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JPL CONTRACT 950670 / SUBNAS 7-100  
PRF-3807-55-285

**PURDUE UNIVERSITY**  
**SCHOOL OF ELECTRICAL ENGINEERING**

**INVESTIGATION OF OPTIMIZATION  
OF ATTITUDE CONTROL SYSTEMS**

**Volume II**

**R. SRIDHAR , G. C. AGARWAL , R. M. BURNS ,  
D. M. DETCHMENDY, E. H. KOPF JR. , R. MUKUNDAN**

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## PREFACE

21764

This volume contains listings of the computer programs used in solving the various example problems. Explanations are given at the beginning of each Appendix concerning the programs contained therein; descriptions of the general makeup of the programs and their interrelations are provided. It is not intended that this volume be independent of the first volume. Rather, use of the program listings contained herein will require that reference be made to the appropriate portions of the first volume.

A handwritten signature in cursive script, appearing to read "J. L. Johnson", with a large arrow pointing upwards and to the right from the end of the signature.

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## APPENDIX G

This appendix contains the program for space vehicle simulation. The standard simulation program is Main Program 1 with the following subroutines:

1. SVAD
2. TRIG
3. TA
4. ATOB
5. TB
6. BTOA
7. EULER
8. CHECK
9. DOT
10. WDOT
11. ANGDOT
12. DATA
13. PARA
14. PRINT

The standard routine AMRK is also required. An option is available which requires the use of an IBM 1620 computer

with a 1443 line printer. For this option Main Program 1 is replaced by Main Program 2 and the Fortran II Routine PUNCH is added. The output from this program is the same as for the standard system with the addition of some BCD punched hollerith cards. These cards are then read into an IBM 1620 under the control of the SPS program "punch routine". This 1620 program will plot the motion of the vehicle roll axis in the initial system and the motion of the sun in the body system.

Another option is available through the replacement of Main Program 1 by Main Program 3. The printout will be similar to Main 1 output except that a plot of the rates and angles will be included. SHARE routine PLT1 must be used with this option.

Conventional sun acquisition is accomplished by control with the following subroutines:

1. CL (conventional sun acquire)
2. SUN
3. GYRO
4. SGN

Cruise mode derived rate control is accomplished by control with:

1. CL (derived rate control)

- 2. SUN
- 3. EARTH
- 4. DR
- 5. SGN

```

* LABEL
C MA1
C1
C2 MAIN PROGRAM NO. 1
C3
4 DIMENSION NOP(4),TOL(7),DUMMY(44),PARAM(15)
5 FORMAT(3I5)
6 COMMON DUMMY,PARAM
7 READ INPUTTAPE 5,5,MROLL,IRUN,NPAR
8 IF(NPAR) 11,11,10
9 FORMAT(3E15.0)
10 READ INPUTTAPE 5,9,(PARAM(I),I=1,NPAR)
11 CALL DATA(PERIOD,TOL,NOP,CODE)
12 CALL PARA(PERIOD,TOL,NOP,CODE)
13 DO 16 I=1,IRUN
14 IN=I-1
15 CALL SVAD(IN,PERIOD,TOL,NOP)
16 CALL PRINT(IN,NOP,MROLL,CODE)
17 CALL EXIT
END
* LABEL
CSVAD
C1
C2 SPACE VEHICLE ATTITUDE DYNAMICS
C3
4 SUBROUTINE SVAD(IN,PERIOD,TOL,NOP)
5 DIMENSION X(7),AN(3),AI(3,3),A(3,3),XD(7),TOL(7),NOP(4),TEMPS(65),
1EUB(6),ELB(6)
6 COMMON NSEQ,X,AN,AI,A,COSPHI,COSTH,COSPSI,SINPHI,SINTH,SINPSI,XD,N
1OP1,NOP2
C7
C8 F CARD
C9
F DOT
C10
C11 FIRST TIME SET-UP
C12
13 IF(IN) 14,14,40
14 NOP1=NOP(1)
15 NOP2=NOP(2)
16 TOL1=TOL(1)
17 TOL2=TOL(2)
18 HMAX=TOL(3)
19 HMIN=TOL(4)
20 EU=TOL(5)
21 EL=TOL(6)
22 HIN=TOL(7)
23 XD(1)=HIN
24 DO 25 I=1,65
25 TEMPS(I)=0.

```



```

C26
C27 DATA CHECK
C28
29 FORMAT(15H1 DATA ERROR)
30 IF(HMAX-HMIN) 35,31,31
31 IF(HMAX-HIN) 35,32,32
32 IF(HIN-HMIN) 35,33,33
33 IF(PERIOD-HIN) 35,34,34
34 IF(EU-EL) 35,35,40
35 WRITE OUTPUT TAPE 6,29
36 CALL EXIT
C37
C38 INTEGRATE
C39
40 T=X(1)
41 IF(IN) 43,43,42
42 IF(NOP(3)) 43,43,44
43 CALL TRIG
44 CALL CHECK(TOL1,TOL2)
45 DO 47 I=1,6
46 EUB(I)=EU*ABSF(X(I+1)) + EU
47 ELB(I)=EL*ABSF(X(I+1)) + EL
48 IF(T+PERIOD-X(1)-XD(1)) 49,50,50
49 XD(1)=T+PERIOD-X(1)
50 IF(NOP1)52,52,51
51 CALL CL
52 CALL DOT
53 CALL AMRK(X,XD,DOT,6,0,EUB,ELB,HMAX,HMIN,TEMPS)
54 IF(T+PERIOD-X(1)) 58,58,43
C55
C56 OPTIONS
C57
58 IF(NOP(3)) 60,60,59
59 CALL TRIG
60 IF(NOP(4)) 950,950,61
61 IF(NSEQ) 62,62,64
62 CALL TA
63 GO TO 950
64 CALL TB
950 RETURN
END
* LABEL
CTRIG
C1
C2 SUBROUTINE FOR EULER ANGLE TRIG FUNCTIONS
C3
4 SUBROUTINE TRIG
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
11,SINTH,SINPSI,XD,NOP1,NOP2

```

```

C7
8   COSPHI=COSF(PHI)
9   COSTH=COSF(THETA)
10  COSPSI=COSF(PHI)
11  SINPHI=SINF(PHI)
12  SINTH=SINF(THETA)
13  SINPSI=SINF(PHI)
950 RETURN
END
* LABEL
CTA
C1
C2   SUBROUTINE FOR MATRIX TRANSFORMATION , SEQ. A
C3
4   SUBROUTINE TA
5   DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7)
6   COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
11, SINTH,SINPSI,XD,NOP1,NOP2
C7
8   A(1,1)=COSPSI*COSPHI-SINPSI*COSTH*SINPHI
9   A(1,2)=COSPSI*SINPHI+SINPSI*COSTH*COSPHI
10  A(1,3)=SINPSI*SINTH
11  A(2,1)=-SINPSI*COSPHI-COSPSI*COSTH*SINPHI
12  A(2,2)=-SINPSI*SINPHI+COSPSI*COSTH*COSPHI
13  A(2,3)=COSPSI*SINTH
14  A(3,1)=SINTH*SINPHI
15  A(3,2)=-SINTH*COSPHI
16  A(3,3)=COSTH
950 RETURN
END
* LABEL
CATOB
C1
C2   CHANGE SEQUENCE A TO B
C3
4   SUBROUTINE ATOB(ER,TOL)
5   DIMENSION W(3),AN(3),AI(3,3),B(3,3),XD(7)
6   COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,B,COSPHI,COSTH,COSPSI,SINPH
11,SINTH,SINPSI,XD,NOP1,NOP2
C7
8   L=1
9   SINTH=B(2,3)
10  COSTH=1.-SINTH**2
11  COSTH=SQRTF(COSTH)
12  SINPSI=-B(1,3)/COSTH
13  SINPHI=-B(2,1)/COSTH
14  COSPSI=B(3,3)/COSTH
15  COSPHI=B(2,2)/COSTH
16  ER=COSPSI*SINPHI+SINPSI*SINTH*COSPHI
17  ER=ABSF(ER-B(1,2))

```

```

18 GO TO (19,30),L
19 IF(TOL-ER) 23,950,950
C20
C21 WRONG QUADRANT
C22
23 SINPHI=-SINPHI
24 COSPHI=-COSPHI
25 SINPSI=-SINPSI
26 COSPSI=-COSPSI
27 COSTH=-COSTH
28 L=2
29 GO TO 16
30 IF(TOL-ER) 31,950,950
31 WRITE OUTPUTTAPE 6,33
32 CALL EXIT
33 FORMAT(16H1 ERROR A TO B)
950 RETURN
END
* LABEL
CEULER
C1
C2 QUADRANT ANALYSIS AND EULERANGLE
C3
4 SUBROUTINE EULER
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
11,SINTH,SINPSI,XD,NOP1,NOP2
C7
8 TPHI=ABSF(SINPHI/COSPHI)
9 TTH=ABSF(SINTH/COSTH)
10 TPSI=ABSF(SINPSI/COSPSI)
11 PHI=ATANF(TPHI)
12 THETA=ATANF(TTH)
13 PSI=ATANF(TPSI)
14 PI=3.1415927
15 S=SINPHI
16 C=COSPHI
17 ANG=PHI
18 L=1
19 IF(S) 23,20,20
20 IF(C) 21,27,27
21 ANG=PI-ANG
22 GO TO 27
23 IF(C) 26,24,24
24 ANG=-ANG
25 GO TO 27
26 ANG=-PI+ANG
27 GO TO (28,34,40),L
28 PHI=ANG
29 L=2

```

```

30 C=COSTH
31 S=SINTH
32 ANG=THETA
33 GO TO 19
34 THETA=ANG
35 L=3
36 C=COSPSI
37 S=SINPSI
38 ANG=PSI
39 GO TO 19
40 PSI=ANG
950 RETURN
END
* LABEL
CTB
C1
C2 SUBROUTINE FOR MATRIX TRANSFORMATION , SEQ B
C3
4 SUBROUTINE TB
5 DIMENSION W(3),AN(3),AI(3,3),B(3,3),XD(7)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,B,COSPHI,COSTH,COSPSI,SINPHI,
SINTH,SINPSI,XD,NOP1,NOP2
C7
8 B(1,1)=COSPSI*COSPHI-SINPSI*SINTH*SINPHI
9 B(1,2)=COSPSI*SINPHI+SINPSI*SINTH*COSPHI
10 B(1,3)=-SINPSI*COSTH
11 B(2,1)=-COSTH*SINPHI
12 B(2,2)=COSTH*COSPHI
13 B(2,3)=SINTH
14 B(3,1)=SINPSI*COSPHI+COSPSI*SINTH*SINPHI
15 B(3,2)=SINPSI*SINPHI-COSPSI*SINTH*COSPHI
16 B(3,3)=COSPSI*COSTH
950 RETURN
END
* LABEL
CBTOA
C1
C2 CHANGE SEQUENCE B TO A
C3
4 SUBROUTINE BTOA(ER,TOL)
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPHI,
SINTH,SINPSI,XD,NOP1,NOP2
C7
8 L=1
9 COSTH=A(3,3)
10 SINTH=1.-COSTH**2
11 SINTH=SQRTF(SINTH)
12 SINPSI=A(1,3)/SINTH
13 COSPSI=A(2,3)/SINTH

```

```

14 SINPHI=A(3,1)/SINTH
15 COSPHI=-A(3,2)/SINTH
16 ER=COSPSI*SINPHI+SINPSI*COSTH*COSPHI
17 ER=ABSF(ER-A(1,2))
18 GO TO (19,30),L
19 IF(TOL-ER)23,950,950
C20
C21 WRONG QUADRANT
C22
23 SINTH=-SINTH
24 COSPHI=-COSPHI
25 SINPHI=-SINPHI
26 COSPSI=-COSPSI
27 SINPSI=-SINPSI
28 L=2
29 GO TO 16
30 IF(TOL-ER) 31,950,950
31 WRITE OUTPUT TAPE 6,33
32 CALL EXIT
33 FORMAT(16H1 ERROR B TO A)
950 RETURN
END
* LABEL
CCHK
C1
C2 SEQUENCE MAINTENANCE AND NORMALIZATION
C3
4 SUBROUTINE CHECK(TOL1,TOL2)
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPHI,
SINTH,SINPSI,XD,NOP1,NOP2
C7
C8 NORMALIZE EULER ANGLES
C9
10 PI=3.1415927
11 L=1
12 ANG=PHI
13 IF(PI-ABSF(ANG)) 14,14,18
14 IF(ANG) 15,15,17
15 ANG=ANG+2.*PI
16 GO TO 18
17 ANG=ANG-2.*PI
18 GO TO (19,23,27),L
19 PHI=ANG
20 L=2
21 ANG=THETA
22 GO TO 13
23 THETA=ANG
24 L=3
25 ANG=PSI

```

```

26 GO TO 13
27 PSI=ANG
C28
C29 CHECK FOR SEQUENCE CHANGE
C30
31 IF(NSEQ) 35,35,49
C32
C33 SEQUENCE A
C34
35 DIF=ABSF(THETA)
36 IF(TOL1-DIF) 37,42,42
37 DIF=PI-DIF
38 IF(TOL1-DIF) 950,42,42
C39
C40 SWITCH TO SEQ. B
C41
42 CALL TA
43 CALL ATOB(ER,TOL2)
44 CALL EULER
45 NSEQ=1
950 RETURN
C46
C47 SEQUENCE B
C48
49 DIF=ABSF(THETA)
50 DIF=ABSF(PI/2.-DIF)
51 IF(TOL1-DIF) 950,55,55
C52
C53 CHANGE TO SEQ. A
C54
55 CALL TB
56 CALL BTOA(ER,TOL2)
57 CALL EULER
58 NSEQ=0
59 GO TO 950
END
* LABEL
CDOT
C1
C2
C3 VECTOR AUXSUB FOR AMRK
C4
C5 NOTE--- X(1) = TIME , XD(1) = H
C6
7 SUBROUTINE DOT
8 DIMENSION X(7),AN(3),AI(3,3),A(3,3),XD(7),WD(3),ADOT(3)
9 COMMON NSEQ,X,AN,AI,A,COSPHI,COSTH,COSPSI,SINPHI,SINTH,SINPSI,XD,N
10P1,NOP2
C10
C11 DETERMINE TORQUES

```

```

C12
13   IF(NOP1) 14,14,18
14   CALL CL
C15
C16   ASSEMBLE BODY RATES
C17
18   CALL WDOT(WD)
19   DO 20 I=1,3
20   XD(I+4)=WD(I)
C21
C22   ASSEMBLE EULER RATES
C23
24   IF(NOP2) 28,28,32
C25
C26   UPDATE TRIG FUNCTIONS
C27
28   CALL TRIG
C29
C30   ENTER AT 32 FOR NO UPDATE
C31
32   CALL ANGDOT(ADOT)
33   DO 34 I=1,3
34   XD(I+1)=ADOT(I)
950  RETURN
      END
*    LABEL
CWDOT
C1
C2   BODY RATES
C3
4    SUBROUTINE WDOT(WD)
5    DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),TS(3,3),WD(3),C(3),EL(3)
501  COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
      11,SINTH,SINPSI,XD,NOP1,NOP2
6    DO 7 I=1,3
7    EL(I)=0.
8    DO 10 I=1,3
9    DO 10 J=1,3
10   EL(I)=EL(I)+AI(I,J)*W(J)
11   L=1
12   DO 14 I=1,3
13   DO 14 J=1,3
14   TS(I,J)=AI(I,J)
15   GO TO (47,32,38),L
16   DI=DET
C17
C18   COMPUTE COLUMN
C19
20   C(1)=AN(1)+W(3)*EL(2)-W(2)*EL(3)
21   C(2)=AN(2)+W(1)*EL(3)-W(3)*EL(1)

```

```

22      C(3)=AN(3)+W(2)*EL(1)-W(1)*EL(2)
C23
C24      WDOTS
C25
26      DO 27 I=1,3
27      TS(I,1)=C(I)
28      L=2
29      GO TO 47
30      WD(1)=DET/DI
31      GO TO 12
32      DO 33 I=1,3
33      TS(I,2)=C(I)
34      L=3
35      GO TO 47
36      WD(2)=DET/DI
37      GO TO 12
38      DO 39 I=1,3
39      TS(I,3)=C(I)
40      L=4
41      GO TO 47
42      WD(3)=DET/DI
950     RETURN
C43
C44
C45      DETERMINANT
C46
47      DET=TS(1,1)*TS(2,2)*TS(3,3)
48      DET=DET+TS(1,2)*TS(2,3)*TS(3,1)
49      DET=DET+TS(1,3)*TS(2,1)*TS(3,2)
50      DET=DET-TS(3,1)*TS(2,2)*TS(1,3)
51      DET=DET-TS(2,1)*TS(1,2)*TS(3,3)
52      DET=DET-TS(1,1)*TS(2,3)*TS(3,2)
53      GO TO (16,30,36,42),L
      END
*      LABEL
CADOT
C1
C2      EULER ANGLE DERIVATIVES (ANGDOT)
C3
4      SUBROUTINE ANGDOT(ADOT)
5      DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),ADOT(3)
6      COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
11, SINTH,SINPSI,XD,NOPI,NOP2
7      IF(NSEQ) 11,11,17
C8
C9      SEQUENCE A
C10
11      ADOT(1)=(W(1)*SINPSI+W(2)*COSPSI)/SINTH
12      ADOT(2)=W(1)*COSPSI-W(2)*SINPSI
13      ADOT(3)=W(3)-ADOT(1)*COSTH

```



```

950 RETURN
C14
C15 SEQUENCE B
C16
17 ADOT(1)=(W(3)*COSPSI-W(1)*SINPSI)/COSTH
18 ADOT(2)=W(3)*SINPSI+W(1)*COSPSI
19 ADOT(3)=W(2)-ADOT(1)*SINTH
951 RETURN
END
* LABEL
CADATA
C1
C2 DATA INPUT ROUTINE
C3
4 SUBROUTINE DATA(PERIOD,TOL,NOP,CODE)
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),NOP(4),TOL(7)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
11,SINTH,SINPSI,XD,NOP1,NOP2
7 FORMAT(A4)
8 FORMAT(3E15.0)
9 FORMAT(4I5)
10 FORMAT(15,2E15.0)
11 READ INPUT TAPE 5,7,CODE
12 READ INPUT TAPE 5,8,((AI(I,J),J=1,3),I=1,3)
13 READ INPUT TAPE 5,8,(W(I),I=1,3)
14 READ INPUT TAPE 5,8,PHI,THETA,PSI
15 READ INPUT TAPE 5,9,(NOP(I),I=1,4)
16 READ INPUT TAPE 5,8,TOL(3),TOL(4),TOL(7)
17 READ INPUT TAPE 5,8,TOL(5),TOL(6),TOL(1)
18 READ INPUT TAPE 5,10,NSEQ,TIME,PERIOD
19 TOL(2)=0.0001
950 RETURN
END
* LABEL
CPARA
C1
C2 PRINT OUT ROUTINE,PARAMETERS
C3
4 SUBROUTINE PARA(PERIOD,TOL,NOP,CODE,NPAR)
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),NOP(4),TOL(7),PARAM(15)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
11,SINTH,SINPSI,XD,NOP1,NOP2,PARAM
7 FORMAT(1H1/16X,15HJOB CODE NAME- ,A4///)
8 FORMAT(12X,27HINERTIA TENSOR (SLUG FT**2)///)
9 FORMAT(8X,3E11.4)
10 FORMAT(//17X,18HINITIAL CONDITIONS//)
11 FORMAT(53H TIME NSEQ PHI THETA PSI W1 W2 W3//)
12 FORMAT(F5.1,14,F8.3,F7.3,F7.3,1X,3F8.5)
13 FORMAT(//22X,7HOPTIONS//)
14 FORMAT(12X,27HNOP(1) NOP(2)NOP(3) NOP(4)//)

```

```

15  FORMAT(12X,14,317//)
16  FORMAT(17X,18HCONTROL PARAMETERS//)
17  FORMAT(52H  HMAX  HMIN  HIN  EU  EL  DZ//)
18  FORMAT(1X,6(1X,E8.2)//)
19  FORMAT(15X,9HPERIOD = ,E14.8)
20  WRITE OUTPUT TAPE 6,7, CODE
21  WRITE OUTPUT TAPE 6,8
22  WRITE OUTPUT TAPE 6,9,((AI(1,J),J=1,3),I=1,3)
23  WRITE OUTPUT TAPE 6,10
24  WRITE OUTPUT TAPE 6,11
25  WRITE OUTPUT TAPE 6,12, TIME, NSEQ, PHI, THETA, PSI, (W(I), I=1,3)
26  WRITE OUTPUT TAPE 6,13
27  WRITE OUTPUT TAPE 6,14
28  WRITE OUTPUT TAPE 6,15, (NOP(I), I=1,4)
29  WRITE OUTPUT TAPE 6,16
30  WRITE OUTPUT TAPE 6,17
31  WRITE OUTPUT TAPE 6,18, TOL(3), TOL(4), TOL(7), TOL(5), TOL(6), TOL(1)
32  WRITE OUTPUT TAPE 6,19, PERIOD
33  FORMAT(/9X,3E15.6)
34  IF(NPAR) 950,950,35
35  WRITE OUTPUT TAPE 6,33, (PARAM(I), I=1, NPAR)
950  RETURN
    END
    LABEL
+
CPRT
C1
C2  PROGRAM FOR PRINT OUT
C3
4  SUBROUTINE PRINT(IN, NOP, MROLL, CODE)
5  DIMENSION W(3), AN(3), AI(3,3), A(3,3), XD(7), NOP(4), TOL(7), Q(3)
6  COMMON NSEQ, TIME, PHI, THETA, PSI, W, AN, AI, A, COSPHI, COSTH, COSPSI, SINPH
11, SINTH, SINPSI, XD, NOP1, NOP2
7  FORMAT(1H1,17X,13HDATA FOR JOB ,A4//)
8  FORMAT(23X,A4,13//)
9  FORMAT(52H ANGULAR VELOCITIES  EULER ANGLES  POSITION OF SUN )
10  FORMAT(48H  (MRAD/SEC)  (RAD)  AXIS //)
11  FORMAT(20X,11HSEQUENCE = ,12//)
12  FORMAT(42H NO ROLL AXIS POSITION AVAILABLE, NOP(4)=0//)
13  FORMAT(7H  W1 = ,F9.4,4X,7HPHI = ,F7.4)
14  FORMAT(7H  W2 = ,F9.4,4X,7HTHETA= ,F7.4)
15  FORMAT(7H  W3 = ,F9.4,4X,7HPSI = ,F7.4//)
16  FORMAT(7H  W1 = ,F9.4,4X,7HPHI = ,F7.4,5X,4HX = ,F6.3)
17  FORMAT(7H  W2 = ,F9.4,4X,7HTHETA= ,F7.4,5X,4HY = ,F6.3)
18  FORMAT(7H  W3 = ,F9.4,4X,7HPSI = ,F7.4,5X,4HZ = ,F6.3//)
19  FORMAT(19X,7HTIME = ,F8.2)
1901 DO 1902 I=1,3
1902 Q(I)=1000.*W(I)
20  IF(IN) 21,21,30
21  IPG=0
22  IF(MROLL) 23,23,25

