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MULTIPLE REGRESSION TECHNIQUE FOR Pth DEGREE POLYNOMIALS WITH AND WITHOUT LINEAR CROSS PRODUCTS

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16. ABSTRACT

A multiple regression technique has been developed by which the nonlinear behavior of specified independent variables can be related to a given dependent variable. The polynomial expression can be of Pth degree and can incorporate N independent variables. Two cases are treated such that mathematical models can be studied both with and without linear cross products. The resulting surface fits can be used to summarize trends for a given phenomenon and provide a mathematical relationship for subsequent analysis.

To implement this technique, separate computer programs have been developed for the case without linear cross products and for the case incorporating such cross products which evaluate the various constants in the model regression equation. In addition, the significance of the estimated regression equation is considered and the standard deviation, the F statistic, the maximum absolute percent error, and the average of the absolute values of the percent of error evaluated.

The computer programs and their manner of utilization are described. Sample problems are included to illustrate the use and capability of the technique which show the output formats and typical plots comparing computer results to each set of input data.

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DEFINITION OF SYMBOLS

Symbol	Definition
A_0	model equation intercept coefficient
B_N	matrix quantity
B_P	model equation coefficients
b_0	defined quantity
b_{NP}	regression model coefficients
$c_{N-1,N}$	regression model cross product coefficients
F	F statistic
K_N	cross product model equation terms
k	cross product technique parameter pertaining to degrees of freedom
N	number of independent variables
n	number of data points
P	degree of equation
R	multiple correlation coefficient
S_{NN}	matrix quantity
S_{NY}	matrix quantity
S_{YY}	total sum of squares
S(Reg)	regression sum of squares
S(Res)	residual sum of squares
s	variance
Y	value of dependent variable

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
y	difference in observed and mean value of dependent variable
Z_N	value of Nth independent variable
z_{NP}	difference in observed and mean values to the Pth power for the Nth independent variable
$\lvert\epsilon\rvert_{AVG}$	average absolute percent error
$\sum_{i=1}$	$\sum_{i=1}^{n}$, summation from the first to the nth term

Superscript

c	calculated value
—	mean value

Subscript

i	$i = 1, 2, \ldots n$
j	$j = 1, 2, \ldots N$

MULTIPLE REGRESSION TECHNIQUE FOR Pth DEGREE POLYNOMIALS WITH AND WITHOUT LINEAR CROSS PRODUCTS

I. INTRODUCTION

The investigation of physical processes and requirements for data analysis methods frequently requires the use of mathematical models which describe the processes. The model can be formulated such that certain variables interact according to physical theories associated with the particular process, or it may contain identified independent variables and unknown parameters. The relationship of the parameters identified in the model can be evaluated using the statistical tool commonly referred to as regression analysis. In principle, it should be possible to establish complex curves or surfaces for higher order multiple variable functions by regression techniques to summarize trends in data and to provide a means of predicting similar phenomena. Furthermore, such a technique might be used to establish unknown laws or relationships.

Most statistical textbooks treat the problem of linear multiple variable regression and of nonlinear regression of one independent variable. However, the study of many complex physical problems requires a method capable of determining nonlinear regression of multiple independent variables. By this means, an analytical representation of the observed data is provided which can be used in subsequent analysis.

The general procedure in regression analysis is to take partial derivatives of a specific model-dependent minimizing function. The set of equations obtained by setting these partial derivatives equal to zero is frequently referred to as the normal equations. If the normal equations are not transcendental in any of the unknown parameters, they can be solved by the usual algebraic methods. It is this situation which is of concern here.

Within this report a method is presented for evaluating multiple variable regression for Pth degree polynomials with and without linear cross products. Implementation of the technique, including error diagnostics has been accomplished on the UNIVAC 1108 computer. Program listings and illustrative example problems are given in the appendices.

The method developed is used in problems requiring the determination of mathematical relationships describing complex curves or surfaces from known physical data. Initial applications of this technique have concerned the external aerodynamics of aircraft and space vehicles and the internal aerodynamics of transonic wind tunnels and have produced excellent results. Applications in almost any other field of study should be expected to be equally as useful.

II. MULTIPLE REGRESSION TECHNIQUE WITHOUT CROSS PRODUCTS

A. Introduction

The derivation of a matrix solution for evaluating multiple variable regression for Pth degree polynomials without cross products is presented in Reference 1. In this section this solution is discussed for the general case where the polynomial expression can be of Pth degree with N independent variables. The mathematical procedures represent a rigorous least squares evolution of input data centered about the mean.

B. Regression Analysis Development

Assuming the observed dependent variable is to be estimated by the following model:

$$Y_i^c = A_0 + (b_{11} Z_{1i} + b_{12} Z_{1i}^2 + \ldots + b_{1P} Z_{1i}^P)$$

$$+ (b_{21} Z_{2i} + b_{22} Z_{2i}^2 + \ldots + b_{2P} Z_{2i}^P)$$

$$+ \ldots + (b_{N1} Z_{Ni} + b_{N2} Z_{Ni}^2 + \ldots + b_{NP} Z_{Ni}^P) \quad (1)$$

where $i = 1, 2, \ldots,$ nth set of data, and the intercept $A_0 = b_{10} + b_{20} + \ldots + b_{N0}$.

For any given independent variable Z_N , the mean value is given by

$$\bar{Z}_N = \frac{\sum Z_{Ni}}{n} \quad (2)$$

If the input data are centered about the mean the model equation (1) becomes

$$Y_i^c = b_0 + (b_{11} z_{11i} + b_{12} z_{12i} + \ldots + b_{1P} z_{1Pi})$$

$$+ (b_{21} z_{21i} + b_{22} z_{22i} + \ldots + b_{2P} z_{2Pi}) \quad (3)$$

$$+ \ldots + (b_{N1} z_{N1i} + b_{N2} z_{N2i} + \ldots + b_{NP} z_{NPi}) \tag{3}$$

(Concluded)

Where, for convenience, the following quantities, which are indicated in equation (3), are defined by

$$\begin{aligned} b_0 = {} & (b_{10} + b_{11} \bar{Z}_1 + b_{12} \bar{Z}_1^2 + \ldots + b_{1P} \bar{Z}_1^P) \\ & + (b_{20} + b_{21} \bar{Z}_2 + b_{22} \bar{Z}_2^2 + \ldots + b_{2P} \bar{Z}_2^P) \\ & + \ldots + (b_{N0} + b_{N1} \bar{Z}_N + b_{N2} \bar{Z}_N^2 + \ldots + b_{NP} \bar{Z}_N^P) \end{aligned} \tag{4}$$

$$\left.\begin{aligned} & \left.\begin{aligned} z_{11i} &= Z_{1i} - \bar{Z}_1 \\ z_{12i} &= Z_{1i}^2 - \bar{Z}_1^2 \\ &\vdots \\ z_{1Pi} &= Z_{1i}^P - \bar{Z}_1^P \end{aligned}\right\} \\ & \qquad \vdots \\ & \left.\begin{aligned} z_{21i} &= Z_{2i} - \bar{Z}_2 \\ z_{22i} &= Z_{2i}^2 - \bar{Z}_2^2 \\ &\vdots \\ z_{2Pi} &= Z_{2i}^P - \bar{Z}_2^P \end{aligned}\right\} \\ & \qquad \vdots \\ & \left.\begin{aligned} z_{N1i} &= Z_{Ni} - \bar{Z}_N \\ z_{N2i} &= Z_{Ni}^2 - \bar{Z}_N^2 \\ &\vdots \\ z_{NPi} &= Z_{Ni}^P - \bar{Z}_N^P \end{aligned}\right\} \end{aligned}\right\} \tag{5}$$

The classical form of the least squares minimizing function is

$$M = \sum (Y_i - Y_i^c)^2 \tag{6}$$

This result follows from the least squares principle that the best representation of the input data is that which makes the sum of the squares of the residuals a minimum. The condition which fulfills this requirement is that the partial derivatives of this function with respect to each of the unknowns be zero. Hence, the following normal equations for i = 1, 2, ... n are written:

$$\left.\begin{array}{l} \frac{\partial M}{\partial b_0} = 0 \\ \\ \frac{\partial M}{\partial b_{11}} = 0 \\ \\ \frac{\partial M}{\partial_{12}} = 0 \\ \vdots \\ \frac{\partial M}{\partial b_{1P}} = 0 \end{array}\right\} \quad \left.\begin{array}{l} \frac{\partial M}{\partial b_{21}} = 0 \\ \\ \frac{\partial M}{\partial b_{22}} = 0 \\ \vdots \\ \frac{\partial M}{\partial b_{2P}} = 0 \end{array}\right\} \quad \cdots \quad \left.\begin{array}{l} \frac{\partial M}{\partial b_{N1}} = 0 \\ \\ \frac{\partial M}{\partial b_{N2}} = 0 \\ \vdots \\ \frac{\partial M}{\partial b_{NP}} = 0 \end{array}\right\} \tag{7}$$

Through the use of the normal equation $\frac{\partial M}{\partial b_0} = 0$ it can be shown that

$$b_0 = \bar{Y} = \frac{\sum Y_i}{m} \tag{8}$$

The following quantity is now defined for the difference in the observed value of Y_i and the mean value of the observed values:

$$y_i = Y_i - \bar{Y} \tag{9}$$

Then, through the use of the remaining normal equations, the following matrix solution for the unknown constant of the independent variables is obtained.

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \cdots & S_{1N} \\ S_{21} & S_{22} \cdots & S_{2N} \\ \vdots & \vdots & \vdots \\ S_{N1} & S_{N2} & S_{NN} \end{bmatrix}^{-1} \begin{bmatrix} S_{1y} \\ S_{2y} \\ \vdots \\ S_{ny} \end{bmatrix} \tag{10}$$

$$\left[[NPx1]\right] \qquad [NPxNP] \qquad [NPx1]$$

where the following matrix quantities are given by:

$$\left.\begin{aligned} B_1 &= \begin{bmatrix} b_{11} \\ b_{12} \\ \vdots \\ b_{1P} \end{bmatrix} \\ B_2 &= \begin{bmatrix} b_{21} \\ b_{22} \\ \vdots \\ b_{2P} \end{bmatrix} \\ &\ \ \vdots \\ B_N &= \begin{bmatrix} b_{N1} \\ b_{N2} \\ \vdots \\ b_{NP} \end{bmatrix} \end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned}
S_{1j} &= \begin{bmatrix} \sum z_{11i}\, z_{j1i} & \sum z_{11i}\, z_{j2i} & \cdots & \sum z_{11i}\, z_{jPi} \\ \sum z_{12i}\, z_{j1i} & \sum z_{12i}\, z_{j2i} & \cdots & \sum z_{12i}\, z_{jPi} \\ \vdots & \vdots & & \vdots \\ \sum z_{1Pi}\, z_{j1i} & \sum z_{1Pi}\, z_{j2i} & \cdots & \sum z_{1Pi}\, z_{jPi} \end{bmatrix} \text{ for } j = 1, 2, \ldots, N \\
S_{2j} &= \begin{bmatrix} \sum z_{21i}\, z_{j1i} & \sum z_{21i}\, z_{j2i} & \cdots & \sum z_{21i}\, z_{jPi} \\ \sum z_{22i}\, z_{j1i} & \sum z_{22i}\, z_{j2i} & \cdots & \sum z_{22i}\, z_{jPi} \\ \vdots & \vdots & & \vdots \\ \sum z_{2Pi}\, z_{j1i} & \sum z_{2Pi}\, z_{j2i} & \cdots & \sum z_{2Pi}\, z_{jPi} \end{bmatrix} \text{ for } j = 1, 2, \ldots, N \\
&\vdots \\
S_{Nj} &= \begin{bmatrix} \sum z_{N1i}\, z_{j1i} & \sum z_{N1i}\, z_{j2i} & \cdots & \sum z_{N1i}\, z_{jPi} \\ \sum z_{N2i}\, z_{j1i} & \sum z_{N2i}\, z_{j2i} & \cdots & \sum z_{N2i}\, z_{jPi} \\ \vdots & \vdots & & \vdots \\ \sum z_{NPi}\, z_{j1i} & \sum z_{NPi}\, z_{j2i} & \cdots & \sum z_{NPi}\, z_{jPi} \end{bmatrix} \text{ for } j = 1, 2, \ldots, N
\end{aligned}\right\} \tag{12}$$

$$
\left.
\begin{aligned}
S_{1y} &= \begin{bmatrix} \sum z_{11i}\, y_i \\ \sum z_{12i}\, y_i \\ \vdots \\ \sum z_{1Pi}\, y_i \end{bmatrix} \\
S_{2y} &= \begin{bmatrix} \sum z_{21i}\, y_i \\ \sum z_{22i}\, y_i \\ \vdots \\ \sum z_{2Pi}\, y_i \end{bmatrix} \\
&\ \vdots \\
S_{Ny} &= \begin{bmatrix} \sum z_{N1i}\, y_i \\ \sum z_{N2i}\, y_i \\ \vdots \\ \sum z_{NPi}\, y_i \end{bmatrix}
\end{aligned}
\right\} \tag{13}
$$

Having determined the matrix solution indicated by equation (10), the intercept A_o of the fitted model expression may be determined as:

$$
\begin{aligned}
A_0 = \overline{Y} - (b_{11}\overline{Z}_1 + b_{12}\overline{Z}_1^{\,2} + \ldots + b_{1P}\overline{Z}_1^{\,P}) - (b_{21}\overline{Z}_2 + b_{22}\overline{Z}_2^{\,2} + \ldots \\
+ b_2\overline{Z}_2^{\,P}) - \ldots - (b_{N1}\overline{Z}_N + b_{N2}\overline{Z}_N^{\,2} + \ldots + b_{NP}\overline{Z}_N^{\,P})
\end{aligned} \tag{14}
$$

C. Discussion

As noted by Graybill [2], there are an infinite number of solutions to the model relationship. However, only one solution must be found to have a useful result. Also note that the solution presented to the higher order multiple regression problem is a function of parameters such as the sum of the squares, cross products, and linear values similar to previously developed solutions for linear regression. In this case, one finds a more complex result with nested matrices, which becomes practical only in combination with digital computer techniques.

As discussed in Appendix A, a computer program has been developed to evaluate the unknown constants in the model equation with the solution specified by equation (10), as well as certain diagnostics reflecting the significance of the estimated regression equation which will be considered in Section IV.

It is required in the application of this technique that the S matrix be nonsingular and that the input data are reasonably well behaved. It is also required that $n \geqq NP$. That is, the number of data points n must be equal to or greater than the number of unknown parameters in the model equation.

Many sets of physical data have been studied using the previously mentioned computer program. Results have generally been excellent and it is felt that the technique can be highly useful in many fields of endeavor.

III. MULTIPLE REGRESSION TECHNIQUE WITH CROSS PRODUCTS

A. Introduction

The regression technique developed in Section II for Pth degree polynomials with N independent variables has proven to be extremely useful in correlating experimental wind tunnel data and aiding subsequent analysis. However, in certain applications the inclusion of cross products for the various independent variables can enhance the value of the technique. Such a case occurs when it is desired to optimize the dependent variable of the fitted mathematical expression with respect to the various independent variables. In such a case the determination of maximum and minimum values by solving for the critical points through the use of the necessary and sufficient conditions (that the first and second partial derivation of the response variable with respect to each of the independent variables be identically equal to zero) can only be determined when cross products are included in the model equation; otherwise the effect of all other independent variables is lost when the partial derivative with respect to a given independent variable is evaluated.

To deal with this and other similar problems, a method is developed in this section which treats all combinations of linear cross products for each of the N independent variables as well as the polynomial terms previously discussed in Section II. Higher order cross product terms could, in principle, be treated, although such a development is beyond the scope of the present work. The method for fitting polynomials with linear cross products is presented without formal derivation. However, the development is similar to that shown in Reference 1.

B. Regression Analysis Development with Linear Cross Products

Consider the following model:

$$
\begin{aligned}
Y_i^c = {} & A_0 + (b_{11} Z_{1i} + b_{12} Z_{1i}^2 + \ldots + b_{1P} Z_{1i}^P) \\
& + (b_{21} Z_{2i} + b_{22} Z_{2i}^2 + \ldots + b_{2P} Z_{2i}^P) \\
& + \ldots + (b_{N1} Z_{Ni} + b_{N2} Z_{Ni}^2 + \ldots + b_{NP} Z_{Ni}^P) \\
& + (C_{12} Z_{1i} Z_{2i} + c_{13} Z_{1i} Z_{3i} + \ldots + C_{1N} Z_{1i} Z_{Ni}) \\
& + (C_{23} Z_{21} Z_{3i} + C_{24} Z_{2i} Z_{4i} + \ldots + C_{2N} Z_{2i} Z_{Ni}) \\
& + \ldots + (C_{N-1,N} Z_{(N-1)i} Z_{Ni})
\end{aligned}
\qquad (15)
$$

where $i = 1, 2, \ldots$, nth set of data, and the intercept $A_0 = b_{10} + b_{20} + \ldots + b_{n0}$. This model represents a polynomial expression of Pth degree with N independent variables and contains all combinations of linear cross products for each of the N independent variables.

If equal quantities are added and subtracted in equation (15), namely, those of the form $b_{NP}\overline{Z}_N^P$ and $C_{ji}\overline{Z_j Z_i}$, (where $j = 1, 2, \ldots, N\text{-}1$) it can be shown that the model equation is equivalent to:

$$Y_i^c = K_{1i} + B_{1i} + K_{2i} + B_{2i} + \ldots + K_{Ni} + B_{Ni} \tag{16}$$

where

$$\left.\begin{aligned}
K_{1i} &= b_{10} + b_{11}\bar{Z}_1 + b_{12}\bar{Z}_1^2 + \ldots + b_{1P}\bar{Z}_1^P + C_{12}\overline{Z_1Z_2} + C_{13}\overline{Z_1Z_3} + \ldots + C_{1N}\overline{Z_1Z_N} \\
K_{2i} &= b_{20} + b_{21}\bar{Z}_2 + b_{22}\bar{Z}_2^2 + \ldots + b_{2P}\bar{Z}_2^P + C_{23}\overline{Z_2Z_3} + C_{24}\overline{Z_2Z_4} + \ldots + C_{2N}\overline{Z_2Z_N} \\
&\vdots \\
K_{Ni} &= b_{N0} + b_{N1}\bar{Z}_N + b_{N2}\bar{Z}_N^2 + \ldots + b_{NP}\bar{Z}_N^P + C_{N-1,N}\overline{Z_{N-1}Z_N}
\end{aligned}\right\} \tag{17}$$

$$\left.\begin{aligned}
B_{1i} &= b_{11}\, z_{11i} + b_{12}\, z_{12i} + \ldots + b_{1P}\, z_{1Pi} + C_{12}\, z^+_{12i} + C_{13}\, z^+_{13i} + \ldots + C_{1N}\, z^+_{1Ni} \\
B_{2i} &= b_{21}\, z_{21i} + b_{22}\, z_{22i} + \ldots + b_{2P}\, z_{2Pi} + C_{23}\, z^+_{23i} + C_{24}\, z^+_{24i} + \ldots + C_{2N}\, z^+_{2Ni} \\
&\vdots \\
B_{Ni} &= b_{N1}\, z_{N1i} + b_{N2}\, z_{N2i} + \ldots + b_{NP}\, z_{NP} + C_{N-1,N}\, z^+_{N-1,N_i}
\end{aligned}\right\} \tag{18}$$

The jth mean value terms in equation (18) are given by

$$\bar{Z}_j = \sum \frac{Z_{ji}}{n} \tag{19}$$

$$\overline{Z_jZ_N} = \sum \frac{Z_{ji}\, Z_{Ni}}{n} \tag{20}$$

and for convenience z and z^+ terms in equation (18) have been defined as:

$$
\left.
\begin{array}{l}
\left.
\begin{aligned}
z_{11i} &= Z_{1i} - \overline{Z}_1 \\
z_{12i} &= Z_{1i}^2 - \overline{Z}_1^2 \\
&\;\vdots \\
z_{1Pi} &= Z_{1i}{}^P - Z_1{}^P
\end{aligned}
\right\} \\[2ex]
\left.
\begin{aligned}
z_{21i} &= Z_{2i} - \overline{Z}_2 \\
z_{22i} &= Z_{2i}^2 - \overline{Z}_2^2 \\
&\;\vdots \\
z_{2Pi} &= Z_{2i}{}^P - Z_2{}^P
\end{aligned}
\right\} \\[2ex]
\left.
\begin{aligned}
z_{N1i} &= Z_{Ni} - Z_N \\
z_{N2i} &= Z_{Ni}^2 - Z_N^2 \\
&\;\vdots \\
z_{NPi} &= Z_{Ni}{}^P - Z_N{}^P
\end{aligned}
\right\}
\end{array}
\right\}
\qquad (21)
$$

$$
\left.
\begin{aligned}
z_{12i}^{+} &= Z_{1i} Z_{2i} - \overline{Z_1 Z_2} \\
z_{13i}^{+} &= Z_{1i} Z_{3i} - \overline{Z_1 Z_3} \\
&\;\vdots \\
z_{1Ni}^{+} &= Z_{1i} Z_{Ni} - \overline{Z_1 Z_N}
\end{aligned}
\right\}
\qquad (22)
$$

$$
\left.\begin{array}{l}
\left.\begin{array}{l}
z^{+}_{23i} = Z_{2i}\,Z_{3i} - \overline{Z_2 Z_3} \\
z^{+}_{24i} = Z_{2i}\,Z_{4i} - \overline{Z_2 Z_4} \\
\vdots \\
z^{+}_{2Ni} = Z_{2i}\,Z_{Ni} - \overline{Z_2 Z_N}
\end{array}\right\} \\
\vdots \\
z^{+}_{N-1,N_i} = Z_{N-1,i}\,Z_{Ni} - \overline{Z_{N-1} Z_N} \quad \}
\end{array}\right\}
\qquad (22)\ \text{(Concluded)}
$$

Regrouping equation (2) the following is obtained

$$Y_i^c = b_0 + B_{1i} + B_{2i} + \ldots B_{Ni} \qquad (23)$$

where

$$b_0 = K_{1i} + K_{2i} + \ldots + K_{Ni} \qquad (24)$$

Through the use of the normal equations resulting from the least squares minimizing function it is found that

$$b_0 = \overline{Y} = \frac{\sum Y_i}{n} \qquad (25)$$

The difference in the observed value and the mean value of the observed values of the dependent variable is again denoted by:

$$y_i = Y_i - \overline{Y} \qquad (26)$$

Then the following matrix solution for the unknown constants of the model equation can be obtained through the use of the normal equations:

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix}^{-1} \begin{bmatrix} S_{1Y} \\ S_{2Y} \\ \vdots \\ S_{NY} \end{bmatrix} \quad (27)$$

$$[M\text{x}1] \qquad [M\text{x}M] \qquad [M\text{x}1]$$

where

$$M = NP + N!/2(N-2)! \quad (28)$$

and the various matrix quantities are given by:

$$\left.\begin{aligned} B_1 &= \begin{bmatrix} b_{11} \\ \vdots \\ b_{1P} \\ \\ c_{12} \\ \vdots \\ c_{1N} \end{bmatrix} \\ B_2 &= \begin{bmatrix} b_{21} \\ \vdots \\ b_{2P} \\ \\ c_{23} \\ \vdots \\ c_{2N} \end{bmatrix} \end{aligned}\right\} \quad (29)$$

$$B_N = \begin{bmatrix} b_{N1} \\ \vdots \\ b_{NP} \\ \\ c_{N-1,N} \end{bmatrix}$$

(29) (Concluded)

$$S_{11} = \begin{bmatrix} \sum z_{11i} z_{11i} & \cdots & \sum z_{11i} z_{1Pi} & \sum z_{11i} z^{+}_{12i} & \cdots & \sum z_{11i} z^{+}_{1Ni} \\ \vdots \\ \sum z_{1Pi} z_{11i} & \cdots & \sum z_{1Pi} z_{1Pi} & \sum z_{1Pi} z^{+}_{12i} & \cdots & \sum z_{1Pi} z^{+}_{1Ni} \\ \sum z^{+}_{12i} z^{+}_{11i} & \cdots & \sum z^{+}_{12i} z^{+}_{1Pi} & \sum z^{+}_{12i} z^{+}_{12i} & \cdots & \sum z^{+}_{12i} z^{+}_{1Ni} \\ \vdots \\ \sum z^{+}_{1Ni} z^{+}_{11i} & \cdots & \sum z^{+}_{1Ni} z^{+}_{1Pi} & \sum z^{+}_{1Ni} z^{+}_{12i} & \cdots & \sum z^{+}_{1Ni} z^{+}_{1Ni} \end{bmatrix}$$

$$S_{21} = \begin{bmatrix} \sum z_{21i} z_{11i} & \cdots & \sum z_{21i} z_{1Pi} & \sum z^{+}_{21i} z^{+}_{12i} & \cdots & \sum z_{21i} z^{+}_{1Ni} \\ \vdots \\ \sum z_{2Pi} z_{11i} & \cdots & \sum z_{2Pi} z_{1Pi} & \sum z_{2Pi} z^{+}_{12i} & \cdots & \sum z_{2Pi} z^{+}_{1Ni} \\ \sum z^{+}_{23i} z^{+}_{11i} & \cdots & \sum z^{+}_{23i} z^{+}_{1Pi} & \sum z^{+}_{23i} z^{+}_{12i} & \cdots & \sum z^{+}_{23i} z^{+}_{1Ni} \\ \vdots \\ \sum z^{+}_{2Ni} z^{+}_{11i} & \cdots & \sum z^{+}_{2Ni} z^{+}_{1Pi} & \sum z^{+}_{2Ni} z^{+}_{12i} & \cdots & \sum z^{+}_{2Ni} z^{+}_{1Ni} \end{bmatrix} \qquad (30)$$

$$\vdots$$

$$S_{N1} = \begin{bmatrix} \sum z_{N1i} z_{11i} & \cdots & \sum z_{N1i} z_{1Pi} & \sum z_{N1i} z^{+}_{12i} & \cdots & \sum z_{N1i} z^{+}_{1Ni} \\ \vdots \\ \sum z_{NPi} z_{11i} & \cdots & \sum z_{NPi} z_{1Pi} & \sum z_{NPi} z^{+}_{12i} & \cdots & \sum z_{NPi} z^{+}_{1Ni} \\ \sum z^{+}_{N-1,Ni} z_{11i} & \cdots & \sum z^{+}_{N-1,Ni} z_{1Pi} & \sum z^{+}_{N-1,Ni} z^{+}_{12i} & \cdots & \sum z_{N-1,Ni} z^{+}_{1Ni} \end{bmatrix}$$

