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The design, construction and preliminary test of the Aero-Thermop

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Rensselaer Polytechnic Institute

THE DESIGN, CONSRUCTION AND PRELIMINARY TEST OF THE AERO-THERMOPREX

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THE DESIGN, CONSTRUCTION AND PRELIMINARY TEST

OF THE AERO-THERMOPREX

by

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Cambridge, Massachusetts

20 May 1949

Professor Joseph S. Newell
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the Degree of Naval Engineer, we submit herewith a thesis entitled, "The Design, Construction and Preliminary Test of the Aero-Thermoprex."

ACKNOWLEDGMENT

The authors wish to express their appreciation to Professor A. H. Shapiro for his valuable advice and assistance, and for his formulation of the problem here under investigation. The authors also wish to express their appreciation to the personnel of the Boston Naval Shipyard, the U. S. Naval Engineering Experiment Station, and the Gas Turbine Laboratory at M. I. T. without whose cooperation and assistance this thesis could not have been undertaken. Also to our co-workers, J. P. Kish and C. A. Templeton we extend our thanks for their invaluable assistance.

TABLE OF CONTENTS

FOREWORD

An investigation of the "Aero-Thermoprex" was conducted as a joint project by Lieutenants R. A. Hawkins, L. V. Mowell, O. A. Templeton, and J. R. Wish. Since the investigation covers many phases, the report has been divided into two sections. The first report, by Hawkins and Mowell, covers the design, construction and preliminary tests of the "Aero-Thermoprex", and includes the theoretical analysis for design, and a modified analysis for the apparatus constructed. The second report, by Templeton and Wish, covers the actual performance of the apparatus and a comparison with the theory to determine the possibilities of the "Aero-Thermoprex" as a pumping device.

NOMENCLATURE

A	Cross-sectional area
c_p	specific heat at constant pressure
D	hydraulic diameter
f	friction coefficient in flow passage
h_L	enthalpy of injected liquid, per unit mass
h_V	enthalpy of the evaporated liquid at the temperature T, per unit mass
k	ratio of specific heats, c_p/c_v
M	Mach number
p	static pressure
p_0	isentropic stagnation pressure
T	absolute temperature
T_0	absolute stagnation temperature
V	velocity of stream
V_L	velocity with which liquid enters main stream
w	mass rate of flow of stream
W	molecular weight
x	distance along duct
y	V_L'/V, where V_L' is forward component of velocity of V_L
$()_m$	refers to mean conditions
$()_1$	refers to section 1, inlet to nozzle
$()_2$	refers to section 2, outlet from diffuser
$()_i$	refers to section i, inlet to water injection section
$()_f$	refers to section f, outlet from water injection section

1 SUMMARY

The purpose of this thesis was to design, build, and conduct preliminary tests of the Aero-Thermoprex, a device for raising the stagnation pressure of a stream of air by injecting and evaporating water into the air. The investigation seemed to divide itself logically into three sections; first, the major design considerations, including the practical aspects of laboratory facilities and equipment readily available, which led to the detail design and actual construction of the apparatus; second, a theoretical analysis of the built machine to determine its approximate operating characteristics for various assumed conditions of friction and evaporation, the results of which would furnish a basis for interpreting the actual performance; and third, a preliminary test to insure that the design specifications had been met, and that all parts of the apparatus performed their assigned functions in a satisfactory manner. The presentation has been organized to show the development, results, and conclusions of the three phases of the investigation.

The apparatus has been described in some detail, with particular attention given to those features which required design study. While the actual design chosen is but one of many possible arrangements, both in detail and general characteristics, it is felt to satisfy the requirements of an experimental study of the basic process. The test runs show that the specifications of design were met. Inlet

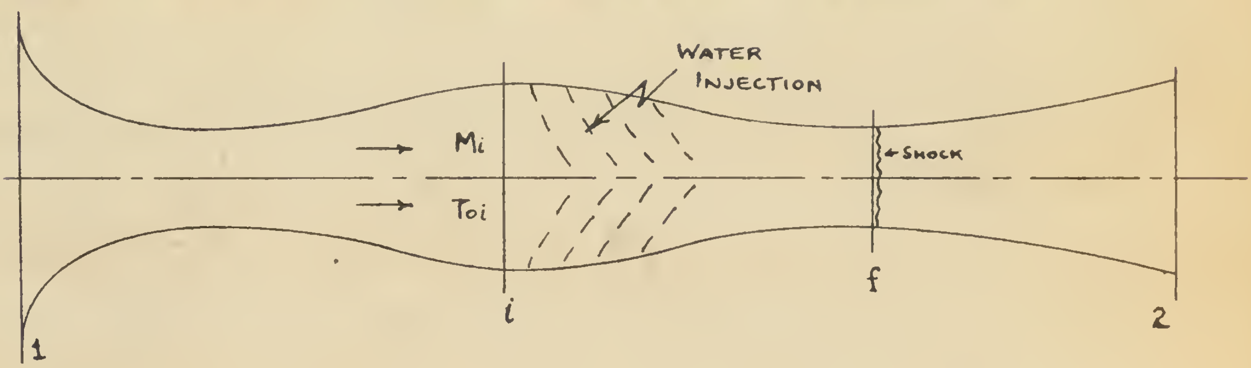
Mach number and stagnation temperature were very close to those chosen for the primary design point, and all parts of the apparatus performed in a satisfactory manner.

A theoretical analysis yielded results that indicated that some stagnation pressure gain over the dry characteristics might be realized if the evaporation of the water injected were at least 50% complete. Results for 50%, 75%, and 100% have been obtained for comparison with experimental data. It has also been demonstrated that any positive "pumping action" in the size apparatus built is extremely unlikely. Some reference has been made to the effects of size on the probable experimental results, but the more complete exposition of this effect has been left for the subsequent work of Templeton and Wish. The material presented here has been intended to be primarily the ground work for an experimental program which may indicate changes in theoretical procedure as well as possible recommendations for future investigation.

II INTRODUCTION

The pumping of a gas is accomplished by raising the stagnation pressure of the gas, ordinarily by employing rotating machinery (Compressors). A novel method, involving no moving parts, has been proposed[1] as an alternative to conventional pumping methods. Raising the stagnation pressure of the gas stream would be accomplished by preliminary heating at low velocity followed by cooling of the gas stream at high velocity. The Reynolds Analogy applied to gas flow with cooling and friction shows that cooling by means of a heat exchanger alone could never produce a net stagnation pressure rise. It has been shown, however, that cooling by evaporation of a liquid into the gas stream gives considerable promise as a pumping scheme, provided that the ratio of latent heat of the liquid to the product of the specific heat at constant pressure and the absolute stagnation temperature is greater than two.

A preliminary investigation of the scheme, utilizing liquid water, was made by Shapiro and Wadleigh[2] using a simple one-dimensional analysis, in which the effects of area change, wall friction, drag of liquid water, evaporation, and changes in molecular weight and specific heat due to evaporation were included. Calculations were made for constant cross-sectional area, constant pressure, constant Mach number, and constant temperature evaporation in the apparatus shown in the sketch below.



These calculations showed that only the constant temperature process gave promise of a stagnation pressure ratio, $\frac{P_{02}}{P_{01}}$, much greater than unity. From the limited calculations made, the scheme was considered to be of marginal promise for certain conditions at the entrance to the evaporation section.

The usefulness of such a pump is readily apparent. One possible application would be for driving large supersonic wind tunnels which are now impractical because of the enormous power and machinery requirements for conventional rotating compressors.

The purpose of this investigation was to design, build, and conduct preliminary tests of the Aero-Thermoprex.* Although some of the infinite number of possible evaporation processes might yield better theoretical results, the laborious computations involved in identifying these processes was not felt to be justified, and the constant temperature process was selected as the basis of design. The examination of the process achieved in the designed apparatus and a comparison of the results with the theoretical predictions is the subject of a companion thesis.[7]

* A pump for raising the stagnation pressure of a gas by cooling through evaporation of a liquid will be called an Aero-Thermoprex.

III DESIGN CONSIDERATIONS

Preliminary Analysis

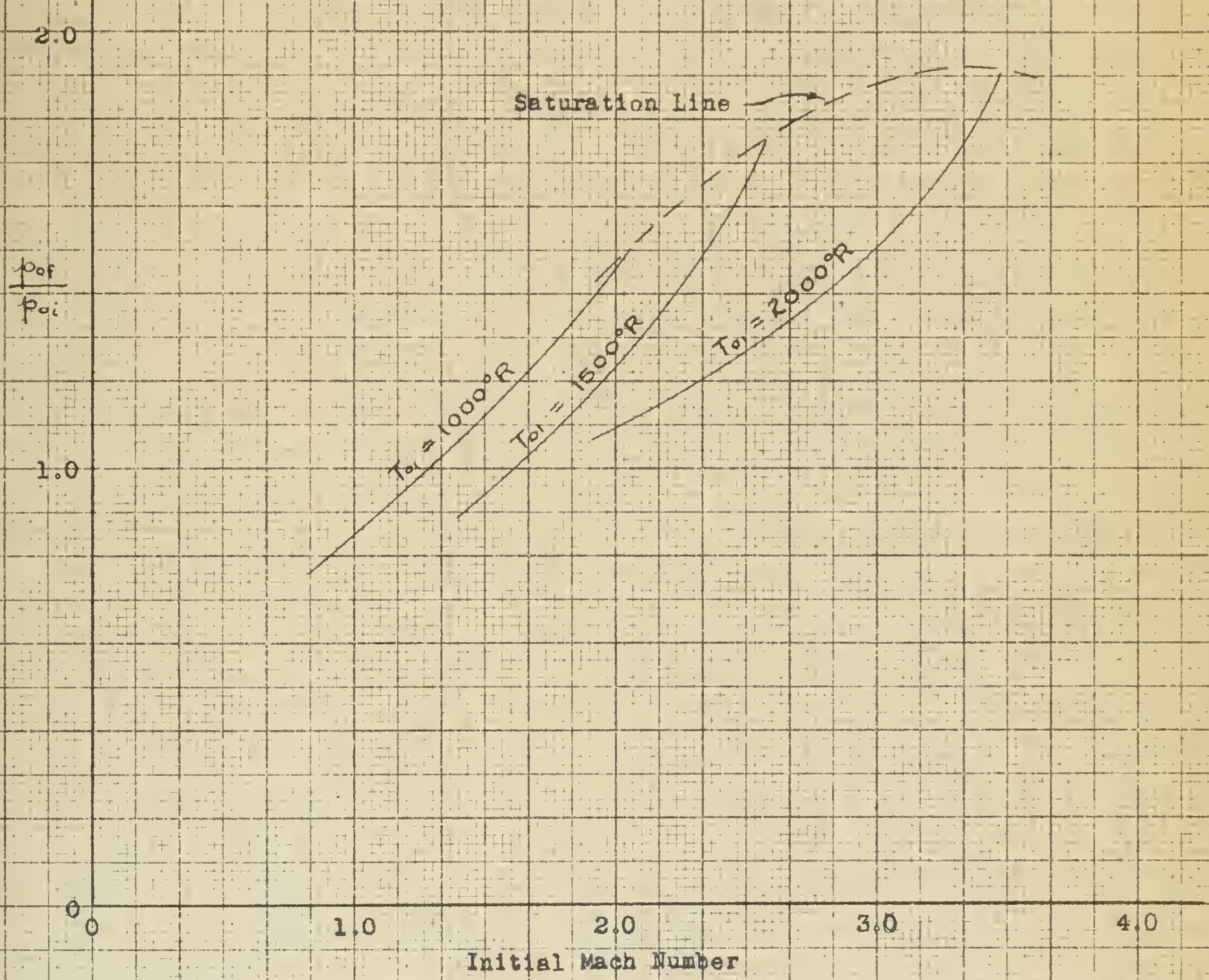
As previously pointed out it was decided to design the experimental apparatus on the basis of constant temperature evaporation. With this end in mind, further calculations were made to supplement and to substantiate the work of Shapiro and Wadleigh. Calculations were made for initial stagnation temperature of 1500°R, and for initial Mach numbers of 2.0, 2.5, and 3.0. For these calculations the same assumptions were made as were made by the above authors. These were: 1) the frictional drag of the wall is zero, 2) the forward momentum of the injected water is zero, and 3) evaporation is instantaneous and complete.

Theoretically, if these assumptions are valid, the total stagnation pressure rise across the water injection section corresponding to a final Mach number of zero would be available. However, the obvious difficulty of cooling a supersonic stream continuously through sonic speed and into the subsonic region would make it unwise to expect the optimum results. It seems reasonable to split the passage into two separate and distinct parts for purposes of analysis; a converging water injection section (supersonic) and a diverging subsonic diffuser, separated by a normal shock. Then the evaporation will be assumed to terminate at the shock, to which a reasonable intensity could be assigned, consistent with stable operation of the diffuser. The shock was assumed to occur at a Mach number of 1.1.

Although evaporation will probably continue after the shock, its

effects will be negligible as shown by curves of stagnation pressure rise obtainable with initial Mach numbers less than one.[1] The total stagnation pressure ratio will then be the product of two ratios; that across the water injection section computed for a final Mach number of 1.1, and the ratio across the normal shock and the subsonic diffuser. Since the second ratio will be determined by the design of the divergent section, it was considered to be essentially constant as regards variation of conditions at inlet to the evaporation section. The merit of any combination of inlet Mach number and stagnation temperature will then be measured only by the stagnation pressure ratio available across the evaporation section.

The method of computation is described in Appendix A, while Appendix B contains a sample calculation for an initial Mach number of 2.5 and an initial stagnation temperature of 1500°R. The results of such calculations are shown plotted in Figure 1. While it was realized that more extensive and detailed computations were desirable, they were not undertaken in view of the labor involved in obtaining solutions without the aid of an automatic computer.