```

```
23   ICK=0
24   GO TO 29
25   IF(NOP(4)) 26,26,28
26   ICK=-1
27   GO TO 29
28   ICK=1
29   ILC=0
2901  GO TO 31
30   IF(8-ILC) 31,31,40
31   ISQ=2
32   IPG=IPG+1
33   ILC=0
34   WRITE OUTPUT TAPE 6,7,CODE
35   WRITE OUTPUT TAPE 6,8,CODE,IPG
36   WRITE OUTPUT TAPE 6,9
37   WRITE OUTPUT TAPE 6,10
38   IF(ICK) 39,40,40
39   WRITE OUTPUT TAPE 6,12
40   IF(ISQ-NSEQ) 41,43,41
41   WRITE OUTPUT TAPE 6,11,NSEQ
42   ISQ=NSEQ
43   IF(ICK) 44,44,49
44   WRITE OUTPUT TAPE 6,19,TIME
45   WRITE OUTPUT TAPE 6,13,Q(1),PHI
46   WRITE OUTPUT TAPE 6,14,Q(2),THETA
47   WRITE OUTPUT TAPE 6,15,Q(3),PSI
48   ILC=ILC+1
950  RETURN
49   WRITE OUTPUT TAPE 6,19,TIME
50   WRITE OUTPUT TAPE 6,16,Q(1),PHI,A(1,3)
51   WRITE OUTPUT TAPE 6,17,Q(2),THETA,A(2,3)
52   WRITE OUTPUT TAPE 6,18,Q(3),PSI,A(3,3)
53   GO TO 48
     END
```

```

* LABEL
CCL1
C1
C2 CONVENTIONAL SUN ACQUIRE
C3
4 SUBROUTINE CL
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),DUMMY(6),ADZ(3),T(3)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,
7 1 SLR,SSV,PYGG,GG,T,ADZ
8 CALL GYRO(PRO,YRO,RR0)
9 CALL SUN(PSS,YSS)
10 L=1
11 ER=-(PRO+PSS)
12 IF(ABSF(ER)-ADZ(L)) 13,13,15
13 AN(L)=0.
14 GO TO 18
15 X=T(L)
16 CALL SGN(X,X,ER)
17 AN(L)=X
18 GO TO (19,22,950),L
19 L=2
20 ER=-(YRO+YSS)
21 GO TO 12
22 L=3
23 ER=-RR0
24 GO TO 12
950 RETURN
END
* LABEL
CSUN
C1
C2 SUN SENSOR ROUTINE
C3
4 SUBROUTINE SUN(PSS,YSS)
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),DUMMY(6),ADZ(3),T(3)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,
7 1 SLR,SSV,PYGG,GG,T,ADZ
7 CALL TRIG
8 IF(NSEQ) 9,9,11
9 CALL TA
10 GO TO 12
11 CALL TB
12 X=A(1,3)
13 Y=A(2,3)
14 Z=A(3,3)
15 L=1
16 ARG=ABSF(Y/Z)
17 PA=ATANF(ARG)
18 ARG=ABSF(X/Z)
19 YA=ATANF(ARG)

```

```

20 IF(Z) 21,23,23
21 PA=3.1415927-PA
22 YA=3.1415927-YA
23 Q=-X
24 CALL SGN(PA,PA,Y)
25 CALL SGN(YA,YA,Q)
26 ANG=PA
27 IF(ABSF(ANG)-SLR) 31,28,28
28 IF(ABSF(ANG)-3.1415927+SLR) 29,29,33
29 CALL SGN(OUT,1.,ANG)
30 GO TO 36
31 OUT=(1./SLR)*ANG
32 GO TO 36
33 Q=3.1415927-ABSF(ANG)
34 CALL SGN(Q,Q,ANG)
35 OUT=(1./SLR)*Q
36 GO TO (37,41),L
37 L=2
38 ANG=YA
39 PSS=SSV*OUT*SQRTF(Y**2+Z**2)
40 GO TO 27
41 YSS=SSV*OUT*SQRTF(X**2+Z**2)
950 RETURN
END

```

CGYRO

```

C1
C2 GYROSCOPE OUTPUTS
C3
4 SUBROUTINE GYRO(PRO,YRO,RRO)
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),DUMMY(6),ADZ(3),T(3)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,
7 1 SLR,SSV,PYGG,RGG,T,ADZ
PRO=PYGG*W(1)
9 YRO=PYGG*W(2)
10 RRO=RGG*W(3)
950 RETURN
END
* LABEL

```

C SIGN

```

C1
C2 SUBROUTINE FOR SIGN TRANSFER
C3
4 SUBROUTINE SGN(X,Y,Z)
5 IF(Z) 7,6,6
6 X=ABSF(Y)
950 RETURN
7 X=-ABSF(Y)
951 RETURN
END

```

```

* LABEL
* LABEL
CDRCL
C1
C2 DERIVED RATE CONTROL
C3
4 SUBROUTINE CL
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),DUMMY(6),XD(7),ER(3),ST(3),TC(
13),TD(3),DRG(3),T(3),ADZ(3)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,SLR,SS
1V,ESV,ESLR,ESA,EAT,TC,TD,DRG,ER,ST,T,LT,ADZ,ONTM
7 CALL SUN(PSS,YSS)
8 IF(EAT-TIME) 9,9,11
9 CALL EARTH(ESO)
10 GO TO 12
11 ESO=0.
12 CALL DR(PRO,YRO,RRO)
13 NOP1=1
14 L=1
15 ER(L)=- (PRO+PSS)
16 IF(ABSF(ER(L))-ADZ(L)) 17,17,1801
17 AN(L)=ST(L)
1701 IF(IND) 22,22,1702
1702 XD(1)=RESTOR
1703 IND=0
18 GO TO 22
1801 IF(IND) 1802,1802,1803
1802 RESTOR=XD(1)
1803 IND=1
19 XD(1)=ONTM
20 CALL SGN(AN(L),T(L),ER(L))
21 AN(L)=AN(L)+ST(L)
22 GO TO (23,26,950),L
23 L=2
24 ER(L)=- (YRO+YSS)
25 GO TO 16
26 L=3
27 ER(L)=- (ESO+RRO)
28 GO TO 16
950 RETURN
END
* LABEL
CDR
C1
C2 DERIVED RATES
C3
4 SUBROUTINE DR(PRO,YRO,RRO)
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),DUMMY(6),XD(7),ER(3),ST(3),TC(
13),TD(3),DRG(3),T(3),ADZ(3)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,SLR,SS

```

```

1V,ESV,ESLR,ESA,EAT,TC,TD,DRG,ER,ST,T,TLT,ADZ
7   L=1
8   VAR=PRO
9   DT=TIME-TLT
10  ERR=AN(L)-ST(L)
11  IF(ABSF(ERR)-0.0001) 12,12,14
12  VAR=VAR-DT*VAR/TD(L)
13  GO TO 18
14  IF(AN(L)) 15,17,17
15  VAR=VAR-VAR*(DT/TC(L))-DRG(L)*(DT/TC(L))
16  GO TO 18
17  VAR=VAR-VAR*(DT/TC(L))+DRG(L)*(DT/TC(L))
18  GO TO(19,23,27),L
19  PRO=VAR
20  L=2
21  VAR=YRO
22  GO TO 10
23  YRO=VAR
24  VAR=RRO
25  L=3
26  GO TO 10
27  RRO=VAR
28  TLT=TIME
950 RETURN
    END
*   LABEL
CEART H
C1
C2   EARTH SENSOR
C3
4   SUBROUTINE EARTH(ESO)
5   DIMENSION W(3),AN(3),AI(3,3),A(3,3),DUMMY(6),XD(7),ER(3),ST(3),TC(
13),TD(3),DRG(3),T(3),ADZ(3)
6   COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,SLR,SS
1V,ESV,ESLR,ESA,EAT,TC,TD,DRG,ER,ST,T,LT,ADZ
7   ARG=ABSF(A(2,1)/A(1,1))
8   EA=ATANF(ARG)
9   IF(A(1,1)) 10,11,11
10  EA=3.1415927-EA
11  Q=-A(2,1)
12  CALL SGN(EA,EA,Q)
13  IF(ESA-ABSF(EA)) 14,16,16
14  ESO=0.
15  GO TO 950
16  IF(ABSF(EA)-ESLR) 19,17,17
17  CALL SGN(ESO,ESV,EA)
18  GO TO 950
19  ESO=(ESV/ESLR)*EA
950 RETURN
    END

```

\* LABEL

CMA2

C1

C2 MAIN PROGRAM NO 2

C3

4 DIMENSION NOP(4),TOL(7),DUMMY(44),PARAM(15)

5 FORMAT(3I5)

6 COMMON DUMMY,PARAM

7 READ INPUT TAPE 5,5,MROLL,IRUN,NPAR

8 IF(NPAR) 11,11,10

9 FORMAT(3E15.0)

10 READ INPUT TAPE 5,9,(PARAM(I),I=1,NPAR)

11 CALL DATA(PERIOD,TOL,NOP,CODE)

12 NOP(3)=1

13 NOP(4)=1

14 CALL PARA(PERIOD,TOL,NOP,CODE)

15 DO 19 I=1,IRUN

16 IN=I-1

17 CALL SVAD(IN,PERIOD,TOL,NOP)

18 CALL PRINT(IN,NOP,MROLL,CODE)

19 CALL PUNCH(IN,IRUN,CODE)

20 CALL EXIT

END

CMA3

C1

C2 MAIN PROGRAM NO. 3

C3

4 DIMENSION NOP(4),TOL(7),DUMMY(43),PARAM(15),T(100),X1(100),X2(100)  
1,X3(100),X4(100),X5(100),X6(100),TS(2020)

5 FORMAT(3I5)

6 COMMON NSEQ,DUMMY,PARAM

7 READ INPUT TAPE 5,5,MROLL,IRUN,NPAR

8 IF(NPAR) 11,11,10

9 FORMAT(3E15.0)

10 READ INPUT TAPE 5,9,(PARAM(I),I=1,NPAR)

11 CALL DATA(PERIOD,TOL,NOP,CODE)

12 CALL PARA(PERIOD,TOL,NOP,CODE,NPAR)

13 DO 23 I=1,IRUN

14 IN=I-1

15 CALL SVAD(IN,PERIOD,TOL,NOP)

16 CALL PRINT(IN,NOP,MROLL,CODE)

17 X1(I)=DUMMY(2)

18 X2(I)=DUMMY(3)

19 X3(I)=DUMMY(4)

20 X4(I)=DUMMY(5)\*1000.

21 X5(I)=DUMMY(6)\*1000.

22 X6(I)=DUMMY(7)\*1000.

23 T(I)=FLOATF(I)

24 FORMAT(1H1//50X,30HW1 = \*, W2 = 0,W3 = X, (MRAD)//)

25 WRITE OUTPUT TAPE 6,24



```

26 M=FLOATF(IRUN)/10. + .5
27 CALL PLT1(IERR,TS,3,M,0.,10.*FLOATF(I),-25.,25.,T,IRUN,X4,T,IRUN,X
15,T,IRUN,X6)
28 FORMAT(1H1//47X,33HPHI = *, THETA = 0, PSI = X (RAD)//)
29 WRITE OUTPUT TAPE 6,28
30 CALL PLT1(IERR,TS,3,M,0.,10.*FLOATF(M),-5.,5.,T,IRUN,X1,T,IRUN,X2
1,T,IRUN,X3)
31 CALL EXIT
END
* LABEL
CPCH
C1
C2 PUNCH ROUTINE 7090/94
C3
4 SUBROUTINE PUNCH(IN,IRUN,CODE)
5 DIMENSION W(3),AN(3),AI(3,3),A(3,3),N(18)
6 COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A
7 FORMAT(A4)
8 FORMAT(I3)
9 FORMAT(I2I4)
10 IF(IN) 11,11,15
11 PUNCH 7,CODE
12 PUNCH 8,IRUN
13 IC3=0
14 IC=0
15 IC=IC+1
16 IC3=IC3+1
17 K=6*(IC3-1)
18 N(K+1)=A(1,3)*25.+ .5
19 N(K+2)=A(2,3)*15.+ .5
20 N(K+3)=A(3,3)*100. + .5
21 N(K+4)=A(3,1)*25.+ .5
22 N(K+5)=A(3,2)*15.+ .5
23 N(K+6)=N(K+3)
24 IF(IRUN-IC) 27,27,25
25 IF(3-IC3) 26,26,950
26 IC3=0
27 L=K+6
28 PUNCH 9,(N(I),I=1,L)
950 RETURN
END
00001* PLOT ROUTINE
00002*
00003ENTRY RACDPA1
00004 TFM A+49,16,10
00005 TFM A+54,ADR1
00006 TDM PA1+7,0
00007 TD PA1+8,RM
00008 TR TITLE2-1,PA1-1
00009 K 0,971

```

00010	K	0,953
00011	TOM	TITLE2+8,0
00012	WA	TITLE1,900
00013	TR	PA1-1,RESET-1
00014	TFM	CTR,0,10
00015A	WA	PA1,900
00016	AM	CTR,1,10
00017	CM	CTR,8,10
00018	BNZ	A
00019	TFM	ADR1,YAW-1
00020	TFM	ADR2,CLR-1
00021	TFM	ADR3,R91-1
00022	BTM	SETUP,0
00023	B	B
00024*		
00025	DS	5
00026SETUP	TF	S1+11,ADR1
00027	TF	S2+11,ADR2
00028	TF	S3+11,ADR1
00029	TF	S4+11,ADR3
00030S1	TR	PA1-1,*
00031S2	TR	PA31-1,*
00032S3	TR	PA61-1,*
00033S4	TR	PA91-1,*
00034	BB	
00035*		
00036B	WA	PA1,900
00037	TFM	ADR1,YONE-1
00038	BTM	SETUP,0
00039	WA	PA1,900
00040*		
00041*		INITIALIZE PRINT AREA
00042*		
00043	TFM	CTR,0,10
00044	TFM	CTR1,0,10
00045	TFM	ADR4,PRINT-1
00046	TFM	ADR1,YMK-1
00047C	BTM	SETUP,0
00048	TF	S5+6,ADR4
00049S5	TR	*,PA1-1
00050	AM	CTR,1,10
00051	AM	CTR1,1,10
00052	AM	ADR4,240
00053	TFM	ADR1,Y-1
00054	CM	CTR,31,10
00055	BZ	D
00056	CM	CTR1,5,10
00057	BNZ	C
00058	TFM	CTR1,0,10
00059	TFM	ADR1,YMK-1

```

00060      B   C
00061*
00062D     TFM ADR1,Y-1
00063     TFM ADR2,PITCH1-1
00064     TFM ADR3,PITCH2-1
00065     BTM SETUP,0
00066     TR  PRINT+3359,PA1-1
00067     TFM ADR1,XL-1
00068     TFM ADR2,XR1-1
00069     TFM ADR3,XR2-1
00070     BTM SETUP,0
00071     TR  PRINT+3599,PA1-1
00072     TFM ADR1,Y-1
00073     TFM ADR2,XONE1-1
00074     TFM ADR3,XONE2-1
00075     BTM SETUP,0
00076     TR  PRINT+3839,PA1-1
00077     TDM PRINT+4080,0
00078      B   G
00079E     TFM CTR,0,10
00080     TFM ADR4,PRINT
00081F     TF  S6+6,ADR4
00082S6    WA  *,900
00083     AM  ADR4,240
00084     AM  CTR,1,10
00085     CM  CTR,31,10
00086     BNZ F
00087     TFM A+49,49,10
00088     TFM A+54,*+24
00089     B   A-24
00090     K   0,952
00091     WA  AC2,900
00092     WA  AC3,900
00093     WA  AC4,900
00094     WA  AC5,900
00095     RCTY
00096     WATYAC
00097     H
00098     BC1 ENTRY
00099     H
00100AC    DAC 25,SWITCH ONE ON FOR REPEAT@
00101AC2   DAC 30,THE --BODY-- PLOT SHOWS THE BO,
00102     DAC 30,DY FIXED AXES WITH THE TIP OF ,
00103     DAC 30,A UNIT LENGTH SUN POINTING VEC,
00104     DAC 30,TOR PLOTTED. THE --INERTIAL-- ,
00105AC3   DAC 30,PLOT SHOWS SIMULTANEOUSLY THE ,
00106     DAC 30,FIXED INERTIAL AXES WITH A UNI,
00107     DAC 30,T LENGTH VECTOR ON THE BODY RO,
00108     DAC 30,LL AXIS PLOTTED. ROLL AXES ARE,
00109AC4   DAC 30,ASSUMED COMING OUT OF THE PAPE,

```



00160	TFM G3+11,PA1
00161	TFM G4+6,PA1+3
00162	TFM CF+6,PA1
00163	TFM CFC,0,10
00164G3	BNF G5,*
00165G4	SF *
00166G5	AM CFC,1,10
00167CF	CF *
00168	AM CF+6,1
00169	AM G3+11,1
00170	CM CFC,3,10
00171	BNZ G3
00172G2	SF *
00173	AM CTR1,1,10
00174	AM G2+6,4
00175	AM G4+6,4
00176	AM CF+6,1
00177	AM G3+11,1
00178	TFM CFC,0,10
00179	CM CTR1,18,10
00180	BNZ G3
00181*	
00182*	
00183*	SET SYMBOLS AND ADRS
00184*	
00185	TFM CTR1,0,10
00186	TFM A1+11,PA1+11
00187	TFM A2+11,PA1+23
00188	TFM A3+11,PA1+3
00189	TFM A4+11,PA1+7
00190	TFM A5+11,PA1+15
00191	TFM A6+11,PA1+19
00192G6	AM CTR,1,10
00193	AM CTR1,1,10
00194	CM CTR,1,10
00195	BNZ G7
00196	TFM TF1+11,S
00197	TFM TF2+11,S
00198	B G10
00199G7	C NO,CTR
00200	BNZ A1
00201	TFM TF1+11,END
00202	TFM TF2+11,END
00203	B G10
00204A1	BNF G8,*
00205	TFM TF1+11,PER
00206	B A2
00207G8	TFM TF1+11,STAR
00208A2	BNF G9,*
00209	TFM TF2+11,PER

00210	B	G10
00211G9	TFM	TF2+11,STAR
00212*		
00213*		
00214*		PLOT POINTS,SET ADRS
00215*		
00216G10	TFM	ADR1,50
00217A3	TF	FAC,*
00218	TF	DV,FAC
00219	A	FAC,DV
00220	A	ADR1,FAC
00221A4	TF	FAC,*
00222	MM	FAC,240,9
00223	SF	95
00224	TF	DV,99
00225	TFM	FAC,PRINT+3600
00226	S	FAC,DV
00227	A	ADR1,FAC
00228	TFM	ADR2,170
00229A5	TF	FAC,*
00230	TF	DV,FAC
00231	A	FAC,DV
00232	A	ADR2,FAC
00233A6	TF	FAC,*
00234	MM	FAC,240,9
00235	SF	95
00236	TF	DV,99
00237	TFM	FAC,PRINT+3600
00238	S	FAC,DV
00239	A	ADR2,FAC
00240*		
00241*		TRANSMIT SYMBOLS
00242*		
00243	TF	TF1+6,ADR1
00244	TF	TF2+6,ADR2
00245TF1	TF	*,*
00246TF2	TF	*,*
00247	AM	A1+11,24
00248	AM	A2+11,24
00249	AM	A3+11,24
00250	AM	A4+11,24
00251	AM	A5+11,24
00252	AM	A6+11,24
00253	C	NO,CTR
00254	BZ	E
00255	CM	CTR1,3,10
00256	BNZ	G6
00257	B	G1
00258DV	DS	5
00259FAC	DS	5

00260NO	DS 3
00261CFC	DS 2
00262STAR	DAC 1,*,
00263PER	DAC 1,X,
00264END	DAC 1,E,
00265S	DAC 1,S,
00266	DENDENTRY

## APPENDIX H

The purpose of this appendix is to describe the computer programs used to obtain some of the data for examples 3.1 and 3.4.

These programs use the following standard subroutines.

INT, INTM: Floating point Adams-Moulton, Runge-Kutta Integration; IBM Share Program No. 602, FAP Coded Subroutine.

This program has two distinct entries, INT to set up the necessary bookkeeping, and INTM for performing the integration steps. The user must supply a FORTRAN subroutine (with the name DAUX) which evaluates the derivatives.

MATINV: Matrix Inversion with Accompanying Solution of Linear Equations (FORTRAN II), IBM Share Program No. 664.

PLT1: Fortran Plotting Subroutine, FAP coded, New York University Library Routine No. O-620424.

Program for Example 3.1

The main program performs the optimization as two one-dimensional problems. The subroutine DAUX describes the system equations. The subroutine PERIN integrates the system equations for given boundary conditions. The SIMPSN integration subroutine is used to calculate the performance



index. The subroutine PARFIT fits a parabolic curve  $Y=AX^2+BX+C$  through the given three points and finds the minimum of the hill.

