


## 

* 


## (6) - 4 n w <br> 等,

 -Ex

## e.

| REPORT DOCUMENTATION PAGE | READ INSTRUCTIONS BEFORE COMPLETUNG FORM |
| :---: | :---: |
|  | 3. RECIPIENT'S CATALOG NUMEER 303 |
| ```4. TITLE (and Subtilfe) THE USER-ASSISTED AUTOMATED EXPERIMENTAL (TEST) DESIGN PROGRAM (AED): VERSION II``` | 5. TYPE OF REPORT A PENOD COVERED <br> Technical Report |
|  | 6. Performing ozg. Report mumben |
| 7. AUTHOR(D) Edwin G. Meyer William H. Rickels Robert G. Mills, PhD* | 8. CONTRACT OR GAANT NUMEER(E) <br> In Part Under F33615-79-C-0505 |
| g. PERFORming organization name and address System Development Corporation 4134 Linden Avenue Suite 305 Dayton, OH 45432 | 10. PROGRAM ELEMENT PROJECT, TASK AREA WORK UNTT' NUMEEAS $62202 F, 7184-00-09$ |
| 11. CONTROLLING OFFICE NAME AND AODRESS <br> Air Force Aerospace Medical Research Laboratory Aerospace Medical Division, Air Force Systems Command, Wright-Patterson Air Force Base, OH 45433 | 12. REPORT DATE January 1983 |
|  | 13. NUMBER OF PAGES 190 |
| 14. MONITORING AGENCY NAME \& ADORESS(Il different from Controline Offico) | 15. SECURITY CLASS. (of thit report) UNCLASSIFIED |
|  | 15e. OECLASSIEICATION/DOWNGRADING |
| 16. DISTRIBUTION STATEMENT (of this Roport) <br> Approved for public release; distribution unlimited. |  |
|  |  |
| 17. DISTRIBUTION STATEMENT (of the ebstract entered in Block 20, /f diflerent fram Report) |  |
| $\cdots$ |  |
| 18. SUPPLEMENTARY NOTES <br> Dr. Robert G. Mills, AFAMRL/HEF, Tele: (513) 255-3481 <br> *Crew Systems Effectiveness Branch <br> Human Engineering Division |  |
|  |  |
| 19. KEY WORDS (Continue on revorsi alde if neceseary mad ldentily by block number) |  |
| Aeronautics Simulators <br> Man-machine Simulation | Experimental Design Mathematics/Statistics |
| 20. ABSTRACT ( Conilinue on revorse aldo if nocosacry end ldentify by block number) |  |
| The large number of factors commonly encountered in simulation and live field test programs dictates that a large (and costly) experimental (test) program be run if the classical full factorial experimental design is employed. Through the use of a screening process involving a sequential design, efficient economical pilot studies can be conducted, leading to a reduction in the number of factors and the size of the experimental space. Fractional factorial designs are used in the pilot studies. The experimental data provide guidance |  |
| DD i JANm 1473 EDItion of inoves is obsolete |  |

## secumity elagsification of This pacerwhen Date ratermy

## 20. ABSTRACT (CONTINUED)

to the experimenter in terms of removing nonsignificant factors and thus reducing the size of the experimental space.

This report documents the second phase of design, development, and use of interactive computer program to aid in the development of fractional factorial experimental designs. Fractional factorial experiments are a special class of experimental procedures that allow the user to perform a smaller number of experiments than would be required in the usual experimental procedures and which maximize information return while minimizing the number of observations (tests) required. The overall experimental design philosophy is described and a brief introduction into the theory of experimental design is presented. The Appendix describes how the computer program was constructed and how it should be used.

AED: Version II includes a mixed level capability in that one set of factors can be set at two levels and a second set of factors can be set at three levels in the same experimental (test) design. Also included is a Central Composite design capability with all factors at five levels.
This version updates and supersedes Version I that was pubilished as AFAMRL-TR-81-100.

## PREFACE

This program was developed for the Air Force Aerospace Medical Research Laboratory under Contract F33615-79-C-0505. Dr. Robert G. Mills, AFAMRL, was the Air Force Program Manager. Mr. Edwin G. Meyer was the SDC Program Manager. Mr. William Rickels and Mrs. C. Hoyland were responsible for algorithm implementation and programming; as well as writing the report.

The authors gratefully acknowledge the assistance of Dr. Carl E. Taylor, SDC Colorado Springs, CO, for his work on the Central Composite Design, Mr. Edwin G. Meyer who developed many of the algorithms and techniques employed, and Dr. Robert G. Mills, AMRL, who provided many valuable suggestions, direction, and guidance in this effort. Each of the individuals connected with this task contributed substantially to its successful completion.


## TABLE OF CONTENTS

PAGE
glossary ..... 4
INTRODUCTION ..... 8
THE PHILOSOPHY DF EXPERIMENTAL DESIGN ..... 11
THE NEED FOR DESIGNED EXPERIMENTS ..... 12
EXPERIMENTAL MODEL ..... 13
NOMENCLATURE--NOTATION AND TERMINOLOGY ..... 14
FRACTIONAL FACTORIAL EXPERIMENTS ..... 15
ALIASING ..... 21
Overview ..... 21
Operations with Aliases ..... 22
Alias Set Generation in Two Level Designs ..... 27
Theoretical "Best" Isolation Designs ..... 29
Realizable Designs ..... 35
Alias Set Modifications and the Generation of New nesigns ..... 36
Alias Summary ..... 39
TESIGN EVALJATION ..... 40
BASIC BLOCK DEFIMITION ..... 40
DATA COLLECTION ..... 45
REFINING DESIGNS ..... 45
IRREGULAR FRACTIONAL FACTORIAL EXPEPIMENTS ..... 47
SCREENING DESIGNS ..... 48
RESPONSE SURFACE DESIGNS ..... 49
APPROXIMATIONS OF RESPONSE SURFACES BY POLYNOMIALS ..... 49
The Response Surface ..... 49
Polynomial Approximations ..... 50
Data Requirements ..... 52
Use of Incomplete Polynomials ..... 53
THE CENTRAL COMPOSITE DESIGN ..... 53
Definition of the Central Composite Design ..... 54
[.- Coded لا...Real World Levels ..... 55
Choice'ofathe Level a ..... 57
Use,of, Multtiple Observations at the Center Point ..... 58
Restricicions, on a Central Composite Design ..... 50
Comparisan, with the three-Level Factorial ..... 52
DatanAnalysis ..... 53
MIXER LEVEL-DESIGNS ..... 64
Combined Qualitative and quantitative Factors ..... 64
2K3L Designs ..... 55
PREDEFINE: DESIGMS ..... 58
USERS A'PPENDIX GUIDE HITH EXAMPLES ..... 73
Introdaction ..... 73
SYSTEM STRUCTIJRE ..... 73
Basic Terminology ..... 73
Problem lefinition ..... 35
Basic Factorial Designs ..... 85
Mixed Lever Designs ..... 114
Central Composite Designs ..... 124
Actual Experimental Design ..... 149
Data Analysis ..... 154
Experimental Refinement ..... 154
Separation of Aliases ..... 154
Exit ..... 182
REFERENCES ..... 185
1 Full Factorial, three-Factor, Two-Level Experiment ..... 14
Full Factorial, three-Factor, Three-Level Experiment ..... 16
MISVAL Example ..... 17
Full Factorial Experiment ..... 17
Full Factorial Experiment Size ..... 18
Effect/Interaction Summary ..... 19
Number of Effects Aliases ..... 24
Total Allas Set ..... 26
$M=1.1$ Member of Alias Set ..... 29
10 $M=2.3$ Members of Alias Set ..... 30
$11 \quad M=3.7$ Members of Alias Set ..... 30
$12 \quad M=4 . \quad 15$ Members of Alias Set ..... 30
$13 M=5 . \quad 31$ Members of Alias Set ..... 31
$14 \quad M=6 . \quad 53$ Members of Alias Set ..... 31
15 $M=7$. 127 Members of Alias Set ..... 31
16 $M=8.255$ Members of Alias Set ..... 32
17
$M=9$. 511 Members of Alias Set ..... 32
18 $M=10.1023$ Members of Alias Set ..... 32
19 $M=11.2047$ Members of Alias Set ..... 33
$M=12.4095$ Members of Alias Set ..... 33
21 Alias Example ..... 41
22 Abbreviated Alias Summary ..... 42
23 Basic Experimental Block ..... 44
24 Basic Block Surmary ..... 45
25 Replicates of Center Point for Nearly Uniform Variance ..... 59
25 Minimum Fractionation Confounding Only Higher Order Effects. ..... 61
27 5 Factor, 2 Level, Mixed Qualitative and quantitative Designs ..... 65
28 5 Factor, 3 Level, Mixed Qualitative and Quantitative nesigns ..... 55

## GLOSSARY

ALIAS

ALPHA

AXIAL POINTS OR STAR POINTS

CENTER POINT

CENTRAL COMPOSITE DESIGN
cooed level

Effect that cannot be distinguished from another effect.

In a central composite design the non-zero coded level value of a factor at an axial point.

In a central composite design for each factor there are two corresponding axial points: The given factor has coded level -ALPHA at one point and +ALPHA at the other, whereas all other factors have coded level zero at these points.

The point in a central composite design where all N factors have coded level zero.

A combination of a full or fractional two-level factorial design and some additional experimental points selected in a particular manner to allow the determination of the quadratic one factor effects. It is specifically intended to allow determination of the constraints used in defining a quadratic approximation of the response surface.

The level of a factor translated from the true quantitative level used for simplifying calculations.

| CONFOUNDING | An experimental arrangement in which <br> certain effects cannot be distin- <br> guished from others. |
| :--- | :--- |
| CORRELATION COEFFICIENT |  |
| (Pearson R) | The square root of the proportion <br> of total variation accounted for by <br> linear regression. |
| CORRELATION INDEX R | The square root of the proportion of <br> total variation accounted for by the <br> regression equation of the degree <br> being fitted to the data. |
| DEFINING CONTRAST | Selection of effects to be con- <br> founded. |
| OEGREES OF FREEDOM | One less than the number of values <br> required to compute the sum of <br> squares. |
| EACTORIAL EXPERIMENT | Change in response caused by a change <br> in the level of a factor. |
| EXPERIMENT MODEL | Hypothesized equation to describe the <br> response as a function of the treatment. |
| An experiment in which all levels of |  |
| each factor in the experiment are |  |
| combined with all levels of every a complete experiment, |  |
| other factor. |  |

FRACTIONAL FACTORIAL

INTERACTION

MIXED LEVEL DESIGN

OBSERVATION VECTOR

REAL WORLD LEVEL

REGRESSION

REPLICATE

An experimental design in which only a fraction of a complete factorial is run.

An interaction between two factors means that a change in response between levels of one factor is not the same for all levels of the other factor.

Sum of squares of the error divided by the number of degrees of freedom for the error ierm.

A full or fractional factorial design where some factors of the design have a different number of levels than other factors of the design.

Planned level of each factor for a single experimental trial.

The true quantitative level of a factor that corresponds to a coded level.

Linear - Response $=A * X 1+B * X 2+C * X 3+\ldots$ 2*XN; Quadratic - Response $=A * X 1+B * X 2+\ldots+C * X 1 X 2+D * X 1 * X 3+\ldots$ $+E * X 1 * * 2+F * X 2 * * 2+.$.

Repetition of observation vectors applied to multiple experimental trials.

RESPONSE FUNCTION

RESPONSE SURFACE

ROOT SUM SQUARE (RSS)

ROTATABLE DESIGN

R-SQUARED

TRIAL

The function $F$ or $Y=F(X 1, X 2, \ldots X N)$ where the levels of the factors are $X 1$, $X 2, \ldots X N$ and the response is $Y$.

The surface in $N+1$ dimensional space represented by the equation $Y=$ F(X1, X2,... XN).

The square root of the sum of the squares represented by the formula:
$\mathrm{N} \quad 21 / 2$
$\sum_{i=1} \quad x_{i}$

A central composite design that leaves the variance of the estimated response to be approximately constant throughout the sphere of radius one.

Small r-Squared--refer to Correlation Coefficient
Big R-Squared--refer to Correlation Index.

A single set of factor values applied to the experimental subject for which the response is measured.

## INTRODUCTION

The Air force Aerospace ine ical Research Labortory (AFAMRL) is engaged in the use of human operators to perform critical systems evaluation. The size and complexity of the various systems preclude the detailed analysis that would enable AMRL to examine each aspect of every system. Large numbers of factors (independent variables) are commonly encountered in real-world simulation or field problems. Complete full factorial experimental designs for problems involving large numbers of factors ( 20 factors are not uncommon) are very costly in time, manpower, and other test resources.

The use of fractional factorial designs permits the experimenter to employ sequential experimental design techniques. See Cochran and Cox (1957) and other references for a complete discussion of fractional factorial experimental designs. In this procedure, the various factors are examined and a potentially significant subset is defined. By using the proper aliasing of effects, a small fractional factorial experiment can be conducted. If additional effects/interactions are identified as being highly significant or if additional interactions are required to be examined, a larger fractional factorial design can be constructed by removing some of the aliasing requirements. This process of designing an experiment, data analysis, and design refinement is the basis of sequential experimental design.

Examples of multivariable design problems can be found in many Air Force and other R\&D programs, e.g., Aume, Mills, et al., 1977. AMRL has been studying these experimental design problems for a number of years, including the studies performed by Simon (1973), Mills \& Williges (1973), and Williges \& Mills (1973, 1979) relating to human factors experimentation. This report represents an effort to implement some of the design strategies previously proposed.

Human factors experimentation is an especially critical area of research because the experimenter must consider the factors in the system being studied and the variations introduced by the presence of a human subject. To overcome these perturbations, the experimental procedure must be rus many times with several different subjects to remove effects caused by the subjects and to
identify variations caused by the parameters being studied. Since this procedure requires a large number of experimental trials (e.g., observations, tests, etc.), it may not be feasible to conduct a study because of cost and time. One way to overcome this problem is to employ a set $0^{\circ}$ experimental designs called fractional factorial experimental designs. Fractional factorial experiments are a special class of reduced data collection designs that allow the user to perform a smaller number of observations than would be required in the usual experimental procedures.

This effort provides the reader with an automated tool to design fractional factorial experiments. A tape of the Isser-Assisted Automated Experimental (Test) Design Program (AED): Version II source listing in FORTRAN 4-Plus can be obtained from AFAMRL/HEF, Attn: Jr. Robert G. : 4 ill , WPAFB, OH 45433. For the purposes of this report, the authors assume that the reader possesses at least a conceptual knowledge of symmetrical experimental design procedures including fractional factorials. This assumption also holds true for the user of the initial version of the computer program which is being described. However, a long range objective of this effort is to eventually develop the program to the extent that, via the interactive mode, the program's user need have only a minimum knowledge of experimental design computational procedures. the primary intent is to develop the computer program such that it can be readily applied by the engineering, etc., community that is involved with performing simulator and live testing of systems. It should also be noted that although the computer program presented herein is designed to assist the sequential experimental design process (i.e., a series of experiments), it can also be used to create a "one-shot" experimental design.
this report provides background on experimental designs and the mathematical formulations implemented in the computer program. A discussion of how an experiment should be conducted is contained in the Philosophy of Experimental Design section. the class of experimental designs known as fractional factorial designs is described along with the terminology involved, the concept of aliasing, the evaluation of designs, and means of defining basic experimental blocks. Screening designs, response surface designs, polynomial approximations to the response surface, central composite designs, and mixed level designs are also
discussed. Brief commentaries on data collections, redesign, and irregular fractional factorial experiments are provided. Some predefined fractional factorial designs including optional aliasing selection to reduce aliasing of main and first-order effects are presented. A selected bibliography of books and reports that present more detailed information on these topics is given in the reference section. An appendix provides a detailed step-by-step description of the computer program.

This report is an update of a previous report and indicates the present status of the automated experimental design program (AED). The AED program contains adequate instructions and text to guide the user in its operation without the assistance of this report. It is provided as an aid to understanding the mathematical formulations and as a source of additional examples (Appendix). For the current version of the program, the following is a summary of its present capabilities.

1. Basic full or fractional designs where
(a) 2 level designs can have up to 20 factors with a maximum of 255 experimental trials.
(b) 3 level designs can have up to 12 factors with a maximum of 243 experimental trials.
(c) 5 level designs can have up to 8 factors with a maximum of 125 experimental trials.
2. $2^{\mathrm{K}} \times 3^{\mathrm{L}}$ mixed level designs where
(a) the combined number of factors for both levels must be less than or equal to $20(K+L \leq 20)$ with a maximum of 256 total experimental trials.
(b) Experimental plans for combining the separate levels have been developed for $1 / 2,1 / 3,1 / 4,1 / 6,1 / 8,1 / 9,1 / 12,1 / 16,1 / 18$, and 1/24 fractionations.
3. Rotatable and non-rotatable central composite designs.
4. 22 predefined stored designs for 2 levels. 19 predefined stored designs for 3 levels.
5. Assistance in generating realizable 2 level designs for $1 / 2,1 / 4,1 / 8$, and 1/16 fractionations.

THE PHILOSOPHY OF EXPERIMENTAL DESIGN

The basis for an experimental design philosophy consists of six steps:

1. Problem recognition and initial study
2. Preliminary model definition
3. Data collection plan development
4. Data collection
5. Data analysis
6. Analysis of results and model reformulation.

In the first step, the experimenter recognizes the existence of a problem. He begins a nreliminary study to identify the problem bounds and its associated parameters. This initial study provides a crude model of the system. In the second step, the experimenter examines this preliminary model and identifies those features that severely affect the performance of the system. He designs a data collection plan that enables him to test the previously hypothesized significant features. Without an adequate data collection plan, the experimenter may arrive at erroneous conclusions.

Once the data collection plan (called the experimental design) is complete, the experimenter "collects" the data. After the data are collected, data analysis is performed. Data analysis consists of the standard analysis methods, e.g., analysis of variance (ANOVA) techniques or regression analysis, if all factors are quantitative. This analysis identifies those factors that account for most of the system variation. According to Pareto's Principle, 80 percent of the variation in a system can be attributed to 20 percent of the factors.

After the data analysis is performed, the experimenter redesigns or refines his system model based on the results of the previous experimentation. This cycle of redesign, data collection, and analysis continues until the experimenter is satisfied with the accuracy of his results. At this point, he draws conclusions about the system based upon the experimentation.

## THE NEED FOR DESIGNED EXPERIMENTS

An experiment is conducted to provide information. An experimenter needs information to identify problem areas, to identify important factors, and to quantify responses. He obtains this information by collecting data. After problem definition is complete, the first step in an experiment is to define questions that need to be answered. Once the questions are identified, an experiment can be designed to aid in answering those questions. The key issue is that an experimenter must design his experiment before any data are collected.

The designer of an experiment must consider the statistical accuracy and the cost of the experiment. Statistical accuracy involves the proper selection of the response to be measured, determination of the number of factors that influence the response, the selection of the subset of these factors to be studied in the experiment being planned, the number of times the basic experiment should be repeated, and the form of the analysis to be conducted.

The cost of an experiment includes, among many other factors, expense incurred by running a single experimental condition (observation), analyzing the data, failing to meet a deadline, availability of subjects, and most importantly, perhaps drawing incorrect conclusions from the experiment. Although cost as a factor is not often discussed in the literature, it is at least as important as considerations of statistical accuracy. In an attempt to minimize the cost of an experiment, the designer usually attempts to choose the simplest experimental design possible, and to use the smallest sample size consistent with satisfactory results. Fortunately, most simple experimental designs are both statistically efficient and economical, so that the designer's efforts to obtain statistical accuracy usually result in economy.

The experiments being studied in a factorial experiment are called fixed effect models. The term fixed effect is related to the predefined levels that the various factors may assume. Consider two factors, $A$ and $B$, which are being studied where there are $N_{A}$ levels for treatment $A$ and $N_{B}$ levels for treatment B. The response in a two-factor experiment may be described by the model:

$$
\begin{aligned}
& x_{i j}=\mu+\alpha_{i}+\beta_{j}+\alpha \beta_{i j}+\varepsilon_{i j} \\
& i=1,2, \ldots, N_{A} \\
& j=1,2, \ldots, N_{B}
\end{aligned}
$$

where
$\mu=$ overall mean effect
$\alpha_{j}=$ true effect of the ith level of factor $A$
$B_{j}=$ true effect of the $j$ th level of factor $B$
$(\alpha \beta)_{i j}=$ effect of the interaction between $\alpha_{i}$ and $B_{j}$
$\varepsilon_{\mathbf{i j}}=$ perimental error
A similar model for three factors may be written as:

$$
x_{i j k}=\mu+\alpha_{i}+\beta_{j}+\gamma_{k}+(\alpha \beta)_{i j}+(\alpha \gamma)_{i k}+(\beta \gamma)_{j k}+(\alpha \beta \gamma)_{i j k}+\varepsilon_{i j k}
$$

The assumptions in a full factorial experiment allow for the examination of each main effect and all interactions. A fractional factorial experiment assumes that the high-order interactions are insignificant. For example, in the three-factor model, if the assumption is made that the interactions ( $\alpha \gamma$ ), ( $B \gamma$ ), and ( $\alpha \beta \gamma$ ) are insignificant, the model becomes:

$$
x_{i j k}=\mu+\alpha_{i}+\beta_{j}+\gamma_{k}+(\alpha \beta)_{i j}+\varepsilon_{i j k}
$$

This permits fewer experimental observations to determine the relative significance of the remaining terms in the model. The effects considered to be insignificant are included in the model error term.

This experimental model can be evaluated using the standard analysis of variance (ANOVA) techniques or a regression analysis may be run, if all factors are quantitative, to determine regression coefficients.

## NOMENCLATURE--NOTATION AND TERMINOLOGY

The previous section showed that responses could be modeled as equations involving true effects of each factor at the level involved, the effects of the interactions among factors, the overall mean effect, and the true test (experimental) error. The techniques for manipulating response data from individual experimental trials to arrive at estimates of the values for each of the terms in the mathematical model involve consideration of response values for various combinations of factors and levels. Two standard means of notation are used to represent these response values. These are illustrated in the following example.

Consider an experiment involving three factors with each factor having two possible levels. If the factors are represented by $a, b$, and $c$, and the levels, by 0 and 1 , the possible trials and notations used to represent the responses are shown in Table 1.

Table 1. Full Factorial, Three-Factor, Two-Level Experiment

| EXPERIMENTAL TRIAL <br> (FACTOR AND LEVEL) | EFFECT OR |  |
| :--- | :--- | :--- |
| INTERACTION | NOTATION |  |
| ${ }^{a_{0} b_{0} c_{0}}$ | 1 | 000 |
| $a_{0} b_{0} c_{1}$ | $C$ | 001 |
| $a_{0} b_{1} c_{0}$ | $B$ | 010 |
| $a_{0} b_{1} c_{1}$ | $A C$ | 011 |
| $a_{1} b_{0} c_{0}$ | $A$ | 100 |
| $a_{1} b_{0} c_{1}$ | $A C$ | 101 |
| $a_{1} b_{1} c_{0}$ | $A B$ | 110 |
| $a_{1} b_{1} c_{1}$ | $A B C$ | 111 |

Main effects are represented by those trials whose notation has a nonzero value in only one column of the notation. Two-factor, or first-order interactions, are represented by those trials whose notation has a nonzero value in two columns. Higher-order interactions are represented by those trials whose notation has a nonzero value in more than two columns. In this example, main effects are $A, B$, and $C$. First-order interactions are $A B, A C$, and $B C$. The only higher-order interaction is ABC.

It is apparent from Table 1 that the notation consists simply of the subscripts representing the levels of the factors in sequential order (thus, trial abblcl has a response notation of 011). The effect or interaction response is represented by the sequence of the factors raised to the power of the level involved (thus, trial aOb1c1 results in the interaction response $A O_{B} 1 C l=B C$ ).

In a similar manner, effects and notations can be defined for the three-level case. Consider an experiment involving three factors with each factor having three possible levels. If the factors are represented by $a, b$, and $c$, and the levels by 0,1 , and 2 , the possible trials and notations used to represent the responses are shown in Table 2. Note that the main effects are $C, C^{2}, B$, $B^{2}, A$, and $A^{2}$ because the notation for these trials has a nonzero value in only one column of the notation.

## FRACTIONAL FACTORIAL EXPERIMENTS

A full factorial experimental design involves an experiment in which every level of each factor is combined with every level of every other factor. If an experiment has $N$ factors and each factor may assume one of $P$ levels, there is a total of PN combinations.

Table 3 is an example of an experiment with three factors at two levels. This example is taken from an AMRL study of the MISVAL program. The term "MISVAL" designates the Missile Launch Envelope Technology Development Program being conducted by the Air Force Wright Aeronautical Laboratories at Wright-Patterson Air Force Base, Ohio. The definitions of factors and levels used in the examples are not considered necessary in order to convey the intent of the examples.