STAGNATION PRESSURE RATIO ACROSS EVAPORATION SECTION

vs

~~INITIAL MACH NUMBER~~

Initial stagnation temperatures: 1000, 1500, 2000R

Final Mach Number 1.10

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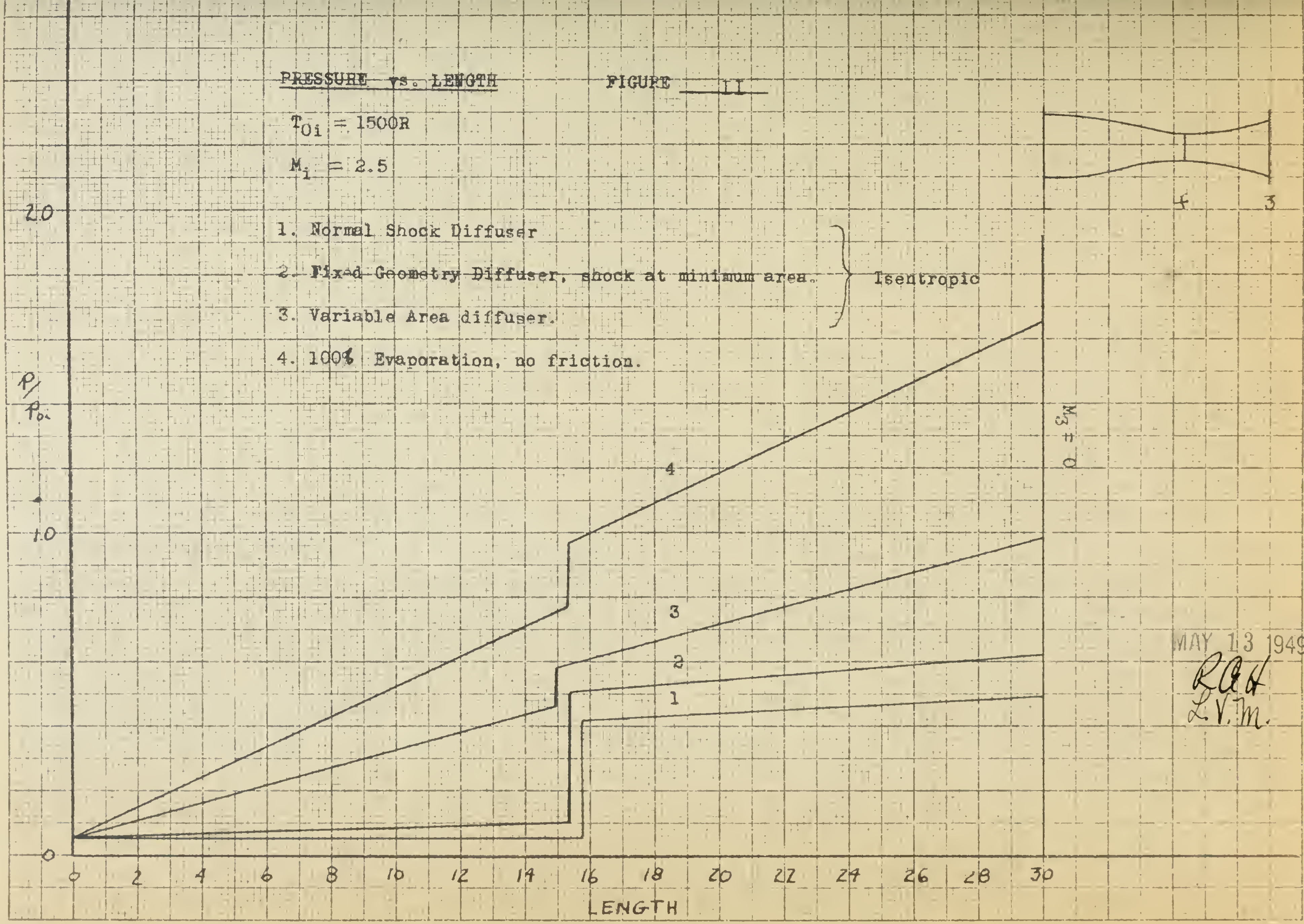
FIGURE I

Practical Considerations

After examination of the data for constant temperature evaporation shown in Figure I, the final design point was selected at an initial stagnation temperature of 1500°R and an initial Mach number of 2.5. This was a compromise selection, dictated by the equipment and materials readily available. For continuous operation an initial stagnation temperature of 1500°R was considered an upper limit. Examination of Figure I shows that an initial Mach number of 2.5 produces nearly optimum results for the temperature chosen. Also, with the pumping capacity available for starting strictly limited, the use of an initial Mach number much greater than 2.5 would increase the starting stagnation pressure ratio to such an extent that the small mass rate of flow permissible would seriously limit the possibility of obtaining measurable results, due to the scale factor effects on friction and evaporation. The effect of scale factor will be discussed more extensively below. Sufficient heating capacity was available to permit tests at higher temperatures after completing investigations at 1500°R. Figure II compares the theoretical performance at the design point with other types of diffusers as indicated.

The design point having been decided upon there remained to be solved three basic problems. These were 1) selection of a method of varying the area of the cross section of the evaporation section in order to be able to pass the flow through the throat in starting, and yet be able to adjust for constant temperature evaporation while injecting water, 2) selection of the method of injecting water, and 3) selection of a length for the evaporation section which would permit reasonably complete evaporation, yet not be so long as to create


PRESSURE vs. LENGTH
FIGURE II
T_{0i} = 1500R
M_i = 2.5
1. Normal Shock Diffuser
2. Fixed Geometry Diffuser, shock at minimum area.
3. Variable Area diffuser.
Isentropic
4. 100% Evaporation, no friction.
2.0
P/P_{0i}
1.0
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
LENGTH
1
2
3
4
$M_3 = 0$
4
3
MAY 13 1949
R.A.H.
L.V.M.

prohibitive frictional effects. Other problems were present, such as the design of the nozzle and of the subsonic diffuser, but since these parts of the apparatus were not directly under investigation no special consideration was given to them.

In solving the problem of varying cross-sectional area, a passage of rectangular section was chosen, having two plane, parallel walls and two curved walls. Variation of throat area is accomplished by movement of the curved walls. Passages of circular cross section were also investigated, but the only feasible method of varying throat area was to install a round core which could be withdrawn in the downstream direction in order to increase the throat area sufficiently to permit starting. For this method the ratio of wall area to cross-sectional area becomes prohibitively large. For the type of passage chosen this ratio is a minimum, and the passage is fairly simple to construct. Its main disadvantage is that the junction of sliding and fixed walls presents problems in sealing.

The solution offered to problems 2 and 3 above was a somewhat arbitrary one due to the almost complete lack of experimental or theoretical information on evaporation rates in a supersonic stream. It can be seen that the two problems are really tied together quite closely. First, it was established that, because of the limited overall size, a length of injection section of roughly 30 inches would produce choking effects associated with friction, and the apparatus could never be started. There were two methods of water injection available; axial injection, and peripheral stepwise injection, which more closely corresponds to the mechanics of the calculations used in analyzing the flow.

Both methods were eventually provided for and presented essentially the same problem of determining the actual distance from the point of injection to the point of completion of evaporation. This distance is a function of the time rate of evaporation, and the acceleration imparted to the water droplets by the moving stream. The evaporation rate depends, in some complex way, on the droplet size, the relative velocity of the stream to the droplet, the vapor pressure in the stream and the temperature differential from droplet to stream. The acceleration of a drop depends mainly on the drag coefficient of the drop, which is a function of the Reynolds number associated with the drop. The process is not a readily predictable one, however, because of its extreme complexity, and the probable absence of equilibrium conditions.

A computation was attempted on the basis of stepwise peripheral injection, in order to determine time rates of evaporation and absolute length of duct required for reasonably complete evaporation. This computation was based on the Colburn analogy[5], and showed that the length of duct required was so sensitive to droplet size that no prediction could be made due to the lack of any exact data on atomization in a supersonic stream. The only experimental data available[6] shows that in a subsonic stream of Mach number 0.49 and initial stagnation temperature 1140R, approximately 50% of evaporation is complete in 20 inches and increases at a very slow rate as length is increased.

Although the validity of extrapolating these results by means of the Colburn analogy is very much open to doubt, it was felt to be worthwhile in obtaining at least order of magnitude results, which was not possible by direct application of theory. Such an extrapolation indicated that at least 50% evaporation might be expected in the neighborhood of

9 inches, for a liquid stream injected through an 0.008" hole into an air stream of Mach number 2.5. The computations are shown in Appendix C. This length, being less than the maximum permissible in view of choking due to friction, was accepted as a reasonable compromise figure. A much greater length could be accomodated with a geometrically similar apparatus of larger gross dimensions, hence more complete evaporation obtained.

Having selected the length, there remained the problem of deciding where to inject the water, and how much at each point in the case of stepwise injection. Curve 1 of Figure X shows the theoretical curve of area corresponding to 100% evaporation with no friction, and represents an evaporation rate linear with distance. Since such an area change would cause prohibitive oblique shocks, and because of the mechanical difficulty of getting the same amount of water into the smallest area as into the maximum area, a modified area curve was drawn, which is Curve 3 in Figure X, the actual area curve for the apparatus as built. Changing the area curve merely means that the axial distance scale has been warped slightly and evaporation is no longer linear with distance. This change is shown in Figure III. The problem then was to inject the water in such a manner that the evaporation would proceed along the modified line. The solution was highly arbitrary, since so little was known about either the rate or degree of evaporation. With the information at hand, it could be equally possible for the axially injected water, where all the liquid is introduced at one location, to follow the desired evaporation curve as for the finite stepwise injection plan which was finally chosen, and indicated in Figure III. In the actual apparatus, the steps could be made no smaller due to the lower limit on

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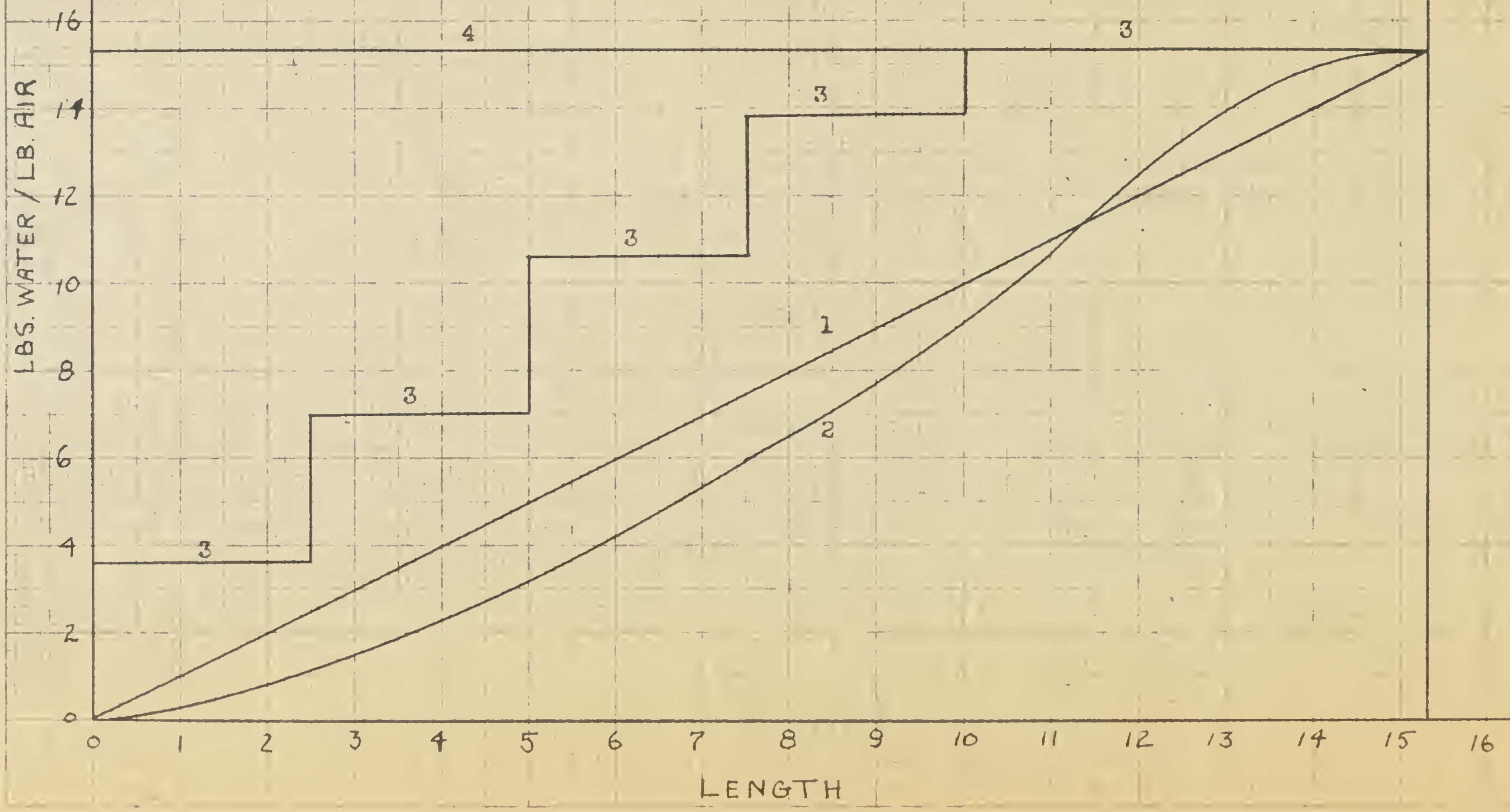
$\frac{\text{POUNDS WATER}}{\text{POUNDS AIR}}$ vs. LENGTH FIGURE III

1. Water Evaporated, evaporation linear with distance.
2. Water Evaporated, evaporation non-linear, corresponding to actual area change.
3. Water Injected, stepwise sideplate injection.
4. Water Injected, axial injection.



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the size of practical injection holes, which were made 0.008". It can be seen, however, from these curves, that in the limit, as the number of steps of injection increase, the evaporation curve must coincide with the desired one if evaporation is complete within each step; whereas with axial injection at one spot, a definite evaporation curve must be accepted, which may differ widely from the desired one. Because of the available control on the progress of evaporation by the stepwise injection, it was felt to be the most desirable provided it could be carried out. Since no conclusion could be reached as to the most advantageous location for axial injection, it was decided to make the point of axial injection a variable.

Application of Theory to the Resultant Design

After final selection of the design point and settling of the major design features had been accomplished, an attempt was made to predict the performance characteristics of the Aero-Thermoprex as designed. As pointed out previously, an investigation of evaporation rates indicated that the assumption of instantaneous evaporation was probably far from the actual condition to be expected in the limited length available in the designed apparatus. Therefore, a re-examination of the assumptions made in solving the basic working equations seemed to be in order, and some quantitative study made of the effect of any changes in the assumptions on the theoretical results.

The assumption that 4f dx/D is counterbalanced by 2y dw/w appears to be extremely optimistic for the scale of the apparatus as designed. In the computations made in the preliminary analysis the value of the primary variable, dw/w, selected for each step was about 0.01. Since about fifteen steps were required to bring the stream to Mach number 1.1, dx is, assuming linear evaporation, one-fifteenth of the total duct length, or roughly 0.5 inches. For the rectangular passage of dimensions t and h, 4f dx/D is equal to $2f\, dx\, \frac{(t+h)}{th}$. For the passage as designed the depth was constant at 1", while the width varied from 2.58" to about 1", corresponding to a variation of 4f dx/D from 1.39f to 2.00f.