#### Program for Example 3.4

The main program sets up the problem. The subroutine OPTMAL finds the optimal solution. The subroutine INTCON obtains the constants in the linear equations in the differential approximation technique for the cases  $U=bx_1$ ,  $U=bx_1 + cx_1^3$  and  $U=bx_1 + cx_2$ . The subroutine UBX1 finds the differential approximation solution for the case  $U=bx_1$  and obtains the exact solution by quasilinearization. The subroutines UBXCX3 and BX1CX2 do the same operations as the subroutine UBX1 for the cases  $U=bx_1 + cx_1^3$  and  $U=bx_1 + cx_2$  respectively. The subroutine DAUX describes the differential equations for all the cases mentioned above. The subroutine SKALE finds the scale factors for the graphs to be plotted. All the graphs are scaled to the interval -1 to +1. The subroutine PLOT sets up the proper variables for the PLT1 subroutine. The subroutine SIMPSN is the regular Simpson rule integration of a tabular function.

```

PROGRAM LISTING FOR EXAMPLE 3.1 SPECIFIC OPTIMAL CONTROL
PROBLEM          PARAMETER OPTIMIZATION METHOD

*   SYMBOL TABLE
*   LABEL
CGRADIENT TECHNIQUE MAIN PROGRAM
C   PLANT DDX+3DX+2X=U U=AX+BDX
C   PARAMETER OPTIMIZATION USING PARABOLIC FITTING METHOD
C   THIS PROGRAM IS FOR TWO DIMENSIONAL HILL.
C   OPTIMIZATION IS CARRIED OUTAS TWO ONE-DIMENSIONAL PROBLEMS.
C
COMMON T,API,BPI,CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI,FUNC,NMAX
DIMENSION T(50),XONE(501),XTWO(501),U(501),FUNC(501)
C
1   READ INPUTTAPE 5,500,CONA,CONB,TT,NMAX,NOPT,NOIT,TOLL
    FN=NMAX-1
    HGRID=TT/FN
2   READ INPUTTAPE 5,510,CONE,CTWO,A,B
148  WRITE OUTPUT TAPE 6,710
    WRITE OUTPUT TAPE 6,520,CONE,CTWO,TT,NMAX,HGRID,A,B,CONA,CONB
    WRITE OUTPUT TAPE 6,740,NOPT,NOIT,TOLL
101  AFB=A
102  BFB=B
145  AMIN=A
146  BMIN=B
    INDEX=0
    API=A
    BPI=B
3   CALL PERIN
4   PIAB=PI
C
C   SEARCH MINIMUM IN A DIRECTION
5   IF(ABSF(A)-0.02) 6,7,7
6   AA=A+0.0004
    GO TO 8
7   AA=A+0.02*A
8   API=AA
    INA=1
    CALL PERIN
9   PIAAB=PI
C   CALCULATE PARTIAL DERIVATIVE OF PI WRT A
10  DPIA=(PIAAB-PIAB)/(AA-A)
    WRITE OUTPUTTAPE 6,670
11  WRITE OUTPUTTAPE 6,530,A,B,PIAB,AA,PIAAB,DPIA
12  IF(PIAAB-PIAB) 13,13,15
15  AA=A
16  PIAAB=PIAB
    GO TO 13
13  AB=AA-DPIA*CONA
    API=AB
    INA=INA+1

```

```

17 CALL PERIN
18 PIABB=PI
WRITE OUTPUT TAPE 6,540,AB,B,PIABB
19 IF(PIABB-PIAAB) 20,21,21
20 DPIA=(PIABB-PIAAB)/(AB-AA)
22 AC=AB-DPIA*CONA
23 API=AC
INA=INA+1
24 CALL PERIN
25 PIACB=PI
26 WRITE OUTPUT TAPE 6,550,DPIA,AC,PIACB
IF(INA-NOPT) 27,29,29
IF(PIACB-PIABB) 28,29,29
27 AA=AB
28 PIAAB=PIABB
30 AB=AC
31 PIABB=PIACB
32 CONA=2.0*CONA
33 WRITE OUTPUT TAPE 6,560,CONA
GO TO 20

C
21 CONA=CONA/2.0
35 AC=AA-DPIA*CONA
36 API=AC
INA=INA+1
37 CALL PERIN
38 PIACB=PI
39 WRITE OUTPUT TAPE 6,570,DPIA,AC,PIACB,CONA
IF(INA-NOPT) 40,29,29
40 IF(PIACB-PIAAB) 29,41,41
41 AB=AC
42 PIABB=PIACB
GO TO 21

C PERFORM PARABOLIC FITTING
29 WRITE OUTPUT TAPE 6,580,AA,PIAAB,AB,PIABB,AC,PIACB
43 CALL PARFIT(AA,PIAAB,AB,PIABB,AC,PIACB,ALPHA,BETA,GAMMA)
44 AMIN=-BETA/(2.0*ALPHA)
45 API=AMIN
46 CALL PERIN
47 PIAMB=PI
48 WRITE OUTPUT TAPE 6,590,AMIN,BMIN,PIAMB
124 IF(PIAMB-PIAAB) 125,125,126
126 PIAMB=PIAAB
127 AMIN=AA
125 IF(PIAMB-PIABB) 128,128,129
129 PIAMB=PIABB
130 AMIN=AB
128 IF(PIAMB-PIACB) 131,131,132
132 PIAMB=PIACB
133 AMIN=AC

```

```

131 WRITE OUTPUT TAPE 6,690,AMIN,BMIN,PIAMB
C
C SEARCH MINIMUM IN B DIRECTION
49 A=AMIN
50 PIAB=PIAMB
51 IF(ABS(F(B)-0.02)52,53,53
52 BA=B+0.0004
GO TO 54
53 BA=B+0.02*B
54 API=A
55 BPI=BA
INB=1
56 CALL PERIN
57 PIABA=PI
C CALCULATE PARTIAL DERIVATIVE OF PI WRT B
58 DPIB=(PIABA-PIAB)/(BA-B)
WRITE OUTPUTTAPE 6,680
59 WRITE OUTPUT TAPE 6,600,A,B,PIAB,BA,PIABA,DPIB
60 IF(PIABA-PIAB) 61,61,62
62 BA=B
63 PIABA=PIAB
GO TO 61
61 BB=BA-DPIB*CONB
64 BPI=BB
INB=INB+1
65 CALL PERIN
66 PIABB=PI
67 WRITE OUTPUTTAPE 6,610,A,BB,PIABB
68 IF(PIABB-PIABA) 69,70,70
69 DPIB=(PIABB-PIABA)/(BB-BA)
71 BC=BB-DPIB*CONB
72 BPI=BC
INB=INB+1
73 CALL PERIN
74 PIABC=PI
75 WRITE OUTPUTTAPE 6,620,DPIB,BC,PIABC
IF(INB-NOPT)76,78,78
76 IF(PIABC-PIABB) 77,78,78
77 BA=BB
79 PIABA=PIABB
80 BB=BC
81 PIABB=PIABC
82 CONB=2.0*CONB
83 WRITE OUTPUT TAPE 6,630,CONB
84 GO TO 69
C
70 CONB=CONB/2.0
85 BC=BA-DPIB*CONB
86 BPI=BC
INB=INB+1

```

```

87 CALL PERIN
88 PIABC=PI
89 WRITE OUTPUT TAPE 6,640,DPIB,BC,PIABC,CONB
   IF(INP-NOPT) 90,78,78
90 IF(PIABC-PIABA) 78,91,91
91 BB=BC
92 PIABB=PIABC
93 GO TO 70
C
C     PERFORM PARABOLICFITTING
78 WRITE OUTPUTTAPE 6,650,BA,PIABA,BB,PIABB,BC,PIABC
95 CALL PARFIT(BA,PIABA,BB,PIABB,BC,PIABC,ALPHA,BETA,GAMMA)
96 BMIN=-BETA/(2.0*ALPHA)
97 BPI=BMIN
98 CALL PERIN
99 PIABM=PI
100 WRITE OUTPUT TAPE 6,660,AMIN,BMIN,PIABM
134 IF(PIABM-PIABA) 135,135,136
136 PIABM=PIABA
137 BMIN=BA
135 IF(PIABM-PIABB) 138,138,139
139 PIABM=PIABB
140 BMIN=BB
138 IF(PIABM-PIABC) 141,141,142
142 PIABM=PIABC
143 BMIN=BC
141 WRITE OUTPUT TAPE 6,700,AMIN,BMIN,PIABM
   INDEX=INDEX+1
C
C     TEST THE SOLUTION
103 ERRA=ABSF(AFB-AMIN)
104 ERRE=ABSF(BFB-BMIN)
105 DELA=TOLL*ABSF(AMIN)
106 DELB=TOLL*ABSF(BMIN)
107 IF(ERRE-DELA) 108,108,110
108 IF(ERRE-DELB) 200,200,110
C
C     NO GOOD GO BACK
110 AFB=AMIN
111 BFB=BMIN
112 A=AMIN
113 B=BMIN
114 PIAB=PIABM
115 IF(INDEX-NOIT) 5,200,200
200 WRITE OUTPUTTAPE 6,720
   API=AMIN
   BPI=BMIN
   CALL PERIN
201 NP=(NMAX-1)/20
202 DO 210 N=1,NMAX,NP

```

```

203 FN=N-1
204 TIME=FN*HGRID
205 WRITE OUTPUT TAPE 6,730,TIME,XONE(N),XTWO(N),U(N)
210 CONTINUE
WRITE OUTPUTTAPE 6,750,AMIN,BMIN,PI
109 GO TO 1
500 FORMAT(2F10.0,F5.0,3I5,F10.0)
510 FORMAT(4F10.0)
520 FORMAT(1H0/39H CONE CTWO TT NMAX HGRID A B CONA CONB /
1 2F10.5,F8.3,15,5F15.8/)
530 FORMAT(/24HABPIAB AA PIAAB DPIA /6F15.8/)
540 FORMAT(/12HAB B PIABB /3F15.8/)
550 FORMAT(/15HDPIA ACPIACB /3F15.8/)
560 FORMAT(/6HCONA /F15.8/)
570 FORMAT(/20HDPIA ACPIACB CONA /4F15.8/)
580 FORMAT(/28HAA PIAAB AB PIABB AC PIACB /6F15.8/)
590 FORMAT(/17HAMIN BMIN PIAMB /3F15.8/)
600 FORMAT(/24HAB PIAB BA PIABA DPIB /6F15.8/)
610 FORMAT(/12HABB PIABB /3F15.8/)
620 FORMAT(/15HDPIB BC PIABC /3F15.8/)
630 FORMAT(/6HCONB /F15.8/)
640 FORMAT(/20HDPIB BCPIABC CONB /4F15.8/)
650 FORMAT(/28HBA PIABA BBPIABB BCPIABC /6F15.8/)
660 FORMAT(/17HAMIN BMIN PIABM /3F15.8/)
670 FORMAT(///31HSEARCH MINIMUM IN A DIRECTION //)
680 FORMAT(///31HSEARCH MINIMUM IN BDIRECTION //)
690 FORMAT(/27HCORRECTED AMIN BMIN PIAMB /3F15.8/)
700 FORMAT(/27HCORRECTED AMIN BMIN PIABM /3F15.8/)
720 FORMAT(/29HSPECIFIC OPTIMAL TRAJECTORY //
1 6X1HT11X4HXONE16X4HXTWO18X1HU/)
730 FORMAT(F10.5,3F20.8)
740 FORMAT(/16HNOPT NOIT TOLL /2I10,F10.5/)
710 FORMAT(1H1/10X92HGRADIENT TECHNIQUE PARAMETEROPTIMIZATION USI
1NG PARABOLICFITTING METHOD G C AGARWAL /
258HSYSTEM DDX+3DX+2X=U,I=INT(0,1)(X**2+DX**2+U**2),U=AX+BDX /)
750 FORMAT(/8HABPI /3F15.8/)
1000 END
* SYMBOL TABLE
* LABEL
CDAUX DAUX SUBROUTINE FOR GRADIENT TECHNIQUE
SUBROUTINE DAUX
C
COMMON T,API,BPI,CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI,FUNC,NMAX
DIMENSION T(50),XONE(501),XTWO(501),U(501),FUNC(501)
C
1 UT=API*T(4)+BPI*T(5)
2 T(6)=T(5)
3 T(7)=-3.0*T(5)-2.0*T(4)+UT
4 RETURN
5 END

```

```

*      SYMBOL TABLE
*      LABEL
C      SUBROUTINE TO CALCULATE PERFORMANCE INDEX
      SUBROUTINE PERIN
C
      COMMON T,API,BPI,CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI,FUNC,NMAX
      DIMENSION T(50),XONE(501),XTWO(501),U(501),FUNC(501)
C
      DO 20 I=1,50
20     T(1)=0.0
1      T(2)=0.0
2      T(3)=HGRID
3      T(4)=CONE
4      T(5)=CTWO
5      CALL INT(T,2,2,0.,0.,0.,0.,0.,0.)
6      XONE(1)=CONE
7      XTWO(1)=CTWO
8      DO 10 I=2,NMAX
9      CALL INTM
11     XONE(I)=T(4)
12     XTWO(I)=T(5)
10     CONTINUE
14     DO 13 I=1,NMAX
15     U(I)=API*XONE(I)+BPI*XTWO(I)
13     FUNC(I)=XONE(I)**2+XTWO(I)**2+U(I)**2
16     CALL SIMPSN(FUNC,HGRID,NMAX,PI)
17     RETURN
18     END
*      SYMBOL TABLE
*      LABEL
C      PARFIT PARABOLIC FITTING SUBROUTINE THROUGH 3 POINTS
      SUBROUTINE PARFIT(A,X,B,Y,C,Z,ALPHA,BETA,GAMMA)
D      DIMENSION PAR(10,10),RPAR(10,1)
1      PAR(1,1)=A**2
2      PAR(1,2)=A
3      PAR(1,3)=1.0
4      PAR(2,1)=B**2
5      PAR(2,2)=B
6      PAR(2,3)=1.0
7      PAR(3,1)=C**2
8      PAR(3,2)=C
9      PAR(3,3)=1.0
10     RPAR(1,1)=X
11     RPAR(2,1)=Y
12     RPAR(3,1)=Z
D      CALL MATINV(PAR,3,RPAR,1,DET)
13     ALPHA=RPAR(1,1)
14     BETA=RPAR(2,1)
15     GAMMA=RPAR(3,1)
16     RETURN

```

```
17  END
*   SYMBOL TABLE
*   LABEL
CSIMPSN SIMPSON INTEGRATION SUBROUTINE
      SUBROUTINE SIMPSN(FUNC,HGRID,NMAX,SIMP)
C     WHERE HGRID=LENGTH OF SUBINTERVAL,NMAX=NUMBER OF POINTS
C     NMAX MUST BE ODD
C
      DIMENSION FUNC(501)
      K=(NMAX-1)/2
      SUMA=0.0
      DO 15 J=1,K
15    SUMA=SUMA+FUNC(2*J)
      SUMB=0.0
      DO 16 J=2,K
16    SUMB=SUMB+FUNC(2*J-1)
      SIMP=(4.*SUMA+2.*SUMB+FUNC(NMAX)+FUNC(1))*HGRID/3.
100  RETURN
      END
```

THIS PROGRAM ALSO REQUIRES SUBROUTINES INT,INTM,MATINV



PROGRAM LISTING FOR EXAMPLE 3.4 SPECIFIC OPTIMAL CONTROL PROB.  
METHOD OF DIFFERENTIAL APPROXIMATION AND QUASILINEARIZATION

```

*      SYMBOL TABLE
*      LABEL
CMAIN  PROGRAM THIS PROGRAM SOLVES THE OPTIMAL SOLUTION THEN FINDS
C      THE FEEDBACK COEFFICIENTS USING DIFF APPROX  FINALLY CHECKSTHE
C      FEEDBACK COEFFICIENTS USING QUASILINEARIZATION  ALL THE RESULTS
C      ARE PLOTTED FOR COMPARISION
C      PLANT DDX+3DX+2X+0.5X**3=U
C
0COMMON T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
0DIMENSION T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),
1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
      NP=51
      TI=0.0
      TT=1.0
      HGRID=0.005
      MBL=5
      NPRNT=20
      MCON=4
      NMAX=201
100  READ INPUT TAPE 5,10,XONE,XTWO,ALMD,AMU
10  FORMAT(18X,2F6.2,2E20.8)
      WRITE OUTPUT TAPE 6,20,XONE,XTWO,ALMD,AMU
20  0FORMAT(1H1/99H SOC PROBLEM SENSITIVITY ANALYSIS PLANT DDX+3DX+2X+0
1.5X**3=U OPTIMAL SOLUTION AND DIFFERENT SOC      ///5H XONE,F6.2/
2  5H XTWO,F6.2/5HALMD,E20.8/5H AMU ,E20.8////)
C
      CONE=XONE
      CTWO=XTWO
      CALL OPTMAL
      ALMDP=ALMD
      AMUP=AMU
C
      CALL INTCON
      CALL UBX1
      CALL UBXCX3
      CALL EX1CX2
      GO TO 100
      END
*      SYMBOL TABLE
*      LABEL
COPTMAL INTEGRATION OF EULER LAGRANGE EQUATIONS WITH GIVEN BOUNDARY
C      CONDITIONS USING QUASILINEARIZATION FOR TPBVP TO FIND OPTIMAL
C      SOLUTION
C      PLANT DDX+3DX+2X+0.5X**3=U OPTIMAL SOLUTION

```

## SUBROUTINE OPTMAL

```

C
ØCOMMON T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
ØDIMENSION T(9ØØ),W(8,2Ø1),H(8,8,2Ø1),P(8,2Ø1),PREV(8),CON(8),
1FUNC(2Ø1),XO(2Ø1),YO(2Ø1),UO(2Ø1),Y1(51),Y2(51),Y3(51),PLT(51),
2TEMP(1Ø25),SCALE(3),AMAX(3),CSOC(1Ø)

C
C      INITIAL APPROXIMATION
C      INTEGRATE NONLINEAR DIFF EQS
C
      IFLAG=1
      DO 2 I=1,9ØØ
C      2 T(I)=Ø.Ø
C      INTEGRATION SETUP
C      1Ø2 T(2)=TI
      T(3)=HGRID
      T(4)=XONE
      T(5)=XTWO
      T(6)=ALMD
      T(7)=AMU
      CALL INT(T,4,2,Ø.,Ø.,Ø.,Ø.,Ø.,Ø.)

C
C      SOLUTION OF NONLINEAR DIFF EQS
      DO 4 N=2,NMAX
      CALL INTM
      DO 3 I=1,4
C      3 W(I,N)=T(I+3)
C      4 CONTINUE

C
C      HOLD GIVEN INITIAL CONDITIONS
      PREV(1)=XONE
      PREV(2)=XTWO
      PREV(3)=ALMD
      PREV(4)=AMU
      WRITE OUTPUT TAPE 6,97,(PREV(I),I=1,4),(W(I,2Ø1),I=1,4)
C      97 ØFORMAT(/52HFOR OPTIMAL SOLUTION INITIAL GUESS BOUNDARYPOINTS
      1/8H INITIAL ,4E2Ø.8/9HTERMINAL ,4E2Ø.8/)

C
C      QUASILINEARIZATION SOLUTION
C      1Ø4 IFLAG=2
C
      ITER=1
C      35Ø DO 6 I=1,9ØØ
      6 T(1)=Ø.Ø
      T(2)=TI
      T(3)=HGRID
      T(4)=1.Ø
      T(9)=1.Ø

```



```

70 CON(I+2)=B(I,1)
C
DO 10 N=2,NMAX
DO 11 I=1,4
W(I,N)=P(I,N)
DO 11 J=1,4
11 W(I,N)=W(I,N)+H(I,J,N)*CON(J)
10 CONTINUE
WRITE OUTPUT TAPE 6,98,(CON(I),I=1,4),(W(I,201),I=1,4)
98 0FORMAT(/27H QL SOLUTION BOUNDARY PTS /8HINITIAL ,4E20.8/
1 9H TERMINAL ,4E20.8/)
C
CHECK THE SOLUTION
ERROR=ABSF(PREV(3)-CON(3))
IF(ERROR-0.000001) 71,71,72
71 ERROR=ABSF(PREV(4)-CON(4))
IF(ERROR-0.000001) 73,73,72
72 IF(ITER-4)300,73,73
300 ITER=ITER+1
DO 13 I=1,4
13 PREV(I)=CON(I)
C DO NEXT ITERATION
GO TO 350
73 W(1,1)=CON(1)
W(2,1)=CON(2)
UO(1)=-0.5*CON(4)
WRITE OUTPUT TAPE 6,99,ITER
99 FORMAT(/20H ITERATION PERFORMED ,15///)
TIME=T1
WRITE OUTPUT TAPE 6,60,TIME,(CON(I),I=1,4),UO(1)
M=M+NPRNT
DO 61 N=2,NMAX
UO(N)=-0.5*W(4,N)
FN=N-1
TIME=FN*HGRID+T1
IF(N-M) 61,62,61
62 WRITE OUTPUT TAPE 6,90,TIME,(W(I,N),I=1,4),UO(N)
M=M+NPRNT
61 CONTINUE
C
C CALCULATE PERFORMANCE INDEX
C
DO 26 N=1,NMAX
26 FUNC(N)=W(1,N)**2+W(2,N)**2+UO(N)**2
CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
200 PERIN=SIMP
WRITE OUTPUT TAPE 6,80,PERIN
C
C SET NEW INITIAL CONDITIONS
XONE=CON(1)
XTWO=CON(2)

```

```

ALMD=CON(3)
AMU=CON(4)
C      HOLD OPTIMAL SOLUTION FOR DIFF APPROX
DO 1000 N=1,NMAX
1000  XO(N)=W(1,N)
      YO(N)=W(2,N)
C      PLOT OPTIMAL SOLUTION
31     J=0
      DO 32 I=1,NMAX,MCON
          J=J+1
          FN=I-1
          PLT(J)=FN*HGRID+TI
          Y1(J)=XO(I)
          Y2(J)=YO(I)
          Y3(J)=UO(I)
32     CONTINUE
C
      CALL SKALE
C
      WRITE OUTPUT TAPE 6,100,XONE,XTWO,TT
C
      CALL PLOT
C
      FORMATS
100  0FORMAT(///82H SENSITIVITY ANALYSIS OPTIMAL SOLUTION GRAPHS IN ORDE
      1R XONE XTWO AND U OPTIMAL /
      225X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4/)
60   0FORMAT(//17H OPTIMAL SOLUTION /
      1 17X1HT10X4HXONE12X4HXTWO12X4HALMD12X3HAMU14X1HU//
      2 10XF10.4,5E16.6)
80   FORMAT(/6HPERIN,F30.8/)
90   FORMAT(10XF10.4,5E16.6)
107  RETURN
      END
*     SYMBOL TABLE
*     LABEL
CINTCON  SUBROUTINE TO CALCULATE THE COEFFICIENTS IN THE LINEAR EQS
C         OBTAINED BY DIFF APPROX METHOD TO FIND THE FEEDBACK COEFFS
      SUBROUTINE INTCON
C
      0COMMON T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
      1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
      2AMAX,IERR,YL,YU,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
      0DIMENSION T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),
      1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
      2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
      DO 10 I=1,NMAX
10     FUNC(I)=UO(I)*XO(I)
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)

```

```

11 CSOC(1)=SIMP
C
DO 20 I=1,NMAX
20 FUNC(1)=X0(I)**2
CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
21 CSOC(2)=SIMP
C
DO 30 I=1,NMAX
30 FUNC(1)=X0(I)**4
CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
31 CSOC(3)=SIMP
C
DO 40 I=1,NMAX
40 FUNC(1)=X0(I)**6
CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
41 CSOC(4)=SIMP
C
DO 50 I=1,NMAX
50 FUNC(1)=U0(I)*X0(I)**3
CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
51 CSOC(5)=SIMP
C
DO 60 I=1,NMAX
60 FUNC(1)=U0(I)*Y0(I)
CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
61 CSOC(6)=SIMP
C
DO 70 I=1,NMAX
70 FUNC(1)=Y0(I)**2
CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
71 CSOC(7)=SIMP
C
DO 80 I=1,NMAX
80 FUNC(1)=X0(I)*Y0(I)
CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
81 CSOC(8)=SIMP
WRITE OUTPUT TAPE 6,200,(CSOC(I),I=1,8)
200 0FORMAT(///68HCoefficients IN THE LINEAR EQUATIONS TO FINDFEEDBAC
1KCOEFFICIENTS /4E20.8/4E20.8///)
90 RETURN
100 END
* SYMBOL TABLE
* LABEL
CUBX1 CASE ONE U=BX1 DIFF APPROXIMATION AND QUASILINEARIZATION
SUBROUTINE UBX1
C
0COMMON T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
1ALMD,AMU,ZETA,ETA,FUNC,X0,Y0,U0,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
0DIMENSION T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),

