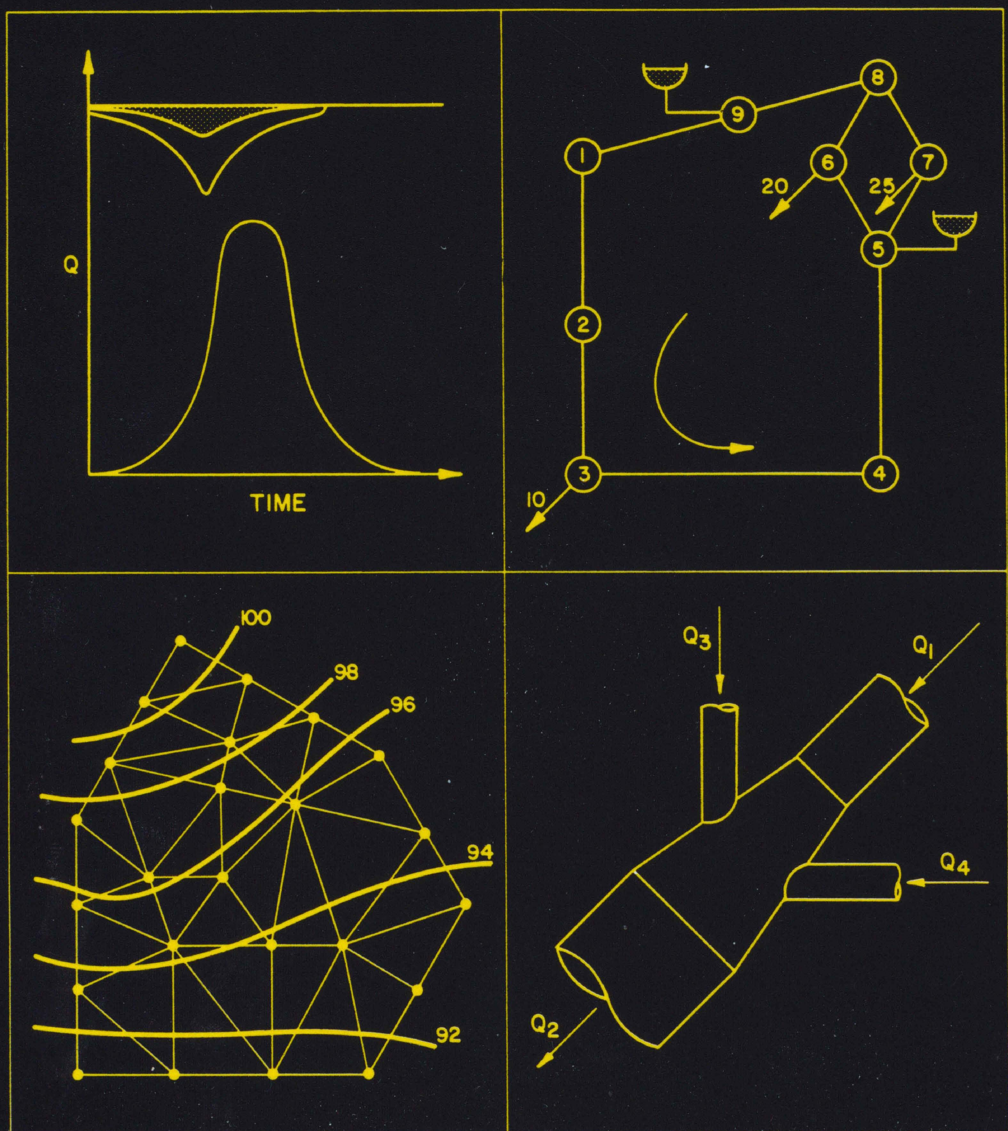
COMPUTER METHODS IN WATER RESOURCES



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CHAPTER 1

INTRODUCTION

1.1. Discussion

Recently there has been a significant increase in the number of practicing civil engineers using computer programs for the preparation of water resource related studies. With the advent of inexpensive microcomputer systems, detailed hydrology and hydraulics studies can be prepared at a fraction of the cost of analysis prepared by hand calculations. Additionally, with the availability of software prepared especially for microcomputers, new advances in water resources are readily distributed for use in practical design.

The main objective of this book is to provide both a summary of the basic principles used in water resources related engineering projects, and a collection of FORTRAN computer programs to apply these principles to a microcomputer system. The book focuses upon the following six major fields:

1. Watershed hydrology for urbanized watersheds,

2. Channel hydraulics for storm drain pipeline systems,

3. Water distribution systems,

4. Unsteady and steady flow in open channels,

5. Groundwater flow modeling,

6. Groundwater contamination modeling.

1.2. Presentation

This book has several features that should appeal to both the student or practicing civil engineer who wishes to advance his skills and prepare a comprehensive software library for use on a microcomputer system which supports FORTRAN. These features include:

1. Review of fundamental principles employed in the subject analysis procedures,

2. FORTRAN computer programs tailored for use on a microcomputer system,

3. Detailed example problems,

4. Comprehensive program documentation including, when appropriate, user-friendly input form sheets for display on the computer terminal.

It is stessed that the included software is currently in extensive use by small and large civil engineering firms, colleges, and public agencies. Several of the programs are utilized for county-wide water resources planning purposes and flood control engineering.

CHAPTER 2

A SYNTHETIC UNIT HYDROGRAPH MODEL

2.1. Introduction

In this chapter, a synthetic unit hydrograph methodology is developed and the associated computer code presented for use on microcomputers. Such a synthetic runoff hydrograph approach may be used for generating estimates of the peak flow rates needed for the design of flood control channels, and estimates for a time distribution of runoff volume for the design of detention basins.

2.1.1. Background

The unit hydrograph method is a synthetic hydrograph approach initially advanced by L. K. Sherman (1932). The keystone of the method is the assumption that watershed discharge is related to the total volume of runoff, and that the time factors which affect the unit hydrograph shape are invariant. The basic unit hydrograph theory was extended by F. F. Snyder (1938) to transpose *storm rainfall-runoff relationships* from gaged watersheds to hydrologically and geographically similar watersheds which lack rainfall-runoff stream gage data. The basic assumptions used in this later work are that the watershed rainfall-runoff relationships are functions of watershed area, slope and certain shape factors. The method is used to estimate a time distribution of runoff accumulating at the watershed downstream point of concentration when stream gage data is either unavailable or inadequate to provide a sound statistical analysis.

To determine the rainfall-runoff relationships to be transposed to ungaged watersheds, stream gage records are studied for various types and sizes of gaged watersheds. For example, the U.S. Army Corps of Engineers (Los Angeles Office) has determined several runoff time-distribution patterns for watersheds in the State of California. These relationships provide a basis for transposing to ungaged watersheds a characteristic time distribution of runoff which is the average distribution for several similar watershed basins. This approach is considered applicable when watersheds are physiographically and hydrologically similar. In Southern California, the counties of Orange, Riverside, and San Bernardino (which together represent a vast spectrum of flood control conditions) have successfully utilized this approach in the development of county-wide flood control facilities. Because the methodology used in the California watersheds would apply elsewhere (with appropriate modifications for local watershed rainfall-runoff relationships), the text and supplemental computer programs are based on the California rainfall-runoff relationships with the intent that the programs can be easily modified by the user to accomodate other regional watershed runoff tendencies.

Although there are several theoretical shortcomings associated with the unit hydrograph approach (such as the assumption of a linear system where runoff hydrographs resulting from a unit period of effective rainfall can be directly summed), the general approach continues to be widely used throughout the United States as a runoff synthesis method for ungaged watersheds. This chapter will examine the general unit hydrograph approach and several currently used variations of the method. A FORTRAN computer program is provided for a severe design storm condition (Hromadka et al., 1983a) which is readily adaptable to currently available microcomputers.

The unit hydrograph approach involves several assumptions which are imprecise approximations of the corresponding hydrologic processes. These basic assumptions are that (1) the critical storm rainfall pattern is uniformly distributed throughout the watershed, (2) there exists a direct proportionality between watershed runoff and the effective rainfall volume, (3) for any volume of effective rainfall occuring within a specified duration, the resulting runoff hydrograph is of a constant duration, and (4) the basin unit hydrograph is invariant throughout the critical design storm. The requirement that the watershed runoff is proportional to the effective rainfall has a direct analogy to a linear systems approach. Consequently, the unit hydrograph method can be considered a black-box modeling approach where the major characteristics of the model are determined by correlating the model output (runoff hydrograph) to the input data (rainfall records). Although the lumped-system model produces only approximations of the complex hydrologic characteristics of the watershed, its use continues to be widespread due to the ease of application. However, all of the several individual processes involved in the total watershed hydrologic system have an associated probabilistic relationship which represents the changing of the variable with respect to time and with respect to the fluctuation of the other variables. Thus, to incorporate the diverse probability effects into a lumped-system or a more sophisticated distributed parameter model, the general model has to be structured towards statistical interpretation of model results and an associated interval of confidence. For studies involving a severe critical design storm over a highly urbanized watershed, however, several of the variables may be eliminated from the hydrologic system model and the corresponding uncertainty is significantly reduced. For this type of condition, the unit hydrograph approach provides a runoff hydrograph which can be successfully used for flood control design purposes in watersheds which are hydrologically similar to gaged watersheds which have had unit hydrographs determined. A detailed analysis of the sensitivity of the unit hydrograph approach to the uncertainty associated to the precise definition of each model input parameter is given in Garen and Burges (1981). In that study, it was shown that the model output of the runoff hydrograph is very sensitive to variations in the input parameter set. The level of discretization of the watershed into smaller subareas to be linked together in a link-node model of the watershed is addressed in Zaghloul (1981) and includes the sensitivity effects of including the various routing processes in a complex model. The basic linearity assumption introduces theoretical difficulties which are discussed in Helweg et al. (1982). That study proposes methods for incorporating model memory and nonlinearity effects into a complex watershed model. Although the statistical component and the nonlinearity element should be included into the general runoff hydrograph generator, the difficulties associated with applying such sophistications in the general flood control study environment usually preclude their inclusion into a local agency's hydrology design criteria.

2.1.2. Terminology

In order to discuss the development of unit hydrographs for both gaged and ungaged watersheds, the following definitions are presented for the several terms:

Effective Rainfall: the total rainfall less infiltration losses, evaporation, transpiration, absorption, and detention. This part of the rainfall runs off the watershed surface in a relatively brief time period.

Unit Hydrograph: the unit hydrograph for a point of concentration on a watershed is a hydrograph showing the time distribution of runoff which results from a unit one inch of effective rainfall over the entire tributary watershed. The unit effective rainfall is assumed to occur as a constant rainfall with respect to both space and time throughout the unit duration. Figure 2.1 illustrates the general formulation of the unit hydrograph.

Distribution Graph: the distribution graph is a unit hydrograph whose ordinates are expressed in terms of percent of ultimate discharge. A distribution graph is generally developed as a block graph with each block representing its associated percent of unit runoff which occurs during the specified unit time period. The unit time used in the distribution graph is identical to the unit time specified for the unit hydrograph.

Summation Hydrograph: the summation hydrograph for a point of concentration is a hydrograph showing the time distribution of the rates of runoff that would result from a continuous series of unit effective rainfalls over the tributary watershed. The ordinates are expressed as rate of runoff in percent of the ultimate rate of runoff.

Lag: the watershed lag is the time (hours) from the beginning of a continuous series of unit effective rainfalls over the watershed to the instant when the rate of resulting runoff at the point of concentration reaches 50 percent of the ultimate rate of runoff. Another definition for lag is the time from the center of mass of the effective rainfall to the peak of the corresponding runoff hydrograph. Some hydrologists define lag as the time from the effective rainfall center of mass to the runoff hydrograph center of mass. In this chapter, however, the first definition of lag will be used for both the theoretical and computer program development.

Ultimate Discharge: the ultimate discharge (or the ultimate rate of runoff) is the maximum rate of watershed runoff which can result from a specified effective rainfall intensity. Ultimate discharge occurs when the rate of runoff on the summation hydrograph is equivalent to the rate of effective rainfall. For a unit effective rainfall intensity of one inch occurring in a unit interval of one hour, the ultimate discharge is 645 cfs for every square mile of watershed.

S–Graph: the S-graph is a summation hydrograph developed by plotting watershed discharge in percent of ultimate discharge as a function of time expressed in percent of watershed lag. Figure 2.1 illustrates each of the above unit hydrograph concepts and definitions.

2.2. Determination of Synthetic Distribution Graphs

In order to develop synthetic distribution graphs, adequate storm rainfall and watershed runoff information is required for several streams in the vicinity of the watershed under study. The distribution graphs for each of the gaged streams can be determined by trial and error attempts to duplicate the runoff hydrographs resulting from severe storm events. The derived distribution graphs are then verified by using them to reproduce runoff hydrographs for other major storm events. In this procedure, the watershed is assumed to be a linear system which ignores the variation of the unit hydrograph during the storm event. Models to approximately include memory and other nonlinearity effects have been studied by Helweg et al. (1982), among others.

It is assumed that the drainage areas within a given region are physiographically and hydrologically similar. Because no two drainage areas have identical hydrologic characteristics, the corresponding rainfall-runoff patterns are dissimilar and the distribution graphs will differ. Therefore, a direct transposition of distribution graphs between watersheds is usually

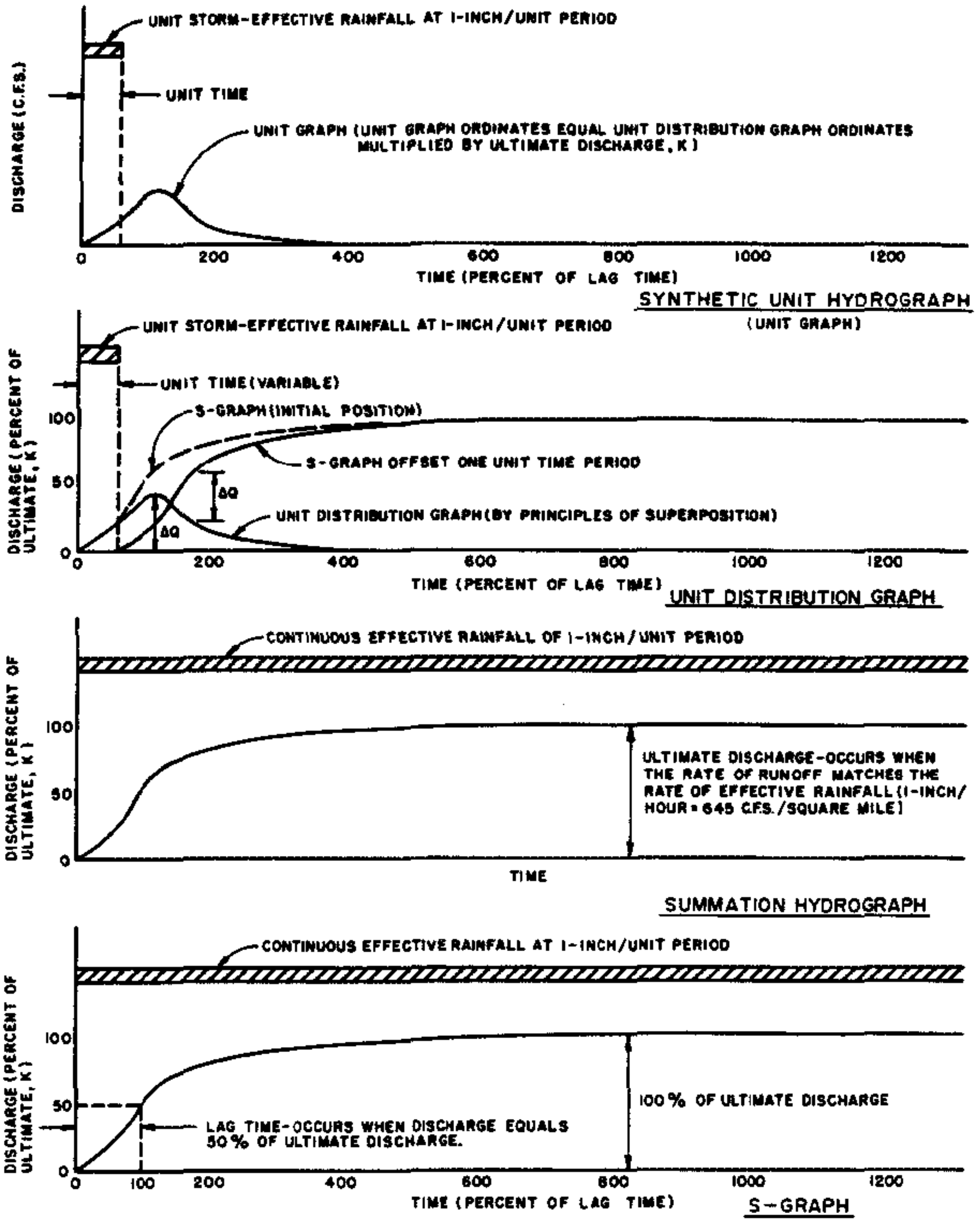


Fig. 2.1. Derivation of a Synthetic Unit Hydrograph.

not feasable. However, most distribution graphs exhibit certain characteristics which appear to be related to the empirical factor of watershed lag. Details for determining watershed lag for watersheds where the time distribution of runoff is known and for the use in developing synthetic distribution graphs are discussed in the following steps:

(1). Summation Hydrograph. The first step in determining watershed lag is the development of a characteristic summation hydrograph, which is the runoff hydrograph that results from a continuous and uniform sequence of unit effective rainfalls. The ordinates of the summation hydrograph are expressed as discharge in percent of ultimate discharge. The summation hydrograph for a point of concentration is also determined by adding a continuous series of identical distribution graphs each offset by one unit period. By examination, the time required to reach the ultimate discharge is equal to the length of the distribution graph base less one unit period.

(2). Lag. Watershed lag can be defined as the time from the beginning of unit effective rainfall to the instant that the summation hydrograph reaches 50 percent of ultimate discharge. When the lags determined from summation hydrographs for several gaged watersheds are correlated to the hydrologic characteristics of the watersheds, an empirical relationship is usually apparent. This relationship can then be used to estimate lag for ungaged watersheds. Analysis of watershed lags estimated for several types of watersheds indicate that a lag factor is expressed by

$$\text{lag} = C_t L^m L_c{}^m S^{-m/2} \tag{2.1}$$

where

- lag = watershed lag in hours
- C_t = a constant
- L = length of longest watercourse (miles)
- L_c = length along longest watercourse, measured from the point of concentration to a point opposite the watershed area centroid
- S = mean watershed slope along watercourse (feet/mile)
- m = a constant

For Southern California watersheds, it is assumed that

$$\begin{aligned} n^* &= \text{a basin factor (see Fig. 2.2)} \\ m &= 0.38 \\ C_t &= 24n^* \end{aligned} \tag{2.2}$$

The basin factors of Fig. 2.2 were determined by the U.S. Army Corps of Engineers by duplicating major storm runoff hydrographs from several Southern California watersheds. The relationship between watershed lag and Eq. (2.1) is illustrated in the plot of Appendix A. From (2.1), the synthetic unit hydrograph approach is a strong function of the watershed lag, and a wide range of results are generated by a variation of the basin factor.

(3). S-Graph. After lag factors are determined for several gaged watersheds, the next step in determining synthetic distribution graphs is the development of S-graphs, which are summation hydrographs modified so that the percent of ultimate discharge is plotted against time expressed in percent of watershed lag. The derivation of a S-graph is identical to the development of a summation hydrograph except that the relationships are now expressed with respect to watershed lag. Four S-graphs assumed applicable in Southern California watersheds are shown in Figs. 2.3a,b,c,d. These S-graphs are for use in watersheds classified as valley, foothill, mountain, or desert. According to the definitions, each S-graph reaches 50 percent of ultimate discharge at 100 percent of watershed lag.

(4). Application of Lag and S-Graphs. To use the S-graphs, a watershed lag is estimated from Figs. 2.1 and 2.2. A unit period is selected (usually at 15 to 25 percent of the watershed lag time) and amassed unit periods are expressed as accumulated percentages of the watershed lag. These percentages of lag are used for superimposing a block graph on the selected S-graph and the resulting block graph pattern is used in determining the accumulated mean percentage of ultimate discharge for each accumulated unit period. Finally, the incremental mean percentage of ultimate discharge for each unit period is estimated by a series of successive subtractions. The following section describes in detail the discussed procedure as applied in developing a watershed unit hydrograph.

2.3. Development of a Synthetic Unit Hydrograph

For watersheds where stream gage data is inadequate for statistical analysis, a synthetic unit hydrograph may be used to approximately describe the time distribution of runoff gathering at the point of concentration. A method for developing a synthetic unit hydrograph is described in the following steps:

(1). Estimate the watershed lag using topographic information, Eq. 2.1, and the appropriate basin factors (such as Fig. 2.2 for Southern California watersheds).

(2). Select a unit period duration. This unit period will be the time base for describing unit rainfalls and the corresponding runoff hydrographs. Usually, 15 to 25 percent of the watershed lag is adequate for computational purposes.

n^* = 0.015

1. Drainage area has fairly uniform, gentle slopes
2. Most watercourses either improved or along paved streets
3. Groundcover consists of some grasses - large % of area impervious
4. Main water course improved channel or conduit

n^*= 0.020

1. Drainage area has some graded and non-uniform, gentle slopes
2. Over half of the area watercourses are improved or paved streets
3. Groundcover consists of equal amount of grasses and impervious area
4. Main watercourse is partly improved channel or conduit and partly greenbelt (see n^*= 0.025)

n^* = 0.025

1. Drainage area is generally rolling with gentle side slopes
2. Some drainage improvements in the area - streets and canals
3. Groundcover consists mostly of scattered brush and grass and small % impervious
4. Main watercourse is straight channels which are turfed or with stony beds and weeds on earth bank (greenbelt type)

n^* = 0.030 (Foothill Area)

1. Drainage area is generally rolling with rounded ridges and moderate side slopes
2. No drainage improvements exist in the area
3. Groundcover includes scattered brush and grasses
4. Watercourses meander in fairly straight, unimproved channels with some boulders and lodged debris

n^* = 0.040 (Foothill Area)

1. Drainage area is composed of steep upper canyons with moderate slopes in lower canyons
2. No drainage improvements exist in the area
3. Groundcover is mixed brush and trees with grasses in lower canyons
4. Watercourses have moderate bends and are moderately impeded by boulders and debris with meandering courses

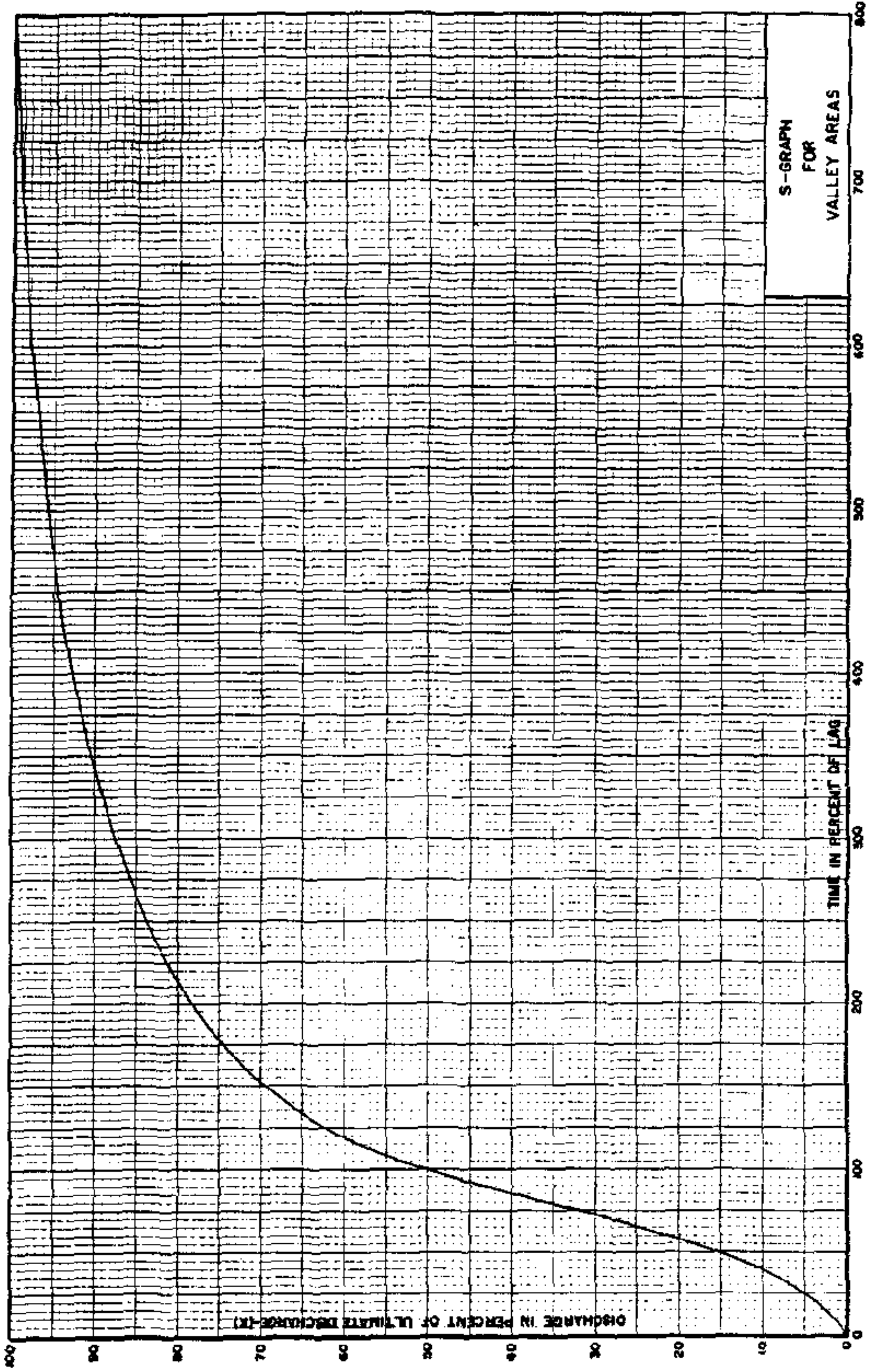
n^* = 0.050 (Mountain Areas)

1. Drainage area is quite rugged with sharp ridges and steep canyons
2. No drainage improvements exist in the area
3. Groundcover, excluding small areas of rock outcrops, includes many trees and considerable underbrush
4. Watercourses meander around sharp bends, over large boulders and considerable debris obstruction

n^* = 0.200

1. Drainage area has comparatively uniform slopes
2. No drainage improvements exist in the area
3. Groundcover consists of cultivated crops or substantial growths of grass and fairly dense small shrubs, cacti, or similar vegetation
4. Surface characteristics are such that channelization does not occur

Fig. 2.2. Basin n Parameter Descriptions.*


S-GRAPH
FOR
VALLEY AREAS
TIME IN PERCENT OF LAG
0
100
200
300
400
500
600
700
800
DISCHARGE IN PERCENT OF ULTIMATE DISCHARGE-(K)
0
10
20
30
40
50
60
70
80
90
100

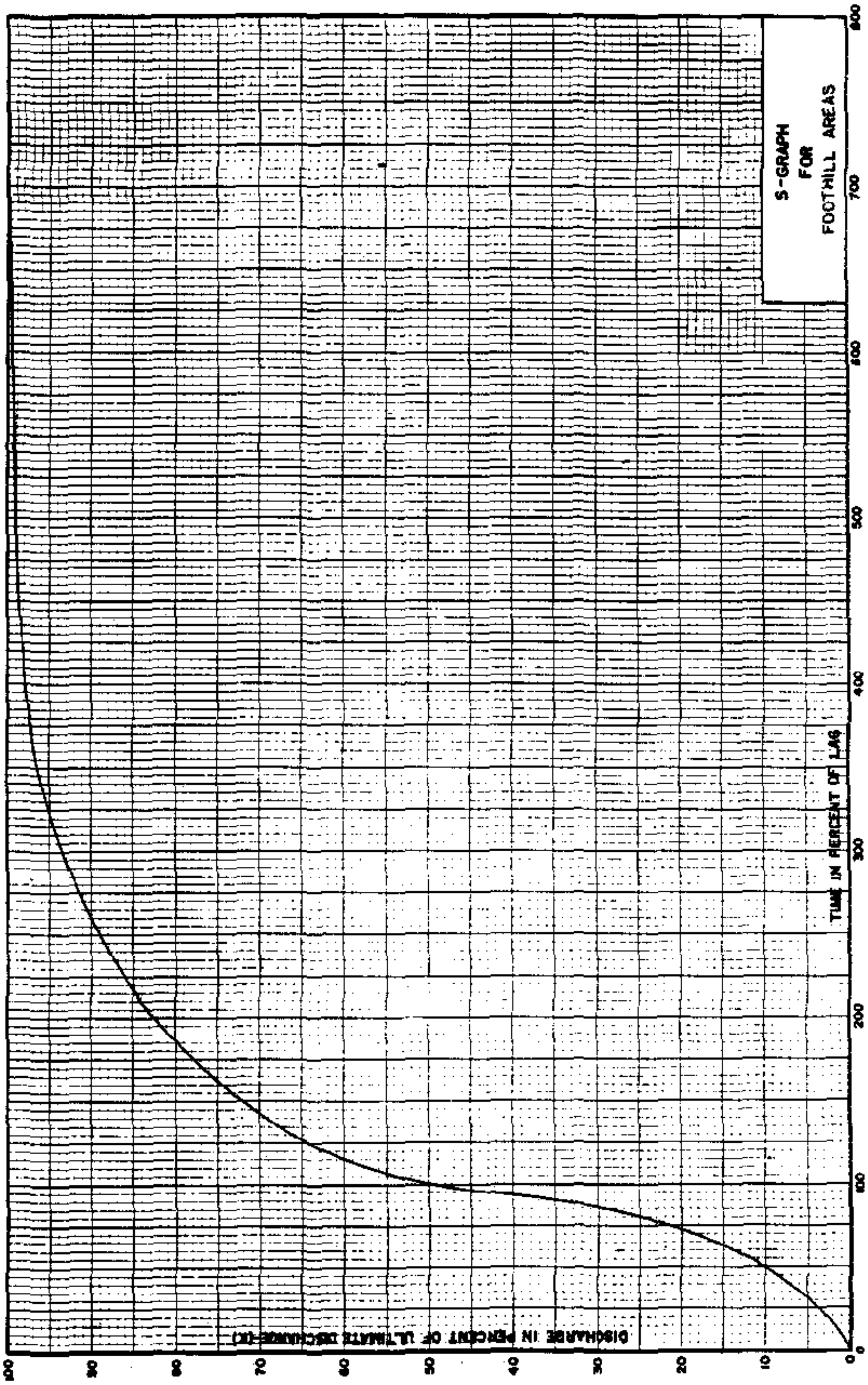

S-GRAPH
FOR
FOOTHILL AREAS
TIME IN PERCENT OF LAG
0
100
200
300
400
500
600
700
800
0
10
20
30
40
50
60
70
80
90
100

Fig. 2.3b.

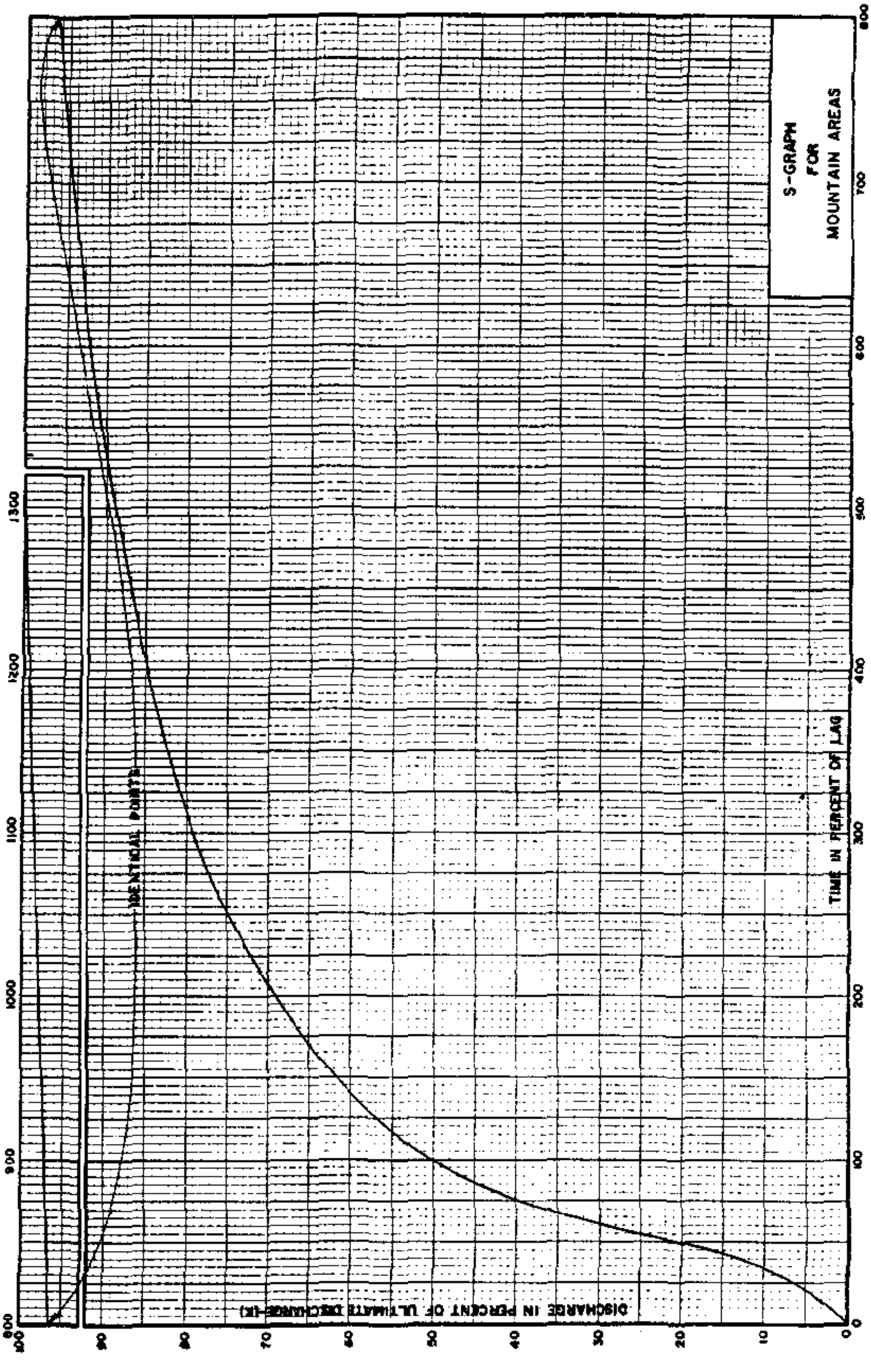

S-GRAPH
FOR
MOUNTAIN AREAS
TIME IN PERCENT OF LAG
DISCHARGE IN PERCENT OF ULTIMATE DISCHARGE-(K)
IDENTICAL POINTS
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
10
20
30
40
50
60
70
80
90
100

Fig. 2.3c.

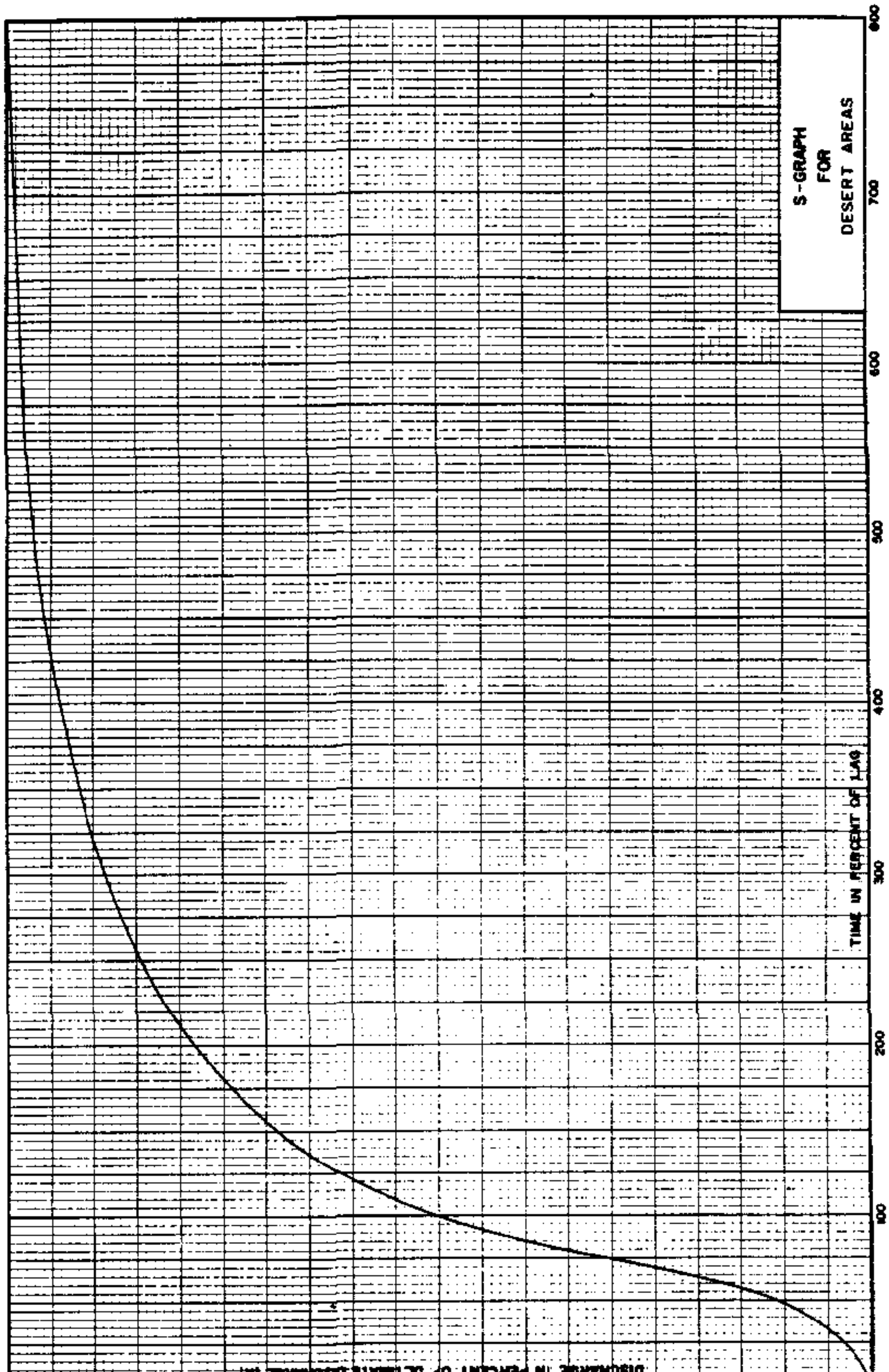


Fig. 2.3d.

(3). Select a watershed S-graph which is determined from a set of gaged watersheds which are physiographically and hydrologically similar to the study watershed.

(4). The S-graph can be approximated by a block graph where the base of each block is the selected unit period (expressed in percentage of watershed lag time) and the ordinate of each block is the time-averaged percentage of ultimate discharge (from the S-graph) for the corresponding unit period. The area of each block equals the area under the S-graph for each respective unit period.

(5). The unit distribution graph is approximated by calculating the difference between the ordinates of the block graph used to approximate the watershed S-graph. This procedure is analogous to calculating the difference between the ordinates of two S-graphs which have been offset by one unit period.

(6). The final step in developing the synthetic unit hydrograph is to multiply the ordinates of the distribution block graph by the ultimate discharge defined by

$$K = 645A/T \tag{2.3}$$

where

K = the watershed ultimate discharge (cfs)
A = drainage area (square miles)
T = unit time period (hours)

A widely used unit hydrograph is the S.C.S. dimensionless unit hydrograph of Fig. 2.4. From the figure, the concept of time to peak (T_p) is used and is shown to be geometrically related to the watershed lag time. The County of San Diego, California (1975) uses the empirical watershed lag formulas of (2.1) and (2.2) plus the relationship

$$T_p = (0.862)\text{lag} \tag{2.4}$$

Equation (2.4) is based on the geometric relationships for the hydrographs shown in Fig. 2.4 which includes the triangular version of the S.C.S. curvilinear unit hydrograph. From Fig. 2.4, it is assumed that

$$\begin{aligned} T_r &= (1.67)T_p \\ T_b &= (2.67)T_p \end{aligned} \tag{2.5}$$

A tabulation of the S.C.S. dimensionless unit hydrograph ratios is given in Table 2.1. From the table, it is seen that whatever the condition or classification of the watershed, the ratio of the time to peak to total unit hydrograph duration is a constant. Additionally, it is seen that the volume of runoff under the rising limb of the unit hydrograph is a constant 37.5

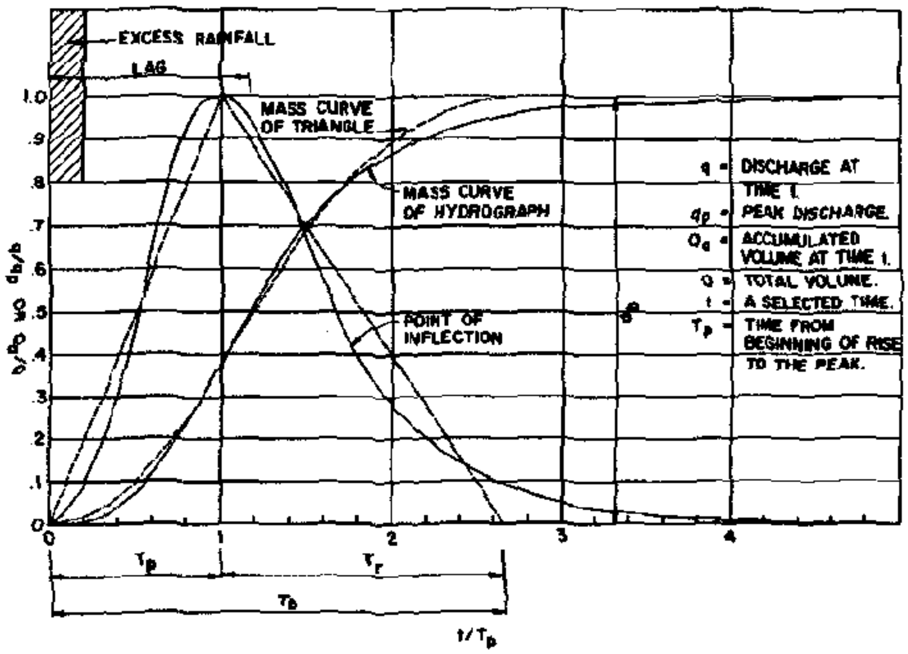


Fig 2.4. *S.C.S. dimensionless unit hydrograph and equivalent triangular hydrograph.*

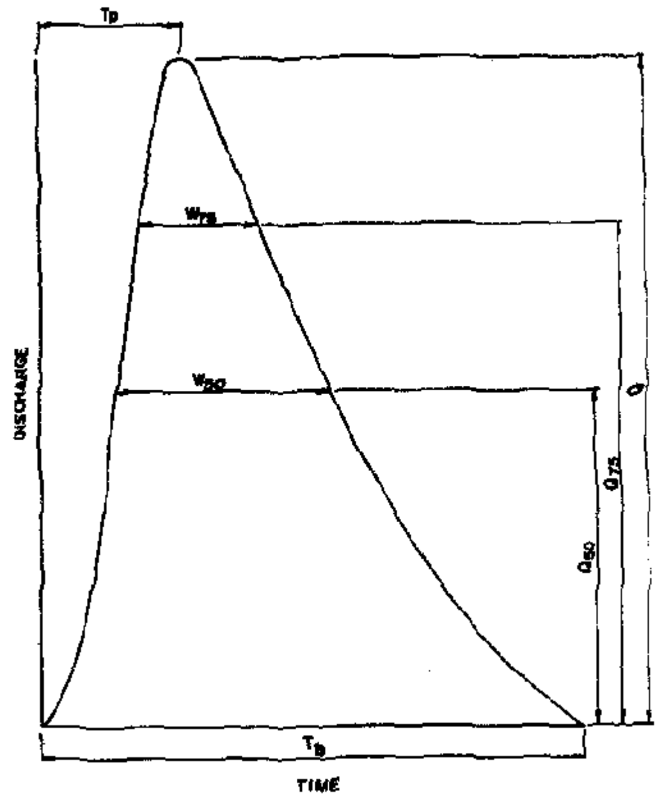


Fig. 2.5. *Definition of Espey 10-minute unit hydrograph parameters.*

percent of the total runoff volume, and that the peak rate factor associated to the unit rainfall is a constant value of 484. The variability of the S.C.S. unit hydrograph with respect to other peak rate factors is investigated by McCuen and Bondelid (1983). In their study, methods to estimate a peak rate factor for ungaged watersheds is proposed using a gamma function distribution for the unit hydrograph and the estimation of approximate watershed storage effects in the determination of the volume of runoff defined under the rising limb. As with many other unit hydrograph schemes, the assumption of linearity is once again utilized to develop a runoff hydrograph. Watershed peak rate factors are determined by time-area analysis and attempts to duplicate runoff hydrographs. The study concluded that the peak rate factor of 484 may be inappropriate for many watersheds.

Possibly, more insight into the mechanics of the unit hydrograph is gained by working with the Espey and Altman (1978) approach. This version of the unit hydrograph involves a set of five parameters which are defined according to Fig. 2.5. The variable relationships are of the form

$$T_p = (3.1)L^{0.23}S^{-0.25}I^{-0.18}K^{1.57}$$

$$T_B = (125890)AQ^{-0.95}$$

$$Q = (31620)A^{0.96}T_p^{-1.07}$$

$$W_{50} = (16220)A^{0.93}Q^{-0.92}$$

$$W_{75} = (3240)A^{0.79}Q^{-0.78} \qquad (2.6)$$

where

L = main channel length (feet)
H = drop in elevation along channel
S = channel slope
I = impervious area fraction
A = watershed area (square miles)
T_p = time to peak (minutes)
T_B = base time (minutes)
K = a dimensionless channel conveyance factor
W_{50}, W_{75} = width at 50 percent and 75 percent Q

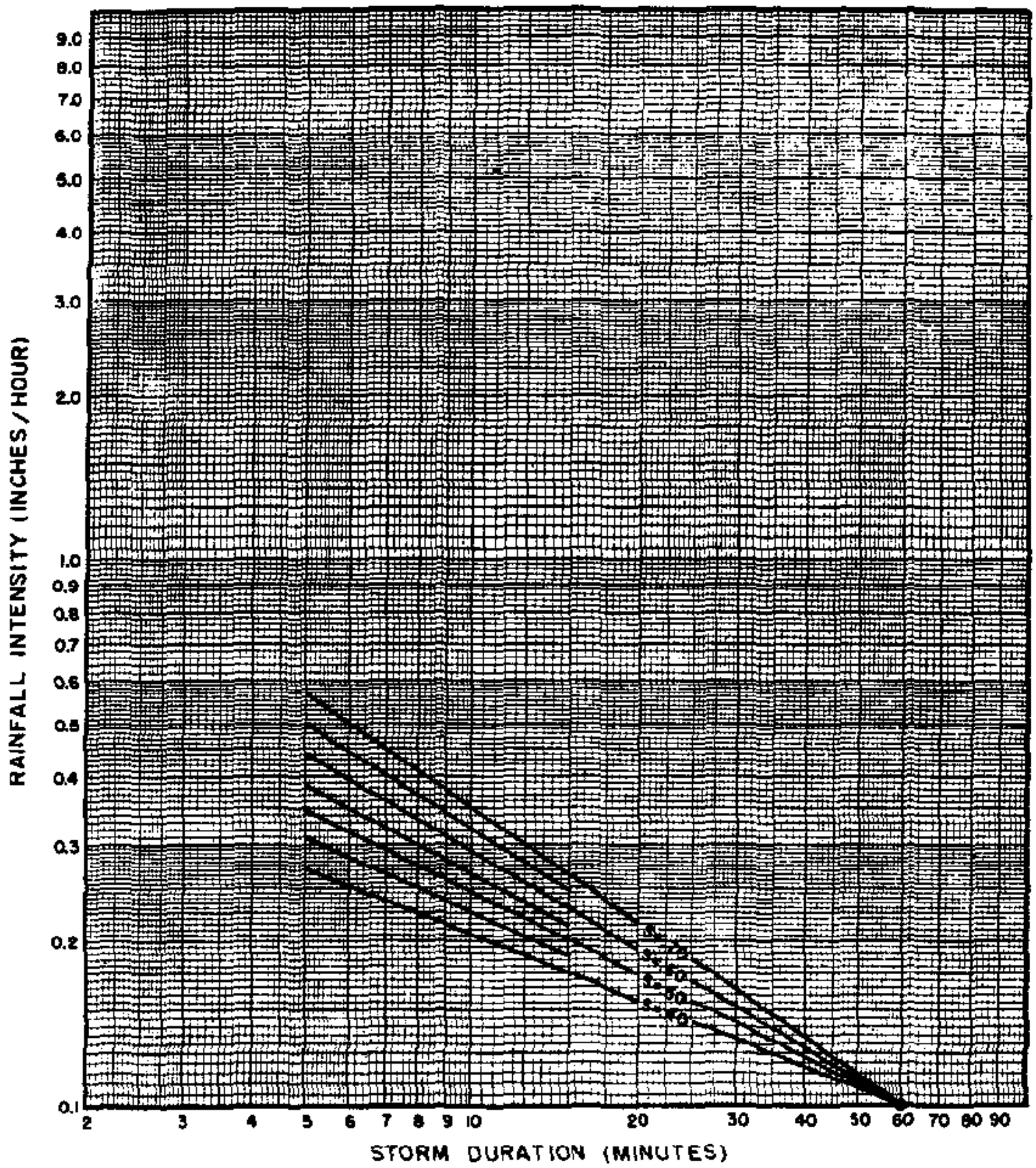
The Espey unit hydrograph is sensitive to the channel conveyance factor (K) in a fashion analogous to the sensitivity of the general unit hydrograph to the basin n^* factor. Both of these methods require some sort of tuning in order to evaluate their respective parameters.

TABLE 2.1. RATIOS FOR THE S.C.S. UNIT HYDROGRAPH

Time Ratio	Discharge Ratio	Mass Curve Ratio
0.00	0.000	0.000
0.10	0.030	0.001
0.20	0.100	0.006
0.30	0.190	0.012
0.40	0.310	0.035
0.50	0.470	0.065
0.60	0.660	0.107
0.70	0.820	0.163
0.80	0.930	0.228
0.90	0.990	0.300
1.00	1.000	0.375
1.10	0.990	0.450
1.20	0.930	0.522
1.30	0.860	0.589
1.40	0.780	0.650
1.50	0.680	0.700
1.60	0.560	0.751
1.70	0.460	0.790
1.80	0.390	0.822
2.00	0.280	0.871
2.20	0.207	0.908
2.40	0.147	0.934
2.60	0.107	0.953
2.80	0.077	0.967
3.00	0.055	0.977
3.20	0.040	0.984
3.40	0.029	0.989
3.60	0.021	0.993
3.80	0.015	0.995
4.00	0.011	0.997
4.50	0.005	0.999
5.00	0.000	1.000

2.4. Intensity-Duration Curves for Design Storm Point Precipitation

Rainfall intensity versus storm duration data is required for use in developing a synthetic design storm. Intensity-duration curves can be developed for a watershed by estimating the area-averaged one hour point precipitation values from synthesized local rain gage data, or from reliable point precipitation maps. Using Fig. 2.6, the one hour point precipitation value is plotted, and a straight line is drawn with the appropriate slope. Generally, slopes vary from about 0.30 to 0.80.



DESIGN STORM FREQUENCY = _____ YEARS

ONE HOUR POINT RAINFALL = _____ INCHES

LOG-LOG SLOPE = _____

PROJECT LOCATION = ____________________

Fig. 2.6. Intensity-duration curves calculations sheet.

2.5. Area-Averaged Point Rainfall

Due to the potential variation of precipitation depth (for a given duration) throughout a watershed, approximate methods are required in order to estimate an area-averaged point rainfall for the entire watershed. The three most often used methods are the arithmetic method, the Thiessen polygon method, and the isohyetal method. The application of these methods is shown in Fig. 2.7.

The arithmetic method is a simple average of point rainfall values. It is the least accurate of the three approaches for area-averaging.

The Thiessen polygon method utilizes an area-averaged value based on polygons associated to each rain gage in the vicinity of the watershed. The polygons are determined by constructing perpendicular bisectors to lines connecting the several rain gages.

Finally, the isohyetal method is a more precise approach in which a map of the interpolated point rainfall isohyetals are plotted on the watershed and the area-averaged point rainfall is directly estimated by contour integration.

2.6. Synthetic Critical Storm Patterns

2.6.1. Design Storm Pattern Approach

The use of computer watershed simulation techniques for hydrologic analysis has become widespread with the advent of inexpensive digital computer availability. Generally speaking, the basis of the hydrologic modeling approach is to study the effects of a severe storm event (or design storm) upon the watershed. Consequently, the decision as to what critical storm to use for study purposes has a significant impact upon the ultimate design objective in providing flood control.

The critical storm approach falls into two categories: (1) a severe storm pattern of record, and (2) a synthetic critical storm pattern. The critical storm pattern of record is a historical rainfall event which is assumed to be associated with the severe flooding event. The storm may have occurred within the vicinity of the watershed, or may have occurred elsewhere but is assumed to be transposable to the study watershed without violating the assumptions of uniformity of hydrologic or geographic characteristics. For example, the U.S. Army Corps of Engineers (Los Angeles, Calif.) utilize for study purposes a critical storm pattern based on a 1943 thunderstorm (Fig. 2.8). From Fig. 2.8, the 3-hour duration storm is composed of twelve 15-minute unit durations of precipitation depths. These unit rainfall values are estimated from peak 15-, 30-, 60-, and 180- minute duration precipitation-depth values as a function of watershed area. The storm pattern is based upon a 3.3-inch total storm rainfall. In transposing the storm to another watershed, the local 3- hour

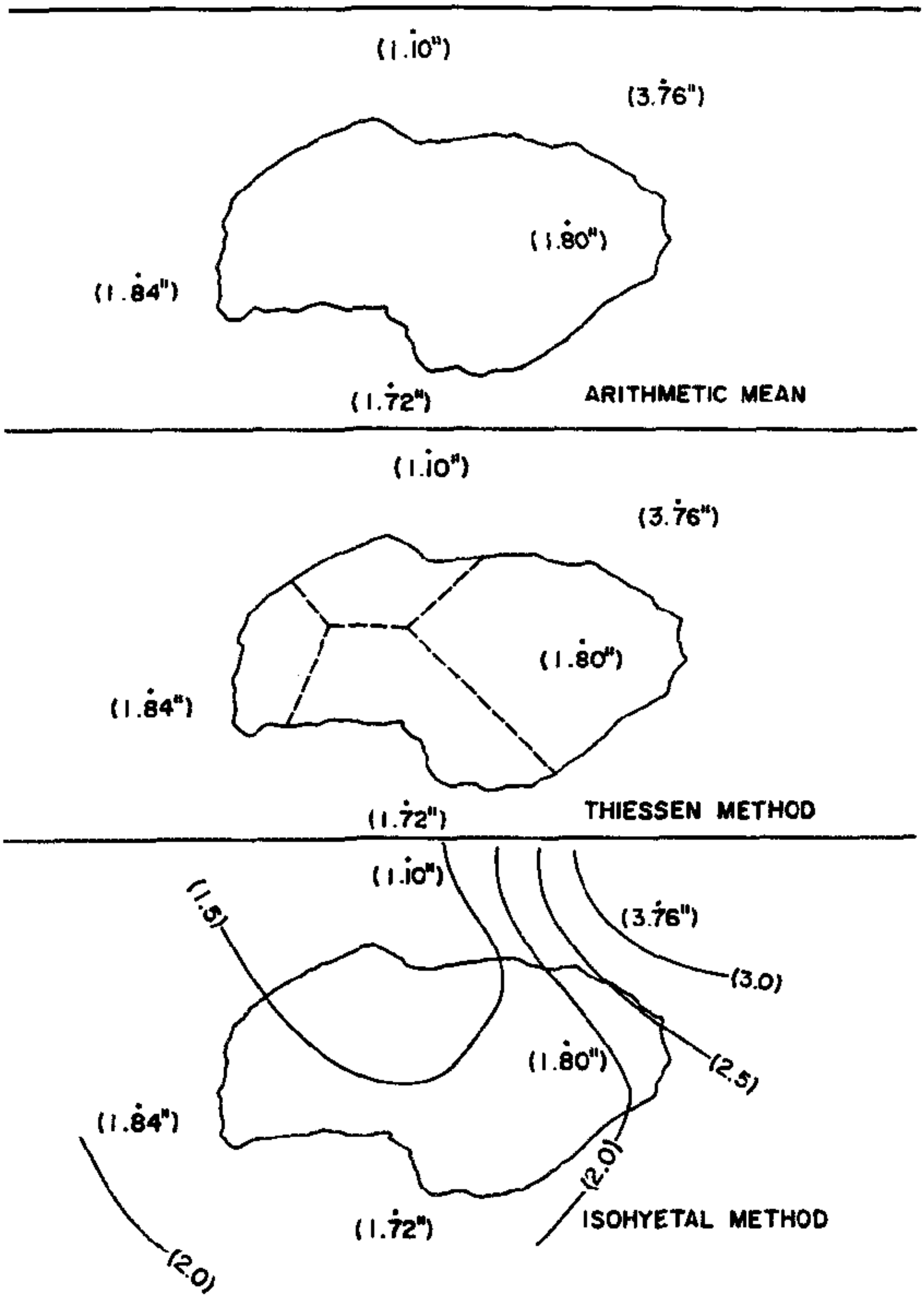
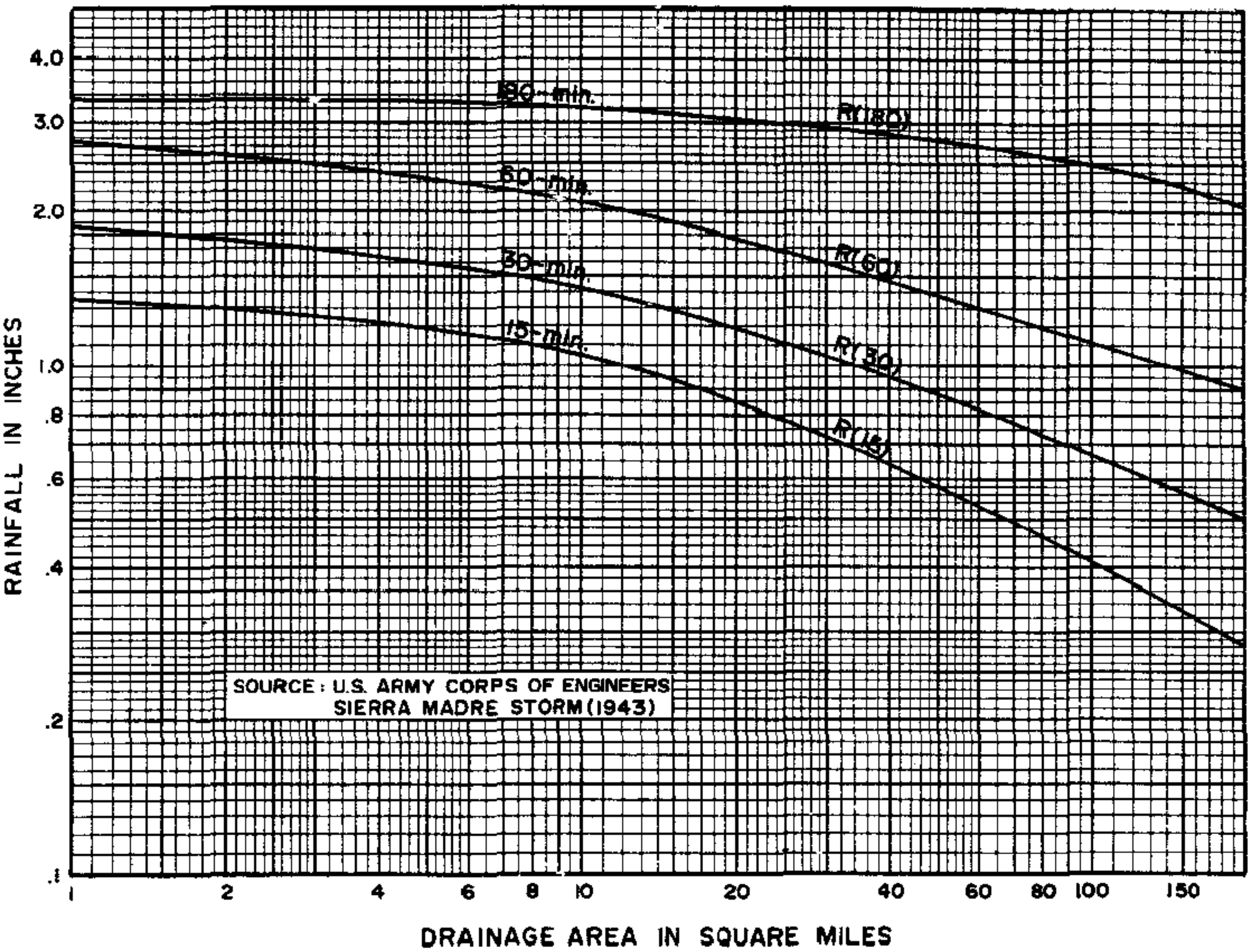


Fig. 2.7. Point rainfall area-averaging methods.



HYETOGRAPH COMPUTATION

UNIT PERIOD	AMOUNT
1	.07 (R(180) - R(60))
2	.05 (R(180) - R(60))
3	.11 (R(180) - R(60))
4	.05 (R(180) - R(60))
5	.20 (R(180) - R(60))
6	.22 (R(180) - R(60))
7	.14 (R(180) - R(60))
8	.16 (R(180) - R(60))
9	.48 (R(60) - R(30))
10	.52 (R(60) - R(30))
11	1.00 (R(15))
12	1.00 (R(30) - R(15))

UNIT PERIOD = 15 MINUTES

Fig. 2.8. *Critical storm depth-area-duration curves.*

duration precipitation-depth is estimated (for the desired return frequency) and the adjusted storm pattern is obtained by a simple ratio. For a depth of 3.3 inches, the resulting storm pattern is shown in Fig. 2.9. *A difficulty with this storm pattern is that* the precipitation-depth ratio of interior durations to the total 3-hour duration may not be appropriate for the local region of interest, and an unacceptably high (or low) precipitation depth may be assigned to the interior unit rainfalls. For example, the design objective may be to provide flood protection for a 100-year return frequency rainfall for the critical duration of every hydraulic structure within a watershed. Then using a 100- year 3-hour depth with the given storm pattern may possibly result in a 150- year (or 50- year) return frequency rainfall depth to be assigned for some critical duration. Designing the structure for such a runoff rate would result in an overdesign (or underdesign).

The second category of critical storm patterns is the synthetic critical storm pattern. This approach is an attempt to accomodate some of the concerns which are associated with using critical storm patterns of record. Several synthetic storm patterns have been proposed, such as Keifer and Chu (1957), Huff (1967), *Terstriep and Stall* (1974), *Pilgrim* and *Cordery* (1975), among others. For example, Hershfield (1962) developed a normalized time distribution of total mass rainfall versus storm duration. This generalized distribution is given in Table 2.2.

TABLE 2.2. HERSHFIELD MASS RAINFALL DISTRIBUTION

Storm Duration	Mass Rainfall
0.00	0.00
0.10	0.06
0.20	0.12
0.30	0.20
0.40	0.29
0.45	0.34
0.50	0.45
0.55	0.63
0.60	0.73
0.65	0.81
0.70	0.86
0.80	0.94
0.90	0.99
1.00	1.00

A widely used set of critical storm patterns was developed by the U.S. Department of Agriculture, Soil Conservation Service (S.C.S., 1973). The synthetic storm patterns are presented in normalized mass rainfall fashion for both a 6- hour and 24- hour duration critical storm event. The 24- hour storm pattern is further classified as a type I or type II, in order to better represent different storm precipitation intensity characteristics as averaged over broad regions of the United States. The S.C.S. developed the dimensionless synthetic storm patterns using the U.S. Weather Service rainfall frequency reports. The patterns are based upon regionally averaged rainfall gage data for watershed areas less than approximately 400 square miles, with durations to 24 hours, and return frequencies from 1 year to 100 years.

These critical storm patterns represent an average of generalized precipitation depth-duration-frequency relationships and, consequently, may be inappropriate for many hydrologic studies where the objective is to design flood control facilities to accomodate the maximum precipitation depth (with a specified return frequency) for the critical duration of each hydraulic structure within the system.

TABLE 2.3. S.C.S. SYNTHETIC STORM PATTERN
(TYPE I, 24 HOUR STORM)

Storm Hour	Mass Rainfall
0.0	0.000
2.0	0.035
4.0	0.076
6.0	0.125
6.0	0.156
8.0	0.194
8.5	0.219
9.0	0.254
9.5	0.303
9.75	0.362
10.0	0.515
11.0	0.624
12.0	0.682
13.0	0.727
14.0	0.767
16.0	0.830
20.0	0.926
24.0	1.000

The storm patterns are constructed by subtracting the peak 30-minute depth from the peak 1- hour depth, and subtracting the peak 1- hour depth from the peak 1.5- hour depth, and so forth until a set of 30-minute unit rainfalls results. The unit rainfalls are arranged such that a nesting of the peak unit rainfalls are formed where the peak 30 minutes occurs within the peak 1- hour, the peak hour occurs within the peak 1.5 hours, and so forth until the 24- hour storm pattern is defined. Because all the critical precipitation depths are contained within the storm pattern, the synthetic storm patterns are generally assumed

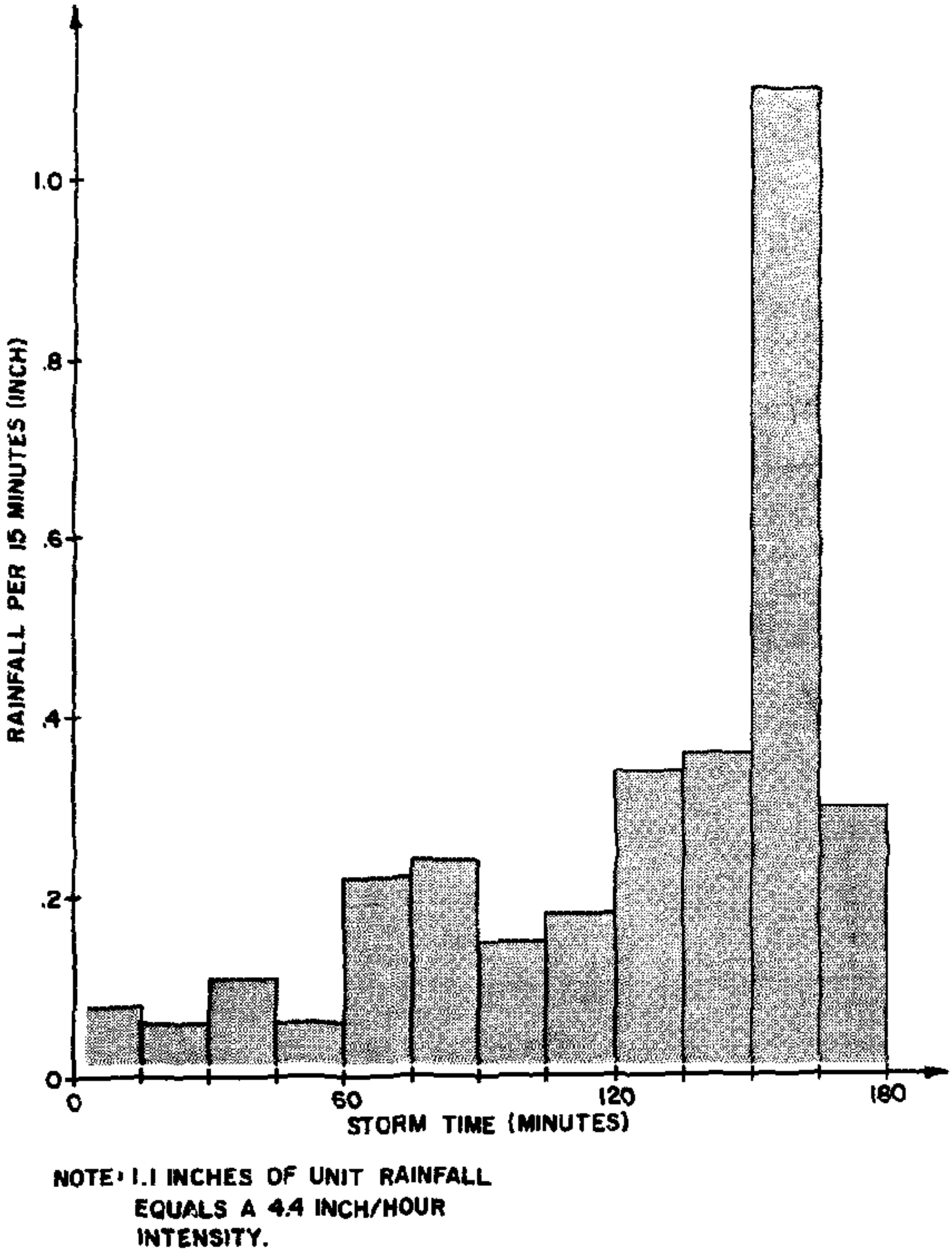


Fig. 2.9. Inch depth critical storm pattern (10 square mile area).

appropriate for design study purposes for both small and large watersheds. The dimensionless S. C. S. 24- hour type I pattern is presented in Table 2.3.

An examination of both the Hershfield and the S.C.S. synthetic storm patterns indicate that neither pattern guarantees a critical precipitation depth for an arbitrary critical duration. That is, both patterns define each interior storm duration as a fixed percentage of total storm mass rainfall. The Composite Method for developing a synthetic storm pattern is analogous to the approach used by the S.C.S. except that the local rain gage data is used to determine the unit rainfalls. This method simply uses the local rainfall intensity-duration curves with the desired return frequency, and incremental unit rainfalls are determined by successive subtractions. The unit rainfalls are then arranged in some nested pattern with the peak intensities usually defined to occur at 0.50, 0.67, or some percentage of the total storm duration. Figures 2.10 a,b,c show a typical composite storm pattern which is used for design hydrology purposes in the County of San Bernardino, California (Hromadka and Guymon, 1983b).

2.6.2. Depth-Area Relationships

A critical storm pattern can be directly applied towards estimating peak flow rates and runoff volumes associated to small watersheds where stream gage records are inadequate for statistical frequency analysis. As the watershed area increases in size, however, studies indicate (Miller et al., 1973, Myers and Zehr, 1980) that the area-averaged point rainfall values used in the design storm pattern should be adjusted. For example, the NOAA Atlas 2 includes point rainfall depth-area reduction relationships (Fig. 2.11) for 30- minute, 1-, 3-, 6-, and 24- hour point rainfall values as a function of watershed size.

2.6.3. Modified Composite Storm Pattern

The composite storm pattern approach can be modified to include the effects of depth-area adjustment. However, due to the adjustment factors being available for only a few durations, an interpolation procedure is required for the remaining durations within the storm pattern. One approach is to plot the adjusted point rainfall values on standard log-log paper and assume a straight line relationship (see Fig. 2.12). Other relationships can be used, but the log-log plot is convenient in that the resulting mass rainfall plot is simple to approximate numerically. Unit rainfalls are determined for the modified composite storm pattern by successive subtractions of unit durations along the adjusted mass rainfall plot. The unit rainfalls are then arranged as is shown in Fig. 2.13.

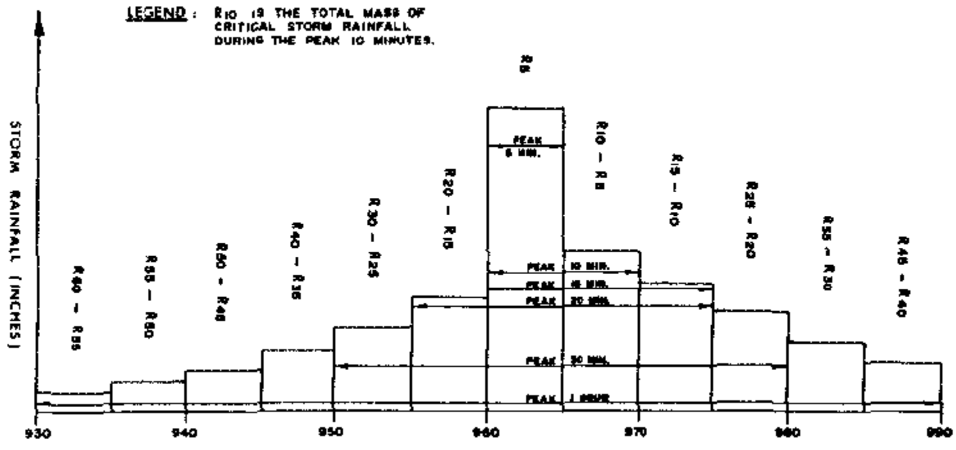


Fig. 2.10a.

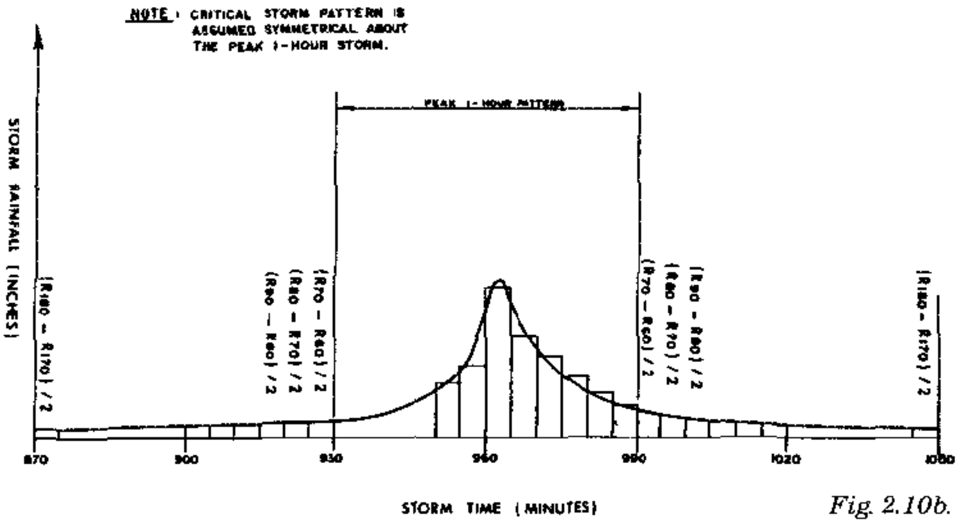


Fig. 2.10b.

