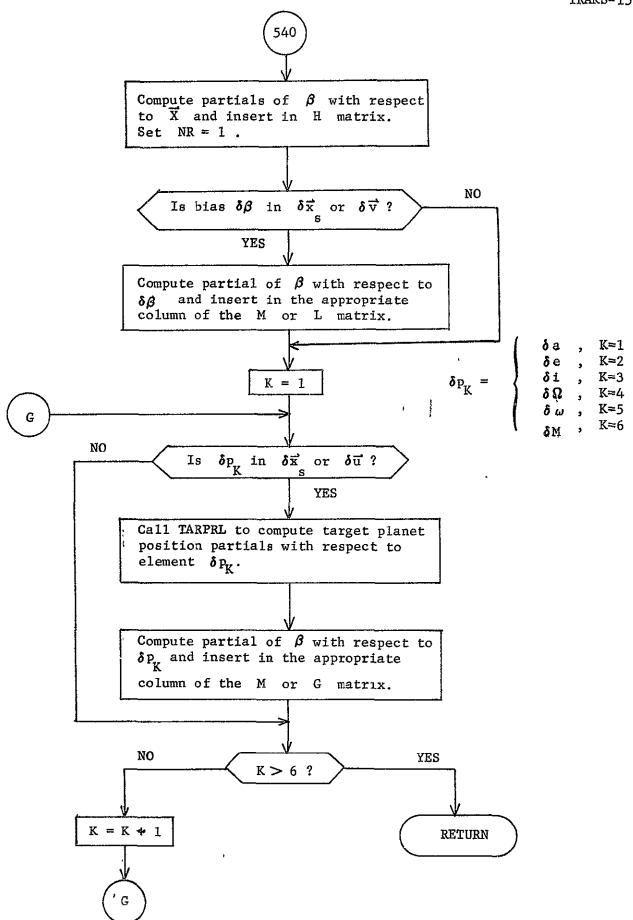












### SUBROUTINE TRANS

PURPOSE: TO PERFORM ONE OF THE FOLLOWING THREE OPTIONS.

- 1. CONVERT FROM GEOCENTRIC EQUATORIAL RECTANG-ULAR COORDINATES TO GEOCENTRIC ELIPTIC COORDINATES
- 2. CONVERT FROM GEOCENTRIC EQUATORIAL COORD-INATES TO HELIOCENTRIC ECLIPTIC COORDINATES
- 3. CONVERT FROM GEOCENTRIC ECLIPTIC COORD-INATES TO HELIOCENTRIC ECLIPTIC COORDINATES

CALLING SEQUENCE: CALL TRANS(ICODE, X, Y, Z, VX, VY, VZ, XE, YE, ZE, VXE, VYE, VZE, EPS, ICODE2)

ARGUMENT: EPS I OBLIQUITY OF EARTH

ICODE I AN INTERNAL CODE THAT DETERMINES IF OPTION 1 OR 2 ABOVE WILL BE EXERCISED

ICODE2 I AN INTERNAL CODE THAT DETERMINES IF OPTION
3 ABOVE IS TO BE EXERCISED

VX I/O X-VELOCITY COMPONENT OF THE VEHICLE

VXE I X-VELOCITY COMPONENT OF EARTH IN HELIOCENTRIC ECLIPTIC COORDINATES

YY I/O Y-VELOCITY COMPONENT OF THE VEHICLE

VYE I Y-VELOCITY COMPONENT OF EARTH

VZ I/O Z-VELOCITY COMPONENT OF THE VEHICLE

VZE I Z-VELOCITY COMPONENT OF EARTH

X I/O X-POSITION COMPONENT OF THE VEHICLE

XE I X-POSITION COMPONENT OF THE EARTH IN HELIOCENTRIC ECLIPTIC COORDINATES

Y I/O Y-POSITION COMPONENT OF THE VEHICLE

YE I Y-POSITION COMPONENT OF THE EARTH

Z I/O Z-POSITION COMPONENT OF THE VEHICLE

ZE I Z-POSITION COMPONENT OF THE EARTH

SUBROUTINES SUPPORTED: DATA DATAS

LOCAL SYMBOLS: CE COSINE OF OBLIQUITY OF EARTH

DUM INTERMEDIATE VARIABLE

# SE SINE OF OBLIQUITY OF EARTH

### TRANS Analysis

Subroutine TRANS transforms the position and velocity components of the spacecraft from one coordinate system to another. The three options available with this subroutine are summarized below.

1) Convert from geocentric equatorial coordinates to geocentric ecliptic coordinates using the following equations:

$$X = X$$
  
 $Y = Y \cos \epsilon + Z \sin \epsilon$   
 $Z = -Y \sin \epsilon + Z \cos \epsilon$   
 $X = X$   
 $Y = Y \cos \epsilon + Z \sin \epsilon$   
 $Z = -Y \sin \epsilon + Z \cos \epsilon$ 

2) Convert from geocentric equatorial coordinates to heliocentric ecliptic coordinates. The same procedure as above is used to convert from geocentric equatorial to geocentric ecliptic. Then translate according to the following equations:

$$X = X + X_{E}$$

$$Y = Y + Y_{E}$$

$$Z = Z + Z_{E}$$

$$X = X + X_{E}$$

$$Y = Y + Y_{E}$$

$$Z = Z + Z_{E}$$

3) Convert from geocentric ecliptic coordinates to heliocentric ecliptic coordinates using the following equations:

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SUBROUTINE TRAPAR

PURPOSE: TO COMPUTE THE FOLLOWING SET OF NAVIGATION PARAMETERS

-- FLIGHT PATH ANGLE, ANGLE BETWEEN RELATIVE VELOCITY

AND PLANE OF THE SKY, GEOCENTRIC DECLINATION, EARTH/
SPACECRAFT/TARGET PLANET ANGLE, ANTENNA AXIS/LIMB OF
SUN ANGLE, AND SPACECRAFT OCCULTATION RATIOS FOR SUN,
MOON, AND PLANETS.

CALLING SEQUENCE: CALL TRAPAR

SUBROUTINES SUPPORTED: PRINT PRINT4 SETVEVS PRINT3 SETEVN

SUBROUTINES REQUIRED: EPHEM TORB PECEQ

LOCAL SYMBOLS & ALFA VECTOR FORMING RIGHT-HANDED ORTHOGONAL

TRIAD WITH XN AND SSS VECTORS FOR

CALCULATION OF ANTENNA AXIS/LIMB OF ANGLE

OF SUN

AMAG MAGNITUDE OF THE ALFA VECTOR

BETA ANTENNA AXIS/EARTH ANGLE

CD COSINE OF GEOCENTRIC DECLINATION

CT INTERMEDIATE VARIABLE FOR ALL CALCULATIONS

CZAE COSINE OF EARTH/SPACECRAFT/TARGET PLANE

ANGLE

DELTA GEOCENTRIC DECLINATION

DS INTERMEDIATE VARIABLE FOR CALCULATION OF

OCCULTATION RATIOS

ECEQP TRANSFORMATION FROM EARTH ECLIPTIC TO

EQUATORIAL FRAME FOR CALCULATION OF

GEOCENTRIC DECLINATION

GAMMA INERTIAL FLIGHT PATH ANGLE

IND LOCATION IN THE F ARRAY OF THE EARTH

POSITION AND VELOCITY IN THE INERTIAL

FRAME

ISAVE SAVES AND RESTORES FIRST ELEMENT OF THE

NO-ARRAY FOR BARYCENTRIC NAVIGATION

JND LOCATION IN THE F ARRAY OF THE TARGET

PLANET POSITION AND VELOCITY IN THE

INERTIAL FRAME

NINETY CONSTANT VALUE. EQUAL TO 90.000

OCCULT OCCULTATION RATIO OF THE I-TH PLANET

PHI ANTENNA AXIS/LIMB OF SUN ANGLE

RDV INTERMEDIATE VARIABLE, DOT PRODUCT OF TWO VECTORS

REMAG MAGNITUDE OF THE EARTH HELIOCENTRIC POSITION

RIMAG MAGNITUDE OF THE POSITION OF THE I-TH PLANET IN THE GEOCENTRIC ECLIPTIC FRAME

RMAG MAGNITUDE OF THE SPACEGRAFT HELIOCENTRIC POSITION

RSS SPACECRAFT HELIOCENTRIC POSITION

SD SINE OF GEOCENTRIC DECLINATION

SKYI ANGLE BETWEEN SPACECRAFT VELOCITY RELATIVE TO EARTH AND PLANE OF THE SKY

SRDV DOT PRODUCT OF SPACECRAFT GEOCENTRIC POSITION AND VELOCITY VECTORS

SRE SPACECRAFT GEOCENTRIC ECLIPTIC POSITION AND VELOCITY

SRMAG MAGNITUDE OF SPACECRAFT GEOCENTRIC POSITION

SRQ SPACECRAFT GEOCENTRIC EQUATORIAL POSITION

SRTMAG MAGNITUDE OF SRTP VECTOR

SRTP SPACECRAFT ECLIPTIC POSITION RELATIVE TO TARGET PLANET

SVMAG MAGNITUDE OF SPACECRAFT GEOCENTRIC VELOCITY

SX INTERMEDIATE VARIABLE FOR ALL CALCULATIONS

SZAE SINE OF EARTH/SPACECRAFT/TARGET PLANET ANGLE

THETA INTERMEDIATE ANGLE USED TO CALCULATE NAVIGATION PARAMETERS

VMAG MAGNITUDE OF SPACECRAFT VELOCITY RELATIVE

# TO INERTIAL FRAME

	XMAG	MAGNITUDE OF THE XN VECTOR BEFORE UNITIZING			RE	
	ХN	CROSS PRODUCT OF SPACECRAFT GEOCENTR POSITION AND SPACECRAFT SPIN AXIS				
	ZAE	EARTH/SF	PACECRAFI	T/TARGET	PLANET	ANGLE
COMMON	COMPUTED/USED:	8	NO	RE		
COMMON	USED 8	F ONE TWO	IBARY PLANET V	NBOD RADIUS XP	NB RAD ZERO	NTP SSS