$$
\left.
\begin{array}{l}
S_{12} = \begin{bmatrix}
\sum z_{11i} z_{21i} & \cdots & \sum z_{11i} z_{2Pi} & \sum z_{11i} z^{+}_{23i} & \cdots & \sum z_{11i} z^{+}_{2Ni} \\
\vdots & & & & & \\
\sum z_{1Pi} z_{21i} & \cdots & \sum z_{1Pi} z_{2Pi} & \sum z_{1Pi} z^{+}_{23i} & \cdots & \sum z_{1Pi} z^{+}_{2Ni} \\
\vdots & & & & & \\
\sum z^{+}_{12i} z_{21i} & \cdots & \sum z^{+}_{12i} z_{1Pi} & \sum z^{+}_{12i} z^{+}_{23i} & \cdots & \sum z^{+}_{12i} z^{+}_{2Ni} \\
\vdots & & & & & \\
\sum z^{+}_{1Ni} z_{21i} & \cdots & \sum z^{+}_{1Ni} z_{2Pi} & \sum z^{+}_{1Ni} z^{+}_{23i} & \cdots & \sum z^{+}_{1Ni} z^{+}_{2Ni}
\end{bmatrix} \cdots \\
\\
S_{22} = \begin{bmatrix}
\sum z_{21i} z_{21i} & \cdots & \sum z_{21i} z_{2Pi} & \sum z_{21i} z^{+}_{23i} & \cdots & \sum z_{21i} z^{+}_{2Ni} \\
\vdots & & & & & \\
\sum z_{2Pi} z_{21i} & \cdots & \sum z_{2Pi} z_{2Pi} & \sum z_{2Pi} z^{+}_{23i} & \cdots & \sum z_{2Pi} z^{+}_{2Ni} \\
\vdots & & & & & \\
\sum z^{+}_{23i} z_{21i} & \cdots & \sum z^{+}_{23i} z_{2Pi} & \sum z^{+}_{23i} z^{+}_{23i} & \cdots & \sum z^{+}_{23i} z^{+}_{2Ni} \\
\vdots & & & & & \\
\sum z^{+}_{2Ni} z_{21i} & \cdots & \sum z^{+}_{2Ni} z_{2Pi} & \sum z^{+}_{2Ni} z^{+}_{23i} & \cdots & \sum z^{+}_{2Ni} z^{+}_{2Ni}
\end{bmatrix} \cdots \\
\vdots \\
S_{N2} = \begin{bmatrix}
\sum z_{N1i} z_{21i} & \cdots & \sum z_{N1i} z_{2Pi} & \sum z_{N1i} z^{+}_{23i} & \cdots & \sum z_{N1i} z^{+}_{2Ni} \\
\vdots & & & & & \\
\sum z_{NPi} z_{21i} & \cdots & \sum z_{NPi} z_{2Pi} & \sum z_{NPi} z^{+}_{23i} & \cdots & \sum z_{NPi} z^{+}_{2Ni} \\
& & & & & \\
\sum z^{+}_{N-1,Ni} z_{21i} & \cdots & \sum z^{+}_{N-1,Ni} z_{2Pi} & \sum z^{+}_{N-1,Ni} z^{+}_{23i} & \cdots & \sum z^{+}_{N-1,Ni} z^{+}_{2Ni}
\end{bmatrix} \cdots
\end{array}
\right\} \quad \begin{array}{c} (30) \\ \text{(Continued)} \end{array}
$$

$$
\left.
\begin{aligned}
S_{1N} &= \begin{bmatrix}
\sum z_{11i} z_{N1i} & \cdots & \sum z_{11i} z_{NPi} & \sum z_{11i} z^{+}_{N-1,Ni} \\
\vdots & & & \\
\sum z_{1Pi} z_{N1i} & \cdots & \sum z_{1Pi} z_{NPi} & \sum z_{1Pi} z^{+}_{N-1,Ni} \\
\sum z^{+}_{12i} z_{N1i} & \cdots & \sum z^{+}_{12i} z_{NPi} & \sum z^{+}_{12i} z^{+}_{N-1,Ni} \\
\vdots & & & \\
\sum z^{+}_{1Ni} z_{N1i} & \cdots & \sum z^{+}_{1Ni} z_{NPi} & \sum z^{+}_{1Ni} z^{+}_{N-1,Ni}
\end{bmatrix} \\
S_{2N} &= \begin{bmatrix}
\sum z_{21} z_{N1i} & \cdots & \sum z_{21i} z_{NPi} & \sum z_{21i} z^{+}_{N-1,Ni} \\
\vdots & & & \\
\sum z_{2Pi} z_{N1i} & \cdots & \sum z_{2Pi} z_{NPi} & \sum z_{2Pi} z^{+}_{N-1,Ni} \\
\sum z^{+}_{23i} z_{N1i} & \cdots & \sum z^{+}_{23i} z_{NPi} & \sum z^{+}_{23i} z^{+}_{N-1,Ni} \\
\vdots & & & \\
\sum z^{+}_{2Ni} z_{N1i} & \cdots & \sum z^{+}_{2Ni} z_{NPi} & \sum z^{+}_{2Ni} z^{+}_{N-1,Ni}
\end{bmatrix} \\
&\vdots \\
S_{NN} &= \begin{bmatrix}
\sum z_{N1i} z_{N1i} & \cdots & \sum z_{N1i} z_{NPi} & \sum z_{N1i} z^{+}_{N-1,Ni} \\
\vdots & & & \\
\sum z_{NPi} z_{N1i} & \cdots & \sum z_{NPi} z_{NPi} & \sum z_{NPi} z^{+}_{N-1,Ni} \\
\sum z^{+}_{N-1,Ni} z_{N1i} & \cdots & \sum z^{+}_{N-1,Ni} z_{NPi} & \sum z^{+}_{N-1,Ni} z^{+}_{N-1,Ni}
\end{bmatrix}
\end{aligned}
\right\}
\qquad
\begin{matrix} (30) \\ \text{(Concluded)} \end{matrix}
$$

$$
\left.\begin{aligned}
S_{1y} &= \begin{bmatrix} \sum z_{11i}\, y_i \\ \vdots \\ \sum z_{1Pi}\, y_i \\ \\ \sum z_{12}^{+}\, y_i \\ \vdots \\ \sum z_{1Ni}^{+}\, y_i \end{bmatrix} \\
S_{2y} &= \begin{bmatrix} \sum z_{21i}\, y_i \\ \vdots \\ \sum z_{2Pi}\, y_i \\ \\ \sum z_{23i}^{+}\, y_i \\ \vdots \\ \sum z_{2Ni}^{+}\, y_i \end{bmatrix} \\
&\ \vdots \\
S_{Ny} &= \begin{bmatrix} \sum z_{N1i}\, y_i \\ \vdots \\ \sum z_{NPi}\, y_i \\ \\ \sum z_{N-1,Ni}^{+}\, y_i \end{bmatrix}
\end{aligned}\right\} \tag{31}
$$

Once the matrix solution given by equation (27) has been determined the intercept A_0 of the fitted model expression may be calculated as

$$A_0 = \bar{Y} - (b_{11}\bar{Z}_1 + b_{12}\bar{Z}_1^2 + \ldots + b_{1P}\bar{Z}_1^{\;P} + c_{12}\overline{Z_1Z_2} + c_{13}\overline{Z_1Z_3} + \ldots + c_{1N}\overline{Z_1Z_N})$$

$$- (b_{21}\bar{Z}_2 + b_{22}\bar{Z}_2^2 + \ldots + b_{2P}\bar{Z}_2^{\;P} + c_{23}\overline{Z_2Z_3} + c_{24}\overline{Z_2Z_4} + \ldots + c_{2N}\overline{Z_2Z_n})$$

$$- \ldots - (b_{N1} + b_{N2}\bar{Z}_N^2 + \ldots + b_{NP}\bar{Z}_N^{\;P} + c_{N-1,N}\overline{Z_{N-1}Z_N}) \tag{32}$$

C. Comments

The solution developed for evaluating multiple variable regression for Pth degree polynomials with linear cross products is similar to that developed in Section II for the case without cross products. However, in this case, the nested matrices contain additional terms to account for the cross products and hence it is required that $n \geqq NP + \frac{N!}{2(N-2)!}$, which again indicates that the number of data points n must be equal to or greater than the number of unknown parameters in the model equation.

As discussed in Appendix B, a computer program has been developed to treat this case. It should be noted that while contrived problems have yielded excellent results, experience with several sets of physical data have generally yielded poorer results. This is evidently due to the larger matrices being manipulated and due to unknowns as to whether true linear cross coupling exists in the given physical process.

IV. SIGNIFICANCE OF THE ESTIMATED REGRESSION EQUATIONS

A. Introduction

To determine if the fitted regression equation obtained from the solution matrix is indeed a useful representation of the input data, it is desirable that certain diagnostics be evaluated. To this end the computer programs discussed in Appendix A and Appendix B determine the standard deviation of the observed data with respect to the fitted equation, the multiple correlation coefficient, and the F statistic as well as the average error and the maximum error of the observed data with respect to the fitted result.

Using these parameters, it is possible to assess the usefulness of the fitted expression for each given application.

B. Analysis of Variance

The significance of the estimated regression equation can be considered from the viewpoint of an analysis of variance as summarized in Table 1, where the total sum of squares is resolved into a component measuring the residual fitting error, and a component which measures the regression variation being tested.

TABLE 1. ANALYSIS OF VARIANCE (WITH AND WITHOUT CROSS PRODUCTS)

A. Analysis of Variance (without Cross Products)

Degrees of Freedom	Type Variation	Sum of Squares (SS)	Mean Square (MS)	F Value
n-1	Total	$S_{yy} = \sum(Y_i - Y)^2$		
NP-1	Residual	$S(RES) = \sum(Y_i - Y_i^c)^2$	$M(RES) = \dfrac{S(RES)}{NP-1}$	
n-NP	Regression	$S(REG) = \sum(Y_i^c - \bar{Y})^2$	$M(REG) = \dfrac{S(REG)}{n-NP}$	$\dfrac{M(REG)}{M(RES)}$

B. Analysis of Variance (with Cross Products)

Degrees of Freedom	Type Variation	Sum of Squares (SS)	Mean Square (MS)	F Value
	Total	$S_{yy} = \sum(Y_i - Y)^2$		
K-1	Residual	$S(RES) = \sum(Y_i - Y_i^c)^2$	$M(RES) = \dfrac{S(REG)}{k-1}$	
n-k	Regression	$S(REG) = \sum(Y_i^c - Y)^2$	$M(REG) = \dfrac{S(RES)}{n-k}$	$\dfrac{M(REG)}{M(RES)}$

where MS = SS/degrees of freedom

$S_{yy} = S(RES) + S(REG)$

$\bar{Y}$ = average of observed values

$k = NP + (N-1) + (N-2) + \ldots + 1$

C. Interpretation Diagnostics

As noted by Smille [3], the results of such an analysis of variance can be used to test the combined effect of all of the independent variables on the dependent variable. That is, the hypothesis that all of the population regression coefficients in the model regression equation are zero can be tested since the ratios of the regression mean square to the residual mean square are distributed in an F distribution as shown below:

$$F = \frac{S(REG)/\text{Regression degrees of freedom}}{S(RES)/\text{Residual degrees of freedom}} \tag{33}$$

where it is assumed that the observations are selected at random- from a normally distributed population with zero mean and constant variance, that $Z_1, Z_2, \ldots Z_N$ are independent variables following χ^2 distributions, and that only random errors are associated with the observations.

The F ratio calculated from equation (33) can be used to test the statistical significance of the regression equation under consideration by comparing it with the appropriate F_{TABLE} value at the desired probability level with the specified numerator and denominator degrees of freedom; that is, the following test of the null hypothesis may be performed:

$$H_0 : b_{10} = b_{12} = \ldots b_{NP} = 0$$

$$\text{accept when} \quad F_c > F_{TABLE} \tag{34}$$

$$\text{reject when} \quad F_c < F_{TABLE}$$

D. Output Data

The desired coefficients of the model equation are outputted in E notation where A_0 is the computed intercept of the fitted polynomial expression and the B coefficients are printed out in ascending order of degree with the first P coefficients indicating b coefficients of the first independent variable, the second set of P coefficients indicating the b coefficient of the second independent variable, etc.

Also included in the output data are the number of independent variables N, the degree of the model equation P, and the number of input data points (sets) n. Further, the significance of the estimated regression equation is indicated by the standard deviation, the multiple correlation coefficient, the F ratio, the maximum percent error, and the average percent error.

The values of the dependent variable are also shown for the input data and for the computed values obtained from the fitted expression for each set of data input, as well as the residual difference in the input values and the computed values.

When called for, plotted results can also be obtained which compare the input and computed dependent variables as ordinates to the point interval along the abscissa (which is normally one of the physical independent variables, but which can be a unit indication of each data set in order of input to program). All computed points are connected by straight lines and are plotted with * symbols. The input points are plotted with + symbols and the points are not connected with lines.

Another useful parameter in testing the significance of the regression equation is the standard deviation which is estimated by:

$$s = \left[\frac{\sum (Y_i - Y_i^c)^2}{\text{Regression degrees of freedom} - 1} \right]^{1/2} = \left[\frac{S(RES)}{\text{Regression degrees of freedom} - 1} \right]^{1/2} \tag{35}$$

This result stems from the work of Junkin [4].

Earlier, the analysis of variance technique was used to test the combined effect of the independent variables on the dependent variable using the F statistic. A closely related statistic is the multiple correlation coefficient. R. Smille [3] defines this statistic as the simple correlation coefficient between the observed values of the dependent variable and those estimated by the multiple regression function as given by:

$$R = \frac{S(REG)}{S_{YY}}^{1/2} \tag{36}$$

If the observed and estimated values are completely unrelated, R will be zero and, if they are identical the multiple correlation coefficient will be unity. Values in between these limits represent different degrees of correlation or the closeness within which the regression equation describes the original data.

Also of interest is the average of the absolute values of the percent error of the dependent variable, considering each fitted observation:

$$|\epsilon|_{AVG} = \frac{1}{n} \sum \left| \frac{Y_i - Y_i^c}{Y_i} \right| \times 100$$

During this calculation the maximum absolute error condition can be determined for evaluation purposes.

V. SUMMARY

A powerful multiple regression technique for Pth degree polynomials with and without linear cross products has been developed with which the nonlinear behavior of identified independent variables can be related to a given dependent variable. The polynomial expression can be of the Pth degree and can incorporate N independent variables. The resulting surface fit can be used to summarize trends for a given phenomenon and the analytic results provide a mathematical basis for subsequent analysis.

It is required in the application of this technique that the S matrix be nonsingular and that the input data are reasonably well behaved. It is also required that the number of data points being fitted be equal to or greater than the number of unknown parameters in the model equation.

Two computer programs have been developed to implement this technique for the cases with and without linear cross products. These programs perform matrix operations in double precision and evaluate the various unknown constants in the model regression equation as well as the standard deviation, the multiple regression coefficient, the F statistic, the maximum absolute percent error, and the average of the absolute values of the percent error. Further included in these programs is the solution for the identity matrix to identify any problems in the original matrix inversion process and a means of obtaining machine plots, comparing the computer results to each set of input data.

Studies of many sets of physical data using the technique without cross products have generally yielded excellent results. However, much less experience has been obtained using the program with cross products, although test programs have shown very good correlation. It is, therefore, felt that the technique developed herein can be highly useful to many fields of endeavor.

APPENDIX A

MULTIPLE REGRESSION PROGRAM FOR CASE WITHOUT CROSS PRODUCTS

A. Input Data

The regression subroutine is called with the following statement:

CALL REGRES(Z,YO,LN,N,IP)

where:

Z is a two-dimensional array containing the independent variables used. The first subscript denotes the independent variable and the second subscript denotes the data set.

YO is an array containing the dependent variables which correspond to the independent variables.

LN is the number of data sets.

N is the number of independent variables used.

IP is the degree of the polynomial curve fit.

These arguments must be assigned a value in a driver program.

The regression subroutine incorporates several special routines: RDWT, IOWR, INVRT, and DMATML.

RDWT is a general FORTRAN I/O package which is capable of reading and/or writing on magnetic tape or drum in FORTRAN or non-FORTRAN format.

IOWR is an assembly language I/O routine called by RDWT.

INVRT is a double precision matrix inversion and simultaneous equation solver. The call to this routine is:

CALL INVRT(A,N,M,DETER)

where:

A is the input matrix for inversion or augmented matrix for simultaneous equations.

N is the order of the coefficient matrix.

M=0 for inversion only; otherwise, M is the number of constant vectors.

DETER is the determinant of the coefficient matrix.

DMATML is a double precision matrix multiplication routine with transpose options. The call to this routine is:

CALL DMATML(C,A,B,M,N,K)

where:

C is the product of matrix A times matrix B,

A is the first input matrix,

B is the second input matrix,

M is the number of rows of A,

N is the number of rows of B, and

K is the number of columns of B.

Certain variables have dimension changes that vary depending on the number of data points (LN), the number of independent variables (N), and the degree of the polynomial curve fit (IP). These will be dimensioned as follows:

DSN(NIP2), DSHAT(NIP,1), DB(NIP,1), Y(LN), RESID(LN), Z(N,LN), SHAT(NIP,NIP), ZBAR(N,IP), YO(LN), BZ(LN), B(N,IP), SVHAT(NIP,1), SH(NIP,NIP), XX(LN), PMAX(LN)

where:

LN,N,IP are as defined above and NIP is computed as: NIP = N*IP; NIP2 is NIP raised to the second power.

SMALLZ, SMALZ1, and SMALZ2 should be dimensioned greater than the number of data points and the dimension is set in a PARAMETER statement; for example:

PARAMETER IDIM = 100

B. Program

In order to run this program on the UNIVAC 1108, the deck is set up as follows:

```
@RUN,//T JBNAME,320590,VDARBYBIN406, 01, 100
@ASG,T 10,F/1/POS/2
@FOR,IS MAIN, MAIN
   (Source deck for inputting data)
@FOR,IS REGRES, REGRES
   (Source deck)
@FOR,IS RDWT, RDWT
   (Source deck)
@ASM,IS IOWR, IOWR
   (Source deck)
@FOR,IS INVRT, INVRT
   (source deck)
@FOR,IS DMATML, DMATML
   (Source deck)
@MAP,I AA, AA
   LIB SYS$*MSFC $.
@XQT AA
   (Input data)
@FIN
```

as illustrated in Figure 1.

A complete program listing for the case without cross products is shown in Table 2 and a flow chart indicating the operation of this computer program is indicated in Figure 2.

C. Output Data

The desired coefficients of the model equation are outputted in E notation where AO is the computed intercept of the fitted polynomial expression and the B coefficients are printed out in ascending order of degree (P) with the first P coefficients indicating the b coefficients of the first independent variable, the second set of P coefficients indicating the b coefficients of the second independent variable, etc.

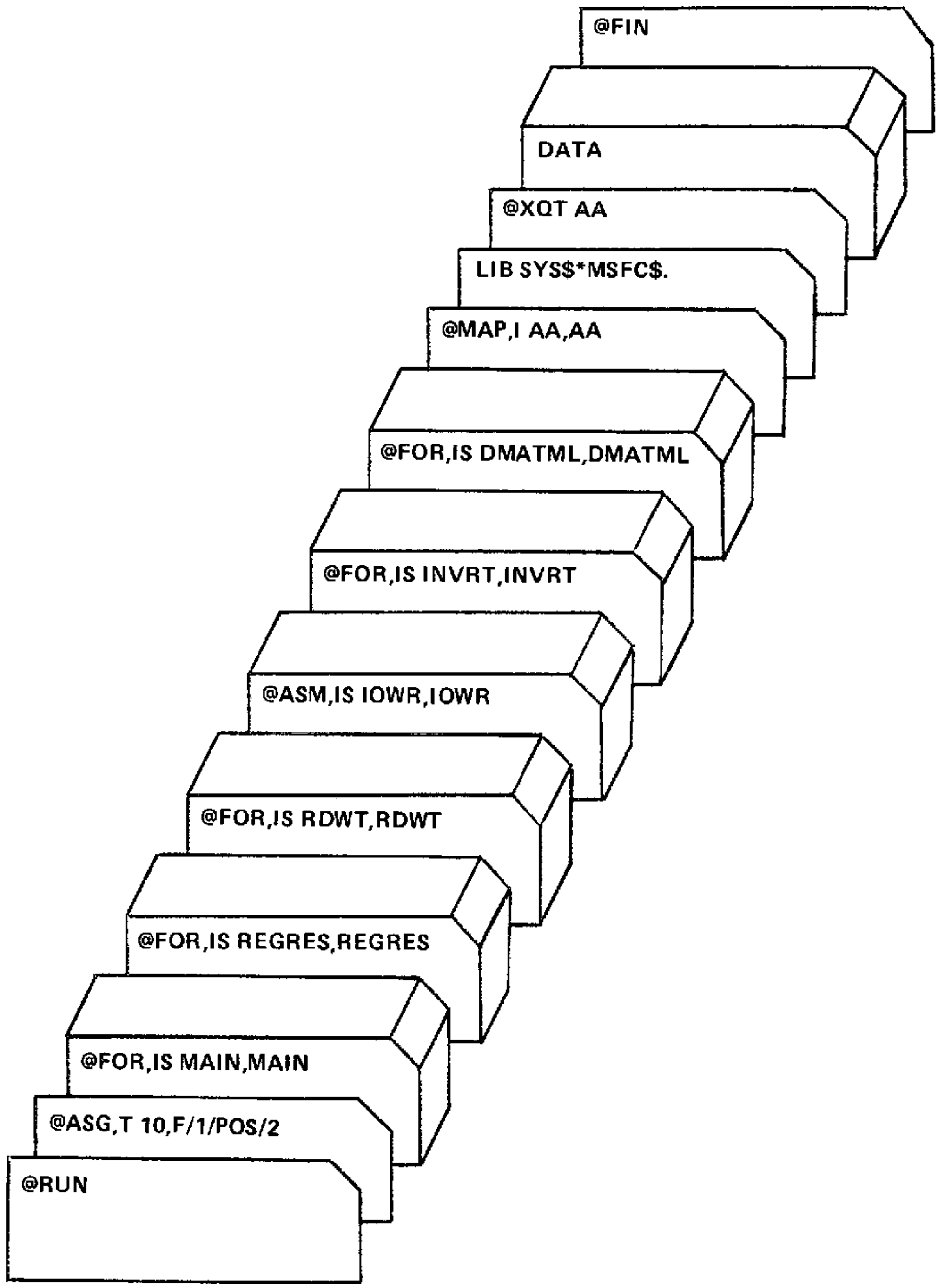


Figure 1. Program deck sequence.

TABLE 2. PROGRAM LISTING (WITHOUT CROSS PRODUCTS)

```
VDARBY
BIN406
011973
040627
```

```
@RUN,ZZE WINHCP,322570,VDARBYBIN406,01,1000
```

TABLE 2. (Continued)

```
RMSG,N                  REMOVE AT CARD READER     01-18-73         CLOCK NO. 0093


 @ASG,I 13,F/1/POS/2

@FOR,IS MAIN,MAIN
RSO 009-01/19-03:56-(,0)

  MAIN PROGRAM

  STORAGE USED: CODE(1) 000021; DATA(0) 000101; BLANK COMMON(2) 000000

  EXTERNAL REFERENCES (BLOCK, NAME)

   0003   REGRES
   0004   NINTR$
   0005   NRNL$
   0006   NSTOP$

  STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

   0000 I 000052 IP      0000 I 000050 LN      0000 I 000051 N      0000   000055 NAM1      0000 R 000036 Y0
   0000 R 000000 Z


00101      1*          DIMENSION Z(3,10),Y0(10)
00103      2*          NAMELIST/NAM1/Z,Y0,LN,N,IP
00104      3*          READ(5,NAM1)
00107      4*          CALL REGRES(Z,Y0,LN,N,IP)
00110      5*          STOP
00111      6*          END
      END OF COMPILATION:       NO  DIAGNOSTICS.
```

TABLE 2. (Continued)

```
@FOR,IS REGRES,REGRES
FOR S029-01/19-00159.(,0)

   SUBROUTINE REGRES     ENTRY POINT 001240

   STORAGE USED: CODE(1) 001310; DATA(0) 001662; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003    IDENT
    0004    IORDAR
    0005    INVRT
    0006    DMATML
    0007    QUIKSL
    0010    CHOJUB
    0011    NRDU$
    0012    NIO2$
    0013    NEXP6$
    0014    SQRT
    0015    NIO1$
    0016    NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0000    001561 100DF     0000    001470 1000DF    0000    001541 10003F    0000    001552 10021F    0001    000075 137G
    0001    000076 142G      0001    000106 150G      0000    001531 1500F     0001    000112 155G      0001    000113 160G
    0001    000123 166G      0001    000127 173G      0001    000144 206G      0001    000146 211G      0001    000150 214G
    0000    001522 2222F     0001    000215 227G      0001    000217 232G      0001    000222 235G      0000    001476 24F
    0001    000264 250G      0000    001526 2500F     0000    001537 2502F     0001    000274 253G      0001    000320 261G
    0001    000330 264G      0001    000350 271G      0001    000422 314G      0001    000423 317G      0001    000451 330G
    0001    000470 340G      0001    000500 343G      0001    000532 350G      0001    000567 364G      0001    000570 367G
    0001    000610 377G      0001    000611 402G      0001    000633 414G      0001    000635 417G      0001    000637 422G
    0001    000667 431G      0001    000725 444G      0001    001004 460G      0001    001100 503G      0001    001101 505G
    0001    001175 505L      0001    001127 521G      0001    001145 536G      0001    001167 546G      0000 R  001331 ADANY
    0000 R  001466 AMAX      0000 R  001455 AO        0000 R  000644 B         0000 R  000632 B2        0000 D  000264 DB
    0000 D  000306 DETER     0000 D  000242 DSHAT     0000 D  000000 DSN       0000 R  001371 FLDX      0000 R  001405 FLDY
    0000 R  001460 FRATIO    0000 I  001421 I         0000 I  001452 ICL       0000 I  001445 ICOL      0000 I  001443 IERR
    0000 I  001446 IK        0000    001611 INJP$     0000 I  001454 IR        0000 I  001444 IROW      0000 I  001442 ISECT
    0000 I  001440 IUNIT     0000 I  001451 I2        0000 I  001430 J         0000 I  001431 K         0000 I  001447 L
    0000 I  001461 LI        0000 I  001462 L1        0000 I  001463 L2        0000 I  001453 M         0000 I  001427 NIP
    0000 I  001450 NR        0000 I  001441 NSECT     0000 I  001422 NWDS      0000 R  001465 PERAVG    0000 R  001467 PERMAX
    0000 R  001357 PMAX      0000 R  000322 RESID     0000 R  001457 RHAT      0000 R  001434 RR        0000 R  001176 SH
    0000 R  000500 SHAT      0000 R  000334 SMALLZ    0000 R  000666 SMALZ1    0000 R  001032 SMALZ2    0000 R  001435 SREG
    0000 R  001437 SRES      0000 R  001456 STADEV    0000 R  001464 SUM       0000 R  000655 SVHAT     0000 R  001436 SYY
    0000 R  001423 XL        0000 R  001424 XR        0000 R  001317 XX        0000 R  000310 Y         0000 R  001425 YB
    0000 R  001432 YBARC     0000 R  001433 YBARO     0000 R  001426 YT        0000 R  000621 ZBAR

00101       1*          SUBROUTINE REGRES(Z,YO,LN,N,IP)
00101       2*    C        THIS SUBROUTINE IS DESIGNED TO DETERMINE SPECIFIED MODEL
```

