The Analysis of Plates on Elastic Foundation by the Boundary Integral Equation Method

DISSERTATION

for the Degree of

Doctor of Philosophy (Applied Mechanics)

JOHN THEODORE KATSIKADELIS

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by

John Theodore Katsikadelis

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Approved:

Department Head

April 15, 1982

Approved by the Guidance Committee

Major: Applied Mechanics

A.E. Armenàkas

Professor of ME and AERO Dept.

Minor: Elasticity

W.P. Vafakos

Professor of ME and AERO Dept.

Minor: Mathematics

S. Preiser

Professor of Mathematics and Computer Science

Additional Committee Member:

J. Kempner

Professor of ME and AERO Dept.

Microfilm or other copies of this dissertation are obtainable from

UNIVERSITY MICROFILMS
300 N. Zeeb Road
Ann Arbor, Michigan, 48106

BIOGRAPHICAL SKETCH OF THE AUTHOR

He was born in Piraeus on Dec. 15 1937. He graduated from the Ionidion-model high school with major in Mathematics. In 1957 he was admitted to the School of Civil Engineering of the National Technical University (N.T.U.) of Athens from where he graduated in 1962. He received his Doctor's Degree (Doctor of Engineering) from the N.T.U. in January 1974.

In September 1974 he was admitted to the P.I.N.Y. where he continued his graduate studies in the Department of Aerospace Engineering and Applied Mechanics. In June 1975 he received his M.S. Degree in Applied Mechanics. In November 1974 he passed the Language examination (German) and in February 1975 he passed the Preliminary Doctoral (Qualifying) Examination. The author worked in his dissertation from September 1975.

From September 1970 the author has been a Lecturer,and from September 1974 a Senior Lecturer in the School of Civil Engineering of N.T.U. In this capacity,he teaches courses in structural Analysis and Mechanics. He has published twelve papers three of which have been presented in international conferences and a book entitled "Theory of Plates Subjected to Inplane Forces". Moreover, he is a licensed professional engineer in Greece.

To my wife

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AN ABSTRACT

THE ANALYSIS OF PLATES ON ELASTIC FOUNDATION
BY THE BOUNDARY INTEGRAL EQUATION METHOD

by

John T. Katsikadelis
Adviser: Prof. A.E. Armenakas

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In this dissertation, the Boundary Integral Equation (BIE) method is developed for plates on elastic foundation. The pertinent, coupled, singular boundary integral equations are established and solved numerically. Moreover, a procedure is presented for obtaining the influence fields of various quantities such as deflections, bending and twisting moments and shearing forces. In this procedure, the influence fields are obtained as deflection surfaces corresponding to appropriately chosen generalized forces. Numerical results for various clamped and simply supported plates under distributed or concentrated loads are also presented and compared with existing results whenever available.

In Part I, the boundary value problem for a thin plate on elastic foundation is formulated in its most general form. The two-dimensional region, occupied by the plate, may be simply or multiply connected and its boundary may have corners. Two versions of the Green identity for the differential self-adjoint operator $(\nabla^4+\kappa^2)$ are established. In the one version, the boundary terms do not have direct physical meaning, whereas, in the other version they have physical significance. Two integral representations for the deflection of the plate are established by using the solution for an infinitely extended plate on elastic foundation, subjected to a concentrated unit load in the two versions of Green's identity. Moreover, two versions of the required boundary integral equations are derived. For the analysis of plates with clamped or simply supported plates, the boundary integral equations resulting from the first version of Green's identity are more suitable than those resulting from the second version. However, the first set of boundary integral equations are not appropriate for the analysis of plates with other boundary conditions.

The existence of the boundary integrals having singular kernels is proved by showing that they behave like single or double layer potentials. Moreover, the jump of the discontinuity of these integrals, whenever their kernel behaves like a Newtonian double layer potential, is evaluated using a suitable procedure.

An elegant procedure, based on the properties of the derivatives of the δ-function, is presented for the numerical evaluation of the influence fields of various field quantities such as deflections, slopes, bending and twisting moments and shearing forces. The influence fields are obtained as the deflection surfaces due to appropriately introduced generalized loads (multipoles),using a generalized form of the reciprocal theorem. Thus, the two sets of the boundary integral equations differ only in the non-homogeneous term,which is evaluated in closed form.

In Part II,a procedure for the numerical solution of the coupled,singular, boundary integral equations for the clamped and simply supported plate is developed. In this procedure,the boundary is divided into a finite number of elements on which the unknown boundary quantities are assumed to vary according to a given law and, thus, the boundary integral equations are approximated by a system of simultaneous linear algebraic equations. The coefficents of the unknowns of this system are evaluated by numerical integration of their expressions on the boundary element. A special technique is applied to overcome the difficulty in the numerical integration on the elements where the integrand is singular. The non-homogeneous terms are double improper integrals on a two-dimensional region with arbitrary shape. A procedure is developed for the numerical evaluation of any such integrals,having a logarithmic or a Cauchy-type singularity. Numerical schemes for the computation of the deflections of the plate,as well as its stress resultants are also presented.

In Part III, a computer program has been written in FORTRAN language and numerical results have been obtained on a CDC/CYBER 171-8 computer for the clamped and simply supported plates. The results are in excellent agreement with those obtained from existing analytical solutions.For small values of the constant of elastic foundation (k=0.1 or 0.01),the results differ negligibly from those of a plate which does not rest on an elastic foundation.

The influence coefficients for the deflection and the stress resultants,at some points of clamped and simply supported circular and rectangular plates,are tabulated for certain values of the dimensionless parameters which characterize the geometry and mechanical properties of the plate and the elasticity of the subgrade.

C O N T E N T S

PARTIAL LIST OF SYMBOLS

w:	Deflection of the middle surface of the plate
$f(p)$:	Loading function.
$D = \dfrac{Eh^3}{12(1-\nu^2)}$:	Flexural rigidity of the plate.
h:	Thickness of the plate
E:	Modulus of elasticity
ν:	Poison's ratio
k:	Constant of the elastic foundation
$\kappa^2 = \dfrac{k}{D}$:	
$\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$:	Harmonic (Laplacian) operator
$\nabla^4 = \dfrac{\partial^4}{\partial x^4} + 2\dfrac{\partial^4}{\partial x^2 \partial y^2} + \dfrac{\partial^4}{\partial y^4}$:	Biharmonic operator
M_x, M_y:	Bending moments per unit length on cross sections of the plate normal to the x and y axes, respectively
M_{xy}	Twisting moment per unit length with respect to the x and y axes.
Q_x,Q_y:	Shearing force per unit length on cross section of the plate normal to the x and y axes, respectively.
M_n,M_t:	Bending moments per unit length in directions normal and tangential to the boundary.

M_{nt}:	Twisting moment with respect to the n and t directions.
V_n:	Effective shearing force (reaction force) per unit length along the boundary of the plate.
M,V:	Differential operators defined by equations (1-2.8).
K(s):	Curvature of the boundary.
I_C:	Additional term of the boundary integral equations for boundary with corners defined by equation (I-2.11).
$\ell=\sqrt[4]{\frac{D}{k}}$:	Parameter having dimensions of length relating the stiffness of the plate to the constant of the elastic foundation.
P,Q,	Points inside the two-dimensional region occupied by the plate.
p,q,	Points on the boundary of the plate
v(P,Q):	Fundamental solution of the problem
$r=\|P-Q\|$:	Distance between the points P and Q.
$\rho=r/\ell$	Dimensionless distance.
ber(ρ), bei(ρ),ker(ρ), kei(ρ):	Kelvin functions of zero order.
δ(P-Q):	Delta function.
$\varphi,\omega=\widehat{r,n}$:	Angle between the direction of the distance r and the normal n to the boundary.

α:	Angle at the corner point of the plate.
Ω, X, Φ, Ψ:	Boundary quantities defined by equations (I-4.26)
$\bar{F}, \bar{G}. \bar{H}$,	Non homogeneous terms of the boundary integral equations defined by equations (I-4.27)
w^*:	Generalized deflection of the plate (influence field).
$N[\delta(P-Q)]$:	Generalized force. N is a linear differential operator.
a:	Radius of a circular plate, or half of the side length of a rectangular plate.
$\lambda = a/\ell$:	Dimensionless parameter relating the geometrical and mechanical properties of the plate to the stiffness of the subgrade.
$\varepsilon = b/a$:	Side ratio of a rectangular plate.

INTRODUCTION

1. Historical developement of the BIE method

The boundary integral methods for the solution of boundary value problems in mathematical physics have their origin in the work of G.Green (1828) [1], who obtained an integral representation for the solution of the Dirichlet and Neumann problems for the Laplace equation. In these problems, a function u(x,y,z) is sought satisfying the Laplace equation at every point of a three dimensional region R. That is,

$$\nabla^2 u = 0 \qquad \text{(H.1)}$$

Moreover, at every point of the boundary S of R, this function assumes either specified values (Direchlet problem) or its derivative in the direction normal to S $(\frac{\partial u}{\partial n})$ assumes specified values (Neumann problem). Green devoloped and employed his well known reciprocal identity, i.e.

$$\iiint_R (u\nabla^2 v - v\nabla^2 u)\,dV = \iint_S \left(u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n}\right) dS \qquad \text{(H.2)}$$

which relates any two functions u(x,y,z) and v(x,y,z) inside the three-dimensional region R with the values of these functions and their normal derivatives[1] on the boundary S of R. The functions u and v must be twice continuously differentiable in R and once on S.

For v, Green chose a singular solution of the Laplace equation [$\nabla^2 v = -4\pi\delta(P-Q)$, where $\delta(P-Q)$ is the Dirac delta-function], i.e.[2]

$$v = 1/r \qquad (H.3)$$

where $r = |P-Q|$ is the distance between any two points P and Q in R. Using equation (H.3), identity (H.2) gives the following integral representation for the solution u(P) of equation (H.1)

$$u(P) = \frac{1}{4\pi}\iint_S \left[\frac{1}{r}\frac{\partial u}{\partial n} - u\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\right] dS \qquad (H.4)$$

Since u and $\frac{\partial u}{\partial n}$ can not both be prescribed on the boundary, the one which is not prescribed should be eliminated from the above relation. To accomplish this, Green introduced certain functions known as the "Green's functions for the surface and the point" [2]. For the Dirichlet problem he introduced the singular function G(P,Q), known as Green's first function, which has the following properties:

(a) G is harmonic inside R except at r=0

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(1) n is the direction of the outward normal to the surface S.

(2) In potential theory the function $v=1/r$ is the three dimensional Newtonian potential at a point P (field point) due to a unit concentrated mass at point Q (source point).

(b) it behaves like $1/r$ at $r=0$ and

(c) it vanishes on S.

These properties imply that $\lim_{r\to 0}(G-\frac{1}{r})=0$ and that the function $(G-\frac{1}{r})$ is harmonic $[\nabla^2(G-\frac{1}{r})=0]$ at all points in R. Consequently, choosing the function $(G-\frac{1}{r})$ for v, relation (H.2) yields

$$0=\iint_S\left[u\frac{\partial}{\partial n}(G-\frac{1}{r})-(G-\frac{1}{r})\frac{\partial u}{\partial n}\right]dS \qquad (H.5)$$

Taking into account that G=0 on S and using equation (H.4), Green obtained the following integral representation for the function u(P)

$$u(P)=-\frac{1}{4\pi}\iint_S u\frac{\partial G}{\partial n}\,dS \qquad (H.6)$$

In an analogous fashion [2], Green expressed the solution of the Neumann problem in terms of a second singular function $\Gamma(P,Q)$ referred to as Green's second function, which has the following properties

(a) Γ is harmonic in R except at the origin $r=0$ and at some point A

(b) it behaves like $1/r$ at $r=0$ and like $-1/r$ at $r=r_A$ and

(c) its normal derivative $\frac{\partial\Gamma}{\partial n}$ vanishes on S.

Choosing for v the function Γ, relation (H.2) yields

$$u(P)=u(A)+\frac{1}{4\pi}\iint_S\Gamma\frac{\partial u}{\partial n}ds \qquad (H.7)$$

where u(A) is the value of the function at point A; that is, the solution of the Neumann problem is obtained to within an arbitrary constant.

The construction of the Green functions for a given boundary is a difficult problem. For this reason, Green's functions have been established only for few surfaces, such as, the plane and the sphere.

Actually, to establish Green functions G and Γ for a given surface S, the following boundary value problems must be solved

$$\begin{aligned} &\nabla^2 G = -4\pi\delta(P-Q) && \text{in } R \\ &G = 0 && \text{on } S \end{aligned} \qquad \text{(H.8)}$$

and

$$\begin{aligned} &\nabla^2 \Gamma = -4\pi[\delta(P-Q) - \delta(P-A)] && \text{in } R \\ &\frac{\partial \Gamma}{\partial n} = 0 && \text{on } S \end{aligned} \qquad \text{(H.9)}$$

where the point P is in R and the point Q is on S. It is apparent that relations (H.6) or (H.7) are obtained from identity (H.2) where it is assumed that the function u satisfies relation (H.1), and for v, the function G (H.8) or Γ (H.9), respectively, is chosen. The constant u(A) in (H.7) results from the fact that the solution of equation (H.9) is obtained to within an arbitrary constant.

From the aforegoing, it is apparent that Green did not actually solve the Dirichlet or the Neumann problem for the

Laplace equation, but rather reduced them to equivalent ones (H.8) or (H.9) with homogenous boundary conditions whose solution depends only on the geometry of the boundary. That is, once Green's function is established for a given boundary surface S, the solution of boundary value problems involving the same surface may be easily established from the integral representation (H.6) or (H.7).

Betti [2,3] presented a general method for integrating the Navier equations of equilibrium of the linear theory of elasticity in the absence of body forces, which may be regarded as a direct extension of the method of Green.

The Navier equations of equilibrium in the absence of body forces are

$$\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{1}{1-2\nu}\frac{\partial e}{\partial x_i} = 0 \qquad (i,j=1,2,3) \qquad \text{(H.10)}$$

where u_i are the components of displacement; $e=\frac{\partial u_i}{\partial x_i}$ is the dilation; and ν is the Poisson ratio of the material. Noting that the dilatation e is a harmonic function, i.e. $\frac{\partial^2 e}{\partial x_j \partial x_j}=0$, Betti wrote equations (H.10) in the following form

$$\frac{\partial^2}{\partial x_j \partial x_j}\left[u_i + \frac{1}{2(1-2\nu)} x_i e\right] = 0 \qquad \text{(H.11)}$$

The normal derivatives of the components of displacement can be expressed as

$$\frac{\partial u_i}{\partial n} = \frac{1}{2\mu} t_i - \frac{\lambda}{2\mu} e\cos(x_i,n) + e_{ijk}\omega_j\cos(x_k,n) \qquad \text{(H.12)}$$

where e_{ijk} is the alternating tensor; ω_i is the rotation vector; and μ,λ the Lamè constants of the material.

Thus, when e is known at every point of the body and the components of the displacement u_i are prescribed on its boundary (first boundary value problem), the determination of the components of displacement u_i is reduced to a Dirichlet problem in potential theory.

Moreover, when e is known at every point of the body, ω_i is known on the boundary, and the components of traction are prescribed on the boundary (second boundary value problem), the values of $\frac{\partial u_i}{\partial n}$ (i=1,2,3) can be established on the boundary and, consequently, the determination of the components of displacement u_i is reduced to a Neumann problem in potential theory.

Betti obtained formulae for the dilatation and the components of rotation in terms of the boundary data by introducing certain functions which are analogous to Green's functions [2]. To accomplish this, Betti employed his reciprocal theorem which can be written as

$$\iint_S (u_i t_i^* - u_i^* t_i) dS + \iiint_R (u_i b_i^* - u_i^* b_i) dV = 0 \qquad (H.13)$$

where u_i, t_i, b_i and u_i^*, t_i^*, b_i^* are the displacement, the traction and the body force vectors corresponding to two states of stress in a body.

Betti's formulae for the dilatation and the components rotation may be established in the following more concise way , than that presented by Love [2].

Let u_i and t_i be the components of displacement and traction in the body under consideration which is subjected only to surface tractions (b_i=0).

If the components of displacement are specified on the boundary of the body (first boundary value problem), in order to express the dilation in terms of the boundary data, the components of displacement u_i^* and of fraction t_i^* in relation (H.13) will be chosen as those in the body under consideration with its surface restrained from moving ($u_i^*=0$ on S), subjected to the following distribution of body forces

$$b_i^* = \frac{\partial}{\partial x_i}\,\delta(Q-P) \quad (1) \qquad \text{(H.14)}$$

That is, the displacement field u_i^* is the solution of the following boundary value problem

$$\frac{\partial^2 u_i^*}{\partial x_j \partial x_j} + \frac{1}{1-2\nu}\frac{\partial e^*}{\partial x_i} + \frac{1}{\mu}\frac{\partial}{\partial x_i}\,\delta(Q-P)=0 \qquad \text{in } R$$

and (H.15)

$$u_i^*=0 \qquad \text{on } S$$

Using equation (H.14) and the second of (H.15) equation (H.13) yields

$$\iiint_R u_i \frac{\partial}{\partial u_i}\,\delta(Q-P)dV = -\iint_S u_i t^* dS$$

(1) This force vector is referred to as a double force without moment (see Love [2]).

The above relation reduces to

$$e(P)= \frac{\partial u_i(P)}{\partial x_i} = \iint_S u_i t_i^* dS \qquad (H.16)$$

Thus, the dilation in the body under consideration with specified components of displacement on its boundary (first boundary value problem) may be found by solving the auxiliary boundary value problem (H.15) and establishing the components of traction corresponding to the displacement field u_i^*. These are the components of traction which must be applied to the surface S of the body in order to restrain it from moving when the body is subjected to the generalized body force (H.14). The functions $u_i^*=u_i^*(P,Q)$ obtained in this way are analogous to Green's function G(P,Q).

If the components of traction are specified on the boundary of the body (second boundary value problem), in order to express the dilatation in terms of the boundary data, the components of the displacement u_i^* and of fraction t_i^* in relation (H.13) will be chosen as those in the body under consideration, subjected to the distribution of body forces given by equation (H.14), while the components of traction vanish ($t_i^*=0$) on the boundary S. That is, the displacement field is the solution of the following boundary value problem

$$\frac{\partial^2 u_i^*}{\partial x_j \partial x_j} + \frac{1}{1-2\nu}\frac{\partial e^*}{\partial x_i} + \frac{1}{\mu}\frac{\partial}{\partial x_i}\delta(Q-P)=0 \qquad \text{in } R$$

and (H.17)

$$t_i^* = \frac{\partial u_i^*}{\partial n} + \frac{\lambda}{2\mu} e^* \cos(x_i,n) + e_{ijk}\omega_j \cos(x_k,n) = 0 \quad \text{on} \quad S$$

Using equation (H.14) and the second of equations (H.17) equation (H.13) yields

$$\iiint_R u_i \frac{\partial}{\partial x_i} \delta(Q-P)dV = \iint_S u_i^* t_i dS$$

The above relation reduces to

$$e(P) = \frac{\partial u_i(P)}{\partial x_i} = -\iint_S u_i^* t_i dS \qquad (H.18)$$

Thus, the dilatation in the body under consideration subjected to specified surface traction (second boundary value problem) may be found by solving the auxiliary boundary value problem (H.17) and establishing the components of the displacement u_i^* on the boundary S. These are the components of displacement which are produced on the boundary when the body is subjected to the generalized body force (H.14) while its surface is traction free. The functions $u_i^* = u_i^*(P,Q)$, obtained in this way, are analogous to Green's function G(P,Q).

If the components of the displacement are specified on the boundary of the body (first boundary value problem), in order to express the component of rotation, say ω_3, in terms of the boundary data, the components of displacement u_i^* and of traction t_i^* in relation (H.13) will be chosen as those in the body under consideration with its surface restrained from moving ($u_i^*=0$ on S), subjected to the following distribution of body forces[1]

$$b_1^* = \frac{\partial}{\partial x_2}\,\delta(Q-P)$$

$$b_2^* = -\frac{\partial}{\partial x_1}\,\delta(Q-P) \qquad \text{(H.19)}$$

$$b_3^* = 0$$

That is, the displacement field u_i^* is the solution of the following boundary value problem

$$\frac{\partial^2 u_i^*}{\partial x_j \partial x_j} + \frac{1}{1-2\nu}\frac{\partial e^*}{\partial x_i} + \frac{1}{\mu}\, b_i^* = 0 \qquad \text{in } R$$

and (H.20)

$$u_i^* = 0 \qquad \text{on } S$$

where b_i^* is given by (H.19).

Using equations (H.19) and the second of equation (H.20), equation (H.13) yields

$$\iiint_R \left[u_1 \frac{\partial}{\partial u_2}\delta(Q-P) - u_2 \frac{\partial}{\partial u_1}\delta(Q-R)\right] dV = -\iint_S u_i t_i^* dS$$

The above relation reduces to

$$\omega_3(P) = \frac{\partial u_1(P)}{\partial u_2} - \frac{\partial u_2(P)}{\partial u_1} = \iint_S u_i t_i^* dS \qquad \text{(H.21)}$$

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(1) This body force vector is due to two equal unit couples about the x_3 axis acting at point P.

Thus, the component of rotation in the body under consideration for the first boundary value problem may be found by solving the auxiliary boundary value problem (H.20) and establishing the components of traction corresponding to the displacement field u_i^*. These are the components of the traction which must be applied to the surface S of the body in order to restrain it from moving when the body is subjected to the generalized body force (H.19). The functions $u_i^*=u_i^*(P,Q)$ obtained in this way, are analogous to Green's function.

If the components of traction are specified on the boundary of the body (second boundary value problem), in order to express the component of rotation, say ω_3, in terms of the boundary data, the components of displacement u_i^* and of traction t_i^* in relation (H.13) will be chosen as those in the body under consideration, subjected to the distribution of body forces given by equation (H.19), while the components of traction vanish on the boundary ($t_i^*=0$ on S). That is, the displacement field is the solution of the following boundary value problem

$$\frac{\partial^2 u_i^*}{\partial x_j \partial x_j} + \frac{1}{1-2\nu}\frac{e^*}{\partial x_i} + \frac{1}{\mu} b_i^*=0 \qquad \text{in R}$$

and (H.22)

$$t_i^* = \frac{\partial u_i^*}{\partial n} + \frac{\lambda}{2\mu} e^* \cos(x_i,n) + e_{ijk}\omega_j \cos(x_k,n) = 0 \qquad \text{on S}$$

where b_i^* is given by equation (H.19).

Using equation (H.19) and the second of equations (H.22), equation (H.13) yields

$$\iiint_R \left[u_1 \frac{\partial}{\partial x_2}\delta(Q-P) - u_2 \frac{\partial}{\partial x_1}\delta(Q-P)\right] dV = \iint u_i^* t_i \, dS$$

The above relation reduces to

$$\omega_3(P) = \frac{\partial u_1(P)}{\partial x_2} - \frac{\partial u_2(P)}{\partial x_1} = -\iint_S u_i^* t_i \, dS \qquad (H.23)$$

Thus, the component of rotation ω_3, in the body under consideration, for the second boundary value problem may be found by solving the auxiliary boundary value problem (H.22) and establishing the components of displacement u_i^* on the boundary S. These are the components of displacement produced on the boundary when the body is subjected to the generalized body force (H.14) while its surface is traction free. Notice, that in this case, the body is not in equilibrium and, consequently, the displacement field can not be uniquely established. To overcome this difficulty, a body force vector opposite to that given by equation (H.19) is applied at some fixed point A. The body is then in equilibrium and the component $\omega_3(P)$ is obtained as

$$\omega_3(P) = \omega_3(A) - \iint_S \tilde{u}_i^* t_i \, ds \qquad (H.24)$$

where $\tilde{u}_i^*$ is the displacement vector on the boundary S produced by the two sets of couples applied at points P and A. Thus, the rotation component is established to within an arbitrary constant. This indeterminacy does not affect the solution. The functions $\tilde{u}_i^* = \tilde{u}_i^*(P,Q)$ obtained in this way are analogous to Green's function $\Gamma(P,Q)$. The components of the rotation ω_1 and ω_2 can be established in an analogous fashion.

On the basis of the aforegoing, it is apparent that the determination of the tractions t_i^* or the displacements u_i^* on the boundary S of the body in terms of the prescribed boundary data is difficult. Thus, Betti's method for integrating the Navier equations of equilibrium has been applied to a limited number of simple cases. For example, Cerruti [4] employed Betti's method to establish the displacement field in a semiinfinite elastic body under given surface tractions or surface displacements.

Another integral representation of the components of displacement in a linear elastic body in terms of the values of the components of traction and displacement at the boundary was obtained by Somigliana [5], who used for u_i^* and t_i^* in Betti's reciprocal formula (H.13) the displacement and traction fields in an elastic body subjected to a concentrated unit body force (Kelvin's problem).

The solution of Kelvin's problem can be written in tensor form as [6]

$$U_{ij} = \frac{1}{4\pi\mu}\left[\frac{3-4\nu}{4(1-\nu)}\,\delta_{ij} + \frac{1}{4(1-\nu)}\,\frac{\partial r}{\partial x_i}\,\frac{\partial r}{\partial x_j}\right]\frac{1}{r} \qquad \text{(H.25)}$$

where $r=|P-Q|$ is the distance between the field point P, and the source point Q_j; δ_{ij} is the Kronecker delta. The component of the tensor U_{ij} denotes the component of displacement at point P in the x_j direction, due to unit force at point Q in the x_i direction. The component of the traction, in the x_j direction, acting on a surface normal to the unit vector n_i at point P, due to a unit load at point Q in the x_i direction, may be obtained from equation (H.25) as

$$T_{ij} = -\frac{1-2\nu}{8\pi(1-\nu)}\left[\frac{\partial r}{\partial n}\left(\delta_{ij} + \frac{3}{1-2\nu}\frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j}\right) - n_j\frac{\partial r}{\partial x_i} + n_i\frac{\partial r}{\partial x_j}\right] \qquad \text{(H.26)}$$

The displacement vector at a point P and the traction acting on a surface normal to the unit vector n at point P, due to a unit force at a point Q acting in the direction specified by the unit vector e_i, are given by

$$u_j = U_{ij}e_i \quad , \qquad t_j = T_{ij}e_i \qquad \text{(H.27)}$$

If u_i, t_i are the components of displacement and traction of the desired state of stress in the body due to a distribution of surface traction only ($b_i=0$) and if u_i^*, t_i^* are the components of the displacement and traction due to a unit body force acting in the e_i direction at a point P inside R, then equation (H.13) may be written as

$$\iint_S (u_i T_{ji} e_j - t_i U_{ji} e_i)dV + \iiint_R u_j b_j^* dV = 0 \qquad \text{(H.28)}$$

Using the Dirac delta-function the body force field b_i^* due to the concentrated unit force may be represented as

$$b_i^*=e_i\delta(Q-P) \qquad \text{(H.29)}$$

Substitution of (H.29) into (H.28) yields[1]

$$u_j(P)=-\iint_S u_i(q)T_{ji}(P,q)dS_q+\iint_S t_i(q)U_{ji}(P,q)dS_q \qquad \text{(H.30)}$$

where P∈R and q∈S. Equation (H.30) is known as the Somigliana identity. It expresses the displacement vector at an interior point of the body in terms of integrals of the boundary data.

For a body of given geometry,if the solution of Kelvin's problem U_{ji} can be chosen (guessed or synthetically constructed) so that it vanishes on the boundary S of the body, equation (H.30) yields

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(1) This result was obtained by Somigliana without using δ-function as following.

In the absence of body forces,Betti's reciprocal theorem (H.13) may be written as

$$\iint_{S+S_o}(u_iT_{ji}-t_iU_{ji})dS=0$$

where S_o is a small sphere surrounding the point P. By letting the radius r_o of S_o approach zero,it can be proven that

$$\lim_{r_o\to 0}\iint_{S_o} t_iU_{ji}dS=0 \quad \text{and} \quad \lim_{r_o\to 0}\iint_{S_o} u_iT_{ji}dS=u_j(P)$$

and,thus,equation (H.30) is obtained.

$$u_j(P)=-\iint_S u_j(q)T_{ji}(P,q)dS_q \qquad (H.31)$$

Equation (H.31) gives the solution of the first boundary value problem for the given body under any prescribed boundary displacements. Similarly, if the solution of Kelvin's problem U_{ji} can be chosen (guessed or synthetically constructed) so that the traction tensor T_{ji} vanishes on the boundary S of the body, equation (H.30) yields

$$u_j(P)=\iint_S t_i(q)U_{ji}(P,q)dS_q \qquad (H.32)$$

Equation (H.32) gives the solution of the second boundary value problem for the given body under any prescribed boundary tractions.

The establishement of Kelvin's solution so that the displacement field or the traction field vanish on the boundary of a body with arbitrary shape, is a very difficult problem. From a mathematical point of view,this requires the establishment of the solution of the Navier equations of equilibrium when the body under consideration is subjected to a unit concentrated force at point P in the x_j (j=1,2,3) direction,while the components of displacement (first boundary value problem) or of traction (second boundary value problem) vanish on the boundary S of the body. This solution has been established for bodies of simple geometries. For example, using this integral approach, Somigliana [5] solved the problem of the semiinfinite body subjected to surface displacements and tractions and obtained the same results with those of Cerruti.

From the aforegoing, it is seen that Betti's and Somigliana's integral method for the solution of the elasticity problems are analogous to Green's method in potential theory. That is, in establishing the unknown field quantities (dilatation and components of rotation in Betti's method or the components of displacement in Somigliana's method) two-point functions have been introduced which are singular solutions of the differential equation of the problem under consideration, with homogeneous boundary conditions. Introducing these functions into the appropriate integral representation of the desired field quantity,the unknown boundary data are eliminated and the field quantity is obtained in an integral form,including only the specified boundary data.

The solution of a boundary value problem by expressing the desired field quantity in an integral form and eliminating the unspecified boundary data from it by introducing appropriate two-point functions is referred to as Green's method. It can be applied to problems governed by linear differential equations.

Instead of trying to eliminate the unspecified data, another approach is to establish it in terms of the specified boundary data. This approach is known as the boundary integral equation (BIE) method. Thus, by allowing the interior point P in the integral representation (H.4) to approach a point p on the boundary S, the following limiting form of Green's integral representation is obtained

$$u(p)=\frac{1}{2\pi}\iint_S\left[\frac{1}{r}\frac{\partial u}{\partial n}-u\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\right]dS \qquad \text{(H.33)}$$

In a well-posed boundary value problem, for the Laplace equation u and $\frac{\partial u}{\partial n}$ are not concurrently prescribed at the points of the boundary S. They must satisfy the relation (H.33) which constitutes a compatibility condition on the boundary S between u and $\frac{\partial u}{\partial n}$. This relation can be used to establish the unknown in terms of the given boundary information. In this case relation (H.33) constitutes a boundary integral equation for the unknown boundary quantity. In as much as the kernels 1/r and $\frac{\partial}{\partial n}(1/r)$ are singular, the integral equation (H.33) is singular.

Similarly, by allowing the interior point P of the body to approach a point p on the boundary S in equation (H.30) the following limiting form of the Somigliana identity, referred to as vector boundary integral equation relating the boundary tractions and displacements, is obtained [6]

$$\frac{1}{2}u_j(p)=-\iint_S u_i(q)T_{ji}(p,q)dS_q+\iint_S t_i(q)U_{ji}(p,q)dS_q \qquad \text{(H.34)}$$

In a well-posed boundary value problem in elasticity, the components of traction and displacement are not concurrently prescribed at the points of the surface of the body .

Fredholm [7] and Lauricella [8] employed the boundary integral equation (H.34) in order to determine the boundary information which is not prescribed. The results are then substituted in the Somigliana identity (H.30) to obtain the displacement field.

Fredholm was the first to use singular boundary integral equations (BIE method) in potential theory [9,10] and in the theory of elasticity [7] to obtain the unknown boundary quantities in terms of the given. Although in potential theory, the boundary integral equation (H.29), which is also well-suited for mixed boundary conditions, can be used to obtain the unknown boundary data, Fredholm [9] used for the solution of the Dirichlet problem, the integral representation of a harmonic function in R as the potential of a double(1) layer

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(1) In the Dirichlet problem the potential of a single layer could be used to represent the harmonic function u(P) i.e.

$$u(P)= \frac{1}{2\pi} \iint_S \mu(q)\frac{1}{r}\, dS$$

In this case, the resulting boundary integral equation obtained by letting $P \to p \in S$ $\left[u(p)= \frac{1}{2\pi} \iint_R \mu(q)\frac{1}{r}\, dS_q\right]$, is a Fredholm equation of the first kind. However, this formulation has not been used in the literature. Jaswon [11] attributes this to the fact that the Fredholm equation of the first kind has not been studied thoroughly.

mass density (mass per unit area) distribution on the boundary S i.e.

$$u(P)=\frac{1}{2\pi}\iint_S \mu(q)\frac{\partial}{\partial n}\left(\frac{1}{r}\right)dS_q \qquad (H.35)$$

where $\mu(q)$ is the boundary mass density to be determined from the prescribed values of the potential u on the boundary; $r=|P-q|$ with $P\in R$ and $q\in S$. Notice, that $\frac{\partial}{\partial n}(1/r)$ is a harmonic function $[\nabla^2\frac{\partial}{\partial n}(1/r)=0]$ because $1/r$ is harmonic.

In Green's integral representation (H.4), the potential u(P) is given as the difference of a single layer potential with density $\frac{\partial u}{\partial n}$ and a double layer potential with density u; that is, the boundary values of $\frac{\partial u}{\partial n}$ and u are analogous to the single layer and double layer densities, respectively. However, in relation (H.35) $\mu(q)$ is the unknown mass density distributed at the boundary of the region R which must be determined from the prescribed values of u on the boundary.

Letting point P in equation (H.35) approach a point p on S, and taking into account that the double layer potential has a discontinuity at the boundary, Fredholm obtained the following relation

$$u(p)=-\mu(p)+\frac{1}{2\pi}\iint_S \mu(q)\frac{\partial}{\partial n}\left(\frac{1}{r}\right)dS_q \qquad (H.36)$$

In the Dirichlet problem, the function u(p) is prescribed. In this case, relation (H.36) provides the necessary integral equation for the determination of the unknown boundary distribution $\mu(q)$. Inasmuch as the kernel $\frac{\partial}{\partial n}(\frac{1}{r})$ in singular, equation (H.36) is a singular integral equation.

For the solution of the Neumann problem, Fredholm used the integral representation of the harmonic function in R as the potential of a single layer mass density distribution on the boundary S, i.e.

$$u(P)= \frac{1}{2\pi} \iint_S \sigma(q)\frac{1}{r}\, dS_q \qquad (H.37)$$

where $\sigma(q)$ is the boundary density to be determined. Differentiating (H.37) with respect to the normal to the boundary n, and letting $P \to p \in S$, the following boundary integral equation is obtained

$$\frac{\partial u}{\partial n_p} = \sigma(p)+ \frac{1}{2\pi} \iint_S \sigma(q)\frac{\partial}{\partial n_p}\left(\frac{1}{r}\right)dS_q \qquad (H.38)$$

from which the unknown boundary distribution $\sigma(p)$ can be determined. Equations (H.36) and (H.38) could be solved analytically only for simple geometries of the boundary.

Thus, the boundary integral equation (BIE) method may be attributed to Fredholm who also applied it to elasticity [7]. Moreover, Fredholm and Lauricella [8] extended the theory of singular integral equations and used it to solve problems in elasticity.

Other integral representations of the displacement field for three dimensional elasticity problems are those proposed by Kupradze [12]. These representations are analogous to those used by Fredholm in the theory of potential.

For the first boundary value problem in three dimensional elasticity, which corresponds to the Dirichlet problem of the potential theory, Kupradze proposed the following integral representation of the displacement field(*) [13]

$$u_i(P)=\iint_S T_{ji}(P,q)\Phi_j(q)dS_q \qquad (H.39)$$

where $\Phi_j(q)$ are unknown functions defined on the boundary S and must be determined from the prescribed boundary values of the displacement field. Inasmuch as the field T_{ji} given by relation (H.26) satisfies the Navier equations of equilibrium (H.10), it is apparent that the displacement field (H.39), also satisfies the same equations. The expression (H.39) is analogous to (H.35) and is referred to as the elastic potential due to the double layer distribution $\Phi_j(q)$.

By letting $P\to p\epsilon S$ in equation (H.39), the following boundary singular integral equations result

$$u_i(p)=-\frac{1}{2}\Phi_i(p)+\iint_S T_{ji}(p,q)\Phi_j(q)dS_q \qquad (H.40)$$

from which the unknown boundary functions $\Phi_i(p)$ can be established when $u_i(p)$ are prescribed on S.

For the second boundary value problem in three

(*) The integral representation $u_i(p)=\iint_S U_{ji}(p,q)\Phi_j(q)dSq$ could also be used for the first boundary value problem in three dimensional elasticity. This would lead to a Fredholm equation of the first kind and it has not been used for the reasons stated in the Footnote on p. 18.

dimensional elasticity, which corresponds to the Neumann problem of potential theory, Kupradze proposed the following integral representation for the displacement field [13]

$$u_i(P)=\iint_S U_{ji}(P,q)\Psi_j(q)dS_q \qquad (H.41)$$

where $\psi_j(q)$ are unknown functions defined on the boundary S and must be determined from the prescribed boundary values of the components of traction. Inasmuch as the field U_{ji} given by relation (H.25) satisfies the Navier equations of equilibrium, it is apparent that the field (H.41) also satisfies the same equations. The expression (H.41) is analogous to (H.37) and is referred to as the elastic potential due to the single layer boundary distribution $\psi_j(q)$.

By introducing (H.41) into the boundary conditions (the tractions in terms of the boundary displacements) and by letting $P \to p \in S$, the following singular boundary integral equations result

$$t_i(p)=\frac{1}{2}\psi_i(p)+\iint_S T_{ji}(p,q)\psi_j(q)dS_q \qquad (H.42)$$

from which the unknown boundary functions $\psi_i(p)$ can be established when $t_i(p)$ are prescribed on S.

Boundary integral equations have also been used by Sherman [14,15], Mikhlin [16], Muskhelishvili [17] and Theocharis [18,19] in treating plane elasticity problems via a complex function approach. As it is known, the plane elasticity problem reduces to the determination of Airy's stress function

$F(x_1,x_2)$ $\left[\tau_{11}=\frac{\partial^2 F}{\partial x_1^2},\ \tau_{12}=-\frac{\partial^2 F}{\partial x_1 \partial x_2},\ \tau_{22}=\frac{\partial^2 F}{\partial x_1^2}\right]$ which is a biharmonic function; i.e., it satisfies inside the two-dimensional region R occupied by the body the equation

$$\nabla^4 F=0 \tag{H.43}$$

and specified conditions on the boundary C of R.

The general solution of (H.43) can be expressed in terms of two analytic functions $\varphi(z)$ and $\chi(z)$, $z=x_1+ix_2$, as

$$F(x_1,x_2)=\text{Re}\left[\bar{z}\varphi(z)+\chi(z)\right] \tag{H.44}$$

Relation (H.44) is known as the Goursat formula [16] and the function $\varphi(z)$ and $\psi(z)=\chi'(z)$ are referred to as the complex potentials. The components of stress and displacement are given in terms of these potentials as [20]

$$\tau_{11}+\tau_{22}=4\text{Re}\left[\varphi'(z)\right] \qquad \text{(a)}$$

$$\tau_{22}-\tau_{11}+2i\tau_{12}=2\left[\bar{z}\varphi''(z)+\psi'(z)\right] \qquad \text{(b)} \qquad \text{(H.45)}$$

$$2G(u_1+iu_2)=\Lambda\varphi(z)-z\overline{\varphi'(z)}-\overline{\psi(z)} \qquad \text{(c)}$$

$$\Lambda=(3-\nu)/(1+\nu)$$

Thus, once the potentials $\varphi(z)$ and $\psi(z)$ are established, the stress and displacement fields can be obtained directly from equations (H.45). The functions $\varphi(z)$ and $\psi(z)$ are established from the boundary conditions of the problem.

Thus, for the first boundary value problem, the components of the displacement are prescribed on C [i.e.

$u_1=g_1(s)$ and $u_2=g_2(s)]$, and from relation (H.45c), the boundary conditions in complex form become

$$\Lambda\varphi(\zeta)-\zeta\overline{\varphi'(\zeta)}-\overline{\psi(\zeta)}=2G(g_1+ig_2) \qquad \text{(H.46)}$$

where $\zeta=x_1+ix_2$ is a point on the boundary C.

For the second boundary value problem, the components of traction are prescribed on the boundary C, i.e. $t_1=t_1(s)$ and $t_2=t_2(s)$. The first derivatives of the stress function $F(x_1,x_2)$ are established to within an arbitrary constant in terms of the boundary components of traction as

$$\frac{\partial F}{\partial x_1}=-\int_0^S t_2(s)ds+d_1, \qquad \frac{\partial F}{\partial x_2}=\int_0^S t_1(s)ds+d_2$$

where d_1 and d_2 are arbitrary constants. Thus

$$\frac{\partial F}{\partial x_1}+i\frac{\partial F}{\partial x_2}=f_1(s)+if_2(s)+d \qquad \text{(H.47)}$$

where it has been set

$$f_1(s)+if_2(s)=i\int_0^S[t_1(s)+it_2(s)]ds$$

and

$$d=d_1+id_2$$

Relation (H.47) constitutes the boundary conditions in complex form for the second boundary value problem which, in terms of the complex potentials $\varphi(z)$ and $\psi(z)$, may be written as

$$\varphi(\zeta)+\zeta\overline{\varphi'(\zeta)}+\psi(\zeta)=f_1+if_2+d \qquad \text{(H.48)}$$

Notice,that the boundary condition (H.48) can be obtained from (H.46) by setting $\Lambda=-1$.

It can be proven that the potentials $\varphi(z)$ and $\psi(z)$ are not independent, and that $\psi(z)$ can be expressed in terms of $\varphi(z)$, or both can be expressed in terms of a function $w(z)$.

Thus, relation (H.46) or (H.48) provides the necessary equation for the determination of the complex potentials $\varphi(z)$ and $\psi(z)$.

Sherman expressed the functions $\varphi(z)$ and $\psi(z)$,in terms of a function $w(z)$,by the following Cauchy integrals

$$\varphi(z)=\frac{1}{2\pi i}\int_C \frac{w(\zeta)}{\zeta-z}d\zeta$$

$$\psi(z)=\frac{1}{2\pi i}\int_C \frac{\overline{w(\zeta)}}{\zeta-z}\,d\zeta-\frac{1}{2\pi i}\int_C \frac{\bar{\zeta}w'(\zeta)}{\zeta-z}\,d\zeta \qquad \text{(H.49)}$$

where $w(\zeta)$ is an unknown function whose derivative satisfies the Hölder(*) condition on the boundary C. For the boundary

(*) A function $w(\zeta)$ is said to satisfy the Hölder condition on the boundary C if the following inequality holds true for any pair of points ζ_1,ζ_2 of C

$$[w(\zeta_2)-w(\zeta_1)]\le A[\zeta_2-\zeta_1]^\alpha$$

where A and α are positive constants and $0<\alpha\le 1$; A is called the Hölder constant and α the Hölder index [17]. For $\alpha=1$,the Hölder condition yields the Lipschitz condition.

values of the function $\varphi(z)$ and $\psi(z)$ we find from (H.49) as $z \to t \in C$.

$$\varphi(t) = \frac{w(t)}{2} + \frac{1}{2\pi i}\int_C \frac{w(\zeta)}{\zeta - t}\, d\zeta \tag{H.50}$$

$$\psi(t) = \frac{\overline{w(t)}}{2} + \frac{1}{2\pi i}\int_C \frac{\overline{w(\zeta)}}{\zeta - t}\, d\zeta - \frac{\bar{t}w'(t)}{2} - \frac{1}{2\pi i}\int_C \frac{\bar{\zeta}w(\zeta)}{\zeta - t}\, d\zeta$$

Substituting equations (H.50) into equations (H.48), and setting $\zeta - t = re^{i\theta}$, the following integral equation in $w(t)$ is obtained

$$w(t) + \frac{1}{\pi}\int_C [w(\zeta) - \overline{w(\zeta)}e^{2i\theta}]\, d\theta = f(t) \tag{H.51}$$

By setting $w(t) = p(s) + iq(s)$, equation (H.51) can be replaced by the following two real boundary integral equations

$$\begin{aligned} p(s) + \frac{1}{\pi}\int_C [p(s')(1-\cos 2\theta) - q(s')\sin 2\theta]\, d\theta &= f_1(s) \\ q(s) + \frac{1}{\pi}\int_C [p(s')\sin 2\theta - q(s')(1+\cos 2\theta)]\, d\theta &= f_2(s) \end{aligned} \tag{H.52}$$

from which the functions $p(s)$ and $q(s)$ can be established.

In the aforementioned references, closed form solution of the boundary integral equations have been obtained only for a few boundary value problems involving simple boundaries. It was not until the beginning of the decade of 1960 that efficient numerical methods for the solution of the singular boundary integral equations have been developed and the numerical solution of the singular integral equations of the BIE method for problems involving more complex boundaries has been programmed on digital computers. Virtually all the numerical methods that have been employed in connection with the BIE method are based on the discretization of the boundary. For instance, in two-dimensional problems,the plane curve boundary is divided into a finite number of line segments. On each segment, the unknown boundary functions are approximated by polynomials of a desired degree, in terms of their values at a finite number of points. The line segment is also approximated by a simple curve (straight line, parabolic arc etc.) and the required integrations are carried out on each boundary segment. Special care is taken for the evaluation of the improper integrals occurring at the segments which include a point where the kernels of the integrals are singular. With this approach, the boundary integral equations are reduced to a system of simultaneous linear algebraic equations whose solution gives the values of the unknown boundary functions at a finite number of points on the boundary. These values are used in evaluating the integral representation of the field function by numerical integration. In three dimensional problems, the boundary is a surface and,thus,it is divided into surface elements. A procedure analogous to that employed in two-dimensional problems is applied to convert the integral equation to simultaneous algebraic equations for the boundary values of the unknown boundary functions.

The simplest boundary value problems to be attacked by the BIE method, using a numerical technique to solve the singular boundary integral equations, were those whose field functions satisfying the Laplace or Poisson equations. In 1963 Jaswon and Ponter [21] applied a numerical technique to solve the boundary integral equation for the classical torsion problem of Saint Venant formulated in terms of the warping function as a Neumann boundary value problem for the Laplace equation. They obtained numerical results for prismatic bars having a variety of cross sections, such as, solid and hollow ellipses, rectangles, equilateral triangles and circles with curved notches. Moreover, they discussed the effectiveness of the BIE method. Mendelson [22] solved the same problem as a Dirichlet boundary value problem in terms of the stress function. He obtained numerical results for prismatic bars with rectangular cross sections. Mendelson [22,23,24,25] also treated the problem of the elastoplastic torsion of prismatic bars as a Dirichlet boundary value problem for the Poisson equation, in terms of the Prandl's stress function.

Symm [26] solved the problem of conformally mapping a given simply connected domain with arbitratry boundary in the complex z-plane, onto the unit circle $|w(z)|=1$, in the complex w-plane. The mapping function w(z) was determined from a Dirichlet problem for the Laplace equation using the BIE method.

Christiansen [27] gives a complete collection of integral equations for solving the Saint Venant torsion problem.

Jaswon [11] presented a brief analytical study of the existence of the solution of the Fredholm integral equations which appear in the formulation (by the BIE method) of problems in potential theory and in the theory of elasticity. Moreover, Symm [28] presented and tested a number of techniques for solving numerically singular integral equations which appear in the formulation by the BIE method of two dimensional problems in potential theory.

The BIE method, with numerical integration of the boundary integral equations, has been applied for the numerical solution of the boundary value problems in two-dimensional elasticity by Rizzo [29] . He formulated the two dimensional counterpart of equations (H.30) and (H.34) using the singular solution for the two-dimensional Navier equations of equilibrium. That is,

$$U_{ij}=\delta_{ij}\ell nr+M\frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j} \qquad (H.53)$$

and the corresponding traction tensor

$$T_{ij}=\frac{\partial}{\partial n}(\ell nr)\left[k\delta_{ij}-4\mu M\frac{\partial r}{\partial x_i}\frac{\partial r}{\partial x_j}\right]+k\left[\frac{\partial}{\partial x_i}(\ell nr)n_j-\frac{\partial}{\partial x_j}(\ell nr)n_i\right] \qquad (H.54)$$

where

$$M=-(\lambda+\mu)/(\lambda+3\mu)$$

$$k=2\mu^2/(\lambda+3\mu)$$

He obtained numerical results for certain simple boundaries and compared them with their analytic solution. Cruse [6] extended Rizzo's work to three dimensional problems by solving numerically the singular boundary integral equations (H.34) and by establishing the displacement field by numerical evaluation of equation (H.30). He obtained numerical results for a number of relatively complicated problems. Dubois and Lachat [30] employed the BIE method for the solution of elastostatic problems. They solved equations (H.34) numerically and obtained numerical results for a number of two dimensional problems. Moreover, they showed that the results obtained by the BIE method were in excellent agreement with those obtained by using the FE (finite element) method as well as the analytical solution, and they discussed the advantages of the BIE method over the FE method. Rizzo and Shippy [31] extended the BIE method to two-dimensional anisotropic elasticity using the singular solution of the Navier equations for the two-dimensional anisotropic elastic body presented by A.E. Green [32]. They obtained equations analogues to (H.30) and (H.34). Moreover, they indicated numerical techniques for the solution of the resulting boundary singular integral equations and analysed several problems for illustration. Vogel and Rizzo [33] constructed the singular solution of the Navier equations for the three-dimensional anisotropic elastic body and extended the BIE method to three dimensional anisotropic elasticity by obtaining equations analogous to (H.30)(Somigliana's) and (H.34). They also discussed techniques for solving numerically the resulting boundary integral equations. Cruse and Rizzo [34] and Cruse [35] employed the BIE method to solve the transient elastodynamic problem, by taking the Laplace transform of the Navier equations of motion and, thus, converting them from hyperbolic to elliptic. Subsequently, they solved them by the BIE method in the transform space. Their results are inverted by a numerical technique. Numerical results were presented for the half-plane subjected to a uniform loading extended on a finite portion of the free boundary.

Ignaczak and Nowacki [36] obtained integral representation for the displacement and temperature fields in three dimensional, simply-connected bodies. Moreover, they formulated the necessary singular boundary integral equations for time harmonic, thermoelastic problems.

In many of the aforementioned references, the problem is formulated in terms of quantities (the components of displacement) which have physical meaning. In these cases, the BIE methods are referred to as direct. In other references, the problems are formulated in terms of unknown but familiar functions, such as, the stress function from which the components of stress are then determined by simple differentiation. In these cases, the BIE methods are referred to assemidirect. Finally, some problems have been formulated in terms of unknown density functions which have no physical significance. However, once these density functions are determined, the components of displacement and stress can be computed directly. In these cases, the BIE methods are referred to as indirect. A lucid presentation of this classification of the BIE methods is given by Medelson [22].

Another family of boundary value problems involving the biharmonic equation

$$\nabla^4 u = f(P) \qquad \text{(H.55)}$$

where

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \qquad \text{(H.56)}$$

such as, two dimensional elasticity problems formulated in terms of Airy's stress function and problems of bending of thin plates have been solved by the BIE method. The integral representation of the biharmonic function is based on the Rayleigh-Green identity [37],[38] [see also relation(I-2.3)]

$$\iint_R (v\nabla^4 u - u\nabla^4 v)d\sigma = \int_C \left(v\frac{\partial}{\partial n}\nabla^2 u - \frac{\partial v}{\partial n}\nabla^2 u - u\frac{\partial}{\partial n}\nabla^2 v + \frac{\partial u}{\partial n}\nabla^2 v\right)ds \qquad \text{(H.57)}$$

where $u=u(x,y)$ and $v=v(x,y)$ are any two functions defined in the two-dimensional region R, bounded by the curve C which are four times continuously differentiable in R, and three times continuously differentiable on C.

If the function v is chosen as a singular, particular solution of the equation

$$\nabla^4 u = \delta(Q-P) \qquad \text{(H.58)}$$

that is,

$$v = \frac{1}{8\pi} r^2 \ell n r \qquad \text{(H.59)}$$

where $r = |P-Q|$ is the distance between any two points P and Q of the plane, and u satisfies equation (H.55), then equation (H.57) yields the following integral representation of the solution of equation (H.55)

$$u(P)=\iint_R fv\,d\sigma-\int_C \left(v\frac{\partial}{\partial n}\nabla^2 u-\frac{\partial u}{\partial n}\nabla^2 u-u\frac{\partial}{\partial n}\nabla^2 v+\frac{\partial u}{\partial n}\nabla^2 v\right)ds \qquad \text{(H.60)}$$

where $v=v(P,q)$ with $P\epsilon R$ and $q\epsilon C$.

In a well-posed boundary value problem involving the biharmonic equation, two conditions must be specified on the boundary. For instance, when u represents the Airy stress function, for the second boundary value problem of plane elasticity, u and its normal derivative $\frac{\partial u}{\partial n}$ must be specified on the boundary C. Thus, two boundary integral equations must be formulated. One of them is derived from equation (H.60) by letting point $P\epsilon R$ approach a point $p\epsilon C$. In taking this limit, the term of the line integral involving $\frac{\partial}{\partial n}(\nabla^2 v)$ behaves like a double layer potential exhibiting a jump equal $\frac{1}{2}u(p)$. Thus, the following singular boundary integral equation is obtained

$$\frac{1}{2}u(p)=\iint_R fv\,d\sigma-\int_C \left(v\frac{\partial}{\partial n}\nabla^2 u-\frac{\partial v}{\partial n}\nabla^2 u-u\frac{\partial}{\partial n}\nabla^2 v+\frac{\partial u}{\partial n}\nabla^2 v\right)ds \qquad \text{(H.61)}$$

where $v=v(p,q)$ with $p,q\epsilon C$.

The second boundary integral equation used in problems involving the biharmonic operator depends on the boundary conditions. A systematic derivation of the second boundary integral equation which can be adopted for all kinds of boundary conditions is given in reference [39].

The integral representation (H.60) has been employed to solve a variety of two-dimensional elastostatic and elasto-plastic problems. Christiansen and Hansen [40] determined the components of stress in an elastic sheet with one or more unloaded holes. Rzasnicki [41] established the stress distribution in an elastoplastic plate with a V-notch subjected to bending. Rzasnicki, Mendelson and Albers [42] established the stress distribution in a plane elastic beam with a V-notch. For these problems, u(P) is the Airy stress function. The integral representation (H.61) has also been used to establish the deflection of the middle surface of thin elastic plates subjected to transverse loading by the direct BIE method. For instance, Segedin and Brickell [43] considered corner-shaped plates. They obtained numerical results and compared them with those obtained from the finite difference method. Maiti and Chakrabarty [44] considered simply supported, polygonal plates and presented numerical results for square, triangular, rhombic, and hexagonal plates. In the aforementioned cases, only straight boundaries and certain boundary conditions were considered. Bezine [45] and Bezine and Gamby [46] considered plates with polygonal boundaries with arbitrary boundary conditions. They obtained numerical results for square plates with various edge conditions and compared the result with those obtained by the FE method or from existing analytical solutions.

The representation of a biharmonic function in terms of two harmonic functions has also been employed in solving two-dimensional, elasticity problems [11,47,48,49,50] and thin elastic plate problems [51,52] by the BIE method. In this approach, the biharmonic function can be expressed as

$$u(P)=r^2\varphi+\psi \tag{H.62}$$

where $\varphi=\varphi(x,y)$ and $\psi(x,y)$ are harmonic functions which, as discussed previously, can be represented as single layer potentials. Thus, we can write

$$\begin{aligned} \varphi(P) &= \int_C \mu(q)\ell nr ds_q \\ \psi(P) &= \int_C \sigma(q)\ell nr ds_q \end{aligned} \tag{H.63}$$

where $\mu(q)$ and $\sigma(q)$ are two unknown simple boundary distributions. Introduction of equations (H.63) into (H.62) yields

$$u(P)=r^2\int_C \mu(q)\ell nr ds_q+\int_C \sigma(q)\ell nr ds_q \tag{H.64}$$

This integral representation of the biharmonic function may be employed to establish the boundary integral equations for boundary value problems involving the homogeneous biharmonic equation. For boundary value problems involving a non homogenous, biharmonic equation, as in the case of bending of thin plates, a particular solution of this equation must be established.

Another differential equation which has been treated by the BIE method is that of Helmholz

$$\nabla^2 u + k^2 u = 0 \qquad \text{(H.65)}$$

Notice, that in case of harmonic waves, the wave equation is reduced to the above. Equation (H.65) is an elliptic partial differential equation and, thus, its solution can be established by using the BIE method. The Green identity for the Helmholz operator is easily obtained from equation (H.2) as

$$\iiint_R [u(\nabla^2 v + k^2 v) - v(\nabla^2 u + k^2 u)]\, dV = \iint_S (u\frac{\partial u}{\partial n} - v\frac{\partial u}{\partial n})\, dS \qquad \text{(H.66)}$$

From this identity, the integral representation for the solution of equation (H.65) may be obtained by taking v as a particular singular solution of equation

$$\nabla^2 v + k^2 v = \delta(Q-P) \qquad \text{(H.67)}$$

For the three dimensional problems, a particular singular solution of (H.67) is

$$v = -\frac{e^{-ikr}}{4\pi r} \qquad \text{(H.68)}$$

where $r=|P-Q|$ is the distance between any two points P and Q. Using relation (H.65) and (H.67), equation (H.66) reduces to

$$u(P)=-\frac{1}{4\pi}\iint_S [u\frac{\partial}{\partial n}(\frac{e^{-ikr}}{r})-\frac{e^{-ikr}}{r}\frac{\partial u}{\partial n}]dS \qquad (H.69)$$

Letting $P \to p \in S$ in (H.69) and taking into account that the first term in the integral has a discontinuity equal to $-2\pi u(p)$, as P approaches p on the boundary S, the following boundary singular integral equation is obtained

$$\frac{1}{2}u(p)=-\frac{1}{4\pi}\iint_S [u\frac{\partial}{\partial n}(\frac{e^{-ikr}}{r})-\frac{e^{-ikr}}{r}\frac{\partial u}{\partial n}]ds \qquad (H.70)$$

where $r=|p-q|$, p, $q \in S$. Equation (H.69) is known as Helmholz's integral equation [53,54].

For two-dimensional problems, the particular singular solution of (H.67) can be expressed in terms of the zero order Bessel function of the second kind $Y_0(kr)$ or the zero order Hankel function of the first kind $H_0^{(1)}(kr)$, i.e.

$$v=\frac{1}{4}Y_0(kr) \qquad (a)$$

or (H.71)

$$v=\frac{1}{4i}H_0^{(1)}(kr) \qquad (b)$$

Thus, the following two integral representations for the solution of equation (H.65) are obtained

$$u(P)=\frac{1}{4}\int_C [u\frac{\partial}{\partial n}Y_0(kr)-Y_0(kr)\frac{\partial u}{\partial n}]ds \qquad (a)$$

or (H.72)

$$u(P)=\frac{1}{4i}\int_C [u\frac{\partial}{\partial n}H_0^{(1)}(kr)-H_0^{(1)}(kr)\frac{\partial u}{\partial n}]ds \qquad (b)$$

Letting $P \to p \epsilon C$ in equations (H.72), and taking into account that the first term in the integral has a discontinuity equal to 2u(p) as P approches p, on the boundary C, the following boundary integral equations are obtained

$$u(p)=\frac{1}{2}\int_C [u \frac{\partial}{\partial n} Y_0(kr)-Y_0(kr)\frac{\partial u}{\partial n}]ds \qquad (a)$$

or (H.73)

$$u(p)=\frac{1}{2i}\int_C [u \frac{\partial}{\partial n} H_0^{(1)}(kr)-H_0^{(1)}(kr)\frac{\partial u}{\partial n}]ds \qquad (b)$$

where $r=|p-q|$, $p,q\epsilon C$. Equation (H.73b) is referred to as the Weber integral equation [53,54].

Equations (H.70) are used for the three-dimensional problem, and equations (H.73) for the two-dimensional problem to establish the boundary values of u for the Neumann problem or of $\frac{\partial u}{\partial n}$ for the Dirichlet problem . The solution of the Helmholz equation (H.65) is then evaluated from equations (H.69) and (H.72).

For the two-dimensional problem, the solution of equation (H.65) also has been represented in a form analogous to (H.37), that is, as a single layer (Bessel) potential of an unknown boundary density $\sigma(q)$

$$u(P)=\int_C \sigma(q)Y_0(kr)ds_q \qquad (H.74)$$

where $r=|P-q|$, $P\epsilon R$ and $q\epsilon C$. Inasmuch as the function $Y_0(kr)$ satisfies equation (H.65), the function given by (H.74) is also a solution of the same equation. The function $\sigma(q)$ is established from the boundary data by solving the boundary integral equation obtained from (H.74) when $P \equiv p \epsilon C$.

For time harmonic vibrations, the solution of the wave equation $[\nabla^2 U - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = 0]$ has the form

$$U = u(x_1, x_2, x_3)e^{-i\omega t} \qquad \text{(H.75)}$$

Thus, the wave equation reduces to Helmholz's equation

$$\nabla^2 u + k^2 u = 0, \quad k^2 = \omega^2/c^2 \qquad \text{(H.76)}$$

A number of problems of harmonic steady state vibrations in acoustic and elastic media have been solved using the BIE method with numerical solution of the integral equations. For instance, the diffraction by arbritrary shaped inclusions of harmonic elastic waves traveling in and infinite medium [55], the determination of the natural frequencies and mode shapes of vibrating membranes [46] and the interaction of waves traveling in an acoustic medium with an elastic obstacle.

Finally, in references [25,46] and [56], the BIE method has been applied to a variety of fields including electrostatics, potential fluid flow, heat conduction, fracture mechanics, rock mechanics, and transient phenomena. In all these problems, the differential equation is either Laplace's, Navier's, Helmholz's or biharmonic.

The BIE method constitutes a powerfull tool in solving problems in continuum mechanics involving elliptic equations. In many cases, its computational efficiency surpasses that of other numerical methods, such as, finite differences or finite elements. In the BIE method, the discretization is restricted only to the boundary of the region. Thus, the dimension of the problem is diminished by one and,consequently, the number of required unknowns is considerable smaller. This can save substantial computer time. The amount of data necessary for the BIE method is comparably less than that required for finite elements. Thus, in the BIE method, the work in the preparation of the data is considerably small and the possibility of error is diminished. The method is well suited to solve problems with an infinite domain where the other numerical methods fail. The BIE method can also be employed for continuum problems with high stress gradients. In the present thesis, the efficiency of the method is shown in the numerical evaluation of the modified singular solutions (influence fields) due to sources of higher order singularity (concentrated moments and generalized loads). Another advantage of the BIE method is that the field quantity in the interior is computed where and when it is needed, and not at prescribed nodal points. Finally, since numerical differentiation is an unstable process, the derivatives of the field quantities are obtained by direct differentiation of the field quantity, without requiring numerical differantiation - a source of error.

From the historical review of the development of the BIE method presented in this Section, it is apparent that problems governed by the following differential equations have been treated :

a) The Lamplace equation

b) The Navier equations of equilibrium for isotropic and anisotropic elastic body

c) The biharmonic equation

d) The Helmholz equation

In this investigation, the integral representation and the singular boundary integral equation for the BIE method are established for the partial differential equation of the fourth order in two dimensions in the form

$$\nabla^4 w + \kappa^2 w = f \qquad (H.77)$$

where the functions w and f are defined in a two-dimensional region R, bounded by a curve C, of arbitrary shape.

2. The essence of the BIE method

In this section, the BIE method will be described for fields satisfying elliptic partial differential equations. For a more concrete presentation, we will specialize our discussion to problems involving fields satisfying the Laplace equation.

Let the function u(P), $P:(x,y,z)\in R$ having continuous m-derivatives in R, satisfy the differential equation of m order

$$Lu(P)=f(P) \qquad p:(x,y,z)\in R \qquad \text{(E.1)}$$

where L is an elliptic differential operator of the form(*)

$$L=\sum_{k=0}^{m}\sum_{p+q+r=k}A_{pqr}(P)\frac{\partial^k}{\partial x^p\partial y^q\partial z^r} \qquad \text{(E.2)}$$

The coefficients $A_{pqr}(P)$ are defined in R. Moreover, the function u(P) satisfies appropriate boundary conditions on the boundary C of the region R.

We can obtain an integral representation of the solution of the differential equation (E1) as follows.

(*) The summation $\sum_{p+q+r=k}$ is extended to all possible terms with subscripts the integers p,q,r the sum of which must be equal to k. Thus, for k=0 we have only one term A_{000}, for k=1 we have three terms $A_{100}, A_{010}, A_{001}$.

Consider the following expression

$$\iiint_R v(P)Lu(P)dV_P \quad (E.3)$$

where dV_P is the volume element at the point P and v(P) has continuous m-derivatives in R. Intregrating expression (E.3) by parts m times all the derivatives of u are removed from the integrand and we obtain an identity of the type [57]

$$\iiint_R [v(P)Lu(P)-u(p)\bar{L}v(P)]dV_P=\iint_C M(u,v)ds \quad (E.4)$$

where $\bar{L}$ is the so-called adjoint differential operator to L and can be written in the form

$$\bar{L}=\sum_{k=o}^{m}(-1)^k \sum_{p+q+r=k} \frac{\partial^k}{\partial x^p \partial y^q \partial z^r}[A_{pqr}(P)\ldots] \quad (E.5)$$

The quantity M(u,v) in relation (E.4) is a bilinear differential expression. That is, it is linear and homogeneous in w and v , while its derivatives are of order $\leq m-1$. The total order of derivatives in u and v occuring in each terms of M(u,v) is, at most, m-1. Notice, that for a given elliptic differential equation, the M(u,v) can be established. Formula (E.4) is known as Green's identity for the operator L.

The function v is chosen to be the fundamental solution of the adjoint differential equation, that is, a singular particular solution of the equation

$$\bar{L}u(Q,P)=\delta(Q-P) \qquad P,Q\epsilon R \qquad (E.6)$$

where $\delta(Q-P)$ is the Dirac δ-function.

The function $v(Q,P)$ is a two-point function which becomes singular when point Q coincides with point P. The differentiation in equation (E.6) is with respect to point Q , retaining point P constant.

If we consider the Green identity (E.4) for the field point Q and substitute equations (E.1) and (E.6) in it, we obtain

$$\iiint_R v(P,Q)f(Q)dV_Q-\iiint_R u(Q)\delta(Q-P)dV_Q=\iint_C M(u,v)ds \qquad (E.7)$$

from which we obtain

$$u(P)=\iiint_R v(P,Q)f(Q)dV_Q-\iint_C M(u,v)ds \qquad (E.8)$$

Thus, we have obtained an integral representation of the solution of the differential equation (E.1). The volume integral in equation (E.8) is a known quantity, while the function u and its derivatives in the boundary terms of M(u,v) are not all known. In a well-posed boundary value problem the number of the unknown boundary quantities is equal to the number of the boundary conditions. A system of simultaneous boundary integral equations is obtained by letting the field point approach the boundary C in the integral representation (E.8) of the field quantity and/or in integral representations of the derivatives of the field quantity. The unknown boundary quantities can then be evaluated by solving the above mentioned system usually numerically.These boundary integral equations, having as kernels the singular solution v and its derivatives, are singular.

On the basis of the aforegoing,it is apparent that in order to solve a boundary value problem by using the BIE method,the following steps must be adhered to :

i) Establish the Green identity for the given linear elliptic operator.

ii) Establish the fundamental solution of the adjoint differential equation.

iii) From the first two steps obtain the integral representation of the solution.

iv) From the integral representation of the solution, establish the necessary boundary integral equations for the determination of the unknown boundary quantities by using the specified boundary conditions.

v) Solve numerically the resulting simultaneous singular boundary integral equations. That is, establish the unknown boundary quantities from the given.

vi) Using the given and computed boundary quantities, obtained in step v evaluate the solution by integrating numerically its integral representation.

In order to make the basic idea of the BIE method more concrete, we will demonstrate how it is applied to two-dimensional boundary value problems for which the governing differential equation involves Laplace's operator i.e. Laplace's equation and Poisson's equation.

We begin with Poisson's equation

$$\nabla^2 u(P)=f(P) \qquad P:(x,y)\epsilon R \tag{E.9}$$

where the Laplace operator ∇^2 in two dimensions is defined as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{E.10}$$

The Laplace equation will result as a particular case of the Poisson's equation with f(P)=0. In general, we wish to find a solution u(p) of the Poisson equation (E.9) in a given region R of the two dimensional space which, on the boundary C, must satisfy one of the following conditions [58]

a) Dirichlet: $u=g(p), \quad p:(x,y)\epsilon C$ (E.11)

b) Neumann: $\frac{\partial u}{\partial n}=h(p), \quad p:(x,y)\epsilon C$ (E.12)

where h(p) satisfies the combatibility condition $\int_C h(p)ds=0$.

c) Mixed: $u=g(p), \quad p\epsilon C_1$

$\frac{\partial u}{\partial n}=h(p), \quad p\epsilon C_2$ $\qquad C_1+C_2=C$ (E.13) g(p) and h(p) are known functions defined on the boundary C and ∂/∂_n denotes differentiation along the outward normal to C.

Conditions (E.11) to (E.13) are particular cases of the more general condition

$$\alpha u+\beta \frac{\partial u}{\partial n}=\gamma(p) \qquad p\epsilon C \qquad (E.14)$$

in which the functions α,β and γ are all known on the boundary C. It can be shown, that for regions bounded by suffienctly smooth boundaries, the solution of Poisson's equation subjected to the condition (E.14) exists, and it is unique for a fairly wide class of functions α,β and γ.

Following the previously mentioned steps, we employ the well known Green identity [59]

$$\iint_R (v\nabla^2 u-u\nabla^2 v)d\sigma=\int_C (v \frac{\partial u}{\partial n} - u \frac{\partial v}{\partial n})ds \qquad (E.15)$$

where u and v have continuous second order derivatives inside the region R. We choose v as the fundamental solution of the equation (E.9). That is, a particular singular solution of the equation

$$\nabla^2 v=\delta(Q-P) , \qquad (E.16)$$

where $\delta(Q-P)$ is the δ-function with the source point at P. A particular singular solution of equation (E.16) is [58]

$$v(P,Q)= \frac{1}{2\pi} \ell nr \qquad (E.17)$$

where

$$r = |Q-P| \tag{E.18}$$

is the distance between the points P and Q.

Introduction of equations (E.9),(E.11),(E.12) and (E.17) into equation (E.15) yields

$$u(P)=\frac{1}{2\pi}\iint_R f(Q)\ln r(P,Q)d\sigma_Q - \frac{1}{2\pi}\int_C [h(q)\ln(P,q)-g(q)\frac{\partial \ln(P,q)}{\partial n_q}]ds_q \tag{E.19}$$

where the subscripts Q or q in $d\sigma$, ds and $\partial/\partial n$ indicate that the integration or the diferrentiation has been done with respect to point $Q\epsilon R$, or point $q\epsilon C$, while point $P\epsilon R$ is retained constant. Equation (E.19) expresses an arbitrary solution u(P);in an integral form containing the fundamental solution $v=\ln r/2\pi$, its normal derivative and the functions g and h appear in both boundary conditions (E.11) and (E.12). In a well-posed boundary value problem involving Poisson's equation, only one of the functions g or h is specified. Thus,we must establish a way to find g or h when h or g, respectively, is specified on C. The required relation which relates the function h and g can be obtained by a limiting process. Thus, by letting point P in equation (E.19) approach some point p on the boundary C, and noting that the double layer potential in the last integral in equation (E.19) exhibits a jump of magnitude g(p)/2 [59] as P tends to $p\epsilon C$,and that W(p)=g(p), equation (E.19) yields

$$g(p)=\frac{1}{\pi}\iint_R \ell f(Q)\ \ell nr(p,Q)d\sigma_Q - \frac{1}{\pi}\int_C [h(q)\ell nr(p,q)-g(q)\frac{\partial \ell nr(p,q)}{\partial n_q}]ds_q \quad \text{(E.20)}$$

Equation (E.20) is a singular boundary integral equation from which the boundary function h(q) or g(q) can be determined. It is a combatibility condition which indicates that both functions g and h can not be arbritrarily prescribed. Once this equation is solved for the one unknown function, the solution of equation (E.9) is given by (E.19). Obviously for f=0, equation (E.20) yields the boundary integral equation for the Laplace equation.

For a boundary C with arbitrary shape, an analytical solution of equation (E.20) is out of question. Thus, numerical techniques have been developed to obtain the solution of this equation, which is a singular, integral Fredholm-type equation, either of the first or the second kind, depending on the given boundary data of the problem. Inasmuch as the integral representation of the solution u(P) is a function of the field point P, its derivatives, when needed, can be evaluated by direct analytical differentiation of equation (E.19).

3. The plate on elastic foundation

Due to mathematical difficulties, analytical solutions for only a few cases of loading of circular or rectangular plates on elastic foundation are available in the literature.

H. Hertz [60] (1884) established the bearing capacity of a floating sheet,subjected to a concentrated force by treating it as an infinite elastic plate of constant thickness,resting on an elastic foundation. A. Föppl [61] (1922) has used Hertz's solution to investigate the circular plate with free boundary,resting on Winkler's elastic foundation,subjected to a central concentrated load. H. Happel [62] (1920) investigated the problem of a rectangular plate resting on an elastic foundation,subjected to a concentrated load by applying Ritz's method. Westergaard [63],using Fourier series, investigated the infinite long plate, the infinitely long strip and the semi-infinite plate under a series of equidistant concentrated loads. F. Schleicher [64] (1926) gave the general solution of the differential equation for the circular plate on elastic foundation under axisymmetric loading. He has considered over forty different cases of loading and boundary conditions and he has given ready to use formulae. Using sine transform, Fletcher and Thorne [65] obtained the deflection of rectangular plates, subjected to a general transverse loading when its deflections and moments are prescribed at two opposite edges of the plate, while at the other two edges, the plate can have any given boundary conditions.They presented numerical results for constant and strip load. H. Reismann [66] obtained a general solution for a circular or a ring-shaped plate, resting on an elastic foundation under general boundary conditions, subjected to arbitrary specified transverse loading. Livesley [67], Kiyoter [68] and Solecki [69,70] investigated the stress and the displacement of semi-infinite, quatrant, and sectorial plates with various boundary and loading conditions. E. Reissner [71] analysed thin plates on Winkler's foundation with various boundary and loading conditions. An extensive literature exists on the application of the theory of plates on elastic foundation in the design of concrete pavements and airfield runways (see for example ref. [72,73,74]). Approximate and numerical methods have been also used for solving plates on elastic foundation. Vin and Elgood [75] employed the Raleigh-Ritz method to a finite rectangular plate with free edges on Winkler's foundation. They obtained numerical results which were compared with those obtained experimentally. Allen and Severn [76] solved the same problem using a relaxation method. Cheung and Zienkiewitz [77]employed the finite element method to analyse plates on elastic foundation. They gave some numerical results for a square plate, subjected to four concentrated loads and compared them with those given in [75].

In this thesis, the BIE method is developed for the solution of the finite plate on an elastic foundation having arbitrary shape and any boundary conditions. The required coupled, singular boundary integral equations are established and numerical techniques for their solution are presented. Moreover, a procedure is presented for obtaining the influence fields of various quantities such as deflections, bending and twisting moments and shearing forces. In this procedure, the influence fields are obtained as deflection surfaces corresponding to appropriately chosen generalized forces. Numerical results for various clamped and simply supported plates, subjected to distributed or concentrated loads are also presented. The results are in excellent agreement with those obtained from existing analytical solutions.

The thesis is divided into three parts and includes a chapter of conclusions and three appendices. Part I is divided into five sections. In Section I-1, the problem for the plate on elastic foundation, having any boundary conditions, is stated in its general form. The two-dimensional region occupied by the plate, may be simply or multiply connected (i.e. it may have holes), and its boundary may have corners. In Section I-2, two versions of Green's identity for the differential self-adjoint operator considered $[\nabla^4+k^2]$ are established. In the first version, the boundary terms do not have direct physical meaning, while in the second version they have physical significance. In Section I-3, the derivation of the fundamental solution is presented together with a systematic procedure for the evaluation of the arbitrary constant of the solution. In Section I-4, the integral equations for plates, with any boundary conditions are derived. Two integral representations and two sets of boundary integral equations are given corresponding to the two versions of the Green identity.

For the analysis of plates with clamped and simply supported edges, the boundary integral equations resulting from the first version of Green's identity are more suitable than those resulting from the second version. However, those resulting from the first version are not appropriate for the analysis of plates with other boundary conditions. The integral equations are formulated by a limiting process in which the field point is let to approach the boundary. For each boundary value problem,a pair of boundary conditions are specified and, consequently,two boundary integral equations are required. It was easier to establish the first boundary integral equation than the second. The latter was derived using a specially developed technique. The existence of the boundary integrals, having singular kernels,is proven by showing that the boundary integrals behave like single or double layer potentials. Moreover, the jump of the discontinuity of these integrals is evaluated using a suitable procedure whenever their kernel behaves like a Newtonian,double-layer potential.

In Section I-5, an elegant procedure based on the properties of the derivatives of the δ-function is presented for the numerical evaluation of the influence fields of various field quantities such as deflections, slopes, bending and twisting moments and shearing forces. The influence fields are obtained as the deflection surfaces,due to appropriately introduced generalized loads (multipoles),using a generalized form of the reciprocal theorem. Thus, for a plate with given boundary conditions,the boundary integral equations differ only in the non-homogeneous term. This term is evaluated in closed form. Results are presented for generalized loads generating the influence fields of the deflections, the bending and twisting moments and the shearing forces.

In Part II,a procedure for the numerical solution of the coupled,singular boundary integral equations for the clamped and simply supported plate is developed. This Part is divided into ten sections. The first five Sections deal with the numerical solution for the clamped plate,and the last five Sections with that for the simply supported plate.

In Section II-1,the boundary integral equations established in Part I for the clamped plate,are approximated by a system of simultaneous linear algebraic equations by a procedure wherein the unknown boundary quantities are assumed to vary according to a given law (step function assumption). In Section II-2,the coefficients of the unknowns of the system of linear algebraic equations, derived in Section II-1 in the form of line integrals on the boundary elements, are evaluated by numerical integration. Special techniques are developed for the numerical integration of these line integrals on the boundary elements where the integrand is singular. In Section II-3,the non-homogeneous terms of the integral equations are evaluated. They are improper,double integrals on a two-dimensional region with arbitrary shape. A procedure is developed for the numerical evaluation of these integrals, which can be used for the numerical evaluation of any double improper integral having a logarithmic or a Cauchy-type singutarity. In Section II-4,a numerical scheme for the computation of the deflections is presented,while in Section II-5 integral expressions for the numerical computations of the stress resultants are derived. These expressions result from direct differentiation of the integral representation for the deflection.

The last five Sections of this Part are devoted to the numerical solution of the singular boundary integral equations for the simply supported plate. Thus, in Section II-6 the boundary integral equations,established in Part I for the simply supported plate,are approximated by a system of simulteneous linear algebraic equations. The technique developed in Section II-1 is also applied here to this approximation. In Section II-7,the coefficients of the unknowns of the system of linear algebraic equations, which were derived in Section II-6 in the form of line integrals on the boundary elements, are evaluated by numerical integration. Special techniques are developed for the elements on which the integrand is singular. In Section II-8,the non homogeneous terms of the integral equations are evaluated using the technique developed in Section II-3. In Section II-9,a numerical scheme for the computation of the deflections is presented,while in Section II-10 tbe integral expressions for the numerical evaluation of the stress resultants are derived by direct differentiation of the integral representation of the deflection.

In Part III, numerical results are presented for clamped and simply supported circular and rectangular plates, as well as for plates with composite geometry subjected to various loading conditions. They have been obtained on a CDC/CYBER 171-8 computer. The computer programs have been written in FORTRAN language. The numerical results are presented in terms of non-dimensional parameters. Moreover, the results for circular and rectangular plates are compared with those obtained from existing analytical solutions, and are in excellent agreement with them. The influence coefficients for the stress resultants are also computed. The effectiveness of the BIE method is confirmed by the fact that, in most cases, accurate results are obtained by subdividing the boundary into less than 40 segments. For small values of the elastic constant of the subgrade, as it was expected, the results differ negligibly from those of plates not resting on an elastic foundation.

Part III is divided into five sections. Section III-1 is introductory. In Sections III-2 and III-3, appropriate dimensionless parameters for circular and rectangular plates, respectively, are established. In Section III-4, the accuracy of the BIE method is discussed and numerical results for certain plates under various loadings are presented. Finally, in Section III-5, tables of dimensionless deflections and stress resultants of circular and rectangular plates are presented.

The thesis also contains a Chapter of Conclusions and three Appendices. Appendix A is divided into two Sections. In Section A-I, certain useful formulae in Cartesian coordinates are derived. These formulae are employed in the differentiation of the kernels of the integral equations and can be used in the derivation of the boundary integral equations for any two-dimensional differential operator. In Section A-II, some relations are derived for differentiation with respect to intrinsic coordinates. In Appendix B, the additional term I_C, appearing in the integral representation of the solution when the boundary has corners, is computed.

Finally, in Appendix C the numerical method, used for the approximation of the Kelvin functions ker(x), kei(x) and their first derivatives ker'(x) and kei(x), is described.

PART I

THE BOUNDARY INTEGRAL EQUATIONS

I-1. Statement of the problem

Consider a thin elastic plate of thickness h, occupying a two dimensional region R, bounded by a curve C, and resting on a linear elastic foundation. The region R may be simply or multiply connected, i.e. the plate may have holes, while the boundary C may have a finite number of corners.

The deflection w(P) of the plate must satisfy the following differential equation at any point P, inside the region R [73].

$$Lw = \frac{f(P)}{D} \qquad \text{(I-1.1)}$$

where f(P) is the distribution of the normal to the surface of the plate external force per unit area; D is the flexural rigidity $[D=Eh^3/12(1-\nu^2)]$ of the plate.

The operator L is defined as

$$L=\nabla^4+\kappa^2 \qquad \text{(I-1.2)}$$

where ∇^4 is the biharmonic operator defined as

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \qquad (I\text{-}1.3)$$

and

$$\kappa^2 = \frac{k}{D}\ ; \qquad (I\text{-}1.4)$$

k is the constant of the elastic foundation

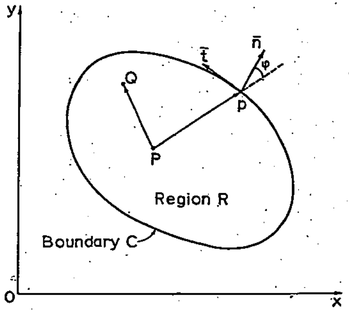

Fig.I-1. Plate occupying the two-dimensional region R bounded by the curve C.

Moreover, the deflection w must satisfy the following conditions on each of the r portions $C^{(i)}$ of the boundary $[C=\sum_{i=1}^{r} C^{(i)}]$

$$\alpha_1^{(i)}(p)w + \alpha_2^{(i)}(p)\ V_n = g_1^{(i)}(p)$$

$$(i=1,2,\ldots r) \qquad (1\text{-}1.5)$$

$$b_1^{(i)}(p)\frac{\partial w}{\partial n} + b_2^{(i)}(p)M_n = g_2^{(i)}(p)$$

where p is a point on the portion $C^{(i)}$ of the boundary C; the functions $g_1^{(i)}(p)$ and $g_2^{(i)}(p)$ and $\alpha_1^{(i)}(p), \alpha_2^{(i)}(p), b_1^{(i)}(p), b_2^{(i)}(p)$ are specified on the portion $C^{(i)}$ of the boundary and depend on the edge conditions of the plate. Thus, we have [37]

$$\alpha_1^{(i)}=1, \quad \alpha_2^{(i)}=0, \quad b_1^{(i)}=1, \quad b_2^{(i)}=0 \qquad \text{(I-1.6a)}$$

if the portion $C^{(i)}$ of the boundary is clamped

$$\alpha_1^{(i)}=1, \quad \alpha_2^{(i)}=0, \quad b_1^{(i)}=0, \quad b_2^{(i)}=1 \qquad \text{(I-1.6b)}$$

if the portion $C^{(i)}$ of the boundary is simply supported

$$\alpha_1^{(i)}=0, \quad \alpha_2^{(i)}=1, \quad b_1^{(i)}=0, \quad b_2^{(i)}=1 \qquad \text{(I-1.6c)}$$

if the portion $C^{(i)}$ of the boundary is free

$$\alpha_1^{(i)}=0, \quad \alpha_2^{(i)}=1, \quad b_1^{(i)}=1, \quad b_2^{(i)}=0 \qquad \text{(I-1.6d)}$$

if the portion $C^{(i)}$ of the boundary is guided.

$\alpha_1^{(i)}, \alpha_2^{(i)}, b_1^{(i)}, b_2^{(i)}$ can have any other given value or be functions of p if the portion $C^{(i)}$ of the boundary is elastically supported.

For plates having the same conditions on the entire boundary, the superscript i in equations (I-1.5) and (I-1.6) will be ommitted.

The effective shearing force V_n and the bending moment M_n, acting on the boundary of the plate, are related to the deflection w by the following relations [73]

$$M_n=-D\left[\nabla^2 w+(\nu-1)\frac{\partial^2 w}{\partial t^2}\right]$$

$$V_n=-D\left[\frac{\partial}{\partial n}\nabla^2 w-(\nu-1)\frac{\partial}{\partial s}\left(\frac{\partial^2 w}{\partial n\partial t}\right)\right] \qquad \text{(I-1.7)}$$

where $\partial/\partial n$ and $\partial/\partial t$ denote differentiation along the outward normal and the tangential direction, respectively, and $\partial/\partial s$ denotes differentiation with respect to the arc length of the boundary.

In the subsequent analysis, it will be more convenient to work with the arc length variable s, rather than t. Thus, t will be eliminated from relation (I-1.7). This can be done by using the following relations, derived in Appendix A

$$\frac{\partial w}{\partial s} \equiv \frac{\partial w}{\partial s}$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial s^2} + K\frac{\partial w}{\partial n} \qquad \text{(I-1.8)}$$

$$\frac{\partial^2 w}{\partial n\partial t} = \frac{\partial^2 w}{\partial s\partial n} - K\frac{\partial w}{\partial s}$$

where K=K(s) is the curvature of the boundary. Using equations (I-1.8), equations (I-1.7) may be written as

$$M_n=-D\left[\nabla^2 w+(\nu-1)\left(\frac{\partial^2 w}{\partial s^2}+K\frac{\partial w}{\partial n}\right)\right]$$

$$V_n=-D\left[\frac{\partial}{\partial n}\nabla^2 w-(\nu-1)\frac{\partial}{\partial s}\left(\frac{\partial^2 w}{\partial s\partial n}-K\frac{\partial w}{\partial s}\right)\right] \qquad \text{(I-1.9)}$$

It is apparent, that for a straight line boundary [K(s)=0, $t\equiv s$], equations (I-1.9) take the form of (I-1.7).

I-2. The Green identity for the differential operator of the problem

In this Section, Green's second formula for the plane is converted into a form directly applicable to the problem at hand.

Consider any two functions u and $\tilde{u}$, which are two times continuously differentiable inside the region R, and once on the boundary C. The region R may be multiply connected and its boundary C may have a finite number of corners i.e. it is piecewise smooth. It can be shown [59] that these functions satisfy the following relation

$$\iint_R (\tilde{u}\nabla^2 u - u\nabla^2\tilde{u})\,d\sigma = \int_C \left(\tilde{u}\frac{\partial u}{\partial n} - u\frac{\partial \tilde{u}}{\partial n}\right) ds \qquad \text{(I-2.1)}$$

Chosing $u=\nabla^2 w$ and $\tilde{u}=v$ relation (I-2.1) yields

$$\iint_R v\nabla^2(\nabla^2 w)\,d\sigma - \iint_R \nabla^2 w\nabla^2 v\,d\sigma = \int_C \left[v\frac{\partial}{\partial n}\nabla^2 w - \nabla^2 w\frac{\partial v}{\partial n}\right] ds \qquad \text{(I-2.2a)}$$

Choosing now $u=w$ and $\tilde{u}=\nabla^2 v$ relation (I-2.1) gives

$$\iint_R \nabla^2 v\nabla^2 w\,d\sigma - \iint_R w\nabla^2(\nabla^2 v)\,d\sigma = \int_C \left[\nabla^2 v\,\frac{\partial w}{\partial n} - w\frac{\partial}{\partial n}\nabla^2 v\right] ds \qquad \text{(I-2.2b)}$$

Addition of equations (I-2.2a) and (I-2.2b) yields

$$\iint_R (v\nabla^4 w - w\nabla^4 v)\,d\sigma = \int_C \left[v\frac{\partial}{\partial n}\nabla^2 w - \frac{\partial v}{\partial n}\nabla^2 w - w\frac{\partial}{\partial n}\nabla^2 v + \frac{\partial w}{\partial n}\nabla^2 v\right] ds$$

(I-2.3)

By adding and substracting $\iint_R \kappa^2 wv d\sigma$ in the left hand side of equation (I-2.3) we get

$$\iint_R [vLw-wLv]d\sigma=\int_C [v\frac{\partial}{\partial n}\nabla^2 w- \frac{\partial v}{\partial n}\nabla^2 w-w\frac{\partial}{\partial n}\nabla^2 v+\frac{\partial w}{\partial n}\nabla^2 v]ds \qquad \text{(I-2.4)}$$

Equation (I-2.4) holds for any two functions, w and v, which have continuous fourth derivatives in R, and continuous third derivatives on C.

The integral in the right hand side of equation (I-2.4) contains the quantities w, $\frac{\partial w}{\partial n}$, $\nabla^2 w$, and $\frac{\partial}{\partial n}(\nabla^2 w)$. When the function w represents the deflection of the middle surface of the plate, $\frac{\partial w}{\partial n}$ is its slope. The other two quantities have a direct physical meaning only in special cases. For example, when the edge of the plate is clamped ($w=0$, $\frac{\partial w}{\partial n}=0$), the condition $w=0$ implies that $\frac{\partial^2 w}{\partial s^2}=0$. Thus, equations (I-1.9) reduce to

$$M_n=-D\nabla^2 w$$
$$\qquad \text{(I-2.5)}$$
$$V_n=-D\frac{\partial}{\partial n}\nabla^2 w$$

that is, the quantities $\nabla^2 w$ and $\frac{\partial}{\partial n}(\nabla^2 w)$ express the bending moment and the reacting force at the boundary, respectively, multiplied by $-1/D$.

As it will be seen later, for the derivation of the boundary integral equations, it is convenient to convert the boundary integral in equation (I-2.4) to an equivalent one containing quantities having a direct physical meaning. For this purpose we may write

$$v\frac{\partial}{\partial n}\nabla^2 w = v\left[\frac{\partial}{\partial n}\nabla^2 w-(\nu-1)\frac{\partial}{\partial s}\left(\frac{\partial^2 w}{\partial n\partial t}\right)\right]+(\nu-1)v\frac{\partial}{\partial s}\left(\frac{\partial^2 w}{\partial n\partial t}\right)=vVw+$$

$$+(\nu-1)v\frac{\partial}{\partial s}\left(\frac{\partial^2 w}{\partial n\partial t}\right)$$

$$-\frac{\partial v}{\partial n}\nabla^2 w=-\frac{\partial v}{\partial n}\left[\nabla^2 w+(\nu-1)\frac{\partial^2 w}{\partial t^2}\right]+(\nu-1)\frac{\partial v}{\partial n}\frac{\partial^2 w}{\partial t^2}=-\frac{\partial v}{\partial n}Mw+(\nu-1)\frac{\partial v}{\partial n}\frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial w}{\partial n}\nabla^2 v=\frac{\partial w}{\partial n}\left[\nabla^2 v+(\nu-1)\frac{\partial^2 v}{\partial t^2}\right]-(\nu-1)\frac{\partial w}{\partial n}\frac{\partial^2 v}{\partial t^2}=\frac{\partial w}{\partial n}Mv-(\nu-1)\frac{\partial w}{\partial n}\frac{\partial^2 v}{\partial t^2}$$

$$-w\frac{\partial}{\partial n}\nabla^2 v=-w\left[\frac{\partial}{\partial n}\nabla^2 v-(\nu-1)\frac{\partial}{\partial s}\left(\frac{\partial^2 v}{\partial n\partial t}\right)\right]-(\nu-1)w\frac{\partial}{\partial s}\left(\frac{\partial^2 v}{\partial n\partial t}\right)=-wVv-$$

$$-(\nu-1)w\frac{\partial}{\partial s}\left(\frac{\partial^2 v}{\partial n\partial t}\right)$$

where the operators M and V are defined as

$$M=\nabla^2+(\nu-1)\frac{\partial^2}{\partial t^2}=\nabla^2+(\nu-1)\left(\frac{\partial^2}{\partial s^2}+K\frac{\partial}{\partial n}\right)$$

$$V=\frac{\partial}{\partial n}\nabla^2-(\nu-1)\frac{\partial}{\partial s}\left(\frac{\partial^2}{\partial n\partial t}\right)=\frac{\partial}{\partial n}\nabla^2-(\nu-1)\frac{\partial}{\partial s}\left(\frac{\partial^2}{\partial s\partial n}-K\frac{\partial}{\partial s}\right)$$

(I-2.8)

By adding relations (I-2.7) and integrating over the boundary we obtain

$$\int_C [v\frac{\partial}{\partial n}\nabla^2 w - \frac{\partial v}{\partial n}\nabla^2 w - w\frac{\partial}{\partial n}\nabla^2 v + \frac{\partial w}{\partial n}\nabla^2 v]ds = \int_C [vVw - wVv - \frac{\partial v}{\partial n}Mw + \frac{\partial w}{\partial n}Mv]ds + I_C \tag{I-2.9}$$

where

$$I_C = (\nu-1)\int_C [v\frac{\partial}{\partial s}(\frac{\partial^2 w}{\partial n\partial t}) - w\frac{\partial}{\partial s}(\frac{\partial^2 w}{\partial n\partial t}) + \frac{\partial v}{\partial n}\frac{\partial^2 w}{\partial t^2} - \frac{\partial w}{\partial n}\frac{\partial^2 v}{\partial t^2}]ds$$

Using intrinsinc coordinates (relations (A-67) and (A-69) in Appendix A),boundary integral I_C may be rewritten as

$$I_C = (\nu-1)\int_C [v\frac{\partial}{\partial s}(\frac{\partial^2 w}{\partial s\partial n} - K\frac{\partial w}{\partial s}) - w\frac{\partial}{\partial s}(\frac{\partial^2 v}{\partial s\partial n} - K\frac{\partial v}{\partial s}) + \frac{\partial v}{\partial n}(\frac{\partial^2 w}{\partial s^2} + K\frac{\partial w}{\partial n}) - \frac{\partial w}{\partial n}(\frac{\partial^2 v}{\partial s^2} + K\frac{\partial v}{\partial n})]ds$$

$$= (\nu-1)\int_C [v\frac{\partial}{\partial s}(\frac{\partial^2 w}{\partial s\partial n}) - v\frac{\partial}{\partial s}(K\frac{\partial w}{\partial s}) - w\frac{\partial}{\partial s}(\frac{\partial^2 v}{\partial s\partial n}) + w\frac{\partial}{\partial s}(K\frac{\partial v}{\partial n}) + \frac{\partial v}{\partial n}\frac{\partial^2 w}{\partial s^2} - \frac{\partial w}{\partial n}\frac{\partial^2 v}{\partial s^2}]ds \tag{I-2.10}$$

Integrating by parts certain terms in the above relation,for a boundary with N corners whose coordinates are s_i (i=1,2,...N), we obtain (see Appendix B)

$$I_C = -(\nu-1)\sum_{i=1}^{N} [\![vTw - wTv + \frac{\partial v}{\partial n}\frac{\partial w}{\partial s} - \frac{\partial w}{\partial n}\frac{\partial v}{\partial s}]\!]_i ds \tag{I-2-11}$$

where $[\![\ldots]\!]_i$ denotes the jump of the function at the point s_i due to the discontinuity of the slope of the boundary of this point. The operator T is defined in Appendix B. It is apparent that for smooth boundaries

$$I_C = 0 \qquad \text{(I-2.12)}$$

Using relation (I-2.9) equation (I-2.4) may be written as

$$\iint_R (vLw - wLv)\,d\sigma = \int_C \left[vVw - wVv - \frac{\partial v}{\partial n} Mw + \frac{\partial w}{\partial n} Mv\right] ds + I_C \qquad \text{(I-2.13)}$$

This form of Green's second formula will be directly applied in deriving the formulas used in the BIE method.

