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THE PERFORMANCE OF THE PANEL STRUCTURES

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Summary

The panels are subject to (a) their dead load, (b) forces exerted from the dish structure on the panel support points, (c) the weight of the surface plates, and (d) snow or wind on the surface plates. In addition to the deformations of the panel structure, we also have the sag and torsion of the surface support bars. All analyses have been done with STRUDL; and a comparison between FRAME and TRUSS has shown good agreement (joint stiffness unimportant).

All panel bars are stable against combined survival loads. The lowest dynamical frequency is 24 cps.

The gravitational rms surface deformation amounts to $\Delta P = 0.00199$ inch = 0.05055 mm, for truss analysis; and $\Delta P = 0.00202$ inch = 0.05588 mm for frame analysis. The thermal deformation is 0.0009 inch at night and 0.0056 inch in full sunshine. The wind deformations of panels and surface plates give at 18 mph an rms pointing error of $\Delta\phi = 0.26$ arcsec, and an rms surface deviation of $\Delta z = 0.0008$ inch.

I. Equivalent Bars for Telescope Analysis

1. Replacement

Since the panels have about 200 members each, they must be replaced in the telescope analysis by some simpler structures; and since they are connected with the dish structure at only a few points, this can be done easily. Their stiffness normal to the surface is 4.2 times smaller than that of the dish; we thus neglect it, replacing the panel only by a planar structure, just connecting all corner points with each other by "equivalent surface bars".

The result is shown in Fig. 1. Panels A and B are replaced by 6 equivalent bars each, and panels C and D by 10 each. Regarding the telescope, two peripheral bars from neighboring panels always add up to a single telescope bar, while all diagonal panel bars enter the telescope unchanged.

Bar areas A_{eq} and material densities ρ_{eq} of these equivalent bars are calculated by a separate program called "Surface". First, the actual panel (200 bars) is analyzed with STRUDL for its reactions under dead loads, and for its corner deflections under the various F-forces of Figure 1 (taken as unit forces). The Surface program then uses an input data the coordinates of the (4 or 5) corner points, and their reactions and deflections from STRUDL. It then makes a "first guess" for all A_{eq} and ρ_{eq}, regarding only one bar at a time. Next, it analyzes the whole equivalent panel (6 or 10 bars), using this first guess. Finally, it applies an iterative method, changing all A_{eq} and ρ_{eq}, for approaching

the reactions and deflections from STRUDL. Mostly, 4 or 5 iterations are enough. The final values A_{eq} and ρ_{eq} are printed and punched, for each equivalent bar of all four panels.

2. Load Conditions

With these equivalent surface bars (61 in 1/4 telescope), all stresses and deformations of the telescope then are analyzed, for the 6 load conditions given in Table 1.

Table 1. Load conditions for telescope analysis.

(z = axis of paraboloid; y = elevation axis)

Ω	Condition	Direction
1	dead load, zenith position	z
2	dead load, horizon position	x
3	wind 18 mph for observation	z
4	dead load plus 20 lb/ft^2 survival snow	z
5	wind 85 mph for survival	x
6	wind 85 mph for survival	y

3. Forces

The telescope analysis yields all forces F exerted on the panels in the directions along those of the equivalent bars. Panels of type A or B have J = 4 possible locations on the telescope (see Fig. 1 of Report 40), panel C has J = 2, and panel D has only J = 1 location. Each

equivalent panel bar thus has J possible locations. The telescope analysis gives as output the stress in all 61 equivalent bars, for all 6 load conditions of Table 1. For each bar we then decide to which panel it belongs, and we call

$$S_{\Omega ij} = \text{stress from load condition } \Omega = 1 \ldots 6 \text{ (Table 1)} \quad (1)$$
$$\text{in equivalent panel bar } i = 1 \ldots I \text{ (Fig. 1)}$$
$$\text{at location } j = 1 \ldots J.$$

For survival, we are interested in the maximum force of all locations, and we call

$$F^{(s)}_{\Omega i} = A_{eq} \text{ Max } \{S_{\Omega i1}, S_{\Omega i2}, \ldots, S_{\Omega iJ}\} \quad (2)$$

For $\Omega = 4$, 5 and 6, and all $i = 1 \ldots I$, for all four panels, we find Max $\{F^{(s)}\} = 30.84$ kip, and Av $\{F^{(s)}\} = 3.76$ kip.

