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DELTA PAC ATTITUDE DETERMINATION AND ERROR PREDICTION

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CONTENTS

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INTRODUCTION

The Delta Packaged control spacecraft (Delta PAC) was an earth satellite designed, built, and tested in house by the Technology Directorate under the leadership of the System Division. The Delta PAC attitude determination and error prediction project was assigned to the author in Jan. 1969, and was completed in six months. It consists of both an analysis and a FORTRAN IV program. The computer program was utilized to evaluate the spacecraft attitude control system after it was launched into a circular orbit.

The Delta PAC control system contains one gimballed fly-wheel scanner with electronic damping that actively controls one axis and passively controls the other two axes. The spacecraft is therefore stabilized about all three axes. The mission objectives of this spacecraft is to test the earth stabilized control system for future spacecraft applications. The evaluation of the behavior of this spacecraft and the flight performance of its control system depends upon a knowledge of the PAC attitude. Therefore, the problem of attitude determination is of great importance.

This report presents the analysis of that project with the FORTRAN IV as its appendix. The analysis includes:

(1) Formulation of attitude determination equations: An approach is developed in which the complete sensor data are used to determine two independent vectors, such as the local vertical and the sun line. These vectors are then applied to obtain the attitude of the spacecraft.

(2) Presentation of special cases: Various cases arising from incomplete sensor output have been considered.

(3) Examination of error propagation: Since measurement errors could exist in the sensor data, each calculation is subjected to errors. The effect of the errors on the determination of orientation of Delta PAC caused by errors of the measurement quantities is also explored.

Coordinate Axes Definitions

For convenience, it is necessary to make the following definitions.

(1) First Reference Axes.-The first coordinate system X_s , Y_s , Z_s has center at the spacecraft's noon position where

X_s axis is along the spacecraft's velocity vector.

Z_s axis is along the negative sun line vector.

Y_s axis is determined by

$$\vec{Y}_s = \vec{Z}_s \times \vec{X}_s$$

(2) Second Reference Axes.-The second coordinate system X_0 , Y_0 , Z_0 has the same center as the first coordinate system with

$$\vec{X}_0 = \vec{X}_s$$

$\vec{Y}_0$ = normal to the orbit plane

$\vec{Z}_0$ = the vector pointing downward toward the local vertical.

(3) Orbital Reference Axes.-The third coordinate system X, Y, Z has its center at the instantaneous location of the spacecraft with

$\overline{X}$ = spacecraft's velocity vector

$\overline{Y} = \overline{Y}_0$

$\overline{Z}$ = the downward local vertical

(4) The Spacecraft Body Axes.-The spacecraft body coordinate system x, y, z is a set of right handed mutually orthogonal axes fixed in the spacecraft with their origin at the center of mass. It is required that the attitude control system maintain the alignment of the spacecraft body axes with the orbital axes system. That is, the yaw axis (z) is pointed toward the downward vertical and the pitch axis (y) is aligned with the normal to the orbit plane, thus causing the roll axis (x) to align with the spacecraft's velocity vector. Any attitude error is detected by a control subsystem which consists of an earth sensor and three sun sensors.

(5) The Sun Sensor Axes System.-There are three sun sensors mounted at 120 degree intervals on the top surface of the Delta PAC. Each sun sensor head i has its own right handed system e_i, f_i, g_i which is oriented such that

(a) The axis g_i is in the x-y plane

(b) The angle between g_i and y is $90° + \xi_i$

(c) The angle between g_1 and x is $180^\circ + \xi_1$

(d) The angle between e_1 and z is $90^\circ + \eta_1$

The nominal constant ξ_1 and η_1 are:

$$\xi_1 = 0, \ \xi_2 = 120^\circ, \ \xi_3 = 240^\circ$$

$$\eta_1 = \eta_2 = \eta_3 = 26^\circ$$

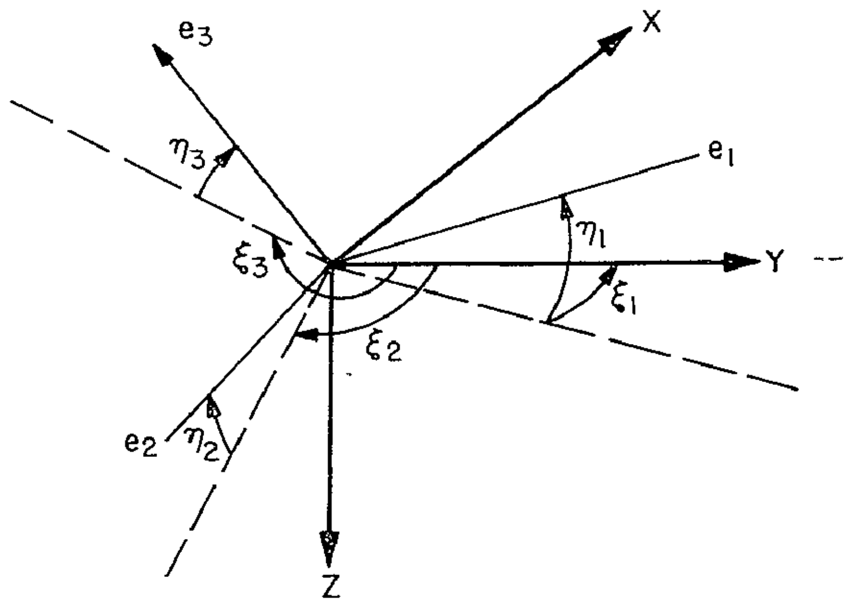


Figure 1 Sun Sensor Zenith Axes e_1, e_2, e_3

The Reference Axes Transformation Equations

If

α = the angle between $\vec{Z}_0$ and $\vec{Z}$

β = the angle between orbit plane and the earth-sun line with positive sign when the sun is above the $\vec{X}_0 - \vec{Z}_0$ plane (see Figure 2), the relationship between the first three reference axes can be expressed as

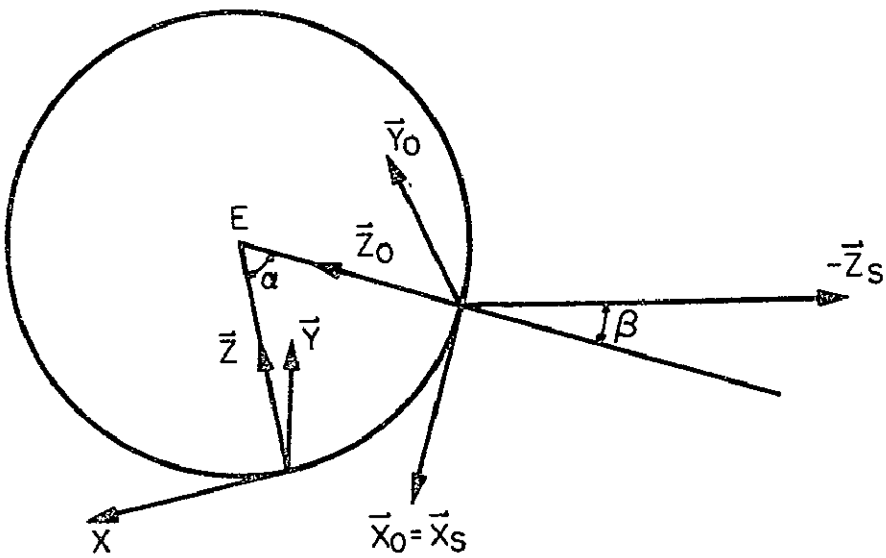


Figure 2. The orbital Reference Axes

$$\begin{bmatrix} \vec{X} \\ \vec{Y} \\ \vec{Z} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} \vec{X}_0 \\ \vec{Y}_0 \\ \vec{Z}_0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \vec{X}_s \\ \vec{Y}_s \\ \vec{Z}_s \end{bmatrix} \qquad \text{(T1)}$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha\sin\beta & \sin\alpha\cos\beta \\ 0 & \cos\beta & -\sin\beta \\ -\sin\alpha & \sin\beta\cos\alpha & \cos\alpha\cos\beta \end{bmatrix} \begin{bmatrix} \vec{X}_s \\ \vec{Y}_s \\ \vec{Z}_s \end{bmatrix}$$

The range of β is $-55° \leq \beta \leq 55°$ for Delta PAC calculated from orbit determination.

Let $\vec{S}$ be the sun line vector, the components of $\vec{S}$ in each of three reference coordinate systems can therefore be determined. It is obvious that

$$\vec{S} = 0 \cdot \vec{X}_s + 0 \cdot \vec{Y}_s - \vec{Z}_s$$

in the X_s, Y_s, Z_s system, and

$$\vec{S} = \sin\beta \, \vec{Y}_0 - \cos\beta \, \vec{Z}_0$$

in the X_0, Y_0, Z_0 system.

Since the unit sun line vector has the coordinate (0,0,-1) in the X_s, Y_s, Z_s system, by using equation (T1) the components of sun line in the X, Y, Z, system are:

$$[S]_0 = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} -\sin\alpha \ \cos\beta \\ \sin\beta \\ -\cos\alpha \ \cos\beta \end{bmatrix} \tag{T2}$$

Therefore

$$\vec{S} = -\sin\alpha \ \cos\beta \cdot \vec{X} + \sin\beta \cdot \vec{Y} - \cos\alpha \ \cos\beta \cdot \vec{Z} \tag{T3}$$

and

$$\vec{S} \cdot \vec{Z} = -\cos\alpha \ \cos\beta = S_3 \tag{T4}$$

If there is no attitude error, the body axes x, y, z should be coincident with the orbital axes X, Y, Z respectively and equation (T2) are the direction cosines of sun line in both axes systems. However, this is no longer true if a misorientation takes place. For that case, the components of sun line in the body axes need to be determined and the sun sensors are used for this purpose.

The Sun Sensor Equations

Let S_x, S_y, S_z be the components of the sun line vector $\vec{S}$ in the body axes system. Then

$$\vec{S} = S_x \vec{x} + S_y \vec{y} + S_z \vec{z} \tag{S1}$$

The method of determining the values of S_x, S_y, and S_z is described as follows.

The body axes x, y, z can be aligned with the sun sensor axes e_1, f_1, g_1 by the sequence of three rotations: a first rotation $R_z\,(90° + \xi_1)$ about axis z through an angle $90° + \xi_1$, a second rotation $R_y\,(\eta_1)$ about displaced y axis through an angle η_1 and a third rotation $R_x\,(-90°)$ about displaced x axis through an angle -90° (See Figure 3). The transformation may be written in the following matrix form.

$$\begin{bmatrix} \vec{e}_1 \\ \vec{f}_1 \\ \vec{g}_1 \end{bmatrix} = R_x\,(-90°)\,R_y\,(\eta_1)\,R_z\,(90° + \xi_1) \begin{bmatrix} \vec{x} \\ \vec{y} \\ \vec{z} \end{bmatrix} \tag{S2}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos\eta_1 & 0 & -\sin\eta_1 \\ 0 & 1 & 0 \\ \sin\eta_1 & 0 & \cos\eta_1 \end{bmatrix} \begin{bmatrix} -\sin\xi_1 & \cos\xi_1 & 0 \\ -\cos\xi_1 & -\sin\xi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{x} \\ \vec{y} \\ \vec{z} \end{bmatrix}$$

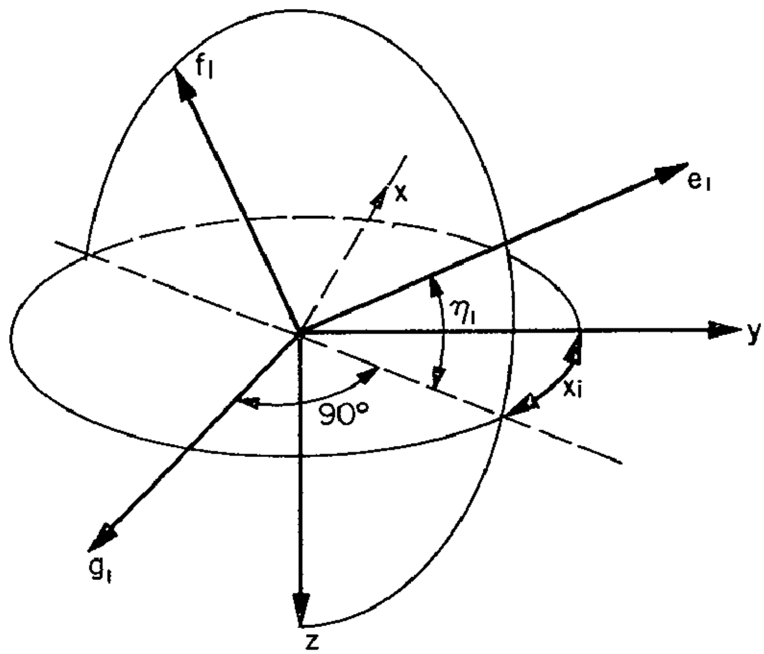


Figure 3 Sensor Frame System e_1, f_1 g_1

$$= \begin{bmatrix} -\cos\eta_1 \sin\xi_1 & \cos\eta_1 \cos\xi_1 & -\sin\eta_1 \\ \sin\eta_1 \sin\xi_1 & -\sin\eta_1 \cos\xi_1 & -\cos\eta_1 \\ -\cos\xi_1 & -\sin\xi_1 & 0 \end{bmatrix} \begin{bmatrix} \vec{x} \\ \vec{y} \\ \vec{z} \end{bmatrix} \tag{S3}$$

In simple notation, that is

$$\begin{bmatrix} \vec{e}_1 \\ \vec{f}_1 \\ \vec{g}_1 \end{bmatrix} = [D] \begin{bmatrix} \vec{x} \\ \vec{y} \\ \vec{z} \end{bmatrix} \tag{S4}$$

Where

$$[D] = R_x(-90)\, R_y(\eta_1)\, R_z(90° + \xi_1)$$

Since [D] is an orthogonal transformation matrix, therefore

$$[D]^{-1} = [D]^T = \begin{bmatrix} -\cos\eta_i \sin\xi_i & \sin\eta_i \sin\xi_i & -\cos\xi_i \\ \cos\eta_i \cos\xi_i & -\sin\eta_i \cos\xi_i & -\sin\xi_i \\ -\sin\eta_i & -\cos\eta_i & 0 \end{bmatrix} \tag{S5}$$

and

$$[D]^{-1} \begin{bmatrix} \vec{e}_i \\ \vec{f}_i \\ \vec{g}_i \end{bmatrix} = [D]^{-1} [D] \begin{bmatrix} \vec{x} \\ \vec{y} \\ \vec{z} \end{bmatrix}$$

or

$$\begin{bmatrix} \vec{x} \\ \vec{y} \\ \vec{z} \end{bmatrix} = [D]^T \begin{bmatrix} \vec{e}_i \\ \vec{f}_i \\ \vec{g}_i \end{bmatrix} = \begin{bmatrix} -\cos\eta_i \sin\xi_i & \sin\eta_i \sin\xi_i & -\cos\xi_i \\ \cos\eta_i \cos\xi_i & -\sin\eta_i \cos\xi_i & -\sin\xi_i \\ -\sin\eta_i & -\cos\eta_i & 0 \end{bmatrix} \begin{bmatrix} \vec{e}_i \\ \vec{f}_i \\ \vec{g}_i \end{bmatrix} \tag{S6}$$

The sensor telemetry yields i (i = 1, 2, or 3) which indicates that the i th sensor receives the greatest sun intensity and is selected to produce the output. The output of the selected sensor is two angles α_i and β_i which define two planes, each containing the sun (Figure 4). The intersection of these two planes determines the sun-line.

Let S_{1i}, S_{2i} and S_{3i} be the direction cosines of sun-line OS with respect to the i th sun sensor system (see Figure 4). Then:

$$\vec{S} = S_{1i}\,\vec{e}_i + S_{2i}\,\vec{f}_i + S_{3i}\,g_i \tag{S7}$$

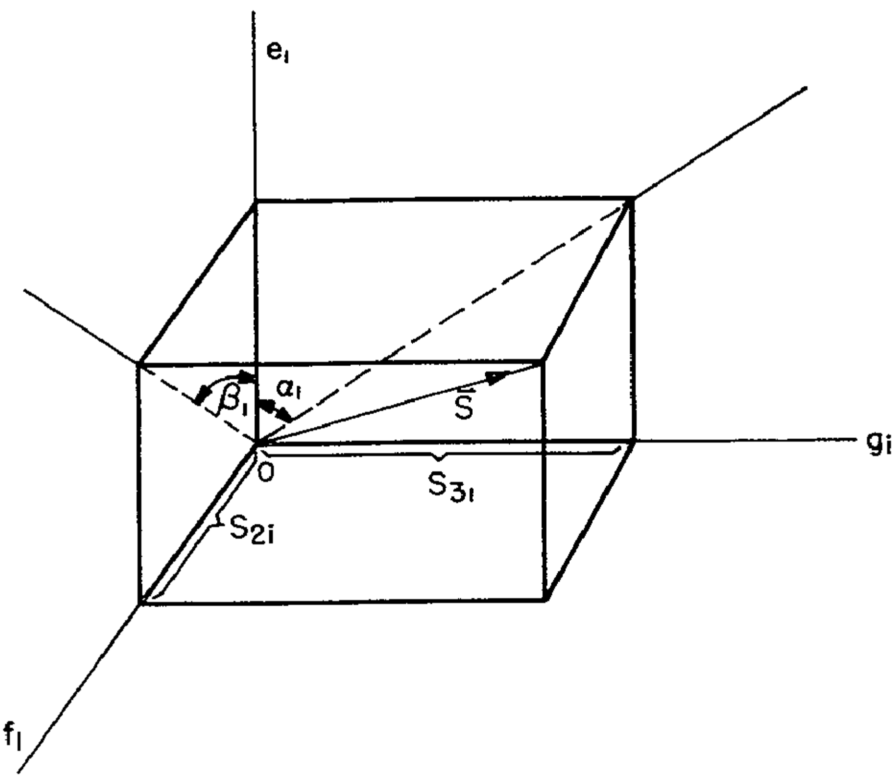


Figure 4 Sun line in e_1, f_1, g_1 system

Since:

$$\left.\begin{aligned} \text{Tan}\ \alpha_1 &= S_{31}/S_{11} \\ \text{Tan}\ \beta_1 &= S_{21}/S_{11} \end{aligned}\right\} \qquad \text{(S8)}$$

and

$$S_{11}^2 + S_{21}^2 + S_{31}^2 = 1$$

giving

$$\left.\begin{aligned} S_{11} &= 1/\sqrt{1 + \text{Tan}^2\ \alpha_1 + \text{Tan}^2\ \beta_1} \\ S_{21} &= \text{Tan}\ \beta_1 / \sqrt{1 + \text{Tan}^2\ \alpha_1 + \text{Tan}^2\ \beta_1} \\ S_{31} &= \text{Tan}\ \alpha_1 / \sqrt{1 + \text{Tan}^2\ \alpha_1 + \text{Tan}^2\ \beta_1} \end{aligned}\right\} \qquad \text{(S9)}$$

Using equation (S7) and equation (S3) or equation (S6) and equation (S7), the components of the sun vector $\vec{S}$ in the body axes system can be found. That is

$$\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \begin{bmatrix} -\cos\eta_1 \sin\xi_1 & \sin\eta_1 \sin\xi_1 & -\cos\xi_1 \\ \cos\eta_1 \cos\xi_1 & -\sin\eta_1 \cos\xi_1 & -\sin\xi_1 \\ -\sin\eta_1 & -\cos\eta_1 & 0 \end{bmatrix} \begin{bmatrix} S_{11} \\ S_{21} \\ S_{31} \end{bmatrix} \tag{S10}$$

In terms of known quantities, equation (S10) becomes

$$[S]_b = \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \frac{1}{\sqrt{1 + \tan^2\alpha_1 + \tan^2\beta_1}} \begin{bmatrix} -\cos\eta_1 \sin\xi_1 & \sin\eta_1 \sin\xi_1 & -\sin\xi_1 \\ \cos\eta_1 \cos\xi_1 & -\sin\eta_1 \cos\xi_1 & -\sin\xi_1 \\ -\sin\eta_1 & -\cos\eta_1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \tan\beta_1 \\ \tan\alpha_1 \end{bmatrix} \tag{S11}$$

This final form and equation (T2) are used to obtain the attitude determination equations.

The Earth Sensor Equations

Since the local vertical $\vec{Z}$ in the orbital axes system has components

$$[Z]_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{E1}$$

the same vector $\vec{Z}$ in the body axes system can be written as

$$\vec{Z} = a_{13}\,\vec{x} + a_{23}\,\vec{y} + a_{33}\,\vec{z} \tag{E2}$$

Here the components

$$[Z]_b = \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} \qquad \text{(E3)}$$

are to be determined by the measured quantities obtained from the earth sensor.

For Delta PAC there is one earth sensor, which is a horizon scanner. The scanner is mounted on a gimbaled platform which rotates about the spacecraft roll axes (x). Eddy currents induced by the motion of the gimbal with respect to the spacecraft provide the means for damping the spacecraft roll-yaw motion. The scanner cone axis (Y_w) coincides with body pitch axis (y) when the gimbal angle γ is zero (see Figure 5). The range of γ is: $-30° \leq \gamma \leq 30°$.

In general, the scan pattern of a single scanner is a cone. As the scan cone cuts the earth, a voltage earth pulse of width 2ρ is emitted by a sensing element called a bolometer. The width of this pulse represents the angular portion of the cone which interesects the earth (see Figure 6).

Let an intermediate parameter C_{23} be calculated first, where

C_{23} = the scalar product of two unit vectors which represent the scan cone axis $\vec{Y}_w$ and the local vertical $\vec{Z}$.