I-3. The fundamental solution of the problem

Inasmuch as the operator L is self-adjoint, the fundamental solution of the problem is a singular particular solution of the following equation

$$Lv = \delta(P-Q)/D \qquad \text{(I-3.1)}$$

where $\delta(P-Q)$ is the Dirac delta function; $P:(x,y)$ is the field point and $Q:(\xi,\eta)$ is the source point. The solution $v=v(P,Q)$ of equation (I-3.1) is a two-point function. Physically, it is the deflection surface of an infinite plate on elastic foundation loaded by a concentrated unit load at point Q. The solution will be axisymmetric with respect to point Q, that is, it will depend only on the radial distance

$r=|P-Q|$. Thus, the Laplace operator in polar coordinates with point Q as its origin, is independent of the angular coordinate, that is

$$\nabla^2 = \frac{d}{dr^2} + \frac{1}{r}\frac{d}{dr} \qquad (I\text{-}3.2)$$

In order to solve equation (I-3.1), we start with its corresponding homogeneous equation, which is valid for all points P of the plane except P=Q. Thus, we have

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\left(\frac{dr^2}{dr^2} + \frac{1}{r}\frac{dv}{dr}\right)+\kappa^2 v=0 \qquad (I\text{-}3.3)$$

introducing the dimensionless independent variable

$$\rho=r/\ell$$

where

$$\ell=\sqrt{1/\kappa}=\sqrt[4]{D/k} \qquad (I\text{-}3.4)$$

equation (I-3.3) becomes

$$\left(\frac{d}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho}\right)\left(\frac{dv^2}{d\rho^2} + \frac{1}{\rho}\frac{dv}{d\rho}\right)+v=0 \qquad (I\text{-}3.5)$$

moreover, introducing the variable

$$x=\rho\sqrt{i}\ , \qquad i=\sqrt{-1} \qquad (I\text{-}3.6)$$

into equation (I-3.5), we obtain

$$\nabla^4 v-v=0 \qquad (I\text{-}3.7)$$

where

$$\nabla^2 = \frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx}$$

is the Laplace operator with respect to the x coordinate.

Equation (I-3.7) can be written in the following two ways [64]

$$\nabla^2(\nabla^2 v+v)-(\nabla^2 v+v)=0$$
$$\nabla^2(\nabla^2 v-v)+(\nabla^2 v-v)=0 \qquad \text{(I-3.8)}$$

Thus, the deflection equation (I-3.7) is satisfied by the solution of the Bessel equations

$$\nabla^2 v+v=0 \qquad \text{(I-3.9)}$$

and

$$\nabla^2 y-y=0 \qquad \text{(I-3.10)}$$

Equation (I-3.10) can be transformed into the equation (I-3.9) by changing the variable from x to xi.

The two linearly independent solutions of equation (I-3.9) are $I_0(\rho\sqrt{i})$ and $K_0(\rho\sqrt{i})$, while those of (I-3.10) are $I_0(i\sqrt{i})$ and $K_0(\rho i\sqrt{i})$. The functions I_0 and K_0 are the Bessel functions of the first and second kind, respectively.

Hence, the general solution of equation (I-3.3) is

$$y=B_1 I_0(\rho\sqrt{i})+B_2 I_0(\rho i\sqrt{i})+B_3 K_0(\rho\sqrt{i})+B_4 K_0(\rho i\sqrt{i}) \qquad \text{(I-3.11)}$$

Inasmuch as ρ is a real variable, all functions in equation (I-3.11) are complex. Consequently, the arbitrary constants B_1, B_2, B_3 and B_4 are also complex. In order to express v in terms of real functions and constants, the Bessel functions I_0 and K_0 are expressed as

$$I_0(\rho\sqrt{\pm i}) = \mathrm{ber}(\rho) \pm i\,\mathrm{bei}(\rho)$$
$$K_0(\rho\sqrt{\pm i}) = \mathrm{ker}(\rho) \pm i\,\mathrm{kei}(\rho) \qquad \text{(I-3.12)}$$

ber(ρ), bei(ρ), ker(ρ), and kei(ρ) are referred to as the Kelvin functions of zero order.

Substituting equations (I-3.12) into solution (I-3.11), we obtain

$$v = C_1\mathrm{ber}(\rho) + C_2\mathrm{bei}(\rho) + C_3\mathrm{kei}(\rho) + C_4\mathrm{ker}(\rho) \qquad \text{(I-3.13)}$$

where C_1, C_2, C_3 and C_4 are arbitrary real constants. The solution (I-3.13) must be finite and must vanish at infinity. Thus, any of the constants C_1, C_2, C_3 or C_4, multiplying a Kelvin function which is not finite for any value of its argument, must be set equal to zero. For small values of their argument, the functions ber(ρ), bei(ρ), ker(ρ), and kei(ρ) can be expanded into the following series [78]

$$\mathrm{ber}(\rho) = 1 - 64(\rho/8)^4 + 113.7777774(\rho/8)^8 - 32.36345652(\rho/8)^{12} + \ldots \qquad \text{(I-3.14a)}$$

$$bei(\rho)=16(\rho/8)^2-113.7777774(\rho/8)^6+72.81777742(\rho/8)^{10}-\ldots \tag{I-3.14b}$$

$$ker(\rho)=-\ell n(\rho/2)ber(\rho)+\frac{\pi}{4}bei(\rho)-.57721566-59.05819744(\rho/8)^4+\ldots \tag{I-3.14c}$$

$$kei(\rho)=-\ell n(\rho/2)bei(\rho)-\frac{\pi}{4}ber(\rho)+6.76454936(\rho/8)^2-$$

$$-142.91827687(\rho/8)^6+\ldots \tag{I-3.14d}$$

For large values of their argument, the functions ber(ρ), bei(ρ), ker(ρ), and kei(ρ) can be approximated by [73]

$$ber(\rho)\sim\frac{e^{\rho/\sqrt{2}}}{\sqrt{2\pi\rho}}\cos(\rho/\sqrt{2}-\pi/8) \tag{I-3.15a}$$

$$bei(\rho)\sim\frac{e^{\rho/\sqrt{2}}}{\sqrt{2\pi\rho}}\sin(\rho/\sqrt{2}-\pi/8) \tag{I-3.15b}$$

$$ker(\rho)\sim\frac{e^{-\rho/\sqrt{2}}}{\sqrt{2\rho/\pi}}\cos(\rho/\sqrt{2}-\pi/8) \tag{I-3.15c}$$

$$kei(\rho)\sim\frac{e^{-\rho/\sqrt{2}}}{\sqrt{2\rho/\pi}}\sin(\rho/\sqrt{2}-\pi/8) \tag{I-3.15d}$$

Thus, the functions ber(ρ) and bei(ρ) become infinitely large for monotonically large values of ρ, while the function ker(ρ) becomes infinitely large for ρ equal to zero. Consequently, the coefficients C_1, C_2, C_4 must vanish and the solution (I-3.13) reduces to

$$v=C_3 kei(\rho) \tag{1-3.16}$$

The function kei(ρ) is finite and vanishes at infinity. The constant C_3 can be evaluated by noting that the resultant of the shearing forces distributed on the circumference of a very small circular element of the plate with center at ρ=0, must approach unity as the radius of the circle tends to zero. However, we will follow a more systematic procedure for the evaluation of the constant C_3, which can also be applied to problems wherein the physical meaning of delta function is not evident.

Integrating both sides of equation (I-3.1) over an arbitrary region Ω, with boundary ∂Ω, we obtain

$$\iint_\Omega (\nabla^4 v+\kappa^2 v)d\sigma=\iint_\Omega \frac{\delta(P-Q)}{D}\, d\sigma=1/D \qquad \text{(I-3.17)}$$

Application of the Green identity (I-2.4) to the functions v and w=1 and use of relation (I-3.17) yields

$$\iint_\Omega (\nabla^4 v+\kappa^2 v)d\sigma=\int_{\partial\Omega}\frac{\partial}{\partial n}\,\nabla^2 v ds=1/D \qquad \text{(I-3.18)}$$

As shown in appendix A, from relation (I-3.16) we obtain

$$\frac{\partial}{\partial n}\nabla^2 v=\frac{C_3}{\ell^3}\text{ker}'(\rho)\cos\varphi \qquad \text{(I-3.19)}$$

where, as shown in Fig. I-2, $\varphi=(\widehat{\bar{r},\bar{n}})$.

Substitution of relation (I-3.19) into (I-3.18) yields

$$\frac{C_3}{\ell^3}\int_{\partial\Omega} \ker'(\rho)\cos\varphi ds=1/D \qquad (I\text{-}3.20)$$

If we choose for Ω, a small circle with center at $\rho=0$ and radius r_0, then $\varphi=0$ and relation (I-3.20) reduces to

$$1/D=\frac{C_3}{\ell^3}\int_{\partial\Omega} \ker'(\rho_0)r_0 d\theta=\frac{C_3}{\ell^2}\ker'(\rho_0)\rho_0 2\pi \qquad (I\text{-}3.21)$$

In obtaining the above result we have taken into account that for small values of ρ, the function $\ker'(\rho)$ behaves like $-1/\rho$ [see equation (I-4.12)] and, consequently, it does not change on the circumference $\partial\Omega$.

If the radius r_0 of the circle tends to zero, we have

$$\lim_{\rho_0\to 0}\rho_0 \ker'(\rho_0)=-1$$

Hence, equation (I-3.21) gives

$$C_3=-\frac{\ell^2}{2\pi D}$$

Thus, the fundamental solution of the differential equation (I-3.1) is

$$v(P,Q)=-\frac{\ell^2}{2\pi D}\ker(\rho) \qquad (I\text{-}3.22)$$

Notice that the value of $v(P,Q)$ does not change if the points P and Q are interchanged. Thus, it is a symmetric function. This implies that the function $v(P,Q)$ also represents the deflection of the plate at the point Q, due to a concentrated unit load at the point P.

I-4. The boundary integral equations

The integral representation of the deflection function w can now be obtained easily. We will find two integral representations. One from the identity (I-2.4) and another from the identity (I-2.13).

Let us consider the functions w(Q) and v(Q,P), satisfying the following differential equations

$$Lw = \frac{f(Q)}{D} \tag{I-4.1}$$

$$Lv = \delta(Q-P)/D \tag{I-4.2}$$

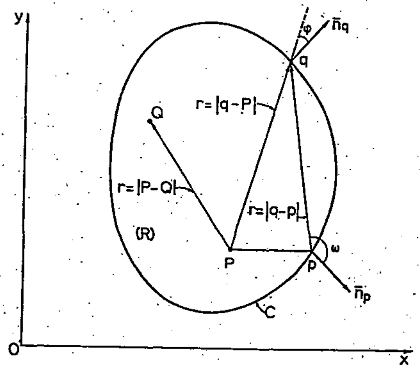

Fig.I-2.

Substituting the functions w and v into identities (I-2.4) and (I-2.13), we get

$$w(P)=\iint_R v(Q,P)f(Q)d\sigma_Q - D\int_C [v(q,P)\frac{\partial}{\partial n_q}\nabla^2 w(q) - w(q)\frac{\partial}{\partial n_q}\nabla^2 v(q,P)$$

$$-\frac{\partial}{\partial n_q}v(q,P)\nabla^2 w(q) + \frac{\partial w(q)}{\partial n_q}\nabla^2 v(q,P)]ds_q$$

(I-4.3)

$$w(P)=\iint_R v(Q,P)f(Q)d\sigma_Q - D\int_C [v(q,P)Vw(q) - w(q)Vv(q,P)$$

$$-\frac{\partial}{\partial n_q}v(q,P)Mw(q) + \frac{\partial w(q)}{\partial n_q}Mv(q,P)]ds_q - DI_C(P,q)$$

(I-4.4)

Notice, that in the above relations, points inside the region R are denoted by capital (Q or P), while points on the boundary C are denoted by small letters (q or p). The subscript of the surface element $d\sigma$, and of the arc element ds, denotes the point which varies during integrations. Also $\frac{\partial}{\partial n_q}$ indicates that the normal derivative is taken at the point q.

Except where they are required for clarification, the arguments of the functions and the indices in equations (I-4.3) and (I-4.4) will be ommitted. Hence, these equations can be written as

$$w(P)=\iint_R vfd\sigma - D\int_C (v\frac{\partial}{\partial n}\nabla^2 w - w\frac{\partial}{\partial n}\nabla^2 v - \frac{\partial v}{\partial n}\nabla^2 w + \frac{\partial w}{\partial n}\nabla^2 v)ds$$

(I-4.5)

$$w(P)=\iint_R vf d\sigma - D\int_C \left(vVw - wVv - \frac{\partial v}{\partial n}Mw + \frac{\partial w}{\partial n}Mv\right)ds - DI_C$$

(I-4.6)

We will first work with equation (I-4.6) in which the terms in the boundary integral have a direct physical meaning. This equation indicates that the deflection w(P) can be evaluated when the loading function f(P) is given at every point in R and the values of the deflection w(p), the slope $\frac{\partial w}{\partial n}$, the bending moment $M_n = Mw$, and the reacting force $V_n = Vw$ are given on the points of the boundary C. However, in a well-posed plate problem , only two of these quantities are prescribed on the boundary [see equation (I-1.5)].Consequently, it becomes necessary to evaluate on the boundary the two unknown quantities in terms of the given. This is done by formulating two coupled boundary integral equations involving the quantities w(p), $\frac{\partial w}{\partial n}$, Mw and Vw. The solution of these equations gives the two unknown quantities.

The first boundary integral equation may be obtained from equation (I-4.6),by letting point P approach a point p on the boundary C. Thus,we obtain

$$w(p)=\iint_R v(Q,p) f d\sigma - D\lim_{P\to p}\int_C \left(vVw - wVv - \frac{\partial v}{\partial n}Mw + \frac{\partial w}{\partial n}Mv\right)ds - DI_C$$

(I-4.7)

Before passing the limit to the integrand of the integral, the existence and continuity of the line integrals must be examined as the $P \to p \in C$. Moreover, if the integral is not continuous as the point P reaches the boundary, its jump must be established.

Substitution of Vv and Mv from equations (I-2.8) into equation (I-4.7) will result in integrals of the following form

$$
\begin{aligned}
I_1(P) &= \int_C \mu_1(s) v\,ds \\
I_2(P) &= \int_C \mu_2(s) \frac{\partial v}{\partial n}\,ds \\
I_3(P) &= \int_C \mu_3(s) \nabla^2 v\,ds \\
I_4(P) &= \int_C \mu_4(s) \frac{\partial v}{\partial s}\,ds = -\int_C \frac{\partial \mu_4}{\partial s} v\,ds \\
I_5(P) &= \int_C \mu_5(s) \frac{\partial^2 v}{\partial s^2}\,ds = \int_C \frac{\partial^2 \mu_5(s)}{\partial s^2} v\,ds \\
I_6(P) &= \int_C \mu_6(s) \frac{\partial^3 v}{\partial s^2 \partial n}\,ds = \int_C \frac{\partial^2 \mu_6(s)}{\partial s^2} \frac{\partial v}{\partial n}\,ds \\
I_7(P) &= \int_C \mu_7(s) \frac{\partial}{\partial n} \nabla^2 v\,ds
\end{aligned}
\qquad (I\text{-}4.8)
$$

where $v(P,q)$.

The function $\mu_4(s)$ is assumed differentiable and the functions $\mu_5(s)$ and $\mu_6(s)$ twice differentiable with respect to s. The second expression for the integrals I_4, I_5, I_6 has resulted by integration by parts. This integration has been performed in order to eliminate the derivatives of the kernels with respect to s, thus, reducing the order of the singularity.

From equation (I-3.22) and appendix A, we have

$$v=-\frac{\ell^2}{2\pi D}\,\mathrm{kei}(\rho)$$

$$\frac{\partial v}{\partial n}=-\frac{\ell}{2\pi D}\,\mathrm{kei}'(\rho)\cos\varphi$$

$$\nabla^2 v=-\frac{1}{2\pi D}\,\mathrm{ker}(\rho) \qquad \text{(I-4.9)}$$

$$\frac{\partial}{\partial n}\nabla^2 v=-\frac{1}{2\pi\ell D}\,\mathrm{ker}'(\rho)\cos\varphi$$

where $\varphi=\widehat{\bar{r},\bar{n}_q}$, $r=[q-P]$

In the above expressions $\mathrm{kei}'(\rho)$ and $\mathrm{ker}'(\rho)$ are the derivatives of the Kelvin functions of zero order $\mathrm{kei}(\rho)$ and $\mathrm{ker}(\rho)$ with respect to their argument ρ. Substituting equations (I-4.9) into (I-4.8) we obtain

$$I_1(P)=\int_C \bar{\mu}_1(s)\,\mathrm{kei}(\rho)\,ds$$

$$I_2(P)=\int_C \bar{\mu}_2(s)\,\mathrm{kei}'(\rho)\cos\varphi\,ds$$

$$I_3(P)=\int_C \bar{\mu}_3(s)\,\mathrm{ker}(\rho)\,ds \qquad \text{(I-4.10)}$$

$$I_4(P)=\int_C \bar{\mu}_4(s)\,\mathrm{kei}(\rho)\,ds$$

$$I_5(P)=\int_C \bar{\mu}_5(s)\,\mathrm{kei}(\rho)\,ds$$

$$I_6(P)=\int_C \bar{\mu}_6(s)\,\mathrm{kei}'(\rho)\cos\varphi\,ds$$

$$I_7(P)=\int_C \bar{\mu}_7(s)\,\mathrm{ker}'(\rho)\cos\varphi\,ds$$

In expressions (I-4.10), the constants in relation (I-4.9) have been incorporated into the functions $\tilde{\mu}_i(s)$.

From equation (I-3.14d), it is apparent that the function $\mathrm{kei}(\rho)$ is not singular. That is,

$$\lim_{\rho\to 0} \mathrm{kei}(\rho) = -\pi/4 \qquad \text{(I-4.11)}$$

Thus, the integrals I_1, I_4, I_5 in equations (I-4.10) exist for $P \equiv p \epsilon C$ and are continuous as $P \to p \epsilon C$. From relation (I-3.14c) we see that the singular term in the series expansion of the $\mathrm{ker}(\rho)$ behaves like $\ln(\rho)$. Consequently, the part of the integral I_3 corresponding to this term represents the logarithmic potential due to a mass distribution $\tilde{\mu}_3(s)$ on the boundary C. This is a single layer potential and, thus, the integral I_3 exists and is continuous as $P \to p \epsilon C$ [9,59]. In this case, the integral is an improper integral and its value is a Cauchy principal value.

The behaviour of the functions $\mathrm{kei}'(\rho)$ and $\mathrm{ker}'(\rho)$, for small values of the argument ρ, can be examined from the following polynomial aproximations [78].

$$\begin{aligned} \mathrm{kei}'(\rho) &= -\ln(\rho/2)\mathrm{bei}'(\rho) - \frac{1}{\rho}\mathrm{bei}(\rho) - \frac{1}{4}\pi \mathrm{ber}'(\rho) \\ &\quad +\rho[.21139217 - 13.39858(\rho/8)^4 + \ldots \\ &= -\ln(\rho/2)\rho[1/2 - 10.66666(\rho/8)^4 + \ldots] \\ &\quad -[\frac{1}{\rho}\, 16(\rho/8)^2 - 113.7777(\rho/8)^6 + \ldots] \qquad \text{(I-4.12)} \\ &\quad - \frac{1}{4}\pi\rho[-4(\rho/8)^2 + 14.2222(\rho/8)^6 - \ldots] \\ &\quad +\rho[.21139217 - 13.39858(\rho/8)^4 + \ldots] \end{aligned}$$

$$\ker'(\rho) = -\ln(\rho/2)\,\mathrm{ber}'(\rho) - \frac{1}{\rho}\,\mathrm{ber}(\rho) + \frac{1}{4}\pi\,\mathrm{bei}'(\rho)$$

$$+\rho[-3.69113(\rho/8)^2+\ldots]$$

$$= -\ln(\rho/2)\rho[-4(\rho/8)^2+14.222(\rho/8)^6-\ldots]$$

$$-\frac{1}{\rho}[1-64(\rho/8)^4+113.7777(\rho/8)^5-\ldots] \qquad \text{(I-4.13)}$$

$$+\frac{1}{4}\pi\rho[\frac{1}{2}-10.666(\rho/8)^4+\ldots]$$

$$+\rho[-3.69113(\rho/8)^2+\ldots]$$

From expression (I-4.13), it is apparent that $\mathrm{kei}'(\rho)$ is not singular. That is,

$$\lim_{\rho\to 0} \mathrm{kei}'(\rho) = 0 \qquad \text{(I-4.14)}$$

Consequently, the integrals I_2 and I_6 in equations (I-4.10) exist for $P \equiv p \in C$ and are continuous as $P \to p \in C$.

From expression (I-4.13) we conclude that for $\rho \to 0$, the function $\ker'(\rho)$ behaves like $1/\rho$. Consequently, the part of the integral I_7 corresponding to this term in equations (I-4.10) represents a double layer potential due to a mass distribution $\tilde{\mu}_7(s)$ on the boundary C. Hence, this integral I_7 exists when $P \equiv p \in C$, but it has a discontinuity as $P \to p \in C$ [9, 59].

The jump of the integral I_7 as $P \to p \in C$ will be established using Green's first formula [59].

$$\iint_R \left(\frac{\partial u}{\partial x}\frac{\partial \bar{v}^2}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial \bar{v}}{\partial y}\right) d\sigma + \iint_R u\nabla^2\bar{v}\, d\sigma = \int_C u\,\frac{\partial \bar{v}}{\partial n}\, ds \qquad \text{(I-4.15)}$$

Choosing $u=\tilde{\mu}(P)$ and $\bar{v}=\nabla^2 v$, where $v=-\frac{\ell^2}{2\pi D}\,\mathrm{kei}(\rho)$ relation (I-4.15) becomes

$$\iint_R \left(\frac{\partial\tilde{\mu}}{\partial x}\frac{\partial(\nabla^2 v)}{\partial x}+\frac{\partial\tilde{\mu}}{\partial y}\frac{\partial(\nabla^2 v)}{\partial y}\right)d\sigma+\iint_R \tilde{\mu}\nabla^4 v\,d\sigma=\int_C \tilde{\mu}\frac{\partial}{\partial n}\nabla^2 v\,ds$$

Adding and substracting $\iint_R \kappa^2\tilde{\mu}v\,d\sigma$ to the left hand side of the above relation, we obtain

$$B(P)+\iint_R \tilde{\mu}(\nabla^4 v+\kappa^2 v)d\sigma=\int_C \tilde{\mu}\,\frac{\partial}{\partial n}(\nabla^2 v)\,ds \qquad \text{(I-4.16)}$$

where

$$B(P)=\iint_R \left(\frac{\partial\tilde{\mu}}{\partial x}\frac{\partial(\nabla^2 v)}{\partial x}+\frac{\partial\tilde{\mu}}{\partial y}\frac{\partial(\nabla^2 v)}{\partial y}\right)d\sigma-\kappa^2\iint\tilde{\mu}v\,d\sigma$$

Using equation (I-3.1), relation (I-4.16) can be written as

$$B(P)+\iint \tilde{\mu}\frac{\delta(q-P)}{D}\,d\sigma=\int_C \tilde{\mu}\,\frac{\partial}{\partial n}(\nabla^2 v)ds \qquad \text{(I-4.17)}$$

This equation for $\tilde{\mu}(P)=\tilde{\mu}_7(P)$ yields[1]

---

(1) Let u(P) be a real function of point P in a plane region R, bounded by a closed curve C, which may be not smooth, that is, it may have a number of corners. Then [59]

$$\iint_R u(Q)\delta(Q-P)d\sigma=u(P) \quad \text{if } P \text{ is inside } R$$

$$\iint_R u(Q)\delta(Q-p)d\sigma=\frac{\alpha}{2\pi}u(p) \quad \text{if } P=p \text{ is on } C$$

$$\iint_R u(Q)\delta(Q-P)d\sigma=0 \quad \text{if } P \text{ is outside } R$$

where α is the angle between the tangents at p (see Fig. I-3); for a smooth boundary curve $\alpha=\pi$.

$$B(P)+\frac{1}{D}\tilde{\mu}_7(P)=\int_C \tilde{\mu}_7 \frac{\partial}{\partial n}\nabla^2 v ds = I_7(P) \qquad \text{if } P\epsilon R$$

$$B(p)+\frac{\alpha}{2\pi D}\tilde{\mu}_7(p)=\int_C \tilde{\mu}_7 \frac{\partial}{\partial n}\nabla^2 v ds = I_7(p) \qquad \text{if } P\equiv p\epsilon C$$

Substracting these two last equations, we obtain

$$I_7(P)-I_7(p)=B(P)-B(p)+\frac{1}{D}\tilde{\mu}_7(P)-\frac{\alpha}{2\pi D}\tilde{\mu}_7(p)$$

Taking the limit of the above equation as $P\to p$, and noting that $B(P)$ and $\tilde{\mu}_7(P)$ are continuous as $P\to p$, the jump of the discontinuity of I_7 as $P\to p\epsilon C$ is

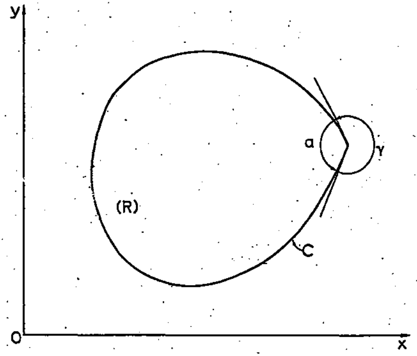

Fig.I-3. Two-dimensional region with a discontinuity of the boundary slope.

$$\lim_{P\to p}[I_7(P)-I_7(p)]=\frac{\gamma}{2\pi D}\tilde{\mu}_7(p) \qquad \text{(I-4.18)}$$

where $\gamma=2\pi-\alpha$ is the angle by which the tangent at point p of the boundary turns (see Fig.I-3). For points where C is smooth

$$\lim_{P\to p}[I_7(P)-I_7(p)]=\frac{1}{2D}\tilde{\mu}_7(p) \qquad \text{(I-4.19)}$$

Thus, all the integrals in equation (I-4.7) exist for $P\equiv p\epsilon C$. Moreover, they are continuous as $P\to p\epsilon C$ except the integral of the term $wV(v)$ which has a finite jump equal to $\frac{\gamma}{2\pi D}w(p)$. Using this fact in equation (I-4.7), we obtain the first boundary integral equation as

$$\frac{\alpha}{2\pi}w(p)=\iint_R vfd\sigma-D\int_C[vVw-wVv-\frac{\partial v}{\partial n}Mw+\frac{\partial w}{\partial n}Mv]ds-DI_C \qquad \text{(I-4.20)}$$

Notice, that $\alpha=\pi$ for points p where the boundary is smooth.

The derivation of the second independent boundary integral equation requires more attention. A general method for deriving systematically the second independent boundary integral equation for any boundary condition has been presented by Katsikadelis et al. [39]. This method has been employed by Bezine [45] to establish the deflection of thin elastic plates supported on their edges.

In this method, the directional derivative in a fixed direction $\bar{m}$ of both sides of equation (I-4.6) is taken as the point P varies.

Thus

$$\frac{\partial w(P)}{\partial m}=\iint_R \frac{\partial v}{\partial m} f d\sigma - D\int_C \left(\frac{\partial v}{\partial m} Vw - w\frac{\partial}{\partial m} Vv - \frac{\partial v^2}{\partial m \partial n} Mw + \frac{\partial w}{\partial n}\frac{\partial}{\partial m} Mv\right) ds - D\frac{\partial I_C}{\partial m}$$

Letting point P approach point pϵC, and choosing the direction $\bar{m}$ as the normal $\bar{n}_p$ to the boundary at point p, we obtain

$$\frac{\partial w}{\partial n_p}=\iint_R \frac{\partial v}{\partial n_p} f d\sigma - D \lim_{P\to p}\int_C \left(\frac{\partial v}{\partial n_p} Vw - w\frac{\partial}{\partial n_p} Vv - \frac{\partial^2 v}{\partial n_p \partial n} Mw + \frac{\partial w}{\partial n}\frac{\partial}{\partial n_p} Mv\right) ds - D\frac{\partial I_C}{\partial n_p} \qquad \text{(I-4.21)}$$

Again, the existence and the continuity of the line integral in the right hand side of equation (I-4.21) as P→ pϵC must be examined.

Substituting Vv and Mv from equations (I-2.8) into equation (I-4.21), and carrying out the appropriate integrations by parts, the resulting relations will involve integrals of the following form

$$\begin{aligned} I_1(P)&=\int_C \mu_1(s)\frac{\partial v}{\partial n_p} ds \\ I_2(P)&=\int_C \mu_2(s)\frac{\partial^2 v}{\partial n_p \partial n} ds \\ I_3(P)&=\int_C \mu_3(s)\frac{\partial}{\partial n_p}(\nabla^2 v) ds \\ I_4(P)&=\int_C \mu_4(s)\frac{\partial}{\partial n_p}\left(\frac{\partial}{\partial n}\nabla^2 v\right) ds \end{aligned} \qquad \text{(I-4.22)}$$

From the Appendix A, we have

$$\frac{\partial v}{\partial n_p} = \frac{\ell}{2\pi D}\,\text{kei}'(\rho)\cos\omega$$

$$\frac{\partial^2 v}{\partial n_p \partial n} = \frac{1}{2\pi D}\left[\text{ker}(\rho)\cos\omega\cos\varphi - \frac{1}{\rho}\,\text{kei}'(\rho)\cos(\varphi+\omega)\right]$$

$$\frac{\partial}{\partial n_p}\nabla^2 v = -\frac{1}{2\pi D}\,\frac{\partial}{\partial n_p}\left[\text{ker}(\rho)\right] \qquad \text{(I-4.23)}$$

$$\frac{\partial}{\partial n_p}\left(\frac{\partial}{\partial n}\nabla^2 v\right) = -\frac{1}{2\pi\ell D}\,\frac{\partial}{\partial n_p}\left[\text{ker}'(\rho)\cos\omega\right]$$

Inasmuch as kei$'(\rho)$ is not a singular function $\left[\lim_{\rho\to 0}\text{kei}'(\rho)=0\right]$, it can be concluded that the integral I_1 in equations (I-4.22) exists for P≡p∊C and it is continuous as P→p∊C. Moreover, we see from equations (I-3.14) and (I-4.13) that for small values of ρ, the functions ker(ρ) and $\frac{1}{\rho}$ kei$'(\rho)$ behave like $\ell n(\rho)$. Consequently, the integral I_2 in (I-4.22) exists for P≡p∊C and is continuous as P→p. It was shown, that for small values of ρ, the function ker(ρ) behaves like $\ell n(\rho)$; hence, the integral I_3 in (I-4.22) is the normal derivative of a single layer potential. It is known [59,79] that this derivative exists for P≡p∊C and it is discontinuous as the point P→∊C. Thus,

$$I_3(p)=\lim_{P\to p\in C}\int_C \mu_3(s)\frac{\partial}{\partial n_p}\nabla^2 v\,ds = -\frac{\gamma}{2\pi D}\,\mu_3(p)+\int_C \mu(s)\frac{\partial}{\partial n_P}\nabla^2 v\,ds \qquad \text{(I-4.24)}$$

The quantity $-\gamma\mu_3(p)/2\pi D$ is the jump of the discontinuity. For a smooth boundary $\gamma=\pi$. Finally, from the last of equations (I-4.23), it is concluded that the integral I_4 in (I-4.22) for small values of the argument ρ behaves like the normal derivative of the double layer potential $\int_C \mu_4(s)\frac{\partial}{\partial n}\nabla^2 v ds$. It is known [59], that this derivative exists for $P\equiv p\epsilon C$ and it is continuous as the point $P\to p\epsilon C$.

Thus, all the integrals in equation (I-4.21) exist for $P\equiv p\epsilon C$. Moreover, they are continuous as $P\to p\epsilon C$ except the integral of the term $\frac{\partial w}{\partial n}\frac{\partial}{\partial n_p}Mv$, which has a finite jump $\frac{\gamma}{2\pi D}\frac{\partial w}{\partial n_p}$. Using these facts in equation (I-4.21) we obtain the second integral equation

$$\frac{\alpha}{2\pi}\frac{\partial w(p)}{\partial n_p}=\iint_R\frac{\partial v}{\partial n_p}f d\sigma-D\int_C\Big(\frac{\partial v}{\partial n_p}Vw-w\frac{\partial}{\partial n_p}Vv-\frac{\partial^2 v}{\partial n_p\partial n}Mv$$

$$+\frac{\partial w}{\partial n}\frac{\partial}{\partial n_p}Mv\Big)ds-D\frac{\partial I_C}{\partial n_p}$$

(I-4.25)

Notice, that $\alpha=\pi$ for points p where the boundary is smooth. From the way the above equation is derived, it can be proven that it is independent from the equation (I-4.21) [45].

We introduce the following notation for the boundary functions

$$\Omega(s)=w(p)$$

$$X(s)=\frac{\partial w}{\partial n}$$

$$\Phi(s)=M\hat{w} \qquad \text{(I-4.26)}$$

$$\Psi(s)=V\hat{w}$$

and

$$\tilde{F}(s)=\iint_R v f d\sigma$$

$$\bar{G}(s)=\iint_R \nabla^2 v f d\sigma \qquad \text{(I-4.27)}$$

$$\tilde{H}(s)=\iint_R \frac{\partial v}{\partial n_p} f d\sigma$$

Using relations (I-4.26) and (I-4.27), the boundary integral equations (I-4.20) and (I-4.25) may be written as

$$\frac{\alpha}{2\pi}\,\Omega=\tilde{F}-D\int_C\left(v\psi-Vv\Omega-\frac{\partial v}{\partial n}\Phi+MvX\right)ds-DI_C$$

$$\frac{\alpha}{2\pi}\,X=\tilde{H}-D\int_C\left(\frac{\partial v}{\partial n_p}\psi+\frac{\partial}{\partial n_p}Vv\Omega-\frac{\partial^2 v}{\partial n_p\partial n}\Phi+\frac{\partial}{\partial n_p}MvX\right)ds-D\frac{\partial I_C}{\partial n_p} \qquad \text{(I-4.28)}$$

For any given boundary value problem, two of the functions (I-4.26) are given [see boundary conditions(I-1.5)]. The other two may be obtained from the solution of the coupled boundary integral equations (I-4.28). In these equations, the terms $\tilde{F}$ and $\tilde{H}$ may be established from the given loading f(P). For certain types of loading, such as concentrated forces or generalised forces [see section I.5], the integrals in (I-4.27) may be evaluated directly, while for other types of loading they must be integrated numerically. Once the functions Ω, X, Φ, Ψ are known, the solution of the boundary value problem (I-1.1), (I-1.5) may be obtained from equation (I-4.6) which, using the notation (I-4.26) and (I-4.27), may be written as

$$w(P)=\tilde{F}(P)-D\int_C \left(v\Psi - Vv\Omega - \frac{\partial v}{\partial n}\Phi + M v X \right) ds - DI_C \qquad \text{(I-4.29)}$$

where $v=v(q,P)$ and $I_C=I_C(q,P)$.

The boundary integral equations (I-4.28) may be used to analyze plates on elastic foundation having any boundary conditions (mixed, homogeneous, non homogeneous). However, these equations require special care when the boundary of the plate is clamped. In this case, the integral equations reduce to Fredholm-type integral equations of the first kind, that is, the unknown functions appear only in the integrals, and as it is known [80], the numerical solution of these equations may not depend continuously on the data; in other words, a small perturbation of the data may give rise to an arbitrary large perturbation of the solution. This difficulty can be overcome by employing special numerical techniques [81]. The terms I_C and $\partial I_C/\partial n_p$, which appear in equations (I-4.29) when the boundary has corners, also require special care during the numerical integration. When we are not interested to investigate the behaviour of the solution in the vicinity of a corner, we can eliminate these terms by smoothing out the boundary at the corner. This can be done by replacing the corner by an arc of known geometry, say a circular arc with a small radius of curvature. Inasmuch as the governing differential equation is elliptic, this replacement changes the solution only near the vicinity of the corner, (in elliptic equations, a disturbance on the boundary does not propagate in the interior of the region).

Boundary integral equations of Fredholm-type of the second kind, can be derived for clamped plates starting with the integral representation (I-4.5). Moreover, boundary integral equations which do not involve the term I_c can be derived for simply supported plates with homogeneous boundary conditions, starting with the integral representation (I-4.5).

For the clamped plate with homogeneous boundary conditions ($w=0$, $\frac{\partial w}{\partial n}=0$ on the boundary), the integral representation (I-4.5) becomes

$$w(P)=\iint vfd\sigma-D\int_C\left(v\frac{\partial}{\partial n}\nabla^2 w-\frac{\partial v}{\partial n}\nabla^2 w\right)ds \qquad \text{(I-4.31)}$$

Letting $P\to p\epsilon C$ in the above equation, and noting that v and $\frac{\partial v}{\partial n}$ are continuous as the point P approaches the boundary, the following boundary integral equation is obtained

$$0=\iint_R vfd\sigma-D\int_C\left(v\frac{\partial}{\partial n}\nabla^2 w-\frac{\partial v}{\partial n}\nabla^2 w\right)ds \qquad \text{(I-4.32)}$$

To obtain the second integral equation, the operator ∇^2 is applied on both sides of equation (I-4.31). Thus

$$\nabla^2 w(P)=\iint_R \nabla^2 vfd\ -D\int_C\left(\nabla^2 v\frac{\partial}{\partial n}\nabla^2 w-\frac{\partial}{\partial n}\nabla^2 v\nabla^2 w\right)ds$$

Letting point $P \to p \epsilon C$ in the above relation, and noting that

$$\lim_{P\to p}\int_C \nabla^2 w\frac{\partial}{\partial n}\nabla^2 v ds = \frac{1}{2D}\nabla^2 w + \int_C \nabla^2 w\frac{\partial}{\partial n}\nabla^2 v ds.$$

we get the following Fredholm-type integral equation of the second kind

$$\frac{1}{2}\nabla^2 w = \iint_C \nabla^2 v f d\sigma - D\int_C (\nabla^2 v\frac{\partial}{\partial n}\nabla^2 w - \frac{\partial}{\partial n}\nabla^2 v \nabla^2 w) ds \qquad \text{(I-4.33)}$$

Referring the relation (I-2.5),and using the notation (I-4.26) and (I-4.27) the boundary integral equations (I-4.32) and (I-4.33) may be written as

$$0 = \tilde{F} - D\int_C (v\Psi - \frac{\partial v}{\partial n}\Phi) ds \qquad \text{(I-4.34)}$$

$$\frac{1}{2}\Phi = \tilde{G} - D\int_C (\nabla^2 v\Psi - \frac{\partial}{\partial n}\nabla^2 v\Phi) ds \qquad \text{(I-4.35)}$$

The boundary condition $w=0$ of simply supported plates implies that $\frac{\partial^2 w}{\partial s^2}=0$ on the boundary. Thus, the first of the equations (I-2.8) becomes

$$M = \nabla^2 + (\nu-1)K\frac{\partial}{\partial n} \qquad \text{(I-4.36)}$$

Adding and substracting the term $(\nu-1)K\frac{\partial v}{\partial n}\frac{\partial w}{\partial n}$ in equation (I-4.5) we obtain

$$w(P) = \iint_R v f d\sigma - D\int_C (v\frac{\partial}{\partial n}\nabla^2 w - w\frac{\partial}{\partial n}\nabla^2 v - \frac{\partial v}{\partial n}Mw + \frac{\partial w}{\partial n}Mv) ds \qquad \text{(I-4.37)}$$

Using the boundary conditions for the simply supported plate, i.e. w=0 and Mw=0 and the notation (I-4.26) and (I-4.27) equation (I-4.37) becomes

$$w(p)=\tilde{F}(s)-D\int_C (v\Psi+MvX)ds \qquad (I-4.38)$$

The first integral equation is obtained by letting point $P\to p\epsilon C$ in equation (I-4.38). Inasmuch as v and Mv are continuous as the point P approaches the boundary, we obtain

$$0=\tilde{F}-D\int_C (v\Psi+MvX)ds \qquad (I-4.39)$$

The second integral equation is obtained by differentiating the integral representation (I-4.38) with respect to a fixed direction $\bar{m}$. Thus,

$$\frac{\partial w(P)}{\partial \bar{m}} =\iint_R \frac{\partial v}{\partial \bar{m}} f d\sigma - D\int_C \left(\frac{\partial v}{\partial \bar{m}}\,\Psi + \frac{\partial}{\partial \bar{m}}\,MvX\right)ds \qquad (I-4.40)$$

Letting, in equation (I-4.40), point $P\to p\epsilon C$ and the direction $\bar{m}$ coincide with the outward normal $\bar{n}_p$ to the boundary at point p and noting that[1]

$$\lim_{P\to p\epsilon C}\int_C \frac{\partial Mv}{\partial n_p} Xds = -\frac{\gamma}{2\pi D}\,X+\int \frac{\partial}{\partial n_p}\,MvXds$$

(1) The integral $\int_C \frac{\partial Mv}{\partial n_p} Xds$ behaves like the normal derivative of a single layer potential [see also equation (I-4.24)].

we obtain

$$\frac{\alpha}{2\pi} X=\tilde{H}-D\int_C\left(\frac{\partial v}{\partial n_p}\Psi+\frac{\partial}{\partial n_p}MvX\right)ds$$

For points p, where the boundary is smooth, it is

$$\alpha=\pi$$

For plates with free or guided edges and for homogeneous boundary conditions, equations (I-4.29) can be used.

On the basis of the aforegoing, it is apparent that depending on the boundary conditions of the plate under consideration, one of the following sets of boundary integral equations are the most suitable for numerical integration

a) Clamped Plate

$\Omega=X=0$, Φ, Ψ: unknowns

$$0=\tilde{F}-D\int_C\left(v\Psi-\frac{\partial v}{\partial n}\Phi\right)ds \quad (a)$$

$$\frac{1}{2}\Phi=\tilde{G}-D\int_C\left(\nabla^2 v\Psi-\frac{\partial}{\partial n}\nabla^2 v\Phi\right)ds \quad (b) \qquad (I-4.41)$$

$$w(P)=\tilde{F}(P)-D\int_C\left(v\Psi-\frac{\partial v}{\partial n}\Phi\right)ds \quad (c)$$

b) Simply supported plate

$\Omega=\Phi=0$, X,Ψ: unknowns

$$0=\tilde{F}-D\int_C(v\Psi+MvX)ds \quad (a) \qquad (I-4.42)$$

$$\frac{1}{2}X=\tilde{H}-D\int_C\left(\frac{\partial v}{\partial n_p}\Psi+\frac{\partial}{\partial n_p}MvX\right)ds \quad (b)$$

$$w(p)=\tilde{F}-D\int_C(v\Psi-MvX)ds \quad (c)$$

c) Plate with free edges

$\Phi=\Psi=0 \quad \Omega, X$:unknowns

$$I_C=-(\nu-1)\sum_{i=1}^{N}|[\nu T\Omega-\Omega T\nu+\frac{\partial v}{\partial n}\frac{\partial\Omega}{\partial s}-X\frac{\partial v}{\partial s}]|$$

$$\frac{\alpha}{2\pi}\Omega=\tilde{F}-D\int_C(-V\nu\Omega+M\nu X)ds-DI_C \qquad \text{(a)}$$

$$\frac{\alpha}{2\pi}X=\tilde{H}-D\int_C(-\frac{\partial}{\partial n_p}V\nu\Omega-\frac{\partial^2 v}{\partial n_p\partial n}\Phi+\frac{\partial}{\partial n_p}M\nu X)ds-\frac{\partial I_C}{\partial n_p} \qquad \text{(b)}$$

$$w(P)=\tilde{F}(P)-D\int_C(-V\nu\Psi+M\nu X)ds-DI_C \qquad \text{(c)}$$

(I-4.43)

d) Plate with guided edges

$X=\Psi=0, \quad \Omega,\Phi$:unknowns

$$I_C=(\nu-1)\sum_{i=1}^{N}[\![\nu T\Omega-\Omega T\nu+\frac{\partial v}{\partial n}\frac{\partial\Omega}{\partial s}]\!]$$

$$\frac{\alpha}{2\pi}\Omega=\tilde{F}-D\int_C(-V\nu\Omega-\frac{\partial v}{\partial n}\Phi)ds-DI_C \qquad \text{(a)}$$

$$0=\tilde{H}-D\int_C(-\frac{\partial}{\partial n_p}V\nu\Omega-\frac{\partial^2 v}{\partial n_p\partial n}\Phi)ds-D\frac{\partial I_C}{\partial n_p} \qquad \text{(b)} \qquad \text{(I-4.44)}$$

$$w(P)=\tilde{F}(P)-D\int_C(-V\nu\Omega-\frac{\partial v}{\partial n}\Phi)ds-DI_C \qquad \text{(c)}$$

where

$$T\Omega=\frac{\partial X}{\partial s}-K\frac{\partial\Omega}{\partial s}$$

The boundary integral equations for plates with free or guided edges,with a finite number of corners,include a number of terms which represent the discontinuity of the functions at the corners of the boundary. To establish these discontinuties the functions $\frac{\partial\Omega}{\partial s}$, $\frac{\partial X}{\partial s}$ at the points $s=s_i^{(-)}$ and $s=s_i^{(+)}$ can be expressed in terms of the values of Ω and X at point s_i and its adjacent nodal points.

I-5. Application to influence fields

In this section,expressions for the non-homogeneous terms $\tilde{F}(p)$, $\tilde{H}(p)$ and $\tilde{G}(p)$ in equations (I-4.29) and (I-4.35) will be established for the application of the BIE method to the numerical evaluation of the influence fields for the deflection, slope and stress resultants (bending moments, twisting moments and shearing forces) of plates on elastic foundation.

For this purpose,the following general form of the reciprocal theorem [B2] will be employed.

Theorem. Let w(Q,P) be the deflection at the point Q of a plate on elastic foundation due to a unit singularity δ(Q-P) at point P. For any linear differential operator N,the quantity N[w(Q,P)] at point Q is equal to the solution w*(P,Q) of the following differential equation

$$\nabla^4 w^* + \kappa^2 w^* = N[\delta(P-Q)]/D \qquad (I-5.1)$$

w*(P,Q) may be regarded as the generalized deflection at point P,due to the generalized load singularity N[δ(P-Q)] at point Q.

Notice that

a. For $N \equiv 1$

the generalized loading $\delta(P-Q)$ is a concentrated unit force at the point Q and $w^*(P,Q)$ is the influence field for the deflection at Q.

b. For $N \equiv \frac{\partial}{\partial x}$ or $\frac{\partial}{\partial y}$

the generalized loading $\frac{\partial}{\partial x}[\delta(P-Q)]$ or $\frac{\partial}{\partial y}[\delta(P-Q)]$ is a concentrated moment at the point Q and $w^*(P,Q)$ is the influence field for the slope $\frac{\partial w}{\partial x}$ or $\frac{\partial w}{\partial y}$ at the point Q.

c. For $N \equiv \frac{\partial^2}{\partial x^2}$ or $\frac{\partial^2}{\partial y^2}$

the generalized loading $\frac{\partial^2}{\partial x^2}[\delta(P-Q)]$ or $\frac{\partial^2}{\partial y^2}[\delta(P-Q)]$ has no physical meaning for the problem at hand (it is a quadrapole in theoretical physics) [83] and $w^*(P,Q)$ is the influence field of the curvature $\frac{\partial^2 w}{\partial x^2}$ or $\frac{\partial^2 w}{\partial y^2}$ at point Q.

d. For $N \equiv \frac{\partial^2}{\partial x \partial y}$

the generalized loading $\frac{\partial^2}{\partial x \partial y}[\delta(P-Q)]$ has no physical meaning for the problem at hand (it is a quadrapole in theoretical physics) and $w^*(P,Q)$ is the influence field for the twist $\frac{\partial w}{\partial x \partial y}$ at the point Q.

e. For $N \equiv -D(\frac{\partial^2}{\partial x^2} + \nu \frac{\partial^2}{\partial y^2})$, $-D(\frac{\partial^2}{\partial y^2} + \nu \frac{\partial^2}{\partial x^2})$ or $D(1-\nu)\frac{\partial^2}{\partial x \partial y}$ the the generalized loadings $-D(\frac{\partial^2}{\partial x^2} + \nu\frac{\partial^2}{\partial y^2})[\delta(P-Q)]$, $-D(\frac{\partial^2}{\partial y^2} + \nu \frac{\partial^2}{\partial x^2})[\delta(P-Q)]$ or $D(1-\nu)\frac{\partial^2}{\partial x \partial y}[\delta(P-Q)]$ have no physical meaning for the problem at hand (they are combinations of quadrapoles) and $w^*(P,Q)$ is the influence field of the bending moments M_x, M_y and the twisting moment M_{xy}, respectively at the point Q.

f. For $N=-D\frac{\partial\nabla^2}{\partial x}$ or $-D\frac{\partial\nabla^2}{\partial y}$
the generalized loading $-D\frac{\partial\nabla^2}{\partial x}[\delta(P-Q)]$ or $-D\frac{\partial\nabla^2}{\partial y}[\delta(P-Q)]$ has no physical meaning for the problem at hand (octapoles in theoretical physics) and $w^*(P,Q)$ is the influence field of the shearing force Q_x or Q_y, respectively, at the point Q.

On the base of the aforegoing, it is apparent that in order to establish the influence surfaces of the deflection, the bending moments, the twisting moments and the shearing forces at a point Q_0, the non homogeneous terms $\tilde{F}(p)$, $\tilde{H}(p)$ and $\tilde{G}(p)$ in equations (I-4.29) and (I-4.35) must be evaluated for $N\equiv 1$, $N\equiv \frac{\partial^2}{\partial x^2}$, $N\equiv \frac{\partial^2}{\partial y^2}$, $N\equiv \frac{\partial^2}{\partial x\partial y}$, $N\equiv \frac{\partial}{\partial x}\nabla^2$, and $N\equiv \frac{\partial}{\partial y}\nabla^2$, respectively.

This can be accomplished by using the following two properties of the Dirac δ-function.

i. For the m-order derivavative of the delta function, the following relation is valid

$$\int_{\alpha}^{b} g(x)\frac{d^m}{dx^m}[\delta(x-\xi_0)]dx=(-1)^m \frac{d^m}{dx^m} g(\xi_0), \qquad \alpha<\xi_0<b$$

ii. The δ-function in two dimensions may be expressed as

$$\delta(P-Q_0)=\delta(x-\xi_0)\delta(y-n_0), \quad P:(x,y), \quad Q_0:(\xi_0,n_0)$$

Thus, for the function $g(P)$ we have

$$\iint_R g(P)\frac{\partial^{m+n}}{\partial x^m\partial y^n}[\delta(P-Q_0)]d\sigma=(-1)^{m+n} \frac{\partial^{m+n}}{\partial x^m\partial x^n} g(Q_0) \qquad \text{(I-5.2)}$$

where $Q_0\epsilon R$

Thus, referring to Fig.I-4 and denoting by

$$\rho_{pQ}=|Q-p|/\ell, \quad \omega_{pQ}=\widehat{\bar{n}_p,\bar{r}_{pQ}}, \quad \rho_{pQ_o}=|Q_o-p|/\ell, \quad \omega=\widehat{\bar{n}_p,\bar{r}_{pQ_o}}$$

$$\text{and} \quad \alpha=\widehat{x,\bar{r}_{pQ}} \qquad \text{(I-5.3)}$$

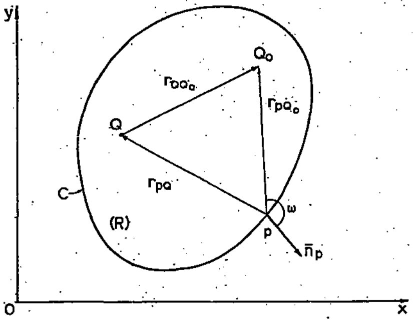

Fig. 1-4. Q_o is the point of application of the generalized force $N[\delta(Q-Q_o)]$

we have

a) $f(Q)=\delta(Q-Q_0)$

$$\tilde{F}(p)=-\frac{\ell^2}{2\pi D}\iint_R \delta(Q-Q_0)\,\mathrm{kei}(\rho_{pQ})\,d\sigma_Q=-\frac{\ell^2}{2\pi D}\mathrm{kei}(\rho_{pQ_0}) \qquad (I\text{-}5.4)$$

$$\tilde{H}(p)=\frac{\ell}{2\pi D}\iint_R \delta(Q-Q_0)\,\mathrm{kei}'(\rho_{pQ_0})\cos\omega_{pQ}\,d\sigma_Q=\frac{\ell}{2\pi D}\mathrm{kei}'(\rho_{pQ_0})\cos\omega \qquad (I\text{-}5.5)$$

$$\tilde{G}(p)=-\frac{1}{2\pi D}\iint_R \delta(Q-Q_0)\,\mathrm{ker}(\rho_{pQ})\,d\sigma_Q=-\frac{1}{2\pi D}\mathrm{ker}(\rho_{pQ_0}) \qquad (I\text{-}5.6)$$

б) $f(Q)=\frac{\partial^2}{\partial x^2}[\delta(Q-Q_0)]$

$$\tilde{F}(p)=-\frac{\ell^2}{2\pi D}\iint_R \mathrm{kei}(\rho_{pQ})\frac{\partial^2}{\partial x^2}[\delta(Q-Q_0)]\,d\sigma_Q$$

$$=-\frac{\ell^2}{2\pi D}\frac{\partial^2}{\partial x^2}[\mathrm{ker}(\rho_{pQ})]_{Q=Q_0}$$

$$=-\frac{1}{2\pi D}\left[\mathrm{ker}(\rho)\cos^2\alpha-\frac{1}{\rho}\mathrm{kei}'(\rho)\cos 2\alpha\right] \qquad (I\text{-}5.7)$$

$$\tilde{H}(p)=\frac{\ell}{2\pi D}\iint_R \mathrm{kei}'(\rho_{pQ})\cos\omega_{pQ}\frac{\partial^2}{\partial x^2}[\delta(Q-Q_0)]\,d\sigma_Q$$

$$=\frac{\ell}{2\pi D}\frac{\partial^2}{\partial x^2}\left[\mathrm{kei}'(\rho_{pQ})\cos\omega_{pQ}\right]_{Q=Q_0}$$

$$=\frac{1}{2\pi\ell D}\left[\mathrm{ker}'(\rho)\cos^2\alpha\cos\omega-\frac{1}{\rho}\left[\mathrm{ker}(\rho)-\frac{2}{\rho}\mathrm{kei}''(\rho)\right]\cos(2\alpha-\omega)\right] \qquad (I\text{-}5.8)$$

$$\tilde{G}(p)=-\frac{1}{2\pi D}\iint_R \mathrm{ker}(\rho_{pQ})\frac{\partial}{\partial x^2}[\delta(Q-Q_0)]\,d\sigma_Q$$

$$=-\frac{1}{2\pi D}\frac{\partial^2}{\partial x^2}[\mathrm{ker}(\rho_{pQ})]_{Q=Q_0}$$

$$=\frac{1}{2\pi\ell^2 D}\left[\mathrm{kei}(\rho)\cos^2\alpha+\frac{1}{\rho}\mathrm{ker}'(\rho)\cos 2\alpha\right] \qquad (I\text{-}5.9)$$

c) $f(Q)=\frac{\partial^2}{\partial y^2}[\delta(Q-Q_o)]$

$$\bar{F}(p)=-\frac{\ell^2}{2\pi D}\iint_R \mathrm{kei}(\rho_{pQ})\frac{\partial^2}{\partial y^2}[\delta(Q-Q_o)]\,d\sigma_Q$$

$$=-\frac{\ell^2}{2\pi D}\frac{\partial^2}{\partial y^2}[\mathrm{kei}(\rho_{pQ})]_{Q=Q_o} \qquad (I\text{-}5.10)$$

$$=-\frac{1}{2\pi D}[\mathrm{ker}(\rho)\sin^2\alpha+\frac{1}{\rho}\,\mathrm{kei}'(\rho)\cos 2\alpha]$$

$$\bar{H}(p)=\frac{\ell}{2\pi D}\iint_R \mathrm{kei}'(\rho_{pQ})\cos\omega_{pQ}\frac{\partial}{\partial y^2}[\delta(Q-Q_o)]\,d\sigma_Q$$

$$=\frac{\ell}{2\pi D}\frac{\partial}{\partial y^2}[\mathrm{kei}'(\rho_{pQ})\cos\omega_{pQ}]_{Q=Q_o} \qquad (I\text{-}5.11)$$

$$=\frac{1}{2\pi\ell D}\left[\mathrm{ker}'(\rho)\sin^2\alpha\cos\omega+\frac{1}{\rho}[\mathrm{ker}(\rho)-\frac{2}{\rho}\mathrm{kei}'(\rho)]\cos(2\alpha-\omega)\right]$$

$$\bar{G}(p)=-\frac{1}{2\pi D}\iint_R \mathrm{ker}(\rho_{pQ})\frac{\partial^2}{\partial y^2}[\delta(Q-Q_o)]\,d\sigma_Q$$

$$=-\frac{1}{2\pi D}\frac{\partial^2}{\partial y^2}[\mathrm{ker}(\rho_{pQ})]_{Q=Q_o} \qquad (I\text{-}5.12)$$

$$=\frac{1}{2\pi\ell^2 D}[\mathrm{kei}(\rho)\sin^2\alpha-\frac{1}{\rho}\mathrm{ker}'(\rho)\cos 2\alpha]$$

d) $f(Q)=\frac{\partial^2}{\partial x\partial y}[\delta(Q-Q_o)]$

$$\bar{F}(p)=-\frac{\ell^2}{2\pi D}\iint_R \mathrm{kei}(\rho_{pQ})\frac{\partial^2}{\partial x\partial y}[\delta(Q-Q_o)]\,d\sigma_Q$$

$$= -\frac{\ell^2}{2\pi D}\frac{\partial^2}{\partial x \partial y}[\mathrm{kei}(\rho_{pQ})]_{Q=Q_0} \qquad (I-5.13)$$

$$= -\frac{1}{4\pi D}[\mathrm{ker}(\rho) - \frac{2}{\rho}\mathrm{kei}'(\rho)]\sin 2\alpha$$

$$\tilde{H}(p) = \frac{\ell}{2\pi D}\iint_R \mathrm{kei}'(\rho_{pQ})\cos\omega_{pQ}\frac{\partial^2}{\partial x \partial y}[\delta(Q-Q_0)]d\sigma_Q$$

$$= \frac{\ell^2}{2\pi D}\frac{\partial^2}{\partial x \partial y}[\mathrm{kei}'(\rho_{pQ})\cos\omega_{pQ}]_{Q=Q_0} \qquad (I-5.14)$$

$$= \frac{1}{4\pi \ell D}[\mathrm{ker}'(\rho)\sin 2\alpha\cos\omega - \frac{2}{\rho}[\mathrm{ker}(\rho) - \frac{2}{\rho}\mathrm{kei}'(\rho)]\sin(2\alpha-\omega)]$$

$$\tilde{G}(p) = -\frac{1}{2\pi D}\iint_R \mathrm{ker}(\rho_{pQ})\frac{\partial^2}{\partial x \partial y}[\delta(Q-Q_0)]d\sigma_Q$$

$$= -\frac{1}{2\pi D}\frac{\partial^2}{\partial x \partial y}[\mathrm{ker}(\rho_{pQ})]_{Q=Q_0} \qquad (I-5.15)$$

$$= +\frac{1}{4\pi \ell^2 D}[\mathrm{kei}(\rho) + \frac{2}{\rho}\mathrm{ker}'(\rho)]\sin 2\alpha$$

e) $f(Q) = \frac{\partial}{\partial x}\nabla^2[\delta(Q-Q_0)]$

$$\tilde{F}(p) = -\frac{\ell^2}{2\pi D}\iint_R \mathrm{kei}(\rho_{pQ})\frac{\partial}{\partial x}\nabla^2[\delta(Q-Q_0)]d\sigma_Q$$

$$= \frac{\ell^2}{2\pi D}\frac{\partial}{\partial x}\nabla^2[\mathrm{kei}(\rho_{pQ})]_{Q=Q_0} \qquad (I-5.16)$$

$$= -\frac{1}{2\pi \ell D}\mathrm{ker}'(\rho)\cos\alpha$$

$$\tilde{H}(p) = \frac{\ell}{2\pi D}\iint_R \mathrm{kei}'(\rho_{pQ})\cos\omega_{pQ}\frac{\partial}{\partial x}\nabla^2[\delta(Q-Q_0)]d\sigma_Q$$

$$= -\frac{\ell}{2\pi D}\frac{\partial}{\partial x}\nabla^2\left[\mathrm{kei}'(\rho_{pQ})\cos\omega_{pQ}\right]_{Q=Q_0} \qquad \text{(I-5.17)}$$

$$= \frac{1}{2\pi\ell^2 D}\left[\frac{1}{\rho}\mathrm{ker}'(\rho)\cos(\alpha-\omega)+\mathrm{kei}(\rho)\cos\omega\cos\alpha\right]$$

$$\tilde{G}(p) = -\frac{1}{2\pi D}\iint_R \mathrm{ker}(\rho_{pQ})\frac{\partial}{\partial x}\nabla^2[\delta(Q-Q_0)]d\sigma_Q$$

$$= \frac{1}{2\pi D}\frac{\partial}{\partial x}\nabla^2[\mathrm{ker}(\rho_{pQ})]_{Q=Q_0} \qquad \text{(I-5.18)}$$

$$= \frac{1}{2\pi\ell^2 D}\mathrm{kei}'(\rho)\cos\alpha$$

f) $f(Q) = \frac{\partial}{\partial y}\nabla^2[\delta(Q-Q_0)]$

$$\tilde{F}(p) = -\frac{\ell^2}{2\pi D}\iint_R \mathrm{kei}(\rho_{pQ})\frac{\partial}{\partial y}\nabla^2[\delta(Q-Q_0)]d\sigma_Q$$

$$= \frac{\ell^2}{2\pi D}\frac{\partial}{\partial y}\nabla^2[\mathrm{kei}(\rho_{pQ})] \qquad \text{(I-5.19)}$$

$$= -\frac{1}{2\pi\ell D}\mathrm{ker}'(\rho)\sin\alpha$$

$$\tilde{H}(p) = +\frac{\ell}{2\pi D}\iint_R \mathrm{kei}'(\rho_{pQ})\cos\omega_{pQ}\frac{\partial}{\partial y}[\delta(Q-Q_0)]d\sigma_Q$$

$$= -\frac{\ell}{2\pi D}\frac{\partial}{\partial y}\left[\mathrm{kei}'(\rho_{pQ})\cos\omega_{pQ}\right]_{Q=Q_0} \qquad \text{(I-5.20)}$$

$$= \frac{1}{2\pi\ell^2 D}\left[\frac{1}{\rho}\mathrm{ker}'(\rho)\sin(\alpha-\omega)+\mathrm{kei}(\rho)\sin\omega\sin\alpha\right]$$

$$\tilde{G}(p) = -\frac{1}{2\pi D}\iint_R \mathrm{ker}(\rho)\frac{\partial}{\partial y}\nabla^2[\delta(Q-Q_0)]d\sigma_Q$$

$$= \frac{1}{2\pi D}\frac{\partial}{\partial y}\nabla^2[\ker(\rho)]_{Q=Q_o} \qquad \text{(I-5.21)}$$

$$= \frac{1}{2\pi \ell^3 D}\,\mathrm{kei}'(\rho)\sin\alpha$$

Notice that the term $\bar{F}(P)$ corresponding to the generalized loads $\frac{\partial^2}{\partial x^2}[\delta(P-Q)]$ or $\frac{\partial}{\partial y^2}[\delta(P-Q)]$ becomes infinite(1) when

---

(1) For $N = \frac{\partial}{\partial x^2}$, referring to relation (I-5.4), we have

$$\bar{F}(P) = -\frac{1}{2\pi D}\left[\ker(\rho)\cos^2\alpha - \frac{1}{\rho}\,\mathrm{kei}'(\rho)\cos 2\alpha\right]$$

where, in this case $\rho = |P-Q_o|/\ell$ and $\alpha = \widehat{n, r}_{PQ}$.
For small values of ρ, using equations (I-3.14c) and (I-4.13) we obtain

$$F(P) = -\frac{1}{2\pi D}\left[-\ell n(\rho)\cos^2\alpha - \frac{1}{\rho}\left[-\frac{1}{\rho}\,\ell n(\rho)\right]\cos 2\alpha + \text{Regular terms}\right]$$

$$= \frac{1}{2\pi D}\,\ell n(\rho)[1 + \text{Regular terms}]$$

Hence

$$\lim_{\rho\to 0}\bar{F}(P) = +\infty$$

For $N = \frac{\partial^2}{\partial y^2}$, referring to equation (I-5.10), we have

$$F(P) = -\frac{1}{2\pi D}\left[\ker(\rho)\sin^2\alpha + \frac{1}{\rho}\,\mathrm{kei}'(\rho)\cos 2\alpha\right]$$

For small values of ρ, using equations (I-3.14c) and (I-4.13) we obtain

$$\bar{F}(P) = -\frac{1}{2\pi D}\left[-\ell n(\rho)\sin^2\alpha + \frac{1}{\rho}\left[-\frac{1}{2}\rho\,\ell n(\rho)\right]\cos 2\alpha + \text{Regular terms}\right]$$

$$\frac{1}{2\pi D}\,\ell n(\rho)[1 + \text{Regular terms}]$$

Hence

$$\lim_{\rho\to 0}\bar{F}(P) = +\infty$$

$P \to Q_0$ $(\rho \to 0)$. Inasmuch as the influence fields of the bending moments M_x or M_y are produced by applying the generalized forces

$$-D\left[\frac{\partial^2}{\partial x^2} + \nu\frac{\partial^2}{\partial y^2}\right]\delta(P-Q_0) \quad \text{or} \quad -D\left[\frac{\partial^2}{\partial y^2} + \nu\frac{\partial^2}{\partial x^2}\right]\delta(P-Q_0),$$ as expected,

$$\lim_{P\to Q_0} M_x = +\infty\ , \qquad \lim_{P\to Q_0} M_y = +\infty \qquad \text{(I-5.22)}$$

Moreover, notice that the term $\bar{F}(P)$ corresponding to the generalized load $\frac{\partial^2}{\partial x \partial y}[\delta(P-Q_0)]$ approaches a finite limit[2] as $P \to Q_0$. Inasmuch as the influence field for the twisting moment M_{xy} is produced by applying the generalized force $D(1-\nu)\frac{\partial^2}{\partial x \partial y}[\delta(P-Q_0)]$, as expected, is the twisting moment at the point of application of the concentrated force is an

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(2) For $N = \frac{\partial^2}{\partial x \partial y}$ referring to relation (I-5.13) we have

$$\bar{F}(P) = -\frac{1}{4\pi D}\left[\ker'(\rho) - \frac{2}{\rho}\,\mathrm{kei}'(\rho)\right]\sin 2\alpha$$

For small values of ρ using equation (I-3.14c) and (I-4.13) we obtain

$$\bar{F}(P) = -\frac{1}{4\pi D}\left[-\ln(\rho) + \ln(\rho) + \text{Regular terms}\right]\sin 2\alpha$$

$$= C\sin 2\alpha$$

where $\lim_{\rho\to 0} C = \text{constant}$

$$\lim_{\rho\to 0} \bar{F}(P) = \text{an indefinite constant}$$

indefinite constant the value of which depends on the rarial direction the point P approaches Q_o. That is

$$\lim_{P \to Q_o} M_{xy} = \text{indefinite constant}$$

Finally, referring to equations (I-5.19), (I-5.20) and (I-4.14), the term $\tilde{F}(P)$ due to the generalized force $\frac{\partial}{\partial x}\nabla^2[\delta(P-Q_o)]$ or $\frac{\partial}{\partial y}\nabla^2[\delta(P-Q_o)]$ becomes infinite as $P \to Q_o$. Thus, as it was expected

$$\lim_{P \to Q_o} Q_x = \pm\infty, \qquad \lim_{P \to Q_o} Q_y = \pm\infty \qquad \text{(I-5.23)}$$

Thus, closed form expressions have been derived for the non homogeneous terms $\tilde{F}(p)$, $\tilde{H}(p)$ and $\tilde{G}(p)$ required for the computation of the influence fields by the BIE method. This renders the BIE method better suited for the numerical evaluation of the influence fields than the other numerical methods (finite differences and finite element method) which may give poor results because of the difficulty in approximating the generalized forces.

PART II

NUMERICAL SOLUTION OF THE INTEGRAL EQUATIONS FOR THE CLAMPED AND SIMPLY SUPPORTED PLATES

II-1. Approximation of the integral equations for the clamped plate by a system of simultaneous linear algebraic equations

Equations (I-4.41a,b) may be written as

$$-\int_C \frac{\partial v}{\partial n}\Phi ds+\int_C v\psi ds=\frac{1}{D}\iint_R vfd\sigma \qquad \text{(II-1.1)}$$

$$\frac{1}{2D}\Phi-\int_C \frac{\partial}{\partial n}\nabla^2 v\Phi ds+\int_C \nabla^2 v\psi ds=\frac{1}{D}\iint_R \nabla^2 vfd\sigma$$

where

$$v=-\frac{\ell^2}{2\pi D}\mathrm{kei}(\rho), \qquad \rho=r/\ell, \qquad r=|p-q| \qquad \text{(II-1.2)}$$

Introducing equation (II-1.2) into equations (II-1.1), and using relations (A-36),(A-8),(A-39) and (A-41) of Appendix A, we obtain

$$-\frac{1}{\ell}\int_C \Phi \mathrm{kei}'(\rho)\cos\varphi ds+\int_C \psi \mathrm{kei}(\rho)ds=\frac{1}{D}\iint_R f\mathrm{kei}(\rho)d\sigma \qquad \text{(II-1.3)}$$

$$-\pi\Phi-\frac{1}{\ell}\int_C \Phi \mathrm{ker}'(\rho)\cos\varphi ds+\int_C \psi \mathrm{ker}(\rho)ds=\frac{1}{D}\iint_R f\mathrm{ker}(\rho)d\sigma$$

From fig.II-1 we have

$$\cos\varphi ds=rd\theta \quad , \qquad \varphi=\widehat{\bar{r},\bar{n}_q} \qquad \text{(II-1.4)}$$

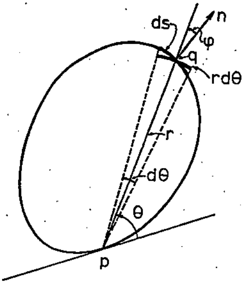

Fig.II-1.

Introducing equation (II-1.4) into equations (II-1.3), we get

$$-\int_C \Phi\rho kei'(\rho)d\theta+\int_C \psi kei(\rho)ds= \frac{1}{D}\iint_R fkei(\rho)d\sigma \qquad \text{(II-1.5)}$$

$$-\pi\Phi-\int \Phi\rho ker'(\rho)d\theta+\int_C \psi ker(\rho)ds= \frac{1}{D}\iint_R fker(\rho)d\sigma \qquad \text{(II-1.6)}$$

This substitution permits the integration of equations (II-1.3) to be performed with respect to the angle θ and ,thus ,the computation of the function $\cos\varphi(s)$ is avoided.

The numerical integration of the boundary integral equations can be carried out by dividing the boundary into a finite number of intervals (Fig.II-2) referred to as boundary elements [46]. The end points of each interval are referred to as dividing points or interval points. The points on which the values of the unknown functions are evaluated are referred to as nodal points or simply nodes. The boundary can be approximated by straight line segments or by curves of higher order (e.g. quadratic). On each interval, the unknown boundary function can be approximated either as constant, or as linearly varying or as quadratically varying [84,85] or by a polynomial of desired degree by applying a Gaussian quadrature. It is apparent,that the integration becomes more complicated with the use of a more refined approximation. Special care must be given to the integration on intervals where the kernel of the integral equation becomes singular. In this case, the Cauchy principal value of the integral must be evaluated. Moreover, each integral requires special treatment depending on the singularity of its kernel.

In this investigation, the unknown functions are assumed constant on each interval (step function assumption). Moreover, the curved boundary is approximated by straight line segments. This is a simple approach and has been proven effective [21,22,23,24].

The boundary is divided into M,not necessarily equal intervals which are numbered,consecutively,clockwise. The centers of these intervals are taken as their nodes. The values of Φ and Ψ are assumed constant on each interval and equal to the values calculated at the nodes. Denoting by Φ_j and Ψ_j the values of Φ and Ψ on the j-integral, equations (II-1.5) and (II-1.6) are transformed to the following 2M simultaneous linear algebraic equations

$$\sum_{j=1}^{M} a_{kj}\Phi_j + \sum_{j=1}^{M} b_{kj}\Psi_j = F_k \qquad (k=1,2,\ldots M)$$

(II-1.7)

$$\sum_{j=1}^{M} (c_{kj} - \pi\delta_{kj})\Phi_j + \sum_{j=1}^{M} d_{kj}\Psi_j = G_k \qquad (k=1,2,\ldots M)$$

where M is the number of the nodal points on the boundary, δ_{kj} is the Kronecker delta and

$$a_{kj} = -\int_j \rho_{kj}\,\mathrm{kei}'(\rho_{kj})\,d\theta$$

$$b_{kj} = \int_j \mathrm{kei}(\rho_{kj})\,ds$$

(II-1.8a,b,c,d)

$$c_{kj} = -\int_j \rho_{kj}\,\mathrm{ker}'(\rho_{kj})\,d\theta$$

$$d_{kj} = \int_j \mathrm{ker}(\rho_{kj})\,ds$$

$$F_k = \frac{1}{D}\iint_R f(Q)\,\mathrm{kei}(\rho_{kQ})\,d\sigma_Q$$

(II-1.8e,f)

$$G_k = \frac{1}{D}\iint_R f(Q)\,\mathrm{ker}(\rho_{kQ})\,d\sigma_Q$$

where

$$\rho_{kQ}=\frac{r_{kQ}}{\ell}=\frac{|Q-p_k|}{\ell}, \qquad Q\epsilon R, \quad p_k\epsilon C$$

In relations (II-1.8), the symbol $\int_j$ denotes the line integral on the j-interval, that is, the interval containing the j nodal point.

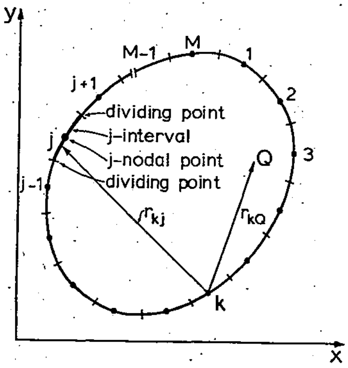

Fig.II-2. Discretization of the boundary.

The system of equations (II-1.7) may be rewritten in matrix form as

$$AY=B \qquad \text{(II-1.9)}$$

where

$$Y=\begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_M \\ \hline \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_M \end{bmatrix} \qquad B=\begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_M \\ \hline G_1 \\ G_2 \\ \vdots \\ G_M \end{bmatrix} \qquad \text{(II-1.10a,b,c)}$$

$$A=\left[\begin{array}{ccc:ccc} a_{11} & a_{12}\cdots & a_{1M} & b_{11} & b_{12}\cdots & b_{1M} \\ a_{21} & a_{22}\cdots & a_{2M} & b_{21} & b_{22}\cdots & b_{2M} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{M1} & a_{M2}\cdots & a_{MM} & b_{M1} & b_{M2}\cdots & b_{MM} \\ \hdashline c_{11}-\pi & c_{12}\cdots & c_{1M} & d_{11} & d_{12}\cdots & d_{1M} \\ c_{21} & c_{22}-\pi\cdots & c_{2M} & d_{21} & d_{22}\cdots & d_{2M} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{M1} & c_{M2}\cdots & c_{MM}-\pi & d_{M1} & d_{M2}\cdots & d_{MM} \end{array}\right]$$

The line integrals (II-1.8a,b,c,d) when $k\neq j$, that is, when $\rho\neq 0$, can be evaluated using any numerical technique. However, in the case $k=j$; some of the kernels become singular and a special technique must be used for the evaluation of the resulting improper integrals. This technique depends, each time, on the specific kernel. The numerical schemes for the computation of these line integrals are given in the following section.

A major problem in the numerical solution is the numerical computation of the double integral (II-1.8f). Inasmuch as $\lim_{\rho \to 0} ker(\rho)=\infty$, this integral must be treated as a double singular (improper) integral. The evaluation of double singular integrals is a complex task which is performed , in this investigation, by employing two different methods. In the first procedure, the singularity is removed by a coordinate transformation, and the resulting double integrals are evaluated numerically by known techniques.This procedure can be equaly applied to any two-dimensional integrals, whose integrand exhibits a logarithmich or a Cauchy-type singularity. The second method is by use of the generalized functions introduced in Chapter I-5 to obtain the influence field $w^*(P,Q)$, which is the Green function of the problem, and then to obtain the solution for any given function $f(Q)$ from the relation

$$w(P)=\iint_R w^*(P,Q)f(Q)d\sigma_Q \qquad \text{(II-1.9)}$$

The function $w^*(P,Q)$ can be evaluated at desired points in the region R and, thus, the integral (II-1.9) can be computed numerically. When the function $w^*(P,Q)$ represents the influence field for the deflection of the plate it does not have singularities. Consequently, in this case, the double integral (II-1.9) can be computed by applying the known techniques for numerical double integration. If the function w*(P,Q) has a singularity, as for example, when it represents the influence field for the bending moment, the first method may be employed to evaluate the integral (II-1.9) numerically.

The second method is preferable to the first since it leads to a simpler computer program for the following reasons.

a) For the generalized loads, the integrals (II-1.8e) and (II-1.8f) are evaluated analytically and they are known functions (see section I.5).

b) For given geometry and boundary conditions, the function w*(P,Q) is computed only once and the value of the integral for any loading f(Q) is obtained by a simple double integration.

II-2. Evaluation of the coefficients a_{kj}, b_{kj}, c_{kj}, d_{kj} for the clamped plate

For the computation of the integrals (II-1.8) on the j-interval, we use either Simpon's rule or, in some cases, the trapezoidal rule. The three points used for Simpson's rule in each interval are the nodal point p_j and its adjacent dividing points q_{j-1} and q_j. Since it is not always simple to find the middle of the arc $q_{j-1}q_j$, we use the unqual spaced Simpson's rule which has the following form

$$I \simeq \frac{1}{6h_1h_2}\left[(g_o-g_1)^2h_2^3+(g_o-g_2)h_1^3+h_1^2h_2(2g_1+3g_o+g_2)+h_1h_2^2(g_1+3g_o+2g_2)\right] \qquad \text{(II-2.1)}$$

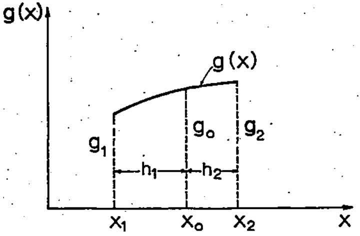

Fig.II-3.

In formula (II-2.1), g_1, g_o, g_2 are the values of the integrand at the points x_1, x_o, x_2, respectively. For $h_1=h_2=h$, the above formula becomes

$$I \simeq \frac{h}{3}(g_1+4g_o+g_2) \qquad \text{(II-2.2)}$$

For the computation of the values g_o, g_1, g_2 and h_1, h_2, we need the values r_1, r_o, r_2, the line segments s_1 and s_2 and the angles θ_1 and θ_2. These quantities are computed from the coordinates of the boundary points which are the only geometrical data of the numerical procedure.

The nodal points are denoted by p_j (j=1,2,...M) and the dividing points by q_j (j=1,2,...M) and their coordinates by x_j, y_j and ξ_j, η_j, respectively (see Fig.II-4).

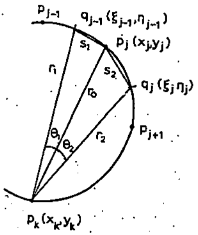

Fig.II-4.

Referring to Fig.II-4, we have

$$
\begin{aligned}
r_1 &= \left[(\xi_{j-1}-x_k)^2+(\eta_{j-1}-y_k)^2\right]^{\frac{1}{2}} \\
r_0 &= \left[(x_j-x_k)^2+(y_j-y_k)^2\right]^{\frac{1}{2}} \\
r_2 &= \left[(\xi_j-x_k)^2+(\eta_j-y_k)^2\right]^{\frac{1}{2}} \qquad \text{(II-2.3)} \\
s_1 &= \left[(x_j-\xi_{j-1})^2+(y_j-\eta_{j-1})^2\right]^{\frac{1}{2}} \\
s_2 &= \left[(x_j-\xi_j)^2+(y_j-\eta_j)^2\right]^{\frac{1}{2}}
\end{aligned}
$$

$$\theta_1=2\arcsin(\sqrt{(\tau_1-r_1)(\tau_1-r_0)/r_1r_0}), \quad \tau_1=(r_1+s_1+r_\sigma)/2$$

$$\theta_2=2\arcsin(\sqrt{(\tau_2-r_2)(\tau_2-r_0)/r_2r_0}), \quad \tau_2=(r_2+s_2+r_0)/2$$

a) Evaluation of the coefficients a_{kj}

Case i $k \neq j$

If both angles θ_1 and θ_2 (see fig.II-4) do not vanish, we can apply Simpson's rule (II-2.1) by setting

$$
\begin{aligned}
&h_1=\theta_1, \quad h_2=\theta_2 \\
&g_1=-\rho_1 \mathrm{kei}'(\rho_1) \\
&g_0=-\rho_0 \mathrm{kei}'(\rho_0) \\
&g_2=-\rho_2 \mathrm{kei}'(\rho_2)
\end{aligned}
\qquad \text{(II-2.4)}
$$

If one of the angles θ_1,θ_2 is equal to zero, say θ_2, then the integral vanishes in the interval p_jq_j, because $\cos\varphi=\cos\pi/2=0$. In the other interval, the integral can be approximated by the trapezoidal rule. Hence,

$$a_{kj} \approx \frac{1}{2}s_i(g_i+g_0) \;, \quad \text{when } \theta_{3-i}=0, \; i=1 \text{ or } 2 \qquad \text{(II-2.5)}$$

If both angles θ_1 and θ_2 vanish

$$a_{kj}=0 \qquad \text{(II-2.6)}$$

Case ii $k=j$

In this case, $r_0=0$ and from equation (I-4.12), we conclude that

$$\lim_{\rho\to 0}[\rho \mathrm{kei}'(\rho)]=0$$

In this limiting case, where $p_k \to p_j$, the angles θ_1 and θ_2 are those between the tangent at the point p_j and the directions

r_1 and r_2, respectively, (see Fig.II-5).

If the slope of the Boundary is approximated by the relation

$$\left.\frac{dy}{dx}\right|_{p_j} = \frac{\eta_j - \eta_{j-1}}{\xi_j - \xi_{j-1}}$$

then the tangent at p_j is parallel to the line $q_{j-1}q_j$ and, to this order of approximation, we have from (Fig.II-5).

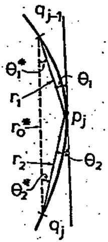

Fig.II-5.

$$\theta_1 \simeq \theta_1^* = 2\arcsin\left(\sqrt{(\tau - r_1)(\tau - r_0^*)/r_1 r_0^*}\right)$$

$$\theta_2 \simeq \theta_2^* = 2\arcsin\left(\sqrt{(\tau - r_2)(\tau - r_0^*)/r_2 r_0^*}\right) \qquad \text{(II-2.7)}$$

where $\tau = (r_1 + r_2 + r_0^*)/2$

The integral is computed from Simpson's rule (II-2.1) by setting

$$h_1=\theta_1,\quad h_2=\theta_2$$
$$g_1=-\rho_1 kei'(\rho_1)$$
$$g_o=0 \qquad \text{(II-2.8)}$$
$$g_2=-\rho_2 kei'(\rho_2)$$

If one of the angles θ_1,θ_2 is zero, then

$$a_{kk}=0 \qquad \text{(II-2.9)}$$

b) <u>Evaluation of the coefficients b_{kj}</u>

<u>Case i</u> $k\neq j$

The integral can be computed by Simpson's rule (II-2.1) by setting

$$h_1=s_1,\quad h_2=s_2$$
$$g_1=kei(\rho_1)$$
$$g_o=kei(\rho_o) \qquad \text{(II-2.10)}$$
$$g_2=kei(\rho_2)$$

<u>Case ii</u> $k=j$

In this case, $r_o=0$ and from equation (I-3.14d), we find that

$$\lim_{\rho\to 0} kei(\rho)=-\pi/4 \qquad \text{(II-2.11)}$$

Again we can apply Simson's rule (II-2.1) by setting

$$h_1=r_1, \quad h_2=r_2$$
$$g_1=kei(\rho_1)$$
$$g_0=-\pi/4 \qquad \text{(II-2.12)}$$
$$g_2=kei(\rho_2)$$

c) <u>Evaluation of the coefficients c_{kj}</u>

<u>Case i</u> $k \neq j$

In this case, the procedure is analogous to that applied for the evaluation of a_{kj}. The integral can be evaluated from Simpson's rule (II-2.1) by setting

$$h_1=\theta_1, \quad h_2=\theta_2 \qquad \theta_1,\theta_2 \neq 0$$
$$\theta_1=-\rho_1 ker'(\rho_1)$$
$$g_0=-\rho_0 ker'(\rho_0) \qquad \text{(II-2.13)}$$
$$g_2=-\rho_2 ker'(\rho_2)$$

When θ_1 or θ_2 is zero, we use the trapezoidal rule (II-2.5).

<u>Case ii</u> $k=j$

In this case, $r_o=0$ and from equation (I-4.13) we conclude that

$$\lim_{\rho \to 0}[\rho ker'(\rho)]=-1 \qquad \text{(II-2.14)}$$

We can apply Simpson's rule (II-2.1) by setting

$$h_1=\theta_1,\ h_2=\theta_2$$
$$g_1=-\rho_1 \ker'(\rho_1)$$
$$g_0=-1 \qquad \text{(II-2.14)}$$
$$g_2=-\rho_2 \ker'(\rho)$$

If one of the angles θ_1,θ_2 is zero, we have

$$c_{kk}=0 \qquad \text{(II-2.15)}$$

d) Evaluation of the coefficients d_{kj}

Case i $k\neq j$

The integral is computed from Simpson's rule (II-2.1) by setting

$$\bar{h}_1=s_1,\ \bar{h}_2=s_2$$
$$g_1=\ker(\rho_1)$$
$$g_0=\ker(\rho_0) \qquad \text{(II-2.16)}$$
$$g_2=\ker(\rho_2)$$

Case ii $k=j$

In this case, $r_o=0$. Moreover, from equation (I-3.14c) we conclude that for small values of ρ the function $\ker(\rho)$ behaves like $-\ln(\rho)$. Thus, we can write

$$d_{kk}=\int_k \ker(\rho)ds=\int_k [\ker(\rho)+\ln(\rho)]ds-\int_k \ln(\rho)ds=d'_{kk}+d''_{kk}$$

(II-2.17)

where

$$d'_{kk}=\int_k [\ker(\rho)+\ln(\rho)]ds \qquad (II-2.18)$$

$$d''_{kk}=-\int_k \ln(\rho)ds \qquad (II-2.19)$$

From equation (I-3.14c) it is apparent that

$$\lim_{\rho\to 0}[\ker(\rho)+\ln(\rho)]=\ln 2-.577217\ldots.$$

Thus, we can approximate the integral d_{kk} using Simpson's rule with

$$\begin{aligned} &h_1=r_1,\ h_2=r_2 \\ &g_1=\ker(\rho_1)+\ln(\rho_1) \\ &g_0=\ln 2-.577217 \\ &g_2=\ker(\rho_2)+\ln(\rho_2) \end{aligned} \qquad (II-2.20)$$

The other integral d''_{kk} can be approximated by direct evaluation of the improper integral [21] on the straight lines p_jq_{j-1} and p_jq_j. Thus,

$$d''_{kk}=-\int_0^{r_1} \ln(\rho)dr-\int_0^{r_2} \ln(\rho)dr$$
$$=-[r_1(\ln\rho_1-1)+r_2(\ln\rho_2-1)] \qquad (II-2.21)$$

II-3. <u>Evaluation of F_k and G_k for the clamped plate</u>

From Fig.II-6, we see that functions kei(ρ) and ker(ρ)

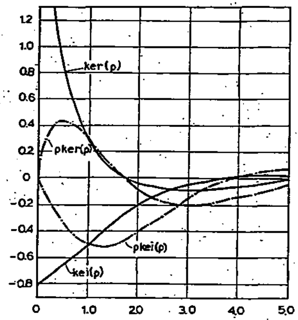

Fig.II-6. Functions kei(ρ),ker(ρ),ρkei(ρ),ρker(ρ).

exhibit a peak at ρ=0. This peak is finite for kei(ρ) (kei(0)= =-0.7854) and infinite for ker(ρ) (ker(0)=+∞). Hence, the integral (II-1.8f) is an improper double integral and special attention must be paid to its numerical computation.

In the sequel, we will present a numerical procedure for the evaluation of the double integral (II-1.8f). This procedure can be equally applied to any two-dimensional integrals,the integrand of which exhibits a logarithmic or Cauchy-type singularity,if the loading function f(Q) is not singular (concentrated load).This latter case of singular loading function has been discussed in Chapter (I-5).Recently [46] Gaussian guadrature formulas for functions with Cauchy-type singularity over triangles and quadrangles have been developed. However, the application of these formulas to an arbitrary area requires subdivision of the area into triangles and quadrangles, which is a tedious task.

Although the integral (II-2.8e) is not improper, its evaluation was not satisfactory using iterated integration with Gaussian quadrature [86]. However, the numerical procedure proposed in this investigation for the numerical evaluation of the double integrals with singular integrand, also yields satisfactory results for the integral (II-1.8e).

In the procedure proposed in this investigation,polar coordinates are employed having the point p_k as the origin, and the tangent line to the boundary at this point as the reference axis for the angles θ (see Fig.II-7). Thus, the integrals (II-1.8e) and (II-1.8f) may be written as

$$\mathcal{F}_k = \frac{1}{D}\int_0^{\pi}\int_0^{r_c(\theta)} f^*(r,\theta)\,\mathrm{kei}(\rho)\,r\,dr\,d\theta \qquad \text{(II-3.1)}$$

$$G_k = \frac{1}{D}\int_0^{\pi}\int_0^{r_c(\theta)} f^*(r,\theta)\,\mathrm{ker}(\rho)\,r\,dr\,d\theta \qquad \text{(II-3.2)}$$

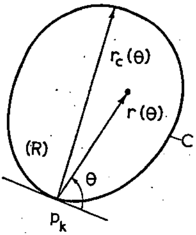

Fig.II-7. Polar coordinates for the functions kei(ρ) and ker(ρ).

From equation (I-3.14c) it is apparent that

$$\lim_{\rho \to 0}[\rho \ker(\rho)]=0$$

Hence, the integrand in the representation for G_k is not singular (see also the plotting of the function ρker(ρ) in Fig.II-6) and the iterated integration with Gaussian quadrature can be employed [86].

The function f(Q) is given with reference to a fixed Cartesian coordinate system.Thus, its conversion to polar coordinates whose origin and reference axis for the angle θ change as the nodal point p_k changes, seems tedious at first sight. However, the value of the function f(Q) at any point (r,θ) can be computed by first establishing the Cartesian coordinates x, y of this point.

If the loading function $f^*(r,\theta)$ is a constant f_0, the integrals (II-3.1) and (II-3.2) can be further simplified [78] as

$$F_k = \frac{f_0 \ell^2}{D}\int_0^{\pi}\left[\int_0^{\rho_c} \rho \operatorname{kei}(\rho)d\rho\right]d\theta = -\frac{f_0\ell^2}{D}\int_0^{\pi}\left[\rho \operatorname{ker}'(\rho)\right]_0^{\rho_c} d\theta \qquad \text{(II-3.3)}$$

$$G_k = \frac{f_0 \ell^2}{D}\int_0^{\pi}\left[\int_0^{\rho_c} \operatorname{kei}(\rho)d\rho\right]d\theta = \frac{f_0\ell^2}{D}\int_0^{\pi}\left[\rho \operatorname{kei}'(\rho)\right]_0^{\rho_c} d\theta \qquad \text{(II-3.4)}$$

taking into account that

$$\lim_{\rho\to 0}[\rho \operatorname{ker}'(\rho)] = -1$$

$$\lim_{\rho\to 0}[\rho \operatorname{kei}'(\rho)] = 0$$

relations (II-3.3) and (II-3.4) reduce to

$$F_k = -\frac{f_0\ell^2}{D}\int_0^{\pi}[\rho_c \operatorname{ker}'(\rho_c)+1]d\theta \qquad \text{(II-3.5)}$$

$$G_k = \frac{f_0\ell^2}{D}\int_0^{\pi}\rho_c \operatorname{kei}'(\rho_c)d\theta \qquad \text{(II-3.6)}$$

These integrals can be evaluated by using Gaussian quadrature.

When the equation of the boundary is given, we can find the function $\rho_c(\theta)=r_c(\theta)/\ell$ for each point p_k (see Fig.II-7). However, the program would be more flexible if we could avoid determining the function $\rho_c(\theta)$ for each point of a given boundary. Thus, in the following, we present a method for computing the integrals (II-3.1),(II-3.2),(II-3.5) and (II-3.6) by using automatically the coordinates of the nodal and of the dividing points of the boundary, instead of using the equation of the

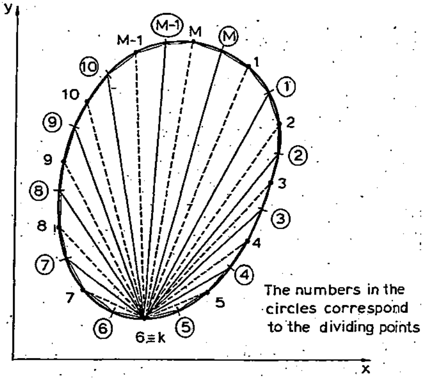

Fig.II-8.

boundary. This method is also valid when the boundary cannot be given by an equation. If we draw all the straight lines from point p_k to the other boundary points (see Fig.II-8), the area is divided into a finite number of sectors and the value of the integrals (II-3.1) and (II-3.2) for the whole area can be found as the sum of their values for each sector.

For the computation of the integrals (II-3.1),(II-3.2) as well as (II-3.5) and (II-3.6), the radial distance $r_c(\theta)$ is required. As shown below, the radial distance $r_c(\theta)$ and the limits of integration for each sector can be computed from the coordinates of the three vertices of the sector. Referring to Fig. II-9 , the equation of the line

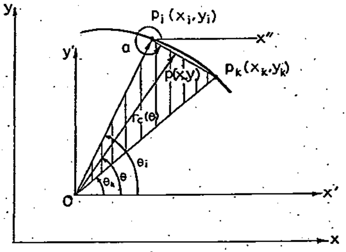

Fig.II-9.

$p_i p_k$ with respect to the axes ox'y' is

$$y'-(y_i-y_o)=\frac{\sin\alpha}{\cos\alpha}\left[x'-(x_i-x_o)\right]$$

or

$$y'\cos\alpha-x'\sin\alpha=(y_i-y_o)\cos\alpha-(x_i-x_o)\sin\alpha \qquad \text{(II-3.7)}$$

moreover,

$$x'=r_c\cos\theta, \quad y'=r_c\sin\theta \qquad \text{(II-3.8)}$$

substitution of (II-3.8) into (II-3.7) yields

$$r_c(\theta)=\frac{(y_i-y_o)\cos\alpha-(x_i-x_o)\sin\alpha}{\cos\alpha\sin\theta-\sin\alpha\cos\theta} \qquad \text{(II-3.9)}$$

The denominator becomes zero when $\alpha=\theta$ or $\alpha=\pi+\theta$. This implies that lines Op and $p_i p_k$ coincide. However, this possibility is excluded because the partial integral is zero in this case.

The angular limits of integration are the angles θ_k and θ_i which are evaluated from the direction cosines of the lines Op_k and Op_i.

The accuracy of the method presented in this investigation for the numerical evaluation of the improper integrals can be improved if the boundary is approximated by parabolic arcs (see Fig.II-10). In this case, the radial distance $r_c(\theta)$ can be evaluated as follows.

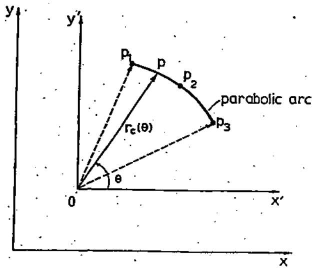

Fig.II-10. Approximation of the boundary elements by parabolic arcs.

The equation of the arc $p_1p_2p_3$ with respect to the axes ox'y' is

$$y'=\alpha_1 x'^2+\alpha_2 x'+\alpha_3 \qquad \text{(II-3.10)}$$

where

$$x'=r_c\cos\theta, \quad y'=r_c\sin\theta \qquad \text{(II-3.11)}$$

and the coefficient $\alpha_1,\alpha_2,\alpha_3$ are given by

$$\begin{bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{bmatrix}=\begin{bmatrix}x_1'^2 & x_1' & 1\\ x_2'^2 & x_2' & 1\\ x_3'^2 & x_3' & 1\end{bmatrix}^{-1}\begin{bmatrix}y_1'\\ y_2'\\ y_3'\end{bmatrix} \qquad \text{(II-3.12)}$$

where $x_i' = x_i - x_o$ $i=1,2,3$

Substitution of (II-3.11) into equation (II-3.10) yields

$$\alpha_1 \cos^2\theta r_c^2 + (\alpha_2\cos\theta - \sin\theta) r_c + \alpha_3 = 0$$

Assuming that $\alpha_1\cos^2\theta \neq 0$, we obtain

$$r_c(\theta) = \frac{-(\alpha_2\cos\theta-\sin\theta)+\sqrt{(\alpha_2\cos\theta-\sin\theta)^2-4\alpha_1\alpha_3\cos^2\theta}}{2\alpha_1\cos^2\theta} \qquad \text{(II-3.13)}$$

If $\alpha_1\cos^2\theta=0$ then

i) Either $\cos\theta=0$ and $\sin\theta\neq0$, thus $r_c(\theta)=\alpha_3/\sin\theta$.

ii)or $\alpha_1=0$, consequently $r_c(\theta)=-\alpha_3/(\alpha_2\cos\theta-\sin\theta)$.