TABLE 2. (Continued)

```
00101    C       PARAMETERS USING REGRESSION TECHNIQUES
00103          PARAMETER IDIM=100
00104          DOUBLE PRECISION DSN(81),DSHAT(9,1),DB(9,1)
00105          DOUBLE PRECISION DETER
00106          DIMENSION Y(10),RESID(10),Z(3,10),SMALLZ(IDIM),SHAT(9,9),
00106         *ZBAR(3,3),YO(10),BZ(10),B(3,3),SVHAT(9,1),
00106         *SMALZ1(IDIM),SMALZ2(IDIM)
00107          DIMENSION SH(9,9),XX(10)
00110          DIMENSION ADARY(22)
00111          DIMENSION PMAX(10)
00112          DIMENSION FLOX(12),FLOY(12)
00113          DATA FLOY/72H                                    Y VALUES
00113         *                     /
00115          DATA FLOX/72H                                 , POINT INTERVALS
00115         *                     /
00117          DATA (ADARY(I),I=1,3)/6HONE    ,6H  COPY,6HFLO   /
00121          CALL IDENT(9,ADARY)
00122          NWDS=IDIM
00123          XL=0.0
00124          XR=11.0
00125          YB=4.0
00126          YT=12.0
00127          WRITE(6,10000) N,IP,LN
00134    10000 FORMAT(1H1,2HN=I4,3X,2HP=I4,3X,3HLN=I4)
00135          NIP=N*IP
00136          DO 1234 I=1,N
00141          DO 1234 J=1,IP
00144     1234 ZBAR(I,J)=0.0
00147          DO 1235 K=1,IDIM
00152     1235 SMALLZ(K)=0.0
00154          DO 1236 I=1,NIP
00157          DO 1236 J=1,NIP
00162     1236 SHAT(I,J)=0.0
00165          DO 1237 I=1,NIP
00170     1237 SVHAT(I,1)=0.0
00172          DO 1238 I=1,LN
00175     1238 BZ(I)=0.0
00177          YBARC=0.0
00200          YBARO=0.0
00201          RR=0.0
00202          SREG=0.0
00203          SYY=0.0
00204          SRES=0.0
00204    C---- DETERMINE THE SMALL Z VALUES FOR THE ASSUMED MODEL FORM
00204    C
00205          DO 225 I=1,N
00210          DO 225 J=1,IP
00213          DO 210 K=1,LN
00216      210 ZBAR(I,J)=ZBAR(I,J)+Z(I,K)**J
00220      225 ZBAR(I,J)=ZBAR(I,J)/LN
00223          IUNIT=10
00224          NSECT=(NWDS*27)/28
00225          ISECT=1
00226          DO 250 I=1,N
00231          DO 250 J=1,IP
00234          DO 245 K=1,LN
```

TABLE 2. (Continued)

```
00237        245 SMALLZ(K)=Z(I,K)**J-ZOAR(I,J)
00241            CALL IORDWR(1,IUNIT,ISECT,SMALLZ,NWDS,IERR)
00242        250 ISECT=ISECT+NSECT
00242      C
00242      C     COMPUTE SHAT MATRICES FOR USE IN THE LEAST SQUARES SOLUTION
00242      C       OF FINDING THE B'S.
00242      C
00242      C
00245            IROW=0
00246            ICOL=1
00247            DO 450 IK=1,N
00252            DO 400 L=1,IP
00255            ISECT=(IK-1)*IP*NSECT+(L-1)*NSECT+1
00256            CALL IORDWR(2,IUNIT,ISECT,SMALZ1,NWDS,IERR)
00257            IROW=IROW+1
00260            DO 350 J=1,N
00263            DO 350 K=1,IP
00266            ISECT=(J-1)*IP*NSECT+(K-1)*NSECT+1
00267            CALL IORDWR(2,IUNIT,ISECT,SMALZ2,NWDS,IERR)
00270            DO 310 I=1,LN
00273        300 SHAT(IROW,ICOL)=SHAT(IROW,ICOL)+SMALZ1(I)*SMALZ2(I)
00274            SH(IROW,ICOL)=SHAT(IROW,ICOL)
00275        310 CONTINUE
00277            IF(ICOL.EQ.N*IP) ICOL=0
00301        350 ICOL=ICOL+1
00304        400 CONTINUE
00306        450 CONTINUE
00310            NR=IROW
00311            I2=1
00312             ICL=N*IP
00313            DO 1322 I=1,ICL
00316            DO 1322 J=1,IROW
00321            DSN(I2)=SH(J,I)
00322       1322 I2=I2+1
00325            M=0
00326            CALL INVRT(DSN,NR,M,DETER)
00327            DO 650 I=1,LN
00332        650 YBARO=YBARO+YO(I)
00334            YBARO=YBARO/LN
00335            NIP=N*IP+1
00336            IROW=1
00337            DO 710 I=1,N
00342            DO 710 J=1,IP
00345            ISECT=(I-1)*IP*NSECT+(J-1)*NSECT+1
00346            CALL IORDWR(2,IUNIT,ISECT,SMALZ2,NWDS,IERR)
00347            DO 700 K=1,LN
00352            SVHAT(IROW,1)=SVHAT(IROW,1)+SMALZ2(K)*(YO(K)-YBARO)
00353        700 DSHAT(IROW,1)=SVHAT(IROW,1)
00355        710 IROW=IROW+1
00355      C     COMPUTE B'S FOR THE ASSUMED MODEL.
00355      C
00360            IR=IROW-1
00361            CALL DMATML(DB,DSN,DSHAT,IR,IR,1)
00362            I2=1
00363            DO 1331 I=1,N
00366            DO 1331 J=1,IP
```

TABLE 2. (Continued)

```
00371              B(I,J)=DB(I2,I)
00372         1331 I2=I2+1
00375              A0=YBARO
00376              DO 23 I=1,N
00401              DO 23 J=1,IP
00404           23 A0=A0-B(I,J)*ZBAR(I,J)
00407              WRITE(6,24) A0
00412           24 FORMAT(1H0,3HA0=,E16.9)
00413              DO 750 I=1,LN
00416              DO 750 J=1,N
00421              DO 750 K=1,IP
00424          750 BZ(I)=BZ(I)+B(J,K)*Z(J,I)**K
00424        C
00424        C     COMPUTE THE Y POINTS
00424        C
00430              DO 775 I=1,LN
00433              Y(I)=A0+BZ(I)
00434              YBARC=YBARC+Y(I)
00435              RESID(I)=YO(I)-Y(I)
00436              RR=RR+RESID(I)**2
00437          775 CONTINUE
00441              YBARC=YBARC/LN
00441        C
00441        C     COMPUTE THE STANDARD DEVIATION.
00441        C
00442              STADEV=SQRT(RR/(LN-N*IP))
00442        C
00442        C     COMPUTE THE MULTIPLE CORRELATION COEFFICIENT,RHAT,AND
00442        C     THE F RATIO,FRATIO.
00442        C
00443              DO 800 I=1,LN
00446              SREG=SREG+(Y(I)-YBARC)**2
00447              SYY=SYY+(YO(I)-YBARO)**2
00450          800 SRES=SRES+RESID(I)**2
00452              RHAT=SQRT(SREG/SYY)
00453              IF(RHAT.GT.1.0) RHAT=2.0-RHAT
00455              FRATIO=(SREG/(N*IP-1))/(SRES/(LN-N*IP))
00456              XX(1)=1.
00457              DO 83 LI=2,LN
00462           83 XX(LI)=XX(LI-1)+1.
00464              L1=-1
00465              L2=0
00466              CALL QUIK3L(L1,XL,XR,YB,YT,40,FLDX,FLDY,-LN,XX,Y)
00467              CALL QUIK3L(L2,XL,XR,YB,YT,34,FLDX,FLDY,LN,XX,YO)
00470              WRITE(6,1000) STADEV,RHAT,FRATIO
00475         1000 FORMAT(1H0,19HSTANDARD DEVIATION=E16.9,4X,33HMULTIPLE CORRELATION
00475             *COEFFICIENT=E16.9,4X,8HF RATIO=E16.9)
00476              WRITE(6,2222)
00500         2222 FORMAT(1H0,'B COEFFICIENTS')
00501              WRITE(6,2500) ((B(I,J),J=1,IP),I=1,N)
00512         2500 FORMAT(1H0,5E21.9/)
00513              WRITE(6,1500)
00515              WRITE(6,10043)
00517         1500 FORMAT(1H0,16X,2HYO,16X,2HYC,15X,5HYO-YC)
00520              DO 2000 I=1,LN
00523              WRITE(6,2502) YO(I),Y(I),RESID(I)
```

TABLE 2. (Continued)

```
 2532 FORMAT(1H0,3E21.9)
 2500 CONTINUE
10003 FORMAT(1H0,12X,8HOBSERVED,12X,8HCOMPUTED,12X,8HRESIDUAL)
      SUM=0.
      DO 501 L=1,LN
      PMAX(L)=ABS((YO(L)-Y(L))/Y(L))
  501 SUM=SUM+PMAX(L)
      PERAVG=(SUM/LN)*100.
      AMAX=PMAX(1)
      DO 505 L=2,LN
      IF(AMAX-PMAX(L)) 506,505,505
  506 AMAX=PMAX(L)
  505 CONTINUE
      PERMAX=AMAX*100.
      WRITE(6,10021) PERMAX,PERAVG
10021 FORMAT(1H0,22HMAXIMUM PERCENT ERROR=E16.9,6X,22HAVERAGE PERCENT ER
     *ROR=E16.9)
      CALL ENDJOB
      RETURN
      END

END OF COMPILATION:        NO  DIAGNOSTICS.
```

TABLE 2. (Continued)

```
WFOR,IS RDWT,RDWT
RSO 009-01/19-01:00-(,0)

   SUBROUTINE RDWT        ENTRY POINT 000270
              REDTPR      ENTRY POINT 000273
              WRITER      ENTRY POINT 000320
              CLOSE       ENTRY POINT 000342
              OPEN        ENTRY POINT 000351
              IORDWR      ENTRY POINT 000360

   STORAGE USED: CODE(1) 000402; DATA(0) 000127; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   IOWR
    0004   NERR2$
    0005   NWDUS
    0006   NIO1$
    0007   NIO2$
    0010   NSTOP$
    0011   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001     000071 10L       0000     000070 100F      0001     000116 20L      0001     000140 21L      0001     000242 215G
    0001     000144 22L       0001     000211 24L       0001     000227 25L      0001     000234 26L      0001     000033 6L
    0001     000035 7L        0001     000050 8L        0000 I   000061 I        0000 I   000065 IBLANK   0000 I   000000 ICWD
    0000 I   000003 IFLD      0000     000111 INJP$     0000 I   000066 IOP      0000 I   000046 IPACKT   0000 I   000067 ITWD
    0000 I   000062 I1        0000 I   000063 I2        0000 I   000064 I3       0000 I   000056 KNTWD    0000 I   000040 KODE


00100      1*    C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
00100      2*    C      GENERAL FORTRAN I/O PACKAGE          EXEC VIII          APRIL,1970
00100      3*    C      READ/WRITE MAG TAPE OR DRUM IN FORTRAN OR NON-FORTRAN FORMAT.
00100      4*    C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
00101      5*           SUBROUTINE RDWT
00103      6*           DIMENSION ARRAY( 1),ICWD(3),IFLD(29),KODE(6),IPACKT(8),KNTWD(3)
00104      7*           DATA (IFLD(I),I=1,29)/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,2H10,
00104      8*          12H11,2H12,2H13,2H14,2H15,2H16,2H17,2H18,2H19,2H20,2H21,2H22,2H23,
00104      9*          22H24,2H25,2H26,2H27,2H28,2H29/
00106     10*           DATA (KODE(I),I=1,6)/000430000000,000200000000,000150000000,
00106     11*          1                    000100000000,000110000000,000400000000/
00110     12*           DATA I1/262144/,I2/1073741824/,I3/0010001/
00114     13*           DATA IBLANK/6H
00114     14*    C*****
00116     15*           ENTRY REDTPR(IUNIT,MODE,IERR,NW,NWDS,ARRAY)
00120     16*           IOP=MODE                    6 =1,READ FTN FORMAT, =2,READ NON-FTN.
00120     17*    C*****
00121     18*           GO TO 10
00122     19*           ENTRY WRITER(IUNIT,MODE,IERR,NWDS,ARRAY)
```

TABLE 2. (Continued)

```
00124            IOP=MODE+2                 @ =3,WRITE FTN FORMAT. =4,WRITE NON-FTN.
00125            KNTWD(1)=NWDS+I1+I3        @ BUILD FORTRAN RECORD-CONTROL WORD.
00126            KNTWD(2)=IBLANK            @ DUMMY CHECKSUM ERROR WORD.
00127            KNTWD(3)=KNTWD(1)
00130            GO TO 10
00130      C*****
00131            ENTRY CLOSE(IUNIT,JOP)
00133            ITWO=1
00134            GO TO (6,6,7,25),JOP       @ JOP=4, NO ACTION,RETURN.
00135          6 ITWO=2                     @ JOP=1,2, WRITE EOF AND REWIND IUNIT.
00136          7 IOP=5                      @ =5, WRITE END-OF-FILE.
00137            GO TO 10
00137      C*****
00140            ENTRY OPEN(IUNIT,JOP)
00142            IF(JOP.GT.1) RETURN        @ JOP=2,3, NO ACTION,RETURN.
00144          8 IOP=6                      @ =6, REWIND IUNIT.
00145            GO TO 10
00145      C*****
00146            ENTRY IORDWR(ICODE,IUNIT,IADDRS,ARRAY,NWDS,IERR)
00150            IF(ICODE.EQ.1) IOP=4       @ =4, SEQUENTIAL WRITE.
00152            IF(ICODE.EQ.2) IOP=2       @ =2, SEQUENTIAL READ.
00154            IPACKT(6)=IADDRS           @ BEGINNING DRUM-SECTOR ADDRESS.
00154      C*****
00155         10 CONTINUE
00156            IPACKT(1)=IFLD(IUNIT)                @ IUNIT IN FIELD-DATA.
00157            IPACKT(2)=IBLANK
00160            IPACKT(3)=0
00161         11 IPACKT(4)=KODE(IOP)                  @ OPERATION CODE TO IOWS.
00162            GO TO (20,21,20,21,22,22),IOP
00163         20 ICWD(1)=   1*I1+LOC(KNTWD(1))        @  FIRST CONTROL ACCESS WORD.
00164 *DIAGNOSTIC* THE VARIABLE ARRAY IS DIMENSIONED.
00164            ICWD(2)=NWDS*I1+LOC(ARRAY)           @ SECOND CONTROL ACCESS WORD.
00165            ICWD(3)=   2*I1+LOC(KNTWD(2))        @  THIRD CONTROL ACCESS WORD.
00166            IPACKT(5)= 3*I1+LOC(ICWD(1))         @ NMBR AND LOC OF 1ST C/A WORD.
00167            GO TO 22
00170 *DIAGNOSTIC* THE VARIABLE ARRAY IS DIMENSIONED.
00170         21 IPACKT(5)=NWDS*I1+LOC(ARRAY)         @ NMBR AND LOC OF DATA WORDS.
00171         22 CALL IOWR(IPACKT)                    @ ROUTINE TO ISSUE EXEC REQUEST.
00172            IF(IOP.EQ.6) RETURN                  @ RETURN AFTER REWINDING IUNIT.
00174            IF(IOP.EQ.5) GO TO (25,8),ITWO       @ ITWO=1,RETURN, =2,REWIND IUNIT.
00176            IERR=IPACKT(4)/I2                    @ ERROR STATUS IN S1 PORTION.
00177            IF(IERR.EQ.0) GO TO 24               @ NORMAL OPERATION,NO ERROR.
00201            IF(IERR.GE.3) GO TO 26               @ TERMINATING ERROR.
00203            IERR=2                               @ END-OF-FILE OR END-OF-TAPE.
00204            RETURN
00205         24 NW=KNTWD(1)/I1                       @ NW FROM FTN CONTROL WORD.
00206            IF(MODE.EQ.1) GO TO 25
00210            NW=IPACKT(4)-IPACKT(4)/I1*I1         @ NW FROM IOWS.
00211         25 IERR=1
00212            RETURN
00213         26 WRITE(6,100) (IPACKT(I),I=1,6)
00221        100 FORMAT(///50H******EXECUTION TERMINATED IN 'RDWT' I/O ROUTINE. //
00221           114H I/O PACKET IS 6O16 )
00222            STOP
00223            END
```

TABLE 2. (Continued)

```
END OF COMPILATION:          2 DIAGNOSTICS.

@ASM,IS IOWR,IOWR
ASM11F-01/19-01:00-(,0)

     1.                                                        $(1)    AXRS
     2.       01    000000  27 00 14 13 0 000000   IOWR*   L        A0,0,X11
     3. U           000001  72 11 00 00 0 000000           ER       IOWS
     4.             000002  74 04 00 13 0 000002           J        2,X11
     5.                                                            END
                   IOWS
 END ASM ERRORS : NONE
```

TABLE 2. (Continued)

```
@FOR,IS INVRT,INVRT
HSO-009-01/19-01:00-(,0)

 SUBROUTINE INVRT      ENTRY POINT 000370

 STORAGE USED: CODE(1) 000416; DATA(0) 000334; BLANK COMMON(2) 000000

 EXTERNAL REFERENCES (BLOCK, NAME)

  0003    HERR3$

 STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

  0001     000016 111G      0001     000144 115L      0001     000027 117G      0001     000034 123G      0001     000042 131G
  0001     000114 162G      0001     000260 181L      0001     000166 201G      0001     000206 207G      0001     000230 220G
  0001     000266 234G      0001     000336 235L      0001     000312 245G      0001     000346 250L      0001     000071 75L
  0001     000073 76L       0001     000110 90L       0000 D 000002 AMAX      0000 I 000273 I         0000 I 000276 IC
  0000 I 000274 IND       0000 I 000100 INDEX     0000 I 000300 IND2      0000   000306 JNJPS     0000 I 000004 IPIV
  0000 I 000275 IR        0000 I 000273 J         0000 I 000272 K         0000 I 000277 L         0000 I 000271 NN
  0000 D 000000 SIGN

00101                 SUBROUTINE INVRT(A,N,M,DETER)
00103                 PARAMETER IDIM=60
00103         C       MATRIX INVERSION AND SIMULTANEOUS EQUATIONS SOLVER
00103         C       A=INPUT MATRIX FOR INVERISON OR AUGUMENTED MATRIX FOR SIME. EQS.  IG500030
00103         C       N=ORDER OF COEFFICIENT MATRIX                                    IG500040
00103         C       M=0 FOR INVERISON ONLY
00103         C       M=NUMBER OF CONSTANT VECTORS
00103         C       DETER=DETERMINANT OF COEFFICIENT MATRIX
00104                 DOUBLE PRECISION A(1),DETER,SIGN,AMAX
00105                 DIMENSION IPIV(IDIM),INDEX(IDIM,2)
00106                 DETER=1.000
00107                 SIGN=1.000
00110                 DO 20 J=1,N
00113              20 IPIV(J)=0
00115                 NN=N+M
00116                 DO 102 K=1,N
00121                 AMAX=0.000
00122              40 DO 76 I=1,N
00125                 IF (IPIV(I)-1)50,76,50
00130              50 DO 75 J=1,N
00133                 IF (IPIV(J)-1)55,75,250
00136              55 IND=(J-1)*N+I
00137                 IF(AMAX-DABS(A(IND))) 60,75,75
00142              60 IR=I
00143                 IC=J
00144                 AMAX=DABS(A(IND))
00145              75 CONTINUE
00147              76 CONTINUE
```

TABLE 2. (Continued)

```
00151     29*            IPIV(IC)=IPIV(IC)+1
00152     30*            IF (IR-IC)90,115,90
00155  *DIAGNOSTIC*  CONTROL CAN NEVER REACH THE NEXT STATEMENT
00155     31*            IF (IR-IC)90,115,90
00160     32*        90  SIGN=-SIGN
00161     33*            DO 110 L=1,NN
00164     34*            IND=(L-1)*N+IR
00165     35*            IND2=(L-1)*N+IC
00166     36*            AMAX=A(IND)
00167     37*            A(IND)=A(IND2)
00170     38*       110  A(IND2)=AMAX
00172     39*       115  INDEX(K,1)=IR
00173     40*            INDEX(K,2)=IC
00174     41*            IND=(IC-1)*N+IC
00175     42*            AMAX=A(IND)
00176     43*            DETER=DETER*AMAX
00177     44*       140  A(IND)=1.0D0
00200     45*            DO 150 L=1,NN
00203     46*            IND=(L-1)*N+IC
00204     47*       150  A(IND)=A(IND)/AMAX
00206     48*            DO 181 L=1,N
00211     49*            IF (L-IC)165,181,165
00214     50*       165  IND=(IC-1)*N+L
00215     51*            AMAX=A(IND)
00216     52*            A(IND)=0.0D0
00217     53*            DO 180 I=1,NN
00222     54*            IND=(I-1)*N+L
00223     55*            IND2=(I-1)*N+IC
00224     56*            A(IND)=A(IND)-A(IND2)*AMAX
00225     57*       180  CONTINUE
00227     58*       181  CONTINUE
00231     59*       182  CONTINUE
00233     60*            DO 235 I=1,N
00236     61*            L=N+1-I
00237     62*            IR=INDEX(L,1)
00240     63*            IC=INDEX(L,2)
00241     64*            IF (IR-IC)210,235,210
00244     65*       210  DO 230 K=1,N
00247     66*            IND=(IR-1)*N+K
00250     67*            IND2=(IC-1)*N+K
00251     68*            AMAX=A(IND)
00252     69*            A(IND)=A(IND2)
00253     70*       230  A(IND2)=AMAX
00255     71*       235  CONTINUE
00257     72*            DETER=SIGN*DETER
00260     73*            RETURN
00261     74*       250  M=-1
00262     75*       255  RETURN
00263     76*            END

       END OF COMPILATION:          1 DIAGNOSTICS.
```

TABLE 2. (Continued)

```
@FOR,IS DMATML,DMATML
H50-009-01/19-01:00-(,0)

 SUBROUTINE DMATML     ENTRY POINT 000140

 STORAGE USED: CODE(1) 000176; DATA(0) 000053; BLANK COMMON(2) 000000

 EXTERNAL REFERENCES (BLOCK, NAME)

  0003   NERR3$

 STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

  0001   000031 1L       0001   000061 130G     0001   000067 135G     0001   000102 143G     0001   000035 2L
  0001   000046 3L       0001   000052 4L       0000 D 000000 CO       0000 I 000005 IA1      0000 I 000006 IA2
  0000 I 000002 IA3      0000 I 000007 IB1      0000 I 000010 IB2      0000 I 000013 IB3      0000 I 000003 IM
  0000 I 000004 IN       0000   000022 INJP$    0000 I 000015 LA       0000 I 000016 LB       0000 I 000012 LC
  0000 I 000011 LM       0000 I 000017 LN       0000 I 000014 LP

00101      1*          SUBROUTINE DMATML(C,A,B,M,N,K)
00101      2*   C*     ABSTRACT
00101      3*   C      GENERAL MATRIX MULTIPLICATION ROUTINE  WITH TRANSPOSE OPTIONS
00101      4*   C       WHERE, M IS THE NUMBER OF ROWS OF (A)
00101      5*   C              N IS THE NUMBER OF ROWS OF (B)
00101      6*   C              K IS THE NUMBER OF COLUMNS OF (B) OR (B)T
00101      7*   C      TRANSPOSE OPTIONS ARE CONTROLLED BY THE SIGNS OF M AND N.
00101      8*   C      THE FOLLOWING PRODUCTS MAY BE OBTAINED
00101      9*   C       (C)=(A)(B)        M AND N POSITIVE
00101     10*   C       (C)=(A)T(B)       M NEGATIVE FOR (A)T
00101     11*   C       (C)=(A)(B)T       N NEGATIVE FOR (B)T
00101     12*   C       (C)=(A)T(B)T      M AND N NEGATIVE
00101     13*   C       WHERE T INDICATES TRANSPOSE
00101     14*   C        IF M IS NEGATIVE, M IS THE NUMBER OF ROWS OF (A)T
00101     15*   C        IF N IS NEGATIVE, N IS THE NUMBER OF ROWS OF (B)T
00101     16*   C
00101     17*   C*     OUTPUT ARGUMENT * C
00103     18*          DIMENSION C(1)
00103     19*   C
00103     20*   C*     INPUT ARGUMENTS * A,B,M,N,K
00104     21*          DIMENSION A(1),B(1)
00104     22*   C
00104     23*   C***********************************************************************
00105     24*          DOUBLE PRECISION CO,C,A,B
00106     25*          IA3=1
00107     26*          IM=IABS(M)
00110     27*          IN=IABS(N)
00111     28*          IF(M .LT. 0)GO TO 1
00113     29*          IA1=IN
00114     30*          IA2=1
```

TABLE 2. (Continued)

```
00115              GO TO 2
00116        1     IA1 = 1
00117              IA2=IN
00120        2     IF(N .LT. 0)GO TO 3
00122              IB1=1
00123              IB2=IN
00124              GO TO 4
00125        3     IB1=K
00126              IB2=1
00127        4     DO 7 LM=1,IM
00132              LC=LM
00133              IB3=1
00134              DO 6 LP=1,K
00137              CD = 0.D00
00140              LA=IA3
00141              LB=IB3
00142              DO 5 LN=1,IN
00145              CD = CD + A(LA)*B(LB)
00146              LA=LA+IA1
00147        5     LB=LB+IB1
00151              C(LC) = CD
00152              LC = LC + IM
00153        6     IB3=IB3+IB2
00155        7     IA3=IA3+IA2
00157              RETURN
00160              END

   END OF COMPILATION:        NO DIAGNOSTICS.
```

TABLE 2. (Continued)

```
@MAP,IS AA,AA
MAP—17H2=01/19-01:00--(,0)

   1.        LIB SYS$*MSFC$.

ADDRESS LIMITS    001000 033207     040000 054642
STARTING ADDRESS  033167

WORDS DECIMAL       13448 IBANK         6563 DBANK

         SEGMENT MAIN            001000 033207      040000 054642
```

NSRTC$/FOR	1	001000 001021		
NRHLK$/MSFCS5	1	001022 001110	0	040000 040001
NEXPA$/MSFCS7	1	001111 001303	2	040002 040053
ALOG$/FOR51	1	001304 001421	2	040054 040114
CSIG6V/SC4020	1	001422 001710	0	040115 040153
			2	BLANK$COMMON
CERMRK/SC4020	1	001711 001740	0	040154 040167
			2	BLANK$COMMON
CLABLV/SC4020	1	001741 003003	0	040170 040276
			2	BLANK$COMMON
NRWND$/FOR50	1	003004 003063	2	040277 040310
NAEF$/MSFCS5	1	003064 003320	2	040311 040331
NFTCH$/FOR57	1	003321 003620	2	040332 040367
CYMODV/SC4020	1	003621 003646	0	040370 040376
			2	BLANK$COMMON
CACCMY/SC4020	1	003647 003670	0	040377 040407
			2	BLANK$COMMON
CXMODV/SC4020	1	003671 003716	0	040410 040416
			2	BLANK$COMMON
CONCAT/MSFC	1	003717 004070	0	040417 040440
SETINT/SC4020	1	004071 004123	0	040441 040446
			2	BLANK$COMMON
CHOLLV/SC4020	1	004124 004206	0	040447 040463
			2	BLANK$COMMON
CHONLN/SC4020	1	004207 004614	0	040464 040535
			2	BLANK$COMMON
CLINRV/SC4020	1	004615 005470	0	040536 040651
	3	GGG	2	BLANK$COMMON
CYSCLV/SC4020	1	005471 005673	0	040652 040700
	3	GGG	2	BLANK$COMMON
CXSCLV/SC4020	1	005674 006076	0	040701 040727
	3	GGG	2	BLANK$COMMON
CLRNEV/SC4020	1	006077 006253	0	040730 040742

TABLE 2. (Continued)