The equation expressing the relationship between the variable C_{23} and the earth pulse half width ρ are presented as follows:

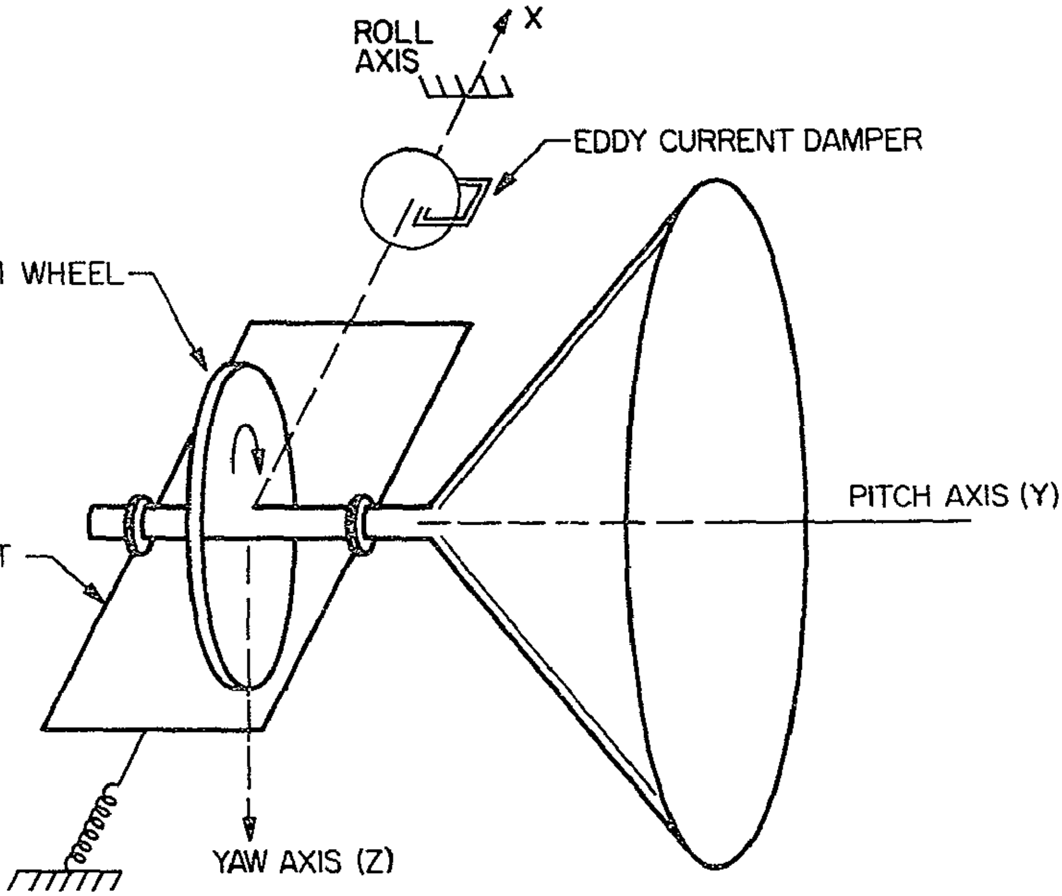


Figure 5 The Horizon Scanner

$$\cos \rho = \frac{\cos \alpha_e - C_{23} \cos \sigma}{\sqrt{1 - C_{23}^2} \sin \sigma} \tag{E4}$$

with

σ = the apex half-angle of the cone

α_e = one-half the angle subtended by the earth.

The valve of α_e is calculated from:

$$\sin a_e = \frac{R_e}{R_e + h}$$

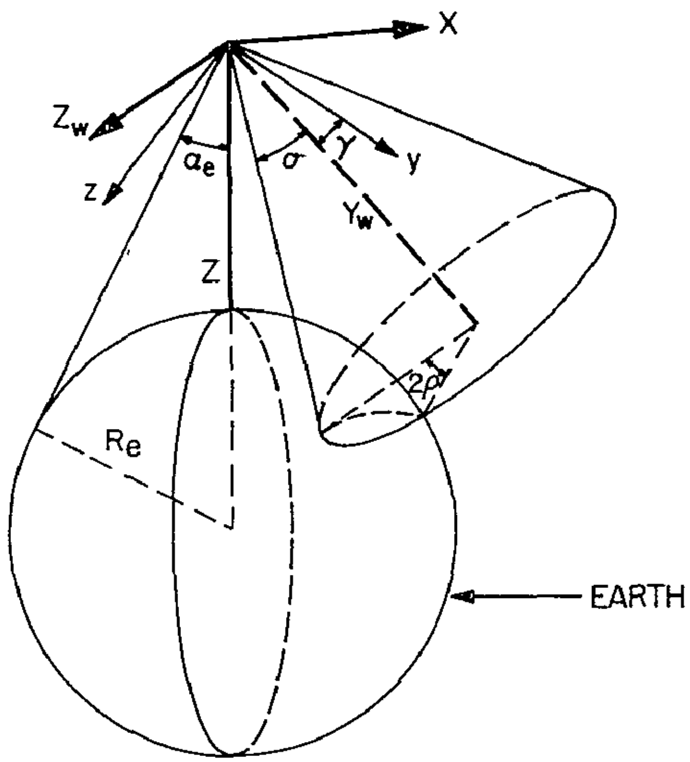


Figure 6. The Scanner Geometry

where

R_e = the earth's radius

h = the orbital altitude

The earth pulse half width ρ is in the range (0, 180). If the scanner does not see the earth, ρ is then zero and the variable C_{23} is less than cos $(\alpha_e + \sigma)$. If the scan cone is completely on the earth, ρ is 180° and C_{23} is larger than cos $(\alpha_e - \sigma)$. The variable C_{23} is computed from equation (E4) only when ρ lies between 0° and 180°. In this case, the result of derivation gives:

$$C_{23} = \frac{\cos\alpha_e \cos\sigma \pm \cos\rho \sin\sigma \sqrt{\sin^2\alpha_e - \sin^2\sigma \sin^2\rho}}{1 - \sin^2\sigma \sin^2\rho} \qquad \text{(E5)}$$

Based on the fact that C_{23} and cos ρ vary inversely, it can be shown that the positive sign in the numerator should be discarded, i.e.,

$$C_{23} = \frac{\cos \alpha_e \cos \sigma - \cos \rho \sin \sigma \sqrt{\sin^2 \alpha_e - \sin^2 \sigma \sin^2 \rho}}{1 - \sin^2 \sigma \sin^2 \rho} \qquad \text{(E6)}$$

for $0 < \rho < 180$

The unit vector representing the scan cone axis can be written as

$$\vec{Y}_W = \cos \gamma \, \vec{y} + \sin \gamma \, \vec{z}$$

where γ is the gimbal angle. Therefore

$$C_{23} = \vec{Z} \cdot \vec{Y}_W = a_{23} \cos \gamma + a_{33} \sin \gamma \qquad \text{(E7)}$$

or

$$a_{33} \sin \gamma = C_{23} - a_{23} \cos \gamma \qquad \text{(E8)}$$

Suppose that a unit vector $\vec{Z}_W$ is defined such that in conjunction with $\vec{X}_W = \vec{x}$ (spacecraft roll axis) and the scanner cone axis $\vec{Y}_W$, a right handed orthogonal coordinate system is formed. Then

$$\vec{Z}_w = \vec{x} \times \vec{Y}_w = -\sin \gamma \, \vec{y} + \cos \gamma \, \vec{z}$$

giving

$$\vec{Z} \cdot \vec{Z}_w = a_{33} \cos \gamma - a_{23} \sin \gamma$$

and

$$\vec{Z} \cdot \vec{X}_w = \vec{Z} \cdot \vec{x} = a_{13}$$

Define the pitch computed angle θ_s measured about the scanner axis Y_w as follows

$$\tan \theta_s = \frac{-\vec{Z} \cdot \vec{X}_w}{\vec{Z} \cdot \vec{Z}_w} = \frac{-a_{13}}{a_{33} \cos \gamma - a_{23} \sin \gamma} \qquad \text{(E9)}$$

This implies

$$\sin \theta_s = -a_{13} \cdot V$$

and

$$\cos \theta_s = (a_{33} \cos \gamma - a_{23} \sin \gamma) \cdot V$$

The variable V, after use of:

$$\sin^2 \theta_s + \cos^2 \theta_s = 1$$

and equation (E7) gives:

$$V = \mp 1/\sqrt{1 - C_{23}^2}$$

Hence,

$$a_{13} = \pm \sqrt{1 - C_{23}^2} \sin \theta_s$$

and

$$a_{33} \cos \gamma - a_{23} \sin \gamma = \mp \sqrt{1 - C_{23}^2} \cos \theta_s \qquad \text{(E10)}$$

Solving equation (E7) and equation (E10), gives:

$$\left.\begin{aligned} a_{23} &= C_{23} \cos\gamma \pm \sqrt{1 - C_{23}^2} \sin\gamma \cos\theta_s \\ a_{33} &= C_{23} \sin\gamma \mp \sqrt{1 - C_{23}^2} \cos\gamma \cos\theta_s \end{aligned}\right\} \qquad \text{(E11)}$$

In addition to providing a measure of the angle ρ, the earth pulse is used to obtain θ_s through the use of signal processing which is described in the following paragraph:

When the scanner passes a reference mark in the yaw-pitch plane of the spacecraft, a reference pulse is generated by a magnetic wedge attached to the pitch wheel. By comparing the reference pulse with the center of the earth pulse, a measure of the angle θ_s is obtained. For 180° of rotation after the reference pulse, the signal processor causes any portion of the earth pulse to contribute to e in a positive sense. For the remaining 180°, of rotation, the earth pulse contributes in a negative sense. If the earth pulse contains the reference mark but not the 180 mark, then it does indeed measure θ_s. If these conditions are not met, the exact relation is summarized mathematically by the following equation:

$$e = (\text{sign}\ \theta_s)\ \text{Min}\ \left\{|\theta_s|,\ |180° - \theta_s|,\ \rho,\ 180° - \rho,\ E\right\} \qquad \text{(E12)}$$

where

E = 45 is the electronic saturation angle

e = telementered scanner pitch signal.

The plot of e vs. θ_s is shown in Figure 7.

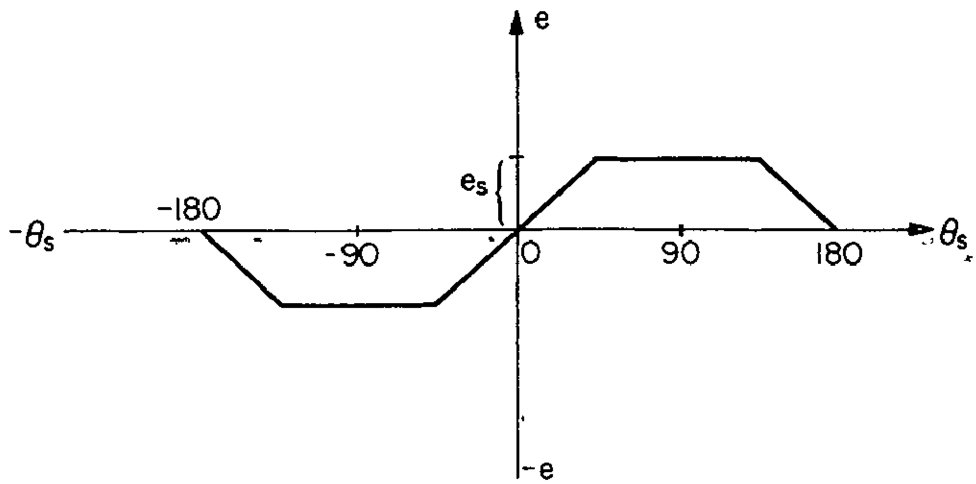


Figure 7 The Error Signal e

For convenience, a new variable, e_s, is defined and equal to:

$$e_s = \text{Min}\ (\rho,\ 180^\circ - \rho,\ E) \tag{E13}$$

Accordingly, the case:

$$|e| \angle e_s$$

and

$$|e| = e_s$$

are to be discussed.

(1) When the absolute value of e is less than e_s, there are possibilities that:

$$\theta_s = e$$

or

$$\theta_s = \pm\ (180^\circ - |e|) \tag{E14}$$

Care must be taken to consider all these possible cases. Four sets of equations are thus obtained to represent $[Z]_b$. They are

$$\left.\begin{aligned}
\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} &= \begin{bmatrix} \sqrt{1 - C_{23}^2}\ \sin e \\ C_{23} \cos\gamma + \sqrt{1 - C_{23}^2}\ \sin\gamma \cos e \\ C_{23} \sin\gamma - \sqrt{1 - C_{23}^2}\ \cos\gamma \cos e \end{bmatrix} \\
\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} &= \begin{bmatrix} -\sqrt{1 - C_{23}^2}\ \sin e \\ C_{23} \cos\gamma + \sqrt{1 - C_{23}^2}\ \sin\gamma \cos e \\ C_{23} \sin\gamma - \sqrt{1 - C_{23}^2}\ \cos\gamma \cos e \end{bmatrix} \\
\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} &= \begin{bmatrix} \sqrt{1 - C_{23}^2}\ \sin e \\ C_{23} \cos\gamma - \sqrt{1 - C_{23}^2}\ \sin\gamma \cos e \\ C_{23} \sin\gamma + \sqrt{1 - C_{23}^2}\ \cos\gamma \cos e \end{bmatrix} \\
\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} &= \begin{bmatrix} -\sqrt{1 - C_{23}^2}\ \sin e \\ C_{23} \cos\gamma - \sqrt{1 - C_{23}^2}\ \sin\gamma \cos e \\ C_{23} \sin\gamma + \sqrt{1 - C_{23}^2}\ \cos\gamma \cos e \end{bmatrix}
\end{aligned}\right\} \qquad \text{(E15)}$$

In order to determine which of these four sets is the correct local vertical, note that

$$[\vec{S} \cdot \vec{Z}]_0 = -\cos\alpha \cos\beta$$

Since the scalar product of two vectors has the same value regardless of what coordinate axis system is used, therefore

$$[\vec{S} \cdot \vec{Z}]_b = S_x\, a_{13} + S_y\, a_{23} + S_z\, a_{33} = -\cos\alpha \cos\beta \qquad \text{(E16)}$$

The representation in equation (E15) which best satisfies this criterion is the proper expression.

(2) When the absolute value of e is equal to e_s, e is saturated. The telemetered error signal e fails to have a definite value and equation (E14) is no longer useful in determining θ_s. However it is still possible to find $[Z]_b$ by making use of ρ and $[S]_b$. The parameter C_{23} determined from ρ in equation (E6) is the cosine of the angle between the sensor cone axis $\vec{Y}_W$ and the local vertical $\vec{Z}$. Thus the local vertical must be on a cone with axis $\vec{Y}_W$. The cosine of the angle between $\vec{Z}$ and $\vec{S}$ is $-\cos\alpha \cos\beta$ and thus $\vec{Z}$ must lie on a cone with axis $\vec{S}$. The determination of the intersection of these two cones is accomplished by solving equation (E16) with equation (E7) to obtain a_{23} and a_{33} in terms of a_{13}. This gives

$$\left.\begin{aligned} a_{23} &= \frac{-\sin\gamma\,(\cos\alpha\cos\beta + S_x\, a_{13}) - C_{23}\, S_z}{S_y \sin\gamma - S_z \cos\gamma} \\ a_{33} &= \frac{\cos\gamma\,(\cos\alpha\cos\beta + S_x\, a_{13}) + C_{23}\, S_y}{S_y \sin\gamma - S_z \cos\gamma} \end{aligned}\right\} \qquad \text{(E17)}$$

Assuming, of course, that:

$$S_y \sin\gamma - S_z \cos\gamma \neq 0$$

If both equations in (E17) are squared and are substituted in the following:

$$a_{23}^2 + a_{33}^2 = 1 - a_{13}^2 \qquad \text{(E18)}$$

a quadratic equation of a_{13} is obtained. That is,

$$E\, a_{13}^2 + F\, a_{13} + G = 0 \tag{E19}$$

where

$$E = S_x^2 + (S_y \sin\gamma - S_z \cos\gamma)^2$$

$$F = 2\, S_x\, (\cos\alpha \cos\beta + C_{23}\, (S_z \sin\gamma + S_y \cos\gamma))$$

and

$$G = -\,(S_y \sin\gamma - S_z \cos\gamma)^2 + 2\, C_{23} \cos\alpha \cos\beta\, (S_z \sin\gamma + S_y \cos\gamma)$$

$$+ \cos^2\alpha \cos^2\beta + C_{23}^2\, (S_y^2 + S_z^2)$$

Solutions of the resulting equation for a_{13} are then substituted in equation (E17) to obtain a_{23} and a_{33}.

In case when

$$\gamma \neq 0$$

$$S_y \sin\gamma - S_z \cos\gamma = 0$$

that is, when the sun is in the $\vec{x}$, $\vec{Y}_w$ plane, then the above expression (E17) can not be used. Since

$$\frac{S_y}{\cos\gamma} = \frac{S_z}{\sin\gamma} = \frac{S_y\, a_{23} + S_z\, a_{33}}{a_{23} \cos\gamma + a_{33} \sin\gamma}$$

or

$$\frac{S_y}{\cos\gamma} = \frac{-\cos\alpha \cos\beta - S_x\, a_{13}}{C_{23}} \tag{E20}$$

Consequently the following equations apply

$$a_{13} = \frac{-\cos\alpha\cos\beta\cos\gamma - S_y C_{23}}{S_x \cos\gamma}$$

$$\left.\begin{array}{l} a_{23} = C_{23}\cos\gamma \pm \tan\gamma \sqrt{(1 - C_{23}^2) S_x^2 \cos^2\gamma - (\cos\alpha\cos\beta\cos\gamma + S_y C_{23})^2} \Big/ S_x \\ a_{33} = C_{23}\sin\gamma \mp \sqrt{(1 - C_{23}^2) S_x^2 \cos^2\gamma - (\cos\alpha\cos\beta\cos\gamma + S_y C_{23})^2} \Big/ S_x \end{array}\right\} \quad \text{(E21)}$$

which apply except where $S_x = 0$ that is when the sun lies on the sensor cone axis. Note that if θ_s can not be determined, then in general two solutions are obtained, from which selection of the correct solution can not be made without additional information.

The Attitude Determination Equations

At this point the direction cosines of the local vertical and the sun line with respect to the body axes system are known. They can be utilized to determine the spacecraft's attitude.

Suppose that the transformation matrix taking the orbital axes system into the body axes system is:

$$\begin{aligned} \vec{x} &= A_{11}\vec{X} + A_{12}\vec{Y} + A_{13}\vec{Z} \\ \vec{y} &= A_{21}\vec{X} + A_{22}\vec{Y} + A_{23}\vec{Z} \\ \vec{z} &= A_{31}\vec{X} + A_{32}\vec{Y} + A_{33}\vec{Z} \end{aligned} \quad \text{(A1)}$$

The coefficient A_{ij} (i, j=1, 2, 3) in equation (A1) defines a matrix

$$[A] = \begin{bmatrix} A_{13} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \qquad \text{(A2)}$$

Equation (A1) implies that any vector V in the X, Y, Z system, written in column form, may be transformed into x, y, z system by the application of the matrix [A]. That is

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = [A] \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad \text{(A3)}$$

where

$$[V]_b = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \quad \text{and} \quad [V]_0 = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

are the components of $\vec{V}$ in the body axes system and in the orbital axes system respectively.

In order to determine the direction cosine matrix [A] it is assumed that $[S]_0$ and $[Z]_0$ are known and that $[S]_b$ and $[Z]_b$ have been determined. By defining

$$\vec{F} = \vec{Z} \times \vec{S} \qquad \text{(A4)}$$

the column matrix $[F]_0$ and $[F]_b$ can be obtained

$$[F]_0 = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} -S_2 \\ S_1 \\ 0 \end{bmatrix} \tag{A5}$$

$$[F]_b = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} S_z a_{23} - S_y a_{33} \\ S_x a_{33} - S_z a_{13} \\ S_y a_{13} - S_x a_{23} \end{bmatrix} \tag{A6}$$

If equation (A3) is applied to the vectors $\vec{Z}, \vec{S}$ and $\vec{F}$ respectively, it produces

$$\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} = [A] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{A7}$$

$$\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = [A] \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} \tag{A8}$$

and

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = [A] \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \tag{A9}$$

Combining these three column matrix equations into a single square matrix equation, yields

$$\begin{bmatrix} a_{13} & S_x & F_x \\ a_{23} & S_y & F_y \\ a_{33} & S_z & F_z \end{bmatrix} = [A] \begin{bmatrix} 0 & S_1 & F_1 \\ 0 & S_2 & F_2 \\ 1 & S_3 & F_3 \end{bmatrix} \tag{A10}$$

It may be written in this simple notation

$$[B] = [A]\ [Q] \tag{A11}$$

Where

$$[Q] = \begin{bmatrix} 0 & S_1 & F_1 \\ 0 & S_2 & F_2 \\ 1 & S_3 & F_3 \end{bmatrix} = \begin{bmatrix} 0 & S_1 & -S_2 \\ 0 & S_2 & S_1 \\ 1 & S_3 & 0 \end{bmatrix} \tag{A12}$$

and

$$[B] = \begin{bmatrix} a_{13} & S_x & F_x \\ a_{23} & S_y & F_y \\ a_{33} & S_z & F_z \end{bmatrix} = \begin{bmatrix} a_{33} & S_x & S_z\, a_{23} - S_y\, a_{33} \\ a_{13} & S_y & S_x\, a_{33} - S_z\, a_{13} \\ a_{23} & S_z & S_y\, a_{13} - S_x\, a_{23} \end{bmatrix} \tag{A13}$$

The determinant of $[Q]$ is

$$\begin{aligned} \det\ [Q] = |Q| &= S_1^2 + S_2^2 \\ &= 1 - S_3^2 \end{aligned} \tag{A14}$$

If $S_3 \neq \pm 1$, then

$$|Q| \neq 0$$

The matrix $[Q]$ is non-singular and possesses an inverse $[Q]^{-1}$ where

$$[Q]^{-1} = \frac{1}{1 - S_3^2} \begin{bmatrix} -S_1 S_3 & -S_2 S_3 & 1 - S_3^2 \\ S_1 & S_2 & 0 \\ -S_2 & S_1 & 0 \end{bmatrix} \quad \text{(A15)}$$

By multiplying the matrix equation (A12) with $[Q]^{-1}$, gives

$$[B]\ [Q]^{-1} = [A]\ [Q]\ [Q]^{-1}$$

$$[A] = [B]\ [Q]^{-1} \quad \text{(A16)}$$

Therefore

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \frac{1}{1 - S_3^2} \begin{bmatrix} S_1 (S_x - a_{13} S_3) - S_2 F_x & S_2 (S_x - a_{13} S_3) + S_1 F_x & a_{13} (1 - S_3^2) \\ S_1 (S_y - a_{23} S_3) - S_2 F_y & S_2 (S_y - a_{23} S_3) + S_1 F_y & a_{23} (1 - S_3^2) \\ S_1 (S_z - a_{33} S_3) - S_2 F_z & S_2 (S_z - a_{33} S_3) + S_1 F_z & a_{33} (1 - S_3^2) \end{bmatrix} \quad \text{(A17)}$$