Notice that, in this case, the parabola has degenerated into a straight line. The possibility $\alpha_2\cos\theta-\sin\theta=0$ is excluded since it implies that the radial direction r_c concides with the line $p_1p_2p_3$.

II-4. Evaluation of the deflections of the clamped plate

Subsequent to the computation of the coefficients a_{kj}, b_{kj}, c_{kj}, d_{kj} and the constants F_k, G_k, the system of the simultaneous equations (II-1.9) is solved and the values of the functions $\Phi(s)$ and $\Psi(s)$ at the nodal points are obtained. These values can be used to obtain the deflection $w(P)$ at any interior point P, as follows.

The deflection $w(P)$ is given by equation (I-4.41c), which may be written as

$$w(P)=\frac{\ell^2}{2\pi}(-I_1+I_2+I_3) \qquad (II-4.1)$$

where

$$I_1=\frac{1}{D}\iint_R f\,kei(\rho)d\sigma \qquad (II-4.2)$$

$$I_2=\int_C \psi\,kei(\rho)ds \qquad (II-4.3)$$

$$I_3=-\int_C \Phi\,kei'(\rho)d\theta \qquad (II-4.4)$$

The integrals (II-4.2),(II-4.3) and (II-4.4) can be approximated by the following sums

$$I_1=\frac{1}{D}\sum_{j=1}^{2M}\iint_j f\,kei(\rho)d\sigma \qquad (II-4.5)$$

$$I_2=\sum_{j=1}^{M}\Psi_j\int_j kei(\rho)ds \qquad (II-4.6)$$

$$I_3=-\sum_{j=1}^{M}\Phi_j\int_j \rho\,kei'(\rho)d\theta \qquad (II-4.7)$$

The integrals $\int_j kei(\rho)ds$ and $\int_j \rho kei'(\rho)d\theta$ are computed as discussed in Sections (II-2a) and (II-2b). The integral $\iint_j f kei(\rho)d\sigma$ is evaluated on the j-sector and computed as discussed in Section II-3. In the integrals (II-4.5) to (II-4.7), the integrand is not singular.

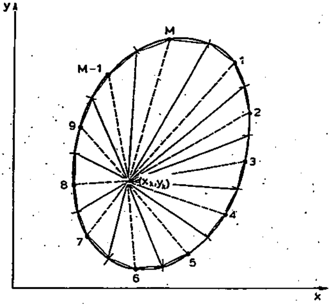

Fig.II-11. Sectors for the evaluation of $\iint_R \rho \operatorname{kei}(\rho)d\sigma$

II-5. Evaluation of the stress resultants for the clamped plate

The bending moments M_n, M_t, the twisting moment M_{nt} and the reaction V_n on the boundary are readily computed by using relations

$$M_n=-D\left[\nabla^2 w+(\nu-1)\left(\frac{\partial^2 w}{\partial s^2}+K\frac{\partial w}{\partial n}\right)\right]$$

$$M_t=-D\left[\nu\nabla^2 w-(\nu-1)\left(\frac{\partial^2 w}{\partial s^2}+K\frac{\partial w}{\partial n}\right)\right]$$

$$M_{nt}=D(1-\nu)\left[\frac{\partial^2 w}{\partial s\partial n}-K\frac{\partial w}{\partial s}\right] \qquad \text{(II-5.1)}$$

$$V_n=-D\left[\frac{\partial}{\partial n}(\nabla^2 w)-(\nu-1)\frac{\partial}{\partial s}\left(\frac{\partial^2 w}{\partial s\partial n}-K\frac{\partial w}{\partial s}\right)\right]$$

Noting that $\frac{\partial w}{\partial s}=\frac{\partial^2 w}{\partial s^2}=\frac{\partial w}{\partial n}=\frac{\partial^2 w}{\partial s\partial n}=0$ and using relations (I-4.26), the above relations become

$$M_n=-D\nabla^2 w=-D\Phi(s) \qquad \text{(II-5.2)}$$

$$M_t=-\nu D\nabla^2 w=\nu M_n \qquad \text{(II-5.3)}$$

$$M_{nt}=0 \qquad \text{(II-5.4)}$$

$$V_n=-D\frac{\partial\nabla^2 w}{\partial n}=-D\Psi(s) \qquad \text{(II-5.5)}$$

Thus, M_n,M_t and V_n are computed directly from the values of Φ and Ψ.

The bending moments M_x,M_y, the twisting moment M_{xy} and the shear forces Q_x and Q_y at any point of the plate are evaluated from the relations

$$M_x=-D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right) \qquad \text{(II-5.6)}$$

$$M_y=-D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right) \qquad \text{(II-5.7)}$$

$$M_{xy}=-M_{yx}=D(1-\nu)\frac{\partial^2 w}{\partial x\partial y} \qquad \text{(II-5.8)}$$

$$Q_x=-D\frac{\partial}{\partial x}\nabla^2 w \qquad \text{(II-5.9)}$$

$$Q_y=-D\frac{\partial}{\partial y}\nabla^2 w \qquad \text{(II-5.10)}$$

As can be seen, it is necessary to evaluate the second and third order partial derivatives of the deflection expression (I-4.41c). One advantage of the Boundary Integral method is that the evaluation of these derivatives can be done analytically.

Instead of evaluating the derivatives $\frac{\partial^2 w}{\partial x^2}$ and $\frac{\partial^2 w}{\partial y^2}$ directly, it is more convenient to first evaluate their sum and their difference.

We introduce the following notation

$$d_{11}=\frac{\partial^2 w}{\partial x^2},\quad d_{22}=\frac{\partial^2 w}{\partial y^2},\quad d_{12}=\frac{\partial^2 w}{\partial x\partial y}$$

$$c_1=\frac{\partial}{\partial x}\nabla^2 w\,,\quad c_2=\frac{\partial}{\partial y}\nabla^2 w \qquad \text{(II-5.11)}$$

From equation (I-4.41c) we obtain

$$d_{11}+d_{22}=\nabla^2 w=\iint_R f\nabla^2 v\,d\sigma-D\int_C[\Psi\nabla^2 v-\Phi\frac{\partial}{\partial n}(\nabla^2 v)]ds \qquad \text{(II-5.12)}$$

$$d_{11}-d_{22}=\iint_R f(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 v}{\partial y^2})d\sigma-D\int_C[\Psi(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 v}{\partial y^2})-\Phi\frac{\partial}{\partial n}(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 v}{\partial y^2})]ds$$

$$\text{(II-5.13)}$$

$$d_{12}=\iint_R f\frac{\partial^2 v}{\partial x\partial y}d\sigma-D\int_C\left[\Psi\frac{\partial^2 v}{\partial x\partial y}-\Phi\frac{\partial}{\partial n}\left(\frac{\partial^2 v}{\partial x\partial y}\right)\right]ds \qquad \text{(II-5.14)}$$

$$c_1=\iint_R f\frac{\partial}{\partial x}\nabla^2 v d\sigma-D\int_C\left[\Psi\frac{\partial}{\partial x}\nabla^2 v-\Phi\frac{\partial}{\partial n}\left(\frac{\partial}{\partial x}\nabla^2 v\right)\right] ds \qquad \text{(II-5.15)}$$

$$c_2=\iint_R f\frac{\partial}{\partial y}\nabla^2 v d\sigma-D\int_C\left[\Psi\frac{\partial}{\partial y}\nabla^2 v-\Phi\frac{\partial}{\partial n}\left(\frac{\partial}{\partial y}\nabla^2 v\right)\right] ds \qquad \text{(II-5.16)}$$

Using the appropriate relations from Appendix A, the above expressions can be written as

$$d_{11}+d_{22}=\frac{1}{2\pi}\left[-\frac{1}{D}\iint_R fK_1(\rho)d\sigma+\int_C\Psi K_1(\rho)ds-\frac{1}{\ell}\int_C\Phi\Lambda_1(\rho)ds\right] \qquad \text{(II-5.17)}$$

$$d_{11}-d_{22}=\frac{1}{2\pi}\left[-\frac{1}{D}\iint_R fK_2(\rho)d\sigma+\int_C\Psi K_2(\rho)ds-\frac{1}{\ell}\int_C\Phi\Lambda_2(\rho)ds\right] \qquad \text{(II-5.18)}$$

$$d_{12}=\frac{1}{4\pi}\left[-\frac{1}{D}\iint_R fK_3(\rho)d\sigma+\int_C\Psi K_3(\rho)ds-\frac{1}{\ell}\int_C\Phi\Lambda_3(\rho)ds\right] \qquad \text{(II-5.19)}$$

$$c_1=\frac{1}{2\pi\ell}\left[\frac{1}{D}\iint_R fK_4(\rho)d\sigma-\int_C\Psi K_4(\rho)ds-\frac{1}{\ell}\int_C\Phi\Lambda_4(\rho)ds\right] \qquad \text{(II-5.20)}$$

$$c_2=\frac{1}{2\pi\ell}\left[\frac{1}{D}\iint_R fK_5(\rho)d\sigma-\int_C\Psi K_5(\rho)ds-\frac{1}{\ell}\int_C\Phi\Lambda_5(\rho)ds\right] \qquad \text{(II-5.21)}$$

where it has been set

$K_1(\rho)=\ker(\rho)$

$A_1(\rho)=\ker'(\rho)\cos\varphi$

$K_2(\rho)=C(\rho)\cos 2\alpha$

$$A_2(\rho)=\ker'(\rho)\cos 2\alpha\cos\varphi-\frac{2}{\rho}C(\rho)\cos(2\alpha-\varphi) \qquad \text{(II-5.22)}$$

$K_3(\rho)=C(\rho)\sin 2\alpha$

$A_3(\rho)=\ker'(\rho)\sin 2\alpha\cos\varphi-\frac{2}{\rho}C(\rho)\sin(2\alpha-\varphi)$

$K_4(\rho)=\ker'(\rho)\cos\alpha$

$A_4(\rho)=\frac{1}{\rho}\ker'(\rho)\cos(\alpha-\varphi)+\ker''(\rho)\cos\alpha\cos\varphi$

$K_5(\rho)=\ker'(\rho)\sin\alpha$

$A_5(\rho)=\frac{1}{\rho}\ker'(\rho)\sin(\alpha-\varphi)+\ker''(\rho)\sin\alpha\sin\varphi$

$C(\rho)=\ker''(\rho)-\frac{2}{\rho}\ker'(\rho)$

Inasmuch as point P, where the quantities (II-5.17) to (II-5.21) are computed, is inside the region R, the argument ρ does not vanish and the kernels (II-5.22) are not singular. Thus, their numerical computation is not difficult. The integral expressions are approximated by the following sums

$$I_i(P)=-\frac{1}{D}\sum_{j=1}^{2M}\iint_j fK_i(\rho)d\sigma+\sum_{j=1}^{M}\Psi_j\int_j K_i(\rho)ds-\frac{1}{\ell}\sum_{j=1}^{M}\Phi_j\int_j A_i(\rho)ds$$

$$(i=1,2,3,4,5) \qquad \text{(II-5.23)}$$

where $\iint_j fK_i(\rho)d\sigma$ is the double integral on the j sector and can be evaluated as discussed in Section II-3. $\int_j K_i(\rho)ds$ and $\int_j \Delta_i(\rho)ds$ are the line integrals extended over the j interval. They can be evaluated by applying Simpson's rule. When the integrals $I_i(P)$ are computed, the derivatives will be given as

$$\begin{aligned} d_{11}+d_{22}&=I_1(P)/2\pi \\ d_{11}-d_{22}&=I_2(P)/2\pi \\ d_{12}&=I_3(P)/4\pi \\ c_1&=I_4(P)/2\pi\ell \\ c_2&=I_5(P)/2\pi\ell \end{aligned} \qquad \text{(II-5.24)}$$

II-6. Approximation of the integral equations for the simply supported plate by a system of simultaneous linear algebraic equations

Equations (I-4.42a,b) may be written as

$$\int_C (v\Psi+MvX)ds=\frac{1}{D}\iint_R vfd\sigma$$

$$\frac{1}{2D}X+\int_C \frac{\partial v}{\partial n_p}\Psi ds+\int_C \frac{\partial}{\partial n_p}MvXds=\frac{1}{D}\iint_R \frac{\partial v}{\partial n_p}fd\sigma \qquad \text{(II-6.1)}$$

where

$$v=-\frac{\ell^2}{2\pi D}\mathrm{kei}(\rho),\quad \rho=r/\ell,\quad r=|q-p| \qquad \text{(II-6.2)}$$

Introducing equation (II-6.2) into equations (II-6.1), and using relations (A-36),(A-17),(A-39),(A-41) with (A-8) and (A-49) we obtain

$$\int_C \Psi \mathrm{kei}(\rho)ds+\int_C X\left[\frac{1}{\ell^2}\mathrm{ker}(\rho)+\frac{\nu-1}{\ell}K(s)\mathrm{kei}'(\rho)\cos\varphi\right]ds=\frac{1}{D}\iint_R f\,\mathrm{kei}(\rho)d\sigma$$

$$\pi X+\int_C X\left[\frac{1}{\ell}\mathrm{ker}'(\rho)\cos\omega+(\nu-1)K(s)\left[\mathrm{ker}(\rho)\cos\omega\cos\varphi-\frac{1}{\rho}\mathrm{kei}'(\rho)\cos(\varphi+\omega)\right]\right]ds$$

$$+\ell\int_C \Psi \mathrm{kei}'(\rho)\cos\omega ds=\frac{\ell}{D}\iint_R f\,\mathrm{kei}'(\rho)\cos\omega d\sigma \qquad \text{(II-6.3)}$$

where $\omega=\widehat{r_{pq},n_p}$ (II-6.4)

The numerical integration of equations (II-6.3) is performed by applying the procedure described in Section II-1. The boundary is divided into M intervals, not necessarily equal and numbered, consecutively, clockwise. The values of Ψ and X are assumed constant on each interval and equal to their values at the nodes of each interval. Denoting by Ψ_j and X_j the values of Ψ and X of the j^{th} node (that is the node of the j-interval) the integral equations (II-6.3) are transformed into the following system of 2M simultaneous algebraic equations

$$\sum_{j=1}^{M} a_{kj}\Psi_j+\sum_{j=1}^{M} b_{kj}X_j=F_k \qquad (k=1,2,\ldots M)$$

$$\text{(II-6.5)}$$

$$\sum_{j=1}^{M} c_{kj}\Psi_j+\sum_{j=1}^{M} (d_{kj}+\pi\delta_{kj})X_j=H_k \qquad (k=1,2,\ldots M)$$

where δ_{kj} is the Kronecker delta and

$$a_{kj}=\int_j kei(\rho_{kj})ds$$

$$b_{kj}=\frac{1}{\ell^2}\int_j ker(\rho_{kj})ds+\frac{\nu-1}{\ell}\int_c Kkei'(\rho_{kj})\cos\varphi ds$$

$$c_{kj}=\ell\int_j kei'(\rho_{kj})\cos\omega ds$$

$$d_{kj}=\frac{1}{\ell}\int_j ker'(\rho_{kj})\cos\omega ds+(\nu-1)\int_j Kker(\rho_{kj})\cos\omega\cos\varphi ds$$

$$-(\nu-1)\int_j K\frac{1}{\rho_{kj}}kei'(\rho_{kj})\cos(\varphi+\omega)ds$$

(II-6.6a,b,c,d,e,f)

$$F_k=\frac{1}{D}\iint_R f(Q)kei(\rho_{kQ})d\sigma$$

$$H_k=\frac{\ell}{D}\iint_R f(Q)kei'(\rho_{kQ})\cos\omega d\sigma$$

where $\rho_{kQ}=\frac{r_{rQ}}{\ell}=\frac{|Q-p_k|}{\ell}$, $Q\epsilon R$, $p_k\epsilon \underline{C}$ (see Fig.II-2)

In the above relations, the symbol $\int_j$ indicates a line integral on the j-interval.

The system of equations (II-6.5) may be written in matrix form as

$$AY=B \qquad (II\text{-}6.7)$$

where

$$Y=\begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_M \\ \hline X_1 \\ X_2 \\ \vdots \\ X_M \end{bmatrix} \qquad B=\begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_M \\ \hline H_1 \\ H_2 \\ \vdots \\ H_M \end{bmatrix}$$

(II-6.8a,b,c)

$$A=\left[\begin{array}{cccc|cccc} a_{11} & a_{12} & \cdots & a_{1M} & b_{11} & b_{12} & \cdots & b_{1M} \\ a_{21} & a_{22} & \cdots & a_{2M} & b_{21} & b_{22} & \cdots & b_{2M} \\ \vdots & & & \vdots & \vdots & & & \vdots \\ a_{M1} & a_{M2} & \cdots & a_{MM} & b_{M1} & b_{M2} & \cdots & b_{MM} \\ \hline c_{11} & c_{12} & \cdots & c_{1M} & d_{11}+\pi & d_{12} & \cdots & d_{1M} \\ c_{21} & c_{22} & \cdots & c_{2M} & d_{21} & d_{22}+\pi & \cdots & d_{2M} \\ \vdots & & & \vdots & \vdots & & & \vdots \\ c_{M1} & c_{M2} & \cdots & c_{MM} & d_{M1} & d_{M2} & \cdots & d_{MM}+\pi \end{array}\right]$$

II-7. Evaluation of the coefficients a_{kj}, b_{kj}, c_{kj}, d_{kj} for the simply supported plate

In what follows, the same notation is employed as in

Section II-2.

a) Evaluation of the coefficients a_{kj}

The evaluation of the integral (II-6.6a) is given in Section II-2b.

b) Evaluation of the coefficients b_{kj}

Equation (II-6.6b) may be written as

$$b_{kj}=\frac{1}{\ell^2}b'_{kj}+(\nu-1)b''_{kj} \qquad \text{(II-7.1)}$$

where

$$b'_{kj}=\int_j \mathrm{ker}(\rho_{kj})ds$$

and

$$b''_{kj}=\int_j K(s)\rho_{kj}\mathrm{kei}'(\rho_{kj})d\theta$$

The numerical technique for the computation of the integral b'_{kj} is described in Section II-2d. Moreover, noting that $\lim_{\rho\to 0}[\rho \mathrm{kei}'(\rho)]=0$, the integral b''_{kj} can be evaluated by employing a procedure analogous to that used in Section II-2a for the evaluation of the integral a_{kj}. Thus, we have:

Case i $k\neq j$

For $\theta_1,\theta_2\neq 0$ we can apply Simpons rule (II-2.1) by setting

$$\begin{aligned} h_1&=\theta_1, \quad h_2=\theta_2 \\ g_1&=K_1\rho_1\mathrm{kei}'(\rho_1) \\ g_0&=K_0\rho_0\mathrm{kei}'(\rho_0) \\ g_2&=K_2\rho_0\mathrm{kei}'(\rho_2) \end{aligned} \qquad \text{(II-7.2)}$$

where K_1, K_0, K_2 are the values of the curvature at the points q_{j-1}, p_j and q_j (see Fig.II-4), respectively.

For θ_1 or $\theta_2 = 0$

$$b''_{kj} = \frac{1}{2} s_i(g_i + g_0), \quad \text{when } \theta_{3-i} = 0, \quad i=1,2 \qquad \text{(II-7.3)}$$

and

$$b''_{kj} = 0, \quad \text{when} \quad \theta_1 = \theta_2 = 0 \qquad \text{(II-7.4)}$$

Case ii k=j

In this case, the integral can be computed by Simpson's rule (II-2.1) setting

$$\begin{aligned} h_1 &= \theta_1, \quad h_2 = \theta_2 \\ g_1 &= K_1 \rho_1 \mathrm{kei}'(\rho_1) \\ g_0 &= 0 \\ g_2 &= K_2 \rho_2 \mathrm{kei}'(\rho_2) \end{aligned} \qquad \text{(II-7.5)}$$

Notice that if one of the angles θ_1, θ_1 is zero, then

$$b''_{kk} = 0 \qquad \text{(II-7.6)}$$

c) Evaluation of the coefficients c_{kj}

Case i k≠j

We use Simpson's rule (II-2.1) setting

$$\begin{aligned} h_1 &= s_1, \quad h_2 = s_2 \\ g_1 &= \ell \,\mathrm{kei}'(\rho_1) \cos\omega_1 \\ g_0 &= \ell \,\mathrm{kei}'(\rho_0) \cos\omega_0 \\ g_2 &= \ell \,\mathrm{kei}'(\rho_2) \cos\omega_2 \end{aligned} \qquad \text{(II-7.7)}$$

Case ii $k=j$

Since $\lim_{\rho\to 0}[\mathrm{kei}'(\rho)]=0$, we can use Simpson's rule with $g_0=0$.

d) Evaluation of the coefficients d_{kj}.

Equation (II-6.6d) may be written as

$$d_{kj}=d'_{kj}+(\nu-1)d''_{kj}-(\nu-1)d'''_{kj}$$

where

$$d'_{kj}=\frac{1}{\ell}\int_j \mathrm{ker}'(\rho_{kj})\cos\omega ds$$

$$d''_{kj}=\int_j K(s)\mathrm{ker}(\rho_{kj})\cos\omega\cos\varphi ds \qquad \text{(II-7.8)}$$

$$d'''_{kj}=\int_j K(s)\frac{1}{\rho_{kj}}\mathrm{ker}'(\rho_{kj})\cos(\varphi+\omega)ds$$

For $k\neq j$, the above integrals can be easily evaluated using Simpson's formula (II-2.1). For the evaluation of the integral d'_{kj} ($k\neq j$) we set

$$g_1=\frac{1}{\ell}\mathrm{ker}'(\rho_1)\cos\omega_1$$

$$g_0=\frac{1}{\ell}\mathrm{ker}'(\rho_0)\cos\omega_0 \qquad \text{(II-7.9)}$$

$$g_2=\frac{1}{\ell}\mathrm{ker}'(\rho_0)\cos\omega_2$$

For the evaluation of the integral d''_{kj} ($k\neq j$), we set

$$g_1=K_1\mathrm{ker}(\rho_1)\cos\omega_1\cos\varphi_1$$

$$g_0=K_0\mathrm{ker}(\rho_0)\cos\omega_0\cos\varphi_0 \qquad \text{(II-7.10)}$$

$$g_2=K_2\mathrm{ker}(\rho_2)\cos\omega_2\cos\varphi_2$$

For the evaluation of the integral d'''_{kj}, we set

$$h_1=s_1, \quad h_2=s_2$$

$$g_1=K_1 \frac{1}{\rho_1}\text{kei}'(\rho_1)\cos(\omega_1+\varphi_1)$$

$$g_0=K_0 \frac{1}{\rho_0}\text{kei}'(\rho_0)\cos(\omega_0+\varphi_0) \qquad \text{(II-7.11)}$$

$$g_2=K_2 \frac{1}{\rho_2}\text{kei}'(\rho_2)\cos(\omega_2+\varphi_2)$$

For k=j, $(\rho\to0)$ and the integrals (II-7.8) require special treatment. The integral d'_{kj} for j=k may be written as

$$d'_{kk}= \frac{1}{\ell}\int_k \left[\text{ker}'(\rho)+ \frac{1}{\rho}\right]\cos\omega- \frac{1}{\ell}\int_k \frac{\cos\omega}{\rho}ds \qquad \text{(II-7.12)}$$

From equation (I-4.13) we conclude that

$$\lim_{\rho\to0}\left[\text{ker}'(\rho)+ \frac{1}{\rho}\right]=0 \qquad \text{(II-7.13)}$$

Hence, for the case $\rho=0$ the first integral in equation (II-7.12) can be evaluated by using Simpson's rule with

$$h_1=s_1, \quad h_2=s_2$$

$$g_1= \frac{1}{\ell}\left[\text{ker}'(\rho_1)+ \frac{1}{\rho_1}\right]\cos\omega_1 \qquad \text{(II-7.14)}$$

$$g_0=0$$

$$g_2= \frac{1}{\ell}\left[\text{ker}'(\rho_2)+ \frac{1}{\rho_2}\right]\cos\omega_2$$

Moreover, referring to Fig.II-12 we have

$$\cos\omega = -\sin\theta$$

Thus, the second integral in equation (II-7.12) may be written as

$$-\frac{1}{\ell}\int_k \frac{\cos\omega}{\rho}ds = \int_k \frac{\sin\theta}{r}ds \qquad \text{(II-7.15)}$$

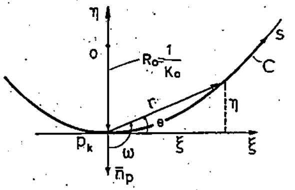

Fig. II-12. The boundary C and the ξ,η local system of axes

In order to use Simpson's rule for the evaluation of the above integral, the following limit must be evaluated

$$\lim_{r\to 0}\frac{\sin\theta}{r} \qquad \text{(II-7.16)}$$

In order to evaluate the above limit, we introduce the coordinate system (ξ,η) [52] having point p_k as its origin [see Fig.II-12], where ξ is the tangential and η is the radial direction at p_k.

The equation of the boundary $\eta=\eta(\xi)$ in the neighbourhood of p_k can be expanded in a Taylor series. Thus,

$$\eta=\alpha_0+\alpha_1\xi+\alpha_2\xi^2+\ldots \qquad \text{(II-7.17)}$$

Noting that $\alpha_0=0$, and that the boundary curve at p_k is symmetric, only even terms will be retained in the above expansion. Thus, we have

$$\eta=\sum_{n=1}^{\infty}\alpha_{2n}\xi^{2n}$$

Since ξ is small, we neglect the terms of order higher than the second and write

$$\eta=\alpha_2\xi^2$$

If K_0 is the curvature at point p_k, we may write

$$\eta=\frac{1}{2}K_0\xi^2 \qquad \text{(II-7.18)}$$

and

$$r^2=\xi^2+\eta^2=\xi^2(1+K_0^2\xi^2/4)$$

$$\sin\theta=\frac{\eta}{r}=\frac{1}{2}\frac{K_0}{\sqrt{1+K_0^2\xi^2/4}}\xi \qquad \text{(II-7.19)}$$

hence,

$$\lim_{r\to 0}\frac{\sin\theta}{r}=K_0/2 \qquad \text{(II-7.20)}$$

Thus , for the case $\rho=0$, the second integral in equation (I-7.12) can be evaluated by using Simpson's rule with

$$h_1=s_1, \quad h_2=s_2$$

$$g_1=-\frac{1}{\ell}\frac{\cos\omega_1}{\rho_1}$$

$$g_0=K_0/2 \qquad \text{(II-7.21)}$$

$$g_2=-\frac{1}{\ell}\frac{\cos\omega_2}{\rho_2}$$

Consider the intagrand of the integral d''_{kj} for the case k=j. Inasmuch as the curvature K is a finite quantity for a smooth boundary, in the limit as $\rho\to0$, we have

$$\lim_{\rho\to0}[K\ker(\rho)\cos\omega\cos\varphi]=K\cos\varphi\lim_{\rho\to0}[\ker(\rho)\cos\omega] \qquad \text{(II-7.22)}$$

The behaviour of $\ker(\rho)$ for small values of the argument is analogous to $\ln(\rho)$ [see equation (I-3.14c)]. Thus, referring to Fig.II-12 relation (II-7.22) becomes

$$\lim_{\rho\to0}[K\ker(\rho)\cos\omega\cos\varphi]=-K\cos\varphi\lim_{\rho\to0}[\sin\theta\ln(r)]$$

Substituting equation (II-7.19) into the above, we obtain

$$\lim_{\rho\to0}[K\ker(\rho)\cos\omega\cos\varphi]=-K\cos\varphi\lim_{\xi\to0}\{\frac{1}{2}\frac{K_0}{\sqrt{1+K_0^2\xi^2/4}}\xi[\ln\xi+\frac{1}{2}\ln(1+K_0^2\xi^2/4)]\}=0$$

(II-7.23)

Thus, for k=j the integral d''_{kj} can be evaluated using Simpson's rule with

$$g_1 = K_1 \ker(\rho_1)\cos\omega_1\cos\varphi_1$$

$$g_0 = 0 \qquad \text{(II-7.24)}$$

$$g_2 = K_2 \ker(\rho_2)\cos\omega_2\cos\varphi_2$$

Finally, for the evaluation of the integral d'''_{kj} in the case k=j, we consider the behaviour of the function kei'(ρ)/ρ as ρ→0. From equation (I-4.12) we have

$$\frac{1}{\rho}\text{kei}'(\rho) = -\ell n(\rho/2)[1/2 - 10.66666(\rho/8)^4 + \ldots]$$
$$- \frac{1}{\rho^2}[16(\rho/8)^2 - 113.77777(\rho/8)^6 + \ldots] \qquad \text{(II-7.25)}$$
$$- \frac{1}{4}\pi[-4(\rho/8)^2 + 14.2222(\rho/8)^6 - \ldots]$$
$$+ [.2113217 - 13.39658(\rho/8)^4 + \ldots]$$

It is apparent that for small values of ρ the function $\frac{1}{\rho}\text{kei}'(\rho)$ behaves like ℓn(ρ). This suggest that for j=k we write the integral d'''_{kk} as

$$d'''_{kk} = \int_k [K\frac{1}{\rho}\text{kei}'(\rho)\cos(\omega+\varphi) + \frac{1}{2}K_0 \ell n(\rho/2)\cos(\varphi_0+\omega_0)]ds$$
$$\qquad \text{(II-7.26)}$$
$$-\frac{1}{2}K_0\cos(\varphi_0+\omega_0)\int_k \ell n(\rho/2)ds$$

Notice that

$$\omega_0 = \lim_{\rho\to 0} \omega = \pm\pi/2$$

$$\varphi_0 = \lim_{\rho\to 0} \varphi = \mp\pi/2 \qquad \text{(II-7.27)}$$

$$\cos(\omega_0+\varphi_0) = -1$$

Moreover, from equation (II-7.25) we have

$$\lim_{\rho\to 0}\left[\frac{1}{\rho}\text{kei}'(\rho) + \frac{1}{2}\ln(\rho/2)\right] = -0.03860783 \qquad \text{(II-7.28)}$$

Thus, the first integral in equation (II-7.26) can be evaluated using Simpson's rule (II-2.1) with

$$h_1 = s_1, \quad h_2 = s_2$$

$$g_1 = K_1\,\frac{1}{\rho_1}\text{kei}'(\rho_1)\cos(\omega_1+\varphi_1) - \frac{1}{2}K_0\ln(\rho_1/2)$$

$$g_0 = 0.03860783K_0$$

$$g_2 = K_2\,\frac{1}{\rho_2}\text{kei}'(\rho_2)\cos(\omega_2+\varphi_2) - \frac{1}{2}K_0\ln(\rho_2/2)$$

The second integral in equation (II-7.26) is approximated by using equation (II-2.21). Thus,

$$\frac{1}{2}K_0\int_k \ln(\rho/2)ds = \frac{1}{2}K_0\left[\int_k \ln(\rho)ds - \ln 2\int_k ds\right]$$

$$= \frac{1}{2}K_0\{r_1[\ln(\rho_1)-1] + r_2[\ln(\rho_2)-1] - \ln 2(r_1+r_2)\}$$

$$\text{(II-7.29)}$$

II-8. Evaluation of F_k and H_k for the simply supported plate

The evaluation of the integral F_k is given in Section (II-3). Thus, in this Section, we present a numerical procedure only for the evaluation of the integral H_k. For the reasons stated in Section (II-3), we use polar coordinates with point p_k as the origin, and the tangent line to the boundary at this point as the reference axis (see Fig.II-13).

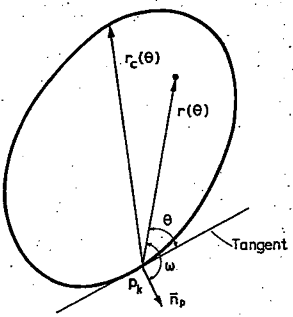

Fig. II-13. Polar coordinates for the function kei'(ρ)cosω

From Fig.II-13, we have $\cos\omega=-\sin\theta$ and thus, the integral (II-6.6f) may be written as

$$H_k=-\frac{\ell}{D}\iint_R f^*(r,\theta)\,\mathrm{kei}'(\rho)\,r\sin\theta\,dr\,d\theta \qquad \text{(II-8.1)}$$

where

$$r=r(\theta) \quad \text{and} \quad \rho=r(\theta)/\ell$$

Since $\lim_{\rho\to 0}[\rho \text{kei}'(\rho)]=0$, the integrand in the integral (II-8.1) is not singular and, thus, it can be integrated using iterated integration with Gaussian quadrature. Thus, we have

$$H_k=-\frac{\ell}{D}\int_0^{\pi}\int_0^{r_c(\theta)} f^*(r,\theta)\text{kei}'(\rho)r\sin\theta\, dr\, d\theta \qquad \text{(II-8.2)}$$

If the loading function is a constant f_0 the integral (II-8.2) can be further simplified

$$H_k=-\frac{f_0\ell^3}{D}\int_0^{\pi}\left[\int_0^{\rho_c}\rho\,\text{kei}'(\rho)d\rho\right]\sin\theta\, d\theta \qquad \text{(II-8.3)}$$

II-9. Evaluation of the deflections of the simply supported plate

The deflection w(P) is given by equation (I-4.42c) which may be written as

$$w(P)=\frac{\ell^2}{2\pi}(-I_1+I_2+I_3+I_4) \qquad \text{(II-9.1)}$$

where

$$I_1=\frac{1}{D}\iint_R f\,\text{kei}(\rho)d\sigma \qquad \text{(II-9.2)}$$

$$I_2=\int_C \Psi\,\text{kei}(\rho)ds \qquad \text{(II-9.3)}$$

$$I_3=\frac{1}{\ell^2}\int_C X\,\text{ker}(\rho)ds \qquad \text{(II-9.4)}$$

$$I_4=(\nu-1)\int_C \mathcal{X}K\text{kei}'(\rho)\rho d\theta \qquad \text{(II-9.5)}$$

The integrals (II-9.3) to (II-9.5) are approximated by the following sums

$$I_2=\sum_{j=1}^{M}\Psi_j\int_j \text{kei}(\rho)ds \qquad \text{(II-9.6)}$$

$$I_3=\frac{1}{\ell^2}\sum_{j=1}^{M}X_j\int_j \text{ker}(\rho)ds \qquad \text{(II-9.7)}$$

$$I_4=(\nu-1)\sum_{j=1}^{M}X_j\int_j K\text{kei}'(\rho)\rho d\theta \qquad \text{(II-9.8)}$$

The numerical procedure for the evaluation of the integral (II-9.2) is given in section II-4, while the numerical procedures for the evaluation of the integrals (II-9.6), (II-9.7) and (II-9.8) are given in sections II-2 and II-7. In the above line integrals, the integrand is not singular because point P lies inside the region R and the distance r_{pq} does not vanish.

The values Ψ_j and X_j of the functions Ψ and X at the nodal points are obtained by solving the system of simultaneous equations (II-6.7) after the coefficients a_{kj}, b_{kj}, c_{kj}, d_{kj} and the constants F_k, H_k have been computed.

II-10. Evaluation of the stress resultants for the simply supported plate

The bending moment M_t, the twisting moment M_{nt}, and the reaction on the boundary are easily computed by using the relations (II-5.1). Noting that $w=\frac{\partial w}{\partial s}=\frac{\partial^2 w}{\partial s^2}=0$ and $M_n=0$ and using relations (I-4.26), relations (II-5.1) yield

$$M_t = -D(1-\nu^2)KX$$

$$M_{nt}=D(1-\nu)\frac{\partial X}{\partial s} \qquad \text{(II-10.1)}$$

$$V_n=-D\left[\Psi-(\nu-1)\frac{\partial^2 X}{\partial s^2}\right]$$

Thus, the bending moment M_t is computed directly from the values of X. The derivatives $\frac{\partial X}{\partial s}$ and $\frac{\partial^2 X}{\partial s^2}$ can be computed either by numerical differentiation with respect to the arc length, using the values of X at the nodal points of the boundary, or by differentiating equation (II-4.40a) with respect to s. That is,

$$\frac{\partial X}{\partial s}=2\iint_R \frac{\partial^2 v}{\partial s \partial n_p} f d\sigma - 2D\int_R \left[\frac{\partial^2 v}{\partial s \partial n_p}\Psi + \frac{\partial^2}{\partial s \partial n_p} Mv\right] ds \qquad \text{(II-10.2)}$$

$$\frac{\partial^2 X}{\partial s^2}=2\iint_R \frac{\partial^3 v}{\partial s^2 \partial n_p} f d\sigma - 2D\int_R \left[\frac{\partial^3 v}{\partial s^2 \partial n_p}\Psi + \frac{\partial^3}{\partial s^2 \partial n_p} Mv\right] ds \qquad \text{(II-10.3)}$$

Equations (II-10.2) and (II-10.3) yield more satisfactory results because they do not require numerical differentiation.

Moreover, using these equations, the derivatives of X can be evaluated at points of the boundary which are not nodal points.

The bending moments M_x, M_y, the twisting moment M_{xy}, and the shear forces Q_x and Q_y are evaluated from relations (II-5.6) to (II-5.10).

As in Section II-5, instead of evaluating directly the derivatives $\frac{\partial^2 w}{\partial x^2}$, $\frac{\partial^2 w}{\partial y^2}$ it is more convenient to evaluate their sum and their difference. Thus, using notation (II-5.11) we have

$$d_{11}+d_{22}=\nabla^2 w=\iint_R f\nabla^2 v d\sigma - D\int_C \left[\Psi\nabla^2 v+\nabla^2 MvX\right]ds \qquad \text{(II-10.4)}$$

$$d_{11}-d_{22}=\iint_R \left(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 v}{\partial y^2}\right)d\sigma - D\int_C \left[\Psi\left(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 v}{\partial y^2}\right)+\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)MvX\right]ds \qquad \text{(II-10.5)}$$

$$d_{12}=\iint_R f\frac{\partial^2 v}{\partial x\partial y}d\sigma - D\int_C \left[\Psi\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 Mv}{\partial x\partial y}X\right]ds \qquad \text{(II-10.6)}$$

$$C_1=\iint_R f\frac{\partial}{\partial x}\nabla^2 v d\sigma - D\int_C \left(\Psi\frac{\partial}{\partial x}\nabla^2 v+\frac{\partial}{\partial x}\nabla^2 MvX\right)ds \qquad \text{(II-10.7)}$$

$$C_2=\iint_R f\frac{\partial}{\partial y}\nabla^2 v d\sigma - D\int_C \left(\Psi\frac{\partial}{\partial y}\nabla^2 v+\frac{\partial}{\partial y}\nabla^2 MvX\right)ds \qquad \text{(II-10.8)}$$

Using the appropriate relations from Appendix A, the above expressions can be written as

$$d_{11}+d_{22}=\frac{1}{2\pi}\left[-\frac{1}{D}\iint_R fK_1(\rho)d\sigma+\int_C \Psi K_1(\rho)-\frac{1}{\ell^2}\int_C XN_1(\rho)ds+\frac{\nu-1}{\ell}\int_C KX\Lambda_1(\rho)ds\right]$$

(II-10.9)

$$d_{11}-d_{22}=\frac{1}{2\pi}\left[-\frac{1}{D}\iint_R fK_2(\rho)d\sigma+\int_C \Psi K_2(\rho)ds-\frac{1}{\ell^2}\int_C XN_2(\rho)ds+\right.$$

$$\left.+\frac{\nu-1}{\ell}\int_C KX\Lambda_2(\rho)ds\right] \quad \text{(II-10.10)}$$

$$d_{12}=\frac{1}{4\pi}\left[-\frac{1}{D}\iint_R fK_3(\rho)d\sigma+\int_C \Psi K_3(\rho)ds-\frac{1}{\ell^2}\int_C XN_3(\rho)ds+\frac{\nu-1}{\ell}\int_C KX\Lambda_3(\rho)ds\right]$$

(II-10.11)

$$C_1=\frac{1}{2\pi\ell}\left[\frac{1}{D}\iint_R fK_4(\rho)d\sigma-\int_C \Psi K_4(\rho)ds+\frac{1}{\ell^2}\int_C XN_4(\rho)ds-\frac{\nu-1}{\ell}\int_C KX\Lambda_4(\rho)ds\right]$$

(II-10.12)

$$C_2=\frac{1}{2\pi\ell}\left[\frac{1}{D}\iint_R fK_5(\rho)d\sigma-\int_C \Psi K_5(\rho)ds+\frac{1}{\ell^2}\int_C XN_5(\rho)-\frac{\nu-1}{\ell}\int_C KX\Lambda_5(\rho)ds\right]$$

(II-10.13)

where the kernels $K_i(\rho)$, $\Lambda_i(\rho)$ $(i=1,\ldots5)$ are given by equations (II-5.22) and

$$\begin{aligned}
N_1(\rho)&=\text{kei}(\rho)\\
N_2(\rho)&=B(\rho)\cos 2\alpha\\
N_3(\rho)&=B(\rho)\sin 2\alpha\\
N_4(\rho)&=\text{kei}'(\rho)\cos\alpha\\
N_5(\rho)&=\text{kei}'(\rho)\sin\alpha\\
B(\rho)&=\text{kei}(\rho)+\frac{2}{\rho}\text{kei}'(\rho)
\end{aligned} \quad \text{(II-10.14)}$$

The numerical procedure for the evaluation of the integrals (II-10.9) to (II-10.13) is the same as that presented in Section II-5.

PART III

NUMERICAL RESULTS

III-1. Introduction

Computer programs have been written for the numerical evaluation of the response of clamped and simply supported plates by integrating the BIE derived in Part I, using the numerical technique developed in Part II. Numerical results have been obtained for circular plates, rectangular plates and a plate with composite shape using a CDC/CYBER 171-8 computer. Whenever possible, the results are compared with those obtained from analytical solutions. It should be mentioned, that analytical results exist only for plates of simple geometry, subjected to simple loading. For rectangular plates, the results are also compared with those available for rectangular plates without elastic foundation by giving small values to the constant of the elastic foundation.

III-2. Dimensionless parameters for the circular plate

a. Circular plate under a concentrated load P at point Q_0 (x_0, x_0):

In this case, the differential equation for the deflection of the plate (I-1.1) is

$$\nabla^4 w + \frac{k}{D} w = \frac{P}{D} \delta(Q-Q_0), \quad Q:(x,y), Q_0(x_0,y_0) \qquad \text{(III-2.1)}$$

Denoting by a the radius of the circular plate, introducing the dimensionless variables

$$\xi = \frac{x}{a}, \qquad \eta = \frac{y}{a} \qquad \text{(III-2.2)}$$

and noting that [90]

$$\delta(Q-Q_0) = \frac{1}{a^2}\,\delta(\tilde{Q}-\tilde{Q}_0)$$

relation (III-2.1) becomes

$$\nabla^4\bar{w} + \lambda^4\bar{w} = \delta(\tilde{Q}-\beta) \qquad \text{(III-2.3)}$$

where

$$\lambda = \frac{a}{\ell}, \quad \beta = \frac{r_0}{a} = \frac{x_0}{a}, \quad \bar{w} = \frac{w}{Pa^2/D}, \quad \tilde{Q}:(\xi,\eta) \qquad \text{(III-2.4)}$$

Referring to the integral equations (II-1.3), it is apparent that for clamped plates the quantity $\bar{w}$ does not depend on Poisson's ratio ν. However, from the integral equations (II-6.3) we conclude that for simply supported plates, when the curvature of the boundary is not zero, their non dismensionless deflection $\bar{w}$ depends on Poisson's ratio. Thus, from equation (III-2.3) it is apparent that for clamped circular plates the dimensionless deflection $\bar{w}$ depends only on the dimensionless parameters λ and β, while for simply supported circular plates it depends on λ,β and ν. The parameter $\lambda=a/\ell=a/\sqrt[4]{D/k}$ includes all the geometrical and mechanical properties of the plate and the mechanical property of the subgrade,while the parameter β characterises the load position.

Thus, for the clamped circular plate, we define the following dimensionless quantities:

$$C_1(\lambda,\beta) = \frac{w}{Pa^2/D}$$

$$C_2(\lambda,\beta) = \frac{M_n}{P} \qquad \text{(III-2,5a,b,c)}$$

$$C_3(\lambda,\beta) = V_n \frac{a}{P}$$

where M_n and V_n are, respectively, the bending moment normal to the boundary and the reaction on the boundary of the plate. The quantities C_1, C_2 and C_3 are tabulated in Table I for various values of λ and β.

For the circular, simply supported plate, we define the following dimensionless quantities:

$$C_4(\lambda,\beta,\nu) = \frac{w}{Pa^2/D}$$

$$C_5(\lambda,\beta,\nu) = \frac{M_t}{P} \qquad \text{(III-2.6 a,b,c)}$$

$$C_6(\lambda,\beta,\nu) = V_n \frac{a}{P}$$

where M_t and V_n are the bending moment along the boundary and the reaction on the boundary, respectively. The quantities C_4, C_5 and C_6 are tabulated in Table II for various values of λ and β for $\nu=0.3$.

To establish the influence fields for the bending moments M_x and M_y, equation (I-5.1) is converted to a dimensionless form. For instance, the influence field for the bending moment M_x is obtained from the following equation.

$$\nabla^4 w^* + \frac{k}{D} w^* = -\left(\frac{\partial^2}{\partial x^2} + \nu \frac{\partial}{\partial y^2}\right)\delta(Q-Q_o)$$

Using the dimensionless coordinates (III-2.2),we obtain

$$\nabla^4 \bar{w}^* + \lambda^4 \bar{w}^* = -\left(\frac{\partial^2}{\partial \xi^2} + \nu \frac{\partial^2}{\partial \eta^2}\right)\delta(\tilde{Q}-\tilde{Q}_o) \qquad \text{(III-2.7)}$$

From equation (III-2.7),we conclude that for circular clamped plates

$$\bar{w}^* = \bar{w}^*(\lambda,\alpha,\beta)$$

and for circular simply supported plates

$$\bar{w}^* = \bar{w}^*(\nu,\lambda,\alpha,\beta)$$

where α denotes the dimensionless position of the bending moment M_x.

The ifluence coefficients for the bending moments M_r and M_t are tabulated in Table III for the clamped plate, and in Table IV for the simply supported plate. Notice, that for the clamped plate the influence coefficients are given for ν=o. This allows us, to establish the influence fields of M_r and M_t for any of the values of ν. However, for the simply supported plate, the influence coefficients are given only for ν=o.3.

b. Circular plates under a distributed load:

Using the dimensionless variables (III-2.2), the differential equation (I-1.1) may be converted to the following dimensionless form

$$\nabla^4\bar{w}+\lambda^4\bar{w}=\varphi_0(\xi,\eta) \qquad \text{(III-2.8)}$$

where it has been set

$$\varphi_0 = \frac{f(\xi,\eta)}{f_0} \qquad \text{(III-2.9)}$$

f_0 being a constant with dimensions of force per unit area and

$$\bar{w}= \frac{w}{f_0 a^4/D} \qquad \text{(III-2.10)}$$

The dimensionless deflection $\bar{w}$ defined by equation (III-2.10) for a specified dimensionless loading φ_0 depends only on the parameter λ for a clamped circular plate and on λ and ν for a circular simply supported plate.

The dimensionless bending moments and the reactions are defined as:

$$d_1 = \frac{M_n}{f_0 a^2}$$

$$d_2 = \frac{V_n}{f_0 a} \qquad \text{(III-2.11)}$$

For the clamped plate,the quantities d_1 and d_2 depend only on the parameter λ, while for the simply supported plate, they depend on λ and ν.

III-3. Dimensionless parameters for the rectangular plate

a. Rectangular plate under a concentrated load P.

For a rectangular plate with 2ax2b dimensions,we choose as dimensionless coordinates

$$-1 \leq \xi = \frac{x}{a} \leq 1, \qquad -\varepsilon \leq \eta = \frac{y}{a} \leq \varepsilon, \qquad \varepsilon = \frac{b}{a} \qquad \text{(III-3.1)}$$

Using these coordinates, the differential equation (III-2.1) takes the following dimensionless form:

$$\nabla^4 \bar{w} + \lambda^4 \bar{w} = \delta(\tilde{Q} - \tilde{Q}_0) \qquad \text{(III-3.2)}$$

where

$$\lambda = \frac{a}{\ell}, \qquad \bar{w} = \frac{w}{Pa^2/D}, \qquad \tilde{Q}:(\xi,\eta), \qquad \tilde{Q}_0:(\xi_0,\eta_0) \qquad \text{(III-3.3)}$$

From equations (III-3.1) and (III-3.2), it is apparent that the dimensionless deflection $\bar{w}$ for a specified load position depends on both the parameter λ and the side ratio $\varepsilon = b/a$ Inasmuch as the curvature of the boundary is zero, $\bar{w}$ does not depend on Poisson's ratio ν even for simply supported plates. The dimensionless bending moments and reacting forces are those defined for circular plates (see equations III-2.5 b,c).

The influence coefficients for bending moments, reactions, and deflections at some characteristic points are tabulated for various values of the parameters λ and ε, and for various dimensionless load positions for clamped plates in Table V, and for simply supported plates in Table VI.

b. Rectangular plate under distributed load:

In this case, the dimensionless quantities defined for the circular plates are used. However, in this case these quantities also depend on the side ratio ε.

III-4. Accuracy of the method and some numerical results

In this section, some numerical results obtained for the axisymmetric loading of the circular plate are compared with those obtained by the existing analytic solution [64].

In Table III-1, the values of the dimensionless deflection $\bar{w}=w(Pa^2/D)$ for a clamped and a simply supported circular plate, subjected to a centrally applied concentrated load, are presented as obtained from the BIE method with 32 boundary nodal points, and from the analytical solution [64].

Table III-1 Deflections of a clamped and a simply supported plate subjected to a centrally applied concentrated load.

$\lambda=1$	Clamped		Simply Supported	
	$\bar{w}=w/(Pa^2/D)$		$\bar{w}=w/(Pa^2/D)$	
r/a	BIE (m=32)	Analytic [64]	BIE (m=32)	Analytic [64]
0	.1972-01	.1973-01	.48688-01	.48689-01
0.2	.1638-01	.1639-01	.44203-01	.44203-01
0.4	.1076-01	.1077-01	.35146-01	.35146-01
0.6	.5351-02	.5357-02	.23978-01	.23977-01
0.8	.1461-02	.1462-02	.11973-01	.11971-01

In Fig. III-1, the percent error in the numerical results for the deflection at points r=0 and r=0.5a and the bending moment

M_n at points r=a of a clamped circular plate, resting on elastic foundation (λ=1), subjected to a centrally applied concentrated load, are plotted versus the number of boundary nodal points.

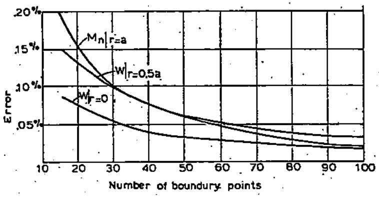

Fig. III-1. Percent error of the deflection $\bar{w}$ at r=o, r=0.5a and the bending moment M_n at r=a of a clamped circular plate on elastic foundation (λ=1) subjected to a concentrated load at its center.

In Fig. III-2 , the percent error in the numerical results for the deflection at r=o and r=0.6a, and the reactive force V_n at r=a of a simply supported plate on elastic foundation (λ=1), subjected to a centrally applied concentrated force at its center, is plotted versus the number of the boundary nodal points.

From Figs. III-1 and III-2, it is apparent that the error is very small. Only few nodal points (30 to 40) on the boundary are sufficient to obtain accurate results. The error increases as the computed deflection approaches the boundary.

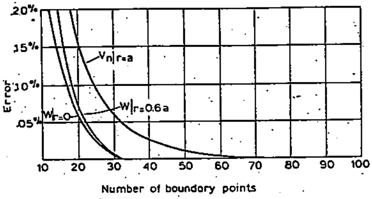

Fig. III-2. Percent error of the deflection at r=o, and r=0.6a and the reactive force of a simply supported circular plate on elastic foundation (λ=1) subjected to a concentrated force at its center.

In this case, it may be necessary to increase the number of the nodal points on the boundary.

It is apparent, that as the constant of the elastic foundation decreases, the deflection obtained for a plate on elastic foundation will approach that of a free plate (i.e. not resting on elastic foundation). This is illustrated in Tables III-2 and III-3

Table III-2. Deflections of a clamped circular plate subjected to a uniformly distributed load q_0

r/a	$\bar{w}_{max} = w_{max}/(q_0 a^4/D)$	
	BIE (m=32) λ=.671	Analytic [64] λ=0
0	.1555-01	.1563-01
.2	.1430-01	.1440-01
.4	.1093-01	.1103-01
.6	.6321-02	.6400-02
.8	.1959-02	.2026-02

Table III-3. Deflections of x=y=0 of simply supported rectangular plate centrally loaded by a concentrated load at its center

b/a	$\bar{w}_{max} = w_{max}/(Pa^2/D)$	
	BIE (m=44) λ=0.212	Analytic(*) λ=0
1.0	0.1160-01	0.1160-01
1.2	0.1355-01	0.1353-01
1.4	0.1486-01	0.1484-01
1.6	0.1569-01	0.1570-01
1.8	0.1620-01	0.1620-01
2.0	0.1650-01	0.1651-01

(*) The analytic solution is obtained from p. 143 of Ref. [73]

In Fig. III-3, the results obtained by the BIE method and those of an analytical solution presented in reference [65] are plotted. These results are in excellent agreement.

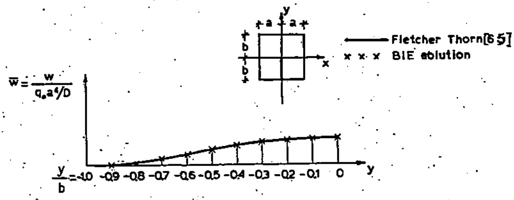

Fig. III-3. Deflection along x=0 of a uniformly loaded clamped square plate on elastic foundation with $\varepsilon=1$, $\lambda=1.1$

In Fig. III-4, the distribution of the deflection, the bending moment M_n and the reactive force V_n along the boundary of a uniformly loaded clamped rectangular (b/a=2) plate for a small value of the elastic constant ($\lambda=0.671$) is presented. The corresponding maxw [Ref. [73] p. 202) is 0.4064×10^{-1} ($\lambda=0$)].

In Fig. III-5, the distribution of the deflections along the x and y axes and of the bending moments M_n, and reactive forces V_n along the boundary of a clamped plate of composite shape, resting on an elastic foundation are presented.

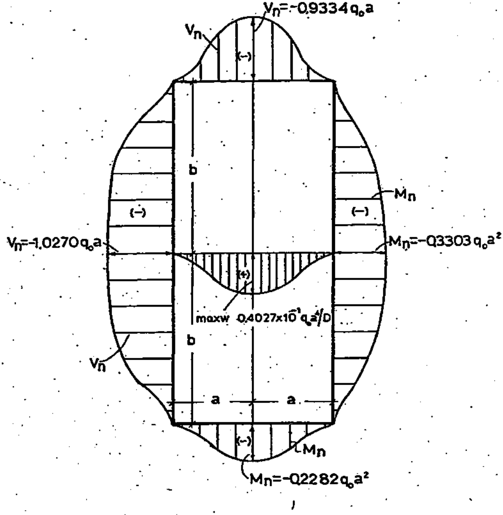

Fig. III-4. Bending moment M_n, reactive force V_n along the boundary and deflection w along y=0 of a uniformly loaded clamped rectangular (ε=2) plate on elastic foundation (λ=0.671)

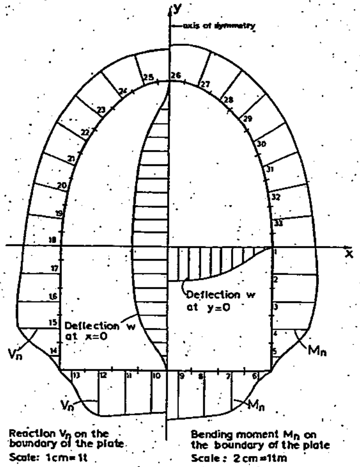

Fig. III-5. Uniformly loaded clamped plate of composite shape resting on elastic foundation k=500, h=.10m, $E=2.1\times10^6$ t/m², $\nu=0.30$, $q_o=.2$t/m²

III-5. Tables for circular and rectangular clamped and simply supported plates resting on elastic foundation.

The constant of the elastic foundation may vary between 0 (free plate) and 20000 t/m³ [73]. Thus, for usual engineering applications, the dimensionless parameter $\lambda (= a/\ell = a/\sqrt[4]{D/k})$ varies between 0.5 (soft subgrade) and 20 (stiff subgrade). For a plate not resting on an elastic foundation it is k=0, and thus, $\lambda=0$. However, this value of *k* results in computational difficulties and, consequently, results for plates not resting on an elastic foundation are obtained using a small value of k (say k=1).(*)

In order to determine for which values of λ should tables for the quantities C_1, C_2, C_3 (defined by equation III-2.5) be given, these quantities are plotted in Fig. III-6. From this figure we see that the change of these quantities is negligible for $0<\lambda<1$ and very small for $\lambda>11$. Thus, the tables will be presented for $\lambda=1,3,5,7,11$.

In Fig. III-7, the bending moment M_x at x=a, y=0 and the deflection at x=0, y=0, as well as the bending moment M_y at x=0, y=b have been plotted versus the side ratio $\varepsilon=b/a$ of a clamped rectangular plate on elastic foundation ($\lambda=2$), subjected to a unit concentrated force at x=y=0. From this Fig., it can be seen that for values of $\varepsilon=b/a>1.8$ the quantities w, M_x and M_y approach the corresponding values of an infinitely long plate clamped at the two (a=constant) opposite edges. Thus, the values of b/a chosen in the tables are $\varepsilon=b/a=1.0$, 1.2, 1.4, 1.6 and 1.8.

(*) For a plate with a=2.5m, h=0.10m, $E=2.1\times10^6 t/m^2$, $\nu=0.3$, and k=0.1 it is $\lambda=0.38$. For the same plate with k=0.01 it is $\lambda=0.21$.

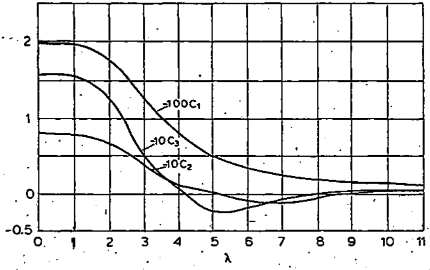

Fig. III-6. Dimensionless parameters C_1,C_2, and C_3 versus λ

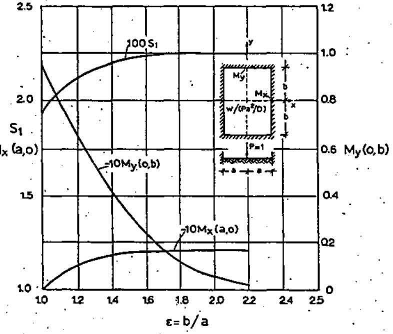

Fig. III-7. Dimensionless parameter S_1,$M_x(a,0)$ and $M_y(a,b)$ versus side ratio ε of a clamped rectangular plate resting on elastic foundation (λ=2).

In Table I, the deflection $c_1=wD/Pa^2$; at points $r=\alpha a$ ($\alpha=0,0.2,0.4,0.6,0.8$) on the radii $\theta=0^o,22.5^o,45^o,67.5^o,90^o$, $112.5^o,135^o,157.5^o,180^o$, the moment $C_2=M_n/P$ and the reacting force $C_3=V_na/P$at points # 1 to # 9 [$\theta=0^o,(22.5^o),180^o$] of the boundary are given for a clamped, circular plate resting on an elastic foundation ($\lambda=1,3,5,7,11$), subjected to a concentrated load P, at points $r=\beta a$($\beta=0,0.2,0.4,0.6,0.8$) of the radius $\theta=0^o$. (See Fig. III-8).

In Table II, the deflection $C_4=wD/Pa^2$, the bending moment $C_5=M_t/P$, and the reactive force $C_6=V_n a/P$for the same points as in Table II, are given for a circular, simply supported plate resting on an elastic foundation ($\lambda=1,3,5,7,11$), subjected to a concentrated load P at points $r=\beta a$ ($\beta=0,0.2,0.4,0.6,0.8$) of the radius $\theta=0^o$ (see Fig. III-8). Notice, that in this case, the results are given for $\nu=0.3$.

In Table III, the influence coefficients for the bending moments M_r and M_t at points $r=\beta a$ ($\beta=0,0.2,0.4,0.6,0.8$) for a clamped, circular plate with λ = 1,3,5,7,11 and $\nu=0$ are given. The values of the influence coefficients have been computed at points $r=\alpha a$ ($\alpha=0,0.2,0.4,0.6,0.8$) of the radii $\theta=0^o,(22.5^o),180^o$.

In Table IV, the influence coefficients of the bending moments M_r and M_t are given for the same values of β, α, and λ as in Table III, for a simply supported circular plate with $\nu=0.3$

In Table V, the influence coefficients for the deflection $S_1=wD/Pa^2$, and the influence coefficients for the bending moments M_x and M_y at x=y=0, as well as the reaction $S_3=V_x a/P$ and the

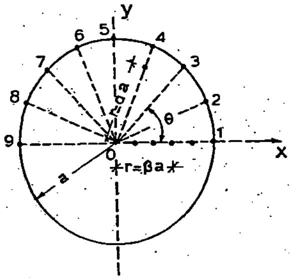

Fig. III-8

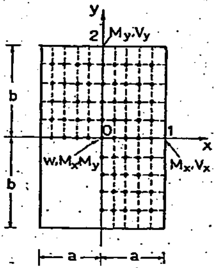

Fig. III-9. Mesh of the points where the influence coefficients are computed bending moment M_x at x=a, y=o, the reaction $S_2=V_y \frac{a}{P}$ and the bending moment M_y at x=o, y=b are given for a clamped rectangular plate having ν=0.3 side ratio ε=b/a =1,1.2,1.4,1.6, 1.8 and resting on an elastic foundation (λ= 1,3,5,7,11). The values of the influence coefficients have been computed at points x/a,y/b=0,±0.2,±0.4,±0.6,±0.8(see Fig. III-9).

Finally, in Table VI, the influence coefficients of corresponding quantities, and for the same values of the parameters as in Table V, are given for a rectangular, simply supported plate resting on an elastic foundation.

TABLE I

Clamped Circular plate							λ = 1
β	θ° \ α	$c_1 = wD/Pa^2$ 0	0.2	0.4	0.6	0.8	$c_2 = M_n/P$	$c_3 = V_n a/P$
0	0	.1973E-01	.1639E-01	.1077E-01	.5354E-02	.1461E-02	-.7866E-01	-.1567E+00
	22.5	.1973E-01	.1639E-01	.1077E-01	.5354E-02	.1461E-02	-.7866E-01	-.1567E+00
	45.	.1973E-01	.1639E-01	.1077E-01	.5354E-02	.1461E-02	-.7866E-01	-.1567E+00
	67.5	.1973E-01	.1639E-01	.1077E-01	.5354E-02	.1461E-02	-.7866E-01	-.1567E+00
	90.	.1973E-01	.1639E-01	.1077E-01	.5354E-02	.1461E-02	-.7866E-01	-.1567E+00
	112.5	.1973E-01	.1639E-01	.1077E-01	.5354E-02	.1461E-02	-.7866E-01	-.1567E+00
	135.	.1973E-01	.1639E-01	.1077E-01	.5354E-02	.1461E-02	-.7866E-01	-.1567E+00
	157.5	.1973E-01	.1639E-01	.1077E-01	.5354E-02	.1461E-02	-.7866E-01	-.1567E+00
	180.	.1973E-01	.1639E-01	.1077E-01	.5354E-02	.1461E-02	-.7866E-01	-.1567E+00
0.2	0	.1639E-01	.1819E-01	.1351E-01	.7142E-02	.2037E-02	-.1137E+00	-.2839E+00
	22.5	.1639E-01	.1758E-01	.1303E-01	.6870E-02	.1952E-02	-.1085E+00	-.2634E+00
	45.	.1639E-01	.1647E-01	.1191E-01	.6209E-02	.1745E-02	-.9596E-01	-.2171E+00
	67.5	.1639E-01	.1527E-01	.1064E-01	.5443E-02	.1507E-02	-.8182E-01	-.1698E+00
	90.	.1639E-01	.1418E-01	.9512E-02	.4761E-02	.1299E-02	-.6968E-01	-.1334E+00
	112.5	.1639E-01	.1328E-01	.8627E-02	.4237E-02	.1141E-02	-.6066E-01	-.1088E+00
	135.	.1639E-01	.1264E-01	.8009E-02	.3878E-02	.1035E-02	-.5465E-01	-.9366E-01
	157.5	.1639E-01	.1225E-01	.7647E-02	.3671E-02	.9742E-03	-.5126E-01	-.8551E-01
	180.	.1639E-01	.1212E-01	.7528E-02	.3603E-02	.9545E-03	-.5016E-01	-.8295E-01
0.4	0	.1077E-01	.1351E-01	.1395E-01	.8520E-02	.2622E-02	-.1552E+00	-.5181E+00
	22.5	.1077E-01	.1303E-01	.1230E-01	.7474E-02	.2281E-02	-.1327E+00	-.3973E+00
	45.	.1077E-01	.1191E-01	.9958E-02	.5687E-02	.1673E-02	-.9378E-01	-.2257E+00
	67.5	.1077E-01	.1064E-01	.7985E-02	.4261E-02	.1199E-02	-.6511E-01	-.1285E+00
	90.	.1077E-01	.9512E-02	.6547E-02	.3309E-02	.8957E-03	-.4780E-01	-.8179E-01
	112.5	.1077E-01	.8627E-02	.5572E-02	.2710E-02	.7200E-03	-.3772E-01	-.5876E-01
	135.	.1077E-01	.8008E-02	.4955E-02	.2351E-02	.6157E-03	-.3199E-01	-.4702E-01
	157.5	.1077E-01	.7646E-02	.4616E-02	.2160E-02	.5613E-03	-.2903E-01	-.4135E-01
	180.	.1077E-01	.7527E-02	.4508E-02	.2100E-02	.5445E-03	-.2812E-01	-.3966E-01
0.6	0.	.5352E-02	.7140E-02	.6518E-02	.8117E-02	.3053E-02	-.2034E+00	-.1022E+01
	22.5	.5352E-02	.6868E-02	.7473E-02	.5787E-02	.2074E-02	-.1293E+00	-.4569E+00
	45.	.5352E-02	.6207E-02	.5686E-02	.3591E-02	.1117E-02	-.6334E-01	-.1418E+00
	67.5	.5352E-02	.5441E-02	.4260E-02	.2346E-02	.6654E-03	-.3583E-01	-.6075E-01
	90.	.5352E-02	.4760E-02	.3308E-02	.1673E-02	.4507E-03	-.2364E-01	-.3433E-01
	112.5	.5352E-02	.4236E-02	.2709E-02	.1301E-02	.3406E-03	-.1761E-01	-.2344E-01
	135.	.5352E-02	.3876E-02	.2350E-02	.1095E-02	.2822E-03	-.1446E-01	-.1833E-01
	157.5	.5352E-02	.3669E-02	.2159E-02	.9902E-03	.2531E-03	-.1292E-01	-.1597E-01
	180.	.5352E-02	.3602E-02	.2099E-02	.9580E-03	.2443E-03	-.1245E-01	-.1527E-01
0.8	0	.1458E-02	.2033E-02	.2618E-02	.3050E-02	.2575E-02	-.2591E+00	-.2621E+01
	22.5	.1458E-02	.1948E-02	.2277E-02	.2070E-02	.9320E-03	-.6361E-01	-.2009E+00
	45.	.1458E-02	.1742E-02	.1669E-02	.1115E-02	.3577E-03	-.2019E-01	-.3403E-01
	67.5	.1458E-02	.1504E-02	.1196E-02	.6639E-03	.1863E-03	-.9934E-02	-.1317E-01
	90.	.1458E-02	.1296E-02	.8974E-03	.4495E-03	.1190E-03	-.6285E-02	-.7371E-02
	112.5	.1458E-02	.1139E-02	.7182E-03	.3398E-03	.8748E-04	-.4479E-02	-.5064E-02
	135.	.1458E-02	.1032E-02	.6141E-03	.2815E-03	.7146E-04	-.3627E-02	-.3952E-02
	157.5	.1458E-02	.9717E-03	.5599E-03	.2525E-03	.6368E-04	-.3217E-02	-.3496E-02
	180.	.1458E-02	.9521E-03	.5431E-03	.2437E-03	.6124E-04	-.3094E-02	-.3350E-02

Clamped Circular plate						$\lambda = 3$	
β	$c_1 = wD/Pa^2$					$c_2 = M_n/P$	$c_3 = V_n a/P$
	$\theta°$ \ α: 0	0.2	0.4	0.6	0.8		

β	$\theta°$	0	0.2	0.4	0.6	0.8	$c_2 = M_n/P$	$c_3 = V_n a/P$
0	0	.1250E-01	.9831E-02	.5969E-02	.2752E-02	.7052E-03	-.3612E-01	-.4922E-01
	22.5	.1250E-01	.9831E-02	.5969E-02	.2752E-02	.7052E-03	-.3612E-01	-.4922E-01
	45	.1250E-01	.9831E-02	.5969E-02	.2752E-02	.7052E-03	-.3612E-01	-.4922E-01
	67.5	.1250E-01	.9831E-02	.5969E-02	.2752E-02	.7052E-03	-.3612E-01	-.4922E-01
	90	.1250E-01	.9831E-02	.5969E-02	.2752E-02	.7052E-03	-.3612E-01	-.4922E-01
	112.5	.1250E-01	.9831E-02	.5969E-02	.2752E-02	.7052E-03	-.3612E-01	-.4922E-01
	135	.1250E-01	.9831E-02	.5969E-02	.2752E-02	.7052E-03	-.3612E-01	-.4922E-01
	157.5	.1250E-01	.9831E-02	.5969E-02	.2752E-02	.7052E-03	-.3612E-01	-.4922E-01
	180	.1250E-01	.9831E-02	.5969E-02	.2752E-02	.7052E-03	-.3612E-01	-.4922E-01
0.2	0	.9831E-02	.1193E-01	.8700E-02	.4433E-02	.1226E-02	-.6700E-01	-.1528E+00
	22.5	.9831E-02	.1135E-01	.8261E-02	.4192E-02	.1153E-02	-.6258E-01	-.1358E+00
	45	.9831E-02	.1031E-01	.7249E-02	.3616E-02	.9773E-03	-.5211E-01	-.9860E-01
	67.5	.9831E-02	.9209E-02	.6131E-02	.2964E-02	.7807E-03	-.4064E-01	-.6258E-01
	90	.9831E-02	.8230E-02	.5161E-02	.2404E-02	.6147E-03	-.3121E-01	-.3702E-01
	112.5	.9831E-02	.7448E-02	.4423E-02	.1987E-02	.4942E-03	-.2452E-01	-.2138E-01
	135	.9831E-02	.6890E-02	.3920E-02	.1710E-02	.4158E-03	-.2025E-01	-.1262E-01
	157.5	.9831E-02	.6559E-02	.3631E-02	.1554E-02	.3724E-03	-.1792E-01	-.8271E-02
	180	.9831E-02	.6449E-02	.3537E-02	.1504E-02	.3586E-03	-.1719E-01	-.6965E-02
0.4	0	.5969E-02	.8700E-02	.1006E-01	.6216E-02	.1905E-02	-.1128E+00	-.3829E+00
	22.5	.5969E-02	.8261E-02	.8484E-02	.5234E-02	.1590E-02	-.9199E-01	-.2720E+00
	45	.5969E-02	.7249E-02	.6317E-02	.3604E-02	.1043E-02	-.5729E-01	-.1213E+00
	67.5	.5969E-02	.6131E-02	.4574E-02	.2365E-02	.6393E-03	-.3323E-01	-.4458E-01
	90	.5969E-02	.5161E-02	.3365E-02	.1588E-02	.4021E-03	-.1988E-01	-.1357E-01
	112.5	.5969E-02	.4423E-02	.2586E-02	.1130E-02	.2706E-03	-.1281E-01	-.1413E-02
	135	.5969E-02	.3920E-02	.2113E-02	.8720E-03	.1997E-03	-.9121E-02	.3386E-02
	157.5	.5969E-02	.3631E-02	.1862E-02	.7408E-03	.1648E-03	-.7345E-02	.5217E-02
	180	.5969E-02	.3537E-02	.1784E-02	.7007E-03	.1543E-03	-.6816E-02	.5691E-02
0.6	0	.2752E-02	.4432E-02	.6216E-02	.6674E-02	.2580E-02	-.1742E+00	-.9133E+00
	22.5	.2752E-02	.4192E-02	.5234E-02	.4412E-02	.1632E-02	-.1025E+00	-.3655E+00
	45	.2752E-02	.3615E-02	.3603E-02	.2368E-02	.7411E-03	-.4123E-01	-.7614E-01
	67.5	.2752E-02	.2964E-02	.2365E-02	.1284E-02	.3510E-03	-.1785E-01	-.1373E-01
	90	.2752E-02	.2403E-02	.1588E-02	.7462E-03	.1841E-03	-.8715E-02	.9358E-03
	112.5	.2752E-02	.1987E-02	.1130E-02	.4754E-03	.1080E-03	-.4791E-02	.4632E-02
	135	.2752E-02	.1710E-02	.8719E-03	.3376E-03	.7173E-04	-.2997E-02	.5482E-02
	157.5	.2752E-02	.1554E-02	.7408E-03	.2720E-03	.5516E-04	-.2199E-02	.5602E-02
	180	.2752E-02	.1504E-02	.7007E-03	.2526E-03	.5035E-04	-.1971E-02	.5598E-02
0.8	0	.7047E-03	.1225E-02	.1904E-02	.2579E-02	.2411E-02	-.2481E+00	-.2571E+01
	22.5	.7047E-03	.1152E-02	.1589E-02	.1632E-02	.7861E-03	-.5436E-01	-.1666E+00
	45	.7047E-03	.9767E-03	.1042E-02	.7407E-03	.2418E-03	-.1320E-01	-.1308E-01
	67.5	.7047E-03	.7802E-03	.6389E-03	.3508E-03	.9416E-04	-.4586E-02	.5050E-03
	90	.7047E-03	.6143E-03	.4018E-03	.1840E-03	.4340E-04	-.1930E-02	.2313E-02
	112.5	.7047E-03	.4939E-03	.2704E-03	.1079E-03	.2287E-04	-.9250E-03	.2369E-02
	135	.7047E-03	.4155E-03	.1996E-03	.7168E-04	.1379E-04	-.5023E-03	.2178E-02
	157.5	.7047E-03	.3721E-03	.1647E-03	.5512E-04	.9841E-05	-.3228E-03	.2027E-02
	180	.7047E-03	.3583E-03	.1542E-03	.5031E-04	.8720E-05	-.2730E-03	.1974E-02

Clamped Circular plate							$\lambda = 5$
β		$c_1 = wD/Pa^2$					$c_2 = M_n/P$	$c_3 = V_n a/P$
	$\theta°$ \ α	0	0.2	0.4	0.6	0.8		
0	0	.4989E-02	.3143E-02	.1291E-02	.3495E-03	.4026E-04	-.1306E-03	.2513E-01
	22.5	.4989E-02	.3143E-02	.1291E-02	.3495E-03	.4026E-04	-.1306E-03	.2513E-01
	45	.4989E-02	.3143E-02	.1291E-02	.3495E-03	.4026E-04	-.1306E-03	.2513E-01
	67.5	.4989E-02	.3143E-02	.1291E-02	.3495E-03	.4026E-04	-.1306E-03	.2513E-01
	90	.4989E-02	.3143E-02	.1291E-02	.3495E-03	.4026E-04	-.1306E-03	.2513E-01
	112.5	.4989E-02	.3143E-02	.1291E-02	.3495E-03	.4026E-04	-.1306E-03	.2513E-01
	135	.4989E-02	.3143E-02	.1291E-02	.3495E-03	.4026E-04	-.1306E-03	.2513E-01
	157.5	.4989E-02	.3143E-02	.1291E-02	.3495E-03	.4026E-04	-.1306E-03	.2513E-01
	180	.4989E-02	.3143E-02	.1291E-02	.3495E-03	.4026E-04	-.1306E-03	.2513E-01
0.2	0	.3143E-02	.4983E-02	.3116E-02	.1230E-02	.2619E-03	-.1142E-01	.7967E-02
	22.5	.3143E-02	.4484E-02	.2808E-02	.1093E-02	.2267E-03	-.9502E-02	.1333E-01
	45	.3143E-02	.3672E-02	.2142E-02	.7866E-03	.1489E-03	-.5389E-02	.2242E-01
	67.5	.3143E-02	.2896E-02	.1485E-02	.4827E-03	.7467E-04	-.1712E-02	.2644E-01
	90	.3143E-02	.2269E-02	.9887E-03	.2637E-03	.2505E-04	.5053E-03	.2497E-01
	112.5	.3143E-02	.1812E-02	.6619E-03	.1309E-03	-.1838E-05	.1528E-02	.2131E-01
	135	.3143E-02	.1511E-02	.4670E-03	.5913E-04	-.1443E-04	.1895E-02	.1790E-01
	157.5	.3143E-02	.1341E-02	.3662E-03	.2517E-04	-.1953E-04	.1990E-02	.1571E-01
	180	.3143E-02	.1287E-02	.3353E-03	.1530E-04	-.2086E-04	.2004E-02	.1497E-01
0.4	0	.1291E-02	.3116E-02	.4862E-02	.2828E-02	.7833E-03	-.4362E-01	-.1248E+00
	22.5	.1291E-02	.2808E-02	.3539E-02	.2088E-02	.5686E-03	-.3006E-01	-.5543E-01
	45	.1291E-02	.2142E-02	.1995E-02	.1018E-02	.2439E-03	-.1074E-01	.1572E-01
	67.5	.1291E-02	.1485E-02	.9903E-03	.3832E-03	.6440E-04	-.1324E-02	.2853E-01
	90	.1291E-02	.9887E-03	.4474E-03	.1008E-03	-.2412E-05	.1476E-02	.2135E-01
	112.5	.1291E-02	.6619E-03	.1824E-03	-.6543E-05	-.2080E-04	.1869E-02	.1348E-01
	135	.1291E-02	.4670E-03	.6150E-04	-.4214E-04	-.2336E-04	.1673E-02	.8528E-02
	157.5	.1291E-02	.3662E-03	.1121E-04	-.5205E-04	-.2238E-04	.1463E-02	.6046E-02
	180	.1291E-02	.3353E-03	-.2362E-05	-.5388E-04	-.2176E-04	.1385E-02	.5309E-02
0.6	0	.3495E-03	.1230E-02	.2828E-02	.4137E-02	.1636E-02	-.1111E+00	-.6156E+00
	22.5	.3495E-03	.1093E-02	.2087E-02	.2168E-02	.8345E-03	-.5127E-01	-.1544E+00
	45	.3495E-03	.7866E-03	.1018E-02	.7127E-03	.2059E-03	-.9339E-02	.2244E-01
	67.5	.3495E-03	.4827E-03	.3832E-03	.1626E-03	.2306E-04	.2568E-03	.2423E-01
	90	.3495E-03	.2636E-03	.1008E-03	-.1037E-05	-.1458E-04	.1475E-02	.1254E-01
	112.5	.3495E-03	.1309E-03	-.6545E-05	-.3829E-04	-.1721E-04	.1186E-02	.5667E-02
	135	.3495E-03	.5912E-04	-.4215E-04	-.4120E-04	-.1393E-04	.8262E-03	.2505E-02
	157.5	.3495E-03	.2516E-04	-.5205E-04	-.3804E-04	-.1130E-04	.6154E-03	.1217E-02
	180	.3495E-03	.1530E-04	-.5388E-04	-.3649E-04	-.1039E-04	.5494E-03	.8744E-03
0.8	0	.4024E-04	.2618E-03	.7832E-03	.1635E-02	.2008E-02	-.2179E+00	-.2385E+01
	22.5	.4024E-04	.2266E-03	.5684E-03	.8344E-03	.4780E-03	-.3348E-01	-.7105E-01
	45	.4024E-04	.1488E-03	.2438E-03	.2059E-03	.6412E-04	-.2504E-02	.1925E-01
	67.5	.4024E-04	.7464E-04	.6438E-04	.2306E-04	-.1066E-05	.5918E-03	.9575E-02
	90	.4024E-04	.2504E-04	-.2414E-05	-.1458E-04	-.7750E-05	.5938E-03	.3552E-02
	112.5	.4024E-04	-.1845E-05	-.2080E-04	-.1720E-04	-.5878E-05	.3551E-03	.1144E-02
	135	.4024E-04	-.1444E-04	-.2336E-04	-.1393E-04	-.3871E-05	.2016E-03	.2666E-03
	157.5	.4024E-04	-.1954E-04	-.2237E-04	-.1130E-04	-.2753E-05	.1281E-03	-.3092E-04
	180	.4024E-04	-.2086E-04	-.2176E-04	-.1039E-04	-.2411E-05	.1070E-03	-.1010E-03

Clamped Circular plate							λ = 7	
β		$c_1 = wD/Pa^2$					$c_2 = M_n/P$	$c_3 = V_n a/P$
	θ° \ α	0	0.2	0.4	0.6	0.8		
0	0	.2550E-02	.1174E-02	.2320E-03	-.1687E-04	-.1979E-04	.1467E-02	.7746E-02
	22.5	.2550E-02	.1174E-02	.2320E-03	-.1687E-04	-.1979E-04	.1467E-02	.7746E-02
	45	.2550E-02	.1174E-02	.2320E-03	-.1687E-04	-.1979E-04	.1467E-02	.7746E-02
	67.5	.2550E-02	.1174E-02	.2320E-03	-.1687E-04	-.1979E-04	.1467E-02	.7746E-02
	90	.2550E-02	.1174E-02	.2320E-03	-.1687E-04	-.1979E-04	.1467E-02	.7746E-02
	112.5	.2550E-02	.1174E-02	.2320E-03	-.1687E-04	-.1979E-04	.1467E-02	.7746E-02
	135	.2550E-02	.1174E-02	.2320E-03	-.1687E-04	-.1979E-04	.1467E-02	.7746E-02
	157.5	.2550E-02	.1174E-02	.2320E-03	-.1687E-04	-.1979E-04	.1467E-02	.7746E-02
	180	.2550E-02	.1174E-02	.2320E-03	-.1687E-04	-.1979E-04	.1467E-02	.7746E-02
0.2	0	.1174E-02	.2549E-02	.1173E-02	.2387E-03	.3908E-05	.1824E-02	.2836E-01
	22.5	.1174E-02	.2129E-02	.9824E-03	.1858E-03	-.3327E-05	.1995E-02	.2633E-01
	45	.1174E-02	.1525E-02	.6123E-03	.8304E-04	-.1544E-04	.2115E-02	.2030E-01
	67.5	.1174E-02	.1022E-02	.3091E-03	.6851E-05	-.2050E-04	.1816E-02	.1251E-01
	90	.1174E-02	.6706E-03	.1291E-03	-.2683E-04	-.1846E-04	.1295E-02	.6195E-02
	112.5	.1174E-02	.4481E-03	.3936E-04	-.3485E-04	-.1411E-04	.8354E-03	.2430E-02
	135	.1174E-02	.3184E-03	-.4429E-06	-.3340E-04	-.1045E-04	.5373E-03	.6034E-03
	157.5	.1174E-02	.2522E-03	-.1614E-04	-.3066E-04	-.8300E-05	.3834E-03	-.1447E-03
	180	.1174E-02	.2320E-03	-.2019E-04	-.2952E-04	-.7612E-05	.3371E-03	-.3399E-03
0.4	0	.2320E-03	.1173E-02	.2547E-02	.1166E-02	.2207E-03	-.8469E-02	.1724E-01
	22.5	.2320E-03	.9824E-03	.1500E-02	.7010E-03	.1195E-03	-.3147E-02	.3605E-01
	45	.2320E-03	.6123E-03	.5594E-03	.1785E-03	.6422E-05	.1741E-02	.3302E-01
	67.5	.2320E-03	.3091E-03	.1344E-03	-.7022E-05	-.1973E-04	.1800E-02	.1342E-01
	90	.2320E-03	.1291E-03	-.4240E-05	-.3445E-04	-.1363E-04	.8350E-03	.2658E-02
	112.5	.2320E-03	.3936E-04	-.3492E-04	-.2537E-04	-.6260E-05	.2648E-03	-.6019E-03
	135	.2320E-03	-.4430E-06	-.3552E-04	-.1565E-04	-.2468E-05	.4286E-04	-.1139E-02
	157.5	.2320E-03	-.1614E-04	-.3168E-04	-.1045E-04	-.9784E-06	-.2648E-04	-.1077E-02
	180	.2320E-03	-.2019E-04	-.3002E-04	-.8926E-05	-.6051E-06	-.4107E-04	-.1020E-02
0.6	0	-.1687E-04	.2386E-03	.1166E-02	.2457E-02	.8978E-03	-.5702E-01	-.2967E+00
	22.5	-.1687E-04	.1858E-03	.7010E-03	.9024E-03	.3247E-03	-.1671E-01	.5384E-02
	45	-.1687E-04	.8303E-04	.1785E-03	.1125E-03	.1167E-04	.1413E-02	.3845E-01
	67.5	-.1687E-04	.6850E-05	-.7022E-05	-.2765E-04	-.1638E-04	.1329E-02	.8595E-02
	90	-.1687E-04	-.2683E-04	-.3445E-04	-.2293E-04	-.6613E-05	.3264E-03	-.1201E-03
	112.5	-.1687E-04	-.3485E-04	-.2537E-04	-.9322E-05	-.1245E-05	-.2885E-05	-.1037E-02
	135	-.1687E-04	-.3340E-04	-.1564E-04	-.2854E-05	.3084E-06	-.6020E-04	-.6966E-03
	157.5	-.1687E-04	-.3066E-04	-.1045E-04	-.6079E-06	.6055E-06	-.5822E-04	-.4331E-03
	180	-.1687E-04	-.2952E-04	-.8926E-05	-.9733E-07	.6339E-06	-.5440E-04	-.3527E-03
0.8	0	-.1979E-04	.3903E-05	.2207E-03	.8977E-03	.1578E-02	-.1793E+00	-.2071E+01
	22.5	-.1979E-04	-.3330E-05	.1195E-03	.3247E-03	.2314E-03	-.1511E-01	.2446E-01
	45	-.1979E-04	-.1544E-04	.6423E-05	.1169E-04	-.3086E-05	.1165E-02	.2084E-01
	67.5	-.1979E-04	-.2050E-04	-.1972E-04	-.1638E-04	-.6876E-05	.4763E-03	.2037E-02
	90	-.1979E-04	-.1846E-04	-.1363E-04	-.6612E-05	-.1440E-05	.4277E-04	-.5486E-03
	112.5	-.1979E-04	-.1411E-04	-.6259E-05	-.1245E-05	.1412E-06	-.3074E-04	-.3928E-03
	135	-.1979E-04	-.1045E-04	-.2468E-05	.3083E-06	.3298E-06	-.2612E-04	-.1573E-03
	157.5	-.1979E-04	-.8299E-05	-.9783E-06	.6055E-06	.2769E-06	-.1723E-04	-.5838E-04
	180	-.1979E-04	-.7611E-05	-.6050E-06	.6339E-06	.2474E-06	-.1421E-04	-.3442E-04

Clamped Circular plate							$\lambda = 11$	
β		$c_1 = wD/Pa^2$					$c_2 = M_n/P$	$c_3 = V_n a/P$
	$\theta°$ \ α	0	0.2	0.4	0.6	0.8		
0	0	.1033E-02	.2123E-03	-.1160E-04	-.5611E-05	.2382E-06	-.6268E-04	-.7061E-03
	22.5	.1033E-02	.2123E-03	-.1160E-04	-.5611E-05	.2382E-06	-.6268E-04	-.7061E-03
	45	.1033E-02	.2123E-03	-.1160E-04	-.5611E-05	.2382E-06	-.6268E-04	-.7061E-03
	67.5	.1033E-02	.2123E-03	-.1160E-04	-.5611E-05	.2382E-06	-.6268E-04	-.7061E-03
	90	.1033E-02	.2123E-03	-.1160E-04	-.5611E-05	.2382E-06	-.6268E-04	-.7061E-03
	112.5	.1033E-02	.2123E-03	-.1160E-04	-.5611E-05	.2382E-06	-.6268E-04	-.7061E-03
	135	.1033E-02	.2123E-03	-.1160E-04	-.5611E-05	.2382E-06	-.6268E-04	-.7061E-03
	157.5	.1033E-02	.2123E-03	-.1160E-04	-.5611E-05	.2382E-06	-.6268E-04	-.7061E-03
	180	.1033E-02	.2123E-03	-.1160E-04	-.5611E-05	.2382E-06	-.6268E-04	-.7061E-03
0.2	0	.2123E-03	.1033E-02	.2123E-03	-.1158E-04	-.5545E-05	.1705E-03	-.1351E-02
	22.5	.2123E-03	.7174E-03	.1431E-03	-.1392E-04	-.4431E-05	.9907E-04	-.1634E-02
	45	.2123E-03	.3677E-03	.4152E-04	-.1390E-04	-.2079E-05	-.2022E-04	-.1756E-02
	67.5	.2123E-03	.1567E-03	-.5701E-05	-.8750E-05	-.3015E-06	-.6891E-04	-.1211E-02
	90	.2123E-03	.5455E-04	-.1475E-04	-.3741E-05	.3465E-06	-.5575E-04	-.5507E-03
	112.5	.2123E-03	.1148E-04	-.1203E-04	-.1078E-05	.3878E-06	-.3113E-04	-.1641E-03
	135	.2123E-03	-.4824E-05	-.8430E-05	-.4590E-07	.2904E-06	-.1539E-04	-.9988E-05
	157.5	.2123E-03	-.1034E-04	-.6290E-05	.2777E-06	.2172E-06	-.8275E-05	.3751E-04
	180	.2123E-03	-.1161E-04	-.5622E-05	.3458E-06	.1932E-06	-.6349E-05	.4719E-04
0.4	0	-.1160E-04	.2123E-03	.1033E-02	.2122E-03	-.8380E-05	.1927E-02	.2096E-01
	22.5	-.1160E-04	.1431E-03	.3561E-03	.6200E-04	-.1270E-04	.1339E-02	.1043E-01
	45	-.1160E-04	.4152E-04	.3069E-04	-.1418E-04	-.6080E-05	.2513E-03	-.1090E-02
	67.5	-.1160E-04	-.5701E-05	-.1473E-04	-.6967E-05	-.3322E-06	-.6662E-04	-.1365E-02
	90	-.1160E-04	-.1475E-04	-.7975E-05	-.6025E-06	.3846E-06	-.3324E-04	-.2023E-03
	112.5	-.1160E-04	-.1203E-04	-.2266E-05	.4633E-06	.1517E-06	-.4066E-05	.5888E-04
	135	-.1160E-04	-.8430E-05	-.2758E-06	.3649E-06	.3071E-07	.2084E-05	.5348E-04
	157.5	-.1160E-04	-.6290E-05	.2498E-06	.2366E-06	-.1861E-08	.2429E-05	.3303E-04
	180	-.1160E-04	-.5622E-05	.3497E-06	.1962E-06	-.7602E-08	.2283E-05	.2655E-04
0.6	0	-.5611E-05	-.1158E-04	.2122E-03	.1032E-02	.2116E-03	-.5826E-02	.3472E-01
	22.5	-.5611E-05	-.1392E-04	.6200E-04	.1310E-03	.1347E-04	.2118E-02	.5150E-01
	45	-.5611E-05	-.1390E-04	-.1418E-04	-.1483E-04	-.7194E-05	.4538E-03	.4329E-03
	67.5	-.5611E-05	-.8750E-05	-.6967E-05	-.2253E-05	.1607E-06	-.6254E-04	-.1008E-02
	90	-.5611E-05	-.3741E-05	-.6025E-06	.4770E-06	.1901E-06	-.8630E-05	.3450E-04
	112.5	-.5611E-05	-.1078E-05	.4633E-06	.2027E-06	.2612E-08	.2413E-05	.4147E-04
	135	-.5611E-05	-.4591E-07	.3649E-06	.3350E-07	-.1412E-07	.1210E-05	.7051E-05
	157.5	-.5611E-05	.2777E-06	.2366E-06	-.5697E-08	-.9219E-08	.4269E-06	-.6962E-06
	180	-.5611E-05	.3458E-06	.1962E-06	-.1159E-07	-.7307E-08	.2536E-06	-.1695E-05
0.8	0	.2382E-06	-.5545E-05	-.8380E-05	.2116E-03	.9185E-03	-.1040E+00	-.1267E+01
	22.5	.2382E-06	-.4431E-05	-.1270E-04	.1348E-04	.2293E-04	.8986E-03	.7986E-01
	45	.2382E-06	-.2080E-05	-.6080E-05	-.7192E-05	-.3557E-05	.2167E-03	-.7160E-03
	67.5	.2382E-06	-.3016E-06	-.3323E-06	.1605E-06	.2581E-06	-.3034E-04	-.3109E-03
	90	.2382E-06	.3464E-06	.3845E-06	.1901E-06	.2701E-07	.8761E-06	.4624E-04
	112.5	.2382E-06	.3878E-06	.1517E-06	.2615E-08	-.1040E-07	.8550E-06	.4957E-05
	135	.2382E-06	.2904E-06	.3072E-07	-.1412E-07	-.2920E-08	.2057E-07	-.2081E-05
	157.5	.2382E-06	.2172E-06	-.1858E-08	-.9219E-08	-.2979E-09	-.7994E-07	-.1378E-05
	180	.2382E-06	.1932E-06	-.7600E-08	-.7307E-08	.1089E-09	-.7865E-07	-.1028E-05

TABLE II

Simply supported circular plate (ν=0.3)								λ=1
β		$c_4=wD/Pa^2$					$c_5=M_t/P$	$c_6=V_n\,a/P$
	θ° \ α	0	0.2	0.4	0.6	0.8		
0	0	.4869E-01	.4420E-01	.3515E-01	.2398E-01	.1197E-01	.5333E-01	-.1495E+00
	22.5	.4869E-01	.4420E-01	.3515E-01	.2398E-01	.1197E-01	.5333E-01	-.1495E+00
	45.	.4869E-01	.4420E-01	.3515E-01	.2398E-01	.1197E-01	.5333E-01	-.1495E+00
	67.5	.4869E-01	.4420E-01	.3515E-01	.2398E-01	.1197E-01	.5333E-01	-.1495E+00
	90.	.4869E-01	.4420E-01	.3515E-01	.2398E-01	.1197E-01	.5333E-01	-.1495E+00
	112.5	.4869E-01	.4420E-01	.3515E-01	.2398E-01	.1197E-01	.5333E-01	-.1495E+00
	135.	.4869E-01	.4420E-01	.3515E-01	.2398E-01	.1197E-01	.5333E-01	-.1495E+00
	157.5	.4869E-01	.4420E-01	.3515E-01	.2398E-01	.1197E-01	.5333E-01	-.1495E+00
	180.	.4869E-01	.4420E-01	.3515E-01	.2398E-01	.1197E-01	.5333E-01	-.1495E+00
0.2	0	.4420E-01	.4581E-01	.3855E-01	.2694E-01	.1361E-01	.6083E-01	-.2317E+00
	22.5	.4420E-01	.4513E-01	.3792E-01	.2648E-01	.1337E-01	.5976E-01	-.2196E+00
	45.	.4420E-01	.4381E-01	.3641E-01	.2533E-01	.1276E-01	.5701E-01	-.1914E+00
	67.5	.4420E-01	.4231E-01	.3459E-01	.2390E-01	.1200E-01	.5355E-01	-.1611E+00
	90.	.4420E-01	.4087E-01	.3285E-01	.2252E-01	.1127E-01	.5020E-01	-.1363E+00
	112.5	.4420E-01	.3965E-01	.3140E-01	.2137E-01	.1065E-01	.4742E-01	-.1196E+00
	135.	.4420E-01	.3873E-01	.3034E-01	.2053E-01	.1021E-01	.4539E-01	-.1070E+00
	157.5	.4420E-01	.3817E-01	.2969E-01	.2002E-01	.9937E-02	.4418E-01	-.1006E+00
	180.	.4420E-01	.3798E-01	.2948E-01	.1985E-01	.9848E-02	.4377E-01	-.9856E-01
0.4	0	.3514E-01	.3854E-01	.3749E-01	.2784E-01	.1439E-01	.6490E-01	-.3807E+00
	22.5	.3514E-01	.3792E-01	.3552E-01	.2636E-01	.1363E-01	.6135E-01	-.3130E+00
	45.	.3514E-01	.3641E-01	.3236E-01	.2347E-01	.1205E-01	.5411E-01	-.2089E+00
	67.5	.3514E-01	.3459E-01	.2932E-01	.2072E-01	.1052E-01	.4704E-01	-.1415E+00
	90.	.3514E-01	.3285E-01	.2680E-01	.1654E-01	.9311E-02	.4153E-01	-.1042E+00
	112.5	.3514E-01	.3140E-01	.2490E-01	.1695E-01	.8450E-02	.3760E-01	-.8360E-01
	135.	.3514E-01	.3034E-01	.2359E-01	.1589E-01	.7881E-02	.3502E-01	-.7178E-01
	157.5	.3514E-01	.2969E-01	.2283E-01	.1528E-01	.7559E-02	.3356E-01	-.6584E-01
	180.	.3514E-01	.2948E-01	.2257E-01	.1509E-01	.7454E-02	.3309E-01	-.6401E-01
0.6	0	.2397E-01	.2693E-01	.2784E-01	.2470E-01	.1370E-01	.6272E-01	-.7060E+00
	22.5	.2397E-01	.2648E-01	.2635E-01	.2165E-01	.1190E-01	.5449E-01	-.3862E+00
	45.	.2397E-01	.2533E-01	.2347E-01	.1785E-01	.9429E-02	.4268E-01	-.1725E+00
	67.5	.2397E-01	.2390E-01	.2072E-01	.1494E-01	.7666E-02	.3440E-01	-.9756E-01
	90.	.2397E-02	.2251E-01	.1854E-01	.1291E-01	.6506E-02	.2904E-01	-.6667E-01
	112.5	.2397E-01	.2136E-01	.1695E-01	.1154E-01	.5754E-02	.2560E-01	-.5155E-01
	135.	.2397E-01	.2052E-01	.1589E-01	.1067E-01	.5286E-02	.2348E-01	-.4361E-01
	157.5	.2397E-01	.2002E-01	.1528E-01	.1019E-01	.5029E-02	.2231E-01	-.3968E-01
	180.	.2397E-01	.1985E-01	.1508E-01	.1004E-01	.4947E-02	.2194E-01	-.3848E-01
0.8	0	.1196E-01	.1360E-01	.1438E-01	.1369E-01	.9921E-02	.4935E-01	-.1707E+01
	22.5	.1196E-01	.1336E-01	.1362E-01	.1190E-01	.7149E-02	.3397E-01	-.2439E+00
	45.	.1196E-01	.1276E-01	.1204E-01	.9425E-02	.5102E-02	.2328E-01	-.8336E-01
	67.5	.1196E-01	.1199E-01	.1051E-01	.7662E-02	.3960E-02	.1780E-01	-.4627E-01
	90.	.1196E-01	.1126E-01	.9305E-02	.6503E-02	.3283E-02	.1466E-01	-.3140E-01
	112.5	.1196E-01	.1065E-01	.8444E-02	.5752E-02	.2867E-02	.1275E-01	-.2416E-01
	135.	.1196E-01	.1020E-01	.7876E-02	.5283E-02	.2614E-02	.1161E-01	-.2038E-01
	157.5	.1196E-01	.9930E-02	.7554E-02	.5026E-02	.2477E-02	.1099E-01	-.1851E-01
	180.	.1196E-01	.9840E-02	.7449E-02	.4944E-02	.2434E-02	.1079E-01	-.1794E-01