As shown by Keenan and Neumann[8] the friction factor for a supersonic stream entering a straight pipe varies considerably for a short distance before a stable boundary layer is formed. Representative values for f are 0.005 at the start of the straight section and 0.002 at an L/D of 6, which is the equivalent L/D of the evaporation section as designed. This gives values of 4 f dx/D ragging from .00695 at entrance to .004 at

exit. Since these are somewhat idealized figures, and the apparent friction factor was in all probability much higher due to the water injection apparatus, leakage, and a much thinner boundary layer than would be encountered in straight pipes, a constant value of .009 was taken for the term 4f dx/D. With this figure, y, or the ratio of the forward velocity of the liquid to the velocity of the main stream, must be 0.45 in order to counterbalance friction. With an initial Mach number of 2.5, and an initial stagnation temperature of 1500°R, this means a liquid velocity of 1425 feet/second, which leads to impossibly small water injection holes, and enormous water pressures. In the apparatus as designed, velocities greater than 200 feet/second cannot be obtained, so that 2y dw/w is less than 10% of 4f dx/D. A sample calculation was made using the value .009 for 4f dx/D and zero for y, still maintaining constant temperature evaporation, (see Appendix B). This produced a more realistic performance curve for complete evaporation for the actual size of test section. From the above it can be seen that, assuming complete evaporation in a length dx, the term 4f dx/D can be made as small as desired merely by increasing the equivalent diameter; a so-called scale effect.

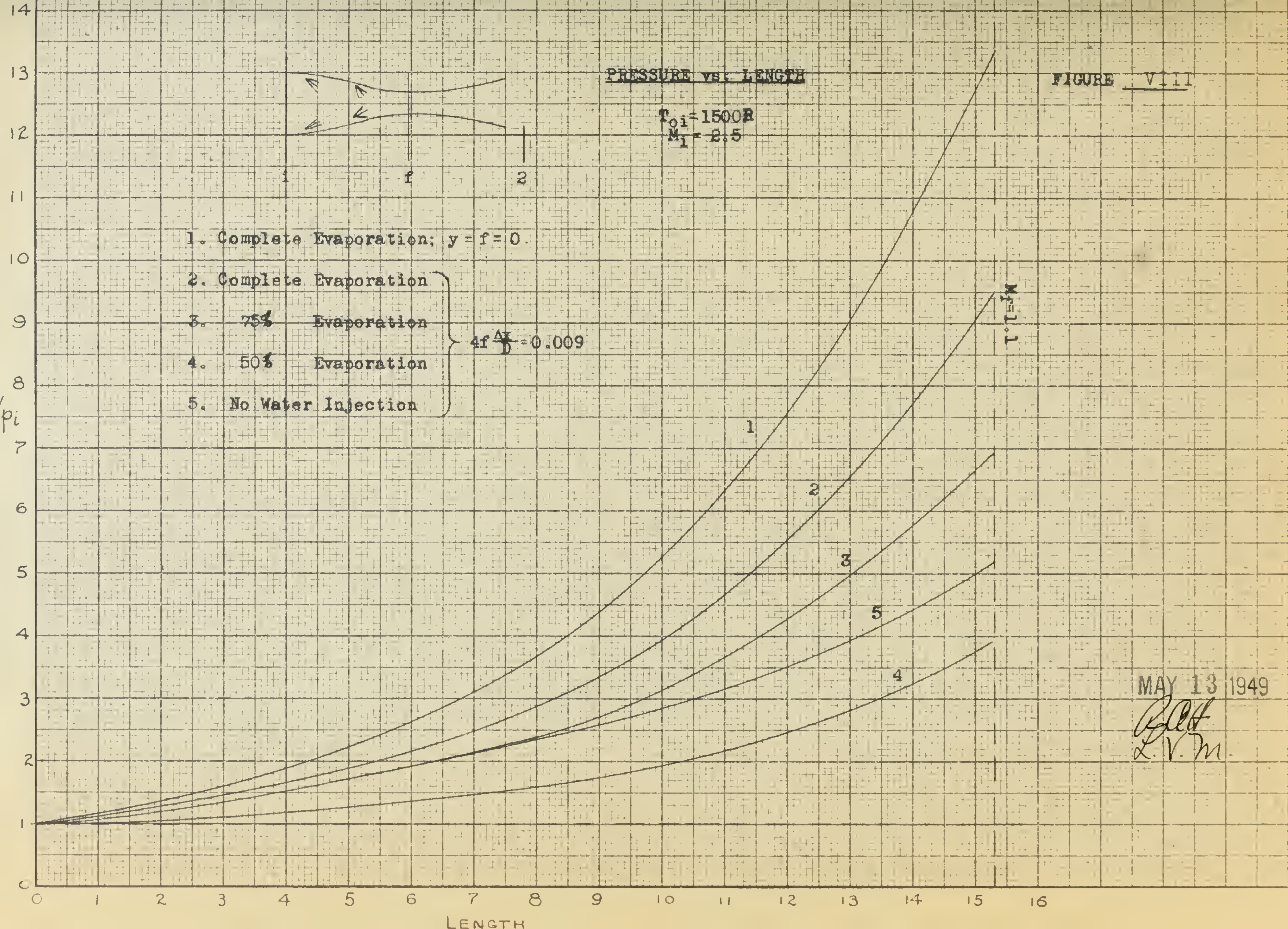
Since it has been shown probable that only fractional evaporation can be obtained in the size apparatus designed, a further refinement can be made on the basic equations to show the effect of this deviation from the preliminary assumption. The net effect on the working equation of partial evaporation is to make the dw associated with stagnation temperature different from the dw' associated with conservation of momentum. For instance, taking dw as one-half of dw' would imply that only one-half of the water injected per step finally evaporated. A question arose as to whether to compute a theoretical result for twice the water injection

required if evaporation were complete, this maintaining a constant temperature down to Mach number 1.1, or for the same total water injection with 50% evaporation. The second method would, if computed for constant temperature, exactly duplicate the first, but would terminate at some Mach number short of 1.1. If computed for the systematic variation of some other parameter, such as area, a final Mach number of 1.1 could be reached, but the complexity of such a computation ruled it out. Therefore, the first method was used, bearing in mind that the results would not differ seriously from those of the second method, if it could be carried out. This can be verified in part by considering the second method carried out at constant temperature until all the water has been injected, then diffusing to Mach number 1.1 by area change only. As shown by Shapiro and Wadleigh, the bulk of the stagnation pressure rise occurs at the higher Mach numbers. Therefore, the result will not differ too radically from those obtained in the actual calculation by the first method. For this calculation c_p and k were based on the constituents of air and water vapor only. This made possible a solution by applying a correction factor to the steps of the computation for complete evaporation (see Appendix B).

It was also felt that some analysis should be made of the flow passage as such, that is without water injection. Such an analysis would be of great use in separating the effects of evaporation from the effects of peculiarities or inadequacies of design. If, for instance, the dry run actually corresponds to the results predicted by the one-dimensional analysis, the wet runs could also be expected to conform. Without such a prediction of dry characteristics, any deviation of wet run results from theoretical results might be attributed to the wrong cause. Furthermore,

such dry characteristics would offer a useful basis of comparison for the merit of the overall wet performance of the apparatus. Such a calculation was made using the same basic one-dimensional approach. k was taken to be the constant, and equal to 1.40, while 4f dx/D was given its previous value of .009. dA/A was taken to be a constant value which would give a final area of about 1.5 square inches, considered to be obtainable operating the evaporation section as a simple variable area diffuser. The method of varying A is arbitrary, since approximately the same end state will be reached as would be reached if area curve of the designed apparatus were used. The computation is included in Appendix B.

Curves of pressure vs. length for the various processes computed are plotted in Figure VIII. Finally, the pressure curves plotted against length must be shifted in order to bring into coincidence the actual and calculated areas at any given point. In Figure XI is shown the shift of the curve of pressure for 100% evaporation without friction.


PRESSURE vs. LENGTH
$T_{oi} = 1500R$
$M_1 = 2.5$
FIGURE VIII
i
f
2
1. Complete Evaporation; y = f = 0
2. Complete Evaporation
3. 75% Evaporation
4. 50% Evaporation
5. No Water Injection
$4f\frac{\Delta X}{D} = 0.009$
$M_f = 1.1$
1
2
3
5
4
p/p_i
LENGTH
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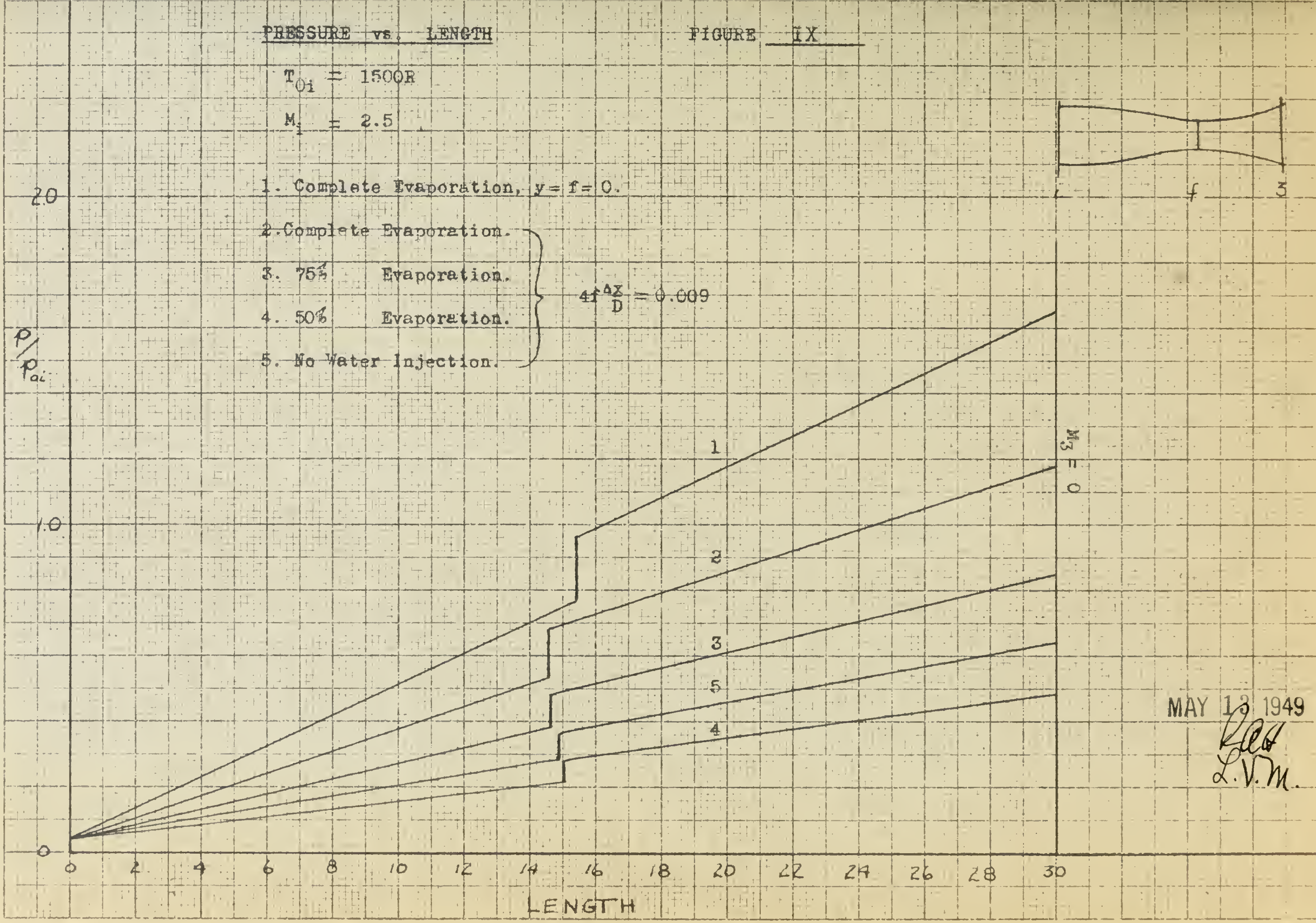
PRESSURE vs. LENGTH
FIGURE IX
T_{0i} = 1500R
M_i = 2.5
1. Complete Evaporation, y = f = 0.
2. Complete Evaporation.
3. 75% Evaporation.
4. 50% Evaporation.
5. No Water Injection.
$4f\frac{\Delta x}{D} = 0.009$
P/P_{oi}
2.0
1.0
0
1
2
3
5
4
$M_3 = 0$
f
3
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
LENGTH
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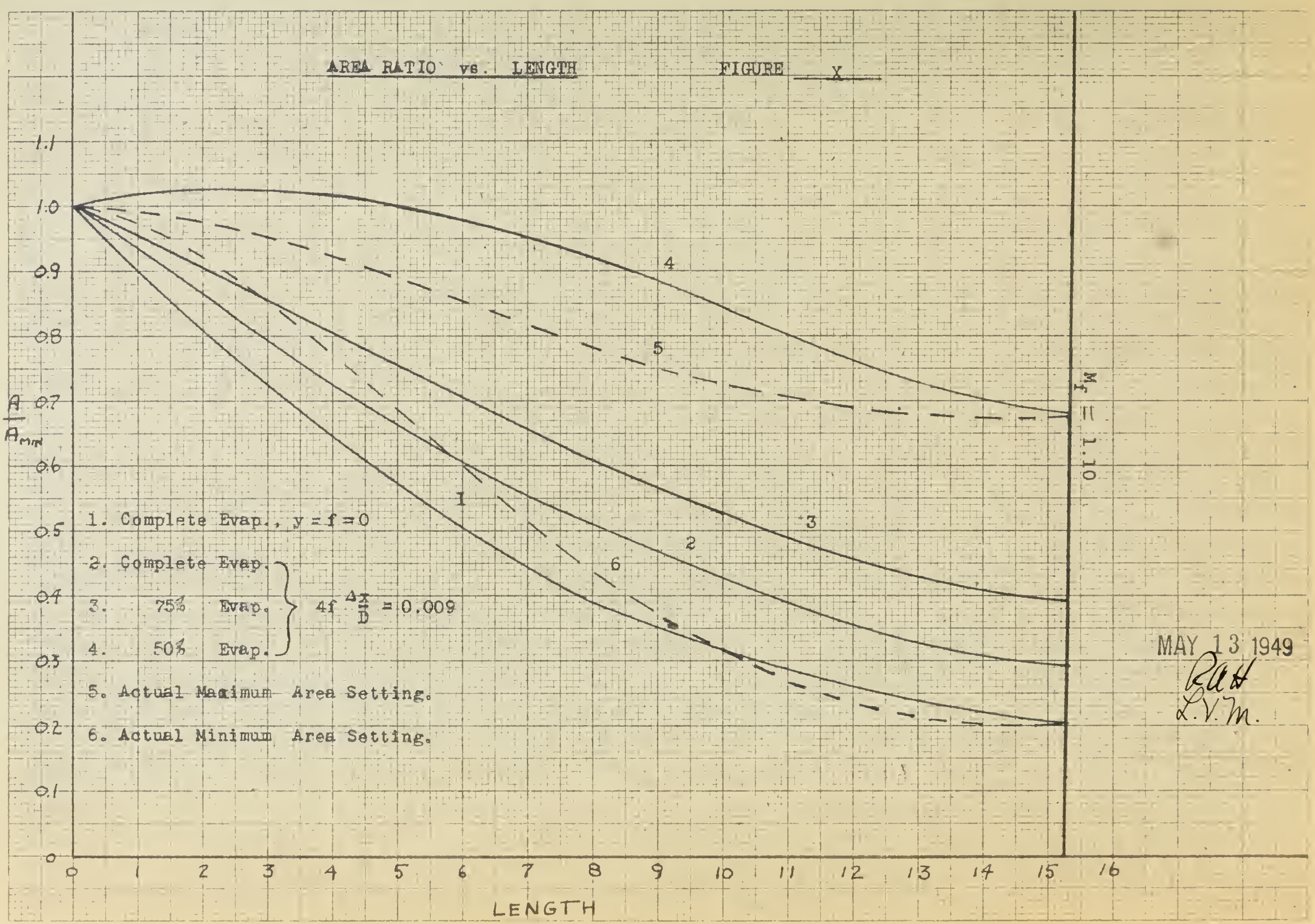