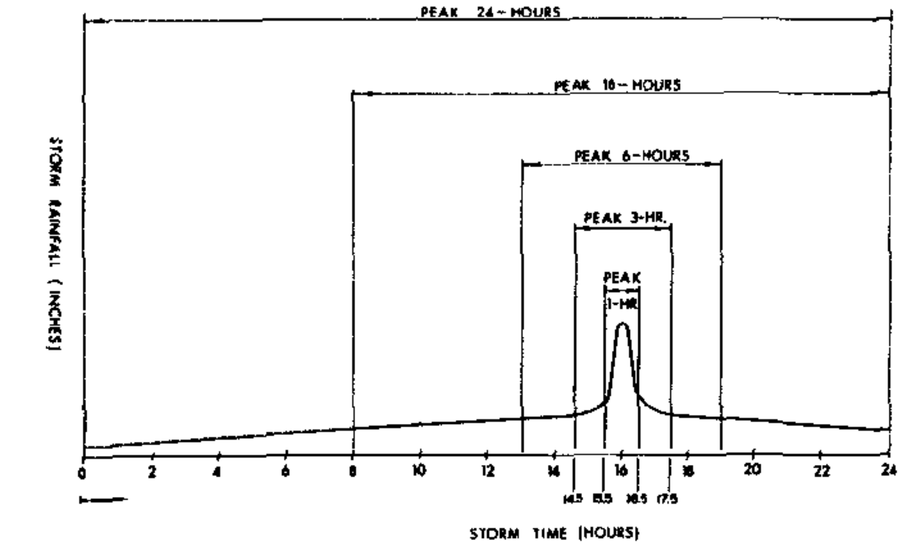


Fig. 2.10c. *Synthetic critical storm peak 24-hour pattern.*

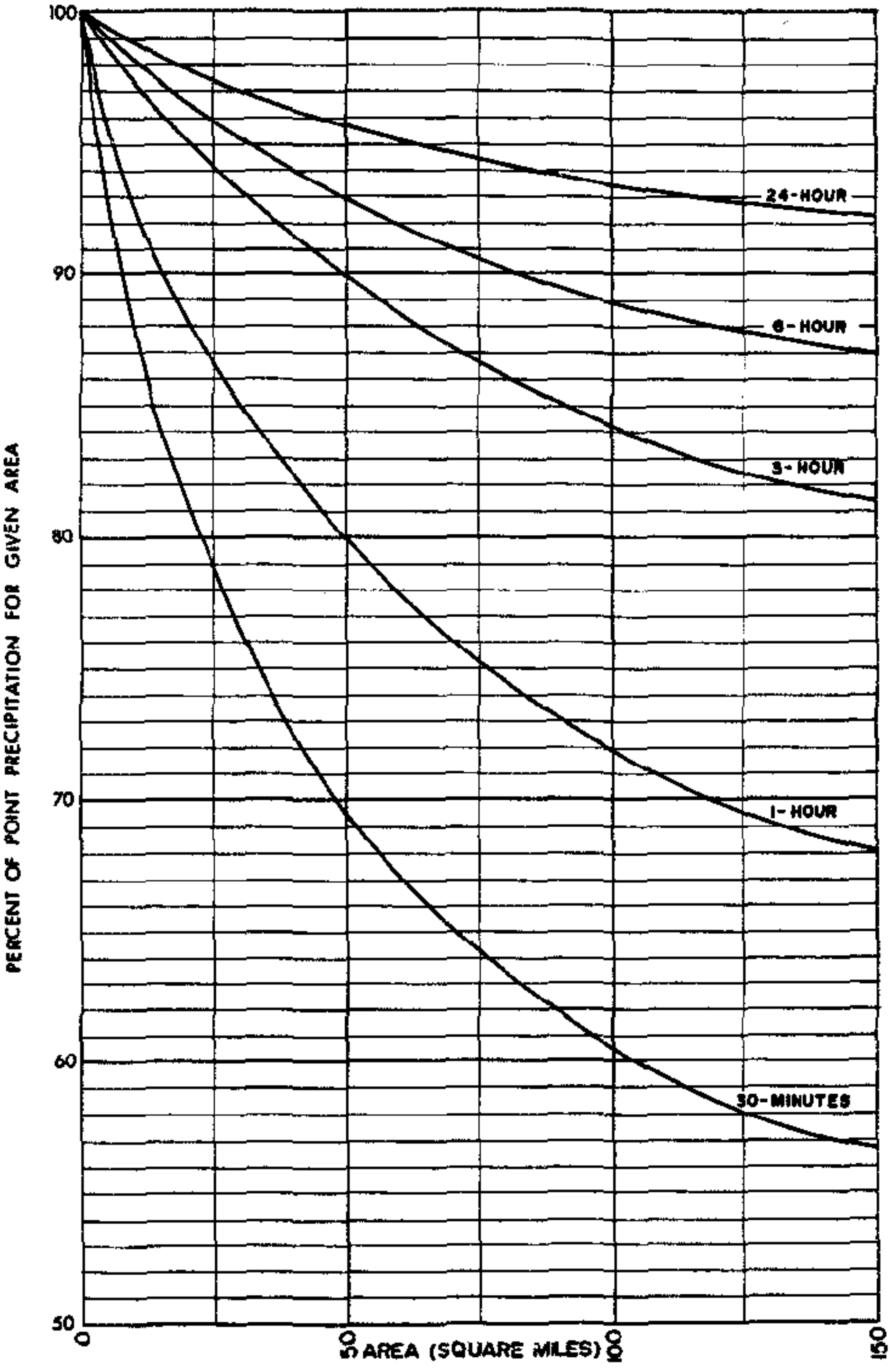


Fig. 2.11. *NOAA Atlas — 2 depth-area curves.*

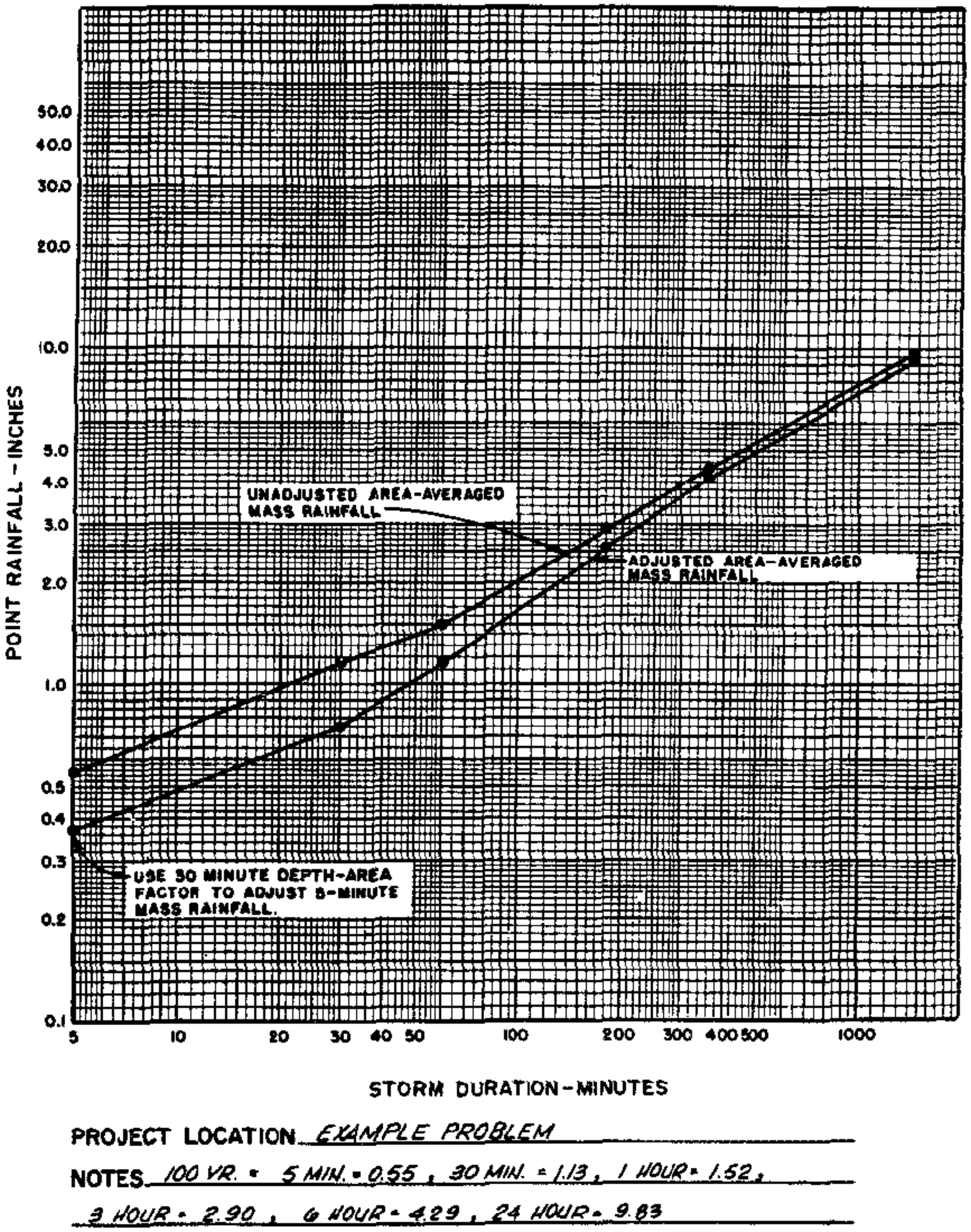


Fig. 2.12. *Area-averaged mass rainfall plotting sheet.*

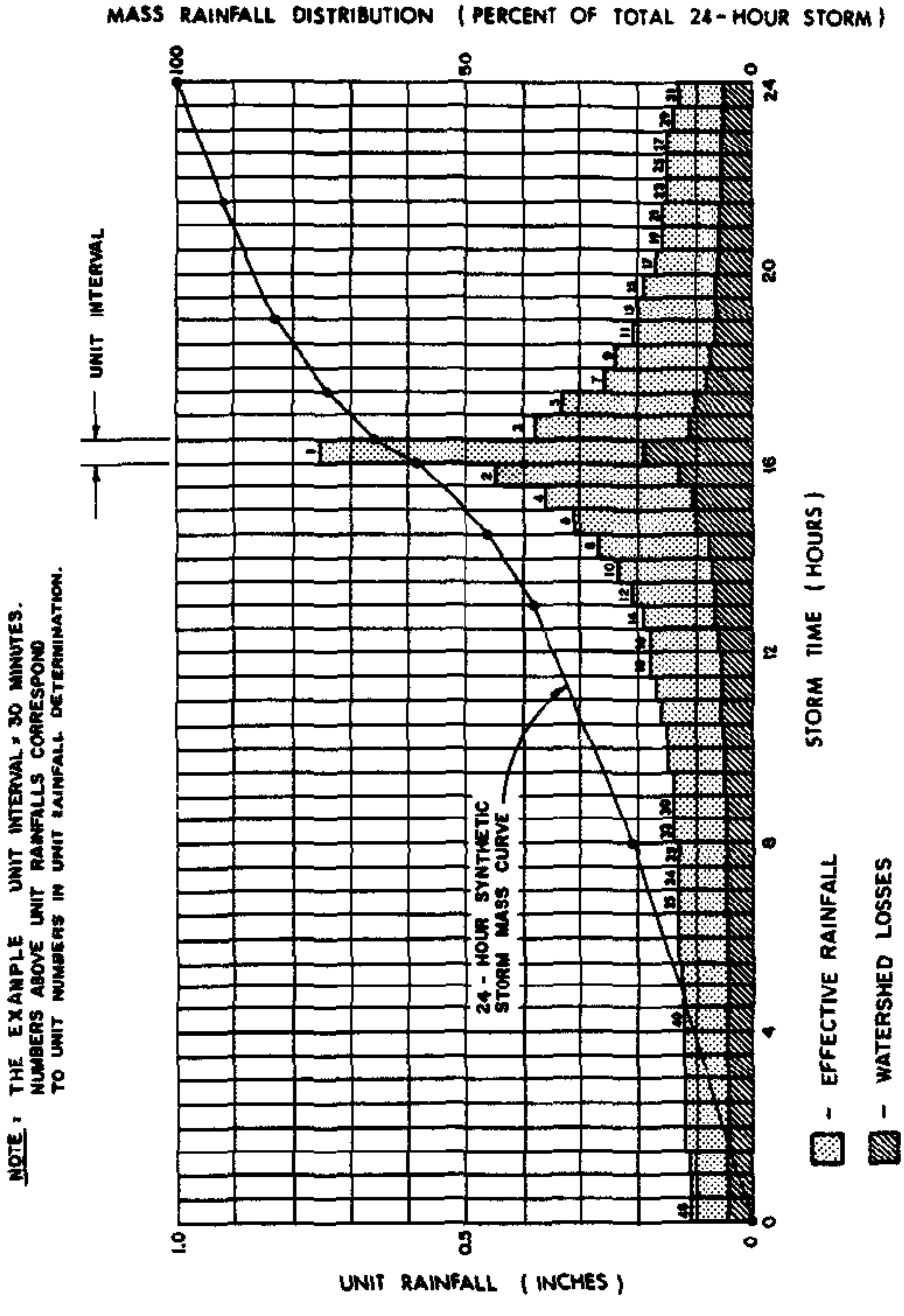


Fig. 2.13. Example synthetic 24-hour critical storm.

2.6.4. Design Storm Point Precipitation

In order to generate a runoff hydrograph, a design storm rainfall pattern needs to be developed which incorporates the local rainfall intensity-duration tendencies. Generally, point precipitation isohyetal information is adequate (e.g. NOAA Atlas 2, 1973) for estimating local rainfall depths of a desired return frequency. Such isohyetal maps can be used to obtain peak point precipitation depths for storm durations from 15 minutes to 24 hours and with return frequencies from 2 years to 100 years. Because additional rainfall data may alter the statistical patterns used to develop the isohyetal maps, such generalized data should always be verified with a recent frequency analysis for all rain gages within the vicinity of the study watershed.

The peak 5-minute, 30-minute, 1-hour, 3-hour, 6-hour, and 24-hour area-averaged point precipitation values should be determined to evaluate the critical design storm pattern used to develop the runoff hydrograph. Extrapolation procedures are often used to estimate intermediate duration point precipitation values. For example, it may be assumed that the peak 3-hour point precipitation value is related to the peak 1-hour and 6-hour point precipitation values by straightline interpolation on log-log paper. Using the appropriate 1-hour point precipitation value, the peak 5-minute and 30-minute point rainfalls can be estimated using Fig. 2.6.

To account for the areal effects on point precipitation values, reduction factors can be used to adjust the area-averaged point rainfalls (e.g. NOAA Atlas 2, or locally determined depth-area curves). In the unit hydrograph approach, the volume of watershed runoff is related to the watershed average depth of precipitation rather than the depth at specific points within the watershed. Depth-area curves approximately relate the average of all point precipitation values for a specific duration and frequency to the average depth of precipitation within the watershed for the same duration and frequency. The NOAA Atlas depth-area curves of Fig. 2.11 provide adjustment factors for the 30-minute, 1-hour, 3-hour, 6-hour, and 24-hour point precipitation values. These curves are an average for the entire United States, however, and local depth-area tendencies may differ substantially.

2.7. Effective Rainfall Estimation

2.7.1. S.C.S. Hydrologic Soil Groups

A major factor affecting infiltration is the condition of the soil. The soil surface characteristics, ability to transmit water through subsurface layers and the available soil storage capacity are all variables in defining the infiltration rate function. The S.C.S. classified more than 4000 soil types into four general categories which provide a classification of infiltration rates and corresponding runoff rates. These soil groups are defined in Table 2.4. The S.C.S. also uses curve numbers (CN) to describe the soil runoff potential. Further studies in California watersheds, however, provide some adjustments to the CN values. Runoff index numbers (RI) are used in this chapter to describe the use of these modified CN values.

Soil infiltration rates can be estimated for each of the soil groups by laboratory studies and measurements. Such measurements show that given a sufficient water supply, an initially dry soil will have an associated infiltration rate which essentially decreases with time. Consequently, if the soil is subjected to a continuous rainfall rate which exceeds the infiltration capacity, the infiltration loss rate will decrease with increasing storm duration. Depending on the condition and cover of the soil, a relative minimum infiltration rate for the S.C.S. soil groups are given in Table 2.5 (e.g. U.S. Bureau of Reclamation, 1973).

TABLE 2.4. S.C.S. HYDROLOGIC SOIL GROUPS

(A): Low runoff potential. Soils having a high infiltration rate even when thoroughly wetted. Consists chiefly of deep, well drained gravels and sands.

(B): Soils having moderate infiltration rates when thoroughly wetted. Consists mainly of moderately deep, well drained soils with moderately fine to moderately coarse texture.

(C): Soils having low infiltration rates when thoroughly wetted and consisting chiefly of soils with a layer that impedes the downward migration of water, or soils with moderately fine to fine texture.

(D): High runoff potential. Soils having a very low rate of infiltration when thoroughly wetted and consisting chiefly of clay soils with a high swelling potential, soils with a high water table (eliminating soil water storage capacity), soils with a claypan or clay layer at or near the soil surface, or shallow soils over a nearly impervious layer.

TABLE 2.5. MINIMUM INFILTRATION RATES

S.C.S. Soil Group	Infiltration Rate (in/hr)
A	0.30 - 0.45
B	0.15 - 0.30
C	0.05 - 0.15
D	0.00 - 0.10

Soil Cover

The type of vegetation or ground cover on a watershed, and the quality or density of that cover, have a major impact on the infiltration capacity of the soil. A widely used system for defining the effects of soil cover was developed by the S.C.S. and is based on the three categories given in Table 2.6.

In most cases, cover type and quality can be readily determined by an inspection of the watershed. U.S. Forest Service and S.C.S. personnel may also be helpful in estimation of such information. Cover types are listed in Fig. 2.14 which relate RI numbers for various cover qualities. General descriptions of cover types are given in Fig. 2.15.

TABLE 2.6. SOIL COVER QUALITY DEFINITIONS

POOR: Heavily grazed or regularly burned areas. Less than 50 percent of the ground surface is protected by plant cover or brush and tree canopy.

FAIR: Moderate cover with 50 percent to 75 percent of the ground surface protected.

GOOD: Heavy or dense cover with more than 75 percent of the ground surface protected.

2.7.2. Antecedent Moisture Condition

In order to accomodate the effects of prior precipitation events, the S.C.S. antecedent moisture condition (AMC) criteria are often used to adjust the infiltration loss rates. AMC curves were determined by analysis of rainfall-runoff data collected from a large number of watersheds where vegetative cover and soil conditions were known. For each watershed, the 24-hour runoff volume was plotted against the 24-hour rainfall. To accomodate scatter in the data, three runoff index numbers (that is, S.C.S. CN values) were associated to each watershed. The RI number which resulted in an equal number of high and low runoff predictions was defined to be an AMC II condition. Lower and upper enveloping RI numbers were associated to AMC I and AMC III conditions, respectively. The AMC designations are defined as follows:

TABLE 2.7. ANTECEDENT MOISTURE CONDITION (AMC) DEFINITIONS

AMC I: Lowest runoff potential. The watershed soils are dry enough to allow satisfactory grading or cultivation to take place.

AMC II: Moderate runoff potential. An intermediate condition which is generally assumed for annual flood studies.

AMC III: Highest runoff potential. The watershed is practically saturated from prior precipitation events. Heavy rainfall (or light rainfall with low temperatures) has occurred within the last five days.

Runoff Index Numbers of Hydrologic Soil-Cover Complexes For Pervious Areas-AMC II					
Cover Type (1)	Quality of Cover (2)	Soil Group A	B	C	D
NATURAL COVERS -					
Barren (Rockland, eroded and graded land)		78	86	91	93
Chaparrel, Broadleaf (Manzonita, ceanothus and scrub oak)	Poor	53	70	80	85
	Fair	40	63	75	81
	Good	31	57	71	78
Chaparrel, Narrowleaf (Chamise and redshank)	Poor	71	82	88	91
	Fair	55	72	81	86
Grass, Annual or Perennial	Poor	67	78	86	89
	Fair	50	69	79	84
	Good	38	61	74	80
Meadows or Cienegas (Areas with seasonally high water table, principal vegetation is sod forming grass)	Poor	63	77	85	88
	Fair	51	70	80	84
	Good	30	58	71	78
Open Brush (Soft wood shrubs - buckwheat, sage, etc.)	Poor	62	76	84	88
	Fair	46	66	77	83
	Good	41	63	75	81
Woodland (Coniferous or broadleaf trees predominate. Canopy density is at least 50 percent.)	Poor	45	66	77	83
	Fair	36	60	73	79
	Good	25	55	70	77
Woodland, Grass (Coniferous or broadleaf trees with canopy density from 20 to 50 percent)	Poor	57	73	82	86
	Fair	44	65	77	82
	Good	33	58	72	79
URBAN COVERS -					
Residential or Commercial Landscaping (Lawn, shrubs, etc.)	Good	32	56	69	75
Turf (Irrigated and mowed grass)	Poor	58	74	83	87
	Fair	44	65	77	82
	Good	33	58	72	79
AGRICULTURAL COVERS -					
Fallow (Land plowed but not tilled or seeded)		77	86	91	94

Fig. 2.14 (1 of 2). *Runoff index numbers for pervious areas.*

Runoff Index Numbers of Hydrologic Soil-Cover Complexes For Pervious Areas-AMC II					
Cover Type (1)	Quality of Cover (2)	Soil Group			
		A	B	C	D
AGRICULTURAL COVERS (Continued)					
Legumes, Close Seeded	Poor	66	77	85	89
(Alfalfa, sweetclover, timothy, etc.)	Good	58	72	81	85
Orchards, Evergreen	Poor	57	73	82	86
(Citrus, avocados, etc.)	Fair	44	65	77	82
	Good	33	58	72	79
Pasture, Dryland	Poor	68	79	86	89
(Annual grasses)	Fair	49	69	79	84
	Good	39	61	74	80
Pasture, Irrigated	Poor	58	74	83	87
(Legumes and perennial grass)	Fair	44	65	77	82
	Good	33	58	72	79
Row Crops	Poor	72	81	88	91
(Field crops - tomatoes, sugar beets, etc.)	Good	67	78	85	89
Small grain	Poor	65	76	84	88
(Wheat, oats, barley, etc.)	Good	63	75	83	87

Notes:

1. All runoff index (RI) numbers are for Antecedent Moisture Condition (AMC) II.

2. Quality of cover definitions:

   Poor-Heavily grazed or regularly burned areas. Less than 50 percent of the ground surface is protected by plant cover or brush and tree canopy.

   Fair-Moderate cover with 50 percent to 75 percent of the ground surface protected.

   Good-Heavy or dense cover with more than 75 percent of the ground surface protected.

Fig. 2.14 (2 of 2). Runoff index numbers for pervious areas.

Residential Landscaping (Lawn, Shrubs, etc.) - The pervious portions of commercial establishments, single and multiple family dwellings, trailer parks and schools where the predominant land cover is lawn, shrubbery and trees.

Row Crops - Lettuce, tomatoes, beets, tulips or any field crop planted in rows far enough apart that most of the soil surface is exposed to rainfall impact throughout the growing season. At plowing, planting and harvest times it is equivalent to fallow.

Small Grain - Wheat, oats, barley, flax, etc. planted in rows close enough that the soil surface is not exposed except during planting and shortly thereafter.

Legumes - Alfalfa, sweetclover, timothy, etc. and combinations are either planted in close rows or broadcast.

Fallow - Fallow land is land plowed but not yet seeded or tilled.

Woodland - grass - Areas with an open cover of broadleaf or coniferous trees usually live oak and pines, with the intervening ground space occupied by annual grasses or weeds. The trees may occur singly or in small clumps. Canopy density, the amount of ground surface shaded at high noon, is from 20 to 50 percent.

Woodland - Areas on which coniferous or broadleaf trees predominate. The canopy density is at least 50 percent. Open areas may have a cover of annual or perennial grasses or of brush. Herbaceous plant cover under the trees is usually sparse because of leaf or needle litter accumulation.

Chaparral - Land on which the principal vegetation consists of evergreen shrubs with broad, hard, stiff leaves such as manzonita, ceanothus and scrub oak. The brush cover is usually dense or moderately dense. Diffusely branched evergreen shrubs with fine needle-like leaves, such as chamise and redchank, with dense high growth are also included in this soil cover.

Annual Grass - Land on which the principal vegetation consists of annual grasses and weeds such as annual bromes, wild barley, soft chess, ryegrass and filaree.

Irrigated Pasture - Irrigated land planted to perennial grasses and legumes for production of forage and which is cultivated only to establish or renew the stand of plants. Dry land pasture is considered as annual grass.

Meadow - Land areas with seasonally high water table, locally called cienegas. Principal vegetation consists of sod-forming grasses interspersed with other plants.

Orchard (Deciduous) - Land planted to such deciduous trees as apples, apricots, pears, walnuts, and almonds.

Orchard (Evergreen) - Land planted to evergreen trees which include citrus and avocados and coniferous plantings.

Turf - Golf courses, parks and similar lands where the predominant cover is irrigated mowed close-grown turf grass. Parks in which trees are dense may be classified as woodland.

Fig. 2.15. S.C.S. cover type descriptions.

As a guide in selecting the watershed AMC condition for the hydrologic analysis, the following agricultural season precipitation limits may be considered:

TABLE 2.8. ANTECEDENT MOISTURE CONDITION CRITERIA

AMC	Total 5-Day Antecedent Rainfall (inch) (dormant season)	(growing season)
I	less than 0.5	less than 1.4
II	0.5 - 1.1	1.4 - 2.1
III	over 1.1	over 2.1

Should the RI number be modified for AMC I or III, the following table can be used to determine an adjusted RI number:

TABLE 2.9. AMC RUNOFF INDEX NUMBER RELATIONSHIPS

RI For AMC II	Corresponding RI for AMC Condition I	III
100	100	100
95	87	99
90	78	98
85	70	97
80	63	94
75	57	91
70	51	87
65	45	83
60	40	79
55	35	75
50	31	70
45	27	65
40	23	60
35	19	55
30	15	50
25	12	45
20	9	39
15	7	33
10	4	26
5	2	17
0	0	0

2.7.3. Impervious Areas

In analysis of urban watersheds, the effects of the impervious surfaces on the assumed area-averaged infiltration rate for the entire watershed must be included. Estimated ranges of impervious percentages for various types of land use development are given in Fig. 2.16. Values given are for the actual percentage of area covered by an impervious surface. However, the actual impervious area needs to be reduced due to the local drainage practices. For example, an impervious surface may drain across a pervious surface where infiltration may take place. To account for this infiltration, the actual impervious area may be reduced by a factor such as 10 percent. This type of adjustment is included in the estimation of watershed loss rates for use in the synthetic unit hydrograph method.

2.7.4. Watershed Development Conditions

Should the flood control facilities be expected to provide for the public protection for an extended period, then the maximum urbanization should be assumed in determining the watershed loss rates. All available long range urbanization master plans should be examined to insure that reasonable land use assumptions are included in the analysis. Particular attention should be directed to local landscape practices. For example, it is common to use ornamental gravels underlain by impervious plastic materials as a substitute for lawns and shrubs, resulting in an increase in the effective impervious percentage.

2.7.5. Estimation of Infiltration Rates

The S.C.S. approach assumes that daily storm rainfall is related to the subsequent runoff by

$$R = (P - Ia)^2 / (P - Ia + S) \qquad (2.7)$$

where

R = total runoff (in.)
P = total precipitation (in.)
Ia = initial abstraction
$S = 1000/RI - 10$

Figure 2.17 provides a plot of Eq. 2.7 as a function of RI numbers. In the figure, it is assumed that the initial abstraction can be estimated by

$$Ia = 0.2S \qquad (2.8)$$

Other studies suggest that the above S.C.S. value of (2.8) is too high and a value of 0.05S to 0.10S is more appropriate (Aron et al., 1977). As discussed previously, severe design storm conditions may reduce the Ia value even further and may even be

ACTUAL IMPERVIOUS COVER		
Land Use (1)	Range-Percent	Recommended Value For Average Conditions-Percent (2)
Natural or Agriculture	0 - 10	0
Single Family Residential: (3)		
100,000 SF (2.5 acre) Lots	5 - 15	10
40,000 SF (1 acre) Lots	10 - 25	20
20,000 SF (1/2 acre) Lots	30 - 45	40
7,200 - 10,000 SF Lots	45 - 55	50
Multiple Family Residential:		
Condominiums	45 - 70	65
Apartments	65 - 90	80
Mobile Home Park	60 - 85	75
Commercial, Downtown Business or Industrial	80 - 100	90

Notes:

1. Land use should be based on ultimate development of the watershed. Long range master plans for the County and incorporated cities should be reviewed to insure reasonable land use assumptions.

2. Recommended values are based on average conditions which may not apply to a particular study area. The percentage impervious may vary greatly even on comparable sized lots due to differences in dwelling size, improvements, etc. Landscape practices should also be considered as it is common in some areas to use ornamental gravels underlain by impervious plastic materials in place of lawns and shrubs. A field investigation of a study area should always be made, and a review of aerial photos, where available, may assist in estimating the percentage of impervious cover in developed areas.

3. For typical horse ranch subdivisions increase impervious area 5 percent over the values recommended in the table above.

Fig. 2.16. Actual impervious cover for developed areas.

entirely eliminated from the rainfall-runoff budget. If it is assumed that Ia is negligible, then (2.7) is simplified to

$$R = P^2/(P + S) \tag{2.9}$$

Letting P_e = P-Ia, both (2.7) and (2.9) can be written as

$$R = P_e^2/(P_e + S) \tag{2.10}$$

where P_e is the precipitation excess in inches.

To estimate infiltration rates from (2.10), the method proposed by Chen (1975) is illustrated by the example calculations shown in Table 2.10.

TABLE 2.10. EXAMPLE COMPUTATION OF INFILTRATION LOSSES

Storm Time (hrs.)	Total Rainfall (in.)	Total Runoff (in.)	Incremental Infiltration (in.)	Infiltration Rate (in/hr)
6.0	2.5	0.46	-	-
6.25	2.60	0.50	0.06	0.24
6.50	2.70	0.55	0.05	0.20
6.75	2.85	0.63	0.07	0.28
7.00	3.00	0.71	0.07	0.28
7.25	3.20	0.83	0.08	0.32
7.50	3.55	1.04	0.12	0.48
7.75	4.05	1.36	0.18	0.72
8.00	4.80	1.89	0.22	0.88

notes:

(i) storm represents the peak 2 hours of a severe historical storm in Southern California

(ii) losses estimated assuming Ia=0.2S, and RI=70 for the pervious portions of the watershed

The incremental computation of the infiltration losses in Table 5.10 is by means of scanning the appropriate rainfall-runoff curve from Fig. 2.17 with respect to incremental precipitation. A difficulty with this procedure is that unacceptably high (and low) infiltration loss rates may be predicted. Given a maximum and minimum infiltration rate, however, the unacceptable incremental infiltration losses can be adjusted and the total infiltration losses distributed throughout the remaining design storm. Rallison (1980) gives a detailed account of the origin of the S.C.S. rainfall-runoff equation. The runoff curve numbers (and the analogous runoff index numbers) were developed by relating daily (24- hour periods) runoff to precipitation records. It is noted

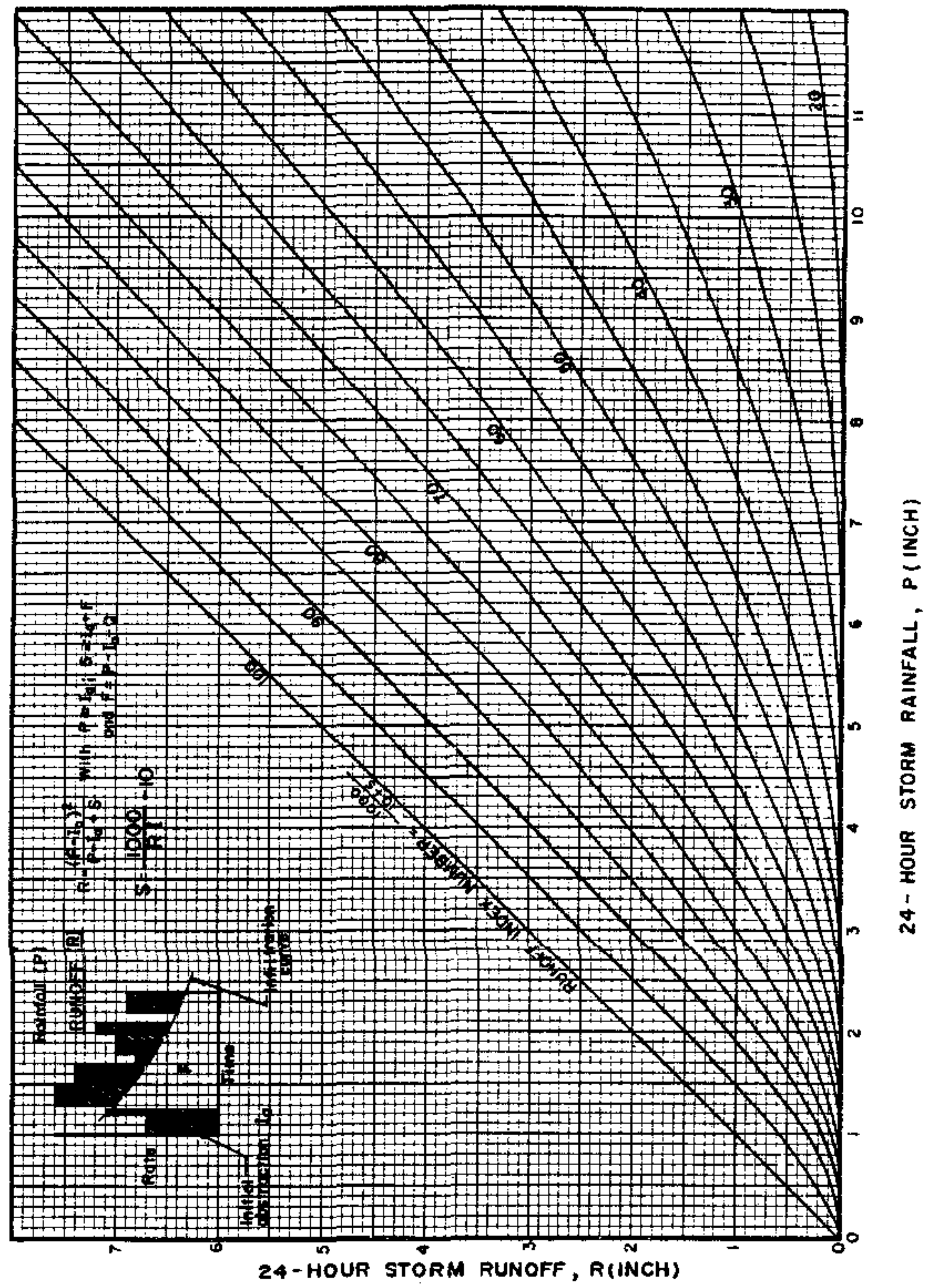


Fig. 2.17. S.C.S. 24-hour storm rainfall-runoff relationships.

that these curves were not designed to be a physical definition of the incremental runoff estimation approach.

In estimating soil infiltration rates, an index of runoff potential is determined for each soil cover complex within the watershed. The RI scale has a range of 0 to 100, where a low RI number indicates a low runoff potential and a high RI number indicates a high runoff potential (an RI of 100 indicates total runoff). Selection of the RI number takes into account several major factors affecting the infiltration rate on pervious surfaces including hydrologic soils group, cover type and quality, and the antecedent moisture condition.

One approach for estimating a maximum limiting value for infiltration losses during the design storm event is utilized by several Southern Californian flood control agencies and is described by the following application problem. A severe design storm condition for many Californian watersheds would be 6 inches of precipitation within a duration of 6 hours. Additionally, the watershed design condition would be such that the initial abstraction Ia term is negligible due to significant prior storms and Eq. 2.9 would apply. For this situation, (2.9) reduces to

$$R = 6^2/(6 + S) \tag{2.11}$$

Assuming that the infiltration rate equals the maximum allowable value throughout this 6 hour severe storm condition gives

$$F_p = (P - R)\,\text{in.}/(6\ \text{hrs}) \tag{2.12}$$

where F_p is the infiltration loss rate for the pervious areas. Rearranging terms results in an estimate for F_p in terms of the pervious area fraction RI number

$$F_p = (1000 - 10RI)/(1000 - 4RI) \tag{2.13}$$

Because this loss rate is for the pervious area only, a composite loss rate needs to be determined which represents both the pervious and impervious surfaces in the watershed. Adjustment of the loss rate for impervious surfaces can be made by

$$F = F_p(1.00 - 0.9A_i) \tag{2.14}$$

where

F = adjusted loss rate (in./hr.)

F_p = loss rate for pervious areas (in./hr.) derived from (2.13)

A_i = impervious area decimal fraction (Fig. 2.16)

The limiting infiltration rate from (2.14) may be used as the upper bound loss rate whenever it is exceeded by the incremental loss rate. Otherwise, the incremental loss rates are used with a lower bound infiltration rate estimated from Table 2.5. A second technique is to define a low loss rate percentage of rainfall to be attributed to infiltration losses during the lower intensity portion of the design storm event.

Assuming that the initial abstraction term (Ia) is negligible, then (2.9) may be used to define

$$Y^* = 1 - P/(P + S) \qquad (2.15)$$

where

Y^* = low loss rate percentage (decimal fraction)
P = 24 hour precipitation (in.)
S = $(1000/RI^*) - 10$
RI^* = area-averaged composite runoff index number

To calculate the area-averaged runoff index used in (2.15), composite runoff index numbers must be computed for each RI number defined within the watershed by

$$RI_j^* = 100\ A_i + (RI_j)A_p \qquad (2.16)$$

where

RI_j^* = composite runoff index number for area j
A_i = fraction of area j that is impervious
A_p = fraction of area j that is pervious

The above formula assumes an RI number of 100 (total runoff) for the impervious area fraction and uses an RI number selected from Fig. 2.14 for the pervious area fraction. The resulting composite runoff index number approximates the S.C.S. curve numbers for the subject development type, cover, perviousness, and soil group. The empirical relation of (2.16) is a simplification of a more complex relationship and, consequently, the RI associated to the impervious area fraction should not be reduced for the effects of effective imperviousness.

From the low loss rate percentage, the corresponding low loss rate, F', is given by

$$F' = Y^* I \qquad (2.17)$$

where I is the rainfall intensity. It should be noted that the adjusted loss rate of (2.14) is an upper bound to the low loss rate of F'.

In the determination of the low loss rate percentage, the area-averaged 24- hour point rainfall (P) is computed and an appropriate composite runoff index number estimated, and both values are substituted into (2.15).

A third method in using a maximum (limiting) infiltration rate is incorporated into the ϕ index approach which holds the loss rate as constant throughout the entire severe design storm event. Table 2.11 compares the three discussed methods in estimating the design storm watershed losses for the application problem presented in Table 2.10. In Table 2.11, it is assumed that the limiting F=0.44 in/hr and Y^*=0.40.

Comparing Tables 2.10 and 2.11 it is noted that limiting the incremental loss rate resulted in an additional 0.19 inches of runoff during the storm's peak 45 minutes of rainfall.

TABLE 2.11. COMPARISON OF INFILTRATION LOSS METHODS

Storm Time (hrs.)	Incremental Rainfall (in.)	Incremental Loss Approach (in.)	Low Loss Approach (in.)	ϕ index Loss (in.)
6.0	0.00	-	-	-
6.25	0.10	0.06	0.04	0.10
6.50	0.10	0.05	0.04	0.10
6.75	0.15	0.07	0.06	0.11
7.00	0.15	0.07	0.06	0.11
7.25	0.20	0.08	0.08	0.11
7.50	0.35	0.11	0.11	0.11
7.75	0.50	0.11	0.11	0.11
8.00	0.75	0.11	0.11	0.11
sum:	(2.30)	(0.66)	(0.61)	(0.86)

2.8. Synthetic Runoff Hydrograph Development

The several hydrologic elements of the critical storm pattern, point rainfall determination, watershed lag estimation, *and the unit hydrograph approximation are combined in the* convolution of effective rainfall with the unit hydrograph. Figure 2.18 illustrates the procedure for developing a synthetic runoff hydrograph. Each unit interval (or unit period) of rainfall from the design storm pattern is split into the effective unit rainfall and the corresponding watershed unit loss. The resulting effective rainfall storm pattern is then combined with the assumed watershed unit hydrograph by a process called convolution to produce the synthetic runoff hydrograph. Figure 2.18 shows a 12 unit period storm pattern and the corresponding unit period runoff hydrographs. Summing together the 12 unit period runoff hydrographs results in the design storm runoff hydrograph.

It should be noted that in the convolution approach, each unit period runoff hydrograph has the same base period regardless of the magnitude of the associated unit rainfall. Also, each unit

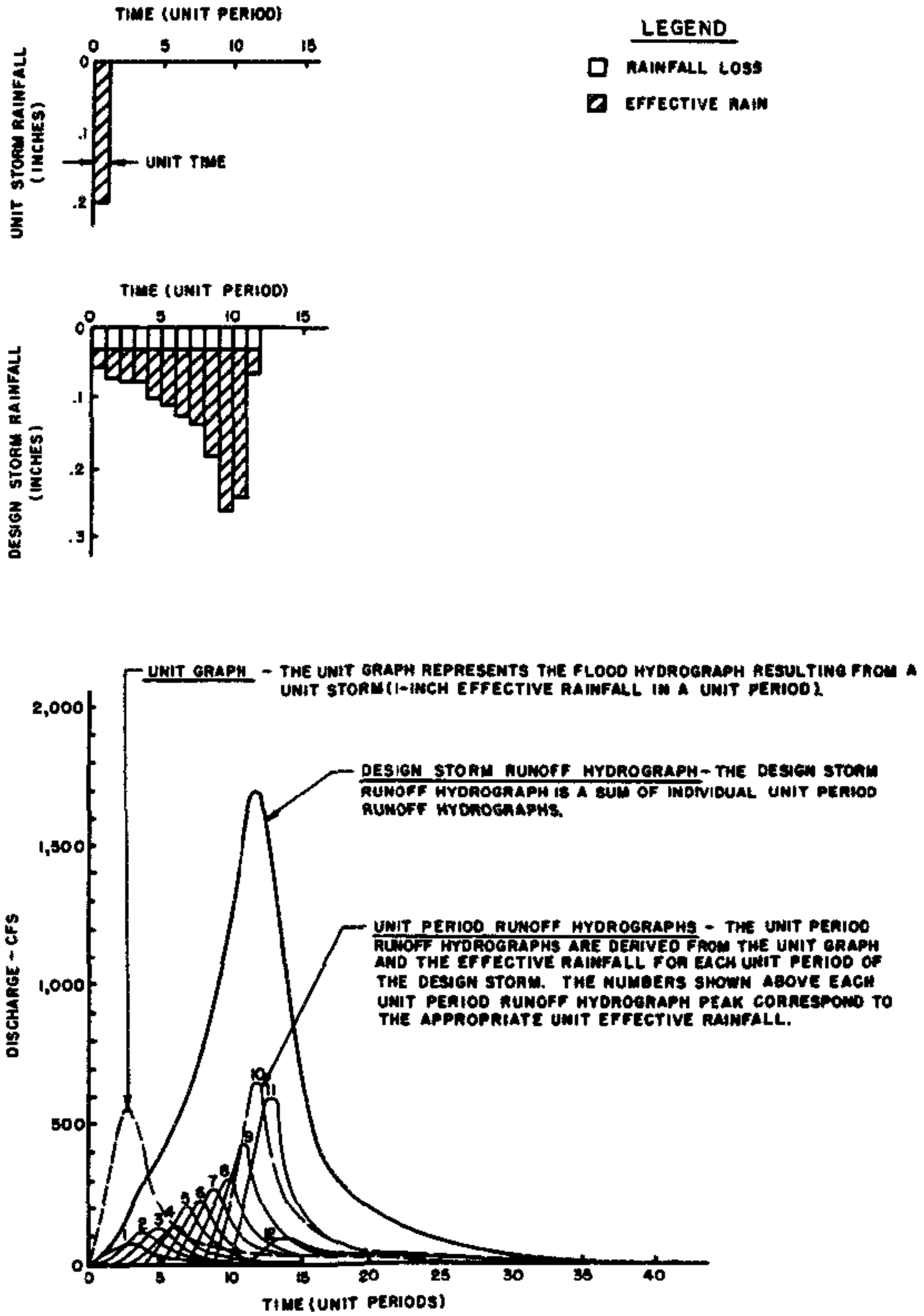


Fig. 2.18. *Derivation of a runoff hydrograph.*

period runoff hydrograph has a constant time to peak duration. Each of the unit period runoff hydrographs are seen to be directly related to the volume of the associated effective rainfalls. Additionally, in the summation process it is assumed that there are negligible restrictions to the watershed runoff flow rates which are represented by the runoff hydrograph. These restrictions would occur due to channel capacity problems and watershed detention effects. Each of these concerns should be addressed after the watershed runoff hydrograph is generated in order to ascertain whether the computed results are reasonable for the watershed conditions being studied. In many study situations, a single runoff hydrograph generation would be inappropriate due to the watershed conditions, and a more complex watershed model is required which includes submodels for channel and basin routing effects. Such a watershed model suitable for microcomputers is given in Hromadka (1983).

Other contributions to the runoff hydrograph such as baseflow and snowmelt can be summed to the synthetic runoff hydrograph. Generally, such considerations are neglected in the study of urban watersheds. However, detailed discussions of ancillary hydrologic processes are contained in texts such as Hjelmfelt and Cassidy (1975).

2.9. Instructions for a Synthetic Runoff Hydrograph Development

In order to describe the salient features of the supplied FORTRAN computer program, a process step chart is provided which describes each operation performed by the program (noted by a superscript star), and information which is to be provided by the program user. The computer program uses watershed S-curves developed for Southern California (U.S. Army Corps of Engineers) and also includes the dimensionless curvilinear S.C.S. unit hydrograph in S-graph form. Additionally, the program uses the NOAA Atlas 2 depth-area relationships of Fig. 2.11. This type of information can be easily replaced with local watershed S-curve information and other depth-area relationships in order to develop a runoff hydrograph generator program which represents local hydrologic characteristics. The basic elements of the program are as follows:

(1). The critical storm pattern is a composite 24 hour duration storm pattern composed of nested point rainfall peak intensities corresponding to each storm duration and assumed return frequency. The point rainfall values are adjusted by the depth-area relationships of the NOAA Atlas 2.

(2). Unit hydrographs are developed from the Southern California S-graphs determined by the U.S. Army Corps of Engineers. Also included is the S.C.S. curvilinear unit hydrograph. The time base of the unit hydrograph is determined by the watershed lag relationship of (2.1) and (2.2).

PROJECT: ______________________ DATE:

ENGINEER: ______________________

1. Enter the length of the longest watercourse (feet) ________
2. Enter the length along the longest watershed watercourse measured from the point of concentration upstream to a point opposite the centroid of the watershed area (feet) ________
3. Enter the difference in elevation between the most remote point in the watershed and the point of concentration (feet) ________
4. Enter the basin factor ________
5. Enter the watershed area (acres) ________
6. Enter baseflow (cfs/square mile) ________
7. Enter S-Graph proportions (decimal)
   - SCS[1] ________
   - Valley ________
   - Foothill ________
   - Mountain ________
   - Desert ________
8. Enter adjusted loss rate (inch/hour) ________
9. Enter low loss rate percentage (decimal) ________
10. Enter watershed area-averaged 5-minute point rainfall (inches)* ________

    Enter watershed area-averaged 30-minute point rainfall (inches)* ________

    Enter watershed area-averaged 1-hour point rainfall (inches)* ________

    Enter watershed area-averaged 3-hour point rainfall (inches)* ________

    Enter watershed area-averaged 6-hour point rainfall (inches)* ________

    Enter watershed area-averaged 24-hour point rainfall (inches)* ________
11. Enter 24-hour storm unit interval (minutes) ________

[1]Note: use either SCS or Valley, Foothill, Mountain, Desert
*Note: enter values unadjusted by depth-area factors

Fig. 2.19. *Program 2.1. Data entry form.*

(3). Watershed losses are assumed equal to infiltration losses and are modeled by use of the adjusted loss rate of (2.14) and the low loss rate of (2.17).

Using the above three model assumptions, the synthetic runoff hydrograph is developed by the following steps:

I. Synthetic Unit Hydrograph Development

A. On a suitable topographic map of the watershed, determine the proposed drainage system boundaries.

B. From the watershed map, determine the following watershed physical factors and enter them on the computer information form of Fig. 2.19:

- A = drainage area (acres)
- L = length of the longest watercourse (feet)
- L_c = length along the longest watercourse, measured upstream to a point opposite the centroid of the watershed area (feet)
- H = elevation difference along watercourse (feet)

C*. Determine the watershed lag from (2.1) and (2.2) where the basin factor is selected from Fig. 2.2. The basin factor is entered on the form of Fig. 2.19.

D. Select a unit time period to be used for the modeling purposes. Usually, a unit period of 15 to 25 percent of the watershed lag time is adequate.

E*. Select a S-graph appropriate for the watershed (see Figs. 2.3 a,b,c,d, or Fig. 2.4). Determine the average percentage of the ultimate discharge for each unit period. In reading the percentage of ultimate discharge from the S-graph, the mean ordinate over the time period should be determined. The S-graphs may be averaged to approximate a combination of watershed runoff characteristics. Enter the desired S-graph proportions on the computer information form.

F*. Compute the unit distribution graph by subtracting successive unit percentages of ultimate discharge.

G*. Compute the ordinates of the synthetic unit hydrograph by multiplying the ordinates of the unit distribution graph by the ultimate discharge defined by

$$K(cfs) = 645A/T$$

where A is the drainage area in square miles, and T is the unit time period in hours.

II. Synthetic Critical Storm Pattern Development

A. Develop point precipitation isohyetal maps for the 1 hour, 6-hour, and 24-hour durations, and calculate an area-averaged value for each of these durations.

B. Unless point precipitation data is available to determine area-averaged point precipitation values for the 5-minute, 30-

minute, and 3-hour durations, use the area-averaged 1 hour value from step II.A to estimate the 5-minute and 30-minute values by plotting on Fig. 2.6 using a slope of the intensity-duration plot which is characteristic of the watershed rainfall records. The 3-hour value can be estimated from the 1-hour and 6-hour values by plotting on Fig. 2.20. Enter the area-averaged point rainfall values on the computer information form.

C*. Adjust all area-averaged point rainfall values for depth-area relationships such as given in the NOAA Atlas 2.

D*. Develop a synthetic critical storm mass rainfall plot such as shown in Fig. 2.12 using Fig. 2.20.

E*. Using the unit time period specified for the unit hydrograph determination, calculate the synthetic storm pattern unit rainfalls by successive subtractions of the mass rainfall curve values separated by a unit time period.

F*. Arrange the unit rainfalls into the critical storm pattern shown in Figs. 2.10a,b,c. It should be noted that the computer stored rainfall pattern sequence places the peak 5-minute unit rainfall at storm time of 16 hours, and essentially distributes the remaining peak rainfalls symmetrically about the peak 5-minute value.

III. Flood Hydrograph Development

A. Find the pervious area fraction mean infiltration loss rate. Adjust the loss rate for the effective impervious area fraction by

$$F = F_p(1.00 - 0.9A_i)$$

where

F = adjusted loss rate (inch/hour)
F_p = loss rate for the pervious area fraction
A_i = impervious area fraction

B. Compute the low loss rate, F', from (2.17). Use the low loss rate for the watershed loss rate whenever the adjusted loss rate exceeds the low loss rate. Enter the adjusted loss rate and the low loss rate percentage of (2.15) on the computer information form.

C*. Compute the unit effective rainfalls by subtracting the unit losses from the unit rainfalls of the synthetic critical storm pattern.

D*. Compute the watershed runoff hydrograph (see Fig. 2.18):

1. Multiply the unit effective rainfall for the first time period by each unit value of the synthetic unit hydrograph to produce the runoff hydrograph corresponding to the unit period.

2. Repeat the above procedure for each unit effective rainfall, advancing the resulting unit period runoff hydrograph by one unit period.
3. Sum the unit period runoff hydrographs to produce the watershed runoff hydrograph.

E*. Add the baseflow to the flood hydrograph. Enter the baseflow on the computer information form.

2.10. A Synthetic Runoff Hydrograph Computer Program

The detailed description of procedures for developing a 24-hour storm runoff hydrograph can be directly formulated into a FORTRAN computer program suitable for currently available microcomputer systems. The following table lists the several subroutines used in the computer program:

TABLE 2.12. RUNOFF HYDROGRAPH PROGRAM SUBROUTINES

Program Number	Program Description
2.1	Main program for developing the effective rainfall storm pattern distribution and the resulting runoff hydrograph.
2.2 (no input)	Synthetic unit hydrograph generator which develops a unit hydrograph from the S-curves of Figs. 2.3a,b,c,d or the S.C.S. unit hydrograph of Fig. 2.4.
2.3 (no input)	Output plotting program which plots the runoff hydrograph flowrate and accumulated runoff volume as a function of storm time.

2.11. PROGRAM 2.1 Data Entry

The computer data entry variables are included in the following computer text pages. These example user-friendly screens show the variable range of values, and also include suggested program commands to provide a user-friendly environment.

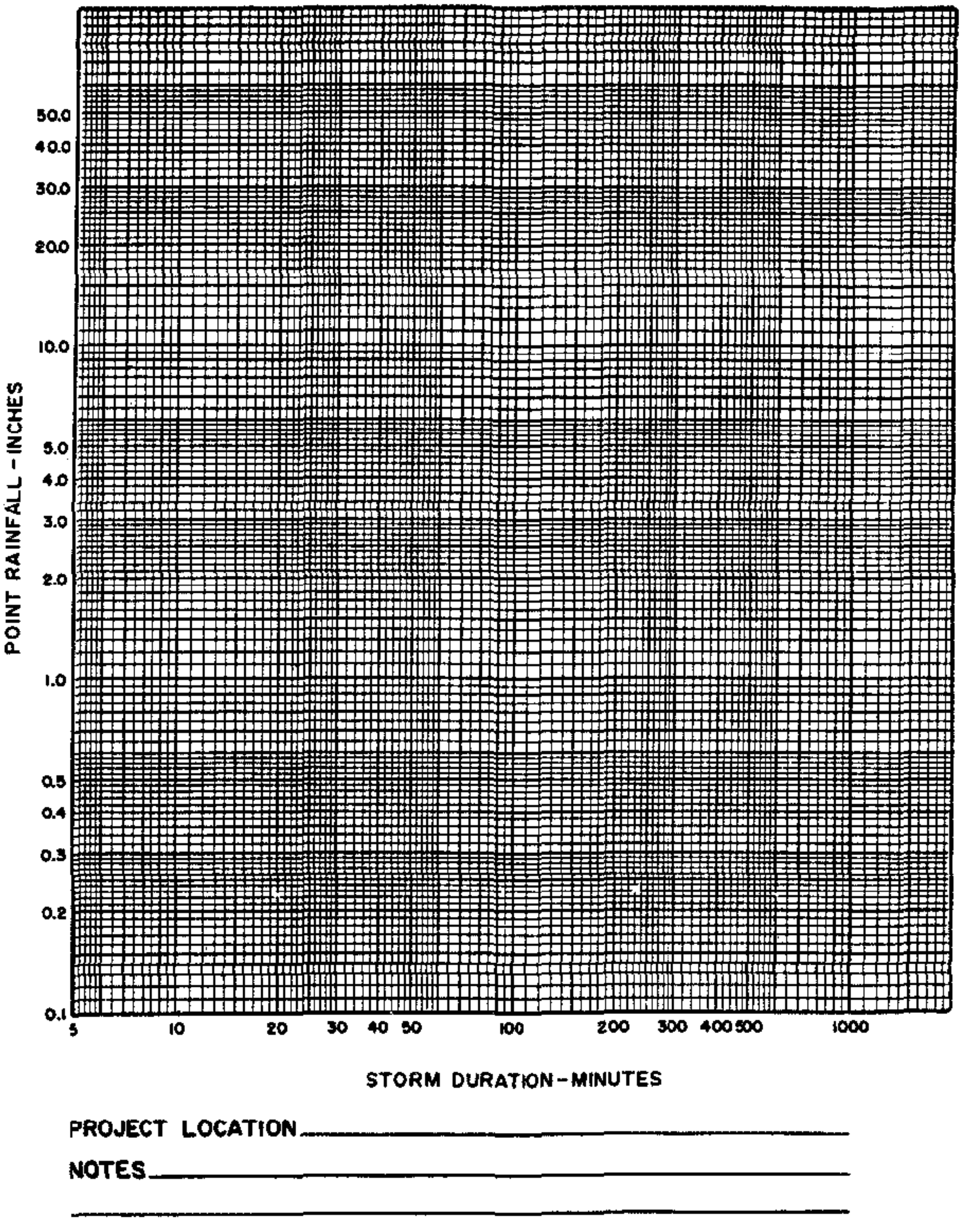


Fig. 2.20. Area-averaged mass rainfall plotting sheet.

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San Diego County, 1981.
San Bernardino County, 1983.
State of Arizona Highway Dept.,1973.

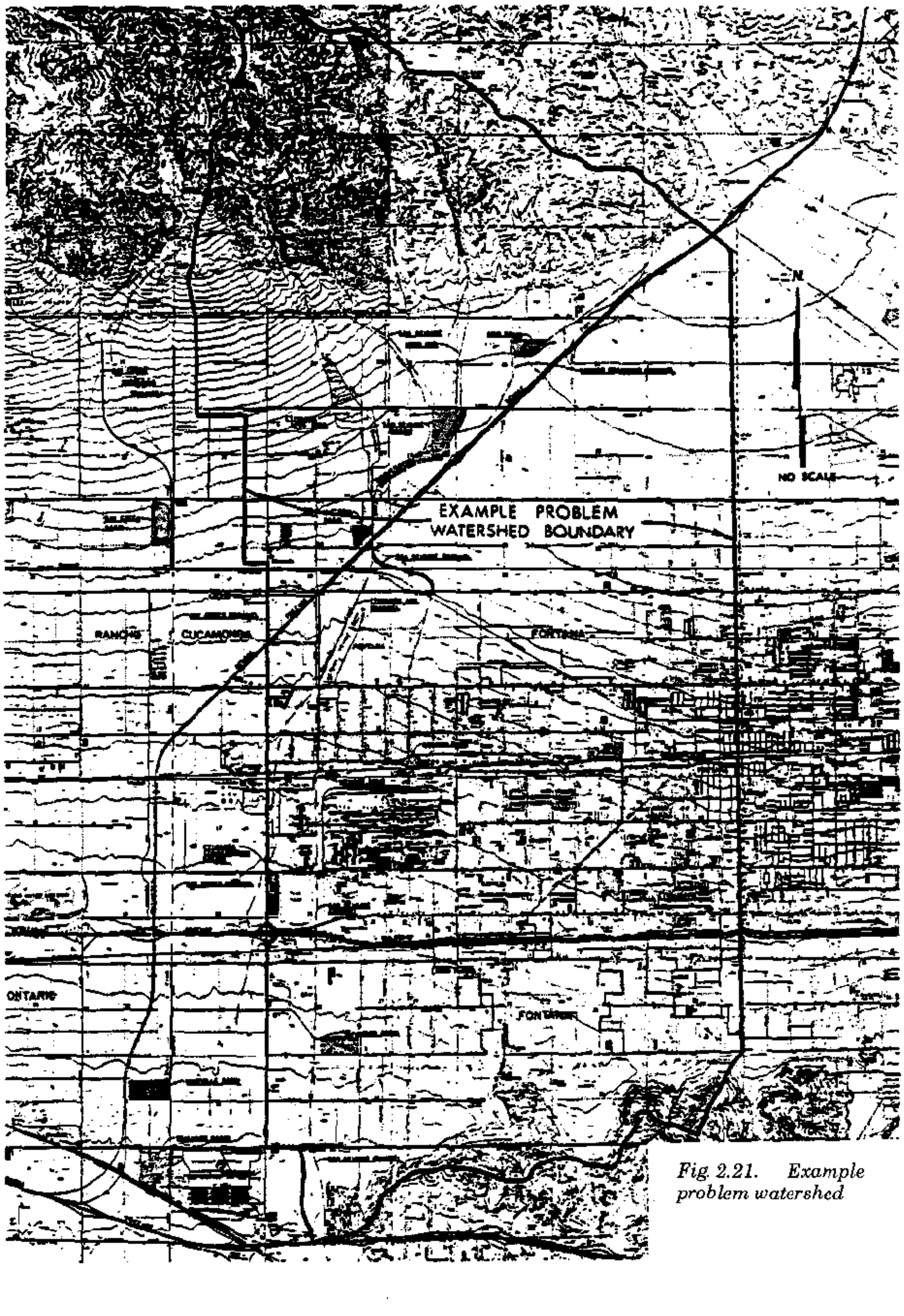

Fig. 2.21. Example problem watershed

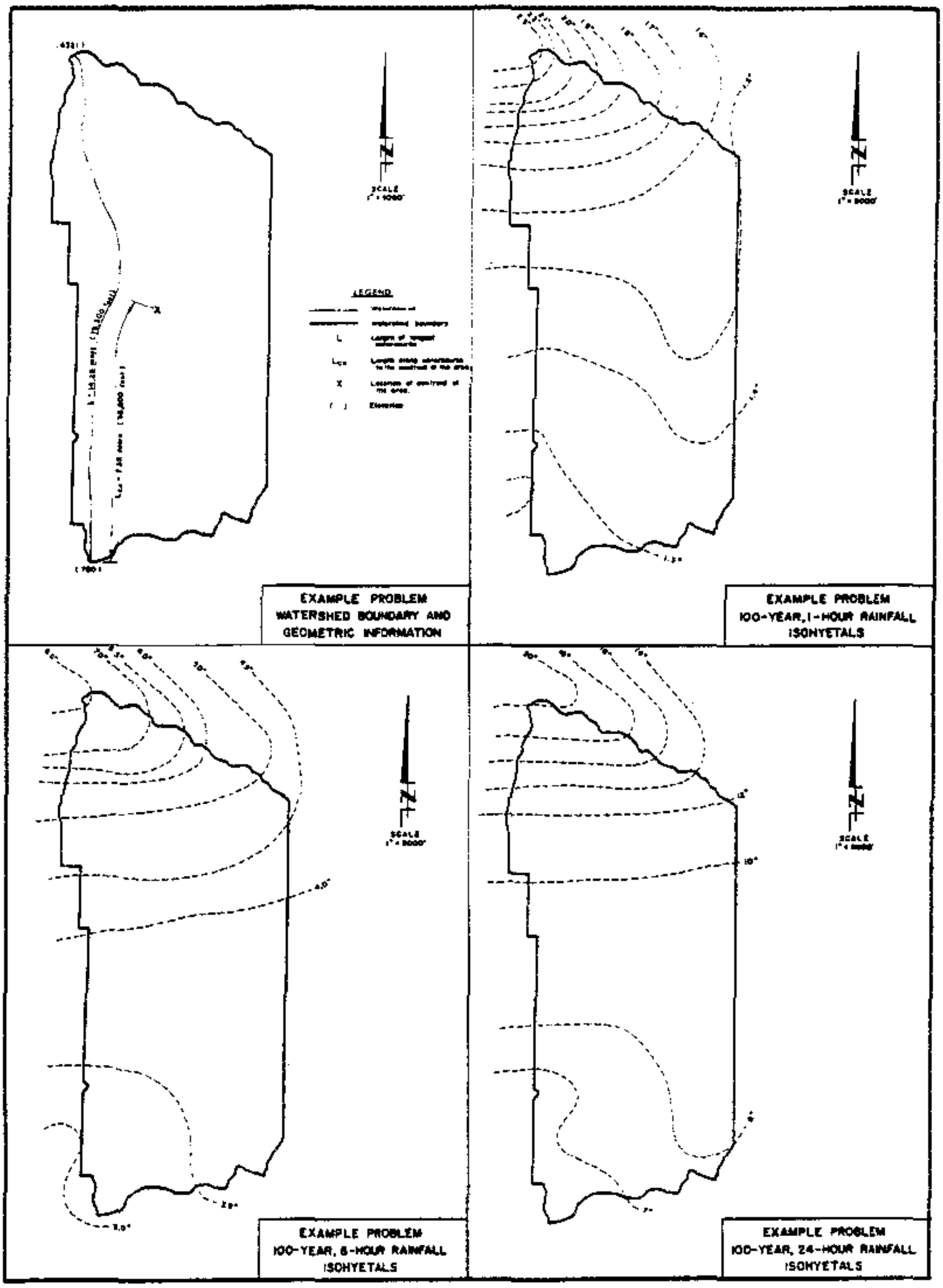

Fig. 2.22. Example problem rainfall isohyetals

PROJECT: *EXAMPLE PROBLEM* DATE: ______

ENGINEER: ______

1.	Enter the length of the longest watercourse (feet)	75,300
2.	Enter the length along the longest watershed watercourse measured from the point of concentration upstream to a point opposite the centroid of the watershed area (feet)	38,800
3.	Enter the difference in elevation between the most remote point in the watershed and the point of concentration (feet)	5,541
4.	Enter the basin factor	0.030
5.	Enter the watershed area (acres)	38,590
6.	Enter baseflow (cfs/square mile)	0.0
7.	Enter S-Graph proportions (decimal)	
	SCS[1]	0.0
	Valley	1.0
	Foothill	0.0
	Mountain	0.0
	Desert	0.0
8.	Enter adjusted loss rate (inch/hour)	0.30
9.	Enter low loss rate percentage (decimal)	0.90
10.	Enter watershed area-averaged 5-minute point rainfall (inches)*	0.56
	Enter watershed area-averaged 30-minute point rainfall (inches)*	1.13
	Enter watershed area-averaged 1-hour point rainfall (inches)*	1.50
	Enter watershed area-averaged 3-hour point rainfall (inches)*	2.90
	Enter watershed area-averaged 6-hour point rainfall (inches)*	4.50
	Enter watershed area-averaged 24-hour point rainfall (inches)*	9.80
11.	Enter 24-hour storm unit interval (minutes)	10.0

[1]Note: use either SCS or Valley, Foothill, Mountain, Desert
*Note: enter values unadjusted by depth-area factors

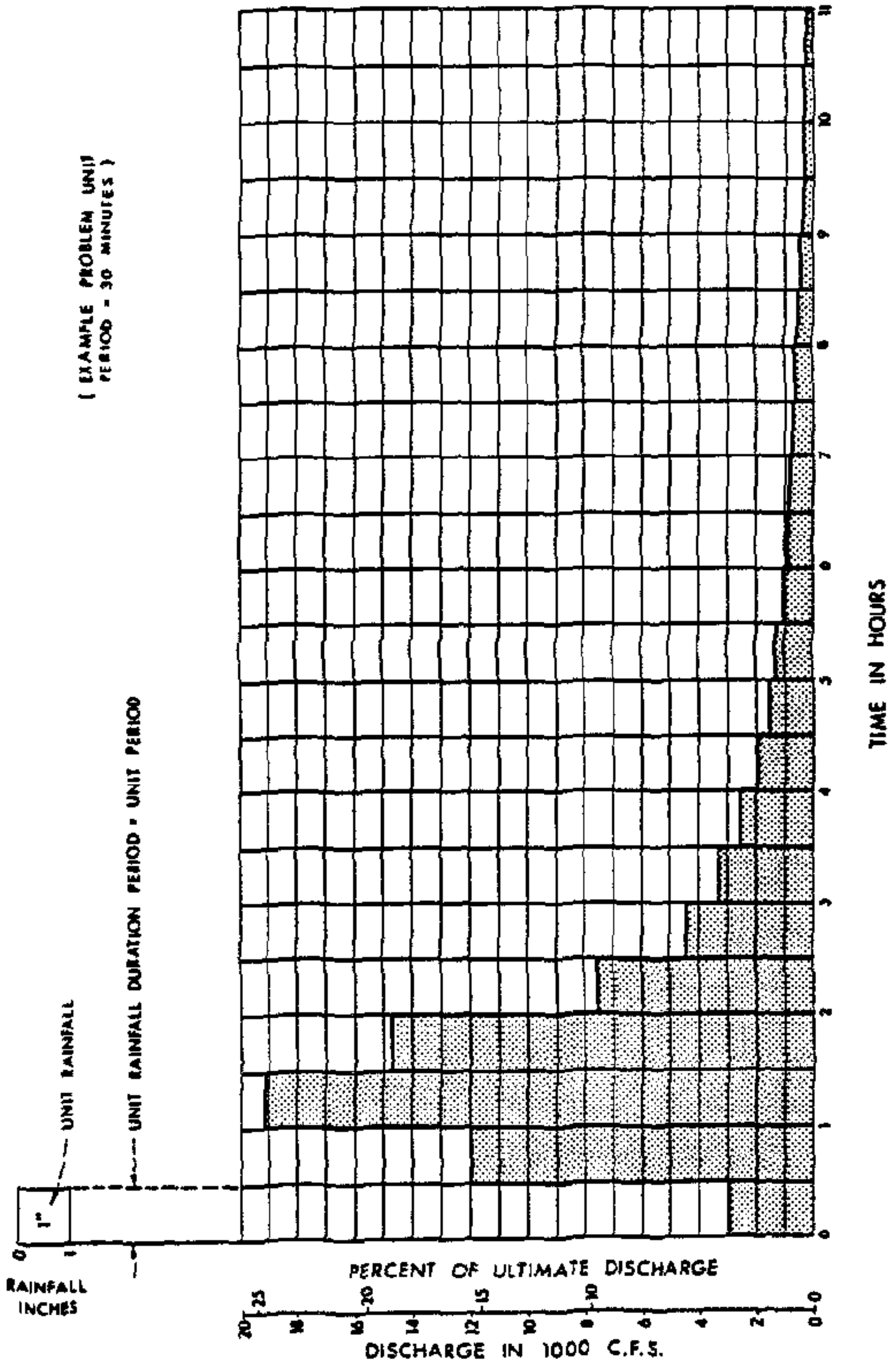

Fig. 2.23. *Example problem unit hydrograph*

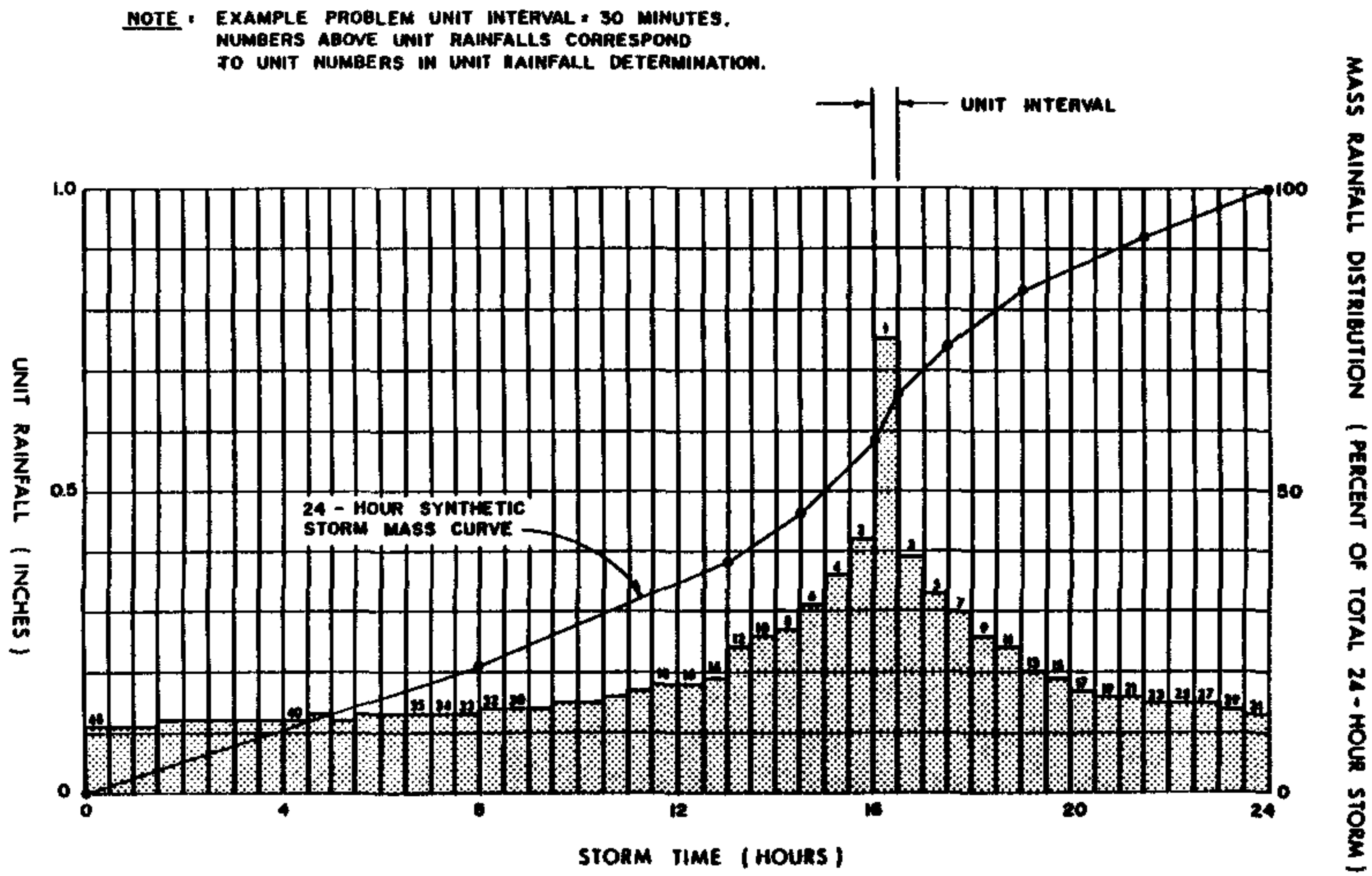

Fig 2.24. Storm rainfall pattern

```
**********DESCRIPTION OF RESULTS*********************************************
*                                                                           *
*                          EXAMPLE PROBLEM                                  *
*                                                                           *
*****************************************************************************

*****************************************************************************
          WATERCOURSE LENGTH =   75300.000 FEET
          LENGTH FROM CONCENTRATION POINT TO CENTROID =   38800.000 FEET
          ELEVATION VARIATION ALONG WATERCOURSE =    5541.000 FEET
          MANNINGS FRICTION FACTOR ALONG WATERCOURSE = .030
          WATERSHED AREA =   38590.000 ACRES
          WATERCOURSE "LAG" TIME =    1.359 HOURS
          UNIT HYDROGRAPH TIME UNIT =  10.000 MINUTES
          UNIT INTERVAL PERCENTAGE OF LAG-TIME =  12.268
          HYDROGRAPH BASEFLOW =     .000 CFS
          UNIFORM MEAN SOIL-LOSS(INCH/HOUR) =   .300
          LOW SOIL-LOSS RATE PERCENT(DECIMAL) =  .900
          VALLEY S-GRAPH SELECTED

          SPECIFIED PEAK 5-MINUTES RAINFALL(INCH)=   .55
          SPECIFIED PEAK 30-MINUTES RAINFALL(INCH) =  1.13
          SPECIFIED PEAK 1-HOUR RAINFALL(INCH) =  1.50
          SPECIFIED PEAK 3-HOUR RAINFALL(INCH) =  2.90
          SPECIFIED PEAK 6-HOUR RAINFALL(INCH) =  4.50
          SPECIFIED PEAK 24-HOUR RAINFALL(INCH)=  9.80

          *HYDROGRAPH MODEL #2 SPECIFIED*

          SAN BERNARDINO COUNTY AREA REDUCTION FACTORS:
            30-MINUTE FACTOR =  .665
             1-HOUR FACTOR =  .780
             3-HOUR FACTOR =  .884
             6-HOUR FACTOR =  .918
            24-HOUR FACTOR =  .948

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<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>

********************************************************************************
                        UNIT HYDROGRAPH DETERMINATION
```

INTERVAL NUMBER	"S" GRAPH MEAN VALUES	"UNIT HYDROGRAPH ORDINATES(CFS)
1	1.063	2481.010
2	3.437	5538.562
3	7.055	8444.191
4	12.251	12124.540
5	18.803	15288.240
6	26.768	18586.040
7	35.415	20178.950
8	44.247	20607.790
9	52.226	18618.550
10	58.433	14485.880
11	63.200	11123.090
12	67.039	8958.375
13	70.172	7309.129
14	72.659	5804.375
15	74.717	4801.316
16	76.542	4259.672
17	78.274	4041.979
18	79.762	3472.066
19	81.181	3311.589
20	82.422	2895.531
21	83.620	2795.193
22	84.710	2542.567
23	85.755	2440.056
24	86.656	2101.976
25	87.517	2009.827
26	88.269	1753.498
27	89.005	1718.568
28	89.741	1716.361
29	90.455	1667.331
30	91.023	1324.016
31	91.562	1258.679
32	92.102	1259.391
33	92.630	1233.398
34	93.130	1166.744
35	93.630	1166.744
36	94.130	1166.744
37	94.630	1166.744
38	95.130	1166.744
39	95.630	1166.744
40	96.130	1166.744
41	96.630	1166.744
42	97.130	1166.744
43	97.630	1166.744
44	98.130	1166.744
45	98.630	1166.744
46	99.130	1166.744
47	99.630	1166.744
48	100.000	862.560

```
********************************************************************************
```

Fig. 2.25. *Example problem program results*

UNIT PERIOD (NUMBER)	UNIT RAINFALL (INCHES)	UNIT SOIL-LOSS (INCHES)	EFFECTIVE RAINFALL (INCHES)
1	.0378	.0340	.0038
2	.0379	.0341	.0038
3	.0380	.0342	.0038
4	.0381	.0343	.0038
5	.0382	.0344	.0038
6	.0383	.0345	.0038
7	.0385	.0346	.0038
8	.0386	.0347	.0039
9	.0387	.0348	.0039
10	.0388	.0349	.0039
11	.0389	.0350	.0039
12	.0390	.0351	.0039
13	.0392	.0353	.0039
14	.0393	.0354	.0039
15	.0394	.0355	.0039
16	.0395	.0356	.0040
17	.0397	.0357	.0040
18	.0398	.0358	.0040
19	.0399	.0359	.0040
20	.0401	.0360	.0040
21	.0402	.0362	.0040
22	.0403	.0363	.0040
23	.0405	.0364	.0040
24	.0406	.0365	.0041
25	.0408	.0367	.0041
26	.0409	.0368	.0041
27	.0410	.0369	.0041
28	.0412	.0371	.0041
29	.0413	.0372	.0041
30	.0415	.0373	.0041
31	.0416	.0375	.0042
32	.0418	.0376	.0042
33	.0419	.0377	.0042
34	.0421	.0379	.0042
35	.0423	.0380	.0042
36	.0424	.0382	.0042
37	.0426	.0383	.0043
38	.0427	.0385	.0043
39	.0429	.0386	.0043
40	.0431	.0388	.0043
41	.0433	.0389	.0043
42	.0434	.0391	.0043
43	.0436	.0393	.0044
44	.0438	.0394	.0044
45	.0440	.0396	.0044
46	.0442	.0397	.0044
47	.0443	.0399	.0044
48	.0440	.0396	.0044
49	.0448	.0403	.0045
50	.0452	.0407	.0045
51	.0456	.0411	.0046
52	.0461	.0415	.0046
53	.0465	.0419	.0047
54	.0470	.0423	.0047
55	.0474	.0427	.0047
56	.0479	.0431	.0048
57	.0484	.0436	.0048
58	.0489	.0440	.0049
59	.0495	.0445	.0049
60	.0500	.0450	.0050
61	.0506	.0455	.0051
62	.0512	.0461	.0051
63	.0518	.0467	.0052
64	.0525	.0472	.0052
65	.0532	.0479	.0053
66	.0539	.0485	.0054
67	.0546	.0492	.0055
68	.0554	.0499	.0055
69	.0563	.0500	.0063
70	.0571	.0500	.0071
71	.0581	.0500	.0081
72	.0590	.0500	.0090
73	.0601	.0500	.0101
74	.0612	.0500	.0112
75	.0623	.0500	.0123
76	.0636	.0500	.0136
77	.0649	.0500	.0149
78	.0663	.0500	.0163
79	.0797	.0500	.0297
80	.0812	.0500	.0312
81	.0828	.0500	.0328
82	.0845	.0500	.0349
83	.0864	.0500	.0364
84	.0885	.0500	.0389
85	.0908	.0500	.0408
86	.0935	.0500	.0435
87	.0964	.0500	.0464
88	.1034	.0500	.0534
89	.1072	.0500	.0572
90	.1117	.0500	.0617
91	.1171	.0500	.0671
92	.1241	.0500	.0742
93	.1334	.0500	.0834
94	.1282	.0500	.0782
95	.1412	.0500	.0912
96	.1228	.0500	.0728
97	.4837	.0500	.4337
98	.1455	.0500	.0955
99	.1479	.0500	.0979
100	.1334	.0500	.0834
101	.1241	.0500	.0741
102	.1171	.0500	.0671
103	.1117	.0500	.0617
104	.1072	.0500	.0572
105	.1034	.0500	.0534
106	.0964	.0500	.0464
107	.0935	.0500	.0435
108	.0908	.0500	.0408
109	.0885	.0500	.0385
110	.0864	.0500	.0364
111	.0845	.0500	.0345
112	.0828	.0500	.0328
113	.0812	.0500	.0312
114	.0797	.0500	.0297
115	.0663	.0500	.0163
116	.0649	.0500	.0149
117	.0636	.0500	.0136
118	.0623	.0500	.0123
119	.0612	.0500	.0112
120	.0601	.0500	.0101
121	.0590	.0500	.0090
122	.0581	.0500	.0081
123	.0571	.0500	.0071
124	.0563	.0500	.0063
125	.0554	.0499	.0055
126	.0546	.0492	.0055
127	.0539	.0485	.0054
128	.0532	.0479	.0053
129	.0525	.0472	.0052
130	.0518	.0467	.0052
131	.0512	.0461	.0051
132	.0506	.0455	.0051
133	.0500	.0450	.0050
134	.0495	.0445	.0049
135	.0489	.0440	.0049
136	.0484	.0436	.0048
137	.0479	.0431	.0048
138	.0474	.0427	.0047
139	.0470	.0423	.0047
140	.0465	.0419	.0047
141	.0461	.0415	.0046
142	.0456	.0411	.0046
143	.0452	.0407	.0045
144	.0448	.0403	.0045

TOTAL SOIL-LOSS VOLUME(ACRE-FEET) = 20411.3600
TOTAL STORM RUNOFF VOLUME(ACRE-FEET) = 9456.9410

HYDROGRAPH POINTS GIVEN IN FIVE-MINUTE INTERVALS

INTERVAL#	VOLUME(AF)	Q(CFS)	0. 5775.0 11550.0 17325.0 23100.0
1	.0645	9.37	Q
2	.1291	9.37	Q
3	.3379	30.32	Q
4	.5467	30.32	Q
5	.9757	62.30	Q
6	1.4048	62.30	Q
7	2.1505	108.28	Q
8	2.8962	108.28	Q
9	4.0418	166.34	Q
10	5.1874	166.34	Q
11	6.8198	237.03	Q
12	8.4522	237.03	Q
13	10.6143	313.94	Q
14	12.7764	313.94	Q
15	15.4810	392.70	Q
16	18.1855	392.70	Q
17	21.3823	464.18	Q
18	24.5792	464.18	Q
19	28.1623	520.27	Q
20	31.7455	520.27	Q
21	35.6287	563.84	Q
22	39.5118	563.84	Q
23	43.6397	599.36	VQ
24	47.7675	599.36	VQ
25	52.0979	628.77	VQ
26	56.4283	628.77	VQ
27	60.9228	652.60	VQ
28	65.4173	652.60	VQ
29	70.0504	672.73	VQ
30	74.6835	672.73	VQ
31	79.4416	690.88	VQ
32	84.1997	690.88	VQ
33	89.0776	708.27	VQ
34	93.9555	708.27	VQ
35	98.9389	723.59	VQ
36	103.9223	723.59	VQ
37	109.0074	738.36	VQ
38	114.0925	738.36	VQ
39	119.2688	751.61	VQ
40	124.4452	751.61	VQ
41	129.7106	764.54	VQ
42	134.9761	764.54	VQ
43	140.3244	776.58	VQ
44	145.6728	776.58	VQ
45	151.1017	788.29	VQ
46	156.5307	788.29	VQ
47	162.0318	798.77	VQ
48	167.5330	798.77	VQ
49	173.1043	808.95	VQ
50	178.6755	808.95	VQ
51	184.3106	818.22	VQ

184	1846.0400	8024.43	V Q
185	1906.0800	8717.90	V Q
186	1966.1210	8717.90	V Q
187	2031.2960	9463.45	V Q
188	2096.4710	9463.45	V Q
189	2167.2440	10276.31	V Q
190	2238.0180	10276.31	V Q
191	2314.4120	11092.45	V Q
192	2390.8060	11092.45	V Q
193	2478.6990	12762.11	V Q
194	2566.5930	12762.11	V Q
195	2667.4470	14644.00	V Q
196	2768.3010	14644.00	V Q
197	2881.5620	16445.54	V Q
198	2994.8230	16445.54	V Q
199	3121.9390	18457.18	V Q
200	3249.0540	18457.18	V Q
201	3387.9010	20160.57	V Q
202	3526.7480	20160.57	V Q
203	3676.7950	21786.85	V Q
204	3826.8420	21786.85	V Q
205	3983.0120	22675.99	V Q
206	4139.1800	22675.99	V Q
207	4297.6090	23004.44	V Q
208	4456.0390	23004.44	V Q
209	4609.8400	22332.24	V Q
210	4763.6410	22332.24	V Q
211	4906.3010	20714.50	V Q
212	5048.9610	20714.50	V Q
213	5180.9100	19160.58	V Q
214	5312.8750	19160.58	V Q
215	5436.0160	17880.52	V Q
216	5559.1560	17880.52	V Q
217	5674.3360	16724.19	V Q
218	5789.5160	16724.19	V Q
219	5896.9690	15602.48	V Q
220	6004.4220	15602.48	V Q
221	6105.3630	14657.08	Q
222	6206.3050	14657.08	QV
223	6301.9060	13881.59	Q V
224	6397.5080	13881.59	Q V
225	6488.8360	13261.05	Q V
226	6580.1640	13261.05	Q V
227	6666.6170	12553.45	Q V
228	6753.0700	12553.45	Q V
229	6835.7730	12008.55	Q V
230	6918.4770	12008.55	Q V
231	6996.7620	11367.36	Q V
232	7075.0470	11367.36	Q V
233	7149.6680	10835.00	Q V
234	7224.2890	10835.00	Q V
235	7294.7110	10225.30	Q V
236	7365.1330	10225.30	Q V
237	7431.6640	9660.36	Q V
238	7498.1950	9660.36	Q V
239	7560.0900	8987.30	Q V
240	7621.9840	8987.30	Q V
241	7679.8120	8397.21	Q V
242	7737.6410	8397.21	Q V
243	7791.0350	7753.07	Q V
244	7844.4300	7753.07	Q V
245	7894.1910	7225.84	Q V
246	7943.9530	7225.84	Q V
247	7990.5740	6769.87	Q V
248	8037.1950	6769.87	Q V

PROGRAM 2.1. DATA ENTRY

```
Enter the length of the longest watercourse...(FEET) ===>    "XL"
:ALLOWABLE VALUES ARE [1000  ] TO [999999]

Enter the length along the longest watershed
watercourse measured from the point of
concentration upstream to a point opposite the
centroid of the watershed area................(FEET) ===>    "XLCA"
:ALLOWABLE VALUES ARE [250   ] TO [800000]

Enter the difference in elevation between
the most remote point in the watershed and
the point of concentration....................(FEET) ===>    "HH"
:ALLOWABLE VALUES ARE [.5    ] TO [100000]

--------------------------------------------------------------------
    TYPE: EXIT to leave program ; TOP  to go to top of page
                                ; BACK to go back one page
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```
BASIN       GENERAL DESCRIPTION OF DRAINAGE AREA
FACTOR      (Ref. U.S. Corps of Engineers)

N= .015 >>> Drainage area has fairly uniform gentle slopes.
            Most watercourses either improved or along paved streets.
N= .020 >>> Drainage has some graded and non-uniform gentle slopes.
            Over half of the area watercourses are improved or paved streets.
N= .025 >>> Drainage area is generally rolling with gentle side slopes.
            Some drainage improvements in the area - streets and canals.
N= .030 >>> Drainage area is generally rolling with rounded ridges and moderate
              side slopes.
            No drainage improvements exist in the area.
N= .040 >>> Drainage area is composed of steep upper canyon with moderate
              slopes in lower canyons.
            No drainage improvements exist in the area.
N= .050 >>> Drainage area is quite rugged with sharp edges and steep canyons.
            No drainage improvements exist in the area.
N= .200 >>> Drainage area has comparatively uniform slopes.
            No drainage improvements exist in the area.
    Enter basin factor (BETWEEN .005 AND .500).......... ===>    "XN"

--------------------------------------------------------------------
    TYPE: EXIT to leave program ; TOP  to go to top of page
                                ; BACK to go back one page
```

```
Enter the watershed area(ACRES)..................... ===>    "A"
:ALLOWABLE VALUES ARE [10    ] TO [96000 ]


Enter baseflow(CFS/SQUARE-MILE)..................... ===>    "BASCON"
:ALLOWABLE VALUES ARE [0] TO [1000  ]


------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
                            ; BACK to go back one page
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```
UNIT-HYDROGRAPH "S" GRAPH OPTIONS:
    1: Valley Zone          2: Foothill Zone          3: Mountain Zone
    4: Desert Zone          5: Combination of options 1 through 4
    6: U.S. Soil Conservation Service (SCS)
Select appropriate S-GRAPH.......................... ===>    "KODE1"

Enter percentage[DECIMAL NOTATION] of watershed
specified with VALLEY "S" curve..................... ===>    "PV"
:ALLOWABLE VALUES ARE [0.00  ] TO [1.00  ]

Enter percentage[DECIMAL NOTATION] of watershed
specified with FOOTHILL "S" curve................... ===>    "PF"
:ALLOWABLE VALUES ARE [0.00  ] TO [1.00  ]

Enter percentage[DECIMAL NOTATION] of watershed
specified with MOUNTAIN "S" curve................... ===>    "PM"
:ALLOWABLE VALUES ARE [0.00  ] TO [1.00  ]

DESERT "S" curve percentage[DECIMAL NOTATION] =         ["PD"]


------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
                            ; BACK to go back one page
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```
Enter adjusted loss rate(INCH/HOUR)................. ===>    "VSL"
:ALLOWABLE VALUES ARE [.01    ] TO [5.0   ]
<RECOMMENDED VALUES ARE [.20] TO [.40]>
NOTE: THE ADJUSTED LOSS RATE INCLUDES THE EFFECTS OF
IMPERVIOUSNESS, INITIAL ABSTRACTION, AND OTHER FACTORS.


Enter low loss rate percentage....[DECIMAL NOTATION] ===>    "SLP"
:ALLOWABLE VALUES ARE [.01    ] TO [.99   ]
NOTE: SUGGESTED VALUES ARE [.10] TO [.50]
NOTE: THE "LOW LOSS RATE PERCENTAGE" IS THE CONSTANT
PERCENTAGE OF RAINFALL TO BE USED FOR WATERSHED LOSSES.
THE MAXIMUM WATERSHED LOSS RATE IS 5.000(INCH/HOUR).

----------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
                            ; BACK to go back one page
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```
PRECIPITATION DEPTH-AREA FACTOR MODELS:

  1: NOAA Atlas II values (30-minute value used
     for 5-minute depth)

  2: User-specified depth-area factors

Select desired model................................ ===>    "DAOPT"

----------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
                            ; BACK to go back one page
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```
TO CONSTRUCT THE SYNTHETIC CRITICAL STORM PATTERN, AREA-AVERAGED
WATERSHED RAINFALL VALUES ARE NEEDED FOR THE PEAK 5-MINUTES,
30-MINUTES,1-,3-,6-,AND 24-HOURS OF RECORDED RAINFALL.

Enter watershed area-averaged  5-minute point
rainfall(INCHES)................................... ===>    "SEE BELOW"
:ALLOWABLE VALUES ARE [  .010] TO [    5.]

Enter  5-minute depth-area adjustment factor........ ===>    "SEE BELOW"
:ALLOWABLE VALUES ARE [  .010] TO [1.    ]

WATERSHED AREA-AVERAGED POINT RAINFALLS     DEPTH-AREA ADJUSTMENT FACTOR

      5-MINUTE    [ "R5"  ] INCHES              5-MINUTE    [ "FX5" ]
     30-MINUTE    [ "R30" ] INCHES             30-MINUTE    [ "FX30"]
        1-HOUR    [ "R1"  ] INCHES                1-HOUR    [ "FX1" ]
        3-HOUR    [ "R3"  ] INCHES                3-HOUR    [ "FX3" ]
        6-HOUR    [ "R6"  ] INCHES                6-HOUR    [ "FX6" ]
       24-HOUR    [ "R24" ] INCHES               24-HOUR    [ "FX24"]

--------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
                            ; BACK to go back one page
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```
24-HOUR STORM UNIT-INTERVAL MODELS:

  MODEL      MODEL UNIT-INTERVAL
 NUMBER        (DURATION)
    1:          5-MINUTES        <=RECOMMENDED MODEL
    2:         10-MINUTES
    3:         15-MINUTES
    4:         20-MINUTES
    5:         30-MINUTES
    6:         60-MINUTES

Select 24-hour storm unit-interval model number..... ===>    "KTYPE"


EFFECTIVE RAINFALL INFORMATION DISPLAY OPTIONS:
    0:        Omit Effective Rainfall information from results
    1:        Include Effective Rainfall information with results
Select display option desired...................... ===>    "KSOIL"



--------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
                            ; BACK to go back one page
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```
HYDROGRAPH RESULTS LISTING OPTIONS:

  1: List ENTIRE runoff-hydrograph

  2: List runoff-hydrograph according to USER-SPECIFIED
     time limits

Select desired option............................. ==>    "SEE BELOW"

Enter model time(HOURS) for BEGINNING of results.... ==>    "TIME1"
:ALLOWABLE VALUES ARE [0.0   ] TO [20.0  ]

Enter model time(HOURS) for END of results.......... ==>    "TIME2"
:ALLOWABLE VALUES ARE [0.0   ] TO [30.0  ]

--------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
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PROGRAM 2.1.

