SSC-229

# EVALUATION AND VERIFICATION OF COMPUTER CALCULATIONS OF WAVE-INDUCED SHIP STRUCTURAL LOADS



SHIP STRUCTURE COMMITTEE

1972

# SHIP STRUCTURE COMMITTEE

AN INTERAGENCY ADVISORY
COMMITTEE DEDICATED TO IMPROVING
THE STRUCTURE OF SHIPS

MEMBER AGENCIES:

UNITED STATES COAST GUARD
NAVAL SHIP SYSTEMS COMMAND
MILITARY SEALIFT COMMAND
MARITIME ADMINISTRATION
AMERICAN BUREAU OF SHIPPING

ADDRESS CORRESPONDENCE TO:

SECRETARY
SHIP STRUCTURE COMMITTEE
U.S. COAST GUARD HEADQUARTERS
WASHINGTON, D.C. 20591

SR-174
1972

Dear Sir:

The wide availability of electronic computers today allows calculations of a detail and accuracy which was impossible a few years ago, but these computer calculations are only as valid as the input data upon which they are based. In the case of ships' hull structures, the increased calculational capability has meant that loads acting on the hull must be known more accurately than ever before.

A major portion of the effort of the Ship Structure Committee research program has been devoted to improving capability of determining hull loads. This report and the two which follow it concern a project directed towards this end, which involved the development of a computer program to calculate these loads.

This report contains a description of the development and verification of the program for predicting hull loads. SSC-230, Program SCORES--Ship Structural Response in Waves, contains the details of the computer program and SSC-231, Further Studies of Computer Simulation of Slamming and Other Wave-Induced Vibratory Structural Loadings on Ships in Waves, contains further details on the use of the analysis method for prediction of other hull loadings.

Comments on this report would be welcomed.

Sincerely,

W. F. REA, III
Rear Admiral, U. S. Coast Guard
Chairman, Ship Structure Committee

SSC-229

Final Report

on

Project SR-174, "Ship Computer Response"

to the

Ship Structure Committee

# EVALUATION AND VERIFICATION OF COMPUTER CALCULATIONS OF WAVE-INDUCED SHIP STRUCTURAL LOADS

by

Paul Kaplan and Alfred I. Raff
Oceanics, Inc.

under

Department of the Navy
Naval Ship Engineering Center
Contract No. N00024-70-C-5076

*This document has been approved for public release and sale; its distribution is unlimited.*

U. S. Coast Guard Headquarters
Washington, D. C.
1972

ABSTRACT

An analytical method for the determination of conventional merchant ship motions and wave-induced moments in a seaway is developed. Both vertical and lateral plane motions and loads are considered for a ship travelling at any heading in regular waves and in irregular long or short crested seas. Strip theory is used and each ship hull cross-section is assumed to be of Lewis form shape for the purpose of calculating hydrodynamic added mass and damping forces in vertical, lateral and rolling oscillation modes. The coupled equations of motion are linear, and the superposition principle is used for statistical response calculations in irregular seas. All three primary ship hull loadings are determined, i.e. vertical bending, lateral bending and torsional moments, as well as shear forces, at any point along the length, with these responses only representing the low frequency slowly varying wave loads directly induced by the waves.

A computer program that carries out the calculations was developed, and is fully documented separately. The results of the method are evaluated by comparison with a large body of model test data. The comparison extends over a wide range of ship speeds, wave angles, wave lengths, and loading conditions, as well as hull forms. The agreement between the calculations and experimental data is generally very good. Thus, a method is available for use in the rational design of the ship hull main girder structure.

CONTENTS

LIST OF FIGURES

## NOMENCLATURE

$a$ = wave amplitude

$a', b, c', d, e, g'$ = coefficients in vertical (heave) equation of motion

$a_{ij}$ = coefficients in lateral plane equations of motion

$\overline{a_i^2}$ = mean squared response amplitude

$A, B, C, D, E, G'$ = coefficients in vertical plane (pitch) equation of motion

$\bar{A}$ = ratio of generated wave to heave amplitude for vertical motion-induced waves

$A'_{33}$ = sectional vertical added mass

$\underline{A}, \underline{B}$ = coefficients in two-parameter spectrum equation

$B^*$ = local waterline beam

$B^*_{⊗}$ = waterline beam amidships

$BM_y$ = lateral bending moment

$BM_z$ = vertical bending moment

$c$ = wave speed (celerity)

$C_s$ = local section area coefficient

$\frac{df_y}{dx}$ = total local lateral loading on ship

$\frac{df_z}{dx}$ = total local vertical loading on ship

$\frac{dm_x}{dx}$ = total local torsional loading on ship

$\frac{dK}{dx}$ = sectional hydrodynamic moment, about x axis, on ship

$\frac{dY}{dx}$ = sectional lateral hydrodynamic force on ship

$\frac{dZ}{dx}$ = sectional vertical hydrodynamic and hydrostatic force on ship

$F_n$ = Froude number

$F_{rs}$ = sectional lateral added mass due to roll motion

$g$ = acceleration of gravity

$G$ = center of gravity of ship

$\overline{GM}$ = initial metacentric height of ship

$\bar{h}$ = mean section draft

$H$ = sectional draft

$H_{1/3}$ = significant wave height

$I_r$ = sectional added mass moment of inertia

$I_x, I_y, I_z$ = mass moments of inertia of ship about x,y,z axes respectively

$I_{xz}$ = mass product of inertia of ship in x-z plane

$k$ = wave number

$K_w$ = wave excitation moment, about x axis, on ship

$L$ = ship length

$m$ = mass of ship

$M_s$ = sectional lateral added mass

$M_w$ = wave excitation moment, about y axis, on ship

$M_{s\phi}$ = sectional added mass moment of inertia due to lateral motion

$N_r$ = sectional roll damping moment coefficient due to wave effects

$N_r^*$ = sectional roll damping moment coefficient due to viscous and bilge keel effects

$N_s$ = sectional lateral damping force coefficient

$N_w$ = wave excitation moment, about z axis, on ship

$N'_{zz}$ = sectional vertical damping force coefficient

$N_{rs}$ = sectional lateral damping force coefficient due to roll motion

$N_{s\phi}$ = sectional damping moment coefficient due to lateral motion

$\overline{OG}$ = vertical distance between waterline and center of gravity, positive up

$S$ = local section area

$S(\omega,\mu)$ = directional spectrum of the seaway

$S_i(\omega,\mu)$ = response spectrum, for a particular response

$S_1(\omega)$ = frequency spectrum

$S_2(\mu)$ = spreading function

$t$ = time

$\tilde{T}$ = mean wave period

$T_1, T_2, \ldots$ = coefficients in lateral plane wave excitation equations

$T_i(\omega,\mu)$ = response amplitude operator

$TM_x$ = torsional moment

$U$ = wind speed

$v_w$ = lateral orbital wave velocity

$V$ = ship forward speed

$x$ = horizontal axis in direction of forward motion of ship (along length of ship)

$x'$ = axis fixed in space

$x_o$ = location along ship length at which moments are determined

$x_s, x_b$ = x coordinates at stern and bow ends of ship, respectively

$y$ = horizontal axis directed to starboard; sway

$Y_w$ = lateral wave excitation force on ship

$z$ = vertical axis directed downwards; heave

$z'$ = vertical space coordinate, from undisturbed water surface, positive downwards

$\bar{z}$ = sectional center of buoyancy, from waterline

$Z_w$ = vertical wave excitation force on ship

$\beta$ = angle between wave propagation direction and ship forward motion

$\gamma$ = local mass gyradius in roll (about x axis)

$\delta, \varepsilon, \sigma, \kappa, \alpha, \nu, \tau, \upsilon$ = phase angles (leads) of heave, pitch, vertical bending moment, sway, yaw, roll, lateral bending moment, torsional moment, respectively

$\delta m$ = local mass

$\zeta$ = local vertical center of gravity, from CG, positive down

$\zeta_\phi$ = fraction of critical roll damping

$\eta$ = surface wave elevation, positive upwards from undisturbed water surface

$\theta$ = pitch angle, positive bow-up

$\lambda$ = wave length

$\mu$ = wave direction relative to predominant direction

$\rho$ = density of water

$\phi$ = roll angle, positive starboard-down

$\psi$ = yaw angle, positive bow-starboard

$\omega$ = circular wave frequency

$\omega_e$ = circular frequency of wave encounter

$\omega_\phi$ = natural roll frequency

Subscripts

avg = average (statistical)

o = amplitude

rms = root-mean-squared

1/3 = significant (average of 1/3 highest)

1/10 = average of 1/10 highest

## SHIP STRUCTURE COMMITTEE

The SHIP STRUCTURE COMMITTEE is constituted to prosecute a research program to improve the hull structures of ships by an extension of knowledge pertaining to design, materials and methods of fabrication.

RADM W. F. Rea, III, USCG, Chairman
Chief, Office of Merchant Marine Safety
U. S. Coast Guard Headquarters

Capt. J. E. Rasmussen, USN
Head, Ship Systems Engineering and Design Department
Naval Ship Engineering Center
Naval Ship Systems Command

Mr. K. Morland, Vice President
American Bureau of Shipping

Mr. E. S. Dillon
Chief
Office of Ship Construction
Maritime Administration

Capt. L. L. Jackson, USN
Maintenance and Repair Officer
Military Sealift Command

## SHIP STRUCTURE SUBCOMMITTEE

The SHIP STRUCTURE SUBCOMMITTEE acts for the Ship Structure Committee on technical matters by providing technical coordination for the determination of goals and objectives of the program, and by evaluating and interpreting the results in terms of ship structural design, construction and operation.

NAVAL SHIP ENGINEERING CENTER

Mr. P. M. Palermo - Chairman
Mr. J. B. O'Brien - Contract Administrator
Mr. G. Sorkin - Member
Mr. H. S. Sayre - Alternate
Mr. I. Fioriti - Alternate

U. S. COAST GUARD

LCDR C. S. Loosmore, USCG - Secretary
CAPT C. R. Thompson, USCG - Member
CDR J. W. Kime, USCG - Alternate
CDR J. L. Coburn, USCG - Alternate

MARITIME ADMINISTRATION

Mr. F. Dashnaw - Member
Mr. A. Maillar - Member
Mr. R. Falls - Alternate
Mr. R. F. Coombs - Alternate

MILITARY SEALIFT COMMAND

Mr. R. R. Askren - Member
LTJG E. T. Powers, USNR - Member

AMERICAN BUREAU OF SHIPPING

Mr. S. G. Stiansen - Member
Mr. F. J. Crum - Member

OFFICE OF NAVAL RESEARCH

Mr. J. M. Crowley - Member
Dr. W. G. Rauch - Alternate

NAVAL SHIP RESEARCH & DEVELOPMENT CENTER

Mr. A. B. Stavovy - Alternate

NATIONAL ACADEMY OF SCIENCES - Ship Research Committee

Mr. R. W. Rumke, Liaison
Prof. R. A. Yagle, Liaison

SOCIETY OF NAVAL ARCHITECTS & MARINE ENGINEERS

Mr. T. M. Buermann, Liaison

BRITISH NAVY STAFF

Dr. V. Flint, Liaison
CDR P. H. H. Ablett, RCNC, Liaison

WELDING RESEARCH COUNCIL

Mr. K. H. Koopman, Liaison
Mr. C. Larson, Liaison

## INTRODUCTION

In order to investigate the utility of a computer simulation approach for determining ship bending moment responses in waves, a research program was instituted under the sponsorship of the Ship Structure Committee with the aid of an advisory panel appointed by the National Academy of Sciences. The original program was considered to be made up of three separate phases of work which include:

1. An assembly of a system of equations that would adequately describe ship structural responses due to the effects of waves.

2. The conversion of these equations to a computer program or to the design of a computer analog.

3. Computer evaluation of the ship response mathematical model, with the verification of the entire procedure provided by such an evaluation.

The first phase of this work, which was the development of a mathematical model, was completed and described in a final report [1]. A mathematical model was developed under that program, where equations for determining wave-induced bending moments in the vertical and lateral planes were established. In addition, a method of treatment for including effects due to slamming was outlined, where the occurrence of slamming was evidenced by "whipping" responses that may be ascribed to nonlinear forces generated due to bow flare.

The second phase of work in this program, which has been completed and described in report form [2], is devoted to the conversion of the equations developed in Phase I into a computer program. The linearized vertical plane motions and vertical bending moment response operators for a ship were determined by a digital computer program for the case of head seas, and this program was then generalized to the case of oblique headings between the ship and seaway. Modifications of the basic head sea program have been carried out under Phase II of this overall program (see [2]), and hence these quantities are amenable to computation by a digital computer. Further modifications to incorporate a given wave spectrum, together with a directional spreading factor to account for short-crestedness, will allow this program to compute the power spectra of vertical bending moments on a ship in irregular short-crested seas.

Since lateral bending moments occur in oblique sea conditions, and since they have significant magnitude in certain cases relative to the vertical bending moment for that same heading, a program for computation of lateral bending moments has also been developed as well in Phase II. While the hydrodynamic data for this particular structural component is not extensively treated in the available literature as is the case of vertical motions and structural responses, there is sufficient basic information that allows a similar treatment to be applied to the lateral loads although no computer programs to calculate the sectional added mass and damping due to lateral and rolling motions had been established previously. Thus lateral bending moment spectra can then also be obtained for a particular input wave spectrum, and these results can be combined with those for the vertical bending moment, if desired.

The work described in the present report treats the analytical determination of one aspect of sea loads, viz. the determination of wave-induced moments that are slowly varying in time and have the same frequency characteristics as the encountered waves. Other sea-induced loadings, such as whipping, slamming and springing, which are of higher frequency, must not be neglected in an overall design, and analytical work to cover these subjects has also been carried out under the present contract, which will be reported separately. Wave-induced moments depend both on the motion responses of the ship and the wave-excitation loads themselves. These factors, in turn, depend on the ship geometry and mass distribution, as well as on the particular wave conditions.

The present report is a continuation of work previously reported in [1] and [2]. While much of the previous analytical results with respect to wave-induced moments are repeated here, analysis procedures for wave-induced moments are slightly expanded and refined in the present report as well as extended to include torsional moments, and the results of more extensive computer calculations based on these procedures are evaluated by comparison with experimental data. The digital computer program (SCORES) developed in the course of this work is fully documented separately [3].

The present results apply to conventional merchant ship hull forms. Consideration is given in the analysis to both vertical and lateral plane motion responses and wave-induced moments, with the ship advancing at any heading with respect to the waves. The wave environment can be represented as either regular sinusoidal waves, a long-crested (unidirectional) seaway of specific spectral form or a fully short-crested seaway, using various wave energy spectral formulations. The three primary ship hull loadings that are considered are, vertical bending moment, lateral bending moment, and torsional moment, with primary emphasis upon vertical and lateral beiding (the related shear forces are also determined in this work).

Since the necessary inputs to the wave-induced moment determination are the rigid body ship motion responses, these must be obtained initially. The equations of ship motion are taken to be linear and coupled only within each plane. That is, heave and pitch motions are coupled in the vertical plane, and sway, yaw, and roll motions are considered coupled in the lateral plane. The equations are solved, or more precisely the terms in the equations are computed by application of "strip" theory, where local forces on each ship section, or strip, are evaluated independently, without allowing for influence, or interaction, among sections. This method was originally derived by Korvin-Kroukovsky [4], and in collaboration with Jacobs [5], for vertical plane motions, and has subsequently been adopted and expanded by many investigators.

The hydrodynamic forces at each station which enter into the equations of motion are obtained by a potential flow solution for an equivalent "Lewis" form section shape [6]. In general, the Lewis form shape, defined simply by two paramters (beam-draft ratio and section area coefficient), is considered to be a fairly close representation of section shapes found in conventional merchant ship hulls, without a large bulb at the bow. The hydrodynamic forces, added mass and damping, are obtained for vertical section oscillations by the method developed by Grim [7], and for lateral and rolling oscillations by the method of Tasai [8].

The present work is aimed at verifying the capability of a digital computer technique in providing valid information for evaluating wave-induced ship structural responses under various environmental conditions, for ships having conventional hull forms. This is achieved by applying the method of computation to a number of particular cases, which represent computer experiments that point out simplications, improvements, etc. that can be incorporated in a final computer program. The program will provide codification of various elemental steps, specific subroutines for computing separate items such as sectional hydrodynamic forces, etc., and the computational experiments are used to establish a final formulation of a complete and efficient digital computer program that will produce structural response information with a minimized computer time and cost. A fully documented computer program, including a description of data input, output forms, flow charts, and the program listing are given in [3]. The results of extensive computations for a number of ships, for which model test data are available, are presented in the present report together with a comparison between the computations and the experiments.

## ANALYTICAL METHOD

The basic analytical procedures for the determination of the wave-induced moments were presented originally in [1]. In the course of the work, certain additions and modifications to the original development have been deemed advisable. Therefore, the full analytical treatment is presented below, with the refinements included.

The coordinate system relationship between the water wave system and the ship coordinate axes is shown in Figure 1. Whereas in the previous work, separate axes conventions were employed for the vertical and lateral motions cases, a single ship axes coordinate system is now used. All the equations of motions are formulated relative to a right-handed cartesian coordinate axes system whose origin is located at the center of gravity of the ship, G, and with the x-axes positive toward the bow (in the direction of forward motion), the y-axis positive to starboard, and the z-axis positive downward. These axis are defined to have a fixed orientation, i.e. they do not rotate with the ship, but they can translate with the ship. The ship angular motions are considered to be small oscillations about the mean position defined by the axes.

The wave propagation, at speed c, is considered fixed in space. The ship then travels, at speed V, at some angle $\beta$ with respect to the wave direction. The wave velocity potential, for simple deep-water waves, is then defined by:

$$\Phi_w = ace^{-kz'}\cos k\ (x' + ct) \tag{1}$$

where a = wave amplitude
c = wave speed
k = wave number = $\frac{2\pi}{\lambda} = \frac{\omega^2}{g}$
$\lambda$ = wave length; $\omega$ = circular wave frequency
z' = vertical coordinate, from undisturbed water surface, positive downwards
x' = axis fixed in space
t = time.

The x' coordinate of a point in the x-y plane can be defined by:

$$x' = -(x+Vt)\cos\beta + y\sin\beta \tag{2}$$

The surface wave elevation $\eta$ (positive upwards) can be expressed as follows:

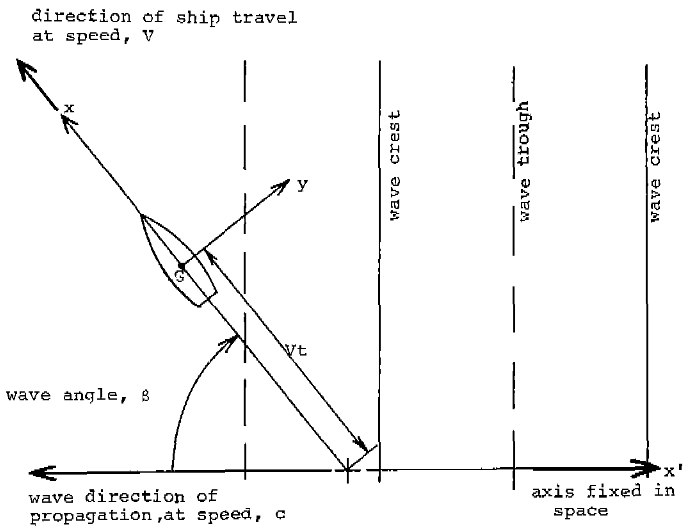



Fig. 1. Wave and Ship Axes Convention

$$\eta = \frac{1}{g}\left(\frac{\partial \Phi_w}{\partial t}\right)_{z'=0} = a \sin k\ (x' + ct) \tag{3}$$

since $c^2 = \frac{g}{k}$

where $g$ = acceleration of gravity.

