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MAGNUS EFFECTS ON

BALLISTIC TRAJECTORIES

THESIS

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Major   USAF

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MAGNUS EFFECTS ON BALLISTIC TRAJECTORIES.

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Master's THESIS

Presented to the Faculty of the School of Engineering

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by

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12 118p. Graduate Astronautical Engineering

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Preface

Most of my time during the past six months has been spent preparing for, working at, and finally completing this project. As time progressed, I managed to develop at least a measure of proficiency in working with the six degree of freedom equations of motion, was introduced to the utility of aerodynamic coefficients, and gained considerable experience working with a computer simulation. This learning experience, unlike most of my rote textbook encounters, was found to be satisfying, rewarding, and real. I also found the professional people I came in contact with during the project to be knowledgable, motivated engineers willing to share their time and talent. My association with my advisor, Dr. D.W. Breuer of the Aeronautics and Astronautics department, Air Force Institute of Technology; Major Ed Mirmak, Air Force Avionics Laboratory who proposed the project; and Captain Bill Miklos, Flight Dynamics Laboratory who provided the computer program, made the project not only possible but enjoyable. An enormous amount of expertise was available to me through these people as well as from those who have previously documented their efforts in this field. This project represents the conclusion to a personal challenge of the first degree with its own reward at the time of completion. If any part of it is ever useful to anyone else, then all the effort is even more worthwhile.

Contents

Contents

List of Figures

List of Tables

List of Symbols

Aerodynamic coefficients		(See Table 3)
C_L, C_M, C_N	Moment coefficient about body X, Y, Z axes	
C_x, C_y, C_z	Force coefficient along inertial axes	
C_X, C_Y, C_Z	Force coefficient along body axes	
d	Maximum projectile diameter	
$\bar{F}$	Net force vector	
F_X, F_Y, F_Z	Components of $\bar{F}$ along body axes	
g	Gravitational acceleration	
$\bar{h}$	Net angular momentum vector	
h_X, h_Y, h_Z	Components of $\bar{h}$ along body axes	
I_X, I_Y, I_Z	Moment of inertia about body axes	
m	Mass of projectile	
$\bar{M}$	Net moment vector	
M_X, M_Y, M_Z	Components of $\bar{M}$ about body axes	
p, q, r	Rate of rotation about body X, Y, Z	
Q	Dynamic pressure	
q"	Magnitude of cross-spin vector	
S	Cross-sectional area of projectile	
t	Time	
t_E	Simulation termination time	
V_a	Speed with respect to air mass	
$\bar{V}$	Net velocity vector	
u, v, w	Components of $\bar{V}$ along body XYZ axes respectively	
y_{min}	Simulation termination altitude	
w	Wedge angle	
x, y, z	Axes of inertial reference frame	

X, Y, Z	Axes of body fixed reference frame
x"	Position of inertial x after second rotation O
z'	New position of inertial z axis after rotation
α	Angle of attack
β	Side slip angle
Δt	Numerical integration step size
ϕ'	Angle from cross-velocity vector to Z fin
ϕ''	Angle from cross-spin vector to Y fin
ψ, θ, ϕ	Rotation about inertial y, z', x" axes
λ	Rotational parameter
η	Number of fins
ρ	Air density
ω^{bi}	Rotation between body and inertial reference frames

Note for hand printed equations: X, Y, Z = X, Y, Z

x, y, z = x, y, z

ABSTRACT

A six degree of freedom computer simulation was used to investigate the lateral progression of a free-fall ballistic trajectory due to spin rate, Magnus aerodynamic coefficients and initial projectile pitching motion. The increased spin rate extends the projectile impact point both downrange and cross range due to a slight increase in time of flight generated by a predominately positive angle of attack. The Magnus force, Magnus moment and side force coefficients,under normal release conditions, presents only a minor influence and can be omitted from the simulation without altering the trajectory appreciably. If the release altitude is sufficiently high, however, the small influence of these coefficients could propagate to a correctable magnitude. Projectile oscillations encountered at release can increase the lateral progression of the trajectory significantly, especially for high speed deliveries. Oscillations induced by an initial pitch rate of the projectile generates considerably more lateral deviation than an initial pitch displacement of equivalent maximum amplitude. This increased lateral displacement causes a corresponding decrease in range impact. Other observations concerning the performance of a projectile following a ballistic trajectory are included as support material in the last section of the study.

MAGNUS EFFECTS ON BALLISTIC TRAJECTORIES

I. Introduction

Purpose

This study uses computer simulations to investigate three factors that effect a free-fall ballistic trajectory. The first objective is to determine the relative influence of Magnus force, Magnus moment, and side force coefficients. The next area of interest is the effect of oscillatory motion induced by initial pitch rate motion compared to initial pitch displacement of the projectile. The third set of simulations determines how spin rate induced by the projectile fins alters the trajectory.

The development of the equations of motion used in the computer program is included in this study, followed by a discussion of the aerodynamic coefficients. The results of the simulations are presented in the last section.

Background

A ballistic projectile is designed to spin so that a predictable trajectory is generated. Without spin, aerodynamic lift created by non-symmetrical shape resulting from manufacturing imperfections would cause the projectile to fly away from the expected ballistic trajectory. Spin is also required for stability considerations. If the projectile has insufficient angular momentum, it will eventually begin precessing and, if the spin rate is in the neighborhood of the natural pitch frequency, it may develop catastrophic yaw. Conversely, with excessive angular momentum, the projectile spin axis tends to become gyroscopically oriented with respect to inertial space and will not track properly throughout the trajectory. Conventional bombs are therefore designed so that a spin rate is achieved that will maintain satisfactory stability within either limit of spin rate (Ref 4:64 & 12:29). This spin, while desirable and necessary for a predictable trajectory, induces lateral errors which are attributable to Magnus lift and a gyroscopically induced lateral angle of attack. These effects are additive and can cause the bomb to progressively deviate laterally.

Assumptions

The set of equations that completely describe the trajectory of a spinning ballistic projectile are fully coupled, non-linear expressions that are developed in dynamics texts (such as Ref 4:99). The solutions to this set of equations requires extensive digital computer capability even with the following simplifying assumptions:

1. The earth curvature, variation in terrain, and earth rotation are neglected. For normal conventional weapon delivery airspeeds and altitudes, no significant error is introduced since the range and time of flight of the projectile is sufficiently small. This assumption eliminates the earth reference and allows the advantage of direct transformations between the body and inertial reference frames (See Fig 1). This greatly decreases the required computer time with no appreciable loss in accuracy.

2. The gravitational field is considered uniform over the entire trajectory.

3. The coriolis acceleration is neglected even though it can have

a small but discernable effect on the projectile impact point. For this investigation, the change in trajectory is of primary interest, not the precise trajectory itself. Coriolis acceleration is addressed in Appendix E but is not written into the computer program.

4. A standard atmosphere is assumed adequate for normal release altitudes. The ARDC 1959 Model Atmosphere was used and no winds were included in the simulation.

II. Mathematical Theory

Reference Frame Transformation Matrix

The orthogonal reference frame used in this development is the body fixed reference frame (XYZ) moving with respect to the inertial frame (xyz). The origin of the body frame is located at the center of gravity of the projectile.

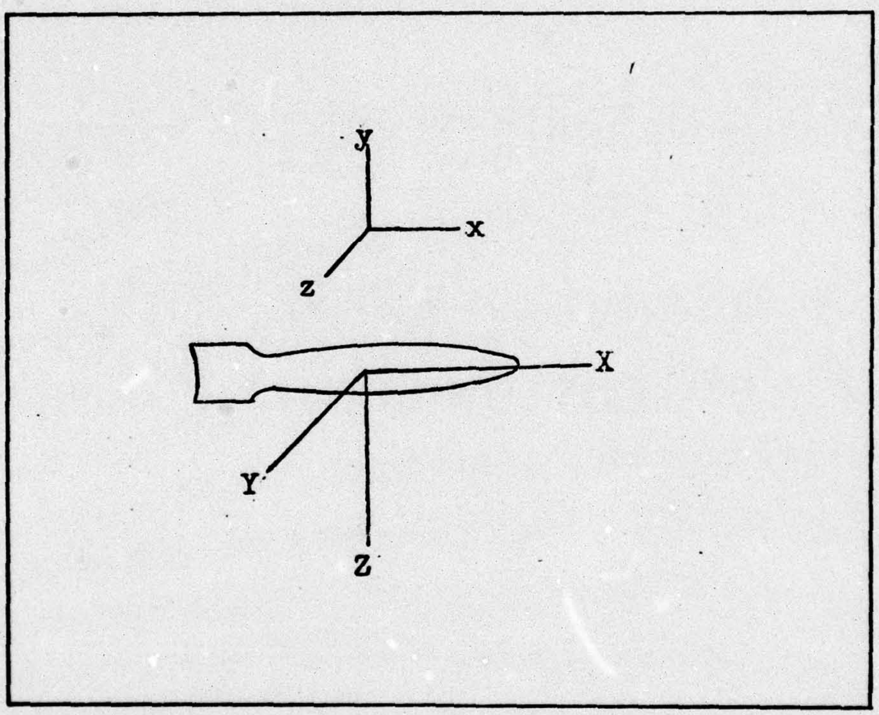


Fig. 1. Reference Frames

To facilitate the direction cosine matrix development, the body reference frame is rotated 90° in the negative direction about the X axis to allign the YZ axes with the corresponding yz axes. The following sequence of Euler angles is then used to generate the direction cosine matrix (Ref 2:8): yaw rotation ψ about the y axis, pitch rotation θ about the new z' axis, then roll rotation ϕ about the new x" axes.

The resulting matrix is:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & C\phi & S\phi \\ 0 & -S\phi & C\phi \end{bmatrix} \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\Psi & 0 & S\Psi \\ 0 & 1 & 0 \\ -S\Psi & 0 & C\Psi \end{bmatrix} \tag{1}$$

which expands to a single Euler angle transformation so that the set of equations describing the velocity vector is (Ref 7:10):

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} C\Psi C\theta & S\theta & C\theta S\Psi \\ -S\theta S\Psi - C\phi C\Psi S\theta & C\phi C\theta & S\phi C\Psi - C\phi S\Psi S\theta \\ -C\phi S\Psi + S\phi C\Psi S\theta & -S\phi C\theta & C\phi C\Psi + S\phi S\Psi S\theta \end{bmatrix}^T \begin{bmatrix} U \\ V \\ W \end{bmatrix} \tag{2}$$

Where $\dot{x}$, $\dot{y}$, $\dot{z}$ are the velocity components in the inertial reference frame and u, v, w are the velocities along the body frame (Fig 2). S, C denotes sine and cosine respectively.

Equations of Motion

The relationship between vector derivatives in any two reference frames (where $\bar{Z}$ denotes any vector) can be expressed as (Ref 6:109):

$$\dot{\bar{Z}}^{i} = \dot{\bar{Z}}^{b} + \bar{\omega}^{bi} \times \bar{Z} \tag{3}$$

Then, for a velocity vector $\bar{V}$

$$\dot{\bar{V}}^{i} = \dot{\bar{V}}^{b} + \bar{\omega}^{bi} \times \bar{V} \tag{4}$$

Consequently, F = ma = mV can be expressed in the reference frame of interest as

$$\bar{F} = m\dot{\bar{V}} + m\bar{\omega} \times \bar{V} \qquad (5)$$

The rotational parameters p, q, and r are defined as the angular velocities about the X, Y, Z body axes respectively and u, v and w are the linear velocity components along the X, Y, Z axes:

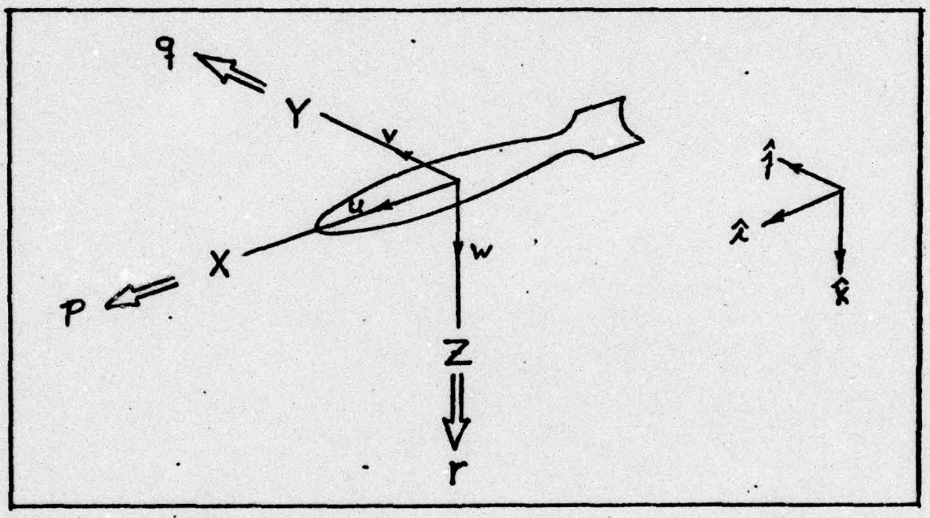


Fig. 2. Rotational and Linear Velocity Axes

The scalar components of equation 5 can be expressed in the body frame as (Ref 4:99):

$$\begin{aligned} F_X &= m(\dot{u} + qw - rv) \\ F_Y &= m(\dot{v} + ru - pw) \\ F_Z &= m(\dot{w} + pv - qu) \end{aligned} \qquad (6)$$

or, equivalently,

$$m\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{bmatrix} = \begin{bmatrix}F_X\\ F_Y\\ F_Z\end{bmatrix} - m\begin{bmatrix}qw - rv\\ ru - pw\\ pv - qu\end{bmatrix} \qquad (7)$$

where the components of $\bar{F}$ are expressed as the combination of aerodynamic and weight forces.

$$\begin{bmatrix}F_X\\ F_Y\\ F_Z\end{bmatrix} = QS\begin{bmatrix}C_X\\ C_Y\\ C_Z\end{bmatrix} - mg\begin{bmatrix}S\theta\\ C\phi C\theta\\ -S\phi C\theta\end{bmatrix} \qquad (8)$$

where $Q = \frac{1}{2}\rho V_a^{\,2}$ = dynamic pressure

S = cross sectional area

C_X, C_Y, C_Z = aerodynamic force coefficients

This set of equations (Ref 2:11) describe the three translational degrees of freedom of a projectile as it proceeds along its path. Three additional equations are needed to account for the spin, pitch and yaw motion that will be encountered.

The expression relating the derivative of the angular momentum vector between the body and inertial frame, from Eq 2, is

$$\dot{\bar{h}}^{i} = \dot{\bar{h}}^{b} + \bar{\omega}^{bi} \times \bar{h} \qquad (9)$$

where

$$\begin{bmatrix}h_X\\ h_Y\\ h_Z\end{bmatrix} = \begin{bmatrix}I_X p\\ I_Y q\\ I_Z r\end{bmatrix} \qquad (10)$$

The rate of change of angular momentum expressed in the body frame, for constant mass projectiles, is

$$\begin{bmatrix} \dot{h}_X \\ \dot{h}_Y \\ \dot{h}_Z \end{bmatrix}_b = \begin{bmatrix} I_X \dot{p} \\ I_Y \dot{q} \\ I_Z \dot{r} \end{bmatrix} \tag{11}$$

The origin of the body frame of reference is located at the center of gravity of the projectile which eliminates the products of inertia since the body frame is located on principal axes and the moment of inertia about the y & z axes are equal. The scalar components of Eq 9 can therefore be written as

$$\begin{aligned} \dot{h}^i_X &= M_X = I_X \dot{p} + qr[I_Z - I_Y] \\ \dot{h}^i_Y &= M_Y = I_Y \dot{q} + pr[I_X - I_Z] \\ \dot{h}^i_Z &= M_Z = I_Z \dot{r} + pq[I_Y - I_X] \end{aligned} \tag{12}$$

Since $I_y = I_z$ for a projectile with rotational mass symmetry, this set is equivalently presented as (Ref 2:11):

$$\begin{bmatrix} I_X \dot{p} \\ I_Y \dot{q} \\ I_Z \dot{r} \end{bmatrix}_b = \begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix} - (I_Y - I_X)\, p \begin{bmatrix} 0 \\ -r \\ q \end{bmatrix} \tag{13}$$

where the M vector is the moment produced by aerodynamic forces expressed as a function of the aerodynamic moment coefficients C_L, C_M, C_N (Ref 2:12):

$$\begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix} = QSd \begin{bmatrix} C_L \\ C_M \\ C_N \end{bmatrix} \tag{14}$$

where d is the diameter of the projectile.

Eqs 7 and 13 completely describe the motion of a six degree-of-freedom ballistic projectile and form the basis of the six degree-of-freedom computer program used in this investigation.

<u>Differential Equations of Motion in Rotational Parameters</u>

Euler angles provide an effective description of body orientation in space and the heading, pitch and roll angles are easily visualized. The transformation matrix of nine direction cosine elements, however, requires considerable computer time to process the numerous trigonometric functions. Also, singularities in the differential equations arise when the pitch angle of $\pm 90^{\circ}$ is reached, as well as inducing unacceptable truncation error when integrating in the neighborhood of the singularity point. The use of Euler rotational parameters (sometimes referred to as quaternions) provide a computational device that avoids the singularity limitation of Euler angles. This system uses four parameters to fix the position of a body in space: three direction cosine angles to specify the orientation of the spin axis, and another rotational parameter to specify the amount of spin about that axis. Instead of working with nine direction cosine elements and six constraint equations from the Euler set, the problem is now reduced to expressions for the rate of change of the four quaternion parameters ($\dot{\lambda}_1$, $\dot{\lambda}_2$, $\dot{\lambda}_3$, & $\dot{\lambda}_0$) and a single constraint equation $\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_0^2 = 1$. The computer time required to solve this set is reduced but still remains extensive. The rotational parameter equivalent of the direction cosine matrix of equation 2 is (Ref 2:7):

$$\begin{bmatrix} 2(\lambda_0^2+\lambda_1^2)-1 & 2(\lambda_1\lambda_2+\lambda_0\lambda_3) & 2(\lambda_1\lambda_3-\lambda_0\lambda_2) \\ 2(\lambda_1\lambda_2-\lambda_0\lambda_3) & 2(\lambda_0^2+\lambda_2^2)-1 & 2(\lambda_2\lambda_3+\lambda_0\lambda_1) \\ 2(\lambda_1\lambda_3+\lambda_0\lambda_2) & 2(\lambda_2\lambda_3-\lambda_0\lambda_1) & 2(\lambda_0^2+\lambda_3^2)-1 \end{bmatrix} \qquad (15)$$

where

$$\begin{aligned} \lambda_0 &= C\tfrac{\phi}{2}\,C\tfrac{\theta}{2}\,C\tfrac{\psi}{2} + S\tfrac{\phi}{2}\,S\tfrac{\theta}{2}\,S\tfrac{\psi}{2} \\ \lambda_1 &= C\tfrac{\phi}{2}\,S\tfrac{\theta}{2}\,S\tfrac{\psi}{2} + S\tfrac{\phi}{2}\,C\tfrac{\theta}{2}\,C\tfrac{\psi}{2} \\ \lambda_2 &= C\tfrac{\phi}{2}\,C\tfrac{\theta}{2}\,S\tfrac{\psi}{2} + S\tfrac{\phi}{2}\,S\tfrac{\theta}{2}\,C\tfrac{\psi}{2} \\ \lambda_3 &= C\tfrac{\phi}{2}\,S\tfrac{\theta}{2}\,C\tfrac{\psi}{2} + S\tfrac{\phi}{2}\,C\tfrac{\theta}{2}\,S\tfrac{\psi}{2} \end{aligned} \qquad (16)$$

The rate of change of these parameters are given by (Ref 2:10):

$$\begin{bmatrix} \dot\lambda_0 \\ \dot\lambda_1 \\ \dot\lambda_2 \\ \dot\lambda_3 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 0 & p & q & r \\ -p & 0 & -r & q \\ -q & r & 0 & -p \\ -r & -q & p & 0 \end{bmatrix}\begin{bmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \qquad (17)$$

The algebra required to develop equations 15 and 16 is quite extensive as shown in Appendix D. The derivation of equation 17 is found in Appendix B of reference 2, page 11.

The differential equations of motion are now in the programmed form and are summarized in Table I.

Table I

Differential Equations of Motion

The kinematic relationships are:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 2(\lambda_0^2+\lambda_1^2)-1 & 2(\lambda_1\lambda_2+\lambda_0\lambda_3) & 2(\lambda_1\lambda_3-\lambda_0\lambda_2) \\ 2(\lambda_1\lambda_2-\lambda_0\lambda_3) & 2(\lambda_0^2+\lambda_2^2)-1 & 2(\lambda_2\lambda_3+\lambda_0\lambda_1) \\ 2(\lambda_1\lambda_3+\lambda_0\lambda_2) & 2(\lambda_2\lambda_3-\lambda_0\lambda_1) & 2(\lambda_0^2+\lambda_3^2)-1 \end{bmatrix}^T \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} \dot{\lambda}_0 \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} 0 & p & q & r \\ -p & 0 & -r & q \\ -q & r & 0 & -p \\ -r & -q & p & 0 \end{bmatrix} \begin{bmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \tag{19}$$

The dynamic relationships are:

$$m \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} F_X \\ F_Y \\ F_Z \end{bmatrix} - m \begin{bmatrix} qw - rv \\ ru - pw \\ pv - qu \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} I_X \dot{p} \\ I_Y \dot{q} \\ I_Z \dot{r} \end{bmatrix} = \begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix} - (I_Y - I_X)\, p \begin{bmatrix} 0 \\ -r \\ q \end{bmatrix} \tag{21}$$

where $\overline{F}$ and $\overline{M}$ are described in Eqs 8 and 14.

III. Method of Problem Solution

The Computer Program

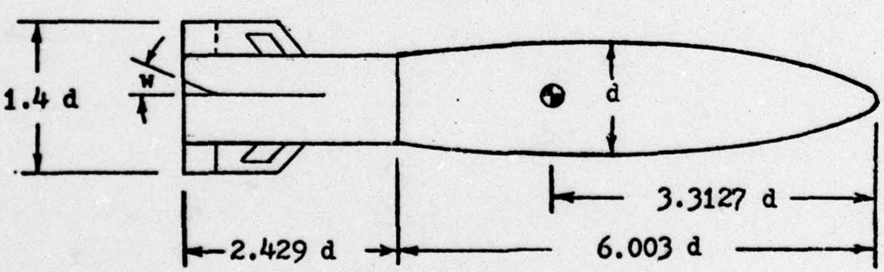
Solution of the set of coupled, non-linear differential equations listed in Table I requires extensive computer capability. The program used in this study was originally coded for the Naval Ordinance Research Computer and has been updated as more advanced generations of computers became operational. The Fortran IV program used for this study was developed by Charles W. Ingram and R.S. Eikenberry using Cohen & Werners' work (Ref 2) as a point of departure. It was written originally for stability analysis applications rather than precision trajectory prediction. The six degree of freedom computer program numerically integrates the set of kinematic and dynamic Euler equations of Table I in the body reference frame and outputs in the inertial reference frame. The accuracy of the program is limited primarily by the quality of the aerodynamic coefficients that are input. The complete program is listed in Appendix F.

Both the Runge-Kutta numerical integration step size (value of A-(20) Table B-2) and the output time increment can be adjusted by the user as a means of tailoring the computer time and cost against the accuracy required. If high projectile spin rates are encountered, coarse step sizes on the order of .01 second could result in erroneous outputs due to insufficient data points to properly resolve the integration problem. The time step should be small enough to provide at least ten integration steps during one revolution of the highest spin rate expected. A step size of .002 second is used for most simulations in this report. Since extensive core memory and computer time is required to process this program, it is written for only a single trajectory per run. A core memory of 100,000 was adequate for all trajectories used in this study. (An integration step size of .001 second requires in excess of 200 seconds of computer time for a 10,000 ft drop.)

Discussion of the computer program input and output is included in Appendices B and C.

Projectile Model Description

The standard Mark 82 warhead with an experimental tail section was the projectile model used to determine the aerodynamic coefficients. The cylindrical aft section has four slotted fins, each fitted with a trailing edge wedge to provide the roll driving moment. The designation of the configuration is FFSW (fin, fixed, slotted, wedge). This particular projectile was selected because of the similarity to existing conventional weapons and, significantly, most of the aerodynamic coefficients were available (Ref 14).



Weight = 525 lbs.

I_X = 1.81 slugs-ft^2

$I_Z = I_Y$ = 46.4 slugs-ft^2

d = 10.75 in.

W = wedge angle

Fig. 3. Projectile Model

The cylindrical tail section provides both a higher drag coefficient and a housing for an inflatable high drag device that may be installed; the slotted fins increase the probability of a well-behaved roll rate throughout the trajectory (Ref 3:1).

A wedge angle of ten degrees is used for most of the simulations in this study. This wedge angle provides sufficient roll acceleration to overcome adverse dynamics encountered during release. The steady state spin rate produced is fast enough to investigate the influence of Magnus coefficients without inducing the type of instabilities that can occur when the spin rate exceeds approximately ten times the nutation frequency (Ref 8; Sec 3,5).

Aerodynamic Coefficients

Values of the aerodynamic coefficients used in equations 8 and 14 must be adequately determined by wind tunnel testing of the projectile model before satisfactory trajectory simulations are possible. The aerodynamic force and moment coefficients can, for most applications, be reduced to the expressions listed in Table II. These expressions include cross-velocity terms in both the force and moment coefficients, but cross spin terms are included only in moment coefficients about pitch and yaw axes. (See Fig 6 for definitions of cross-velocity and cross spin terms.) Table III describes the terms which make up the total aerodynamic force and moment coefficients. These mathematical expressions were determined in Cohen and Werners' work (Ref 2:15-19).

In order to illustrate the utility of the aerodynamic coefficient terms, the expression for induced roll moment is used as an example (See Table III):

$$C_{\ell_0} + C_{\ell_7} \sin \eta \phi' + C_{\ell_8} \cos \eta \phi' \tag{22}$$

where

ϕ' = Angle to cross velocity vector (See Fig 6)

C_{ℓ_0} = Initial curve shift (See Fig 5)

C_{ℓ_7}, C_{ℓ_8} = Amplitude of curve

η = Number of fins

The C_{ℓ_0} term is the residual rolling moment that remains after the projectile has been aligned in the wind tunnel at the angle of attack of interest with no fin deflection; that is, an index of rolling moment due to body shape.

The next set of terms, $C_{\ell_7} \sin \eta \phi' + C_{\ell_8} \cos \eta \phi'$, is the mathematical format required to describe the data curve obtained from the wind tunnel measurements. This application can best be appreciated by the illustration of a spinning projectile at various stages of roll moving away at an angle of attack sufficient to cause a vortex to trail along the top of the missile as in Figure 4.

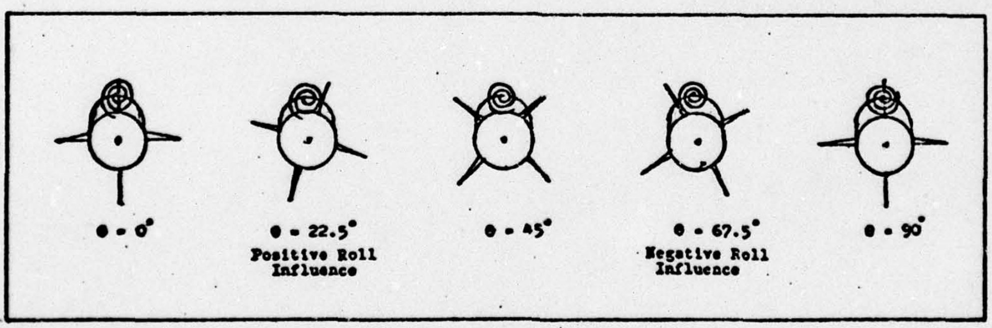


Fig. 4. Periodic Change in Aerodynamic Coefficient C_{λ_0}

At the beginning of the sequence, the trailing vortex is cut symmetrically by the fin, so no net roll influence is imposed by the dynamic pressure change on the fin due to the vortex. After 22.5 degrees of roll, the vortex is in a position to exert a maximum rolling influence. At the 45° angle, a condition of symmetry is again encountered. For this type of oscillating characteristic, C_{ℓ} could be graphed for the entire sequence as the solid curve of Fig 5. The graph obtained from the actual wind tunnel data for the coefficient of interest, however, is generally skewed from the theoretical curve as indicated by the dotted curve.

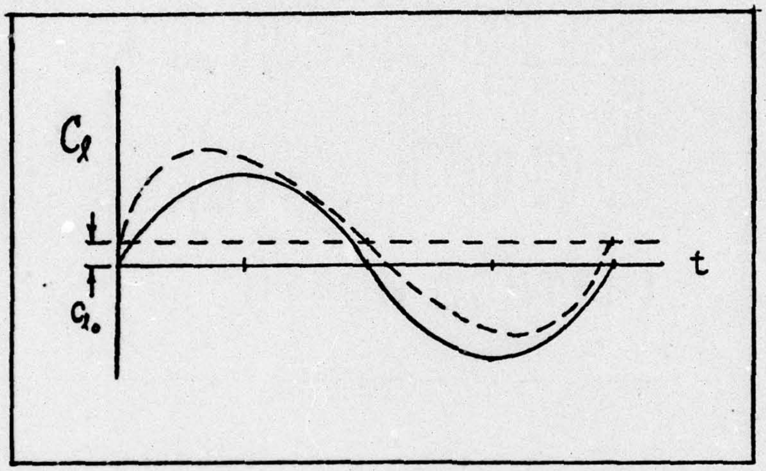


Fig. 5. Theoretical vs Experimental Data Curves

The three terms, $C_{\ell_0} + C_{\ell_7} \sin \eta \phi' + C_{\ell_8} \cos \eta \phi'$, are therefore used in whatever combination of magnitudes and sinusoidals that produce the best simulation of the actual wind tunnel data. The relative magnitude of C_{ℓ_0} versus the amplitude of the sinusoidals often makes one or the other of the terms negligible.

The remaining trigonometric expressions in Table II not defined in Table III are attributable to transformations between reference frames.