For observation, we want the rms force over all locations, and we call

$$F^{(o)}_{\Omega i} = A_{eq} \text{ rms } \{S_{\Omega i1}, S_{\Omega i2}, \ldots, S_{\Omega iJ}\} \quad (3)$$

For $\Omega = 1$ and 2, all $i = 1 \ldots I$, and all four panels, we find Max $\{F^{(s)}\} = 13.44$ kip, and Av $\{F^{(o)}\} = 4.32$ kip.

During the design phase of the panels, several "cycles" between telescope and panels were needed. Present panel design gives certain A_{eq} and ρ_{eq}. These enter the telescope structure, and the telescope ananlysis yields all F-forces. These forces enter the panel analysis, for stability as well as deformations. If any of these two is unsatisfactory, the panel design must be changed, and the whole cycle must be repeated. This was actually done three times until all agreed.

II. Surface Support Bars

The surface plates are supported by the tangential surface bars of the panels; and for better beam action, these bars are of rectangular tubing, 2 x 4 inch, the longer side vertical, see Fig. 2.

If the panel looks at <u>zenith</u>, the bars will sag in z-direction under their dead load plus the weight of the plates. What counts is the rms $(\Delta z - \overline{\Delta z})$ over all points of plate support. This was calculated and found as

$$b_1 = \text{rms } (\Delta z - \overline{\Delta z}) = 0.0012 \text{ inch.} \tag{4}$$

If the panel looks at <u>horizon</u>, we have first a sag in x-direction, and second a torsion of the bar about its axis, since the plate corners will be mounted on studs and pedestals, at most 7 inch above the bar axis. Both sag and torsion add up to a displacement Δx of the plates. If everything were aligned perfectly, this would just result in a parallel shift Δx of the surface which does not matter. Actually, the support points have a (necessary) offset of about 4°, see Fig. 2. In addition, we allow a

$$\text{misalighnment} = \pm\ 10° \text{ Max,} \tag{5}$$

or $\pm$ 4° rms. Offset and misalignment add up quadratically to an rms total of $\pm$ 5.7°, and the result is a deformation normal to the surface, of $\Delta z = \Delta x \sin 5.7°$. Over all support points, the calculations gave

$$b_2 = \text{rms}(\Delta z) = 0.0010 \text{ inch.} \tag{6}$$

III. Joint Stiffness

At all joints of the panels, the pipes are just welded together (Report 40), and the joints will have some limited stiffness, somewhere in between pin joints and stiff joints. Since the actual stiffness is difficult to estimate, this would be bad if it did matter. In order to find out how much it matters, all panels were analyzed with the STRUDL program, both with TRUSS (pin joints) and FRAME (stiff joints).

Table 2. Difference (in per cent) between TRUSS and FRAME deformations. s = sag; $F_1 \ldots F_I$ see Fig. 1; ΔP from (23).

Panel	s	Av $\{F_1 \ldots F_I\}$	ΔP
A	1.1	1.0	2.4
B	2.4	.7	1.2
C	.9	2.5	4.8
D	3.9	.0	2.1
all	1.3	1.0	1.5

The result is shown in Table 2. All differences between FRAME and TRUSS are only a few per cent. What counts is the average of ΔP, which is 1.5% and can be completely neglected. Thus, the actual joint stiffness does not matter.

IV. Stability and Dynamics

1. Stability

Each panel is analyzed with STRUDL under 8 load conditions for panels A and B, and 12 for C and D. From Fig. 1, we call I = number of

force directions. From the analysis results we call, for each <u>actual</u> panel bar,

S_d = stress from dead loads and plates;

S_2 = stress from 21 lb/ft^2 of snow on plates;

$\sigma_1 \ldots \sigma_I$ = stress from unit forces $F_1 \ldots F_I$ of Fig. 1.

With the maximum forces $F^{(s)}_{\Omega i}$ from (2), we find for each actual panel bar its maximum stress for snow load as

$$S_s = \left| S_d + S_2 + \sum_{i=1}^{I} F^{(s)}_{4i} \sigma_i \right| \tag{7}$$

and for survival winds as

$$S_w = \left| S_d + \sum_{i=1}^{I} F^{(s)}_{1i} \sigma_i \right| + \sqrt{\left(\sum_{i=1}^{I} F^{(s)}_{5i} \sigma_i\right)^2 + \left(\sum_{i=1}^{I} F^{(s)}_{6i} \sigma_i\right)^2} \tag{8}$$

To be on the safe side, we consider always S_s and S_w as being in compression. For each bar, we then calculate

S_a = maximum allowable stress, as a function of $K\ell/r$, with $K = 0.8$. (8)

For stability, we demand for each bar, first,

$$Q_s = S_s / S_s \leq 1; \tag{10}$$

second,

$$Q_w = S_w / S_a \leq 1; \tag{11}$$

and third,

$$K/r = \leq 200, \text{ with } K = 0.8. \tag{12}$$

During the design phase, the panel bars were changed until finally all bars of each panel fulfilled all three demands. These final bars are given in Report 40.