Table 2. Full Factorial, Three-Factor, Three-Level Experiment

| EXPERHENTAL THIAL (FACTOR AND LEVEL) | EFFECT On INTERACTION | notation |
| :---: | :---: | :---: |
| ${ }^{4} 0^{6} 0_{0} 0$ | 1 | 000 |
| ${ }^{2} 0^{6} 0^{4}$ | c | 001 |
| ${ }_{0} b_{0} c_{2}$ | $c^{2}$ | 002 |
| $00^{6} 19$ | B | 010 |
| ${ }^{0} b_{1} c_{1} c_{1}$ | ${ }^{\text {c }}$ | 011 |
| $a_{0} b_{1} c_{2}$ | $\mathrm{cc}^{2}$ | 012 |
| $\mathrm{Ba}_{0} \mathrm{~b}^{\text {c }} 0$ | $8^{2}$ | 020 |
| $0_{0} \mathrm{~b}^{c_{1}{ }^{1} 1}$ | $\mathrm{s}^{2} \mathrm{c}$ | 021 |
| $0_{0}{ }^{\text {b }}{ }^{\text {c }}$ | $\mathrm{B}^{2} c^{2}$ | 022 |
| ${ }^{1} 10_{0} c_{0}$ | A | 100 |
| ${ }^{1} \mathrm{~b}_{0} \mathrm{c}_{1}$ | AC | 101 |
| $a_{1} b_{0} c_{2}$ | $A C^{2}$ | 102 |
|  | AB | 110 |
| $01^{6}, c_{1}$ | ABC | 111 |
| ${ }^{1} \mathrm{l}^{\mathrm{b}, \mathrm{c}_{2}}$ | $\mathrm{AECO}^{2}$ | 112 |
| ${ }^{1} 1^{b_{2} c_{0}}$ | $A^{2}$ | 120 |
| ${ }^{1} 1^{b_{2} c_{1} c_{1}}$ | $A B^{2} C$ | 121 |
| $0_{1} b_{2} c_{2}$ | $A B^{2} C^{2}$ | 122 |
| ${ }_{2}{ }^{6} 00_{0}$ | $A^{2}$ | 200 |
| ${ }_{2} 2^{\text {b }}{ }^{c_{1}}$ | $A^{2} \mathrm{C}$ | 201 |
| $22^{b_{0} c_{2}}$ | $A^{2} c^{2}$ | 202 |
|  | $A^{2} 8$ | 210 |
| $22^{\text {b }} 1^{\text {c }} 1$ | $A^{2} \mathrm{BC}$ | 211 |
| $22^{\text {b }} 1{ }^{5}$ | $A^{2} \mathrm{BC}^{2}$ | 212 |
| $\mathrm{a}_{2} \mathrm{~b}_{2} \mathrm{c}_{0}$ | $A^{2} \mathrm{~B}^{2}$ | 220 |
| $2^{4} 2^{c_{1}{ }_{1}}$ | $A^{2} \mathrm{~B}^{2} \mathrm{C}$ | 221 |
| $\mathrm{a}_{2} \mathrm{~B}_{2} \mathrm{C}_{2}$ | $A^{2} \mathrm{~s}^{2} \mathrm{C}^{2}$ | 222 |

Table 3. MISVAL Example

| castl | Facton | LOW LEVEL (0) | HIOM LEVEL (1) |
| :---: | :---: | :---: | :---: |
| A | MLE CONCETT | fanc concerr | GD COMCEPT |
| E | PILOT FUNCTION | FUNCTION 1 | FUNCTION 2 |
| c | Mhesile TYPE | Alm-7F | AMM-9F |

The $23=8$ combinations of these factors that would comprise a full factorial experiment for the MISVAL program are given in Table 4.

Table 4. Full Factorial Experiment

| EXPERIMENTAL UNIT | LABEL | NOTATION | MLE CONCEPT | PILOT FUNCTION | Missile <br> TYPE |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 000 | FAAC | 1 | AIM-7F |
| 2 | c | 001 | FAAC | 1 | AlM-9F |
| 3 | 8 | 010 | FAAC | 2 | AIM-7F |
| 4 | Ec | 011 | FAAC | 2 | AIM-9F |
| 5 | A | 100 | GD | 1 | AIM-7F |
| 6 | AC | 101 | GD | 1 | AlM-9F |
| 7 | A8 | 110 | GD | 2 | Alm-7F |
| 8 | ABC | 111 | CD | 2 | AlMM-9F |

Table 5 shows the number of observations required in a full factorial experiment for experiments with 2 to 10 factors at 2 or 3 levels. Note that the number of observations required rises drastically as the number of factors and/or levels increases.

A full factorial experimental design provides an estimate of every possible effect, i.e., one is able to estimate those effects caused by all combinations of factors. In many experiments, interactions among factors may be insignificant. Interactions involving two factors are called first-order interactions, and interactions among three factors are called second-order interactions.

Table 5. Full Factorial Experiment Size

| N | P = 2 |  |
| :---: | :---: | :---: |
| NUMEER OF FACTOAS | LEVELS PER FACTOAS | LEVELS PER FACTOAS |
| 2 | 4 | 9 |
| 3 | 8 | 27 |
| 4 | 16 | 81 |
| 5 | 32 | 243 |
| 6 | 64 | 729 |
| 7 | 128 | 2597 |
| 8 | 256 | 6061 |
| 9 | 512 | 19883 |
| 10 | 1024 | 59690 |

In many human factors experiments, the assumption that second and higher-order interactions are insignificant is reasonable (Simon, 1973).

The total number of effects and interactions is given by $\mathrm{PN}-1$. The number of main effects is given by ( $P-1$ )N. The number of first-order interactions is given by:

$$
\frac{(P-1)^{2}}{2} N(N-1)
$$

Thus, the number of higher-order interactions is given by:

$$
P^{N}-1-(P-1) N-\frac{(P-1)^{2}}{2} N(N-1)
$$

Table 6 shows the number of main, first-order, and higher-order effects for a variety of factorial experiments.

Figure 1 shows examples of the groupings of main, first-order, and higher-order effects for three factors at two and three levels.

Table 6. Effect/Interaction Summary

| N <br> NUMBER OF FACTORS | $p=2$ |  |  | $\mathrm{P}=3$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | LEVELS PER FACTOR |  |  | LEVELS PER FACTOR |  |  |
|  | MAIN EFFECTS | $\begin{aligned} & \text { 1ST } \\ & \text { ORDER } \end{aligned}$ | HIGHER ORDER | MAIN EFFECTS | $\begin{gathered} \text { 1ST } \\ \text { ORDER } \end{gathered}$ | HIGHER ORDER |
| 2 | 2 | 1 | 0 | 4 | 4 | 0 |
| 3 | 3 | 3 | 1 | 6 | 12 | 8 |
| 4 | 4 | 6 | 5 | 8 | 24 | 48 |
| 5 | 5 | 10 | 16 | 10 | 40 | 192 |
| 6 | 6 | 15 | 42 | 12 | 60 | 656 |
| 7 | 7 | 21 | 99 | 14 | 84 | 2088 |
| 8 | 8 | 28 | 219 | 16 | 112 | 6432 |
| 9 | 9 | 36 | 468 | 18 | 144 | 19620 |
| 10 | 10 | 45 | 968 | 20 | 180 | 58848 |

A fractional factorial design, sometimes called a fractional replication, is a portion or a fraction of a complete factorial experiment. In a fractional replicate, certain interactions cannot be separated from other interactions. This is the price that is paid for reducing the number of experimental trials. Interactions or effects that cannot be separated are said to be aliased or confounded.

The use of fractional factorial experiments is based on the assumption that higher-order interactions above first-order are insignificant and need not be examined in detail. For example, consider an experiment in which main effect $A$ is aliased with interaction $B C D$. When data are collected, the experimenter estimates the response caused by effect A and BCD together. There is no way to know if the response is due only to $A$ or if effect $B C D$ plays a significant part in the response. Thus, effects $A$ and $B C D$ are not separable. The assumption in a fractional factorial experiment is that the contribution caused by BCD would be negligible.

THO LEVELS PEA FACTOR
$\left.\begin{array}{l}\left.\begin{array}{l}\text { A } \\ \text { B }\end{array}\right\} \text { MAN EFFECTS } \\ \text { AB } \\ \text { AC } \\ \text { AC } \\ \text { AEC }\end{array}\right\}$ HICTMONDER INTERACTIONS
THAEE LEVELS PER FACTOR


Figure 1. Main Effects and Interations

A full factorial experiment is useful when an experimenter requires that:

1. Every main effect of every factor be estimated independently of every other one.
2. The dependence of the effect of every factor upon the levels of the others (the interactions) be determined.
3. The effects be determined with maximum precision.

If an experimenter does not require this level of detail, or if faced with time or budget limitations that prohibit a full factorial experiment, fractional factorial designs are available. The primary assumption in the use of a fractional factorial experiment is that higher-order interactions are insignificant. If this assumption is not valid for a particular experiment, a fractional factorial design should not be used. In most human factors experiments, however, this is a reasonable assumption and can result in a significant reduction in the number of experimental (test) trials or observations required.

Interactions that are assumed to be insignificant can be used to define the aliasing or confounding used in the fractional factorial design. The concept of aliasing is discussed in the next section.

ALIASING

## OVERVIEW

Each successive step in fractionating a full factorial design, or dividing it into blocks, requires that an additional effect referred to as a defining contrast be defined for the fractional factorial design. A defining contrast is a selected observation vector whose factor combinations will not be important to the experimentation. Defining contrasts are then used in generating the alias set. The alias set is composed of all factor combinations of the defining contrasts. Thus, a two-level, one-half design requires one defining contrast to be defined by the experimenter; while a one-fourth design requires two defining contrasts, and so on. The defining contrasts must be selected by the experimenter to meet the requirements of the design.

The selection of defi.,ing contrasts is important in the design of a fractional factorial experiment. In a given experiment, the value of aliased terms cannot be estimated; thus, no term of interest to the experimenter should be selected as a defining contrast.

Defining contrasts are usually selected to avoid aliasing main effects with other main effects. In a sequential design, however, it may be desirable to alias two main effects. For example, if the experimenter suspects that two factors, A and $E$, are not significant, he might design the first pilot experiment so that $A$ and $E$, are confounded. If the data from this pilot experiment show that the estimated values of $A$ and $E$ are not significant, then the experimenter's suspicions are confirmed. These two factors can be dropped, thus reducing the experiment size for the next pilot experiment.

The defining contrasts can be described by identities that determine which effects will be confounded. the experimenter does not have a completely free hand in the selection of these defining contrasts. Defining contrasts are linearly independent if one defining contrast is not a factor combination of another. Unless the selected defining contrasts are linearly independent, some factor combination of one defining contrasts will be the same as a factor combination of the nonindependent defining contrasts selected. This indicates that the nunindependent defining contrasts is redundant, and the experimenter has selected fewer defining contrasts than planned. This results in a larger experiment block size than desired. In this case, it is necessary to redefine the nonindependent defining contrast so that a set of independent defining contrasts is selected.

The following paragraphs provide background for the generation of the alias set and development of alias summaries. These operations are performed within the computer proqram, and understanding of this material is not necessary to use the program.

## OPERATIONS WITH ALIASES

Assuming each of N factors will be varied over ? levels, the set of fractional factorial experiments considered here is the $1 / p^{M}$ designs, where $M$ is a
positive integer. Thus two-level designs might be $1 / 2,1 / 4,1 / 8,1 / 16$, etc. Three-level designs might be $1 / 3,1 / 9,1 / 27,1 / 81$, etc. In general, a $1 / p^{M}$ design requires $M$ defining contrasts. Specifying the defining contrasts is an important problem in designing fractional factorial experiments. One way to specify the defining contrasts is to describe which effects are to be confounded.

Although confounding two factor interactions with other two factor interactions is not always desirable in a good experimental design, this will be done in the following examples for ease of calculation and demonstration. If effects $A B$ and $C D$ are to be confounded, the user may specify the defining contrast as $A 8=C D$. Defining contrasts may also be described in terms of the identity effect, I. This is accomplished by multiplying both sides of the equation in this example by $A B$, yielding $A A^{2} 2=A B C D$. Assuming a two-level problem, apply modulo 2 arithmetic to the exponents of the factors $A^{2} B^{2}=A O_{B}=I=A B C D$. If the effects to be confounded are $A^{2} B=C D$ in a three-level problem, first multiply both sides by $A^{2} B$ using module 3 arithmetic on the exponents. Modulo $P$ is merely the remainder when the number is divided by $P$ (e.g., 4 modulo $3=1$, 12 modulo $3=0$, 13 modulo $3=1$, etc.).

This gives

$$
\begin{aligned}
& \left(A^{2} B\right)\left(A^{2} B\right)=A^{2} B C D \\
& A^{4} B^{2}=A B^{2}=A^{2} B C D
\end{aligned}
$$

Multiply both sides again by $A^{2} B$, giving $\left(A^{2}\right)\left(A^{2} B\right)=\left(A^{2} B C D\right)\left(A^{2} B\right)$

$$
\begin{aligned}
& \text { or } A^{3} B^{3}=I=A^{4} B^{2} C D=A B^{2} C D \\
& \text { or } I=A B^{2} C D .
\end{aligned}
$$

Each effectwill be aliased with ( $\mathrm{p}^{M}-1$ ) other effects. For $1 / p^{M}$ designs, $M$ must be less than or equal to ( $N-1$ ). As the number of defining contrasts increases, the number of effects aliased with each effect increases rapidly as shown in Table 7.

Table 7. Number of Effects Aliases

| DESIGN | P = <br> NO. OF LEVELS | $M$ | NO. OF <br> DEFINING <br> CONTRASTS | NO. OF EFFECTS <br> ALIASED WITH EACH EFFECT |
| :--- | :---: | :---: | :---: | :---: |
| $1 / 2$ | 2 | 1 | 1 | 1 |
| $1 / 4$ | 2 | 2 | 2 | 3 |
| $1 / 8$ | 2 | 3 | 3 | 7 |
| $1 / 16$ | 2 | 4 | 4 | 15 |
| $1 / 32$ | 2 | 5 | 5 | 31 |
| $1 / 84$ | 2 | 6 | 6 | 63 |
| $1 / 128$ | 2 | 7 | 7 | 127 |
| $1 / 3$ | 3 | 1 | 1 | 2 |
| $1 / 9$ | 3 | 2 | 2 | 8 |
| $1 / 27$ | 3 | 3 | 3 | 26 |
| $1 / 81$ | 3 | 4 | 4 | 80 |
| $1 / 243$ | 3 | 5 | 5 | 242 |
| $1 / 729$ | 3 | 6 | 6 | 728 |
| $1 / 2187$ | 3 | 7 | 7 | 2186 |

The number of effects aliased with each effect (such as a main effect) increases rapidly as smaller fractional factorial designs are considered. Thus, as the number of defining contrasts ( $M$ ) increases without a corresponding increase in the number of factors ( $N$ ), it becomes difficult to select defining contrasts that avoid aliasing main effects with other main effects in this case.

The total alias combination set may be generated by considering all combinations of all powers of the individual defining contrasts from 1 to the $(P-1)$ th power. Thus, if a two-level experiment is being considered ( $P=2$ ), only the first power of the defining contrasts is considered in deriving the alias set. For a three-level experiment ( $P=3$ ), both first and second powers of the defining contrasts are considered in deriving the alias set.

For example, consider a three-level experiment involving four factors $(P=3, N=4)$. Suppose $M$ is selected as a value of 3 , resulting in a $1 / 27$
design. From the tabulation, we find that three defining contrasts are required and each effect will be aliased with 26 effects or interactions. The alias set may be derived by representing all the integers from 1 to ( $p^{M}-1$ ) in base $p$ arithmetic representation ( 1 to 26 in this example expressed in base 3 arithmetic).

| $1=001$ | $10=101$ | $19=201$ |
| :--- | :--- | :--- |
| $2=002$ | $11=102$ | $20=202$ |
| $3=010$ | $12=110$ | $21=210$ |
| $4=011$ | $13=111$ | $22=211$ |
| $5=012$ | $14=112$ | $23=212$ |
| $6=020$ | $15=120$ | $24=220$ |
| $7=021$ | $16=121$ | $25=221$ |
| $8=022$ | $17=122$ | $26=222$ |
| $9=100$ | $18=200$ |  |

Each digit of the base 3 representation is used as the power to which each of the three defining contrasts is raised. For example, the combination 121 indicates that the first and third defining contrasts are raised to the first power while the second defining contrast is squared.

From this list, only those combinations that are in standard form are used, since the other combinations will result in duplications. A combination is in standard form if the leading nonzero exponent is 1. Thus, 120 is in standard form whereas 210 is not.

In our example ( $P=3, N=4, M=3$ ), let us specify the defining contrasts selected as $I=A B C D=B^{2} C 2 D=A^{2} B$. The tabulation of combinations is shown in Table 8. Note that the exponents are reduced modulo $P$ to arrive at the final alias combinations. Thus, $\left(A A^{2}\right)^{2}=A^{4} B^{2}=A B^{2}$ modulo 3.

In our example for our experiment design, we assigned three defining contrasts. The total alias set was then derived ( 26 in this case), which represents the combinations applicable to this design. The experimenter must now be concerned with how the individual effects and interactions are aliased.

Table 8．Total Alias Set

| Powen ste | DEFINIMG CONTRAST combination | TOTAL ALIAB SET |
| :---: | :---: | :---: |
| 001 | $A^{2}$ | $A^{2} 8$ |
| 002 | $\left.\left(A^{2}\right)^{2}\right)^{2}$ | $A N S^{2}$ |
| 010 | $s^{2} C^{2} 0$ | $8^{2} C^{2} 0$ |
| 011 | $\left(B^{2} c^{2} 0\right)\left(A^{2} a\right)$ | $A^{2} C^{2} D$ |
| 012 | $\left(8^{2} c^{2} 014 a^{2} B\right)^{2}$ | $\mathrm{AcC}^{2} \mathrm{O}$ |
| 020 | $\left(8^{2} c^{2} 0\right)^{2}$ | $\operatorname{ecos}^{2}$ |
| 021 | （ $\left.\left.\mathrm{B}^{2} \mathrm{c}^{2} 0\right)^{2}\left(A^{2}\right)^{2}\right)$ | $A^{2} \mathrm{O}^{2} \mathrm{CO} \mathrm{C}^{2}$ |
| 022 | $\left(\theta^{2} c^{2} 0\right)^{2}\left(A^{2} \theta\right)^{2}$ | $A C D^{2}$ |
| 100 | AECD | AECD |
| 101 | （ascoila ${ }^{2}$ ） | $\mathrm{s}^{2} \mathrm{CD}$ |
| 102 | （алсD ［a $\left.^{2}\right)^{2}$ | $A^{2} \mathrm{CD}$ |
| 110 | （ACCDI ${ }^{2} \mathrm{C}^{2} \mathrm{O}$ ） | $A 0^{2}$ |
| 111 |  | $80^{2}$ |
| 112 |  | $\mathrm{A}^{2} \mathrm{O}^{2}$ |
| 120 | （AACDIN $\left.{ }^{2} \mathrm{C}^{2} \mathrm{O}\right)^{2}$ | $A 日^{2} C^{2}$ |
| 121 | （ $\left.A B C D \\| 8^{2} C^{2} 01^{2}\left(A^{2}\right)^{\prime}\right)$ | $c^{2}$ |
| 122 | （ABCDI的 $\left.c^{2} 0\right)^{2}\left(A^{2}\right)^{2}$ | $A^{2} x^{2}$ |
| 200 | （ascol ${ }^{2}$ | $A^{2} 8^{2} c^{2} D^{2}$ |
| 201 | $\left.(A B C D)^{2}\left(A^{2}\right)^{1}\right)$ | $A C S^{2} O^{2}$ |
| 202 | ASCDI ${ }^{2}\left(A^{2} A\right)^{2}$ | $x^{2} 0^{2}$ |
| 210 | （ $A C C O)^{2}\left(E^{2} c^{2} D\right)$ | $A^{2} \mathrm{ec}$ |
| 211 | （ACCD）${ }^{2}\left(B^{2} C^{2} 01 / A^{2} B\right)$ | $A 日^{2} C$ |
| 212 | $(A B C D)^{2}\left(A^{2} c^{2} 0 \\|\left(A^{2}\right)^{2}\right.$ | c |
| 220 | （ACCD）${ }^{2}\left(Q^{2} C^{2} D\right)^{2}$ | $\mathrm{A}^{2} \mathrm{O}$ |
| 221 | （ABCD）${ }^{2}\left(a^{2} c^{2} 0\right)^{2}\left(a^{2}\right)$ | 1.80 |
| 222 | $(A B C D)^{2}\left(\mathrm{~s}^{2} c^{2} 0\right)^{2}\left(A^{2} \mathrm{~s}\right)^{2}$ | $\mathrm{a}^{2} 0$ |

## ALIAS SET GENERATION IN TWO LEVEL DESIGNS

The alias set for a given fractional factorial design may be selected to meet the objectives of the experimenter within certain constraints. For example, if it is known that two factors, $A$ and $B$, are not significant, the experimenter may choose to deliberately confound these effects and select one defining contrast as AB. Obviously, it is not possible to develop guidelines for every case which might arise. Since a large portion of possible experimentation is directed toward screening or the identification of significant factors, emphasis has been placed on examining the selection of aliases for isolation designs in which factors of interest are not confounded with other factors of interest so their significance can be determined.

Generally, in a two-level experiment involving $N$ factors, it is desired to isolate all single factors and two factor interactions. This is under the assumption that three factor and higher order interactions are insignificant. For this situation to exist, all members of the alias set must contain a minimum of five factors. There will be cases in which this cannot be achieved (e.g., an experiment which involves fewer than five factors).

To obtain the largest degree of isolation for a given experiment, all members of the alias set should contain approximately the same number of factors that are at their high level.

$$
\begin{aligned}
\text { Given: } & \begin{aligned}
N= & \text { number of factors } \\
M & =\text { fractionation measure (fraction }=\frac{1}{2^{M}} \text { ) } \\
\text { Then: } \quad S= & 2^{M}-1 \\
\Sigma= & \text { maximum number of factors which the summation of all members } \\
& \quad \text { of the alias set can contain } \\
= & 2^{M-1} N .
\end{aligned} .
\end{aligned}
$$

To illustrate, consider a $1 / 8$ fractionation of a 25 experiment.
Here $N=5, M=3 \quad\left(\frac{1}{2^{3}}=\frac{1}{8}\right)$

$$
\begin{aligned}
& S=23-1=7 \text { members of alias set } \\
& \Sigma=22 \times 5=4 \times 5=20 \text { factors }
\end{aligned}
$$

In this example, the seven members of the alias set will contain a maximum of 20 factors. The alias set can consist of either $S$ members, each of which has an even number of factors, or $\frac{S+1}{2}$ members containing an odd number of factors and $\frac{S-1}{2}$ members containing an even number of factors. In our example, the total alias set may thus consist of either:
(a) 7 members each containing an even number of factors. The summation should equal 20. An example would be five members of 2 factors, one member of 4 factors, and one member of 6 factors. This is represented as:

$$
5(2)+(4)+(6)=20
$$

A better example for this case, in terms of having all members of the alias set with as close to the same number of factors as possible would be

$$
4(2)+3(4)=20
$$

(b) 4 members containing an odd number of terms, and 3 containing an even number of terms. For our example, this would be $2(2)+4(3)+1(4)=20$.

For our sample case, the alias set design in (b) is better than that in (a) in that (b) only has two 2 factor members while (a) has four 2 factor members. The quality of an alias set design may be quantified by determining the root sum square (RSS) of the departure from the mean:

$$
\begin{aligned}
& \text { Mean }=\frac{20 \text { factors }}{7 \text { members }}=2.857 \text { factors/member } \\
& \text { Quality }=\sum_{i=1}^{S}(x-\bar{x})^{2} 1 / 2, \text { where } x_{i}=\text { number of factors for the } i \text { th case }
\end{aligned}
$$

$$
\begin{aligned}
& \overline{\mathrm{x}}=\text { means, and } \mathrm{S}=\text { number of members of the alias set. } \\
& \text { Quality }(\mathrm{a})=4(2-2.857)^{2}+3(4-2.857)^{2} \quad 1 / 2=6.856 \quad 1 / 2=2.619 \\
& \text { Quality }(\mathrm{b})=2(2-2.857)^{2}+4(3-2.857)^{2}+1(4-2.857)^{2} \quad 1 / 2=2.856 \mathrm{l}^{1 / 2} \\
& \quad=1.690
\end{aligned}
$$

Thus the members of alias set design (b) are clustered nearer the mean than the members of (a) are.

## THEORETICAL "BEST" ISOLATION DESIGNS

From the rules in the preceding section, it is possible to formulate the "best" theoretical isolation designs. "Best," in this context, means that the members of the alias set contain as close to the same number of high level factors as possible. These theoretical designs are presented in Tables 9 through 20 for the range of values of $M$ and $N$ which are included in the two level design portion of the AED program.