Most of the aerodynamic coefficient arrays used in this computer program had been previously determined through a joint effort between the Naval Surface Weapons Center, Dahlgren Laboratory, and the Aero-

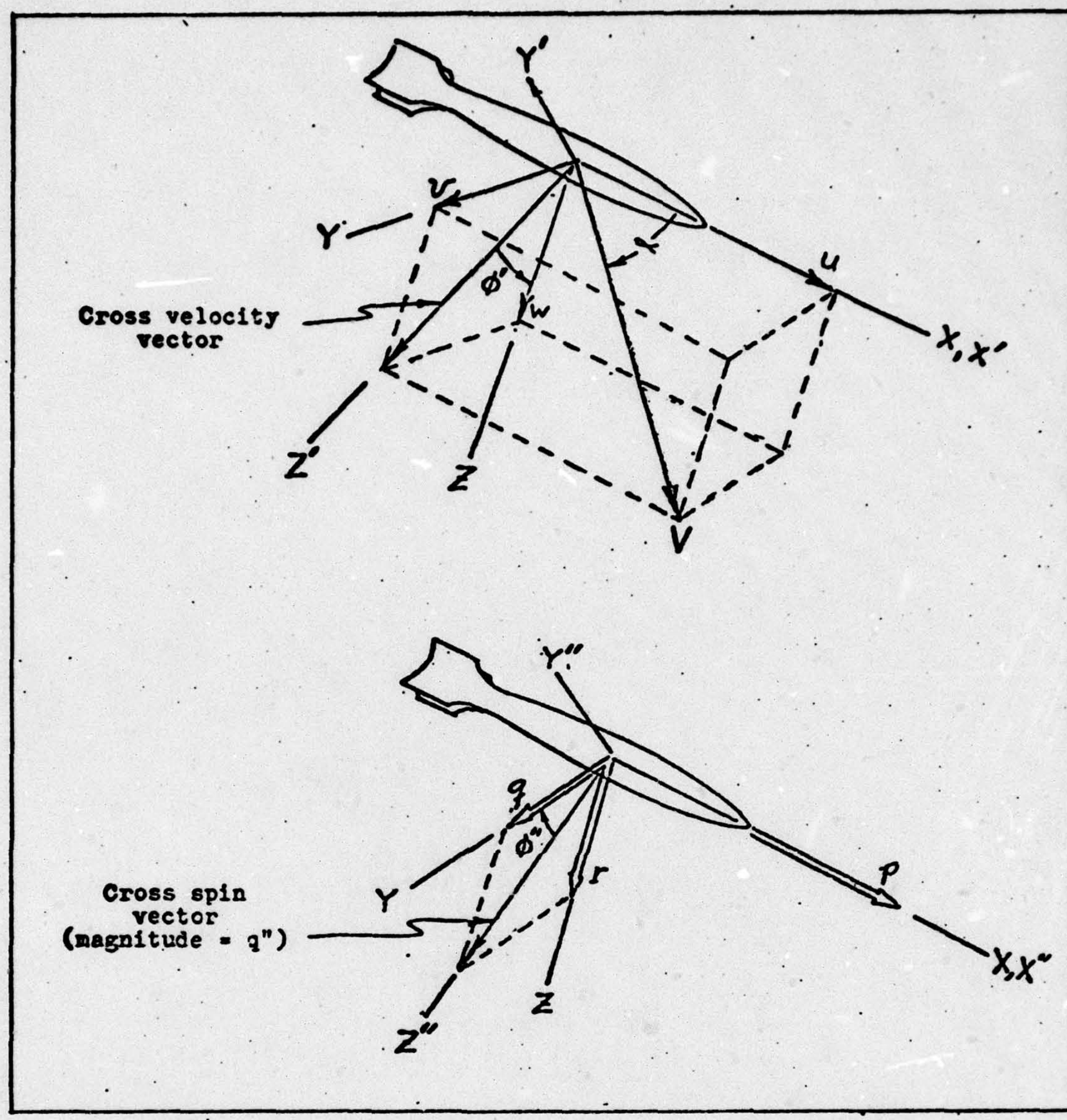


Fig. 6 . Definition of Cross-Spin/Velocity Terms.

where (Ref 2)

$q'' = \sqrt{q^2 + r^2}$, magnitude of cross-spin (rad/sec)

$\alpha = \arctan \sqrt{\dfrac{v^2 + w^2}{u}}$, magnitude of yaw or angle of attack (deg)

$\phi' = \arctan \dfrac{v}{w}$ (angle about X-axis from cross-velocity vector of the center of gravity of missile to Z fin) (deg)

$\phi'' = \arctan \dfrac{(-r)}{q}$ angle about the X-axis from cross-spin vector to Y fin) (deg)

$$C_X = C_x$$

$$C_Y = \left[C_{y_0} + C_{y_7}\sin\eta\phi' + C_{y_8}\cos\eta\phi' + \frac{Pd}{2V_a}C_{y_p}\right]\cos\phi' + \left[C_{z_0} + C_{z_7}\sin\eta\phi' + C_{z_8}\cos\eta\phi'\right]\sin\phi' + C_y(\epsilon)$$

$$C_Z = \left[C_{y_0} + C_{y_7}\sin\eta\phi' + C_{y_8}\cos\eta\phi' + \frac{Pd}{2V_a}C_{y_p}\right](-\sin\phi') + \left[C_{z_0} + C_{z_7}\sin\eta\phi' + C_{z_8}\cos\eta\phi'\right]\cos\phi' + C_Z(\epsilon)$$

$$C_L = C_{\ell_0} + C_{\ell}(\delta w) + C_{\ell_7}\sin\eta\phi' + C_{\ell_8}\cos\eta\phi' + \frac{Pd}{2V_a}C_{\ell_p} + \alpha\left(C_{\ell_{\epsilon\tilde{\alpha}_0}} + C_{\ell_{\epsilon\tilde{\alpha}_1}}\sin\phi' + C_{\ell_{\epsilon\tilde{\alpha}_2}}\cos\phi'\right)$$

$$C_M = \left[C_{m_0} + C_{m_7}\sin\eta\phi' + C_{m_8}\cos\eta\phi'\right]\cos\phi' + \left[C_{n_0} + C_{n_7}\sin\eta\phi' + C_{n_8}\cos\eta\phi' + \frac{Pd}{2V_a}C_{n_p}\right]\sin\phi'$$
$$+\left[C_{mq_0} + C_{mq_7}\sin\eta\phi'' + C_{m_8}\cos\eta\phi''\right]\frac{q''d}{2V_a}\cos\phi'' + C_m(\epsilon)$$

$$C_N = \left[C_{m_0} + C_{m_7}\sin\eta\phi' + C_{m_8}\cos\eta\phi'\right](-\sin\phi') + \left[C_{n_0} + C_{n_7}\sin\eta\phi' + C_{n_8}\cos\eta\phi' + \frac{Pd}{2V_a}C_{n_p}\right]\cos\phi'$$
$$+\left[C_{mq_0} + C_{mq_7}\sin\eta\phi'' + C_{n_8}\cos\eta\phi''\right]\frac{q''d}{2V_a}(-\sin\phi'') + C_n(\epsilon)$$

TABLE II

Force And Moment Coefficients (Adapted from Ref 2)

TABLE III

Aerodynamic Coefficients
(Adapted from Ref 2)

Description of Coefficient		Array Symbol *
C_x	Axial force	CX
$C_{y_o} + C_{y_7}\sin\eta\phi' + C_{y_8}\cos\eta\phi'$	Induced side force	CY7
C_{y_p}	Magnus force	CYP
$C_{z_o} + C_{z_7}\sin\eta\phi' + C_{z_8}\cos\eta\phi'$	Normal force	CZ
$C_y(\epsilon)$	Trim force along Y-axis	N/A
$C_z(\epsilon)$	Trim force along Z-axis	N/A
$C_{l_o} + C_{l_7}\sin\eta\phi' + C_{l_8}\cos\eta\phi'$	Induced roll moment	CLGA
$C_l(\delta w)$	roll moment due to fin cant	CLDW
C_{l_p}	Roll damping moment	CLP
$C_{m_o} + C_{m_7}\sin\eta\phi' + C_{m_8}\cos\eta\phi'$	Restoring moment	CM
$\left[C_{m_{q_o}} + C_{m_{q_7}}\sin\eta\phi'' + C_{m_{q_8}}\cos\eta\phi''\right]\frac{q''d}{2V_a}$	Damping moment	CMQ
$C_{n_o} + C_{n_7}\sin\eta\phi' + C_{n_8}\cos\eta\phi'$	Induced side moment	CNGA
C_{n_p}	Magnus moment	CNP
$C_m(\epsilon)$	Trim moment about y-axis	CMDE
$C_n(\epsilon)$	Trim moment about z-axis	N/A
$\epsilon\,(C_{l_{\epsilon\alpha_o}} + C_{l_{\epsilon\alpha_1}}\sin\phi' + C_{l_{\epsilon\alpha_2}}\cos\phi')$	Roll moment due to eccentric tail	N/A

* See Appendix G

space Research Laboratory, Wright-Patterson Air Force Base and were available for this project (See Appendix G). The Magnus force coefficient, however, was not available and had to be determined so that its effect on lateral error could be studied. The Magnus force coefficient is determined from the definition

$$C_{y_p} = \frac{\partial C_y}{\partial (pd/2V)} \tag{23}$$

where p = spin rate, d = projectile diameter, V = wind tunnel velocity.

This relation states that the slope of a line fit through a plot of C_y versus pd/2V should yield the nominal value of C_{y_p} for a specific angle of attack and Mach number. Tables A-I and A-II show the wind tunnel data taken at Mach 0.8 at an angle of attack of 1.87 degrees. C_y was plotted against pd/2V and then a "best fit" line was drawn through the data points (Fig 8). Greater weight was given to the lower values pd/2V so that the cluster of data points in the neighborhood of the steady state spin condition would not overly influence the slope. No attempt was made to force the line through the origin. As long as the axis intercept remains small, the error in the wind tunnel measuring instruments will dominate. The primary objective is to provide a best estimate of the most representative slope of the plot. The slope of the resulting line produced the nominal value of C_{y_p} that was entered as a single element in the CYP array corresponding to Mach = 0.8 and $\alpha = 2$ (Ref Table IV). The remaining 71 elements of the 8 x 9 matrix were determined in the same manner. Note that the Mach = 0.4 values are used to fill the Mach = 0 row. This is done because, at lower Mach numbers, the compressibility effects diminish and the coefficient remains nearly constant. Fig. 7 shows a typical force coefficient progression.

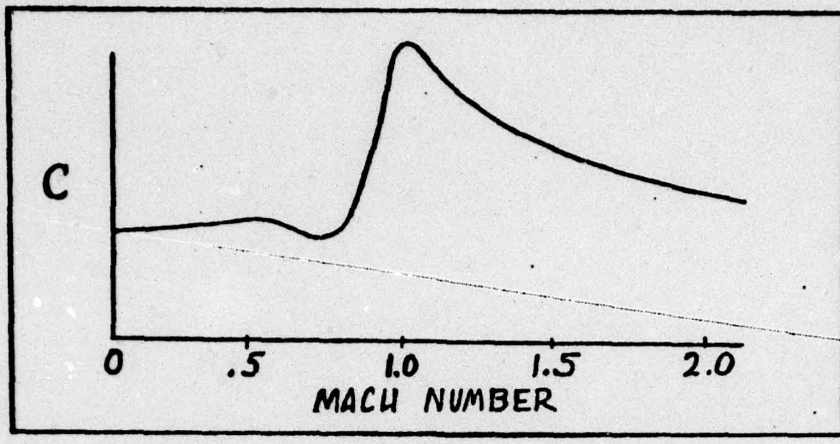


Fig. 7. Force Coefficient vs Mach Number

The alpha = 50^{o} column contains unreliable data but is used only to provide a value for the program to interpolate against in the event $\alpha = 25^{o}$ is exceeded. The computer program is written so that interpolation is accomplished for any intermediate value of angle of attack and Mach. The angles of attack used in this study were well below 25 degrees so this ill-defined edge of the array was not encountered.

Approach to the Investigation

The primary objective of this study is to investigate the relative contributions of the Magnus coefficients to lateral deflection of a spinning projectile following a ballistic trajectory. The general approach is to isolate the effect of the Magnus force coefficient C_{y_p}, the induced side coefficient C_{y_η}, and the Magnus moment coefficient C_{n_p} on the trajectories initiated from identical release conditions. Each coefficient is studied individually, and in pairs; then compared to the performance of all three coefficients together. The initial conditions include a range of airspeed, altitudes and attitude pertubations.

The Magnus force coefficient, C_{y_p}, is of interest because it is

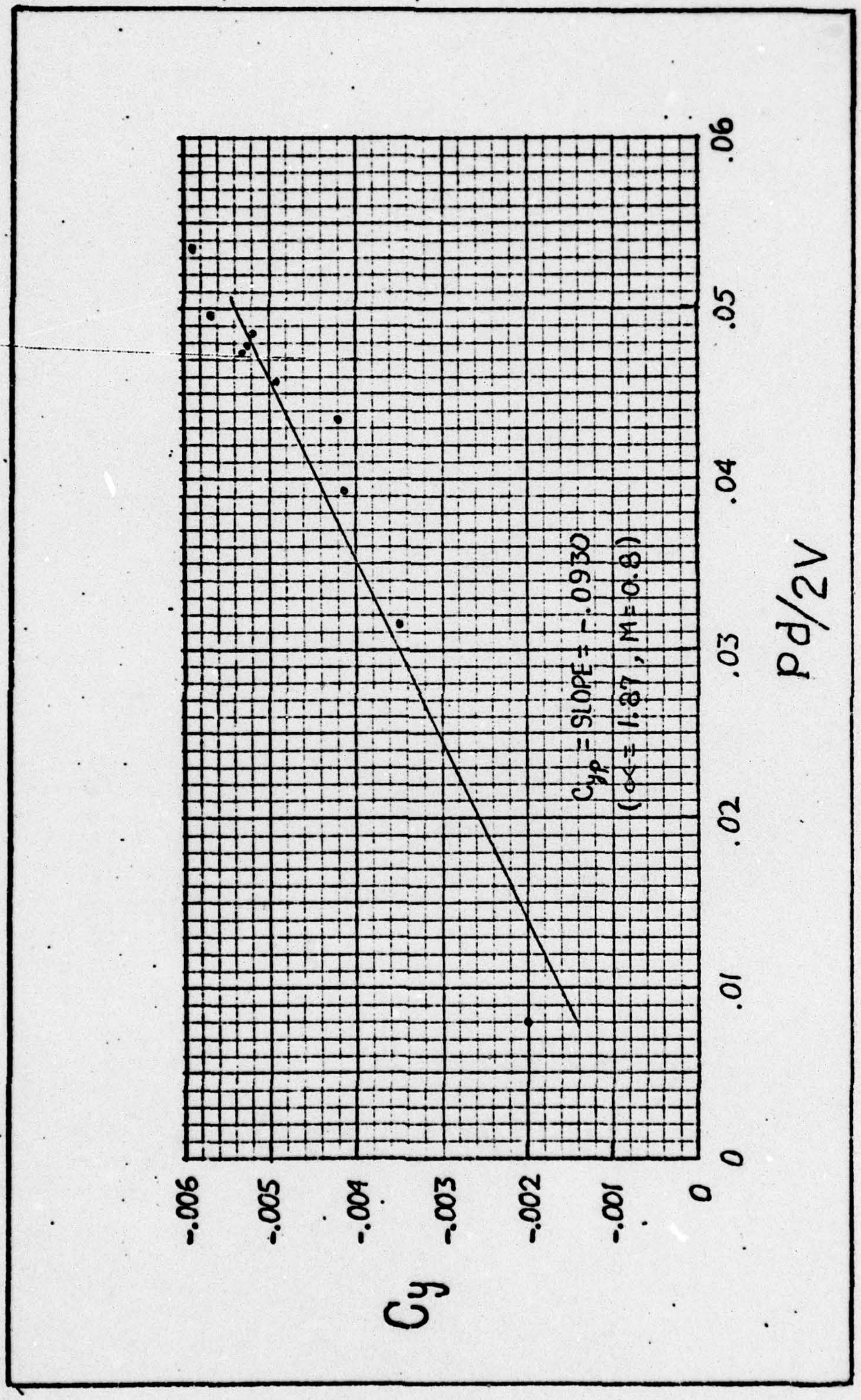


Fig. 8. Change in Magnus Force Coefficient

TABLE IV

Magnus Force Aerodynamic Coefficient Array
(Extracted from Appendix G)

CYP C(5)

	ALPHA	0.0000	2.0000	4.0000	8.0000	10.0000	15.0000	20.0000	25.0000	90.0000
MACH	0.0000	0.0000	-.0300	-.1350	-.5350	-.7750	-2.1830	-5.9180	-12.5000	-12.5000
MACH	.4000	0.0000	-.0300	-.1350	-.5350	-.7750	-2.1830	-5.9180	-12.5000	-12.5000
MACH	.8000	0.0000	-.0930	-.1750	-.4100	-.4600	-1.1650	-2.8900	-6.1500	-6.1500
MACH	.9000	0.0000	-.1050	-.1850	-.3500	-.4400	-1.1000	-2.8000	-5.0000	-5.0000
MACH	1.0000	0.0000	-.0700	-.1630	-.2950	-.3300	-.7800	-2.3720	-3.4500	-3.4500
MACH	1.1000	0.0000	-.0800	-.1500	-.3120	-.2540	-.6000	-2.4630	-3.4000	-3.4000
MACH	1.2000	0.0000	-.0800	-.1800	-.3100	-.2650	-.6750	-1.7700	-2.3300	-2.3300
MACH	2.0000	0.0000	-.0800	-.1800	-.3100	-.2650	-.6750	-1.7700	-2.3300	-2.3300

the source of Magnus lift that is generated by a spinning body subject to a relative wind component acting perpendicular to the body spin axis. This force will therefore be generated anytime the spinning projectile is falling with an angle of attack. The direction of the lateral Magnus lifting force is determined by the direction of spin and the cross-velocity component w. The spin rate, p, causes a velocity differential to be produced as shown in Fig. 9. A corresponding pressure differential is generated which produces lift in the lateral direction.

The side force coefficient, C_{y_7}, contributes to the deflection of a projectile by influencing side lift due to a lateral angle of attack.

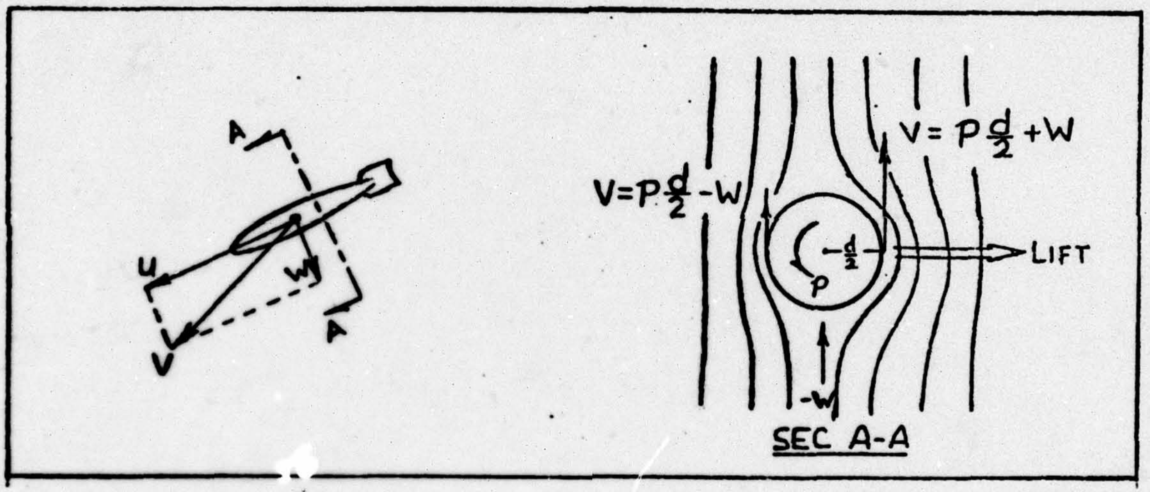


Fig. 9. Magnus Lift

The Magnus moment coefficient, C_{n_p}, can directly effect lateral progression of a projectile following a ballistic trajectory. Magnus moment is generated when the projectile, at a sufficiently high angle of attack, trails a wake vortex that blanks out the rolling lift of the fin passing through it. This causes a change in roll torque and a fin force imbalance that produces a yaw moment on the projectile (Fig 10). The value of the aerodynamic coefficient is a measure of the sensitivity of the projectile to this effect.

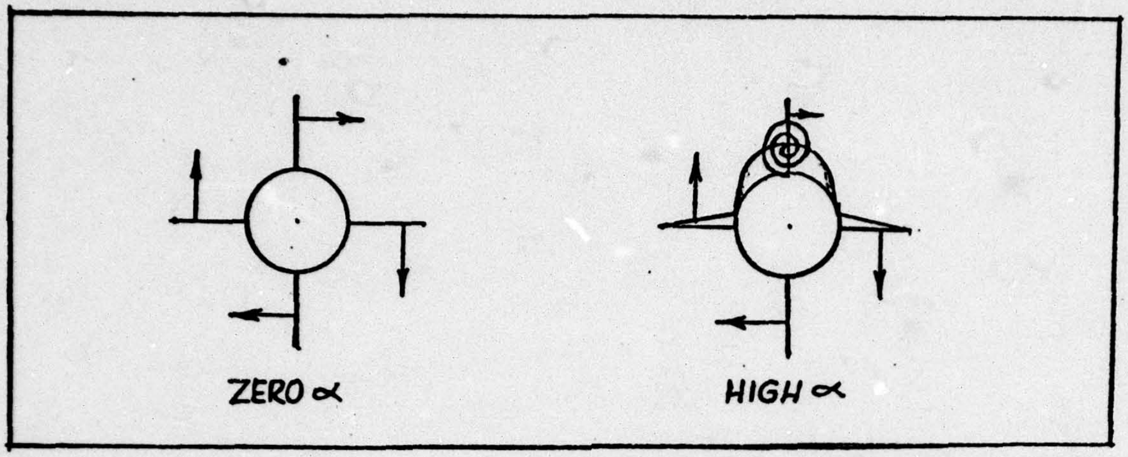


Fig. 10. Magnus Moment

The release velocities used as initial conditions are 600 ft/sec for a lower limit (approximately 350 knots) and 1000 ft/sec for the high speed delivery. The lower limit was selected as a representative velocity for a slow delivery. The 1000 ft/sec velocity remains in the subsonic region and provides adequate velocity spread for comparisons. A velocity of 1200 ft/sec was used, in some cases, to investigate the effect of a supersonic delivery (Mach 1.1 at 10,000 ft).

The 10,000 foot altitude was selected for most of the simulations. This altitude provides sufficient time of flight to identify any significant trajectory trends and does not require prohibitive computer time to process. An altitude of 20,000 feet was used when a lower air density trajectory was desired for comparison.

All releases are from a level flight path. The projectile is, however, perturbed initially under three conditions: zero perturbation, a nose-down rate motion, and a nose-down displacement. The zero perturbation condition simulates a projectile perfectly aligned with the velocity vector at release. The pitch rate motion of $q = -.5$ radians/sec is used to investigate the effect of motion that could be induced by the bomb ejection mechanism. The initial angle of attack of $\alpha_o =$ -7.75 degrees, the third condition, is approximately the same maximum amplitude achieved by the preceding initial motion, but obtained in a different manner.

Spin rate of the projectile directly influences Magnus lift and, consequently, lateral progression of the trajectory. Wedge angles of 5, 10, and 15 degrees were used in the simulations to investigate the relative influence of the different wedges.

The parameters studied under various initial conditions are listed an an overview in Table V.

TABLE V

Parameters Investigated

Parameters		Initial Conditions											
		Altitude		Velocity			α_o		q_o		v_o		
		10K	20K	600	1000	1200	0	-7.75	0	-.5	-.5	0	+5.
All Coefficients		x	x	x	x	x	x	x	x	x	x	x	x
C_{n_p}	(Omitted)	x		x									
C_{y_p}	"	x		x									
C_{y_7}	"	x		x									
C_{y_p}, C_{n_p}	"	x		x									
C_{y_p}, C_{y_7}	"	x		x									
C_{n_p}, C_{y_7}	"	x		x									
C_{y_p}, C_{n_p}, C_{y_7}	"	x		x	x	x	x	x	x	x			
C_Z	"	x		x				x		x			
5^o		x		x	x								
10^o		x	x	x	x	x		x		x			
15^o		x		x	x								
z		x	x	x	x	x	x	x	x	x	x	x	x
t		x		x	x			x		x			

IV. Results of the Simulations

Effect of Magnus Coefficients

The first simulations were run to determine the relative influence of the Magnus coefficients. Initial release conditions of level flight at an altitude of 10,000 feet and 600 feet per second, using all twelve available aerodynamic coefficient arrays produced the result shown in Fig. 11. The ground zero impact point for the trajectory using all coefficients was 57.73 feet to the left of the inertial x-y plane containing the release point. With three coefficients removed that influence lateral displacement (C_{n_p}, C_{y_7}, C_{y_p}), the impact point was changed to 56.58 feet. This total spread of only 1.15 feet, generated over a relatively long trajectory, is lost in the accuracy limitations of the aerodynamic coefficients and the simplifying assumptions used in the equation of motion development. This small, Magnus induced lateral motion is initiated early and propagates slowly throughout the trajectory.

A second set of simulations were run at a velocity of 1200 ft/sec to determine if higher velocity would cause these coefficients to generate a more significant change in the trajectories. The net difference in the trajectories with and without the three coefficients remains on the order of only a few feet.

There are a number of possible explanations why the Magnus influence is so small. Magnus lift depends on both the relative velocity component acting on the projectile due to angle of attack and the spin rate of the projectile. For normal weapon deliveries, the angle of attack is relatively small on release and dampens toward zero early in the fall. During the time the angle of attack is at a maximum, shortly after release, high spin rate has not yet developed. After the projectile

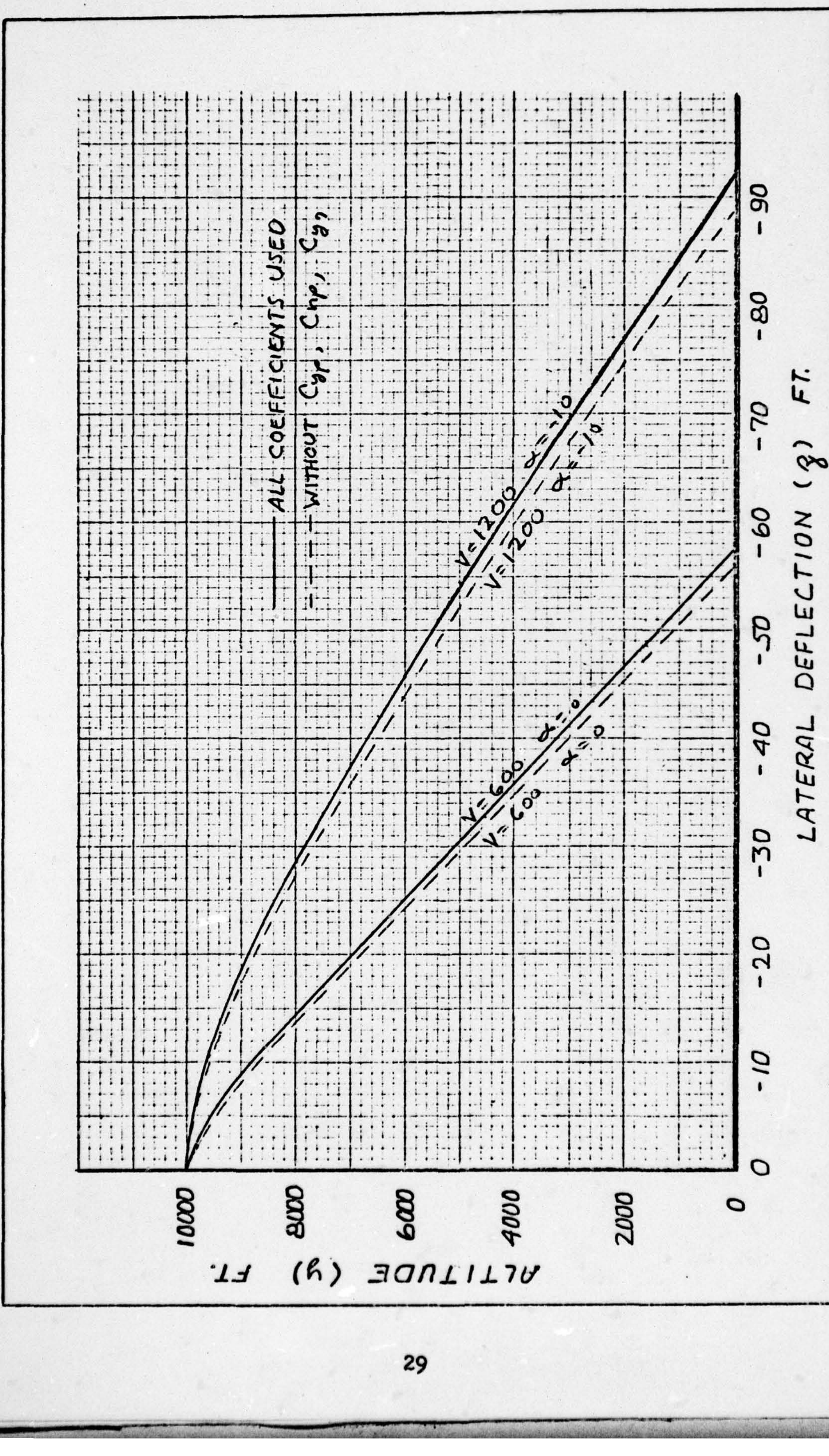


Fig. 11. Effect of Magnus & Side Force Coefficients on Lateral Displacement

has fallen long enough to generate a high spin rate, the angle of attack is negligible. The Magnus side force due to body lift is negligible because the lateral angle of attack is small. The Magnus moment is also small since there is no significant wake generated at small angles of attack to produce a lift differential in the fins.

If oscillatory motion is encountered during release, the projectile angle of attack alternates from positive to negative. The Magnus lift will therefore alternate from positive to negative and average the net influence to zero.

Effect of Initial Pitch Motion

To simulate an initial release condition more realistic than perfect alignment of the projectile with the velocity vector, an initial angle of attack of -7.75 degrees was used as an perturbation. The same release initial conditions were used as before and simulations were run first using all coefficients, and then with C_{y_p}, C_{n_p}, C_{y_7} removed. There was essentially no difference in the resulting trajectories for $\alpha_o = -7.75$ degrees compared to $\alpha_o = 0$ as shown in Table VI).

An initial pitch rotation of the projectile at release, as might be induced by an ejector mechanism, was then implemented by entering a value of $q_o = -.5$ rad/sec. This initial rotation produced the same maximum amplitude of oscillation as the $\alpha_o = -7.75^o$ case, but a considerable difference in the resulting impact point of the projectile was generated. The increased lateral displacement caused a corresponding decrease in range which is consistent with the principle of conservation of energy. The oscillation induced by an initial angular displacement produced a trajectory that impacted at 14,165 feet downrange and 57.73 feet cross

TABLE VI (a)

Effect of Magnus Coefficients

Velocity = 600 ft/sec
Altitude = 10,000 ft
Wedge angle = 10^{o}
Level flight release

Aerodynamic Coefficient	Initial Pitch Motion						t
	$q_o = 0$, $\alpha_o = 0^o$		$q_o = 0$, $\alpha_o = -7.75^o$		$q_o = -.5$, $\alpha_o = 0^o$		time of flight
	X	Z	X	Z	X	Z	Sec
All coefficients	14165.16	-57.73	14179.60	-56.44	14124.86	-73.02	25.725
Less C_{y_p}	14165.48	-57.64					25.727
Less C_{y_7}	14166.83	-56.54	14178.94	-55.86			25.729
Less C_{n_p}	14178.31	-56.60					25.729
Less C_{y_p} & C_{y_7}	14167.15	-56.45					25.729
Less C_{y_p} & C_{n_p}	14165.98	-57.73					25.726
Less C_{y_7} & C_{n_p}	14167.34	-56.66					25.728
Less C_{y_p}, C_{y_7} & C_{n_p}	14167.67	-56.58	14179.45	-55.82	14124.47	-67.68	25.728
Less C_Z, C_{y_p}, C_{y_7} & C_{n_p}			14191.70	+ 2.53	14188.68	+ 3.46	
Less C_Z			14191.06	+ 1.79			

X = Range (Ft)

Z = Lateral displacement (Ft)

TABLE VI (b)

Effect of Magnus Coefficients

Velocity = 600 ft/sec
Altitude = 10,000 ft
Wedge angle = 10^0
Level flight release

Aerodynamic Coefficient	Initial Pitch Motion					
	$q_o = 0$, $\alpha_o = 0^0$		$q_o = 0$, $\alpha_o = -7.75^0$		$q_o = -.5$, $\alpha_o = 0^0$	
	V_f	p_f	V_f	p_f	V_f	p_f
All coefficients	889.06	62.9	889.37	62.9	888.55	62.8
Less C_{y_p}	889.06	62.9				
Less C_{y_7}	889.05	62.9	889.36	62.9		
Less C_{n_p}	889.37	62.9				
Less C_{y_p} & C_{y_7}	889.05	62.9				
Less C_{y_p} & C_{n_p}	889.10	62.9				
Less C_{y_7} & C_{n_p}	889.10	62.9				
Less C_{y_p}, C_{y_7} & C_{n_p}	889.10	62.9	889.38	62.9	888.63	62.8
Less C_Z, C_{y_p}, C_{y_7} & C_{n_p}			889.76	63.0	889.66	63.0
Less C_Z			889.76	63.0		

Impact velocity V_f in ft/sec
Impact spin rate p_f in rad/sec

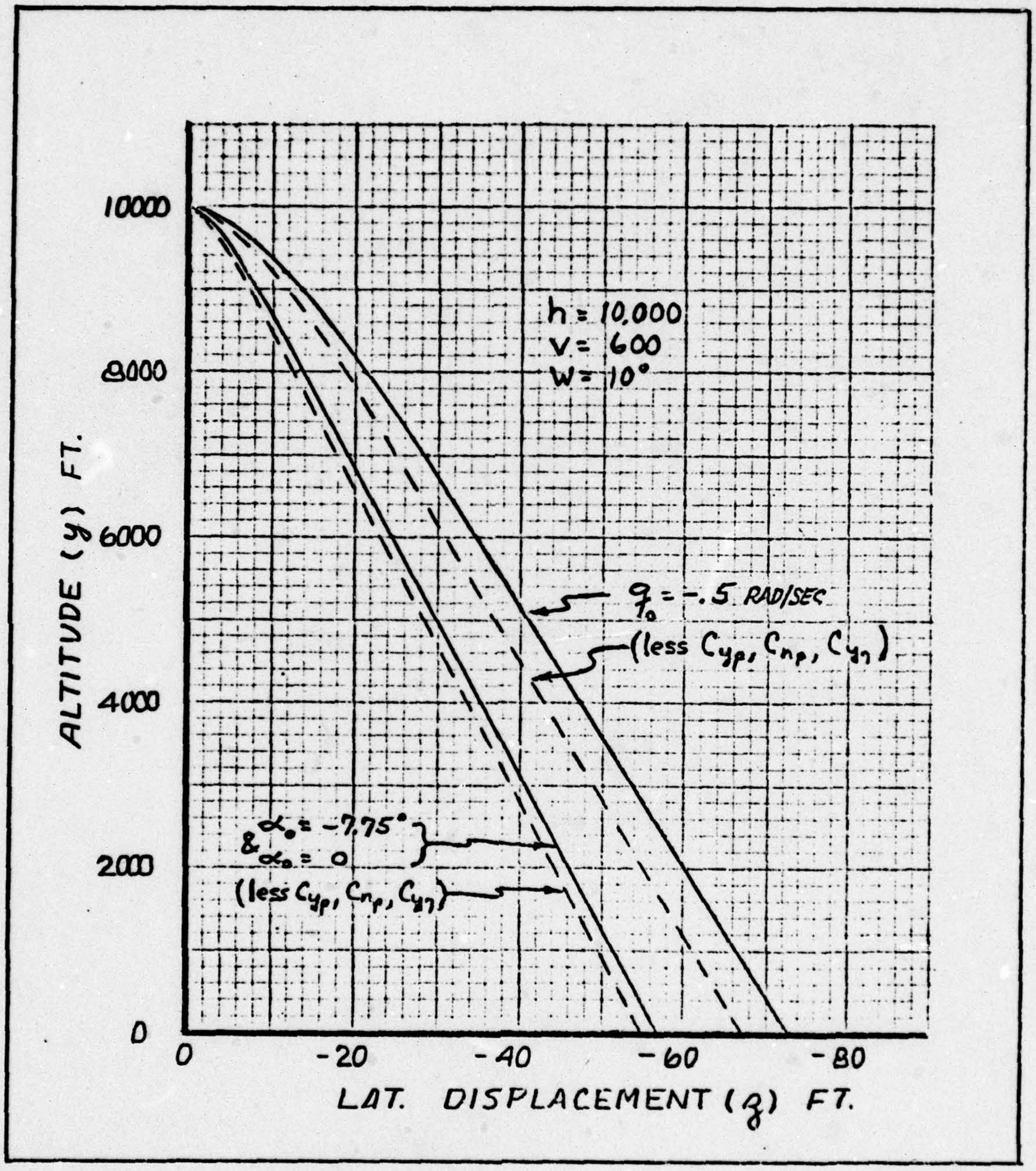


Fig. 12. Effect of Pitch Motion and Magnus Coefficients on Lateral Displacement

range to the left. The rate induced oscillation, with the same release conditions, impacted at 14,179 feet downrange and 73.02 feet cross range (See Table VI and Fig 12).

