

MULTIVARIABLE DIGITAL FLIGET CONTROL DESIGN FOR TEE FPCC AIRCRAFT THESIS

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# MULTIVARIABLE DIGITAL FLIGRT CONTROL DESIGN FOR THE FPCC AIRCRAFT 

THESIS

Presented to the Faculty of the School of Engineering of the Air Force Institute of Technology<br>Air University<br>in Partial Fulfiliment of the Requirements for the Degree of<br>Master of Science

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## PREFACE

This thesis follows up work done by Joseph Smyth and Jon Bauschlicher in their thesis efforts at the Air Force Institute of Technology. It was supported by the Air Force Fifght Dynamics Laboratory/Control Division at Wright-Patterson Air Force Base.

I thank my thesis advisor, Dr. John J. D'Azzo, without whose help this thesis would not have been possible. I also wish to thank my colleagues Captain William Locken, Lt Marc Hoffman, Lt Roger Feldmann, and Lt Brian Mayhew, all of whom contributed to this thesis.

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This thesis details the application of the Porter Method to the design of a digital tracker controller for the FPCC aircraft. The aircraft is simulated with software models supplied by Lockheed, and incorporates horizontal and vertical canards, along with maneuver flaps, allerons, rudder, and fet flaps for vectored thrust.

Separate lateral and longitudinal designs are accomplished at 0.15 mach, sea level. Longitudinal designs for 0.6 mach, sea level, and 0.9 mach at 30,000 ft. are also completed. For the one lateral design the following maneuvers are performed:

Yaw pointing
Flat turn
Horizontal translation
For the three longitudinal designs the following maneuvers are performed:

Pitch pointing
Direct ift
Vertical translation
One coupled maneuver, a jink maneuver incorporating both vertical and horizontal translation demonstrates the combined lateral and longitudinal controller at 0.15 mach. A "universal" controller is found for the 0.6 and 0.9 mach flight conditions and is tested with the same longitudinal maneuvers. All of the controllers are again tested with the addition of a one sampling period delay

## CHAPTER I

## INTRODUCTION

## Background

Digital computers and electical wiring are rapidiy replacing analog controllers and mechanical linkages in the design of aircraft flight control systems. This is true for both military and civilian aviation.

The digital fiight control system must hande multiple inputs from the pilot and also command multiple outputs simultaneously, ensuring optimal aircraft performance for varied flight conditions. The digital flight control system (DFCS) can control more flight control surfaces than a pilot, and can also control them simultaneously. Sensing elements such as the gyros and the accelerometers allow the DFCS to use detailed information about the aircraft's attitude to command fight control surface deflections more efficiently than a pilot.

Using a DFCS allows the pilot to command a paiticular maneuver, instead of the pilot having to control the fight control surfaces directly. Thus, the pilot's workload is reduced, increasing pilot efficiency and allowing the pilot more time to concentrate on other vital tasks. In the combat arena, with the corresponding pressures that are applied to the pilot, the possibility of error should be reduced and the chances for survival should increase.

The DFCS generates the signals necessary to deflect the control surfaces in response to a commanded maneuver from the pilot. Consequentiy, classical single input single output (SISO)
design techniques are not well suited to this multiple input multiple output (MIMO) problem. Other techniques, such as LQG, that require all states to be either accessable or estimated lead to complicated calculations due to the lack of full state information on the typical aircraft. An alternative design method has been developed by Professor Brain Porter and associates at the University of Salford, U.K.

The Porter method proposes a straight forward approach to the MIMO control system design that does not require complicated mathematics nor does it require considering each output-input pair separately (Ref l). Previous AFIT master's theses have demonstrated the use of the method (Ref 2 and 3).

A hypothetical aircraft called the FPCC, for fight propulsion control coupling, has been proposed by pratt-Whitney, Lockheed, and Honeywell. The aircraft's main feature is the use of thrust vectoring using jet flaps, hence the FPCC label. The aircraft also incorporates horizontal and vertical canards, along with maneuver flaps. The FPCC is longitudinally statically unstable, adding to the aircraft's maneuverability (Ref 4). Resulting from these features is the aircrafts ability to perform decoupled six degree of freedom (6DOF) maneuvers.

Design of a DFCS for this aircraft is alded by two computer programs, a flight simulation program for the FPCC provided by the contractors (Ref 5), and an interactive design program incorporating the Porter design method called MULTI (Ref 6).

This thesis extends previous design work done on a DFCS for the FPCC aircraft (Ref 2), by developing 3 multi-variable tracker control laws for each of 3 fifgh conditions, using the Porter method. Each controller performs coupled or decoupled, lateral or longitudinal maneuvers. Attempting to demonstrate robustars, a universal controller for two of the filght conditions is designed, and its performance evaluated compared to the individual controllers.

In an attempt to partially validate the short takeoff and landing (STOL) capability of the FPCC, the first fight condition is at 0.15 mach, with low angle of attack, at sea level. The other two flight conditions are identical to those from Bauschlichers thesis, 0.6 mach at sea level, and 0.9 mach at 30,000 feet. This thesis repeats designs at these fight conditions because the longitudinal aircraft model has been changed. Therefore, the designs for these filght conditions are in the longitudinal mode only.

The responses for each design are then reaccomplished incorporating a time delay of one sampling period between the output of the controller and the input of the aircraft plant model, demonstrating any dettabilizing effects from this lag. The designs are redone, as necessary, to compensatefor these effects. The simulations are also repeated with certain controller matrix elements set to zero, demonstrating posible simplifications that can be made to thesematrices.

Additionally, this thesis includes two improvements to MULTI, the computer program used for controller design based on
the Porter method. An additional option was added that calculates the figures of merit resulting from the time simulation used in the program. Specifically, the maximum, minimum, and final values are displayed, along with the times at which the maximum and minimum occur. The option then allows the user to use the default value of within $2 \%$ of the final value, or the user may specify the value, which is to be used to calculate the settling time.

The other modification to MULTI incorporates the option of command line file names when invoking the program. The files named contain system and design data to be used within the program.

## Current Knowledge

Papers published by Professor Porter and associates detail the design technique that is used in this thesis (Ref 6). Also, previous theses have both used and explained in detail the porter method. This thesis is the first to investigate the STOL capabilities of the FPCC aircraft, and extends upon the work done by Jon Bauschlicher completed in December, 1982. Bauschlicheres thesis designed multi-variable controllers for fifgt conditions reflecting high subsonic, transonic, and supersonic speeds. The MULTI modification in this thesis extends the work started by Douglas Porter as published in his thesis of December, 1981 (Ref 3). Bauschlicher's thesis and the reports by Lockheed constitute the background information on the FPCC aircraft and the simulation program (Ref 4,5).

Support
Computers used include the CDC mainframes (ASD) at WrightPatterson Air Force Base, and the author's microcomputer. Computer support facilities used include the RJE (Remote Job Entry) sites at both AFIT and the Air Force Fight Dynamics Laboratory (AFFDL), along with the main site at the ASD Computer Center.

## Overview

Chapter I is the general introduction, with Chapter II being an introduction to the Porter method of controller design. Chapter III is a description of the FPCC aircraft with a sumary of the aircraft's behaviour within the STOL regime, presenting previously unpublished tabular analysis of the aircraft's response to flight control surfaces at slow sped and low altitude. Chapter IV describes the use of the FPCC simulation program, along with explanations of the input and output of the program.

Chapter IV details the design and testing of the controllers for each flight condition. Responses of the controllers to commanded inputs for six decoupled and one coupled maneuver are given.

Chapter $V$ describes the design of a that can be used for the two non-STOL fight conditions. The effects of a $\begin{gathered}\text { e } \\ \text { ampling period time delay in the output of the }\end{gathered}$ PI controller is studied for all the flight conditions and the universal controller. Finally, certain elements of the



DESIGN OF LINEAR MULTIVARIABLE TRACKING SYSTEMS BASED UPON SINGULAR PERTURBATION METHODS

## Introduction

The methods used in this thesis for the design of the controllers were developed by Professor brain Porter, University of Salford, England, and his associates (Ref $1,8,9$ ). The references
 the design methodology commonly called the Porter method, but two of the references are more useful as sumaries of the method. The first, (Ref l), details the approach for both regular and irregular known plants using a proportional plus integral (PI) cascade compensator which is either analog or digital. The second (Ref 8), briefly explains the approach for an unknown plant with an analog $P I$ compensator. This chapter summarizes the material from these references. Note that for the known plants only the digital controller is considered, since that is the type of controller used in this thesis and in the program MULTI.

If additional decoupling andor reduction in the initial undershooting of the outputs is required, then a technique based upon the $B^{*}$ approach is used, which is also included in this chapter. If the $\underline{B}^{*}$ method falls, then there is another alternative, which is not explained in this thesis but is referenced (Ref 9).

Throughout this thesis the description of the continuous time plant is given by:

$$
\begin{equation*}
\underline{\dot{x}}(t)=\underline{A} \underline{x}(t)+\underline{B} \underline{u}(t) \tag{1}
\end{equation*}
$$

where

```
A = continuous time plant matrix (nxn)
B=continuous time control matrix (nxm)
C= continuous time output matrix (Ixn)
D = continuous time feed forward matrix (1xm)
```

and
n a nuber of plant states
1 = number of outputs
$m=$ number of inputs (sometimes called $p$ )
The PI controller performs both the tracking and disturbance rejection tasks, and as stated, is the discrete time type for this thesis. The equations governing the controller are:

$$
\begin{align*}
& \underline{x}[(k+1) T]=\exp \{\underline{A} T\} * x(k T)+\int_{0}^{T} \exp \{\underline{A} T\} * \underline{B} d t * \underline{u}(k T)  \tag{3}\\
& \underline{y}[k T]=C * \underline{x}(k T)+\underline{D} * \underline{u}(k T) \tag{4}
\end{align*}
$$

where

$$
\begin{aligned}
& \exp \{\underline{A T}\}=\text { sampled data plant matrix } \\
& \int_{0}^{T} \exp \{\underline{A T}\} * \underline{B d t} \text { ampled data control matrix } \\
& \underline{C}=\text { sampled data output marix } \\
& \underline{D}=\text { sampled data feed forward matrix } \\
& T \quad=s a m p l i n g ~ t i m e ~
\end{aligned}
$$

Note that the $u(k T)$ is piecewise constant over the sample period, resulting in its removal from inside the integral. The separate nanes designating the sampled data matrices are not used in this thesis since these marices are not explicitly used.

Figure 1 shows the $P I$ controller used in this thesis. Since the computer aided design program MULTI is used to assist in the



#### Abstract

design of the controller, the placement of the sampler is dictated by MULTI's implementation of the controller. MULTI actually implements two samplers, one in the feedback loop from the plant, and the other in the commanded input. Mathematically, the one sampler shown in figure 1 is equivalent to the other two. The zero order sample and hold device shown is indicative of the way MULTI passes the control input to the plant, with the duration of the hold controlled by an option within MULTI. The integral action shown in the figure is currently implemented in MULTI using an approximation to the $1 / s$ function.

Since the choice of which design method to use, Unknown, Known/Regular, or Known/Irregular, is based upon the plant parameters, the rest of this chapter explains how the choice is made and summarizes the method used in each case. At the end of the chapter the $\underline{B}^{*}$ method is explained.


Unknown Plant (Ref 8)
This approach applies typically to an industrial process where little is known about the plant model, or in the situation where developing the state model is not desired. The problem is to develop the controller without knowledge of the $\mathbb{A}, \underline{B}, \underline{C}$, or $\underline{D}$ matrices.

The first step is to isolate the plant and determine its steady state transfer function $\mathbf{G}^{(0)}$. This is posible as long as the plant is asymptotically stable. Since the transfer function

$$
\underline{G}(\lambda)=\underline{C}\left(\lambda \underline{I}_{n}-\underline{A}\right)^{-1} \quad \underline{B}
$$

then

$$
\begin{equation*}
\underline{G}(0)=-\underline{C} \underline{A}^{-1} \quad \underline{B} \tag{5}
\end{equation*}
$$

implying that this approach could be used if the plant dynamics were known.

Since the addition of the $P I$ controller must preserve stability, the rank of $G(0)$ must be equal to 1 , the number of outputs. This requirement means that the number of outputs must be less than or equal to the number of inputs, and that $G(\lambda)$ must have no transmission zeros either at the origin or in the right half plane.

$$
\begin{align*}
& \text { The control law for the PI controller is: } \\
& \underline{u}(t)=\propto \in \underline{K} e(t)+\epsilon \underline{K} \underline{z}(t) \tag{6}
\end{align*}
$$

with

$$
\begin{align*}
& \underline{e}(t)=\underline{v}(t)-\underline{y}(t)  \tag{7}\\
& \underline{z}(t)=\int_{0}^{T} e(t) d t \tag{8}
\end{align*}
$$

where
$\underline{u}(t)=$ control input to the plant
$\alpha \quad$ ratio of error to the integral of error
$\epsilon \quad=$ scaler multiplier
$\mathrm{K} \quad=$ the controller matrix
$e(t)=e r r o r ~ v e c t o r ~$
$\underline{v}(t)=$ command input vector
$y(t)=$ output vector
$z(t)=$ integral of error vector
Since the system has a cascade vector integrator, a constant step command input vector yields zero steady state error and the output vector follows the input vector, hence tracking and disturbance rejection result.

Manipulation of the above equations, along with the fact the
steady state error equals zero yields the following:

$$
\begin{equation*}
\underline{K}=\underline{G}(0)^{\top}\left[\underline{G}(0) \underline{G}(0)^{\top}\right]^{-1} \tag{9}
\end{equation*}
$$

which, if the number of outputs equals the number of inputs means that $G(0)$ is square and
$\underline{K}=\underline{G}(0)^{-1}$
The above development can be extended to the digital PI controller by writing the control law equations as:
$\underline{u}(k T)=T\left[\underline{K}_{0} \underline{e}(k T)+\underline{K}_{i} \underline{z}(k T)\right]$
$\underline{e}(k T)=\underline{v}(k T)-\underline{y}(k T)$
$\underline{z}[(k+1) T]=\underline{z}(k T)+T e(k T)$
where
$\underline{K}_{0}=\alpha \underline{K}_{1}$
$\underset{-1}{K}=\underline{K}$
Note that Equation (13) is an approximation to Equation (8).
Following the same basic reasoning usid above to derive Equation (9), the controller matrices $\underline{K}_{0}$ and $\underline{K}_{\text {, }}$ can be found from: $\underline{K}=\left\{\underline{G}(0)^{\top}\left[\underline{G}(0) \underline{G}(0)^{\top}\right]^{-1}\right\}$
where $\sum$ is a weighting matrix of scalar diagonal entries chosen by the designer.

The choice of the diagonal elements of $E$ and of the sampling time $T$ will affect the output response. This means that fine tuning of the output time responses can be accomplished by adjusting these values.

Unfortunately, this method is only valid for a plant with negative eigenvalues, since $G(0)$ must be stable. This precludes its use with statically unstable aircraft such as the FPCC. Additionally, if an angle is used in the state vector then the derIvative of that angle (a rate) cannot be used in the output
vector since that would generate a transmision zero at the origin.

Reference 2 discusses the effects of varying the parameters 1a the above equations. Increasing an elementinthe $\underline{\underline{E}}$ matrix apeeds up the corresponding output response. Multiplying the $\Sigma$ matrix by acalar greater than one increases the $P$ controller inputs and outputs, while multiplying by acalar less than one decreases the inputs and outputs. Decreasing the sampling time T increases the response of the controller outputs. Changingo varies the amount of over or undershoot in the outputs.

## Known Plant (Ref 1)

Eigure 2 shows the block diagram of the plant with known $A$, B, C, and D matrices. Note that the $\underline{D}$ matrix is missing, which is due to requirement that the system representation not contain a matrix. If there is a matrix, then either the outputs or state representation must be changed to eliminate the feed forward matrix. A $\quad$ matrix occurs whenever there is an acceleration termin the output vector. The plant equations must be expressed in the following format to use the Porter method:

$$
\begin{align*}
& {\left[\begin{array}{l}
\dot{x}_{1}(t) \\
\dot{x}_{2}(t)
\end{array}\right]=\left[\begin{array}{ll}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{array}\right]\left[\begin{array}{l}
x_{1}(t) \\
\underline{x}_{2}(t)
\end{array}\right]+\left[\begin{array}{l}
0 \\
u(t) \\
\underline{B}_{2}
\end{array}\right]} \\
& \text { with } \\
& \underline{y}(t)=\left[\begin{array}{ll}
\underline{C}_{1} & \underline{C}_{2}
\end{array}\right]\left[\begin{array}{l}
\underline{x}_{1}(t) \\
\underline{x}_{2}(t)
\end{array}\right] \tag{18}
\end{align*}
$$

The A matrix of equation (l) has been partitioned into the following sections:


$$
\begin{array}{ll}
A_{11} & (n-1) \times(n-1) \\
A_{12} & (n-1) \times 1 \\
A_{21} & 1 \times(n-1) \\
A_{22} & 1 \times 1
\end{array}
$$

The $\underline{B}_{\text {matrix }}$ has been also partitioned as:
B $_{2} \quad 1 \times 1$ (must have full rank)
with the $C$ matrix partitioned as:
C, $\quad 1 \times(n-1)$
$\mathrm{C}_{2} \quad 1 \times 1$
Note that in these equations, as in all that follow (and also in MULTI), the number of outputs equals the number of inputs, i.e. 1=m. Therefore the representation of the matrix partitions uses 1 only.

Equations (1) and (2) may be put into the format of (17) and (18) with a similarity transformation. This transformation yields the transmission zeros directly, but does not actually have to be performed to apply the design method.

Implementation of the Porter method requires that the plant be controllable ( $\underset{A}{A}, \underline{B}$ be a controllable pair), and also observable ( $\underline{A}, \underline{C}$ be an observable pair), along with the requirement that the matrix

have full rank of $n+1$ to ensure controllability using the $P I$ controller.

After these requirements have been satisfied, the next step is to calculate the rank of the first Markov parameter, which is
 method to use, either regular or irregular, is based upon the rank of $C$ B. If this rank is less than 1 , then the irregular method must be used, while if this raak is equal to then the regular method can be used.

## Known/Regular Plant

If the product $C B\left(0 r{\underset{2}{ }}^{B_{2}}\right.$ ) has full rank, then the plant is considered regular and $C$ B can be inverted, which is a necessary condition to use this method. The control law for the digital controller is given by:

$$
\begin{equation*}
\underline{u}(k T)=(1 / T)\left[{\underset{\sim}{K}}_{0} e(k T)+\underline{K}_{1} \underline{z}(k T)\right] \tag{19}
\end{equation*}
$$

with

$$
\begin{equation*}
\underline{e}(k T)=\underline{v}(k T)-\underline{y}(k T) \tag{20}
\end{equation*}
$$

and

$$
\begin{equation*}
2[(k+1) T]=2(k T)+T_{e}(k T) \tag{21}
\end{equation*}
$$

The controller matrices are found:

$$
\begin{align*}
& \underline{X}_{0}=\alpha\left(\underline{C}_{2} \underline{B}_{2}\right)^{-1} \underline{E} E  \tag{22}\\
& \underline{K}_{1}=\left(\underline{C}_{2} \underline{B}_{2}\right)^{-1} \underline{E} E \tag{23}
\end{align*}
$$

where
$\underline{\Sigma}=$ diagonal matrix $\left\{\sigma_{1}, \sigma_{2}, \ldots, \sigma_{\ell}\right\}$
constrained by
$-1<\left(1-\sigma_{i}\right)<1$
Note that in an actual design, the value of epsilon can be changed to obtain a required range of values for the elements in the controller matrices, and thus the magnitudes of the entries in the sigma matix can be kept within the required range.

Once again the cascade vector integrator drives the stady
state error vector to zerofor a constant comand input vector, which ensures tracking and disturbance rejection.