			2	BLANK$COMMON
CERRLN/SC4020	1	006254 006363	0	040743 040760
			2	BLANK$COMMON
CSETCV/SC4020	1	006364 006424	0	040761 040771
			2	BLANK$COMMON
CSETMV/SC4020	1	006425 006503	0	040772 041006
			2	BLANK$COMMON
CXAXIS/SC4020	1	006504 006707	0	041007 041042
			2	BLANK$COMMON
VCHARV/SC4020	1	006710 007153	0	041043 041062
RITE2V/SC4020	1	007154 007404	0	041063 041111
BPLOTK/SC4020	1	007405 007465	0	041112 041471
			2	BLANK$COMMON
CFRAM/SC4020	1	007466 007705	0	041472 041556
	3	GGG	2	BLANK$COMMON
CCAMRA/SC4020	1	007706 007757	0	041557 041570
	3	GGG	2	BLANK$COMMON
TABLIV/SC4020			0	041571 042111
NBDCV$/FOR57	1	007760 010113	2	042112 042151
NFTY$/FOR	1	010114 010136		
NCLOS$/MSFC57	1	010137 010305	2	042152 042177
NBBLK$/MSFC57	1	010306 010474	0	042200 042201
NBSBL$/FOR	1	010475 010532		
NUPOA$/FOR	1	010533 010566		
NBFOO$/FOR			2	042202 044403
NCNVT$/FOR57	1	010567 011021	2	044404 044473
NININ$/MSFC55	1	011022 011253	2	044474 044515
CPLOTV/SC4020	1	011254 011436	0	044516 044554
	3	GGG	2	BLANK$COMMON
CLINEV/SC4020	1	011437 012022	0	044555 044633
			2	BLANK$COMMON
YSCLVI/SC4020	1	012023 012141	0	044634 044644
	3	GGG	2	BLANK$COMMON
XSCLVI/SC4020	1	012142 012257	0	044645 044655
	3	GGG	2	BLANK$COMMON
CAPLOT/SC4020	1	012260 012533	0	044656 044731
	3	GGG	2	BLANK$COMMON
CAPRNV/SC4020	1	012534 012616	0	044732 044750
			2	BLANK$COMMON
CPRNTV/SC4020	1	012617 013210	0	044751 045005
	3	GGG	2	BLANK$COMMON
CGRDIV/SC4020	1	013211 014115	0	045006 045121
	3	GGG	2	BLANK$COMMON
COXDYV/SC4020	1	014116 014710	0	045122 045206
			2	BLANK$COMMON
CBRITV/SC4020	1	014711 015012	0	045207 045222
	3	GGG	2	BLANK$COMMON
CMARGN/SC4020	1	015013 015071	0	045223 045264
			2	BLANK$COMMON
CNULNK/SC4020	1	015072 015141	0	045265 045300
			2	BLANK$COMMON
BMDV/MSFC	1	015142 015222	0	045301 045311
			2	BLANK$COMMON
TRACE	1	015223 015346	0	045312 045317
			2	045320 045402
SPCFRM/MSFC			0	045403 045456

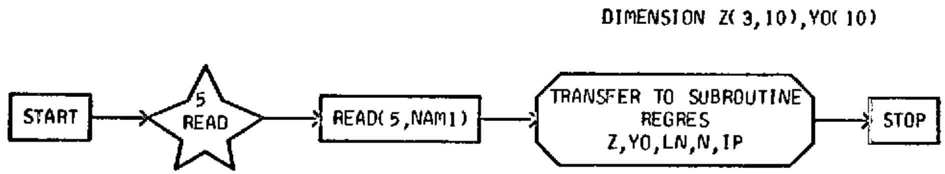
TABLE 2. (Concluded)

CTOBCU/MSFC	1	015347	015645	0	045457	045630
BOXLTR/CSC	1	015646	016500	0	045631	045647
CIDENT/SC4020	1	016501	017653	0	045650	046033
	3	GGG		2	BLANK$COMMON	
NOTIMS/MSFC55	1	017654	020204	2	046034	046044
NOUTS/MSFC57	1	020205	021166	2	046045	046076
NFMTS/MSFC57	1	021167	022074	2	046077	046115
NIOERS/MSFC57	1	022075	022254	2	046116	046240
NFCHK$/MSFC57	1	022255	023073	2	046241	046415
				4	046416	046467
NTABS/MSFC55				2	046470	046556
NLINPS/MSFC57	1	023074	024640	2	046557	046770
SQRTS/FOR55	1	024641	024700	2	046771	047002
NEXP55/FOR57	1	024701	024764	2	047003	047012
GGG (COMMON BLOCK)					047013	047146
CQUIKL/SC4020	1	024765	025402	0	047147	047231
	3	GGG		2	BLANK$COMMON	
IDENT/SC4020	1	025403	026645	0	047232	051027
H$MONITOR/MSFC55	1	026646	027755	2	051030	051614
NIERS/FOR52	1	027756	030040	2	051615	051744
NOBUFS/FOR51	1	030041	030100			
ERUS/MSFC55						
NERRS/FOR57	1	030101	030435	2	051745	052121
BLANK$COMMON (COMMON BLOCK)						
DMATML	1	030436	030633	0	052122	052174
				2	BLANK$COMMON	
INVRT	1	030634	031251	0	052175	052530
				2	BLANK$COMMON	
IOWR	1	031252	031254			
RDWT	1	031255	031656	0	052531	052657
				2	BLANK$COMMON	
REGRES	1	031657	033166	0	052660	054541
				2	BLANK$COMMON	
MAIN	1	033167	033207	0	054542	054642
				2	BLANK$COMMON	

SYS$*RLIB$. LEVEL MS7-5

END OF COLLECTION - TIME 3.518 SECONDS

@XQT AA



D I M E N S I O N E D   V A R I A B L E S

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
Y	10	RESID	10	Z	3,10	SMALLZ	IDIM	SHAT	9,9
ZBAR	3,3	YO	10	BZ	10	B	3,3	SVHAT	9,1
SMALZ1	IDIM	SMALZ2	IDIM	SH	9,9	XX	10	ADARY	22
PMAX	10	FLDX	12	FLDY	12				

Figure 2. Operation of the computer program (noncross product).

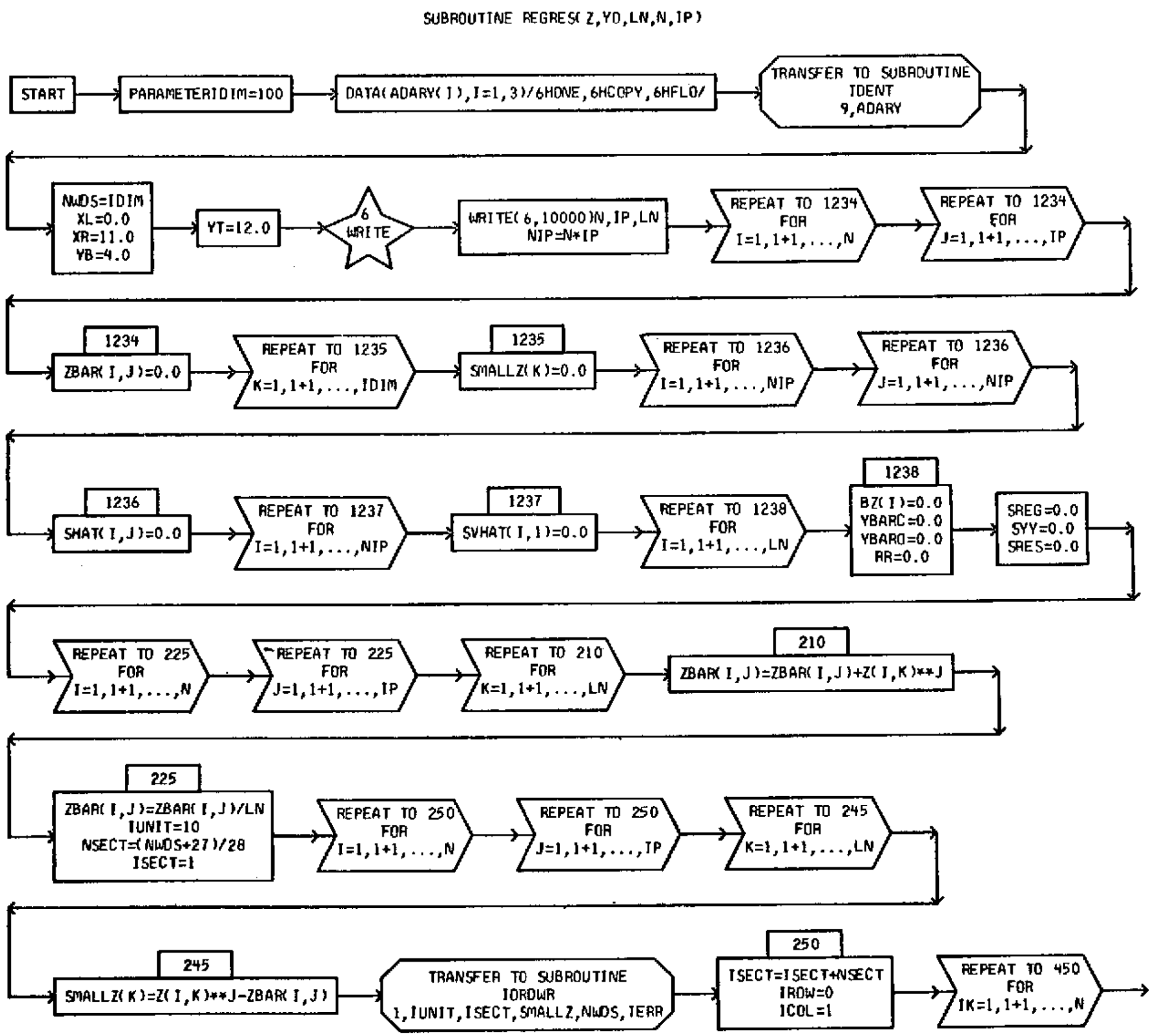


Figure 2. (Continued)

SUBROUTINE REGRES(Z,Y0,LN,N,IP)

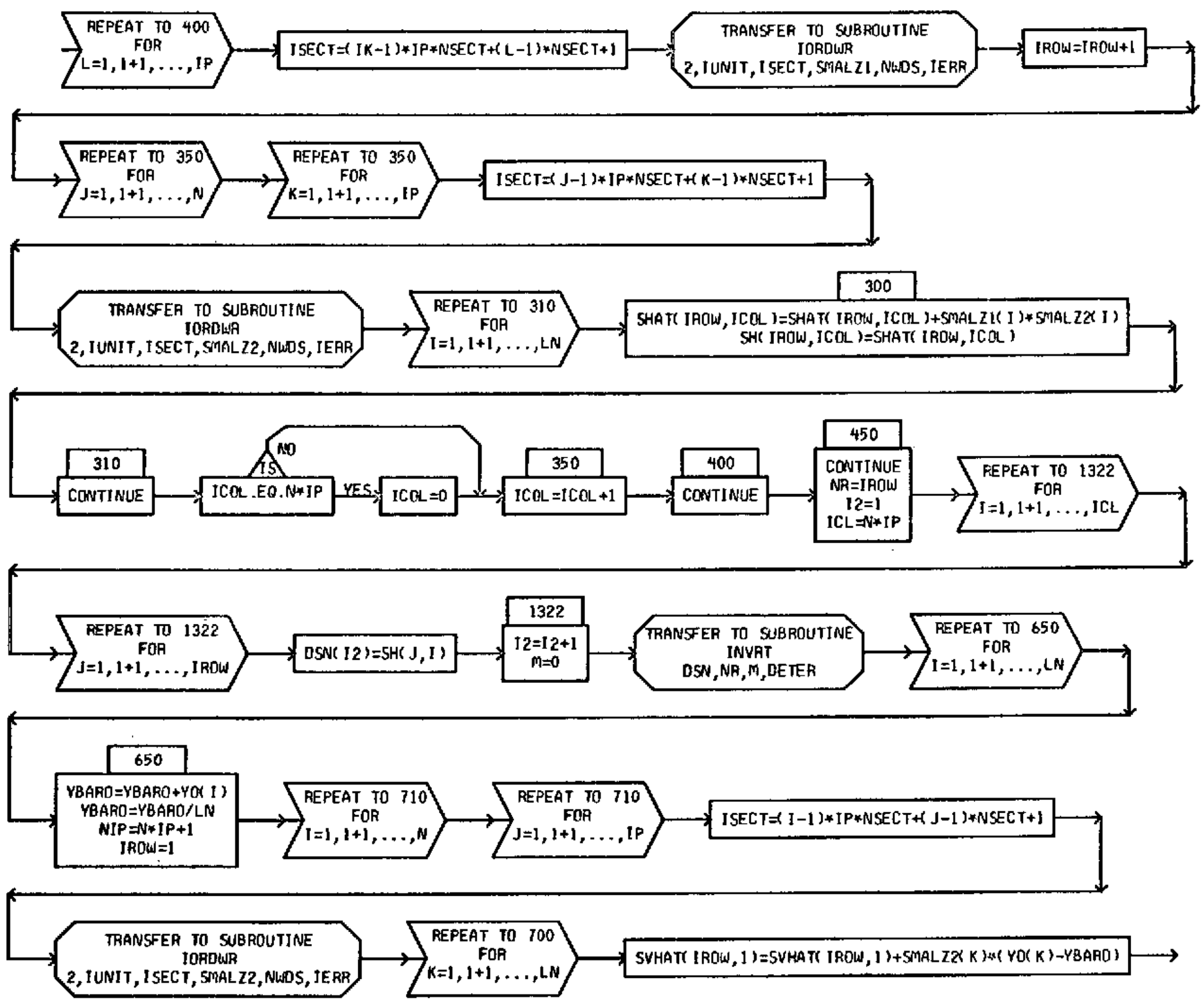


Figure 2. (Continued)

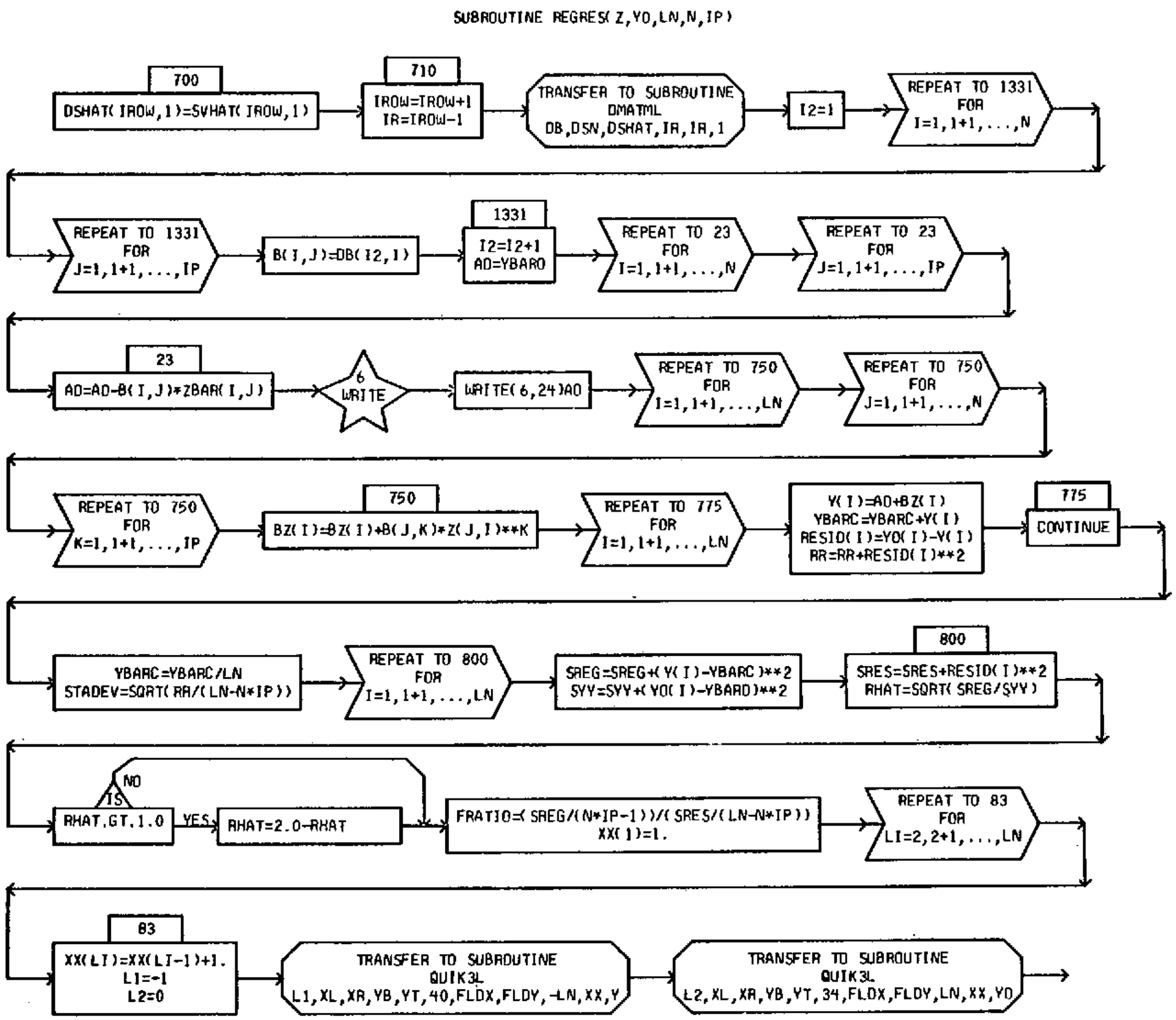


Figure 2. (Continued)

SUBROUTINE REGRES(Z,YO,LN,N,IP)

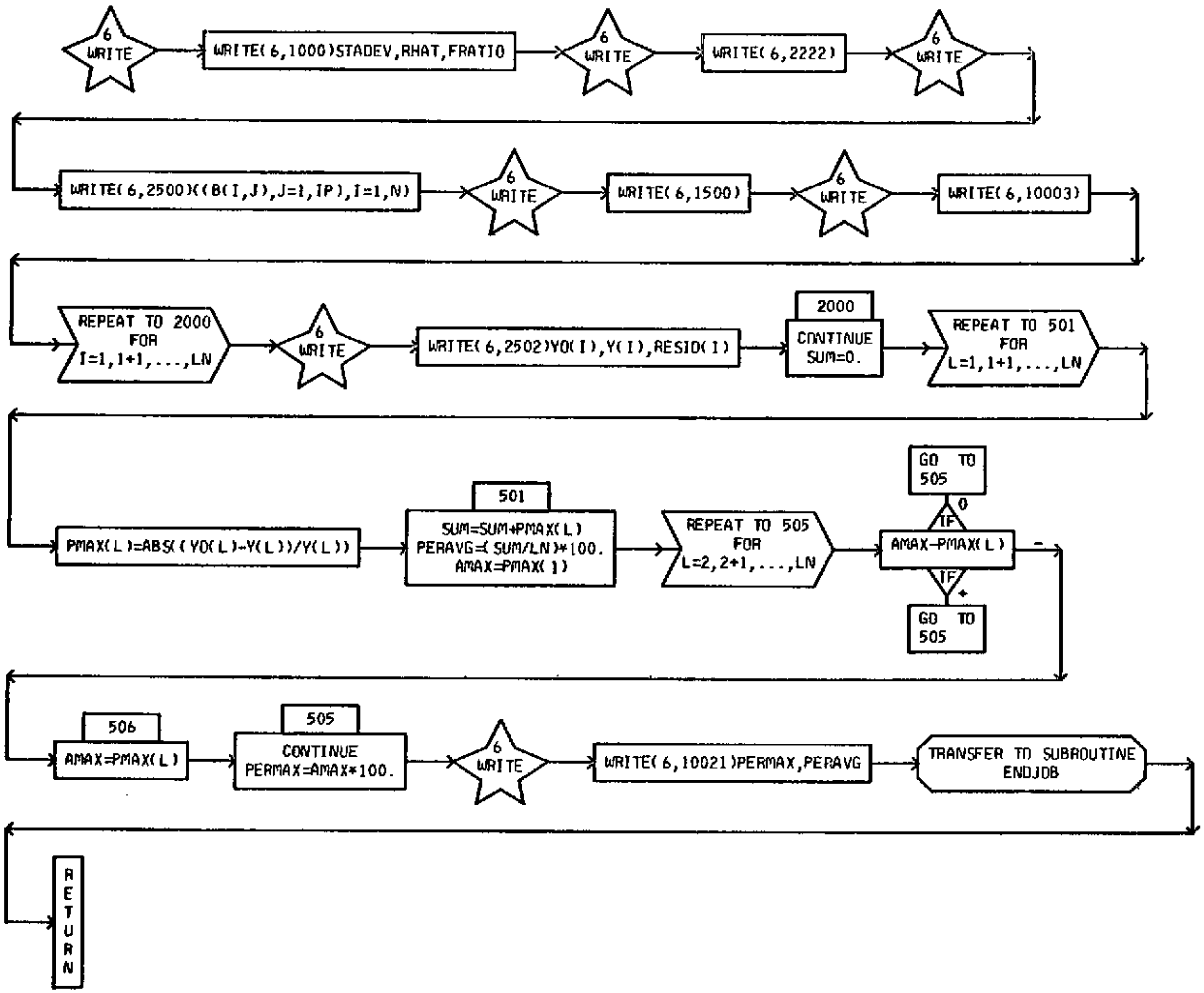


Figure 2. (Continued)

GENERAL FORTRAN I/O PACKAGE   EXEC VIII   APRIL,1970

READ/WRITE MAG TAPE OR DRUM IN FORTRAN OR NON-FORTRAN FORMAT.

D I M E N S I O N E D   V A R I A B L E S

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
ARRAY	1	ICWD	3	IFLD	29	KODE	6	IPACKT	8
NTWD	3								

Figure 2. (Continued)

SUBROUTINE RDWT

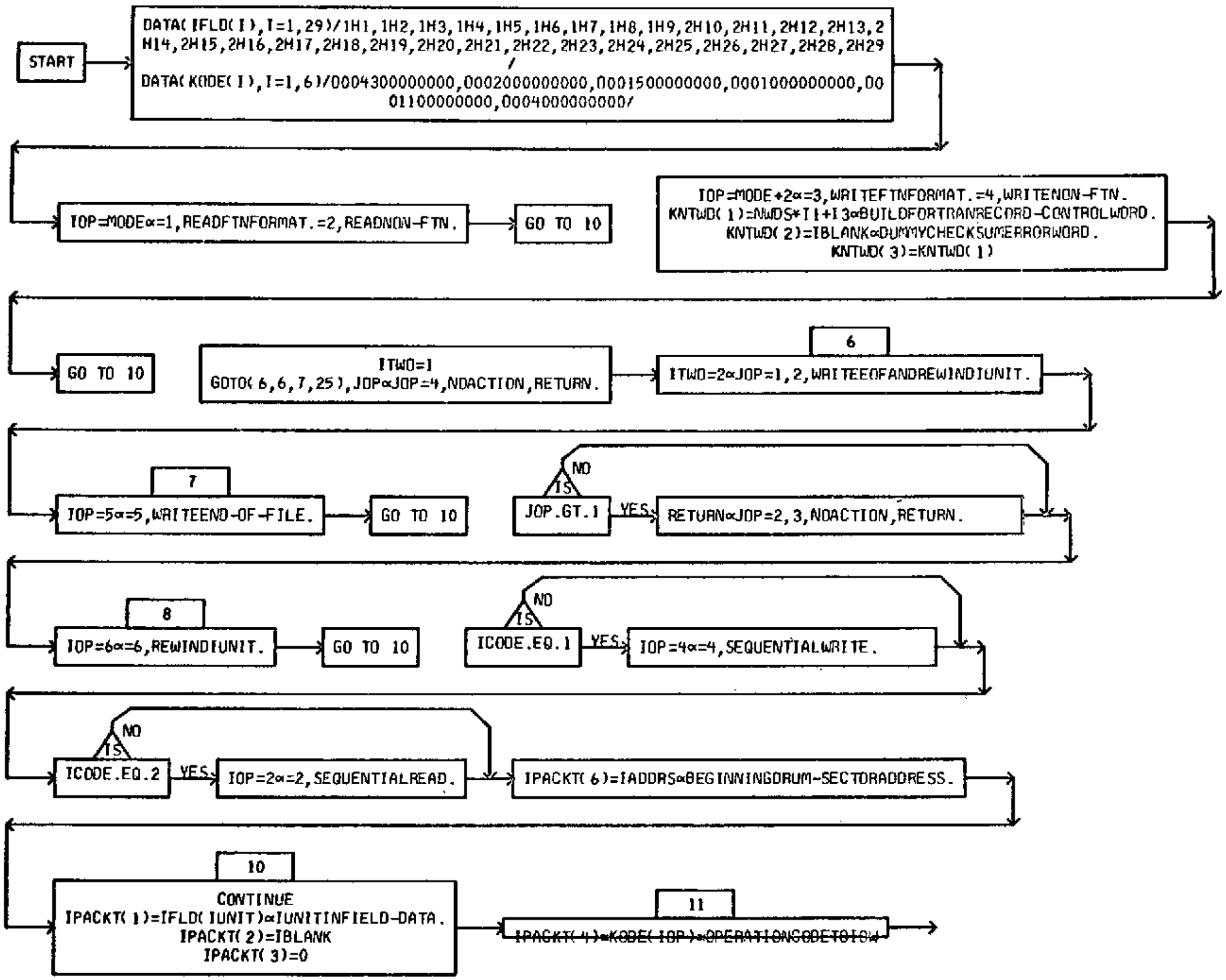


Figure 2. (Continued)

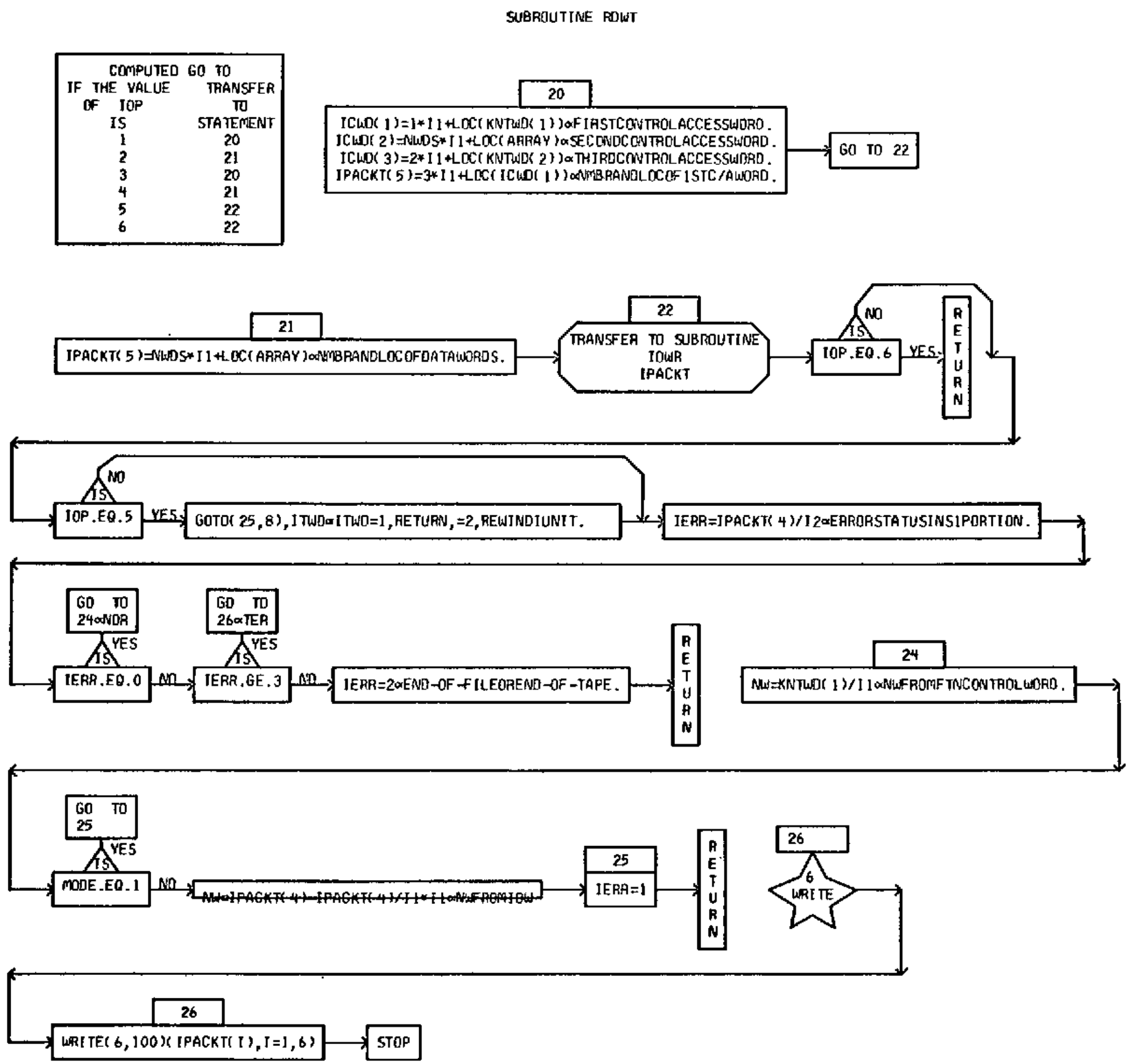


Figure 2. (Continued)