Table 9. $M=1.1$ Member of Alias Set

| $N$ | 2 | 3 | 4 | 5 | 1 | 7 | 1 | 9 | 10 | 19 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 1 | 1 |  |  |  |  |  |  |  |  |
| 3 |  | 1 |  |  |  |  |  |  |  |  |
| 4 |  |  | 1 |  |  |  |  |  |  |  |
| 3 |  |  |  | 1 |  |  |  |  |  |  |
| 1 |  |  |  |  | 1 |  |  |  |  |  |
| 0 |  |  |  |  |  | 1 |  |  |  |  |
| 9 |  |  |  |  |  |  | 1 |  |  |  |

Table 10. $M=2.3$ Members of Alias Set

| $N$ | 2 | 3 | 4 | 5 | 1 | 1 | 6 | 2 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 3 |  |  |  |  |  |  |  |  |  |
| 4 | 1 | 2 |  |  |  |  |  |  |  |  |
| 5 |  | 2 | 1 |  |  |  |  |  |  |  |
| 0 |  |  | 3 |  |  |  |  |  |  |  |
| 7 |  |  | 1 | 2 |  |  |  |  |  |  |
| 0 |  |  |  | 2 | 1 |  |  |  |  |  |
| 10 |  |  |  |  | 3 |  |  |  |  |  |

Table 11. $M=3.7$ Members of Alias Set

| $N$ | 2 | 3 | 4 | 5 | 0 | 7 | 8 | 9 | 10 | 11 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | 6 |  | 1 |  |  |  |  |  |  |  |
| 5 | 2 | 4 | 1 |  |  |  |  |  |  |  |
| 6 |  | 4 | 3 |  |  |  |  |  |  |  |
| 7 |  |  | 7 |  |  |  |  |  |  |  |
| 8 |  |  | 3 | 4 |  |  |  |  |  |  |
| 9 |  |  | 1 | 4 | 2 |  |  |  |  |  |
| 10 |  |  |  | 3 | 3 | 1 |  |  |  |  |
| 11 |  |  |  | 1 | 3 | 3 |  |  |  |  |

Table 12. $M=4$. 15 Members of Alias Set

| $N$ | 2 | 3 | 4 | 5 | 6 | 7 | 1 | 9 | 10 | 11 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | 6 | 1 | 1 |  |  |  |  |  |  |  |
| 0 | 2 | 1 | 3 |  |  |  |  |  |  |  |
| 7 |  | 0 | 7 | 2 |  |  |  |  |  |  |
| 1 |  | 2 | 7 | 0 |  |  |  |  |  |  |
| 0 |  |  | 5 | 1 | 2 |  |  |  |  |  |
| 10 |  |  | 1 | 1 | 1 |  |  |  |  |  |
| 11 |  |  |  | 5 | 7 | 3 |  |  |  |  |
| 12 |  |  |  | 1 | 7 | 7 |  |  |  |  |

Table 13. $M=5$. 31 Members of Alias Set

| $\omega$ | 2 | 3 | 4 | 5 | 6 | 7 | 6 | 0 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 6 | 16 | 0 |  |  |  |  |  |  |  |
| 7 |  | 14 | 15 | 2 |  |  |  |  |  |  |
| 6 |  |  | 20 |  | 2 |  |  |  |  |  |
| 9 |  |  | 13 | 16 | 2 |  |  |  |  |  |
| 10 |  |  | 3 | 10 | 10 |  |  |  |  |  |
| 11 |  |  |  | 13 | 18 | 3 |  |  |  |  |
| 12 |  |  |  | 5 | 15 | 11 |  |  |  |  |
| 13 |  |  |  |  | 12 | 10 | 3 |  |  |  |

Table 14. $M=6$. 63 Members of Alias Set

| $N$ | 2 | 3 | 4 | 5 | 6 | 7 | 1 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 |  |  | 30 | 31 | 2 |  |  |  |  |  |
| 9 |  |  | 14 | 31 | 18 |  |  |  |  |  |
| 9 |  |  |  | 29 | 32 | 2 |  |  |  |  |
| 10 |  |  |  | 13 | 32 | 18 |  |  |  |  |
| 11 |  |  |  |  | 29 | 31 | 3 |  |  |  |
| 12 |  |  |  |  | 13 | 31 | 19 |  |  |  |
| 13 |  |  |  |  |  | 21 | 32 | 3 |  |  |
| 14 |  |  |  |  |  | 12 | 32 | 19 |  |  |

Table 15. $M=7 . \quad 127$ Members of Alias Set

| $N$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8 |  | 30 | 63 | 35 |  |  |  |  |  |  |
| 9 |  |  | 61 | 64 | 2 |  |  |  |  |  |
| 10 |  |  | 29 | 64 | 34 |  |  |  |  |  |
| 11 |  |  |  | 61 | 63 | 3 |  |  |  |  |
| 12 |  |  |  | 29 | 63 | 35 |  |  |  |  |
| 13 |  |  |  |  | 60 | 64 | 3 |  |  |  |
| 14 |  |  |  |  | 28 | 64 | 35 |  |  |  |
| 18 |  |  |  |  |  | 60 | 63 | 4 |  |  |

Table 16. $M=8$. 255 Members of Alias Set

| $N$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :--- |
| 9 |  | 2 | 125 | 126 |  |  | 2 |  |  |  |
| 10 |  |  | 61 | 128 | 66 |  |  |  |  |  |
| 11 |  |  |  | 125 | 127 | 3 |  |  |  |  |
| 12 |  |  |  | 61 | 127 | 67 |  |  |  |  |
| 13 |  |  |  |  | 124 | 128 | 3 |  |  |  |
| 14 |  |  |  |  | 60 | 128 | 67 |  |  |  |
| 15 |  |  |  |  |  | 124 | 127 | 4 |  |  |
| 16 |  |  |  |  |  | 60 | 127 | 68 |  |  |

Table 17. $M=9.511$ Members of Alias Set

| $N$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 10 |  |  | 125 | 256 | 130 |  |  |  |  |  |
| 11 |  |  |  | 253 | 255 | 3 |  |  |  |  |
| 12 |  |  |  | 125 | 255 | 131 |  |  |  |  |
| 13 |  |  |  |  | 252 | 256 | 3 |  |  |  |
| 14 |  |  |  |  | 124 | 256 | 131 |  |  |  |
| 15 |  |  |  |  |  | 252 | 255 | 4 |  |  |
| 16 |  |  |  |  |  | 124 | 255 | 132 |  |  |
| 17 |  |  |  |  |  |  |  |  |  |  |

Table 18. $M=10.1023$ Members of Alias Set

| $N$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 |  |  | 48 | 462 | 462 | 50 | 1 |  |  |  |
| 12 |  |  |  | 253 | 511 | 259 |  |  |  |  |
| 13 |  |  |  |  | 508 | 512 | 3 |  |  |  |
| 14 |  |  |  |  | 252 | 512 | 259 |  |  |  |
| 15 |  |  |  |  |  | 508 | 511 | 4 |  |  |
| 16 |  |  |  |  |  | 252 | 511 | 280 |  |  |
| 17 |  |  |  |  |  |  | 507 | 512 | 4 |  |
| 18 |  |  |  |  |  |  | 251 | 512 | 250 |  |

Table 19. $M=11 . \quad$ 2047 Members of Alias Set

| N | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12 |  |  |  | 508 | 1023 | 515 |  |  |  |  |
| 13 |  |  |  |  | 1020 | 1024 | 3 |  |  |  |
| 14 |  |  |  |  | 508 | 1024 | 515 |  |  |  |
| 15 |  |  |  |  |  | 1020 | 1023 | 4 |  |  |
| 16 |  |  |  |  |  | 508 | 1023 | 516 |  |  |
| 17 |  |  |  |  |  |  | 1019 | 1024 | 4 |  |
| 18 |  |  |  |  |  |  | 507 | 1024 | 516 |  |
| 19 |  |  |  |  |  |  |  | 1019 | 1023 | 5 |

Table 20. $M=12.4095$ Members of Alias Set

| $N$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 13 |  |  |  |  | 2044 | 2048 | 3 |  |  |  |
| 14 |  |  |  |  | 1020 | 2048 | 1027 |  |  |  |
| 15 |  |  |  |  |  | 2044 | 2047 | 4 |  |  |
| 18 |  |  |  |  |  | 1020 | 2047 | 1028 |  |  |
| 17 |  |  |  |  |  |  | 2043 | 2048 | 4 |  |
| 18 |  |  |  |  |  |  | 1019 | 2048 | 1028 |  |
| 19 |  |  |  |  |  |  |  | 2043 | 2047 | 5 |
| 20 |  |  |  |  |  |  |  | 1019 | 2047 | 1029 |

To illustrate the use of the tables, consider the previous example with $M=3$ and $N=5$. In Table 11 headed $M=3$, in the row for $N=5$, we find the number 2 under the column headed "2," the number 4 under the column headed "3," and the number 1 under the column headed "4." This is read as

$$
2(2)=4(3)+1(4)
$$

The optimum isolation design for this case would result in an alias set conaining two members consisting of two factors, four members consisting of three factors,
and one member consisting of four factors. Since the alias set contains two factor terms, it is not possible to isolate main effects.

From inspection of Tables 9 through 20, it can be seen that within the constraints of the AED program (number of trials $\leqq 256$ ) and realizability, for a given value of $M$, the number of factors must equal or exceed the number shown in the following table to permit isolating all main and two factor effects:

| $M$ | $\underline{N} \geqq$ |
| ---: | ---: |
| 1 | 5 |
| 2 | 8 |
| 3 | 10 |
| 4 | 11 |
| 5 | 11 |
| 6 | 11 |
| 7 | 11 |
| 8 | 11 |
| 9 | 11 |
| 10 | 12 |
| 11 | 12 |
| 12 | 13 |

Alternatively, for a given value of $N$ (number of factors), the allowable values of $M$ are shown in the following table to permit complete isolation of main and two factor interactions:

| $\frac{N}{2}$ | M |
| :--- | :--- |
| 3 | Not possible |
| 4 | Not possible |
| 5 | Not possible |
| 6 | $M=1$ |
| 7 | $M=1$ |
| 8 | $M=1$ |
| $M=1,2$ |  |


| 9 | $M=1,2$ |
| :--- | :--- |
| 10 | $M=2,3$ |
| 11 | $M=3,4,5,6,7,8,9,10$ |
| 12 | $M=4,5,6,7,8,9,10,11$ |
| 13 | $M=5,6,7,8,9,10,11,12$ |
| 14 | $M=6,7,8,9,10,11,12$ |
| 15 | $M=7,8,9,10,11,12$ |
| 16 | $M=8,9,10,11,12$ |
| 17 | $M=9,10,11,12$ |
| 18 | $M=10,11,12$ |
| 19 | $M=11,12$ |
| 20 | $M=12$ |

Similar tables could be constructed for isolation of main effects only (all members of alias set contain at least four factors), isolation of main, two factor, and three factor interactions (all members of the alias set contain at least six factors), etc.

## REALIZABLE DESIGNS

All of the designs presented in the Tables 9 through 20 are not realizable, particularly in those cases in which $M$ approaches $N$. However, inspection of the theoretically best isolation designs will show if the desired results in terms of isolation could be obtained if the design can be found. If the design is not adequate, perhaps more trials should be added (reduce $M$ ) to improve the isolation. If the table design is adequate, the next step is to determine if the design is stored in the AED program. The designs stored in the AED program are the best realizable isolation designs that have been found to date.

If the desired design does not reside in the AED program, a realizable design can be constructed following the procedure described in the next section. Variants of this procedure can also be used to modify stored designs to meet special needs of the experimenter.

## ALIAS SET MODIFICATIONS AND THE GENERATION OF NEW DESIGNS

The alias set is generated by developing all of the possible combinations of the defining contrasts. As an example, if three defining contrasts, $(M=3)$, say $A B, B C$, and $C D$ are used in a four factor problem ( $N=4$ ), the resulting alias set is generated as follows:

| 1 | Defining Contrast No. 1 | $=A B$ | 1100 |
| ---: | :--- | ---: | :--- |
| 2 | 2 factors |  |  |
| 2 | Defining Contrast No. 2 | $=B C$ | 0110 |
| 2 factors |  |  |  |
| $1 \times 2$ | Product of 1 and 2 | $=A C$ | 1010 |
| 2 factors |  |  |  |
| 3 | Defining Contrast No. 3 | $=C D$ | 0011 |
| $2 \times 3$ factors |  |  |  |
| $2 \times 3$ Product of 1 and 3 | $=A B C D$ | 1111 | 4 factors |
| $1 \times 2 \times 3$ Product of 1,2 and 3 | $=B D$ | 0101 | 2 factors 3 |

This is a $6(2)+1(4)$ design

A new design for $\mathrm{N}=5$ can be built from this design by adding another column. The goal should be to increase the two factor terms and not increase the four factor (1 $\times 3$ ) term.

This can be done as follows:

| Contrast Product | Original $\qquad$ | Added Column |  |
| :---: | :---: | :---: | :---: |
| 1 | 1100 | 1 | 3 factors |
| 2 | 0110 | 1 | 3 factors |
| Contrast Product | Original Design | Added <br> Column |  |
| 12 | 1010 | 0 | 2 factors |
| 3 | 0011 | 1 | 3 factors |
| 13 | 1111 | 0 | 4 factors |
| 23 | 0101 | 0 | 2 factors |
| 123 | 1001 | 1 | 3 factors |

This results in a $2(2)+4(3)+1(4)$ design for $N=5$ which means the best theoretical design from Table 11 for $N=5$ and $M=3$.

The defining contrasts for this design are:

$$
\begin{aligned}
& 11001=A B E \\
& 01101=B C E \\
& 00111=C D E
\end{aligned}
$$

Now assume we wish to build a design for $N=6$. This can ' done by generating another new column. The new column should increase the two factor terms ( $1 \times 2$ and $2 \times 3$ ) and not increase the four factor term ( $1 \times 3$ ). This will result in the following design:

| Contrast <br> Product | 5 Factor <br> Design |  | Added <br> Column |  |
| :---: | :--- | :---: | :--- | :--- |
| 1 | 11001 |  | 0 | 3 factors |
| 2 | 01101 |  | 1 | 4 factors |
| 12 | 10100 |  | 1 | 3 factors |
| 3 | 00111 | 0 | 3 factors |  |
| 13 | 11110 | 0 | 4 factors |  |
| 23 | 01010 |  | 1 | 3 factors |
| 123 | 10011 |  | 1 | 4 factors |

This results in 4(3) + 3(4) design, which again is a best isolation design. In general, as $M$ increases, this procedure will not result in best isolation designs, but all designs will be achievable.

The process can be used in reverse by removing columns. In fact, a poor design can be improved by removing columns, then adding columns by paying attention to the numbers of factors in the rows. To illustrate, assume the following ten factor design is available:

| Contrast Product | 10 Factor Design |  |
| :---: | :---: | :---: |
| 1 | 1111110000 | 6 factors |
| 2 | 0001111100 | 5 factors |
| 12 | 1110001100 | 5 factors |
| 3 | 1101010011 | 6 factors |
| 13 | 0010100011 | 4 factors |
|  |  | -37- |

## 7 factors

123
001101111

## 7 factors

This is a $1(4)+2(5)+2(6)+2(7)$ design. The first step is to reduce this to a nine factor design by removing a column. Columns 3,5,9, and 10 should not be removed since this would reduce the four factor member ( $1 \times 3$ ). In the same manner, columns 3,4 , and 6 should not be removed since the seven factor term ( $2 \times 3$ ) would not be reduced. Also columns 1,2 , and 5 should not be removed since this would not reduce the seven factor term $1 \times 2 \times 3$. This leaves columns 7 and 8 as candidates for removal. Removing column 7, we are left with a nine factor design. A new column can then be added as shown, which now results in a best ten factor design.

| Contrast Product | Resulting 9 Factor Design | Factors | Added Column | Factors |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 111111000 | 6 | 0 | 5 |
| 2 | 050111100 | 4 | 1 | 5 |
| 12 | 111000100 | 4 | 1 | 5 |
| 3 | 11010101 | 5 | 1 | 7 |
| 13 | 001010011 | 4 | 1 | 5 |
| 23 | 11001011 | 6 | 0 | 6 |
| 123 | ווווטו1ו00 | 6 | 0 | 6 |

the nine factor desigr is a $3(4)+4(6)$ design. The new ten factor design is a $3(5)+3(6)+1(7)$ design as compared with our starting point of a $1(4)+$ $2(5)+2(6)+2(7)$ design. Thus, we have arrived at a much improved design.

## ALIAS SUMMARY

The primary concern of the experimenter is directed toward examining how all main effects and first-order interactions are aliased. The aliasing for any effect can be found by multiplying the effect by the total alias set (applying modulo $P$ arithmetic to the factor exponents).

For example, consider a design consisting of 6 factors each at 2 levels with an M of 2. this is a one-fourth design, and two defining contrasts must be specified by the experimenter. Assume that the identities selected are $I=A B C E=A B D F$. Following the rules given in the previous sections, we find that each effect is aliased with three effects; and the complete alias set consists of ABCE, ABDF, and CDEF. The main effects and first-order interactions and how they are aliased are illustrated in Table 21.

Individual effects or interactions can be examined by multiplying the particular effect by each member of the alias set. For example, with $I=A B C E=A B D F=$ CDEF, the interaction EF can be found to be aliased as:

$$
\begin{aligned}
& (I)(E F)=(A B C D)(E F)=(A B D F)(E F)=(C D E F)(E F) \\
& E F=A B C E^{2} F=A B D E F^{2}=C D E^{2} F^{2} \\
& E F=A B C F=A B D E=C D
\end{aligned}
$$

when all exponents are reduced modulo 2.

If the experiment becomes large, it may be difficult to read the alias summary as shown in Table 21. Therefore, an abbreviated summary (Table 22) is provided to show how much main effect and first-order interaction is aliased with main effects and first- and higher-order interactions. An exmaination of this table shows that all main effects are aliased only with higher-order interactions. This is an acceptable design for a fractional factorial experiment. Refer to Table 9 to find the specific aliased terms. The particular design acceptability criteria depend upon the problem being studied.

## dESIGN EVALUATION

Once the alias summary has been generated, the experimenter must determine if the design is acceptable. The acceptability of a design depends upon the aliasing of those effects assumed to be significant by the experimenter. If an insignificant effect is aliased with a significant effect, the design is considered to be a good design. If a few significant effects are aliased, however, the user may define another design or he might use the current design and let the data analysis indicate if an effect that consists of the combination of two potentially significant effects is significant. If a combined effect is significant, the design can be refined to perform the required effect separation (refer to Section 8, Refining Designs).

Once an acceptable design has been generated, the specific observation vectors used to collect data must be found. This collection of observation vectors is called the basic experimental block.

## BASIC BLOCK DEFINITION

Once the total alias set has been defined, the specific treatment combinations used to collect data must be found. The details of the construction of this block may be skipped by the novice user.

The $M$ members of the defining contrast set are used to generate the block of treatments and are consequently called generators. If the generators are denoted by

Table 21. Alias Example

| Effect | Aliased mith: |  |  |
| :---: | :---: | :---: | :---: |
| $\wedge$ | ece | 80\% | acder |
| - | ace | a ${ }^{\text {d }}$ | ceder |
| c | ABE | Ascdr | OEF |
| 0 | ABCDE | A AFP | cef |
| E | asc | abdef | CDF |
| F | ABCEF | ABD | CDE |
| AE | CE | DF | abcdef |
| AC | BE | bcof | adef |
| AD | bcde | BF | acef |
| AE | BC | boter | acde |
| AF | bcef | 80 | acde |
| ec | AE | acof | edef |
| so | acde | AF | bcef |
| BE | AC | adef | ectaf |
| BF | acef | AD | ecde |
| CD | Abot | ABCF | EF |
| CE | AB | abcdef | OF |
| cF | abef | ABCD | DE |
| DE | AECD | Abef | CF |
| DF | abcoef | AB | CE |
| EF | ABCF | abot | CD |

Table 22. Abbreviated Alias Summary

| EFFECT | MAIN | 1ST ORDER | higher order |
| :---: | :---: | :---: | :---: |
| A | 0 | 0 | 3 |
| 8 | 0 | 0 | 3 |
| c | 0 | 0 | 3 |
| D | 0 | 0 | 3 |
| E | 0 | 0 | 3 |
| F | 0 | 0 | 3 |
| AB | 0 | 2 | 1 |
| AC | 0 | 1 | 2 |
| AD | 0 | 1 | 2 |
| AE | 0 | 1 | 2 |
| AF | 0 | 1 | 2 |
| BC | 0 | 1 | 2 |
| 80 | 0 | 1 | 2 |
| 8E | 0 | 1 | 2 |
| BF | 0 | 1 | 2 |
| co | 0 | 1 | 2 |
| CE | 0 | 2 | 1 |
| cF | 0 | 1 | 2 |
| DE | 0 | 1 | 2 |
| DF | 0 | 2 | 1 |
| EF | 0 | 1 | 2 |

$\qquad$

$$
G_{i}=A^{a j} B_{B i c c i} \ldots i=1,2, \ldots, M,
$$

then the levels in the factorial combination $\times 1 \times 2 \times 3 \ldots$ selected for the block satisfy simultaneously the $M$ equations

$$
a_{i} x_{1}+b_{j} \times 2+c ; x_{3}+\ldots=0(\operatorname{modulo} P)
$$

$$
\mathfrak{i}=1,2, \ldots, M
$$

Similar equations in which $1, \ldots,(P-1)$ is used in place of 0 are equally valid; however, the set of treatments defined using the 0 , called the Basic Block or the principal block, will be used here.

Consider the one-fourth replicate of an experiment with six factors at two levels, where the defining contrasts are $1=A B C D=A B D F$. (The total alias set is $1=A B C D=A B D F=C D E F$.) The defining contrasts define two generating equations:

$$
\begin{aligned}
& \mathrm{G}_{1}=A^{1} 1_{B}^{1} C^{1} D_{D E} 1_{F} 0 \\
& \mathrm{G}_{2}=A_{B}{ }_{B} C_{C O D} 1_{E} O_{F}
\end{aligned}
$$

The simultaneous equations to be solved are:

$$
\begin{array}{ll}
x_{1}+x_{2}+x_{3}+x_{5} & =0 \text { modulo } 2 \\
x_{1}+x_{2}+x_{4}+x_{6} & =0 \text { modulo } 2
\end{array}
$$

Each of the $26=64$ factor combinations is evaluated using this system of equations, and those combinations satisfying the equations form the basic experimental block. Table 23 shows all 26 treatment combinations, and the 16 that form the basic block are shown with an asterisk.

The basic block (or observation vector) for the example is given in Table 24. Note that 0 indicates the factor is at its low level whereas a 1 indicates a factor is at its highest level. Once the basic block is defined, the experimenter must collect the experimental data. Data collection procedures are discussed in the next section.
Table 23. Basic Experimental Block



Table 24. Basic Block Summary

| EXPERIMENTALUNIT | Facton level |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | 8 | c | D | E | $F$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 1 | 0 | 1 |
| 3 | 0 | 0 | 1 | 0 | 1 | 0 |
| 4 | 0 | 0 | 1 | 1 | 1 | 1 |
| 5 | 0 | 1 | 0 | 0 | 1 | 1 |
| 6 | 0 | 1 | 0 | 1 | 1 | 0 |
| 7 | 0 | 1 | 1 | 0 | 0 | 1 |
| 8 | 0 | 1 | 1 | 1 | 0 | 0 |
| 9 | 1 | 0 | 0 | 0 | 1 | 1 |
| 10 | 1 | 0 | 0 | 1 | 1 | 0 |
| 11 | 1 | 0 | 1 | 0 | 0 | 1 |
| 12 | 1 | 0 | 1 | 1 | 0 | 0 |
| 13 | 1 | 1 | 0 | 0 | 0 | 0 |
| 14 | 1 | 1 | 0 | 1 | 0 | 1 |
| 18 | 1 | 1 | 1 | 0 | 1 | 0 |
| 18 | 1 | 1 | 1 | 1 | 1 | 1 |

## dATA COLLECTION

Using the observation vectors defined in the basic experimental block, the data collection process is relatively straightforward. The different combinations in the basic block are tested, and the response value is measured.

No consideration has been given to the specific order in which the observation vectors are to be run. The general procedure is to select random combinations until each of the observation vectors in the basic block has been run. This is acceptable unless the experimenter considers a change in the system over time. A system change can be overcome by dividing the basic block into smaller blocks. The assumption is made that the system is relatively homogeneous within each block. Techniques for the construction of blocks are identical to those used to build the alias set. An effect is selected, and the effect equation is generated. For example, consider effect AC. The value of the effect equation for AC equals 0 modulo 2 goes into one block, and the effect equation for $A C$ equals 1 modulo 2 goes into a second block. This causes effect $A C$ to be no longer measurable, i.e., the experimenter cannot know if a response is due to the interaction $A C$ or to a change between blocks.

Blocking procedures are not included in this program. This capability will be added at a later date. The inclusion of this feature, however, requires a trained, experienced user.

Once the data have been collected, they must be analyzed to identify significant effects. This analysis consists of an analysis of variance (ANOVA) or of a regression analysis. Because details concerning ANOVA and regression analysis methods may be found in any statistical analysis text, they have not been included here.

## REFINING DESIGNS

When data have been collected and analyzed from a fractional factorial experiment, the experimenter may determine that he wishes to further examine certain effects or interactions that were confounded in the original design. Finding
a new design in which these effects are separated is called refining the design. This is accomplished by dividing one of the effects to be separated by the other to yield a member of the alias set. This division is defined as a subtraction of corresponding exponents where an exponent is increased by $P$ whenever the subtrahend would have been larger than the minuend. For example, if $P=3$,


The member of the power set that was used to generate this member of the alias set is examined for non-zero columns. Removal of any one of the defining contrasts represented by these columns results in separating the two effects, (i.e., if the member of the power series was found to be 210 , removal of either the first or the second defining contrast would accomplish the desired separation).

Removal of a defining contrast results in doubling the total number of trials in a two-level experiment, and tripling the total number of trials in a three-level experiment, although the first portion of the experiment may have already been completed before removal of the defining contrast. This is the price paid for reducing the confounding.

## IRREGULAR FRACTIONAL FACTORIAL EXPERIMENTS

The generation of fractional experiments uses a $1 /{ }^{( } N$ design, i.e., a $1 / 8$, a $1 / 27$, or a $1 / 64$ design. Although any fraction such as $k / P^{N}$ may be constructed, these designs have many problems. Consider the case with five factors at two levels, but with only 24 observation vectors available. One possibility would be a $3 / 4$ design based on using a $1 / 2$ and $1 / 4$ replicate design. If this is used, the total design will have highly correlated estimates that could lead to extremely difficult tests of significance.

Another approach would be to use a $1 / 2$ replicate and a $1 / 2$ replicate of the unused portion of the larger design. Because of confounding, however, this design provides less information to the experimenter than the $1 / 2$ replicate alone. For these reasons, fractional designs other than a $1 / \mathrm{PN}$ are not advantageous.

## SCREENING DESIGNS

One of the common information objectives of human factors engineering research (and indeed of many experiments in general) is to determine the factors that produce a certain result and the relative importance of these factors.

Before a formal experiment can ever be conducted, an investigator is generally required to cull the list of possibly hundreds of potential factors by considering such criteria as:

Information gained from related research
Practical constraints of time and money
Customer interest
Rational analysis.