As the sampling time $T$ approaches zero the transfer function matrix $G(\lambda)$ approaches an asymptotic form:

$$
\begin{equation*}
\Gamma(\lambda)=\tilde{\Gamma}(\lambda)+\hat{\Gamma}(\lambda) \tag{24}
\end{equation*}
$$

where

$$
\begin{align*}
& \tilde{\Gamma}(\lambda)=\underline{C}_{0}\left(\lambda I_{n}-I_{n}-T_{-} A_{0}^{-1} T \underline{B}_{0}\right.  \tag{25}\\
& \hat{\Gamma}(\lambda)=\left(\lambda I_{2}-I_{2}+\underline{C}_{2} \underline{B}_{2} \underline{K}_{0}\right)^{-1} \quad \underline{C}_{2} \underline{B}_{2} \underline{K}_{0} \tag{26}
\end{align*}
$$

with

$$
\begin{align*}
& \underline{C}_{0}=\left[\begin{array}{lll}
\mathbf{R}_{0}^{-1} & \underline{R} & \underline{0}
\end{array}\right] \tag{28}
\end{align*}
$$

Equation (25) corresponds to what are called the slow modes and (26) corresponds to what are called the fast modes. Professor Porter shows that as $T$ approaches zero the slow modes become both uncontrollable and unobservable while the fast modes remain both observable and controllable. Consequently, as approaches zero, The slow modes vanish and the closed loop transfer function becomes:

$$
\begin{equation*}
\Gamma(\lambda)=\underline{\Gamma}(\lambda)=\left(\lambda I_{-2}-I_{2}+\underline{C}_{2} B_{2} \underline{R}_{0}\right)^{-1} \quad C_{2} B_{2} K_{0} \tag{30}
\end{equation*}
$$

This means that if $K$ is chosen such that:

$$
\begin{equation*}
\underline{C}_{2} \underline{B}_{2} \underline{R}_{0}=\text { diagonal matrix }\left\{\sigma_{1}, \sigma_{2}, \ldots, \sigma_{\ell}\right\} \tag{31}
\end{equation*}
$$

chen
$\Gamma(\lambda)=d 1 a g o n a l \operatorname{matrix}\left\{\sigma_{1} / \lambda-1+\sigma_{1}, \sigma_{2} / \lambda-1+\sigma_{2}, \ldots, \sigma_{l} / \lambda-1+\sigma_{l}\right\}(32)$

As stated earlier, the fast mode roots and the transmision zeros must lie within the unit circle to ensure stability. The transmission zeros are a subset of the slow modes and are given by:

$$
\begin{equation*}
\underline{z}_{t}(\lambda)=\left\{\left|\lambda I \quad-I \quad-T A_{11}+T A_{12} \underline{C}_{2}^{-1} \underline{C}_{1}\right|=0\right\} \tag{33}
\end{equation*}
$$

as long as the state equations are in the form of (17).
From Equation (32) the conclusion drawn is that as $T$ gees to zero decoupling is achieved. However, the requirement that the transmission zeros lie within the unit circle in the z domain may dictate a change in the $C$ matrix. This means a redefining of the output variables, which could entail a major change in the approach to the design, since changing $C$ must not affect the controllability of the system.

Known/Irregular Plant (Ref 1 )
When the product $\underset{-}{\mathbf{B}}$ does not have full rank, and thus is not invertible, the plant is called irregular. In this case extra plant output measurements are introduced and incorporated with inner loop compensators. This results in the feedback vector:

$$
\begin{align*}
& \mathrm{X}_{2}(\mathrm{t}) \tag{35}
\end{align*}
$$

with

$$
\begin{array}{ll}
M & 1 \times(n-1) \\
{\underset{F}{1}} & 1 \times(n-1) \\
{\underset{E}{2}} & 1 \times 1
\end{array}
$$

which is shown in Figure 3. The choice of the measurement matrix
 method for the irregular plant is just like that for the regular plant, with $\underset{\text { F }}{ }$ replacing C. The control law equation for the PI controller is the same as for the regular plant, but the error vector becomes:

$$
\begin{equation*}
\underline{e}(k T)=\underline{v}(k T)-\underline{w}(k T) \tag{36}
\end{equation*}
$$

Since

$$
\lim _{t \rightarrow \infty}\left[\underline{A}_{11} x_{1}(t)+A_{12} x_{2}(t)\right]=\underline{0}
$$

then

$$
\begin{align*}
\lim _{k \rightarrow \infty} e(k T)= & \lim _{k \rightarrow \infty}\left\{\underline{v}(k T)-\underline{y}(k T)-\underline{M}\left[A_{1,}, x_{1}(t)+A_{12} x_{2}(t)\right]\right\} \\
& =\lim _{k \rightarrow \infty}\{\underline{v}(k T)-\underline{y}(k T)\} \\
& =\underline{0} \tag{37}
\end{align*}
$$



Consequently the steady state error vector is zerofor a constant command input vector. Once again tracking is achieved.

As in the case of the regular plant, the closed loop transfer function matrix approaches an asyptotic value as the gain 1/T goes to infinity, of:

$$
\begin{equation*}
\Gamma(\lambda)=\tilde{\Gamma}(\lambda)+\underline{\Gamma}(\lambda) \tag{38}
\end{equation*}
$$

where

$$
\begin{align*}
& \underline{\Gamma}(\lambda)=\underline{C}_{2} \underline{F}_{2}^{-1}\left(\lambda \underline{I}_{\ell}-I_{2}+\underline{F}_{2} \underline{B}_{2} \underline{K}_{0}\right)^{-1} \quad \underline{F}_{2} \underline{B}_{2} \underline{K}_{0} \tag{39}
\end{align*}
$$

Once again the $\tilde{\Gamma}(\lambda)$ contains the slow modes and $\hat{\Gamma}(\lambda)$ contains the fast modes. The transmission zeros are given by (see equation (33)):

$$
\begin{equation*}
\underline{z}_{t}(\lambda)=\left\{\left|\lambda I_{n-l}-\underline{I}_{n-l}-T A_{11}+T A_{12} \underline{F}_{2}^{-1} \underline{F}_{1}\right|=0\right\} \tag{41}
\end{equation*}
$$

The transmision zeros must life in the left half side of the s plane or within the unit circle in the 2 domain.

Unlike the regular plant case, $\tilde{\Gamma}(\lambda)$ does not approach zero as the sampling time decreases, so that for the ir regular plant both $\tilde{\Gamma}(\lambda)$ and $\hat{\Gamma}(\lambda)$ must be diagonal for decoupling. The choice of the measurement matrix $\underline{M}$ and the control law matrices $\underline{K}_{\text {。 and }} \underline{K}$, to achieve decoupling requires that the following conditions be satisfied:

1. Mis chosen such that $\underline{F}_{2}$ and ${\underset{F}{2}}^{B_{2}}$ have full rank $=1$
2. All closed loop poles must lie in the left half plane
3. $\underline{\underline{F}}_{2} \underline{B}_{2} \underline{X}_{0}=\left(\underline{C}_{2}+\underline{M A}_{12}\right) \underline{B}_{2} \underline{R}_{0}=$ diagonal matrix $\left\{\sigma_{1}, \sigma_{2}, \ldots, \sigma_{2}\right\}$
4. $-1<\left(1-\sigma_{i}\right)<1$ (roots must lie within unit circle)

The standard procedure for the selection of the measurement matrix is finding the sparsest $\underline{M}$ that yields an $\mathrm{F}_{2}$ of full rank, while also diagonalizing the product ${\underset{-}{2}}_{-F_{2}^{-1}}^{-1}$. Although this tech-
nique is straightforward, it is not always possible to generate a diagonal transfer function matrix ( 1 ).

Another problem with this method is that the possible presence in the output of the modes corresponding to the transmission zeros prevents the output response from being improved beyond a certain point with the reduction of $T$.

Once the measurement matrix is chosen, then the controller matrices are chosen such that:

$$
\begin{equation*}
\underline{K}_{0}=\alpha \underline{K}_{1}=\left(\underline{E}_{2} \underline{B}_{2}\right)^{-1} \text { diag }\left\{\sigma_{1}, \sigma_{2}, \ldots, \sigma_{2}\right\} \tag{42}
\end{equation*}
$$

B* Design (Ref 1 and 9)
This design technique supplements the other methods when they do not achieve the desired decounling and/or too much undershoot occurs in the output response. Additionally, under certain conditions the $\underline{B}^{*}$ matrix can be used to help pick the elements in the $\underline{M}$ matrix, which will be discussed at the end of this section.

The first step is to form the B* matrix using the following $^{*}$ formula:

$$
\underline{B}^{*}=\left[\begin{array}{cc}
c_{1}^{\top} A^{d_{1}} & \underline{B}  \tag{43}\\
c_{2}^{\top} A^{d_{2}} & \underline{B} \\
\vdots & \\
c_{2}^{\top} A^{d_{l}} & \underline{B}
\end{array}\right]
$$

where $C_{i}$ is the ith row of $C$, and $d_{i}$ is the smallestinteger such that the ith row of $B^{*}$ contains at least one non-zero element. If, however, there is no value for di that yields a row not equal to all zeros, then $d_{i}=n-1$ and the ith row of $\underline{B}^{*}$ is set to all
zeros. When this happens $B^{*}$ is not of full rank and this method cannot be used. However, under certain constraints another approach can be used and is discussed in the last paragraph of this section.

When $B^{*}$ has full rank the controller matrices can be deternined from:

$$
\begin{align*}
& \underline{K}_{0}=\underline{B}^{-1} \sum_{1}  \tag{44}\\
& \underline{K}_{1}=\underline{G}(0)^{-1} \underline{E}_{2} \tag{45}
\end{align*}
$$

where $\sum_{1}$ and $\underline{\Sigma}_{2}$ are diagonal $1 \times 1$ matrices chosen by the designer, based on the concepts that $\leq$ controls the initial output response and $\sum_{2}$ affects the steady-state response. As with the other methods the control law equations are:

$$
\begin{equation*}
\underline{u}(k T)=(1 / T)\left[\underline{K}_{0} e(k T)+\underline{K}_{1} \underline{z}(k T)\right] \tag{46}
\end{equation*}
$$

and

$$
\begin{equation*}
e(k T)=\underline{v}(k T)-\underline{y}(k T) \tag{47}
\end{equation*}
$$

If $B^{*}$ is singular, the above method cannot be used, but there is another possibility. If the row dimension of the $C$ matrix (Equation (18)) is greater than the row dimension of $A, ~(E-$ quation (17)), then $B^{*}$ can be used to help pick the elements in the - matrix (using the irregular plant method). When using this approach, the $\underline{B}^{*}$ matrix must be formed by using
$\underset{C}{C}=\underset{1}{ }$
A=A.1.
$B=A_{12}$
1n Equation (43).
The technique is continued by forming:

$$
{\underset{F}{2}}^{C_{2}}+\underline{M A}_{12}
$$

and assigning values to those entries in $\underline{F}_{2}$ that correspond to
the non-zero elements of $B^{*}$. The design is continued along the lines of the irregular plant method, with further refinement of $\underline{M}$ based upon the fact that $\mathrm{C}_{2} \mathrm{~F}_{2}$ must be diagonal, $\mathrm{F}_{\mathrm{z}}$ must have full rank and $M$ should be as sparse as possible.

## Summary

The design methods presented in this chapter form the basis for the Porter method, as developed by Professor Brian Porter and his associates. The computer program MULTI incorporates these concepts into an interactive tool used to design controllers such as those designed for this thesis. Although the presentation in this chapter specified digital PI controllers, since that is how MULTI is written, Professor Porters publications include similar design techniques for analog controllers.

## Introduction

This thesis designs controllers for the FPCC aircraft, an aircaft which exists in concept only. This "paper airplane" was conceived through the combined efforts of Lockheed, Pratt and Whitney, and Honeywell. FPCC designates Fifght Propulsion Control Coupling, which summarizes the most striking feature of this plane, the ability to vector thrust with the jet flaps. This capability, taken with the other flight control surfaces, increases the control vector's dimensions, allowing for more commanded quantities in the input vector.

The contractors developed the model and simulation program between June 1976 and June 1977. At that time the techniques developed by Professor Porter had not yet been published, and the Air Force was not involved in designing aircraft fiight control sygtems for aircraft capable of decoupled six degree of freedom maneuvers. But the FPCC aircraft was desigred with the extra flight control surfaces needed for such maneuvers, so that as the Air Force developed an interest in decoupled maneuvers the FPCC becare a logical plane to test the ideas on. In keeping with the concept of using the FPCC to test the Porter method of controller design, this thesis extends the work of Jon Bauschlicher (Ref 2), continuing to use the maneuvers similar to those currentiy being tested on the AFTI F-16 alrcraft. Additional information on the FPCC aircraft can be found in References 4 and 10 .

## Aircraft Description

Figure 4 shows the FPCC aircraft. This aircraft is a single seat supersonic fighter with a primary mission of air superiority fighter, and a secondary mission of ground attack. The airframe has the following dimensions:
A1rcraft Length=60.0 ft
A1rcraft Length=60.0 ft
Wing Span (B)=54.25 ft
Wing Span (B)=54.25 ft
Wing Mean Aerodynamic Chord=14.00 ft
Wing Mean Aerodynamic Chord=14.00 ft
Wing Planform Area (S) = 654.00 ft
Wing Planform Area (S) = 654.00 ft
Aspect Ratio (B*B/S)=4.50
Aspect Ratio (B*B/S)=4.50
Takeoff Mass=1055.9 slugs
Takeoff Mass=1055.9 slugs
Maneuver Mass=900.62 slugs
Maneuver Mass=900.62 slugs
Maximum Angle of Attack= +23 degrees
Maximum Angle of Attack= +23 degrees
Minimum Angle of Attack=-11.5 degrees
Minimum Angle of Attack=-11.5 degrees
The FPCC has standard flight control surfaces in aileron and rudder, along with non-standard surfaces in its horizontal and vertical canards and maneuver and jet flaps. The vertical canards along with the rudder give the FPCC the opposing lateral moments which, when combined with the aileron, allow decoupled lateral maneuvers. Likewise, the jet flaps, maneuver flaps, and horizontal canards supply the opposing longitudinal moments necessary to perform vecoupled longitudinal maneuvers. There are no elevators.

Table I shows a sumary of the FPCC aircraft design guideifnes and constraints, and Table II shows the control surface limits along with the actuator and sensor dynamics for the control surfaces. Note that although these are the same actuator


## TABLE I <br> SUMMARY LISTING OF INITIAL FPCC REFERENCE AIRCRAFT POINT DESIGN GUIDELINES AND CONSTRAINNS (REF 2)

1. Primary aircraft design mission -- supersonic/transonic air sliperiority
Secondary aircraft design mission -- transonic close air support
Alrcraft dash speed capability - within the range of Mach 2.2 through 3.0
2. Advanced CTOL or STOL design emphasizing air combat maneuvering and air combat tracking requirements for high subsonic/transonic air-to-air and air-to-ground tasks
3. Mission range, payload, takeoff and landing distance requirements are to be considered of secondary importance
4. Aircraft and propulsion design not constrained to any specification except for those noted in the RF? Statement of Work
5. Flight control designs may include any advanced feature such as Relaxed Static Staibility (RSS) and Direct Side Force Control (DSFC)
6. Aircraft design may include modulatory type Aerodynamic/ Propulsive Interactive Force (APIF) systems
7. Digital control and ily-by-wire assumed
8. The supersonic engine air inlet shall be of the external or mixed compression type consistent with the selected aircraft dash speed Mach number. Inlet bypass ai= may be used for aircraft maneuvering augmentation
9. Aircraft design thrust to weight (T/W) ratio shall be higher than 1.0 with dry engine operation. However, engines shall incorporate afterburning. Engine control may inciude

- variable fan guide vane angle
- variable compressor stator angle
- variable turbine area
- variable exhaust nozzle area

Engine bleed capabilities may be considered consistent with APIF system
10. The Lockheed reference aircraft point design will incorporate the current state of the art Pratt and Whitney aircraft F-100-PW-100 afterburning turbofan engine and controls
dynamics used in the program MULTI, these are not the same sensor dynamics. MULTI sensors refer to the sensors used to sense the alrcraft states, not the flight control surfaces. The only possible overlap for this aircraft would be if the total thrust were used as both a flight control and a state.

The original design for the aircraft stipulated that the jet flaps would be limited to only positive deflection (trailing edge down) of 0 to 90 degrees, and also restricted the maneuver flaps to $\pm 15$ degrees. Jon Bauschlicher had the simulation program changed to reflect the values listed for these control surfaces in Table II, and this author has let this modification stand.

More detailed information about the workings of the control surfaces is found in the next section on the simulation program.

FPCC Aircraft Simulation Program
In effect, the simulation program is the aircraft. All of the control surface limits depicted in Table II are set within the program, as are the actual implementations of these surfaces. The current version of the program deflects the ailerons and rudder in the standard manner (ailerons together in opposite directions), and deflects the maneuver flaps, the horizontal canards, the vertical canards, and the jet flaps symmetrically. The program allows asymmetric thrust, but this feature is not used in this thesis.

All of the matrices used in this thesis are generated by the simulation program, as are the initial state vectors. The simulation program is availablefrom the Flight Dynamics Lab at Wright-Patterson Air Force Base, and was developed by Lockheed

## TABLE II

CONTROL SURFACE LIMITS AND DYNANICS

| Control Surface | $\begin{gathered} \text { Deflection } \\ M<1.0 \end{gathered}$ | $\begin{array}{r} \text { Limits } \\ M>1.0 \end{array}$ | Servo Dynamics | Sensor Dynamics |
| :---: | :---: | :---: | :---: | :---: |
| $\underset{\substack{\text { Maneuver } \\\left(\delta_{\mathrm{MF}}\right)}}{\text { Flap }}$ | $\pm 30^{\circ}$ | $\pm 15^{\circ}$ | $\frac{25}{s+25}$ | $\frac{100}{3+100}$ |
| $\left(\begin{array}{c} \text { Jet Flap } \\ \left(\delta_{j}\right) \end{array}\right.$ | $\pm 90^{\circ}$ | $\pm 90^{\circ}$ | $\frac{25}{s+25}$ | $\frac{100}{s+100}$ |
| Horizontal Canard ( $\delta_{\mathrm{HC}}$ ) | $\pm 20^{\circ}$ | $\pm 10^{\circ}$ | $\frac{25}{s+25}$ | $\frac{100}{s+100}$ |
| $\begin{array}{\|l\|l\|}  & \text { Thrust } \\ \left(F_{\text {TOT }}\right) \end{array}$ | - | - | $\frac{0.574}{s+0.574}$ | $\frac{100}{s+100}$ |
| $\begin{aligned} & \text { A1leron } \\ & \left(\delta_{a}\right) \end{aligned}$ | $\begin{gathered} \pm 20^{\circ} \\ \text { (per side) } \end{gathered}$ | $\begin{aligned} & \pm 20^{\circ} \\ & (\text { per } \\ & \text { side) } \end{aligned}$ | $\frac{25}{s+25}$ | $\frac{100}{3+100}$ |
| ${ }_{\left(\delta_{r}\right)}^{\text {Ruder }}$ | $\pm 30^{\circ}$ | $\pm 15^{\circ}$ | $\frac{25}{3+25}$ | $\frac{100}{s+100}$ |
| $\begin{aligned} & \text { Vertical } \\ & \text { Canard } \\ & \left(\delta_{\text {vc }}\right) \end{aligned}$ | $\pm 35^{\circ}$ | $\pm 15^{\circ}$ | $\frac{25}{s+25}$ | $\frac{100}{s+100}$ |

under contract to the Air force. Runing in batch mode, the program outputs the data used for this thesis in the form of Equations (48) and (49).

$\left.\begin{array}{ll}0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -.00000326 & .00000325 \\ -.000410 & -.000415 \\ 0 & .00000935 \\ 0 & 0 \\ 0 & 0 \\ q \\ \psi \\ w \\ y \\ q \\ \theta \\ h \\ x\end{array}\right]$


The output equation is


Note that these equations contain the output for the trim condition of 0.15 mach at sea level, which is the first fight condition used. Tables III and IV show the input and state vectors, which are basically the same as those used in this thesis, while Table $V$ list the output vector which is not the same as used in this thesis. While the simulation program generates data that is used for the controller design, the form of the matrix equations is changed silightiy.