D I M E N S I O N E D   V A R I A B L E S

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
IPIV	IDIM	INDEX	IDIM,2						

Figure 2. (Continued)

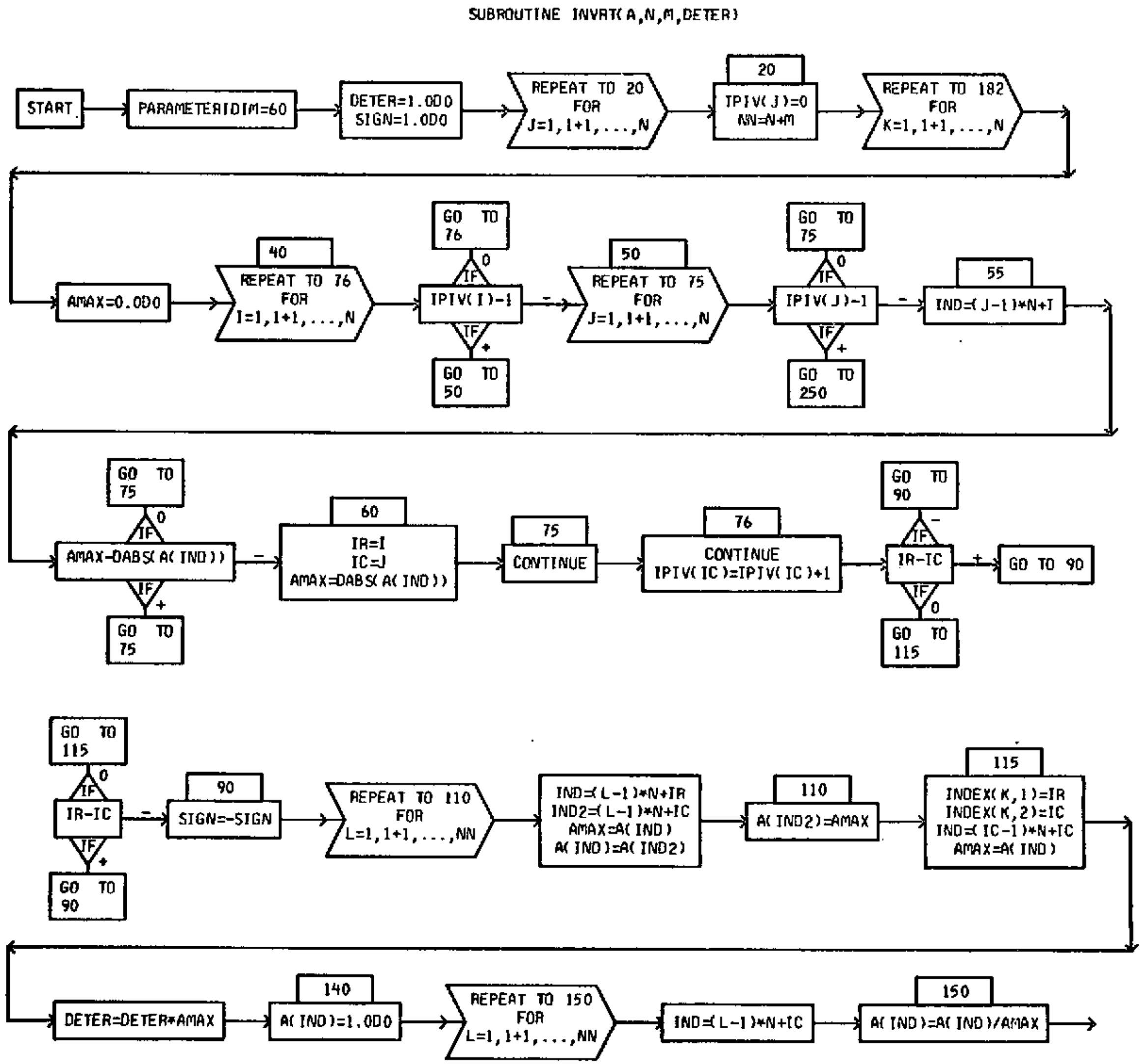


Figure 2. (Continued)

SUBROUTINE INVRT(A,N,M,DETER)

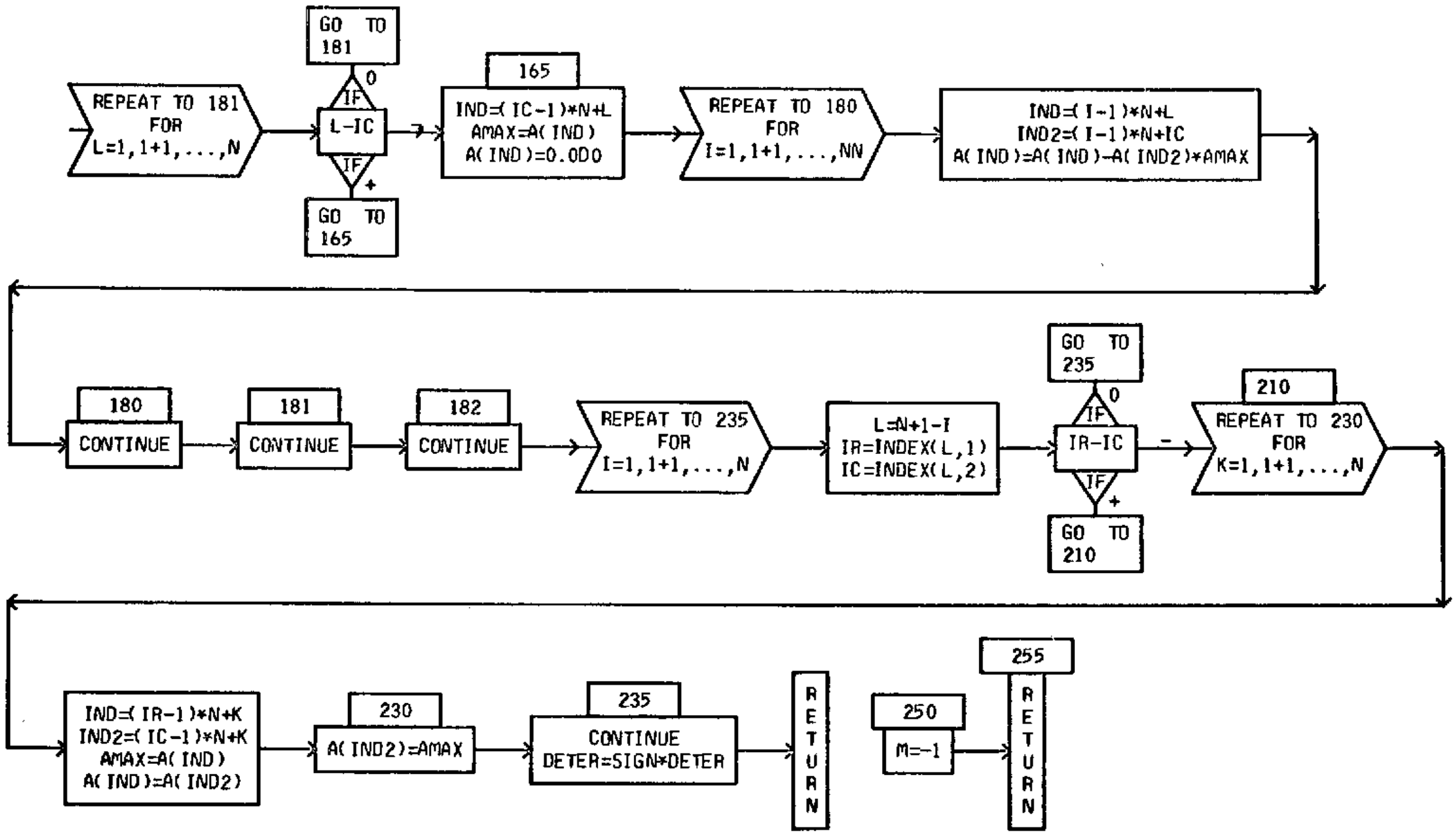


Figure 2. (Continued)

D I M E N S I O N E D   V A R I A B L E S

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
C	1	A	1	B	1				

SUBROUTINE DMATML(C,A,B,M,N,K)

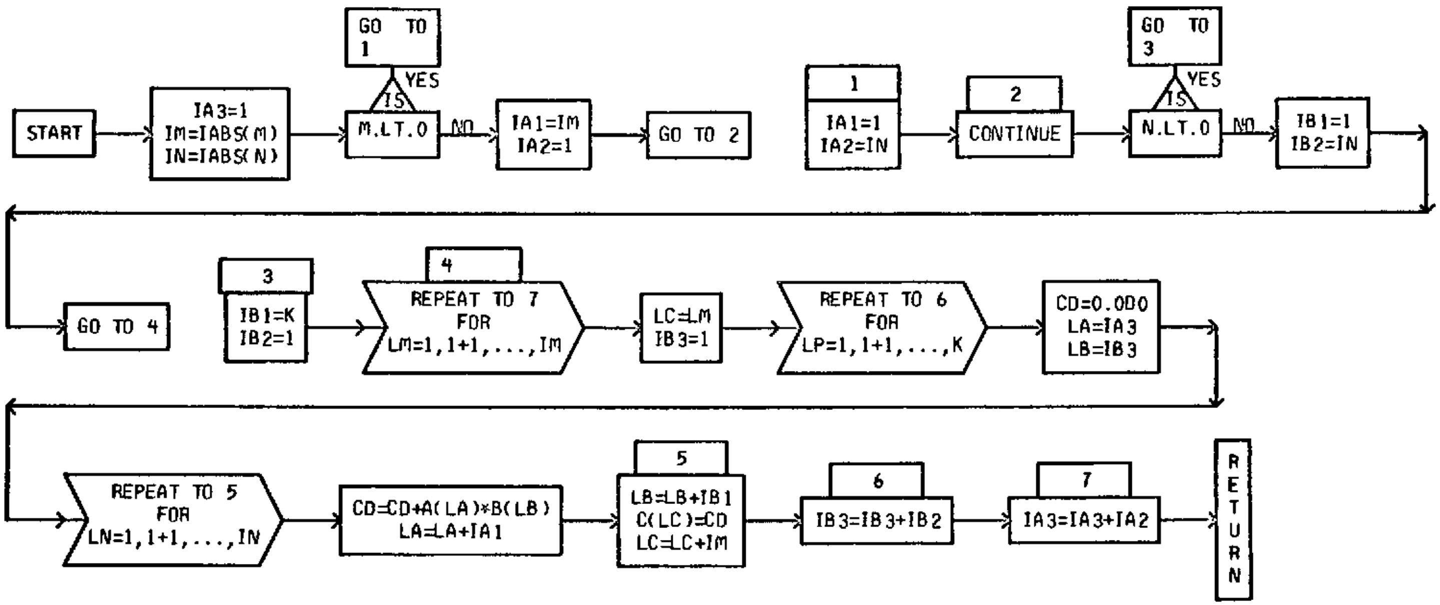


Figure 2. (Concluded)

Also included in the output are the number of independent variables N, the degree of the model equation P, and the number of input data points (sets) n denoted in the output as LN. Further, the significance of the estimated regression equation is indicated by the standard deviation, the multiple correlation coefficient, the F ratio, the maximum percent error, and the average percent error.

The values of the dependent variable are also shown for the input data and for the computed values obtained from the fitted expression for each set of data input, as well as for the residual difference in the input values and the computed values.

Plotted results are also obtained which compare the input and the computed dependent variables as ordinates to the point interval along the abscissa (which is normally one of the physical independent variables, but which can be a unit indication of each data set in order of input to the program). The computed points are connected by straight lines and are plotted with * symbols. The input points are plotted with + symbols and these points are not connected with lines.

D. Illustrative Problem

For the purpose of illustrating the use of the program and its capabilities the following arbitrary equation was selected for use:

$$
\begin{aligned}
Y = 9 &- 6X_1 + 7X_1^2 + 5X_1^3 \\
&- 4X_2 + 1X_2^2 + 7X_2^3 \\
&- 6X_3 + 7X_3^2 - 6X_3^3
\end{aligned}
\qquad (37)
$$

From equation (37) the following set of input data (Table 3) was developed containing exact dependent variable values (to three decimal places) for arbitrary values of the three independent variables.

TABLE 3. INPUT DATA, DEPENDENT VARIABLE VALUES

Data Point Number	Y	X_1	X_2	X_3
1	5.719	0.5	0.4	0.4
2	7.111	0.6	0.3	0.2
3	5.216	0.2	0.6	0.6
4	5.592	0.2	0.6	0.5
5	10.915	0.9	0.0	0.5
6	7.569	0.7	0.0	0.6
7	7.450	0.3	0.9	0.7
8	9.618	0.9	0.3	0.6
9	5.019	0.5	0.6	0.7
10	8.021	0.3	0.8	0.2

The input for this sample problem was read in through namelist NAM1 as follows:

```
Col. 2
     $NAM1
          Z = 0.5, 0.4, ..., 0.2,
          YO = 5.719, 7.111, ..., 8.021,
          LN = 10,
          N = 3,
          IP = 3,
     $
```

The computed results are shown in Table 4 for this example problem and the plotted results are shown in Figure 3.

The plot graphs the point intervals on the X axis against the exact and computed dependent variables. The exact dependent variables are plotted with (+) and the points are not connected with lines. The computed dependent variables are plotted with (*) and the points are connected by straight line segments.

Inspection of the digital and plotted results reveals that the computed regression relation yields an almost exact representation of the input data for this arbitrary case. Experience with many sets of physical data have also shown excellent results.

TABLE 4. OUTPUT DATA LISTING FOR ILLUSTRATIVE PROBLEM (CASE WITHOUT CROSS PRODUCTS)

```
N=   3    P=   3    LN=  10

A0=   .899989665+01

STANDARD DEVIATION=   .149373568-04    MULTIPLE CORRELATION COEFFICIENT=   .999999970+00    F RATIO=   .190450291+11

B COEFFICIENTS

     -.599952072+01     .699866408+01     .500090784+01    -.400001669+01     .999836586+00

     .700009459+01     -.599887073+01     .699684674+01    -.599740386+01
```

YO OBSERVED	YC COMPUTED	YO-YC RESIDUAL
.571899998+01	.571900237+01	-.238418579-05
.711100000+01	.711100280+01	-.280141830-05
.521600002+01	.521600103+01	-.101327896-05
.559200001+01	.559199882+01	.119209290-05
.109150000+02	.109150084+02	-.846385956-05
.756900001+01	.756899226+01	.774860382-05
.744999999+01	.745000219+01	-.220537186-05
.961800003+01	.961799359+01	.643730164-05
.501899999+01	.501900327+01	-.327825546-05
.802100003+01	.802099574+01	.429153442-05

MAXIMUM PERCENT ERROR=   .102372490-03   AVERAGE PERCENT ERROR=   .517097560-04

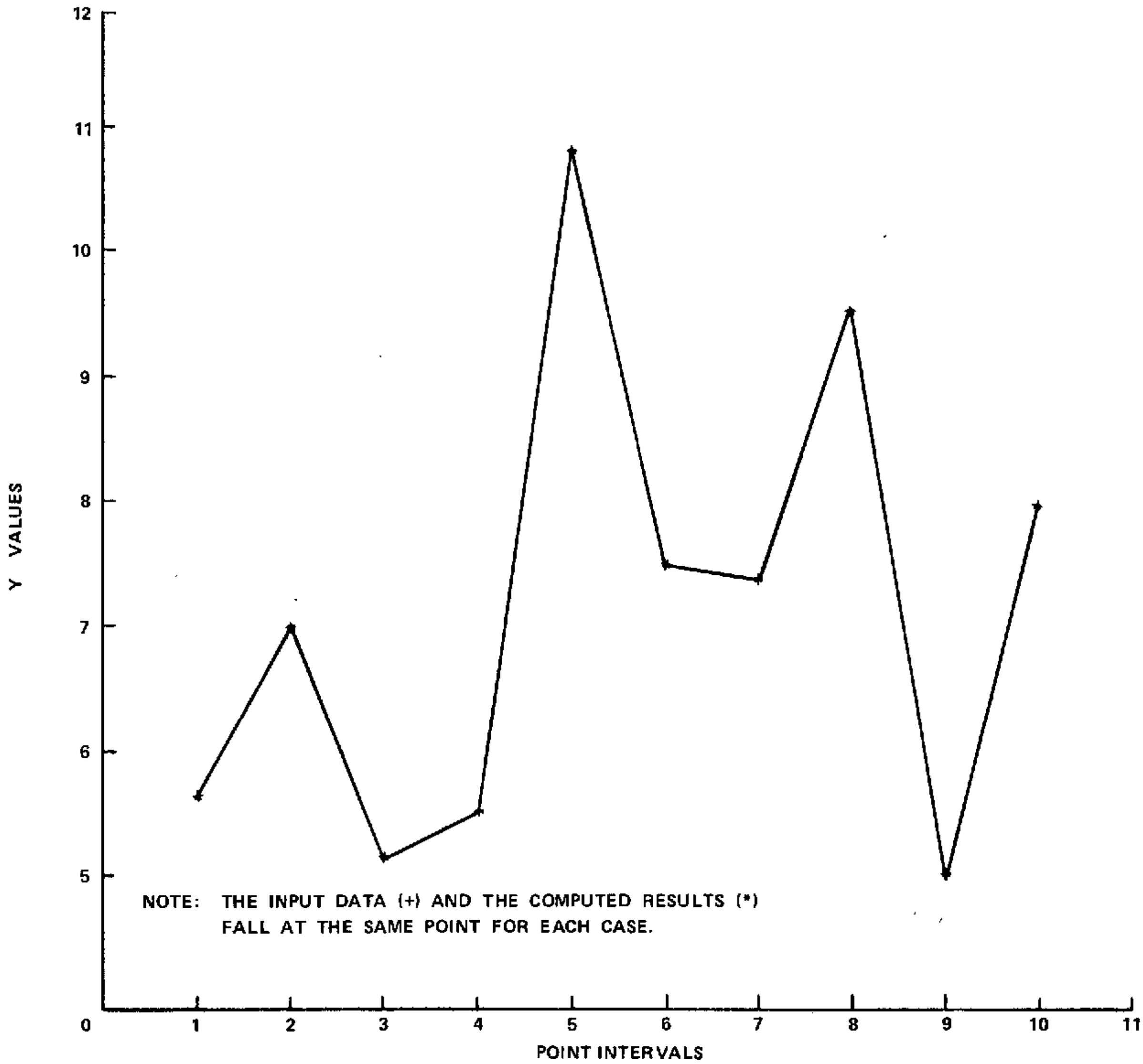


Figure 3. Comparison of input and computed values for illustrative problem without cross products.

APPENDIX B

MULTIPLE REGRESSION PROGRAM FOR CASE WITH CROSS PRODUCTS

A. Input Data

The regression subroutine is called with the following statement:

CALL REGRES(Z, YO, LN, N, IP)

where:

Z is a two-dimensional array containing the independent variables used. The first subscript denotes the independent variable and the second subscript denotes the data set.

YO is an array containing the dependent variables which correspond to the independent variables.

LN is the number of data sets.

N is the number of independent variables used.

IP is the degree of the polynomial curve fit.

These arguments must be assigned a value in a driver program.

The regression subroutine incorporates several special routines: RDWT, IOWR, INVRT, and DMATML.

RDWT is a general FORTRAN I/O package which is capable of reading and/or writing on magnetic tape or drum in FORTRAN or non-FORTRAN format.

IOWR is an assembly language I/O routine used by RDWT.

INVRT is a double precision matrix inversion and simultaneous equations solver. The call to this routine is:

CALL INVRT(A,N,M,DETER)

where:

A is the input matrix for inversion or augmented matrix for simultaneous equations.

N is the order of the coefficient matrix.

M=0 for inversion only; otherwise, M is the number of constant vectors.

DETER is the determinant of the coefficient matrix.

DMATML is a double precision matrix multiplication routine with transpose options. The call to this routine is:

CALL DMATML(C,A,B,M,N,K)

where:

C is the product of matrix A times matrix B.

A is the first input matrix,

B is the second input matrix,

M is the number of rows of A,

N is the number of rows of B, and

K is the number of columns of B.

Certain variables have dimension changes that vary depending on the number of data points (LN), the number of independent variables (N), and the degree of the polynomial curve fit (IP). These will be dimensioned as follows:

DSN(NCL2), DSVHAT(NCL,1), DB(NCL,1), X(LN), Z(N,LN), ZBAR(N,IP), ZZBAR(NN,IP), SHAT(NCL,NCL), YO(LN), SVHAT(NCL,1), YC(LN), BZ(LN), RESID(LN), B(N,IP), A(NN,IP)

where:

LN,N,IP are as defined above and
NCL is computed as follows:

NCL = N*IP + N!/[2*(N-2)!]

NCL2 is NCL raised to the second power.
NN is N minus 1.

SMALLZ, SMALZ1, and SMALZ2 should be dimensioned greater than the number of data points and the dimension is set in a PARAMETER statement; for example,

PARAMETER IDIM = 100

B. Program

In order to run this program on the UNIVAC 1108, the deck is set up as follows:

```
@RUN,//T JBNAME,320590,UDARBYBIN406,01.100
@ASG,T 10,F/1/POS/2
@FOR,IS MAIN, MAIN
    (Source deck for inputting data)
@FOR,IS REGRES, REGRES
    (Source deck)
@FOR,IS RDWT, RDWT
    (Source deck)
@ASM,IS IOWR, IOWR
    (Source deck)
@FOR,IS INVRT, INVRT
    (Source deck)
@FOR,IS DMATML,DMATML
    (Source deck)
@MAP,I AA, AA
    LIB SYS$*MSFC$.
@XQT AA
    (Input data)
@FIN
```

as illustrated in Figure 1.

A complete program listing for the case with linear cross products is shown in Table 5 and a flow chart indicating the operation of this computer program is indicated in Figure 4.

C. Output Data

The desired coefficients of the model equation are outputted in E notation where BHAT is the computed intercept A_O of the fitted polynomial expression and the B coefficients are printed out in ascending order of degree (P) with the first P coefficients indicating the b coefficients of the first independent variable, the second set of P coefficients indicating the b coefficients of the second independent variable, etc. The A coefficient designation represents the C coefficients of the various cross products in the following order:

$C_{12}, C_{13}, \ldots, C_{1N}$

$C_{23}, C_{24}, \ldots, C_{2N}$

$C_{N-1,N}$

Also included in the output are the number of independent variables N, the degree of the model equation P, and the number of input data points (sets) n denoted in the output as LN. Further, the significance of the estimated regression equation is indicated by the standard deviation, the multiple correlation coefficient, the F ratio, the maximum percent error, and the average percent error.

The values of the dependent variable are also shown for the input data and for the computed values obtained from the fitted expression for each set of data input, as well as for the residual difference in the input values and the computed values.

Plotted results are also obtained which compare the input and the computed dependent variables as ordinates to the point interval along the abscissa (which is normally one of the physical independent variables, but which can be a unit indication of each data set in order of input to the program). The computed points are connected by straight lines and are plotted with * symbols. The input points are plotted with + symbols and the points are not connected with lines.