2. Dynamics

All panels were analyzed with an IBM-dynamics program for their lowest dynamical mode, which is a vibration up and down in z-direction. The resulting lowest frequencies are given in Table 3. All frequencies are so high that we can completely neglect their contribution to the telescope dynamics.

Table 3. Lowest dynamical frequency of panels.

Panel	cps
A	26
B	38
C	24
D	28

V. Gravitational Deformations

1. Single Contributions

We must add the rms deformations from sag under dead loads, from the F-forces of the dish structure, and from sag and torsion of the surface support bars; for both zenith and horizon position.

The <u>sag</u> for horizon position was once calculated and its z-component was found negligible. For zenith position, we call s the sag of each surface point of a panel, $\overline{s}$ the average over the panel, and $\overline{s_o}$ the average over all panels (weighted by their number). What counts is the deviation from a parallel shift,

$$d = \text{rms}\ (s - \overline{s_o}). \tag{13}$$

Each <u>F-force</u> from (3) gives a surface deviation f,

$f_{\Omega i}$ = rms(Δs) of surface points, for force $F_{\Omega i}^{(o)}$ according to (3), from load condition Ω in direction i. (14)

For zenith position, Ω = 1, and horizon position, Ω = 2, we calculate

$$f_\Omega = \sqrt{\sum_{i=1}^{I} F_{\Omega i}^2}\ \Bigg|\ I = 6 \text{ or } 10, \text{ Fig. 1} \tag{15}$$

If the telescope were adjusted in the absence of gravity, and gravity were then "switched on", the panel deviation in zenith position then would be

$$\Delta P_1 = \sqrt{d^2 + f_1^2 + b_1^2} \tag{16}$$

and in horizon position

$$\Delta P_2 = \sqrt{f_2^2 + b_2^2}\ . \tag{17}$$

2. Adjustment for 30° Zenith Angle

In Report 33 it was decided to adjust the telescope surface for a zenith distance of $\theta = 30°$. And since points beyond the zenith can be observed by turning 180° in azimuth, whereas the atmosphere will limit or prevent observations close to horizon, it was decided to regard

$$0 \leq \phi \leq 60° \tag{18}$$

as the range of zenith angle ϕ for observations at shortest wavelengths.

If the telescope is adjusted at zenith angle θ and observes at zenith angle ϕ, the deviation ΔP_ϕ from a best-fit paraboloid is in general, for the panel contribution,

$$\Delta P_\phi = \sqrt{\Delta P_1^2 (\cos\phi - \cos\theta)^2 + \Delta P_2^2 (\sin\phi - \sin\theta)^2}. \tag{19}$$

With $\theta = 30°$ we then have, at the limits of the observing range (18),

$$\Delta P_0 = 0.500 \sqrt{(.268\ \Delta P_1)^2 + \Delta P_2^2} \tag{20}$$

and

$$\Delta P_{60} = 0.366 \sqrt{\Delta P_1^2 + \Delta P_2^2}. \tag{21}$$

What finally counts is the larger of the two,

$$\Delta P = \mathrm{Max}\,\{\Delta P_0, \Delta P_{60}\}\ . \tag{22}$$

Table 4. Gravitational deformations of panels (in 10^{-3} inch).

( ) = equation of definition.