This directly indicates:

$$A_{11} = \frac{1}{1 - S_3^2} (S_1 (S_x - a_{13} S_3) - S_2 (S_z a_{23} - S_y a_{33})) \quad \text{(A18)}$$

$$A_{12} = \frac{1}{1 - S_3^2} (S_2 (S_x - a_{13} S_3) + S_1 (S_z a_{23} - S_y a_{33})) \quad \text{(A19)}$$

$$A_{13} = a_{13} \quad \text{(A20)}$$

$$A_{21} = \frac{1}{1 - S_3^2} (S_1 (S_y - a_{23} S_3) - S_2 (S_x a_{33} - S_z a_{13})) \quad \text{(A21)}$$

$$A_{22} = \frac{1}{1 - S_3^2} (S_2 (S_y - a_{23} S_3) + S_1 (S_x a_{33} - S_z a_{13})) \quad \text{(A22)}$$

$$A_{23} = a_{23} \quad \text{(A23)}$$

$$A_{31} = \frac{1}{1 - S_3^2} (S_1 (S_z - a_{33} S_3) - S_2 (S_y a_{13} - S_x a_{23})) \tag{A24}$$

$$A_{32} = \frac{1}{1 - S_3^2} (S_2 (S_z - a_{33} S_3) + S_1 (S_y a_{13} - S_x a_{23})) \tag{A25}$$

$$A_{33} = a_{33} \tag{A26}$$

Euler Angle Transformations

Any attitude errors of the body axes relative to the desired orientation (that is, any deviation of the x, y, z, axes from the X, Y, Z axes) can be represented by Euler angle rotations: yaw angle ψ around the Z axis, roll angle ϕ about the displaced X axis, and pitch angle θ about the doubly displaced Y axis (see Figure 8). The notation to be used for the matrix which performs each of these three Euler rotations is

$$R_z(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

Given any vector V with components V_1, V_2, V_3, in the Orbital axes system, its components v_x, v_y, v_z in the body axes system are

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = R_y(\theta)\, R_x(\phi)\, R_z(\psi) \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\psi - \sin\theta\sin\phi\sin\psi & \cos\theta\sin\psi + \sin\theta\sin\phi\cos\psi & -\sin\theta\cos\phi \\ -\cos\phi\sin\psi & \cos\phi\cos\psi & \sin\phi \\ \sin\theta\cos\psi - \cos\theta\sin\phi\sin\psi & \sin\theta\sin\psi - \cos\theta\sin\phi\cos\psi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Comparing this equation with matrix equation (A3) gives

$$[A] = \begin{bmatrix} \cos\theta\cos\psi - \sin\theta\sin\phi\sin\psi & \cos\theta\sin\psi + \sin\theta\sin\phi\cos\psi & -\sin\theta\cos\phi \\ -\cos\phi\sin\psi & \cos\phi\cos\psi & \sin\phi \\ \sin\theta\cos\psi - \cos\theta\sin\phi\sin\psi & \sin\theta\sin\psi - \cos\theta\sin\phi\cos\psi & \cos\theta\cos\phi \end{bmatrix}$$

The roll angle ϕ, pitch angle θ, and yaw angle ψ can thus be derived:

$$a_{13} = -\sin\theta\cos\phi$$

$$a_{23} = \sin\phi$$

$$a_{33} = \cos\theta\cos\phi$$

and

$$A_{21} = -\sin\psi\cos\phi$$

Thus

$$\phi = \sin^{-1}(a_{23})$$

$$\theta = \tan^{-1}\left(-\frac{a_{13}}{a_{33}}\right)$$

If the angle of ϕ is limited to $-90 \leq \phi \leq 90$, then $\cos\phi$ is always positive and

$$\sin\psi = -\frac{A_{21}}{\cos\phi} = -\frac{A_{21}}{\sqrt{1 - A_{23}^2}}$$

Therefore

$$\psi = \sin^{-1} \; -\frac{A_{21}}{\sqrt{1 - A_{23}^2}}$$

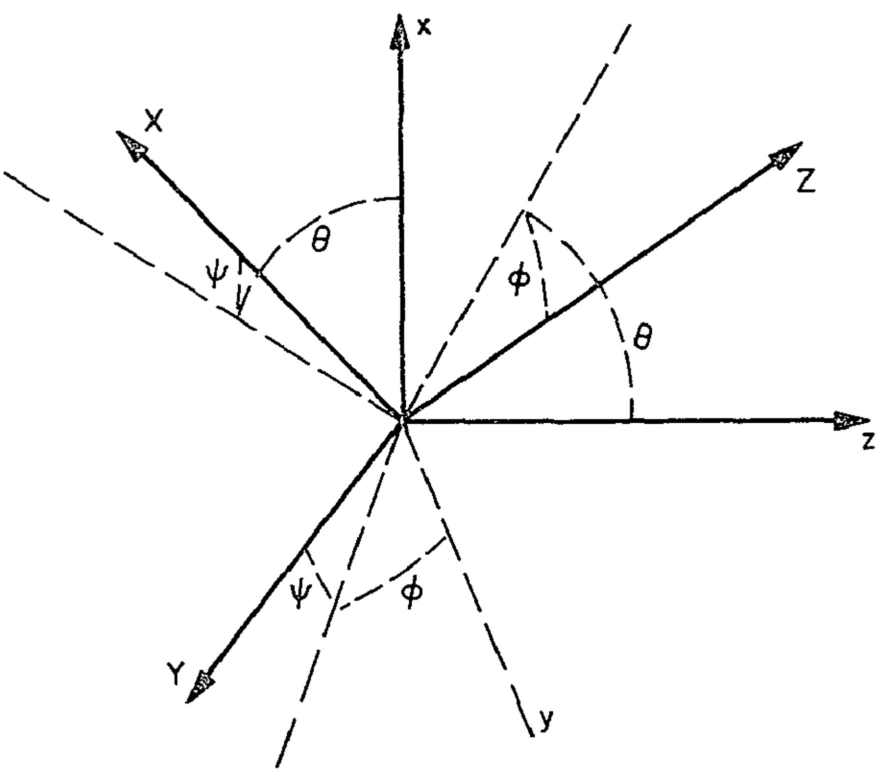


Figure 8. Attitude Error Representation Roll ϕ Pitch θ, Yaw ψ

Presentation of Special Cases

Several cases are presented in this report to illustrate the conditions in which the earth sensor or the sun sensor or both fails to provide the sufficient information. If any of these cases does occur, it is convenient to use a graphical representation of the curves related a_{13}, a_{23} and a_{33} on a unity sphere which is defined as a sphere in a_{13}, a_{23}, a_{33} space of unit radius and fixed to the spacecraft axis system with center at the origin. Since

$$a_{13}^2 + a_{23}^2 + a_{33}^2 = 1$$

or

$$a_{33}^2 = 1 - a_{13}^2 - a_{23}^2 \tag{D1}$$

the local vertical certainly lies on the surface of this sphere at any moment and provides a convenient means for visualizing the motion of the local vertical with respect to body axes. It is useful to indicate the geometric region for displaying the local vertical on such a unity sphere and project it on the a_{13}, a_{23} plane for positive and negative a_{33}.

(A) Case 1

First, a worst case is considered if the spacecraft is in the earth's shadow and if the earth sensor doesn't cut the earth. Evidently, the local vertical and the sun-line cannot be determined. Since the earth sensor does not see the earth, it can be concluded that the local vertical must lie within one of two possible regions in which body axes vary as the gimbal angle γ changes. If the equation (E8) and the equation (D1) are combined to eliminate a_{33}, after some algebraic rearrangement, the result gives:

$$\left(\frac{a_{23} - C_{23} \cos \gamma}{\sqrt{1 - C_{23}^2} \sin \gamma}\right)^2 + \left(\frac{a_{13}}{\sqrt{1 - C_{23}^2}}\right)^2 = 1 \qquad \text{(D2)}$$

for

$$\gamma \neq 0$$

Equation (D2) represents an ellipse within the a_{13} versus a_{23} plane unity circles with center at $E(C_{23} \cos \gamma, 0)$, with major axis $1/2 \sqrt{1 - C_{23}^2}$ and minor axis $1/2 \sqrt{1 - C_{23}^2} \sin \gamma$. There are two possible sets of curves:

(A_a) When the scan cone does not cut earth, $\rho = 0$ and $C_{23} < \cos(\alpha_e + \sigma)$ then

$$\left(\frac{a_{23} - C_{23} \cos \gamma}{\sin(\alpha_e + \sigma) \sin \gamma}\right)^2 + \left(\frac{a_{13}}{\sin(\alpha_e + \sigma)}\right)^2 < 1 \qquad \text{(D3)}$$

(A_b) When the entire scan cone intersects the earth, i.e., $\rho = 180$ and $C_{23} > \cos(\alpha_e - \sigma)$, then:

$$\left(\frac{a_{23} - C_{23} \cos \gamma}{\sin(\alpha_e - \sigma) \sin \gamma}\right)^2 + \left(\frac{a_{13}}{\sin(\alpha_e - \sigma)}\right)^2 > 1 \qquad \text{(D4)}$$

The shaded areas in Figure 9 and 10 represent respectively the required projections of region on the $a_{23} - a_{13}$ plane indicated by equation (D3) and equation (D4).

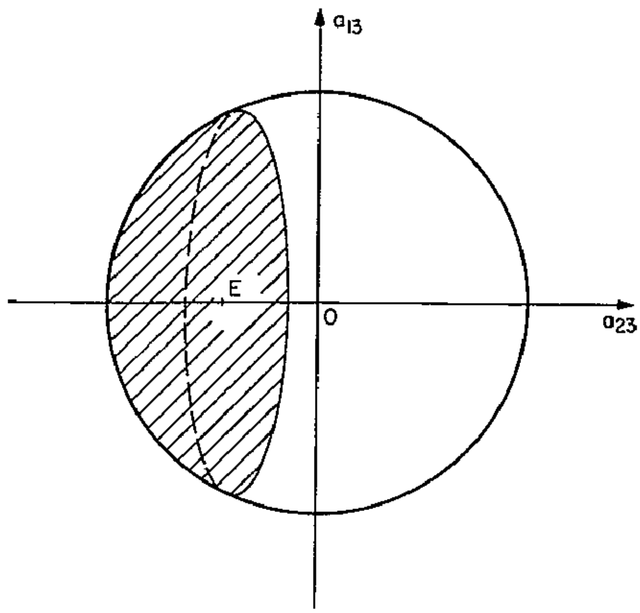


Figure 9. Projection of Error Region in $a_{23} - a_{13}$ plane[1] when $\rho = 0$, $\gamma \neq 0$

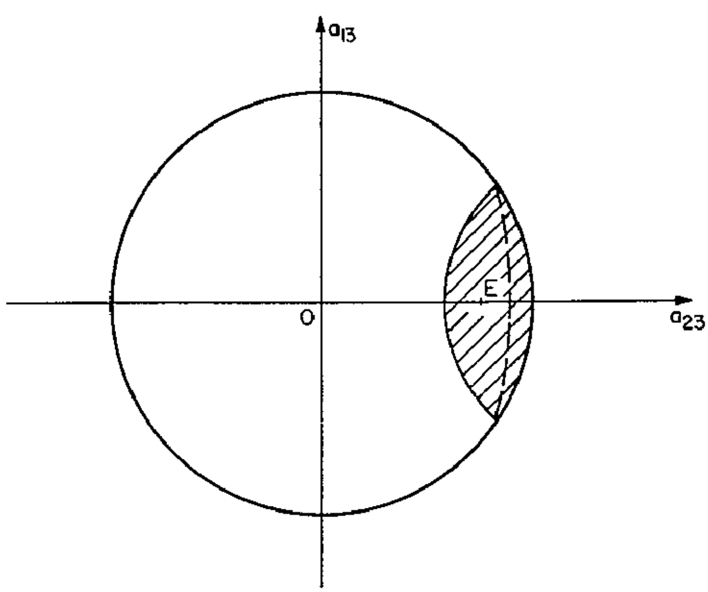


Figure 10. Projection of Error Region in $a_{23} - a_{23}$ plane when $\rho = 180$, $\gamma \neq 0$

If $\gamma = 0$, equation (E8) becomes:

$$a_{23} = C_{23} \tag{D5}$$

Therefore, the ellipse represented by equation (D2) degenerates into a straight line, the location of the line depends on the value of $\rho = 0$, or $\rho = 180$, when

$$\rho = 0$$

then

$$a_{23} < \cos(\alpha_e + \sigma) \tag{D6}$$

when

$$\rho = 180$$

then

$$a_{23} > \cos(\alpha_e - \sigma) \tag{D7}$$

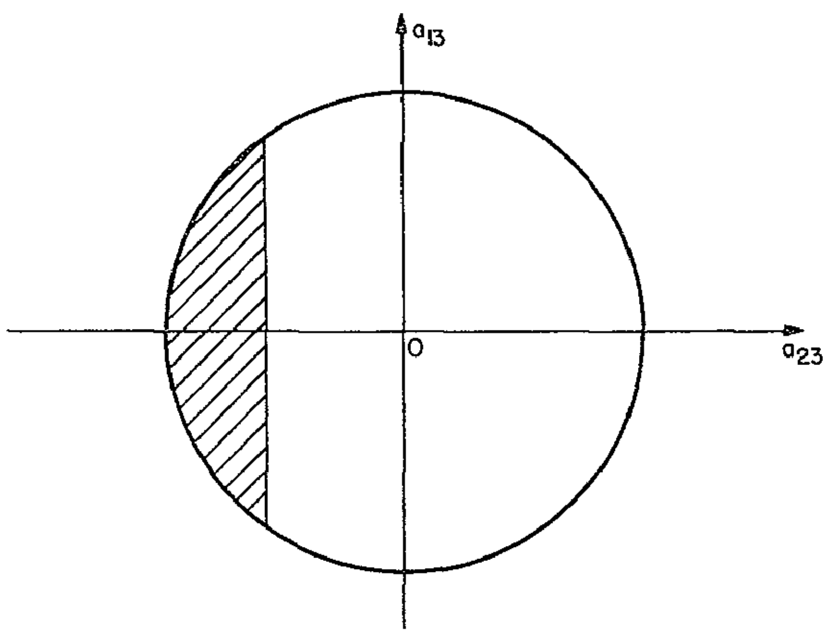


Figure 11 Error Region in $a_{23} - a_{13}$ plane when $\rho = 0$, $\gamma = 0$

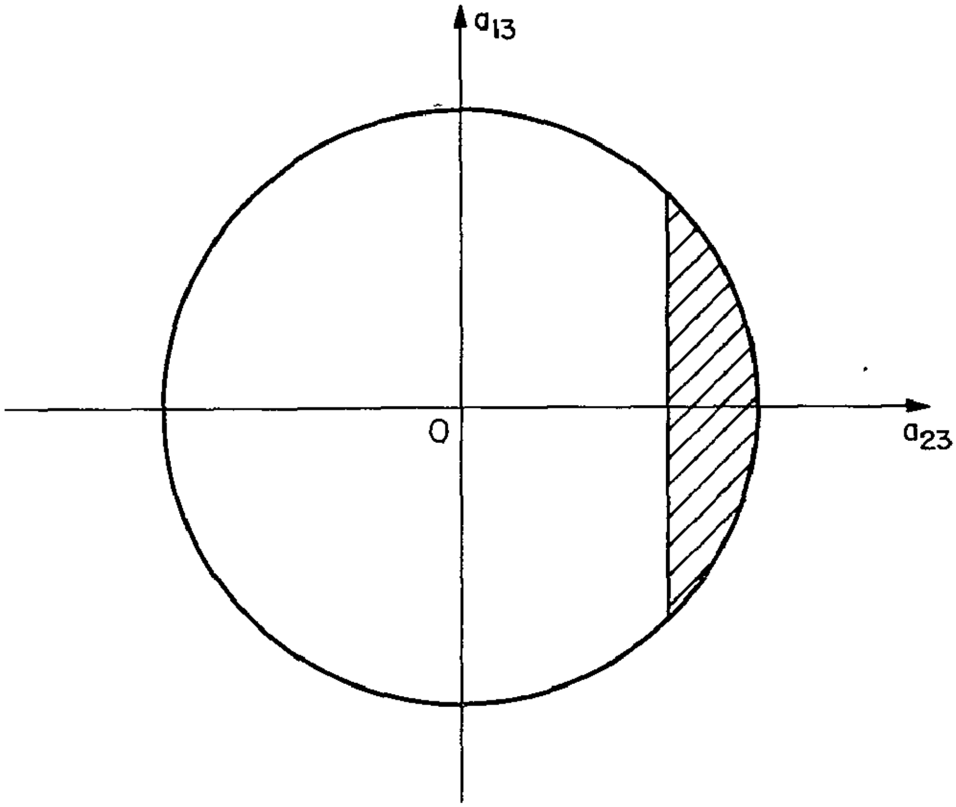


Figure 12. Error Region in $a_{23} - a_{13}$ plane when $\rho = 180$, $\gamma = 0$

The above two cases are illustrated in Figure 11 and Figure 12, respectively.

(B) Case 2.

Second, consider the case when the sun sensors do receive the sunlight, but the half earth pulse provided by the earth sensor shows its value of 0 or 180 degrees. In addition to the knowledge that the local vertical lies within specified regions as noted previously, it is also known that it must lie somewhere on a specified curve. From the information given by the sun sensor, equation (D1) and equation (E16) are useful for specifying this curve. Rewrite equation (E16) gives:

$$S_z a_{33} = S_3 - S_x a_{13} - S_y a_{23} \qquad \text{(D7)}$$

If $S_z = 0$, equation (D7) represents a straight line in a_{23} -a_{13} plane.

If $S_z \neq 0$, combining equation (D1) and equation (D7) to remove the a_{33} term, yields:

$$(S_x^2 + S_z^2)\, a_{13}^2 + 2\, S_x\, S_y\, a_{13}\, a_{23} + (S_y^2 + S_z^2)\, a_{23}^2 - 2\, S_3\, (S_x\, a_{13} + S_y\, a_{23}) = S_z^2 - S_3^2 \qquad \text{(D8)}$$

This is the general form of an ellipse with the center at C ($S_3 S_y$, $S_3 S_x$). If the origin is translated to the center of ellipse, the two first degree terms of equation (D8) can be removed. Then rotate the axes through an angle λ ($\lambda \leq 90$) where

$$\tan 2\lambda = \frac{2\, S_x\, S_y}{S_y^2 - S_x^2}$$

Equation (D8) is thus transformed into a standard equation of an ellipse having the foci on the new a_{13} axis with semi-axes $\sqrt{1 - S_3^2}$, and $S_z\sqrt{1 - S_3^2}$.

When either S_x or S_y is zero, it is obvious that no rotation of the axes is necessary.

When both S_x and S_y are zero, Equation (D8) represents a circle of radius $\sqrt{1 - S_3^2}$, a point, if $S_3 = \pm 1$.

Within the $a_{13} - a_{13}$ plane, the projection of the curves defined by equation (D7) and (D1) to represent the direction cosines of the body axes with respect to the local vertical also must satisfy the condition mentioned in section (A) Case 1.

Hence the portion of the projection presented graphically on the $a_{23} - a_{13}$ plane within the region described in the previous section (A) according to $\rho = 0$ or $\rho = 180$ is all that can be obtained to meet the required conditions.

It is helpful to generate another view of the sphere and plot the locus of the orbital Y axis (A_{12}, A_{22}, A_{32}). Obviously, it is a curve which lies on the surface of the A_{12} (i = 1, 2, 3) direction unity sphere since

$$A_{12}^2 + A_{22} + A_{32}^2 = 1 \tag{D9}$$

The projection of this curve on the $A_{32} - A_{12}$ plane is given for both $A_{22} > 0$ and $A_{22} < 0$.

Using equation (T3) to form the scalar product $\vec{S} \cdot \vec{Y}$, yields:

$$\vec{S} \cdot \vec{Y} = \sin\beta \tag{D10}$$

For convenience, the above equation is therefore written as:

$$S_x A_{12} + S_y A_{22} + S_z A_{32} = \sin\beta \tag{D11}$$

If $S_y = 0$, then equation (D11) defines a straight line on the $A_{32} - A_{12}$ plane assuming $S_y \neq 0$ and solving the equation (D11) for A_{22}, gives:

$$A_{22} = \frac{\sin\beta - S_x A_{12} - S_z A_{32}}{S_y} \tag{D12}$$

From equation (D9) and equation (D12) we have

$$\begin{aligned}(S_y^2 + S_z^2) A_{32}^2 + 2 S_x S_z A_{32} A_{12} + (S_x^2 + S_y^2) A_{12}^2 - 2 S_z \sin\beta A_{32} \\ - 2 S_x \sin\beta A_{12} = S_y^2 - \sin^2\beta\end{aligned} \tag{D13}$$

It can be seen easily that the above equation represents a circle of $S_y = 1$. However, if S_y is neither equal to 0 nor equal to 1, (D13) is the general equation of an ellipse of which the axes of symmetry are:

$$a_{32} = S_z \sin \beta$$

$$a_{12} = S_x \sin \beta$$

and the center of symmetry is ($S_z \sin \beta$, $S_x \sin \beta$). Like equation (D8) by a proper translation of the axes to remove the a_{32} term and the a_{12} term and then by a rotation of the axes with a chosen angle λ' for the removal of the $a_{32} a_{12}$ term where

$$\tan 2\lambda' = \frac{2 S_x S_z}{S_z^2 - S_x^2}$$

The equation (D13) is reduced to a standard form with the foci on the new a_{12} axis and with semi axes $\cos\beta$ and $S_y \cos \beta$.

From the above discussion, it is believed that the graphical techniques are useful to solve the algebraic system of relationships. This approach certainly limits the region from which the error of the attitudes of Delta PAC can be pictured. Such graphical representation can be plotted easily if a computer is used. A FORTRAN program subroutine plot is written for this purpose (see Appendix).

(C) Case 3

Third, for the case when the earth sensor does intersect the earth but the spacecraft enters into 'the earth's shadow, the informations from the earth sensor will indicate one of the following possibilities

(A) $0 < \rho < 180°$ and e is not saturated

This case appears as if there were no sun sensors aboard in the spacecraft at that moment. The given informations can only indicate the possible locations of the local vertical which are represented by four sets of equations as shown in equations (E15). No selection can be made from these four sets of equations to represent the true local vertical because the information about the sun line is lacking. However if the locations of the local vertical determined at previous sequences of time during the same orbit are examined, it is always possible to make the proper selection.