The simulation program actually trims the aircraft given a mach number, altitude, and settings for the maneuver flaps and jet flaps. The plane is then trimmed using the horizontal canards and the thrust. The ailerons, vertical canards, and the rudder are not deflected. Trimming is successful only if the limits on the filght control surfaces, thrust, and angle of at-

TABLE III
INPUT VECTOR, $\underline{u}^{*}$

| Variatie | Trits | Description |
| :---: | :---: | :---: |
| $\delta \mathrm{a}$ | deg | sum of both alleron deflections |
| $\delta \mathrm{c}$ | deg | horizontal canard deflection |
| $\delta \mathrm{mf}$ | deg | maneuver flap deflection |
| $\delta r$ | deg | rudder deflection |
| $\delta \mathrm{VC}$ | deg | vertical canard deflection |
| $\delta j$ | deg | Jet flap deflection wrT fuselage ref line |
| $\mathrm{F}_{1}$ | lbs | Net thrust engine $1^{-}$ |
| $\mathrm{F}_{2}$ | 1bs | Net thrust engine 2 |
| $\mathrm{CDI}_{1}$ | - | Inlet drag coefficient engine 1 |
| $\mathrm{CDI}_{2}$ | - | Inlet drag coeficient engine 2 |

TABLE IV
STATE VECTOR, $x^{*}$

| Varlable | Units | Description |
| :---: | :---: | :---: |
| $p$ | $\mathrm{rad} / \mathrm{sec}$ | Body axis roll rate |
| $r$ | rad/sec | Body axis yaw rate |
| $v$ | $\mathrm{ft} / \mathrm{sec}$ | Body axis side velocity |
| ¢ | rad | Roll angle |
| $\pm$ | rad | Yaw angle |
| y | ft | Cross-range position reference to initial body axis |
| q | $\mathrm{rad} / \mathrm{sec}$ | Body axis pitch rate |
| w | $\mathrm{ft} / \mathrm{sec}$ | Body axis vertical velocity |
| $u$ | $\mathrm{ft} / \mathrm{sec}$ | Body axis forward velocity |
| $\theta$ | rad | Pitch angle |
| h | ft | Altitude |
| x | ft | Down-range position referenced to initial body axis |

TABLE V
OUTPUT VECTOR, I* $^{*}$

| Variable | Units | Description |
| :---: | :---: | :---: |
| $a^{\circ}$ | rad | Angle-of-attaci |
| $\beta$ | rad | Sideslip angle |
| VEL | $\mathrm{ft} / \mathrm{sec}$ | Total airspeed |
| ACGX | $\mathrm{ft} / \mathrm{sec}^{2}$ | Accel. of c.g. along x-body axis |
| ACGY | $\mathrm{ft} / \mathrm{sec}^{2}$ | Accel. of c.g. along y-body |
| ACGZ | $\mathrm{ft} / \mathrm{sec}^{2}$ | axis <br> Accel. of c.g. along z-body |
| ACG2 | c/sec | axis |

tack are not exceeded. The filght conditions used in this thesis
are:
1). 0.15 mach, $0 f t$ altitude
2). 0.60 mach, $0 f t$ altitude
3). 0.90 mach, $30,000 \mathrm{ft}$ altitude

Since the first condition represents a landing configuration, the angle of attack is restricted for pilot visibility of the runway. The simulation program trimmed the aircraft to ll degrees angle of attack with the maneuver flaps set for +25 degrees and the jet flaps set at +15 degrees, and the horizontal canard trimed to +0.5 degrees.

The second fifght condition represents a posible ground attack mode, high subsonic speed at low altitude. The third flight condition is a candidate for aerial combat, just below supersonic at high altitude.

Table VI shows the results of several different flight conditions run on the FPCC simulation program. These conditions were tested to observe the interaction between the trimmed angle of attack, horizontal canard setting, thrust, and the given settings of maneuver flaps and jet flaps at low speds at sea level.

Table VI clearly shows the relationship between the angle of attack and the jet flap setting as slow speeds. However, to counter the moment caused by a large jet flap setting, the horizontal canards must be deflected possibly beyond their limits. To offset this drawback, the horizontal canards need more effectiveness, perhaps through increased area. This would allow for a smaller angle of attack at even slower airspeds than used in this thesis, thus permitting a more STOL-like operation.

The values in Table VI often exceed the limits listed in Table II because the simulation program does not actually impose

FPCC AIRCRAFT RESPONSE AT SEA LEVEL

| Mach* | $\alpha$ | $\delta_{G}$ | $\delta_{m f}$ | $\delta_{J}$ | $F_{\text {TOT }}$ |
| :--- | :--- | ---: | :--- | ---: | :--- |
| $0.10 *$ | 65.62 | -17.18 | 30 | 0 | 16,668 |
| $0.10 *$ | 57.81 | -1.79 | 30 | 10 | 15,564 |
| $0.10 *$ | 51.42 | 5.36 | 30 | 15 | 15,366 |
| $0.10 *$ | 44.22 | 12.34 | 30 | 20 | 15,296 |
| 0.10 | 33.04 | 27.16 | 0 | 30 | 16,586 |
| $0.10 *$ | 30.17 | 25.49 | 30 | 30 | 15,210 |
| $0.10 *$ | 10.39 | 74.00 | 30 | 60 | 16,504 |
| $0.15 *$ | 13.44 | -9.20 | 30 | 0 | 8,826 |
| 0.15 | 13.44 | -9.20 | 15 | 0 | 8,826 |
| $0.15 *$ | 11.48 | -2.52 | 30 | 10 | 7,730 |
| $0.15 *$ | 10.78 | 0.51 | 30 | 15 | 7,468 |
| $0.15 *$ | 10.78 | 0.51 | 25 | 15 | 7,468 |
| 0.20 | 5.92 | -2.31 | 15 | 0 | 6,846 |
| $0.20 *$ | 5.92 | -2.31 | 30 | 0 | 6,846 |
| $0.20 *$ | 4.84 | 2.07 | 30 | 10 | 6,702 |
| $0.20 *$ | 4.30 | 4.26 | 30 | 15 | 6,884 |

* Represents a flight condition with a column of all zeros in the $B$ matrix corresponding to the maneuver flaps.
these limits on the aircraft during trimming. However, the program indicates whenever the limits have been exceeded, and then continues with the generation of the linear dynamics

Several of the flight conditions yield matrices that have a column of zeros in the colum corresponding to the manuever flaps. If the program is correct ingenerating these zeros, then the maneuver flaps have no effect (for small pertubations) at these flight conditions. This is the case for the flight condition at 0.15 mach used in this thesis, which is why the maneuver flaps are removed from the longitudinal model (Chapter IV). However, since the simulation program sems to be erroneous in certain other entries within the $\underline{B}^{\text {matrix (Appendix } A), ~ t h e n ~}$ suspicion has to fall on all of the matrix, including these colums of zeros. The position taken for this thesis is that these columns of zeros are correct, and the longitudinal model adjusted accordingly. This position resulted more from a lack of time to prove otherwise, than any other consideration.

An interesting aside is that stol operation at very slow speeds is going to require large amounts of thrust, which may not be a desirable trait.

This thesis assumes that the maximum combined thrust is 30,000 pounds, based upon the specification of at least a 1.0 thrust to weight ratio and the weight of the aircraft.

## Introduction

This chapter details the design of a complete controller for conbined lateral and longitudinal maneuvers for the STOL like flight condition (0.15 mach, sea level), along with longitudinal controllers for two other fifght conditions (0.6 mach, sea level and 0.9 mach, 30,000 ft.). The 0.15 mach fight condition has not previously been studied, but the other two have been (Ref 2). However, this thesis uses a different longitudinal model than ang previous efforts (Ref 2 and 3 ), which led to the redesigning of the longitudinal controllers at the 0.6 and 0.9 mach fifght conditions.

The change in the longitudinal model resultsfrom a forced change at the 0.15 mach fight condition which is then carried over to the other two filght conditions. This forced change results from a column of zeros in the betrix at 0.15 mach corresponding to the maneuver flaps (Equation (48)). If this column of zeros is correct, then that means that the maneuver flaps have no effect upon the motion of the aircraft (for small pertubations) at this fifght condition. Unfortunately, since the aircraft does notexist, there is no way to verify this. Considering that there are at least two other errors in this same B matrix (Appendix A), this columa of zeros would have to be considered uspect. This thesis is based upon the idea of using just thre of the longitudinal fifght control surfaces to execute the maneuvers, and the decision to leave out the maneuver flaps was
based upon this column of zeros. If this zero column were erroneous, the impact on these results would be minimal.

Since the maneuver flaps have no apparent effect at 0.15 mach, they can be eliminated from the longitudinal model for the purposes of performing the maneuvers. However, the maneuver flaps are needed to trim the aircraft, since without them the low angle of attack needed for landing would not be possible. Based on these considerations, it was decided to model the aircraft as using the maneuver flaps for triming only and not using them for any of the maneuvers. Since this was contrary to previous efforts, the 0.6 mach and 0.9 mach flight conditions from Bauschlichers thesis were reaccomplished using this new longitudinal model. This would allow comparisons between the different longitudinal models, along with drawing conclusions as to which model generates the best aircraft performance. Unfortunately, a design for the 2.3 mach fight condition was not found. Part of the reason for this was the limited time avallable to the author, but there had to be other reasons also, since the other designs were accomplished in a relatively short time compared to the time spent attempting to find a controller for 2.3 mach. One of the reasons might be the fact that the eigenvalues of the system matrix for 2.3 mach are different than the other flight conditions. As stated later in this chapter, the short period roots for all of the fight conditions consist of two real roots, one in the left half s plane and one in the right half splane. In fact, the short period roots for the 2.3 mach flight condition are very nearly identical to those
for the 0.9 mach flight condition. The phugoid roots for 2.3 mach, however, have become real with both lying in the left half s plane. This is the only flight condition studied for which this occurs.

The design of a complete 12 state combined lateral and longitudinal controller would be beyond the capabilities of MULTI (currently limited to 10 states), and perhaps the author also. Consequently, the design is broken down into two parts, the longitudinal and lateral modes. After the lateral and longitudinal controllers are designed and tested independently, the lateral and longitudinal models are combined and a combined controller is then tested based on the combination of the separate controllers. The combined controller is tested using one coupled maneuver, to verify that lateral and longitudinal commands can be executed simultaneously. Although this phase is simplified by the decoupled nature of the original equations of motion (as represented by the matrices in equations (48) and (49)), the procedure should still be the same if the equations were not decoupled.

The computer program MULTI is used extensively in the design of the controllers for this thesis, and is the sole tool used to test the designs via simulation. MULTI was originally written by $A F I T$ students in 1981 and was based upon a simulation program written by Professor Brain Porter (Ref 7). The program has undergone many alterations since then, including the addition of an option to calculate and display the figures of merit, and passing of the input data local filenames thru command line arguments into the program. Both were written by this author and
detailed in Appendix C. Appendix $D$ lists the addition of program code that would allow faster design times for the experienced MULTI user, and would also allow the true implementation of $\quad$ D feedforward matrix. Although MULTI currently asks for a matrix, the simulation does not include a matrix in the calculations of the output used for feedback. A completedescription of MULTI and its options can be obtained from the AFIr Electrical Engineering Department.

This chapter contains the design and testing of the separate lateral and longitudinal controllers for the 0.15 mach, sea level flight condition, along with the longitudinal controllers for the 0.6 mach, sea level and 0.9 mach, $30,000 \mathrm{ft}$. flight conditions. The approach used for each flight condition is that the controller designed will be used for all of the maneuvers, instead of trying to find a different, possibly more optimized controller for each maneuver. Finally, the iateral and longitudinal controllers for the 0.15 mach flight condition are combined and tested with one coupled lateral/longitudinal maneuver. Chapter $V$ investigates using one controller for allof the fight conditions and the effect of delaying the control input to the actuators.

## Lateral Controller

The lateral controller design is accomplished only for the first fiight condition, 0.15 mach, sea level. The lateral controllers for the other flight conditions used in this thesis can be found in Bauschlicher's thesis (Ref 2).

Manuever Descriptions. Four maneuvers are used in

Bauschlicher's thesis to test the lateral controller, the flat turn, yaw pointing, horizontal translation, and the rollover. The same four maneuvers were attempted at 0.15 mach with the controller designed for this thesis, with the result that the rollover could not be performed at all. This is probably due to the very slow sped of the aircraft and subsequent lack of aerodynamic forces exerted on the flight control surfaces. Consequently, only the first three maneuvers, the flat turn, yaw pointing, and horizontal translation, are tested in this thesis.

The flat turn maneuver is commanded by deciding on what $g$ forces are desired in the $x y$ plane (body axes), and calculating the necessary yaw rate from:

$$
\begin{equation*}
A y=w^{2} * d / 2=U * r \tag{50}
\end{equation*}
$$

where

$$
\begin{aligned}
\text { Ay } & =\text { acceleration in } x y \text { plane } \\
w & =r \text { (body yaw rate) } \\
w * d / 2 & =U \text { (body velocity) }
\end{aligned}
$$

and with d/2 used instead of $r$ for radius. The body axis side velocity $v i s$ commanded to zero, along with the euler roll angle $\boldsymbol{\phi}$. Henceforth, all coordinate systems are body axis, except for the euler angles, $\theta(p i t c h), \psi(y a w), \phi(r o l l)$.

Yaw pointing is accomplished by deciding upon the angle of pointing desired, then commanding the sidesifipangle $\beta$ to that value via the relationship

$$
\begin{equation*}
\beta=v / U \tag{51}
\end{equation*}
$$

which is good for small values of sideslip. Roll angle is commanded to zero, and the command for $r$ is based upon a curve
whose integral (area under curve) will be equal to the sideslip angle. Note that an angle of less than lo degrees insures an error of less than one percent in equation (51).

Horizontal translation consists of commanding the side velocity $v$ depending upon desired g forces, while commanding both roll angle and yaw rate to zero.

Although the rollover maneuver is not tested in this thesis, it is performed by commanding both yaw rate and side velocity to zero while commanding the desired roll angle.

Lateral Model. These sections detailing the design of the lateral model and controller use the flight condition of 0.15 mach at sea level for the numerical examples to illustrate the procedure. The same techniques are used to develop the model and controllers for the other two flight conditions, with the actual matrices used listed in Appendix D.

The first step is to reduce the system model from its current form of equation (48). The six lateral states are p, r, $v, y, \phi$, and $\Psi$. The lateral inputs are $\delta a, \delta r, \delta v c, F_{1}, F_{2}$, $C \cap I_{1}$, and $\mathrm{CDI}_{2}$. To reduce the number of states without eliminating crucial information about the aircraft, the eigenvalues can be used. The complete set of eigenvalues for the lateral model of equation (48) are:

$$
\begin{aligned}
& -0.1990 \pm j 0.9563 \text { (dutch roll roots) } \\
& -0.6927 \text { (roll subsidence) } \\
& 0.01069 \text { (spiral divergence) } \\
& \pm 0.1471 \text { E-06 }
\end{aligned}
$$

The two roots near zero contain no essential aircraftinformation and so can be eliminated from the model. Based on previous
experience, the choice is made to eliminate y and from the state model. Computing the eigenvalues of this new four state A matrix yields:
$-0.6927$
0.01069
and the important aircraft characteristics have been preserved. Now that the state model has been reduced to four states, the inputs are going to have to be reduced to at most three (and still have the correct form for an irregular design). This is done by observing that the columns in the $B$ matrix corresponding to $F_{1}$ and $F_{2}$ have equal and opposite values (in the lateral part of the $B$ matrix). This means that as long as the two engines generate equal thrust these terms cancel out to zero, having no effect on the model. Likewise, since the terms corresponding to the two coefficients of drag for the inlets are also equal and opposite, they have no effect with equal thrust in the two engines. This reduces the inputs to three for the lateral model, $\delta a, \delta r$, and $\delta v e$. This model is satisfactory as long as the two engines produce the same thrust. Imposing this constraint would require extra circuitry in addition to the controller. Quite possibly, this constraint on the thrust would be valid even without any additional engine control.

The choice of the three outputs, $v, \phi$, and $r$, is dictated by Bauschlicher's thesis, since one of the purposes of this thesis is to compare results with Bauschlicher. This choice of outputs satisfies the requirement for the number of inputs to
equal the number of outputs.
These outputs result in completely different output matrices than those in equation (49). Since there are no accelerations in the output, there is no feedforward matrix, and the $C$ matrix is also different from equation (49).

These changes result in a new set of matrices describing the linear dynamics of the aircraft and are given by:

$$
\begin{align*}
& {\left[\begin{array}{l}
\dot{\phi} \\
\dot{v} \\
\dot{r} \\
\dot{p}
\end{array}\right]=\left[\begin{array}{llll}
0 & 0 & .19 & 1 \\
31.6 & -.172 & -164 & 31.5 \\
0 & .00415 & -.212 & .0654 \\
0 & -.00818 & .364 & -.696
\end{array}\right]\left[\begin{array}{l}
\phi \\
\mathrm{v} \\
\mathrm{r} \\
\mathrm{p}
\end{array}\right]+} \\
& {\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & .111 & .0363 \\
.00114 & -.0112 & .0022 \\
.0122 & .0118 & -.00212
\end{array}\right]\left[\begin{array}{c}
\delta_{a} \\
\delta_{r} \\
\delta_{v c}
\end{array}\right]}  \tag{52}\\
& {\left[\begin{array}{l}
v \\
\phi \\
r
\end{array}\right]=\left[\begin{array}{llll}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{array}\right]\left[\begin{array}{l}
\phi \\
v \\
r \\
p
\end{array}\right]} \tag{53}
\end{align*}
$$

The above set of equations represents the lateral model for the FPCC aircraft at 0.15 mach at sea level. This system is checked for decoupling zeros, and since there aren't any it is both controllable and observable, two necessary requirements. Although Bauschlicher stated that the transmission zeros must also lie in the left half plane, this testis performed on the
 this substitution and then checking for transission zeros yields none in the right half plane, another requirement. (Although it may be possible to design with transmission zeros at the origin).
 does not have full rank of 1 , the design is irregular. Consequently, an matrix must be found which meets the requirements specified in Chapter II. This measurement matrix supplies information to the feedback loop on the derivative of $\boldsymbol{\phi}$, which is not necessarily equal to the roll rate. The following section details the development of the matrix.

Measurement Matrix Determination. The first step is partitioning the matrices of equations (52) and (53) so the metrods of Chapter II can be used to find the $\underline{B}^{*}$ matrix. The partitioned matrices used in equation (43) are:

$$
\begin{align*}
& \underline{C}=\underline{C}=\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right]  \tag{54}\\
& \underline{A}=\underline{A}_{11}=\left[\begin{array}{l}
0
\end{array}\right]  \tag{55}\\
& \underline{B}=\underline{A}_{12}=\left[\begin{array}{lll}
0 & 0.19 & 1
\end{array}\right] \tag{56}
\end{align*}
$$

The resulting $B^{*}$ matrix is found to be:

$$
\underline{B}^{*}=\left[\begin{array}{lll}
0 & 0 & 0  \tag{57}\\
0 & 0.19 & 1 \\
0 & 0 & 0
\end{array}\right]
$$

with $\left\{d_{1}, d_{2}, d_{3},\right\}=\{3,0,3\}$ as used in equation (43).
Next, the measurement matrix $M$ must be found, so that $\underset{\sim}{ }$ can be calculated. Since the dimensions of $M$ are $1 \times(n-1)$, which for this system is $3 \times 1$, M can be generalized as:

$$
\underline{M}=\left[\begin{array}{l}
m_{1}  \tag{58}\\
m_{2} \\
m_{3}
\end{array}\right]
$$

Using $\mathrm{F}_{2}=\underline{C}_{2}+$ MA $_{1,2}$ and

$$
\underline{c}_{2}=\left[\begin{array}{lll}
1 & 0 & 0  \tag{59}\\
0 & 0 & 0 \\
0 & 1 & 0
\end{array}\right]
$$

yields

$$
\underline{F}_{2}=\left[\begin{array}{lll}
1 & 0.19 m_{1} & m_{1}  \tag{60}\\
0 & 0.19 m_{2} & m_{2} \\
0 & 1+0.19 m_{3} & m_{3}
\end{array}\right]
$$

Since the rank of must be equal to 1 , or three, and the allowable non zero elements of ${\underset{F}{z}}^{(d u e ~ t o ~ m u s t ~ h a v e ~ a ~}$ corresponding non zero element in $B^{*}$, the choice of must be:

$$
\underline{M}=\left[\begin{array}{l}
0  \tag{61}\\
\mathbf{m}_{2} \\
0
\end{array}\right]
$$

which yields

$$
\underline{F}_{2}=\left[\begin{array}{lll}
1 & 0 & 0  \tag{62}\\
0 & 0.19 m_{2} & m_{2} \\
0 & 1 & 0
\end{array}\right]
$$

Using the above information the asymptotic transfer function from equation (38) can be derived:

$$
\begin{aligned}
& \underline{F}_{2}^{-1}=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 / m & -0.19
\end{array}\right] \\
& {\underset{-2}{2}-2}_{F_{-2}^{-1}}=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{array}\right]
\end{aligned}
$$

which is in the desired diagonal form.

$$
\underline{\underline{F}}_{1}=\underline{c}_{1}+\underline{M A}_{\underline{A}_{1}}=\underline{\underline{c}}=\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right]
$$