Analysis	Panel	$\bar{s}$ Sag	d (13)	f_1 (15)	f_2 (15)	ΔP_1 (16)	ΔP_2 (17)	ΔP (22)	ΔP ruled by (20) or (21)	Weight
Frame	A	19.22	5.94	1.65	1.10	6.61	2.56	2.08	(21)	16/44
	B	10.04	4.46	1.68	1.08	5.26	2.64	1.91	(20)	16/44
	C	11.06	6.02	1.81	1.08	6.54	2.89	2.18	(20)	8/44
	D	18.64	5.50	1.26	0.97	6.17	1.88	1.90	(21)	4/44
	All	14.35	5.38	1.65	1.08	6.07	2.58	2.02	--	44/44
Truss	A	19.01	6.10	1.67	1.11	6.77	2.60	2.13	(21)	16/44
	B	9.78	4.50	1.71	1.15	5.31	2.68	1.73	(20)	16/44
	C	11.00	6.35	1.88	1.12	6.88	2.99	2.27	(20)	8/44
	D	18.35	5.38	1.30	1.00	6.10	1.93	1.89	(21)	4/44
	All	14.14	5.50	1.69	1.12	6.20	2.64	1.99	--	44/44

Table 4 shows the results obtained from the STRUDL analysis. The last line gives the rms over all panels, weighted by their number. What finally matters is ΔP of the last line,

$$\begin{aligned} \Delta P &= 0.00199 \text{ inch} = 0.05055 \text{ mm for truss analysis} \\ \Delta P &= 0.00202 \text{ inch} = 0.05588 \text{ mm for frame analysis} \end{aligned} \qquad (23)$$

which is the contribution of the panels (plus surface support bars) to the total non-homologous deformation of the telescope surface.

VI. Other Deformations

1. Thermal Deformations

The thermal deformations of the panels are already described in Report 37. We used $\Delta T = 1.5°F$ at night, and $\Delta T = 9.0°F$ in sunshine, for the thermal z-gradient; and $\dot{T} = 1.5°F/h$ at night, and $\dot{T} = 8.6°F/h$ after sunset and sunrise, for the thermal lag; all values hold for at least 95% of all time, according to a summary of many measurements at Green Bank and Kitt Peak. The computer analysis then yielded, for the rms deformation of all panels from ΔT and $\dot{T}$ together,

$$\text{rms}(\Delta z) = \begin{cases} .0009 \text{ inch at night,} \\ .0056 \text{ inch in full sun.} \end{cases} \tag{24}$$

2. Wind Deformations

With a wind velocity v in mph, and a shape factor C_s, the pressure in lb/ft^2 is

$$p = .00256\ C_s\ v^2. \tag{25}$$

With v = 18 mph (third quartile at Green Bank, gust factor included, 150 ft. above ground), and with a shape factor of $C_s = 1.56$ for the average pressure on a face-on paraboloid, we find

$$p = 1.29\ lb/ft^2. \tag{26}$$

With this pressure, all panels were analyzed for their surface deformation Δz. In addition, we have the beam action of the surface support bars, to be added linearly for $\overline{\Delta z}$, and quadratically for rms $(\Delta z - \overline{\Delta z})$, which both add up quadratically to rms (Δz). In principle, we should now

add the deformations caused by the F-forces from the dish structure; this was neglected, however, since for wind load (26) these forces are in the average 12.5 times smaller than those used for the gravitational deformations.

Table 5. Wind deformations of panels, in 10^{-3} inch, for panel structure plus surface support bars. For pressure $p = 1.29$ lb/ft^2.

Panel	$\overline{\Delta z}$	rms(Δz)	rms($\Delta z - \overline{\Delta z}$)
A	3.06	3.13	.67
B	1.98	2.07	.60
C	1.88	1.99	.66
D	2.96	3.01	.55
All	2.44	2.58	.84

The results of the STRUDL analysis are given in Table 5. The last line is the weighted average $\overline{\Delta z}_o$ of $\overline{\Delta z}$, the weighted rms of rms(Δz), and the quadratic difference of both which is the rms($\Delta z - \overline{\Delta z}_o$) of all panels.

In the following we want to include the wind deformations of the surface plates, too, from Report 38. The total is given in Table 6, where $\overline{\Delta z}$ is added linearly, rms($\Delta z - \overline{\Delta z}$) quadratically, and both results are added quadratically to obtain rms (Δz).

<u>Table 6</u>. Wind deformations (10^{-3} inch) of panels and plates; $p = 1.29\ \mathrm{lb/ft}^2$.