In x-y coordinates, relative to the ship, we have:

$$\eta = a \sin k\ [-x \cos\beta + y \sin\beta + (c-V \cos\beta)t], \tag{4}$$

$$\dot{\eta} = \frac{D\eta}{Dt} = \left(\frac{\partial}{\partial t} - V \frac{\partial}{\partial x}\right) \eta\ (x,t)$$

$$= akc \cos k\ [-x \cos\beta + y \sin\beta + (c-V \cos\beta)t] \tag{5}$$

and $\ddot{\eta} = \frac{D\dot{\eta}}{Dt}$ $= -akg \sin k\ [-x \cos\beta + y \sin\beta + (c-V \cos\beta)t]$ (6)

The results of the equations of motion and the wave-induced moments will be referenced to the wave elevation $\eta$ at the origin of the x-y axes, which is

$$\eta = a \sin k\ (c-V \cos\beta)\ t \tag{7}$$

or $\eta = a \sin \omega_e t$

where $\omega_e = \frac{2\pi}{\lambda}(c-V \cos\beta)$ (8)

and $\omega_e$ is known as the circular frequency of encounter. The quantity $\omega_e$ is generally positive, and only for following waves ($90° < \beta < 90°$), where the ship is overtaking the waves, is $\omega_e$ negative.

## Vertical Plane Equations

The coupled equations of motion in the vertical plane for heave, z (positive downwards), and pitch, $\theta$ (positive bow-up), in keeping with the revised axes convention, are given as:

$$m\ddot{z} = \int_{x_s}^{x_b} \frac{dZ}{dx}\, dx + Z_w \tag{9}$$

$$I_y\ddot{\theta} = -\int_{x_s}^{x_b} \frac{dZ}{dx}\, x\, dx + M_w \tag{10}$$

where $m$ = mass of ship
$I_y$ = mass moment of inertia of ship about y axis
$\frac{dZ}{dx}$ = local sectional vertical hydrodynamic and hydrostatic force on ship
$x_s, x_b$ = coordinates of stern and bow ends of ship, respectively
$Z_w, M_w$ = wave excitation force and moment on ship.

The general hydrodynamic and hydrostatic force is taken to be:

$$\frac{dZ}{dx} = -\frac{D}{Dt}\left[A'_{33}(\dot{z}-x\dot{\theta}+V\theta)\right] - N'_z(\dot{z}-x\dot{\theta}+V\theta) - \rho g B^*(z-x\theta) \tag{11}$$

where $\rho$ = density of water
$A'_{33}$ = local sectional vertical added mass
$N'_z$ = local sectional vertical damping force coefficient
$B^*$ = local waterline beam

and $$N'_z = \rho g^2 \bar{A}^2 |\omega_e|^{-3} \tag{12}$$

with $\bar{A}$ = ratio of generated wave to heave amplitude for vertical motion-induced wave.

Values of the two-dimensional $A'_{33}$ and $\bar{A}$ terms are calculated by the method of Grim [7] for the equivalent Lewis forms at each section.

Expanding the derivative in Eq. (11), we obtain:

$$\frac{dZ}{dx} = -A'_{33}\,(\ddot{z}-x\ddot{\theta}+2V\dot{\theta}) - \left(N'_z - V\frac{dA'_{33}}{dx}\right)(\dot{z}-x\dot{\theta}+V\theta) - \rho g B^*(z-x\theta) \tag{13}$$

The equations of motion, (9) and (10) are transformed into the familiar form as follows:

$$a'\ddot{z} + b\dot{z} + c'z - d\ddot{\theta} - e\dot{\theta} - g'\theta = Z_w \tag{14}$$

$$A\ddot{\theta} + B\dot{\theta} + C\theta - D\ddot{z} - E\dot{z} - G'z = M_w \tag{15}$$

The coefficients on the left hand sides are then defined by:

$$\left.\begin{aligned}
a' &= m + \int A'_{33}dx \;, \quad b = \int N'_z dx - V \int d(A'_{33}) \\
c' &= \rho g \int B^* dx \;, \quad d = D = \int A'_{33} x dx \\
e &= \int N'_z x dx - 2V \int A'_{33} dx - V \int x d(A'_{33}) \\
g' &= \rho g \int B^* x dx - Vb \;, \quad A = I_y + \int A'_{33} x^2 dx \\
B &= \int N'_z x^2 dx - 2V \int A'_{33} x dx - V \int x^2 d(A'_{33}) \\
C &= \rho g \int B^* x \, dx - VE, \; E = \int N'_z x dx - V \int x d(A'_{33}), \\
G' &= \rho g \int B^* x dx
\end{aligned}\right\} \tag{16}$$

where all the indicated integrations are over the length of the ship.

The wave excitations, the right hand sides of Eqs. (14) and (15), are given by:

$$Z_w = \int_{x_s}^{x_b} \frac{dZ_w}{dx} dx \tag{17}$$

$$M_w = -\int_{x_s}^{x_b} \frac{dZ_w}{dx} x dx \tag{18}$$

The local sectional vertical wave force acting on the ship section is represented by:

$$\frac{dZ_w}{dx} = - \left[ \rho g B^* \eta + \left( N_z' - V \frac{dA_{33}'}{dx} \right) \dot{\eta} + A_{33}' \ddot{\eta} \right] e^{-k\bar{h}} \qquad (19)$$

where $\bar{h}$ = mean section draft. Substituting the expressions for $\eta$, $\dot{\eta}$ and $\ddot{\eta}$ from Eqs. (4), (5) and (6), with y=0, and incorporating an approximate factor for short wave lengths (by carrying out an integration over the lateral extent of the ship, in terms of the y-coordinate extending from $-\frac{B^*}{2}$ to $\frac{B^*}{2}$), leads to

$$\frac{dZ_w}{dx} = -ae^{-k\bar{h}} \left\{ \left[ (\rho g B^* - A_{33}' kg) \sin(-kx \cos\beta) + kc \left( N_z' - V \frac{dA_{33}'}{dx} \right) \cos(-kx \cos\beta) \right] \cos\omega_e t + \left[ (\rho g B^* - A_{33}' kg) \cos(-kx \cos\beta) - kc \left( N_z' - V \frac{dA_{33}'}{dx} \right) \sin(-kx \cos\beta) \right] \sin\omega_e t \right\} \frac{\sin\left( \frac{\pi B^*}{\lambda} \sin\beta \right)}{\frac{\pi B^*}{\lambda} \sin\beta} \qquad (20)$$

where the latter factor in Eq. (20) represents this short wave length factor. The value of $\bar{h}$ is approximated by:

$$\bar{h} = HC_s \qquad (21)$$

where $H$ = local section draft
$C_s$ = local section area coefficient.

The steady-state solution of the equations of motion, at each particular regular wave length, is obtained by conventional methods for second order ordinary differential equations (using complex notation). The solutions are expressed as:

$$z = z_o \sin(\omega_e t + \delta)$$
$$\theta = \theta_o \sin(\omega_e t + \varepsilon) \qquad (22)$$

where the zero subscripted quantities are the motion response amplitudes and $\delta$, $\varepsilon$ are the phase angle differences, i.e. leads with respect to the wave elevation in Eq. (7).

Having obtained solutions for the motions in the vertical plane, the wave-induced vertical bending moment can then be calculated. The bending moment is found from the total loading at each section. This is made up of the loads due to inertia (ship mass), hydrodynamic and hydrostatic forces, and the direct wave loads. The total local vertical loading is then given by:

$$\frac{df_z}{dx} = - \delta m\ (\ddot{z}-x\ddot{\theta}) + \frac{dZ}{dx} + \frac{dZ_w}{dx} \tag{23}$$

where $\delta m$ = local mass.
Eq. (23) is simply the summation of inertial, hydrodynamic, hydrostatic and wave excitation forces. The latter terms are given in Eqs. (13) and (20). The vertical bending moment at any location $x_o$ along the ship length is then given by:

$$BM_z(x_o) = \left[ \int_{x_s}^{x_o} \text{ or } \int_{x_o}^{x_b} \right] (x-x_o)\ \frac{df_z}{dx}\ dx \tag{24}$$

and is expressed in a form similar to the motions, i.e.

$$BM_z = BM_{zo} \sin\ (\omega_e t+\sigma) \tag{25}$$

<u>Lateral Plane Equations</u>

The coupled equations of motion in the lateral plane for sway, y (positive to starboard), yaw, $\psi$ (positive bow-starboard), and roll, $\phi$ (positive starboard-down), are given as:

$$m\ddot{y} = \int_{x_s}^{x_b} \frac{dY}{dx}\ dx+Y_w \tag{26}$$

$$I_z\ddot{\psi} - I_{xz}\ddot{\phi} = \int_{x_s}^{x_b} \frac{dY}{dx}\ xdx+N_w \tag{27}$$

$$I_x\ddot{\phi} - I_{xz}\ddot{\psi} = \int_{x_s}^{x_b} \frac{dK}{dx}\ dx-mg\ \overline{GM}\ \phi+K_w \tag{28}$$

where $I_z$ = mass moment of inertia of ship about z axis
$I_x$ = mass moment of inertia of ship about x axis
$I_{xz}$ = mass product of inertia of ship in x-z plane

$\frac{dY}{dz}$ = local sectional lateral hydrodynamic force on ship

$\frac{dK}{dx}$ = local sectional hydrodynamic rolling moment on ship

$Y_w$, $N_w$, $K_w$ = wave excitation force and moments on ship

$\overline{GM}$ = initial metacentric height of ship (hydrostatic).

The cross inertial terms, involving $I_{xz}$, the mass product of inertia, are usually small but necessary for the equilibrium balance of forces and moments. The hydrodynamic force and moment in the above equations are given by:

$$\frac{dY}{dx} = -\frac{D}{Dt}\left[M_s(\dot{y}+x\dot{\psi}-V\psi)-F_{rs}\dot{\phi}\right]-N_s(\dot{y}+x\dot{\psi}-V\psi) + N_{rs}\dot{\phi}$$

$$+ \overline{OG}\frac{D}{Dt}(M_s\dot{\phi}) + \overline{OG}\,N_s\dot{\phi} \tag{29}$$

$$\frac{dK}{dx} = -\frac{D}{Dt}\left[I_r\dot{\phi}-M_{s\phi}(\dot{y}+x\dot{\psi}-V\dot{\psi})\right]-N_r\dot{\phi}+N_{s\phi}(\dot{y}+x\dot{\psi}-V\psi)$$

$$-\overline{OG}\frac{D}{Dt}(M_{s\phi}\dot{\phi}) - \overline{OG}\,N_{s\phi}\dot{\phi} - \overline{OG}\frac{dY}{dx} \tag{30}$$

where $\overline{OG}$ = distance of ship C.G. from waterline, positive up

$M_s$ = sectional lateral added mass

$N_s$ = sectional lateral damping force coefficient

$M_{s\phi}$ = sectional added mass moment of inertia due to lateral motion

$N_{s\phi}$ = sectional damping moment coefficient due to lateral motion

$I_r$ = sectional added mass moment of inertia

$N_r$ = sectional damping moment coefficient

$F_{rs}$ = sectional lateral added mass due to roll motion

$N_{rs}$ = sectional lateral damping force coefficient due to roll motion

and the sectional added mass moments and damping moment coefficients are taken with respect to an axis at the waterline. Values of these sectional hydrodynamic properties for the equivalent Lewis form at each section, as functions of the frequency of oscillation, can be calculated by the method of Tasai [8] based on the potential theory solution. It has been shown by Vugts [9] that such potential theory results for the lateral and rolling modes, which ignore viscous and surface tension effects, are in good agreement with experimental results except for the roll damping moment. In addition, the influence of bilge keels, which are usually used but not considered up to this point, is expected to be primarily upon the roll damping moment.

In order to account for the above effects, that is the viscous effect and the bilge keel effect upon the roll damping moment, an adjustment is made to the potential theory result. Roll motion is generally, for conventional merchant hull forms, a very lightly damped response. This means that at resonance, i.e. at the natural roll response frequency, the damping value is important in limiting large roll responses, but that at frequencies away from resonance the amount of damping hardly affects the roll response at all. Thus it is most important to determine the proper value of the roll damping moment at the resonant frequency, while at other frequencies away from resonance its influence is almost negligible. The adjustment, or addition, to the roll damping moment is made so that at the resonant frequency the total roll damping is a particular fraction of the critical roll damping. This fraction is estimated, or known by experimentation, to produce the proper roll response at resonance. This approach was employed by Vugts [10] and verified experimentally for the rolling motions of a cylinder of rectangular cross-section in regular beam waves. Therefore, we have:

$$N_r^* = \zeta_\phi C_c/L - N_r(\omega_\phi) \tag{31}$$

where
$N_r^*$ = sectional damping moment coefficient due to viscous and bilge keel effects
$\zeta_\phi$ = fraction of critical roll damping (empirical data)
$C_c$ = critical roll damping
$L$ = ship length ($L = x_b - x_s$)
$\omega_\phi$ = natural roll (resonant) frequency
$N_r(\omega_\phi)$ = value of $N_r$ at frequency of $\omega_\phi$.

This procedure is still linear, with the empirical value of the damping at resonance reflecting an average or equivalent linear value that can be applied in an approximate manner. Since the main concern of this study is determining structural loads, and the influence of roll motion per se must be explored in the investigation itself, the use of this method of representation is considered sufficiently valid for this purpose. The critical roll damping can be expressed in terms of the natural roll frequency as follows:

$$C_c = 2mg\,\overline{GM}\,\omega_\phi^{-1}$$

with

$$\omega_\phi = \left[\frac{mg\,\overline{GM}}{I_x + \int I_r(\omega_\phi)\,dx}\right]^{1/2} \tag{32}$$

where the integral is over the ship length.

Expanding the derivatives in Eqs. (29) and (30), and including the above additional roll damping moment, we obtain:

$$\frac{dY}{dx} = - M_s(\ddot{y}+x\ddot{\psi}-2V\dot{\psi}) + \left(V\frac{dM_s}{dx} - N_s\right)(\dot{y}+x\dot{\psi}-V\psi) + \left(F_{rs} + \overline{OG}\,M_s\right)\ddot{\phi}$$

$$+ \left[N_{rs} + \overline{OG}\,N_s - V\left(\frac{dF_{rs}}{dx} + \overline{OG}\,\frac{dM_s}{dx}\right)\right]\dot{\phi} \tag{33}$$

$$\frac{dK}{dx} = - \left[I_{r\phi} + \overline{OG}\left(M_{s\phi} + F_{rs} + \overline{OG}\,M_s\right)\right]\ddot{\phi} + \left[V\left(\frac{dI_r}{dx} + \overline{OG}\,\frac{dM_{s\phi}}{dx}\right)\right.$$

$$\left. - \overline{OG}\left(N_{rs} + N_{s\phi} + \overline{OG}\,N_s\right) + \overline{OG}\,V\left(\frac{dF_{rs}}{dx} + \overline{OG}\,\frac{dM_s}{dx}\right) - N_r - N_r^*\right]\dot{\phi}$$

$$+ \left(M_{s\phi} + \overline{OG}\,M_s\right)(\ddot{y}+x\ddot{\psi}-2V\dot{\psi}) + \left[N_{s\phi} + \overline{OG}\,N_s - V\left(\frac{dM_{s\phi}}{dx} + \overline{OG}\,\frac{dM_s}{dx}\right)\right]$$

$$(\dot{y}+x\dot{\psi}-V\psi) \tag{34}$$

The equations of motion, (26), (27) and (28) are transformed into this familiar form:

$$\left.\begin{aligned}
&a_{11}\ddot{y}+a_{12}\dot{y}+a_{14}\ddot{\psi}+a_{15}\dot{\psi}+a_{16}\psi+a_{17}\ddot{\phi}+a_{18}\dot{\phi} = Y_w \\
&a_{21}\ddot{y}+a_{22}\dot{y}+a_{24}\ddot{\psi}+a_{25}\dot{\psi}+a_{26}\psi+a_{27}\ddot{\phi}+a_{28}\dot{\phi} = N_w \\
&a_{31}\ddot{y}+a_{32}\dot{y}+a_{34}\ddot{\psi}+a_{35}\dot{\psi}+a_{36}\psi+a_{37}\ddot{\phi}+a_{38}\dot{\phi}+a_{39}\phi = K_w
\end{aligned}\right\} \tag{35}$$

The coefficients on the left-hand sides are then defined by:

$$\left.\begin{aligned}
&a_{11} = m + \int M_s dx \quad , \quad a_{12} = \int N_s dx - V\int d(M_s) \quad , \\
&a_{14} = \int M_s x dx \quad , \quad a_{15} = \int N_s x dx - 2V\int M_s dx - V\int x d(M_s) \quad , \\
&a_{16} = -Va_{12} \quad , \quad a_{17} = -\int F_{rs} dx - \overline{OG}\int M_s dx \quad , \\
&a_{18} = -\int N_{rs} dx + \overline{OG}V\int d(M_s) - \overline{OG}\int N_s dx + V\int d(F_{rs})
\end{aligned}\right\} \tag{36}$$

$$\left.\begin{aligned}
a_{21} &= \int M_s x\,dx \quad , \quad a_{22} = \int N_s x\,dx - V\int x\,d(M_s) \quad , \\
a_{24} &= I_z + \int M_s x^2 dx \quad , \quad a_{25} = \int N_s x^2 dx - 2V\int M_s x\,dx - V\int x^2 d(M_s) \, , \\
a_{26} &= -Va_{22} \quad , \quad a_{27} = -I_{xz} - \int F_{rs}\; x\,dx - \overline{OG}\int M_s x\,dx \quad , \\
a_{28} &= -\int N_{rs} x\,dx + \overline{OG}\,V\int x\,d(M_s) - \overline{OG}\int N_s x\,dx + V\int x\,d(F_{rs}) \quad .
\end{aligned}\right\} (37)$$

$$\left.\begin{aligned}
a_{31} &= -\int M_{s\phi}dx - \overline{OG}\int M_s dx \quad , \\
a_{32} &= -\int N_{s\phi}dx - \overline{OG}\int N_s dx + V\int d(M_{s\phi}) + V\,\overline{OG}\int d(M_s) \quad , \\
a_{34} &= -I_{xz} - \int M_{s\phi}x\,dx - \overline{OG}\int M_s x\,dx \quad , \\
a_{35} &= -\int N_{s\phi}x\,dx - \overline{OG}\int N_s x\,dx + V\int x\,d(M_{s\phi}) + V\,\overline{OG}\int x\,d(M_s) - 2Va_{31} \, , \\
a_{36} &= -Va_{32} , \\
a_{37} &= I_x + \int I_r dx + \overline{OG}\int M_{s\phi}dx + \overline{OG}\int F_{rs}dx + \overline{OG}^2\int M_s dx \quad , \\
a_{38} &= \int (N_r + N_r^*)dx + \overline{OG}\int N_{s\phi}dx + \overline{OG}\int N_{rs}dx + \overline{OG}^2\int N_s dx \\
&\quad - V\left[\int d(I_r) + \overline{OG}\int d(M_{s\phi}) + \overline{OG}\int d(F_{rs}) + \overline{OG}^2\int d(M_s)\right] \quad , \\
a_{39} &= mg\;\overline{GM}
\end{aligned}\right\} (38)$$

where all the indicated integrations are over the ship length.