D. Illustrative Problem

For the purposes of illustrating the use of the cross product program and its capabilities, the following arbitrary equation was selected for use:

$$\begin{aligned} Y = {} & 9 - 6X_1 + 7X_1^2 + 5X_1^3 \\ & - 4X_2 + 1X_2^2 + 7X_2^3 \\ & - 6X_3 + 7X_3^2 - 6X_3^3 \\ & + 5X_1X_2 + 5X_1X_3 - 6X_2X_3 \end{aligned} \tag{38}$$

TABLE 5. PROGRAM LISTING (WITH CROSS PRODUCTS)

```
VDARBY
BIN406
011673
020543
```

6RUN,771 [illegible]CF,320570,VDARBYBIN406,03,1000

TABLE 5. (Continued)

```
@MSG,N              REMOVE AT CARD READER          01-16-73       CLOCK NO.   9353

@ASG,T 10,F/1/POS/2

@FOR,IS MAIN,MAIN
FOR 009-01/16-13:51 (,0)

MAIN PROGRAM

STORAGE USED: CODE(1) 000021; DATA(0) 000125; BLANK COMMON(2) 000000

EXTERNAL REFERENCES (BLOCK, NAME)

 0003    REGRES
 0004    NINTR$
 0005    NFNL$
 0006    NSTOP$

STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

 0000 I 000076 IP       0000 I 000074 LN       0000 I 000075 N        0000    000101 NAM1      0000 R 000055 YO
 0000 R 000000 Z


00101       1*          DIMENSION Z(3,15),YO(15)
00103       2*          NAMELIST/NAM1/Z,YO,LN,N,IP
00104       3*          READ(5,NAM1)
00107       4*          CALL REGRES(Z,YO,LN,N,IP)
00110       5*          STOP
00111       6*          END
     END OF COMPILATION:        NO  DIAGNOSTICS.
```

TABLE 5. (Continued)

```
@FOR,IS REGRES,REGRES
MSG 009-01/16-13:51 (,0)

   SUBROUTINE REGRES     ENTRY POINT 001617

   STORAGE USED: CODE(1) 001665; DATA(0) 002140; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003    IDENT
    0004    IORDER
    0005    INVRT
    0006    DMATML
    0007    NOTK3L
    0010    ENWJ$B
    0011    NWDU$
    0012    NIO2$
    0013    NEXP6$
    0014    SQRT
    0015    NIO1$
    0016    NSTOP$
    0017    NERR3$

   STORAGE ASSIGNMENT   (BLOCK, TYPE, RELATIVE LOCATION, NAME)
```

0000 001744 10000F	0000 001752 10006F	0000 001755 10009F	0000 001776 10010F	0000 002002 10011F
0000 002005 10012F	0000 002011 10014F	0000 002013 10026F	0000 002024 10027F	0000 002027 10033F
0001 000115 142G	0001 000116 145G	0001 000126 153G	0001 000127 156G	0001 000137 164G
0001 000143 171G	0001 000144 174G	0001 000154 202G	0001 000160 207G	0001 000200 225G
0001 000202 230G	0001 000204 233G	0001 000240 244G	0001 000267 250G	0001 000270 253G
0001 000327 266G	0001 000331 271G	0001 000334 274G	0001 000375 306G	0001 000422 312G
0001 000424 315G	0001 000462 327G	0001 000505 340G	0001 000542 351G	0001 000556 357G
0001 000575 363G	0001 000623 376G	0001 000640 405G	0001 000654 411G	0001 000676 422G
0001 000677 425G	0001 000742 442G	0001 000743 445G	0001 000764 456G	0001 000777 462G
0001 001015 472G	0001 001016 475G	0001 001033 504G	0001 001235 505L	0001 001046 510G
0001 001316 513L	0001 001066 522G	0001 001070 525G	0001 001072 530G	0001 001121 537G
0001 001126 543G	0001 001147 547G	0001 001167 556G	0001 001205 567G	0001 001227 577G
0001 001246 611G	0001 001240 671G	0001 001333 634G	0001 001422 660G	0001 001423 662G
0001 001444 675G	0001 001457 701G	0001 001501 716G	0001 001521 734G	0000 R 001653 A
0000 R 000522 ARARY	0000 R 001731 AMAX	0000 R 001642 B	0000 R 001726 BHAT	0000 R 001604 BZ
0000 D 000472 DF	0000 D 000000 DETER	0000 D 000002 DSN	0000 D 000442 DSVHAT	0000 R 000567 FLDX
0000 R 000603 FLDY	0000 R 001737 FRATIO	0000 I 001662 I	0000 I 001720 ICOL	0000 I 001711 IERR
0000 002066 INJP$	0000 I 001717 IROW	0000 I 001740 IS	0000 I 001710 ISECT	0000 I 001721 ISECT1
0000 I 001706 IUNIT	0000 I 001661 IZERO	0000 I 001722 I2	0000 I 001724 I21	0000 I 001672 J
0000 I 001704 K	0000 I 001734 KHAT	0000 I 001712 KK	0000 I 001705 L	0000 I 001743 LI
0000 I 001741 L1	0000 I 001742 L2	0000 I 001725 M	0000 I 001716 NCOL	0000 I 001713 NFACT
0000 I 001714 NFACT2	0000 I 001733 NI	0000 I 001671 NIP	0000 I 001670 NN	0000 I 001723 NR
0000 I 001707 NSECT	0000 I 001663 NADS	0000 I 001715 N2	0000 R 001730 PERAVG	0000 R 001732 PERMAX
0000 R 000550 PMAX	0000 R 001701 R	0000 R 001623 RESID	0000 R 001736 RHAT	0000 R 001674 RR
0000 R 001331 SHAT	0000 R 000655 SMALLZ	0000 R 001021 SMALZ1	0000 R 001165 SMALZ2	0000 R 001675 SREG
0000 R 001676 SRES	0000 R 001703 SS	0000 R 001735 STADEV	0000 R 001727 SUM	0000 R 001551 SVHAT
0000 R 001700 SYY	0000 R 001677 VAR	0000 R 000617 X	0000 R 001664 XL	0000 R 001665 XR

TABLE 5. (Continued)

```
0000 R 001666 YB        0000 R 001673 YBAR      0000 R 001565 YC        0000 R 001667 YT        0000 R 001702 YY
0000 R 000636 ZBAR      0000 R 000647 ZZBAR

00101            SUBROUTINE REGRES(Z,YO,LN,N,IP)
00103            PARAMETER IDIM=100
00104            DOUBLE PRECISION DETER
00105            DOUBLE PRECISION DSN(144),DSVHAT(12,1),DB(12,1)
00106            DIMENSION ADARY(22)
00107            DIMENSION PMAX(15)
00110            DIMENSION FLOX(12),FLOY(12)
00111            DIMENSION X(15)
00112            DIMENSION Z(3,15),ZBAR(3,3),ZZBAR(2,3),SMALLZ(IDIM),
00112           *SMALZ1(IDIM),SMALZ2(IDIM),SHAT(12,12),YO(15),SVHAT(12,1),
00112           *YC(15),BZ(15),RESID(15),B(3,3),A(2,3)
00113            DATA IZERO/0/
00115            DATA FLOY/72H                                    Y VALUES
00115           *                  /
00117            DATA FLOX/72H                                    POINT INTERVALS
00117           *                  /
00121            DATA (ADARY(I),I=1,3)/6HONE   ,6H  COPY,6HFLO   /
00123            CALL IDENT(9,ADARY)
00124            NWDS=IDIM
00125            XL=0.
00126            XR=16.
00127            YB=1.
00130            YT=17.
00131            WRITE(6,10000) N,IP,LN
00136      10000 FORMAT(1H1,2HN=I4,3X,2HP=I4,3X,3HLN=I4)
00136      C     INITIALIZE VARIABLES
00137            NN=N-1
00140            NIP=N*IP
00141            DO 15 I=1,N
00144            DO 15 J=1,IP
00147         15 ZBAR(I,J)=0.0
00152            DO 16 I=1,NN
00155            DO 16 J=1,IP
00160         16 ZZBAR(I,J)=0.0
00163            DO 7 I=1,IDIM
00166          7 SMALLZ(I)=0.0
00170            DO 8 I=1,NIP
00173            DO 8 J=1,NIP
00176          8 SHAT(I,J)=0.0
00201            DO 9 I=1,NIP
00204          9 SVHAT(I,1)=0.0
00206            DO 5 I=1,LN
00211          5 BZ(I)=0.0
00213            YBAR=0.0
00215            RR=0.0
00215            SPLG=0.0
00216            SRES=0.0
00217            VAR=0.0
00220            SYY=0.0
00221            N=0.0
```

TABLE 5. (Continued)

```
00222             YY=0.0
00223             SS=0.0
00223       C     COMPUTE ZBAR
00224             DO 125 I=1,N
00227             DO 125 J=1,IP
00232             DO 100 K=1,LN
00235         100 ZBAR(I,J)= ZBAR(I,J)+ Z(I,K)**J
00237         125 ZBAR(I,J)= ZBAR(I,J)/LN
00237       C     COMPUTE ZZBAR
00242             K=1
00243             DO 150 I=1,NN
00246             K=I+1
00247             DO 150 J=K,N
00252             DO 130 L=1,LN
00255         130 ZZBAR(I,J)=ZZBAR(I,J)+Z(I,L)*Z(J,L)
00257         150 ZZBAR(I,J)=ZZBAR(I,J)/LN
00257       C     COMPUTE SMALLZ'S
00262             IUNIT=10
00263             NSECT=(NWDS+27)/28
00264             ISECT = 1
00265             DO 200 I=1,N
00270             DO 200 J=1,IP
00273             DO 175 K=1,LN
00276         175 SMALLZ(K)=Z(I,K)**J-ZBAR(I,J)
00300             CALL IORDWR(1,IUNIT,ISECT,SMALLZ,NWDS,IERR)
00301         200 ISECT=ISECT+NSECT
00304             KK=1
00305             DO 230 I=1,NN
00310             KK=KK+1
00311             DO 230 J=KK,N
00314             DO 225 K=1,LN
00317         225 SMALLZ(K)=Z(I,K)*Z(J,K)-ZZBAR(I,J)
00321             CALL IORDWR(1,IUNIT,ISECT,SMALLZ,NWDS,IERR)
00322         230 ISECT=ISECT+NSECT
00322       C     COMPUTE NUMBER OF COLUMNS
00325             NFACT=1
00326             DO 250 I=1,N
00331         250 NFACT=NFACT*I
00333             NFACT2=1
00334             N2=N-2
00335             IF(N2.EQ.0) N2=1
00337             DO 275 I=1,N2
00342         275 NFACT2=NFACT2*I
00344             NCOL=N*IP+(NFACT/(2*NFACT2))
00344       C     COMPUTE ELEMENTS OF THE SHAT MATRIX
00345             ISECT=1
00346             NSECT=(NWDS + 27)/28
00347             IROW=1
00350             DO 310 J=1,NCOL
00353             CALL IORDWR(2,IUNIT,ISECT,SMALZ1,NWDS,IERR)
00354             ICOL=1
00355             ISECT1=1
00356             DO 305 K=1,NCOL
00361             CALL IORDWR(2,IUNIT,ISECT1,SMALZ2,NWDS,IERR)
00362             DO 300 I=1,LN
00365         300 SHAT(IROW,ICOL)=SHAT(IROW,ICOL)+SMALZ1(I)*SMALZ2(I)
```

TABLE 5. (Continued)

```
00367          ICOL=ICOL+1
00370      305 ISECTI=ISECTI+NSECT
00372          IROW=IROW+1
00373      310 ISECT=ISECT+NSECT
00373    C     COMPUTE SVHAT
00375          DO 325 I=1,LN
00400      325 YBAR=YBAR+YO(I)
00402          YBAR=YBAR/LN
00403          ISECT=1
00405          DO 350 K=1,NCOL
00407          CALL IGRDAR(2,IUNIT,ISECT,SMALZI,NWDS,IERR)
00410          DO 330 I=1,LN
00413          SVHAT(K,1)=SVHAT(K,1)+SMALZI(I)*(YO(I)-YBAR)
00414      330 DSVHAT(K,1)=SVHAT(K,1)
00416      350 ISECT=ISECT+NSECT
00420          I2=1
00421          DO 375 I=1,NCOL
00424          DO 375 J=1,NCOL
00427          DSN(I2)=SHAT(I,J)
00430      375 I2=I2+1
00433          NR=NCOL
00434          I21=I2-1
00434    C     FIND SHAT MATRIX INVERSE
00435          M=0
00436          CALL INVRT(DSN,NR,M,DETER)
00436    C     COMPUTE B AND A COEFFICIENTS FOR THE ASSUMED MODEL
00437          CALL DMATML(DB,DSN,DSVHAT,NR,NR,1)
00440          I2=1
00441          DO 400 I=1,N
00444          DO 400 J=1,IP
00447          B(I,J)=DB(I2,1)
[illegible]      400 I2=I2+1
00453          I21=I2
00454          K=1
00455          DO 410 I=1,NN
00460          K=K+1
00461          DO 410 J=K,N
00464          A(I,J)=DB(I21,1)
00465      410 I21=I21+1
00470          BHAT=YBAR
00471          DO 420 I=1,N
00474          DO 420 J=1,IP
00477      420 BHAT=BHAT-B(I,J)*ZBAR(I,J)
00502          K=1
00503          DO 425 I=1,NN
00506          K=K+1
00507          DO 425 J=K,N
00512      425 BHAT=BHAT-A(I,J)*ZZBAR(I,J)
00515          WRITE(6,10006) BHAT
00520    10006 FORMAT(1H0,5HBHAT=E16.9)
00521          DO 450 I=1,LN
00524          DO 450 J=1,N
00527          DO 450 K=1,IP
00532      450 BZ(I)=BZ(I)+B(J,K)*Z(J,I)**K
00536          DO 475 I=1,LN
00541          KK=1
```

TABLE 5. (Continued)

```
00542               DO 475 J=1,NN
00545               KK=KK+1
00546               DO 475 L=KK,N
00551           475 BZ(I)=BZ(I)+A(J,L)*Z(J,I)*Z(L,I)
00551         C     COMPUTE THE Y POINTS
00555               DO 500 I=1,LN
00560               YC(I)=BHAT+BZ(I)
00561               RESID(I)=YO(I)-YC(I)
00562               RR=RR+RESID(I)**2
00563           500 CONTINUE
00565               SUM=0.
00566               DO 501 L=1,LN
00571               PMAX(L)=ABS((YO(L)-YC(L))/YC(L))
00572           501 SUM=SUM+PMAX(L)
00574               PERAVG=(SUM/LN)*100.
00575               AMAX=PMAX(1)
00576               DO 505 L=2,LN
00601               IF(AMAX-PMAX(L)) 506,505,505
00604           506 AMAX=PMAX(L)
00605           505 CONTINUE
00607               PERMAX=AMAX*100.
00610               DO 507 L=1,LN
00613               R=R+ABS(RESID(L))
00614               YY=YY+YO(L)
00615           507 CONTINUE
00616         C     COMPUTE STANDARD DEVIATION
00617               N2=0
00620               DO 510 I=1,NN
00623               N1=N-I
00624           510 N2=N2+N1
00626.              KHAT=N*IP+N2
00627               IF((LN-KHAT-1).LE.0) GO TO 513
00631               STADEV=SQRT(RR/(LN-KHAT-1))
00632           513 CONTINUE
00632         C     COMPUTE THE MULTIPLE CORRELATION COEFFICIENT, AND FRATIO
00633               DO 525 I=1,LN
00636               SREG=SREG+(YC(I)-YBAR)**2
00637               SYY=SYY+(YO(I)-YBAR)**2
00640               SS=SS+(YC(I)-YBAR)*(YO(I)-YC(I))
00641           525 SRES=SRES+RESID(I)**2
00643               RHAT=SREG/SYY
00644               FRATIO=(SREG/(KHAT-1))/(SRES/(LN-KHAT))
00645               WRITE(6,10009) STADEV,RHAT,FRATIO
00652         10009 FORMAT(1H0,19HSTANDARD DEVIATION=E16.9,4X,33HMULTIPLE CORRELATION
00652              *COEFFICIENT=E16.9,4X,8HF RATIO=E16.9)
00653               WRITE(6,10010)
00655         10010 FORMAT(1H0,14HB COEFFICIENTS)
00656               WRITE(6,10011) ((B(I,J),J=1,IP),I=1,N)
00667         10011 FORMAT(1H ,5E21.9/)
00670               WRITE(6,10012)
00672         10012 FORMAT(1H0,14HA COEFFICIENTS)
00673               K=1
00674               DO 530 I=1,NN
00677               K=K+1
00700               DO 530 J=K,N
00703               WRITE(6,10014) A(I,J)
```

TABLE 5. (Continued)

```
00706        10014 FORMAT(1H ,E21.9)
00707          530 CONTINUE
00712              WRITE(6,10026)
00714        10026 FORMAT(1H0,12X,8HOBSERVED,12X,8HCOMPUTED,12X,8HRESIDUAL)
00715              DO 1233 IS=1,LN
00720              WRITE(6,10027) YO(IS),YC(IS),RESID(IS)
00725         1233 CONTINUE
00727        10027 FORMAT(1H ,3E21.9/)
00730              L1=-1
00731              L2=0
00732              X(1)=1.
00733              DO 649 LI=2,LN
00736          649 X(LI)=X(LI-1)+1.
00740              CALL QUIK3L(L1,XL,XR,YB,YT,40,FLDX,FLDY,-LN,X YC)
00741              CALL QUIK3L(L2,XL,XR,YB,YT,34,FLDX,FLDY,LN,X,YU)
00742              WRITE(6,10033) PERMAX,PERAVG
00746        10033 FORMAT(1H0,22HMAXIMUM PERCENT ERROR=E16.9,6X,22HAVERAGE PERCENT ER
00746             *ROR=E16.9)
00747         1234 CONTINUE
00750              CALL ENDJOB
00751              STOP
00752              END

      END OF COMPILATION:        NO DIAGNOSTICS.
```

TABLE 5. (Continued)

```
@FOR,IS RDAT,RDAT
HSO 009-01/16-13:51 (,0)

   SUBROUTINE RDAT        ENTRY POINT 000270
              REDTPR      ENTRY POINT 000273
              WRITER      ENTRY POINT 000320
              CLOSE       ENTRY POINT 000342
              OPEN        ENTRY POINT 000351
              IORDAR      ENTRY POINT 000360

   STORAGE USED: CODE(1) 000404; DATA(0) 000127; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   IOTR
    0004   NERR2$
    0005   NADD$
    0006   NI01$
    0007   NI02$
    0010   NSTOP$
    0011   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000071 10L      0000   000070 100F     0001   000116 20L      0001   000140 21L      0001   000242 215G
    0001   000144 22L      0001   000211 24L      0001   000227 25L      0001   000234 26L      0001   000033 6L
    0001   000035 7L       0001   000257 9L       0000 I 000061 I        0000 I 000065 IBLANK   0000 I 000000 ICWD
    0000 I 000002 IFLD     0000   000111 INJP$    0000 I 000066 IOP      0000 I 000046 IPACKT   0000 I 000067 ITWO
    0000 I 000062 I1       0000 I 000063 I2       0000 I 000064 I3       0000 I 000056 KNTWO    0000 I 000040 KODE


00100    C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
00100    C      GENERAL FORTRAN I/O PACKAGE         EXEC VIII           APRIL,1970
00100    C      READ/WRITE MAG TAPE OR DRUM IN FORTRAN OR NON-FORTRAN FORMAT.
00100    C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
00101           SUBROUTINE RDAT
00103           DIMENSION ARRAY(1),ICWD(3),IFLD(29),KODE(6),IPACKT(8),KNTWO(3)
00104           DATA (IFLD(I),I=1,29)/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,2H10,
00104          12H11,2H12,2H13,2H14,2H15,2H16,2H17,2H18,2H19,2H20,2H21,2H22,2H23,
00104          22H24,2H25,2H26,2H27,2H28,2H29/
00106           DATA (KODE(I),I=1,6)/000430000000,000200000000,000150000000,
00106          1                        000100000000,000110000000,000400000000/
00110           DATA I1/262144/,I2/10737418247,I3/00100001/
00114           DATA IBLANK/6H      /
00114    C*****
00116           ENTRY REDTPR(IUNIT,MODE,IERR,NW,NADS,ARRAY)
00120           IOP=MODE                  $ =1,READ FTN FORMAT, =2,READ NON-FTN.
00120    C*****
00121           GO TO 10
00122           ENTRY WRITER(IUNIT,MODE,IERR,NADS,ARRAY)
```

TABLE 5. (Continued)

```
00124          IOP=MODE+2               @ =3,WRITE FTN FORMAT. =4,WRITE NON-FTN.
00125          KNTWD(1)=NWDS*I1+I3      @ BUILD FORTRAN RECORD-CONTROL WORD.
00125          KNTWD(2)=IBLANK          @ DUMMY CHECKSUM ERROR WORD.
00127          KNTWD(3)=KNTWD(1)
00130          GO TO 10
00130    C*****
00131          ENTRY CLOSE(IUNIT,JOP)
00133          ITWO=1
00134          GO TO (6,6,7,25),JOP     @ JOP=4, NO ACTION,RETURN.
00135        6 ITWO=2                   @ JOP=1,2, WRITE EOF AND REWIND IUNIT.
00136        7 IOP=5                    @ =5, WRITE END-OF-FILE.
00137          GO TO 10
00137    C*****
00140          ENTRY OPEN(IUNIT,JOP)
00142          IF(JOP.GT.1) RETURN      @ JOP=2,3, NO ACTION,RETURN.
00144        8 IOP=6                    @ =6, REWIND IUNIT.
00145          GO TO 10
00145    C*****
00146          ENTRY FORDWR(ICODE,IUNIT,IADDRS,ARRAY,NWDS,IERR)
00150          IF(ICODE.EQ.1) IOP=4     @ =4, SEQUENTIAL WRITE.
00152          IF(ICODE.EQ.2) IOP=2     @ =2, SEQUENTIAL READ.
00154          IPACKT(6)=IADDRS         @ BEGINNING DRUM-SECTOR ADDRESS.
00154    C*****
00155       10 CONTINUE
00155          IPACKT(1)=IFLD(IUNIT)              @ IUNIT IN FIELD-DATA.
00157          IPACKT(2)=IBLANK
00160          IPACKT(3)=0
00161       11 IPACKT(4)=KODE(IOP)                @ OPERATION CODE TO IOWS.
00162          GO TO (20,21,20,21,22,22),IOP
00163       20 ICWD(1)=   1*I1+LOC(KNTWD(1))      @ FIRST CONTROL ACCESS WORD.
00164 *DIAGNOSTIC* THE VARIABLE ARRAY IS DIMENSIONED.
00164          ICWD(2)=NWDS*I1+LOC(ARRAY)         @ SECOND CONTROL ACCESS WORD.
00165          ICWD(3)=   2*I1+LOC(KNTWD(2))      @  THIRD CONTROL ACCESS WORD.
00166          IPACKT(5)= 3*I1+LOC(ICWD(1))       @ NMBR AND LOC OF 1ST C/A WORD.
00167          GO TO 22
00170 *DIAGNOSTIC* THE VARIABLE ARRAY IS DIMENSIONED.
00170       21 IPACKT(5)=NWDS*I1+LOC(ARRAY)       @ NMBR AND LOC OF DATA WORDS.
00171       22 CALL IOWR(IPACKT)                  @ ROUTINE TO ISSUE EXEC REQUEST.
00172          IF(IOP.EQ.6) RETURN                @ RETURN AFTER REWINDING IUNIT.
00174          IF(IOP.EQ.5) GO TO (25,8),ITWO     @ ITWO=1,RETURN, =2,REWIND IUNIT.
00176          IERR=IPACKT(4)/I2                  @ ERROR STATUS IN S1 PORTION.
00177          IF(IERR.EQ.0) GO TO 24             @ NORMAL OPERATION,NO ERROR.
00201          IF(IERR.GE.3) GO TO 26             @ TERMINATING ERROR.
00203          IERR=2                             @ END-OF-FILE OR END-OF-TAPE.
00204          RETURN
00205       24 NW=KNTWD(1)/I1                     @ NW FROM FTN CONTROL WORD.
00205          IF(MODE.EQ.1) GO TO 25
00210          NW=IPACKT(4)-IPACKT(4)/I1*I1       @ NW FROM IOWS.
00211       25 IERR=1
00212          RETURN
00213       26 WRITE(6,100) (IPACKT(I),I=1,6)
00221      100 FORMAT(///50H*****.EXECUTION TERMINATED IN 'RDWT' I/O ROUTINE. //
00221         1I4H I/O PACKET IS 6O16 )
00222          STOP
00223          END
```

TABLE 5. (Continued)

```
END OF COMPILATION:            2 DIAGNOSTICS.

@ASM,IS IOAR,IOWR
ASM11F 01/16-13:51-(,0)

    1.                                                $(1)    AXR$
    2.        01    000000  27 00 14 13 0 000000   IOWR*   L        A0,0,X11
    3. U            000001  72 11 00 00 0 000000           ER       IOA$
    4.              000002  74 04 00 13 0 000002           J        2,X11
    5.                                                     END
                      IOA$
 END ASM ERRORS : NONE
```

TABLE 5. (Continued)

```
@FOR,IS INVRT,INVRT
HSO 009-01/16-13:51 (,0)

   SUBROUTINE INVRT      ENTRY POINT 000364

   STORAGE USED: CODE(1) 000412; DATA(0) 000202; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001    000016 111G    0001   000140 115L    0001   000027 117G    0001   000034 123G    0001   000042 131G
    0001    000110 157G    0001   000162 176G    0001   000254 181L    0001   000202 204G    0001   000224 215G
    0001    000262 231G    0001   000332 235L    0001   000306 242G    0001   000342 250L    0001   000071 75L
    0001    000073 76L     0000 D 000002 AMAX    0000 I 000141 I      0000 I 000144 IC     0000 I 000142 IND
    0000 I  000042 INDEX   0000 I 000146 IND2    0000   000154 INJP$   0000 I 000004 IPIV   0000 I 000143 IR
    0000 I  000136 J       0000 I 000140 K       0000 I 000145 L      0000 I 000137 NN     0000 D 000000 SIGN
```

```
00101              SUBROUTINE INVRT(A,N,M,DETER)
00103              PARAMETER IDIM=30
00103       C      MATRIX INVERSION AND SIMULTANEOUS EQUATIONS SOLVER
00103       C      A=INPUT MATRIX FOR INVERISON OR AUGUMENTED MATRIX FOR SIME. EQS.   16500030
00103       C      N=ORDER OF COEFFICIENT MATRIX                                       16500040
00103       C      M=0 FOR INVERISON ONLY
00103       C      M=NUMBER OF CONSTANT VECTORS
00103       C      DETER=DETERMINANT OF COEFFICIENT MATRIX
00104              DOUBLE PRECISION A(1),DETER,SIGN,AMAX
00105              DIMENSION IPIV(IDIM),INDEX(IDIM,2)
00106              DETER=1.0D0
00107              SIGN=1.0D0
00110              DO 20 J=1,N
00113           20 IPIV(J)=0
00115              NN=N+M
00115              DO 182 K=1,N
00121              AMAX=0.0D0
00122           40 DO 76 I=1,N
00125              IF (IPIV(I)-1)50,76,50
00130           50 DO 75 J=1,N
00133              IF (IPIV(J)-1)55,75,250
00136           55 IND=(J-1)*N+I
00137              IF(AMAX-DABS(A(IND))) 60,75,75
00142           60 IR=I
00143              IC=J
00144              AMAX=DABS(A(IND))
00145           75 CONTINUE
00147           76 CONTINUE
00151              IPIV(IC)=IPIV(IC)+1
```

TABLE 5. (Continued)

```
00152            IF (IR-IC)90,115,90
00155       90   SIGN=-SIGN
00156            DO 110 L=1,NN
00161            IND=(L-1)*N+IR
00162            IND2=(L-1)*N+IC
00163            AMAX=A(IND)
00164            A(IND)=A(IND2)
00165      110   A(IND2)=AMAX
00167      115   INDEX(K,1)=IR
00170            INDEX(K,2)=IC
00171            IND=(IC-1)*N+IC
00172            AMAX=A(IND)
00173            DETER=DETER*AMAX
00174      140   A(IND)=1.0D0
00175            DO 150 L=1,NN
00200            IND=(L-1)*N+IC
00201      150   A(IND)=A(IND)/AMAX
00203            DO 181 L=1,N
00206            IF (L-IC)165,181,165
00211      165   IND=(IC-1)*N+L
00212            AMAX=A(IND)
00213            A(IND)=0.0D0
00214            DO 180 I=1,NN
00217            IND=(I-1)*N+L
00220            IND2=(I-1)*N+IC
00221            A(IND)=A(IND)-A(IND2)*AMAX
00222      180   CONTINUE
00224      181   CONTINUE
00226      182   CONTINUE
00230            DO 235 I=1,N
00233            L=N+1-I
00234            IR=INDEX(L,1)
00235            IC=INDEX(L,2)
00236            IF (IR-IC)210,235,210
00241      210   DO 230 K=1,N
00244            IND=(IR-1)*N+K
00245            IND2=(IC-1)*N+K
00246            AMAX=A(IND)
00247            A(IND)=A(IND2)
00250      230   A(IND2)=AMAX
00252      235   CONTINUE
00254            DETER=SIGN*DETER
00255            RETURN
00256      250   M=-1
00257      255   RETURN
00260             END

     END OF COMPILATION:        NO  DIAGNOSTICS.
```

TABLE 5. (Continued)

```
@FOR,IS DMATML,DMATML
HSO 009-01/16-13:51 (,0)

   SUBROUTINE DMATML    ENTRY POINT 000160

   STORAGE USED: CODE(1) 000176; DATA(0) 000053; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000031 1L        0001   000061 130G      0001   000067 135G      0001   000102 143G      0001   000035 2L
    0001   000046 3L        0001   000052 4L        0000 D 000000 CB        0000 I 000005 IA1       0000 I 000006 IA2
    0000 I 000002 IA3       0000 I 000007 IB1       0000 I 000010 IB2       0000 I 000013 IB3       0000 I 000003 IM
    0000 I 000004 IN        0000   000022 INJP$     0000 I 000015 LA        0000 I 000016 LB        0000 I 000012 LC
    0000 I 000011 LM        0000 I 000017 LN        0000 I 000014 LP


00101         SUBROUTINE DMATML(C,A,B,M,N,K)
00101   C*    ABSTRACT
00101   C     GENERAL MATRIX MULTIPLICATION ROUTINE  WITH TRANSPOSE OPTIONS
00101   C      WHERE, M IS THE NUMBER OF ROWS OF (A)
00101   C             N IS THE NUMBER OF ROWS OF (B)
00101   C             K IS THE NUMBER OF COLUMNS OF (B) OR (B)T
00101   C     TRANSPOSE OPTIONS ARE CONTROLLED BY THE SIGNS OF M AND N.
00101   C     THE FOLLOWING PRODUCTS MAY BE OBTAINED
00101   C      (C)=(A)(B)       M AND N POSITIVE
00101   C      (C)=(A)T(B)      M NEGATIVE FOR (A)T
00101   C      (C)=(A)(B)T      N NEGATIVE FOR (B)T
00101   C.     (C)=(A)T(B)T     M AND N NEGATIVE
00101   C      WHERE T INDICATES TRANSPOSE
00101   C      IF M IS NEGATIVE, M IS THE NUMBER OF ROWS OF (A)T
00101   C      IF N IS NEGATIVE, N IS THE NUMBER OF ROWS OF (B)T
00101   C
00101   C*    OUTPUT ARGUMENT * C
00103         DIMENSION C(1)
00103   C
00103   C*    INPUT ARGUMENTS * A,B,M,N,K
00104         DIMENSION A(1),B(1)
00104   C
00104   C*************************************************************************
00105         DOUBLE PRECISION CB,C,A,B
00106         IA3=1
00107         IM=IABS(M)
00110         IN=IABS(N)
00111         IF(M .LT. 0)GO TO 1
00113         IA1=IM
00114         IA2=1
```