	Panels (Table 5)	Plates (Report 38)	Total
$\overline{\Delta z}_o$	2.44	.95	3.39
$\mathrm{rms}(\Delta z)$	2.58	1.03	3.52
$\mathrm{rms}(\Delta z - \overline{\Delta z}_o)$	.84	.47	.96

Next, we use measurements (by Cohen and Vellozzi, see Fig. 3) of pressure distributions $C_{\alpha x}$ across a parabolic surface ($f/D = 0.5$) as a function of the yaw angle α (between wind and telescope rim plane). For a round dish, we use in the following averages a weight function (-1 ($-1 \leq x \leq +1$, from rim to rim):

$$w(x) = \frac{2}{\pi} \sqrt{1 - x^2} \ . \tag{27}$$

For each of the 13 curves of Fig. 3, we find the slope

$$\gamma = 4\ \overline{xC} \tag{28}$$

of the best-fitting straight line

$$C_o = \overline{C} + \gamma\, x \tag{29}$$

and the residual R, the rms deviation between curve and line, given by

$$R = \mathrm{rms}(C - C_o) = \sqrt{\overline{C^2} - \overline{C}^{\,2} - \gamma^2/4}. \tag{30}$$

The wind deformations of panels and plates, at 18 mph, then has two effects. First, a tilt of the telescope axis (pointing error) of

$$\Delta\phi = \frac{\gamma}{1.56}\,\frac{\overline{\Delta z}_o}{D/2}\;2.06 \text{ arcsec}, \qquad (31)$$

where $1.56 = C_s$ is the shape factor used for Tables 5 and 6; $\overline{\Delta z}_o - 3.39 \times 10^{-3}$ inch from Table 6; and $D/2 = 1280$ inch = telescope radius. The tilt then is

$$\Delta\phi = 0.350\;\gamma \text{ arcsec}. \qquad (32)$$

The second effect is a surface deviation which has two components. First, the rms deviation of the average surface from the tilted parabola, $R\;\overline{\Delta z}_o/1.56 = 2.17\;R$; second, the rms deviation between local and average surface, rms(C) rms$(\Delta z - \overline{\Delta z}_o)/1.56 = 0.62$ rms(C); both contributions add up quadratically to

$$\Delta z = \sqrt{4.72\;R^2 + 0.38\;\overline{C^2}}\;. \qquad (33)$$

All this was done for the 13 yaw angles α of Fig. 3; and Fig. 4 shows $\overline{C}$, rms(C), R and γ as functions of α. Tilt and residual are largest for $\alpha = 15°$, where we find

$$\left.\begin{aligned}\Delta\phi &= 0.40 \text{ arcsec}\\ \Delta z &= 1.24 \times 10^{-3} \text{ inch}\end{aligned}\right\}\ \text{maximum, for } \alpha = 15°. \qquad (34)$$

Actually, we should take the rms values over all yaw angles, using $1/2 \cos\alpha$ as a weight function. This yields

$$\text{rms(C)} = 1.01 \qquad (35)$$

$$\mathrm{rms}(\gamma) = .74 \tag{36}$$

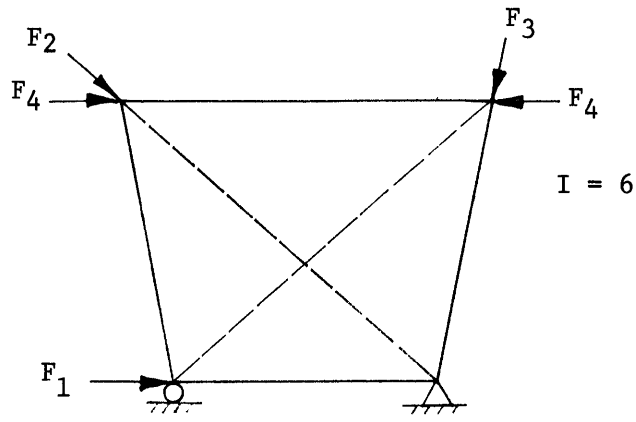
$$\mathrm{rms}(R) = .234 \tag{37}$$

and finally, for pointing error and surface deviation from panels and plates:

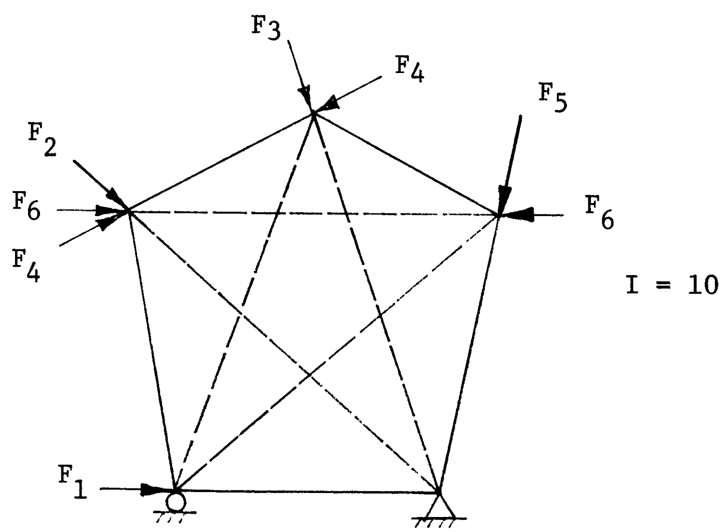
$$\mathrm{rms}(\Delta\phi) = 0.26 \text{ arcsec} \tag{38}$$