```
C  ---------------------------------------------------------------
      SUBROUTINE UNITH
C  ---------------------------------------------------------------
C
C  UNIT HYDROGRAPH
C
C
      DIMENSION H(440)
      DIMENSION UH(150),PERCNT(150),R(288),IOFILE(16)
      DIMENSION DYR(48)
      COMMON/NUT/NUT
C
      DATA DYR/0.,20.,40.,60.,80.,100.,125.,150.,100.,89.4,82.3,78.,
     C 74.8,72.4,70.2,68.5,100.,94.8,91.2,88.4,86.3,84.5,82.7,81.3,
     C 100.,96.2,93.8,91.8,90.3,89.,87.8,86.7,100.,97.4,95.8,94.8,
     C 93.9,93.2,92.8,92.3,100.,82.,72.5,66.6,63.,60.8,59.,57.5/
      DATA UH/150*0./
      DATA PERCNT/150*0./
      DATA R/288*0./
      DATA H/440*0./
C
C  FUNCTION
C
      RR(T)=EXP(A*ALOG(T)+B)
C
C..READ DATA INPUT
      READ FREE(5)KTYPE,XL,XLCA,HH,XN,AREA,VSL,KODE1,BASCON,SLP,
     . R5,R30,R1,R3,R6,R24,SS,KSTORM,KSOIL,PV,PF,PM,PD,NUT,
     . FX5,FX30,FX1,FX3,FX6,FX24,DAOPT,TIME1,TIME2
      AX=AREA
      XLX=XL
      XLCAX=XLCA
C
      FX=1.
C
      IF(DAOPT.EQ.2)GO TO 183
C
      AREA=AREA/640.
      FX1=.651
      FX3=.78
      FX6=.831
      FX24=.91
      IF(AREA.GE.350.)GOTO 180
C
      DO 100 I=1,13
      IF(AREA.LE.DYR(I))GOTO 110
100   CONTINUE
C
110   I=I-1
      DX=DYR(I+1)-DYR(I)
      FACT=AREA-DYR(I)
      FX1=(DYR( 8+I)-FACT*(DYR( 8+I)-DYR( 9+I))/DX)/100.
      FX3=(DYR(16+I)-FACT*(DYR(16+I)-DYR(17+I))/DX)/100.
      FX6=(DYR(24+I)-FACT*(DYR(24+I)-DYR(25+I))/DX)/100.
      FX24=(DYR(32+I)-FACT*(DYR(32+I)-DYR(33+I))/DX)/100.
      FX30=(DYR(40+I)-FACT*(DYR(40+I)-DYR(41+I))/DX)/100.
      FX5=FX30
180   CONTINUE
      AREA=AREA*640.
```

```
C
C
183     CONTINUE
        WRITE(NUT,5000)
        WRITE(NUT,181)
181     FORMAT(/,14X,'U N I T - H Y D R O G R A P H   A N A L Y S I S',
     .  /)
        WRITE(NUT,5000)
6666    FORMAT(/,10X,'*USER SPECIFIED PRECIPITATION DEPTH-AREA ',
     .  'REDUCTION FACTORS:')
6665    FORMAT(/,10X,'PRECIPITATION DEPTH-AREA REDUCTION FACTORS:')
6664    FORMAT(
     .  12X,'5-MINUTE FACTOR = ',F5.3,/,12X,'30-MINUTE FACTOR = ',
     .   F5.3,/,
     C  12X,'1-HOUR FACTOR = ',F5.3,/,12X,'3-HOUR FACTOR = ',F5.3,/,
     C  12X,'6-HOUR FACTOR = ',F5.3,/,12X,'24-HOUR FACTOR = ',F5.3)
C
C  DETERMINE HYDROGRAPH FACTORS
C
        XL=XL/5280.
        XLCA=XLCA/5280.
      S=HH/XL
      XLAGX=1.2*(XL*XLCA/S**0.5)**0.38
      XLAG=20.0*XN*XLAGX
C
C  DESIGNATE UNIT INTERVALS
C
        IF(KTYPE.EQ.1)UNIT=5.
        IF(KTYPE.EQ.2)UNIT=10.
        IF(KTYPE.EQ.3)UNIT=15.
        IF(KTYPE.EQ.4)UNIT=20.
        IF(KTYPE.EQ.5)UNIT=30.
        IF(KTYPE.EQ.6)UNIT=60.
C
        TIMLAG=100.*UNIT/60./XLAG
        SQMI=AREA*43560./5280./5280.
        BASFLO=BASCON*SQMI
        XK=645.*SQMI*60./UNIT
        XK=XK/100.
C
C  RETURN XL AND XLCA TO UNITS OF FEET
C
        XL=XL*5280.
        XLCA=XLCA*5280.
5000    FORMAT(1X,76('*'))
C
        WRITE(NUT,5000)
        WRITE(NUT,5001)XLX,XLCAX,HH,XN,AX,XLAG,UNIT,TIMLAG,
     C  BASCON,VSL,SLP
5001    FORMAT(10X,'WATERCOURSE LENGTH = ',F11.3,' FEET',/,
     C10X,'LENGTH FROM CONCENTRATION POINT TO CENTROID = ',
     CF11.3,' FEET',/,10X,'ELEVATION VARIATION ALONG ',
     C 'WATERCOURSE = ',F11.3,' FEET',/,10X,
     C 'BASIN FACTOR = ',F5.3,/,
     C 10X,'WATERSHED AREA = ',F11.3,' ACRES',
     C/,10X,'WATERCOURSE "LAG" TIME = ',F8.3,' HOURS',/,
     C10X,'UNIT HYDROGRAPH TIME UNIT = ',F7.3,' MINUTES',/,
     C10X,'UNIT INTERVAL PERCENTAGE OF LAG-TIME = ',F7.3,/,
     C10X,'BASEFLOW = ',F7.3,' CFS/SQUARE-MILE',/,10X,
     C  'MAXIMUM WATERSHED LOSS RATE(INCH/HOUR) = ',F6.3,/,
```

```
     C  10X,'LOW LOSS RATE PERCENTAGE(DECIMAL) = ',F5.3)
        IF(KODE1.EQ.1)WRITE(NUT;8200)
        IF(KODE1.EQ.2)WRITE(NUT,8202)
        IF(KODE1.EQ.3)WRITE(NUT,8204)
        IF(KODE1.EQ.4)WRITE(NUT,8205)
        IF(KODE1.EQ.5)WRITE(NUT,82041)PV,PF,PM,PD
        IF(KODE1.EQ.6)WRITE(NUT,82042)
82041   FORMAT(10X,'VALLEY "S"-CURVE PERCENTAGE(DECIMAL NOTATION) = ',
     C  F6.3,/,10X,'FOOTHILL "S"-CURVE PERCENTAGE(DECIMAL NOTATION) = '
     C  ,F6.3,/,10X,'MOUNTAIN "S"-CURVE PERCENTAGE(DECIMAL NOTATION) = '
     C  ,F6.3,/,10X,'DESERT "S"-CURVE PERCENTAGE(DECIMAL NOTATION) = ',
     C  F6.3,/)
8205    FORMAT(10X,'DESERT S-GRAPH SELECTED',/)
8200    FORMAT(10X,'VALLEY S-GRAPH SELECTED',/)
8202    FORMAT(10X,'FOOTHILL S-GRAPH SELECTED',/)
8204    FORMAT(10X,'MOUNTAIN S-GRAPH SELECTED',/)
82042   FORMAT(10X,'U.S. SOIL CONSERVATION SERVICE S-GRAPH SELECTED',/)
        WRITE(NUT,8209)R5,R30
8209    FORMAT(10X,'SPECIFIED PEAK 5-MINUTES RAINFALL(INCH)= ',F5.2,/,
     C  10X,'SPECIFIED PEAK 30-MINUTES RAINFALL(INCH) = ',F5.2)
        WRITE(NUT,8206)R1,R3,R6,R24,KTYPE
8206    FORMAT(10X,'SPECIFIED PEAK 1-HOUR RAINFALL(INCH) = ',F5.2,/,
     C  10X,'SPECIFIED PEAK 3-HOUR RAINFALL(INCH) = ',F5.2,/,
     C  10X,'SPECIFIED PEAK 6-HOUR RAINFALL(INCH) = ',F5.2,/,
     C  10X,'SPECIFIED PEAK 24-HOUR RAINFALL(INCH)= ',F5.2,//,
     C  10X,'*HYDROGRAPH MODEL #',I1,' SPECIFIED*')
        IF(DAOPT.EQ.1.)WRITE(NUT,6665)
        IF(DAOPT.EQ.2.)WRITE(NUT,6666)
        WRITE(NUT,6664)FX5,FX30,FX1,FX3,FX6,FX24
C
C  USER TIME LIMITS
        IF(TIME2.EQ.31.)GO TO 957
        WRITE(NUT,958)TIME1,TIME2
958     FORMAT(/10X,'RUNOFF HYDROGRAPH LISTING LIMITS:',
     .  /10X,'MODEL TIME(HOURS) FOR BEGINNING OF RESULTS',
     .  ' = ',F5.2,
     .  /10X,'MODEL TIME(HOURS) FOR END OF RESULTS ',
     .  '= ',F5.2)
C
        XR=VSL*UNIT/60.
C
C  DETERMINATION OF UNIT HYDROGRAPH ORDINATES
C
        IF(KODE1.LT.5 .OR. KODE1.EQ.6)GOTO 94
C
C=======LINEAR WEIGHTING OF S-CURVES
        NLAG=0
        DO 98 KLAG=1,4
        PLAG=PV
        IF(KLAG.EQ.2)PLAG=PF
        IF(KLAG.EQ.3)PLAG=PM
        IF(KLAG.EQ.4)PLAG=PD
        IF(KLAG.EQ.0)GOTO 98
        CALL SUBSB(TIMLAG,PERCNT,KLAG,NUMBER)
        IF(NUMBER.GT.NLAG)NLAG=NUMBER
        DO 99 I=1,150
        IF (PERCNT(I).EQ.0.)PERCNT(I)=100.
        H(I)=H(I)+PERCNT(I)*PLAG
99      PERCNT(I)=0.
98      CONTINUE
```

```
C
C  UNIT HYDROGRAPH DETERMINATION
C
        NUMBER=NLAG
C
        DO 97 I=1,150
        IF(I.LE.NLAG)PERCNT(I)=H(I)
97      H(I)=0
        GOTO 96
C
94      CALL SUBSB(TIMLAG,PERCNT,KODE1,NUMBER)
C
96    SUM=0.
        IF(NUMBER.GE.150)WRITE(NUT,8207)
8207    FORMAT(3X,'UNIT HYDROGRAPH TERMINATED AFTER 150 UNIT INTERVALS',/)
        IF(NUMBER.GT.150)NUMBER=150
C
        DO 8208 I=1,NUMBER
        UH(I)=(PERCNT(I)-SUM)*XK
        SUM=PERCNT(I)
        IF(UH(I).LT.0.)UH(I)=0.
8208    CONTINUE
C
        WRITE(NUT,5002)
5002    FORMAT(23X,'UNIT HYDROGRAPH DETERMINATION',/)
        WRITE(NUT,5003)
5003    FORMAT(76('-'))
        WRITE(NUT,5004)
5004    FORMAT(5X,'INTERVAL',10X,'"S" GRAPH',10X,'UNIT HYDROGRAPH',/,
     C6X,'NUMBER',10X,'MEAN VALUES',10X,'ORDINATES(CFS)',)
        WRITE(NUT,5003)
        WRITE(NUT,5005)(I,PERCNT(I),UH(I),I=1,NUMBER)
5005    FORMAT(7X,I3,15X,F7.3,13X,F10.3)
C
        WRITE(NUT,5000)
5050    FORMAT(A1)
        IF(KSOIL.EQ.1)WRITE(NUT,5000)
        IF(KSOIL.EQ.1)WRITE(NUT,5020)
        IF(KSOIL.EQ.1)WRITE(NUT,5003)
5020    FORMAT(10X,'UNIT',14X,'UNIT',12X,'UNIT',14X,'EFFECTIVE',/,
     C  9X,'PERIOD',11X,'RAINFALL',7X,'SOIL-LOSS',12X,'RAINFALL',/,
     C  8X,'(NUMBER)',10X,'(INCHES)',8X,'(INCHES)',12X,'(INCHES)')
C
C
C  24-HOUR STORM RAINFALL PATTERN
C
        R5A=R5*FX5
        R30A=R30*FX30
        R1A=R1*FX1
        R3A=R3*FX3
        R6A=R6*FX6
        R24A=R24*FX24
C
C
        A=(ALOG(R30A)-ALOG(R5A))/(ALOG(.5)-ALOG(.0833))
        B=ALOG(R5A)-A*ALOG(.0833)
        R(193)=RR(.0833)
        R(194)=RR(.1667)-RR(.0833)
        R(195)=RR(.25)-RR(.1667)
        R(192)=RR(.3333)-RR(.25)
```

```
      R(196)=RR(.41667)-RR(.3333)
      R(191)=RR(.5)-RR(.41667)
      A=(ALOG(R1A)-ALOG(R30A))/(ALOG(1.)-ALOG(.5))
      B=ALOG(R30A)-A*ALOG(.5)
      R(197)=RR(.5833)-RR(.5)
      R(190)=RR(.6667)-RR(.5833)
      R(198)=RR(.75)-RR(.6667)
      R(189)=RR(.8333)-RR(.75)
      R(188)=RR(.9167)-RR(.8333)
      R(187)=RR(1.)-RR(.9167)
C.......REMAINING PART OF PEAK 3-HOUR STORM
      A=(ALOG(R3A)-ALOG(R1A))/(ALOG(3)-ALOG(1))
      B=ALOG(R1A)-A*ALOG(1)
      RRSAVE=R1A
C
      DO 1001 J=1,12
      XJ=J
      DT=XJ*.1667
      T=1.+DT
      RRNEW=RR(T)
      DR=(RRNEW-RRSAVE)/2.
      R(198+J)=DR
      R(187-J)=DR
1001  RRSAVE=RRNEW
C
C  REMAINING PART OF PEAK 6-HOUR STORM
C
      A=(ALOG(R6A)-ALOG(R3A))/(ALOG(6)-ALOG(3))
      B=ALOG(R3A)-A*ALOG(3)
      RRSAVE=R3A
C
      DO 1010 J=1,18
      XJ=J
      DT=XJ*.1667
      T=3.+DT
      RRNEW=RR(T)
      DR=(RRNEW-RRSAVE)/2.
      R(210+J)=DR
      R(175-J)=DR
1010  RRSAVE=RRNEW
C
C  REMAINING PART OF PEAK 24-HOUR STORM
      A=(ALOG(R24A)-ALOG(R6A))/(ALOG(24)-ALOG(6))
      B=ALOG(R6A)-A*ALOG(6)
      RRSAVE=R6A
C
      DO 1020 J=1,60
      XJ=J
      DT=XJ*.1667
      T=6.+DT
      RRNEW=RR(T)
      DR=(RRNEW-RRSAVE)/2.
      R(228+J)=DR
      R(157-J)=DR
1020  RRSAVE=RRNEW
C
      DO 1030 J=1,96
      XJ=J
      DT=XJ*.08333
      T=16.+DT
```

```
          RRNEW=RR(T)
          DR=RRNEW-RRSAVE
          R(97-J)=DR
1030      RRSAVE=RRNEW
C
C   ADJUST R-ARRAY FOR LARGER UNIT INTERVALS
C
C
C   NI = NUMBER OF STORM HYDROGRAPH INTERVALS
          NI=288
          IF(KTYPE.EQ.1)GOTO 1050
          K=KTYPE
          IF(KTYPE.EQ.5)K=6
          IF(KTYPE.EQ.6)K=12
          NI=288/K
          DO 1040 I=1,NI
          TEMP=0.
          II=(I-1)*K
          DO 1035 J=1,K
1035      TEMP=TEMP+R(II+J)
1040      R(I)=TEMP
1050      CONTINUE
C
C   ADJUST FOR CONSTANT SOIL LOSS
C
          XTOTAL=0.
          TEMPS=UNIT/60.
          XRA=XR
          DO 300 I=1,NI
          XLOSS=R(I)*SLP
          IF(XLOSS.GT.XRA) XLOSS=XRA
          TEMP=R(I)
          R(I)=R(I)-XLOSS
          XTOTAL=XTOTAL+XLOSS
C
          IF(KSOIL.EQ.1)WRITE(NUT,5021)I,TEMP,XLOSS,R(I)
5021      FORMAT(10X,I3,13X,F6.4,11X,F6.4,14X,F6.4)
  300 CONTINUE
C
C DETERMINE STORM RUNOFF HYDROGRAPH
C
          INTERV=NUMBER+NI-1
          IF(INTERV.GT.440)WRITE(NUT,432)
          IF(INTERV.GT.439)INTERV=439
432       FORMAT(3X,'RUNOFF HYDROGRAPH TERMINATED AFTER 440 UNIT',/,
     C    3X,'INTERVALS. SUGGEST USING A LARGER UNIT INTERVAL.')
          DO 600 I=1,INTERV
          M=I
          N=1+M
          DO 500 J=1,M
          K=N-J
          IF(J.GT.NI)GO TO 500
          IF(K.GT.NUMBER) GO TO 500
          H(M)=H(M)+R(J)*UH(K)
  500 CONTINUE
          H(M)=H(M)+BASFLO
  600 CONTINUE
C
C   COMPUTE SUMMED HYDROGRAPH
C
```

```
      SUM=0.0
      XMAX=0.
      DO 700 I=1,INTERV
      IF(H(I).LT.0.)H(I)=0.
      IF(H(I).GT.XMAX)XMAX=H(I)
      SUM=SUM + H(I)
 700  CONTINUE
      SUM=SUM*UNIT*60./43560.
      XTOTAL=XTOTAL/12.*AREA
C
      WRITE(NUT,5003)
      WRITE(NUT,6008)XTOTAL,SUM
6008  FORMAT(5X,'TOTAL SOIL-LOSS VOLUME(ACRE-FEET) = ',F12.4,/,
     C 5X,'TOTAL STORM RUNOFF VOLUME(ACRE-FEET) = ',F12.4)
C
      IF(XMAX.LT.100.)GOTO 8000
      I=XMAX/100.
      II=I+1
      XMAX=II
      XMAX=XMAX*100.
      GOTO 8100
8000  I=XMAX/10.
      II=10*(I+1)
      XMAX=II
8100  CONTINUE
      WRITE(NUT,5003)
C
      CALL OASB(KTYPE,H,INTERV,XMAX,UNIT,SUM,TIME1,TIME2)
C
10000 CONTINUE
C
      RETURN
      END
```

PROGRAM 2.2.

```
C -----------------------------------------------------------------
      SUBROUTINE SUBSB(TIMLAG,PERCNT,KODE1,NUMBER)
C -----------------------------------------------------------------
C
C     THIS SUBROUTINE DETERMINES THE PERCNTAGES OF DISCHARGE
C     FACTORS FOR THE VARIOUS ZONE CLASSIFICATIONS.
C
C
C
      REAL MNT
      DIMENSION PERCNT(150),VAL(31,2),FOOT(31,2),MNT(33,2),SCS(33,2)
      DIMENSION DESERT(33,2)
      COMMON/NUT/NUT
C
      DATA VAL/0.,15.0,25.,35.0,50.,65.0,75.,100.,115.,125.,
     C140.0,150.0,165.0,175.0,200.0,225.0,250.,275.,300.,325.,350.,
     C375.,400.,450.,500.,550.,600.,650.,700.,750.,99999.,0.0,2.6,
     C5.0,8.6,15.5,25.0,32.0,50.0,57.9,62.0,66.8,69.5,72.6,74.3,
     C78.0,81.0,83.5,85.7,87.5,89.0,90.5,91.6,92.7,94.3,95.8,
     C96.9,97.8,98.5,99.0,99.5,100. /
      DATA FOOT/0.,15.0,25.,35.0,50.,60.0,75.,85.0,90.0,95.0,
     C100.0,110.0,125.0,140.0,150.0,175.0,200.0,225.0,250.0,
     C275.0,300.0,325.0,350.0,375.0,400.0,450.0,500.0,550.0,
     C600.0,650.0,99999.,0.0,1.9,3.8,6.0,10.3,14.0,21.7,29.0,
     C34.2,43.3,50.0,56.9,63.8,69.0,71.9,77.8,82.4,86.0,89.0,
     C91.4,93.4,95.0,96.2,97.2,97.9,98.5,99.0,99.3,99.7,
     C99.9,100. /
      DATA MNT/0.,15.0,25.,35.0,40.0,50.,65.0,75.0,90.0,
     C100.0,115.0,125.0,140.0,150.0,175.0,200.0,225.0,250.0,
     C275.0,300.0,325.0,350.0,375.0,400.0,450.0,500.0,
     C550.0,600.0,650.0,700.0,750.0,800.0,99999.0,0.0,
     C3.3,6.7,10.6,13.4,21.0,33.0,39.3,46.3,50.0,54.2,
     C56.7,59.8,61.8,65.8,69.2,72.2,74.8,77.0,79.0,80.7,
     C82.2,83.5,84.8,86.9,88.9,90.5,92.0,93.3,94.5,95.5,
     C96.4,100. /
      DATA DESERT/0.,12.5,25.0,37.5,50.0,62.5,75.0,87.5,100.,
     C112.5,125.,137.5,150.,162.5,175.,187.5,200.,225.,250.,275.,
     C300.,325.,350.,375.,400.,450.,500.,550.,600.,700.,800.,1000.,
     C9999.,0.,1.1,3.2,6.3,10.5,18.5,31.3,42.0,50.0,56.5,61.3,65.2,
     C68.5,71.5,74.0,76.2,78.3,81.6,84.3,86.7,88.7,90.2,91.6,92.8,
     C93.9,95.6,96.9,97.8,98.3,99.5,99.9,99.99,100./
      DATA SCS/0.,8.6,17.2,25.9,34.5,43.1,51.7,60.3,69.,77.6,
     C 86.2,94.8,103.4,112.1,120.7,129.3,137.9,146.5,155.2,
     C 163.8,172.4,189.6,206.9,224.1,241.4,258.6,275.8,293.1,
     C 310.3,327.6,344.8,387.9,999.,0.,.1,.6,1.2,3.5,6.5,10.7,
     C 16.3,22.8,30.,37.5,45.,52.2,58.9,65.,70.,75.1,79.,82.2,
     C 84.9,87.1,90.8,93.4,95.3,96.7,97.7,98.4,98.9,99.3,99.5,99.7,
     C 99.9,100./
C
C     KODE1=1: VALLEY
C     KODE1=2: FOOTHILL
C     KODE1=3: MOUNTAIN
C     KODE1=4: DESERT
C     KODE1=5: NOT USED DIRECTLY.  LINEAR INTERPOLATION OF 1-4
C     KODE1=6: SCS
C     TIMLAG=LAG
C     TIME=INCREMENTS OF LAG
C     NUMBER=INDEX OF PERCNT VECTOR
C
```

```
      ANEW=0.
      AOLD=0.
      K=1
      NUMBER=1
      TIME=TIMLAG
10    K=K+1
      IF(NUMBER.GE.151)GO TO 1000
      N=K-1
C
      GO TO(100,200,300,350,355,355)KODE1
C
100   CONTINUE
C
C     INTEGRATE "S" GRAPH IN ORDER TO DETERMINE UH(I)
C
      TEMP=0.5*(VAL(K,2)+VAL(N,2))*
     C (VAL(K,1)-VAL(N,1))
      ANEW=ANEW+TEMP
      IF(TIME.GT.VAL(K,1))GO TO 10
      Y=VAL(K,2)
      X=VAL(N,2)
      DEL=(TIME-VAL(N,1))/(VAL(K,1)-VAL(N,1))
      B=VAL(K,1)-VAL(N,1)
      GO TO 400
200   CONTINUE
      TEMP=0.5*(FOOT(K,2)+FOOT(N,2))*
     C (FOOT(K,1)-FOOT(N,1))
      ANEW=ANEW+TEMP
      IF(TIME.GT.FOOT(K,1))GO TO 10
C
      Y=FOOT(K,2)
      X=FOOT(N,2)
      DEL=(TIME-FOOT(N,1))/(FOOT(K,1)-FOOT(N,1))
      B=FOOT(K,1)-FOOT(N,1)
      GO TO 400
300   CONTINUE
      TEMP=0.5*(MNT(K,2)+MNT(N,2))*
     C (MNT(K,1)-MNT(N,1))
      ANEW=ANEW+TEMP
      IF(TIME.GT.MNT(K,1))GO TO 10
      Y=MNT(K,2)
      X=MNT(N,2)
      DEL=(TIME-MNT(N,1))/(MNT(K,1)-MNT(N,1))
      B=MNT(K,1)-MNT(N,1)
C
      GO TO 400
350   CONTINUE
      TEMP=0.5*(DESERT(K,2)+DESERT(N,2))*
     C (DESERT(K,1)-DESERT(N,1))
      ANEW=ANEW+TEMP
      IF(TIME.GT.DESERT(K,1))GO TO 10
      Y=DESERT(K,2)
      X=DESERT(N,2)
      DEL=(TIME-DESERT(N,1))/(DESERT(K,1)-DESERT(N,1))
      B=DESERT(K,1)-DESERT(N,1)
      GO TO 400
C
355   CONTINUE
C
C SCS METHOD
```

```
C
        TEMP=0.5*(SCS(K,2)+SCS(N,2))*(SCS(K,1)-SCS(N,1))
        ANEW=ANEW+TEMP
        IF(TIME.GT.SCS(K,1)) GO TO 10
        Y=SCS(K,2)
        X=SCS(N,2)
        DEL=(TIME-SCS(N,1))/(SCS(K,1)-SCS(N,1))
        B=SCS(K,1)-SCS(N,1)
C
400     CONTINUE
        DEL=DEL*(Y-X)
        XX=X+DEL
C
C       ADJUST INTEGRATION FOR INTERPOLATION
C
        DELA=0.5*(Y+XX)*(1.-DEL/(Y-X))*B
        ANEW=ANEW-DELA
        PERCNT(NUMBER)=(ANEW-AOLD)/TIMLAG
        NUMBER=NUMBER+1
        AOLD=ANEW
        ANEW=ANEW-.5*(X+XX)*DEL/(Y-X)*B
        TIME=TIME+TIMLAG
        K=K-1
        IF(NUMBER.EQ.2)GO TO 10
        DELX=PERCNT(NUMBER-1)-PERCNT(NUMBER-2)
        IF(DELX.LE..51)GO TO 1000
        GO TO 10
1000    CONTINUE
        NUMBER=NUMBER-1
        IF(NUMBER.GE.150)WRITE(NUT,1001)NUMBER
1001    FORMAT(7X,'UNIT-HYDROGRAPH TERMINATED AFTER',I4,' INTERVALS')
C
C  FINISH-OFF HYDROGRAPH
C
        IF(NUMBER.GE.150)GOTO 1250
        NNUM=150-NUMBER
        XNUM=NNUM
        REM=100.-PERCNT(NUMBER)
        REM1=REM/XNUM
        DELX=.5
        IF(REM1.LT.DELX)GOTO 1150
        DO 1140 K=1,NNUM
1140    PERCNT(NUMBER+K)=PERCNT(NUMBER+K-1)+REM1
        GOTO 1200
1150    XNUM=REM/DELX+1.
        NNUM=XNUM
        DO 1160 K=1,NNUM
1160    PERCNT(NUMBER+K-1)=PERCNT(NUMBER+K-2)+DELX
        NNUM=NNUM-1
1200    NUMBER=NUMBER+NNUM
        IF(PERCNT(NUMBER).GE.100.)PERCNT(NUMBER)=100.
        IF(NUMBER.GE.100)GOTO 1250
        IF(PERCNT(NUMBER).GE.100.)GOTO 1250
        NUMBER=NUMBER+1
        PERCNT(NUMBER)=100.
1250    CONTINUE
C
        RETURN
        END
```

PROGRAM 2.3.

```
C    ------------------------------------------------------------------
          SUBROUTINE OASB(KTYPE,H,INTERV,XMAX,UNIT,SUM,TIME1,TIME2)
C    ------------------------------------------------------------------
C
          COMMON/NUT/NUT
          DIMENSION H(440)
          INTEGER LETM,BLANK,DOT,CROSS,DASH,LINE(41)
          DATA LETM,BLANK,DOT,CROSS,DASH,LINE/'V',' ','.','Q','I',41*' '/
C
C
5050      FORMAT(1X,A1)
          WRITE(NUT,101)
          WRITE(NUT,10)
10        FORMAT(/,25X,'2 4 - H O U R    S T O R M',/,
          21X,'R U N O F F    H Y D R O G R A P H',/)
100       CONTINUE
          WRITE(NUT,101)
101       FORMAT(76('*'))
102       FORMAT(76('-'))
          STEP=5.*60./43560.
          WRITE(NUT,103)
103       FORMAT(18X,'HYDROGRAPH IN FIVE-MINUTE INTERVALS(CFS)',/)
          WRITE(NUT,102)
          F=40./XMAX
          FMASS=40./SUM
          T0=0.0
          T1=XMAX/4.
          T2=2.*T1
          T3=3.*T1
          WRITE(NUT,105)T0,T1,T2,T3,XMAX
105       FORMAT(2X,'TIME(HRS) VOLUME(AF)    Q(CFS) ',F2.0,1X,4F10.1)
          WRITE(NUT,102)
          XMASS=0.
C
          GO TO(141,142,143,146,147,148),KTYPE
141       KNUM=1
          GOTO 144
142       KNUM=2
          GOTO 144
143       KNUM=3
          GO TO 144
146       KNUM=4
          GO TO 144
147       KNUM=6
          GO TO 144
148       KNUM=12
144       TIME=0.
C
C  OUTPUT GRAPH LOOP
C
          DO 200 I=1,INTERV
          TEST=H(I)*F
          XMASS=XMASS+H(I)*STEP
          LINE(1)=DOT
          LINE(11)=DOT
          LINE(21)=DOT
          LINE(31)=DOT
          LINE(41)=DOT
          J=TEST+1
```

```
      JMASS=XMASS*FMASS+1
      LINE(JMASS)=LETM
      LINE(J)=CROSS
      IF(KNUM.EQ.1)GOTO 400
C
320   DO 350 K=1,KNUM
      TIME=TIME+.083333
      IF(K.EQ.1)GOTO 349
      XMASS=XMASS+H(I)*STEP
      JMASS=XMASS*FMASS+1
      LINE(JMASS)=LETM
      LINE(J)=CROSS
349   IF(TIME.GE.TIME1.AND.TIME.LE.TIME2)WRITE(NUT,210)TIME,XMASS,
     . H(I),LINE
350   LINE(JMASS)=BLANK
      GOTO 215
C
400   CONTINUE
      TIME=TIME+.083333
      IF(TIME.GE.TIME1.AND.TIME.LE.TIME2)WRITE(NUT,210)TIME,XMASS,
     . H(I),LINE
210   FORMAT(2X,F7.3,F12.4,1X,F9.2,2X,41A1)
215   LINE(J)=BLANK
      LINE(JMASS)=BLANK
200   CONTINUE
      WRITE(NUT,101)
C
      RETURN
      END
```

CHAPTER 3

OPEN CHANNEL FLOW HYDRAULICS

3.1. Introduction

The study of open channel flow hydraulics requires an understanding of the fundamental principles embodied in the conservation of mass, momentum, and energy. Consequently, the basic definitions and equations need to be presented prior to developing the detailed computer software which can be applied to solving engineering problems. In the following, the necessary fundamentals of open channel hydraulics is briefly reviewed. These concepts will then be extended towards the development of comprehensive microcomputer software for the analysis of steady and unsteady flow in open channels.

3.2. Conservation of Mass, Momentum, and Energy

The study of open channel flow hydraulics is based upon the three conservation laws of mass, momentum, and energy. These laws are applicable to a specified quantity of matter (or system) which preserves its identity while undergoing a change in position, energy level, or other conditions.

The usual application of these laws is to develop integral equations which express the fundamental principles with respect to fluid flow through a control volume. The integral equations can be directly applied to flow problems or rewritten in terms of partial differential equations to analyze the interior of the fluid.

3.2.1. Conservation of Mass

For a fixed control volume Ω enclosed by the surface Γ, the integral form of the conservation of mass is given in vector notation by

$$\int_{\Gamma} \rho \mathbf{V} \cdot d\mathbf{A} + \frac{\partial}{\partial t} \int_{\Omega} \rho \, d\Omega = 0 \tag{3.1}$$

where $\mathbf{V}$ is the velocity vector with respect to the Cartesian coordinate system, and $d\mathbf{A}$ is the outward normal vector to Γ with magnitude dA. For steady flow the time derivative is zero, giving

$$\int_{\Gamma} \rho \mathbf{V} \cdot d\mathbf{A} = 0 \tag{3.2}$$

For incompressible flow

$$\int_{\Gamma} \mathbf{V} \cdot d\mathbf{A} = 0 \tag{3.3}$$

The differential equation form of mass conservation is often used in open channel flow hydraulics. This form is obtained by application of Gauss' theorem to (3.1) giving

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \tag{3.4}$$

where (u,v,w) are the (x,y,z) directional flow velocities. For steady, incompressible flow (3.4) reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{3.5}$$

3.2.2. Conservation of Momentum

Newton's second law of motion relates the net force **F** acting upon a system to the change in momentum **M** by

$$\mathbf{F} = \frac{d\mathbf{M}}{dt} \tag{3.6}$$

With respect to the fixed control volume Ω , (3.6) can be written in integral form as

$$\int_{\Gamma} \mathbf{V}\rho\mathbf{V} \cdot d\mathbf{A} + \frac{\partial}{\partial t}\int_{\Omega} \mathbf{V}\rho d\Omega = \mathbf{F} \tag{3.7}$$

The **F** vector is composed of pressure and shear forces acting upon the surface of the system($\mathbf{F}_S$), and the body force vector (**B**) which relates body forces (such as gravity) per unit volume of the system. Using $\mathbf{F}_S$ and **B** , (3.7) is rewritten as

$$\int_{\Gamma} \mathbf{V}\rho\mathbf{V} \cdot d\mathbf{A} + \frac{\partial}{\partial t}\int_{\Omega} \mathbf{V}\rho d\Omega = \mathbf{F}_S + \int_{\Omega} \mathbf{B} d\Omega \tag{3.8}$$

For steady flow, (3.8) becomes

$$\int_{\Gamma} \mathbf{V}\rho\mathbf{V} \cdot d\mathbf{A} = \mathbf{F}_S + \int_{\Omega} \mathbf{B} d\Omega \tag{3.9}$$

An important application of (3.9) is when the fluid crosses Γ at only one point of entrance (point 1) and exit (point 2). Assuming that the fluid density and flow velocity are constant over the entrance and exit areas, then (3.9) becomes

$$\Sigma F_x = \dot{M}(u_2 - u_1)$$

$$\Sigma F_y = \dot{M}(v_2 - v_1) \qquad (3.10)$$

$$\Sigma F_z = \dot{M}(w_2 - w_1)$$

where $\dot{M}$ is the mass flowrate through Ω .

3.2.3. Conservation of Energy

The first law of thermodynamics is used to develop the integral equation form of the conservation of energy. The conservation law is given by

$$dE = Q - W \qquad (3.11)$$

where dE is the change in the energy of the system, Q is the heat added to the system, and W is the work done by the system. The energy E is written in terms of several contributions by

$$E = U + \frac{1}{2} mV^2 + mgZ \qquad (3.12)$$

where U is the internal energy, m is the system mass, $mV^2/2$ is the kinetic energy, and mgZ is the potential energy. For $e = E/m$, (3.11) is written in integral equation form with respect to time by

$$\int_\Gamma e\rho\mathbf{V}\cdot d\mathbf{A} + \frac{\partial}{\partial t}\int_\Omega e\rho d\Omega = \frac{dQ}{dt} - \frac{dW}{dt} \qquad (3.13)$$

Flow work done on Γ due to normal stresses (hydrostatic pressure) can be isolated from the W term and (3.13) rewritten as

$$\int_\Gamma (e + p/\rho)\ \rho\mathbf{V}\cdot d\mathbf{A} + \frac{\partial}{\partial t}\int_\Omega e\rho d\Omega = \frac{dQ}{dt} - \frac{dW^*}{dt} \qquad (3.14)$$

where p is the fluid pressure and W^* is the work term W less the flow work contribution.

For steady flow, (3.14) reduces to

$$\int_\Gamma (e + p/\rho)\ \rho\mathbf{V}\cdot d\mathbf{A} = \frac{dQ}{dt} - \frac{dW^*}{dt} \qquad (3.15)$$

For one entrance (point 1) and exit (point 2) associated to Γ , and constant e, p, ρ over the entrance and exit areas,

$$\dot{M}(e_2 - e_1) + \dot{M}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right) = \frac{dQ}{dt} - \frac{dW^*}{dt} \qquad (3.16)$$

Noting e = E/m, and $\dot{M}$ being the mass flow rate through Ω gives

$$\dot{M}\left[\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right) + \left(i_2 - i_1\right) + \left(\frac{V_2^2 - V_1^2}{2}\right) + g\left(Z_2 - Z_1\right)\right] = \frac{dQ}{dt} - \frac{dW^*}{dt} \qquad (3.17)$$

where i = U/m. Letting

$$gH_L = (i_2 - i_1) - \frac{dQ}{dt} \Big/ \dot{M} \qquad (3.18)$$

further reduces (3.17) for zero system work to

$$\frac{(p_2 - p_1)}{\rho} + \frac{(V_2^2 - V_1^2)}{2} + g(Z_2 - Z_1) + g\,H_L = 0 \qquad (3.19)$$

or in terms of length units (or head)

$$\frac{(p_2 - p_1)}{\gamma} + \frac{(V_2^2 - V_1^2)}{2g} + (Z_2 - Z_1) + H_L = 0 \qquad (3.20)$$

where γ is the fluid specific weight, and H_L is the head loss.

3.3. Fundamentals of Hydraulics

3.3.1. Hydraulic Grade Line and Energy Grade Line

For any point in the fluid, the summation of the elevation plus the pressure head is known as the piezometric head. The piezometric head represents the level to which liquid will rise in a piezometer tube where a line drawn through the tops of a series of piezometer columns is known as the hydraulic grade line (HGL). The energy grade line (EGL) is determined by the sum of the HGL and the velocity head ($V^2/2g$) such as is shown in Fig. 3.1.

3.3.2. Specific Energy

In open channel flow, the specific energy, S_E, is given by

$$S_E = y\cos^2\theta + cV^2/2g \qquad (3.21)$$

where

y = vertical depth of flow
θ = angle of the longitudinal bed profile with respect to the horizontal. (In most cases θ is small, therefore $\cos^2\theta = 1$)

c = kinetic energy correction factor. This is equal to one when the velocity distribution is uniform.
V = average flow velocity
g = gravitational acceleration

Given the flow rate (Q), and cross section flow area (A), and for $\cos^2\theta = 1$,

$$S_E = y + Q^2/2gA^2 \tag{3.22}$$

$$(S_E - y)A^2 = Q^2/2g = \text{constant} \tag{3.23}$$

From Equation (3.23), it is clear that the specific energy curve of Fig. 3.2 has the two asymptotes of $y = S_E$, and $y = 0$.

Alternate depths are defined as the two possible depths of flow for a given Q and S_E, and represent the two possible regimes of flow. For a point on the upper limb of the curve (Fig. 3.2), flow has a higher depth and thus a lower velocity. In this case, the flow is known as subcritical. On the lower limb of the curve the flow has a lower depth and thus a high velocity. This flow is classified as supercritical. When $dS_E/dy = 0$, the flow is critical (the location of this condition is at the crest of the curve). The depth relating to critical flow is known as the critical depth, y_c.

3.3.3. The Specific Force

Consider a steady, uniform, incompressible flow in an open channel between channel section A to section B, and apply Newton's second law of motion. The second law of motion states that the change of momentum per unit time in the body is equal to the resultant of all the external forces that are acting on the body. Thus for a fixed control volume,

$$Q(\beta_B V_B - \beta_A V_A) = P_A - P_B + W\sin\theta - F_f \tag{3.24}$$

where

β = momentum correction factor
P_A and P_B = resultant pressures acting on section A and B, respectively
W = Equivalent weight of the fluid pressure enclosed between sections A and B
F_f = Total external forces (including friction) along the wetted boundary of the channel between section A and section B
θ = angle of channel slope with respect to the horizontal

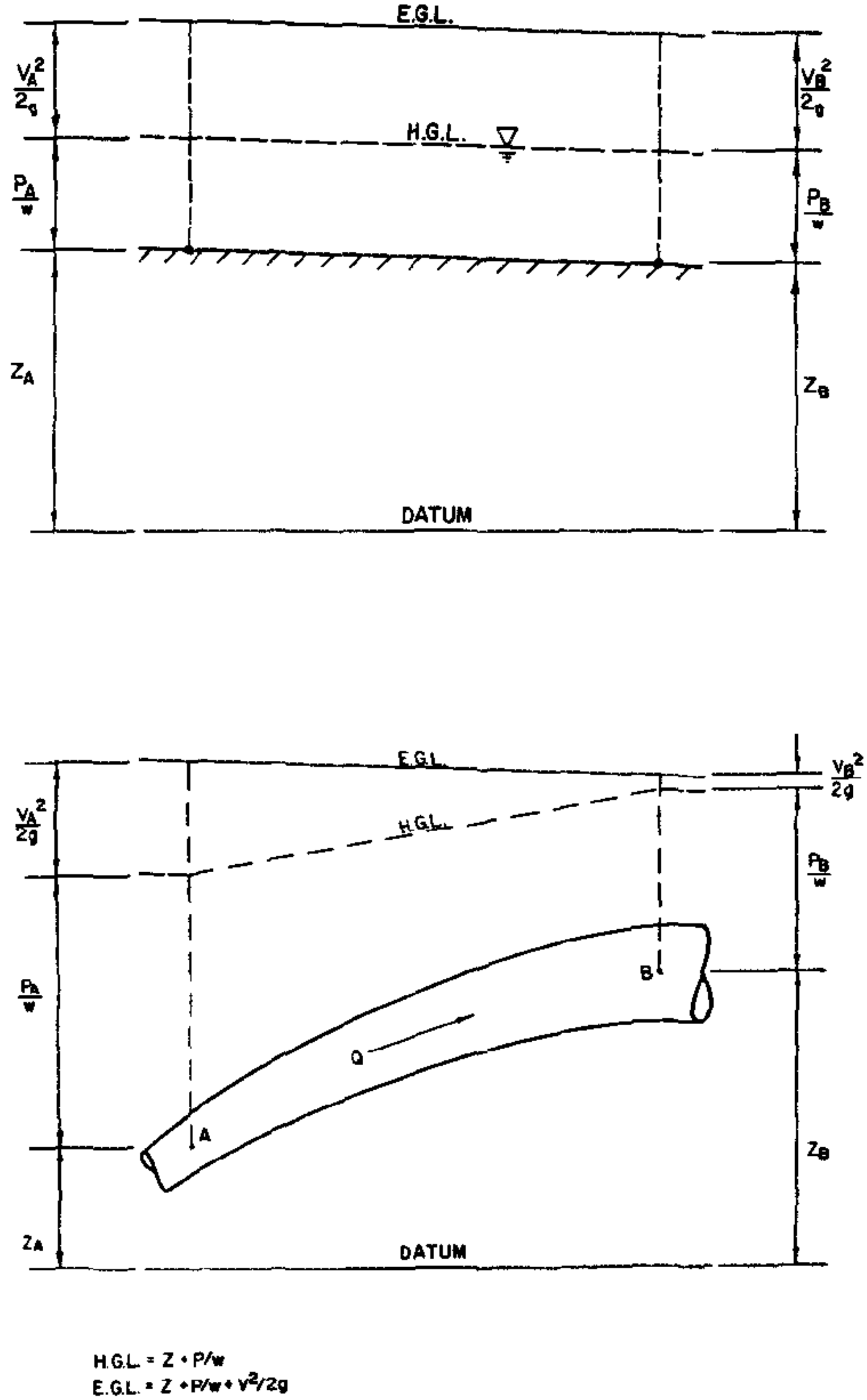


Fig. 3.1. *Open channel flow energy balance.*

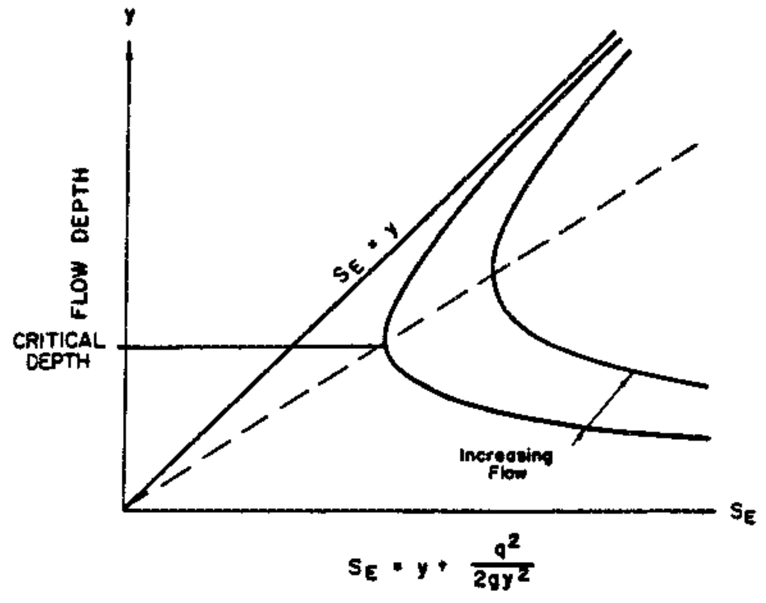


Fig. 3.2. *The specific energy curve.*

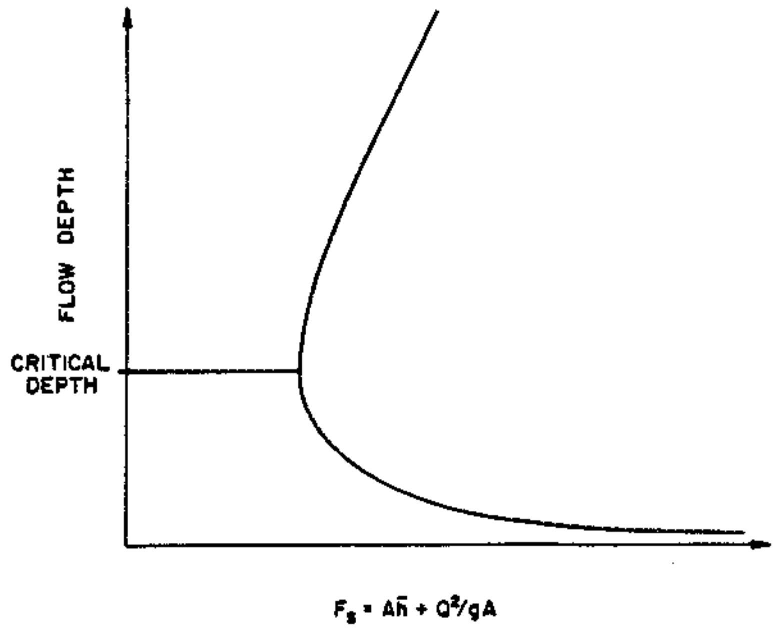


Fig. 3.3. *The specific force curve.*

The pressure forces are calculated by

$$P_A = wA_A\bar{h}_A \ , \ P_B = wA_B\bar{h}_B \qquad (3.25)$$

where $\bar{h}$ = the distance to the centroid of the cross section below the water surface

If the difference of $W\sin\theta - F_f$ can be neglected and $\beta_1 = \beta_2 = 1$, then equation (3.24) can be simplified as

$$A_A\bar{h}_A + Q^2/gA_A = A_B\bar{h}_B + Q^2/gA_B \qquad (3.26)$$

Both sums of the terms in (3.26) involve identical components, and can be grouped together as the specific force, F_S. That is,

$$F_S = A\bar{h} + Q^2/gA \qquad (3.27)$$

The specific force curve (Fig. 3.3) is similar in some of its characteristics to the specific energy curve (Fig. 3.2). Both the specific force and specific energy are asymptotic to the y = 0 axis. However, the specific force curve is not asymptotic to the 45° line.

3.3.4. The Hydraulic Jump in a Rectangular Channel

Solution of the continuity and momentum equations for the special case of a rectangular channel leads to the following relation for the initial (y_1) and sequential depths (y_2) of a hydraulic jump on a horizontal floor:

$$y_2/y_1 = 1/2\ ((1 + 8F_1^2)^{.5} - 1) \qquad (3.28)$$

and

$$y_1/y_2 = 1/2\ ((1 + 8F_2^2)^{.5} - 1) \qquad (3.29)$$

In the above, F_1 and F_2 are the Froude numbers corresponding to depths y_1 and y_2, respectively. Substituting these values into the energy equation gives the energy loss in the jump

$$dE = (y_2 - y_1)^3/4y_1y_2 \qquad (3.30)$$

The jump efficiency E_2/E_1 can be expressed as

$$E_2/E_1 = ((8F_1^2 + 1)^{3/2} - 4F_1^2 + 1)/8F_1^2(2 + F_1^2) \qquad (3.31)$$

The relative height of the jump $(y_2 - y_1)/E_1$ can be expressed as

$$(y_2 - y_1)/E_1 = ((1 + 8F_1^2)^{.5} - 3)/(2 + F_1^2) \qquad (3.32)$$

The U.S. Bureau of Reclamation has classified various types of hydraulic jumps based on the Froude number, F. Their results are summarized below:

TABLE 3.1. HYDRAULIC JUMP CLASSIFICATIONS

F	Classification
1 to 1.7	undular jump
1.7 to 2.5	weak jump
2.5 to 4.5	oscillating jump
4.5 to 9.0	steady jump
> 9.0	strong jump

3.4. Gradually Varied Flow

Gradually varied flow in a prismatic channel can be modeled by the one-dimensional differential equation

$$dy/dx = (S_o - S_f)/(1 - F^2) \tag{3.33}$$

where

y = flow depth
S_o = the bed slope
S_f = the friction slope
F = the Froude number
x = coordinate along channel bottom

When S_f approaches S_o, dy/dx approaches zero. Therefore, water surface profiles approach the normal depth of flow asymptotically.

If F approaches unity, dy/dx approaches infinity. Therefore, by (3.33), the water surface becomes nearly vertical.

3.4.1. S Profiles

A channel is classified as steep for a discharge when the normal depth is less than the critical depth, and is mild when the normal depth is greater than the critical. When the normal flow is rapid (normal depth less than critical) in a channel, the resulting profiles S_1, S_2 and S_3 are known as the steep profiles. The S_1 profile approximates gradually varied flow which is above the normal and critical depths, S_2 represents the flow profile occurring between the critical and normal depths, and S_3 occurs below the normal depth, (Fig. 3.4).

For the S_1 curve, both the numerator and denominator of (3.33) are positive and the depth increases downstream approaching a horizontal asymptote. An example is a steep canal emptying into a pool of high elevation.

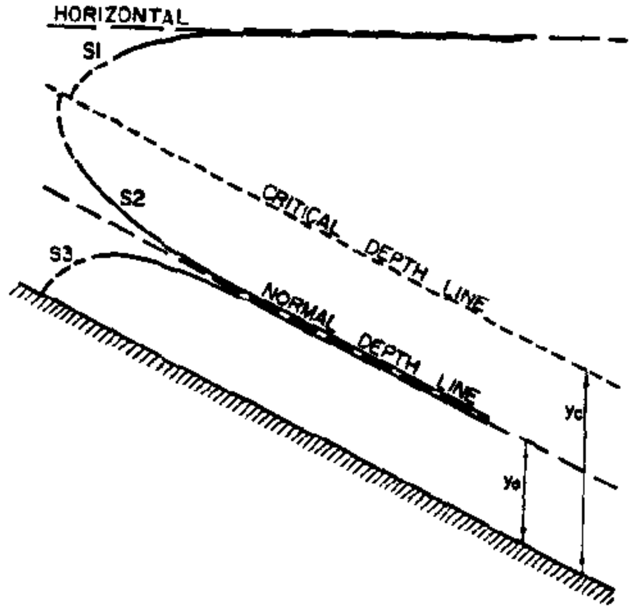


Fig. 3.4. *Gradually varied flow profiles for steep slopes.*

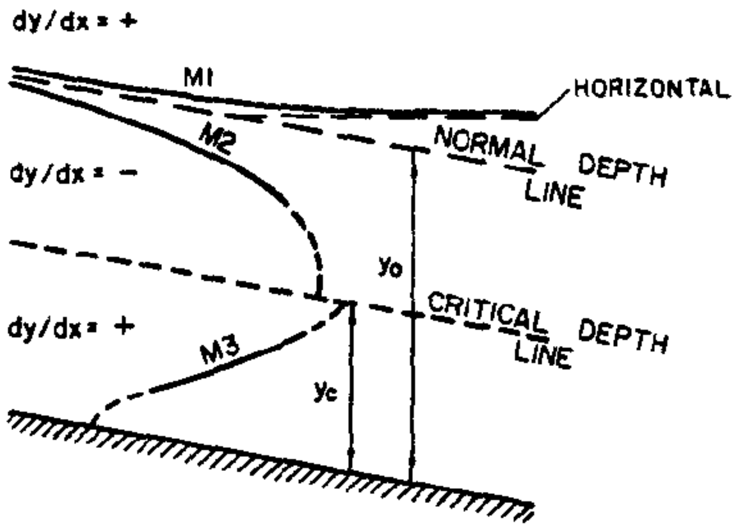


Fig. 3.5. *Gradually varied flow profiles for mild slopes.*

For the S_2 curve, the numerator of (3.33) is negative and the denominator is positive (but approaches zero at $y = y_o$). This curve approaches the normal depth asymptotically. An example is the profile formed on the downstream side of an enlargement of a channel section.

In the S_3 curve, both the numerator and denominator of (3.33) are negative. An example is the water surface profile as the slope changes from a steep to a milder (but steep) slope.

3.4.2. M Profiles

A mild slope is one where the normal flow is tranquil (i.e., normal depth, y_o, is greater than the critical depth, y_c). Three profiles may occur, and are classified as M_1, M_2, and M_3, for flow depths above normal depth, below normal and above critical depths, and below critical depth, respectively, (Fig. 3.5).

For the M_1 profile ($y>y_o>y_c$), the upstream end of the flow profile is tangent to the normal-depth line, since $dy/dx = 0$ as $y = y_o$. The downstream end is tangent to the horizontal because $dy/dx = S_o$ as y approaches infinity. A typical example in this case is the profile behind a dam in a mildly flowing river.

For the M_2 profile ($y_o>y>y_c$), the upstream end of the flow profile is tangent to the normal depth line, since $dy/dx = 0$ as $y = y_o$. The downstream end of the flow profile is less than the normal depth but above (or equal to) the critical depth. A typical example of this profile occurs at the upstream side of a sudden enlargement of a mild channel cross-section.

For the M_3 profile ($y<y_c<y_o$), the upstream flow depth is modeled to begin as an acute angle. The downstream flow terminates with a hydraulic jump. The most upstream flow depth is modeled as $y = 0$, and has an associated infinite flow velocity. The typical example of this profile is when a supercritical flow enters a mild channel.

3.4.3. C Profiles

When the normal depth and the critical depth are equal, the profiles resulting from this are labeled C_1 and C_3. C_1 occurs when the flow depth is above the critical depth, and C_3 occurs when the flow depth is below the critical depth. These profiles represent the transition conditions between M (mild) and S (steep) flow profiles. The C_2 profile is usually associated to the case of uniform critical flow, (Fig. 3.6).

3.4.4. The Standard Step Method

Gradually varied flow profiles are generally computed by using any of three popular methods. Namely, the graphical-integration method, the direct-integration method, and the

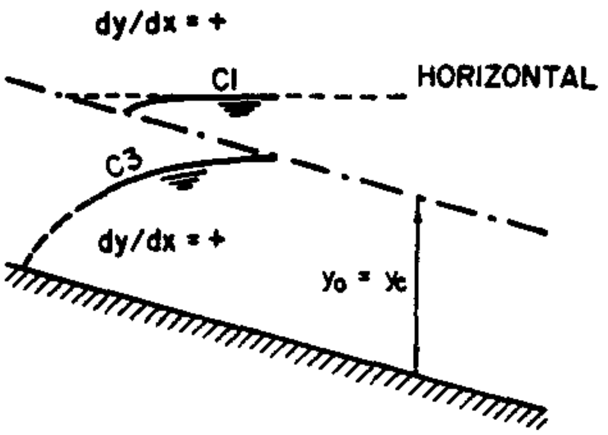


Fig. 3.6. Gradually varied flow profiles for critical slopes.

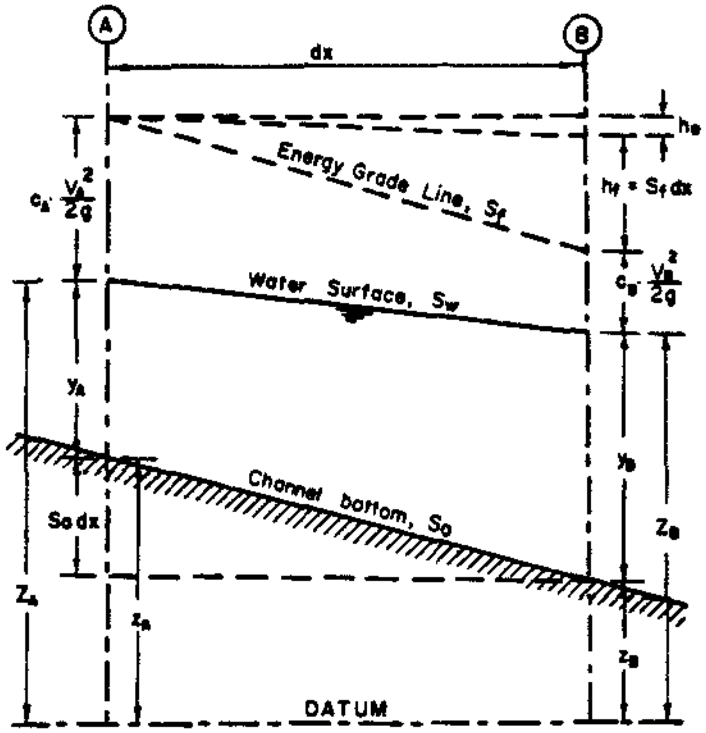


Fig. 3.7. Channel reach used for derivation of standard step method.

standard step method. The standard step method continues to be the most commonly used.

In the standard step method, the computation of the flow depth is carried out on a station to station basis where the hydraulic characteristics are known. The computation procedure is a trial and error method to balance the energy equation.

For convenience, the position of the water surface is measured with respect to a horizontal datum. The water surface elevations above the datum at the two end sections can be expressed as (Fig. 3.7)

$$Z_A = y_A + z_A \tag{3.34}$$

and

$$Z_B = y_B + z_B \tag{3.35}$$

The friction losses are estimated between points A and B by

$$h_f = S_f \, dx = (S_A + S_B) dx/2 \tag{3.36}$$

where S_f can be taken as the average of the friction slopes at the two end sections. The total head at sections A and B can be equated by the energy equation

$$S_o dx + y_A + c_A V_A^2/2g = y_B + c_B V_B^2/2g + S_f dx + h_e \tag{3.37}$$

By substitution, the following is written

$$Z_A + c_A V_A^2/2g = Z_B + c_B V_B^2/2g + h_f + h_e \tag{3.38}$$

where h_e is the eddy loss defined by

$$h_e = k(cV^2/2g)$$

and k is defined by

- k = 0 to 0.1 for gradually converging reaches
- k = 0 to 0.2 for gradually diverging reaches
- k = 0.5 for abrupt expansion and contraction
- k = 0 for prismatic and regular channel

The total heads at the two end sections A and B are

$$H_A = Z_A + c_A V_A^2/2g \tag{3.39a}$$

and

$$H_B = Z_B + c_B V_B^2/2g \tag{3.39b}$$

Using (3.39a,b), equation (3.38) can be expressed as

$$H_A = H_B + h_f + h_e \tag{3.40}$$

Given the values of H_A (or H_B), the energy head for H_B (or H_A) is computed by estimating possible flowdepths until the governing energy equation is satisfied.

3.5. PROGRAM 3.1. Irregular Channel Backwater Curve Analysis

The Standard Step Method provides for the computation of an upstream water surface and EGL given the current values of the water surface elevation, EGL, flow rate, and other flow and channel characteristics. Consequently, the main thrust of the computer program is to develop backwater and drawdown curves for channel flow situations which can be considered mild (i.e., Froude numbers are all less than 1, and the normal depths are greater than the critical depths).

The program data entry requirements fall into two categories:

(1) preparation of channel cross section information
(2) definition of uniform channel flowrate, energy balance locations along the channel, and downstream hydraulic control information

The preparation of channel cross section information entails the definition of up to 20 cross sections along the channel. Each cross section is defined to have up to 20 (x,y) coordinate pairs, a Manning's friction factor, a kinetic energy correction factor, and an eddy loss factor. Each cross section is assumed to have a single flowline such that the section begins on one bank, decreases in elevation constantly to the flowline, and then increases in elevation until the other bank elevation is reached. The section coordinate data is entered with the first x-coordinate being defined as x= 0.0. Thus the coordinate data must be entered consistently in order to scan the cross sections properly. That is, all sections should have the coordinate data prepared from left-to-right (or right-to-left). The cross section data entry begins with the most downstream section, with subsequent section data entered in the upstream direction.

The second set of data entry requirements includes the constant flowrate to be used through the entire study, the definition of the downstream hydraulic control water surface elevation, and the definition of energy balance locations along the channel. PROGRAM 3.1 computes the critical depth at each energy balance location. Consequently, should the specified control water surface elevation correspond to a channel flowdepth less than critical depth, the program will redefine the control to equal the critical depth. The Standard Step Method computes the water surface profile by balancing the energy losses (between energy balance locations) to the change in the EGL. Thus the engineer locates those points where the energy balance computations are to occur within the study.

As the study proceeds in the upstream direction, the upstream water surface and EGL are computed at energy balance locations by linearly interpolating all cross section geometric and hydraulic information. Should the water surface fall below the corresponding critical depth, the channel flowdepth is redefined to equal the critical depth (Froude number equals 1, modified by the kinetic energy correction factor). In reaches where normal depth flows could be supercritical, the word STEEP is noted in the right hand column of the computer solution tabulation. Other program output features are listed in the program description page which is included in the program output.

PROGRAM 3.1 is designed to tabulate the hydraulic computations according to the form given in Chow (1959, pg. 269). The computer results also include a report description page which briefly describes the computer program operation and the interpretation of the final results. It should be noted that PROGRAM 3.1 models the gradually varied flow hydraulics as a one-dimensional channel reach, ignoring all additional losses due to bends, angle points, and so forth. However, the eddy loss factor represents a portion of the velocity head to be used for energy losses, and can be used to include any other additional losses as a function of the flow velocity head, Hv. Should channel flows exceed either channel bank, the model assumes that a vertical line extends upwards from both channel banks, with zero friction assumed along the imaginary boundary. In this flooding situation, the word FLOOD is printed in the computer results.

3.5.1. Data Entry for PROGRAM 3.1.

In order to determine a water surface profile in a subcritical channel flow, the channel needs to be defined by cross-section coordinate pairs and associated flow parameters. This cross-section information is entered as one travels upstream through the channel. The program input variable names and descriptions are provided below.

PROGRAM CONTROL DATA ENTRY:

Variable Name	Description
N	Total number of cross-sections
QQ	Channel flow (cfs)
TOL	Tolerance in computation (feet)
YCON	Downstream hydraulic control water surface elevation (feet)

CROSS-SECTION DATA ENTRY (I = 1,N):

M(I)	Number of coordinate pairs
XMAN(I)	Manning's friction factor
ALPHA(I)	Kinetic energy correction factor
EDDY(I)	Eddy loss coefficient
X(I,J),Y(I,J)	(X,Y) coordinates for cross-section number I
DISTS(I)	Distance to hydraulic control (feet)

ENERGY BALANCE LOCATION DATA ENTRY:

Program 3.1 uses straight line interpolation between cross-sections to define the irregular channel geometry and flow parameter values. To approximately solve the energy equation, locations are needed where the energy balance is to be made.

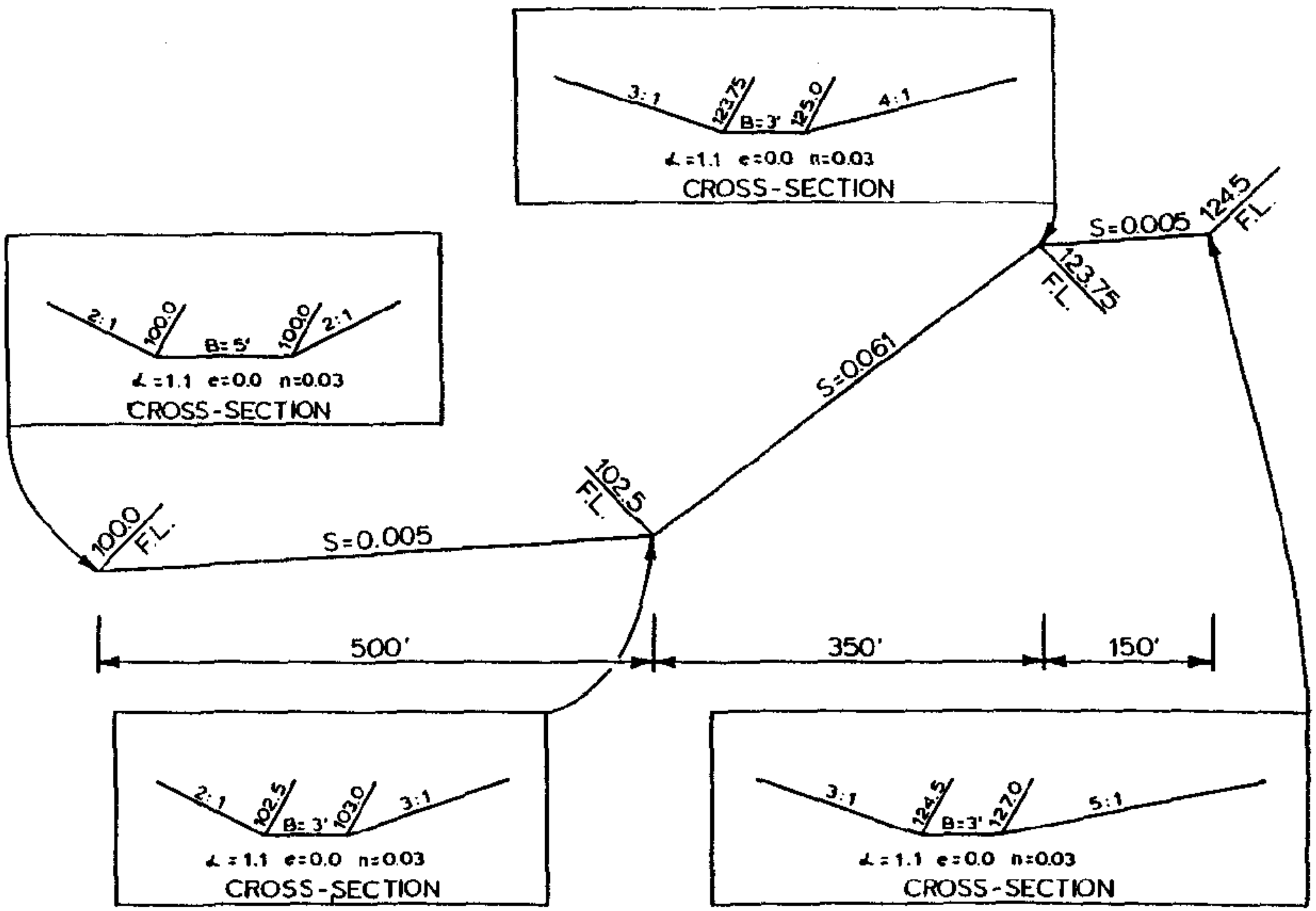
NE	Total number of energy balance locations
DIST(I)	Distance from hydraulic control where energy balance is to be made (I = 1, NE)

3.6. Unsteady Flow Analysis

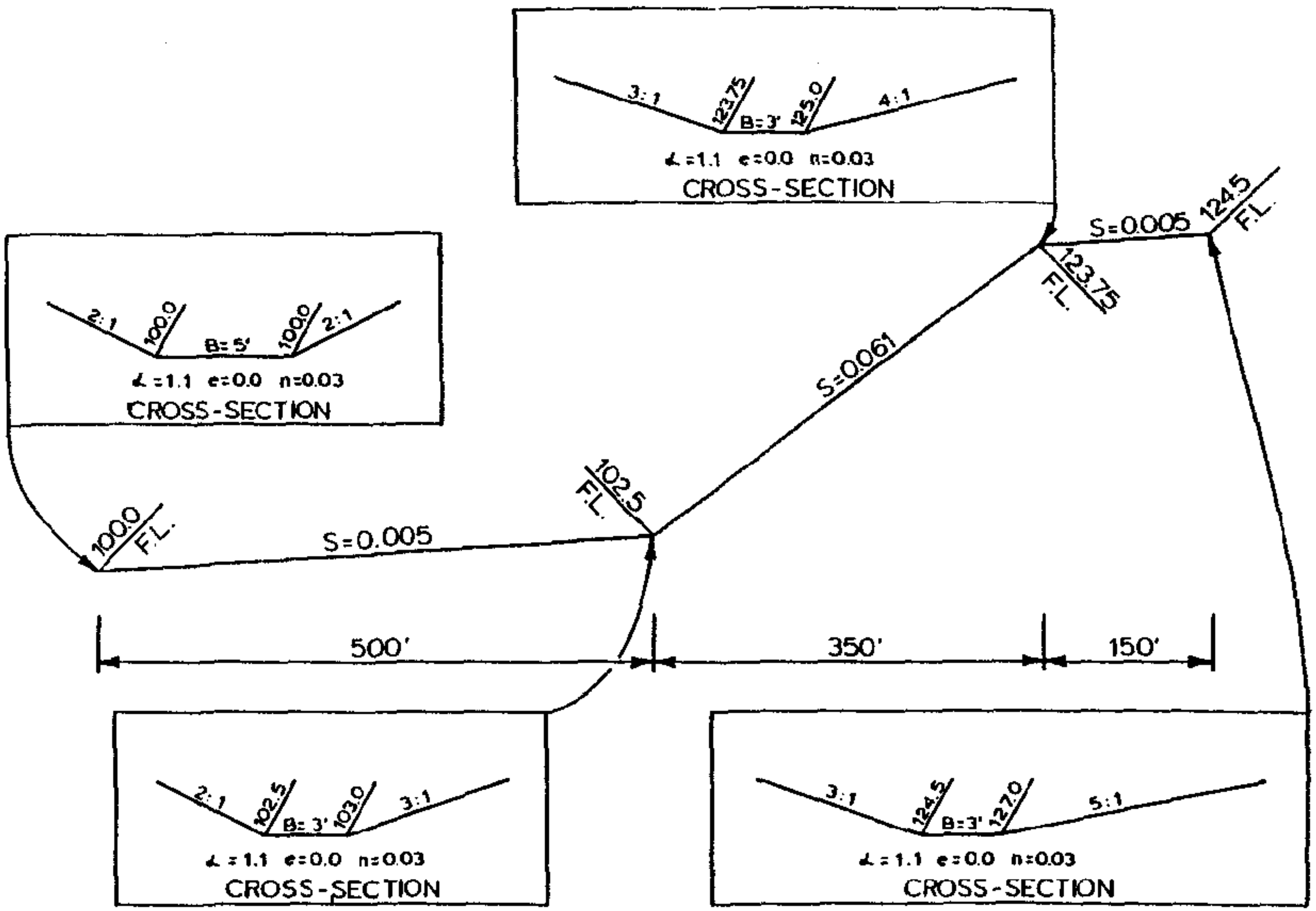
The equations which describe one-dimensional flow in an open channel are a pair of partial differential equations which can normally be solved only by means of a digital computer, typically a main-frame computer. Provided here is a program which runs very conveniently on a microcomputer system. It simulates a single reach of trapezoidal channel. The program can be altered to allow other channel geometries to be used, or it can be be structured to deal with multiple reaches.

The solution of the full equations describing unsteady channel flow is required for a number of situations where approximate solutions do not give accurate results. These situations include flood routing in rivers subject to tidal effects and unsteady flows in irrigation canals or drainage systems.

This section provides a derivation of the equations of one-dimensional, unsteady open channel flow, a description of a numerical procedure for solving the equations, the computer code for a single reach unsteady flow simulation program, and two example problems analyzed by this program.



EXAMPLE PROBLEM - IRREGULAR CHANNEL PROFILE AND CROSS-SECTIONS



EXAMPLE PROBLEM - IRREGULAR CHANNEL PROFILE AND CROSS-SECTIONS