The wave excitation, the right-hand sides of Eqs. (35), is given by:

$$Y_w = \int_{x_s}^{x_b} \frac{dY_w}{dx}\, dx \tag{39}$$

$$N_w = \int_{x_s}^{x_b} \frac{dY_w}{dx}\, x\, dx \qquad (40).$$

$$K_w = \int_{x_s}^{x_b} \frac{dK_w}{dx}\, dx \qquad (41)$$

The local sectional lateral force and roll moment due to the waves acting on the ship are represented as:

$$\frac{dY_w}{dx} = \left[ (\rho S + M_s) \frac{Dv_w}{Dt} - Vv_w \frac{dM_s}{dx} + N_s v_w + k \left( -M_{s\phi} \frac{Dv_w}{Dt} + V \frac{dM_{s\phi}}{dx} v_w \right) \right] \frac{\sin\left(\frac{\pi B^*}{\lambda} \sin\beta\right)}{\frac{\pi B^*}{\lambda} \sin\beta} \qquad (42)$$

$$\frac{dK_w}{dx} = \left[ -\frac{D}{Dt}(M_{s\phi} v_w) + \rho \left( \frac{B^{*3}}{12} - S\bar{z} \right) \frac{Dv_w}{Dt} - N_{s\phi} v_w \right] \frac{\sin\left(\frac{\pi B^*}{\lambda} \sin\beta\right)}{\frac{\pi B^*}{\lambda} \sin\beta} - \overline{OG} \frac{dY_w}{dx} \qquad (43)$$

where $v_w$ = lateral orbital wave velocity

$S$ = local section area

$\bar{z}$ = local sectional center of buoyancy, from waterline.

The lateral wave orbital velocity is obtained as follows:

$$v_w = - \frac{\partial \Phi_w}{\partial y}$$

$$v_w = - akc\, e^{-k\bar{h}} \sin\beta \sin k \left[ -x \cos\beta + y \sin\beta + (c - V\cos\beta) t \right] \qquad (44)$$

and then we have:

$$\frac{Dv_w}{Dt} = - akg\ e^{-k\bar{h}} \sin\beta \cos k \left[-x \cos\beta + y \sin\beta + (c-V \cos\beta)t\right] \quad (45)$$

After substituting these expressions and expanding terms, we obtain for the lateral plane wave excitation force and moment:

$$\frac{dY_w}{dx} = T_1 \cos \omega_e t + T_2 \sin \omega_e t \quad (46)$$

with

$$T_1 = T_3 \left[gT_4 \cos T_6 + c\ T_5 \sin T_6\right]$$

$$T_2 = T_3 \left[-gT_4 \sin T_6 + c\ T_5 \cos T_6\right]$$

$$T_3 = - ake^{-k\bar{h}} \sin\beta \left[ \frac{\sin\left(\frac{\pi B^*}{\lambda} \sin\beta\right)}{\frac{\pi B^*}{\lambda} \sin\beta} \right]$$

$$T_4 = \rho S + M_s - kM_{s\phi}$$

$$T_5 = N_s - V \frac{dM_s}{dx} + k\ V \frac{dM_{s\phi}}{dx}, \quad T_6 = -kx \cos\beta$$

and

$$\frac{dK_w}{dx} = T_7 \cos \omega_e t + T_8 \sin \omega_e t \quad (47)$$

with

$$T_7 = T_3 \left[g\ T_9 \cos T_6 + c\ T_{10} \sin T_6\right]$$

$$T_8 = T_3 \left[-g\ T_9 \sin T_6 + c\ T_{10} \cos T_6\right]$$

$$T_9 = \rho \left(\frac{B^{*3}}{12} - S\bar{z}\right) - M_{s\phi} - \overline{OG}\ T_4$$

$$T_{10} = N_{s\phi} + V \frac{dM_{s\phi}}{dx} - \overline{OG}\ T_5$$

The steady-state solution of the lateral plane equations of motion, at each particular regular wave length, are expressed as:

$$y = y_o \sin(\omega_e t + \kappa) \tag{48}$$

$$\psi = \psi_o \sin(\omega_e t + \alpha) \tag{49}$$

$$\phi = \phi_o \sin(\omega_e t + \nu) \tag{50}$$

where the zero-subscripted quantities are the motion response amplitudes and $\kappa$, $\alpha$ and $\nu$ are phase angle leads with respect to the wave elevation.

The local lateral (force) and rotational (moment) loadings derived in a manner similar to the vertical loading, are given by:

$$\frac{df_y}{dx} = -\delta m\,(\ddot{y}+x\ddot{\psi}-\zeta\ddot{\phi}) + \frac{dY}{dx} + \frac{dY_w}{dx} \tag{51}$$

$$\frac{dm_x}{dx} = -\delta m.\gamma^2\ddot{\phi} + \delta m\zeta(\ddot{y}+x\ddot{\psi}) + \rho g\left(\frac{B^{*3}}{12} - S\bar{z} - S\overline{OG}\right)\phi - g\delta m\zeta\phi + \frac{dK}{dx} + \frac{dK_w}{dx} \tag{52}$$

where $\zeta$ = local center of gravity (relative to ship C.G.), positive down

$\gamma$ = local mass gyradius in roll

and the hydrodynamic and wave excitation terms are given in Eqs. (33), (34), (46), and (47). While the local lateral loading is directly comparable to the local vertical loading, including inertial, hydrodynamic and wave excitation forces, the local rotational, or torsional, loading must in addition account for the static rotational moment, due to the initial metacentric height taken on a local (sectional) basis.

Finally, the wave-induced lateral bending moment and torsional moment at any location $x_o$ along the ship length are then given by:

$$BM_y(x_o) = \left[\int_{x_s}^{x_o} \text{ or } \int_{x_o}^{x_b}\right] (x-x_o)\ \frac{df_y}{dx}\ dx \tag{53}$$

$$TM_x(x_o) = \left[\int_{x_s}^{x_o} \text{ or } \int_{x_o}^{x_b}\right] \frac{dm_x}{dx}\ dx \tag{54}$$

and again they are expressed in this form:

$$\begin{aligned} BM_y &= BM_{yo} \sin\ (\omega_e t + \tau) \\ TM_x &= TM_{xo} \sin\ (\omega_e t + \nu) \end{aligned} \tag{55}$$

The parameters defining the ship mass distribution must meet certain constraints. The requirement on $\zeta$, the local vertical mass center, is:

$$\int_{x_s}^{x_b} \delta m \zeta\ dx = 0 \tag{56}$$

since $\zeta$ is measured relative to the ship C.G., and all first moments about that point must sum to zero, by definition. Similarly, the requirement on $\gamma$, the local roll gyradius, is:

$$\int_{x_s}^{x_b} \delta m \gamma^2 dx = I_x \tag{57}$$

The product of inertia in the x-z plane is then defined by:

$$I_{xz} = \int_{x_s}^{x_b} \delta m x \zeta dx \tag{58}$$

We should note here that it is usual practice in model test work that each overall segment, or portion, of the model is ballasted to the same overall specified V.C.G. and roll ryradius. However, data concerning the variation of $\zeta$ and $\gamma$ with length is usually not available.

Irregular Sea Equations

All of the results obtained in the preceeding analyses have been appropriate to conditions of regular sinusoidal unidirectional waves, which occur only in model test tanks. In a realistic seaway, waves appear randomly, and the motions and structural responses of a ship in such waves also have a random nature. In order to characterize the random ship responses, the energy spectra of the responses are employed. Each spectrum is a measure of the variation of the squares of the amplitudes of the various sinusoidal components of the particular random response, presented as a function of the wave frequency. The spectral technique for analyzing random irregular time histories of motion and structural response is applicable to linear systems only, since in that case a unique response amplitude operator is obtained. The spectral techniques evolve as a result of linear superposition, as originally developed in [11], of the responses to individual frequency components contained in the wave excitation.

The surface wave system, which is defined by the wave energy spectrum, is considered to be a separable function of wave frequency and direction, with limits, as follows:

$$S\,(\omega,\mu) = S_1(\omega)\;S_2(\mu) \qquad \text{for } 0\le\omega\le\infty \quad \text{and } -\frac{\pi}{2} \le \mu \le \frac{\pi}{2} \tag{59}$$

where $S\,(\omega,\mu)$ = directional spectrum of the seaway (short crested sea spectrum)
$\omega$ = circular wave frequency
$\mu$ = wave direction relative to predominent direction
$S_1(\omega)$ = frequency spectrum (long crested sea spectrum)
$S_2(\mu)$ = spreading function.

The mean squared wave amplitude is a basic measure of the total energy, or intensity, of the particular sea spectrum. It is obtained simply as the integral of all the various components, in continuous form, as:

$$\overline{a^2} = \int_0^\infty \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} S(\omega,\mu)\; d\omega\; d\mu \tag{60}$$

where $\overline{a^2}$ = mean squared wave amplitude, or variance of the wave time-history record. Since the spreading function depends on relative wave direction only, it is usual to impose the following constraint:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} S_2(\mu)\, d\mu = 1.0 \tag{61}$$

Therefore, we can define the mean squared wave amplitude in terms of the long crested sea spectrum as:

$$\overline{a^2} = \int_0^{\infty} S_1(\omega)\, d\omega \tag{62}$$

Other statistical parameters of interest for the sea spectrum, and similarly for any response spectrum, can be obtained from the mean squared amplitude, or variance, of the particular random variable. For the waves, we have:

$$a_{rms} = (\overline{a^2})^{1/2} \tag{63}$$

$$a_{avg} = 1.25\ a_{rms} \tag{64}$$

$$a_{1/3} = 2.0\ a_{rms} \tag{65}$$

$$a_{1/10} = 2.55\ a_{rms} \tag{66}$$

where
$a_{rms}$ = root-mean-squared wave amplitude
$a_{avg}$ = average (statistical) wave amplitude
$a_{1/3}$ = significant (average of 1/3 highest) wave amplitude
$a_{1/10}$ = average of 1/10 highest wave amplitude.

Various long crested, or unidirectional, sea spectra have been proposed over the years as representative of realistic conditions at sea. Three spectral formulations in popular usage among various investigators in the field are given below, for reference.

Neumann Spectrum (1953): This frequency spectrum [12] can be specified by:

$$S_1(\omega) = 0.000827\ g^2\pi^3\omega^{-6}e^{-2g^2\omega^{-2}U^{-2}} \tag{67}$$

where U = wind speed. The constant given here is one half that originally specified by Neumann, so that this spectrum satisfies Eq. (62). Thus, originally the Neumann spectrum required only a factor of $\sqrt{2}$ in Eq. (65), instead of 2.0.

Pierson-Moskowitz Spectrum (1964): This is given [13] by:

$$S_1(\omega) = 0.0081\ g^2\omega^{-5}e^{-.74g^4\omega^{-4}U^{-4}} \tag{68}$$

and was derived on the basis of fully arisen seas.

Two Parameter Spectrum (1967): This spectrum is intended for use in conjunction with "observed" wave height and period, which are then taken to be the significant height and mean period. This spectrum is similar to that adopted by the I.S.S.C. (1967) [14] as "nominal", except that it is expressed here in circular wave frequency instead of frequency in cycles per second:

$$S_1(\omega) = \underline{A}\cdot\underline{B}\omega^{-5}e^{-\underline{B}\omega^{-4}} \tag{69}$$

where

$$\underline{A} = 0.25\ (H_{1/3})^2$$

$$\underline{B} = (0.817\ \frac{2\pi}{\tilde{T}})^4$$

$$H_{1/3} = \text{significant wave height } (=2.0a_{1/3})$$

$$\tilde{T} = \text{mean wave period.}$$

The spreading function can be expressed for long crested, or unidirectional, seas as follows:

$$S_2(\mu) = \delta(\mu) \tag{70}$$

where $\delta(\ )$ = delta function.

For short crested seas, various spreading functions have been suggested and developed. Perhaps the most widely used, and a compromise among the proposed forms, is the cosine-squared spreading, expressed as:

$$S_2(\mu) = \frac{2}{\pi}\cos^2\mu \tag{71}$$

Having defined the wave excitation, or sea spectrum, in the forms as given above, the energy spectrum of the motions or structural responses can be calculated. In line with the linear assumption for all responses, and employing the principle of wave superposition, a response spectrum is obtained by:

$$S_i(\omega,\mu) = \left(T_i(\omega,\mu)\right)^2 S(\omega,\mu) \tag{72}$$

where $S_i(\omega,\mu)$ = response spectrum, for a particular response

$T_i(\omega,\mu)$ = response amplitude operator (amplitude of i-th response per unit wave amplitude).

We then have, similar to the wave amplitude:

$$\overline{a_i^2} = \int_0^\infty \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} S_i(\omega,\mu)\; d\omega \quad \mu$$

$$\overline{a_i^2} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} S_2(\mu) \left[ \int_0^\infty \left(T_i(\omega,\mu)\right)^2 S_1(\omega)\; d\omega \right] d\mu \tag{73}$$

where $\overline{a_i^2}$ = mean squared response amplitude. The term in square brackets in the integral above is the mean squared response amplitude for long crested seas at a particular heading $\mu$, relative to the predominant wave direction. The other statistical parameters of interest for each response can be obtained from the mean squared amplitude by use of equations similar to Eqs. (63) to (66).

## EXPERIMENTAL DATA

In order to evaluate the analytical methods presented for the calculation of wave-induced moments, the results of such calculations are to be compared with experimental results. Experimentation on ship models, under controlled laboratory conditions, for the determination of wave-induced moments is a relatively new procedure. Lewis [15] first presented such results for vertical bending in 1954. These initial tests were limited to head and following seas directions. The tests measured midship bending moments only.

Subsequently, the experimental procedures were expanded to cover a greater degree and range of relevent parameters, approaching description of responses under various realistic conditions. Among the earliest tests conducted at oblique wave headings in order to yield both vertical and lateral bending moments, and torsional moments as well, were those of Numata [16] conducted at Davidson Laboratory on a T-2 tanker model. This work was quickly followed by an extensive series of tests on Series 60 models by Vossers, et al, [17] at the Netherlands Ship Model Basin (NSMB), reported in 1961. In addition, tests also have been conducted for determination of wave-induced loads at points along the hull other than midships. However, at this time the total amount of all such data is not very large and some experimental problems still exist. Very little data has been collected with regard to torsional moments, and therefore the emphasis in the comparison to follow will be upon the vertical and lateral bending moments at midships.

A fairly intensive test series was reported by Wahab [18] in 1967. These tests of only one Series 60 hull form, with block coefficient of 0.80, were conducted over a large and exhaustive range of regular wave lengths and wave angles. Measurements included vertical and lateral bending moments, plus vertical and lateral shears and torsional moment, all at midships. Recently the Ship Structure Committee has supported additional experimental work at Davidson Laboratory that is related to other full-scale measurement projects. The model tests have been reported by Chiocco and Numata [19] for the "Wolverine State," and by Numata and Yonkers [20] for the Mariner-class "California Bear."

With regard to the comparison between such experimental data and the projected calculations, certain conditions of the model tests should be recognized. The bulk of the test data to be used in this comparison are the results of model tests in regular waves at oblique headings, referenced above. Such tests are conducted by using a fairly free-running self-propelled model. The model must then have an operational rudder which is used to maintain the model along the prescribed wave-to-course angle. In more recent tests of this type, the rudder is controlled by an automatic procedure and/or device based on yaw and sway motions that are sensed by elements on the model, while in some early tests the rudder was controlled manually. In regular wave tests, it would appear that the rudder movements could contain significant encounter frequency content. In any event it is clear that the rudder action influences the model motion responses under such conditions. Furthermore, the rudder forces generated in this manner contribute directly to the total loading distribution on the hull, which is assumed to be in equilibrium. Since the lever arm of the rudder forces is large for moments at midships, it appears that rudder forces can significantly affect the lateral bending and torsional moments. To the extent that the use of the rudder affects the overall ship motion response in oblique seas, the vertical bending moment also can be influenced, but to a much smaller degree. The calculations, based on the analytical method presented earlier, do not include any rudder force and moment effects.

The above discussion of rudder effects only points out a difference between experimental conditions and the proposed calculations. Another point, and one of perhaps equal importance but not directly bearing on the subject comparison at hand, is whether such model scale rudder forces and control techniques are representative of full-scale effects. Questions of scale effect and response times enter into this problem, and will not be considered here. The point is, however, that the calculations are being compared with experiments which include additional unaccounted effects, which are not necessarily realistic with regard to full-scale behavior.

Another aspect of the experimental conditions also is significant with regard to the comparison with calculations. In the experiments at oblique wave angles, it is noted that the model's mean heading angle differs from the mean wave-to-course angle, the difference being referred to as the leeway angle. The leeway angle appears to be due to the non-zero mean lateral forces and moments imposed by the waves. It is greatest at low speeds in relatively short wavelengths. Thus, for example, in an experiment at a wave-to-course angle of 120°, bow seas, the actual average heading of the hull to the waves may be as high as 135° [16]. The analytical methods take no account of such mean, or drift, forces and moments, so that in the calculations the leeway angle is assumed to be zero. Since no account of the leeway angle is made in the computations, and the wave-to-course angle used in the computations is the nominal value prescribed in the tests, the influence of the actual heading of the ship relative to the waves is not accounted for properly. In the model tests, the mean wave forces and moments which cause the leeway angle, and the mean hydrodynamic forces and moments resulting thereby, are supposedly in balance with the force and moment from a non-zero mean rudder angle. That is, it is usually necessary to apply a mean rudder angle in order to keep the model on a prescribed mean course, but with a particular resulting leeway angle.

Obviously, these forces and moments have some effect upon the motion responses of the model and therefore upon the measured moments. The extent and nature of such effects are unknown, although the only important effects will be those forces at the frequency of encounter in the regular wave tests. However, in the reports of the experimental work little or no significance is given to these forces. The details of the rudder and control system are not described. Rudder motion is not given, and even leeway angle is not always reported. Thus, at this point in the development, the experimental inputs for comparison with a full analytical treatment of rudder forces and mean wave forces and moments, if such were desired, are not yet available. The effects then of leeway angle and rudder forces may turn out to be small in many cases, but they must still be recognized as an unknown element in the comparison.

## COMPARISON OF CALCULATIONS WITH EXPERIMENT

All of the calculations of wave-induced moments were done by use of a digital computer program developed in the course of this work, and fully described elsewhere [3]. The program follows the analytical methods presented in this report and its predecessors [1, 2]. The calculations of the midship wave-induced moments were carried out for hull forms, mass distributions and test conditions corresponding to the bulk of the experimental data cited previously. In general, sufficient data was available in the model test reports with regard to the full description of the necessary significant parameters for input to the computer calculation. However, as pointed out previously, no data was available with regard to the longitudinal distribution of $\zeta$, the local vertical center of gravity, and $\gamma$, the local roll gyradius. These parameters can be expected to affect the lateral bending moment in the region of roll resonance only, and also for the torsional moment. In some cases a reasonable approximation to the vertical center of gravity distribution was employed, corresponding to the usual model test ballasting methods. In these cases, the lateral bending moment calculation results were seen (via numerical tests), to be sensitive to this distribution in the region of roll resonance. The use of a reasonable approximation generally yielded results which were in better agreement with the experimental results.

In order to simplify the presentation of the results of the computations, and comparison with model test data, Table 1 has been prepared. It lists the calculations to be presented herein, together with the reference for the experimental results. For each of the five sets of calculations, Tables 2 to 6 give the basic hull form and mass distribution data used, based on the input values specified and inherent assumptions in the computer program. Also shown in Table 1 are the roll damping fractions used in the computations for lateral plane motions, and the figure numbers which give the results, including comparison with corresponding experimental data.