To investigate the cause of the difference in trajectory, the nose track, viewing the projectile from behind, was plotted for the two different inputs in Fig. 13. The beta versus alpla plot represents a close approximation of the nose track of the projectile early in the trajectory before the body reference frame and the projectile velocity vector rotate a significant amount with respect to the inertial frame. The motion of the projectile nose for the two conditions is quite different. Fig 13a represents single arm coning motion caused by nutation. No precession is evident and the damping is relatively constant. The nose track in Fig 13b shows less coning and displays two arm motion with both nutation and precession. This oscillation dampens faster in pitch than in yaw. The direction of nutation is opposite to the projectile spin rate while the precession is in the same direction. Figs 13c, d, e, f were plotted to identify any parameters that could cause the difference in trajectories. These plots show that the difference in spin rate buildup and velocity progression is very small. A slight phase shift in yaw oscillation is apparent, and the initial slopes of the β versus t plots are noticably different. The last plot shows a significantly different behavior in the angle of attack oscillations between the two initial conditions.

The lateral displacement for both initial conditions is plotted against time in Fig. 14 to further define the projectile motion shortly after release. The linearized average slope of these plots produces a measure of the initial lateral average velocity. The slope for the

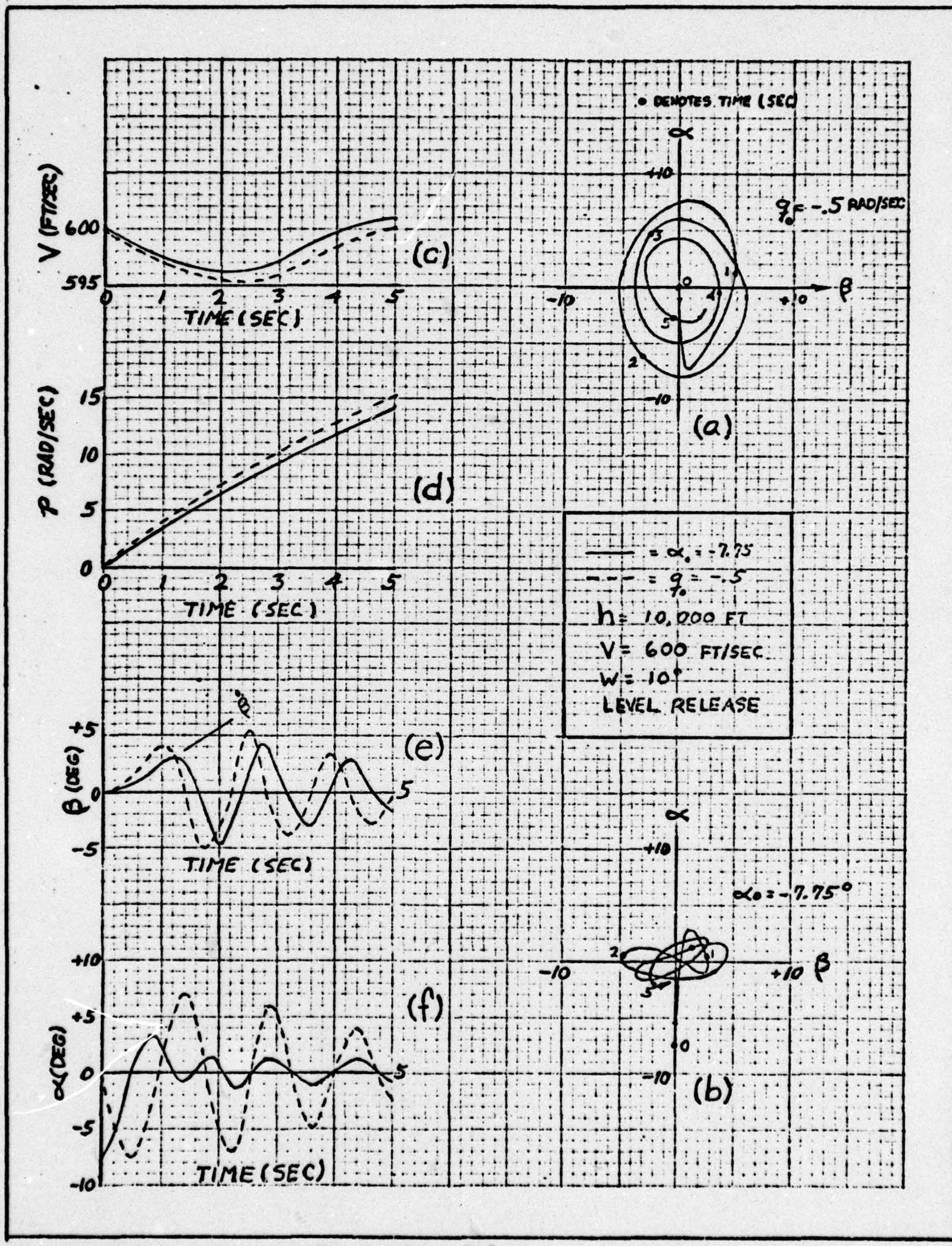


Fig. 13. Projectile Nose Track After Release

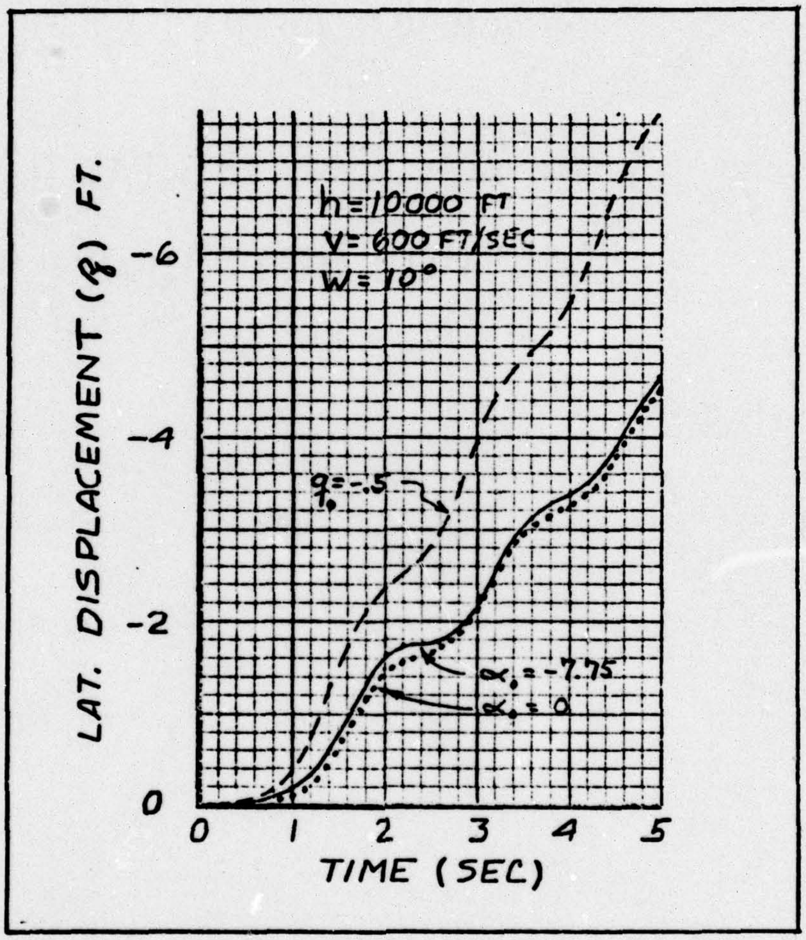


Fig. 14. Initial Rate of Lateral Displacement

α=-7.75° initial condition is approximately 1 ft/sec; for q = -.5 rad/sec, the slope is close to 1.5 ft/sec. These values of slope are significantly different and can be modeled as an initial lateral velocity of the projectile at release. The initial lateral velocity, v_o, can be input into the simulation to determine its effect. Fig. 15 shows the lateral progression of the trajectories resulting from an initial side velocity of -5, 0, +5 ft/sec. The remaining initial release conditions were the same for all three trajectories. As the plots show, the magnitude of v_o directly influences the overall trajectory. An equation for this lateral initial velocity is given by (Ref 1:53-68):

$$V_z = -\left(\frac{C_{Z_\alpha} A\ q}{M}\right)\left(\frac{T}{1 - \omega^2 T^2}\right)(2\beta + T\dot{\beta}) \qquad (24)$$

where C_{Z_α} = the normal force coefficient (acting in plane containing the relative wind).

A = frontal area of the projectile.

q = dynamic pressure.

M = mass

T = time constant for oscillation damping.

ω = frequency of oscillation.

β = yaw angle of attack.

$\dot{\beta}$ = yaw rate.

If the influence of the displacement term is considered small with respect to the rate term, the expression reduces to:

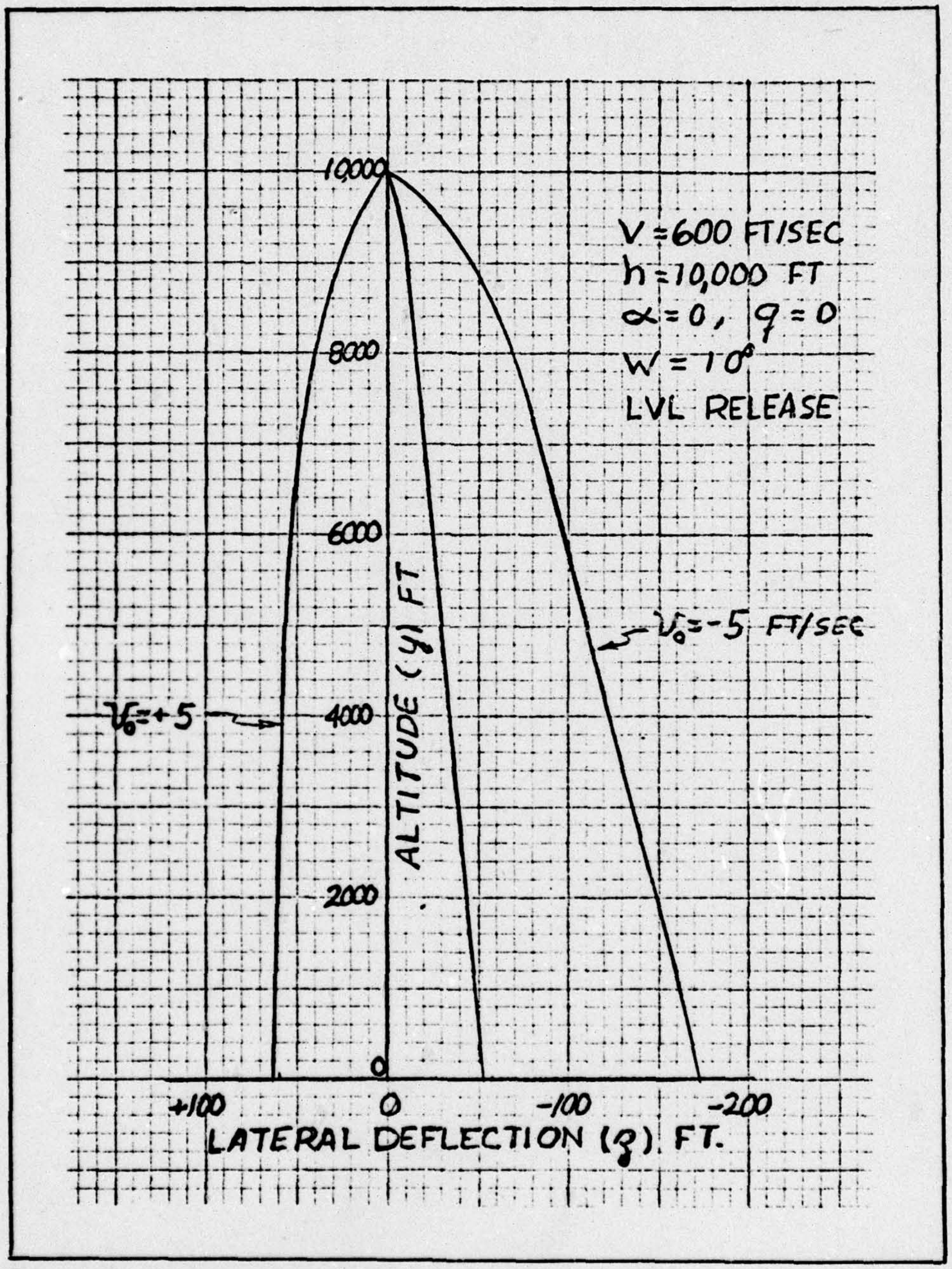


Fig. 15. Effect of Initial Side Velocity on Lateral Progression

$$V_z = -\frac{C_{Z_{\bar{\alpha}}}}{C_{M_{\bar{\alpha}}}} \frac{I}{M\,d} \tag{25}$$

where $C_{M_{\bar{\alpha}}}$ = moment coefficient (with respect to total angle of attack).

I = transverse moment of inertia.

d = max diameter of projectile.

and the following relation has been used:

$$\frac{T^2}{1 + \omega^2 T^2} = \frac{I}{C_{M_{\bar{\alpha}}} A\,q\,d} \tag{26}$$

Equation 25 shows that the lateral velocity induced by oscillation of the projectile is sensitive to the normal force coefficient C_Z, the moment coefficient C_M, and the yaw rate $\dot{\beta}$.

The magnitude of the side velocity predicted by this closed form approximation is given by (Ref 1:65)

$$V_z = \frac{\dot{\beta}}{\omega} (.362) \text{ ft/sec/degree} \tag{27}$$

where the constant, 0.362, is the result of equation 24 (displacement term neglected) using a velocity of 750 ft/sec, an altitude of 5000 ft, and coefficients of a projectile similiar to that in this investigation (coefficients used in this derivation are determined with respect to the total angle of attack instead of pd/2V). These same initial conditions were used in a six degree of freedom simulation and the lateral displacement and yaw oscillations were plotted in Fig. 16. From these plots, a representative yaw rate and frequency of oscillation were

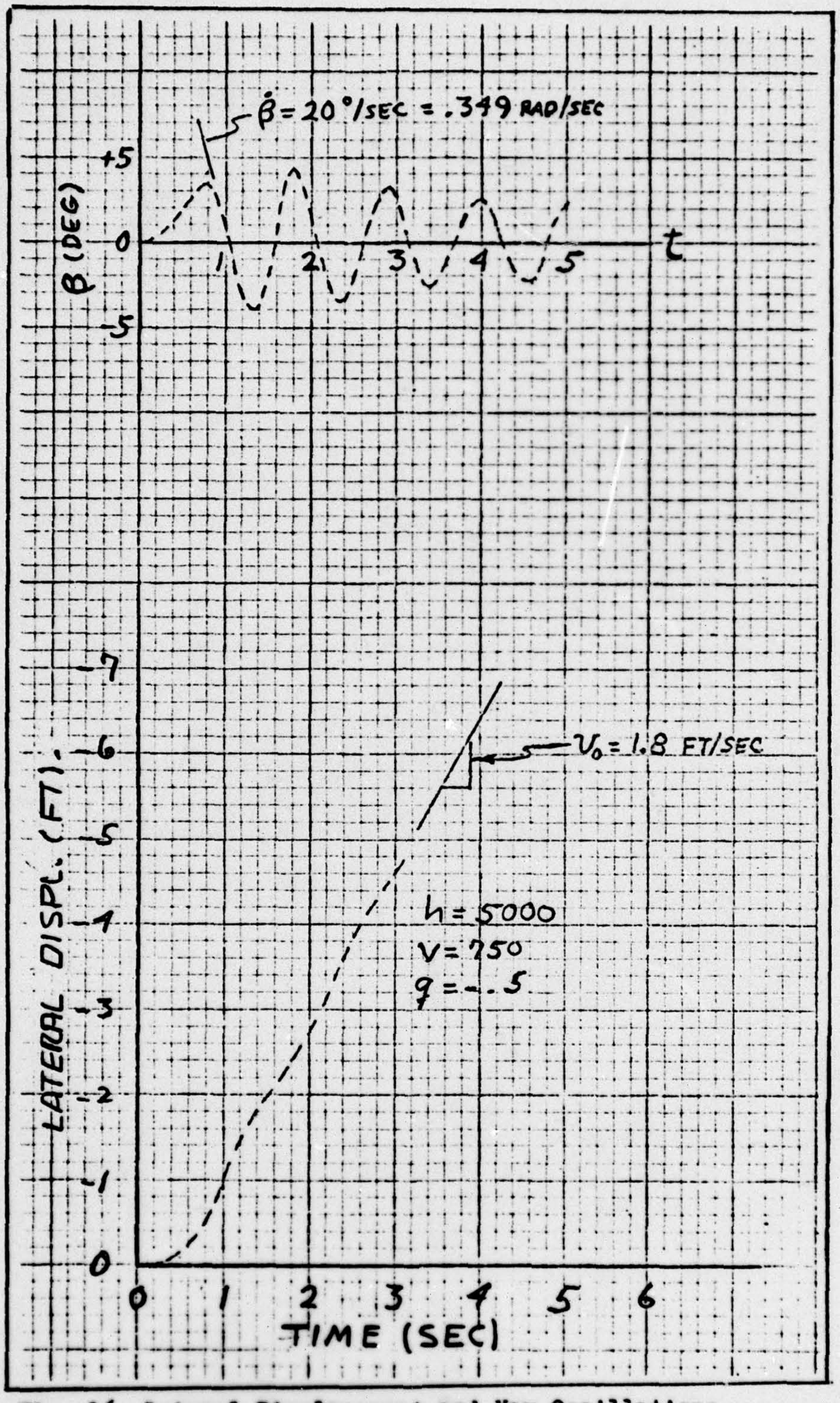


Fig. 16. Lateral Displacement and Yaw Oscillations

estimated as 0.349 radians/sec and 5.34 radians/sec respectively. The closed form approximation (Eqn 27) then produces a value of $V_z = 1.36$ feet per second.

The plot of lateral displacement versus time indicates the initial lateral velocity, according to the simulation, is approximately 1.8 feet per second. This is on the order of 25% difference between the six degree of freedom simulation results and the closed form expression. This difference can be attributed to the more extensive simplifying assumptions used to make the closed form expression possible.

Effect of Spin Rate

The effect of spin rate induced by various wedge angles is investigated at different airspeeds and release perturbations (Table VII). The spin rate, induced by the wedge angle of fifteen degrees, produces the most nominally linear lateral deviation throughout the entire trajectory. This pattern holds for both velocities investigated (Fig 17).

Table VII also reveals that increasing the spin rate of the projectile causes the trajectory to impact longer and wider. This apparent contradiction to the principle of conservation of energy was initially thought to be an error caused by using an integration step size that was too large. A step size of .002 second, however, is adequate for the spin rates encountered.

Another possible explanation is that the projectile developed gyroscopic rigidity and was starting to fly at a net angle of attack rather than averaging to zero angle of attack in its oscillations. This concept is supported by the simulation readout which shows the angle of attack of the projectile with a five degree wedge angle oscillates on the

TABLE VII

Effect of Spin Rate

Alt = 10,000 ft, level flt release, all coefficients included.

	V = 600, α = 0, q = 0					V = 600, α = -10, q = 0				
Wedge	X	Z	V_f	p_f	t_f	X	Z	V_f	p_f	t_f
5^o	14159	-37.4	888.5	31.4	25.72	14165	-38.9	888.6	31.4	25.70
10^o	14165	-57.7	889.1	62.9	25.72	14174	-59.1	889.2	62.9	25.71
15^o	14185	-65.6	889.4	94.4	25.73	14199	-66.1	889.6	94.4	25.72
	V = 1000, α = 0, q = 0					V = 1000, α = -10, q = 0				
5^o	22668	-30.7	1029.6	38.0	26.11	22669	-41.1	1029.3	38.3	26.10
10^o	22716	-79.7	1030.3	76.1	26.13	22718	-79.6	1030.5	76.1	26.12
15^o	22746	-87.2	1030.6	114.2	26.14	22738	-93.2	1030.7	114.1	26.13

X = Range (ft)

Z = Lat displacement (ft/sec)

p_f = Impact spin rate (rad/sec)

t_f = Time to impact

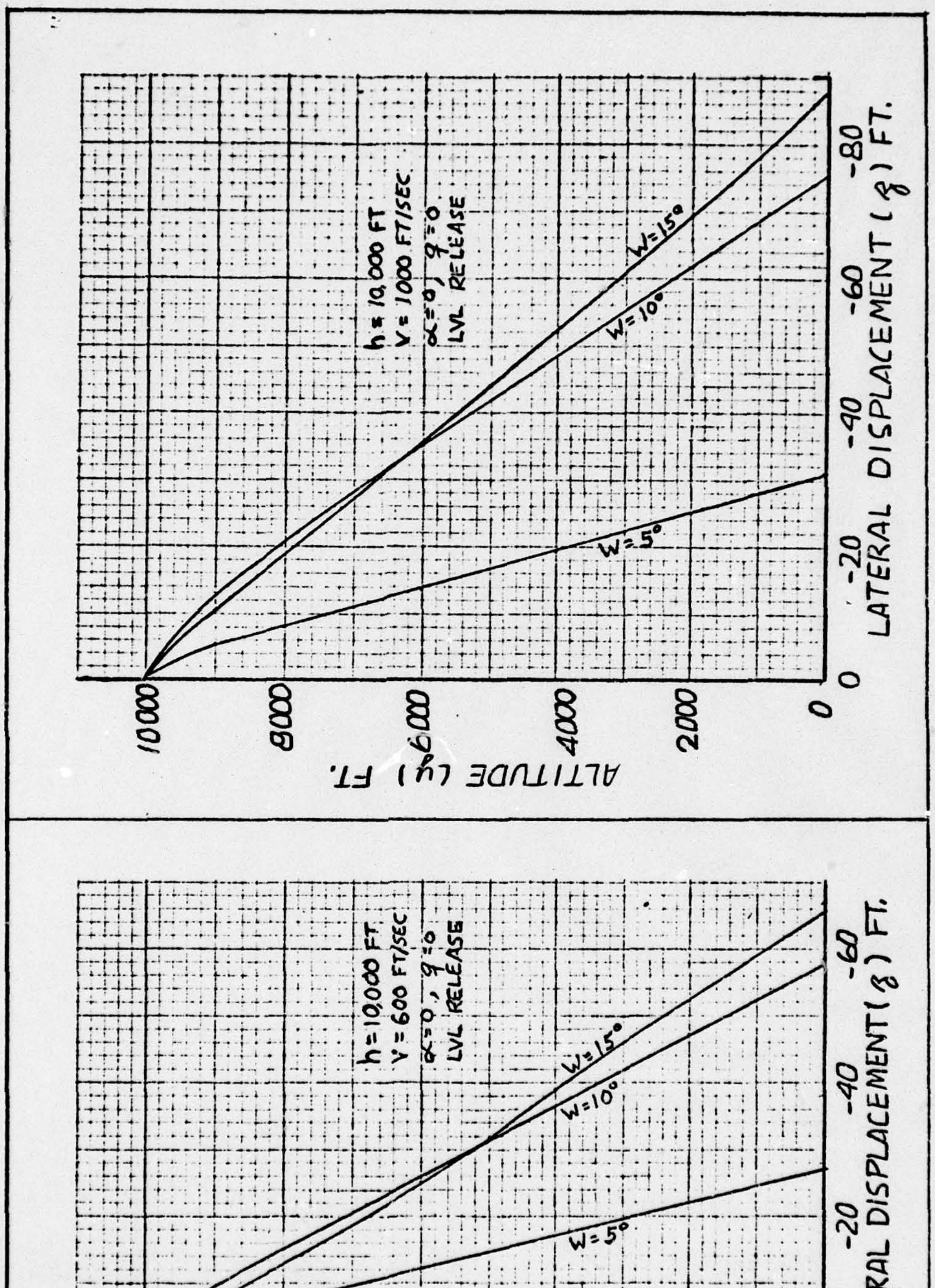


Fig. 17. Effect of Spin Rate on Lateral Displacement

order of ± 0.1 degree in the latter half of the trajectory. The fifteen degree wedge readout, however, shows only about half the magnitude of oscillation but the oscillations are predominantly positive in the last half of the trajectory. In fact, the angle of attack does not go negative at all during the final ten seconds of flight. Also, Table VII indicates the total time of flight of the projectile spinning at the faster rates is increased very slightly (on the order of 0.02 second). The downrange velocity just prior to impact is approximately 750 ft/sec which produces an opportunity for the projectile to impact 15 feet longer in the extra 0.02 second. This is compatible with the data shown in Table VII.

Other Observations

Fig. 18 shows the aerodynamic coefficients, C_{y_7}, C_{y_p} & C_{n_p}, have no discernable effect on the projectile velocity. The velocity progression graphs remained the same regardless of whether or not these coefficients were included in the simulation.

The wedge angle did not change the velocity progression significantly. Simulations modeling spin rates as 5, 10, or 15° wedge angle indicates the velocity progression is not influenced by spin rate. This observation is subject to the projectile spin rate remaining low enough to preclude instability.

The higher dynamic pressure encountered during release at high airspeeds causes a significantly greater velocity change of the projectile after release. The magnitude of the difference in the release and steady state velocity of the projectile governs how the velocity will progress throughout the trajectory. The projectile shows significant velocity change throughout the trajectory when released from a low speed condition.

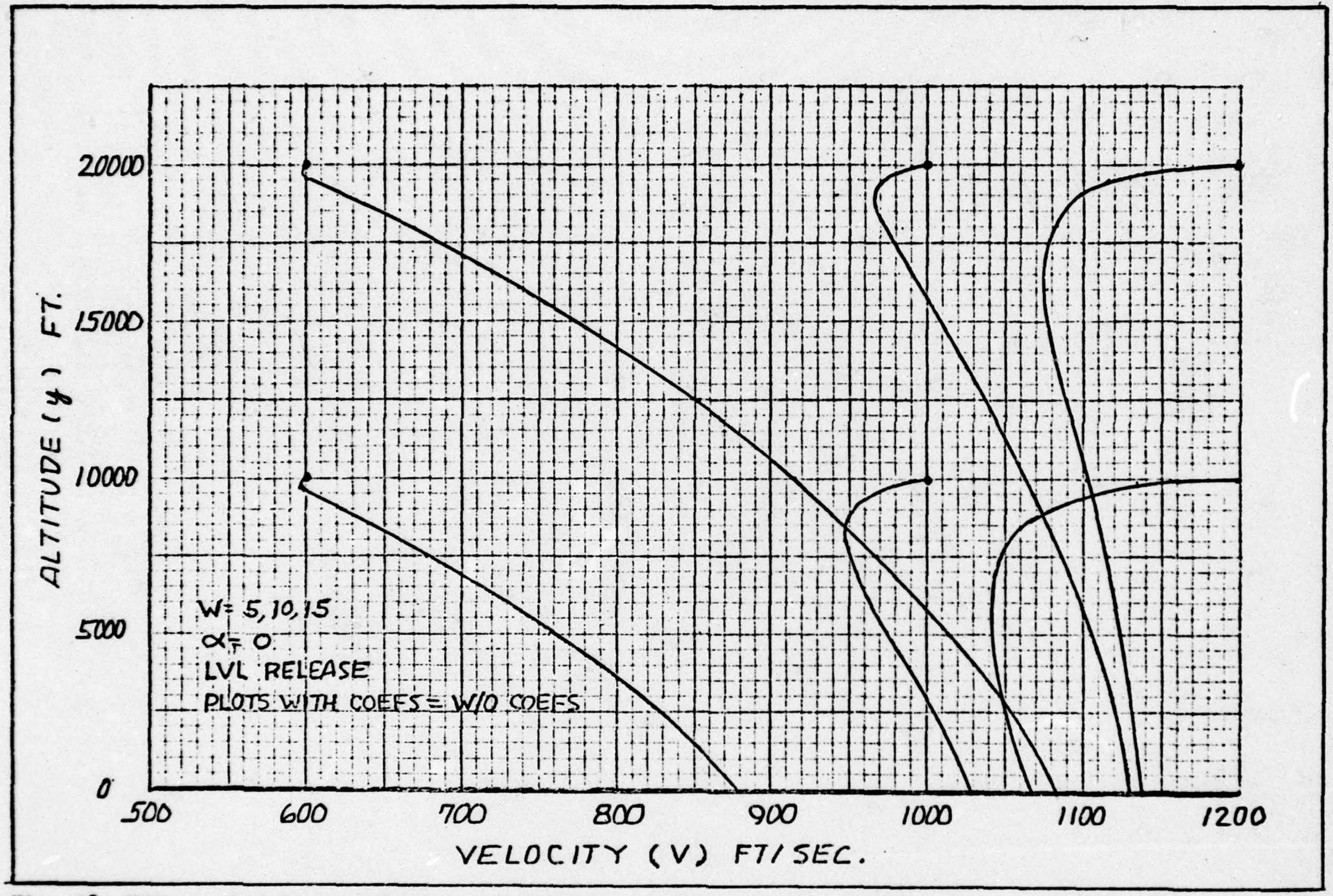


Fig. 18. Effect of Release Velocity and Altitude on Velocity Progression

The projectile shows much less acceleration when released from a high dynamic pressure condition.

Figure 19 investigates projectile acceleration from release to impact. The slope of the velocity vs time curve shows that maximum deceleration is proportional to the drag force encountered at release and, as a result, velocity loss is greater at high release velocities. The time required for transition from deceleration to acceleration also increases with higher release velocities. The same plots apply for each of the projectile fin wedge angles investigated. Also, there are no significant changes in the graphs for an initial angle of attack of $\alpha_o = 0$ versus $\alpha_o = -10$.

Fig. 20 shows the lateral displacement progression of a projectile subjected to two different release perturbations ($\alpha_o = -10^o$ and $q_o = -.5$ rad/sec) at two different release velocities. The slope of a pair of the curves is nearly identical at any given time but the magnitude of the lateral displacement attained in the same given time can be significantly different at the lower airspeed.

Fig. 21 shows that the rate of altitude loss does not change significantly with initial release airspeeds. For the 10,000 ft drop, time of flight through a standard atmosphere is increased on the order of one second compared to the time of flight through a vacuum.

The effect of release velocity and altitude is shown as a composite in Fig. 22. The set of trajectories initiated at 10,000 ft is essentially a duplication of Fig. 11 and is included with the 20,000 ft set to give an indication of the effect of air density on the lateral deflection.

