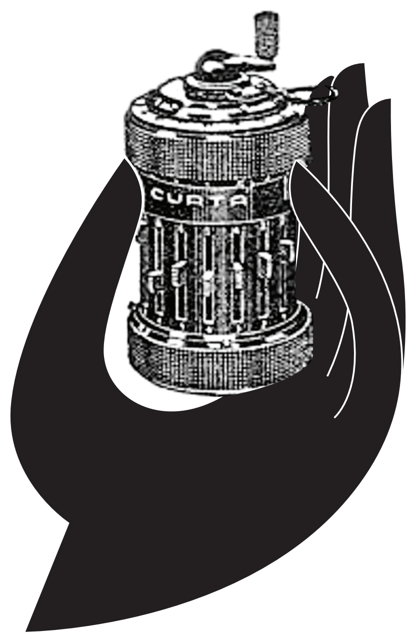
CURTA

CALCULATING TECHNIQUES



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PREFACE

CALCULATION WITH THE CURTA

This manual is primarily intended to supplement the Manufacturers' little handbook *'Instructions for the use of the Curta Calculating Machine'* which should first be read by the Curta user. It is hoped however, that it will prove useful to calculating machine users generally.

The operation of the machine itself, whether Model I or Model II, is so simple that many users may not at first appreciate the large variety of calculations which can be solved with the Curta. Basically, the Curta only performs repeated addition, but there are few operations of numerical analysis and virtually no commercial calculations which cannot be reduced to a process of repeated addition.

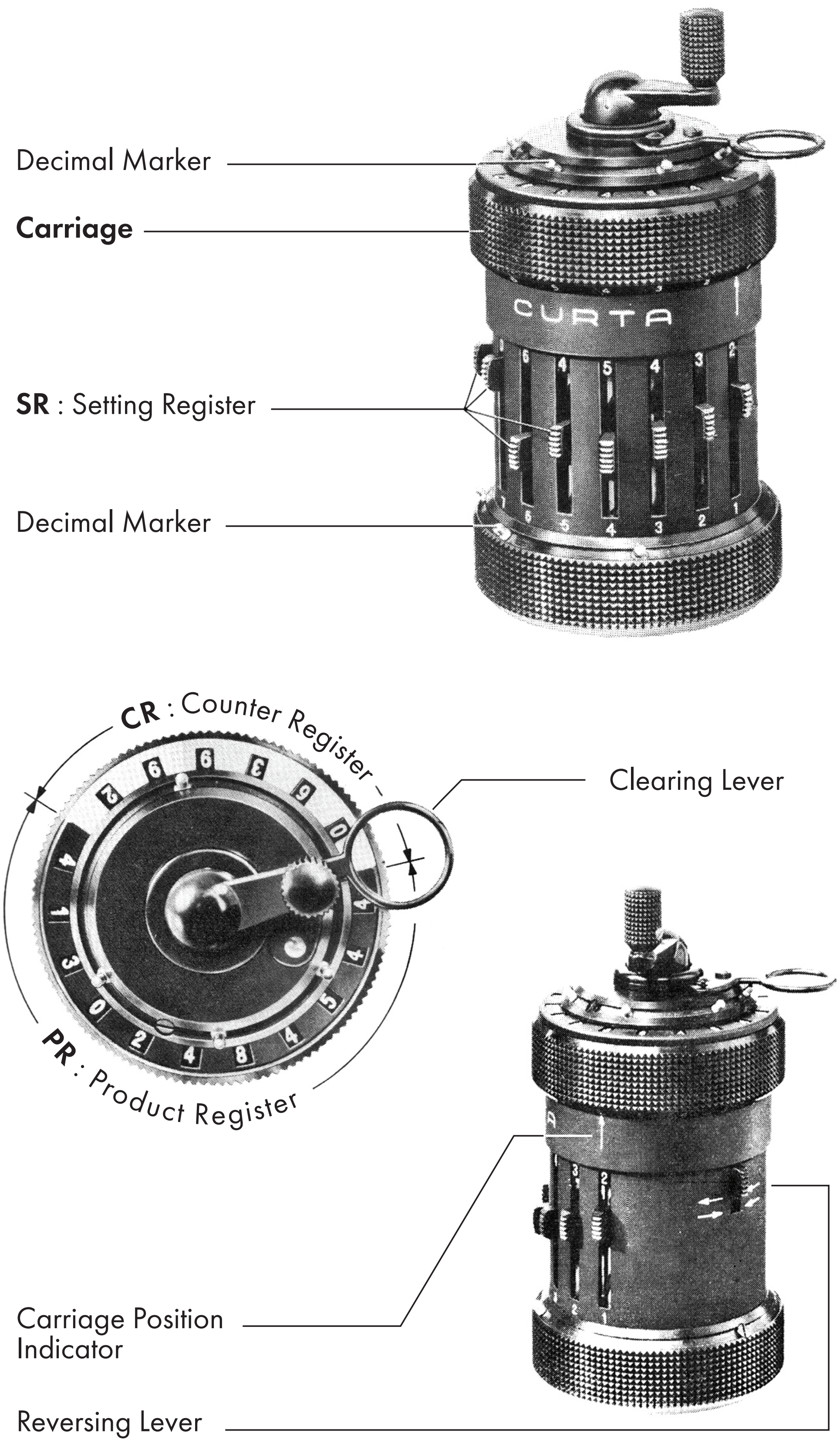
It is the purpose of this manual to show the most suitable method or 'programme' by which common commercial and scientific calculations can best be adapted for solution by the Curta. Further instruction sheets will be made available from time to time and it is hoped that readers will send us a note of any methods, not so far indicated, which they have found useful for special purposes. In this way, a really comprehensive 'library' can be built up. Every six months we will circularise users with a copy of the latest Index so that they may apply for any new instruction sheets likely to be of interest to them.

Anyone who has attempted to put instructions for calculating procedure on paper will appreciate the difficulty of explaining a technique clearly, yet concisely. Calculations which take only a few seconds to perform are liable to take as many minutes to describe, and Examples are more useful than detailed descriptions. For these reasons we have evolved an abbreviated notation for describing the process of solution which we hope will be readily understood.

We strongly recommend that the examples should be worked out on a Curta when the instructions are first read. Generally, a Model I will be sufficient unless otherwise indicated.

The various uses of the Curta will be indexed by a letter showing the general type of calculation and a serial number indicating the particular calculation described.

CURTA TYPE 1


Decimal Marker
Carriage
SR : Setting Register
Decimal Marker
CR : Counter Register
Clearing Lever
PR : Product Register
Carriage Position Indicator
Reversing Lever

ABBREVIATED NOTATION

The Curta has 3 Registers, or groups of dials, which will be indicated by their initials:-

Setting Register:	**SR**
Counter Register:	**CR**
Product Register:	**PR**

A carriage position will be indicated thus:-

Carriage to position 3:	**C3**

The carriage position indicated is normally that for the 1st step of the calculation.

The position of the Reversing Lever, that is the Lever which reverses the gearing of **CR**, will be indicated by an ***R*** immediately after the Carriage Position when the reversing lever is down, i.e. in the Reverse Position (to count negative turns). If the lever is up, the ***R*** will be omitted.

The position of decimal markers will be indicated as follows:-

Decimal place:	**.**
Decimal places where markers are also set for other purposes:	**:**
Marker used for other purpose:	**,**
Where two markers are required, e.g. to indicate a split **SR**:	**,,**

In order to indicate the position of a setting or result on the corresponding register and to avoid printing unnecessary zeros, an oblique stroke **/** will be printed either to the left or right of a number indicating that the dials on the left or right will read **0**.

To indicate that all the dials to the left of a number will read 9, a nine followed by an oblique stroke **/** will be placed on the left of the number, thus **9/**.

Examples:

A setting of 1.534/ indicates that 1.534 has been set on the four left hand dials of **SR**.

9/7,641:27 on **PR** of Model I indicates that the figures 999997,641:27 appear in the **PR** dials. One decimal marker has been set to indicate the 'thousands' and two decimal markers to indicate the decimal place.

In showing the steps of a calculation, **SR** will always be placed on the left, **CR** in the centre and **PR** on the right.

An asterisk will be placed on the right of the result. Thus, if 1.534 is to be multiplied by 37.2 and the result shown positively in the extreme left hand side of **PR** in Model I, the abbreviated description will be:-

	SR	CR	PR
C6	**/1.53400**	**37.2/**	**57.0648/***

This means that we start with the carriage in position 6, the Reversing Lever up, set 1,534 in **SR** dials 6 to 3, multiply by 37.2 from the left and obtain the result 57.0648 in the left hand dials of **PR**.

It is sometimes desirable to set a complement into the machine or to convert a number, read from **PR** or **CR**, into its complement. This can very easily be done mentally by deducting each digit from 9, except the right hand digit, which should be deducted from 10.

	5390724
Example: (i) **Set complement as**	**9/4609276**
	9/400912
Example: (ii) **Read complement as**	**599088**

It will be found with a little practice that complements can be set or read almost as easily as the numbers themselves.

There are several techniques in which the **CR** dials are not used and where it is not necessary to set the Reversing Lever down. In the examples, however, the figures appearing in the **CR** dials and the position of the Reversing Lever are generally given and the **CR** dials are cleared in order to show the turns of the handle which are made at each step. Of course full advantage should be taken of short cuts so that the number of turns actually made will be many less than the reading on the **CR** dials.

HANDLING THE CURTA

There are a few points in handling the Curta which we do not think have been made sufficiently clear in the Makers' Handbook.

1. When holding the machine in the left hand, the third and little fingers should be clear of the setting levers to avoid moving these levers by mistake during operation.

2. The carriage should **not** be gripped **tightly** by the thumb and forefinger. When the carriage has been lifted by thumb and forefinger, pressure of the forefinger should be instinctively released and the carriage rotated by the thumb only so that it must drop into the next position. During a calculation the carriage should only be stepped one place at a time, even if no turns are to be made in a position. In this way, the carriage can be moved without looking at the machine and there will be no danger of moving two steps instead of one.

3. The crank handle should be held **lightly** between the thumb and forefinger of right hand. This will avoid the danger of passing over the zero position by mistake.

4. Left handed operators will generally find it more convenient to hold the Curta in their right hand, since they will wish to hold their pencil in the left hand whilst operating the machine.

5. Normally, the clearing ring should be left in the right hand position, i.e. between the first dial of **PR** and the last dial of **CR**. It is possible to clear a portion of **PR** or **CR** if desired and this can be very useful for certain calculations. If the left hand side of **PR** is to be cleared after each calculation, it will of course be necessary to leave the clearing ring in the left hand position.

6. The use of short cut methods of multiplication and division cannot be too strongly stressed. The operator should get accustomed to short cutting when multiplying from the left or right and when using such special techniques as successive and transfer multiplication described in *Section A*.

   Time saved in rotating the handle is not the only advantage of short cutting in multiplication. It also minimizes mistakes in counting turns of the handle as the number of turns in any one position of the carriage (other than the left hand position) need seldom be more than five.

7. When not in use, the Curta should be replaced in its rubber-sealed metal case, both to avoid accidental damage and the collection of dust and dirt on the setting levers, resulting in stiffness of these levers. If the levers should become at all stiff, the Curta should be returned to *Automatic Business Machines Ltd., 15 Cromwell Road, London SW7.*

SUCCESSIVE MULTIPLICATION

We give this name to multiplication of several factors, e.g. $a \times b \times c \times \ldots\ldots$ It is, of course possible to re-set each partial product but this wastes time and re-setting or writing down partial products may lead to errors.

Having obtained the first partial product on **PR**, set the next factor on **SR** WITH THE RIGHT HAND OPERATIVE DIGIT (digit other than zero) reduced by 1. Step the carriage so that this digit is below the left hand digit in **PR** and rotate the handle through the number of turns corresponding to that digit of **PR**. Step the carriage one turn to the left and turn the handle through the next digit and so on.

What we are really doing is to add $ab(c-1)$ to ab, getting abc.

Example (i) 123 x .345 x 567

	SR	CR	PR
C1	**/.345**	**/123**	**/42.435***
C5	**/566**	**/42435**	**/24060.645***

Answer: 24,060.645

Example (ii) 24,060.645 x 192.1, where the first figure is a product already produced on **PR**.

If we have more than the **CR** capacity set on **PR**, we turn the carriage to its extreme left hand position, 6 with Model I Curta, and set the new factor with the last digit reduced by 1 immediately under the left hand digit of **PR**, thus:-

	SR	CR	PR
C6	**/192.000**	**240606**	**4622041.2645***

Answer = 4,622,041.2645

Note that we set .0 under the left hand digit of **PR**.

Note also that we have not been able to step the carriage so as to multiply by the two right hand digits, 4 and 5, and the result will therefore not be exact. In fact, it will have an error up to 5 in the sixth place of the answer. If we want an exact answer we must either re-set the last factor on the right of **SR** and multiply by the remaining digits, having stepped the carriage to the left hand of these digits, or we must reset the last partial product and multiply by the last factor in the normal manner.

Thus, with the former method, we have

	SR	CR	PR
C2	**/1920**	**/45**	**4622049.9045***

Answer = 4,622,049.9045

An objection may be made to this method of successive multiplication on the grounds that we cannot check the multiplier, unless we write down the partial product and clear **CR**. A method of overcoming this is to reduce, not the right hand operative digit, but the next zero on the right, by one.

Thus, taking the same calculation as before, viz 123 x .345 x 567

	SR	CR	PR	
C1	**/.345**	**/123**	**/42.435***	**Clear CR**
C5	**/566.9**	**/42435**	**/24060.6450***	

Notice that as we rotate the handle through the number of terms corresponding to the digit of **PR** immediately above '.9', this digit is reduced to zero and therefore there is no danger that we turn through the wrong number of turns. Notice also that if we wish to use the decimal markers we must move the marker in **PR** one place to the left in the second stage as there is then one decimal place in **SR**.

There are occasions when it is required to multiply a negative quantity on **PR** (appearing as a complement) by a positive quantity, producing the product as a complement on **PR**; or by a negative quantity, producing a positive amount on **PR**; or to multiply a positive quantity on **PR** by a negative quantity, producing a negative amount on **PR**.

A variation of the Successive Multiplication technique is used and is applicable to such problems as evaluation of Polynomials (see **M 18**) where some of the terms are negative.

Example (i) -123 x 47

Where the complement, 9/877 is set on **PR**. Set /46 on **SR**. Carriage to **C4** so that right hand figure '6' on **SR** comes below the right hand'9' on **PR**. Make one negative turn. Carriage to **C3** and proceed as for normal successive multiplication, thus:-

	SR	CR	PR
C4	**/46**	**9/877**	**9/4219***

The amount showing on **PR** is the complement of 123 x 47, i.e. of 5781.

Example (ii) -123 x -47, where 9/877 is set on **PR**.

Set /48 on **SR** and proceed as before but making one positive turn in position **C4** and negative turns thereafter, thus:-

		SR	CR	PR
C4	***R***	**/48**	**9/877**	**/5781 ***

Example (iii) 123 x -47, where /123 is set on **PR**.

Set /48 on SR, carriage to **C3** and proceed as in normal successive multiplication but making subtractive, instead of additive, turns.