Since $A_{11}=[0]$

$$
\begin{align*}
& \underline{C}_{1}-\underline{\underline{G}}_{2} \underline{F}_{2}^{-1} \underline{F}_{1}=\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right] \\
& {\left[\begin{array}{lll}
T_{12} & F_{2}^{-1}
\end{array}\right]=\left[\begin{array}{lll}
T & \left(\begin{array}{lll}
0 & 1 / m_{2} & 0
\end{array}\right]
\end{array}\right]} \\
& {\left[\lambda \underline{I}_{n-2}-\underline{I}_{n-2}-T \underline{A}_{11}+T \underline{I}_{12} \underline{F}_{2}^{-1} \underline{F}_{1}\right]^{-1} \quad=\left[\lambda-1+T / m_{2}\right]^{-1}} \\
& =\left[1 /\left(\lambda-1+T / m_{2}\right)\right] \tag{63}
\end{align*}
$$

which yields:

$$
\begin{aligned}
\tilde{\Gamma} & =\left[\underline{C}_{1}-\underline{C}_{2} \underline{F}_{2}^{-1} \underline{F}_{1}\right]\left[\lambda \underline{I}_{n-2}-\underline{I}_{n-2}-T \underline{A}_{11}+\underline{T A}_{12} \underline{F}_{2}^{-1}{\underset{-1}{1}}^{-1}\left[T \underline{A}_{12} \underline{F}_{2}^{-1}\right]\right. \\
& =\left[\begin{array}{lll}
0 & 0 & 0 \\
0 & T / m_{2} & 0 \\
0 & 0 & 0
\end{array}\right] \quad\left[1 /\left(\lambda-1+T / m_{2}\right)\right]
\end{aligned}
$$

This has the desired diagonal form.

$$
\begin{aligned}
& \underline{F}_{2} \underline{B}_{2} \underline{K}_{0}=\left[\begin{array}{lll}
\sigma_{1} & 0 & 0 \\
0 & \sigma_{2} & 0 \\
0 & 0 & \sigma_{3}
\end{array}\right] \\
& {\left[\lambda \underline{I}_{2}-\underline{I}_{2}+\underline{E}_{2} \underline{B}_{2} \underline{K}_{0}\right]^{-1} }=\left[\begin{array}{ccc}
\lambda-1+\sigma_{1} & 0 & 0 \\
0 & \lambda-1+\sigma_{2} & 0 \\
0 & 0 & \lambda-1+\sigma_{3}
\end{array}\right]^{-1} \\
&=\left[\begin{array}{llll}
1 /\left(\lambda-1+\sigma_{1}\right) & 0 & 0 \\
0 & 1 /\left(\lambda-1+\sigma_{2}\right) & 0 \\
0 & 0 & 1 /\left(\lambda-1+\sigma_{3}\right)
\end{array}\right] \\
& {\left[\underline{C}_{2} \underline{F}_{2}^{-1}\right]\left[\lambda \underline{I}_{2}-\underline{I}_{2}+\underline{F}_{2} \underline{B}_{2} \underline{K}_{0}\right]^{-1}\left[\underline{F}_{2} \underline{B}_{2} \underline{K}_{0}\right]=\left[\begin{array}{lll}
\sigma_{1} /\left(\lambda-1+\sigma_{1}\right) & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \sigma_{3} /\left(\lambda-1+\sigma_{3}\right)
\end{array}\right] }
\end{aligned}
$$

which is also in the desired diagonal form. Now, since $\underset{\sim}{\underline{I}}+\hat{\underline{T}}$

$$
I=\left[\begin{array}{lll}
\sigma_{1} /\left(\lambda-1+\sigma_{1}\right) & 0 & 0  \tag{64}\\
0 & \left(T / m_{2}\right) /\left(\lambda-1+T / m_{2}\right) & 0 \\
0 & 0 & \sigma_{3} /\left(\lambda-1+\sigma_{3}\right)
\end{array}\right]
$$

The asymptotic transfer function in equation (64) indicates that the desired design goal of output decoupling with infinite gain has been achieved. Note that a decrease in sampling time corresponds to an increase in gain, so that saying the sampling time goes to zero is equivalent to stating that the gain goes to infinity. From equation (64) the closed loop eigenvalues for this system are:

$$
\begin{aligned}
& \lambda_{1}=1-\sigma_{1} \\
& \lambda_{2}=1-T / m_{2} \\
& \lambda_{3}=1-\sigma_{3}
\end{aligned}
$$

These eigenvalues indicate that the time response of the outputs are greatly affected by the values of $\sigma_{1}, m_{2}$, and $\sigma_{3}$.

Going back to an intermediate stepin the derivation of equation (64), the one (n-1) transmission zerofor this system is (equation (63)):
$Z_{t}=1-T / m_{2}$
Controller Design. Detailed below is the basic algorithm used for the controller design of all of the controllers designed for this thesis. This procedure is based mostly on the controllers within this thesis and as such should not be considered well tested. However, there is no reason not to expect this technique
to help the designer get started with a design. Implementing this process depends upon avallable design tools, and the one used for this thesis is the interactive program MULTI. Without MULTI this thesis would not have been possible. The design process follows these steps:

1. Set all of the design parameters equal to one: Mele ments; $\sum$ elements; $\boldsymbol{\alpha}$; and $\in$.
2. Calculate $\underline{K}_{0}$ and $\underline{K}_{1}$. Reduce until the largest order of magnitude of elements in either controller matrix is $10 \mathrm{f}+\mathrm{O}$.
3. Run a simulation and use option \#28 in MULTI (figures of merit) to check for a bounded (stable) response. Continue reducing $\epsilon$, but by relatively small increments, until the response is stable. Experience will best answer the question of what reiatively small increments are, but a good starting guideline would be factors of two. (Based uponthe designs in this thesis, step three will reduce the largestorder of magnitude to $10 E+02$, but information obtained from other thesis efforts suggests the above guideline of $10 E+03$ in step two)
4. Further mold the controller matrices using the elements of $\sum_{\infty}$ Increase or decrease the individual elements so as to increase or decrease the corresponding colums in the controller matrices by the same factor until the orders of magnitude lie within $10 E-02$ to $10 E+02$ (or as close as posible). Generally, numbers that are smaller than the suggested values are more desirable than numbers that are larger than suggested.
5. Run a simulation to observe which outputs need improved response. Generally, increasing an element in the $\underline{L}$ matrix will increase the sped of response of some of the outputs, but with
the resulting problem of increased overshoot. Experimentation will show which $£$ element affects which output. Repeat this process until the best time responses are achieved.
6. Experiment with the value of $\propto$ while observing output response. In this thesis increasing $\propto$ sometimes reduced overshoot on the control surface deflections with little degredation in output response, but sometimes this parameter had little noticable effect.
7. Now vary the Melements to see if they will improve output response. Generally the Melements directly affect the corresponding output response depending upon which column they re in. For example, decreasing an entry in the second column might increase the sped of response of the state whose derivative is measured by that column. Most designs, including those from other theses, have values for $\underline{M}$ elements within the range of 0.25 to 1.0.
8. Repeat steps 5 thru 7 until the best responses are achieved, keping in mind that tradeoffs will always be made betwen desirable and undesirable output responses and control surface deflections. Even though option \#27 in MULTI does limit the control surface deflections based on physical limitations, a design that reduces the number of times a surface "hits the stops" is considered superior to one that doesn't.

Obviously different plants are going to respond in different ways to changes in the design parameters. This means that a satisfactory design is more the result of trial and error than any "cookbook" approach. One conclusion tentatively drawn from

$$
\begin{align*}
& \therefore \quad \therefore \quad-r=1, \quad T=0.01  \tag{66}\\
& E=0.1  \tag{67}\\
& \underline{M}=\left[\begin{array}{l}
0 \\
0.25 \\
0
\end{array}\right]  \tag{68}\\
& \underline{K}_{0}=\left[\begin{array}{lll}
-0.00382 & 29.9 & 15.5 \\
0.0505 & 1.90 & -13.0 \\
0.259 & -5.82 & 39.6
\end{array}\right]  \tag{69}\\
& \underline{K}_{1}=\underline{K}_{0} \tag{70}
\end{align*}
$$

Figures 5 through 11 show the time responses for the outputs and the flight control surface deflections for the 0.15 mach, sea level filght condition. The figures of merit and commanded inputs for each maneuver are listed in Table VII.


1.5 G HORIZONTAL TRANSLATION 10.15 MACH. O FTI

Figure 5. 1.5 g Horizontal Translation ( 0.15 Mach)

1.5 G HORIZONTAL TRANSLATION (D. O. 15 MACH. O FT)

1.5 G HORIZONTAL TRANSLATION (D. O. 15 MACH. O FT)

Figure 6. 1.5 g Horizontal Translation (2.15 Mach)


1 G FLAT TURN ( 0.15 MACH. O FT)


Figure 7. 1 g Flat $\operatorname{Turn}(0.15$ Mach)


1 G FLAT TURN 10.0 .15 MACH. O FT)


Figure 8. 1 g Flat Turn (0.15 Mach)



3 DEGREE YAW POINTING 10.15 MACH. O FTI

Figure 9. 3 Degree Yaw Pointing (0.15 Mach)


3 degree yaw pointing io. 0.15 Mach, oft


3 degree yan polnting (0. 0.15 mach. oft)
Figure 10. 3 Degree Yaw Pointing ( 0.15 Mach)

## SUMMARY OF OUTPUT RESPONSES FOR LATERAL CONTROLLER

| F1t. Cond. | Maneuver | Command, V | Overshoot | Rise Time |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 0.15 \text { Mach } \\ & 0 \text { ft. } \end{aligned}$ | Yaw | 1,-11.5,20,20 | none | 1 sec |
|  | Pointing | $0,0,0,0$ | -0.001 rad |  |
|  |  | . $2, .0524,1,1.2$ | 8.2\% | 0.27 sec |
|  | Flat | 0,0,0,0 | -38.9 ft/sec |  |
|  | TArn | 0, 0, 0, 0 | .068 rad |  |
|  |  | 1,.195,20,20 | none | * |
|  | Horiz. | 1,48.3,20,20 | none | 10 sec |
|  | Transl. | $0,0,0,0$ | .052 rad |  |
|  |  | $0,0,0,0$ | -. $12 \mathrm{rad} / \mathrm{sec}$ |  |

T
Note: The command vector is $\left[\begin{array}{l}\mathrm{v} \\ \mathrm{t}\end{array}\right]$.
Overshoot is the first peak for non zero commanded outputs or is the greatest deviation from zerofor zero commanded outputs. Rise time is from initial value to $90 \%$ of commanded value.

* For this maneuver the output did not reach $90 \%$ of the command.


3 DEGREE YAN POINTING (D. O. 15 MACH. O FT)

Figure 11. 3 Degree Yaw Pointing (0.15 Mach)

Longitudinal Controller
Maneuver Description. Continuing with the concept of direct comparison with Bauschlicher's results, the same three longitudinal maneuvers that he used are chosen for this thesis. These maneuvers are the pitch pointing, vertical translation, and direct lift. These maneuvers are typically claimed to be very useful in the combat arena, both for air-to-air and air-to-ground combat.

Pitch pointing involves equal changes in the angle of attack and the pitch angle, resulting in the flight path angle $\gamma(\theta-\infty)$ remaining constant. Therefore, the nose pitches while the aircraft remains on the current trajectory. Note that using the euler pitch angle implies that the aircraft has zero initial roll angle. To generalize this maneuver for any initial attitude, the integral of pitch rate could be used instead of $\theta$. The change in velocity is commanded to zero.

Vertical translation results in the alrcraft moving in the vertical direction (along $z$ axis) with no change in the pitch angle. Consequently, the pitch angle change is commanded to zero, as is the change in velocity. The angle of attack is commanded so as to actually command a change in $w$, the velocity in the $z$ direction. Since
$\alpha=w / U$
for small angles of attack, changes in $\alpha$ equate to changes in w (and thus commanded $g$ forces in the $z d i r e c t i o n)$.

Previous thesis students have called the third maneuver a direct lift, even though direct lift has typically been used to describe a different maneuver while the constant g pull up has
been used to describe this last maneuver. This thesis will follow previous theses in calling this maneuver the direct lift. To command this maneuver, equation (50) can be rewritten as

$$
\begin{equation*}
A z=q^{*} U \tag{72}
\end{equation*}
$$

from which the pitch angle command necessary to generate the desired acceleration in the z direction (Az) can be found. Both the change in $u$ and angle of attack are commanded to zero, so that the flight path angle follows the pitch angle.

Longitudinal Model. Just as with the lateral controller, modifications to the data generated by the FPCC simulation program are required. The modifications made to equations (48) and (49) in this chapter apply to the 0.15 mach fifghtondition, but the same changes are also made to the data for the other flight conditions (Appendix D).

The longitudinal states usedin equation (48) are q, w, u, $\theta$, $h$, and $x$. Just as with the lateral model, an analysis of the eigenvalues of the A matrix can be used to reduce the system state model. The eigenvalues of the original six state matrix are:

```
-0.02010 土 f0.2034 (phugoid roots)
1.166 (unstable short period root)
-1.887 (stable short period root)
0.0001981
-0.001131
```

Since this aircraft is longitudinally unstable, one of the short period roots has migrated over into the right half slane. The two roots near zero supply no essential information and so can be
removed from the model. Once again from experience the decision is made to eliminate the two states $h$ and $x$ yielding the following eigenvalues:
$-0.02059 \pm j 0.2040$
1.166
$-1.887$

So the plant model has been reduced to four state, q, w, u, and $\theta$ with no loss of essential aircraft characteristics.

From equation (49) there are seven longitudinal control inputs, $\delta c, \delta_{j}, \delta_{m f}, F_{1}, F_{2}, C D I_{1}$, and $C D I_{\alpha}$. However, for this particular flight condition there is a column of zeros in the matrix corresponding to the maneuver flaps, implying they have no effect upon the aircraft in this configuration. of the remaining six control inputs, the two engine thrusts can be combined into one input called frot, by simply adding those two columns of the B matrix together. Once again the contributions of the terms in the equations due to $C D I_{,}$and $C D I_{2}$ are minimal and can be dropped. This leaves just three control inputs, $\delta c, \delta j$, and $F_{\text {tor }}$.

Since there are three inputs, there can be at most three outputs. The outputs chosen are the same as Bauschlicher used except for thrust. These outputs are $\theta$, $u$, and $\alpha$. Since angle of attack isn't one of the states, it is generated as an output by assuming that $\alpha$ is equal to $w / U$. This means that in each $C$ matrix used for this thesis this entry is the reciprocal of the airspeed at trin.

The relationship between the input command to the thrust and the engine thrust is modeled as an actuator in this thesis. This is preferable to inbedding the engine thrust into the plant A
matrix. Imbedding the thrust (or any other actuator) means that the model assumes instantaneous aircraft response to the change in thrust (or fight control surface), whereas keeping the actuator dynamics separate and using option \#4 in MULTI means the model incorporates a (more realistic) one sampling period time delay between actuator movement and aircraft response.

With these new states, inputs, and outputs the longitudinal model used in this thesis is expressed by the following equations:
$\left[\begin{array}{l}\dot{\theta} \\ \dot{u} \\ \dot{w} \\ \dot{q}\end{array}\right]=\left[\begin{array}{llll}0 & 0 & 0 & 1 \\ -31.6 & -.0214 & .0867 & -31.5 \\ -6.01 & -.215 & -.638 & 166 \\ 0 & -.000156 & .0134 & -.103\end{array}\right]\left[\begin{array}{l}\theta \\ u \\ w \\ q\end{array}\right]+$

An interesting aside is that the system above yielded no transmission zeros and no decoupling zeros.

The above equations represent the longitudinal model for this aircraft at 0.15 mach, sea level. This system is checked (with $E$ substituted for C) for transmision zeros and none are found in the right half plane. No decoupling zeros are found,

Indicating that the system is controllable and observable.
Since the matrix C $\underset{\text { B }}{ }$ does not have full rank l, the system is irregular and the measurement matrix must be found. The concept behind the ir egular design is explained in Chapter II.

Measurement Matrix Development. Since $n-1$ is one, the M matrix is once again $3 x 1$, this time measuring the derivative of the pitch angle. The first step is to find $\boldsymbol{B H}^{*}$, which can be derived from:

$$
\begin{align*}
& \underline{C}_{1}=\left[\begin{array}{l}
1 \\
0 \\
0
\end{array}\right]  \tag{75}\\
& \underline{A}_{11}=\left[\begin{array}{l}
0
\end{array}\right]  \tag{76}\\
& {\underset{A}{12}}=\left[\begin{array}{lll}
0 & 0 & 1
\end{array}\right] \tag{77}
\end{align*}
$$

yielding

$$
\underline{B}^{*}=\left[\begin{array}{lll}
0 & 0 & 1  \tag{78}\\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right]
$$

with

$$
\begin{equation*}
\left\{d_{1}, d_{2}, d_{3}\right\}=\{0,3,3\} \tag{79}
\end{equation*}
$$

Using the general form for $M$ :

$$
\underline{M}=\left[\begin{array}{l}
m_{1}  \tag{80}\\
m_{2} \\
m_{3}
\end{array}\right]
$$

and

$$
\underline{\mathrm{c}}_{2}=\left[\begin{array}{lll}
0 & 0 & 0  \tag{81}\\
1 & 0 & 0 \\
0 & .00607 & 0
\end{array}\right]
$$

then

$$
\underline{F}_{\alpha}=\left[\begin{array}{lll}
0 & 0 & m_{1}  \tag{82}\\
1 & 0 & m_{\bar{\alpha}} \\
0 & .00607 & m_{3}
\end{array}\right]
$$

For $E_{2}$ to have full rank, yet be as sparse as posibile, and using the guidelines of Chapter II in choosing the M elements based upon B*, the obvious choice for M is: $^{\text {m }}$

$$
\underline{M}=\left[\begin{array}{l}
m_{1}  \tag{83}\\
0 \\
0
\end{array}\right]
$$


which gields

$$
\underline{F}_{2}=\left[\begin{array}{lll}
0 & 0 & m_{1}  \tag{84}\\
1 & 0 & 0 \\
0 & .00607 & 0
\end{array}\right]
$$

The asymptotic transfer function matrix can be derived as in the lateral model earlier in this chapter and is found to be:

$$
\Gamma=\left[\begin{array}{lll}
\left(T / m_{1}\right) /\left(\lambda-1+T / m_{1}\right) & 0 & 0  \tag{86}\\
0 & \sigma_{2} /\left(\lambda-1+\sigma_{2}\right) & 0 \\
0 & 0 & \sigma_{3} /\left(\lambda-1+\sigma_{3}\right)
\end{array}\right]
$$

From equation (86) the transmission zero is

$$
\begin{equation*}
z_{t}=1-T / m_{1} \tag{87}
\end{equation*}
$$

Equation (86) shows that the outputs become decoupled as the gain goes to infinity (or sampling time $T$ goes to zero). Also, $m_{1}, \sigma_{2}$, and $\sigma_{3}$ have the greatest impact on the output time responses.

Controller Design. The same algorithm listed under lateral controller design is used for the longitudinal controller design. Actually, this algorithm is used for every controller design for this thesis. The choice of one set of controller matrices for all three maneuvers for each flight condition requires compromises in the performance of each maneuver. The design values

$$
\begin{align*}
& \text { used are listed below. } \\
& \underline{\Sigma}=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0.025
\end{array}\right]  \tag{88}\\
& \alpha=2  \tag{90}\\
& \epsilon=0.1, \quad T=0.01  \tag{91}\\
& \underline{M}=\left[\begin{array}{l}
0.25 \\
0 \\
0
\end{array}\right]
\end{align*}
$$

with

$$
\begin{align*}
& \underline{K}_{0}=\left[\begin{array}{lll}
16.856 & .00589 & -1.340 \\
-18.518 & -.564 & -5.0981 \\
427.964 & 89.785 & -100.0318
\end{array}\right]  \tag{92}\\
& \underline{K}_{1}=\left[\begin{array}{lll}
8.428 & .00295 & -.670 \\
-9.259 & -.282 & -2.549 \\
213.982 & 44.893 & -50.0159
\end{array}\right] \tag{93}
\end{align*}
$$

Appendix $D$ lists all of the design parameters for the other flight conditions.

Figures 12 through 20 show the time responses for the outputs and control inputs for the three longitudinal maneuvers at 0.15 mach, sea level. Figures 21 through 39 show the same time responses for the same maneuvers at the other two flight



1. 75 DEGREE PITCH POINTING (O. $15 \mathrm{MACH} . \mathrm{C} F \mathrm{~F}$ )

Figure 12. 1.75 Degree Pitch Pointing (0.15 Mach)



Figure 14. 1.75 Degree Pitch Pointing (0.15 Mach)

0.5 G DIRECT LIFT ( 0.15 MACH. O FT)

Figure 15. 0.5 g Direct Lift ( 0.15 Mach)



0.5 OIRECT LIFT ( 0.15 MACH . O FT)

Figure 17. 0.5 g Direct Lift (0.15 Mach)


O.5 G VERTICAL TRANSLATION (0.15 MACH. O FT)

Figure 20. 0.5 gertical Translation (0.15 Mach)

1.75 DEGREE PITCH POINTING (O.6 MACH. O FT)

Figure 21. 1.75 Degree Pitch Pointing (0.6 Mach)



1. 75 DEGREE PETCH PO!NTING 0.6 MACH . O FTI

Figure 22. 1.75 Degree Pitch Pointing ( 0.6 Mach)



1. 75 DEGREE D!TSH PO!NT!NG 10.5 MACH. C FT!