AREA RATIO vs. LENGTH
FIGURE X
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
A / A_MIN
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
LENGTH
M_f = 1.10
1. Complete Evap., y = f = 0
2. Complete Evap.
3. 75% Evap.
4. 50% Evap.
4f Δx/D = 0.009
5. Actual Maximum Area Setting.
6. Actual Minimum Area Setting.
1
2
3
4
5
6
MAY 13 1949
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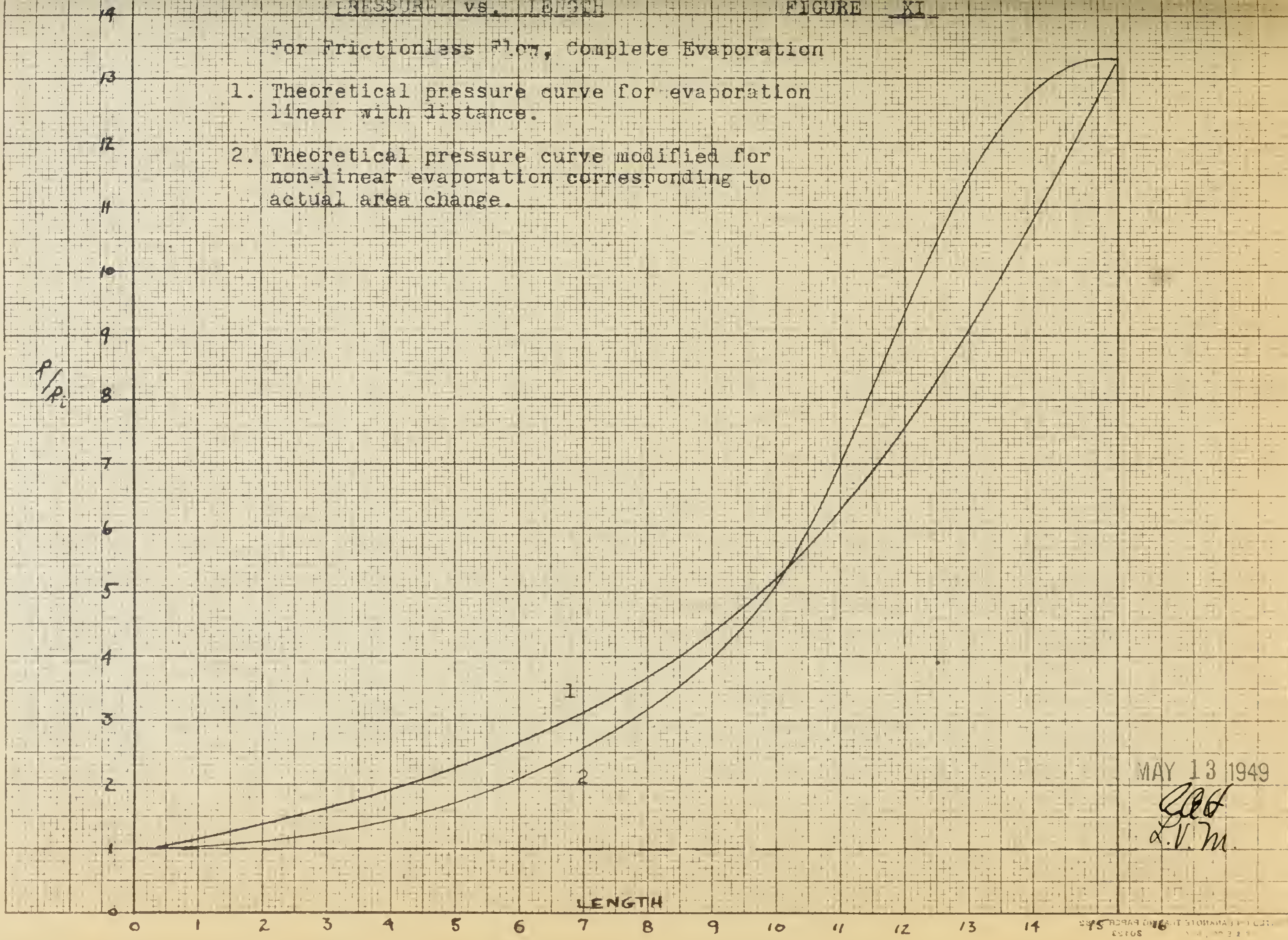
PRESSURE vs. LENGTH FIGURE XI

For Frictionless Flow, Complete Evaporation

1. Theoretical pressure curve for evaporation linear with distance.

2. Theoretical pressure curve modified for non-linear evaporation corresponding to actual area change.



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IV DESCRIPTION OF EQUIPMENT

The elements of the Aero-Thermoprex are shown in the schematic sketch, Figure IV, and in the photograph, Figure IV-A. The legend of Figure IV lists the basic elements of the apparatus. The air ejector supplies whatever pumping action is required to maintain the desired back pressure in the exhaust receiver. Air enters the furnace and leaves with products of combustion at 1500°R. In the nozzle the gas stream is accelerated to Mach number 2.5, and into the supersonic stream water is injected by either of two methods: 1) axially in the direction of flow, and 2) peripherally with a moderate downstream component.

Description of the Elements

A. Air Heating Apparatus

A propane furnace is used to heat the air; the products of combustion pass out and go through the test section.

B. Nozzle-Water Injection-Diffuser Section

The nozzle and diffuser were made by shaping flat stainless steel blocks, 1 inch thick, and fixing them to stainless steel side plates to form a rectangular flow passage cross-section. (See Figure V.) The nozzle was designed to give the largest throat area possible in accordance with the air ejector capacity available in the Gas Turbine Laboratory at M. I. T. This value proved to be 0.91 in^2, which corresponds to a mass flow of 620 lbs/hour at 1500R stagnation temperature. The nozzle exit area was designed for a Mach number of 2.5, with provisions made to vary the Mach number from about 2.4 to 2.8 by shimming the nozzle blocks. The design of the water injection and diffuser sections was complicated, as

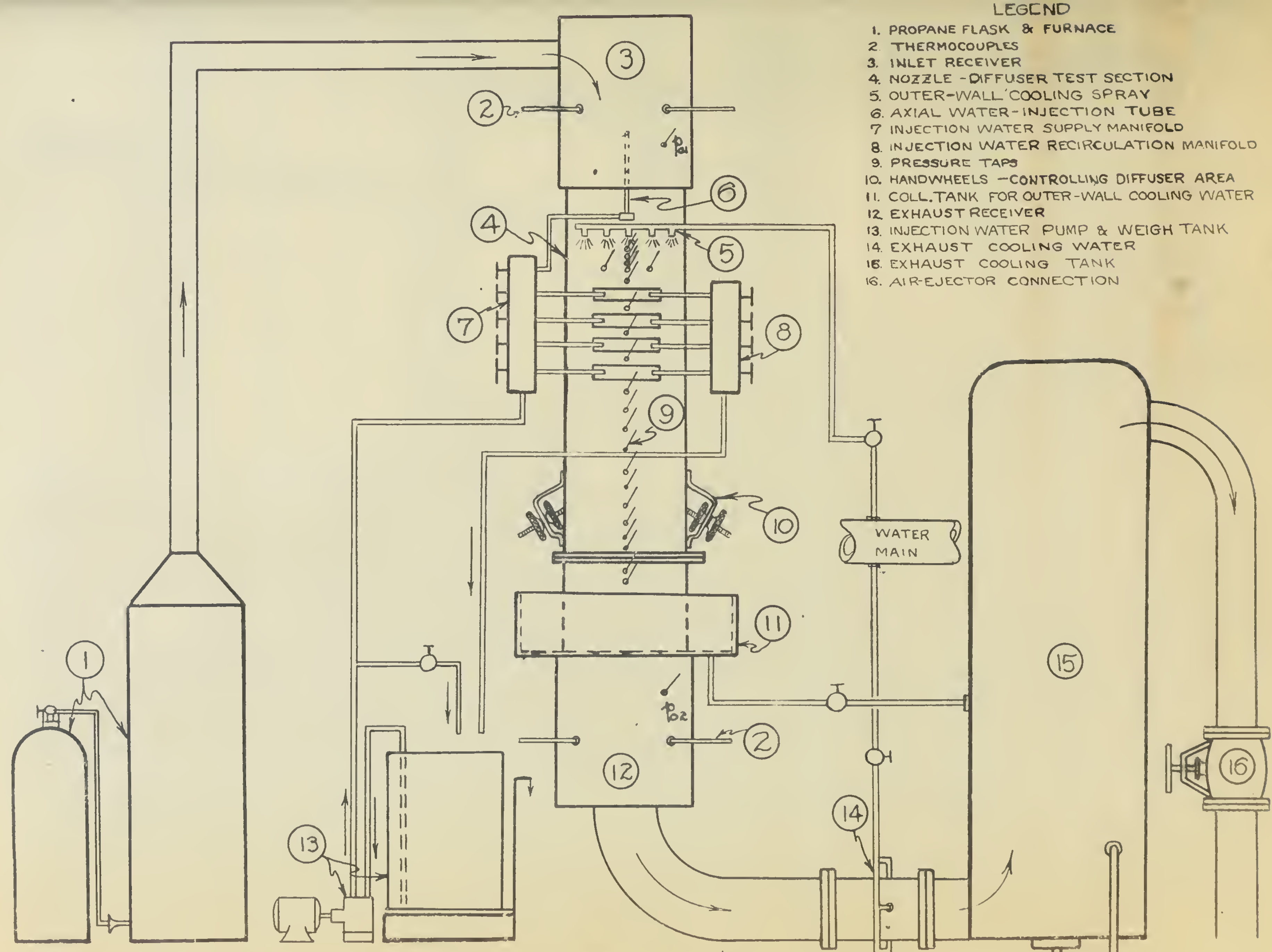
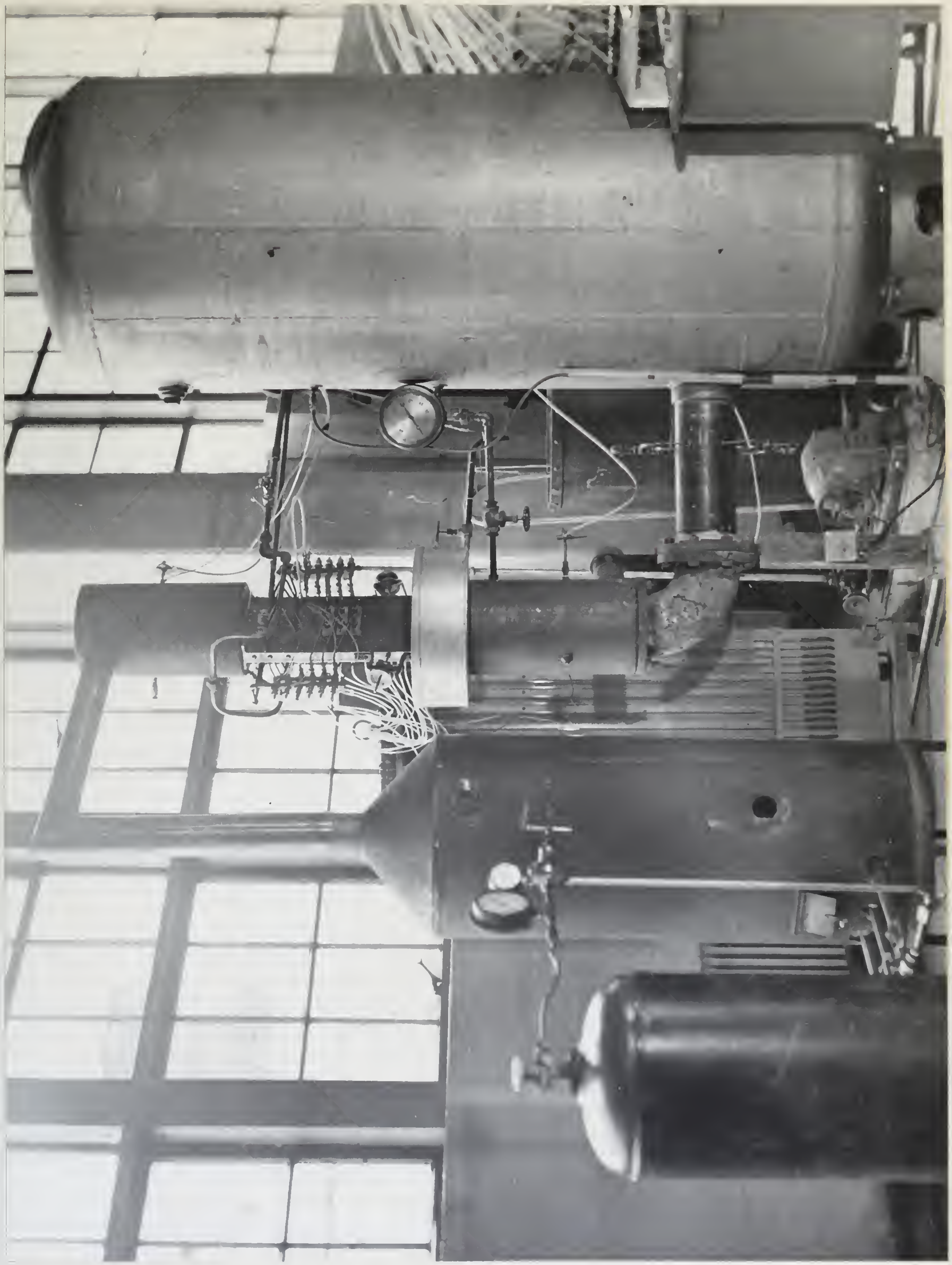