		SR	CR	PR
C3	***R***	**/48**	**/123**	**9/4219***

We can thus formulate the following rules.

(1) When multiplying by a negative quantity, add '1' to last operative figure set on **SR**, instead of subtracting '1', and rotate the handle in subtractive position through the number of turns corresponding to the digits on **PR**.

(2) If a complement is set on **PR**, step the carriage so that the right hand figure on **SR** comes below the right hand 9/ on **PR**, turning the handle in the opposite direction to that used for the other digits of **PR**.

TRANSFER MULTIPLICATION

We give this name to a method of multiplying a figure produced in **CR** by a figure set on **SR**, without re-setting the **CR** figure. This method can be very useful in series of multiplications and divisions, such as

$$\frac{a \times b \times c}{d \times e}$$

This calculation is made in stages - first $a \times b = f$ on **PR**, then $f \div d = g$ on **CR**, then $g \times c = h$ on **PR**, finally $h \div e = i$ on **CR**.

Example: $$\frac{123 \times 345 \times 567}{234 \times 456}$$

	SR	CR	PR	
C6	**/345**	**123/**	**/42435000***	**Clear CR**
C6 *R*	**/234**	**181346***	**/36**	**Clear PR**
C1 *R*	**/567**	**000000**	**/102823182***	
C6 *R*	**/456**	**225489***	**/198**	

Note that in the 3rd stage, when we have to multiply a figure on **CR**, 181346 by 567, we merely set /567 on **SR** and using positive turns, reduce each digit of **CR**,in turn, to zero. This is very quickly done, with virtually no risk of mistake, by positive turns. Since the Reversing Lever is down, the respective figures in **CR** will be reduced by the number of turns made. At the same time the figure set in **SR** is multiplied by the number of turns made.

We can, theoretically, carry on like this indefinitely, alternately multiplying and dividing. After the first stage the Reversing Lever can be left down.

Generally it is not worth while setting decimal markers, unless a number of very similar calculations have to be made, and the decimal markers can be left in position for all stages.

If the number of factors in the numerator exceed those in the denominator by more than one, we can use successive multiplication, per method **A 2**, at one or more stages. If the factors are identical in number, we can start with a division. If the factors in the denominator exceed those in the numerator, we must either multiply some of these together or re-set the quotient at some stage, or use the method of successive division described in **A 3**.

SUCCESSIVE DIVISION

Suppose we have a figure in **PR** at some stage of calculation, and we want to divide it by some divisor and obtain the quotient in **PR**, perhaps because we want to divide the quotient by another figure. We use a method which we have called 'Successive Division', because of the analogy to successive multiplication. The method is sometimes called 'Complementary Division'.

Example: Divide 567 (already produced on **PR** as, say,56700000)
by 4560

Set the complement of 456 on **SR**, preceded by a figure 9, thus

	SR	CR	PR
C6	**/9544**	**124342**	**/124342(0048)***

Answer = /.124342

Note that although extra figures may be produced in **PR**, the quotient can only be obtained accurately to the same number of figures as the capacity of **CR**.

Note that the quotient is also produced in **CR** and this can he used as a check that the calculation has been performed accurately.

Successive division can be very useful but takes a little practice. The operator should carry the two left hand figures of the divisor in his mind during the operation.

Since the quotient remains in the **PR**, it can be divided by a further divisor, either by subtractive division or by successive division.

COMPLEMENTARY DIVISION

In a division calculation, instead of setting the dividend on **PR** and proceeding by Subtractive Division, there may, in some cases, be an advantage in setting the complement of the dividend on **PR** and building up **PR** to zero with the divisor set in **SR**. This is particularly the case with such calculations as $\frac{a \times b}{c}$ carried out in one operation, when we wish to obtain the maximum capacity of which the machine is capable.

Example: $\frac{123 \times 456789}{234567}$ the answer required to 6 places of decimals

Using a Model II,

		SR	CR	PR	
C6	***R***	**/456789**	**/100000**	**9/54321100000**	**Clear CR**
C6		**01230234567**	**/194737**	**239526509973879**	

Answer: = 239527

We use the right hand side of **SR**, 234567 to build up the right hand side of **PR** to as near zero as possible. Meanwhile, the left hand side of **SR**, 0123, is multiplied automatically by the quotient of 456789/234567 and the product shown in the left hand side of **PR**.

Note: The positioning of the Reversing Lever and the clearing of **CR** are actually unnecessary since the **CR** dial is not used, but they are specified here to show that the handle has been given one negative turn in the 6th position of the carriage for the 1st operation and the quotient is shown on **CR** in the 2nd operation.

Note also that if we had made the calculation by setting 01230234567 in **SR** and building up to 456789 in **PR**, we should have had the wrong answer, 239572, because the left hand portion of the multiplicand, 456789, would have overlapped the correct answer.

A 5. 1.

ACCUMULATION of QUOTIENTS

Quotients can be accumulated on **CR** by build-up division, the reversing lever being pushed up for positive terms and down tor negative terms. To set the decimal marker in **PR**, add the maximum number of decimals in the denominators to the number of decimal places required in the answer, plus 1. Decimal markers should always be set when accumulating quotients.

Example (i) $$\frac{32.45}{1.39} + \frac{69.8}{7.465} - \frac{101.34}{11.7}$$

The answer is required to three places of decimals.
Decimal markers are set before positions 3, 7 and 4 in **SR**, **PR** and **CR**.

	SR	CR	PR	
C6	**/1.390**	**/23.3453***	**/32.4499670**	**Clear PR**
C6	**/7.456**	**/32.6956***	**/369.7999895**	**Clear PR**
C6 ***R***	**/11.700**	**/24.0341***	**/101.3395500**	

Answer = 24.034

When the numerators are the sum or differences of other numbers the accumulation can still be carried out in the above way, but using subtractive division, provided care is taken to correct **CR** as in the example below. In calculating the position for the decimal marker in **PR**, we must add to the maximum number of decimal places in denominators or numerators, the number of decimal places required in the answer, plus 1.

Example (ii) $\dfrac{13.475 - 5.75}{6.29} + \dfrac{17.24 + 3.92}{7.86} - \dfrac{18.715}{9.5}$

The answer is required to three decimal places.
Decimal markers are set before positions 3 , 7 and 4 of SR, **PR** and **CR**.

	SR	CR	PR	
C5 *R*	**/13,475**	**99/**	**/13.4750000***	
R	**/5.750**	**0**	**/7.7250000***	
C5 *R*	**/6.290**	**/1.2281***	**/0.0002510**	**Clear PR**
C5 *R*	**/17.240**	**/.2281**	**/17.2400000**	
R	**/3.920**	**9/.2281**	**/21.1600000**	

Now clear **SR** and make two negative turns to correct **CR**.

	SR	CR	PR	
C5 *R*	**0**	**/1.2281**	**/21.1600000**	
C5 *R*	**/7.860**	**/3.9202**	**/00.0000940**	**Clear PR**

The last quotient can be obtained by building-up division, but we keep the Reversing Lever down because it has to be subtracted.

	SR	CR	PR
C5 *R*	**/9.500**	**/1.9502**	**/18.7150000**

Answer = 1.950

STERLING

(a) Decimalising

Sterling calculations are normally carried out by converting shillings and pence into decimals of £1, or pence into decimals of one shilling. The latter is used mainly for small amounts.

Curta Tables 1 and *2* show the decimal equivalents of pence and fractions of pence expressed as decimals of £1 and one shilling respectively.

Curta Table 5 shows shillings and pence as decimals of £1; shillings can, however, easily be expressed as decimals of £1 by dividing by 2. Thus 17/- = .85 of £1

Example (i) Find the cost of 455 articles @ 3/6½d. each.

From *Table 2*, 6½ is equivalent to .54167 of 1/-
We therefore set 3.54167 on **SR** and multiply by 455 in the normal manner.

	SR	CR	PR
C1	**/3.54167**	**/455**	**/1,611.45985***

Mentally divide 1,611 shillings by 20 obtaining £80. 11/- ... Looking up the nearest figure to .45985 on *Table 2* we obtain .4583 or 5½d., so that the answer is:- **£80. 11. 5½d.**

The pence may alternatively be found by mentally multiplying the decimal by 12 or by memorising the first two decimal places, if much of this work is being done.

Example (ii) Find the cost of 215 shares @ £1. 3. 7½d. each.

Set the pounds on **SR** followed by the shillings divided by 2 with, say, 5 decimal places, thus 1.15000

From *Table 1*, 7½d. is equivalent to .03125 of £1.
We therefore add this to the previous setting, so that 1.18125 is set on **SR**.

	SR	CR	PR
C1	**/1.18125**	**/215**	**/253.96875***

The answer is thus **£253.96875** or **£253. 18. 4½d.**

The shillings can be found by reducing the first two decimal figures to a multiple of 5, in this case 95, and multiplying by 2. The remainder, .01875, can be looked up in *Table 1*, giving 4½d., or we can regard the 2nd and 3rd place of decimals, in this case 18, as farthings, mentally dividing by 4 to obtain 4½d. If the pence figure thus found comes to 6d. or over we deduct a farthing. Thus, for .037, which lies between 8½ and 9d., we obtain 9¼d. less ¼d., equals 9d.

Example (iii) Find the cost of 5,150 articles at £3. 17. 7d.

From *Table 5*, we obtain .8791667 for 17/7d.

	SR	CR	PR
C1	**3.8791667**	**/5,150**	**19,977.708505***

Answer = £19,977. 14. 2d.

To decide the number of decimal places to be set on **SR**, it is advisable to add the number of digits in the multiplier to 3, if decimals of £1 are to be set, or 2 if decimals of 1/- are set. This will enable the answer to be given lo the nearest farthing. Thus, in the above example, we had 4 figures in the multiplier so that we set 7 decimal places of £1.

(b) Pence Transfer

The disadvantages of decimalising sterling are that a table is required, unless a skilled operator is employed who can memorise the decimal equivalents, and, if the multiplier is large, a considerable number of decimal places have fo be set. An alternative method, which is especially useful when the sum, only, of a number of sterling products is required, is to set the shillings and pence, separated by several zeros. After multiplication, the **PR** shows the answer in shillings and pence. The surplus pence (in excess of 11d.) can be converted into shillings by an adaptation of the method of Successive Division given in **A 3**.

Example (i) Check the following Invoices in total.

125 dozen @ 3/7d. per dozen	22. 7. 11d.
65 articles@ 13/11d. each	45. 4. 7d.
75 lbs. @ £1. 3. 5d. per lb.	87. 16. 3d.
	£155. 8. 9d.

	SR	CR	PR	
C1	**/3,000007**	**/125**	**/375,000875***	**Clear CR**
C1	**13,000011**	**/65**	**/1220,001550***	**Clear CR**
C1	**23,000005**	**/75**	**/2945,001965***	**Clear CR**

Now follows the transfer of the surplus pences

	SR	CR	PR
C3	**/,999988**	**/163**	**/3108,000009***

Answer = 3108 shillings and ninepence = £155. 8. 9d.

As the operation is one of checking only, it is not really necessary to Clear **CR**. The number of zeros to be left between shillings and pence depends on the quantity of items and number of entries on the invoice. Here we could have reduced the number of zeros by 2. If, at some stage of the calculation, the pence threaten to flow over to the shillings in the **PR**, we can always make a pence transfer and then continue with the other items on the invoice. In making the pence transfer, we set 88 under the right hand pence figures and 9s up to the shillings figure.

We can deal with fractions of pence by decimalising them, as this is easy.

Example (ii)

38 @ 2/7¾d.	5. 0. 6½d.
150 @ 13/½d.	99. 1. 3d.
144 @ 5/11¼d.	42. 15. 0d.
	£146. 16. 9½d

	SR	CR	PR
C1	**/2,0007.75**	**/38**	**/76,0294.50***
C1	**13,0002.50**	**/188**	**/2026,0669.50***
C1	**/5,0011.25**	**/332**	**/2746,2289.50***
C3	**/,9988.00**	**/522**	**/2936,0009.50***

Answer = £146. 16. 9½d.

This method of checking invoices is very speedy and can be carried out by operators with little training. It gives a more independent check than checking each item. The same method may be used with advantage in checking stock sheets, in bills of quantity and even in adding figures in sterling, in which case the figures of pounds can also be separated from the shillings on Curta Model II.

CURTA TABLE No. 1   PENCE to DECIMALS of 1/ - INCHES to DECIMALS of 1 FT.

Pence	d	¼ d	½ d	¾ d	Pence
0	.000	.020 83	.0416	.062 50	0
1	.083	.104 16	.1250	.145 83	1
2	.166	.187 50	.2083	.229 16	2
3	.250	.270 83	.2916	.312 50	3
4	.333	.354 16	.3750	.395 83	4
5	.416	.437 50	.4583	.479 16	5
6	.500	.520 83	.5416	.562 50	6
7	.583	.604 16	.6250	.645 83	7
8	.666	.687 50	.7083	.729 16	8
9	.750	.770 83	.7916	.812 50	9
10	.833	.854 16	.8750	.895 83	10
11	.916	.937 50	.9583	.979 16	11

Note. The final figures '3' and '6' are to be repeated for as many decimal places as are required, with the last '6' replaced by a '7'.
Thus:- 3¼d = .2708333 and 6½d = .5416667 to 7 decimal places.

CURTA TABLE No. 2 PENCE to DECIMALS of £1

Pence	d	¼ d	½ d	¾ d	Pence
	.0	.0	.0	.0	
0	000 0	010 416	020 833	031 250	0
1	041 6	052 083	062 500	072 916	1
2	083 3	093 750	104 166	114 583	2
3	125 0	135 416	145 833	156 250	3
4	166 6	177 083	187 500	197 916	4
5	208 3	218 750	229 166	239 583	5
6	250 0	260 416	270 833	281 250	6
7	291 6	302 083	312 500	322 916	7
8	333 3	343 750	354 166	364 583	8
9	375 0	385 416	395 833	406 250	9
10	416 6	427 083	437 500	447 916	10
11	458 3	468 750	479 166	489 583	11

Note. The final figures '3' and '6' are to be repeated for as many decimal places as are required, with the last '6' replaced by a '7'.

Thus:- 2/3¼d = .11354167 and 3/6½d = .17708333 to 8 decimal places.

CURTA TABLE No. 3 QUARTERS, STONES and POUNDS as DECIMALS of 1 CWT

QUARTERS	0		1		2		3		EXTRA
STONE	0	1	2	3	4	5	6	7	FIGURES
lbs.									
0	.000	.125	.250	.375	.500	.625	.750	.875	000
1	.008	.133	.258	.383	.508	.633	.758	.883	929
2	.017	.142	.267	.392	.517	.642	.767	.892	857
3	.026	.151	.276	.401	.526	.651	.776	.901	786
4	.035	.160	.285	.410	.535	.660	.785	.910	714
5	.044	.169	.294	.419	.544	.669	.794	.919	643
6	.053	.178	.303	.428	.553	.678	.803	.928	571
7	.062	.187	.312	.437	.562	.687	.812	.937	500
8	.071	.196	.321	.446	.571	.696	.821	.946	429
9	.080	.205	.330	.455	.580	.705	.830	.955	357
10	.089	.214	.339	.464	.589	.714	.839	.964	286
11	.098	.223	.348	.473	.598	.723	.848	.973	214
12	.107	.232	.357	.482	.607	.732	.857	.982	143
13	.116	.241	.366	.491	.616	.741	.866	.991	071

Example:- 5 stone 6 lbs = .678571 to 6 places of decimals.