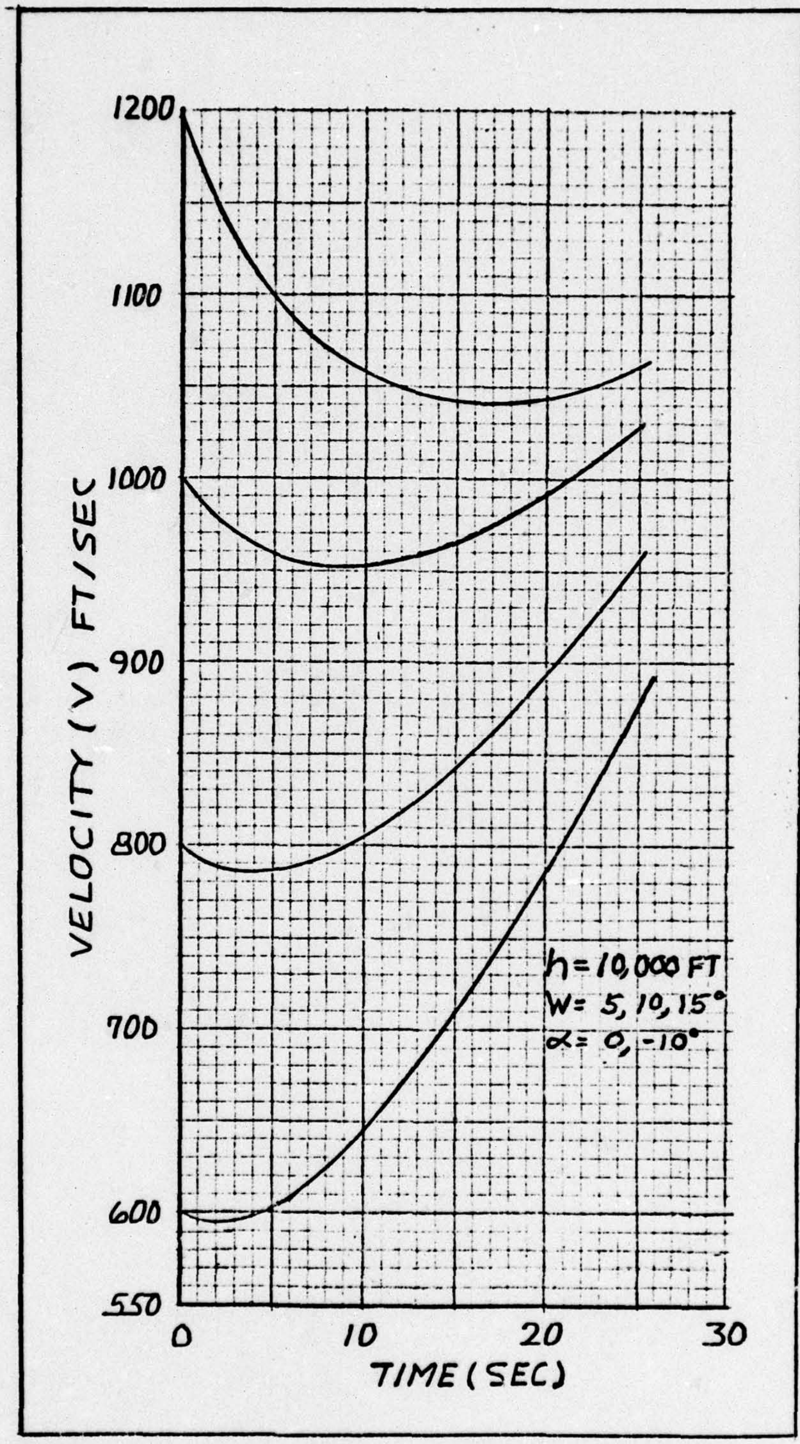


Fig. 19. Effect of Release Velocity on Total Projectile Acceleration

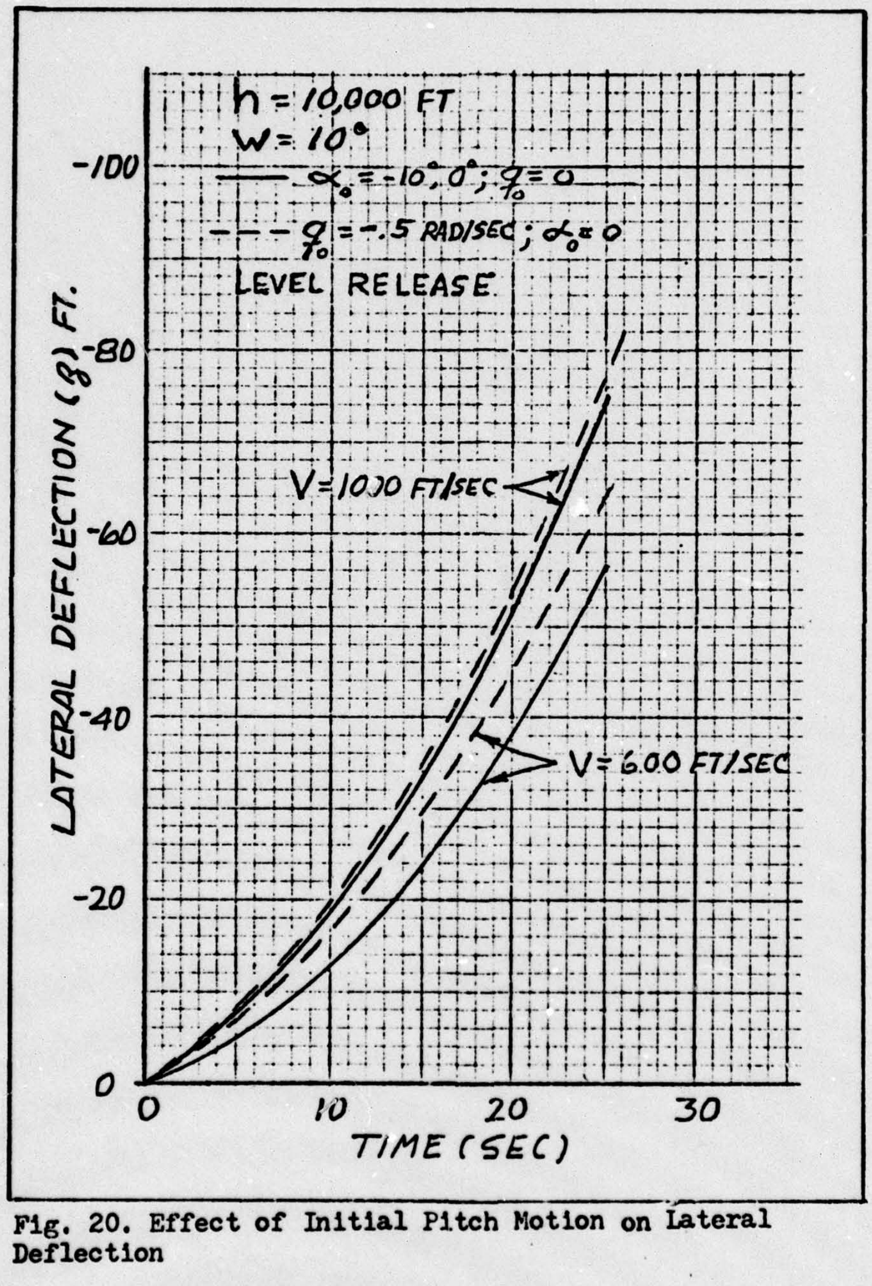


Fig. 20. Effect of Initial Pitch Motion on Lateral Deflection

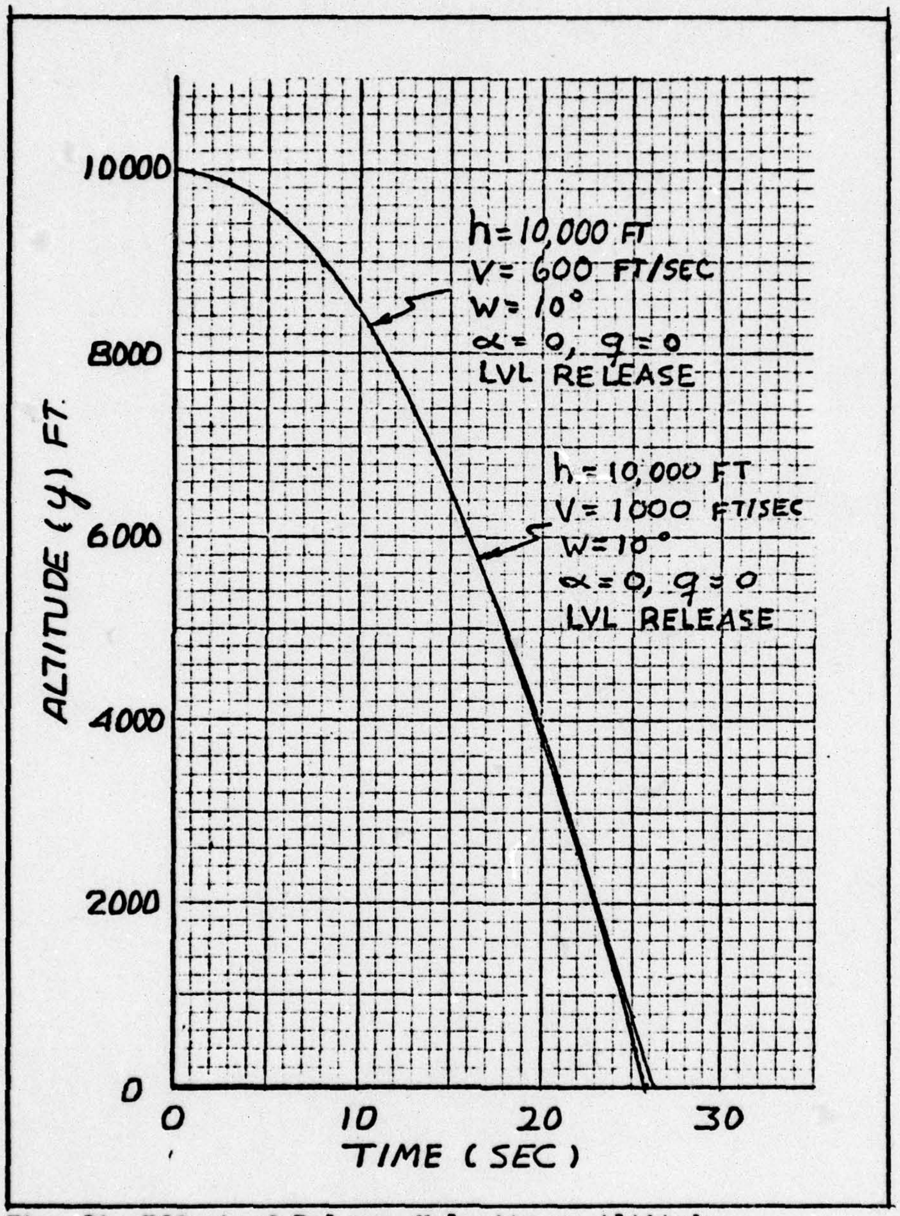


Fig. 21. Effect of Release Velocity on Altitude Progression

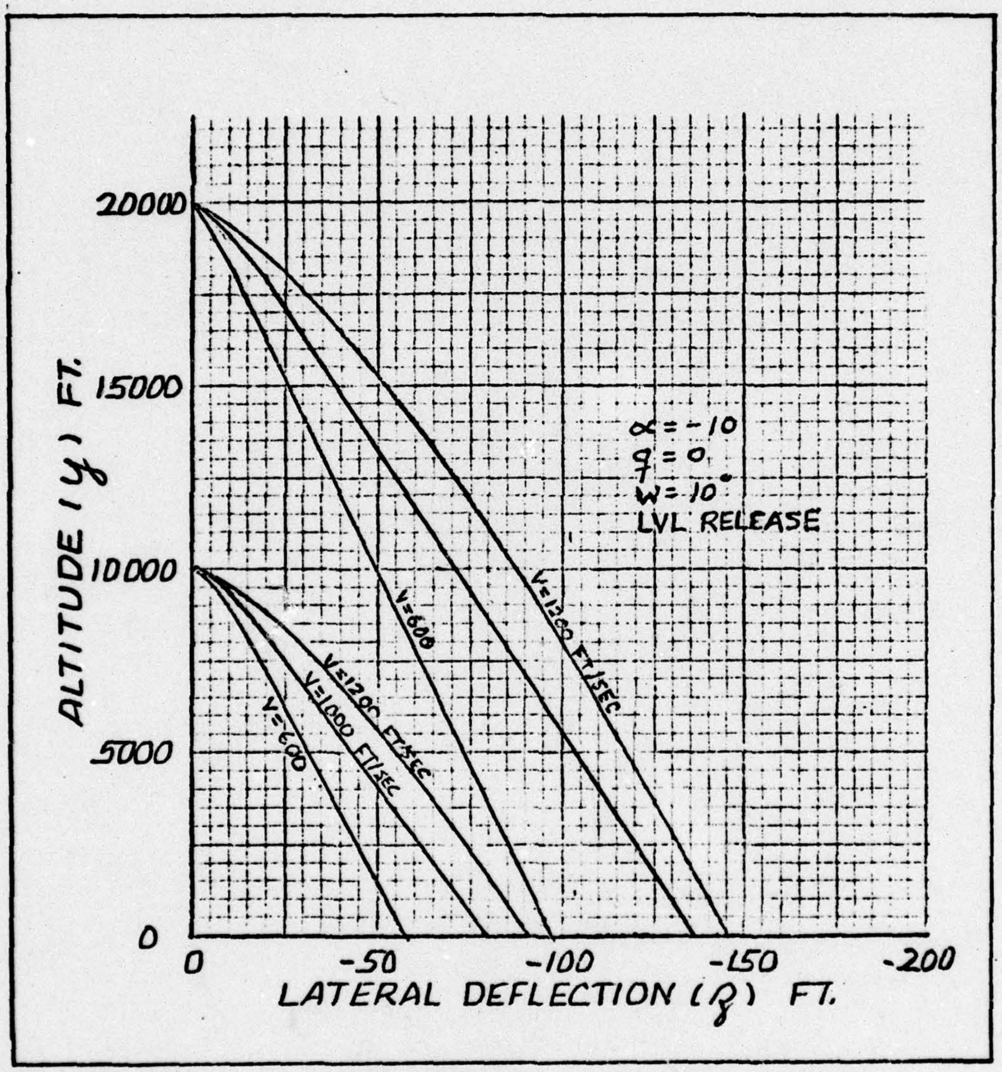


Fig. 22. Effect of Release Velocity and Altitude on Lateral Displacement

V. Conclusions

The Magnus force, Magnus moment, and side force coefficients, for a projectile of the type modeled in the simulations, and for normal release conditions, exert only a very small influence on the impact point. The cross range displacement attributable to these coefficients, for a trajectory initiated at 10,000 feet, are on the order of only a few feet. The influence on the downrange component of the trajectory is also very small. These coefficients could be influential under conditions of high angle of attack and high spin rates which, for normal releases, are not present simultaneously. When the projectile oscillation and the corresponding angle of attack is maximum following release, the spin rate has not yet developed. As the projectile progresses along the trajectory, the spin rate builds up but the oscillations dampen to a very small value. Also, the oscillations alternate about the velocity vector of the center of gravity of the projectile so the Magnus force direction, being dependent on the direction of the angle of attack, also alternates and tends to average to zero effect.

The lateral displacement of the trajectory is, in general, increased with increasing spin rate. Insufficient spin rate can be a stability factor as can excessive spin rate. Low values of angular momentum can allow the projectile to develop catastrophic yaw. High spin rates induce gyroscopic resistance to change in projectile orientation, allowing a positive angle of attack to build up as the trajectory progresses. A net lift is then developed which increases the time of flight of the projectile and, consequently, the cross range and downrange impact point.

A projectile released at zero angle of attack with no initial perturbations produces the least lateral displacement of the trajectory. If the projectile is released with an initial negative angle of attack, the lateral progression is increased only slightly. If the projectile is released with a rate rotational motion, as could be induced by the ejector mechanism or an initial positive angle of attack, the lateral progression is significantly increased. This increase is directly proportional to the amplitude of the projectile oscillations. The difference in lateral progression between the rate induced and displacement induced oscillation is less at higher airspeeds. Also, as oscillations increase the cross range impact point of the trajectory, there is, in general, a corresponding decrease in the downrange impact point. These results suggest the following conclusions:

1. Magnus force, as such, exerts no significant influence on the trajectory of a projectile such as the Mk-82 bomb. For most release airspeeds and altitudes, compensation for this specific effect in a weapons delivery system would not be cost effective.
2. The projectile should be designed to spin as slow as possible consistent with stability considerations.
3. Release mechanisms should be designed so no significant moment is applied. If assisted separation of the projectile from the delivery aircraft is desired, a slight negative angle of attack should be used instead of a moment.

Bibliography

1. Ausman, J.S. Theoretical Analysis and Experimental Measurements of Separation Disturbances in Weapon Delivery Systems. Aircraft/Stores Compatability Symposium Proceedings, JTCG/MD WP No. 12:53-85 (September, 1975).

2. Cohen, C.J. and D. Werner. A Formulation of the Equations of Motion of a Four-Finned Missile. Defense Documentation Center Reference AD112006, September 1956.

3. Daniels, P. Minimization of Lock-In Roll Moment on Missiles via Slots. Defense Documentation Center Reference AD010130, January 1975.

4. Etkin, B. Dynamics of Flight. New York: John Wiley and Sons, Inc., 1959.

5. Ingram, C.W. A Computer Program for Integrating the Six Degree of Freedom Equations of Motion of a Symmetrical Missile. Unpublished notes, Aerospace Department, University of Notre Dame.

6. Likins, P.W. Elements of Engineering Mechanics. New York: McGraw-Hill Book Company, 1973.

7. Meirovitch, L. Methods of Analytical Dynamics. New York: McGraw-Hill Book Company, 1970.

8. Miklos, W. Analysis of the Stability and Flight Performance of the Candidate Replacement for the MK-82 Snakeye Bomb. Aif Force Flight Dynamics Laboratory Report Number AFFDL-TR-77-108, June 1976.

9. Murphy, C.H. Free Flight Motion of Symmetric Missiles. Defense Documentation Center Reference AD442757, July 1963.

10. Naveh, B.Z.N. Untitled, unpublished notes, Aerospace Department, University of Notre Dame, 1974.

11. Nicolaides, J.D. Missile Flight and Astrodynamics. Defense Documentation Center Reference AD287400.

12. Vaughn, H.R. A Detailed Development of the Tricycle Theory. Sandia Laboratory Report SC-M-67-2933, February 1968.

13. Whittaker, E.T. A Treatise on the Analytical Dynamics of Particles and Rigid Bodies. Dover Publications, 1944.

14. Air Force Armament Laboratory Report. AFATL MK 82/84 Air Inflatable Retarder (Magnus Phase) in the Aerodynamic Wind Tunnel (4T) Off-Line Data, parts 6 through 153. P41C-86A, TC387 Arnold Air Force Station, Tennessee. (July 1975).

Appendix A

Typical Wind Tunnel Data

The aerodynamic coefficients for a specific projectile model are normally obtained from wind tunnel tabulated data. Table A-I shows the spin-up data of a projectile model at a specific Mach and angle of attack. The data measurements were made as the projectile was progressively accelerated from a zero roll rate to the steady state spin rate (P_{ss}). Table A-II shows the spin-down data taken as the projectile was progressively decelerated from a high spin rate down to P_{ss}. Both of these figures were obtained from reference 11, part 31.

TABLE A-I

Spin-up Data

TEST	PART	POINT	TIME	DATE	DATA SET	DAY	DATA MODE	AEDC PROPULSION WIND TUNNEL
TC-387	31	5	35206	07-15-75	1	143	92	TRANSONIC 4T

MI	PTI	QI	PI	RX10-6	PIA-1	PIB-2	PCA-1	PCB-2	PE	TTA-1	TTB-2	TM-W	TAU-I	TAU-B	TAU-S	TAU-N	TAUBR
0.8005	1197.1	352.1	784.9	2.269	1197.1	1197.3	791.7	792.2	1010.0	92.8	92.6	0.00	4.97	4.97	5.02	4.97	4.98

MC	TPR	ALFI	PHII	TOPC	PMC	SCX100	SCHED	MB	DM	ER CODE	WIND-OFF
0.7918	1.1852	2.00	0.0	39.9	0.0	-62.197	B	0.800	0.010	0	27/ -3

CONFIGURATION	MACH NO	PO	TO	P-INF	T-INF	Q-INF	V-INF	RE/INX10-6	REDX10-6	RELX10-6	CYP	CLNP
FFS#1	0.8005	6.3	552.5	5.451	489.7	2.445	868.808	0.189	0.898	7.573	-0.10974	0.2636

SAMPLE	ALFA-M	CN	CLM	CY	CLN	XCP/D	XCY/D	TAVG	P(C)	RPM	P	PD/2V
1	1.87	0.145	-0.160	-0.00206	0.00458	-1.108	-2.224	1.7107	79.8	756.	79.2	0.0180
2	1.87	0.145	-0.162	-0.00261	0.00638	-1.118	-2.447	2.7401	103.6	984.	103.1	0.0235
3	1.87	0.147	-0.167	-0.00315	0.00818	-1.137	-2.592	3.7694	123.2	1172.	122.7	0.0280
4	1.87	0.147	-0.165	-0.00351	0.00775	-1.127	-2.207	4.7986	139.3	1325.	138.8	0.0316
5	1.87	0.145	-0.163	-0.00380	0.00936	-1.124	-2.460	5.8279	152.6	1452.	152.1	0.0346
6	1.87	0.145	-0.166	-0.00359	0.00949	-1.142	-2.645	6.8572	163.5	1556.	163.0	0.0371
7	1.87	0.145	-0.166	-0.00418	0.01006	-1.142	-2.404	7.8865	172.6	1643.	172.1	0.0392
8	1.87	0.146	-0.168	-0.00479	0.01030	-1.152	-2.152	8.9158	180.0	1714.	179.5	0.0409
9	1.87	0.146	-0.168	-0.00476	0.01124	-1.152	-2.363	9.9450	186.1	1774.	185.8	0.0423
10	1.87	0.147	-0.171	-0.00420	0.01150	-1.161	-2.740	10.9743	191.1	1824.	191.0	0.0435
11	1.87	0.148	-0.171	-0.00488	0.01157	-1.159	-2.373	12.0036	195.3	1864.	195.2	0.0445
12	1.87	0.147	-0.171	-0.00515	0.01163	-1.161	-2.259	13.0328	198.7	1895.	198.5	0.0452
13	1.87	0.147	-0.172	-0.00499	0.01230	-1.167	-2.466	14.0620	201.5	1922.	201.3	0.0458
14	1.87	0.146	-0.170	-0.00498	0.01262	-1.163	-2.534	15.0913	203.8	1944.	203.6	0.0464
15	1.87	0.146	-0.171	-0.00519	0.01223	-1.169	-2.354	16.1205	205.7	1961.	205.4	0.0468
16	1.87	0.146	-0.170	-0.00533	0.01219	-1.164	-2.284	17.1498	207.3	1976.	207.0	0.0471
17	1.87	0.146	-0.171	-0.00531	0.01288	-1.169	-2.427	18.1791	208.6	1989.	208.2	0.0474
18	1.87	0.146	-0.171	-0.00518	0.01262	-1.169	-2.435	19.2083	209.7	1999.	209.3	0.0477
19	1.87	0.146	-0.171	-0.00526	0.01220	-1.169	-2.319	20.2375	210.6	2006.	210.1	0.0479
20	1.87	0.146	-0.170	-0.00547	0.01222	-1.164	-2.233	21.2668	211.3	2014.	210.9	0.0480
21	1.87	0.146	-0.171	-0.00531	0.01296	-1.169	-2.442	22.2961	211.9	2019.	211.4	0.0482
22	1.87	0.147	-0.172	-0.00517	0.01300	-1.173	-2.517	23.3254	212.4	2025.	212.0	0.0483
23	1.87	0.146	-0.171	-0.00543	0.01330	-1.169	-2.449	24.3546	212.8	2029.	212.5	0.0486

CYO	CLNO	CLO	CLP	PSS	(PSS)D/2V	CL-DEL	PSSF	LOTARE	LPTARE
-0.00116	0.01256	0.0543	-1.0330	230.61	0.05253	0.00362	214.65	0.0	-0.0003

FINAL CY COEFFICIENTS

1.0487E-03 -2.4294E-01 4.9148E 00 -5.4547E 01

FINAL CLN COEFFICIENTS

-7.6932E-03 1.0542E 00 -2.5169E 01 2.5193E 02

00163

TABLE A-II

Spin-down Data

TEST	PART	POINT	TIME	DATE	DATA SET	DAY	DATA MODE	AEDC PROPULSION WIND TUNNEL
TC-347	31	6	35350	07-15-75	1	143	92	TRANSONIC 4T

MI	PTI	QI	PI	RX10-6	PTA-1	PTB-2	PCA-1	PCB-2	PE	TTA-1	TTB-2	TM-W	TAU-T	TAU-B	TAU-S	TAU-N	TAUBR
0.7987	1198.0	351.4	786.9	2.268	1198.0	1197.5	793.7	794.4	1008.7	92.8	92.6	0.00	4.97	4.97	5.02	4.97	4.98

MC	TPR	ALFI	PHII	TUPC	PMC	SCX100	SCHED	MB	DM	ER CODE	WIND-OFF
0.7900	1.1877	2.00	0.0	40.1	0.0	-62.197	8	0.800	0.010	0	27/ -3

CONFIGURATION	MACH NO	PO	TO	P-INF	T-INF	Q-INF	V-INF	RE/INX10-6	REDX10-6	RELX10-6	CYP	CLNP
FFS#1	0.7987	6.3	552.5	5.465	490.0	2.440	867.054	0.189	0.898	7.570	-0.10974	0.2636

SAMPLE	ALFA-M	CN	CLM	CY	CLN	XCP/D	XCY/D	TAVG	P(C)	RPM	P	PD/2V
1	1.87	0.137	-0.148	-0.00711	0.01388	-1.081	-1.953	1.7126	412.5	3963.	415.0	0.0947
2	1.87	0.154	-0.202	-0.00927	0.02201	-1.307	-2.373	2.7418	377.3	3612.	378.3	0.0863
3	1.87	0.153	-0.197	-0.00865	0.02047	-1.288	-2.366	3.7711	348.3	3326.	348.3	0.0795
4	1.87	0.152	-0.191	-0.00881	0.02013	-1.258	-2.286	4.8003	324.5	3098.	324.4	0.0740
5	1.87	0.151	-0.189	-0.00802	0.01918	-1.249	-2.390	5.8296	304.9	2910.	304.7	0.0695
6	1.87	0.150	-0.184	-0.00714	0.01711	-1.230	-2.395	6.8589	288.8	2756.	288.6	0.0659
7	1.87	0.151	-0.187	-0.00689	0.01641	-1.237	-2.383	7.8881	275.6	2630.	275.4	0.0629
8	1.87	0.151	-0.185	-0.00727	0.01745	-1.220	-2.401	8.9173	264.8	2527.	264.6	0.0604
9	1.87	0.151	-0.182	-0.00678	0.01594	-1.210	-2.352	9.9466	255.8	2442.	255.7	0.0584
10	1.87	0.149	-0.180	-0.00693	0.01566	-1.203	-2.261	10.9759	248.5	2373.	248.5	0.0567
11	1.87	0.148	-0.177	-0.00606	0.01505	-1.195	-2.482	12.0051	242.5	2317.	242.6	0.0554
12	1.87	0.148	-0.177	-0.00617	0.01401	-1.194	-2.272	13.0344	237.5	2269.	237.6	0.0542
13	1.87	0.147	-0.175	-0.00598	0.01547	-1.191	-2.586	14.0636	233.5	2230.	233.5	0.0533
14	1.87	0.147	-0.174	-0.00612	0.01357	-1.185	-2.219	15.0929	230.1	2197.	230.1	0.0525
15	1.87	0.147	-0.173	-0.00575	0.01432	-1.181	-2.490	16.1221	227.4	2171.	227.3	0.0519
16	1.87	0.147	-0.174	-0.00606	0.01522	-1.179	-2.510	17.1514	225.1	2148.	225.0	0.0514
17	1.87	0.147	-0.172	-0.00621	0.01301	-1.174	-2.096	18.1806	223.3	2130.	223.0	0.0509
18	1.87	0.147	-0.172	-0.00551	0.01325	-1.175	-2.402	19.2100	221.8	2114.	221.4	0.0505
19	1.87	0.147	-0.173	-0.00555	0.01416	-1.181	-2.551	20.2392	220.5	2102.	220.1	0.0502
20	1.87	0.147	-0.174	-0.00576	0.01416	-1.179	-2.461	21.2685	219.5	2090.	218.9	0.0500
21	1.87	0.147	-0.172	-0.00579	0.01307	-1.174	-2.256	22.2978	218.6	2083.	218.1	0.0498
22	1.87	0.147	-0.172	-0.00570	0.01396	-1.175	-2.450	23.3271	217.9	2076.	217.4	0.0496
23	1.87	0.147	-0.172	-0.00525	0.01311	-1.175	-2.499	24.3563	217.4	2072.	216.9	0.0495

CYO	CLNO	CLO	CLP	PSS	(PSS)D/2V	CL-DEL	PSSF	LOTARE	LPTARE
-0.00116	0.01256	0.0551	-1.0465	230.49	0.05261	0.00367	214.73	0.0	-0.0003

FINAL CY COEFFICIENTS
-3.3368E-02 1.5233E 00 -2.6386E 01 1.3948E 02

FINAL CLN COEFFICIENTS
1.2107E-01 -5.5611E 00 9.2358E 01 -4.8057E 02

00C1

Appendix B

Computer Program Data Inputs

Coefficient Arrays

A single 000005 card precedes all of the coefficient matrix input decks. This card designates the total number of aerodynamic coefficient arrays used in the program and must match the actual number of arrays that have been input. The last digit of the array designator integer must be in column 12. For example, if ten coefficients are used, the lead card is:

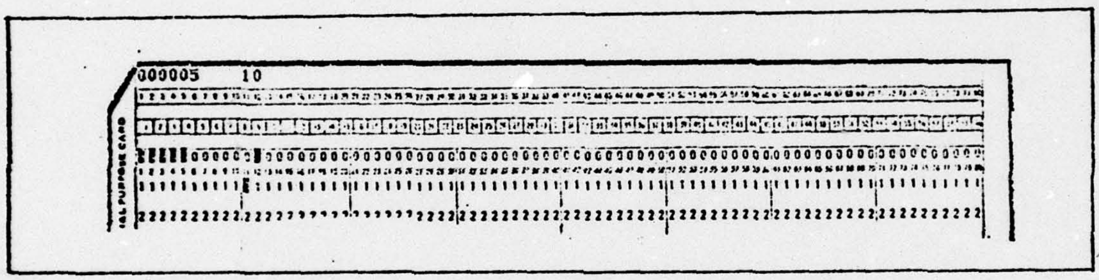


Fig. B-1a. Computer Input Card

Each individual coefficient array is then preceded by a card that specifies the desired coefficient symbol, the location of the array within the input deck in accordance with Table B-I, and the number of angle of attack columns and Mach number rows that will make up the array. A 20x20 matrix is the maximum size of the input array.

The first four spaces on the card shown in Fig. B-1b are available for the desired coefficient symbol. The last digit of the remaining three integers must lie on space 7, 9, and 11.