$$\mathrm{rms}(\Delta z) = 0.79 \times 10^{-3} \text{ inch.} \tag{39}$$

FIG. 1 FORCES F_i EXERTED FROM DISH STRUCTURE TO PANEL SUPPORT POINTS



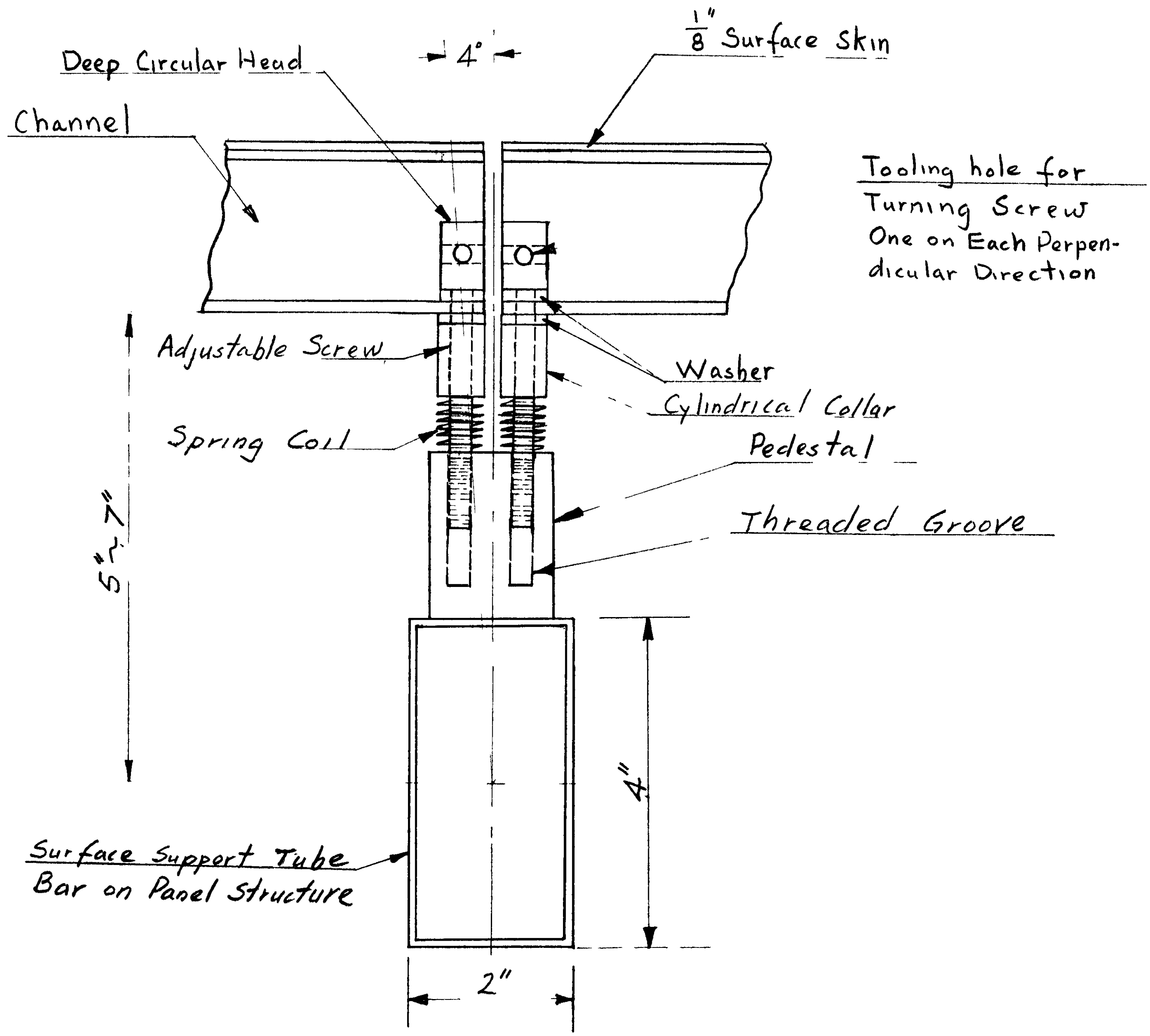
PANEL A & B



PANEL C & D

—— Peripheral equivalent bars, replacing panel in telescope analysis.
- - - Diagonal equivalent bars, replacing panel in telescope analysis.
I = Number of equivalent bars and of force directions.