(B) $0 < \rho < 180°$ and e is saturated

This one is similar to case 1 discussed previously. As before the information given by the earth sensor can only be used to obtain equation (D2) for $\gamma \neq 0$ and equation (D5) for $\gamma = 0$. The only difference is that the variable C_{23} in this case has a definite value instead of having limited range. Therefore if the curve which relates a_{13}, a_{23} and a_{33} on the surface of the unit sphere is projected on the $a_{23} - a_{13}$ plane, the projection expresses exactly the relationship between a_{23} and a_{13}. Clearly the projection is either an ellipse represented by equation (D2) for $\gamma \neq 0$ or a straight line given by equation (D5) for $\gamma = 0$.

Error Propagation

It is a known fact that measurements are always accompanied by errors. However small errors are more frequent than large ones since the goal of the sensor design is to keep errors as small as possible. Therefore it is reasonable to assume that the measured quantities which indirectly determine the PAC attitude contain errors but small ones. Because of these errors, the predicted attitudes might not be the true values but the difference between them is expected to be small also.

For Delta PAC there are five quantities being measured by using the sun sensors and the earth sensor, namely: the angles α_i and β_i which determine the direction cosines of the sun line with respect to the i th sensor system, the gimbal angle γ, the half earth pulse ρ, and the telemetered pitch error e. The corresponding errors are defined by the symbols $\Delta\alpha_i$, $\Delta\beta_i$, $\Delta\gamma$, $\Delta\rho$ and Δe which are assumed to have the standard deviation σ_{α_i}, σ_{β_i}, σ_γ, σ_ρ and σ_e respectively. All measurement errors due to unknown cause are of random type, and are independent to each other. They are assumed to follow the laws of probability as given by the normal distribution.

It should be noted that the two angles α and β defined in the sun sensor section are not measured quantities. At any time their values will be determined by orbit calculation and are assumed to contain no errors.

In this report the methods of probability are applied to analyze the effect of measurement errors. It would be of interest to present the results in terms of error ellipses for a given attitude, the error ellipses are ellipses of equal probability which are similarly placed and are centered at the predicted position (h, k). They are determined by the formula

$$\frac{1}{2(1-r^2)}\left\{\left(\frac{u-h}{\sigma_u}\right)^2 - 2r\left(\frac{u-h}{\sigma_u}\right)\left(\frac{w-k}{\sigma_w}\right) + \left(\frac{w-k}{\sigma_w}\right)^2\right\} = L = \text{const} \qquad \text{(R1)}$$

for $0 < r < 1$ where σ_u, σ_w and r are respectively standard deviation of two variables u and w and their correlation coefficients. If u and w are not correlated variables then the value of r is equal to 0 and equation (R1) becomes

$$\left(\frac{u-h}{\sigma_u}\right)^2 + \left(\frac{w-k}{\sigma_w}\right)^2 = 2L \qquad \text{(R2)}$$

For each ellipse there is a definite probability that a true position (u, w) will lie within the ellipse. The probability for (u, w) to belong to the ellipse L given by either equation (R1) or (R2) is

$$P = 1 - e^{-L} \qquad \text{(R3)}$$

and with L = 1/2, 4/2, and 9/2 the probabilities are 0.4, 0.8, and 0.99 respectively. Of course, formula (R3) can be inverted to find the value of L corresponding to a given value of P.

It is essential that the determination of error ellipses for two variables u and w depend upon their predicted values of h and k and their standard deviations of σ_u and σ_w if r = 0. Therefore in considering the error ellipses of a_{23} and a_{13} and also of a_{32} and a_{12} for delta PAC, the former can be defined either at $0 < \rho < 180^\circ$ and $|e| < e_s$ without sun sensor information, or at $0 < \rho < 180^\circ$ and $|e| = e_s$ in conjunction with the sun's direction; the later can only be presented when the local vertical and the sun line are known but not parallel. The calculation of prediction errors for PAC attitude is thus limited to the case when $0 < \rho < 180$.

If the attitude control system of Delta PAC works well, the values of variables a_{23} and a_{13} are expected to be much less than 1. That is, the projection of the true local vertical in $a_{23} - a_{13}$ plane should stay well within $a_{23} - a_{13}$ unity circle. For this reason, the correlation coefficient of a_{23} and a_{13} can be assumed to have the value of 0, and equation (R2) is used to obtain error ellipse for a_{23} and a_{13}.

Similarly, equation (R2) can be applied to obtain the error ellipse for a_{32} and a_{12}. For Delta PAC each of these ellipses are centered at the prediction point with the probability of 0.99 that a true point falls within it. The lengths of the semi-axes are equal to $3(\sigma a_{13})$, i = 2, 1 for the $a_{23} - a_{13}$ ellipse and $3(\sigma a_{j2})$, (j = 3, 1) for the $a_{32} - a_{12}$ ellipse where $\sigma a_{13}, \sigma a_{23}, \sigma a_{32}$, and σa_{12} are the standard deviations of a_{13}, a_{23}, a_{32} and a_{12}, respectively.

If equations (S9) are differentiated with respect to α_1 and β_1 the following equations are obtained:

$$
\left.
\begin{aligned}
\frac{\partial s_{1i}}{\partial \alpha_i} &= - s_{1i}^2 \, s_{3i} \sec^2 \alpha_i \\
\frac{\partial s_{1i}}{\partial \beta_i} &= - s_{1i}^2 \, s_{2i} \sec^2 \beta_i \\
\frac{\partial s_{2i}}{\partial \alpha_i} &= - s_{1i} \, s_{2i} \, s_{3i} \sec^2 \alpha_i \\
\frac{\partial s_{2i}}{\partial \beta_i} &= \quad (1 - s_{2i}^2) \, s_{1i} \sec^2 \beta_i \\
\frac{\partial s_{3i}}{\partial \alpha_i} &= \quad (1 - s_{3i}^2) \, s_{1i} \sec^2 \alpha_i \\
\frac{\partial s_{3i}}{\partial \beta_i} &= - s_{1i} \, s_{2i} \, s_{3i} \sec^2 \beta_i
\end{aligned}
\right\} \qquad \text{(R4)}
$$

By differentiating equations (S10) with respect to s_{1i} , s_{2i} and s_{3i} respectively, yield

$$
\left.
\begin{aligned}
\frac{\partial S_x}{\partial s_{1i}} &= - \cos \eta_i \sin \xi_i \\
\frac{\partial S_x}{\partial s_{2i}} &= \quad \sin \eta_i \sin \xi_i \\
\frac{\partial S_x}{\partial s_{3i}} &= - \cos \xi_i \\
\frac{\partial S_y}{\partial s_{1i}} &= \quad \cos \eta_i \cos \xi_i \\
\frac{\partial S_y}{\partial s_{2i}} &= - \sin \eta_i \cos \xi_i \\
\frac{\partial S_y}{\partial s_{3i}} &= - \sin \xi_i \\
\frac{\partial S_z}{\partial s_{1i}} &= - \sin \eta_i \\
\frac{\partial S_z}{\partial s_{2i}} &= - \cos \eta_i \\
\frac{\partial S_z}{\partial s_{3i}} &= \quad 0
\end{aligned}
\right\} \qquad \text{(R5)}
$$

Since S_x, S_y and S_z are functions of variables s_{1i} , s_{2i} and s_{3i} where each variable is a function of the independent variables α_i and β_i therefore:

$$\left.\begin{aligned}\frac{\partial S_x}{\partial \alpha_i} &= \frac{\partial S_x}{\partial s_{1i}}\frac{\partial s_{1i}}{\partial \alpha_i} + \frac{\partial S_x}{\partial s_{2i}}\frac{\partial s_{2i}}{\partial \alpha_i} + \frac{\partial S_x}{\partial s_{3i}}\frac{\partial S_{3i}}{\partial \alpha_i}\\ &= -\{S_x\, s_{3i} + \cos\xi_i\}\, s_{1i}\, \sec^2\alpha_i\\ \frac{\partial S_x}{\partial \beta_i} &= \frac{\partial S_x}{\partial s_{1i}}\frac{\partial s_{1i}}{\partial \beta_i} + \frac{\partial S_x}{\partial s_{2i}}\frac{\partial s_{2i}}{\partial \beta_i} + \frac{\partial S_x}{\partial s_{3i}}\frac{\partial s_{3i}}{\partial \beta_i}\\ &= -\{S_x\, s_{2i} - \sin\eta_i\, \sin\xi_i\}\, s_{1i}\, \sec^2\beta_i\end{aligned}\right\} \quad \text{(R6)}$$

A similar method is applied to calculate the derivatives of

$$\frac{\partial S_y}{\partial \alpha_i},\ \frac{\partial S_y}{\partial \beta_i},\ \frac{\partial S_z}{\partial \alpha_i} \text{ and } \frac{\partial S_z}{\partial \beta_i}$$

with the aid of equations (R4) and equations of (R5) the results are:

$$\frac{\partial S_y}{\partial \alpha_i} = -\{S_y\, s_{3i} + \sin\xi_i\}\, s_{1i}\, \sec^2\alpha_i \quad \text{(R7)}$$

$$\frac{\partial s_i}{\partial \beta_i} = -\{S_y\, s_{2i} + \sin\eta_i\, \cos\xi_i\}\, s_{1i}\, \sec^2\beta_i \quad \text{(R8)}$$

$$\frac{\partial S_z}{\partial \alpha_i} = -\ S_z\, s_{1i}\, s_{3i}\, \sec^2\alpha_i \quad \text{(R9)}$$

$$\frac{\partial S_z}{\partial \beta_i} = -\{S_z\, s_{2i} + \cos\eta_i\}\, s_{1i}\, \sec^2\beta_i \quad \text{(R10)}$$

Consider the equation (E6) where C_{23} is a function of ρ only. The error of C_{23} can be expressed in terms of measurement error of half earth pulse as

$$\Delta C_{23} = \frac{d\,C_{23}}{d\,\rho}\Delta\rho \quad \text{(R11)}$$

The standard deviation of C_{23} is given by

$$\sigma C_{23} = \left| \frac{d\, C_{23}}{d\, \rho} \right| \sigma \rho \tag{R12}$$

where $\sigma \rho$ is the standard deviation of ρ and

$$\frac{d\, C_{23}}{d\rho} = - \frac{(1 - C_{23}^2) \sin \sigma \sin \rho}{C_{23} \sin \sigma \cos \rho - \cos \sigma \sqrt{1 - C_{23}^2}} \tag{R13}$$

For Delta PAC, $\sigma = 45°$, thus, it can be shown that $C_{23} \cos \rho$ is less than $\sqrt{1 - C_{23}^2}$ whenever $0 < \rho < 180$. The differentiation of equation (E6) with respect to ρ in order to obtain equation (R12) is always possible.

As mentioned before, there are two different cases which satisfy the condition $0 < \rho < 180$, but require separate approaches. They are:

(A) $0 < \rho < 180^\circ$ and $|e| < e_s$

In this case, the local vertical is determined by the earth sensor equation and is not affected by the measurement errors from the sun sensors. Therefore,

$$\left. \begin{aligned} \frac{\partial a_{13}}{\partial \alpha_1} &= \frac{\partial a_{13}}{\partial \beta_1} = 0 \\ \frac{\partial a_{23}}{\partial \alpha_1} &= \frac{\partial a_{23}}{\partial \beta_1} = 0 \\ \frac{\partial a_{23}}{\partial \alpha_1} &= \frac{\partial a_{33}}{\partial \beta_1} = 0 \end{aligned} \right\} \tag{R14}$$

Evidently, the components of the local vertical depend only on the values of γ, ρ, and e as shown in equation (E15). The standard deviation of a_{13}, a_{23}, and a_{33} can be expressed in terms of σ_γ, σ_ρ and σ_e as follows

$$\left.\begin{aligned} \sigma^2_{a_{13}} &= \left(\frac{\partial a_{13}}{\partial \gamma}\right)^2 \sigma^2_\gamma + \left(\frac{\partial a_{13}}{\partial \rho}\right)^2 \sigma^2_\rho + \left(\frac{\partial a_{13}}{\partial e}\right)^2 \sigma^2_e \\ \sigma^2_{a_{23}} &= \left(\frac{\partial a_{23}}{\partial \gamma}\right)^2 \sigma^2_\gamma + \left(\frac{\partial a_{23}}{\partial \rho}\right)^2 \sigma^2_\rho + \left(\frac{\partial a_{23}}{\partial e}\right)^2 \sigma^2_e \\ \sigma^2_{a_{33}} &= \left(\frac{\partial a_{33}}{\partial \gamma}\right)^2 \sigma^2_\gamma + \left(\frac{\partial a_{33}}{\partial \rho}\right)^2 \sigma^2_\rho + \left(\frac{\partial a_{33}}{\sigma e}\right)^2 \sigma^2_e \end{aligned}\right\} \qquad \text{(R15)}$$

As noted at the end of the earth sensor equation section, there are four sets of equations, (see equation E13), representing the position of the local vertical. Only one set of these equations give the correct components of the local vertical and it is selected. The technique applied for the selection was mentioned before. The partial derivatives

$$\frac{\partial a_{i3}}{\partial \gamma}, \ \frac{\partial a_{i3}}{\partial \rho}, \ \text{and} \ \frac{\partial a_{i3}}{\partial e}, \ (i = 1, 2, 3)$$

which appear in this case should be calculated from the equations of that selected set.

Since the components of the sun line are independent from the errors of the earth sensor, this implies

$$\left.\begin{aligned}\frac{\partial S_x}{\partial \gamma} = \frac{\partial S_x}{\partial \rho} = \frac{\partial S_x}{\partial e} = 0 \\ \frac{\partial S_y}{\partial \gamma} = \frac{\partial S_y}{\partial \rho} = \frac{\partial S_y}{\partial e} = 0 \\ \frac{\partial S_z}{\partial \gamma} = \frac{\partial S_z}{\partial \rho} = \frac{\partial S_z}{\partial e} = 0\end{aligned}\right\} \quad \text{(R16)}$$

Moreover, since S_1, S_2, and S_3 depend only on the value of α and β which are assumed to contain no errors. It is obvious

$$\left.\begin{aligned}\frac{\partial S_1}{\partial \alpha_1} = \frac{\partial S_2}{\partial \alpha_1} = \frac{\partial S_3}{\partial \alpha_1} = 0 \\ \frac{\partial S_1}{\partial \beta_1} = \frac{\partial S_2}{\partial \beta_1} = \frac{\partial S_3}{\partial \beta_1} = 0 \\ \frac{\partial S_1}{\partial \gamma} = \frac{\partial S_2}{\partial \gamma} = \frac{\partial S_3}{\partial \gamma} = 0 \\ \frac{\partial S_1}{\partial \rho} = \frac{\partial S_2}{\partial \rho} = \frac{\partial S_3}{\partial \rho} = 0 \\ \frac{\partial S_1}{\partial e} = \frac{\partial S_2}{\partial e} = \frac{\partial S_3}{\partial e} = 0\end{aligned}\right\} \quad \text{(R17)}$$

By the use of above equations the partial derivatives of A_{12} and A_{32} with respect to α_1, β_1, γ, ρ and e are:

$$\left.\begin{aligned}
\frac{\partial A_{12}}{\partial \alpha_1} &= \frac{1}{1 - S_3^2}\left\{S_2\left(\frac{\partial S_x}{\partial \alpha_1}\right) + S_1\left(a_{23}\frac{\partial S_z}{\partial \alpha_1} - a_{33}\frac{\partial S_y}{\partial \alpha_1}\right)\right\} \\
\frac{\partial A_{12}}{\partial \beta_1} &= \frac{1}{1 - S_3^2}\left\{S_2\left(\frac{\partial S_x}{\partial \beta_1}\right) + S_1\left(a_{23}\frac{\partial S_z}{\partial \beta_1} - a_{33}\frac{\partial S_y}{\partial \beta_1}\right)\right\} \\
\frac{\partial A_{12}}{\partial \gamma} &= \frac{1}{1 - S_3^2}\left\{-(S_2)(S_3)\left(\frac{\partial a_{13}}{\partial \gamma}\right) + S_1\left(S_z\frac{\partial a_{23}}{\partial \gamma} - S_y\frac{\partial a_{33}}{\partial \gamma}\right)\right\} \\
\frac{\partial A_{12}}{\partial \rho} &= \frac{1}{1 - S_3^2}\left\{-(S_2)(S_3)\left(\frac{\partial a_{13}}{\partial \rho}\right) + S_1\left(S_z\frac{\partial a_{23}}{\partial \rho} - S_y\frac{\partial a_{33}}{\partial \rho}\right)\right\} \\
\frac{\partial A_{12}}{\partial e} &= \frac{1}{1 - S_3^2}\left\{-(S_2)(S_3)\left(\frac{\partial a_{13}}{\partial e}\right) + S_1\left(S_z\frac{\partial a_{23}}{\partial e} - S_y\frac{\partial a_{33}}{\partial e}\right)\right\}
\end{aligned}\right\} \quad \text{(R18)}$$

and

$$\begin{aligned}
\frac{\partial A_{32}}{\partial \alpha_1} &= \frac{1}{1 - S_3^1}\left\{S_2\left(\frac{\partial S_z}{\partial \alpha_1}\right) + S_1\left(a_{13}\frac{\partial S_y}{\partial \alpha_1} - a_{23}\frac{\partial S_x}{\partial \alpha_1}\right)\right\} \\
\frac{\partial A_{32}}{\partial \beta_1} &= \frac{1}{1 - S_3^2}\left\{S_2\left(\frac{\partial S_z}{\partial \beta_1}\right) + S_1\left(a_{13}\frac{\partial S_y}{\partial \beta_1} - a_{23}\frac{\partial S_x}{\partial \beta_1}\right)\right\} \\
\frac{\partial A_{32}}{\partial \gamma} &= \frac{1}{1 - S_3^2}\left\{-(S_2)\left(S_3\frac{\partial a_{33}}{\partial \gamma}\right) + S_1\left(S_y\frac{\partial a_{13}}{\partial \gamma} - S_x\frac{\partial a_{23}}{\partial \gamma}\right)\right\} \\
\frac{\partial A_{32}}{\partial \rho} &= \frac{1}{1 - S_3^2}\left\{-(S_2)\left(S_3\frac{\partial a_{33}}{\partial \rho}\right) + S_1\left(S_y\frac{\partial a_{13}}{\partial \rho} - S_x\frac{\partial a_{23}}{\partial \rho}\right)\right\} \\
\frac{\partial A_{32}}{\partial e} &= \frac{1}{1 - S_3^2}\left\{-(S_2)\left(S_3\frac{\partial a_{33}}{\partial e}\right) + S_1\left(S_y\frac{\partial a_{13}}{\partial e} - S_x\frac{\partial a_{23}}{\partial e}\right)\right\}
\end{aligned} \qquad \text{(R19)}$$

In terms of known partial derivatives the standard deviation of A_{12} and A_{32} can be obtained from the following

$$\begin{aligned}
\sigma_{A_{12}}^2 &= \left(\frac{\partial A_{12}}{\partial \alpha_1}\right)^2 \sigma_{\alpha_1}^2 + \left(\frac{\partial A_{12}}{\partial \beta_1}\right)^2 \sigma_{\beta_1}^2 + \left(\frac{\partial A_{12}}{\partial \gamma}\right)^2 \sigma_{\gamma}^2 + \left(\frac{\partial A_{12}}{\partial \rho}\right)^2 \sigma_{\rho}^2 + \left(\frac{\partial A_{12}}{\partial e}\right)^2 \sigma_{e}^2 \\
\sigma_{A_{32}}^2 &= \left(\frac{\partial A_{12}}{\partial \beta_1}\right)^2 \sigma_{\alpha_1}^2 + \left(\frac{\partial A_{32}}{\partial \beta_1}\right)^2 \sigma_{\beta_1}^2 + \left(\frac{\partial A_{32}}{\partial \gamma}\right)^2 \sigma_{\gamma}^2 + \left(\frac{\partial A_{32}}{\partial \rho}\right)^2 \sigma_{\rho}^2 + \left(\frac{\partial A_{32}}{\partial e}\right)^2 \sigma_{e}^2
\end{aligned} \qquad \text{(R20)}$$

(B) $0 < \rho < 180$ and $|e| = e_s$

As stated in the earth sensor equation section the components of the local vertical are determined by employing the following three equations for case in which $0 < \rho < 180$ and $|e| = e_s$

$$\left.\begin{aligned} &S_x a_{13} + S_y a_{23} + S_z a_{33} = -\cos\alpha\cos\beta \\ &\cos\gamma\, a_{23} + \sin\gamma\, a_{33} = C_{23} \\ &a_{13}^2 + a_{23}^2 + a_{33}^2 = 1 \end{aligned}\right\} \qquad \text{(R21)}$$

In view of the above equations, it is apparent that the variables a_{13}, a_{23}, and a_{33} are now dependent on the values of S_x, S_y, and S_z as well as γ and ρ. Differentiating the equations (R21) with respect to S_x, S_y, S_z, γ, and ρ respectively and making use of the equations (R16) in addition to

$$\left.\begin{aligned} &\frac{\partial\gamma}{\partial S_x} = \frac{\partial\gamma}{\partial S_y} = \frac{\partial\gamma}{\partial S_z} = 0 \\ &\frac{\partial C_{23}}{\partial S_x} = \frac{\partial C_{23}}{\partial S_y} = \frac{\partial C_{23}}{\partial S_z} = 0 \end{aligned}\right\} \qquad \text{(R22)}$$

give the following five sets of equations

$$\left.\begin{aligned} &S_x \frac{\partial a_{13}}{\partial S_x} + S_y \frac{\partial a_{23}}{\partial S_x} + S_z \frac{\partial a_{33}}{\partial S_x} = -a_{13} \\ &\cos\gamma \frac{\partial a_{23}}{\partial S_x} + \sin\gamma \frac{\partial a_{33}}{\partial S_x} = 0 \\ &a_{13} \frac{\partial a_{13}}{\partial S_x} + a_{23} \frac{\partial a_{23}}{\partial S_x} + a_{33} \frac{\partial a_{33}}{\partial S_x} = 0 \end{aligned}\right\} \qquad \text{(R23)}$$

$$\left.\begin{aligned} S_x \frac{\partial a_{13}}{\partial S_y} + S_y \frac{\partial a_{23}}{\partial S_y} + S_z \frac{\partial a_{33}}{\partial S_y} &= - a_{23} \\ \cos\gamma \frac{\partial a_{23}}{\partial S_y} + \sin\gamma \frac{\partial a_{33}}{\partial S_y} &= 0 \\ a_{13} \frac{\partial a_{13}}{\partial S_y} + a_{23} \frac{\partial a_{23}}{\partial S_y} + a_{33} \frac{\partial a_{33}}{\partial S_y} &= 0 \end{aligned}\right\} \qquad \text{(R24)}$$

$$\left.\begin{aligned} S_x \frac{\partial a_{13}}{\partial S_z} + S_y \frac{\partial a_{23}}{\partial S_z} + S_z \frac{\partial a_{33}}{\partial S_z} &= - a_{33} \\ \cos\gamma \frac{\partial a_{23}}{\partial S_z} + \sin\gamma \frac{\partial a_{33}}{\partial S_z} &= 0 \\ a_{13} \frac{\partial a_{13}}{\partial S_z} + a_{23} \frac{\partial a_{23}}{\partial S_z} + a_{33} \frac{\partial a_{33}}{\partial S_z} &= 0 \end{aligned}\right\} \qquad \text{(R25)}$$

$$\left.\begin{aligned} S_x \frac{\partial a_{13}}{\partial \gamma} + S_y \frac{\partial a_{23}}{\partial \gamma} + S_z \frac{\partial a_{33}}{\partial \gamma} &= 0 \\ \cos\gamma \frac{\partial a_{23}}{\partial \gamma} + \sin\gamma \frac{\partial a_{33}}{\partial \gamma} &= a_{23} \sin\gamma - a_{33} \cos\gamma \\ a_{13} \frac{\partial a_{13}}{\partial \gamma} + a_{23} \frac{\partial a_{23}}{\partial \gamma} + a_{33} \frac{\partial a_{33}}{\partial \gamma} &= 0 \end{aligned}\right\} \qquad \text{(R26)}$$

and

$$\left.\begin{aligned} S_x \frac{\partial a_{13}}{\partial \rho} + S_y \frac{\partial a_{23}}{\partial \rho} + S_z \frac{\partial a_{33}}{\partial \rho} &= 0 \\ \cos\gamma \frac{\partial a_{23}}{\partial \rho} + \sin\gamma \frac{\partial a_{33}}{\partial \rho} &= \frac{d C_{23}}{d \rho} \\ a_{13} \frac{\partial a_{13}}{\partial \rho} + a_{23} \frac{\partial a_{23}}{\partial \rho} + a_{33} \frac{\partial a_{33}}{\partial \rho} &= 0 \end{aligned}\right\} \qquad \text{(R27)}$$

where $dC_{23}/d\rho$ is given in equation (R13).