```

```

1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
C      CASE ONE U=BX
1000 BFC=CSOC(1)/CSOC(2)
1500 WRITE OUTPUT TAPE 6,200,CONE,CTWO,TT,BFC
200 0FORMAT(////16H CASE ONE U=BX1 /5HXONE,F6.2/5H XTWO,F6.2/
13H TT,F6.2/20H BY DIFF APPROX BFC ,E20.8/)
C      SPECIFIC OPTIMAL TRAJECTORY DIFF APP
C
      IFLAG=3
      DO 2 I=1,900
2 T(1)=0.0
  ZETA=0.0
C      INTEGRATION SETUP
      T(2)=T1
      T(3)=HGRID
      T(4)=CONE
      T(5)=CTWO
      T(6)=BFC
      T(7)=ALMDP
      T(8)=AMUP
      T(9)=ZETA
      CALL INT(T,6,2,0.,0.,0.,0.,0.,0.)
      U=BFC*T(4)
      WRITE OUTPUTTAPE 6,50,T(2),T(4),T(5),U
C
C      SOLUTION OF NONLINEAR DIFF EQS
      M=1+NPRNT
      DO 4 N=2,NMAX
      CALL INTM
      DO 3 I=1,6
3 W(1,N)=T(1+3)
  IF(N-M) 4,5,4
5 U=BFC*T(4)
  WRITE OUTPUTTAPE 6,70,T(2),T(4),T(5),U
  M=M+NPRNT
4 CONTINUE
      W(1,1)=CONE
      W(2,1)=CTWO
      W(3,1)=BFC
      W(4,1)=ALMDP
      W(5,1)=AMUP
      W(6,1)=ZETA
      DO 45 N=1,NMAX
45 FUNC(N)=(1.0+BFC**2)*W(1,N)**2+W(2,N)**2
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITE OUTPUT TAPE 6,80,PERIN
C      PLOT SPECIFIC OPTIMAL TRAJECTORY

```

```

46      J=0
        DO 47 I=1, NMAX,MCON
          J=J+1
          Y1(J)=W(1,I)
          Y2(J)=W(2,I)
47      Y3(J)=BFC*W(1,I)
        WRITE OUTPUTTAPE 6,48,CONE,CTWO,TT,BFC
C
        CALL PLOT
C
C          OBTAIN EXACT SOLUTION BY QUASILINEARIZATION
C
        IFLAG=4
        PREV(1)=CONE
        PREV(2)=CTWO
        PREV(3)=BFC
        PREV(4)=ALMDP
        PREV(5)=AMUP
        PREV(6)=ZETA
C      INTEGRATION SETUP
C
        ITER=1
350     DO 20 I=1,900
20      T(1)=0.0
        T(2)=T1
        T(3)=HGRID
        T(4)=1.0
        T(11)=1.0
        T(18)=1.0
        T(25)=1.0
        T(32)=1.0
        T(39)=1.0
        XONE=PREV(1)
        XTWO=PREV(2)
        BFC=PREV(3)
        ALMD=PREV(4)
        AMU=PREV(5)
        ZETA=PREV(6)
        CALL INT(T,42,2,0.,0.,0.,0.,0.,0.)
        N=1
        L=3
        DO 21 I=1,6
          DO 21 J=1,6
            L=L+1
21      H(I,J,N)=T(L)
          DO 22 I=1,6
            L=L+1
22      P(I,N)=T(L)
C
C      INTEGRATE P AND H

```



```

C
DO 24 N=2,NMAX
XONE=W(1,N)
XTWO=W(2,N)
BFC=W(3,N)
ALMD=W(4,N)
AMU=W(5,N)
ZETA=W(6,N)
CALL INTM
L=3
DO 23 I=1,6
DO 23 J=1,6
L=L+1
23 H(I,J,N)=T(L)
DO 24 I=1,6
L=L+1
24 P(I,N)=T(L)
C
C      DETERMINE CONSTANTS TO MATCH THE GIVEN BOUNDARY CONDITIONS
C      USES MATINV DOUBLE PRECISION ACCURACY
C
D      DIMENSION A(10,10),B(10,1)
N=NMAX
CON(1)=PREV(1)
CON(2)=PREV(2)
CON(6)=PREV(6)
DO 31 J=1,3
DO 31 I=1,3
31 A(I,J)=H(I+3,J+2,N)
DO 32 I=1,3
B(I,1)=-P(I+3,N)-H(I+3,6,N)*CON(6)
DO 32 J=1,2
32 B(I,1)=B(I,1)-H(I+3,J,N)*CON(J)
D      CALL MATINV(A,3,B,1,DET)
DO 33 I=1,3
33 CON(I+2)=B(I,1)
C      NEW VARIABLES
C
DO 7 N=2,NMAX
DO 6 I=1,6
W(I,N)=P(I,N)
DO 6 J=1,6
6 W(I,N)=W(I,N)+H(I,J,N)*CON(J)
7 CONTINUE
9 DO 10 I=1,6
10 PREV(I)=CON(I)
WRITE OUTPUTTAPE 6,98,(CON(I),I=1,6),(W(I,201),I=1,6)
98 0FORMAT(/28HQL SOLUTION BOUNDARY PTS /8H INITIAL,
13E20.8/3E20.8/9H TERMINAL,3E20.8/3E20.8/)
C

```

```

C      CHECK THE SOLUTION
      ERROR=ABSF(BFC-CON(3))
      IF(ERROR-0.00001)61,61,62
62     IF(ITER-4)300,61,61
300    ITER=ITER+1
      BFC=CON(3)
C     DO NEXT ITERATION
      GO TO 350

C
C     WRITE AND PLOT QL TRAJ
61     BFC=CON(3)
      U=BFC*CON(1)
      WRITE OUTPUT TAPE 6,99,ITER
99     FORMAT(//20HITERATION PERFORMED ,15//)
      WRITE OUTPUT TAPE 6,63,TI,CON(1),CON(2),U
63     0FORMAT(///24HSPE OPT TRAJ QL U=BX1 /
117X1HT10X4HXONE12X4HXTWO12X5HU=BX1 //
210XF10.4,3E15.6)
      DO 64 N=21,NMAX,NPRNT
      FN=N-1
      TIME=FN*HGRID+TI
      U=BFC*W(1,N)
64     WRITE OUTPUT TAPE 6,65,TIME,W(1,N),W(2,N),U
65     FORMAT(10XF10.4,3E15.6)
C     CALCULATE PERFORMANCE INDEX
C
      W(1,1)=CON(1)
      W(2,1)=CON(2)
      DO 26 N=1,NMAX
26     FUNC(N)=(1.0+BFC**2)*W(1,N)**2+W(2,N)**2
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITE OUTPUT TAPE 6,80,PERIN
80     FORMAT(/6HPERIN,F30.8/)
C
50     0FORMAT(/48HSPECIFIC OPTIMAL TRAJECTORY DIFF APPROX U=BX1 /
117X1HT10X4HXONE12X4HXTWO12X5HU=BX1 //10XF10.4,3E15.6)
70     FORMAT(10XF10.4,3E15.6)
48     0FORMAT(///60HSPE OPT TRAJ DIFF APPU=BX1 GRAPHS XONE XTWO AND U S
1PE OPT /25X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC,
2 F10.6/)
C     PLOT QL TRAJ
71     J=0
      DO 72 I=1,NMAX,MCON
      J=J+1
      Y1(J)=W(1,I)
      Y2(J)=W(2,I)
      Y3(J)=BFC*W(1,I)
72     CONTINUE
      WRITE OUTPUT TAPE 6,73,CONE,CTWO,TT,BFC

```

```

C
CALL PLOT
73 0FORMAT(///55HSPE OPT TRAJ QL U=BX1 GRAPHS XONE XTWO AND U SPE OPT
1 /25X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC,F10.6/)
RETURN
END
* SYMBOL TABLE
* LABEL
CUBXCX3 CASE TWO U=BX1+CX1**3 DIFF APPROX AND QL
SUBROUTINE UBXCX3
C
0COMMON T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
0DIMENSION T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),
1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
C CASE TWO U=BX+CX**3
1000 DENO=CSOC(2)*CSOC(4)-CSOC(3)**2
BFC=(CSOC(1)*CSOC(4)-CSOC(3)*CSOC(5))/DENO
CFC=(CSOC(2)*CSOC(5)-CSOC(3)*CSOC(1))/DENO
1100 WRITE OUTPUTTAPE 6,1200,CONE,CTWO,TT,BFC,CFC
1200 0FORMAT(///23HCASE TWO U=BX1+CX1**3 /5HXONE,F6.2/5H XTWO,
1 F6.2/3H TT,F6.2/20H BY DIFF APPROX BFC ,E20.8,10X3HCFC,
2 E20.8/)
C
C SPECIFIC OPTIMAL TRAJECTORY DIFF APP
IFLAG=5
DO 2 I=1,900
2 T(I)=0.0
C INITIAL APPROXIMATION
C INTEGRATION SETUP
ZETA=0.0
ETA=0.0
T(2)=TI
T(3)=HGRID
T(4)=CONE
T(5)=CTWO
T(6)=BFC
T(7)=CFC
T(8)=ALMDP
T(9)=AMUP
T(10)=ZETA
T(11)=ETA
CALL INT(T,8,2,0.,0.,0.,0.,0.,0.)
U=BFC*CONE+CFC*CONE**3
WRITE OUTPUT TAPE 6,50,T(2),T(4),T(5),U
50 0FORMAT(/39H SPE OPT TRAJ DIFF APPROX U=BX1+CX1**3 /
1 17X1HT10X4HXONE12X4HXTWO12X13HU=BX1+CX1**3 //10XF10.4,3E15.6)

```

```

C
C      SOLUTION OF NONLINEAR DIFF EQS
C
      M=1+NPRNT
      DO 4 I=2,NMAX
      CALL INTM
      DO 3 I=1,8
3     W(I,N)=T(I+3)
      IF(N-M) 4,5,4
5     U=BFC*T(4)+CFC*T(4)**3
      WRITE OUTPUTTAPE 6,70,T(2),T(4),T(5),U
70    FORMAT(10XF10.4,3E15.6)
      M=M+NPRNT
4     CONTINUE
      W(1,1)=CONE
      W(2,1)=CTWO
      W(3,1)=BFC
      W(4,1)=CFC
      W(5,1)=ALMDP
      W(6,1)=AMUP
      W(7,1)=ZETA
      W(8,1)=ETA
      DO 45 N=1,NMAX
45    FUNC(N)=W(1,N)**2+W(2,N)**2+(BFC*W(1,N)+CFC*W(1,N)**3)**2
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITE OUTPUT TAPE 6,80,PERIN
80    FORMAT(/ 6HPERIN,F30.8/)
C      PLOT SPE OPT TRAJ
46    J=0
      DO 47 I=1,NMAX,MCON
      J=J+1
      Y1(J)=W(1,I)
      Y2(J)=W(2,I)
      Y3(J)=BFC*W(1,I)+CFC*W(1,I)**3
47    CONTINUE
      WRITE OUTPUTTAPE 6,48,CONE,CTWO,TT,BFC,CFC
      CALL PLOT
48    FORMAT(///67HSPE OPT TRAJ DIFF APP U=BX1+CX1**3 GRAPHS XONE XTWO
1AND U SPE OPT /10X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC
2,F10.6,5X3HCFC,F10.6/)
C      QUASILINEARIZATION SOLUTION
      IFLAG=6
      PREV(1)=CONE
      PREV(2)=CTWO
      PREV(3)=BFC
      PREV(4)=CFC
      PREV(5)=ALMDP
      PREV(6)=AMUP
      PREV(7)=ZETA

```

```

PREV(8)=ETA
C
C INTEGRATION SETUP
ITER=1
350 DO 20 I=1,900
20 T(1)=0.0
T(2)=TI
T(3)=HGRID
T(4)=1.0
T(13)=1.0
T(22)=1.0
T(31)=1.0
T(40)=1.0
T(49)=1.0
T(58)=1.0
T(67)=1.0
XONE=PREV(1)
XTWO=PREV(2)
BFC=PREV(3)
CFC=PREV(4)
ALMD=PREV(5)
AMU=PREV(6)
ZETA=PREV(7)
ETA=PREV(8)
CALL INT(T,72,2,0.,0.,0.,0.,0.,0.)
N=1
L=3
DO 21 I=1,8
DO 21 J=1,8
L=L+1
21 H(I,J,N)=T(L)
DO 22 I=1,8
L=L+1
22 P(I,N)=T(L)
C
C C INTEGRATE P AND H
DO 24 N=2,NMAX
XONE=W(1,N)
XTWO=W(2,N)
BFC=W(3,N)
CFC=W(4,N)
ALMD=W(5,N)
AMU=W(6,N)
ZETA=W(7,N)
ETA=W(8,N)
CALL INTM
L=3
DO 23 I=1,8
DO 23 J=1,8

```

```

L=L+1
23 H(I,J,N)=T(L)
DO 24 I=1,8
L=L+1
24 P(I,N)=T(L)
C
C DETERMINE CONSTANTS TO MATCH THE GIVEN BOUNDARY CONDITIONS
C USES MATINV DOUBLE PRECISION ACCURACY
C DIMENSION A(10,10),B(10,1)
D N=NMAX
CON(1)=PREV(1)
CON(2)=PREV(2)
CON(7)=PREV(7)
CON(8)=PREV(8)
DO 31 J=1,4
DO 31 I=1,4
31 A(I,J)=H(I+4,J+2,N)
DO 32 I=1,4
B(I,1)=-P(I+4,N)-H(I+4,7,N)*CON(7)-H(I+4,8,N)*CON(8)
DO 32 J=1,2
32 B(I,1)=B(I,1)-H(I+4,J,N)*CON(J)
D CALL MATINV(A,4,B,1,DET)
DO 33 I=1,4
33 CON(I+2)=B(I,1)
C
C NEW VARIABLES
C
DO 7 N=2,NMAX
DO 6 I=1,8
W(I,N)=P(I,N)
DO 6 J=1,8
6 W(I,N)=W(I,N)+H(I,J,N)*CON(J)
7 CONTINUE
9 DO 10 I=1,8
10 PREV(I)=CON(I)
WRITE OUTPUTTAPE 6,98,(CON(I),I=1,3),(W(I,201),I=1,8)
98 @FORMAT(/28HQL SOLUTION BOUNDARY PTS /6H INITIAL,
14E20.8/4E20.8/9HTERMINAL,4E20.8/4E20.8/)
C
C CHECK THE SOLUTION
ERROR=ABSF(BFC-CON(3))
IF(ERROR-0.00001) 61,61,62
61 ERROR=ABSF(CFC-CON(4))
IF(ERROR-0.00001) 63,63,62
62 IF(ITER-4) 300,63,63
300 ITER=ITER+1
BFC=CON(3)
CFC=CON(4)
C DO NEXT ITERATION
GO TO 350

```

```

63   BFC=CON(3)
      CFC=CON(4)
      WRITE OUTPUTTAPE 6,99,ITER
99   FORMAT(//20ITERATION PERFORMED ,15//)
C    WRITE QL TRAJ
      U=BFC*CONE+CFC*CONE**3
      WRITE OUTPUT TAPE 6,64,TI,CONE,CTWO,U
64   0FORMAT(///30HSPE OPT TRAJ QL U=BX1+CX1**3 /17X1HT10X4HXONE12X
14HXTWO12X13HU=BX1+CX1**3 /10XF10.4,3E15.6)
      DO 65 N=21,NMAX,NPRNT
      FN=N-1
      TIME=FN*HGRID+TI
      U=BFC*W(1,N)+CFC*W(1,N)**3
65   WRITE OUTPUT TAPE 6,66,TIME,W(1,N),W(2,N),U
66   FORMAT(10XF10.4,3E15.6)
C
C    PERFORMANCE INDEX
C
      W(1,1)=CON(1)
      W(2,1)=CON(2)
      DO 26 N=1,NMAX
26   FUNC(N)=W(1,N)**2+W(2,N)**2+(BFC*W(1,N)+CFC*W(1,N)**3)**2
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITE OUTPUT TAPE 6,80,PERIN
C
C    PLOT QL TRAJ
71   J=0
      DO 72 I=1,NMAX,MCON
      J=J+1
      Y1(J)=W(1,I)
      Y2(J)=W(2,I)
72   Y3(J)=BFC*W(1,I)+CFC*W(1,I)**3
      WRITE OUTPUT TAPE 6,73,CONE,CTWO,TT,BFC,CFC
      CALL PLOT
73   FORMAT(///61HSPE OPT TRAJ QL U=BX1+CX1**3 GRAPHS XONE XTWOAND U
1SPE OPT /10X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC,
2 F10.6,5X3HCFC,F10.6/)
      RETURN
      END
*    SYMBOL TABLE
*    LABEL
CBX1CX2 CASE THREE U=BX1+CX2 DIFF APP AND QL
      SUBROUTINE BX1CX2
C
C
0COMMON T,W,H,P,HGRID,NMAX,PREV,CON,I FLAG,NPRNT,XONE,XTWO,BFC,CFC,
1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
2AMAX,IERR,YL,YU,TT,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
0DIMENSION T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),

```

```

1FUNC(201),X0(201),Y0(201),U0(201),Y1(51),Y2(51),Y3(51),PLT(51),
2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
C      CASE THREE  U=BX1+CX2
2000  DENO=CSOC(2)*CSOC(7)-CSOC(8)**2
      BFC=(CSOC(1)*CSOC(7)-CSOC(6)*CSOC(8))/DENO
      CFC=(CSOC(2)*CSOC(6)-CSOC(8)*CSOC(1))/DENO
2100  WRITE OUTPUTTAPE 6,2200,CONE,CTWO,TT,BFC,CFC
2200  FORMAT(///22HCASE THREE U=BX1+CX2 /5HXONE,F6.2/5HXTWO,
1 F6.2/3H TT,F6.2/20HBY DIFF APPROX BFC ,E20.8,10X3HCFC,
2 E20.8/)
C
C      SPECIFIC OPTIMAL TRAJECTORY DIFF APP
C      INTEGRATE NONLINEAR DIFF EQS
C
      IFLAG=7
      DO 2 I=1,900
2 T(1)=0.0
C      INTEGRATION SETUP
      ZETA=0.0
      ETA=0.0
102 T(2)=T1
      T(3)=HGRID
      T(4)=CONE
      T(5)=CTWO
      T(6)=BFC
      T(7)=CFC
      T(8)=ALMDP
      T(9)=AMUP
      T(10)=ZETA
      T(11)=ETA
      CALL INT(T,8,2,0.,0.,0.,0.,0.,0.)
      U=BFC*CONE+CFC*CTWO
      WRITE OUTPUT TAPE 6,50,T(2),T(4),T(5),U
50 0FORMAT(/39HSPE OPT TRAJ DIFF APPROX U=BX1+CX2 /
1 17X1HT10X4HXONE12X4HXTW012X13HU=BX1+CX2 //10XF10.4,3E15.6)
C
C      SOLUTION OF NONLINEAR DIFF EQS
      M=1+NPRNT
      DO 4 N=2,NMAX
      CALL INTM
      DO 3 I=1,8
3 W(I,N)=T(I+3)
      IF(N-M) 4,5,4
5 U=BFC*T(4)+CFC*T(5)
      WRITE OUTPUT TAPE 6,70,T(2),T(4),T(5),U
70 FORMAT(10XF10.4,3E15.6)
      M=M+NPRNT
4 CONTINUE
C

```



```

C      HOLD GIVEN INITIAL CONDITIONS
      W(1,1)=CONE
      W(2,1)=CTWO
      W(3,1)=BFC
      W(4,1)=CFC
      W(5,1)=ALMDP
      W(6,1)=AMUP
      W(7,1)=ZETA
      W(8,1)=ETA
      DO 45 N=1,NMAX
45     FUNC(N)=W(1,N)**2+W(2,N)**2+(BFC*W(1,N)+CFC*W(2,N))**2
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITE OUTPUT TAPE 6,80,PERIN
80     FORMAT(/ 6HPERIN,F30.8/)
C      PLOT SPE OPT TRAJ
46     J=0
      DO 47 I=1,NMAX,MCON
          J=J+1
          Y1(J)=W(1,I)
          Y2(J)=W(2,I)
          Y3(J)=BFC*W(1,I)+CFC*W(2,I)
47     CONTINUE
      WRITE OUTPUT TAPE 6,48,CONE,CTWO,TT,BFC,CFC
      CALL PLOT
48     OFORMAT(///67HSPE OPT TRAJ DIFF APP U=BX1+CX2      GRAPHS XONE XTWO
1AND U SPE OPT      /10X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC
2,F10.6,5X3HCFC,F10.6/)
C
C      QUASILINEARIZATION SOLUTION
      IFLAG=8
      PREV(1)=CONE
      PREV(2)=CTWO
      PREV(3)=BFC
      PREV(4)=CFC
      PREV(5)=ALMDP
      PREV(6)=AMUP
      PREV(7)=ZETA
      PREV(8)=ETA
      ITER=1
C
350    DO 6 I=1,900
6      T(1)=0.0
      T(2)=T1
      T(3)=HGRID
      T(4)=1.0
      T(13)=1.0
      T(22)=1.0
      T(31)=1.0
      T(40)=1.0

```

```

T(49)=1.0
T(58)=1.0
T(67)=1.0
XONE=PREV(1)
XTWO=PREV(2)
BFC=PREV(3)
CFC=PREV(4)
ALMD=PREV(5)
AMU=PREV(6)
ZETA=PREV(7)
ETA=PREV(8)

```

C

```

CALL INT(T,72,2,0.,0.,0.,0.,0.,0.)
N=1
L=3
DO 21 I=1,8
DO 21 J=1,8
L=L+1
21 H(I,J,N)=T(L)
DO 22 I=1,8
L=L+1
22 P(I,N)=T(L)

```

C  
C  
C

INTEGRATE P AND H

```

DO 7 N=2,NMAX
XONE=W(1,N)
XTWO=W(2,N)
BFC=W(3,N)
CFC=W(4,N)
ALMD=W(5,N)
AMU=W(6,N)
ZETA=W(7,N)
ETA=W(8,N)
CALL INTM
L=3
DO 8 I=1,8
DO 8 J=1,8
L=L+1
8 H(I,J,N)=T(L)
DO 9 I=1,8
L=L+1
9 P(I,N)=T(L)
7 CONTINUE

```

C  
C  
C  
D

DETERMINE CONSTANTS TO MATCH THE GIVEN BOUNDARY CONDITIONS  
USES MATINV DOUBLE PRECISION ACCURACY

```

DIMENSION A(10,10),B(10,1)
N=NMAX