CURTA TABLE No. 4 QUARTERS and POUNDS as DECIMALS of 1 TON

Qrs.	0	1	2	3	Qrs.	0	1	2	3	EXTRA FIGURES
lbs.	.0	.0	.0	.0	lbs.	.0	.0	.0	.0	
0	0000	1250	2500	3750	14	0625	1875	3125	4375	000
1	0044*	1294*	2544*	3794*	15	0669*	1919*	3169*	4419*	64
2	0089	1339	2589	3839	16	0714	1964	3214	4464	29
3	0133*	1383*	2633*	3883*	17	0758*	2008*	3258*	4508*	93
4	0178*	1428*	2678*	3928*	18	0803*	2053*	3303*	4553*	57
5	0223	1473	2723	3973	19	0848	2098	3348	4598	21
6	0267*	1517*	2767*	4017*	20	0892*	2142*	3392*	4642*	86
7	0312*	1562*	2812*	4062*	21	0937*	2187*	3437*	4687*	50
8	0357	1607	2857	4107	22	0982	2232	3482	4732	14
9	0401*	1651*	2901*	4151*	23	1026*	2276*	3526*	4776*	79
10	0446	1696	2946	4196	24	1071	2321	3571	4821	43
11	0491	1741	2991	4241	25	1116	2366	3616	4866	07
12	0535*	1785*	3035*	4285*	26	1160*	2410*	3660*	4910*	71
13	0580	1830	3080	4330	27	1205	2455	3705	4955	36

Note. For each cwt. add .05. If only 5 decimal places are required, figures marked with an asterisk should be rounded up 1.
Example:- 4 cwt 3 qrs 9 lbs = .2415179 to 7 decimal places. 3 cwt 2 qrs 20 lbs = .18393 to 5 decimal places.

WAGES

The Curta can be used with advantage in the Wages Office, either as the sole means of calculation, or to supplement existing calculating machines at peak periods. For this purpose it has the advantages of low capital cost and simplicity of operation enabling it to be used by comparatively untrained staff.

Example: Assuming a standard working week of 44 hours, calculate the wage earned in the following cases.

Employee	Standard Wage	Equivalent Number of Hours Worked	Wage Earned
A. Smith	6. 15. 0.	47½	7. 5. 9.
W. Jones	"	49	7. 10. 5.
H Brown	"	58¾	9. 2. 7.
R. Robinson	7. 2. 6.	37¼	6. 0. 8.
S. Green	"	59	9. 11. 1.

It is best to use a Curta II.

The standard hours are set on the right of **SR** and not altered. The standard wage is set on the left of **SR** and only altered when the standard wage group alters. We build up the standard hours to the equivalent number of hours worked on the right of **PR** and the wage earned is shown on the left of **PR**. We only clear the **CR** and **PR** dials when the standard wage changes.

	SR	CR	PR	
C5	**/6.750000044**	**/10795**	**/7.2866250,,47.4980***	
C3		**/11136**	**/7.5168000,,48.9984***	
C4		**/13352**	**/9.0126000,,58.7488***	**Clear CR & PR**
C5	**/7.125000044**	**/8466**	**/6.0320250,,37.2504***	
C4		**/13409**	**/9.5539125,,58.9996***	

The answers, as given on the left of **PR**, are written down on the wages sheet, as above, usually rounded off to the nearest penny above the figure calculated.

PERCENTAGES

Percentages are used in a great variety of ways in Commerce and Industry. A few of these are dealt with in *'Instructions for Use of the Curta'.* The object or these notes is to summarise the various types of percentage calculations and to show the quickest way of obtaining the answers on the Curta.

Example:

(1) $a\%$ on $b = \dfrac{a \times b}{100}$

(2) Add $a\%$ to $b = b + \dfrac{a \times b}{100}$

(3) Rebate or Discount of $a\%$ on $b = b - \dfrac{a \times b}{100}$

(4) a expressed as %age of $b = \dfrac{a \times 100}{b}$

(5) a, b and c expressed as %ages of their sum, $a + b + c$

$$= \frac{a \times 100}{a + b + c} \text{ , etc.}$$

(6) Selling price to give a percentage profit of $a\%$ on sale for

a cost price $b = \dfrac{b}{1 - \dfrac{a}{100}}$

(7) Increase or Decrease expressed as a percentage of the larger or smaller figure. If b is greater than a

% increase or decrease on $a = \dfrac{(b - a)}{a} \times 100$

% increase or decrease on $b = \dfrac{(b - a)}{b} \times 100$

(8) Successive Percentage corrections, $d + a\% + b\% - c\%$, etc.

A percentage is normally required to a certain standard number of decimal places, so it is advisable to set the decimal markers, once and for all, before the set or calculations is commenced. This will reduce the work to the minimum.

(1) $a\%$ on b = $\frac{a \times b}{100}$

We merely set *b*, multiply by *a* and mark off 2 additional decimal places on **PR**.

Example (i) Calculate a dividend of 4½% on £1, 115. 10. 6. stock

	SR	CR	PR
C1	**/1,115.525**	**/4.5**	**/50.198625***

Answer = £50. 3. 11¾d.

Note that there are 2 more decimal places in **PR** than in **SR** + **CR**. Unless the percentage is more than 100, it is normally never necessary to set more than 3 decimal places in **SR** for sterling amounts.

(2) Add $a\%$ to b = $b + \frac{a \times b}{100}$

In such cases, as in the extension of invoices, it is often desired to know both the percentage addition and the total. If we only have one such calculation to make we can set *b* and multiply first by $\frac{a}{100}$ and then by 1, without clearing **PR**. If we have a number of such calculations to perform with the same percentage, we can set $\frac{a}{100}$ on the right of **SR**, $1 + \frac{a}{100}$ on the left and multiply by *b*, obtaining the percentage on the right of PR and the total on the left.

Example (ii) Add 17½% to an invoice for £13. 14. 7.

	SR	CR	PR
C1	**/13.729**	**/17.5**	**/2.402575***
C4		**/117.5 /**	**16.131575***

Answer =	Net amount of invoice	**£13. 14. 7.**
	Add 17½%	**2. 8. 0½.**
		£16. 2. 7½.

Example (iii) Add 12½% to a number of invoices.
Using a Curta II. First invoice for £18. 4. 9.

	SR	CR	PR
C1	**1.1250000.125**	**/18.237**	**20.516625:2.279625***

Answer = Net amount of invoice **£18. 4. 9.**
Add 12% **2. 5. 7.**
£20. 10. 4.

This method is obviously impractical if the amount of the invoice is large or there are too many significant figures in the percentage. The setting will be left for the next calculation.

(3) Rebate or Discount of *a*% on $b = b - \dfrac{a \times b}{100}$

Precisely the same considerations apply as for (2)

Example (iv) Deduct a discount of 2½% from £136. 15. 4.

	SR	CR	PR
C1	**/136.766**	**/2.5**	**/3.419150***
C4		**/97.5**	**/133.346850***

Answer = Gross amount of invoice **£136. 15. 4.**
Less 2½% discount **3. 8. 4½.**
£133. 6. 11½.

(4) a expressed as %age of $b = \dfrac{a \times 100}{b}$

This is simply a division sum and is best carried out by build up division.

Example (v) A mixture containing 1.137 litres includes 39 centilitres of alcohol. What is the percentage of alcohol to 3 places of decimals.

Set 3 places of decimals in **CR** (for the answer). There are 3 decimal places in **SR**, so there will be 3 + 3 + 2 (for %age) = 8 in **PR**. Set 1.137 and build up to .39 in **PR**.

	SR	CR	PR
C5	**/1.137**	**/34.301***	**/.39000237**

Answer = 34.301%

(5) a, b and c expressed as %ages of their sum, $a + b + c$

We set decimal markers on **SR**, **CR** and **PR**, adding 2 for **PR** as we require the answers as percentages.

We set $a + b + c$ on **SR** and build up in turn to a, b and c in **PR**, without clearing any of the dials.

Example (vi) The cost of a batch of articles is given as follows:-

Direct Wages	£139. 14. 0.
Materials	57. 16. 6.
Overheads	112. 10. 0.
	£310. 0. 6.

Compute the percentage of each on the total cost, to 2 places of decimals.

	SR	CR	PR
C4	**/310.025**	**/45.06***	**/139.6972650**
		/18.65*	**/57.8196625**
		/36.29*	**/112.5080725**

Answer =	Direct Wages	45.06%
	Materials	18.65%
	Overheads	36.29%
		100.00%

The carriage position is simply chosen by eye to enable the dividend to be built up correctly. In this case 4 happens to be the best position to start the build up in each case. None of the dials are cleared during the operation. The total of the percentages should, of course, equal 100% with .01% either way.

If there is a large number of constituent parts, e.g. more than 5, it is worth while computing the reciprocal of the sums and multiplying each part by this reciprocal in turn.

Thus, in this example we might compute 100/310.025 by build up division to 100, obtaining .32255. Set this figure on **SR** and multiply in turn by the three constituent parts, 139.7, 57.825 and 112.5.

(6) Selling price to give a percentage profit of a% on sale for

$$a \text{ cost price } b = \frac{b}{1 - \frac{a}{100}}$$

Example (vii) A number of articles costing 22/9, 25/6, 31/3, etc. have to be written up to their selling prices to the nearest penny to give a profit of 22½% on the selling price.

If there are a large number of articles, we compute $\frac{100}{100 - a}$

i.e. $\frac{100}{77.5}$

Set the quotient on **SR** and multiply by each cost price in turn. It is better to work in decimals as if we require 2 places of decimals in the answer to give the nearest penny, and as there are at most 2 digits in the shillings, it is sufficient to have 4 decimal places in the quotient. We therefore set the carriage in position 5 for evaluating $\frac{100}{77.5}$

	SR	CR	PR	
C5	**/77.5**	**/1.2903***	**/.9999825**	**Clear CR & PR**
C1	**/1.2903**	**/22.75**	**/29.354325***	**- do -**
C2		**/25.50**	**/32.902650***	**- do -**
C3		**/31.25**	**/40.321875***	

Answer = 29/4, 32/11, 40/4, etc.

(7) Increase or Decrease expressed as a percentage of the larger or smaller figure. If b is greater than a

$$\text{\% increase or decrease on } a = \frac{b - a}{a} \times 100 = \left(\frac{b}{a} - 1\right) 100$$

$$\text{\% increase or decrease on } b = \frac{b - a}{b} \times 100 = \left(1 - \frac{a}{b}\right) 100$$

We have merely to divide ***b*** by ***a*** or ***a*** by ***b***, either by subtractive division or by build up division. We can make use of the Reversing Lever, either to subtract one from the quotient in the first case, or to give the complement, of $\frac{a}{b}$ on **CR** in the second case.

Example (viii) The turnover of a business increases from £567,450 to £786,584. Find the %age increase to two places of decimals.

We require $\frac{b - a}{a}$ x 100. Using subtractive division we set 786584 and make one positive turn with Reversing Lever down.

	SR	CR	PR	
C5 *R*	**/78684**	**99/**	**0786584/**	**Do not clear dials**
C5 *R*	**/567450**	**/38.62***	**9/848100**	

Answer = 38.62%

The above operation can also be appreciated in this way. After 567450 has been set and one subtractive turn has been made, we are left with 0 on **CR** and *b - a* on **PR**. We proceed to divide this by a so that we obtain $\frac{b - a}{a}$ on **CR**.

Example (ix) The turnover of a business decreases from £786,584 to £672,410. Find the %age decrease to two places of decimals.

We require $\frac{(b - a)}{b}$ x 100. Using build up division, we set 786584 and build up to a with reversing lever down.

	SR	CR	PR
C5	**/786584**	**9/14.52***	**/6723720032**

Answer = 14.52%

The rule is therefore: Set the higher of the two figures, with the reversing lever down, and place carriage in position to give required decimal places in answer. If the change is an increase, make one positive turn, set the lower figure and reduce **PR** to zero. If the change is a decrease, build up to the lower figure.

(8) Successive Percentage corrections, $d + a\% + b\% - c\%$, etc. Such calculations are common where prices are fluctuating too rapidly for catalogues to be amended.

Example (x) £5. 14. 6 + 5% + 12½% - 2½%

We can set 5.725 and multiply successively by 1.05, 1.125 and .975. Using Curta II and method given in **A 1**.

	SR	CR	PR	
C1	**/5. 725**	**/1.05**	**/6.01125***	**Clear CR**
C6	**/1.124**	**/6.01125**	**/6.76265625***	**Clear CR**
C8	**/9.740**	**/6.7626562**	**/6.59358979505***	

Answer = £6. 11. 10½.

Note: It is not necessary to clear **CR** nor to write anything down except the answer. Nor is it necessary to proceed with the last step beyond 4 places on **CR** in this particular case.

If such successive percentages have to be applied to a number of prices, it is better to work out the equivalent percentage once and for all. Thus, if we multiply 1.05 x 1.125 x .975 we obtain 1.15171875 so that + 5% + 12½% - 2½% is equivalent to + 15.172%. If we add this percentage to £5. 14. 6. we get 6.593597 or £6. 11. 10½. as before.

AMORTISATION of DEBTS by ANNUITIES

Loans may be repayable in a number of ways but a common method is by equal payments at regular intervals. Such payments may be calculated to repay the loan over a fixed period or the annuities may be a fixed proportion of the loan.

(a) Repayable over a fixed period.

Part of the annuity represents interest and part repayment of capital, A schedule is required showing the amounts outstanding at the end of each period and the interest and capital portions for the next period. The amount of the annuity can be found from an annuity table.

Example:

A loan of £750,000 on 1.1.54, carrying interest at 3½ is to be repaid by 10 equal payments at half-yearly intervals. The amount is the same as if a 1¾% loan were repayable at yearly intervals. The amortisation factor found from the table is .1098753442. Multiplying this on the Curta by 75, we obtain the half yearly payment £82,406.51. We shall leave the figures in decimals.

We enter the half-yearly payment on **PR** and set the rate of interest on **SR**. We multiply this negatively by the amount of the loan with the Revering Lever down, so that we subtract the interest for the 1st half year from the half-yearly payment, leaving the capital portion in **PR**, whilst **CR** a shows the amount of the loan. The former is noted down and used to multiply the interest rate, positively, and added to **PR**.

In this way the **PR** shows the capital value of the annuity for each half year and the **CR** shows the outstanding amount of the loan at the beginning of the half-year.

The operations on the Curta II are as follows, **PR** and **CR** being scheduled at each step. The setting remains constant at the interest rate.