If

$$\Delta = \begin{vmatrix} S_x & S_y & S_z \\ 0 & \cos\gamma & \sin\gamma \\ a_{13} & a_{13} & a_{33} \end{vmatrix} \neq 0 \qquad \text{(R28)}$$

then each set of equations (R23), (R24), (R25), (R26), and (R27) can be solved to obtain the derivatives

$$\frac{\partial a_{i3}}{\partial x}, \frac{\partial a_{i3}}{\partial y}, \frac{\partial a_{i3}}{\partial z}, \frac{\partial a_{i3}}{\partial \gamma} \quad \text{and} \quad \frac{\partial a_{i3}}{\partial \rho}, (i = 1, 2, 3.)$$

Let

$$P = a_{23} \sin\gamma - a_{33} \cos\gamma$$

Then

$$\left.\begin{aligned} \frac{\partial a_{13}}{\partial S_x} &= \frac{a_{13} P}{\Delta} \\ \frac{\partial a_{23}}{\partial S_x} &= \frac{-a_{13}^2 \sin\gamma}{\Delta} \\ \frac{\partial a_{33}}{\partial S_x} &= \frac{a_{13}^2 \cos\gamma}{\Delta} \end{aligned}\right\} \qquad \text{(R29)}$$

$$\left.\begin{aligned} \frac{\partial a_{13}}{\partial S_y} &= \frac{a_{23} P}{\Delta} \\ \frac{\partial a_{23}}{\partial S_y} &= \frac{-a_{13} a_{23} \sin\gamma}{\Delta} \\ \frac{\partial a_{33}}{\partial S_y} &= \frac{a_{13} a_{23} \cos\gamma}{\Delta} \end{aligned}\right\} \qquad \text{(R30)}$$

$$\left.\begin{aligned}
\frac{\partial a_{13}}{\partial S_z} &= \frac{a_{33}\, P}{\Delta} \\
\frac{\partial a_{23}}{\partial S_z} &= \frac{-\, a_{13}\, a_{33} \sin \gamma}{\Delta} \\
\frac{\partial a_{33}}{\partial S_z} &= \frac{a_{13}\, a_{33} \cos \gamma}{\Delta}
\end{aligned}\right\} \qquad \text{(R31)}$$

$$\left.\begin{aligned}
\frac{\partial a_{13}}{\partial \gamma} &= (a_{23}\, S_z - a_{33}\, S_y)\, P/\Delta \\
\frac{\partial a_{23}}{\partial \gamma} &= (S_x\, a_{33} - S_z\, a_{13})\, P/\Delta \\
\frac{\partial a_{33}}{\partial \gamma} &= (S_y\, a_{13} - S_x\, a_{23})\, P/\Delta
\end{aligned}\right\} \qquad \text{(R32)}$$

and

$$\left.\begin{aligned}
\frac{\partial a_{13}}{\partial \rho} &= (S_z\, a_{23} - S_y\, a_{33})\, \frac{d\, C_{23}}{d\, \rho} \Big/ \Delta \\
\frac{\partial a_{23}}{\partial \rho} &= (S_x\, a_{33} - S_z\, a_{13})\, \frac{d\, C_{23}}{d\, \rho} \Big/ \Delta \\
\frac{\partial a_{33}}{\partial \rho} &= (S_y\, a_{13} - S_x\, a_{23})\, \frac{d\, C_{23}}{d\, \rho} \Big/ \Delta
\end{aligned}\right\} \qquad \text{(R33)}$$

With the use of the equations (R29) to (R33) and the equations (R6) to (R10), the following partial derivatives can be evaluated.

$$\begin{aligned}
\frac{\partial a_{13}}{\partial \alpha_1} &= \frac{\partial a_{13}}{\partial S_x}\, \frac{\partial S_x}{\partial \alpha_1} + \frac{\partial a_{13}}{\partial S_y}\, \frac{\partial S_y}{\partial \alpha_1} + \frac{\partial a_{13}}{\partial S_z}\, \frac{\partial S_z}{\partial \alpha_1} \\
&= -\,(S_3\, s_{31} + a_{13} \cos \xi_1 + a_{23} \sin \xi_1)\, (s_{11} \sec^2 \alpha_1)\, P/\Delta
\end{aligned} \qquad \text{(R34)}$$

$$\frac{\partial a_{13}}{\partial \beta_1} = \frac{\partial a_{13}}{\partial S_x}\frac{\partial S_x}{\partial \beta_1} + \frac{\partial a_{13}}{\partial S_y}\frac{\partial S_y}{\partial \beta_1} + \frac{\partial a_{13}}{\partial S_z}\frac{\partial S_z}{\partial \beta_1}$$

$$= (-S_3 s_{21} + a_{13}\sin\eta_1 \sin\xi_1 - a_{23}\sin\eta_1 \cos\xi_1 - a_{33}\cos\eta_1)(s_{21}\sec^2\beta_1 P)/\Delta \tag{R35}$$

$$\frac{\partial a_{23}}{\partial \alpha_1} = \frac{\partial a_{23}}{\partial S_x}\frac{\partial S_x}{\partial \alpha_1} + \frac{\partial a_{23}}{\partial S_y}\frac{\partial S_y}{\partial \alpha_1} + \frac{\partial a_{23}}{\partial S_z}\frac{\partial S_z}{\partial \alpha_1}$$

$$= (S_3 s_{31} + a_{13}\cos\xi_1 + a_{23}\sin\xi_1)(s_{11}\sec^2\alpha_1) a_{13}\sin\gamma/\Delta \tag{R36}$$

$$\frac{\partial a_{23}}{\partial \beta_1} = \frac{\partial a_{23}}{\partial S_x}\frac{\partial S_x}{\partial \beta_1} + \frac{\partial a_{23}}{\partial S_y}\frac{\partial S_y}{\partial \beta_1} + \frac{\partial a_{23}}{\partial S_z}\frac{\partial S_z}{\partial \beta_1}$$

$$= (S_3 s_{21} - a_{13}\sin\eta_1 \sin\xi_1 + a_{23}\sin\eta_1 \cos\xi_1 + a_{33}\cos\eta_1)(s_{11}\sec^2\beta_1 a_{13}\sin\gamma)/\Delta \tag{R37}$$

Hence

$$\left.\begin{aligned}
\sigma^2_{a_{13}} &= \left(\frac{\partial a_{13}}{\partial \alpha_1}\right)^2 \sigma^2_{\alpha_1} + \left(\frac{\partial a_{13}}{\partial \beta_1}\right)^2 \sigma^2_{\beta_1} + \left(\frac{\partial a_{13}}{\partial \gamma}\right)^2 \sigma^2_{\gamma} + \left(\frac{\partial a_{13}}{\partial \rho}\right)^2 \sigma^2_{\rho} \\
\sigma^2_{a_{23}} &= \left(\frac{\partial a_{23}}{\partial \alpha_1}\right)^2 \sigma^2_{\alpha_1} + \left(\frac{\partial a_{23}}{\partial \beta_1}\right)^2 \sigma^2_{\beta_1} + \left(\frac{\partial a_{23}}{\partial \gamma}\right)^2 \sigma^2_{\gamma} + \left(\frac{\partial a_{23}}{\partial \rho}\right)^2 \sigma^2_{\rho}
\end{aligned}\right\} \tag{R38}$$

Also the partial derivatives of A_{12} and A_{32} with respect to α_1, β_1, γ, and ρ can be derived from the équations (A19) and (A25) and then are simplified as follows

$$\left.\begin{aligned}
\frac{\partial A_{12}}{\partial \alpha_1} &= \frac{1}{1-S_3^2}\left(S_2\left(\frac{\partial S_x}{\partial \alpha_1} - S_3\,\frac{\partial a_{13}}{\partial \alpha_1}\right) + S_1\left(a_{23}\,\frac{\partial S_z}{\partial \alpha_1} + S_z\,\frac{\partial a_{23}}{\partial \alpha_1} - a_{33}\,\frac{\partial S_y}{\partial \alpha_1} - S_y\,\frac{\partial a_{33}}{\partial \alpha_1}\right)\right)\\
\frac{\partial A_{12}}{\partial \beta_1} &= \frac{1}{1-S_3^2}\left(S_2\left(\frac{\partial S_x}{\partial \beta_1} - S_3\,\frac{\partial a_{13}}{\partial \beta_1}\right) + S_1\left(a_{23}\,\frac{\partial S_z}{\partial \beta_1} + S_z\,\frac{\partial a_{23}}{\partial \beta_1} - a_{33}\,\frac{\partial S_y}{\partial \beta_1} - S_y\,\frac{\partial a_{33}}{\partial \beta_1}\right)\right)\\
\frac{\partial A_{12}}{\partial \gamma} &= \frac{1}{1-S_3^2}\left(-S_2 S_3\,\frac{\partial a_{13}}{\partial \gamma} + S_1\left(S_z\,\frac{\partial a_{23}}{\partial \gamma} - S_y\,\frac{\partial a_{33}}{\partial \gamma}\right)\right)\\
\frac{\partial A_{12}}{\partial \rho} &= \frac{1}{1-S_3^2}\left(-S_2 S_3\,\frac{\partial a_{13}}{\partial \rho} + S_1\left(S_z\,\frac{\partial a_{23}}{\partial \rho} - S_y\,\frac{\partial a_{33}}{\partial \rho}\right)\right)\\
&\text{and}\\
\frac{\partial A_{32}}{\partial \alpha_1} &= \frac{1}{1-S_3^2}\left(S_2\left(\frac{\partial S_z}{\partial \alpha_1} - S_3\,\frac{\partial a_{33}}{\partial \alpha_1}\right) + S_1\left(a_{13}\,\frac{\partial S_y}{\partial \alpha_1} + S_y\,\frac{\partial a_{13}}{\partial \alpha_1} - a_{23}\,\frac{\partial S_x}{\partial \alpha_1} - S_x\,\frac{\partial a_{23}}{\partial \alpha_1}\right)\right)\\
\frac{\partial A_{32}}{\partial \beta_1} &= \frac{1}{1-S_3^2}\left(S_2\,\frac{\partial S_z}{\partial \beta_1} - S_3\,\frac{\partial a_{33}}{\partial \beta_1}\right) + S_1\left(a_{13}\,\frac{\partial S_y}{\partial \beta_1} + S_y\,\frac{\partial a_{13}}{\partial \beta_1} - a_{23}\,\frac{\partial S_x}{\partial \beta_1} - S_x\,\frac{\partial a_{23}}{\partial \beta_1}\right)\Big)\\
\frac{\partial A_{32}}{\partial \gamma} &= \frac{1}{1-S_3^2}\left(-S_2 S_3\,\frac{\partial a_{33}}{\partial \gamma} + S_1\left(S_y\,\frac{\partial a_{13}}{\partial \gamma} - S_x\,\frac{\partial a_{23}}{\partial \gamma}\right)\right)\\
\frac{\partial A_{32}}{\partial \rho} &= \frac{1}{1-S_3^2}\left(-S_2 S_3\,\frac{\partial a_{33}}{\partial \rho} + S_1\left(S_y\,\frac{\partial a_{13}}{\partial \rho} - S_x\,\frac{\partial a_{23}}{\partial \rho}\right)\right)
\end{aligned}\right\} \qquad \text{(R39)}$$

In terms of the above partial derivatives, the standard deviation of a_{12} and a_{32} can thus be calculated. They are

$$\begin{aligned}
\sigma_{A_{12}}^2 &= \left(\frac{\partial A_{12}}{\partial \alpha_1}\right)^2 \sigma_{\alpha_1}^2 + \left(\frac{\partial A_{12}}{\partial \beta_1}\right)^2 \sigma_{\beta_1}^2 + \left(\frac{\partial A_{12}}{\partial \gamma}\right)^2 \sigma_{\gamma}^2 + \left(\frac{\partial A_{12}}{\partial \rho}\right)^2 \sigma_{\rho}^2\\
\sigma_{A_{32}}^2 &= \left(\frac{\partial A_{32}}{\partial \alpha_1}\right)^2 \sigma_{\alpha_1}^2 + \left(\frac{\partial A_{32}}{\partial \beta_1}\right)^2 \sigma_{\beta_1}^2 + \left(\frac{\partial A_{32}}{\partial \gamma}\right)^2 \sigma_{\gamma}^2 + \left(\frac{\partial A_{32}}{\partial \rho}\right)^2 \sigma_{\rho}^2
\end{aligned} \qquad \text{(R40)}$$

Conclusion

This report contains the formulation and the derivation of the attitude determination equations, the calculation of the required elements of error ellipses, and the discussion of various possible special cases for the spacecraft Delta PAC. Based on the analysis developed in this report, A FORTRAN IV program was written to compute the spacecraft attitude with a subroutine to output the plots of the error ellipses and to indicate the region of possible attitude solution for incomplete sensor data. The program was written for the SDS-9300 digital computer and was utilized with actual flight data obtained from the Delta PAC which was launched on Aug. 9, 1969. In the course of examining the flight attitude data, the program was exercised for all modes of data processing including no sun information, with sun information, etc. The results yielded by the use of this analysis have determined the attitude behavior of Delta PAC and evaluated the spacecraft attitude control system. The program is essential to prove that the flight performance of Delta PAC has successfully met its mission objectives.

REFERENCES

1. Thomson, W. T., "Introduction to Space Dynamics," New York: John Wiley and Sons, Inc., 1961

2. Shapiro, I. I., "The Prediction of Ballistic Missile Trajectories from Radar Observations," New York: McGraw-Hill Book Company, Inc., April 1958

3. TRW System, "Nimbus Type Control Configuration Study," Feb. 1967

APPENDIX A

INSTRUCTION FOR DIGITAL PROGRAM

The first input card for the digital program contains ten input values in which the first one, ISUN, indicates the sun sensor information. All initial conditions for the earth sensor equations are specified on this card. If ISUN is zero, the spacecraft is in the earth shadow and a run is initiated immediately. IF ISUN is not zero, a second input card is needed to read in the values of α_1, β_1, α, and β. The value of ISUN determines whether one or two input cards is required for each run. The run is terminated when a blank card is read in. A summary of the total runs is then listed in the output.

APPENDIX B

```
C************************LISTING OF FORTRAN PROGRAM*****************************
C      ATTITUDE DETERMINATION FOR DELTA PAC
C      BY   A. C. FANG
C
       DIMENSION ETA(3)   ,SETA(3) ,CETA(3) ,SE(3)   ,SSE(3)  ,CSE(3)
       DIMENSION AA11(2)  ,AA21(2) ,AA31(2) ,AA12(2) ,AA22(2) ,AA32(2)
       DIMENSION AA13(4)  ,AA23(4) ,AA33(4) ,TH(4)   ,TE(4)   ,PH(4)
       DIMENSION NT(2)    ,DVS(4)  ,DAA12(2),DAA32(2)
       DIMENSION DAE13(2),DAG13(2),DAR13(2),DAE23(2),DAG23(2),DAR23(2)
       DIMENSION DAE33(2),DAG33(2),DAR33(2),DAA13(4),DAA23(4),DAA33(4)
       DIMENSION DAI13(2),DAI23(2),DAI33(2),DBI13(2),DBI23(2),DBI33(2)
       DIMENSION AW(15)
       DIMENSION IA(90,55),IB(90,55)
C
       COMMON C23,IA,IB,ROMD,IDAY,ITIM
C
C*****DEFINE THE VARIABLES
C
C******************************************************************************
C               VAR 1                          VAR 2                    UNIT
C******************************************************************************
C          ,RADIUS OF EARTH           ORBITAL ATTITUDE                  N.M.
C                 RE                           H
C******************************************************************************
C            GIMBAL ANGLE            SCAN CONE HALF ANGLE               DEGREE
C                GAMD                        SIGMAD
C******************************************************************************
C          HALF EARTH PULSE          ELECTRONIC SATURATION              DEGREE
C                ROMD                          ED
C******************************************************************************
C   TELEMETERED PITCH ERROR  GEOMETRICAL PITCH COMPUTED ERROR      DEGREE
C                 ETD                        THETAS
C******************************************************************************
C   ORBITAL POSITION ANGLE     EARTH SUBTENDED ANGLE (HALF)         DEGREE
C               DALFA                        ALFAE
C******************************************************************************
C               ANGLE BETWEEN ORBIT PLANE AND EARTH SUN LINE        DEGREE
C                                            DBETA
C******************************************************************************
C               AZIMUTH                    ELEVATION                DEGREE
C                ALID                        BTID
C******************************************************************************
C
C       CONTINUE
C
C******************************************************************************
C******************************************************************************
C                           BODY AXES :
C             X                  Y                      Z
C******************************************************************************
C           RIGHT HANDED SYSTEM FOR SUN SENSOR HEAD I, I=1,2,3.
C             EI                 FI                     GI
C******************************************************************************
```