### TRAPAR Analysis

The coordinate systems and variables required for the derivation of the first four navigation parameters are shown in Figure 1. The inertial coordinate system XYZ may be heliocentric or barycentric ecliptic. The position and velocity of the earth in inertial space is given by  $\vec{r}_E$  and  $\vec{v}_E$ ; that of the spacecraft, by  $\vec{r}$  and  $\vec{v}_E$ ; and that of the target planet (or moon), by  $\vec{r}_{TP}$  and  $\vec{v}_{TP}$ . The xyz coordinate system is geocentric equatorial.

1. Flight path angle, ^ .

Let  $\theta$  denote the angle between  $\vec{r}$  and  $\vec{v}$ , so that

$$\cos \theta = \frac{\overrightarrow{r} \cdot \overrightarrow{v}}{r \cdot v}$$
 and  $\sin \theta = + \left[1 - \cos^2 \theta\right]^{\frac{1}{2}}$ .

Then

$$\gamma = \frac{\pi}{2} - \theta.$$

2. Angle between relative velocity and plane of the sky, i.

The plane of the sky is defined as the plane perpendicular to the vector  $\overrightarrow{r}-\overrightarrow{r}_E$ . Let  $\theta'$  denote the angle between  $\overrightarrow{r}-\overrightarrow{r}_E$  and  $\overrightarrow{v}-\overrightarrow{v}_E$ , so that

$$\cos \theta^{!} = \frac{(\vec{r} - \vec{r}_{E}) \cdot (\vec{v} - \vec{v}_{E})}{\left| \vec{r} - \vec{r}_{E} \right| \cdot \left| \vec{v} - \vec{v}_{E} \right|} \quad \text{and} \quad \sin \theta^{!} = + \left[ 1 - \cos^{2} \theta^{1} \right]^{\frac{1}{2}}$$

Then

$$i' = \frac{\tau}{2} - \theta'.$$

Note that, is not defined if the relative velocity  $\overrightarrow{v} - \overrightarrow{v_E}$  is zero.

Geocentric declination, δ.

Let (x, y, z) denote the geocentric equatorial components of  $\overrightarrow{r} - \overrightarrow{r}$ . Then

$$\delta = \tan^{-1} \left( \frac{z}{\sqrt{x^2 + y^2}} \right) .$$

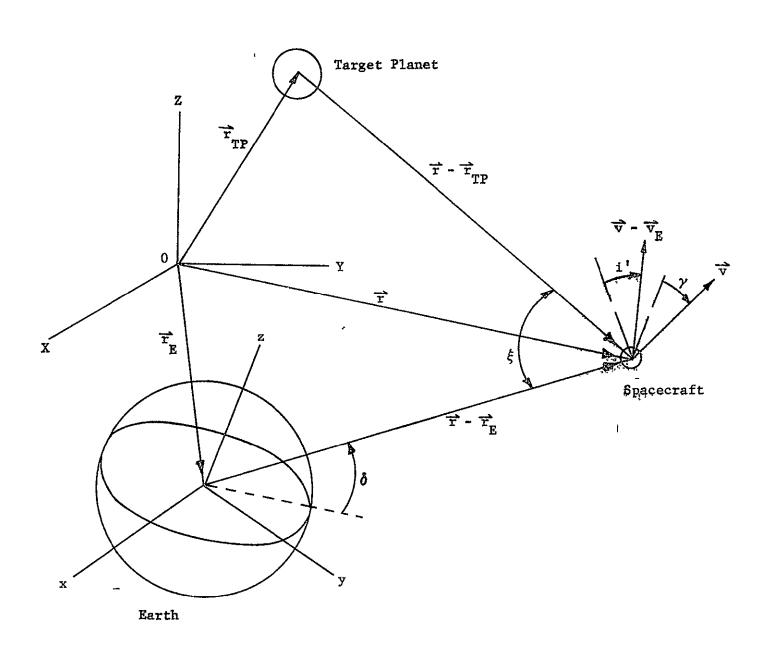


Figure 1

4. Earth/spacecraft/target planet angle,  $\xi$ .

The angle  $\xi$  is the angle between the vectors  $\vec{r} - \vec{r}_E$  and  $\vec{r} - \vec{r}_{TP}$ , so that

$$\cos \xi = \frac{(\vec{r} - \vec{r}_E) \cdot (\vec{r} - \vec{r}_{TP})}{|\vec{r} - \vec{r}_E| \cdot |\vec{r} - \vec{r}_{TP}|}$$

and

$$\sin \xi = + \left[1 - \cos^2 \xi\right]^{\frac{1}{2}}$$

The next two navigation parameters relate to the spacecraft antenna axis. The pertinent geometry is shown in Figure 2. The antenna axis  $\alpha$  is defined as the intersection between the antenna plane (the plane perpendicular to the spacecraft spin axis  $\overline{s}$ ) and the plane formed by the  $\overline{r} - \overline{r}_E$  and  $\overline{s}$  vectors. The vector  $\overline{\rho}$  originates from the limb of the sun and lies in the  $\overline{r}$ ,  $\overline{\alpha}$  plane.

5. Antenna axis/Earth angle, 2.

Let  $\psi$  denote the angle between the unit spin axis vector  $\vec{s}$  and  $\vec{r} - \vec{r}_E$ , so that

$$\cos \psi = \frac{\vec{s} \cdot (\vec{r} - \vec{r}_E)}{|\vec{r} - \vec{r}_E|} \quad \text{and} \quad \sin \psi = + \left[1 - \cos^2 \psi\right]^{\frac{1}{2}}.$$

Then

$$\beta = \frac{\pi}{2} - \psi .$$

Note that the antenna axis is not uniquely defined when the angle  $\psi=0$ .

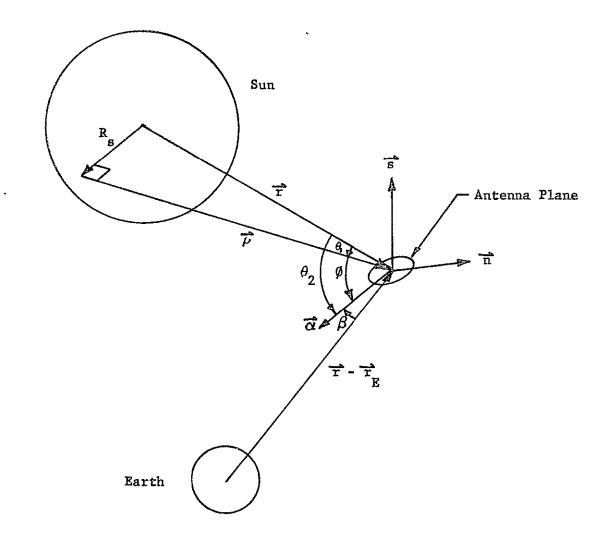


Figure 2. Antenna Axis Geometry

6. Antenna axis/limb of Sun angle,  $\emptyset$ .

The unit vector  $\vec{n}$  normal to the  $\vec{s}$ ,  $\vec{r}$  -  $\vec{r}_E$  plane is given by

$$\vec{n} = \frac{(\vec{r} - \vec{r}_E) \times \vec{s}}{\left| (\vec{r} - \vec{r}_E) \times \vec{s} \right|}.$$

Then the unit antenna axis vector  $\overrightarrow{lpha}$  is given by

$$\vec{\alpha} = \vec{n} \times \vec{s}$$
.

The angle  $\theta_2$  denotes the angle between the vectors  $\overrightarrow{r}$  and  $\overrightarrow{\alpha}$ , so that

$$\cos \theta_2 = -\frac{\overrightarrow{r} \cdot \overrightarrow{\alpha}}{r}$$
 and  $\sin \theta_2 = +\left[1 - \cos^2 \theta_2\right]^{\frac{1}{2}}$ .