```

```

CON(1)=PREV(1)
CON(2)=PREV(2)
CON(7)=PREV(7)
CON(8)=PREV(8)
DO 31 J=1,4
DO 31 I=1,4
31 A(I,J)=H(I+4,J+2,N)
DO 32 I=1,4
B(I,1)=-P(I+4,N)-H(I+4,7,N)*CON(7)-H(I+4,8,N)*CON(8)
DO 32 J=1,2
32 B(I,1)=B(I,1)-H(I+4,J,N)*CON(J)
D CALL MATINV(A,4,B,1,DET)
DO 33 I=1,4
33 CON(I+2)=B(I,1)
C
C NEW VARIABLES
DO 10 N=2,NMAX
DO 11 I=1,8
W(I,N)=P(I,N)
DO 11 J=1,8
11 W(I,N)=W(I,N)+H(I,J,N)*CON(J)
10 CONTINUE
C
C HOLD NEW INITIAL CONDITIONS
DO 13 I=1,8
13 PREV(I)=CON(I)
WRITE OUTPUT TAPE 6,98,(CON(I),I=1,8),(W(I,201),I=1,8)
98 @FORMAT(/28HQL SOLUTION BOUNDARY PTS /8H INITIAL,
14E20.8/4E20.8/9HTERMINAL,4E20.8/4E20.8/)
C
C CHECK THE SOLUTION
ERROR=ABSF(BFC-CON(3))
IF(ERROR-0.00001)61,61,62
61 ERROR=ABSF(CFC-CON(4))
IF(ERROR-0.00001)63,63,62
62 IF(ITER-4)300,63,63
300 ITER=ITER+1
BFC=CON(3)
CFC=CON(4)
C DO NEXT ITERATION
GO TO 350
63 BFC=CON(3)
CFC=CON(4)
WRITE OUTPUTTAPE 6,99,ITER
99 FORMAT(/20HITERATION PERFORMED ,15///)
C WRITE QL TRAJ
U=BFC*CONE+CFC*CTWO
WRITE OUTPUTTAPE 6,64,T1,CONE,CTWO,U
64 @FORMAT(/30HSPE OPT TRAJ QL U=BX1+CX2 /17X1HT10X4HXONE12X
14HXTWO12X13HU=BX1+CX2 /10XF10.4,3E15.6)

```

```

DO 65 N=21,NMAX,NPRNT
FN=N-1
TIME=FN*HGRID+ T1
U=BFC*W(1,N)+CFC*W(2,N)
65 WRITE OUTPUT TAPE 6,66,TIME,W(1,N),W(2,N),U
66 FORMAT(10XF10.4,3E15.6)
C     CALCULATE PERFORMANCE INDEX
C
W(1,1)=CON(1)
W(2,1)=CON(2)
DO 26 N=1,NMAX
26 FUNC(N)=W(1,N)**2+W(2,N)**2+(BFC*W(1,N)+CFC*W(2,N))**2
CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
200 PERIN=SIMP
WRITE OUTPUT TAPE 6,80,PERIN
C
C     PLOT THE QL TRAJ
C
71 J=0
DO 72 I=1,NMAX,MCON
J=J+1
Y1(J)=W(1,I)
Y2(J)=W(2,I)
72 Y3(J)=BFC*W(1,I)+CFC*W(2,I)
WRITE OUTPUT TAPE 6,73,CONE,CTWO,TT,BFC,CFC
CALL PLOT
73 FORMAT(///61H SPE OPT TRAJ QL U=BX1+CX2   GRAPHS XONE XTWO AND U
1SPE OPT /10X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC,
2 F10.6,5X3HCFC,F10.6/)
107 RETURN
END
*     SYMBOL TABLE
*     LABEL
CDAUX SUBROUTINE DAUX DESCRIBING DIFFERENTIAL EQUATIONS FOR PLANT
C     DDX+3DX+2X+0.5X**3=U OPTIMAL U=BX1 U=BX1+CX1**3 U=BX1+CX2
SUBROUTINE DAUX
C
0COMMON T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
0DIMENSION T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),
1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
C     PHASE DECISION
GO TO (1,2,3,4,5,6,7,8),IFLAG
C
C     OPTIMAL SOLUTION
NONLINEAR EQUATIONS
1 T(8)=T(5)

```

```

T(9)=-2.*T(4)-3.*T(5)-0.5*T(7) -0.5*T(4)**3
T(10)=-2.*T(4)+2.*T(7) +1.5*T(7)*T(4)**2
T(11)=-2.*T(5)+3.*T(7)-T(6)
100 RETURN
C
C      LINEAR EQUATIONS      OPTIMAL SOLUTION
2      AA=-1.5*XONE**2
      AB=3.0*AMU*XONE
      DO 20 I=1,4
      T(1+23)=T(1+7)
      T(1+27)=-2.*T(1+3)-3.*T(1+7)-0.5*T(1+15) +AA*T(1+3)
      T(1+31)=-2.*T(1+3)+2.*T(1+15)+AB*T(1+3)-AA*T(1+15)
20     T(1+35)=-2.*T(1+7)-T(1+11)+3.*T(1+15)
      T(40)=T(21)
      T(41)=-2.*T(20)-3.*T(21)-0.5*T(23)+XONE**3 +AA*T(20)
      T(42)=-2.*T(20)+2.*T(23) -3.0*AMU*XONE**2 +AB*T(20)-AA*T(23)
      T(43)=-2.*T(21)-T(22)+3.*T(23)
200 RETURN
C
C      CASE ONE U=BX
C      NONLINEAR EQUATIONS
3      T(10)=T(5)
      T(11)=T(6)*T(4)-2.0*T(4)-3.0*T(5)-0.5*T(4)**3
      T(12)=0.0
      0T(13)=-2.0*T(4)*T(6)**2-2.0*T(4)-T(6)*T(8)+2.0*T(8)+
1      1.5*T(8)*T(4)**2
      T(14)=-2.0*T(5)-T(7)+3.0*T(8)
      T(15)=-2.0*T(6)*T(4)**2-T(8)*T(4)
300 RETURN
C
C      LINEAR EQUATIONS U=BX1
4      AA=BFC-2.0 -1.5*XONE**2
      BB=-2.0*BFC**2-2.0+3.0*AMU*XONE
      CC=-4.0*BFC*XONE-AMU
      DD=-2.0*XONE**2
      DO 40 I=1,6
      T(1+45)=T(1+9)
      T(1+51)=AA*T(1+3)-3.0*T(1+9)+XONE*T(1+15)
      T(1+57)=0.0
      T(1+63)=BB*T(1+3)+CC*T(1+15)-AA*T(1+27)
      T(1+69)=-2.0*T(1+9)-T(1+21)+3.0*T(1+27)
40     T(1+75)=CC*T(1+3)+DD*T(1+15)-XONE*T(1+27)
      T(82)=T(41)
      T(83)=AA*T(40)-3.0*T(41)+XONE*T(42)-BFC*XONE +XONE**3
      T(84)=0.0
      T(85)=BB*T(40)+CC*T(42)-AA*T(44)-CC*BFC -3.0*AMU*XONE**2
      T(86)=-2.0*T(41)-T(43)+3.0*T(44)
      T(87)=CC*T(40)+DD*T(42)-XONE*T(44)-CC*XONE
400 RETURN
C

```

```

C      CASE TWO U=BX+CX**3
C      NONLINEAR EQUATIONS
5      XONEQ=T(4)**3
        T(12)=T(5)
        T(13)=-2.*T(4)-3.*T(5)+T(6)*T(4)+T(7)*XONEQ-0.5*XONEQ
        T(14)=0.
        T(15)=0.
        ØT(16)=-2.*T(4)-2.*T(4)*T(6)**2-8.*T(6)*T(7)*XONEQ+2.*T(9)
1      -(6.*T(7)**2)*XONEQ*T(4)**2-3.*T(9)*T(7)*T(4)**2-T(6)*T(9)
2      +1.5*T(9)*T(4)**2
        T(17)=-2.*T(5)-T(8)+3.*T(9)
        T(18)=-2.*T(6)*T(4)**2-2.*T(7)*T(4)*XONEQ-T(9)*T(4)
        T(19)=-2.*T(6)*T(4)*XONEQ-2.*T(7)*XONEQ**2-T(9)*XONEQ
500    RETURN
C
C      FOR U=B*XONE+C*XONE**3
C      LINEAR EQUATIONS
6      XONEQ=XONE**3
        AA=-2.+BFC+3.*CFC*XONE**2 -1.5*XONE**2
        ØBB=-2.-2.*BFC**2-24.*BFC*CFC*XONE**2-30.*CFC**2*XONE**4
1      -6.*AMU*CFC*XONE +3.0*AMU*XONE
        CC=-4.*BFC*XONE-8.*CFC*XONEQ-AMU
        DD=-8.*BFC*XONEQ-12.*CFC*XONEQ*XONE**2-3.*AMU*XONE**2
        EE=-2.*XONE**2
        FF=EE*XONE**2
        GG=FF*XONE**2
        DO 60 I=1,8
        T(I+75)=T(I+11)
        T(I+83)=AA*T(I+3)-3.*T(I+11)+XONE*T(I+19)+XONEQ*T(I+27)
        T(I+91)=0.0
        T(I+99)=0.0
        T(I+107)=BB*T(I+3)+CC*T(I+19)+DD*T(I+27)-AA*T(I+43)
        T(I+115)=-2.*T(I+11)-T(I+35)+3.*T(I+43)
        T(I+123)=CC*T(I+3)+EE*T(I+19)+FF*T(I+27)-XONE*T(I+43)
60     T(I+131)=DD*T(I+3)+FF*T(I+19)+GG*T(I+27)-XONEQ*T(I+43)
        T(140)=T(69)
        ØT(141)=AA*T(68)-3.*T(69)+XONE*T(70)+XONEQ*T(71)-BFC*XONE-
1      3.*CFC*XONEQ +XONEQ
        T(142)=0.0
        T(143)=0.0
        ØT(144)=BB*T(68)+CC*T(70)+DD*T(71)-AA*T(73)
1      +(32.*BFC*CFC+36.*CFC**2*XONE**2)*XONEQ
2      +9.*AMU*CFC*XONE**2+4.*BFC**2*XONE+AMU*BFC-3.0*AMU*XONE**2
        T(145)=-2.*T(69)-T(72)+3.*T(73)
        ØT(146)=CC*T(68)+EE*T(70)+FF*T(71)-XONE*T(73)+AMU*XONE+
1      (4.*BFC+8.*CFC*XONE**2)*XONE**2
        ØT(147)=DD*T(68)+FF*T(70)+GG*T(71)-XONEQ*T(73)+
1      (8.*BFC*XONE+12.*CFC*XONEQ+3.*AMU)*XONEQ
600    RETURN
C

```

C  
C  
C  
7CASE THREE U=BX1+CX2  
NONLINEAR EQUATIONS

T(12)=T(5)  
 T(13)=-2.\*T(4)-3.\*T(5)+T(6)\*T(4)+T(7)\*T(5) -0.5\*T(4)\*\*3  
 T(14)=0.0  
 T(15)=0.0  
 0T(16)=-2.\*T(4)-2.\*T(6)\*\*2\*T(4)-2.\*T(6)\*T(7)\*T(5)+2.\*T(9)-T(6)\*T(9)  
 1 +1.5\*T(9)\*T(4)\*\*2  
 0T(17)=-2.\*T(5)-2.\*T(6)\*T(7)\*T(4)-2.\*T(7)\*\*2\*T(5)-T(8)+3.\*T(9)  
 1 -T(7)\*T(9)  
 T(18)=-2.\*T(6)\*T(4)\*\*2-2.\*T(7)\*T(5)\*T(4)-T(9)\*T(4)  
 T(19)=-2.\*T(6)\*T(4)\*T(5)-2.\*T(7)\*T(5)\*\*2-T(9)\*T(5)  
 RETURN

700  
C  
C  
8

## LINEAR EQUATIONS U=BX1+CX2

AA=-2.+BFC -1.5\*XONE\*\*2  
 AB=-3.+CFC  
 AC=-2.-2.\*BFC\*\*2 +3.0\*AMU\*XONE  
 AD=-2.\*BFC\*CFC  
 AE=-4.\*BFC\*XONE-2.\*CFC\*XTWO-AMU  
 AF=-2.\*BFC\*XTWO  
 AG=-2.-2.\*CFC\*\*2  
 AH=-2.\*CFC\*XONE  
 AI=-2.\*BFC\*XONE-4.\*CFC\*XTWO-AMU  
 AJ=-2.\*XONE\*\*2  
 AK=-2.\*XONE\*XTWO  
 AL=-2.\*XTWO\*\*2  
 AM=-BFC\*XONE-CFC\*XTWO  
 AN=4.\*BFC\*\*2\*XONE+4.\*BFC\*CFC\*XTWO+AMU\*BFC  
 AO=4.\*BFC\*CFC\*XONE+4.\*CFC\*\*2\*XTWO+AMU\*CFC  
 AP=4.\*BFC\*XONE\*\*2+AMU\*XONE+4.\*CFC\*XONE\*XTWO  
 AQ=4.\*BFC\*XONE\*XTWO+4.\*CFC\*XTWO\*\*2+AMU\*XTWO  
 DO 80 I=1,8  
 T(I+75)=T(I+11)  
 T(I+83)=AA\*T(I+3)+AB\*T(I+11)+XONE\*T(I+19)+XTWO\*T(I+27)  
 T(I+91)=0.0  
 T(I+99)=0.0  
 T(I+107)=AC\*T(I+3)+AD\*T(I+11)+AE\*T(I+19)+AF\*T(I+27)-AA\*T(I+43)  
 0T(I+115)=AD\*T(I+3)+AG\*T(I+11)+AH\*T(I+19)+AI\*T(I+27)-T(I+35)  
 1 -AB\*T(I+43)  
 T(I+123)=AE\*T(I+3)+AH\*T(I+11)+AJ\*T(I+19)+AK\*T(I+27)-XONE\*T(I+43)  
 T(I+131)=AF\*T(I+3)+AI\*T(I+11)+AK\*T(I+19)+AL\*T(I+27)-XTWO\*T(I+43)  
 T(140)=T(69)  
 T(141)=AA\*T(68)+AB\*T(69)+XONE\*T(70)+XTWO\*T(71)+AM +XONE\*\*3  
 T(142)=0.0  
 T(143)=0.0  
 0T(144)=AC\*T(68)+AD\*T(69)+AE\*T(70)+AF\*T(71)-AA\*T(73)+AN  
 1 -3.0\*AMU\*XONE\*\*2  
 T(145)=AD\*T(68)+AG\*T(69)+AH\*T(70)+AI\*T(71)-T(72)-AB\*T(73)+AO

80

```

      T(146)=AE*T(68)+AH*T(69)+AJ*T(70)+AK*T(71)-XONE*T(73)+AP
      T(147)=AF*T(68)+AI*T(69)+AK*T(70)+AL*T(71)-XTWO*T(73)+AQ
800  RETURN
      END
*     SYMBOL TABLE
*     LABEL
CSKALE  FIND THE SCALE FACTORS FOR THE GRAPH TO BE PLOTTED USING
C       PLOTTING SUBROUTINE PLOT
C       ALL GRAPHS ARE SCALED TO THE INTERVAL -1 TO +1 SCALEFACTORS
C       ARE 1,2,5,10,10**N AND RECIPROCAL OF THESE NUMBERS
C       SUBROUTINE SCALE
C
0COMMON  T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
2AMAX,IERR,YL,YU,TI,TT,NP,HCON,CSOC,SIMP,CONE,CTWO,ALNDP,AMUP
0DIMENSION  T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),
1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
      AMAX(1)=ABSF(Y1(1))
      DO 10 I=1,NP
      IF(ABSF(Y1(I))-AMAX(1)) 10,10,11
11  AMAX(1)=ABSF(Y1(I))
10  CONTINUE
      AMAX(2)=ABSF(Y2(1))
      DO 20 I=1,NP
      IF(ABSF(Y2(I))-AMAX(2)) 20,20,21
21  AMAX(2)=ABSF(Y2(I))
20  CONTINUE
      AMAX(3)=ABSF(Y3(1))
      DO 30 I=1,NP
      IF(ABSF(Y3(I))-AMAX(3)) 30,30,31
31  AMAX(3)=ABSF(Y3(I))
30  CONTINUE
90  DO 40 I=1,3,
      XMUL=1.0
      DIV=1.0
      SFI=0.0
      Y=AMAX(1)
      IF(10.-Y) 50,51,52
51  SCALE(1)=10.0
      GO TO 40
50  DIV=DIV*10.0
      Y=Y/DIV
      IF(1.0-Y) 53,54,54
54  SCALE(1)=DIV
      GO TO 40
53  Y=AMAX(1)
      GO TO 50
52  IF(1.0-Y) 41,42,43

```



```

42 SCALE(1)=1.0
   GO TO 40
41 SFI=SF1+1.0
   DIV=SF1**2+1.0
   Y=Y/DIV
   IF(1.0-Y) 44,45,45
44 Y=AMAX(1)
   GO TO 41
45 SCALE(1)=DIV
   GO TO 40
43 IF(0.001-Y) 61,60,60
60 SCALE(1)=0.001
   GO TO 40
61 IF(0.1-Y) 62,63,64
63 SCALE(1)=0.1
   GO TO 40
64 XMUL=XMUL*10.0
   Y=Y*XMUL
   IF(1.0-Y) 65,67,66
66 Y=AMAX(1)
   GO TO 64
67 SCALE(1)=1.0/XMUL
   GO TO 40
65 SCALE(1)=10.0/XMUL
   GO TO 40
62 SFI=SF1+1.0
   XMUL=SF1**2+1.0
   Y=Y*XMUL
   IF(1.0-Y) 70,71,72
72 Y=AMAX(1)
   GO TO 62
71 SCALE(1)=1.0/XMUL
   GO TO 40
70 SFI=SF1-1.0
   XMUL=SF1**2+1.0
   SCALE(1)=1.0/XMUL
   GO TO 40
40 CONTINUE
80 RETURN

```

\* SYMBOL TABLE

\* LABEL

C PLOT PLOTTINGSUBROUTINE TO PLOT THREE CURVES WITH GIVEN SCALE  
 C FACTORSUSING NYU PLT1 SUBROUTINE

C SUBROUTINE PLOT

```

0COMMON T,W,H,P,HGRID,NMAX,PREV,CON,I FLAG,NPRNT,XONE,XTWO,BFC,CFC,
1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
0DIMENSION T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),

```

```

1FUNC(201),X0(201),Y0(201),U0(201),Y1(51),Y2(51),Y3(51),PLT(51),
2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
5 WRITE OUTPUTTAPE 6,10,(SCALE(1),I=1,3)
10 FORMAT(/25HSCALE FACTOR MULTIPLY BY /
1 15HFIRST CURVE * ,E15.6,10X,15HSECOND CURVE 0 ,E15.6,10X,
2 14HTHIRD CURVEX ,E15.6///)
DO 20 I=1,NP
Y1(I)=Y1(I)/SCALE(1)
Y2(I)=Y2(I)/SCALE(2)
Y3(I)=Y3(I)/SCALE(3)
20 CONTINUE
30 YL=-1.0
YU=1.0
40 CALL PLT1(IERR,TEMP,3,MBL,TI,TT,YL,YU,PLT,NP,Y1,PLT,NP,Y2,PLT,
1 NP,Y3)
50 RETURN
END
* SYMBOL TABLE
* LABEL
CSIMPSN SIMPSON INTEGRATION SUBROUTINE
SUBROUTINE SIMPSN(FUNC,HGRID,NMAX,SIMP)
C INTEGRATION OF TABULAR FUNCTION,FUNC,BY SIMPSON RULE
C WHERE HGRID=LENGTH OF SUBINTERVAL,NMAX=NUMBER OF POINTS
C NMAX MUSTBE ODD
DIMENSION FUNC(201)
K=(NMAX-1)/2
SUMA=0.0
DO 15 J=1,K
15 SUMA=SUMA+FUNC(2*J)
SUMB=0.0
DO 16 J=2,K
16 SUMB=SUMB+FUNC(2*J-1)
SIMP=(4.*SUMA+2.*SUMB+FUNC(NMAX)+FUNC(1))*HGRID/3.
100 RETURN
END

```

THIS PROGRAM ALSO REQUIRES SUBROUTINES INT,INTM,MATINV,PLT1

## APPENDIX I

The program listings contained in this appendix have been used to produce the data for examples in chapter 4. The first program listed produces the optimum control function  $u^*(t)$  and the optimum control trajectory  $x^*(t)$  by the "approximation in policy space" approach for example 4.3. Similar programs can be utilized for examples 4.1, 4.2, and 4.4 to produce the optimal control and trajectories. This program also requires the subroutines INT, INTM (integration routines, details in Appendix H) and MATINV (matrix inversion). The main program uses quasilinearization and "approximation in policy space" to solve the optimum control problem. The subroutine INSTRT starts the procedure by providing good initial approximations for the trajectories to start the quasilinearization technique. Subroutine CNSTNT uses MATINV to solve for the missing initial conditions. DAUX is a required subroutine for INT and INTM. SIMSPN is a simple Simpson's rule integration routine for calculating the performance index.

The second program listed produces the specific controller parameters for examples 4.2 & 4.3. by a gradient

technique. The main program optimizes the controller parameters by a gradient search. This program requires also INT, INTM and MATINV. DAUX is a derivative routine. PARFIT is a parabolic fitting technique to find a minimum. PERIN calculates the performance index and uses SIMSPN.