Primarily the comparison is made for the Wolverine State data [19] and the Series 60, block 0.80 hull data of Wahab [18]. These represent more recent tests of this type, where experimental procedures are perhaps more refined compared to earlier tests. The Wolverine State comparison is for two different hull loadings, two speeds, and over a fairly wide range of wave angles and wavelengths. The Series 60, block 0.80 hull comparison is at one loading and speed, but the experimental data cover a wide range of wave lengths and angles more intensively. The comparison is also presented for the Series 60, block 0.70 hull data (NSMB, 1961) and the T-2 Tanker Model (Davidson Lab., 1960) so that a wider range of hull forms and test conditions can be covered. From Tables 2 to 6, it can be seen that twenty stations along the ship's length were generally used to define the hull form and mass distribution. This is considered an appropriate number, compared with other numerical aspects

Table 1. Calculations Reference Data

| | Model Description | Model Test Reference | Basic Data | $\zeta_\phi$ | Calculation Results |
|---|---|---|---|---|---|
| 1 | Wolverine State, Full Load | [19] | Table 2 | 0.05 | Figs. 2, 3 |
| 2 | Wolverine State, Light Load | [19] | Table 3 | 0.05 | Figs. 2, 3 |
| 3 | Series 60, Block .80 Hull | [18] | Table 4 | 0.10 | Figs. 4-7 |
| 4 | Series 60, Block .70 Hull | [17] | Table 5 | 0.05 | Figs. 8-11 |
| 5 | T-2 Tanker Model | [16] | Table 6 | 0.05 | Figs. 12 |

Table 2. Basic Data for WOLVERINE STATE, Full Load

WOLVERINE STATE. FULL LOAD, DAVIDSON LAB. TEST CONDITION - OCEANICS PROJECT 1093

OPTION CONTROL TAGS - A B C D E F G H I J
1 2 1 0 1 0 0 1 0 -1 NO. OF STATIONS = 20

BASIC INPUT DATA

LENGTH = 496.00 DENSITY = .028570
DISPL. = 19875.00 GRAVITY = 32.175000

| STATION | BEAM | AREA COEF. | DRAFT | Z-BAR | WEIGHT | ZETA | GYR.ROLL |
|---|---|---|---|---|---|---|---|
| 0.00 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 104.0000 | -21.0000 | 26.6000 |
| 1.00 | 11.2000 | .8060 | 30.0000 | 13.0600 | 306.0000 | -20.8366 | 26.6000 |
| 2.00 | 24.4000 | .8480 | 30.0000 | 13.4800 | 532.0000 | -20.0000 | 26.6000 |
| 3.00 | 38.6500 | .8840 | 30.0000 | 13.8400 | 862.0000 | -17.0000 | 26.6000 |
| 4.00 | 51.4000 | .9130 | 30.0000 | 14.1300 | 1060.0000 | -12.0000 | 26.6000 |
| 5.00 | 61.3000 | .9330 | 30.0000 | 14.3300 | 1201.0000 | -6.0000 | 26.6000 |
| 6.00 | 67.4000 | .9570 | 30.0000 | 14.5700 | 1310.0000 | 2.0000 | 26.6000 |
| 7.00 | 70.5000 | .9750 | 30.0000 | 14.7500 | 1399.0000 | 8.0000 | 26.6000 |
| 8.00 | 71.5000 | .9840 | 30.0000 | 14.8400 | 1418.0000 | 10.0000 | 26.6000 |
| 9.00 | 71.5000 | .9890 | 30.0000 | 14.8900 | 1428.0000 | 12.0000 | 26.6000 |
| 10.00 | 71.5000 | .9890 | 30.0000 | 14.8900 | 1442.0000 | 12.0000 | 26.6000 |
| 11.00 | 71.5000 | .9840 | 30.0000 | 14.8400 | 1446.0000 | 12.0000 | 26.6000 |
| 12.00 | 71.5000 | .9670 | 30.0000 | 14.6700 | 1395.0000 | 10.0000 | 26.6000 |
| 13.00 | 71.5000 | .9310 | 30.0000 | 14.3100 | 1296.0000 | 8.0000 | 26.6000 |
| 14.00 | 71.2000 | .8630 | 30.0000 | 13.6300 | 1079.0000 | 2.0000 | 26.6000 |
| 15.00 | 69.4000 | .7720 | 30.0000 | 12.7200 | 791.0000 | -6.0000 | 26.6000 |
| 16.00 | 63.4000 | .6730 | 30.0000 | 11.7300 | 716.0000 | -12.0000 | 26.6000 |
| 17.00 | 52.3000 | .5830 | 30.0000 | 10.8300 | 772.0000 | -17.0000 | 26.6000 |
| 18.00 | 37.4000 | .4950 | 30.0000 | 9.9500 | 593.0000 | -19.0000 | 26.6000 |
| 19.00 | 21.9500 | .3860 | 30.0000 | 8.8600 | 513.0000 | -20.0000 | 26.6000 |
| 20.00 | 6.2500 | .5000 | 3.0300 | 1.0100 | 212.0000 | -21.1840 | 26.6000 |

OG = -4.500 GYRADIUS.ROLL = 26.600

CALCULATE MOMENTS AT STATION 10

DERIVED RESULTS

DISPL.(WTS.) = 19875.00

LONG. C.B. = 2.966 (FWD. OF MIDSHIPS) DISPL.(VOL.) = 19862.03

LONG. C.G. = 3.212 (FWD. OF MIDSHIPS) LONG. GYRADIUS = 116.689 GM = 3.722

## Table 3. Basic Data for WOLVERINE STATE, Light Load

WOLVERINE STATE. LIGHT LOAD. DAVIDSON LAB. TEST CONDITION - OCEANICS PROJ. 1093

OPTION CONTROL TAGS - A B C D E F G H I J
0 2 1 0 1 0 0 1 0 1 NO. OF STATIONS = 10

BASIC INPUT DATA

LENGTH = 496.00 DENSITY = .028570
DISPL. = 12105.00 GRAVITY = 32.175000

| STATION | BEAM | AREA COEF. | DRAFT | Z-BAR | WEIGHT | ZETA | GYR.ROLL |
|---|---|---|---|---|---|---|---|
| .50 | 9.7500 | .8400 | 17.3200 | 7.7363 | 551.0000 | -24.0010 | 26.4500 |
| 1.50 | 36.5000 | .8700 | 17.7600 | 8.1104 | 823.0000 | -10.0010 | 26.4500 |
| 2.50 | 60.1000 | .9100 | 18.2000 | 8.5540 | 1670.0000 | -3.9210 | 26.4500 |
| 3.50 | 70.6000 | .9550 | 18.6400 | 9.0404 | 1970.0000 | 7.9990 | 26.4500 |
| 4.50 | 71.5000 | .9750 | 19.0800 | 9.3810 | 1067.0000 | 11.4990 | 26.4500 |
| 5.50 | 71.5000 | .9700 | 19.5200 | 9.5648 | 1600.0000 | 11.4990 | 26.4500 |
| 6.50 | 71.0000 | .9000 | 19.9600 | 9.3147 | 1390.0000 | 7.4990 | 26.4500 |
| 7.50 | 62.8500 | .7700 | 20.4000 | 8.6360 | 1256.0000 | -1.3010 | 26.4500 |
| 8.50 | 37.0000 | .6900 | 20.8400 | 8.2665 | 1077.0000 | -10.0010 | 26.4500 |
| 9.50 | 8.6000 | .7400 | 21.2800 | 8.7957 | 685.0000 | -24.0010 | 26.4500 |

OG = 3.200 GYRADIUS.ROLL = 26.450
CALCULATE MOMENTS AT STATION 5

DERIVED RESULTS

DISPL.(WTS.) = 12089.00

LONG. C.B. = .701 (FWD. OF MIDSHIPS) DISPL.(VOL.) = 12147.21

LONG. C.G. = .601 (FWD. OF MIDSHIPS) LONG. GYRADIUS = 123.391 GM = 7.332

## Table 4. Basic Data for SERIES 60, BLOCK .80 Hull

SERIES 60 HULL FORM. 0.80 BLOCK (TNO RPT. NO. 100 S) OCEANICS PROJECT NO. 1093

OPTION CONTROL TAGS - A B C D E F G H I J
1 2 1 3 1 0 2 0 1 1 NO. OF STATIONS = 20

BASIC INPUT DATA

LENGTH = 193.00 DENSITY = 1.025000
DISPL. = 48126.40 GRAVITY = 9.806650

| STATION | BEAM | AREA COEF. | DRAFT | Z-BAR | WEIGHT | ZETA | GYR.ROLL |
|---|---|---|---|---|---|---|---|
| 0.00 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 240.6000 | 0.0000 | 8.9602 |
| 1.00 | 14.3900 | .8720 | 11.0300 | 5.0444 | 481.3000 | 0.0000 | 8.9602 |
| 2.00 | 22.8800 | .8940 | 11.0300 | 5.1253 | 1203.2000 | 0.0000 | 8.9602 |
| 3.00 | 26.5800 | .9290 | 11.0300 | 5.2540 | 2406.3000 | 0.0000 | 8.9602 |
| 4.00 | 27.5400 | .9700 | 11.0300 | 5.4047 | 3850.1000 | 0.0000 | 8.9602 |
| 5.00 | 27.5700 | .9910 | 11.0300 | 5.4819 | 4090.7000 | 0.0000 | 8.9602 |
| 6.00 | 27.5700 | .9940 | 11.0300 | 5.4929 | 4331.4000 | 0.0000 | 8.9602 |
| 7.00 | 27.5700 | .9940 | 11.0300 | 5.4929 | 4331.4000 | 0.0000 | 8.9602 |
| 8.00 | 27.5700 | .9940 | 11.0300 | 5.4929 | 3368.8000 | 0.0000 | 8.9602 |
| 9.00 | 27.5700 | .9940 | 11.0300 | 5.4929 | 1684.4000 | 0.0000 | 8.9602 |
| 10.00 | 27.5700 | .9940 | 11.0300 | 5.4929 | 1684.4000 | 0.0000 | 8.9602 |
| 11.00 | 27.5700 | .9940 | 11.0300 | 5.4929 | 1443.8000 | 0.0000 | 8.9602 |
| 12.00 | 27.5700 | .9930 | 11.0300 | 5.4893 | 2195.8000 | 0.0000 | 8.9602 |
| 13.00 | 27.5700 | .9890 | 11.0300 | 5.4746 | 3290.7000 | 0.0000 | 8.9602 |
| 14.00 | 27.5700 | .9680 | 11.0300 | 5.3973 | 3633.6000 | 0.0000 | 8.9602 |
| 15.00 | 27.2400 | .9210 | 11.0300 | 5.2245 | 3465.1000 | 0.0000 | 8.9602 |
| 16.00 | 25.9400 | .8510 | 11.0300 | 4.9672 | 3146.3000 | 0.0000 | 8.9602 |
| 17.00 | 23.4600 | .7580 | 11.0300 | 4.6252 | 1955.1000 | 0.0000 | 8.9602 |
| 18.00 | 19.6300 | .6270 | 11.0300 | 4.1436 | 721.9000 | 0.0000 | 8.9602 |
| 19.00 | 13.8700 | .4190 | 11.0300 | 3.3789 | 481.3000 | 0.0000 | 8.9602 |
| 20.00 | 4.4100 | .5300 | 1.1000 | .3777 | 120.3000 | 0.0000 | 8.9602 |

OG = -1.099 GYRADIUS.ROLL = 8.960
CALCULATE MOMENTS AT STATION 10

DERIVED RESULTS

DISPL.(WTS.) = 48126.50

LONG. C.B. = 4.716 (FWD. OF MIDSHIPS) DISPL.(VOL.) = 48077.53

LONG. C.G. = 4.825 (FWD. OF MIDSHIPS) LONG. GYRADIUS = 46.159 GM = 1.378

## Table 5. Basic Data for SERIES 60, BLOCK .70 Hull

SERIES 60 HULL FORM, .70 BLOCK, L/B = 7.0, L/H = 17.5 (NSMB TESTS) OCEANICS 1093

OPTION CONTROL TAGS - A B C D E F G H I J
1 2 1 0 1 0 2 1 1 1 NO. OF STATIONS = 20

BASIC INPUT DATA

LENGTH = 400.00 DENSITY = .028571
DISPL. = 10456.00 GRAVITY = 32.175000

| STATION | BEAM | AREA COEF. | DRAFT | Z-BAR | WEIGHT | ZETA | GYR.ROLL |
|---|---|---|---|---|---|---|---|
| 0.00 | .1700 | .9000 | 1.0000 | .4667 | 52.3000 | -9.1075 | 21.3500 |
| 1.00 | 9.4300 | .8486 | 22.8600 | 10.2763 | 104.6000 | -6.6075 | 21.3500 |
| 2.00 | 21.8800 | .8341 | 22.8600 | 10.1658 | 209.1000 | -4.6075 | 21.3500 |
| 3.00 | 34.5100 | .8619 | 22.8600 | 10.3777 | 366.0000 | -2.6075 | 21.3500 |
| 4.00 | 44.9700 | .8908 | 22.8600 | 10.5979 | 679.6000 | -1.1075 | 21.3500 |
| 5.00 | 52.0000 | .9210 | 22.8600 | 10.8280 | 815.6000 | -.0075 | 21.3500 |
| 6.00 | 55.6000 | .9526 | 22.8600 | 11.0688 | 1045.6000 | .8925 | 21.3500 |
| 7.00 | 56.9100 | .9761 | 22.8600 | 11.2479 | 993.3000 | 1.3925 | 21.3500 |
| 8.00 | 57.1400 | .9850 | 22.8600 | 11.3157 | 313.7000 | 1.8925 | 21.3500 |
| 9.00 | 57.1400 | .9860 | 22.8600 | 11.3233 | 313.7000 | 1.8925 | 21.3500 |
| 10.00 | 57.1400 | .9860 | 22.8600 | 11.3233 | 334.6000 | 1.8925 | 21.3500 |
| 11.00 | 57.1400 | .9860 | 22.8600 | 11.3233 | 444.4000 | 1.8925 | 21.3500 |
| 12.00 | 57.1400 | .9830 | 22.8600 | 11.3005 | 627.4000 | 1.8925 | 21.3500 |
| 13.00 | 57.0800 | .9672 | 22.8600 | 11.1801 | 836.5000 | 1.3925 | 21.3500 |
| 14.00 | 56.6800 | .9363 | 22.8600 | 10.9446 | 836.5000 | .8925 | 21.3500 |
| 15.00 | 55.3700 | .8812 | 22.8600 | 10.5247 | 862.6000 | -.0075 | 21.3500 |
| 16.00 | 52.1100 | .8109 | 22.8600 | 9.9891 | 784.2000 | -1.1075 | 21.3500 |
| 17.00 | 46.6800 | .7133 | 22.8600 | 9.2453 | 470.5000 | -2.6075 | 21.3500 |
| 18.00 | 38.4000 | .5869 | 22.8600 | 8.2822 | 209.1000 | -4.6075 | 21.3500 |
| 19.00 | 26.4000 | .3884 | 22.8600 | 6.7696 | 104.6000 | -6.6075 | 21.3500 |
| 20.00 | 8.9100 | .5060 | 2.2800 | .7646 | 52.3000 | -9.1075 | 21.3500 |

OG = -2.554 GYRADIUS.ROLL = 21.350

CALCULATE MOMENTS AT STATION 10

DERIVED RESULTS

DISPL.(WTS.) = 10456.20

LONG. C.B. = -2.463 (FWD. OF MIDSHIPS) DISPL.(VOL.) = 10444.72

LONG. C.G. = -2.000 (FWD. OF MIDSHIPS) LONG. GYRADIUS = 95.980 GM = 2.857

## Table 6. Basic Data for T-2 Tanker Model

T-2 TANKER MODEL, DAVIDSON LAB. (NUMATA) TEST CONDITIONS - OCEANICS PROJECT 1093

OPTION CONTROL TAGS - A B C D E F G H I J
1 2 0 0 1 0 2 1 0 1 NO. OF STATIONS = 20

BASIC INPUT DATA

LENGTH = 4.80 DENSITY = 62.500000
DISPL. = 41.02 GRAVITY = 32.175000

| STATION | BEAM | AREA COEF. | DRAFT | Z-BAR | WEIGHT | ZETA | GYR.ROLL |
|---|---|---|---|---|---|---|---|
| 0.00 | 0.0000 | 0.0000 | .2860 | .0477 | .1313 | 0.0000 | .2550 |
| 1.00 | .1670 | .8375 | .2860 | .1275 | .3243 | 0.0000 | .2550 |
| 2.00 | .3450 | .8716 | .2860 | .1308 | 1.2278 | 0.0000 | .2550 |
| 3.00 | .4870 | .8616 | .2860 | .1298 | 1.5598 | 0.0000 | .2550 |
| 4.00 | .5810 | .8967 | .2860 | .1332 | 2.3243 | 0.0000 | .2550 |
| 5.00 | .6330 | .9390 | .2860 | .1372 | 2.5946 | 0.0000 | .2550 |
| 6.00 | .6480 | .9766 | .2860 | .1408 | 2.6795 | 0.0000 | .2550 |
| 7.00 | .6480 | .9820 | .2860 | .1413 | 2.8726 | 0.0000 | .2550 |
| 8.00 | .6480 | .9820 | .2860 | .1413 | 3.1351 | 0.0000 | .2550 |
| 9.00 | .6480 | .9820 | .2860 | .1413 | 2.8108 | 0.0000 | .2550 |
| 10.00 | .6480 | .9820 | .2860 | .1413 | 2.7799 | 0.0000 | .2550 |
| 11.00 | .6480 | .9820 | .2860 | .1413 | 2.7799 | 0.0000 | .2550 |
| 12.00 | .6480 | .9820 | .2860 | .1413 | 2.7876 | 0.0000 | .2550 |
| 13.00 | .6480 | .9659 | .2860 | .1397 | 2.7568 | 0.0000 | .2550 |
| 14.00 | .6480 | .9389 | .2860 | .1372 | 2.7027 | 0.0000 | .2550 |
| 15.00 | .6410 | .9000 | .2860 | .1335 | 2.2780 | 0.0000 | .2550 |
| 16.00 | .6140 | .8428 | .2860 | .1280 | 2.0386 | 0.0000 | .2550 |
| 17.00 | .5500 | .7438 | .2860 | .1186 | 1.3745 | 0.0000 | .2550 |
| 18.00 | .4350 | .5787 | .2860 | .1028 | 1.0425 | 0.0000 | .2550 |
| 19.00 | .2830 | .4324 | .2860 | .0889 | .4788 | 0.0000 | .2550 |
| 20.00 | .1020 | .5100 | .0520 | .0175 | .3398 | 0.0000 | .2550 |

OG = -.070 GYRADIUS.ROLL = .255

CALCULATE MOMENTS AT STATION 10

DERIVED RESULTS

DISPL.(WTS.) = 41.02

LONG. C.B. = .020 (FWD. OF MIDSHIPS) DISPL.(VOL.) = 40.94

LONG. C.G. = .021 (FWD. OF MIDSHIPS) LONG. GYRADIUS = 1.090 GM = .052

of the computer program, in order to obtain reasonable results at the shorter wavelengths of interest.

The results of the calculations are presented in the same form as the experimental data in the various sources. For the Wolverine State, the results are given for the full-scale ship. For the T-2 Tanker, model-scale results are shown. For the Series 60 hulls, results are shown in non-dimensional form, as follows:

Froude Number: $F_n = V/\sqrt{gL}$

Non-dimensional wave frequency $= \omega\sqrt{L/g}$

Non-dimensional moment $= \dfrac{BM_z \text{ (or } BM_y \text{ or } TM_x)}{\rho g\, B^*_{⊠}\, L^2 a}$

Non-dimensional shear $= \dfrac{\text{Shear}}{\rho g B^*_{⊠}\, La}$

where $B^*_{⊠}$ = waterline beam amidships.

The comparison between calculations of vertical and lateral bending moments and the experimental results for the Wolverine State, shown in Figures 2 and 3, indicates generally very good agreement. This holds for both loading conditions, both speeds, and over the range of wave angle and wavelength. The experimental results shown for lateral bending moment in head and following seas, where lateral motions and loads should be zero as in the calculations, are regarded as indicative of the possible error, or range of discrepancy, to be expected between calculations and experimental results. These loads are believed to arise in the model tests due to its free-running, but rudder controlled, condition. That is, the model may undergo small lateral motions, with rudder corrections to keep course, which leads to the measured lateral bending moments.