	SR	CR	PR	
C1	**/1098753442**	**/75**	**/82,406.508150***	**Clear CR and PR**

This gives us the ½ yearly payment, £82,406,51

C1	**/82406.510000**	**/1**	**/82406.510000**	**Clear CR**

Date of Payment			Outstanding Loan	½ Yearly Amortisation	Interest (obtained Later)
		SR	CR	PR	
1.7.54.	**C8 *R***	**/1. 75**	**750,000.00**	**69,281.51**	**13,125.00**
1.1.55.	**C1 *R***		**680,718.49**	**70,493.93**	**11,912.58**
1.7.55.	**C7 *R***		**610,224.56**	**71,727.58**	**10,678.93**
1.1.56.	**C1 *R***		**538,496.98**	**72,982.81**	**9,423.70**
1.7.56.	**C7 *R***		**465,514.17**	**74,260.01**	**8,146.50**
1.1.57.	**C1 *R***		**391,254.16**	**75,559.56**	**6,846.95**
1.7.57.	**C7 *R***		**315,694.60**	**76,881.85**	**5,524.66**
1.1.58.	**C1 *R***		**238,812.75**	**78,227.29**	**4,179.22**
1.7.58.	**C7 *R***		**160,585.46**	**79,596.26**	**2,810.25**
1.1.59.	**C1 *R***		**80,989.20**	**80.989.20**	**1,417.31**
				£750,000.00	£74,065.10

After the first 9 repayments the ½ yearly amortisation figure,£80,989.20, should equal the outstanding amount of the loan thus proving that the calculations have been made correctly and that the ½ yearly amortisation has been written down correctly. There may be a small discrepancy due to rounding off.

To prove the correct writing down of the outstanding amount of the loan, the ½ yearly amortisation figures are accumulated, starting with the last and checked with the loan figures, thus the total of the amortisation column should amount to the total of the loan.

The interest figures are obtained by setting the ½ yearly payment on **PR** and deducting each amortisation figure in turn, restoring **PR** by a positive turn each time, before setting the next amortisation figure.

If the amount of the loan is not more than 7 figures, these two operations can be performed with a single setting of each amortisation figure, using the one side of **PR** to accumulate the outstanding loan figures and the other to obtain the interest.

The total of the Interest column should equal the total amount of the amortisation, i.e. £824,065.10 is less the total of the loan, £750,000, that is £74,065.10.

GEAR RATIOS

These notes do not attempt to go into the problems of gear train design, but merely to show the application of the Curta to one type of calculation arising therefrom.

A required ratio may be obtained in the form of a relationship between two quantities, i.e. a proper fraction, e.g. $\frac{27}{167}$, or a decimal fraction, e.g. .16168. It is necessary to express the decimal fraction as a proper fraction to obtain the number of teeth in each gear of the train and, even if the ratio is already expressed as a proper fraction, the denominator or numerator may not factorise and may be too large for practical purposes, as is the denominator in this case, or may require a gear which is not available. Consequently it may be necessary to find other factors, each within the required accuracy, one of which will factorise to a convenient number of teeth.

If the accuracy required is not great, the appropriate ratios can be found from a table of ratios, known as a *Brocot Table*. A table to the base 100 gives the decimal equivalents of all fractions from 99/100 to 1/100 in descending decimal order. A table to any base could, of course, be Constructed but would be cumbersome if an accuracy, say, of 1 in 100,000 were required. There are methods of obtaining additional accuracy from tables of base 100 but they involve the making of a number of separate calculations. Fortunately a very simple method is provided by the Curta for finding a set of fractions to any required degree of accuracy with but a single setting on the Curta.

Any fraction can be expressed as a 'Continued Fraction' and a sufficient number of terms may be taken to give any required accuracy. For instance, the fractions given above can be expressed as

$$\cfrac{1}{6 + \cfrac{1}{5 + \cfrac{1}{2 + \text{etc.}}}}$$

The successive terms give equivalent proper fractions which are alternately greater and less than the required fraction, until, if the fraction is determinate, a zero remainder is obtained.

It can be shown that, between successive terms, there is no fraction with a smaller denominator, so that, if we proceed to evaluate the continued fraction until the required accuracy is obtained, we can be sure that we have not left out a fraction as accurate but with a smaller denominator. In this case the successive fractions are 1/6, 5/31, 11/68, etc.

The method which we are going to describe is simply a way of finding the successive continued fractions automatically with a single setting on the Curta, and at the same time giving the appropriate accuracy check figure so that we can compare it with the 'Remainder' of the numerator and see whether we have obtained the necessary accuracy. A Curta I can be used, but the method will be a little clearer if we use a Curta II.

Example (i) Find a gear train to give the ratio .16168 With an accuracy of ± .00002

Set decimal markers before the 5th, 8th and 10th dials of **PR**. Set the decimal equivalent of the required fraction on left of **SR** and the accuracy on the right of **SR**. (With a Curta I we set decimal markers before the 3rd, 6th and 8th dials. In setting the accuracy, we must leave out the first 2 zeros.) As we turn the handle, the numerator appears on the left of **PR** and the denominator on **CR**. The right side of **PR** shows the accuracy multiplied by the denominator which we will here call the 'accuracy check'. We must turn until the numerator is sufficiently near to a whole number to give the required accuracy, i.e. until either the Remainder given in **PR** dials 6 - 10 is less than the accuracy check or they are together equal to more than 1.

Proceed as follows, first making 1 turn at a time until a carry over is made to **PR** dial 11.

	SR	CR	PR	No. of turns at a time
C1	**/.16168.00002**	**/7**	**/1.13,176,00014**	*1*

Note the denominator shown in **CR** and deduct the number of turns made at a time, i.e 1, giving 6.

We now proceed to make 6 turns at a time. The Remainder will decrease from .13176 and we continue until the Remainder has been reduced below zero.

	CR	PR	No. of turns at a time
C1	**/37**	**/5, 98,216.00074**	*6*

Write down the denominator **37**

and deduct the previous number 6 obtaining for the next number of turns 31, i.e. 1 turn in **C1** and 3 turns in **C2**. The Remainder will now increase.

Thus we proceed with the remainder alternately increasing and decreasing towards zero until either the Remainder is less than the accuracy check or the sum is greater than 1. It is clear that **PR** dials 9 and 10 must either show 00 or 99, and if this occurs, we check dials 6, 7 and 8 with 1, 2 and 3,

	CR	PR	
C1	**/99**	**/16.00,632,00198**	*31*
			99
C1	**/167***	**/27*.00,056.00334**	*68*

We now have sufficient accuracy since 56 is less than 334 so that 27/167 will give the required accuracy, but we must go on, since 167 is a prime number.

	CR	PR	
C1	**/235***	**/37*.99,480.00470**	*68*
			235
C1	**/402***	**/64*.99,536.00804**	*167*
C1	**/569***	**/91*.99,592.01138**	*167*
C1	**/736***	**/118*.99,648.01472**	*167*

and so on.

We thus obtain a series of fractions, all sufficiently accurate, $\frac{27}{167}$, $\frac{65}{402}$, $\frac{92}{569}$, $\frac{119}{736}$, etc.

The second, $\frac{65}{402}$ factorises as $\frac{5 \times 13}{2 \times 3 \times 67}$ so that we might use as our gear train say $\frac{5}{6} \times \frac{13}{67}$.

If this is not suitable, e.g. because there are too few teeth in the first pair, it is generally not desirable to multiply top and bottom by a common factor and we may require a further fraction. The denominator 569 is a prime number, as can be found from a table of factors, but $\frac{119}{736}$ factorises into $\frac{7 \times 17}{2^5 \times 23}$

We might therefore use $\frac{7}{23} \times \frac{17}{32}$ or $\frac{14}{23} \times \frac{17}{64}$, etc.

Note that the number of turns made at a time, shown in the right hand column above, are the denominators of the successive continued fractions, $\frac{1}{6}$, $\frac{5}{31}$, $\frac{11}{68}$, $\frac{27}{167}$.

Example (ii) Find a gear train to give the ratio .2905 within an accuracy of ± .00003

	SR	CR	PR	No. of turns at a time
C1	**/.29050.00003**	**/4**	**/1.16,200.00012**	*1*
				4
		/10	**/2.90,500.00030**	*3*
				10
		/31	**/9.00,550.00053**	*7*
				31
		/55	**/15.97,750.00165**	*24*
				55
		/179*	**/51*.99,950,00537**	*31*
		/210*	**/61*.00,500.00630**	*31*
				210
		/389*	**/113*.00,450.01167**	*179*
				etc.

We thus have the fractions $\frac{52}{179}$, $\frac{61}{210}$, $\frac{113}{389}$, etc.

As 179 is a prime number and too high, the first is no use.

The second factorises as $\frac{61}{2 \times 3 \times 5 \times 7}$,

so that we might use $\frac{61}{100} \times \frac{10}{21}$ and various other combinations.

It is desirable to write down the denominators when the Remainders change sign, deducting the previous number of turns required to be made in order to find the next number of turns required to be made at a time, as in the right hand column above, but no other figures need be written down, except the accurate fractions.

With a little practice, a series of fractions can be obtained with great rapidity. Short-cutting should be used and where a Remainder approaches zero slowly, we may turn through a multiple of the indicated number of turns; e.g. where 8 turns at a time are indicated, we might turn immediately through 4 turns in the 2nd position of the carriage (40 turns), proceeding thereafter with 8 turns in the 1st position.

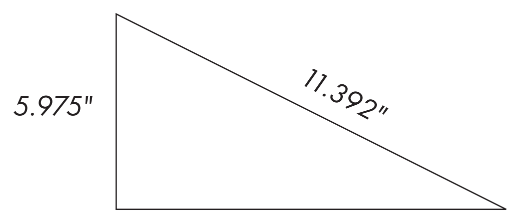
E 2. 1.

SIDES of a TRIANGLE - PYTHAGORAS

Draughtsmen and Designers frequently require to find the third side of a right angled triangle, the other two sides of which are known. This can of course be evaluated with a slide rule or by log tables, or tables of squares and square roots, but greater speed with less effort and less danger of error can be achieved with the Curta.

The operation consists of finding $C = \sqrt{(a^2 \pm b^2)}$. We set 'a' on **SR** and square. We now set 'b' on **SR** and square with either positive or negative turns of the handle. This gives us the radicand, $a^2 \pm b^2$, on **PR**. We find the square root by one of the methods given in Section **M 1**. If we already know the approximate value of C, Sabielny's method (3) is probably the quickest, otherwise method (5), using *Table 101*, will probably be found the simplest.

Example (i) Find the third side of the triangle given below



	SR	CR	PR	
C1	**/11.392**	**/11.392**	**/129.777664***	**Clear CR**
C1 *R*	**/5.975**	**/5.975**	**/94.077039***	**Clear CR**

Using *Table 101*, set the nearest figure for R^2

	SR	CR	PR
C1	**/94.259600**	**/1**	**188.336639***

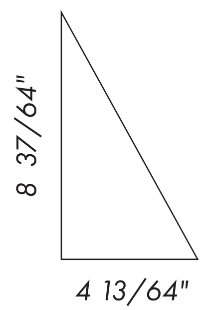
Multiply **PR** by *575*

	SR	CR	PR
C6	**0.0514/**	**188337**	**9.6993554639***

Answer = 9.699"

When the measurements are expressed in fractions of an inch, e.g. 8 37/64", we merely convert them to 64ths, thus, multiply 8 mentally by 64 and set on **SR**, then add 37 by changing the setting levers.

Example (ii) Find the third side of the triangle given below



	SR	CR	PR	
C1	**/549**	**/549**	**/301401***	**Clear CR**
C1	**/269**	**/269**	**/373762***	**Clear CR**

Find square root by *Table 101*.

	SR	CR	PR	
C1	**/372711**	**/1**	**/746473***	**Clear CR**
C6	**/818**	**/7464.73**	**/611.361 387***	**Clear CR**

Now divide by 64.

	SR	CR	PR
C1	**64/**	**/9***	**/35361387**

Answer = 9 35/64"

E 3. 1.

HEAT INSULATION

There are several calculations involved in Heat Insulation, problems which can be solved speedily on the Curta Model II, using a method similar to those set out in **A 5** for the accumulation of quotients. For instance, the heat loss, q , in BTU's per foot run of pipe, ignoring resistance of the steel pipe, is given by

$$q = \frac{\pi(t_1 - t_2)}{\frac{1}{2K_1}\log_e\frac{d_1}{d} + \frac{1}{2K_2}\log_e\frac{d_2}{d_1} + \frac{1}{2K_n}\log_e\frac{D}{d_{n-1}} + \frac{1}{Œ \times D}}$$

Where t_1 and t_2 are the temperatures of steam and air.
K_1, K_2 etc. are the thermal conductivities of layers 1, 2 etc.
d, d_1, d_2 etc. are the diameters of the pipe and of layers 1, 2 etc.
and D is the overall diameter.
$Œ$ is the film conductance between insulation surface and outside air.

K_1, K_2 etc. can be obtained from a graph which should be calibrated in terms of $2K_1$ etc.

$Œ$ can be obtained from a graph or tables which should be calibrated or tabulated in terms of $\frac{1}{Œ}$

Example: $t_1 = 650°F$, $t_2 = 70°F$

Insulation of 6½" outside diameter pipe is ½" H. T. R., 2" Nag and ½" H.S., so that $d = 6.5$, $d_1 = 7.5$, $d_2 = 11.5$, $D = 12.5$, and $2K_1 = .096$, $2K_2 = .082$ and $2K_3 = .150$.

$$\frac{1}{Œ} = 6.67$$

Thus the amount to be computed is:

$$q = \frac{580\pi}{\frac{1}{.096}\log_e\frac{7.5}{6.5} + \frac{1}{.082}\log_e\frac{11.5}{7.5} + \frac{1}{.150}\log_e\frac{12.5}{11.5} + \frac{6.67}{12.5}}$$

We set log_e 6.5 on left of **SR** with carriage in position 1 and turn one subtractive turn. Then set log_e 7.5 and one positive turn bringing $log_e \frac{d_1}{d}$ on to **PR**. We set $2K_1$, i.e. .096 on right of **SR** and divide by subtraction with Reversing Lever down bringing quotient on **CR**. In this way we sum the quotient of the K terms, then set Reversing Lever up and add on the last quotient by setting D on **SR** and building up to 6.667. Finally we reset the accumulated quotients on right of **SR** with $t_1 - t_2$, i.e. 580, set on the left and build up to π, obtaining the answer on the left of **PR**, thus:-

	SR	CR	PR	
C1 R	**01.8718/**	**/1**	**9/8128200000***	
C1 R	**02.0149/**	**/0**	**/143100000***	
C7 R	**02.0149//.096**	**01.491/***	**....9/64000**	**Clear PR**
C1 R	**02.0149/**	**01.491//1**	**9/7985100000***	
C1 R	**02.4424/**	**01.491/**	**/427500000***	
C7 R	**02.4424//.082**	**06.704/***	**.../34000**	**Clear PR**
C1 R	**02.4424/**	**06.704//1**	**9/7557600000***	
C1 R	**02. 5257/**	**06.704/**	**/83300000***	
C6 R	**02.5257/.150**	**07.259/***	**..../50000**	**Clear PR**
C6	**/12.500**	**07.792***	**/6.662500000**	

Reset **CR** on right of **SR**, clear **CR** and **PR**, set 580 on left of **SR** and build up to to π (3.1416) on right of **PR**

C4	**580//7.792**	**/.4032**	**233856031417344**

Answer = 234 BTU's.