Once the least interesting factors have been pared from the list of potentials, fractional factorial designs used as sequential experiments can be of real value.

The term sequential design is generally used to describe experiments in which the yield or response on any unit is known before the experimenter treats the next unit. When an experiment is sequential, the experimenter can stop after every observation or group of observations and examine the results to date before deciding how or whether to continue the experiment.

The term screening design is generally used to describe experiments where an investigator starts with a relatively large number of factors, and by performing a small number of trials, (using a highly fractionated design) can determine that some of the factors are of little significance. These factors can then
be eliminated or screened from further consideration, and another experiment can be designed from the remaining factors. Once the factors that are of importance relative to a given error of prediction have been identified, it may be necessary to perform detailed work on these factors, perhaps using more levels per factor, or creating a mixed level design, or using a central composite design.

## RESPONSE SURFACE DESIGNS

In many investigations, the final goal is to study how the response varies in an experimental region and perhaps to determine the factor combination yielding an optimum response (if an optimum response is suspected to exist). If the $N$ significant factors are all quantitative with their levels denoted $X_{1}, X_{2}$, $\ldots, X_{N}$ and the response is quantitative with its level denoted $Y$, then $Y$ is a function of the $x_{1}, x_{2}, \ldots, X_{N}$; i.e., $Y=\emptyset\left(x_{1}, x_{2}, \ldots, x_{N}\right)$. The function $\square$ is called the response function or response surface. Then the final goal of the investigation is to determine the behavior of the function $\varnothing$ in the experimental region.

APPROXIMATIONS OF RESPONSE SURFACES BY POLYNOMIALS

## THE RESPONSE SURFACE

Suppose all $N$ factors and the response are quantitative and continuous. Denote the levels of the factors by $X_{1}, X_{2}, \ldots, X_{N}$ and denote the level of the response by $Y$. Then $Y$ is a function $\emptyset$ of $X_{1}, X_{2}, \ldots, X_{N}$ and we can write $Y=\varnothing\left(X_{1}\right.$, $X_{2}, \ldots, X_{N}$ ). The function $\emptyset$ is called the response function. We can also interpret $Y=0\left(X_{1}, X_{2}, \ldots, X_{N}\right)$ as the equation of a surface in $N+1$ dimensional space, called the response surface. The terms response surface and response function will be used interchangeably.

Note that we have restricted the concept of a response surface to the case where the response and all factors are quantitative. If only some of the factors are quantitative, say the first $K$, then one could fix the levels of the qualitative factors, so that they become constants instead of factors. One could then talk about the response function $Y=\emptyset\left(X_{1}, X_{2}, \ldots X_{K}\right)$. Whether
or not consideration of this restricted response function is worthwhile, and if so, at which levels the qualitative factors should be fixed, must be determined by the investigator.

## POLYNOMIAL APPROXIMATIONS

In most investigations the nature of the response surface (i.e., the form of the response function) is unknown, so a goal of actually determining the true response function is unattainable. Consequently, we will be satisfied if we can find an approximating function such that the value of the approximating function, in the experimental region under investigation, is sufficiently close to the observed response for our purposes.

The only approximating functions we will consider will be polynomials in the levels of the factors. Polynomials are relatively easy to work with, and in a given region any continuous function can be approximated to any desired degree of accuracy by a polynomial of sufficiently high degree.

For example, suppose an investigation involves three factors. Then some of the possible polynomial approximations are as follows:

1. A linear approximation is a polynomial of degree one and is of the form:

$$
Y=B_{0}+B_{1} x_{1}+B_{2} x_{2}+B_{3} x_{3}
$$

2. A quadratic approximation is a polynomial of degree two and is of the form:

$$
\begin{aligned}
Y= & B_{0}+B_{11} x_{1}^{2}+B_{22} x_{2}^{2}+B_{3} x_{3} \\
& +B_{11} x_{1}^{2}+B_{22} x_{2}^{2}+B_{33} x_{3}^{2} \\
& +B_{12} x_{1} x_{2}+B_{13} x_{1} x_{3}+B_{23} x_{2} x_{3}
\end{aligned}
$$

3. A cubic approximation is a polynomial of degree three and is of the form:

$$
\begin{aligned}
Y= & B_{0}+B_{1} x_{1}+B_{2} x_{2}+B_{3} x_{3}+B_{11} x_{1}^{2}+B_{22} x_{2}^{2} \\
& +B_{33} x_{3}^{2}+B_{12} x_{1} x_{2}+B_{13} x_{1} x_{3}+B_{23} x_{2} x_{3} \\
& +B_{111} x_{1}^{3}+B_{222} x_{2}^{3}+B_{333} x_{3}^{3}+B_{112} x_{1}^{2}+B_{122} x_{1} x_{2}^{2} \\
& +B_{113} x_{1}^{2} x_{3}+B_{133} x_{1} x_{3}^{2}+B_{223} x_{2}^{2} x_{3}+B_{233} x_{2} x_{3}^{2}+B_{123} x_{1} x_{2} x_{3}
\end{aligned}
$$

A polynomial of degree greater than three is a straightforward extension of the above cases. However, such polynomials are relatively unlikely to be used for approximating response surfaces.

Note that each of the above polynomials is a complete polynomial in that it contains all possible terms of degree less than or equal to the degree of the polynomial. Also, in the above polynomials each term can be interpreted as a contribution to the response level $Y$ due to a certain one-factor effect or to a factor interaction. For instance, in the quadratic case
$B_{0}$ is the base level of the response (the response level when all factors are at level zero)
$B_{i} X_{i}$ is the linear effect of the $i$-th factor
$B_{i j} X_{i}^{2}$ is the quadratic effect of the $i-t h$ factor
$B_{i j} X_{i} X_{j}$ is the two-factor interaction effect of the $\boldsymbol{i}$-th and $\boldsymbol{j}$-th factors.
When we choose the degree of the approximating polynomial, we make certain implicit assumptions about the significance of certain effects. If we choose a linear approximation, we are assuming that all two-factor interactions and
all quadratic one-factor effects are not significant; if we choose a quadratic approximation, we are assuming that all three-factor interactions and all cubic one-factor effects are not significant. If these implicit assumptions are incorrect, the approximating polynomial is likely to fit the observed responses poorly.

The polynomial degree chosen may also vary with the purpose of the approximation and may be different at different phases of a sequential design. For instance, in an investigation whose goal is to determine an optimum response we would normally use linear approximations at each step where we use the method of steepest ascent (see Cochran \& Cox, 1957) but then go to a quadratic approximation at the point where the method of steepest ascent finally breaks down.

## DATA REQUIREMENTS

Having decided what degree of polynomial approximation to use, we must design an experiment that will yield data sufficient to allow a determination of the coefficients of the polynomial. In particular, the following restrictions apply.

1. The number of levels at which the i-th factor occurs must be at least one greater than the hightest power of $X_{\mathfrak{i}}$ in the polynomial.
2. The total number of distinct factor combinations (experimental trials) must be at least as large as the number of polynomial coefficients to be determined.
3. The design of the experiment should not allow any two terms in the polynomial to have their corresponding factor combinations aliased with one another.

Even these three conditions do not guarantee that the experimental data are sufficient to allow determination of all polynomial coefficients. However, an appropriate central composite design (discussed later) will always be sufficient to determine all coefficients of the desired quadratic polynomial.

## USE OF INCOMPLETE POLYNOMIALS

To this point it has been assumed that the approximating polynomial is a complete polynomial, i.e., that it contains all possible terms of degree less than or equal to the degree of the polynomial. This need not be the case. From a complete polynomial we could omit those terms corresponding to effects that are known to be not significant.

For example, suppose there are 6 factors $A, B, C, D, E$, and $F$ but that it is known that the only significant interactions are the two-factor interactions between $A, B$, and $C$. Then instead of the complete quadratic polynomial, which contains 28 terms, one could use the following as an approximating polynomial:

$$
\begin{aligned}
Y & =B_{0}+\sum_{i=1}^{6} B_{i} X_{i}+\sum_{i=1}^{6} B_{i i} X^{2} \\
& +B_{12} X_{1} x_{2}+B_{13} x_{1} x_{3}+B_{23} x_{2} x_{3}
\end{aligned}
$$

This incomplete polynomial contains only 16 terms; twelve two-factor interaction terms have been omitted. The possible advantage of an incomplete polynomial is that one may be able to generate sufficient data (to determine the polynomial coefficients) from a smaller experiment than is required for the complete polynomial. The notation $P^{N-M}$ where $P$ is the number of levels, $N$ is the number of factors, and $M$ is the number of defining contrasts will be used in the following examples to describe the factorial case under consideration. In the above example, a $2^{6-1}$ factorial (e.g., with $I=A B C D E F$ ) would be required to determine all two-factor interactions in the complete polynomial, but a $2^{6-2}$ factorial (e.g., with $I=A B C D=A B E F$ ) would allow determination of all necessary two-factor interactions in the complete polynomial.

THE CENTRAL COMPOSITE DESIGN

The central composite design is specifically intended to allow determination of a quadratic approximation of the response surface. It is a composite or combination of a full or fractional two-level factorial design and some additional experimental points selected in a particular manner so as to allow a good determination of the quadratic one factor effects. Thus the central
composite design is often appropriate when one suspects that the relationship between the response and the level of a factor is non-linear. In addition, the central composite design should give a good estimate of the response mean and provide an estimate of the precision of the mean.

The central composite design is often especially suitable in a sequential design process. It is sometimes a less costly alternative to three or fivelevel factorial designs. Some examples of situations where the central composite design might be used are as follows:

1. One has already run a full or fractional factorial experiment and now wants information about possible non-linearity and the shape of the response surface, plus a better estimate of the mean, with a minimum number of additional trials.
2. An investigation involves a small number of continuous quantitative factors and one wants as much information as possible quickly and at low cost.
3. One has already run a full or fractional two-level experiment and wants to expand the experimental region at low cost.

## DEFINITION OF THE CENTRAL CO!?POSITE DESIGY

An N -factor central composite design consists of three components.

1. A full or $1 / 2^{M}$ replicate of a $2^{N}$ factorial design, where for each factor the two coded levels are -1 and 1.
2. 2 N star or axial points. For each factor there are two corresponding axial points; the given factor has coded level $-\alpha$ at one point and $+\alpha$ at the other, whereas all other factors have coded level zero at both points.
3. The center point, where all N factors have coded level zero.


Figure 2. Central Composite Design

For example, a three-factor central composite design would be as follows, where the experimental point with first factor at coded level $X_{1}$, second factor at coded level $X_{2}$, and third factor at coded level $x_{3}$ is represented by the vector $\left(x_{1}, x_{2}, x_{3}\right)$.

1. A full $2^{3}$ factorial: $(-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,1,1)$.

$$
(1,-1,-1),(1,-1,-1),(1,1,-1),(1,1,1)
$$

2. 6 axial points: $(-\alpha, 0,0),(\alpha, 0,0),(0,-\alpha, 0),(0, \alpha c)$.

$$
(0,0,-\alpha),(0,0, \alpha)
$$

3. The center point: $(0,0,0)$

## CODED VS. REAL WORLD LEVELS

In the central composite design the coded or formal levels of each factor are $-\alpha,-1,0,1$, and $\alpha$. The real world levels of a factor (i.e., the true quantitative levels of the factor that correspond to the coded levels) must be determined by the researcher. In determining the real world experimental range of levels for a factor, whether for a central composite design or some other design, the following should be considered.

1. If the chosen range is too small, the researcher may erroneously conclude that the factor is not significant (e.g., in a pilot study to determine significant factors), that the one-factor effect of that factor is linear, or that the factor does not interact with other factors.
2. If the chosen range is too large, effects of order higher than the degree of the approximating polynomial may become significant. This could result in a poor fit between the approximating polynomial and the observed data.

Having decided upon the real world experimental range to be used for a particular factor, we can use the following linear transformation to determine the real world level corresponding to any formal level.

$$
r=\frac{\left(r_{U}-r_{L}\right)}{2 \alpha} f+\frac{\left(r_{U}+r_{L}\right)}{2}
$$

Here $r$ is the real world level, $f$ is the formal or coded level, and $r_{L}$ and $r_{U}$ are the lower and upper value in the chosen real world range.

If a full or fractional 2 level factorial experiment has already been completed, we can use the following linear transformation to determine the real world level corresponding to any formal level.

$$
r=\frac{\left(r_{+1}-r_{-1}\right)}{2} f+\frac{\left(r_{+1}+r_{-1}\right)}{2}
$$

Here $r$ is the real world level, $f$ is the formal or coded level, and $r_{-1}$ and $r_{+1}$ are the real world levels of the previously defined 2 level factorial experiment.

In certain instances one might replace the real world levels of a factor by a function of the levels. For instance, if one is using the real world range .001 to .1 but wishes to consider .01 as the midpoint of the range, then one could replace the raw values by their logarithms. This would give $r_{L}=\log$ $(.001)=-3, r_{U}=\log (.1)=-1$, and $\log (.01)=-2$ is midway between $r_{L}$ and $r_{U}$

The linear transformation would then be:

$$
r=-2+\frac{f}{a}
$$

The 5 levels of that factor occurring in the central composite design would be:

| coded or formal level | $-\alpha$ | -1 | 0 | 1 | $\alpha$ |
| :--- | :--- | ---: | :---: | :---: | :---: |
| real world level (log) | -3 | $-2-1 / \alpha$ | -2 | $-2+1 / \alpha$ | -1 |
| real world raw value | .001 | $10-2-1 / \alpha$ | .01 | $10-2+1 / \alpha$ | .1 |

## CHOICE OF THE LEVEL $\boldsymbol{\alpha}$

A rotatable design is one that leaves the variance of the estimated response unchanged when the design is rotated about the center. This means that the variance of the estimated response is the same at all points equidistant from the center $(0,0, \ldots, 0)$ of the design. Rotatable designs are desirable in situations where the researcher has no advance knowledge of the response surface or how it is oriented relative to the factor axes and thus has no knowledge of how the variance of the estimated response will vary along the surface.

Box and Hunter (1957) have shown that if the factorial part of the central composite design is a $2^{\mathrm{N}}-\mathrm{M}$ factorial in which one-factor effects and twofactor interactions are aliased only with higher order effects, then choosing $\alpha=2(N-M) / 4$ will make the design rotatable. Although rotatability is not critical, unless we have a reason for doing otherwise we would normally place all $2 N$ axial points at the distance $\alpha=2(N-M) / 4$ from the center point so that the central composite design is rotatable.

In certain situations, such as in a sequential design process where a $1 / 2^{\mathrm{M}}$ fractionation resulted from a prior experiment, there may be one or more factors for which the levels $\pm_{2}(N-M) / 4$ are not feasible. In such situations we might choose a value of $\alpha$ different from $2(N-M) / 4$. Alternatively, the axial points for those factors only could be chosen at a distance other than $\alpha$ from the center; in fact the two axial points for a factor could be at unequal distances from the center point.

The purpose of each pair of axial points is to increase the precision of the estimate of the corresponding quadratic term $B_{i j} X_{\}}$. Increasing the distances of these axial points from the center will decrease the variance of this estimated one-factor quadratic effect but will also increase its correlation with other such effects and will increase the danger of bias from higher order effects. On the other hand, if these axial points are within or very near the range from -1 to 1 then they may not significantly increase the precision of the estimate of the quadratic effect $B_{i j} X_{i}$.

## USE OF MULTIPLE OBSERVATIONS AT THE CENTER POINT

Using multiple regression techniques on the experimental data resulting from the central composite design or some other appropriate design, we can derive a polynomial least squares approximation to the response surface. From this process we would also like estimates of the lack of fit and of the experimental error.

1. The lack of fit estimate should indicate the significance of the totality of all effects without corresponding terms in the polynomial. If the approximation is a complete quadratic polynomial these would be the effects represented by terms of degree 3 or greater; e.g.,
$B_{i j i} X^{3}{ }_{i}, B_{i j j} X_{i}{ }^{2} X_{j}, B_{i j k} X_{i} X_{j} X_{k}$, etc.
2. The experimental error estimate is just an estimate of the experimental error variance, i.e., of the standard error of the estimated response.

In the central composite design we can provide for an estimate of the experimental error by repeating observations at the center point. If there are $n_{0}$ replications of the center point, they provide $n_{0}-1$ degrees of freedom for estimating the experimental error. Box and Hunter (1957) determined values of $n_{0}$ for which the variance of the estimated response is approximately the same at the center and at unity distance from the center of a rotatable central composite design. In this case the standard error is roughly the same at all points within the sphere of radius one and can be approximated by the standard error at the center point; the graph in Box and Hunter indicates that the
standard error increases fairly rapidly outside the sphere of radius one. From Box and Hunter's results it follows that the formula for such a value of $n_{0}$ is

$$
n_{0}=\left(2^{N-M}+4+4.2^{(N-M / 2}\right)\left(\frac{N+3+9 N^{2}+14 N-7}{4 N+8}\right)-2^{N-M}-2 N
$$

Table 25 gives such values of $n_{0}$ (rounded to the nearest integer) for rotatable central composite designs.

Table 25. Replicates of Center Point for Nearly Uniform Variance.

| $M$ | $M=0$ | $M=1$ | $M=2$ | $M=3$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 5 |  |  |  |
| 3 | 6 |  |  |  |
| 4 | 7 |  |  |  |
| 5 | 10 | 6 |  |  |
| 6 | 15 | 9 |  |  |
| 7 | 21 | 14 |  |  |
| 8 | 28 | 20 | 13 |  |
| 9 | 37 | 25 | 16 |  |
| 10 | 43 | 38 | 29 | 18 |

After deriving an approximate polynomial, one can do an analysis of variance. By partitioning the total sum of squares into a number of parts, each the contribution due to a source of interest, one can get a lack of fit estimate. For instance, for a quadratic approximation resulting from a central composite design one could partition the sum of squares into four parts:
(a) sum of squares due to first order terms
(b) sum of squares due to second order terms
(c) sum of squares due to experimental error (from the $n_{0}$ replications of the center point)
(d) sum of squares due to lack of fit.

The sum of squares in (d) is found by subtracting from the total sum of squares the sum or squares in (a), (b) and (c). By comparing the mean squares for the various parts in the partition, one can determine the relative significance of each part of the partition.

## RESTRICTIONS ON A CENTRAL COMPOSITE DESIGN

The following comments are appropriate to a discussion of the restrictions which must be placed on a central composite design in order to allow determination of a quadratic approximation of a response surface.

1. In that part of the central composite design consisting only of the axial and center points, each factor must appear at three separate levels $-\alpha, 0$, and $\alpha$, with the levels of all other factors fixed at 0 . Thus the basic level $B_{0}$, the linear one-factor terms $B_{i} X_{i}$, and the quadratic one-factor terms $B_{i j} x_{i}^{2}$ could be estimated from these points alone.
2. The two-factor interaction terms $B_{i j} X_{i} X_{j}$ must be estimated from the factorial part of the central composite design, since in the center and axial points portion of the central composite design only one factor at a time is being varied.

From these two comments one can see that when using a central composite design to fit a quadratic polynomial to a response surface, the basic restriction on the central composite design is imposed by the requirement that the factorial part of the central composite design allow estimation of all two-factor interaction terms in the polynomial. In particular, when fitting a complete quadratic polynomial this requirement is that no two-factor effect be aliased with I or any other two-factor effect. In practice, we strengthen this requirement to "no one-factor effect or two-factor interaction can be aliased with 1, a one-factor effect, or a two-factor interaction."

When fitting an incomplete quadratic polynomial this restriction can be relaxed. Then we usually require that each of the main effects and two-factor interaction effects with a corresponding term in the polynomial not be aliased with another of these effects.

To have one-factor effects and two-factor interactions aliased only with higher order effects it is necessary and sufficient that the non-I terms of the total alias set all involve at least five factors. For a given number of factors, Table 26 gives the smallest possible fractional factorial with this property.

Table 26. Minimum Fractionation Confounding Only Higher Order Effects

| NUMBER N OF FACTORS | MINIMULM FRACTIONATION | NUMBER OF OBSERVATION VECTORS IN THE FRACTIONATION |
| :---: | :---: | :---: |
| 2 | FULL | 4 |
| 3 | full | 8 |
| 4 | full | 16 |
| 5 | 1/2 | 16 |
| 6 | 1/2 | 32 |
| 7 | 1/2 | 64 |
| 8 | $1 / 4$ | 64 |
| 9 | 1/4 | 128 |
| 10 | 1/8 | 128 |
| 11 | 1/16 | 128 |
| 12 | 1/16 | 256 |
| 13 | 1/32 | 256 |
| 14 | 1/64 | 256 |
| 15 | 1/128 | 256 |
| 16 | 1/128 | 512 |
| 17 | 1/256 | 512 |

As can be seen from the above table, when six or more factors are involved the central composite design requires a fairly large number of factor combinations even for a minimum fractional factorial. Thus, when feasible, it may be worthwhile to use an incomplete quadratic polynomial. For instance, if the only significant interactions are two-factor interactions among just four of the factors, then a 16 unit fractional factorial will suffice for 6,7 , or 8 factors and a 32 unit fractional factorial will suffice for 9 or 10 factors. Predefined Designs 2.6.16, 2.7.16, 2.8.16, and 2.9.32 (see section 15, Predefined Designs) are examples of such fractional factorials if the only significant interactions are taken to be two-factor interactions among $A, C, D$, and $E$.

## COMPARISON WITH THE THREE-LEVEL FACTORIAL

The central composite design is specifically designed to allow a quadratic approximation of a response surface. Other designs can also be used for this purpose. The most obvious alternate design is the three-level factorial. For example, if we want a complete quadratic approximation of a five-factor response surface, then we could either use a central composite design involving a 25-1 factorial or use a 35-1 factorial. The central composite design will often have a number of advantages over the three level factorial.

1. The central composite design is often more economical than alternate designs. In the above example the central composite design requires only 27 points whereas the 35-1 design requires 81 points, where neither count includes multiple replicates of the center point.
2. The central composite design is often appropriate in a sequential design process. If one has already run a $2 \mathrm{~N}-\mathrm{M}$ factorial experiment from which one has determined the desirability of finding a quadratic approximation of the response surface, then it may be sufficient to collect additional observations on the axial and central points to complete the central composite design.
3. In the central composite design every factor appears at five levels whereas in the 3 N factorial every factor appears at only three levels. Thus the central composite design should give better relative precision of the one-factor quadratic terms $\mathrm{B}_{\mathbf{i}} \times \mathbf{} \mathbf{2}$.

If it turns out that a quadratic approximation yields a poor fit, then the 3level factorial may be more advantageous. A complete $3^{N}$ factorial allows determination of third degree terms with each factor exponent $\leq 2$, whereas the central composite design involving a complete $2^{N}$ factorial allows determination of third and fourth degree one-factor terms and third degree interaction terms with all factor exponents $\leq 1$. Thus if we want to go to a cubic approximation without additional experimental points, whether or not the terms $B_{i j i} X_{j}^{3}$ are more significant than the terms $B_{i j j} X_{i}^{2} X_{j}$ determines whether or not the central composite design allows a better fit than the 3 N factorial. In
any case, as $N$ increases, the $3 N$ factorial becomes much more expensive than the central composite design.

## DATA ANALYSIS

Once the experimental data have been collected according to the central composite design, the data must be analyzed. The first step in the analysis process is to use multiple regression techniques to derive the polynomial (of the chosen form) which provides a least squares best fit to the experimental data; that is, the polynomial which minimizes the sum of the squares of the differences between the observed response level and the response level predicted by the polynomial. The details of this derivation are documented in many of the references and in most statistics texts and are not presented here. From this derivation process we can also get estimates of $\sigma^{2}$ (the variance of the estimated response) and of the variances and covariances of the estimates of the coefficients of the derived polynomial.

The second step in the analysis process is an analysis of variance. Here we tabulate the degrees of freedom, the sum of squares, and the mean square for each part of the partition of sum of squares. A comparison of the mean squares indicates the relative significance of each part. In particular, a mean square corresponding to lack of fit that is substantially larger than the mean square corresponding to experimental error indicates that the derived polynomial does not fit the true response surface very well.

If one is satisfied that the derived polynomial provides an adequate fit, then there are several additional analysis steps one might consider.

1. One could try to find a stationary point of the quadratic surface; i.e., a point where the partial derivatives of the polynomial with respect to the factor levels are all zero.
2. If one does find a stationary point, one could perform a translation of axes and a rotation of axes to transform the quadratic function to its canonical form $Y=Y_{0}+\gamma_{1} Z_{1}^{2}+\gamma_{2} Z_{2}^{2}+\ldots+\gamma_{N} Z_{N}^{2}$. In this canonical form the origin is at the stationary point and the principal
axes of the quadratic surface are also coordinate axes, so it is relatively easy to determine the shape of the surface.
3. One can determine whether the stationary point is a maximum point or a saddle point.
4. If the stationary point is a saddle point, one can determine the directions of most promise for a further search for an optimum point.

These steps are appropriate mainly in those situations where one might suspect the existence of an optimum response. In particular these steps depend on the existence of a stationary point, which in many situations may not exist.

In any analysis of the derived polynomial, it should be kept in mind that even if the polynomial provides a good fit the polynomial should be considered to be a good approximation only within the region bounded by the experimental ranges of the factors. If one extrapolates beyond that region then any conclusions drawn may be suspect. For instance, if the analysis of the quadratic function indicates a stationary point is a maximum point, but the stationary point is outside the experimental region, then the stationary point may not be very close to a real maximum. In this situation it might be best to do further investigation by moving toward this calculated maximum point and running additional observations.