2

```
C        ANGLE BETWEEN Y-Z PLANE AND PLANE CONTAINING E1 AND Z
C            SE1                   SE2                         SE3              DEGREE
C*********************************************************************************
C                          ANGLE BETWEEN E1 AND X-Y PLANE
C           ETAD1                  ETAD2                      ETAD3           . DEGREE
C*********************************************************************************
C    COSINE OF ANGLE BETWEEN SCANNER CONE AXIS AND LOCAL VETICAL.
C                                  C23
C*********************************************************************************
C    DIRECTION COSINES OF THE LOCAL VETICAL W.R.T. THE BODY AXES.
C           AA13                   AA23                       AA33           ,
C*********************************************************************************
C    DIRECTION COSINES OF SUN-LINE W.R.T. I TH SUN SENSOR SYSTEM.
C            ST                    SU                         SV
C*********************************************************************************
C    DIRECTION COSINES OF SUN-LINE W.R.T. THE LOCAL VERTICAL SYSTEM
C            S1                    S2                         S3             ,
C*********************************************************************************
C           DIRECTION COSINES OF SUN-LINE W.R.T. BODY AXES
C            SX                    SY                         SZ
C*********************************************************************************
C      DIRECTION COSINES OF ORBIT NORMAL W.R.T. THE BODY AXES
C            A12                   A22                      A32
C*********************************************************************************
C      DIRECTION COSINES OF VELOCITY VECTOR W.R.T. THE BODY AXES
C            A11                   A21                      A31
C*********************************************************************************
C       ROLL  ANGLE         PITCH  ANGLE                    YAW  ANGLE
C            TH                    TE                         PH
C*********************************************************************************
C          ERROR ESTIMATE IN ETD,GAMAD,ROMD,ALID,BTID    ARE
C          DET                  DGA                         DRM
C          DALI                 DBTI                                            .
C*********************************************************************************
C
C*****DERIVATIVES
C
C*********************************************************************************
C          VAR 1                VAR 2                       VAR 3               NOTE
C*********************************************************************************
C          PARTIAL DERIVATIVE OF SX,SY AND SZ W.R.T. ALID
C          DSXA                 DSYA                        DSZA
C*********************************************************************************
C          PARTIAL DERIVATIVE OF SX,SY AND SZ W.R.T. BTID
C          DSXB                 DSYB                        DSZB
C*********************************************************************************
C      PARTIAL DERIVATIVE OF A13(1),A23(1),A33(1) W.R.T. SX ARE
C          AX131                AX231                       AX331
C*********************************************************************************
C      PARTIAL DERIVATIVE OF A13(2),A23(2),A33(2) W.R.T. SX ARE
C          AX132                AX232                       AX332
C*********************************************************************************
C      PARTIAL DERIVATIVE OF A13(1),A23(1),A33(1) W.R.T. SY ARE
C          AY131                AY231                       AY331
```

```
C***************************************************************************
C      PARTIAL DERIVATIVE OF A13(2),A23(2),A33(2) W.R.T. SY ARE
C          AY132               AY232                  AY332
C***************************************************************************
C      PARTIAL DERIVATIVE OF A13(1),A23(1),A33(1) W.R.T. SZ ARE
C          AZ131               AZ231                  AZ331
C***************************************************************************
C      PARTIAL DERIVATIVE OF A13(2),A23(2),A33(2) W.R.T. SZ ARE
C          AZ132               AZ232                  AZ332
C***************************************************************************
C          PARTIAL DERIVATIVE OF A13(I),A23(I),A33(I)  W.R.T. ALID
C          DAI13(I)          DAI23(I)               DDAI33(I)
C***************************************************************************
C          PARTIAL DERIVATIVE OF A13(I),A23(I),A33(I)  W.R.T. BTID
C          DBI13(I)           DBI23(I)               DBI33(I)
C***************************************************************************
C    PARTIAL DERIVATIVE OF A12 W.R.T. ALID,BTID,ETD,GAMAD,ROMD, ARE:
C          DA12               DB12                   DE12
C          DG12               DR12
C***************************************************************************
C    PAPTIAL DERIVATIVE OF A32 W.R.T. ALID,BTID,ETD,GAMAD,ROMD, ARE:
C          DA32               DB32                   DE32
C          DG32               DR32
C***************************************************************************
C      PARTIAL DERIVATIVE OF A13(I),A23(I),A33(I) W.R.T. ETD
C          DAE13(I)           DAE23(I)               DAE33(I)
C***************************************************************************
C      PARTIAL DERIVATIVE OF A13(I),A23(I),A33(I) W.R.T. GAMAD
C          DAG13(I)          DAG23(I)               DAG33(I)
C      PARTIAL DERIVATIVE OF A13(I),A23(I),A33(I) W.R.T. ROMD
C          DAR13(I)          DAR23(I)               DAR33(I)
C***************************************************************************
C******
C*****STORE INPUT DATA .AND START THE CALCULATION.
C******
      DATA RE/3448.0/
      DATA RAD,PI/0.01745329,3.14159265/
      DATA SIGMAD,ED/45.0,45.0/
      DATA ETAD1,ETAD2,ETAD3/26.08,26.06,26.25/
      DATA SE1,SE2,SE3/0.07,120.06,239.87/
      DATA DET,DGA,DRM,DALI,DBTI/.017453,.00873,.017453,.017453,.017453/
      MRUN=0
      TDIF=0.0001
      E=ED*RAD
      SIGMA=SIGMAD*RAD
      SE(1)=SE1*RAD
      SE(2)=SE2*RAD
      SE(3)=SE3*RAD
      ETA(1)=ETAD1*RAD
      ETA(2)=ETAD2*RAD
      ETA(3)=ETAD3*RAD
      SNS = SIN (SIGMA)
      CSS=COS(SIGMA)
      DO 10 I=1,3
```

```
      SSE(I)=SIN(SE(I))
      CSE(I)=COS(SE(I))
      SETA(I)=SIN(ETA(I))
      CETA(I)=COS(ETA(I))
   10 CONTINUE
C******
   20 READ 30,ISUN,GAMAD,ETD,ROMD,IDAY,IHR,MIN,ISEC,IO,H
C******
   30 FORMAT(I2,F12.8,F14.8,F14.8,I4,I4,I4,I4,I6,F8.3)
      IF (IO.EQ.0) GO TO 990
      IF(MRUN.EQ.0) GO TO 40
      IF (ISR .NE.IO) GO TO 990
   40 ITIM=IHR*10000+MIN*100+ISEC
      PRINT 60
      PRINT 50,IDAY,IHR,MIN,ISEC,IO,H
   50 FORMAT(1X,$  DAY = $,I4,$,    HR=$,I4,$,    MIN=$,I4,$,    SEC=$,I4,
     1$,   ORBIT=$,I4,1X,$, H= $,F10.4,/)
      AW(1)=IDAY
      AW(2)=IHR
      AW(3)=MIN
      AW(4)=ISEC
      AW(5)=IO
      AW(6)=GAMAD
      AW(7)=ROMD
      AW(8)=0.0
      AW(9)=0.0
      AW(10)=0.0
      AW(11)=0.0
      AW(12)=0.0
      AW(13)=0.0
      ALFAE = ASIN (RE/(RE+H))
      ALFAD=ALFAE/RAD
      SNA=SIN(ALFAE)
      CSA = COS (ALFAE)
      CAPS=COS(ALFAE+SIGMA)
      CAMS=COS(ALFAE-SIGMA)
      PRINT 60
   60 FORMAT(1X,$************$,/)
      PRINT 70
   70 FORMAT(1X,$EARTH SENSOR INFORMATION GIVES:$,/)
      PRINT 80,GAMAD,ETD,ROMD
   80 FORMAT(1X,$  GAMA(IN DEGREE)=$,F10.6,$,    ET(IN DEGREE)=$,
     1F10.6,$,  ROM(IN DEGREE)=$,F11.6,/)
      GAMA=GAMAD*RAD
      ET=ETD*RAD
      ROM=ROMD*RAD
      CSG = COS (GAMA)
      SNG=SIN(GAMA)
      SNR=SIN(ROM)
      CSR=COS(ROM)
      TNG=SNG/CSG
      DO 90 I=1,2
      DAE13(I)=0.0
      DAE23(I)=0.0
```

```
      DAE33(I)=0.0
      DAG13(I)=0.0
      DAG23(I)=0.0
      DAG33(I)=0.0
      DAR13(I)=0.0
      DAR23(I)=0.0
      DAR33(I)=0.0
      DAI13(I)=0.0
      DAI23(I)=0.0
      DAI33(I)=0.0
      DBI13(I)=0.0
      DBI23(I)=0.0
      DBI33(I)=0.0
      DAA13(I)=0.0
      DAA23(I)=0.0
      DAA12(I)=0.0
      DAA32(I)=0.0
   90 CONTINUE
      CT1=0.0
      CT2=0.0
      FAL=0.0
      FAL=0.0
      FBT=0.0
      FBT=0.0
      LE=0
C***********************************************************************
C*PART  1..........APPLICATION OF EARTH SENSOR
C***********************************************************************
      AEI=ABSF(EI)
      IF(ROMD.EQ.180.0) GO TO 160
      IF (ROMD.EQ.0.0) GO TO 170
  100 CB=CSS*CSA
      CD=SNS*SNS*SNR*SNR
      CA=1.0-CD
      CC=SNS*CSR*SQRTF(SNA*SNA-CD)
      C23=(CB-CC)/CA
      RRC23=1-C23*C23
      SRC23=SQRTF(RRC23)
      DNC23=-RRC23*SNS*SNR*DRM
      DDC23=CSR*SNS*C23-CSS*SRC23
      LE=0
      IF(DDC23 .NE. 0.0) GO TO 110
      LE=1
      GO TO 120
  110 DC23=DNC23/DDC23
  120 ES=AMIN (ROM,PI-ROM,L)
      IF(AEI.LT.ES) GO TO 130
C************WHEN ABSF(COS(ROM))<1 BUT AEI=ES PUT LP=0
      LP=0
      MLP=0
      GO TO 180
C************WHEN ABSF(COS(ROM))<1 AND AEI<ES PUT LP=3
  130 LP=3
      MLP=3
```

```
C*****CALCULATION OF A13,A23,A33 WHEN  (ET) IS NOT EQUAL TO 0
      THETAS=ET
C     THETAS=GEOMETRICAL PITCH COMPUTED ERROR(-180<THETAS<180)
      SNT=SIN(THETAS)
      CNT=COS(THETAS)
      AA13(1)=SNT*SRC23
      AA13(2)=-AA13(1)
      AA23(1)=C23*CSG+SRC23*SNG*CNT
      AA23(2)=C23*CSG-SRC23*SNG*CNT
      AA33(1)=C23*SNG-SRC23*CSG*CNT
      AA33(2)=C23*SNG+SRC23*CSG*CNT
      AA13(3)=AA13(1)
      AA13(4)=AA13(2)
      AA23(3)=AA23(2)
      AA23(4)=AA23(1)
      AA33(3)=AA33(2)
      AA33(4)=AA33(1)
      DO 140 I=1,4
      TH(I)=ASIN(AA23(I))/RAD
      TE(I)=ATAN (-AA13(I),AA33(I))/RAD
  140 CONTINUE
      IF(LE .EQ. 1) GO TO 180
C*****CALCULATE STANDARD DEVIATION OF A13,A23,A33
      DAE13(1)=SRC23*CNT*DET
      DAE13(2)=-DAE13(1)
      DAG13(1)=0.0
      DAG13(2)=0.0
      DAR13(1)=-C23*SNT*DC23/SRC23
      DAR13(2)=-DAR13(1)
      DAE23(1)=-SRC23*SNG*SNT*DET
      DAE23(2)=-DAE23(1)
      DAG23(1)=-(AA33(1))*DGA
      DAG23(2)=-(AA33(2))*DGA
      DAR23(1)=(CSG-C23*SNG*CNT/SRC23)*DC23
      DAR23(2)=(CSG+C23*SNG*CNT/SRC23)*DC23
      DAE33(1)=SRC23*CSG*SNT*DET
      DAE33(2)=-DAE33(1)
      DAG33(1)=(AA23(1))*DGA
      DAG33(2)=(AA23(2))*DGA
      DAR33(1)=(SNG+C23*CSG*CNT/SRC23)*DC23
      DAR33(2)=(SNG-C23*CSG*CNT/SRC23)*DC23
      DO 150 I=1,2
      DAA13(I)=SQRTF(DAE13(I)**2+DAR13(I)**2+DAG13(I)**2)
      DAA23(I)=SQRTF(DAE23(I)**2+DAR23(I)**2+DAG23(I)**2)
      DAA33(I)=SQRTF(DAE33(I)**2+DAR33(I)**2+DAG33(I)**2)
      DAI13(I)=0.0
      DBI13(I)=0.0
      DAI23(I)=0.0
      DBI23(I)=0.0
      DAI33(I)=0.0
      DBI33(I)=0.0
  150 CONTINUE
      DAA23(3)=DAA23(2)
      DAA23(4)=DAA23(1)
```

```
      DAA33(3)=DAA33(2)
      DAA33(4)=DAA33(1)
      GO TO 180
C*****
C******** *** WHEN COS(ROM)=-1, PUT LP=2 TO PLOT THE REGION
  160 LP=2
      MLP=2
      C23=CAMS
      GO TO 180
C******** *** WHEN COS(ROM)=1 PUT LP=1 TO PLOT THE REGION
  170 LP=1
      MLP=1
      C23=CAPS
      GO TO 180
C**********************************************************************
C*PART 2...........SUN SENSOR APPLICATION
C**********************************************************************
  180 IF (ISUN.EQ.0) GO TO 590
C******
      READ 190,ALID,BTID,DALFA,DBETA
C*****
  190 FORMAT(6F14.8)
      ALI=ALID*RAD
      BTI=BTID*RAD
      TAI=(SIN(ALI))/COS(ALI)
      TBI=(SIN(BTI))/COS(BTI)
      TAB=SQRTF(1.0+(TAI**2)+(TBI**2))
      ST=1.0/TAB
      SU=TBI/TAB
      SV=TAI/TAB
      PRINT 60
      PRINT 200,ISUN
  200 FORMAT(1X,$ISUN=$,I2,$,  SUN SENSOR INFORMATION GIVES:$,/)
      PRINT  210,ALID,BTID
  210 FORMAT(3X,$AZIMUTH=$,F10.5,$  DEG,  ELEVATION=$,F10.5,$  DEG.$,/)
      PRINT 220,ST,SU,SV,DALFA,DBETA
  220 FORMAT(3X,$S11=$,F9.6,$,  S21=$,F9.6,$,  S31=$,F9.6,
     1$,  ALFA(IN DEGREE)=$,F11.6,$,  BETA(IN DEGREE)=$,F11.6,/)
      ALFA=DALFA*RAD
      BETA=DBETA*RAD
C*****CALCULATE THE DIRECTION COSINES OF SUN LINE W.R.T. BODY AXIS
      J=ISUN
      SX=-ST*CETA(J)*SSE(J)+SU*SETA(J)*SSE(J)-SV*CSE(J)
      SY=ST*CETA(J)*CSE(J)-SU*SETA(J)*CSE(J)-SV*SSE(J)
      SZ=-ST*SETA(J)-SU*CETA(J)
      PRINT  230,SX,SY,SZ
  230 FORMAT(2X,$ THE DIRECTION COSINES OF SUN LINE W.R.T. SPACECRAFT
     1AXES ARE  $,F12.8,$,   $,F12.8,$,   $,F12.8,/)
C*****CALCULATE PARTIAL DERIVATIVE OF SX,SY,AND SZ W.R.T.ALID AND BTID
      SEAI=1.0+TAI*TAI
      SEBI=1.0+TBI*TBI
      STSAI=ST*SEAI*DALI
      STSBI=ST*SEBI*DBTI
      DSXA=(-SX*SV-CSE(J))*STSAI
```

```
      DSXB=(-SX*SU+SETA(J)*SSE(J))*STSBI
      DSYA=(-SY*SV-SSE(J))*STSAI
      DSYB=(-SY*SU-SETA(J)*CSE(J))*STSBI
      DSZA=-SZ*SV*STSAI
      DSZB=(-SZ*SU-CETA(J))*STSBI
*****CALCULATE THE DIRECTION COSINES OF SUN LINE W.R.T. GEOCENTRIC AXIS
      S1=-COS(BETA)*(SIN(ALFA))
      S2=SIN(BETA)
      S3=-COS(BETA)*(COS(ALFA))
      PRINT 240,S1,S2,S3
      PRINT 60
  240 FORMAT(3X,$THE DIRECTION COSINES OF SUN LINE W.R.T.GEOCENTRIC AXIS
     1:$,F12.8,$,$,F12.8,$,$,F12.8,/)
      IF(LP.EQ.0)GO TO 250
      GO TO (610,610,450),LP
*****CALCULATE THE LOCAL VERTICAL WHEN EI=ES.
  250 SYSG=SY*SNG
      SYCG=SY*CSG
      SZSG=SZ*SNG
      SZCG=SZ*CSG
      FCC=SYSG-SZCG
      IF (FCC.EQ.0.0)GO TO 280
      FCR=FCC**2
      FA=SX*SX+FCR
      FB=SX*(-S3+C23*(SZSG+SYCG))
      FC=FCR+2.0*C23*S3*(SZSG+SYCG)-S3*S3-C23*C23*(1.0-SX*SX)
      ROT=FB*FB+FA*FC
      IF(ROT .LT. 0.0) GO TO 260
      AA13(1)=(-FB+SQRTF(ROT))/FA
      AA13(2)=(-FB-SQRTF(ROT))/FA
      AA23(1)=(SNG*(S3-SX*AA13(1))-C23*SZ)/FCC
      AA23(2)=(SNG*(S3-SX*AA13(2))-C23*SZ)/FCC
      AA33(1)=(CSG*(-S3+SX*AA13(1))+C23*SY)/FCC
      AA33(2)=(CSG*(-S3+SX*AA13(2))+C23*SY)/FCC
      GO TO 320
  260 PRINT 270
  270 FORMAT(3X,$A13,A23,A33 ARE NOT REAL NUMBER. THEREFORE$)
      GO TO 420
  280 PRINT 290
  290 FORMAT(3X,$SY*SIN(GAMA)-SZ*COS(GAMA)=0$,/)
      IF (SX.EQ.0.0)GO TO 420
C     TRY TO USE OTHER METHOD  FOR SEEKING THE SOLUTION OF A13,A23,A33
      AA13(1)=(S3*CSG-SY*C23)/SX*CSG
      AA13(2)=AA13(1)
      ROOT=1.0-AA13(1)*AA13(1)-C23*C23
      IF(ROOT.LT.0.0)GO TO 260
      SQRR=SQRTF(ROOT)
      AA23(1)=C23*CSG+SNG*SQRR
      AA23(2)=C23*CSG-SNG*SQRR
      AA33(1)=C23*SNG-CSG*SQRR
      AA33(2)=C23*SNG+CSG*SQRR
      PRINT 300, AA13(1),AA23(1),AA33(1)
  300 FORMAT(3X,$A13(1)=$,F11.8,$,  A23(1)=$,F11.8,$,  A33(1)=$,F11.8,/)
      PRINT 310,AA13(2),AA23(2),AA33(2)
```

```
  310 FORMAT(3X,$A13(2)=$,F11.8,$,  A23(2)=$,F11.8,$,  A33(2)=$,F11.8,/)
C*****
C      TEST IF A13 SATISFY THE CONDITION FOR THE REQUIRED REGION
C      TL,TM ARE THE LOWER LIMITS FOR THE TWO POSSIBLE REGIONS
C      TU,TN ARE THE UPPER LIMITS FOR THE TWO POSSIBLE REGIONS
  320 TL=SRC23*SIN(ES)
      TU=SRC23
      TM=-TU
      TN=-TL
      DO 340 I=1,2
      CK=AA13(I)
      IF((CK.GE.TL.AND.CK.LE.TU).OR.(CK.GE.TM.AND.CK.LE.TN))GO TO 330
      NT(I)=0
      GO TO 340
  330 NT(I)=I
  340 CONTINUE
      MNT=AMIN (NT(1),NT(2))
      MAT=AMAX (NT(1),NT(2))
      PRINT 300, AA13(1),AA23(1),AA33(1)
      PRINT 310,AA13(2),AA23(2),AA33(2)
      PRINT 60
      PRINT 350,TL,TU,TM,TN
  350 FORMAT(3X,$NOTE : A13 HAS TO SATISFY THE REQUIRED CONDITION FOR
     1THE LIMITED REGION,$,F8.5,$ < A13 < $,F8.5,$, OR,$,F8.5,
     2$ < A13 < $,F8.5,/)
      IF(MAT.NE.MNT)GO TO 370
      PRINT 360
  360 FORMAT(3X,$THE TWO SOLUTIONS OF A13 DO NOT SATISFY THE CONDITION
     1FOR THE REQUIRED REGION $,/)
      GO TO 420
  370 TH(1)=ASIN(AA23(1))/RAD
      TH(2)=ASIN(AA23(2))/RAD
      TE(1)=ATAN (-AA13(1),AA33(1))/RAD
      TE(2)=ATAN (-AA13(2),AA33(2))/RAD
      PRINT 380, TH(1),TE(1)
  380 FORMAT(3X,$ROLL ANGLE TH(1)=$,F10.5,$ DEG., PITCH ANGLE TE(1)=$,
     1F10.5,$DEG.$,/)
      PRINT 390,TH(2),TE(2)
  390 FORMAT(3X,$ROLL ANGLE TH(2)=$,F10.5,$ DEG.,PITCH ANGLE TE(2)=$,
     1F10.5,$DEG.$,/)
      IF(LE .EQ. 1) GO TO 450
C*****CALCULATE PARTIAL DIFFERENTIATE OF A13,A23,A33 W.R.T.SX,SY,SZ,ETC
C*****AND CALCULATE STANDARD DEVIATION OF A13,A23,A33
      SA131=AA13(1)*AA13(1)
      SA132=AA13(2)*AA13(2)
      CFR1=SNG*AA23(1)-CSG*AA33(1)
      CFR2=SNG*AA23(2)-CSG*AA33(2)
      CDN1=SX*(-CFR1)+FCC*AA13(1)
      CDN2=SX*(-CFR2)+FCC*AA13(2)
      IF(CDN1 .EQ.0.0 ) GO TO 400
      AX131=CFR1*AA13(1)/CDN1
      AX231=-SNG*SA131/CDN1
      AX331= CSG*SA131/CDN1
      AY131=CFR1*AA23(1)/CDN1
```

```
      AY231=-SNG*AA13(1)*AA23(1)/CDN1
      AY331= CSG*AA13(1)*AA23(1)/CDN1
      AZ131=CFR1*AA33(1)/CDN1
      AZ231=-SNG*AA13(1)*AA33(1)/CDN1
      AZ331= CSG*AA13(1)*AA33(1)/CDN1
      RG131=(SZ*AA23(1)-SY*AA33(1))/CDN1
      RG231=(SX*AA33(1)-SZ*AA13(1))/CDN1
      DAG13(1)=CFR1*RG131*DGA
      DAG23(1)=CFR1*RG231*DGA
      DAR13(1)=RG131*DC23
      DAR23(1)=RG231*DC23
      DAI13(1)= AX131*DSXA+AY131*DSYA+AZ131*DSZA
      DAI23(1)= AX231*DSXA+AY231*DSYA+AZ231*DSZA
      DAI33(1)= AX331*DSXA+AY331*DSYA+AZ331*DSZA
      DBI13(1)= AX131*DSXB+AY131*DSYB+AZ131*DSZB
      DBI23(1)= AX231*DSXB+AY231*DSYB+AZ231*DSZB
      DBI33(1)= AX331*DSXB+AY331*DSYB+AZ331*DSZB
      DAA13(1)=SQRTF(DAI13(1)**2+DBI13(1)**2+DAG13(1)**2+DAR13(1)**2)
      DAA23(1)=SQRTF(DAI23(1)**2+DBI23(1)**2+DAG23(1)**2+DAR23(1)**2)
      DAE13(1)=0.0
      DAE23(1)=0.0
      DAE33(1)=0.0
      GO TO 410
C*****
  400 LE=2
  410 IF(CDN2 .EQ.0.0) GO TO 440
      RG132=(SZ*AA23(2)-SY*AA33(2))/CDN2
      RG232=(SX*AA33(2)-SZ*AA13(2))/CDN2
      DAG13(2)=CFR2*RG132*DGA
      DAG23(2)=CFR2*RG232*DGA
      DAR13(2)=RG132*DC23
      DAR23(2)=RG232*DC23
      AX132= CFR2*AA13(2)/CDN2
      AX232=-SNG*SA132/CDN2
      AX332= CSG*SA132/CDN2
      AY132=CFR2*AA23(2)/CDN2
      AY232=-SNG*AA13(2)*AA23(2)/CDN2
      AY332= CSG*AA13(2)*AA23(2)/CDN2
      AZ132=CFR2*AA33(2)/CDN2
      AZ232=-SNG*AA13(2)*AA33(2)/CDN2
      AZ332= CSG*AA13(2)*AA33(2)/CDN2
      DAI13(2)= AX132*DSXA+AY132*DSYA+AZ132*DSZA
      DAI23(2)= AX232*DSXA+AY232*DSYA+AZ232*DSZA
      DAI33(2)= AX332*DSXA+AY332*DSYA+AZ332*DSZA
      DBI13(2)= AX132*DSXB+AY132*DSYB+AZ132*DSZB
      DBI23(2)= AX232*DSXB+AY232*DSYB+AZ232*DSZB
      DBI33(2)= AX332*DSXB+AY332*DSYB+AZ332*DSZB
      DAA13(2)=SQRTF(DAI13(2)**2+DBI13(2)**2+DAG13(2)**2+DAR13(2)**2)
      DAA23(2)=SQRTF(DAI23(2)**2+DBI23(2)**2+DAG23(2)**2+DAR23(2)**2)
      DAE13(2)=0.0
      DAE23(2)=0.0
      DAE33(2)=0.0
      PRINT  480, DAA13(1), DAA13(2)
      PRINT  490, DAA23(1), DAA23(2)
```

```
      PRINT 60
      GO TO 450
  420 PRINT 430
  430 FORMAT(3X,$NO SOLUTION FOR A13,A23,A33. TRY TO USE PLOT$,/)
      MLP=1
      GO TO 610
  440 LE=LE+3
C*****
C*****CHECKING WHICH ONE OF THE LOCAL VERTICAL SHOULD BE SELECTED
C      K=1 OR 2 MEANS ONE ANSWER FOR LOCAL VERTICAL. K=3 MEANS TWO.
  450 IDO=2+(LP+1)/2
      DO 460 I=1,IDO
      CHECK1=(AA13(I))**2+(AA23(I))**2+(AA33(I))**2-1.0
      CHECK2=(AA23(I))*CSG+(AA33(I))*SNG-C23
      CHECK3=SX*AA13(I)+SY*AA23(I)+SZ*AA33(I)-S3
      DVS(I)=AMAX(ABSF(CHECK1),ABSF(CHECK2),ABSF(CHECK3))
  460 CONTINUE
      PRINT 60
      ASV=AMIN (DVS(1),DVS(2))
      IF(LP.EQ.0) GO TO 510
      ASW=AMIN (DVS(3),DVS(4))
      IF(ASV.LE.ASW) GO TO 470
      AA23(1)=AA23(3)
      AA23(2)=AA23(4)
      AA33(1)=AA33(3)
      AA33(2)=AA33(4)
      TH(1)=TH(3)
      TH(2)=TH(4)
      TE(1)=TE(3)
      TE(2)=TE(4)
      ASV=ASW
      DVS(1)=DVS(3)
      DVS(2)=DVS(4)
  470 PRINT 300, AA13(1),AA23(1),AA33(1)
      PRINT 310,AA13(2),AA23(2),AA33(2)
      PRINT 380, TH(1),TE(1)
      PRINT 390,TH(2),TE(2)
      PRINT 60
      IF(LE.EQ.1) GO TO 510
      PRINT 60
      PRINT  480,DAA13(1),DAA13(2)
  480 FORMAT(3X,$DEVIATION IN A13,    DA13(1)=$,F8.5,$,   DA13(2)=$,F8.5)
      PRINT  490, DAA23(1),DAA23(2)
  490 FORMAT(3X,$DEVIATION IN A23,    DA23(1)=$,F8.5,$,   DA23(2)=$,F8.5)
      PRINT 500,DAA33(1),DAA33(2)
  500 FORMAT(3X,$DEVIATION IN A33,    DA33(1)=$,F8.5,$,   DA33(2)=$,F8.5)
      PRINT 60
  510 DIF=ABSF(DVS(1)-DVS(2))
      IF(DIF.LE.TDIF) GO TO 550
      IF(ASV.EQ.DVS(1)) GO TO 520
      K=2
      GO TO 530
  520 K=1
  530 IF(LP.EQ. 3)GO TO 570
```

```
      IF(NT(K).EQ.K) GO TO 570
      AW(8)=TH(1)
      AW(9)=TE(1)
      AW(11)=TH(2)
      AW(12)=TE(2)
      PRINT 540
  540 FORMAT(3X,$THE POSITION OF A13 DOES NOT SATISFY THE REQUIRED LIMIT
     1REGION$,/)
      GO TO 420
  550 K=3
      IF(LP .EQ. 3) GO TO 660
      IF(MNT.NE.0)GO TO 610
      IF(NT(1).EQ.0) GO TO 560
      K=1
      GO TO 570
  560 K=2
  570 A13=AA13(K)
      A23=AA23(K)
      A33=AA33(K)
      VAE13=DAE13(K)
      VAE23=DAE23(K)
      VAE33=DAE33(K)
      VAG13=DAG13(K)
      VAR13=DAR13(K)
      VAG23=DAG23(K)
      VAR23=DAR23(K)
      VAG33=DAG33(K)
      VAR33=DAR33(K)
      VAI13=DAI13(K)
      VAI23=DAI23(K)
      VAI33=DAI33(K)
      VBI13=DBI13(K)
      VBI23=DBI23(K)
      VBI33=DBI33(K)
      VAA13=DAA13(K)
      VAA23=DAA23(K)
      PRINT 60
      PRINT 580,A13,A23,A33
  580 FORMAT(3X,$DIRECTION COSINES OF LOCAL VERTICAL ARE:$,F12.8,F12.8,
     1F12.8)
      IF(LP .EQ. 0) GO TO 610
      GO TO 660
  590 PRINT 600,ISUN
  600 FORMAT(1X,$ISUN=$,I2,$,  THE SPACECRAFT IS IN EARTH SHADOW. THE
     1DIRECTION COSINES OF SUN LINE CAN NOT BE DETERMINED$,/)
      IF(LP .EQ.0) GO TO 650
      GO TO (630,640,860),LP
  610 PRINT 620
  620 FORMAT(1H1)
      WHEN LP IS LESS THAN 3, CT1, CT2, EAL,EBT, , ARE INPUT DATA TO
      SUBROUTINE FOR PRINTING PURPOSE.
      CT1=DALFA
      CT2=DBETA
      EAL=ALID
```

```
      EBT=BTID
      IF (LP.EQ.0) GO TO 650
      GO TO(630,640),LP
  630 CALL PLOT(LP,GAMA,ISUN,SX,SY,SZ,S2,S3,CT1,CT2,EAL,EBT)
      GO TO 970
  640 CALL PLOT(LP,GAMA,ISUN,SX,SY,SZ,S2,S3,CT1,CT2,EAL,EBT)
      GO TO 970
  650 CALL PLOT(LP,GAMA,ISUN,SX,SY,SZ,S2,S3,CT1,CT2,EAL,EBT)
      IF(ISUN .EQ. 0) GO TO 970
      IF(MLP .EQ. 1) GO TO 970
C*********************************************************************
C*PART 3..........DETERMINATION OF BODY AXIS
C*********************************************************************
  660 IF(K .EQ. 3) GO TO 670
      I=K
      GO TO 680
  670 DO 830 I=1,2
      A13=AA13(I)
      A23=AA23(I)
      A33=AA33(I)
      VAE13=DAE13(I)
      VAE23=DAE23(I)
      VAE33=DAE33(I)
      VAG13=DAG13(I)
      VAR13=DAR13(I)
      VAG23=DAG23(I)
      VAR23=DAR23(I)
      VAG33=DAG33(I)
      VAR33=DAR33(I)
      VAI13=DAI13(I)
      VAI23=DAI23(I)
      VAI33=DAI33(I)
      VBI13=DBI13(I)
      VBI23=DBI23(I)
      VBI33=DBI33(I)
      VAA13=DAA13(I)
      VAA23=DAA23(I)
  680 Q=S3
      D=Q*Q-1.0
      IF(ABSF(D).LT.(.001**2 )) GO TO 840
      A11=(S1*(A13*Q-SX)+S2*(SZ*A23-SY*A33))/D
      A21=(S1*(A23*Q-SY)+S2*(SX*A33-SZ*A13))/D
      A31=(S1*(A33*Q-SZ)+S2*(SY*A13-SX*A23))/D
      A12=(S2*(A13*Q-SX)-S1*(SZ*A23-SY*A33))/D
      A22=(S2*(A23*Q-SY)-S1*(SX*A33-SZ*A13))/D
      A32=(S2*(A33*Q-SZ)-S1*(SY*A13-SX*A23))/D
      THI=ASIN(A23)/RAD
      THE=ATAN ( -A13,A33)/RAD
      IF(ABSF(A23).GE.1.0) GO TO 700
      QSA23=SQRTF(1.0-A23*A23)
      PHE=ASIN(-A21/QSA23)/RAD
      GO TO 720
  700 PRINT 710
  710 FORMAT(3X,$YAW ANGLE CAN NOT BE DETERMINED,SET IT TO 50 DEG.$,/)
```

```
      PHE=50.0
  720 TH(I)=THI
      TE(I)=THE
      PH(I)=PHE
      LU=10+(I-1)*3
      AW(LU)=PHE
      LV=9+(I-1)*3
      AW(LV)=THE
      LW=8+(I-1)*3
      AW(LW)=THI
      AA11(I)=A11
      AA21(I)=A21
      AA31(I)=A31
      AA12(I)=A12
      AA22(I)=A22
      AA32(I)=A32
      AA13(I)=A13
      AA23(I)=A23
      AA33(I)=A33
      IF(LE .EQ. 0) GO TO 730
      IF((LE.EQ.1). OR.(LE.EQ.5)) GO TO 870
      IF(((LE-I).EQ.2).OR.((LE-I).EQ.0)) GO TO 730
      GO TO 870
C*****
C     PLOT THE ERROR ELLIPSE OF A23,A13
  730 LP=3
      CT1=-A23
      CT2=-A13
      EAL=3.0*VAA23
      EBT=3.0*VAA13
      CALL PLOT(LP,GAMA,ISUN,SX,SY,SZ,S2,S3,CT1,CT2,EAL,EBT)
C*****
C*****CALCULATE THE PARTIAL DERIVATIVES OF Q W.R.T.ALID,BTID,GAMA,ETC.
C*****CALCULATE THE STANDARD DEVIATION OF A12,A22,A32
      IF(MLP.EQ.0)GO TO 740
      DA12=(S2*(-DSXA)-S1*(A23*DSZA-A33*DSYA))/D
      DB12=(S2*(-DSXB)-S1*(A23*DSZB-A33*DSYB))/D
      DE12=(S2*Q*VAE13-S1*(SZ*VAE23-SY*VAE33))/D
      DG12=(S2*Q*VAG13-S1*(SZ*VAG23-SY*VAG33))/D
      DR12=(S2*Q*VAR13-S1*(SZ*VAR23-SY*VAR33))/D
      DDA12=SQRTF(DA12**2+DB12**2+DE12**2+DG12**2+DR12**2)
      DA32=(S2*(-DSZA)-S1*(A13*DSYA-A23*DSXA))/D
      DB32=(S2*(-DSZB)-S1*(A13*DSYB-A23*DSXB))/D
      DE32=(S2*Q*VAE33-S1*(SY*VAE13-SX*VAE23))/D
      DG32=(S2*Q*VAG33-S1*(SY*VAG13-SX*VAG23))/D
      DR32=(S2*Q*VAR33-S1*(SY*VAR13-SX*VAR23))/D
      DDA32=SQRTF(DA32**2+DB32**2+DE32**2+DG32**2+DR32**2)
      GO TO 750
  740 DK12= S2*(Q*VAI13-DSXA)-S1*(SZ*VAI23+A23*DSZA-SY*VAI33-A33*DSYA)
      DK32= S2*(Q*VAI33-DSZA)-S1*(SY*VAI13+A13*DSYA-SX*VAI23-A23*DSXA)
      DL12= S2*(Q*VBI13-DSXB)-S1*(SZ*VBI23+A23*DSZB-SY*VBI33-A33*DSYB)
      DL32= S2*(Q*VBI33-DSZB)-S1*(SY*VBI13+A13*DSYB-SX*VBI23-A23*DSYB)
      DA12=DK12/D
      DA32=DK32/D
```

```
      DB12=DL12/D
      DB32=DL32/D
      DG12=(S2*Q*VAG13-S1*(SZ*VAG23-SY*VAG33))
      DG32=(S2*Q*VAG33-S1*(SY*VAG13-SX*VAG23))/D
      DR12=(S2*Q*VAR13-S1*(SZ*VAR23-SY*VAR33))
      DR32=(S2*Q*VAG33-S1*(SY*VAG13-SX*VAG23))/D
      DDA12=SQRTF(DA12**2+DB12**2+DR12**2+DG12**2)
      DDA32=SQRTF(DA32**2+DB32**2+DG32**2+DR32**2)
  750 DAA12(I)=DDA12
      DAA32(I)=DDA32
      PRINT 60
      PRINT 760,I,AA13(I),I,AA23(I),I,AA33(I)
  760 FORMAT(3X,$A13($,I1,$)=$,F11.8,2X,$A23($,I1,$)=$,F11.8,
     12X,$A33($,I1,$)=$,F11.8,/)
      PRINT 770,I,AA11(I),I,AA21(I),I,AA31(I)
  770 FORMAT(3X,$A11($,I1,$)=$,F11.8,2X,$A21($,I1,$)=$,F11.8,
     12X,$A31($,I1,$)=$,F11.8,/)
      PRINT 780,I,AA12(I),I,AA22(I),I,AA32(I)
  780 FORMAT(3X,$A12($,I1,$)=$,F11.8,2X,$A22($,I1,$)=$,F11.8,
     12X,$A32($,I1,$)=$,F11.8,/)
      PRINT 790,I,TH(I),I,TE(I),I,PH(I)
  790 FORMAT(3X,$ROLL ANGLE  TH($,I1,$)=$,F10.5,$ DEG.,PITCH ANGLE TE($,
     1I1,$)=$,F10.5,$ DEG.,  YAW ANGLE PH($,I1,$)=$,F10.5,$ DEG.$,/)
      PRINT 60
      PRINT 60
      PRINT 820,I,DAA12(I),I,DAA32(I)
  820 FORMAT(3X,$STANDARD DEVIATION OF A12,A32 ARE,DA12($,I1,
     1$)=$,F8.5,$, DA32($,I1,$)=$,F9.6,/)
C******
      LP=4
      DT1=-A32
      DT2=-A12
      FAL=3.0*DDA32
      FBT=3.0*DDA12
      CALL PLOT(LP,GAMA,ISUN,SX,SY,SZ,S2,S3,DT1,DT2,FAL,FBT)
  830 CONTINUE
      GO TO 970
C*********
  840 PRINT 850
  850 FORMAT(1X,$SUNLINE AND LOCAL VERTICAL ARE IN THE SAME LINE,NO
     1FURTHER CALCULATION FOR A12,A22,A32,ETC.$,/)
      IF(LE.NE.0)GO TO 870
      IF(K .EQ. 3) GO TO 940
      KA1=K
      KA2=K
      GO TO 950
  860 PRINT 300, AA13(1),AA23(1),AA33(1)
      PRINT 310,AA13(2),AA23(2),AA33(2)
      PRINT 380, TH(1),TE(1)
      PRINT 390,TH(2),TE(2)
      PRINT 60
      AW(8)=TH(1)
      AW(9)=TE(1)
      AW(11)=TH(2)
```

```
      AW(12)=TE(2)
      IF(LE.EQ. 0) GO TO 940
  870 GO TO (880,900,920,920,920),LE
  880 PRINT 890
  890 FORMAT(3X,$DEVIATION OF C23 IS TOO LARGE,HENCE THERE IS NO ERROR
     1CALCULATION FOR A13,A23,A33$,/)
      GO TO 970
  900 PRINT 910
  910 FORMAT(3X,$DAG13(1),DAG23(1) CAN NOT BE DETERMINED. NO FURTHER
     1CALCULATION FOR ERROR ELLIPSE $,/)
      GO TO 970
  920 PRINT 930
  930 FORMAT(3X,$DAG23(1),DAG23(2) CAN NOT BE DETERMINED. NO FURTHER
     1CALCULATION FOR ERROR ELLIPSE $,/)
      GO TO 970
  940 PRINT  480,DAA13(1),DAA13(2)
      PRINT  490, DAA23(1),DAA23(2)
      PRINT 60
      KA1=1
      KA2=2
  950 DO 960  I=KA1,KA2
      CT1=-AA23(I)
      CT2=-AA13(I)
      EAL=3.0*DAA23(I)
      EBT=3.0*DAA13(I)
      CALL PLOT(LP,GAMA,ISUN,SX,SY,SZ,S2,S3,CT1,CT2,EAL,EBT)
  960 CONTINUE
C***********************************************************************
C*PART 4...........SUMMARY OF RUNS
C***********************************************************************
  970 MRUN=MRUN+1
      WRITE OUTPUT TAPE 2, 980,(AW(I),I=1,13)
  980 FORMAT (1X,I3,13(1X,F8.2),/)
      IOR=IO
      GO TO 20
  990 DO 1000 J=1, MRUN
 1000 BACKSPACE 2
      PRINT 1010,IOR
 1010 FORMAT(40X,$.....SUMMARY OF RUNS FOR ORBIT NO.$,F8.2,2X,$.....$)
      PRINT 60
      PRINT 1020
 1020 FORMAT(2X,$NO.$,3X,$DATE$,7X,$HR.$,5X,$MIN$,7X,$SEC$,
     14X,$ORBIT$,5X,$GAMA$,4X,$ROMD$,7X,$THI$,5X,$THETA$,
     25X,'PHI',5X,'THI',5X,'THETA',6X,'PHI',/)
      DO 1030 JI=1, MRUN
      READ INPUT TAPE 2, 980,(AW(I),I=1,13)
      PRINT 980, JI,(AW(I),I=1,13)
 1030 CONTINUE
      PRINT 60
      MRUN=0
      IOR=IO
      PRINT 620
      IF(IO.NE.0) GO TO 40
      END
```

```
      SUBROUTINE PLOT (LP, GA, ISUN, SX, SY, SZ, S2,S3,CT1,CT2,EAL,EBT)
      COMMON C23,IA(90,55),IB(90,55),ROMD,IDAY,ITIM
C
C     THIS PROGRAM IS USING POLAR COORDINATES FOR PLOT
C     AE AND BE ARE THE MAJOR AND MINOR AXIS OF ELLIPSE
C     IA REPRESENTS THE PLOT FOR A13 AND A23.
C     IB REPRESENTS THE PLOT FOR A12 AND A32.
C     WHEN LP=0, CC=C23,WHERE C23 IS CALCULATE FROM MAIN PROGRAM
C     WHEN LP=1, C23 IS LESS THAN -0.3733(PUT CC=-0.3733
C     WHEN LP=2, C23 IS GREATER THAN 0.9277(PUT CC=0.9277)
C
      SNG=SIN(GA)
      CNG=COS(GA)
      GAD=GA/0.01745329
      CC=C23
      RC=1.0-CC*CC
      CCN=CC*CNG
      SRC=SQRTF(RC)
      AE=SRC*SNG
      BE=SRC
      CE=-CCN
      IXC=(CC+1)*42+1.5
      IYM=SRC*25+26.5
      DO 10 I=1,90
      DO 10 J=1,55
      IA(I,J)=1H
      IB(I,J)=1H
   10 CONTINUE
C     PLOT Y AXIS
      DO  20 J=1,53
      IA(43,J)=1H*
      IB(43,J)=1H*
   20 CONTINUE
C     PLOT X-AXIS
      DO  30 J=1,87,2
      IA(J,26)=1H*
      IB(J,26)=1H*
   30 CONTINUE
C     LABEL FOR A23 AXIS
      IA(90,26)=1HA
      IA(89,26)=1H2
      IA(88,26)=1H3
      IA(87,26)=1H+
C     LABEL FOR A32 AXIS
      IB(90,26)=1HA
      IB(89,26)=1H3
      IB(88,26)=1H2
      IB(87,26)=1H+
      DO 40 J=1,85,21
      IA(J,26)=1HI
      IB(J,26)=1HI
   40 CONTINUE
C******
C     LABEL FOR THE CENTER O OF CELESTIAL SPHERE
```

```
      IA(43,26)=1H0
      IB(43,26)=1H0
C     AL REPRESENTS POLAR ANGLE
      AL=0
      RP=1.0
      CALL POINT(AL,RP,X1,Y1,IX1,IY1)
C     DAL IS THE INCREMENT OF ANGLE FOR POLAR COORDINATES METHOD
      DAL=5.0*0.01745329
      IF(LP.LT.3)GO TO 100
C******
C     PLOT ERROR ELLIPSE WHEN LP IS GREATER THAN 3
      SNU=0.0
      CSU=1.0
   50 CALL ELIPS(SNU,CSU,AL,EAL,EBT,CT1,CT2,XER,YER,IXR,IYR)
      IA(IX1,IY1)=1H*
      IB(IX1,IY1)=1H*
      IF(ABSF(XER).GT.1.0)GO TO 60
      YEM=SQRTF(1.0-XER*XER)
      IF(YER.GT.YEM)GO TO 60
      IF((IXR.LT.0. OR.IXR.GT.85).OR.(IYR.LT.0. OR.IYR.GT.52))GO TO 60
      IA(IXR,IYR)=1HR
      IB(IXR,IYR)=1HR
   60 AL=AL+DAL
      CALL POINT(AL,RP,X1,Y1,IX1,IY1)
      IF(AL.LE.6.3) GO TO 50
      CT1=-CT1
      CT2=-CT2
      IXER=(CT1+1)*42+1.5
      IYEP=CT2*25+26.5
      IA(IXER,IYER )=1HR
      IB(IXER,IYER )=1HR
      PRINT 510
      IF(LP.EQ. 4)GO TO 80
      PRINT 70,CT1,CT2,EAL,EBT
   70 FORMAT(1X,$ERROR ELLIPSE OF A23,A13:  CENTER AT A23=$,F8.5,$,  A13
     1=$,F8.5,$, AXIS PARALLEL TO A23=$,F8.5,$, AXIS PARALLEL TO A13=$,
     2F8.5)
      GO TO 660
   80 PRINT 90,CT1,CT2,EAL,EBT
   90 FORMAT(1X,$ERROR ELLIPSE OF A32,A12:  CENTER AT A32=$,F8.5,$,  A12
     1=$,F8.5,$, AXIS PARALLEL TO A32=$,F8.5,$, AXIS PARALLEL TO A12=$,
     2F8.5)
      GO TO 530
C******
  100 PRINT 110
  110 FORMAT(20X,$SUN SENSOR INFORMATION$,25X,$EARTH SENSOR INFORMATION$
     1,/)
C     ISUN=0 MEANS NO INFORMATION OF SUN LINE DIRECTION
C     ISUN=1,2,3, MEANS INFORMATION OF SUN LINE IS AVAILABLE
      IF(ISUN.EQ.0) GO TO 360
C     INFORMATION FROM SUN SENSOR AVAILABL.PLOT THE REGION
      IXA=(S3+1)*42+1.5
      IXB=(S2+1)*42+1.5
      SR=SQRTF(ABSF(1.0-S2*S2))
```

```
      ST=SQRTF(ABSF(1.0-S3*S3))
C     AP AND BP ARE THE MAJOR AND MINOR AXIS OF ELLIPSE IN A13,A23,PLOT.
      AP=ABSF(SZ*ST)
      BP=ST
C     AQ AND BQ ARE THE MAJOR AND MINOR AXIS OF ELLIPSE IN A12,A32 PLOT.
      AQ=ABSF(SY*SR)
      BQ=SR
C     (EX,EY),(FX,FY) ARE CENTER OF EACH ELLIPSE.
      EX=-S3*SY
      EY=-S3*SX
      FX=-SZ*S2
      FY=-SX*S2
      RXY=SQRTF(SX*SX+SY*SY)
      RXZ=SQRTF(SX*SX+SZ*SZ)
C******
      IF(SX .EQ. 0.0) GO TO 120
      SM=-SY/SX
      SN=-SZ/SX
      SK=S3/SX
      SL=S2/SX
  120 CALL TEST(SX,SY,SZ,IS)
      CALL TEST(SX,SZ,SY,LS)
      IF(LS .NE. 1) GO TO 130
      SNF=SX/RXZ
      CSF=SZ/RXZ
      FL=ASIN(SNF)
  130 GO TO (160,180,140,150),IS
C******
  140 IY=IY1
      IX=IXA
      X3=S3
      Y3=Y1
      GO TO 190
C******
  150 Y3=SM*X1+SK
      IY =Y3*25+26.5
      IX=IX1
      X3=X1
      GO TO 190
C******
  160 SNE=SX/RXY
      CSE=SY/RXY
      EL=ASIN(SNE)
  170 ANGLE=AL+EL
      CALL ELIPS(SNE,CSE, ANGLE, AP,BP,EX,EY,X3,Y3,IX,IY)
      GO TO 190
C*****
C     CALCULATE THE COORDINATES OF CIRCLE
  180 CALL POINT(AL,ST,X3,Y3,IX,IY)
  190 IF((IX.LT.0. OR.IX.GT.85).OR.(IY.LT.0. OR.IY.GT.52)) GO TO 250
      IA(IX,IY)=1H=
      GO TO (200,200,230,230),IS
  200 A3=(S3-X3*SY-Y3*SX )/SZ
      PRINT 210,X3,Y3,A3
```

```
  210 FORMAT(1X,$A23=$,F9.6,10X,$A13=$,F9.6,10X,$A33=$,F9.6)
      IF( A3 .LT. 0.0) GO TO 220
      IA(IX,IY)=1HP
      GO TO 250
  220 IA(IX,IY)=1HN
      GO TO 250
  230 PRINT 240,X3,Y3
  240 FORMAT(11X,$A23=$,F9.6, 8X,$A13=$,F9.6, 8X,$SIGN OF A33 UNKNOWN$)
  250 GO TO (260,270,280,290),LS
C*****CALCULATE A12,A32 REGION BY SUN SENSOR INFORMATION
  260 ANGLF=AL+FL
      CALL ELIPS(SNF,CSF, ANGLF, AQ,BQ,FX,FY,X2,Y2,IX,IY)
      GO TO 300
  270 CALL POINT(AL,SR,X2,Y2,IX,IY)
      GO TO 300
  280 IX=IXB
      IY=IY1
      Y2=Y1
      X2=S2
      GO TO 300
  290 Y2=SN*X1+SL
      IY=Y2*25+26.5
      X2=X1
      IX=IX1
  300 IF((IX.LT.0. OR.IX.GT.85).OR.(IX.LT.0. OR.IY.GT.52)) GO TO 360
      IB(IX,IY)=1H=
C******
      GO TO(310,310,340,340),LS
  310 A2=(S2-X2*SZ-Y2*SX)/SY
      PRINT 320,X2,Y2,A2
  320 FORMAT(1X,$A32=$,F9.6, 8X,$A12=$,F9.6,8X,$A22=$,F9.6,/)
      IF( A2 .LT. 0.0) GO TO 330
      IB(IX,IY)=1HP
      GO TO 360
  330 IB(IX,IY)=1HN
      GO TO 360
  340 PRINT 350,X2,Y2
  350 FORMAT(1X,$A32=,$,F9.6,10X,$A12=$,F9.6,9X,$SIGN OF A22 UNKNOWN$)
C*****
C      INFORMATION FROM EARTH SENSOR ONLY
  360 IF(GA .NE. 0.0) GO TO 370
      NS=1
      GO TO 380
  370 NS=2
C      PLOT THE CELESTIAL SPHERE.
  380 IA(IX1,IY1)=1H*
      IB(IX1,IY1)=1H*
      GO TO ( 390,410),NS
C      IF GAMA IS ZERO, THE REGION IS A STRAIGHT LINE, A23=CC
C      PLOT THIS LINE
  390 IX=IXC
      IF(IY1.GT.IYM)GO TO 470
      IY=IY1
      X=CC
```

```
      Y=Y1
      PRINT  400,X,Y
  400 FORMAT(50X,$A23=$,F9.6, 8X,$A13=$,F9.6,6X,$SIGN OF A33 UNKNOWN$)
      GO TO 430
C*****
C     IF IT IS ANOTHER ELLIPSE (BY EARTH SENSOR INFORMATION)
C     CALCULATE THE COORDINATES OF THIS ELLIPSE
  410 CALL ELIPS(0.0,1.0,AL,AE,BE,CE,0.0,X,Y,IX,IY)
      AZ3=(CC-X*CNG)/SNG
      PRINT 420, X,Y,AZ3
  420 FORMAT(65X,$A23=$,F9.6, 8X,$A13=$,F9.6,6X,$A33=$,F9.6)
  430 IF((IX.LT.0. OR.IX.GT.85).OR.(IY.LT.0. OR.IY.GT.52))GO TO 470
      IA(IX,IY)=1HI
      IF(GA.EQ.0.0) GO TO 470
      IF(LP.EQ.0) GO TO 450
      IF(AZ3.GE.0) GO TO 440
      GO TO 470
  440 IA(IX,IY)=1H.
      GO TO 470
  450 IF(AZ3.GT.0)GO TO 460
      IA(IX,IY)=1H-
      GO TO 470
  460 IA(IX,IY)=1H+
C******
  470 AL=AL+DAL
      RP=1
      CALL POINT(AL,RP,X1,Y1,IX1,IY1)
      IF(AL.GT.6.3) GO TO 480
      IF(ISUN.EQ.0 ) GO TO 380
      GO TO (170,180,140,150),IS
  480 IF(ISUN.EQ.0) GO TO 570
      IF(IS.NE.1) GO TO 490
      IX=(-EX+1)*42+1.5
      IY=-EY*25+26.5
      IA(IX,IY)=1HE
  490 IF(LS.NE.1) GO TO 500
      IX=(-FX+1)*42+1.5
      IY=-FY*25+26.5
      IB(IX,IY)=1HF
  500 PRINT 510
  510 FORMAT(1H1)
      PRINT 520,ISUN,CI1,CI2,EAL,EBI
  520 FORMAT(1X,$ISUN=$,I2,$, ALFA=$,F8.3,$ DEG, BETA=$,F8.3,$ DEG, AZ=$
     1,F8.3,$DEG, EL=$,F8.3,$ DEG. P FOR+A22(OR+A33),N FOR-A22(OR-A33)$)
  530 DO 550 J=1,52
      PRINT 540,(IB(I,J),I=90,1,-1)
  540 FORMAT(20X,100A1)
  550 CONTINUE
      PRINT 560,IDAY,ITIM
  560 FORMAT(64X,$A12+$,35X,I4,$,  $,I6,/)
      IF(LP .GE. 3) GO TO 690
      GO TO 590
  570 PRINT 510
      PRINT 580
```

```
580 FORMAT(1X,$NO SUN SENSOR INFORMATUON,NO PLOT FOR A12,A32$)
590 PRINT 510
    IF(LP.GT.0) GO TO 610
    PRINT 600,GAD,CC,ROMD
600 FORMAT(1X,$GAMA=$,F6.2,$ DEG,  C23=$,F7.4,$  ROM=$,F7.3,$ DEG, IF
   1GAMA NOT 0, + FOR +A33,- FOR -A33; IF GAMA=0,1 FOR BOTH$)
    GO TO 660
610 GO TO (620,640),LP
620 PRINT 630,GAD,CC
630 FORMAT(1X,$GAMA=$,F6.2,$  DEG, ROM=0, C23<$,F6.3,$,  . IS FOR +A33
   1I FOR - A33 IF GAMA NOT 0; WHEN GAMA=0, I IS FOR EITHER$)
    GO TO 660
640 PRINT 650,GAD,CC
650 FORMAT(1X,$GAMA=$,F6.2,$ DEG, ROM=180 DEG,C23>$,F6.3,$,  . FOR +A33
   1I FOR -A33 IF GAMA NOT 0; WHEN GAMA=0, I FOR EITHER$)
660 DO 670 J=1,52
    PRINT 540,(IA(I,J),I=90,1,-1)
670 CONTINUE
    PRINT 680,IDAY,ITIM
680 FORMAT(64X,$A13+$,35X,I4,$,  $,I6,/)
690 RETURN
    END
    SUBROUTINE TEST(TX,TY,TZ,JS)
C
C     CONDITION FOR ELLIPSE, CIRCLE, OR STRAIGHT LINE IN PLOT
C       (1) WHEN JS =1, IT IS AN ELLIPSE
C       (2) WHEN JS=2, IT IS A  CIRCLE
C       (3) WHEN JS =3, IT IS A STRAIGHT LINE PARALLEL TO Y AXIS
C       (4) WHEN JS=4, IT IS A STRAIGHT LINE WITH SLOPE - TY/TX.
C
    ATZ=ABS (TZ)
    IF(ATZ.NE.0.0) GO TO 20
    IF(TX.NE.0.0) GO TO 10
    JS=3
    GO TO 40
 10 JS=4
    GO TO 40
 20 IF (ATZ.EQ.1) GO TO 30
    JS=1
    GO TO 40
 30 JS=2
 40 RETURN
    END
    SUBROUTINE ELIPS(SNU,CSU,ANGL,A,B,CX,CY,XE,YE,IX,IY)
C
C     CALCULATE THE COORDINATES OF ELLIPSE.
C     A AND B ARE THE HALF LENGTH OF MAJOR AND MINOR AXIS
C     ANGL REPRESENTS CURRENT POLAR ANGLE
C     (CX,CY) REPRESENTS THE CENTER OF ELLIPSE
C     (XE,YE)ARE CURRENT POINT REPRESENTED AS (IX,IY) FOR PLOT
C
    SAN=SIN(ANGL)
    CAN=COS(ANGL)
    XEX=A*CAN
```