The angle  $\theta_1$  denotes the angle between the vectors  $\overrightarrow{\rho}$  and  $\overrightarrow{r}$ , so that

$$\theta_1 = \sin^{-1}\left(\frac{R_s}{r}\right), \quad 0 \le \theta_1 \le \frac{\pi}{2}$$

where R is the radius of the Sun.

Then

$$\emptyset = \theta_2 - \theta_1$$
.

The final set of navigation parameters relate to spacecraft occultation ratios for the Sun and all other celestial bodies assumed in the dynamic model. The pertinent geometry is shown in Figure 3. The position of the i-th celestial body relative to the Sun is denoted by  $\overline{r}_i$ . Occultation parameters d and d are defined as the minimal distances from the centers of the Sun and i-th body, respectively, to the Earth/spacecraft vector  $\overline{r} - \overline{r}_E$ .

7. Spacecraft occultation ratio for the Sun.

The occultation ratio for the Sun is defined as  $\frac{d}{s}/R_s$ , where  $R_s$  is the Sun radius. As long as the occultation ratio is greater than one, the spacecraft is neither being occulted by the Sun nor passing in front of the Sun. The occultation ratio is computed only when the angle between the  $\overline{r} - \overline{r}_s$  and  $\overline{r}_s$  vectors is less than or equal to 90 degrees, or, equivalently, when

$$\vec{r}_{R} \cdot (\vec{r} - \vec{r}_{R}) \leq 0.$$

If this condition is satisfied, the occultation ratio is computed using the equations

$$d_{s} = \left[r_{E}^{2} - b^{2}\right]^{\frac{1}{2}}$$

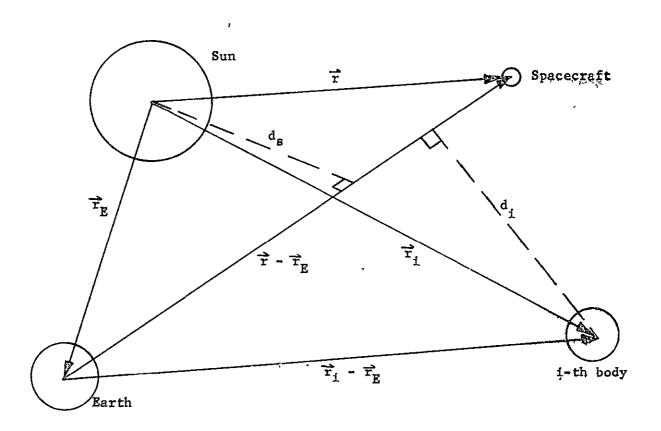


Figure 3. Occultation Geometry

and

$$b = \frac{-\vec{r}_E \cdot (\vec{r} - \vec{r}_E)}{\left| \vec{r} - \vec{r}_E \right|} .$$

Occultation occurs if  $\frac{d_s}{R_s} \leq 1$  and  $|\vec{r} - \vec{r}_E| \geq r_E$ ; if  $\frac{d_s}{R_s} \leq 1$  and  $|\vec{r} - \vec{r}_E| \leq r_E$ , then the spacecraft is passing in front of the Sun.

8. Spacecraft occultation ratios for other celestial bodies. The occultation ratio for the i-th celestial body is defined as  $\frac{d}{i}/R_i$ , where  $R_i$  is the radius of the i-th body. The occultation ratio is

computed only when

$$(\vec{r} - \vec{r}_E) \cdot (\vec{r}_1 - \vec{r}_E) \ge 0$$
.

If this conditions is satisfied, the occultation ratio is computed using the equations

$$d_{i} = \begin{bmatrix} a_{i}^{2} - b_{i}^{2} \end{bmatrix}^{\frac{1}{2}}$$

$$a_{i} = \begin{vmatrix} \overrightarrow{r}_{i} - \overrightarrow{r}_{E} \end{vmatrix}$$

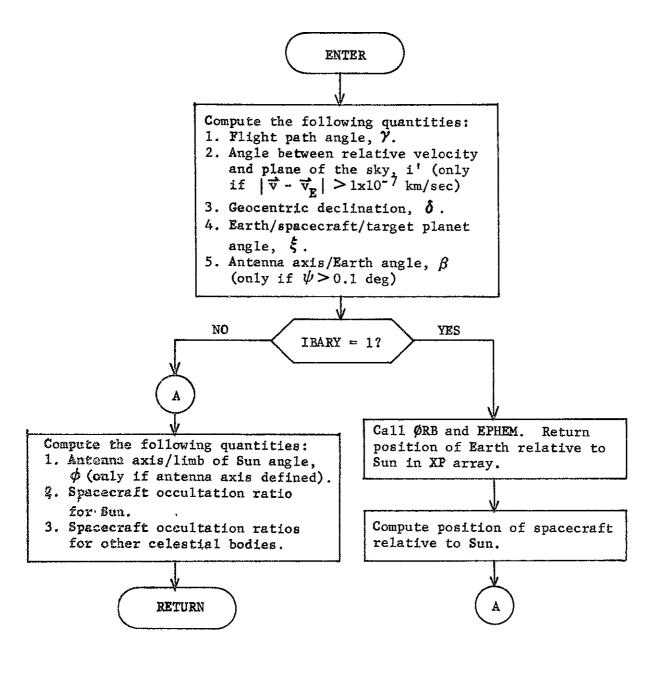
$$b_{i} = \frac{\begin{vmatrix} \overrightarrow{r} - \overrightarrow{r}_{E} \end{vmatrix} \cdot \langle \overrightarrow{r}_{i} - \overrightarrow{r}_{E} \rangle}{\begin{vmatrix} \overrightarrow{r} - \overrightarrow{r}_{E} \end{vmatrix}}.$$

anđ

Occultation occurs if  $\frac{d_i}{R_i} \neq 1$  and  $|\vec{r} - \vec{r}_E| \geq |\vec{r}_i - \vec{r}_E|$ ; if

 $\frac{d_i}{R_i} \leq 1$  and  $|\vec{r} - \vec{r}_E| < |\vec{r}_i - \vec{r}_E|$ , then the spacecraft is passing is front of the i-th celestial body.

#### TRAPAR Flow Chart



SUBROUTINE TRUTRY

PURPOSES TO DETERMINE THE TIME OF THE NEXT GUIDANCE EVENT AND INTEGRATE THE NOMINAL TRAJECTORY FROM THE PREVIOUS EVENT TIME TO THE NEXT TIME.

CALLING SEQUENCE: CALL TRUTRY

SUBROUTINES SUPPORTED: NOMNAL

SUBROUTINES REQUIRED: VMP

LOCAL SYMBOLS: ACK ACCURACY USED TO INTEGRATE THE NOMINAL TRAJECTORY

DELMIN TIME(DAYS) BETWEEN THE LAST EVENT AND THE NEXT EVENT

DELTM SAME AS DELMIN -THE TIME VMP IS TO INTEGRATE THE TRAJECTORY UNLESS ANOTHER

STOPPING CONDITION OCCURS

DELT SAME AS DELMIN

ERROR MINIMUM ALLOWABLE VALUE OF DELMIN

ISP2 FLAG TO CONTROL STOPPING CONDITION

=1 STOP AT SHPERE-OF-INFLUENCE

=0 DO NOT STOP AT SHPERE-OF-INFLUENCE

I INDEX

J INDEX

RSF SPACECRAFT STATE AT FINAL TIME

COMMON COMPUTED/USED: D1 ICL2 ICL ISPH KSICA KTIM RIN TIMG TRTM IPRINT COMMON COMPUTED: DELTP IEPHEM INPR KUR COMMON USEDS ACKT KGYD NCPR NOGYD **TMPR** 

1

#### TRJTRY Analysis

TRJTRY determines the time of the next guidance event and integrates the nominal trajectory from the previous event time to the next time.

Special provisions must be made in determining the next guidance event because of the flexibility permitted in specifying the times of those guidance events. For every guidance event i, parameters KTIM(i) and TIMG(i) will have been set before entering TRJTRY. KTIM(i) prescribes the epoch to which the guidance event i is referenced with KTIM(i) = 1,2,3 corresponding to epochs of initial time, sphere of influence (SOI) intersection, and closest approach (CA) passage respectively. TIMG(i) then specifies the time interval (days) from the epoch to the guidance event. The guidance events do not need to be arranged chronologically. After execution of each guidance event i the flag KTIM(i) is set equal to 0.

The first computational procedure in TRJTRY is the sequencing loop. Here a search determines the minimum value of TIMG(i) over all values of i such that KTIM(i) = 1. The time interval  $\Delta t$  between that time and the current time is then computed. If  $\Delta t$  is less than an allowable tolerance  $\epsilon$  (=10⁻⁵ days) the program returns to NOMNAL for the processing of the current event.

If  $\Delta t \ge \epsilon$  TRJTRY must perform an integration to the next guidance event. TRJTRY first sets up flags controlling integration stopping conditions depending upon the current value of KSICA. The flag KSICA determines the current phase of the trajectory. KSICA is initially set equal to 1 (PRELIM). When the target planet SOI is encountered KSICA is set to 2. Finally when CA to the target planet occurs it is set to 3.