Figure 23. 1.75 Degree Pitch Pointing (0.6 Mach)



!. 3 G D!PECT LIFT ( $0.6 \mathrm{MACH} . \mathrm{CFT}$
0.8 G VERTICAL TRANSLATION $(0.6$ MACH. O FT)

Figure 27. 0.8 g Vertical Translation ( 0.6 Mach)
C.B G VERT!CAL TRANSLATION (O. 5 MAC4. C FT)

O.E G VERT!CAL TPANSLATION :O.S MACH. G FTI

Figure 28. 0.8 gertical Translation (0.6 Mach)



Figure 29.



3 DEGREE PITCH PO!NTING (O.9 MACH. O ETI
$\square$
3 Degree pitch pointing (0.9 mach)
Figure 30. 3 Degree Pitch Pointing ( 0.9 Mach)



3 DEGREE PITCH POINTING (O. Q MACH. O FT)
Figure 31. 3 Degree Pitch Pointing ( 0.9 Mach)





( G VERTICAL TRANSLATION (O.9 MACH)


1 G VERTICAL TRANELAT!ON (0.9 MACH)
Figure 38. 1 g Vertical Translation (0.9 Mach)

conditions, 0.6 mach, sea level, and 0.9 mach, $30,000 \mathrm{ft}$. Table VIII shows the figures of merit from these time responses along with the commanded inputs for each maneuver.

Controller Response Analysis
Lateral Controller Response Analysis. Figure 5 shows the side velocity response for the horizontal translation maneuver, the only non-zero commanded input. Obviousiy the response does not follow the command, either in the transient stage or in the steady state (defined here as ten seconds after initial command input). Experimenting with other commanded g levels resulted in similar tise responses of the side velocity. optimizing the controller for just this one maneuver did not significanty improve the response. This may be due to the presence of alow mode, due to a transmission zero, in the output.

Figure 7 shows the time response of the yaw rate for the flat turn, the only non-zero commanded input. For this flight condition, this is the worst responding maneuver of the three. Apparenty the aircraft tracked the commanded yaw rate fairly well until one second, when the command was leveled off to a constant value. At this time the yaw rate falls off rapidiy while the roll angle increases as does the side velocity, until in the steady state the aircraft is turning and sliding sideways through the airatream.

Figure 8 shows the control surface deflections, indicating that the control surfaces are being taxed due to the slow airsped. The vertical canard runs immediately to full deflection, and the ailerons are run from full deflection in one
direction to full deflection in the other in just one second.

Longitudinal Controller Response Analysis. For the 0.15 mach flight condition, Figure 12 shows the output responses for the 1.75 degree pitch pointing maneuver. Although the transient response is fairly acceptable, the steady state response is probably not, with alpha less than theta in magnitude. This results in a maximum flight path angle of about 0.9 degrees instead of the commanded zero degrees.

Figure 13 shows the oscillations in thrust and jet flap deflection, which have the same frequency as the oscillations in forward velocity. Here is the first indication that the jet flap is not going to work as well as the maneuver flap did for Bauschlicher in performing as an elevator. The problem is that deflecting the fet flap to induce a pitching moment also deflects the thrust and affects the forward velocity. The horizontal canard must deflect both to control any pitching moment from jet flap deflection and also to control changes in lift due to deflection of thrust (and consequent changes in velocity). The deflection rates of the horizontal canards and jet flaps present the least desirable (and possibly obtainable) aspects of this maneuver.

Figure 16 indicates that the aircraft response to the direct lift maneuver followed the commanded inputs quite well. However, the 0.5 commanded was about the maximum possible using this controller. Even optimizing the controller for this maneuver did not add significantiy to the aircraft performance. The apparent instability at the end of the simulation time probably results
from either "wind up" or from an induced instability in the controller due to ag in the integration routine. Appendix $C$ contains more detail concerning this problem.

Another possible problem is the initial jet flap deflection needed to trim the aircraft at 0.15 mach, sea level. Obviously this adds complications not present with zero initial jet flap deflection, such as in Bauschlicheres thesis.

Figure 18 shows the output responses for the $y$ tical translation maneuver at 0.15 mach, sea level. The out sesultin a lower grating than commanded, butitis stil a vertical translation with almost no change in pitch angle. -ace again an instability occurs at the end of the simulation. Appendix $C$ contains more information on the causes and cures for this instability.

From the pitch pointing and vertical translation maneuvers, one conclusion is that the alpha response does not track commands as well as does the theta response.

## Combined Lateral and Longitudinal Controller

Development. The last section of this chapter shows how the longitudinal and lateral models can be combined along with the lateral and longitudinal controllers. Since the system matrix for the 0.15 mach, sea levelfight condition is decoupled (equation (48)), as is the $\underline{B}$ matrix after dropping the colums corresponding to $C D I_{1}$ and $C D I_{2}$, combining the separate models together poses no problems. The original columns for each thrust input in the $B$ matrix are added together, just as for the longitudinal model. Combining the lateral and longitudinal

## $\therefore$ system equations into one set of system equations yields: <br> 0

$$
\sqrt{0}=4040, x \cdot E \cdot \theta \cdot \omega,
$$

1101010
0010101
$+\left[\begin{array}{lllllll}0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -.0765 & -.0211 & .0021 & 0 & 0 & 0 \\ -.111 & -.118 & -.000734 & 0 & 0 & 0 \\ .039 & -.00908 & -.0000596 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .111 & .0363 \\ 0 & 0 & 0 & 0.00114 & -.0112 & .0022 \\ 0 & 0 & 0 & 0122 & .0118 & -.00212\end{array}\right]\left[\begin{array}{l}\delta_{c} \\ \delta_{j} \\ F \\ \delta_{a} \\ \delta_{r} \\ \delta_{v c}\end{array}\right]$
$\left[\begin{array}{l}\theta \\ u \\ \alpha \\ \phi \\ r\end{array}\right]=\left[\begin{array}{llllllll}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .00607 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\end{array}\right]\left[\begin{array}{l}\theta \\ \phi \\ u \\ w \\ q \\ \mathbf{v} \\ \mathbf{r} \\ p\end{array}\right]$

The combined system shown in equation (94) is both observable and controllable, and has no transmission zeros in the right half s plane. After substituting the $\underset{\sim}{\text { fatrix for the }}$ matrix the transmision zeros are found to be in the left half plane and are:

$$
\begin{equation*}
z_{t}=\left\{1-T / m_{1}, 1-T / m_{2}\right\} \tag{95}
\end{equation*}
$$

with

$$
\underline{M}=\left[\begin{array}{ll}
.25 & 0  \tag{96}\\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & .25 \\
0 & 0
\end{array}\right]
$$

The $\Gamma, \underline{\Sigma}, \underline{K}_{0}$, and $\underline{K}_{\text {, }}$ matrices are just the block diagonal combinations of the corresponding matrices for the individual designs with the off block diagonal terms equal to zero. However, the $\alpha$ scaler has now become a matrix and is given by:

$$
\alpha=\left[\begin{array}{llllll}
2 & 0 & 0 & 0 & 0 & 0  \tag{97}\\
0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
$$

To demonstrate the decoupling of the combined system, the separate lateral inputs for the horizontal translation and the longitudinal inputs for the vertical transiation are combined to command a jink maneuver. As can be seen in figures 40 through 43, the responses to the combined jink maneuver are nothing more



JINK MRNEUVER ( 0.15 MACH, O FTI
Figure 40. Jink Maneuver (0.15 Mach)

JINK MANEUVER ( 0.15 MACH, 0 FT)


JINK MRNEUVER $10.15 \mathrm{MACH}, \mathrm{O}$ FTI
Figure 41 . Jink Maneuver ( 0.15 Mach)



Jink maneuver (0.15 Mach. O fT)
Figure 42. Jink Maneuver ( 0.15 Mach )

SUMMARY OF OUTPUT RESPONSES FOR LONGITUDINAL CONTROLLERS

| Flt. Cond. | Maneuver | Command, V | Overshoot | Rise Time |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 0.15 \mathrm{Mach} \\ & 0 \mathrm{ft} . \end{aligned}$ | Pitch | .5,.0305,20,20 | 4\% | . 5 sec |
|  | Pointing | 0,0,0,0 | -. $385 \mathrm{ft} / \mathrm{sec}$ |  |
|  |  | .5,.0305,20,20 | 3\% | . 75 sec |
|  | Direct | 10,.976, 20, 20 | none | 9.9 sec |
|  | Lift | 0,0,0,0 | -.861 ft/sec |  |
|  |  | 0,0,0,0 | . 07 rad |  |
|  | Vertical | 0,0,0,0 | . 009 rad |  |
|  | Transl. | 0,0,0,0 | $.323 \mathrm{ft} / \mathrm{sec}$ |  |
|  |  | 1,.0977,20,20 |  | 9 sec |
| $\begin{aligned} & 0.6 \mathrm{Mach} \\ & 0 \mathrm{ft} . \end{aligned}$ | Pitch | .5,.0305, 20, 20 | 48\% | . 8 sec |
|  | Pointing | 0,0,0,0 | . $348 \mathrm{ft} / \mathrm{sec}$ |  |
|  |  | .5,.0305, 20,20 | none | ** |
|  | Direct | 10,.865,20,20 | none | ** |
|  | Lift | 0,0,0,0 | -. 324 |  |
|  |  | $0,0,0,0$ | . 0132 rad |  |
|  | Vertical | 0,0,0,0 | . 0239 rad |  |
|  | Transl. | 0,0,0,0 | . $339 \mathrm{ft} / \mathrm{sec}$ |  |
|  |  | 1,.0384, 20, 20 | none | 9.1 sec |
| $\begin{aligned} & 0.9 \mathrm{Mach} \\ & 30,000 \mathrm{ft} . \end{aligned}$ | Pitch | .5,.0524, 20,20 | 12.2\% | 1.5 sec |
|  | Pointing | $0,0,0,0$ | -1.39 ft/sec |  |
|  |  | . 5,.0524, 20, 20 | none | ** |
|  | Direct | 10,.865, 20, 20 | none | 10 sec |
|  | Lift | 0,0,0,0 | -3.74 |  |
|  |  | 0,0,0,0 | . 0416 rad |  |
|  | Vertical | 0,0,0,0 | -. 00459 rad |  |
|  | Transl. | 0,0,0,0 | -. $283 \mathrm{ft/sec}$ |  |
|  |  | 1,-.036,20,20 | none | ** |

Note: Overshoot is the first peak for non zero commanded outputs and is the greatest deviation from zero for zero commanded outputs. Rise time is from initial value to $90 \%$ of the commanded value. $T$ Command vector is $[\theta$ u $\alpha]$.

* For this maneuver the shape of the curve did not yield an obvious value for overshoot.
** For this maneuver the output did not reach $90 \%$ of the commaded value.


JINK MANEUVER (0.15 MACH, O FT)
Figure 43. Jink Maneuver ( 0.15 Mach, 0 ft.)
than the responses to the vertical and horizontal transiation commands (see Figures 5 through 11 and 12 through 20). However, if the aircraft were not decoupled, then this would not be true and the response to coupled maneuver would be different than that obtained by commanding the lateral and longitudinal inputs separately. In this case the controller may have to be modified to compensate for coupling in the system matrices.

## Conclusions

Not unexpectedly, the alrcraft response to the commanded inputs is generally better for the lateral design than for the longitudinal designs. This may be due to the use of the jet flaps to induce a pitching moment (instead of maneuver flaps) which also vectors the thrust. Obviously, having one control surface affect two control inputs is not as efficient as separating the control inputs. Comparing these results to Bauschlicher's (using maneuver flaps), the conclusion is that using the maneuver flaps for longitudinal maneuvers yields significantly increased aircraft performance.

Designing one controller laterally and longitudinally for each flight condition that functioned nearly as well as separate controllers designed for each maneuver further demonstrates the power of the Porter method, and is an improvement over the current practice of gain scheduling by maneuver. Extending this concept further to one controller for multiplefight conditions is examined in the following chapter.
ROBUSTNESS TESTING OF LONGITUDINAL CONTROLLER
AND
COMPENSATION FOR EFFECTS OF CONTROLLER DELAY

## Introduction

This chapter details the design and tesing of aniversal controller" that can be used for multiplefight conditions. The obvious advantage to uch a robust controlier is that the controller matrices gains do not have to be changed as often as if this robustness was not possible, a practice called gain scheduling. Any valid clains for robustass using the Porter method would have to be substantiated with more complete robustness testing than is posible for this thesis. The attempt here is fust to demonstrate that some robustiess does exist and how to test robustness. This chapter also details the effects of a one sampling period time delayin the output cf the PI controller. This time delay represents the time delay that would actually exist in the hardware realization of the design due to the sampe and hold devices (A/D converters) and the processing delay. Another aspect of gain reduction is finding certain gains within the controller matrices that can be set to zero for wultiple flight conditions. Unfortunately, this approach did not Yield the hoped for results with this aircraft, but the limited resulta achieved are summarized in this chapter.

## Universal Controller

The approach used for this thesis in searching for a universal controller was to test the controllers for each fight condition
the 0.9 mach, 30,000 ft. filght condition. The same maneuvers from Chapter IV are used and consist of:

Pitch Pointing
Direct Lift
Vertical Translation
Figures 44 through 52 show the results of these maneuvers, along with the commanded inputs and control surface deflections. Note that the title for each plot uses the abbreviation "U" to designate universal controller testing. Table IX summarizes the results of these plots.

Results. Generally, the results of the responses for the universal controller are fairly good. This section details the resulte for each maneuver.

Figures 44 through 46 show the responses for the 1.75 degree pitch pointing command. Comparing these plots with the ones for the responses of the aircraft with the controller designed for this flight condition (Figures 21 through 23), the worst feature of this maneuver is the decreased tracking of the angle of attack. However, there is less overshoot in the theta response but with greater error after about five seconds. The deviation of the flight path angle is greater with the universal controller than without. The velocity deviation from zero is greater with the universal controller, but the frequency of oscillation is much less, and the damping is greater. The control surface deflections aren't much different with the universal controller, and there is a significant decrease in both the required magnitude of thrust and its rate of oscillation.

Making the same comparisons of the universal controller

1.75 DEGREE PITCH POINTING (U. O.6 MACH. O FT)

1.75 DEGREE PITCH POINTING 10.6 MACH. O FTI

Figure 44. 1.75 Degree Pitch Pointing With Universal Controller (0.6 Mach, 0 Ft.)


1. 75 DEGREE PITCH POINTING (U. O. 6 MACH. O FT)

1.75 DEGREE PITCH POINTING (U. O. 6 MRCH. O FTI

Pigure 45. 1.75 Degree Pitch Pointing With Universal Controller (0.6 Mach, 0 Ft.)

1.75 DEGREES PITCH POINTING (U. 0.6 MACH .0 FT$)$

Figure 46. 1.75 Degree Pitch Pointing With Universal Controller (0.6 Mach, 0 Ft.)

1.8 G DIRECT LIFT ( $0.6 \mathrm{MACH}, \mathrm{OFT}$ )

Figure 47. 1.8 G Direct Lift With Universal Controller (0.6 Mach, 0 Ft.)



1.8 G DIRECT LIFT (U. $0.6 \mathrm{MACH}, 0 \mathrm{FT}$ )

Figure 49. 1.8 G Direct Lift With Universal Controller (0.6 Mach, 0 Ft.)

O. 8 G VERTICAL TRANSLATION (U. O. 6 MACH . O FT)

0.8 G VERTICAL TRANSLATION ( $0.6 \mathrm{MACH}, \mathrm{O}$ FT)

Figure 50. 0.8 G Vertical Transiation $W i t h$ Universal Controller ( 0.6 Mach, 0 Ft.)

0.8 G VERTICAL TRANSLATION (U. $0.6 \mathrm{MACH}, \mathrm{O} F \mathrm{I}$

O.B G VERTICAL TRANSLATION (U. 0.6 MACH. O FT)

Figure 51. 0.8 G Vertical Translation With Universal Controller (0.6 Mach, 0 Ft.)

TABLE IX

SUMMARY OF OUTPUT RESPONSES FOR UNIVERSAL CONTROLLER
Flt. Cond. Maneuver Command, V
0.60 Mach

Pitch .5,.0305,20,20
$\begin{array}{ll}\text { Pointing } \quad 0,0,0,0 \\ & .5, .0305,20,20\end{array}$
Peak Time To
Final
Value

0 ft .

| Direct | $10, .865,20,20$ | .776 | 10 sec | .776 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Lift | $0,0,0,0$ | -3.93 | 9.6 sec | -3.83 |
|  | $0,0,0,0$ | .0237. | 1.9 sec | .0168 |
|  |  |  |  |  |
| Vertical | $0,0,0,0$ | .00828 | 2.4 sec | .00597 |
| Transl. | $0,0,0,0$ | .403 | 5 sec | .203 |
|  | $1, .0384,20,20$ | .0189 | 10 sec | .0189 |

Note: The peak value is the greatest deviation from the initial value of zero. The command vector is composed of:
$\underline{V}=\left[\begin{array}{ll}\theta & (r a d) \\ u & (f t / s e c) \\ \alpha & (r a d)\end{array}\right]$
6

O.8 G VERTICAL TRANSLATION (U. O. 6 MACH. O FT)

Figure 52. 0.8 G Vertical Translation With Universal Controller (0.6 Mach, 0 Ft.)
response (Figures 47 through 49) with the responses using the controller designed for this flight condition (figures 24 through 26) for a 1.8 g direct lift maneuver, several conclusions can be drawn. The theta response with the universal controller is practically identical to the dedicated controller response, but once again the velocity response has greater magnitude of deviation but with less frequency of oscillation. The angle of attack response is also almost identical for both controllers. The control surface deflections along with the thrust required are not too dissimilar. Of the three maneuvers, this is the one with the best response using the unviversal controller.

Comparing the responses of the universal controller to a 0.8 g vertical transiation (Figures 50 through 52) with those using the dedicated controller (figures 27 through 29) yields the worst performance for the universal controller. Even though the theta response has improved, the angle of attack response has significantly degraded. Once again the forward velocity response with the universal controller exhibits greater deviation from the commanded zero level but has greater damping and lower frequency. And as with the other maneuvers the control surface deflections have not changed much, although the thrust, in addition to being smaller in magnitude, also has a marked change in the basic shape of the time response plot.

Conclusions. Even though the controller from the 0.9 mach fifght condition proved to yield the best results of any of the controllers tested in the search for a universal controller, its performance is probably not adequate enough for a fighter
aircraft, such as the FPCC. However, there is some degree of robustness to the universal controller, and perhaps this type of approach would yield an adequate universal controller for a transport type aircraft.

## Effects of Controller Delay

This section details the testing of the effects of introducing a one sampling period time delay into the output of the PI controller. This delay represents a real world delay that would exist in the output of a digital controller using a sampe and hold device (such as an analog to digital converter) to produce the inputerror signal. This is implemented in the simulation within the program MULTI by simply delaying the output of the controller one sampling period, i.e. the current control input to the actuators (or plant) is the previously calculated value, while the most recently calculated value is saved for the next sampling period (which is the "outer loop" within the multi simulation). MULT also has the capability of allowing the analog plant (the aircraft in this thesis) to continue calculating new states and outputs by setting the calculation step size smaller than the sampling period. Using this feature would certainly make the simulation more true to life and should be considered for future thesis work.

Results. Figures 53 through 79 show the results of performing the longitudinal maneuvers for all of the flight conditions from Chapter $I V$ with a one sampling period time delay in the output of the $P I$ controller. Note that this delay is only valid for the case of a digital controller. Table $x$ ummarize;

1.75 DEGREE PITCH POINTING (D. O. 15 MACH. O FTI

1.75 DEGREE PITCH POINTING 10.15 MACH. O FTI

Figure 53. 1.75 Degree Pitch Pointing $W i t h$ Delay (0.15 Mach, OFt.)

1.75 DEGREE PITCH POINTING $1 \mathrm{D} .0 .15 \mathrm{MACH}, \mathrm{OFT}$

1.75 DEGREE PITCH POINTING (D. O. 15 MACH. G FYI

Figure 54. 1.75 Degree Pitch Pointing With Delay (0.15 Mach, OFt.)