```
      ICOUNT=1
      DCON=YCON
      ITYPE=BLANK
      CALL PROP4(0,YC,YN,J,K,NT,A,V,R,FR,ERROR)
      IF(YC.GE.YCON)DCON=YC
      IF(YC.GE.YCON)ITYPE=NOTE
      IF(YC.GE.YCON)GOTO 10
      CALL PROP3(1,YCON,A,V,P,DH,TW,R,FR,1.,NT,ERROR)
10    WRITE(NT,1000)IFF
1000  FORMAT(1X,A1,//,50X,'CUT ALONG OUTSIDE BORDER* *',/,
     C 76X,'*',/,76X,'*',/,76X,'*',/,76X,'*',/,76X,'V')
      WRITE(NT,1001)
1001  FORMAT(10X,'+',109('-'),'+')
      WRITE(NT,1002)
1002  FORMAT(2(10X,'|',109X,'|',/),10X,'|',109X,'|',/,
     . 10X,'|*** IRREGULAR CHANNEL SUBCRITICAL FLOW MODEL ***',7X,
     . '(C) Copyright 1983 Advanced Engineering Software [AES]',
     . '|',/,
     C 10X,'|Standard Step Method irregular channel',
     C ' analysis. Based on development in "OPEN CHANNEL'
     C ' HYDRAULICS",CHOW(1959)|')
      WRITE(NT,1010)Q
1010  FORMAT(10X,'|  STUDY NAME:',20X,'Channel Flow = ',F9.2,' cfs',
     C 16X,'PAGE NUMBER:',20X,'|')
      WRITE(NT,1001)
      WRITE(NT,1003)
1003  FORMAT(10X,'| LENGTH| WATER | FLOW|  FLOW | FLOW|',
     C '  2    |  TOTAL | HYDR |FRICTION| AVERAGE| REACH|',
     C ' LOSS|EDDY|  TOTAL |      |',/,10X,'|  from |SURFACE|',
     C 'DEPTH|  AREA |  V  |aV /2g|  HEAD  |RADIUS|  SLOPE |',
     C '  REACH |LENGTH|  Hf |LOSS|  HEAD  |  Fr |',/,
     C 10X,'|CONTROL|(elev.)| (ft)|(ft*ft)|(fps)| (ft) |  (ft)  |',
     C ' (ft) |   Sf   |   Sf   | (ft) | (ft)|(ft)|  (ft)  |     |')
      WRITE(NT,1004)
1004  FORMAT(10X,'|-------|-------|-----|-------|-----|',
     C '------|--------|------|--------|--------|------|',
     C '-----|----|--------|-----|')
C
      SF=XMAN(1)*XMAN(1)*V*V/2.2082/R**1.3333
      AHV=ALPHA(1)*V*V/64.4
      H1=DCON+AHV
      H2=H1
      NODE=NFL(1)
      YY=DCON-Y(1,NODE)
      WRITE(NT,1005)DISTS(1),DCON,YY,A,V,AHV,H1,R,SF,H2,ITYPE,FR
1005  FORMAT(10X,'|'F7.1,'|',F7.2,'|',F5.2,'|',F7.1,'|',
     C F5.2,'|',F6.3,'|',F8.3,'|',F6.2,'|',F8.6,'|        |',
     C '      |     |    |',F8.3,'|',A1,F4.2,'|')
      DO 13 IJ=1,5
13    IWTS(IJ)=BLANK
      DO 131 IJ=1,12
131   IWT(IJ)=BLANK
      IF(ERROR.EQ.0.)GOTO 15
      DO 14 IJ=1,5
14    IWT(IJ)=IST(IJ)
      DO 141 IJ=6,12
141   IWT(IJ)=IST(10+IJ)
15    CONTINUE
      IF(YN.GT.YC)GOTO 17
      DO 16 IJ=1,5
```

```
16        IWTS(IJ)=IST(IJ+5)
          IF(IFIRST.NE.0)GOTO 79
          DO 78 IJ=1,5
78        IWTS(IJ)=IST(IJ+10)
79        IFIRST=IFIRST+1
17        WRITE(NT,1011)(IWT(IJ),IJ=1,5),(IWT(IJ),IJ=6,12),ALPHA(1),
     C    XMAN(1),(IWTS(IJ),IJ=1,5)
1011      FORMAT(10X,'|',1X,5A1,1X,'|',7A1,'|'5X,'|',7X,'|',5X,'|a=',
     C    F4.2,'|',8X,'|',6X,'|n=',F6.4,'|',8X,'|',6X,'|',
     C    5X,'|',4X,'|',8X,'|',5A1,'|')
          WRITE(NT,1004)
C.......INITIALIZE SF1
          SF1=SF
C.......MAIN LOOP
          DO 100 L=1,NE
          CALL PROF4(L,YC,YN,J,K,NT,A,V,R,FR,ERROR)
          YMIN=YC
          YMAX=YC+100.
          R=(DIST(L)-DISTS(J))/(DISTS(K)-DISTS(J))
          R1=1.-R
          JFL=NFL(J)
          KFL=NFL(K)
          YMINJ=Y(J,JFL)
          YMINK=Y(K,KFL)
          YFL=R*YMINK+R1*YMINJ
C.......ESTIMATE SECTION BANK-ELEVATIONS
          XLBNK=R*Y(K,1)+R1*Y(J,1)
          MMJ=M(J)
          MMK=M(K)
          XRBNK=R*Y(K,MMK)+R1*Y(J,MMJ)
          DO 50 LL=1,15
          YTEST=.5*(YMIN+YMAX)
          DY=YTEST-YFL
          YJ=YMINJ+DY
          YK=YMINK+DY
          CALL PROF3(J,YJ,AJ,V,PJ,DHJ,TWJ,XR,FR,1.,NT,ERROR)
          CALL PROF3(K,YK,AK,V,PK,DHK,TWK,XR,FR,1.,NT,ERROR)
          A=R*AK+R1*AJ
          V=Q/A
          AL=R*ALPHA(K)+R1*ALPHA(J)
          XN=R*XMAN(K)+R1*XMAN(J)
          AHV=AL*V*V/64.36
          TW=R*TWK+R1*TWJ
          P=R*PK+R1*PJ
          RH=A/P
          DH=A/TW
          SF=XN*XN*V*V/2.22/RH**1.3333
          DX=DIST(1)
          IF(L.GT.1)DX=DIST(L)-DIST(L-1)
          SFM=.5*(SF1+SF)
          HF=DX*SFM
          RDXE=DX/2./(DISTS(K)-DISTS(J))
          ED=(R-RDXE)*EDDY(K)+(R1+RDXE)*EDDY(J)
          HE=ED*V*V/64.36
          H2T=H2+HE+HF
          H1T=YTEST+AHV
          IF(H2T-H1T)30,60,40
30        YMAX=YTEST
          GOTO 50
40        YMIN=YTEST
```

```
50      CONTINUE
60      H1=H1T
        H2=H1
C.......OUTPUT RESULTS
        DO 61 IJ=1,5
61      IWTS(IJ)=BLANK
        DO 611 IJ=1,12
611     IWT(IJ)=BLANK
        IF(YTEST.LT.XLBNK.AND.YTEST.LT.XRBNK)GOTO 70
        DO 62 IJ=1,5
62      IWT(IJ)=IST(IJ)
70      CONTINUE
        FR=SQRT(AL*V*V/DH/32.18)
        ITYPE=BLANK
        TEST=ABS(1.-FR)
        IF(TEST.LT..01)ITYPE=NOTE
        SF1=SF
        IF(YN.GT.YC)GOTO 77
        DO 75 IJ=1,5
75      IWTS(IJ)=IST(IJ+5)
77      WRITE(NT,1006)DIST(L),YTEST,DY,A,V,AHV,H1,RH,SF,SFM,DX,HF,
     C  HE,H2T,ITYPE,FR
1006    FORMAT(10X,'|',F7.1,'|',F7.2,'|',F5.2,'|',F7.1,'|',
     C  F5.2,'|',F6.3,'|',F8.3,'|',F6.2,'|',F8.6,'|',F8.6,'|',
     C  F6.1,'|',F5.3,'|',F4.2,'|',F8.3,'|',A1,F4.2,'|')
C.......SEE IF LOCATION =CROSS-SECTION
        IF(DIST(L).EQ.DISTS(J).OR.DIST(L).EQ.DISTS(K))GOTO 82
        GOTO 85
82      DO 83 IJ=6,12
83      IWT(IJ)=IST(10+IJ)
85      CONTINUE
        WRITE(NT,1012)(IWT(IJ),IJ=1,5),(IWT(IJ),IJ=6,12),
     C  AL,XN,ED,(IWTS(IJ),IJ=1,5)
1012    FORMAT(10X,'|',1X,5A1,1X,'|',7A1,'|',5X,'|',7X,'|',5X,'|a=',
     C  F4.2,'|',8X,'|',6X,'|n=',F6.4,'|',8X,'|',6X,'|',5X,
     C  'e=',F4.2,8X,'|',5A1,'|')
        WRITE(NT,1004)
C.......PAGE COSMETICS
        ICOUNT=ICOUNT+1
        IF(ICOUNT.LT.13)GOTO 100
        WRITE(NT,1001)
        IF(NE.EQ.L)GOTO 200
        WRITE(NT,1000)
        WRITE(NT,1001)
        WRITE(NT,1002)
        WRITE(NT,1010)Q
        WRITE(NT,1001)
        WRITE(NT,1003)
        WRITE(NT,1004)
        ICOUNT=0
100     CONTINUE
C.......END OF STUDY, BUT IN MIDDLE OF PAGE....FINISH OFF PAGE
        NREM=13-ICOUNT
        DO 150 K=1,NREM
        WRITE(NT,1007)
        WRITE(NT,1007)
1007    FORMAT(10X,'|',7X,'|',7X,'|',5X,'|',7X,'|',5X,'|',6X,
     C  '|',8X,'|',6X,'|',8X,'|',8X,'|',6X,'|',5X,'|',4X,'|',
     C  8X,'|',5X,'|')
        WRITE(NT,1004)
```

```
150     CONTINUE
        WRITE(NT,1001)
200     CONTINUE
        WRITE(NT,2000)
2000    FORMAT(/)
C
        RETURN
        END
C
C
C -----------------------------------------------------------------
        SUBROUTINE PROP3(L,YDEPTH,A,V,P,DH,TW,R,FR,S,NT,ERROR)
C -----------------------------------------------------------------
C
C
        COMMON/BLK 1/X(20,20),Y(20,20),NPL(20),M(20)
        COMMON/BLK 2/DISTS(20),ALPHA(20),XMAN(20),EDDY(20)
        COMMON/BLK 3/TOL,Q,N,DIST(50),NE
C
C
        HYPOT(B,C)=((B*B)+(C*C))**.5
        ERROR=0.
        I=L
        P=0.
        V=0.
        TW=0.
        A=0.
        KK=M(I)-1
        DO 200 J=1,KK
        IF((YDEPTH.LE.Y(I,J)).AND.(YDEPTH.LE.Y(I,(J+1))))GOTO 200
        XDIF=X(I,(J+1))-X(I,J)
        YDIF=Y(I,(J+1))-Y(I,J)
        IF(YDIF)150,150,160
150     YDIF=-YDIF
        YUP=Y(I,J)
        YLOW=Y(I,(J+1))
        GOTO 170
160     YUP=Y(I,(J+1))
        YLOW=Y(I,J)
170     TEMP=YUP-YDEPTH
        IF(TEMP.LE.0.)GOTO 155
        GOTO 165
155     AA=((YDEPTH-YUP)*XDIF)+(.5*XDIF*YDIF)
        PP=HYPOT(XDIF,YDIF)
        TWTW=XDIF
        GOTO 198
165     IF(ABS(YDIF).LT..001)GOTO 155
        YDEEP=YDEPTH-YLOW
        TWTW=XDIF*(YDEEP/YDIF)
        AA=.5*YDEEP*TWTW
        PP=(YDEEP/YDIF)*HYPOT(XDIF,YDIF)
198     A=A+AA
        P=P+PP
        TW=TW+TWTW
200     CONTINUE
        IF(P.LE..001)WRITE(NT,220)
220     FORMAT(1X,'UNACCEPTABLE CHANNEL CONFIGURATION')
        IF(P.LE..001)GOTO 180
        R=A/P
        V=Q/A
```

```
      DH=A/TW
      FR=SQRT(ALPHA(I)*V*V/32.18/DH)
180   CONTINUE
C.......CHECK FOR OVERFLOW
      KK=KK+1
      IF(YDEPTH.GT.Y(L,1).OR.YDEPTH.GT.Y(L,KK))ERROR=1.
C
      RETURN
      END
C
C
C ------------------------------------------------------------------------
      SUBROUTINE PROF4(I,YC,YN,J,K,NT,A,V,RH,FR,ERROR)
C ------------------------------------------------------------------------
C
C
C  DETERMINES RELATIVE DC ELEVATION AND SURROUNDING
C  SECTIONS "J" AND "K"
C  I IS INDEX TO ENERGY-BALANCE LOCATION
C  I=0 IMPLIES CONTROL SECTION ANALYSIS
C
C  FIND J AND K:
C
C
      COMMON/BLK 1/X(20,20),Y(20,20),NFL(20),M(20)
      COMMON/BLK 2/DISTS(20),ALPHA(20),XMAN(20),EDDY(20)
      COMMON/BLK 3/TOL,Q,N,DIST(50),NE
C
C
      DD=.01
      L=2
      IF(I.EQ.0)GOTO 15
      IF(I.GT.0)DD=DIST(I)
      DO 10 L=1,N
      IF(DD.GT.DISTS(L))GOTO 10
      GOTO 15
10    CONTINUE
15    J=L-1
      K=L
C.......GET LENGTH RATION R
      R=(DD-DISTS(J))/(DISTS(K)-DISTS(J))
      R1=1.-R
C.......FIND YC
      JFL=NFL(J)
      KFL=NFL(K)
      YMINJ=Y(J,JFL)
      YMINK=Y(K,KFL)
      YMIN=R*YMINK+R1*YMINJ
      YFL=YMIN
      YMAX=YMIN+100.
      AL=R*ALPHA(K)+R1*ALPHA(J)
      DO 100 L=1,15
      YTEST=.5*(YMIN+YMAX)
      DY=YTEST-YFL
      YJ=YMINJ+DY
      YK=YMINK+DY
      CALL PROF3(J,YJ,AJ,V,PJ,DHJ,TWJ,XR,FR,1.,NT,ERROR)
      CALL PROF3(K,YK,AK,V,PK,DHK,TWK,XR,FR,1.,NT,ERROR)
      A=R*AK+R1*AJ
      TW=R*TWK+R1*TWJ
```

```
      P=R*PK+R1*PJ
      RH=A/P
      DH=A/TW
      V=Q/A
      FR=(AL*V*V/32.18/DH)
      IF(FR-1.)20,120,40
20    YMAX=YTEST
      GOTO 100
40    YMIN=YTEST
100   CONTINUE
120   YC=YTEST
      FR=SQRT(FR)
C.......GET YN
      XN=R*XMAN(K)+R1*XMAN(J)
      SLOPE=(YMINK-YMINJ)/(DISTS(K)-DISTS(J))
      YN=0.
      IF(I.EQ.0)GOTO 300
      IF(SLOPE.LT..00001)GOTO 300
      FACT=1.486/XN*SQRT(SLOPE)
      YMIN=R*YMINK+R1*YMINJ
      YMAX=YMIN+100.
      DO 200 L=1,17
      YTEST=.5*(YMIN+YMAX)
      DY=YTEST-YFL
      YJ=YMINJ+DY
      YK=YMINK+DY
      CALL PROF3(J,YJ,AJ,V,PJ,DHJ,TWJ,XR,FR,1.,NT,ERROR)
      CALL PROF3(K,YK,AK,V,PK,DHK,TWK,XR,FR,1.,NT,ERROR)
      A=R*AK+R1*AJ
      P=R*PK+R1*PJ
      RH=A/P
      QTEST=FACT*A*RH**.6667
      IF(QTEST-Q)150,220,180
150   YMIN=YTEST
      GOTO 200
180   YMAX=YTEST
200   CONTINUE
220   YN=YTEST
300   CONTINUE
C
      RETURN
      END
C
C
C  ------------------------------------------------------------------
      SUBROUTINE PROF5(NT)
C  ------------------------------------------------------------------
      COMMON/BLK 3/TOL,Q,N,DIST(50),NE
C
      WRITE(NT,10)
10    FORMAT(//,35X,'(AES):',/,30X,'Irregular Channel',/,
     C 30X,'Subcritical Flow',/,34X,'Analysis',/,5X,
     C 'Study Name',30('_'),5X,'Page Number',15('_'),//)
      WRITE(NT,20)
20    FORMAT(5X,'The following study is based on the well known',/,
     C 5X,'STANDARD STEP METHOD to analyse subcritical flow in an',/,
     C 5X,'irregular channel. Energy-head losses and corresponding',/,
     C 5X,'notation used in the program are as follows:',/,
     C 5X,'    FRICTION LOSSES:   n = Mannings friction parameter',/,
     C 5X,'    EDDY LOSSES:       e = eddy loss coefficient',/,
```

```
     C  5X,'    KINETIC ENERGY',/,
     C  5X,'    CORRECTION FACTOR: a = correction factor')
        WRITE(NT,30)
30      FORMAT(/,5X,'The PROGRAM determines subcritical flow water',/,
     C  5X,'surface elevations by balancing the classical energy',/,
     C  5X,'equation between specified channel locations. All',/,
     C  5X,'geometric and parameter information is averaged between',/,
     C  5X,'defined channel cross-sections by straight-line',/,
     C  5X,'interpolation.')
        WRITE(NT,40)
40      FORMAT(/,5X,'THE ONLY LOSSES INCLUDED ARE FRICTION AND',/,
     C  5X,'EDDY LOSSES. The analysis formulation and presentation',/,
     C  5X,'of results follow the development given in',/,
     C  5X,'"OPEN CHANNEL HYDRAULICS", by Chow(1959).',//,
     C  5X,'The PROGRAM will default to a minimum flow-depth',/,
     C  5X,'of CRITICAL DEPTH, where the Froude number (Fr)',/,
     C  5X,'equals one (1). Therefore, supercritical water ')
        WRITE(NT,50)
50      FORMAT(5X,'surface information is not computed in this program.'
     C  ,/,5X,'In reaches of supercritical flow, critical depth is',/,
     C  5X,'assumed as a minimum flow depth which exceeds the actual',/,
     C  5X,'supercritical flowdepth in the channel.')
        WRITE(NT,60)Q,N,NE
60      FORMAT(/,5X,'For this study, the following information is used:'
     C  ,/,10X,'Channel flow(cfs) = ',F8.1,/,10X,
     C  'Number of channel cross-sections = ',I2,/,10X,
     C  'Number of Energy-Balance locations = ',I2)
        WRITE(NT,70)
70      FORMAT(//,5X,'Special notation given in the computer results',
     C  /,5X,'are as follows:',/,
     C  7X,'(1)FLOOD....this word appears in the first column',/,
     C  7X,'           whenever the estimated flowdepth exceeds either',/,
     C  7X,'           bank of the channel section.',/,
     C  7X,'(2)STEEP....this word appears in the last column',/,
     C  7X,'           whenever the channel Critical Depth exceeds',/,
     C  7X,'           the Normal Depth. Because this program')
        WRITE(NT,80)
80      FORMAT(7X,'           is intended only for subcritical flow,',/,
     C  7X,'           supercritical flow is modeled with flowdepths',/,
     C  7X,'           equal to or greater than Critical depth.',/,
     C  7X,'           This simplification results in conservative',/,
     C  7X,'           channel flowdepth estimations. However, supercritical',/,
     C  7X,'           flow effects including hydraulic jumps are',/,
     C  7X,'           NOT MODELED ACCURATELY.')
        WRITE(NT,90)
90      FORMAT(7X,'(3)SECTION..this word appears in the second column',/,
     C  7X,'           whenever the energy-balance channel location',/,
     C  7X,'           occurs at one of the defined channel cross-sections.')
C
        RETURN
        END
C
C
C  ------------------------------------------------------------------
        SUBROUTINE PROF6(NT)
C  ------------------------------------------------------------------
C
C
        COMMON/BLK 1/X(20,20),Y(20,20),NFL(20),M(20)
        COMMON/BLK 2/DISTS(20),ALPHA(20),XMAN(20),EDDY(20)
```

```
          COMMON/BLK 3/TOL,QQ,N,DIST(50),NE
C
C
C.......OUTPUT CROSS-SECTIONS
C
          IFF=12
C
          WRITE(NT,10)IFF
10        FORMAT(1X,A1,/)
          WRITE(NT,5)
5         FORMAT(76('*'))
C
          WRITE(NT,161)
          WRITE(NT,5)
C
          WRITE(NT,7)
7         FORMAT(5X,'CROSS-SECTION INFORMATION:')
          WRITE(NT,6)
C
C
          DO 50 K=1,N
          WRITE(NT,20)K,XMAN(K),ALPHA(K),EDDY(K),DISTS(K)
20        FORMAT(5X,'INFORMATION FOR CROSS-SECTION NUMBER:',I3,/,
     C    5X,'MANNINGS FRICTION FACTOR = ',F6.5,/,
     C    5X,'KINETIC ENERGY CORRECTION FACTOR = ',F5.3,/,
     C    5X,'EDDY LOSS FACTOR = ',F5.3,/,
     C    5X,'DISTANCE(ft.) TO CROSS-SECTION #1 = ',F8.2,/)
          MM=M(K)
          WRITE(NT,25)
25        FORMAT(5X,'NODAL POINT COORDINATE INFORMATION:',/,
     C    5X,'NODE NO.',5X,'X(ft.)',4X,'Y(elev.)')
          DO 30 KK=1,MM
          WRITE(NT,31)KK,X(K,KK),Y(K,KK)
31        FORMAT(7X,I3,7X,F7.2,5X,F7.2)
30        CONTINUE
          WRITE(NT,6)
6         FORMAT(76('='))
50        CONTINUE
          WRITE(NT,163)
163       FORMAT(//)
          WRITE(NT,5)
          WRITE(NT,60)
60        FORMAT(5X,'USER-SPECIFIED ENERGY-BALANCE CHANNEL LOCATIONS:'
     C    ,/,10X,'ENERGY BALANCE          DISTANCE TO',/,
     C    10X,   'LOCATION NUMBER      CROSS-SECTION #1')
          WRITE(NT,61)(I,DIST(I),I=1,NE)
61        FORMAT(18X,I2,13X,F8.2)
          WRITE(NT,5)
          CONTINUE
C
C  FORMATS
C
161       FORMAT(16X,'IRREGULAR CHANNEL SUBCRITICAL FLOW ANALYSIS')
162       FORMAT(76('-'))
C
          RETURN
          END
```

(AES):
Irregular Channel
Subcritical Flow
Analysis

Study Name________________________ Page Number____________

The following study is based on the well known STANDARD STEP METHOD to analyse subcritical flow in an irregular channel. Energy-head losses and corresponding notation used in the program are as follows:

```
   FRICTION LOSSES:    n = Mannings friction parameter
   EDDY LOSSES:        e = eddy loss coefficient
   KINETIC ENERGY
   CORRECTION FACTOR: a = correction factor
```

The PROGRAM determines subcritical flow water surface elevations by balancing the classical energy equation between specified channel locations. All geometric and parameter information is averaged between defined channel cross-sections by straight-line interpolation.

THE ONLY LOSSES INCLUDED ARE FRICTION AND EDDY LOSSES. The analysis formulation and presentation of results follow the development given in "OPEN CHANNEL HYDRAULICS", by Chow(1959).

The PROGRAM will default to a minimum flow-depth of CRITICAL DEPTH, where the Froude number (Fr) equals one (1). Therefore, supercritical water surface information is not computed in this program. In reaches of supercritical flow, critical depth is assumed as a minimum flow depth which exceeds the actual supercritical flowdepth in the channel.

For this study, the following information is used:

```
     Channel flow(cfs) =   1000.0
     Number of channel cross-sections =  4
     Number of Energy-Balance locations = 11
```

Special notation given in the computer results are as follows:

(1)FLOOD....this word appears in the first column whenever the estimated flowdepth exceeds either bank of the channel section.

(2)STEEP....this word appears in the last column whenever the channel Critical Depth exceeds the Normal Depth. Because this program is intended only for subcritical flow, supercritical flow is modeled with flowdepths equal to or greater than Critical depth. This simplification results in conservative channel flowdepth estimations. However, supercritical flow effects including hydraulic jumps are NOT MODELED ACCURATELY.

(3)SECTION..this word appears in the second column whenever the energy-balance channel location occurs at one of the defined channel cross-sections.

```
****************************************************************************
      CROSS-SECTION INFORMATION:
****************************************************************************
    INFORMATION FOR CROSS-SECTION NUMBER:  1
    MANNINGS FRICTION FACTOR = .03000
    KINETIC ENERGY CORRECTION FACTOR = 1.100
    EDDY LOSS FACTOR =  .000
    DISTANCE(ft.) TO CROSS-SECTION #1 =       .00

    NODAL POINT COORDINATE INFORMATION:
    NODE NO.      X(ft.)     Y(elev.)
       1             .00      110.00
       2           20.00      100.00
       3           25.00      100.00
       4           45.00      110.00
----------------------------------------------------------------------------
    INFORMATION FOR CROSS-SECTION NUMBER:  2
    MANNINGS FRICTION FACTOR = .03000
    KINETIC ENERGY CORRECTION FACTOR = 1.100
    EDDY LOSS FACTOR =  .000
    DISTANCE(ft.) TO CROSS-SECTION #1 =   500.00

    NODAL POINT COORDINATE INFORMATION:
    NODE NO.      X(ft.)     Y(elev.)
       1             .00      112.00
       2           19.00      102.50
       3           22.00      103.00
       4           46.00      111.00
----------------------------------------------------------------------------
    INFORMATION FOR CROSS-SECTION NUMBER:  3
    MANNINGS FRICTION FACTOR = .03000
    KINETIC ENERGY CORRECTION FACTOR = 1.100
    EDDY LOSS FACTOR =  .000
    DISTANCE(ft.) TO CROSS-SECTION #1 =   850.00

    NODAL POINT COORDINATE INFORMATION:
    NODE NO.      X(ft.)     Y(elev.)
       1             .00      135.10
       2           34.00      123.75
       3           37.00      125.00
       4           77.00      135.00
----------------------------------------------------------------------------
    INFORMATION FOR CROSS-SECTION NUMBER:  4
    MANNINGS FRICTION FACTOR = .03000
    KINETIC ENERGY CORRECTION FACTOR = 1.100
    EDDY LOSS FACTOR =  .000
    DISTANCE(ft.) TO CROSS-SECTION #1 =  1000.00

    NODAL POINT COORDINATE INFORMATION:
    NODE NO.      X(ft.)     Y(elev.)
       1             .00      136.00
       2           34.50      124.50
       3           37.50      127.00
       4           82.50      136.00
----------------------------------------------------------------------------
****************************************************************************
    USER-SPECIFIED ENERGY-BALANCE CHANNEL LOCATIONS:
         ENERGY BALANCE          DISTANCE TO
         LOCATION NUMBER       CROSS-SECTION #1
                1                   100.00
                2                   200.00
                3                   300.00
                4                   400.00
                5                   500.00
                6                   600.00
                7                   700.00
                8                   800.00
                9                   850.00
               10                   900.00
               11                  1000.00
****************************************************************************
```

*** (AES): IRREGULAR CHANNEL SUBCRITICAL FLOW MODEL ***
Standard Step Method irregular channel analysis. Based on development in "OPEN CHANNEL HYDRAULICS",CHOW(1959)
STUDY NAME: Channel Flow = 1000.00 cfs PAGE NUMBER:

LENGTH from CONTROL	WATER SURFACE (elev.)	FLOW DEPTH (ft)	FLOW AREA (ft*ft)	FLOW V (fps)	2 aV /2g (ft)	TOTAL HEAD (ft)	HYDR RADIUS (ft)	FRICTION SLOPE Sf	AVERAGE REACH Sf	REACH LENGTH (ft)	LOSS Hf (ft)	EDDY LOSS (ft)	TOTAL HEAD (ft)	Fr
.0	105.91	5.91	99.4	10.06	1.729 a=1.10	107.639	3.16	.008885 n= .0300					107.639	1.00 GIVEN
100.0	107.21	6.71	123.5	8.10	1.121 a=1.10	108.335	3.47	.005061 n= .0300	.006973	100.0	.697	.00 e= .00	108.336	.77
200.0	107.73	6.73	123.6	8.09	1.119 a=1.10	108.845	3.42	.005155 n= .0300	.005108	100.0	.511	.00 e= .00	108.846	.78
300.0	108.26	6.76	124.4	8.04	1.105 a=1.10	109.362	3.38	.005177 n= .0300	.005166	100.0	.517	.00 e= .00	109.362	.78
400.0	108.79	6.79	125.4	7.97	1.087 a=1.10	109.880	3.34	.005171 n= .0300	.005174	100.0	.517	.00 e= .00	109.879	.78
500.0	109.33 SECTION	6.83	126.5	7.91	1.068 a=1.10	110.398	3.30	.005158 n= .0300	.005164	100.0	.516	.00 e= .00	110.397	.78
600.0	114.63	6.06	105.4	9.49	1.538 a=1.10	116.170	2.89	.008861 n= .0300	.007009	100.0	.701	.00 e= .00	111.099	*1.00 STEEP
700.0	120.62	5.98	107.4	9.31	1.482 a=1.10	122.106	2.81	.008875 n= .0300	.008868	100.0	.887	.00 e= .00	117.057	*1.00 STEEP
800.0	126.62	5.91	109.3	9.15	1.431 a=1.10	128.054	2.73	.008892 n= .0300	.008884	100.0	.888	.00 e= .00	122.994	*1.00 STEEP
850.0	129.63 SECTION	5.88	110.3	9.06	1.404 a=1.10	131.032	2.70	.008866 n= .0300	.008879	50.0	.444	.00 e= .00	128.498	*1.00 STEEP
900.0	130.24	6.24	119.3	8.38	1.200 a=1.10	131.438	2.76	.007355 n= .0300	.008110	50.0	.406	.00 e= .00	131.437	.91
1000.0	130.98 SECTION	6.48	118.4	8.45	1.220 a=1.10	132.202	2.65	.007902 n= .0300	.007628	100.0	.763	.00 e= .00	132.201	.93

The development of mathematical procedures for describing the unsteady flow of water in open channels has gone on for many years. For example, wave motion was described in general terms about 200 years ago by Laplace and Lagrange. Lagrange developed a formula for the computation of the celerity of small waves in shallow water which is still in general use today.

A more detailed decription of the form of partial differential equations describing unsteady flow in open channels was first presented by Barre' de Sainte-Venant in 1871 in the Transactions of the French Academy of Sciences. The equations presented by de Saint-Venant are essentially the same as the present-day forms of the one-dimensional unsteady flow equations, and the set of equations which describe one-dimensional, gradually varied unsteady flow in open channels are commonly called the St. Venant Equations.

To solve the general problem of unsteady open channel flow, the equations must supply a numerical quantification of flow and depth. The mathematical relationships used should provide sufficient detail to give an accurate flow analysis, but too much detail may mean that excessive computational work is required.

Figure 3.8 shows a generalized sketch of an open channel. The flow is changing with time, and the cross section of the channel can vary along its length. Flows can enter and leave laterally. The flow is considered to be 'one-dimensional' (i.e., properties such as flow and depth vary only in one direction - the direction of flow).

3.7. Derivation of the St. Venant Equations

The physical laws which govern the flow of water in a channel are:

1) the principle of conservation of mass, and
2) the principle of conservation of momentum.

These laws can be expressed mathematically in the form of partial differential equations such as those presented in section 3.2. The differential equations can be solved using finite difference approximations to the derivatives in the equations to give quantitative solutions to unsteady flow problems. To permit these solutions the mathematical relationships must be developed in sufficient detail to permit accurate flow analyses. Too much detail, however, makes the computation procedures unnecessarily complicated and produces little improvement in accuracy of the results.

Consider the channel shown schematically in Fig. 3.8. Assume that the flow is changing with time, and also, that the dimensions and cross sectional shape of the channel vary along its length. Consider a short reach or element of the channel in which the depth and flow changes are small and have approximately linear variations along the length of the element. As shown in Fig. 3.8,

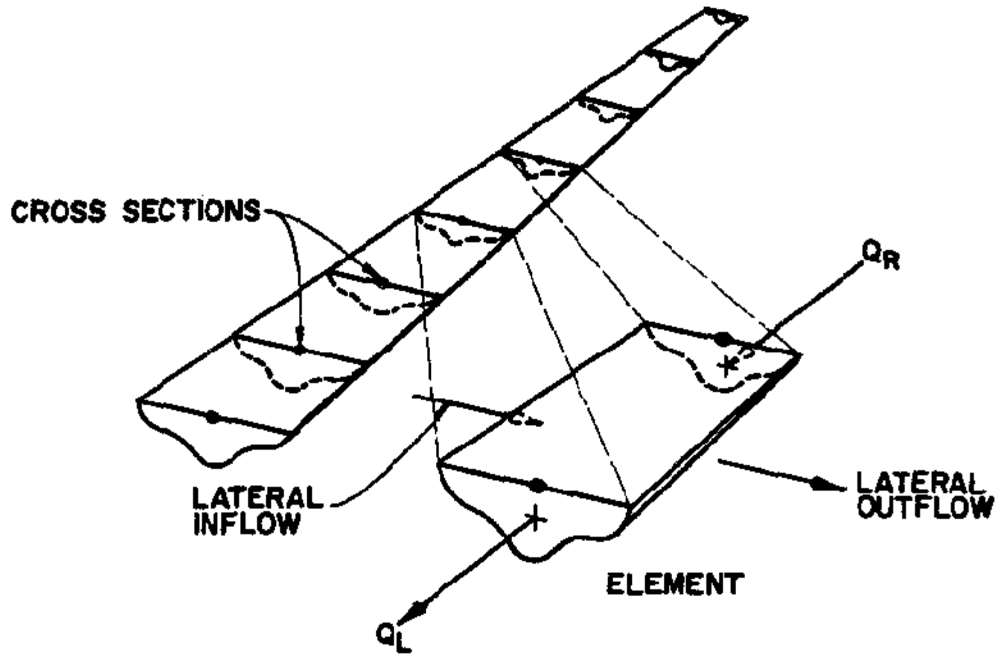


Fig. 3.8. *Generalized sketch of open channel and channel element.*

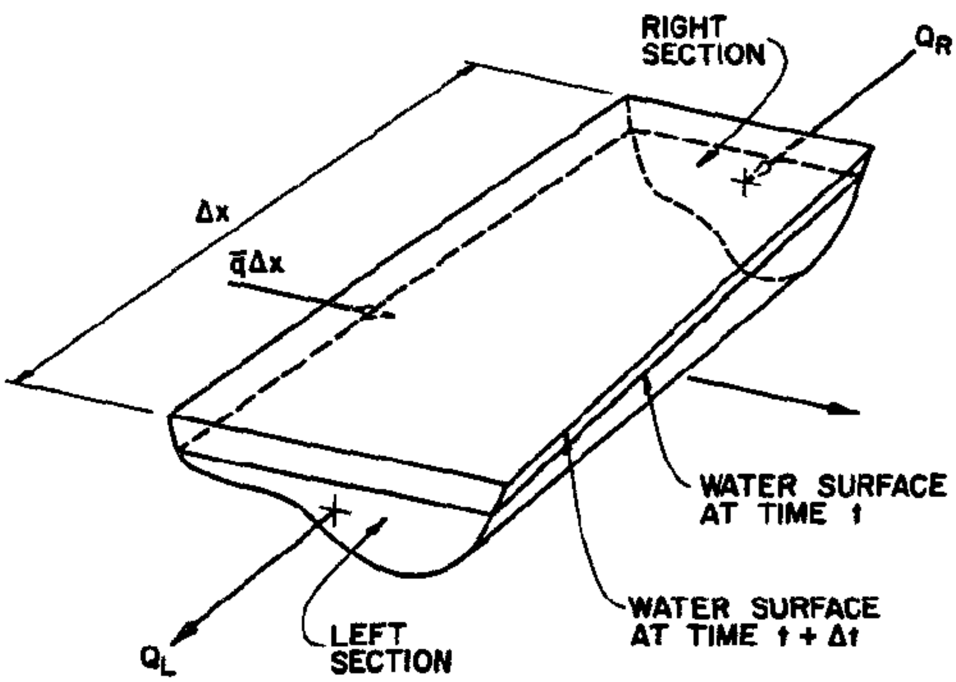


Fig. 3.9. *Detailed sketch of channel element.*

there can be lateral inflows and outflows in addition to the flow into and out of the element in the direction of the channel axis. Also, at any time the sums of the inflows and outflows will not be the same, thus the volume of water in the element will change with time. This means that the changes in water surface elevation and in water surface slope with time must also be accounted for.

3.7.1. Continuity Equation

Consider the channel element shown in Fig. 3.9. Flow enters the element at the right (Section R) and leaves at the left (Section L). Flow can also enter or leave the element laterally either through the channel bed or as lateral inflow or outflow at the surface. The solid boundaries are fixed, so that any change in volume of water between Section R and Section L requires a change in the water surface elevation. The flows through the right and left section boundaries are Q_R and Q_L, respectively. The lateral flow is taken as an average value along the length of the element, Δx, thus, the total lateral flow into the element (or, if negative, out of the element) is $q\Delta x$.

At some time t_1, the flow into the element at Section R can be designated as Q_{R_1}. At some increment of time later, $t_2 = t_1 + \Delta t$, and the flow through Section R is Q_{R_2}. The original volume of water in the element is given by

$$[(A_{R_1} + A_{L_1})/2]\ \Delta x.$$

During the time interval the volume has changed.

The principle of conservation of mass states that

(Inflow − Outflow)=(Change in volume during the time interval)(3.41)

so

$$[(Q_{R_1} + q_1\ \Delta x - Q_L) + (Q_{R_2} + q_2\ \Delta x - Q_L)]/2$$
$$= [(A_{R_2} + A_{L_2}) - (A_{R_1} + A_{L_1})]\ \frac{\Delta x}{2\Delta t} \tag{3.42}$$

This algebraic equation can be expressed as a differential equation by reducing the time and length increments so that in the limit they approach zero. Thus if we let $\Delta x \rightarrow 0$ and $\Delta t \rightarrow 0$ then

$Q_{R_2} \to Q_{R_1} \to Q_R$ $\qquad$ $A_{R_2} \to A_{L_2} \to A_2$

$Q_{L_2} \to Q_{L_1} \to Q_L$ $\qquad$ $A_{R_1} \to A_{L_1} \to A_1$

$A_{R_2} \to A_{R_1} \to A_R$

$A_{L_2} \to A_{L_1} \to A_L$

$q_2 \to q_1 \to q$

So Eq. (3.42) becomes

$$\frac{1}{2\Delta x}[Q_R + q\,\Delta x - Q_L + Q_R + q\,\Delta x - Q_L] = [(A_2 + A_2) - (A_1 + A_1)]\frac{1}{2\Delta t}$$

Simplifying gives

$$\frac{Q_R - Q_L}{\Delta x} + q - \frac{A_2 - A_1}{\Delta t} = 0 \qquad (3.43)$$

In the limit as $\Delta x \to 0$:

$$\frac{Q_L - Q_R}{\Delta x} = \frac{\Delta Q}{\Delta x} = \frac{\partial Q}{\partial x}$$

and as $\Delta t \to 0$:

$$\frac{A_2 - A_1}{\Delta t} = \frac{\Delta A}{\Delta t} = \frac{\partial A}{\partial t}$$

Thus the differential equation describing conservation of mass is:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \qquad (3.44)$$

3.7.2. Equation of Motion

In Equation (3.44), there are two unknowns: Q and A. Because there are two unknowns, a second equation is required for the solution of the problem. This can be obtained through the principle of conservation of linear momentum. By Newton's second law

$$\Sigma F = \frac{dM}{dt} \tag{3.45}$$

This law states that the sum of all the external forces acting on a fluid element is equal to the change in momentum with time. The momentum (a vector quantity) is given by $M = mV$ (fluid mass times the velocity vector in the direction of flow). If we consider the forces which act in the direction of flow (the x-direction), there are three major forces:

(1) Pressure forces (P)

(2) Forces due to the weight of the water (W), and

(3) Fluid drag at the solid boundaries (D)

hence,

$$\Sigma F_x = P_x + W_x + D_x = \frac{dM_x}{dt} \tag{3.46}$$

Momentum Change in Element— If it is assumed that the change in momentum added by the lateral inflow can be neglected, then the change in momentum can be considered as consisting of two parts:

(1) Local (or temporal) momentum change, and

(2) Convective (or spatial) momentum change.

The local momentum of the water in the element is $\rho Q\ \Delta x$, where ρ = fluid density. The time rate of change of momentum in the element is

$$\frac{\partial}{\partial t}(\rho Q\ \Delta x) = \rho \partial x \frac{\partial Q}{\partial t} \tag{3.47}$$

The momentum entering the element is ρQV (or $\rho Q^2/A$, since $V = Q/A$). The flow momentum also changes with distance, and the momentum of the flow leaving the element can be expressed in terms of the entering momentum as:

$$\rho\ Q^2/A + \frac{\partial}{\partial x}\left(\rho \frac{Q^2}{A}\right) \Delta x$$

The difference between the momentum entering and leaving is:

$$\rho \; \Delta x \frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right)$$

The total change in momentum is the sum of the temporal and spatial changes:

$$\frac{dM}{dt} = \rho \Delta x \frac{\partial Q}{\partial t} + \rho \; \Delta x \frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right) \tag{3.48}$$

Pressure Forces-- Assume that the pressures are hydrostatic. Therefore, at any cross section the pressure varies linearly with water depth. The total pressure is the sum (or integration) of the unit pressures acting on the cross section. Pressures act at right angles to the surfaces of the element. If the right and left faces of the element are relatively close together, $A_R \approx A_L$, and the area of both faces can be represented by an average area $\bar{A}$. However, water depths are different at the two faces, and the difference in levels Δy can be determined by multiplying the change in depth with distance along the channel $\partial y/\partial x$ by the distance between the right and left faces of the element, Δx.

In this analysis we are considering the general case where the channel width varies, therefore, (as is illustrated in Fig. 3.10), $A_R = A_L$. In this case pressure forces on the side walls of the channel also have to be taken into account because the channel sides are at an angle to the flow direction, x. The channel sides exert a force on the water element equal and opposite to the water pressure forces. The pressure is at right angles to the boundary, and thus there is also a force component in the x-direction. Recalling from the principles of hydrostatics that the x-direction pressure force component can be computed using the projected area of the surface in the x-direction, the total pressure force (see Fig. 3.10) is:

$$\Delta P_x = \left[P_L + P_{W_x} \right] - P_R \tag{3.49}$$

where P_{W_x} is the x-direction component of forces on channel walls. The sum of A_L and the projected area is equal to A_R, so that total area on which the pressures are acting can be represented by an average area A. The pressure difference is due to the difference in water depth Δy. Thus the sum of the pressure forces on the element can be computed directly from this difference:

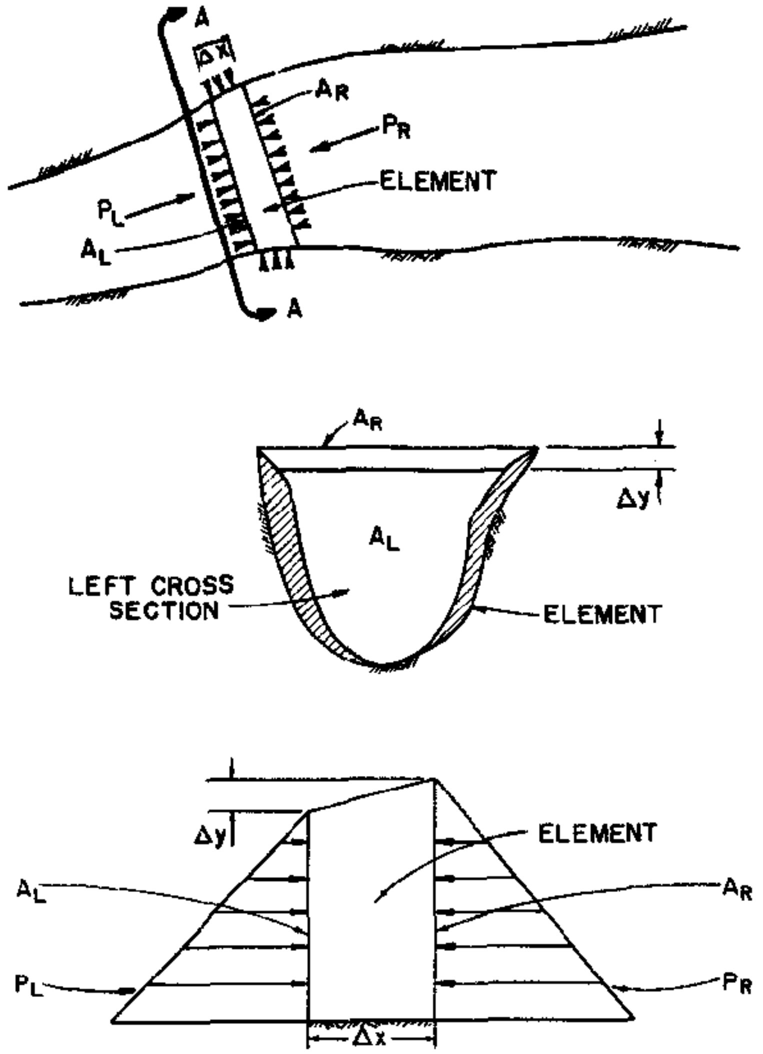


Fig. 3.10. Pressure force on channel element.

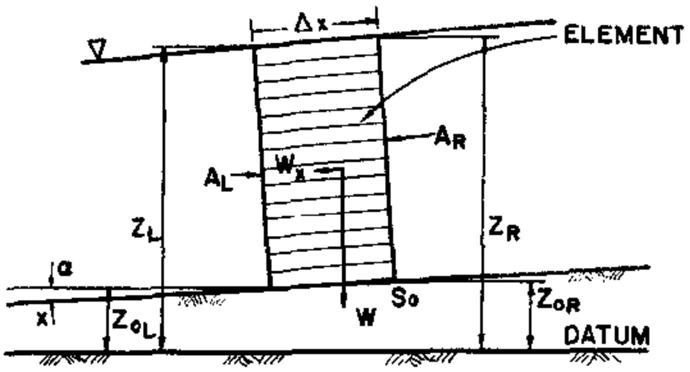


Fig. 3.11. Sketch illustrating fluid weight component.

$$\Sigma P_x = \gamma \bar{A} \frac{\partial y}{\partial x} \Delta x \tag{3.50}$$

Force Due to Fluid Weight—Another force acting on the element is due to fluid weight W_x. The x-direction component of total weight of the element (see Fig. 3.11) is:

$$W_x = \gamma \bar{A} \, \Delta x \, \sin \alpha \quad \bar{A} \, \Delta x \left[\frac{Z_{o_R} - Z_{o_L}}{\Delta x} \right] \tag{3.51}$$

where Z_o is the elevation of the channel bottom.

For convenience the pressure force and weight force terms can be combined:

$$\Sigma P_x + W_x = \gamma \, \Delta x \left[-\bar{A} \left(\frac{y_L - y_R}{\Delta x} \right) \Delta x + \bar{A} \left(\frac{Z_{o_R} - Z_{o_L}}{\Delta x} \right) \right]$$

$$= \gamma \bar{A} \, (Z_L - Z_R) = \gamma \bar{A} \, \Delta Z \tag{3.52}$$

where Z is the water surface elevation, $Z = y + Z_o$.

Drag Force of Channel Walls on Fluid in Element— The total drag force on the element D_x can be related to the velocity by means of a drag coefficient. The drag force is proportional to the square of the velocity, and can be expressed as:

$$D_x = \gamma C_D \, \bar{Q} |\bar{Q}| \cdot \Delta x \tag{3.53}$$

where C_D is a drag coefficient which depends on the roughness of the bed and the cross section geometry.

This drag coefficient can be determined from the Chezy relationship

$$C_D = 1/(C^2 \, AR) \tag{3.54}$$

where C = Chezy's coefficient, and R is the hydraulic radius (area divided by wetted perimeter). When Manning's formula is used, $C = (1.486/n) \, R^{1/6}$. Thus

$$D_x = \gamma \left[\frac{n}{1.486 \, R^{1/6}} \right]^2 \frac{\bar{Q}|\bar{Q}|}{AR} \Delta x \tag{3.55}$$

Since by Manning's Equation

$$S_f = \frac{Q^2}{A^2}\left[\frac{n}{1.486\ R^{2/3}}\right]^2 \qquad (3.56)$$

then

$$D_x = -\gamma S_f\ A\ \Delta x \qquad (3.57)$$

The negative sign is due to taking S_f as positive when the slope in the x-direction is downward.

Combining Terms in Equation of Motion-- Substituting the above terms into the momentum equation (Eq. 3.48) we get

$$-\gamma A\ \Delta Z - \gamma A\ \Delta x\ S_f = \rho\ \Delta x \frac{\partial Q}{\partial t} + \rho \Delta x \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) \qquad (3.58)$$

Dividing through by $\rho \Delta x$ gives

$$-gA\frac{\Delta Z}{\Delta x} - gA\ S_f = \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)$$

This equation of motion can be written as

$$\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{\partial Z}{\partial x} + S_f = 0 \qquad (3.59)$$

which along with the continuity equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \qquad (3.44)$$

make up the pair of equations known as the St. Venant Equations. As they are written above, they are in what is termed momentum-conservation and mass-conservation form.

3.7.3. Assumptions Used in the Derivation of the St. Venant Equations

1. The pressure distribution along a vertical line in the flow is hydrostatic.

2. Friction losses in unsteady flow can be computed using steady-flow friction loss formulas.

3. The velocity distribution across a channel section does not affect wave propagation.

4. The water surface across section is horizontal.

5. The average channel bottom slope is small.

3.7.4. Meaning of the Various Terms in the St. Venant Equations

The water surface elevation Z is the sum of the bottom elevation Z_o and the water depth y: thus the water surface slope term $\partial Z/\partial x = \partial Z_0/\partial x + \partial y/\partial x$. In a channel with constant bottom slope S_o, $\partial Z_0/\partial x = -S_0$. Using this expression, and including the effects of losses due to expansions and contraction (S_e), motion can be written as:

$$\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{\partial y}{\partial x} - S_0 + S_f + S_e = 0 \qquad (3.60)$$

Also, in the most general case, there may be areas of off-channel storage where the flow velocity can be considered negligible. If A is the flow cross sectional area, and A_o is the off-channel storage cross sectional area, then the continuity equation can be written in the form:

$$\frac{\partial Q}{\partial x} + \frac{\partial (A + A_0)}{\partial t} - q = 0 \qquad (3.61)$$

The meaning of the various terms in the equation of motion is as follows:

$\frac{1}{gA}\frac{\partial Q}{\partial t}$ = local acceleration of the flow

$\frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)$ = convective acceleration of the flow

$\frac{\partial y}{\partial x}$ = pressure gradient term (variation of water depth with distance)

S_0 = bottom slope term

S_e = expansion or contraction loss term

3.8. Unsteady Flow Profiles by the Implicit Method with Double Sweep

Consider the case of unsteady flow with a continuous profile as described by Equations (3.59) and (3.60). Assume no lateral inflow or outflow occurs, (i.e., q = 0). At any channel section the area of flow is designated by A and the channel top width by B as shown in Fig. 3.12. Also shown in Fig. 3.12 is a longitudinal section along the channel center line. The elevation of the water surface is designated by the symbol Z, the water depth by y, and the discharge at any point by Q. Subscripts are used to indicate the location at which the variables are defined.

3.8.1. Continuity Equation

The continuity relationship is given by Equation 3.44. With q = 0, it can be written as

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad (3.62)$$

where Q and A are functions of time t and distance along the channel x. This equation can also be written as

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial z} \; \frac{\partial z}{\partial t} = 0 \qquad (3.63)$$

and because in a prismatic channel

$$B = \frac{\partial A}{\partial z} \qquad (3.64)$$

the continuity relationship can be expressed as

$$\frac{\partial Q}{\partial x} + B\frac{\partial z}{\partial t} = 0 \qquad (3.65)$$

3.8.2. Equation of Motion

The equation of motion for unsteady flow with a continuous profile is given by Equation (3.59). Using the mean velocity V = Q/A, and expressing the friction slope S_f in terms of K, the 'conveyance' of the channel g, the equation can be written as

$$\frac{\partial Z}{\partial x} + \frac{\partial}{\partial x}\left(\frac{V^2}{2g}\right) + \frac{Q|Q|}{K^2} + \frac{1}{g}\;\frac{\partial V}{\partial t} = 0 \qquad (3.66)$$

The conveyance is a function of the channel roughness and shape and is related to the slope of the energy grade line (friction slope) by the following equation:

$$Q = KS^{1/2} \qquad (3.67)$$

3.9. Finite Differences

Because of the difficulty in solving the St. Venant equations in closed form, it is advantageous to set them up in a form which will permit their solution by numerical methods. Consider the two sections in the channel separated by some length Δx as shown in Fig. 3.12. At some time t, the discharge at Section 1 is

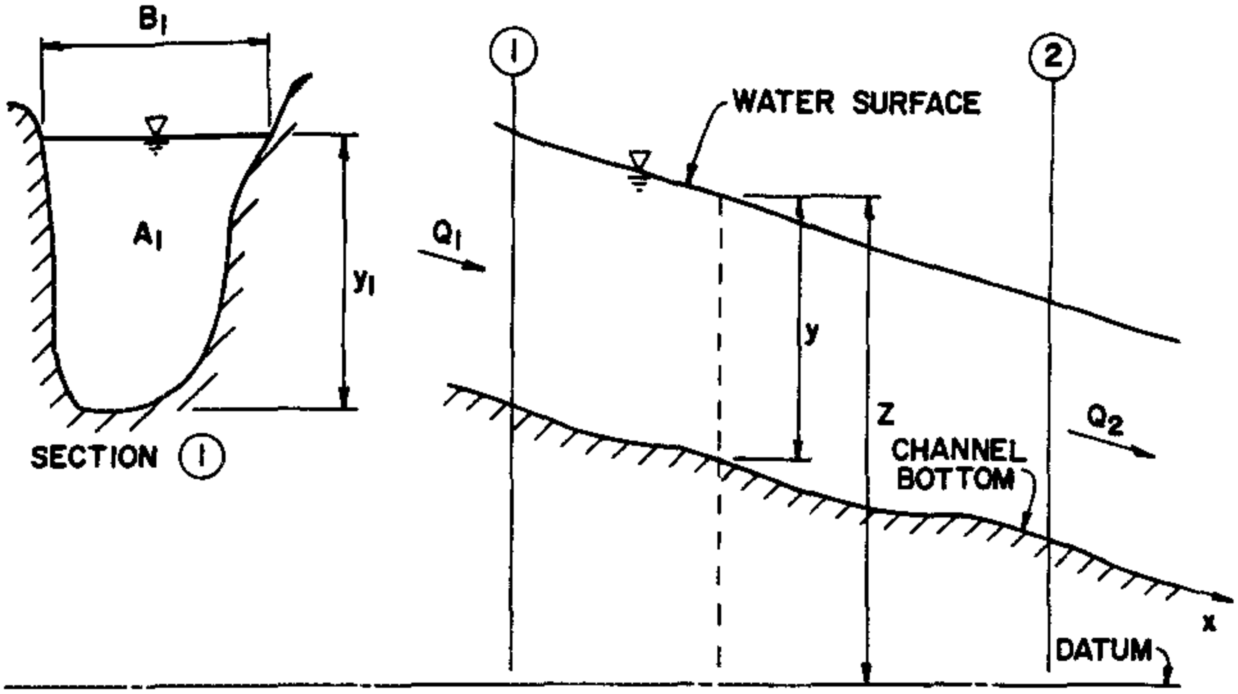


Fig. 3.12. Definition sketch of unsteady flow solution.

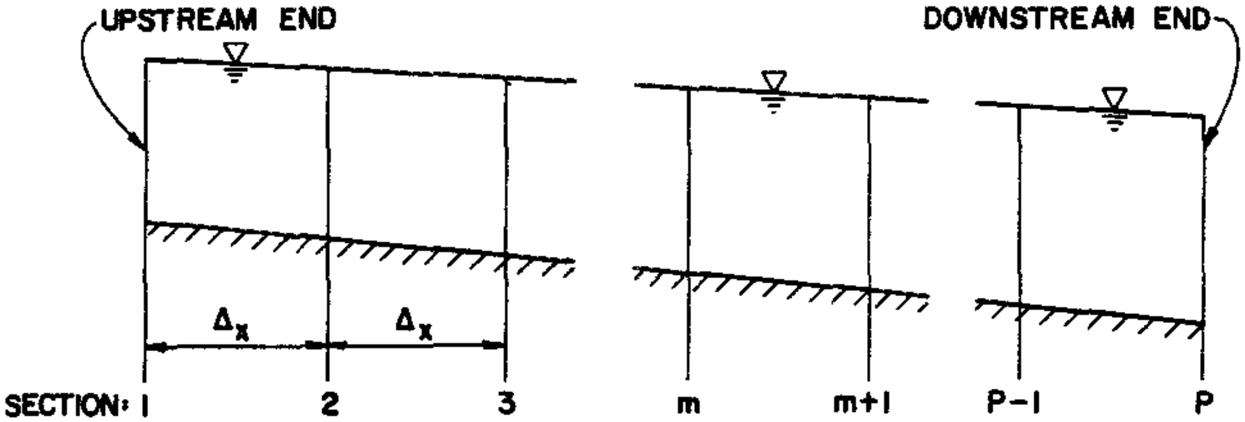


Fig. 3.13. Notation

designated as Q_1: the discharge at 2 is Q_2. At some new time Δt later $(t + \Delta t)$, the new discharges at these points are given by $Q_1 + q_1$ at Section 1 and $Q_2 + q_2$ at Section 2: in this nomenclature $q = \Delta Q$, the change in flow in the time interval.

The change in water surface elevation over the time interval is given in a similar way, with $z = \Delta Z$. Therefore, at time $t + \Delta t$, the water surface elevation at Section 1 is $Z_1 + z_1$, and at Section 2 is $Z_2 + z_2$.

It is also of interest to consider intermediate times within the interval $(t,\ t + \Delta t)$, and these times are designated by $t + \theta\Delta t$ where $\theta\Delta t$ is some fraction of the time interval. Thus

$$0 \leq \theta \leq 1$$

The assumption is now made that at time $t + \theta\Delta t$ the flows at Sections 1 and 2 are $Q_1 + \theta q_1$ and $Q_2 + \theta q_2$, and the water surface elevations at these sections are $Z_1 + \theta z_1$ and $Z_2 + \theta z_2$. It has been found that for the computations using this method to converge, the value of θ must be taken such that

$$0.5 \leq \theta \leq 1$$

3.9.1. Continuity Equation

Equation 3.65 (the continuity equation) can now be written in the following finite difference form:

$$\frac{(Q_2 + \theta q_2) - (Q_1 + \theta q_1)}{\Delta x} + \frac{1}{2}\left[\frac{B_1\ \theta z_1}{\theta \Delta t} + \frac{B_2\ \theta z_2}{\partial \Delta t}\right] = 0 \qquad (3.68)$$

This can be simplified somewhat to

$$-\theta q_1 + \theta q_2 + \frac{\Delta x}{2\Delta t} B_1\ z_1 + \frac{\Delta x}{2\Delta t} B_2\ z_2 + Q_2 - Q_1 = 0 \qquad (3.69)$$

Thus, the continuity equation is now of the form:

$$C_1 q_1 + C_2 q_2 + C_3 z_1 + C_4 z_2 + C_5 = 0 \qquad (3.70)$$

where

$$C_1 = -\theta \qquad C_3 = \frac{\Delta x}{2\Delta t} B_1$$

$$C_5 = Q_2 - Q_1$$

$$C_2 = \theta \qquad C_4 = \frac{\Delta x}{2\Delta t} B_2$$

3.9.2. Equation of Motion

The equation of motion, Equation 3.66, can be similarly expressed in finite difference form. The equation is first multiplied by Δx :

$$\left[\frac{\partial Z}{\partial x} + \frac{\partial}{\partial x}\left(\frac{V^2}{2g}\right) + \frac{Q^2}{K^2}\right]\Delta x + \frac{\Delta x}{g}\frac{\partial V}{\partial t} = 0 \tag{3.71}$$

The finite difference form of the first term evaluated at time $t + \theta\Delta t$ is:

$$\frac{\partial Z}{\partial x}\Delta x = Z_2 + \theta z_2 - Z_1 - \theta z_1 \tag{3.72}$$

For the second term, the substitution V = Q/A is made, giving:

$$\frac{\partial}{\partial x}\left(\frac{V^2}{2g}\right)\Delta x = \frac{1}{2g}\left\{\frac{Q_2^2}{A_2^2} + \frac{2Q_2}{A_2^2}\theta q_2 - \frac{2Q_2^2}{A_2^3}\frac{dA_2}{dZ}\theta z_2 - \frac{Q_1^2}{A_1^2} - \frac{2Q_1}{A_1^2}\theta q_1 + \frac{2Q_1^2}{A_1^3}\frac{dA_1}{dZ}\theta z_1\right\} \tag{3.73}$$

For the third term,

$$\frac{Q^2}{K^2}\Delta x = \frac{\Delta x}{2}\left\{\frac{Q_1^2}{K_1^2} + \frac{2Q_1}{K_1^2}\theta q_1 - \frac{2Q_1^2}{K_1^3}\frac{dK_1}{dZ}\theta z_1 + \frac{Q_2^2}{K_2^2} + \frac{2Q_2}{K_2^2}\theta q_2 - \frac{2Q_2^2}{K_2^3}\frac{dK_2}{dZ}\theta z_2\right\} \tag{3.74}$$

And for the last term, again making the substitution V = Q/A

$$\frac{\Delta x}{g}\frac{\partial V}{\partial t} = \frac{1}{2}\frac{\Delta x}{g\theta\Delta t}\left\{\left[\left[\frac{Q_1}{A_1} + \frac{\theta q_1}{A_1} - \frac{Q_1}{A_1^2}\frac{dA_1}{dz}\theta z_1\right] - \frac{Q_1}{A_1}\right] + \left[\left[\frac{Q_2}{A_2} + \frac{\theta q_2}{A_2} - \frac{Q_2}{A_2}\frac{dA_2}{dz}\theta z_2\right] - \frac{Q_2}{A_2}\right]\right\} \tag{3.75}$$

Substituting these expressions back into Eq. 3.71 produces an equation of the form:

$$D_1 q_1 + D_2 q_2 + D_3 z_1 + D_4 z_2 + D_5 = 0 \tag{3.76}$$

where

$$D_1 = -\frac{Q_1\theta}{gA_1^2} + \frac{\Delta x\, Q_1\theta}{K_1^2} + \frac{\Delta x}{2gA_1\Delta t} \tag{3.77}$$

$$D_2 = \frac{Q_2\theta}{gA_2^2} + \frac{\Delta x\ Q_2\theta}{K_2^2} + \frac{\Delta x}{2gA_2\ \Delta t} \tag{3.78}$$

$$D_3 = -\theta + \frac{Q_1^2 B_1\theta}{gA_1^3} - \frac{\Delta x\ Q_1^2}{K_1^3}\frac{dK_1}{dZ}\theta - \frac{\Delta x\ B_1 Q_1}{2gA_1^2\,\Delta t} \tag{3.79}$$

$$D_4 = \theta - \frac{Q_2^2 B_2\theta}{gA_2^3} - \frac{\Delta x\ Q_2^2}{K_2^3}\frac{dK_2}{dZ}\theta - \frac{\Delta x\ B_2 Q_2}{2gA_2^2\,\Delta t} \tag{3.80}$$

and

$$D_5 = z_2 - z_1 + \frac{Q_2^2}{2gA_2^2} - \frac{Q_1^2}{2gA_1^2} + \frac{\Delta x}{2}\left(\frac{Q_1^2}{K_1^2} + \frac{Q_2^2}{K_2^2}\right) \tag{3.81}$$

Equations 3.70 and 3.76 are two simultaneous equations in four unknowns. If two of the unknowns are specified as boundary conditions, the equations can be solved. Additional points can also be considered, giving two additional equations and two additional unknowns for each additional point.

This system of equations could be solved by the usual methods of solving simultaneous equations, however, as the number of equations increases, this becomes tedious. The 'double-sweep method' described below provides a more efficient procedure of solution of this type of system of equations, and fits in well with solutions by computer.

3.9.3. Double-Sweep Method

Assume that the channel is divided into P - 1 equally spaced segments, as shown in Fig. 3.13. For this method, the following assumption is made concerning the discharge increment q_m at some point m:

$$q_m = E_m z_m + F_m \tag{3.82}$$

where E_m and F_m are constants at a particular point and time. For the segment (m, m+1), we can write the equation of motion (Eq. 3.76) as:

$$D_{1_m} q_m + D_{2_m} q_{m+1} + D_{3_m} z_m + D_{4_m} z_{m+1} + D_{5_m} = 0 \qquad (3.83)$$

The variable q_m can be eliminated from Equation 3.83 by using Eq. 3.82 and thus

$$z_m = L_m z_{m+1} + M_m q_{m+1} + N_m \qquad (3.84)$$

where

$$L_m = D_{4_m} / G_m \qquad (3.85)$$

$$M_m = D_{2_m} / G_m \qquad (3.86)$$

$$N_m = \left(D_{1_m} F_m + D_{5_m} \right) / G \qquad (3.87)$$

and

$$G_m = - \left(D_{1_m} E_m + D_{3_m} \right) \qquad (3.88)$$

Applying the continuity relationship (Eq. 3.70) to the segment (m, m+1),

$$C_{1_m} q_m + C_{2_m} q_{m+1} + C_{3_m} z_m + C_{4_m} z_{m+1} + C_{5_m} = 0 \qquad (3.89)$$

We can now elmiminate q_m and z_m from Eq. 3.89 by use of Eqs. 3.82 and 3.84. This gives

$$q_{m+1} = E_{m+1}\, z_{m+1} + F_{m+1} \qquad (3.90)$$

where

$$E_{m+1} = \left(C_{1_m} E_m L_m + C_{3_m} L_m + C_{4_m} \right) / R_m \tag{3.91}$$

$$F_{m+1} = \left[C_{1_m} \left(E_m N_m + F_m \right) + C_{3_m} N_m + C_{5_m} \right] / R_m \tag{3.92}$$

and

$$R_m = - \left(C_{1_m} E_m M_m + C_{2_m} + C_{3_m} M_m \right) \tag{3.93}$$

Note that the coefficients E_{m+1} and F_{m+1} are completely specified in terms of coefficients which are determined from known conditions at point m.

In addition to the above relationships, boundary conditions must be specified for the upstream and downstream ends of the reach. Either the discharge Q or the water surface elevation Z must be given as a function of time at these points.

3.9.4. Upstream Boundary Conditions

Consider first the upstream boundary with Q(t) known. The flow at some time t at the upstream end of the reach is designated as Q_1^t. The change in flow between times t and t+1 is, therefore,

$$q_1 = Q_1^{t+1} - Q_1^t \tag{3.94}$$

In terms of the relationship bewteen q and z given by Eq. 3.82, this corresponds to a situation where $E_1 = 0$ and

$$F_1 = Q_1^{t+1} - Q_1^t.$$

When Z(t) known is the upstream boundary condition, the situation is a little less straightforward. Because

$$Z_1^{t+1} = Z_1^t + z_1 \tag{3.95}$$

then

$$z_1 + Z_1^t - Z_1^{t+1} = 0 \tag{3.96}$$

Let us, however, make the approximation

$$z_1 + Z_1^t - Z_1^{t+1} \simeq \varepsilon <<< 1 \qquad (3.97)$$

then

$$q_1 \simeq N \left[z_1 + Z_1^t - Z_1^{t+1} \right] \qquad (3.98)$$

where N is a large number (say of the order 10^5). In terms of the variables used in Eq. 3.82, $E_1 = N$, and $F_1 = N(Z_1{}^t - Z_1{}^{t+1})$.

At the first point, q_1 is given by the upstream boundary condition and is computed from either Eqs. 3.94 or 3.98. In the following sections it is assumed that Q(t) is known, and q_1 is computed from Eq. 3.94. If Z(t) is the known quantity, a value of N is assumed for each time increment and the computations proceed with the value of q_1 calculated from Eq. 3.98. After both the forward and backward sweeps are made, a check of q must be made and the procedure repeated with a new value of N (and q_1) if necessary.

3.9.5. Forward Sweep Computations

Equation 3.93, the equation of motion, can be written as

$$D_1(Q_1{}^{t+1} - Q_1{}^t) + D_2q_2 + D_3z_1 + D_4z_2 + D_5 = 0$$

This equation can also be written in the form of Eq. 3.84,

$$z_1 = L_1z_2 + M_1q_2 + N_1 \qquad (3.99)$$

where the coefficients can be evaluated from Eqs. 3.85, 3.86 and 3.87 which are in terms of the Q's and Z's at time t and are known.

By the use of the continuity equation (Eq. 3.89) q_1 and z_1 are eliminated from Eq. 3.97, giving an equation in terms of z_2 and q_2 only (Eq. 3.90):

$$q_2 = E_2z_2 + F_2 \qquad (3.100)$$

The values of E_2 and F_2 are computed from Eqs. 3.91 and 3.92 which are in terms of the known quantities E_1, F_1, L_1, M_1, N_1, and the Q's and Z's at time t.

The process is now repeated for point 3, and E_3 and F_3 are computed. As for the first step, these quantities depend only on values computed at the previous point. Computation of the various E and F values then proceeds until the last point P is reached.

3.9.6. Downstream Boundary Conditions

At the downstream end of the channel either $Q_p(t)$ or $Z_p(t)$ are specific. In addition, from the forward sweep the coefficients E_p and F_p are now known. Because

$$q_p = E_p z_p + F_p \tag{3.101}$$

and if $Q_p(t)$ is known, then also

$$q_p = Q_p^{t+1} - Q_p^t \tag{3.102}$$

and z_p is computed from Eq. 3.101.

If $Z_p(t)$ is known, then

$$z_p = Z_p^t - Z_p^{t+1} \tag{3.103}$$

and q_p is computed from Eq. 3.101.

3.9.7. Backward Sweep

For the point $p - 1$, the value of z_{p-1} can be computed from Eq. 3.84 and the known values of L, M, and N:

$$z_{p-1} = L_{p-1} \; z_p + M_{p-1} \; q_p + N_{p-1} \tag{3.104}$$

Also, q_{p-1} can be computed from Eq. 3.84 using the known values of E and F:

$$q_{p-1} = E_{p-1} z_{p-1} + F_{p-1} \tag{3.105}$$

This allows us now to compute the values of Z and Q at time t + 1 from z and q at point p-1, because

$$Z_{p-1}^{t+1} = Z_{p-1}^t + z_{p-1} \tag{3.106}$$

and

$$Q_{p-1}^{t+1} = Q_{p-1}^t + q_{p-1} \tag{3.107}$$

By repeating this process for the point p − 2 and successive points, the Q's and Z's at the new time can be calculated. When the upstream boundary is reached the solution can then be advanced to the next time and the process repeated. If, however, $Z_1(t)$ is being used for the upstream boundary condition, it is at this point that the check is made to see if the correct value of N was used.

This computational procedure lends itself well to use with a microcomputer for several reasons:

1. Storage requirements are relatively small, only the coefficients for one time increment need to be retained.

2. The solution does not require repeated trials, except in the case when $Z_1(t)$ is specified for the upstream boundary conditions.

3. The solution procedure is inherently stable, and no limitations are imposed on the size of the Δt and Δx used in the problem.

3.10. PROGRAM 3.2. Unsteady Flow Analysis

The procedure described above is given in the form of a FORTRAN computer program as listed in the following section. The program is written to represent a single-reach trapezoidal channel and boundary conditions are given by specifying discharge as a function of time.

The initial condition is a steady-state water surface profile. This profile is computed from a specified depth at the downstream end of the reach. A separate subroutine is used to compute the steady state profile, using a procedure which is compatible with the unsteady flow calculations which follow.

Because the geometry and hydraulic parameters of the trapezoidal canal can be easily described, the input requirements for this program are minor. Five groups of data are required for each run: additional runs can be made by using repeated sets of data. The first line of input data provides alpha-numeric information for identifying the run, the second two lines give data for the initial condition, as well as unsteady flow parameters. The final two sets of data are used to specify the upstream and downstream boundary conditions.

3.10.1. Program Structure

This program consists of a main program and five subroutines. One subroutine (STEADY) computes a steady-state water surface profile to provide the initial condition for the unsteady flow computations. The unsteady flow computations are done in the subroutine named UNSTDY. Both of these routines use a subroutine named GEOM to calculate the flow section properties of top width, area, wetted perimeter, hydraulic radius, and hydraulic depth at each computational point. The upstream and downstream boundary values are computed at each computation time value in subroutines BCUP and BCDOWN respectively. A message is printed if the boundary condition data do not have all time values in increasing order. The initial section of the program deals with data input. The data are read from a file named CANAL.DAT. The output data are written to the printer.

3.10.2. PROGRAM 3.2. Application

This program can be used to evaluate unsteady flow conditions in a water conveyance canal. For example, irrigation canals usually have control gates at intervals along the canal to control the flow into and out of a canal reach. With this program the gates themselves are not modeled, but the flow into and out of the reach through the gates is specified.

Channels used for conveying flow in an irrigation system must respond to variations in the flow which result from changes in the deliveries from the canal. Hydraulic transients which occur when the flow is changed must be kept within specified limits to prevent damage to the canal which can result if the water level becomes too great or if the water level is drawn down too rapidly.

This program permits the flow changes to be simulated, and the effects of the change on the water levels can be evaluated. Various operation procedures can be evaluated with this computer model, and inappropriate procedures can be weeded out. By comparing various types of operations, the 'best' operation scheme can be selected for the actual canal operation.

In this example the channel is assumed to be bounded at its upstream and downstream ends by gates which permit the flow to be regulated in such a way that the boundary conditions can be specified by giving discharge as a function of time. If other types of boundary conditions exist, the subroutines in the program which deal with what is happening at the boundaries must be revised. The subroutines are quite simple, and such changes can be made easily.

EXAMPLE: This example run is given to illustrate how the program can be applied to the analysis of a canal experiencing a major flow change. The input data for this example are printed in Table 3.2. The program output is shown in Table 3.3.

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Henderson, F. M., Open Channel Flow, MacMillan, New York, 1966.

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Strelkoff, T., 'The One-Dimensional Equations of Open Channel Flow', Journal of the Hydraulics Division, ASCE, Vol. 95, No. HY3, May, 1969.

U. S. Corps of Engineers, "Hydraulic Design of Flood Control Channels", Engineering Manual No. EM 1110-02-1601, July, 1970.

TABLE 3.2. INPUT DATA FOR EXAMPLE FOR PROGRAM 3.2.