The last program listed produces the parameters of specific controller (ii) for Examples 4.2 and 4.3. The main program performs the quasilinearization operations for solving the two-point boundary value problem resulting from the optimization of the controller. INSTRT starts the quasilinearization by providing good initial guesses. CNSTNT solves for the unknown initial conditions. DAUX is the usual derivative routine. SIMSPN is again for calculating performance index. This program also needs INT, INTM and MATINV.

```

* LABEL
CRML00 SECOND ORDER OPTIMAL SOLUTION USING P.H.P. AND Q.L.
C OPTIMAL SOLUTION
C MAIN PROGRAM
COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
1XONE,XTWO,ALMD,AMU,FUNC,NT,NU,DELU
DIMENSION T(250),W(5,101),H(4,4,101),P(4,101),PREV(4),CON(4),
1FUNC(101)
C
C INPUT AND START
1 CALL INSTRT
C
C K ITERATIONS
C
DO 97 N=1,NMAX
W(5,N)=0.0
97 CONTINUE
NT=1
DO 99 K=1,KMAX
INTEGRATION SETUP
C
DO 2 I=1,250
2 T(1)=0.0
T(2)=TI
T(3)=HGRID
T(4)=1.0
T(9)=1.0
T(14)=1.0
T(19)=1.0
XONE=PREV(1)
XTWO=PREV(2)
ALMD=PREV(3)
AMU=PREV(4)
CALL INT(T,20,N1,0.,0.,0.,0.,0.)
N=1
L=3
DO 21 I=1,4
DO 21 J=1,4
L=L+1
21 H(I,J,N)=T(L)
DO 22 I=1,4
L=L+1
22 P(I,N)=T(L)
C
C INTEGRATE P AND H
C
DO 4 I=2,NMAX
NT=N
XONE=W(1,N)
XTWO=W(2,N)

```

```

ALMD=W(3,N)
AMU=W(4,N)
CALL INTM
L=3
DO 3 I=1,4
DO 3 J=1,4
L=L+1
3 H(I,J,N)=T(L)
DO 4 I=1,4
L=L+1
4 P(I,N)=T(L)
C
C DETERMINE CONSTANTS,
C OR INITIAL VALUES
C
CALL CNSTNT
TIME=TI
IF(ABS(0.5*CON(4))-1.0) 60,60,62
60 W(5,1)=-0.5*CON(4)
GO TO 76
62 IF(CON(4)) 64,60,66
64 W(5,1)=1.0
GO TO 76
66 W(5,1)=-1.0
WRITE OUTPUTTAPE 6,50,K,TIME,(CON(I),I,1,4),W(5,1)
C
C NEW VARIABLES
C
M=1+NPRNT
DO 7 N=2,NMAX
DO 6 I=1,4
W(I,N)=P(I,N)
DO 6 J=1,4
6 W(I,N)=W(I,N)+H(I,J,N)*CON(J)
IF(ABS(0.5*W(4,N))-1.0) 30,30,40
30 W(5,N)=-0.5*W(4,N)
GO TO 28
40 IF(W(4,N)) 32,30,42
32 W(5,N)=1.0
GO TO 28
42 W(5,N)=-1.0
28 FN=N-1
TIME=FN*HGRID+TI
IF(N-M) 7,15,7
15 WRITE OUTPUTTAPE 6,70,TIME,(W(I,N),I=1,5)
M=M+NPRNT
7 CONTINUE
9 DO 10 I=1,4
10 PREV(I)=CON(I)
W(1,1)=CON(1)

```

```

W(2,1)=CON(2)
DO 26 N=1,NMAX
26  FUNC(N)=W(1,N)**2+W(2,N)**2+W(5,N)**2
    CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
    PERIN=SIMP
    WRITEOUTPUT TAPE 6,80,PERIN
C
C  DO NEXT ITERATION
C
99  CONTINUE
    GO TO 1
C  FORMATS
50  FORMAT(1H0//59X9HITERATION,13///
117X1HT10X4HXONE12X4HXTWO12X4HALMD12X3HAMU14X1HU//
210XF10.4,5E16.6)
70  FORMAT(10XF10.4,5E16.6)
80  FORMAT(/6HPERIN,F30.8/)
    END
*  LABEL
CRM001 INPUT-STARTSECOND ORDER OPTIMAL SOLUTION
    SUBROUTINE INSTRT
    COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
1XONE,XTWO,ALMD,AMU,FUNC,NT,NU,DELU
    DIMENSION T(250),W(5,101),H(4,4,101),P(4,101),PREV(4),CON(4),
1FUNC(101)
C
C  INPUT
    READ INPUT TAPE 5,110,N1,KMAX,NMAX,NPRNT,TI
C
C  INITIAL CONDITIONS
    READ INPUT TAPE 5,120,HGRID,XONE,XTWO,ALMD,AMU
    WRITE OUTPUT TAPE 6,10,N1,KMAX,NMAX,NPRNT,HGRID
C
C  START
    IFLAG=1
    DO 2 I=1,250
2    T(I)=0.0
C  INITIAL APPROXIMATION
C  INTEGRATION SETUP
    T(2)=TI
    T(3)=HGRID
    T(4)=XONE
    T(5)=XTWO
    T(6)=ALMD
    T(7)=AMU
    CALL INT(T,4,N1,0.,0.,0.,0.,0.,0.)
    WRITE OUTPUT TAPE 6,50,T(2),(T(1),1,4,7)
C
C  SOLUTION OF NONLINEAR DIFF EQNS
C

```

```

M=1+NPRNT
DO 4 N=2,NMAX
CALL INTM
DO 3 I=1,4
3 W(I,N)=T(I+3)
IF(N-M) 4,5,4
5 WRITE OUTPUT TAPE 6,70,T(2),(T(I),I=4,7)
M=M+NPRNT
4 CONTINUE
C INITIALIZATION FOR SECOND STAGE
IFLAG=2
PREV(1)=XONE
PREV(2)=XTWO
PREV(3)=ALMD
PREV(4)=AMU
RETURN
10 FORMAT(1H1/4HSECOND ORDER LINEAR SYSTEM OPTIMAL PROBLEM//
13HN1,110/5HKMAX,110/5HNMAX,110/6H NPRNT,110/6H HGRID,F15.6///
220X16HOPTIMAL SOLUTION///)
110 FORMAT(4I10,F10.0)
120 FORMAT(5F10.0)
50 FORMAT(/22HINITIAL APPROXIMATION/
117X1HT10X4HXONE12X4HXTWO12X4HALMD12X3HAMU//
210XF10.4,4E16.6)
70 FORMAT(10XF10.4,4E16.6)
END
* LABEL
CRM002 SOLVE CONSTANTSOR INITIAL CONDITION
C USES MATINV DOUBLE PRECISION ACCURACY
SUBROUTINE CNSTNT
COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
1XONE,XTWO,ALMD,AMU,FUNC,NT,NU,DELU
DIMENSION T(250),W(5,101),H(4,4,101),P(4,101),PREV(4),CON(4),
1FUNC(101)
D DIMENSION A(10,10.),B(10,1)
N=NMAX
CON(1)=PREV(1)
CON(2)=PREV(2)
DO 1 J=1,2
DO 1 I=1,2
1 A(I,J)=H(I+2,J+2,N)
DO 2 I=1,2
2 B(I,1)=-P(I+2,N)-H(I+2,1,N)*CON(1)-H(I+2,2,N)*CON(2)
WRITE OUTPUT TAPE 6,80,((A(I,J),J=1,2),I=1,2)
WRITE OUTPUT TAPE 6,90,(B(I,1),I=1,2)
D CALL MATINV(A,2,B,1,DET)
WRITE OUTPUT TAPE 6,100,DET
DO 70 I=1,2
70 CON(I+2)=B(I,1)
RETURN

```



```

80  FORMAT(/9H MATRIX A/(30X,2E16.6))
90  FORMAT(/9HCOLUMN B/30X,2E16.6)
100 FORMAT(/12HDETERMINANT/30X,E16.6/)
    END
*   LABEL
CRM003  DAUX SECOND ORDER LINEAR SYSTEM OPTIMAL SOLUTION
        SUBROUTINE DAUX
          COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
          1XONE,XTWO,ALMD,AMU,FUNC,NT,NU,DELU
          DIMENSION T(250),W(5,101),H(4,4,101),P(4,101),PREV(4),CON(4),
          1FUNC(101)

C
C   PHASE DECISION
C   GO TO (1,2),IFLAG

C
C   NONLINEAR EQUATIONS
1     T(8)=T(5)
      T(9)=-2.*T(4)-3.*T(5)-0.5*(T(4)**3)
      T(10)=-2.*T(4)+2.*T(7)+1.5*T(7)*(T(4)**2)
      T(11)=-2.*T(5)-T(6)+3.*T(7)
      RETURN

C
C   LINEAR EQUATIONS
2     DO 10 I=1,4
      T(1+23)=T(1+7)
      T(1+27)=-2.*T(1+3)-3.*T(1+7)-1.5*(XONE**2)*T(1+3)
      T(11+31)=-2.*T(1+3)+2.*T(1+15)+3.*AMU*XONE*T(1+3)+1.5*(XONE**2)
10    1T(1+15)
      T(1+35)=-2.*T(1+7)-T(1+11)+3.*T(1+15)
      T(40)=T(21)
      T(41)=-2.*T(20)-3.*T(21)+W(5,NT)-1.5*(XONE**2)*T(20)+XONE**3
      T(42)=-2.*T(20)+2.*T(23)
      1+3.*AMU*XONE*T(20)+1.5*(XONE**2)*T(23)-3.*AMU*(XONE**2)
      T(43)=-2.*T(21)-T(22)+3.*T(23)
      RETURN
    END
*   LABEL
CRM004  SIMPSON INTEGRATION
        SUBROUTINE SIMPSN(FUNC,HGRID,NMAX,SIMP)
C       INTEGRATION OF TABULAR FUNCTION ,FUNC,BY SIMPSON RULE
C       WHERE HGRID=LENGTH OF SUBINTERVAL,NMAX=NUMBER OF POINTS
        DIMENSION FUNC(101)
        K=(NMAX-1)/2
        SUMA=0.0
        DO 15 J=1,K
15     SUMA=SUMA+FUNC(2*J)
        SUMB=0.0
        DO 16 J=2,K
16     SUMB=SUMB+FUNC(2*J-1)
        SIMP=(4.*SUMA+2.*SUMB+FUNC(NMAX)+FUNC(1))*HGRID/3.

```

RETURN  
END

```

* LABEL
CGRADIENT TECHNIQUE MAIN PROGRAM
C      U=SAT(AX+BDX)
C      PARAMETER OPTIMIZATION USING PARABOLIC FITTING METHOD
C      THIS PROGRAM IS FOR TWO DIMENSIONAL HILL.
C      OPTIMIZATION IS CARRIED OUT AS TWO ONE-DIMENSIONAL PROBLEMS.
COMMON T,API,BPI,CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI,FUNC,NMAX
DIMENSION T(50),XONE(501),XTWO(501),U(501),FUNC(501)
C
1      READ INPUT TAPE 5,500,CONA,CONB,TT,NMAX,NOPT,NOIT,TOLL
      FN=NMAX-1
      HGRID=TT/FN
2      READ INPUT TAPE 5,510,CONE,CTWO,A,B
148     WRITE OUTPUT TAPE 6,710
      WRITE OUTPUT TAPE 6,520,CONE,CTWO,TT,NMAX,HGRID,A,B,CONA,CONB
      WRITE OUTPUT TAPE 6,740,NOPT,NOIT,TOLL
101     AFB=A
102     BFB=B
145     AMIN=A
146     BMIN=B
      INDEX=0
      API=A
      BPI=B
3      CALL PERIN
4      PIAB=PI
C
C      SEARCH MINIMUM IN A DIRECTION
5      IF(ABSF(A)-0.02) 6,7,7
6      AA=A+0.0004
      GO TO 8
7      AA=A+0.02*A
8      API=AA
      INA=1
      CALL PERIN
9      PIAAB=PI
C      CALCULATE PARTIAL DERIVATIVE OF PI WRT A
10     DPIA=(PIAAB-PIAB)/(AA-A)
      WRITE OUTPUT TAPE 6,670
11     WRITE OUTPUT TAPE 6,530,A,B,PIAB,AA,PIAAB,DPIA
12     IF(PIAAB-PIAB) 13,13,15
15     AA=A
16     PIAAB=PIAB
      GO TO 13
13     AB=AA-DPIA*CONA
      API=AB
      INA=INA+1
17     CALL PERIN
18     PIABB=PI
      WRITE OUTPUT TAPE 6,540,AB,B,PIABB
19     IF(PIABB-PIAAB) 20,21,21

```

```

20  DPIA=(PIABB-PIAAB)/(AB-AA)
22  AC=AB-DPIA*CONA
23  API=AC
    INA=INA+1
24  CALL PERIN
25  PIACB=PI
26  WRITE OUTPUT TAPE 6,550,DPIA,AC,PIACB
    IF(INA-NOPT) 27,29,29
27  IF(PIACB-PIABB) 28,29,29
28  AA=AB
30  PIAAB=PIABB
31  AB=AC
32  PIABB=PIACB
33  CONA=2.0*CONA
    WRITE OUTPUT TAPE 6,560,CONA
34  GO TO 20
C
21  CONA=CONA/2.0
35  AC=AA-DPIA*CONA
36  API=AC
    INA=INA+1
37  CALL PERIN
38  PIACB=PI
39  WRITE OUTPUT TAPE 6,570,DPIA,AC,PIACB,CONA
    IF(INA-NOPT) 40,29,29
40  IF(PIACB-PIAAB) 29,41,41
41  AB=AC
42  PIABB=PIACB
    GO TO 21
C      PERFORM PARABOLIC FITTING
29  WRITE OUTPUT TAPE 6,580,AA,PIAAB,AB,PIABB,AC,PIACB
43  CALL PARFIT(AA,PIAAB,AB,PIABB,AC,PIACB,ALPHA,BETA,GAMMA)
44  AMIN=-BETA/(2.0*ALPHA)
45  API=AMIN
46  CALL PERIN
47  PIAMB=PI
48  WRITE OUTPUT TAPE 6,590,AMIN,BMIN,PIAMB
124 IF(PIAMB-PIAAB) 125,125,126
126 PIAMB=PIAAB
127 AMIN=AA
125 IF(PIAMB-PIABB) 128,128,129
129 PIAMB=PIABB
130 AMIN=AB
128 IF(PIAMB-PIACB) 131,131,132
132 PIAMB=PIACB
133 AMIN=AC
131 WRITE OUTPUT TAPE 6,690,AMIN,BMIN,PIAMB
C
C      SEARCH MINIMUM IN B DIRECTION
49  A=AMIN

```

```

50  PIAB=PIAMB
51  IF(ABS(F(B)-0.02) 52,53,53
52  BA=B+0.0004
    GO TO 54
53  BA=B+0.02*B
54  API=A
55  BPI=BA
    INB=1
56  CALL PERIN
57  PIABA=PI
C    CALCULATE PARTIAL DERIVATIVE OF PI WRT B
58  DPIB=(PIABA-PIAB)/(BA-B)
    WRITE OUTPUT TAPE 6,680
59  WRITE OUTPUT TAPE 6,600,A,B,PIAB,BA,PIABA,DPIB
60  IF(PIABA-PIAB) 61,61,62
62  BA=B
63  PIABA=PIAB
    GO TO 61
61  BB=BA-DPIB*CONB
64  BPI=BB
    INB=INB+1
65  CALL PERIN
66  PIABB=PI
67  WRITE OUTPUT TAPE 6,610,A,BB,PIABB
68  IF(PIABB-PIABA) 69,70,70
69  DPIB=(PIABB-PIABA)/(BB-BA)
71  BC=BB-DPIB*CONB
72  BPI=BC
    INB=INB+1
73  CALL PERIN
74  PIABC=PI
75  WRITE OUTPUT TAPE 6,620,DPIB,BC,PIABC
    IF(INB-NOPT)76,78,78
76  IF(PIABC-PIABB) 77,78,78
77  BA=BB
79  PIABA=PIABB
80  BB=BC
81  PIABB=PIABC
82  CONB=2.0*CONB
83  WRITE OUTPUT TAPE 6,630,CONB
84  GO TO 69
C
70  CONB=CONB/2.0
85  BC=BA-DPIB*CONB
86  BPI=BC
    INB=INB+1
87  CALL PERIN
88  PIABC=PI
89  WRITE OUTPUT TAPE 6,640,DPIB,BC,PIABC,CONB
    IF(INB-NOPT) 90,78,78

```

```

90 IF(PIABC-PIABA) 78,91,91
91 BB=BC
92 PIABB=PIABC
93 GO TO 70
C
C PERFORM PARABOLIC FITTING
78 WRITE OUTPUT TAPE 6,650,BA,PIABA,BB,PIABB,BC,PIABC
95 CALL PARFIT(BA,PIABA,BB,PIABB,BC,PIABC,ALPHA,BETA,GAMMA)
96 BMIN=-BETA/(2.0*ALPHA)
97 BPI=BMIN
98 CALL PERIN
99 PIABM=PI
100 WRITE OUTPUT TAPE 6,660,AMIN,BMIN,PIABM
134 IF(PIABM-PIABA) 135,135,136
136 PIABM=PIABA
137 BMIN=BA
135 IF(PIABM-PIABB) 138,138,139
139 PIABM=PIABB
140 BMIN=BB
138 IF(PIABM-PIABC) 141,141,142
142 PIABM=PIABC
143 BMIN=BC
141 WRITE OUTPUT TAPE 6,700,AMIN,BMIN,PIABM
INDEX=INDEX+1
103 ERRA=ABSF(AFB-AMIN)
104 ERRB=ABSF(BFB-BMIN)
105 DELA=TOLL*ABSF(AMIN)
106 DELB=TOLL*ABSF(BMIN)
107 IF(ERRA-DELA) 108,108,110
108 IF(ERRB-DELB) 200,200,110
110 AFB=AMIN
111 BFB=BMIN
112 A=AMIN
113 B=BMIN
114 PIAB=PIABM
115 IF(INDEX-NOIT) 5,200,200
200 WRITE OUTPUT TAPE 6,720
API=AMIN
BPI=BMIN
CALL PERIN
201 NP=(NMAX-1)/20
202 DO 210 N=1,NMAX,NP
203 FN=N-1
204 TIME=FN*HGRID
205 WRITE OUTPUT TAPE 6,730,TIME,XONE(N),XTWO(N),U(N)
210 CONTINUE
WRITE OUTPUT TAPE 6,750,AMIN,BMIN,PI
109 GO TO 1
500 FORMAT(2F10.0,F5.0,3I5,F10.0)
510 FORMAT(4F10.0)

```

```

520 0FORMAT(1H0/39H CONE CTWO TT NMAX HGRID A B CONA CONB /
1 2F10.5,F8.3,15,5F15.8/)
530  FORMAT(/24H A B PIAB AA PIAAB DPIA /6F15.8/)
540  FORMAT(/12HAB B PIABB /3F15.8/)
550  FORMAT(/15HDPIA AC PIACB /3F15.8/)
560  FORMAT(/6HCONA /F15.8/)
570  FORMAT(/20HDPIA AC PIACB CONA /4F15.8/)
580  FORMAT(/28HAA PIAAB AB PIABB AC PIACB /6F15.8/)
590  FORMAT(/17H AMIN BMIN PIAMB /3F15.8/)
600  FORMAT(/24HA B PIAB BA PIABA DPIB /6F15.8/)
610  FORMAT(/12HA BB PIABB /3F15.8/)
620  FORMAT(/15HDPIB BC PIABC /3F15.8/)
630  FORMAT(/6HCONB /F15.8/)
640  FORMAT(/20HDPIB BC PIABC CONB /4F15.8/)
650  FORMAT(/28H BA PIABA BBPIABB BC PIABC /6F15.8/)
660  FORMAT(/17HAMIN BMIN PIABM /3F15.8/)
670  FORMAT(///31HSEARCH MINIMUM IN A DIRECTION //)
680  FORMAT(///31H SEARCH MINIMUM IN B DIRECTION //)
690  FORMAT(/27HCORRECTED AMIN BMIN PIAMB /3F15.8/)
700  FORMAT(/27HCORRECTED AMIN BMIN PIABM /3F15.8/)
720 0FORMAT(/29HSPECIFIC OPTIMAL TRAJECTORY //
1 6X1HT11X4HXONE16X4HX TWO18X1HU/)
730  FORMAT(F10.5,3F20.8)
740  FORMAT(/16HNOPT NOIT TOLL /2110,F10.5/)
710 0FORMAT(1H1/10X92HGRADIENT TECHNIQUE PARAMETER OPTIMIZATION USI
ING PARABOLIC FITTING METHOD R.MUKUNDAN. /
262H SYSTEM DDX+3DX+2X=U, I=INT(%,1)(X**2+DX**2+U**2),U=SAT(AX+BDX)
3 /)
750  FORMAT(/8HA B PI /3F15.8/)
1000  END
* SYMBOL TABLE
* LABEL
CDAUX  DAUX SUBROUTINE FOR GRADIENT TECHNIQUE
SUBROUTINE DAUX
COMMON T,API,BPI, CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI, FUNC,NMAX
DIMENSION T(50),XONE(501),XTWO(501),U(501),FUNC(501)
C
Z=API*T(4)+BPI*T(5)
IF(ABS(Z)-1.0) 20,20,21
20  UT=Z
GO TO 2
21  IF(Z) 22,23,24
22  UT=-1.0
GO TO 2
23  UT=0.0
GO TO 2
24  UT=1.0
GO TO 2
2  T(6)=T(5)
3  T(7)=-3.*T(5)-2.*T(4)-0.5*(T(4)**3)+UT

```

```

4      RETURN
5      END
*      SYMBOL TABLE
*      LABEL
CPARFIT  PARABOLIC FITTING SUBROUTINE THROUGH 3 POINTS
SUBROUTINE PARFIT(A,X,B,Y,C,Z,ALPHA,BETA,GAMMA)
D      DIMENSION PAR(10,10),RPAR(10,1)
1      PAR(1,1)=A**2
2      PAR(1,2)=A
3      PAR(1,3)=1.0
4      PAR(2,1)=B**2
5      PAR(2,2)=B
6      PAR(2,3)=1.0
7      PAR(3,1)=C**2
8      PAR(3,2)=C
9      PAR(3,3)=1.0
10     RPAR(1,1)=X
11     RPAR(2,1)=Y
12     RPAR(3,1)=Z
D      CALL MATINV(PAR,3,RPAR,1,DET)
13     ALPHA=RPAR(1,1)
14     BETA=RPAR(2,1)
15     GAMMA=RPAR(3,1)
16     RETURN
17     END
*      SYMBOL TABLE
*      LABEL
CPERIN  SUBROUTINE TO CALCULATE PERFORMANCE INDEX
SUBROUTINE PERIN
COMMON T,API,BPI,CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI,FUNC,NMAX
DIMENSION T(50),XONE(501),XTWO(501),U(501),FUNC(501)
C
DO 30 I=1,50
30    T(1)=0.0
1    T(2)=0.0
2    T(3)=HGRID
3    T(4)=CONE
4    T(5)=CTWO
5    CALL INT(T,2,2,0.,0.,0.,0.,0.,0.)
6    XONE(1)=CONE
7    XTWO(1)=CTWO
8    DO 10 I=2,NMAX
9    CALL INTM
11   XONE(I)=T(4)
12   XTWO(I)=T(5)
10  CONTINUE
14  DO 13 I=1,NMAX
Z=API*XONE(I)+BPI*XTWO(I)
IF(ABS(Z)-1.0) 20,20,21
20  U(I)=Z