## MIXED LEVEL DESIGNS

## COMBINED QUALITATIVE AND QUANTITATIVE FACTORS

The simplest form of combined level design involves an experiment which includes both qualitative and quantitative factors with all factors involving the same number of levels. As an example, consider an experiment involving five factors. Factors $A, B$, and $C$ are quantitative and $D$ and $E$ are qualitative. A two level design is to be used. This may be expressed as a $2^{322}$ design. Fractionation should not be applied to qualitative variables, so only the $2^{3}$ portion of this design can be fractionated. Possible fractionations for this design are as shown in Table 27.

Table 27. 5 Factor, 2 Level, Mixed Qualitative and Quantitative Designs

| FRACTIONATION | NO. OF TRIALS | DEFINING CONTRASTS | ALIAS SET |
| :---: | :---: | :---: | :--- |
| NONE | 32 | NONE | NONE |
| $1 / 2$ | 16 | $1=A B C$ | ABC |
| $1 / 4$ | 8 | $1=A B, 1=B C$ | AB,BC,AC |

As another example, consider the same five factor problem in which a three level design is to be used. This is shown in Table 28.

Table 28. 5 Factor, 3 Level, Mixed Qualitative and Quantitative Designs

| FRACTIONATION | NO. OF TRIALS | DEFINING CONTRASTS | ALIAS SET |
| :--- | :---: | :--- | :--- |
| NONE | 243 | NONE | NONE |
| $1 / 3$ | 81 | $1=A B C$ | ABC,A2B2C2 |
| $1 / 9$ | 27 | $1=A B, I=B C$ | $A B, A 2 B 2, B C, B 2 C 2$ |
|  |  |  | AC,AC2,A2C,A2BC2 |

Thus any grouping of factors at a given level which consists of both qualitative and quantitative factors should be expressed as

$$
\begin{aligned}
P Q P R \text { where } P & =\text { no of levels } \\
Q & =\text { no of quantitative factors } \\
R & =\text { no of qualitative factors }
\end{aligned}
$$

Fractionation should only be applied to the $P Q$ portion of this expression.

## $2 \mathrm{~K}_{3} \mathrm{~L}$ DESIGNS

If an experiment involves only quantitative factors, some at one set of levels (two levels for example) and the other at a different number of levels (e.g., three levels) it requires a mixed level design. A mixed level design of this type is most simply approached by considering each portion of the design separately, assigning defining contrasts for each portion of the design, and performing an optimum fractionation for each portion of the design followed by a cross multiplication of the resultants for each portion of the design.

As an example, a $2^{K} 3 \mathrm{~L}$ design could be defined for a $1 / 2$ fraction by defining the basic block for a $1 / 2$ fraction of the $2^{K}$ which would be assembled with the complete 3 L to arrive at the design.

If $\mathrm{K}=3$ and $\mathrm{L}=2$, the resultant component would be as follows:

| $\underline{2}$ Level | 3 Level | Design |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 000 | 00 | 00000 | 01100 | 10100 | 11000 |
| 011 | 01 | 00001 | 01101 | 10101 | 11001 |
| 101 | 02 | 00002 | 01102 | 10102 | 11002 |
| 110 | 10 | 00010 | 01110 | 10110 | 11010 |
|  | 11 | 00011 | 01111 | 10111 | 11011 |
|  | 12 | 00012 | 01112 | 10112 | 11012 |
|  | 20 | 00020 | 01120 | 10120 | 11020 |
|  | 21 | 00021 | 01120 | 10121 | 11021 |
|  | 22 | 00022 | 01122 | 10122 | 11022 |

Upon examining the resulting design, we find that the levels are not "mixed" as well as they might be. As an example, when the first two (2) level factors are at the same level (either both low or both high), although the three level term is exercised at all levels, the third 2 level term is always at its low level.

An approach to improving the "mixing" of levels in the design is to perform a fractionation upon all portions of the design and assemble the resulting fractions to arrive at the desired fraction. In our sample case this would be done as follows:

If a $1 / 2$ fraction is desired, the two level portion is divided into two parts $S_{1}$ and $S_{2} . S_{1}$ is the basic block and $S_{2}$ is the remaining block. The three level term is divided into $S_{1}^{\prime}, S_{2}^{\prime}$, and $S_{3}^{\prime}$ where $S_{1}^{\prime}=0(\bmod 3), S_{2}^{\prime}=1(\bmod$ 3) and $S_{3}^{\prime}=2(\bmod 3)$. The treatment combinations are then:

## 2 Level

3 Level

| $\frac{S_{1}}{000}$ | $\frac{S_{2}}{001}$ | $\frac{S_{1}^{\prime}}{00}$ | $\frac{S_{2}^{\prime}}{01}$ | $\frac{S_{3}^{\prime}}{02}$ |
| :--- | :--- | :--- | :--- | :--- |
| 011 | 010 | 12 | 10 | 11 |
| 101 | 100 | 21 | 22 | 20 |
| 110 | 111 |  |  |  |

The experimental plan can then be formulated as:

$$
S_{1} S_{1}^{\prime}+S_{2} S_{3}^{\prime}+S_{1} S_{3}^{\prime} \text { or fractionally } \frac{1}{2}=\frac{1}{2} \cdot \frac{1}{3}+\frac{1}{2} \cdot \frac{1}{3}+\frac{1}{2} \cdot \frac{1}{3}
$$

The resulting design is now:

| 00000 | 00101 | 00002 |
| :--- | :--- | :--- |
| 01100 | 01001 | 01102 |
| 10100 | 10001 | 10102 |
| 11000 | 11101 | 11002 |
| 00012 | 00110 | 00011 |
| 01112 | 01010 | 01111 |
| 10112 | 10010 | 10111 |
| 11012 | 11110 | 11011 |
| 00021 | 00122 | 00020 |
| 01121 | 01022 | 01120 |
| 10121 | 10022 | 10120 |
| 11021 | 11122 | 11020 |

This design results in a better mixing than the original design. A symmetrical design, in which each fraction for a portion of the design is used the same number of times as every other fraction for that portion of the experimental design, would provide the optimum level of mixing. However, this cannot be achieved except in a few cases.

The first step in the process of formulating a mixed level design is to determine the number of components in the experimental plan. This will be illustrated for the $2 \mathrm{~K}_{3} \mathrm{~L}$ design, $1 / 2$ fractionation.

The experimental plan will consist of:

$$
\begin{aligned}
1 / 2=G \frac{1}{2^{M}} \cdot \frac{1}{3^{P}} \text { where } G & =3^{P} 2^{M-1} \\
\text { and } P & \leqq L \\
M & \leq K
\end{aligned}
$$

If $K=3, L=2\left(233^{2}\right)$, the possibilities are:

$$
1 / 2=3 \frac{1}{2} \cdot \frac{1}{3}=6 \frac{1}{4} \cdot \frac{1}{3}=12 \frac{1}{8} \cdot \frac{1}{3}
$$

Experimental plans corresponding to these are:

$$
S_{1} S_{1}^{\prime}+S_{2} S_{2}^{\prime}+S_{1} s_{3}^{\prime}
$$

or

$$
S_{1} S_{1}^{\prime}+S_{2} S_{2}^{\prime}+S_{3} S_{3}^{\prime}+S_{4} S_{1}^{\prime}+S_{2} S_{3}^{\prime}
$$

or

$$
\begin{aligned}
& S_{1} S_{1}^{1}+S_{2} S_{2}^{\prime}+S_{3} S_{3}^{\prime}+S_{4} S_{1}^{\prime}+S_{5} S_{2}^{\prime}+S_{6} S_{3}^{\prime}+S_{7} S_{1}^{\prime}+S_{8} S_{2}^{1} \\
& +S_{1} S_{3}^{\prime}+S_{2} S_{1}^{\prime}+S_{3} S_{2}^{\prime}+S_{4} S_{3}^{\prime}
\end{aligned}
$$

For simplicity, the 3 element plan would be selected since the more complex plans do not offer significant advantages.

## PREDEFINED DESIGNS

An experimenter frequently requires a design for which he is unable to find the appropriate aliasing. This section includes a set of predefined designs that allow for all main effects to be measurable. Main effects are aliased with only high-order interactions. Also, most two-factor interactions are measurable.

To aid the user, the different experimental designs are identified by the notation L.F.S., where $L$ is the number of levels per factor, 2 or $3 ; F$ is the number of factors; and $S$ is the number of observation vectors in the fractional design (e.g., 2.4.8 $=2$ levels, 4 factors, and 8 units). the sources of these predefined designs are C\&C (Cochran and Cox, 1957) NBS \#48 and NBS \#54. Some predefined designs were developed as a result of this study.

Design 2.4.8
$2^{4}$ factorial in 8 units
1/2 replicate
$\mathrm{I}=\mathrm{ABCD}$
C\&C, p. 276

Design 2.5.8
$2^{5}$ factorial in 8 units
1/4 replicate
$\mathrm{I}=\mathrm{ABE}=\mathrm{CDE}$
C\&C, p. 277

Design 2.5.15
$2^{5}$ factorial in 16 units
1/2 replicate
$\mathrm{I}=\mathrm{ABCDE}$
C\&C, p. 277

Design 2.7.8
$2^{7}$ factorial in 8 units
1/16 replicate
$I=A B G=A C E=A D F=B C F$
C\&C, p. 280

Design 2.7.26
$2^{7}$ factorial in 16 units
1/8 replicate
$I=A B C D=A B E F=A C E G$
C\&C, p. 280

Design 2.6.8
$2^{6}$ factorial in 8 units
1/8 replicate
$\mathrm{I}=\mathrm{ACE}=\mathrm{ADF}=\mathrm{BCF}$
C\&C, p. 278

Design 2.6.16
$2^{6}$ factorial in 16 units
1/4 replicate
$\mathrm{I}=\mathrm{ABCE}=\mathrm{ABDF}$
C\&C, p. 278

Design 2.5.3?
$2^{6}$ factorial in 32 units
1/2 replicate
$\mathrm{I}=\mathrm{ABCDEF}$
C\&C, p. 279

Design 2.8.32
$2^{8}$ factorial in 32 units
1/8 replicate
$I=B C D H=B D F G=A B C E F$
C\&C, p. 285
$\frac{\text { Design 2.8.64 }}{2^{8} \text { factorial in }}$
1/4 replicate
$I=A B C E G=A B D F H$
C\&C, p. 287

Design 2.7.32
27 factorial in 32 units
1/4 replicate
$I=A B C D E=A B C F G$
C\&C, p. 281

Design 2.7.64
27 factorial in 64 units
1/2 replicate
$\mathrm{I}=\mathrm{ABCDEFG}$
C\&C, P. 283

Design 2.8.16
$2^{8}$ factorial in 16 units
1/16 replicate
I = ABCD = ABEF = ABGH = ACEH
C\&C, p. 285

Design 2.9.128
29 factorial in 128 units
1/4 replicate
$\mathrm{I}=\mathrm{ABCEG} . \mathrm{i}=\mathrm{ABDFHJ}$
NBS \#48, p. 24

Design 2.9.256
29 factorial in 256 units
1/2 replicate
$1=A B D E F G H J$
NBS \#48, p. 17

Design 2.10.64
210 factorial in 64 units
1/16 replicate
$I=A B C D J K=A B E F J=B C E G J K=A B C D E F G H$
NBS \#48, p. 44

Design 2.8.128
28 factorial in 128 units
1/2 replicate
I = ABCDEFGH
C\&C, p. 288

Design 2.9.32
29 factorial in 32 units
1/16 replicate
$I=A B C D=A B E F=B C E G=E F G H J$
NBS \#48, p. 43

Design 2.9.64
29 factorial in 64 units
1/8 replicate
$\downarrow=$ ABEGHJ $=$ ACFGJ $=$ ABCD
NBS \#48, p. 33

Design 2.10.512
210 factorial in 512 units
1/2 replicate
$\mathrm{I}=\mathrm{ABCDEFGHJK}$

Design 3.3.9
33 factorial in 9 units
1/3 replicate
$\mathrm{I}=\mathrm{ABC}$
Developed Design

Design 3.4.9
34 factorial in 9 units
1/9 replicate
$I=A B C=B C 2 D$
Developed Design

Design 2.10.128
210 factorial in 128 units 1/8 replicate
$I=A B E G H J=A C F G J K=A B C D K$
NBS \#48, p. 36

Design 2.10.256
$2^{10}$ factorial in 256 units
1/4 replicate
$I=A B C O E F G=A B C D H J K$
NBS \#48, p. 29

Design 3.5.81
35 factorial in 81 units
1/3 replicate
$\mathrm{I}=\mathrm{ABCDE}$
NBS \#54, p. 11
Design 3.6.27
36 factorial in 27 units
1/27 replicate
$I=A B C D E F 2=B^{2} 2 E F 2=A B C E$
Developed Design

Design 3.6.81
$3^{6}$ factorial in 81 units
1/9 replicate
$I=A C D E=B C 2 D E 2 F$
NBS \#54, p. 19

Design 3.4.27
34 factorial in 27 units
1/3 replicate
$I=A B C D$
NBS \#54, p. 11

Design 3.5.27
35 factorial in 27 units
1/9 replicate
$I=A B C D E=A B C^{2}$
Developed Design

Design 3.7.243
37 factorial in 243 units
1/9 replicate
$I=A B C D E=C D^{2} E F^{2} G^{2}$
NBS \#54, p. 20

Design 3.7.729
37 factorial in 729 units
1/3 replicate
$I=A B^{2} C_{D E}^{2 F G}$
NBS \#54, p. 17

Design 3.8.81
38 factorial in 81 units
1/81 replicate
$I=B C D E F G=A C D E 2 F 2 H=A C^{2} D^{2} F G=$
$B C^{2} F^{2} C$
Developed Design

Design 3.6.243
36 factorial in 243 units
1/3 replicate
$I=A B^{2} C D E^{2} F$
NBS

Design 3.7.27
$3^{7}$ factorial in 27 units
1/81 replicate
$1=A C D E F 2=B C{ }^{2} E F^{2} G=$
$A B C E G^{2}=A B^{2} C^{2} E^{2} F^{2}{ }^{2}{ }^{2}$
Developed Design

Design 3.7.81
37 factorial in 81 units
1/27 replicate
$1=$ ACDEF $^{2} G=$ BC $^{2}{ }^{2}{ }^{2}{ }^{2} G=A B C E G 2$
NBS \#54, p. 23

Design 3.9.243
39 factorial in 243 units
1/81 replicate
$I=B C D E F G=A C D E 2 F 2 H=$ $A D^{2} E^{2} F J=A B C C^{2} F^{2}$

NBS $\boldsymbol{*} 54$, p. 31

Design 3.10.243
310 factorial in 243 units
1/243 replicate
$1=\operatorname{BCDEFG}=A C D E 2 F 2 H=$ $A D^{2} E^{2} F J=A B C 2 E F^{2}=$ $A B^{2} C^{2} D F K$

Developed Design

Design 3.8.243
38 factorial in 243 units
1/27 replicate
$I=B C D E F G=A C D E 2 F 2 H=A B D E^{2}{ }^{2} F$
NBS \#54, p. 25

Design 3.8.729
$3^{8}$ factorial in 729 units
1/9 replicate
$\mathrm{I}=\mathrm{ABCDE} \mathrm{H}^{2}=\mathrm{CD}^{2} \mathrm{EFF}^{2} \mathrm{G}^{2}$
NBS \#54, p. 23

Design 3.9.81
39 factorial in 81 units
1/243 replicate
$\mathrm{I}=\mathrm{BCDEFG}=\mathrm{ACDE}{ }^{2} \mathrm{~F}^{2} \mathrm{H}=\mathrm{ABD}^{2} \mathrm{E}^{2} \mathrm{FJ}=$ $A B^{2} C^{2} D F=A B C 2 E F 2$
NBS \#54, p. 36

Design 3.9.729
39 factorial in 729 units
1/27 replicate
$I=B C D E F G=A C D E 2 F 2 H=A B D 2 E 2 F J$
NBS \#54, p. 26

## USER'S APPENDIX GUIDE WITH EXAMPLES

## INTRODUCTION

This appendix describes the procedures for using the AED computer program. the "conversational mode" of the program operation is illustrated. the functions of each of the program segments are explained, and a listing is included of the program input and output for an example from each segment.

## SYSTEM STRUCTURE

The automated experimental design program is divided into five program segments. Upon execution of the program, a display menu is presented. The user may select one of the five by inputting the number of the segment he wishes. The program echoes the input and requests the user to input an identification label to identify the run. The I.D. label may be any alphanumberic string.

The menu, entry request, and I.D. label request occur whenever a segment is completed and the program is ready to enter another segment. The actual test displayed and a system "walk-through" will be included with the explanation of the use of each segment in the following sections.

## BASIC TERMINOLOGY

Segment 1--Basic Terminology--provides the user with a basic introduction to the process of experimental design. A description of the program assumptions, vocabulary, and a discussion of the rationale behind experimental designs are provided. This material is essentially a tutorial for the user who is unfamiliar with experimental design. Once the user is acquainted with the design process, there should be no need to enter this segment except for an occasional review session. User input is explained in this segment.

ACOTONIWOEL JISVA

1
welcore to tie automated experimental design program.
you may enter one of the program segments:

1. BASIC TERMINOLOGY PROBLEM DEFINITION DESICN actual experimental inesici experimental repinement
nanlay dit any invm nox yagnin inakogs alli yaina 1

## YOUR ENTRY WAS: 1--BASIC TERMINOLOGY

please enter this run problem i.d. basic terminology test run
helcome to the automated experimental design program. tHIS PROGRAM PROVIDES THE OPTION FOR FORMULATING SCREENING desicns or response surface designs. ing designs assume that your objective is to the deterinne factors that produce a certain result and the By cervitio and Int of those factors that may possibly influence the outcome OF THE EXPERIMENT. IN THIS CASE, FRACTIONAL DESIGNS ARE OF real value. they may be used to determine which of the pOSSIble factors are of tmportance relative to a given error of prediction. once these factors have been discovered, it is necessary to perform detailed hork on the factors, possibly
even one at a time in order to formulate a law relating response to the level for each factor.

## hit return hhen ready to continue.

## basic terhinology test run



> SCREENING DESICNS -- A CLASS OF FRACTIONAL FACTORIALS -ARE SYSTEMYTIC DATA COLLECTION PLANS THAT ENABLE THE EFFECTS OF A VERY LARGE NUMBER OF FACTORS TO BE ESTIMATED ECONOMICALLY. SCREENING DESIGNS ARE USED PRIMARILY IN THE SECOND PHASE OF A TOTAL RESEARCH PROGRAM WHERE THEY ARE INTENDED TO DETERMINE WHICH OF THE GREAT MANY FACTORS HAVE NON-TRIVIAL EFFECTS ON THE PERFORMNCE OF A PARTICULAR TASK. SCRENING DESIGNS ARE TO BE USED TO IDENTIY IMPORTANT FACTORS, NOT TO OBIIN AN ACCURATE REPRESENTATION OF THE EXPERIMENTAL SPACE, THIS LATTER OPERATION WILL OCCUR IN SUBSEQUENT PHASES OF THE RESEARCH PROGRAM.

## hit return to continue

BASIC TERMINOLOGY TEST RUN THE SCREENING DESIGNS PROVIDE A MEANS OF EXAMINING A GREAT NUMBER
OF FACTORS WITH THE MAXIMUM AMOUNT OF INFORATION WITH A MINIMUM
AMOUNT OF REDUNDANCE AND RELATIVELY FEW TRALS. WHAT THE RESULTS
FROM MANY LITTLE TRADITIONAL EXPERIMENTS CANNOT DO, BUT WHICH RESULTS
FROM THE SCREENING DESIGN CAN DO IS TO ORDER THE FACTORS ACCORDING TO
SIZE OF THEIR EFFECTS AND TO DISCOVER INTERACTIONS AMONG FACTORS THAT
APPEAR WITHIN THE SAME EXPERIMENT. SCREENING DESIGNS DO ALL THIS
ECONOMICALLY FOR THEY CAN BE USED TO STUDY FACTORS WITH A SMALL NUMBER
OF TRIALS (ALTHOUGH THE SIZE OF THE DESIGNS IN THIS PROGRAM WILL ALL BE
EQUL TO A POWER OF 2 OR 3 OR 5). THE EFFECTS OBTAINED FROH SCREENING STUDIES
NOT ONLY PERIT THE RANING OF FACTOR EFFECTS ON A QUANTTATIVE SCALE,
BUT CAN PROVIDE AN EQUATION APPROXIMATING THE EXPERIMENTAL SPACE IF
THAT SPACE CAN BE REPRESENTED BY A LINEAR MODEL.
hit return to continue
the beauty of using a screening design is that once the important factors have been identified (Step one), the same data can be used if supplemented by relatively few additional trials at new experimental CONDITIONS, TO COMPLETE A RESPONSE SURFACE (STEP TNO) CAPABle Of accurately approximating the experimental space defined by the original set of factors.
in using a screening design, the experimenter must ask certain questions about EACH FACTOR.
> does the factor llave an effect on performancer does the factor account for a meaningrul proportion of the variance in tile experiment? does including the factor materially improve the ability to predict performance under oprrational CONDITIONS? COULD THE OBSERVED EFFECT HAVE bEEN DUE TO CHANCE? Can the cuiulative effects of a large number of non-Critical factors be ignored? do you hant to review this discussion of screening designs? No
BASIC TERMINOLOGY TEST RUN ---RESPONSE SURFACE DESIGNS---
 THE FINAL COAL IN MANY INVESTICATIONS IS TO DETERTME the final goal in many investications is to determine the response surface in an experimental region. usually the nature of tie response surface (i.e., the form of tile response function) is unknonn. since actually determining the true response function is unattainable, an approximating function must be used. polynomials in the levels of the factors are suitable approximating functions. polynomials are relatively easy to work hith, and in a given region any continuous function can be approximated to any desired level of accuracy by a polynomial of sufficiently high degree. basic factorial designs arid mixed level designs can provide experimental data sufficient to determine the polynomial coefficients, and appropriate central Composite designs hill alhays be sufficient to determine all the coefficients of
a quadratic polynomial.
basic terminology test run
 -- CENTRAL COMPOSITB DRSIGNS-ュー
THE CENTRAL COMPOSITE DESIGN IS AN EXPERIMENTAL DESIGN INTENDED TO ALLOW A QUADRATC polynomial approximation of a response function mien all factors and the response are quantitative tue central corposite desicy consists on a dind the response two-level factorial design plus additional axial points and a genter point necessary
ro approximate the response surface.

2. only a small nutiber of factors is involved and tie EXPERIMENTER WANTS AS MUCH INFORMATION ABOUT THE behavior of the response as possible, quickly and at low cost.
3. THE EXPERIMENTER HAS ALREADY RUN A TWO-LEVEL FACTORIAL EXPERIMENT AND NOW WANT TO EXPaND THE EXPERIMENTAL region as cheaply as possible.

## hit return to continue

BASIC TERMINOLOGY TEST RUN
the central cotposite design should not be used in the following situations.

1. all that is desired is a screen for significant factors.
2. SOME FACTORS ARE QUALITATIVE.

## 3. INTERACTION EFFECTS ARE NON-LINEAR.

DO YOU WANT TO REVIEW THIS DISCUSSION OF RESPONSE SURFACE DESIGNS? no
BASIC TERMINOLOGY TEST RUN

you hill be asked a series of questions. SONE OF THESE REQUIRE ONLY A YES OR NO RESPONSE. YOU may enter a y or yes or $n$ OR No to these questions do you want to continue with these basic instructions? yes
-79-
The user may review these basic instructiona A "NO" response would skip the remainder of this segment.
BASIC TERMINOLOGY TEST RUN

THESE INSTRUCTIONS CONSIST OF A DEFINITION OF YOUR RESPONSES for numbers and an explanation of the various terms used in the design of experiments.
wilin a number is requested for input, you must enter the value AS +1 or -2 or 96 , ETC., OR 11.67 or $\mathbf{- 3 . 2 6 .}$ the number cannot be input as a fraction (1/2). CORRECT IT If NECESSARY.
input a number of your own chosce as a test
112.345
IS THIS INPUT CORRECT: 112.345
NO No
i hill echo each value input and give you the opportunity to
Input a nurber of your own chosce as a tes
input a nuriber of your own choice as a test 99.76
IS THIS INPUT CORRECT: 99.76 YES

Data input is critical to the use of this program. Data are echoed and the user may change/correct his response. Note that the user had the option to change the input response until satisfied. The program recognizes incorrectly formatted input and continues to prompt the user until a valid response is made.
BASIC TERPSNOLOGY TEST RUN

'NAISAG TVLNGHIBGdXG NI GGSO SWAGL dO LSIT V SI ONIMOT'IOA 3HI
ALIAS
EFFECT OF A FACTOR WHICH CANNOT BE DISTINGUISHED FROM THAT OF ANOTHER FACTOR.
In a CENTRAL COHPOSITE DESIGN THE NON-ZERO CODED LEVEL VALUE OF A FACTOR AT AN AXIAL POINT.
AXIAL POINTS OR STAR POINTS ARE THO CORRESPONDING AXIAL POINTS: IIAS CODED LEVEL -ALPHA AT ONE POINT AND +ALPIIA AT THE OTHER, WHEREAS ALL OTHER FACTORS HAVE CODED LEVEL ZERO AT THESE POINTS.