1. 75 Legree pitch pointing (D. O.!5 MRCH. OFT)

Figure 55. 1.75 Degree Pitch Pointing With Delay (0.15 Mach, 0 Ft.)

0.5 G DIRECT LIFT $(0.15 \mathrm{MACH}, \mathrm{O} F \mathrm{~T})$

Figure 56. 0.5 G Direct Lift With Delay (0.15 Mach, Oft.)

O.5 G DIRECT LIFT (D. 0. 15 MACH. O FTI

0.5 G OIRECT LIFT (O. 0.15 MACH. O FT)
Figure 57. 0.5 G Direct Lift With Delay (0.15 Mach, Oft.)


0.5 G DIRECT LIFT ( 0.0 .15 MACH .0 FT )
Figure 58. 0.5 G Direct Lift With Delay (0.15 Mach, Oft.)


O.5 G VERTICAL TRANSLATION (0.15 MACH. O FT)

Figure 59. 0.5 G Vertical Translation With Delay (0.15 Mach, OFt.)

0.5 G VERTICAL TRANSLATION (D. O. 15 MACH .0 FT )

0.5 G VERTICAL TRANSLATION

Figure 60. 0.5 G Vertical Translation With Delay (0.15 Mach, 0 Ft.)

TABLE X
SUMMARY OF OUTPUT RESPONSES FOR LONGITUDINAL CONTROLLERS WITH

## DELAY

| Flt. Cond. | Maneuver | Command, V | Peak Value | $\begin{aligned} & \text { Time To } \\ & \text { Peak } \end{aligned}$ | Final Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 0.15 \mathrm{Mach} \\ & 0 \mathrm{ft} . \end{aligned}$ | Pitch | . 5,.0305, 20, 20 | . 0351 | 1 sec | .0310 |
|  | Pointing | $0,0,0,0$ | -. 685 | 0.7 sec | -. 0497 |
|  |  | . 5,.0305, 20,20 | . 0336 | 0.9 sec | . 0238 |
|  | Direct | 10,.976, 20, 20 | . 950 | 10 sec | . 950 |
|  | Lift | 0, 0, 0, 0 | -1.58 | 0.9 sec | -. 651 |
|  |  | $0,0,0,0$ | . 0924 | 3.4 sec | . 0141 |
|  | Vertical | $0,0,0,0$ | . 00244 | 2.6 sec | .000663 |
|  | Transl. | 0,0,0,0 | . 289 | 1.2 sec | -. 000325 |
|  |  | 1,.0977,20,20 | . 0824 | 10 sec | . 0824 |

(The following flight conditions reflect delay compensation)

| 0.6 Mach | Pitch | .5,.0305, 20, 20 | . 0571 | 1.6 sec | . 0416 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 ft . | Pointing | $0,0,0,0$ | -. 370 | 0.8 sec | . 0893 |
|  |  | . $5, .0305,20,20$ | . 0248 | 1 sec | . 0236 |
|  | Direct | 10,.865,20,20 | . 774 | 10 sec | . 774 |
|  | Lift | 0, 0, 0, 0 | -. 583 | 8.5 sec | -. 540 |
|  |  | $0,0,0,0$ | .0183 | 1.5 sec | . 00723 |
|  | Vertical | $0,0,0,0$ | . 0329 | 2 sec | . 0149 |
|  | Transl. | $0,0,0,0$ | . 242 | 4.1 sec | . 0362 |
|  |  | 1,.0384, 20,20 | . 0299 | 10 sec | . 0299 |
| $\begin{aligned} & 0.9 \mathrm{Mach} \\ & 30,000 \mathrm{ft} . \end{aligned}$ | Pitch | . 5,.0524, 20, 20 | .0635 | 2.1 sec | . 0569 |
|  | Pointing | 0,0,0,0 | -1.516 | 1.9 sec | .110 |
|  |  | . $5, .0524,20,20$ | . 0267 | 1.2 sec | .0153 |
|  | Direct | 10, .865, 20,20 | . 784 | 10 sec | . 784 |
|  | Lift | $0,0, C, 0$ | -5.50 | 9.9 sec | -5.49 |
|  |  | $0,0,0,0$ | . 0459 | 2.4 sec | . 0361 |
|  | Vertical | $0,0,0,0$ | -. 005 | 2.9 sec | -. 00407 |
|  | Transl. | 0, 0, 0, 0 | -. 187 | 4.9 sec | -. 0852 |
|  |  | $1,-.036,20,20$ | -. 0107 | 10 sec | -. 0107 |

Note: The delay compensation is describedin text. The peak value is the greatest deviation from the initial value of zero. The command vector is composed of :
$\underline{v}=\left[\begin{array}{l}\theta(\mathrm{rad}) \\ u(\mathrm{ft} / \mathrm{sec}) \\ \alpha(\mathrm{rad})\end{array}\right]$

TABLE XI

SUMMARY OF OUTPUT RESPONSES FOR UNIVERSAL CONTROLLER WITH DELAY

| F1t. Cond. | Maneuver | Command, V | Peak <br> Value | $\begin{aligned} & \text { Time To } \\ & \text { Peak } \end{aligned}$ | $\begin{aligned} & \text { Final } \\ & \text { Value } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 0.60 \mathrm{Mach} \\ & 0 \mathrm{ft} . \end{aligned}$ | Pitch Pointing | $\begin{aligned} & .5, .0305,20,20 \\ & 0,0,0,0 \\ & .5, .0305,20,20 \end{aligned}$ |  | 2.1 sec <br> 2.3 sec <br> 1 sec | $\begin{aligned} & .0358 \\ & -.0398 \\ & .010 \end{aligned}$ |
|  | Direct Lift | $\begin{aligned} & 10, .865,20,20 \\ & 0,0,0,0 \\ & 0,0,0,0 \end{aligned}$ |  | 10 sec <br> 10 sec <br> 2.1 sec | $\begin{array}{r} .781 \\ -5.60 \\ .0212 \end{array}$ |
|  | Vertical | $0,0,0,0$ | . 00906 | 2.6 sec | . 00749 |
|  | Transl. | 0, 0, 0, 0 | -. 312 | 2 sec | . 0586 |
|  |  | 1,.0384, 20,20 | . 0126 | 10 sec | . 0126 |

Note: Figures shown represent compensated results, with compensation used explained in the text. The peak value is defined as greatest deviation from the initial zero value. The command vector is composed of:
$\underline{V}=\left[\begin{array}{ll}\theta & (r a d) \\ u & (f t / s e c) \\ \alpha & (r a d)\end{array}\right]$

O.5 G VERTICAL TRANSLATION (D. O.!5 MACH. O FT)

Figure 61. 0.5 G Vertical Translation With Delay (0.15 Mach, OFt.)

1.75 DEGREE PITCH POINTING (D. ALPHA=2. 0.6 MACH)

1.75 DEGREE PITCH POINTING (D. ALPHA=2, 0.6 MACH)

Figure 62. 1.75 Degree Pitch Pointing With Delay, Control Ratio Of Two ( 0.6 Mach, 0 Ft.)

1.75 DEGREE PITCH POINTING (D. ALPHA=2. 0.6 MACH)

1.75 DEGREE P[TCH POINTING (D. ALPHA=2. 0.6 MACH)

Figure 63. 1.75 Degree Pitch Pointing With Delay, Control Ratio Of Two ( $0.6 \mathrm{Mach}, 0 \mathrm{Ft}$ )

1.75 DEGREE PITCH POINTING (D. ALPHA=3, 0.6 MACH)

1.75 DEGREE PITCH POINTING (D. ALPHA=3. 0.6 MACH)

Figure 64. 1.75 Degree Pitch Pointing With Delay, Control Ratio Of Three ( 0.6 Mach, 0 Ft.)

1.75 DEGREE PITCH POINTING (D. ALPHA=3. 0.6 MACH)

1.75 DEGREE PITCH POINTING (D. ALPHA=3. 0.6 MACH)

Figure 65. 1.75 Degree Pitch Pointing With Delay, Control Ratio Of Three ( 0.6 Mach, 0 Ft.)



1.8 OIRECT LIFT (D. ALPHA=2. 0.6 MACH)

Figure 67. 1.8 G Direct Lift $W$ ith Delay, Control Ratio of Two (0.6 Mach, O Ft.)

1.8 G DIRECT LIFT (D. ALPHA=3. 0.6 MACH)

1.8 O DIRECT LIFT (D. ALPHA=3. 0.6 MACH)

Figure 68. 1.8 G Direct Lift With Delay, Control Ratio of Three (0.6 Mach, 0 Ft.)


Figure 69. 1.8 G Direct Lift With Delay, Control Ratio of Three (0.6 Mach, 0 Ft.)

0.8 G VERTICAL TRANSLATION (D. ALPHA=2, 0.6 MACH)

0.8 G VERTICAL TRANSLATION (D. ALPHA=2. 0.6 MACH)

Pigure 70. 0.8 G Vertical Translation With Delay, Control Ratio Of Two ( 0.6 Mach, 0 Ft.)

0.8 G VERTICAL TRANSLATION (D. ALPHA=2, 0.6 MACH)

0.80 VERTICAL TRANSLATION (D. ALPHR=2. 0.6 MACH)

Figure 71 . 0.8 Gertical Translation With Delay, Control Ratio Of Two ( $0.6 \mathrm{Mach}, 0 \mathrm{Ft}$.)


O.B G VERT!CAL TRANSLAT!ON (D. QLPHA=3. 0.6 MACH )

Figure 72. 0.8 G Vertical Translation With Delay, Control Ratio
$\quad$ Of Three ( 0.6 Mach, 0 Ft.)

O.B G VERTICAL TRANSLATION ( $0.9 L P H A=3.0 .6 \mathrm{MACH}$ )

O.B G VERTISAL TRANSLATICN (D. ALPHA=3. ©. 6 MACH)

Figure 73. 0.8 G Vertical Translation With Delay, Control Ratio Of Three ( 0.6 Mach, 0 Ft.)



3 DEGREE PITCH POINTING (D. ALPHA=4. 0.9 MACH)



3 Degree pitch pointing (D. alpha $=4,0.9$ MACH)


3 OEGREE PITCH POINTING (D. ALPHA $=4$. 0.9 maCH)
Figure 75. $\quad$ 3 Degree Pitch Pointing With Delay, Control Ratio
Of Four ( 0.9 Mach, 30000 Ft )


2.40 DIRECT LIFT (D. ALPHA=4. 0.9 MACH)

Figure 76. 2.4 G Direct Lift With Delay, Control Ratio of Four (0.9 Mach, $30000 \mathrm{Ft}$. )

$\omega$

2.4 O OIRECT LIFT (D. ALPHA=4. O.9 MACH)

Figure 77. 2.4 G Direct Lift $W$ ith Delay, Control Ratio of four (0.9 Mach, 30000 Ft .)



10 VERTICAL TRANSLATION (D. ALPHA=4, 0.9 MACH)
Figure 78. 1 G Vertical Translation With Delay, Control Ratio Of Four (0.9 Mach, 30000 Ft.)



10 VERTICAL TRANSLATION (D. ALPHA=4. O.9 HACH)
Figure 79. 1 G Vertical Translation With Delay, Control Ratio Of Four ( 0.9 Mach, 30000 Ft.)
these results.
Starting with the pitch pointing maneuver at the 0.15 mach flight condition (Figures 53 through 55), the effects of the delay are most obvious in the angle of attach response. Both theta and alpha remain essentially unchanged in their response during the first 1.5 seconds when compared with the corresponding responses for no delay (Figures 12 through 14). After the inftial l.5 seconds the alpha response falls off considerably. The forward velocity, control surface deflections, and total thrust do not change much with the addition of the delay.

For the direct lift maneuver at 0.15 mach the introduction of the delay causes almost no change in the output responses (Figures 56 through 58) as compared to the same responses with no delay (figures 15 through 17). For the control surfaces the effect of the delay is minimal, as is the effect of the delay upon the total thrust. Interestingly, this maneuver exhibits both the best responses with and without the delay.

Incorporation of the delay into the vertical translation maneuver at 0.15 mach (Figures 59 through 61) does degrade the alpha response significantly, although the theta response is essentially unchanged when compared to the responses without the delay (figures 18 through 20). The forward velocity, control surface deflections, and total thrust responses have not changed much with the added delay.

Figures 62 through 63 show the results of adding the delay to the pitch pointing maneuver for the 0.6 mach, sea level flight condition. The added instability from the delay has caused the
theta and alpha response to become oscillatory, and the forward velocity response has become unstable. The control surface deflections show the oscillatory nature and the total thrust demonstrates instability. Obviousiy these results are unacceptable compared to those without any delay (Figures 21 through 23).

However, if the ratio of proportional to integral controller matrices is increased to three from the original value of two, then most of the effect of the delay can be removed. Figures 64 through 65 show the responses for the pitch pointing maneuver at 0.6 mach, but with this ratio set to three. Compared with the undelayed responses the theta and alpha plots are not changed much, nor is the forward velocity response. The control surface deflections haven't changed much either, but the total thrust response has changed. The combination of the delay and increasing the proportional to integral control ratio has yiflded a much more acceptable thrust response. It's important to point out that increasing the control ratio without the delay does not yield satisfactory results for all of the maneuvers at this flight condition.

The destabilizing effect of the delay is also apparent in the responses for the direct lift maneuver at 0.6 mach (Figures 66 through 67). The theta response has become oscillatory, with the forward velocity once again showing instability. The jet flaps also demonstrate an oscillatory response, with the total thrust apparently unstable. These results would have to be considered unsatisfactory when compared with the undelayed responses (Figures 24 through 26).

Increasing the ratio of proportional to integral control once again compensates for practically all of the deleterious effects of the delay. Figures 68 through 69 show the responses for the direct lift maneuver at 0.6 mach with the control ratio increased to three from the original value of two. The theta, alpha, and forward velocity responses are nearly identical to the undelayed responses, and the control surface deflections are very similar. The thrust response is actually improved, showing less oscillatory nature.

The vertical translation maneuver at 0.6 mach also follows right along with the previous results. Figures 70 through 71 show the responses with the control ratio unchanged but with the addition of the delay. The theta, alpha, and forward velocity curves show unstable oscillations as do the control surface deflections and the total thrust plots. These results are also unsatisfactory when compared to the original responses (figures 27 through 29).

Unfortunately, increasing the control ratio to three from its initial value of two does not have quite the same impact as with the prior maneuvers (Figures 72 and 73). The theta and alpha responses have improved, but not to the point where they coulu be called similar to the undelayed responses. They are not very dissimilar either, but the effects of delay are not as completely compensated for as before. The forward velocity hasn't changed much, nor have the control surface deflections. But once again the thrust response has actually improved. Figures 74 through 79 show the responses for the
longitudinal maneuvers at the 0.9 mach, 30,000 ft. flight condition with attempted compensation thru increasing the ratio of proportional to integral control. These results can be compared to the undelayed responses for this flight condition (Figures 30 through 39). The pitch pointing maneuver is compensted very well, with an actual improvement in the forward velocity and thrust responses. The direct lift maneuver is also well compensated, but without any obvious improvement in the thrust or forward velocity responses. The vertical transiation also is well compensated without any marked difference in any of the responses. Unfortunately, this last maneuver is one of the worst responding ones, even undelayed.

Conclusions. Due to time constraints in preparing the results of the effects of and compensating for time delay, all of the longitudinal maneuvers were tested instead of the lateral maneuvers at the 0.15 mach flight condition. This was done so that tentative conclusions could be drawn regarding the effect of delay on a $u$ iversal controller.

Obviously, introducing a one sampling period time delay into the output of the $P I$ controller introduces oscillations and instabilities into the time responses. The effect of the delay seems to increase with increasing speed and altitude, although this is a tenuous conclusion at best. For this aircraft, compensating for this time delay is effectively achieved through increasing the ratio of proportional to integral control in the PI controller matrices. Time limitations precluded rigorous searching for other methods of compensating for this delay. One of the most interesting results of compensating for the delay is
an improvement of the velocity and/or thrust responses for some of the maneuvers. Overall, this method of compensation seems very promising.

Universal Controller Incorporating Delay
Extending these results to compensating a universal type controller, the logical conclusion would be that increasing the ratio of proportional to integral control should work, with the actual increase probably only found experimentally. To test this hypothesis, the universal controller discussed at the beginning of this chapter is tested by introducing the time delay. Figures 80 through 85 show the time responses for the universal controller using the longitudinal maneuvers at the 0.6 mach flight condition. Limited experimentation showed that a proportional to integral control ratio of four was close to the best value, with increasing ratios degrading the angle of attack response and decreasing ratios not compensating the delay enough. This is the same ratio used for the 0.9 mach fiight condition, further reinforcing the choice of the design parameters from the 0.9 mach filght condition for the universal controller. Table XI summarizes the results.

Results. Figures 80 through 81 show the time responses of the universal controller to the 1.75 degree pitch pointing command. Comparing these to the results for the undelayed responses (Figures 44 through 46 ) shows no major changes in the time responses. The theta response has more overshoot, while the angle of attack shows greater error at the end of ten seconds. The cransient response of alpha is almost unchanged. The forward


3 DEGREE PITCH POINTING (U. D. ALPHA=4. 0.6 M$)$


3 DEGREE PITCH POINTING (U. D. ALPHA=4, 0.6 M)



3 DEGREE PITCH POINTING (U, D. ALPHA=4, 0.6 M)


3 DEGREE PITCH POINTING (U. D. PLPHA=4. 0.6 MACH)

Figure 81. 1.75 Degree Pitch Pointing With Delay, Universal Controller ( 0.6 Mach, 0 ft )


Figure 82. 1.8 G Direct Lift With Delay, Universal Controller (0.6 Mach, 0 ft.)


2.4 G DIRECT LIFT (U. D. ALPHA=4, 0.6 M)

Figure 83. 1.8 G Direct Lift With Delay, Universal Controller (0.6 Mach, 0 ft.)


1 G VERTICAL TRANSLATION (U. D. ALPHA=4. 0.6 M)


1 G VERTICAL TRANSLATION (U, D. ALPHA=4, 0.6 M)
Figure 84. $\begin{aligned} & 0.8 \text { G Vertical Translation With Delay, Universal } \\ & \text { Controller (0.6 Mach, } 0 \text { ft.) }\end{aligned}$


1 G VERTICAL TRANSLRTION (U. D. RLPHA=4. 0.6 M)
10


1 G VERTICAL TRANSLATION (U. D. ALPHA=4. 0.6 M)
Figure 85. 0.8 G Vertical Translation With Delay, Universal Controller ( $0.6 \mathrm{Mach}, 0 \mathrm{ft}$ )
velocity shows increased damping but with about the same magnitude of deviation from zero. The control surface deflection for the horizontal canards hasn't changed much, while the jet flaps are not deflected as much. Total thrust requirements are reduced slightly.

From Figures 82 through 83 comparisons can be made with the undelayed responses (Figures 47 through 49) for the 1.8 g direct lift command. Once again, the responses have changed little with the introduction of the delay and subsequent compensation for the delay. The theta and alpha responses are practically identical, with only minor changes in the forward velocity response. As with the pitch pointing command, the canard deflection has not changed much, and the jet flaps are deflected less. The total thrust responses are very similar.

The final maneuver, the 0.8 g vertical translation (Figures 84 and 85 ), yields the most dissimilar results. The alpha response changes quickly after the first second, and after ten seconds is much less than for the undelayed response. The theta response shows more overshoot, but otherwise is very similar. Although the magnitude of the forward velocity response is about the same, the damping is much greater for the compensated delayed plot. Just as with the other maneuvers the canard deflection is basically unchanged while the jet flaps are deflected less. The thrust requirements are only slightly reduced.

Conclugions. Once again the overriding conclusion is that 1: the maneuver performs well without the delay, then the delay can be effectively compensated for by increasing the ratio of


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proportional to integral control, whereas a maneuver's performance is not going to improve with the inclusion of the delay. As with other results from this thesis, the angle of attack response seems to be the limiting factor in trying to improve aircraft response. This is most evidentin the pitch pointing maneuver where increasing the control ratio decreases the alpha response.