```



```
GO TO 13
21 IF(Z) 22,23,24
22 U(1)=-1.0
GO TO 13
23 U(1)=0.0
GO TO 13
24 U(1)=1.0
GO TO 13
13 FUNC(1)=XONE(1)**2+XTWO(1)**2+U(1)**2
16 CALL SIMPSN(FUNC,HGRID,NMAX,PI)
17 RETURN
18 END
```

```

* LABEL
CRM000 SECOND ORDER SOFT CONSTRAINT SYSTEM
C SOLUTION
C MAIN PROGRAM
COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
1XONE,XTWO,BTRI,AMOO,ALAM,SIGM,U,FUNC,ALPHA
DIMENSION T(600),W(6,251),H(6,6,251),P(6,251),PREV(6),U(251),
1FUNC(251),CON(6)

C
C INPUT AND START
C 1 CALL INSTRT
C
C DO K ITRS.
C
C DO 99 K=1,KMAX
C INTEGRATION SETUP
C
DO 2 I=1,600
2 T(1)=0.0
T(2)=TI
T(3)=HGRID
T(4)=1.0
T(11)=1.0
T(18)=1.0
T(25)=1.0
T(32)=1.0
T(39)=1.0
XONE=PREV(1)
XTWO=PREV(2)
BTRI=PREV(3)
AMOO=PREV(4)
ALAM=PREV(5)
SIGM=PREV(6)
CALL INT(T,42,N1,0.,0.,0.,0.,0.,0.)
N=1
L=3
DO 21 I=1,6
DO 21 J=1,6
L=L+1
21 H(I,J,N)=T(L)
DO 22 I=1,6
22 P(I,N)=T(L)
C
C INTEGRATE P AND H
C
DO 4 N=2,NMAX
XONE=W(1,N)
XTWO=W(2,N)
BTRI=W(3,N)
AMOO=W(4,N)

```

```

ALAM=W(5,N)
SIGM=W(6,N)
CALL INTM
L=3
DO 3 I=1,6
DO 3 J=1,6
L=L+1
3 H(I,J,N)=T(L)
DO 4 I=1,6
L=L+1
4 P(I,N)=T(L)
C
C DETERMINE CONSTANTS
C
CALL CNSTNT
TIME=T1
U(1)=CON(1)*CON(3)
WRITE OUTPUT TAPE 6,50,K,TIME,(CON(I),I=1,6),U(1)
C
C NEW VARIABLES
C
M=1+NPRNT
DO 7 N=2,NMAX
DO 6 I=1,6
W(I,N)=P(I,N)
DO 6 J=1,6
6 W(I,N)=W(I,N)+H(I,J,N)*CON(J)
U(N)=W(1,N)*W(3,N)
FN=N-1
TIME=FN*HGRID+T1
IF(N-1) 7,15,7
15 WRITE OUTPUT TAPE 6,70,TIME,(W(I,N),I=1,6),U(N)
M=M+NPRNT
7 CONTINUE
9 DO 10 I=1,6
10 PREV(I)=CON(I)
W(1,1)=CON(1)
W(2,1)=CON(2)
W(6,1)=CON(6)
DO 26 N=1,NMAX
26 FUNC(N)=W(1,N)**2+W(2,N)**2+U(N)**2
CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
PERIN=SIMP
WRITE OUTPUT TAPE 6,80,PERIN
C
C DO NEXT OPERATION
C
99 CONTINUE
GO TO 1
C FORMATS

```

```

50  FORMAT(1H0//59X9HITERATION,13///
14X1HT9X4HXONE9X4HXTWO9X4HBTRI9X4HAMOO10X4HALAM9X4HSIGM10X1HU///
2F8.4,1P7E14.6)
70  FORMAT(F8.4,1P7E14.6)
80  FORMAT(//19H PERFORMANCE INDEX=,F30.8//)
END
* LABEL
CRM001 INPUT START SOFT CONSTRAINT SYSTEM
SUBROUTINE INSTRT
COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
1XONE,XTWO,BTRI,AMOO,ALAM,SIGM,U,FUNC,ALPHA
DIMENSION T(600),W(6,251),H(6,6,251),P(6,251),PREV(6),U(251),
1FUNC(251),CON(6)
C
C INPUT
C READ INPUT TAPE 5,110,N1,KMAX,NMAX,NPRNT,TI
C
C INITIAL CONDITIONS
C READ INPUT TAPE 5,120,HGRID,XONE,XTWO,BTRI
C READ INPUT TAPE 5,130,AMOO,ALAM,SIGM,ALPHA
C WRITE OUTPUT TAPE 6,10,N1,KMAX,NMAX,NPRNT,HGRID,ALPHA
C
C START
C IFLAG=1
C DO 2 I=1,600
2 T(I)=0.0
C INITIAL APPROXIMATION
C INTEGRATION SETUP
C T(2)=TI
C T(3)=HGRID
C T(4)=XONE
C T(5)=XTWO
C T(6)=BTRI
C T(7)=AMOO
C T(8)=ALAM
C T(9)=SIGM
C CALL INT(T,6,N1,0.,0.,0.,0.,0.,0.)
C WRITE OUTPUT TAPE 6,50,T(2),(T(I),I=4,9)
C
C SOLUTION OF NONLINEAR DIFFERENTIAL EQUATIONS
C
C M=1+NPRNT
C DO 4 N=2,NMAX
C CALL INTM
C DO 3 I=1,6
3 W(I,N)=T(I+3)
C IF(N-M) 4,5,4
5 WRITE OUTPUT TAPE 6,70,T(2),(T(I),I=4,9)
C M=M+NPRNT
4 CONTINUE

```

```

C      INITIALIZATION OF SECOND STAGE
      IFLAG=2
      PREV(1)=XONE
      PREV(2)=XTWO
      PREV(3)=BTRI
      PREV(4)=AMOO
      PREV(5)=ALAM
      PREV(6)=SIGM
      RETURN
10     FORMAT(1H1/36H SECOND ORDER SOFT CONSTRAINT SYSTEM//
13H N1,110/5HKMAX,110/5HNMAX,110/6H NPRNT,110/6H HGRID,F15.6/
26H ALPHA,F15.6///20X18HSPECIFIC SOLUTION///)
110    FORMAT(4I10,F10.0)
120    FORMAT(4F10.0)
130    FORMAT(4F10.0)
50     FORMAT(//22HINITIAL APPROXIMATION/
14X1HT9X4HXONE9X4HXTW09X4HBTRI10X4HAM009X4HALAM10X4HSIGM///
2F8.4,1P6E14.6)
70     FORMAT(F8.4,1P6E14.6)
      END
*      LABEL
CRM002 SOLVE CONSTANTS
C      USES DOUBLE PRECISION ACCURACY
      SUBROUTINE CNSTNT
      COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
1XONE,XTWO,BTRI,AMOO,ALAM,SIGM,U,FUNC,ALPHA
      DIMENSION T(600),W(6,251),H(6,6,251),P(6,251),PREV(6),U(251),
1FUNC(251),CON(6)
D      DIMENSION A(10,10),B(10,1)
      N=NMAX
      CON(1)=PREV(1)
      CON(2)=PREV(2)
      CON(6)=PREV(6)
      DO 1 J=1,3
      DO 1 I=1,3
1      A(I,J)=H(I+3,J+2,N)
      DO 2 I=1,3
2      B(I,1)=-P(I+3,N)-H(I+3,1,N)*CON(1)-H(I+3,2,N)*CON(2)
      WRITE OUTPUT TAPE 6,80,((A(I,J),J=1,3),I=1,3)
      WRITE OUTPUT TAPE 6,90,(B(I,1),I=1,3)
D      CALL MATINV(A,3,B,1,DET)
      WRITE OUTPUT TAPE 6,100,DET
      DO 70 I=1,3
70     CON(I+2)=B(I,1)
80     FORMAT(/9H MATRIX A/(30X,3E16.6))
90     FORMAT(/9HCOLUMN B/30X,3E16.6)
100    FORMAT(/12HDETERMINANT/30X,E16.6)
      RETURN
      END
*      LABEL

```

CRM003 DAUX2

```

SUBROUTINE DAUX
COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
1XONE,XTWO,BTRI,AMOO,ALAM,SIGM,U,FUNC,ALPHA
DIMENSION T(600),W(6,251),H(6,6,251),P(6,251),PREV(6),U(251),
1FUNC(251),CON(6)

```

C

C

```

PHASE DECISION
GO TO (1,2),IFLAG

```

C

C

1

NONLINEAREQUATIONS

```

T(10)=T(5)
T(11)=- (2.-T(6))*T(4)-3.*T(5)-0.5*(T(4)**3)
T(12)=0.0
T(13)=- (2.+4.*ALPHA*(T(4)**2)*(T(6)**4))*T(4)+(2.-T(6))*T(8)
1+1.5*T(8)*(T(4)**2)
T(14)=-2.*T(5)-T(7)+3.*T(8)
T(15)=-4.*ALPHA*(T(6)**3)*(T(4)**4)-T(4)*T(8)
RETURN

```

C

C

2

LINEAR EQUATIONS

```

DO 10 I=1,6
AA=ALPHA*(BTRI**4)*(XONE**2)
BB=AA*XONE/BTRI
T(1+45)=T(1+9)
T(1+51)=- (2.-BTRI)*T(1+3)-3.*T(1+9)+XONE*T(1+15)-1.5*(XONE**2)*T
1(1+3)
T(1+57)=0.0
T(1+63)=- (2.+12.*AA)*T(1+3)-(ALAM+16.*BB)*T(1+15)+(2.-BTRI)
1*T(1+27)+3.*ALAM*XONE*T(1+3)+1.5*(XONE**2)*T(1+27)
T(1+69)=-2.*T(1+9)-T(1+21)+3.*T(1+27)
10 T(1+75)=- (ALAM+16.*BB)*T(1+3)-12.*XONE*(BB/BTRI)*T(1+15)-XONE
1*T(1+27)
T(82)=T(41)
T(83)=- (2.-BTRI)*T(40)-3.*T(41)+XONE*T(42)-XONE*BTRI
1-1.5*(XONE**2)*T(40)+XONE**3
T(84)=0.0
T(85)=- (2.+12.*AA)*T(40)-(ALAM+16.*BB)*T(42)+(2.-BTRI)*T(44)+
124.*AA*XONE+ALAM*BTRI+3.*ALAM*XONE*T(40)+1.5*(XONE**2)*T(44)
2-3.*ALAM*(XONE**2)
T(86)=-2.*T(41)-T(43)+3.*T(44)
T(87)=- (ALAM+16.*BB)*T(40)-12.*XONE*(BB/BTRI)*T(42)-XONE*T(44)
1+ALAM*XONE+24.*BB*XONE
RETURN
END

```

```
* LABEL
CRMO04 SIMPSON'S RULE
SUBROUTINE SIMPSN(FUNC,HGRID,NMAX,SIMP)
DIMENSION FUNC(251)
K=(NMAX-1)/2
SUMA=0.0
DO 15 J=1,K
15  SUMA=SUMA+FUNC(2*J)
SUMB=0.0
DO 16 J=2,K
16  SUMB=SUMB+FUNC(2*J-1)
SIMP=(4.*SUMA+2.*SUMB+FUNC(NMAX)+FUNC(1))*HGRID/3.
RETURN
END
```

## APPENDIX J

The purpose of this appendix is to give a brief description of the programs which were used to produce some of the data displayed in Chapter 5. Specifically this appendix will limit itself to those programs concerned with estimation of state variables and control, i.e., rate reduction, using the estimated state for the rotational motion of a rigid body.

The program labeled MJPL8 is the main program. The program labeled DVJPL8 is a subroutine used to compute the derivative for the system and state estimator equations. The program labeled DT8 is a subroutine used to generate the disturbance inputs.

The following variables are of interest:

VAR (2 to 4) - true angular velocities about the body principal axes, stored in X (1 to 3, I)  
where I is for the time increment

VAR (5 to 7) - estimated angular velocities, stored in XE (1 to 3, I)

VAR (8 to 16) - the  $P_{ij}$ 's with  $P_{11} = \text{VAR}(8)$ ,  $P_{21} = \text{VAR}(9)$ , etc., stored in P(L,N,I)



- CC (1 to 3) - the constants in the system equations  
(5.8.2) i.e.  $CC(1) = \frac{I_2 - I_3}{I_1}$  etc.
- CC (4 to 6) - the corresponding constants in the  
state estimate equations (5.8.3)
- HV (1 to 3) - the actual linear combination measured,  
i.e.  $output = h_1 \omega_1 + h_2 \omega_2 + h_3 \omega_3$
- HV (4 to 6) - the corresponding constants in the  
estimator equations
- ALPHA (1 to 3) - used to adjust the acceleration con-  
stants for the control portion for  
example considering axis 1, then the  
control input is  
 $u_1 = - ALPHA (1) \cdot SGN (\omega_1)$
- ALPHA (4 to 6) - used to adjust the error in the input,  
for example for axis 1 the error is =  
 $ALPHA(4) * SGN(\omega_1)$
- TIC - the initial value of time
- XO (1 to 3) - the initial conditions for the system  
equations
- XOE (1 to 3) - the initial estimates for the states
- PO (I, J) - the initial condition for the P equa-  
tions

- H - step size used to integrate the differential equations
- NS - the number of steps, of size H, taken between points plotted
- NOPT - (a) if = 0 then 171 points will be pointed out, (aa) if  $\neq$  0 then NOPT points will be printed out (usually NOPT = 0 or = 101)
- BETA (I) - the constant forcing term in the equation for  $P_{ii}$
- KSTART - the starting integer for the random sequence used for the noise
- PER1, PER2 - the parameters P1 and P2 in the noise model given by equation (5.8.1)
- YL, YU - the lower and upper values of the dependent variable for the plotting routine PLT1.
- XM (I) = stored value of the measured output
- XLC (I) = stored value of the actual output
- XLCE (I) = stored value of the estimated output

The variables X, XE, P, XM, XLC, and XLCD are used to store the data for use in the plotting routine.

The main program uses the standard subroutines AMRK and PLT1. The function RRN is used to generate sequences of random numbers uniformly distributed between 0 and 1.

```

*      SYMBOL TABLE
*      LABEL
CMJPL8
C
C      CONTROL OF ROTATIONAL MOTION OF RIGID BODY USING ESTIMATED STATE
C
      COMMON VAR,DER,CC,U1,U2,U3,UD1,UD2,UD3,Y,BETA,HV,HM
DIMENSION VAR(16),DER(16),CC(6),ALPHA(6),EU(15),EL(15),TEMPS(3500)
1,X(3,171),XE(3,171),P(3,3,171),T(171),Y1(171),Y2(171),Y3(171),
2Y4(171),PO(3,3),XO(3),XOE(3),UD(3,172),XM(171)
3,XLC(171),XLCE(171),HV(6),HM(3,3)
F
C      DVJPL
C
C      READ IN PROBLEM PARAMETERS
C
1      WRITE OUTPUT TAPE 6,98
      READ INPUT TAPE 5,99,PER1,PER2,KSTART,NS,H,YL,YU,BETA,NOPT
      WRITE OUTPUT TAPE 6,100,PER1,PER2,KSTART,NS,H,BETA
      READ INPUT TAPE 5,101,(CC(J),J=1,6)
      READ INPUT TAPE 5,101,(HV(J),J=1,6)
      WRITE OUTPUT TAPE 6,121,(CC(J),J=1,6)
      WRITE OUTPUT TAPE 6,122,(HV(J),J=1,6)
      READ INPUT TAPE 5,101,(ALPHA(J),J=1,6)
      WRITE OUTPUT TAPE 6,123,(ALPHA(J),J=1,6)
C
C      GENERATE DISTURBANCE TORQUES
C
      CALL TORQUE(H,NS,UD)
      Y=1.0
      DV1=SIGNF(Y,ALPHA(4))
      DV2=SIGNF(Y,ALPHA(5))
      DV3=SIGNF(Y,ALPHA(6))
      Y=NS
      HS=Y*H
C
C      READ IN I.C.
C
      READ INPUT TAPE 5,101,XO(1),XO(2),XO(3),TIC
      READ INPUT TAPE 5,101,(XOE(J),J=1,3)
      WRITE OUTPUT TAPE 6,115,(XO(J),J=1,3)
      WRITE OUTPUT TAPE 6,116,(XOE(J),J=1,3)
      DO 2 J=1,3
2      READ INPUT TAPE 5,101,(PO(J,K),K=1,3)
      NBLKS=17
      NPTS=171
      IF(NOPT) 301,301,3
3      NPTS=NOPT
      NBLKS=10
301     CONTINUE
      Y=RRN(KSTART)

```

```

KRUN=0
NE=15
DO 4 I=1,3
DO 4 J=1,3
4   HM(I,J)=HV(I+3)*HV(J+3)
C
C   SET UP I.C.
C
DO 10 J=1,3
VAR(J+1)=X0(J)
VAR(J+4)=XOE(J)
DO 10 I=1,3
JK=3*J+I+4
10  VAR(JK)=PO(I,J)
DER(1)=H
VAR(1)=0.0
Z=HV(1)*VAR(2)+HV(2)*VAR(3)+HV(3)*VAR(4)
Y=Z+2.0*PER1*(RRN(KRUN)-0.5)*ABSF(Z)+2.0*PER2*(RRN(KRUN)-0.5)
DO 11 J=1,20
11  TEMPS(J)=0.0
DO 20 I=1,NPTS
C
C   STORE SOLUTIONS
C
DO 12 K=1,3
X(K,I)=VAR(K+1)
XE(K,I)=VAR(K+4)
DO 12 L=1,3
JK=3*L+K+4
12  P(K,L,I)=VAR(JK)
XM(I)=Y
XLC(I)=HV(1)*VAR(2)+HV(2)*VAR(3)+HV(3)*VAR(4)
XLCE(I)=HV(4)*VAR(5)+HV(5)*VAR(6)+HV(6)*VAR(7)
C
C   INTEGRATE EQUATIONS
C
TT=VAR(1)
DO 20 J=1,NS
UINT=(VAR(1)-TT)/HS
U1=-SIGNF(ALPHA(1),VAR(5))
U2=-SIGNF(ALPHA(2),VAR(6))
U3=-SIGNF(ALPHA(3),VAR(7))
UD1=UD(1,I)+(UD(1,I+1)-UD(1,I))*UINT+DV1*SIGNF(ALPHA(4),VAR(5))
UD2=UD(2,I)+(UD(2,I+1)-UD(2,I))*UINT+DV2*SIGNF(ALPHA(5),VAR(6))
UD3=UD(3,I)+(UD(3,I+1)-UD(3,I))*UINT+DV3*SIGNF(ALPHA(6),VAR(7))
CALL DVJPL
CALL AMRK(VAR,DER,DVJPL,NE,0,EU,EL,H,H,TEMPS)
Z=HV(1)*VAR(2)+HV(2)*VAR(3)+HV(3)*VAR(4)
Y=Z+2.0*PER1*(RRN(KRUN)-0.5)*ABSF(Z)+2.0*PER2*(RRN(KRUN)-0.5)
20  CONTINUE

```

```

C
C
C
DATA PRINTOUT
T(1)=TIC
DO 22 J=2,171
22 T(J)=T(J-1)+HS
WRITE OUTPUT TAPE 6,117,(X(J,NPTS),J=1,3)
WRITE OUTPUT TAPE 6,118,(XE(J,NPTS),J=1,3)
WRITE OUTPUT TAPE 6,112
M=1
DO 28 J=1,18
28 WRITE OUTPUT TAPE 6,113,(P(1,K,M),P(2,K,M),P(3,K,M),K=1,3)
M=M+10
C
C
C
PLOTTING ROUTINE
XL=TIC
XU=T(NPTS)
NPLOT=4
DO 40 J=1,3
DO 30 I=1,NPTS
Y1(I)=X(J,I)
Y2(I)=P(J,1,I)*HV(4)+P(J,2,I)*HV(5)+P(J,3,I)*HV(6)
30 Y3(I)=XE(J,I)
Y4(I)=P(J,J,I)
M=J
WRITE OUTPUT TAPE 6,114,M
ØCALL PLT1(IERR,TEMPS,NPLOT,NBLKS,XL,XU,YL,YU,T,NPTS,Y1,T,NPTS,Y2,
40 1T,NPTS,Y3,T,NPTS,Y4)
CONTINUE
WRITE OUTPUT TAPE 6,124
ØCALL PLT1(IERR,TEMPS,3,NBLKS,XL,XU,YL,YU,T,NPTS,XLC,T,NPTS,XM,
1T,NPTS,XLCE)
!F(NOPT) 42,42,50
42 WRITE OUTPUT TAPE 6,119
WRITE OUTPUT TAPE 6,120
DO 44 J=1,57
44 ØWRITE OUTPUT TAPE 6,113,T(J),XLC(J),XM(J),T(J+57),XLC(J+57),
1XM(J+57),T(J+114),XLC(J+114),XM(J+114)
50 GO TO 1
C
98 ØFORMAT(6ØH1SATELLITEATTITUDE CONTROL WITH N.L. FILTER STATE ESTIM
1ATOR//)
99 FORMAT(2F10.6,2I5,4F10.6,15)
100 FORMAT(3ØHPER1 PER2 KSTART NS H BETA = 2F7.3,2I5,2F10.6//)
101 FORMAT(6F10.6)
112 ØFORMAT(10ØHP11 P21 P31 P12 P22
1 P32 P13 P23 P33//)
113 FORMAT(9E12.4//)
114 FORMAT(46H1*=XN 0 = HV(J)*PNJ X = XEN H = PNN N = 15//)

```

```

115 FORMAT(10HXO(1) = 3F10.6//)
116 FORMAT(10H XOE(1) = 3F10.6//)
117 FORMAT(10HXO(T) = 3F10.6//)
118 FORMAT(10HXOE(T) = 3F10.6//)
119 ØFORMAT(55H1X1 AND X1 MEASURED CORRESPONDING TO THE POINTS PLOTTED/
1/)
120 ØFORMAT(107H TIME          X1          X1 MEASURED TIME          X1
1   X1 MEASURED TIME          X1          X1 MEASURED//)
121 FORMAT(9H CC(J) = 6F10.6//)
122 FORMAT(9HHV(J) = 6F10.6//)
123 FORMAT(12H ALPHA(J) = 6F10.6//)
124 FORMAT(28H1*= XLC  0 = XLCM  X = XLCE//)

```

```

END
* SYMBOL TABLE
* LABEL

```

CDVJPL8

```

SUBROUTINE DVJPL
COMMON VAR, DER, CC, U1, U2, U3, UD1, UD2, UD3, Y, BETA, HV, HM
ØDIMENSION VAR(16), DER(16), CC(6), A(3,3), B(3,3), C(3,3), G(3,3)
1, HV(6), HM(3,3)
DER(2)=CC(1)*VAR(3)*VAR(4)+U1+UD1
DER(3)=CC(2)*VAR(2)*VAR(4)+U2+UD2
DER(4)=CC(3)*VAR(2)*VAR(3)+U3+UD3
Z=Y-HV(4)*VAR(5)-HV(5)*VAR(6)-HV(6)*VAR(7)
ØDER(5)=CC(4)*VAR(6)*VAR(7)+2.Ø*(HV(4)*VAR(8)+HV(5)*VAR(11)+
1HV(6)*VAR(14))*Z+U1
ØDER(6)=CC(5)*VAR(5)*VAR(7)+2.Ø*(HV(4)*VAR(9)+HV(5)*VAR(12)+
1HV(6)*VAR(15))*Z+U2
ØDER(7)=CC(6)*VAR(5)*VAR(6)+2.Ø*(HV(4)*VAR(10)+HV(5)*VAR(13)+
1HV(6)*VAR(16))*Z+U3
G(1,1)=Ø.Ø
G(1,2)=CC(4)*VAR(7)
G(1,3)=CC(4)*VAR(6)
G(2,1)=CC(5)*VAR(7)
G(2,2)=Ø.Ø
G(2,3)=CC(5)*VAR(5)
G(3,1)=CC(6)*VAR(6)
G(3,2)=CC(6)*VAR(5)
G(3,3)=Ø.Ø
DO 2 I=1,3
DO 2 J=1,3
A(I,J)=Ø.Ø
B(I,J)=Ø.Ø
C(I,J)=Ø.Ø
DO 2 L=1,3
DO 2 M=1,3
JK1=3*M+4+I
JK2=3*J+4+L
2 A(I,J)=A(I,J)+HM(M,L)*VAR(JK1)*VAR(JK2)
DO 4 I=1,3

```

```

DO 4 J=1,3
DO 4 K=1,3
JK1=3*J+K+4
B(I,J)=B(I,J)+G(I,K)*VAR(JK1)
JK2=3*K+I+4
4 C(I,J)=C(I,J)+G(J,K)*VAR(JK2)
DO 6 I=1,3
DO 6 J=1,3
JK=3*J+I+4
6 DER(JK)=-2.0*A(I,J)+B(I,J)+C(I,J)
DER(8)=DER(8)+BETA
DER(12)=DER(12)+BETA
DER(16)=DER(16)+BETA
50 RETURN
END
* SYMBOL TABLE
* LABEL
CDT8
SUBROUTINE TORQUE(H,NS,UD)
COMMON VAR,DER,CC,U1,U2,U3,UD1,UD2,UD3,Y,BETA,HV,HM
DIMENSION VAR(16),DER(16),CC(6),HV(6),HM(3,3),UD(3,172)
WRITE OUTPUT TAPE 6,100
DO 10 I=1,3
DO 10 J=1,172
10 UD(I,J)=0.0
NNS=NS
HH=H
100 FORMAT(34HDISTURBANCE TORQUES UD1,2,3 = 0.0//)
50 RETURN
END

```



## APPENDIX K

This appendix contains listings of the Fortran programs used to solve the problem of example 6.2. The main program is listed first; it is divided into several different parts for performing the various steps in the min-max solution.