```
        CANAL.DAT
 1   DATA FOR IMPLICIT UNSTEADY FLOW COMPUTATION
 18000.  10.         2.         0.001       1200.       6.42       .014
       19 60.        900.       0.7         120.                   1
        2 0.         1200.      5.          1400.       100.       1400.
        2 0.         1200.      100.        1200.
-1      END OF DATA
```

Description of Data

Record No. 1:

NRUN (I2) = Run Number (negative to end run)
A (I) (13A4) = Alpha numeric data describing run

Record No. 2:

XL (F10.4) = Length of Canal (ft)
B (F10.4) = Bottom Width
ZZ (F10.4) = Canal Side Slope (ZZ horizontal to 1 vertical)
SO (F10.4) = Canal Bottom Slope
QIN (F10.4) = Initial (Steady State) Canal Discharge (cfs)
YGIVEN (F10.4) = Initial Depth at Downstream End (ft)
MANEN (F10.4) = Manning roughness coefficient

Record No. 3:

IP (I10) = Number of Cross Sections Used in Computations
(IP - 1 = No. of DX increments)
DT (F10.4) = Maximum Simulation Time (sec)
THETA (F10.4) = Weighting Factor Used in Implicit Solution Scheme, (0.5< THETA < 1.0, typically THETA = 0.6 to 0.7)
T1 (F10.4) = Print Control Parameter (sec) Detailed Results are printed for all times less than T1.
NDT (I10) = Control Parameter for Printing Output at a Multiple of Computational Increment DT

Record No. 4:

NUP (I10) = Number of pair of points for upstream boundary data
TU (I) (F10.2) = Time (sec.)
QUP(I) (F10.2) = Discharge (cfs) - up to 10 pairs may be used

Record No. 5:

NDOWN (I10) Number of pairs of points for downstream boundary data
TD (I) (F10.2) = Time (sec.)
QD (I) (F10.2) = Discharge (cfs) - up to 10 pairs of points may be used

TABLE 3.3.A. OUTPUT FOR EXAMPLE FOR PROGRAM 3.2. (Part A)

```
   UNSTEADY WATER SURFACE PROFILE -- TRAPEZOIDAL CHANNEL

     DATA FOR IMPLICIT UNSTEADY FLOW COMPUTATION

INPUT DATA--

   CHANNEL DIMENSIONS

                    LENGTH           = 18000.00  FT
                    BOTTOM WIDTH     =    10.00  FT
                    SIDE SLOPE       =     2.00
                    INVERT SLOPE     =      .0010000

   FLOW CONDITIONS

                    DISCHARGE        =  1200.00  CFS
                    DOWNSTREAM DEPTH=      6.42  FT
                    MANNINGS N       =      .0140

    RUN DATA

                    NO. OF STATIONS =      19
                    DX               =  1000.0   FT
                    TIME INCREMENT   =    60.00  SEC
                    MAX. TIME        =   900.00  SEC
                    T1               =   120.00  SEC
                    NDT              =       1

                    THETA            =      .70

                    (V+C)*DT/DX      =     1.181

   TIME=      0.0
```

I	X	Y	W S ELEV	E.G.L.	Q
1	0.00	6.42	24.42	25.46	1200.00
2	1000.00	6.42	23.42	24.46	1200.00
3	2000.00	6.42	22.42	23.46	1200.00
4	3000.00	6.42	21.42	22.46	1200.00
5	4000.00	6.42	20.42	21.46	1200.00
6	5000.00	6.42	19.42	20.46	1200.00
7	6000.00	6.42	18.42	19.46	1200.00
8	7000.00	6.42	17.42	18.46	1200.00
9	8000.00	6.42	16.42	17.46	1200.00
10	9000.00	6.42	15.42	16.46	1200.00
11	10000.00	6.42	14.42	15.46	1200.00
12	11000.00	6.42	13.42	14.46	1200.00
13	12000.00	6.42	12.42	13.46	1200.00
14	13000.00	6.42	11.42	12.46	1200.00
15	14000.00	6.42	10.42	11.46	1200.00
16	15000.00	6.42	9.42	10.46	1200.00
17	16000.00	6.42	8.42	9.46	1200.00
18	17000.00	6.42	7.42	8.46	1200.00
19	18000.00	6.42	6.42	7.46	1200.00

TABLE 3.3.B. OUTPUT FOR EXAMPLE FOR PROGRAM 3.2. (Part B)

TIME= 120.0

I	X	Y	W S ELEV	E.G.L.	Q
1	0.00	6.77	24.77	25.97	1400.00
2	1000.00	6.70	23.70	24.88	1364.20
3	2000.00	6.53	22.53	23.62	1262.42
4	3000.00	6.46	21.46	22.51	1218.34
5	4000.00	6.43	20.43	21.47	1203.75
6	5000.00	6.42	19.42	20.46	1199.50
7	6000.00	6.42	18.42	19.46	1198.24
8	7000.00	6.42	17.42	18.46	1198.02
9	8000.00	6.42	16.42	17.46	1197.82
10	9000.00	6.42	15.42	16.46	1197.96
11	10000.00	6.42	14.42	15.46	1197.75
12	11000.00	6.42	13.42	14.46	1198.00
13	12000.00	6.42	12.42	13.46	1197.70
14	13000.00	6.42	11.42	12.46	1198.03
15	14000.00	6.42	10.42	11.46	1197.74
16	15000.00	6.42	9.42	10.46	1197.88
17	16000.00	6.42	8.42	9.46	1198.14
18	17000.00	6.43	7.43	8.46	1196.98
19	18000.00	6.40	6.40	7.45	1200.00

	*** UPSTREAM ***		***DOWNSTREAM***	
TIME (MIN)	W.S. EL (FT)	Q (CFS)	W.S. EL (FT)	Q (CFS)
2.000	24.766	1400.00	6.396	1200.00
3.000	24.786	1400.00	6.385	1200.00
4.000	24.822	1400.00	6.375	1200.00
5.000	24.838	1400.00	6.364	1200.00
6.000	24.855	1400.00	6.354	1200.00
7.000	24.866	1400.00	6.344	1200.00
8.000	24.875	1400.00	6.335	1200.00
9.000	24.882	1400.00	6.326	1200.00
10.000	24.888	1400.00	6.322	1200.00
11.000	24.892	1400.00	6.330	1200.00
12.000	24.896	1400.00	6.363	1200.00
13.000	24.899	1400.00	6.436	1200.00
14.000	24.902	1400.00	6.553	1200.00
15.000	24.904	1400.00	6.702	1200.00
16.000	24.906	1400.00	6.868	1200.00

PROGRAM 3.2.

```
C
C*********************************************************************
C                                                                   *
C     PROGRAM   :    IMPLICIT                                       *
C     COMPUTATION OF UNSTEADY WATER SURFACE PROFILES FOR TRAPEZOIDAL *
C     OPEN CHANNELS    ---   SINGLE REACH                           *
C                                                    J. DE VRIES *
C*********************************************************************
C
C
C
      REAL MANEN, A1(13), A2(13)
C
      COMMON /U1/ B,XL,SO
      COMMON /U2/ MANEN,G,GAMMA,IP,THETA,DX,DT,TMAX,NDT,T1,ZZ
      COMMON /U3/ Q(51),Y(51),Z(51),ZO(51)
      COMMON /B1/ QU(10),TU(10),NUP
      COMMON /B2/ QD(10),TD(10),NDOWN
      CALL OPEN (5,'CANAL   DAT',1)
C
C ********************************************************************
C                                                                   *
C     LIST OF INPUT VARIABLES:                                      *
C                                                                   *
C     XL  = LENGTH OF CANAL (FT)      YGIVEN = CANAL DOWNSTEAM DEPTH *
C     B   = BOTTOM WIDTH (FT)         MANEN  = MANNINGS N           *
C     QIN = STEADY STATE FLOW (CFS)   IP-1   = NO. OF X INCREMENTS  *
C     ZZ  = CANAL SIDE SLOPE          SO     = INVERT SLOPE         *
C     NDT = NO. OF DT FOR PRINTOUT    T1     = ALL RESULTS BEFORE THIS *
C                                            TIME ARE PRINTED       *
C                                                                   *
C ********************************************************************
C
    1 READ(5,501) NRUN,(A1(I),I=1,13)
      IF (NRUN .LT. 0) GO TO 99
      WRITE(1,601) (A1(I),I=1,13)
      READ(5,502) XL,B,ZZ,SO,QIN,YGIVEN,MANEN
      READ(5,503) IP,DT,TMAX,THETA,T1,NDT
      READ(5,504) NUP, (TU(I), QU(I), I = 1, NUP)
      READ(5,504) NDOWN, (TD(I), QD(I), I = 1, NDOWN)
C
C *** PHYSICAL CONSTANTS**********************************************
C
      G     = 32.174
      GAMMA = 62.45
C
C ********************************************************************
C
C *** W.S. PROFILE IS COMPUTED FOR CONSTANT INCREMENTS OF X
C
      DX = XL/FLOAT(IP - 1)
      CALL GEOM(YGIVEN,T,A,R)
      ALPHA = (QIN/A + SQRT(G*A/T))*DT/DX
      WRITE(1,602) XL,B,ZZ,SO, QIN,YGIVEN,MANEN
      WRITE(1,604) IP,DX,DT,TMAX,T1,NDT
```

```
      WRITE(1,608) THETA
      WRITE(1,605) ALPHA
      VC=ALPHA*DX/DT
      WRITE(1,609) VC
      WRITE(1,606) (TU(I), QU(I), I = 1, NUP)
      WRITE(1,607) (TD(I), QD(I), I = 1, NDOWN)
C
      DO 2 I=1,IP
    2 Q(I)=QIN
C
C *** COMPUTE (INITIAL CONDITION) STEADY STATE WATER SURFACE PROFILE****
C
      CALL STEADY(YGIVEN,QIN)
C
C *** PERFORM UNSTEADY FLOW CALCULATIONS
C
      CALL UNSTDY
      GO TO 1
   99 WRITE(1,603) (A1(I), I=1,13)
C
C *** FORMATS FOR INPUT AND OUTPUT***************************************
C
  501 FORMAT(I2,20A4)
  502 FORMAT(7F10.4)
  503 FORMAT(I10, 4F10.4, I10)
  504 FORMAT(I10,(7F10.2)/8F10.2)
  601 FORMAT(1H1, 58H    UNSTEADY WATER SURFACE PROFILE -- TRAPEZOIDAL C
     1HANNEL    //  5X, 20A4////)
  602 FORMAT(1H0,12HINPUT DATA-- // 5X, 18HCHANNEL DIMENSIONS  //
     1 20X, 17HLENGTH           =,F9.2, 5H  FT   /
     2 20X, 17HBOTTOM WIDTH     =,F9.2, 5H  FT   /
     3 20X, 17HSIDE SLOPE       =,F9.2   /
     4 20X, 17HINVERT SLOPE     =,F14.7, //5X,15HFLOW CONDITIONS //
     5 20X, 17HDISCHARGE        =,F9.2, 5H  CFS  /
     6 20X, 17HDOWNSTREAM DEPTH=,F9.2, 5H  FT   /
     7 20X, 17HMANNINGS N       =,F11.4          /)
  603 FORMAT(1H1, 20A4)
  604 FORMAT(1H0, 5X, 8HRUN DATA  //
     1 20X,17HNO. OF STATIONS =,I7             /
     2 20X,17HDX              =,F8.1, 5H   FT  /
     3 20X,17HTIME INCREMENT  =,F9.2, 5H  SEC  /
     5 20X,17HMAX. TIME       =,F9.2, 5H  SEC  /
     6 20X,17HT1              =,F9.2, 5H  SEC  /
     7 20X,17HNDT             =,I7             /  )
  605 FORMAT(1H0, 19X, '(V+C)*DT/DX      =', F10.3 / )
  606 FORMAT(1H0, 'BOUNDARY CONDITIONS' // 5X,
     1'UPSTREAM END OF CHANNEL',/20X, 'TIME IN SEC      Q IN CFS',//
     2 9(15X, 2F14.3/))
  607 FORMAT(1H0,/ 5X, 'DOWNSTREAM END OF CHANNEL',/ 20X,
     1'TIME IN SEC      Q IN CFS',// 9(15X, 2F14.3/)//////////)
  608 FORMAT( 20X,17HTHETA            =,F9.2 /)
  609 FORMAT(1H0,19X,'V+C               =',F10.3,'  FT/SEC')
      END
```

```
      SUBROUTINE BCDOWN(TIME, QDOWN)
C
C***********************************************************************
C                                                                      *
C     DOWNSTREAM BOUNDARY CONDITIONS - VALUES OBTAINED BY LINEAR INTERP*
C                                                                      *
C***********************************************************************
C
      COMMON /B2/ QD(10), TD(10), NDOWN
      IF(TIME .GE. TD(NDOWN)) GO TO 2
      DO 1  I = 1, NDOWN
      IP1 = I + 1
      IF((TIME .GE. TD(I)) .AND. (TIME .LE. TD(IP1))) GO TO 4
    1 CONTINUE
      WRITE(1, 65) TIME
    2 QDOWN = QD(NDOWN)
      RETURN
    4 QDOWN = QD(I) + (TIME-TD(I))*(QD(IP1)-QD(I))/(TD(IP1)-TD(I))
      RETURN
   65 FORMAT(1H0, 'ERROR IN SPECIFICATION OF DOWNSTREAM BOUNDARY CONDITI
     1ON', / 10X 'TIME =', F8.1, // )
      END

      SUBROUTINE BCUP(TIME, QUP)
C
C***********************************************************************
C                                                                      *
C     UPSTREAM BOUNDARY CONDITIONS - VALUES OBTAINED BY LINEAR INTERP. *
C                                                                      *
C***********************************************************************
C
      COMMON /B1/ QU(10), TU(10), NUP
      IF(TIME .GE. TU(NUP)) GO TO 4
      DO 5 I = 1, NUP
      IP1 = I + 1
      IF((TIME .GE. TU(I)) .AND. (TIME .LE. TU(IP1))) GO TO 2
    5 CONTINUE
      WRITE(1, 61) TIME
    4 QUP = QU(NUP)
      RETURN
    2 QUP = QU(I) + (TIME - TU(I))*(QU(IP1)-QU(I))/(TU(IP1)-TU(I))
      RETURN
   61 FORMAT(1H0, 'ERROR IN SPECIFICATION OF UPSTREAM BOUNDARY CONDITION
     1 ', / 10X, 'TIME =', F8.1// )
      END
```

```
      SUBROUTINE GEOM(DEPTH, T, AREA, HYRAD)
C
C*********************************************************************
C                                                                    *
C     COMPUTATIONS OF HYDRAULIC DEPTH, HYD. RADIUS, FLOW  AREA       *
C     FROM FLOW DEPTH AND CHANNEL GEOMETRY (TRAPEZOIDAL CANAL)       *
C     B=BOTTOM WIDTH           ZZ=SIDE SLOPE                         *
C     T=TOP WIDTH              WETP=WETTED PERIMETER                 *
C     HYRAD= HYD. RADIUS       HYDEP=HYDRAULIC DEPTH                 *
C                                                                    *
C*********************************************************************
C
      REAL MANEN
      COMMON /U1/ B,XL,SO
      COMMON /U2/ MANEN,G,GAMMA,IP,THETA,DX,DT,TMAX,NDT,T1,ZZ
C     TOP WIDTH
      T=B+2.*ZZ*DEPTH
      AREA= DEPTH*(B+T)/2.
      HYDEP=AREA/T
      WETP=B+2.*DEPTH*SQRT( 1.+ZZ*ZZ)
      HYRAD=AREA/WETP
      RETURN
      END
      SUBROUTINE STEADY(YGIVEN, QIN)
C
C*********************************************************************
C                                                                    *
C     CALCULATION OF STEADY STATE W. S. PROFILE FROM INITIAL DISCHARGE *
C     AND GIVEN DOWNSTREAM DEPTH                                     *
C     PROFILE COMPUTED FOR CONSTANT INCREMENTS OF LENGTH (DX)        *
C                                                                    *
C*********************************************************************
      DIMENSION EGL(51),V(51),X(51)
      REAL MANEN
      COMMON/U1/B,XL,SO
      COMMON/U2/MANEN,G,GAMMA,IP,THETA,DX,DT,TMAX,NDT,T1,ZZ
      COMMON/U3/ Q(51),Y(51),Z(51),ZO(51)
      NUM=IP
      X(1)= 0.0
      DO 1 I=2,IP
      IM1 = I - 1
    1 X(I)=X(IM1) + DX
C
C   COMPUTE CONDITIONS AT DOWNSTREAM END
C
      CALL GEOM(YGIVEN,T,AREA,HYRAD)
      VIN=QIN/AREA
      HYDEP=AREA/T
      I=NUM
      V(I)=VIN
      Y(I)=YGIVEN
   11 ITRY=1
```

```
C
C     MAKE INITIAL ASSUMPTION OF DEPTH AT DX UPSTREAM = DOWNSTR DEPTH
C
      INO=I-1
      Y(INO)=Y(I)
    3 WHY=Y(INO)
      SF= MANEN**2*V(I)**2/(2.22*HYRAD**1.3333333)
      DYDXUP=(SO - SF)/(1.-QIN*QIN/(G*HYDEP*AREA*AREA))
      IF(ITRY .GT. 1)  GOTO 2
      DYDXD=DYDXUP
    2 DY=(DYDXUP+DYDXD)/2. *DX
      Y(INO)=Y(I)-DY
C
C     TEST FOR CONVERGENCE
C
      IF( ABS(Y(INO) -WHY) .LE. .0001)  GOTO  10
      ITRY=ITRY+1
      IF( ITRY .GT. 30)   GOTO 98
      CALL GEOM(Y(INO),T,AREA,HYRAD)
      HYDEP = AREA/T
      GOTO 3
   10 V(INO)=QIN/AREA
      I=INO
      IF ( I-1 )  12,12,11
   12 CONTINUE
C
C     COMPUTE ELEVATIONS AND PRINT RESULTS
C     INVERT AT DOWNSTREAM END = 0.0 FT.  (ZEND)
C
      ZEND=0.
      ZO(1)=ZEND + XL*SO
      DO 20 I=2,NUM
   20 ZO(I)=ZO(1) - SO*X(I)
C
C     COMPUTATION OF WATER SURFACE ELEVATION AND ENERGY GRADE LINE (EGL)
C
      DO 21 I=1,NUM
      Z(I)=Y(I)+ZO(I)
   21 EGL(I)=   Z(I)+V(I)*V(I)/(2.*G)
C
C     WRITE OUT STEADY STATE DEPTH AND ELEVATIONS
C
C     BYPASS OF PRINT OUT OF STEADY STATE PROFILE
C     GO TO 98
C
      WRITE(1,601)
  601 FORMAT(1H1, 2H I,7X,1HX,9X,1HY,4X, 8HW S ELEV,4X, 6HE.G.L.,7X, 1HV
     *   /)
      DO 22 I=1,NUM
   22 WRITE(1,602)I,X(I),Y(I),Z(I),EGL(I),V(I)
  602 FORMAT(1H0, I2, 5F10.3)
   98 CONTINUE
      RETURN
      END
C
```

```
C
C
      SUBROUTINE UNSTDY
C
C***********************************************************************
C                                                                      *
C     DETERMINATION OF UNSTEADY FLOW PROFILES BY THE IMPLICIT METHOD   *
C                WITH DOUBLE SWEEP (PREISSMANN METHOD)                 *
C                                                                      *
C***********************************************************************
C
      REAL QUE(51),ZEE(51),QNEW(51),ZNEW(51),A(51),E(51),F(51),R(51),
     1 T(51),DTDZ(51),K(51),DKDZ(51),EL(51),EM(51),
     2 EN(51),PHI
      INTEGER P
      DIMENSION ZU(901),ZD(901),TZ(901)
      COMMON /U1/ B,XL,SO
      COMMON /U2/ MANEN,G,GAMMA,P,THETA,DX,DT,TMAX,NDT,T1,ZZ
      COMMON /U3/ Q(51),Y(51),Z(51),ZO(51)
      EQUIVALENCE(YWIND,YW)
      WRITE(1,6)
    6 FORMAT(/// ' SOLUTION BY PREISSMANN SCHEME WITH DOUBLE SWEEP' //)
C  INITIALIZATION
      TIME=0.
      HR=IHR
      KT=0
      KZ=0
      ILINE=0
      CW=0.
      VW=0.
      RHOAIR=0.
  100 DO 101 I=1,P
      Y(I)=Z(I)-ZO(I)
      CALL GEOM(Y(I),T(I),A(I),R(I) )
      K(I)=(1.486/MANEN)*A(I)*(R(I)**0.6666667)
      DTDZ(I)=2.*ZZ
  101 DKDZ(I)=(K(I)/A(I))*(5.*T(I)-4.*R(I)*SQRT(1.+ZZ*ZZ) )/3.
C
C WRITE OUT RESULTS OF COMPUTATIONS
C
      IF (TIME .EQ.0) GO TO 110
      IF (KT .LT.NDT) GO TO 103
C
      KT=0
      KS=KS+1
C
C
      IF (TIME .GT. T1) GOTO 104
  110 KS=50
      WRITE(1,601) TIME
  601 FORMAT(1H0,5X,'TIME=',F8.1///'  I',  7X,  'X'
     1     ,9X,'Y     W S ELEV    E.G.L.',7X, 'Q'/)
      DO 302 I=1,P
      X=DX*(I-1)
      V=Q(I)/A(I)
```

```
      EGL=Z(I)+V*V/64.4
  302 WRITE(1,602) I,X,Y(I),Z(I),EGL,Q(I)
  602 FORMAT(1H ,I2,5F10.2)
  104 IF(KS .LT.50)GO TO 105
      WRITE(1,606)
  606 FORMAT(1H1 //13X,'*** UPSTREAM ***',4X,'***DOWNSTREAM***'/
     1 5X,' TIME     W.S. EL',5X,'Q',6X,'W.S. EL',6X,'Q'/
     2 5X,'(MIN)      (FT)       (CFS)     (FT)        (CFS)      '/)
      KS=0
      KS=0
  105 TM=TIME/60.
      WRITE(1,605) TM,Z(1),Q(1),Z(P),Q(P)
  605 FORMAT(1H ,F9.3,  2(F10.3   ,F10.2))
      KZ=KZ+1
      ZU(KZ)=Z(1)
      ZD(KZ)=Z(P)
      TZ(KZ)=TN
C     GO TO 103
C
C ILINE CONTROLS THE HEADER AND SPACES TO A NEW PAGE
C
      ILINE=ILINE+1
      IF (ILINE .LT.50)GO TO 155
      ILINE=0
  155 CONTINUE
  103 CONTINUE
C
      IF (TIME .GT.TMAX)GO TO 400
C
C UPSTREAM BOUNDARY CONDITIONS
C     Q SPECIFIED AS A FUNCTION OF TIME
      TNEW=TIME + DT
      CALL BCUP(TNEW,QUP)
C     WRITE (1,698) TIME, QUP
C 698 FORMAT(5X,'TIME = ',F9.3,5X,'QUP = ',F9.3)
      E(1)=0.
      QUE(1)=QUP-Q(1)
      F(1)=QUE(1)
C
C   FORWARD SWEEEP
C
      M=1
  200 N=M+1
      D1=-Q(M)*THETA/(G*A(M)*A(M))+DX*Q(M)*THETA/(K(M)*K(M))
     1 +DX/(2.*G*A(M)*DT)
      D2= Q(N)*THETA/(G*A(N)*A(N))+DX*Q(N)*THETA/
     1 (K(N)*K(N))+DX/(2.*G*A(N)*DT)
      D3=THETA*(-1.+Q(M)*Q(M)*T(M)/(G*A(M)**3.)-DX
     2 *Q(M)*ABS(Q(M))*DKDZ(M)/K(M)**3.)-DX*T(M)*Q(M)
     3 /(2.*G*A(M)*A(M)*DT)
     4 -DX*CW*RHOAIR*VW*ABS(VW)*THETA/(2.*GAMMA)*(DTDZ(M)/
     5 A(M)-(T(M)/A(M))**2.)
```

```
      D4=THETA*(1.-Q(N)*Q(N)*T(N)/(G*A(N)**3.)-DX*Q(N)*ABS(Q(N))
     1 *DKDZ(N)/K(N)**3.) - DX*T(N)*Q(N)/(2.*G*A(N)*A(N)*DT)
     2 -DX*CW*RHOAIR*VW*ABS(VW)*THETA/(2.*GAMMA)
     3 *(DTDZ(N)/A(N)-(T(N)/A(N))**2.)
      D5=Z(N)-Z(M)+((Q(N)/A(N))**2.-(Q(M)/A(M))**2.)/(2.*G)
     1 +(DX/2.)*(Q(M)*ABS(Q(M))/(K(M)**2.) +Q(N)*ABS(Q(N))
     2 /(K(N)**2.)) -DX*CW*RHOAIR*VW**ABS(VW)/(2.*GAMMA)
     3 *(T(N)/A(N)+T(M)/A(M))
C
      C1=-THETA
      C2=THETA
      C3=DX*T(M)/(2.*DT)
      C4=DX*T(N)/(2.*DT)
      C5=Q(N)-Q(M)
C
      SW1=DX*CW*RHOAIR*VW*ABS(VW)/(2.*GAMMA)*(T(N)/A(N)+T(M)/A(M))
      SF1=(DX/2.)*(Q(M)*ABS(Q(M))/K(M)**2. +Q(N)*ABS(Q(N))/K(N)**2.)
      GEE=-D1*E(M)-D3
      EL(M)=D4/GEE
      EM(M)=D2/GEE
      EN(M)=(D1*F(M) + D5)/GEE
C
      S=-C1*EM(M)*EM(M) - C2 - C3*EM(M)
      E(M+1)=(C1*EL(M)*E(M) + C3*EL(M) + C4)/S
      F(M+1)=(C1*(EN(M)*E(M) + F(M)) + C3*EN(M) + C5)/S
C
C  INCREMENT AND TEST TO SEE IF AT DOWNSTREAM END OF CHANNEL (M = P)
C
      M=M+1
      IF (M .LT. P) GO TO 200
C
C  DOWNSTREAM BOUNDARY CONDITIONS
C  Q SPECIFIED AS A FUNCTION OF TIME
C
      CALL BCDOWN(TNEW,QDOWN)
      QUE(P)=QDOWN-Q(P)
      ZEE(P)=(QUE(P) - F(P))/E(P)
C
C  BACKWARD SWEEP
C
      QNEW(P)=Q(P)+QUE(P)
      ZNEW(P)=Z(P)+ZEE(P)
      J=P-1
  300 ZEE(J)=EL(J)*ZEE(J+1) + EM(J)*QUE(J+1) + EN(J)
      QUE(J)=E(J)*ZEE(J) + F(J)
C
      ZNEW(J)=Z(J) + ZEE(J)
      QNEW(J)=Q(J) + QUE(J)
C
      J=J-1
      IF (J .GT. 0) GO TO 300
C
```

```
      DO 301 I=1,P
      Z(I)=ZNEW(I)
      Q(I)=QNEW(I)
  301 CONTINUE
      TIME=TIME+DT
      KT=KT+1
C
C  RETURN TO BEGINNING AND REPEAT PROCESS AT NEXT TIME
C
      GO TO 100
C
  400 CONTINUE
      TD=DT*NDT/60.
      WRITE(1,604)
  604 FORMAT(1H0, 'END OF RUN  --  IMPLICIT UNSTEADY OPEN CHANNEL FLOW')
      RETURN
      END
```

CHAPTER 4

MODELING GROUNDWATER FLOW

4.1. Introduction

Mathematical models of groundwater flow are now routinely used by hydrologists to solve groundwater problems. Models have been used for many years, but until the last decade, their application has been limited to those situations where analytical solutions were available, such as the discharge of a well in an infinite two-dimensional aquifer, which is the well-known Theis solution (Theis, 1935). However, the widespread availability of computers and the concomitant development of efficient numerical methods for the solution of partial differential equations has expanded the utilization of models to more general applications. The present computational capability is to solve almost any problem, including the solution of practical problems in saturated-unsaturated flow. The limitations are neither theoretical nor computational, but important limitations result from the availability of data to be used in model characterization and parameter identification.

The objective of this chapter is to provide the theoretical background and applied tools necessary for the modeling of simple problems of groundwater flow. This is accomplished by first developing the partial differential equation of groundwater flow in two dimensions. Then the finite-element method for the numerical solution of that equation is developed, including the application of the finite-element method to one-dimensional and two-dimensional sample problems. Next, a generalized FORTRAN algorithim is described for the solution of two-dimensional problems of groundwater resource appraisal. Finally, the algorithim is applied to the solution of a regional groundwater problem.

The scope of the chapter is somewhat limited. The subject of groundwater hydrology is well described in recent books, such as those by Freeze and Cherry (1979) and Bear (1979), and it will not be developed here. Also, several excellent texts are available on the numerical solution of partial differential equations, such as Pinder and Gray (1972), Lapidus and Pinder (1982), Zienkiewicz (1977), Botha and Pinder (1983), Huyakara and Pinder (1983). In general, the following is limited to the application of the finite-element method to the solution of two-dimensional problems. Concepts of groundwater hydrology are utilized to the extent necessary to achieve that purpose, and the broad subject of numerical methods is limited to the finite element method.

4.2. Equation of Groundwater Flow

4.2.1. Continuity Equation

The flow of groundwater can be described by a statement for the conservation of mass and a constitutive relation for groundwater flow. For a control volume as shown in Fig. 4.1, the conservation of mass requires that the mass inflow to less the mass outflow from the volume equals the rate of storage change within the control volume. Stated mathematically, this principle is given by the relation

$$I - O = \frac{\partial V}{\partial t} \tag{4.1}$$

where I is the inflow rate, O is the outflow rate, V is storage, and t is time. For an incompressible fluid, which will be assumed throughout this chapter, I, O, and V can be expressed in volumetric terms.

Considering again the control volume shown in Fig. 4.1, the inflow is

$$I = q_x \Delta y + q_y \Delta x + N \Delta x \Delta y \tag{4.2}$$

where q_x and q_y are discharge in the aquifer per unit width, N is recharge rate per unit area, and Δx and Δy are widths of the control volume. Outflow from the control volume is

$$O = \left(q_x + \Delta x \frac{\partial q_x}{\partial x}\right) \Delta y + \left(q_y + \Delta y \frac{\partial q_y}{\partial y}\right) \Delta x \tag{4.3}$$

The rate of storage change can be expressed in terms of the storage coefficient of an aquifer, which is the volume of water taken into storage with a unit increase in hydraulic head in the aquifer. By this approach,

$$\frac{\partial V}{\partial t} = S \frac{\partial h}{\partial t} \Delta x \Delta y \tag{4.4}$$

where S is the storage coefficient and h is the hydraulic head in the aquifer.

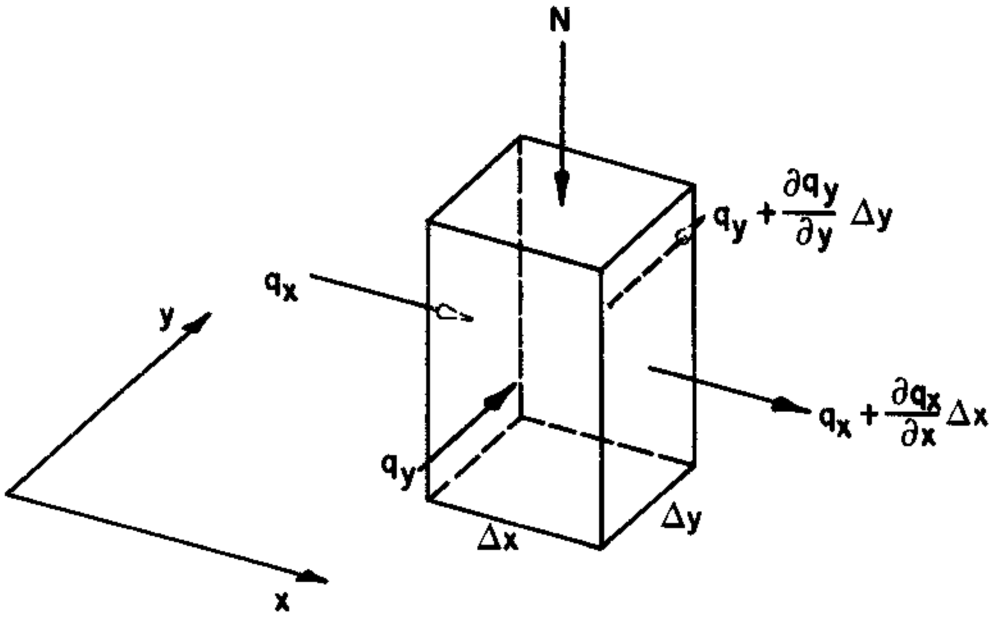


Fig. 4.1. *Central volume extending through entire thickness of an aquifer.*

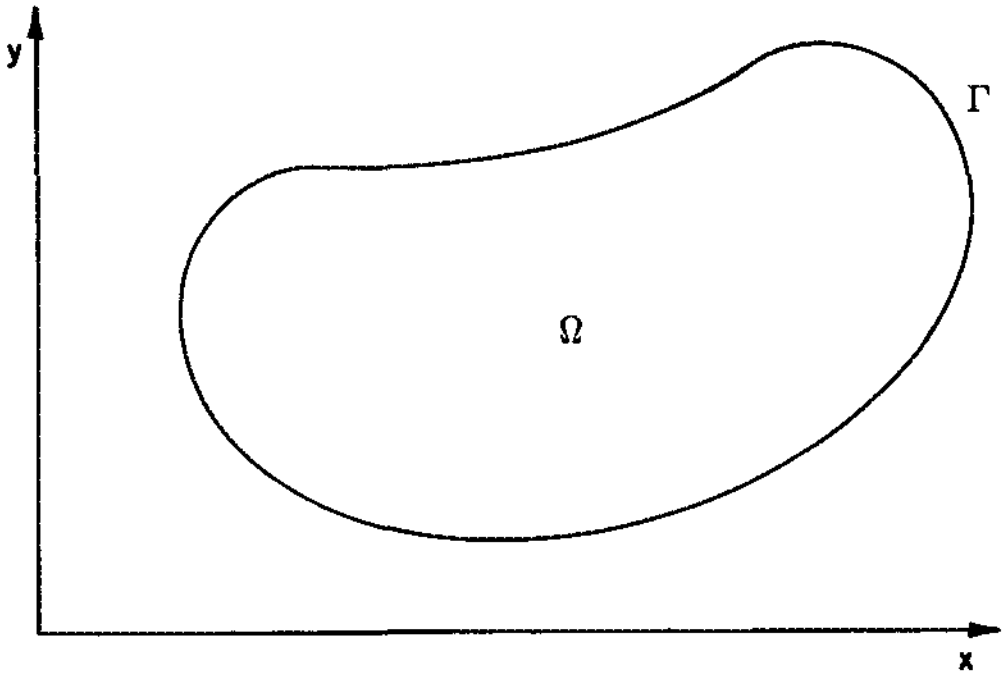


Fig. 4.2. *Flow domain for solution of groundwater problem.*

A partial-differential equation stating the conservation of mass for an aquifer can now be constructed. Equations 4.2- 4.4 can be substituted into Eq. 4.1 to obtain

$$q_x \, \Delta y + q_y \, \Delta x + N \, \Delta x \, \Delta y - q_x \, \Delta y - \frac{\partial q_x}{\partial x} \Delta x \, \Delta y - q_y \, \Delta x - \frac{\partial q_y}{\partial y} \Delta x \, \Delta y = S \frac{\partial h}{\partial t} \Delta x \, \Delta y \tag{4.5}$$

or by collecting terms,

$$- \frac{\partial q_x}{\partial x} \Delta x \, \Delta y - \frac{\partial q_y}{\partial y} \Delta x \, \Delta y + N \, \Delta x \, \Delta y = S \frac{\partial h}{\partial t} \Delta x \, \Delta y \tag{4.6}$$

Dividing through by $\Delta x \, \Delta y$ produces the equation

$$- \frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} + N = S \frac{\partial h}{\partial t} \tag{4.7}$$

which is the intended partial-differential equation.

However, Eq. 4.7 is expressed in terms of three dependent variables, and two additional equations relating these variables are needed. These relations are obtained from the constitutive equation of groundwater flow, which is Darcy's law (Bear, 1972). By Darcy's law,

$$q_x = - T \frac{\partial h}{\partial x} \tag{4.8a}$$

and

$$q_y = - T \frac{\partial h}{\partial y} \tag{4.8b}$$

where T is the transmittancy of the aquifer. Substitution of Eq. 4.8 into Eq. 4.7 yields the governing equation of groundwater flow

$$\frac{\partial}{\partial x} \left(T \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(T \frac{\partial h}{\partial y} \right) + N = S \frac{\partial h}{\partial t} \tag{4.9}$$

which is the transient-state form. The steady-state form of the equation of groundwater flow is obtained when $\partial h / \partial t = 0$, or

$$\frac{\partial}{\partial x} \left(T \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left(T \frac{\partial h}{\partial y} \right) + N = 0 \tag{4.10}$$

4.2.2. Mathematical Definition of the Groundwater Problem

Equations 4.9 and 4.10 describe the conservation of mass at a point in the groundwater system. To solve either of these partial-differential equations, additional specification of the groundwater problem is needed. In particular, the solution of the transient-state and steady-state forms depends on definition of the solution domain, aquifer parameters, sources and sinks, and boundary conditions. For the transient-state form, the initial conditions also need to be specified.

Solution Domain

The solution domain is the spacial domain of a steady-state problem or the space-time domain of a transient-state problem for which a solution of Eqs. 4.10 or 4.9 is to be obtained. The spacial, or flow, domain is the geographic area included in the groundwater model. Mathematically, this is an area Ω defined in the $x - y$ plain and surrounded by the boundary line Γ (Fig. 4.2). The boundary of the flow domain is usually selected to correspond with natural boundaries of the groundwater system. A boundary might be selected where a permeability contrast occurs or where groundwater levels are maintained at a known level by a lake or stream. In situations for which a natural boundary is not suitable, the boundary of the flow domain might be established at sufficient distance from a local area of interest such that conditions on the boundary do not effect the solution in the local area. The time domain is the period over which a transient-state solution is to be obtained.

Aquifer Parameters

Aquifer parameters are the transmissivity and storage coefficient distributions over the flow domain. These values are estimated in part from interpretation of aquifer tests, geologic logs for wells, and other geohydrologic data and information concerning the groundwater system. However, the available data seldom completely define the distributions of transmissivity and storage coefficient, and techniques of parameter identification (Beck and Arnold, 1977) are sometimes used to better define these distributions. Parameter identification involves the process of estimating values for the aquifer parameters such that the model reproduces by some measure the observed groundwater levels for some historical condition of the groundwater system. This is accomplished by trial and error methods or by mathematical procedures of parameter identification, such as a least-squares method of parameter identification.

Boundary and Initial Conditions

Boundary conditions are a mathematical statement of the groundwater condition at the boundary of the flow domain. In general, these conditions are of two types: specified-flux and specified-head boundaries. A specified-flux boundary is described by the relation

$$T \frac{\partial h}{\partial n} = q_b\ (\Gamma, t) \tag{4.11}$$

where $\partial/\partial n$ is the outward-pointing normal derivative to the boundary and q_b is the specified discharge into the flow domain per unit length of the boundary, and the boundary discharge can be a function of position and time. The specified-head boundary condition is given by the relation

$$h = h_b\ (\Gamma, t) \tag{4.12}$$

where h_b is the specified head, and the specified head on the boundary can be a function of position and time.

These boundary conditions occur naturally in groundwater systems. Specified-flux boundary conditions occur on a boundary that corresponds to a permeability contrast. A typical situation is the boundary between alluvial deposits and lower permeability consolidated rocks, where the flow domain includes only the alluvial deposits. Groundwater conditions in the alluvial deposits often will have little impact on groundwater movement in the consolidated rocks, and this situation can be characterized by the specified-flux boundary condtion. Specified-head boundary conditions occur on a boundary that corresponds to a river or lake. Here, exchanges of water between the aquifer and surface-water body will tend to maintain water levels in the aquifer at values similar to those observed for the surface-water body.

Initial conditions are the specification of heads over the flow domain at the start of the time domain. For each point in the flow domain, heads in the groundwater system are specified. These heads are sometimes obtained from maps showing contours of equal groundwater levels based upon water-level measurements. In other cases, initial heads are those computed from a steady-state simulation of the groundwater system. This is a useful approach when a model is being used to simulate both pre-development and post-development conditions. In other situations, initial conditions can be assigned everywhere zero. This approach is useful if the transient-state simulation is intended to compute only water-level changes.

The transient-state and steady-state forms of the groundwater flow equation respectively represent general classes of partial-differential equations referred to as parabolic or elliptic equations. Each class requires particular boundary and initial conditions. The transient-state equation requires that initial conditions be specified within the flow domain and that boundary conditions be specified on the entire boundary as either the specifed-flux or specified-head type. The steady-state equation requires only boundary conditions. They must be specified on the entire boundary, and the boundary condition locally may be one of the two types. However, if only specified-flux boundaries are utilized, the discharge across the boundary must balance the sources and sinks within the flow domain. This requirement can be expressed as

$$\int_{\Gamma} T \frac{\partial h}{\partial n} d\Gamma - \int_{\Omega} N \, d\Omega = 0 \tag{4.13}$$

which is satisfied automatically if some segment of Γ is a specified-head boundary.

Steady-state problems with only specifed-flux boundaries present additional difficulties. For this situation, a unique solution to Eq. 4.10 exists only if a head is specified at some point in the flow domain or on its boundary. Without this specification, the relative distribution of head will be obtained by a solution to Eq. 4.10, but the datum is undefined. If a numerical method is employed, the computed head obtained will depend on factors such as the initial guess of heads used in an iterative method.

Sources and Sinks

Sources and sinks are the specification of recharge and discharge for the groundwater system over the time domain to be simulated. While simple in concept, these values are difficult to obtain in practical applications. Recharge occurs by the typical mechanisms of streamflow infiltration, deep percolation of precipitation, return of applied irrigation water, and return of domestic or other waste water. Discharge occurs by the typical mechanisms of pumpage, groundwater discharge to streams or lakes, and water-use by phreatophytes. Adequate data are seldom available on these quantities. The groundwater modeler often must rely on very uncertain estimates. Consequently, recharge and discharge are often treated as structural parameters of the model to be estimated by a parameter-identification technique, given specification of the aquifer parameters.

4.3. Finite-Element Method

4.3.1. Introduction

The finite-element method for solving the partial-differential equation of groundwater flow involves several steps. First, a trial solution to the equation and its specified boundary and initial conditions is selected. The trial solution is formulated such that it contains unknown coefficients. Second, the trial solution is substituted into the original partial-differential equation, and the equation is transformed such that a system of algebraic equations in the unknown coefficients is produced. Third, that system of equations is solved for the unknown coefficients of the trial solution.

The fundamental idea of the method is to replace the exact continuous solution of the original partial-differential equation by an approximate piecewise-continuous solution. The piecewise-continuous function is specified by coefficients of the function that are specified at a finite number of discrete points called nodes. Functional values between these points are calculated using the piecewise-continuous interpolating functions defined over a finite number of subdomains called elements. The interpolating functions will provide an exact representation as the element size approaches zero in the limit. For a finite number of nodes, the approximation will not exactly satisfy the original partial-differential equation, and there will be a residual. The finite-element method forces this residual to zero, in an average sense, through selection of coefficients for the interpolating functions.

4.3.2. Galerkin Method of Weighted Residuals

Let the function h be the exact solution of the partial-differential equation

$$f(h) = 0 \tag{4.14}$$

where h represents the solution in the domain Ω, which is enclosed by the boundary $\Gamma = \Gamma_1 \cup \Gamma_2$. The boundary conditions are

$$T \frac{\partial h}{\partial n} = q_b(\Gamma_1) \text{ on } \Gamma_1 \tag{4.15a}$$

and

$$h = h_b(\Gamma_2) \text{ on } \Gamma_2 \tag{4.15b}$$

where $\partial/\partial n$ is the outward normal derivative on the boundary.

To solve f(h) = 0, we assume trial solutions of the form

$$h \simeq \hat{h} = \sum_{i=1}^{n} H_i \phi_i \tag{4.16}$$

where $\hat{h}$ is a series approximation to h, ϕ_i are linearly independent trial functions defined over the domain Ω, and H_i are unknown coefficients. The functions ϕ_i are known functions in the domain Ω . Therefore, the problem of finding a solution to the partial-differential equation $f(h) = 0$ is reduced to the problem of finding values for the coefficients H_i.

The series approximation to the Eq. 4.14 will provide an exact solution as η approaches infinity. For a finite series, however, the approximation will not exactly satisfy Eq. 4.14, and there will be a residual R. The residual is defined by

$$R = f(\hat{h}) - f(h) \qquad (4.17)$$

However, because $f(h) = 0$, R is then given by the expression

$$R = f(\hat{h}) \qquad (4.18)$$

If the trial solution $\hat{h}$ were the exact solution, the residual would vanish. We attempt to force this residual to zero, in an average over the domain Ω , through the selection of the unknown coefficients H_i. This might be accomplished with a relation such as

$$\int_{\Omega} R d\Omega = 0 \qquad (4.19)$$

However, this yields only one equation, and η equations are needed to obtain the η values of H_i. To obtain the η equations, the H_i are calculated by setting the weighted integrals of the residual to zero by an expression of the form

$$\int_{e} R w_i \, d\Omega = 0 \quad (i = 1,2,\cdots,n) \qquad (4.20)$$

where the w_i are linearly independent weighting functions.

Equation 4.20 is a general expression for the method of weighted residuals, which includes various specific methods that are distinguished by the use of different weighting functions. In the subdomain method, for example, the domain Ω is divided into subdomains Ω_i , and the residual is integrated over each subdomain and set to zero. By this method, the weighting function is given by

$$w_i(x) = \begin{cases} 1 & x \text{ in } \Omega_i \\ 0 & x \text{ out of } \Omega_i \end{cases} \qquad (4.21)$$

where x is a location in Ω . In the collocation method, a set of points, called collocation points, are established within the domain Ω , and the residual is evaluated at each collocation point. By this method, the weighting function is given by the Dirac delta function

$$w_i(x) = \delta(x - x_i) \tag{4.22}$$

where x_i is the location of a collocation point.

In the Galerkin method, the trial functions themselves are used as weighting functions, or

$$w_i(x) = \phi_i(x) \tag{4.23}$$

Introducing this relation into Eq. 4.20 yields

$$\int_\Omega R\phi_i \, d\Omega = 0 \quad (i = 1, 2, \cdots, n) \tag{4.24}$$

From Eq. 4.24, we obtain n equations, which can be solved for the n values of H_i.

4.3.3. Trial Functions in One Dimension

In order to carry out the integrations indicated by Eq. 4.24, the domain is divided into subregions Ω^e called elements, and the trial functions are defined piecewise over those elements. For example, Fig. 4.3a shows a one-dimensional domain, which is divided into four elements. Also shown are trial functions ϕ_i, which are defined by the generalized expression

$$\phi_i = \begin{cases} 0 & x < x_{i-1} \\ \dfrac{x - x_{i-1}}{x_i - x_{i-1}} & x_{i-1} \le x \le x_i \\ \dfrac{x_{i+1} - x}{x_{i+1} - x_i} & x_i \le x \le x_{i+1} \\ 0 & x_{i+1} < x \end{cases} \tag{4.25}$$

By these expressions, $\phi_i = 1$ at the node i, varies linearly from 1 to zero from nodes i to i − 1 and from nodes i to i + 1, and is zero elsewhere.

Using these linear trial functions, the function $\hat{h}$ is expressed as the linear combination of the trial functions. For example, in the internal $x_3 \le x \le x_4$, the trial solution is given by

$$\hat{h} = H_3 \phi_3 + H_4 \phi_4 \tag{4.26}$$

as shown in Fig. 4.3b. Equation 4.26 can be expanded to obtain

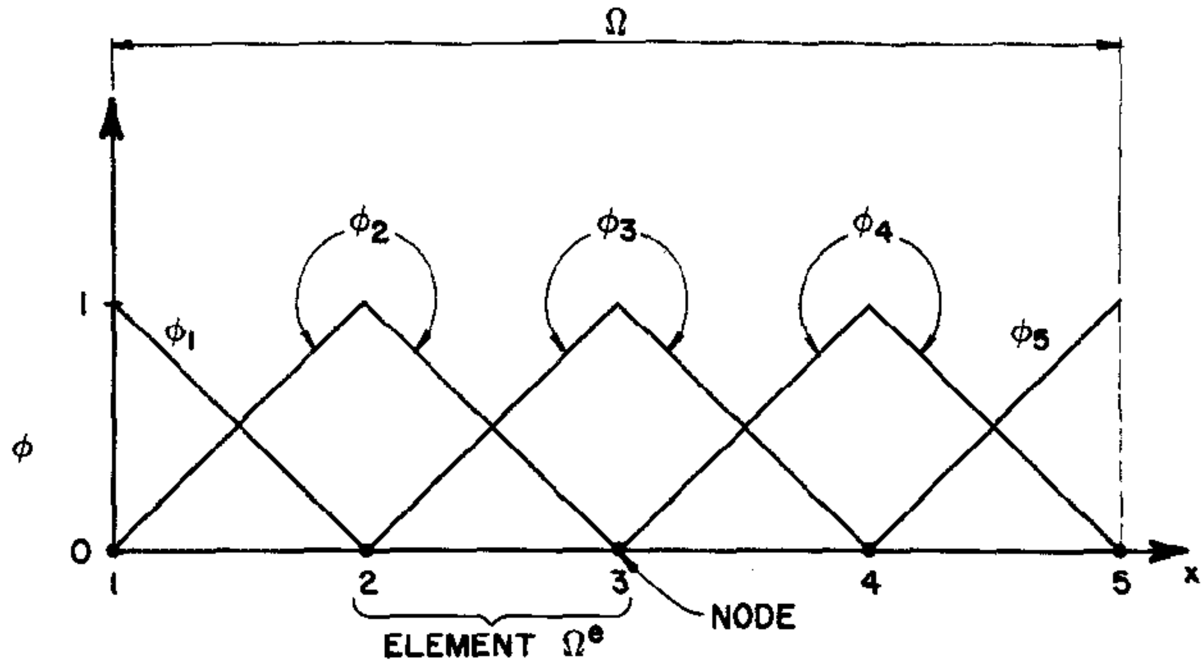


Fig. 4.3.(a). One-dimensional linear trial functions.

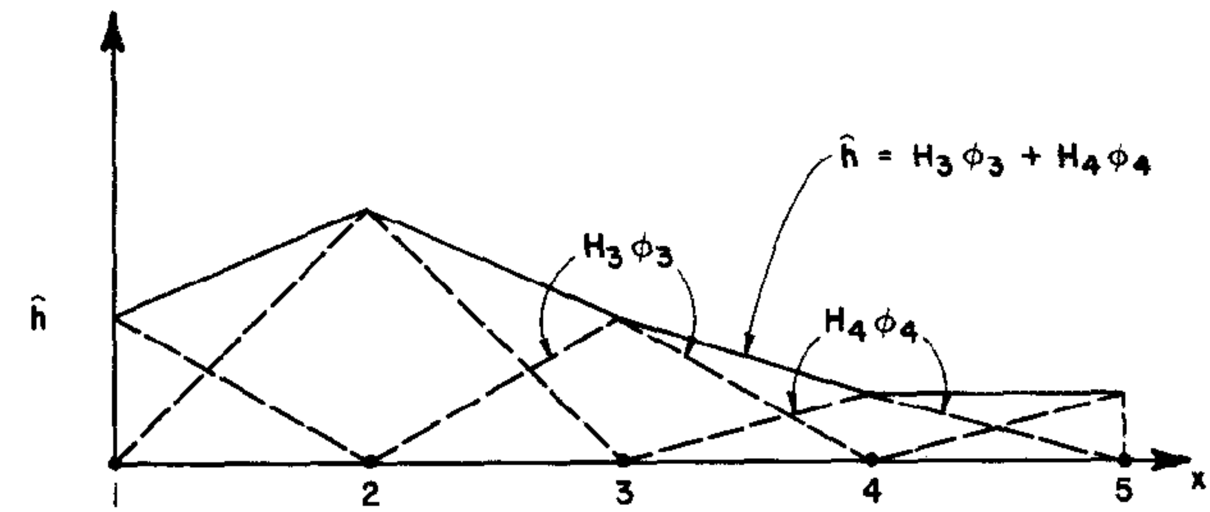


Fig. 4.3.(b). Combination of trial functions to form trial solution.

$$\hat{h}(x) = H_3 \frac{x_4 - x}{x_4 - x_3} + H_4 \frac{x - x_3}{x_4 - x_3} \qquad (4.27)$$

which is shown in Fig. 4.3b. Because the trial function ϕ_i has the value of unity at node i and is zero at all other nodes, the function $\hat{h}$ has the value H_i at node i.

4.3.4. Application to One-Dimensional Problem

To demonstrate the use of linear trial functions in the Galerkin method, the method is used to solve the one-dimensional equation of groundwater flow

$$T \frac{\partial^2 h}{\partial x^2} + N = 0 \qquad (4.28)$$

in the domain $0 \le x \le b$ for the boundary conditions

$$T \frac{\partial h}{\partial x} = q_b \text{ at } x = 0 \qquad (4.29a)$$

and

$$h = h_b \text{ at } x = b \qquad (4.29b)$$

This is the steady-state groundwater flow equation, where h is head, T is transmissivity, and N is recharge rate. The condition at the left boundary is the specified discharge q_b into the flow domain. The condition at the right boundary is a specified head h_b.

Galerkin Method

Equation 4.28 is to be solved in the domain $0 \le x \le 2$, which is divided into two elements by three nodes, as shown in Fig. 4.4. The first step in obtaining a solution is to define the trial solution

$$h \simeq \hat{h} = \sum_{i=1}^{3} H_i \phi_i \qquad (4.30)$$

The second step is to formulate the integral equations by substituting

$$R = T \frac{\partial^2 h}{\partial x^2} + N \qquad (4.31)$$

into Eq. 4.24 to obtain

$$\int_0^2 \left(T \frac{\partial^2 \hat{h}}{\partial x^2} + N \right) \phi_i \, dx = 0 \qquad (4.32)$$

$$(i = 1,2,3)$$

Equation 4.32 cannot be solved using linear trial functions because of the second-order term. The second derivative is zero. However, the order can be reduced by applying integration by parts. In its general form integration by parts is expressed as

$$\int_a^b u\,dv = uv\Big]_a^b - \int_a^b v\,du \qquad (4.33)$$

If $u = \phi_i$, $du = \frac{\partial \phi_i}{\partial x} dx$, $v = \frac{\partial^2 \hat{h}}{\partial x^2}$, and $dv = \frac{\partial^2 \hat{h}}{\partial x^2} dx$, then Eq. 4.33 becomes

$$\int_0^2 \frac{\partial^2 \hat{h}}{\partial x^2} \phi_i \, dx = \phi_i \frac{\partial \hat{h}}{\partial x}\Big]_0^2 - \int_0^2 \frac{\partial \phi_i}{\partial x} \frac{\partial \hat{h}}{\partial x} dx \qquad (4.34)$$

Substitution of Eq. 4.34 into Eq. 4.32 yields

$$\int_0^2 T \frac{\partial \phi_i}{\partial x} \frac{\partial \hat{h}}{\partial x} dx - T \phi_i \frac{\partial \hat{h}}{\partial x}\Big]_0^2 - \int_0^2 N \phi_i \, dx = 0 \qquad (4.35)$$

$$(i = 1, 2, 3)$$

Equation 4.35 can be expanded further by substitution of Eq. 4.30 for h in Eq. 4.35 to obtain

$$\sum_{j=1}^{3} \int_0^2 T \frac{\partial \phi_i}{\partial x} \frac{\partial \phi_j}{\partial x} H_j \, dx - T \phi_i \frac{\partial \hat{h}}{\partial x}\Big]_0^2 - \int_0^2 N \phi_i \, dx = 0 \qquad (4.36)$$

$$(i = 1, 2, 3)$$

which can be expressed in matrix form as

$$\begin{bmatrix} T\int_0^2 \frac{\partial \phi_1}{\partial x}\frac{\partial \phi_1}{\partial x} dx & T\int_0^2 \frac{\partial \phi_1}{\partial x}\frac{\partial \phi_2}{\partial x} dx & T\int_0^2 \frac{\partial \phi_1}{\partial x}\frac{\partial \phi_3}{\partial x} dx \\ T\int_0^2 \frac{\partial \phi_2}{\partial x}\frac{\partial \phi_1}{\partial x} dx & T\int_0^2 \frac{\partial \phi_2}{\partial x}\frac{\partial \phi_2}{\partial x} dx & T\int_0^2 \frac{\partial \phi_2}{\partial x}\frac{\partial \phi_3}{\partial x} dx \\ T\int_0^2 \frac{\partial \phi_3}{\partial x}\frac{\partial \phi_1}{\partial x} dx & T\int_0^2 \frac{\partial \phi_3}{\partial x}\frac{\partial \phi_2}{\partial x} dx & T\int_0^2 \frac{\partial \phi_3}{\partial x}\frac{\partial \phi_3}{\partial x} dx \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix}$$

$$= \begin{bmatrix} \displaystyle\int_0^2 N\,\phi_1\,dx + \left[T\,\phi_1\,\frac{\partial\hat{h}}{\partial x}\right]_0^2 \\ \displaystyle\int_0^2 N\,\phi_2\,dx + \left[T\,\phi_2\,\frac{\partial\hat{h}}{\partial x}\right]_0^2 \\ \displaystyle\int_0^2 N\,\phi_3\,dx + \left[T\,\phi_3\,\frac{\partial\hat{h}}{\partial x}\right]_0^2 \end{bmatrix} \tag{4.37}$$

Equation 4.37 can be expressed in matrix notation as

$$[A]\,\{H\} = \{F\} \tag{4.38}$$

where $[A]$ is a $[n\times n]$ matrix and $\{H\}$ and $\{F\}$ are $[n\times 1]$ vectors. Typical elements of $[A]$ and $\{F\}$ are

$$A_{ij} = T\int_0^2 \frac{\partial\phi_i}{\partial x}\,\frac{\partial\phi_j}{\partial x}\,dx \tag{4.39a}$$

and

$$F_i = \int_0^2 N\,\phi_i\,dx + \left[T\,\phi_i\,\frac{\partial\hat{h}}{\partial x}\right]_0^2 \tag{4.39b}$$

The matrix $[A]$ is referred to as the stiffness matrix. This is a term of structural engineering, where, in structural problems, the matrix $[A]$ expresses the stiffness of a structure. From a similar background, the vector $\{F\}$ is referred to as the load vector.

Assembly of Solution

Evaluation of the integrals appearing in Eq. 4.37 for $T = 1$, $N = 2$, $q_b = 0$, and $H_b = 1$ yields

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{4.40}$$

However, H_3 is known from the specified boundary conditions (Eq. 4.15b) and substitution of $H_3 = 1$ into Eq. 4.40 gives

$$\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad (4.41)$$

The solution of Eq. 4.40 then is

$$\{H\} = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix} \quad (4.42)$$

which are also values of $\hat{h}$ at the nodal points.

Comparison of the numerical solution to the analytical solution of this problem indicates close agreement. The analytical solution is

$$h = h_b + \frac{N}{2T}(L^2 - x^2) + \frac{q_b}{T}(L - x) \quad (4.43)$$

where L is length of the flow domain. Equation 4.43 gives values of $h(0) = 5$, $h(1) = 4$, and $h(2) = 1$. In this particular case, the finite-element method agrees exactly with the analytical solution.

4.3.5. Trial Functions in Two Dimensions

Triangular elements are commonly used to describe irregular two-dimensional solution domains as shown in Fig. 4.5a. While many other element geometries can be used, the triangular elements with straight sides can easily describe complex geometries. Additionally, if linear trial functions are utilized in conjunction with the triangular elements, integrations over the elements are evaluated by simple integration formula.

These linear trial functions are shown in Fig. 4.5b. The trial functions form a pyramid at each node with a polygonal base. The base is formed by the triangular elements that share the particular node. At the node i in Fig. 4.5b, the trial function has a value of unity. At all other nodes in the solution domain, the trial function has a value of zero. Within each of the triangular elements common to node i, the trial function varies linearly from unity at node i to zero at the other nodes.

Over a particular element, this trial function is described by a plane that passes through the three points $(x_i, y_i, 1)$ $(x_j, y_j, 0)$, and $(x_m, y_m, 0)$. The plain is described by the linear function

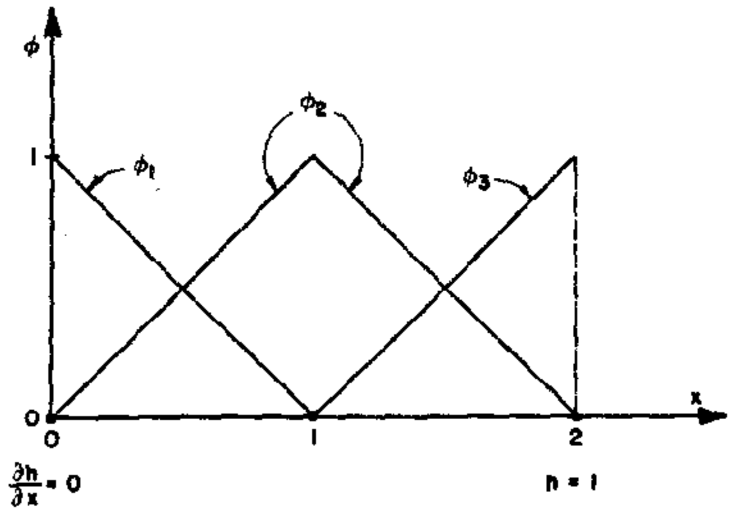


Fig. 4.4. *Flow domain for one-dimensional steady-state problem.*

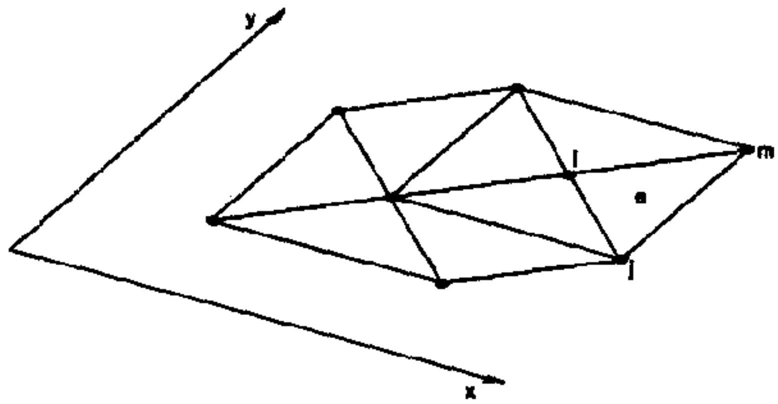


Fig. 4.5.(a). *Two-dimensional flow domain divided into triangular elements.*

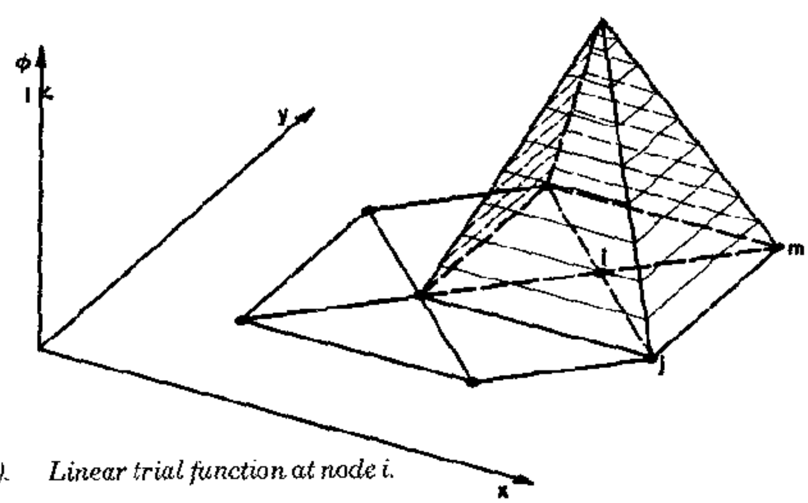


Fig. 4.5.(b). *Linear trial function at node i.*

$$\phi_i^e = a_i + b_i x + c_i y \qquad (4.44)$$

where a_i, b_i, and c_i are coefficients to be determined from knowledge of the nodal coordinates. The superscript e indicates that ϕ_i^e describes that part of ϕ_i associated with element e (Fig. 4.6). The complete trial function is formed by the union of all elemental trial functions associated with node i.

The three coefficients of Eq. 4.44 can be obtained by writing an equation for each node of element e. In particular, the equations

$$\begin{aligned} 1 &= a_i + b_i x_i + c_i y_i \\ 0 &= a_i + b_i x_j + c_i y_j \\ 0 &= a_i + b_i x_m + c_i y_m \end{aligned} \qquad (4.45)$$

can be written for the nodes i, j, and m, which in matrix notation can be expressed as

$$\begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_m & y_m \end{bmatrix} \begin{bmatrix} a_i \\ b_i \\ c_i \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad (4.46)$$

Equation 4.46 can solved to obtain the following expressions for the coefficients:

$$a_i = \frac{1}{2A^e}(x_j y_m - x_m y_j) \qquad (4.47a)$$

$$b_i = \frac{1}{2A^e}(y_j - y_m) \qquad (4.47b)$$

and

$$c_i = \frac{1}{2A^e}(x_m - x_j) \qquad (4.47c)$$

where

$$2A^e = (x_i y_j - x_j y_i) + (x_m y_i - x_i y_m) + (x_j y_m - x_m y_j) \qquad (4.48)$$

and A^e is the area of the triangular element.

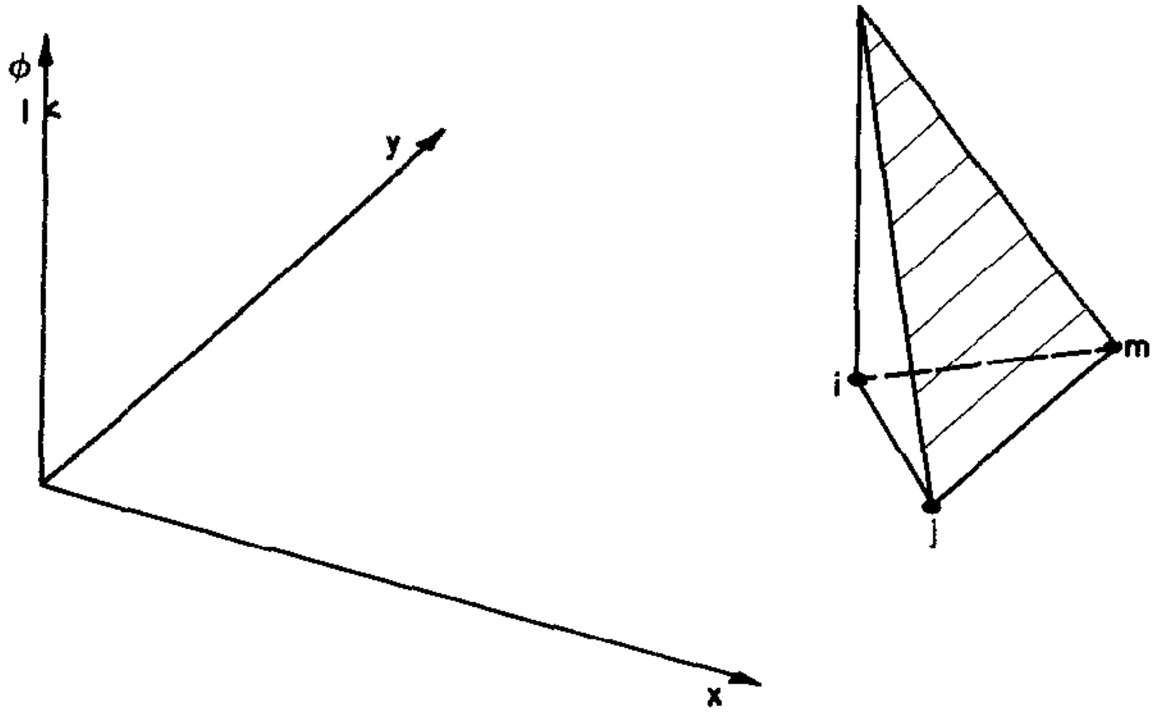


Fig. 4.6. Elemental trial function.

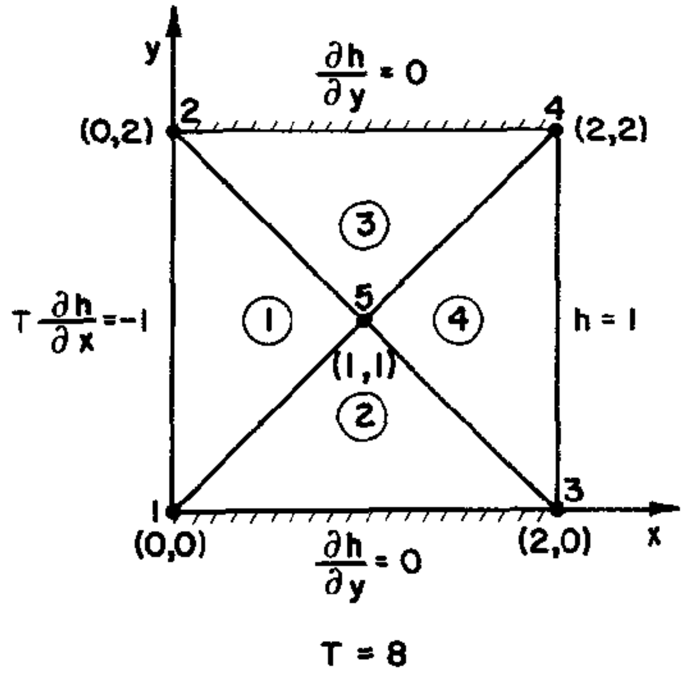


Fig. 4.7. Two-dimensional steady-state example problem.

Substitution of Eq. 4.47 into Eq. 4.44 defines the elemental trial function at the node i as

$$\phi_i^e = \frac{1}{2A^e}\left[(x_j y_m - x_m y_j) + (y_j - y_m)\, x + (x_m - x_j)\, y\right] \quad (4.49a)$$

Similarly, at the nodes j and m of element e, the trial functions are defined as

$$\phi_y^e = \frac{1}{2A^e}\left[(x_m y_i - x_i y_m) + (y_m - y_i)\, x + (x_i - y_m)\, y\right] \quad (4.49b)$$

and

$$\phi_m^e = \frac{1}{2A^e}\left[(x_i y_j - x_j y_i) + (y_i - y_j)\, x + (x_j - x_i)\, y\right] \quad (4.49c)$$

4.3.6. Application to Two-dimensional Problem

To demonstrate the use of triangular elements, the Galerkin method is used to solve the two-dimensional equation of groundwater flow:

$$\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial h}{\partial y}\right) + N = 0 \quad (4.50)$$

in the domain $0 \le x \le 2$ and $0 \le y \le 2$ for the boundary conditions

$$T\frac{\partial h}{\partial x} = -1 \text{ for } x = 0 \text{ and } 0 \le y \le 2 \quad (4.51a)$$

$$\frac{\partial h}{\partial n} = 0 \text{ for } \begin{cases} y = 2 \text{ and } 0 \le x \le 2 \\ y = 0 \text{ and } 0 \le x \le 2 \end{cases} \quad (4.51b)$$

and

$$h = 1 \text{ for } x = 2 \text{ and } 0 \le y \le 2 \quad (4.51c)$$

Galerkin Method

Equation 4.50 is to be solved in the flow domain indicated in Fig. 4.7, which shows the domain represented by four triangular elements. The first step in the solution is to define the trial solution

$$n \simeq \hat{h} = \sum_{i=1}^{5} H_i \phi_i \quad (4.52)$$

where ϕ_i is defined piecewise by Eq. 4.49. The second step is to formulate the integral equations by substituting

$$R = \frac{\partial}{\partial x}\left(T\frac{\partial \hat{h}}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial \hat{h}}{\partial y}\right) + N \tag{4.53}$$

into Eq. 4.24 to obtain

$$\int_0^2\int_0^2\left[\frac{\partial}{\partial x}\left(T\frac{\partial \hat{h}}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial \hat{h}}{\partial y}\right) + N\right]\phi_i dx\, dy = 0 \tag{4.54}$$

$$(i = 1, 2, 3, 4, 5)$$

As in the case of the one-dimensional problem, Eq. 4.54 cannot be solved using linear trial functions because of the second-order terms. However, the order can be reduced by applying integration by parts in the form

$$\iint_\Omega\left[\frac{\partial^2\hat{h}}{\partial x^2}\phi_i + \frac{\partial^2 h}{\partial y^2}\phi_i\right]dx\, dy =$$

$$-\iint_\Omega\left[\frac{\partial \hat{h}}{\partial x}\frac{\partial \phi_i}{\partial x} + \frac{\partial \hat{h}}{\partial y}\frac{\partial \phi_i}{\partial y}\right]dx\, dy$$

$$+\int_\Gamma\left[\frac{\partial \hat{h}}{\partial x}n_x + \frac{\partial \hat{h}}{\partial y}n_y\right]\phi_i\, d\Gamma\ , \tag{4.55}$$

where Γ is the boundary of the domain Ω and n_x and n_y are the x and y components of the unit outward vector normal to Γ. Substitution of Eq. 4.55 into Eq. 4.54 yields

$$\int_0^2\int_0^2\left[T\frac{\partial \phi_i}{\partial x}\frac{\partial \hat{h}}{\partial x} - T\frac{\partial \phi_i}{\partial y}\frac{\partial \hat{h}}{\partial y}\right]dx\, dy$$

$$-\int_0^2\int_0^2 N\,\phi_i\, dx\, dy - \int_\Gamma\left[T\frac{\partial h}{\partial x}n_x + T\frac{\partial \hat{h}}{\partial y}n_y\right]\phi_i\, d\Gamma = 0 \tag{4.56}$$

$$(i = 1, 2, 3, 4, 5)$$

Equation 4.56 can be expanded further by substitution of Eq. 4.52 for $\hat{h}$ in Eq. 4.56 to obtain

$$\sum_{j=1}^{5}\int_0^2\int_0^2\left[T\frac{\partial\phi_i}{\partial x}\frac{\partial\phi_j}{\partial x}+T\frac{\partial\phi_i}{\partial y}\frac{\partial\phi_j}{\partial y}\right]H_j\,dx\,dy$$

$$-\int_0^2\int_0^2 N\,\phi_i\,dx\,dy$$

$$-\int_\Gamma\left[T\frac{\partial\hat{h}}{\partial x}n_x+T\frac{\partial\hat{h}}{\partial y}n_y\right]\phi_i\,d\Gamma=0 \tag{4.57}$$

$$(i = 1, 2, 3, 4, 5)$$

which can be expressed in matrix form as

$$[A]\,\{H\} = \{F\} \tag{4.58}$$

The typical elements of [A] and {F} are

$$A_{ij}=\iint_\Omega\left[T\frac{\partial\phi_i}{\partial x}\frac{\partial\phi_j}{\partial x}+T\frac{\partial\phi_i}{\partial y}\frac{\partial\phi_j}{\partial y}\right]dx\,dy \tag{4.59a}$$

and

$$F_i=\iint_\Omega N\,\phi_i\,dx\,dy$$

$$+\int_\Gamma\left[T\frac{\partial\hat{h}}{\partial x}n_x+T\frac{\partial\hat{h}}{\partial y}n_y\right]\phi_i\,d\Gamma \tag{4.59b}$$

Assembly of Solution

In order to generate the set of algebraic equations represented by Eq. 4.58, it is necessary to perform integrations of the trial functions of the form

$$\iint_{\Omega} \frac{\partial \phi_i}{\partial x} \frac{\partial \phi_j}{\partial x} \, dx \, dy \,, \qquad \iint_{\Omega} \phi_i \, dx \, dy \,,$$

and

$$\int_{\Gamma} \phi_i \, d\Gamma \,.$$

To facilitate these integrations, they are performed on an elemental basis. Elemental matrices are generated, and the results are then transferred to the global matrix. Because there are three nodes in an element, each elemental matrix will be of order three.

Stiffness Matrix [A].— A typical elemental stiffness matrix $[A^e]$ will be of the form

$$[A^e] = T^e \iint_{\Omega} \begin{bmatrix} \frac{\partial \phi_1^e}{\partial x}\frac{\partial \phi_1^e}{\partial x} & \frac{\partial \phi_1^e}{\partial x}\frac{\partial \phi_2^e}{\partial x} & \frac{\partial \phi_1^e}{\partial x}\frac{\partial \phi_3^e}{\partial x} \\ \frac{\partial \phi_2^e}{\partial x}\frac{\partial \phi_1^e}{\partial x} & \frac{\partial \phi_2^e}{\partial x}\frac{\partial \phi_2^e}{\partial x} & \frac{\partial \phi_2^e}{\partial x}\frac{\partial \phi_3^e}{\partial x} \\ \frac{\partial \phi_3^e}{\partial x}\frac{\partial \phi_1^e}{\partial x} & \frac{\partial \phi_3^e}{\partial x}\frac{\partial \phi_2^e}{\partial x} & \frac{\partial \phi_3^e}{\partial x}\frac{\partial \phi_3^e}{\partial x} \end{bmatrix} dx \, dy$$

$$+ \, T^e \iint_{\Omega} \begin{bmatrix} \frac{\partial \phi_1^e}{\partial y}\frac{\partial \phi_1^e}{\partial y} & \frac{\partial \phi_1^e}{\partial y}\frac{\partial \phi_2^e}{\partial y} & \frac{\partial \phi_1^e}{\partial y}\frac{\partial \phi_3^e}{\partial y} \\ \frac{\partial \phi_2^e}{\partial y}\frac{\partial \phi_1^e}{\partial y} & \frac{\partial \phi_2^e}{\partial y}\frac{\partial \phi_2^e}{\partial y} & \frac{\partial \phi_2^e}{\partial y}\frac{\partial \phi_3^e}{\partial y} \\ \frac{\partial \phi_3^e}{\partial y}\frac{\partial \phi_1^e}{\partial y} & \frac{\partial \phi_3^e}{\partial y}\frac{\partial \phi_2^e}{\partial y} & \frac{\partial \phi_3^e}{\partial y}\frac{\partial \phi_3^e}{\partial y} \end{bmatrix} dx \, dy \qquad (4.60)$$

where T^e is the transmissivity within an element. The transmissivity is moved outside the integrand because it is assumed to be constant over an element. The indices used in the elemental stiffness matrix are local, and they pertain to nodes numbered counterclockwise around a triangular element.

The global stiffness matrix is obtained by summing, for given global node, the contribution to that node from each elemental stiffness matrix. For example, if nodes i and j in the local elemental nodal system correspond to nodes p and q in the global nodal system, the A^e_{ij} in the elemental stiffness matrix is added to A_{pq} in the global stiffness matrix. This operational procedure is repeated for each node in an element and for all elements in the domain Ω.

The integrations of Eq. 4.60, however, are performed in the global coordinate system. First, derivatives of the elemental trial functions are taken to obtain

$$\frac{\partial \phi_i^e}{\partial x} = \frac{\partial}{\partial x}(a_i + b_i\ c + c_i y) \qquad (4.61)$$

However,

$$\frac{\partial \phi_i^e}{\partial x} = b_i \qquad (4.62)$$

or

$$\frac{\partial \phi_i^e}{\partial x} = \frac{1}{2A^e}(y_j - y_m) \qquad (4.63)$$

Similarly,

$$\frac{\partial \phi_i^e}{\partial y} = \frac{1}{2A^e}(x_m - x_j) \qquad (4.64)$$

Second, Eqs. 4.63 and 4.64 are substituted into Eq. 4.59 to obtain

$$A_{ij}^e = T^e \iint_e b_i\ b_j\ dx\ dy + T^e \iint_e c_i\ c_j\ dx\ dy \qquad (4.65)$$

However, because the b_i's and c_i's are constant over an element,

$$A_{ij}^e = T^e\ b_i\ b_j \iint_e dx\ dy + T^e\ c_i\ c_j \iint_e dx\ dy \qquad (4.66)$$

or

$$A_{ij}^e = \frac{T^e}{4A^e}\left[(y_j - y_m)(y_m - y_i) + (x_m - x_j)(x_i - x_m)\right] \quad (4.67)$$

For the domain shown in Fig. 4.7, the elemental stiffness matrices are

$$[A^1] = \begin{bmatrix} 4 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 4 \end{bmatrix} \quad (4.68a)$$

$$[A^2] = \begin{bmatrix} 4 & 0 & -4 \\ 0 & 4 & -4 \\ -4 & -4 & 8 \end{bmatrix} \quad (4.68b)$$

$$[A^3] = \begin{bmatrix} 8 & -4 & -4 \\ -4 & 4 & 0 \\ -4 & 0 & 4 \end{bmatrix} \quad (4.68c)$$

and

$$[A^4] = \begin{bmatrix} 4 & 0 & -4 \\ 0 & 4 & -4 \\ -4 & -4 & 8 \end{bmatrix} \quad (4.68d)$$

Then, the global stiffness matrix is

$$[A] = \begin{bmatrix} 8 & 0 & 0 & 0 & -8 \\ 0 & 8 & 0 & 0 & -8 \\ 0 & 0 & 8 & 0 & -8 \\ 0 & 0 & 0 & 8 & -8 \\ -8 & -8 & -8 & -8 & 32 \end{bmatrix} \quad (4.69)$$

Load vector {F}. -- The load vector arises because of sources, sinks, and specified-flux boundary conditions. The term N in Eq. 4.50 can represent distributed sources or sinks such as areal recharge from precipitation, but it can also represent point sources or sinks such as discharge from a well. In this latter case, N is described by the relation

$$N(x,y) = - \sum_k Q_k \; \delta(x - x_k,\; y - y_k) \tag{4.70}$$

where Q_k is the discharge from a well, x_k and y_k are coordinates of the well location, and δ is the Dirac delta function. The minus sign in Eq. 4.70 arises because Q_k is defined as an extraction from the aquifer.

The Dirac delta function has special properties that facilitate integration of the source-sink term. First, the function has a value of unity at the point (x_k, y_k) and has a value of zero elsewhere. Mathematically, this is expressed as

$$\delta(x - z_k,\; y - y_k) = \begin{cases} 1 \text{ at } (x_k,\ y_k) \\ 0 \text{ elsewhere} \end{cases} \tag{4.71}$$

The Dirac delta function has the additional property that

$$\iint_\Omega \delta(x - x_k,\; y - y_k)\; dx\; dy = 1 \tag{4.72}$$

where the point (x_k, y_k) is located within the domain Ω.

Because of these properties of the Dirac delta function, the integral

$$\iint_\Omega N \,\phi_i \; dx \; dy$$

in Eq. 4.59b is equal to Q_k, if x_k and y_k are the coordinates of a node. Then, for sources and sinks, the global force vector is assembled by simply adding Q_k to F_i, where Q_k is located at the node i.

The specified-flux boundary condition appears in the load vector as a result of integration by parts to reduce the order of terms in the original partial-differential equation. This is the term

$$\int_\Gamma \left(T \frac{\partial \hat{h}}{\partial x} n_x + T \frac{\partial \hat{h}}{\partial y} n_y \right) \phi_i \; d\Gamma$$

that appears in Eq. 4.59b. This term can be restated as

$$\int_{\Gamma} T \frac{\partial \hat{h}}{\partial n} \phi_i \, d\Gamma$$

where $\partial/\partial n$ is the outward-pointing normal derivative on the boundary Γ. However, by Darcy's law,

$$T \frac{\partial \hat{h}}{\partial n} = q_b \tag{4.73}$$

where q_b is the inward groundwater discharge into the flow domain normal to the boundary (Fig. 4.8). Consequently,

$$\int_{\Gamma} \left(T \frac{\partial \hat{h}}{\partial x} n_x + T \frac{\partial \hat{h}}{\partial y} n_y \right) \phi_i \, d\Gamma = \int_{\Gamma} q_b \, \phi_i \, d\Gamma \tag{4.74}$$

The evaluation of the term

$$\int_{\Gamma} q_b \, \phi_i \, d\Gamma$$

is easily performed on an elemental basis. Elemental load vectors are generated, and the results are then transferred to the global load vector. Because two nodes define a boundary segment, each elemental vector will be of order two. The typical elemental load vector $\{F^e\}$ will be of the form

$$\{F^e\} = q_b \int_0^L \begin{bmatrix} \phi_1^e \\ \phi_2^e \end{bmatrix} d\Gamma \tag{4.75}$$

where L is the length of the boundary segment. The boundary discharge of Eq. 4.75 is moved outside the integrand because it is assumed constant over the boundary segment.

In the integrations of Eq. 4.75, the trial functions are evaluated along the boundary segment. Along the boundary, $\phi_e^e(x,y)$ is represented by the one-dimensional function $\phi_i^e(\Gamma)$ This function (Fig. 4.8) has a value of unity at node i and varies linearly to zero at node j. Similarly, $\phi_j^e(\Gamma)$ has a value of unity at node j and zero at node i. Consequently,

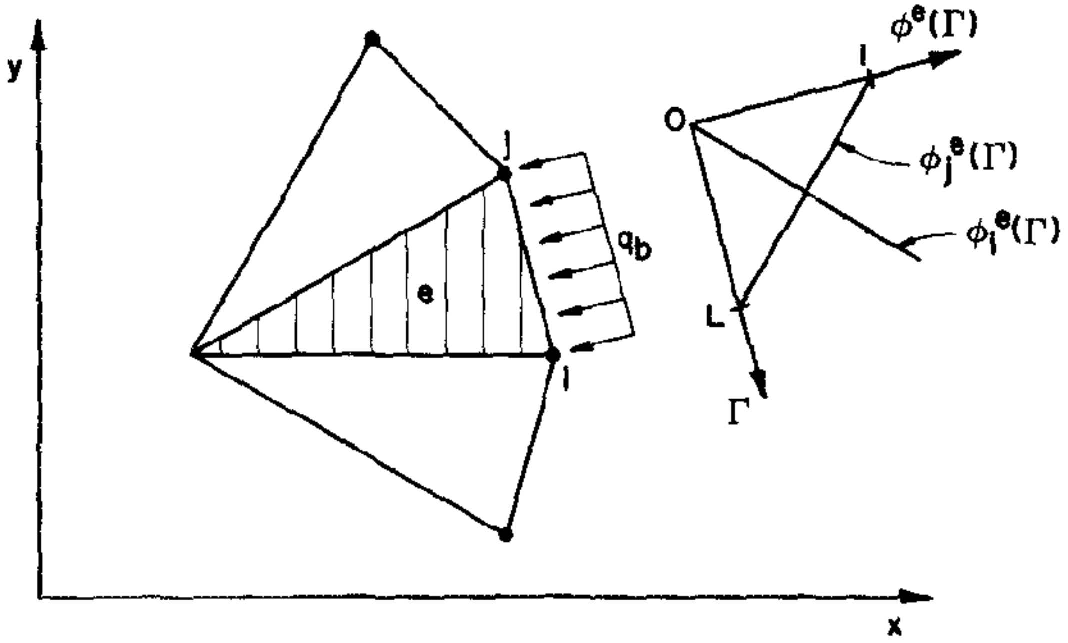


Fig. 4.8. *Definition of boundary-flux form.*

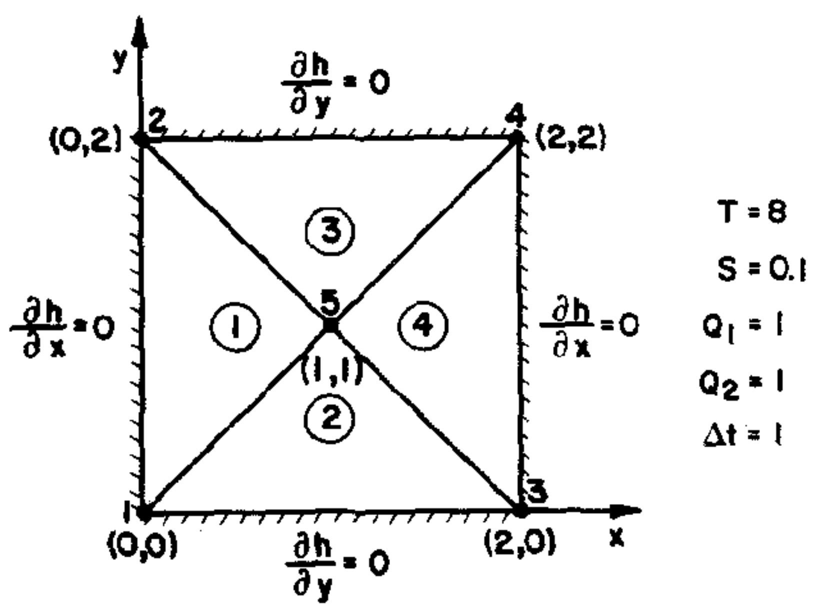


Fig. 4.9. *Two-dimensional transient-state example problem.*

$$\int_0^L \phi_i^e \, d\Gamma = \frac{L}{2} \tag{4.76}$$

and

$$\{F^e\} = \begin{bmatrix} q_b \frac{L}{2} \\ \\ q_b \frac{L}{2} \end{bmatrix} \tag{4.77}$$

The quantity q_b L/2 is one-half the discharge across the boundary segment.

For the specified-flux boundary condition, the global force vector is obtained by summing, for a given global node, the contributions from each elemental load vector. For example, if the node i in the elemental nodal system corresponds to node p in the global nodal system, the boundary discharge contribution to the global load vector is obtained by adding F_i^e in the element load vector to F_p.

For the problem shown in Fig. 4.7, the global load vector is

$$\{F\} = \{Q\} + \int_\Gamma \{q_b\} \, d\Gamma \tag{4.78}$$

or

$$\{F\} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.79}$$

Specified-Head Boundary

The specified-head boundary condition is imposed by substituting the known values of head into Eq. 4.57. For the problem defined by Eqs. 4.50 and 4.51 and shown in Fig. 4.7, Eqs. 4.70 and 4.79 can be substituted into Eq. 4.58 to obtain:

$$\begin{bmatrix} 8 & 0 & 0 & 0 & -8 \\ 0 & 8 & 0 & 0 & -8 \\ 0 & 0 & 8 & 0 & -8 \\ 0 & 0 & 0 & 8 & -8 \\ -8 & -8 & -8 & -8 & 32 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.80}$$

However, H_3 and H_4 are known from the specified-head boundary condition (Eq. 4.51b), and substitution of $H_3 = 1$ and $H_4 = 1$ into Eq. 4.80 gives

$$\begin{bmatrix} 8 & 0 & -8 \\ 0 & 8 & -8 \\ -8 & -8 & 32 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 16 \end{bmatrix} \quad (4.81)$$

Then, the solution of Eq. 4.80 for the boundary condition is

$$\{H\} = \begin{bmatrix} 1.25 \\ 1.25 \\ 1 \\ 1 \\ 1.125 \end{bmatrix} \quad (4.82)$$

which also are the values of $\hat{h}$ at the nodal points.

4.3.7. Transient-State Problem in Two Dimensions

The transient-state two-dimensional form of the equation of groundwater flow is

$$\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial h}{\partial y}\right) + N - S\frac{\partial h}{\partial t} = 0 \quad (4.83)$$

where h is hydraulic head, T is transmissivity, S is storage coefficient, x and y are Cartesian coordinates, and t is time. This equation can be solved by the Galerkin method using finite elements in space and time. In that case, the trial solution can be of the form

$$\hat{h}\ (x,y,t) = \sum_{i=1}^{n} H_i \phi_i(x,y,t) \quad (4.84)$$

Alternatively, time dependence can be introduced through the coefficients of the series approximation. In that case, the trial solution is of the form

$$\hat{h}\ (x,y,t) = \sum_{i=1}^{n} H_i(t)\ \phi_i(x,y) \quad (4.85)$$

In the latter case, application of the Galerkin method in space produces a system of ordinary differential equations, which can be solved in time by finite-difference methods. This approach is developed in the following.

Galerkin Method

As in the steady-state problem, the first step is to formulate the integral equations by application of the Galerkin method to produce

$$\iint_{\Omega} \left[\frac{\partial}{\partial x}\left(T \frac{\partial \hat{h}}{\partial x}\right) + \frac{\partial}{\partial y}\left(T \frac{\partial \hat{h}}{\partial y}\right) + N - S \frac{\partial \hat{h}}{\partial t} \right] \phi_i \, dx\, dy = 0 \qquad (4.86)$$

$$(i = 1, 2, \cdots, n)$$

where Ω is the solution domain and n is the number of nodes. The second step is to apply integration by parts to eliminate the second-order term. This yields the equation

$$\iint_{\Omega} \left[T \frac{\partial \phi_i}{\partial x} \frac{\partial \hat{h}}{\partial x} + T \frac{\partial \phi_i}{\partial y} \frac{\partial \hat{h}}{\partial y} \right] dx\, dy - \iint_{\Omega} N \phi_i \, dx\, dy - \int_{\Gamma} T \frac{\partial \hat{h}}{\partial n} \phi_i \, d\Gamma + \iint_{\Omega} S \frac{\partial \hat{h}}{\partial t} \phi_i \, dx\, dy = 0 \qquad (i = 1, 2, \cdots, n) \qquad (4.87)$$

where Γ is the boundary of the solution domain.

Equation 4.87 can be expanded by utilization of the relations

$$\frac{\partial \hat{h}}{\partial x} = \sum_{j=1}^{n} H_j(t) \frac{\partial}{\partial x} \phi_j(x,y) \tag{4.88a}$$

$$\frac{\partial \hat{h}}{\partial y} = \sum_{j=1}^{n} H_j(t) \frac{\partial}{\partial y} \phi_j(x,y) \tag{4.88b}$$

and

$$\frac{\partial \hat{h}}{\partial t} = \sum_{j=1}^{n} \frac{dH_j}{dt}(t)\ \phi_j(x,y) \tag{4.88c}$$

where the ordinary differential dH_j/dt and not the partial differential occurs because H_j is a function only of t. Substitution of these relations into Eq. 4.87 yields

$$\sum_{j=1}^{n} \iint_{\Omega} \left(T \frac{\partial \phi_i}{\partial x} \frac{\partial \phi_j}{\partial x} + T \frac{\partial \phi_i}{\partial y} \frac{\partial \phi_j}{\partial y} \right) H_j \, dx\, dy - \iint_{\Omega} N \phi_i \, dx\, dy - \int_{\Gamma} T \frac{\partial \hat{h}}{\partial n} \phi_i \, d\Gamma + \sum_{j=1}^{n} \iint_{\Omega} S \frac{dH_j}{dt} \phi_i \phi_j \, dx\, dy = 0 \tag{4.89}$$

$$(i = 1, 2, \cdots, n)$$

Equation 4.89 can be expressed in matrix notation as

$$[A]\ \{H\} + [B] \left\{ \frac{dh}{dt} \right\} = \{F\} \tag{4.90}$$

where [A] and [B] are [n × n] dimensional matrices and $\left\{ \frac{dH}{dt} \right\}$ $\{H\}$ and $\{F\}$ are [n × 1] dimensional vectors. Using terminology from structural engineering, [A] is the stiffness matrix, [B] is the mass matrix, and $\{F\}$ is the load vector. The term mass matrix is used because in structural engineering the matrix [B] arise in vibrational problems, and represents in part this mass of the structure.

Typical elements of [A], [B], and $\{F\}$ are

$$A_{ij} = \iint_{\Omega} \left(T \frac{\partial \phi_i}{\partial x} \frac{\partial \phi_j}{\partial x} + T \frac{\partial \phi_i}{\partial y} \frac{\partial \phi_j}{\partial y} \right) dx\, dy \tag{4.91a}$$

$$B_{ij} = \iint_{\Omega} S\, \phi_i\, \phi_j\, dx\, dy \tag{4.91b}$$

and

$$F_i = \iint_{\Omega} N\, \phi_i\, dx\, dy + \int_{\Gamma} T \frac{\partial \hat{h}}{\partial n} \phi_i\, d\Gamma \tag{4.91c}$$

Assembly of Solution

Mass matrix [B].— The integrations of Eq. 4.91 are performed on *an elemental basis.* Elemental stiffness and mass matrices are generated, and the results are then transferred to the global matrices. For triangular elements, Eq. 4.67 gives the integration formula for the elemental stiffness matrices. Likewise, Eqs. 4.78 and 4.79 give the integration formula for the load vector. For the elemental mass matrices, integrations of the form

$$\iint_{e} S\, \phi_i^e\, \phi_j^e\, dx\, dy$$

are required, where e indicates integration over an element.

A typical elemental mass matrix $[B^e]$ will be of the form

$$[B^e] = S^e \iint \begin{bmatrix} \phi_1^e\phi_1^e & \phi_1^e\phi_2^e & \phi_1^e\phi_3^e \\ \phi_2^e\phi_1^e & \phi_2^e\phi_2^e & \phi_2^e\phi_3^e \\ \phi_3^e\phi_1^e & \phi_3^e\phi_2^e & \phi_3^e\phi_3^e \end{bmatrix} dx\, dy \tag{4.92}$$

where S^e *is the storage coefficient* within an element. The storage coefficient can be moved outside the integrand if it is assumed constant over an element.

The integrations in Eq. 4.93 can be performed by the formula (Zienkiewicz, 1977)

$$\iint_{e} \phi_i^e\, \phi_j^e\, dx\, dy = \frac{A^e}{12} \text{ for } i \neq j \tag{4.93a}$$

and

$$\iint_{e} \phi_i^e\, \phi_i^e\, dx\, dy = \frac{A^e}{6} \tag{4.93b}$$

Therefore, Eq. 4.92 can be restated as

$$[B^e] = S^e A^e \begin{bmatrix} \frac{1}{6} & \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{6} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{6} \end{bmatrix} \tag{4.94}$$

Finite-Differences in Time.--Although the matrices [A] and [B] and the vector {F} of Eq. 4.90 can now be evaluated, we still must solve the system of ordinary-differential equations

$$[A]\ \{H\} + [B]\left\{\frac{dH}{dt}\right\} = \{F\} \tag{4.95}$$

To do this, the time derivative can be approximated using the finite-difference approximation

$$\left\{\frac{dH}{dt}\right\} = \frac{1}{\Delta t}\left\{ H^{t+\Delta t} - H^t \right\} \tag{4.96}$$

Then, substitution of Eq. 4.96 into Eq. 4.95 yields the first-order correct, implicit finite-difference scheme

$$[A]\left\{ H^{t+\Delta t} \right\} + \frac{1}{\Delta t}[B]\left\{ H^{t+\Delta t} - H^t \right\} = \{F\} \tag{4.97}$$

which can be rearranged to obtain

$$\left([A] + \frac{1}{\Delta t}[B]\right)\left\{H^{t+\Delta t}\right\} = \frac{1}{\Delta t}[B]\left\{H^t\right\} + \{F\} \tag{4.98}$$

However, Eq. 4.98 can be restated as

$$[L]\left\{H^{t+\Delta t}\right\} = \{R\} \tag{4.99}$$

where

$$[L] = [A] + \frac{1}{\Delta t}[B] \tag{4.100}$$

and

$$[R] = \frac{1}{\Delta t} [B] \left\{H^t\right\} + \{F\} \tag{4.101}$$

A solution to the original partial-differential equation can be obtained by marching the solution through time. At any time t, [A], [B], and $\{F\}$ are evaluated. Using head at time t, $1/\Delta t[B]\{H^t\}$ is evaluated, and Eq. 4.99 is solved for $\{H^{t+\Delta t}\}$. The time index is updated by Δt, and the process is repeated until the required time domain is simulated. At the start of the simulation, $\{H^t\}$ is the initial condition $\{H^o\}$.

Application to Transient State Problem

The procedure described in the preceeding section is illustrated solving Eq. 4.81 in the domain shown in Fig. 4.9. The boundary condition on the flow domain is

$$\frac{\partial h}{\partial n} = 0 \text{ on } \Gamma \tag{4.102a}$$

The initial conditions are

$$h(x,y,0) = 0 \tag{4.102b}$$

The sources and sinks are

$$N(x,y,t) = Q_1\delta(x,y) + Q_2\delta(x,y-2) \tag{4.103}$$

where $Q_1 = 1$, and $Q_2 = 1$. Additionally the aquifer parameters are $T = 8$ and $S = 0.1$. The time domain is the interval $0 \le t \le 2$, which is divided into two time steps, where $\Delta t = 1$.

For this problem, by Eq. 4.91a,

$$[A] = \begin{bmatrix} 8 & 0 & 0 & 0 & -8 \\ 0 & 8 & 0 & 0 & -8 \\ 0 & 0 & 8 & 0 & -8 \\ 0 & 0 & 0 & 8 & -8 \\ -8 & -8 & -8 & -8 & 32 \end{bmatrix} \tag{4.104a}$$

and by Eq. 4.91b,

$$[B] = \frac{1}{120}\begin{bmatrix} 4 & 1 & 1 & 0 & 2 \\ 1 & 4 & 0 & 1 & 2 \\ 1 & 0 & 4 & 1 & 2 \\ 0 & 1 & 1 & 4 & 1 \\ 2 & 2 & 2 & 2 & 8 \end{bmatrix} \qquad (4.104b)$$

The load vector, by Eq. 4.91c, is

$$\{F\} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (4.104c)$$

The vector of initial conditions is

$$\{H^0\} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad (4.104d)$$

For the first time step, $t = 0$ and $t + \Delta t = 1$. Then, Eqs. 4.104 can be substituted into Eq.4.100 to obtain

$$\begin{bmatrix} 8.034 & 0.008 & 0.008 & 0 & -7.983 \\ 0.008 & 8.034 & 0 & 0.008 & -7.983 \\ 0.008 & 0 & 8.034 & 0.008 & -7.983 \\ 0 & 0.008 & 0.008 & 8.034 & -7.983 \\ -7.983 & -7.983 & -7.983 & -7.983 & 32.07 \end{bmatrix} \begin{bmatrix} H_1^1 \\ H_2^1 \\ H_3^1 \\ H_4^1 \\ H_5^1 \end{bmatrix} = \begin{bmatrix} 1.067 \\ 1.067 \\ 0.067 \\ .067 \\ 0.134 \end{bmatrix}$$

(4.105)

The solution for this system of equations is

$$\{H^1\} = [6.038,\ 6.038,\ 5.913,\ 5.913,\ 5.955]^T \tag{4.106}$$

At the second time step, t = 1 and $t+\Delta t = 2$. Then, Eq. 4.96 becomes

$$\begin{bmatrix} 8.034 & 0.008 & 0.008 & 0 & -7.983 \\ 0.008 & 8.034 & 0 & 0.008 & -7.983 \\ 0.008 & 0 & 8.034 & 0.008 & -7.983 \\ 0 & 0.008 & 0.008 & 8.034 & -7.983 \\ -7.983 & -7.983 & -7.983 & -7.983 & 32.07 \end{bmatrix} \begin{bmatrix} H_1^2 \\ H_2^2 \\ H_3^2 \\ H_4^2 \\ H_5^2 \end{bmatrix} = \begin{bmatrix} 1.403 \\ 1.403 \\ 0.398 \\ 0.398 \\ 0.800 \end{bmatrix} \tag{4.107}$$

The solution for this system of equations is

$$\{H^2\} = [11.007,\ 11.007,\ 10.882,\ 10.882,\ 10.924]^T \tag{4.108}$$

4.4. Computer Program for Finite-Element Analysis

In this section a computer program is described that can be used to solve two-dimensional steady-state or transient-state problems of ground water flow in unconfined aquifers. The program utilizes triangular elements and linear trial functions. Aquifer parameters are assumed constant over an element but may vary from element to element. Allowed boundary conditions include both specifed-flux and specified-head boundaries. sources and sinks are allowed at nodal points, and rates may vary from time-step to time-step.

4.4.1. Program Structure

The program has three basic modules: a main driver module, a data-input module, and a solution and output module. These modules are shown schematically in Fig. 4.10. The main driver program FLOW calls subroutine FDATA, which reads data defining the geometry of the flow domain, initial conditions, boundary condtions, aquifer parameters, and sources and sinks. This information is stored in common blocks, and control is returned to the main program. The main program in turn calls subroutine FMODEL, which is the solution and output module. This subroutine marches the solution through time by at each time step assembling the stiffness and mass matrices and then assembling the load

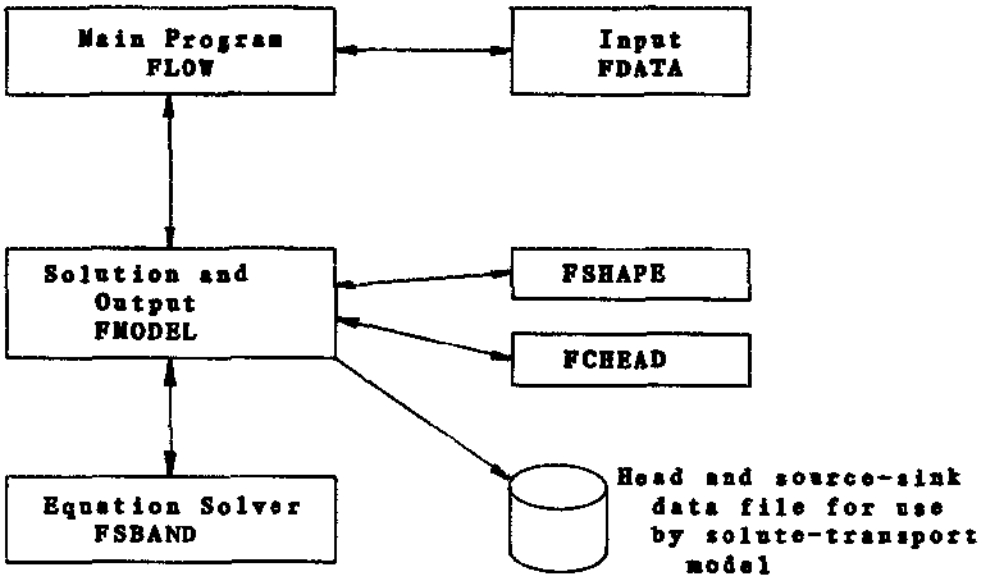


Fig. 4.10. *General configuration of finite-element program.*

TABLE 4.1. VARIABLES FOR SUBROUTINE FDATA

Varisble	Type	Definition
BASE	Real array	Altitude of aquifer base at nodes
CHEAD	Real array	Water levels at constant-head nodes
CHNODE	Integer array	Constant-head nodes
CLOSE	Real scalar	Closure criterion for iterations, which is compared with the maximum for all nodes of the difference between heads at two successive iterations
DELT	Real scalar	Time step
HO	Real array	Altitude of initial water level at nodes
IN	Integer array	Element incidence
KO	Real array	Hydraulic conductivity over elements
MAXIT	Integer scalar	Maximum number of iterations per time step
MAXKNS	Integer scalar	Number of time steps
NCHN	Integer scalar	Number of constant-head nodes
NE	Integer scalar	Number of elements
NN	Integer scalar	Number of nodes
NQSET	Integer scalar	Number of source-sink data sets
QSET	Real array	Recharge or discharge at nodes
TITLE	Real array	Identification of problem
X	Real array	x - coordinate of node
Y	Real array	y - coordinate of node

vector. The resulting system of linear equations is solved by subroutine FSBAND. Then, the discharge at a constant-head nodes is computed by FCHEAD.

The program modules and submodules are described in following sections. In each case, the important variables are identified, the program logic is described, and a source-code listing is provided. All source-code is written in standard FORTRAN IV. No machine-dependent extensions are utilized, and the code should run successfully on a wide range of computer hardware.

Main Program FLOW

The purpose of the main program is to call subroutines FDATA and FMODEL, and the module consists essentially of call statements to these subroutines. A listing of program FLOW is shown in Fig. 4.11.

Subroutine FDATA

The purpose of subroutine FDATA (Table 4.1) is to input data that describes the ground water problem to be solved. The input data occurs in three basic groups: definition of the basic solution domain, specification of constant heads, and specification of sources and sinks. The important variables in each of these groups is described in Table 4.1, the units of measure and input formats are indicated by comment statements in the source listing. A source listing of subroutine FDATA is given in Fig. 4.12.

In the first data-input group, the flow domain is described. This is accomplished by establishing a grid of triangular elements over the flow domain. The grid is defined by specifying the number of nodes in the grid NN and the x - y coordinates of each node X(I) and Y(I). The association of nodes to elements is specified by the element incidences IN (I,J). The incidences indicate the nodal numbers of the three nodes of a triangular element. These are listed in a counter-clockwise direction around an element starting with any node. For example, for the grid shown in Fig. 4.13a, the element incidences of element 4 are IN(4,1) = 5, IN(4,2) = 6, and IN(4,3) = 3.

Another important feature of the grid is the half-band width that it generates. The Galerkin method generates symmetric stiffness and mass matrices, and for a properly designed grid, these matrices are also banded. This property is utilized to decrease computer memory requirements by storing in memory only the half-band. The half-band width depends on the numbering of nodes in the grid, and it is determined by the numerical difference plus one between the nodes in an element with the highest and lowest nodal number. The half-band width is produced

Fig. 4.11. Source listing for program FLOW.