In this method, each logarithm has only to be set once and no figures need be written down except the answer. To check the calculation, it should be performed again in the reverse order. The right hand quotient will be obtained by build-up division, then the Reversing Lever will be pushed down, $log_e D$ set with one positive turn and $log_e d_{n-1}$ subtracted, the difference divided by $2K_n$ and so on for the other terms.

Finally we have the accumulation of the quotients on **CR**. Set the answer, already obtained, on **SR** and bring **CR** to zero by positive turns. Divide by 580 and check that we have π on **CR**.

SQUARE ROOTS

There are various ways of finding square roots on the Curta, either with or without some other aid such as tables or a slide rule. Each method has merits according to circumstances.

(1) Toplers Method - Using the arithmetic series 1 + 3 + 5 + 7 + ... This method is described in '*Instructions for the Use of the Curta*'. It is comparatively slow but is simple and requires no outside aids. We give here one example in our abbreviated notation.

Example: *457.315½*

In all methods we divide the Radicand into groups of 2 digits from the decimal point. We build up to the first group, 4, by setting 1 and then 3 on **SR** and making one turn for each. We then increase **SR** by 1 and move the carriage down one place and commence building by the series again. Thus, with Model I :-

	SR	CR	PR	
C6	**001/**	**1/***	**1/**	
	003/	**2/***	**4/**	
C5	**0041/**	**21/***	**441/**	
C4	**0042.1/**	**21.1/***	**445.21/**	
	0042.3/	**21.2/***	**449.44/**	
	0042.5/	**21.3/***	**453.69/**	
C3	**0042.61/**	**21.31/***	**454.1161/**	and so on, adding 2 to **SR** at each step
	....	....	....	
	0042.75/	**21.38/***	**457.1C44/**	
C2	**0042.761/**	**21.381/***	**457.147161/**	
	....	....	....	
	0042767/	**21.384/***	**457.275456/**	
C1	**0042.7681/**	**21.3841/***	**457.27973281**	
	....	....	....	
	0042.7697	**21.3849***	**457.31394801**	

Thus the answer to six significant figures is **21.3849.**

Notice that if we add a further 1 to **SR**, **SR** is twice **CR**. This proves that we have made the calculation correctly, and should always, be checked.

After the first 4 digits, 2138, i.e. half the significant figures required plus 1, have been obtained, it is no longer necessary to proceed with the series 1 + 3 + 5 etc., as the **SR** is sufficiently accurate. We may proceed simply to build up the **PR** as nearly as possible to the Radicand, moving the carriage as necessary, thus:

	SR	CR	PR
C2	**0042.76/**	**21.384/ ***	**457.27544/**
C1		**21.3849/ ***	**457.313924/**

Answer = 21.3849

(2) Herrmann's Method

This is also given in '*Instructions for Use of the Curta*'. If R is an approximate root of N^2, we may put $N = R + d$ where d is small.
Hence $N^2 = R^2 + 2Rd + d^2$ or ignoring d^2 and substituting $N - R$ for d,
we get $N = (N^2 + R^2)/2R$

We may also express this as $N = R + \dfrac{N^2 - R^2}{2R}$

or as $N = ½\left(\dfrac{N^2}{R} + R\right)$

In each case N gives us a second order approximation, accurate to twice as many figures as R.

Herrmann's Method uses the second of these expressions.

If we set R on **SR**, square it, obtaining R^2 in **PR** and R in **CR**, and then set $2R$ on **SR**, building up (or reducing) **PR** to N^2 (the Radicand),
we shall have $R + \dfrac{N^2 - R^2}{2R}$ on **CR**

Example: 457.315½

Suppose we find the root of 457, on a slide rule, to be 21.4,

	SR	CR	PR	
C6	**0021.4/**	**21.4/**	**457.96/ ***	
C3	**0042.8/**	**21.3849/ ***	**457.31372/**	starting with a subtractive turn

Answer = 21.3849

In the same way we could find 12 significant figures by squaring 21.3849, obtaining 457.31394801, set 42.7698 and build up the 6 right hand digits to 500000.

On Curta I it is necessary to clear **PR** and re-set 394801/.

With Curta II the steps are

	SR	CR	PR	
C8	**21.3849/**	**21.3849/**	**.31394801/ ***	**Clear CR**
C7	**42. 7696**	**/2459656***	**.314999999951888**	

Answer = 21. 3849245966

This method is quicker than Method (1), though perhaps it is rather easier to make a mistake.

(3) Sabielny's Method

This uses the expression $N = ½ (\frac{N^2}{R} + R)$

A root R is guessed, or found on a slide rule or from tables, and this is divided into N^2. The mean of the quotient and R gives the second order approximation.

Example: 457.315½

Obtain 214 as an approximate root, as in method (1)

	SR	CR	PR
C6	**/214000**	**213699***	**45731586/**

Answer = 21.3849

This is slightly quicker than Hermann's Method but needs perhaps a little more mental effort.

(4) Classical Method

This is the method normally taught in elementary school arithmetic. The method is speedy and no outside aids are required, but it needs rather more mental effort than the foregoing methods.

We set the radicand on **PR**, set the approximate root on the left hand group, or the nearest figure below it, say x, and subtract it x times. We add x, move the carriage to the next position and set a figure y in the next column so that if we subtract y times, we shall not reduce **PR** below zero. Then we add y and carry on as before.

Example: 457.315½

	SR	CR	PR	
C6	**/457.315**	**1/**	**457.315/**	**Clear CR**
	/20.0000 *R*	**2/***	**057.315/**	
C5	**/41.0000 *R***	**21/***	**016.315/**	
C4	**/42.3000 *R***	**21.3/***	**003.625/**	
C3	**/42.6800 *R***	**21.38/***	**000.2106/**	

As in method (1) the setting on **SR** is sufficiently accurate and we need merely reduce **PR** to zero. Notice that if we add the last setting, 8, the **SR** will be twice **CR**, as in Method (1).

	SR	CR	PR
C2	**/42.7600 R**	**21.3849***	**/107600**

Answer = 21.3849

(5) Using *Curta Table 101*

This method uses the expression $N = \frac{N^2 - R^2}{2R}$

Table 101 gives a column for R^2 and one for $\frac{1}{2R}$, the latter being spaced at intervals calculated to give 5 figure accuracy to the answer.

We set the radicand on **PR**, add the nearest figure to the radicand, taken from *Table 101*, thus obtaining $N^2 + R^2$ on **PR**, and multiply this by the corresponding value of $\frac{1}{2R}$ by the method of successive multiplication.
See **A 1**.

Example: 457.315½

	SR	CR	PR	
C1	**/457.315**	**/1**	**/457.315**	
	/456.571	**/2**	**/913.886**	**Clear CR**
C6	**/.0233**	**913.886**	**/21.3849324***	

Answer = 21.385

This method is very rapid if a 5 figure root is sufficient and requires little mental effort. In practise it is unnecessary to clear **CR**.

CURTA TABLE No. 101

SQUARE ROOTS $N = (N^2 + R^2)/2R$ for 5-FIGURE ROOT

R^2	$1/2R$	R^2	$1/2R$	R^2	$1/2R$
1.00 00 0	500	3.50 68 5	267	8.45 05 1	172
1.03 70 0	491	3.58 70 1	264	8.65 05 2	170
1.07 60 8	482	3.66 99 4	261	8.75 31 9	169
1.11 74 2	473	3.75 57 8	258	8.85 77 1	168
1.16 11 9	464	3.84 46 8	255	8.96 41 1	167
1.20 22 9	456	3.93 67 6	252	9.07 24 3	166
1.24 56 1	448	4.03 21 9	249	9.18 27 4	165
1.29 13 2	440	4.13 11 4	246	9.29 50 6	164
1.33 95 9	432	4.23 37 7	243	9.40 94 6	163
1.38 40 8	425	4.34 02 8	240	9.52 59 9	162
1.43 08 3	418	4.45 08 5	237	9.64 46 9	161
1.47 99 8	411	4.56 57 1	234	9.76 56 3	160
1.53 17 1	404	4.68 50 7	231	9.88 88 5	159
1.58 62 0	397	4.80 91 7	228	10.01 44	158
1.64 36 5	390	4.93 82 7	225	10.14 24	157
1.69 54 2	384	5.07 26 4	222	10.27 29	156
1.74 96 7	378	5.16 52 9	220	10.40 58	155
1.80 65 7	372	5.26 05 0	218	10.54 14	154
1.86 62 8	366	5.35 83 7	216	10.67 97	153
1.92 90 1	360	5.45 89 9	214	10.82 06	152
1.99 49 6	354	5.56 24 8	212	10.96 44	151
2.05 25 3	349	5.66 89 3	210	11.11 11	150
2.11 26 3	344	5.77 84 8	208	11.26 07	149
2.17 54 1	339	5.89 12 2	206	11.41 34	148
2.24 10 3	3.4	6.00 73 0	204	11.56 93	147
2.30 96 6	329	6.12 68 5	202	11.72 83	146
2.38 15 0	324	6.25 00 0	200	11.89 06	145
2.45 67 4	319	6.37 69 0	198	12.05 63	144
2.53 56 0	314	6.50 77 0	196	12.22 55	143
2.60 14 6	310	6.64 25 8	194	12.39 83	142
2.66 99 1	306	6.78 16 8	192	12.57 48	141
2.74 11 1	302	6.92 52 1	190	12.75 51	140
2.81 51 9	298	7.07 33 4	188	12.93 93	139
2.89 23 1	294	7.22 62 7	186	13.12 75	138
2.97 26 5	290	7.38 42 2	184	13.31 98	137
3.05 63 8	286	7.54 74 0	182	13.51 64	136
3.14 37 1	282	7.71 60 5	180	13.71 74	135
3.23 48 2	278	7.89 04 2	178	13.92 29	134
3.32 99 6	274	8.07 07 6	176	14.13 31	133
3.42 93 6	270	8.25 73 7	174	14.34 80	132

R^2	1/2R	R^2	1/2R	R^2	1/2R
14.56 79	131	31.00 18	898	60.27 93	644
14.79 29	130	31.49 09	891	61.03 52	640
15.02 31	129	31.99 16	884	61.80 53	636
15.25 88	128	32.50 43	877	62.59 01	632
15.50 00	127	33.02 95	870	63.39 00	628
15.74 70	126	33.56 75	863	64.20 53	624
16.00 00	125	34.11 87	856	65.03 64	620
16.25 91	124	34.68 36	849	65.88 38	616
16.52 46	123	35.17 91	843	66.74 78	612
16.79 66	122	35.68 53	837	67.62 90	608
17.07 53	121	36.20 24	831	68.52 77	604
17.36 11	120	36.73 09	825	69.44 44	600
17.65 41	119	37.27 11	819	70.37 97	596
17.95 46	118	37.82 33	813	71.33 40	592
18.26 28	117	38.38 78	807	72.30 78	588
18.57 91	116	38.96 50	801	73.30 17	584
18.90 36	115	39.55 54	795	74.06 07	581
19.23 67	114	40.15 93	789	74.83 15	578
19.57 87	113	40.77 71	783	75.61 44	575
19.92 99	112	41.40 93	777	76.40 96	572
20.29 06	111	42.05 63	771	77.21 75	569
20.66 12	110	42.71 86	765	78.03 82	566
21.04 20	109	43.28 25	760	78.87 21	563
21.43 35	108	43.85 77	755	79.71 94	560
21.83 60	107	45.65 38	740	80.58 04	557
22.20 80	1061	46.27 70	735	81.45 55	554
22.58 96	1052	46.91 31	730	82.34 49	551
22.98 11	1043	47.56 24	725	83.24 90	548
23.38 29	1034	48.22 53	720	84.16 80	545
23.79 54	1025	48.90 21	715	85.10 23	542
24.21 88	1016	49.59 33	710	86.05 23	539
24.65 36	1007	50.29 93	705	87.01 83	536
25.10 03	998	51.02 04	700	88.00 06	533
25.50 76	990	51.75 72	695	88.99 96	530
25.92 49	982	52.51 00	690	90.01 58	527
26.35 25	974	53.27 93	685	91.04 95	524
26.79 08	966	54.06 57	680	92.10 10	521
27.24 01	958	54.70 75	676	93.17 09	518
27.70 08	950	55.36 07	672	94.25 96	515
28.17 33	942	56.02 57	668	95.36 74	512
28.65 80	934	56.70 27	664	96.49 49	509
29.15 53	926	57.39 21	660	97.64 25	506
29.60 12	919	58.09 41	656	98.81 07	503
30.05 73	912	58.80 91	652	99.20 48	502
30.52 41	905	59.53 74	648	100.00 00	500

CUBE ROOTS

The method of finding cube roots on the Curta is very similar to Method (5) for square roots. We use *Curta Table 102*.

If R is an approximate cube root of N^3, we may put

$N = R + d$, where d is small. Thus $N^3 = R^3 + 3R^2d + 3Rd^2 + d^3$

and if we ignore d^2 & d^3 and substitute $N - R$ for d we get

$N = \dfrac{N^3 + R^2}{3R^3}$ as a second order approximation for N.

Table 102 gives a column for N^3 and one for $\dfrac{1}{3R^2}$, the latter being spaced at intervals calculated to give 5 figure accuracy to the answer. The table is arranged in groups of 3 figures from the decimal point.