The first part of the main program generates the optimum solution to the problem as posed, using quasilinearization. The SHARE subroutine AMRK is used for the integration, and DERR is the derivative subroutine used by AMRK. The matrix inversion is done by SHARE subroutine MATINV. Next the initial guess on the solution is obtained by integrating the exact system equations with assumed initial conditions. The derivative routine used here is YDOT. The next part of the main program generates the approximation to the solution using the "quasilinear" equations. AMRK is used for the integration, along with derivative subroutines DERH and DERIH.

The next part of the program sets up the linear programming problem in standard form for a simplex method solution. The subroutines SMPLX, PIVOT, and PRINT are used to solve the linear programming problem.

A plot of the results is obtained in the final part of the main program. The SHARE subroutine PLT1 is used for plotting.

Listings of the programs follow.

```

*      SYMBOL TABLE
*      LABEL
C      TEST PROGRAM FOR SOC SOLUTION USING MIN-MAX CRITERION IN DIFF.
C      APPROX. USE 10 POINTS ON X1.
      DIMENSION ARRAY(52,253),IROW(50),XOPT(4,101),Y(20,101),VAR(21),DER
1(21),EU(20),EL(20),TEMPS(1020),POINTS(10),X(3,101),PART(3,101),
2XONE(51),YONE(51),YTWO(51)
      COMMON ARRAY,IROW,AFC,X,H,VAR,DER
D      DIMENSION A(10,10),B(10,1)
F      DERR,YDOT,DERH,DERIH
C      READ IN INITIAL CONDITIONS, TIME INTERVAL, AND STEP SIZE.
      READ INPUT TAPE 5,1,C1,C2,T,H
1  FORMAT (2F10.6/F10.6/F10.6)
C      READ IN NUMBER OF ITERATIONS OF OL/LP DESIRED.
      READ INPUT TAPE 5,2,NUMI
2  FORMAT (I2)
C
C      SOLVE FOR OPTIMUM OPEN-LOOP SOLUTION WITH Q.L.
C
      DO 3 I= 1,191
3  TEMPS(I) = 0.
      DO 4 I = 1,20
4  Y(I,1) = 0.
      DO 5 I = 1,16,5
5  Y(I,1) = 1.
      VAR(1) = 0.
      DER(1) = H
      IT = T/H+0.5
      DO 3 M = 1,IT
      DO 6 I = 1,20
6  VAR(I+1) = Y(I,M)
      CALL DERR
      CALL AMRK(VAR,DER,DERR,20,4,EU,EL,H,H,TEMPS)
      DO 7 I = 1,20
7  Y(I,M+1) = VAR(I+1)
8  CONTINUE
C      LINEAR EQUATIONS
      DO 9 I = 1,4
      IND = 1+(I-1)*4
      A(1,I) = Y(IND,1)
9  A(2,I) = Y(IND+1,1)
      DO 10 I = 1,4
      IND = 3+(I-1)*4
      A(3,I) = Y(IND,101)
10 A(4,I) = Y(IND+1,101)
      B(1,1) = C1-Y(17,1)
      B(2,1) = C2-Y(18,1)
      B(3,1) = -Y(19,101)
      B(4,1) = -Y(20,101)
D      DET = 0.

```

```

CALL MATINV (A 4,B,1,DET)
DO 11 I= 1,4
DO 11 J= 1,101
11 XOPT(I J) = B(1)*Y(I,J)+B(2)*Y(I+4,J)+B(3)*Y(I+8,J)+B(4)*Y(I+12,J)
1+Y(I+16,J)
WRITE OUTPUTTAPE 6,110
110 FORMAT(1H1,4X,26HOPTIMUM OPEN-LOOP SOLUTION//)
WRITE OUTPUT TAPE 6,111,((XOPT(I,J),I=1,4),J=1,101,2)
111 FORMAT (5X,24HX1, X2, LAMBDA1, LAMBDA2//(1P4E16.7))
DO 112 I = 1,51
IND = 1 + (I-1)*2
112 YONE(I) = XOPT(1,IND)
XONE(I) = 0.
DO 113 I= 2,51
113 XONE(I) = XONE(I-1)+2.*H
WRITE OUTPUT TAPE 6,114
114 FORMAT(5X,53HPOINTS TO BE FITTED ARE TAKEN EVERY TENTH OFASECOND
1)
C OPTIMAL SOLUTION NOW STORED IN XOPT. TAKE SELECTED POINTS AND PLA
C CE IN POINTS.
DO 12 I = 1,10
IND = 11+(I-1)*10
12 POINTS(I) = XOPT(1,IND)
C PHASE ONE COMPLETED. OBSERVED POINTS STORED IN POINTS.
C
C BEGIN PHASE TWO.
C GENERATE INITIAL GUESSUSING ASSUMED VALUE OF PARAMETER, WHICH
C IS READ IN.
INDEX = 0
READ INPUT TAPE 5,13,AFC
13 FORMAT (F10.6)
DO 14 I = 1,38
14 TEMPS(I) = 0.
VAR(1)= 0.
DER(1) = H
X(1,1)= C1
X(2,1) = C2
X(3,1) = AFC
DO 17 M = 1,1T
DO 15 I = 1,3
15 VAR(I+1) = X(I,M)
CALL YDOT
CALL AMRK(VAR,DER,YDOT,3,0,EU,EL,H,H,TEMPS)
DO 16 I = 1,3
16 X(I,M+1) = VAR(I+1)
17 CONTINUE
C INITIAL GUESS NOW STORED IN X. SUBSEQUENT RESULTS OF ITERATIONS
C WILL BE PLACED IN X.
C
C PROCEED TO Q.L. SOLUTION.

```



```

      ARRAY(1+10,1+22) = 1.0
32  ARRAY(1+20,1+32)= 1.0
      DO 33 I = 11,30
33  ARRAY(1,72) = 1.0
      DO 34 I = 1,10
      IND = 11+(I-1)*10
34  ARRAY(1,74)=POINTS(1)-PART(1,IND)-C1*Y(1,IND)-C2*Y(4,IND)
C   MATRIX OF ORIGINAL EQUATIONS NOW IN ARRAY.  ELIMINATE Z FROM ALL
C   EQUATIONS BUT LAST.
      DO 35 I = 1,29
      DO 35 J = 1,74
35  ARRAY(1,J) = ARRAY(1,J)-ARRAY(30,J)
C   CHECK TO INSURE NON-NEGATIVITY OF CONSTANTS
      DO 351 I = 1,29
      IF(ARRAY(1,74)) 350,351,351
350 DO 352 J = 1,74
352 ARRAY(1,J) = -ARRAY(1,J)
351 CONTINUE
C   ALL CONSTANTS ARE NOW NON-NEGATIVE
C   NUMBERS IN ARRAY ARE NOW IN PROPER FORM FOR APPLICATION OF
C   SIMPLEX METHOD.
      NADM = 42
      NEQ = 29
      WRITE OUTPUT TAPE 6,36
36  FORMAT(72H1  IN OUTPUT FROM SIMPLEX METHOD SOLUTION, VALUE OF FEED
1BACK COEFFICIENT/72H  WILL BE X(1) MINUS X(2).  THE VALUE OF THE
2OBJECTIVE FORM WILL BE THE /34H  VALUE OF THE MAXIMUM DEVIATION.
3///)
      CALL SMPLX (NADM,NEQ)
      AP = 0.
      APP = 0.
      DO 40 I = 1,NEQ
      IF (IROW(I)-1) 38,37,38
37  AP = ARRAY(1,74)
      GO TO 40
38  IF (IROW(I)-2) 40,39,40
39  APP = ARRAY(1,74)
40  CONTINUE
      A1 = AP-APP
      WRITE OUTPUT TAPE 6,41,INDEX,A1 -
41  FORMAT(23H0  FOR ITERATION NUMBER,13/16H  VALUE OF A IS,1PE16.7)
42  AFC = A1
C   CONSTRUCT NEW X.
      DO 43 I = 1,3
      DO 43 J = 1,101
43  X(I,J) = C1*Y(1,J)+C2*Y(1+3,J)+AFC*Y(1+6,J)+PART(1,J)
      WRITE OUTPUT TAPE 6,430,INDEX
430 FORMAT(29H1  RESULTS OF ITERATION NO.,13)
      WRITE OUTPUT TAPE 6,431,((X(I,J),I=1,2),J=1,101,2)
431 FORMAT(5X,6HX1, X2//(1P2E16.7))

```

```

DO 432 I = 1,51
  IND = 1 + (I-1)*2
432 YTWO(I) = X(1,IND)
  WRITE OUTPUT TAPE 6,430,INDEX
  WRITE OUTPUT TAPE 6,433
433 FORMAT(5X,54H* IS OPTIMAL TRAJECTORY   0 IS TRAJECTORY FROM MIN-MA
  1X//)
  IERR = 0
  CALL PLT1(IERR,TEMPS,2,5,0.,1.,-6.,0.,XONE,51,YONE,XONE,51,YTWO)
  IF(NUM1-INDEX) 44,44,18
44 CALL EXIT
  END
*   SYMBOL TABLE
*   LABEL
CYDOT
  SUBROUTINE YDOT
C   DERIVATIVES FOR SYSTEM WITH FEEDBACK
  DIMENSION VAR(21),DER(21),ARRAY(52,253),IROW(50),X(3,101)
  COMMON ARRAY,IROW,A,X,M,VAR,DER
  DER(2) = VAR(3)
  DER(3) = (-2.+VAR(4))*VAR(2)-3.*VAR(3)
  DER(4) = 0.
  RETURN
  END
*   SYMBOL TABLE
*   LABEL
CDERR
  SUBROUTINE DERR
C   DERIVATIVES FOR EXACT OPTIMAL SOLUTION
  DIMENSION VAR(21),DER(21),ARRAY(52,253),IROW(50),X(3,101)
  COMMON ARRAY,IROW,A,X,M,VAR,DER
  DO 1 I = 2,18,4
  DER(I) = VAR(I+1)
  DER(I+1) = -2.*VAR(I)-3.*VAR(I+1)-0.5*VAR(I+3)
  DER(I+2) = -2.*VAR(I)+2.*VAR(I+3)
  1 DER(I+3) = -2.*VAR(I+1)-VAR(I+2)+3.*VAR(I+3)
  RETURN
  END
*   SYMBOL TABLE
*   LABEL
CDERH
  SUBROUTINE DERH
C   HOMOGENEOUS DERIVATIVES FOR Q.L. SOLUTION
  DIMENSION VAR(21),DER(21),ARRAY(52,253),IROW(50),X(3,101)
  COMMON ARRAY,IROW,A,X,M,VAR,DER
  DO 1 I = 2,8,3
  DER(I) = VAR(I+1)
  DER(I+1) = (-2.+X(3,M))*VAR(I)-3.*VAR(I+1)+X(1,M)*VAR(I+2)
  1 DER(I+2) = 0.
  RETURN

```

```
      END
*     SYMBOL TABLE
*     LABEL
C     CDERIH
      SUBROUTINE DERIH
C     INHOMOGENEOUS DERIVATIVES FOR Q.L. SOLUTION
      DIMENSION VAR(21),DER(21),ARRAY(52,253),IROW(50),X(3,101)
      COMMON ARRAY,IROW,A,X,M,VAR,DER
      DER(2) = VAR(3)
      DER(3) = (-2.+X(3,M))*VAR(2)-3.*VAR(3)+X(1,M)*VAR(4)-X(1,M)*X(3,M)
      DER(4) = 0.
      RETURN
      END
```



\* SYMBOL TABLE  
\* LABEL

CSMPLX

SUBROUTINE SMPLX(NADM,NEQ)  
C MAIN PROGRAM OF PACKAGE CONTAINING SMPLX, PIVOT, AND PRINT FOR  
C SOLUTION OF LINEAR PROGRAMMING PROBLEM. BASIC CAPABILITY IS 50  
C EQUATIONS IN 200 VARIABLES. PROGRAM EMPLOYS SIMPLEX ALGORITHM  
C A LA DANTZIG TO OBTAIN A BASIC FEASIBLE SOLUTION, THEN PROCEEDS TO  
C MINIMIZE THE OBJECTIVE FORM USING SIMPLEX ALGORITHM.  
C IN THIS PROGRAM, ARRAY IS THE MATRIX OF COEFFICIENTS AND CONSTANTS  
C NADM IS THE NUMBER OF ADMISSIBLE VARIABLES, NEQ IS THE NUMBER OF  
C EQUATIONS, WHICH IS ALSO THE NUMBER OF ARTIFICIAL VARIABLES TO BE  
C INTRODUCED. IROW IS A ONE-DIMENSIONAL ARRAY CONTAINING THE SUB-  
C SCRIPTS OF THE ELEMENTS IN THE BASIC SOLUTION.

C DIMENSION ARRAY(52,253),IROW(50)

C COMMON ARRAY, IROW

C MAIN PROGRAM PLACES COEFFICIENTS OF PROBLEM IN STANDARD FORM IN  
C ARRAY.

C DEFINE CERTAIN USEFUL CONSTANTS

K1 = NEQ+1

K2 = NEQ+2

K3 = NADM + NEQ

K4 = K3 + 3

C SET IN COEFFICIENTS OF ARTIFICIAL VARIABLES, -Z, AND -W

DO 1 I = 1,K2

K5 = NADM + 1

1 ARRAY(1,K5) = 1.

C MAKE SYSTEM CANONICAL IN ARTIFICIAL VARIABLES, -Z, AND -W.

DO 3 I = 1,NADM

SUM = 0.

DO 2 J = 1,NEQ

2 SUM = SUM + ARRAY(J, I)

3 ARRAY(K2, I) = -SUM

SUM = 0.

DO 4 J = 1,NEQ

4 SUM = SUM + ARRAY(J, K4)

ARRAY(K2, K4) = -SUM

C STORE SUBSCRIPTS OF BASIS ELEMENTS IN IROW.

DO 5 I = 1,NEQ

5 IROW(I) = NADM + I

C

C

C

51 M = 1

D = ARRAY(K2, 1)

C SEARCH FOR COLUMN CONTAINING PIVOT ELEMENT.

DO 7 I = 2, K3

IF(D-ARRAY(K2, I)) 7, 7, 6

```

6 D = ARRAY(K2,1)
  M = 1
7 CONTINUE
C CHECK TO SEE IF D IS NON-NEGATIVE. IF YES, CHECK INFEASIBILITY FO
C RM. IF NO, PROCEED WITH ITERATION.
  IF (D+0.00001) 11,8,8
8 W = ABSF(ARRAY(K2,K4))
  IF (0.00001-W) 9,12,12
9 WRITE OUTPUTTAPE 6,10
10 FORMAT (5X,29HTHERE IS NO FEASIBLE SOLUTION)
  RETURN
C CHECK INDICATES IMPROVEMENT IS POSSIBLE. NOW NECESSARY TO
C REPLACE ONE ELEMENT IN BASIC SOLUTION WITH ANOTHER. SUBROUTINE
C PIVOT DOES THIS.
11 CALL PIVOT(2,M,NADM,NEQ)
  GO TO 51
C
C BEGIN PHASETWO. CEASE CONSIDERATION OF ARTIFICIAL VARIABLES.
12 M = 1
  D = ARRAY(K1,1)
  DO 14 I = 2,NADM
    IF(D-ARRAY(K1,I)) 14,14,13
13 D = ARRAY(K1,I)
  M = 1
14 CONTINUE
C D NOW CONTAINS MINIMUM COEFFICIENT IN OBJECTIVE FORM. CHECK SIGN
C OF D.
  IF (D+0.00001) 16,15,15
15 CALL PRINT(NADM,NEQ)
  RETURN
16 II = 0
  DO 18 I = 1,NEQ
    IF(ARRAY(I,M)) 17,17,18
17 II = II+1
18 CONTINUE
  IF(NEQ - II) 19,19,21
19 WRITE OUTPUT TAPE 6,20
20 FORMAT (35H OBJECTIVE FORM HAS NO LOWER BOUND)
  RETURN
21 CALL PIVOT (1,M,NADM,NEQ)
  GO TO 12
  END
* SYMBOL TABLE
* LABEL
CPIVOT
C
C SUBROUTINE PIVOT(JJ,M,NADM,NEQ)
C
C PROGRAM FOR FINDING PIVOT ELEMENT AND REPLACING A BASIC VARIABLE
C WITH A NEW BASIC VARIABLE.

```

```

C      JJ = 1 FOR PHASE TWO, JJ = 2 FORPHASE ONE.
C
C      DIMENSION ARRAY(52,253),IROW(50),TEMP1(253),TEMP2(52)
C      COMMON ARRAY,IROW
C
C      M IS THE COLUMN CONTAINING PIVOT ELEMENT, BROUGHT IN FROM CALLING
C      PROGRAM. M1 WILL BE ROW OF PIVOT ELEMENT.
C      FIRST CHOOSE ROW M1.
C      D1 = 0.1E06
C      M1 = 1
C      K1 = NADM + NEQ + 3
C      IJ = 0
C      K3 = NEQ + 2
C      DO 1 I= 1,K1
1     TEMP1(I) = 0.
C      DO 6 I = 1,NEQ
C      IF (ARRAY(I,M)) 6,6,2
2     A = ARRAY(I,K1)/ARRAY(I,M)
C      IF (A-0.000001) 3,3,4
3     IJ = IJ + 1
4     IF (D1-A) 6,6,5
5     D1 = A
C      M1 = I
6     CONTINUE
C      CHECK FOR MULTIPLE DEGENERACY. IF MORE THAN ONE CONSTANT IS ZERO,
C      PICK FOR PIVOT ELEMENT THE ONE WITH LARGEST ARRAY(I,M) AND ZERO
C      CONSTANT.
C      IF (IJ-1) 12,12,7
7     DO 11 I = 1,NEQ
C      IF (ARRAY(I,K1)-0.000001) 8,8,11
8     IF (ARRAY(I,M)) 11,11,9
9     IF (ARRAY(I,M)-A) 11,11,10
10    M1 = I
C      A = ARRAY(I,M)
11    CONTINUE
C      ARRAY(M1,M) IS PIVOT ELEMENT.
12    A= ARRAY(M1,M)
C      IROW(M1) = M
C      DO 13 I= 1,K1
13    TEMP1(I) = ARRAY(M1,I)/A
C      DO 14 I= 1,K3
14    TEMP2(I) = ARRAY(I,M)
C      K2 = NEQ + JJ
C      DO 15 I = 1,K2
C      DO 15 J = 1,K1
15    ARRAY(I,J) = ARRAY(I,J)-TEMP2(I)*TEMP1(J)
C      DO 16 I = 1,K1
16    ARRAY(M1,I) = TEMP1(I)
C      RETURN
C      END

```

```

*      SYMBOL TABLE
*      LABEL
CPRINT
C
      SUBROUTINE PRINT(NADM,NEQ)
C
C      PROGRAM FOR PRINTING RESULTS OF SIMPLEX METHOD SOLUTION OF
C      LINEAR PROGRAMMING PROBLEM.
C
      DIMENSION ARRAY(52,253),IROW(50)
      COMMON ARRAY, IROW
      K1 = NADM + NEQ + 3
      WRITE OUTPUT TAPE 6,1
1  FORMAT(61H1      SOLUTION TO LINEAR PROGRAMMING PROBLEM BYSIMPLEX M
1ETHOD////)
      WRITE OUTPUT TAPE 6,2,NEQ
2  FORMAT (3X,22HNUMBER OF EQUATIONS = , 13///)
      WRITE OUTPUT TAPE 6,3,NADM
3  FORMAT (3X,22HNUMBER OF VARIABLES = , 13///)
      WRITE OUTPUT TAPE 6,30
30 FORMAT(5X,92HVALUES OF BASIC VARIABLES IN OPTIMUM SOLUTION FOLLOW.
1  SOLUTION APPEARS AS PAIRS OF NUMBERS./5X,91HTHE FIRST NUMBER IS
2THE SUBSCRIPT OF THE COMPONENT APPEARING IN THE BASIC SOLUTION, AN
3D THE/5X,88HSECOND IS ITS VALUE. THE CORRESPONDENCE BETWEEN THE S
4UBSCRIPTS AND THE VARIABLES OF THE/5X,55HORIGIAL PROBLEMMUST BE
5ESTABLISHED BY THE PROGRAMMER.///)
      WRITE OUTPUT TAPE 6,4,(IROW(I),ARRAY(I,K1)),I=1,NEQ)
4  FORMAT (7(14,1H,,1PE12.5))
      WRITE OUTPUT TAPE 6,5
5  FORMAT(1H0,39H      ALL OTHER VARIABLES HAVE VALUE ZERO///)
      K2 = NEQ+1
      Z = -ARRAY(K2,K1)
      WRITE OUTPUT TAPE 6,6,Z
6  FORMAT (39H      OPTIMUM VALUE OF OBJECTIVE FORM = ,1PE12.5///)
      RETURN
      END

```