Simply supported circular plate ($\nu = 0.3$) $\lambda = 3$

β	$\theta°$ \ α	$c_4 = wD/Pa^2$: 0	0.2	0.4	0.6	0.8	$c_5 = M_t/P$	$c_6 = V_n a/P$
0	0	.1498E-01	.1233E-01	.8470E-02	.5053E-02	.2304E-02	.9941E-02	.2194E-01
0	22.5	.1498E-01	.1233E-01	.8470E-02	.5053E-02	.2304E-02	.9941E-02	.2194E-01
0	45.	.1498E-01	.1233E-01	.8470E-02	.5053E-02	.2304E-02	.9941E-02	.2194E-01
0	67.5	.1498E-01	.1233E-01	.8470E-02	.5053E-02	.2304E-02	.9941E-02	.2194E-01
0	90.	.1498E-01	.1233E-01	.8470E-02	.5053E-02	.2304E-02	.9941E-02	.2194E-01
0	112.5	.1498E-01	.1233E-01	.8470E-02	.5053E-02	.2304E-02	.9941E-02	.2194E-01
0	135.	.1498E-01	.1233E-01	.8470E-02	.5053E-02	.2304E-02	.9941E-02	.2194E-01
0	157.5	.1498E-01	.1233E-01	.8470E-02	.5053E-02	.2304E-02	.9941E-02	.2194E-01
0	180.	.1498E-01	.1233E-01	.8470E-02	.5053E-02	.2304E-02	.9941E-02	.2194E-01
0.2	0	.1233E-01	.1491E-01	.1208E-01	.7838E-02	.3746E-02	.1642E-01	-.2469E-01
0.2	22.5	.1233E-01	.1429E-01	.1156E-01	.7486E-02	.3571E-02	.1564E-01	-.1704E-01
0.2	45.	.1233E-01	.1314E-01	.1033E-01	.6614E-02	.3133E-02	.1368E-01	-.5159E-03
0.2	67.5	.1233E-01	.1188E-01	.8913E-02	.5573E-02	.2605E-02	.1132E-01	.1469E-01
0.2	90.	.1233E-01	.1073E-01	.7624E-02	.4620E-02	.2121E-02	.9165E-02	.2438E-01
0.2	112.5	.1233E-01	.9790E-02	.6601E-02	.3871E-02	.1742E-02	.7479E-02	.2927E-01
0.2	135.	.1233E-01	.9101E-02	.5879E-02	.3349E-02	.1481E-02	.6318E-02	.3128E-01
0.2	157.5	.1233E-01	.8686E-02	.5454E-02	.3046E-02	.1330E-02	.5649E-02	.3193E-01
0.2	180.	.1233E-01	.8547E-02	.5315E-02	.2947E-02	.1281E-02	.5432E-02	.3206E-01
0.4	0	.8469E-02	.1208E-01	.1428E-01	.1077E-01	.5464E-02	.2439E-01	-.1559E+00
0.4	22.5	.8469E-02	.1156E-01	.1251E-01	.9478E-02	.4828E-02	.2155E-01	-.1003E+00
0.4	45.	.8469E-02	.1033E-01	.9845E-02	.7109E-02	.3568E-02	.1584E-01	-.2349E-01
0.4	67.5	.8469E-02	.8912E-02	.7480E-02	.5038E-02	.2445E-02	.1074E-01	.1457E-01
0.4	90.	.8469E-02	.7623E-02	.5681E-02	.3551E-02	.1655E-02	.7174E-02	.2727E-01
0.4	112.5	.8469E-02	.6600E-02	.4427E-02	.2575E-02	.1149E-02	.4912E-02	.2975E-01
0.4	135.	.8469E-02	.5878E-02	.3622E-02	.1978E-02	.8477E-03	.3573E-02	.2906E-01
0.4	157.5	.8469E-02	.5454E-02	.3177E-02	.1659E-02	.6895E-03	.2874E-02	.2800E-01
0.4	180.	.8469E-02	.5314E-02	.3035E-02	.1559E-02	.6403E-03	.2658E-02	.2756E-01
0.6	0	.5050E-02	.7835E-02	.1076E-01	.1189E-01	.6899E-02	.3177E-01	-.4976E+00
0.6	22.5	.5050E-02	.7483E-02	.9475E-02	.9056E-02	.5259E-02	.2431E-01	-.2018E+00
0.6	45.	.5050E-02	.6611E-02	.7107E-02	.5826E-02	.3154E-02	.1431E-01	-.2427E-01
0.6	67.5	.5050E-02	.5570E-02	.5036E-02	.3624E-02	.1827E-02	.8113E-02	.1701E-01
0.6	90.	.5050E-02	.4618E-02	.3550E-02	.2277E-02	.1075E-02	.4675E-02	.2401E-01
0.6	112.5	.5050E-02	.3869E-02	.2574E-02	.1484E-02	.6568E-03	.2796E-02	.2299E-01
0.6	135.	.5050E-02	.3347E-02	.1977E-02	.1036E-02	.4296E-03	.1787E-02	.2074E-01
0.6	157.5	.5050E-02	.3044E-02	.1658E-02	.8085E-03	.3173E-03	.1292E-02	.1909E-01
0.6	180.	.5050E-02	.2945E-02	.1558E-02	.7390E-03	.2834E-03	.1144E-02	.1852E-01
0.8	0	.2300E-02	.3740E-02	.5456E-02	.6891E-02	.6275E-02	.3260E-01	-.1542E+01
0.8	22.5	.2300E-02	.3565E-02	.4821E-02	.5254E-02	.3609E-02	.1781E-01	-.1359E+00
0.8	45.	.2300E-02	.3127E-02	.3563E-02	.3150E-02	.1811E-02	.8373E-02	-.4412E-02
0.8	67.5	.2300E-02	.2600E-02	.2442E-02	.1826E-02	.9435E-03	.4217E-02	.1268E-01
0.8	90.	.2300E-02	.2117E-02	.1652E-02	.1074E-02	.5110E-03	.2226E-02	.1378E-01
0.8	112.5	.2300E-02	.1739E-02	.1147E-02	.6560E-03	.2881E-03	.1222E-02	.1209E-01
0.8	135.	.2300E-02	.1478E-02	.8462E-03	.4290E-03	.1729E-03	.7107E-03	.1040E-01
0.8	157.5	.2300E-02	.1327E-02	.6882E-03	.3168E-03	.1178E-03	.4686E-03	.9324E-02
0.8	180.	.2300E-02	.1278E-02	.6391E-03	.2830E-03	.1015E-03	.3972E-03	.8963E-02

Simply supported circular plate (ν=0.3)							λ = 5	
β		$c_4 = wD/Pa^2$					$c_5 = M_t/P$	$c_6 = V_n a/P$
	θ° \ α	0	0.2	0.4	0.6	0.8		
0	0	.4989E-02	.3143E-02	.1292E-02	.3515E-03	.4277E-04	.2013E-04	.2576E-01
	22.5	.4989E-02	.3143E-02	.1292E-02	.3515E-03	.4277E-04	.2013E-04	.2576E-01
	45.	.4989E-02	.3143E-02	.1292E-02	.3515E-03	.4277E-04	.2013E-04	.2576E-01
	67.5	.4989E-02	.3143E-02	.1292E-02	.3515E-03	.4277E-04	.2013E-04	.2576E-01
	90.	.4989E-02	.3143E-02	.1292E-02	.3515E-03	.4277E-04	.2013E-04	.2576E-01
	112.5	.4989E-02	.3143E-02	.1292E-02	.3515E-03	.4277E-04	.2013E-04	.2576E-01
	135.	.4989E-02	.3143E-02	.1292E-02	.3515E-03	.4277E-04	.2013E-04	.2576E-01
	157.5	.4989E-02	.3143E-02	.1292E-02	.3515E-03	.4277E-04	.2013E-04	.2576E-01
	180.	.4989E-02	.3143E-02	.1292E-02	.3515E-03	.4277E-04	.2013E-04	.2576E-01
0.2	0	.3143E-02	.5006E-02	.3188E-02	.1369E-02	.4277E-03	.1540E-02	.4639E-01
	22.5	.3143E-02	.4505E-02	.2871E-02	.1214E-02	.3696E-03	.1309E-02	.4526E-01
	45.	.3143E-02	.3687E-02	.2186E-02	.6659E-03	.2390E-03	.7913E-03	.4070E-01
	67.5	.3143E-02	.2904E-02	.1905E-02	.5171E-03	.1109E-03	.2920E-03	.3267E-01
	90.	.3143E-02	.2269E-02	.9902E-03	.2646E-03	.2374E-04	-.3649E-04	.2382E-01
	112.5	.3143E-02	.1807E-02	.6512E-03	.1126E-03	-.2324E-04	-.2023E-03	.1657E-01
	135.	.3143E-02	.1502E-02	.4503E-03	.3215E-04	-.4421E-04	-.2682E-03	.1171E-01
	157.5	.3143E-02	.1321E-02	.3470E-03	-.4838E-05	-.5214E-04	-.2880E-03	.9052E-02
	180.	.3143E-02	.1276E-02	.3154E-03	-.1535E-04	-.5401E-04	-.2916E-03	.8220E-02
0.4	0	.1292E-02	.3188E-02	.5093E-02	.3290E-02	.1357E-02	.5549E-02	.2347E-01
	22.5	.1292E-02	.2871E-02	.3735E-02	.2467E-02	.1020E-02	.4148E-02	.4216E-01
	45.	.1292E-02	.2185E-02	.2116E-02	.1227E-02	.4670E-03	.1821E-02	.5120E-01
	67.5	.1292E-02	.1505E-02	.1027E-02	.4522E-03	.1259E-03	.4034E-03	.3515E-01
	90.	.1292E-02	.9901E-03	.4485E-03	.5759E-04	-.1205E-04	-.1376E-03	.1825E-01
	112.5	.1292E-02	.6512E-03	.1692E-03	-.3425E-04	-.5025E-04	-.2631E-03	.7965E-02
	135.	.1292E-02	.4502E-03	.3685E-04	-.7207E-04	-.5351E-04	-.2541E-03	.2946E-02
	157.5	.1292E-02	.3469E-03	-.1347E-04	-.8063E-04	-.4945E-04	-.2254E-03	.8612E-03
	180.	.1292E-02	.3154E-03	-.2657E-04	-.8113E-04	-.4741E-04	-.2134E-03	.3059E-03
0.6	0	.3512E-03	.1368E-02	.3289E-02	.5101E-02	.2825E-02	.1286E-01	-.2116E+00
	22.5	.3512E-03	.1213E-02	.2466E-02	.2902E-02	.1697E-02	.7671E-02	-.3201E-02
	45.	.3512E-03	.8554E-03	.1226E-02	.1049E-02	.5283E-03	.2279E-02	.5504E-01
	67.5	.3512E-03	.5127E-03	.4519E-03	.2472E-03	.8392E-04	.2950E-03	.3005E-01
	90.	.3512E-03	.2643E-03	.5747E-04	-.1177E-04	-.3068E-04	-.1731E-03	.1091E-01
	112.5	.3512E-03	.1124E-03	-.3430E-04	-.6906E-04	-.4332E-04	-.2017E-03	.2458E-02
	135.	.3512E-03	-.3202E-04	-.7308E-04	-.6875E-04	-.3453E-04	-.1502E-03	-.5262E-03
	157.5	.3512E-03	-.4936E-05	-.8063E-04	-.5984E-04	-.2651E-04	-.1102E-03	-.1393E-02
	180.	.3512E-03	-.1544E-04	-.8113E-04	-.5602E-04	-.2368E-04	-.9664E-04	-.1560E-02
0.8	0	.4238E-04	.4263E-03	.1354E-02	.2881E-02	.3731E-02	.2021E-01	-.1257E+01
	22.5	.4238E-04	.3684E-03	.1017E-02	.1695E-02	.1440E-02	.7475E-02	.1480E-01
	45.	.4238E-04	.2381E-03	.4657E-03	.5275E-03	.3282E-03	.1519E-02	.4335E-01
	67.5	.4238E-04	.1104E-03	.1255E-03	.8379E-04	.3290E-04	.1209E-03	.1713E-01
	90.	.4238E-04	.2345E-04	-.1214E-04	-.3064E-04	-.2267E-04	-.1122E-03	.4599E-02
	112.5	.4238E-04	-.2338E-04	-.5023E-04	-.4326E-04	-.2244E-04	-.9957E-04	.2995E-03
	135.	.4238E-04	-.4436E-04	-.5345E-04	-.3448E-04	-.1480E-04	-.6113E-04	-.8517E-03
	157.5	.4238E-04	-.5236E-04	-.4938E-04	-.2646E-04	-.9694E-05	-.3733E-04	-.1049E-02
	180.	.4238E-04	-.5401E-04	-.4734E-04	-.2364E-04	-.8058E-05	-.2991E-04	-.1057E-02

β	θ° \ α	$c_4 = wD/Pa^2$: 0	0.2	0.4	0.6	0.8	$c_5 = M_t/P$	$c_6 = V_n\, a/P$
Simply supported circular plate (ν = 0.3) λ = 7								
0	0	.2551E-02	.1175E-02	.2317E-03	-.2217E-04	-.3111E-04	-.1458E-03	.6494E-03
	22.5	.2551E-02	.1175E-02	.2317E-03	-.2217E-04	-.3111E-04	-.1458E-03	.6494E-03
	45.	.2551E-02	.1175E-02	.2317E-03	-.2217E-04	-.3111E-04	-.1458E-03	.6494E-03
	67.5	.2551E-02	.1175E-02	.2317E-03	-.2217E-04	-.3111E-04	-.1458E-03	.6494E-03
	90.	.2551E-02	.1175E-02	.2317E-03	-.2217E-04	-.3111E-04	-.1458E-03	.6494E-03
	112.5	.2551E-02	.1175E-02	.2317E-03	-.2217E-04	-.3111E-04	-.1458E-03	.6494E-03
	135.	.2551E-02	.1175E-02	.2317E-03	-.2217E-04	-.3111E-04	-.1458E-03	.6494E-03
	157.5	.2551E-02	.1175E-02	.2317E-03	-.2217E-04	-.3111E-04	-.1458E-03	.6494E-03
	180.	.2551E-02	.1175E-02	.2317E-03	-.2217E-04	-.3111E-04	-.1458E-03	.6494E-03
0.2	0	.1175E-02	.2550E-02	.1172E-02	.2318E-03	-.1048E-04	-.1841E-03	.1586E-01
	22.5	.1175E-02	.2130E-02	.9818E-03	.1786E-03	-.1851E-04	-.1973E-03	.1694E-01
	45.	.1175E-02	.1526E-02	.6116E-03	.7570E-04	-.3111E-04	-.2059E-03	.1023E-01
	67.5	.1175E-02	.1022E-02	.3085E-03	.4129E-06	-.3420E-04	-.1784E-03	.3786E-02
	90.	.1175E-02	.6717E-03	.1287E-03	-.3158E-04	-.2855E-04	-.1294E-03	-.9534E-04
	112.5	.1175E-02	.4451E-03	.3927E-04	-.3796E-04	-.2078E-04	-.8459E-04	-.1674E-02
	135.	.1175E-02	.3153E-03	-.3916E-06	-.3540E-04	-.1479E-04	-.5476E-04	-.2059E-02
	157.5	.1175E-02	.2530E-03	-.1603E-04	-.3206E-04	-.1140E-04	-.3912E-04	-.2052E-02
	180.	.1175E-02	.2328E-03	-.2007E-04	-.3076E-04	-.1034E-04	-.3438E-04	-.2019E-02
0.4	0	.2317E-03	.1173E-02	.2551E-02	.1188E-02	.2738E-03	.7370E-03	.5951E-01
	22.5	.2317E-03	.9817E-03	.1502E-02	.7135E-03	.1452E-03	.3227E-03	.5130E-01
	45.	.2317E-03	.6116E-03	.5584E-03	.1762E-03	-.1754E-06	-.1259E-03	.2524E-01
	67.5	.2317E-03	.3085E-03	.1326E-03	-.1335E-04	-.3244E-04	-.1707E-03	.5127E-02
	90.	.2317E-03	.1287E-03	-.5295E-05	-.3827E-04	-.2082E-04	-.8771E-04	-.1433E-02
	112.5	.2317E-03	.3926E-04	-.2514E-04	-.2666E-04	-.8791E-05	-.2961E-04	-.2003E-02
	135.	.2317E-03	-.3962E-06	-.3538E-04	-.1580E-04	-.2957E-05	-.5458E-05	-.1408E-02
	157.5	.2317E-03	-.1604E-04	-.3147E-04	-.1029E-04	-.8152E-06	.2250E-05	-.9715E-03
	180.	.2317E-03	-.2008E-04	-.2982E-04	-.8708E-05	-.3079E-06	.3875E-05	-.8227E-03
0.6	0	-.2219E-04	.2317E-03	.1188E-02	.2598E-02	.1219E-02	.4778E-02	-.6900E-02
	22.5	-.2219E-04	.1785E-03	.7133E-03	.9845E-03	.4889E-03	.1932E-02	.7843E-01
	45.	-.2219E-04	.7563E-04	.1761E-03	.1227E-03	.2615E-04	.2916E-04	.3609E-01
	67.5	-.2219E-04	.3764E-06	-.1327E-04	-.3506E-04	-.2677E-04	-.1308E-03	.3562E-02
	90.	-.2219E-04	-.3160E-04	-.3827E-04	-.2675E-04	-.1089E-04	-.4221E-04	-.1879E-02
	112.5	-.2219E-04	-.3797E-04	-.2665E-04	-.9809E-05	-.1574E-05	-.1911E-05	-.1196E-02
	135.	-.2219E-04	-.3540E-04	-.1579E-04	-.2415E-05	.8666E-06	.6233E-05	-.4403E-03
	157.5	-.2219E-04	-.3206E-04	-.1028E-04	-.1190E-06	.1169E-05	.6219E-05	-.1394E-03
	180.	-.2219E-04	-.3075E-04	-.8704E-05	.3517E-06	.1149E-05	.5770E-05	-.7025E-04
0.8	0	-.3107E-04	-.1064E-04	.2709E-03	.1217E-02	.2341E-02	.1280E-01	-.9079E+00
	22.5	-.3107E-04	-.1863E-04	.1446E-03	.4879E-03	.5302E-03	.2857E-02	.1113E+00
	45.	-.3107E-04	-.3116E-04	-.2182E-06	.2607E-04	.1705E-04	.6213E-04	.3042E-01
	67.5	-.3107E-04	-.3419E-04	-.3241E-04	-.2671E-04	-.1454E-04	-.6859E-04	.1414E-02
	90.	-.3107E-04	-.2851E-04	-.2078E-04	-.1086E-04	-.3655E-05	-.1286E-04	-.1143E-02
	112.5	-.3107E-04	-.2074E-04	-.8767E-05	-.1567E-05	.4377E-06	.3548E-05	-.4227E-03
	135.	-.3107E-04	-.1476E-04	-.2944E-05	.8674E-06	.8427E-06	.3981E-05	-.3933E-04
	157.5	-.3107E-04	-.1127E-04	-.8076E-06	.1168E-05	.6287E-06	.2567E-05	.5623E-04
	180.	-.3107E-04	-.1021E-04	-.3017E-06	.1148E-05	.5301E-06	.2039E-05	.6924E-04

Simply supported circular plate (ν=0.3)							λ = 11	
β		$c_4 = wD/Pa^2$					$c_5 = M_t/P$	$c_6 = V_n\, a/P$
	θ° \ α	0	0.2	0.4	0.6	0.8		
0	0	.1033E-02	.2123E-03	-.1161E-04	-.5611E-05	.3673E-06	.3857E-05	-.2164E-03
	22.5	.1033E-02	.2123E-03	-.1161E-04	-.5611E-05	.3673E-06	.3857E-05	-.2164E-03
	45.	.1033E-02	.2123E-03	-.1161E-04	-.5611E-05	.3673E-06	.3857E-05	-.2164E-03
	67.5	.1033E-02	.2123E-03	-.1161E-04	-.5611E-05	.3673E-06	.3857E-05	-.2164E-03
	90.	.1033E-02	.2123E-03	-.1161E-04	-.5611E-05	.3673E-06	.3857E-05	-.2164E-03
	112.5	.1033E-02	.2123E-03	-.1161E-04	-.5611E-05	.3673E-06	.3857E-05	-.2164E-03
	135.	.1033E-02	.2123E-03	-.1161E-04	-.5611E-05	.3673E-06	.3857E-05	-.2164E-03
	157.5	.1033E-02	.2123E-03	-.1161E-04	-.5611E-05	.3673E-06	.3857E-05	-.2164E-03
	180.	.1033E-02	.2123E-03	-.1161E-04	-.5611E-05	.3673E-06	.3857E-05	-.2164E-03
0.2	0	.2123E-03	.1033E-02	.2123E-03	-.1159E-04	-.5882E-05	-.1018E-04	-.2770E-02
	22.5	.2123E-03	.7174E-03	.1431E-03	-.1392E-04	-.4634E-05	-.6010E-05	-.2486E-02
	45.	.2123E-03	.3677E-03	.4152E-04	-.1389E-04	-.2051E-05	.1073E-05	-.1645E-02
	67.5	.2123E-03	.1567E-03	-.5707E-05	-.8744E-05	-.1681E-06	.4131E-05	-.6865E-03
	90.	.2123E-03	.5455E-04	-.1475E-04	-.2739E-05	.4606E-06	.3415E-05	-.1116E-03
	112.5	.2123E-03	.1148E-04	-.1204E-04	-.1079E-05	.4536E-06	.1937E-05	.8607E-04
	135.	.2123E-03	-.4824E-05	-.8433E-05	-.4653E-07	.3234E-06	.9649E-06	.1158E-03
	157.5	.2123E-03	-.1034E-04	-.6291E-05	.2773E-06	.2351E-06	.5197E-06	.1063E-03
	180.	.2123E-03	-.1161E-04	-.5623E-05	.3455E-06	.2069E-06	.3984E-06	.1004E-03
0.4	0	-.1161E-04	.2123E-03	.1032E-02	.2123E-03	-.1221E-04	-.1142E-03	.6148E-02
	22.5	-.1161E-04	.1431E-03	.3562E-03	.6195E-04	-.1538E-04	-.8114E-04	-.2043E-02
	45.	-.1161E-04	.4152E-04	.2076E-04	-.1414E-04	-.6729E-05	-.1743E-04	-.3225E-02
	67.5	-.1161E-04	-.5707E-05	-.1472E-04	-.5934E-05	-.2312E-06	.3478E-05	-.8877E-03
	90.	-.1161E-04	-.1475E-04	-.7981E-05	-.5997E-06	.4583E-06	.2102E-05	.6460E-04
	112.5	-.1161E-04	-.1204E-04	-.2270E-05	.4611E-06	.1630E-06	.2969E-06	.9600E-04
	135.	-.1161E-04	-.8433E-05	-.2761E-06	.3642E-06	.2699E-07	-.1186E-06	.3846E-04
	157.5	-.1161E-04	-.6291E-05	.2504E-06	.2366E-06	-.6842E-08	-.1490E-06	.1417E-04
	180.	-.1161E-04	-.5623E-05	.3464E-06	.1963E-06	-.1236E-07	-.1413E-06	.8649E-05
0.6	0	-.5610E-05	-.1159E-04	.2121E-03	.1033E-02	.2193E-03	.2898E-03	.8928E-01
	22.5	-.5610E-05	-.1392E-04	.6194E-04	.1307E-03	.1150E-04	-.9569E-04	.3446E-01
	45.	-.5610E-05	-.1389E-04	-.1414E-04	-.1493E-04	-.8576E-05	-.3376E-04	-.3207E-02
	67.5	-.5610E-05	-.9743E-05	-.6934E-05	-.2189E-05	.2545E-06	.3138E-05	-.6089E-03
	90.	-.5610E-05	-.3739E-05	-.5995E-06	.4821E-06	.2196E-06	.6674E-06	.1114E-03
	112.5	-.5610E-05	-.1078E-05	.4611E-06	.1997E-06	-.2095E-08	-.1334E-06	.2419E-04
	135.	-.5610E-05	-.4641E-07	.3642E-06	.2325E-07	-.1676E-07	-.7618E-07	-.2739E-05
	157.5	-.5610E-05	.2773E-06	.2366E-06	-.5573E-08	-.1013E-07	-.2739E-07	-.4239E-05
	180.	-.5610E-05	.3456E-06	.1962E-06	-.1152E-07	-.7870E-08	-.1626E-07	-.3827E-05
0.8	0	.3684E-06	-.5871E-05	-.1224E-04	.2189E-03	.1052E-02	.5052E-02	-.2445E+00
	22.5	.3684E-06	-.4624E-05	-.1539E-04	.1140E-04	.3917E-04	.1873E-03	.1047E+00
	45.	.3684E-06	-.2045E-05	-.6718E-05	-.8557E-05	-.5317E-05	-.2371E-04	-.7314E-03
	67.5	.3684E-06	-.1653E-06	-.2285E-06	.2558E-06	.3598E-06	.1952E-05	-.8820E-04
	90.	.3684E-06	.4612E-06	.4580E-06	.2191E-06	.3510E-07	.1564E-07	.5905E-04
	112.5	.3684E-06	.4533E-06	.1626E-06	-.2160E-08	-.1499E-07	-.6399E-07	-.1992E-05
	135.	.3684E-06	.3230E-06	.7685E-07	-.1674E-07	-.3145E-08	-.1438E-08	-.2706E-05
	157.5	.3684E-06	.2347E-06	-.6902E-08	-.1013E-07	.6852E-10	.5984E-08	-.7719E-05
	180.	.3684E-06	.2065E-06	-.1240E-07	-.7857E-09	.4674E-09	.5739E-08	-.3766E-05

TABLE III

Clamped circular plate — λ = 1

β	θ° \ a	Influence surface of M_r (ν=0) 0	0.2	0.4	0.6	0.8	Influence surface of M_t (ν=0) 0	0.2	0.4	0.6	0.8
0	0	.1000E+31	.5254E-01	.1095E-01	-.1358E-02	-.1796E-02	.1000E+31	.1259E+00	.6707E-01	.3121E-01	.8516E-02
	22.5	.1000E+31	.6328E-01	.1917E-01	.3412E-02	-.2862E-03	.1000E+31	.1151E+00	.5885E-01	.2644E-01	.7005E-02
	45.	.1000E+31	.8920E-01	.3901E-01	.1493E-01	.3360E-02	.1000E+31	.8920E-01	.3901E-01	.1493E-01	.3360E-02
	67.5	.1000E+31	.1151E+00	.5885E-01	.2644E-01	.7005E-02	.1000E+31	.6328E-01	.1917E-01	.3412E-02	-.2862E-03
	90.	.1000E+31	.1259E+00	.6707E-01	.3121E-01	.8516E-02	.1000E+31	.5254E-01	.1095E-01	-.1358E-02	-.1796E-02
	112.5	.1000E+31	.1151E+00	.5885E-01	.2644E-01	.7005E-02	.1000E+31	.6328E-01	.1917E-01	.3412E-02	-.2862E-03
	135.	.1000E+31	.8920E-01	.3901E-01	.1493E-01	.3360E-02	.1000E+31	.8920E-01	.3901E-01	.1493E-01	.3360E-02
	157.5	.1000E+31	.6328E-01	.1917E-01	.3412E-02	-.2862E-03	.1000E+31	.1151E+00	.5885E-01	.2644E-01	.7005E-02
	180.	.1000E+31	.5254E-01	.1095E-01	-.1358E-02	-.1796E-02	.1000E+31	.1259E+00	.6707E-01	.3121E-01	.8516E-02
0.2	0	.4788E-01	.1000E+31	.4854E-01	.9010E-02	-.4027E-03	.1274E+00	.1000E+31	.1149E+00	.5110E-01	.1423E-01
	22.5	.4788E-01	.1943E+00	.7109E-01	.2101E-01	.3603E-02	.1274E+00	.1222E+00	.7343E-01	.3385E-01	.9039E-02
	45.	.4788E-01	.1319E+00	.8338E-01	.3535E-01	.9038E-02	.1274E+00	.8030E-01	.2677E-01	.8175E-02	.9500E-03
	67.5	.4788E-01	.9045E-01	.6806E-01	.3371E-01	.9378E-02	.1274E+00	.6681E-01	.1344E-01	-.1148E-02	-.2045E-02
	90.	.4788E-01	.3900E-01	.4386E-01	.2227E-01	.6155E-02	.1274E+00	.6478E-01	.1787E-01	.1845E-02	-.8024E-03
	112.5	.4788E-01	.3344E-01	.2197E-01	.9970E-02	.2453E-02	.1274E+00	.6724E-01	.2703E-01	.8732E-02	.1611E-02
	135.	.4788E-01	.1911E-01	.6159E-02	.7463E-03	-.3226E-03	.1274E+00	.7062E-01	.3512E-01	.1460E-01	.3618E-02
	157.5	.4788E-01	.9536E-02	-.3072E-02	-.4635E-02	-.1922E-02	.1274E+00	.7315E-01	.4022E-01	.1833E-01	.4816E-02
	180.	.4788E-01	.6390E-02	-.6078E-02	-.6377E-02	-.2436E-02	.1274E+00	.7406E-01	.4193E-01	.1953E-01	.5205E-02
0.4	0	-.6922E-02	.3295E-01	.1000E+31	.3626E-01	.3190E-02	.7234E-01	.1194E+00	.1000E+31	.9266E-01	.2552E-01
	22.5	-.6922E-02	.3208E-01	.1234E+00	.5905E-01	.1284E-01	.7234E-01	.1015E+00	.5717E-01	.2807E-01	.7987E-02
	45.	-.6922E-02	.2382E-01	.6208E-01	.4310E-01	.1302E-01	.7234E-01	.7581E-01	.2632E-01	-.4713E-03	-.2888E-02
	67.5	-.6922E-02	.1137E-01	.2669E-01	.1927E-01	.5995E-02	.7234E-01	.6019E-01	.2153E-01	.1458E-02	-.1481E-02
	90.	-.6922E-02	.1248E-03	.5871E-02	.4282E-02	.1175E-02	.7234E-01	.5220E-01	.2276E-01	.6936E-02	.9275E-03
	112.5	-.6922E-02	-.8155E-02	-.5743E-02	-.3432E-02	-.1183E-02	.7234E-01	.4837E-01	.2439E-01	.9675E-02	.2251E-02
	135.	-.6922E-02	-.1351E-01	-.1185E-01	-.7093E-02	-.2224E-02	.7234E-01	.4641E-01	.2536E-01	.1117E-01	.2833E-02
	157.5	-.6922E-02	-.1644E-01	-.1474E-01	-.8679E-02	-.2647E-02	.7234E-01	.4547E-01	.2578E-01	.1177E-01	.3057E-02
	180.	-.6922E-02	-.1737E-01	-.1558E-01	-.9119E-02	-.2760E-02	.7234E-01	.4518E-01	.2589E-01	.1193E-01	.3114E-02
0.6	0	-.3874E-01	-.3310E-01	.3146E-02	.1000E+31	.1618E-01	.4025E-01	.6108E-01	.9999E-01	.1000E+31	.5436E-01
	22.5	-.3874E-01	-.3268E-01	.4580E-02	.6177E-01	.2680E-01	.4025E-01	.5617E-01	.5830E-01	.1266E-01	-.2634E-02
	45.	-.3874E-01	-.3247E-01	-.8191E-02	.1058E-01	.6015E-02	.4025E-01	.4651E-01	.3011E-01	.3706E-02	-.3020E-02
	67.5	-.3874E-01	-.3286E-01	-.1785E-01	-.6354E-02	-.1495E-02	.4025E-01	.3829E-01	.2157E-01	.6310E-02	.6166E-03
	90.	-.3874E-01	-.3315E-01	-.2164E-01	-.1084E-01	-.3105E-02	.4025E-01	.3278E-01	.1842E-01	.7357E-02	.1658E-02
	112.5	-.3874E-01	-.3305E-01	-.2238E-01	-.1150E-01	-.3250E-02	.4025E-01	.2937E-01	.1668E-01	.7320E-02	.1827E-02
	135.	-.3874E-01	-.3271E-01	-.2207E-01	-.1121E-01	-.3112E-02	.4025E-01	.2719E-01	.1555E-01	.7012E-02	.1788E-02
	157.5	-.3874E-01	-.3240E-01	-.2164E-01	-.1087E-01	-.2983E-02	.4025E-01	.7603E-01	.1488E-01	.6760E-02	.1732E-02
	180.	-.3874E-01	-.3227E-01	-.2147E-01	-.1074E-01	-.2936E-02	.4025E-01	.2566E-01	.1466E-01	.6648E-02	.1709E-02
0.8	0	-.6121E-01	-.7734E-01	-.8287E-01	-.5637E-01	.1000E+31	.1756E-01	.2580E-01	.3791E-01	.6378E-01	.1000E+31
	22.5	-.6121E-01	-.7484E-01	-.7439E-01	-.4332E-01	-.1282E-02	.1756E-01	.2430E-01	.2858E-01	.1780E-01	-.5911E-02
	45.	-.6121E-01	-.6867E-01	-.6005E-01	-.3556E-01	-.1066E-01	.1756E-01	.2100E-01	.1783E-01	.7658E-02	.7581E-03
	67.5	-.6121E-01	-.6135E-01	-.4758E-01	-.2627E-01	-.7622E-02	.1756E-01	.1769E-01	.1248E-01	.5700E-02	.1340E-02
	90.	-.6121E-01	-.5461E-01	-.3833E-01	-.1971E-01	-.5430E-02	.1756E-01	.1511E-01	.9684E-02	.4493E-02	.1140E-02
	112.5	-.6121E-01	-.4927E-01	-.3208E-01	-.1569E-01	-.4183E-02	.1756E-01	.1329E-01	.8039E-02	.3694E-02	.9417E-03
	135.	-.6121E-01	-.4551E-01	-.2818E-01	-.1336E-01	-.3494E-02	.1756E-01	.1211E-01	.7059E-02	.3203E-02	.8129E-03
	157.5	-.6121E-01	-.4330E-01	-.2605E-01	-.1215E-01	-.3145E-02	.1756E-01	.1145E-01	.6534E-02	.2940E-02	.7432E-03
	180.	-.6121E-01	-.4258E-01	-.2538E-01	-.1177E-01	-.3038E-02	.1756E-01	.1124E-01	.6368E-02	.2857E-02	.7213E-03

Clamped circular plate — $\lambda = 3$

β	θ \ a	Mr (ν=0): 0	Mr: 0.2	Mr: 0.4	Mr: 0.6	Mr: 0.8	Mt (ν=0): 0	Mt: 0.2	Mt: 0.4	Mt: 0.6	Mt: 0.8
0	0	.1000E+31	.2217E-01	-.9439E-02	-.1136E-01	-.4438E-02	.1000E+31	.9432E-01	.4398E-01	.1862E-01	.4814E-02
	22.5	.1000E+31	.3273E-01	-.1616E-02	-.6970E-02	-.3083E-02	.1000E+31	.8375E-01	.3616E-01	.1423E-01	.3460E-02
	45.	.1000E+31	.5824E-01	.1727E-01	.3630E-02	.1884E-03	.1000E+31	.5824E-01	.1727E-01	.3630E-02	.1884E-03
	67.5	.1000E+31	.8375E-01	.3616E-01	.1423E-01	.3460E-02	.1000E+31	.3273E-01	-.1616E-02	-.6970E-02	-.3083E-02
	90.	.1000E+31	.9432E-01	.4398E-01	.1862E-01	.4814E-02	.1000E+31	.2217E-01	-.9439E-02	-.1136E-01	-.4438E-02
	112.5	.1000E+31	.8375E-01	.3616E-01	.1423E-01	.3460E-02	.1000E+31	.3273E-01	-.1616E-02	-.6970E-02	-.3083E-02
	135.	.1000E+31	.5824E-01	.1727E-01	.3630E-02	.1884E-03	.1000E+31	.5824E-01	.1727E-01	.3630E-02	.1884E-03
	157.5	.1000E+31	.3273E-01	-.1616E-02	-.6970E-02	-.3083E-02	.1000E+31	.8375E-01	.3616E-01	.1423E-01	.3460E-02
	180.	.1000E+31	.2217E-01	-.9439E-02	-.1136E-01	-.4438E-02	.1000E+31	.9432E-01	.4398E-01	.1862E-01	.4814E-02
0.2	0	.1992E-01	.1000E+31	.2462E-01	-.4205E-02	-.4217E-02	.9489E-01	.1000E+31	.9037E-01	.3711E-01	.9963E-02
	22.5	.1992E-01	.1644E+00	.4737E-01	.7750E-02	-.2856E-03	.9489E-01	.9105E-01	.4976E-01	.2066E-01	.5111E-02
	45.	.1992E-01	.1031E+00	.6050E-01	.2235E-01	.5136E-02	.9489E-01	.5006E-01	.4904E-02	-.3388E-02	-.2287E-02
	67.5	.1992E-01	.6315E-01	.4693E-01	.2178E-01	.5792E-02	.9489E-01	.3755E-01	-.6776E-02	-.1148E-01	-.4759E-02
	90.	.1992E-01	.3360E-01	.2521E-01	.1211E-01	.3184E-02	.9489E-01	.3629E-01	-.1406E-02	-.7690E-02	-.3341E-02
	112.5	.1992E-01	.1200E-01	.5988E-02	.1795E-02	.1851E-03	.9489E-01	.3921E-01	.8009E-02	-.8284E-03	-.9930E-03
	135.	.1992E-01	-.2658E-02	-.7532E-02	-.5719E-02	-.1989E-02	.9489E-01	.4279E-01	.1597E-01	.4825E-02	.8635E-03
	157.5	.1992E-01	-.1111E-01	-.1524E-01	-.9975E-02	-.3198E-02	.9489E-01	.4538E-01	.2087E-01	.8190E-02	.1935E-02
	180.	.1992E-01	-.1386E-01	-.1771E-01	-.1133E-01	-.3577E-02	.9489E-01	.4630E-01	.2249E-01	.9281E-02	.2276E-02
0.4	0	-.1817E-01	.1620E-01	.1000E+31	.2428E-01	-.7286E-03	.4602E-01	.9237E-01	.1000E+31	.7893E-01	.2112E-01
	22.5	-.1817E-01	.1591E-01	.1067E+00	.4775E-01	.9103E-02	.4602E-01	.7492E-01	.3577E-01	.1561E-01	.4174E-02
	45.	-.1817E-01	.9204E-02	.4793E-01	.3383E-01	.9971E-02	.4602E-01	.5027E-01	.7090E-02	-.1079E-01	-.5794E-02
	67.5	-.1817E-01	-.1120E-02	.1592E-01	.1271E-01	.3943E-02	.4602E-01	.3584E-01	.4194E-02	-.7398E-02	-.3880E-02
	90.	-.1817E-01	-.1010E-01	-.1618E-02	.2646E-03	.3487E-04	.4602E-01	.2897E-01	.6679E-02	-.1579E-02	-.1276E-02
	112.5	-.1817E-01	-.1636E-01	-.1054E-01	-.5495E-02	-.1654E-02	.4602E-01	.2586E-01	.9054E-02	.1902E-02	.1120E-03
	135.	-.1817E-01	-.2015E-01	-.1473E-01	-.7846E-02	-.2267E-02	.4602E-01	.2443E-01	.1044E-01	.3594E-02	.7165E-03
	157.5	-.1817E-01	-.2212E-01	-.1649E-01	-.8694E-02	-.2456E-02	.4602E-01	.2378E-01	.1107E-01	.4232E-02	.9501E-03
	180.	-.1817E-01	-.2272E-01	-.1697E-01	-.8898E-02	-.2496E-02	.4602E-01	.2360E-01	.1124E-01	.4410E-02	.1010E-02
0.6	0	-.2986E-01	-.2824E-01	.2690E-02	.1000E+31	.1428E-01	.2230E-01	.4199E-01	.8318E-01	.1000E+31	.5053E-01
	22.5	-.2986E-01	-.2751E-01	.5001E-02	.5952E-01	.2544E-01	.2230E-01	.3740E-01	.4241E-01	.2961E-02	-.5679E-02
	45.	-.2986E-01	-.2645E-01	-.5779E-02	.1069E-01	.5753E-02	.2230E-01	.2855E-01	.1601E-01	-.4122E-02	-.5257E-02
	67.5	-.2986E-01	-.2577E-01	-.1338E-01	-.4242E-02	-.9252E-03	.2230E-01	.2132E-01	.9082E-02	-.2870E-03	-.1222E-02
	90.	-.2986E-01	-.2505E-01	-.1559E-01	-.7455E-02	-.2075E-02	.2230E-01	.1674E-01	.7128E-02	.1521E-02	.3822E-04
	112.5	-.2986E-01	-.2413E-01	-.1531E-01	-.7414E-02	-.1999E-02	.2230E-01	.1402E-01	.6219E-02	.1979E-02	.3478E-03
	135.	-.2986E-01	-.2324E-01	-.1441E-01	-.6778E-02	-.1763E-02	.2230E-01	.1244E-01	.5634E-02	.1989E-02	.4008E-03
	157.5	-.2986E-01	-.2260E-01	-.1368E-01	-.6284E-02	-.1596E-02	.2230E-01	.1160E-01	.5283E-02	.1917E-02	.3971E-03
	180.	-.2986E-01	-.2237E-01	-.1342E-01	-.6109E-02	-.1539E-02	.2230E-01	.1134E-01	.5165E-02	.1885E-02	.3921E-03
0.8	0	-.3364E-01	-.4984E-01	-.6096E-01	-.4405E-01	.1000E+31	.8716E-02	.1619E-01	.2922E-01	.5779E-01	.1000E+31
	22.5	-.3364E-01	-.4754E-01	-.5274E-01	-.3100E-01	.2305E-02	.8716E-02	.1486E-01	.2034E-01	.1254E-01	-.7618E-02
	45.	-.3364E-01	-.4193E-01	-.3920E-01	-.2367E-01	-.7065E-02	.8716E-02	.1198E-01	.1050E-01	.3403E-02	-.5105E-03
	67.5	-.3364E-01	-.3340E-01	-.2791E-01	-.1528E-01	-.4343E-02	.8716E-02	.9202E-02	.6058E-02	.2182E-02	.3242E-03
	90.	-.3364E-01	-.2957E-01	-.1995E-01	-.9718E-02	-.2517E-02	.8716E-02	.7147E-02	.3984E-02	.1490E-02	.2926E-03
	112.5	-.3364E-01	-.2507E-01	-.1485E-01	-.6556E-02	-.1573E-02	.8716E-02	.5775E-02	.2874E-02	.1044E-02	.2058E-03
	135.	-.3364E-01	-.2198E-01	-.1182E-01	-.4844E-02	-.1096E-02	.8716E-02	.4920E-02	.2256E-02	.7837E-03	.1484E-03
	157.5	-.3364E-01	-.2021E-01	-.1023E-01	-.4001E-02	-.8717E-03	.8716E-02	.4456E-02	.1941E-02	.6502E-03	.1186E-03
	180.	-.3364E-01	-.1964E-01	-.9736E-02	-.3748E-02	-.8054E-03	.8716E-02	.4309E-02	.1844E-02	.6093E-03	.1094E-03

Column groups: Influence surface of M_r ($\nu=0$) and Influence surface of M_t ($\nu=0$), each for a = 0, 0.2, 0.4, 0.6, 0.8.

Clamped circular plate — λ = 5

β	θ° \ a	M_r (v=0): 0	M_r: 0.2	M_r: 0.4	M_r: 0.6	M_r: 0.8	M_t (v=0): 0	M_t: 0.2	M_t: 0.4	M_t: 0.6	M_t: 0.8
0	0	.1000E+31	-.1039E-01	-.2741E-01	-.1373E-01	-.3901E-02	.1000E+31	.5596E-01	.1742E-01	.4822E-02	.8589E-03
	22.5	.1000E+31	-.6721E-03	-.1743E-01	-.1102E-01	-.3204E-02	.1000E+31	.4625E-01	.1144E-01	.2104E-02	.1618E-03
	45.	.1000E+31	.2279E-01	-.2995E-02	-.4457E-02	-.1521E-02	.1000E+31	.2279E-01	-.2995E-02	-.4457E-02	-.1521E-02
	67.5	.1000E+31	.4625E-01	.1144E-01	.2104E-02	.1618E-03	.1000E+31	-.6721E-03	-.1743E-01	-.1102E-01	-.3204E-02
	90.	.1000E+31	.5596E-01	.1742E-01	.4822E-02	.8589E-03	.1000E+31	-.1039E-01	-.2341E-01	-.1373E-01	-.3901E-02
	112.5	.1000E+31	.4625E-01	.1144E-01	.2104E-02	.1618E-03	.1000E+31	-.6721E-03	-.1743E-01	-.1102E-01	-.3204E-02
	135.	.1000E+31	.2279E-01	-.2995E-02	-.4457E-02	-.1521E-02	.1000E+31	.2279E-01	-.2995E-02	-.4457E-02	-.1521E-02
	157.5	.1000E+31	-.6721E-03	-.1743E-01	-.1102E-01	-.3204E-02	.1000E+31	.4625E-01	.1144E-01	.2104E-02	.1618E-03
	180.	.1000E+31	-.1039E-01	-.2341E-01	-.1373E-01	-.3901E-02	.1000E+31	.5596E-01	.1742E-01	.4822E-02	.8589E-03
0.2	0	-.1064E-01	.1000E+31	-.8792E-02	-.1957E-01	-.7624E-02	.5593E-01	.1000E+31	.5594E-01	.1699E-01	.3671E-02
	22.5	-.1064E-01	.1273E+00	.1389E-01	-.8723E-02	-.4386E-02	.5593E-01	.5065E-01	.1920E-01	.4249E-02	.3720E-03
	45.	-.1064E-01	.6413E-01	.2850E-01	.4916E-02	.1512E-03	.5593E-01	.1360E-01	-.1764E-01	-.1248E-01	-.4074E-02
	67.5	-.1064E-01	.2934E-01	.1984E-01	.6552E-02	.1251E-02	.5593E-01	.5045E-02	-.2289E-01	-.1545E-01	-.4695E-02
	90.	-.1064E-01	.6206E-02	.5026E-02	.1998E-02	.3261E-03	.5593E-01	.6428E-02	-.1492E-01	-.1048E-01	-.3083E-02
	112.5	-.1064E-01	-.8853E-02	-.5008E-02	-.2401E-02	-.6574E-03	.5593E-01	.1048E-01	-.5974E-02	-.4975E-02	-.1463E-02
	135.	-.1064E-01	-.1798E-01	-.1144E-01	-.4955E-02	-.1171E-02	.5593E-01	.1420E-01	.3681E-03	-.1368E-02	-.4899E-03
	157.5	-.1064E-01	-.2277E-01	-.1454E-01	-.6069E-02	-.1352E-02	.5593E-01	.1655E-01	.3751E-02	.4329E-03	-.4388E-04
	180.	-.1064E-01	-.2424E-01	-.1544E-01	-.6361E-02	-.1390E-02	.5593E-01	.1733E-01	.4789E-02	.9581E-03	.7864E-04
0.4	0	-.2441E-01	-.1025E-01	.1000E+31	.3152E-02	-.9398E-02	.1730E-01	.5592E-01	.1000E+31	.5413E-01	.1250E-01
	22.5	-.2441E-01	-.8581E-02	.7200E-01	.2241E-01	.7117E-03	.1730E-01	.4026E-01	.4912E-02	-.2834E-02	-.1466E-02
	45.	-.2441E-01	-.1009E-01	.2270E-01	.1541E-01	.3687E-02	.1730E-01	.1970E-01	-.1436E-01	-.1986E-01	-.7470E-02
	67.5	-.2441E-01	-.1364E-01	.1893E-02	.3226E-02	.8730E-03	.1730E-01	.9560E-02	-.1063E-01	-.1181E-01	-.4127E-02
	90.	-.2441E-01	-.1589E-01	-.5860E-02	-.1823E-02	-.4461E-03	.1730E-01	.5978E-02	-.5017E-02	-.4946E-02	-.1549E-02
	112.5	-.2441E-01	-.1655E-01	-.7751E-02	-.2793E-02	-.5909E-03	.1730E-01	.4971E-02	-.1572E-02	-.1733E-02	-.5061E-03
	135.	-.2441E-01	-.1634E-01	-.7577E-02	-.2490E-02	-.4310E-03	.1730E-01	.4740E-02	.3971E-04	-.5227E-03	-.1729E-03
	157.5	-.2441E-01	-.1595E-01	-.7056E-02	-.2101E-02	-.2905E-03	.1730E-01	.4690E-02	.6660E-03	-.1320E-03	-.8403E-04
	180.	-.2441E-01	-.1577E-01	-.6831E-02	-.1947E-02	-.2410E-03	.1730E-01	.4679E-02	.8221E-03	-.4640E-04	-.6781E-04
0.6	0	-.1580E-01	-.2449E-01	-.1001E-01	.1000E+31	.4049E-02	.4655E-02	.1719E-01	.5480E-01	.1000E+31	.4109E-01
	22.5	-.1580E-01	-.2272E-01	-.4084E-02	.4396E+01	.1783E-01	.4655E-02	.1384E-01	.1832E-01	-.1325E-01	-.1086E-01
	45.	-.1580E-01	-.1911E-01	-.7274E-02	.5309E-02	.3082E-02	.4655E-02	.7945E-02	-.7041E-03	-.1224E-01	-.6937E-02
	67.5	-.1580E-01	-.1552E-01	-.8195E-02	-.2481E-02	-.5403E-03	.4655E-02	.3952E-02	-.2316E-02	-.4770E-02	-.1964E-02
	90.	-.1580E-01	-.1241E-01	-.6422E-02	-.2391E-02	-.5231E-03	.4655E-02	.2061E-02	-.1276E-02	-.1602E-02	-.5210E-03
	112.5	-.1580E-01	-.9920E-02	-.4359E-02	-.1310E-02	-.1854E-03	.4655E-02	.1266E-02	-.5726E-03	-.5785E-03	-.1632E-03
	135.	-.1580E-01	-.8159E-02	-.2900E-02	-.5737E-03	.8335E-05	.4655E-02	.9266E-03	-.2807E-03	-.2788E-03	-.7888E-04
	157.5	-.1580E-01	-.7129E-02	-.2106E-02	-.2147E-03	.8905E-04	.4655E-02	.7793E-03	-.1834E-03	-.1968E-03	-.5982E-04
	180.	-.1580E-01	-.6792E-02	-.1860E-02	-.1114E-03	.1099E-03	.4655E-02	.7368E-03	-.1619E-03	-.1802E-03	-.5646E-04
0.8	0	-.6455E-02	-.1752E-01	-.3218E-01	-.2866E-01	.1000E+31	.7656E-03	.4303E-02	.1466E-01	.4448E-01	.1000E+31
	22.5	-.6455E-02	-.1589E-01	-.2485E-01	-.1504E-01	.6354E-02	.7656E-03	.3547E-02	.7721E-02	.2979E-02	-.1089E-01
	45.	-.6455E-02	-.1215E-01	-.1402E-01	-.8715E-02	-.2460E-02	.7656E-03	.2079E-02	.1461E-02	-.1892E-02	-.1939E-02
	67.5	-.6455E-02	-.8236E-02	-.6645E-02	-.3274E-02	-.7690E-03	.7656E-03	.9361E-03	-.1068E-03	-.8513E-03	-.4131E-03
	90.	-.6455E-02	-.5193E-02	-.2658E-02	-.7466E-03	-.3351E-04	.7656E-03	.3157E-03	-.3163E-03	-.3865E-03	-.1334E-03
	112.5	-.6455E-02	-.3186E-02	-.7976E-03	.1302E-03	.1447E-03	.7656E-03	.2986E-04	-.2933E-03	-.2159E-03	-.6192E-04
	135.	-.6455E-02	-.2009E-02	-.1578E-04	.3734E-03	.1640E-03	.7656E-03	-.9187E-04	-.2519E-03	-.1456E-03	-.3733E-04
	157.5	-.6455E-02	-.1413E-02	.2793E-03	.4204E-03	.1511E-03	.7656E-03	-.1396E-03	-.2250E-03	-.1153E-03	-.2766E-04
	180.	-.6455E-02	-.1233E-02	.3533E-03	.4241E-03	.1445E-03	.7656E-03	-.1521E-03	-.2161E-03	-.1067E-03	-.2500E-04

Clamped circular plate — λ = 7

β	θ° \ a	Influence surface of M_r (ν=0): 0	0.2	0.4	0.6	0.8	Influence surface of M_t (ν=0): 0	0.2	0.4	0.6	0.8
0	0	.1000E+31	-.2164E-01	-.1759E-01	-.5447E-02	-.6511E-03	.1000E+31	.3521E-01	.6429E-02	.6482E-03	-.5891E-04
	22.5	.1000E+31	-.1332E-01	-.1407E-01	-.4554E-02	-.5644E-03	.1000E+31	.2688E-01	.2908E-02	-.2444E-03	-.1456E-03
	45.	.1000E+31	.6782E-02	-.5583E-02	-.2399E-02	-.3550E-03	.1000E+31	.6782E-02	-.5583E-02	-.2399E-02	-.3550E-03
	67.5	.1000E+31	.2688E-01	.2900E-02	-.2444E-03	-.1456E-03	.1000E+31	-.1332E-01	-.1407E-01	-.4554E-02	-.5644E-03
	90.	.1000E+31	.3521E-01	.6425E-02	.6482E-03	-.5891E-04	.1000E+31	-.2164E-01	-.1759E-01	-.5447E-02	-.6511E-03
	112.5	.1000E+31	.2688E-01	.2908E-02	-.2444E-03	-.1456E-03	.1000E+31	-.1332E-01	-.1407E-01	-.4554E-02	-.5644E-03
	135.	.1000E+31	.6782E-02	-.5583E-02	-.2399E-02	-.3550E-03	.1000E+31	.6782E-02	-.5583E-02	-.2399E-02	-.3550E-03
	157.5	.1000E+31	-.1332E-01	-.1407E-01	-.4554E-02	-.5644E-03	.1000E+31	.2688E-01	.2908E-02	-.2444E-03	-.1456E-03
	180.	.1000E+31	-.2164E-01	-.1759E-01	-.5447E-02	-.6511E-03	.1000E+31	.3521E-01	.6425E-02	.6482E-03	-.5891E-04
0.2	0	-.2167E-01	.1000E+31	-.2161E-01	-.1723E-01	-.4457E-02	.3520E-01	.1000E+31	.3520E-01	.6447E-02	.7045E-03
	22.5	-.2167E-01	.9648E-01	-.5274E-03	-.9481E-02	-.2753E-02	.3520E-01	.2700E-01	.4735E-02	-.9971E-03	-.5760E-03
	45.	-.2167E-01	.4201E-01	.1305E-01	-.2774E-03	-.5596E-03	.3520E-01	-.3852E-02	-.2066E-01	-.8645E-02	-.1751E-02
	67.5	-.2167E-01	.1307E-01	.7803E-02	.1187E-02	-.2181E-04	.3520E-01	-.7143E-02	-.1908E-01	-.7483E-02	-.1224E-02
	90.	-.2167E-01	-.3155E-02	.3945E-03	-.9284E-04	-.9383E-04	.3520E-01	-.3138E-02	-.1056E-01	-.3731E-02	-.4090E-03
	112.5	-.2167E-01	-.1164E-01	-.3709E-02	-.7922E-03	-.5809E-04	.3520E-01	.1316E-02	-.4254E-02	-.1348E-02	-.5087E-04
	135.	-.2167E-01	-.1561E-01	-.5144E-02	-.7833E-03	.6592E-04	.3520E-01	.4395E-02	-.1025E-02	-.4130E-03	-.3480E-05
	157.5	-.2167E-01	-.1719E-01	-.5407E-02	-.5871E-03	.1643E-03	.3520E-01	.5943E-02	.2902E-03	-.1627E-03	-.3923E-04
	180.	-.2167E-01	-.1760E-01	-.5404E-02	-.4967E-03	.1985E-03	.3520E-01	.6420E-02	.6252E-03	-.1245E-03	-.5844E-04
0.4	0	-.1768E-01	-.2187E-01	.1000E+31	-.1995E-01	-.1214E-01	.6393E-02	.3515E-01	.1000E+31	.3503E-01	.5774E-02
	22.5	-.1768E-01	-.1819E-01	.4781E-01	.6314E-02	-.3186E-02	.6393E-02	.2212E-01	-.1167E-01	-.1042E-01	-.3070E-02
	45.	-.1768E-01	-.1474E-01	.9019E-02	.5038E-02	.4855E-03	.6393E-02	.6963E-02	-.1778E-01	-.1534E-01	-.4236E-02
	67.5	-.1768E-01	-.1281E-01	-.1673E-02	.1693E-03	-.5240E-04	.6393E-02	.1394E-02	-.8679E-02	-.5615E-02	-.1149E-02
	90.	-.1768E-01	-.1054E-01	-.3028E-02	-.5631E-03	-.6256E-04	.6393E-02	.3823E-03	-.2998E-02	-.1221E-02	-.4969E-04
	112.5	-.1768E-01	-.8472E-02	-.2085E-02	-.1320E-03	.8267E-04	.6393E-02	.4554E-03	-.8516E-03	-.1514E-03	.7143E-04
	135.	-.1768E-01	-.6779E-02	-.1132E-02	.2044E-03	.1422E-03	.6393E-02	.5794E-03	-.2580E-03	-.2981E-04	.2630E-04
	157.5	-.1768E-01	-.5754E-02	-.5975E-03	.3402E-03	.1487E-03	.6393E-02	.6123E-03	-.1429E-03	-.5926E-04	-.4843E-05
	180.	-.1768E-01	-.5416E-02	-.4351E-03	.3716E-03	.1464E-03	.6393E-02	.6156E-03	-.1314E-03	-.7522E-04	-.1338E-04
0.6	0	-.5516E-02	-.1791E-01	-.2166E-01	.1000E+31	-.9375E-02	.5696E-03	.6308E-02	.3496E-01	.1000E+31	.3041E-01
	22.5	-.5516E-02	-.1560E-01	-.1133E-01	.2564E-01	.8452E-02	.5696E-03	.4401E-02	.5609E-02	-.2135E-01	-.1250E-01
	45.	-.5516E-02	-.1094E-01	-.6898E-02	.7445E-03	.6822E-03	.5696E-03	.1521E-02	-.4112E-02	-.9988E-02	-.4474E-02
	67.5	-.5516E-02	-.6724E-02	-.3587E-02	-.9743E-03	-.1895E-03	.5696E-03	.1582E-03	-.2092E-02	-.1989E-02	-.4521E-03
	90.	-.5516E-02	-.3781E-02	-.1220E-02	-.1076E-03	.5226E-04	.5696E-03	-.1558E-03	-.5976E-03	-.1665E-03	.6071E-04
	112.5	-.5516E-02	-.1998E-02	-.1044E-03	.2358E-03	.9653E-04	.5696E-03	-.1703E-03	-.1573E-03	.3669E-04	.4220E-04
	135.	-.5516E-02	-.1032E-02	.2817E-03	.2569E-03	.6693E-04	.5696E-03	-.1530E-03	-.7833E-04	.1340E-04	.1257E-04
	157.5	-.5516E-02	-.5778E-03	.3806E-03	.2169E-03	.4215E-04	.5696E-03	-.1460E-03	-.7337E-04	-.6443E-05	-.2888E-05
	180.	-.5516E-02	-.4466E-03	.3959E-03	.1988E-03	.3410E-04	.5696E-03	-.1447E-03	-.7479E-04	-.1150E-04	.1094E-05
0.8	0	-.2880E-03	-.5257E-02	-.1833E-01	-.2494E-01	.1000E+31	-.1764E-03	.5101E-03	.5886E-02	.3147E-01	.1000E+31
	22.5	-.2880E-03	-.4357E-02	-.1210E-01	-.9165E-02	.5815E-02	-.1764E-03	.2699E-03	.1713E-02	-.2636E-02	-.1239E-01
	45.	-.2880E-03	-.2500E-02	-.4357E-02	-.2929E-02	-.7708E-03	-.1764E-03	-.8804E-04	-.6585E-03	-.2240E-02	-.1434E-02
	67.5	-.2880E-03	-.9329E-03	-.7683E-03	-.1852E-03	.4802E-04	-.1764E-03	-.2191E-03	-.4236E-03	-.3643E-03	-.6869E-04
	90.	-.2880E-03	-.5677E-04	.2561E-03	.7695E-03	.1024E-03	-.1764E-03	-.1968E-03	-.1388E-03	-.1468E-04	.2173E-04
	112.5	-.2880E-03	.3053E-03	.3649E-03	.1800E-03	.3888E-04	-.1764E-03	-.1463E-03	-.4309E-04	.1447E-04	.1120E-04
	135.	-.2880E-03	.4125E-03	.2836E-03	.8319E-04	.6368E-05	-.1764E-03	-.1103E-03	-.1824E-04	.9153E-05	.4574E-05
	157.5	-.2880E-03	.4287E-03	.2150E-03	.3807E-04	-.3934E-05	-.1764E-03	-.9143E-04	-.1173E-04	.5760E-05	.2548E-05
	180.	-.2880E-03	.4276E-03	.1918E-03	.2611E-04	-.5994E-05	-.1764E-03	-.8571E-04	-.1029E-04	.4916E-05	.2138E-05

Clamped circular plate — λ = 11.

β	θ° \ a	Mr (ν=0): 0	0.2	0.4	0.6	0.8	Mt (ν=0): 0	0.2	0.4	0.6	0.8
0	0	.1000E+31	-.2304E-01	-.4279E-02	.3423E-03	.1306E-03	.1000E+31	.1375E-01	.3706E-03	-.1058E-03	-.7357E-05
	22.5	.1000E+31	-.1765E-01	-.3598E-02	.2767E-03	.1104E-03	.1000E+31	.8365E-02	-.3103E-03	-.4021E-04	.1285E-04
	45.	.1000E+31	-.4642E-02	-.1954E-02	.1182E-03	.6163E-04	.1000E+31	-.4642E-02	-.1954E-02	.1182E-03	.6163E-04
	67.5	.1000E+31	.8365E-02	-.3103E-03	-.4021E-04	.1285E-04	.1000E+31	-.1765E-01	-.3598E-02	.2767E-03	.1104E-03
	90.	.1000E+31	.1375E-01	.3706E-03	-.1058E-03	-.7357E-05	.1000E+31	-.2304E-01	-.4279E-02	.3423E-03	.1306E-03
	112.5	.1000E+31	.8365E-02	-.3103E-03	-.4021E-04	.1285E-04	.1000E+31	-.1765E-01	-.3598E-02	.2767E-03	.1104E-03
	135.	.1000E+31	-.4642E-02	-.1954E-02	.1182E-03	.6163E-04	.1000E+31	-.4642E-02	-.1954E-02	.1182E-03	.6163E-04
	157.5	.1000E+31	-.1765E-01	-.3598E-02	.2767E-03	.1104E-03	.1000E+31	.8365E-02	-.3103E-03	-.4021E-04	.1285E-04
	180.	.1000E+31	-.2304E-01	-.4279E-02	.3423E-03	.1306E-03	.1000E+31	.1375E-01	.3706E-03	-.1058E-03	-.7357E-05
0.2	0	-.2304E-01	.1000E+31	-.2303E-01	-.4278E-02	.2650E-03	.1375E-01	.1000E+31	.1375E-01	.3719E-03	-.9940E-04
	22.5	-.2304E-01	.6383E-01	-.7076E-02	-.2143E-02	.2135E-03	.1375E-01	-.4648E-03	-.4044E-02	-.8092E-03	.1240E-04
	45.	-.2304E-01	.1809E-01	.2141E-02	-.2063E-03	.5604E-04	.1375E-01	-.1697E-01	-.1181E-01	-.8479E-03	.2116E-03
	67.5	-.2304E-01	.5224E-03	.6684E-03	-.1334E-03	-.1544E-04	.1375E-01	-.1143E-01	-.5944E-02	.1186E-03	.2096E-03
	90.	-.2304E-01	-.5124E-02	-.4090E-03	-.3495E-04	.2288E-05	.1375E-01	-.5124E-02	-.1651E-02	.3375E-03	.9140E-04
	112.5	-.2304E-01	-.5894E-02	-.2664E-03	.9310E-04	.1059E-04	.1375E-01	-.1644E-02	-.2789E-03	.1803E-03	.2124E-04
	135.	-.2304E-01	-.5228E-02	.7991E-04	.1382E-03	.2943E-05	.1375E-01	-.2075E-03	-.5975E-04	.5457E-04	.2670E-05
	157.5	-.2304E-01	-.4539E-02	.2833E-03	.1364E-03	-.4531E-05	.1375E-01	.2690E-03	-.8481E-04	.4801E-05	.1360E-05
	180.	-.2304E-01	-.4279E-02	.3427E-03	.1315E-03	-.7060E-05	.1375E-01	.3705E-03	-.1059E-03	-.6469E-05	.1943E-05
0.4	0	-.4279E-02	-.2304E-01	.1000E+31	-.2302E-01	-.4294E-02	.3704E-03	.1375E-01	.1000E+31	.1376E-01	.4240E-03
	22.5	-.4279E-02	-.1711E-01	.2255E-01	-.1526E-02	-.1011E-02	.3704E-03	.5984E-02	-.2249E-01	-.8975E-02	-.8162E-03
	45.	-.4279E-02	-.9307E-02	.6915E-03	.1304E-03	-.8294E-04	.3704E-03	-.3662E-03	-.9749E-02	-.2974E-02	.2010E-03
	67.5	-.4279E-02	-.4373E-02	-.6568E-03	-.8598E-04	-.1583E-04	.3704E-03	-.9038E-03	-.1492E-02	.2414E-03	.2097E-03
	90.	-.4279E-02	-.1652E-02	.3253E-04	.6435E-04	.7087E-05	.3704E-03	-.4091E-03	.3287E-04	.1820E-03	.2561E-04
	112.5	-.4279E-02	-.3729E-03	.2355E-03	.4371E-04	-.3555E-05	.3704E-03	-.1723E-03	.7240E-04	.3339E-04	-.3876E-05
	135.	-.4279E-02	.1348E-03	.2017E-03	.7792E-05	-.6392E-05	.3704E-03	-.1145E-03	.1620E-04	.3799E-05	-.1732E-05
	157.5	-.4279E-02	.3049E-03	.1503E-03	-.6607E-05	-.5327E-05	.3704E-03	-.1063E-03	-.3079E-05	.1793E-05	-.1214E-06
	180.	-.4279E-02	.3428E-03	.1317E-03	-.9715E-05	-.4711E-05	.3704E-03	-.1059E-03	-.6437E-05	.2117E-05	.1893E-06
0.6	0	.3458E-03	-.4280E-02	-.2312E-01	.1000E+31	-.2056E-01	-.1057E-03	.3702E-03	.1375E-01	.1000E+31	.1354E-01
	22.5	.3458E-03	-.3006E-02	-.9130E-02	.7432E-02	.1792E-03	-.1057E-03	.4980E-04	-.1442E-02	-.1876E-01	-.6830E-02
	45.	.3458E-03	-.9709E-03	-.1613E-02	-.3063E-03	-.1287E-03	-.1057E-03	-.1825E-03	-.1247E-02	-.1421E-02	.9370E-04
	67.5	.3458E-03	.9475E-04	.1157E-03	.7390E-04	.1261E-04	-.1057E-03	-.1064E-03	.4049E-04	.2379E-03	.1115E-03
	90.	.3458E-03	.3401E-03	.1828E-03	.3407E-04	-.1437E-05	-.1057E-03	-.3478E-04	.6484E-04	.2449E-04	-.2747E-05
	112.5	.3458E-03	.2890E-03	.6326E-04	-.5984E-05	-.3607E-05	-.1057E-03	-.1385E-04	.1410E-04	-.1618E-05	-.2318E-05
	135.	.3458E-03	.2026E-03	.8477E-05	-.8577E-05	-.9795E-06	-.1057E-03	-.9009E-05	.3045E-05	-.8362E-06	-.2618E-06
	157.5	.3458E-03	.1488E-03	-.7009E-05	-.5822E-05	.3289E-07	-.1057E-03	-.7092E-05	.2077E-05	.3986E-07	-.4838E-07
	180.	.3458E-03	.1318E-03	-.9829E-05	-.4773E-05	.2219E-06	-.1057E-03	-.6439E-05	.2112E-05	.1702E-06	-.5006E-07
0.8	0	.1358E-03	.3523E-03	-.4394E-02	-.2328E-01	.1000E+31	-.5046E-05	-.1037E-03	.3601E-03	.1348E-01	.1000E+31
	22.5	.1358E-03	.3806E-03	-.1597E-02	-.4348E-02	.1377E-02	-.5046E-05	-.8232E-04	-.2439E-03	-.3195E-02	-.8448E-02
	45.	.1358E-03	.3494E-03	.2206E-03	.6811E-05	-.2018E-04	-.5046E-05	-.3467E-04	-.3594E-04	-.1158E-04	.1449E-03
	67.5	.1358E-03	.2179E-03	.1763E-03	.7477E-04	.1255E-04	-.5046E-05	-.3238E-05	.3926E-04	.6094E-04	.2235E-04
	90.	.1358E-03	.9390E-04	.2369E-04	-.5603E-05	-.3954E-05	-.5046E-05	.4252E-05	.8605E-05	-.3360E-06	-.2619E-05
	112.5	.1358E-03	.2642E-04	-.1013E-04	-.5253E-05	-.3969E-06	-.5046E-05	.3747E-05	.4736E-06	-.1236E-05	-.2954E-06
	135.	.1358E-03	-.9103E-07	-.9213E-05	-.1002E-05	.3218E-06	-.5046E-05	.2772E-05	.1049E-07	-.2049E-06	.1691E-07
	157.5	.1358E-03	-.8360E-05	-.6004E-05	.1530E-06	.2370E-06	-.5046E-05	.2302E-05	.9093E-07	-.6750E-07	.2072E-08
	180.	.1358E-03	-.1907E-04	-.4935E-05	.3300E-06	.1876E-06	-.5046E-05	.2171E-05	.1036E-06	-.5995E-07	-.3234E-09

TABLE IV

Simply supported circular plate							$\lambda = 1$				
β		Influence surface of M_r (ν=0.3)					Influence surface of M_t (ν=0.3)				
	θ° \ α	0	0.2	0.4	0.6	0.8	0	0.2	0.4	0.6	0.8
0	0	.1000E+31	.1608E+00	.9074E-01	.5092E-01	.2288E-01	.1000E+31	.2137E+00	.1356E+00	.8333E-01	.3971E-01
	22.5	.1000E+31	.1686E+00	.9732E-01	.5567E-01	.2535E-01	.1000E+31	.2060E+00	.1291E+00	.7859E-01	.3725E-01
	45.	.1000E+31	.1873E+00	.1132E+00	.6713E-01	.3130E-01	.1000E+31	.1873E+00	.1132E+00	.6713E-01	.3130E-01
	67.5	.1000E+31	.2060E+00	.1291E+00	.7859E-01	.3725E-01	.1000E+31	.1686E+00	.9732E-01	.5567E-01	.2535E-01
	90.	.1000E+31	.2137E+00	.1356E+00	.8333E-01	.3971E-01	.1000E+31	.1608E+00	.9074E-01	.5092E-01	.2288E-01
	112.5	.1000E+31	.2060E+00	.1291E+00	.7859E-01	.3725E-01	.1000E+31	.1686E+00	.9732E-01	.5567E-01	.2535E-01
	135.	.1000E+31	.1873E+00	.1132E+00	.6713E-01	.3130E-01	.1000E+31	.1873E+00	.1132E+00	.6713E-01	.3130E-01
	157.5	.1000E+31	.1686E+00	.9732E-01	.5567E-01	.2535E-01	.1000E+31	.2060E+00	.1291E+00	.7859E-01	.3725E-01
	180.	.1000E+31	.1608E+00	.9074E-01	.5092E-01	.2288E-01	.1000E+31	.2137E+00	.1356E+00	.8333E-01	.3971E-01
0.2	0	.1606E+00	.1000E+31	.1539E+00	.7984E-01	.3494E-01	.2160E+00	.1000E+31	.2017E+00	.1154E+00	.5382E-01
	22.5	.1606E+00	.3074E+00	.1641E+00	.8758E-01	.3898E-01	.2160E+00	.2558E+00	.1654E+00	.9860E-01	.4626E-01
	45.	.1606E+00	.2318E+00	.1619E+00	.9514E-01	.4412E-01	.2160E+00	.1937E+00	.1183E+00	.7029E-01	.3265E-01
	67.5	.1606E+00	.1847E+00	.1405E+00	.8894E-01	.4265E-01	.2160E+00	.1660E+00	.9613E-01	.5434E-01	.2452E-01
	90.	.1606E+00	.1505E+00	.1137E+00	.7366E-01	.3574E-01	.2160E+00	.1530E+00	.9059E-01	.5126E-01	.2312E-01
	112.5	.1606E+00	.1252E+00	.8967E-01	.5716E-01	.2752E-01	.2160E+00	.1474E+00	.9217E-01	.5428E-01	.2509E-01
	135.	.1606E+00	.1075E+00	.7168E-01	.4395E-01	.2072E-01	.2160E+00	.1454E+00	.9557E-01	.5850E-01	.2766E-01
	157.5	.1606E+00	.9708E-01	.6078E-01	.3574E-01	.1644E-01	.2160E+00	.1448E+00	.9828E-01	.6160E-01	.2952E-01
	180.	.1606E+00	.9363E-01	.5714E-01	.3299E-01	.1500E-01	.2160E+00	.1447E+00	.9928E-01	.6271E-01	.3018E-01
0.4	0	.8992E-01	.1532E+00	.1000E+31	.1369E+00	.5554E-01	.1450E+00	.2092E+00	.1000E+31	.1765E+00	.7715E-01
	22.5	.8992E-01	.1461E+00	.2239E+00	.1392E+00	.6057E-01	.1450E+00	.1902E+00	.1715E+00	.1123E+00	.5418E-01
	45.	.8992E-01	.1273E+00	.1485E+00	.1094E+00	.5392E-01	.1450E+00	.1595E+00	.1155E+00	.6735E-01	.3098E-01
	67.5	.8992E-01	.1060E+00	.1040E+00	.7647E-01	.3890E-01	.1450E+00	.1371E+00	.9340E-01	.5364E-01	.2406E-01
	90.	.8992E-01	.8768E-01	.7480E-01	.5237E-01	.2628E-01	.1450E+00	.1230E+00	.8372E-01	.4994E-01	.2302E-01
	112.5	.8992E-01	.7366E-01	.5577E-01	.3674E-01	.1795E-01	.1450E+00	.1144E+00	.7907E-01	.4888E-01	.2311E-01
	135.	.8992E-01	.6403E-01	.4397E-01	.2732E-01	.1293E-01	.1450E+00	.1093E+00	.7666E-01	.4845E-01	.2325E-01
	157.5	.8992E-01	.5844E-01	.3755E-01	.2233E-01	.1032E-01	.1450E+00	.1067E+00	.7545E-01	.4822E-01	.2330E-01
	180.	.8992E-01	.5662E-01	.3552E-01	.2077E-01	.9508E-02	.1450E+00	.1058E+00	.7508E-01	.4814E-01	.2331E-01
0.6	0	.4951E-01	.7836E-01	.1359E+00	.1000E+31	.1001E+00	.1040E+00	.1358E+00	.1905E+00	.1000E+31	.1257E+00
	22.5	.4951E-01	.7534E-01	.1201E+00	.1563E+00	.8786E-01	.1040E+00	.1296E+00	.1444E+00	.1063E+00	.5316E-01
	45.	.4951E-01	.6739E-01	.8673E-01	.8438E-01	.4903E-01	.1040E+00	.1163E+00	.1022E+00	.6454E-01	.2890E-01
	67.5	.4951E-01	.5744E-01	.5994E-01	.4914E-01	.2673E-01	.1040E+00	.1035E+00	.8142E-01	.5121E-01	.2376E-01
	90.	.4951E-01	.4819E-01	.4226E-01	.3071E-01	.1575E-01	.1040E+00	.9372E-01	.7046E-01	.4500E-01	.2148E-01
	112.5	.4951E-01	.4091E-01	.3128E-01	.2079E-01	.1019E-01	.1040E+00	.8692E-01	.6407E-01	.4130E-01	.1996E-01
	135.	.4951E-01	.3586E-01	.2479E-01	.1543E-01	.7310E-02	.1040E+00	.8252E-01	.6023E-01	.3898E-01	.1892E-01
	157.5	.4951E-01	.3294E-01	.2138E-01	.1277E-01	.5914E-02	.1040E+00	.8005E-01	.5814E-01	.3767E-01	.1832E-01
	180.	.4951E-01	.3199E-01	.2031E-01	.1196E-01	.5496E-02	.1040E+00	.7926E-01	.5747E-01	.3725E-01	.1812E-01
0.8	0	.2177E-01	.3359E-01	.5424E-01	.9927E-01	.1000E+31	.7531E-01	.9224E-01	.1129E+00	.1505E+00	.1000E+31
	22.5	.2177E-01	.3232E-01	.4911E-01	.7442E-01	.7401E-01	.7531E-01	.8949E-01	.9890E-01	.8875E-01	.4437E-01
	45.	.2177E-01	.2903E-01	.3704E-01	.3966E-01	.2640E-01	.7531E-01	.8305E-01	.7829E-01	.5696E-01	.2766E-01
	67.5	.2177E-01	.2489E-01	.2589E-01	.2188E-01	.1227E-01	.7531E-01	.7591E-01	.6437E-01	.4436E-01	.2186E-01
	90.	.2177E-01	.2103E-01	.1832E-01	.1335E-01	.6869E-02	.7531E-01	.6977E-01	.5563E-01	.3749E-01	.1849E-01
	112.5	.2177E-01	.1797E-01	.1368E-01	.9057E-02	.4428E-02	.7531E-01	.6511E-01	.5002E-01	.3326E-01	.1635E-01
	135.	.2177E-01	.1587E-01	.1097E-01	.6826E-02	.3231E-02	.7531E-01	.6190E-01	.4651E-01	.3066E-01	.1503E-01
	157.5	.2177E-01	.1465E-01	.9568E-02	.5740E-02	.2666E-02	.7531E-01	.6004E-01	.4456E-01	.2923E-01	.1430E-01
	180.	.2177E-01	.1425E-01	.9134E-02	.5413E-02	.2498E-02	.7531E-01	.5943E-01	.4394E-01	.2877E-01	.1407E-01

Simply supported circular plate — λ = 3

β	θ° \ α	Influence surface of M_r (ν=0.3): 0	0.2	0.4	0.6	0.8	Influence surface of M_t (ν=0.3): 0	0.2	0.4	0.6	0.8
0	0	.1000E+31	.4819E-01	-.4717E-04	-.1106E-01	-.7935E-02	.1000E+31	.9984E-01	.4158E-01	.1742E-01	.6292E-02
	22.5	.1000E+31	.5577E-01	.6055E-02	-.6891E-02	-.5850E-02	.1000E+31	.9230E-01	.3549E-01	.1325E-01	.4210E-02
	45.	.1000E+31	.7406E-01	.2078E-01	.3181E-02	-.8195E-03	.1000E+31	.7406E-01	.2078E-01	.3181E-02	-.8195E-03
	67.5	.1000E+31	.9230E-01	.3549E-01	.1325E-01	.4210E-02	.1000E+31	.5577E-01	.6053E-02	-.6891E-02	-.5850E-02
	90.	.1000E+31	.9984E-01	.4158E-01	.1742E-01	.6292E-02	.1000E+31	.4819E-01	-.4717E-04	-.1106E-01	-.7935E-02
	112.5	.1000E+31	.9230E-01	.3549E-01	.1325E-01	.4210E-02	.1000E+31	.5577E-01	.6055E-02	-.6891E-02	-.5850E-02
	135.	.1000E+31	.7406E-01	.2078E-01	.3181E-02	-.8195E-03	.1000E+31	.7406E-01	.2078E-01	.3181E-02	-.8195E-03
	157.5	.1000E+31	.5577E-01	.6055E-02	-.6891E-02	-.5850E-02	.1000E+31	.9230E-01	.3549E-01	.1325E-01	.4210E-02
	180.	.1000E+31	.4819E-01	-.4717E-04	-.1106E-01	-.7935E-02	.1000E+31	.9984E-01	.4158E-01	.1742E-01	.6292E-02
0.2	0	.4839E-01	.1000E+31	.5352E-01	.7611E-02	-.2037E-02	.1001E+00	.1000E+31	.1019E+00	.4319E-01	.1654E-01
	22.5	.4839E-01	.1931E+00	.6453E-01	.1589E-01	.2232E-02	.1001E+00	.1413E+00	.6700E-01	.2799E-01	.9996E-02
	45.	.4839E-01	.1195E+00	.6487E-01	.2530E-01	.8306E-02	.1001E+00	.8107E-01	.2346E-01	.3434E-02	-.1261E-02
	67.5	.4839E-01	.7553E-01	.4769E-01	.2275E-01	.8776E-02	.1001E+00	.5573E-01	.5370E-02	-.8508E-02	-.6982E-02
	90.	.4839E-01	.4492E-01	.2624E-01	.1227E-01	.4604E-02	.1001E+00	.4503E-01	.3191E-02	-.8576E-02	-.6679E-02
	112.5	.4839E-01	.2325E-01	.7719E-02	.8778E-03	-.6538E-03	.1001E+00	.4126E-01	.7048E-02	-.3796E-02	-.3816E-02
	135.	.4839E-01	.8714E-02	-.5562E-02	-.7937E-02	-.4892E-02	.1001E+00	.4050E-01	.1176E-01	.1250E-02	-.8954E-03
	157.5	.4839E-01	.3522E-03	-.1333E-01	-.1321E-01	-.7457E-02	.1001E+00	.4070E-01	.1511E-01	.4646E-02	.1040E-02
	180.	.4839E-01	-.2375E-02	-.1587E-01	-.1494E-01	-.8300E-02	.1001E+00	.4086E-01	.1629E-01	.5812E-02	.1699E-02
0.4	0	.6458E-03	.5403E-01	.1000E+31	.6314E-01	.1621E-01	.4273E-01	.1031E+00	.1000E+31	.1031E+00	.3840E-01
	22.5	.6458E-03	.4813E-01	.1311E+00	.6778E-01	.2253E-01	.4273E-01	.8503E-01	.7698E-01	.4195E-01	.1728E-01
	45.	.6458E-03	.3290E-01	.6185E-01	.4409E-01	.1945E-01	.4273E-01	.5705E-01	.2631E-01	.2963E-02	-.2098E-02
	67.5	.6458E-03	.1652E-01	.2555E-01	.1919E-01	.9186E-02	.4273E-01	.3804E-01	.1021E-01	-.4797E-02	-.5460E-02
	90.	.6458E-03	.3487E-02	.4811E-02	.3129E-02	.1292E-02	.4273E-01	.2732E-01	.5667E-02	-.3873E-02	-.3901E-02
	112.5	.6458E-03	-.5592E-02	-.6843E-02	-.5700E-02	-.3132E-02	.4273E-01	.2163E-01	.4903E-02	-.1711E-02	-.2077E-02
	135.	.6458E-03	-.1132E-01	-.1307E-01	-.1012E-01	-.5286E-02	.4273E-01	.1871E-01	.5131E-02	-.7320E-04	-.8684E-03
	157.5	.6458E-03	-.1441E-01	-.1604E-01	-.1209E-01	-.6209E-02	.4273E-01	.1734E-01	.5432E-02	.8410E-03	-.2345E-03
	180.	.6458E-03	-.1539E-01	-.1691E-01	-.1264E-01	-.6460E-02	.4273E-01	.1693E-01	.5549E-02	.1128E-02	-.4095E-04
0.6	0	-.9860E-02	.8683E-02	.6372E-01	.1000E+31	.6529E-01	.2030E-01	.4706E-01	.1067E+00	.1000E+31	.8944E-01
	22.5	-.9860E-02	.6887E-02	.5070E-01	.9877E-01	.5561E-01	.2030E-01	.4179E-01	.6297E-01	.4361E-01	.1946E-01
	45.	-.9860E-02	.2222E-02	.2419E-01	.3511E-01	.2225E-01	.2030E-01	.3106E-01	.2625E-01	.8462E-02	-.3167E-03
	67.5	-.9860E-02	-.3267E-02	.5624E-02	.8746E-02	.5553E-02	.2030E-01	.2155E-01	.1149E-01	.1361E-02	-.1653E-02
	90.	-.9860E-02	-.7834E-02	-.4363E-02	-.2099E-02	-.8873E-03	.2030E-01	.1512E-01	.5771E-02	.3500E-05	-.1161E-02
	112.5	-.9860E-02	-.1094E-01	-.9123E-02	-.6296E-02	-.3156E-02	.2030E-01	.1122E-01	.3432E-02	-.1922E-03	-.7547E-03
	135.	-.9860E-02	-.1278E-01	-.1118E-01	-.7767E-02	-.3863E-02	.2030E-01	.8992E-02	.2415E-02	-.1800E-03	-.5295E-03
	157.5	-.9860E-02	-.1370E-01	-.1196E-01	-.8204E-02	-.4035E-02	.2030E-01	.7860E-02	.1976E-02	-.1517E-03	-.4223E-03
	180.	-.9860E-02	-.1398E-01	-.1215E-01	-.8289E-02	-.4059E-02	.2030E-01	.7514E-02	.1852E-02	-.1410E-03	-.3908E-03
0.8	0	-.6689E-02	-.8548E-03	.1699E-01	.6551E-01	.1000E+31	.1203E-01	.2530E-01	.4960E-01	.9974E-01	.1000E+31
	22.5	-.6689E-02	-.1422E-02	.1359E-01	.4343E-01	.5562E-01	.1203E-01	.2323E-01	.3721E-01	.4061E-01	.1809E-01
	45.	-.6689E-02	-.2857E-02	.5690E-02	.1430E-01	.1231E-01	.1203E-01	.1857E-01	.2047E-01	.1376E-01	.4939E-02
	67.5	-.6689E-02	-.4503E-02	-.6732E-03	.1956E-02	.1780E-02	.1203E-01	.1382E-01	.1094E-01	.5843E-02	.2061E-02
	90.	-.6689E-02	-.5813E-02	-.3978E-02	-.2306E-02	-.1051E-02	.1203E-01	.1013E-01	.6135E-02	.2739E-02	.8809E-03
	112.5	-.6689E-02	-.6629E-02	-.5319E-02	-.3579E-02	-.1765E-02	.1203E-01	.7626E-02	.3652E-02	.1303E-02	.3186E-03
	135.	-.6689E-02	-.7051E-02	-.5737E-02	-.3841E-02	-.1874E-02	.1203E-01	.6067E-02	.2355E-02	.6018E-03	.4495E-04
	157.5	-.6689E-02	-.7230E-02	-.5814E-02	-.3827E-02	-.1843E-02	.1203E-01	.5225E-02	.1725E-02	.2769E-03	-.7968E-04
	180.	-.6689E-02	-.7277E-02	-.5815E-02	-.3800E-02	-.1821E-02	.1203E-01	.4959E-02	.1537E-02	.1822E-03	-.1156E-03

Simply supported circular plate — λ = 5

β	θ° \ α	M_r (ν=0.3): 0	0.2	0.4	0.6	0.8	M_t (ν=0.3): 0	0.2	0.4	0.6	0.8
0	0	.1000E+31	.6082E-02	-.1942E-01	-.1481E-01	-.6663E-02	.1000E+31	.5269E-01	.9985E-02	-.1058E-04	-.1034E-02
	22.5	.1000E+31	.1293E-01	-.1511E-01	-.1264E-01	-.5838E-02	.1000E+31	.4588E-01	.5683E-02	-.2175E-02	-.1858E-02
	45.	.1000E+31	.2943E-01	-.4710E-02	-.7404E-02	-.3847E-02	.1000E+31	.2943E-01	-.4710E-02	-.7404E-02	-.3847E-02
	67.5	.1000E+31	.4588E-01	.5683E-02	-.2175E-02	-.1858E-02	.1000E+31	.1293E-01	-.1511E-01	-.1264E-01	-.5838E-02
	90.	.1000E+31	.5269E-01	.9985E-02	-.1058E-04	-.1034E-02	.1000E+31	.6082E-02	-.1942E-01	-.1481E-01	-.6663E-02
	112.5	.1000E+31	.4588E-01	.5683E-02	-.2175E-02	-.1858E-02	.1000E+31	.1293E-01	-.1511E-01	-.1264E-01	-.5838E-02
	135.	.1000E+31	.2943E-01	-.4710E-02	-.7404E-02	-.3847E-02	.1000E+31	.2943E-01	-.4710E-02	-.7404E-02	-.3847E-02
	157.5	.1000E+31	.1293E-01	-.1511E-01	-.1264E-01	-.5838E-02	.1000E+31	.4588E-01	.5683E-02	-.2175E-02	-.1858E-02
	180.	.1000E+31	.6082E-02	-.1942E-01	-.1481E-01	-.6663E-02	.1000E+31	.5269E-01	.9985E-02	-.1058E-04	-.1034E-02
0.2	0	.6131E-02	.1000E+31	.5711E-02	-.1916E-01	-.1237E-01	.5268E-01	.1000E+31	.5258E-01	.1018E-01	.7508E-03
	22.5	.6131E-02	.1367E+00	.1773E-01	-.1125E-01	-.8805E-02	.5268E-01	.8708E-01	.2249E-01	.4220E-04	-.2664E-02
	45.	.6131E-02	.6777E-01	.2198E-01	-.9409E-03	-.3265E-02	.5268E-01	.3251E-01	-.1038E-01	-.1369E-01	-.7394E-02
	67.5	.6131E-02	.3064E-01	.1235E-01	.9504E-03	-.1057E-02	.5268E-01	.1349E-01	-.1836E-01	-.1644E-01	-.7940E-02
	90.	.6131E-02	.8047E-02	.1010E-02	-.1779E-02	-.1302E-02	.5268E-01	.7989E-02	-.1435E-01	-.1221E-01	-.5683E-02
	112.5	.6131E-02	-.5735E-02	-.7075E-02	-.4580E-02	-.1956E-02	.5268E-01	.7635E-02	-.8186E-02	-.7138E-02	-.3259E-02
	135.	.6131E-02	-.1373E-01	-.1165E-01	-.6145E-02	-.2286E-02	.5268E-01	.8722E-02	-.3451E-02	-.3612E-02	-.1675E-02
	157.5	.6131E-02	-.1781E-01	-.1377E-01	-.6757E-02	-.2352E-02	.5268E-01	.9719E-02	-.7708E-03	-.1768E-02	-.8937E-03
	180.	.6131E-02	-.1905E-01	-.1437E-01	-.6897E-02	-.2347E-02	.5268E-01	.1009E-01	.7244E-04	-.1215E-02	-.6687E-03
0.4	0	-.1923E-01	.5938E-02	.1000E+31	.8510E-02	-.1194E-01	.9953E-02	.5256E-01	.1000E+31	.5361E-01	.1167E-01
	22.5	-.1923E-01	.2999E-02	.7179E-01	.1843E-01	-.3267E-02	.9953E-02	.3738E-01	.2575E-01	.2536E-02	-.2900E-02
	45.	-.1923E-01	-.4503E-02	.1754E-01	.8275E-02	.6315E-03	.9953E-02	.1629E-01	-.9034E-02	-.1870E-01	-.1118E-01
	67.5	-.1923E-01	-.1093E-01	-.1663E-02	-.8419E-03	-.8828E-03	.9953E-02	.5084E-02	-.1157E-01	-.1400E-01	-.7662E-02
	90.	-.1923E-01	-.1415E-01	-.7562E-02	-.3767E-02	-.1534E-02	.9953E-02	.9254E-03	-.7782E-02	-.7323E-02	-.3615E-02
	112.5	-.1923E-01	-.1504E-01	-.8341E-02	-.3680E-02	-.1233E-02	.9953E-02	-.1304E-03	-.4370E-02	-.3360E-02	-.1463E-02
	135.	-.1923E-01	-.1486E-01	-.7613E-02	-.2868E-02	-.7597E-03	.9953E-02	-.1671E-03	-.2359E-02	-.1531E-02	-.5789E-03
	157.5	-.1923E-01	-.1446E-01	-.6847E-02	-.2233E-02	-.4410E-03	.9953E-02	-.1777E-04	-.1413E-02	-.8140E-03	-.2725E-03
	180.	-.1923E-01	-.1428E-01	-.6554E-02	-.2009E-02	-.3352E-03	.9953E-02	.5039E-04	-.1144E-02	-.6323E-03	-.2018E-03
0.6	0	-.1440E-01	-.1852E-01	.9113E-02	.1000E+31	.2196E-01	-.9299E-04	.1020E-01	.5384E-01	.1000E+31	.5430E-01
	22.5	-.1440E-01	-.1785E-01	.3728E-02	.4464E-01	.2079E-01	-.9299E-04	.7246E-02	.1797E-01	.1449E-02	-.3022E-02
	45.	-.1440E-01	-.1626E-01	-.6191E-02	.3874E-02	.3256E-02	-.9299E-04	.2148E-02	-.3698E-02	-.1356E-01	-.1090E-01
	67.5	-.1440E-01	-.1415E-01	-.8579E-02	-.3733E-02	-.1361E-02	-.9299E-04	-.9398E-03	-.5959E-02	-.8079E-02	-.5035E-02
	90.	-.1440E-01	-.1181E-01	-.6959E-02	-.3259E-02	-.1183E-02	-.9299E-04	-.1899E-02	-.3980E-02	-.3551E-02	-.1797E-02
	112.5	-.1440E-01	-.9665E-02	-.4753E-02	-.1757E-02	-.4587E-03	-.9299E-04	-.1864E-02	-.2220E-02	-.1389E-02	-.5474E-03
	135.	-.1440E-01	-.8036E-02	-.3137E-02	-.7354E-03	-.1389E-04	-.9299E-04	-.1587E-02	-.1237E-02	-.5315E-03	-.1363E-03
	157.5	-.1440E-01	-.7050E-02	-.2245E-02	-.2349E-03	.1792E-03	-.9299E-04	-.1369E-02	-.7929E-03	-.2336E-03	-.1905E-04
	180.	-.1440E-01	-.6724E-02	-.1968E-02	-.9130E-04	.2298E-03	-.9299E-04	-.1292E-02	-.6698E-03	-.1636E-03	.4124E-05
0.8	0	-.6164E-02	-.1146E-01	-.1077E-01	.2282E-01	.1000E+31	-.1150E-02	.1116E-02	.1322E-01	.5708E-01	.1000E+31
	22.5	-.6164E-02	-.1077E-01	-.9578E-02	.1063E-01	.3027E-01	-.1150E-02	.4584E-03	.5661E-02	.7822E-02	-.2885E-02
	45.	-.6164E-02	-.9072E-02	-.7830E-02	-.1983E-02	.1391E-02	-.1150E-02	-.7048E-03	-.1372E-02	-.4466E-02	-.4847E-02
	67.5	-.6164E-02	-.7059E-02	-.5502E-02	-.2964E-02	-.1127E-02	-.1150E-02	-.1361E-02	-.2441E-02	-.3067E-02	-.2029E-02
	90.	-.6164E-02	-.5223E-02	-.3250E-02	-.1548E-02	-.5469E-03	-.1150E-02	-.1434E-02	-.1704E-02	-.1378E-02	-.6928E-03
	112.5	-.6164E-02	-.3806E-02	-.1696E-02	-.5074E-03	-.7416E-04	-.1150E-02	-.1246E-02	-.9830E-03	-.5335E-03	-.1950E-03
	135.	-.6164E-02	-.2856E-02	-.8239E-03	-.1059E-04	.1201E-03	-.1150E-02	-.1035E-02	-.5619E-03	-.1892E-03	-.3145E-04
	157.5	-.6164E-02	-.2326E-02	-.4140E-03	.1794E-03	.1780E-03	-.1150E-02	-.8936E-03	-.3654E-03	-.6626E-04	.1566E-04
	180.	-.6164E-02	-.2158E-02	-.2968E-03	.2258E-03	.1888E-03	-.1150E-02	-.8458E-03	-.3098E-03	-.3676E-04	.2506E-04