```
C        ------------
         PROGRAM FLOW
C        ------------
C
         CALL FDATA
         CALL FMODEL
         STOP
         END
```

Fig. 4.12. Source listing for subroutine FDATA.

```
C       ---------------
        SUBROUTINE FDATA
C       ---------------
C
C.....DIMENSIONED FOR THE FOLLOWING PROBLEM SIZE:
C          NUMBER OF NODES = 100
C          NUMBER OF ELEMENTS = 150
C          NUMBER OF CONSTANT-HEAD NODES = 20
C          NUMBER OF TIME STEPS = 30
C          NUMBER OF SOURCE-SINK DATA SETS = 4
C
      COMMON /MODEL/ NN,NE,NB,MAXKNS,MAXIT,NCHN,NQSET,NSET,
     1   CLOSE,DELT,X,Y,IN,KO,SO,BASE,H,HL,HO,
     2   FACTOR,CHNODE,CHEAD,CCHRH,CCHLH,
     3   QSET,FACSET,ISAVE
C
      INTEGER IN(150,3),CHNODE(20),NSET(30)
      REAL X(100),Y(100),BASE(100),H(100),HL(100),HO(100)
      REAL KO(150),SO(150)
      REAL CHEAD(20),CCHRH(100),CCHLH(100)
      REAL QSET(100,4),FACSET(30)
C
      REAL TITLE(20)
C
C     LOGICAL UNIT NUMBERS
C
      IR=5
      IW=6
C
C.....BASIC DEFINITION OF SOLUTION DOMAIN
C
C     BASIN NAME
C
      READ(IR,900) (TITLE(I),I=1,20)
      WRITE(IW,901) (TITLE(I),I=1,20)
  900 FORMAT(20A4)
  901 FORMAT(1H1,10X,20A4/1H ,10X,80('-')/)
C
C     TIME STEP (DAYS), NUMBER OF TIME STEPS, NUMBER OF ITERATIONS,
C        CLOSURE CRITERION ON ITERATIONS (FEET), AND SWITCH FOR SAVING
C        HEADS AND SOURCE-SINK FLUXES(1=YES AND 0=NO)
C
      READ(IR,902) DELT,MAXKNS,MAXIT,CLOSE,ISAVE
      WRITE(IW,903) DELT,MAXKNS,MAXIT,CLOSE,ISAVE
      DELT=DELT*3600.0*24.0
  902 FORMAT(F12.0,2I6,F12.0,I6)
  903 FORMAT(1H0,10X,'TIME PARAMETERS'/1H ,10X,15('-')/1H0,10X,
     1   'TIME STEP (DAYS)',21X,F8.1/1H ,10X,'NUMBER OF TIME STEPS',
     2   21X,I4/
     3   1H ,10X,'NUMBER OF ITERATIONS',20X,I5/
     4   1H ,10X,'CLOSURE CRITERION FOR INERATIONS (FEET)',F6.3/
     5   1H ,10X,'SAVE HEADS AND SOURCE-SINK FLUXES',6X,I6)
C
C     NUMBER OF NODES AND ELEMENTS
C
      READ(IR,904) NN,NE
      WRITE(IW,905) NN,NE
  904 FORMAT (2I6)
```

```
  905  FORMAT(1H0,10X,'FINITE-ELEMENT DATA'/1H ,10X,19('-')/1H0,10X,
      1    'NUMBER OF NODES    ',I10/1H ,10X,'NUMBER OF ELEMENTS',I10)
C
C      NODE COORDINATES (FEET)
C
       READ(IR,906) FACX,FACY
       READ(IR,907) (I,X(I),Y(I),N=1,NN)
       DO 100 I=1,NN
       X(I)=X(I)*FACX
       Y(I)=Y(I)*FACY
  100  CONTINUE
       WRITE(IW,908) FACX,FACY
       WRITE(IW,909) (I,X(I),Y(I),I=1,NN)
  906  FORMAT(2E12.0)
  907  FORMAT(3(I6,2F6.0,6X))
  908  FORMAT(1H0,10X,'NODE COORDINATES (FEET)'/1H ,10X,23('-')/
      1    1H0,10X,'FACTOR FOR X-COORDINATE',1PE11.3/
      2    1H ,10X,'FACTOR FOR Y-COORDINATE',1PE11.3/
      3    1H0,10X,3('NODE',11X,'X',11X,'Y',5X)/)
  909  FORMAT((1H ,10X,3(I4,F12.1,F12.1,5X)))
C
C      ELEMENT INCIDENCES (COUNTER CLOCKWISE)
C
       READ(IR,910) (I,(IN(I,J),J=1,3),N=1,NE)
       WRITE(IW,911)
       WRITE(IW,912) (I,(IN(I,J),J=1,3),I=1,NE)
  910  FORMAT(2(4I6,6X))
  911  FORMAT(1H0,10X,'ELEMENT INCIDENCES (COUNTER CLOCKWISE)'/
      1    1H ,10X,38('-')/
      2    1H0,10X,3('ELEM',13X,'CORNERS',9X)/)
  912  FORMAT((1H ,10X,3(I4,2X,3I6,9X)))
C
C      FIND HALF-BAND WIDTH
C
       NB=0
       DO 102 L=1,NE
       IMAX=0
       IMIN=10000
       DO 101 I=1,3
       II=IN(L,I)
       IMAX=MAX0(IMAX,II)
       IMIN=MIN0(IMIN,II)
  101  CONTINUE
       NB=MAX0((IMAX-IMIN+1),NB)
  102  CONTINUE
       WRITE(IW,913) NB
  913  FORMAT(1H0,10X,'HALF-BAND WIDTH',I9)
C
C      HYDRAULIC CONDUCTIVITY (FEET PER DAY)
C
       READ(IR,914) FACK
       READ(IR,915) (K0(I),I=1,NE)
       DO 103 I=1,NE
       K0(I)=K0(I)*FACK
  103  CONTINUE
       WRITE(IW,917) FACK
       WRITE(IW,918) (I,K0(I),I=1,NE)
       DO 104 I=1,NE
       K0(I)=K0(I)/(3600.0*24.0)
  104  CONTINUE
  914  FORMAT(E12.0)
  915  FORMAT(10F6.0)
  917  FORMAT(1H0,10X,'HYDRAULIC CONDUCTIVITY (FEET PER DAY)'/
      1    1H ,10X,37('-')/
      2    1H0,10X,'FACTOR FOR CONDUCTIVITY',1PE11.3/
      3    1H0.10X.5('ELEM'.7X.'VALUE'.4X)/)
```

```
  918  FORMAT((1H ,10X,5(I4,3X,F9.1,4X)))
C
C      SPECIFIC YIELD (DIMENSIONLESS)
C
       READ(IR,919) FACS
       READ(IR,920) (SO(I),I=1,NE)
       DO 105 I=1,NE
       SO(I)=SO(I)*FACS
  105  CONTINUE
       WRITE(IW,922) FACS
       WRITE(IW,923) (I,SO(I),I=1,NE)
  919  FORMAT(E12.0)
  920  FORMAT(10F6.0)
  922  FORMAT(1H0,10X,'SPECIFIC YIELD (DIMENSIONLESS)'/
      1    1H ,10X,31('-')/
      2    1H0,10X,'FACTOR FOR STORAGE',1PE11.3/
      3    1H0,10X,5('ELEM',7X,'VALUE',4X)/)
  923  FORMAT((1H ,10X,5(I4,3X,F9.1,4X)))
C
C      ALTITUDE FOR BASE OF AQUIFER (FEET ABOVE DATUM)
C
       READ(IR,925) (BASE(I),I=1,NN)
       WRITE(IW,926)
       WRITE(IW,927) (I,BASE(I),I=1,NN)
  925  FORMAT(10F6.0)
  926  FORMAT(1H0,10X,'BASE OF AQUIFER (FEET ABOVE DATUM)'/
      1    1H ,10X,34('-')/
      2    1H0,10X,5('NODE',7X,'VALUE',4X)/)
  927  FORMAT((1H ,10X,5(I4,3X,F9.1,4X)))
C
C      INITIAL WATER LEVELS (FEET)
C
       READ(IR,928) (HO(I),I=1,NN)
       WRITE(IW,929)
       WRITE(IW,9301) (I,HO(I),I=1,NN)
  928  FORMAT(10F6.0)
  929  FORMAT(1H0,10X,'INITIAL WATER LEVELS (FEET ABOVE DATUM)'/
      1    1H ,10X,39('-')/
      2    1H0,10X,5('NODE',7X,'VALUE',4X)/)
 9301  FORMAT((1H ,10X,5(I4,3X,F9.1,4X)))
C
C.....DEFINITION OF CONSTANT-HEAD BOUNDARIES
C
C
C      NUMBER OF CONSTANT-HEAD NODES, LEAKANCE FACTOR (FEET SQUARED PER
C         DAY), AND CONSTANT-HEAD VALUES (FEET ABOVE DATUM)
C
       READ(IR,930) NCHN,FACTOR
       IF(NCHN.EQ.0) GO TO 3001
       READ(IR,931) (CHNODE(I),CHEAD(I),I=1,NCHN)
       WRITE(IW,932) FACTOR
       WRITE(IW,933) (CHNODE(I),CHEAD(I),I=1,NCHN)
  930  FORMAT(I6,E12.0)
  931  FORMAT(I6,F6.0)
  932  FORMAT(1H0,10X,'CONSTANT-HEAD NODES'/1H ,10X,19('-')/
      1    1H0,10X,'LEAKANCE FACTOR (FEET SQUARED PER DAY)',1PE12.3/
      2    1H0,10X,'NODE',8X,'HEAD (FEET ABOVE DATUM)'/)
  933  FORMAT(1H ,10X,I4,F12.1)
C
C      INITIALIZE MATRIX DIAG(CCHLH) AND VECTOR (CCHRH)
C
 3001  DO 300 I=1,NN
       CCHRH(I)=0.0
       CCHLH(I)=0.0
  300  CONTINUE
C
```

```
      IF (NCHN.EQ.0) GO TO 3011
C
C     COMPUTE LEFT-HAND AND RIGHT-HAND COEFFICIENT
C        MATRIX DIAG(CCHLH) AND VECTOR (CCHRH)
C
      DO 301 I=1,NCHN
      J=CHNODE(I)
      CCHLH(J)=FACTOR
      CCHRH(J)=FACTOR*CHEAD(I)
 301  CONTINUE
 3011 CONTINUE
C
C.....DEFINITION OF SOURCES AND SINKS
C
C
C     NUMBER OF SOURCE-SINK PERIODS AND FACTOR FOR SOURCE-SINK STRENGTHS
C
      READ(IR,940) NQSET,FACQ
      IF(NQSET.EQ.0) GO TO 4012
      WRITE(IW,941) NQSET,FACQ
 940  FORMAT(I6,E12.0)
 941  FORMAT(1H0,10X,'SOURCES AND SINKS (CUBIC FEET PER SECOND)'/
     1   1H ,10X,40('-')/
     2   1H0,10X,'NUMBER OF SOURCE-SINK DATA SETS',I5/
     3   1H ,10X,'FACTOR FOR SOURCES AND SINKS',10X,1PE11.3)
C
C     SOURCE-SINK DATA SETS (CUBIC FEET PER SECOND) WITH IN (+) AND OUT (-)
C
      DO 401 J=1,NQSET
      DO 400 I=1,NN
      QSET(I,J)=0.0
 400  CONTINUE
 401  CONTINUE
      DO 403 J=1,NQSET
      READ(IR,952) NQ
      READ(IR,953) (I,QSET(I,J),K=1,NQ)
      DO 402 I=1,NN
      QSET(I,J)=QSET(I,J)*FACQ
 402  CONTINUE
      WRITE(IW,954) J
      WRITE(IW,955) (I,QSET(I,J),I=1,NN)
 403  CONTINUE
 952  FORMAT(I6)
 953  FORMAT(5(I6,F6.0))
 954  FORMAT(1H0,10X,'SOURCE-SINK DATA SET',I5/1H ,10X,20('-')/1H0,10X,
     1   5('NODE',7X,'VALUE',4X)/)
 955  FORMAT((1H ,10X,5(I4,3X,F9.4,4X)))
C
C     TIME-STEP INDICATORS AND MULTIPLIERS
C
      READ(IR,956) (I,NSET(I),FACSET(I),J=1,MAXKNS)
      WRITE(IW,957)
      WRITE(IW,958) (I,NSET(I),FACSET(I),I=1,MAXKNS)
 956  FORMAT(I6,I6,F6.0)
 957  FORMAT(1H0,10X,'TIME-STEP INDICATORS AND MULTIPLIERS'/1H ,10X,
     1   37('-')/1H0,10X,'STEP',7X,'SET',4X,'FACTOR (DIMENSIONLESS)'/)
 958  FORMAT((1H ,10X,I4,7X,I3,4X,F6.3))
 4012 CONTINUE
C
C     END DATA INPUT
C
      RETURN
      END
```

by the element with the greatest difference. For example, the grid shown in Fig. 4.13a has a half-band width of 5. However, the grid shown in Fig. 4.13b is numbered differently, and it has a half-band width of 6.

Subroutine FMODEL

The purpose of subroutine FMODEL (Table 4.2) is to march the solution through time. To accomplish this, the subroutine includes three basic loops, as is shown schematically in Fig. 4.14. The outer most is the time-step loop, which controls the march through time. Next is the interation loop, which controls the interative update of transmissivity. For the water table problem, the transmissivity is a function in part of the hydraulic head in the aquifer, which makes Eq. 4.99 nonlinear because [A] is now a function of $\{H^{t+\Delta t}\}$. To solve the nonlinear system of equations, an iterative procedure is employed. The inner most loop is the element loop. In this loop, the integrations of Eq. 4.59 are evaluated on an elemental basis, and the elemental contributions are made to the stiffness and mass matrices [A] and [B]. A source listing of subroutine FMODEL is given in Fig. 4.15.

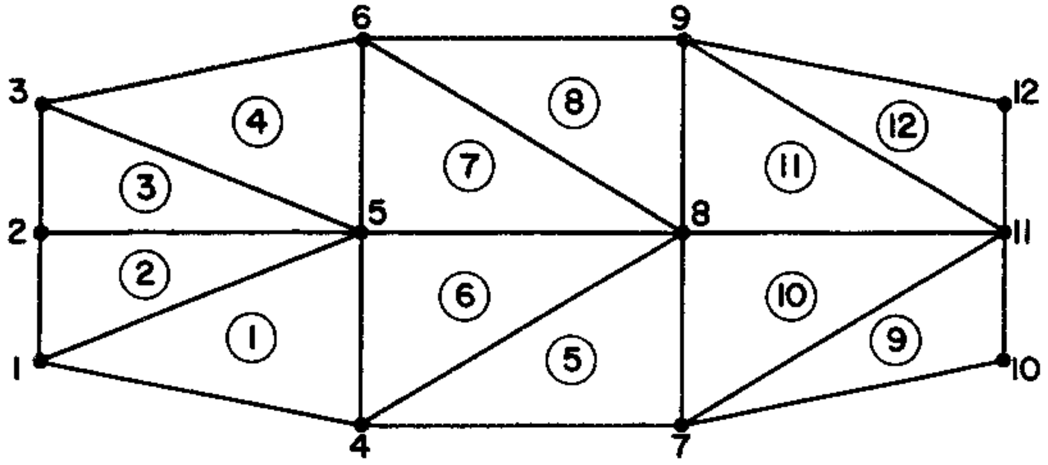
A compressed storage is used in subroutine FMODEL for the matrices [A], [B], and [L]. first, as was described earlier, these matrices are symmetric. This property can be used to reduce the storage requirement because it is only necessary to store either the upper or lower half of a symmetric matrix. The other half can be reconstructed at any time from the relation

$$A_{ij} = A_{ji} \tag{4.109}$$

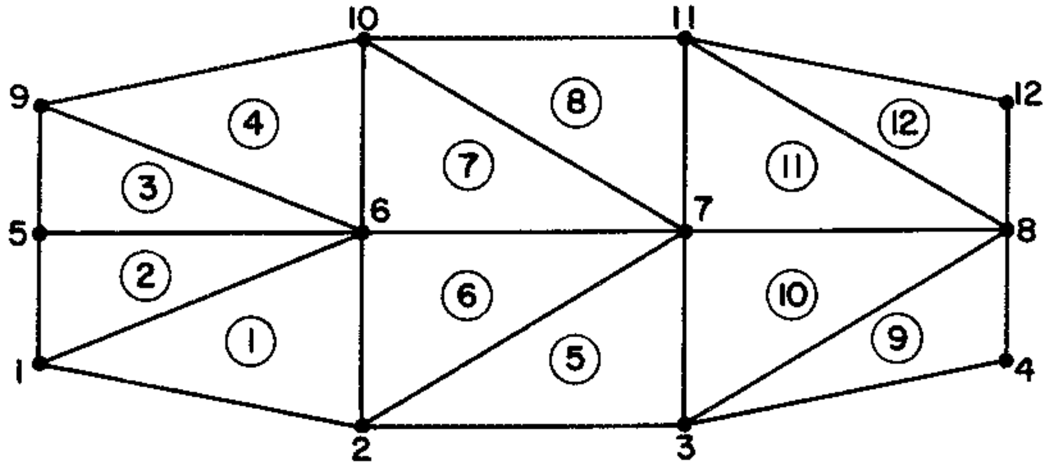
which holds for symmetric matrices. Second, the matrices [A], [B], and [L] are banded. If nodes are properly numbered in the finite-element grids storage requirements can be reduced by storing only values within the band. Figure 4.16 shows how the symmetric and banded structure can be used to reduce storage requirements.

Subroutine FSHAPE

The purpose of subroutine FSHAPE (Table 4.3) is to construct the elemental stiffness and mass matrices $[A^e]$ and $[B^e]$. The coordinates of elemental nodes transmissivity, and storage coefficient are passed to subroutine FSHAPE from subroutine FMODEL. Subroutine FSHAPE returns $[A^e]$ and $[B^e]$ to subroutine FMODEL. A source listing of subroutine FSHAPE is given in Fig. 4.17.



HALF-BAND WIDTH 5



HALF-BAND WIDTH 6

Fig. 4.13a & 4.13b. *Numbering of finite-element grid.*

TABLE 4.2. VARIABLES FOR SUBROUTINE FMODEL

Variable	Type	Definition
A	Real array	Global stiffness matrix
AE	Real array	Elemental stiffness matrix
B	Real array	Global mass matrix
BE	Real array	Elemental mass matrix
CCHLH	Real array	Left-hand-side constant-head coefficient
CCHRH	Real array	Right-hand-side constant-head coefficient
H	Real array	Computed water level at time t + Δt
HL	Real array	Computed water level at time t
HLIT	Real array	Computed water level at time t + Δt for previous iteration
KNS	Integer scalar	Time step
L	Real array	Left-hand-side matrix
R	Real array	Right-hand-side vector
THK	Real array	Aquifer thickness

TABLE 4.3. VARIABLES FOR SUBROUTINES FSHAPE

Variable	Type	Definition
AREA	Real scalar	Area for element
BB	Real array	Coefficient b of trial function
CC	Real array	Coefficient c of trial function
THKE	Real scalar	Average aquifer thickness for element
XE	Real array	x - coordinate for node
YE	Real array	y - coordinate for node

TIME-STEP LOOP

ITERATION LOOP

Update Transmissivity

ELEMENT LOOP

Construct $[A^e]$ and $[B^e]$

Add contribution to [A] and [B]

Assemble left and right sides

$$[L] = [A] + \frac{1}{\Delta t}[B]$$

$$\{R\} = \frac{1}{\Delta t}[B]\{H^t\} + \{F\}$$

Solve equation

$$[L]\{H^{t+\Delta t}\} = \{R\}$$

Display results

Fig. 4.14. Generalized flow diagram for subroutine FMODEL.

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} & 0 & 0 & 0 \\ L_{21} & L_{22} & L_{23} & L_{24} & 0 & 0 \\ L_{31} & L_{32} & L_{33} & L_{34} & L_{35} & 0 \\ 0 & L_{42} & L_{43} & L_{44} & L_{45} & L_{46} \\ 0 & 0 & L_{53} & L_{54} & L_{55} & L_{56} \\ 0 & 0 & 0 & L_{64} & L_{65} & L_{66} \end{bmatrix}$$

Natural Storage

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{22} & L_{23} & L_{24} \\ L_{33} & L_{34} & L_{35} \\ L_{44} & L_{45} & L_{46} \\ L_{55} & L_{56} & 0 \\ L_{66} & 0 & 0 \end{bmatrix}$$

Compressed Storage

Fig. 4.16. Band storage of matricies.

Fig. 4.15. Source listing for subroutine FMODEL.

```
C      ------------------
       SUBROUTINE FMODEL
C      ------------------
C
C.....DIMENSIONED FOR THE FOLLOWING PROBLEM SIZE:
C          NUMBER OF NODES = 100
C          NUMBER OF ELEMENTS = 150
C          NUMBER OF CONSTANT-HEAD NODES = 20
C          NUMBER OF TIME STEPS = 30
C          MAXIMUM HALF-BAND WIDTH = 15
C
       COMMON /MODEL/ NN,NE,NB,MAXKNS,MAXIT,NCHN,NQSET,NSET,
     1    CLOSE,DELT,X,Y,IN,KO,SO,BASE,H,HL,HO,
     2    FACTOR,CHNODE,CHEAD,CCHRH,CCHLH,
     3    QSET,FACSET,ISAVE
C
       INTEGER IN(150,3),CHNODE(20),NSET(30)
       REAL X(100),Y(100),BASE(100),H(100),HL(100),HO(100)
       REAL KO(150),SO(150)
       REAL CHEAD(20),CCHRH(100),CCHLH(100)
       REAL QSET(100,4),FACSET(30)
C
       REAL L(100,15),A(100,15),B(100,15),R(100)
       REAL AE(3,3),BE(3,3)
       REAL THK(100),Q(100),HLIT(100)
       REAL K(150),S(150)
       EQUIVALENCE (K,KO)
       EQUIVALENCE (S,SO)
C
C      LOGICAL UNIT NUMBERS
C
       IR=5
       IW=6
       IW2=10
       IF(ISAVE.EQ.1) REWIND IW2
C
C      INITIAL WATER LEVELS
C
       DO 400 I=1,NN
       HL(I)=HO(I)
       H(I)=HO(I)
 400   CONTINUE
C
C.....BEGIN TIME-STEP LOOP
C
       KNS=0
 401   KNS=KNS+1
       IF(KNS.GT.MAXKNS) RETURN
C
C      INTERCHANGE WATER LEVELS
C
       DO 402 I=1,NN
       HL(I)=H(I)
 402   CONTINUE
C
C      COMPUTE VALUES FOR SOURCE/SINK VECTOR (Q) AND
```

```
C        COMPUTE CONSTANT-HEAD COEFFICIENTS (CCHRH) AND (CCHLH)
C
      DO 505 I=1,NN
      Q(I)=0.0
 505  CONTINUE
      IF(NQSET.EQ.0) GO TO 506
      JJ=NSET(KNS)
      DO 504 I=1,NN
      Q(I)=QSET(I,JJ)*FACSET(KNS)
 504  CONTINUE
 506  CONTINUE
C
C.....BEGIN ITERATION LOOP
C
      DO 4171 I=1,NN
      HLIT(I)=HL(I)
 4171 CONTINUE
C
      DO 421 ITER=1,MAXIT
C
C
C     UPDATE AQUIFER-THICKNESS VALUES
C
      DO 404 I=1,NN
      THK(I)=H(I)-BASE(I)
      IF(THK(I).LE.10.0) THK(I)=10.0
 404  CONTINUE
C
C     INITIALIZE MATRICIES (A) AND (B)
C
      DO 406 I=1,NN
      DO 405 J=1,NB
      A(I,J)=0.0
      B(I,J)=0.0
 405  CONTINUE
 406  CONTINUE
C
C.....BEGIN LOOP OVER ELEMENTS
C
      DO 412 LL=1,NE
C
C     ELEMENT STIFFNESS MATRIX [AE] AND DYNAMIC MATRIX [BE]
C
      CALL FSHAPE(LL,IN,X,Y,THK,K,S,AE,BE)
C
C     GLOBAL STIFFNESS MATRIX [A] AND MASS MATRIX [B]
C
      DO 411 I=1,3
      II=IN(LL,I)
      DO 410 J=1,3
      JJ=IN(LL,J)-II+1
      IF(JJ.LT.1) GO TO 410
      A(II,JJ)=A(II,JJ)+AE(I,J)
      B(II,JJ)=B(II,JJ)+BE(I,J)
 410  CONTINUE
 411  CONTINUE
 412  CONTINUE
C
      DO 414 I=1,NN
      DO 413 J=1,NB
      B(I,J)=B(I,J)/DELT
 413  CONTINUE
 414  CONTINUE
C
C.....END LOOP OVER ELEMENTS
C
```

```
C       MULTIPLICATION OF GLOBAL MASS MATRIX [B] AND
C          LAST-WATER-LEVEL VECTOR {HL} TO PRODUCE VECTOR {R}
C
        DO 417 I=1,NN
        R(I)=B(I,1)*HL(I)
        DO 416 J=2,NB
        II=I+J-1
        IF (II.GT.NN) GO TO 415
        R(I)=R(I)+B(I,J)*HL(II)
  415   II=I-J+1
        IF(II.LT.1) GO TO 416
        R(I)=R(I)+B(II,J)*HL(II)
  416   CONTINUE
  417   CONTINUE
C
C       CONSTRUCT LEFT-HAND-SIDE MATRIX [L]=[A]+[B]+DIAG{CCHLH}
C
        DO 419 I=1,NN
        DO 418 J=1,NB
        L(I,J)=A(I,J)+B(I,J)
  418   CONTINUE
        L(I,1)=L(I,1)+CCHLH(I)
  419   CONTINUE
C
C       CONSTRUCT RIGHT-HAND-SIDE VECTOR {R}=[B]{HL}+{Q}+{CCHRH}
C
        DO 420 I=1,NN
        R(I)=R(I)+Q(I)+CCHRH(I)
  420   CONTINUE
C
C       COMPUTE NEW WATER-LEVEL VECTOR {H}
C
        CALL FSBAND(L,R,H,NN,NB)
C
C       CHECK FOR CLOSURE
C
        DELMAX=0.0
        DO 4201 I=1,NN
        DELH=ABS(H(I)-HLIT(I))
        IF(DELH.GT.DELMAX) DELMAX=DELH
        HLIT(I)=H(I)
 4201   CONTINUE
        IF(DELMAX.LE.CLOSE) GO TO 4211
  421   CONTINUE
        IF(ITER.GT.MAXIT) ITER=ITER-1
 4211   CONTINUE
C
C.....END ITERATION LOOP
C
C       DISPLAY WATER LEVELS AND AQUIFER THICKNESS
C
  423   CONTINUE
        TIME=KNS*DELT/(3600.0*24.0)
        WRITE(IW,980) KNS,TIME,ITER,DELMAX
        WRITE(IW,981) (I,H(I),I=1,NN)
        WRITE(IW,982)
        WRITE(IW,981) (I,THK(I),I=1,NN)
  982   FORMAT(1H0,10X,'AQUIFER THICKNESS (FEET)'/1H ,10X,24('-')/
     1     1H ,10X,5('NODE',7X,'VALUE',4X)/)
  980   FORMAT(1H1,10X,'TIME STEP',I4/1H ,10X,13('-')/
     1     1H0,10X,'ELAPSED TIME (DAYS)',F10.1/
     2     1H ,10X,'NUMBER OF ITERATIONS',I9/
     3     1H ,10X,'WATER-LEVEL CHANGE ON LAST INERATION (FEET)',F10.4/
     4     1H0,10X,'COMPUTED WATER LEVELS (FEET ABOVE DATUM)'/
     5     1H ,10X,38('-')/1H0,10X,5('NODE',7X,'VALUE',4X)/)
  981   FORMAT((1H ,10X,5(I4,F12.3,4X)))
```

```
C
C      DISPLAY CONSTANT-HEAD FLUXES
C
       CALL FCHEAD(CCHRH,CCHLH,Q,H,CHNODE,NCHN)
C
C      SAVE HEADS AND SOURCE-SINK FLUXES ON FILE FOR FUTURE USE WITHIN
C         TRANSPORT PROBLEM.
C
       IF(ISAVE.NE.1) GO TO 500
       WRITE(IW2,966) (H(I),I=1,NN)
       WRITE(IW2,967) (Q(I),I=1,NN)
 966   FORMAT(5F12.1)
 967   FORMAT(5F12.5)
 500   CONTINUE
C
C.....END TIME-STEP LOOP
C
       GO TO 401
       END
```

Subroutine FSBAND

The purpose of subroutine FSBAND (Table 4.4) is to solve the system of linear equations

$$[L]\ \{H^{t+\Delta t}\} = \{R\} \tag{4.110}$$

for $\{H^{t+\Delta t}\}$. The matrix [L] and the vector $\{R\}$ are passed to subroutine FSBAND, and the solution vector $\{H^{t+\Delta t}\}$ is passed back to the calling subroutine FMODEL. A source listing of subroutine FSBAND is shown in Fig. 4.18.

The Cholesky method (Churchhouse, 1981) is used to solve the system of equations. The method is based upon the fact that, for a symmetrical matrix [L], there exists a matrix $[\ell]$ such that

$$[L] = [\ell]\ [\ell]^T, \tag{4.111}$$

where $[\ell]$ is a lower triangular matrix. A lower triangular matrix has coefficients of zero above the diagonal. The general ability to factorize a matrix into upper and lower triangular matrices, is the essential idea in all elimination schemes for solving systems of linear equations, and the Cholesky method is one particular scheme of many.

Setting aside the problem of finding $[\ell]$, equation 4.111 can be substituted into Eq. 4.110 to obtain

$$[\ell]\ [\ell]^T\ \{H^{t+\Delta t}\} = \{R\} \tag{4.112}$$

When $[\ell]$ is determined, an intermediate solution $\{Y\}$ can be defined as

$$[\ell]\ \{Y\} = \{R\} \tag{4.113}$$

Because of the triangular structure of $[\ell]$, Eq. 4.113 can be readily solved for $\{Y\}$ by direct back substitution. However, Eq. 4.113 can be substituted into Eq. 4.112 to obtain

$$[\ell]\ [\ell]^T\ \{H^{t+\Delta t}\} = [\ell]\ \{Y\} \tag{4.114}$$

By premultiplying by $[\ell]^{-1}$, Eq. 4.114 can be reduced to

$$[\ell]^T\ \left\{H^{t+\Delta t}\right\} = \{Y\} \tag{4.115}$$

Again, because of this triangular structure of $[\ell]$, Eq. 4.115 can be solved for $\left\{H^{t+\Delta t}\right\}$ by direct back substitution.

Expression for the coefficients of $[\ell]$ can be obtained by first writing expressions for the expansion of Eq. 4.111 by the definition of matrix multiplication and then by solving for the coefficients of $[\ell]$. Expansion of Eq. 4.111 yields

TABLE 4.4. VARIABLES FOR SUBROUTINE FSBAND

Variable	Type	Definition
H	Real array	Solution vector and intermediate-solution vector
L	Real array	Left-hand-side matrix
R	Real array	Right-hand-side vector

TABLE 4.5. VARIABLES FOR SUBROUTINE FCHEAD

Variable	Type	Definition
CCHLH	Real array	Left-hand-side constant-head coefficient
CCHRH	Real array	Right-hand-side constant-head coefficient
QCH	Real array	Discharge at constant-head node

$$L_{jj} = \ell^2_{j1} + \ell^2_{j2} + \cdots, \ell^2_{jj} \tag{4.116}$$

on the diagonal, and

$$L_{ij} = \ell_{i1}\,\ell_{j1} + \ell_{i2}\,\ell_{j2} + \cdots + \ell_{ij}\,\ell_{jj} \tag{4.117}$$

below the diagonal.

Equations 4.116 and 4.117 can be rearranged to obtain

$$\ell_{jj} = \left(L_{jj} - \sum_{k=1}^{j-1} \ell^2_{jk}\right)^{1/2} \tag{4.118}$$

on the diagonal, and

$$\ell_{ij} = \left(L_{ij} - \sum_{k=1}^{j-1} \ell_{ik}\,\ell_{jk}\right) / \ell_{jj} \tag{4.119}$$

below the diagonal. Equations 4.118 and 4.119 are used in subroutine FSBAND to lower triangularize matrix [L].

Subroutine FCHEAD

The purpose of subroutine FCHEAD (Table 4.5) is to compute the flux rate at constant-head nodes. The constant-head nodes are treated in the model as leakage against a specified head. The relation used in the model is

$$Q_{b_i} = C\left(H_i^{t+\Delta t} - H_{b_i}\right) \delta\,(x - x_i,\ y - y_i) \tag{4.120}$$

where Q_{bi} is the flux rate at node i, C is a constant, H_i is the head in the aquifer, H_{bi} is the specified constant-head value, and δ is the Dirac delta function. For large C, the head in the aquifer H_i will be close to the specified head H_{bi}. A source listing of subroutine FCHEAD is shown in Fig. 4.19.

From integration over the flow domain in the Galerkin method, constant-head nodes are incorporated into the model by adding

$$Q_{b_i} = C\left(H_i^{t+\Delta t} - H_{b_i}\right) \tag{4.121}$$

to the vector $\{R\}$ of Eq. 4.97. However, Eq. 4.114 includes the dependent variable, and the part including the dependent variable is moved to the matrix [L] of Eq. 4.97. By this operation, the coefficient L_{ii} is replaced by

$$L_{ii} + C$$

```
C      ----------------------------------------------
       SUBROUTINE FSHAPE(LL,IN,X,Y,THK,K,S,AE,BE)
C      ----------------------------------------------
C
C.....DIMENSION FOR THE FOLLOWING PROBLEM SIZE:
C          NUMBER OF NODES = 100
C          NUMBER OF ELEMENTS = 150
C
       INTEGER IN(150,3)
       REAL X(100),Y(100),THK(100)
       REAL K(150),S(150)
       REAL AE(3,3),BE(3,3),XE(3),YE(3),BB(3),CC(3)
C
C.....NODAL COORDINATES AND ELEMENTAL THICKNESS
C
       THKE=0.0
       DO 407 I=1,3
       II=IN(LL,I)
       XE(I)=X(II)
       YE(I)=Y(II)
       THKE=THKE+THK(II)
 407   CONTINUE
       THKE=THKE/3.0
C
C.....COEFFICIENTS OF THE TRIAL FUNCTIONS
C
       BB(1)=YE(2)-YE(3)
       BB(2)=YE(3)-YE(1)
       BB(3)=YE(1)-YE(2)
       CC(1)=XE(3)-XE(2)
       CC(2)=XE(1)-XE(3)
       CC(3)=XE(2)-XE(1)
C
C      AREA OF ELEMENT
C
       AREA=(BB(1)*XE(1)+BB(2)*XE(2)+BB(3)*XE(3))/2.0
C
C.....AREA INTEGRATIONS
C
       DO 409 I=1,3
       DO 408 J=1,3
       AE(I,J)=THKE*K(LL)*BB(I)*BB(J)/(4.0*AREA)
     1    +THKE*K(LL)*CC(I)*CC(J)/(4.0*AREA)
       BE(I,J)=S(LL)*AREA/12.0
       IF(I.EQ.J) BE(I,J)=BE(I,J)*2.0
 408   CONTINUE
 409   CONTINUE
C
       RETURN
       END
```

Fig. 4.17. *Source listing for subroutine FSHAPE.*

Fig. 4.18. Source listing for subroutine FSBAND.

```
C      ------------------------------
       SUBROUTINE FSBAND(L,R,H,NN,NB)
C      ------------------------------
C
C.....DIMENSIONED FOR THE FOLLOWING PROBLEM SIZE:
C         NUMBER OF NODES = 100
C         MAXIMUM HALF-BAND WIDTH = 15
C
       REAL L(100,15),R(100),H(100)
C
C      LOGICAL UNIT NUMBERS
C
       IR=5
       IW=6
C
C.....UPPER TRINGULARIZE MATRIX OF COEFFICIENTS [L]
C         UPPER TRIANGULARIZED MATRIX TEMPORARILY STORED IN [L]
C
       DO 221 I=1,NN
       IP=NN-I+1
       IF(NB.LT.IP) IP=NB
       DO 220 J=1,IP
       IQ=NB-J
       IF((I-1).LT.IQ) IQ=I-1
       SUM=L(I,J)
       IF(IQ.LT.1) GO TO 450
       DO 440 K=1,IQ
       II=I-K
       JZ=J+K
       SUM=SUM-L(II,K+1)*L(II,JZ)
  440  CONTINUE
  450  IF(J.NE.1) GO TO 230
       IF(SUM.LE.0.0) GO TO 260
       TEMP=1.0/SQRT(SUM)
       L(I,J)=TEMP
       GO TO 220
  230  L(I,J)=SUM*TEMP
  220  CONTINUE
  221  CONTINUE
C
       GO TO 261
  260  WRITE(IW,910) I
       STOP
  910  FORMAT(1H1,10X,'UPPER TRIANGULARIZATION FAILS AT ROW',I4)
C
C.....INTERMEDIATE SOLUTION VECTOR {Y}
C         VECTOR {Y} TEMPORARILY STORED IN {H}
C
  261  CONTINUE
       DO 320 I=1,NN
       J=I-NB+1
       IF((I+1).LE.NB) J=1
       SUM=R(I)
       K1=I-1
       IF(J.GT.K1) GO TO 340
       DO 330 K=J,K1
       II=I-K+1
       SUM=SUM-L(K,II)*H(K)
```

```
 330   CONTINUE
 340   H(I)=SUM*L(I,1)
 320   CONTINUE
C
C.....BACK SUBSTITUTION TO OBTAIN VECTOR (H)
C
       DO 540 I1=1,NN
       I=NN-I1+1
       J=I+NB-1
       IF(J.GT.NN) J=NN
       SUM=H(I)
       K2=I+1
       IF(K2.GT.J) GO TO 250
       DO 550 K=K2,J
       KK=K-I+1
       SUM=SUM-L(I,KK)*H(K)
 550   CONTINUE
 250   H(I)=SUM*L(I,1)
 540   CONTINUE
C
       RETURN
       END
```

```
C     ----------------------------------------------------------
      SUBROUTINE FCHEAD(CCHRH,CCHLH,Q,H,CHNODE,NCHN)
C     ----------------------------------------------------------
C
C.....DIMENSIONED FOR THE FOLLOWING PROBLEM SIZE:
C         NUMBER OF NODES = 100
C         NUMBER OF CONSTANT-HEAD NODES = 20
C
      INTEGER CHNODE(20)
      REAL CCHRH(100),CCHLH(100),Q(100),H(100)
      REAL QCH(20)
C
C     LOGICAL UNIT NUMBERS
C
      IR=5
      IW=6
C
C.....COMPUTE CONSTANT-HEAD FLUXES
C         AND UPDATE SOURCE-SINK VECTOR
C
      IF(NCHN.EQ.0) RETURN
      SUMCH=0.0
      DO 150 I=1,NCHN
      J=CHNODE(I)
      QCH(I)=CCHRH(J)-CCHLH(J)*H(J)
      SUMCH=SUMCH+QCH(I)
      Q(J)=Q(J)+QCH(I)
 150  CONTINUE
C
C.....PRINT CONSTANT-HEAD FLUXES
C
      WRITE(IW,950) SUMCH
      WRITE(IW,951) (CHNODE(I),QCH(I),I=1,NCHN)
 950  FORMAT(1H0,10X,'CONSTANT-HEAD NODES'/1H ,10X,19('-')/1H0,10X,
     1   'CUMULATIVE RATE (CUBIC FEET PER SECOND)',F9.4/1H0,10X,
     2   'NODE',8X,'RATE (CUBIC FEET PER SECOND)'/)
 951  FORMAT(1H ,10X,I4,F12.4)
C
      RETURN
      END
```

Fig 4.19. Source listing for subroutine FCHEAD.

and the coefficient R_i is replaced by

$$R_i + CH_{b_i}$$

Once the head in the aquifer is computed, Eq. 4.114 can be used to evaluate the flux rate at constant-head nodes. These flux rates are computed in subroutine CHEAD.

4.4.2. Application to a Simple Problem

The example problem of Fig. 4.20 is used to demonstrate the application of the finite-element program. Figure 4.21 shows input file for the problem, and Fig. 4.22 shows the output file.

4.5. Regional Groundwater Problem

Groundwater models can be used to solve problems of different space and time scales, and each requires somewhat different approaches. In each case, however, the model is formulated mathematically from the one-, two- or three- dimensional partial differential equation of groundwater flow and the associated requirement for specification of boundary conditions and, for transient-state simulations, initial conditions. Most likely, the partial-differential equation is solved by numerical methods, of which the finite-element method is one of current popularity. For a problem of smaller time and space scales, such as the design or evaluation of a well field, a finite-element grid with small elements would be utilized, with dimensions measured in hundreds of feet. Time steps would be correspondingly small, with steps measured in perhaps hours, and the simulation might include several years.

In contrast, however, the problem of evaluating or managing the resources of a regional groundwater system would be approached from the perspective of large space and time scales. A finite-element grid for such a problem would have elements with dimensions of thousands of feet or miles, and time steps would be specified in months or years. Furthermore, the total simulation period of interest might be a century.

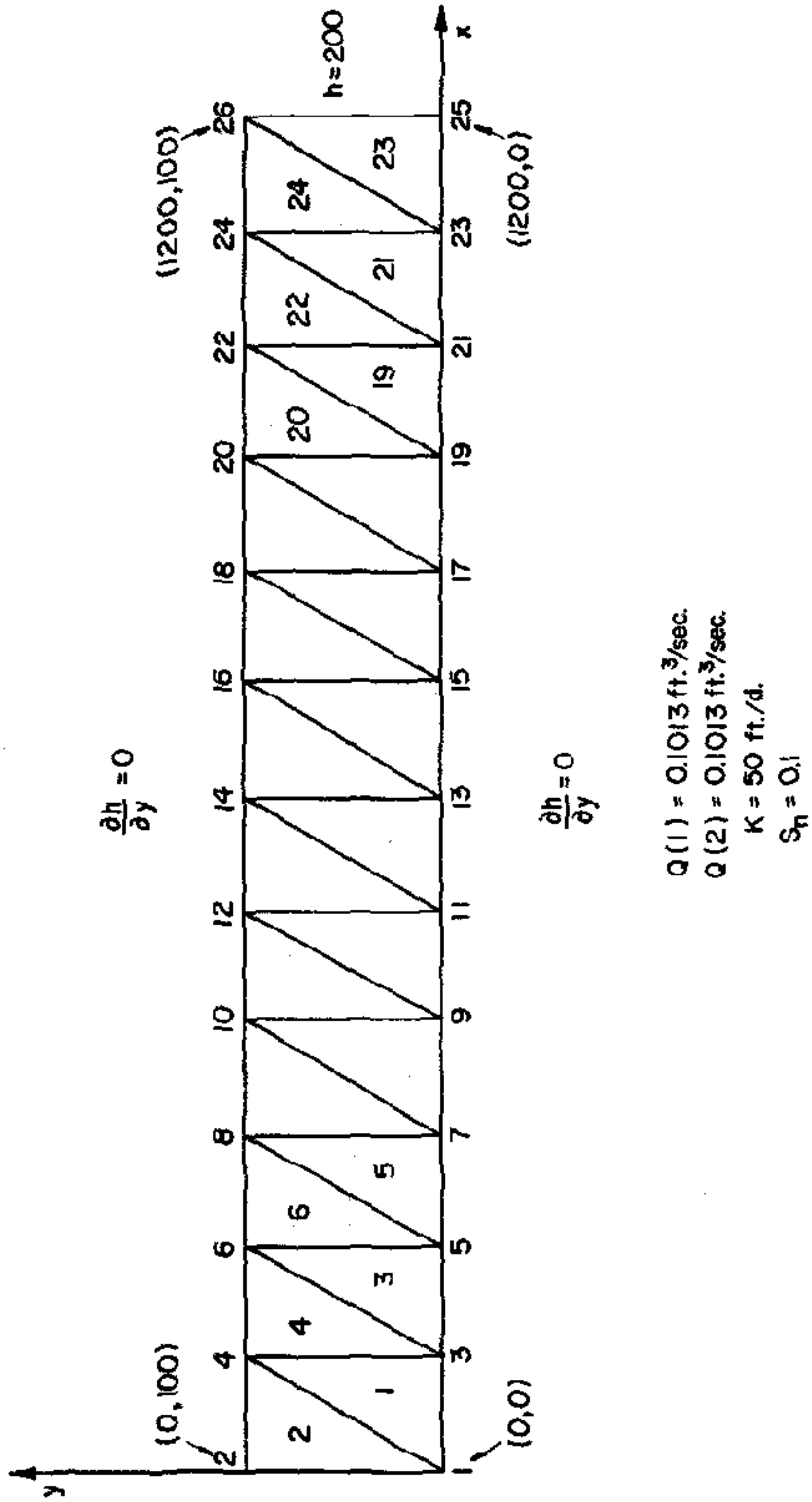


Fig. 4.20. *Problem for demonstration of finite-element program.*

```
GROUNDWATER-FLOW PROBLEM OF FIGURE 4.20
      30.0        2        8            0.1
  26    24
   1.0E+02        1.0E+02
   1    0.0     0.0                2    0.0     1.0              3    1.0     0.0
   4    1.0     1.0                5    2.0     0.0              6    2.0     1.0
   7    3.0     0.0                8    3.0     1.0              9    4.0     0.0
  10    4.0     1.0               11    5.0     0.0             12    5.0     1.0
  13    6.0     0.0               14    6.0     1.0             15    7.0     0.0
  16    7.0     1.0               17    8.0     0.0             18    8.0     1.0
  19    9.0     0.0               20    9.0     1.0             21   10.0     0.0
  22   10.0     1.0               23   11.0     0.0             24   11.0     1.0
  25   12.0     0.0               26   12.0     1.0
   1      1       3       4               2       1       4       2
   3      3       5       6               4       3       6       4
   5      5       7       8               6       5       8       6
   7      7       9      10               8       7      10       8
   9      9      11      12              10       9      12      10
  11     11      13      14              12      11      14      12
  13     13      15      16              14      13      16      14
  15     15      17      18              16      15      18      16
  17     17      19      20              18      17      20      18
  19     19      21      22              20      19      22      20
  21     21      23      24              22      21      24      22
  23     23      25      26              24      23      26      24
   1.0E+00
  50.0   50.0   50.0   50.0   50.0   50.0   50.0   50.0   50.0   50.0
  50.0   50.0   50.0   50.0   50.0   50.0   50.0   50.0   50.0   50.0
  50.0   50.0   50.0   50.0   50.0   50.0
   1.0E+00
   0.1    0.1    0.1    0.1    0.1    0.1    0.1    0.1    0.1    0.1
   0.1    0.1    0.1    0.1    0.1    0.1    0.1    0.1    0.1    0.1
   0.1    0.1    0.1    0.1    0.1    0.1
 142.0  142.0  138.5  138.5  135.0  135.0  131.5  131.5  128.0  128.0
 124.5  124.5  121.0  121.0  117.5  117.5  114.0  114.0  110.5  110.5
 107.0  107.0  103.5  103.5  100.0  100.0
 200.0  200.0  200.0  200.0  200.0  200.0  200.0  200.0  200.0  200.0
 200.0  200.0  200.0  200.0  200.0  200.0  200.0  200.0  200.0  200.0
 200.0  200.0  200.0  200.0  200.0  200.0
   2            1.0
  25  200.0
  26  200.0
   1            1.0
   2
   1  .1013       2  .1013
   1      1     1.0
   2      1     1.0
```

Fig. 4.21. *Input file for example problem.*

1 GROUNDWATER-FLOW PROBLEM OF FIGURE 4.20

0 TIME PARAMETERS

0	TIME STEP (DAYS)	30.0
	NUMBER OF TIME STEPS	2
	NUMBER OF ITERATIONS	8
	CLOSURE CRITERION FOR INERATIONS (FEET)	0.100
	SAVE HEADS AND SOURCE-SINK FLUXES	0

0 FINITE-ELEMENT DATA

0	NUMBER OF NODES	26
	NUMBER OF ELEMENTS	24

0 NODE COORDINATES (FEET)

0	FACTOR FOR X-COORDINATE	1.000E+02
	FACTOR FOR Y-COORDINATE	1.000E+02

0

NODE	X	Y	NODE	X	Y	NODE	X	Y
1	0.0	0.0	2	0.0	100.0	3	100.0	0.0
4	100.0	100.0	5	200.0	0.0	6	200.0	100.0
7	300.0	0.0	8	300.0	100.0	9	400.0	0.0
10	400.0	100.0	11	500.0	0.0	12	500.0	100.0
13	600.0	0.0	14	600.0	100.0	15	700.0	0.0
16	700.0	100.0	17	800.0	0.0	18	800.0	100.0
19	900.0	0.0	20	900.0	100.0	21	1000.0	0.0
22	1000.0	100.0	23	1100.0	0.0	24	1100.0	100.0
25	1200.0	0.0	26	1200.0	100.0			

0 ELEMENT INCIDENCES (COUNTER CLOCKWISE)

0

ELEM	CORNERS			ELEM	CORNERS			ELEM	CORNERS		
1	1	3	4	2	1	4	2	3	3	5	6
4	3	6	4	5	5	7	8	6	5	8	6
7	7	9	10	8	7	10	8	9	9	11	12
10	9	12	10	11	11	13	14	12	11	14	12
13	13	15	16	14	13	16	14	15	15	17	18
16	15	18	16	17	17	19	20	18	17	20	18
19	19	21	22	20	19	22	20	21	21	23	24
22	21	24	22	23	23	25	26	24	23	26	24

0 HALF-BAND WIDTH 4

0 HYDRAULIC CONDUCTIVITY (FEET PER DAY)

0 FACTOR FOR CONDUCTIVITY 1.000E+00

0

ELEM	VALUE	ELEM	VALUE	ELEM	VALUE	ELEM	VALUE	ELEM	VALUE
1	50.0	2	50.0	3	50.0	4	50.0	5	50.0
6	50.0	7	50.0	8	50.0	9	50.0	10	50.0
11	50.0	12	50.0	13	50.0	14	50.0	15	50.0
16	50.0	17	50.0	18	50.0	19	50.0	20	50.0
21	50.0	22	50.0	23	50.0	24	50.0		

Fig. 4.22. Output file for example problem.

0 SPECIFIC YIELD (DIMENSIONLESS)

0 FACTOR FOR STORAGE 1.000E+00

ELEM	VALUE	ELEM	VALUE	ELEM	VALUE	ELEM	VALUE	ELEM	VALUE
1	0.1	2	0.1	3	0.1	4	0.1	5	0.1
6	0.1	7	0.1	8	0.1	9	0.1	10	0.1
11	0.1	12	0.1	13	0.1	14	0.1	15	0.1
16	0.1	17	0.1	18	0.1	19	0.1	20	0.1
21	0.1	22	0.1	23	0.1	24	0.1		

0 BASE OF AQUIFER (FEET ABOVE DATUM)

NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	142.0	2	142.0	3	138.5	4	138.5	5	135.0
6	135.0	7	131.5	8	131.5	9	128.0	10	128.0
11	124.5	12	124.5	13	121.0	14	121.0	15	117.5
16	117.5	17	114.0	18	114.0	19	110.5	20	110.5
21	107.0	22	107.0	23	103.5	24	103.5	25	100.0
26	100.0								

0 INITIAL WATER LEVELS (FEET ABOVE DATUM)

NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	200.0	2	200.0	3	200.0	4	200.0	5	200.0
6	200.0	7	200.0	8	200.0	9	200.0	10	200.0
11	200.0	12	200.0	13	200.0	14	200.0	15	200.0
16	200.0	17	200.0	18	200.0	19	200.0	20	200.0
21	200.0	22	200.0	23	200.0	24	200.0	25	200.0
26	200.0								

0 CONSTANT-HEAD NODES

0 LEAKANCE FACTOR (FEET SQUARED PER DAY) 1.000E+00

NODE	HEAD (FEET ABOVE DATUM)
25	200.0
26	200.0

0 SOURCES AND SINKS (CUBIC FEET PER SECOND)

0 NUMBER OF SOURCE-SINK DATA SETS 1
FACTOR FOR SOURCES AND SINKS 1.000E+00

0 SOURCE-SINK DATA SET 1

NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	0.1013	2	0.1013	3	0.0000	4	0.0000	5	0.0000
6	0.0000	7	0.0000	8	0.0000	9	0.0000	10	0.0000
11	0.0000	12	0.0000	13	0.0000	14	0.0000	15	0.0000
16	0.0000	17	0.0000	18	0.0000	19	0.0000	20	0.0000
21	0.0000	22	0.0000	23	0.0000	24	0.0000	25	0.0000
26	0.0000								