Minimizing Controller Matrices Elements
This section describes the efforts to reduce the number of elements needed in the controller matrices by finding common elements to be set equal to zero. If the same elements in either controller matix can be set to zero for all maneuvers at a given flight condition, then for that flight condition fewer memory locations need to be accessed, increasing computational speed. This is especially true in consideration of the fact that a memory access is one of the slowest microprocessor instructions. The greatest benefit would occur if the same elements could be eliminated for all flight conditions (or perhaps all common flight conditions). Unfortunately, the later case was not true for this thesis effort. The best results obtained were a reduction of two elements from the controller matrices for the longitudinal maneuvers at the 0.15 mach fifght condition, and a reduction of one element for the lateral maneuvers at the same flight condition. This means that a combined controller for the 0.15 mach fiight condition would need three fewer elements. For the other fight conditions no elements were found that could be set to zerofor all of the maneuvers. In fact, for the 0.9 mach
flight condition, there were no elements that could be set to zerofor even two of the maneuvers (which was possible at 0.6 mach). Consequently, the following discussion covers only the controllers for the 0.15 mach flight condition.

Lateral Controller. For the lateral controller, the following matrix is used for the proportional and integral controller matices (which are the same). The zero element represents an element that could be removed from the algorithm used within the PI processor.

$$
\underline{K}_{1}=\underline{K}_{0}=\left[\begin{array}{lll}
-.3823 \mathrm{E}-02 & .2993 \mathrm{E}+02 & 0  \tag{98}\\
.5047 \mathrm{E}-01 & .1904 \mathrm{E}+01 & -.1296 \mathrm{E}+02 \\
.2589 \mathrm{E}+00 & -.5821 \mathrm{E}+01 & .3962 \mathrm{E}+02
\end{array}\right]
$$

Figures 86 through 87 show the responses of the aircraft to the 1.5 ghorizontal translation command using the controller matrix from equation (98). Comparing these to the original responses to this command using the full controller matrices (Figures 5 through 6), yields some interesting observations. First, the side velocity response is almost exactly identical with the original and the yaw rate response has changed very little. However, the roll angle response has improved considerably. The control surface deflections show very similar characteristics also. This means that the maneuver actually improved by eliminating the element in the controller matrices.

From Figures 88 through 89 similar observations are made comparing the results of the reduced controller matrices responses to the 1 fiat turn to the original unmodified responses (Figures 7 through 8). The side velocity and yaw rate responses have changed very little while the roll angle

1.5 G HORIZONTAL TRANSLATION (Z. O.15 MACHI

1.5 G HORIZONTAL TRANSLATION (O. 15 MACH)

Figure 86. 1.5 G Horizontal Translation With Reduced Controller Matrices (0.15 Mach, Oft.)

1.5 G HORIZONTAL TRANSLATION (Z. O. 15 MACH)

Figure 87. 1.5 G Horizontal Translation With Reduced Controller Matrices (0.15 Mach, 0 ft.)


## 1 G FLAT TURN (Z. 0.15 MACH)

Figure 88. 1 G Fiat Turn With Reduced Controller Matrices (0.15 Mach, Oft.)



1 O FLAT TURN (Z. O. 15 MACH)
Figure 89. 1 G Flat Turn $W i t h$ Reduced Controller Matrices (0.15 Mach, Oft.)



3 OEGREE YAW POINTING (Z. 0.15 MACH)
Figure 90. 3 Degree Yaw Pointing With Reduced Controller Matrices (0.15 Mach, Oft.)


3 DEGREE YAW POINTING (Z. 0.15 MACH)

Figure 91. 3 Degree Yaw Pointing With Reduced Controller Matrices ( $0.15 \mathrm{Mach}, 0 \mathrm{ft}$ )

1.75 DEGREE PITCH POINTING (Z. O. 15 MRCH. O FT)

Figure 92. 1.75 Degree Pitch Pointing With Reduced Controller Matrices (0.15 Mach, Oft.)

1.75 DEGREE PITCH POINTING (Z. 0.15 MACH$)$

1.75 DEGREE PITCH POINTING (Z. O. 15 MACH)

Figure 93. 1.75 Degree Pitch Pointing With Reduced Controller Matrices (0.15 Mach, $0 f$ )

1.75 DEGREE PITCH POINTING (Z. 0. 15 MACH)

Figure 94. 1.75 Degree Pitch Pointing With Reduced Controller Matrices ( 0.15 Mach, $0 f t$.


Figure 95. 0.5 G Direct Lift With Reduced Controller Matrices (0.15 Mach, 0 ft.)



Figure 96. 0.5 G Direct Lift With Reduced Controller Matrices (0.15 Mach, 0 ft.)

0.5 G DIRECT LIFT (E. O. 15 MRCH)

Figure 97. 0.5 G Direct Lift With Reduced Controller Matrices (0.15 Mach, Oft.)

0.5 G VERTICAL TRANSLATION (Z. 0. 15 MACH)

Figure 98. 0.5 G Vertical Translation With Reduced Controller Matrices (0.15 Mach, 0 ft.)


## O.5 G VERTICAL TRANSLATION (Z. 0.15 MACH)


0.5 G VERTICAL TRANSLATION (Z. 0. 15 MACH)

Figure 99. 0.5 G Vertical Translation With Reduced Controller Matrices (0.15 Mach, 0 ft.)

O.5 G VERTICAL TRANSLATION (Z. 0.15 MACH)

Figure 100. 0.5 G Vertical Transiation With Reduced Controller Matrices ( $0.15 \mathrm{Mach}, 0 \mathrm{ft}$ )
response has been improved quite dramatically. Once again the control surface deflections have not changed except for their transient response. This maneuver has also been improved merely by introucing a zero element in the controller matrices.

For the final maneuver, the 3 degree yaw pointing, Figures 90 through 91 show that very little changes have occured in any of the responses when compared with the original unmodified curves (figures 9 through 11). In fact, the roll angle response has degraded just slightly.

Since the element set to zero would contribute to the alleron deflection depending on the error in yaw rate, then the aileron deflection would be based more upon roll angle error. This should reduce the roll angle error since roll angle is mainly governed by aileron deflection. The above results certainly seem a logical extension of this analysis. Unfortunately, this author found no apriori method of predicting which elements of the controller matrices could be set to zero based upon this type of analysis.

Longitudinal Controller. The following shows which elements of the controller matrices have been set to zero. Zero elements can be removed from the algorithm that is used within the PI controller to calculate the control inputs to the actuators.
$\underline{K}_{0}=\left[\begin{array}{lll}.1686 \mathrm{E}+02 & 0 & 0 \\ -.1852 \mathrm{E}+02 & -.5640 \mathrm{E}+00 & -.5098 \mathrm{E}+01 \\ .4280 \mathrm{E}+03 & .8979 \mathrm{E}+02 & -.1000 \mathrm{E}+03\end{array}\right]$
$X_{1}=\underline{K}_{0} / 2$
Figures 92 through 94 show the responses of the reduced
element controller to the 1.75 degree pitch pointing command. Comparing these results with the original responses with no modifications (Figures 12 through 14), a couple of observations are possible. The wost striking is that although the responses of theta, alpha, forward velocity, and control surface deflections are almost identical with the original plots, the total thrust is not. In fact, with the introduction of the two zero elements, the maximum required thrust has been reduced by about a factor of four. Why this would be with no apparent changes in any of the other responses sems to have no obvious logical explanation.

Figures 95 through 97 show the responses for the reduced element controller to the 0.5 g direct lift command. Comparing these to the responses for the original unmodified controller (Figures 15 through 17), some observations can be made. Once again the theta and alpha response are essentially unchanged, but now the forward velocity shows a surprising change. Whereas with the original responses the velocity response seemed unstable after about six seconds, now the velocity response shows no such instability. There seems to be no obvious logical reason for this. The control surface deflections seem to have changed very little, but close inspection of the curves from figure 96 shows that they also differ from the original curves after about six seconds. The most obvious difference is the thrust response, which has a marked lack of the oscillatory and posible unstable response of the original response. This maneuver has definitely been improved in the steady-staie response by introducing the two zeros.

Figures 98 through 100 show that the instabilities present in the original responses to the 0.5 gertical translation command (Figures 18 through 20) have not disappeared with the two zero elements in the controller matrices. The theta, alpha, and forward velocity responses haven't changed very much, although the velocity response shows slightly smaller peak values. The control surface deflections show little change, but the thrust response has changed significantly. Unfortunately, this maneuver shows no signs of improved response with the two zero elements, although there appears to be no degradation of the responses.

The two zero elements reduce the contributions to the horizontal canard deflection to just the pitch angle error. Following the train of thought from the lateral case, it would be expected that if anything would be improved it would be the theta response, and yet this did not happen. However, the one zero that corresponds to velocity error contributions to canard deflections is an obvious candidate for elimination. But the other zero, the angle of attackerror effect on canard deflection 1s not such an obvious choice. Also, other seemingly obvious choices for zero elements did not work out.

Conclusions. Even though there seems to be no apriori way to pick the zero elements in the controller matrices, the possible choices will always be finite and can be tested using intelligent software. The expected benefits from doing this include possibly improved responses and the reductions of memory accesses needed in the controller algorithm. These factors would definitely recommend this technique for "polishing" the design.

RESULTS, CONCLUSIONS, AND RECOMMENDATIONS

## Introduction

This thesis incorporates Professor Porter's multivariable design method into the design of flight controllers for three flight conditions for the FPCC aircraft. The first flight condition represents a possible STOL-like operation, while the other two are more representative of normal operation and were also studied by Jon Bauschlicher in a previous thesis (Ref 2). One of the purposes of this thesis was to design longitudinal controllers for these two flight conditions using a different longitudinal model than Baschlicher so as to make comparisons. For the first flight condition, 0.15 mach at sea level, a complete controller is designed by decoupling the aircraft model into a longitudinal and a lateral model. The other two flight conditions, 0.6 mach at sea level, and 0.9 mach at $30,000 \mathrm{ft}$, are decoupled into a lateral and longitudinal model, but only the longitudinal deaign is accomplished because Bauschlicher had already designed controllers for the lateral model. An attempt to find a "universal" controller capable of performing all the maneuvers for all of the flight conditions failed, but the controller for the 0.9 mach fight condition is found to yield marginal results at the 0.6 mach fight condition.

A digital implementation of the PI controller would introduce a delay in the output of the controller, and the effects of this delay are studied for all of the maneuvers at all of the flight conditions. Finally, an attempt to find common


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elements of the controller matrices that can be eliminated is


 partially successful.Additionally, the design tool MULTI, a program based upon Professor Porter"s design method and capable of simulating aircraft response to commanded maneuvers, is upgraded with two more options added.

## Results

Figures 5 through 11 show the time responses for the outputs and the flight control surface deflections for the 0.15 mach flight condition lateral controller. The figures of merit and the commanded inputs for each maneuver are listed in Table VII. One controller was used for all of the maneuvers yielding results that were not much different than if an optimum controller had been used for each maneuver. Of the three maneuvers, the yaw pointing showed the best results, with the horizontal transiation fairly acceptable. The flat turn responses would have to be considered unacceptable.

Figures 12 through 20 show the time responses for the outputs and surface deflections for the three longitudinal maneuvers at 0.15 mach, sea level. Table VIII shows the figures of merit and commanded inputs for the maneuvers. Once again one controller was found for all three maneuvers, and once again the results aremixed. The pitch pointing maneuver is classified fair, with the direct lift considered very good. But the vertical transiation would have to be termed poor.

Figures 21 through 39 show the same curves for the other two flight conditions, 0.6 mach at sea level, and 0.9 mach at 30,000 feet. Table VIII also lists the figures of merit and commanded
inputs for these longitudinal maneuvers. At 0.6 mach, the direct lift maneuver shows the best results, which would have to considered very good. But the pitch pointing maneuver would be called poor, at best, and the vertical transiation is certainly unacceptable. At 0.9 mach the results are about the same, with the direct lift showing very good results but the vertical translation and pitch pointing showing unacceptable results.

The general conclusion regarding the comparison with Bauschlicher's results is that the longitudinal model used in this thesis did not perform nearly as well as the one Bauschlicher used.

An attempt to find a universal controller failed, and the attempt to find a longitudinal controller that would work for two of the three flight conditions yielded marginal results. Figures 44 through 52 show the responses for the 0.6 mach filight condition using the 0.9 mach controller. The direct ift showed very good results, but the other two would have to be considered very poor. Table IX summarizes the results.

All of the longitudinal maneuvers for all three fifght conditions were repeated with a one sampling period time delay added to the output of the PI controller. Figures 53 through 61 show these results for the 0.15 mach fight condition. As can be seen from all of the maneuvers, the addition of the delay introduces instability into the responses.

Figures 62 through 73 show the effects of delay on the longitudinal maneuvers for the 0.6 mach fifght condition both with and without compensation. The compensation is an increase
in the ratio of proportional to integral control within the PI controller. The uncompensated curves show quite dramatically the unstabilizing effect of the delay. From the compensated plots the conclusion is that the delay can be effectively cancelled out.

From Figures 74 through 79 the conclusion is that the delay can also be compensated for in the longitudinal maneuvers at the 0.9 mach flight condition. Table $X$ sumarizes these results for all of the longitudinal maneuvers at the three fight conditions.

Extending this testing to the semi-universal controller yields the results shown in Figures 80 through 85. Once again the conclusion is that the increase of the proportional to integral control ratio effectively compensates for the addition of the delay. Table XI summarizes these results.

The final testing performed is an attempt to reduce the number of gains needed in the controller matrices. Unfortunately, there were no common gains found that could be eliminated from all of the controller matrices for all three flight conditions. In fact, there were no gains that could be eliminated from the matrices for all three maneuvers at either the 0.6 or 0.9 mach flight condition. There were gains that could be removed from both the longitudinal and lateral controllers for the 0.15 mach flight condition. Figures 86 through 100 show that these reduced gain controllers show no degradation in response and even sometimes improve the response.

Conclusions
Summarizing the above results, the conclusions drawn are
these:

1. Removing the maneuver flaps from the longitudinal model definitely degrades the responses to the commanded maneuvers.
2. Adding a delay to the output of the PI controller adds instability to the responses.
3. The effects of the delay can be effectively compensated for by increasing the ratio of proportional to integral contrcl.
4. Setting certain elements of the controller matrices to zero can yield improved responses.

In addition to these conclusions, there are two important results worth summarizing:

1. A universal controller that yielded acceptable results compared to the individual controller responses was not found.
2. The aircraft responses were typically limited by the angle of attack response.

## Recommendations

This section describes the recommendations this author has for future efforts involving both the design method and the design tool MULTI.

Design Method. The following is a list of the suggested areas for future work on the Porter method of controller design:

1. Investigate the transformation from the analog to the discete $P I$ controller more closely. The current equations for the discete controlier implement the integration via a first order rectangular integration approximation. Perhaps reworking these equations to yield more accurate integration approximations would be valuable.
2. More information about the relationship between the transmission zeros and the types of expected results is needed. Although Professor Porter's theory implies that the regular design should not have any limitations in the responses due to the transmision zeros, this author found thatin general an irregular design yielded better results. If irregular designs are (and so far they do seem to be) going to be more prevelant in fifght control design, then insight into how transmision zeros affect responses would certainly be helpful. An example of the type of questions that could be answered is: How do transmission zeros in the right half plane or at the origin affect controller design and expected responses?
3. Since irregular designs have been the most prevelant to date, most of the design work has centered around the use of angles instead of rates as outputs. This author feels more designs incorporating rates (and therefore typically regular designs) should be attempted. Reasons for this suggestion include the two considerations that current pilot controls command rates, and that regular designs have no inherent limitation upon the output responses due to the slow modes (as do irregular designs). Another factor is that for an initial aircraft attitude that is not straight and level, the use of angles is unappropriate.

Design Tool, MULTI. The following is a list of suggestions for improvements to the design tool MULTI:

1. In conjuction with the suggestion for improving the integration approximation in the design method, implement an improved integration routine within the simulation in MULTI.

Appendix $B$ lists the program code for one such improvement.
2. Consideration should be given to implementing the exact discrete differential equations instead of using the current analog integration approximations, accomplished with the system library $0 D E$ (ordinary differential equation solver). This would not be a formidable task because the system marices are all time invariant (constant coefficients). Students receive the background necessary for this type of implementation in courses such as EE 7.12 and EE 6.44. If the simulation were written in this fashion and then segmented as a separate overlay it would be useful as a general purpose discrete simulation of analog systems.
3. The calculation of actuator (or control input) rates should be added to Option \#28 in MULTI, along with a toggle switch that the user can use to specify either modified spline or linear curve fitting on the CALCOMP plots. These two features used together would allow the user to decide when to use linear curve fitti.ig based on curve slopes. The program code to implement these two features is included in Appendix $C$.
4. MULTI needs to be completely rewritten in a language that has both dynamic memory allocation and more structure than FORTRAN. The dynamic memory allocation should allow for larger numbers of states, in addition to being more efficient in general. Using a language that causes program code to have more structure should help prevent what has happened to the current version of MULTI, which is a proliferation of GO TO statements. Because of this unstructured nature of FORTRAN, MULTI has evolved
into a program that is very hard to understand and maintain.
5. The addition of sensors and actuators to MULTI (Options \#4 and \#5) has caused severe problems. Some of these problems were corrected by this author, but a couple still remain. A quick summary of the known remaining problems are (all are within the simulation):
a. Formation of the $\underline{y}$ measurement vector $Y M$ is no longer correct and should be eliminated.
b. The use of $Y M$ to form the error vector $E$ is also no longer correct and the $Y M$ should be replaced with $Q$, which is the output vector incorporating sensors (if present). This is currently causing erroneous data for a regular design that includes sensors.
c. The call to subroutine YOUT results in the actual values of the outputs to be plotted instead of the sensed values if there are sensors. However, this is not clear to either the programmer or the iser. Addtionally, this means that there is no current way to plot the sensed values of the outputs. Something should be done, even if it is just improved documentation of the program code.
6. MULTI should be changed to allow the user to specify both the number and placement of the columns in the matrix. This would permit the designer to chose which state derivatives should be measured (or estimated). Additionally, the provision for an Matrix should be added to the options for a regular design.

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## APPENDIX A

JUSTIFICATION FOR ELIMINATION OF JET FLAPS FROM LATERAL MODEL

## Introduction

This appendix derives the relationship between the jet flaps and the derivatives of the lateral states $p$ and $r$, showing that the jet flaps can be removed from the lateral model used in this thesis. This relationship is expressed by two terms in the $\underline{B}$ matrix from the system equation

$$
\underline{\dot{x}}=\underline{A} \underline{x}+\underline{B} \underline{u}
$$

with $x$ being the state vector. The two terms are in the first and second row, sixth column of the original 12 state system $B$ matrix given in equation (48) from Chapter III. From equation (48) these two numbers are given as:

$$
\begin{aligned}
& .00939 \\
& -.000238
\end{aligned}
$$

This appendix demonstrates that these numbers should both be zero, removing the jet flaps frow the lateral partition of the $B$ matrix.

Derivation (Ref 4)
Starting with the rotation versus moment equations

$$
\begin{aligned}
& \dot{p}=L_{B} / I x x+(n o n ~ l i n e a r ~ t e r m e s) \\
& \dot{r}=N_{B} / I_{z}+(n o n ~ l i n e a r ~ t e r m s)
\end{aligned}
$$

and substituting

$$
\begin{align*}
& L_{B}=L_{S} \cos \alpha-N_{S} \sin \alpha+F_{y_{B}} \Delta z+F z_{B} \Delta y-Y_{\text {engr }} Z_{\text {eng. }}  \tag{A-3}\\
& -Y_{\text {engr }} z_{\text {engr }}+Z_{\text {eng }} Y_{\text {engr }}+Z_{\text {engr }} Y_{\text {eng } 2} \\
& N_{B}=L_{s} \operatorname{in\alpha }+N_{s} \cos \alpha+\mathrm{Fx}_{B} \Delta y+\mathrm{Fy}_{B} \Delta x-X_{e_{n y \prime}} Y_{\text {engin }}  \tag{A-4}\\
& -X_{\text {eng } 2} Y_{\text {engr }}+Y_{\text {engr }} x_{\text {engr }}+Y_{\text {eng } 21} X_{\text {eng } 2}
\end{align*}
$$

where $L_{G}$ and $N_{S}$ represent the aerodynamic moments in stability axes. Pulling out the only terms that are dependent upon jet flap setting:

$$
\begin{aligned}
& F y_{B} \Delta z=0 \\
& F z_{B} \Delta y=0 \\
& F x_{B} \Delta y=0 \\
& F y_{B} \Delta x=0
\end{aligned}
$$

since $\Delta x, \Delta y$, and $\Delta z$, the components of the center of gravity offset from the moment reference point are all set to zero in the FPCC simulation program. Continuing with the terms that are dependent upon jet flap setting, the direct propulsion force terms are:

$$
\begin{aligned}
& X_{e n g 1}^{\text {are: }}=\eta_{F} F_{1} \cos \delta_{j} \cos Y_{\text {eng }}-C D I_{1} 1_{2} \rho V_{T}^{2} S \\
& X_{e n y 2}=\eta_{F} F_{2} \cos \delta_{j} \cos \Psi_{\text {engr }}-\operatorname{cDI} I_{2} \rho V_{T}^{2} S \\
& Y_{\text {enl }}=\eta_{F} F_{1} \cos \delta_{j} \sin Y_{\text {eng }} \\
& Y_{\operatorname{eng} 2}=-\eta_{F} F_{2} \cos \delta_{j} \sin Y_{\text {engr }} \\
& Z_{\text {eng }}=-\eta_{F} F_{1} \sin \delta_{j} \\
& Z_{\text {engr }}=-\eta_{F} F_{2} \sin \delta_{j}
\end{aligned}
$$

Assuming the following equalities (which are true in the FPCC simulation program)

$$
\begin{aligned}
F_{1} & =F_{2} \\
Y_{\text {eng 1 }} & =Y_{\text {eng } 2} \\
C D I_{1} & =C D I_{2} \\
X_{\text {eng 1 }} & =X_{\text {eng 2 }} \\
Y_{\text {eng 1 }} & =Y_{\text {eng } 2}
\end{aligned}
$$

$$
z_{\operatorname{lng} 1}=z_{\operatorname{lng} 2}
$$

with
Peng, = yaw offset of engine one with respect to aircraft

$$
Y_{\operatorname{eng}}=y a w \text { offset of engine two with respect to aircraft }
$$

$$
\text { Mf } \quad \text { thrust adjustment factor (0.98) }
$$

$$
V_{\gamma} \text { scaler total velocity (square root of the sum of the }
$$ squares of the body axis velocities)


$S \quad$ alanform area

Substituting these terns back into equations (A-3) and (A-4) results in neither $L_{5}$ nor $_{5}$ having any dependency on the jet flaps. This means that the jet flap deflection (considering the above stated symmetry and engines of equal thrust) has no effect on yaw or roll rates, a logical conclusion.