LEGEND
1. PROPANE FLASK & FURNACE
2. THERMOCOUPLES
3. INLET RECEIVER
4. NOZZLE - DIFFUSER TEST SECTION
5. OUTER-WALL COOLING SPRAY
6. AXIAL WATER-INJECTION TUBE
7. INJECTION WATER SUPPLY MANIFOLD
8. INJECTION WATER RECIRCULATION MANIFOLD
9. PRESSURE TAPS
10. HANDWHEELS - CONTROLLING DIFFUSER AREA
11. COLL. TANK FOR OUTER-WALL COOLING WATER
12. EXHAUST RECEIVER
13. INJECTION WATER PUMP & WEIGH TANK
14. EXHAUST COOLING WATER
15. EXHAUST COOLING TANK
16. AIR-EJECTOR CONNECTION
WATER MAIN
p_{01}
p_{02}

FIG. IV

FIGURE IV-A



Arrangement of Apparatus

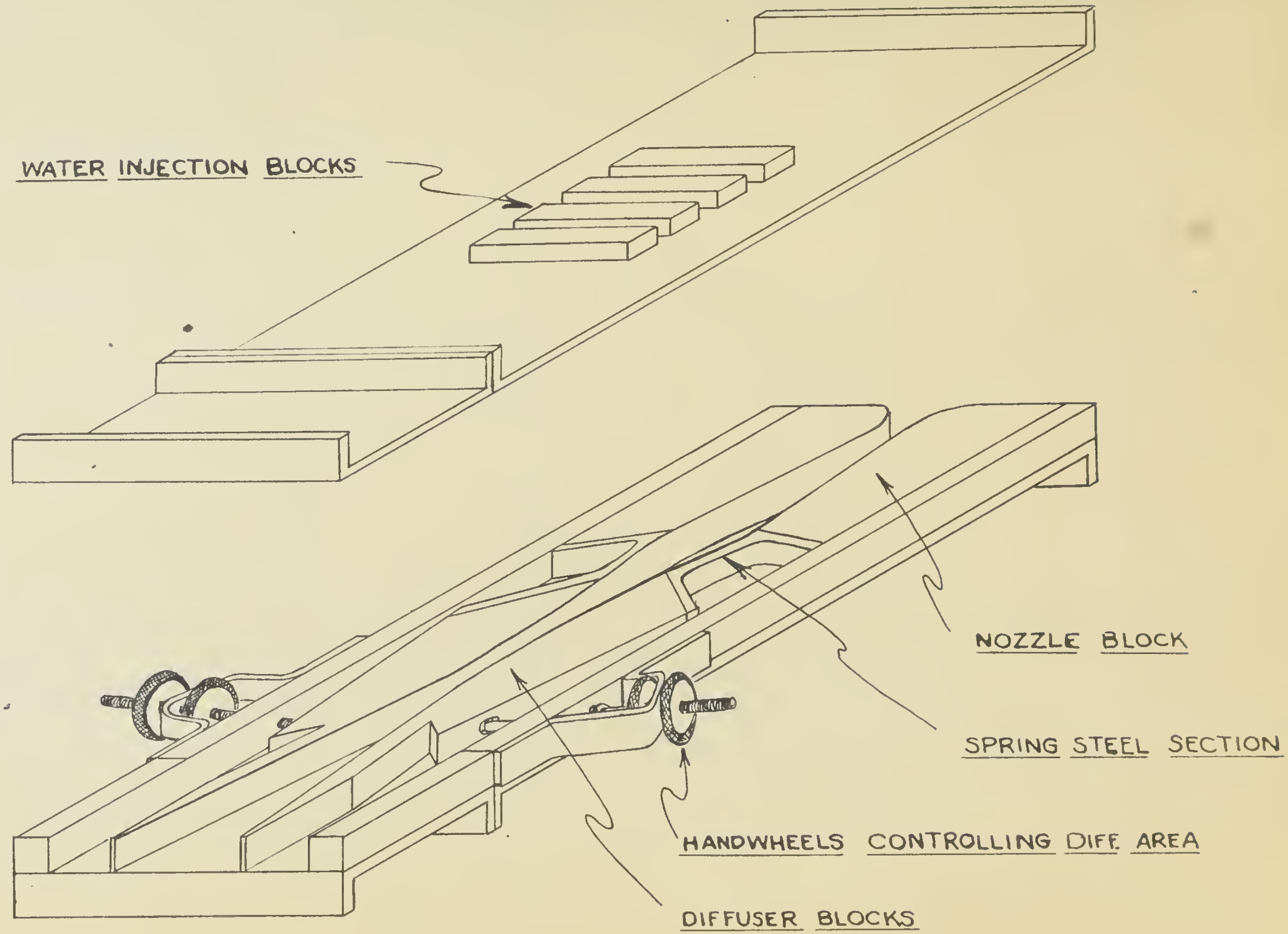


FIG. V NOZZLE - DIFFUSER TEST SECTION

FIG. VI METHOD OF INJECTING WATER AXIALLY

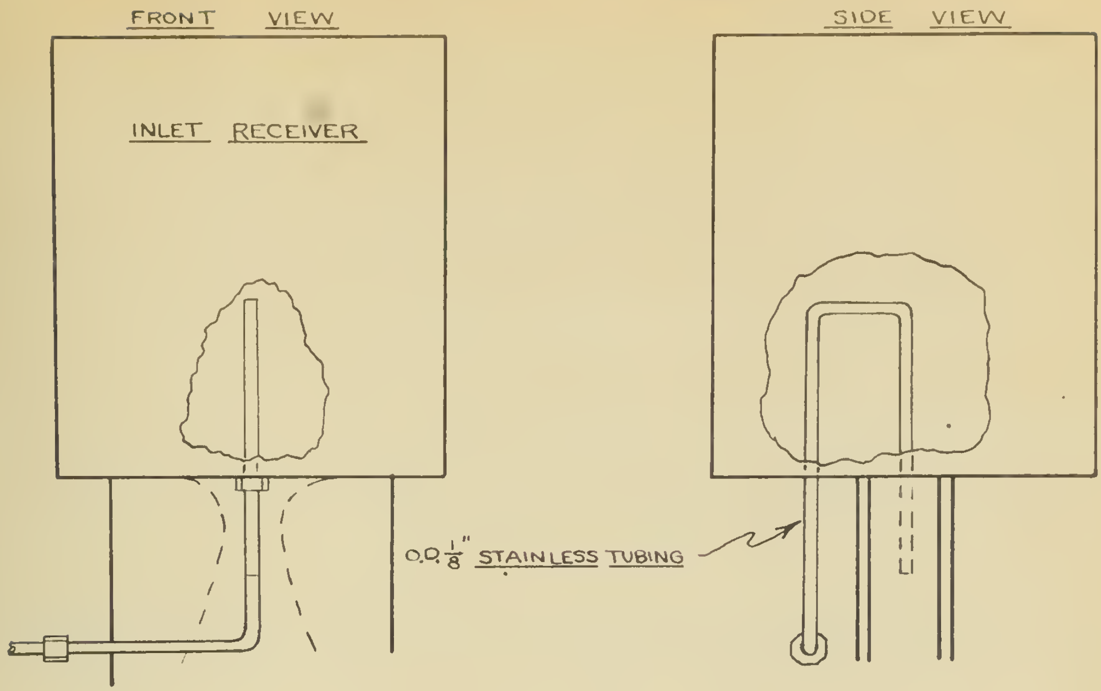
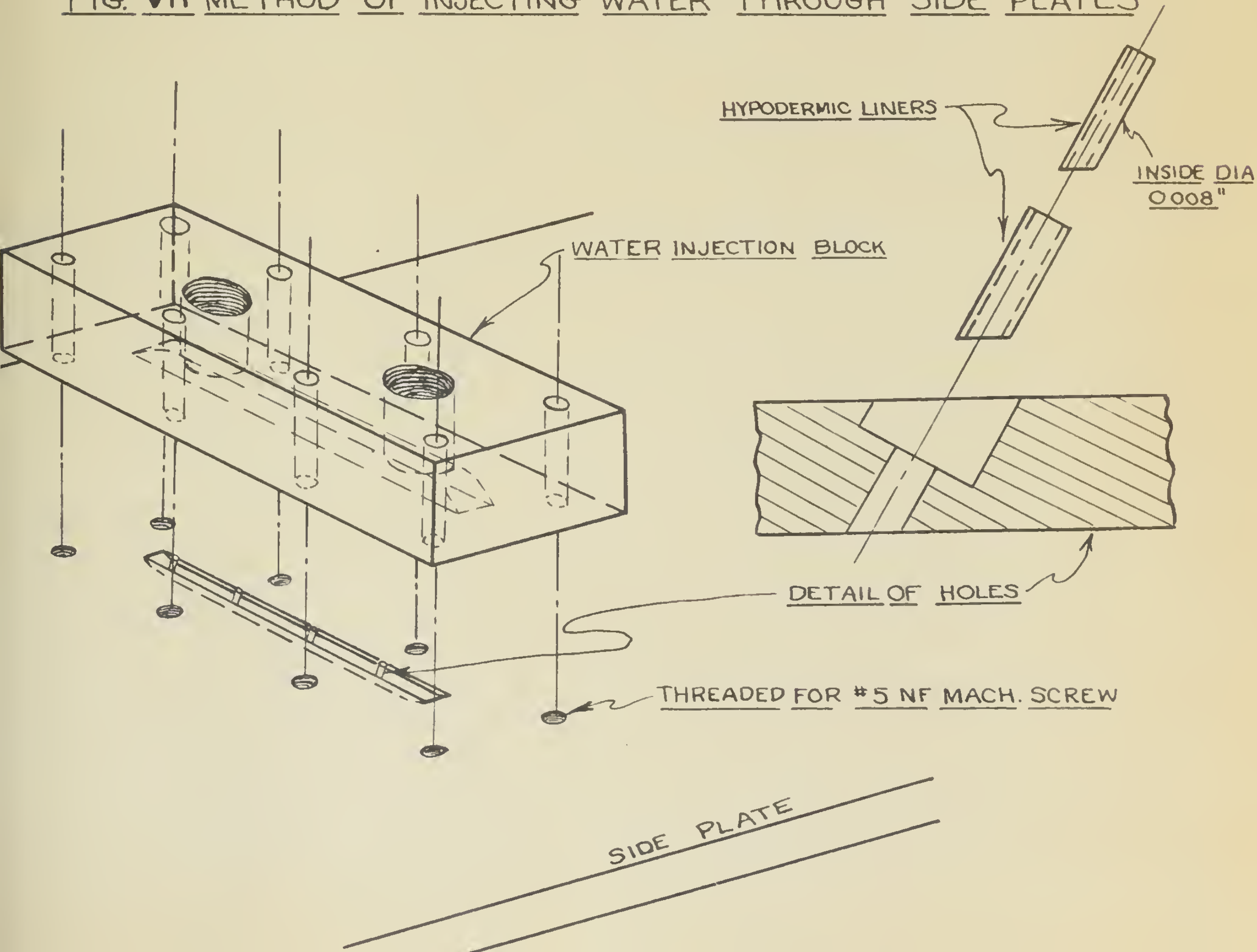


FIG. VII METHOD OF INJECTING WATER THROUGH SIDE PLATES



pointed out, by the fact that it is necessary to provide a larger diffuser throat area for starting than for operation. This requirement of variable area was met by securing rigidly only the nozzle blocks, attaching them to the diffuser blocks with spring steel bars, and providing draw-rods with handwheels to move the diffuser blocks in and out. (See Figure V.) The shape of the diffuser blocks was designed such that in the fully closed position, the area would conform to the modified theoretical curve shown in Figure X, curve 6.

C. Water Injection Apparatus

Water is pumped from a weigh tank and injected into the stream by two methods.

1. An axial injection tube of stainless steel, 0.05" inside diameter, was installed in the inlet receiver as shown in Figure VI. The position of the tube is adjustable, allowing water injection at various points along the flow passage.

2. Peripheral injection holes with cover blocks were installed in the sides of the two flat stainless steel plates as shown in Figure VII. Because of the difficulty of drilling holes of the designed diameter, (0.008") it was necessary first to drill larger size holes and line them with hypodermic tubing, which was in turn lined with smaller tubing of the required inside diameter. To prevent the water from boiling under the injection cover blocks, it is circulated through the blocks and returned to the weigh tank via the recirculation manifold. A supply manifold allows any desired combination of blocks to be used for water injection. Two #200 mesh wire screens were installed in the supply line to safeguard the small injection holes. Pump pressure is regulated by a by-pass valve.

D. Test Section Outer Wall Cooling Apparatus

It was found to be necessary to cool the outer surfaces of the test section to secure protection for the operator of the diffuser area handwheels against the high local temperature. Water is piped from the city main and is flooded down the outer walls of the test section. It is caught in a pan at the bottom of the test section, and discharged into the exhaust cooling tank. (See schematic sketch Figure IV.)

E. Exhaust Cooling Apparatus

It is necessary to cool the exhaust gases further after they leave the test section in order to reduce the temperature to allowable limits for passing through the air ejector. Water from the city main is injected into the exhaust gases and separated in the cooling tank, as shown in the schematic sketch, Figure IV. The problem of draining this cooling water from the tank, in which there is a high vacuum, was simplified somewhat by taking advantage of a 25 foot drop to the sump tank. A water-jet eductor was installed in the drain line.

F. Measuring Apparatus

1. Pressure taps are spaced as shown in Figure XIII. Traps were installed in the pressure leads to prevent water from accumulating in the mercury manometer columns.

2. Chromel-Alumel, silver shielded thermocouples were installed in the Inlet and Exhaust Receivers to measure the temperature at these points.

3. The amount of water injected is measured from weigh tank readings.

V RESULTS AND DISCUSSION

The selection of the design point for the Aero-Thermoprex was a compromise between the limitations imposed by the facilities available, and the optimum operating point as indicated by the preliminary analysis. The design point selected was at initial Mach number 2.5 and initial stagnation temperature 1500°R for constant temperature evaporation. Provision was made for the variation of inlet Mach number within fairly narrow limits, while inlet temperature could be varied rather widely. The mechanical system as built is believed to be adequate for the purpose of investigating the fundamental problem of raising the stagnation pressure of a high temperature air stream by evaporating water into the air stream. The theoretical stagnation pressure ratio available across the evaporation section for frictionless flow and 100% evaporation at the design point is 1.66.