```
0        TIME-STEP INDICATORS AND MULTIPLIERS
         ------------------------------------
0        STEP        SET    FACTOR (DIMENSIONLESS)

            1          1      1.000
            2          1      1.000
1        TIME STEP   1
         -------------
0        ELAPSED TIME (DAYS)        30.0
         NUMBER OF ITERATIONS          4
         WATER-LEVEL CHANGE ON LAST INERATION (FEET)     0.0262
0        COMPUTED WATER LEVELS (FEET ABOVE DATUM)
         ----------------------------------------
0        NODE        VALUE     NODE        VALUE     NODE        VALUE     NODE        VALUE     NODE        VALUE

            1      233.973        2      233.999        3      230.294        4      230.297        5      226.821
            6      226.820        7      223.545        8      223.544        9      220.450       10      220.449
           11      217.517       12      217.516       13      214.729       14      214.728       15      212.068
           16      212.067       17      209.517       18      209.516       19      207.058       20      207.056
           21      204.673       22      204.672       23      202.348       24      202.347       25      200.066
           26      200.065
0        AQUIFER THICKNESS (FEET)
         ------------------------
         NODE        VALUE     NODE        VALUE     NODE        VALUE     NODE        VALUE     NODE        VALUE

            1       91.998        2       92.025        3       91.801        4       91.805        5       91.819
            6       91.819        7       92.040        8       92.038        9       92.443       10       92.442
           11       93.011       12       93.010       13       93.724       14       93.723       15       94.565
           16       94.563       17       95.514       18       95.513       19       96.556       20       96.554
           21       97.672       22       97.671       23       98.848       24       98.846       25      100.066
           26      100.065
0        CONSTANT-HEAD NODES
         -------------------
0        CUMULATIVE RATE (CUBIC FEET PER SECOND)   -0.1312
0        NODE         RATE (CUBIC FEET PER SECOND)

           25      -0.0657
           26      -0.0654
1        TIME STEP   2
         -------------
0        ELAPSED TIME (DAYS)        60.0
         NUMBER OF ITERATIONS          4
         WATER-LEVEL CHANGE ON LAST INERATION (FEET)     0.0114
0        COMPUTED WATER LEVELS (FEET ABOVE DATUM)
         ----------------------------------------
0        NODE        VALUE     NODE        VALUE     NODE        VALUE     NODE        VALUE     NODE        VALUE

            1      240.134        2      240.138        3      236.588        4      236.589        5      233.083
            6      233.083        7      229.619        8      229.619        9      226.197       10      226.197
           11      222.817       12      222.817       13      219.477       14      219.477       15      216.175
           16      216.175       17      212.907       18      212.907       19      209.669       20      209.669
           21      206.458       22      206.457       23      203.267       24      203.267       25      200.092
           2[illegible]
```

0 AQUIFER THICKNESS (FEET)

NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	98.145	2	98.150	3	98.094	4	98.096	5	98.085
6	98.086	7	98.119	8	98.119	9	98.197	10	98.197
11	98.317	12	98.316	13	98.477	14	98.477	15	98.675
16	98.674	17	98.907	18	98.907	19	99.170	20	99.169
21	99.458	22	99.458	23	99.767	24	99.767	25	100.092
26	100.092								

0 CONSTANT-HEAD NODES

0 CUMULATIVE RATE (CUBIC FEET PER SECOND) -0.1837

0

NODE	RATE (CUBIC FEET PER SECOND)
25	-0.0919
26	-0.0918

Nevertheless, the basic approach is the same for problems of extremely different space and time scales, and this approach is illustrated in the following by the example of a regional appraisal of the upper Coachella Valley groundwater basin in California. Swain (1978) developed a model of that basin in order to evaluate the impact of artifical recharge on historically declining groundwater levels. To do that, he used a finite-element algoritm developed by G.F. Pinder (written commun., 1974, cited in Swain, 1978). To demonstrate utilization of the computer algorithm developed in the preceeding section, Swain's model of the upper Coachella Valley groundwater basin was reconstructed using that algorithm.

4.5.1. Geohydrologic Silting

The upper Coachella Valley (Fig. 4.23) is a 300 square mile area in Riverside County, Calif., in the northwestern part of the Salton Trough. However, The model was limited to the Whitewater River and Garnet Hill subbasins, which have an area of about 200 square miles.

As generalized by Tyley (1974), the lithology of the study area may be divided into three categories: unconsolidated deposits, semi-consolidated deposits, and consolidated rocks (Fig. 4.23). The consolidated rocks are non-water-bearing, and they form the base and southwestern boundary of the groundwater basin. The principal semiconsolidated deposits in the study area underlies the Indio Hills. These deposits generally have low permeability, yield only small quantities of water to well, and are considered poor aquifers. In constrast, the unconsolidated deposits yeild as much as 250 gal/min per foor of drawdown. The unconsolidated depostis consist of alluvial materials derived from the nearby terrain, and they are as much as 3,000 feet thick.

Faults along the northeast boundary of the study area separate te Whitewater River subbasin from other parts of the upper Coachella Valley groundwater basin. The Garnet Hill Fault is a barrier to groundwater movement, and water level differentials of as much as 400 ft. occur across the fault (Fig. 4.24). Likewise, water level differentials of as much as 280 feet occur across the San Andreas Fault. In each case, these faults effectively isolate adjacent subbasins.

Recharge to the groundwater basin is principally by streamflow infiltration and discharge is by pumping and underflow across the southern boundary of the study area. Because the average rainfall over the groundwater basin is less than 3 in/yr and potential evaporation is more than 9 ft/yr, no significant recharge occurs by direct infiltration of precipitation. However, higher precipitation in the adjacent mountain areas generates runoff that eventally flows onto the valley floor. In turn, deep percolation of seepage losses from stream channels produces 35,000 acre-ft/yr of groundwater recharge. Net pumpage (pumpage minus

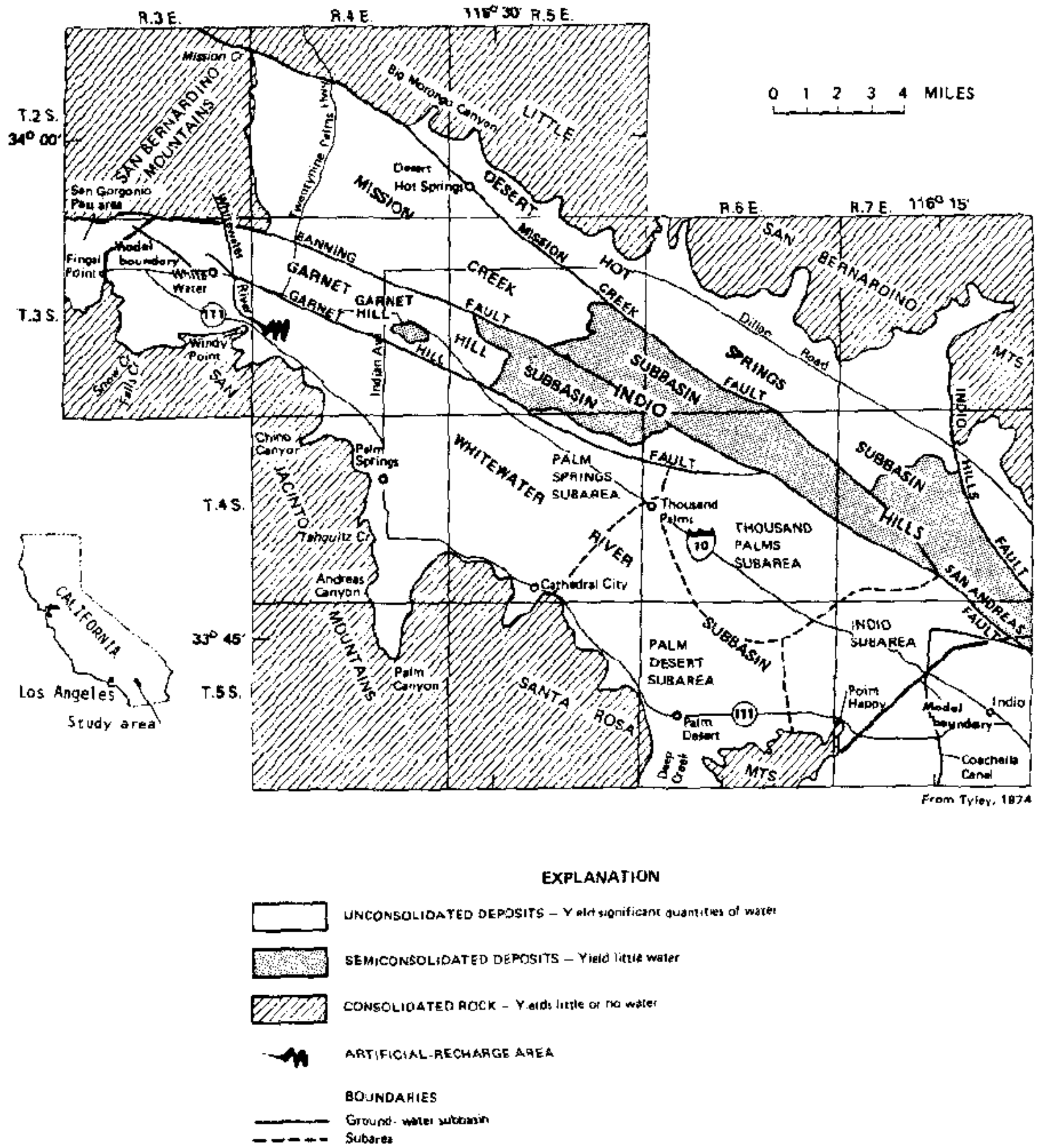


Fig. 4.23. Groundwater subbasins and generalized geology of the upper Coachella Valley, CA. (from Swain, 1978).

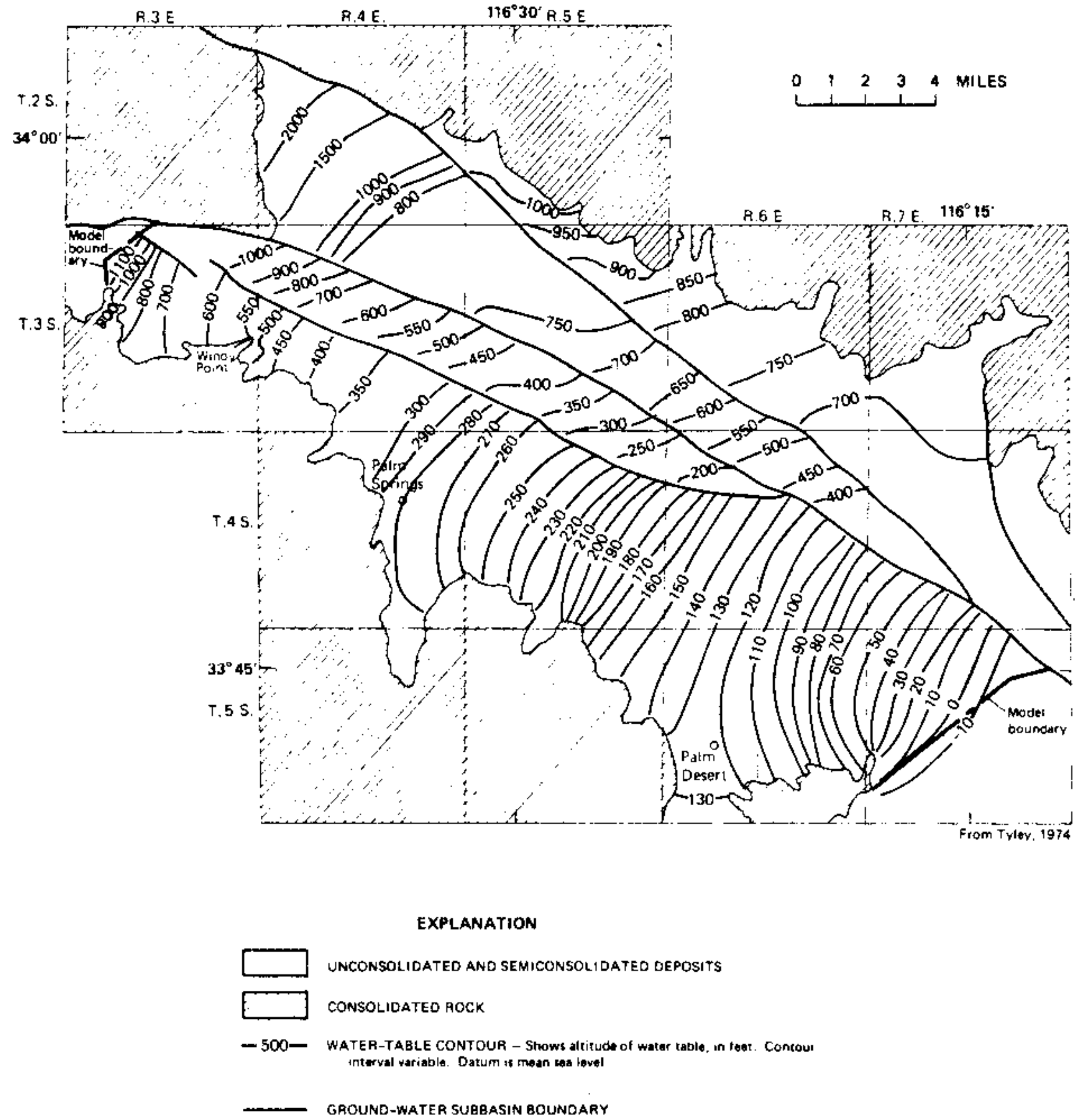


Fig. 4.24. *Water table contours, 1936, (from Swain, 1978).*

returns) has ranged from 5,000 acre-ft/yr in 1936 to 53,000 acre-ft/yr in 1973. Under flow across the northern boundary of the basin is 30,000 acre-ft/yr.

In response to increased pumping rates, groundwater levels declined more than 100 feet from 1936 to 1973 in some areas of the upper Coachella Valley. In an effort to counteract these delining water levels, local water agencies decided to artifically recharge the ground water basin with water imported from the Colorado River. Since 1973 this imported water has been recharged by discharging water into the usually dry channel of the Whitewater River, where it infiltrates into the permeable channel bed and eventually becomes groundwater recharge. The purpose of the groundwater model developed by Swain (1978) was to evaluate the impacts on groundwater levels of this recharge.

4.5.2. Groundwater Model

The upper Coachella groundwater basin was represented by the two-dimensional from of the equation of ground water flow, which is

$$\frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial h}{\partial y}\right) + N = S\frac{\partial h}{\partial t} \qquad (4.122)$$

where h is hydraulic head, T is transmissivity, S is storage coefficient, N is recharge or pumping, x and y are Cartesian coordinates, and t is time. This equation was solved by the finite-element algorithm described previously using the grid shown in Fig. 4.25. The grid includes 209 nodes and 524 elements.

The boundary conditions on the flow domain (Fig. 4.25), are specified flux and specified head. These conditions are expressed functionally as

$$\frac{\partial h}{\partial n} = 0 \qquad \text{on } \Gamma_1 \qquad (4.123a)$$

$$T\frac{\partial h}{\partial n} = q_b \qquad \text{on } \Gamma_2 \qquad (4.123b)$$

and

$$h = h_b \qquad \text{on } \Gamma_3 \qquad (4.123c)$$

where $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3$.

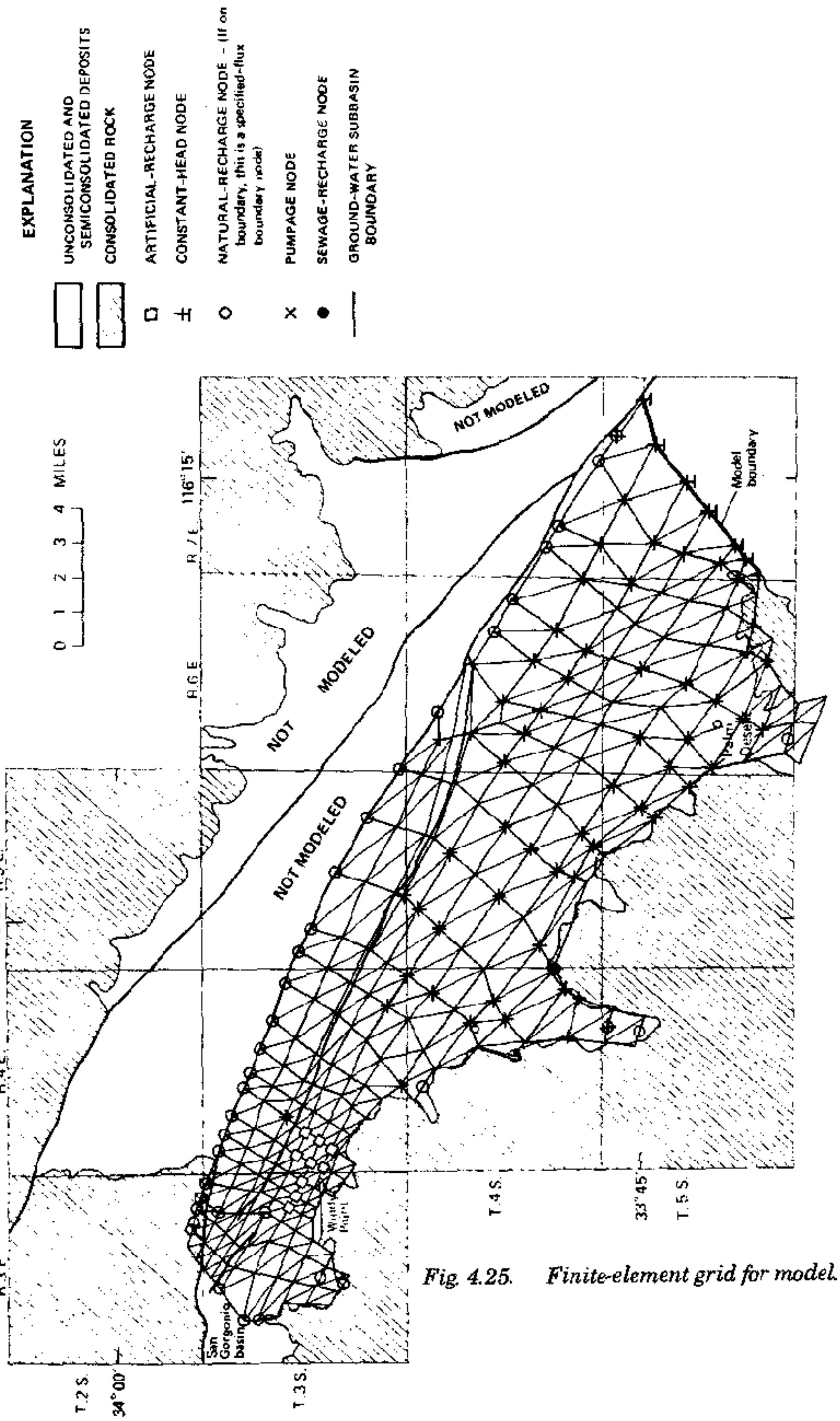


Fig. 4.25. Finite-element grid for model.

The no-flow boundary condition on Γ_1 represents the permeability contrast of the uconsolidated deposits with the non-water-bearing consolidated rock. The constant-discharge boundary condition Γ_2, represents underflow from the San Gorgonio basin or flow across the Banning and San Andreas Faults. The constant-head boundary condition on Γ_3 represents maintenance of groundwater levels by irrigation returns and leakage from canals.

Initial conditions for the model were derived from the model itself. As background, the model was calibrated on the steady-state condition that existed in 1936 and on the transient-state condition for the period 1936-1973. Then, the model was used to make predictions for the period 1973-2000. Computed steady-state water levels were used as the initial condition for the 1936-1974 simulation, and computed water levels for 1974 were used as initial conditions for the 1974-2000 simulation.

Swain (1978) used a subjective trial-and-error procedure to identify the transmissivity and storage coefficient distributions for this model. The steady-state condition was used to identify transmissivitiy given natural recharge and net pumping for the groundwater basin. By adjusting the transmissivity distribution, simulated water levels were within 4 feet of observed values throughout the flow domain of the model. Once transmissivity had been identified, the storage coefficient distribution was identified from the 1936-1973 transient-state condition. Figure 4.26 shows a comparison of hydrographs for simulated and observed water levels for the final values of aquifer parameters.

Having achieved a reasonable fit of the model to historically observed water levels in the groundwater basin, the model was used to examine the inpacts of artificial recharge. Fig.4.27 shows simulated water level declines for the period 1974-2000 without artificial recharge, and Fig. 4.28 shows simulated water level declines with artificial recharge of as much as 60,000 acre-ft/yr. As indicated by the model, water level declines are as much as 200 feet without artificial recharge, and water levels would rise as much as 320 feet with recharge.

4.5.3. Modeling Approaches

The modeling approach used by Swain (1978) for the upper Coachella Valley involved three basic steps. The first step is model characterization, which involved the basic definition of the groundwater model. An important decision in this regard was the selection of a two-dimensional representation of the ground water system. While the basin deposits are as much as 3,000 ft. in thickness, heads in general do not vary with depth. Another important decision was the selection of boundary conditions, which followed from an understanding of the geologic controls acting on the groundwater system. The selection of the equation to be solved, definition of the flow domain, and specification of boundary conditions constitute the characterization of the model.

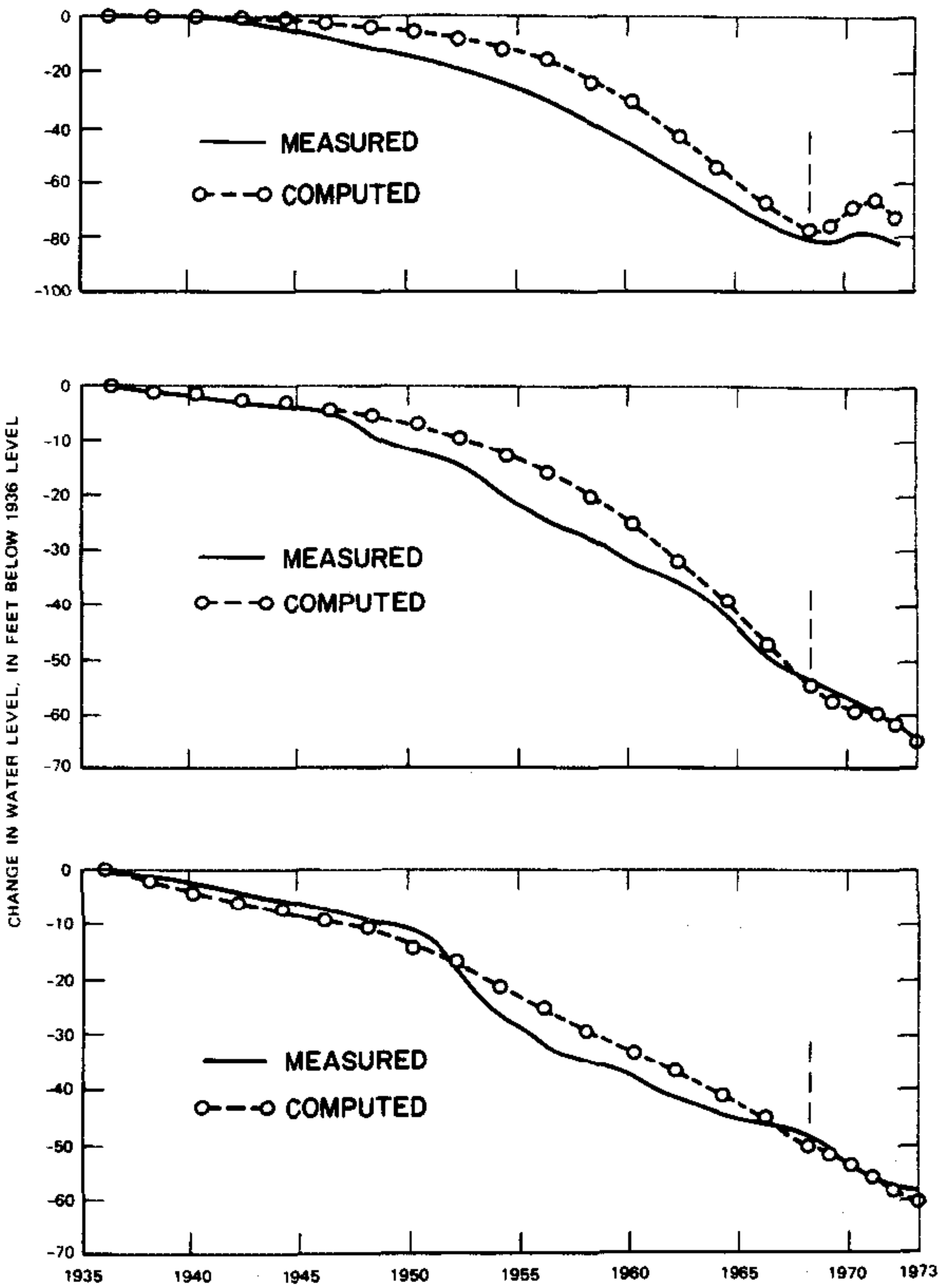


Fig. 4.26. Comparison of computed and measured hydrographs.

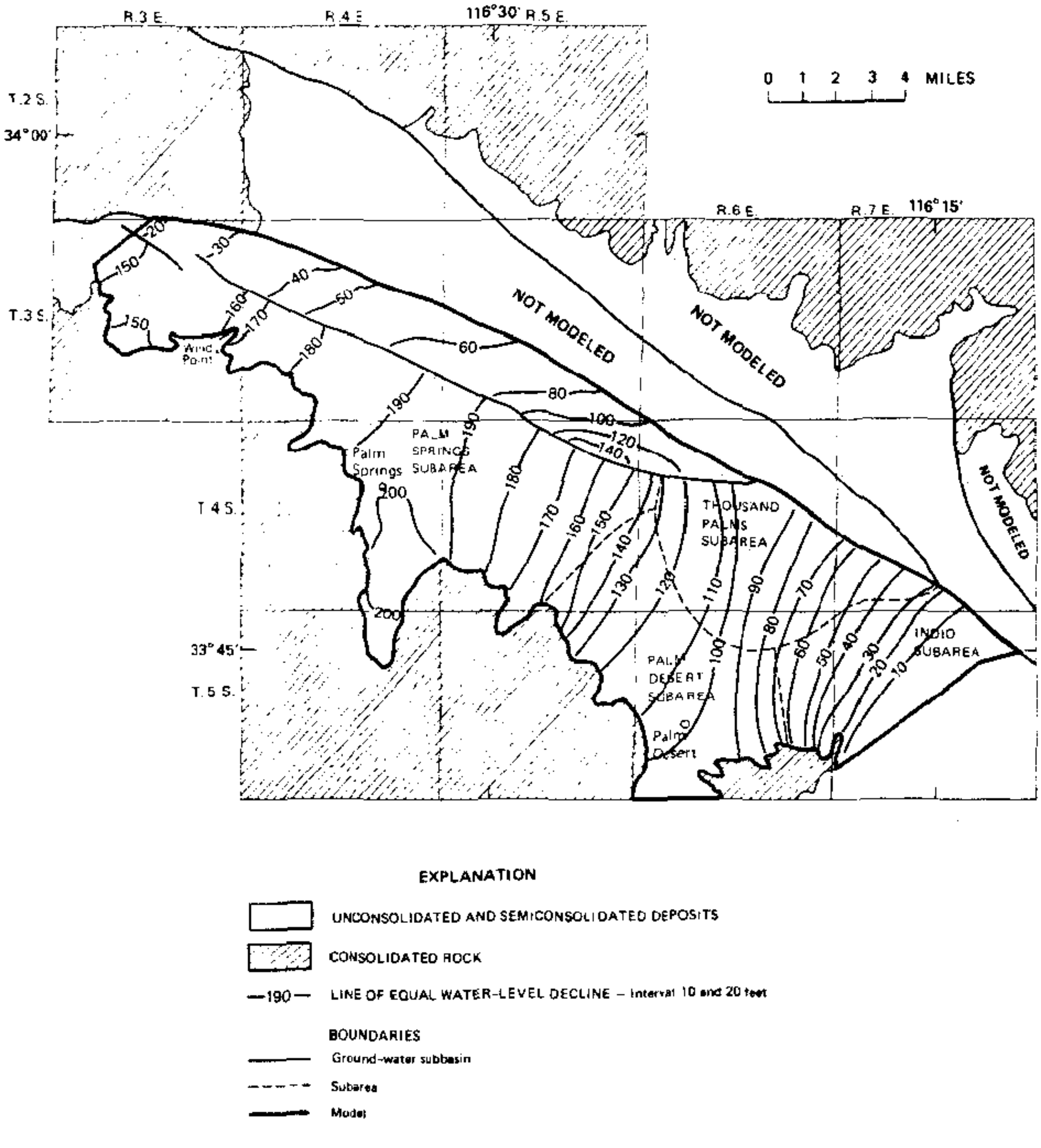


Fig. 4.27. Water level change, 1974-2000, without artificial recharge.

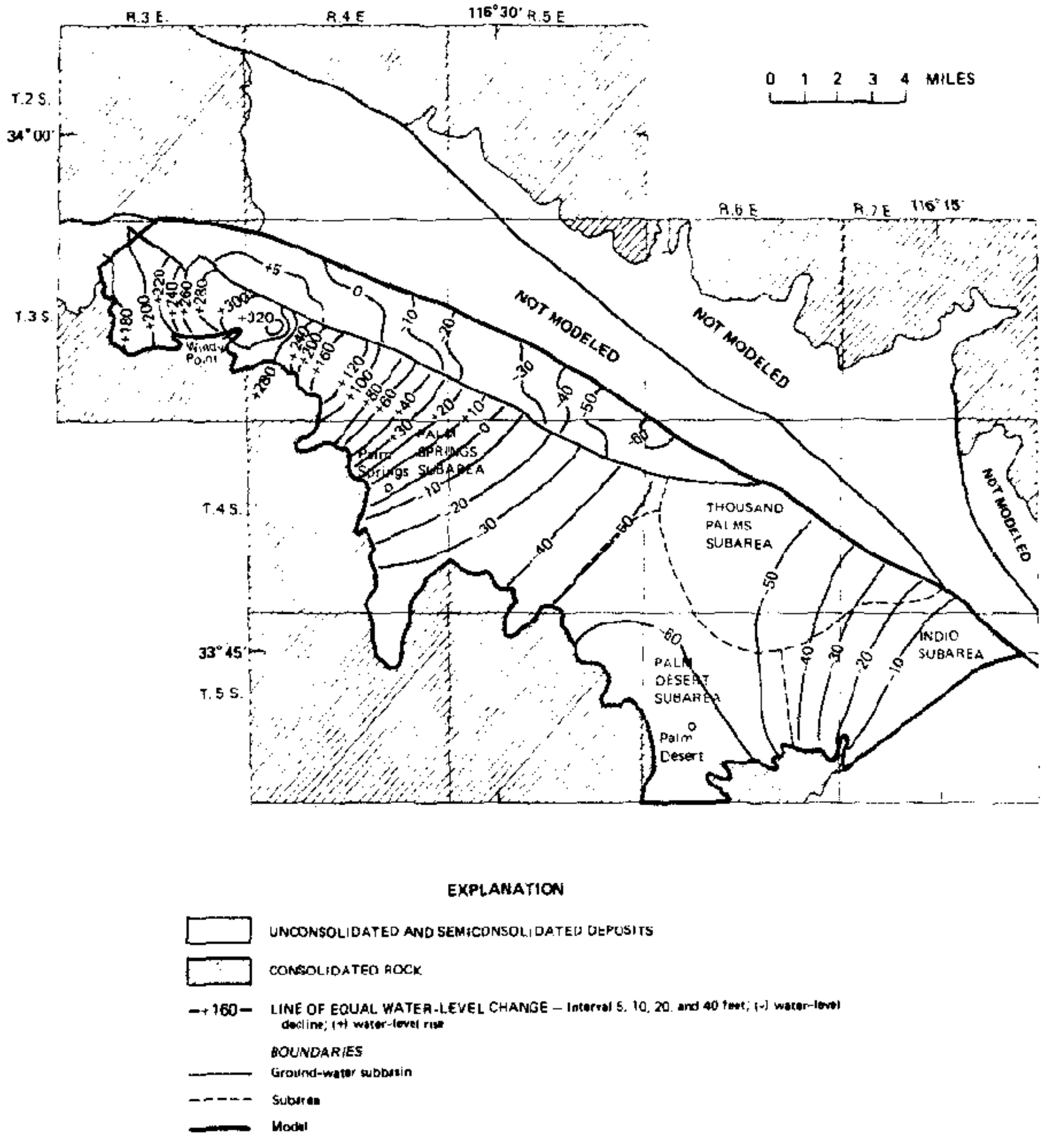


Fig. 4.28. Water level change, 1974-2000, with artificial recharge and projected pumpage.

Model characterization results in the specification of the basic form of the model, and the next step is to identify values for parameters of the model.

Therefore, the second step of model development is parameter identification, which involves estimating the geographic distribution of transmissivity and storage coefficient for the model. For the upper Coachella Valley, transmissivity was estimated from the pre-development steady-state condition. A trial and error subjective procedure was used to identify transmissivities such that the model reproduces as best as possible the observed steady-state water levels. To do this, information was needed on the recharge and discharge from the basin under steady-state conditions. Similarly, the post-development transient-state condition was used to identify storage coefficients by a trial and error procedure. To identify storage coefficients, information was needed on the recharge and pumpage for the groundwater basin over the period used for parameter identification.

The last step is to use the model to make predictions about the future under different management alternatives. For the upper Coachella Valley, the model was used to simulate water levels with or without artificial recharge. Once the model is constructed, it can be used to simulate conditions under a large number of different conditions. The alternatives that are actually simulated usually are limited only by the interests of the groundwater manager.

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CHAPTER 5

MODELING GROUNDWATER TRANSPORT

5.1. Introduction

Mathematical models of solute transport can be used to predict the movement of a contaminant in groundwater, and these models can be diffenentiated by the absence or occurrence of chemical or physical reactions. Most commonly, these models are used to simulate the movement of conservative constituents such as total dissolved solids or chlorides. In these cases, the assumption is made that chemical reactions of the solute occur with neither other dissolved constituents nor with the aquifer solid matrix. Models can and have been developed to simulate the movement of nonconservative constituents such as an unstable isotope that is subject to radioactive decay or a trace element that might be absorbed onto the aquifer matrix. These chemical reactions are incorporated into the model by a term of the transport equation that represents mathematically the governing chemistry of the situation.

Models of solute transport might also be differentiated by the degree of coupling between transport and hydrodynamic processes. That coupling occurs through the effects of differences in fluid density resulting from the presences of a dissolved constituent. For a dilute solution, the density of water and solute is essentially unaffected by the solute concentration, and the transport and hydrodynamic processes are completely decoupled. In that case, models of groundwater flow and solute transport can be developed separately. However, computations of groundwater velocity based upon the hydraulic model are used as input to the transport model. For concentrated solutions, groundwater velocities depend upon spatial varriations of fluid density caused by the solute concentration, and the hydraulic and transport equations must be solved simultaneously.

The objective of this chapter, which utilizes the concepts developed in Chapter 4, is to provide the theoretical background and applied tools necessary for the modeling of dilute solutions of conservative constituents for problems that can be treated in two dimensions. This is accomplished by first developing the partial-differential equation of solute transport in two dimensions. Then the finite-element method for the numerical solution of that equation is developed. Next, a generalized FORTRAN program is described for the finite-element method. Finally, the algorithm is applied to the solution of a regional problem of solute transport.

5.2. Equation of Solute Transport

5.2.1. Continuity Equation

Conservation of Mass

The transport of a solute in groundwater can be described by a statement for the conservation of mass for the solute and a constitutive relation for solute fluxes. For a control volume as shown in Fig. 5.1, the conservation of mass requires that the mass inflow to less the mass outflow from the volume equals the rate of mass accumulation within the control volume. This principal is given by the relation

$$I - 0 = \frac{\partial M}{\partial t} \tag{5.1}$$

where I is the mass inflow rate for the solute, 0 is the outflow rate, M is the mass of solute within the control volume, and t is time.

Considering again the control volume shown in Fig. 5.1, the inflow is

$$I = J_x \Delta y + J_y \Delta x + Nc' \Delta x \Delta y \tag{5.2}$$

where J_x and J_y are mass inflow rates per unit width of aquifer, N is the recharge rate to the aquifer, c' is the solute concentration of the recharge water, and Δx and Δy are widths of the control volume. Outflow from the control volume is

$$0 = \left(J_x + \frac{\partial J_x}{\partial x} \Delta x \right) \Delta y + \left(J_y + \frac{\partial J_y}{\partial y} \Delta y \right) \Delta x \tag{5.3}$$

The rate of storage change can be expressed in terms of the volume of the control volume and the concentration of solute per unit volume by the relation

$$\frac{\partial M}{\partial t} = \Delta x \Delta y \ell \partial \frac{\partial c}{\partial t} \tag{5.4}$$

where ℓ is the height of the control volume, c is the solute concentration, and θ is the porosity of the aquifer. The term $\Delta x \Delta y \ell \theta$ is the volume of water-filled matrix voids within the control volume.

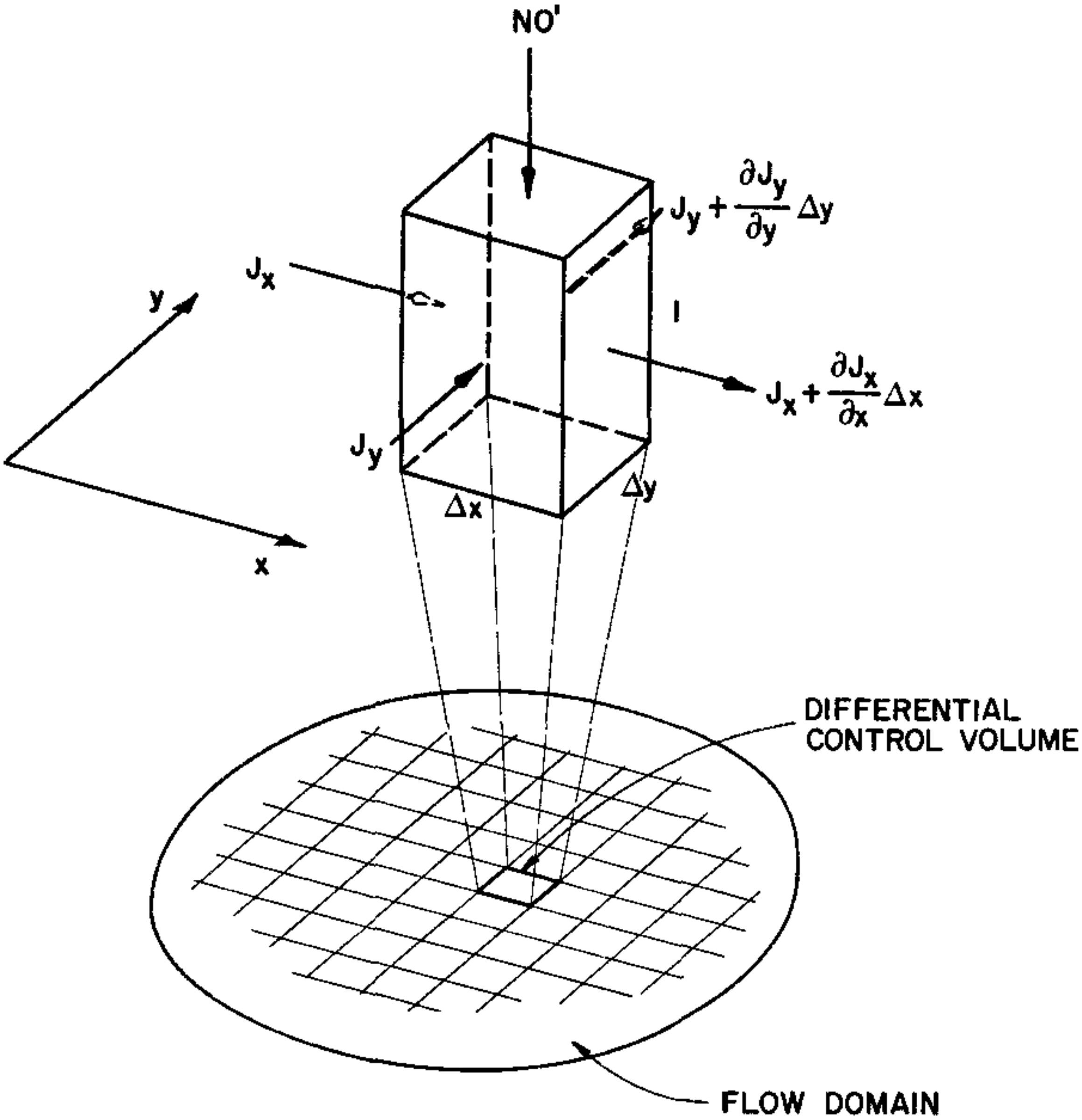

Fig. 5.1. Central volume extending through entire thickness of aquifer.

A partial differential equation stating the conservation of mass now can be constructed. Equations 5.2 to 5.4 can be substituted into Eq.5.1 to obtain

$$\begin{aligned} & J_x \Delta y + J_y \Delta x + Nc' \ \Delta x \ \Delta y \\ & - J_x \Delta y - \frac{\partial J_x}{\partial x} \Delta x \ \Delta y \\ & - J_y \Delta x - \frac{\partial J_y}{\partial y} \Delta x \ \Delta y = \Delta x \ \Delta y \ \ell\theta \frac{\partial c}{\partial t} \end{aligned} \qquad (5.5)$$

or by collecting terms,

$$- \frac{\partial J_x}{\partial x} \Delta x \ \Delta y - \frac{\partial J_y}{\partial y} \Delta x \ \Delta y + Nc' \ \Delta x \ \Delta y = \Delta x \ \Delta y \ \ell\theta \frac{\partial c}{\partial t} \qquad (5.6)$$

Dividing through by $\Delta x \ \Delta y$ produces the equation

$$- \frac{\partial J_x}{\partial x} - \frac{\partial J_y}{\partial y} + Nc' = \ell\theta \frac{\partial c}{\partial t} \qquad (5.7)$$

which is the desired equation.

However, Eq. 5.7 is expressed in terms of the three dependent variables J_x, J_y, and c, and two additional equations relating these variables are needed.

Solute Flux

These relations can be obtained by considering that solutes are transported in groundwater by the processes of advection and dispersion. Advection is the transport of solute by the moving groundwater. By this process, the soluted is moved from one position in the aquifer to another with the moving groundwater according to a trajectory determined by the velocity field within the aquifer. The concentration of solute remains unchanged within a moving elemental volume of water. In contrast, dispersion is the mixing of solute between elemental volumes of water owing to the combined processes of molecular diffusion and hydrodynamic dispersion. The concentration of solute within a moving elemental volume of water changes during the translation from one position to another. The flux of solute into or out of the elemental volume is proportional to the concentration gradient at the element boundaries.

In formalizing these concepts, it is helpful to consider first the one-dimensional case and then to extend the results to the two-dimensional case. For the one-dimensional case, the advective flux of solute is described by the term

$$Qc$$

where Q is the volumetric flux of water through a unit width of aquifer and c is the mass concentration of solute per unit volume of water. The dispersive flux of solute is described by the term

$$-\ell\theta D\frac{\partial c}{\partial x}$$

where D is the dispersion coefficient, ℓ is the aquifer thickness, θ is aquifer porosity, and $\partial c/\partial x$ is the concentration gradient. The negative sign arises because the flux of solute is in the direction of down the concentration gradient. The terms for advective and dispersive flux can be combined to obtain

$$J = Qc - \ell\theta D\frac{\partial c}{\partial x} \qquad (5.8)$$

As stated before, dispersion occurs by the combined processes of molecular diffusion and hydrodynamic dispersion. These processes are additive, and the dispersion coefficient of Eq. 5.8 can be expressed as

$$D = D_h + D_\tau \qquad (5.9)$$

where D_h is the coefficient of hydrodynamic dispersion and D_τ is the coefficient of molecular diffusion. The value of D_τ depends on the small-scale structure of the aquifer solid matrix. The value of D_h depends on the larger scale properties of the aquifer matrix and on the velocity of groundwater movement. Accordingly, D_h can be expressed as

$$D_h = \alpha|v| \qquad (5.10)$$

where α is the dispersivity of the aquifer matrix and v is groundwater velocity. In turn, the groundwater velocity is given by

$$v = \frac{Q}{\ell\theta} \qquad (5.11)$$

where v is the groundwater velocity within the pores of the aquifer matrix, Q is the discharge, ℓ is aquifer thickness, and θ is aquifer porosity. Equations 5.9 and 5.10 can be combined to obtain the expression

$$D = \alpha|V| + D_T \qquad (5.12)$$

In the two-dimensional case, it must be considered that the dispersion coefficient is a second rank tensor (Bear, 1972 and 1979) with principal direction parallel and perpendicular to the direction of groundwater flow. Concomitantly, Eq. 5.8 can be restated for the two-dimensional case as

$$J_x = Q_x\, c - \ell\theta\, D_{xx} \frac{\partial c}{\partial x} - \ell\theta\, D_{xy} \frac{\partial c}{\partial y} \qquad (5.13a)$$

and

$$J_y = Q_y\, c - \ell\theta\, D_{yy} \frac{\partial c}{\partial y} - \ell\theta\, D_{yx} \frac{\partial c}{\partial x} \qquad (5.13b)$$

where Q_x and Q_y are the x and y components of groundwater discharge per unit width of aquifer and D_{xx}, D_{xy}, D_{yx}, and D_{yy} are components of the dispersion tensor.

From properties of second rank tensors (Black, 1962), if the coordinate system is aligned with the local vector of the groundwater velocity field, then $D_{xy} = 0$ and $D_{yx} = 0$. In this case the dispersion coefficient tensor is given by

$$[D'] = \begin{bmatrix} D_L & 0 \\ 0 & D_T \end{bmatrix} \qquad (5.14)$$

where the velocity vector is in the direction of the positive x axis of the coordinate system, D_L is the longitudinal dispersion coefficient, and D_T is the transverse dispersion coefficient. The prime indicates that the coordinate system has been rotated to be aligned with the local velocity vector.

The tensor [D'] represents a statement of the basic physics of the situation. Those physics must be invariant by the choice of a coordinate system, and this logic can be used to derive [D] from [D'] as follows: the dispersion flux of solute in the x' – y' coordinate system is given by

$$\begin{bmatrix} J^*_{x'} \\ J^*_{y'} \end{bmatrix} = -\,[D'] \begin{bmatrix} \dfrac{\partial c}{\partial x'} \\ \dfrac{\partial c}{\partial y'} \end{bmatrix} \qquad (5.15)$$

where $J^*_{x'}$ and $J^*_{y'}$ are dispersive fluxes in the x' − y' coordinate system. However, the flux and concentration gradient of Eq. 5.15 can be expressed in terms of the x − y coordinate system as

$$\begin{bmatrix} J^*_{x'} \\ J^*_{y'} \end{bmatrix} = [R] \begin{bmatrix} J^*_{x} \\ J^*_{y} \end{bmatrix} \tag{5.16}$$

and

$$\begin{bmatrix} \dfrac{\partial c}{\partial x'} \\ \dfrac{\partial c}{\partial y'} \end{bmatrix} = [R] \begin{bmatrix} \dfrac{\partial c}{\partial x} \\ \dfrac{\partial c}{\partial y} \end{bmatrix} \tag{5.17}$$

where [R] is simply the rotation matrix for the rotation from the x − y to the x' − y ' coordinates. The rotation matrix is given by

$$[R] = \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix} \tag{5.18}$$

where β is the counter clockwise angle from the x − y to the x' − y' system. Equations 5.16 and 5.17 can be substituted into Eq. 5.15 to obtain

$$[R] \begin{bmatrix} J^*_{x} \\ J^*_{y} \end{bmatrix} = - [D'] \; [R] \begin{bmatrix} \dfrac{\partial c}{\partial x} \\ \dfrac{\partial c}{\partial y} \end{bmatrix} \tag{5.19}$$

By rearrangement,

$$\begin{bmatrix} J^*_{x} \\ J^*_{y} \end{bmatrix} = - [R]^{-1} \; [D'] \; [R] \begin{bmatrix} \dfrac{\partial c}{\partial x} \\ \dfrac{\partial c}{\partial y} \end{bmatrix} \tag{5.20}$$

Then, to preserve the invariant physics,

$$[D] = [R]^{-1}[D'] \ [R] \tag{5.21}$$

where the inverse of the rotation matrix is

$$[R]^{-1} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \tag{5.22}$$

Equation 5.21 is the desired expression for [D] in terms of [D'].

A working expression for [D] can be obtained by substitution of Eqs. 5.14, 5.18, and 5.22 into Eq. 5.21. That substitution yields

$$D_{xx} = D_L \cos^2\beta + D_T \sin^2\beta \tag{5.23a}$$

$$D_{yy} = D_L \sin^2\beta + D_T \cos^2\beta \tag{5.23b}$$

$$D_{xy} = (D_L - D_T) \sin\beta \cos\beta \tag{5.23c}$$

and

$$D_{yx} = D_{xy} \tag{5.23d}$$

However,

$$\cos\beta = \frac{V_x}{V} \tag{5.24a}$$

and

$$\sin\beta = \frac{V_y}{V} \tag{5.24b}$$

Therefore, Eqs. 5.23 a-d can be restated as

$$D_{xx} = D_L \frac{V_x V_x}{V^2} + D_T \frac{V_y V_y}{V^2} \tag{5.25a}$$

$$D_{yy} = D_L \frac{V_y V_y}{V^2} + D_T \frac{V_x V_x}{V^2} \tag{5.25b}$$

$$D_{xy} = (D_L - D_T)\frac{v_x\ v_y}{v^2} \tag{5.25c}$$

and

$$D_{yx} = D_{xy} \tag{5.25d}$$

where, with analogy to Eq. 5.12,

$$D_L = \alpha_L |v| + D_\tau \tag{5.26a}$$

and

$$D_T = \alpha_T |v| + D_\tau \tag{5.26b}$$

where α_L and α_T are the longitudinal and transverse dispersivities of the aquifer matrix.

Governing Equation

Returning to Eq. 5.7, which was

$$-\frac{\partial J_x}{\partial x} - \frac{\partial J_y}{\partial y} + Nc' = \ell\theta\frac{\partial c}{\partial t} \tag{5.7}$$

Eqs. 5.13 can used to obtain the governing equation for solute transport in groundwater. By substituting Eqs. 5.13 into Eq. 5.7, we obtain

$$\frac{\partial}{\partial x}\left(\ell\theta D_{xx}\frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial x}\left(\ell\theta D_{xy}\frac{\partial c}{\partial y}\right)$$

$$+ \frac{\partial}{\partial y}\left(\ell\theta D_{yy}\frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial y}\left(\ell\theta D_{yx}\frac{\partial c}{\partial x}\right)$$

$$- \frac{\partial}{\partial x}(Q_x\ c) - \frac{\partial}{\partial y}(Q_y\ c) \tag{5.27}$$

$$+ Nc' = \ell\theta\frac{\partial c}{\partial t}$$

This is the transport equation for the advective-dispersive transport of a dilute dissolved solute in moving groundwater.

5.2.2. Mathematical Definition of the Transport Problem

Equation 5.27 describes the conservation of solute mass at a point in the groundwater system. To solve this equation, additional specification of the transport problem is needed. As for the groundwater problem, the solution of the transport problem requires definition of the solution domain, aquifer parameters, solute sources and sinks, and boundary and initial conditions.

Solution Domains

The solution domain is the space-time domain within which the solute-transport problem is to be solved. In general, the transport problem will have the same solution domain as established for the corresponding groundwater problem, as described in Section 4.2.2.

Aquifer Parameters

Aquifer parameters are the aquifer porosity, dispersivity, thickness, and groundwater velocity distributions over the flow domain. Porosity is usually estimated from analysis of geologic logs from wells. However, very often initial estimates of porosity from field data are modified during the process of calibrating the transport model. By this process, porosity values used in the model are modified so that the model provides a better representation of field observations of solute concentrations. Dispersivity values also can be estimated from field tests, but in most cases better values are obtained from the model calibration. The problem in estimating dispersivity from field tests is that dispersivity is a scale-dependent property of an aquifer. Consequently, results obtained from tracer tests or column experiments cannot be interpreted in terms of larger scale processes occurring in an aquifer.

Aquifer thickness is generally estimated from field data. Thickness can be estimated from geologic logs on wells that penetrate the entire thickness of the aquifer. However, in many situations such data are not adequate to define the geographic distribution of thickness. When this is the case, aquifer thickness can be estimated by other methods such as gravity surveys, electrical-resistivity soundings, and seismic soundings (Dobrin, 1960).

Groundwater velocity is obtained computationally from the groundwater model. Groundwater discharge and velocity within an aquifer are related to water-level gradients and transmissivity by the relations

$$Q_x = - K\ell \frac{\partial h}{\partial x} \tag{5.28a}$$

$$Q_y = -K\ell\frac{\partial h}{\partial y} \tag{5.28b}$$

$$v_x = \frac{Q_x}{\theta\ell} \tag{5.29a}$$

$$v_y = \frac{Q_y}{\theta\ell} \tag{5.29b}$$

where Q_x and Q_y are the x and y components of groundwater discharge, v_x and v_y are the x and y components of groundwater velocity, K is hydraulic conductivity, ℓ is aquifer thickness, θ is porosity, and $\partial h/\partial x$ and $\partial h/\partial y$ are the x and y components of the groundwater hydraulic gradient. The groundwater gradient conductivity, and thickness can be obtained from values used in or computed from the groundwater model.

Boundary and Initial Conditions

As for the transient-state groundwater equation, the transport equation is a parabolic partial-differential equation. As such, a solution to Eq. 5.27 can be obtained only if boundary conditions are specified everywhere on the boundary of the flow domain, and initial conditions are specified everywhere within the flow domain.

Boundary conditions are a mathematical statement of the conditions on the boundary of the flow domain. As for the groundwater problem, these conditions are of two types: specified-flux and specified-concentration boundaries. A specified-flux boundary is described by the relation

$$\left(Q_x\, c - D_{xx}\frac{\partial c}{\partial x} - D_{xy}\frac{\partial c}{\partial y}\right) n_x + \left(Q_y\, c - D_{yy}\frac{\partial c}{\partial y} - D_{yx}\frac{\partial c}{\partial x}\right) n_y = J_b\,(\Gamma,\ t) \tag{5.30}$$

where J_b is the solute flux normal to the boundary. The boundary solute flux can be a function of position and time. The specified-concentration boundary condition is given by the relation

$$c = c_b(\Gamma, t) \tag{5.31}$$

where c_b is the specified concentration, and the specified concentration on the boundary can be a function of position and time.

The specified-flux boundary condition is most often encountered in field applications. From the model of groundwater flows, the fluxes of water are known for the boundary of the flow domain. This is the case even if specified-head boundaries are prescribed for the groundwater model. Given the water flux for the boundary, the solute flux can be determined by specifying the solute concentration in the water crossing the boundary. If water moves out of the flow domain, the specified concentration is the solute in the groundwater within the flow domain. If water moves into the flow domain, the specified concentration is the solute concentration in the water moving into the flow domain.

By this approach, the specified-flux boundary is given by the expression

$$Q_n c_b = J_b(\Gamma, t) \tag{5.32}$$

where Q_n is the water flux normal to the boundary, c_b is the specified solute concentration in groundwater crossing the boundary, and J_b is the solute flux normal to the boundary. Equation 5.32 assumes that the diffusive flux across the boundary is zero. In practical applications, this assumption is useful for simplifying the specification of boundary conditions, and the assumption usually has little impact on the computed concentrations within the flow domain.

Initial conditions are the specification of solute concentrations over the flow domain at the start of the time domain. For each point in the flow domain, solute concentrations must be specified. These concentrations are sometimes obtained from maps showing contours of equal concentrations in groundwater based upon chemical analyses of water samples taken from wells. However, it is often convenient and appropriate to assume equal solute concentation over the flow domain. Consequently, simulated concentration changes result only from exogenous sources and sinks acting after the start of the simulation period.

Sources and Sinks

Sources and sinks are the specification of solute flux into or from the flow domain with the recharge or discharge of water to or from the groundwater system. For groundwater recharge, the solute flux is given by the source term of Eq. 5.27, which is

$$Nc'$$

where N is the recharge rate and c' is the solute concentration of the recharge water. For groundwater discharge, the solute flux is given by the term

$$Nc$$

where c is the solute concentration of water in the aquifer. Water discharging from the aquifer in a well, for example, will discharge water with a solute concentration equal to the ambient concentration in the aquifer.

5.3. Finite-Element Method

5.3.1. Galerkin Method

Equation 5.27 is to be solved by the Galerkin method. The first step in the solution is to define the trial solution

$$c \approx \hat{c} = \sum_{i=1}^{n} C_i \phi_i \tag{5.33}$$

where $\hat{c}$ is a series approximation to c, ϕ_i are linearly independent trial functions that are defined piecewise on triangular elements as described in Chapter 4 by Eqs. 4.49, and C_i are unknown coefficients. The second step is to formulate the integral equations by substituting

$$R = \frac{\partial}{\partial x}\left(\ell\theta D_{xx}\frac{\partial \hat{c}}{\partial x}\right) + \frac{\partial}{\partial x}\left(\ell\theta D_{xy}\frac{\partial \hat{c}}{\partial y}\right)$$

$$+ \frac{\partial}{\partial y}\left(\ell\theta D_{yy}\frac{\partial \hat{c}}{\partial y}\right) + \frac{\partial}{\partial y}\left(\ell\theta D_{yx}\frac{\partial \hat{c}}{\partial x}\right)$$

$$- \frac{\partial}{\partial x}(Q_x \hat{c}) - \frac{\partial}{\partial y}(Q_y \hat{c}) \tag{5.34}$$

$$+ Nc' - \ell\theta\frac{\partial \hat{c}}{\partial t}$$

into Eq. 4.24 to obtain

$$\iint_{\Omega} \Bigg[\frac{\partial}{\partial x} \left(\ell\theta D_{xx} \frac{\partial \hat{c}}{\partial x} \right) + \left(\ell\theta D_{xy} \frac{\partial \hat{c}}{\partial y} \right) + \frac{\partial}{\partial y} \left(\ell\theta D_{yy} \frac{\partial \hat{c}}{\partial y} \right) + \frac{\partial}{\partial y} \left(\ell\theta D_{yx} \frac{\partial \hat{c}}{\partial x} \right) - \frac{\partial}{\partial x} (Q_x\ \hat{c}) - \frac{\partial}{\partial y} (Q_y\ \hat{c}) + Nc' - \ell\theta \frac{\partial \hat{c}}{\partial t} \Bigg] \phi_i \, dx\, dy = 0 \qquad (5.35)$$

$$(i = 1,2,\cdots,n)$$

The third step is to apply integration by parts so as to reduce the order of Eq. 5.35. Integration by parts can be applied to the diffusive terms in the form

$$\iint_{\Omega} \Bigg[D_{xx} \frac{\partial^2 \hat{c}}{\partial x^2} \phi_i + D_{xy} \frac{\partial^2 \hat{c}}{\partial x \partial y} \phi_i + D_{yy} \frac{\partial^2 \hat{c}}{\partial y^2} \phi_i + D_{yx} \frac{\partial^2 \hat{c}}{\partial y \partial x} \phi_i \Bigg] dxdy =$$

$$- \iint_{\Omega} \Bigg[D_{xx} \frac{\partial \hat{c}}{\partial x} \frac{\partial \phi_i}{\partial x} + D_{xy} \frac{\partial \hat{c}}{\partial x} \frac{\partial \phi_i}{\partial y} + D_{yy} \frac{\partial \hat{c}}{\partial y} \frac{\partial \phi_i}{\partial y} + D_{yx} \frac{\partial \hat{c}}{\partial y} \frac{\partial \phi_i}{\partial x} \Bigg] dxdy$$

$$+ \int_{\Gamma} \Bigg[\left(D_{xx} \frac{\partial \hat{c}}{\partial x} + D_{xy} \frac{\partial \hat{c}}{\partial y} \right) n_x + \left(D_{yy} \frac{\partial \hat{c}}{\partial y} + D_{yx} \frac{\partial \hat{c}}{\partial x} \right) n_y \Bigg] d\Gamma \qquad (5.36)$$

where the boundary integral represents the diffusive solute flux across the boundary. Integration by parts may also be applied to the advective terms of Eq. 5.35 in the form

$$\iint_{\Omega} \left[\frac{\partial}{\partial x} (Q_x\ \hat{c}) - \frac{\partial}{\partial y} (Q_y\ \hat{c}) \right] \phi_i \ dxdy =$$

$$- \iint_{\Omega} \left[Q_x\ \hat{c} \frac{\partial \phi_i}{\partial x} + Q_y\ \hat{c} \frac{\partial \phi_i}{\partial y} \right] dxdy \qquad (5.37)$$

$$+ \int_{\Gamma} (Q_x\ \hat{c}\ n_x + Q_y\ \hat{c}\ n_y)\ d\Gamma$$

where the boundary integral represents the advective flux across the boundary.

Equations 5.36 and 5.37 can be used to reduce the order of Eq. 5.35 by their substitution into Eq. 5.35. That substitution yields

$$\iint_{\Omega} \ell\theta \left[D_{xx} \frac{\partial \hat{c}}{\partial x} \frac{\partial \phi_i}{\partial x} + D_{xy} \frac{\partial \hat{c}}{\partial x} \frac{\partial \phi_i}{\partial y} \right.$$

$$\left. + D_{yy} \frac{\partial \hat{c}}{\partial y} \frac{\partial \phi_i}{\partial y} + D_{yx} \frac{\partial \hat{c}}{\partial y} \frac{\partial \phi_i}{\partial x} \right] dxdy - \iint_{\Omega} \left[Q_x\ \hat{c} \frac{\partial \phi_i}{\partial x} + Q_y\ \hat{c} \frac{\partial \phi_i}{\partial y} \right] dxdy$$

$$- \iint_{\Omega} Nc'\phi_i \ dxdy + \iint_{\Omega} \ell\phi \frac{\partial \hat{c}}{\partial t} \phi_i \ dxdy$$

$$- \int_{\Gamma} \left[\left(D_{xx} \frac{\partial \hat{c}}{\partial x} + D_{yx} \frac{\partial \hat{c}}{\partial y} \right) n_x + \left(D_{yy} \frac{\partial \hat{c}}{\partial y} + D_{yx} \frac{\partial \hat{c}}{\partial x} \right) n_y \right] \phi_i d\Gamma$$

$$+ \int_{\Gamma} (Q_x\ \hat{c}\ n_x + Q_y\ \hat{c}\ n_y)\ \phi_i d\Gamma = 0$$

$$(i = 1, 2, \cdots, n) \qquad (5.38)$$

The combined diffusive and advective solute flux across the boundary appears in the two line integrals.

The fourth step is to substitute the trial solution

$$\hat{c} = \sum_{j=1}^{n} C_j \phi_j \qquad (5.33)$$

into Eq. 5.38. That substitution yields

$$\sum_{j=1}^{n} \iint_{\Omega} \ell\theta \left[D_{xx} \frac{\partial \phi_j}{\partial x} \frac{\partial \phi_i}{\partial x} + D_{xy} \frac{\partial \phi_j}{\partial x} \frac{\partial \phi_i}{\partial y} + D_{yy} \frac{\partial \phi_j}{\partial y} \frac{\partial \phi_i}{\partial x} + D_{yx} \frac{\partial \phi_j}{\partial y} \frac{\partial \phi_i}{\partial x} \right] c_j \, dxdy$$

$$- \sum_{j=1}^{n} \iint_{\Omega} \left[Q_x \phi_j \frac{\partial \phi_i}{\partial x} + Q_y \phi_j \frac{\partial \phi_i}{\partial y} \right] c_j \, dxdy$$

$$- \iint_{\Omega} Nc' \phi_i \, dxdy + \sum_{j=1}^{n} \iint_{\Omega} \ell\theta \frac{\partial c_j}{\partial t} \phi_j \phi_i \, dxdy$$

$$- \int_{\Gamma} \left[\left(D_{xx} \frac{\partial \hat{c}}{\partial x} + D_{yx} \frac{\partial \hat{c}}{\partial y} \right) n_x + \left(D_{yy} \frac{\partial \hat{c}}{\partial y} + D_{yx} \frac{\partial \hat{c}}{\partial x} \right) n_y \right] \phi_i d\Gamma$$

$$+ \int_{\Gamma} (Q_x \hat{c}\, n_x + Q_y \hat{c}\, n_y) \, \phi_i d\Gamma = 0 \qquad (5.39)$$

$$(i = 1,2,\cdots,n)$$

However, Eq. 5.39 can be expressed in matrix notation as

$$[A]\,\{C\} + [B] \left\{ \frac{dC}{dt} \right\} = \{F\} \qquad (5.40)$$

where [A] and [B] are [n × n] dimensional matrices and $\{C\}$, and $\{\partial C/\partial t\}$ are [n × 1] dimensional vectors. Typical elements of [A], [B], and $\{F\}$ are

$$A_{ij} = \iint_{\Omega} \ell\theta \left[D_{xx} \frac{\partial\phi_j}{\partial x} \frac{\partial\phi_i}{\partial x} + D_{xy} \frac{\partial\phi_j}{\partial x} \frac{\partial\phi_i}{\partial y} \right. \tag{5.41a}$$

$$\left. + D_{yy} \frac{\partial\phi_j}{\partial y} \frac{\partial\phi_i}{\partial y} + D_{yx} \frac{\partial\phi_j}{\partial y} \frac{\partial\phi_i}{\partial x} \right] dxdy$$

$$- \iint_{\Omega} \left[Q_x\phi_j \frac{\partial\phi_i}{\partial x} + Q_y\phi_j \frac{\partial\phi_i}{\partial y} \right] dxdy$$

$$B_{ij} = \iint_{\Omega} \ell\theta\phi_j\phi_i dxdy \tag{5.41b}$$

and

$$F_i = \iint_{\Omega} Nc'\phi_i dxdy + \int_{\Gamma} \left[\left(D_{xx} \frac{\partial\hat{c}}{\partial x} + D_y \frac{\partial\hat{c}}{\partial y} \right) n_x \right.$$

$$\left. + \left(D_{yy} \frac{\partial\hat{c}}{\partial y} + D_{yx} \frac{\partial\hat{c}}{\partial x} \right) n_y \right] \phi_i d\Gamma$$

$$- \int_{\Gamma} \left(Q_x\hat{c}\, n_x + Q_y\hat{c}\, n_y \right) \phi_i d\Gamma \tag{5.41c}$$

5.3.2. Assembly of Solution

In order to generate the set of ordinary-differential equations represented by Eq. 5.40, it is necessary to perform integrations of the trial functions of the form

$$\iint_\Omega \frac{\partial \phi_j}{\partial x} \frac{\partial \phi_i}{\partial x} dxdy \ ,$$

$$\iint_\Omega \phi_j \frac{\partial \phi_i}{\partial x} dxdy \ ,$$

$$\iint_\Omega \phi_j \phi_i dxdy \ ,$$

and

$$\iint_\Gamma \phi_i \phi \Gamma \ .$$

In the following these integrations are described on an elemental basis for linear trial functions on triangular elements. The trial functions are described in Chapter 4 by Eqs. 4.49. Also described in Chapter 4 is the rationale for performing the integrations on an elemental basis.

Stiffness matrix [A].— A typical elemental stiffness matrix $[A^e]$ will be of the form

$$[A^e] = \ell\theta D_{xx} \iint_e \begin{bmatrix} \frac{\partial \phi_1^e}{\partial x}\frac{\partial \phi_1^e}{\partial x} & \frac{\partial \phi_1^e}{\partial x}\frac{\partial \phi_2^e}{\partial x} & \frac{\partial \phi_1^e}{\partial x}\frac{\partial \phi_3^e}{\partial x} \\ \frac{\partial \phi_2^e}{\partial x}\frac{\partial \phi_1^e}{\partial x} & \frac{\partial \phi_2^e}{\partial x}\frac{\partial \phi_2^e}{\partial x} & \frac{\partial \phi_2^e}{\partial x}\frac{\partial \phi_3^e}{\partial x} \\ \frac{\partial \phi_3^e}{\partial x}\frac{\partial \phi_1^e}{\partial x} & \frac{\partial \phi_3^e}{\partial x}\frac{\partial \phi_2^e}{\partial x} & \frac{\partial \phi_3^e}{\partial x}\frac{\partial \phi_3^e}{\partial x} \end{bmatrix} dxdy$$

$$+ \ell\theta \, D_{xy} \iint_e \begin{bmatrix} \frac{\partial\phi_1^e}{\partial x}\frac{\partial\phi_1^e}{\partial y} & \frac{\partial\phi_1^e}{\partial x}\frac{\partial\phi_2^e}{\partial y} & \frac{\partial\phi_1^e}{\partial x}\frac{\partial\phi_3^e}{\partial y} \\ \frac{\partial\phi_2^e}{\partial x}\frac{\partial\phi_1^e}{\partial y} & \frac{\partial\phi_2^e}{\partial x}\frac{\partial\phi_2^e}{\partial y} & \frac{\partial\phi_2^e}{\partial x}\frac{\partial\phi_3^e}{\partial y} \\ \frac{\partial\phi_3^e}{\partial x}\frac{\partial\phi_1^e}{\partial y} & \frac{\partial\phi_3^e}{\partial x}\frac{\partial\phi_2^e}{\partial y} & \frac{\partial\phi_3^e}{\partial x}\frac{\partial\phi_3^e}{\partial y} \end{bmatrix} dxdy$$

$$+ \ell\theta \, D_{yy} \iint_e \begin{bmatrix} \frac{\partial\phi_1^e}{\partial y}\frac{\partial\phi_1^e}{\partial y} & \frac{\partial\phi_1^e}{\partial y}\frac{\partial\phi_2^e}{\partial y} & \frac{\partial\phi_1^e}{\partial y}\frac{\partial\phi_3^e}{\partial y} \\ \frac{\partial\phi_2^e}{\partial y}\frac{\partial\phi_1^e}{\partial y} & \frac{\partial\phi_2^e}{\partial y}\frac{\partial\phi_2^e}{\partial y} & \frac{\partial\phi_2^e}{\partial y}\frac{\partial\phi_3^e}{\partial y} \\ \frac{\partial\phi_3^e}{\partial y}\frac{\partial\phi_1^e}{\partial y} & \frac{\partial\phi_3^e}{\partial y}\frac{\partial\phi_2^e}{\partial y} & \frac{\partial\phi_3^e}{\partial y}\frac{\partial\phi_3^e}{\partial y} \end{bmatrix} dxdy$$

$$+ \ell\theta \, D_{yx} \iint_e \begin{bmatrix} \frac{\partial\phi_1^e}{\partial y}\frac{\partial\phi_1^e}{\partial x} & \frac{\partial\phi_1^e}{\partial y}\frac{\partial\phi_2^e}{\partial x} & \frac{\partial\phi_1^e}{\partial y}\frac{\partial\phi_3^e}{\partial x} \\ \frac{\partial\phi_2^e}{\partial y}\frac{\partial\phi_1^e}{\partial x} & \frac{\partial\phi_2^e}{\partial y}\frac{\partial\phi_2^e}{\partial x} & \frac{\partial\phi_2^e}{\partial y}\frac{\partial\phi_3^e}{\partial x} \\ \frac{\partial\phi_3^e}{\partial y}\frac{\partial\phi_1^e}{\partial x} & \frac{\partial\phi_3^e}{\partial y}\frac{\partial\phi_2^e}{\partial x} & \frac{\partial\phi_3^e}{\partial y}\frac{\partial\phi_3^e}{\partial x} \end{bmatrix} dxdy$$

(5.42 cont.)

$$- Q_x \iint_e \begin{bmatrix} \phi_1 \frac{\partial \phi_1^e}{\partial x} & \phi_1 \frac{\partial \phi_2^e}{\partial x} & \phi_1 \frac{\partial \phi_3^e}{\partial x} \\ \phi_2 \frac{\partial \phi_1^e}{\partial x} & \phi_2 \frac{\partial \phi_2^e}{\partial x} & \phi_2 \frac{\partial \phi_3^e}{\partial x} \\ \phi_3 \frac{\partial \phi_1^e}{\partial x} & \phi_3 \frac{\partial \phi_2^e}{\partial x} & \phi_3 \frac{\partial \phi_3^e}{\partial x} \end{bmatrix} dxdy$$

$$- Q_y \iint_e \begin{bmatrix} \phi_1 \frac{\partial \phi_1^e}{\partial y} & \phi_1 \frac{\partial \phi_2^e}{\partial y} & \phi_1 \frac{\partial \phi_3^e}{\partial y} \\ \phi_2 \frac{\partial \phi_1^e}{\partial y} & \phi_2 \frac{\partial \phi_2^e}{\partial y} & \phi_2 \frac{\partial \phi_3^e}{\partial y} \\ \phi_3 \frac{\partial \phi_1^e}{\partial y} & \phi_3 \frac{\partial \phi_2^e}{\partial y} & \phi_3 \frac{\partial \phi_3^e}{\partial y} \end{bmatrix} dxdy \qquad (5.42)$$

where ℓ, θ, D_{xx}, D_{xy}, D_{yx}, Q_x, and Q_y are the average values of these quantities over the element. The indicies used in the elemental stiffness matrix are local, and they pertain to nodes numbered counter clockwise around a triangular element. The assembly of the global stiffness matrix [A] from the elemental stiffness matrices $[A^e]$ is the same as described in Chapter 4 for the groundwater flow problem.

The integrations of the diffusive terms of Eq. 5.42 can be performed by an expression similar to Eq.4.66. The integration formulas are

$$\iint_e \frac{\partial \phi_i^e}{\partial x} \frac{\partial \phi_j^e}{\partial x} dxdy = \frac{b_i b_j}{4A^e} \qquad (5.43a)$$

$$\iint_e \frac{\partial \phi_i^e}{\partial x} \frac{\partial \phi_j^e}{\partial y} dxdy = \frac{b_i c_j}{4A^e} \qquad (5.43b)$$

$$\iint_e \frac{\partial \phi_i^e}{\partial y} \frac{\partial \phi_j^e}{\partial y} dxdy = \frac{c_i c_j}{4A^e} \qquad (5.43c)$$

and

$$\iint_e \frac{\partial \phi_i^e}{\partial y} \frac{\partial \phi_j^e}{\partial x} \, dxdy = \frac{c_i b_j}{4A^e} \tag{5.43d}$$

where the b's and c's are given by Eqs. 4.47 and A^e is the area of the element.

The integrations of the advective terms of Eq. 5.42 can be performed by the formulas

$$\iint_e \phi_i^e \frac{\partial \phi_j^e}{\partial x} \, dxdy = \frac{b_j}{6} \tag{5.44a}$$

$$\iint_e \phi_i^e \frac{\partial \phi_j^e}{\partial y} \, dxdy = \frac{c_j}{6} \tag{5.44b}$$

Mass matrix [B].— A typical elemental mass matrix $[B^e]$ will be of the form

$$[B^e] = \ell\theta \iint \begin{bmatrix} \phi_1^e\phi_1^e & \phi_1^e\phi_2^e & \phi_1^e\phi_3^e \\ \phi_2^e\phi_1^e & \phi_2^e\phi_2^e & \phi_2^e\phi_3^e \\ \phi_3^e\phi_1^e & \phi_3^e\phi_2^e & \phi_3^e\phi_3^e \end{bmatrix} dxdy \tag{5.45}$$

where ℓ and θ are average values of these quantities over the element. The integrations in Eq. 5.45 can be performed by formulas similar to Eqs. 4.94. The integration formulas are

$$\iint_e \phi_i^e \, \phi_j^e \, dxdy = \frac{A^e}{12} \quad \text{for } i \neq j \tag{5.46a}$$

and

$$\iint_e \phi_i^e \, \phi_i^e \, dxdy = \frac{A^e}{6} \tag{5.46b}$$

Load vector [F].— The load vector arises because of solute sources, sinks, and specified-flux boundary conditions. The term

Nc' in Eq. 5.41c can represent distributed solute sources such as areal irrigation returns or point solute sources such as an injection well. In the latter case, the source is described by the expression

$$Nc' = \sum_k Q_k c'_k \delta(x - x_k, y - y_k) \tag{5.47}$$

where Q_k is the water recharge at a point k, c'_k is the solute concentration of the recharge water, x_k and y_k are coordinates of the recharge point, and δ is the Dirac delta function.

Because of the properties of the Dirac delta function as described in Section 4.3.6, the integral

$$\iint_\Omega Nc'\phi_i \, dxdy$$

in Eq. 5.41c is equal to $Q_k c'_k$, if x_k and y_k are the coordinates of a node. Then for sources, the global force vector is assembled by adding $Q_k c'_k$ to F_i, where Q_k is located at the node i. For sinks, the integral is equal to $Q_k c$, if x_k and y_k are the coordinates of a node. However, c is the dependent variable of equation 5.40. Consequently, the quantity $Q_k c$ does not appear in the load vector, but Q_k is added to the diagonal of the stiffness matrix [A]. Concomitantly, Q_k is added to A_{ii} where Q_k is located at the node i.

The load vector also includes the advective and diffusive flux across the boundary. In practical applications where the boundary solute flux is non-zero, the diffusive component is often assumed to be zero, and the boundary solute flux is assumed to occur by only the advective process. This is the term

$$\int_\Gamma (Q_x \hat{c}\, n_x + Q_y \hat{c}\, n_y)\, \phi_i d\Gamma$$

in Eq. 5.42c. However, by arguments parallel to those that led to Eq. 4.78 for groundwater flow, the elemental load vector contribution for solute flux is given by the relation

$$\{F^e\} = \begin{bmatrix} q_b c' \frac{L}{2} \\ \\ q_b c' \frac{L}{2} \end{bmatrix} \tag{5.48}$$

where L is the length of the elemental boundary segment, q_b is the groundwater flux into the flow domain normal to the boundary, and c' is the solute concentration.

The solute concentration depends on the direction of groundwater flow across the boundary. If the flow is into the flow domain, the concentration c' is an exogenous solute concentration. *If the direction of flow is out of the flow domain*, the moving groundwater carries solute at the concentration in the groundwater system immediately adjacent to the boundary. In that case, the solute concentration c' is equal to the concentration in the aquifer. Parallel to the handling of sinks, the quantity $q_b L/2$ is added to the diagonal of matrix [A] for each boundary segment associated with a particular node.

5.3.3. Solution of the System of Equations

From the preceeding section, the coefficients of the matrices [A] and [B] and the vector {F} of Eq. 5.40 can now be evaluated. However, the system of ordinary differential equations

$$[A]\ \{C\} + [B] \left\{ \frac{dC}{dt} \right\} = \{F\} \qquad (5.40)$$

must be solved. However, these equations can be solved by replacing the time derivative with the finite-difference approximation

$$\left\{ \frac{dC}{dt} \right\} = \frac{1}{\Delta t} \left\{ C^{t+\Delta t} - C^{t} \right\} \qquad (5.49)$$

to obtain

$$[A] \left\{ C^{t+\Delta t} \right\} + \frac{1}{\Delta t} [B] \left\{ C^{t+\Delta t} - C^{t} \right\} = \{F\} \qquad (5.50)$$

which is a first-order correct implicit finite-difference scheme. Equation 5.50 can be rearranged to obtain

$$\left([A] + \frac{1}{\Delta t} [B] \right) \left\{ C^{t+\Delta t} \right\} = \frac{1}{\Delta t} [B] \left\{ C^{t} \right\} + \{F\} \qquad (5.51)$$

Equation 5.51 can be restated as

$$[L] \left\{ C^{t+\Delta t} \right\} = \{R\} \qquad (5.52)$$

where

$$[L] = [A] + \frac{1}{\Delta t} [B] \qquad (5.53)$$

and
$$\{R\} = \frac{1}{\Delta t} [B] \left\{C^t\right\} + \{F\} \tag{5.54}$$

A solution to the original partial differential equation can be obtained by marching the solution through time. At any time step t, [A], [B], and $\{F\}$ are evaluated. Using the solute concentration at time t, $/\Delta t [B]\{C^t\}$ is evaluated, and Eq. 5.51 is solved for $\{C^{t+\Delta t}\}$. The time index is updated by Δt , and the process is repeated until the required time domain is simulated at the start of the simulation, $\{C^t\}$ is the initial conditions $\{C^\circ\}$.

5.4. Computer Program for Finite-Element Analysis

In this section, a computer program is described that can be used to solve two-dimensional transient-state problems of solute transport in groundwater. The program utilizes triangular elements and linear trial functions. Aquifer parameters are assumed constant over an element but may vary from element to element. Boundary conditions permitted include only the specified solute-flux condition. Sources and sinks are allowed only at nodal points.

This program is a companion to the previously described program for the finite-element analysis of groundwater flow. The program for solute-transport utilizes the computed heads from the groundwater flow model for the computation of groundwater discharge, aquifer thickness, and groundwater velocity. Additionally, the solute-transport model utilizes the source-sink strengths from the groundwater flow model for the water-discharge component of the solute source-sink strengths of the solute-transport model. Both models make computations on the same finite-element grid and use the same values for aquifer parameters. In this companionship, the groundwater flow model is executed first, and the computed heads and source-sink strengths are saved to a file. The solute-transport model is then executed, and these quantities are retrieved from that file.

5.4.1. Program Structure

Parallel to the groundwater flow model, the solute-transport program has three basic modules: a main driver module, a data-input module, and a solution and output module. These modules are shown schematically in Fig. 5.2. The solution and output module is in turn divided into submodules that perform limited functions. The main driver program TRANS calls subroutine TDATA, which reads data defining the geometry of the flow domain, initial conditions, aquifer parameters, and sources and sinks. This information is stored in common blocks, and control is returned to the main

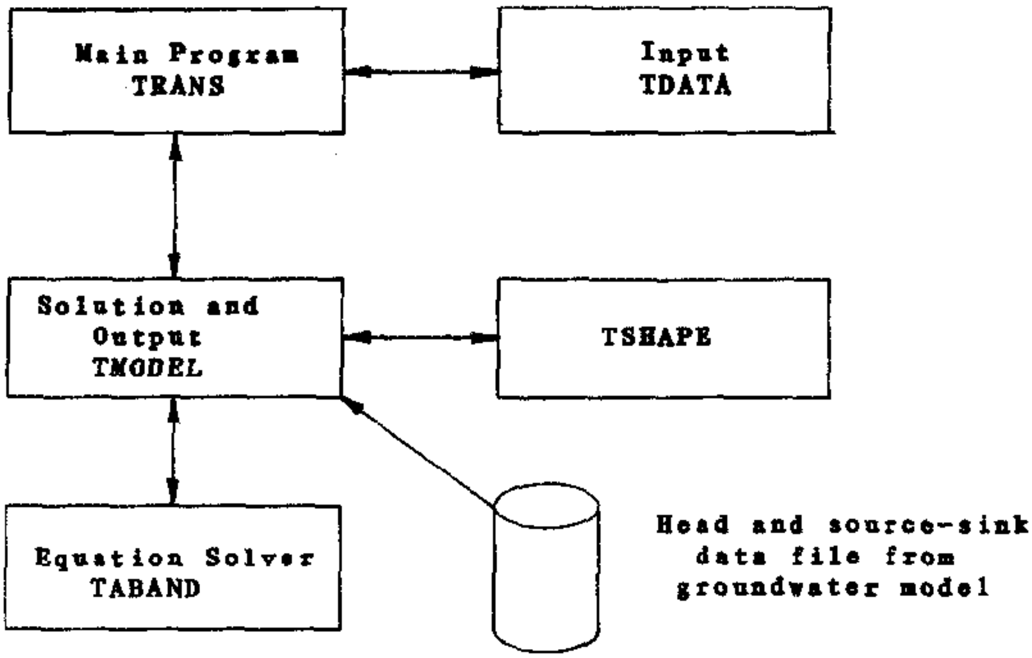

Fig. 5.2. *General configuration of finite-element program.*

TABLE 5.1. VARIABLES FOR SUBROUTINE TDATA

Variable	Type	Definition
TITLE	Real array	Identification of problem
DELT	Real scalar	Hydraulic time step
MAXKNS	Integer scalar	Number of hydraulic time steps
NN	Integer scalar	Number of nodes
NE	Integer scalar	Number of elements
X	Real array	X-coordinate of node
Y	Real array	Y-coordinate of node
IN	Integer array	Element incidence
K	Real array	Hydraulic conductivity over element
BASE	Real array	Altitude of aquifer base at node
NDELT	Integer scalar	Number of solute time steps per hydraulic time steps
POR	Real array	Porosity over element
CO	Real array	Initial concentration at node
DL	Real scalar	Longitudinal dispersivity
DT	Real scalar	Transverse dispersivity
NSS	Integer scalar	Number of source-sink data-sets
CSET	Real array	Concentration of recharge and discharge at nodes
NSET	Integer array	Source-sink data-set indicators
FACSET	Real array	Multipliers for source-sink data-sets

program. The main program in turn calls subroutine TMODEL, which is the solution and output module.

Subroutine TMODEL marches the solution through time. In this process the time domain is first divided into time steps of the groundwater flow model, which in turn are divided into smaller time steps of the solute-transport model. At each flow timestep, groundwater discharge and velocity for each element are computed from computed heads obtained from the groundwater flow model. Subroutine TSHAPE is called to generate the global stiffness and mass matrices. An upper triangularization is performed on the matrix

$$[A] + \frac{1}{\Delta t}[B]$$

by subroutine TABAND. Then for each solute time step, which is embedded in the flow time step, the vector

$$\frac{1}{\Delta t}[B]\{C^t\} + \{F\}$$

is computed, and solute concentration at the new time step is computed by a back substitution of the vector into the upper triangularized left-hand matrix. When each solute time step has been completed, the procedure is repeated for a new flow time step.

The program modules and submodules that perform these procedures are described in the following sections. In each case, the important variables are identified, the program logic is described, and a source-code listing is provided. Information on data input is provided by comment statements in subroutine TDATA. Otherwise, specific information on input variables and formats is not given, in as much as the input data are not complicated.

Main Program TRANS

The purpose of the main program is to call subroutines TDATA and TMODEL, and the module consists essentially of call statements to those subroutines. A source listing of program TRANS is shown in Fig. 5.3.

Subroutine TDATA

The purpose of subroutine TDATA (Table 5.1) is to input data that describes the solute-transport problem to be solved. The input data occurs in two basic groups: definition of the corresponding groundwater flow problem and definition of the solute-transport problem. The important variables in both of these groups are described in Table 5.1. The units of measure and imput formats are indicated by comment statements in the source listing. That source listing of subroutine TDATA is given in Fig. 5.4.

Fig. 5.3. Source listing for program TRANS.