## HIT RETURN TO CONTINUE

BASIC TERMINOLOGY TEST RUN

CENTER POINT
THE POINT IN A CENTRAL COMPOSITE DESIGN GHERE ALL N FACTORS havf coded level zero.
A CONBINATION OF A FULL OR FRACTIONAL TWO-LEVEL FACTORIAL
DESIGN AND SOME ADDITIONAL EXPERIMENTAL POINTS SELECTED IN A PARTICULAR MANNER TO ALLOW THE DETERMINATION OF TUE qUADRATIC ONE FACTOR EFFECTS. IT IS SPECIFICAILLY INTENDED TO ALLOH DETEPIIINATION OF THE CONSTRAINTS USED IN DEFININS, A QUADRATIC APPROMIMATION OF THE RESPONSE SURFACE.
COJJED LEVEL
TIIE IEVEL OF A FACTOR TKANSLATED FROM THE TRUE QUANTITATIVE LEVEL USEI) FOR SIMTLIFYING CALCILATIONS.
CONFOUNDING
D FROH OTHERS.
llit peturn to confinue
BASIC TERMINOLOGY TEST RUN

CORRELATION COEFFICIENT (PEARSON R)
THE SQUARE ROOT OF THE TOTAL VARIATION ACCOUNTED
FOR BY LINEAR REGRESSION.
TIIE SQUARE ROOT OF TIIE PROPORTION OF TOTAL VARIATION ACCOUNTED
TIIE SQUARE ROOT OF TIIE PROPORTION OF TOTAL VARIATION ACCOUNTED
FOR BY THE REGRESSION EQUATION OF THE DEGREE BEING FITTED TO THE DEFINING CONTRAST
SEIECTION OF EFFECTS TO BE CO!NPOINDED.
DESREES OF FREEDON
IN TIIS PROGRAII, ONE LESS THAN THE N!IMBER OF VALUES TO COMPUTE TIIE SIM OF SQUARES.
EFFECC
cliance in response due to a change in tile level of a factor.

## HIT RETURN TO CONTINUE

BASIC :EERMINOLOGY TEST RUN


## EXPERIMENTAL HODEL

hypotiesized equation to describe the response as a
FUNCTION OF TIIF TREATMERT.
EXPERIMENTAL TRIAL
ONE UNIT OF A COMPLETE EXPERIMENT, CONDUCTED WITI FACTORS AT
LEVELS DEFINLD BY A SINGLE OBSERVATION VECTOR.
FACTURIAI, EXPERIAENT
AN EXPERIMENI IN WIIICII ALL LEVELS OF EACII FACTOR IN THE
AN EXPERIMIENT IN NIICII ALL IEVELS OF EACI FACTOR IN THE
EXIERIMENT ARE COMBINED WITH ALL LEVELS OF EVERY OTHER FACTOR.
FRACTIONAL FACTORIAL.
AN EXPERIMENTAL DESIGN IN WHICH ONLY A FRACTION OF A COHPIETE FACTORIAL IS RUN.
hit Return to continie.
BASIC TEMMINOLOCY TEST RIIN


## INTERACTION

an tnteraction between two factors means that a change IN RESPONSE BETHEEN LEVELS OF ONE FACTOR IS NOT THE Same for all levels of the other factor.

## mean square error

SUM OF SQUARES OF THE ERROR DIVIDED BY THE NUMRER OF DEGREES OF FREEUON FOR TIIE ERROR TERM.
MIXED LEVEL DESIGN
A FULL OR FRACTIONAL FACTORIAL DESIGN WHERE SOME FACTORS OF
OI the design have a different number of levels than other FACTORS OF THE DESIGN.
OBSERVATION VECTOR
Planned level of eacil factor for a single eyperimental trial.
IIIT RETURN TO CONTINUE
BASIC TERMINOLOGY TEST RUN

REAL WORLD LEVEL
THE TRIJE QUANTITATIVE LEVEL OF A FACTOR THAT CORPFiSPONDS TO A CODED LEVEL.
REGRESSION
LINEAR -RESPONSE $=A * X 1+B * X 2+C * x 3+\ldots+2 * x N$
QUADRATIC - RESPONSE $=A * X 1+B X 2+\ldots+C * X 1 * X 2+D * X 1 * X 3+\ldots+E X 1 * * 2+F * X 2 * * 2+$ ..$+2 * X N * * 2$.
repetition of observation vectors applied to multiple
EXPERIMEHTAL TRIALS.
RESPONSE FUNCTION
OF THE FACTORS ARE $X 1, \times 2, \ldots$, XN AND THE RESPONSE IS $Y$.
RESPONSE SURFACE
replicate
THE SURFACE IN N+1 dimensional space represented by the EQUATION Y $=F(X 1, X 2, \ldots, X N)$

## \& hit return to continue

[^0]4
\#


## PROBLEM DEFINITION

Segment 2--Problem Definition--is used by the experimenter to define the particular problem being studied. A new menu is displayed allowing the user to select one of four options.

## BASIC FACTORIAL DESIGNS

Option 1--Basic Factorial Designs--allows the user to define full or fractional factorial designs for 2, 3, or 5 levels. This definition includes: (1) the number of factors, (2) the number of levels per each factor, (3) the number of experimental trials available, and (4) aliasing information.

Since the process of specifying the defining contrasts that determine the aliasing is critical, the user is offered assistance in the definition of these aliases. In this help section, the program can provide certain predefined designs (see example 1). These designs are described in Volume I of this report.

If the predefined alias set is unacceptable or if one is not available, the user is given help (for 2 level designs only) in developing a "good" alias set by taking the predefined design and either by deleting factors (see example 2) or by adding factors (see example 3) arrives at a new option design. If the user is still not satisfied with the design, the defining contrasts can be specified as input values.

Care should be taken that the members of the alias set are linearly independent, since if any member of the alias set is a linear combination of the other members, the experimental block will contain more than the desired number of observations. The program will check for independence of the alias set, and give the user an opportunity to redefine the set.

The user may also have the total alias set displayed. (The total alias set is constructed by forming all possible combinations of the original defining contrasts specified by the user.) The alias set may be redefined until the design is acceptable to the user.

The user may also have the total alias set displayed. (The total alias set is constructed by forming all possible combinations of the original defining contrasts specified by the user.) The alias set may be redefined until the design is acceptable to the user.

1
z
basic factorial designs

$$
-88-
$$

$$
\text { EXAMPLE } 1
$$

WELCOHE TO TIIE AUTOHATED EXPERIMENTAL DESIGN PROGRAM.
YOU MLAY ENTER ONE OF THE PROGRAM SEGMENTS:
ENTER THE SEGMENT NUMBER YOU WANT AND HIT RETURN
YOUR ENTRY WAS: 2--PROBLEM DEFINITION
PLEASE ENTER THIS RUN PROBLEM I.D.
PROBLEM DEFINITION AND DESIGN TEST
PROBLEM DEFINITION AND DESIGN TEST *********************************************************
YOU MAY ENTER ONE OF THE PROBLEN DEFINITION OPTIONS: BASIC FACTORIAL DESIGNS MIXED LEVEL DESIGNS
CENTRAL COMPOSITE DESIGNS EXIT
$\therefore$ ~~் BASIC TERHINOLOGY DESIGN EXPERIMENTAL REFINEMENT EXIT PROBLEM DEFINITION CIAL EXPERIMENT

- iviポ
HON MANY EXPERIMENTAL TRIALS ARE AVAILABLE?
16
2 LEVELS FOR THE FACTORS. IS TIIIS CORRECT?
5 factors are present. IS TIIS correct?
16 EXPERIMENTAL TRIALS ARE AVAILABLE. IS THIS CORRECT?
31 IS THE TOTAL NUMBER OF EFFECTS AND INTERACTIONS.
OF TIIESE:
5 ARE MAIN EFFECTS.
10 ARE FIRST ORDER INTERACTIONS.
16 ARE HIGHER ORDER INTERACTIONS.
hit return when ready to continue
PROBLEM DFFINITION AND DESIGN TEST

HOW MANY EXPERIMENTAL TRIALS ARE TO BE RUN?
THIS MUST BE SELECTED FROM THE VALUES
$\begin{array}{lllll}2 & 4 & 8 & 16 & 32\end{array}$
IS TIIS NIAT YOU WENT: 16
TIIS IS A $1 / 2$ FRACIIONAL FAC'TORIAL DESIGN.
ac means factor a at its high level, factor b at its low level, and factor C at its hich level.
based on the experimental description you have given about the number of HOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?
YES
PROBLEM DEFINITION AND DESIGN TEST
$* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * ~$ HOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?
YES
PROBLEM DEFINITION AND DESIGN TEST
$* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * ~$ aND FACTOR C AT ITS HIGH LEVEL.
the defining contrast set defined contains 1 Independent member ( S )
do you hant to see the total alias set?
YES
total alias set
ABCDE
hit return when ready to continue.
would you like to redefine the defining contrasts?
problem definition completed.
hit return wien ready to continue.


WELCOME TO THE AUTOHATED EXPERIMENTAL DESIGN PROGRAM.
HON MANY EXPERIMENTAL TRIALS ARE AVAILABLE?
2 LEVELS FOR THE FACTORS. IS THIS CORRECT?
3 FACTORS ARE PRESENT. IS THIS CORRECT?
2 EXPERIMENTAL TRIALS ARE AVAILABLE. IS THIS CORRECT?
7 IS THE TOTAL NUNBER OF effects AND INTERACTIONS.
OF THESE:
3 ARE MAIN EFFECTS.
3 ARE FIRST ORDER INTERACTIONS.
hit return hhen reany to continue.
HELP DEMONSTRATION

HOW MANY EXPERIMENTAL TRIALS ARE TO BE RUN? THIS RUST BE SELECTED FROM THE VALUES?
8
IS THIS WHAT YOU WANT: 2
THIS IS A $1 / 4$ FRACTIONAL FACTORIAL DESIGN.
FOR AN EXPERIMENT OF THIS SIZE, YOU WIIL NEED 2
LINEARLY INDEPENDENT DEFINLNG CONTRAST (s).
THE FACTORS IN THE DESIGN MUST BE DESIGNATED AS A,B,C,D, ETC. THE LETTER I IS RESERVED FOR THE ILENTITY EFFECT.
SINCE EACH FACTOR MUST APPEAR IN A TREATMENT, THE FOLLONING NOTATION IS USED TO DESCRIBE THE FACTORS IN A PARTICULAR TRIAL OR ALIAS DEFINITION.
FOR A 2 LEVEL PER FACTOR EXPERIMENT, THE ABSENCE OF A LETTER INDICATES THE FACTOR IS AT ITS LOW LEVEL.
THE PRESENCE OF A LETTER INDICATES THE FACTOR IS AT ITS HIGIL LEVEL. AC means factor a at its higli level, factor b at its low level, AND FACTOR C AT ITS HIGH LEVEL.
WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?

HELP DEMONSTRATION

BASED ON THE EXPERIMENTAL DESCRIPTION YOU HAVE GIVEN ABOUT THE NUMBER OF FACTORS, THE FACTOR LEVELS, AND THE NUMBER OF TRIALS TO BE USED, NO PREDIFINED DESIGN IS AVAILABLE.
SEE "INTRODUCTION TO EXPERIMENTAL DESIGN AND THE AUTOMATED EXPFRIMENTAL DESIGN PROGRAM: VOL. I, SECTION 4 FOR MORE DETAIL. DO yOU WANT help in generating a rralizable
2 level alias set?
yes DO you Want help in generating a rralizable
2 level alias set?
yes DO YOU WANT help in generating a realizable
2 LEVEL ALIAS SET?
YES PREDIFINED DESIGN IS AVAILABLE.
HELP DEMONSTRATION

GENERALLY, IN A TWO LEVEL EXPERIMENT INVOLVING ANY NUMBER OF FACTORS
(REPRESENTED BY THE LETTER N) IT IS DESIRED TO ISOLATE ALL SINGLE FACTORS AND TWO FACTOR INTERACTIONS. THIS IS UNDER THE ASSUMPTION THAT THREE FACTOR AND HIGHER ORDER INTERACTIONS ARE INSIGNIFICANT.
TO OBTAIN THE LARGEST DEGREE OF ISOLATION FOR A GIVEN EXPERIMENT, ALL MEMBERS Of the alias set should contain approximately the same number of high level factors.
HIT RETURN TO CONTINUE
IIELP DEMONSTRATION
 A USEFULL HOTATION HAS BEEN DEVELOPED TO DESCRIBE THE COMPOSITION OF THE TOTAL ALIAS SET. AN EXANPLE OF THIS NOTATION POLLONS:
FOR A $1 / 4$ REPLICATE OF A 2 LEVEL DESIGN WITH 5 FACTORS, THE TOTAL ALIAS SET OF THIS DESIGN MIGHT BE:


## ABE <br> ABCD

THIS ALIAS SET CONTAINS THREE (3) MEMBERS, TWO (2) MEMBERS UITII
THREE (3) HIGU LEVEL FACTORS EACH AND ONE (1) MENBER HITH FOUR (4)
HICH LEVEL FACTORS.
IN NOTATION FORH THIS BECOHES:
$2(3)+1(4)$
HOULD YOU LIKE TO SEE AN EXAMPLE OF GENERATING A NEW DESIGN?
HELP DEMONSTRATION

TO MODIFY THE ALIAS SET ABD CRBATE A NEN DESIGN, THE ORIGINAL ALIAS SET MUST PIRST BE CENERATED BY DEVELOPING ALL OF THE POSSIBLE CORBINATIONS OF THE DEFINING CONTRASTS. AS AN EXAMPLE, IF TWO (2) DEFINING CONTRASTS, ABCDE AND ABCFG, ARE USED IN A SEVEN (7) FACTOR PROBLEM, THE FOLLOWING ALIAS SET IS GENERATED:


THIS IS A $1(4)+2(5)$ DESIGN.
hit Return to ccatinue.
HELP DEAYONSTRATION


## a Nen design for six（6）factors can be generated from this design

 by deleting one of the columns．the best theoretical design for SIX（6）FACTORS IS A 3（4）DESIGN．KNOWING THIS WILL ALLOW US TO make a better judgement about hhich coluna to delete．removing anyone of the colunas shown hill result in the corresponding design．
hit return to continue
HELP DEMONSTRATION

BY REMOVING COLUMN 1,2 ，or 3 THE BEST THEORETICAL DESIGN WILL BE OBTAINED． COLUNN NUMBER $\begin{array}{llllll}1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1\end{array}$
4 FACTORS
4 FACTORS
4 FACTORS
hit return to continue
help demonstration
 as anotier example, if two (2) defining contrasts, abe and cde, are used in a five (5) factor problem, the following alias set IS GENERATED:
COLUMN NUMBER 3FACTORS 3factors 4 FACTORS

| 1 | 1 | 1 |
| :--- | :--- | :--- | :--- |
|  | 0 | 1 |
| 5 | 7 | $\varepsilon$ |

$\begin{array}{llllll}1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0\end{array}$

THIS IS A $2(3)+1(4)$ DESIGN
hit return to continue

HELP DEMONSTRATION
1
2
$\times 2$
g
0
0
0
0
0
旨
0
added columa
$\begin{array}{llllll}1 & 2 & 3 & 4 \\ 1 & 5 & \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0\end{array}$ BY ADDING ANOTHER COLNSN. THE BEST THEORETICAL DESIGN FOR SIX (6) factors is a 3(4) design. knowing this will allow us to make a better judgement about which rows should be increased.

## ORIGINAL DESIGN

## this micht be done as follons: <br> defining contrast products

A REN DESIGN FOR SIX (6) FACTORS CAN BE GENERATED FROM THIS DESIGN
NUMBER OF FACTORS PER RON 4 PACTORS

0
1
1 1
hit return to continue.
HELP DBHORSTRATION

BY INCREASINC DEFINING CONTRAST 1 AND 2, THE BEST THEORETICAL DESIGN WILL BE OBTAINED.

THIS IS A 3(4) DESICN.
HIT RETURN TO CONTINUE
HELLP DEMONSTRATION

YOUR PRESENT DESIGN CONSISTS OF 2 LEVELS,
3 FACTORS, AND 2 DEFINING CONTRASTS.
A REALIZABLE DESIGN CAN BE CONSTRUCTED BY STARTING WITH A STORED DESIGN,
STORED DESIGNS FOR THE PRESENT NURBER OF DEFINING CONTRASTS ARE AS FOLLONS:

NO. OF DEFINING CONTRASTS = 2 STORED DESIGNS FOR N (NO. OF FACTORS)
YOU MAY ENTER A VALUE OF N FROM THE STORED DESIGNS SMALLER THAN YOUR DESIRED VAlue of n and build up to the desired value of $n$ or enter a larger value of A FROM THE STORED DESIGNS AND REDUCE TO THE DESIRED VALUE OF N. EATER THE STORED DESICN VALUE OF N YOU WISH TO START WITH.
IS THIS THE NUMBER YOU WANT? 5
$Y$

DO YOU WISH TO REDUCE N?
HELP DEMONSTRATION

THE BEST THEORETICAL ALIAS SET WOULD CONTAIN:
$1(2)+2(3)$
an alternative best theoretical alias set hould contain:
2(2) $+1(4)$
the best theoretical design may or may not be achievable.
removing any givfn column will result in the following designs:
RESULTING DESIGN
$1(2)+2(3)$
$1(2)+2(3)$
$1(2)+2(3)$
$1(2)+2(3)$
-101-

5

$$
(y) I+(z) z
$$

lasnohsy usim nox od nhoto hoilm
1 :INVM nox yaghn till sihi si
help demonstration


the alias set of the present design consists of: $1(2)+2(3)$

## y y YOU WISH TO REDUCE N?

## help demonstration

*****************************************************************
the dest theoretical alias set would contain:
3(2) FACTOR TERM

$$
\begin{aligned}
& 1 \\
& 1
\end{aligned}
$$

LInaOyd LSviINOD ONINIABa

$$
\begin{aligned}
& \text { TOTAL ALIAS SET } \\
& \text { COLURN NUMBER }
\end{aligned}
$$

removing any given colunn will result in the following designs: RESULTING DESIGN
$1(1)+1(2)+1(3)$
$3(2)$
$3(2)+1(2)+1(3)$
$1(1)+1(2)$
 colunn removed
 $\underset{* * * * * * * * * * * * * * * * * * *}{\text { HELP }}$
*
total alias set COLIMN NuMber $\begin{array}{lll}1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0\end{array}$
the alins set of the present design consists of: (2)

$$
N N
$$

dEFINING CONTRAST PRODUCT
-103-
WOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS?
N
PROBLEM DEFINITION COMPLETED.
hit return wien ready to continue.

EXAMPLE 3
-105-

复


you may enter one of the program seghents:
BASIC TERMINOLOGY
actual experimental design
EXPERIMENTAL REFINEMENT
EXIT

enter the segment number you hant and ilit return
2

## YOUR ENTRY WAS: 2--PROBLEM DEFINITION

please enter this run problem i.d.

## help demonstration

IIELP DEMONSTRATION

you may enter one of the problem definition options
bASIC factorial desicns
mixed level desicns
CENTRAL COHPOSITE DESIGNS
EXIT
$\therefore \dot{\sim} \dot{\sim}$
enter the option nurber you hant and hit return
Your entry was: 1--basic factorial designs
how he must define the specific problem to be analyzed in this run.
-106-
how hiny level.s for the factors: (2,3, or 5)
HON HANY l.EvEL.S FOR THE FACTORS:
bow hany factors are involven?
YOU MAY CHCOSE UP TO 20.
how hany experthental trials are available?
16 levels for tue factors. is tuis correct?
$y 2$ JLVES HOR
6 FACTORS ARE PRESENT. IS THIS CORRECT?
Y 16 EXPERIMENTAL TRIALS ARE AVAILABLE. IS THIS CORRECT?
63 IS TIIE TOTAL !uniber of effects and interactions.
OF tilesf:
6 are main effects
15 ARE FIRST ORDER INTERACTIONS
41 ARE HIGHER ORDER INTERACTIONS
hit return haen ready to continue
-107-

## help dehonstration

 HOW MANY EXPERIMENTAL TRIALS ARE TO BE RUN? THIS MUST BE SELECTED FROM THE VALUES
64


S THIS WHAT YOU WANT: 16 tuis size, you will need 2 linearly imdependent defining contrast(s).
tie factors in the design must be designated as a,b,C,D, etc. THE LETTER I IS RESERVED FOR THE IDENTITY EFFECT.
hotation is used to describe the factors in a particular trial or alias definition.
for a 2 level per factor experiment, the absence of a letter indicates the factor is at its low level.
the presence of a letter indicates the factor is at its high level.
a c means factor a at its high level, factor b at its low level, and factor C at its high level.
would you like to use a previously stored desicn?
$\mathbf{Y}$
help demonstration
-108-
based on the experimental description you have given about tie number of factors, the factor levels, and the number of trials THE NUMBER OF FACTORS, THE FACTOR LEVELS, AND THE NUMBER OF TRIALS
TO BE USED, THE FOLLONING DESIGN IS FEASIBLE: DESIGN $2.6 .161 / 4$ R alias definition:
$I=A B C E=A B D F$

do you hant help in generating a realizable 2 level alias set?
help demonstration

generally, in a two level experiment involving any number of factors
(represented by tue letter n) it is desired to isolate all single factors and two pactor interactions. this is under the assur HIGIER ORDER INTERACTIONS ARE INSIGNIPICANT.
to obtain the largest degree of isolation for a given experiment, all members of the alias set should contain approximately the same number of high level FACTORS.

> hit Return to continue
HELP DEHONSTRATION


## O A USEFUL HOTATION HAS BEEN DEVELOPED TO DESCRIBE THE COHPOSITION

 of tie total alias set. an exanple of this notation follohs:for a $1 / 4$ replicate of a 2 level design with 5 factors, the total alias set of this desicn might be:
ABE
ABCD
THIS ALIAS SET CONTAINS THREE (3) MEMBERS, TWO (2) MEMBERS WITH
THREE (3) HICH LEVEL FACTORS EACH AND ONR, (1) MEMBER WITH FOUR (4) hich level factors.
IN MOTATION FORM THIS BECOMES:
hould you like to see an example of generating a new desicn?
HELP IEMONSTRATION

YOUR PRESENT DESIGN CONSISTS OF 2 LEVELS, 6 FACTORS, AND 2 DEFINING CONTRASTS.
STORED DESIGN. STORED DESIGNS FOR THE PRESENT NURBER OF DEFINING CONTRASTS
ARE AS FOLLOWS:

NO. OF DEFINING CONTRASTS $=2$ STORED DESIGNS FOR N (NO, OF FACTORS )
YOU MAY ENTER A VALUE OF N FROM THE STORED DESIGNS SMALLER THAN YOUR DESIRED VALUE OF N AND BUILD UP TO THE DESIRED VALUE OF $N$ OR ENTER A LARGER VALUE OF N FROM THE STORED DESIGNS AND REDUCE TO THE DESIRED VALUE OR N.
ENTER THE STORED DESIGN VALUE OF N YOU NISH TO START WITH.
IS THIS THE NUMBER YOU WANT? 5
$Y$
HELP DEMONSTRATION
-110-
DO YOU WISH TO REDUCE N?
DO YOU WISH TO INCREASE N? YES
HELP DEAONSTRATION

TIIE BEST THEORETICAL DESIGN MAY OR MAY NOT BE ACHEIVABLE.
select a defining contrast product you wish to change. if you
IF YOU do NOT HISH TO INCREASE IT ENTER A O
defining Contrast products must be of til form:
DEFINIMG CONTRAST PRODUCT $=0$ or 1
enter the defining contrast product. be sure to leave a space betheen
nuibers as in the following example: 123
-111-
NOW ENTER A $O$ OR 1 FOR THE ABOVE dEFINING CONTRAST PRODUCT.
-
IS TUIS CORRECT?
YES
MLLP DEHONSTATION

$i$
NUMBER OF FACTORS PER RON

4
3
4
nIMBER OF PACTORS PER RON
added colurn


ENTER THE DEFINING CONTRAST PRODUCT. BE SURE TO LEAVE


-112-
HELP DEHONSTRATION


the alias set of the present design consists of:

$$
\begin{aligned}
& 3(4) \\
& \text { DO YOU WISH TO REDUCE N? } \\
& \text { NO YOU WISH TO INCREASE N? } \\
& \text { W NO YO }
\end{aligned}
$$

THE DEFINING CONTRAST SET DEFINED CONTAINS 2 INDEPENDENT MEMBER (s). do you want to see the total alias set?

## Lכnaoyd LSVAINOD ONINIAJQ

number of factors per row
5
$\begin{array}{ll}2 & 1 \\ 2 & 1\end{array}$
proalem definition cohpleted.
hit return hhen ready to continue.

MIXED LEVEL DESIGNS
Option 2--Mixed Level Designs allow the user to define full or fractional factorial designs where some factors of the design have a different number of levels than other factors of the design. A menu is displayed, but at this time only the 2 level crossed with the 3 level case has been developed (see the following example). The mixed level design is treated like two separate designs, one for the factors at 2 levels and one for the factors at 3 levels. Then the resulting obeservation blocks from the two separate designs are combined to produce a mixed level observation block.

The displayed menu also allows the user to exit if the $2 \times 3$ mixed level is not desired.

welcohe to the automated experimental design program.
THE EASIEST APPROACH TO COMBINING DESICNS OF different levels is to consider each level independently by performinc an optimu fractionation for that level FOLLOWED BY A CROSSING hULTIPLICATION OF THE OBSERVATION blocks for each level.

## THE AVAILABLE DESIGNS ARE:

## 1. $2 \times 3$ 2. EXIT

enter the design nurber you want and hit return.

## YOUR EMTRY HAS: $1-2 \times 3$

 how many factors are involved in the 2 level portion of the MIXED LEvEL DESIGN?YOU MAY CHOOSE UP TO 18. MIXED LEvEL DESIGN?
YOU MAY CHOOSE UP TO 18. YOU HAY CIKOSE UR YO 1 O.
for 2 levels, there are 1 factors. IS THIS WIAT YOU WANT? Yes
how many factors are involved in the 3 level portion of the MIXED LEVEL DESIGN?
YOU MAY CHOOSE UP TO 11. 2
(2G many total experimental trials are THIS hUST be selected from the values:
6

IS THIS WHAT YOU WANT: 6
YES
YES
THIS IS A $1 / 3$ FRACTIONAL FACTORIAL DESIGN.
hit return to continue
MIXED LEVEL DESIGN DEMONSTRATION

FOR A MIXED LEVEL DESIGN YOU WILL HAVE TO DEFINE THE alias set for one level at a time.