Further investigation of the designed Aero-Thermoprex by one-dimensional theory showed that:

1. The assumption that y (which is V_L'/V) is equal to zero is not in serious error.
2. The assumption that f is equal to zero is not valid. Using an empirical friction factor for the apparatus as constructed, the available stagnation pressure across the evaporation section is reduced from 1.66 to 1.18 for complete evaporation.
3. The assumption that evaporation is complete is probably not valid. For 75% evaporation the stagnation pressure ratio available across the evaporation section is reduced

from 1.18 to 0.83, while for 50% evaporation this figure is reduced to 0.49, which is less than the corresponding ratio for zero water injection, or a dry run.

4. The theoretical computation for a dry run shows that a stagnation pressure ratio of 0.64 is obtained across the evaporation section. This is much greater than the ratio obtained in any actual supersonic diffuser at Mach number 2.5, which leads to the conclusion that, for converging supersonic passages, a one-dimensional treatment with the use of a reasonable friction factor will give results that are optimistic for any assumed rate of evaporation.

5. The accurate determination of the rate of evaporation in the designed apparatus is impossible. As a result of comparison with other experimental work it may be expected to be between 50% and 75% complete.

Preliminary tests of the Aero-Thermoprex were not performed with the goal in mind of producing an increase of stagnation pressure, but rather of determining the degree to which the individual components met the requirements which grew out of the design study. Data from preliminary tests is shown in Figures XII and XIII, Appendix D.

The flow passage behaved about as was expected, producing an average Mach number at the maximum area section of 2.49 as determined by calculation, a sample of which is included in Appendix D. The average Mach number was determined only for a stagnation temperature of $1500^{\circ}R$. The diffuser throat was capable of adjustment between the limits shown in Figure X. Upon disassembly after several hours of operation at temperatures above 1500R, the stainless steel surfaces showed only slight discoloration. The sections of the wall at the maximum area section which were made of cutlery spring steel were

blackened somewhat, and were beginning to show minute localized pitting, which was not felt to be serious enough to introduce any new factors into the analysis. The cutlery steel retained its elasticity throughout the test. The leakage past the variable area diffuser blocks was apparently not significant.

Water injection apparatus was served with the required water flow up to 60 psig. The axial injection tube could be adjusted so as to discharge water ranging from three inches upstream of the nozzle throat to three inches downstream of the nozzle throat, and at rates varying from zero to 175 pounds per hour. Stepwise water injection from zero to 175 pounds per hour was possible. The furnace which heated the air flowing to the inlet receiver was capable of raising the stagnation temperature to the upper level indicated in the design point studies. The highest inlet temperature obtained was 1850R. Thermocouples provided for the measurement of inlet and outlet stagnation temperatures had previously been calibrated by the U. S. Naval Engineering Experiment Station but were checked against one another to insure that all were in agreement. One thermocouple failed as a result of oxidation under the silver shield after about four hours at elevated temperature.

The pressure taps installed along the wall provided the only available measure of the stream properties in the high speed regions. By observing the readings of the mercury manometers it was possible to follow the axial movement of shocks, and of the diffuser throat as the boundary conditions imposed on the stream changed.

In the size constructed any positive "pumping" action by the Aero-Thermoprex is extremely improbable. Only if evaporation is something more than 50% complete will the device perform more efficiently with water injection than without. However, complete testing of the apparatus was felt to be desirable, both for the purpose of substantiating the theoretical results and to provide a basis for further investigation. Some information on evaporation rates is also to be gained from such testing. Such information has previously been non-existent. If good correlation between actual and calculated characteristics is obtained, the Aero-Thermoprex could become a positive "pumping" device in larger sizes since, as has been pointed out, the absolute size of the flow passage has so great an influence on the effect of friction and on the completeness of evaporation. Complete testing of the apparatus is the subject of a companion thesis.

APPENDIX A

ANALYSIS OF VARIOUS EVAPORATION PROCESSES AT CONSTANT TEMPERATURE

BASIC EQUATIONS

For analysis of one-dimensional evaporation processes at constant temperature the following equations are used:

Equation (65) of reference 2 is written in the form

$$M_2^2 = \frac{(K_1-1)}{(K_2-1)}\frac{c_{p_1}}{c_{p_2}} M_1^2 - \frac{2(h_v - h_L)}{c_{p_2}T(K_2-1)}\frac{\Delta w}{w_m} \qquad \text{A-1}$$

and is integrated stepwise to obtain the Mach number at each point in the passage as a function of $\Delta w/w_m$ only.

Equation (10) of reference 2 is written

$$\frac{dM^2}{M^2} = -\frac{2\left(1+\frac{K-1}{2}M^2\right)}{1-M^2}\frac{dA}{A} - \frac{1+KM^2}{1-M^2}\,\frac{(h_v-h_L)+\frac{V^2-V_L^2}{2}}{c_pT}\,\frac{dw}{w} - \frac{dK}{K}$$
$$+\frac{KM^2\left(1+\frac{K-1}{2}M^2\right)}{1-M^2}\left[4f\frac{dx}{D} - 2y\frac{dw}{w}\right] + \frac{2(1+KM^2)\left(1+\frac{K-1}{2}M^2\right)}{1-M^2}\frac{dw}{w} - \frac{1+K}{1-M^2}\frac{dW}{W},$$

solved for dA/A, and then written in the finite difference form

$$\frac{\Delta A}{A_m} = \left(\frac{KM^2}{2}\right)_m\left[4f\frac{dx}{D} - 2y\frac{dw}{w}\right]_m + (1+KM^2)_m\frac{\Delta w}{w_m} - \frac{(1+KM^2)_m}{2\left(1+\frac{K-1}{2}M^2\right)_m}\left[\frac{h_v - h_L + \frac{V^2-V_L^2}{2}}{c_pT}\right]\frac{\Delta w}{w_m} \qquad \text{A-2}$$
$$-\frac{(1+KM^2)_m}{2\left(1+\frac{K-1}{2}M^2\right)_m}\frac{\Delta W}{W_m} - \frac{(1-M^2)_m}{2\left(1+\frac{K-1}{2}M^2\right)_m}\frac{\Delta K}{K_m} - \frac{(1-M^2)_m}{2\left(1+\frac{K-1}{2}M^2\right)_m}\frac{\Delta M^2}{M_m^2},$$

after which it is integrated stepwise as shown, depending on the assumptions made.

In order to compute stream pressures and stagnation pressures

at any point, relations (66) and (38) of reference 2 are written

$$\frac{p_2}{p_1} = \frac{\omega_2 A_1 M_1}{\omega_1 A_2 M_2}\sqrt{\frac{K_1 W_1}{K_2 W_2}} \qquad \text{A-3}$$

and

$$\frac{p_{01}}{p_{02}} = \frac{p_1}{p_2}\,\frac{\left(1+\frac{K_{01}-1}{2}M_1^2\right)^{\frac{K_{01}}{K_{01}-1}}}{\left(1+\frac{K_{02}-1}{2}M_2^2\right)^{\frac{K_{02}}{K_{02}-1}}} \qquad \text{A-4}$$

Numerical integrations are shown for the following cases with assumptions as indicated.

1. Frictionless Flow, Complete Evaporation
   - a. $f = y = 0$
   - b. $V^2 - V_L^2$ neglected. (see note)
   - c. Evaporation Complete and Instantaneous
2. Flow with Friction, Complete Evaporation
   - a. $y = 0$
   - b. $4f\ dx/D = 0.009$ constant
   - c. $V^2 - V_L^2$ neglected. (see note)
   - d. Evaporation Complete and Instantaneous
3. Flow with Friction, 75% Evaporation
   - a. $y = 0$
   - b. $4f\ dx/D = 0.009 =$ constant
   - c. $V^2 - V_L^2$ neglected. (see note)
   - d. $\Delta w/w_m$ in second term of A-2 becomes $\Delta w'/w_m$ and is four-thirds of $\Delta w/w_m$.

4. Flow with Friction, 50% Evaporation

Same as 3 except $\Delta w'/w_m$ is twice $\Delta w/w_m$

5. Flow with Friction, No Water Injected

a. $y = \Delta w/w_m = \Delta W/W_m = \Delta k/k_m = 0$

b. For unit step $4f\, dx/D = 0.009$

c. For unit step $\Delta A/A_m = 0.070$

d. A - 2 is solved for $\Delta M^2/M^2$ and integrated stepwise as shown.

Note: Assumption that $y = 0$ means $V_L = 0$. $V^2/2g$ varies from about 10% of $h_v - h_L$ at start to 1% at end of process. If included it would present a slightly more optimistic rise in stream and stagnation pressures.

APPENDIX B

NUMERICAL INTEGRATION

100% Evaporation — Friction Absent — M = 2.5

	Step		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		M_1	2.5000	2.4224	2.3436	2.2648	2.1830	2.1011	2.0176	1.9326	1.8465	1.7562	1.6641	1.5685	1.4690	1.3644	1.2536	1.1390	1.0095
2		M^2	6.2500	5.8664	5.4911	5.1241	4.7655	4.4148	4.0709	3.7349	3.4063	3.0842	2.7691	2.4603	2.1579	1.8616	1.5715	1.2973	1.0189
3		$\Delta W/W_m$	.00995	.00985	.00976	.00966	.00957	.00949	.00939	.00930	.00922	.00913	.00905	.00897	.00889	.00881	.00873	.00865	.00857
4	[illegible]		0	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13	.14	.15	.16
		k	1.3990	1.3983	1.3975	1.3968	1.3960	1.3953	1.3945	1.3938	1.3930	1.3923	1.3915	1.3908	1.3900	1.3893	1.3886	1.3879	1.3872
		k_2	1.3983	1.3975	1.3968	1.3960	1.3953	1.3945	1.3938	1.3930	1.3923	1.3915	1.3908	1.3900	1.3893	1.3886	1.3879	1.3872	
		W_1	28.97	28.86	28.75	28.64	28.53	28.42	28.31	28.20	28.09	27.98	27.87	27.76	27.65	27.54	27.43	27.32	27.21
		W_2	28.86	28.75	28.64	28.53	28.42	28.31	28.20	28.09	27.98	27.87	27.76	27.65	27.54	27.43	27.32	27.21	
5		k_1-1	.3990	.3983	.3975	.3968	.3960	.3953	.3945	.3938	.3930	.3923	.3915	.3908	.3900	.3893	.3886	.3879	.3872
6		k_2-1	.3983	.3975	.3968	.3960	.3953	.3945	.3938	.3930	.3923	.3915	.3908	.3900	.3893	.3886	.3879	.3872	
7	⑤/⑥		1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002	1.002
8		c_{p_1}	.2411	.2432	.2453	.2474	.2495	.2516	.2536	.2557	.2578	.2599	.2620	.2641	.2662	.2683	.2704	.2725	.2746
9		c_{p_2}	.2432	.2453	.2474	.2495	.2516	.2536	.2557	.2578	.2599	.2620	.2641	.2662	.2683	.2704	.2725	.2746	
10	⑧/⑨		.9914	.9914	.9915	.9916	.9917	.9917	.9918	.9919	.9919	.9920	.9920	.9921	.9921	.9922	.9922	.9923	
11	⑦ × ⑩ × ②		6.2086	5.8276	5.4553	5.0912	4.7354	4.3869	4.0456	3.7121	3.3855	3.0656	2.7524	2.4457	2.1451	1.8508	1.5624	1.2899	
12	[③/(⑥ × ⑨)] × C		.3422	.3365	.3312	.3257	.3206	.3160	.3107	.3058	.3013	.2965	.2921	.2878	.2835	.2793	.2751	.2710	
13	⑪ − ⑫	M_2^2	5.8664	5.4911	5.1241	4.7655	4.4148	4.0709	3.7349	3.4063	3.0842	2.7691	2.4603	2.1579	1.8616	1.5715	1.2973	1.0189	
14		$1+kM^2$	9.7437	9.2029	8.6738	8.1573	7.6526	7.1608	6.6769	6.2057	5.7450	5.2941	4.8532	4.4218	3.9995	3.5863	3.1822	2.8005	2.4134
15		$1+\frac{k-1}{2}M^2$	2.2469	2.1683	2.0914	2.0166	1.9436	1.8726	1.8030	1.7354	1.6693	1.6050	1.5421	1.4807	1.4208	1.3624	1.3053	1.2516	1.1973
16		$1-M^2$	−5.2500	−4.8664	−4.4911	−4.1241	−3.7655	−3.4148	−3.0709	−2.7349	−2.4063	−2.0842	−1.7691	−1.4603	−1.1579	−0.8616	−0.5715	−0.2973	−0.0189
17	⑭ × ⑮ / ⑯		−4.1701	−4.1005	−4.0392	−3.9888	−3.9410	−3.9268	−3.9202	−3.9378	−3.9854	−4.0769	−4.2307	−4.4836	−4.9076	−5.6708	−7.2681	−11.7779	−152.887
18	⑭/⑯		−1.8559	−1.8911	−1.9310	−1.9780	−2.0323	−2.0970	−2.1742	−2.2691	−2.3875	−2.5401	−2.7433	−3.0280	−3.4541	−4.1623	−5.5682	−9.4103	−127.693
19	⑮/⑯		−.42798	−.44556	−.46568	−.48898	−.51616	−.54838	−.58712	−.63454	−.69372	−.77008	−.87169	−1.01397	−1.22705	−1.58124	−2.28398	−4.20564	−63.349
20	Σ ⑰		−8.2706	−8.1397	−8.0280	−7.9298	−7.8678	−7.8470	−7.8580	−7.9232	−8.0623	−8.3076	−8.7143	−9.3912	−10.5784	−12.9389	−19.0460	−164.6649	
21	[Σ ⑱]/2		−1.8735	−1.9111	−1.9545	−2.0052	−2.0646	−2.1356	−2.2216	−2.3283	−2.4638	−2.6417	−2.8856	−3.2410	−3.8082	−4.8652	−7.4892	−68.551	
22	Σ ⑲		−.87354	−.91124	−.95466	−1.00514	−1.06454	−1.13550	−1.22166	−1.32826	−1.46380	−1.64177	−1.88566	−2.24102	−2.80829	−3.86522	−6.48962	−67.554	
23		$\Delta W/W_m$	−.003804	−.003819	−.003833	−.003848	−.003863	−.003878	−.003893	−.003908	−.003923	−.003939	−.003954	−.003969	−.003986	−.004002	−.004018	−.004034	
24		$\Delta k/k_m$	−.000536	−.000537	−.000537	−.000537	−.000537	−.000537	−.000538	−.000538	−.000538	−.000539	−.000539	−.000539	−.000540	−.000540	−.000540	−.000541	
25		$\Delta M^2/M^2_m$	−.06332	−.06609	−.06933	−.07252	−.07640	−.08105	−.08609	−.09203	−.09925	−.10766	−.11810	−.13096	−.14743	−.16900	−.19116	−.24039	
26	⑧ + ⑨		.4843	.4885	.4927	.4969	.5011	.5052	.5093	.5135	.5177	.5219	.5261	.5303	.5345	.5387	.5429	.5471	
27	C/㉕		6.8789	6.8197	6.7616	6.7044	6.6483	6.5943	6.5412	6.4877	6.4351	6.3833	6.3323	6.2822	6.2328	6.1842	6.1364	6.0893	
28	㉑ × ㉗		−12.8876	−13.0331	−13.2155	−13.4437	−13.7261	−14.0828	−14.5319	−15.1053	−15.8548	−16.8628	−18.2750	−20.3606	−23.7357	−30.0874	−45.9567	−417.4276	
29	[⑳ − ㉘] × ③		.04594	.04820	.05063	.05326	.05606	.05918	.06267	.06679	.07185	.07811	.08652	.09840	.11697	.15108	.23493	2.18640	
30	㉑ × ㉓		.007127	.007298	.007492	.007716	.007976	.008282	.008649	.009098	.009665	.010406	.011410	.012864	.015179	.019471	.030092	.20653	
31	㉙ − ㉚ − ㉔ − ㉕		.10267	.10753	.11348	.11860	.12502	.13249	.14065	.15026	.16197	.17590	.19375	.21704	.24976	.30115	.39659	2.22080	
32	㉛/㉒	$\Delta A/A_m$	−.1175	−.1180	−.1189	−.1180	−.1174	−.1167	−.1151	−.1131	−.1107	−.1071	−.1027	−.0968	−.0889	−.0779	−.0611	−.03287	
33	[2 − ㉜] ÷ [2 + ㉜]	A_1/A_2	1.1248	1.1254	1.1264	1.1254	1.1247	1.1239	1.1221	1.1199	1.1172	1.1132	1.1085	1.1017	1.0930	1.0810	1.0630	1.0334	
34		k_1/k_2	1.0006	1.0006	1.0006	1.0006	1.0006	1.0006	1.0006	1.0006	1.0006	1.0006	1.0006	1.0006	1.0006	1.0006	1.0006	1.0006	
35		W_1/W_2	1.00381	1.00383	1.00384	1.00386	1.00387	1.00389	1.00390	1.00392	1.00393	1.00395	1.00396	1.00398	1.00399	1.00401	1.00402	1.00403	
36	$\sqrt{㉞ \times ㉟}$		1.0022	1.0022	1.0022	1.0022	1.0022	1.0022	1.0023	1.0023	1.0023	1.0023	1.0023	1.0023	1.0023	1.0023	1.0023	1.0023	
37		w_2/w_1	1.01000	1.00990	1.00980	1.00971	1.00962	1.00952	1.00943	1.00935	1.00926	1.00916	1.00909	1.00901	1.00893	1.00885	1.00877	1.00869	
38	㊱ × ㊲ × ㉝ × M_1/M_2 =	p_2/p_1	1.1750	1.1774	1.1796	1.1815	1.1824	1.1842	1.1852	1.1858	1.1882	1.1883	1.1895	1.1896	1.1900	1.1897	1.1829	1.1788	
		p/p_{min}	1.0000	1.1750	1.3834	1.6319	1.9281	2.2798	2.6997	3.1997	3.7942	4.5082	5.3571	6.3723	7.5805	9.0208	10.7320	12.6949	14.9647
		A/A_0	1.0000	.8890	.7899	.7013	.6231	.5540	.4930	.4393	.3923	.3511	.3154	.2845	.2583	.2363	.2186	.2056	.1990