The comparison for the Series 60, block 0.80 hull shown in Figure 4 for vertical and lateral bending moments indicates excellent agreement, in general. Similar results were also shown for this hull by Faltinsen [21] based on a new strip theory of Salvesen, et al [22]. Figure 5 shows the torsional moment comparison, while in Figure 6 the vertical and lateral shear forces, which were also measured by Wahab [18], are shown. The agreement for torsional moments is only fair and indicates excessive response at roll resonance conditions. The agreement for the shear forces is quite good, in general, with the exception of some deviation in lateral shear at 110° wave angle. However, the shear forces are generally small at midships, and should really be investigated at the quarter-length points. Vertical and lateral bending moment responses in irregular seas are shown in Figure 7. The experimental results

---

Note: Figures 2--12 are grouped at end of report beginning with page 33.

are not from direct irregular wave testing, but rather are calculated from the regular wave unidirectional data, using the particular sea spectrum indicated. The difference between long crested and short crested seas results are particularly interesting for the lateral bending moment. They show that while the response is minimal in unidirectional beam seas, compared to the peaks at bow and stern quartering headings, the short crested seas response is maximum in beam seas.

Figures 8 and 11 show the comparison for vertical and lateral bending moments for the Series 60, block 0.70 hull form. A wide range of ship speed is covered in this data. The T-2 Tanker model comparison is shown in Figure 12. For the 150 and 120 degree wave angles, experimental data and calculations are shown over a range of speed for two wavelengths, i.e. a wavelength equal to model length and a wavelength such that its "effective length" is equal to model length. In the latter case, the actual wavelength equalled the model length times the cosine of the wave angle. This data covers vertical and lateral bending, and torsional moments. In general, the agreement is fairly satisfactory, considering the factors involved in the experimental comparison. With regard to this point, consider the double peak calculated vertical bending moment response for the T-2 Tanker at 120° wave heading and 1.65 fps model speed (Figure 12h). While the corresponding experimental data do not indicate such a response similar double peaked responses for vertical bending are confirmed by experimental results for Wolverine State, full load (Figure 2c), and the Series 60, block 0.80 hull (Figure 4b). The greater resolution of the test data due to testing at more wavelength conditions for these latter cases tends to produce such results, thereby limiting the utility of the experimental points for the T-2 Tanker as a complete measure of bending moment variation.

The preceding comparisons have demonstrated the capability of the present analysis and its computer implementation to provide valid predictions of wave-induced structural loads on conventional ship hull forms. As discussed previously, the technique used is based upon a sectional representation with Lewis forms, and hence bulbous bows cannot be represented accurately (i.e. in matching the desired sectional form with the resulting shape obtained by the Lewis form fit). However some limited results obtained by comparing the outputs from a Lewis form representation with that from an accurate "close fit" technique (see [23]) showed little effect on the resulting motions of heave and pitch when using either method of determining the two-dimensional sectional added mass and damping, although the inability to match the section form was demonstrated. This result would appear to imply that the use of the Lewis form fit produced sufficiently useful data for sectional forces that would manifest whatever influence was exhibited by the bulbous bow form, or possibly that such a localized force did not have a significant influence on the overall body motions. In either case the same characteristics would be expected to carry over as well to the case of the computation of bending moments, and hence the presently developed technique can also be used for predictions for the case of bulbous bow hulls. Since the computation of the sectional added mass and damping are determined by a specific subroutine in the overall computer program in [3], and only a limited portion of the hull (at the bow) is affected by the bulb, the use of a specialized procedure for that region can be adapted if desired, based on the methods and computer program used in [9], for example, or any other simple computer program developed to encompass bulbous bow hulls.

## CONCLUSIONS

An analytical method for the determination of wave-induced moments on ships has been developed, implemented (via computer program), and successfully evaluated by comparison with a large body of model-scale experimental data. It should prove to be a valuable aid to, and integral element of, the fundamental and rational ship structural design approach. It can be used to predict the ship motions and wave-induced vertical and lateral bending moments, and torsional moment, at any station along the length, for a ship traveling at any heading relative to long or short crested seas.

The computer program, which embodies the developed method, is documented in complete detail in [3]. It can be used in the basic ship design process for the prediction or determination of both ship motions and the wave-induced structural loads. The approach and implementation are straight-forward, and the program is efficient in regard to computer time usage.

While the possibilities for use of the analytical method appear great, some additional development work would seem to be in order. The influence of rudder effects should be investigated. The effect of the rudder and control system upon ship motions and loads needs some careful attention. In addition, the effects due to mean drift forces and moments, manifested by leeway angles and mean rudder angles, ought to be determined. The present evaluation of the method indicates that such effects are relatively small, since the responses of interest are those of oscillatory nature with a frequency equal to the encounter frequency in regular waves, but a fuller understanding of their influence is nevertheless required.

REFERENCES

1. Kaplan, P., "Development of Mathematical Models for Describing Ship Structural Response in Waves," Ship Structure Committee Report SSC-193, January 1969 (AD 682591)

2. Kaplan, P., T.P. Sargent and A.I. Raff, "An Investigation of the Utility of Computer Simulation to Predict Ship Structural Response in Waves," Ship Structure Committee Report SSC-197, June 1969 (AD 690229)

3. Raff, A.I., "Program SCORES - Ship Structural Response in Waves," Oceanics, Inc. Tech. Report No. 70-75, August 1970

4. Korvin-Kroukovsky, B.V., "Investigation of Ship Motions in Regular Waves," Trans. SNAME, Vol. 63, 1955

5. Korvin-Kroukovsky, B.V. and W.R. Jacobs, "Pitching and Heaving Motions of a Ship in Regular Waves," Trans. SNAME, Vol. 65, 1957

6. Lewis, F.M., "The Inertia of Water Surrounding a Vibratory Ship," Trans. SNAME, 1929

7. Grim, O., "Die Schwingungen von schwimmeden, zweidimensionalen Korpern," HSVA Report No. 1171, September 1959

   Grim, O. and M. Kirsch, private communication, September 1967

8. Tasai, F., "Hydrodynamic Force and Moment Produced by Swaying and Rolling Oscillation of Cylinders on the Free Surface," Reports of Research Institute for Applied Mechanics, Kyushu University, Japan, Vol. IX, No. 35, 1961

9. Vugts, J.H., "The Hydrodynamic Coefficients for Swaying, Heaving and Rolling Cylinders in a Free Surface," Laboratorium voor Scheepsbouwkunde, Technische Hogeschool Delft, Report No. 194, January 1968

10. Vugts, J.H., "The Coupled Roll-Sway-Yaw Performance in Oblique Waves," Laboratorium voor Scheepsbouwkunde, Technische Hogeschool Delft, Report 245, July 1969

11. St. Denis, J. and W.J. Pierson, Jr., "On the Motions of Ships in Confused Seas," Trans. SNAME, Vol. 61, 1953

12. Neumann, G., "On Ocean Wave Spectra and a New Method of Forecasting Wind Generated Sea," Tech. Mem. 43, Beach Erosion Board, Corps of Engineers, 1953

13. Pierson, W.J., and L. Moskowitz, "A Proposed Spectral Form For Fully Developed Wind Seas Based on the Similarity Theory of S. A. Kitaigorodskii," J. Geophys. Res., Vol. 69, No. 24, 1964

14. Report of Committee I, "Proceedings of the Third International Ship Structures Congress," issued by Det Norske Veritas, Olso, 1967

15. Lewis, E.V., "Ship Model Tests to Determine Bending Moments in Waves," Trans. SNAME, Vol. 62, 1954

16. Numata, E., "Longitudinal Bending and Torsional Moments Acting on a Ship Model at Oblique Headings to Waves," J. Ship Res., Vol. 4, No. 1, 1960

17. Vossers, G., W. A. Swaan and H. Rijken, "Vertical and Lateral Bending Moment Measurements on Series 60 Models," Int. Shipbuilding Progress, Vol. 8, No. 83, 1961

18. Wahab, R., "Admidships Forces and Moments on a $C_B$=0.80 "Series 60" Model in Waves from Various Directions," Netherlands Ship Research Centre TNO, Report No. 100S, November 1967

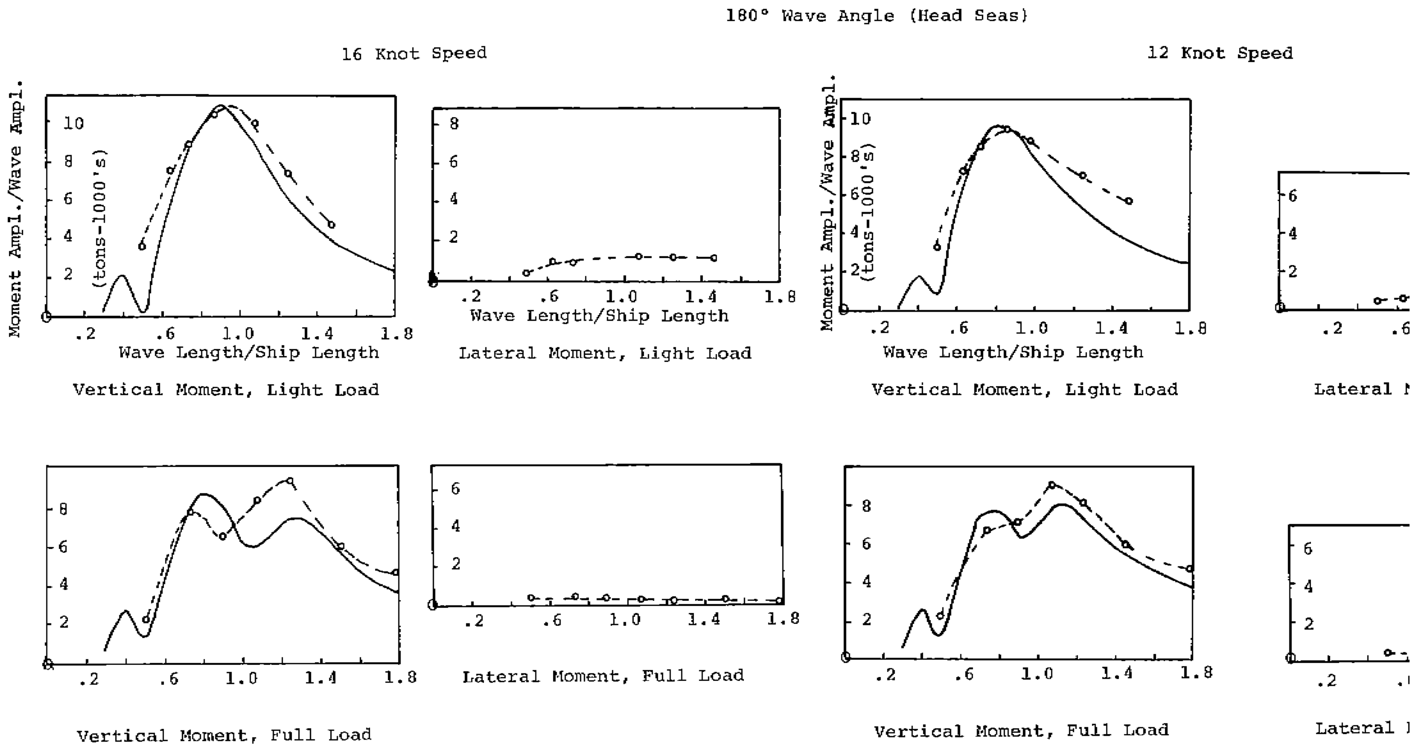
19. Chiocco, M.J. and E. Numata, "Midship Wave Bending Moments in a Model of the Cargo Ship "Wolverine State" Running at Oblique Headings in Regular Waves," Ship Structure Committee Report SSC-201, September 1969 (AD 695123)

20. Numata, E. and W.F. Yonkers, "Midship Wave Bending Moments in a Model of the Mariner-class Cargo Ship "California Bear" Running at Oblique Headings in Regular Waves," Ship Structure Committee Report SSC-202, November 1969 (AD 698847)

21. Faltinsen, O., "Comparison Between Theory and Experiments of Wave Induced Loads for Series 60 Hull with $C_B$=0.80," Det Norske Veritas, Res. Dept., Rpt. No. 70-27-S, July 1970

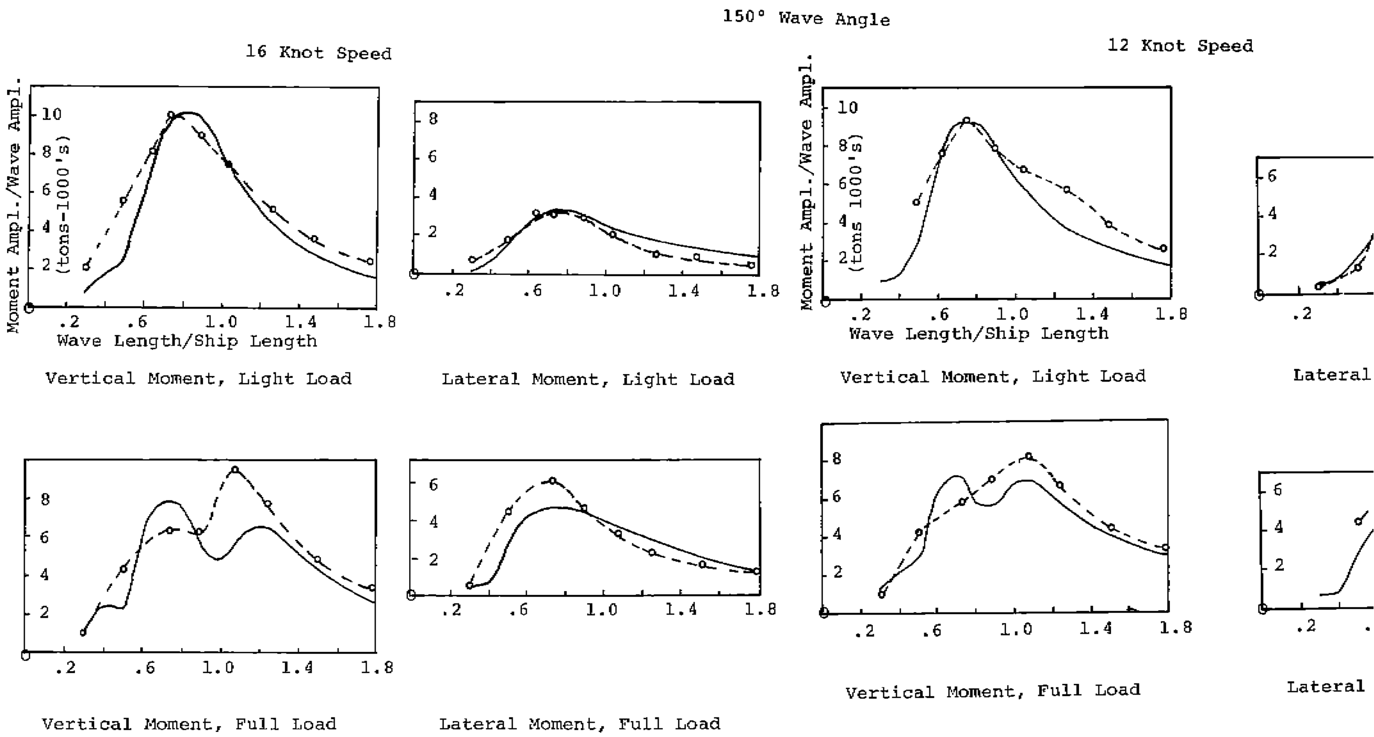
22. Salvesen, N., E. O. Tuck and O. Faltinsen, "Ship Motions and Sea Loads," Trans. SNAME, Vol. 78, 1970

23. Frank, W. and N. Salvesen, "The Frank Close-Fit Ship-Motion Computer Program," NSRDC Report 3289, 1970.




Figs. 2a and 3a. Midship Wave Bending Moments on WOLVERINE STATE

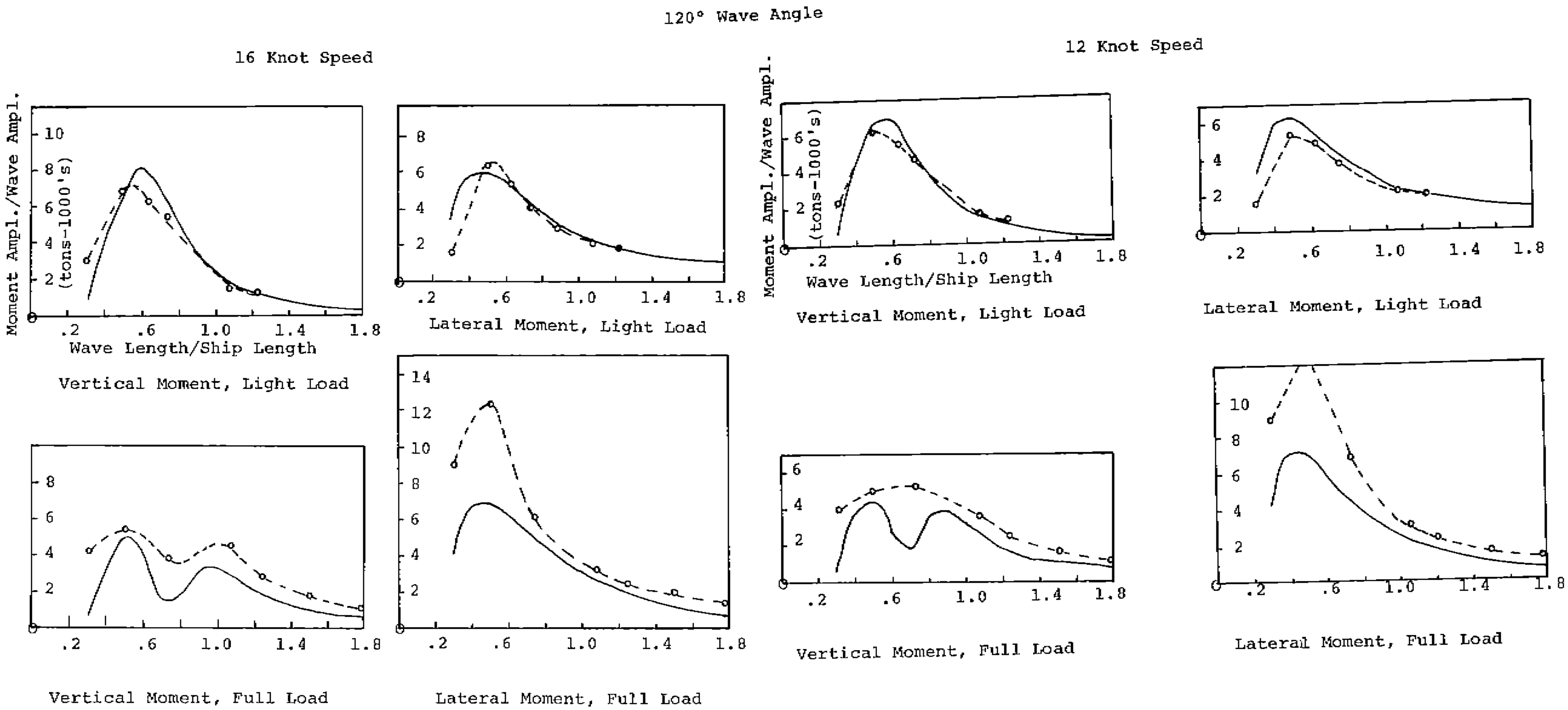
o- - -o- - -o Experimental Result
_____________ Calculation