FIG. 2 SURFACE PLATE CORNER SUPPORT



Scale : 1/2" = 1"

Alternatives

1. Cylindrical deep nut in lieu of spring coil.

2. Regular screw head with adjustment from top of dish surface.

This spring coil
might destroy
the entire precision
of deflection control

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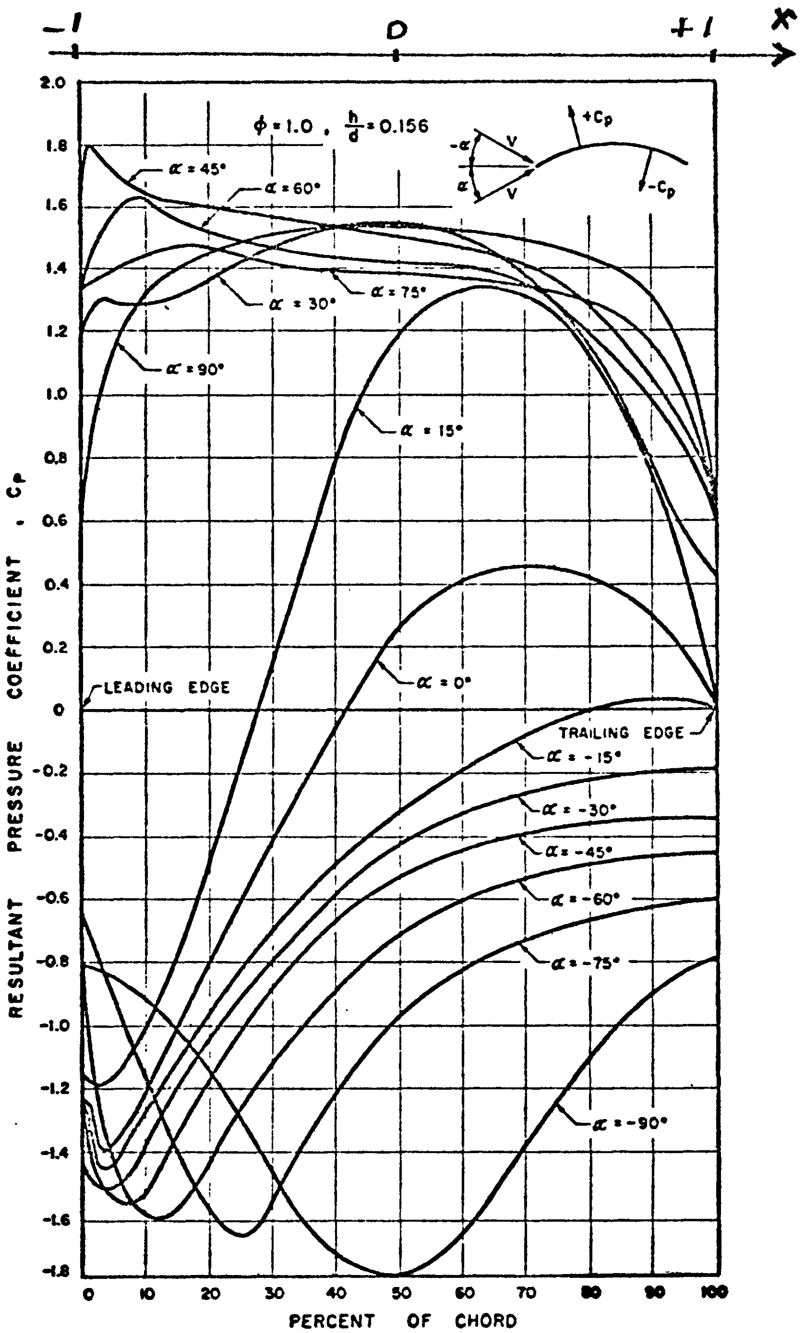

FIGURE 25. Chordwise pressure profiles. E. Cohen and J. Vellozzi

Fig. 3. Pressure distribution across telescope surface.