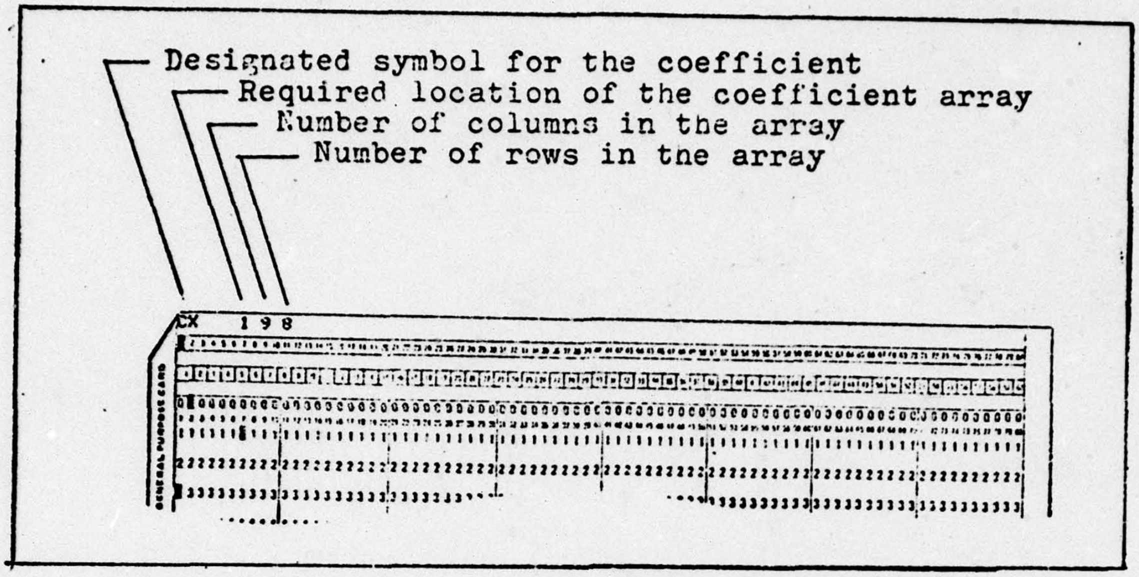


Fig. B-1b. Computer Input Card

The specific values of the input array are entered by rows. For example, the nine angle of attack elements that make up the heading row of the C_x array (Appendix G) is entered as:

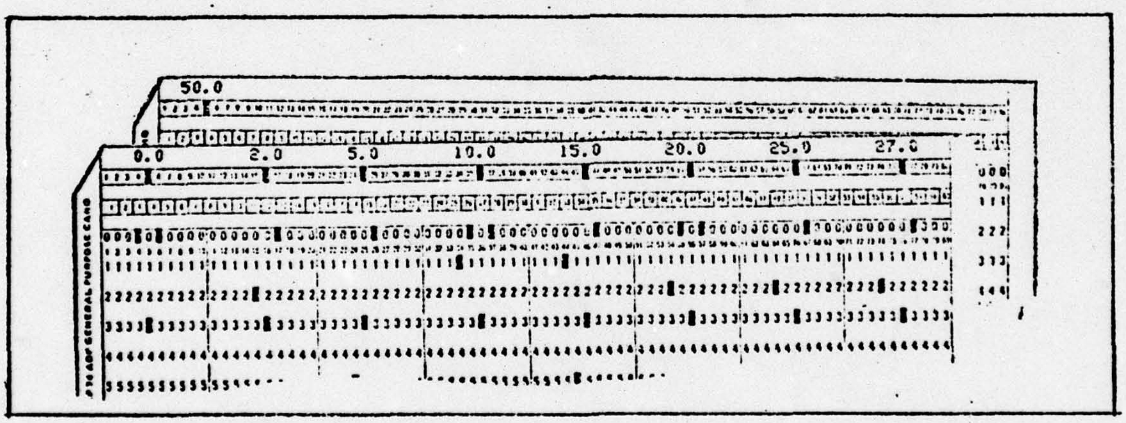


Fig. B-1c. Computer Input Card

and, in successive rows, values of the array elements are entered; such as the zero angle of attack row:

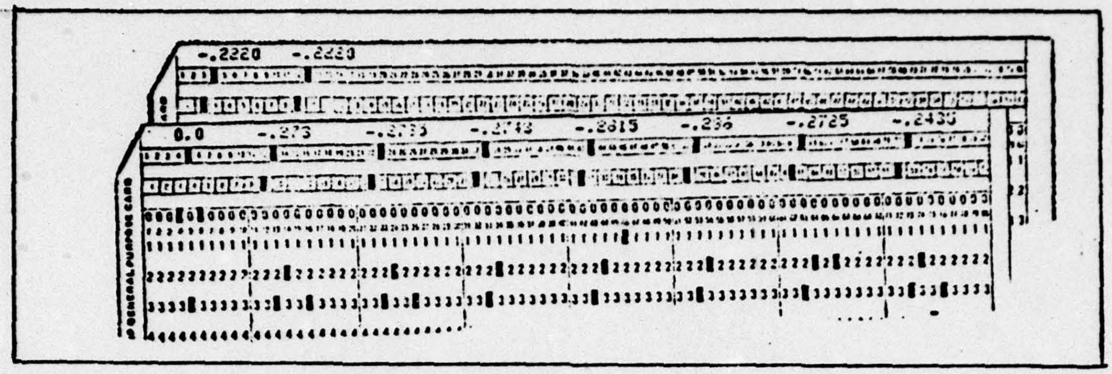


Fig. B-1d. Computer Input Card

The specific values of each element may be punched anywhere within successive ten-block segments of the card.

The remaining values of each element in the coefficient array are similarly entered until the matrix is complete (See Appendix G).

Each individual aerodynamic coefficient array used in the program is input in a similar manner. The sequence of these completed arrays must be in the order indicated in Table B-I. Only the desired aerodynamic coefficients need be input; zero entries are not required. The program will accept a maximum of thirty coefficient arrays.

Initial Conditions

The initial condition input deck is preceded with the desired title card. This title will appear as the heading of the computer output.

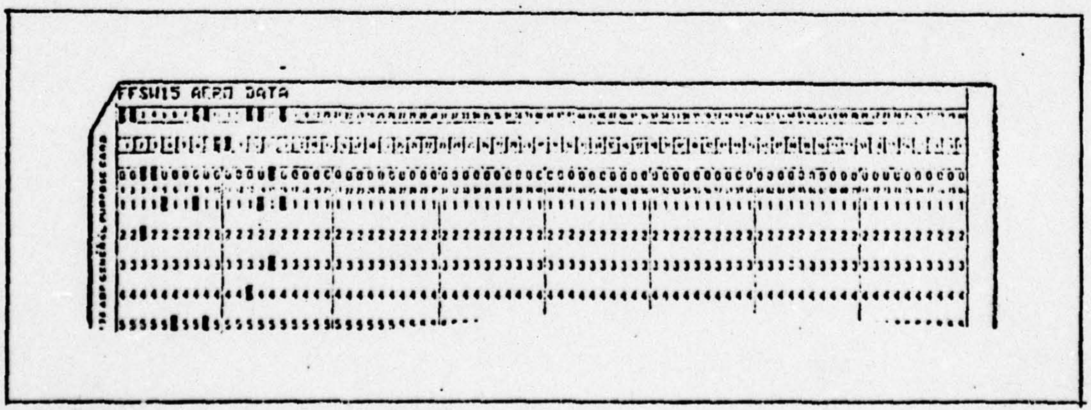


Fig. B-1e. Computer Input Card

The next card (Fig B-1f) identifies which of the option parameters available in the program are to be used. These parameters are (Ref 5:23):

NORC: Gravity designator

NORC=1 denotes constant gravity used.
NORC=0 denotes variable gravity will be input.

IPRN: Output time increment control

IPRN x Δt = output increment that will be printed. Δt is the A(20) value (Table B-2II).

NALL $\equiv$ 1 for this program.

IPUN: Punch option for angle of attack (α) and sideslip (β) data

IPUN=1 implements the option.
IPUN=0 option is not used.

NBODY: Reference frame output designator

NBODY=1 output prints and/or punches α & β in body axes.

NBODY=0 output prints and/or punches α & β aeroballistic axes.

ISCALE: Coefficient scaling option.

ISCALE=1 implements option
ISCALE=0 option is not used

ITRST: Thrust option

ITRST=1 implements thrust equation into program (See Ref 5: 11, 12)
ITRST=0 for unpowered projectile

The last digit of these integers must fall on every tenth space.

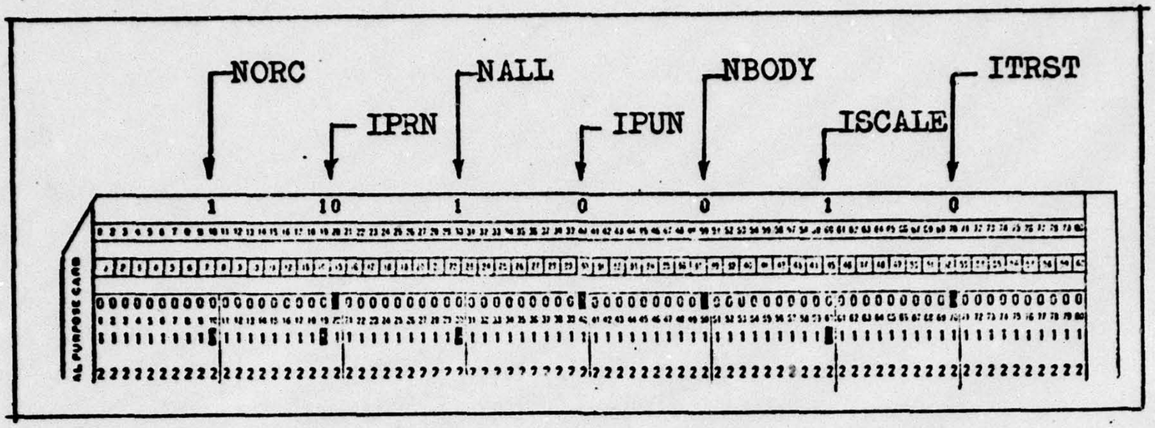


Fig. B-1f. Computer Input Card

The next three cards specify the desired initial conditions and projectile mass parameters in the sequence outlined by Table B-2II. All values are entered anywhere within successive ten-space segments.

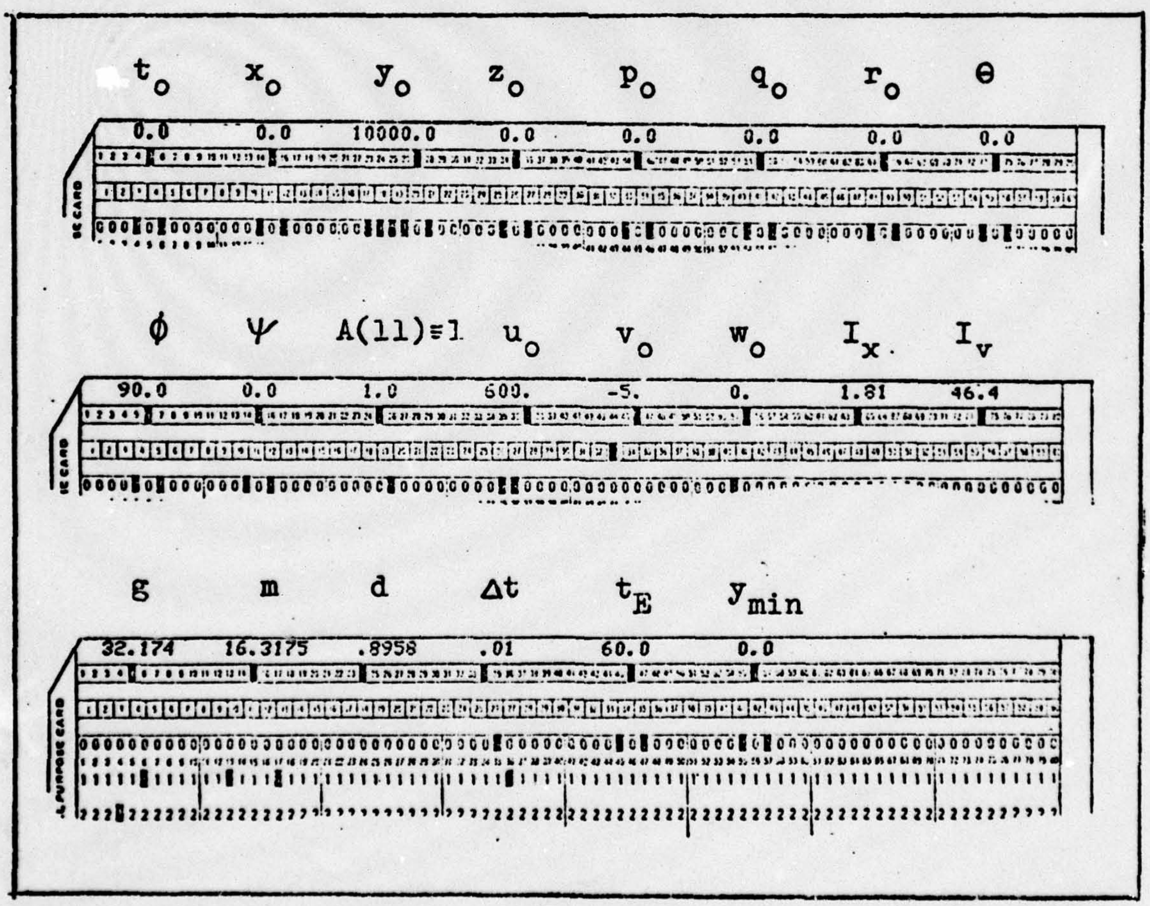


Fig. B-1g. Computer Input Cards

The units of the initial conditions are:

t_o, Δt, t_E	Sec
x_o, y_o, z_o, d, y_{min}	Ft
p_o, q_o, r_o	Rad/Sec
θ, ϕ, ψ	Degrees
u_o, v_o, w_o	Ft/Sec
I_x, I_y	Lb-Ft-Sec^2
g	Ft/Sec^2
m	Slugs

The initial values of θ and α (which is input implicitly) as well as u_o & w_o are determined according to the following scheme:

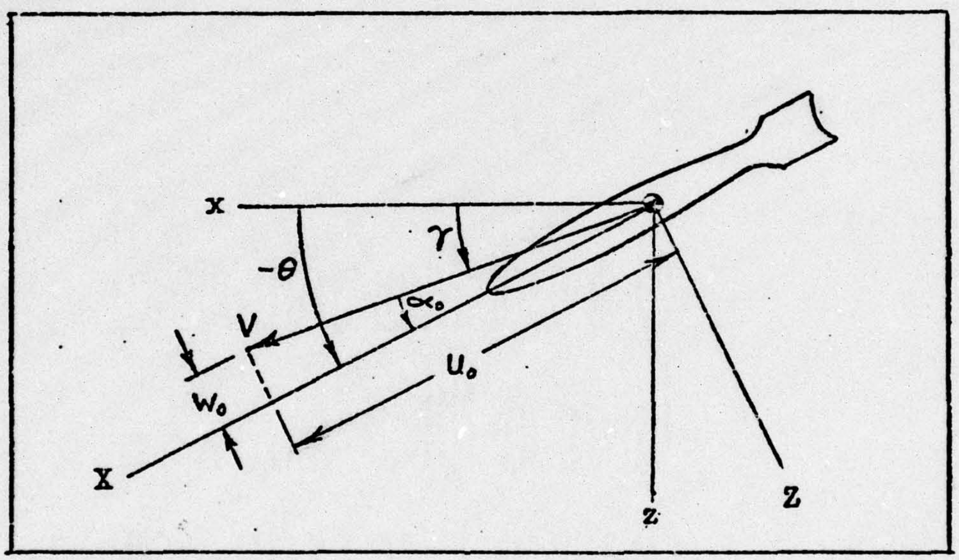


Fig. B-2. Flight Path/Perturbation Angle Input

where

V = velocity vector of the center of gravity of the projectile.

θ = total angle between the body fixed and inertial frame.

γ = flight path angle = angle between the inertial frame and velocity vector V.

α_o= initial pertubation angle from the initial flight path angle.

w_o = V sinα_o = projection of the velocity vector on the body Z axis.

u_o = V cosα_o = projection of the velocity vector on the body X axis.

For a projectile in a 30° initial dive angle at 800 ft/sec subjected to a 5° initial perturbation upon release, with no out-of-plane components, the following values would be input:

$\theta = -35.0$, $\phi = 90.0$, $\psi = 0.0$

$u_o = 800 \cos 10^\circ = 787.846$

$v_o = 0.0$

$w_o = -800 \sin 10^\circ = -138.918$

Note: $\theta \equiv 90^\circ$ to allign the inertial frame with the body fixed frame (See Fig 1).

Scale Factors

The remaining four cards are for scale factors (applicable only if ISCALE was previously designated as 1). The scale factor changes the corresponding coefficient of Table B-I by the multiple of the scaling factor.

The only scaling factor used in this study was $C_\ell\ (\delta w)$. $C_\ell\ (\delta w)$ is the coefficient for roll due to fin wedge angle. To scale the 15 degree wedge angle values of the coefficient array to, say, 10 degrees; the value .6666 is the appropriate scaling factor to enter in the ninth block of the scaling cards (Fig B-3).

The integer 1.0 is the required entry when no scaling of the corresponding coefficient array is desired.

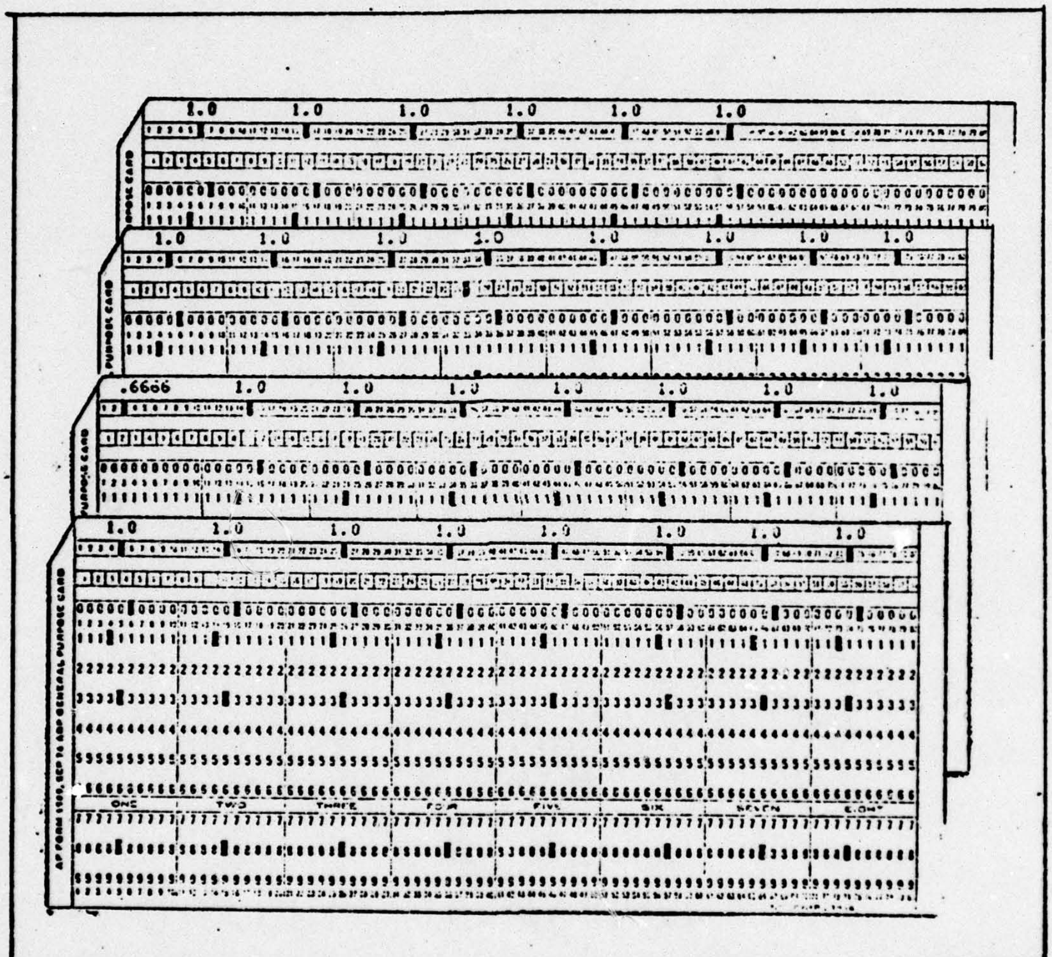


Fig. B-3. Scale Factor Inputs C(1) thru C(30)

TABLE B-I

Aerodynamic Coefficient Array
(Adapted from Ref 5)

C(I, J, K) Aerodynamic coefficient array

where: I Mach number array

J Angle of attack array

K Location of aerodynamic coefficient

C(I, J, 1)	C_x	C(I, J, 2)	C_{y_o}
C(I, J, 3)	C_{y_7}	C(I, J, 4)	C_{y8}
C(I, J, 5)	C_{yp}	C(I, J, 6)	C_{z_o}
C(I, J, 7)	C_{z7}	C(I, J, 8)	C_{z8}
C(I, J, 9)	$C_{\ell}\,(\delta w)$	C(I, J, 10)	C_{ℓ_7}
C(I, J, 11)	C_{ℓ_8}	C(I, J, 12)	C_{ℓ_p}
C(I, J, 13)	C_{m_o}	C(I, J, 14)	C_{m_7}
C(I, J, 15)	C_{m_8}	C(I, J, 16)	C_{n_o}
C(I, J, 17)	C_{n_7}	C(I, J, 18)	C_{n_8}
C(I, J, 19)	C_{n_p}	C(I, J, 20)	C_{m_ϵ}
C(I, J, 21)	C_{n_ϵ}	C(I, J, 22)	C_{y_ϵ}
C(I, J, 23)	C_{z_ϵ}	C(I, J, 24)	$C_{\ell_\epsilon \alpha p}$
C(I, J, 25)	$C_{\ell_\epsilon \alpha_1}$	C(I, J, 26)	$C_{\ell_\epsilon \alpha_2}$
C(I, J, 27)	$C_{m_{qo}}$	C(I, J, 28)	C_{mq7}
C(I, J, 29)	C_{mq_8}	C(I, J, 30)	C_{ℓ_o}

TABLE B-II

Computer Input Array
(Adapted from Ref 5)

A(1)	t_o	A(12)	u_o
A(2)	x_o	A(13)	v_o
A(3)	y_o	A(14)	w_o
A(4)	z_o	A(15)	I_x
A(5)	p_o	A(16)	I_y
A(6)	q_o	A(17)	g
A(7)	r_o	A(18)	m
A(8)	θ	A(19)	d
A(9)	ϕ	A(20)	Δt
A(10)	ψ	A(21)	t_E
A(11)	1.0	A(22)	y_{min}

Appendix C

Computer Output Example

The computer output parameters are shown in Table C-I.

TIME is measured from release.

RANGE, ALT, Z correspond to x, y, z of the inertial reference frame.

V = total velocity of the projectile with respect to the air mass.

p = spin rate.

$\overline{\mathrm{ALPHA}} = \sqrt{\alpha^2+\beta^2}$ (where $-\!-\!=$ denotes vector).

M = mach number.

PHI = roll angle.

ALPHA = angle of attack.

BETA = yaw angle.

The next set of parameters are useful for stability analysis applications. The outputs are based on linearized equations for the tricyclic response of a projectile to an initial disturbance (Ref 12, 8 or 11):

L-N = λ_n = damping factor for nutation mode.

L-P = λ_p = damping factor for precession mode.

W-N = ω_n = nutation frequency.

W-P = ω_p = precession frequency.

S = s = stability factor.

TAU = τ = dynamic weight factor.

K-T = K_t = trim arm.

TABLE C-I

Sample Computer Readout

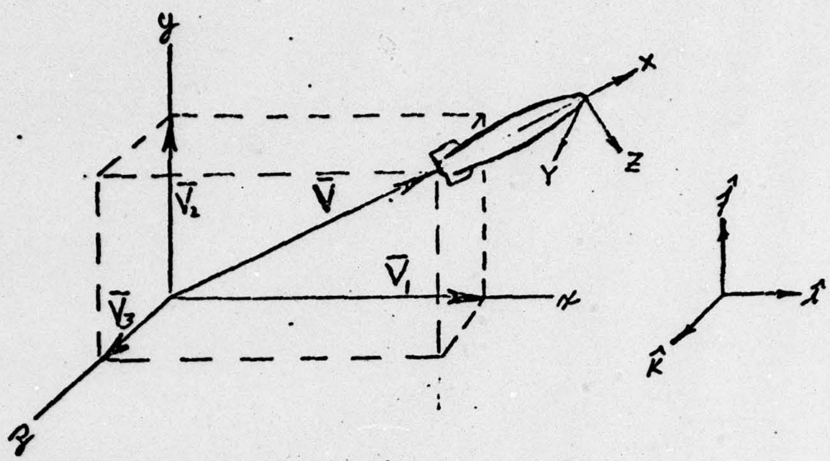
FFSW15 AERO DATA

PAGE 1

TIME SEC	RANGE FEET	ALT FEET	Z FEET	V FT/SEC	P RA/SEC	--* ALPHA DEG	M	PHI DEG	ALPHA DEG	BETA DEG	L-N 1/SEC	L-P 1/SEC	W-N RA/SEC	W-P RA/SEC	S	TAU	K-T DEG
0.0000	0.00	10000.00	0.00	600.02	0.0	.48	.56	90.00	.00	.48	0.00	0.00	0.0	0.0	0.000	0.00	0.000
.1000	59.98	9999.84	.50	599.69	.4	.46	.55	73.61	.14	.44	-.22	-.22	4.0	-4.0	-.000	.00	-22.344
.2000	119.93	9999.36	.99	599.37	.7	.34	.56	104.96	-.07	.34	-.22	-.22	4.0	-4.0	-.000	.00	-2.121
.3000	179.85	9998.55	1.48	599.07	1.0	.50	.56	167.97	-.55	.21	-.22	-.22	4.0	-4.0	-.000	.01	-2.203
.4000	239.73	9997.42	1.97	598.79	1.4	1.25	.56	190.57	-1.25	.12	-.22	-.22	4.0	-4.0	-.000	.01	-2.327
.5000	299.57	9995.96	2.46	598.52	1.7	2.01	.56	201.95	-2.01	.10	-.22	-.23	4.0	-4.0	-.000	.01	-2.508
.6000	359.38	9994.16	2.94	598.27	2.1	2.71	.56	211.11	-2.71	.22	-.23	-.23	4.0	-4.0	-.000	.01	-2.779
.7000	419.16	9992.01	3.42	598.03	2.4	3.23	.56	219.72	-3.19	.51	-.23	-.23	4.0	-4.0	-.000	.01	-3.186
.8000	478.89	9989.51	3.90	597.80	2.8	3.49	.55	227.78	-3.36	.96	-.23	-.23	4.1	-3.9	-.000	.01	-3.788
.9000	538.59	9986.67	4.36	597.59	3.1	3.47	.55	234.47	-3.12	1.53	-.23	-.23	4.1	-3.9	-.000	.02	-4.703
1.0000	598.26	9983.48	4.82	597.40	3.4	3.25	.55	239.08	-2.45	2.13	-.23	-.23	4.1	-3.9	-.000	.02	-6.268
1.1000	657.88	9979.94	5.25	597.23	3.7	3.00	.55	236.52	-1.37	2.67	-.23	-.23	4.1	-3.9	-.000	.02	-9.551
1.2000	717.47	9975.07	5.56	597.09	4.0	3.02	.55	230.61	.03	3.02	-.23	-.23	4.1	-3.9	-.000	.02	-16.974
1.3000	777.02	9971.89	6.05	596.95	4.3	3.47	.55	227.03	1.62	3.07	-.23	-.23	4.1	-3.9	-.000	.02	-13.187
1.4000	836.54	9967.39	6.41	596.83	4.6	4.19	.55	230.49	3.20	2.70	-.23	-.23	4.1	-3.9	-.001	.02	-7.012
1.5000	896.03	9962.61	6.74	596.71	4.9	4.88	.55	239.96	4.51	1.88	-.23	-.24	4.1	-3.9	-.001	.02	-4.560
1.6000	955.48	9957.55	7.05	596.60	5.1	5.31	.55	253.10	5.27	.66	-.23	-.24	4.2	-4.0	-.001	.02	-3.500
1.7000	1014.89	9952.21	7.36	596.49	5.4	5.34	.55	267.52	5.28	-.78	-.23	-.24	4.2	-4.0	-.001	.03	-2.908
1.8000	1074.27	9946.61	7.67	596.41	5.7	4.99	.55	280.09	4.42	-2.31	-.23	-.24	4.1	-3.9	-.001	.03	-2.236
1.9000	1133.61	9940.72	8.00	596.34	6.1	4.55	.55	287.52	2.67	-3.68	-.23	-.24	4.1	-3.9	-.001	.03	-1.676
2.0000	1192.92	9934.54	8.37	596.30	6.4	4.50	.55	290.28	.23	-4.49	-.23	-.24	4.1	-3.9	-.001	.03	-1.369
2.1000	1252.20	9928.03	8.78	596.27	6.8	5.07	.55	295.73	-2.52	-4.41	-.23	-.24	4.2	-3.9	-.001	.03	-1.136
2.2000	1311.43	9921.19	9.22	596.25	7.2	5.95	.55	308.84	-4.98	-3.26	-.23	-.24	4.2	-3.9	-.001	.03	-.997
2.3000	1370.61	9913.99	9.69	596.23	7.4	6.53	.55	328.53	-6.51	-1.24	-.23	-.24	4.2	-3.9	-.001	.04	-.891
2.4000	1429.75	9906.41	10.17	596.21	7.7	6.71	.55	352.77	-6.63	.97	-.23	-.24	4.2	-3.9	-.001	.04	-.824
2.5000	1488.83	9898.46	10.65	596.23	8.0	6.05	.55	18.03	-5.34	2.84	-.23	-.24	4.2	-3.9	-.001	.04	-.733
2.6000	1547.89	9890.15	11.10	596.28	8.4	4.98	.55	39.19	-2.95	4.02	-.23	-.24	4.2	-3.8	-.002	.04	-.655
2.7000	1606.85	9881.50	11.51	596.37	8.5	4.19	.55	53.71	-.15	4.19	-.23	-.24	4.2	-3.8	-.002	.04	-.590
2.8000	1665.84	9872.53	11.89	596.48	8.9	4.14	.55	68.25	2.30	3.45	-.23	-.24	4.2	-3.8	-.002	.04	-.552
2.9000	1724.77	9863.26	12.23	596.61	9.1	4.52	.55	92.97	3.93	2.23	-.23	-.24	4.2	-3.8	-.002	.04	-.515
3.0000	1783.67	9853.72	12.55	596.74	9.4	4.81	.55	127.78	4.72	.93	-.23	-.24	4.2	-3.8	-.002	.05	-.472
3.1000	1842.53	9843.89	12.87	596.89	9.7	4.87	.55	168.95	4.86	-.23	-.23	-.24	4.2	-3.8	-.002	.05	-.444
3.2000	1901.37	9833.80	13.19	597.05	10.0	4.66	.55	212.98	4.50	-1.21	-.23	-.24	4.2	-3.8	-.002	.05	-.412
3.3000	1960.17	9823.42	13.52	597.23	10.3	4.23	.55	255.08	3.62	-2.18	-.23	-.24	4.2	-3.8	-.002	.05	-.369
3.4000	2018.95	9812.77	13.86	597.44	10.5	3.65	.55	291.17	2.12	-2.97	-.22	-.24	4.2	-3.8	-.003	.05	-.365
3.5000	2077.69	9801.92	14.24	597.66	10.8	3.26	.55	317.67	.04	-3.26	-.22	-.24	4.2	-3.8	-.003	.05	-.343

APPENDIX D

ROTATIONAL PARAMETER TRANSFORMATION (REF 10)



$x\,y\,z \equiv$ INERTIAL FRAME

$\hat{i}\,\hat{j}\,\hat{k} \equiv$ INERTIAL UNIT VECTORS

$X\,Y\,Z \equiv$ BODY FIXED FRAME

THE UNIT VECTOR $\bar{V}$ IS MADE UP OF THE THREE COMPONENTS $\bar{V}_1$ $\bar{V}_2$ $\bar{V}_3$ SO THAT

$$\bar{V} = \bar{V}_1\,\hat{i} + \bar{V}_2\,\hat{j} + \bar{V}_3\,\hat{k} \qquad (1)D$$

NOW DEFINE

$$\begin{aligned} \bar{V}_1 &= \lambda_1 / \sin\alpha/2 \\ \bar{V}_2 &= \lambda_2 / \sin\alpha/2 \\ \bar{V}_3 &= \lambda_3 / \sin\alpha/2 \end{aligned} \qquad (2)D$$

WHERE λ_1 λ_2 λ_3 AND α ARE UNKNOWN PARAMETERS.