FOR THE 2 LEVEL PORTION OF THE MIXED LEVEL DBSIGN, YOU WILL NEED 1 LINEARLY INDEPENDENT DEFINING CONTRAST (S)

## HIT RETURN TO CONTINUE

-118-

FOR A 2 LEVEL PER FACTOR EXPERIMENT, THE ABSENCE OF A LETTER INDICATES
THE FACTOR IS AT ITS LON LEVEL.
THE PRESENCE OF A LETTER INDICATES THE FACTOR IS AT ITS HIGH LEVEL.
THE FACTORS IN THE DESIGN MUST BE DESIGNATED AS A,B,C,D, ETC. THE LETTER I IS RESERVED FOR THE IDENTITY EFFECT.
Since each factor must appear in a treatment, the following NOTATION IS USED TO DESCRIBE THE FACTORS IN A PARTICULAR TRIAL OR AlIAS DEFINITION.

ac heans factor a at its high level, factor b at its low level, and factor c at its high level.
WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?
NO
> you fay define contrasts by describing hhich effects art. TO BE CONFOUNDED. FOR EXAMPLE, AB=CD. ANOTHER (hort: cormon) way is to define the aliasing in terms of the identity effect. for example, $1=$ abcd
remehber that the defining contrast must be of the form effect $1=$ effect 2 the first contrast effect is input then the second.

## dEFINING CONTRAST NO. 1 FIRST TERM

defining contrast no. 1 second term A
 IS tIIIS CORRECT?
tIIE defining contrast set defined contains 1 independent member(s).
do you want to see the total alias set?
YES
-119-
IIt return when ready to continue.
hOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS? NO
this is a sumpary of the number of main effic ts and first order INTERACTIONS CONFOUNDED WITH EACH MAIN EFFECT AND FIRST ORDER interactions. main effects are marked with tile letter m, and those hain effects that are confounded with other main effects are flagged With an *.
hit return then ready to continue.
EXPERIAENTAL. DESICN SUHIARY


WHICII EFFECT/INTERACTION DO YOU WANT TO SEE? HIT RETURN HHEN READY TO CONTINUE.
MIXED LEVEL DESIGN DERIONSTRATION
 MAIN 1-ST HIGHER 0 M END OF SURMARY TABLE I. HIT RETURN TO CONTINUE.
O MIXE! LEVEL DESIGN DEMO
MIXED LEVEL DESIGN DEMONSTRATION ENTER THE EFFECT dESCRIPTION. ENTER A BLANK TO EXIT.
EXAMPLE: ENTER ab tO SEE WHAT IS ALIASED WITH EFFECT AB.
 SET FOR ONE LEVEL AT A TIME. FOR THE 3 LEVEL PORTION OF THE MIXED LEVEL DESIGN, YOU WILL NEED 1 LINEARLY INDEPENDENT DEFINING CONTRAST(s).

## HIT RETURN TO CONTINUE.

MIXPD LEVEL DESIGN DEMONSTRATION
THE FACTORS IN THE DESIGN MUST BE DESIGNATED AS A,B,C,D, ETC.
THE LETTER I IS RESERUED FOR THE IDENTITY EFFECT THE LETTER I IS RESERVED FOR THE IDENTITY EFFECT. SINCE EACH
MUST APPEAR IN A TREATMENT, THE FOLLOWING NOTATION IS USED TO TIIE FACTORS IN A PARTICULAR TRIAL OR ALIAS DEFINITION.
for a 3 level per factor experiment, the absence of a letter is the ion level, THE PRRSENCE OF TIIE LETTER INDICATES THE INTERAEDIATE LEVE1., HHILE THE LETTER FOLLONED BY A 2 INDIGATES TIE HICH LEVEI.
azC heans factor a at its hich leven, factor b at its low level, and factor c
AT ITS INTERMEDIATE IEVEL.
WOULD YOU LIKE TO USE A PREVIOUSIIY STORED DESIGN?
YOU MAY dEFINE CONTRASTS BY dESCRIBING WHICH EFFECTS ARE TO BE CONFOUNDED.
FOR EXAMPLE, AB $=$ CD.
ANOTHER (MORE COMYON) WAY IS TO DEFINE THE ALIASING IN TERMS OF THE
IDENTITY EFFECT. FOR EXAHPLE, I = ABCD.
REMENBER THAT TIIE DEFINING CONTRAST MUST BE OF THE FORM EEFECT $1=$ EFFECT? TIE FIRST CONTRAST EFFECT IS INPUT THEN TIE SECOND.
DEFINING CONTRAST NO. 1 FIRST TERM
DEFINING CONTRAST NO. 1 SECOND TERM
A2B2
IS THIS CORRECT?
YES

- (s) yGquak inadnadauni I SNIVLnOD uanisad las lSvaino oninisaa aill
dO YOU WANT TO SEE THE TOTAL ALIAS SET?
YES
total alias set
A2B2
HIT RETURN HIIEN READY TO CONTIHUE.
WOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS?
RO
( ARE MALKIII WITII TIIE SETTER MFECT AND FIRST ORDER INTERACTION. MAIN EFFECTS

hit return hilen ready to continue.
EXPERIMENTAL DESIGN SUMMARY

hit return hien ready to continue
mixed level design demonstration
***************************************k******************
wilich effect/interaction do you want to sees enter the effect description. enter a blank to exit. example: enter ab to see what is aliased hitil effect ab.
basic experimental block/OBSERVATION VECTOR these are the experimental observations to be run in the data collection process.
for tie 2 level per factor portion on the experiment,
0 and 1 represent the low and hicil factor values.
FOR THE 3 level per factor portion of tie experiment,
O , 1, AND 2 represent the Low, intermediate, and high factor values.
hould you like to save a copy of this observation vector TO BE PRINTED ON THE LINE PRINTER?
hit return to continue.



## CENTRAL COMPOSITE DESIGNS

Option 3--Central Composite Designs allow the user to add more observation vectors to the basic factorial experiments in order that a quadratic approximation to the response surface of the experimental region can be made. The user can choose a rotatable or non-rotatable central composite design. There are also two methods for specifying the real world levels. One is to specify the real world range for each factor. The other is to specify the real world levels of each factor in the basic factorial design.

Central composite designs can be defined in one of two ways. The user may use program segment 2 - option 1 and program segment 3 to define the basic factorial portion of the central composite design, then enter segment 2 option 3 to complete the additional experiments (see example 1).

Or the user may enter directly into segment 2 - option 3 in which case, the program will prompt the user for the basic factorial definition (see example 2).

## EXIT

Option 4--Exit allows the user to exit from the problem definition segment.

CENTRAL COMPOSITE DESIGNS
-125-

$$
\text { EXAMPLE } 1
$$

welcoie to tue autorated experimental design program.

## YOU MAY ENTER ONE OF THE PROGRAM SEGMENTS: <br> BASIC TERHINOLOGY PRODLEK DEFENITION DESIGN <br> ACTVAL EAPERI NEFINEMENT 5. EXIT <br> enter the segment number you want and hit return

YOUR ENTRY WAS: 2--PROBLEM DEFINITION

## PLEASE ENTER THIS RUN PROBLEM I.D.

CENTRAL COMPOSITE DEHONSTRATION

YOU MAY ENTER ONE OF THE PROBLEM DEFIAITION OPTIONS:
BASIC FACTORIAL DESIGNS
MIXED LEVEL DESIGNS
CENTRAL COMPOSITE DESIGNS

## EXIT

1. 
2. 
3. 

ENTER THE OPTION NUMBER YOU WANT AND HIT RETURN
YOUR ENTRY WAS: 1--BASIC FACTORIAL DESIGNS
NOW WE MUST DEFINE THE SPECIFIC PROBLEM TO BE AMALYZED IN THIS RUN. HON MAIIY LEVELS FOR THE FACTORS? (2,3, OR 5)
how mary factors are involved? YOU MAY CIIOOSE UP TO 20.
how many experimental trials are available?
${ }_{8}^{110 \mathrm{~W}}$

| how many experimental trials are available? |
| :--- |
| B |
| y levels for the factors. is this correct? |
| Y $\quad 3$ factors are present. is tis correct? |

8 experimental trials are available. IS this correct?
7 IS THE TOTAL NUHBER OF EFFECTS AND INTERACTIONS.
OF THESE:
3 ARE MAIH EFFRCTS.
3 ARE FIRST ORDER INTERACTIONS.
1 ARE HIGIER ORDER INTERACTIONS.

## ilt return when ready to continue

CENTRAL COHPOSITEE DEMONSTRATION

(10) MANY EXPERIMENTAL TRIALS are to
tuis must be selected from the values:
$8 \quad 2 \quad 4 \quad 8$
y this hhat you want:
THIS IS A FULL FACTORIAL DESIGN. FOR AN EXPERIMENT OF THIS SIZE, YOU WILL NEED
O LINEARLY INDEPENDENT DEFINING CONTRAST(s).
PROBLEM DEFINITION COMPLETED.
hit return hien ready to continue.
C:ENTRAL CORPOSITE DEMONSTRATION

you hay enfer une of tie probley definition options:
basic factorial desicns
Mixed level designs
enter the option number you want afid hit return
YOUR ENTRY WAS: 4--ixit
you hay enter one of the program segments:

1. BASIC TERMINOLOGY 2. PROBLEM DEFINITION 4. EXPERIMENTAL REFINEMENT
2. EXIT
enter the segment numiber you want and hit return
your entry was: 3--ACTUAL EXPERIMENTAL DESIGN would you like to change tie run i.d.?
NO AliASINg is used in this desicn.
basic experimental biock/ObSERVATION VECTOR
these are tie experlmental observations to be run in the data collection process. for this 2 level. per factor experiment, 0 and 1 represent the loh and hich factor values. hould you like to save a copy of this observation vector to br printed on the line printert

## hit return hhen ready to continue.

CENTRAL COMPOSITE DEMONSTRATION

basic experimental block/OBSERVATION VECTORS

-129-
100
1101
111
hit return hilen ready to continue.
CENTRAL COHPOSITB DEMONSTRATION

you hay enter one of the program segments: $\begin{array}{ll}\text { 1. } & \text { BASIC TERMINOLOGY } \\ \text { 2. } & \text { PROBLEM DEFINITION }\end{array}$ 2. PROBLEM DEFINITION
actual experimental design
4. EXPERIMENTAL REFINEMENT
enter the segment number you want and hit return
CENTPAL COHPOSITE DEMONSTRATION

you hay enter one of tilf problem definition options:
basic factorial designs
MIXED LEVEL DESIGNS
CENTRAL COMPOSITE DESIGNS
EXIT
1.
3.
4.
enter the option nanber you want and hit return
-130-
YOUR ENTRY WAS: 3--CENTRAL COHPOSITE DESIGNS

AN N-FACTOR CENTRAL COMPOSITE DESIGN CONSISTS OF THREE COMPONENTS.

1. A FULL OR FRACTIONAL REPLICATE OF A TWO-LEVEL FACTORIAL DESIGN, WIIERE FOR EACII FACTOR TIIE TWO CODED IEEVELS ARE -1 and 1 .
2. TWO CORRESPONDING AXIAL POINTS FOR EACH FACTOR WITI CODED l,EVEL -ALPIIA AT ONE POINT, HALPHA AT THE, OTHER AND WITH ALJ OTHER FACTORS IIAVING CODED LEVEL ZERO AT THESE POINTS.
3. THE CENTER POINT OF EACH OF THE FACTORS HAVING
CODED LEVEL ZERO.
hit Return to continue.
CENTRAL COMPOSITE DEMONSTRATION
************************************************************ FIRST CENTRAL CORTPOSITE DESIGNS REQUIRE A TWO (2)
LEVEL FNLL OR FRACTIONAL DESIGN WIIERE NO ONE-FACTOR
EFFECTS OR THO-FACTOR INTERACTIONS CAN BE ALIASED
WITH I, ANOTIER ONE-FACTOR EFFECT, OR ANOTHER TNO
FACTOR INTERACTION. HAVE YOU ALREADY DEVELOPED A
TWO (2) LEVEL FACTORIAL DESIGN IN THE PROBLEM
DEFINITION (SEGENT 2-OPTION 1) MEETING THE ABOVE
CONDITIONS?
YES
if you hant a rotatable central composite design, THE CODED LEVEL OF ALPHA WILL BE COMPUTED FOR yOU, or you may choose to supply your own value for the Coded level of alpha. do you want a rotatable DESIGN?
tuis is the value of alpha necessary for a rotatable CENTRAL COMPOSITE DESIGN.

## ALPHA $=1.68$

## hit return to continue

## CENTRAL COMPOSITE DEMONSTRATION


TWO methods are available for specifying real world levels.

1. the first is to specify the real world range that
you hant for each factor the coded levels -alpha and talpiald correspond to evels -alpha and talpha. this method
would be appropriate hien generating a central COHPOSITE DESIGN FROM SCRATCH.
2. THE SECOND IS TO SPECIFY THE REAL WORLD LEVELS OF
EACH FACTOR CORRESPONDING TO THE CODED LEVELS -1
AND 1. THIS METHOD WOULD BE APPROPRIATE IN A
SERUENTIAL DESIGN WHERE THE FACTORIAL PART OF THE
EXPERIMENT HAS ALREADY BEEN COMPLETED.
WHICH METHOD HOULD YOU LIKE? ENTER -- 1 or 2 .
2
ENTER THE REAL HORLD VALUE CORRESPONDING TO THE CODED ENTER THE REAL HORLD VALUE CORRESPONDING TO THE CODED
LEVEL -1 FOR FACTOR NUMBER 1 48.24
enter the real horld value corresponbing to the coded LEVEL +1 FOR FACTOR NUABER 1 101.76 48.24 101.76
are these the values that you hant?

## yes

 - 1 Level = +1 Level -133-> ARE TIESE TIIE VALUES TIAAT YOU WANT?
YES
$\begin{aligned} & \text { CENTRAL CORTOSITE DEMONSTRATION } \\ & * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *\end{aligned}$
> Lever tie real horld value corresponding to the coded
5.98
> LEvEL the real horld value corresponding to the coded
10.02
> $\begin{array}{rrr}-1 & \text { LEVEL }= & 5.98 \\ +1 & \text { LEVEL }= & 10.02\end{array}$
> ENTER

 CODED VALUES AND THEIR CORRESPONDING REAL WORLD values for each factor of the central composite


$$
\begin{aligned}
& \begin{array}{c}
1 \\
101.76 \\
101.76 \\
1 \\
101.76 \\
101.76
\end{array} \\
& \begin{array}{l}
5 \\
6 \\
7 \\
8
\end{array} \\
& \text { FACTOR NO. } \\
& \text { TRIAL NO. } \\
& \text { factor no. } \\
& \text { trial No. } \\
& \text { 'ON YOLJVA } \\
& \text { trial No. } \\
& \text { FACTOR NO. } \\
& \text { TRIAL HO. } \\
& \text { hit return to continue } \\
& \text { CENTRAL COMPOSITE DEMONSTRATION }
\end{aligned}
$$

> 6 additional axial points of the central composite design (EXPRESSED IN REAL hORLD LEVELS) $\begin{array}{rrrr}8 & 8 & 8 & 8 \\ m & m \infty & m & m \infty\end{array}$ ~ $\begin{gathered}1 \\ 120.00 \\ 1 \\ 75.00 \\ 1 \\ 75.00 \\ 1 \\ 30.00\end{gathered}$ 1 -
$\stackrel{1}{8 . e}$

FACROR NO.
POTNT NO.
FACTOR NO.
POINE NO.
HIT RETIRN TO CONTINUE
CENTRAL COHPOSITE DEMONSTRATION


[^1]EXAMPLE 2

Enter tile segment number you bant and hit return
2
YOIR EMTRY WAS: 2--PROBLEM DEFINITION
hOULD YOU LIKE TO CHANGE THE RUN I.D.?
CENTRAL COHPOSITE DEMOHSTRATION
YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS: BASIC FACTORIAL DESIGNS MIXED LEVEL DESIGNS

CENTRAL COMPOSITE DESIGNS
EXIT
ENTER THE OPTION NUHBER YOU WANT AND HIT RETURN
YOUR ENTRY HAS: 3--CENTRAL COIIPOSITE DESIGNS
CENTRAL COMPOSITE DEMONSTRATION

AN N-FACTOR CENTRAL COAIPOSITE DESIGN CONSISTS OF THREE COMPONENTS. 1. A FULL OR FRACTIONAL REPLICATE OF A TWO-LEVEL
FACTORIAL DESIGN, WHERE FOR EACH FACTOR THE TWO CODED
LEVELS ARE -1 and 1 .
2. TWO CORRESPONDING AXIAL POINTS FOR EACH FACTOR
WITI CODED LEVEL -ALPHA AT ONE POINT, +ALPHA AT
THE OTHER AND WITH ALL OTHER FACTORS HAVING CODED
LEVEL ZERO AT THESE POINTS.
3. THE CENTER POINT OF EACH OF THE FACTORS HAVING
CODED LEVEL ZERO.
IIIT RETURN TO CONTINUE
CENTRAL COMPOSITE DEMONSTRATION

FIRST CENTRAL COAROSITE DESIGNS REQUIRE A TWO (2) LEVEL FULL OR FRACTIONAL DESIGN WHERE NO ONE-FACTOR
EFFECTS OR TWO-FACTOR INTERACTIONS CAN BE ALIASED
WITH I, ANOTHER ONE-FACTOR EFFECT, OR ANOTHER TWO
FACTOR INTERACTION. HAVE YOU ALREADY DEVELOPED
TWO (2) LEVEL FACTORIAL DESSIGN IN THE PROBLEM DEFINITION (SEGMENT 2 - OPTION 1) MEETING THE NO

[^2]NOW WE RUST DEFINE THE SPECIFIC PROBLEM TO BE ANALYZED IN THIS RUN.

HIT RETURN WIEN READY TO CONTINUE.
CENTRAL COMPOSITE DEMONSTRATION


this is a full factorial design. for an experiment of this size, you will need 0 LINEARLY INDEPENDENT DEFINING PROBLEM DEFINITION COMPLETED.
ilit return when ready to continue.
CENTRAL COMPOSITE DEMONSTRATION ***********************k************************************

## no aliasing is used in this design

basic experimental block/ObSERVation vector for this ? level per factor experiment, 0 and 1 represent the low and hich factor values. hould you like to save a copy of dE PRINTED ON the line printer?
ilit return wien ready to continue.
CENTRAL COMPOSITE DEMONSTRATION

basic experimental block/OBSERVATION vectors
hit return hien ready to continue.

## 

ABC
000
0011
010
011
100
1110
1111
CENTRAL COMPOSITE DEMONSTRATION

## basic factorial part of central composite design corpleted.

 THE CODEd LLEVEL OF alpha will be computed for you,OR you may choose to Supply your Own value for the coded level of alpha.
do you want a rotatable design?
yes
tilis is tie value of appia necessary for a rotatable. tiils is tile value of appia necessary for a rotatable
central cohposite design.

## $\mathrm{AL} . \mathrm{PHA}=1.68$

## ilit return to continue

CENTRAL COAPOSITE DEMONSTRATION
two methods are available for specifying real horld levels.

1. tile first is to specify the real horld range that YOU WANT FOR EACII FACTOR. THIS WOULD CORRESPOND TO the coded levels -alpha and +alpila. this hethod COAPOSITE DESIGN fROM SCRATCH.
2. tile second is to specify the real world levels of 2.
EACII FACTOR CORRESPONDING TO THE CODED LEVELS -1
AND 1. THIS METHOD WOULD BE APPROPRIATE IN A
SEQUENTIAL DESIGN WIERE THE FACTORIAL PART OF THE
EXPERIMENT HAS ALREADY BEEN COMPLETED.
hiICI method hould you like? enter --1 or 2.
enter tie loner real horld range corresponding to tue coded 1 yganon yolova yos vhatv- tansi 30.00
enter tie upper real horld range corresponding to tie coned LEVEL +ALPHA FOR FACTOR HUMBER 1. 120.00

## LONER LEVEL $=30.00$

UPPER LEVEL $=120.00$
are these the values that you hant?
enter the lower real horld range corresponding to tie coded level -alpin for factor number 2
0.10
enter the upper real hrold range corresponding to the coded level +alpil for factor number 2 0.50
LOWER LEVEL $=0.10$ UPPER LEVEL $=0.50$
are these the values that you want?
enter the loner real world range corresponding to the coded LEVEL -ALPHA FOR FACTOR NUMBER 3 4.60
enter the upper real horld range corresponding to the coded Level +alpha for factor number 3 11.40
LONER LEVEL $=4.60$
UPPER LEVEL $=11.40$
are tiese the values that you want?
yes
CENTRAL COMPOSITE DRMONSTRATION

---CENTRAL COMPOSITE (C-C) DESIGN---
PROBLEM DEFINITION SUPMARY
NO. OF BASIC FACTORIAL LEVELS = 2 NO. OF BASIC FACTORIAL FACTORS $=3$
NO. OF BASIC FACTORIAL TRIALS $=8$
$=1.68$
CENTRAL COMPOSITE DEMONSTRATION

annilno 0 OL NUnIay IIH
 CODED VALUES and their corresponding real world
values for each factor of tile central composite design
C-C
C-C
0.0 75.00
0.30 0
$?$
0
$\infty$

 8.00
C-C
LEVEL
1.68
120.00
0.50
11.40


3
8.00
3
4.60
~ $\stackrel{\circ}{0}$ ~ 1
75.00
1
75.00
n $\quad 0$ FACTOR NO.
POINT NO. FACTOR NO.
POINT NO. factor no. POINT NO. hit RETURN

CENTRAL COMPOSITE DEMONSTRATION

$\therefore$
this central cotposite design is rotatable, if 6 replicates of the center point are made, the Variance of the estimated response will be approximately tile same at all points hithin tile sphere of radius one.
hit return to continue.

## ACTUAL EXPERIMENTAL DESIGN

Segment 3--Actual Experimental Design--uses the previously specified experimental definition (Segment 2 - Option 1) to construct the set of experimental treatments to be run. This set is called the basic experimental block/observation vectors. To aid the experimenter in deciding if the experiment has an acceptable structure, an alias summary is displayed that shows how main effects and first-order interactions are aliased. The user may also have the aliasing of any specific effect displayed.

If the design is unacceptable, the experimenter can rerun the problem definition phase.

The following is an example of the experimental design of the problem defined in Segment 2 - Option 1 - Example 1.
you may enter one of the program segments:
enter the segrent number you hant and hit return

## BASIC TERMIN

 PROBLEM DEFINITION DESIGN Prgtual experimental ACTUAL EXPERIMENTAL DESIEXPERIMENTAL REFINEMENT EXIT
1.
2.
3.
4.
5.
YOUR ENTRY WAS: 3--ACTUAL EXPERIMENTAL DESIGN hould you like to change the run I.d.? YES
please enter this run probleh i.d.
actual experimental design demonstration
this is a sumtiary of the number of main effects
AND FIRST ORDER INTERACTIONS CONFOUNDED WITH
MAIN TAINECTS ARE MARKED HITH THE LETTER M, AND THOSE main effects that are confounded with other main effects are flagged with an *.
hit return when ready to continue
EXPERIMENTAL DESIGN SUMMARY
EFFECT

$\begin{array}{cc}\text { 1-ST } & \text { HIGHER } \\ 0 & 1 \\ 0 & \text { M } \\ 0 & 1 \\ 0 & 1\end{array}$
MAIN
-151.


| $\Sigma$ | 2 | 2 |
| :---: | :---: | :---: | 00000000000 00000000000

end of sufplary table

## hit return hien ready to continue. <br> HIT RETURN HIEN READ

If any of the main effects had been confounded with another main effect,
it would have been flagged with an *.
Note that in this design, none of the main effects or first-order interactions are confounded with any other main effects or first-order
interactions. actions are confounded with any other main effects or first-order
interactions.

## ACTUAL EXPERIMENTAL DESICN DEMONSTRATION

 -152- WIICH EFFECT/INTERACTION DO YOU HANT TO SEE?
ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.
EXAMPLE: ENTER AB TO SEE WLAT IS ALIASED WITH EFFECT AB.
A WIICH EFFECT/INTERACTION DO YOU HANT TO SEE?
ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.
EXAMPLE: ENTER AB TO SEE WLAT IS ALIASED WITH EFFECT AB
A WIICH EFFECT/INTERACTION DO YOU WANT TO SEE?
ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.
EXAMPLE: ENTER AB TO SEE WLAT IS ALIASED WITH EFFECT AB.
A BCDE
WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?
ENTER THE EFFECT DESCRIPTION. ENTER A BLANR TO EXIT BCDE
WHICH EFFECT/INTERACTION DO YOU WANT TO SEE?
ENTER THE EFFECT DESCRIPTION. ENTER A BLANR TO EXIT BCDE
WHICH EFFECT/INTERACTION DO YOU HANT TO SEE?
ENTER THE EFFECT DESCRIPTION. ENTER A BLANR TO EXIT. EXANPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB
CD ABE
WIICH EFFECT/INTERACTION DO YOU WANT TO SEE? ABE
WIICH EFFECT/INTERACTION DO YOU WANT TO SEE? ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT. EXAMPLE: ENTER AB TO SEE WIAT IS ALIASED WITH EFFECT AB.
 A析
BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTOR THESE ARE THE EXPERIMENTAL OBSERVATIONS TO BE RUN IN THE DATA COLLECTION PROCESS.
FOR THIS 2 LEVEL PER FACTOR EXPERIMENT, 0 AND 1 REPRESENT the Low and high factor values.
hould you like to save a copy of this observation vector TO EE PRINTED ON THE LINE PRINTERT
hit return when ready to continue.
actual exprithertal design dehonstration


[^3]hit return hhen ready to continue.