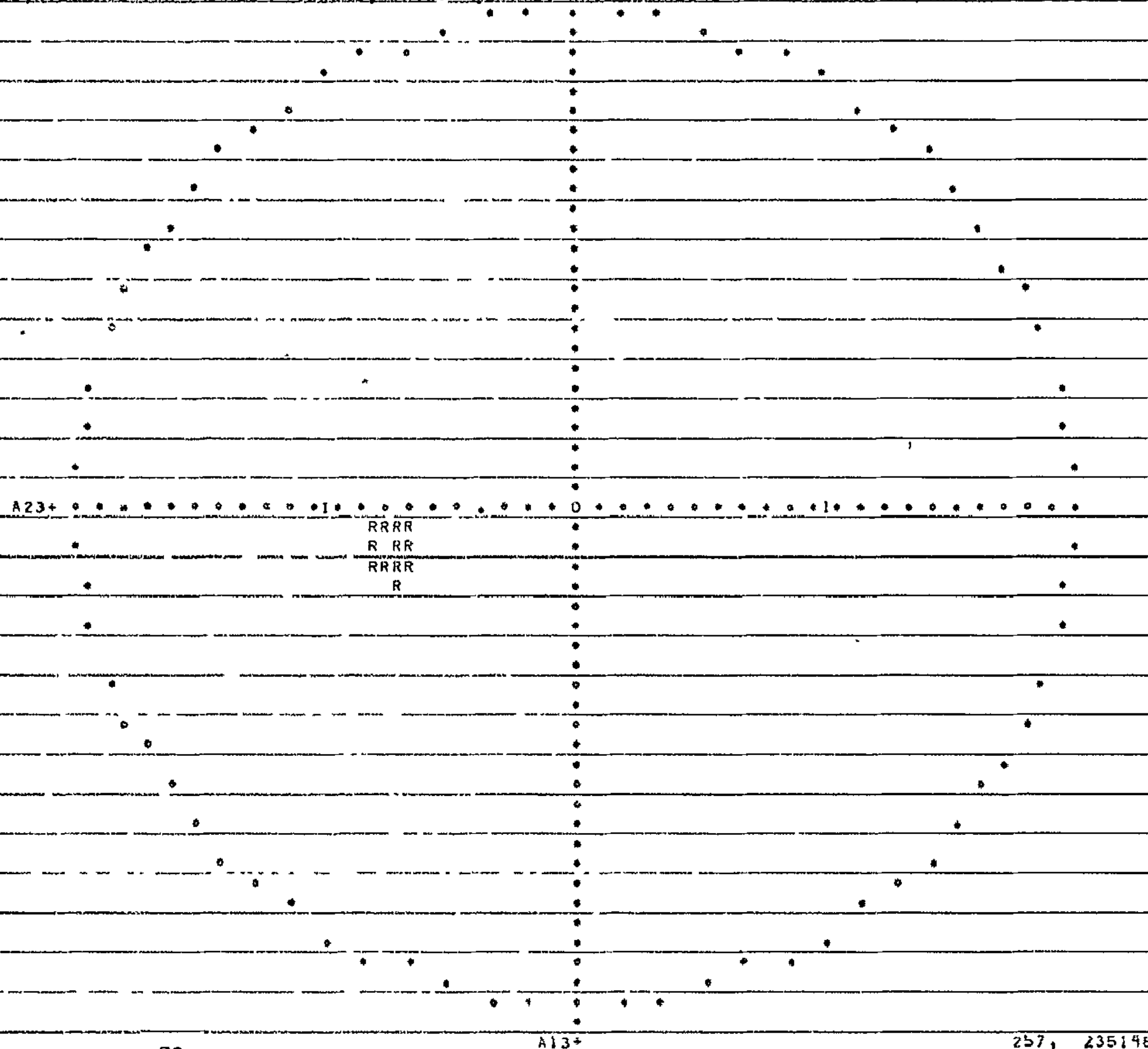
```
      YEY=B*SAN
      XP=XEX*CSU-YEY*SNU
      YP=XEX*SNU+YEY*CSU
      XE=XP-CX
      YE=YP-CY
      IX=(XE+1)*42+1.5
      IY=YE*25+26.5
      RETURN
      END
      SUBROUTINE POINT(AL,RP,X,Y,IX,IY)
C
C     AL REPRESENTS POLAR ANGLE
C     (X,Y) REPRESENTS CURRENT POINT OF A CIRCLE.
C     RR REPRESENTS RADIUS
C
      SNL=SIN(AL)
      CSL=COS(AL)
      X=RP*CSL
      Y=RP*SNL
      IX=(X+1)*42+1.5
      IY=Y*25+26.5
      RETURN
      END
```