Figs. 2b and 3b. Midship Wave Bending Moments on WOLVERINE STATE

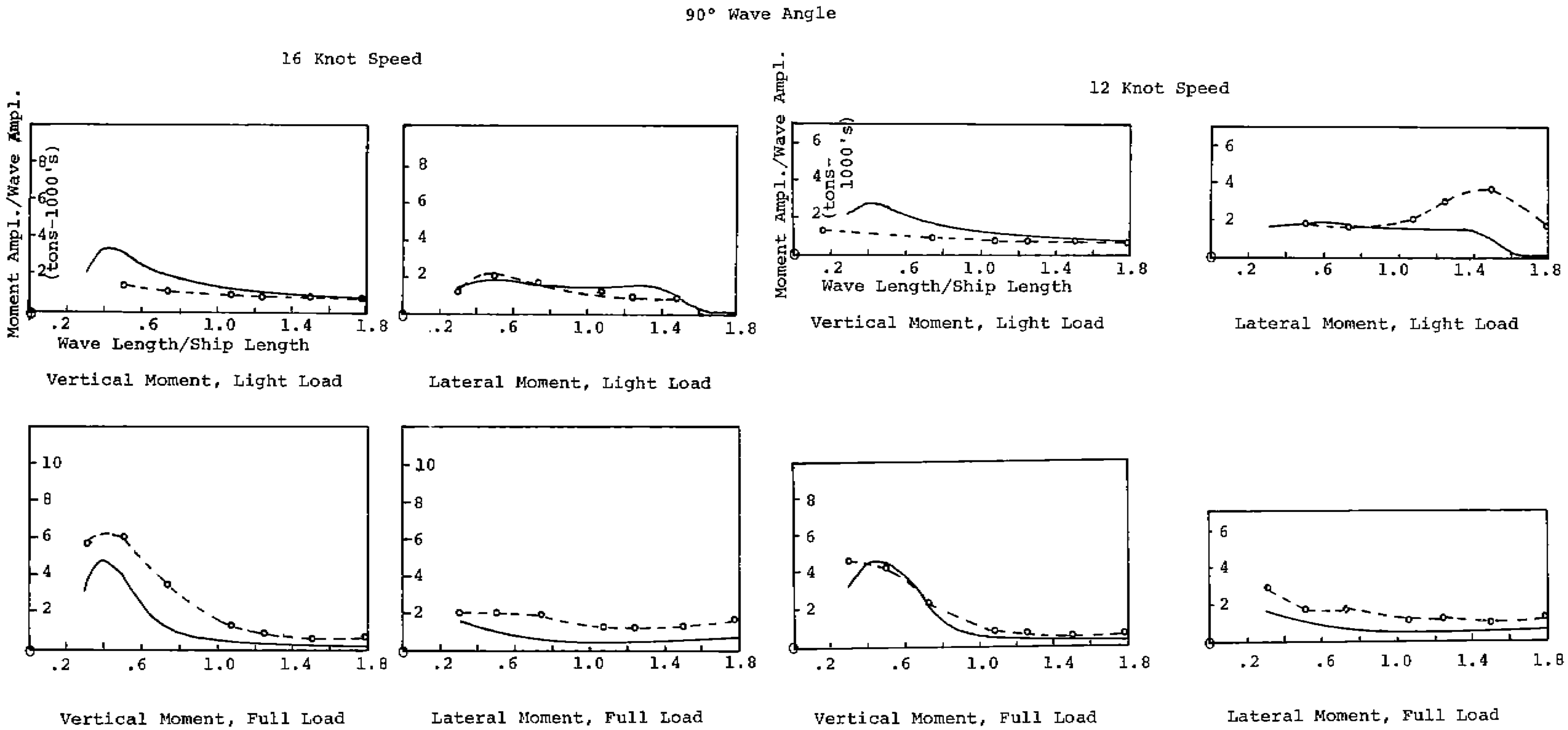
0- - -0- - -0 Experimental Result

____________ Calculation



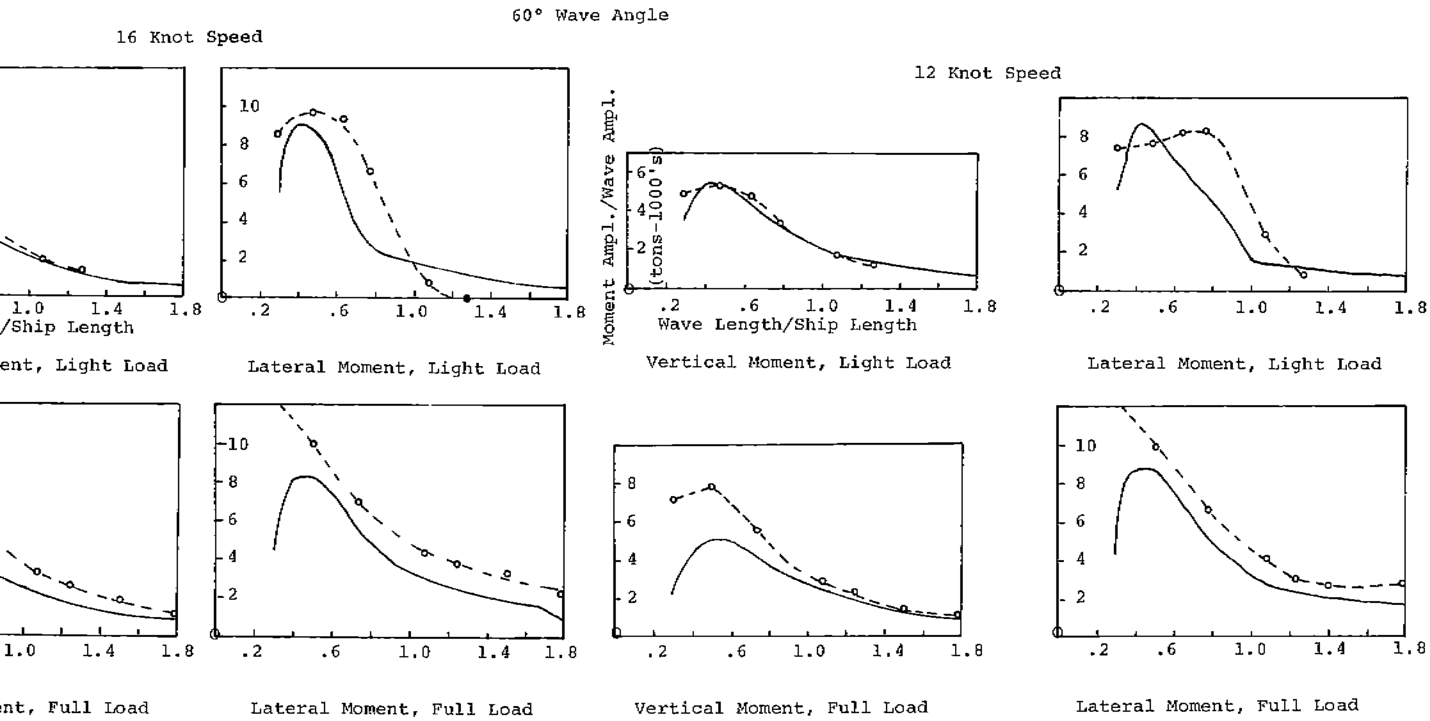

Figs. 2c and 3c. Midship Wave Bending Moments on WOLVERINE STATE

0- - -0- - -0 Experimental Result
____________ Calculation



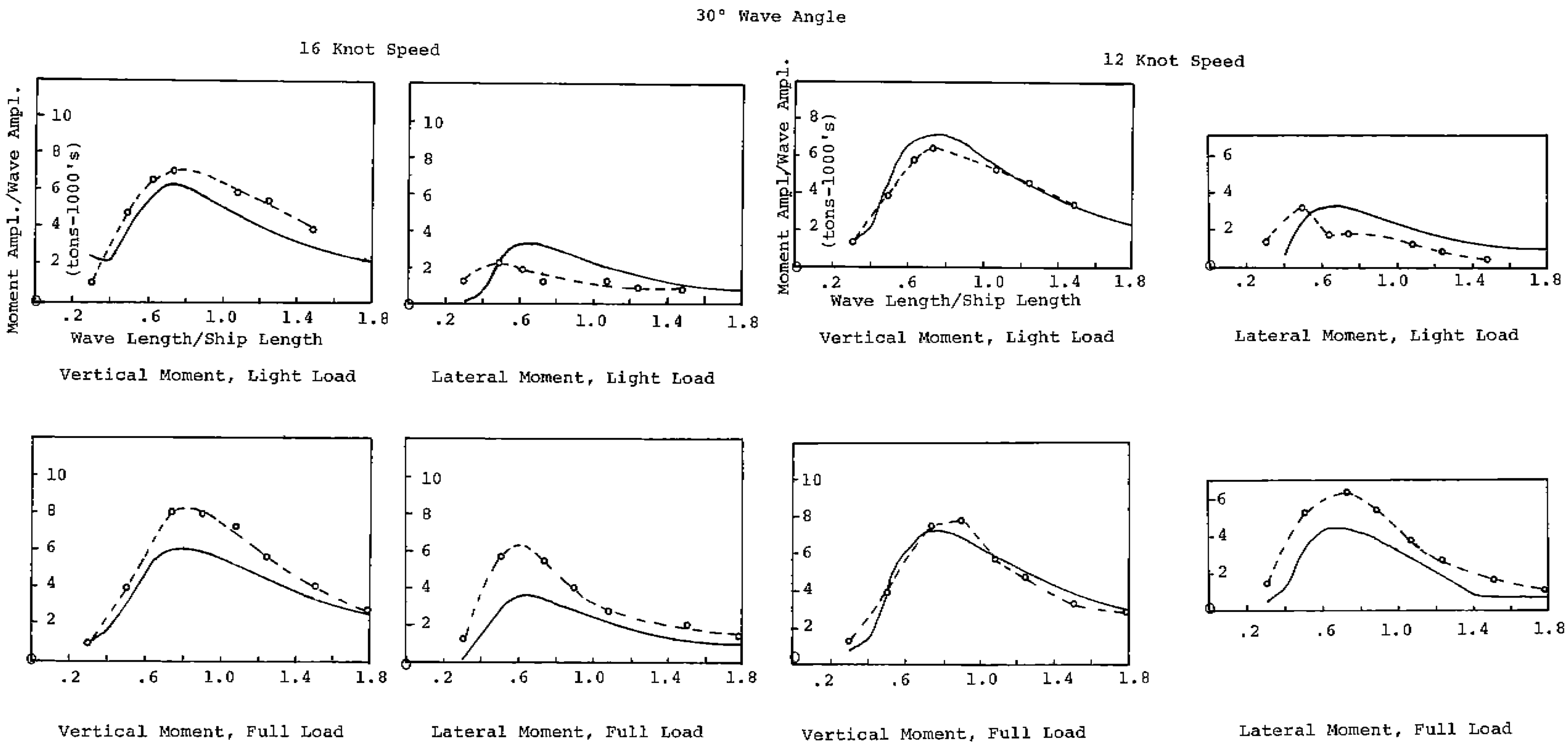

Figs. 2d and 3d. Midship Wave Bending Moments on WOLVERINE STATE

0- - -0- - -0 Experimental Result
____________ Calculation



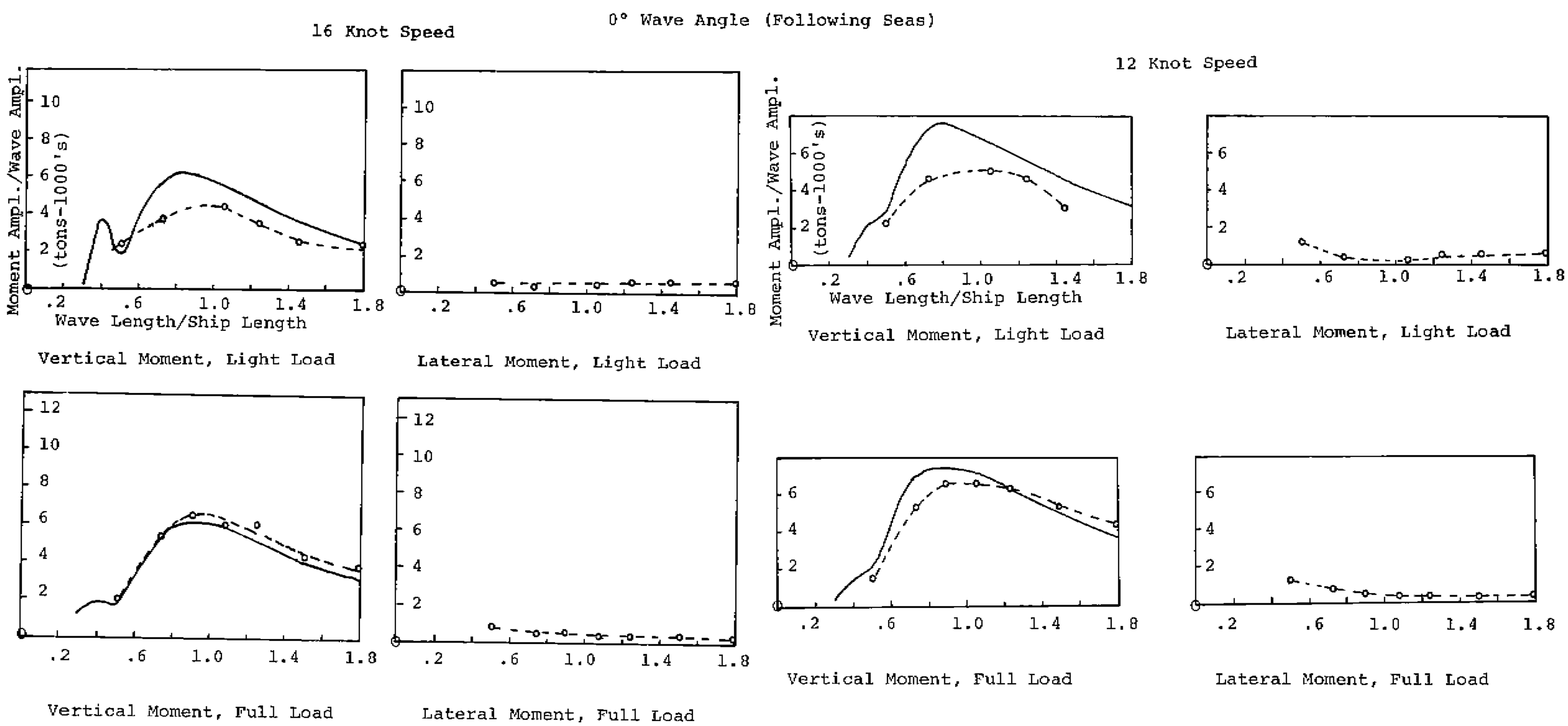

Figs. 2e and 3e. Midship Wave Bending Moments on WOLVERINE STATE

O- - -O- - -O Experimental Result
____________ Calculation




Figs. 2f and 3f. Midship Wave Bending Moments on WOLVERINE STATE

0- - -0- - -0 Experimental Result
———— Calculation




Figs. 2g and 3g. Midship Wave Bending Moments on WOLVERINE STATE

O- - -O- - -O Experimental Result
____________ Calculation

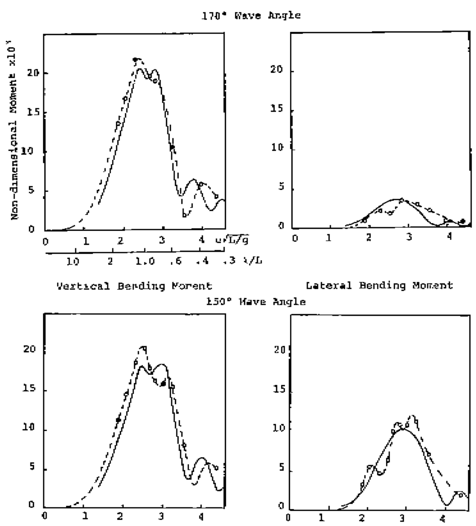



Fig. 4a.

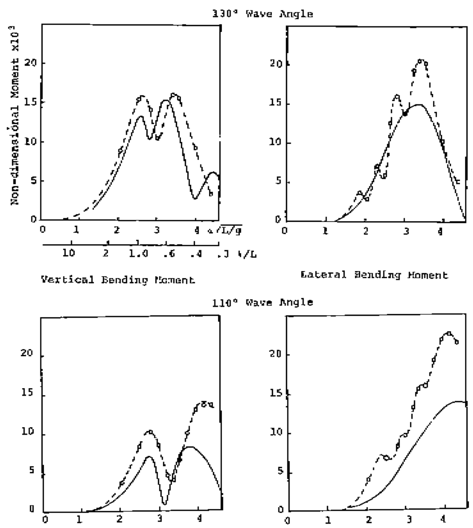



Fig. 4b.

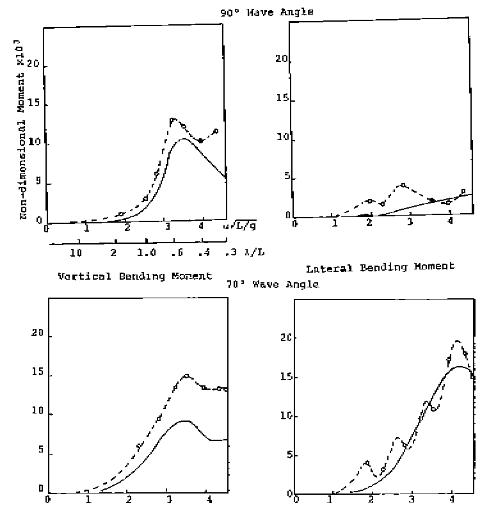



Fig. 4c.



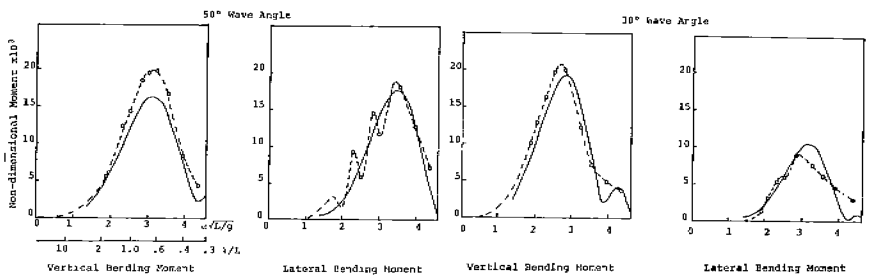



Fig. 4d.

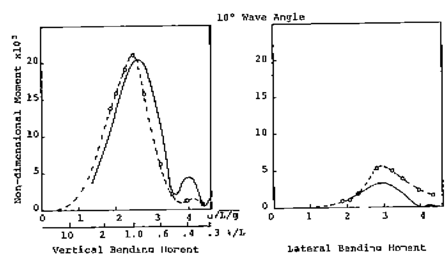



Fig. 4e.