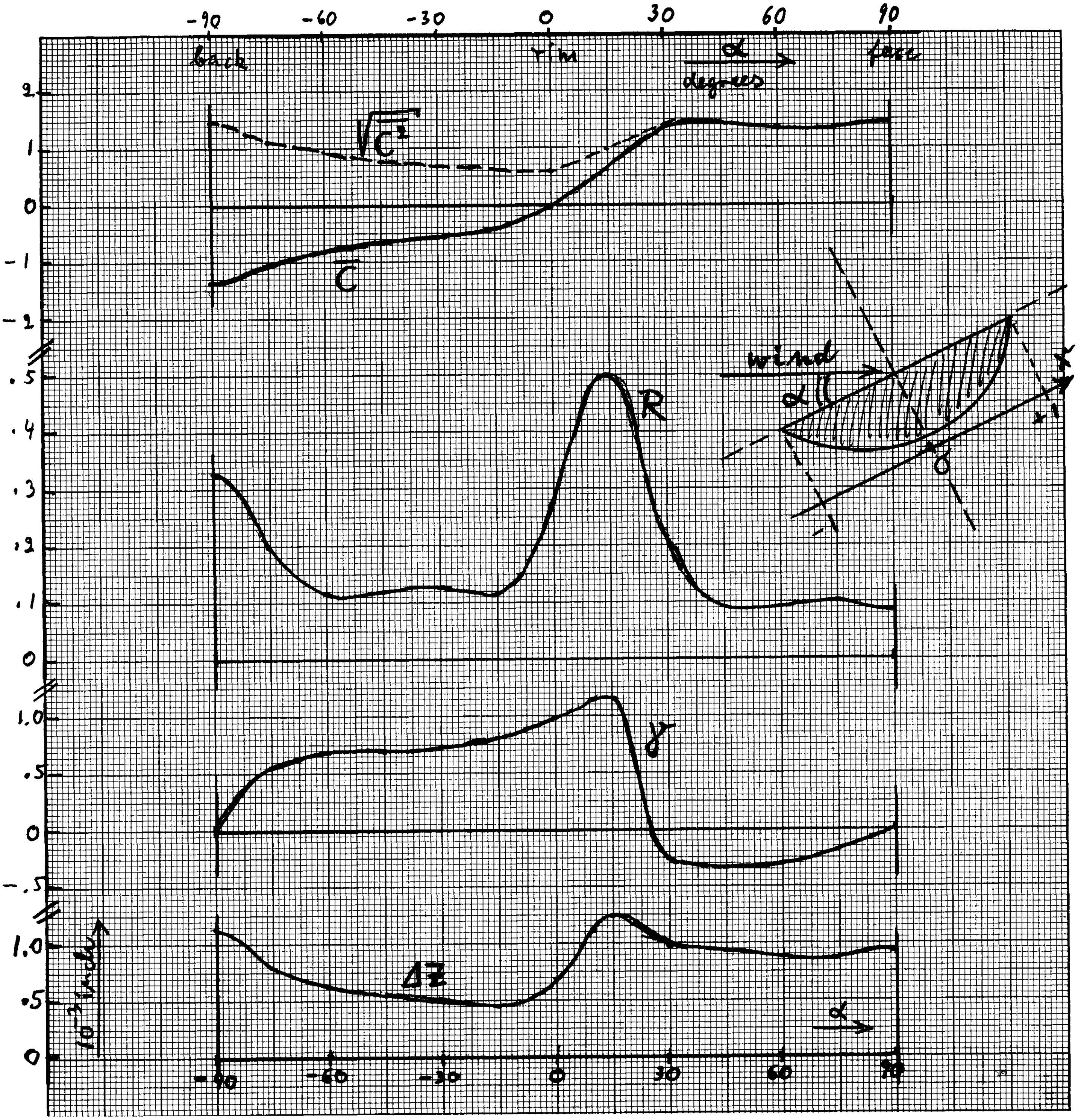

Fig. 4. Pressure coefficients C as function of yaw angle α.

C defined by pressure $p = .00256\ C\ v^2$;

γ = slope of best-fitting straight line, $C_o = \overline{C} + \gamma x$;

R = residual = rms$(C - C_o)$;

Δz = rms surface deviation of panels and plates.