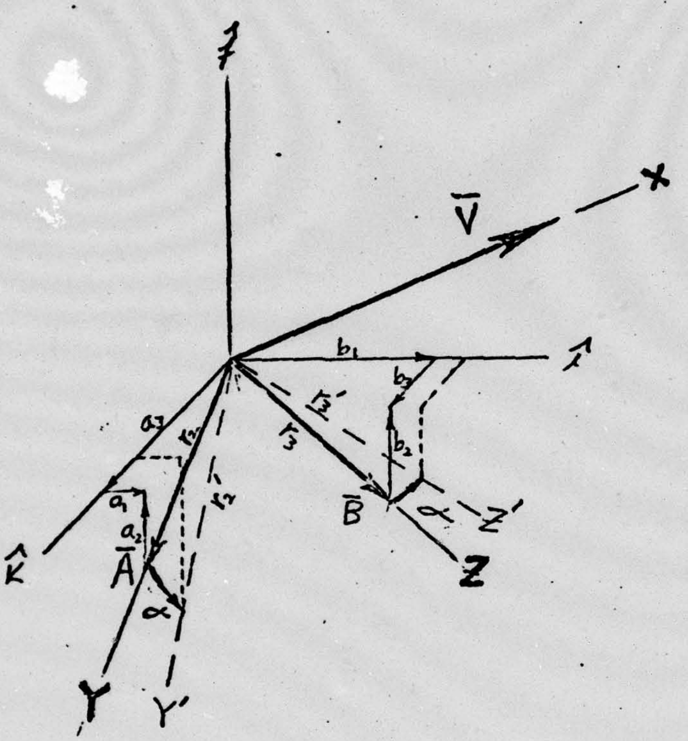
USING THE RELATION FOR DIRECTION COSINES THAT THE SUM OF THE SQUARES OF THE COMPONENTS OF A UNIT VECTOR MUST EQUAL ONE GIVES

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = \sin^2\alpha/2 \qquad (3)D$$

NOW DEFINE ANOTHER PARAMETER $\lambda_0^2 = \cos^2\alpha/2$. SO THE EXPRESSION CAN BE WRITTEN AS

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_0^2 = 1 \qquad (4)D$$

THE BODY FIXED FRAME CAN BE RESOLVED INTO INERTIAL COMPONENTS AS FOLLOWS:



LET $\bar{A}$ BE A UNIT VECTOR ALONG OY AND $\bar{B}$ ALONG OZ SO THAT

$$\bar{A} = a_1 \hat{\imath} + a_2 \hat{\jmath} + a_3 \hat{k} \qquad (5)D$$
$$\bar{B} = b_1 \hat{\imath} + b_2 \hat{\jmath} + b_3 \hat{k}$$

WHERE $a_1\, a_2\, a_3$, $b_1\, b_2\, b_3$ ARE INERTIAL FRAME COMPONENTS. ALSO LET α BE THE AMOUNT OF ROTATION OF THE PROJECTILE ABOUT THE OX AXIS WHERE r IS A VECTOR FIXED ON THE BODY BEFORE ROTATION AND r' IS THE POSITION AFTER ROTATION. THE DIRECTION COSINE MATRIX FOR

THIS ROTATION IS

$$\begin{bmatrix} r_1' \\ r_2' \\ r_3' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\alpha & -S\alpha \\ 0 & S\alpha & C\alpha \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \qquad (6)_D$$

OR

$$\begin{aligned} r_2' &= r_2 C\alpha - r_3 S\alpha \\ r_3' &= r_2 S\alpha + r_3 C\alpha \end{aligned} \qquad (7)_D$$

WHERE $r_1\ r_2\ r_3$ ARE COMPONENTS OF r;
$C \equiv COS$, $S \equiv SIN$;
NEGATIVE ROTATION ABOUT X WAS USED SO EQN 12 SIMPLIFIES.

EXPRESSING r's COMPONENTS IN THE INERTIAL FRAME:

$$\begin{aligned} r_1\bar{V} &= \frac{\lambda_1}{SIN\,\alpha/2}\hat{\imath} + \frac{\lambda_2}{SIN\,\alpha/2}\hat{\jmath} + \frac{\lambda_3}{SIN\,\alpha/2}\hat{k} \\ r_2\bar{A} &= a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k} \\ r_3\bar{B} &= b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k} \end{aligned} \qquad (8)_D$$

COMBINING EQN 7 & 8 IN TERMS OF THE BODY FRAME:

$$\begin{aligned} r_1'\bar{V} &= \frac{\lambda_1}{SIN\,\alpha/2}\hat{\imath} + \frac{\lambda_2}{SIN\,\alpha/2}\hat{\jmath} + \frac{\lambda_3}{SIN\,\alpha/2}\hat{k} \\ r_2'\bar{A} &= (a_1 \cos\alpha - b_1 SIN\alpha)\hat{\imath} + (a_2 COS\alpha - b_2 SIN\alpha)\hat{\jmath} + (a_3 COS\alpha - b_3 SIN\alpha)\hat{k} \\ r_3'\bar{B} &= (a_1 SIN\alpha - b_1 \cos\alpha)\hat{\imath} + (a_2 SIN\alpha + b_2 COS\alpha)\hat{\jmath} + (a_3 SIN\alpha + b_3 COS\alpha)\hat{k} \end{aligned} \qquad (9)_D$$

DENOTING THE TRANSFORMATION MATRIX BY [C], THE EXPRESSION THAT TRANSFORMS THE COMPONENTS OF THE MOVING BODY AXES SYSTEM TO INERTIAL REFERENCE IS:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \frac{\lambda_1}{\sin\alpha/2} & a_1 & b_1 \\ \frac{\lambda_2}{\sin\alpha/2} & a_2 & b_2 \\ \frac{\lambda_3}{\sin\alpha/2} & a_3 & b_3 \end{bmatrix} = \begin{bmatrix} \frac{\lambda_1}{\sin\alpha/2}, & a_1 C\alpha - b_1 S\alpha, & a_1 S\alpha + b_1 C\alpha \\ \frac{\lambda_2}{\sin\alpha/2}, & a_2 C\alpha - b_2 S\alpha, & a_2 S\alpha + b_2 C\alpha \\ \frac{\lambda_3}{\sin\alpha/2}, & a_3 C\alpha - b_3 S\alpha, & a_3 S\alpha + b_3 C\alpha \end{bmatrix}$$

EQN (10)D

POST MULTIPLYING BY THE INVERSE (TRANSPOSE FOR THE ORTHOGONAL SYSTEM):

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} \frac{\lambda_1}{\sin\alpha/2}, & a_1 C\alpha - b_1 S\alpha, & a_1 S\alpha + b_1 C\alpha \\ \frac{\lambda_2}{\sin\alpha/2}, & a_2 C\alpha - b_2 S\alpha, & a_2 S\alpha + b_2 C\alpha \\ \frac{\lambda_3}{\sin\alpha/2}, & a_3 C\alpha - b_3 S\alpha, & a_3 S\alpha + b_3 C\alpha \end{bmatrix} \begin{bmatrix} \frac{\lambda_1}{\sin\alpha/2}, & \frac{\lambda_2}{\sin\alpha/2}, & \frac{\lambda_3}{\sin\alpha/2} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

EQN(11)D

MULTIPLYING IT OUT:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} \frac{\lambda_1^2}{S^2\alpha/2} + (a_1^2 + b_1^2) C\alpha, \; \frac{\lambda_1\lambda_2}{S^2\alpha/2} + (a_1 b_2 - a_2 b_1) S\alpha + (a_1 a_2 + b_1 b_2) C\alpha, \\ \frac{\lambda_1\lambda_3}{S^2\alpha/2} + (a_1 b_3 - b_1 a_3) S\alpha + (b_1 b_3 + a_3 a_1) C\alpha \\ \frac{\lambda_1\lambda_2}{S^2\alpha/2} + (a_1 a_2 + b_1 b_2) C\alpha + (b_1 a_2 - a_1 b_2) S\alpha, \; \frac{\lambda_2^2}{S^2\alpha/2} + (a_2^2 + b_2^2) C\alpha, \\ \frac{\lambda_2\lambda_3}{S^2\alpha/2} + (b_3 a_2 - b_2 a_3) S\alpha + (a_2 a_3 + b_2 b_3) C\alpha \\ \frac{\lambda_1\lambda_3}{S^2\alpha/2} + (a_1 a_3 + b_1 b_3) C\alpha + (b_1 a_3 - a_1 b_3) S\alpha, \\ \frac{\lambda_3\lambda_2}{S^2\alpha/2} + (a_3 a_2 + b_3 b_2) C\alpha + (b_2 a_3 - b_3 a_2) S\alpha, \; \frac{\lambda_3^2}{S^2\alpha/2} + (a_3^2 + b_3^2) C\alpha \end{bmatrix}$$

EQN(12)D

THE FOLLOWING RELATIONSHIPS BETWEEN DIRECTION COSINES APPLY TO AN ORTHOGONAL, RIGHT-HAND SYSTEM:

$$a_1^2 + a_2^2 + a_3^2 = 1 \qquad (13)_D$$

$$b_1^2 + b_2^2 + b_3^2 = 1 \qquad (14)_D$$

$$a_1\lambda_1 + a_2\lambda_2 + a_3\lambda_3 = 0 \qquad (15)_D$$

$$b_1\lambda_1 + b_2\lambda_2 + b_3\lambda_3 = 0 \qquad (16)_D$$

$$a_1 b_1 + a_2 b_2 + a_3 b_3 = 0 \qquad (17)_D$$

$$b_1 = \frac{1}{S\alpha/2}(\lambda_2 a_3 - \lambda_3 a_2) \qquad (18)_D$$

$$b_2 = \frac{1}{S\alpha/2}(\lambda_3 a_1 - \lambda_1 a_3) \qquad (19)_D$$

$$b_3 = \frac{1}{S\alpha/2}(\lambda_1 a_2 - \lambda_2 a_1) \qquad (20)_D$$

$$\begin{vmatrix} \frac{\lambda_1}{S\alpha/2} & \frac{\lambda_2}{S\alpha/2} & \frac{\lambda_3}{S\alpha/2} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 1 \qquad (21)_D$$

EQUATIONS (15) AND (16) CAN BE WRITTEN AS:

$$a_2 = -\frac{1}{\lambda_2}(a_1\lambda_1 + a_3\lambda_3) \qquad (22)_D$$

$$b_2 = -\frac{1}{\lambda_2}(b_1\lambda_1 + b_3\lambda_3) \qquad (23)_D$$

SUBSTITUTING (22) & (23) INTO (17)

$$a_1 b_1 + \frac{1}{\lambda_2^2}(a_1\lambda_1 + a_3\lambda_3)(b_1\lambda_1 + b_3\lambda_3) + a_3 b_3 = 0$$

$$(24)_D$$

$$a_1 b_1(\lambda_1^2 + \lambda_2^2) + a_3 b_3(\lambda_2^2 + \lambda_3^2) + (a_3 b_1 + b_3 a_1)\lambda_1\lambda_3 = 0$$

SUBSTITUTING (22) INTO (13) AND (23) INTO (14)

$$a_1^2(\lambda_1^2+\lambda_2^2)+a_3^2(\lambda_2^2+\lambda_3^2)=\lambda_2^2-2\lambda_1\lambda_3\,a_1a_3 \qquad (25)_D$$

$$b_1^2(\lambda_1^2+\lambda_2^2)+b_3^2(\lambda_2^2+\lambda_3^2)=\lambda_2^2-2\lambda_1\lambda_3\,b_1b_3 \qquad (26)_D$$

EQUATIONS (22),(23) INTO (21)

$$-\frac{\lambda_1 b_3}{\lambda_2}(a_1\lambda_1+\cancel{a_3\lambda_3})+\lambda_2 b_1 a_3-\frac{\lambda_3 a_1}{\lambda_2}(\cancel{b_1\lambda_1}+b_3\lambda_3)$$

$$+\frac{\lambda_3 b_1}{\lambda_2}(\cancel{a_1\lambda_1}+a_3\lambda_3)-\lambda_2 a_1 b_3+\frac{\lambda_1 a_3}{\lambda_2}(b_1\lambda_1+\cancel{b_3\lambda_3})=S\frac{\alpha}{2}$$

AFTER CANCELLATIONS AND REGROUPING

$$-b_3a_1[\lambda_1^2+\lambda_2^2+\lambda_3^2]+b_1a_3[\lambda_1^2+\lambda_2^2+\lambda_3^2]=\lambda_2 S^{\alpha/2}$$

AND SINCE $\lambda_1^2+\lambda_2^2+\lambda_3^2=1$

$$b_3=\frac{1}{a_1}\left[b_1a_3-\lambda_2/S^{\alpha/2}\right] \qquad (27)_D$$

EQUATION (27) INTO (24)

$$a_1b_1(\lambda_1^2+\lambda_2^2)+\frac{a_3}{a_1}(\lambda_1^2+\lambda_3^2)(b_1a_3-\frac{\lambda_2}{S^{\alpha/2}})+\lambda_1\lambda_3[2a_3b_1-\frac{\lambda_2}{S^{\alpha/2}}]=0$$

$$a_1^2b_1(\lambda_1^2+\lambda_2^2)+b_1a_3^2(\lambda_2^2+\lambda_3^2)+a_3[2\lambda_1\lambda_3\,a_1b_1-\frac{\lambda_2}{S^{\alpha/2}}(\lambda_2^2+\lambda_3^2)]=\frac{a_1}{S^{\alpha/2}}\lambda_1\lambda_2\lambda_3 \qquad (28)_D$$

EQUATION (27) INTO (26)

$$b_1^2(\lambda_1^2+\lambda_2^2)+\frac{1}{a_1^2}[b_1a_3-\frac{\lambda_2}{S^{\alpha/2}}]^2(\lambda_2^2+\lambda_3^2)=\lambda_2^2-2\frac{b_1}{a_1}(b_1a_3-\frac{\lambda_2}{S^{\alpha/2}})\lambda_1\lambda_3$$

$$a_1^2b_1^2(\lambda_1^2+\lambda_2^2)+a_3^2b_1^2(\lambda_2^2+\lambda_3^2)+a_3b_1[2\lambda_1\lambda_3\,a_1b_1-\frac{\lambda_2}{S^{\alpha/2}}(\lambda_2^2+\lambda_3^2)]=$$
$$\lambda_2^2a_1^2+2b_1a_1\frac{\lambda_1\lambda_2\lambda_3}{S^{\alpha/2}}-\frac{\lambda_2^2}{S^{2\alpha/2}}(\lambda_2^2+\lambda_3^2)+a_3b_1\frac{\lambda_2}{S^{\alpha/2}}(\lambda_2^2+\lambda_3^2) \qquad (29)_D$$

LEFT SIDE OF $(28)\times b_1$ = LEFT SIDE OF (29) THEREFORE

$$-\frac{a_1 b_1}{S\alpha/2}\lambda_1\lambda_2\lambda_3 = \lambda_2^2 a_1^2 + 2b_1 a_1 \frac{\lambda_1\lambda_2\lambda_3}{S\alpha/2} + a_3 b_1 \frac{\lambda_2}{S\alpha/2}(\lambda_2^2+\lambda_3^2) - \frac{\lambda_2^2}{S^2\alpha/2}(\lambda_2^2+\lambda_3^2)$$

MULTIPLYING THRU BY $\frac{S\alpha/2}{b_1}$

$$a_3 = \left\{\left[\frac{\lambda_2}{S\alpha/2}(\lambda_2^2+\lambda_3^2) - S\alpha/2\,\lambda_2 a_1^2\right]\frac{1}{b_1} - a_1\lambda_1\lambda_3\right\}\frac{1}{\lambda_2^2+\lambda_3^2} \qquad (30)D$$

EQUATION (18) AND (29)

$$b_1 = \frac{1}{S\alpha/2}\left[\lambda_2 a_3 + \lambda_3\frac{\lambda_1 a_1 + \lambda_3 a_3}{\lambda_2}\right] = \frac{\lambda_1\lambda_3}{S\alpha/2\,\lambda_2}a_1 + \left(\frac{\lambda_2}{S\alpha/2} + \frac{\lambda_3^2}{S\alpha/2\,\lambda_2}\right)a_3 \qquad (31)D$$

EQUATION (30) INTO (33)

$$b_1 = \frac{\lambda_1\lambda_3}{S\alpha/2\,\lambda_2}a_1 + \left(\frac{\lambda_2}{S\alpha/2} + \frac{\lambda_3^2}{S\alpha/2\,\lambda_2}\right)\left(\frac{1}{\lambda_2^2+\lambda_3^2}\right)\left\{\left[\frac{\lambda_2}{S\alpha/2}(\lambda_2^2+\lambda_3^2) - a_1^2\lambda_2 S\alpha/2\right]\frac{1}{b_1} - a_1\lambda_1\lambda_3\right\}$$

$$= \frac{\lambda_1\lambda_3}{S\alpha/2\,\lambda_2}a_1 + \left(\frac{\lambda_2^2+\lambda_3^2}{S\alpha/2\,\lambda_2}\right)\left(\frac{1}{\lambda_2^2+\lambda_3^2}\right)\left\{\left[\frac{\lambda_2}{S\alpha/2}(\lambda_2^2+\lambda_3^2) - (a_1^2\lambda_2 S\alpha/2)\right]\frac{1}{b_1} - \lambda_1\lambda_3 a_1\right\}$$

$$= \cancel{\frac{\lambda_1\lambda_3}{S\alpha/2\,\lambda_2}a_1} + \frac{1}{S\alpha/2\,\lambda_2}\left\{\left[\frac{\lambda_2}{S\alpha/2}(\lambda_2^2+\lambda_3^2) - (a_1^2\lambda_2 S\alpha/2)\right]\frac{1}{b_1} - \cancel{\lambda_1\lambda_3 a_1}\right\}$$

$$b_1 = \frac{1}{S\alpha/2\,\lambda_2}\left[\frac{\lambda_2}{S\alpha/2}(\lambda_2^2+\lambda_3^2) - a_1^2\lambda_2 S\alpha/2\right]\frac{1}{b_1}$$

$$b_1^2 + a_1^2 = \frac{1}{S^2\alpha/2}(\lambda_2^2+\lambda_3^2) \qquad (32)D$$

EQUATION (32) IS USED TO EVALUATE C_{11} IN (12)

$$C_{11} = \frac{\lambda_1^2}{S^2\alpha/2} + (a_1^2+b_1^2)\cos\alpha$$

FROM THE DEFINITIONS $\lambda_0^2 = \cos^2\alpha/2$ AND $\cos 2\alpha = 2\cos^2\alpha - 1$

$$C_{11} = \frac{1}{S^2\alpha/2}\left[\lambda_1^2 + (\lambda_2^2+\lambda_3^2)(2\lambda_0^2-1)\right]$$

$$= \frac{1}{S^2\alpha/2}\left[\lambda_1^2 + (1-\lambda_0^2-\lambda_1^2)(2\lambda_0^2-1)\right]$$

$$= \frac{1}{S^2\alpha/2}\left[\lambda_1^2 + 2\lambda_0^2 - 1 - 2\lambda_0^2 + \lambda_0^2 - 2\lambda_1^2\lambda_0^2 + \lambda_1^2\right]$$

$$= \frac{1}{S^2\alpha/2}\left[2(\lambda_1^2+\lambda_2^2) - 2(\lambda_0^2+\lambda_1^2)\lambda_0^2 + \lambda_0^2 - 1\right]$$

$$= \frac{1}{S^2\alpha/2}\left[2(\lambda_1^2+\lambda_0^2)(1-\lambda_0^2) - (1-\lambda_0^2)\right]$$

$$= \frac{(1-\lambda_0^2)}{S^2\alpha/2}\left[2(\lambda_1^2+\lambda_0^2) - 1\right]$$

USING $S^2\alpha/2 = 1 - C^2\alpha/2 = 1-\lambda_0^2$

$$\underline{C_{11} = 2(\lambda_1^2+\lambda_0^2) - 1} \qquad (33)_D$$

SIMILARLY

$$\underline{C_{22}} = \frac{\lambda_2^2}{S^2\alpha/2} + (a_2^2+b_2^2)\cos\alpha = \underline{2(\lambda_2^2+\lambda_0^2)-1} \qquad (34)_D$$

$$\underline{C_{33}} = \frac{\lambda_3^2}{S^2\alpha/2} + (a_3^2+b_3^2)\cos\alpha = \underline{2(\lambda_3^2+\lambda_0^2)-1} \qquad (35)_D$$

FROM EQUATION (27)

$$b_1a_3 - b_3a_1 = \frac{\lambda_2}{S\alpha/2} \qquad (36)_D$$

BY ANALOGY AND WITH (21)

$$b_2a_1 - b_1a_2 = \frac{\lambda_3}{S\alpha/2} \qquad (37)_D$$

$$b_3a_2 - b_2a_3 = \frac{\lambda_1}{S\alpha/2} \qquad (38)_D$$

EVALUATE $a_1a_3 + b_1b_3$ BY USING (27)

$$a_1a_3 + b_1b_3 = a_1a_3 + b_1\frac{1}{a_1}\left(b_1a_3 - \frac{\lambda_2}{S\alpha/2}\right)$$

$$= \frac{a_3}{a_1}(a_1^2+b_1^2) - \frac{b_1}{a_1}\frac{\lambda_2}{S\alpha/2}$$

SUB EQUATION (32):

$$= \frac{a_3}{a_1}\frac{\lambda_2^2+\lambda_3^2}{S^2\alpha/2} - \frac{b_1}{a_1}\frac{\lambda_2}{S\alpha/2}$$

SUB EQUATION (30):

$$= \frac{1}{S^2\alpha/2\, a_1}\left\{\frac{1}{b_1}\left[\frac{\lambda_2}{S\alpha/2}(\lambda_2^2+\lambda_3^2) - S\alpha/2\,\lambda_2\, a_1^2\right] - a_1\lambda_1\lambda_3\right\} - \frac{b_1}{a_1}\frac{\lambda_2}{S\alpha/2}$$

$$= \frac{\lambda_2}{S^3\alpha/2\, a_1 b_1}(\lambda_2^2+\lambda_3^2) - \frac{\lambda_2\, a_1^2}{S\alpha/2\, b_1 a_1} - \frac{\lambda_1\lambda_3}{S^2\alpha/2} - \frac{b_1^2}{a_1}\,\frac{\lambda_2}{b_1 S\alpha/2}$$

$$= \frac{\lambda_2}{S^3\alpha/2\, a_1 b_1}(\lambda_2^2+\lambda_3^2) - \frac{\lambda_2}{S\alpha/2\, a_1 b_1}(a_1^2+b_1^2) - \frac{\lambda_1\lambda_3}{S^2\alpha/2}$$

SUB EQUATION (32)

$$a_1 a_3 + b_1 b_3 = -\frac{\lambda_1\lambda_3}{S^2\alpha/2} \qquad (39)_D$$

SIMILARLY

$$a_1 a_2 + b_1 b_2 = -\frac{\lambda_1\lambda_2}{S^2\alpha/2} \qquad (40)_D$$

$$a_2 a_3 + b_2 b_3 = -\frac{\lambda_2\lambda_3}{S^2\alpha/2} \qquad (41)_D$$

EVALUATING C_{12} FROM EQN (12) WITH (40) AND (37)

$$C_{12} = \frac{\lambda_1\lambda_2}{S^2\alpha/2} + (a_1a_2+b_1b_2)\cos\alpha + (a_1b_2 - a_2b_1)\sin\alpha$$

$$= \frac{\lambda_1\lambda_2}{S^2\alpha/2} - \frac{\lambda_1\lambda_2}{S^2\alpha/2}(2\lambda_0^2-1) + \frac{\lambda_3}{S\alpha/2}\,2\,S\alpha/2\,\lambda_0$$

WHERE $\cos 2(\alpha/2) = 2(\cos^2\alpha/2) - 1$, $\sin 2(\alpha/2) = 2(\sin\alpha/2)(\cos\alpha/2)$; $\cos\alpha/2 = \lambda_0$

$$= \lambda_1\lambda_2\left[\frac{2\cancel{(1-\lambda_0^2)}}{\cancel{S^2\alpha/2}}\right] + 2\lambda_0\lambda_3$$

$$\underline{C_{12} = 2[\lambda_1\lambda_2 + \lambda_0\lambda_3]} \qquad (42)_D$$

IN THE SAME MANNER, USING EQUATIONS (32) (37) (38) (39) (40) (41), THE

REMAINING COMPONENTS ARE

$$C_{21} = 2\left[\lambda_1\lambda_2 - \lambda_0\lambda_3\right] \quad (43)_D$$

$$C_{13} = 2\left[\lambda_1\lambda_3 - \lambda_0\lambda_2\right] \quad (44)_D$$

$$C_{31} = 2\left[\lambda_1\lambda_3 - \lambda_0\lambda_2\right] \quad (45)_D$$

$$C_{23} = 2\left[\lambda_2\lambda_3 - \lambda_0\lambda_1\right] \quad (46)_D$$

$$C_{32} = 2\left[\lambda_2\lambda_3 - \lambda_0\lambda_1\right] \quad (47)_D$$

SO THAT THE DIRECTION COSINE MATRIX BETWEEN THE INERTIAL AND BODY FIXED REFERENCE FRAMES AFTER ROTATION α IS

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} 2(\lambda_1^2+\lambda_0^2)-1 & 2(\lambda_1\lambda_2+\lambda_0\lambda_3) & 2(\lambda_1\lambda_3-\lambda_0\lambda_2) \\ 2(\lambda_1\lambda_2-\lambda_0\lambda_3) & 2(\lambda_2^2+\lambda_0^2)-1 & 2(\lambda_2\lambda_3+\lambda_0\lambda_1) \\ 2(\lambda_1\lambda_3+\lambda_0\lambda_2) & 2(\lambda_2\lambda_3-\lambda_0\lambda_1) & 2(\lambda_3^2+\lambda_0^2)-1 \end{bmatrix}$$

EQN $(48)_D$

WHICH IS EQUIVALENT TO THE EULER ANGLE FORM OF THE DIRECTION COSINE MATRIX

$$\begin{bmatrix} C\theta\, C\psi & S\theta & S\psi\, C\theta \\ -S\psi\, S\phi - C\psi\, S\theta\, C\phi & C\theta\, C\phi & C\psi\, S\phi - S\psi\, S\theta\, C\phi \\ -S\psi\, C\phi + C\psi\, S\theta\, S\phi & -C\theta\, S\phi & C\psi\, C\phi + S\psi\, S\theta\, S\phi \end{bmatrix}$$

EQN $(49)_D$

COMPARING COEFFICIENTS

$$C_{11} = 2(\lambda_0^2 + \lambda_1^2) - 1 = C\psi C\theta \qquad (50)_D$$

$$C_{22} = 2(\lambda_0^2 + \lambda_2^2) - 1 = C\phi C\theta \qquad (51)_D$$

$$C_{33} = 2(\lambda_0^2 + \lambda_3^2) - 1 = C\psi C\phi + S\psi S\theta S\phi \qquad (52)_D$$

FROM EQ'N (4)

$$\lambda_0^2 = 1 - (\lambda_1^2 + \lambda_2^2 + \lambda_3^2) \qquad (53)_D$$

EQUATION (53) INTO (50)(51)(52)

$$1 - 2(\lambda_2^2 + \lambda_3^2) = C\psi C\theta \qquad (54)_D$$

$$1 - 2(\lambda_1^2 + \lambda_3^2) = C\phi C\theta \qquad (55)_D$$

$$1 - 2(\lambda_2^2 + \lambda_1^2) = C\psi C\phi + S\psi S\theta S\phi \qquad (56)_D$$

EQN (56) − (55)

$$-2(\lambda_2^2 + \lambda_3^2) = C\psi C\phi + S\psi S\theta S\phi - C\phi C\theta \qquad (57)_D$$

EQN (54) + (57)

$$1 - 4\lambda_2^2 = C\psi(C\theta + C\phi) + S\psi S\theta S\psi - C\phi C\theta$$

$$4\lambda_2^2 = 1 - C\psi(C\theta + C\phi) + C\phi C\theta - S\psi S\theta S\phi \qquad (58)_D$$

USING THE IDENTITY $C\theta + C\phi = 2\,C(\frac{\theta+\phi}{2})\,C(\frac{\theta-\phi}{2})$

$$4\lambda_2^2 = 1 - 2C\psi\left[C\frac{\theta+\phi}{2}\,C\frac{\theta-\phi}{2}\right] + C\theta C\phi - S\psi S\theta S\phi$$

USING THE IDENTITY $C2\psi = C^2\psi - S^2\psi$

$$4\lambda_2^2 = 1 - 2\left(C^2\frac{\psi}{2} - S^2\frac{\psi}{2}\right)\left(C^2\frac{\theta}{2}\,C^2\frac{\phi}{2} - S^2\frac{\theta}{2}\,S^2\frac{\phi}{2}\right) + C\phi C\theta - S\psi S\theta S\phi$$

$$4\lambda_2^2 = 1 + 2\left[C^2\frac{\psi}{2}\,S^2\frac{\theta}{2}\,S^2\frac{\phi}{2} + S^2\frac{\psi}{2}\,C^2\frac{\theta}{2}\,C^2\frac{\phi}{2}\right] + C\phi C\theta - S\psi S\theta S\phi +$$
$$-2\left[C^2\frac{\psi}{2}\,C^2\frac{\theta}{2}\,C^2\frac{\phi}{2} + S^2\frac{\psi}{2}\,S^2\frac{\theta}{2}\,S^2\frac{\phi}{2}\right]$$

USING IDENTIES $C2\theta = 2C^2\theta - 1$ & $C2\theta = 1 - 2S^2\theta$

$$4\lambda_2^2 = 2\left[C^2\tfrac{\psi}{2}\,S^2\tfrac{\theta}{2}\,S^2\tfrac{\phi}{2} + S^2\tfrac{\psi}{2}\,C^2\tfrac{\theta}{2}\,C^2\tfrac{\phi}{2}\right] + 1 +$$
$$-\frac{2}{8}\Big[(C\psi+1)(C\theta+1)(C\phi+1) + (1-C\psi)(1-C\theta)(1-C\phi)\Big] +$$
$$+ C\phi C\theta - S\psi S\theta S\phi$$

$$4\lambda_2^2 = 2\left[C^2\tfrac{\psi}{2}\,S^2\tfrac{\theta}{2}\,S^2\tfrac{\phi}{2} + S^2\tfrac{\psi}{2}\,C^2\tfrac{\theta}{2}\,C^2\tfrac{\phi}{2}\right] + 1 +$$
$$-\frac{1}{2}\Big[C\psi C\theta + C\psi C\phi + C\theta C\psi + 1\Big] +$$
$$+ C\phi C\theta - S\psi S\theta S\phi$$

$$4\lambda_2^2 = 2\left[C^2\tfrac{\psi}{2}\,S^2\tfrac{\theta}{2}\,S^2\tfrac{\psi}{2} + S^2\tfrac{\psi}{2}\,C^2\tfrac{\theta}{2}\,C^2\tfrac{\psi}{2}\right] +$$
$$+\frac{1}{2}\Big[1 - C\psi(C\theta + C\phi) + C\phi C\theta - S\psi S\theta S\phi\Big]$$
$$-\frac{S\psi S\theta S\phi}{2}$$

$$4\lambda_2^2 = 2\left[C^2\tfrac{\psi}{2}\,S^2\tfrac{\theta}{2}\,S^2\tfrac{\phi}{2} + S^2\tfrac{\psi}{2}\,C^2\tfrac{\theta}{2}\,C^2\tfrac{\phi}{2}\right] + 2\lambda_2^2 - \frac{1}{2}S\psi S\theta S\phi$$

USING THE IDENTITY $S2\theta = 2S\theta C\theta$

$$\lambda_2^2 = C^2\tfrac{\psi}{2}\,S^2\tfrac{\theta}{2}\,S^2\tfrac{\phi}{2} + S^2\tfrac{\psi}{2}\,C^2\tfrac{\theta}{2}\,C^2\tfrac{\phi}{2} - 2\,C\tfrac{\psi}{2}\,S\tfrac{\psi}{2}\,C\tfrac{\theta}{2}\,S\tfrac{\theta}{2}\,C\tfrac{\phi}{2}\,S\tfrac{\phi}{2}$$

WHICH FACTORS INTO

$$\lambda_2^2 = \left(C\tfrac{\psi}{2}\,S\tfrac{\theta}{2}\,S\tfrac{\phi}{2} - S\tfrac{\psi}{2}\,C\tfrac{\theta}{2}\,C\tfrac{\phi}{2}\right)^2$$

$$\underline{\underline{\lambda_2 = C\tfrac{\psi}{2}\,S\tfrac{\theta}{2}\,S\tfrac{\phi}{2} - S\tfrac{\psi}{2}\,C\tfrac{\theta}{2}\,C\tfrac{\phi}{2}}} \qquad (59)_D$$

BY ANALOGY

$$\underline{\underline{\lambda_1 = -C\tfrac{\phi}{2}\,S\tfrac{\theta}{2}\,S\tfrac{\psi}{2} + S\tfrac{\phi}{2}\,C\tfrac{\theta}{2}\,C\tfrac{\psi}{2}}} \qquad (60)_D$$

EQN (57) - (59)

$$4\lambda_3^2 - 1 = C\psi C\phi + S\psi S\theta S\phi - C\phi C\theta - C\psi C\theta$$

$$4\lambda_3^2 = 1 - C\theta(C\psi + C\phi) + C\phi C\psi + S\psi S\theta S\phi$$

THE SOLN TO THIS EQN IS SIMILIAR TO EQN 58:

$$\underline{\underline{\lambda_3 = C\tfrac{\theta}{2} S\tfrac{\psi}{2} S\tfrac{\phi}{2} + S\tfrac{\theta}{2} C\tfrac{\psi}{2} C\tfrac{\phi}{2}}} \qquad (61)_D$$