Example: 457.315½

	SR	CR	PR	
C1	**/457.315**	**/1**	**/457.315**	
	/456787	**/3**	**/1370.889**	**Clear CR**
C6	**/.005610**	**1370.89**	**/7.70440179**	

Answer = 7.7044

CURTA TABLE No. 102

CUBE ROOTS - $N = (N^3 + R^3) / 3R^2$ for 5-FIGURE ROOT

R^3	$1/3R^2$	R^3	$1/3R^2$	R^3	$1/3R^2$
1.001 50	(.)333	4.407 42	124	15.294 0	541
1.043 52	316	4.516 25	122	15.683 8	532
1.083 40	324	4.629 63	120	16.090 4	523
1.125 88	308	4.747 83	118	16.514 8	514
1.171 22	300	4.871 14	116	16.908 0	506
1.219 67	292	4.999 89	114	17.317 1	498
1.264 88	285	5.134 42	112	17.742 9	490
1.312 96	278	5.275 08	110	18.186 4	482
1.364 16	271	5.422 28	108	18.648 8	474
1.418 77	264	5.576 47	106	19.131 1	466
1.468 55	258	5.738 10	104	19.570 4	459
1.521 31	252	5.907 69	102	20.026 7	452
1.577 30	246	6.085 80	100	20.501 1	445
1.636 82	240	6.273 05	(.0)980	20.994 6	438
1.700 18	234	6.470 10	960	21.508 1	431
1.756 16	229	6.667 05	941	22.042 9	424
1.815 28	224	6.874 19	922	22.600 3	417
1 877 81	219	7.080 52	904	23.097 0	411
1.944 00	214	7.297 39	886	23.612 1	405
2.014 18	209	7.512 57	869	24.146 7	399
2.088 68	204	7.738 53	852	24.701 8	393
2.151 66	200	7.961 75	836	25.278 5	387
2.217 86	196	8.195 91	820	25.878 O	381
2.287 53	192	8.441 78	804	26.501 5	375
2.360 92	188	8.683 65	789	27.150 5	369
2.438 32	184	8.937 30	774	27.711 9	364
2.520 05	180	9.203 55	759	28.292 8	359
2.606 45	176	9.464 20	745	28.894 4	354
2.697 90	172	9.737 38	731	29.517 6	349
2.794 82	168	10.024 0	717	30.163 4	344
2.871 39	165	10.302 9	704	30.833 2	339
2.951 52	162	10.595 O	691	31.528 1	334
3.035 45	159	10.901 2	678	32.249 6	329
3.123 43	156	11.197 1	666	32.999 0	324
3.215 74	153	11.506 7	654	33.777 8	319
3.312 69	150	11.830 8	642	34.423 3	315
3.411 62	147	12.141 5	631	35.089 5	311
3.521 88	144	12.466 1	620	35.777 5	307
3.634 87	141	12.805 4	609	36.488 3	303
3.754 04	138	13.160 3	598	37.223 0	299
3.879 87	135	13.497 5	588	37.982 6	295
4.012 89	132	13.849 3	578	38.768 4	291
4.105 84	130	14.216 6	568	39.581 7	287
4.202 46	128	11.561 3	559	40.423 9	283
4.302 91	126	14.920 2	550	41.296 4	279

R^3		$1/3R^2$	R^3	$1/3R^2$	R^3	$1/3R^2$	R^3	$1/3R^2$
42.200	6	(.0)275	106.887	148	216.324	925	489.054	537
43.138	4	271	107.980	147	219.880	915	495.965	532
44.111	5	267	109.091	146	223.535	905	503.040	527
44.865	4	264	110.222	145	227.292	895	510.285	522
45.641	2	261	111.372	144	231.155	885	517.705	517
46.438	6	258	112.542	143	235.129	875	525.307	512
47.261	5	255	113.733	142	238.803	866	533.097	507
48.108	0	252	114.944	141	242.575	857	541.081	502
48.980	0	249	116.179	140	246.447	848	547.614	498
49.878	7	246	117.434	139	250.423	839	554.278	494
50.805	2	243	118. 713	138	254.507	830	561.078	490
51.760	8	240	120.015	137	258.704	821	568.020	486
52.746	8	237	121.342	136	263.017	812	575.106	482
53.746	3	234	122.692	135	266.952	804	582.340	478
54.815	1	231	124.068	134	270.987	796	589.726	474
55.900	5	228	125.470	133	275.124	788	597.271	470
57.022	2	225	126.898	132	279.367	780	604.978	466
58.182	0	222	128.354	131	283.121	772	612.851	462
59.381	6	219	129.838	130	288.189	764	620.897	458
60.623	0	216	131.351	129	292.776	756	629.122	454
61.908	2	213	132.893	128	296.890	749	637.528	450
63.239	6	210	134.466	127	301.101	742	646.124	446
64.153	9	208	136.070	126	305.412	735	654.914	442
65.090	4	206	137.706	125	309.828	728	663.907	438
66:050	0	204	139.375	124	314.351	721	673.106	434
67.033	4	202	141.078	123	318.985	714	682.520	430
68.041	4	200	142.816	122	323.734	707	692.155	426
69.074	9	198	144.590	121	328.602	700	702.020	422
70.134	8	196	146.401	120	333.594	693	709.573	419
71.222	2	194	148.250	119	338.713	686	717.263	416
72.338	0	192	150.139	118	343.964	679	725.092	413
73.483	1	190	152.068	117	349.352	672	733.064	410
74.658	9	188	154.039	116	354.883	665	741.184	407
75.866	3	186	156.053	115	359.741	659	749.455	404
77.106	6	184	158.111	114	364.710	653	757.882	401
78.381	0	182	160.214	113	369.795	647	766.467	398
79.691	0	180	162.265	112	375.000	641	775.215	395
81.037	9	178	164.564	111	380.327	635	784.131	392
82.423	1	176	166.813	110	385.782	629	793.220	389
83.848	3	174	169.113	109	391.369	623	802.485	386
85.315	0	172	171.467	108	397.091	617	811.932	383
86.825	0	170	173.877	107	402.955	611	821.566	380
88.380	1	168	176.343	106	408.964	605	831.392	377
89.982	1	166	178.868	105	415.124	599	841.416	374
91.633	1	164	181.454	104	421.440	593	851.642	371
93.335	2	162	184.103	103	427.918	587	862.078	368
95.090	7	160	186.817	102	433.445	582	872.727	365
96.901	9	158	189.599	101	439.091	577	883.599	362
98.771	4	156	192.450	100	444.861	572	894.698	359
100.702		154	195.669	(.00)989	450.758	567	906.031	356
101.692		153	198.980	978	456.787	562	917.605	353
102.696		152	202.385	967	462.952	557	929.429	350
103.718		151	205.888	956	469.256	552	941.508	347
104.756		150	209.493	945	475.704	547	953.851	344
105.813		149	212.863	935	482.302	542	966.466	341
							979.361	338
							992.546	335

FRACTIONAL POWERS of NUMBERS

Fractional Powers, other than square and cube roots, fourth roots, etc., for practical purposes, cannot be found directly on a calculating machine. However, the Curta will be found very useful when used in conjunction with log tables.

For instance, if a number of quantities have to be raised to the same fractional power, the power can be set on **SR**. The log of each number in turn is looked up in the tables and turned into the **CR**. The antilog of the resultant **PR** is looked up in the tables.

Example: $5.395^{1.42125}$ Using Curta II

	SR	CR	PR
C1	**/1.42125**	**.7319914/**	**01.04034277725/ ***

Answer (from 7 figure log tables) = **10.97344**

The advantages of using a calculating machine are that the power can be set on the machine once for all similar computations and only the answer need be written down. The special advantage of tho Curta is that it can be held aver the log tables during the work facilitating the reading of the antilog and reducing the chance of error.

QUADRATIC EQUATIONS

Any quadratic equation can be expressed in the form

$x^2 + ax + b = 0,$

the roots of which are:-

$\frac{1}{2}(-a \pm (a^2 - 4b)^{\frac{1}{2}})$

The square root of $a^2 - 4b$ can, of course, be found on a Curta by one of the methods indicated in **M 1** and the roots of the quadratic computed in this manner. If, however, a^2 is very large in relation to $4b$, one of the roots will be the difference between two nearly equal quantities and too few significant figures may be given unless the square root of $a^2 - 4b$ is computed to a very large number of figures.

In such cases, the root can be found very quickly to 6 significant figures on Curta I and 8 significant figures on Curta II merely by a process of division.

If the roots are real, $-a$ is one approximate root and $-b \div a$ the other. Second approximations are given by $-a + b \div a$ and $\frac{b}{-a + b \div a}$

Example: $x^2 - 152x + 3 = 0$

Using Curta II

One approximate root is 3 ÷ 152

Another is 152 – 3 ÷ 152

A closer approximation of the 1st root is 3 ÷ (152 – 3 ÷ 152) Using build up division but with Reversing Lever down to give complement of quotient.

	SR	CR	PR	
C7 *R*	**/ 152**	**98026/ ***	**/ 300048000**	**Clear dials**
C8	**/ 151.98026**	**19739406***	**300000005612556**	

One root is therefore .019739406
The other is 152, less the above root, i.e. 151.980260594

CUBIC EQUATIONS

Using similar notation to that used for square and cube roots in **M 1** and **2**, any cubic equation can be reduced to the form

$N^3 + aN = b$ .... (1)

If R is an approximate solution so that $N = R + d$, where d is small,

$(R + d)^3 + a(R + d) = b$, and ignoring $d^2 + d^3$

$R^3 + 3R^2d + aN = b$

Substituting $N - R$ for d, $R^3 + 3R^2N - 3R^3 + aN = b$,

and $N = \dfrac{b + 2R^3}{a + 3R^3}$ .... (2)

Thus, if an approximate root, R, has been found, a second order approximation can be found from equation (2) and thus any desired accuracy can be obtained by successive approximations.

Example: Find the real roots of $N^3 + 2N = 8$ .... (3)

An approximate root lies between 1.5 and 2, say 1.75
Set 8 on **PR**, 2 on **SR** and multiply by -1.75 obtaining for N^3, 4.50

From a table of squares and cubes, e.g. Barlows', we find N^3 should be 5.359375, so that 1.75 is too large.

We therefore turn the handle positively until we get 4.660 on **PR** for 1.670 on **CR** for which N^3, i.e. $1.67^3 = 4.657463$. This is as near as we can get to 4 significant figures.

From equation (2) we have a second approximation

$$N = \frac{8 + 2(1.670)^3}{2 + 3(1.670)^2} = \frac{8 + 2 \times 4.657463}{2 + 3 \times 2.7889} \quad \text{(from the tables)}$$

$$= 1.6702447$$

Dividing $N^3 + 2N - 8$ by N - 1.6702447,
we get N^2 + 1.6702447N + 4.7897174 = 0
the roots of which are imaginary.

So our only real root is 1.6702447.

Using a Curta II, the operations are as follows:-

	SR	CR	PR	
C8	**0008/**	**1/**	**8/***	**Clear CR**
C8 *R*	**0002/**	**1.75/***	**4.50**	**Clear CR & PR**
C4	**8/**	**1/1000**	**08/***	
	4.657463/	**/3000**	**17.314926/***	
C1	**/2.7889**	**/3003**	**17.314926:008.3667***	
	/2.0000	**/3004**	**7.314926:010.3667***	**Clear CR**
C8 *R*	**/10.366700**	**/1.6702447***	**/2788767**	

Answer = 1.6702447

In making the above calculation we have used each side of the **PR** to accumulate numerator and denominator of N. The **CR** figures are meaningless after the 2nd step until the last step as there is no necessity to clear **CR**.

SUMMATION of SQUARES

In many statistical and other problems it is necessary to sum the squares of a number of quantities, Σx^2. Frequently it is also necessary to sum the quantities themselves, Σx. Both summations can be performed simultaneously on the Curta, preferably on model II, since on that machine the multiplier can be checked at each step of the operation.

We set the figure '1' on the left hand setting dial and x on the right of **SR**. Multiply by x so that x^2 appears on the right side of **PR** and x, both on the left of **PR** and on **CR**. We can either check and clear **CR**, accumulating Σx on the left of **PR**, or check and clear the left side of **PR** (returning the clearing lever to the left side of **PR**) accumulating Σx on **CR**. In either case Σx^2 is accumulated on the right of **PR**. Generally the second method is the best if a large number of quantities has to be squared and summed, since it is easier to check the multiplier on the left of **PR**, whilst **CR** provides a larger capacity for summing the quantities. On the other hand, when further operations have to be performed on Σx^2, the former method may have advantages.

Example: Find the standard deviation of the following quantities: -
239, 1347, 1196, 0985, 1082.

The standard deviation is given by: $S = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n}}$,where $\bar{x}$ is the arithmetic mean of the quantities. This is more conveniently evaluated as $S = \dfrac{1}{n}\sqrt{n\Sigma x^2 - (\Sigma x)^2}$, so that we must find Σx^2 and Σx.

The steps on Curta II are as follows:-

	SR	CR	PR	
C1	**1//1239**	**/1239**	**01239//1535121 ***	**Clear CR**
C1	**1//1347**	**/1347**	**02586//3349.530***	**Clear CR**
C1	**1//1196**	**/1196**	**03782//4779946***	**Clear CR**
C1	**1//0985**	**/0985**	**04767//5750171 ***	**Clear CR**
C1	**1//1082**	**/1082**	**05849//6920895***	**Clear CR**

We now multiply the right side of **PR** by 5, by successive multiplication.

	SR	CR	PR	
C7	**/4**	**/6920895**	**05849//34604475***	**Clear CR**

Then deduct the square of 5849 from right side of *PR*

		SR	CR	PR	
C4	***R***	**1//5849**	**/5849**	**/393674***	**Clear CR**

Next find the square root of 393674 by method **M 1**.(5) using *Table 101*.

	SR	CR	PR	
C1	**/395554**	**/1**	**/789228***	**Clear CR**
C6	**/794**	**/789228**	**/627.436260***	**Clear CR**

Now divide by 5

	SR	CR	PR
C8	**/5**	**125.487/***	**/1260**

Note: Since the original observations included no decimal places, we cannot get greater accuracy in the standard deviation, unless of course the quantities were integers. We can thus take the standard deviation as 125.

Of course the number of squares to be summed will normally be many more than 5 but the principle is identical.

ACCUMULATION of PRODUCTS

There are many problems in which it is necessary to sum the products $\sum xy$ and $\sum xz$ of two sets of quantities y and z multiplied by a third set of quantities x, which also has to be summed. All three sums can be evaluated simultaneously on the Curta. We set y on the left of **SR**, z on the right and multiply by x. If none of the dials are cleared, we obtain $\sum xy$ on the left of **PR**, $\sum xz$ on the right and $\sum x$ on **CR**. It is desirable, of course, to check the latter with the sum of the x's found by some other means, e.g. mentally or by adding-listing machine.

Example: Evaluate E given by the following equation:-

$$E = \frac{30260\,\Sigma(f_\lambda E_\lambda) + 9{,}531\,\Sigma(\lambda E_\lambda) - 27047{,}861\,\Sigma(E_\lambda)}{8566.7}$$

where λ, f_λ, and E_λ, are given by the following table

λ	f_λ	E_λ
300	.553	1.34
305	.669	1.54
310	.811	1.76
315	.893	2.03
320	.936	2.16
325	1.000	2.26
330	.982	2.22
335	.888	2.03
340	.790	1.80
345	.667	1.57
350	.517	1.34

Note: This example has been kindly supplied by the Physical Department of Beecham Research Laboratories Ltd. as a sample of the usefulness of the Curta Model II in accumulating two products and one multiplier simultaneously. The computation is used in the calculation of vitamin A content in percomorph liver oil, from ultra violet absorption spectra. Any readers who would like to examine the complete system of analysis computation may obtain full details from London Automatic Business Machines Ltd., 15 Cromwell Road, London SW7.

The operations on the Curta II are:-

	SR	CR	PR
C1	300//.553	/1.34*	0402.00//0.74102*
C1	305//.669	/2.88*	00871.70//1.77128*
C1	310//.811	/4.64*	01417.30//3.19864*
C1	315//.893	/6.67*	02056.75//5.01143*
C1	320/.936	/8.83*	02747.95//7.03319*
C1	326//1.000	/11.09*	03484.71//9.29319*
C1	330//.982	/13.31*	04217.31//11.47323*
C1	335//.888	/15.34*	04897.36//13.27587*
C2	340//.790	/17.14*	05509.36//14.69787
C1	345//.667	/18.71*	06051.01//15.74506*
C1	350//.517	/20.05*	06520.01//16.43784*

Write down $\Sigma(f_\lambda E_\lambda)$ = 16.43784

$\Sigma(\lambda E_\lambda)$ = 6520.01

$\Sigma(E_\lambda))$ = 20.05 and **Clear dials**

	SR	CR	PR	
C8	/16.43784	30260/	0497409.0384/	Clear CR
C1	/6520.01000	/9.531	0559551.25371/	Clear CR
C2 *R*	/27047.86100	/20.050	0017241.64066/	Clear CR

Now we divide by 8566.7

	SR	CR	PR
C8 *R*	/8566.7	2.0126/	/30024000

Answer = 2.013

EVALUATION of a POLYNOMIAL or of a POWER SERIES

A series $a_0 + a_f x + a_2 x^2 + \ldots$ for any given value of x could be computed by raising x to the successive powers of 2, 3, etc., writing down the answers and accumulating the products of these answers and the appropriate coefficients a_2, a_3, etc. It is speedier and less liable to cause error, however, if we re-write the series in a form which will enable us to carry out the computation in a continuous operation, without writing down anything except the answer.