APPENDIX C SAMPLE OF OUTPUT

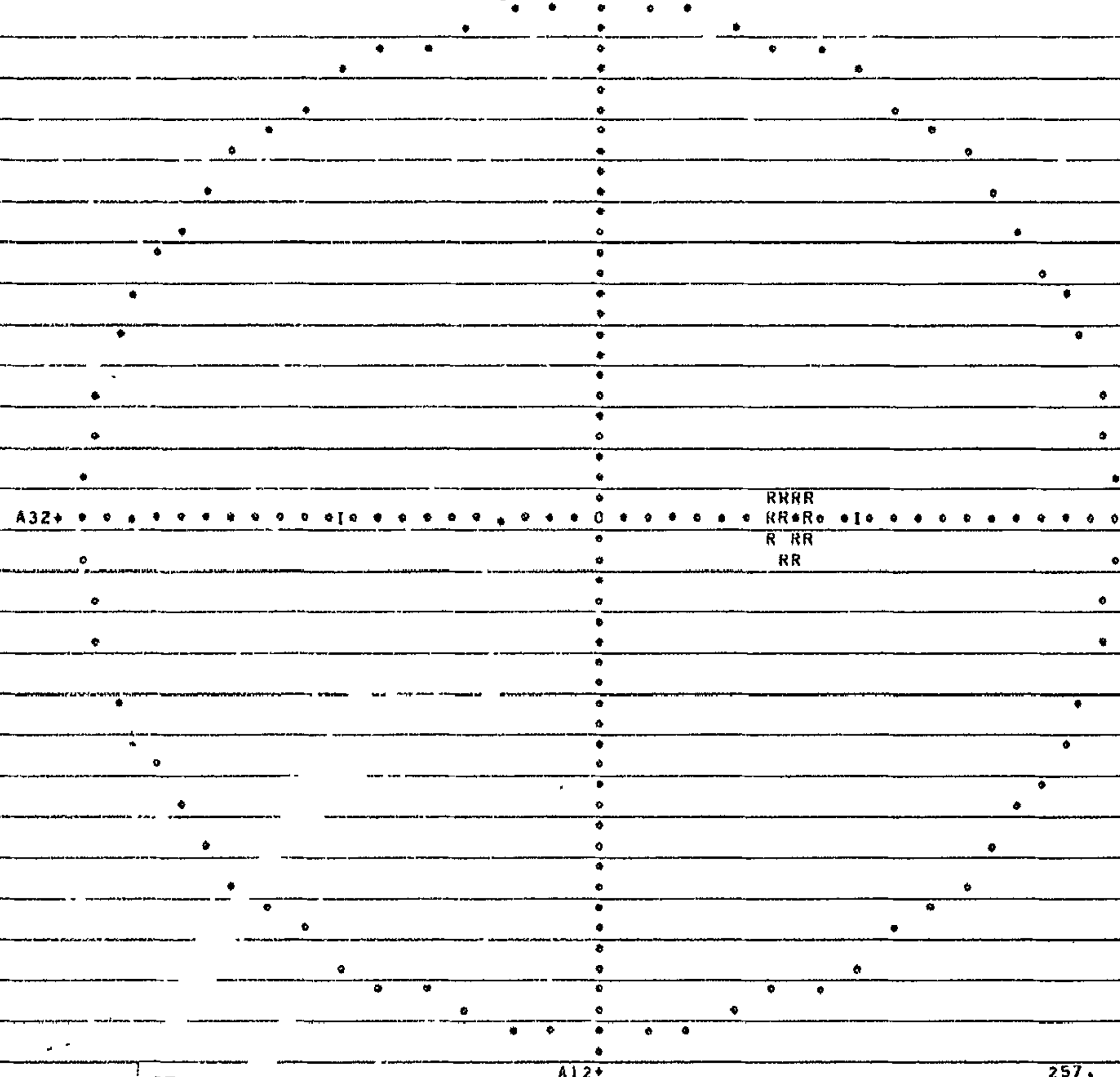
```
DAY =  257,   HR=  23,   MIN=  51,   SEC=  48,   ORBIT= 556 , H=   267.1000
***********
EARTH SENSOR INFORMATION GIVES:
 GAMA(IN DEGREE)=-23.570000,   ET(IN DEGREE)= -5.050000,  ROM(IN DEGREE)=  56.000000
***********
ISUN= 2,  SUN SENSOR INFORMATION GIVES:
 AZIMUTH= -43.00000  DEG,   ELEVATION= -39.90000  DEG.
 S1I=  .623941,  S2I= -.521696,  S3I= -.581834,  ALFA(IN DEGREE)=  95.080000,  BETA(IN DEGREE)=   1.185000
 THE DIRECTION COSINES OF SUN LINE W.R.T. SPACECRAFT  AXES ARE    -.97492669,      .10801531,      .19455232
 THE DIRECTION COSINES OF SUN LINE W.R.T.GEOCENTRIC AXIS:  -.99585902,   .02068067,   .08852740
***********
***********
 A13(1)= -.08797894,  A23(1)= -.42776804,  A33(1)= -.8995.668
 A13(2)=  .08797894,  A23(2)=  .36844860,  A33(2)=  .9254.572
 ROLL ANGLE TH(1)= -25.32600 DEG., PITCH ANGLE TE(1)= 174.41437DEG.
 ROLL ANGLE TH(2)=  21.61997 DEG.,PITCH ANGLE TE(2)=  -5.43042DEG.
***********
***********
 DEVIATION IN A13,   DA13(1)=  .01738,  DA13(2)=  .01738
 DEVIATION IN A23,   DA23(1)=  .01506,  DA23(2)=  .01549
 DEVIATION IN A33,   DA33(1)=  .00729,  DA33(2)=  .00632
***********
***********
 DIRECTION COSINES OF LOCAL VERTICAL ARE:   .08797894   .36844860   .92547572
```

```
A13(2)=  .08797894  A23(2)=  .36844860  A33(2)=  .925475,2
A11(2)=  .98696566  A21(2)= -.05651465  A31(2)= -.120727,9
A12(2)=  .00790463  A22(2)=  .92438405  A32(2)= -.367739,1
ROLL ANGLE  TH(2)=  21.61997 DEG.,PITCH ANGLE TE(2)=  -5.43042 DEG.,  YAW ANGLE PH(2)=   3.48524 DEG.
************
************
STANDARD DEVIATION OF A12,A32 ARE,DA12(2)=  .01936, DA32(2)=  .015313
```

ERROR ELLIPSE OF A23,A13: CENTER AT A23= .36845, A13= .08798, AXIS PARALLEL TO A23= .04647, AXIS PARALLEL TO A13= .05213



ERROR ELLIPSE OF A32,A12: CENTER AT A32= -.36774, A12= .00790, AXIS PARALLEL TO A32= .04594, AXIS PARALLEL TO A12= .05807



257, 235148

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.....SUMMARY OF RUNS FOR ORBIT NO. 556.00 .. ..

NO.	DATE	HR.	MIN	SEC	ORBIT	GAMA	ROMD	THI	THETA	PHI	THI	THETA	PHI
1	257.00	23.00	49.00	10.00	556.00	-22.82	54.40	.00	.00	.00	-25.96	-176.34	-2.37
2	257.00	23.00	51.00	48.00	556.00	-23.57	56.00	.00	.00	.00	21.62	-5.43	2.47
3	257.00	23.00	54.00	6.00	556.00	-22.82	57.60	.00	.00	.00	22.11	-7.22	3.32
4	257.00	23.00	56.00	17.00	556.00	-20.59	56.00	-22.29	174.54	.00	18.73	-5.33	.00
5	257.00	23.00	58.00	12.00	556.00	-18.36	57.60	-18.72	175.53	.00	17.90	-4.45	.00

****