Simply supported circular plate

λ = 7

β	θ° \ α	Influence surface of M_r (ν=0.3) 0	0.2	0.4	0.6	0.8	Influence surface of M_t (ν=0.3) 0	0.2	0.4	0.6	0.8
0	0	.1000E+31	-.1106E-01	-.1566E-01	-.5276E-02	-.7178E-03	.1000E+31	.2869E-01	.1141E-02	-.1021E-02	-.3282E-03
	22.5	.1000E+31	-.5220E-02	-.1320E-01	-.4652E-02	-.6604E-03	.1000E+31	.2289E-01	-.1315E-02	-.1643E-02	-.3850E-03
	45.	.1000E+31	.8856E-02	-.7252E-02	-.3146E-02	-.5225E-03	.1000E+31	.8856E-02	-.7252E-02	-.3146E-02	-.5225E-03
	67.5	.1000E+31	.2289E-01	-.1315E-02	-.1643E-02	-.3850E-03	.1000E+31	-.5220E-02	-.1320E-01	-.4652E-02	-.6604E-03
	90.	.1000E+31	-.2869E-01	.1141E-02	-.1021E-02	-.3282E-03	.1000E+31	-.1106E-01	-.1566E-01	-.5276E-02	-.7178E-03
	112.5	.1000E+31	.2289E-01	-.1315E-02	-.1643E-02	-.3850E-03	.1000E+31	-.5220E-02	-.1320E-01	-.4652E-02	-.6604E-03
	135.	.1000E+31	.8856E-02	-.7252E-02	-.3146E-02	-.5225E-03	.1000E+31	.8856E-02	-.7252E-02	-.3146E-02	-.5225E-03
	157.5	.1000E+31	-.5220E-02	-.1320E-01	-.4652E-02	-.6604E-03	.1000E+31	.2289E-01	-.1315E-02	-.1643E-02	-.3850E-03
	180.	.1000E+31	-.1106E-01	-.1566E-01	-.5276E-02	-.7178E-03	.1000E+31	.2869E-01	.1141E-02	-.1021E-02	-.3282E-03
0.2	0	-.1106E-01	.1000E+31	-.1115E-01	-.1599E-01	-.5747E-02	.2869E-01	.1000E+31	.2866E-01	.1040E-02	-.1100E-02
	22.5	-.1106E-01	.1044E+00	.8483E-03	-.1031E-01	-.4044E-02	.2869E-01	.5604E-01	.4570E-02	-.4108E-02	-.1977E-02
	45.	-.1106E-01	.4089E-01	.6831E-02	-.3115E-02	-.1581E-02	.2869E-01	.8821E-02	-.1677E-01	-.8988E-02	-.2489E-02
	67.5	-.1106E-01	.1099E-01	.2089E-02	-.1136E-02	-.5597E-03	.2869E-01	-.3168E-02	-.1675E-01	-.7265E-02	-.1515E-02
	90.	-.1106E-01	-.4047E-02	-.2755E-02	-.1226E-02	-.2624E-03	.2869E-01	-.4046E-02	-.1093E-01	-.3770E-02	-.4568E-03
	112.5	-.1106E-01	-.1121E-01	-.4969E-02	-.1175E-02	-.3570E-04	.2869E-01	-.2152E-02	-.5347E-02	-.1547E-02	.6053E-05
	135.	-.1106E-01	-.1428E-01	-.5440E-02	-.8579E-03	.1681E-03	.2869E-01	-.3148E-03	-.2556E-02	-.6143E-03	.8132E-04
	157.5	-.1106E-01	-.1540E-01	-.5311E-02	-.5698E-03	.2937E-03	.2869E-01	.7895E-03	-.1329E-02	-.3212E-03	.4687E-04
	180.	-.1106E-01	-.1567E-01	-.5208E-02	-.4625E-03	.3336E-03	.2869E-01	.1143E-02	-.9965E-03	-.2626E-03	.2618E-04
0.4	0	-.1566E-01	-.1113E-01	.1000E+31	-.1167E-01	-.1538E-01	.1151E-02	.2867E-01	.1000E+31	.2853E-01	.1336E-02
	22.5	-.1566E-01	-.1135E-01	.4405E-01	.1670E-02	-.7080E-02	.1151E-02	.1676E-01	.2603E-02	-.9475E-02	-.5749E-02
	45.	-.1566E-01	-.1252E-01	.3684E-02	.1574E-05	-.1602E-02	.1161E-02	.2619E-02	-.1516E-01	-.1455E-01	-.5665E-02
	67.5	-.1566E-01	-.1226E-01	-.4197E-02	-.1563E-02	-.5670E-03	.1151E-02	-.2374E-02	-.9189E-02	-.5813E-02	-.1629E-02
	90.	-.1566E-01	-.1043E-01	-.3863E-02	-.8987E-03	-.6493E-04	.1151E-02	-.2749E-02	-.3866E-02	-.1370E-02	-.3238E-04
	112.5	-.1566E-01	-.8274E-02	-.2306E-02	-.1348E-03	.1785E-03	.1151E-02	-.2042E-02	-.1441E-02	-.1362E-03	.1843E-03
	135.	-.1566E-01	-.6568E-02	-.1196E-02	.2331E-03	.2300E-03	.1151E-02	-.1428E-02	-.5811E-03	.5976E-04	.1177E-03
	157.5	-.1566E-01	-.5545E-02	-.6369E-03	.3527E-03	.2171E-03	.1151E-02	-.1093E-02	-.3187E-03	.5270E-04	.6300E-04
	180.	-.1566E-01	-.5210E-02	-.4743E-03	.3762E-03	.2072E-03	.1151E-02	-.9919E-03	-.2625E-03	.4109E-04	.4675E-04
0.6	0	-.5271E-02	-.1992E-01	-.1150E-01	.1000E+31	-.5504E-02	-.1012E-02	.1041E-02	.2853E-01	.1000E+31	.2997E-01
	22.5	-.5271E-02	-.1417E-01	-.9793E-02	.1819E-01	.3004E-02	-.1012E-02	-.1829E-03	.2126E-02	-.1464E-01	-.1290E-01
	45.	-.5271E-02	-.1038E-01	-.8096E-02	-.2428E-02	-.1036E-02	-.1012E-02	-.1680E-02	-.6358E-02	-.1095E-01	-.6740E-02
	67.5	-.5271E-02	-.6586E-02	-.4145E-02	-.1583E-02	-.4348E-03	-.1012E-02	-.1784E-02	-.3200E-02	-.2568E-02	-.9335E-03
	90.	-.5271E-02	-.3761E-02	-.1351E-02	-.1238E-03	.1138E-03	-.1012E-02	-.1223E-02	-.9119E-03	-.1537E-03	.1404E-03
	112.5	-.5271E-02	-.2004E-02	-.1280E-03	.2766E-03	.1621E-03	-.1012E-02	-.7164E-03	-.1449E-03	.1556E-03	.1275E-03
	135.	-.5271E-02	-.1049E-02	.2656E-03	.2739E-03	.9812E-04	-.1012E-02	-.4248E-03	.2444E-04	.1051E-03	.5097E-04
	157.5	-.5271E-02	-.6018E-03	.3580E-03	.2178E-03	.5422E-04	-.1012E-02	-.2919E-03	.4473E-04	.5912E-04	.1989E-04
	180.	-.5271E-02	-.4731E-03	.3708E-03	.1954E-03	.4099E-04	-.1012E-02	-.2550E-03	.4402E-04	.4581E-04	-.1301E-04
0.8	0	-.7112E-03	-.5578E-02	-.1486E-01	-.4788E-02	.1000E+31	-.3596E-03	-.1174E-02	.1382E-02	.3055E-01	.1000E+31
	22.5	-.7112E-03	-.4778E-02	-.1076E-01	-.4571E-02	.1192E-01	-.3596E-03	-.1172E-02	-.1669E-02	-.4513E-02	-.1217E-01
	45.	-.7112E-03	-.3044E-02	-.4810E-02	-.3328E-02	-.1009E-02	-.3596E-03	-.9984E-03	-.2304E-02	-.4241E-02	-.3654E-02
	67.5	-.7112E-03	-.1449E-02	-.1320E-02	-.6332E-03	-.1643E-03	-.3596E-03	-.6320E-03	-.8614E-03	-.7259E-03	-.3087E-03
	90.	-.7112E-03	-.4452E-03	-.1080E-04	.1679E-03	.1182E-03	-.3596E-03	-.2974E-03	-.1060E-03	.7327E-04	.9243E-04
	112.5	-.7112E-03	.4457E-04	.2749E-03	.1868E-03	.6886E-04	-.3596E-03	-.1010E-03	.7092E-04	.9654E-04	.4978E-04
	135.	-.7112E-03	.2384E-03	.2625E-03	.1016E-03	.1787E-04	-.3596E-03	-.1343E-04	.7501E-04	.4594E-04	.1331E-04
	157.5	-.7112E-03	.3003E-03	.2155E-03	.5285E-04	-.1654E-05	-.3596E-03	.1845E-04	.5825E-04	.2141E-04	.2097E-05
	180.	-.7112E-03	.3130E-03	.1971E-03	.3914E-04	-.5889E-05	-.3596E-03	.2599E-04	.5169E-04	.1535E-04	.8221E-07

Simply supported circular plate											λ = 11
β	θ° \ ρ	Influence surface of M_r (ν=0.3)					Influence surface of M_t (ν=0.3)				
		0	0.2	0.4	0.6	0.8	0	0.2	0.4	0.6	0.8
0	0	.1000E+31	-.1891E-01	-.4172E-02	.3099E-03	.1359E-03	.1000E+31	.6823E-02	-.9152E-03	-.3380E-05	.3480E-04
	22.5	.1000E+31	-.1512E-01	-.3693E-02	.2641E-03	.1211E-03	.1000E+31	.3074E-02	-.1390E-02	.4263E-04	.4958E-04
	45.	.1000E+31	-.6003E-02	-.2539E-02	.1535E-03	.8529E-04	.1000E+31	-.6003E-02	-.2539E-02	.1535E-03	.8529E-04
	67.5	.1000E+31	.3074E-02	-.1390E-02	.4263E-04	.4958E-04	.1000E+31	-.1512E-01	-.3693E-02	.2641E-03	.1211E-03
	90.	.1000E+31	-.6823E-02	-.9152E-03	-.3380E-05	.3480E-04	.1000E+31	-.1891E-01	-.4172E-02	.3099E-03	.1359E-03
	112.5	.1000E+31	.3074E-02	-.1390E-02	.4263E-04	.4958E-04	.1000E+31	-.1512E-01	-.3693E-02	.2641E-03	.1211E-03
	135.	.1000E+31	-.6003E-02	-.2539E-02	.1535E-03	.8529E-04	.1000E+31	-.6003E-02	-.2539E-02	.1535E-03	.8529E-04
	157.5	.1000E+31	-.1512E-01	-.3693E-02	.2641E-03	.1211E-03	.1000E+31	.3074E-02	-.1390E-02	.4263E-04	.4958E-04
	180.	.1000E+31	-.1891E-01	-.4172E-02	.3099E-03	.1359E-03	.1000E+31	.6823E-02	-.9152E-03	-.3380E-05	.3480E-04
0.2	0	-.1891E-01	.1000E+31	-.1891E-01	-.4165E-02	.3261E-03	.6823E-02	.1000E+31	.6822E-02	-.9131E-03	-.6634E-06
	22.5	-.1891E-01	.6350E-01	-.8270E-02	-.2384E-02	.2839E-03	.6823E-02	.1876E-01	-.6147E-02	-.1451E-02	.1079E-03
	45.	-.1891E-01	.1300E-01	-.1406E-02	-.5611E-03	.1467E-03	.6823E-02	-.1150E-01	-.1117E-01	-.9396E-03	.2673E-03
	67.5	-.1891E-01	-.2891E-02	-.1118E-02	-.9872E-04	.5506E-04	.6823E-02	-.1125E-01	-.5749E-02	.7746E-04	.2270E-03
	90.	-.1891E-01	-.6650E-02	-.9044E-03	.6607E-04	.3168E-04	.6823E-02	-.6650E-02	-.1776E-02	.3262E-03	.9658E-04
	112.5	-.1891E-01	-.6383E-02	-.3501E-03	.1471E-03	.1613E-04	.6823E-02	-.3407E-02	-.3589E-03	.2079E-03	.2288E-04
	135.	-.1891E-01	-.5291E-02	.6157E-04	.1544E-03	.1293E-05	.6823E-02	-.1775E-02	-.3580E-04	.9593E-04	.1850E-05
	157.5	-.1891E-01	-.4462E-02	.2573E-03	.1379E-03	-.7160E-05	.6823E-02	-.1095E-02	.6966E-07	.4575E-04	-.9965E-06
	180.	-.1891E-01	-.4172E-02	.3103E-03	.1296E-03	-.9620E-05	.6823E-02	-.9150E-03	-.3249E-05	.3301E-04	-.8978E-06
0.4	0	-.4172E-02	-.1891E-01	.1000E+31	-.1890E-01	-.4328E-02	-.9152E-03	.6822E-02	.1000E+31	.6825E-02	-.9590E-03
	22.5	-.4172E-02	-.1530E-01	.1577E-01	-.4207E-02	-.1279E-02	-.9152E-03	.8518E-03	-.1570E-01	-.9422E-02	-.1127E-02
	45.	-.4172E-02	-.9411E-02	-.2231E-02	-.7600E-03	.2515E-05	-.9152E-03	-.3153E-02	-.9541E-02	-.2933E-02	.2555E-03
	67.5	-.4172E-02	-.4644E-02	-.1105E-02	-.1393E-04	.5499E-04	-.9152E-03	-.2214E-02	-.1691E-02	.2148E-03	.2341E-03
	90.	-.4172E-02	-.1776E-02	.4191E-04	.1186E-03	.1472E-04	-.9152E-03	-.9045E-03	.4167E-04	.2000E-03	.2727E-04
	112.5	-.4172E-02	-.4260E-03	.2571E-03	.5363E-04	-.6289E-05	-.9152E-03	-.2844E-03	.1430E-03	.4631E-04	-.6868E-05
	135.	-.4172E-02	.9938E-04	.2066E-03	.8960E-05	-.7892E-05	-.9152E-03	-.7422E-04	.7680E-04	.6192E-05	-.4265E-05
	157.5	-.4172E-02	.2722E-03	.1495E-03	-.6028E-05	-.5834E-05	-.9152E-03	-.1500E-04	.4206E-04	-.1651E-06	-.1908E-05
	180.	-.4172E-02	.3103E-03	.1299E-03	-.9043E-05	-.4974E-05	-.9152E-03	-.3241E-05	.3312E-04	-.7880E-06	-.1336E-05
0.6	0	.3098E-03	-.4165E-02	-.1889E-01	.1000E+31	-.1939E-01	-.3591E-05	-.9126E-03	.6828E-02	.1000E+31	.6765E-02
	22.5	.3098E-03	-.2989E-02	-.9481E-02	.1819E-02	-.2622E-02	-.3591E-05	-.8510E-03	-.4144E-02	-.1655E-01	-.7429E-02
	45.	.3098E-03	-.1028E-02	-.1965E-02	-.6956E-03	-.9662E-04	-.3591E-05	-.4742E-03	-.1720E-02	-.1492E-02	.1015E-03
	67.5	.3098E-03	.5854E-04	.1267E-03	.1467E-03	.5275E-04	-.3591E-05	-.7953E-04	.7471E-04	.2632E-03	.1380E-03
	90.	.3098E-03	.3262E-03	.2002E-03	.4333E-04	-.3579E-05	-.3591E-05	.6585E-04	.1183E-03	.4313E-04	-.5480E-05
	112.5	.3098E-03	.2829E-03	.6711E-04	-.6461E-05	-.4794E-05	-.3591E-05	.7204E-04	.3284E-04	-.3480E-05	-.3941E-05
	135.	.3098E-03	.1990E-03	.9513E-05	-.8728E-05	-.1009E-05	-.3591E-05	.5140E-04	.5658E-05	-.3329E-05	-.5347E-06
	157.5	.3098E-03	.1463E-03	-.6227E-05	-.5775E-05	.1334E-06	-.3591E-05	.3738E-04	.4550E-07	-.1690E-05	-.6506E-08
	180.	.3098E-03	.1296E-03	-.9047E-05	-.4706E-05	.3192E-06	-.3591E-05	.3298E-04	-.7758E-06	-.1257E-05	.4228E-07
0.8	0	.1353E-03	.3184E-03	-.4319E-02	-.1921E-01	.1000E+31	.3496E-04	.1047E-05	-.9604E-03	.6771E-02	.1000E+31
	22.5	.1353E-03	.3542E-03	-.1691E-02	-.5302E-02	-.9348E-03	.3496E-04	.3210E-04	-.7174E-03	-.4672E-02	-.1001E-01
	45.	.1353E-03	.3393E-03	.2046E-03	-.1104E-04	-.5526E-05	.3496E-04	.7143E-04	.4934E-04	.1898E-04	.1310E-03
	67.5	.1353E-03	.2180E-03	.1890E-03	.9463E-04	.2359E-04	.3496E-04	.6258E-04	.9840E-04	.9521E-04	.4019E-04
	90.	.1353E-03	.9600E-04	.2686E-04	-.5607E-05	-.5539E-05	.3496E-04	.3196E-04	.1508E-04	-.3401E-05	-.5544E-05
	112.5	.1353E-03	.2807E-04	-.9903E-05	-.5657E-05	-.4937E-06	.3496E-04	.1112E-04	-.3097E-05	-.3038E-05	-.4454E-06
	135.	.1353E-03	.1007E-05	-.9241E-05	-.1076E-05	.3977E-06	.3496E-04	.2389E-05	-.2839E-05	-.4695E-06	.1672E-06
	157.5	.1353E-03	-.7525E-05	-.6019E-05	.1241E-06	.2645E-06	.3496E-04	-.4067E-06	-.1690E-05	-.4539E-08	.8314E-07
	180.	.1353E-03	-.9308E-05	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE V

Clamped rectangular plate — b/a = 1.0 — λ = 1

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2038E-02	.1830E-02	.1295E-02	.6479E-03	.2261E-03
0.6	.6796E-02	.6116E-02	.4393E-02	.2328E-02	.6479E-03
0.4	.1283E-01	.1148E-01	.8195E-02	.4393E-02	.1295E-02
0.2	.1877E-01	.1646E-01	.1148E-01	.6116E-02	.1830E-02
0	.2220E-01	.1877E-01	.1283E-01	.6796E-02	.2038E-02
-0.2	.1877E-01	.1646E-01	.1148E-01	.6116E-02	.1830E-02
-0.4	.1283E-01	.1148E-01	.8195E-02	.4393E-02	.1295E-02
-0.6	.6796E-02	.6116E-02	.4393E-02	.2328E-02	.6479E-03
-0.8	.2038E-02	.1830E-02	.1295E-02	.6479E-03	.2261E-03

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1623E-02	.1770E-02	.1698E-02	.9712E-03	.3174E-03
0.6	.1064E-01	.1084E-01	.9466E-02	.5561E-02	.1502E-02
0.4	.3531E-01	.3494E-01	.2779E-01	.1536E-01	.4439E-02
0.2	.9553E-01	.8747E-01	.5692E-01	.2812E-01	.8018E-02
0	.1000E+31	.1470E+00	.7536E-01	.3466E-01	.9735E-02
-0.2	.9553E-01	.8747E-01	.5692E-01	.2812E-01	.8018E-02
-0.4	.3531E-01	.3494E-01	.2779E-01	.1536E-01	.4439E-02
-0.6	.1064E-01	.1084E-01	.9466E-02	.5561E-02	.1502E-02
-0.8	.1623E-02	.1770E-02	.1698E-02	.9712E-03	.3174E-03

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2892E+01	-.9510E+00	.1753E-01	-.4425E-01	.1086E-01
0.6	-.1550E+01	-.9789E+00	-.3073E+00	-.8608E-01	-.1401E-01
0.4	-.9519E+00	-.7435E+00	-.3807E+00	-.1432E+00	-.3225E-01
0.2	-.6159E+00	-.5229E+00	-.3253E+00	-.1477E+00	-.3821E-01
0	-.3966E+00	-.3492E+00	-.2371E+00	-.1182E+00	-.3262E-01
-0.2	-.2439E+00	-.2181E+00	-.1539E+00	-.7977E-01	-.2237E-01
-0.4	-.1362E+00	-.1224E+00	-.8735E-01	-.4545E-01	-.1243E-01
-0.6	-.6228E-01	-.5593E-01	-.3956E-01	-.1996E-01	-.4940E-02
-0.8	-.1676E-01	-.1494E-01	-.1023E-01	-.4666E-02	-.1470E-02

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2957E+00	-.1534E+00	-.4228E-01	-.1785E-01	-.2500E-02
0.6	-.2711E+00	-.2063E+00	-.1064E+00	-.4464E-01	-.1135E-01
0.4	-.2232E+00	-.1902E+00	-.1214E+00	-.5859E-01	-.1637E-01
0.2	-.1724E+00	-.1530E+00	-.1067E+00	-.5572E-01	-.1630E-01
0	-.1245E+00	-.1123E+00	-.8129E-01	-.4398E-01	-.1307E-01
-0.2	-.8288E-01	-.7521E-01	-.5518E-01	-.3013E-01	-.8881E-02
-0.4	-.4905E-01	-.4454E-01	-.3265E-01	-.1765E-01	-.5032E-02
-0.6	-.2344E-01	-.2122E-01	-.1536E-01	-.8027E-02	-.2082E-02
-0.8	-.6515E-02	-.5856E-02	-.4111E-02	-.1957E-02	-.7050E-03

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.3174E-03	.1502E-02	.4439E-02	.8018E-02	.9735E-02	.8018E-02	.4439E-02	.1502E-02	.3174E-03
0.6	.9712E-03	.5561E-02	.1536E-01	.2812E-01	.3466E-01	.2812E-01	.1536E-01	.5561E-02	.9712E-03
0.4	.1698E-02	.9466E-02	.2779E-01	.5692E-01	.7536E-01	.5692E-01	.2779E-01	.9466E-02	.1698E-02
0.2	.1770E-02	.1084E-01	.3494E-01	.8747E-01	.1470E+00	.8747E-01	.3494E-01	.1084E-01	.1770E-02
0	.1623E-02	.1064E-01	.3531E-01	.9553E-01	.1000E+31	.9553E-01	.3531E-01	.1064E-01	.1623E-02

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.7050E-03	-.2082E-02	-.5032E-02	-.8881E-02	-.1307E-01	-.1630E-01	-.1637E-01	-.1135E-01	-.2500E-02
0.6	-.1957E-02	-.8027E-02	-.1765E-01	-.3013E-01	-.4398E-01	-.5572E-01	-.5859E-01	-.4464E-01	-.1785E-01
0.4	-.4111E-02	-.1536E-01	-.3265E-01	-.5518E-01	-.8129E-01	-.1067E+00	-.1214E+00	-.1064E+00	-.4228E-01
0.2	-.5856E-02	-.2122E-01	-.4454E-01	-.7521E-01	-.1123E+00	-.1530E+00	-.1902E+00	-.2063E+00	-.1534E+00
0	-.6515E-02	-.2344E-01	-.4905E-01	-.8288E-01	-.1245E+00	-.1724E+00	-.2232E+00	-.2711E+00	-.2957E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1470E-02	-.4940E-02	-.1243E-01	-.2237E-01	-.3262E-01	-.3821E-01	-.3225E-01	-.1401E-01	.1086E-01
0.6	-.4666E-02	-.1996E-01	-.4545E-01	-.7977E-01	-.1182E+00	-.1477E+00	-.1432E+00	-.8608E-01	-.4425E-01
0.4	-.1023E-01	-.3956E-01	-.8735E-01	-.1539E+00	-.2371E+00	-.3253E+00	-.3807E+00	-.3073E+00	.1753E-01
0.2	-.1494E-01	-.5593E-01	-.1224E+00	-.2181E+00	-.3492E+00	-.5229E+00	-.7435E+00	-.9789E+00	-.9510E+00
0	-.1676E-01	-.6228E-01	-.1362E+00	-.2439E+00	-.3966E+00	-.6159E+00	-.9519E+00	-.1550E+01	-.2892E+01

Clamped rectangular plate — b/a = 1.2 — λ = 1

y/b \ x/a	Influence coefficient $s_1 = wD/Pa^2$ at x=y=0: 0	0.2	0.4	0.6	0.8	Influence surface of M_y at x=y=0 (ν=0.3): 0	0.2	0.4	0.6	0.8
0.8	.2149E-02	.1940E-02	.1389E-02	.7050E-03	.1997E-03	-.5449E-03	-.3901E-03	-.1677E-03	-.1437E-03	-.1015E-03
0.6	.7256E-02	.6574E-02	.4795E-02	.2581E-02	.7331E-03	.3487E-02	.3783E-02	.3622E-02	.2157E-02	.4880E-03
0.4	.1403E-01	.1266E-01	.9203E-02	.5013E-02	.1487E-02	.2244E-01	.2251E-01	.1877E-01	.1075E-01	.3086E-02
0.2	.2113E-01	.1876E-01	.1332E-01	.7184E-02	.2151E-02	.7815E-01	.7370E-01	.5062E-01	.2562E-01	.7327E-02
0	.2554E-01	.2182E-01	.1513E-01	.8089E-02	.2424E-02	.1000E+31	.1468E+00	.7522E-01	.3458E-01	.9691E-02
-0.2	.2113E-01	.1876E-01	.1332E-01	.7184E-02	.2151E-02	.7815E-01	.7370E-01	.5062E-01	.2562E-01	.7327E-02
-0.4	.1403E-01	.1266E-01	.9203E-02	.5013E-02	.1487E-02	.2244E-01	.2251E-01	.1877E-01	.1075E-01	.3086E-02
-0.6	.7256E-02	.6574E-02	.4795E-02	.2581E-02	.7331E-03	.3487E-02	.3783E-02	.3622E-02	.2157E-02	.4880E-03
-0.8	.2149E-02	.1940E-02	.1389E-02	.7050E-03	.1997E-03	-.5449E-03	-.3901E-03	-.1677E-03	-.1437E-03	-.1015E-03

y/b \ x/a	Influence coefficient $s_2 = V_y a/P$ at x=0, y=b: 0	0.2	0.4	0.6	0.8	Influence surface of M_y at x=0, y=b: 0	0.2	0.4	0.6	0.8
0.8	-.2519E+01	-.1016E+01	-.6423E-01	-.4336E-01	.5277E-02	-.2967E+00	-.1740E+00	-.5873E-01	-.2317E-01	-.4444E-02
0.6	-.1265E+01	-.8929E+00	-.3636E+00	-.1150E+00	-.2229E-01	-.2544E+00	-.2057E+00	-.1185E+00	-.5299E-01	-.1408E-01
0.4	-.7391E+00	-.6126E+00	-.3600E+00	-.1542E+00	-.3815E-01	-.1960E+00	-.1719E+00	-.1169E+00	-.5960E-01	-.1715E-01
0.2	-.4477E+00	-.3926E+00	-.2642E+00	-.1305E+00	-.3572E-01	-.1395E+00	-.1257E+00	-.9067E-01	-.4890E-01	-.1447E-01
0	-.2649E+00	-.2373E+00	-.1683E+00	-.8794E-01	-.2484E-01	-.9198E-01	-.8355E-01	-.6147E-01	-.3369E-01	-.9965E-02
-0.2	-.1477E+00	-.1333E+00	-.9608E-01	-.5083E-01	-.1423E-01	-.5570E-01	-.5068E-01	-.3736E-01	-.2040E-01	-.5916E-02
-0.4	-.7440E-01	-.6713E-01	-.4823E-01	-.2514E-01	-.6743E-02	-.3010E-01	-.2733E-01	-.1999E-01	-.1071E-01	-.2985E-02
-0.6	-.3088E-01	-.2770E-01	-.1948E-01	-.9646E-02	-.2294E-02	-.1330E-01	-.1201E-01	-.8611E-02	-.4416E-02	-.1118E-02
-0.8	-.7676E-02	-.6796E-02	-.4532E-02	-.1926E-02	-.4412E-03	-.3493E-02	-.3119E-02	-.2140E-02	-.9714E-03	-.2594E-03

Influence surface of M_x at x=y=0 (ν=0.3)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.4634E-03	.2090E-02	.4900E-02	.7729E-02	.8952E-02	.7729E-02	.4900E-02	.2090E-02	.4634E-03
0.6	.1787E-02	.7630E-02	.1726E-01	.2755E-01	.3224E-01	.2755E-01	.1726E-01	.7630E-02	.1787E-02
0.4	.3103E-02	.1348E-01	.3273E-01	.5769E-01	.7121E-01	.5769E-01	.3273E-01	.1348E-01	.3103E-02
0.2	.3358E-02	.1593E-01	.4342E-01	.9451E-01	.1416E+00	.9451E-01	.4342E-01	.1593E-01	.3358E-02
0	.3126E-02	.1562E-01	.4421E-01	.1074E+00	.1000E+31	.1074E+00	.4421E-01	.1562E-01	.3126E-02

Influence surface of M_x at x=a, y=0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.7101E-03	-.2808E-02	-.6325E-02	-.1034E-01	-.1383E-01	-.1541E-01	-.1361E-01	-.8261E-02	-.2100E-02
0.6	-.2768E-02	-.1064E-01	-.2214E-01	-.3532E-01	-.4746E-01	-.5447E-01	-.5120E-01	-.3458E-01	-.1110E-01
0.4	-.5622E-02	-.2035E-01	-.4140E-01	-.6611E-01	-.9097E-01	-.1102E+00	-.1142E+00	-.8980E-01	-.3662E-01
0.2	-.7940E-02	-.2823E-01	-.5718E-01	-.9215E-01	-.1303E+00	-.1673E+00	-.1949E+00	-.1942E+00	-.1157E+00
0	-.8832E-02	-.3126E-01	-.6332E-01	-.1025E+00	-.1467E+00	-.1934E+00	-.2394E+00	-.2804E+00	-.3050E+00

Influence coefficient $s_3 = V_x a/P$ at x=a, y=0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1303E-02	-.5614E-02	-.1299E-01	-.2110E-01	-.2693E-01	-.2665E-01	-.1796E-01	-.5037E-02	.1291E-02
0.8	-.5780E-02	-.2303E-01	-.4901E-01	-.7898E-01	-.1051E+00	-.1152E+00	-.9544E-01	-.4645E-01	-.4579E-03
0.4	-.1263E-01	-.4705E-01	-.9865E-01	-.1628E+00	-.2318E+00	-.2890E+00	-.2996E+00	-.2113E+00	-.7432E-01
0.2	-.1869E-01	-.6833E-01	-.1438E+00	-.2442E+00	-.3705E+00	-.5239E+00	-.6954E+00	-.8103E+00	-.4363E+00
0	-.2113E-01	-.7692E-01	-.1624E+00	-.2792E+00	-.4351E+00	-.6503E+00	-.9760E+00	-.1570E+01	-.3041E+01

Clamped rectangular plate	b/a = 1.4	λ = 1

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2000E-02	.1806E-02	.1293E-02	.6561E-03	.1812E-03
0.6	.6913E-02	.6278E-02	.4604E-02	.2490E-02	.7107E-03
0.4	.1389E-01	.1259E-01	.9235E-02	.5064E-02	.1500E-02
0.2	.2186E-01	.1953E-01	.1400E-01	.7598E-02	.2274E-02
0	.2723E-01	.2338E-01	.1629E-01	.8734E-02	.2620E-02
-0.2	.2186E-01	.1953E-01	.1400E-01	.7598E-02	.2274E-02
-0.4	.1389E-01	.1259E-01	.9235E-02	.5064E-02	.1500E-02
-0.6	.6913E-02	.6278E-02	.4604E-02	.2490E-02	.7107E-03
-0.8	.2000E-02	.1806E-02	.1293E-02	.6561E-03	.1812E-03

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2371E-02	-.2153E-02	-.1629E-02	-.1000E-02	-.3585E-03
0.6	-.2685E-02	-.2196E-02	-.1332E-02	-.8071E-03	-.4176E-03
0.4	.1089E-01	.1135E-01	.1010E-01	.5950E-02	.1624E-02
0.2	.6204E-01	.5973E-01	.4293E-01	.2220E-01	.6374E-02
0	.1000E+31	.1450E+00	.7387E-01	.3384E-01	.9491E-02
-0.2	.6204E-01	.5973E-01	.4293E-01	.2220E-01	.6374E-02
-0.4	.1089E-01	.1135E-01	.1010E-01	.5950E-02	.1624E-02
-0.6	-.2685E-02	-.2196E-02	-.1332E-02	-.8071E-03	-.4176E-03
-0.8	-.2371E-02	-.2153E-02	-.1629E-02	-.1000E-02	-.3585E-03

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2209E+01	-.1041E+01	-.1415E+00	-.4903E-01	.6428E-04
0.6	-.1048E+01	-.7970E+00	-.3843E+00	-.1376E+00	-.2990E-01
0.4	-.5768E+00	-.4952E+00	-.3165E+00	-.1479E+00	-.3899E-01
0.2	-.3196E+00	-.2850E+00	-.1999E+00	-.1032E+00	-.2900E-01
0	-.1679E+00	-.1514E+00	-.1092E+00	-.5785E-01	-.1634E-01
-0.2	-.8060E-01	-.7281E-01	-.5249E-01	-.2755E-01	-.7492E-02
-0.4	-.3388E-01	-.3045E-01	-.2157E-01	-.1088E-01	-.2705E-02
-0.6	-.1135E-01	-.1006E-01	-.6804E-02	-.3087E-02	-.5939E-03
-0.8	-.2221E-02	-.1905E-02	-.1122E-02	-.3164E-03	-.2902E-04

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2933E+00	-.1888E+00	-.7389E-01	-.2886E-01	-.6201E-02
0.6	-.2357E+00	-.1981E+00	-.1231E+00	-.5816E-01	-.1604E-01
0.4	-.1676E+00	-.1496E+00	-.1056E+00	-.5577E-01	-.1634E-01
0.2	-.1079E+00	-.9788E-01	-.7174E-01	-.3924E-01	-.1161E-01
0	-.6318E-01	-.5750E-01	-.4243E-01	-.2323E-01	-.6795E-02
-0.2	-.3348E-01	-.3042E-01	-.2232E-01	-.1204E-01	-.3397E-02
-0.4	-.1567E-01	-.1417E-01	-.1024E-01	-.5363E-02	-.1419E-02
-0.6	-.5981E-02	-.5361E-02	-.3757E-02	-.1841E-02	-.4267E-03
-0.8	-.1375E-02	-.1209E-02	-.7834E-03	-.3085E-03	-.7332E-04

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.5030E-03	.2088E-02	.4507E-02	.6719E-02	.7621E-02	.6719E-02	.4507E-02	.2088E-02	.5030E-03
0.6	.2028E-02	.7916E-02	.1646E-01	.2463E-01	.2810E-01	.2463E-01	.1646E-01	.7916E-02	.2028E-02
0.4	.3712E-02	.1489E-01	.3320E-01	.5401E-01	.6419E-01	.5401E-01	.3320E-01	.1489E-01	.3712E-02
0.2	.4158E-02	.1841E-01	.4704E-01	.9485E-01	.1326E+00	.9485E-01	.4704E-01	.1841E-01	.4158E-02
0	.3820E-02	.1796E-01	.4843E-01	.1131E+00	.1000E+31	.1131E+00	.4843E-01	.1796E-01	.3820E-02

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.7308E-03	-.2909E-02	-.6327E-02	-.9890E-02	-.1245E-01	-.1287E-01	-.1046E-01	-.5838E-02	-.1391E-02
0.6	-.2998E-02	-.1123E-01	-.2269E-01	-.3478E-01	-.4436E-01	-.4771E-01	-.4159E-01	-.2593E-01	-.7680E-02
0.4	-.6157E-02	-.2207E-01	-.4397E-01	-.6815E-01	-.9012E-01	-.1037E+00	-.1006E+00	-.7286E-01	-.2807E-01
0.2	-.8886E-02	-.3141E-01	-.6274E-01	-.9907E-01	-.1365E+00	-.1699E+00	-.1902E+00	-.1782E+00	-.9186E-01
0	-.9993E-02	-.3515E-01	-.7038E-01	-.1121E+00	-.1574E+00	-.2035E+00	-.2472E+00	-.2848E+00	-.3012E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1024E-02	-.4548E-02	-.1018E-01	-.1551E-01	-.1791E-01	-.1531E-01	-.8085E-02	-.6530E-03	.1582E-02
0.6	-.5274E-02	-.2059E-01	-.4236E-01	-.6474E-01	-.7995E-01	-.7917E-01	-.5728E-01	-.2238E-01	.4133E-02
0.4	-.1247E-01	-.4604E-01	-.9430E-01	-.1502E+00	-.2034E+00	-.2364E+00	-.2213E+00	-.1350E+00	-.5877E-01
0.2	-.1967E-01	-.7145E-01	-.1483E+00	-.2470E+00	-.3656E+00	-.5003E+00	-.6321E+00	-.6684E+00	-.2316E+00

Clamped rectangular plate — $b/a = 1.6$ — $\lambda = 1$

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1698E-02	.1532E-02	.1093E-02	.5518E-03	.1493E-03
0.6	.6103E-02	.5546E-02	.4073E-02	.2204E-02	.6301E-03
0.4	.1297E-01	.1178E-01	.8682E-02	.4774E-02	.1404E-02
0.2	.2163E-01	.1941E-01	.1400E-01	.7623E-02	.2279E-02
0	.2799E-01	.2407E-01	.1680E-01	.9016E-02	.2712E-02
-0.2	.2163E-01	.1941E-01	.1400E-01	.7623E-02	.2279E-02
-0.4	.1297E-01	.1178E-01	.8682E-02	.4774E-02	.1404E-02
-0.6	.6103E-02	.5546E-02	.4073E-02	.2204E-02	.6301E-03
-0.8	.1698E-02	.1532E-02	.1093E-02	.5518E-03	.1493E-03

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.3555E-02	-.3271E-02	-.2510E-02	-.1486E-02	-.4887E-03
0.6	-.7050E-02	-.6377E-02	-.4753E-02	-.2840E-02	-.1025E-02
0.4	.1877E-02	.2625E-02	.3106E-02	.1931E-02	.3799E-03
0.2	.4851E-01	.4749E-01	.3557E-01	.1881E-01	.5416E-02
0	.1000E+31	.1435E+00	.7277E-01	.3324E-01	.9350E-02
-0.2	.4851E-01	.4749E-01	.3557E-01	.1881E-01	.5416E-02
-0.4	.1877E-02	.2625E-02	.3106E-02	.1931E-02	.3799E-03
-0.6	-.7050E-02	-.6377E-02	-.4753E-02	-.2840E-02	-.1025E-02
-0.8	-.3555E-02	-.3271E-02	-.2510E-02	-.1486E-02	-.4887E-03

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1951E+01	-.1038E+01	-.2094E+00	-.5948E-01	-.5093E-02
0.6	-.8780E+00	-.7026E+00	-.3806E+00	-.1510E+00	-.3552E-01
0.4	-.4499E+00	-.3946E+00	-.2658E+00	-.1315E+00	-.3598E-01
0.2	-.2235E+00	-.2010E+00	-.1439E+00	-.7587E-01	-.2141E-01
0	-.1008E+00	-.9106E-01	-.6579E-01	-.3472E-01	-.9652E-02
-0.2	-.3904E-01	-.3512E-01	-.2498E-01	-.1271E-01	-.3241E-02
-0.4	-.1165E-01	-.1033E-01	-.6972E-02	-.3167E-02	-.5752E-03
-0.6	-.1828E-02	-.1503E-02	-.7386E-03	-.4021E-04	.1491E-03
-0.8	.2029E-03	.2445E-03	.3226E-03	.3125E-03	.1145E-03

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2876E+00	-.1987E+00	-.8718E-01	-.3458E-01	-.7997E-02
0.6	-.2162E+00	-.1865E+00	-.1222E+00	-.6032E-01	-.1712E-01
0.4	-.1406E+00	-.1266E+00	-.9141E-01	-.4933E-01	-.1455E-01
0.2	-.8061E-01	-.7331E-01	-.5407E-01	-.2967E-01	-.8696E-02
0	-.4092E-01	-.3721E-01	-.2737E-01	-.1485E-01	-.4278E-02
-0.2	-.1818E-01	-.1646E-01	-.1194E-01	-.6301E-02	-.1703E-02
-0.4	-.6775E-02	-.6085E-02	-.4296E-02	-.2146E-02	-.4986E-03
-0.6	-.1886E-02	-.1660E-02	-.1092E-02	-.4613E-03	-.7288E-04
-0.8	-.2683E-03	-.2195E-03	-.1006E-03	.6574E-05	.8952E-05

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.4355E-03	.1768E-02	.3695E-02	.5381E-02	.6049E-02	.5381E-02	.3695E-02	.1768E-02	.4355E-03
0.6	.1909E-02	.7158E-02	.1428E-01	.2065E-01	.2325E-01	.2065E-01	.1428E-01	.7158E-02	.1909E-02
0.4	.3800E-02	.1468E-01	.3113E-01	.4823E-01	.5603E-01	.4823E-01	.3113E-01	.1468E-01	.3800E-02
0.2	.4524E-02	.1941E-01	.4787E-01	.9158E-01	.1222E+00	.9158E-01	.4787E-01	.1941E-01	.4524E-02
0	.4107E-02	.1892E-01	.5018E-01	.1154E+00	.1000E+31	.1154E+00	.5018E-01	.1892E-01	.4107E-02

Influence surface of M_x at $x = a$; $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.6398E-03	-.2561E-02	-.5462E-02	-.8299E-02	-.1003E-01	-.9849E-02	-.7552E-02	-.3973E-02	-.8885E-03
0.6	-.2813E-02	-.1037E-01	-.2061E-01	-.3078E-01	-.3786E-01	-.3891E-01	-.3215E-01	-.1894E-01	-.5243E-02
0.4	-.6024E-02	-.2160E-01	-.4234E-01	-.6471E-01	-.8330E-01	-.9239E-01	-.8528E-01	-.5813E-01	-.2203E-01
0.2	-.9131E-02	-.3225E-01	-.6402E-01	-.1001E+00	-.1360E+00	-.1659E+00	-.1803E+00	-.1604E+00	-.7174E-01
0	-.1050E-01	-.3676E-01	-.7354E-01	-.1161E+00	-.1619E+00	-.2077E+00	-.2505E+00	-.2864E+00	-.2951E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$; $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.5285E-03	-.2648E-02	-.6023E-02	-.8804E-02	-.9205E-02	-.6463E-02	-.1791E-02	.1611E-02	.1697E-02
0.6	-.3904E-02	-.1533E-01	-.3112E-01	-.4588E-01	-.5337E-01	-.4830E-01	-.3036E-01	-.8014E-02	.6270E-02
0.4	-.1094E-01	-.4058E-01	-.8215E-01	-.1278E+00	-.1666E+00	-.1827E+00	-.1563E+00	-.8542E-01	-.5189E-01
0.2	-.1930E-01	-.7007E-01	-.1447E+00	-.2386E+00	-.3479E+00	-.4647E+00	-.5618E+00	-.5398E+00	-.8845E-01
0	-.2330E-01	-.8389E-01	-.1750E+00	-.2962E+00	-.4544E+00	-.6694E+00	-.9923E+00	-.1565E+01	-.2800E+01

Clamped rectangular plate — b/a = 1.8 — λ = 1

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1338E-02	.1205E-02	.8540E-03	.4273E-03	.1146E-03
0.6	.5097E-02	.4632E-02	.3398E-02	.1832E-02	.5248E-03
0.4	.1167E-01	.1062E-01	.7841E-02	.4314E-02	.1252E-02
0.2	.2092E-01	.1883E-01	.1365E-01	.7449E-02	.2222E-02
0	.2829E-01	.2434E-01	.1700E-01	.9128E-02	.2761E-02
-0.2	.2092E-01	.1883E-01	.1365E-01	.7449E-02	.2222E-02
-0.4	.1167E-01	.1062E-01	.7841E-02	.4314E-02	.1252E-02
-0.6	.5097E-02	.4632E-02	.3398E-02	.1832E-02	.5248E-03
-0.8	.1338E-02	.1205E-02	.8540E-03	.4273E-03	.1146E-03

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.4080E-02	-.3755E-02	-.2865E-02	-.1655E-02	-.5203E-03
0.6	-.9584E-02	-.8785E-02	-.6694E-02	-.3967E-02	-.1344E-02
0.4	-.4600E-02	-.3648E-02	-.2023E-02	-.1089E-02	-.5611E-03
0.2	.3748E-01	.3724E-01	.2898E-01	.1568E-01	.4519E-02
0	.1000E+31	.1427E+00	.7214E-01	.3292E-01	.9301E-02
-0.2	.3748E-01	.3724E-01	.2898E-01	.1568E-01	.4519E-02
-0.4	-.4600E-02	-.3648E-02	-.2023E-02	-.1089E-02	-.5611E-03
-0.6	-.9584E-02	-.8785E-02	-.6694E-02	-.3967E-02	-.1344E-02
-0.8	-.4080E-02	-.3755E-02	-.2865E-02	-.1655E-02	-.5203E-03

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1735E+01	-.1016E+01	-.2657E+00	-.7320E-01	-.1044E-01
0.6	-.7409E+00	-.6143E+00	-.3615E+00	-.1554E+00	-.3885E-01
0.4	-.3491E+00	-.3103E+00	-.2160E+00	-.1107E+00	-.3086E-01
0.2	-.1524E+00	-.1376E+00	-.9934E-01	-.5268E-01	-.1469E-01
0	-.5677E-01	-.5121E-01	-.3672E-01	-.1903E-01	-.5153E-02
-0.2	-.1580E-01	-.1407E-01	-.9649E-02	-.4546E-02	-.9688E-03
-0.4	-.1493E-02	-.1174E-02	-.4428E-03	.1716E-03	.2924E-03
-0.6	.1565E-02	.1512E-02	.1312E-02	.9056E-03	.3360E-03
-0.8	.8533E-03	.8026E-03	.6520E-03	.4089E-03	.1206E-03

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2805E+00	-.2047E+00	-.9833E-01	-.4008E-01	-.9786E-02
0.6	-.1966E+00	-.1726E+00	-.1174E+00	-.5996E-01	-.1742E-01
0.4	-.1158E+00	-.1049E+00	-.7670E-01	-.4188E-01	-.1232E-01
0.2	-.5835E-01	-.5310E-01	-.3918E-01	-.2142E-01	-.6162E-02
0	-.2506E-01	-.2274E-01	-.1660E-01	-.8863E-02	-.2520E-02
-0.2	-.8793E-02	-.7917E-02	-.5636E-02	-.2863E-02	-.7153E-03
-0.4	-.2147E-02	-.1892E-02	-.1252E-02	-.5382E-03	-.5393E-04
-0.6	-.8926E-04	-.4730E-04	.4313E-04	.1010E-03	.5798E-04
-0.8	.1477E-03	.1468E-03	.1381E-03	.1032E-03	.2982E-04

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.3286E-03	.1332E-02	.2767E-02	.4002E-02	.4486E-02	.4002E-02	.2767E-02	.1332E-02	.3286E-03
0.6	.1613E-02	.5934E-02	.1161E-01	.1648E-01	.1842E-01	.1648E-01	.1161E-01	.5934E-02	.1613E-02
0.4	.3585E-02	.1360E-01	.2789E-01	.4182E-01	.4788E-01	.4182E-01	.2789E-01	.1360E-01	.3585E-02
0.2	.4674E-02	.1966E-01	.4723E-01	.8667E-01	.1117E+00	.8667E-01	.4723E-01	.1966E-01	.4674E-02
0	.4227E-02	.1926E-01	.5082E-01	.1163E+00	.1000E+31	.1163E+00	.5082E-01	.1926E-01	.4227E-02

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.4926E-03	-.1980E-02	-.4199E-02	-.6277E-02	-.7382E-02	-.6985E-02	-.5127E-02	-.2572E-02	-.5224E-03
0.6	-.2394E-02	-.8754E-02	-.1723E-01	-.2529E-01	-.3031E-01	-.3012E-01	-.2392E-01	-.1347E-01	-.3479E-02
0.4	-.5516E-02	-.1993E-01	-.3900E-01	-.5854E-01	-.7382E-01	-.7951E-01	-.7053E-01	-.4598E-01	-.1770E-01
0.2	-.9026E-02	-.3191E-01	-.6316E-01	-.9814E-01	-.1321E+00	-.1587E+00	-.1683E+00	-.1426E+00	-.5498E-01
0	-.1073E-01	-.3736E-01	-.7444E-01	-.1177E+00	-.1636E+00	-.2093E+00	-.2518E+00	-.2866E+00	-.2875E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.2932E-04	-.7140E-03	-.2063E-02	-.2959E-02	-.2431E-02	-.3996E-03	.1953E-02	.2731E-02	.1741E-02
0.6	-.2316E-02	-.9504E-02	-.1944E-01	-.2796E-01	-.3067E-01	-.2494E-01	-.1239E-01	.5760E-03	.7149E-02
0.4	-.8883E-02	-.3352E-01	-.6748E-01	-.1031E+00	-.1300E+00	-.1350E+00	-.1062E+00	-.5410E-01	-.4848E-01
0.2	-.1844E-01	-.6701E-01	-.1380E+00	-.2260E+00	-.3254E+00	-.4250E+00	-.4921E+00	-.4280E+00	.1139E-01
0	-.2351E-01	-.8417E-01	-.1756E+00	-.2971E+00	-.4557E+00	-.6712E+00	-.9931E+00	-.1551E+01	-.2672E+01

Clamped rectangular plate   b/a = 1.0   λ = 3

Influence coefficient $s_1 = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.8447E-03	.7620E-03	.5121E-03	.2373E-03	.4903E-04
0.6	.3098E-02	.2734E-02	.1866E-02	.9210E-03	.2373E-03
0.4	.6394E-02	.5581E-02	.3748E-02	.1866E-02	.5121E-03
0.2	.1026E-01	.8664E-02	.5581E-02	.2734E-02	.7620E-03
0	.1293E-01	.1026E-01	.6394E-02	.3098E-02	.8647E-03
-0.2	.1026E-01	.8664E-02	.5581E-02	.2734E-02	.7620E-03
-0.4	.6394E-02	.5581E-02	.3748E-02	.1866E-02	.5121E-03
-0.6	.3098E-02	.2734E-02	.1866E-02	.9210E-03	.2373E-03
-0.8	.8647E-03	.7620E-03	.5121E-03	.2373E-03	.4903E-04

Influence surface of M_y at x = y = 0 (ν = 0.3)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.3580E-02	-.2966E-02	-.1771E-02	-.8444E-03	-.2839E-03
0.6	-.6766E-02	-.5056E-02	-.2366E-02	-.9761E-03	-.3899E-03
0.4	.2826E-02	.5279E-02	.5633E-02	.2911E-02	.6259E-03
0.2	.4975E-01	.4581E-01	.2593E-01	.1065E-01	.2569E-02
0	.1000E+31	.1004E+00	.4082E-01	.1521E-01	.3640E-02
-0.2	.4975E-01	.4581E-01	.2593E-01	.1065E-01	.2569E-02
-0.4	.2826E-02	.5279E-02	.5633E-02	.2911E-02	.6259E-03
-0.6	-.6766E-02	-.5056E-02	-.2366E-02	-.9761E-03	-.3899E-03
-0.8	-.3580E-02	-.2966E-02	-.1771E-02	-.8444E-03	-.2839E-03

Influence coefficient $s_2 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2794E+01	-.8705E+00	.6664E-01	-.2041E-01	.1708E-01
0.6	-.1325E+01	-.7819E+00	-.1739E+00	-.1743E-01	.5973E-02
0.4	-.6561E+00	-.4787E+00	-.1915E+00	-.4114E-01	-.1393E-02
0.2	-.3146E+00	-.2499E+00	-.1249E+00	-.3662E-01	-.4018E-02
0	-.1360E+00	-.1117E+00	-.6052E-01	-.1925E-01	-.2078E-02
-0.2	-.4758E-01	-.3890E-01	-.2014E-01	-.4822E-02	.5191E-03
-0.4	-.9542E-02	-.6900E-02	-.1425E-02	.2171E-02	.1699E-02
-0.6	.2012E-02	.2564E-02	.3451E-02	.3218E-02	.1447E-02
-0.8	.1883E-02	.1923E-02	.1868E-02	.1401E-02	.5495E-03

Influence surface of M_y at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2757E+00	-.1359E+00	-.3005E-01	-.1137E-01	-.1020E-02
0.6	-.2191E+00	-.1595E+00	-.7235E-01	-.2593E-01	-.3604E-02
0.4	-.1489E+00	-.1225E+00	-.7105E-01	-.3019E-01	-.7469E-02
0.2	-.9123E-01	-.7874E-01	-.5084E-01	-.2386E-01	-.6242E-02
0	-.5021E-01	-.4420E-01	-.2986E-01	-.1456E-01	-.3803E-02
-0.2	-.2424E-01	-.2145E-01	-.1459E-01	-.7018E-02	-.1702E-02
-0.4	-.9718E-02	-.8533E-02	-.5619E-02	-.2460E-02	-.4505E-03
-0.6	-.2840E-02	-.2420E-02	-.1412E-02	-.4077E-03	.5132E-04
-0.8	-.4026E-03	-.3087E-03	-.9390E-04	.8367E-04	.9083E-04

Influence surface of M_x at x = y = 0 (ν = 0.3)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.2839E-03	-.3899E-03	.6259E-03	.2569E-02	.3640E-02	.2569E-02	.6259E-03	-.3899E-03	-.2839E-03
0.6	-.8444E-03	-.9761E-03	.2911E-02	.1065E-01	.1521E-01	.1065E-01	.2911E-02	-.9761E-03	-.8444E-03
0.4	-.1771E-02	-.2366E-02	.5633E-02	.2593E-01	.4082E-01	.2593E-01	.5633E-02	-.2366E-02	-.1771E-02
0.2	-.2966E-02	-.5056E-02	.5279E-02	.4581E-01	.1004E+00	.4581E-01	.5279E-02	-.5056E-02	-.2966E-02
0	-.3580E-02	-.6766E-02	.2826E-02	.4975E-01	.1000E+31	.4975E-01	.2826E-02	-.6766E-02	-.3580E-02

Influence surface of M_x at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.9083E-04	.5132E-04	-.4505E-03	-.1702E-02	-.3803E-02	-.6242E-02	-.7469E-02	-.5604E-02	-.1020E-02
0.6	.8367E-04	-.4077E-03	-.2460E-02	-.7018E-02	-.1456E-01	-.2386E-01	-.3019E-01	-.2593E-01	-.1137E-01
0.4	-.9390E-04	-.1412E-02	-.5619E-02	-.1459E-01	-.2986E-01	-.5084E-01	-.7105E-01	-.7235E-01	-.3005E-01
0.2	-.3087E-03	-.2420E-02	-.8533E-02	-.2145E-01	-.4420E-01	-.7874E-01	-.1225E+00	-.1595E+00	-.1359E+00
0	-.4026E-03	-.2840E-02	-.9718E-02	-.2424E-01	-.5021E-01	-.9123E-01	-.1489E+00	-.2191E+00	-.2757E+00

Influence coefficient $s_3 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.5495E-03	.1447E-02	.1699E-02	.5191E-03	-.2078E-02	-.4018E-02	-.1393E-02	.5973E-02	.1708E-01
0.6	.1401E-02	.3218E-02	.2171E-02	-.4822E-02	-.1925E-01	-.3662E-01	-.4114E-01	-.1743E-01	-.2041E-01
0.4	.1868E-02	.3451E-02	-.1425E-02	-.2014E-01	-.6052E-01	-.1249E+00	-.1915E+00	-.1739E+00	.6664E-01
0.2	.1923E-02	.2564E-02	-.6900E-02	-.3890E-01	-.1117E+00	-.2499E+00	-.4787E+00	-.7819E+00	-.8705E+00
0	.1883E-02	.2012E-02	-.9542E-02	-.4758E-01	-.1360E+00	-.3146E+00	-.6561E+00	-.1325E+01	-.2794E+01

Clamped rectangular plate — b/a = 1.2 — λ = 3

Influence coefficient $s_1 = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.6254E-03	.5531E-03	.3726E-03	.1694E-03	.3190E-04
0.6	.2453E-02	.2181E-02	.1509E-02	.7508E-03	.1920E-03
0.4	.5613E-02	.4949E-02	.3389E-02	.1709E-02	.4686E-03
0.2	.9922E-02	.8479E-02	.5547E-02	.2741E-02	.7627E-03
0	.1327E-01	.1058E-01	.6622E-02	.3217E-02	.8955E-03
-0.2	.9922E-02	.8479E-02	.5547E-02	.2741E-02	.7627E-03
-0.4	.5613E-02	.4949E-02	.3389E-02	.1709E-02	.4686E-03
-0.6	.2453E-02	.2181E-02	.1509E-02	.7508E-03	.1920E-03
-0.8	.6254E-03	.5531E-03	.3726E-03	.1694E-03	.3190E-04

Influence surface of M_y at x = y = 0 (ν = 0.3)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.4788E-02	-.4240E-02	-.2955E-02	-.1568E-02	-.4720E-03
0.6	-.1173E-01	-.1010E-01	-.6684E-02	-.3488E-02	-.1128E-02
0.4	-.8322E-02	-.5596E-02	-.2239E-02	-.1032E-02	-.4836E-03
0.2	.3176E-01	.3143E-01	.1916E-01	.7912E-02	.1853E-02
0	.1000E+31	.9812E-01	.3917E-01	.1433E-01	.3395E-02
-0.2	.3176E-01	.3143E-01	.1916E-01	.7912E-02	.1853E-02
-0.4	-.8322E-02	-.5596E-02	-.2239E-02	-.1032E-02	-.4836E-03
-0.6	-.1173E-01	-.1010E-01	-.6684E-02	-.3488E-02	-.1128E-02
-0.8	-.4788E-02	-.4240E-02	-.2955E-02	-.1568E-02	-.4720E-03

Influence coefficient $s_2 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2392E+01	-.9101E+00	.2136E-02	-.1121E-01	.1357E-01
0.6	-.9992E+00	-.6582E+00	-.2006E+00	-.2958E-01	.2831E-02
0.4	-.4253E+00	-.3293E+00	-.1538E+00	-.4110E-01	-.3693E-02
0.2	-.1617E+00	-.1322E+00	-.7088E-01	-.2241E-01	-.2526E-02
0	-.4499E-01	-.3652E-01	-.1834E-01	-.3817E-02	.8044E-03
-0.2	-.8263E-03	.8238E-03	.3880E-02	.4800E-02	.2382E-02
-0.4	.9989E-02	.9738E-02	.8572E-02	.5894E-02	.2189E-02
-0.6	.7838E-02	.7382E-02	.5979E-02	.3722E-02	.1249E-02
-0.8	.2723E-02	.2538E-02	.1991E-02	.1168E-02	.3480E-03

Influence surface of M_y at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2698E+00	-.1503E+00	-.4205E-01	-.1432E-01	-.2025E-02
0.6	-.1902E+00	-.1476E+00	-.7585E-01	-.2930E-01	-.6773E-02
0.4	-.1123E+00	-.9554E-01	-.5969E-01	-.2714E-01	-.6981E-02
0.2	-.5699E-01	-.5004E-01	-.3360E-01	-.1632E-01	-.4277E-02
0	-.2393E-01	-.2116E-01	-.1437E-01	-.6893E-02	-.1667E-02
-0.2	-.7248E-02	-.6300E-02	-.4000E-02	-.1595E-02	-.2090E-03
-0.4	-.5592E-03	-.3410E-03	.1280E-03	.4329E-03	.2882E-03
-0.6	.9924E-03	.9881E-03	.9205E-03	.6837E-03	.2765E-03
-0.8	.5208E-03	.5002E-03	.4266E-03	.2836E-03	.1005E-03

Influence surface of M_x at x = y = 0 (ν = 0.3)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1784E-03	-.2469E-03	.3089E-03	.1291E-02	.1802E-02	.1291E-02	.3089E-03	-.2469E-03	-.1784E-03
0.6	-.5264E-03	-.4722E-03	.2147E-02	.6733E-02	.9217E-02	.6733E-02	.2147E-02	-.4722E-03	-.5264E-03
0.4	-.1291E-02	-.1350E-02	.5440E-02	.2014E-01	.2964E-01	.2014E-01	.5440E-02	-.1350E-02	-.1291E-02
0.2	-.2660E-02	-.4201E-02	.6312E-02	.4310E-01	.8444E-01	.4310E-01	.6312E-02	-.4201E-02	-.2660E-02
0	-.3504E-02	-.6522E-02	.3308E-02	.5044E-01	.1000E+31	.5044E-01	.3308E-02	-.6522E-02	-.3504E-02

Influence surface of M_x at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1134E-03	.1554E-03	-.1762E-03	-.1042E-02	-.2354E-02	-.3573E-02	-.3741E-02	-.2307E-02	-.4431E-03
0.6	.1323E-03	-.1959E-03	-.1854E-02	-.5440E-02	-.1088E-01	-.1664E-01	-.1914E-01	-.1450E-01	-.4510E-02
0.4	-.6770E-04	-.1297E-02	-.5221E-02	-.1330E-01	-.2630E-01	-.4269E-01	-.5582E-01	-.5206E-01	-.2360E-01
0.2	-.3403E-03	-.2560E-02	-.8807E-02	-.2165E-01	-.4366E-01	-.7592E-01	-.1144E+00	-.1405E+00	-.9630E-01
0	-.4658E-03	-.3120E-02	-.1037E-01	-.2536E-01	-.5171E-01	-.9279E-01	-.1500E+00	-.2198E+00	-.2824E+00

Influence coefficient $s_3 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.6481E-03	.2034E-02	.3387E-02	.4348E-02	.5241E-02	.7091E-02	.1031E-01	.1171E-01	.6094E-02
0.6	.1774E-02	.4921E-02	.6819E-02	.5669E-02	.1574E-02	-.1810E-02	.2642E-02	.1546E-01	.1935E-01
0.4	.2333E-02	.5496E-02	.3994E-02	-.7862E-02	-.3474E-01	-.7476E-01	-.1067E+00	-.8197E-01	-.2882E-01
0.2	.2251E-02	.3968E-02	-.3480E-02	-.3181E-01	-.9735E-01	-.2199E+00	-.4103E+00	-.6060E+00	-.3569E+00
0	.2106E-02	.2938E-02	-.7589E-02	-.4445E-01	-.1316E+00	-.3091E+00	-.6501E+00	-.1328E+01	-.2938E+01

Clamped rectangular plate — $b/a = 1.4$ — $\lambda = 3$

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.3688E-03	.3237E-03	.2113E-03	.8757E-04	.1139E-04
0.6	.1720E-02	.1531E-02	.1058E-02	.5187E-03	.1267E-03
0.4	.4616E-02	.4093E-02	.2831E-02	.1434E-02	.3891E-03
0.2	.9270E-02	.7988E-02	.5282E-02	.2624E-02	.7297E-03
0	.1334E-01	.1063E-01	.6663E-02	.3238E-02	.9016E-03
-0.2	.9270E-02	.7988E-02	.5282E-02	.2624E-02	.7297E-03
-0.4	.4616E-02	.4093E-02	.2831E-02	.1434E-02	.3891E-03
-0.6	.1720E-02	.1531E-02	.1058E-02	.5187E-03	.1267E-03
-0.8	.3688E-03	.3237E-03	.2113E-03	.8757E-04	.1139E-04

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.4805E-02	-.4337E-02	-.3150E-02	-.1710E-02	-.4985E-03
0.6	-.1330E-01	-.1185E-01	-.8410E-02	-.4590E-02	-.1450E-02
0.4	-.1456E-01	-.1190E-01	-.7244E-02	-.3711E-02	-.1254E-02
0.2	.1874E-01	.2027E-01	.1356E-01	.5695E-02	.1289E-02
0	.1000E+31	.9735E-01	.3863E-01	.1407E-01	.3344E-02
-0.2	.1874E-01	.2027E-01	.1356E-01	.5695E-02	.1289E-02
-0.4	-.1456E-01	-.1190E-01	-.7244E-02	-.3711E-02	-.1254E-02
-0.6	-.1330E-01	-.1185E-01	-.8410E-02	-.4590E-02	-.1450E-02
-0.8	-.4805E-02	-.4337E-02	-.3150E-02	-.1710E-02	-.4985E-03

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2054E+01	-.9100E+00	-.5729E-01	-.7340E-02	.1169E-01
0.6	-.7552E+00	-.5358E+00	-.1992E+00	-.3861E-01	-.1812E-03
0.4	-.2683E+00	-.2152E+00	-.1101E+00	-.3332E-01	-.3854E-02
0.2	-.7299E-01	-.5991E-01	-.3188E-01	-.8969E-02	-.1920E-03
0	-.4462E-02	-.2216E-02	.2160E-02	.4232E-02	.2334E-02
-0.2	.1162E-01	.1123E-01	.9695E-02	.6529E-02	.2379E-02
-0.4	.1005E-01	.9377E-02	.7420E-02	.4485E-02	.1462E-02
-0.6	.5120E-02	.4719E-02	.3600E-02	.2050E-02	.6129E-03
-0.8	.1409E-02	.1285E-02	.9448E-03	.4952E-03	.1220E-03

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2596E+00	-.1588E+00	-.5263E-01	-.1746E-01	-.2978E-02
0.6	-.1622E+00	-.1314E+00	-.7367E-01	-.3042E-01	-.7390E-02
0.4	-.8164E-01	-.7085E-01	-.4634E-01	-.2201E-01	-.5774E-02
0.2	-.3276E-01	-.2896E-01	-.1971E-01	-.9588E-02	-.2418E-02
0	-.8999E-02	-.7862E-02	-.5090E-02	-.2146E-02	-.3550E-03
-0.2	-.1090E-03	.7290E-04	.4422E-03	.6120E-03	.3426E-03
-0.4	.1833E-02	.1754E-02	.1487E-02	.9934E-03	.3657E-03
-0.6	.1337E-02	.1250E-02	.9943E-03	.6061E-03	.1998E-03
-0.8	.4411E-03	.4082E-03	.3143E-03	.1794E-03	.5193E-04

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1381E-03	-.2627E-03	-.7756E-04	.3246E-03	.5379E-03	.3246E-03	-.7756E-04	-.2627E-03	-.1381E-03
0.6	-.3797E-03	-.4383E-03	.1019E-02	.3513E-02	.4817E-02	.3513E-02	.1019E-02	-.4383E-03	-.3797E-03
0.4	-.9672E-03	-.8814E-03	.4371E-02	.1465E-01	.2075E-01	.1465E-01	.4371E-02	-.8814E-03	-.9672E-03
0.2	-.2420E-02	-.3597E-02	.6651E-02	.3919E-01	.7086E-01	.3919E-01	.6651E-02	-.3597E-02	-.2420E-02
0	-.3512E-02	-.6513E-02	.3337E-02	.5049E-01	.1000E+31	.5049E-01	.3337E-02	-.6513E-02	-.3512E-02

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1399E-03	.3039E-03	.2388E-03	-.1492E-03	-.7467E-03	-.1203E-02	-.1096E-02	-.4233E-03	.9370E-04
0.6	.2223E-03	.2188E-03	-.7390E-03	-.3021E-02	-.6370E-02	-.9506E-02	-.1020E-01	-.6899E-02	-.1624E-02
0.4	.3332E-04	-.8570E-03	-.4038E-02	-.1064E-01	-.2094E-01	-.3304E-01	-.4100E-01	-.3530E-01	-.1544E-01
0.2	-.3004E-03	-.2394E-02	-.8358E-02	-.2059E-01	-.4131E-01	-.7099E-01	-.1045E+00	-.1221E+00	-.7198E-01
0	-.4685E-03	-.3139E-02	-.1043E-01	-.2547E-01	-.5189E-01	-.9305E-01	-.1504E+00	-.2200E+00	-.2774E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.7152E-03	.2491E-02	.4739E-02	.7169E-02	.9774E-02	.1241E-01	.1387E-01	.1154E-01	.4788E-02
0.6	.2098E-02	.6472E-02	.1102E-01	.1455E-01	.1715E-01	.2036E-01	.2530E-01	.2745E-01	.1951E-01
0.4	.2758E-02	.7446E-02	.9267E-02	.3868E-02	-.1137E-01	-.3281E-01	-.4331E-01	-.1973E-01	-.1960E-01
0.2	.2485E-02	.5060E-02	-.5838E-03	-.2542E-01	-.8392E-01	-.1912E+00	-.3470E+00	-.4681E+00	-.1559E+00
0	.2185E-02	.3336E-02	-.6687E-02	-.4309E-01	-.1302E+00	-.3083E+00	-.6506E+00	-.1323E+01	-.2820E+01

Clamped rectangular plate

$b/a = 1.6$ $\lambda = 3$

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1585E-03	.1348E-03	.7748E-04	.2020E-04	-.4793E-05
0.6	.1074E-02	.9516E-03	.6467E-03	.3031E-03	.6607E-04
0.4	.3644E-02	.3241E-02	.2252E-02	.1138E-02	.3031E-03
0.2	.8544E-02	.7412E-02	.4947E-02	.2469E-02	.6861E-03
0	.1334E-01	.1064E-01	.6667E-02	.3240E-02	.9041E-03
-0.2	.8544E-02	.7412E-02	.4947E-02	.2469E-02	.6861E-03
-0.4	.3644E-02	.3241E-02	.2252E-02	.1138E-02	.3031E-03
-0.6	.1074E-02	.9516E-03	.6467E-03	.3031E-03	.6607E-04
-0.8	.1585E-03	.1348E-03	.7748E-04	.2020E-04	-.4793E-05

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.4043E-02	-.3669E-02	-.2689E-02	-.1453E-02	-.4083E-03
0.6	-.1256E-01	-.1134E-01	-.8265E-02	-.4580E-02	-.1426E-02
0.4	-.1736E-01	-.1493E-01	-.9924E-02	-.5247E-02	-.1697E-02
0.2	.9092E-02	.1155E-01	.8796E-02	.3773E-02	.7943E-03
0	.1000E+31	.9722E-01	.3855E-01	.1405E-01	.3356E-02
-0.2	.9092E-02	.1155E-01	.8796E-02	.3773E-02	.7943E-03
-0.4	-.1736E-01	-.1493E-01	-.9924E-02	-.5247E-02	-.1697E-02
-0.6	-.1256E-01	-.1134E-01	-.8265E-02	-.4580E-02	-.1426E-02
-0.8	-.4043E-02	-.3669E-02	-.2689E-02	-.1453E-02	-.4083E-03

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1769E+01	-.8826E+00	-.1073E+00	-.7944E-02	.1004E-01
0.6	-.5691E+00	-.4250E+00	-.1808E+00	-.4239E-01	-.2463E-02
0.4	-.1617E+00	-.1322E+00	-.7095E-01	-.2247E-01	-.2565E-02
0.2	-.2430E-01	-.1905E-01	-.7955E-02	.2663E-03	.1612E-02
0	.9911E-02	.9889E-02	.9149E-02	.6594E-02	.2548E-02
-0.2	.1092E-01	.1022E-01	.8126E-02	.4950E-02	.1633E-02
-0.4	.5724E-02	.5269E-02	.4008E-02	.2276E-02	.6781E-03
-0.6	.1976E-02	.1793E-02	.1302E-02	.6778E-03	.1733E-03
-0.8	.3627E-03	.3200E-03	.2101E-03	.8436E-04	.7673E-05

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2471E+00	-.1625E+00	-.6130E-01	-.2058E-01	-.3913E-02
0.6	-.1361E+00	-.1135E+00	-.6774E-01	-.2955E-01	-.7447E-02
0.4	-.5700E-01	-.5004E-01	-.3362E-01	-.1634E-01	-.4280E-02
0.2	-.1678E-01	-.1481E-01	-.9959E-02	-.4641E-02	-.1031E-02
0	-.1631E-02	-.1279E-02	-.4824E-03	-.1740E-03	.2465E-03
-0.2	.1886E-02	.1813E-02	.1555E-02	.1054E-02	.3939E-03
-0.4	.1595E-02	.1487E-02	.1173E-02	.7076E-03	.2305E-03
-0.6	.7352E-03	.6761E-03	.5123E-03	.2886E-03	.8505E-04
-0.8	.1813E-03	.1643E-03	.1186E-03	.5997E-04	.1385E-04

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1226E-03	-.3031E-03	-.3450E-03	-.2549E-03	-.1930E-03	-.2549E-03	-.3450E-03	-.3031E-03	-.1226E-03
0.6	-.3222E-03	-.5536E-03	.6687E-04	.1278E-02	.1917E-02	.1278E-02	.6687E-04	-.5536E-03	-.3222E-03
0.4	-.7427E-03	-.6890E-03	.3124E-02	.1013E-01	.1406E-01	.1013E-01	.3124E-02	-.6890E-03	-.7427E-03
0.2	-.2182E-02	-.3050E-02	.6715E-02	.3503E-01	.5954E-01	.3503E-01	.6715E-02	-.3050E-02	-.2182E-02
0	-.3527E-02	-.6523E-02	.3327E-02	.5048E-01	.1000E+31	.5048E-01	.3327E-02	-.6523E-02	-.3527E-02

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1562E-03	.4227E-03	.5880E-03	.5720E-03	.4324E-03	.3273E-03	.3768E-03	.4608E-03	-.2981E-03
0.6	.3076E-03	.6280E-03	.3487E-03	-.7910E-03	-.2558E-02	-.4095E-02	-.4199E-02	-.2394E-02	-.1118E-03
0.4	.1477E-03	-.3490E-03	-.2699E-02	-.7770E-02	-.1556E-01	-.2414E-01	-.2866E-01	-.2312E-01	-.1046E-01
0.2	-.2493E-03	-.2168E-02	-.7751E-02	-.1922E-01	-.3848E-01	-.6544E-01	-.9416E-01	-.1045E+00	-.5221E-01
0	-.4704E-03	-.3140E-02	-.1043E-01	-.2548E-01	-.5193E-01	-.9316E-01	-.1506E+00	-.2199E+00	-.2707E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.6886E-03	.2552E-02	.5114E-02	.7985E-02	.1079E-01	.1281E-01	.1270E-01	.9117E-02	.3439E-02
0.6	.2210E-02	.7208E-02	.1324E-01	.1926E-01	.2485E-01	.2974E-01	.3229E-01	.2836E-01	.1698E-01
0.4	.3016E-02	.8792E-02	.1318E-01	.1288E-01	.6506E-02	-.2243E-02	-.2057E-02	.1186E-01	-.1968E-01
0.2	.2620E-02	.5784E-02	.1613E-02	-.1997E-01	-.7145E-01	-.1635E+00	-.2868E+00	-.3493E+00	-.1810E-01
0	.2168E-02	.3337E-02	-.6605E-02	-.4297E-01	-.1303E+00	-.3089E+00	-.6515E+00	-.1314E+01	-.2694E+01

Clamped rectangular plate — b/a = 1.8 — λ = 3

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1722E-04	.8350E-05	-.1067E-04	-.2236E-04	-.1399E-04
0.6	.5793E-03	.5072E-03	.3293E-03	.1373E-03	.2041E-04
0.4	.2789E-02	.2484E-02	.1727E-02	.8660E-03	.2236E-03
0.2	.7821E-02	.6822E-02	.4592E-02	.2302E-02	.6390E-03
0	.1334E-01	.1064E-01	.6669E-02	.3242E-02	.9077E-03
-0.2	.7821E-02	.6822E-02	.4592E-02	.2302E-02	.6390E-03
-0.4	.2789E-02	.2484E-02	.1727E-02	.8660E-03	.2236E-03
-0.6	.5793E-03	.5072E-03	.3293E-03	.1373E-03	.2041E-04
-0.8	.1722E-04	.8350E-05	-.1067E-04	-.2236E-04	-.1399E-04

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2951E-02	-.2676E-02	-.1949E-02	-.1030E-02	-.2752E-03
0.6	-.1060E-01	-.9617E-02	-.7086E-02	-.3936E-02	-.1205E-02
0.4	-.1798E-01	-.1582E-01	-.1098E-01	-.5923E-02	-.1885E-02
0.2	.1712E-02	.4610E-02	.4651E-02	.2015E-02	.3293E-03
0	.1000E+31	.9724E-01	.3858E-01	.1408E-01	.3378E-02
-0.2	.1712E-02	.4610E-02	.4651E-02	.2015E-02	.3293E-03
-0.4	-.1798E-01	-.1582E-01	-.1098E-01	-.5923E-02	-.1885E-02
-0.6	-.1060E-01	-.9617E-02	-.7086E-02	-.3936E-02	-.1205E-02
-0.8	-.2951E-02	-.2676E-02	-.1949E-02	-.1030E-02	-.2752E-03

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1529E+01	-.8375E+00	-.1461E+00	-.1175E-01	.8363E-02
0.6	-.4255E+00	-.3294E+00	-.1540E+00	-.4128E-01	-.3735E-02
0.4	-.9072E-01	-.7460E-01	-.4018E-01	-.1204E-01	-.7820E-03
0.2	-.6855E-04	.1511E-02	.4382E-02	.5073E-02	.2471E-02
0	.1212E-01	.1146E-01	.9378E-02	.5935E-02	.2058E-02
-0.2	.7007E-02	.6472E-02	.4970E-02	.2870E-02	.8822E-03
-0.4	.2319E-02	.2107E-02	.1539E-02	.8105E-03	.2099E-03
-0.6	.3623E-03	.3137E-03	.1926E-03	.6354E-04	-.2845E-05
-0.8	-.3931E-04	-.4286E-04	-.4802E-04	-.4243E-04	-.1957E-04

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2334E+00	-.1625E+00	-.6787E-01	-.2346E-01	-.4799E-02
0.6	-.1123E+00	-.9557E-01	-.5973E-01	-.2718E-01	-.7039E-02
0.4	-.3798E-01	-.3354E-01	-.2281E-01	-.1113E-01	-.2842E-02
0.2	-.7031E-02	-.6102E-02	-.3852E-02	-.1512E-02	-.1737E-03
0	.1314E-02	.1327E-02	.1275E-02	.9788E-03	.4113E-03
-0.2	.1804E-02	.1689E-02	.1349E-02	.8297E-03	.2779E-03
-0.4	.8668E-03	.7979E-03	.6067E-03	.3444E-03	.1019E-03
-0.6	.2469E-03	.2231E-03	.1599E-03	.8094E-04	.1951E-04
-0.8	.3065E-04	.2619E-04	.1513E-04	.3688E-05	-.1224E-05

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1077E-03	-.3066E-03	-.4538E-03	-.5088E-03	-.5163E-03	-.5088E-03	-.4538E-03	-.3066E-03	-.1077E-03
0.6	-.2925E-03	-.6491E-03	-.5474E-03	-.7339E-04	.1992E-03	-.7339E-04	-.5474E-03	-.6491E-03	-.2925E-03
0.4	-.5878E-03	-.6364E-03	.1980E-02	.6642E-02	.9172E-02	.6642E-02	.1980E-02	-.6364E-03	-.5878E-03
0.2	-.1946E-02	-.2546E-02	.6593E-02	.3094E-01	.5009E-01	.3094E-01	.6593E-02	-.2546E-02	-.1946E-02
0	-.3547E-02	-.6530E-02	.3325E-02	.5048E-01	.1000E+31	.5048E-01	.3325E-02	-.6530E-02	-.3547E-02

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1537E-03	.4659E-03	.7620E-03	.9543E-03	.1038E-02	.1047E-02	.9772E-03	.7411E-03	.3300E-03
0.6	.3577E-03	.9056E-03	.1125E-02	.7964E-03	.5162E-04	-.6234E-03	-.6582E-03	.1744E-04	.5985E-03
0.4	.2469E-03	.1086E-03	-.1477E-02	-.5174E-02	-.1082E-01	-.1668E-01	-.1909E-01	-.1465E-01	-.7461E-02
0.2	-.1979E-03	-.1937E-02	-.7117E-02	-.1778E-01	-.3551E-01	-.5970E-01	-.8383E-01	-.8029E-01	-.3619E-01
0	-.4758E-03	-.3134E-02	-.1045E-01	-.2552E-01	-.5200E-01	-.9329E-01	-.1507E+00	-.2193E+00	-.2628E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.5641E-03	.2193E-02	.4516E-02	.7079E-02	.9341E-02	.1051E-01	.9628E-02	.6259E-02	.2321E-02
0.6	.2076E-02	.7000E-02	.1332E-01	.1997E-01	.2610E-01	.3046E-01	.3075E-01	.2463E-01	.1381E-01
0.4	.3098E-02	.9485E-02	.1562E-01	.1898E-01	.1683E-01	.1797E-01	.2210E-01	.2499E-01	-.2280E-01
0.2	.2716E-02	.6348E-02	.3493E-02	-.1495E-01	-.5944E-01	-.1367E+00	-.2309E+00	-.2494E+00	.7585E-01
0	.2141E-02	.3238E-02	-.6774E-02	-.4326E-01	-.1308E+00	-.3098E+00	-.6513E+00	-.1299E+01	-.2565E+01

Clamped rectangular plate — b/a = 1.0 — λ = 5

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.3914E-04	.2356E-04	-.5190E-05	-.1875E-04	-.1073E-04
0.6	.3475E-03	.2619E-03	.9861E-04	-.5311E-05	-.1875E-04
0.4	.1290E-02	.9881E-03	.4467E-03	.9861E-04	-.5190E-05
0.2	.3143E-02	.2269E-02	.9881E-03	.2619E-03	.2356E-04
0	.4990E-02	.3143E-02	.1290E-02	.3475E-03	.3914E-04
-0.2	.3143E-02	.2269E-02	.9881E-03	.2619E-03	.2356E-04
-0.4	.1290E-02	.9881E-03	.4467E-03	.9861E-04	-.5190E-05
-0.6	.3475E-03	.2619E-03	.9861E-04	-.5311E-05	-.1875E-04
-0.8	.3914E-04	.2356E-04	-.5190E-05	-.1875E-04	-.1073E-04

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.3976E-02	-.3291E-02	-.1901E-02	-.7618E-03	-.1508E-03
0.6	-.1271E-01	-.1029E-01	-.5803E-02	-.2433E-02	-.6045E-03
0.4	-.1842E-01	-.1338E-01	-.6865E-02	-.3078E-02	-.8812E-03
0.2	.6376E-02	.8328E-02	.1408E-02	-.1196E-02	-.6573E-03
0	.1000E+31	.5281E-01	.1029E-01	.5345E-03	-.4116E-03
-0.2	.6376E-02	.8328E-02	.1408E-02	-.1196E-02	-.6573E-03
-0.4	-.1842E-01	-.1338E-01	-.6865E-02	-.3078E-02	-.8812E-03
-0.6	-.1271E-01	-.1029E-01	-.5803E-02	-.2433E-02	-.6045E-03
-0.8	-.3976E-02	-.3291E-02	-.1901E-02	-.7618E-03	-.1508E-03

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2506E+01	-.6661E+00	.1519E+00	.6606E-02	.2048E-01
0.6	-.8455E+00	-.4015E+00	.2534E-01	.5460E-01	.1999E-01
0.4	-.2160E+00	-.1117E+00	.2314E-01	.4561E-01	.1775E-01
0.2	-.1612E-01	.4802E-02	.3256E-01	.3068E-01	.1129E-01
0	.2359E-01	.2557E-01	.2552E-01	.1730E-01	.5618E-02
-0.2	.1780E-01	.1681E-01	.1332E-01	.7544E-02	.2069E-02
-0.4	.7326E-02	.6591E-02	.4599E-02	.2116E-02	.3573E-03
-0.6	.1631E-02	.1361E-02	.7015E-03	.4828E-04	-.1628E-03
-0.8	.2029E-04	-.3082E-04	-.1413E-03	-.2030E-03	-.1217E-03

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2320E+00	-.1004E+00	-.9714E-02	-.2506E-02	.1061E-02
0.6	-.1299E+00	-.8321E-01	-.2370E-01	-.2883E-02	.6220E-03
0.4	-.5196E-01	-.3742E-01	-.1352E-01	-.1335E-02	.7304E-03
0.2	-.1369E-01	-.9757E-02	-.2716E-02	.1009E-02	.9003E-03
0	-.1164E-03	.6159E-03	.1775E-02	.1849E-02	.8038E-03
-0.2	.2457E-02	.2420E-02	.2145E-02	.1440E-02	.5071E-03
-0.4	.1704E-02	.1589E-02	.1245E-02	.7307E-03	.2224E-03
-0.6	.6867E-03	.6254E-03	.4582E-03	.2380E-03	.5623E-04
-0.8	.1402E-03	.1244E-03	.8264E-04	.3232E-04	-.2251E-06

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1508E-03	-.6045E-03	-.8812E-03	-.6573E-03	-.4116E-03	-.6573E-03	-.8812E-03	-.6045E-03	-.1508E-03
0.6	-.7618E-03	-.2433E-02	-.3078E-02	-.1196E-02	.5345E-03	-.1196E-02	-.3078E-02	-.2433E-02	-.7618E-03
0.4	-.1901E-02	-.5803E-02	-.6865E-02	.1408E-02	.1029E-01	.1408E-02	-.6865E-02	-.5803E-02	-.1901E-02
0.2	-.3291E-02	-.1029E-01	-.1338E-01	.8328E-02	.5281E-01	.8328E-02	-.1338E-01	-.1029E-01	-.3291E-02
0	-.3976E-02	-.1271E-01	-.1842E-01	.6376E-02	.1000E+31	.6376E-02	-.1842E-01	-.1271E-01	-.3976E-02

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.2251E-06	.5623E-04	.2224E-03	.5071E-03	.8038E-03	.9003E-03	.7304E-03	.6220E-03	.1061E-02
0.6	.3232E-04	.2380E-03	.7307E-03	.1440E-02	.1849E-02	.1009E-02	-.1335E-02	-.2883E-02	-.2506E-02
0.4	.8264E-04	.4582E-03	.1245E-02	.2145E-02	.1775E-02	-.2716E-02	-.1352E-01	-.2370E-01	-.9714E-02
0.2	.1244E-03	.6254E-03	.1589E-02	.2420E-02	.6159E-03	-.9757E-02	-.3742E-01	-.8321E-01	-.1004E+00
0	.1402E-03	.6867E-03	.1704E-02	.2457E-02	-.1164E-03	-.1369E-01	-.5196E-01	-.1299E+00	-.2320E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1217E-03	-.1628E-03	.3573E-03	.2069E-02	.5618E-02	.1129E-01	.1775E-01	.1999E-01	.2048E-01
0.6	-.2030E-03	.4828E-04	.2116E-02	.7544E-02	.1730E-01	.3068E-01	.4561E-01	.5460E-01	.6606E-02
0.4	-.1413E-03	.7015E-03	.4599E-02	.1332E-01	.2552E-01	.3256E-01	.2314E-01	.2534E-01	.1519E+00
0.2	-.3082E-04	.1361E-02	.6591E-02	.1681E-01	.2557E-01	.4802E-02	-.1117E+00	-.4015E+00	-.6661E+00
0	.2029E-04	.1631E-02	.7326E-02	.1780E-01	.2359E-01	-.1612E-01	-.2160E+00	-.8455E+00	-.2506E+01

Clamped rectangular plate	b/a=1.2	λ=5

Influence coefficient $s_1 = wD/Pa^2$ at x=y=0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2901E-04	-.3150E-04	-.3344E-04	-.2622E-04	-.1030E-04
0.6	.8551E-04	.5078E-04	-.1285E-04	-.4209E-04	-.2366E-04
0.4	.8020E-03	.6140E-03	.2648E-03	.3889E-04	-.1560E-04
0.2	.2706E-02	.1990E-02	.8795E-03	.2300E-03	.1802E-04
0	.4992E-02	.3146E-02	.1292E-02	.3488E-03	.3978E-04
-0.2	.2706E-02	.1990E-02	.8795E-03	.2300E-03	.1802E-04
-0.4	.8020E-03	.6140E-03	.2648E-03	.3889E-04	-.1560E-04
-0.6	.8551E-04	.5078E-04	-.1285E-04	-.4209E-04	-.2366E-04
-0.8	-.2901E-04	-.3150E-04	-.3344E-04	-.2622E-04	-.1030E-04

Influence surface of M_y at x=y=0 (ν=0.3)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2128E-02	-.1788E-02	-.1049E-02	-.3925E-03	-.5186E-04
0.6	-.9051E-02	-.7546E-02	-.4470E-02	-.1884E-02	-.4348E-03
0.4	-.1822E-01	-.1417E-01	-.7666E-02	-.3283E-02	-.8686E-03
0.2	-.4193E-02	.3741E-03	-.1050E-02	-.1715E-02	-.7169E-03
0	.1000E+31	.5290E-01	.1037E-01	.5967E-03	-.3837E-03
-0.2	-.4193E-02	.3741E-03	-.1050E-02	-.1715E-02	-.7169E-03
-0.4	-.1822E-01	-.1417E-01	-.7666E-02	-.3283E-02	-.8686E-03
-0.6	-.9051E-02	-.7546E-02	-.4470E-02	-.1884E-02	-.4348E-03
-0.8	-.2128E-02	-.1788E-02	-.1049E-02	-.3925E-03	-.5186E-04

Influence coefficient $s_2 = V_y a/P$ at x=0, y=b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2045E+01	-.6561E+00	.1149E+00	.2513E-01	.1869E-01
0.6	-.5143E+00	-.2639E+00	.1709E-01	.5322E-01	.1999E-01
0.4	-.6644E-01	-.2522E-01	.3093E-01	.3661E-01	.1392E-01
0.2	.2156E-01	.2530E-01	.2780E-01	.1976E-01	.6393E-02
0	.1780E-01	.1681E-01	.1332E-01	.7554E-02	.2077E-02
-0.2	.5641E-02	.5019E-02	.3376E-02	.1424E-02	.1679E-03
-0.4	.2249E-03	.8287E-04	-.2285E-03	-.4257E-03	-.2699E-03
-0.6	-.7015E-03	-.6935E-03	-.6381E-03	-.4706E-03	-.1887E-03
-0.8	-.2947E-03	-.2817E-03	-.2375E-03	-.1547E-03	-.515,1E-04

Influence surface of M_y at x=0, y=b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2147E+00	-.1049E+00	-.1530E-01	-.2457E-02	.8715E-03
0.6	-.9346E-01	-.6386E-01	-.2093E-01	-.2592E-02	.6256E-03
0.4	-.2510E-01	-.1829E-01	-.6323E-02	.1908E-03	.8500E-03
0.2	-.1597E-02	-.5116E-03	.1317E-02	.1812E-02	.8436E-03
0	.2460E-02	.2423E-02	.2147E-02	.1440E-02	.5068E-03
-0.2	.1483E-02	.1373E-02	.1059E-02	.6071E-03	.1787E-03
-0.4	.4106E-03	.3672E-03	.2531E-03	.1145E-03	.1761E-04
-0.6	.2547E-05	-.4937E-05	-.2148E-04	-.3151E-04	-.1964E-04
-0.8	-.3483E-04	-.3436E-04	-.3172E-04	-.2390E-04	-.9890E-05

Influence surface of M_x at x=y=0 (ν=0.3)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.4790E-04	-.2648E-03	-.4791E-03	-.5305E-03	-.5082E-03	-.5305E-03	-.4791E-03	-.2648E-03	-.4790E-04
0.6	-.4147E-03	-.1427E-02	-.2028E-02	-.1513E-02	-.9462E-03	-.1513E-02	-.2028E-02	-.1427E-02	-.4147E-03
0.4	-.1412E-02	-.4347E-02	-.5245E-02	-.4396E-03	.4239E-02	-.4396E-03	-.5245E-02	-.4347E-02	-.1412E-02
0.2	-.3024E-02	-.9410E-02	-.1188E-01	.7038E-02	.3900E-01	.7038E-02	-.1188E-01	-.9410E-02	-.3024E-02
0	-.3958E-02	-.1269E-01	-.1839E-01	.6407E-02	.1000E+31	.6407E-02	-.1839E-01	-.1269E-01	-.3958E-02

Influence surface of M_x at x=a, y=0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.8617E-05	.2001E-04	.1430E-03	.3891E-03	.7255E-03	.1044E-02	.1218E-02	.1116E-02	.5570E-03
0.6	.1170E-04	.1606E-03	.5755E-03	.1265E-02	.1969E-02	.2173E-02	.1665E-02	.1159E-02	.1083E-02
0.4	.6122E-04	.3841E-03	.1113E-02	.2063E-02	.2237E-02	-.3285E-03	-.6722E-02	-.1205E-01	-.7522E-02
0.2	.1112E-03	.5839E-03	.1521E-02	.2389E-02	.8918E-03	-.8285E-02	-.3227E-01	-.6822E-01	-.6374E-01
0	.1320E-03	.6631E-03	.1667E-02	.2411E-02	-.1590E-03	-.1370E-01	-.5190E-01	-.1300E+00	-.2388E+00

Influence coefficient $s_3 = V_x a/P$ at x=a, y=0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1573E-03	-.3518E-03	-.1584E-03	.9604E-03	.3560E-02	.7654E-02	.1160E-01	.1138E-01	.4746E-02
0.6	-.2994E-03	-.3865E-03	.1002E-02	.5380E-02	.1403E-01	.2674E-01	.4003E-01	.4411E-01	.2795E-01
0.4	-.2453E-03	.2644E-03	.3577E-02	.1171E-01	.2476E-01	.3821E-01	.4614E-01	.5497E-01	.2786E-01
0.2	-.9676E-04	.1106E-02	.6064E-02	.1617E-01	.2612E-01	.1219E-01	-.7673E-01	-.2659E+00	-.1840E+00

Clamped rectangular plate   b/a = 1.4   λ = .5

Influence coefficient $s_1 = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.4189E-04	-.4011E-04	-.3367E-04	-.2159E-04	-.7199E-05
0.6	-.3650E-04	-.4647E-04	-.6021E-04	-.5250E-04	-.2216E-04
0.4	.4520E-03	.3395E-03	.1275E-03	-.5189E-05	-.2234E-04
0.2	.2297E-02	.1714E-02	.7669E-03	.1965E-03	.1201E-04
0	.4994E-02	.3147E-02	.1293E-02	.3497E-03	.4008E-04
-0.2	.2297E-02	.1714E-02	.7669E-03	.1965E-03	.1201E-04
-0.4	.4520E-03	.3395E-03	.1275E-03	-.3189E-05	-.2234E-04
-0.6	-.3650E-04	-.4647E-04	-.6021E-04	-.5250E-04	-.2216E-04
-0.8	-.4189E-04	-.4011E-04	-.3367E-04	-.2159E-04	-.7199E-05

Influence surface of M_y at x = y = 0 (ν = 0.3)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.6786E-03	-.5483E-03	-.2655E-03	-.3192E-04	.3808E-04
0.6	-.5276E-02	-.4437E-02	-.2643E-02	-.1063E-02	-.2056E-03
0.4	-.1570E-01	-.1264E-01	-.7129E-02	-.3022E-02	-.7638E-03
0.2	-.1111E-01	-.5544E-02	-.3205E-02	-.2211E-02	-.7858E-03
0	.1000E+31	.5291E-01	.1038E-01	.6053E-03	-.3815E-03
-0.2	-.1111E-01	-.5544E-02	-.3205E-02	-.2211E-02	-.7858E-03
-0.4	-.1570E-01	-.1264E-01	-.7129E-02	-.3022E-02	-.7638E-03
-0.6	-.5276E-02	-.4437E-02	-.2643E-02	-.1063E-02	-.2056E-03
-0.8	-.6786E-03	-.5483E-03	-.2655E-03	-.3192E-04	.3808E-04

Influence coefficient $s_2 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1658E+01	-.6123E+00	.8194E-01	.3882E-01	.1927E-01
0.6	-.2947E+00	-.1546E+00	.2012E-01	.4847E-01	.1873E-01
0.4	-.5658E-03	.1390E-01	.3226E-01	.2781E-01	.9979E-02
0.2	.2194E-01	.2139E-01	.1808E-01	.1093E-01	.3242E-02
0	.7241E-02	.6510E-02	.4531E-02	.2075E-02	.3489E-03
-0.2	.2130E-03	.7187E-04	-.2369E-03	-.4307E-03	-.2718E-03
-0.4	-.7478E-03	-.7277E-03	-.6441E-03	-.4534E-03	-.1733E-03
-0.6	-.3172E-03	-.2979E-03	-.2386E-03	-.1439E-03	-.4369E-04
-0.8	-.4666E-04	-.4234E-04	-.2995E-04	-.1293E-04	-.5351E-06