The stopping condition flags are ISP2 and ICL2. The flag ISP2 determines whether the integration should be stopped at SOI if encountered (ISP2 = 1) or not (ISP2 = 0). The flag ICL2 determines whether the integration should be stopped at CA if encountered (ICL2 = 1) or not (ICL2 = 0).

Therefore if KSICA = 1, TRJTRY sets ISP2 = 1 so that the integration will stop at the guidance event time only if that time occurs before SOI. But if the SOI is encountered before the event time, all times referenced to the SOI must be updated before determining the next event. Similarly when KSICA = 2 TRJTRY sets ICL2 = 1 so that times referenced to CA may be updated when CA occurs. Of course when KSICA = 3, all times have been updated (referenced to initial time) and neither ISP2 nor ICL2 need be set to 1.

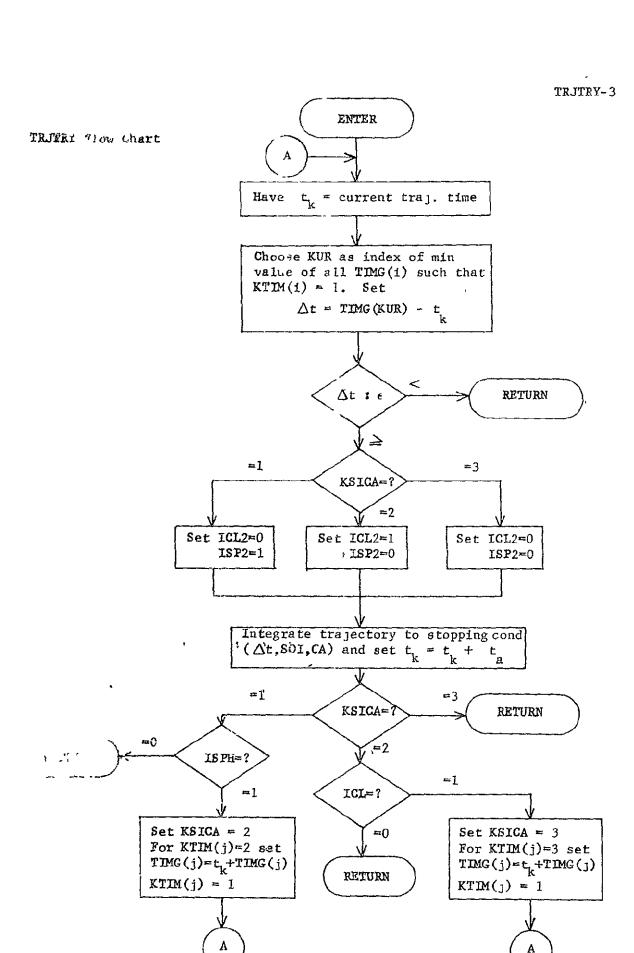
Having set the stopping condition flags, TRJTRY now calls VMP for the propagation of the trajectory to the required stopping condition. At the end of the integration it records the current trajectory time and state.

TRJTRY now sorts again on KSICA. If KSICA = 3, the trajectory has been integrated to the time of the current event and so control may be returned to NOMNAL.

If KSICA = 1 the SOI had not yet been reached at the previous event. TRJTRY then checks the flag ISPH. The flag ISPH reveals whether the current trajectory intersected the target planet SOI (ISPH = 1) or did not (ISPH = 0). Therefore if ISPH = 0, the current guidance event occurred before the trajectory intersected the SOI and thus the current state corresponds to the time of the guidance event. Therefore the return is made to NCMNAL.

If however KSICA = 1 and ISPH = 1 the trajectory integration was stopped at the SOI. TRJTRY now sets KSICA = 2 and updates all times referenced to the SOI so that they are now referenced to initial time (KTIM(i) = 1). It reenters the sequencing loop to determine the time of the next guidance event where the candidate events now include those originally referenced to SOI.

Similar steps are made when KSICA = 2. The flag ICL designates whether the current trajectory had a CA (ICL = 1) or not (ICL = 0). If KSICA = 2 and ICL = 0, the trajectory encountered the guidance event before reaching a CA so the return is made to NOMNAL. If KSICA = 2 and ICL = 1, the final time and state of the trajectory refer to closest approach. In this case TRJTRY sets KSICA = 3 and updates to initial time.all times originally referenced to CA. It then returns to the sequencing loop.



SUBROUTINE USCALE

PURPOSE: SCALES LENGTH OF A THREE-VECTOR TO SPECIFIED NUMBER

ARGUMENT: A I LENGTH OF VECTOR TO BE OUTPUT

U I VECTOR TO BE SCALED

V O SCALED OUTPUT VECTOR

SUBROUTINES SUPPORTED: CAREL IMPCT KTROL STIMP SUBSOL

SUBROUTINE UXV

PURPOSE: CALCULATE VECTOR CROSS PRODUCT OF TWO VECTORS

APGUMENT: UV O CROSS PRODUCT OF U AND V

U I INPUT VECTOR

V I INPUT VECTOR

SUBROUTINES SUPPORTED: CAREL IMPCT KTROL STIMP SUBSOL

#### SUBROUTINE VARADA

PURPOSE COMPUTE VARIATION MATRIX FOR THREE-VARIABLE 8-PLANE GUIDANCE POLICY IN THE ERROR ANALYSIS PROGRAM

CALLING SEQUENCE: CALL VARADA(RI, XSIP, XSIV, TEVN, TSI, ADA, BS, BDTS, BDRS)

ARGUMENT &	ADA	n	VARIATION	MATRTY
J-1/00112-1414	AUR	•	TRIVARIATION	ICMINEA

BS I B OF THE NOMINAL TRAJECTORY

BDRS I B DOT R OF THE NOMINAL TRAJECTORY

BDTS I B DOT T OF THE NOMINAL TRAJECTORY

RI I POSITION AND VELOCITY OF THE VEHICLE AT THE TIME OF GUIDANCE EVENT

TEVN I TRAJECTORY TIME OF THE GUIDANCE EVENT

TSI I TRAJECTORY TIME AT WHICH THE VEHICLE
REACHED THE SPHERE OF INFLUENCE ON THE
NOMINAL TRAJECTORY

XSIP I POSITION OF THE VEHICLE AT THE SPHERE OF INFLUENCE ON THE NOMINAL TRAJECTORY

XSIV I VELOCITY OF THE VEHICLE AT THE SPHERE OF INFLUENCE ON THE NOMINAL TRAJECTORY

#### SUBROUTINES SUPPORTED: GUID

SUBROUTINES REQUIRED: NTM

LOCAL SYMBOLS: BDR1 TEMPORARY STORAGE FOR BDR

BDT1 TEMPORARY STORAGE FOR BDT

B1 TEMPORARY STORAGE FOR B

DSI1 TEMPORARY STORAGE FOR DSI

IPR TEMPORARY STORAGE FOR IPRINT

ISP TEMPORARY STORAGE FOR ISP2

IPO TEMPORARY STORAGE FOR IPRINT

RF ALTERED FINAL STATE OF VEHICLE

TSI1 TEMPORARY STORAGE FOR TSI

	ХC	ALTERED	INITIAL	STATE OF	F VEHICLE	
COMMON	COMPUTED/USED8	BDR ISP2	вот	DSI	IPRINT	ISPH
COMMON	COMPUTEDS	8	DELTH	RSI	TRTM1	VŞĪ
COMMON	USED:	DATEJ NTP	FACP	FACV	FNTH	ntmc

# VARADA Analysis

Subroutine VARADA employs numerical differencing to compute the variation matrix  $\eta$  for the three-variable B-plane guidance policy in the guidance event of the error analysis mode. See subroutine VARSIM Analysis for further analytical details, since the only difference between VARADA and VARSIM is that VARADA computations are based on the most recent targeted nominal, while VARSIM computations are based on the most recent nominal. The VARADA flow chart is identical to that of VARSIM except for the fact that in VARADA the nominal position/velocity state at  $t_{\rm SI}$  is saved prior to calling VARADA, while in VARSIM it is saved locally.