EQN (59) INTO (53)

$$\lambda_0^2 = 1 - [\lambda_1^2 + \tfrac{1}{2}(1 - C\psi C\theta)] = \tfrac{1}{2}(1 + C\psi C\theta) - \lambda_1^2$$

$$\lambda_0^2 = \tfrac{1}{2} + \tfrac{1}{2} C\psi C\theta - C^2\tfrac{\phi}{2} S^2\tfrac{\theta}{2} S^2\tfrac{\psi}{2} - S\tfrac{\phi}{2} C\tfrac{\psi}{2} C\tfrac{\theta}{2} + {}$$
$$+ 2C\tfrac{\phi}{2} S\tfrac{\phi}{2} C\tfrac{\psi}{2} S\tfrac{\psi}{2} C\tfrac{\theta}{2} S\tfrac{\theta}{2}$$

$$\lambda_0^2 = \tfrac{1}{2}(1 + C\psi C\theta) - (1 - S^2\tfrac{\phi}{2}) S^2\tfrac{\phi}{2} S^2\tfrac{\psi}{2} - S^2\tfrac{\phi}{2} C^2\tfrac{\psi}{2} C^2\tfrac{\theta}{2} + {}$$
$$+ 2C\tfrac{\phi}{2} S\tfrac{\phi}{2} C\tfrac{\psi}{2} S\tfrac{\psi}{2} C\tfrac{\theta}{2} S\tfrac{\theta}{2}$$

$$\lambda_0^2 = \tfrac{1}{2}\left[1 + (2C^2\tfrac{\psi}{2} - 1)(2C^2\tfrac{\theta}{2} - 1)\right] + S^2\tfrac{\phi}{2} S^2\tfrac{\theta}{2} S^2\tfrac{\psi}{2} + {}$$
$$- S^2\tfrac{\theta}{2} S^2\tfrac{\psi}{2} - S^2\tfrac{\phi}{2} C^2\tfrac{\psi}{2} C^2\tfrac{\theta}{2} + 2C\tfrac{\phi}{2} S\tfrac{\phi}{2} C\tfrac{\psi}{2} S\tfrac{\psi}{2} C\tfrac{\theta}{2} S\tfrac{\theta}{2}$$

$$\lambda_0^2 = 1 + C^2\tfrac{\psi}{2} C^2\tfrac{\theta}{2}(1 - S^2\tfrac{\phi}{2}) + S^2\tfrac{\phi}{2} S^2\tfrac{\theta}{2} S^2\tfrac{\psi}{2} + C^2\tfrac{\psi}{2} C^2\tfrac{\theta}{2} + {}$$
$$- C^2\tfrac{\psi}{2} - C^2\tfrac{\theta}{2} - S^2\tfrac{\theta}{2} S^2\tfrac{\psi}{2} + 2C\tfrac{\phi}{2} S\tfrac{\phi}{2} C\tfrac{\psi}{2} S\tfrac{\psi}{2} C\tfrac{\theta}{2} S\tfrac{\theta}{2}$$

NOTE THAT

$$1 + C^2\tfrac{\psi}{2} C^2\tfrac{\theta}{2} - C^2\tfrac{\psi}{2} - C^2\tfrac{\theta}{2} - S^2\tfrac{\theta}{2} S^2\tfrac{\psi}{2} =$$
$$1 + C^2\tfrac{\theta}{2}(-S^2\tfrac{\psi}{2}) - C^2\tfrac{\psi}{2} - S^2\tfrac{\theta}{2} S^2\tfrac{\psi}{2} =$$
$$1 - S^2\tfrac{\psi}{2} - C^2\tfrac{\psi}{2} = 0$$

THEREFORE

$$\lambda_0^2 = C^2\tfrac{\psi}{2} C^2\tfrac{\theta}{2} C^2\tfrac{\phi}{2} + S^2\tfrac{\phi}{2} S^2\tfrac{\theta}{2} S^2\tfrac{\psi}{2} + 2C\tfrac{\phi}{2} S\tfrac{\phi}{2} C\tfrac{\psi}{2} S\tfrac{\psi}{2} C\tfrac{\theta}{2} S\tfrac{\theta}{2}$$

$$\underline{\underline{\lambda_0 = C\tfrac{\psi}{2} C\tfrac{\phi}{2} C\tfrac{\theta}{2} + S\tfrac{\psi}{2} S\tfrac{\phi}{2} S\tfrac{\theta}{2}}} \qquad (62)_D$$

THE SET OF ROTATIONAL PARAMETERS ARE

$$\boxed{\begin{aligned}
\lambda_0 &= C\tfrac{\psi}{2}\,C\tfrac{\phi}{2}\,C\tfrac{\theta}{2} + S\tfrac{\psi}{2}\,S\tfrac{\phi}{2}\,S\tfrac{\theta}{2} \\
\lambda_1 &= -C\tfrac{\phi}{2}\,S\tfrac{\theta}{2}\,S\tfrac{\psi}{2} + S\tfrac{\phi}{2}\,C\tfrac{\theta}{2}\,C\tfrac{\psi}{2} \\
\lambda_2 &= C\tfrac{\psi}{2}\,S\tfrac{\theta}{2}\,S\tfrac{\phi}{2} - S\tfrac{\psi}{2}\,C\tfrac{\theta}{2}\,C\tfrac{\phi}{2} \\
\lambda_3 &= C\tfrac{\theta}{2}\,S\tfrac{\psi}{2}\,S\tfrac{\phi}{2} + S\tfrac{\theta}{2}\,C\tfrac{\psi}{2}\,C\tfrac{\phi}{2}
\end{aligned}}$$

Appendix E

Coriolis

From the kinematical expression for motion in terms of moving reference frames (Ref 7:111):

$$\ddot{r}^{i} = \ddot{r}^{e} + 2\omega^{ei} \times \dot{r}^{e} + \dot{\omega} \times r^{e} + \omega^{ei} \times (\omega^{ei} \times r^{e}) \qquad (1)E$$

where the superscripts denote the applicable frame of reference.

The term $2\,\omega^{ei} \times \dot{r}$ is the coriolis acceleration and, for the application of a projectile moving with a velocity (V) over the earth rotating at rate (Ω), the coriolis expression becomes

$$2\,\Omega \times V \qquad (2)E$$

Define the body frame (XYZ) moving globally with respect to a north-east-down reference frame (NED):

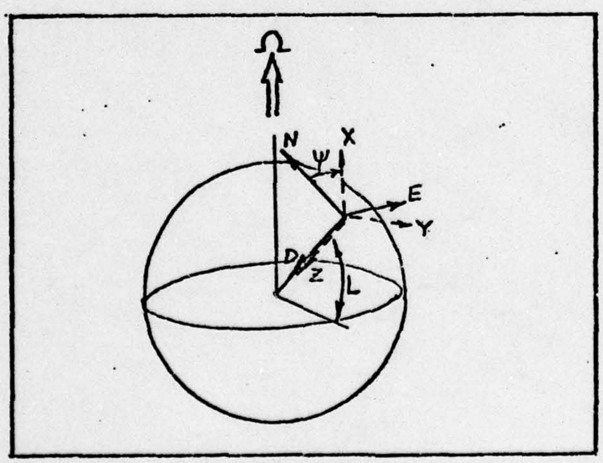


Fig. E-1. Local Level Reference Frame

where ψ is the heading with respect to north and L denotes latitude.

The transformation between the body frame and north frame is

$$C_N^b = \begin{bmatrix} \cos\Psi & \sin\Psi & 0 \\ -\sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3E}$$

The earth rate component as seen by the body frame is then

$$\Omega^b = C_N^b \Omega^N = \begin{bmatrix} \cos\Psi & \sin\Psi & 0 \\ -\sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Omega \cos L \\ 0 \\ -\Omega \sin L \end{bmatrix}$$

$$= \begin{bmatrix} \Omega \cos L \cos\Psi \\ -\Omega \cos L \sin\Psi \\ -\Omega \sin L \end{bmatrix} \tag{4E}$$

so that the coriolis expression becomes

$$2\Omega \times V = 2 \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \Omega CL\, C\Psi & -\Omega CL S\Psi & -\Omega SL \\ V_N & V_E & V_D \end{bmatrix} \tag{5E}$$

where V_N, V_E, V_D are the N, E, D velocity components. The scalar components of coriolis acceleration then, are

$$\begin{aligned} (2\Omega\times V)_x &= -2V_D \Omega\, CL S\Psi + 2V_E \Omega\, SL \\ (2\Omega\times V)_y &= -2V_D \Omega\, CL\, C\Psi - 2V_N \Omega\, SL \\ (2\Omega\times V)_z &= 2V_E \Omega\, CL\, C\Psi + 2V_N \Omega\, CL S\Psi \end{aligned} \tag{6E}$$

AD-A048 966 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OHIO SCH--ETC F/G 19/4
MAGNUS EFFECTS ON BALLISTIC TRAJECTORIES.(U)
DEC 77 J D SCHNEIDER
UNCLASSIFIED AFIT/GA/AA-77D-8 NL

2 of 2
ADA048 966

END
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Appendix F

Six Degree of Freedom Computer Program

```
PROGRAM SIXDOF      74/74    OPT=1                              FTN 4.5+414        09/12/77  17.09.00

              PROGRAM SIXDOF (INPUT,OUTPUT,TAPE5=INPUT)
              EXTERNAL ARDCHA
              CALL CFRO
              CALL INIT6
              CALL SIXDEG (ARDCHA)
              STOP
              END
```

```
SUBROUTINE ARDCMA      74/74    OPT=1                        FTN 4.5+414       09/12/77  17.09.

          SUBROUTINE ARDCMA(H)
    C NOBBLE 250   R. S. EIKENBERRY, AERO-SPACE ENGR.                    WBL01256
    C              UNIVERSITY OF NOTRE DAME, NOTRE DAME, IND.            WBL01257
    C                                                                    WBL01258
    C ARDC 1959 MODEL ATMOSPHERE SUBROUTINE                               WBL01259
    C                                                                    WBL01260
          COMMON /AIRBLK/ T,P,R,VK,VS                                    WBL01261
          DIMENSION A(3),B(3),C(3),D(3)                                  WBL01262
          DIMENSION HB(12),W1(11),W2(11),W3(11),TB(11),PB(11),RB(11)     WBL01263
    C                                                                    WBL01264
          DATA HB/0.,11000.,25000.,47000.,53000.,79000.,90000.,105000.,16000
         $0.,170000.,200000.,700000./
          DATA W1/-.225569E-4,0.,.138466E-4,0.,-.159202E-4,0.,.241459E-4,
         $.895289E-4,.754341E-5,.350716E-5,.222129E-5/
          DATA W2/-.525612E1,0.,.113887E2,.0,-.759218E1,0.,.854120E1,
         $.170924E1,.341643E1,.687236E1,.976137E1/
          DATA W3/0.,.157689E-3,0.,.120969E-3,0.,.206234E-3,0.,0.,0.,0.,0./
          DATA TB/.518688E3,.389988E3,.389988E3,.508788E3,.508788E3,
         $.298188E3,.298188E3,.405188E3,.238618E4,.256618E4,.293618E4/
          DATA PB/.2116217E4,.47273E3,.51979E2,.25155E1,.12181E1,.2108E-1,
         $.21809E-2,.15562E-3,.7578E-5,.58954E-5,.29759E-5/
          DATA RB/.237692E-2,.7062E-3,.7765E-4,.28304E-5,.139468E-5,
         $.41189E-7,.4261E-8,.2232E-9,.1845E-11,.1338E-11,.6113E-12/
          DATA CON1,CON2,CON3,CON4,CON5,CON6/.3048,5356766.,49.020576,0.0226WBL01278
         $388E-05,198.72,90000./                                         WBL01279
          DATA A/1.,.759511,.335787/
          DATA B/0.,.174164,.273966/
          DATA C/0.,.22E6,.18E6/
          DATA D/0.,.25E5,.14E6/
    C                                                                    WBL01282
          HGP=CON1*H/(1.+(CON1*H/CON2))                                  WBL01283
          IF(HGP.LT.0.) HGP=0.                                           WBL01284
          DO 1002 M=1,11                                                 WBL01285
          IF(HGP-HB(M))1003,1004,1002                                    WBL01286
     1002 CONTINUE                                                       WBL01287
          IF ((HGP-HB(12)).GT.0.) GO TO 1052                             WBL01288
          M=12                                                           WBL01289
     1003 M=M-1                                                          WBL01290
     1004 TM=TB(M)*(1.+W1(M)*(HGP-HB(M)))                                WBL01291
          IF ((HGP-90000.).GT.0.) GO TO 1006                             WBL01292
          T=TM                                                           WBL01293
          GO TO 1070                                                     WBL01294
     1006 IF ((HGP-180000.).GT.0.) GO TO 1009                            WBL01295
          I=2                                                            WBL01296
          GO TO 1007                                                     WBL01297
     1009 I=3                                                            WBL01298
     1007 T=TM*(A(I)-B(I)*ATAN((HGP-C(I))/D(I)))                         WBL01299

     1070 IF (W2(M))1011,1020,1011                                       WBL01300
     1011 TEMP=1.+W1(M)*(HGP-HB(M))                                      WBL01301
          P=PB(M)/TEMP**W2(M)                                            WBL01302
          R=RB(M)/TEMP**(1.+W2(M))                                       WBL01303
          GO TO 1030                                                     WBL01304
     1020 TEMP=EXP(-W3(M)*(HGP-HB(M)))                                   WBL01305
          P=PB(M)*TEMP                                                   WBL01306
          R=RB(M)*TEMP                                                   WBL01307
     1030 IF ((HGP-CON6).GT.0.) GO TO 1032                               WBL01308
          VS=CON3*SQRT(TM)                                               WBL01309
          VK=CON4*(T**1.5/((T+CON5)*P))                                  WBL01310
          RETURN                                                         WBL01311
     1052 T=0.                                                           WBL01312
          P=0.                                                           WBL01313
          R=0.                                                           WBL01314
     1032 VS=0.                                                          WBL01315
          VK=0.                                                          WBL01316
          RETURN                                                         WBL01317
          END                                                            WBL01318
```

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```
SUBROUTINE CFRD        74/74    OPT=1                                FTN 4.5+414        09/12/77  17.09.

      SUBROUTINE CFRD                                                   00000070
C                                                                       00000080
C SIXDEG  50    C. W. INGRAM, AERO-SPACE ENGR.                          00000090
C              UNIVERSITY OF NOTRE DAME, NOTRE DAME, IND.               00000100
C                                                                       00000110
C READS COEFFICIENTS IN TABULAR FORM AS A FUNCTION OF ANGLE OF          00000120
C ATTACK AND MACH NUMBER                                                00000130
C MAXIMUM NUMBER OF ANGLES OF ATTACK = 20, MAXIMUM NUMBER OF MACH
C NUMBERS = 20
C                                                                       00000150
 1001 FORMAT(A4,1X,3I2)                                                 00000160
 1002 FORMAT(//14X,*ALPHA*, 2X,10F10.4/21X,10F10.4)
 1003 FORMAT(/5X,*MACH*,1F10.4,2X,10F10.4/21X,10F10.4)
 1004 FORMAT(/1XA6,3X2HC(I2,1H))                                        00000190
 1006 FORMAT(8F10.4)
 1007 FORMAT(8F10.4)
      COMMON /COEF/ AMCH(20,30),ALPHA(20,30),C(20,20,30),NOA(30),NOM(30)00000200
      COMMON /COEP1/ NPLY                                               00000210
      COMMON /POL/ CE(30), ALPHAR,CMA,CMPA,CZA                          00000220
      COMMON /BLK1/ RUN
      DIMENSION AMC(600),ALP(600),CC(12000)                             00000230
      EQUIVALENCE (AMC,AMCH),(ALP,ALPHA),(CC,C)                         00000240
C                                                                       00000250
C SET ARRAY TABLE TO ZERO                                               00000260
      NPLY=0                                                            00000270
      DO 110 I=1,600                                                    00000280
      AMC(I)=0.                                                         00000290
  110 ALP(I)=0.                                                         00000300
      DO 111 I=1,12000                                                  00000310
  111 CC(I)=0.                                                          00000320
      DO 112 I=1,30                                                     00000330
      NOA(I)=0                                                          00000340
      NOM(I)=0                                                          00000350
  112 CONTINUE                                                          00000360
      CZA=0.0                                                           00000370
      CMA=0.0                                                           00000380
      CMPA=0.0                                                          00000390
C                                                                       00000400

C READ NUMBER OF AERODYNAMIC COEFFICIENTS                               00000410
      READ(5,1005) RUN,NC
1005  FORMAT(A6,4X,I2)
      PRINT 1008,RUN,NC
1008  FORMAT(1H1,2X,A6,2X,I2 // )
C FOR CONSTANT ANGLE OF ATTACK   NO ANGLE OF ATTACK TABLES ARE READ     00000430
C FOR CONTANT MACH NUMBER   ONE TABLE OF MACH NUMBER IS READ            00000440
      DO 130 M=1,NC                                                     00000450
      READ(5,1001) NAME,K,NOA(K),NOM(K)                                 00000460
      PRINT 1004, NAME,K                                                00000470
      NM=NOM(K)                                                         00000480
      IF(NM.EQ.0) NM=1                                                  00000490
      NA=NOA(K)                                                         00000500
      IF(NA.EQ.0) NA=1                                                  00000510
      IF(NA.EQ.1) GO TO 119
       READ(5,1006) (ALPHA(I,K),I=1,NA)
      PRINT 1002, (ALPHA(I,K),I=1,NA)                                   00000540
119   DO 120 I=1,NM
  120 READ(5,1007) AMCH(I,K),(C(I,J,K),J=1,NA)
      DO 125 I=1,NM                                                     00000560
  125 PRINT 1003,AMCH(I,K),(C(I,J,K),J=1,NA)                            00000570
  130 CONTINUE                                                          00000580
      RETURN                                                            00000590
      END
```

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```
SUBROUTINE ED          74/74    OPT=1                         FTN 4.5+414        09/12/77  17.09.

                 SUBROUTINE ED                                                          00004560
                 REAL MASS,IX,IY,L
                 INTEGER PG                                                             00004590
                 COMMON /AIRBLK/ TEM,PRES,RHO,KVIS,VA                                   00004600
                 COMMON /COEF/ AMCH(20,30),ALPHA(20,30),C(20,20,30),NOA(30),NOM(30)00004610
                 COMMON /INDBLK/ IDEN,IWIND,ITER,IPRN,IALL,NORC,IPUN,NBODY,ITRST,NT00004620
                $RST                                                                    00004630
                 COMMON /INIT/ TITLE(12),SCALE(30),A(22)                                00004640
                 COMMON/THRUST/TIM(15),TRST(15),MASS(15),CG(15),IX(15),IY(15),CGP(100004550
                $5)                                                                     00004650
                 COMMON/TSTDAT/EPS,THTC,ETA,L  ,BL                                      00004670
                 COMMON /WINBLK/ NOW,YW(128),W(128,3)                                   00004690
                 COMMON/COEP1/ NPLY                                                     00004690
                 COMMON /POL/ CE(30), ALPHAP,CMA,CMPA,CZA                               00004700
                 COMMON/EDBK/D1(14),D2(14),DE1(14),DE2(14),WIN(3),DAL(10),DBE(10)       00004710
                 COMMON/EDBK2/V,Q,S,OP,ALPHAN,AMCHN,PG,LN                               00004720
                 COMMON/EDBK3/AXX,AXY,AXZ,AYX,AYY,AYZ,AZX,AZY,AZZ                       00004730
                 COMMON/EDBK4/ LCOUNT
                 COMMON /BLK1/ RUN
                 DATA COND/57.295779/
            1000 FORMAT(56X,*--=*)
            1001 FORMAT(1X,T4,*TIME*,5X,*RANGE*,6X,*ALT*,7X,*Z*,9X,*V*,7X,*P*,3X,
                1*ALPHA*,3X,*M*,5X,*PHI*,2X,*ALPHA*,2X,*BETA*,2X,*L-N*,3X,*L-P*,
                24X,*W-N*,4X,*W-P*,5X,*S*,5X,*TAU*,3X,*K-T*)
            1002 FORMAT(1X,T4,*SEC*,7X,*FEET*,6X,*FEET*,5X,*FEET*,4X,*FT/SEC*,2X,
                1*RA/SEC*,2X,*DEG*,10X,*DEG*,3X,*DEG*,4X,*DEG*,1X,*1/SEC*,1X,
                2*1/SEC*,1X,*RA/SEC*,1X,*RA/SEC*,16X,*DEG*/)
            1003 FORMAT(1X,1F8.4,2F10.2,2F9.2,1F7.1,2F6.2,1F7.2,4F6.2,2F7.1,
                11F8.3,1F6.2,1F7.3)
            1010 FORMAT(1H112A4,30X4HPAGEI4)                                            00004840
            1090 FORMAT (10F7.3,2X,1HA)                                                 00004850
            1091 FORMAT (10F7.3,2X,1HB)                                                 00004850
                 IF (NBODY. EQ. 1) GO TO 821                                            00004870
                 BP=V*(2.*AXZ*D2(2)-2.*AXX*D2(4))                                       00004880
                 AP=2.*AXY*V*V-D2(3)*D1(12)                                             00004890
                 XI=ARCTAN(BP,AP)                                                       00004900
                 BETA=-ALPHAN*SIN(XI)                                                   00004910
                 ALPH= ALPHAN*COS(XI)                                                   00004920
                 GO TO 841                                                              00004930
             821 ALPH=ATAN(D1(14)/D1(12))*COND                                          00004940
                 BETA=ATAN(D1(13)/D1(12))*COND                                          00004950
             841 PP=OP*COND                                                             00004960
                 IF(MOD(LN,50).NE.0) GO TO 2                                            00004970
                 PG=PG+1                                                                00004980
                 PRINT 1010,TITLE,PG                                                    00004990
                 PRINT 1000                                                             00005000
                 PRINT 1001                                                             00005010
                 PRINT 1002                                                             00005020
               2 P= D1(5)                                                               00005030
                 IF(P.EQ.0.) GO TO 4752                                                 00005040
                 SSRC=(D1(5)*D1(5)*A(15)*A(15))/(Q*S*A(19)*CMA*A(16)*4.)                00005050

                 TAU = 1./SQRT(1. - 1./SSRC)                                            00005060
                 D2M= A(19)*A(19)*A(19)                                                 00005070
                 CLAMP =(Q*S)/(2.*A(19)*V)*(CZA*(1.+TAU)+ D2M/(2.*A(16))*(CE(27)        00005080
                $*(1.-TAU)) - D2M/A(15)*CMPA*TAU)                                       00005090
                 CLAMN =(Q*S)/(2.*A(19)*V)*(CZA*(1.-TAU)+ D2M/(2.*A(16))*(CE(27)        00005100
```

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SUBROUTINE ED          74/74    OPT=1                        FTN 4.5+414        09/12/77  17.09.0C

       $*(1.+TAU)) + D2M/A(15)*CMPA*TAU)                                        00005110
        WLAMP= (P*A(15))/(2.*A(15))*(1.- 1./TAU)                                00005120
        AA= CLAMN*CLAMP-(P-WLAMN)*(P-WLAMP)                                     00005130
        WLAMN= (P*A(15))/(2.*A(15))*(1.+ 1./TAU)                                00005140
        BB=-CLAMP*(P-WLAMN)-CLAMN*(P-WLAMP)                                     00005150
        AK3= (2*S*A(19)*DE(20)/(A(15)*SQRT(AA**2 + BB**2)))*COND                00005160
        GO TO 4763                                                              00005170
  4762  CLAMP=0.                                                                00005180
        CLAMN=0.                                                                00005190
        WLAMP=0.                                                                00005200
        WLAMN=0.                                                                00005210
        AK3=0.                                                                  00005220
        TAU=0.                                                                  00005230
        SSBC=0.0
  4763  CONTINUE                                                                00005250
            PRINT 1003, (D1(I),I=1,4),V,D1(5),ALPHAN,AMCHN,PP,ALPH,BETA,        00005260
       1CLAMN,CLAMP,WLAMN,WLAMP,SSBC,TAU,AK3                                    00005270
        IF(IPUN.EQ.0) GO TO 1113                                                00005280
        LCOUNT=LCOUNT+1                                                         00005290
        IF(LCOUNT.EQ.10) GO TO 1112                                             00005300
        DAL(LCOUNT)=ALPH                                                        00005310
        DBE(LCOUNT)=BETA                                                        00005320
        GO TO 1113                                                              00005330
  1112  DAL(LCOUNT)=ALPH                                                        00005340
        DBE(LCOUNT)=BETA                                                        00005350
        LCOUNT=0                                                                00005380
  1113  LN=LN+1                                                                 00005390
        RETURN                                                                  00005400
        END                                                                     00005410
```

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```
SUBROUTINE INIT6       74/74    OPT=1                              FTN 4.5+414        09/12/77  17.09.0

               SUBROUTINE INIT6                                                        00000610
         C                                                                             00000620
         C SIXDEG 100    C. W. INGRAM, AERO-SPACE ENGR.                                00000630
         C                UNIVERSITY OF NOTRE DAME, NOTRE DAME, IND.                   00000640
         C                                                                             00000650
         C INITIALIZATION PROGRAM FOR N-DEGREE OF FREEDOM MOTION   (3+N+6)             00000660
         C                                                                             00000670
               REAL MASS,IX,IY,L                                                       00000680
          1001 FORMAT(12A4)                                                            00000690
          1002 FORMAT (////1X,23HECHO OF INITIALIZATION INPUT'//)                      00000700
          1003 FORMAT (4(6(1X,2HA(,I2,4H) = ,F11.4)/))                                 00000710
          1004 FORMAT (1X,7HNORD = ,I1,3X,7HIPRN = ,I2,3X,7HNALL = ,I1,3X,             00000720
              $7HIPUN = ,I1,3X,7HNBODY = ,I1,3X,9HISCALE = ,I1,3X,8HITRST = ,I1,/00000730
              $/)                                                                      00000740
          1005 FORMAT (//5(6(1X,6HSCALE(,I2,4H) = ,E8.2)/))                            00000750
          1006 FORMAT(/,5X,4HTIME,6X,6HTHRUST,7X,4HMASS,5X,11HCG POSITION,7X,5HCG00000760
              $-AC,2HIX,7X,2HIY,/)                                                     00000770
          1007 FORMAT(4X,F5.2,6X,F5.0,6X,F5.3,6X,F8.2,8X,F6.2,4XF6.3,4X,F6.3)          00000780
          1008 FORMAT(5X,13HTHRUST DATA ',3X,6HEPS = ,F5.2,3X,9HTHETAC = ,F5.2,3X00000790
              $,6HETA = ,F5.2,3X,4HL = ,F5.2,//)                                       00000800
          1009 FORMAT(//,5X,14HBODY LENGTH = ,F4.2,/)                                  00000810
         1010  FORMAT(7I10)
         1011  FORMAT(I10,3E10.4,I10,E10.4)
         1012  FORMAT(8E10.4 / 8E10.4 / 6E10.4)
         1013  FORMAT(6E10.4)
         1014  FORMAT(2E10.4)
         1015  FORMAT(3(8E10.4 /), 6E10.4)
               LOGICAL LSCALE,IALL,LTRST                                               00000820
               COMMON /WINBLK/ NOW,YW(128),W(128,3)                                    00000830
               COMMON /AIRBLK/ TEM,PRES,RHO,KVIS,VA                                    00000840
               COMMON /DENBLK/ NOD,YD(128),DEN(128),TEMP(128)                          00000850

               COMMON/COEP1/ NPLY                                                      00000860
               COMMON /POL/ CE(30), ALPHAR,CMA,CMPA,CZA                                00000870
               COMMON /INDBLK/ IDEN,IWIND,ITER,IPRN,IALL,NORD,IPUN,NBODY,ITRST,NT00000880
              $RST                                                                     00000890
               COMMON/INIT/TITLE(12),SCALE(30),A(22)                                   00000900
               COMMON/THRUST/TIM(15),TRST(15),MASS(15),CG(15),IX(15),IY(15),CGP(100000910
              $5)                                                                      00000920
               COMMON/TSTDAT/EPS,THTC,ETA,L ,RL                                        00000930
               READ(5,1001) TITLE                                                      00000940
         C                                                                             00000950
               DO 10 I=1,15                                                            00000960
               TIM(I)=0.0                                                              00000970
               TRST(I)=0.0                                                             00000980
               MASS(I)=0.0                                                             00000990
               CG(I)=0.0                                                               00001000
               IX(I)=0.0                                                               00001010
               IY(I)=0.0                                                               00001020
               CGP(I)=0.0                                                              00001030
            10 CONTINUE                                                                00001040
               IDEN=0
               IWIND=0
               DO 11 J=1,3                                                             00001050
               DO 11 I=1,128                                                           00001060
               W(I,J)=0.0                                                              00001070
               YW(I)=0.0                                                               00001080
```

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SUBROUTINE INIT6        74/74    OPT=1                          FTN 4.5+414        09/12/77  17.09

               VD(I)=0.0                                                            00001090
               DEN(I)=0.0                                                           00001100
               TEMP(I)=0.0                                                          00001110
            11 CONTINUE                                                             00001120
         C                                                                          00001130
         C READ CONTROL PARAMETERS                                                  00001140
                READ(5,1010) NORC,IPRN,NALL,IPUN,NBODY,ISCALE,ITRST
               LSCALE=ISCALE.EQ.1                                                   00001160
               IALL=NALL.EQ.1                                                       00001170
               LTRST=ITRST.EQ.0                                                     00001180
         C                                                                          00001190
         C READ INITIAL CONDITIONS                                                  00001200
               IF(LTRST) GO TO 4                                                    00001210
                 READ(5,1011) NTRST,EPS,THTC,ETA,L,BL
               EPS=EPS/57.295779                                                    00001230
               THTC=THTC/57.295779                                                  00001240
               GO TO 7                                                              00001250
             4 CONTINUE                                                             00001260
               NTRST=1                                                              00001270
             7 CONTINUE                                                             00001280
                READ(5,1012) (A(I),I=1,22)
               IF(LSCALE) GO TO 1                                                   00001300
               DO 2 I=1,30                                                          00001310
             2 SCALE(I)=1.                                                          00001320
               GO TO 3                                                              00001330
             1 READ(5,1015) (SCALE(I),I=1,30)
             3 CONTINUE                                                             00001350
               IF(LTRST) GO TO 5                                                    00001360
                 READ(5,1013) TIM(1),MASS(1),CG(1),IX(1),IY(1),CGP(1)
                READ(5,1013) TIM(IO),MASS(IO),CG(IO),IX(IO),IY(IO),CGP(IO)
                READ(5,1014) (TIM(I),TRST(I),I=1,NTRST)
             5 CONTINUE                                                             00001400
         C                                                                          00001410
         C ECHO OUTPUT                                                              00001420
               PRINT 1002                                                           00001430
               PRINT 1004, NORC,IPRN,NALL,IPUN,NBODY,ISCALE,ITRST                   00001440
               PRINT 1003, (I,A(I),I=1,22)                                          00001450
               PRINT 1005, (I,SCALE(I),I=1,30)                                      00001450
               IF(LTRST) GO TO 6                                                    00001470
               EEPS=EPS*57.295779                                                   00001480
               TTHTC=THTC*57.295779                                                 00001490
               PRINT 1008,EEPS,TTHTC,ETA,L                                          00001500
               PRINT 1009 ,BL                                                       00001510
               PRINT 1006                                                           00001520
               PRINT 1007,(TIM(I),TRST(I),MASS(I),CGP(I),CG(I),IX(I),IY(I),I=1,NT00001530
              $RST)                                                                 00001540
             6 CONTINUE                                                             00001550
               RETURN                                                               00001560
               END                                                                  00001570
```

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```
SUBROUTINE SIXDEG      74/74    OPT=1                       FTN 4.5+414        09/12/77  17.09.

          SUBROUTINE SIXDEG(ATMOS)                                              00001580
C                   UNIVERSITY OF NOTRE DAME, NOTRE DAME, IND.                  00001590
 1004 FORMAT(12H ALPHA ERROR2XE14.8)                                            00001600
 1005 FORMAT(11H WIND ERROR)                                                    00001610
 1006 FORMAT(11H MACH ERROR2XE14.8)                                             00001620

 1157 FORMAT (1X,3F20.5)                                                        00001630
 8589 FORMAT(/// 5X,*----ALPHA *,1F10.4,* DEGREES EXCEEDS MAXIMUM*,
     1* ALPHA IN ARRAY----*)
 1666 FORMAT(//5X,*----MACH *,1F10.4,* EXCEEDS MAXIMUM MACH *,
     1*NUMBER IN ARRAY----*)
      REAL MASS,IX,IY,MTX,MTY,MTZ,L,MYJD,MZJD                                   00001640
      REAL MDOT                                                                 00001650
      INTEGER PG                                                                00001660
      LOGICAL LCZA,LCMA,LCMPA                                                   00001830
      LOGICAL LTRST                                                             00001840
      COMMON /AIRBLK/ TEM,PRES,RHO,KVIS,VA                                      00001670
      COMMON /COEF/ AMCH(20,30),ALPHA(20,30),C(20,20,30),NOA(30),NOM(30) 00001680
      COMMON /INDBLK/ IDEN,IWIND,ITER,IPRN,IALL,NORC,IPUN,NBODY,ITRST,NT00001690
     $RST                                                                       00001700
      COMMON /INIT/ TITLE(12),SCALE(30).A(22)                                   00001710
      COMMON/THRUST/TIM(15),TRST(15).MASS(15),CG(15),IX(15),IY(15),CGP(100001720
     $5)                                                                        00001730
      COMMON/TSTDAT/EPS,THTC,ETA,L  ,RL                                         00001740
      COMMON /WINBLK/ NOW,YW(128),W(128,3)                                      00001750
      COMMON/COEP1/ NPLY                                                        00001760
      COMMON /POL/ CE(30),  ALPHAR,CMA,CMPA,CZA                                 00001770
      COMMON /DENBLK/ NOD,YD(128),DEN(128),TEMP(128)                            00001780
      COMMON/EDBK/D1(14),D2(14),DE1(14),DE2(14),WIN(3),DAL(10),DBE(10)          00001790
      COMMON/EDBK2/V,Q,S,QP,ALPHAN,AMCHN,PG,LN                                  00001800
      COMMON/EDBK3/AXX,AXY,AXZ,AYX,AYY,AYZ,AZX,AZY,AZZ                          00001810
      COMMON/EDBK4/ LCOUNT
      DATA COND,RAD/57.295779,.017453293/                                       00001820
      LTRST=ITRST.EQ.0                                                          00001850
      ITER=0                                                                    00001860
      LN=0                                                                      00001870
      PG=0                                                                      00001880
      KOUNT=0                                                                   00001890
      LCOUNT=0                                                                  00001900
      M4=1                                                                      00001910
      D2(1)=1.                                                                  00001920
      S=3.1415927*A(19)**2/4.                                                   00001930
      DO 20 I=1,14                                                              00001940
   20 D1(I)=A(I)                                                                00001950

      RAD2=RAD/2.                                                               00001960
      D=D1(8)*RAD2                                                              00001970
      E=D1(9)*RAD2                                                              00001980
      F=D1(10)*RAD2                                                             00001990
      CP=COS(E)                                                                 00002000
      SP=SIN(E)                                                                 00002010
      CT=COS(D)                                                                 00002020
      ST=SIN(D)                                                                 00002030
      CS=COS(F)                                                                 00002040
      SS=SIN(F)                                                                 00002050
C VERSOR COMPONENTS (5A)                                                        00002060
      D1(4)=-CP*ST*SS+SP*CT*CS                                                  00002070
```