Influence surface of M_y at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1939E+00	-.1041E+00	-.1959E-01	-.2545E-02	.8040E-03
0.6	-.6404E-01	-.4554E-01	-.1614E-01	-.1827E-02	.6871E-03
0.4	-.9516E-02	-.6582E-02	-.1329E-02	.1317E-02	.9031E-03
0.2	.2130E-02	.2260E-02	.2282E-02	.1689E-02	.6347E-03
0	.1710E-02	.1593E-02	.1247E-02	.7310E-03	.2228E-03
-0.2	.4083E-03	.3651E-03	.2515E-03	.1136E-03	.1740E-04
-0.4	-.3714E-04	-.4085E-04	-.4722E-04	-.4424E-04	-.2226E-04
-0.6	-.6106E-04	-.5825E-04	-.4913E-04	-.3274E-04	-.1192E-04
-0.8	-.1740E-04	-.1628E-04	-.1292E-04	-.7631E-05	-.2171E-05

Influence surface of M_x at x = y = 0 (ν = 0.3)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.6619E-05	-.6450E-04	-.1814E-03	-.2608E-03	-.2833E-03	-.2608E-03	-.1814E-03	-.6450E-04	.6619E-05
0.6	-.1820E-03	-.7326E-03	-.1191E-02	-.1187E-02	-.1054E-02	-.1187E-02	-.1191E-02	-.7326E-03	-.1820E-03
0.4	-.9932E-03	-.3125E-02	-.3935E-02	-.1363E-02	.1052E-02	-.1363E-02	-.3935E-02	-.3125E-02	-.9932E-03
0.2	-.2752E-02	-.8501E-02	-.1046E-01	.5501E-02	.2860E-01	.5501E-02	-.1046E-01	-.8501E-02	-.2752E-02
0	-.3963E-02	-.1268E-01	-.1838E-01	.6416E-02	.1000E+31	.6416E-02	-.1838E-01	-.1268E-01	-.3963E-02

Influence surface of M_x at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1554E-04	-.1600E-04	.4847E-04	.2090E-03	.4591E-03	.7303E-03	.8882E-03	.7636E-03	.3243E-03
0.6	-.8423E-05	.7437E-04	.3725E-03	.9339E-03	.1642E-02	.2203E-02	.2353E-02	.2062E-02	.1319E-02
0.4	.4010E-04	.3019E-03	.9413E-03	.1868E-02	.2405E-02	.1267E-02	-.2048E-02	-.4509E-02	-.3527E-02
0.2	.1020E-03	.5501E-03	.1459E-02	.2354E-02	.1164E-02	-.6787E-02	-.2715E-01	-.5483E-01	-.4231E-01
0	.1304E-03	.6571E-03	.1655E-02	.2392E-02	-.1879E-03	-.1375E-01	-.5201E-01	-.1300E+00	-.2335E+00

Influence coefficient $s_3 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1585E-03	-.4425E-03	-.5560E-03	-.1476E-03	.1107E-02	.3101E-02	.4723E-02	.4092E-02	.1400E-02
0.6	-.3413E-03	-.6956E-03	-.5700E-04	.2780E-02	.8888E-02	.1804E-01	.2680E-01	.2754E-01	.1724E-01
0.4	-.3078E-03	-.8349E-04	.2551E-02	.9625E-02	.2210E-01	.3809E-01	.5396E-01	.6453E-01	.1312E-01
0.2	-.1253E-03	.9624E-03	.5687E-02	.1562E-01	.2654E-01	.1892E-01	-.4568E-01	-.1658E+00	-.8205E-02
0	-.2202E-04	.1473E-02	.7022E-02	.1738E-01	.2313E-01	-.1652E-01	-.2164E+00	-.8479E+00	-.2535E+01

Clamped rectangular plate — $b/a = 1.6$, $\lambda = 5$

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2973E-04	-.2764E-04	-.2151E-04	-.1242E-04	-.3576E-05
0.6	-.7185E-04	-.7173E-04	-.6574E-04	-.4611E-04	-.1708E-04
0.4	.2166E-03	.1527E-03	.3316E-04	-.3391E-04	-.2563E-04
0.2	.1921E-02	.1450E-02	.6538E-03	.1619E-03	.5693E-05
0	.4995E-02	.3148E-02	.1293E-02	.3499E-03	.4009E-04
-0.2	.1921E-02	.1450E-02	.6538E-03	.1619E-03	.5693E-05
-0.4	.2166E-03	.1527E-03	.3316E-04	-.3391E-04	-.2563E-04
-0.6	-.7185E-04	-.7173E-04	-.6574E-04	-.4611E-04	-.1708E-04
-0.8	-.2973E-04	-.2764E-04	-.2151E-04	-.1242E-04	-.3576E-05

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.6328E-04	.9173E-04	.1416E-03	.1448E-03	.7211E-04
0.6	-.2512E-02	-.2094E-02	-.1185E-02	-.3949E-03	-.2833E-04
0.4	-.1245E-01	-.1025E-01	-.5952E-02	-.2517E-02	-.6095E-03
0.2	-.1544E-01	-.9686E-02	-.4964E-02	-.2642E-02	-.8458E-03
0	.1000E+31	.5291E-01	.1038E-01	.6032E-03	-.3835E-03
-0.2	-.1544E-01	-.9686E-02	-.4964E-02	-.2642E-02	-.8458E-03
-0.4	-.1245E-01	-.1025E-01	-.5952E-02	-.2517E-02	-.6095E-03
-0.6	-.2512E-02	-.2094E-02	-.1185E-02	-.3949E-03	-.2833E-04
-0.8	.6328E-04	.9178E-04	.1416E-03	.1448E-03	.7211E-04

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1335E+01	-.5488E+00	.5580E-01	.4782E-01	.2012E-01
0.6	-.1533E+00	-.7633E-01	.2621E-01	.4262E-01	.1657E-01
0.4	.2155E-01	.2529E-01	.2778E-01	.1975E-01	.6569E-02
0.2	.1323E-01	.1222E-01	.9197E-02	.4868E-02	.1192E-02
0	.1330E-02	.1070E-02	.4488E-03	-.1219E-03	-.2240E-03
-0.2	-.7672E-03	-.7544E-03	-.6851E-03	-.4982E-03	-.1977E-03
-0.4	-.3151E-03	-.2958E-03	-.2367E-03	-.1425E-03	-.4303E-04
-0.6	-.1886E-04	-.1580E-04	-.7605E-05	.1668E-05	.4455E-05
-0.8	.1623E-04	.1570E-04	.1387E-04	.1003E-04	.4084E-05

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1720E+00	-.9938E-01	-.2237E-01	-.2694E-02	.7579E-03
0.6	-.4153E-01	-.3016E-01	-.1095E-01	-.8147E-03	.7761E-03
0.4	-.1597E-02	-.5120E-03	.1317E-02	.1813E-02	.8448E-03
0.2	.2312E-02	.2206E-02	.1829E-02	.1151E-02	.3823E-03
0	.6907E-03	.6268E-03	.4540E-03	.2313E-03	.5314E-04
-0.2	-.1185E-04	-.1844E-04	-.3229E-04	-.3017E-04	-.2182E-04
-0.4	-.6209E-04	-.5918E-04	-.4981E-04	-.3310E-04	-.1199E-04
-0.6	-.1598E-04	-.1480E-04	-.1137E-04	-.6328E-05	-.1622E-05
-0.8	-.7405E-08	.9695E-07	.3606E-06	.5812E-06	.4086E-06

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.2386E-04	.2393E-04	-.1642E-04	-.6006E-04	-.7735E-04	-.6006E-04	-.1642E-04	.2393E-04	.2386E-04
0.6	-.5181E-04	-.3120E-03	-.6069E-03	-.7284E-03	-.7321E-03	-.7284E-03	-.6069E-03	-.3120E-03	-.5181E-04
0.4	-.6624E-03	-.2170E-02	-.2900E-02	-.1667E-02	-.4684E-03	-.1667E-02	-.2900E-02	-.2170E-02	-.6624E-03
0.2	-.2472E-02	-.7591E-02	-.9158E-02	.3941E-02	.2070E-01	.3941E-02	-.9158E-02	-.7591E-02	-.2472E-02
0	-.3973E-02	-.1269E-01	-.1838E-01	.6416E-02	.1000E+31	.6416E-02	-.1838E-01	-.1269E-01	-.3973E-02

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1738E-04	-.3650E-04	-.2054E-04	.5441E-04	.1894E-03	.3437E-03	.4279E-03	.3436E-03	.1341E-03
0.6	-.2171E-04	.5978E-05	.1886E-03	.5825E-03	.1141E-02	.1683E-02	.1950E-02	.1739E-02	.1057E-02
0.4	.2178E-04	.2243E-03	.7642E-03	.1619E-02	.2354E-02	.2147E-02	.7043E-03	-.5437E-03	-.2026E-02
0.2	.9394E-04	.5190E-03	.1398E-02	.2320E-02	.1442E-02	-.5276E-02	-.2214E-01	-.4269E-01	-.2556E-01
0	.1307E-03	.6571E-03	.1654E-02	.2389E-02	-.1978E-03	-.1378E-01	-.5206E-01	-.1296E+00	-.2266E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1248E-03	-.4043E-03	-.6734E-03	-.7410E-03	-.4369E-03	.1918E-03	.6757E-03	.4073E-03	.9381E-04
0.6	-.3266E-03	-.8174E-03	-.7598E-03	.6885E-03	.4266E-02	.9720E-02	.1453E-01	.1452E-01	.1017E-01
0.4	-.3430E-03	-.3551E-03	.1609E-02	.7439E-02	.1849E-01	.3415E-01	.5099E-01	.5643E-01	-.3489E-02
0.2	-.1493E-03	.8317E-03	.5315E-02	.1502E-01	.2680E-01	.2507E-01	-.1791E-01	-.8455E-01	.1050E+00
0	-.1988E-04	.1483E-02	.7036E-02	.1739E-01	.2310E-01	-.1661E-01	-.2158E+00	-.8362E+00	-.2406E+01

Clamped rectangular plate — b/a = 1.8 — λ = 5

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1412E-04	-.1285E-04	-.9343E-05	-.4706E-05	-.9881E-06
0.6	-.6636E-04	-.6326E-04	-.5255E-04	-.3340E-04	-.1130E-04
0.4	.6836E-04	.3479E-04	-.2542E-04	-.4956E-04	-.2606E-04
0.2	.1583E-02	.1205E-02	.5442E-03	.1276E-03	-.5869E-06
0	.4995E-02	.3148E-02	.1294E-02	.3499E-03	.4002E-04
-0.2	.1583E-02	.1205E-02	.5442E-03	.1276E-03	-.5869E-06
-0.4	.6836E-04	.3479E-04	-.2542E-04	-.4956E-04	-.2606E-04
-0.6	-.6636E-04	-.6326E-04	-.5255E-04	-.3340E-04	-.1130E-04
-0.8	-.1412E-04	-.1285E-04	-.9343E-05	-.4706E-05	-.9881E-06

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2762E-03	.2681E-03	.2354E-03	.1638E-03	.6264E-04
0.6	-.8493E-03	-.6688E-03	-.2854E-03	.1020E-04	.6981E-04
0.4	-.9244E-02	-.7707E-02	-.4560E-02	-.1912E-02	-.4354E-03
0.2	-.1787E-01	-.1234E-01	-.6272E-02	-.2974E-02	-.8881E-03
0	.1000E+31	.5291E-01	.1038E-01	.6021E-03	-.3860E-03
-0.2	-.1787E-01	-.1234E-01	-.6272E-02	-.2974E-02	-.8881E-03
-0.4	-.9244E-02	-.7707E-02	-.4560E-02	-.1912E-02	-.4354E-03
-0.6	-.8493E-03	-.6688E-03	-.2854E-03	.1020E-04	.6981E-04
-0.8	.2762E-03	.2681E-03	.2354E-03	.1638E-03	.6264E-04

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1067E+01	-.4761E+00	.3719E-01	.5294E-01	.2093E-01
0.6	-.6647E-01	-.2526E-01	.3089E-01	.3658E-01	.1399E-01
0.4	.2338E-01	.2334E-01	.2058E-01	.1288E-01	.3918E-02
0.2	.5644E-02	.5020E-02	.3372E-02	.1415E-02	.1558E-03
0	-.5923E-03	-.6270E-03	-.6677E-03	-.5710E-03	-.2627E-03
-0.2	-.4461E-03	-.4221E-03	-.3461E-03	-.2178E-03	-.7183E-04
-0.4	-.3549E-04	-.3125E-04	-.1958E-04	-.5093E-05	.2717E-05
-0.6	.2300E-04	.2204E-04	.1892E-04	.1313E-04	.5152E-05
-0.8	.6141E-05	.5742E-05	.4548E-05	.2683E-05	.7781E-06

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1505E+00	-.9207E-01	-.2367E-01	-.2816E-02	.7296E-03
0.6	-.2510E-01	-.1829E-01	-.6324E-02	.1919E-03	.8606E-03
0.4	.1674E-02	.1945E-02	.2231E-02	.1780E-02	.6952E-03
0.2	.1483E-02	.1374E-02	.1060E-02	.6069E-03	.1773E-03
0	.1298E-03	.1096E-03	.5921E-04	.7045E-05	-.1257E-04
-0.2	-.7091E-04	-.6833E-04	-.5930E-04	-.4129E-04	-.1595E-04
-0.4	-.2019E-04	-.1878E-04	-.1464E-04	-.8422E-05	-.2319E-05
-0.6	.1103E-05	.1151E-05	.1237E-05	.1153E-05	.6187E-06
-0.8	.1309E-05	.1242E-05	.1033E-05	.6768E-06	.2407E-06

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.2216E-04	.4475E-04	.4366E-04	.3074E-04	.2401E-04	.3074E-04	.4366E-04	.4475E-04	.2216E-04
0.6	.9526E-05	-.8592E-04	-.2468E-03	-.3572E-03	-.3883E-03	-.3572E-03	-.2468E-03	-.8592E-04	.9526E-05
0.4	-.4155E-03	-.1452E-02	-.2087E-02	-.1601E-02	-.1046E-02	-.1601E-02	-.2087E-02	-.1452E-02	-.4155E-03
0.2	-.2192E-02	-.6704E-02	-.7993E-02	.2509E-02	.1470E-01	.2509E-02	-.7993E-02	-.6704E-02	-.2192E-02
0	-.3987E-02	-.1269E-01	-.1838E-01	.6415E-02	.1000E+31	.6415E-02	-.1838E-01	-.1269E-01	-.3987E-02

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1462E-04	-.3897E-04	-.4974E-04	-.3117E-04	.2019E-04	.8542E-04	.1209E-03	.8657E-04	.3964E-04
0.6	-.2733E-04	-.3608E-04	.5548E-04	.2968E-03	.6722E-03	.1067E-02	.1283E-02	.1159E-02	.7411E-03
0.4	.6242E-05	.1544E-03	.5937E-03	.1346E-02	.2148E-02	.2467E-02	.2033E-02	.1150E-02	-.1747E-02
0.2	.8555E-04	.4863E-03	.1333E-02	.2276E-02	.1703E-02	-.3821E-02	-.1745E-01	-.3205E-01	-.1261E-01
0	.1310E-03	.6577E-03	.1654E-02	.2388E-02	-.2034E-03	-.1380E-01	-.5202E-01	-.1288E+00	-.2185E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.7545E-04	-.2826E-03	-.5528E-03	-.7931E-03	-.9084E-03	-.8723E-03	-.7913E-03	-.7649E-03	-.1365E-03
0.6	-.2751E-03	-.7823E-03	-.1064E-02	-.5848E-03	.1119E-02	.3864E-02	.6219E-02	.6618E-02	.6109E-02
0.4	-.3613E-03	-.5668E-03	.7761E-03	.5327E-02	.1449E-01	.2812E-01	.4235E-01	.4163E-01	-.1709E-01
0.2	-.1764E-03	.6882E-03	.4903E-02	.1431E-01	.2678E-01	.3023E-01	.5506E-02	-.2154E-01	.1752E+00
0	-.1985E-04	.1487E-02	.7043E-02	.1739E-01	.2310E-01	-.1647E-01	-.2140E+00	-.8191E+00	-.2277E+01

Clamped rectangular plate — b/a = 1.0 — λ = 7

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2091E-04	-.1942E-04	-.1429E-04	-.6973E-05	-.1432E-05
0.6	-.1742E-04	-.2728E-04	-.3479E-04	-.2338E-04	-.6973E-05
0.4	.2323E-03	.1295E-03	-.4004E-05	-.3479E-04	-.1429E-04
0.2	.1174E-02	.6713E-03	.1295E-03	-.2728E-04	-.1942E-04
0	.2550E-02	.1174E-02	.2323E-03	-.1742E-04	-.2091E-04
-0.2	.1174E-02	.6713E-03	.1295E-03	-.2728E-04	-.1942E-04
-0.4	.2323E-03	.1295E-03	-.4004E-05	-.3479E-04	-.1429E-04
-0.6	-.1742E-04	-.2728E-04	-.3479E-04	-.2338E-04	-.6973E-05
-0.8	-.2091E-04	-.1942E-04	-.1429E-04	-.6973E-05	-.1432E-05

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.6707E-03	-.4343E-03	-.5069E-04	.1062E-03	.6417E-04
0.6	-.5257E-02	-.3756E-02	-.1363E-02	-.1485E-03	.7460E-04
0.4	-.1567E-01	-.1043E-01	-.3864E-02	-.8954E-03	-.6501E-04
0.2	-.1107E-01	-.4056E-02	-.2756E-02	-.1205E-02	-.2173E-03
0	.1000E+31	.2868E-01	.1140E-02	-.9942E-03	-.2629E-03
-0.2	-.1107E-01	-.4056E-02	-.2756E-02	-.1205E-02	-.2173E-03
-0.4	-.1567E-01	-.1043E-01	-.3864E-02	-.8954E-03	-.6501E-04
-0.6	-.5257E-02	-.3756E-02	-.1363E-02	-.1485E-03	.7460E-04
-0.8	-.6707E-03	-.4343E-03	-.5069E-04	.1062E-03	.6417E-04

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2082E+01	-.4238E+00	.1857E+00	-.1937E-02	.1210E-01
0.6	-.3999E+00	-.1100E+00	.9146E-01	.3717E-01	.5893E-02
0.4	-.9364E-03	.3194E-01	.4952E-01	.2233E-01	.3292E-02
0.2	.3010E-01	.2860E-01	.1956E-01	.6595E-02	.2351E-03
0	.9738E-02	.7839E-02	.3420E-02	-.2077E-03	-.8332E-03
-0.2	.1509E-03	-.2771E-03	-.1057E-02	-.1262E-02	-.6126E-03
-0.4	-.1049E-02	-.1040E-02	-.9376E-03	-.6396E-03	-.2206E-03
-0.6	-.4103E-03	-.3752E-03	-.2758E-03	-.1389E-03	-.2503E-04
-0.8	-.4878E-04	-.3969E-04	-.1739E-04	.5181E-05	.1133E-04

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1836E+00	-.6698E-01	.2080E-02	-.4676E-04	.9723E-03
0.6	-.6376E-01	-.3335E-01	-.2045E-02	.2210E-02	.8339E-03
0.4	-.9612E-02	-.4405E-02	.1744E-02	.2099E-02	.6500E-03
0.2	.2056E-02	.2322E-02	.2189E-02	.1202E-02	.2930E-03
0	.1661E-02	.1461E-02	.9352E-03	.3631E-03	.4388E-04
-0.2	.3829E-03	.3038E-03	.1291E-03	-.1077E-04	-.3650E-04
-0.4	-.4520E-04	-.5555E-04	-.7167E-04	-.6611E-04	-.3039E-04
-0.6	-.6103E-04	-.5815E-04	-.4850E-04	-.3126E-04	-.1050E-04
-0.8	-.1631E-04	-.1487E-04	-.1095E-04	-.5676E-05	-.1052E-05

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.6417E-04	.7460E-04	-.6501E-04	-.2173E-03	-.2629E-03	-.2173E-03	-.6501E-04	.7460E-04	.6417E-04
0.6	.1062E-03	-.1485E-03	-.8954E-03	-.1205E-02	-.9942E-03	-.1205E-02	-.8954E-03	-.1485E-03	.1062E-03
0.4	-.5069E-04	-.1363E-02	-.3864E-02	-.2756E-02	.1140E-02	-.2756E-02	-.3864E-02	-.1363E-02	-.5069E-04
0.2	-.4343E-03	-.3756E-02	-.1043E-01	-.4056E-02	.2868E-01	-.4056E-02	-.1043E-01	-.3756E-02	-.4343E-03
0	-.6707E-03	-.5257E-02	-.1567E-01	-.1107E-01	.1000E+31	-.1107E-01	-.1567E-01	-.5257E-02	-.6707E-03

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1052E-05	-.1050E-04	-.3039E-04	-.3650E-04	.4388E-04	.2930E-03	.6500E-03	.8339E-03	.9723E-03
0.6	-.5676E-05	-.3126E-04	-.6611E-04	-.1077E-04	.3631E-03	.1202E-02	.2099E-02	.2210E-02	-.4676E-04
0.4	-.1095E-04	-.4850E-04	-.7167E-04	.1291E-03	.9352E-03	.2189E-02	.1744E-02	-.2045E-02	.2080E-02
0.2	-.1487E-04	-.5815E-04	-.5555E-04	.3038E-03	.1461E-02	.2322E-02	-.4405E-02	-.3335E-01	-.6698E-01
0	-.1631E-04	-.6103E-04	-.4520E-04	.3829E-03	.1661E-02	.2056E-02	-.9612E-02	-.6376E-01	-.1836E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1133E-04	-.2503E-04	-.2206E-03	-.6126E-03	-.8332E-03	.2351E-03	.3292E-02	.5893E-02	.1210E-01
0.6	.5181E-05	-.1389E-03	-.6396E-03	-.1262E-02	-.2077E-03	.6595E-02	.2233E-01	.3717E-01	-.1937E-02
0.4	-.1739E-04	-.2758E-03	-.9376E-03	-.1057E-02	.3420E-02	.1956E-01	.4952E-01	.9146E-01	.1857E+00
0.2	-.3969E-04	-.3752E-03	-.1040E-02	-.2771E-03	.7839E-02	.2860E-01	.3194E-01	-.1100E+00	-.4238E+00
0	-.4878E-04	-.4103E-03	-.1049E-02	.1509E-03	.9738E-02	.3010E-01	-.9364E-03	-.3999E+00	-.2082E+01

Clamped rectangular plate — b/a = 1.2 — λ = 7

Influence coefficient $s_1 = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1138E-04	-.9798E-05	-.5943E-05	-.2034E-05	-.2769E-07
0.6	-.3600E-04	-.3476E-04	-.2742E-04	-.1433E-04	-.3473E-05
0.4	.7725E-04	.3067E-04	-.2783E-04	-.3213E-04	-.1136E-04
0.2	.9111E-03	.5322E-03	.9726E-04	-.2989E-04	-.1860E-04
0	.2551E-02	.1174E-02	.2325E-03	-.1732E-04	-.2083E-04
-0.2	.9111E-03	.5322E-03	.9726E-04	-.2989E-04	-.1860E-04
-0.4	.7725E-04	.3067E-04	-.2783E-04	-.3213E-04	-.1136E-04
-0.6	-.3600E-04	-.3476E-04	-.2742E-04	-.1433E-04	-.3473E-05
-0.8	-.1138E-04	-.9798E-05	-.5943E-05	-.2034E-05	-.2769E-07

Influence surface of M_y at x = y = 0 (ν = 0.3)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1988E-03	.2172E-03	.2189E-03	.1530E-03	.5410E-04
0.6	-.1715E-02	-.1159E-02	-.2529E-03	.1521E-03	.1117E-03
0.4	-.1111E-01	-.7744E-02	-.2910E-02	-.5767E-03	.3925E-05
0.2	-.1657E-01	-.8272E-02	-.3535E-02	-.1223E-02	-.1966E-03
0	.1000E+31	.2868E-01	.1136E-02	-.9967E-03	-.2636E-03
-0.2	-.1657E-01	-.8272E-02	-.3535E-02	-.1223E-02	-.1966E-03
-0.4	-.1111E-01	-.7744E-02	-.2910E-02	-.5767E-03	.3925E-05
-0.6	-.1715E-02	-.1159E-02	-.2529E-03	.1521E-03	.1117E-03
-0.8	.1988E-03	.2172E-03	.2189E-03	.1530E-03	.5410E-04

Influence coefficient $s_2 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1574E+01	-.3765E+00	.1618E+00	.1413E-01	.8915E-02
0.6	-.1553E+00	-.2394E-01	.7210E-01	.3332E-01	.5177E-02
0.4	.3272E-01	.3650E-01	.3019E-01	.1187E-01	.1263E-02
0.2	.1331E-01	.1105E-01	.5497E-02	.5404E-03	-.7556E-03
0	.1491E-03	-.2789E-03	-.1058E-02	-.1263E-02	-.6137E-03
-0.2	-.9482E-03	-.9191E-03	-.7884E-03	-.5091E-03	-.1645E-03
-0.4	-.2013E-03	-.1770E-03	-.1136E-03	-.3892E-04	.5235E-05
-0.6	.3405E-04	.3615E-04	.3907E-04	.3482E-04	.1714E-04
-0.8	.2068E-04	.1986E-04	.1707E-04	.1166E-04	.4149E-05

Influence surface of M_y at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1576E+00	-.6472E-01	-.2506E-03	.7586E-03	.8634E-03
0.6	-.3431E-01	-.1844E-01	-.4076E-03	.2313E-02	.8001E-03
0.4	.4375E-04	.1287E-02	.2410E-02	.1600E-02	.4341E-03
0.2	.1965E-02	.1762E-02	.1182E-02	.4963E-03	.7955E-04
0	.3829E-03	.3038E-03	.1292E-03	-.1067E-04	-.3629E-04
-0.2	-.6533E-04	-.7025E-04	-.7489E-04	-.6124E-04	-.2606E-04
-0.4	-.4291E-04	-.3990E-04	-.3116E-04	-.1823E-04	-.5252E-05
-0.6	-.3993E-05	-.3221E-05	-.1356E-05	.4823E-06	.9327E-06
-0.8	.1923E-05	.1911E-05	.1804E-05	.1423E-05	.6360E-06

Influence surface of M_x at x = y = 0 (ν = 0.3)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.4115E-04	.8640E-04	.6757E-04	.1168E-04	-.1612E-04	.1168E-04	.6757E-04	.8640E-04	.4115E-04
0.6	.1120E-03	.9502E-04	-.2152E-03	-.4946E-03	-.5461E-03	-.4946E-03	-.2152E-03	.9502E-04	.1120E-03
0.4	.4525E-04	-.7111E-03	-.2300E-02	-.2165E-02	-.7047E-03	-.2165E-02	-.2300E-02	-.7111E-03	.4525E-04
0.2	-.3535E-03	-.3244E-02	-.8874E-02	-.3586E-02	.1819E-01	-.3586E-02	-.8874E-02	-.3244E-02	-.3535E-03
0	-.6716E-03	-.5258E-02	-.1567E-01	-.1107E-01	.1000E+31	-.1107E-01	-.1567E-01	-.5258E-02	-.6716E-03

Influence surface of M_x at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.2063E-06	-.5491E-05	-.2300E-04	-.4579E-04	-.3950E-04	.4549E-04	.2002E-03	.2810E-03	.1348E-03
0.6	-.2965E-05	-.2269E-04	-.6090E-04	-.6266E-04	.1223E-03	.6400E-03	.1345E-02	.1690E-02	.1198E-02
0.4	-.8643E-05	-.4259E-04	-.7545E-04	.5782E-04	.6900E-03	.1864E-02	.2413E-02	.1141E-02	-.4817E-03
0.2	-.1398E-04	-.5625E-04	-.5894E-04	.2718E-03	.1375E-02	.2374E-02	-.2767E-02	-.2427E-01	-.3566E-01
0	-.1614E-04	-.6061E-04	-.4489E-04	.3824E-03	.1660E-02	.2056E-02	-.9614E-02	-.6411E-01	-.1906E+00

Influence coefficient $s_3 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1696E-04	.1411E-04	-.1044E-03	-.4379E-03	-.9221E-03	-.1139E-02	-.6896E-03	-.2026E-03	-.4304E-03
0.6	.2017E-04	-.6038E-04	-.4588E-03	-.1191E-02	-.1361E-02	.1345E-02	.8625E-02	.1549E-01	.1316E-01
0.4	-.3397E-05	-.2139E-03	-.8411E-03	-.1266E-02	.1657E-02	.1413E-01	.4012E-01	.7071E-01	.3766E-01
0.2	-.3462E-04	-.3548E-03	-.1022E-02	-.4229E-03	.7101E-02	.2768E-01	.4103E-01	-.2773E-01	-.4240E-02
0	-.4871E-04	-.4089E-03	-.1046E-02	.1530E-03	.9743E-02	.3010E-01	-.1483E-02	-.4103E+00	-.2232E+01

Clamped rectangular plate — b/a = 1.4 — λ = 7

Influence coefficient $s_1 = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2415E-05	-.1813E-05	-.5042E-06	.4835E-06	.4838E-06
0.6	-.2548E-04	-.2272E-04	-.1524E-04	-.6528E-05	-.1049E-05
0.4	-.5537E-06	-.1818E-04	-.3620E-04	-.2702E-04	-.8438E-05
0.2	.6861E-03	.4052E-03	.6634E-04	-.3221E-04	-.1775E-04
0	.2551E-02	.1174E-02	.2325E-03	-.1733E-04	-.2087E-04
-0.2	.6861E-03	.4052E-03	.6634E-04	-.3221E-04	-.1775E-04
-0.4	-.5537E-06	-.1818E-04	-.3620E-04	-.2702E-04	-.8438E-05
-0.6	-.2548E-04	-.2272E-04	-.1524E-04	-.6528E-05	-.1049E-05
-0.8	-.2415E-05	-.1813E-05	-.5042E-06	.4835E-06	.4838E-06

Influence surface of M_y at x = y = 0 (ν = 0.3)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2515E-03	.2296E-03	.1690E-03	.9037E-04	.2450E-04
0.6	-.1241E-03	.2224E-04	.2299E-03	.2390E-03	.1004E-03
0.4	-.6942E-02	-.4936E-02	-.1821E-02	-.2601E-03	.6101E-04
0.2	-.1875E-01	-.1062E-01	-.4030E-02	-.1200E-02	-.1698E-03
0	.1000E+31	.2868E-01	.1136E-02	-.9968E-03	-.2637E-03
-0.2	-.1875E-01	-.1062E-01	-.4030E-02	-.1200E-02	-.1698E-03
-0.4	-.6942E-02	-.4936E-02	-.1821E-02	-.2601E-03	.6101E-04
-0.6	-.1241E-03	.2224E-04	.2299E-03	.2390E-03	.1004E-03
-0.8	.2515E-03	.2296E-03	.1690E-03	.9037E-04	.2450E-04

Influence coefficient $s_2 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1164E+01	-.3098E+00	.1392E+00	.2576E-01	.8045E-02
0.6	-.3306E-01	.2113E-01	.5662E-01	.2619E-01	.3967E-02
0.4	.2621E-01	.2384E-01	.1512E-01	.4549E-02	-.1550E-03
0.2	.2488E-02	.1610E-02	-.1864E-03	-.1225E-02	-.7641E-03
0	-.1050E-02	-.1041E-02	-.9386E-03	-.6403E-03	-.2212E-03
-0.2	-.2014E-03	-.1771E-03	-.1136E-03	-.3881E-04	.5352E-05
-0.4	.3975E-04	.4044E-04	.4005E-04	.3282E-04	.1501E-04
-0.6	.1445E-04	.1360E-04	.1101E-04	.6771E-05	.1982E-05
-0.8	-.4256E-06	-.4917E-06	-.6433E-06	-.7357E-06	-.4865E-06

Influence surface of M_y at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1310E+00	-.5879E-01	-.1788E-02	.1363E-02	.8690E-03
0.6	-.1571E-01	-.7986E-02	.1174E-02	.2210E-02	.7102E-03
0.4	.2357E-02	.2385E-02	.1970E-02	.1006E-02	.2282E-03
0.2	.7923E-03	.6625E-03	.3608E-03	.8580E-04	-.2008E-04
0	-.4537E-04	-.5570E-04	-.7176E-04	-.6614E-04	-.3040E-04
-0.2	-.4295E-04	-.3993E-04	-.3119E-04	-.1824E-04	-.5258E-05
-0.4	-.1247E-05	-.7318E-06	.4472E-06	.1398E-05	.1116E-05
-0.6	.2219E-05	.2133E-05	.1845E-05	.1290E-05	.4991E-06
-0.8	.3062E-06	.2810E-06	.2097E-06	.1059E-06	.1244E-07

Influence surface of M_x at x = y = 0 (ν = 0.3)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1673E-04	.4687E-04	.5909E-04	.5213E-04	.4590E-04	.5213E-04	.5909E-04	.4687E-04	.1673E-04
0.6	.8324E-04	.1405E-03	.4358E-04	-.1050E-03	-.1640E-03	-.1050E-03	.4358E-04	.1405E-03	.8324E-04
0.4	.9968E-04	-.2789E-03	-.1254E-02	-.1512E-02	-.1067E-02	-.1512E-02	-.1254E-02	-.2789E-03	.9968E-04
0.2	-.2696E-03	-.2728E-02	-.7399E-02	-.3343E-02	.1106E-01	-.3343E-02	-.7399E-02	-.2728E-02	-.2696E-03
0	-.6712E-03	-.5258E-02	-.1567E-01	-.1107E-01	.1000E+31	-.1107E-01	-.1567E-01	-.5258E-02	-.6712E-03

Influence surface of M_x at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1048E-05	-.6447E-06	-.1089E-04	-.3137E-04	-.5152E-04	-.4944E-04	-.1879E-04	.2314E-05	-.4263E-05
0.6	-.6404E-06	-.1315E-04	-.4546E-04	-.7441E-04	-.1754E-04	.2244E-03	.6038E-03	.8177E-03	.6534E-03
0.4	-.6455E-05	-.3577E-04	-.7382E-04	.1322E-05	.4601E-03	.1441E-02	.2364E-02	.2303E-02	-.6943E-04
0.2	-.1323E-04	-.5444E-04	-.6224E-04	.2381E-03	.1277E-02	.2388E-02	-.1298E-02	-.1678E-01	-.1892E-01
0	-.1618E-04	-.6058E-04	-.4460E-04	.3831E-03	.1660E-02	.2053E-02	-.9619E-02	-.6386E-01	-.1851E+00

Influence coefficient $s_3 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1630E-04	.3593E-04	.8053E-06	-.1735E-03	-.5342E-03	-.9742E-03	-.1267E-02	-.1293E-02	-.6783E-03
0.6	.2726E-04	.2708E-05	-.2549E-03	-.8778E-03	-.1559E-02	-.1212E-02	.1042E-02	.3526E-02	.5553E-02
0.4	.7426E-05	-.1548E-03	-.7189E-03	-.1352E-02	.1892E-03	.8791E-02	.2833E-01	.4903E-01	.5971E-02
0.2	-.3013E-04	-.3353E-03	-.1003E-02	-.5801E-03	.6245E-02	.2616E-01	.4667E-01	.2312E-01	.1243E+00
0	-.4901E-04	-.4091E-03	-.1044E-02	.1596E-03	.9750E-02	.3010E-01	-.1095E-02	-.4023E+00	-.2110E+01

Clamped rectangular plate — $b/a = 1.6$ — $\lambda = 7$

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.8125E-06	.8984E-06	.1011E-05	.8757E-06	.3944E-06
0.6	-.1221E-04	-.1037E-04	-.5980E-05	-.1719E-05	.1905E-06
0.4	-.3113E-04	-.3521E-04	-.3442E-04	-.2068E-04	-.5720E-05
0.2	.4998E-03	.2950E-03	.3855E-04	-.3388E-04	-.1673E-04
0	.2551E-02	.1174E-02	.2325E-03	-.1734E-04	-.2093E-04
-0.2	.4998E-03	.2950E-03	.3855E-04	-.3388E-04	-.1673E-04
-0.4	-.3113E-04	-.3521E-04	-.3442E-04	-.2068E-04	-.5720E-05
-0.6	-.1221E-04	-.1037E-04	-.5980E-05	-.1719E-05	.1905E-06
-0.8	.8125E-06	.8984E-06	.1011E-05	.8757E-06	.3944E-06

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1099E-03	.9585E-04	.6169E-04	.2534E-04	.3037E-05
0.6	.3361E-03	.3411E-03	.3080E-03	.1996E-03	.6755E-04
0.4	-.3785E-02	-.2680E-02	-.8916E-03	-.5425E-05	.9863E-04
0.2	-.1882E-01	-.1149E-01	-.4227E-02	-.1136E-02	-.1381E-03
0	.1000E+31	.2868E-01	.1136E-02	-.9969E-03	-.2641E-03
-0.2	-.1882E-01	-.1149E-01	-.4227E-02	-.1136E-02	-.1381E-03
-0.4	-.3785E-02	-.2680E-02	-.8916E-03	-.5425E-05	.9863E-04
-0.6	.3361E-03	.3411E-03	.3080E-03	.1996E-03	.6755E-04
-0.8	.1099E-03	.9585E-04	.6169E-04	.2534E-04	.3037E-05

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.8406E+00	-.2378E+00	.1199E+00	.3305E-01	.7603E-02
0.6	.1842E-01	.3713E-01	.4271E-01	.1855E-01	.2539E-02
0.4	.1331E-01	.1105E-01	.5495E-02	.5345E-03	-.7792E-03
0.2	-.8631E-03	-.1024E-02	-.1241E-02	-.1063E-02	-.4381E-03
0	-.4063E-03	-.3707E-03	-.2705E-03	-.1344E-03	-.2369E-04
-0.2	.3464E-04	.3670E-04	.3951E-04	.3514E-04	.1725E-04
-0.4	.1445E-04	.1359E-04	.1100E-04	.6767E-05	.1983E-05
-0.6	-.1330E-05	-.1335E-05	-.1306E-05	-.1116E-05	-.5801E-06
-0.8	-.5476E-06	-.5194E-06	-.4326E-06	-.2823E-06	-.9402E-07

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1058E+00	-.5080E-01	-.2501E-02	.1784E-02	.8854E-03
0.6	-.5164E-02	-.1765E-02	.2133E-02	.1954E-02	.5800E-03
0.4	.1965E-02	.1762E-02	.1182E-02	.4958E-03	.7782E-04
0.2	.1262E-03	.8456E-04	-.1641E-05	-.5611E-04	-.3920E-04
0	-.6192E-04	-.5889E-04	-.4888E-04	-.3132E-04	-.1034E-04
-0.2	-.3892E-05	-.3124E-05	-.1272E-05	.5433E-06	.9526E-06
-0.4	.2221E-05	.2134E-05	.1846E-05	.1291E-05	.4989E-06
-0.6	.1406E-06	.1236E-06	.7786E-07	.1860E-07	-.1787E-07
-0.8	-.7750E-07	-.7438E-07	-.6452E-07	-.4620E-07	-.1894E-07

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.3119E-05	.1430E-04	.2414E-04	.2797E-04	.2843E-04	.2797E-04	.2414E-04	.1430E-04	.3119E-05
0.6	.4944E-04	.1087E-03	.9805E-04	.4206E-04	.1272E-04	.4206E-04	.9805E-04	.1087E-03	.4944E-04
0.4	.1188E-03	-.2585E-04	-.5959E-03	-.9315E-03	-.8683E-03	-.9315E-03	-.5959E-03	-.2585E-04	.1188E-03
0.2	-.1884E-03	-.2231E-02	-.6059E-02	-.3173E-02	.6269E-02	-.3173E-02	-.6059E-02	-.2231E-02	-.1884E-03
0	-.6707E-03	-.5258E-02	-.1567E-01	-.1107E-01	.1000E+31	-.1107E-01	-.1567E-01	-.5258E-02	-.6707E-03

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1145E-05	.1804E-05	-.1806E-05	-.1270E-04	-.2953E-04	-.4423E-04	-.4892E-04	-.4572E-04	-.1504E-04
0.6	.7830E-06	-.5650E-05	-.2848E-04	-.6204E-04	-.6786E-04	.1040E-04	.1666E-03	.2900E-03	.3503E-03
0.4	-.4486E-05	-.2886E-04	-.6870E-04	-.3974E-04	.2642E-03	.1004E-02	.1889E-02	.2060E-02	-.6962E-03
0.2	-.1240E-04	-.5241E-04	-.6552E-04	.2020E-03	.1168E-02	.2357E-02	-.4327E-04	-.1054E-01	-.6780E-02
0	-.1620E-04	-.4050E-04	-.4422E-04	.3838E-03	.1660E-02	.2051E-02	-.9589E-02	-.6328E-01	-.1780E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1071E-04	.3390E-04	.4547E-04	-.1843E-06	-.1511E-03	-.3961E-03	-.6317E-03	-.7137E-03	-.1893E-03
0.6	.2688E-04	.3764E-04	-.9611E-04	-.5213E-03	-.1210E-02	-.1726E-02	-.1530E-02	-.3936E-03	.3072E-02
0.4	.1606E-04	-.1003E-03	-.5847E-03	-.1323E-02	-.8548E-03	.4317E-02	.1695E-01	.2716E-01	-.1574E-01
0.2	-.2498E-04	-.3130E-03	-.9793E-03	-.7414E-03	.5310E-02	.2417E-01	.4954E-01	.5760E-01	.1943E+00
0	-.4894E-04	-.4081E-03	-.1040E-02	.1634E-03	.9748E-02	.3013E-01	-.7972E-04	-.3896E+00	-.1982E+01

Clamped rectangular plate | b/a = 1.8 | λ = 7

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.9391E-06	.8979E-06	.7579E-06	.4993E-06	.1743E-06
0.6	-.3667E-05	-.2825E-05	-.9720E-06	.4765E-06	.5933E-06
0.4	-.3625E-04	-.3495E-04	-.2749E-04	-.1431E-04	-.3414E-05
0.2	.3498E-03	.2031E-03	.1495E-04	-.3473E-04	-.1557E-04
0	.2551E-02	.1174E-02	.2325E-03	-.1736E-04	-.2101E-04
-0.2	.3498E-03	.2031E-03	.1495E-04	-.3473E-04	-.1557E-04
-0.4	-.3625E-04	-.3495E-04	-.2749E-04	-.1431E-04	-.3414E-05
-0.6	-.3667E-05	-.2825E-05	-.9720E-06	.4765E-06	.5933E-06
-0.8	.9391E-06	.8979E-06	.7579E-06	.4993E-06	.1743E-06

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1746E-04	.1312E-04	.3510E-05	-.4112E-05	-.4280E-05
0.6	.3227E-03	.2969E-03	.2223E-03	.1212E-03	.3432E-04
0.4	-.1697E-02	-.1143E-02	-.2420E-03	.1571E-03	.1136E-03
0.2	-.1761E-01	-.1131E-01	-.4156E-02	-.1034E-02	-.1029E-03
0	.1000E+31	.2868E-01	.1136E-02	-.9971E-03	-.2645E-03
-0.2	-.1761E-01	-.1131E-01	-.4156E-02	-.1034E-02	-.1029E-03
-0.4	-.1697E-02	-.1143E-02	-.2420E-03	.1571E-03	.1136E-03
-0.6	.3227E-03	.2969E-03	.2223E-03	.1212E-03	.3432E-04
-0.8	.1746E-04	.1312E-04	.3510E-05	-.4112E-05	-.4280E-05

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.5902E+00	-.1695E+00	.1042E+00	.3667E-01	.7346E-02
0.6	.3271E-01	.3650E-01	.3016E-01	.1181E-01	.1219E-02
0.4	.4346E-02	.3165E-02	.6454E-03	-.1050E-02	-.8366E-03
0.2	-.9479E-03	-.9186E-03	-.7875E-03	-.5073E-03	-.1584E-03
0	-.2597E-04	-.1608E-04	.6755E-05	.2518E-04	.2008E-04
-0.2	.2408E-04	.2298E-04	.1941E-04	.1292E-04	.4480E-05
-0.4	-.9329E-06	-.9798E-06	-.1066E-05	-.1028E-05	-.5857E-06
-0.6	-.5477E-06	-.5187E-06	-.4299E-06	-.2785E-06	-.9227E-07
-0.8	.5077E-07	.4929E-07	.4463E-07	.3541E-07	.1744E-07

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.8319E-01	-.4202E-01	-.2520E-02	.2060E-02	.5009E-03
0.6	.4353E-04	.1287E-02	.2409E-02	.1597E-02	.4336E-03
0.4	.1056E-02	.8982E-03	.5215E-03	.1584E-03	-.6409E-05
0.2	-.6531E-04	-.7022E-04	-.7486E-04	-.6123E-04	-.2587E-04
0	-.1819E-04	-.1634E-04	-.1145E-04	-.5311E-05	-.6925E-06
-0.2	.2678E-05	.2625E-05	.2392E-05	.1803E-05	.7654E-06
-0.4	-.2718E-06	.2467E-06	.1772E-06	.8038E-07	.1703E-08
-0.6	-.8295E-07	-.7948E-07	-.6854E-07	-.4861E-07	-.1983E-07
-0.8	-.2691E-08	-.2385E-08	-.1493E-08	-.1868E-09	.6045E-09

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1279E-05	.1771E-06	.3549E-05	.6157E-05	.7042E-05	.6157E-05	.3549E-05	.1771E-06	-.1279E-05
0.6	.2366E-04	.6332E-04	.7710E-04	.6505E-04	.5563E-04	.6505E-04	.7710E-04	.6332E-04	.2366E-04
0.4	.1132E-03	.9792E-04	-.2115E-03	-.4913E-03	-.5433E-03	-.4913E-03	-.2115E-03	.9792E-04	.1132E-03
0.2	-.1133E-03	-.1772E-02	-.4879E-02	-.2990E-02	.3126E-02	-.2990E-02	-.4879E-02	-.1772E-02	-.1133E-03
0	-.6702E-03	-.5258E-02	-.1567E-01	-.1107E-01	.1000E+31	-.1107E-01	-.1567E-01	-.5258E-02	-.6702E-03

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.7841E-06	.2025E-05	.1943E-05	-.1434E-05	-.8924E-05	-.1832E-04	-.2558E-04	-.2755E-04	.2411E-05
0.6	.1405E-05	-.8168E-06	-.1441E-04	-.4109E-04	-.6604E-04	-.6021E-04	-.1097E-04	.7701E-04	.2227E-03
0.4	-.2662E-05	-.2202E-04	-.6041E-04	-.6419E-04	.1138E-03	.6150E-03	.1280E-02	.1306E-02	-.1456E-02
0.2	-.1150E-04	-.5007E-04	-.6832E-04	.1650E-03	.1049E-02	.2279E-02	.9494E-03	-.5609E-02	.1751E-02
0	-.1622E-04	-.6037E-04	-.4371E-04	.3845E-03	.1659E-02	.2051E-02	-.9510E-02	-.6233E-01	-.1699E+00

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.4517E-05	.2004E-04	.4175E-04	.5264E-04	.2667E-04	-.4789E-04	-.1490E-03	-.2232E-03	.1333E-03
0.6	.2214E-04	.4009E-04	.2626E-05	-.2315E-03	-.7144E-03	-.1297E-02	-.1581E-02	-.7050E-03	.2414E-02
0.4	.2256E-04	-.5151E-04	-.4439E-03	-.1190E-02	-.1439E-02	.1088E-02	.7951E-02	.1032E-01	-.2730E-01
0.2	-.1937E-04	-.2891E-03	-.9474E-03	-.8930E-03	.4337E-02	.2179E-01	.4976E-01	.7811E-01	.2258E+00
0	-.4877E-04	-.4066E-03	-.1035E-02	.1689E-03	.9730E-02	.3018E-01	.1549E-02	-.3730E+00	-.1855E+01

Clamped rectangular plate — b/a = 1.0 — λ = 11

Influence coefficient $s_1 = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2480E-06	.3550E-06	.3890E-06	.1884E-06	.2245E-07
0.6	-.5618E-05	-.3747E-05	-.6067E-06	.4724E-06	.1884E-06
0.4	-.1160E-04	-.1475E-04	-.7978E-05	-.6067E-06	.3890E-06
0.2	.2123E-03	.5455E-04	-.1475E-04	-.3747E-05	.3550E-06
0	.1033E-02	.2123E-03	-.1160E-04	-.5618E-05	.2480E-06
-0.2	.2123E-03	.5455E-04	-.1475E-04	-.3747E-05	.3550E-06
-0.4	-.1160E-04	-.1475E-04	-.7978E-05	-.6067E-06	.3890E-06
-0.6	-.5618E-05	-.3747E-05	-.6067E-06	.4724E-06	.1884E-06
-0.8	.2480E-06	.3550E-06	.3890E-06	.1884E-06	.2245E-07

Influence surface of M_y at x = y = 0 (ν = 0.3)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1291E-03	.9226E-04	.2699E-04	-.4164E-05	-.4240E-05
0.6	.3097E-03	.3263E-03	.2004E-03	.4351E-04	-.2738E-05
0.4	-.4172E-02	-.1775E-02	.4214E-04	.1187E-03	.1458E-04
0.2	-.1891E-01	-.6650E-02	-.9043E-03	.6610E-04	.2983E-04
0	.1000E+31	.6823E-02	-.9151E-03	-.3409E-05	.3211E-04
-0.2	-.1891E-01	-.6650E-02	-.9043E-03	.6610E-04	.2983E-04
-0.4	-.4172E-02	-.1775E-02	.4214E-04	.1187E-03	.1458E-04
-0.6	.3097E-03	.3263E-03	.2004E-03	.4351E-04	-.2738E-05
-0.8	.1291E-03	.9226E-04	.2699E-04	-.4164E-05	-.4240E-05

Influence coefficient $s_2 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1118E+01	-.4901E-01	.1086E+00	-.1988E-01	.4039E-02
0.6	.3029E-01	.6470E-01	.2864E-01	-.4087E-02	-.1020E-02
0.4	.2324E-01	.1433E-01	.2930E-03	-.2423E-02	-.3999E-03
0.2	-.1026E-02	-.1666E-02	-.1771E-02	-.6255E-03	.2951E-04
0	-.7611E-03	-.6000E-03	-.2250E-03	.4703E-04	.6093E-04
-0.2	.3677E-04	.5164E-04	.6655E-04	.4737E-04	.1370E-04
-0.4	.2829E-04	.2417E-04	.1350E-04	.2489E-05	-.1969E-05
-0.6	-.1316E-05	-.1749E-05	-.2415E-05	-.2243E-05	-.1000E-05
-0.8	-.1082E-05	-.9715E-06	-.6681E-06	-.2941E-06	-.3289E-07

Influence surface of M_y at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.9764E-01	-.2212E-01	.5137E-02	-.7835E-03	.2402E-03
0.6	-.5975E-02	.2640E-03	.2004E-02	.7279E-04	-.4494E-04
0.4	.2066E-02	.1561E-02	.4596E-03	-.5718E-04	-.3595E-04
0.2	.1887E-03	.7745E-04	-.6110E-04	-.5451E-04	-.9626E-05
0	-.6631E-04	-.5889E-04	-.3487E-04	-.8390E-05	.1521E-05
-0.2	-.6989E-05	-.4336E-05	.5226E-06	.2606E-05	.1403E-05
-0.4	.2400E-05	.2240E-05	.1662E-05	.7902E-06	.1251E-06
-0.6	.2784E-06	.2024E-06	.3911E-07	-.7751E-07	-.6834E-07
-0.8	-.8293E-07	-.8015E-07	-.6801E-07	-.4400E-07	-.1532E-07

Influence surface of M_x at x = y = 0 (ν = 0.3)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.4240E-05	-.2738E-05	.1458E-04	.2983E-04	.3211E-04	.2983E-04	.1458E-04	-.2738E-05	-.4240E-05
0.6	-.4164E-05	.4351E-04	.1187E-03	.6610E-04	-.3409E-05	.6610E-04	.1187E-03	.4351E-04	-.4164E-05
0.4	.2699E-04	.2004E-03	.4214E-04	-.9043E-03	-.9151E-03	-.9043E-03	.4214E-04	.2004E-03	.2699E-04
0.2	.9226E-04	.3263E-03	-.1775E-02	-.6650E-02	.6823E-02	-.6650E-02	-.1775E-02	.3263E-03	.9226E-04
0	.1291E-03	.3097E-03	-.4172E-02	-.1891E-01	.1000E+31	-.1891E-01	-.4172E-02	.3097E-03	.1291E-03

Influence surface of M_x at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1532E-07	-.6834E-07	.1251E-06	.1403E-05	.1521E-05	-.9626E-05	-.3595E-04	-.4494E-04	.2402E-03
0.6	-.4400E-07	-.7751E-07	.7902E-06	.2606E-05	-.8390E-05	-.5451E-04	-.5718E-04	.7279E-04	-.7835E-03
0.4	-.6801E-07	.3911E-07	.1662E-05	.5226E-06	-.3487E-04	-.6110E-04	.4596E-03	.2004E-02	.5137E-02
0.2	-.8015E-07	.2024E-06	.2240E-05	-.4336E-05	-.5889E-04	.7745E-04	.1561E-02	.2640E-03	-.2212E-01
0	-.8293E-07	.2784E-06	.2400E-05	-.6989E-05	-.6631E-04	.1887E-03	.2066E-02	-.5975E-02	-.9764E-01

Influence coefficient $s_3 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.3289E-07	-.1000E-05	-.1969E-05	.1370E-04	.6093E-04	.2951E-04	-.3999E-03	-.1020E-02	.4039E-02
0.6	-.2941E-06	-.2243E-05	.2489E-05	.4737E-04	.4703E-04	-.6255E-03	-.2423E-02	-.4087E-02	-.1988E-01
0.4	-.6681E-06	-.2415E-05	.1350E-04	.6655E-04	-.2250E-03	-.1771E-02	.2930E-03	.2864E-01	.1086E+00
0.2	-.9715E-06	-.1749E-05	.2417E-04	.5164E-04	-.6000E-03	-.1666E-02	.1433E-01	.6470E-01	-.4901E-01
0	-.1082E-05	-.1316E-05	.2829E-04	.3677E-04	-.7611E-03	-.1026E-02	.2324E-01	.3029E-01	-.1118E+01

Clamped rectangular plate — b/a = 1.2 — λ = 11

Influence coefficient $s_1 = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2851E-06	.2385E-06	.1225E-06	.1861E-07	-.1212E-07
0.6	-.6342E-06	-.1500E-06	.4345E-06	.3425E-06	.6974E-07
0.4	-.1392E-04	-.1140E-04	-.4154E-05	.1112E-06	.3241E-06
0.2	.1198E-03	.2549E-04	-.1445E-04	-.3073E-05	.3804E-06
0	.1033E-02	.2123E-03	-.1160E-04	-.5617E-05	.2567E-06
-0.2	.1198E-03	.2549E-04	-.1445E-04	-.3073E-05	.3804E-06
-0.4	-.1392E-04	-.1140E-04	-.4154E-05	.1112E-06	.3241E-06
-0.6	-.6342E-06	-.1500E-06	.4345E-06	.3425E-06	.6974E-07
-0.8	.2851E-06	.2385E-06	.1225E-06	.1861E-07	-.1212E-07

Influence surface of M_y at x = y = 0 (ν = 0.3)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1355E-06	-.4698E-05	-.9445E-05	-.6590E-05	-.1731E-05
0.6	.2680E-03	.2062E-03	.8007E-04	.4944E-05	-.5511E-05
0.4	-.1087E-02	-.2994E-03	.2201E-03	.9635E-04	.6446E-05
0.2	-.1684E-01	-.6527E-02	-.7462E-03	.8671E-04	.2802E-04
0	.1000E+31	.6823E-02	-.9151E-03	-.3416E-05	.3208E-04
-0.2	-.1684E-01	-.6527E-02	-.7462E-03	.8671E-04	.2802E-04
-0.4	-.1087E-02	-.2994E-03	.2201E-03	.9635E-04	.6446E-05
-0.6	.2680E-03	.2062E-03	.8007E-04	.4944E-05	-.5511E-05
-0.8	-.1355E-06	-.4698E-05	-.9445E-05	-.6590E-05	-.1731E-05

Influence coefficient $s_2 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.6444E+00	-.2201E-02	.8923E-01	-.1351E-01	.1717E-02
0.6	.5254E-01	.4530E-01	.1209E-01	-.3350E-02	-.8413E-03
0.4	.3170E-02	.5497E-03	-.2192E-02	-.1258E-02	-.8370E-04
0.2	-.1095E-02	-.9130E-03	-.4243E-03	-.2187E-05	.7052E-04
0	.3678E-04	.5164E-04	.6655E-04	.4734E-04	.1332E-04
-0.2	.1788E-04	.1460E-04	.6726E-05	-.3797E-06	-.2111E-05
-0.4	-.2264E-05	-.2236E-05	-.1977E-05	-.1309E-05	-.4427E-06
-0.6	-.1727E-06	-.1246E-06	-.1595E-07	.7102E-07	.6838E-07
-0.8	.6144E-07	.5778E-07	.4665E-07	.2937E-07	.9741E-08

Influence surface of M_y at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.6835E-01	-.1660E-01	.4195E-02	-.4002E-03	.1166E-03
0.6	.1215E-02	.2212E-02	.1795E-02	.2540E-04	-.4761E-04
0.4	.7233E-03	.4484E-03	.3127E-04	-.7136E-04	-.1942E-04
0.2	-.7289E-04	-.6987E-04	-.4789E-04	-.1481E-04	.6859E-06
0	-.6988E-05	-.4336E-05	.5232E-06	.2605E-05	.1378E-05
-0.2	.1954E-05	.1754E-05	.1177E-05	.4704E-06	.3357E-07
-0.4	-.2391E-07	-.4750E-07	-.8835E-07	-.9405E-07	-.4833E-07
-0.6	-.3785E-07	-.3321E-07	-.2138E-07	-.8216E-08	-.9678E-11
-0.8	.3493E-08	.3569E-08	.3511E-08	.2827E-08	.1320E-08

Influence surface of M_x at x = y = 0 (ν = 0.3)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1421E-05	-.3839E-05	-.3166E-05	.3307E-06	.2132E-03	.3307E-06	-.3166E-05	-.3839E-05	-.1421E-05
0.6	-.6140E-05	.4571E-05	.4286E-04	.6015E-04	.5351E-04	.6015E-04	.4286E-04	.4571E-05	-.6140E-05
0.4	.9294E-05	.1267E-03	.1611E-03	-.2346E-03	-.4169E-03	-.2346E-03	.1611E-03	.1267E-03	.9294E-05
0.2	.7886E-04	.3163E-03	-.1195E-02	-.4780E-02	.2144E-02	-.4780E-02	-.1195E-02	.3163E-03	.7886E-04
0	.1291E-03	.3097E-03	-.4172E-02	-.1891E-01	.1000E+31	-.1891E-01	-.4172E-02	.3097E-03	.1291E-03

Influence surface of M_x at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.3282E-08	-.4999E-07	-.7010E-07	.4743E-06	.1921E-05	.1518E-05	-.5424E-05	-.1553E-04	-.1302E-04
0.6	-.2667E-07	-.9943E-07	.3232E-06	.2181E-05	-.5814E-07	-.2476E-04	-.6256E-04	-.4321E-04	.1089E-03
0.4	-.5933E-07	-.2146E-07	.1329E-05	.1879E-05	-.2336E-04	-.7413E-04	.1511E-03	.9578E-03	.2771E-03
0.2	-.7939E-07	.1710E-06	.2185E-05	-.3296E-05	-.5589E-04	.3961E-04	.1366E-02	.1715E-02	-.4298E-02
0	-.8394E-07	.2762E-06	.2426E-05	-.6928E-05	-.6688E-04	.1876E-03	.2079E-02	-.6188E-02	-.1041E+00

Influence coefficient $s_3 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.7266E-07	-.4048E-06	-.2293E-05	-.1272E-06	.2528E-04	.7497E-04	.6827E-04	-.6500E-04	-.9461E-04
0.6	-.8590E-07	-.1674E-05	-.1542E-05	.2602E-04	.8197E-04	-.1118E-03	-.1042E-02	-.2047E-02	.9573E-03
0.4	-.5163E-06	-.2497E-05	.8836E-05	.6374E-04	-.8878E-04	-.1384E-02	-.2205E-02	.6628E-02	-.5482E-02
0.2	-.9391E-06	-.1929E-05	.2281E-04	.5727E-04	-.5432E-03	-.1855E-02	.1137E-01	.7042E-01	.1833E+00
0	-.1100E-05	-.1346E-05	.2870E-04	.3745E-04	-.7709E-03	-.1029E-02	.2353E-01	.2555E-01	-.1249E+01

Clamped rectangular plate

$b/a = 1.4$ $\lambda = 11$

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2592E-07	.1359E-07	-.8513E-08	-.1663E-07	-.8362E-08
0.6	.4625E-06	.4677E-06	.3576E-06	.1342E-06	.5373E-08
0.4	-.8401E-05	-.5967E-05	-.1461E-05	.4291E-06	.2344E-06
0.2	.5796E-04	.5249E-05	-.1348E-04	-.2386E-05	.4000E-06
0	.1033E-02	.2123E-03	-.1160E-04	-.5617E-05	.2478E-06
-0.2	.5796E-04	.5249E-05	-.1348E-04	-.2386E-05	.4000E-06
-0.4	-.8401E-05	-.5967E-05	-.1461E-05	.4291E-06	.2344E-06
-0.6	.4625E-06	.4677E-06	.3576E-06	.1342E-06	.5373E-08
-0.8	.2592E-07	.1359E-07	-.8513E-08	-.1663E-07	-.8362E-08

Influence surface of M_y at $x = y = 0$ $(\nu = 0.3)$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.9228E-05	-.8043E-05	-.4868E-05	-.1553E-05	.3484E-08
0.6	.7705E-04	.5086E-04	.7851E-05	-.8187E-05	-.4103E-05
0.4	.1151E-03	.2597E-03	.2312E-03	.6128E-04	-.2719E-06
0.2	-.1343E-01	-.5564E-02	-.5417E-03	.1040E-03	.2552E-04
0	.1000E+31	.6823E-02	-.9151E-03	-.3415E-05	.3210E-04
-0.2	-.1343E-01	-.5564E-02	-.5417E-03	.1040E-03	.2552E-04
-0.4	.1151E-03	.2597E-03	.2312E-03	.6128E-04	-.2719E-06
-0.6	.7705E-04	.5086E-04	.7851E-05	-.8187E-05	-.4103E-05
-0.8	-.9228E-05	-.8043E-05	-.4868E-05	-.1553E-05	.3484E-08

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.3301E+00	.3486E-01	.7070E-01	-.9139E-02	.5423E-03
0.6	.3393E-01	.2298E-01	.2833E-02	-.2779E-02	-.5564E-03
0.4	-.1601E-02	-.1781E-02	-.1396E-02	-.3916E-03	.5223E-04
0.2	-.1074E-03	-.5162E-04	.4500E-04	.7160E-04	.3059E-04
0	.2829E-04	.2417E-04	.1350E-04	.2491E-05	-.1896E-05
-0.2	-.2264E-05	-.2236E-05	-.1977E-05	-.1308E-05	-.4500E-06
-0.4	-.3113E-07	-.2577E-08	.5736E-07	.9267E-07	.6423E-07
-0.6	.2876E-07	.2563E-07	.1752E-07	.7892E-08	.4732E-09
-0.8	-.3158E-08	-.3079E-08	-.2756E-08	-.2045E-08	-.9056E-09

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.4437E-01	-.1086E-01	.3444E-02	-.1471E-03	.5278E-04
0.6	.2358E-02	.1969E-02	.6973E-03	-.3629E-04	-.4167E-04
0.4	.4905E-04	-.1132E-04	-.7235E-04	-.4357E-04	-.6143E-05
0.2	-.2513E-04	-.1934E-04	-.7033E-05	.1370E-05	.2014E-05
0	.2401E-05	.2241E-05	.1662E-05	.7904E-06	.1302E-06
-0.2	-.2391E-07	-.4750E-07	-.8833E-07	-.9399E-07	-.4872E-07
-0.4	-.2275E-07	-.1924E-07	-.1073E-07	-.2208E-08	.1793E-08
-0.6	.3006E-08	.2844E-08	.2340E-08	.1530E-08	.5504E-09
-0.8	-.1061E-09	-.1189E-09	-.1403E-09	-.1353E-09	-.7831E-10

Influence surface of M_x at $x = y = 0$ $(\nu = 0.3)$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.3693E-07	-.8567E-06	-.1835E-05	-.2169E-05	-.2138E-05	-.2169E-05	-.1835E-05	-.8567E-06	-.3693E-07
0.6	-.3811E-05	-.5164E-05	.5481E-05	.1831E-04	.2200E-04	.1831E-04	.5481E-05	-.5164E-05	-.3811E-05
0.4	-.1265E-05	.6622E-04	.1449E-03	.2302E-04	-.8596E-04	.2302E-04	.1449E-03	.6622E-04	-.1265E-05
0.2	.6494E-04	.2975E-03	-.7202E-03	-.3355E-02	-.6461E-04	-.3355E-02	-.7202E-03	.2975E-03	.6494E-04
0	.1291E-03	.3097E-03	-.4172E-02	-.1891E-01	.1000E+31	-.1891E-01	-.4172E-02	.3097E-03	.1291E-03

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.2543E-08	-.1633E-07	-.7819E-07	-.2707E-07	.5590E-06	.1637E-05	.1778E-05	-.6699E-06	.1988E-05
0.6	-.1193E-07	-.8183E-07	.3325E-07	.1234E-05	.2492E-05	-.4705E-05	-.2555E-04	-.2898E-04	.9053E-04
0.4	-.4850E-07	-.4393E-07	.9557E-06	.2497E-05	-.1272E-04	-.6398E-04	-.1853E-04	.2762E-03	-.7132E-03
0.2	-.7675E-07	.1405E-06	.2070E-05	-.2318E-05	-.5118E-04	.7517E-05	.1126E-02	.2270E-02	.2277E-02
0	-.8319E-07	.2779E-06	.2406E-05	-.6976E-05	-.6643E-04	.1886E-03	.2070E-02	-.6020E-02	-.5890E-01

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.7483E-07	.2791E-07	-.1020E-05	-.3036E-05	.4146E-06	.2117E-04	.5054E-04	.3761E-04	.7530E-04
0.6	.4095E-07	-.9526E-06	-.2858E-05	.8428E-05	.5919E-04	.8218E-04	-.1523E-03	-.4015E-03	.1702E-02
0.4	-.3516E-06	-.2337E-05	.4304E-05	.5333E-04	.1295E-04	-.8689E-03	-.2660E-02	-.2374E-02	-.2087E-01
0.2	-.8721E-06	-.2055E-05	.2055E-04	.6074E-04	-.4625E-03	-.1920E-02	.7998E-02	.6198E-01	.1999E+00
0	-.1087E-05	-.1325E-05	.7837E-04	.3691E-04	-.7631E-03	-.1025E-02	.2332E-01	.2941E-01	-.1141E+01

Clamped rectangular plate — $b/a = 1.6$ — $\lambda = 11$

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1638E-07	-.1567E-07	-.1241E-07	-.6598E-08	-.1494E-08
0.6	.2876E-06	.2394E-06	.1206E-06	.1605E-07	-.1340E-07
0.4	-.3372E-05	-.2064E-05	-.4737E-07	.4581E-06	.1429E-06
0.2	.1972E-04	-.7040E-05	-.1195E-04	-.1724E-05	.4088E-06
0	.1033E-02	.2123E-03	-.1160E-04	-.5618E-05	.2489E-06
-0.2	.1972E-04	-.7040E-05	-.1195E-04	-.1724E-05	.4088E-06
-0.4	-.3372E-05	-.2064E-05	-.4737E-07	.4581E-06	.1429E-06
-0.6	.2876E-06	.2394E-06	.1206E-06	.1605E-07	-.1340E-07
-0.8	-.1638E-07	-.1567E-07	-.1241E-07	-.6598E-08	-.1494E-08

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1517E-05	-.1084E-05	-.2230E-06	.2979E-06	.2398E-06
0.6	-.1619E-05	-.5951E-05	-.1015E-04	-.6786E-05	-.1723E-05
0.4	.3611E-03	.3178E-03	.1604E-03	.2794E-04	-.4305E-05
0.2	-.9849E-02	-.4245E-02	-.3231E-03	.1159E-03	.2233E-04
0	.1000E+31	.6823E-02	-.9151E-03	-.3414E-05	.3212E-04
-0.2	-.9849E-02	-.4245E-02	-.3231E-03	.1159E-03	.2233E-04
-0.4	.3611E-03	.3178E-03	.1604E-03	.2794E-04	-.4305E-05
-0.6	-.1619E-05	-.5951E-05	-.1015E-04	-.6786E-05	-.1723E-05
-0.8	-.1517E-05	-.1084E-05	-.2230E-06	.2979E-06	.2398E-06

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1353E+00	.5747E-01	.5423E-01	-.6428E-02	-.1810E-03
0.6	.1433E+01	.7840E-02	-.1222E-02	-.2037E-02	-.2787E-03
0.4	-.1095E-02	-.9130E-03	-.4242E-03	-.2181E-05	.6535E-04
0.2	.5961E-04	.5834E-04	.4686E-04	.2306E-04	.3985E-05
0	-.1320E-05	-.1753E-05	-.2418E-05	-.2254E-05	-.1071E-05
-0.2	-.1727E-06	-.1247E-06	-.1589E-07	.7129E-07	.6268E-07
-0.4	.2876E-07	.2564E-07	.1753E-07	.8013E-08	.1887E-08
-0.6	-.2542E-08	-.2422E-08	-.2049E-08	-.1426E-08	-.6454E-09
-0.8	.1429E-09	.1435E-09	.1385E-09	.1143E-09	.5352E-10

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2644E-01	-.5917E-02	.2866E-02	-.3020E-05	.1096E-04
0.6	.1606E-02	.1134E-02	.2701E-03	-.6957E-04	-.3076E-04
0.4	-.7289E-04	-.6987E-04	-.4789E-04	-.1480E-04	.4409E-06
0.2	.8359E-06	.1588E-05	.2553E-05	.2104E-05	.7667E-06
0	.2786E-06	.2026E-06	.3922E-07	-.7794E-07	-.7222E-07
-0.2	-.3788E-07	-.3324E-07	-.2140E-07	-.8218E-08	-.3821E-09
-0.4	.3007E-08	.2845E-08	.2341E-08	.1537E-08	.6204E-09
-0.6	-.1469E-09	-.1496E-09	-.1481E-09	-.1239E-09	-.6762E-10
-0.8	-.8634E-14	.9378E-12	.2953E-11	.4264E-11	.2952E-11

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1325E-06	.1116E-06	-.1335E-06	-.3862E-06	-.4802E-06	-.3862E-06	-.1335E-06	.1116E-06	.1325E-06
0.6	-.1418E-05	-.3944E-05	-.3431E-05	-.1829E-07	.1771E-05	-.1829E-07	-.3431E-05	-.3944E-05	-.1418E-05
0.4	-.5700E-05	.2607E-04	.9064E-04	.7854E-04	.3917E-04	.7854E-04	.9064E-04	.2607E-04	-.5700E-05
0.2	.5120E-04	.2705E-03	-.3622E-03	-.2280E-02	-.9146E-03	-.2280E-02	-.3622E-03	.2705E-03	.5120E-04
0	.1291E-03	.3097E-03	-.4172E-02	-.1891E-01	.1000E+31	-.1891E-01	-.4172E-02	.3097E-03	.1291E-03

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.2658E-09	.9492E-09	-.2849E-07	-.8340E-07	-.4395E-07	.2460E-06	.5292E-06	-.6879E-07	.4953E-05
0.6	-.2408E-08	-.5078E-07	-.8251E-07	.4392E-06	.1955E-05	.2155E-05	-.1951E-05	.4703E-05	.7860E-04
0.4	-.3690E-07	-.8755E-07	.6069E-06	.2503E-05	-.4838E-05	-.4408E-04	-.7608E-04	-.7076E-04	-.1063E-02
0.2	-.7338E-07	.1092E-06	.1931E-05	-.1374E-05	-.4574E-04	-.1896E-04	.8873E-03	.2418E-02	.5692E-02
0	-.8212E-07	.2799E-06	.2378E-05	-.7026E-05	-.6575E-04	.1889E-03	.2049E-02	-.5814E-02	-.5308E-01

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.3455E-07	.1164E-06	-.9497E-07	-.1368E-05	-.3368E-05	-.2626E-05	.1870E-05	-.3064E-05	.8011E-04
0.6	.8881E-07	-.3703E-06	-.2398E-05	-.8229E-06	.2512E-04	.8333E-04	.1193E-03	.2577E-03	.1390E-02
0.4	-.2007E-06	-.2014E-05	.8006E-06	.3936E-04	.6628E-04	-.4222E-03	-.2060E-02	-.4700E-02	-.2094E-01
0.2	-.7952E-06	-.2156E-05	.1803E-04	.6284E-04	-.3771E-03	-.1895E-02	.5005E-02	.5077E-01	.1845E+00
0	-.1070E-05	-.1303E-05	.2793E-04	.3645E-04	-.7517E-03	-.1033E-02	.2289E-01	.3315E-01	-.1039E+01

Clamped rectangular plate — b/a = 1.8 — λ ≐ 11

Influence coefficient $s_1 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.4669E-08	-.3845E-08	-.1900E-08	-.1278E-09	.4189E-09
0.6	.6362E-07	.4363E-07	.4460E-08	-.1630E-07	-.1071E-07
0.4	-.6339E-06	-.1497E-06	.4349E-06	.3427E-06	.6810E-07
0.2	-.1589E-05	-.1308E-04	-.1004E-04	-.1123E-05	.4046E-06
0	.1033E-02	.2123E-03	-.1160E-04	-.5618E-05	.2498E-06
-0.2	-.1589E-05	-.1308E-04	-.1004E-04	-.1123E-05	.4046E-06
-0.4	-.6339E-06	-.1497E-06	.4349E-06	.3427E-06	.6810E-07
-0.6	.6362E-07	.4363E-07	.4660E-08	-.1630E-07	-.1071E-07
-0.8	-.4669E-08	-.3845E-08	-.1900E-08	-.1278E-09	.4189E-09

Influence surface of M_y at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.4088E-06	.4118E-06	.3700E-06	.2393E-06	.8031E-07
0.6	-.1113E-04	-.1012E-04	-.6768E-05	-.2570E-05	-.2193E-06
0.4	.2682E-03	.2064E-03	.8018E-04	.4998E-05	-.5567E-05
0.2	-.6686E-02	-.2917E-02	-.1210E-03	.1209E-03	.1854E-04
0	.1000E+31	.6823E-02	-.9151E-03	-.3415E-05	.3214E-04
-0.2	-.6686E-02	-.2917E-02	-.1210E-03	.1209E-03	.1854E-04
-0.4	.2682E-03	.2064E-03	.8018E-04	.4998E-05	-.5567E-05
-0.6	-.1113E-04	-.1012E-04	-.6768E-05	-.2570E-05	-.2193E-06
-0.8	.4088E-06	.4118E-06	.3700E-06	.2393E-06	.8031E-07

Influence coefficient $s_2 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2468E-01	.6632E-01	.4021E-01	-.4893E-02	-.5762E-03
0.6	.3171E-02	.5501E-03	-.2191E-02	-.1260E-02	-.7637E-04
0.4	-.2581E-03	-.1704E-03	-.1890E-05	.7629E-04	.3571E-04
0.2	.1788E-04	.1460E-04	.6737E-05	-.3117E-06	-.1704E-05
0	-.1107E-05	-.9859E-06	-.6603E-06	-.2742E-06	-.1780E-07
-0.2	.6233E-07	.5843E-07	.4675E-07	.2892E-07	.6535E-08
-0.4	-.3186E-08	-.3101E-08	-.2762E-08	-.1988E-08	-.3660E-09
-0.6	.1436E-09	.1441E-09	.1381E-09	.1076E-09	.3320E-10
-0.8	-.5127E-11	-.5325E-11	-.5511E-11	-.4905E-11	-.5335E-11

Influence surface of M_y at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1400E-01	-.2195E-02	.2403E-02	.6104E-04	-.1507E-04
0.6	.7233E-03	.4484E-03	.3131E-04	-.7143E-04	-.1901E-04
0.4	-.3838E-04	-.3098E-04	-.1395E-04	-.5586E-06	.1860E-05
0.2	.1955E-05	.1754E-05	.1178E-05	.4735E-06	.4848E-07
0	-.9152E-07	-.8754E-07	-.7247E-07	-.4558E-07	-.1557E-07
-0.2	.3722E-08	.3773E-08	.3653E-08	.2890E-08	.1176E-08
-0.4	-.1115E-09	-.1239E-09	-.1439E-09	-.1341E-09	-.4965E-10
-0.6	.9462E-13	.1030E-11	.2979E-11	.3933E-11	.2000E-11
-0.8	.3589E-12	.3039E-12	.1713E-12	.3059E-13	-.1764E-12

Influence surface of M_x at $x = y = 0$ ($\nu = 0.3$)

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.4520E-07	.1017E-06	.1157E-06	.9631E-07	.8305E-07	.9631E-07	.1157E-06	.1017E-06	.4520E-07
0.6	-.1986E-06	-.1417E-05	-.2516E-05	-.2565E-05	-.2336E-05	-.2565E-05	-.2516E-05	-.1417E-05	-.1986E-06
0.4	-.6128E-05	.4603E-05	.4289E-04	.6019E-04	.5356E-04	.6019E-04	.4289E-04	.4603E-05	-.6128E-05
0.2	.3825E-04	.2373E-03	-.1138E-03	-.1482E-02	-.1070E-02	-.1482E-02	-.1138E-03	.2373E-03	.3825E-04
0	.1291E-03	.3097E-03	-.4172E-02	-.1891E-01	.1000E+31	-.1891E-01	-.4172E-02	.3097E-03	.1291E-03

Influence surface of M_x at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1116E-08	.3534E-08	-.7072E-09	-.2644E-07	-.6877E-07	-.8379E-07	-.6647E-07	-.1776E-07	.4727E-05
0.6	.2109E-08	-.2245E-07	-.8775E-07	.1816E-07	.7905E-06	.2151E-05	.3553E-05	.1199E-04	.4761E-04
0.4	-.2567E-07	-.9296E-07	.3207E-06	.2097E-05	-.3669E-07	-.2412E-04	-.7169E-04	-.1725E-03	-.9759E-03
0.2	-.6928E-07	.7883E-07	.1772E-05	-.5252E-06	-.3987E-04	-.3842E-04	.6686E-03	.2294E-02	.7027E-02
0	-.8067E-07	.2821E-06	.2339E-05	-.7071E-05	-.6475E-04	.1886E-03	.2010E-02	-.5627E-02	-.8716E-01

Influence coefficient $s_3 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.5354E-08	.6101E-07	.1323E-06	-.9884E-07	-.1126E-05	-.2761E-05	-.4054E-05	-.3552E-05	.6413E-04
0.6	.8312E-07	-.2513E-07	-.1334E-05	-.3273E-05	.3617E-05	.3596E-04	.9694E-04	.2475E-03	.7216E-03
0.4	-.7814E-07	-.1580E-05	-.1423E-05	.2516E-04	.7972E-04	-.1199E-03	-.1207E-02	-.3927E-02	-.1529E-01
0.2	-.7106E-06	-.2219E-05	.1537E-04	.6310E-04	-.2928E-03	-.1785E-02	.2590E-02	.3916E-01	.1551E+00
0	-.1048E-05	-.1282E-05	.2735E-04	.3611E-04	-.7357E-03	-.1048E-02	.2223E-01	.3594E-01	-.9484E+00

TABLE VI

Simply supported rectangular plate (ν=0.3)	b/a = 1.0	λ = 1

Influence coefficient $s_4 = wD/Pa^2$ at $x=y=0$ | **Influence surface of M_y at $x=y=0$**

y/b \ x/a	0	0.2	0.4	0.6	0.8	0	0.2	0.4	0.6	0.8
0.8	.1129E-01	.1065E-01	.8893E-02	.6322E-02	.3275E-02	.1768E-01	.1743E-01	.1593E-01	.1229E-01	.6673E-02
0.6	.2224E-01	.2093E-01	.1737E-01	.1227E-01	.6322E-02	.4044E-01	.3989E-01	.3599E-01	.2702E-01	.1436E-01
0.4	.3233E-01	.3026E-01	.2482E-01	.1737E-01	.8893E-02	.7580E-01	.7450E-01	.6422E-01	.4546E-01	.2323E-01
0.2	.4061E-01	.3754E-01	.3026E-01	.2093E-01	.1065E-01	.1429E+00	.1338E+00	.9992E-01	.6423E-01	.3130E-01
0	.4481E-01	.4061E-01	.3233E-01	.2224E-01	.1129E-01	.1000E+31	.1958E+00	.1207E+00	.7296E-01	.3473E-01
-0.2	.4061E-01	.3754E-01	.3026E-01	.2093E-01	.1065E-01	.1429E+00	.1338E+00	.9992E-01	.6423E-01	.3130E-01
-0.4	.3233E-01	.3026E-01	.2482E-01	.1737E-01	.8893E-02	.7580E-01	.7450E-01	.6422E-01	.4546E-01	.2323E-01
-0.6	.2224E-01	.2093E-01	.1737E-01	.1227E-01	.6322E-02	.4044E-01	.3989E-01	.3599E-01	.2732E-01	.1436E-01
-0.8	.1129E-01	.1065E-01	.8893E-02	.6322E-02	.3275E-02	.1768E-01	.1743E-01	.1593E-01	.1228E-01	.6673E-02

Influence coefficient $s_6 = V_y a/P$ at $x=0, y=b$ | **Influence surface of M_x at $x=0, y=0$**

y/b \ x/a	0	0.2	0.4	0.6	0.8	0	0.2	0.4	0.6	0.8
0.8	-.1925E+01	-.8753E+00	-.1933E+00	-.8202E-01	-.3053E-01	.3473E-01	.3130E-01	.2323E-01	.1436E-01	.6673E-02
0.6	-.1001E+01	-.7295E+00	-.3471E+00	-.1527E+00	-.5980E-01	.7296E-01	.6423E-01	.4546E-01	.2702E-01	.1228E-01
0.4	-.6341E+00	-.5343E+00	-.3402E+00	-.1820E+00	-.7747E-01	.1207E+00	.9992E-01	.6422E-01	.3599E-01	.1593E-01
0.2	-.4353E+00	-.3892E+00	-.2836E+00	-.1724E+00	-.7932E-01	.1958E+00	.1338E+00	.7450E-01	.3989E-01	.1743E-01
0	-.3079E+00	-.2831E+00	-.2209E+00	-.1447E+00	-.7018E-01	.1000E+31	.1429E+00	.7580E-01	.4044E-01	.1768E-01
-0.2	-.2174E+00	-.2028E+00	-.1641E+00	-.1122E+00	-.5618E-01	.1958E+00	.1338E+00	.7450E-01	.3989E-01	.1743E-01
-0.4	-.1485E+00	-.1396E+00	-.1152E+00	-.8063E-01	-.4115E-01	.1207E+00	.9992E-01	.6422E-01	.3599E-01	.1593E-01
-0.6	-.9266E-01	-.8746E-01	-.7294E-01	-.5172E-01	-.2668E-01	.7296E-01	.6423E-01	.4546E-01	.2702E-01	.1228E-01
-0.8	-.4454E-01	-.4213E-01	-.3533E-01	-.2523E-01	-.1311E-01	.3473E-01	.3130E-01	.2323E-01	.1436E-01	.6673E-02

Influence coefficient $s_5 = V_x a/P$ at $x=a, y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1311E-01	-.2668E-01	-.4115E-01	-.5618E-01	-.7018E-01	-.7932E-01	-.7747E-01	-.5980E-01	-.3053E-01
0.6	-.2523E-01	-.5172E-01	-.8063E-01	-.1122E+00	-.1447E+00	-.1724E+00	-.1820E+00	-.1527E+00	-.8202E-01
0.4	-.3533E-01	-.7294E-01	-.1152E+00	-.1641E+00	-.2209E+00	-.2836E+00	-.3402E+00	-.3471E+00	-.1933E+00
0.2	-.4213E-01	-.8746E-01	-.1396E+00	-.2028E+00	-.2831E+00	-.3592E+00	-.5343E+00	-.7295E+00	-.8753E+00
0	-.4454E-01	-.9266E-01	-.1485E+00	-.2174E+00	-.3079E+00	-.4353E+00	-.6341E+00	-.1001E+01	-.1925E+01

Influence surface of M_{xy} at $x=0, y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1971E-01	-.3571E-01	-.4154E-01	-.2925E-01	0.	.2925E-01	.4154E-01	.3571E-01	.1971E-01
0.6	-.3571E-01	-.6762E-01	-.8445E-01	-.6435E-01	0.	.6435E-01	.8445E-01	.6762E-01	.3571E-01
0.4	-.4154E-01	-.8445E-01	-.1209E+00	-.1117E+00	0.	.1117E+00	.1209E+00	.8445E-01	.4154E-01
0.2	-.2925E-01	-.6435E-01	-.1117E+00	-.1599E+00	0.	.1599E+00	.1117E+00	.6435E-01	.2925E-01
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.2925E-01	.6435E-01	.1117E+00	.1599E+00	0.	-.1599E+00	-.1117E+00	-.6435E-01	-.2925E-01
-0.4	.4154E-01	.8445E-01	.1209E+00	.1117E+00	0.	-.1117E+00	-.1209E+00	-.8445E-01	-.4154E-01
-0.6	.3571E-01	.6762E-01	.8445E-01	.6435E-01	0.	-.6435E-01	-.8445E-01	-.6762E-01	-.3571E-01
-0.8	.1971E-01	.3571E-01	.4154E-01	.2925E-01	0.	-.2925E-01	-.4154E-01	-.3571E-01	-.1971E-01

Simply supported rectangular plate ($\nu = 0.3$) $b/a = 1.2$ $\lambda = 1$

Influence coefficient $s_4 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1230E-01	.1166E-01	.9816E-02	.7047E-02	.3672E-02
0.6	.2449E-01	.2316E-01	.1941E-01	.1386E-01	.7195E-02
0.4	.3617E-01	.3405E-01	.2826E-01	.1999E-01	.1032E-01
0.2	.4624E-01	.4306E-01	.3512E-01	.2453E-01	.1257E-01
0	.5166E-01	.4712E-01	.3787E-01	.2626E-01	.1340E-01
-0.2	.4624E-01	.4306E-01	.3512E-01	.2453E-01	.1257E-01
-0.4	.3617E-01	.3405E-01	.2826E-01	.1999E-01	.1032E-01
-0.6	.2449E-01	.2316E-01	.1941E-01	.1386E-01	.7195E-02
-0.8	.1230E-01	.1166E-01	.9816E-02	.7047E-02	.3672E-02

Influence surface of M_y at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1132E-01	.1115E-01	.1025E-01	.8032E-02	.4432E-02
0.6	.2817E-01	.2781E-01	.2543E-01	.1956E-01	.1060E-01
0.4	.5865E-01	.5788E-01	.5128E-01	.3744E-01	.1950E-01
0.2	.1225E+00	.1171E+00	.9099E-01	.5960E-01	.2928E-01
0	.1000E+31	.1932E+00	.1185E+00	.7139E-01	.3393E-01
-0.2	.1225E+00	.1171E+00	.9099E-01	.5960E-01	.2928E-01
-0.4	.5865E-01	.5788E-01	.5128E-01	.3744E-01	.1950E-01
-0.6	.2817E-01	.2781E-01	.2543E-01	.1956E-01	.1060E-01
-0.8	.1132E-01	.1115E-01	.1025E-01	.8032E-02	.4432E-02

Influence coefficient $s_6 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1647E+01	-.8742E+00	-.2445E+00	-.9843E-01	-.3733E-01
0.6	-.8260E+00	-.6498E+00	-.3590E+00	-.1727E+00	-.6997E-01
0.4	-.5100E+00	-.4482E+00	-.3138E+00	-.1835E+00	-.8238E-01
0.2	-.3403E+00	-.3119E+00	-.2416E+00	-.1571E+00	-.7583E-01
0	-.2330E+00	-.2177E+00	-.1767E+00	-.1213E+00	-.6099E-01
-0.2	-.1591E+00	-.1500E+00	-.1246E+00	-.8789E-01	-.4515E-01
-0.4	-.1053E+00	-.9965E-01	-.8371E-01	-.5988E-01	-.3110E-01
-0.6	-.6402E-01	-.6072E-01	-.5128E-01	-.3693E-01	-.1928E-01
-0.8	-.3026E-01	-.2873E-01	-.2433E-01	-.1758E-01	-.9206E-02

Influence surface of M_x at $x = 0$, $y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.3333E-01	.3079E-01	.2431E-01	.1615E-01	.7920E-02
0.6	.7032E-01	.6390E-01	.4862E-01	.3117E-01	.1493E-01
0.4	.1172E+00	.1018E+00	.7121E-01	.4285E-01	.1986E-01
0.2	.1917E+00	.1426E+00	.8562E-01	.4834E-01	.2190E-01
0	.1000E+31	.1568E+00	.8757E-01	.4895E-01	.2214E-01
-0.2	.1917E+00	.1426E+00	.8562E-01	.4834E-01	.2190E-01
-0.4	.1172E+00	.1018E+00	.7121E-01	.4285E-01	.1986E-01
-0.6	.7032E-01	.6390E-01	.4862E-01	.3117E-01	.1493E-01
-0.8	.3333E-01	.3079E-01	.2431E-01	.1615E-01	.7920E-02

Influence coefficient $s_5 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1248E-01	-.2493E-01	-.3713E-01	-.4821E-01	-.5643E-01	-.5906E-01	-.5337E-01	-.3899E-01	-.1973E-01
0.6	-.2471E-01	-.4987E-01	-.7559E-01	-.1010E+00	-.1234E+00	-.1373E+00	-.1336E+00	-.1042E+00	-.5391E-01
0.4	-.3580E-01	-.7309E-01	-.1133E+00	-.1571E+00	-.2040E+00	-.2492E+00	-.2783E+00	-.2564E+00	-.1457E+00
0.2	-.4384E-01	-.9037E-01	-.1427E+00	-.2045E+00	-.2805E+00	-.3774E+00	-.5015E+00	-.6399E+00	-.6046E+00
0	-.4681E-01	-.9687E-01	-.1541E+00	-.2236E+00	-.3140E+00	-.4408E+00	-.6395E+00	-.1011E+01	-.1990E+01

Influence surface of M_{xy} at $x = 0$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1807E-01	-.3120E-01	-.3392E-01	-.2250E-01	0.	.2250E-01	.3392E-01	.3120E-01	.1807E-01
0.6	-.3536E-01	-.6356E-01	-.7335E-01	-.5159E-01	0.	.5159E-01	.7335E-01	.6356E-01	.3536E-01
0.4	-.4566E-01	-.8846E-01	-.1164E+00	-.9587E-01	0.	.9587E-01	.1164E+00	.8846E-01	.4566E-01
0.2	-.3539E-01	-.7561E-01	-.1241E+00	-.1564E+00	0.	.1564E+00	.1241E+00	.7561E-01	.3539E-01
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.3539E-01	.7561E-01	.1241E+00	.1564E+00	0.	-.1564E+00	-.1241E+00	-.7561E-01	-.3539E-01
-0.4	.4566E-01	.8846E-01	.1164E+00	.9587E-01	0.	-.9587E-01	-.1164E+00	-.8846E-01	-.4566E-01
-0.6	.3536E-01	.6356E-01	.7335E-01	.5159E-01	0.	-.5159E-01	-.7335E-01	-.6356E-01	-.3536E-01
-0.8	.1807E-01	.3120E-01	.3392E-01	.2250E-01	0.	-.2250E-01	-.3392E-01	-.3120E-01	-.1807E-01

Simply supported rectangular plate (ν = 0.3) b/a = 1.4 λ = 1

Influence coefficient $s_4' = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1240E-01	.1176E-01	.9952E-02	.7183E-02	.3756E-02
0.6	.2502E-01	.2371E-01	.1999E-01	.1437E-01	.7490E-02
0.4	.3774E-01	.3563E-01	.2978E-01	.2122E-01	.1100E-01
0.2	.4937E-01	.4617E-01	.3792E-01	.2664E-01	.1370E-01
0	.5601E-01	.5126E-01	.4139E-01	.2882E-01	.1474E-01
-0.2	.4937E-01	.4617E-01	.3792E-01	.2664E-01	.1370E-01
-0.4	.3774E-01	.3563E-01	.2978E-01	.2122E-01	.1100E-01
-0.6	.2502E-01	.2371E-01	.1999E-01	.1437E-01	.7490E-02
-0.8	.1240E-01	.1176E-01	.9952E-02	.7183E-02	.3756E-02

Influence surface of M_y at x = y = 0.