T = Constant = 665.5° F abs. $\quad C = \frac{2(h_v - h_L)}{T} = \frac{2(1148.2 - 38.9)}{665.5} = 3.33144$

$$\frac{p_{of}}{p_{oi}} = \frac{p_f}{p_i} \frac{\left(1+\frac{k_{of}-1}{2}M_f^2\right)^{k_{of}/k_{of}-1}}{\left(1+\frac{k_{oi}-1}{2}M_i^2\right)^{k_{oi}/k_{oi}-1}} = 13.35\left(\frac{2.14}{17.20}\right) = 1.6610 \quad (\text{For } M_f = 1.1,\ k_{of} = 1.3691,\ k_{oi} = 1.3500)$$

NUMERICAL INTEGRATION
Friction Present – M = 2.5

100% Evaporation

Step		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
39 [Σ (14)] − 2	$2[KM^2]_m$	16.9466	15.8767	14.8311	13.8099	12.8134	11.8377	10.8826	9.9507	9.0391	8.1473	7.2750	6.4213	5.5858	4.7685	3.9827	3.2139	
40 (39) × .00225	$\Delta(\Delta A/A_m)$	.03813	.03572	.03337	.03107	.02883	.02663	.02449	.02239	.02034	.01833	.01637	.01445	.01257	.01073	.00896	.00723	
41 (32) + (40)	$(\Delta A/A_m)_f$	−.0794	−.0823	−.0855	−.0869	−.0886	−.0901	−.0906	−.0907	−.0904	−.0888	−.0863	−.0824	−.0763	−.0672	−.0521	−.0256	
42 [2 − (41)] ÷ [2 + (41)]	$(A_1/A_2)_f$	1.0827	1.0858	1.0893	1.0908	1.0927	1.0943	1.0949	1.0950	1.0947	1.0929	1.0902	1.0859	1.0793	1.0695	1.0535	1.0259	
43 (38) × (42) ÷ (33)	p_2/p_1	1.1310	1.1360	1.1407	1.1452	1.1488	1.1530	1.1565	1.1594	1.1643	1.1666	1.1699	1.1725	1.1751	1.1770	1.1723	1.1702	
44	p/p_{min}	1.0000	1.1310	1.2848	1.4656	1.6784	1.9281	2.2231	2.5710	2.9809	3.4706	4.0488	4.7367	5.5538	6.5263	7.6804	9.0049	10.5376
45	A/A_0	1.0000	.9236	.8506	.7809	.7151	.6551	.5987	.5468	.4993	.4561	.4174	.3828	.3525	.3266	.3054	.2899	.2826

For a Final Mach No. Of 1.1 :– $\frac{p_{of}}{p_{oi}} = \frac{2.14}{17.20}(9.50) = 1.1819$

75% Evaporation

Step		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
59 (54) − ⅓ (27)	$\Delta A/A_m$	−.04796	−.05292	−.05233	−.06136	−.06513	−.06813	−.07060	−.07220	−.07334	−.07338	−.07247	−.06984	−.06519	−.05737	−.04367	−.01799	
60 [2 − (59)] ÷ [2 + (59)]	A_1/A_2	1.0491	1.0544	1.0537	1.0633	1.0673	1.0705	1.0732	1.0749	1.0761	1.0762	1.0752	1.0724	1.0674	1.0591	1.0446	1.0182	
61 (38) × (60) ÷ (33)	p_2/p_1	1.0958	1.1031	1.1034	1.1079	1.1220	1.1280	1.1336	1.1382	1.1445	1.1488	1.1538	1.1579	1.1621	1.1656	1.1624	1.1614	
62	p/p_{min}	1.0000	1.0958	1.2088	1.3337	1.4777	1.6580	1.8702	2.1201	2.4131	2.7617	3.1727	3.6607	4.2387	4.9258	5.7415	6.6739	7.7511
63	A/A_0	1.0000	.95320	.90402	.85795	.80687	.75599	.70621	.65804	.61219	.56889	.52861	.49164	.45845	.42950	.40553	.38821	.38128

For a Final Mach No. Of 1.1 :– $\frac{p_{of}}{p_{oi}} = \frac{2.14}{17.20}(6.90) = 0.8585$

50% Evaporation

Step		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
46 Σ (14)		18.9466	17.8767	16.8311	15.8099	14.8134	13.8377	12.8826	11.9507	11.0391	10.1473	9.2750	8.2413	7.5858	6.7685	5.9827	5.2139	
47 (46) × (3)		.18852	.17609	.16427	.15272	.14176	.13132	.12097	.11114	.10178	.09264	.08385	.07554	.06744	.05963	.05223	.04510	
48 (14) − (15)		4.3365	4.2443	4.1474	4.0451	3.9373	3.8240	3.7032	3.5759	3.4416	3.2985	3.1471	2.9863	2.8150	2.6323	2.4379	2.2375	2.0157
49 ¼ Σ (48)		2.1452	2.0979	2.0481	1.9956	1.9403	1.8818	1.8918	1.7544	1.6850	1.6115	1.5334	1.4503	1.3618	1.2676	1.1688	1.0633	
50 C ÷ (26)		6.8788	6.8197	6.7616	6.7044	6.6483	6.5943	6.5412	6.4877	6.4351	6.3833	6.3323	6.2822	6.2328	6.1842	6.1364	6.0893	
51 (49) × (50) × (3)		.14683	.14092	.13156	.12924	.12345	.11776	.11177	.10585	.09997	.09392	.08788	.08173	.07546	.06906	.06261	.05601	
52 (49) × (23)		−.008160	−.008012	−.007850	−.007679	−.007495	−.007298	−.006939	−.006856	−.006610	−.006348	−.006063	−.005756	−.005428	−.005073	−.004696	−.004289	
53 [(24) + (25)] ÷ (22)		.07310	.07312	.07368	.07268	.07270	.07185	.07091	.06969	.068170	.06590	.06292	.05868	.05269	.04386	.02954	.003566	
54 (40) + (47) − (51) − (52) − (53)	$\Delta A/A_m$	+.01488	+.00577	+.00243	−.01045	−.01807	−.02436	−.03028	−.03515	−.03941	−.04250	−.04452	−.04466	−.04271	−.03749	−.02626	−.00296	
55 [2 − (54)] ÷ [2 + (54)]	A_1/A_2	.9852	.9942	.9976	1.0105	1.0182	1.0247	1.0307	1.0358	1.0402	1.0434	1.0455	1.0457	1.0436	1.0382	1.0266	1.0030	
56 (38) × (55) ÷ (33)	p_2/p_1	1.0291	1.0401	1.0447	1.0609	1.0704	1.0797	1.0887	1.0968	1.1063	1.1138	1.1219	1.1291	1.1362	1.1426	1.1424	1.1441	
57	p/p_{min}	1.0000	1.0291	1.0704	1.1182	1.1863	1.2698	1.3710	1.4926	1.6371	1.8112	2.0173	2.2632	2.5553	2.9034	3.3174	3.7898	4.3359
58	A/A_0	1.0000	1.0150	1.0209	1.0233	1.01267	.99457	.97060	.94169	.90914	.87400	.83767	.80199	.76633	.73431	.70729	.68896	.68690

For a Final Mach No. Of 1.1 :– $\frac{p_{of}}{p_{oi}} = \frac{2.14}{17.20}(4.000) = 0.4977$

For All Integrations Above:

T = Constant = 665.5° F abs. ; $C = \frac{2(h_v - h_c)}{T} = 3.33144$; AND $4f\frac{\Delta x}{D} = 0.009$ For Each Step Has Been Assumed.

NUMERICAL INTEGRATION

Diffusion Of Air With Friction

Initial $M = 2.5$

$k = 1.40$

	Step		0	1	2	3	4	5	6	7	8	9	10	11	12
1		M_1^2	6.250	5.863	5.496	5.148	4.816	4.498	3.905	3.354	2.833	2.325	1.800	1.495	1.201
2		$F_{A1} \times \Delta A/A_m$	-.18750	-.1839	-.1796	-.1763	-.1738	-.3419	-.3352	-.3333	-.3389	-.3599	-.2147	-.1368	
3		$F_{F1} \times 4f\frac{\Delta x}{D}$	-.2109	-.1936	-.1776	-.1634	-.1503	-.2766	-.2356	-.2012	-.1712	-.1506	-.0693	-.0369	
4	②+③	ΔM^2	-.3984	-.3775	-.3572	-.3397	-.3241	-.6185	-.5708	-.5345	-.5101	-.5105	-.2840	-.1737	
5	①+½④	M_m^2	6.051	5.674	5.317	4.978	4.654	4.189	3.620	3.087	2.578	2.070	1.658	1.408	
6		$F_{Am} \times \Delta A/A_m$	-.1853	-.1814	-.1779	-.1748	-.1722	-.3380	-.3335	-.3351	-.3466	-.3821	-.2345	-.1547	
7		$F_{Fm} \times 4f\frac{\Delta x}{D}$	-.2019	-.1853	-.1702	-.1568	-.1459	-.2549	-.2173	-.1862	-.1609	-.1425	-.0704	-.0392	
8	⑥+⑦	ΔM^2	-.3872	-.3667	-.3481	-.3316	-.3181	-.5929	-.5508	-.5213	-.5075	-.5246	-.3049	-.1939	
9	①+⑧	M_2^2	5.863	5.496	5.148	4.816	4.498	3.905	3.354	2.833	2.325	1.800	1.495	1.201	
10	①+½⑧	M_m^2	6.056	5.679	5.322	4.982	4.657	4.202	3.630	3.094	2.579	2.063	1.648	1.348	
11		$1-M_m^2$	-5.056	-4.679	-4.322	-3.982	-3.657	-3.202	-2.630	-2.094	-1.579	-1.063	-.648	-0.348	
12		kM_m^2	8.478	7.951	7.4508	6.9748	6.5198	5.8828	5.0820	4.3316	3.6106	2.8882	2.3072	1.8872	
13	⑫ ÷ ⑪ x($\Delta A/A_m$)		.0587	.0595	.0603	.0613	.0624	.1286	.1353	.1448	.1601	.1901	.1246	.0949	
14	½ ⑫ ÷ ⑪ x (⑪+⑫)		2.869	2.780	2.697	2.621	2.552	2.463	2.369	2.314	2.323	2.479	2.953	4.173	
15	⑭ x $4f\frac{\Delta x}{D}$		.0258	.0250	.0243	.0236	.0229	.0444	.0426	.0416	.0418	.0446	.0266	.0188	
16	⑬+⑮	$\Delta p/p_m$	.0845	.0845	.0846	.0849	.0853	.1730	.1779	.1864	.2019	.2347	.1512	.1137	
17	[⑯+2] ÷ [⑯-2]	p_2/p_1	1.088	1.085	1.088	1.089	1.089	1.189	1.195	1.205	1.224	1.266	1.163	1.120	
18		p/p_{min}	1.000	1.088	1.184	1.288	1.403	1.528	1.817	2.171	2.616	3.202	4.054	4.715	5.281

← Unit Steps → ← Double Steps → ← Unit Step → ← Half Step →

FOR UNIT STEP:-

$$\frac{\Delta A}{A_m} = 0.035 \; ; \; 4f\frac{\Delta x}{D} = 0.009 \text{ (Assumed).}$$

$$\frac{p_{o12}}{p_{oo}} = \frac{p_{12}}{p_o}\left[\frac{1+\frac{k-1}{2}M_{12}^2}{1+\frac{k-1}{2}M_o^2}\right]^{\frac{k}{k-1}} = 5.281\left(\frac{.05853}{.46035}\right) = 0.650$$

MAY 13

L.V.M.