SUBROUTINE WARSIM

PURPOSE COMPUTE VARIATION MATRIX FOR THREE-VARIABLE B-PLANE GUIDANCE POLICY IN THE SIMULATION PROGRAM

CALLING SEQUENCE: CALL VARSIM(RI1, TEVN, TSI, ADA)

ARGUHENT8 ADA O VARIATION MATRIX

RI1 I VEHICLE POSITION/VELOCITY ON MOST RECENT NOMINAL TRAJECTORY AT TIME OF THE GUIDANCE EVENT

TEVN I TRAJECTORY TIME OF GUIDANCE EVENT

TSI I TRAJECTORY TIME AT SPHERE OF INFLUENCE

SUBROUTINES SUPPORTED: GUISS

SUBROUTINES REQUIRED: NTMS

LOCAL SYMBOLS: BDRS TEMPORARY STORAGE FOR BDR

BDTS TEMPORARY STORAGE FOR BDT

BS TEMPORARY STORAGE FOR B

IPR TEMPORARY STORAGE FOR IPRINT

ISPS TEMPORARY STORAGE FOR ISP2

RF1 ALTERED FINAL STATE OF VEHICLE ON MOST

RECENT NOMINAL

RSIS TEMPORARY STORAGE FOR RSI

TSI1 TEMPORARY STORAGE FOR TSI

VSIS TEMPORARY STORAGE FOR VSI

XC ALTERED INITIAL STATE OF VEHICLE ON MOST

RECENT NOMINAL

COMMON COMPUTED/USED: BOR BOT B : OSI IPRINT

ISPH ISP2 RSI VSI

COMMON COMPUTED: TRIM1

COMMON USED: DATEJ FACP FACV NTMC NTP

# VARSIM Analysis

Subroutine VARSIM employs numerical differencing to compute the variation matrix  $\eta$  for the three-variable B-plane guidance policy in the guidance event of the simulation mode. This variation matrix relates deviations in the position/velocity state at t to deviations in B·T, B·R, and  $t_{ST}$ :

$$\begin{bmatrix} \boldsymbol{\delta} \, \mathbf{B} \cdot \mathbf{T} \\ \boldsymbol{\delta} \, \mathbf{B} \cdot \mathbf{R} \\ \boldsymbol{\delta} \, \mathbf{t}_{\mathbf{S} \, \mathbf{I}} \end{bmatrix} = \eta \begin{bmatrix} \boldsymbol{\delta} \, \mathbf{R} \\ \boldsymbol{\delta} \, \mathbf{V}_{\mathbf{k}} \end{bmatrix} = \eta \boldsymbol{\delta} \, \mathbf{X}_{\mathbf{k}}$$

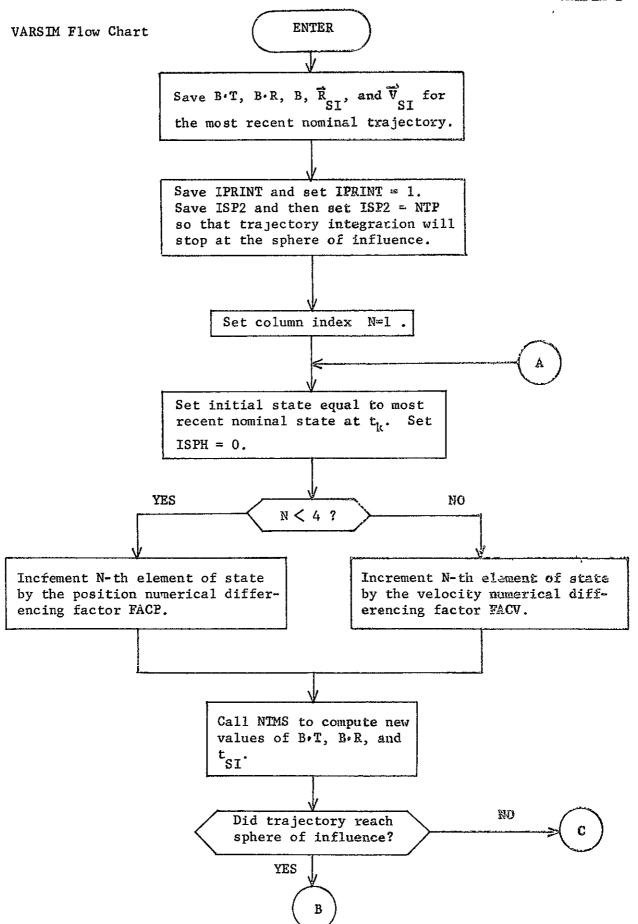
Since no good analytical formulas which relate  $\delta$  t_{SI} to  $\delta$   $\overrightarrow{R}$  and  $\delta$   $\overrightarrow{V}$  exist, numerical differencing must be employed to compute  $\eta$ .

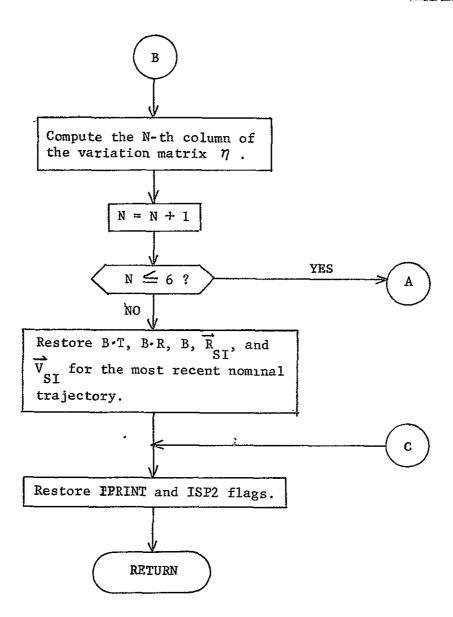
Let  $\overrightarrow{\eta}$  be the j-th column of the matrix  $\eta$ , and assume (most recent) nominal B·T*, B·R*,  $t_{ST_k}$ , and  $\overrightarrow{X_k}$  are available. To obtain  $\overrightarrow{\eta}$  we increment the j-th element of  $\overrightarrow{X_k}$  by the numerical differencing factor  $\Delta X$  and numerically integrate the spacecraft equations of motion from to the sphere of influence of the target planet to obtain the new values of B·T, B·R, and  $t_{ST}$ . Then

$$\overline{\eta_{j}} = \begin{bmatrix} \frac{B \cdot T - B \cdot T^{*}}{\triangle X_{j}}, & \frac{B \cdot R - B \cdot R^{*}}{\triangle X_{j}}, & \frac{t_{SI} - t_{SI}^{*}}{\triangle X_{j}} \end{bmatrix}^{T}$$

$$j = 1, 2, \ldots, 6$$

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SUBROUTINE VECTOR

PURPOSES TO COMPUTE THE VECTOR ORBITAL ELEMENTS K (ANGULAR

MOMENTUM VECTOR), E (ECCENTRICITY VECTOR TOWARD

PERIHELION), TO COMPUTE THE SPACECRAFT FINAL POSITION ON THE ORBIT TO ACCURATELY APPROXIMATE THE DESIRED TIME

INTERVAL, AND TO COMPUTE THE CONIC SECTION TIME OF

FLIGHT.

CALLING SEQUENCES CALL VECTOR

SUBROUTINES SUPPORTED: VMP

SUBROUTINES REQUIRED: SPACE

LOCAL SYMBOLS DUM INTERMEDIATE VARIABLE

COMMON COMPUTED/USED: V

COMMON COMPUTED 8 KOUNT

COMMON USED 8 HALF ITRAT ONE PI THREE

TWOPI TWO

VECTØR Analysis

The Kepler vector  $\vec{k}$  representing twice the areal rate of the spacecraft with respect to the virtual mass to be used during the current interval is computed from

$$\vec{k} = \vec{r}_{VS_B} \times \vec{r}_{VS_B}$$
 (1)

where the position and velocity vectors are referenced to the virtual mass at the beginning of the interval. The eccentricity vector for the interval is given by

$$\vec{e} = -\frac{\vec{r}_{VS}}{r_{VS_R}} - \frac{\vec{k} \times \vec{r}_{VS}}{\vec{\mu}_{V}}$$
 (2)

where  $\overline{\mu}_{_{\mbox{\scriptsize V}}}$  is the average value of the virtual mass during the interval.

The current time interval is computed from

$$\Delta \tau = \Delta t + \kappa \Delta t^2 \tag{3}$$

where the factor K was precomputed during the previous iterations. The direction of the final position  $\overline{\sigma}$  is determined from

$$\vec{\sigma} = \vec{r}_{VS_B} + \Delta \tau \vec{r}_{VS_B}$$
 (4)

The magnitude factor B is chosen to force the final position to satisfy the orbit equation (  $\vec{e} \cdot \vec{r} = -r + k^2/\mu$  )

$$B = \frac{k^2/\overline{\mu}}{\overrightarrow{e} \cdot \sigma + |\sigma|}$$
 (5)

The position and velocity vectors of the spacecraft relative to the virtual mass at the end of the interval are then

$$\frac{\vec{r}}{vs}_{E} = B \vec{\sigma}^{1}$$

$$\frac{\vec{r}}{vs}_{E} = \frac{\overline{\mu}_{V}}{k^{2}} \left[ \frac{\vec{k}}{k} \times (\vec{e} + \frac{\vec{v}_{S}}{r_{VS}}) \right]$$
(6)

The final position and velocity of the spacecraft in the reference inertial coordinates are computed from

$$\vec{r}_{S_{E}} = \vec{r}_{VS_{E}} + \vec{r}_{V}$$

$$\vec{r}_{S_{E}} = \vec{r}_{VS_{E}} + \vec{r}_{V}$$
(7)

The exact conic section time of flight is now computed. The in-plane normal to the major axis is

$$\vec{n} = \frac{\vec{k} \times \vec{e}}{k e} \qquad e \neq 0$$

$$\vec{k} \times \vec{r}_{VS_B} \qquad e = 0$$
(8)