## DATA ANALYSIS

Once the data have been collected, it must be analyzed to identify significant effects. this analysis could consist of an analysis of variance (ANOVA) or of a regression analysis.

The capability to perform the data analysis has not been included in the AED program at this time since a program to analyze a fractional factorial experiment with both regression analysis and ANOVA techniques would be a full-time project in itself. The program user should analyze his data with existing routines available at AMRL or other computer facilities.

## EXPERIMENTAL REFINEMENT

Once the experimenter has conducted a fraction of a full factorial experiment, an analysis of the basic block data may provide sufficient information to preclude additional data collection. If one or more factors produce significant results, all further work may be confined to studying these factors in detail. the experiment may be redesigned with fewer factors or with other factors added. Additional work is required if:

1. The main effects are not given with sufficient precision.
2. Some main effects may be confounded with two-factor interactions and may require separation.
3. Some two-factor interactions may require separation.
4. Additional factors may need to be included in the design.

## SEPARATION OF ALIASES

Separation of aliases assumes that a fractional design has been executed and that we wish to collect additional data in order to obtain a higher degree of precision and/or to separate the main effects from two-factor interactions.

A $1 / 2$ factorial or a $1 / 3$ factorial will require a full factorial to separate aliases. thus, this assumes that a $1 / 4$ or $1 / 9$ (etc.) fractional factorial
has been performed and the problem is to separate effect $X$ from effect $Y$, i.e., $X$ and $Y$ are in the same aliasing group, $X=Y$.

The user will define the aliased terms and the program will tell the user which of his original alias terms may be deleted to remove this aliasing.

The following is an example of the separation of alias feature.

The problem can be defined in one of two ways. The user may use program segments 2 - options 1 and 3 to define the problem, then enter segment 4 to perform the refinement (see example 1).

Or the user may enter directly into segment 4 in which case, the program will prompt him for the program definition (see example 2).


WFLCOME TO THE AUTOMATED EXPERIMENTAL DESIGN PROGRAM.
yOU tuy enter one of the program segments:

## bASIC TERMINOLOGY

PROBLEM DEFINITION DESIGN
ACTUAL EXPERIMENTAL REFINEMENT
EXIT
1.
2.
3.
4.
5.
enter the segment number you want and hit return
2

## YOUR ENTRY WAS: 2--PROBLEM DEFINITION

PLEASE ENTER THIS RUN PROBIEM I.D.

## EXPERIMENTAL REFINEMENT DEMONSTRATION

EXPERIMENTAL REFINEMENT DEMONSTRATION


YOU MAY ENTER ONE OF THE PROBLEM DEFINITION OPTIONS: 1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGUS
4. EXIT 1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT 1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT 1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGNS
4. EXIT 1. BASIC FACTORIAL DESIGNS
2. MIXED LEVEL DESIGNS
3. CENTRAL COMPOSITE DESIGAS
4. EXIT

ENTER TIIE OPTION NUMBER YOU WANT AND HIT RETURN

YOUR EATRY WAS: 1--BASIC FACTORIAL DESIGNS HOW WE fllist define tie specific problem to be ANAIYZED IN THIS RUN.

HOW MANY LEVELS FOR THE FACTORS? (2, 3, or 5)
HON ILAHY FACTORS ARE INVOLVED? YOIJ MAY CHOOSE UP TO 20.
HOW MANY EXPERIMENTAL TRIALS ARE AVAILABLE?
20

This is an example of usirg segments 2 - option 1 and 3 to define the problem.

## EXPERIMERTAL REFINEMENT DEMONSTRATION



## 20 EXPERIMENTAL TRIALS ARE AVAILABLE. IS IIIIS CURKRU'゙1'? <br> 63 IS TIIE TOTAL NUMBER OF EFFECTS AND INTERACTIONS. <br> OF 6 ARE MAIN EFFRCTS <br> HIT RETURN WHEN READY TO CONTINUE

SINCE EACH FACTOR MUST APPEAR IN A TREATMENT, THE FOLLONING NOTATION IS USED TO describe tile factors in a particular trial or alias definition.

FOR A 2 LEVEL PER FACTOR EXPERIMENT, THE ABSENCE OF A LETTER INDICATRS
TIIE FACTOR IS AT ITS LOW LEVEL.
TIIE FACTOR IS AT ITS LOW LEVEL.
THE PRESENCE OF A LETTER INDICAT
THE PRESENCE OF a LETTER INDICATES THE factor IS at ITS hich level.
AC heans factor at at its high level, factor b at its low level,
AC MEANS FACTOR AT AT ITS HICH LEVEL, FACTOR B AT ITS LOW LEVEL,
AND FACTOR C AT ITS HIGH LEVEL.
WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?
NO.
AC MEANS FACTOR AT AT ITS HICH LEVEL, FACTOR B AT ITS LOW LEVEL,
AND FACTOR C AT ITS HIGH LEVEL.
WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?
NO.
AC MEANS FACTOR AT AT ITS HICH LEVEL, FACTOR B AT ITS LOW LEVEL,
AND FACTOR C AT ITS HIGH LEVEL.
WOULD YOU LIKE TO USE A PREVIOUSLY STORED DESIGN?
NO.
YOU
YOU MAY DEFINE CONTRASTS BY DESCRIEING WIIICH EFFECTS ARE TO BE CONFOUNDED. FOR EXAMPLE, AB=CD. ANOTHER (MORE COMMON) WAY IS TO DEFINE THE ALIASING
IN TERMS OF THE IDENTITY EFFECT FOR EXAMPLE, I=ABCD.

IN TERMS OF THE IDENTITY EFFECT. FOR EXAIPLE, I=ABCD.
REMEABER THAT THE DEFINING CONTRAST MUST BE OF THE FORM EFFECT 1 - EFFECT 2 THE FIRST CONTRAST EFFECT IS INPUT THEN THE SECOND DEFINING CONTRAST NO. 1 FIRST TERM

DEFI
DEFINING CONTRAST NO. 1 SECOND TERM
ABCD
IS TUIS CORRECT?
$=\mathrm{ABCD}$
DEFINLNG CONTRAST NO. 2 FIRST TERM
DEFINING CONTRAST NO. 2 SECOND TERM
ACDEF
IS THIS CORRECT?

- ACDEF
-160-

This is an example of how the alias set is input if the predefined (stored) alias set was unacceptable or unavailable.
THE DEFINING CONTRAST SET DEFINED CONTAINS 2 Independent member(s).
do you want to see the total alias set?
yes

## total alias set

ACDEF
ABDC
BEF
hit return hien ready to continue:
WOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS?
PROBLEM DEFINITION COMPLETED.
hit return hien ready to continue
-161-
you may enter one of the problem definition options: basic factorial designs
Mixed level designs central contposite designs EXIT
experimental. rhfinientint

1.
2.
3.
4.
enter tile option number you hant and ilit return
YOUR ENTRY WAS: 4--EXIT
WElCOME to the screening designs program. you may enter one of the program segments: 1. BASIC TERMINOLOGY 3. ACTUAL EXPERIMENTAL DESIGN 4. EXPERIMENTAL REFINEMENT 5. EXIT
enter the segment number you want and hit return
Your entry was: 3--ACTUAL EXPERIMENTAL DESIGN.
would you like to change the run i.d.? NO
this is a sidmary of tie number of main effects and first order interactions confounded With each main effect and first order interaction. hain effects are marked with the letter m, and those main effects that are confounded with other main effects are FLAgGED WITH AN *.
hit return when ready to continue. EXPERIMENTAL DESIGN SUMMARY
Effect
A
AB
AC
AD
AE
AF
B
l-ST hicher
$\Sigma$

O--MOO~
-162-

| 0 | 1 | 2 |  | ${ }^{\text {bC }}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 2 |  | nn |
| 1 | 0 | 2 |  | 日 |
| 1 | 0 | 2 |  | ${ }^{\text {BF }}$ |
| 0 | 0 | 3 | M | c |
| 0 | 1 |  |  | CD |
| 0 | 0 | 3 |  | CE |
| 0 | 0 | 3 |  | CF |
| 0 | 0 | 3 | M | D |
| 0 | 0 | 3 |  | DE |
| 0 | 0 | 3 |  | DF |
| 0 | 1 | 2 | M | E |
| hit return wien ready to continue |  |  |  |  |
| 1 | 0 | 2 |  | Ef |
| 0 | 1 | 2 | M | F |

$\Sigma$
N

$$
\begin{aligned}
& 0 \rightarrow 0 \\
& -0
\end{aligned}
$$

hit return wien ready to continue



> WIICII EFEECT/INTERACTION DO YOU WANT TO SEE?
ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT. example: enter ab to see what is aliased with effect ab. A

ABEF
CDEF
BCD


ICH EFFECT/INTERACTION DO yOU WANT TO SEE? example: enter ab to see what is aliased with effect ab. ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.

hit Return when ready to continue.
BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTORS
ABCDEF
000000
000011
001100
001111
010101
010110
011001
011010
100100
100111
101000
101011
110001
110010
111101
111110
EXPERIMENTAL REFINEMENT DEMONSTRATION



5 to separate these two effects with effect ABEP. The user may want to use aegent
you may enter one of the program segments:
basic terminology PROBLEK DEFINITION actual experimental desich EXPERIMENTAL REFINEMENT EXIT

- ~~~~~
~~~~~


ENTER THE SEGMENT MUMBER YOU HANT AND IIIT RETURN

[^4]-165-

$\begin{array}{lll}\text { MAY DELETE ALIAS NUMBER 1: } & I=A B C D \\ \text { MAY DELETE ALIAS NUMBER 2: } & I=A C D E F\end{array}$

## WHICII ALIAS NUMBER WOULD YOU LIIKE TO <br> WHICII ALIAS NUMBER WOULD YOU LIKE TO ELIMINATE?

The separation can be accomplished by deleting
or number 2. In this case, number 2 is chosen.
The separation can be accomplished by deleting either alias number 1 chosen.

[^5] EFFECTS TIIAT ARE CONFOUNDED WITH OTIIER MAIN EFFEGTS
HIT RETURN WIEN READY TO CONIINUE
岛

EXPERIMIFNTAL DESIGN SUPRIARY

## i $0-1+000-1000-000000$

$\frac{3}{3} 0000000000000000000$
hit return hien ready to continue
00

## a7gyi xyviwns 10 ONA

davad NaHM NynLay IIH
1 M
EXPERIMENTAL REFINERRENT DEMONSTRATION
 HIICH EFFECT/INTERACTION DO YOU WANT TO SEE?
ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT. EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.
BCD
WHICH EFFECT/INTERACTION DO YOU HANT TO SEE? ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT. EXAMPLE: ENTER AB TO SEE WILAT IS ALIASED WITH EFFECT AB.
Note: Effect A is no longer confounded with ABEF.
Note: Effect A is no longer confounded with ABEF.
BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTOR
THESE ARE THE EXPERIMENTAL OBSERVATIONS TO BE RUN
IN THE DATA COLLECTION PROCESS.
FOR THIS 2 LEVEL PER FACTOR EXPERIMENT, 0 AND 1 REPRESENT THE LON AND HIGH FACTOR VALUES.
NEW TRIALS (NOT PART OF PREVIOUS BLOCK)
ARE MARKED WITH THE LETTER N
WOULD YOU LIKE TO SAVE A COPY OF THIS OBSERVATION VECTOR TO BE PRINTED ON THE LINE PRINTER?
hit return when ready to continue.
EXPERIMENTAL REFINEMENT DEMONSTRATION
BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTORS

|  | ABCDEF |
| :---: | :---: |
|  | 000000 |
| $N$ | 000001 |
| N | 000010 |
|  | 000011 |
|  | 001100 |
| N | 001101 |
| $N$ | 001110 |
|  | 00111 |
| $N$ | 010100 |
|  | 010101 |
|  | 010110 |
| N | 010111 |
| $N$ | 011000 |
|  | 011001 |
|  | 011010 |
| $N$ | 011011 |
|  | 100100 |
| N | 100101 |
| N | 100110 |
|  | 100111 |

$$
\begin{aligned}
& \begin{array}{l}
\text { ABCDEF } \\
101001 \\
101010 \\
101011 \\
110000 \\
110001 \\
110010 \\
110011 \\
111100 \\
111101 \\
11110 \\
111111
\end{array} \\
& \geq \geq 2 \\
& \geq=2 \\
& N \\
& \text { hit return when ready to continue. } \\
& \text { Assuming that the previous block of experiments had already been performed, } \\
& \text { only those trials marked with an } N \text { (new) need to run now. }
\end{aligned}
$$

EXAMPLE 2
WElCOME TO THE AUTOLATED EXPERIMENTAL DESIGN PROGRAM
you may enter one of the program segients:

$$
\begin{aligned}
& \text { 1. BASIC TERMINOLOGY } \\
& \text { 2. PROBLEM DEFINITION } \\
& \text { 3. ACTUAL EXPERTMENTAL DESIGN } \\
& \text { 4. EXPERIMENTAL REFINEHENT } \\
& \text { 5. EXIT }
\end{aligned}
$$

enter the segment number you want and i!it return
4
YOUR EVTRY WAS: 4--EXPERIMENTAL REFINEMENT
PLEASE ENTER THIS RUN PROBLEM I.D.
EXPERIMENTAL REFINEMENT
This is an example of entering directly into segment number 4, without
first entering segment 2 - option 1.
EXPERIMENTAL REFINEMENT

THIS REQUIRES THAT THE EXPERIMENTER DESCRIBE THE ORIGINAL DESIGN. TIIE MOST EFFICIENT HAY TO DESCRIBE THE ORIGINAL DESIGN IS TO USE DESIGN DEFINITION CAPABILITY AS IN SEGMENTS 2 - OPTION 1 and 3.
YOU MUST SPECIFY THE DESIGN IN THE SAME WAY AS IN THE ORIGINAL RUN.
NOW WE MUST DEFINE THE SPECIFIC PROBLEM TO DE ANALYZED IN THIS RUN.
how many levels for the factors? (2,3, or 5)
2
the factors in the desicn must be desicmated as a,b,C,D, etc.
the letter I is reserved for the identity effect.
since each factor must appear in a treathent, the folloning notation is used to describe the factors in a particular trial or alias definition.
for a 2 level. per factor experiment, the absence of a letter indicates for factor is at its lon level.
the presemce of a letter indicates the factor is at its high level.
ac heans factor a at its high level, factor b at its lof level, and factor c at its hich level. hould you like to use a previously stored design? ल
you may define contrasts by describing which effects are to be confounded. for exarple, ab-cd. another (hore common) hay is to define the aliasing IN TERHS OF THE IDENTITY EFFECT. FOR EXAHPLE, I=ABCD.
rektheer that the defining contrast hist be of theform- effect 1 - efpect 2 the first contrast bffect is input then the second.
defining contrast no. 1 first term
ABCD
defining contrast mo. 1 second term
IS this correct?
-ABCD
acder is this correct?
=ACDEF
the defining contrast set defined contains 2 independent member(s).
dO YOU WANT TO SEE THE TOTAL ALIAS SET?
YES
WOULD YOU LIKE TO REDEFINE THE DEFINING CONTRASTS? NO

[^6]
this is a surmary of the number of main effects AND FIRST ORDER INTERACTIONS CONFOUNDED WITH AND FIRST ORDER INTERACTIONS CONFOUNDED WITH EACH MAIN EFFECT AND FIRST ORDER INTERACTION. main effects tiat are confounded with other main effects are flagged with an *.
hit return hhen ready to continue.
EXPERIMENTAL DESIGN SUMMARY 1-ST HIGHER O-N-00-1-1000-2000000000-1000

EXPERIMENTAL REFINEMENT WIICI EFFECT/INTERACTION DO YOU WANT TO SEE?
ENTER TIIE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.
EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.
A $\quad$ CDEF
BCD
ABEF
WIII EFFECT/INTERACTION DO YOU WANT TO SEE?
ENTER THE EFFECT DESCRIPTION. ENTER A BLANK TO EXIT.
EXAMPLE: ENTER AB TO SEE WHAT IS ALIASED WITH EFFECT AB.
BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTOR
THESE ARE TIIE EXPERIMENTAL OBSERVATIONS TO BE RUN
IN THE DATA COLLECION PROCESS.
FOR THIS 2 LEVEL PER FACTOR EXPERIMENT, O AND 1 REPRESENT
TIUE LOW AND HIGH FACTOR VALUES.
-175-
HOULD YOU LIKE TO SAVE A COPY OF THIS OBSERVATION VECTOR TO BE PRINTED ON THE LINE PRINTER?
I DID NOT UNDERSTARD YOUR RESPONSE.
PLEASE ANSWER THE QUESTION WITH A YES OR NO RESPONSE.
hit return when ready to continue.
EXPERIMENTAL REFINEMENT
BASIC EXPERIMENTAL BLOCK/OBSERVATION VECTORS
hit return when ready to continue.
EXPERIMENTAL REFINEPENT

I have the basic experimental block as in the original problem. NOW, YOU RUST SPECIFY THE THO ALIASED EFFECTS YOU WISH TO have separated.
WHAT IS THE FIRST EFFECT:
hhat is the second effect:
ABEF
ALIASED EFFECT: A
are these the aliased effects? YES
hit return hhen ready to continue
may delete alias mubber 1: I=ABCD may delete alias number 2: I=ACDEf WHICH ALIAS NURBER WOULD YOU LIKE TO ELIMINATE?
this is a sumarary of the number of main effects AND FIRST ORDER INTERACTIONS CONFOUNDED WITH EACH MAIN EFFECT AND FIRST ORDER INTERACTION. main effects that are confounded with other main epfects are flagged hith an *.
hit return hhen ready to continue.
囟

EXPERIMENTAL REFINEMENT

Which effect/interaction do you want to see?

basic experimental block/ObSERVation Vector
THESE ARE THE EXPERIMENTAL OBSERVATIONS TO BE RUN
IN THE DATA COLLECTION PROCESS.
FOR THIS 2 LEVEL PER FACTOR EXPE
FOR THIS 2 level per factor experiment, 0 and 1 represent
the low and high factor values.
new trials (not part of previous block)
are marked with the letter n.
hould you like to save a copy of this observation vector TO BE PRINTED ON THE LINE PRINTER?
hit return when ready to continue.
EXPERIMENTAL REFINEMENT

basic expertmental block/observation vectors
ABCDEF
000000
000001
000010
000011
001100
001101
001110
001111


## EXIT

Segment 5--Exit is the segment to enter from the AED program.

The following is an example of its use.

EXIT
welcohe to the automated experimental design program.

$$
\begin{aligned}
& \text { YOU HAY ENTER ONE OF THE PROGRAM SEGMENTS: } \\
& \text { 1. BASIC TERMINOLOGY } \\
& \text { 2. PROBLEM DEFINITION } \\
& \text { 3. ACTUAL EXPERIHENTAL DESIGN } \\
& \text { 4. EXPERIMENTAL REFINEMENT } \\
& \text { 5. EXIT } \\
& \text { ENTER THE SEGMENT NUHBER YOU WANT AND HIT RETURN } \\
& \text { 5 } \\
& \text { YOUR ENTRY WAS: S--EXIT PROGRAM } \\
& \hline \begin{array}{l}
\text { YOUTINE WRAPUP ENTERED -- } \\
\text { MO INTERNAL PROCESSING REQUIRED BY THIS SYSTEM. } \\
\text { FORTRAN STOP }
\end{array}
\end{aligned}
$$

## REFERENCES

Anderson, V.L. and McLean, R.A., Design of Experiments: A Realistic Approach, M. Dekker, 1974.

Aume, N.M., Milles, R.G., et al., Summary Report of AMRL Remotely piloted Vehicle (RPV) System Simulation Study $V$ Results, Aerospace Medical Research Laboratory, Wright-Patterson AFB, Ohio, AMRL-TR-13, April 1977.

Box, G.E.P. and Hunter, J.S., MMulti-Factor Experimental Designs for Exploring Response Surfaces," Annals of Mathematical Statistics, 1957, 28, pp. 195-241.

Box, E.P., Hunter, W.G., and Hunter, J.S., Statistics for Experiments, John Wiley \& Sons, Inc., 1978.

Clark, C. and Williges, R.C., "Response Surface Methodology Central-Composite Design Modifications for Human Performance Research, " Human Factors, 1973, 15(4), pp. 295-310.

Cochran, W.G. and Cox, G.M., Experimental Design, John Wiley \& Sons, Inc. 1957.
Cox, D.R., Planning of Experiments, John Wiley \& Sons, Inc., 1958.

Davies, O.L., The Design and Analysis of Industrial Experiments, Longman Group Limited, 1978.

Edwards, A.L., Experimental Design in Psychological Research, Holt, Rinehart, and Winston, 1960.

Finn, J.D., A General Model for Multivariate Analysis, Holt, Rinehart, and Winston, 1974.

Hicks, C.R., Fundamental Concepts in the Design of Experiments, Holt, Rinehart, and Winston, 1973.

Hope, K., Methods of Multivariate Analysis, University of London, 1968.

Kempthorne, 0. , Design and Analysis of Experiments, Robert E. Krieger Publishing Company, 1979.

Kirk, R.E., Experimental Design: Procedures for the Behavioral Sciences, Brooks/Cole Publishing Company, 1968.

Mendenhall, W., Introduction to Linear Models and the Design and Analysis of Experiments, Duxbury Press, 1968.

Meyers, J.L., Fundamentals of Experimental Design, Allyn and Bacon, 1972.

Mills, R.G. and Willigies, R.C., "Prediction of Operator Performance in a Single-Operator Simulated Surveillance System, " Human Factors, 1973, 15(4), pp. 337-348.

Namboadiri, K.N., Carter, L.F., and Blalock, H.M., Applied Multivariate Analysis and Experimental Design, McGraw-Hill, 1975.

NBS \#48, "Fractional Factorial Experiment Designs for Factors at Two Levels," National Bureau of Standards Applied Mathematics Series 48, 15 April 1957.

NBS \#54, "Fractional Factorial Experiment Designs for Factors at Three Levels," National Bureau of Standards Applied Mathematics Series 54, 1 May 1959.

Patel, M.S., "Investigations of Factorial Designs," Ph.D. Dissertation, the University of North Carolina, 1961.

Peng, K.D., The Design and Analyses of Industrial Experiments, Addison Wesley, 1967.

Shannon, R.E., Systems Simulation: The Art and Science, Prentice-Hall, Inc., 1975.

Simon, C., "Economical Multifactor Designs for Human Factors Engineering Experiments," Hughes Aircraft Company, Technical Report No. P73-326, June 1973.

Simon, C.W., "The use of Central-Composite Designs in Human Factors Engineering Experiments," Hughes Aircraft Company, Technical Report No. AFOSR-70-6 December 1970.

TM-HU-0070/001/03, "Introduction to Experimental Designs and the Automated Experimental Design Program," System Development Corporation, Technical Memo, October 1981.

TM-HU-0070/002/01, "The Automated Sequential Design Program User's Manual," System Development Corporation, Technical Memo, January 1981.

TM-HU-7743/000/00, "The Use of Central Composite Designs in Quadratic Approximation of Response Surfaces, System Development Corporation, Technical Memo, November 30, 1981.

Williges, R.C., "Working Paper on Research Methodologies for System Experimentation," Final Report, Texas A\&M Research Foundation, December 1979.

Williges, R.C. and Mills, R.G., "Predictive Validity of Central Composite Design Regression Equations," Human Factors, 1973, 12(4), pp. 349-354.

Winer, B.J., Statistical Principles in Experimental Design, McGraw-Hill, 1962.



[^0]:    ROOT SUM SQUARE (RSS)

    ## BASIC TERMIHOLOGY TEST RUN

    tile soluare root of the sum of the squares
    R-SQuARED
    SMALL R-SQuared -see correlation coefficient.
    bic r-squared - see correlation index.
    A single set of factor values applied to the experimental SUBJECT FOR WIIICH THE RESPONSE IS MEASURED.
    do you hant to revien these definitions?
    $\stackrel{\text { no }}{\text { Bas }}$
    BASIC TRRHITNOLOGY TEST RUN
    $\Delta * * * * * * * * * * * * * * * * * * * * * * ~$
    no Yol want to rekun this seghent?

[^1]:    THIS CENTRAL COMPOSITE DESIGN IS ROTATABLE,
    If 6 REPLICATES OF TIE CENTER POINT ARE mADE,
    the center point of the central composite design
    

    CENTER POINT
    Variance of the estimated response will be approximately the same at all points within the sphere of radius one.
    hit return to continue.

[^2]:    this requires tilat the experimenter describe a new design. THE MOST EFFICIENT WAY TO DESCRIBE THE NEW DESIGN IS TO USE THE
    PROGRAM DESIGN DEFINITION CAPABILITY AS IN SEGYENTS 2 - OPTION PROGRAM DESIGN DEFINITION CAPABILITY AS IN SEGMENTS 2 - OPTION 1 and

[^3]:    basic experinental block/ObSERVATION vectors

[^4]:    I have the basic experimental block as in the original problem. NOW, YOU MUST SPECIFY THE TWO ALIASED EFFECTS YOU WISH TO AhVE SEPARATED.
    hilat is the first effect:
    hiat is the second effect: ABEF
    aliased effect: a
    aliased effect: abef
    are these tile aliased effects?
    hit return hilen ready to continue.

[^5]:    MAIN EFFECT AND FIRST ORDER THOSE
    THIS IS A SURMARY OF THE NUMBER OF MAIN EFFECTS AND FIRST ORDER
    fFFEGS THAT ARE CONFOUNDED WITH OTHER MAIN EFFEGTS ARE FLAGGED WI'TH AN *

[^6]:    EXPERIMENTAL REFINEMENT