APPENDIX C

CALCULATION FOR THEORETICAL EVAPORATION RATES

The basic equation used was taken from Sherwood[5]. It was derived by Chilton and Colburn by analogy of mass transfer to heat transfer, similar to the Reynolds analogy of friction and heat transfer.

$$1. \quad K_G = \frac{h}{c_p P_A W_A} \left(\frac{c_p \rho_A D}{k} \right)^{\frac{2}{3}}$$

From McAdams, for a sphere

$$2. \quad \frac{h D_s}{k} = 0.33 \left(\frac{D_s G}{\mu_f} \right)^{0.6}$$

Combining equations 1 and 2

$$3. \quad K_G P_A W_w = \frac{k}{D_s c_p} \quad \frac{W_w}{W_A} \left(\frac{c_p \rho_A D}{k} \right)^{\frac{2}{3}} \left(0.33 \left[\frac{D_s G}{\mu_f} \right]^{0.6} \right)$$

At 200F

$c_p = 0.2392$ $\qquad$ $k = 0.0180$

$W_A = 28.97$ $\qquad$ $\frac{c_p \rho_A D}{k} = 1.280$

$W_w = 18.02$ $\qquad$ $\mu_f = 0.0432$

Equation 3 then reduces to

$$K_G P_A W_w = 0.117 \frac{G^{0.6}}{D_s^{0.4}} \quad \frac{\text{lbs}}{\text{hr ft}^2}$$

Or

$$\frac{dw}{dt} = 0.117 \quad \frac{G^{0.6}}{D_s^{0.4}} \times \text{AREA} = 0.368\, G^{0.6} D_s^{1.6}$$

But

$$\frac{dw}{dt} = -\rho_w \frac{\pi}{2} D_s^2 \frac{dD_s}{dt} = -98.0 D_s^2 \frac{dD_s}{dt}$$

Therefore

$$dt = -266 \frac{D_s^{0.4}}{G^{0.6}} dD_s$$

Integrating this expression, the time of evaporation is obtained

$$4. \quad t = 190\,\frac{D_{S1}^{1.4}}{G^{0.6}}\ \text{hrs} = 0.685\,\frac{D_{S1}^{1.4}}{G^{0.6}} \times 10^{6}\ \text{seconds}$$

If the average velocity during evaporation is $V_a/2$, that is, evaporation is complete at moment drop is accelerated to stream speed

$$L = V_{AVE}\,t = 0.3425 \times 10^{6}\,\frac{V_a D_{S1}^{1.4}}{G^{0.6}}\ \text{ft} \quad ; \quad G = \rho_A V_A$$

$$L = 4.11 \times 10^{6}\,\frac{D_{S1}^{1.4}\,V_A^{0.4}}{\rho_A^{0.6}}\ \text{inches}$$

And

$$M_1 = 2.5 \quad , \quad T_{01} = 1500\,R \quad ; \quad P_{01} = 14.7\ \text{psia}$$

$$\rho_A = 0.00346 \ ; \ V_a = 3170$$

$$L = 3030 \times 10^{6}\,D_{S1}^{1.4}$$

Lengths of duct required for drop sizes of 10^{-5}, 10^{-6}, and 10^{-7} feet are shown below.

$L_{D_{S1}=10^{-6}}$ = 12.1 inches

$L_{D_{S1}=10^{-5}}$ = 303 inches

$L_{D_{S1}=10^{-7}}$ = 0.481 inches

From this it can be seen that the drop size must be known with great accuracy, which with present information is impossible. However, if it is assumed that with lateral injection into the stream, the drop size, D_{S1}, is inversely proportional to the velocity of the stream, and proportional to the diameter of the injection hole,

A comparison may be made to the measured results of Curry, in a much lower speed stream. This assumption seems to logically follow the theory of the formation of drops by a shearing phenomena as postulated by many investigators of atomization of liquids in a subsonic stream.

The proportionality equation then becomes

$$L \propto \frac{D_{ini}}{V_A^{0.6}\,\rho_A^{0.6}}$$

In a typical Curry run

$$\left.\begin{array}{l} T_{01} = 1140\ R \\ M_1 = 0.49 \\ P_1 = 278\ psia \\ D_{ini} = 0.02'' \end{array}\right\} \begin{array}{l} V_A = 795 \\ \rho_A = 0.0585 \end{array}$$

$$\frac{L}{L_c} = \left(\frac{V_{Ac}}{V_A}\right)^{0.6}\left(\frac{D_{ini}}{D_{inic}}\right)^{1.4}\left(\frac{\rho_{Ac}}{\rho_A}\right)^{0.6}$$

$$\frac{L}{L_c} = \left(\frac{795}{3170}\right)^{0.6}\left(\frac{008}{02}\right)^{1.4}\left(\frac{.0585}{.00346}\right)^{0.6} = 0.485$$

For an average run, Curry obtained 50% evaporation in about 20 inches, after which evaporation proceeded very slowly with increasing length. The above comparison would indicate that at least this amount of evaporation could be expected in about 9.5 inches.

SYMBOLS USED

1. K_G diffusion rate, lb mol/hr ft^2 atmos.
2. h heat transfer coefficient for similar situation, BTU/hr ft^2 F
3. C_p specific heat of air at constant pressure, BTU/lb F
4. P_A partial pressure of air in main stream

5. W_A molecular weight of air

6. W_w molecular weight of water

7. ρ_a density of air, lbs/ft^3

8. ρ_w density of water, lbs/ft^3

9. D diffusivity constant for air

10. K thermal conductivity of air, BTU/hr ft^2 F/ft.

11. μ_f viscosity of air, lbs/hr ft.

12. D_s diameter of water drop, ft.

13. D_{s1} initial value of D_s, ft.

14. D_{inj} diameter of water injection hole, ft.

15. V_A air velocity, ft/sec.

16. G mass flow, lbs/ft^2 sec.

17. M Mach number

APPENDIX D

DETERMINATION OF MACH NUMBER AT THE SECTION OF MAXIMUM AREA

In order to judge whether or not the Mach number attained at the maximum area corresponded to the design value, pressure taps were fixed to the first row of water injection tubes, providing a pressure traverse of the section of maximum area.

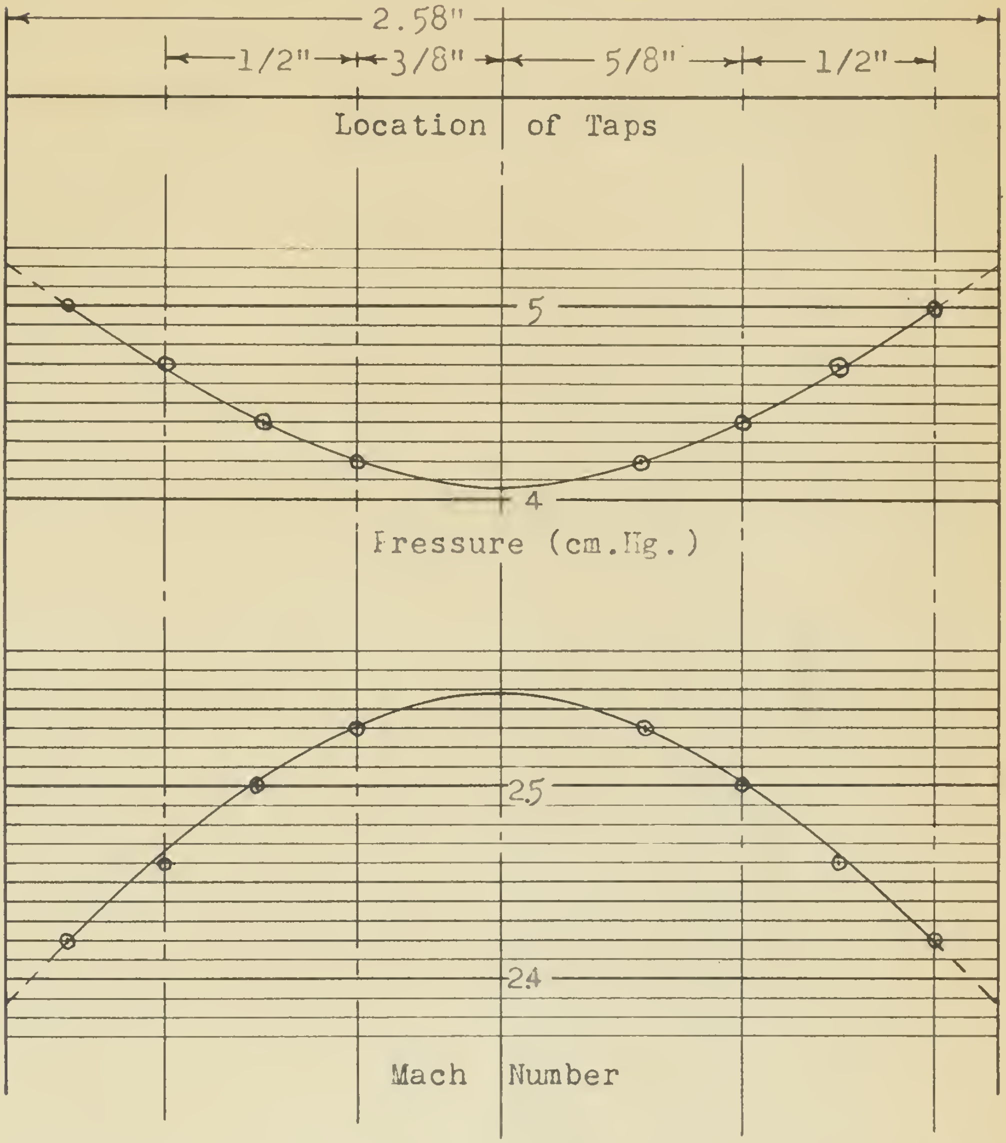
Results of a typical test are shown in Figure XII. The average Mach number was estimated first by determining the integrated mean pressure across the maximum area section, from which p_m/p_0 and thus the corresponding Mach number could be determined. In a similar method the Mach number corresponding to each pressure measured was determined, and the curve of Mach numbers was plotted. As shown the integrated mean of the Mach numbers agrees very closely with that determined from the average pressure.

In making the calculations described above, the stagnation pressure was assumed constant along each streamline, throughout the expansion to the maximum area section. Such an assumption is very accurate except for the portions of the stream very close to the walls, which are subjected to high shearing forces. It was also assumed that the velocity vectors are all parallel to the axis of the stream in integrating the curve of Mach numbers.

Since pressures could not be read more accurately than to the nearest millimeter of mercury, there exists a possible error of about two percent in p/p_{01}, with a corresponding error in Mach number of about 0.014. As a result, there were some differences in the Mach numbers estimated for various runs. For the tests with inlet stagnation temperature of 1500R, the average Mach number for all tests was 2.49.

Figure XII

Plot of Typical Pressure Traverse and Determination of Mach Number at Section of Maximum Area



p_o = 75.70 cm.Hg.

T_o = 1513°R

T ≈ 670°R

k(mean) = 1.375

Integrated Average p 4.492 cm.Hg.

$\frac{p_m}{p_o} = \frac{4.492}{7.570} = 0.5394$ from which

it is found $M_{av.} = 2.487$

Integrated Average M = 2.489

MAY 13 1949

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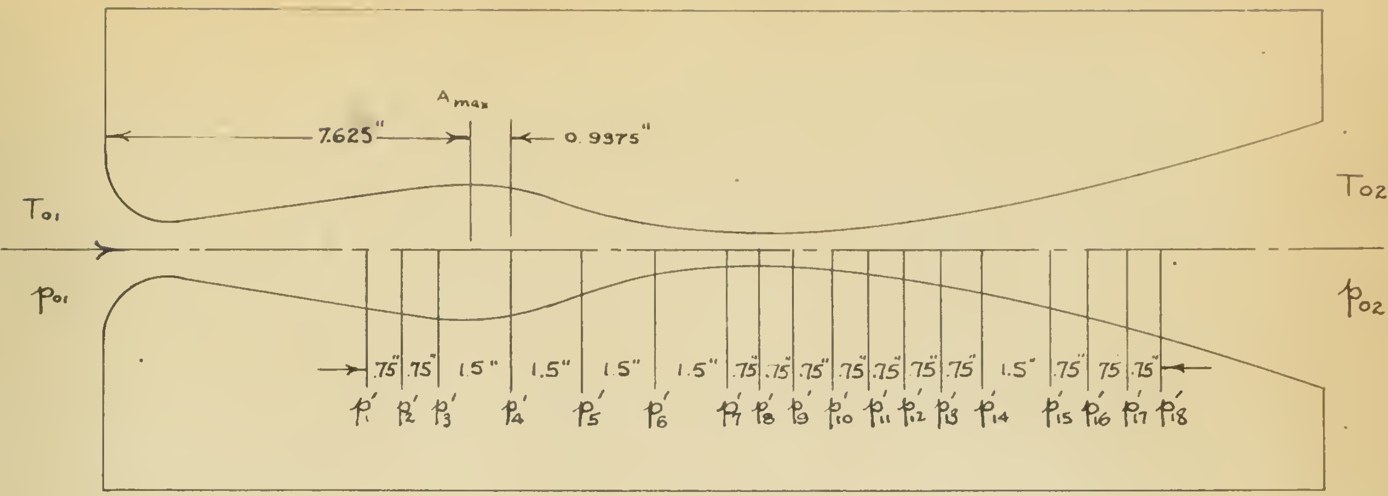
FIG. XIII - TEST DATA FOR DRY RUNS



	A	B
DIFF. THROAT AREA	1.484	1.655
T_{01} °R	1509	1513
T_{02} °R	1190	1100
p'_1 cm Hg	6.1	6.1
p'_2	5.2	5.3
p'_3	4.4	4.5
p'_4	3.7	3.7
p'_5	8.2	8.4
p'_6	12.5	10.5
p'_7	13.1	13.7
p'_8	13.7	17.2
p'_8	18.6	20.2
p'_{10}	21.9	21.0
p'_{11}	22.6	21.5
p'_{12}	23.4	21.6
p'_{13}	23.5	21.8
p'_{14}	23.7	21.9
p'_{15}	23.8	21.9
p'_{16}	24.0	–
p'_{17}	24.1	–
p'_{18}	24.3	–
p_{01}	76.1	75.7
p_{02}	24.6	22.5

A – MIN. AREA TO RUN.

B – MIN. AREA TO START.

MAY 13 1949

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APPENDIX E

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