Fig. 4. Midship Wave Moments on SERIES 60, BLOCK .80 Hull

Fn = 0.15

0- - -0- - -0 Experimental Results
____________ Calculation

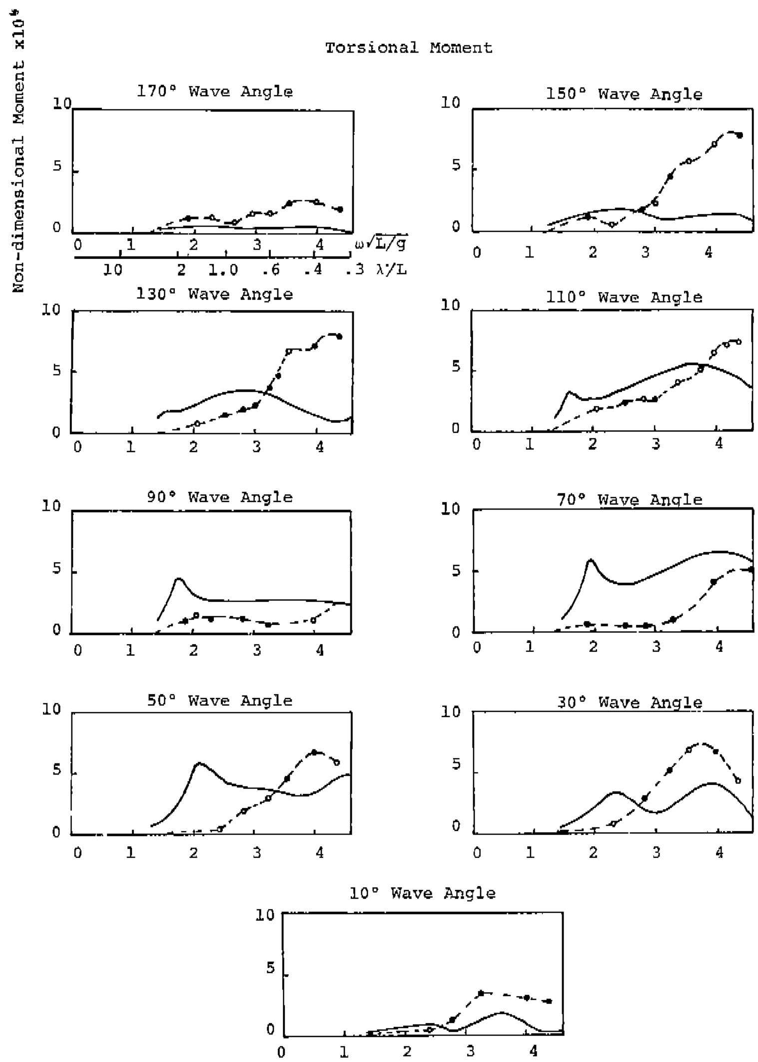



Fig. 5. Midship Wave Moments on SERIES 60, BLOCK .80 Hull

Fn = 0.15

0- - -0- - -0 Experimental Results

____________ Calculation

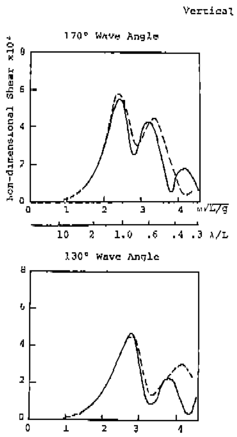



Fig. 6a.

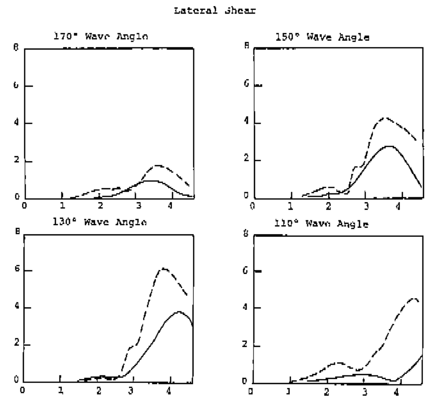



Fig. 6b.

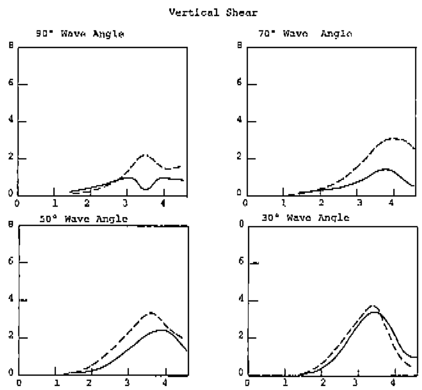



Fig. 6c.

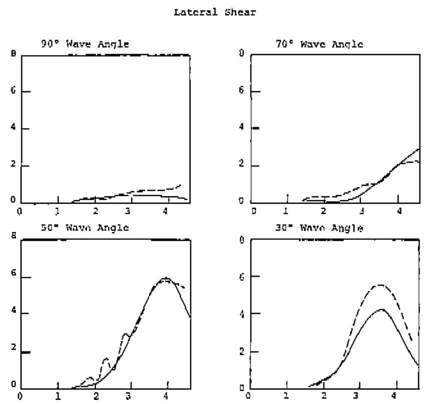



Fig. 6d.

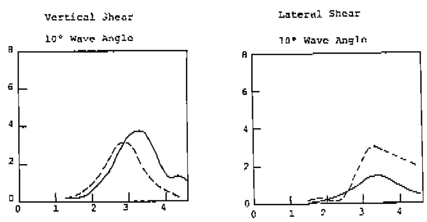



Fig. 6e.

Fig. 6. Midship Wave Shear Forces on SERIES 60, BLOCK .80 Hull

Fn = 0.15

--------------- Experimental Results

_______________ Calculation

Irregular Seas

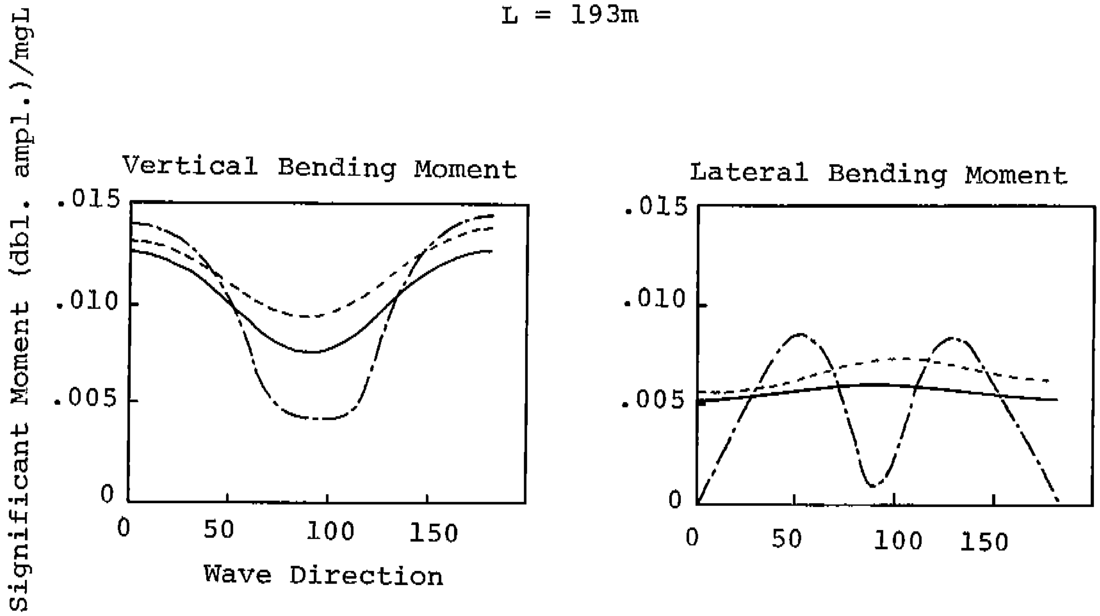
Two Parameter Spectrum

$H_{1/3} = 8.4m$

$\tilde{T} = 10.0$ sec.

Cosine-squared Spreading Function

L = 193m




- - - - - - - - - - Calculated from Experimental Results, Short Crested Seas [18]

———————— Calculation, Short Crested Seas

-·-·-·-·-·-·-·-·- Calculation, Long Crested Seas

Fig. 7. Midship Wave Moments on SERIES 60, BLOCK .80 Hull

Fn = 0.15

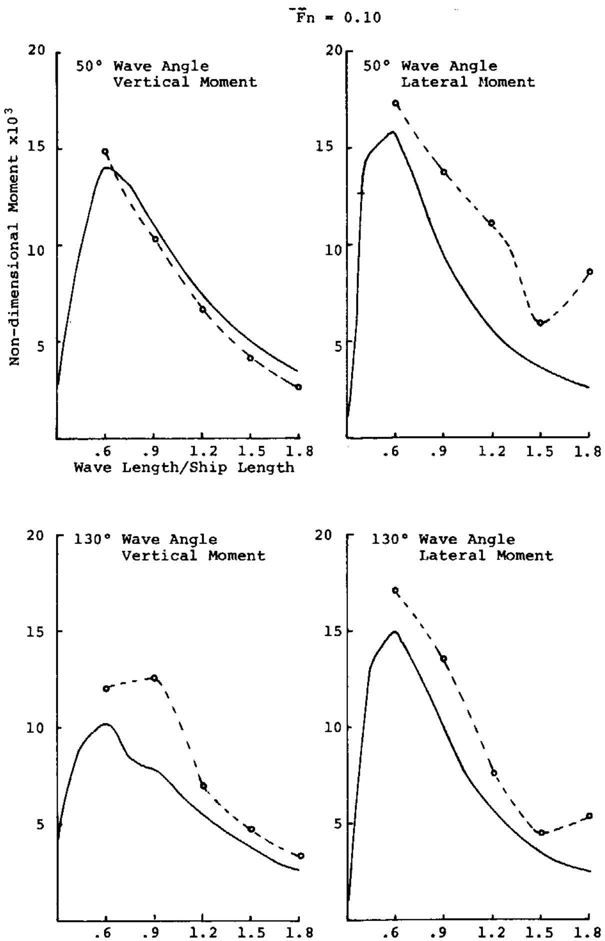



Fig. 8a.

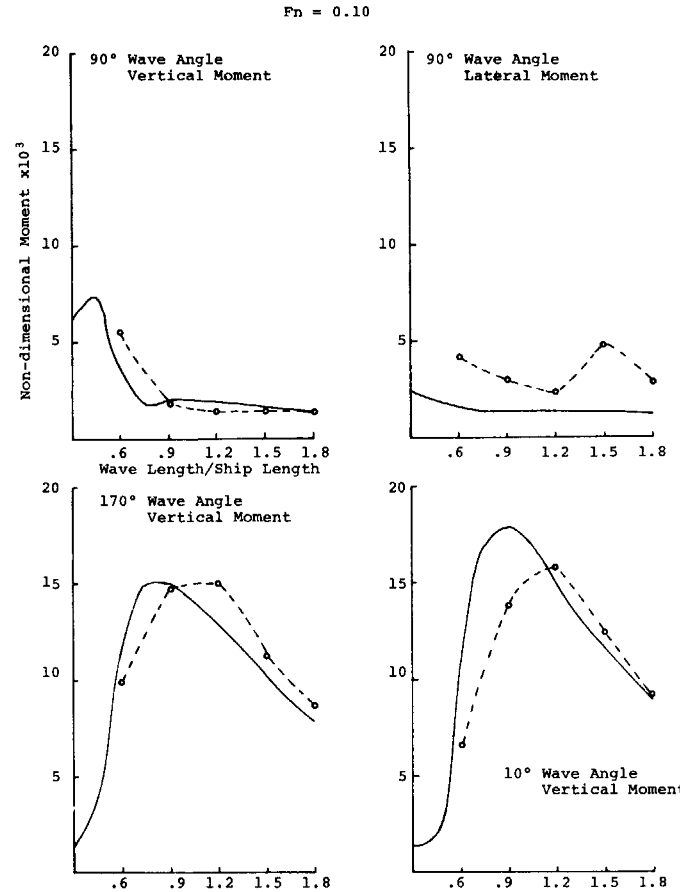



Fig. 8b.

Fig. 8. Midship Wave Bending Moments on SERIES 60, BLOCK .70 Hull

0- - -0- - -0 Experimental Results

____________ Calculation

Calculation

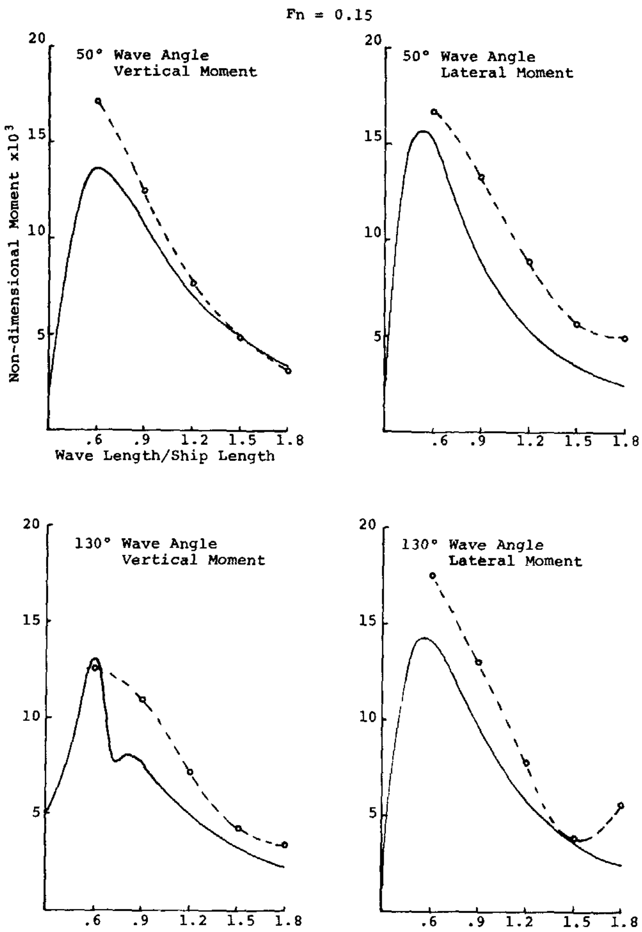



Fig. 9a.

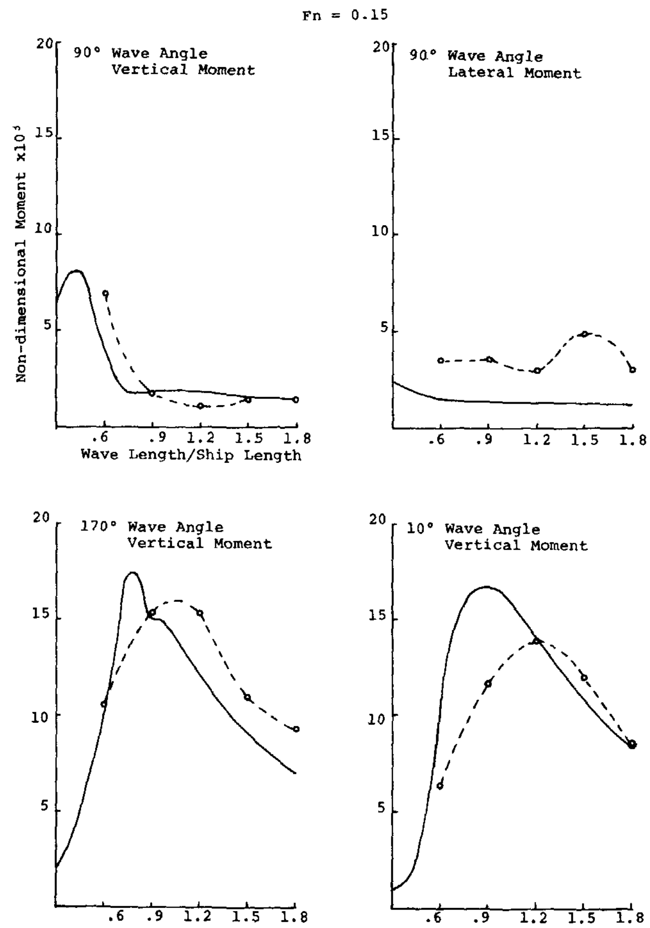



Fig. 9b.

Fig. 9. Midship Wave Bending Moments on SERIES 60, BLOCK .70 Hull

0- - -0- - -0 Experimental Results

Calculation

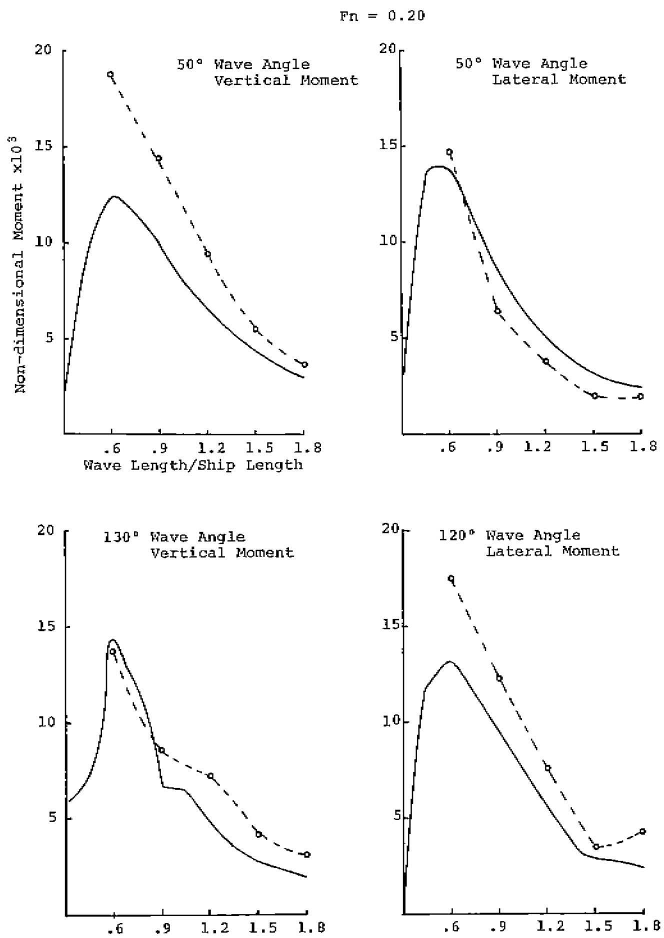



Fig. 10a.

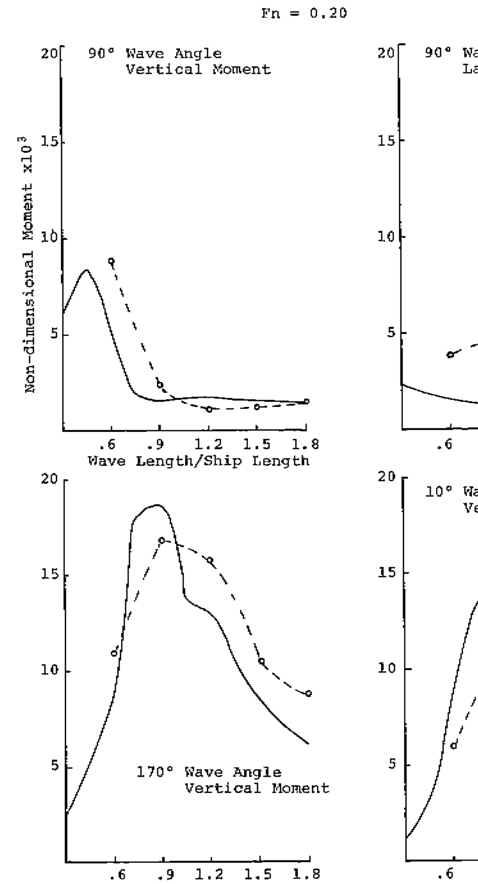



Fig. 10b.

Fig. 10. Midship Wave Bending Moments on SERIES 60, BLOCK .70 Hull

0- - -0- - -0 Experimental Results

———————— Calculation

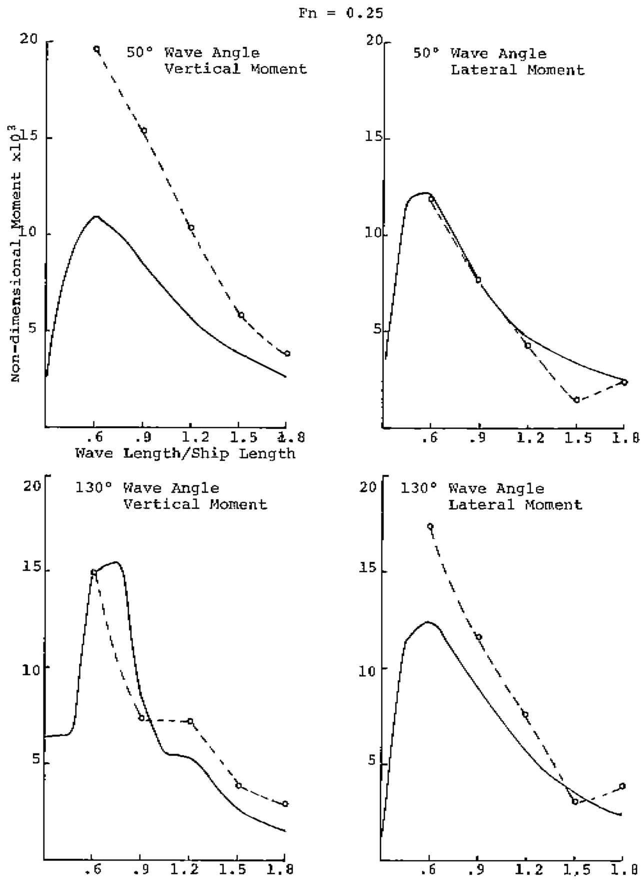



Fig. 11a.