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SUBROUTINE SIXDEG 74/74 OPT=1 FTN 4.5+414 09/12/77 17.09.0

```
      D1(9)=-CP*CT*SS+SP*ST*CS
      D1(10)=CP*ST*CS+SP*CT*SS
      D1(11)=CP*CT*CS+SP*ST*SS
      AMP=A(18)
      M = A(20)
 25   IF(A(20)*(D1(1)-A(21)).GT.0.)GO TO 597
      M1=1
   55 Y=D1(3)
      IF(LTRST) GO TO 56
      X=D1(1)
      IF(X.GT.TIM(NTRST)) GO TO 56
      I=1
   58 IF(X.LT.TIM(I+1)) GO TO 57
      I=I+1
      GO TO 58
   57 CONTINUE
      Y1=TRST(I)
      Y2=TRST(I+1)
      Y=Y1+(X-TIM(I))*((Y2-Y1)/(TIM(I+1)-TIM(I)))
      ATRST=Y

      Y1=MASS(1)
      Y2=MASS(NTRST)
      Y=Y1+(X-TIM(1))*((Y2-Y1)/(TIM(NTRST)-TIM(1)))
      A(18)=Y
      MDOT=(AMP-A(18))/A(20)
      AMP=A(18)
      Y1=CG(1)
      Y2=CG(NTRST)
      Y=Y1+(X-TIM(1))*((Y2-Y1)/(TIM(NTRST)-TIM(1)))
      ACG=Y
      Y1=IX(1)
      Y2=IX(NTRST )
      Y=Y1+(X-TIM(1))*((Y2-Y1)/(TIM(NTRST)-TIM(1)))
      A(15)=Y
      Y1=IY(1)
      Y2=IY(NTRST)
      Y=Y1+(X-TIM(1))*((Y2-Y1)/(TIM(NTRST)-TIM(1)))
      A(16)=Y
      Y1=CGP(1)
      Y2=CGP(NTRST)
      Y=Y1+(X-TIM(1))*((Y2-Y1)/(TIM(NTRST)-TIM(1)))
      CGL=Y
   56 CONTINUE
      IF(X.GT.TIM(NTRST)) MDOT=0.
      SCL=1./SQRT(D1(8)**2+D1(9)**2+D1(10)**2+D1(11)**2)
      D1(8)=SCL*D1(8)
      D1(9)=SCL*D1(9)
      D1(10)=SCL*D1(10)
      D1(11)=SCL*D1(11)
      CALL ATMOS(Y)
C MATRIX OF DIRECTION COSINES
      AXX=D1(8)**2+D1(11)**2-.5

      AXY=D1(8)*D1(9)+D1(10)*D1(11)
      AXZ=D1(8)*D1(10)-D1(11)*D1(9)
      AYX=D1(8)*D1(9)-D1(11)*D1(10)
```

```
      AYY=D1(9)**2+D1(11)**2-.5
      AYZ=D1(9)*D1(10)+D1(11)*D1(8)
      AZX=D1(8)*D1(10)+D1(11)*D1(9)
      AZY=D1(9)*D1(10)-D1(11)*D1(8)
      AZZ=D1(10)**2+D1(11)**2-.5
      IF(IWIND.EQ.0) GO TO 2104
C COMPUTE WIND
      IF(Y.LT.YW(1)) GO TO 101
      I=2
  100 IF(YW(I).GE.Y) GO TO 210
      I=I+1
      GO TO 100
  101 PRINT 1005
      RETURN
  210 FASE=(Y-YW(I-1))/(YW(I)-YW(I-1))
      DO 220 K=1,3
  220 WIN(K)=W(I-1,K)+FASE*(W(I,K)-W(I-1,K))
C WIND COMPONENTS
      WX=2.*(WIN(1)*AXX+WIN(2)*AXY+WIN(3)*AXZ)
      WY=2.*(WIN(1)*AYX+WIN(2)*AYY+WIN(3)*AYZ)
      WZ=2.*(WIN(1)*AZX+WIN(2)*AZY+WIN(3)*AZZ)
C VELOCITY VECTOR
      D1(12)=D1(12)-WX
      D1(13)=D1(13)-WY
      D1(14)=D1(14)-WZ
C SPEED
 2104 V=SQRT(D1(12)**2+D1(13)**2+D1(14)**2)
C ALPHA PRIME - YAW - ARCTAN(SQRT(V*V+W*W)/U))
      ALPHAR=ARCTAN(SQRT(D1(13)**2+D1(14)**2),D1(12))

      ALPHAN=ALPHAR*COND
C DYNAMIC PRESSURE
      Q=.5*RHO*V*V
C CROSS-SPIN
      QPP=SQRT(D1(6)**2+D1(7)**2)
C ROLL ANGLES
      OP=ARCTAN(D1(13),D1(14))
      OPP=ARCTAN(-D1(7),D1(6))
C PD/2V AND QD/2V
      IF(V.GT.0.) GO TO 2214
    2 CONTINUE
      PD=0.
      QD=0.
      AMCHN=0.
      GO TO 2215
 2214 PD=(D1(5)*A(19))/(2.*V)
      QD=(QPP*A(19))/(2.*V)
      AMCHN=V/VA
 2215 QSD=Q*S*A(19)
      QSM=(Q*S)*1./A(19)
C INTERPOLATE COEFFICIENT TABLES
      DO 150 K=1,30
      IF((NOA(K).EQ.0).AND.(NOM(K).EQ.1)) GO TO 145
      I=1
      J=1
      IF(NOA(K).EQ.0) GO TO 120
      IF(ALPHAN.LT.ALPHA(1,K)) GO TO 111
```

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```
SUBROUTINE SIXDEG      74/74    OPT=1                    FTN 4.5+414        09/12/77  17.09.

            N=NOA(K)                                                          00003200
            IF(ALPHAN.GT.ALPHA(N,K)) GO TO 8888
            J=2                                                               00003210
        110 IF(ALPHA(J,K).GE.ALPHAN) GO TO 115                                00003220
            J=J+1                                                             00003230
            GO TO 110                                                         00003240
        111 PRINT 1004,ALPHAN                                                 00003250
            RETURN                                                            00003260
       8888 PRINT 8889, ALPHAN
            RETURN
        115 FRA=(ALPHAN-ALPHA(J-1,K))/(ALPHA(J,K)-ALPHA(J-1,K))               00003270
        120 IF(NOM(K).EQ.0) GO TO 140                                         00003280
            IF(AMCHN.LT.AMCH(1,K)) GO TO 131                                  00003290
            N=NOM(K)                                                          00003300

            IF(AMCHN.GT.AMCH(N,K)) GO TO 1131
            I=2                                                               00003310
        130 IF(AMCH(I,K).GE.AMCHN) GO TO 135                                  00003320
            I=I+1                                                             00003330

            GO TO 130                                                         00003340
        131 PRINT 1006,AMCHN                                                  00003350
            RETURN                                                            00003360
       1131 PRINT 1665, AMCHN
            RETURN
        135 FRM=(AMCHN-AMCH(I-1,K))/(AMCH(I,K)-AMCH(I-1,K))                   00003370
            CE(K)=C(I-1,J,K)+FRM*(C(I,J,K)-C(I-1,J,K))                        00003380
            IF(NOA(K).EQ.0) GO TO 150                                         00003390
            T1=C(I-1,J-1,K)+FRM*(C(I,J-1,K)-C(I-1,J-1,K))                     00003400
            LCZA=K.EQ.6                                                       00003410
            LCMA=K.EQ.13                                                      00003420
            LCMPA=K.EQ.19                                                     00003430
            IF(LCZA ) CZA=(CE(K)-T1)/(ALPHA(J,K)-ALPHA(J-1,K))*COND*          00003440
           $SCALE(K)                                                          00003450
            IF(LCMA ) CMA=(CE(K)-T1)/(ALPHA(J,K)-ALPHA(J-1,K))*COND*          00003460
           $SCALE(K)                                                          00003470
            IF(LCMPA) CMPA=(CE(K)-T1)/(ALPHA(J,K)-ALPHA(J-1,K))*COND*         00003480
           $SCALE(K)                                                          00003490
            CE(K)=T1+FRA*(CE(K)-T1)                                           00003500
            GO TO 150                                                         00003510
        140 CE(K)=C(I,J-1,K)+FRA*(C(I,J,K)-C(I,J-1,K))                        00003520
            GO TO 150                                                         00003530
        145 CE(K)=C(1,1,K)                                                    00003540
        150 CF(K)=CE(K)*SCALE(K)                                              00003550
      C COMPUTE COEFFICIENTS CX, CY, CZ, CL, CM, CN  (TABLE 3)                00003560
       1511 CONTINUE                                                          00003570
            IF(NPLY.NE.1) GO TO 1512                                          00003580
            DO 1513 KK=1,30                                                   00003590
       1513 CE(KK)=CE(KK)*SCALE(KK)                                           00003600
       1512 CONTINUE                                                          00003610
            SP= SIN (OP)                                                      00003620
            CP=COS(OP)                                                        00003630
            SP4=SIN(4.*OP)                                                    00003640
            CP4=COS(4.*OP)                                                    00003650
            SPP4=SIN(4.*OPP)                                                  00003660
            CPP4=COS(4.*OPP)                                                  00003670
            CX=CE(1)                                                          00003680
```

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SUBROUTINE SIXDEG   74/74   OPT=1   FTN 4.5+414   09/12/77 17.09.

```
      CY=(CE(2)+CE(3)*SP4+CE(4)*CP4+PD*CE(5))*CP+(CE(6)+CE(7)*SP4+CE(8)*
     $CP4)*SP+CE(22)
      CZ=(CE(2)+CE(3)*SP4+CE(4)*CP4+PD*CE(5))*(-SP)+(CE(6)+CE(7)*SP4+CE(
     $8)*CP4)*CP+CE(23)
      CL=CE(9)+CE(30)+CE(10)*SP4+CE(11)*CP4+PD*CE(12)+ALPHAR*(CE(24)+CE(
     $25)*SP+CE(26)*CP)
      CM=(CE(13)+CE(14)*SP4+CE(15)*CP4)*CP+(CE(16)+CE(17)*SP4+CE(18)*CP4
     $+PD*CE(19))*SP+(CE(27)+CE(28)*SPP4+CE(29)*CPP4)*QD*COS(OPP)+CE(20)
      CN=(CE(13)+CE(14)*SP4+CE(15)*CP4)*(-SP)+(CE(16)+CE(17)*SP4+CE(18)*
     $CP4+PD*CE(19))*CP+(CE(27)+CE(28)*SPP4+CE(29)*CPP4)*QD*(-SIN(OPP))+
     $CE(21)
      IF(IWIND.EQ.0) GO TO 151
C ADD WIND BACK
      D1(12)=D1(12)+WX
      D1(13)=D1(13)+WY
      D1(14)=D1(14)+WZ
C KINEMATICAL RELATIONSHIPS
C * VELOCITY (9)
  151 D2(2)=2.*(D1(12)*AXX+D1(13)*AYX+D1(14)*AZX)
      D2(3)=2.*(D1(12)*AXY+D1(13)*AYY+D1(14)*AZY)
      D2(4)=2.*(D1(12)*AXZ+D1(13)*AYZ+D1(14)*AZZ)
C * MOMENTS
      D2(5)=1./A(15)*(QSD*CL)
      D2(6)=1./A(16)*(QSD*CM+D1(5)*D1(7)*(A(16)-A(15)))
      D2(7)=1./A(16)*(QSD*CN-D1(5)*D1(5)*(A(16)-A(15)))
C * VERSOR (10)
      D2(8)=-.5*(-D1(5)*D1(11)-D1(7)*D1(9)+D1(6)*D1(10))
      D2(9)=-.5*(-D1(6)*D1(11)+D1(7)*D1(8)-D1(5)*D1(10))
      D2(10)=-.5*(-D1(7)*D1(11)-D1(5)*D1(8)+D1(5)*D1(9))
      D2(11)=-.5*(D1(5)*D1(8)+D1(6)*D1(9)+D1( 7)*D1(10))
      IF(NORC.NE.0) GO TO 230
      G=2.*(A(17)-.30783368E-5*D1(3))

      GO TO 250
  230 G=2.*A(17)
C * FORCE
  250 D2(12)=QSM*CX-G*AXY-(D1(6)*D1(14)-D1(7)*D1(13))
      D2(13)=QSM*CY-G*AYY-(D1(7)*D1(12)-D1(5)*D1(14))
      D2(14)=QSM*CZ-G*AZY-(D1(5)*D1(13)-D1(6)*D1(12))
      IF(LTPST) GO TO 251
      TX=ATRST*COS(EPS)
      TY=ATRST*SIN(EPS)*SIN(THTC)
      TZ=-ATRST*SIN(EPS)*COS(THTC)
      MTX=ETA*TX
      MTY=-(BL-CGL)*TZ
      MTZ=-(BL-CGL)*TY
      MYJD=MDOT*D1(6)*L**2
      MZJD=MDOT*D1(7)*L**2
      D2(5)=D2(5)+MTX
      D2(6)=D2(6)+(MTY+MYJD+Q*S*CZ*ACG)/A(16)
      D2(7)=D2(7)+(MTZ+MZJD+Q*S*CY*ACG)/A(16)
      D2(12)=D2(12)+TX/A(18)
      D2(13)=D2(13)+TY/A(18)
      D2(14)=D2(14)+TZ/A(18)
  251 CONTINUE
      IF(LN.EQ.0) CALL ED
```

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```
SUBROUTINE SIXDEG      74/74    OPT=1                         FTN 4.5+414         09/12/77  17.09.0

        C RUNGE-KUTTA CONSTANTS                                                       00004240
              DO 298 I=1,14                                                           00004250
              GO TO (294,295,296,297,70),M1                                           00004260
          294 DE1(I)=D1(I)                                                            00004270
              DE2(I)=D2(I)*H                                                          00004280
              E1=DE2(I)*.5                                                            00004290
              GO TO 298                                                               00004300
          295 E1=D2(I)*.5*H                                                           00004310
              DE2(I)=DE2(I)+4.*E1                                                     00004320
              GO TO 298                                                               00004330
          296 E1=D2(I)*H                                                              00004340
              DE2(I)=DE2(I)+E1+E1                                                     00004350
              GO TO 298                                                               00004360
          297 E1=(DE2(I)+D2(I)*H)/6.                                                  00004370
          298 D1(I)=DE1(I)+E1                                                         00004380
              M1=M1+1                                                                 00004390
              GO TO 55                                                                00004400
           70 GO TO (540,550),M4                                                      00004410
          540 KOUNT=KOUNT+1                                                           00004420
              IF(D1(3)-A(22)) 550,597,580                                             00004430
          550 IF(ABS(D1(3)-A(22)).LT..01) GO TO 597                                   00004440
              H=-(D1(3)-A(22))/D2(3)                                                  00004450
              M4=2                                                                    00004460
              M1=1                                                                    00004470
        C NEXT TIME LINE                                                              00004480
              GO TO 55                                                                00004490
          580 IF(MOD(KOUNT,IPRN).NE.0) GO TO 25                                       00004500
              CALL ED                                                                 00004510
              GO TO 25                                                                00004520
          597 CALL ED                                                                 00004530
              RETURN                                                                  00004540
              END                                                                     00004550
```

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```
FUNCTION ARCTAN     74/74    OPT=1                              FTN 4.5+414        09/12/77  17.09.

            FUNCTION ARCTAN(P1,P2)                                ARC00000
            IF(P2) 2,1,2                                          ARC00010
          1 IF(P1) 3,8,4                                          ARC00020
          3 ARCTAN=4.7123890                                      ARC00030
            RETURN                                                ARC00040
          4 ARCTAN=1.5707963                                      ARC00050
            RETURN                                                ARC00060
          2 IF(P1) 5,6,5                                          ARC00070
          6 IF(P2.GT.0.) GO TO 8                                  ARC00080
            ARCTAN=3.1415927                                      ARC00090
            RETURN                                                ARC00100
          8 ARCTAN=.0                                             ARC00110
            RETURN                                                ARC00120
          5 ARC=ATAN(P1/P2)                                       ARC00130
            IF(P2) 10,9,9                                         ARC00140
         10 ARCTAN=ARC+3.1415927                                  ARC00150
            RETURN                                                ARC00160
          9 IF(ARC) 11,12,12                                      ARC00170
         11 ARCTAN=ARC+6.2831853                                  ARC00180
            RETURN                                                ARC00190
         12 ARCTAN=ARC                                            ARC00200
            RETURN                                                ARC00210
            END                                                   ARC00220
```

Appendix G

Coefficient Arrays Input

000005 12

CX C(1)

	ALPHA	0.0000	2.0000	5.0000	10.0000	15.0000	20.0000	25.0000	27.0000	50.0000
MACH	0.0100	-.2730	-.2735	-.2748	-.2815	-.2860	-.2725	-.2435	-.2220	-.2220
MACH	.4100	-.2730	-.2735	-.2748	-.2815	-.2860	-.2725	-.2435	-.2220	-.2220
MACH	.8100	-.2918	-.2930	-.3010	-.3225	-.3175	-.3110	-.2393	-.2630	-.2630
MACH	.9100	-.3090	-.3110	-.3200	-.3450	-.3489	-.3430	-.3250	-.2910	-.2910
MACH	1.0100	-.3890	-.3965	-.4200	-.4520	-.4610	-.4550	-.4150	-.3750	-.3750
MACH	1.1100	-.5050	-.5055	-.5120	-.5420	-.5700	-.5660	-.5290	-.5110	-.5110
MACH	1.2100	-.5210	-.5218	-.5260	-.5560	-.5840	-.5790	-.5470	-.5430	-.5430
MACH	2.0100	-.5210	-.5218	-.5260	-.5560	-.5840	-.5790	-.5470	-.5430	-.5430

CYZ C(3)

	ALPHA	0.0000	2.0000	4.0000	8.0000	10.0000	15.0000	20.0000	25.0000	50.0000
MACH	0.0100	.0213	.0043	.0208	.0441	.0631	.1343	.2663	.1906	.2000
MACH	.4100	.0213	.0043	.0208	.0441	.0631	.1343	.2563	.1905	.2000
MACH	.8100	.0012	.0013	.0033	.0450	.0870	.1652	.1945	.2000	.2000
MACH	.9100	-.0009	-.0036	.0025	.0528	.0987	.1914	.2227	.3340	.1000
MACH	1.0100	-.0009	-.0012	-.0013	.0641	.1129	.2410	.2903	.1543	.2000
MACH	1.1100	.0050	.0051	.0109	.0570	.1141	.2395	.2807	.0965	.1000
MACH	1.2100	.0056	.0023	.0076	.0585	.1031	.2235	.2170	.0319	.1000
MACH	2.0100	.0056	.0023	.0076	.0586	.1031	.2235	.2170	.0319	.1000

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CYP C(5)

	ALPHA	0.0000	2.0000	4.0000	8.0000	10.0000	15.0000	20.0000	25.0000	50.0000
MACH	0.0000	0.0000	-.0300	-.1350	-.5350	-.7750	-2.1830	-5.9180	-12.5000	-12.5000
MACH	.4000	0.0000	-.0300	-.1350	-.5350	-.7750	-2.1830	-5.9180	-12.5000	-12.5000
MACH	.8000	0.0000	-.0930	-.1750	-.4100	-.4600	-1.1650	-2.8900	-6.1500	-6.1500
MACH	.9000	0.0000	-.1050	-.1850	-.3500	-.4400	-1.1000	-2.8000	-5.0000	-5.0000
MACH	1.0000	0.0000	-.0700	-.1630	-.2950	-.3300	-.7800	-2.3720	-3.4500	-3.4500
MACH	1.1000	0.0000	-.0800	-.1600	-.3120	-.2540	-.6000	-2.4630	-3.4000	-3.4000
MACH	1.2000	0.0000	-.0800	-.1800	-.3100	-.2650	-.6750	-1.7700	-2.3300	-2.3300
MACH	2.0000	0.0000	-.0800	-.1800	-.3100	-.2650	-.6750	-1.7700	-2.3300	-2.3300

CZ C(6)

	ALPHA	0.0000	5.0000	10.0000	15.0000	20.0000	25.0000	27.0000	50.0000
MACH	0.0000	0.0000	-.3800	-.7500	-1.1900	-1.7100	-2.4600	-2.7800	-2.7800
MACH	.4000	0.0000	-.3800	-.7500	-1.1900	-1.7100	-2.4600	-2.7800	-2.7800
MACH	.8000	0.0000	-.4190	-.8000	-1.2900	-1.8750	-2.5400	-2.9150	-2.9150
MACH	.9000	0.0000	-.3800	-.8400	-1.3800	-1.9600	-2.6700	-3.0900	-3.0900
MACH	1.0000	0.0000	-.4100	-.8800	-1.4600	-2.1600	-3.0600	-3.6600	-3.6600
MACH	1.1000	0.0000	-.4100	-.8800	-1.4600	-2.1800	-3.3160	-3.9100	-3.9100
MACH	1.2000	0.0000	-.4100	-.9000	-1.5270	-2.4000	-3.6800	-4.2600	-4.2600
MACH	2.0000	0.0000	-.4100	-.9000	-1.5270	-2.4000	-3.6800	-4.2600	-4.2600

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CLOW S(9)

	ALPHA	0.0000	2.0000	4.0000	8.0000	10.0000	15.0000	20.0000	25.0000	50.0000
MACH	0.0000	.0470	.0470	.0480	.0495	.0504	.0490	.0471	.0415	.0415
MACH	.4000	.0470	.0470	.0480	.0495	.0504	.0490	.0471	.0415	.0415
MACH	.8000	.0553	.0627	.0634	.0665	.0690	.0704	.0677	.0596	.0596
MACH	.9000	.0567	.0562	.0561	.0585	.0606	.0610	.0582	.0361	.0361
MACH	1.0000	.0525	.0552	.0542	.0574	.0593	.0622	.0560	.0489	.0489
MACH	1.1000	.0536	.0548	.0541	.0555	.0565	.0604	.0576	.0500	.0500
MACH	1.2000	.0546	.0543	.0547	.0552	.0567	.0582	.0530	.0444	.0444
MACH	2.0000	.0545	.0543	.0547	.0552	.0567	.0582	.0530	.0444	.0444

CLGA S(10)

	ALPHA	0.0000	2.0000	4.0000	8.0000	10.0000	15.0000	20.0000	25.0000	27.0000	50.0000
MACH	0.0000	0.0000	0.0000	.0060	.0123	.0251	.0343	.0311	.0534	.0620	.0620
MACH	.4000	0.0000	0.0000	.0060	.0123	.0251	.0343	.0311	.0534	.0620	.0620
MACH	.8000	0.0000	.0031	.0055	.0147	.0246	.0544	.0962	.1486	.1486	.1486
MACH	.9000	0.0000	.0031	.0068	.0166	.0246	.0543	.1043	.1725	.1725	.1725
MACH	1.0000	0.0000	.0042	.0099	.0187	.0274	.0494	.0886	.1459	.1459	.1459
MACH	1.1000	0.0000	.0040	.0084	.0217	.0285	.0576	.0907	.1060	.1060	.1060
MACH	1.2000	0.0000	.0050	.0059	.0168	.0259	.0543	.0860	.0940	.0940	.0940
MACH	2.0000	0.0000	.0050	.0059	.0168	.0259	.0543	.0860	.0940	.0940	.0940

CLP C(12)

	ALPHA	0.0000	2.0000	4.0000	8.0000	10.0000	15.0000	20.0000	25.0000	50.0000
MACH	0.0100	-1.0789	-1.0500	-1.0415	-1.0518	-1.0014	-.8497	-1.0173	-.9832	-.9832
MACH	.4100	-1.0789	-1.0500	-1.0415	-1.0518	-1.0014	-.8497	-1.0173	-.9932	-.9932
MACH	.8100	-1.0916	-1.0426	-1.0569	-1.0947	-1.0994	-1.0569	-1.0088	-.9868	-.9868
MACH	.9100	-1.0964	-1.0694	-1.0738	-1.1036	-1.1083	-1.1149	-1.0869	-.9498	-.9498
MACH	1.0100	-1.0413	-1.0893	-1.0978	-1.1634	-1.1900	-1.2546	-1.2537	-1.4388	-1.4388
MACH	1.1100	-1.1042	-1.1130	-1.1261	-1.1753	-1.1878	-1.2629	-1.2605	-1.3964	-1.3964
MACH	1.2100	-1.1117	-1.1008	-1.1316	-1.2086	-1.2152	-1.2192	-1.4101	-1.4405	-1.4405
MACH	2.0100	-1.1117	-1.1008	-1.1316	-1.2086	-1.2152	-1.2192	-1.4101	-1.4405	-1.4405

CM C(13)

	ALPHA	0.0000	5.0000	7.0000	10.0000	15.0000	20.0000	22.0000	25.0000	27.0000	50.0000
MACH	0.0100	0.0000	-.3450	-.4850	-.7200	-1.2500	-1.7450	-1.9600	-2.3190	-2.4930	-2.4930
MACH	.4100	0.0000	-.3450	-.4850	-.7200	-1.2500	-1.7450	-1.9600	-2.3190	-2.4930	-4.9500
MACH	.8100	0.0000	-.3980	-.5700	-.8590	-1.4000	-1.9820	-2.2280	-2.5800	-2.8200	-5.5500
MACH	.9100	0.0000	-.4400	-.6400	-.9600	-1.5190	-2.0900	-2.2600	-2.4850	-2.6300	-2.6300
MACH	1.0100	0.0000	-.4850	-.6750	-.9700	-1.5900	-2.1600	-2.1650	-2.1700	-2.1900	-2.1900
MACH	1.1100	0.0000	-.4160	-.6050	-.8800	-1.4500	-1.9000	-1.9050	-1.9100	-1.9500	-1.9500
MACH	1.2100	0.0000	-.3950	-.5780	-.8350	-1.3700	-1.8200	-1.9000	-2.0500	-2.1400	-2.1400
MACH	2.0100	0.0000	-.3950	-.5780	-.8350	-1.3700	-1.8200	-1.9100	-2.0500	-2.1400	-2.1400

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CNGA C(17)

	ALPHA	0.0000	2.0000	4.0000	8.0000	10.0000	15.0000	20.0000	25.0000	27.0000	50.0000
MACH	0.0000	0.0000	.0025	.0249	.1364	.2148	.5089	.9434	.9261	1.0689	1.0689
MACH	.4000	0.0000	.0025	.0249	.1364	.2148	.5089	.9434	.9261	1.0589	1.0689
MACH	.5000	0.0000	.0035	.0037	.1727	.3312	.7229	.9509	.5038	.2629	.2629
MACH	.9000	0.0000	.0031	.0057	.2021	.3780	.8345	1.0400	.6287	.3916	.3916
MACH	1.0000	0.0000	.0009	.0073	.2511	.4548	1.0030	1.2600	.9800	.3099	.3099
MACH	1.1000	0.0000	.0281	.0450	.2755	.4570	1.0010	1.2840	.8332	.6044	.6044
MACH	1.2000	0.0000	.0008	.0176	.2233	.3931	.9091	1.0051	.5373	.6214	.6214
MACH	2.0000	0.0000	.0008	.0176	.2233	.3931	.9091	1.0051	.5373	.6214	.6214

CNP C(19)

	ALPHA	0.0000 50.0000	5.0000	8.0000	10.0000	12.0000	14.0000	16.0000	20.0000	22.0000	25.0000
MACH	0.0000	0.0000 35.8000	.2300	.5300	.9200	1.5200	2.6000	4.6000	12.5000	19.5000	35.8000
MACH	.4000	0.0000 35.8000	.2300	.5300	.9200	1.5200	2.6000	4.6000	12.5000	19.5000	35.8000
MACH	.8000	0.0000 15.4000	.4700	.5000	.5000	.8300	1.5500	2.7000	6.6400	9.5000	15.4000
MACH	.9000	0.0000 12.0000	.5500	.4200	.2400	.5000	1.1400	2.2000	5.6000	8.0000	12.0000
MACH	1.0000	0.0000 6.7000	.1400	.1100	0.0000	0.0000	.4600	1.3500	5.0000	6.0000	6.7000
MACH	1.1000	0.0000 6.7100	.3000	.3000	-.3200	-.5000	-.3000	.6900	4.1500	5.4100	6.7100
MACH	1.2000	0.0000 4.7400	.4500	.3200	-.5000	-.5000	0.0000	.8000	3.3000	4.1000	4.7400
MACH	2.0000	0.0000 4.7400	.4500	.3200	-.5000	-.6000	0.0000	.8000	3.3000	4.1000	4.7400

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CMPE C(20)

	ALPHA	0.0000	50.0000
MACH	0.0000	.0714	.0714
MACH	.4000	.0714	.0714
MACH	.8000	.0818	.0818
MACH	.9000	.0900	.0900
MACH	1.0000	.0973	.0973
MACH	1.1000	.0800	.0800
MACH	1.2000	.0727	.0727
MACH	2.0000	.0727	.0727

CMO C(27)

	ALPHA	0.0000	2.0000	4.0000	8.0000	15.0000	25.0000	50.0000
MACH	0.0000	-111.3300	-115.0000	-119.0000	-126.8000	-138.5000	-154.0000	-154.0000
MACH	.4000	-111.3300	-115.0000	-119.0000	-126.8000	-138.5000	-154.0000	-154.0000
MACH	.8000	-138.8200	-141.8000	-145.4000	-152.6000	-164.0000	-178.3000	-178.3000
MACH	.9000	-135.6200	-145.0000	-154.5000	-169.5000	-187.5000	-210.0000	-210.0000
MACH	1.0000	-105.7500	-130.0000	-147.0000	-165.5000	-192.0000	-215.0000	-215.0000
MACH	1.1000	-121.1800	-128.0000	-134.0000	-144.4000	-158.4000	-175.0000	-175.0000
MACH	1.2000	-143.7600	-131.0000	-123.0000	-111.8000	-97.8000	-83.5000	-83.5000
MACH	2.0000	-143.7600	-131.0000	-123.0000	-111.8000	-97.8000	-83.5000	-83.5000

Vita

James D. Schneider graduated from Oklahoma State University in 1962 with a Bachelor of Science degree in Mechanical Engineering. He entered pilot training, after receiving his commission through Officer Training School, and earned his aeronautical rating in February 1964. He accumulated twelve years of flying experience including instructor pilot in both the T-37 and T-38 trainers, aircraft commander in the F-4D, and operations pilot in the T-39. After completing an assignment to Headquarters Air Training Command as Operations Center Staff Officer, Major Schneider entered the Air Force Institute of Technology for study in Astronautical Engineering.

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→ The effect of Magnus coefficients, projectile spin rate, and pitching motion on lateral displacement of a ballistic trajectory is investigated by use of a six degree of freedom simulation. The coefficients had no significant effect. Increased spin rate extends the projectile impact point both down range and cross range. Oscillations induced by rate motion increased the lateral deviation but oscillations induced by an initial displacement did not. ←

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