Since equations ( $A-3$ ) and ( $A-4$ ) substituted into ( $A-1$ ) and (A-2) show that the derivatives of $p$ and $r$ have no dependency on the jet flap deflection, then these corresponding entries in the system $\underline{B}$ matrix are set to zero for the purposes of this thesis. Since the $B$ matrix from equation (48) shows these numbers incorrectly as being non zero, then there is an error in the FPCC simulation program. Unfortunately, this means that any other

$$
\begin{aligned}
& X_{\text {eng }}=x \text { distance from center of gravity to engine one } \\
& \text { Xeng2 }=x \text { distance from center of gravity to engine two } \\
& \text { Yengi }=y \text { distance from center of gravity to engine one } \\
& \text { eng }=y \text { distance fro center of gravity to engine two } \\
& \text { Zen, }=\text { distance from center of gravity to engine one } \\
& \text { Zeng2 }=z \text { distance from center of gravity to engine two }
\end{aligned}
$$

data produced by the program is also suspect and so, consequent$1 y$, are the results of this thesis.

## APPENDIX B

## IMPLEMENTING A FIRST FORWARD DIFFERENCE APPROXIMATION TO THE 1/S ROUTINE WITHIN THE PI CONTROLLER IN MULTI

## Introduction

The simulation program MULTI currently uses the first backward difference approximation for the integration routine within the proportional plus integral (PI) controller. However, due to certain results in this thesis (see Figures 16,19 , and 20) there seemed to be a problem with the integration within the PI controller. The referenced plots demonstrate a feature that appears to be the result of windup, implying that the integration needed improvement. One simple way of improving a digital integration is to use the first forward difference approxima= tion, which was tried with the routine within the pi controller with good results.

## Inplementation

The current integration within the $P$ controlier in MULTI is done with the first backward difference approximation as expressed in the ine of FORTRAN code as:

$$
\begin{equation*}
Z(I)=Z(I)+S A M P T * E(I) \tag{B-1}
\end{equation*}
$$

which is the representation of:

$$
\begin{equation*}
Z[(k+1) T]-Z(k T)+T \in[(k-1) T] \tag{B-2}
\end{equation*}
$$

Since the line of code (B-1) appears after the calculation of the control input which is based upon (kT) in

$$
\begin{equation*}
\underline{u}(k T)=\left\{\underline{R}_{0} e(k T)+R_{i} \underline{z}(k T)\right\} / T \tag{B-3}
\end{equation*}
$$

the error used in equation ( $B-2$ ) is actually the previous error
and thus the e[(k-1)T] shown in equation (B-2). There is some question as to whether equation (B-2) is actually equivalent to Professor Porter's expression (Ref xx):

$$
\begin{equation*}
Z[(k+1) T]=Z(k T)+T[v(k T)-y(k T)] \tag{B-4}
\end{equation*}
$$

Close examination of expression (B-4) shows that it is equivalent to expression (B-2) if the next iteration of kT starts at the sample and hold device of Figure 1. However, this would seem to conflict withexpression (B-3) where the error used to generate the control input is in the same iteration as the 2 . One way to resolve this problem would be to say that the kT iterations start with the sample and hold device and then to define:

$$
Z[(k+1) T]=Z(k T)+T e(k T)
$$

which is a first forward difference approximation.
What was tested first though, was an average of the first forward and first backward differences, which is still a rectangular approximation, thus keeping the "spirit" of Professor Porter"s work. This averaged first difference can be expressed as:

$$
\begin{equation*}
Z[(k+1) T]=Z(k T)+T\{e(k T)+e[(k-1) T]\} / 2 \tag{B-5}
\end{equation*}
$$

The FORTRAN code implementation of expression (B-5) appeared in MULTI as:

$$
\begin{equation*}
Z(I)=Z(I)+S A M P T *(E(I)+P R E V E(I)) / 2.0 \tag{B-6}
\end{equation*}
$$

Note that in expresions ( $B-1$ ) and ( $B-6$ ) the runing variable I is used to position within the vectors involved, not as a kT iteration counter, and that PREVE is the previous error.

With the code $(B-6)$ subsituted into the pI controller routine the plots for the longitudinal maneuvers at the 0.15 mach flight condition were regenerated. The only noticable changes were in the plots for the forward velocity with the direct lift
command, and in the forward velocity and total thrust curves with the vertical transiation command. Comparing the new curves (Figures $B-1, \quad B-2$, and $B-3$ ) with the originals (Figures 16, 19, and 20), the effect of using the new integration approximation is to eliminate the instabilities near the end of the simulation. After testing the averaged first difference approximation, a first forward difference approximation was implemented by changing ( $B-6$ ) to

$$
\begin{equation*}
Z(I)=Z(I)+S A M P T * E(I) \tag{B-7}
\end{equation*}
$$

This line of code was also moved before the calculation of the control input so that it does implement the first forward difference approximation. Limited testing indicates that this change also removes the instabilities due to using the first backward difference approximation.


FIGURE B-1
Figure B-1. Forward Velocity Response to 0.5 Girect Lift Command ( $0.15 \mathrm{Mach}, 0 \mathrm{ft}$.)


FIGURE B-2
Figure B-2. Forward Velocity Response to 0.5 G Vertical Translation Command ( $0.15 \mathrm{Mach}, 0 \mathrm{ft}$ )


FIGURE B-3
Figure B-3. Total Thrust Response To 0.5 G Vertical Transiation Command ( 0.15 Mach, 0 ft.)

APPENDIX C

## ADDITIONS TO DESIGN PROGRAM MULTI

## Introduction

This appendix briefly describes the two additions made to the design program MULT by this author. The first addition allows the use of the local file names for the system, design, and simulation data files as command line arguments. Within MULTI, these command line arguments are then used in an option \$199 which performs the same functions as options \#9, \#19, and *29. The second addition is 0ption \#28, which calculates and displays several figures of merit from the latest simulation. At the end of this appendix is a short section on the addition of program code that would calculate and display maximum actuator rates. The final section is a description of another option that would allow the user to toggle the curve fitting for the CALCOMP plots from modified spline to linear and back.

Command Line Argurents and Option 199
The concept behind this addition is to allow the user to specify the local file names of the threefiles containing the systen, design, and sinulation parameters as command line arguments to be used with Option 199 . For example, if the user had these three files named MEMO, MEM10, and MEM2O, then the command (in response to Cyber's Command query)

COMMAND> MULTI(, , MEMO, MEM10, MEM20)
will etart up MULT execution in the normal manner, but will allow the user to type (in response to the option query)

```
OPTION, PLEASE>#199
```

invoking Option $\# 199$. The user must type in all three local file names to use Option \#199. If not, then a FORTRAN file read error upon attempting Option \#199 will cause the user to be "bombed out" of MULTI and back into the Cyber comand mode. of course, the three local files must contain the data in the format necessary to satisfy MULTIs requirements. This currently means that the files must be in the format specified in the MULTI Users Manual for Options \#9, \#19, and \#29 mis manual can be obtained from the Electrical Engineering Def rment, Air Force Institute of Technology, Wright Patterson, Torce Base, Ohio, 45433. Invoking Option $\# 199$ actually causes modified versions of Options ${ }^{(19,}$, and 29 to be sequentially executed. This means that the user can use Option $\$ 199$ whenever the other three would be used. The following shows what the user will see after specifying Option 199:

```
OPTION, PLEASE > #199
```

DATA COPY COMPLETE FOR OPTIONS *2, \#3, \#4

DATA COPY COMPLETE FOR OPTIONS *11, \#12, \#13, *14, \#16, \#18

DATA COPY COMPLETE FOR OPTIONS *21, \#22, *23, *24, \#25, *27

OPTION, PLEASE > *
At this point the user may continue as if Options *9, *l9, and *29 had just been completed.

Local file names as command line arguments is not currenty supported in CDC versions of FORTRAN 4. If MULTI were recompiled in FORTRAN 4, changing the program deciaration ine in the main

```
program to read:
```

PROGRAM EXEC (PLOT,TAPE9=PLOT)
would be sufficient (not considering other syntax problems) to disable the command line argument and Option \#199 feature. The code for implementing Option 199 would remain, so that changing the program declaration back to:

PROGRAM EXEC (PLOT,TAPE9=PLOT,TAPE10,TAPE 35,TAPE40)
would then reactivate command line arguments and Option fig9 capability.

Option \#28, Figures of Merit
The second addition made to MULTI by this author is the calculation and display of several figures of merit using option *28. Specifically, this option calculates the peak value, time to peak value, minimum value, time to minimum value, final value, and settifng time. All nf the calculations are performed upon the packed data used for the plots. Additionally, Option \#28 checks to see if the simulation has been run prior to its execution, and if not, then informs the user and returns to the option query. Assuming the simulation has been performed, then using Option *28 would produce:

OPTION, PLEASE> $\geqslant 28$
THIS OPTION CALCULATES THE FIGURES OF MERIT CONTINGENT UPON COMPLETION OF SIMULAIION

HON MANY SEQUENTIAL OUTPUTS (STARTING WITH YI)
DO YOU WISH FIGURES OF MERIT FOR? >1
Y1 PEAR=42.98407352055
II TIME TO PEAK=9.999999999995
I1 MINIMUM=0.
Y1 TIME TO MINIMUM=0.

DO YOU WISH TO CONTINUE WITH SETTLING TIME
CALCULATIONS? (1 FOR YES/O FOR NO) $>1$

```
FINAL VALUE OF Y1=42.98407352055
DO YOU WISH TO USE DEFAULT VALUE OF WITHIN 2%
OF THIS VALUE FOR CALCULATION OF SETTLING TIME?
(1 FOR YES/O FOR NO)>1
```

SETTLING TIME FOR YI=6.499999999999

The user in this example only specified one output, but up to as many as there actualy are can be specified, with the values given above being repeated for each output. Additionally, the user does not have to continue with setting time calculations, in which case the user is returned to the option mode. If the user answers " $0^{\prime \prime}$ to the question about using the default values for the settling time calculations, then this example would continue:

DO YOU WISH TO USE DEFAULT VALUE OF WITHIN 2\%
OF THIS VALUE FOR CALCULATION OF SETTLING TIME?
(1 FOR YES/O FOR NO) $>0$
ENTER UPPER BOUND FOR SETTLING VALUE $>40$

ENTER LOWER BOUND ROR SETTLING VALUE >39

SETTLING TIME FOR Y1=4.399999999999
If the response does not settle down within either the user specified band, or within the $\pm 2 \%$ of final value range, then the message

SIMULATION DID NOT REACH A SETTLING TIME is displayed, and the user is returned to Option query.

Almost all of the added code for Option \#28 ia contained in the last overlay in MULTI, OVERLAY(21,0).

Suggested Addition To Option ${ }^{\text {\# } 28}$
This section describes program code that would calculate and
display the maximum positive and negative actuator rates.
First, two common blocks have to be added to OVERLAY(21,0):
COMMON /B 6 / DMATRIX,ACT,SEN
COMMON /B 13A/ UP(101,11)
Next, right before the common block declaration area add:
CHARACTER DMATRIX,ACT,SEN
At the end of the DIMENSION statements add:
DIMENSION RATEMAX(10), RATEMIN(10)
At the end of the DATA statements add:
DATA RATEMAX,RATEMIN/20*0.0/
Add the following to the indicated DO loop:
DO $1400 \mathrm{I}=1, \mathrm{P}$
DO $1400 \mathrm{~J}=1$, LCOUNT

IF (J.NE.1) THEN

$$
\begin{aligned}
& \text { RATE=(UP(J,I+1)-UP(J-1,I+1))/(UP(J,1)-UP(J-1,1))} \\
& I P(R A T E \cdot G T \cdot R A T E M A X(I)) \text { RATEMAX(I)=RATE } \\
& I F(R A T E \cdot L T \cdot R A T E M I N(I)) \text { RATEMIN(I)=RATE }
\end{aligned}
$$

## ENDIF

1400 CONTINUE

RATEMAX and RATEMIN are used to store the maximum positive and maximum negative values of the rates, respectively. RATE is used to calculate each rate, with $U P(J, I+1)$ storing the current value of the actuator deflection and $U P(J-1, I+1)$ storing the previous value of actuator deflection. $U P(J, l)$ and $U P(J-1, l)$ store the time of the current and the time of the previous actuator deflections respectively. To display the results, the following

```
code is added to the end of the overlay:
    DO 1600 I=1,P
    IF(RATEMIN(I).LT.0) THEN
        IF(ACT.EQ. 'Y`) THEN
        PRINT*, MAXIMUM NEGATIVE ACTUATOR RATE *', I, " = , RATEMIN(I)
        ELSE
```


RATEMIN(I)
ENDIF
ENDIF
IF (ACT.EQ. ${ }^{-} Y^{-}$) THEN

ELSE

RATEMAX(I)
ENDIF
PRINT*, -
1600 CONTINUE
This code correctiy informs the user that the rates are for
control inputs when there are no actuators.
This code has been tested on a copy of MULTI and seems to
work correctig.

Suggested Addition, Option ${ }^{\text {\#38 }}$
This section lists the code that would implenent option 38 , an option to toggle between modified spife and inear curve fitting for the CALCOMP plots. For more information on how this code works, the user is referred to the CALCOMP USER'S GUIDE avallable from either the AFIT Engineering Building (Bidg 640) or
from the ASD Computer Center, Building 676.
The first step is to add the following to the common block statements in the main section of the program:

COMMON /B 13A/ UP(101,11),LINEFIT
and also add the following to the DATA list in the same section: DATA LINEFIT/-1/

Next, add the following to $\operatorname{OVERLAY}(10,0)$ :

First, to the common block list:
COMMON /B 13A/ UP(101,11),LINEFIT
To the listing of options:
PRINT*, - 38. TOGGLE CALCOMP PLOT CURVE FITTING*
To the sequential option code listings
C OPTION *38
2038 LINEFIT=LINEFIT* (-1)
PRINT*, "CALCOMP PLOT CURVE FITTING IS" IF (LINEFIT.EQ.1) PRINT*, ${ }^{\prime}$ LINEAR" IF(LINEFIT.EQ.-1) PRINT*, $\mathrm{MODIFIEDSPLINE"}$ $\mathrm{NOPT}=0$ GO TO 8010

Then add the following to OVERLAY(12,0):
To the common block 11st:

COMMON /B 13A/ UP(101,11),LINEFIT
In two places in the code section replace
CALL FLINE (XAXIS, YAXIS, $-\mathbb{N}, 1,1,72$ )
with
CALL FLINE (XAXIS, YAXIS,N*LINEFIT,1,1,72)
This option has been added to a copy of MULTI and tested with no problems. Option 38 acts like a switch, and is designed

```
to be used whenever the modified Option \#28 (with rates, above) displays rates that the user thinks are too high for the modified spline fit. These two options were used in the authors thesis to prevent actuator deflection plots from assuming non-functional shapes.
```

SYSTEM MATRICES, CONTROLLER MATRICES, AND DESIGN PARAMETERS FOR 0.6 MACH, SEA LEVEL, AND FOR 0.9 MACH, $30,000 \mathrm{FEET}$

## Introduction

This appendix lists the system matrices, the design matrices, and the design parameters for the two fight conditions not discussed in detail in the text of this thesis, 0.6 mach at sea level, and 0.9 mach at 30,000 feet. The complete system matrices in the form of equation (48) from Chapter III can be found in Jon Bauschlicher's thesis (Ref 2).
0.6 Mach, Sea Level Filght Condition
$A=\left[\begin{array}{llll}0 & 0 & 0 & 1 \\ -32.1 & -.0297 & .105 & -12.1 \\ -.577 & -.0568 & -2.82 & 674 \\ 0 & -.0011 & .0639 & -.463\end{array}\right]$
$\underline{B}=\left[\begin{array}{lll}0 & 0 & 0 \\ -1.43 & -.616 & -.000069 \\ .640 & -.043 & -.00000448\end{array}\right]$
$\underline{C}=\left[\begin{array}{llll}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & .00149 & 0\end{array}\right]$
$M=\left[\begin{array}{l}1 \\ 0 \\ 0\end{array}\right]$
$x=2, E=.1, T=0.01$


$$
\begin{aligned}
& \underline{K}_{0}=\left[\begin{array}{lll}
.350 & -.0000258 & -.219 \\
-.469 & -.00155 & -3.56 \\
-57.8 & 9.18 & 34.9
\end{array}\right] \\
& \underline{K}_{1}=\underline{X}_{0} / 2
\end{aligned}
$$

Jeffrey A Simmers was born in Harrisburg, Pennsylvania on January 9, 1953. He spent most of his secondary school years in Schenectady, New York. After graduating from high school he attended Texas A\&M University for two years, then enlisted into the Air Force. Attending the University of Ilifnois through the Airman Education and Conmissioning Program (AECP), he graduated with a B.S. degree in electrical engineering in December of 1979. After attending oTS he was commissioned as an officer in April of 1980.

His first assignment as an officer was as an analyst at the Foreign Technology Division (FTD) at Wright-Patterson AFB, Ohio until 1982, when he started at the Air Force Institute of Technology. His next assignment is with a detatchment of the Air Force Operational Test and Evaluation Center at Eglin AFB, Florida.

19. ABt 由ACT (Continue on reverse if neceseery and identify by Gock number)

Title: Multivariable Digital Flight Control Design for the FPCC Aircraft

Thesis Chair. n: Dr. John J. D'Azzo
Deputy Department Head
Electrical Engineering Department
hir Force Institute of Technology

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| 22.. NAME OF NESPONSIBLE INDIVIDUAL <br> Dr. John J. N'Azzo | 22b. TELEPHONE NUMEEA Include tree Codel <br> $517-55 \cdot 35,76$ | 22c. OFFICE SYMBOL AFIT/FIIC. |
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SECUAITY CLASSIFICATION OF THIS PAGE

Multivariable design techniques developed by Professor Brian Porter of the University of Salford, England, are used to develop disital control laws for the Ily!ht Propulsion Goatrol Loupling (FPCC) Aircrift. Control laws are developed for each of tiree fli:! $:$ conditions. A design tool, the computire program iUlTI, is molified to calculate and display the figures of merit for each sinulation.

The controllers developed utilize output feedback with proportional plus integrai (PI) control. Due to the nature of the system model, certain derivatives of the states are measured and added to the feedtack.

A robust controller is tested by performing specific maneuvers for multiple flight conditions, and these results are compared to those obtained with the controllers designed for each flight condition.

The individual controllers and the robust controller are tested with the addition of a delay that represents the processing delay within the proportional plus integiral implementation. The results with delay are compared to those without delay.

All of the designs accomplished included first order modela for the control surface actuators and assumed perfect knowledge of all aircraft states. The designs used reduced order state models with decoupled lateral and longitudinal models.