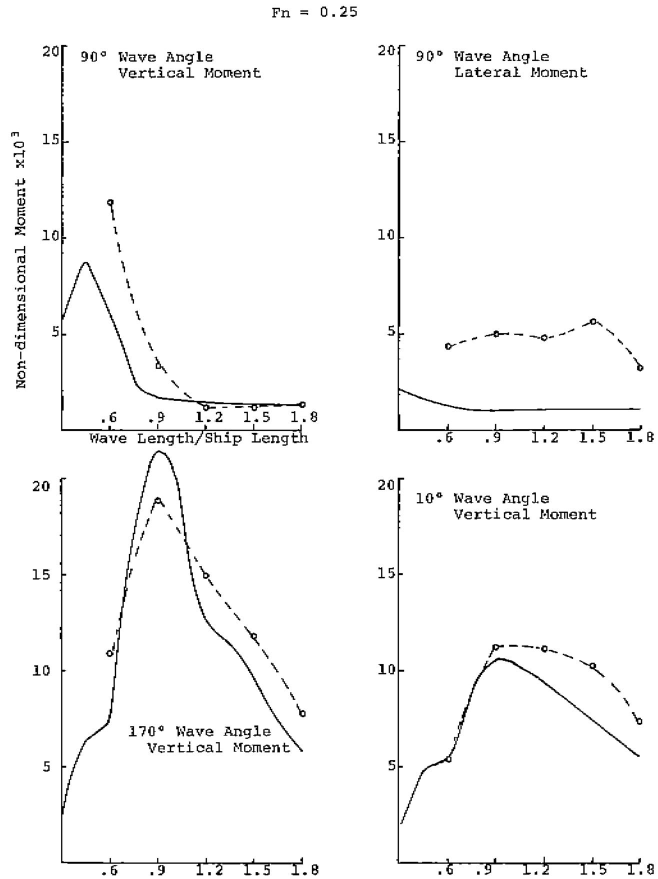



Fig. 11b.

Fig. 11. Midship Wave Bending Moments on SERIES 60, BLOCK .70 Hull

O- - -O- - -O Experimental Results

———————— Calculation

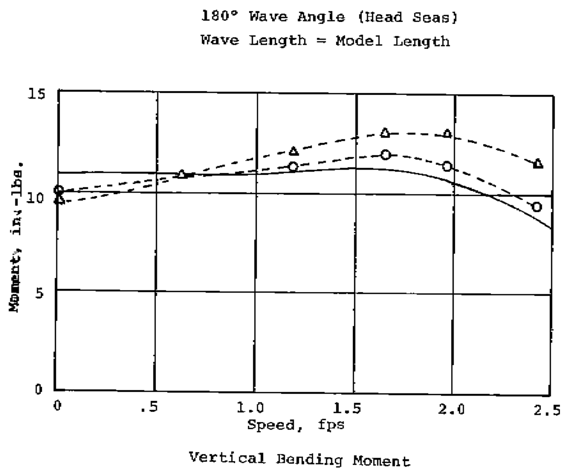



Fig. 12a.

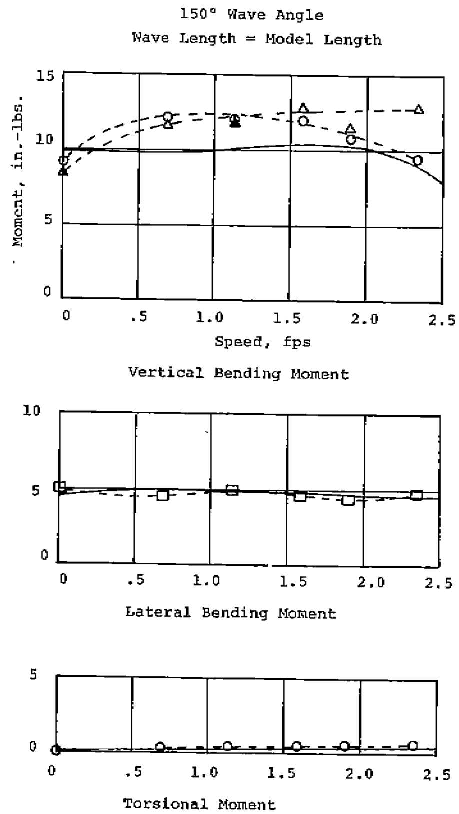



Fig. 12b.

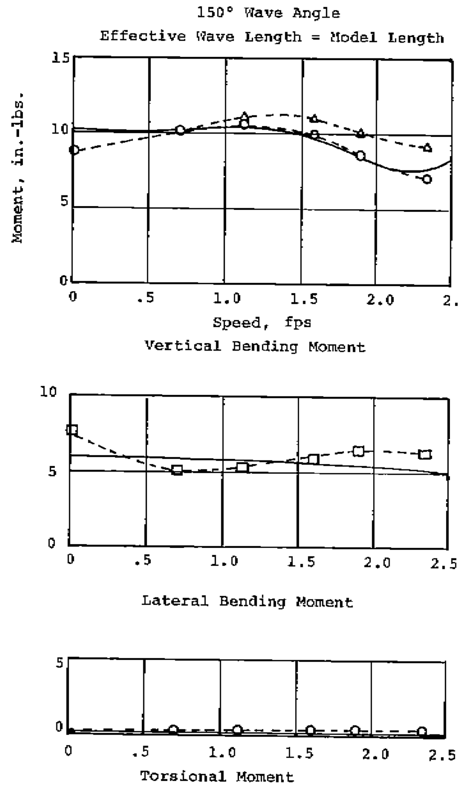



Fig. 12c.

Fig. 12. Midship Wave Moments on T-2 TANKER Model

Legend:

--Δ---Δ--
--O---O--
--□---□-- Experimental Results
--∇---∇--

__________ Calculation

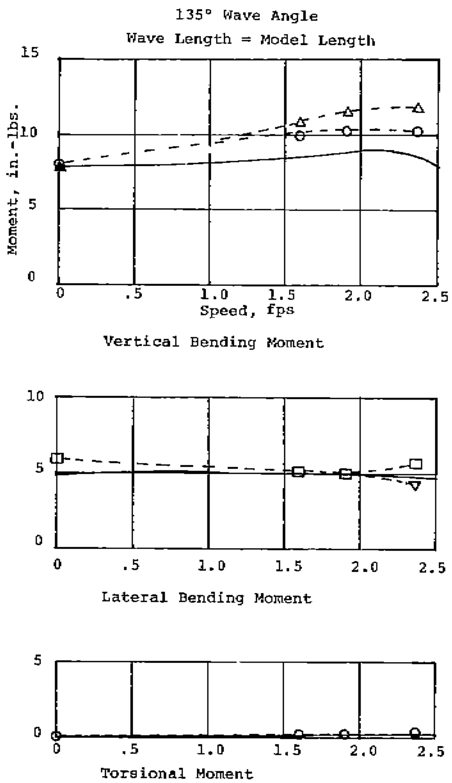



Fig. 12d.

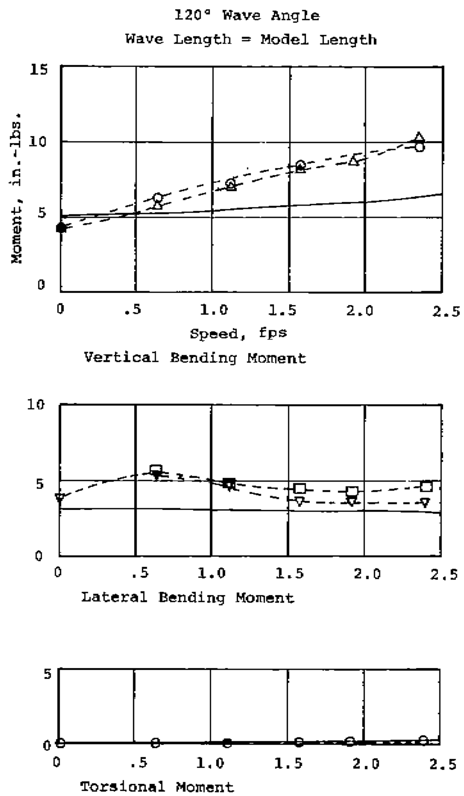



Fig. 12e.

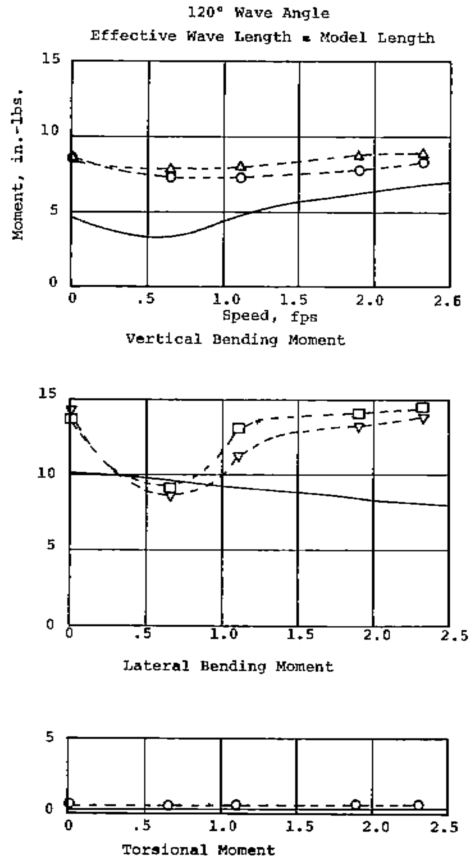



Fig. 12f.

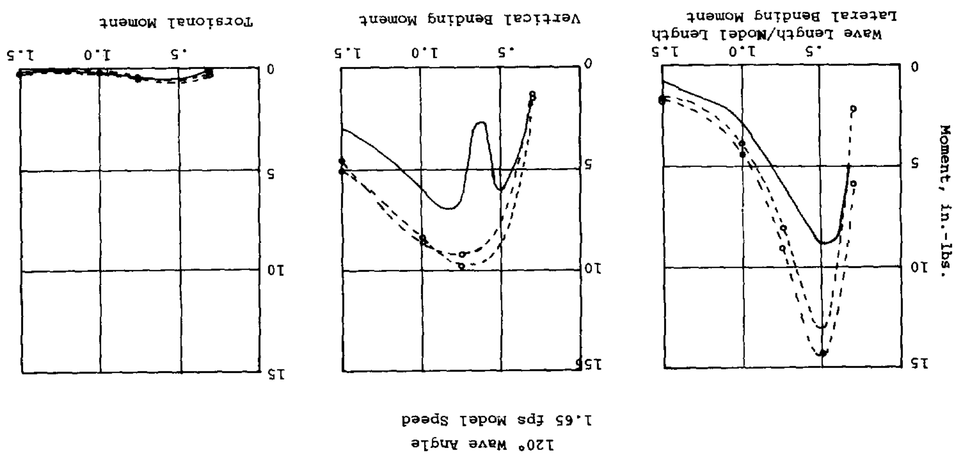



Fig. 12h.

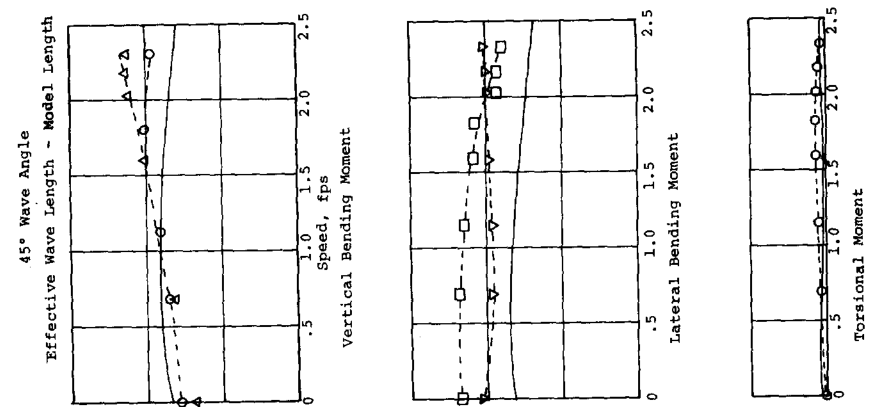



Fig. 12g.

UNCLASSIFIED

Security Classification

**DOCUMENT CONTROL DATA - R & D**

*(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)*

| | |
|---|---|
| 1. ORIGINATING ACTIVITY *(Corporate author)*<br>Oceanics, Inc.<br>Plainview, New York | 2a. REPORT SECURITY CLASSIFICATION<br>Unclassified<br>2b. GROUP |

3. REPORT TITLE

Evaluation and Verification of Computer Calculations of Wave-Induced Ship Structural Loads

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

5. AUTHOR(S) *(First name, middle initial, last name)*

Paul Kaplan and A. I. Raff

| 6. REPORT DATE | 7a. TOTAL NO. OF PAGES | 7b. NO. OF REFS |
|---|---|---|
| July 1972 | 50 | 23 |

| | |
|---|---|
| 8a. CONTRACT OR GRANT NO<br>N00024-70-C-5076<br>b. PROJECT NO<br>c.<br>d. | 9a. ORIGINATOR'S REPORT NUMBER(S)<br><br>9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*<br>SSC-229 |

10. DISTRIBUTION STATEMENT

Unlimited

| 11. SUPPLEMENTARY NOTES | 12. SPONSORING MILITARY ACTIVITY |
|---|---|
| | Naval Ship Systems Command |

13. ABSTRACT


An analytical method for the determination of conventional merchant ship motions and wave-induced moments in a seaway is developed. Both vertical and lateral plane motions and loads are considered for a ship travelling at any heading in regular waves and in irregular long or short crested seas. Strip theory is used and each ship hull cross-section is assumed to be of Lewis form shape for the purpose of calculating hydrodynamic added mass and damping forces in vertical, lateral and rolling oscillation modes. The coupled equations of motion are linear, and the superposition principle is used for statistical response calculations in irregular seas. All three primary ship hull loadings are determined, i.e. vertical bending, lateral bending and torsional moments, as well as shear forces, at any point along the length, with these responses only representing the low frequency slowly varying wave loads directly induced by the waves.

A computer program that carries out the calculations was developed, and is fully documented separately. The results of the method are evaluated by comparison with a large body of model test data. The comparison extends over a wide range of ship speeds, wave angles, wave lengths, and loading conditions, as well as hull forms. The agreement between the calculations

| 14. KEY WORDS | LINK A | | LINK B | | LINK C | |
|---|---|---|---|---|---|---|
| | ROLE | WT | ROLE | WT | ROLE | WT |
| | | | | | | |

DD FORM 1 NOV 65 1473 (BACK)
(PAGE 2)

GPO 938-963

SHIP RESEARCH COMMITTEE
Maritime Transportation Research Board
National Academy of Sciences-National Research Council

The Ship Research Committee has technical cognizance of the inter-agency Ship Structure Committee's research program:

PROF. R. A. YAGLE, Chairman, *Prof. of Naval Architecture, University of Michigan*

DR. H. N. ABRAMSON, *Director, Dept. of Mechanical Sciences, Southwest Research Inst.*

MR. W. H. BUCKLEY, *Coordinator of Hydrofoil Struc. Res., Naval Ship R & D Center*

MR. E. L. CRISCUOLO, *Senior Non-destructive Testing Spec., Naval Ordnance Lab.*

DR. W. D. DOTY, *Research Consultant, U. S. Steel Corporation*

PROF. J. E. GOLDBERG, *School of Civil Engineering, Purdue University*

PROF. W. J. HALL, *Prof. of Civil Engineering, University of Illinois*

MR. J. E. HERZ, *Chief Structural Design Engineer, Sun Shipbuilding & Dry Dock Co.*

MR. G. E. KAMPSCHAEFER, JR., *Manager, Application Engineering, ARMCO Steel Corporation*

MR. R. C. STRASSER, *Director of Research, Newport News Shipbuilding & Dry Dock Co.*

CAPT R. M. WHITE, USCG, *Chief, Applied Engineering Section, U. S. Coast Guard Academy*

MR. R. W. RUMKE, *Executive Secretary, Ship Research Committee*

Advisory Group I, "Ship Response and Load Criteria" prepared the project prospectus and evaluated the proposals for this project.

DR. H. N. ABRAMSON, Chairman, *Director, Dept. of Mech. Sciences, Southwest Res. Inst.*

MR. W. H. BUCKLEY, *Coordinator of Hydrofoil Struc. Res., Naval Ship R & D Center*

PROF. A. M. FREUDENTHAL, *College of Engineering, George Washington University*

MR. R. G. KLINE, *Associate Research Consultant, U. S. Steel Corporation*

DR. M. K. OCHI, *Research Scientist, Naval Ship R & D Center*

MR. R. C. STRASSER, *Director of Research, Newport News Shipbuilding & Dry Dock Co.*

CAPT R. M. WHITE, USCG, *Chief, Applied Engineering Section, U. S. Coast Guard Academy*

The SR-174 Project Advisory Committee provided the liaison technical guidance, and reviewed the project reports with the investigator.

DR. H. N. ABRAMSON, Chairman, *Director, Dept. of Mech. Sciences, Southwest Res. Inst.*

MR. M. K. OCHI, *Research Scientist, Naval Ship R & D Center*

MR. R. C. STRASSER, *Director of Research, Newport News Shipbuilding & Dry Dock Co.*

## SHIP STRUCTURE COMMITTEE PUBLICATIONS

*These documents are distributed by the National Technical Information Service, Springfield, Va. 22151. These documents have been announced in the Clearinghouse Journal U.S. Government Research & Development Reports (USGRDR) under the indicated AD numbers.*

SSC-219, *Crack Propagation and Arrest in Ship and Other Steels* by G. T. Hahn, R. G. Hoagland, P. N. Mincer, A. R. Rosenfield, and M. Sarrate. 1971.

SSC-220, *A Limited Survey of Ship Structural Damage* by S. Hawkins, G. H. Levine, and R. Taggart. 1971.

SSC-221, *Response of the Delta Test to Specimen Variables* by L. J. McGeady. 1971.

SSC-222, *Catamarans - Technological Limits to Size and Appraisal of Structural Design Information and Procedures* by N. M. Maniar and W. P. Chiang. 1971.

SSC-223, *Compressive Strength of Ship Hull Girders - Part II - Stiffened Plates* by H. Becker, A. Colao, R. Goldman, and J. Pozerycki. 1971.

SSC-224, *Feasibility Study of Glass Reinforced Plastic Cargo Ship* by R. J. Scott and J. H. Sommella. 1971.

SSC-225, *Structural Analysis of Longitudinally Framed Ships* by R. Nielson, P. Y. Chang, and L. C. Deschamps. 1972.

SSC-226, *Tanker Longitudinal Strength Analysis - User's Manual and Computer Program* by R. Nielson, P. Y. Chang, and L. C. Deschamps. 1972.

SSC-227, *Tanker Transverse Strength Analysis - User's Manual* by R. Nielson, P. Y. Chang, and L. C. Deschamps. 1972.

SSC-228, *Tanker Transverse Strength Analysis - Programmer's Manual* by R. Nielson, P. Y. Chang, and L. C. Deschamps. 1972.

SSC-229, *Evaluation and Verification of Computer Calculations of Wave-Induced Ship Structural Loads* by P. Kaplan and A. I. Raff. 1972.

SSC-230, *Program SCORES -- Ship Structural Response in Waves* by A. I. Raff. 1972.

SSC-231, *Further Studies of Computer Simulation of Slamming and Other Wave-Induced Vibratory Structural Loadings on Ships in Waves* by P. Kaplan and T. P. Sargent. 1972.