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.6054E-02	.5987E-02	.5575E-02	.4445E-02	.2487E-02
0.6	.1786E-01	.1767E-01	.1636E-01	.1283E-01	.7067E-02
0.4	.4389E-01	.4345E-01	.3926E-01	.2939E-01	.1558E-01
0.2	.1045E+00	.1013E+00	.8140E-01	.5434E-01	.2691E-01
0	.1000E+31	.1902E+00	.1160E+00	.6956E-01	.3297E-01
-0.2	.1045E+00	.1013E+00	.8140E-01	.5434E-01	.2691E-01
-0.4	.4389E-01	.4345E-01	.3926E-01	.2939E-01	.1558E-01
-0.6	.1786E-01	.1767E-01	.1636E-01	.1283E-01	.7067E-02
-0.8	.6054E-02	.5987E-02	.5575E-02	.4445E-02	.2487E-02

Influence coefficient $s_6 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1432E+01	-.8510E+00	-.2853E+00	-.1144E+00	-.4377E-01
0.6	-.6951E+00	-.5754E+00	-.3531E+00	-.1836E+00	-.7717E-01
0.4	-.4167E+00	-.3758E+00	-.2798E+00	-.1742E+00	-.8161E-01
0.2	-.2681E+00	-.2493E+00	-.2002E+00	-.1357E+00	-.6756E-01
0	-.1760E+00	-.1658E+00	-.1375E+00	-.9679E-01	-.4961E-01
-0.2	-.1150E+00	-.1089E+00	-.9157E-01	-.6559E-01	-.3410E-01
-0.4	-.7296E-01	-.6925E-01	-.5859E-01	-.4229E-01	-.2213E-01
-0.6	-.4284E-01	-.4070E-01	-.3453E-01	-.2501E-01	-.1311E-01
-0.8	-.1978E-01	-.1880E-01	-.1597E-01	-.1158E-01	-.6083E-02

Influence surface of M_x at x = 0, y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.3091E-01	.2892E-01	.2359E-01	.1629E-01	.8226E-02
0.6	.6584E-01	.6087E-01	.4824E-01	.3231E-01	.1597E-01
0.4	.1112E+00	.9930E-01	.7329E-01	.4613E-01	.2200E-01
0.2	.1849E+00	.1453E+00	.9183E-01	.5343E-01	.2466E-01
0	.1000E+31	.1652E+00	.9472E-01	.5413E-01	.2486E-01
-0.2	.1849E+00	.1453E+00	.9183E-01	.5343E-01	.2466E-01
-0.4	.1112E+00	.9930E-01	.7329E-01	.4613E-01	.2200E-01
-0.6	.6584E-01	.6087E-01	.4824E-01	.3231E-01	.1597E-01
-0.8	.3091E-01	.2892E-01	.2359E-01	.1629E-01	.8226E-02

Influence coefficient $s_5 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1066E-01	-.2096E-01	-.3034E-01	-.3786E-01	-.4216E-01	-.4171E-01	-.3570E-01	-.2507E-01	-.1251E-01
0.6	-.2213E-01	-.4410E-01	-.6538E-01	-.8463E-01	-.9915E-01	-.1046E+00	-.9582E-01	-.7079E-01	-.3564E-01
0.4	-.3390E-01	-.6866E-01	-.1050E+00	-.1429E+00	-.1804E+00	-.2118E+00	-.2232E+00	-.1899E+00	-.1023E+00
0.2	-.4333E-01	-.8904E-01	-.1399E+00	-.1991E+00	-.2705E+00	-.3583E+00	-.4635E+00	-.5598E+00	-.4567E+00
0	-.4699E-01	-.9717E-01	-.1543E+00	-.2237E+00	-.3136E+00	-.4399E+00	-.6375E+00	-.1005E+01	-.1938E+01

Influence surface of M_{xy} at x = 0, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1479E-01	-.2485E-01	-.2605E-01	-.1675E-01	0.	.1675E-01	.2605E-01	.2485E-01	.1479E-01
0.6	-.3159E-01	-.5489E-01	-.6034E-01	-.4050E-01	0.	.4050E-01	.6034E-01	.5489E-01	.3159E-01
0.4	-.4601E-01	-.8595E-01	-.1063E+00	-.8104E-01	0.	.8104E-01	.1063E+00	.8595E-01	.4601E-01
0.2	-.3996E-01	-.8358E-01	-.1309E+00	-.1484E+00	0.	.1484E+00	.1309E+00	.8358E-01	.3996E-01
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.3996E-01	.8358E-01	.1309E+00	.1484E+00	0.	-.1484E+00	-.1309E+00	-.8358E-01	-.3996E-01
-0.4	.4601E-01	.8595E-01	.1063E+00	.8104E-01	0.	-.8104E-01	-.1063E+00	-.8595E-01	-.4601E-01
-0.6	.3159E-01	.5489E-01	.6034E-01	.4050E-01	0.	-.4050E-01	-.6034E-01	-.5489E-01	-.3159E-01
-0.8	.1479E-01	.2485E-01	.2605E-01	.1675E-01	0.	-.1675E-01	-.2605E-01	-.2485E-01	-.1479E-01

Simply supported rectangular plate (ν = 0.3) | b/a = 1.6 | λ = 1

Influence coefficient $s_4 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1185E-01	.1125E-01	.9542E-02	.6907E-02	.3617E-02
0.6	.2435E-01	.2310E-01	.1954E-01	.1410E-01	.7366E-02
0.4	.3769E-01	.3565E-01	.2993E-01	.2143E-01	.1116E-01
0.2	.5072E-01	.4756E-01	.3926E-01	.2770E-01	.1428E-01
0	.5863E-01	.5375E-01	.4351E-01	.3036E-01	.1554E-01
-0.2	.5072E-01	.4756E-01	.3925E-01	.2770E-01	.1428E-01
-0.4	.3769E-01	.3565E-01	.2993E-01	.2143E-01	.1116E-01
-0.6	.2435E-01	.2310E-01	.1954E-01	.1410E-01	.7366E-02
-0.8	.1185E-01	.1125E-01	.9542E-02	.6907E-02	.3617E-02

Influence surface of M_y at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2064E-02	.2094E-02	.2064E-02	.1740E-02	.1008E-02
0.6	.9764E-02	.9744E-02	.9217E-02	.7416E-02	.4160E-02
0.4	.3183E-01	.3162E-01	.2904E-01	.2223E-01	.1198E-01
0.2	.8940E-01	.8741E-01	.7229E-01	.4919E-01	.2458E-01
0	.1000E+31	.1878E+00	.1140E+00	.6810E-01	.3218E-01
-0.2	.8940E-01	.8741E-01	.7229E-01	.4919E-01	.2458E-01
-0.4	.3183E-01	.3162E-01	.2904E-01	.2223E-01	.1198E-01
-0.6	.9764E-02	.9744E-02	.9217E-02	.7416E-02	.4160E-02
-0.8	.2064E-02	.2094E-02	.2064E-02	.1740E-02	.1008E-02

Influence coefficient $s_6 = V_y a/P$ at $x = 0,\ y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1260E+01	-.8162E+00	-.3158E+00	-.1295E+00	-.4991E-01
0.6	-.5936E+00	-.5090E+00	-.3371E+00	-.1872E+00	-.8148E-01
0.4	-.3441E+00	-.3156E+00	-.2448E+00	-.1594E+00	-.7714E-01
0.2	-.2121E+00	-.1990E+00	-.1633E+00	-.1135E+00	-.5771E-01
0	-.1325E+00	-.1254E+00	-.1051E+00	-.7506E-01	-.3888E-01
-0.2	-.8214E-01	-.7795E-01	-.6593E-01	-.4757E-01	-.2487E-01
-0.4	-.4954E-01	-.4707E-01	-.3995E-01	-.2894E-01	-.1520E-01
-0.6	-.2786E-01	-.2649E-01	-.2251E-01	-.1634E-01	-.8575E-02
-0.8	-.1250E-01	-.1188E-01	-.1010E-01	-.7338E-02	-.3853E-02

Influence surface of M_x at $x = 0,\ y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2789E-01	.2628E-01	.2182E-01	.1539E-01	.7899E-02
0.6	.6026E-01	.5628E-01	.4571E-01	.3146E-01	.1586E-01
0.4	.1038E+00	.9435E-01	.7217E-01	.4689E-01	.2292E-01
0.2	.1765E+00	.1441E+00	.9467E-01	.5623E-01	.2624E-01
0	.1000E+31	.1701E+00	.9885E-01	.5714E-01	.2643E-01
-0.2	.1765E+00	.1441E+00	.9467E-01	.5623E-01	.2624E-01
-0.4	.1038E+00	.9435E-01	.7217E-01	.4689E-01	.2282E-01
-0.6	.6026E-01	.5628E-01	.4571E-01	.3146E-01	.1586E-01
-0.8	.2789E-01	.2628E-01	.2182E-01	.1539E-01	.7899E-02

Influence coefficient $s_5 = V_x a/P$ at $x = a,\ y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.8376E-02	-.1624E-01	-.2295E-01	-.2772E-01	-.2966E-01	-.2811E-01	-.2311E-01	-.1579E-01	-.7794E-02
0.6	-.1865E-01	-.3677E-01	-.5351E-01	-.6743E-01	-.7626E-01	-.7707E-01	-.6742E-01	-.4794E-01	-.2368E-01
0.4	-.3096E-01	-.6227E-01	-.9422E-01	-.1261E+00	-.1553E+00	-.1760E+00	-.1763E+00	-.1407E+00	-.7419E-01
0.2	-.4198E-01	-.8612E-01	-.1349E+00	-.1911E+00	-.2576E+00	-.3366E+00	-.4243E+00	-.4859E+00	-.3430E+00
0	-.4651E-01	-.9626E-01	-.1530E+00	-.2220E+00	-.3116E+00	-.4375E+00	-.6341E+00	-.9957E+00	-.1883E+01

Influence surface of M_{xy} at $x = 0,\ y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1125E-01	-.1864E-01	-.1916E-01	-.1211E-01	0.	.1211E-01	.1916E-01	.1864E-01	.1125E-01
0.6	-.2646E-01	-.4500E-01	-.4794E-01	-.3128E-01	0.	.3128E-01	.4794E-01	.4500E-01	.2646E-01
0.4	-.4395E-01	-.7975E-01	-.9405E-01	-.6796E-01	0.	.6796E-01	.9405E-01	.7975E-01	.4395E-01
0.2	-.4338E-01	-.8898E-01	-.1335E+00	-.1385E+00	0.	.1385E+00	.1335E+00	.8898E-01	.4338E-01
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.4338E-01	.8898E-01	.1335E+00	.1385E+00	0.	-.1385E+00	-.1335E+00	-.8898E-01	-.4338E-01
-0.4	.4395E-01	.7975E-01	.9405E-01	.6796E-01	0.	-.6796E-01	-.9405E-01	-.7975E-01	-.4395E-01
-0.6	.2646E-01	.4500E-01	.4794E-01	.3128E-01	0.	-.3128E-01	-.4794E-01	-.4500E-01	-.2646E-01
-0.9	.1125E-01	.1864E-01	.1916E-01	.1211E-01	0.	-.1211E-01	-.1916E-01	-.1864E-01	-.1125E-01

Simply supported rectangular plate ($\nu = 0.3$) | $b/a = 1.8$ | $\lambda = 1$

Influence coefficient $s_4 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1091E-01	.1037E-01	.8801E-02	.6381E-02	.3342E-02
0.6	.2292E-01	.2176E-01	.1844E-01	.1334E-01	.6976E-02
0.4	.3660E-01	.3467E-01	.2920E-01	.2097E-01	.1096E-01
0.2	.5091E-01	.4784E-01	.3964E-01	.2805E-01	.1449E-01
0	.6015E-01	.5520E-01	.4474E-01	.3125E-01	.1599E-01
-0.2	.5091E-01	.4784E-01	.3964E-01	.2805E-01	.1449E-01
-0.4	.3660E-01	.3467E-01	.2920E-01	.2097E-01	.1096E-01
-0.6	.2292E-01	.2176E-01	.1844E-01	.1334E-01	.6976E-02
-0.8	.1091E-01	.1037E-01	.8801E-02	.6381E-02	.3342E-02

Influence surface of M_y at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.7565E-03	-.6466E-03	-.3911E-03	-.1467E-03	-.2293E-04
0.6	.3720E-02	.3836E-02	.3887E-02	.3332E-02	.1945E-02
0.4	.2223E-01	.2218E-01	.2070E-01	.1618E-01	.8866E-02
0.2	.7674E-01	.7549E-01	.6398E-01	.4438E-01	.2239E-01
0	.1000E+31	.1862E+00	.1126E+00	.6709E-01	.3162E-01
-0.2	.7674E-01	.7549E-01	.6398E-01	.4438E-01	.2239E-01
-0.4	.2223E-01	.2218E-01	.2070E-01	.1618E-01	.8866E-02
-0.6	.3720E-02	.3836E-02	.3887E-02	.3332E-02	.1945E-02
-0.8	-.7565E-03	-.6466E-03	-.3911E-03	-.1467E-03	-.2293E-04

Influence coefficient $s_6 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1121E+01	-.7760E+00	-.3372E+00	-.1433E+00	-.5572E-01
0.6	-.5126E+00	-.4507E+00	-.3160E+00	-.1851E+00	-.8318E-01
0.4	-.2862E+00	-.2655E+00	-.2117E+00	-.1424E+00	-.7060E-01
0.2	-.1681E+00	-.1585E+00	-.1319E+00	-.9321E-01	-.4801E-01
0	-.9937E-01	-.9422E-01	-.7952E-01	-.5721E-01	-.2978E-01
-0.2	-.5806E-01	-.5516E-01	-.4679E-01	-.3388E-01	-.1776E-01
-0.4	-.3307E-01	-.3143E-01	-.2671E-01	-.1938E-01	-.1020E-01
-0.6	-.1770E-01	-.1683E-01	-.1431E-01	-.1039E-01	-.5454E-02
-0.8	-.7666E-02	-.7290E-02	-.6200E-02	-.4504E-02	-.2363E-02

Influence surface of M_x at $x = 0$, $y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2462E-01	.2328E-01	.1953E-01	.1395E-01	.7217E-02
0.6	.5422E-01	.5094E-01	.4202E-01	.2943E-01	.1502E-01
0.4	.9585E-01	.8813E-01	.6914E-01	.4599E-01	.2275E-01
0.2	.1675E+00	.1406E+00	.9533E-01	.5756E-01	.2709E-01
0	.1000E+31	.1728E+00	.1012E+00	.5884E-01	.2730E-01
-0.2	.1675E+00	.1406E+00	.9533E-01	.5756E-01	.2709E-01
-0.4	.9585E-01	.8813E-01	.6914E-01	.4599E-01	.2275E-01
-0.6	.5422E-01	.5094E-01	.4202E-01	.2943E-01	.1502E-01
-0.8	.2462E-01	.2328E-01	.1953E-01	.1395E-01	.7217E-02

Influence coefficient $s_5 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.6101E-02	-.1170E-01	-.1619E-01	-.1900E-01	-.1985E-01	-.1796E-01	-.1427E-01	-.9509E-02	-.4634E-02
0.6	-.1500E-01	-.2931E-01	-.4194E-01	-.5162E-01	-.5662E-01	-.5519E-01	-.4650E-01	-.3210E-01	-.1560E-01
0.4	-.2767E-01	-.5528E-01	-.8281E-01	-.1091E+00	-.1314E+00	-.1440E+00	-.1379E+00	-.1045E+00	-.5519E-01
0.2	-.4041E-01	-.8276E-01	-.1293E+00	-.1824E+00	-.2439E+00	-.3142E+00	-.3859E+00	-.4196E+00	-.2556E+00
0	-.4593E-01	-.9523E-01	-.1515E+00	-.2201E+00	-.3095E+00	-.4350E+00	-.6303E+00	-.9855E+00	-.1827E+01

Influence surface of M_{xy} at $x = 0$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.8158E-02	-.1342E-01	-.1364E-01	-.8543E-02	0.	.8543E-02	.1364E-01	.1342E-01	.8158E-02
0.6	-.2126E-01	-.3564E-01	-.3721E-01	-.2384E-01	0.	.2384E-01	.3721E-01	.3564E-01	.2126E-01
0.4	-.4053E-01	-.7178E-01	-.8162E-01	-.5673E-01	0.	.5673E-01	.8162E-01	.7178E-01	.4053E-01
0.2	-.4585E-01	-.9230E-01	-.1331E+00	-.1280E+00	0.	.1280E+00	.1331E+00	.9230E-01	.4585E-01
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.4585E-01	.9230E-01	.1331E+00	.1280E+00	0.	-.1280E+00	-.1331E+00	-.9230E-01	-.4585E-01
-0.4	.4053E-01	.7178E-01	.8162E-01	.5673E-01	0.	-.5673E-01	-.8162E-01	-.7178E-01	-.4053E-01
-0.6	.2126E-01	.3564E-01	.3721E-01	.2384E-01	0.	-.2384E-01	-.3721E-01	-.3564E-01	-.2126E-01
-0.8	.8158E-02	.1342E-01	.1364E-01	.8543E-02	0.	-.8543E-02	-.1364E-01	-.1342E-01	-.8158E-02

Simply supported rectangular plate ($\nu = 0.3$) — $b/a = 1.0$ — $\lambda = 3$

Influence coefficient $s_4 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2125E-02	.1956E-02	.1527E-02	.9971E-03	.4810E-03
0.6	.4697E-02	.4287E-02	.3281E-02	.2098E-02	.9971E-03
0.4	.7989E-02	.7167E-02	.5296E-02	.3281E-02	.1527E-02
0.2	.1178E-01	.1021E-01	.7167E-02	.4287E-02	.1956E-02
0	.1441E-01	.1178E-01	.7989E-02	.4697E-02	.2125E-02
-0.2	.1178E-01	.1021E-01	.7167E-02	.4287E-02	.1956E-02
-0.4	.7989E-02	.7167E-02	.5296E-02	.3281E-02	.1527E-02
-0.6	.4697E-02	.4287E-02	.3281E-02	.2098E-02	.9971E-03
-0.8	.2125E-02	.1956E-02	.1527E-02	.9971E-03	.4810E-03

Influence surface of M_y at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.8939E-02	-.7936E-02	-.5439E-02	-.3138E-02	-.1399E-02
0.6	-.1247E-01	-.1025E-01	-.6259E-02	-.3321E-02	-.1459E-02
0.4	-.1483E-02	.1431E-02	.2995E-02	.1755E-02	.5551E-03
0.2	.4684E-01	.4334E-01	.2464E-01	.1084E-01	.3738E-02
0	.1000E+31	.9846E-01	.4007E-01	.1597E-01	.5371E-02
-0.2	.4684E-01	.4334E-01	.2464E-01	.1084E-01	.3738E-02
-0.4	-.1483E-02	.1431E-02	.2995E-02	.1755E-02	.5551E-03
-0.6	-.1247E-01	-.1025E-01	-.6259E-02	-.3321E-02	-.1459E-02
-0.8	-.8939E-02	-.7836E-02	-.5439E-02	-.3138E-02	-.1399E-02

Influence coefficient $s_6 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1707E+01	-.6906E+00	-.5498E-01	.9402E-02	.1374E-01
0.6	-.6682E+00	-.4256E+00	-.1107E+00	.4450E-02	.1773E-01
0.4	-.2638E+00	-.1903E+00	-.6413E-01	.5861E-02	.1659E-01
0.2	-.8418E-01	-.6027E-01	-.1434E-01	.1446E-01	.1551E-01
0	-.7652E-02	-.4221E-03	.1344E-01	.2026E-01	.1442E-01
-0.2	.1910E-01	.2059E-01	.2259E-01	.2055E-01	.1239E-01
-0.4	.2296E-01	.2269E-01	.2113E-01	.1690E-01	.9505E-02
-0.6	.1754E-01	.1697E-01	.1507E-01	.1149E-01	.6273E-02
-0.8	.9097E-02	.8742E-02	.7634E-02	.5721E-02	.3086E-02

Influence surface of M_x at $x = 0$, $y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.5371E-02	.3738E-02	.5551E-03	-.1459E-02	-.1399E-02
0.6	.1597E-01	.1084E-01	.1755E-02	-.3321E-02	-.3138E-02
0.4	.4007E-01	.2464E-01	.2995E-02	-.6259E-02	-.5439E-02
0.2	.9846E-01	.4334E-01	.1431E-02	-.1025E-01	-.7836E-02
0	.1000E+31	.4684E-01	-.1483E-02	-.1247E-01	-.8939E-02
-0.2	.9846E-01	.4334E-01	.1431E-02	-.1025E-01	-.7836E-02
-0.4	.4007E-01	.2464E-01	.2995E-02	-.6259E-02	-.5439E-02
-0.6	.1597E-01	.1084E-01	.1755E-02	-.3321E-02	-.3138E-02
-0.8	.5371E-02	.3738E-02	.5551E-03	-.1459E-02	-.1399E-02

Influence coefficient $s_5 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.3086E-02	.6273E-02	.9505E-02	.1239E-01	.1442E-01	.1551E-01	.1659E-01	.1773E-01	.1374E-01
0.6	.5721E-02	.1149E-01	.1690E-01	.2055E-01	.2026E-01	.1446E-01	.5861E-02	.4450E-02	.9402E-02
0.4	.7634E-02	.1507E-01	.2113E-01	.2259E-01	.1344E-01	-.1434E-01	-.6413E-01	-.1107E+00	-.5498E-01
0.2	.8742E-02	.1697E-01	.2269E-01	.2059E-01	-.4221E-03	-.6027E-01	-.1903E+00	-.4256E+00	-.6906E+00
0	.9097E-02	.1754E-01	.2296E-01	.1910E-01	-.7652E-02	-.8418E-01	-.2638E+00	-.5692E+00	-.1707E+01

Influence surface of M_{xy} at $x = 0$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1413E-01	-.2636E-01	-.3175E-01	-.2296E-01	0.	.2296E-01	.3175E-01	.2636E-01	.1413E-01
0.6	-.2636E-01	-.5180E-01	-.6766E-01	-.5341E-01	0.	.5341E-01	.6766E-01	.5180E-01	.2636E-01
0.4	-.3175E-01	-.6766E-01	-.1027E+00	-.9961E-01	0.	.9961E-01	.1027E+00	.6766E-01	.3175E-01
0.2	-.2296E-01	-.5341E-01	-.9961E-01	-.1515E+00	0.	.1515E+00	.9961E-01	.5341E-01	.2296E-01
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.2296E-01	.5341E-01	.9961E-01	.1515E+00	0.	-.1515E+00	-.9961E-01	-.5341E-01	-.2296E-01
-0.4	.3175E-01	.6766E-01	.1027E+00	.9961E-01	0.	-.9961E-01	-.1027E+00	-.6766E-01	-.3175E-01
-0.6	.2636E-01	.5180E-01	.6766E-01	.5341E-01	0.	-.5341E-01	-.6766E-01	-.5180E-01	-.2636E-01
-0.8	.1413E-01	.2636E-01	.3175E-01	.2296E-01	0.	-.2296E-01	-.3175E-01	-.2636E-01	-.1413E-01

Simply supported rectangular plate ($\nu = 0.3$) | $b/a = 1.2$ | $\lambda = 3$

Influence coefficient $s_4 = wD/Pa^2$ at $x=y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1342E-02	.1242E-02	.9824E-03	.6509E-03	.3168E-03
0.6	.3355E-02	.3082E-02	.2393E-02	.1553E-02	.7441E-03
0.4	.6558E-02	.5929E-02	.4444E-02	.2784E-02	.1303E-02
0.2	.1089E-01	.9506E-02	.6728E-02	.4034E-02	.1840E-02
0	.1424E-01	.1162E-01	.7852E-02	.4596E-02	.2072E-02
-0.2	.1089E-01	.9506E-02	.6728E-02	.4034E-02	.1840E-02
-0.4	.6558E-02	.5929E-02	.4444E-02	.2784E-02	.1303E-02
-0.6	.3355E-02	.3082E-02	.2393E-02	.1553E-02	.7441E-03
-0.8	.1342E-02	.1242E-02	.9824E-03	.6509E-03	.3168E-03

Influence surface of M_y at $x=y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1036E-01	-.9451E-02	-.7236E-02	-.4643E-02	-.2219E-02
0.6	-.1706E-01	-.1512E-01	-.1089E-01	-.6644E-02	-.3101E-02
0.4	-.1166E-01	-.8683E-02	-.4673E-02	-.2640E-02	-.1344E-02
0.2	.3023E-01	.3015E-01	.1854E-01	.8176E-02	.2699E-02
0	.1000E+31	.9755E-01	.3931E-01	.1543E-01	.5094E-02
-0.2	.3023E-01	.3015E-01	.1854E-01	.8176E-02	.2699E-02
-0.4	-.1166E-01	-.8683E-02	-.4673E-02	-.2640E-02	-.1344E-02
-0.6	-.1706E-01	-.1512E-01	-.1089E-01	-.6644E-02	-.3101E-02
-0.8	-.1036E-01	-.9451E-02	-.7236E-02	-.4643E-02	-.2219E-02

Influence coefficient $s_6 = V_y a/P$ at $x=0$, $y=b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1398E+01	-.6365E+00	-.8062E-01	.9601E-02	.1525E-01
0.6	-.4657E+00	-.3179E+00	-.9689E-01	.3300E-02	.1739E-01
0.4	-.1395E+00	-.1020E+00	-.3221E-01	.1075E-01	.1579E-01
0.2	-.1814E-01	-.8803E-02	.9327E-02	.1933E-01	.1459E-01
0	.1892E-01	.2042E-01	.2245E-01	.2044E-01	.1234E-01
-0.2	.2292E-01	.2249E-01	.2061E-01	.1624E-01	.9049E-02
-0.4	.1677E-01	.1613E-01	.1414E-01	.1063E-01	.5741E-02
-0.6	.9647E-02	.9224E-02	.7954E-02	.5875E-02	.3128E-02
-0.8	.4162E-02	.3969E-02	.3402E-02	.2494E-02	.1321E-02

Influence surface of M_x at $x=0$, $y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1620E-02	.8876E-03	-.5725E-03	-.1446E-02	-.1140E-02
0.6	.7920E-02	.5173E-02	-.8918E-05	-.3042E-02	-.2606E-02
0.4	.2712E-01	.1730E-01	.1815E-02	-.5584E-02	-.4839E-02
0.2	.8101E-01	.3926E-01	.1467E-02	-.9928E-02	-.7667E-02
0	.1000E+31	.4622E-01	-.2020E-02	-.1287E-01	-.9156E-02
-0.2	.8101E-01	.3926E-01	.1467E-02	-.9928E-02	-.7667E-02
-0.4	.2712E-01	.1730E-01	.1815E-02	-.5584E-02	-.4839E-02
-0.6	.7920E-02	.5173E-02	-.8918E-05	-.3042E-02	-.2606E-02
-0.8	.1620E-02	.8876E-03	-.5725E-03	-.1446E-02	-.1140E-02

Influence coefficient $s_5 = V_x a/P$ at $x=a$, $y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.3242E-02	.6812E-02	.1090E-01	.1544E-01	.2010E-01	.2431E-01	.2695E-01	.2553E-01	.1645E-01
0.6	.5987E-02	.1239E-01	.1926E-01	.2592E-01	.3111E-01	.3376E-01	.3418E-01	.3296E-01	.2419E-01
0.4	.7907E-02	.1596E-01	.2348E-01	.2807E-01	.2530E-01	.1013E-01	-.1697E-01	-.3635E-01	-.1700E-01
0.2	.8936E-02	.1757E-01	.2417E-01	.2377E-01	.6068E-02	-.4665E-01	-.1587E+00	-.3404E+00	-.4257E+00
0	.9241E-02	.1796E-01	.2388E-01	.2071E-01	-.5374E-02	-.8167E-01	-.2625E+00	-.6741E+00	-.1770E+01

Influence surface of M_{xy} at $x=0$, $y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1128E-01	-.1998E-01	-.2242E-01	-.1524E-01	0.	.1524E-01	.2242E-01	.1998E-01	.1128E-01
0.6	-.2357E-01	-.4391E-01	-.5296E-01	-.3859E-01	0.	.3859E-01	.5296E-01	.4391E-01	.2357E-01
0.4	-.3270E-01	-.6659E-01	-.9320E-01	-.8078E-01	0.	.8078E-01	.9320E-01	.6659E-01	.3270E-01
0.2	-.2671E-01	-.6073E-01	-.1079E+00	-.1454E+00	0.	.1454E+00	.1079E+00	.6073E-01	.2671E-01
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.2671E-01	.6073E-01	.1079E+00	.1454E+00	0.	-.1454E+00	-.1079E+00	-.6073E-01	-.2671E-01
-0.4	.3270E-01	.6659E-01	.9320E-01	.8078E-01	0.	-.8078E-01	-.9320E-01	-.6659E-01	-.3270E-01
-0.6	.2357E-01	.4391E-01	.5296E-01	.3859E-01	0.	-.3859E-01	-.5296E-01	-.4391E-01	-.2357E-01
-0.8	.1128E-01	.1998E-01	.2242E-01	.1524E-01	0.	-.1524E-01	-.2242E-01	-.1998E-01	-.1128E-01

Simply supported rectangular plate ($\nu = 0.3$) $b/a = 1.4$ $\lambda = 3$

Influence coefficient $s_4 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.7079E-03	.6559E-03	.5200E-03	.3449E-03	.1677E-03
0.6	.2210E-02	.2037E-02	.1594E-02	.1041E-02	.5012E-03
0.4	.5251E-02	.4774E-02	.3621E-02	.2290E-02	.1079E-02
0.2	.1004E-01	.8831E-02	.6307E-02	.3799E-02	.1737E-02
0	.1416E-01	.1155E-01	.7786E-02	.4549E-02	.2047E-02
-0.2	.1004E-01	.8831E-02	.6307E-02	.3799E-02	.1737E-02
-0.4	.5251E-02	.4774E-02	.3621E-02	.2290E-02	.1079E-02
-0.6	.2210E-02	.2037E-02	.1594E-02	.1041E-02	.5012E-03
-0.8	.7079E-03	.6559E-03	.5200E-03	.3449E-03	.1677E-03

Influence surface of M_y at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.9349E-02	-.8667E-02	-.6902E-02	-.4628E-02	-.2254E-02
0.6	-.1733E-01	-.1576E-01	-.1200E-01	-.7702E-02	-.3697E-02
0.4	-.1671E-01	-.1398E-01	-.9140E-02	-.5369E-02	-.2545E-02
0.2	.1828E-01	.1993E-01	.1355E-01	.6131E-02	.1980E-02
0	.1000E+31	.9772E-01	.3946E-01	.1554E-01	.5155E-02
-0.2	.1828E-01	.1993E-01	.1355E-01	.6131E-02	.1980E-02
-0.4	-.1671E-01	-.1398E-01	-.9140E-02	-.5369E-02	-.2545E-02
-0.6	-.1733E-01	-.1576E-01	-.1200E-01	-.7702E-02	-.3697E-02
-0.8	-.9349E-02	-.8667E-02	-.6902E-02	-.4628E-02	-.2284E-02

Influence coefficient $s_8 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1155E+01	-.6050E+00	-.9848E-01	.8693E-02	.1637E-01
0.6	-.3207E+00	-.2284E+00	-.7573E-01	.4462E-02	.1680E-01
0.4	-.6330E-01	-.4424E-01	-.7276E-02	.1580E-01	.1523E-01
0.2	.1186E-01	.1494E-01	.2030E-01	.2080E-01	.1321E-01
0	.2311E-01	.2283E-01	.2125E-01	.1698E-01	.9545E-02
-0.2	.1641E-01	.1580E-01	.1385E-01	.1043E-01	.5629E-02
-0.4	.8376E-02	.8005E-02	.6895E-02	.5084E-02	.2706E-02
-0.6	.3273E-02	.3119E-02	.2668E-02	.1952E-02	.1031E-02
-0.8	.9219E-03	.8776E-03	.7484E-03	.5452E-03	.2868E-03

Influence surface of M_x at $x = 0$, $y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.3518E-03	-.6427E-03	-.1195E-02	-.1398E-02	-.9403E-03
0.6	.3024E-02	.1579E-02	-.1215E-02	-.2785E-02	-.2137E-02
0.4	.1798E-01	.1162E-01	.7674E-03	-.4861E-02	-.4161E-02
0.2	.6724E-01	.3520E-01	.1745E-02	-.9268E-02	-.7289E-02
0	.1000E+31	.4610E-01	-.2129E-02	-.1295E-01	-.9198E-02
-0.2	.6724E-01	.3520E-01	.1745E-02	-.9268E-02	-.7289E-02
-0.4	.1798E-01	.1162E-01	.7674E-03	-.4861E-02	-.4161E-02
-0.6	.3024E-02	.1579E-02	-.1215E-02	-.2785E-02	-.2137E-02
-0.8	-.3518E-03	-.6427E-03	-.1195E-02	-.1398E-02	-.9403E-03

Influence coefficient $s_5 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.2992E-02	.6309E-02	.1035E-01	.1508E-01	.2012E-01	.2457E-01	.2672E-01	.2410E-01	.1470E-01
0.6	.5609E-02	.1182E-01	.1894E-01	.2669E-01	.3422E-01	.4034E-01	.4367E-01	.4131E-01	.2734E-01
0.4	.7603E-02	.1558E-01	.2364E-01	.3009E-01	.3179E-01	.2557E-01	.1293E-01	.6232E-02	.1130E-01
0.2	.8694E-02	.1723E-01	.2410E-01	.2476E-01	.9881E-02	-.3590E-01	-.1309E+00	-.2712E+00	-.2867E+00
0	.9006E-02	.1756E-01	.2345E-01	.2039E-01	-.5375E-02	-.8110E-01	-.2606E+00	-.6668E+00	-.1717E+01

Influence surface of M_{xy} at $x = 0$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.7865E-02	-.1351E-01	-.1454E-01	-.9547E-02	0.	.9547E-02	.1454E-01	.1351E-01	.7865E-02
0.6	-.1895E-01	-.3403E-01	-.3895E-01	-.2701E-01	0.	.2701E-01	.3895E-01	.3403E-01	.1895E-01
0.4	-.3117E-01	-.6115E-01	-.8033E-01	-.6436E-01	0.	.6436E-01	.8033E-01	.6115E-01	.3117E-01
0.2	-.2944E-01	-.6566E-01	-.1116E+00	-.1355E+00	0.	.1355E+00	.1116E+00	.6566E-01	.2944E-01
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.2944E-01	.6565E-01	.1116E+00	.1355E+00	0.	-.1355E+00	-.1115E+00	-.6566E-01	-.2944E-01
-0.4	.3117E-01	.6115E-01	.8033E-01	.6436E-01	0.	-.6436E-01	-.8033E-01	-.6115E-01	-.3117E-01
-0.6	.1895E-01	.3403E-01	.3895E-01	.2701E-01	0.	-.2701E-01	-.3895E-01	-.3403E-01	-.1895E-01
-0.8	.7865E-02	.1351E-01	.1454E-01	.9547E-02	0.	-.9547E-02	-.1454E-01	-.1351E-01	-.7865E-02

Simply supported rectangular plate (ν = 0.3) | b/a = 1.6 | λ = 3

Influence coefficient $s_4 = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.8914E-05	.4421E-05	-.5221E-05	-.1183E-04	-.9783E-05
0.6	.7123E-03	.6551E-03	.5085E-03	.3271E-03	.1545E-03
0.4	.3184E-02	.2917E-02	.2251E-02	.1447E-02	.6894E-03
0.2	.8494E-02	.7566E-02	.5510E-02	.3365E-02	.1550E-02
0	.1414E-01	.1152E-01	.7769E-02	.4537E-02	.2039E-02
-0.2	.8494E-02	.7566E-02	.5510E-02	.3365E-02	.1550E-02
-0.4	.3184E-02	.2917E-02	.2251E-02	.1447E-02	.6894E-03
-0.6	.7123E-03	.6551E-03	.5085E-03	.3271E-03	.1545E-03
-0.8	.8914E-05	.4421E-05	-.5221E-05	-.1183E-04	-.9783E-05

Influence surface of M_y at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.4992E-02	-.4685E-02	-.3850E-02	-.2680E-02	-.1358E-02
0.6	-.1256E-01	-.1166E-01	-.9312E-02	-.6273E-02	-.3107E-02
0.4	-.1911E-01	-.1705E-01	-.1249E-01	-.7764E-02	-.3689E-02
0.2	.1780E-02	.4699E-02	.4806E-02	.2257E-02	.5718E-03
0	.1000E+31	.9804E-01	.3973E-01	.1575E-01	.5263E-02
-0.2	.1780E-02	.4699E-02	.4806E-02	.2257E-02	.5718E-03
-0.4	-.1911E-01	-.1705E-01	-.1249E-01	-.7764E-02	-.3689E-02
-0.6	-.1256E-01	-.1166E-01	-.9312E-02	-.6273E-02	-.3107E-02
-0.8	-.4992E-02	-.4685E-02	-.3850E-02	-.2680E-02	-.1358E-02

Influence coefficient $s_6 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.8002E+00	-.4851E+00	-.1124E+00	.5666E-02	.1752E-01
0.6	-.1395E+00	-.1020E+00	-.3219E-01	.1077E-01	.1574E-01
0.4	.6709E-02	.1090E-01	.1854E-01	.2075E-01	.1364E-01
0.2	.2264E-01	.2222E-01	.2039E-01	.1607E-01	.8977E-02
0	.1263E-01	.1211E-01	.1051E-01	.7824E-02	.4181E-02
-0.2	.4160E-02	.3967E-02	.3399E-02	.2491E-02	.1314E-02
-0.4	.4971E-03	.4734E-03	.4040E-03	.2942E-03	.1543E-03
-0.6	-.3969E-03	-.3776E-03	-.3216E-03	-.2342E-03	-.1234E-03
-0.8	-.3052E-03	-.2903E-03	-.2471E-03	-.1798E-03	-.9452E-04

Influence surface of M_x at x = 0, y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1277E-02	-.1279E-02	-.1230E-02	-.1016E-02	-.5821E-03
0.6	-.1058E-02	-.1404E-02	-.2028E-02	-.2151E-02	-.1389E-02
0.4	.6979E-02	.4259E-02	-.8279E-03	-.3673E-02	-.2957E-02
0.2	.4673E-01	.2724E-01	.2050E-02	-.7762E-02	-.6355E-02
0	.1000E+31	.4614E-01	-.2095E-02	-.1293E-01	-.9172E-02
-0.2	.4673E-01	.2724E-01	.2050E-02	-.7762E-02	-.6355E-02
-0.4	.6979E-02	.4259E-02	-.8279E-03	-.3673E-02	-.2957E-02
-0.6	-.1058E-02	-.1404E-02	-.2028E-02	-.2151E-02	-.1389E-02
-0.8	-.1277E-02	-.1279E-02	-.1230E-02	-.1016E-02	-.5821E-03

Influence coefficient $s_5 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1601E-02	.3529E-02	.5989E-02	.8970E-02	.1211E-01	.1463E-01	.1538E-01	.1320E-01	.7672E-02
0.6	.3858E-02	.8360E-02	.1393E-01	.2056E-01	.2765E-01	.3384E-01	.3678E-01	.3315E-01	.2034E-01
0.4	.6437E-02	.1349E-01	.2139E-01	.2944E-01	.3615E-01	.4006E-01	.4141E-01	.4116E-01	.2834E-01
0.2	.8217E-02	.1647E-01	.2367E-01	.2614E-01	.1653E-01	-.1612E-01	-.8077E-01	-.1582E+00	-.1059E+00
0	.8751E-02	.1713E-01	.2294E-01	.2000E-01	-.5312E-02	-.7987E-01	-.2560E+00	-.6484E+00	-.1605E+01

Influence surface of M_{xy} at x = 0, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.2910E-02	-.4862E-02	-.5032E-02	-.3197E-02	0.	.3197E-02	.5032E-02	.4862E-02	.2910E-02
0.6	-.1016E-01	-.1745E-01	-.1880E-01	-.1235E-01	0.	.1235E-01	.1880E-01	.1745E-01	.1016E-01
0.4	-.2483E-01	-.4583E-01	-.5491E-01	-.3981E-01	0.	.3981E-01	.5491E-01	.4583E-01	.2483E-01
0.2	-.3278E-01	-.7021E-01	-.1096E+00	-.1125E+00	0.	.1125E+00	.1096E+00	.7021E-01	.3278E-01
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.3278E-01	.7021E-01	.1096E+00	.1125E+00	0.	-.1125E+00	-.1096E+00	-.7021E-01	-.3278E-01
-0.4	.2483E-01	.4583E-01	.5491E-01	.3981E-01	0.	-.3981E-01	-.5491E-01	-.4583E-01	-.2483E-01
-0.6	.1016E-01	.1745E-01	.1880E-01	.1235E-01	0.	-.1235E-01	-.1880E-01	-.1745E-01	-.1016E-01
-0.8	.2910E-02	.4862E-02	.5032E-02	.3197E-02	0.	-.3197E-02	-.5032E-02	-.4862E-02	-.2910E-02

Simply supported rectangular plate ($\nu = 0.3$)					b/a = 1.8		$\lambda = 3$			
Influence coefficient $s_4 = wD/Pa^2$ at $x=y=0$					Influence surface of M_y at $x=y=0$					
y/b \ x/a	0	0.2	0.4	0.6	0.8	0	0.2	0.4	0.6	0.8
0.8	.2690E-03	.2473E-03	.1916E-03	.1227E-03	.5743E-04	-.7259E-02	-.6784E-02	-.5512E-02	-.3785E-02	-.1901E-02
0.6	.1333E-02	.1229E-02	.9626E-03	.6285E-03	.3016E-03	-.1538E-01	-.1417E-01	-.1111E-01	-.7342E-02	-.3590E-02
0.4	.4127E-02	.3769E-02	.2885E-02	.1841E-02	.8724E-03	-.1880E-01	-.1640E-01	-.1151E-01	-.6969E-02	-.3284E-02
0.2	.9248E-02	.8189E-02	.5906E-02	.3581E-02	.1643E-02	.9072E-02	.1159E-01	.8992E-02	.4183E-02	.1289E-02
0	.1414E-01	.1153E-01	.7769E-02	.4537E-02	.2040E-02	.1000E+31	.9794E-01	.3963E-01	.1569E-01	.5230E-02
-0.2	.9248E-02	.8189E-02	.5906E-02	.3581E-02	.1643E-02	.9072E-02	.1159E-01	.8992E-02	.4183E-02	.1289E-02
-0.4	.4127E-02	.3769E-02	.2885E-02	.1841E-02	.8724E-03	-.1880E-01	-.1640E-01	-.1151E-01	-.6969E-02	-.3284E-02
-0.6	.1333E-02	.1229E-02	.9626E-03	.6285E-03	.3016E-03	-.1538E-01	-.1417E-01	-.1111E-01	-.7342E-02	-.3590E-02
-0.8	.2690E-03	.2473E-03	.1916E-03	.1227E-03	.5743E-04	-.7259E-02	-.6784E-02	-.5512E-02	-.3785E-02	-.1901E-02
Influence coefficient $s_6 = V_y a/P$ at $x=0, y=b$					Influence surface of M_x at $x=0, y=0$					
y/b \ x/a	0	0.2	0.4	0.6	0.8	0	0.2	0.4	0.6	0.8
0.8	-.9595E+00	-.5460E+00	-.1087E+00	.7217E-02	.1711E-01	-.1152E-02	-.1239E-02	-.1361E-02	-.1248E-02	-.7590E-03
0.6	-.2157E+00	-.1571E+00	-.5308E-01	.7245E-02	.1622E-01	.2904E-03	-.4416E-03	-.1843E-02	-.2489E-02	-.1735E-02
0.4	-.1821E-01	-.8860E-02	.9280E-02	.1930E-01	.1458E-01	.1153E-01	.7374E-02	-.1174E-03	-.4212E-02	-.3518E-02
0.2	.2221E-01	.2272E-01	.2271E-01	.1931E-01	.1126E-01	.5604E-01	.3117E-01	.1975E-02	-.8509E-02	-.6833E-02
0	.1882E-01	.1819E-01	.1611E-01	.1226E-01	.6662E-02	.1000E+31	.4612E-01	-.2115E-02	-.1294E-01	-.9189E-02
-0.2	.9242E-02	.8838E-02	.7625E-02	.5634E-02	.2999E-02	.5604E-01	.3117E-01	.1975E-02	-.8509E-02	-.6833E-02
-0.4	.3003E-02	.2863E-02	.2450E-02	.1792E-02	.9477E-03	.1153E-01	.7374E-02	-.1174E-03	-.4212E-02	-.3518E-02
-0.6	.3822E-03	.3640E-03	.3105E-03	.2262E-03	.1187E-03	.2904E-03	-.4416E-03	-.1843E-02	-.2489E-02	-.1735E-02
-0.8	-.2007E-03	-.1910E-03	-.1628E-03	-.1187E-03	-.6280E-04	-.1152E-02	-.1239E-02	-.1361E-02	-.1248E-02	-.7590E-03

Influence coefficient $s_5 = V_x a/P$ at $x=a, y=0$									
y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.2340E-02	.5068E-02	.8444E-02	.1247E-01	.1674E-01	.2031E-01	.2164E-01	.1894E-01	.1122E-01
0.6	.4822E-02	.1031E-01	.1687E-01	.2442E-01	.3227E-01	.3909E-01	.4267E-01	.3933E-01	.2482E-01
0.4	.7059E-02	.1464E-01	.2276E-01	.3028E-01	.3504E-01	.3501E-01	.3132E-01	.2975E-01	.2391E-01
0.2	.8424E-02	.1680E-01	.2382E-01	.2539E-01	.1318E-01	-.2588E-01	-.1050E+00	-.2106E+00	-.1831E+00
0	.8821E-02	.1723E-01	.2304E-01	.2002E-01	-.5541E-02	-.8072E-01	-.2586E+00	-.6584E+00	-.1661E+01

Influence surface of M_{xy} at $x=0, y=0$									
y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.4976E-02	-.8394E-02	-.8818E-02	-.5671E-02	0.	.5671E-02	.8818E-02	.8394E-02	.4976E-02
0.6	-.1422E-01	-.2488E-01	-.2748E-01	-.1845E-01	0.	.1845E-01	.2748E-01	.2488E-01	.1422E-01
0.4	-.2831E-01	-.5376E-01	-.6712E-01	-.5079E-01	0.	.5079E-01	.6712E-01	.5376E-01	.2831E-01
0.2	-.3144E-01	-.6872E-01	-.1118E+00	-.1241E+00	0.	.1241E+00	.1118E+00	.6872E-01	.3144E-01
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.3144E-01	.6872E-01	.1118E+00	.1241E+00	0.	-.1241E+00	-.1118E+00	-.6872E-01	-.3144E-01
-0.4	.2831E-01	.5376E-01	.6712E-01	.5079E-01	0.	-.5079E-01	-.6712E-01	-.5376E-01	-.2831E-01
-0.6	.1422E-01	.2488E-01	.2748E-01	.1845E-01	0.	-.1845E-01	-.2748E-01	-.2488E-01	-.1422E-01
-0.8	.4976E-02	.8394E-02	.8818E-02	.5671E-02	0.	-.5671E-02	-.8818E-02	-.8394E-02	-.4976E-02

Simply supported rectangular plate (ν = 0.3)	b/a = 1.0	λ = 5

Influence coefficient $s_4 = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.3051E-04	.1112E-04	-.2524E-04	-.4307E-04	-.3098E-04
0.6	.3398E-03	.2515E-03	.8162E-04	-.2887E-04	-.4307E-04
0.4	.1287E-02	.9832E-03	.4367E-03	.8162E-04	-.2524E-04
0.2	.3144E-02	.2268E-02	.9832E-03	.2515E-03	.1112E-04
0	.4992E-02	.3144E-02	.1287E-02	.3398E-03	.3051E-04
-0.2	.3144E-02	.2268E-02	.9832E-03	.2515E-03	.1112E-04
-0.4	.1297E-02	.9832E-03	.4367E-03	.8162E-04	-.2524E-04
-0.6	.3398E-03	.2515E-03	.5162E-04	-.2887E-04	-.4307E-04
-0.8	.3051E-04	.1112E-04	-.2524E-04	-.4307E-04	-.3098E-04

Influence surface of M_y at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.6434E-02	-.5475E-02	-.3454E-02	-.1687E-02	-.6178E-03
0.6	-.1446E-01	-.1188E-01	-.7039E-02	-.3336E-02	-.1233E-02
0.4	-.1914E-01	-.1406E-01	-.7492E-02	-.3721E-02	-.1518E-02
0.2	.6270E-02	.8195E-02	.1170E-02	-.1639E-02	-.1230E-02
0	.1000E+31	.5285E-01	.1017E-01	.1627E-03	-.9425E-03
-0.2	.6270E-02	.8195E-02	.1170E-02	-.1639E-02	-.1230E-02
-0.4	-.1914E-01	-.1406E-01	-.7492E-02	-.3721E-02	-.1518E-02
-0.6	-.1446E-01	-.1188E-01	-.7039E-02	-.3336E-02	-.1233E-02
-0.8	-.6434E-02	-.5475E-02	-.3454E-02	-.1687E-02	-.6178E-03

Influence coefficient $s_6 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1354E+01	-.4263E+00	.7572E-01	.5799E-01	.2574E-01
0.6	-.2878E+00	-.1179E+00	.5742E-01	.6787E-01	.3272E-01
0.4	-.6241E-02	.2332E-01	.5717E-01	.5040E-01	.2497E-01
0.2	.3894E-01	.4121E-01	.4009E-01	.2903E-01	.1392E-01
0	.2545E-01	.2405E-01	.1947E-01	.1248E-01	.5603E-02
-0.2	.9170E-02	.8257E-02	.5902E-02	.3170E-02	.1150E-02
-0.4	.9729E-03	.6800E-03	.2315E-04	-.4939E-03	-.4831E-03
-0.6	-.1288E-02	-.1317E-02	-.1325E-02	-.1147E-02	-.6832E-03
-0.8	-.9956E-03	-.9735E-03	-.8871E-03	-.6972E-03	-.3897E-03

Influence surface of M_x at x = 0, y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.9425E-03	-.1230E-02	-.1518E-02	-.1233E-02	-.6178E-03
0.6	.1627E-03	-.1639E-02	-.3721E-02	-.3336E-02	-.1687E-02
0.4	.1017E-01	.1170E-02	-.7492E-02	-.7039E-02	-.3454E-02
0.2	.5285E-01	.8195E-02	-.1406E-01	-.1188E-01	-.5475E-02
0	.1000E+31	.6270E-02	-.1914E-01	-.1446E-01	-.6434E-02
-0.2	.5285E-01	.8195E-02	-.1406E-01	-.1188E-01	-.5475E-02
-0.4	.1017E-01	.1170E-02	-.7492E-02	-.7039E-02	-.3454E-02
-0.6	.1627E-03	-.1639E-02	-.3721E-02	-.3336E-02	-.1687E-02
-0.8	-.9425E-03	-.1230E-02	-.1518E-02	-.1233E-02	-.6178E-03

Influence coefficient $s_5 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.3897E-03	-.6832E-03	-.4831E-03	.1150E-02	.5603E-02	.1392E-01	.2497E-01	.3272E-01	.2574E-01
0.6	-.6972E-03	-.1147E-02	-.4939E-03	.3170E-02	.1248E-01	.2903E-01	.5040E-01	.6787E-01	.5799E-01
0.4	-.8871E-03	-.1325E-02	.2315E-04	.5902E-02	.1947E-01	.4009E-01	.5717E-01	.5742E-01	.7572E-01
0.2	-.9735E-03	-.1317E-02	.6800E-03	.8257E-02	.2405E-01	.4121E-01	.2332E-01	-.1179E+00	-.4263E+00
0	-.9956E-03	-.1288E-02	.9729E-03	.9170E-02	.2545E-01	.3894E-01	-.6241E-02	-.2878E+00	-.1354E+01

Influence surface of M_{xy} at x = 0, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1844E-02	-.4905E-02	-.8047E-02	-.7063E-02	0.	.7063E-02	.8047E-02	.4905E-02	.1844E-02
0.6	-.4905E-02	-.1363E-01	-.2445E-01	-.2373E-01	0.	.2373E-01	.2445E-01	.1363E-01	.4905E-02
0.4	-.8047E-02	-.2445E-01	-.5190E-01	-.6314E-01	0.	.6314E-01	.5190E-01	.2445E-01	.8047E-02
0.2	-.7063E-02	-.2373E-01	-.6314E-01	-.1236E+00	0.	.1236E+00	.6314E-01	.2373E-01	.7063E-02
0	0.	0.	0.	0.	0.	0.	0.	.	0.
-0.2	.7063E-02	.2373E-01	.6314E-01	.1236E+00	0.	-.1236E+00	-.6314E-01	-.2373E-01	-.7063E-02
-0.4	.8047E-02	.2445E-01	.5190E-01	.6314E-01	0.	-.6314E-01	-.5190E-01	-.2445E-01	-.8047E-02
-0.6	.4905E-02	.1363E-01	.2445E-01	.2373E-01	0.	-.2373E-01	-.2445E-01	-.1363E-01	-.4905E-02
-0.8	.1844E-02	.4905E-02	.8047E-02	.7063E-02	0.	-.7063E-02	-.8047E-02	-.4905E-02	-.1844E-02

Simply supported rectangular plate (ν = 0.3) b/a = 1.2 λ = 5

Influence coefficient $s_4 = wD/Pa^2$ at x=y=0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.5917E-04	-.6121E-04	-.6168E-04	-.5162E-04	-.2936E-04
0.6	.6846E-04	.3313E-04	-.3271E-04	-.6599E-04	-.4840E-04
0.4	.7980E-03	.6087E-03	.2551E-03	.2163E-04	-.3778E-04
0.2	.2708E-02	.1991E-02	.8758E-03	.2200E-03	.4726E-05
0	.4995E-02	.3148E-02	.1290E-02	.3422E-03	.3181E-04
-0.2	.2708E-02	.1991E-02	.8758E-03	.2200E-03	.4726E-05
-0.4	.7980E-03	.6087E-03	.2551E-03	.2163E-04	-.3778E-04
-0.6	.6845E-04	.3313E-04	-.3271E-04	-.6599E-04	-.4840E-04
-0.8	-.5917E-04	-.6121E-04	-.6168E-04	-.5162E-04	-.2936E-04

Influence surface of M_y at x=y=0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2834E-02	-.2416E-02	-.1500E-02	-.6758E-03	-.2086E-03
0.6	-.9425E-02	-.7900E-02	-.4799E-02	-.2227E-02	-.7696E-03
0.4	-.1830E-01	-.1428E-01	-.7873E-02	-.3670E-02	-.1381E-02
0.2	-.4160E-02	.3627E-03	-.1210E-02	-.2121E-02	-.1281E-02
0	.1000E+31	.5291E-01	.1022E-01	.1943E-03	-.9270E-03
-0.2	-.4160E-02	.3627E-03	-.1210E-02	-.2121E-02	-.1281E-02
-0.4	-.1830E-01	-.1428E-01	-.7873E-02	-.3670E-02	-.1381E-02
-0.6	-.9425E-02	-.7900E-02	-.4799E-02	-.2227E-02	-.7696E-03
-0.8	-.2834E-02	-.2416E-02	-.1500E-02	-.6758E-03	-.2086E-03

Influence coefficient $s_6 = V_y a/P$ at x=0, y=b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1018E+01	-.3654E+00	.6671E-01	.6435E-01	.2869E-01
0.6	-.1259E+00	-.3916E-01	.5905E-01	.6227E-01	.3065E-01
0.4	.3385E-01	.4194E-01	.4832E-01	.3733E-01	.1821E-01
0.2	.2921E-01	.2795E-01	.2320E-01	.1522E-01	.6955E-02
0	.9202E-02	.8288E-02	.5929E-02	.3190E-02	.1162E-02
-0.2	.2191E-03	.1924E-05	-.4612E-03	-.7576E-03	-.5816E-03
-0.4	-.1361E-02	-.1352E-02	-.1278E-02	-.1043E-02	-.5970E-03
-0.6	-.7911E-03	-.7532E-03	-.6730E-03	-.5100E-03	-.2769E-03
-0.8	-.2417E-03	-.2307E-03	-.1980E-03	-.1453E-03	-.7667E-04

Influence surface of M_x at x=0, y=0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.7304E-03	-.7540E-03	-.7072E-03	-.4939E-03	-.2250E-03
0.6	-.1023E-02	-.1634E-02	-.2292E-02	-.1901E-02	-.9431E-03
0.4	.4276E-02	-.5082E-03	-.5654E-02	-.5291E-02	-.2634E-02
0.2	.3907E-01	.6946E-02	-.1250E-01	-.1091E-01	-.5087E-02
0	.1000E+31	.6293E-02	-.1911E-01	-.1444E-01	-.6422E-02
-0.2	.3907E-01	.6946E-02	-.1250E-01	-.1091E-01	-.5097E-02
-0.4	.4276E-02	-.5082E-03	-.5654E-02	-.5291E-02	-.2634E-02
-0.6	-.1023E-02	-.1634E-02	-.2292E-02	-.1901E-02	-.9431E-03
-0.8	-.7304E-03	-.7540E-03	-.7072E-03	-.4939E-03	-.2250E-03

Influence coefficient $s_5 = V_x a/P$ at x=a, y=0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.3303E-03	-.7825E-03	-.1010E-02	-.4746E-03	.1753E-02	.6359E-02	.1246E-01	.1625E-01	.1226E-01
0.6	-.6851E-03	-.1295E-02	-.1257E-02	.8814E-03	.7303E-02	.1962E-01	.3614E-01	.4821E-01	.3880E-01
0.4	-.8696E-03	-.1425E-02	-.5446E-03	.4238E-02	.1611E-01	.3603E-01	.5855E-01	.7405E-01	.7028E-01
0.2	-.9404E-03	-.1308E-02	.5222E-03	.7708E-02	.2303E-01	.4107E-01	.3203E-01	-.6535E-01	-.1951E+00
0	-.9538E-03	-.1223E-02	.1020E-02	.9124E-02	.2515E-01	.3300E-01	-.9054E-02	-.2982E+00	-.1421E+01

Influence surface of M_{xy} at x=0, y=0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.3419E-03	-.1298E-02	-.2416E-02	-.2173E-02	0.	.2173E-02	.2416E-02	.1298E-02	.3419E-03
0.6	-.2579E-02	-.7358E-02	-.1281E-01	-.1175E-01	0.	.1175E-01	.1281E-01	.7358E-02	.2579E-02
0.4	-.6791E-02	-.2013E-01	-.3987E-01	-.4353E-01	0.	.4353E-01	.3987E-01	.2013E-01	.6791E-02
0.2	-.7713E-02	-.2558E-01	-.6527E-01	-.1134E+00	0.	.1134E+00	.6527E-01	.2558E-01	.7713E-02
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.7713E-02	.2558E-01	.6527E-01	.1134E+00	0.	-.1134E+00	-.6527E-01	-.2558E-01	-.7713E-02
-0.4	.6791E-02	.2013E-01	.3987E-01	.4353E-01	0.	-.4353E-01	-.3987E-01	-.2013E-01	-.6791E-02
-0.6	.2579E-02	.7358E-02	.1281E-01	.1175E-01	0.	-.1175E-01	-.1281E-01	-.7358E-02	-.2579E-02
-0.8	.3419E-03	.1298E-02	.2416E-02	.2173E-02	0.	-.2173E-02	-.2416E-02	-.1298E-02	-.3419E-03

Simply supported rectangular plate (ν = 0.3) | b/a = 1.4 | λ = 5

Influence coefficient $s_4 = wD/Pa^2$ at $x=y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.6049E-04	-.5820E-04	-.5067E-04	-.3738E-04	-.1970E-04
0.6	-.4391E-04	-.5479E-04	-.7178E-04	-.6987E-04	-.4252E-04
0.4	.4516E-03	.3376E-03	.1203E-03	-.2143E-04	-.4505E-04
0.2	.2299E-02	.1715E-02	.7629E-03	.1856E-03	-.2641E-05
0	.4996E-02	.3148E-02	.1291E-02	.3426E-03	.3205E-04
-0.2	.2299E-02	.1715E-02	.7629E-03	.1856E-03	-.2641E-05
-0.4	.4516E-03	.3376E-03	.1203E-03	-.2143E-04	-.4505E-04
-0.6	-.4391E-04	-.5479E-04	-.7178E-04	-.6987E-04	-.4252E-04
-0.8	-.6049E-04	-.5820E-04	-.5067E-04	-.3738E-04	-.1970E-04

Influence surface of M_y at $x=y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.6697E-03	-.5314E-03	-.2363E-03	-.3021E-05	.6107E-04
0.6	-.5255E-02	-.4430E-02	-.2681E-02	-.1178E-02	-.3581E-03
0.4	-.1568E-01	-.1266E-01	-.7253E-02	-.3331E-02	-.1199E-02
0.2	-.1109E-01	-.5567E-02	-.3376E-02	-.2624E-02	-.1354E-02
0	.1000E+31	.9289E-01	.1020E-01	.1822E-03	-.9337E-03
-0.2	-.1109E-01	-.5567E-02	-.3376E-02	-.2624E-02	-.1354E-02
-0.4	-.1568E-01	-.1266E-01	-.7253E-02	-.3331E-02	-.1199E-02
-0.6	-.5255E-02	-.4430E-02	-.2681E-02	-.1178E-02	-.3581E-03
-0.8	-.6697E-03	-.5314E-03	-.2363E-03	-.3021E-05	.6107E-04

Influence coefficient $s_6 = V_y a/P$ at $x=0, y=b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.7606E+00	-.2973E+00	.6079E-01	.6806E-01	.3091E-01
0.6	-.3426E-01	.8650E-02	.5863E-01	.5463E-01	.2709E-01
0.4	.3808E-01	.3877E-01	.3574E-01	.2518E-01	.1196E-01
0.2	.1483E-01	.1362E-01	.1031E-01	.6089E-02	.2520E-02
0	.1020E-02	.7251E-03	.6237E-04	-.4645E-03	-.4666E-03
-0.2	-.1356E-02	-.1347E-02	-.1273E-02	-.1039E-02	-.5951E-03
-0.4	-.6489E-03	-.6247E-03	-.5480E-03	-.4127E-03	-.2231E-03
-0.6	-.8946E-04	-.8478E-04	-.7141E-04	-.5113E-04	-.2641E-04
-0.8	.3847E-04	.3706E-04	.3258E-04	.2465E-04	.1344E-04

Influence surface of M_x at $x=0, y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.3010E-03	-.2793E-03	-.2064E-03	-.1023E-03	-.2629E-04
0.6	-.1006E-02	-.1172E-02	-.1280E-02	-.9659E-03	-.4567E-03
0.4	.1128E-02	-.1376E-02	-.4232E-02	-.3857E-02	-.1936E-02
0.2	.2866E-01	.5415E-02	-.1105E-01	-.9916E-02	-.4686E-02
0	.1000E+31	.6290E-02	-.1912E-01	-.1444E-01	-.6425E-02
-0.2	.2866E-01	.5415E-02	-.1105E-01	-.9916E-02	-.4686E-02
-0.4	.1128E-02	-.1376E-02	-.4232E-02	-.3857E-02	-.1936E-02
-0.6	-.1006E-02	-.1172E-02	-.1280E-02	-.9659E-03	-.4567E-03
-0.8	-.3010E-03	-.2793E-03	-.2064E-03	-.1023E-03	-.2629E-04

Influence coefficient $s_5 = V_x a/P$ at $x=a, y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.2928E-03	-.6739E-03	-.1095E-02	-.1258E-02	-.6310E-03	.1222E-02	.3934E-02	.5758E-02	.4430E-02
0.6	-.5837E-03	-.1211E-02	-.1557E-02	-.6307E-03	.3157E-02	.1106E-01	.2191E-01	.2941E-01	.2298E-01
0.4	-.8095E-03	-.1418E-02	-.9134E-03	.2833E-02	.1279E-01	.3060E-01	.5328E-01	.7199E-01	.6428E-01
0.2	-.9172E-03	-.1305E-02	.3875E-03	.7242E-02	.2221E-01	.4129E-01	.4035E-01	-.2498E-01	-.8465E-01
0	-.9401E-03	-.1197E-02	.1057E-02	.9189E-02	.2533E-01	.3859E-01	-.7025E-02	-.2901E+00	-.1367E+01

Influence surface of M_{xy} at $x=0, y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.3159E-03	.2643E-03	-.8631E-04	-.2553E-03	0.	.2553E-03	.8631E-04	-.2643E-03	-.3159E-03
0.6	-.9533E-03	-.3158E-02	-.5722E-02	-.5188E-02	0.	.5188E-02	.5722E-02	.3158E-02	.9533E-03
0.4	-.5315E-02	-.1538E-01	-.2878E-01	-.2903E-01	0.	.2903E+01	.2878E-01	.1538E-01	.5315E-02
0.2	-.8141E-02	-.2648E-01	-.6454E-01	-.1007E+00	0.	.1007E+00	.6454E-01	.2648E-01	.8141E-02
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.8141E-02	.2648E-01	.6454E-01	.1007E+00	0.	-.1007E+00	-.6454E-01	-.2648E-01	-.8141E-02
-0.4	.5315E-02	.1538E-01	.2878E-01	.2903E-01	0.	-.2903E-01	-.2878E-01	-.1538E-01	-.5315E-02
-0.6	.9533E-03	.3158E-02	.5722E-02	.5188E-02	0.	-.5188E-02	-.5722E-02	-.3158E-02	-.9533E-03
-0.8	-.3159E-03	-.2643E-03	.8631E-04	.2553E-03	0.	-.2553E-03	-.8631E-04	.2643E-03	.3159E-03

Simply supported rectangular plate ($\nu=0.3$)					b/a = 1.6			$\lambda=5$		
	Influence coefficient $s_4=wD/Pa^2$ at x=y=0					Influence surface of M_y at x=y=0				
y/b \ x/a	0	0.2	0.4	0.6	0.8	0	0.2	0.4	0.6	0.8
0.8	-.3609E-04	-.3399E-04	-.2807E-04	-.1952E-04	-.9818E-05	.2082E-03	.2313E-03	.2656E-03	.2479E-03	.1512E-03
0.6	-.7324E-04	-.7420E-04	-.7185E-04	-.5819E-04	-.3245E-04	-.2454E-02	-.2047E-02	-.1172E-02	-.4267E-03	-.7377E-04
0.4	.2172E-03	.1517E-03	.2660E-04	-.4986E-04	-.4835E-04	-.1243E-01	-.1026E-01	-.6057E-02	-.2777E-02	-.9703E-03
0.2	.1923E-02	.1450E-02	.6491E-03	.1499E-03	-.1050E-04	-.1542E-01	-.9712E-02	-.5134E-02	-.3050E-02	-.1410E-02
0	.4996E-02	.3148E-02	.1291E-02	.3425E-03	.3201E-04	.1000E+31	.5288E-01	.1020E-01	.1787E-03	-.9354E-03
-0.2	.1923E-02	.1450E-02	.6491E-03	.1499E-03	-.1050E-04	-.1542E-01	-.9712E-02	-.5134E-02	-.3050E-02	-.1410E-02
-0.4	.2172E-03	.1517E-03	.2660E-04	-.4986E-04	-.4835E-04	-.1243E-01	-.1026E-01	-.6057E-02	-.2777E-02	-.9703E-03
-0.6	-.7324E-04	-.7420E-04	-.7185E-04	-.5819E-04	-.3245E-04	-.2454E-02	-.2047E-02	-.1172E-02	-.4267E-03	-.7377E-04
-0.8	-.3609E-04	-.3399E-04	-.2807E-04	-.1952E-04	-.9818E-05	.2082E-03	.2313E-03	.2656E-03	.2479E-03	.1512E-03
	Influence coefficient $s_6=V_y a/P$ at x=0, y=b					Influence surface of M_x at x=0, y=0				
y/b \ x/a	0	0.2	0.4	0.6	0.8	0	0.2	0.4	0.6	0.8
0.8	-.5615E+00	-.2307E+00	.5780E-01	.6957E-01	.3231E-01	-.3477E-04	-.1889E-04	.1893E-04	.4968E-04	.4319E-04
0.6	.1323E-01	.3315E-01	.5489E-01	.4606E-01	.2273E-01	-.6668E-03	-.6842E-03	-.6264E-03	-.4152E-03	-.1747E-03
0.4	.2921E-01	.2795E-01	.2320E-01	.1522E-01	.6946E-02	-.3898E-03	-.1661E-02	-.3120E-02	-.2726E-02	-.1366E-02
0.2	.4977E-02	.4352E-02	.2815E-02	.1200E-02	.2506E-03	.2077E-01	.3863E-02	-.9710E-02	-.8915E-02	-.4268E-02
0	-.1266E-02	-.1296E-02	-.1306E-02	-.1132E-02	-.6732E-03	.1000E+31	.6288E-02	-.1912E-01	-.1445E-01	-.6426E-02
-0.2	-.7691E-03	-.7421E-03	-.6546E-03	-.4962E-03	-.2689E-03	.2077E-01	.3863E-02	-.9710E-02	-.8915E-02	-.4268E-02
-0.4	-.9042E-04	-.8568E-04	-.7212E-04	-.5157E-04	-.2648E-04	-.3898E-03	-.1661E-02	-.3120E-02	-.2726E-02	-.1366E-02
-0.6	.4757E-04	.4569E-04	.3987E-04	.2989E-04	.1616E-04	-.6668E-03	-.6842E-03	-.6264E-03	-.4152E-03	-.1747E-03
-0.8	.2344E-04	.2240E-04	.1929E-04	.1422E-04	.7556E-05	-.3477E-04	-.1889E-04	.1893E-04	.4968E-04	.4319E-04

Influence coefficient $s_5=V_x a/P$ at x=a, y=0									
y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1742E-03	-.4498E-03	-.8509E-03	-.1254E-02	-.1379E-02	-.9408E-03	.2357E-04	.9075E-03	.8939E-03
0.6	-.4474E-03	-.1002E-02	-.1515E-02	-.1366E-02	.5124E-03	.5003E-02	.1137E-01	.1566E-01	.1212E-01
0.4	-.7407E-03	-.1372E-02	-.1184E-02	.1600E-02	.9589E-02	.2461E-01	.4484E-01	.6176E-01	.5163E-01
0.2	-.9057E-03	-.1326E-02	.2113E-03	.6692E-02	.2120E-01	.4097E-01	.4681E-01	.6864E-02	-.8955E-02
0	-.9423E-03	-.1203E-02	.1050E-02	.9207E-02	.2548E-01	.3926E-01	-.4611E-02	-.2805E+00	-.1310E+01
Influence surface of M_{xy} at x=0, y=0									
y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.4195E-03	.6104E-03	.5154E-03	.2654E-03	0.	-.2654E-03	-.5154E-03	-.6104E-03	-.4195E-03
0.8	-.3169E-04	-.8086E-03	-.1945E-02	-.1892E-02	0.	.1892E-02	.1945E-02	.8086E-03	.3169E-04
0.4	-.3814E-02	-.1092E-01	-.1962E-01	-.1873E-01	0.	.1973E-01	.1962E-01	.1092E-01	.3814E-02
0.2	-.8300E-02	-.2645E-01	-.6158E-01	-.8739E-01	0.	.8739E-01	.6158E-01	.2645E-01	.8300E-02
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.8300E-02	.2645E-01	.6198E-01	.8739E-01	0.	-.8739E-01	-.6158E-01	-.2645E-01	-.8300E-02
-0.4	.3814E-02	.1092E-01	.1962E-01	.1873E-01	0.	-.1873E-01	-.1962E-01	-.1092E-01	-.3814E-02
-0.6	.3169E-04	.8086E-03	.1945E-02	.1892E-02	0.	-.1892E-02	-.1945E-02	-.8086E-03	-.3169E-04
-0.8	-.4195E-03	-.6104E-03	-.5154E-03	-.2654E-03	0.	.2654E-03	.5154E-03	.6104E-03	.4195E-03

Simply supported rectangular plate (ν = 0.3) | b/a = 1.8 | λ = 5

Influence coefficient $s_4 = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1459E-04	-.1350E-04	-.1061E-04	-.6895E-05	-.3238E-05
0.6	-.6589E-04	-.6374E-04	-.5612E-04	-.4180E-04	-.2214E-04
0.4	.6895E-04	.3375E-04	-.3183E-04	-.6504E-04	-.4799E-04
0.2	.1584E-02	.1204E-02	.5390E-03	.1147E-03	-.1822E-04
0	.4996E-02	.3148E-02	.1290E-02	.3424E-03	.3200E-04
-0.2	.1584E-02	.1204E-02	.5390E-03	.1147E-03	-.1822E-04
-0.4	.6895E-04	.3375E-04	-.3183E-04	-.6504E-04	-.4799E-04
-0.6	-.6589E-04	-.6374E-04	-.5612E-04	-.4180E-04	-.2214E-04
-0.8	-.1459E-04	-.1350E-04	-.1061E-04	-.6895E-05	-.3238E-05

Influence surface of M_y at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.3727E-03	.3619E-03	.3230E-03	.2465E-03	.1337E-03
0.6	-.8165E-03	-.6404E-03	-.2685E-03	.1849E-04	.8785E-04
0.4	-.9228E-02	-.7714E-02	-.4644E-02	-.2117E-02	-.7148E-03
0.2	-.1785E-01	-.1236E-01	-.6436E-02	-.3371E-02	-.1442E-02
0	.1000E+31	.5288E-01	.1020E-01	.1783E-03	-.9351E-03
-0.2	-.1785E-01	-.1236E-01	-.6436E-02	-.3371E-02	-.1442E-02
-0.4	-.9228E-02	-.7714E-02	-.4644E-02	-.2117E-02	-.7148E-03
-0.6	-.8165E-03	-.6404E-03	-.2685E-03	.1849E-04	.8785E-04
-0.8	.3727E-03	.3619E-03	.3230E-03	.2465E-03	.1337E-03

Influence coefficient $s_6 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.4072E+00	-.1702E+00	.5698E-01	.6938E-01	.3292E-01
0.6	.3385E-01	.4193E-01	.4832E-01	.3734E-01	.1818E-01
0.4	.1815E-01	.1681E-01	.1301E-01	.7929E-02	.3381E-02
0.2	.2174E-03	.1212E-05	-.4621E-03	-.7595E-03	-.5875E-03
0	-.1132E-02	-.1103E-02	-.9967E-03	-.7758E-03	-.4287E-03
-0.2	-.1966E-03	-.1874E-03	-.1602E-03	-.1169E-03	-.6092E-04
-0.4	.4510E-04	.4340E-04	.3806E-04	.2873E-04	.1574E-04
-0.6	.2188E-04	.2091E-04	.1802E-04	.1330E-04	.7075E-05
-0.8	.1174E-05	.1125E-05	.9750E-06	.7203E-06	.3728E-06

Influence surface of M_x at x = 0, y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.6035E-04	.6670E-04	.7810E-04	.7682E-04	.4920E-04
0.6	-.3364E-03	-.3166E-03	-.2393E-03	-.1188E-03	-.2895E-04
0.4	-.9707E-03	-.1581E-02	-.2241E-02	-.1857E-02	-.9185E-03
0.2	.1477E-01	.2443E-02	-.8504E-02	-.7930E-02	-.3842E-02
0	.1000E+31	.6289E-02	-.1912E-01	-.1445E-01	-.6424E-02
-0.2	.1477E-01	.2443E-02	-.8504E-02	-.7930E-02	-.3842E-02
-0.4	-.9707E-03	-.1581E-02	-.2241E-02	-.1857E-02	-.9185E-03
-0.6	-.3364E-03	-.3166E-03	-.2393E-03	-.1188E-03	-.2895E-04
-0.8	.6035E-04	.6670E-04	.7810E-04	.7682E-04	.4920E-04

Influence coefficient $s_5 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.7137E-04	-.2263E-03	-.5076E-03	-.8776E-03	-.1199E-02	-.1287E-02	-.1053E-02	-.6260E-03	-.2570E-03
0.6	-.3153E-03	-.7609E-03	-.1298E-02	-.1568E-02	-.8970E-03	.1313E-02	.4691E-02	.7067E-02	.5583E-02
0.4	-.6693E-03	-.1305E-02	-.1383E-02	.5287E-03	.6626E-02	.1862E-01	.3525E-01	.4867E-01	.3813E-01
0.2	-.8961E-03	-.1353E-02	.1049E-04	.6066E-02	.1997E-01	.4004E-01	.5117E-01	.3054E-01	.4114E-01
0	-.9464E-03	-.1216E-02	.1030E-02	.9207E-02	.2563E-01	.3998E-01	-.1860E-02	-.2697E+00	-.1254E+01

Influence surface of M_{xy} at x = 0, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.3081E-03	.4858E-03	.4659E-03	.2768E-03	0.	-.2768E-03	-.4659E-03	-.4858E-03	-.3081E-03
0.6	.3762E-03	.2794E-03	-.1956E-03	-.3975E-03	0.	.3975E-03	.1956E-03	-.2794E-03	-.3762E-03
0.4	-.2463E-02	-.7165E-02	-.1262E-01	-.1163E-01	0.	.1163E-01	.1262E-01	.7165E-02	.2463E-02
0.2	-.8206E-02	-.2562E-01	-.5706E-01	-.7465E-01	0.	.7465E-01	.5706E-01	.2562E-01	.8206E-02
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.8206E-02	.2562E-01	.5706E-01	.7465E-01	0.	-.7465E-01	-.5706E-01	-.2562E-01	-.8206E-02
-0.4	.2463E-02	.7165E-02	.1262E-01	.1163E-01	0.	-.1163E-01	-.1262E-01	-.7165E-02	-.2463E-02
-0.6	-.3762E-03	-.2794E-03	.1956E-03	.3975E-03	0.	-.3975E-03	-.1956E-03	.2794E-03	.3762E-03
-0.8	-.3081E-03	-.4858E-03	-.4659E-03	-.2768E-03	0.	.2768E-03	.4659E-03	.4858E-03	.3081E-03

Simply supported rectangular plate ($\nu = 0.3$)	$b/a = 1.0$	$\lambda = 7$

Influence coefficient $s_4 = wD/Pa^2$ at $x=y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.3023E-04	-.2772E-04	-.2010E-04	-.1030E-04	-.3298E-05
0.6	-.2115E-04	-.3056E-04	-.3724E-04	-.2981E-04	-.1030E-04
0.4	.2322E-03	.1293E-03	-.4426E-05	-.3724E-04	-.2010E-04
0.2	.1175E-02	.6719E-03	.1293E-03	-.3056E-04	-.2772E-04
0	.2551E-02	.1175E-02	.2322E-03	-.2115E-04	-.3023E-04
-0.2	.1175E-02	.6719E-03	.1293E-03	-.3056E-04	-.2772E-04
-0.4	.2322E-03	.1293E-03	-.4426E-05	-.3724E-04	-.2010E-04
-0.6	-.2115E-04	-.3056E-04	-.3724E-04	-.2581E-04	-.1030E-04
-0.8	-.3023E-04	-.2772E-04	-.2010E-04	-.1030E-04	-.3298E-05

Influence surface of M_y at $x=y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.6716E-03	-.4086E-03	.1959E-04	.1920E-03	.1342E-03
0.6	-.5255E-02	-.3744E-02	-.1332E-02	-.1025E-03	.1314E-03
0.4	-.1567E-01	-.1043E-01	-.3859E-02	-.8854E-03	-.4919E-04
0.2	-.1107E-01	-.4058E-02	-.2760E-02	-.1218E-02	-.2468E-03
0	.1000E+31	.2868E-01	.1135E-02	-.1015E-02	-.3120E-03
-0.2	-.1107E-01	-.4058E-02	-.2760E-02	-.1218E-02	-.2468E-03
-0.4	-.1567E-01	-.1043E-01	-.3859E-02	-.8854E-03	-.4919E-04
-0.6	-.5255E-02	-.3744E-02	-.1332E-02	-.1025E-03	.1314E-03
-0.8	-.6716E-03	-.4086E-03	.1959E-04	.1920E-03	.1342E-03

Influence coefficient $s_6 = V_y a/P$ at $x=0$, $y=b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.9539E+00	-.1854E+00	.1188E+00	.3897E-01	.7583E-02
0.6	-.3376E-01	.4385E-01	.7860E-01	.3379E-01	.6224E-02
0.4	.5525E-01	.5307E-01	.3613E-01	.1258E-01	.1028E-02
0.2	.2087E-01	.1701E-01	.8065E-02	.5565E-03	-.1583E-02
0	.1467E-02	.5288E-03	-.1266E-02	-.2132E-02	-.1491E-02
-0.2	-.1799E-02	-.1808E-02	-.1693E-02	-.1283E-02	-.6617E-03
-0.4	-.8331E-03	-.7625E-03	-.5715E-03	-.3305E-03	-.1309E-03
-0.6	-.9517E-04	-.7167E-04	-.1774E-04	.2774E-04	.3292E-04
-0.8	.5934E-04	.6149E-04	.6358E-04	.5637E-04	.3406E-04

Influence surface of M_x at $x=0$, $y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.3120E-03	-.2468E-03	-.4919E-04	.1314E-03	.1342E-03
0.6	-.1015E-02	-.1218E-02	-.8854E-03	-.1025E-03	.1920E-03
0.4	.1139E-02	-.2760E-02	-.3859E-02	-.1332E-02	.1959E-04
0.2	.2868E-01	-.4058E-02	-.1043E-01	-.3744E-02	-.4086E-03
0	.1000E+31	-.1107E-01	-.1567E-01	-.5255E-02	-.6716E-03
-0.2	.2868E-01	-.4058E-02	-.1043E-01	-.3744E-02	-.4086E-03
-0.4	.1135E-02	-.2760E-02	-.3859E-02	-.1332E-02	.1959E-04
-0.6	-.1015E-02	-.1218E-02	-.8854E-03	-.1025E-03	.1920E-03
-0.8	-.3120E-03	-.2468E-03	-.4919E-04	.1314E-03	.1342E-03

Influence coefficient $s_5 = V_x a/P$ at $x=a$, $y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.3406E-04	.3292E-04	-.1309E-03	-.6617E-03	-.1491E-02	-.1583E-02	.1028E-02	.6224E-02	.7583E-02
0.6	.5637E-04	.2774E-04	-.3305E-03	-.1283E-02	-.2132E-02	.5565E-03	.1258E-01	.3379E-01	.3897E-01
0.4	.6358E-04	-.1774E-04	-.5715E-03	-.1693E-02	-.1266E-02	.8065E-02	.3613E-01	.7860E-01	.1188E+00
0.2	.6149E-04	-.7167E-04	-.7625E-03	-.1808E-02	.5288E-03	.1701E-01	.5307E-01	.4385E-01	-.1854E+00
0	.5934E-04	-.9517E-04	-.8331E-03	-.1799E-02	.1467E-02	.2087E-01	.5525E-01	-.3376E-01	-.9539E+00

Influence surface of M_{xy} at $x=0$, $y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.6014E-03	.8783E-03	.4456E-03	-.1901E-03	0.	.1901E-03	-.4456E-03	-.8783E-03	-.6014E-03
0.6	.8783E-03	.2788E-03	-.3263E-02	-.5888E-02	0.	.5888E-02	.3263E-02	-.2798E-03	-.8783E-03
0.4	.4456E-03	-.3263E-02	-.1752E-01	-.3198E-01	0.	.3198E-01	.1752E-01	.3263E-02	-.4456E-03
0.2	-.1901E-03	-.5888E-02	-.3198E-01	-.9202E-01	0.	.9202E-01	.3195E-01	.5888E-02	.1901E-03
0	0.		0.	0.	0.	0.	0.	0.	0.
-0.2	.1701E-03	.5888E-02	.3198E-01	.9202E-01	0.	-.9202E-01	-.3198E-01	-.5889E-02	-.1901E-03
-0.4	-.4456E-03	.3263E-02	.1752E-01	.3198E-01	0.	-.3198E-01	-.1752E-01	-.3263E-02	.4456E-03
-0.6	-.8783E-03	-.2788E-03	.3263E-02	.5888E-02	0.	-.5888E-02	-.3263E-02	.2788E-03	.8783E-03
-0.8	-.6014E-03	-.8783E-03	-.4456E-03	.1901E-03	0.	-.1901E-03	.4456E-03	.8783E-03	.6014E-03

Simply supported rectangular plate (ν=0.3) | b/a = 1.2 | λ =.7

Influence coefficient $s_4 = wD/Pa^2$ at x=y=0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1315E-04	-.1125E-04	-.6694E-05	-.2258E-05	-.9640E-07
0.6	-.3633E-04	-.3498E-04	-.2757E-04	-.1498E-04	-.4961E-05
0.4	.7750E-04	.3093E-04	-.2793E-04	-.3399E-04	-.1595E-04
0.2	.9114E-03	.5325E-03	.9693E-04	-.3316E-04	-.2658E-04
0	.2551E-02	.1175E-02	.2321E-03	-.2121E-04	-.3025E-04
-0.2	.9114E-03	.5325E-03	.9693E-04	-.3316E-04	-.2658E-04
-0.4	.7750E-04	.3093E-04	-.2793E-04	-.3399E-04	-.1595E-04
-0.6	-.3633E-04	-.3498E-04	-.2757E-04	-.1498E-04	-.4961E-05
-0.8	-.1315E-04	-.1125E-04	-.6694E-05	-.2258E-05	-.9640E-07

Influence surface of M_y at x=y=0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.3216E-03	.3309E-03	.3094E-03	.2187E-03	.1029E-03
0.6	-.1686E-02	-.1132E-02	-.2286E-03	.1843E-03	.1646E-03
0.4	-.1112E-01	-.7748E-02	-.2910E-02	-.5671E-03	.3225E-04
0.2	-.1657E-01	-.8275E-02	-.3539E-02	-.1233E-02	-.2190E-03
0	.1000E+31	.2868E-01	.1132E-02	-.1017E-02	-.3126E-03
-0.2	-.1657E-01	-.8275E-02	-.3539E-02	-.1233E-02	-.2190E-03
-0.4	-.1112E-01	-.7748E-02	-.2910E-02	-.5671E-03	.3225E-04
-0.6	-.1686E-02	-.1132E-02	-.2286E-03	.1843E-03	.1646E-03
-0.8	.3215E-03	.3309E-03	.3094E-03	.2187E-03	.1029E-03

Influence coefficient $s_6 = V_y a/P$ at x=0, y=b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.6225E+00	-.1215E+00	.1112E+00	.4178E-01	.8015E-02
0.6	.4018E-01	.6333E-01	.6119E-01	.2498E-01	.4106E-02
0.4	.3501E-01	.3000E-01	.1673E-01	.3965E-02	-.9586E-03
0.2	.3591E-02	.2243E-02	-.4541E-03	-.2050E-02	-.1627E-02
0	-.1797E-02	-.1808E-02	-.1693E-02	-.1284E-02	-.6625E-03
-0.2	-.6295E-03	-.5679E-03	-.4070E-03	-.2171E-03	-.7615E-04
-0.4	.1335E-04	.2449E-04	.4751E-04	.5905E-04	.4181E-04
-0.6	.5429E-04	.5295E-04	.4792E-04	.3733E-04	.2065E-04
-0.8	.1184E-04	.1101E-04	.8770E-05	.5809E-05	.2787E-05

Influence surface of M_x at x=0, y=0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.5112E-05	.3543E-04	.9684E-04	.1226E-03	.8134E-04
0.6	-.5430E-03	-.4906E-03	-.2049E-03	.1284E-03	.1811E-03
0.4	-.7081E-03	-.2168E-02	-.2296E-02	-.6799E-03	.1237E-03
0.2	.1819E-01	-.3589E-02	-.8874E-02	-.3230E-02	-.3183E-03
0	.1000E+31	-.1108E-01	-.1567E-01	-.5256E-02	-.6725E-03
-0.2	.1819E-01	-.3589E-02	-.8874E-02	-.3230E-02	-.3183E-03
-0.4	-.7081E-03	-.2168E-02	-.2296E-02	-.6799E-03	.1237E-03
-0.6	-.5430E-03	-.4906E-03	-.2049E-03	.1284E-03	.1811E-03
-0.8	.5112E-05	.3543E-04	.9684E-04	.1226E-03	.8134E-04

Influence coefficient $s_5 = V_x a/P$ at x=a, y=0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.3117E-04	.5731E-04	.4594E-05	-.2973E-03	-.9773E-03	-.1760E-02	-.1816E-02	-.7000E-03	.3172E-03
0.6	.5408E-04	.6476E-04	-.1517E-03	-.8878E-03	-.1977E-02	-.1657E-02	.3366E-02	.1291E-01	.1539E-01
0.4	.6253E-04	.9947E-05	-.4549E-03	-.1524E-02	-.1726E-02	.4665E-02	.2645E-01	.6281E-01	.7826E-01
0.2	.5981E-04	-.6570E-04	-.7314E-03	-.1781E-02	.2473E-03	.1559E-01	.5149E-01	.6473E-01	-.7737E-02
0	.5674E-04	-.1003E-03	-.8360E-03	-.1782E-02	.1522E-02	.2082E-01	.5391E-01	-.4296E-01	-.1021E+01

Influence surface of M_{xy} at x=0, y=0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.3478E-03	.6411E-03	.6853E-03	.4080E-03	.	-.4080E-03	-.6853E-03	-.6411E-03	-.3478E-03
0.6	.7960E-03	.9840E-03	-.1036E-03	-.1209E-02	0.	.1209E-02	.1036E-03	-.9840E-03	-.7960E-03
0.4	.6896E-03	-.1550E-02	-.1035E-01	-.1757E-01	0.	.1757E-01	.1035E-01	.1550E-02	-.6896E-03
0.2	-.1183E-03	-.5921E-02	-.3131E-01	-.7976E-01	0.	.7976E-01	.3131E-01	.5921E-02	.1183E-03
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.1183E-03	.5921E-02	.3131E-01	.7976E-01	0.	-.7976E-01	-.3131E-01	-.5921E-02	-.1183E-03
-0.4	-.6896E-03	.1550E-02	.1035E-01	.1757E-01	0.	-.1757E-01	-.1035E-01	-.1550E-02	.6896E-03
-0.6	-.7960E-03	-.9840E-03	.1036E-03	.1209E-02	0.	-.1209E-02	-.1036E-03	.9840E-03	.7960E-03
-0.8	-.3478E-03	-.6411E-03	-.6853E-03	-.4080E-03	0.	.4080E-03	.6853E-03	.6411E-03	.3478E-03

Simply supported rectangular plate ($\nu = 0.3$) $b/a = 1.4$ $\lambda = 7$

Influence coefficient $s_4 = wD/Pa^2$ at $x=y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2132E-05	-.1493E-05	-.1227E-06	.8736E-06	.8330E-06
0.6	-.2539E-04	-.2260E-04	-.1513E-04	-.6696E-05	-.1564E-05
0.4	-.3941E-06	-.1800E-04	-.3629E-04	-.2844E-04	-.1188E-04
0.2	.6864E-03	.4055E-03	.6602E-04	-.3530E-04	-.2523E-04
0	.2551E-02	.1175E-02	.2321E-03	-.2121E-04	-.3026E-04
-0.2	.6864E-03	.4055E-03	.6602E-04	-.3530E-04	-.2523E-04
-0.4	-.3941E-06	-.1800E-04	-.3629E-04	-.2844E-04	-.1188E-04
-0.6	-.2539E-04	-.2260E-04	-.1513E-04	-.6696E-05	-.1564E-05
-0.8	-.2132E-05	-.1493E-05	-.1227E-06	.8736E-06	.8380E-05

Influence surface of M_y at $x=y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2925E-03	.2660E-03	.1954E-03	.1101E-03	.4302E-04
0.6	-.1206E-03	.2528E-04	.2350E-03	.2575E-03	.1421E-03
0.4	-.6945E-02	-.4938E-02	-.1820E-02	-.2446E-03	.1013E-03
0.2	-.1875E-01	-.1062E-01	-.4031E-02	-.1206E-02	-.1831E-03
0	.1000E+31	.2968E-01	.1133E-02	-.1016E-02	-.3124E-03
-0.2	-.1875E-01	-.1062E-01	-.4031E-02	-.1206E-02	-.1831E-03
-0.4	-.6945E-02	-.4938E-02	-.1820E-02	-.2446E-03	.1013E-03
-0.6	-.1206E-03	.2528E-04	.2350E-03	.2575E-03	.1421E-03
-0.8	.2925E-03	.2660E-03	.1954E-03	.1101E-03	.4302E-04

Influence coefficient $s_6 = V_y a/P$ at $x=0, y=b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.3869E+00	-.6290E-01	.1033E+00	.4213E-01	.8064E-02
0.6	.5725E-01	.5941E-01	.4406E-01	.1638E-01	.1968E-02
0.4	.1508E-01	.1194E-01	.4953E-02	-.5428E-03	-.1720E-02
0.2	-.1507E-02	-.1715E-02	-.1978E-02	-.1749E-02	-.9874E-03
0	-.8349E-03	-.7642E-03	-.5730E-03	-.3317E-03	-.1316E-03
-0.2	.1344E-04	.2457E-04	.4757E-04	.5910E-04	.4185E-04
-0.4	.4595E-04	.4442E-04	.3933E-04	.2987E-04	.1620E-04
-0.6	.3042E-05	.2523E-05	.1618E-05	.6047E-06	.5351E-07
-0.8	-.2469E-05	-.2414E-05	-.2201E-05	-.1733E-05	-.9691E-06

Influence surface of M_x at $x=0, y=0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.5531E-04	.6192E-04	.6817E-04	.5840E-04	.3207E-04
0.6	-.1648E-03	-.1058E-03	.4675E-04	.1616E-03	.1316E-03
0.4	-.1070E-02	-.1515E-02	-.1250E-02	-.2466E-03	.1798E-03
0.2	.1106E-01	-.3345E-02	-.7397E-02	-.2709E-02	-.2250E-03
0	.1000E+31	-.1107E-01	-.1567E-01	-.5256E-02	-.6720E-03
-0.2	.1106E-01	-.3345E-02	-.7397E-02	-.2709E-02	-.2250E-03
-0.4	-.1070E-02	-.1515E-02	-.1250E-02	-.2466E-03	.1798E-03
-0.6	-.1648E-03	-.1058E-03	.4675E-04	.1616E-03	.1316E-03
-0.8	.5631E-04	.6192E-04	.6817E-04	.5840E-04	.3207E-04

Influence coefficient $s_8 = V_x a/P$ at $x=a, y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.2004E-04	.5103E-04	.6546E-04	-.3267E-04	-.3697E-03	-.9292E-03	-.1400E-02	-.1364E-02	-.7983E-03
0.6	.4393E-04	.7468E-04	-.2782E-04	-.5122E-03	-.1467E-02	-.2148E-02	-.8483E-03	.2745E-02	.4279E-02
0.4	.6010E-04	.3253E-04	-.3436E-03	-.1328E-02	-.2024E-02	.1677E-02	.1660E-01	.4270E-01	.5079E-01
0.2	.6057E-04	-.5433E-04	-.6954E-03	-.1775E-02	-.1570E-03	.1378E-01	.4860E-01	.7484E-01	.5907E-01
0	.5723E-04	-.9941E-04	-.8378E-03	-.1798E-02	.1481E-02	.2087E-01	.5501E-01	-.3551E-01	-.9662E+00

Influence surface of M_{xy} at $x=0, y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1214E-03	.2573E-03	.3268E-03	.2327E-03	0.	-.2327E-03	-.3268E-03	-.2573E-03	-.1214E-03
0.6	.5695E-03	.9368E-03	.7483E-03	.2098E-03	0.	-.2098E-03	-.7483E-03	-.9368E-03	-.5695E-03
0.4	.8398E-03	-.2056E-03	-.5114E-02	-.8747E-02	0.	.8747E-02	.5114E-02	.2056E-03	-.8388E-03
0.2	-.3774E-05	-.5582E-02	-.2894E-01	-.6624E-01	0.	.6624E-01	.2894E-01	.5592E-02	.3774E-05
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	.3774E-05	.5582E-02	.2894E-01	.6624E-01	0.	-.6624E-01	-.2894E-01	-.5582E-02	-.3774E-05
-0.4	-.8388E-03	.2056E-03	.5114E-02	.8747E-02	0.	-.8747E-02	-.5114E-02	-.2056E-03	.8388E-03
-0.6	-.5695E-03	-.9368E-03	-.7483E-03	-.2098E-03	0.	.2098E-03	.7483E-03	.9368E-03	.5695E-03
-0.8	-.1214E-03	-.2573E-03	-.3268E-03	-.2327E-03	0.	.2327E-03	.3268E-03	.2573E-03	.1214E-03

Simply supported rectangular plate (ν = 0.3) b/a = 1.6 λ = 7

Influence coefficient $s_4 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1054E-05	.1136E-05	.1239E-05	.1112E-05	.6595E-06
0.6	-.1218E-04	-.1031E-04	-.5899E-05	-.1691E-05	.1937E-06
0.4	-.3100E-04	-.3505E-04	-.3444E-04	-.2166E-04	-.8113E-05
0.2	.5001E-03	.2952E-03	.3826E-04	-.3674E-04	-.2367E-04
0	.2551E-02	.1175E-02	.2321E-03	-.2121E-04	-.3027E-04
-0.2	.5001E-03	.2952E-03	.3826E-04	-.3674E-04	-.2367E-04
-0.4	-.3100E-04	-.3505E-04	-.3444E-04	-.2166E-04	-.8113E-05
-0.6	-.1218E-04	-.1031E-04	-.5899E-05	-.1691E-05	.1937E-06
-0.8	.1054E-05	.1136E-05	.1239E-05	.1112E-05	.6595E-06

Influence surface of M_y at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1122E-03	.9730E-04	.6177E-04	.2618E-04	.5862E-05
0.6	.3350E-03	.3401E-03	.3095E-03	.2121E-03	.9680E-04
0.4	-.3786E-02	-.2681E-02	-.8885E-03	.1351E-04	.1455E-03
0.2	-.1882E-01	-.1149E-01	-.4228E-02	-.1138E-02	-.1425E-03
0	.1000E+31	.2868E-01	.1133E-02	-.1016E-02	-.3124E-03
-0.2	-.1882E-01	-.1149E-01	-.4228E-02	-.1138E-02	-.1425E-03
-0.4	-.3786E-02	-.2681E-02	-.8885E-03	.1351E-04	.1455E-03
-0.6	.3350E-03	.3401E-03	.3095E-03	.2121E-03	.9680E-04
-0.8	.1122E-03	.9730E-04	.6177E-04	.2618E-04	.5862E-05

Influence coefficient $s_6 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2217E+00	-.1515E-01	.9522E-01	.4057E-01	.7742E-02
0.6	.4981E-01	.4550E-01	.2886E-01	.9226E-02	.2204E-03
0.4	.3591E-02	.2243E-02	-.4538E-03	-.2049E-02	-.1627E-02
0.2	-.1519E-02	-.1454E-02	-.1225E-02	-.8346E-03	-.3960E-03
0	-.9764E-04	-.7416E-04	-.2017E-04	.2571E-04	.3164E-04
-0.2	.5418E-04	.5284E-04	.4781E-04	.3720E-04	.2049E-04
-0.4	.3078E-05	.2657E-05	.1647E-05	.6210E-06	.4428E-07
-0.6	-.2083E-05	-.2028E-05	-.1828E-05	-.1422E-05	-.7879E-06
-0.8	-.1221E-06	-.1145E-06	-.9374E-07	-.6431E-07	-.3105E-07

Influence surface of M_x at $x = 0$, $y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2934E-04	.2843E-04	.2415E-04	.1570E-04	.6625E-05
0.6	.1113E-04	.4036E-04	.9892E-04	.1217E-03	.7960E-04
0.4	-.8707E-03	-.9336E-03	-.5917E-03	.5190E-05	.1944E-03
0.2	.6257E-02	-.3175E-02	-.6056E-02	-.2209E-02	-.1351E-03
0	.1000E+31	-.1107E-01	-.1567E-01	-.5255E-02	-.6715E-03
-0.2	.6267E-02	-.3175E-02	-.6056E-02	-.2209E-02	-.1351E-03
-0.4	-.8707E-03	-.9336E-03	-.5917E-03	.5190E-05	.1944E-03
-0.6	.1113E-04	.4036E-04	.9892E-04	.1217E-03	.7960E-04
-0.8	.2934E-04	.2843E-04	.2415E-04	.1570E-04	.6625E-05

Influence coefficient $s_5 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.8485E-05	.3022E-04	.6179E-04	.6373E-04	-.3592E-04	-.2746E-03	-.5574E-03	-.6675E-03	-.4548E-03
0.6	.3200E-04	.6859E-04	.4126E-04	-.2295E-03	-.9038E-03	-.1741E-02	-.1902E-02	-.8770E-03	.1924E-03
0.4	.5676E-04	.9158E-04	-.2355E-03	-.1096E-02	-.2074E-02	-.4167E-03	.8772E-02	.2561E-01	.2893E-01
0.2	.6164E-04	-.4086E-04	-.6517E-03	-.1752E-02	-.5602E-03	.1181E-01	.4459E-01	.7837E-01	.9486E-01
0	.5786E-04	-.9783E-04	-.8380E-03	-.1814E-02	.1430E-02	.2090E-01	.5616E-01	-.2736E-01	-.9117E+00

Influence surface of M_{xy} at $x = 0$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1397E-04	.4951E-04	.8261E-04	.6765E-04	0.	-.6765E-04	-.8261E-04	-.4951E-04	-.1397E-04
0.6	.3335E-03	.6260E-03	.6806E-03	.4106E-03	0.	-.4106E-03	-.6805E-03	-.6260E-03	-.3335E-03
0.4	.8668E-03	.6172E-03	-.1850E-02	-.3773E-02	0.	.3773E-02	.1850E-02	-.6172E-03	-.8668E-03
0.2	.1369E-03	-.4966E-02	-.2551E-01	-.5328E-01	0.	.5328E-01	.2551E-01	.4966E-02	-.1369E-03
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	-.1369E-03	.4966E-02	.2551E-01	.5328E-01	0.	-.5328E-01	-.2551E-01	-.4966E-02	.1369E-03
-0.4	-.8668E-03	-.6172E-03	.1850E-02	.3773E-02	0.	-.3773E-02	-.1850E-02	.6172E-03	.8668E-03
-0.6	-.3335E-03	-.6260E-03	-.6806E-03	-.4106E-03	0.	.4106E-03	.6806E-03	.6260E-03	.3335E-03
-0.8	-.1397E-04	-.4951E-04	-.8261E-04	-.6765E-04	0.	.6785E-04	.8261E-04	.4951E-04	.1397E-04