Suppose the powers in excess of 4 can be ignored,
we may re-write the series:- $a_0 + x\,(a_1 + x\,(a_2 + x\,(a_3 + xa_4\,)\,)\,)$

Thus, we can set a_4 on **SR**, multiply by x; add a_3, multiply by x; add a_2, multiply by x; add a_2, multiply by x and add a_0. We then obtain the required value on **PR**.

Example: With a Curta Model II

Evaluate $1 + 5x + 9x^2 + 13x^3 + 17x^4 + 21x^5 + \ldots$
to 5 places of decimals where $x = .0925$

We can ignore the terms containing powers of x greater than 6.

	SR	CR	PR	
C3	**/21**	**/.0925**	**/1.9425***	**Clear CR**
C5	**/17**	**/1.0000**	**/18.9425***	**Clear CR**
C6	**/.0924**	**/18.9425**	**/1.75218125***	**Clear CR**
C8	**/13.0**	**1/**	**/14.75218125***	**Clear CR**
C8	**/.092400**	**/14.752181**	**/1.364576742525***	**Clear CR**
C8	**/9.00000**	**1/**	**/10.364576742525***	
C8	**/10.36458**	**0**	**9/6742525***	**Clear PR**
C1		**/925**	**/.95872.3650***	**Clear CR**
C8	**/5.00**	**1/**	**/5.958723650***	**Clear CR**
C8	**/.092400**	**5.9587236**	**/.5511819330050***	

Answer = 1.55118

EVALUATION of SERIES

Convergent series of form $\frac{a_1}{a_2} + \frac{a_1}{a_2} , \frac{b_1}{b_2} + \frac{a_1}{a_2} , \frac{b_1}{b_2} , \frac{c_1}{c_2}$,etc. can be evaluated on Curta II in a continuous operation.

We split **SR** and **PR**, and set a_1 on the left of **SR** and a_2 on the right. If we build up to 1 on the left of **PR**, we shall obtain $\frac{a_1}{a_2}$ on the right. We may either clear the left side of **PR** or build up to 1 to the left of the 15th dial of **PR**, i.e. off the register. So we now have 0 on the left and the 1st term of the series on the right of **PR**.

We change the settings to b_1 and b_2 and build up the left hand side of **PR** to the amount which showed on the right hand side, i.e. $\frac{a_1}{a_2}$. Thus we add $\frac{a_1}{a_2} \times \frac{b_1}{b_2}$ to the right hand side. We now have the 1st term on the left and the sum of the first two terms on the right.

We proceed in this way until the left and right hand sides of **PR** agree to the required number of significant figures, obtaining the sum of n terms on the left and $n + 1$ terms on the right.

In the following example we shall **omit CR**, which is not used in this type of calculation, and show the 2 sides of **SR** and **PR** separately.

Example: Evaluate the following series to six places of decimals.

$$.3 + \frac{.3 \times .5}{2\,!} + \frac{.3 \times .5 \times .7}{3\,!} + \ldots .$$

Split **SR** and **PR** between the 8th and 9th dials.

	L SR	*R*	*L* PR	*R*
C8	**/1**	**/3**	**.0/**	**.3/**
C7	**/2**	**/5**	**.3/**	**.375/**
C6	**/3**	**/7**	**.375/**	**.3925/**
C5	**/4**	**/9**	**.3925/**	**.3964375/**
C5	**/5**	**/11**	**.3964375**	**.39730375**
C4	**/6**	**/13**	**.3973039**	**.39749147**
C3	**/7**	**/15**	**.3974915**	**.39753167**
C2	**/8**	**/17**	**.3975315**	**.39754017**
C2	**/9**	**/19**	**.3975405**	**.39754207**
C1	**/10**	**/21**	**.3975415**	**.39754228**
C1	**/11**	**/23**	**.3975426**	**.39754251**

Answer = .397542

CONSTRUCTION of an n^{th} ORDER POLYNOMIAL from n^{th} DIFFERENCES

If we have a polynomial $a_0 x^n + a_1 x^{n-1} + \ldots$, then the n^{th} differences are constant and equal to $a_0 n\ (sx)^2$, where sx is the interval of x. To construct the polynomial for successive values of x we require 'n' consecutive values of the polynomial and the next value as a numerical check. We calculate the successive differences in these values and then build up from the constant n^{th} difference.

Example: $y = x^4 + 3x^3 + 7x^2 + 5x + 2$ from 0 with intervals of .1

The 4th difference is $24x\,(.1)^4 = .0024$

Taking as the 1st five terms -.2, -.1, 0, .1 and .2, for ease of calculation we get:-

x	y	sy	s^2y	s^3y	s^4y
-.2.	1.2576				
		.3095			
-.1	1.5671		.1234		
		.4329		.0168	
0	2.0000		.1402		.0024
		.5731		.0192	
.1	2.5731		.1594		
		.7325			
.2	3.3056				

We shall use the symbols y_1, $sy_{1½}$, $s^3y_{½}$, etc. to represent respectively the value of y corresponding to $x = .1$, the value of the second difference between sy_1 and sy_2, the value of the third difference between s^2y_0 and s^2y_1, etc.

We require a Curta Model II.

Set s^2y_1 on left of **SR** and y_1 on the right.
Turn the handle once.
Set $s^3y_{½}$ on left of **SR** and $sy_{1½}$ on the right.
Add .0024 to left hand side of **SR** and turn handle once.
Check that right side of **PR** reads y_2

From now on, after each turn of the handle, increase left side of **PR** by 24 and add to right side the amount showing in the left of **PR**. To avoid mistakes it will be as well to move the carriage to position 8 so that right side of **SR** comes under left side of **PR**. Alter the setting dials and return the carriage to position 1 before making the next turn, thus:-

	SR	CR	PR
C1	.1594,,002.5731	/.1	/.1594,,002.5731 *
	s^3y sy	x	s^2y y
C1	.0216,,000.1325	/.2	/.1810,,003.3056*
C8	.0240,,000.9135		
C1		/.3	/.2050,,004.2191 *
C8	.0264,,001.1185		
C1		/.4	/.2314,,005.3376*
C8	.0288,,001.3499		
C1		/.5	/. 2602,,006.6875*
C8	.0312, ,001.6101		
C1		/.6	/.2914,,008.2976*
C8	.0336,,001.9015		
C1		/.7	/.3250,,010.1991 *
C8	.0360,,002.2265		
C1		/.8	/.3610,,012.4256*
C8	.0384,,002.5875		
C1		/.9	/.3994,,015.0131 *
C8	.0408,,002. 9869		
C1		/1.0	/.4402,,018.0000*

The above table gives the values of y and the successive differences, in this order, s^3y, sy, s^2y and y. The final value of y should be checked. This will prove that the calculation has been made correctly, but not that the values of y have been written down correctly. This is proved by differencing the last column and checking the first differences with the second column of figures.

Owing to the compactness of the Curta it will be found a simple matter to add or subtract figures from the setting dials. For polynomials of degree higher than 4, the calculation will have to be performed in 2 or more parts; for instance, with a polynomial of order 6, we can find the 2nd difference, as above, and build up the functions themselves from the second differences.

If a large volume of such work has to be done, it is best to use a National 3000 Class Accounting Machine, or, if the polynomial is of order greater than 6, a LogAbax Accounting Machine.

INTERPOLATION

Interpolation is the process of finding the value of a function corresponding to a value or the argument between two values for which the value of the function is given.

There are many methods of interpolation, some of which are of little practical value and others which are appropriate for differing circumstances.

Here we confine ourselves to indicating the Curta technique applicable to the five methods most commonly useful.

1. Linear Interpolation
2. Neville's Method
3. Lagrange's Formula
4. Everett's Formula
5. Bessel's Formula

Of these, linear interpolation is used wherever it gives sufficient accuracy. In other cases, Bessel's is the most useful but requires tables of Bessel's coefficients and the computation of successive differences to the order necessary to give the required accuracy. Everett's method is useful where differences of order more than four are necessary, since only even order differences are involved.

Lagrange's method also uses a table of coefficients but requires no differencing and Neville's method requires no table of coefficients and gives an automatic check on the accuracy of the computation but is rather more labourous than the last three.

Notation.

Various notations are used in textbooks and tables, and our reason for using that given below is that it appears to us to be short, unambiguous and comparatively easy to type on an ordinary typewriter.

If we have a function $y = f(x)$ and require the value f_n
corresponding to $x_n = x_0 + nSx$ where Sx is the interval in the argument,
the central difference table is

Argument	$f(x)$	Differences			
		1st	2nd	3rd	4th
x_{-2}	f_{-2}				
		$S_{-1½}$			
x_{-1}	f_{-1}		S^2_{-1}		
		$S_{-½}$		$S^3_{-½}$	
x_0	f_0		S^2_0		S^4_0
		$S_{½}$		$S^3_{½}$	
x_1	f_1		S^2_1		
		$S_{1½}$			
x_2	f_2				

Examples: In the examples given here for each method we shall use the same set of figures from a table of *sin x* for *x* given in radians at intervals of 0.1. In this way the reader will be able to compare the time taken and accuracy of the different methods.

	x	$sin\ x$	S	S^2	S^3	S^4	S^5
x-2	4.0	-0.7560					
			-.06148				
x-1	4.1	.81828		+0.00818			
			.05330		+0.00053		
x0	4.2	87158		871		-0.00008	
			.04459		45		-0.00003
x1	4.3	.91617		916		11	
			.03543		34		
x2	4.4	.95160		950			
			.02593				
x3	4.5	.97753					

If the maximum contribution to an interpolate from the lowest order of differences is not to exceed .5 in the last significant figure, the limiting values for the differences ignored are given in *'Interpolation and Allied Tables'* (H.M. Stationery Office) as follows:-

Difference Neglected.	Limiting Value.
2nd	4
3	60
4	20
5	500
6	100
7	3500
8	400
9	20.000
10	2.000

Thus, in the above example we can ignore differences over the 2nd. If we ignore 2nd differences we can only rely on the first two decimal places in the interpolate, though the third decimal place is not likely to be inaccurate by more than about 1 unit in the 3rd decimal place.

1. Linear Interpolation.

The simplest form of interpolation is 'linear'. This assumes that a function $y = f(x)$ can be represented between points x_0 and x_1 by a straight line through these points, i.e. $f(x)$ can be represented by a polynomial of degree 1. We will call this polynomial

$$f_{0.1} = \frac{f_0\ (x_1 - x_n) + f_1\ (x_n - x_0)}{x_1 - x_0}$$

For unit interval in the argument, this reduces to

$$\begin{aligned} f_{0.1} &= f_0 + (x_n - x_0)(f_n - f_0) \\ &= f_0 + nS_{½} \end{aligned}$$

Example (i) Find the value of *sin* 4.215 radians by linear interpolation. There is no object in using more than 3 figures in *sin x* as we can only rely on two decimal places in the answer.

We could set $-f_0$, i.e. .872, and multiply by $\dfrac{x_1 - x_n}{x_1 - x_0}$ that is .85

Then set $-f_1$, i.e. .916, and multiply by $\dfrac{x_n - x}{x_1 - x_0}$ that is .15

obtaining .87860 in the product dials, so that the answer is .88 to two places of decimals. Actually the 1st 3 figures .879 are in this case, correct.

In order to check the settings, however, it is generally considered preferable to set f_0 and turn into **PR**. Then set S_n and turn into **PR**, checking that we have $f_1 = f_0 + S_n$ in **PR**. If we now make one negative turn to restore f_0 in **PR** and then turn through .15 we set .87860 on **PR**, as before, thus:-

	SR	CR	PR	
C3	**/.872**	**/100**	**/.87200***	
	/.044	**/200**	**/.91600***	**Check PR**
		/100	**/.87200***	
		/115	**/.87860***	

Answer .879

Note: If we were to set the full 5 figure values we should get

	SR	CR	PR	
C3	**/.87158**	**/100**	**/.8715800***	
	/.04459	**/200**	**/.9161700***	**Check PR**
		/100	**/.8715800**	
C2		**/115**	**/.8782685***	

As the correct 5 figure interpolate is .87883, we are not gaining in accuracy by working to more than 3 figures.

2. Neville's Method.

If linear interpolation gives insufficient accuracy, we can represent the function by a polynomial of higher degree. Both Aitken's and Neville's methods are based on using a polynomial of degree sufficient to give the required accuracy but Neville's method is slightly preferable for use with the Curta.

Suppose the 1st degree polynomial between points x_0 and x_1 is represented by

$$f_{0,1} = \frac{-f_{-1}\ (x_1 - x_n) + 1(x_0 - x_n)}{x_1 - x_0}$$

We can obtain a second degree polynomial, represented by $f_{-1,0,1}$, between x_{-1}, x_0 and x_1 formed in the same way from $f_{0,-1}$ and $f_{0,1}$, thus:-

$$f_{-1,0,1} = \frac{f_{0,-1}\ (x_1 - x_n) - f_{0,1}\ (x_{-1} - x_n)}{x_1 - x_{-1}}$$

We can set out the scheme of computation thus for a 4th degree polynomial: -

x_{-2}	$x_{-2} - x_n$	f_{-2}			
			$f_{-2,-1}$		
x_{-1}	$x_{-1} - x_n$	f_{-1}		$f_{-2,-1,0}$	
			$f_{-1,0}$		$f_{-2,-1,0,1}$
x_0	$x_0 - x_n$	f_0		$f_{-1,0,1}$	
			$f_{0,1}$		$f_{-1,0,1,2}$
x_1	$x_1 - x_n$	f_1		$f_{0,1,2}$	
			$f_{1,2}$		
x_2	$x_2 - x_n$	f_2			

Example (ii) Find the value of sin 4.215 radians, from the table already given, by Neville's method.

The only figures which it is necessary to write down are:-

x	$x - x_n$	1st degree	2nd degree	3rd degree
4.0	-215			
		-0.888982		
4.1	-115		-0.878869	
		79575		-0.878836
4.2	-015		23	
		78268		4
4.3	+085		52	
		86054		
4.4	+185			

The polynomials have been computed to 6 figures but we can only get 5 figure accuracy in the answer which may thus be taken as -0.87883.