TABLE 5. (Continued)

```
00115           GO TO 2
00116      1    IA1 = 1
00117           IA2=IN
00120      2    IF(N .LT. 0)GO TO 3
00122           IB1=1
00123           IB2=IN
00124           GO TO 4
00125      3    IB1=K
00126           IB2=1
00127      4    DO 7 LM=1,IM
00132           LC=LM
00133           IB3=1
00134           DO 6 LP=1,K
00137           CD = 0.0D0
00140           LA=IA3
00141           LB=IB3
00142           DO 5 LN=1,IN
00145           CD = CD + A(LA)*B(LB)
00146           LA=LA+IA1
00147      5    LB=LB+IB1
00151           C(LC) = CD
00152           LC = LC + IM
00153      6    IB3=IB3+IB2
00155      7    IA3=IA3+IA2
00157           RETURN
00160           END
     END OF COMPILATION:        NO  DIAGNOSTICS.
```

TABLE 5. (Continued)

```
GMAP,I AA,AA
MAP 17M2-01/16-13:51 -(,0)

   1.        LIB SYS$*MSFCS.

ADDRESS LIMITS     001000 033560     040000 055012
STARTING ADDRESS   033540
WORDS DECIMAL      13681 IBANK       6667 DBANK
```

SEGMENT MAIN		001000 033560		040000 055012
NSQTC$/FOR	1	001000 001021		
NKBLK$/MSFC55	1	001022 001110	0	040000 040001
NEXP6$/MSFC57	1	001111 001303	2	040002 040053
ALOG$/FOR51	1	001304 001421	2	040054 040114
CSIG6V/SC4020	1	001422 001710	0	040115 040153
			2	BLANK$COMMON
CENFRK/SC4020	1	001711 001740	0	040154 040167
			2	BLANK$COMMON
CLABLV/SC4020	1	001741 003003	0	040170 040276
			2	BLANK$COMMON
NRANDS/FOR50	1	003004 003063	2	040277 040310
NVEFS/MSFC55	1	003064 003320	2	040311 040331
NFTCH$/FOR57	1	003321 003620	2	040332 040367
CYMODV/SC4020	1	003621 003646	0	040370 040376
			2	BLANK$COMMON
CACCBY/SC4020	1	003647 003670	0	040377 040407
			2	BLANK$COMMON
CAHCDV/SC4020	1	003671 003716	0	040410 040416
			2	BLANK$COMMON
CONCAT/MSFC	1	003717 004070	0	040417 040440
SETIMT/SC4020	1	004071 004123	0	040441 040446
			2	BLANK$COMMON
CHOLLV/SC4020	1	004124 004206	0	040447 040463
			2	BLANK$COMMON
CHONLN/SC4020	1	004207 004614	0	040464 040535
			2	BLANK$COMMON
CLINRV/SC4020	1	004615 005470	0	040536 040651
	3	GGG	2	BLANK$COMMON
CYSCLV/SC4020	1	005471 005673	0	040652 040700
	3	GGG	2	BLANK$COMMON
CXSCLV/SC4020	1	005674 006076	0	040701 040727
	3	GGG	2	BLANK$COMMON
CERNLV/SC4020	1	006077 006253	0	040730 040742

TABLE 5. (Continued)

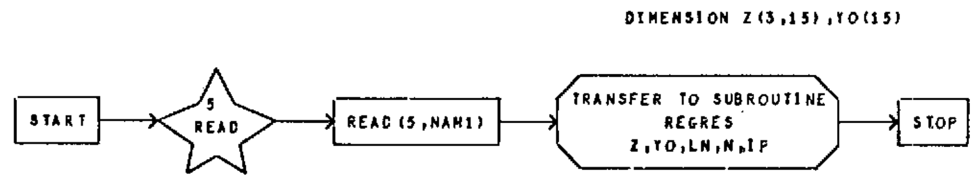
			2	BLANK$COMMON
CERKLN/SC4020	1	006254 006363	0	040743 040760
			2	BLANK$COMMON
CSETCV/SC4020	1	006364 006424	0	040761 040771
			2	BLANK$COMMON
CSETMV/SC4020	1	006425 006503	0	040772 041006
			2	BLANK$COMMON
CXAXIS/SC4020	1	006504 006707	0	041007 041042
			2	BLANK$COMMON
VCHARV/SC4020	1	006710 007153	0	041043 041062
RITE2V/SC4020	1	007154 007404	0	041063 041111
BPLOTK/SC4020	1	007405 007465	0	041112 041471
			2	BLANK$COMMON
CFRAM/SC4020	1	007466 007705	0	041472 041556
	3	GGG	2	BLANK$COMMON
CCAMRA/SC4020	1	007706 007757	0	041557 041570
	3	GGG	2	BLANK$COMMON
TABLIV/SC4020			0	041571 042111
NBUCV$/FOR57	1	007760 010113	2	042112 042151
NFTV$/FOR	1	010114 010136		
VCLBS$/MSFC57	1	010137 010305	2	042152 042177
NBLKS$/MSFC57	1	010306 010474	0	042200 042201
NOSBL$/FOR	1	010475 010532		
NUROA$/FOR	1	010533 010566		
NBRUO$/FOR			2	042202 044403
NCNVT$/FOR57	1	010567 011021	2	044404 044473
MINIUS/MSFC55	1	011022 011253	2	044474 044515
CPLOTV/SC4020	1	011254 011436	0	044516 044554
	3	GGG	2	BLANK$COMMON
CLINEV/SC4020	1	011437 012022	0	044555 044633
			2	BLANK$COMMON
YSCLVI/SC4020	1	012023 012141	0	044634 044644
	3	GGG	2	BLANK$COMMON
XSCLVI/SC4020	1	012142 012257	0	044645 044655
	3	GGG	2	BLANK$COMMON
CAPLOT/SC4020	1	012260 012533	0	044656 044731
	3	GGG	2	BLANK$COMMON
CAPRNV/SC4020	1	012534 012616	0	044732 044750
			2	BLANK$COMMON
CPRNTV/SC4020	1	012617 013210	0	044751 045005
	3	GGG	2	BLANK$COMMON
CGRDIV/SC4020	1	013211 014115	0	045006 045121
	3	GGG	2	BLANK$COMMON
COXOYV/SC4020	1	014116 014710	0	045122 045206
			2	BLANK$COMMON
CBRITV/SC4020	1	014711 015012	0	045207 045222
	3	GGG	2	BLANK$COMMON
CMARGN/SC4020	1	015013 015071	0	045223 045264
			2	BLANK$COMMON
CNBLNK/SC4020	1	015072 015141	0	045265 045300
			2	BLANK$COMMON
BNOV/MSFC	1	015142 015222	0	045301 045311
			2	BLANK$COMMON
TRACE	1	015223 015316	0	045312 045317
			2	045320 045402
SPCFRM/MSFC			0	045403 045456

TABLE 5. (Concluded)

CTOBCD/MSFC	1	015347 015645	0	045457 045630
BOXLTR/CSC	1	015646 016500	0	045631 045647
CIDENT/SC4020	1	016501 017653	0	045650 046033
	3	GGG	2	BLANK$COMMON
NOTIN$/MSFC55	1	017654 020204	2	046034 046044
NOUT$/MSFC57	1	020205 021166	2	046045 046076
NFMT$/MSFC57	1	021167 022074	2	046077 046115
NIOER$/MSFC57	1	022075 022254	2	046116 046240
NFCHK$/MSFC57	1	022255 023073	2	046241 046415
			4	046416 046467
NTAB$/MSFC55			2	046470 046556
NLIOP$/MSFC57	1	023074 024640	2	046557 046770
SQRT$/FOR55	1	024641 024700	2	046771 047002
NEXP5$/FOR57	1	024701 024764	2	047003 047012
GGG (COMMON BLOCK)				047013 047146
CQUIKL/SC4020	1	024765 025402	0	047147 047231
	3	GGG	2	BLANK$COMMON
IDSKT/SC4020	1	025403 026645	0	047232 051027
MONITOR/MSFC55	1	026646 027755	2	051030 051614
NTER$/FOR52	1	027756 030040	2	051615 051744
NUBUF$/FOR51	1	030041 030100		
ERO$/MSFC55				
NERR$/FOR57	1	030101 030435	2	051745 052121
BLANK$COMMON (COMMON BLOCK)				
DMATML	1	030436 030633	0	052122 052174
			2	BLANK$COMMON
INVRT	1	030634 031245	0	052175 052376
			2	BLANK$COMMON
IOUR	1	031246 031250		
ROOT	1	031251 031652	0	052377 052525
			2	BLANK$COMMON
REGRES	1	031653 033537	0	052526 054665
			2	BLANK$COMMON
MAIN	1	033540 033560	0	054666 055012
			2	BLANK$COMMON

SYS$*RLIB$. LEVEL MS7-Q
END OF COLLECTION - TIME 3.887 SECONDS

@XQT AA



D I M E N S I O N E D   V A R I A B L E S

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
ADARY	22	PMAX	15	FLDX	12	FLDY	12	X	15
Z	3,15	ZBAR	3,3	ZZBAR	2,3	SMALLZ	IDIM	SMALZ1	IDIM
SMALZ2	IDIM	SHAT	12,12	YO	15	SVHAT	12,1	YC	15
BZ	15	RESID	15	B	3,3	A	2,3		

Figure 4. Operation of the computer program (cross product).

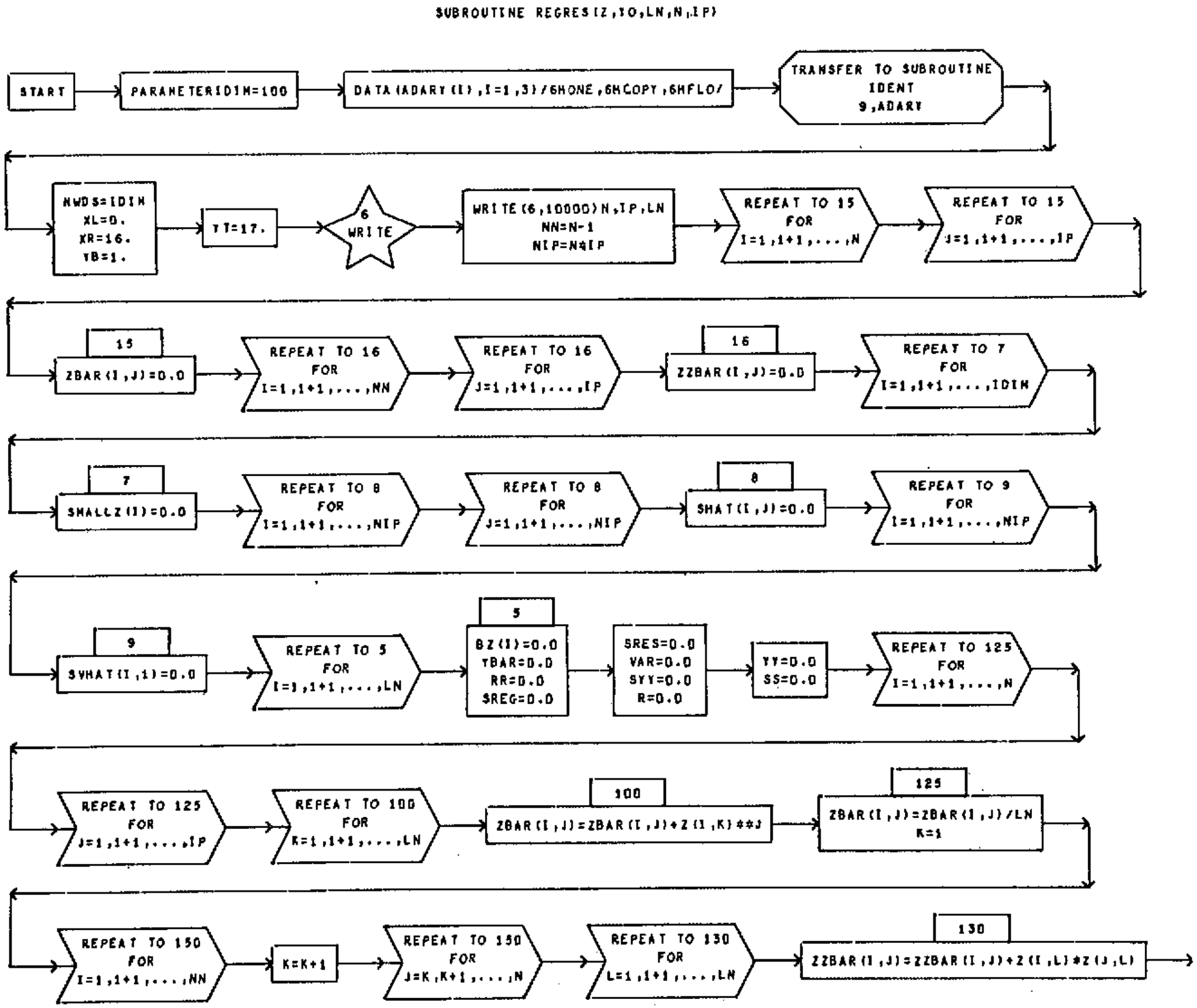


Figure 4. (Continued)

SUBROUTINE REGRES(Z,YO,LN,N,IP)

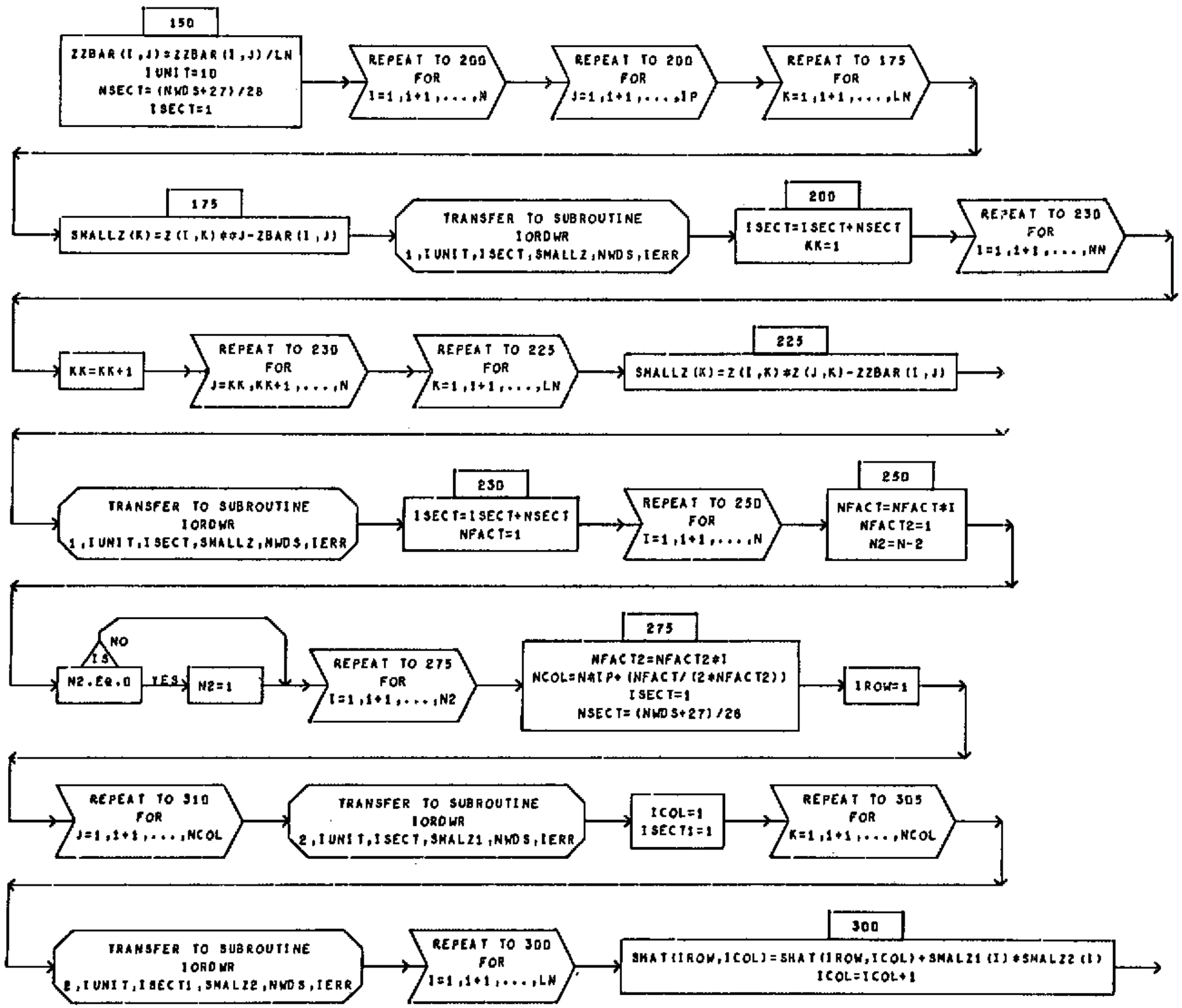


Figure 4. (Continued)

SUBROUTINE REGRES(Z,YO,LN,N,IP)

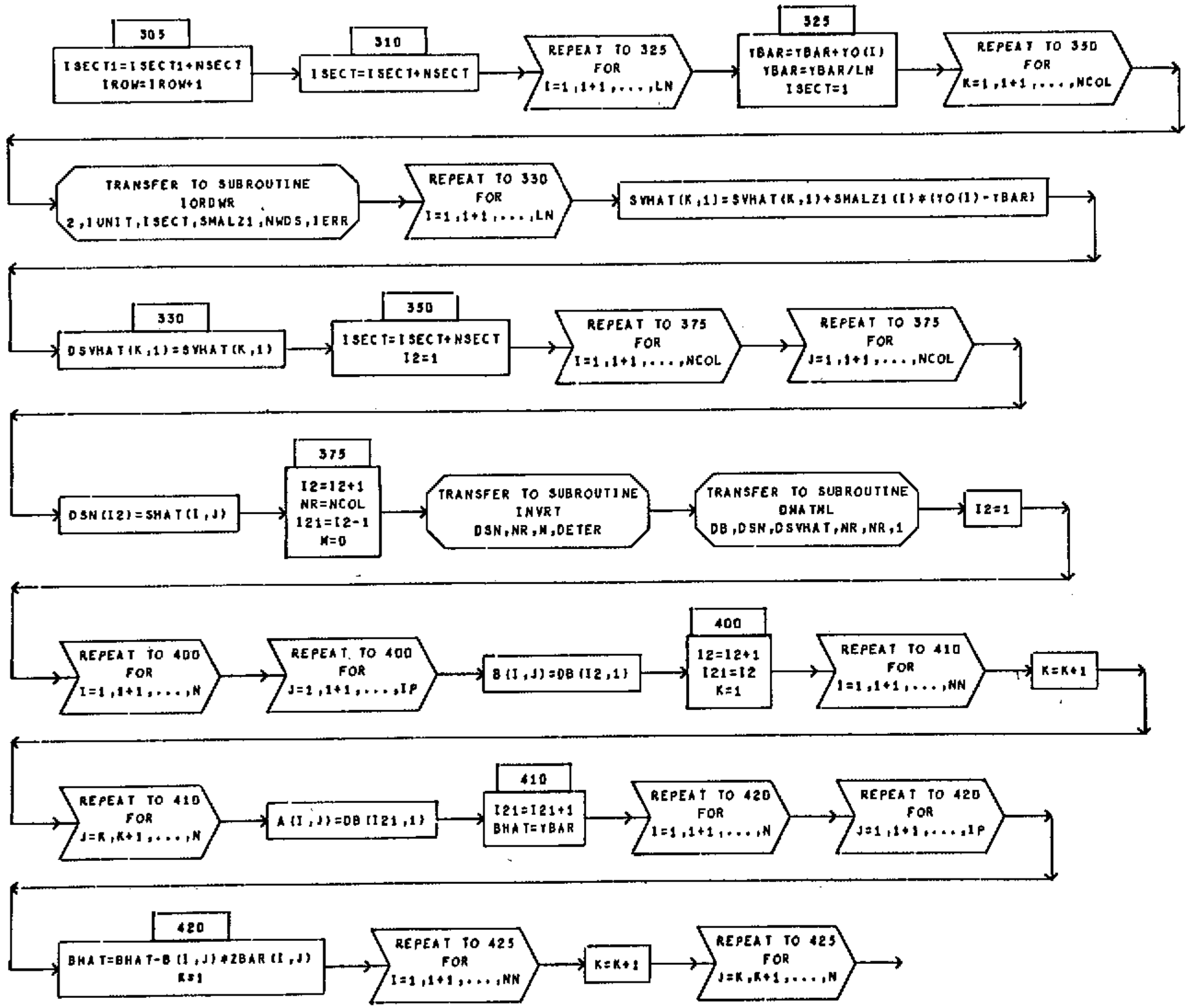


Figure 4. (Continued)

SUBROUTINE REGRES(Z,YO,LN,N,IP)

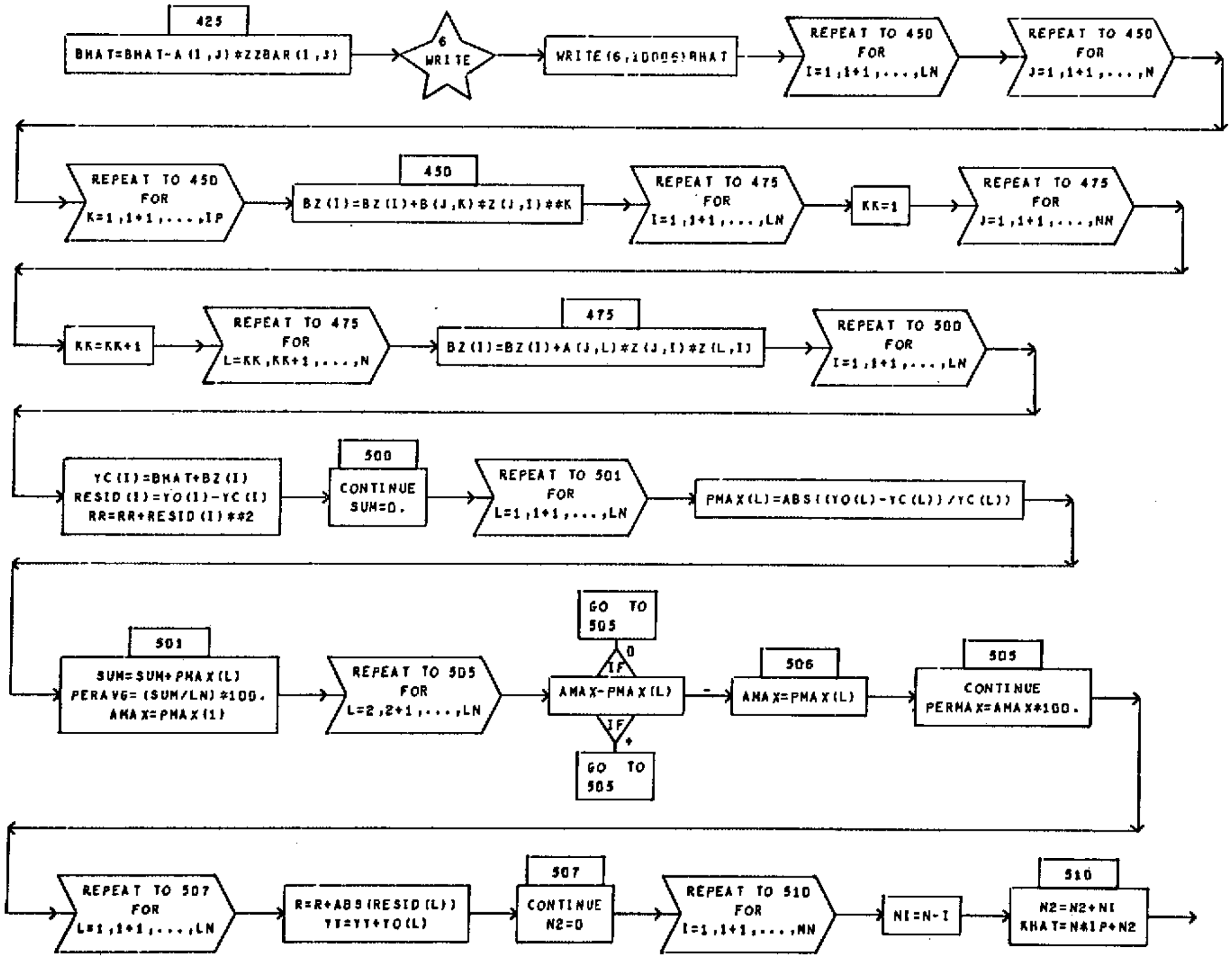


Figure 4. (Continued)

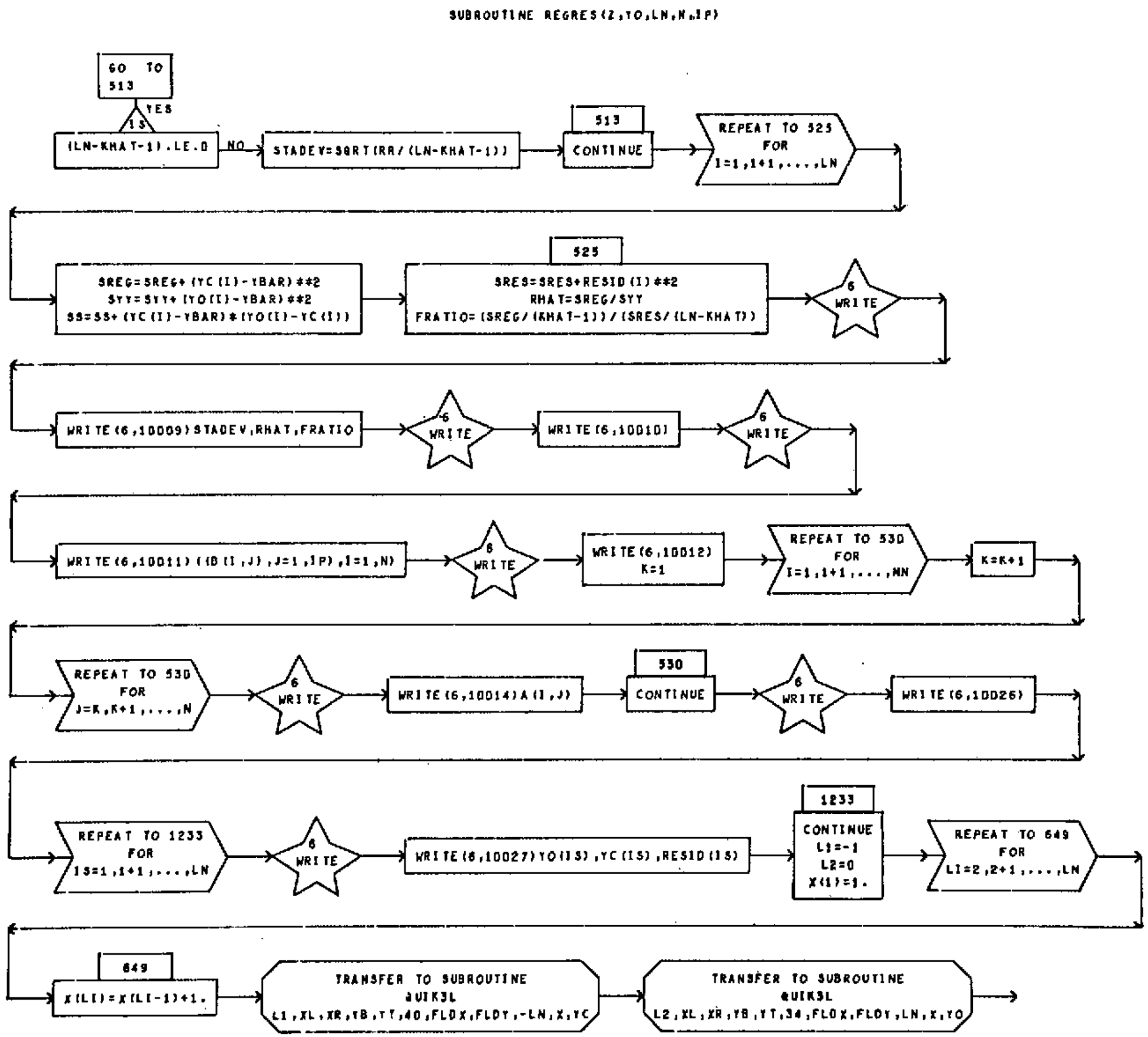
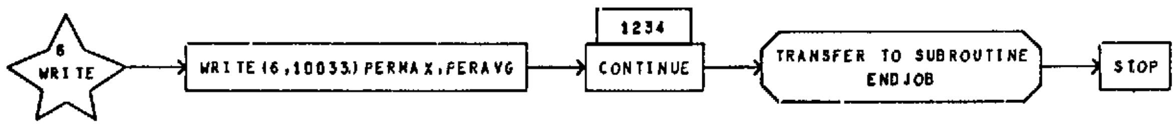


Figure 4. (Continued)

SUBROUTINE REGRES(Z,YO,LN,N,IP)



GENERAL FORTRAN I/O PACKAGE ,EXEC VIII ,APRIL,1970

READ/WRITE MAG TAPE OR DRUM IN FORTRAN OR NON-FORTRAN FORMAT.