The length of the semi-major axis is given by

$$a = \frac{k^{2}}{\overline{\mu}_{V} | 1-e^{2}|^{\frac{1}{2}}} \qquad e \neq 1$$

$$a_{1} = \frac{2}{r_{VS_{1}} - k^{2}/\overline{\mu}_{V}} \qquad e = 1, 1 = B,E$$
(9)

The projection of the radius vector orthogonal to the major axis divided by a is given by

$$X_{i} = \frac{\vec{n} \cdot \vec{r}_{VS}}{a_{i}} \qquad i = B, E \qquad (10)$$

The mean angular rate is

$$\overline{\omega} = \frac{\overline{\mu}_{V}(1-e^{2})}{ka} \qquad e \neq 1$$

$$= \frac{k}{2} \qquad e = 1$$
(11)

where  $\omega < 0$  for hyperbolic orbits. The eccentric anomaly is given by

$$E_{i} = \sin^{-1} X_{i} \qquad e < 1$$

$$= \frac{k^{2}/\overline{\mu}_{V} X_{i}}{3} \qquad e = 1$$

$$= \sinh^{-1} X_{i} \qquad e > 1$$
(12)

Then  $M_i = E_i - e X_i$  i = B, E (13)

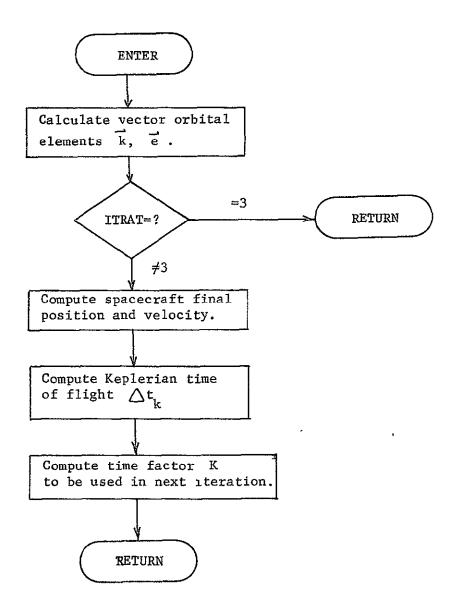
and the actual conic time of flight is

$$\triangle t = t_{E} - t_{B} = \frac{M_{E} - M_{B}}{\overline{\omega}}$$
 (14)

The value of the time factor K to be used in the following interval is then computed

$$K = \frac{\Delta \tau - \Delta t}{(\Delta t)^2}$$
(15)

# VECTOR Flow Chart



SUBROUTINE VMASS

PURPOSE: TO COMPUTE THE POSITION, VELOCITY, MAGNITUDE, AND MAGNITUDE RATE OF THE VIRTUAL MASS.

CALLING SEQUENCE: CALL YMASS

SUBROUTINES SUPPORTED: VMP

SUBROUTINES REQUIRED: NONE

COMMON COMPUTED/USED8 F V

COMMON USED: NBODY THREE ZERO

# VMASS Analysis

The current virtual mass data is computed by VMASS. The magnitude and position of the virtual mass is given by

$$\mu_{V} = r_{VS}^{3} M_{S} \tag{1}$$

$$\vec{r}_{V} = \frac{\vec{M}}{M_{S}} \tag{2}$$

where the intermediate variables are given by

$$\vec{M} = \sum_{i=1}^{n} \frac{\mu_i \vec{r}_i}{3}$$

$$\vec{r}_{is}$$
(3)

$$M_{S} = \sum_{i=1}^{n} \frac{\mu_{i}}{3}$$

$$i=1 \quad r_{iS}$$

$$(4)$$

and of course  $r = |\vec{r} - \vec{r}|$  and  $\vec{r} = |\vec{r} - \vec{r}|$  where  $\vec{r}$  represents the inertial position vector of the i-th body.

The time derivatives of these variables are given by

$$\dot{\mu}_{V} = \mu_{V} \left(\alpha_{V} + \frac{\dot{M}_{S}}{M}\right) \tag{5}$$

$$\frac{\dot{\vec{r}}}{\dot{\vec{r}}_{V}} = \frac{\dot{\vec{M}} - \dot{\vec{r}}_{V} \dot{M}_{S}}{M_{S}}$$
 (6)

$$\frac{\dot{\mathbf{m}}}{\mathbf{m}} = \sum_{\mathbf{i}=1}^{n} \frac{\mu_{\mathbf{i}}}{3} \begin{bmatrix} \dot{\mathbf{r}} & \dot{\mathbf{r}} \\ \dot{\mathbf{r}} & \dot{\mathbf{r}} \end{bmatrix} \qquad (7)$$

$$\dot{M}_{S} = -\sum \frac{\mu_{i}}{r_{iS}} \alpha_{iS}$$
 (8)

VMASS-2

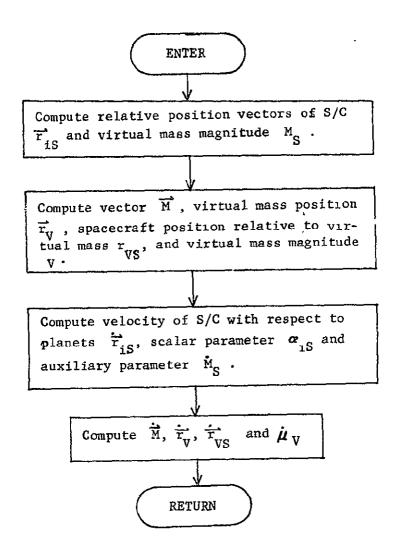
where

$$\alpha_{iS} = \frac{3 \vec{r}_{iS} \cdot \vec{r}_{iS}}{r_{iS}}$$
(9)

Finally, the velocity of the spacecraft with respect to the virtual mass is

$$\frac{r}{r_{VS}} = \frac{r}{r_{S}} - \frac{r}{r_{V}} \tag{10}$$

# VMASS Flow Chart



SUBROUTINE VMP

PURPOSE PROVIDE LOGIC TO GENERATE VIRTUAL MASS TRAJECTORY

CALLING SEQUENCE: CALL VMP(RS, ACC, D1, TRTM, DELTM, RSF, ISP2)