The operations on the Curta are are follows:-

		SR	CR	PR	
(a)	**C1**	**/. 75680**	**9/885**	**9/ 1296800**	
	C1	**/.81828**	**/100**	**/8889820***	**Clear CR & PR**
	C1		**9/85**	**9/8772580**	
	C1	**/.87158**	**/100**	**/8795750***	**- do -**
	C1		**/85**	**/7408430**	
	C1	**/.91617**	**/100**	**/8782685***	**- do -**
	C1		**/185**	**/16949145**	
	C1	**/.95160**	**/100**	**/8860545***	

Note that since the interval between the x's is .1 the figure in **PR** can be divided mentally by $x_{-1} - x_2$ etc. to obtain $f_{-2,-1}$, etc.

The **CR** is only cleared after each pair of multiplications, when it should read 100. Note also that it is only necessary to set each value of sin x once.

		SR	CR	PR	
(b)	**C1**	**88982**	**9/85**	**9/8665279**	
	C1	**79575**	**/200**	**/15773895**	**Divide by 2 & Clear CR & PR**
	C1		**/85**	**/6763875**	
	C1	**78268**	**/200**	**/15764695**	**- do -**
			/185	**/141.79580**	
	C1	**86054**	**/200**	**/15770390**	**- do -**

Note that it is unnecessary to set the first 8 which is common to all the values of the 1st degree polynomial.

		SR	CR	PR	
(c)	**C1**	**69**	**/85**	**/5865**	
	C1	**23**	**/300**	**/10810**	**Divide by 3 & Clear CR & PR**
	C1		**/185**	**/4255**	
	C1	**52**	**/300**	**/10235**	**- do -**

It is only necessary to set the last 2 figures of the 2nd degree polynomial.

Since the two values of the 3rd degree polynomial agree to 5 decimal places, it is unnecessary to proceed further.

3. Lagrange's Formula.

If we extend the value of the four point polynomial $f_{-1, 0, 1, 2}$ in terms of f_{-1}, f_0, f_1, and f_2, we have an expression

$$fn = f_{-1} L_{-1}(n) + f_0 L_0(n) + f_1 L_1(n) + f_2 L_2(n)$$

where L_{-1}, L_0, L_1 and L_2 are 3rd degree polynomials in n, and are known as Lagrange Interpolation Coefficients. They can be evaluated and tabulated for various values of n and for various degrees of the polynomial, see Chambers' 6 figure tables which give the coefficients for 4 and 6 point interpolation. A table for four point interpolation and two figure values for n is given in *Curta Table No.103*.

The value of fn can be found by accumulating the products $f_{-1} L_{-1}(n)$ etc.

Example (iii) Find the value of sin 4.215 radians by Lagrange's Formula.

For n = .15 and a 4-point polynomial the Lagrange coefficients (see *Curta Table No. 103*) are: -

L_{-1}	-.03931
L_0	+.90419
L_1	+.15956
L_2	-.02444

All we need to do is to set the values of f_{-1} etc. in turn, multiply by the coefficients and accumulate the products thus:-

	SR	CR	PR
C1	**.81828**	**9/6069**	**9/678334132***
C1	**.87158**	**/.86488**	**/7559073334***
C1	**.91617**	**1.02444**	**/9020914186***
C1	**.95160**	**1.00000**	**/8788343146***

Answer = -0.87883

Note that it is not necessary to clear **CR** as the sum of the Lagrange coefficients should be 1.0 and the multipliers can be checked in this way.

4. Everett's Formula.

We can express Lagrange's Formula for a 4-point polynomial in terms of f_0 and 1st and 2nd differences, by taking

$$f_{-1} = f_0 - S_{½} + S_0^2, \quad f_1 = f_0 + S_{½}, \quad f_2 = f_0 + 2S_{½} + S_1^2$$

Lagrange's Formula then becomes

$$f_n = f_0 \; L + S_{½}(2L_2 + L_1 - L_{-1}) + L_{-1}S_0^2 + L_2 S_1^2$$

Now L = 1 and $2L_2 + L_1 - L_{-1} = n$

So $f_n = f_0 + n\,S_{½} + L_{-1}S_0^2 + L_2 S_1^2$

This is Everett's formula for a 4-point polynomial. L_{-1} and L_2 are usually written E_0^{ii} and E_1^{ii} respectively. The full formula is

$$f_n = f_0 + n\,S_{½} + E_0^{ii}\; S_0 + E_1^{ii}\; S_1^2 + E_0^{iv}\,S_0^4 + E_1^{iv}\,S_1^4 + \text{ etc.,}$$

and it will be noticed that all differences after the first are even-order.

Everett's interpolation coefficients are tabulated in '*Interpolation and Allied Tables*'. The 'throwback' can be used as with Bessel's Formula.

Curta Table No. 103 gives values of E_0^{ii} and E_1^{ii} for 2 figure values of n.

Example (iv) Find the value of sin 4.215 radians, by Everett's Formula. Again we can ignore differences greater than the 2nd.

From *Curta Table No. 103*.

E_0^{ii} = -.03931

E_1^{ii} = -.02444

	SR	CR	PR	
C6	**.87158**	**1.00000.**	**/.8715800000***	**Clear CR**
C5	**.04459**	**/.15000**	**/.8782685000***	**Clear CR**
C4	**.00871**	**/.03931**	**/.8786108901***	**Clear CR**
C4	**.00916**	**/.02444**	**/.8788347605**	**Clear CR**

Answer = -0.87883

CURTA TABLE No.103.1.

LAGRANGE & EVERETT INTERPOLATION COEFFICIENTS.

n	$L_{-1} = E_0^{ii}$	L_0	L_1	$L_2 = E_1^{ii}$	n
	–	+	+	–	
0.00	0.00000	1.00000	0.00000	0.00000	1.00
.01	.00328	.99490	.01005	.00167	0.99
.02	.00647	.98960	.02020	.00333	.98
.03	.00955	.98411	.03044	.00500	.97
.04	.01254	.97843	.04077	.00666	.96
0.05	0.01544	0.97256	0.05119	0.00831	0.95
.06	.01824	.96651	.06169	.00996	.94
.07	.02094	.96027	.07228	.01161	.93
.08	.02355	.95386	.08294	.01325	.92
.09	.02607	.94726	.09369	.01488	.91
0.10	0.02850	0.94050	0.10450	0.01650	0.90
.11	.03084	.93357	.11538	.01811	.89
.12	.03309	.92646	.12634	.01971	.88
.13	.03525	.91920	.13735	.02130	.87
.14	.03732	.91177	.14843	.02288	.86
0.15	0.03931	0./90419	0.15956	0.02444	0.85
.16	.04122	.89645	.17075	.02598	.84
.17	.04304	.88856	.18199	.02751	.83
.18	.04477	.88052	.19328	.02903	.82
.19	.04643	.87233	.20462	.03052	.81
0.20	0.04800	0.86400	0.21600	0,03200	0.80
.21	.04949	.85553	.22742	.03346	.79
.22	.05091	.84692	.23888	.03489	.78
.23	.05224	.83818	.25037	.03631	.77
.24	.05350	.82931	.26189	.03770	.76
0.25	0.05469	0.82031	0.27344	0.03906	0.75

CURTA TABLE No.103.2.

LAGRANGE & EVERETT INTERPOLATION COEFFICIENTS

n	$L_{-1} = E_0^{ii}$	L_0	L_1	$L_2 = E_1^{ii}$	n
	–	+	+	–	
0.25	0.05469	0.82031	0.27344	0.03906	0.75
.26	.05580	.81119	.28501	.04040	.74
.27	.05683	.80194	.29661	.04172	.73
.28	.05779	.79258	.30822	.04301	.72
.29	.05868	.78309	.31986	.04427	.71
0.30	0.05950	0.7350	0.33150	0.04550	0.70
.31	.06025	.76380	.34315	.04670	.69
.32	.06093	.75398	.35482	.04787	.68
.33	.06154	.74407	.36648	.04901	.67
.34	.06208	.73405	.37815	.05012	.66
0.35	0.06256	0.72394	0.38981	0.05119	0.65
.36	.06298	.71373	.40147	.05222	.64
.37	.06333	.70343	.41312	.05322	.63
.38	.06361	.69304	.42476	.05419	.62
.39	.06384	.68256	.43639	.05511	.61
0.40	0.06400	0.67200	0.44800	0.05600	0.60
.41	.06410	.66136	.45959	.05685	.59
.42	.06415	.65064	.47116	.05765	.58
.43	.06413	.63985	.48270	.05842	.57
.44	.06406	.62899	.49421	.05914	.56
0.45	0.06394	0.61806	0.50569	0.05981	0.55
.46	.06376	.60707	.51713	.06044	.54
.47	.06352	.59601	.52854	.06103	.53
.48	.06323	.58490	.53990	.06157	.52
.49	.06289	.57372	.55123	.06206	.51
0.50	0.06250	0.56250	0.56250	0.06250	0.50

5. Bessel's Formula.

This formula expresses the interpolated value of f in terms of central differences, thus:-

$$f_n = f_0 + n\,S_{½} + B^{ii}\,(S_0^{\;2} + S_1^{\;2}) + B^{iii}\,S_{½}^{\;3} + B^{iv}(S_0^{\;4} + S_1^{\;4}) + \text{etc.}$$

where B^{ii}, B^{iii}, B^{iv}, etc. are known as Bessel's Interpolation Coefficients dependent on the value or n, and are tabulated, as for instance in *'Interpolation and Allied Tables'*. A table of B^{ii} and B^{iii} to 3 decimal places is given in *Curta Table No. 104*.

A good approximation is given by applying Bessel's formula correct to 2nd or 3rd differences only and modifying S^2 by subtracting .18484 This is known as 'throwback' from 4th to 2nd differences and may be used if the 4th difference does not exceed 1000.

Example (v) Find the value of sin 4.215 radians by Bessel's Formula.

In this case we can ignore differences greater than the 2nd, as already stated. Therefore all we have to do is to look up the value of B^{ii} for n in the tables, e.g. *Curta Table No. 104* gives -.03187 for B^{ii}. We set f_0 and transfer to **PR**, set $S_{½}^{\;1}$ and multiply by n, and set $S_0^{\;2} + S^2$ and multiply by .032, thus:-

	SR	CR	PR	
C6	**.87158**	**1.00000**	**/.8715800000***	**Clear CR**
C5	**.04459**	**/.15000**	**/.8782685000***	**Clear CR**
C4	**.01787**	**/.03187**	**/.8788380169***	

Answer = -.87884

Note that $S_0^{\;4} + S_1^{\;4}$ = .00019 so that the 'throwback' from 4th to 2nd difference could only amount to .00019 x .184 x .03187 = .0000011141752

CURTA TABLE No. 104

BESSEL INTERPOLATION COEFFICIENTS

n	B^{ii} -0.0	B^{iii} +0.0-	n	n	B^{ii} -0.0	B^{iii} +0.0-	n
0.00	0000	0000	1.00	0.25	4687	0781	0.75
.01	0247	0081	.99	6	4810	0770	.74
.02	0490	0157	.98	7	4927	0756	.73
.03	0727	0228	.97	8	5040	0739	.72
.04	0960	0294	.96	9	5147	0721	.71
0.05	1187	0356	0.95	0.30	5250	0700	0.70
.06	1410	0414	.94	1	5347	0677	.69
.07	1627	0467	.93	2	5440	o653	.68
.08	1840	0515	.92	3	5527	0626	.67
.09	2047	0560	.91	4	5610	0598	.66
0.10	2250	0600	0.90	0.35	5687	0569	0.65
.11	2447	0636	.89	6	5760	0538	.64
.12	2640	0669	.88	7	5827	0505	.63
.13	2827	0697	.87	8	5890	0471	.62
.14	3010	0722	.86	9	5947	0436	.61
0.15	3187	0744	0.85	0.40	6000	0400	0.60
.16	3360	0762	.84	1	6047	0363	.59
.17	3527	0776	.83	2	6090	0325	.58
.18	3690	0787	.82	3	6127	0286	.57
.19	3847	0795	.81	4	6160	0246	.56
0.20	4000	0800	0,80	0.45	6187	0206	0.55
.21	4147	0802	.79	6	6210	0166	.54
.22	4290	0801	.78	7	6227	0125	.53
.23	4427	0797	.77	8	6240	0083	.52
.24	4560	0790	.76	9	6247	0042	.51
0.25	4687	0781	0.75	0.50	6250	0000	

Examples.

n = 0.15 B^{ii} = -0.03187 B^{iii} = +0.00744

n = 0.55 B^{ii} = -0.06187 B^{iii} = -0.00206

T1. 1

COTTON CLOTH CALCULATIONS

Calculations of warp and weft weight in lbs. involve division by the 'Hank Length' and the 'Counts' of yarn and weft. This may be carried out on the Curta by the method of transfer multiplication given in **A 2** but, since Hank Length is a constant, 840, a little time will be saved by using a table of reciprocals of Hank Length x Counts - *Curta Table 20*.

(a) Warp weight in lbs. =

$$\frac{\text{No. of ends in width} \times \text{Tape Length in yards}}{\text{Hank Length} \times \text{Counts of yarn}}$$

Example (i)

No. of ends	21424
Tape Length	106 yards
Counts	38

From *Table 20* the reciprocal of 38 x 840 is .00003133, so all we have to do is to evaluate 21424 x 106 x .00003133 by successive multiplication (**A 1**), setting eight decimal places.

	SR	CR	PR	
C1	**/3133**	**/106**	**/332098**	**Clear CR**
C6	**/21423**	**/332098**	**/7114867552**	

Answer = 71.15 lbs. or 71 lbs. 2 oz.

(b) Weft weight in lbs. =

$$\frac{\text{Cloth Length in yds.} \times \text{P. P. inches} \times \text{width in Reed}}{\text{Hank Length} \times \text{Counts of weft}}$$

Example (ii)

Cloth Length	124 yds.
P.P. inches	64
Width in Reed	36
Counts of weft	34

From *Table 20* the reciprocal of 34 x 840 is .00003501
Evaluating 124 x 64 x 36 x 3501 and resetting eight decimal places, we get by successive multiplication 10.002

	SR	CR	PR	
C1	**/124**	**/64**	**/7936**	**Clear CR**
C4	**/35**	**/7936**	**/285696**	
C6	**/3500**	**/285696**	**/10.00221696**	

Answer = 10 lbs.

CURTA TABLE No. 20 RECIPROCALS of COUNTS of "HANK LENGTH"

Count	Reciprocal	Count	Reciprocal	Count	Reciprocal	Count	Reciprocal
4	29762	30	3968	62	1920	94	1266
5	23810	32	3720	64	1860	91	1240
6	19841	34	3501	66	1804	98	1215
7	17007	36	3307	68	1751	100	1190
8	14881	38	3133	70	1701	102	1167
9	13228	40	2976	72	1653	104	1145
10	11905	42	2834	74	1609	106	1123
12	9921	44	2706	76	1566	108	1102
14	8503	46	2588	78	1526	110	1082
16	7440	48	2480	80	1488	112	1063
18	6614	50	2381	82	1452	114	1044
20	5952	52	2289	84	1417	116	1026
22	5411	54	2205	86	1384	118	1009
24	4960	56	2126	88	1353	120	992
26	4579	58	2053	90	1323	122	976
28	4252	60	1984	92	1294	124	960

The figures given above are the last figures of eight decimal places