D I M E N S I O N E D   V A R I A B L E S

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
ARRAY	1	ICWD	3	IFLD	29	KODE	6	IPACKT	6
NTWD	3								

Figure 4. (Continued)

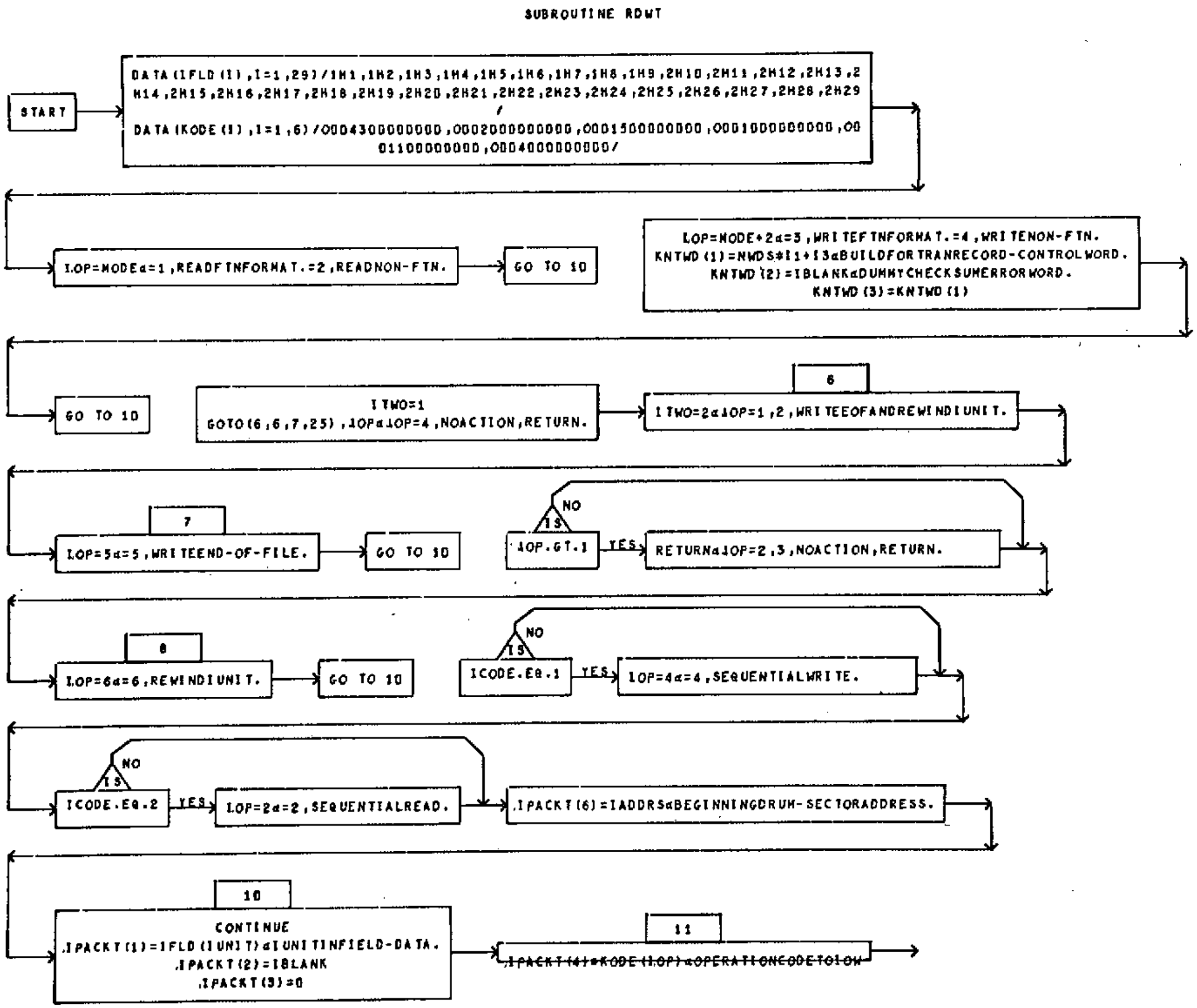


Figure 4. (Continued)

SUBROUTINE RDWT

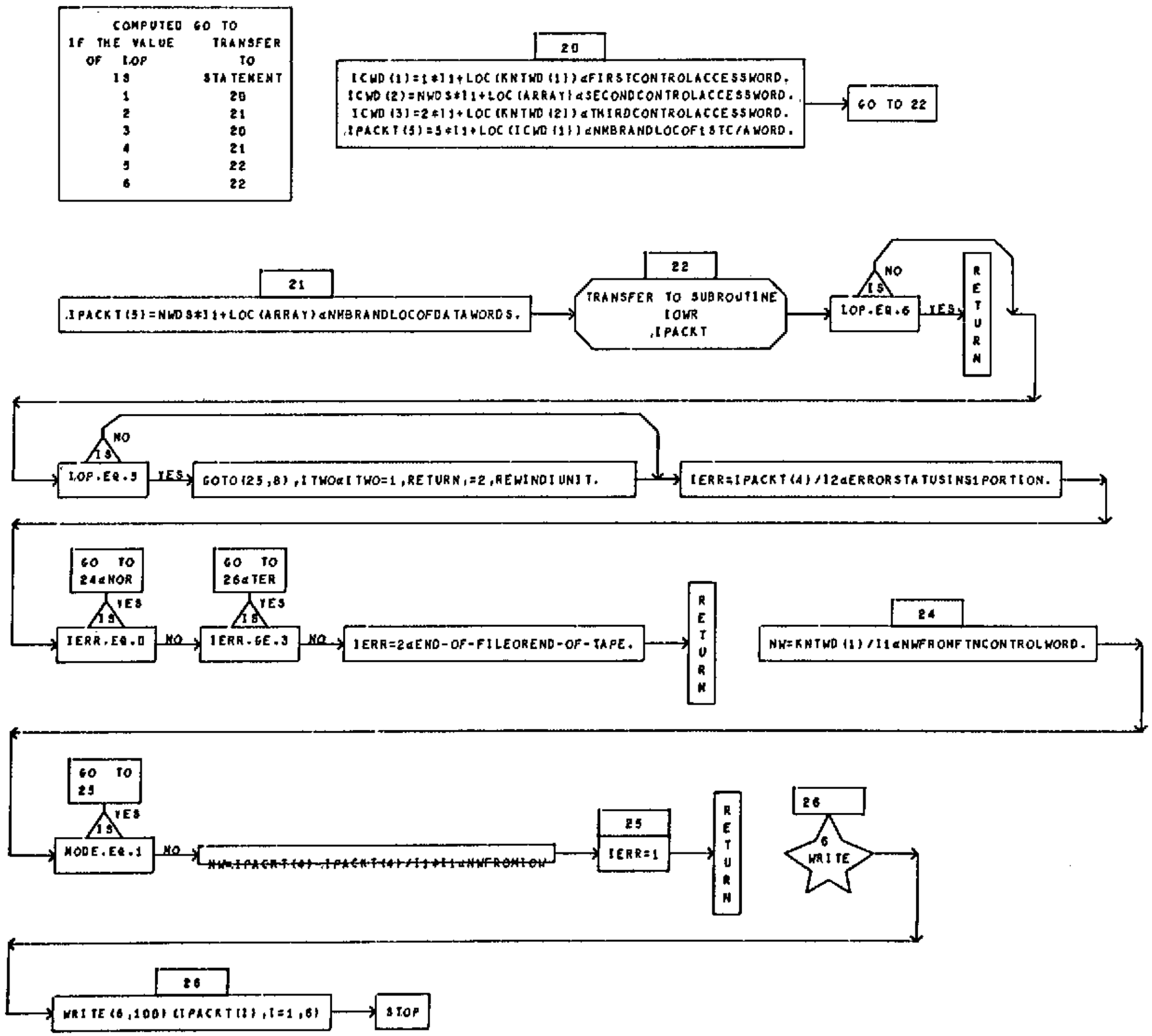


Figure 4. (Continued)

D I M E N S I O N E D   V A R I A B L E S

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
IPIV	IDIM	INDEX	IDIM,2						

Figure 4. (Continued)

SUBROUTINE INVRT(A,N,M,DETER)

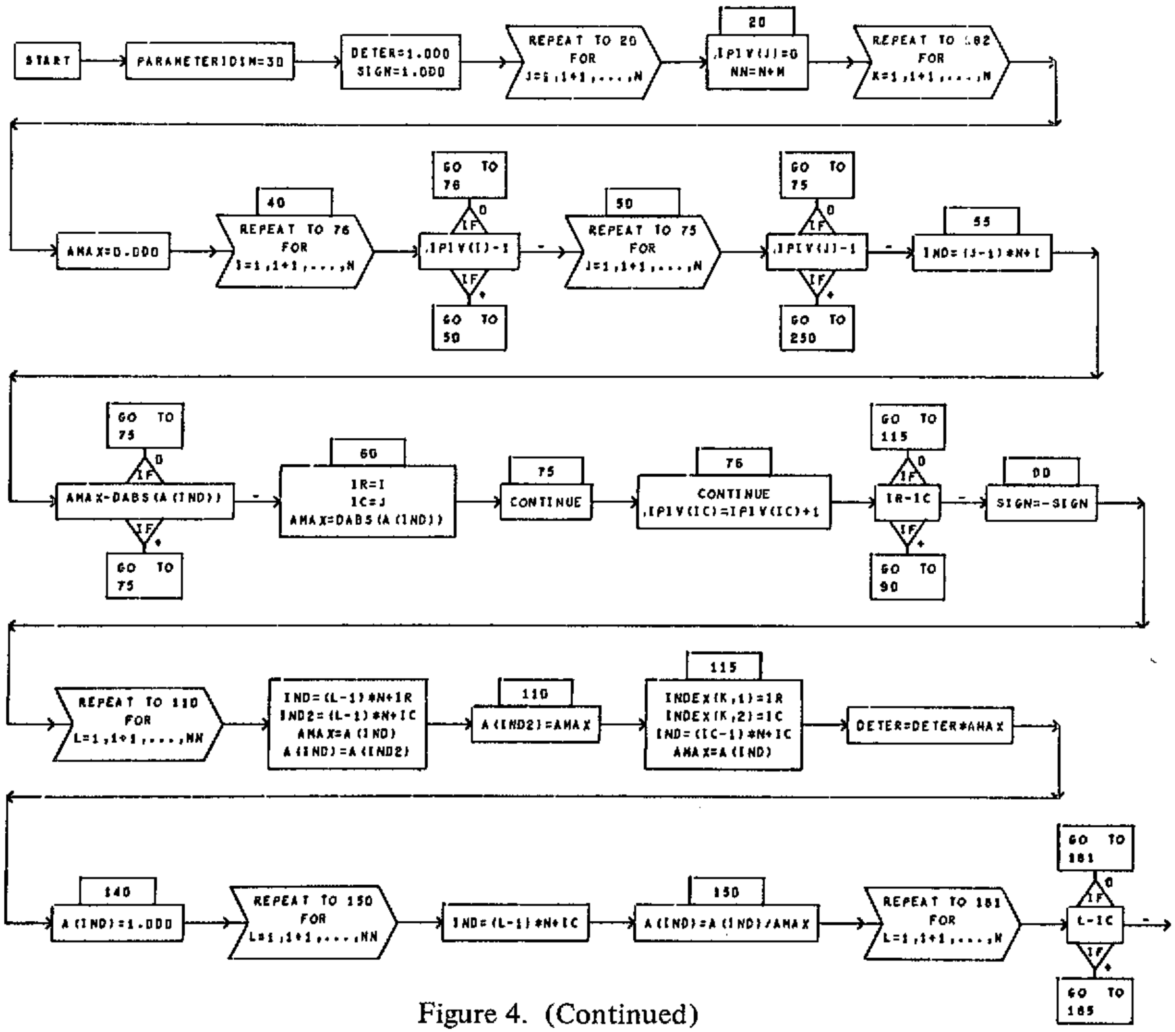


Figure 4. (Continued)

SUBROUTINE INVRT(A,N,M,DETER)

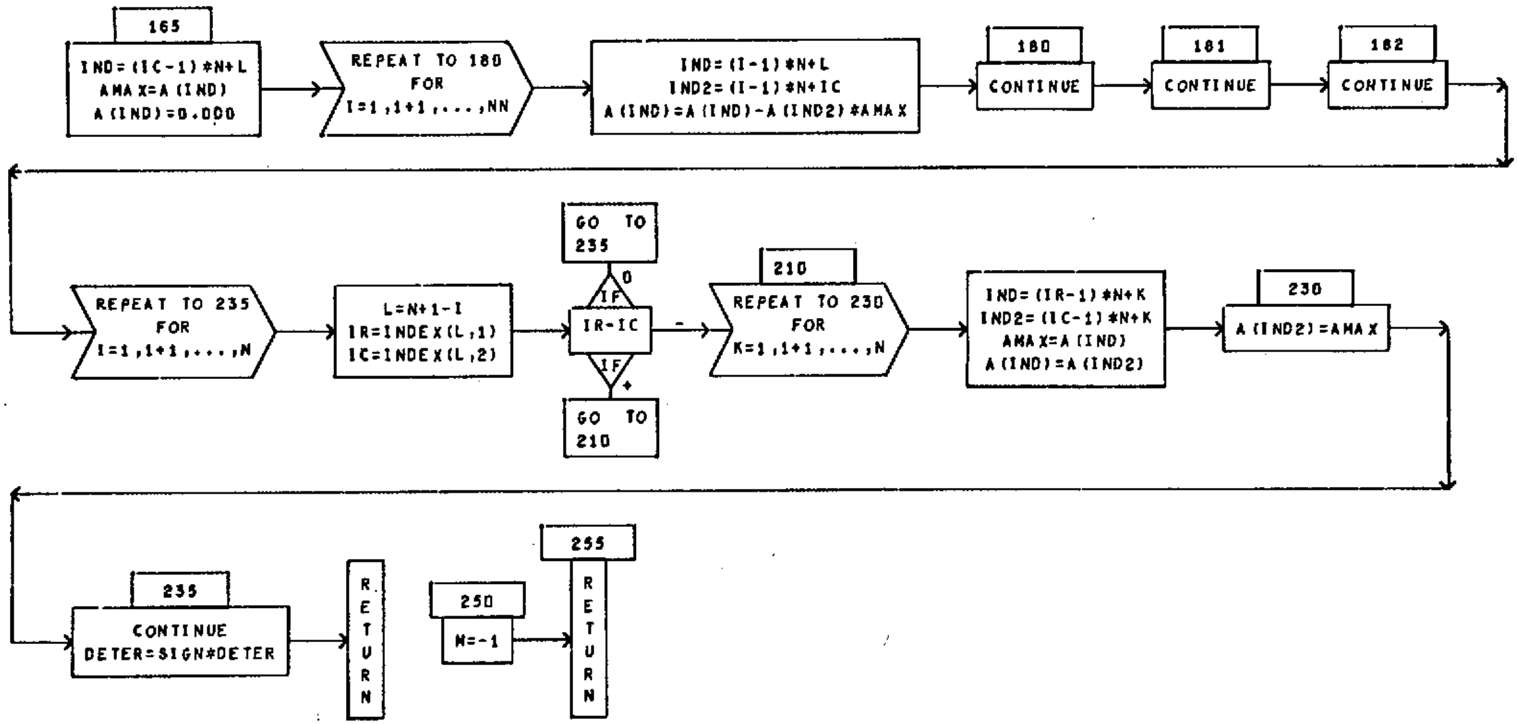


Figure 4. (Continued)

DIMENSIONED VARIABLES

SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES	SYMBOL	STORAGES
C	1	A	1	B	1				

SUBROUTINE DMATML(C,A,B,M,N,K)

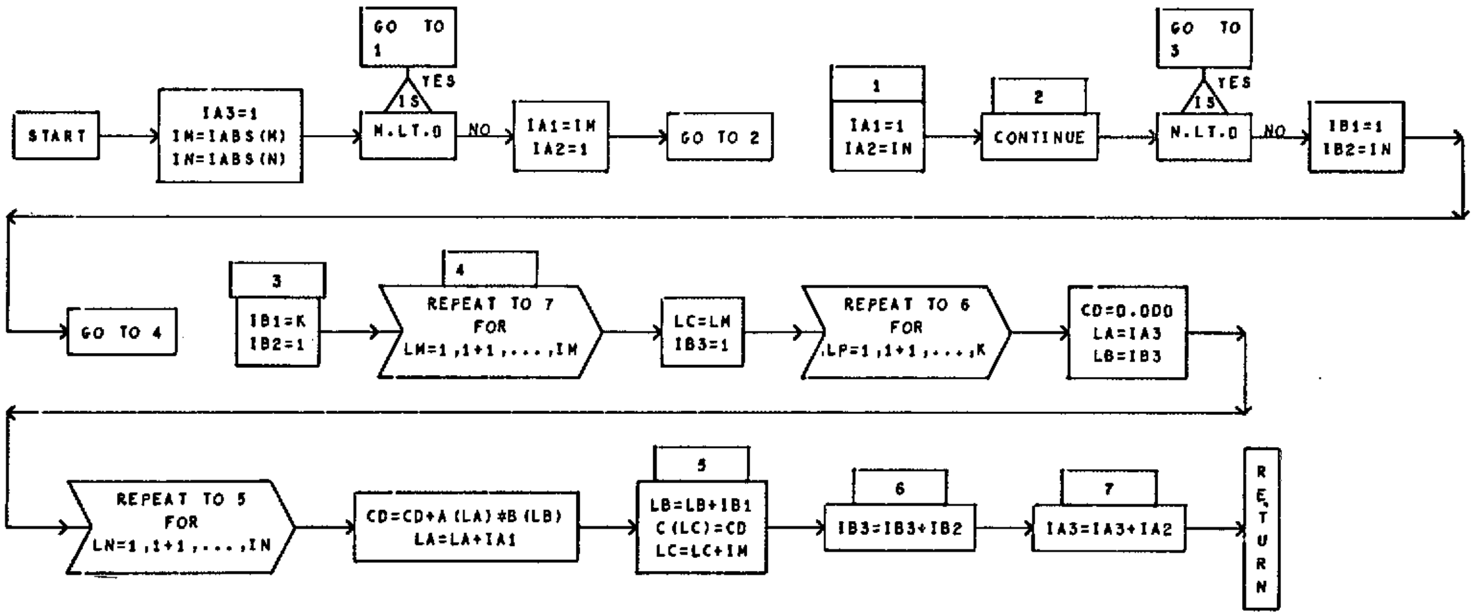


Figure 4. (Concluded)

From equation (38) the following set of input data (Table 6) was developed, containing exact dependent variable values (to three decimal places) for arbitrary values of the three independent variables. In as far as was possible the problem selected in Appendix A was duplicated for comparison purposes.

TABLE 6. INPUT DATA, DEPENDENT VARIABLE VALUES

Data Point Number	Y	X_1	X_2	X_3
1	6.759	0.5	0.4	0.4
2	8.251	0.6	0.3	0.2
3	4.256	0.2	0.6	0.6
4	4.892	0.2	0.6	0.5
5	13.165	0.9	0.0	0.5
6	9.669	0.7	0.0	0.6
7	6.070	0.3	0.9	0.7
8	12.588	0.9	0.3	0.6
9	5.749	0.5	0.6	0.7
10	8.561	0.3	0.8	0.2
11	11.333	0.2	0.9	0.0
12	10.589	0.8	0.5	0.6
13	16.117	0.9	0.8	0.3
14	3.525	0.1	0.4	0.7
15	2.333	0.0	0.5	0.8

The input for this sample program was read in through namelist NAM1 as follows:

```
Col. 2
$NAM1

    Z = 0.5, 0.4, ..., 0.8,
    YO = 6.759, 8.251, ..., 2.333,
    LN = 15,
    N = 3,
    IP = 3,
$
```

The computed results are shown in Table 7 for this example problem and the plotted results are shown in Figure 5.

TABLE 7. OUTPUT DATA LISTING FOR ILLUSTRATIVE PROBLEM (CASE WITH CROSS PRODUCTS)

```
N=   3   P=   3   LN=  15

BHAT=   .900033772+01

STANDARD DEVIATION=   .945426711-05   MULTIPLE CORRELATION COEFFICIENT=   .100000000+01   F RATIO=   .335245672+12

B COEFFICIENTS
      -.600040036+01       .699976504+01       .500027460+01      -.400052220+01       .100033142+01
       .699985909+01      -.600018770+01       .699929118+01      -.599732355+01

A COEFFICIENTS
       .300028425+01
       .500044143+01
      -.599971730+01
         OBSERVED             COMPUTED             RESIDUAL
       .675900000+01       .675899339+01        .661611557-05
       .825100005+01       .825100231+01       -.226477650-05
       .425599998+01       .425599325+01        .673532486-05
       .489200002+01       .489205270+01       -.268220901-05
       .131650000+02       .131649994+02        .596046448-06
       .966900003+01       .966900027+01       -.238418579-06
       .606999999+01       .607000053+01       -.536441803-06
       .125880001+02       .125880008+02       -.715255737-06
       .574900001+01       .574900067+01       -.655651093-06
       .856099999+01       .856100559+01       -.560283661-05
       .113329999+02       .113329972+02        .274181366-05
       .105590000+02       .105890003+02       -.357627869-06
       .161170001+02       .161169991+02        .953674316-06
       .352500001+01       .352500546+01       -.545382500-05
       .233300000+01       .233299756+01        .244379044-05

MAXIMUM PERCENT ERROR=   .158255063-03      AVERAGE PERCENT ERROR=   .486492650-04
```

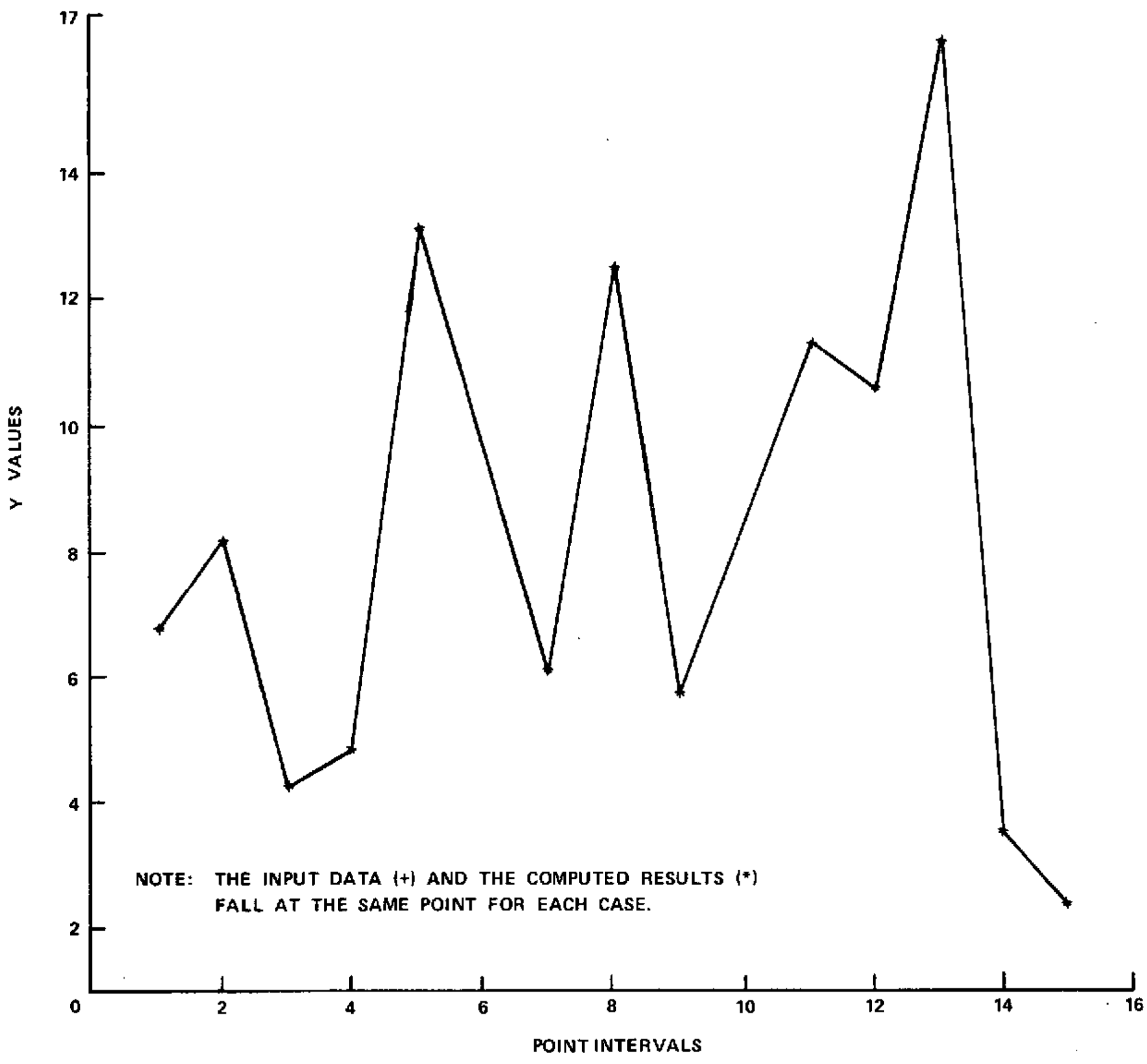


Figure 5. Comparison of input and computed values for illustrative problem with cross products.

The plot graphs the point intervals on the X axis against the exact and computed dependent variables. The exact dependent variables are plotted with (+) and the points are not connected by line segments. The computed dependent variables are plotted with (*) and the points are connected by straight line segments.

Inspection of the digital and plotted results reveals that the computed regression relation yields an almost exact representation of the input data for this arbitrary case. However, experience with several sets of physical data have generally yielded poorer results evidently due to the larger matricies being manipulated and due to unknowns as to whether true linear cross coupling exists in the physical process.

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NASA-Langley, 1973 —— 19