ARGUMENTS RS(6) I INERTIAL POSITION AND VELOCITY OF S/C AT INITIAL TIME

ACC I ACCURACY USED IN INTEGRATION

D1 I JULIAN DATE, EPOCH 1900, OF INITIAL TIME

TRIM I TRAJECTORY TIME (DAYS) AT INITIAL TIME

DELTM I TIME INTERVAL IN DAYS OVER WHICH THE TRAJECTORY IS TO BE PROPAGATED UNLESS A STOPPING CONDITION IS REACHED

RSF(6) O INERTIAL POSITION AND VELOCITY OF S/C AT FINAL TIME

ISP2 I SPHERE OF INFLUENCE STOPPING FLAG
= 0 00 NOT STOP AT SOI

=1 STOP AT SOI IF INTERSECTED BEFORE FINAL TIME

SUBROUTINES SUPPORTED: CASCAD NTMS GIDANS TARGET TARMAX

NTM TRJTRY DESENT

TPPROP TPRTRG MPPROP

SUBROUTINES REQUIRED: CAREL ELCAR EPHEM IMPACT ORB
PECEQ TIME ESTMT INPUTZ PRINT
SPACE VECTOR VMASS

LOCAL SYMBOLS AU NOT USED

CXI COSINE OF THE TRAJECTORY INCLINATION AT CLOSEST APPROACH

D INTERMEDIATE DATE FOR PRINTOUT PURPOSES

DELR INTERMEDIATE VARIABLE FOR INTERSECTION OF SPHERE-OF-INFLUENCE

DELT INTERMEDIATE TIME INCREMENT FOR INTER-POLATED SPHERE-OF-INFLUENCE POSITION

ECEQP TRANSFORMATION FROM ECLIPTIC TO EQUATORIAL SYSTEM FOR TARGET PLANET

ICUT CUTOFF FLAG USED WHEN CLOSEST APPROACH CUTOFF WAS DESIRED BUT NO VALID CLOSEST

APPROACH FOUND

IDAY PRINTOUT CALENDAR DAY

IHR PRINTOUT CALENDAR HOUR

IMO PRINTOUT CALENDAR MONTH

IP NUMBER OF PLANET, USED IN PRINTOUT

ISPHI INDICATOR FOR CALCULATION OF SPECIAL COMPUTING INTERVAL NEAR TARGET PLANET SPHERE-OF-INFLUENCE

IYR PRINTOUT CALENDAR YEAR

JJ COUNTER FOR NUMBER OF ITERATIONS FOR INTERPOLATED SPHERE OF INFLUENCE

LARCA INDICATOR FOR CALCULATION OF PSUEDO CLOSEST APPROACH

MIN PRINTOUT CALENDAR MINUTES

NTPI INDEX OF THE SPACECRAFT VECTORS IN THE F-ARRAY WITH RESPECT TO THE TARGET PLANET

RCM MAGNITUDE OF POSITION OF VEHICLE RELATIVE TO TARGET PLANET AT CLOSEST APPROACH

RCM1 PREVIOUS RADIUS OF VEHICLE RELATIVE TO TARGET PLANET

RCM2 PRESENT RADIUS OF VEHICLE RELATIVE TO TARGET PLANET

RDT NOT USED

RTEMP SPACECRAFT POSITION AT INTERPOLATED CLOSEST APPROACH IN THE TARGET PLANET EQUATORIAL SYSTEM

SEC PRINTOUT CALENDAR SECONDS

TIMOR TIME INCREMENT USED FOR INTERPOLATED CLOSEST APPROACH

TIMIN TOTAL TIME USED IN ONE INTEGRATED TRAJECTORY

TIM1 CLOCK TIME AT BEGINNING OF TRAJECTORY

TIM2 CLOCK TIME AT END OF TRAJECTORY

	TMU	GRAVITAT	IONAL CO	INSTANT C	F TARGET	PLANET
	TP	SPECIAL		G INTERV	AL NEAR	ATION OF SPHERE-OF-
	TTG	GRAVITAT	TIONAL CO SEC**2)	INSTANT C	F TARGET	PLANET
	VCA		MAGNITU AT INTERF			TO TARGET Approach
	VCM	TO TARGE	DE OF VEL ET PLANEI INTERPOLA	AT CLOS		E RELATIVE ROACH
	VQ	-	PLANET AT		_	RELATIVE TO CH BEFORE
	VTEMP	CLOSEST	AFT VELOC APPROACH IAL SYSTE	IN THE		
	XI		POLATED E T PURPOSE	•	AL INCLIM	NATION FOR
	XMAG	INTERME!	DIATE VAF	RIABLE FO	OR CALCUL	ATION OF
	XN	TARGET I	NORMAL TO PLANET EC FION OF )	QUATORIAL		
	хQ	TO TARGE	TAL SPACE ET PLANET APPROACE	F AT UNI		
	ZM		DIATE VAR			ATION OF NCLINATION
	ZTEMP	CALCULA	NORMAL TO TION OF I H INCLINA	INTERPOLA		
COMMON	COMPUTED/USED:	CAINC INCMT NO VSI	DG ISPH RCA V	DSI ITRAT RC	ICL KOUNT RSI	INCMNT NBODYI TIMINT
COMMON	COMPUTEQ#	DELTH	INCPR	RE	RTP	RVS
, <del>I</del>						

VMU

COMMON USED:	ALNGTH EM7 IEPHEM NTP	BDR EM8 INPR ONE	BDT F IPRINT PLANET	B HALF NBOO PMASS	DELTP ICL2 NB
	,		PLANET	PMASS	RADIUS
	RAD	SPHERE	TM	TWO	17FP0

# VMP Analysis

VMP provides the logic to integrate an N-body trajectory from an initial spacecraft state  $(\bar{r}_S, \bar{v}_S)$  at time  $t_B$  to one of the following stopping conditions.

- Target planet sphere of influence (SOI) is reached (ISP2 ≠ 0).
- The closest approach to the target planet has been reached (ICL2 = 1).
- 3. The preset final trajectory time  $t_p$  has been exceeded.

The integration logic is controlled by ITRAT

- - 2 Second pass through computation cycle (excluding ephemeris).
  - 3 Initialization flag.

To start the integration, appropriate variables are initialized (PRINTZ) and ITRAT is set equal to 3. The state of all gravitational bodies at  $t_B$  are found (ORB, EPHEM). The initial virtual mass position  $\tilde{r}_{V_B}$ , velocity  $\tilde{v}_{V_B}$ , magnitude  $\mu_{V_B}$  and magnitude rate  $\dot{\mu}_{V_B}$  are found by VMASS. Virtual mass dependent values are then initilized

$$\mu_{V_{AVE}} = \mu_{V_E} = \mu_{V_B} \tag{1}$$

$$\dot{\bar{r}}_{V_{AVE}} = \dot{\bar{r}}_{V_B} \tag{2}$$

$$\vec{r}_{VS_E} = \vec{r}_{VS_B} \tag{3}$$

$$\dot{\bar{r}}_{VS_E} = \dot{\bar{r}}_{VS_B} \tag{4}$$

$$\left(\Delta t\right)^2 = 1 \tag{5}$$

$$ISPH1 = 0 (6)$$

At this point the standard integration routine is entered by calling VECTOR.

In the standard integration routine, a new increment is initiated by calling ESTMT which:

- 1. Initializes all appropriate variables at the beginning of the increment (subscript B) to equal their values at the end of the previous increment.
- 2. Computes a  $\Delta t$  for the increment based on a modified true anomaly passage.
- Computes the time at the end of the increment t_k.
- 4. Estimates the final (subscript E) position  $\bar{r}_{_{V_E}}$  and magnitude  $\mu_{_{V_E}}$  of the virtual mass.

Based on these estimates, the average magnitude and velocity of the virtual mass is computed

$$\mu_{V_{AVE}} = 1/2 (\mu_{V_B} + \mu_{V_E})$$
 (7)

$$\vec{v}_{V_{AVE}} = (\vec{r}_{V_E} - \vec{r}_{V_B})/\Delta t$$
 (8)

Subroutine VECTOR then computes the orbit relative to the virtual mass based on these estimates. It also refines the estimate of the spacecraft final state ( $\tilde{r}_S$ ,  $\tilde{v}_S$ ). ORB and EPHEM are called to determine the state at t_E of all gravitational bodies being considered. The virtual mass position  $\tilde{r}_V$ , velocity  $\tilde{v}_V$ , magnitude  $\mu_V$  and magnitude rate  $\tilde{\mu}_V$  are determined by VMASS.

Using these refined values, the virtual mass average magnitude  $\mu_{\text{AVE}}$  and velocity  $\bar{\mathbf{v}}_{\text{V}}$  are recomputed using equations (7) and (8). At this point a second pass is made through VECTOR to compute the spacecraft final state ( $\bar{\mathbf{r}}_{\text{S}}$ ,  $\bar{\mathbf{v}}_{\text{S}}$ ) which will be used in all subsequent calculations. What is again called to make a final determination of the virtual mass

position, velocity, magnitude and magnitude rate at the end of the increment.

The virtual mass average accelerations are then computed

$$\ddot{\mu}_{V_{AVE}} = \left[ \mu_{V_E} - \mu_{V_B} - \dot{\mu}_{V_B} (\Delta t) \right] / (\Delta t)^2$$
 (9)

$$\dot{\bar{\mathbf{v}}}_{\mathbf{V}_{\mathbf{AVE}}} = \left[\bar{\mathbf{r}}_{\mathbf{V}_{\mathbf{E}}} - \bar{\mathbf{r}}_{\mathbf{V}_{\mathbf{E}}} - \bar{\mathbf{v}}_{\mathbf{V}_{\mathbf{E}}} - \bar{\mathbf{v}}_{\mathbf{V}_{\mathbf{E}}}\right] / (\Delta \mathbf{t})^{2}$$
 (10)

These values are subsequently used by ESTMT to estimate the final position  $r_{VE}$  and magnitude  $\mu_{VE}$  of the virtual mass for the next increment.

Tests are now made to determine whether the vehicle is inside a planetocentric sphere of radius 1.025 times larger than that of the SØI. If it is not, integration goes on as usual. If it is and yet is still outside the SØI, the integration step size is reduced to obtain an integration state near enough the SØI to permit accurate extrapolation to it. Finally, if the vehicle is inside of the sphere of influence, a refined sphere of influence (SØI) state is constructed by fitting an osculating planetocentric conic to the current state and extrapolating it to the sphere. The entire refinement process is carried out in subroutine SØIPS.

The refined state at the SØI is then used by IMPACT to compute  $B \cdot T$  and  $B \cdot R$ .

If trajectory data are to be printed at this point, the orbit inclination (assuming a hyperbolic orbit about the planet) is computed by first determining the "Kepler vector"

$$k = \bar{r}_{ST} \times \bar{v}_{ST}$$
 (11)

in planetocentric equatorial coordinates. Then

$$\cos i = \frac{k_z}{|\vec{k}|} \tag{12}$$

where i = orbit inclination and k = component of k normal to planet equatorial plane.