Simply supported rectangular plate ($\nu = 0.3$) | $b/a = 1.8$ | $\lambda = 7$

Influence coefficient $s_4 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.9850E-06	.9423E-06	.8060E-06	.5783E-06	.2940E-06
0.6	-.3661E-05	-.2801E-05	-.9073E-06	.5930E-06	.8102E-06
0.4	-.3615E-04	-.3482E-04	-.2746E-04	-.1492E-04	-.4934E-05
0.2	.3500E-03	.2034E-03	.1470E-04	-.3735E-04	-.2192E-04
0	.2551E-02	.1175E-02	.2321E-03	-.2121E-04	-.3025E-04
-0.2	.3500E-03	.2034E-03	.1470E-04	-.3735E-04	-.2192E-04
-0.4	-.3615E-04	-.3482E-04	-.2746E-04	-.1492E-04	-.4934E-05
-0.6	-.3661E-05	-.2801E-05	-.9073E-06	.5930E-06	.8102E-06
-0.8	.9850E-06	.9423E-06	.8060E-06	.5783E-06	.2940E-06

Influence surface of M_y at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1470E-04	.1029E-04	.7637E-06	-.6265E-05	-.6066E-05
0.6	.3219E-03	.2961E-03	.2230E-03	.1288E-03	.5139E-04
0.4	-.1698E-02	-.1143E-02	-.2388E-03	.1765E-03	.1614E-03
0.2	-.1761E-01	-.1131E-01	-.4156E-02	-.1032E-02	-.9837E-04
0	.1000E+31	.2868E-01	.1133E-02	-.1016E-02	-.3122E-03
-0.2	-.1761E-01	-.1131E-01	-.4156E-02	-.1032E-02	-.9837E-04
-0.4	-.1698E-02	-.1143E-02	-.2388E-03	.1765E-03	.1614E-03
-0.6	.3219E-03	.2961E-03	.2230E-03	.1288E-03	.5139E-04
-0.8	.1470E-04	.1029E-04	.7637E-06	-.6265E-05	-.6066E-05

Influence coefficient $s_6 = V_y a/P$ at $x = 0,\ y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1056E+00	.2021E-01	.8702E-01	.3762E-01	.7105E-02
0.6	.3501E-01	.3000E-01	.1673E-01	.3962E-02	-.9596E-03
0.4	-.9579E-03	-.1339E-02	-.1952E-02	-.1944E-02	-.1158E-02
0.2	-.6296E-03	-.5679E-03	-.4070E-03	-.2167E-03	-.7475E-04
0	.6541E-04	.6695E-04	.6750E-04	.5849E-04	.3472E-04
-0.2	.1042E-04	.9630E-05	.7543E-05	.4849E-05	.2201E-05
-0.4	-.2365E-05	-.2312E-05	-.2109E-05	-.1660E-05	-.9314E-06
-0.6	-.1176E-06	-.1101E-06	-.8972E-07	-.6095E-07	-.2907E-07
-0.8	.7125E-07	.6856E-07	.6008E-07	.4522E-07	.2433E-07

Influence surface of M_x at $x = 0,\ y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.6309E-05	.5231E-05	.2394E-05	-.6011E-06	-.1540E-05
0.6	.5448E-04	.6371E-04	.7707E-04	.7030E-04	.3994E-04
0.4	-.5455E-03	-.4933E-03	-.2079E-03	.1255E-03	.1798E-03
0.2	.3123E-02	-.2992E-02	-.4876E-02	-.1747E-02	-.5273E-04
0	.1000E+31	-.1107E-01	-.1567E-01	-.5255E-02	-.6710E-03
-0.2	.3123E-02	-.2992E-02	-.4876E-02	-.1747E-02	-.5273E-04
-0.4	-.5455E-03	-.4933E-03	-.2079E-03	.1255E-03	.1798E-03
-0.6	.5948E-04	.6371E-04	.7707E-04	.7030E-04	.3994E-04
-0.8	.6309E-05	.5231E-05	.2394E-05	-.6011E-06	-.1540E-05

Influence coefficient $s_5 = V_x a/P$ at $x = a,\ y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1162E-05	.1163E-04	.3552E-04	.6208E-04	.6060E-04	.9120E-06	-.1019E-03	-.1712E-03	-.1300E-03
0.6	.2091E-04	.5465E-04	.6889E-04	-.4914E-04	-.4429E-03	-.1079E-02	-.1564E-02	-.1436E-02	-.7396E-03
0.4	.5199E-04	.6498E-04	-.1378E-03	-.8456E-03	-.1907E-02	-.1606E-02	.3387E-02	.1329E-01	.1411E-01
0.2	.6246E-04	-.2664E-04	-.6022E-03	-.1712E-02	-.9371E-03	.9781E-02	.3978E-01	.7678E-01	.1097E+00
0	.5844E-04	-.9600E-04	-.8376E-03	-.1831E-02	.1365E-02	.2089E-01	.5726E-01	-.1907E-01	-.8603E+00

Influence surface of M_{xy} at $x = 0,\ y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.1396E-04	-.1405E-04	-.3187E-05	.3645E-05	0.	-.3645E-05	.3187E-05	.1405E-04	.1396E-04
0.6	.1554E-03	.3230E-03	.4009E-03	.2794E-03	0.	-.2794E-03	-.4009E-03	-.3230E-03	-.1554E-03
0.4	.7915E-03	.9709E-03	-.1162E-03	-.1216E-02	0.	.1216E-02	.1162E-03	-.9709E-03	-.7915E-03
0.2	.2908E-03	-.4166E-02	-.2156E-01	-.4176E-01	0.	.4176E-01	.2156E-01	.4166E-02	-.2908E-03
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	-.2908E-03	.4166E-02	.2156E-01	.4176E-01	0.	-.4176E-01	-.2156E-01	-.4166E-02	.2908E-03
-0.4	-.7915E-03	-.9709E-03	.1162E-03	.1216E-02	0.	-.1216E-02	-.1162E-03	.9709E-03	.7915E-03
-0.6	-.1554E-03	-.3230E-03	-.4009E-03	-.2794E-03	0.	.2794E-03	.4009E-03	.3230E-03	.1554E-03
-0.8	.1396E-04	.1405E-04	.3187E-05	-.3645E-05	0.	.3645E-05	-.3187E-05	-.1405E-04	-.1396E-04

Simply supported rectangular plate ($\nu = 0.3$) | $b/a = 1.0$ | $\lambda = 11$

Influence coefficient $s_4 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.3589E-06	.4522E-06	.4489E-06	.2084E-06	.2577E-07
0.6	-.5615E-05	-.3744E-05	-.6074E-06	.4706E-06	.2084E-06
0.4	-.1161E-04	-.1475E-04	-.7983E-05	-.6074E-06	.4489E-06
0.2	.2123E-03	.5455E-04	-.1475E-04	-.3744E-05	.4522E-06
0	.1033E-02	.2123E-03	-.1161E-04	-.5615E-05	.3589E-06
-0.2	.2123E-03	.5455E-04	-.1475E-04	-.3744E-05	.4522E-06
-0.4	-.1161E-04	-.1475E-04	-.7983E-05	-.6074E-06	.4489E-06
-0.6	-.5615E-05	-.3744E-05	-.6074E-06	.4706E-06	.2084E-06
-0.8	.3589E-06	.4522E-06	.4489E-06	.2084E-06	.2577E-07

Influence surface of M_y at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1344E-03	.9523E-04	.2629E-04	-.5990E-05	-.5784E-05
0.6	.3100E-03	.3263E-03	.2003E-03	.4330E-04	-.3800E-05
0.4	-.4172E-02	-.1775E-02	.4212E-04	.1187E-03	.1445E-04
0.2	-.1891E-01	-.6650E-02	-.9043E-03	.6615E-04	.3130E-04
0	.1000E+31	.6823E-02	-.9151E-03	-.3320E-05	.3433E-04
-0.2	-.1891E-01	-.6650E-02	-.9043E-03	.6615E-04	.3130E-04
-0.4	-.4172E-02	-.1775E-02	.4212E-04	.1187E-03	.1445E-04
-0.6	.3100E-03	.3263E-03	.2003E-03	.4330E-04	-.3800E-05
-0.8	.1344E-03	.9523E-04	.2629E-04	-.5990E-05	-.5784E-05

Influence coefficient $s_6 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2708E+00	.8534E-01	.5946E-01	-.3809E-03	-.1077E-02
0.6	.8508E-01	.5971E-01	.1166E-01	-.2958E-02	-.9670E-03
0.4	.6836E-02	.1891E-02	-.3116E-02	-.1894E-02	-.2056E-03
0.2	-.2621E-02	-.2293E-02	-.1243E-02	-.2066E-03	.1039E-03
0	-.2414E-03	-.1336E-03	.5122E-04	.1096E-03	.5634E-04
-0.2	.9406E-04	.8703E-04	.6192E-04	.2612E-04	.3226E-05
-0.4	.9499E-05	.6531E-05	.3007E-06	-.3843E-05	-.3368E-05
-0.6	-.3571E-05	-.3387E-05	-.2720E-05	-.1624E-05	-.6060E-06
-0.8	-.4068E-06	-.3102E-06	-.8740E-07	.1020E-06	.1269E-06

Influence surface of M_x at $x = 0$, $y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.3433E-04	.3130E-04	.1445E-04	-.3800E-05	-.5784E-05
0.6	-.3320E-05	.6615E-04	.1187E-03	.4330E-04	-.5990E-05
0.4	-.9151E-03	-.9043E-03	.4212E-04	.2003E-03	.2629E-04
0.2	.6823E-02	-.6650E-02	-.1775E-02	.3263E-03	.9523E-04
0	.1000E+31	-.1891E-01	-.4172E-02	.3100E-03	.1344E-03
-0.2	.6923E-02	-.6650E-02	-.1775E-02	.3263E-03	.9523E-04
-0.4	-.9151E-03	-.9043E-03	.4212E-04	.2003E-03	.2629E-04
-0.6	-.3320E-05	.6615E-04	.1187E-03	.4330E-04	-.5990E-05
-0.8	.3433E-04	.3130E-04	.1445E-04	-.3800E-05	-.5784E-05

Influence coefficient $s_5 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1269E-06	-.6060E-06	-.3368E-05	.3226E-05	.5634E-04	.1039E-03	-.2056E-03	-.9670E-03	-.1077E-02
0.6	.1020E-06	-.1624E-05	-.3843E-05	.2612E-04	.1096E-03	-.2066E-03	-.1894E-02	-.2958E-02	-.3809E-03
0.4	-.8740E-07	-.2720E-05	.3007E-06	.6192E-04	.5122E-04	-.1243E-02	-.3116E-02	.1166E-01	.5946E-01
0.2	-.3102E-06	-.3387E-05	.6531E-05	.8703E-04	-.1336E-03	-.2293E-02	.1891E-02	.5971E-01	.8534E-01
0	-.4068E-06	-.3571E-05	.9499E-05	.9406E-04	-.2414E-03	-.2621E-02	.6836E-02	.8508E-01	-.2708E+00

Influence surface of M_{xy} at $x = 0$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.2225E-04	-.2495E-04	.4739E-04	.1051E-03	0.	-.1051E-03	-.4739E-04	.2495E-04	.2225E-04
0.6	-.2495E-04	.1484E-03	.6133E-03	.6096E-03	0.	-.6096E-03	-.6133E-03	-.1484E-03	.2495E-04
0.4	.4739E-04	.6133E-03	.7033E-03	-.3635E-02	0.	.3635E-02	-.7033E-03	-.6133E-03	-.4739E-04
0.2	.1051E-03	.6096E-03	-.3635E-02	-.4065E-01	0.	.4065E-01	.3635E-02	-.6096E-03	-.1051E-03
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	-.1051E-03	-.6098E-03	.3635E-02	.4065E-01	0.	-.4065E-01	-.3635E-02	.6096E-03	.1051E-03
-0.4	-.4739E-04	-.6133E-03	-.7033E-03	.3635E-02	0.	-.3635E-02	.7033E-03	.6133E-03	.4739E-04
-0.6	.2495E-04	-.1494E-03	-.6133E-03	-.6096E-03	0.	.6096E-03	.6133E-03	.1484E-03	-.2495E-04
-0.8	.2225E-04	.2495E-04	-.4739E-04	-.1051E-03	0.	.1051E-03	.4739E-04	-.2495E-04	-.2225E-04

Simply supported rectangular plate ($\nu = 0.3$)						b/a = 1.2				λ = 11

Influence coefficient $s_4 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2920E-06	.2431E-06	.1223E-06	.1553E-07	-.1662E-07
0.6	-.6344E-06	-.1503E-06	.4338E-06	.3413E-06	.7535E-07
0.4	-.1392E-04	-.1140E-04	-.4156E-05	.1111E-06	.3677E-06
0.2	.1198E-03	.2549E-04	-.1446E-04	-.3071E-05	.4728E-06
0	.1033E-02	.2123E-03	-.1161E-04	-.5615E-05	.3589E-06
-0.2	.1198E-03	.2549E-04	-.1446E-04	-.3071E-05	.4728E-06
-0.4	-.1392E-04	-.1140E-04	-.4156E-05	.1111E-06	.3677E-06
-0.6	-.6344E-06	-.1503E-06	.4338E-06	.3413E-06	.7535E-07
-0.8	.2920E-06	.2431E-06	.1223E-06	.1553E-07	-.1662E-07

Influence surface of M_y at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2048E-05	-.6376E-05	-.1050E-04	-.7011E-05	-.2040E-05
0.6	.2682E-03	.2063E-03	.8019E-04	.4953E-05	-.6422E-05
0.4	-.1087E-02	-.2994E-03	.2202E-03	.9629E-04	.5759E-05
0.2	-.1684E-01	-.6527E-02	-.7462E-03	.8674E-04	.2919E-04
0	.1000E+31	.6823E-02	-.9151E-03	-.3120E-05	.3435E-04
-0.2	-.1684E-01	-.6527E-02	-.7462E-03	.8674E-04	.2919E-04
-0.4	-.1087E-02	-.2994E-03	.2202E-03	.9629E-04	.5759E-05
-0.6	.2682E-03	.2063E-03	.8019E-04	.4953E-05	-.6422E-05
-0.8	-.2048E-05	-.6376E-05	-.1050E-04	-.7011E-05	-.2040E-05

Influence coefficient $s_6 = V_y a/P$ at $x = 0, y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.5215E-01	.1028E+00	.4887E-01	-.7152E-03	-.1177E-02
0.6	.4496E-01	.2704E-01	.1796E-02	-.2977E-02	-.6556E-03
0.4	-.2730E-02	-.3022E-02	-.2332E-02	-.7007E-03	.4888E-04
0.2	-.5305E-03	-.3611E-03	-.4283E-04	.1096E-03	.7285E-04
0	.9406E-04	.8703E-04	.6192E-04	.2611E-04	.3239E-05
-0.2	.2489E-05	.7423E-06	-.2612E-05	-.4158E-05	-.2811E-05
-0.4	-.2108E-05	-.1882E-05	-.1272E-05	-.5512E-06	-.9628E-07
-0.6	.1273E-06	.1388E-06	.1549E-06	.1414E-06	.8448E-07
-0.8	.3359E-07	.2897E-07	.1750E-07	.5541E-08	-.5354E-09

Influence surface of M_x at $x = 0, y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.1679E-05	-.1193E-06	-.3524E-05	-.4019E-05	-.1731E-05
0.6	.5354E-04	.6019E-04	.4294E-04	.4531E-05	-.7479E-05
0.4	-.4169E-03	-.2346E-03	.1611E-03	.1265E-03	.7708E-05
0.2	.2144E-02	-.4780E-02	-.1195E-02	.3163E-03	.8105E-04
0	.1000E+31	-.1891E-01	-.4172E-02	.3100E-03	.1345E-03
-0.2	.2144E-02	-.4780E-02	-.1195E-02	.3163E-03	.8105E-04
-0.4	-.4159E-03	-.2346E-03	.1611E-03	.1265E-03	.7708E-05
-0.6	.5354E-04	.6019E-04	.4294E-04	.4531E-05	-.7479E-05
-0.8	.1679E-05	-.1193E-06	-.3524E-05	-.4019E-05	-.1731E-05

Influence coefficient $s_5 = V_x a/P$ at $x = a, y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1266E-06	-.1690E-07	-.1919E-05	-.4026E-05	.1238E-04	.6502E-04	.9634E-04	-.1236E-04	-.1111E-03
0.6	.1590E-06	-.8815E-06	-.4033E-05	.9104E-05	.8106E-04	.5850E-04	-.6701E-03	-.1944E-02	-.1763E-02
0.4	-.9243E-08	-.2321E-05	-.1671E-05	.4653E-04	.9399E-04	-.7818E-03	-.3171E-02	.1478E-02	.1840E-01
0.2	-.2841E-06	-.3306E-05	.5630E-05	.8388E-04	-.1009E-03	-.2152E-02	.5177E-03	.5084E-01	.1370E+00
0	-.4180E-06	-.3563E-05	.9771E-05	.9364E-04	-.2497E-03	-.2600E-02	.7193E-02	.8379E-01	-.3177E+00

Influence surface of M_{xy} at $x = 0, y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.6370E-05	-.1890E-04	-.2160E-04	-.8320E-05	0.	.8320E-05	.2160E-04	.1890E-04	.6370E-05
0.6	-.2838E-04	.5116E-05	.1873E-03	.2751E-03	0.	-.2751E-03	-.1873E-03	-.5116E-05	.2838E-04
0.4	.9837E-05	.4226E-03	.1005E-02	-.2069E-03	0.	.2069E-03	-.1005E-02	-.4226E-03	-.9837E-05
0.2	.1051E-03	.6833E-03	-.2635E-02	-.2977E-01	0.	.2977E-01	.2635E-02	-.6833E-03	-.1051E-03
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	-.1051E-03	-.6833E-03	.2635E-02	.2977E-01	0.	-.2977E-01	-.2635E-02	.6833E-03	.1051E-03
-0.4	-.9837E-05	-.4226E-03	-.1005E-02	.2069E-03	0.	-.2069E-03	.1005E-02	.4226E-03	.9837E-05
-0.6	.2838E-04	-.5116E-05	-.1873E-03	-.2751E-03	0.	.2751E-03	.1873E-03	.5116E-05	-.2838E-04
-0.8	.6370E-05	.1890E-04	.2160E-04	.8320E-05	0.	-.8320E-05	-.2160E-04	-.1890E-04	-.6370E-05

Simply supported rectangular plate ($\nu = 0.3$) | $b/a = 1.4$ | $\lambda = 11$

Influence coefficient $s_4 = wD/Pa^2$ at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.2406E-07	.1190E-07	-.9686E-08	-.1738E-07	-.1011E-07
0.6	.4628E-06	.4680E-06	.3575E-06	.1334E-06	.4109E-09
0.4	-.8401E-05	-.5967E-05	-.1462E-05	.4285E-06	.2622E-06
0.2	.5796E-04	.5248E-05	-.1349E-04	-.2384E-05	.4848E-06
0	.1033E-02	.2123E-03	-.1161E-04	-.5615E-05	.3589E-06
-0.2	.5796E-04	.5248E-05	-.1349E-04	-.2384E-05	.4848E-06
-0.4	-.8401E-05	-.5967E-05	-.1462E-05	.4285E-06	.2622E-05
-0.6	.4628E-06	.4680E-06	.3575E-06	.1334E-06	.4109E-08
-0.8	.2406E-07	.1190E-07	-.9686E-08	-.1738E-07	-.1011E-07

Influence surface of M_y at $x = y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.9334E-05	-.8114E-05	-.4866E-05	-.1501E-05	.4034E-07
0.6	.7706E-04	.5087E-04	.7879E-05	-.8202E-05	-.4667E-05
0.4	.1151E-03	.2597E-03	.2312E-03	.6120E-04	-.1240E-05
0.2	-.1343E-01	-.5564E-02	-.5417E-03	.1040E-03	.2635E-04
0	.1000E+31	.6823E-02	-.9151E-03	-.3323E-05	.3434E-04
-0.2	-.1343E-01	-.5564E-02	-.5417E-03	.1040E-03	.2635E-04
-0.4	.1151E-03	.2597E-03	.2312E-03	.6120E-04	-.1240E-05
-0.6	.7706E-04	.5087E-04	.7879E-05	-.8202E-05	-.4667E-05
-0.8	-.9334E-05	-.8114E-05	-.4866E-05	-.1501E-05	.4034E-07

Influence coefficient $s_6 = V_y a/P$ at $x = 0$, $y = b$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.5495E-01	.1047E+00	.3808E-01	-.1347E-02	-.1206E-02
0.6	.1551E-01	.7226E-02	-.2456E-02	-.2328E-02	-.3396E-03
0.4	-.2058E-02	-.1701E-02	-.7975E-03	-.5782E-04	.1077E-03
0.2	.9377E-04	.1030E-03	.9981E-04	.5941E-04	.1703E-04
0	.9499E-05	.6531E-05	.3003E-06	-.3844E-05	-.3367E-05
-0.2	-.2108E-05	-.1882E-05	-.1273E-05	-.5515E-06	-.9573E-07
-0.4	.1500E-06	.1506E-06	.1433E-06	.1136E-06	.6145E-07
-0.6	.5152E-08	.3184E-08	-.1268E-08	-.4720E-08	-.4228E-08
-0.8	-.2320E-08	-.2126E-08	-.1617E-08	-.9792E-09	-.4140E-09

Influence surface of M_x at $x = 0$, $y = 0$

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.2177E-05	-.2199E-05	-.1833E-05	-.8197E-06	-.2512E-07
0.6	.2200E-04	.1832E-04	.5522E-05	-.5166E-05	-.4519E-05
0.4	-.8596E-04	.2303E-04	.1450E-03	.6607E-04	-.3093E-05
0.2	-.6461E-04	-.3355E-02	-.7203E-03	.2974E-03	.6631E-04
0	.1000E+31	-.1891E-01	-.4172E-02	.3100E-03	.1344E-03
-0.2	-.6461E-04	-.3355E-02	-.7203E-03	.2974E-03	.6631E-04
-0.4	-.8596E-04	.2303E-04	.1450E-03	.6607E-04	-.3093E-05
-0.6	.2200E-04	.1832E-04	.5522E-05	-.5166E-05	-.4519E-05
-0.8	-.2177E-05	-.2199E-05	-.1833E-05	-.8197E-06	-.2512E-07

Influence coefficient $s_5 = V_x a/P$ at $x = a$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.6376E-07	.1596E-06	-.4442E-06	-.2878E-05	-.3050E-05	.1263E-04	.4522E-04	.6056E-04	.3835E-04
0.6	.1565E-06	-.3119E-06	-.3075E-05	-.8191E-06	.4024E-04	.1058E-03	-.5029E-04	-.5682E-03	-.7051E-03
0.4	.7078E-07	-.1827E-05	-.3248E-05	.3314E-04	.1101E-03	-.3647E-03	-.2375E-02	-.2487E-02	.2886E-02
0.2	-.2331E-06	-.3194E-05	.4184E-05	.7948E-04	-.5551E-04	-.1949E-02	-.8885E-03	.3907E-01	.1242E+00
0	-.4069E-06	-.3572E-05	.9548E-05	.9400E-04	-.2431E-03	-.2618E-02	.6909E-02	.8495E-01	-.2785E+00

Influence surface of M_{xy} at $x = 0$, $y = 0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.2499E-06	-.3185E-05	-.7720E-05	-.6802E-05	0.	.6802E-05	.7720E-05	.3185E-05	-.2499E-06
0.6	-.1719E-04	-.2828E-04	.9193E-05	.5144E-04	0.	-.5144E-04	-.9193E-05	.2828E-04	.1719E-04
0.4	-.1689E-04	.2288E-03	.7852E-03	.6045E-03	0.	-.6045E-03	-.7852E-03	-.2288E-03	.1689E-04
0.2	.9765E-04	.7214E-03	-.1536E-02	-.2014E-01	0.	.2014E-01	.1536E-02	-.7214E-03	-.9765E-04
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	-.9765E-04	-.7214E-03	.1536E-02	.2014E-01	0.	-.2014E-01	-.1536E-02	.7214E-03	.9765E-04
-0.4	.1689E-04	-.2288E-03	-.7852E-03	-.6045E-03	0.	.6045E-03	.7852E-03	.2288E-03	-.1689E-04
-0.6	.1719E-04	.2828E-04	-.9193E-05	-.5144E-04	0.	.5144E-04	.9193E-05	-.2828E-04	-.1719E-04
-0.8	-.2499E-06	.3185E-05	.7720E-05	.6802E-05	0.	-.6802E-05	-.7720E-05	-.3185E-05	.2499E-06

Simply supported rectangular plate (ν = 0.3) | b/a = 1.6 | λ = 11

Influence coefficient $s_4 = wD/Pa^2$ at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1649E-07	-.1574E-07	-.1237E-07	-.6516E-08	-.1761E-08
0.8	.2876E-06	.2394E-06	.1206E-06	.1552E-07	-.1610E-07
0.4	-.3372E-05	-.2064E-05	-.4840E-07	.4572E-06	.1582E-06
0.2	.1973E-04	-.7040E-05	-.1195E-04	-.1723E-05	.4851E-06
0	.1033E-02	.2128E-03	-.1161E-04	-.5615E-05	.3585E-06
-0.2	.1973E-04	-.7040E-05	-.1199E-04	-.1723E-05	.4851E-06
-0.4	-.3372E-05	-.2064E-05	-.4840E-07	.4572E-06	.1582E-06
-0.6	.2876E-06	.2394E-06	.1206E-05	.1552E-07	-.1610E-07
-0.8	-.1649E-07	-.1574E-07	-.1237E-07	-.6516E-08	-.1761E-08

Influence surface of M_y at x = y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.1492E-05	-.1062E-05	-.2058E-06	.3154E-06	.2854E-05
0.8	-.1624E-05	-.5954E-05	-.1014E-04	-.6777E-05	-.1945E-05
0.4	.3611E-03	.3178E-03	.1604E-03	.2787E-04	-.5308E-05
0.2	-.9849E-02	-.4245E-02	-.3231E-03	.1159E-03	.2283E-04
0	.1000E+31	.6823E-02	-.9151E-03	-.3326E-05	.3432E-04
-0.2	-.9849E-02	-.4245E-02	-.3231E-03	.1159E-03	.2283E-04
-0.4	.3611E-03	.3178E-03	.1604E-03	.2787E-04	-.5308E-05
-0.6	-.1624E-05	-.5954E-05	-.1014E-04	-.6777E-05	-.1945E-05
-0.8	-.1492E-05	-.1062E-05	-.2058E-06	.3154E-06	.2854E-06

Influence coefficient $s_6 = V_y a/P$ at x = 0, y = b

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	.9541E-01	.9480E-01	.2795E-01	-.2033E-02	-.1174E-02
0.6	.1451E-02	-.1180E-02	-.3165E-02	-.1456E-02	-.9557E-04
0.4	-.5305E-03	-.3611E-03	-.4282E-04	.1096E-03	.7248E-04
0.2	.5408E-04	.4633E-04	.2645E-04	.6108E-05	-.2590E-05
0	-.3576E-05	-.3392E-05	-.2726E-05	-.1627E-05	-.6076E-06
-0.2	.1275E-06	.1389E-06	.1550E-06	.1412E-06	.8348E-07
-0.4	.5148E-08	.3183E-08	-.1260E-08	-.4700E-08	-.4219E-08
-0.6	-.1393E-08	-.1243E-08	-.8647E-09	-.4377E-09	-.1334E-09
-0.8	.1429E-09	.1361E-09	.1161E-09	.8428E-10	.4345E-10

Influence surface of M_x at x = 0, y = 0

y/b \ x/a	0	0.2	0.4	0.6	0.8
0.8	-.4744E-06	-.3810E-06	-.1278E-06	.1239E-06	.1629E-06
0.6	.1768E-05	-.1912E-07	-.3409E-05	-.3920E-05	-.1675E-05
0.4	.3916E-04	.7855E-04	.9069E-04	.2596E-04	-.7375E-05
0.2	-.9146E-03	-.2280E-02	-.3622E-03	.2704E-03	.5184E-04
0	.1000E+31	-.1891E-01	-.4172E-02	.3100E-03	.1344E-03
-0.2	-.9146E-03	-.2280E-02	-.3622E-03	.2704E-03	.5184E-04
-0.4	.3916E-04	.7855E-04	.9069E-04	.2596E-04	-.7375E-05
-0.6	.1768E-05	-.1912E-07	-.3409E-05	-.3920E-05	-.1675E-05
-0.8	-.4744E-06	-.3810E-06	-.1278E-06	.1239E-06	.1629E-06

Influence coefficient $s_5 = V_x a/P$ at x = a, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.1465E-07	.1079E-06	.1024E-06	-.8006E-06	-.2868E-05	-.2766E-05	.4046E-05	.1292E-04	.1192E-04
0.6	.1204E-06	.2042E-07	-.1739E-05	-.3884E-05	.1140E-04	.6429E-04	.9004E-04	-.1551E-04	-.9612E-04
0.4	.1264E-06	-.1312E-05	-.3952E-05	.1930E-04	.1008E-03	-.8828E-04	-.1421E-02	-.2835E-02	-.1773E-02
0.2	-.1797E-06	-.3042E-05	.2721E-05	.7380E-04	-.1309E-04	-.1707E-02	-.1921E-02	.2816E-01	.1013E+00
0	-.3986E-06	-.3573E-05	.9279E-05	.9410E-04	-.2349E-03	-.2626E-02	.6562E-02	.8517E-01	-.2474E+00

Influence surface of M_{xy} at x = 0, y = 0

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.7690E-06	.7475E-06	-.1662E-06	-.6871E-06	0.	.6871E-06	.1662E-06	-.7475E-06	-.7690E-06
0.6	-.6143E-05	-.1846E-04	-.2113E-04	-.8039E-05	0.	.8039E-05	.2113E-04	.1846E-04	.6143E-05
0.4	-.2875E-04	.8519E-04	.4469E-03	.5176E-03	0.	-.5176E-03	-.4469E-03	-.8519E-04	.2875E-04
0.2	.8418E-04	.7205E-03	-.5468E-03	-.1260E-01	0.	.1260E-01	.5468E-03	-.7205E-03	-.8418E-04
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	-.8418E-04	-.7205E-03	.5468E-03	.1260E-01	0.	-.1260E-01	-.5468E-03	.7205E-03	.8418E-04
-0.4	.2875E-04	-.8519E-04	-.4469E-03	-.5176E-03	0.	.5176E-03	.4469E-03	.8519E-04	-.2875E-04
-0.6	.6143E-05	.1846E-04	.2113E-04	.8039E-05	0.	-.8039E-05	-.2113E-04	-.1846E-04	-.6143E-05
-0.8	-.7690E-06	-.7475E-06	.1662E-06	.6871E-06	0.	-.6871E-06	-.1662E-06	.7475E-06	.7690E-06

Simply supported rectangular plate (ν = 0.3) b/a = 1.8 λ = 11

y/b \ x/a	Influence coefficient $s_4 = wD/Pa^2$ at $x=y=0$: 0	0.2	0.4	0.6	0.8	Influence surface of M_y at $x=y=0$: 0	0.2	0.4	0.6	0.8
0.8	-.4655E-08	-.3831E-08	-.1874E-08	-.7561E-10	.5031E-09	.4099E-06	.4123E-06	.3689E-06	.2397E-06	.9475E-07
0.6	.6362E-07	.4366E-07	.4730E-08	-.1649E-07	-.1256E-07	-.1113E-04	-.1012E-04	-.6760E-05	-.2553E-05	-.2505E-06
0.4	-.6338E-06	-.1496E-06	.4343E-05	.3417E-06	.7505E-07	.2692E-03	.2064E-03	.8020E-04	.4952E-05	-.6434E-05
0.2	-.1589E-05	-.1908E-04	-.1005E-04	-.1122E-05	.4724E-06	-.6686E-02	-.2917E-02	-.1211E-03	.1209E-03	.1877E-04
0	.1033E-02	.2123E-03	-.1161E-04	-.5615E-05	.3575E-05	.1000E+31	.6823E-02	-.9151E-03	-.3330E-05	.3430E-04
-0.2	-.1589E-05	-.1308E-04	-.1005E-04	-.1122E-05	.4724E-06	-.6685E-02	-.2917E-02	-.1211E-03	.1209E-03	.1877E-04
-0.4	-.6338E-06	-.1496E-06	.4343E-06	.3417E-06	.7505E-07	.2682E-03	.2064E-03	.8020E-04	.4952E-05	-.6434E-05
-0.6	.6362E-07	.4366E-07	.4730E-08	-.1649E-07	-.1256E-07	-.1113E-04	-.1012E-04	-.6760E-05	-.2553E-05	-.2505E-06
-0.8	-.4655E-08	-.3831E-08	-.1874E-08	-.7561E-10	.5031E-09	.4099E-06	.4123E-06	.3689E-06	.2397E-06	.9475E-07

y/b \ x/a	Influence coefficient $s_6 = V_y a/P$ at $x=0, y=b$: 0	0.2	0.4	0.6	0.8	Influence surface of M_x at $x=0, y=0$: 0	0.2	0.4	0.6	0.8
0.8	.9909E-01	.7534E-01	.1904E-01	-.2601E-02	-.1090E-02	.8345E-07	.9622E-07	.1147E-06	.1023E-06	.5463E-07
0.6	-.2730E-02	-.3022E-02	-.2332E-02	-.7009E-03	.4806E-04	-.2337E-05	-.2566E-05	-.2505E-05	-.1394E-05	-.2428E-06
0.4	.4711E-04	.7634E-04	.1076E-03	.7899E-04	.2752E-04	.5356E-04	.6020E-04	.4294E-04	.4536E-05	-.7459E-05
0.2	.2489E-05	.7416E-06	-.2612E-05	-.4149E-05	-.2785E-05	-.1070E-02	-.1482E-02	-.1139E-03	.2372E-03	.3833E-04
0	-.3915E-06	-.2980E-06	-.8277E-07	.9932E-07	.1224E-06	.1000E+31	-.1891E-01	-.4172E-02	.3100E-03	.1343E-03
-0.2	.3340E-07	.2885E-07	.1757E-07	.5739E-08	-.4412E-09	-.1070E-02	-.1482E-02	-.1139E-03	.2372E-03	.3633E-04
-0.4	-.2323E-08	-.2132E-08	-.1625E-08	-.9853E-09	-.4040E-09	.5356E-04	.6020E-04	.4294E-04	.4536E-05	-.7459E-05
-0.6	.1432E-09	.1365E-09	.1165E-09	.8439E-10	.4325E-10	-.2337E-05	-.2566E-05	-.2505E-05	-.1394E-05	-.2428E-06
-0.8	-.7973E-11	-.7781E-11	-.7061E-11	-.5515E-11	-.3018E-11	.8345E-07	.9622E-07	.1147E-06	.1023E-06	.5463E-07

Influence coefficient $s_5 = V_x a/P$ at $x=a, y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	-.3857E-08	.3434E-07	.1269E-06	.3721E-07	-.7387E-06	-.2066E-05	-.2531E-05	-.1390E-05	-.1483E-06
0.6	.7261E-07	.1420E-06	-.6642E-06	-.3218E-05	-.1148E-05	.2187E-04	.6010E-04	.6701E-04	.3256E-04
0.4	.1538E-06	-.8374E-06	-.3889E-05	.8593E-05	.7734E-04	.5318E-04	-.6639E-03	-.1917E-02	-.1967E-02
0.2	-.1266E-06	-.2853E-05	.1331E-05	.6706E-04	.2330E-04	-.1446E-02	-.2563E-02	.1891E-01	.7649E-01
0	-.3875E-06	-.3561E-05	.8981E-05	.9383E-04	-.2258E-03	-.2619E-02	.6196E-02	.8445E-01	-.2251E+00

Influence surface of M_{xy} at $x=0, y=0$

y/b \ x/a	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8
0.8	.2254E-06	.4480E-06	.4791E-06	.2782E-06	0.	-.2782E-06	-.4791E-06	-.4480E-06	-.2254E-05
0.6	-.5667E-06	-.5831E-05	-.1138E-04	-.9046E-05	0.	.9046E-05	.1138E-04	.5831E-05	.5667E-06
0.4	-.2838E-04	.5123E-05	.1873E-03	.2751E-03	0.	-.2751E-03	-.1873E-03	-.5123E-05	.2838E-04
0.2	.6664E-04	.6823E-03	.2118E-03	-.7213E-02	0.	.7213E-02	-.2118E-03	-.6823E-03	-.6664E-04
0	0.	0.	0.	0.	0.	0.	0.	0.	0.
-0.2	-.6664E-04	-.6823E-03	-.2118E-03	.7213E-02	0.	-.7213E-02	.2118E-03	.6823E-03	.6664E-04
-0.4	.2838E-04	-.5123E-05	-.1873E-03	-.2751E-03	0.	.2751E-03	.1873E-03	.5123E-05	-.2838E-04
-0.6	.5557E-06	.5831E-05	.1138E-04	.9046E-05	0.	-.9046E-05	-.1138E-04	-.5831E-05	-.5667E-06
-0.8	-.2254E-06	-.4480E-06	-.4791E-06	-.2782E-06	0.	.2782E-06	.4791E-06	.4480E-06	.2254E-06

C O N C L U S I O N S

In this investigation,the BIE method has been successfully developed for the two-dimensional boundary value problem governing the deflection of thin elastic plates,resting on Winkler's type elastic foundation. The integral representation of the solution and the coupled,boundary,singular,integral equations are established and a procedure of their numerical solution is presented. Numerical results are also obtained for plates of various shapes. The accuracy of the method is discussed and its effectiveness is demonstrated. The method is adjusted for the evaluation of influence fields (Green's functions) of various field quantities.

In Part I of this investigation the following has been done:

a) The problem has been stated in its most general form including all possible boundary conditions mixed or not.

b) The region occupied by the plate may be multiply connected.

c) Two versions of Green's identity of the problem under consideration are established. In the first version, the boundary terms do not have physical significance. It can be applied to derive boundary integral equations only for the clamped and simply supported plate. The second version of Green's identity is obtained by modifying the first so that all the boundary terms have physical significance. This allows the derivation of boundary integral equations for all kinds of boundary conditions (geometric or natural) regardless of whether they are homogeneous or mixed and, thus, the problem can be treated in a unified form. From the computational point of view, one set of boundary integral equations for each boundary value problem is proposed which are very suitable for numerical solution, as substantiated in this investigation.

d) The boundary may have corners. In this case, an additional term appears in the integral representation of the solution and the boundary integral equations. This term results from the discontinuity of the slope of the boundary.

e) The fundamental solution used is that of an infinitely extended plate on elastic foundation, under a concentrated load. In presenting the derivation of the fundamental solution, a systematic procedure for the evaluation of the arbitrary constant of the solution is described.

f) The boundary integral equations were formulated by a limiting process in which the field point is let to approach the boundary. The first boundary integral equation was established in a straight forward manner. However, a special technique was developed in order to obtain a second boundary integral equation.

g) The existence of the boundary integrals having singular kernels as the field point approached the boundary was proved. Moreover, a procedure was presented for computing the jump of the discontinuity of the boundary integrals whenever the kernel behaved like a Newtonian double layer potential. Higher order singularities were reduced to that of a double layer potential by appropriate integration, by parts, along the boundary.

h) By examining the behaviour of the Kelvin functions and their derivatives, it is shown that the boundary integrals behave like single and double layer potential and, hence, theorems valid for the potential theory can be applied to the derivation of the boundary integral equations.

i) It is shown that the BIE method is very well suited for the numerical evaluation of influence fields of various quantities (such as deflections, slopes, bending and twisting moments, and shearing forces). In this case, an approach based on the properties of the δ-function is employed. Generalized loads are introduced which are actually derivatives of δ-function. In the problem at hand, these generalized loads have not direct physical meaning (they are combinations of multipoles of theoretical physics). The reciprocal theorem is used in a generalized form and the influence field is the deflection surface produced by the generalized force. A major advantage of this approach is that the non homogenous terms in the boundary integral equations are readily obtained analytically for plates with any shape and, thus, the numerical evaluation of improper double integral is avoided. Another advantage of this approach is that it results in considerable saving of computer time, because the influence field is obtained by solving once the boundary value problem. When the influence field of a quantity is obtained by placing the unit load at various positions, a boundary value problem is solved for every position of the unit load. An additional advantage of the BIE method is that a concentrated load does not have to be approximated by an equivalent distributed load on a small area, as in the case of the finite difference method, or to apply it only at nodal points, as in the case of the finite element method.

In Part II of this investigation, the following has been done:

a) The boundary integral equations are approximated by a system of simultaneous, linear, algebraic equations. The approximation is based on the discretization of the boundary into a finite number of elements on each of which the unknown boundary quantities are assumed to be constant. The coefficients of the system are evaluated by numerical integration on the boundary element. Special numerical schemes are developed for the evaluation of line integrals on the elements where the integrand is singular.

b) A numerical procedure is developed which can be employed for the numerical evaluation of improper double integrals having a logarithmic or a Cauchy-type singularity. This procedure is employed for the numerical evaluation of improper double integrals which are present in the boundary integrals equations.

c) The numerical schemesfor the computation of the deflections at any desired point are given. Numerical schemes for the evaluation of the stress resultants are also presented. They are computed at any desired point without requiring numerical differentiation.

In Part III of this investigation,numerical results are obtained for clamped and simply supported plates of various geometry. From these results,the following conclusions can be drawn:

a) The numerical results are presented in terms of appropriate,non-dimensional parameters for circular and rectangular plates. Tables are given for the analysis of circular and rectangular plates of a wide range of dimensions.

b) Inasmuch as in the BIE method only the boundary is discretized, in this method, less input data is required for the computer program than for other numerical methods, such as the finite element and the finite difference method in which the whole two-dimensional area is discretized. This saves time and reduces the risk of error.

c) For relatively smooth boundaries (without notches) only a few nodal points on the boundary can give accurate results. As the number of nodal points on the boundary is increased, the error of the results approaches zero.

d) The step function assumption for the unknown boundary quantities gives satisfactory results.

e) As it was anticipated, for small values of the constant of the elastic foundation, the results approach those for a plate not resting on elastic foundation.

f) The accuracy of the established quantities is greater for points located away from the boundary. In order to improve the accuracy for points near the boundary, the number of nodal points must be increased.

The formulae established in Appendix A may be used in developing the BIE method for other two-dimensional differential operators.

From Appendix C, it is concluded that the values of the Kelvin functions obtained by expanding them in Chebyshef series are a better approximation than those available in the IMSL library. The latter have been computed by Burgoyne using Lanczos' economization procedure.

A P P E N D I X A

In this appendix certain formulas are derived which facilitate the differentiations of the kernels of the internal equations.

A-I. Cartesian coordinates

The points of the region R are denoted by $P(x,y)$, while the points on its boundary are denoted by $q(\xi,\eta)$. The angle between the positive x axis and the vector $\vec{Pq}=\bar{r}$ is denoted by α. The angle between the positive x axis and the normal to the boundary, at the point q, is denoted by β. Finally, the angle between the positive x axis and an arbitrary direction $\bar{m}$ is denoted by γ (see Fig.A-1).

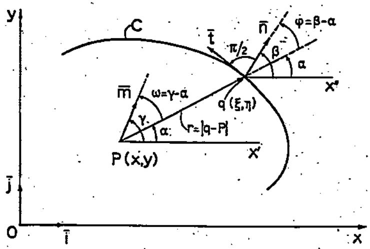

Fig.A-1.

From Fig.A-1 we have

$$\alpha=(\widehat{\bar{i},\bar{r}})$$
$$\beta=(\widehat{\bar{i},\bar{n}})$$
$$\gamma=(\widehat{\bar{i},\bar{m}}) \qquad \text{(A-1)}$$
$$\varphi=\beta-\alpha$$
$$\omega=\gamma-\alpha$$

The angles α,β and γ are positive when the x axis rotates counterclockwise to reach the directions $\vec{Pq},\bar{n}$ and $\bar{m}$, respectively.

Referring to Fig. A-1, we have

$$\cos\alpha=\frac{\xi-x}{r} \qquad \text{(A-2)}$$

$$\sin\alpha=\frac{\eta-y}{r} \qquad \text{(A-3)}$$

$$r=[(\xi-x)^2+(\eta-y)^2]^{\frac{1}{2}} \qquad \text{(A-4)}$$

Differentiating (A-4) we obtain

$$\frac{\partial r}{\partial x}=-\frac{\partial r}{\partial \xi}=-\frac{\xi-x}{r}=-\cos\alpha \qquad \text{(A-5)}$$

$$\frac{\partial r}{\partial y}=-\frac{\partial r}{\partial \eta}=-\frac{\eta-y}{r}=-\sin\alpha \qquad \text{(A-6)}$$

$$\frac{\partial r}{\partial n_p}=\frac{\partial r}{\partial x}\cos\beta+\frac{\partial r}{\partial y}\sin\beta=$$
$$=-(\cos\alpha\cos\beta+\sin\alpha\sin\beta)=-\cos(\beta-\alpha) \qquad \text{(A-7)}$$
$$=-\cos\varphi$$

$$\frac{\partial r}{\partial n_q} = \frac{\partial r}{\partial \xi}\cos\beta + \frac{\partial r}{\partial \eta}\sin\beta = -\frac{\partial r}{\partial n_p} = \cos\varphi^{(*)} \qquad \text{(A-8)}$$

Differentiating (A-2) and (A-3) we get

$$\frac{\partial(\cos\alpha)}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\xi - x}{r}\right)$$

$$= \frac{-r+(\xi-x)\partial r/\partial x}{r^2} \qquad \text{(A-9)}$$

$$= -\frac{\sin^2\alpha}{r}$$

$$\frac{\partial(\cos\alpha)}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\xi - x}{r}\right)$$

$$= (\xi - x)\frac{-1}{r^2}\frac{\partial r}{\partial y} \qquad \text{(A-10)}$$

$$= \frac{\cos\alpha\sin\alpha}{r}$$

$$\frac{\partial(\sin\alpha)}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\eta - y}{r}\right)$$

$$= (\eta - y)\frac{-1}{r^2}\frac{\partial r}{\partial x} \qquad \text{(A-11)}$$

$$= \frac{\cos\alpha\sin\alpha}{r}$$

(*) Here the subscript p or q denotes that the normal derivative $\frac{\partial}{\partial n}$ is taken assuming the points q or p is fixed, respectively.

$$\frac{\partial(\sin\alpha)}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\eta-y}{r}\right)$$

$$= \frac{-r-(\eta-y)\partial r/\partial y}{r^2} \qquad \text{(A-12)}$$

$$= -\frac{\cos^2\alpha}{r}$$

$$\frac{\partial(\cos\alpha)}{\partial n_p} = \frac{\partial(\cos\alpha)}{\partial x}\cos\beta + \frac{\partial(\cos\alpha)}{\partial y}\sin\beta$$

$$= -\frac{\sin^2\alpha}{r}\cos\beta + \frac{\cos\alpha\sin\alpha}{r}\sin\beta \qquad \text{(A-13)}$$

$$= \frac{\sin\alpha\sin\varphi}{r}$$

$$\frac{\partial(\sin\alpha)}{\partial n_p} = \frac{\partial(\sin\alpha)}{\partial x}\cos\beta + \frac{\partial(\sin\alpha)}{\partial y}\sin\beta$$

$$= \frac{\sin\alpha\cos\alpha}{r}\cos\beta - \frac{\cos^2\alpha\sin\beta}{r} \qquad \text{(A-14)}$$

$$= -\frac{\cos\alpha\sin\varphi}{r}$$

moreover,

$$\frac{\partial(\cos 2\alpha)}{\partial n_p} = \frac{\partial(2\cos^2\alpha-1)}{\partial n_p}$$

$$= 4\cos\alpha\,\frac{\partial(\cos\alpha)}{\partial n_p} \qquad \text{(A-15)}$$

$$= 4\cos\alpha\,\frac{\sin\alpha\sin\varphi}{r}$$

$$= \frac{2\sin 2\alpha\sin\varphi}{r}$$

$$\frac{\partial(\sin 2\alpha)}{\partial n_p} = 2\left[\sin\alpha\,\frac{\partial(\cos\alpha)}{\partial n_p} + \cos\alpha\,\frac{\partial(\sin\alpha)}{\partial n_p}\right]$$

$$= 2\left[\sin\alpha\,\frac{\sin\alpha\sin\varphi}{r} - \cos\alpha\,\frac{\cos\alpha\sin\varphi}{r}\right]$$

$$= -\frac{2(\cos^2\alpha - \sin^2\alpha)\sin\varphi}{r}$$

$$= -\frac{2\cos 2\alpha\sin\varphi}{r} \qquad \text{(A-16)}$$

Referring to Fig. (A-1), and using relations (A-5) and (A-6) we get

$$\frac{\partial r}{\partial m_p} = \frac{\partial r}{\partial x}\cos\gamma + \frac{\partial r}{\partial y}\sin\gamma$$

$$= -(\cos\alpha\cos\gamma + \sin\alpha\sin\gamma) \qquad \text{(A-17)}$$

$$= -\cos(\gamma - \alpha)$$

$$= -\cos\omega$$

$$\frac{\partial r}{\partial t_q} = \frac{\partial r}{\partial \xi}\cos\left(\frac{\pi}{2} + \beta\right) + \frac{\partial r}{\partial \eta}\sin\left(\frac{\pi}{2} + \beta\right)$$

$$= -\cos\alpha\sin\beta + \sin\alpha\cos\beta \qquad \text{(A-18)}$$

$$= -\sin(\beta - \alpha)$$

$$= -\sin\varphi\ .$$

moreover,

$$\frac{\partial(\cos\alpha)}{\partial t_q} = \frac{\partial(\cos\alpha)}{\partial \xi}\cos\left(\frac{\pi}{2} + \beta\right) + \frac{\partial(\cos\alpha)}{\partial \eta}\sin\left(\frac{\pi}{2} + \beta\right)$$

$$= -\frac{\sin^2\alpha}{r}\sin\beta - \frac{\cos\alpha\sin\alpha}{r}\cos\beta \qquad \text{(A-19)}$$

$$= -\frac{\sin\alpha\cos\varphi}{r}$$

$$\frac{\partial(\sin\alpha)}{\partial t_q} = \frac{\partial(\sin\alpha)}{\partial\xi}\cos(\frac{\pi}{2}+\beta) + \frac{\partial(\sin\alpha)}{\partial\eta}\sin(\frac{\pi}{2}+\beta)$$

$$= \frac{\cos\alpha\sin\alpha}{r}\sin\beta + \frac{\cos^2\alpha}{r}\cos\beta \qquad \text{(A-20)}$$

$$= \frac{\cos\alpha\cos\varphi}{r}$$

$$\frac{\partial}{\partial t_q}(\sin\varphi) = \frac{\partial}{\partial t_q}[\sin(\beta-\alpha)]$$

$$= \sin\beta\frac{\partial(\cos\alpha)}{\partial t_q} - \cos\beta\frac{\partial(\sin\alpha)}{\partial t_q} \qquad \text{(A-21)}$$

$$= \sin\beta\frac{-\sin\alpha\cos\varphi}{r} - \cos\beta\frac{\cos\alpha\cos\varphi}{r}$$

$$= -\frac{\cos^2\varphi}{r}$$

$$\frac{\partial}{\partial t_q}(\cos\varphi) = \frac{\partial}{\partial t_q}[\cos(\beta-\alpha)]$$

$$= \cos\beta\frac{\partial(\cos\alpha)}{\partial t_q} + \sin\beta\frac{\partial(\sin\alpha)}{\partial t_q} \qquad \text{(A-22)}$$

$$= \cos\beta - \frac{\sin\alpha\cos\varphi}{r} + \sin\beta\frac{\cos\alpha\cos\varphi}{r}$$

$$= \frac{\cos\varphi\sin\varphi}{r}$$

$$\frac{1}{2}\frac{\partial}{\partial t_q}(\sin 2\varphi) = [\sin\varphi\frac{\partial}{\partial t_q}(\cos\varphi) + \cos\varphi\frac{\partial}{\partial t_q}(\sin\varphi)] \qquad \text{(A-23)}$$

$$= \sin\varphi\frac{\cos\varphi\sin\varphi}{r} - \cos\varphi\frac{\cos^2\varphi}{r}$$

$$= -\frac{\cos\varphi\cos 2\varphi}{r}$$

$$\frac{\partial^2 r}{\partial t_q^2} = \frac{\partial}{\partial t_q}\left(\frac{\partial r}{\partial t_q}\right) = \frac{\partial}{\partial t_q}(-\sin\varphi) \qquad \text{(A-24)}$$

$$= -\frac{\cos^2\varphi}{r}$$

$$\frac{\partial^2 r}{\partial t_q \partial n_q} = \frac{\partial}{\partial t_q}\left(\frac{\partial r}{\partial n_q}\right) = \frac{\partial}{\partial t_q}(\cos\varphi) \qquad \text{(A-25)}$$

$$= \frac{\cos\varphi\sin\varphi}{r}$$

Differentiating relation (A-17) and referring to Fig.A-1, we get

$$\frac{\partial^2 r}{\partial m_p \partial n_q} = \frac{\partial}{\partial n_q}\left(\frac{\partial r}{\partial m_p}\right) = \frac{\partial}{\partial n_q}(-\cos\omega)$$

$$= -\frac{\partial}{\partial n_q}\cos(\gamma-\alpha)$$

$$= -\left[\cos\gamma\frac{\partial(\cos\alpha)}{\partial n_q} + \sin\gamma\frac{\partial(\sin\alpha)}{\partial n_q}\right] \qquad \text{(A-26)}$$

$$= \cos\gamma\frac{\sin\alpha\sin\varphi}{r} - \sin\gamma\frac{\cos\alpha\sin\varphi}{r}$$

$$= -\frac{\sin\varphi\sin\omega}{r}$$

Consider the differential equation

$$w'' + \frac{1}{\rho}w' - iw = 0 \qquad \text{(A-27)}$$

Its solution is [76]

$$w = \mathrm{ker}(\rho) + i\,\mathrm{kei}(\rho) \qquad \text{(A-28)}$$

From equation (A-27) we have

$$w'' = -\frac{1}{\rho}w' + iw \qquad \text{(A-29)}$$

Substituting equation (A-28) into equation (A-I.29), and separating the real and imaginary parts, we obtain

$$\mathrm{kei}''(\rho) = -\frac{1}{\rho}\,\mathrm{kei}'(\rho) + \mathrm{ker}(\rho) \qquad \text{(A-30)}$$

$$\mathrm{ker}''(\rho) = -\frac{1}{\rho}\,\mathrm{ker}'(\rho) - \mathrm{kei}(\rho) \qquad \text{(A-31)}$$

In what follows, we will express certain higher order derivatives of the functions kei(ρ) and ker(ρ) in terms of these functions and their first derivatives.

Denoting by $u=\mathrm{kei}(\rho)$ and $z=\mathrm{ker}(\rho)$ relations (A-30) and (A-31) can be written as

$$u'' = -\frac{1}{\rho}\,u' + z \qquad \text{(A-32)}$$

$$z'' = -\frac{1}{\rho}\,z' - u \qquad \text{(A-33)}$$

where the prime denotes differentiation with respect to the argument ρ.

Referring to relations (I-3.4), (A-5), (A-6) and (A-7) we get

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial r}\frac{\partial r}{\partial x} = -\frac{1}{\ell}\,u'\cos\alpha \qquad \text{(A-34)}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial r}\frac{\partial r}{\partial y} = -\frac{1}{\ell}\,u'\sin\alpha \qquad \text{(A-35)}$$

$$\frac{\partial u}{\partial n_p} = \frac{\partial u}{\partial x}\cos\beta + \frac{\partial u}{\partial y}\sin\beta = -\frac{1}{\ell}\,u'\cos\varphi \qquad \text{(A-36)}$$

Differentiating (A-34) and using relations (A-9) and (A-5) we obtain

$$\frac{\partial^2 u}{\partial x^2} = -\frac{1}{\ell}\frac{\partial}{\partial x}(u'\cos\alpha) = -\frac{1}{\ell}\left[\frac{\partial u'}{\partial x}\cos\alpha + u'\frac{\partial(\cos\alpha)}{\partial x}\right]$$

$$= \frac{1}{\ell^2}\left[u''\cos^2\alpha + \frac{1}{\rho}u'\sin^2\alpha\right] \qquad \text{(A-37)}$$

$$= \frac{1}{\ell^2}\left[z\cos^2\alpha - \frac{1}{\rho}u'\cos 2\alpha\right]$$

In obtaining the last result, relation (A-32) has been employed.

Differentiating (A-35) and using relations (A-12) and (A-6), we obtain

$$\frac{\partial^2 u}{\partial y^2} = -\frac{1}{\ell}\frac{\partial}{\partial y}(u'\sin\alpha) = -\frac{1}{\ell}\left[\frac{\partial u'}{\partial y}\sin\alpha + u'\frac{\partial(\sin\alpha)}{\partial y}\right]$$

$$= \frac{1}{\ell^2}\left[u''\sin^2\alpha + \frac{1}{\rho}u'\cos^2\alpha\right] \qquad \text{(A-38)}$$

$$= \frac{1}{\ell^2}\left[z\sin^2\alpha + \frac{1}{\rho}u'\cos 2\alpha\right]$$

In obtaining the last result relation (A-32) has been employed.

Using relations (A-37) and (A-38) we obtain

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{\ell^2} z \qquad \text{(A-39)}$$

Differentiation (A-34) and using relations (A-6), (A-10) and (A-32) we get

$$\frac{\partial u}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = -\frac{1}{\ell}\frac{\partial}{\partial y}(u'\cos\alpha)$$

$$= -\frac{1}{\ell}\left[\frac{\partial' u}{\partial y}\cos\alpha + u'\frac{\partial(\cos\alpha)}{\partial y}\right]$$

$$= -\frac{1}{\ell}\left(-\frac{1}{\ell}u''\sin\alpha\cos\alpha + u'\frac{\cos\alpha\sin\alpha}{r}\right) \qquad (A-40)$$

$$= \frac{1}{2\ell^2}\left(u'' - \frac{1}{\rho}u'\right)\sin 2\alpha$$

$$= \frac{1}{2\ell^2}\left(z - \frac{2}{\rho}u'\right)\sin 2\alpha$$

Differentiating (A-39) and using relation (A-8) we get

$$\frac{\partial}{\partial n_q}(\nabla^2 u) = \frac{1}{\ell^3}z'\cos\varphi \qquad (A-41)$$

Moreover, from relations (A-37) and (A-38) we obtain

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = \frac{1}{\ell^2}\left(z - \frac{2}{\rho}u'\right)\cos 2\alpha \qquad (A-42)$$

Differentiating (A-42) and using relations (A-8) and (A-32) we get

$$\frac{\partial}{\partial n_q}\left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}\right) = \frac{1}{\ell^2}\frac{\partial}{\partial n_q}\left[\left(u'' - \frac{1}{\rho}u'\right)\cos 2\alpha\right]$$

$$= \frac{1}{\ell^2}\left[\cos 2\alpha\frac{\partial}{\partial n_q}\left(u'' - \frac{1}{\rho}u'\right) + \left(u'' - \frac{1}{\rho}u'\right)\frac{\partial(\cos 2\alpha)}{\partial n_q}\right]$$

$$= \frac{1}{\ell^3}\left[\left(u''' - \frac{1}{\rho}u'' + \frac{1}{\rho^2}u'\right)\cos\varphi\cos 2\alpha - \frac{2}{\rho}\left(u'' - \frac{1}{\rho}u'\right)\sin\varphi\sin 2\alpha\right]$$

$$= \frac{1}{\ell^3}\left[z'\cos 2\alpha\cos\varphi - \frac{2}{\rho}\left(z - \frac{2}{\rho}u'\right)\cos(2\alpha - \varphi)\right] \qquad (A-43)$$

Differentiating (A-40) and using relations (A-8) and (A-32) we get

$$\frac{\partial}{\partial n_q}\left(\frac{\partial^2 u}{\partial x \partial y}\right)= \frac{1}{2\ell^2}\frac{\partial}{\partial n_q}\left[(z-\frac{2}{\rho}u')\sin 2\alpha\right]$$

$$=\frac{1}{2\ell^2}\left[\sin 2\alpha\frac{\partial}{\partial n_q}(z-\frac{2}{\rho}u')+(z-\frac{2}{\rho}u')\frac{\partial}{\partial n_q}(\sin 2\alpha)\right]$$

$$=\frac{1}{2\ell^2}\left[\sin 2\alpha\frac{1}{\ell}(z-\frac{2}{\rho}u')'\cos\varphi+(z-\frac{2}{\rho}u')\frac{2\cos 2\alpha\sin\varphi}{r}\right]$$

$$=\frac{1}{2\ell^3}\left[\sin 2\alpha(z'-\frac{2}{\rho}u''+\frac{2}{\rho^2}u')\cos\varphi+\frac{2}{\rho}(z-\frac{2}{\rho}u')\cos 2\alpha\sin\varphi\right]$$

$$=\frac{1}{2\ell^3}\left[z'\sin 2\alpha\cos\varphi-\frac{2}{\rho}(z-\frac{2}{\rho}u')\sin(2\alpha-\varphi)\right] \qquad \text{(A-44)}$$

Differentiating (A-39) and using relations (A-5) and (A-6) we obtain

$$\frac{\partial}{\partial x}(\nabla^2 u)=-\frac{1}{\ell^3}z'\cos\alpha \qquad \text{(A-45)}$$

$$\frac{\partial}{\partial y}(\nabla^2 u)=-\frac{1}{\ell^3}z'\sin\alpha \qquad \text{(A-46)}$$

Differentiating (A-45) and (A-46) and using relations (A-8) and (A-33) we obtain

$$\frac{\partial}{\partial n_q}\left[\frac{\partial}{\partial x}(\nabla^2 u)\right]=-\frac{1}{\ell^3}\left[\frac{\partial}{\partial n_q}(z')\cos\alpha+z'\frac{\partial}{\partial n_q}(\cos\alpha)\right]$$

$$=-\frac{1}{\ell^4}\left[z''\cos\varphi\cos\alpha-\frac{1}{\rho}z'\sin\alpha\sin\varphi\right]$$

$$=\frac{1}{\ell^4}\left[u\cos\varphi\cos\alpha+\frac{1}{\rho}z'\cos(\alpha-\varphi)\right] \qquad \text{(A-47)}$$

$$\frac{\partial}{\partial n_q}\left[\frac{\partial}{\partial y}(\nabla^2 u)\right]=-\frac{1}{\ell^3}\left[\frac{\partial}{\partial n_q}(z')\sin\alpha+z'\frac{\partial}{\partial n_q}(\sin\alpha)\right]$$

$$= -\frac{1}{\ell^4}\left[z\cos\varphi\sin\alpha + \frac{1}{\rho}z'\cos\alpha\sin\varphi\right] \qquad (A-48)$$

$$= \frac{1}{\ell^4}\left[u\sin\varphi\cos\alpha + \frac{1}{\rho}\, z'\sin(\alpha-\varphi)\right]$$

Moreover, using relations (A-25),(A-17),(A-8) and (A-32) we obtain

$$\frac{\partial^2 u}{\partial m_p \partial n_q} = \frac{\partial}{\partial m_p}\left(\frac{\partial u}{\partial n_q}\right) = \frac{\partial}{\partial m_p}\left[\frac{1}{\ell}\, u'\frac{\partial r}{\partial n_q}\right]$$

$$= \frac{1}{\ell}\left[\frac{\partial u'}{\partial m_p}\frac{\partial r}{\partial n_q} + u'\frac{\partial^2 r}{\partial m_p \partial n_q}\right]$$

$$= -\frac{1}{\ell}\left[u''\cos\omega\cos\varphi + \frac{1}{\rho}\, u'\sin\varphi\sin\omega\right] \qquad (A-49)$$

$$= -\frac{1}{\ell^2}\left[z\cos\omega\cos\varphi - \frac{1}{\rho}\, u'\cos(\varphi+\omega)\right]$$

By substituting -u for z, -u' for z', z for u and z' for u' in equations (A-34) to (A-49) we obtain the respective derivatives for the function z.

Thus

$$\frac{\partial z}{\partial x} = -\frac{1}{\ell}\, z'\cos\alpha \qquad (A-50)$$

$$\frac{\partial z}{\partial y} = -\frac{1}{\ell}\, z'\sin\alpha \qquad (A-51)$$

$$\frac{\partial z}{\partial n_p} = -\frac{1}{\ell}\, z'\cos\varphi \qquad (A-52)$$

$$\frac{\partial^2 z}{\partial x^2} = -\frac{1}{\ell^2}\left[u\cos^2\alpha + \frac{1}{\rho}\, z'\cos 2\alpha\right] \qquad (A-53)$$

$$\frac{\partial^2 z}{\partial y^2} = -\frac{1}{\ell^2}\left[u\sin^2\alpha - \frac{1}{\rho}\dot{z}'\cos 2\alpha\right] \qquad \text{(A-54)}$$

$$\nabla^2 z = -\frac{1}{\ell^2}u \qquad \text{(A-55)}$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{2\ell^2}(u + \frac{2}{\rho}z')\sin 2\alpha \qquad \text{(A-56)}$$

$$\frac{\partial}{\partial n_q}(\nabla^2 u) = -\frac{1}{\ell^3}u'\cos\varphi \qquad \text{(A-57)}$$

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = -\frac{1}{\ell^2}(u + \frac{2}{\rho}z')\cos 2\alpha \qquad \text{(A-58)}$$

$$\frac{\partial}{\partial n_q}\left(\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2}\right) = \frac{1}{\ell^3}\left[-u'\cos 2\alpha\cos\varphi + \frac{2}{\rho}(u + \frac{2}{\rho}z')\cos(2\alpha - \varphi)\right] \qquad \text{(A-59)}$$

$$\frac{\partial}{\partial n_q}\left(\frac{\partial^2 z}{\partial x \partial y}\right) = \frac{1}{2\ell^3}\left[-u'\sin 2\alpha\cos\varphi + \frac{2}{\rho}(u + \frac{2}{\rho}z')\sin(2\alpha - \varphi)\right] \qquad \text{(A-60)}$$

$$\frac{\partial}{\partial x}(\nabla^2 z) = \frac{1}{\ell^3}u'\cos\alpha \qquad \text{(A-61)}$$

$$\frac{\partial}{\partial y}(\nabla^2 z) = \frac{1}{\ell^3}u'\sin\alpha \qquad \text{(A-62)}$$

$$\frac{\partial}{\partial n_q}\left[\frac{\partial}{\partial x}(\nabla^2 z)\right] = \frac{1}{\ell^4}\left[z\cos\varphi\cos\alpha - \frac{1}{\rho}u'\cos(\alpha - \varphi)\right] \qquad \text{(A-63)}$$

$$\frac{\partial}{\partial n_q}\left[\frac{\partial}{\partial y}\nabla^2 z\right] = \frac{1}{\ell^4}\left[z\sin\varphi\cos\alpha - \frac{1}{\rho}u'\sin(\alpha - \varphi)\right] \qquad \text{(A-64)}$$

$$\frac{\partial^2 z}{\partial m_p \partial n_q} = \frac{1}{\ell^2}\left[u\cos\omega\cos\varphi + \frac{1}{\rho}z'\cos(\varphi + \omega)\right] \qquad \text{(A-65)}$$

A-II. Intrinsic Coordinates

It is often necessary to use the intrinsic coordinates s and n, that is, the arc length of the boundary and the length along the normal n to the boundary. In this case, the derivative with respect to s, generally, is not identified with that corresponding to the tangential direction t. For a function w, we have

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial t} \qquad \text{(A-66)}$$

Referring to Fig. A-2, we obtain

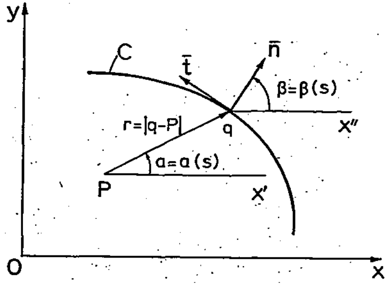

Fig.A-2.

$$\frac{\partial^2 w}{\partial s \partial n} = \frac{\partial}{\partial s}\left(\frac{\partial w}{\partial x}\cos\beta + \frac{\partial}{\partial y}\sin\beta\right)$$

$$= \frac{\partial}{\partial s}\left(\frac{\partial w}{\partial x}\right)\cos\beta + \frac{\partial}{\partial s}\left(\frac{\partial w}{\partial y}\right)\sin\beta + \frac{\partial w}{\partial x}\frac{\partial}{\partial s}(\cos\beta) + \frac{\partial w}{\partial y}\frac{\partial}{\partial s}(\sin\beta)$$

$$= \frac{\partial}{\partial t}\left(\frac{\partial w}{\partial x}\cos\beta + \frac{\partial w}{\partial y}\sin\beta\right) + \left(-\frac{\partial w}{\partial x}\sin\beta + \frac{\partial w}{\partial y}\cos\beta\right)\frac{\partial \beta}{\partial s}$$

$$= \frac{\partial^2 w}{\partial t \partial n} + K\frac{\partial w}{\partial t}$$

where

$$K = K(s) = \frac{\partial \beta}{\partial s}$$

is curvature of the boundary.

Thus, we obtain the following two relations

$$\frac{\partial^2 w}{\partial s \partial n} = \frac{\partial^2 w}{\partial t \partial n} + K\frac{\partial w}{\partial t} \qquad \text{(A-67)}$$

$$\frac{\partial^2 w}{\partial t \partial n} = \frac{\partial^2 w}{\partial s \partial n} - K\frac{\partial w}{\partial s} \qquad \text{(A-68)}$$

Moreover, using (A-66) we obtain

$$\frac{\partial^2 w}{\partial s^2} = \frac{\partial}{\partial s}\left(\frac{\partial w}{\partial t}\right) = \frac{\partial}{\partial s}\left(-\frac{\partial w}{\partial x}\sin\beta + \frac{\partial w}{\partial y}\cos\beta\right)$$

$$= -\frac{\partial}{\partial s}\left(\frac{\partial w}{\partial x}\right)\sin\beta + \frac{\partial}{\partial s}\left(\frac{\partial w}{\partial y}\right)\cos\beta - \frac{\partial w}{\partial x}\frac{\partial(\sin\beta)}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial}{\partial s}(\cos\beta)$$

$$= \frac{\partial}{\partial t}\left[-\frac{\partial w}{\partial x}\sin\beta + \frac{\partial w}{\partial y}\cos\beta\right] - \left[\frac{\partial w}{\partial x}\cos\beta + \frac{\partial w}{\partial y}\sin\beta\right]\frac{\partial \beta}{\partial s}$$

$$= \frac{\partial^2 w}{\partial t^2} - K\frac{\partial w}{\partial n}$$

hence,

$$\frac{\partial^2 w}{\partial s^2} = \frac{\partial^2 w}{\partial t^2} - K\frac{\partial w}{\partial n} \qquad \text{(A-69)}$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial s^2} + K\frac{\partial w}{\partial n} \qquad \text{(A-70)}$$

$$\nabla^2 w = \frac{\partial^2 w}{\partial n^2} + \frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial n^2} + \frac{\partial^2 w}{\partial s^2} + K\frac{\partial w}{\partial n} \qquad \text{(A-71)}$$

Similarly, using relations (A-67), (A-69), (A-18) and (A-25) we get

$$\frac{\partial(\cos\varphi)}{\partial s} = \frac{\partial}{\partial s}\left(\frac{\partial r}{\partial n_q}\right) = \frac{\partial^2 r}{\partial t_q \partial n_q} + K\frac{\partial r}{\partial t_q} \qquad \text{(A-72)}$$

$$= \frac{\cos\varphi\sin\varphi}{r} - K\sin\varphi$$

$$\frac{\partial(\sin\varphi)}{\partial s} = -\frac{\partial}{\partial s}\left(\frac{\partial r}{\partial t_q}\right) = -\frac{\partial^2 r}{\partial s^2} = -\frac{\partial^2 r}{\partial t_q^2} + K\frac{\partial r}{\partial n_q} \qquad \text{(A-73)}$$

$$= \frac{\cos^2\varphi}{r} + K\cos\varphi$$

For rectilinear boundaries $K(s)=0$ and it is always $\frac{\partial}{\partial s} \equiv \frac{\partial}{\partial t}$.

A P P E N D I X B

Computation of the term I_c for non-smooth boundaries

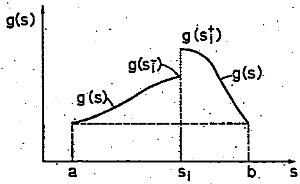

Fig.B-1. Discontinuous function with g(a)=g(b).

Consider a function g(s),defined on an interval (a,b), which at a point s_i has a discontinuity with a jump $[\![g]\!]_i = g(s_i^+) - g(s_i^-)$. Moreover, assume that

$$g(a)=g(b) \tag{B-1}$$

If $\varphi(s)$ is a continuous function defined on the interval (a,b) with

$$\varphi(a)=\varphi(b) \tag{B-2}$$

we have

$$\int_a^b g\frac{d\varphi}{ds}\,ds = \int_a^{s_i^-} g\frac{d\varphi}{ds}\,ds + \int_{s_i^+}^b g\frac{d\varphi}{ds}\,ds$$

$$= \left[g\varphi\right]_\alpha - \int_a^{s_i^-}\frac{dg}{ds}\,\varphi ds + \left[g\varphi\right]_{s_i^+}^b - \int_a^b \frac{dg}{ds}\,\varphi ds$$

$$=(g\varphi)_{s_i^-}-(g\varphi)_a+(g\varphi)_b-(g\varphi)_{s_i^+}-\int_a^b \frac{dg}{ds}\varphi ds$$

$$=(g\varphi)_{s_i^-}-(g\varphi)_{s_i^+}-\int_a^b \frac{dg}{ds}\varphi ds$$

$$=-[\![g\varphi]\!]_i-\int_a^b \frac{dg}{ds}\varphi ds \qquad \text{(B-3)}$$

When g(s) and φ(s) are defined on a closed curve C equation (B-1) is satisfied and relation (B-3) becomes

$$\int_C g\frac{d\varphi}{ds}ds=-[\![g\varphi]\!]_i-\int_C \frac{dg}{ds}\varphi ds \qquad \text{(a)}$$

(B-4)

or

$$\int_C \varphi\frac{dg}{ds}ds=-[\![g\varphi]\!]_i-\int_C \frac{d\varphi}{ds}g ds \qquad \text{(b)}$$

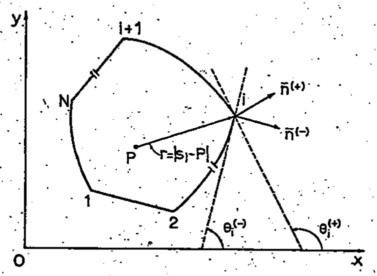

Fig.B-2. Boundary having N corners.

Notice, that when g is continuous $[\![g\varphi]\!]_i = 0$. Relation (B-4) and (B-5) can be used to carry out integration by parts when one of the functions in the integrand is discontinuous.

Referring to Fig. B-2, we see that the curvature of the boundary $K(s)$ and the normal derivatives $\frac{\partial w}{\partial n}$ and $\frac{\partial v}{\partial n}$ are discontinuous at corner points s_i. Thus, relations (B-4) can be used in integrating, by parts, the following terms of the boundary integral (I-2.10).

$$\int_C \frac{\partial w}{\partial n}\frac{\partial^2 v}{\partial s^2}ds = -\left[\!\!\left[\frac{\partial w}{\partial n}\frac{\partial v}{\partial s}\right]\!\!\right]_i - \int_C \frac{\partial^2 w}{\partial s \partial n}\frac{\partial v}{\partial s}ds$$

$$= -\left[\!\!\left[\frac{\partial w}{\partial n}\frac{\partial v}{\partial s}\right]\!\!\right]_i + \left[\!\!\left[v\frac{\partial^2 w}{\partial s\partial n}\right]\!\!\right]_i + \int_C v\frac{\partial}{\partial s}\left(\frac{\partial^2 w}{\partial s \partial n}\right)ds \qquad \text{(B-5a)}$$

$$\int_C \frac{\partial v}{\partial n}\frac{\partial^2 w}{\partial s^2}ds = -\left[\!\!\left[\frac{\partial v}{\partial n}\frac{\partial w}{\partial s}\right]\!\!\right]_i + \left[\!\!\left[w\frac{\partial^2 v}{\partial s\partial n}\right]\!\!\right]_i + \int_C w\frac{\partial}{\partial s}\left(\frac{\partial^2 v}{\partial s \partial n}\right)ds \qquad \text{(B-5b)}$$

$$\int_C w\frac{\partial}{\partial s}\left(K\frac{\partial v}{\partial s}\right)ds = -\left[\!\!\left[wK\frac{\partial v}{\partial s}\right]\!\!\right]_i - \int_C \frac{\partial w}{\partial s}K\frac{\partial v}{\partial s}ds \qquad \text{(B-5c)}$$

$$\int_C v\frac{\partial}{\partial s}\left(K\frac{\partial w}{\partial s}\right)ds = -\left[\!\!\left[vK\frac{\partial w}{\partial s}\right]\!\!\right]_i - \int_C \frac{\partial v}{\partial s}K\frac{\partial w}{\partial s}ds \qquad \text{(B-5d)}$$

Substituting equations (B-5a) to (B-5d) into relation (I-2.10) and denoting by

$$T = \frac{\partial^2}{\partial s \partial n} - K\frac{\partial}{\partial s} \qquad \text{(B-6)}$$

we get for a boundary with N corners

$$I_C = -(\nu-1)\sum_{i=1}^{N}\left[\!\!\left[vTw - wTv + \frac{\partial v}{\partial n}\frac{\partial w}{\partial s} - \frac{\partial w}{\partial n}\frac{\partial v}{\partial s}\right]\!\!\right]_i \qquad \text{(B-7)}$$

A P P E N D I X C.

Evaluation of the Kelvin functions

A method for approximating the Kelvin functions ker(x), kei(x) and their first derivatives ker'(x), kei'(x) has been presented by F.D.Burgoyne [87]. According to this method, the Kelvin functions are approximated to at least nine significant figures(*). This accuracy did not meet our computation needs. In order to increase the accuracy, the Kelvin functions are approximated with their finite expansions in Chebyshev polynomials [88]. Thus,

a) for $0<x<8$

$$\mathrm{ker}(x)=-(\gamma+\ln x/2)\mathrm{ber}(x)+(\pi/4)\mathrm{bei}(x)-(x/8)^4\sum_{n=0}^{\infty} e_n T_{2n}(x^2/64)$$

$$\mathrm{kei}(x)=-(\gamma+\ln x/2)\mathrm{bei}(x)-(\pi/4)\mathrm{ber}(x)+(x/8)^2\sum_{n=0}^{\infty} f_n T_{2n}(x^2/64)$$

(C-1)

$$\mathrm{ker}'(x)=-(\gamma+\ln x/2)\mathrm{ber}'(x)-x^{-1}\mathrm{ber}(x)+(\pi/4)\mathrm{bei}'(x)-$$

$$-(x/8)^3\sum_{n=0}^{\infty} g_n T_{2n}(x^2/64)$$

$$\mathrm{kei}'(x)=-(\gamma+\ln x/2)\mathrm{bei}'(x)-x^{-1}\mathrm{bei}(x)-(\pi/4)\mathrm{ber}'(x)+$$

$$+(x/8)\sum_{n=0}^{\infty} h_n T_{2n}(x^2/64)$$

(*) The IMSL Library uses this approximation for the Kelvin functions.

where

$$ber(x)=\sum_{n=0}^{\infty} a_n T_{2n}(x^2/64) \qquad bei(x)=\sum_{n=0}^{\infty} b_n T_{2n+1}(x^2/64)$$

$$ber'(x)=(x/8)\sum_{n=0}^{\infty} c_n T_{2n+1}(x^2/64) \qquad bei'(x)=(x/8)\sum_{n=0}^{\infty} d_n T_{2n}(x^2/64) \qquad \text{(C-2)}$$

$T_{2n}(x)$ and $T_{2n+1}(x)$ are the even and odd Chebyshef polynomials, respectively. The coefficients $a_n, b_n, c_n, d_n, e_n, f_n, g_n$ and h_n are given in Table C-1.

The Chebyshef polynomials are evaluated from the recursive formula.

$$T_n(x)=2xT_{n-1}(x)-T_{n-2}(x) \qquad n\geq 2$$

$$T_0=1 \qquad \text{(C-3)}$$

$$T_1=x$$

b) for $5\leq x$

$$ker(x)=Re\left[K_0(xe^{-i\pi/4})\right]$$

$$kei(x)=-Im\left[K_0(xe^{-i\pi/4})\right] \qquad \text{(C-4)}$$

$$ker'(x)=Re\left[e^{3i\pi/4}K_1(xe^{-i\pi/4})\right]$$

$$kei'(x)=-Im\left[e^{3i\pi/4}K_1(xe^{-i\pi/4})\right]$$

where

$$K_0(xe^{-i\pi/4})=(\pi/2x)^{\frac{1}{2}}e^{i(\pi/8+u)}e^{-u}\sum_{n=0}^{\infty}p_nT_n^*(5/x)$$

$$K_1(xe^{-i\pi/4})=(\pi/2x)^{\frac{1}{2}}e^{i(\pi/8+u)}e^{-u}\sum_{n=0}^{\infty}q_nT_n^*(5/x)$$

(C-5)

$$u=2^{-\frac{1}{2}}x, \quad x\geq 5$$

$$p_n=R(p_n)+iI(p_n), \quad q_n=R(q_n)+iI(q_n)$$

$T_n^*(x)$ are the Chebyshev polynomials of the second kind and are related to T_n with the relation

$$T_n^*=T_n(2x-1) \tag{C-6}$$

The complex coefficients p_n and q_n are given in Table C-I.$R(p_n)$ and $I(p_n)$ denote the real and imaginary part of p_n.

TABLE C-I

CHEBYSHEV COEFFICIENTS FOR ber(x), bei(x), ker(x), kei(x), AND THEIR DERIVATIVES

n	a_n	n	b_n
0	2.25521 15482 79523 90138	0	−29.34949 10970 21269 22722
1	10.84058 01738 13068 20665	1	−8.98868 87413 38297 57684
2	8.71271 74101 86675 55916	2	3.46690 09758 41511 39894
3	−0.85344 63696 95052 22986	3	−0.14735 80153 21209 28048
4	0.01904 82639 34734 39291	4	0.00192 21031 54268 04953
5	−0.00015 59976 15956 17446	5	−0.00001 04178 99277 03635
6	0.00000 05829 62923 95910	6	0.00000 00277 43180 21356
7	−0.00000 00011 36930 89629	7	−0.00000 00000 40549 17690
8	0.00000 00000 01270 22191	8	0.00000 00000 00035 27916
9	−0.00000 00000 00000 87119	9	−0.00000 00000 00000 01933
10	0.00000 00000 00000 00039	10	0.00000 00000 00000 00001

n	c_n	n	d_n
0	25.78109 24425 89600 75371	0	−9.99884 34643 81679 05729
1	14.94051 22687 76532 54706	1	−5.32294 13802 52723 50097
2	−2.48492 25515 96818 57127	2	8.16009 17317 54580 36429
3	0.07541 65574 88338 11832	3	−0.50716 07078 49198 13307
4	−0.00077 64982 50599 45331	4	0.00859 23457 75034 34863
5	0.00000 34898 29181 41896	5	−0.00005 71184 19171 87424
6	−0.00000 00079 48362 33566	6	0.00000 01800 26851 82568
7	0.00000 00000 10153 93838	7	−0.00000 00003 03819 56248
8	−0.00000 00000 00007 83687	8	0.00000 00000 00299 26429
9	0.00000 00000 00000 00387	9	−0.00000 00000 00000 18356
		10	0.00000 00000 00000 00007

TABLE C-I (*Continued*)

n	e_n	n	f_n
0	5.03749 13279 40243 09626	0	−34.11314 87924 14490 76243
1	−82.13362 54977 30465 74995	1	−33.37426 03178 96596 83482
2	8.61760 65894 62441 70191	2	15.96104 66759 83989 93950
3	−0.20943 20427 43605 42250	3	−0.76688 42692 52450 82998
4	0.00183 72709 07812 58606	4	0.01089 43699 35866 72405
5	−0.00000 72563 87142 14307	5	−0.00006 29825 20880 94100
6	0.00000 00148 11082 80775	6	0.00000 01765 55262 95937
7	−0.00000 00000 17194 29932	7	−0.00000 00002 69199 99368
8	0.00000 00000 00017 18762	8	0.00000 00000 00242 40280
9	−0.00000 00000 00000 00557	9	−0.00000 00000 00000 13718
10	0.00000 00000 00000 00000	10	0.00000 00000 00000 00005

n	g_n	n	h_n
0	−30.24095 21143 27190 84320	0	−10.38306 52726 52518 42353
1	−65.51939 55565 94624 78265	1	4.63400 89303 76357 74285
2	12.30542 66464 04620 11634	2	17.69391 91618 10822 91434
3	−0.41155 11950 45437 01405	3	−1.29878 38466 03610 97755
4	0.00455 69840 80054 95188	4	0.02419 84772 00411 21282
5	−0.00002 16804 15192 22712	5	−0.00017 21444 83306 79904
6	0.00000 00517 18527 18270	6	0.00000 05719 22142 65045
7	−0.00000 00000 68679 74648	7	−0.00000 00010 07561 98801
8	0.00000 00000 00054 79415	8	0.00000 00000 01028 97780
9	−0.00000 00000 00000 02784	9	−0.00000 00000 00000 65109
10	0.00000 00000 00000 00001	10	0.00000 00000 00000 00027

n	$R(p_n)$	n	$I(p_n)$
0	0.99125 27590 13757 71500	0	−0.00790 65568 61206 00284
1	−0.00870 45775 92248 60627	1	−0.00761 52111 06332 59121
2	0.00004 88558 00279 13133	2	0.00028 01428 53466 02342
3	0.00000 54968 42797 59179	3	−0.00001 08679 91455 73912
4	−0.00000 06378 02857 72398	4	0.00000 03430 71791 35686
5	0.00000 00537 88925 26933	5	0.00000 00047 78727 85761
6	−0.00000 00038 03665 36911	6	−0.00000 00028 77466 56739
7	0.00000 00001 80405 87493	7	0.00000 00004 46278 05555
8	0.00000 00000 07409 37643	8	−0.00000 00000 53463 29543
9	−0.00000 00000 04904 46940	9	0.00000 00000 05345 74325
10	0.00000 00000 00807 04345	10	−0.00000 00000 00376 17314
11	−0.00000 00000 00126 91627	11	−0.00000 00000 00005 44569
12	0.00000 00000 00016 78693	12	0.00000 00000 00009 39965
13	−0.00000 00000 00001 74350	13	−0.00000 00000 00002 53051
14	0.00000 00000 00000 07994	14	0.00000 00000 00000 50860
15	0.00000 00000 00000 02512	15	−0.00000 00000 00000 08650
16	−0.00000 00000 00000 01072	16	0.00000 00000 00000 01240
17	0.00000 00000 00000 00280	17	−0.00000 00000 00000 00130
18	−0.00000 00000 00000 00060	18	0.00000 00000 00000 00001
19	0.00000 00000 00000 00011	19	0.00000 00000 00000 00005
20	−0.00000 00000 00000 00002	20	−0.00000 00000 00000 00002

n	$R(q_n)$	n	$I(q_n)$
0	1.02638 45771 27877 23444	0	0.02493 11563 84580 59657
1	0.02632 27379 40162 54838	1	0.02443 02125 32635 26335
2	−0.00007 11145 12850 82471	2	−0.00048 50015 54807 11200
3	−0.00000 83456 68118 73449	3	0.00001 55186 22052 84493
4	0.00000 08572 29001 62198	4	−0.00000 04354 08068 17209
5	−0.00000 00676 79783 56927	5	−0.00000 00075 27108 70232
6	0.00000 00045 60014 86316	6	0.00000 00036 02315 70219
7	−0.00000 00002 03852 67136	7	−0.00000 00005 34668 13801
8	−0.00000 00000 09832 00251	8	0.00000 00000 62439 08190
9	0.00000 00000 04715 61149	9	−0.00000 00000 06062 51798
10	−0.00000 00000 00924 49058	10	0.00000 00000 00411 47493
11	0.00000 00000 00142 72392	11	0.00000 00000 00008 52051
12	−0.00000 00000 00018 56911	12	−0.00000 00000 00010 79488
13	0.00000 00000 00001 88902	13	0.00000 00000 00002 83532
14	−0.00000 00000 00000 07963	14	−0.00000 00000 00000 56177
15	−0.00000 00000 00000 02892	15	0.00000 00000 00000 09441
16	0.00000 00000 00000 01189	16	−0.00000 00000 00000 01336
17	−0.00000 00000 00000 00307	17	0.00000 00000 00000 00138
18	0.00000 00000 00000 00065	18	0.00000 00000 00000 00000
19	−0.00000 00000 00000 00012	19	−0.00000 00000 00000 00005
20	0.00000 00000 00000 00002	20	0.00000 00000 00000 00002
		21	−0.00000 00000 00000 00001

In table C-II the values of the functions ker(x), kei(x), ker′(x), kei′(x) are given as computed from the two different methods of approximation. For comparison the corresponding values are listed as they are given in the "Tables of the Bessel-Kelvin functions Ber,Bei,Ker,Kei and their derivatives" by H.H. Lowell [89]. Lowell has used normal series (ascending powers of the argument) for ker and kei and their derivatives over the argument range o to 8.89; beyond that he has used asymptotic series. Lowell's Tables are to the author's knowledge the most accurate. As we can see from the Table C-II there is an excellent agreement between the values computed by using Chebyshef series and these from Lowell's tables.

TABLE C-II

Values of the functions ker(x),kei(x),ker′(x),kei′(x). Computed from Chebyshef series expansion, from Lowell's Tables and Burgoyne's approximation.

	Chebyshef	Lowell	Burgoyne
x	ker(x)		
1	.28670 62087 283+00	.28670 62087 283+00	.28670 62087 280+00
2	-.41664 51399 151-01	-.41664 51399 151-01	-.41664 51399 039-02
3	-.67029 23330 380-01	-.67029 23330 380-01	-.67029 23330 399-01
4	-.36178 84789 955-01	-.36178 84789 95 -01	-.36178 84790 288-01
5	-.11511 72719 949-01	-.11511 72719 95 02	-.11511 72720 023-01
6	-.65303 75083 473-03	-.65303 75084 -03	-.65303 75043 060-03
7	.19220 21568 665-02	.19220 21569 -02	.19220 21563 925-02
8	.14858 34068 519-02	.14858 34068 -02	.14858 34073 692-02
9	.63716 41911 213-03	.63716 4190 -03	.63716 41897 911-03
10	.12946 63302 148-03	.12946 63302 -03	.12946 63247 367-03
91	-.49043 87640 680-29	-.49043 87640 680-29	-.49043 87641 642-29
92	-.63049 96901 550-29	-.63049 96901 549-29	-.63049 96901 908-29
93	-.35219 43633 927-29	-.35219 43633 927-29	-.35219 43633 964-29
94	-.11098 55929 518-29	-.11098 55929 512-29	-.11098 55929 494-29
95	.19523 67685 910-31	.19523 67685 908-31	.19523 67691 084-31
96	.28156 48455 459-30	.28156 48455 458-30	.28156 48455 704-30
97	.20530 26701 591-30	.20530 26701 591-30	.20530 26701 650-30
98	.85377 17615 684-31	.85377 17615 684-31	.85377 17615 523-31
99	.14277 06180 246-31	.14277 06180 246-31	.14277 06179 996-31
100	-.98984 17996 731-32	-.98984 17996 731-32	-.98984 17998 226-32

x	kei(x)		
1	-.49499 46365 187+00	-.49499 46365 187+00	-.49499 46365 167+00
2	-.20240 00677 647+00	-.20240 00677 647+00	-.20240 00677 632+00
3	-.51121 88404 599-01	-.51121 88404 599-01	-.51121 88404 613-01
4	.21983 99294 973-02	.21983 99295 0 -02	.21983 99292 972-02
5	.11187 58650 987-01	.11187 58650 99 01	.11187 58650 948-01
6	.72164 91544 425-02	.72164 91544 4 -02	.72164 91546 180-02
7	.27003 65107 986-02	.27003 65108 -02	.27003 65105 588-02
8	.36958 39561 260-03	.36958 3956 -03	.36958 39579 366-03
9	-.31915 29161 913-03	-.31915 2917 -03	-.31915 29166 270-03
10	-.30752 45690 881-03	-.30752 45690 -03	-.30752 45703 589-03
91	-.14051 81565 231-28	-.14051 15652 312-28	-.14051 81563 232-28
92	-.36762 39700 148-29	-.36762 39700 147-29	-.36762 39699 845-29
93	.63812 42733 258-30	.63812 42733 258-30	.63812 42725 516-30
94	.13600 68579 577-29	.13600 68579 577-29	.13600 68579 675-29
95	.86077 16465 875-30	.86077 16465 875-30	.86077 16466 043-30
96	.31475 70142 482-30	.31475 70142 482-30	.31475 70142 380-30
97	.27652 61087 606-31	.27652 61087 606-31	.27652 61086 452-31
98	-.55113 93389 310-31	-.55113 93389 310-31	-.55113 93388 923-31
99	-.47764 74436 860-31	-.47764 74436 859-31	-.47764 74437 039-31
100	-.22363 35526 041-31	-.22365 35526 041-31	-.22365 35526 028-31

x	ber'(x)		
1	-.69460 38911 007+00	-.69460 38911 006+00	-.69460 38910 982+00
2	-.10660 09658 811+00	-.10660 09658 810+00	-.10660 09658 802+00
3	.21476 18686 772-01	.21476 18686 773-01	.21476 18686 767-01
4	.31478 48982 209-01	.31478 48982 21 -01	.31478 48982 185-01
5	.17193 40382 841-01	.17193 40382 84 -01	.17193 40382 834-01
6	.56317 09259 650-02	.56317 09259 6 -02	.56317 09259 745-02
7	.42050 94672 958-03	.42050 9467 -03	.42050 94673 965-03
8	-.87972 40992 200-03	-.87972 4099 -03	-.87972 40990 988-03
9	-.71123 08637 467-03	-.71123 0864 -03	-.71123 08636 005-03
10	-.31559 69344 209-03	-.31558 69345 -03	-.31559 69342 580-03
91	-.64414 61211 989-29	-.64414 61211 989-29	-.64414 61212 723-29
92	.18929 74692 904-29	.18929 74692 904-29	.18929 74692 329-29
93	.29605 17845 398-29	.29605 17845 398-29	.29605 17845 147-29
94	.17524 06160 934-29	.17524 06160 933-29	.17524 06160 885-29
95	.59475 78851 601-30	.59475 78851 600-30	.59475 78851 833-30
96	.22009 59877 176-31	.22009 59877 176-31	.22009 59880 065-31
97	-.12667 36384 517-30	-.12667 36384 516-30	-.12667 36384 358-30
98	-.99777 52881 588-31	-.99777 52881 588-31	-.99777 52881 101-31
99	-.43942 58417 050-31	-.43942 58417 049-31	-.43942 58417 066-31
100	-.87662 46185 882-32	-.87662 46185 883-32	-.87662 46187 160-32

x	kei'(x)		
1	.35236 99133 362+00	.35236 99133 361+00	.35236 99133 359+00
2	.21980 79099 196+00	.21980 79099 196+00	.21980 79099 193+00
3	.92043 05048 029-01	.92043 05048 030-01	.92043 05048 032-01
4	.23910 61379 169-01	.23910 61379 17 01	.23910 61379 230-01
5	-.81998 65436 219-03	-.81998 65436 -03	-.81998 65435 512-03
6	-.52239 20860 983-02	-.52239 20861 0 -02	-.52239 20861 787-02
7	-.34595 08641 021-02	-.34595 08641 -02	-.34595 08640 209-02
8	-.13363 12914 860-02	-.13363 12915 -02	-.13363 12915 687-02
9	-.20807 94171 277-03	-.20807 9418 -03	-.20807 94165 675-03
10	.14091 38376 126-03	.14091 38377 -02	.14091 38392 258-03
91	.13481 16925 980-28	.13481 16925 980-28	.13481 16925 885-28
92	.70778 06107 907-29	.70778 06107 906-29	.70778 06107 785-29
93	.20357 79421 732-29	.20357 79421 731-29	.20357 79421 869-29
94	-.18413 69291 682-30	-.18413 69291 683-30	-.18413 69290 376-30
95	-.62698 49293 957-30	-.62698 49293 956-30	-.62698 49293 317-30
96	-.42330 22331 147-30	-.42330 22331 146-30	-.42330 22330 984-30
97	-.16486 84392 566-30	-.16486 84392 566-30	-.16486 84392 598-30
98	-.21119 43197 352-31	-.21119 43197 352-31	-.21119 43197 971-31
99	.23920 04588 527-31	.23920 04588 526-31	.23920 04588 143-31
100	.22925 64824 625-31	.22925 64824 625-31	.22925 64824 489-31

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