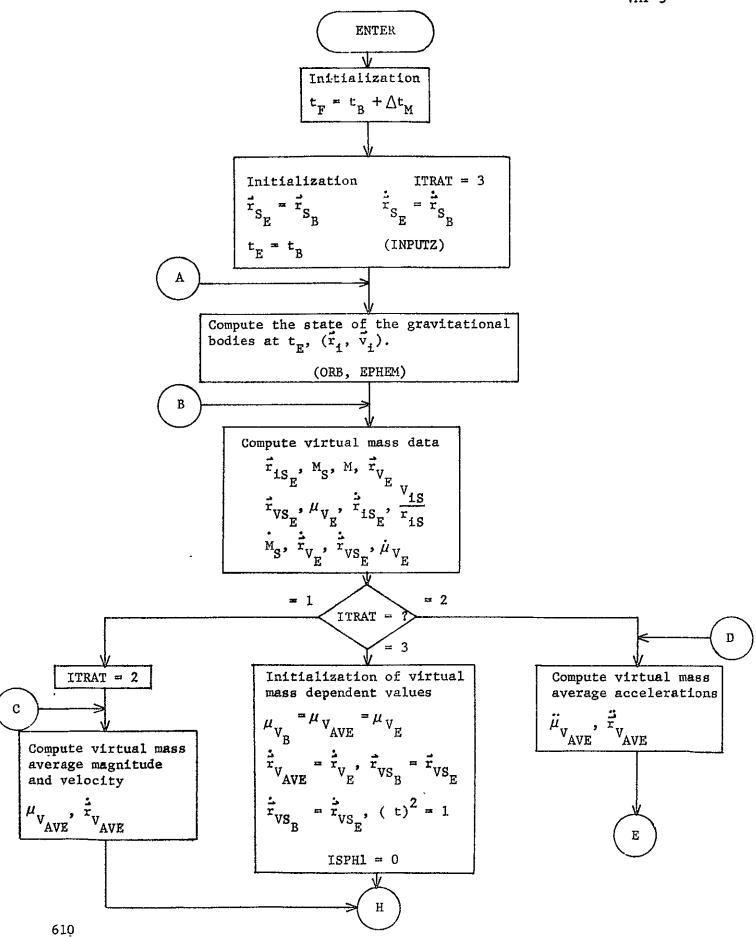
Tests are now made to determine if the spacecraft has reached a closest approach to the target planet. If it has, the interpolated state at closest approach  $(\bar{r}_{CA}, \bar{v}_{CA})$  is computed by calking CAREL with the spacecraft state just following closest approach. CAREL returns the element of the near planet conic. ELCAR is then called with these conic elements and returns the interpolated state at closest approach.

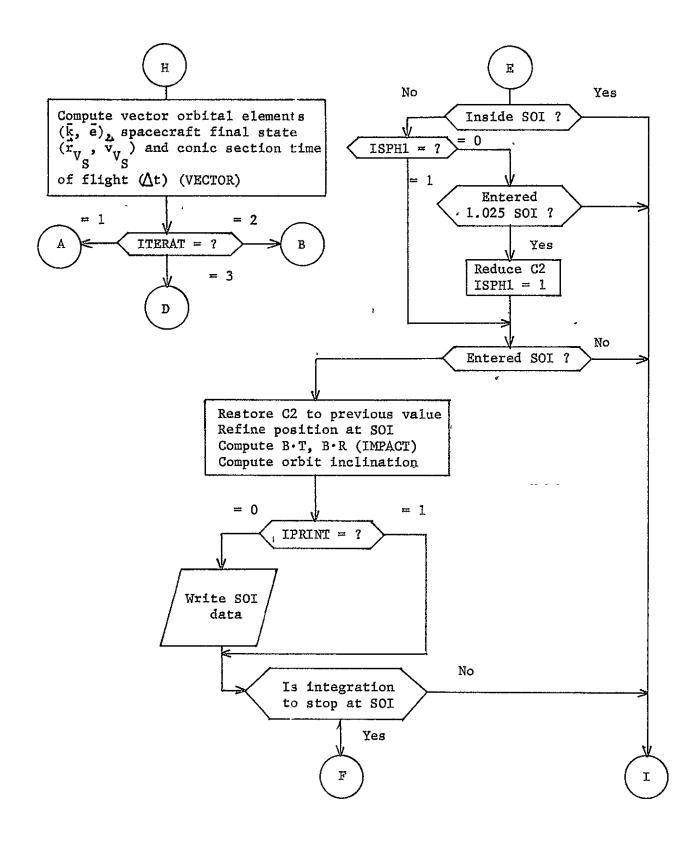
If the spacecraft is not within 10 SOI of the target planet, printout of closest approach data may occur; however, integration continues.

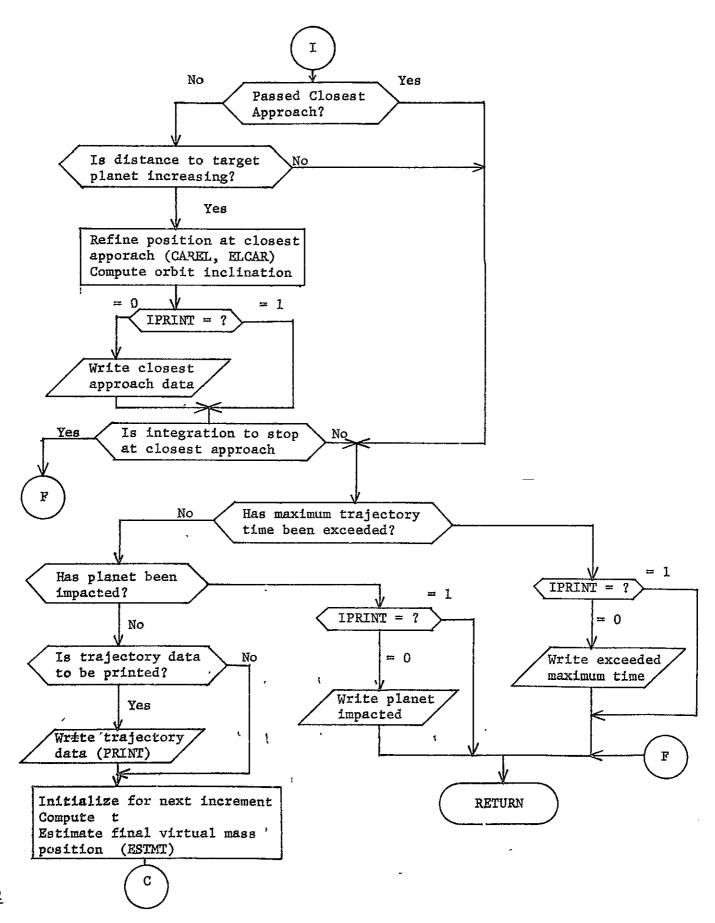
The final tests before starting a new integration increment determine if the maximum trajectory time  $t_{\rm p}$  has been exceeded or

a planet has been impacted. If the latter has occurred, the actual impact state is determined by fitting an osculating planetocentric conic to the current state and extrapolating to the planet surface. As was the case earlier at the SOI, this procedure is carried out in the subroutine SØIPS. If the impacted planet is the object of any type of probe targeting, the planet radius used in the impact state computation is the probesphere value input by the user. If these final two tests are passed, a new integration cycle is initiated by calling ESTMT.









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# SUBROUTINE ZERIT

PURPOSE: TO COMPUTE THE COMPUTATION OF THE ZERO ITERATE VALUES OF TIME, POSITION VECTOR, AND VELOCITY VECTOR.

CALLING SEQUENCES CALL ZERIT

SUBROUTINES SUPPORTED: PRELIM GIDANS

SUBROUTINES REQUIRED: HELIO LUNA

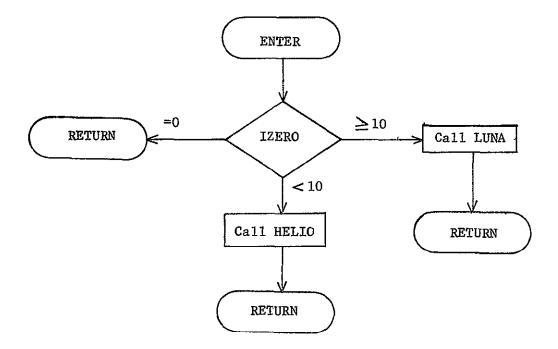
COMMON USED: IZERO LTARG

# ZERIT Analysis

ZERIT is the executive subroutine handling the computation of the zero iterate values of time, position vector, and velocity vector.

The flag IZERO controls the operation of ZERIT. If IZERO = 0, no zero iterate computation is needed and so ZERIT is exited. If IZERO < 10, the zero iterate is to be computed for a interplanetary trajectory so HELIO is called before returning. If IZERO  $\geq$  10, the zero iterate is to be computed for a lunar trajectory so LUNA is called for that computation.

#### ZERIT Flow Chart



#### SUBROUTINE ZRANS

PURPOSE: TO CALCULATE THE TRANSCENDENTAL FUNCTIONS USED IN THE UNIVERSAL FORM OF KEPLER-S EQUATION

CALLING SEQUENCE: CALL ZRANS(X,S,C)

ARGUMENTS: X I ANGLE

S O SIN OR SINH OF X

C O COS OR COSH OF X

SUBROUTINES SUPPORTED: BATCON

LOCAL SYMBOLS: CH INTERMEDIATE VARIABLE

SH INTERMEDIATE VARIABLE

T1 INTERMEDIATE VARIABLE

T2 INTERMEDIATE VARIABLE

Y INTERMEDIATE VARIABLE

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