```
C       -------------
        PROGRAM TRANS
C       -------------
        CALL TDATA
        CALL TMODEL
        STOP
        END
```

Fig. 5.4. Source listing for subroutine TDATA.

```
C       ----------------
        SUBROUTINE TDATA
C       ----------------
C
C.....DIMENSIONED FOR THE FOLLOWING PROBLEM SIZE:
C          NUMBER OF NODES = 100
C          NUMBER OF ELEMENTS = 150
C          NUMBER OF TIME STEPS = 30
C          NUMBER OF SOURCE-SINK DATA SETS = 4
C
        COMMON /MODEL/ MAXKNS,NN,NE,NB,NBM1,NB2,NDELT,NSS,
     1                 DL,DT,DELT,
     2                 X,Y,CO,BASE,
     3                 K,POR,IN,
     4                 CSET,NSET,FACSET
C
        INTEGER IN(150,3),NSET(30)
        REAL X(100),Y(100),CO(100),BASE(100)
        REAL K(150),POR(150)
        REAL CSET(100,4),FACSET(30)
        REAL TITLE(20)
C
C       LOGICAL UNIT NUMBERS
C
        IR=5
        IW=6
C
C.....INPUT HYDRAULIC DATA
C
C       BASIN NAME
C
        READ(IR,900) (TITLE(I),I=1,20)
        WRITE(6,901) (TITLE(I),I=1,20)
 900    FORMAT(20A4)
 901    FORMAT(1H1,10X,20A4/1H ,10X,80('-')/)
C
C       HYDRAULIC TIME STEP (DAYS) AND NUMBER OF TIME STEPS
C
        READ(IR,902) DELT,MAXKNS
        WRITE(6,903) DELT,MAXKNS
        DELT=DELT*3600.0*24.0
 902    FORMAT(F12.0,2I6,F12.0)
 903    FORMAT(1H0,10X,'TIME PARAMETERS'/1H ,10X,15('-')/1H0,10X,
     1    'TIME STEP (DAYS)',21X,F8.1/1H ,10X,'NUMBER OF TIME STEPS',
     2    21X,I4)
C
C       NUMBER OF NODES AND ELEMENTS
C
        READ(IR,904) NN,NE
        WRITE(IW,905) NN,NE
 904    FORMAT (2I6)
 905    FORMAT(1H0,10X,'FINITE-ELEMENT DATA'/1H ,10X,19('-')/1H0,10X,
     1    'NUMBER OF NODES    ',I10/1H ,10X,'NUMBER OF ELEMENTS',I10)
C
C       NODE COORDINATES (FEET)
C
        READ(IR,906) FACX,FACY
        READ(IR,907) (I,X(I),Y(I),N=1,NN)
```

```
      DO 100 I=1,NN
      X(I)=X(I)*FACX
      Y(I)=Y(I)*FACY
  100 CONTINUE
      WRITE(IW,908) FACX,FACY
      WRITE (6,909) (I,X(I),Y(I),I=1,NN)
  906 FORMAT(2E12.0)
  907 FORMAT(3(I6,2F6.0,6X))
  908 FORMAT(1H0,10X,'NODE COORDINATES (FEET)'/1H ,10X,23('-')/
     1   1H0,10X,'FACTOR FOR X-COORDINATE',1PE11.3/
     2   1H ,10X,'FACTOR FOR Y-COORDINATE',1PE11.3/
     3   1H0,10X,3('NODE',11X,'X',11X,'Y',5X)/)
  909 FORMAT((1H ,10X,3(I4,F12.1,F12.1,5X)))
C
C     ELEMENT INCIDENCES (COUNTER CLOCKWISE)
C
      READ(IR,910) (I,(IN(I,J),J=1,3),N=1,NE)
      WRITE(IW,911)
      WRITE(IW,912) (I,(IN(I,J),J=1,3),I=1,NE)
  910 FORMAT(2(4I6,6X))
  911 FORMAT(1H0,10X,'ELEMENT INCIDENCES (COUNTER CLOCKWISE)'/
     1   1H ,10X,38('-')/
     2   1H0,10X,3('ELEM',13X,'CORNERS',9X)/)
  912 FORMAT((1H ,10X,3(I4,2X,3I6,9X)))
C
C     FIND HALF-BAND WIDTH
C
      NB=0
      DO 102 L=1,NE
      IMAX=0
      IMIN=10000
      DO 101 I=1,3
      II=IN(L,I)
      IMAX=MAX0(IMAX,II)
      IMIN=MIN0(IMIN,II)
  101 CONTINUE
      NB=MAX0((IMAX-IMIN+1),NB)
  102 CONTINUE
      NBM1=NB-1
      NB2=NB*2-1
      WRITE(IW,913) NB
  913 FORMAT(1H0,10X,'HALF-BAND WIDTH',I9)
C
C     HYDRAULIC CONDUCTIVITY (FEET PER DAY)
C
      READ(IR,914) FACK
      READ(IR,915) (K(I),I=1,NE)
      DO 103 I=1,NE
      K(I)=K(I)*FACK
  103 CONTINUE
      WRITE(IW,917) FACK
      WRITE(IW,918) (I,K(I),I=1,NE)
      DO 104 I=1,NE
      K(I)=K(I)/(3600.0*24.0)
  104 CONTINUE
  914 FORMAT(E12.0)
  915 FORMAT(10F6.0)
  917 FORMAT(1H0,10X,'HYDRAULIC CONDUCTIVITY (FEET PER DAY)'/
     1   1H ,10X,37('-')/
     2   1H0,10X,'FACTOR FOR CONDUCTIVITY',1PE11.3/
     3   1H0,10X,5('ELEM',7X,'VALUE',4X)/)
  918 FORMAT((1H ,10X,5(I4,3X,F9.1,4X)))
C
C     ALTITUDE FOR BASE OF AQUIFER (FEET ABOVE DATUM)
C
      READ(IR,925) (BASE(I),I=1,NN)
```

```
      WRITE(IW,926)
      WRITE(IW,927) (I,BASE(I),I=1,NN)
  925 FORMAT(10F6.0)
  926 FORMAT(1H0,10X,'BASE OF AQUIFER (FEET ABOVE DATUM)'/
     1   1H ,10X,34('-')/
     2   1H0,10X,5('NODE',7X,'VALUE',4X)/)
  927 FORMAT((1H ,10X,5(I4,3X,F9.1,4X)))
C
C.....SOLUTE-TRANSPORT DATA
C
      READ(IR,952) NDELT
      WRITE(IW,953) NDELT
      DELT2=DELT/NDELT
  952 FORMAT(2I6)
  953 FORMAT(1H0,10X,'TIME PARAMETERS'/1H ,10X,15('-')/
     2   1H ,10X,'NUMBER OF SOLUTE TIME STEPS',2X,I10)
C
C     AQUIFER POROSITY
C
      READ(IR,9401) FACPOR
      READ(IR,940) (POR(I),I=1,NE)
      DO 1001 I=1,NE
      POR(I)=POR(I)*FACPOR
 1001 CONTINUE
      WRITE(IW,941) FACPOR
      WRITE(IW,942) (I,POR(I),I=1,NE)
  940 FORMAT(10F6.0)
 9401 FORMAT(E12.0)
  941 FORMAT(1H0,10X,'AQUIFER POROSITY'/1H ,10X,16('-')/
     1   1H0,10X,'FACTOR FOR POROSITY',1PE11.3/
     2   1H0,10X,5('ELEM',7X,'VALUE',4X)/)
  942 FORMAT((1H ,10X,5(I4,3X,F9.3,4X)))
C
C     INITIAL CONCENTRATION (ANY CONSISTENT UNITS)
C
      READ(IR,921) (CO(I),I=1,NN)
      WRITE(IW,922)
      WRITE(IW,923) (I,CO(I),I=1,NN)
  921 FORMAT(10F6.0)
  922 FORMAT(1H0,10X,'INITIAL CONCENTRATION'/1H ,10X,21('-')/
     1   1H0,10X,5('NODE',7X,'VALUE',4X)/)
  923 FORMAT((1H ,10X,5(I4,3X,F9.1,4X)))
C
C     LONGITUDINAL AND TRANSVERSE DISPERSIVITY (FEET)
C
      READ(IR,935) DL,DT
      WRITE(IW,936) DL,DT
  935 FORMAT(2F12.0)
  936 FORMAT(1H0,10X,'DISPERSIVITY'/1H ,10X,12('-')/
     1   1H0,10X,'LONGITUDINAL',F10.2/
     2   1H ,10X,'TRANSVERSE',F12.2)
C
C     NUMBER OF SOURCE-SINK DATA SETS AND FACTOR FOR CONCENTRATIONS
C
      READ(IR,964) NSS,FACC
      WRITE(IW,965) NSS,FACC
  964 FORMAT(I6,2F12.0)
  965 FORMAT(1H0,10X,'RECHARGE AND DISCHARGE'/1H ,10X,22('-')/
     1   1H0,10X,'NUMBER OF DATA SETS',I10/
     2   1H ,10X,'FACTOR FOR C'',7X,1PE10.3)
C
C     SOURCES (+) AND SINKS (-) STRENGTH (CUBIC FEET PER SECOND)
C        AND CONCENTRATION (ANY CONSISTENT UNITS)
C
      DO 105 J=1,NSS
      DO 106 I=1.NN
```

```
      CSET(I,J)=0.0
 106  CONTINUE
 105  CONTINUE
      DO 107 J=1,NSS
      READ(IR,976) NQ
      READ(IR,977) (I,CSET(I,J),N=1,NQ)
      DO 1061 I=1,NN
      CSET(I,J)=CSET(I,J)*FACC
 1061 CONTINUE
      WRITE(IW,980) J
      WRITE(IW,981) (I,CSET(I,J),I=1,NN)
 107  CONTINUE
 976  FORMAT(I6)
 977  FORMAT(5(I6,F6.0))
 980  FORMAT(1H0,10X,'CONCENTRATION SET',I5/1H ,10X,22('-')/
     1   1H0,10X,5('NODE',7X,'VALUE',4X)/)
 981  FORMAT((1H ,10X,5(I4,3X,F9.1,4X)))
C
C     TIME-STEP INDICATORS AND MULTIPLIERS
C
      READ(IR,932) (I,NSET(I),FACSET(I),N=1,MAXKNS)
      WRITE(IW,933)
      WRITE(IW,934) (I,NSET(I),FACSET(I),I=1,MAXKNS)
 932  FORMAT(2I6,F6.0)
 933  FORMAT(1H0,10X,'TIME-STEP INDICATORS AND MULTIPLIERS'/
     1   1H ,10X,37('-')/1H0,10X,'STEP',7X,'SET',4X,'FACTOR'/)
 934  FORMAT((1H ,10X,I4,7X,I3,4X,F6.3))
C
C     END DATA INPUT
C
      RETURN
      END
```

In the first data-input group, the space-time domain of the groundwater flow problem is described. These data include the flow time step, number of flow time steps, number of nodes and elements, nodal coordinates, element incidences, hydraulic conductivity, and altitude for base of the aquifer. The same values are used in the transport problem as were used in the corresponding groundwater flow problem, and additional information of these inputs are described in Chapter 4 under the section on subroutine FDATA.

In the second data input group, the solute-transport problem is described. This description includes data on the number of solute-transport time steps per groundwater flow time step, the aquifer porosity, initial concentration, longitudinal and transverse dispersivity, and data on source-sink concentrations. With regard to sources and sink, data are to be provided for the nodes corresponding to the groundwater flow problem. That includes the recharge and discharge nodes and the constant-head nodes. The water fluxes at these nodes are saved from the groundwater flow model at each time step, and values for this quantity are input from a file at each groundwater-flow time step of the solute-transport model.

Subroutine TMODEL

The purpose of subroutine TMODEL (Table 5.2) is to march the solution through time. This is accomplished by three basic loops, as is shown schematically in Fig. 5.5. The outermost loop is the groundwater flow time-step loop, which controls the basic march through time. Within this loop, hydraulic heads and the water source-sink strengths are input from the groundwater flow model and the solute source-sink strengths are computed using the source-sink concentrations. Next is a loop over each of the elements by which groundwater velocities, aquifer dispersivities, and global stiffness and mass matricies are computed. Next, the left-hand side matrix is upper triangularized. The solute-transport time-step loop is then used to compute solute concentrations for each solute-transport time step by back substitution into the upper triangularized left-hand side matrix. A source listing of subroutine TMODEL is given in Fig. 5.6.

A compressed storage is used in subroutine TMODEL for the matrices [A], [B], and [L] in a manner similar to that described for the groundwater flow model. However, for the solute-transport model, the matrices are not symmetric, and both the uppper and lower half bands must be stored. Nevertheless, the matrices are banded for an efficiently organized problem. Chapter 4 gives some guidance in the generation of a finite-element grid that tends to enhance the banded structure of the matrices [A], [B], and [L]. Figure 5.7 shows how the banded structure can be used to reduce storage requirements.

TABLE 5.2. VARIABLES FOR SUBROUTINE TMODEL

Variable	Type	Definition
KNS	Integer scalar	Time step
H	Real array	Water levels for hydraulic time step
Q	Real array	Source-sink strengths for hydraulic time step
C	Real array	Computed concentration at time t + Δt
CL	Real array	Computed concentration at time t
A	Real array	Global stiffness matrix
B	Real array	Global mass matrix
AE	Real array	Elemental stiffness matrix
BE	Real array	Elemental mass matrix
THK	Real array	Aquifer thickness
L	Real array	Left-hand-side matrix
R	Real array	Right-hand-side vector

Groundwater Time-step Loop

Reads heads and source-sink strengths

ELEMENT LOOP

Compute groundwater velocity and dispersion tensor

Construct $[A^e]$ and $[B^e]$

Add contribution to [A] and [B]

Assemble left-hand-side matrix

$$[L] = [A] + \frac{1}{\Delta t}[B]$$

Upper triangularize [L]

SOLUTE-TRANSPORT TIME-STEP LOOP

Assemble right-hand-side vector

$$\{R\} = \frac{1}{\Delta t}[B]\{c^t\} + \{F\}$$

Solve for $\{c^{t+\Delta t}\}$ by backsubstitution into triangularized matrix

Fig. 5.5. Generalized flow diagram for subroutine TMODEL.

Fig. 5.6. Source listing for subroutine TMODEL.

```
C     ------------------
      SUBROUTINE TMODEL
C     ------------------
C
C.....DIMENSIONED FOR THE FOLLOWING PROBLEM SIZE:
C        NUMBER OF NODES = 100
C        NUMBER OF ELEMENTS = 150
C        NUMBER OF TIME STEPS = 30
C        NUMBER OF SOURCE-SINK DATA SETS = 4
C
      COMMON /MODEL/ MAXKNS,NN,NE,NB,NBM1,NB2,NDELT,NSS,
     1               DL,DT,DELT,
     2               X,Y,CO,BASE,
     3               K,POR,IN,
     4               CSET,NSET,FACSET
C
      INTEGER IN(150,3),NSET(30)
      REAL X(100),Y(100),CO(100),BASE(100)
      REAL K(150),POR(150)
      REAL CSET(100,4),FACSET(30)
C
      REAL AE(3,3),BE(3,3)
      REAL A(100,29),B(100,29)
      REAL Q(100),QC(100),C(100),CL(100),P(100),H(100),F(100),
     1   THK(100)
C
C     LOGICAL UNIT NUMBERS
C
      IR=5
      IW=6
      IR2=10
      REWIND IR2
C
C     INITIAL CONCENTRATION
C
      DO 2001 I=1,NN
      CL(I)=CO(I)
      C(I)=CO(I)
 2001 CONTINUE
      SKNS2=0
      DELT2=DELT/FLOAT(NDELT)
C
C.....BEGIN HYDRAULIC TIME-STEP LOOP
C
      KNS=0
  400 KNS=KNS+1
      IF(KNS.GT.MAXKNS) RETURN
C
C     HYDRAULIC HEADS AND SOURCE-SINK STRENGTHS FROM
C        HYDRAULIC MODEL
C
      READ(IR2,926)  (H(I),I=1,NN)
      READ(IR2,926)  (Q(I),I=1,NN)
  926 FORMAT(5E12.5)
C
C     THICKNESS OF AQUIFER
C
      DO 325 I=1,NN
```

```
      THK(I)=H(I)-BASE(I)
      IF(THK(I).LT.10.0) THK(I)=10.0
 325  CONTINUE
C
C     INITIALIZE MATRICIES (A) AND (B)
C
      DO 4102 I=1,NN
      DO 4101 J=1,NB2
      A(I,J)=0.0
      B(I,J)=0.0
 4101 CONTINUE
      F(I)=0.0
 4102 CONTINUE
C
C     SOURCE (QC) AND SINK (Q) VECTORS
C
      JJ=NSET(KNS)
      DO 455 I=1,NN
      QC(I)=Q(I)*CSET(I,JJ)*FACSET(KNS)
 455  CONTINUE
      WRITE(IW,951) KNS
 951  FORMAT(1H0,10X,'HYDRAULIC PARAMETERS',/1H ,10X,20('-')/
     1   1H0,10X,'HYDRAULIC TIME STEP',I6/
     2   1H0,10X,'ELEM',13X,'QX',13X,'QY',12X,'DXX',12X,'DXY',12X,
     3   'DYY'/)
C
C.....BEGIN LOOP OVER ELEMENTS
C
      DO 412 LL=1,NE
      CALL TSHAPE(LL,IN,X,Y,THK,POR,K,H,DL,DT,AE,BE)
C
C     GLOBAL STIFFNESS MATRIX (A) AND DYNAMIC MATRIX (B)
C
      DO 411 I=1,3
      II=IN(LL,I)
      DO 410 J=1,3
      JJ=IN(LL,J)-II+NB
      A(II,JJ)=A(II,JJ)+AE(I,J)
      B(II,JJ)=B(II,JJ)+BE(I,J)/DELT2
 410  CONTINUE
 411  CONTINUE
 412  CONTINUE
C
C.....END LOOP OVER ELEMENTS
C
C
C     CONSTRUCT LEFT-HAND-SIDE MATRIX (A) AND UPPER TRIANGULARIZE
C
      DO 430 I=1,NN
      IF(Q(I).LE.0.0) A(I,NB)=A(I,NB)-Q(I)
      DO 429 J=1,NB2
      A(I,J)=A(I,J)+B(I,J)
 429  CONTINUE
 430  CONTINUE
      CALL TABAND(1,A,F,NN,NBM1)
C
C.....BEGIN SOLUTE-TRANSPORT TIME-STEP LOOP
C
      KNS2=0
 300  KNS2=KNS2+1
      IF(KNS2.GT.NDELT) GO TO 400
      SKNS2=SKNS2+1
C
C     MULTIPLICATION OF DYNAMIC MATRIX (B) AND
C        LAST-CONCENTRATION VECTOR (CL) AND ADD TO RIGT-HAND-SIDE
C        VECTOR (F)
```

```
C
      DO 419 I=1,NN
      J1=1
      IF(I-NB.GE.0) GO TO 416
      J1=NB-I+1
  416 J2=NB2
      IF(I+NB-1.LE.NN) GO TO 417
      J2=NB+NN-I
  417 P(I)=0.0
      DO 418 J=J1,J2
      P(I)=P(I)+B(I,J)*CL(I-NB+J)
  418 CONTINUE
      F(I)=P(I)
  419 CONTINUE
C
C     ADD SOURCE-SINK LOAD TO RIGT-HAND-SIDE VECTOR (F)
C
      DO 420 I=1,NN
      IF(QC(I).GT.0.0) F(I)=F(I)+QC(I)
  420 CONTINUE
C
C     SOLVE FOR NEW CONCENTRATION VECTOR (C)
C
      CALL TABAND(2,A,F,NN,NBM1)
C
C     INTERCHANGE CONCENTRATIONS
C
      DO 310 I=1,NN
      C(I)=F(I)
      CL(I)=C(I)
  310 CONTINUE
C
C     DISPLAY RESULTS
C
      TIME=SKNS2*DELT2/(3600.0*24.0)
      WRITE(IW,980) SKNS2,TIME
      WRITE(IW,981) (I,C(I),I=1,NN)
  980 FORMAT(1H1,10X,'TIME STEP',I4/1H ,10X,13('-')/
     1   1H0,10X,'ELAPSED TIME',F10.2,1X,'DAYS'/
     2   1H0,10X,'COMPUTED CONCENTRATION'/1H ,10X,22('-')/
     3   1H0,10X,5('NODE',7X,'VALUE',4X)/)
  981 FORMAT((1H ,10X,5(I4,3X,F9.1,4X)))
      GO TO 300
C
C.....END SOLUTE-TRANSPORT TIME-STEP LOOP
C.....END HYDRAULIC TIME-STEP LOOP
C
      END
```

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} & & & \\ L_{21} & L_{22} & L_{23} & L_{24} & & \\ L_{31} & L_{32} & L_{33} & L_{34} & L_{35} & \\ & L_{42} & L_{43} & L_{44} & L_{45} & L_{46} \\ & & L_{53} & L_{54} & L_{55} & L_{56} \\ & & & L_{64} & L_{65} & L_{66} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & L_{11} & L_{12} & L_{13} \\ 0 & L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} & L_{35} \\ L_{42} & L_{43} & L_{44} & L_{45} & L_{46} \\ L_{53} & L_{54} & L_{55} & L_{56} & 0 \\ L_{64} & L_{65} & L_{66} & 0 & 0 \end{bmatrix}$$

Fig. 5.7. Band storage of matrices.

Subroutine TSHAPE

The purpose of subroutine TSHAPE (Table 5.3) is to construct the elemental stiffness and mass matrices $[A^e]$ and $[B^e]$. The coordinates of an element, aquifer thickness, hydraulic conductivity, and dispersivities are passed to subroutine TSHAPE from subroutine *TMODEL*. Subroutine *TSHAPE* returns $[A^e]$ and $[B^e]$ to subroutine TMODEL. A source listing of subroutine TSHAPE is given in Fig. 5.8a.

Subroutine TABAND

The purpose of subroutine TABAND (Table 5.4) is to solve the linear equation

$$\{R\} = \frac{1}{\Delta t}[B]\{C^t\} + \{F\} \qquad (5.54)$$

for $\{C^{t+\Delta t}\}$. Subroutine TABAND solves Eq. 5.52 in two passes. First, the matrix [L] is upper triangularized and returned. Second, the upper triangularized form of [L] is passed to subroutine TABAND along with the vector $\{R\}$ and a solution vector is returned in $\{R\}$. In this second pass, a solution is obtained by back substituting $\{R\}$ into the uppper triangularized matrix. A *source listing of subroutine TABAND is given in Fig. 5.8b.*

The Doolittle method is used to solve the system of equations. The method is based upon the fact that, for an asymmetric matrix, there exists matrices $[\ell]$and$[u]$ such that

$$[\ell]\ [u] = [L] \qquad (5.55)$$

where $[\ell]$ is a lower triangular matrix and $[u]$ is an upper triangular matrix. As for the Cholesky method that was described in Chapter 4, an intermediate solution $\{Y\}$ can be defined as

$$[\ell]\ \{Y\} = \{R\} \qquad (5.56)$$

Because of the triangular structure of $[\ell]$, Eq. 5.56 can be readily solved for $\{Y\}$ by direct back substitution. However, Eqs. 5.55 and 5.56 can be substituted into Eq. 5.52 to obtain

$$[\ell]\ [u]\ \left\{C^{t+\Delta t}\right\} = [\ell]\ \{Y\} \qquad (5.57)$$

Premultiplication by $[\ell]^{-1}$ yields

$$[u]\ \left\{C^{t+\Delta t}\right\} = \{Y\} \qquad (5.58)$$

Once $[\ell]$ and $[u]$ are known, a solution to Eq. 5.52 can be obtained. Equation 5.56 is used to obtain $\{Y\}$. Then Eq. 5.58 is used to solve for $\left\{C^{t+\Delta t}\right\}$. The problem is to obtain the matrices

TABLE 5.3. VARIABLES FOR SUBROUTINE TSHAPE

Variable	Type	Definition
XE	Real array	X-coordinate of node
YE	Real array	Y-coordinate
BB	Real array	Coefficient b of trial function
CC	Real array	Coefficient c of trial function
AREA	Real scalar	Area of element
QX	Real scalar	Groundwater discharge in x direction
QP	Real scalar	Groundwater discharge in y direction
VX	Real scalar	Groundwater velocity in x direction
VY	Real scalar	Groundwater velocity in y direction
DXX	Real scalar	Components of dispersion tensor
DXY		
DYX		
DYY		
POR	Real array	Aquifer porosity
THKE	Real scalar	Average aquifer thickness over element

TABLE 5.4. VARIABLES FOR SUBROUTINE TABAND

Variable	Type	Definition
L	Real array	Left-hand-side matrix
R	Real array	Right-hand-side sector
C	Real array	Solution vector and intermediate solution vector

```
C     ------------------------------------------------------------------
      SUBROUTINE TSHAPE(LL,IN,X,Y,THK,POR,K,H,DL,DT,AE,BE)
C     ------------------------------------------------------------------
C
C.....DIMENSIONED FOR THE FOLLOWING PROBLEM SIZE:
C        NUMBER OF NODES = 100
C        NUMBER OF ELEMENTS = 150
C
      INTEGER IN(150,3)
      REAL X(100),Y(100),THK(100),H(100)
      REAL POR(150),K(150)
      REAL AE(3,3),BE(3,3),XE(3),YE(3),BB(3),CC(3)
C
C.....ELEMENT COORDINATES, GROUND-WATER VELOCITY, AND
C        HYDRODYNAMIC DISPERSION
C
      DO 407 I=1,3
      II=IN(LL,I)
      XE(I)=X(II)
      YE(I)=Y(II)
  407 CONTINUE
      BB(1)=YE(2)-YE(3)
      BB(2)=YE(3)-YE(1)
      BB(3)=YE(1)-YE(2)
      CC(1)=XE(3)-XE(2)
      CC(2)=XE(1)-XE(3)
      CC(3)=XE(2)-XE(1)
      AREA=(BB(1)*XE(1)+BB(2)*XE(2)+BB(3)*XE(3))*0.5
C
C     GROUND-WATER VELOCITY
C
      QX=0.0
      QY=0.0
      THKE=0.0
      DO 380 I=1,3
      II=IN(LL,I)
      QX=QX+H(II)*BB(I)
      QY=QY+H(II)*CC(I)
      THKE=THKE+THK(II)
  380 CONTINUE
      THKE=THKE/3.0
      T=THKE*K(LL)
      QX=-QX*T*0.5/AREA
      QY=-QY*T*0.5/AREA
      VX=ABS(QX/THKE)
      VY=ABS(QY/THKE)
      VV=SQRT(VX*VX+VY*VY)
C
C     DISPERSION COEFFICIENTS
C
      DXX=DT*VV+(DL-DT)*VX*VX/VV
      DXY=(DL-DT)*VX*VY/VV
      DYX=DXY
      DYY=DT*VV+(DL-DT)*VY*VY/VV
C
      WRITE(6,952) LL,QX,QY,DXX,DXY,DYY
  952 FORMAT(1H ,10X,I4,5(1PE15.3))
C
```

Fig. 5.8a. Source listing for subroutine TSHAPE.

```
C.....ELEMENT STIFFNESS MATRIX [AE] AND DYNBAMIC MATRIX [BE]
C
      DO 409 I=1,3
      DO 408 J=1,3
      AE(I,J)=-QX*BB(I)/6.0-QY*CC(I)/6.0
     1    +THKE*POR(LL)*DXX*BB(J)*BB(I)/(4.0*AREA)
     2    +THKE*POR(LL)*DXY*CC(J)*BB(I)/(4.0*AREA)
     3    +THKE*POR(LL)*DYX*BB(J)*CC(I)/(4.0*AREA)
     4    +THKE*POR(LL)*DYY*CC(J)*CC(I)/(4.0*AREA)
      BE(I,J)=POR(LL)*THKE*AREA/12.0
      IF(I.EQ.J) BE(I,J)=BE(I,J)*2.0
  408 CONTINUE
  409 CONTINUE
C
C
      RETURN
      END
```

```
C     ------------------------------------
      SUBROUTINE TABAND(KKK,L,R,NEQ,IHALFB)
C     ------------------------------------
C
C.....DIMENSIONED FOR THE FOLLOWING PROBLEM SIZE:
C         NUMBER OF NODES = 100
C         FULL-BAND WIDTH = 29
C
      DIMENSION L(100,29),R(100)
      NRS=NEQ-1
      IHBP=IHALFB+1
      IF (KKK.EQ.2) GO TO 30
C
C.....TRINGULARIZE MATRIX A USING A DOOLITTLE METHOD
C
      DO 10 K=1,NRS
      PIVOT=L(K,IHBP)
      KK=K+1
      KC=IHBP
      DO 20 I=KK,NEQ
      KC=KC-1
      IF(KC.LE.0) GO TO 10
      C=-L(I,KC)/PIVOT
      L(I,KC)=C
      KI=KC+1
      LIM=KC+IHALFB
      DO 20 J=KI,LIM
      JC=IHBP+J-KC
20    L(I,J)=L(I,J)+C*L(K,JC)
10    CONTINUE
      GO TO 100
C
C.....MODIFY LOAD VECTOR R
C
30    NN=NEQ+1
      IBAND=2*IHALFB+1
      DO 40 I=2,NEQ
      JC=IHBP-I+1
      JI=1
      IF (JC.LE.0) GO TO 50
      GO TO 60
50    JC=1
      JI=I-IHBP+1
60    SUM=0.0
      DO 70 J=JC,IHALFB
      SUM=SUM+L(I,J)*R(JI)
70    JI=JI+1
40    R(I)=R(I)+SUM
C
C.....BACK SOLUTION
C
      R(NEQ)=R(NEQ)/L(NEQ,IHBP)
      DO 80 IBACK=2,NEQ
      I=NN-IBACK
      JP=I
      KR=IHBP+1
      MR=MIN0(IBAND,IHALFB+IBACK)
```

Fig. 5.8b. *Source listing for subroutine TABAND.*

```
      SUM=0.0
      DO 90 J=KR,MR
      JP=JP+1
90    SUM=SUM+L(I,J)*R(JP)
80    R(I)=(R(I)-SUM)/L(I,IHBP)
100   RETURN
      END
```

[ℓ] and [u]. The procedure by which this is accomplished by the Doolittle method is described in Pinder and Gray (1977).

5.4.2. Application to a Simple Problem

The example problem as shown in Fig. 5.9 is used to demonstrate the application of the finite-element program. For the problem, steady-state groundwater flow is used. The no-flow boundary condition is specified for the upper and lower boundaries. The transient-state transport is simulated for a solute introduced to the flow domain with groundwater moving across the left boundary. Figures 5.10 and 5.11 shows the input file for the problem, and Fig. 5.12 shows the output file.

5.5. Regional Solute-Transport Problem

5.5.1. Hydrologic Silting

In Chapter 4, a model of the upper Coachella Valley groundwater basin was used to demonstrate the application of the finite-element method to a regional groundwater problem. In this section, the upper Coachella Valley is used to demonstrate the application of the finite-element method to a regional solute-transport problem. The groundwater model was used to predict the impacts on water levels of artificial recharge of imported water. However, the imported water was of higher dissolved solids than the native groundwater. The imported water has dissolved solids of about 650 mg/l, and the native water has dissoved solids of about 200 mg/l. A solute-transport model was used by Swain (1978) to predict the mixing with the groundwater basin of imported water.

The use of the transport model was to predict the movement of imported water for 1973 to 2000. During that period, however, the imported water was expected to move no more than about 6 miles from the lower end of the recharge area. Concommitantly, the flow domain for the solute-transport model was of a much more limited area than used for the regional groundwater model, because there was no need to extend the flow domain beyond the area over which transport might be expected. The location of the flow domain and its finite-element grid is shown in Fig. 5.13. To provide groundwater velocities for the transport model, a corresponding groundwater problem was also solved for the flow domain shown in Fig. 5.13.

5.5.2. Transport Model

A steady-state model was used to represent groundwater flow (Eq. 4.10). The boundary conditions for this problem included specified-head and specified-flux boundaries. As shown in Fig. 5.10, specified-head boundaries were used to represent the

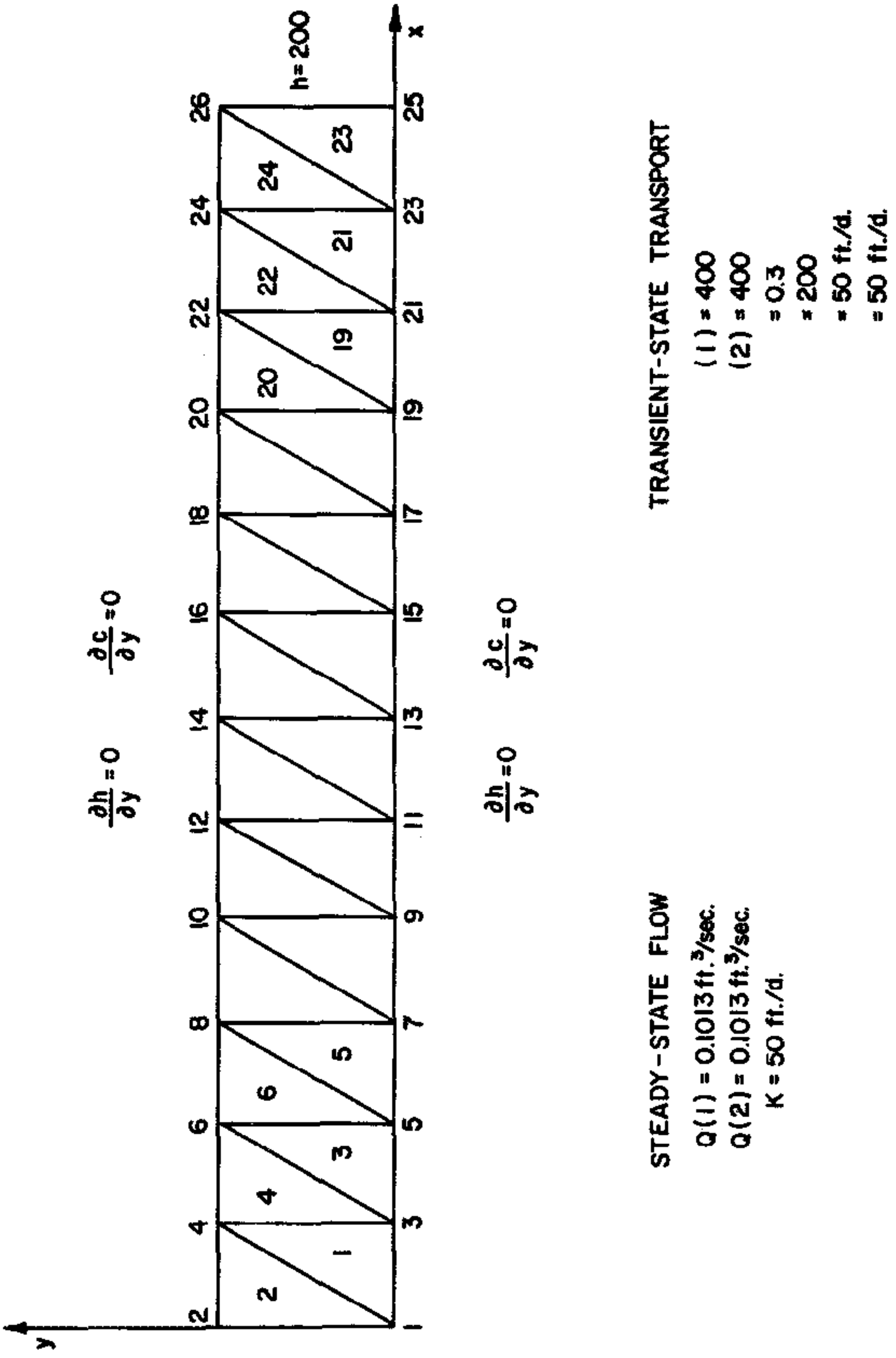

Fig. 5.9. Problem for demonstration of finite-element program.

```
GROUNDWATER-TRANSPORT PROBLEM OF FIGURE 5.9
        30.0      2      8          0.1
  26     24
   1.0E+02      1.0E+02
   1    0.0    0.0           2    0.0    1.0           3    1.0    0.0
   4    1.0    1.0           5    2.0    0.0           6    2.0    1.0
   7    3.0    0.0           8    3.0    1.0           9    4.0    0.0
  10    4.0    1.0          11    5.0    0.0          12    5.0    1.0
  13    6.0    0.0          14    6.0    1.0          15    7.0    0.0
  16    7.0    1.0          17    8.0    0.0          18    8.0    1.0
  19    9.0    0.0          20    9.0    1.0          21   10.0    0.0
  22   10.0    1.0          23   11.0    0.0          24   11.0    1.0
  25   12.0    0.0          26   12.0    1.0
   1      1      3      4          2      1      4      2
   3      3      5      6          4      3      6      4
   5      5      7      8          6      5      8      6
   7      7      9     10          8      7     10      8
   9      9     11     12         10      9     12     10
  11     11     13     14         12     11     14     12
  13     13     15     16         14     13     16     14
  15     15     17     18         16     15     18     16
  17     17     19     20         18     17     20     18
  19     19     21     22         20     19     22     20
  21     21     23     24         22     21     24     22
  23     23     25     26         24     23     26     24
   1.0E+00
 50.0  50.0  50.0  50.0  50.0  50.0  50.0  50.0  50.0  50.0
 50.0  50.0  50.0  50.0  50.0  50.0  50.0  50.0  50.0  50.0
 50.0  50.0  50.0  50.0  50.0  50.0
142.0 142.0 138.5 138.5 135.0 135.0 131.5 131.5 128.0 128.0
124.5 124.5 121.0 121.0 117.5 117.5 114.0 114.0 110.5 110.5
107.0 107.0 103.5 103.5 100.0 100.0
  30
        1.0
  0.3    0.3    0.3    0.3    0.3    0.3    0.3    0.3    0.3    0.3
  0.3    0.3    0.3    0.3    0.3    0.3    0.3    0.3    0.3    0.3
  0.3    0.3    0.3    0.3    0.3
200.0 200.0 200.0 200.0 200.0 200.0 200.0 200.0 200.0 200.0
200.0 200.0 200.0 200.0 200.0 200.0 200.0 200.0 200.0 200.0
200.0 200.0 200.0 200.0 200.0 200.0
      150.0         50.0
   1           1.0          1.0
   4
   1 400.0      2 400.0     25 400.0     26 400.0
   1      1    1.0
   2      1    1.0
```

Fig. 5.10 and 5.11   *Input file for example problem.*

242.0	242.0	238.5	238.5	235.0
235.0	231.5	231.5	228.0	228.0
224.5	224.5	221.0	221.0	217.5
217.5	214.0	214.0	210.5	210.5
207.0	207.0	203.5	203.5	200.0
200.0				
0.1013	0.1013	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	-0.1013
-0.1013				
242.0	242.0	238.5	238.5	235.0
235.0	231.5	231.5	228.0	228.0
224.5	224.5	221.0	221.0	217.5
217.5	214.0	214.0	210.5	210.5
207.0	207.0	203.5	203.5	200.0
200.0				
0.1013	0.1013	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	-0.1013
-0.1013				

1 GROUNDWATER-TRANSPORT PROBLEM OF FIGURE 5.9

0 TIME PARAMETERS

0 TIME STEP (DAYS) 30.0
NUMBER OF TIME STEPS 2

0 FINITE-ELEMENT DATA

0 NUMBER OF NODES 26
NUMBER OF ELEMENTS 24

0 NODE COORDINATES (FEET)

0 FACTOR FOR X-COORDINATE 1.000E+02
FACTOR FOR Y-COORDINATE 1.000E+02

0

NODE	X	Y	NODE	X	Y	NODE	X	Y
1	0.0	0.0	2	0.0	100.0	3	100.0	0.0
4	100.0	100.0	5	200.0	0.0	6	200.0	100.0
7	300.0	0.0	8	300.0	100.0	9	400.0	0.0
10	400.0	100.0	11	500.0	0.0	12	500.0	100.0
13	600.0	0.0	14	600.0	100.0	15	700.0	0.0
16	700.0	100.0	17	800.0	0.0	18	800.0	100.0
19	900.0	0.0	20	900.0	100.0	21	1000.0	0.0
22	1000.0	100.0	23	1100.0	0.0	24	1100.0	100.0
25	1200.0	0.0	26	1200.0	100.0			

0 ELEMENT INCIDENCES (COUNTER CLOCKWISE)

0

ELEM	CORNERS			ELEM	CORNERS			ELEM	CORNERS		
1	1	3	4	2	1	4	2	3	3	5	6
4	3	6	4	5	5	7	8	6	5	8	6
7	7	9	10	8	7	10	8	9	9	11	12
10	9	12	10	11	11	13	14	12	11	14	12
13	13	15	16	14	13	16	14	15	15	17	18
16	15	18	16	17	17	19	20	18	17	20	18
19	19	21	22	20	19	22	20	21	21	23	24
22	21	24	22	23	23	25	26	24	23	26	24

0 HALF-BAND WIDTH 4

0 HYDRAULIC CONDUCTIVITY (FEET PER DAY)

0 FACTOR FOR CONDUCTIVITY 1.000E+00

0

ELEM	VALUE	ELEM	VALUE	ELEM	VALUE	ELEM	VALUE	ELEM	VALUE
1	50.0	2	50.0	3	50.0	4	50.0	5	50.0
6	50.0	7	50.0	8	50.0	9	50.0	10	50.0
11	50.0	12	50.0	13	50.0	14	50.0	15	50.0
16	50.0	17	50.0	18	50.0	19	50.0	20	50.0
21	50.0	22	50.0	23	50.0	24	50.0		

Fig. 5.12. Output file for example problem.

0 BASE OF AQUIFER (FEET ABOVE DATUM)

0

NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	142.0	2	142.0	3	138.5	4	138.5	5	135.0
6	135.0	7	131.5	8	131.5	9	128.0	10	128.0
11	124.5	12	124.5	13	121.0	14	121.0	15	117.5
16	117.5	17	114.0	18	114.0	19	110.5	20	110.5
21	107.0	22	107.0	23	103.5	24	103.5	25	100.0
26	100.0								

0 TIME PARAMETERS

NUMBER OF SOLUTE TIME STEPS 30

0 AQUIFER POROSITY

0 FACTOR FOR POROSITY 1.000E+00

0

ELEM	VALUE	ELEM	VALUE	ELEM	VALUE	ELEM	VALUE	ELEM	VALUE
1	0.300	2	0.300	3	0.300	4	0.300	5	0.300
6	0.300	7	0.300	8	0.300	9	0.300	10	0.300
11	0.300	12	0.300	13	0.300	14	0.300	15	0.300
16	0.300	17	0.300	18	0.300	19	0.300	20	0.300
21	0.300	22	0.300	23	0.300	24	0.300		

0 INITIAL CONCENTRATION

0

NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	200.0	2	200.0	3	200.0	4	200.0	5	200.0
6	200.0	7	200.0	8	200.0	9	200.0	10	200.0
11	200.0	12	200.0	13	200.0	14	200.0	15	200.0
16	200.0	17	200.0	18	200.0	19	200.0	20	200.0
21	200.0	22	200.0	23	200.0	24	200.0	25	200.0
26	200.0								

0 DISPERSIVITY

0 LONGITUDINAL 50.00
TRANSVERSE 50.00

0 RECHARGE AND DISCHARGE

0 NUMBER OF DATA SETS 1
FACTOR FOR C 1.000E+00

0 CONCENTRATION SET 1

0

NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	400.0	2	400.0	3	0.0	4	0.0	5	0.0
6	0.0	7	0.0	8	0.0	9	0.0	10	0.0
11	0.0	12	0.0	13	0.0	14	0.0	15	0.0
16	0.0	17	0.0	18	0.0	19	0.0	20	0.0
21	0.0	22	0.0	23	0.0	24	0.0	25	400.0
26	400.0								

0 TIME-STEP INDICATORS AND MULTIPLIERS

0

STEP	SET	FACTOR
1	1	1.000
2	1	1.000

0 HYDRAULIC PARAMETERS

0 HYDRAULIC TIME STEP 1

0 ELEM	QX	QY	DXX	DXY	DYY
1	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
2	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
3	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
4	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
5	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
6	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
7	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
8	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
9	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
10	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
11	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
12	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
13	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
14	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
15	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
16	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
17	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
18	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
19	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
20	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
21	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
22	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
23	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03
24	2.025E-03	0.000E-01	1.013E-03	0.000E-01	1.013E-03

1 TIME STEP 1

0 ELAPSED TIME 1.00 DAYS

0 COMPUTED CONCENTRATION

0 NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	228.0	2	250.5	3	199.8	4	186.9	5	199.7
6	201.5	7	200.1	8	199.9	9	200.0	10	200.0
11	200.0	12	200.0	13	200.0	14	200.0	15	200.0
16	200.0	17	200.0	18	200.0	19	200.0	20	200.0
21	200.0	22	200.0	23	200.0	24	200.0	25	200.0
26	200.0								

1 TIME STEP 2

0 ELAPSED TIME 2.00 DAYS

0 COMPUTED CONCENTRATION

0 NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	254.3	2	288.7	3	199.2	4	180.4	5	199.9
6	201.5	7	200.0	8	200.0	9	200.0	10	200.0
11	200.0	12	200.0	13	200.0	14	200.0	15	200.0
16	200.0	17	200.0	18	200.0	19	200.0	20	200.0
21	200.0	22	200.0	23	200.0	24	200.0	25	200.0
26	200.0								

1 TIME STEP 57

0 ELAPSED TIME 57.00 DAYS

0 COMPUTED CONCENTRATION

NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	399.2	2	396.6	3	385.5	4	384.2	5	355.0
6	356.0	7	311.0	8	313.4	9	265.8	10	267.8
11	231.6	12	231.9	13	212.0	14	211.2	15	203.6
16	202.5	17	200.8	18	200.1	19	200.1	20	199.8
21	200.0	22	199.9	23	200.0	24	200.0	25	199.9
26	199.9								

1 TIME STEP 58

0 ELAPSED TIME 58.00 DAYS

0 COMPUTED CONCENTRATION

NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	399.3	2	396.7	3	386.2	4	384.9	5	356.8
6	357.6	7	313.6	8	316.0	9	268.5	10	270.5
11	233.6	12	234.1	13	213.2	14	212.4	15	204.1
16	203.0	17	201.0	18	200.2	19	200.2	20	199.8
21	200.0	22	199.9	23	200.0	24	200.0	25	199.9
26	199.9								

1 TIME STEP 59

0 ELAPSED TIME 59.00 DAYS

0 COMPUTED CONCENTRATION

NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	399.4	2	396.9	3	386.9	4	385.6	5	358.5
6	359.2	7	316.2	8	318.5	9	271.2	10	273.3
11	235.7	12	236.3	13	214.3	14	213.7	15	204.5
16	203.5	17	201.1	18	200.4	19	200.2	20	199.8
21	200.0	22	199.9	23	200.0	24	200.0	25	199.9
26	199.9								

1 TIME STEP 60

0 ELAPSED TIME 60.00 DAYS

0 COMPUTED CONCENTRATION

NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE	NODE	VALUE
1	399.5	2	397.0	3	387.6	4	386.2	5	360.2
6	360.8	7	318.8	8	320.9	9	273.9	10	276.0
11	237.8	12	238.5	13	215.6	14	215.0	15	205.1
16	204.1	17	201.3	18	200.5	19	200.3	20	199.8
21	200.0	22	199.9	23	200.0	24	199.9	25	199.9
26	199.9								

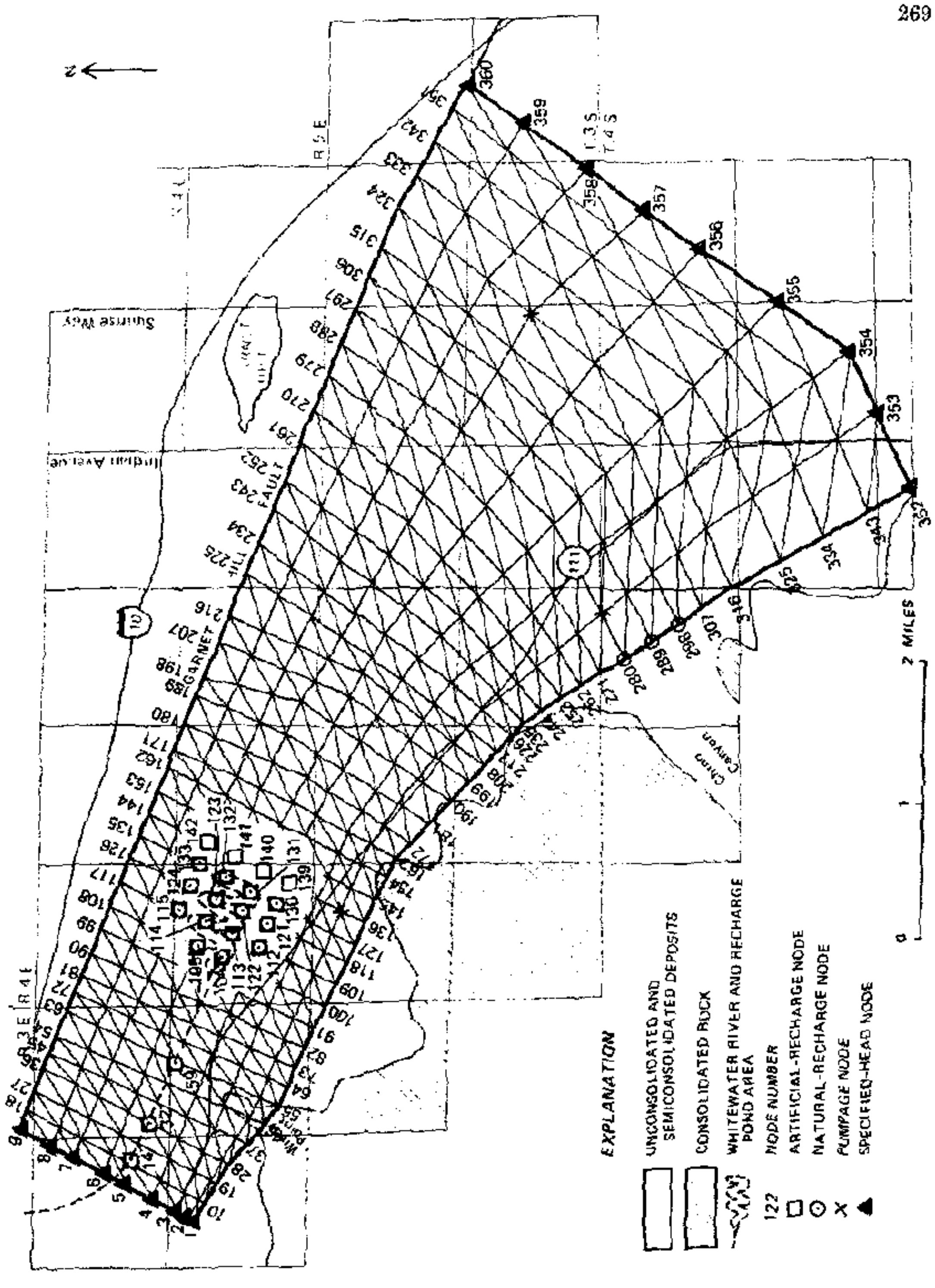

Fig. 5.13. Finite-element grid used for the solute-transport model.

northwest and southeast boundaries of the flow domain. Regional groundwater movement occurs across these boundaries, and head values were derived from the regional groundwater model. No-flow boundaries were used to represent the northeast and southwest boundaries of the flow domain. Respectively, these boundaries represent the Garnet Hill Fault and the contact of alluvial deposits with non-water-bearing consolidated rocks. The steady-state model of groundwater flow does not utilize initial conditions.

A transient-state model was used to represent solute transport in the groundwater system. The boundaries for this problem were specified-flux boundaries. The northeast and southwest boundaries were no-flux boundaries. There the advective and diffusive flux was zero. On the northwest and southeast boundaries, the advective flux was determined by the discharge of groundwater in the aquifer and the dissolved solids concentration in the moving groundwater. On the northwest boundary, groundwater moved into the flow domain, and the concentration was assumed to be 200 mg/l. On the southeast boundary, groundwater moves out of the flow domain, and the concentration was computed by the transport model. For the transport problem, initial conditions were a uniform concentration of 200 mg/l.

The aquifer parameters for the solute-transport model were derived from the literature and other sources. Longitudinal and transverse dispersivities for the model were 100 and 33 ft., respectively. These values were used because of their use in other studies (Robson, 1974, Konikow and Bredehoeft, 1974, Bredehoeft and Pinder, 1973). Because groundwater velocity is a direct function of porosity, field data were collected by determining porosity of cores obtained from drill holes (Swain, 1978). The saturated thickness was assumed to be 1,000 ft. over the flow domain, which was an estimate based on a gravity survey (Biehler, 1964). These aquifer parameters were used without modification by model calibration, because data were not available to perform a calibration (Swain,1978).

The model predicted that the dissolved-solids plume from the recharge of imported water moved were about five miles downstream during 1973, when recharge began, to 1991. These results are indicated by Figs. 5.14 to 5.16, which show contours of equal dissolved solids for 1981, 1991, and 2000.

5.5.3. Modeling Approach

The modeling approach used by Swain (1978) included the three basic steps of system characterization, parameter identification, and model use. The important decision in the characterization phase was the selection of a two-dimensional representation of the groundwater system, which is a vertically-averaged representation. The adequacy of the two-dimensional model depends on the correspondence of the prototype or data on the prototype with its

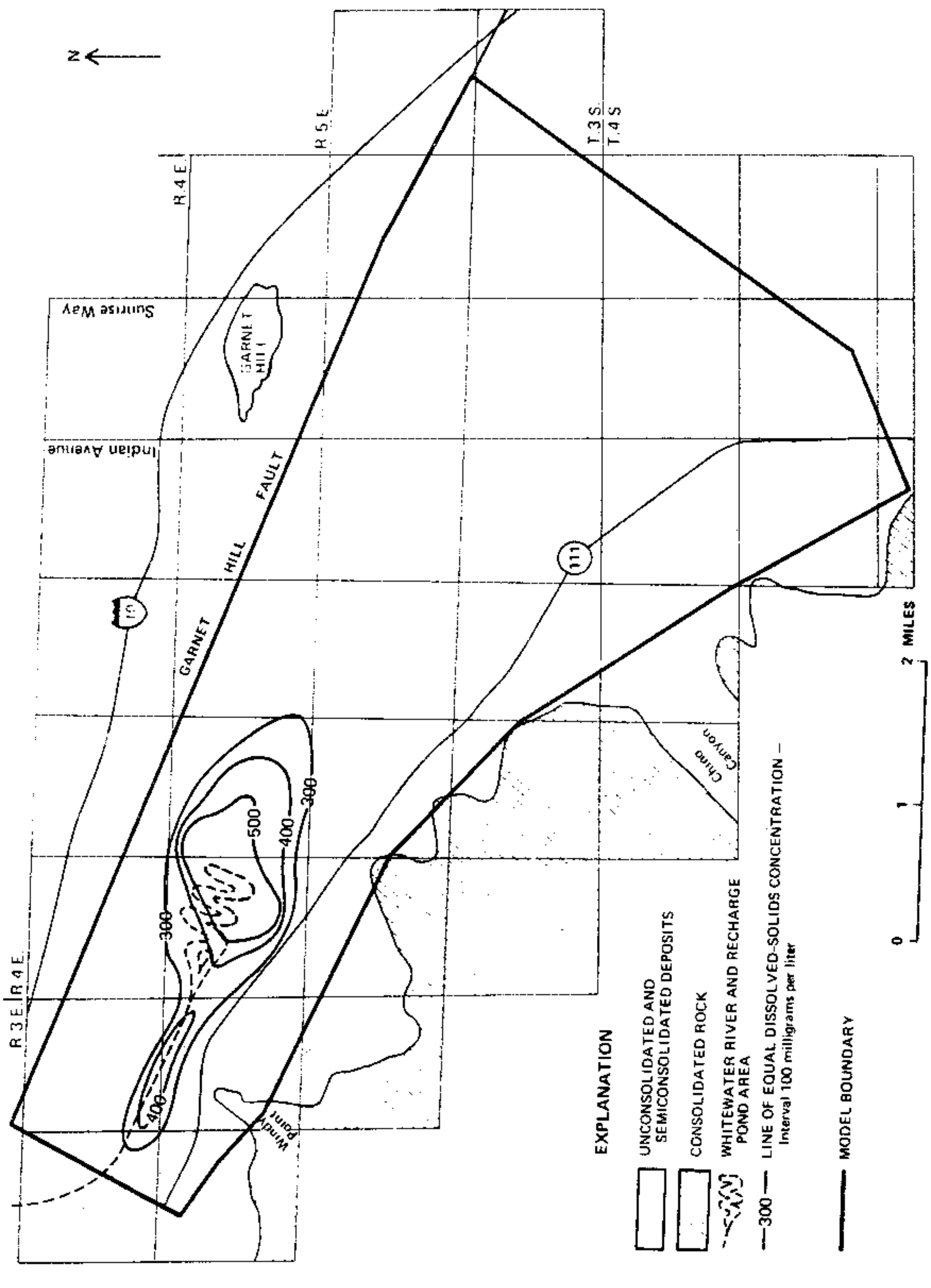

Fig. 5.14. Model-predicted dissolved solids for 1981.

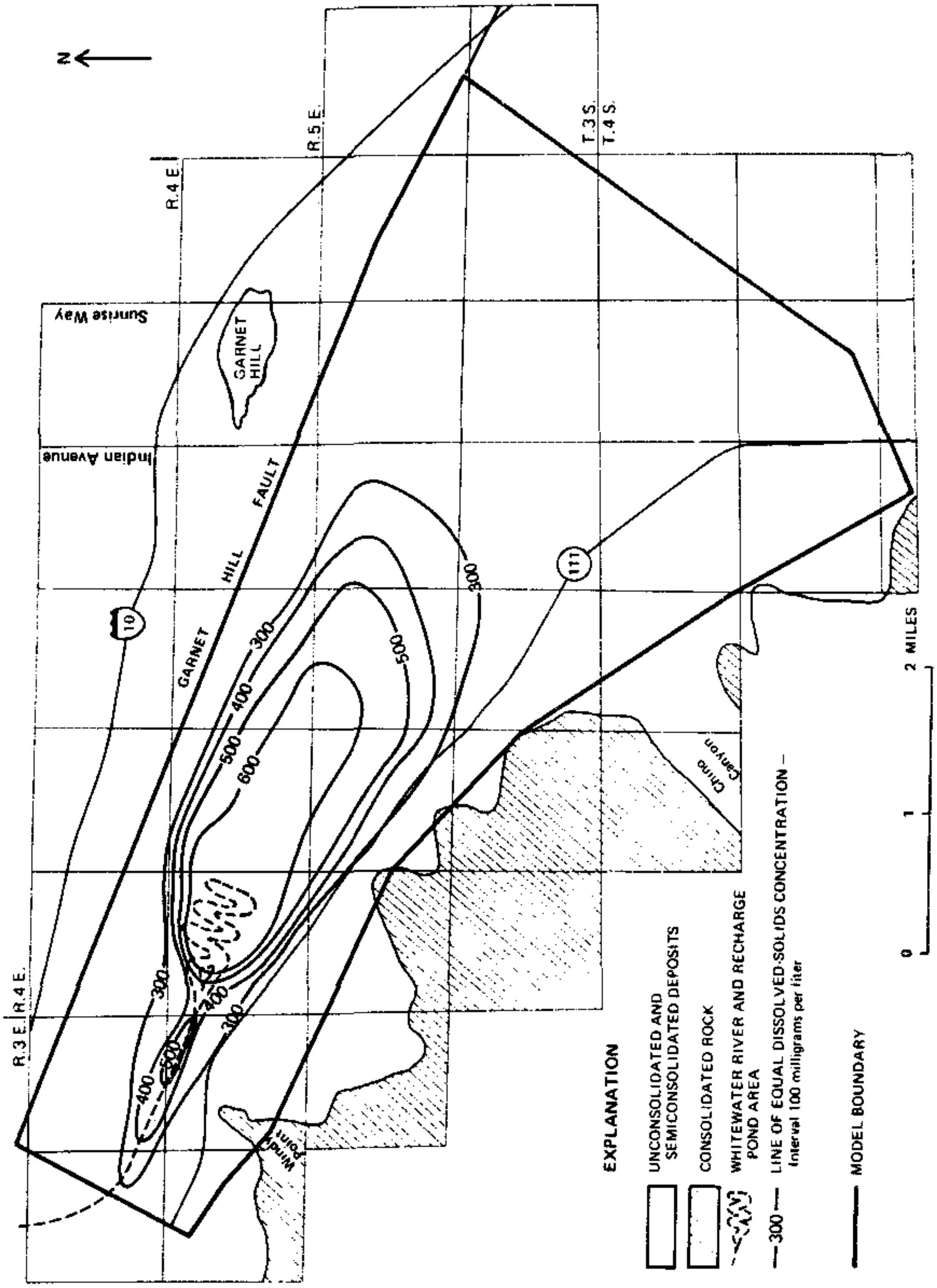

Fig. 5.15. Model-predicted dissolved solids for 1991.

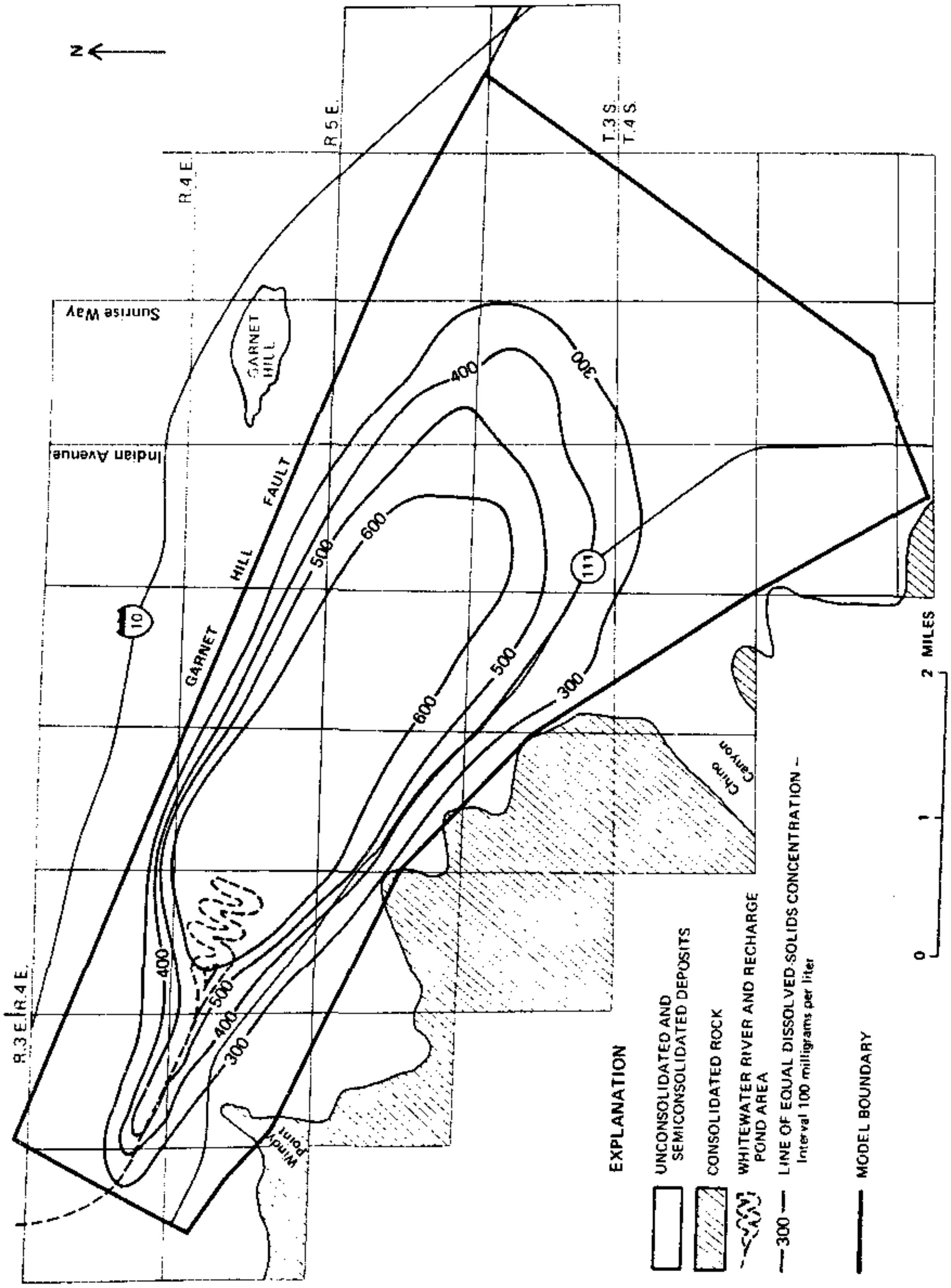

Fig. 5.16. Model-predicted dissolved solids for 2000.

assumptions. Obviously, the two-dimensional representation is adequate if the solute is well mixed over depth in the aquifer. Even if this criterion is not satisfied, the two-dimensional representation can be adequate if the velocity profile is uniform with depth. In this case data on the prototype should be collected so as to produce depth-averaged estimates of the solute concentration, such as might be obtained by water samples drawn from a well that is perforated over the entire saturated thickness of an aquifer. At sufficient distance from the recharge basins in the upper Coachella Valley, field data suggests that the solute concentrations are approximately uniform with depth. However, the groundwater velocity profile probably exhibits decreasing velocity with depth owing to possibly decreasing permeability of the alluvial deposits with depth. Nevertheless, the first condition justified use of a two-dimensional depth-averaged model.

For the upper Coachella Valley, parameter identification involvedonly the selection of values for aquifer porosity, thickness, and dispersivity from existing data. Data were not available for modifying these values such that the model provided a better representation of the time-space distribution of solute concentration in the groundwater system. Had data been available, the transport model would have been modified by a subjective or objective calibration procedure. Nevertheless, groundwater-quality data collected after the model was developed verified that Swain (1978) selected appropriate values for aquifer parameters.

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CHAPTER 6

WATER SYSTEMS

6.1. Flow in Pipes and Pipe Networks

6.1.1. Introduction

Fluid flow in pipes and pipe systems is very common in engineering applications. Pipes that flow completely full are covered in this chapter . There are also many situations where pipes flow partially full, such as in culverts and sewers, however, these flows are in the category of open channel flows and are not treated here. Chapter 7 deals with problems of this type.

Pipe flow problems can be solved by application of the principles of conservation of energy and mass conservation (continuity). Energy relationships are affected by fluid resistance. Resistance to flow in pipes is due primarily to frictional drag at the pipe walls (boundary resistance). Flow resistance is also produced at pipe fittings and other changes in pipe geometry where turbulence is generated. The dissipation of this turbulence gives rise to energy losses in the pipe flow.

In the discussion which follows it is assumed that the fluid is incompressible, and in the examples in this chapter it is assumed that the fluid is water, since this is the fluid of interest in hydraulic engineering and hydrology.

6.1.2. Basic Equations for Pipe Flow Analysis

The energy equation for incompressible pipe flow (Bernoulli equation) is

$$\frac{\alpha_1 V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{\alpha_2 V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_{l_{1-2}} \qquad (6.1)$$

where
α_i = kinetic energy correction factor (α is usually taken as 1.0 for pipeflow)
V_i = mean velocity (fps)
p_i = pressure (lb/ft^2)
z_i = elevation above a datum (ft)
g = acceleration due to gravity (ft/sec^2)
γ = unit weight of water (lb/ft^3)
h_l = energy loss (ft)

The units of energy in Eq. 6.1 are actually ft-lb/lb. As long as branching flows do not have to be considered, it is most convenient to express energy in units of length (ft). Thus, the individual terms in the energy equation are referred to as 'heads.' The term $V^2/2g$ (the velocity head) represents the kinetic energy of the flow, while the pressure head (p/γ) and elevation head (z) terms represent potential energy of the flow. Potential energy can be readily converted to kinetic energy, however, it is more difficult to convert kinetic energy back to potential energy (velocity head recovery) due to losses related to turbulence which occur when high velocities are changed to low velocities.

The velocity at various points in the pipe system can be evaluated using the principle of conservation of mass (continuity principle). In the most general case this principle states that the difference between the flow into the system and the flow out is equal to the change in storage in a given period of time:

$$Q\ in - Q\ out = \Delta S/\Delta t \tag{6.2}$$

where Q in = inflow into system (cfs)
Q out = outflow from system (cfs)
ΔS = change in storage (ft^3)
and Δt = time interval (sec)

In most pipes systems, the boundaries are fixed so that there is no change in water stored in the system. If the discharge is constant, then at each flow cross section

$$Q = A_1V_1 = A_2V_2 = \ldots = A_nV_n \tag{6.3}$$

where A_i = cross sectional area of pipe, $A_i = \pi d^2_i/4$ (ft^2)
d_i = diameter of pipe (ft)
V_i = mean pipe flow velocity (fps)

6.2. Water Properties

Some of the physical properties of water vary with temperature to a great enough extent that this variation must be taken into account in the solution of pipe problems. Table 6.1 lists commonly used properties of water in the lb-sec-ft system of units.

TABLE 6.1. PROPERTIES OF WATER

Temperature ^oF	Specific Gravity	Specific Weight (γ) lb-ft^3	Absolute Viscosity (μ) lb-sec-ft^2	Kinematic Viscosity (ν) ft^2-sec
32	0.9999	62.42	3.746×10^{-5}	1.931×10^{-5}
39	1.0000	62.43	3.274 "	1.687 "
50	0.9997	62.41	2.735 "	1.410 "
60	0.9990	62.37	2.359 "	1.217 "
70	0.9980	62.30	2.050 "	1.059 "
80	0.9966	62.22	1.799 "	0.930 "
100	0.9931	62.00	1.424 "	0.739 "

6.3. Flow Classification— Reynolds Number

The Reynolds number (N_R) is used to delineate regimes of laminar and turbulent flow in pipes. For pipe flow the Reynolds number is defined using the pipe diameter d (in ft) as the characteristic length and the mean velocity V (in ft/sec) as the characteristic velocity. The kinematic viscosity ν(in ft^2-sec) is also used in the definition of the Reynolds number

$$N_R = Vd/\nu \tag{6.4}$$

For circular pipes flow will be laminar if $N_R < 2100$. When the Reynolds number is above 4000 the flow is turbulent. Flows with Reynolds numbers between 2100 and 4000 lie in a transition zone between laminar and turbulent flow. Care should be taken when selecting energy loss parameters for flows in this region.

6.4. Pipe Friction Losses

6.4.1. The Darcy-Weisbach Equation

Experiments show that in turbulent flow the head loss depends on the water velocity (V), pipe length (L), and pipe diameter (d). The Darcy-Weisbach equation relates these terms to the energy loss due to friction (h_f) by a friction factor term (f). The Darcy-Weisbach equation is written as

$$h_f = fL/d(V^2/2g) \tag{6.5}$$

Experimential observations show that f is related to pipe roughness and also to the fluid properties of viscosity (μ) and density (ρ). If the pipe roughness can be expressed as a representative roughness length e (ft), dimensional analysis can be used to show that the friction factor f can be directly related to a 'relative roughness' e/d and to the Reynolds number. Values of e are listed in Table 6.2 for several common types of pipe.

TABLE 6.2. EQUIVALENT ROUGHNESS, e, FOR VARIOUS TYPES OF PIPES

Type of Pipe	e in ft
Drawn Tubing	0.000 005
PVC (Plastic) Pipe	0.000 007
New Steel	0.000 15
New Cast Iron	0.000 85
Concrete	0.001 to 0.005
Riveted Steel	0.003 to 0.01
Old Cast Iron	0.01 to 0.03

Most pipe flows of engineering interest occur in the transition and fully-turbulent flow ranges where the relationship between f and N_R and e/d has been determined from many experiments. This complex relationship is usually presented in the form of the 'Moody diagram', shown here in Fig. 6.1. Because f usually changes with N_R, pipe flow problems usually require solution by an iterative or trial procedure. This type of solution is illustrated by the following example:

Example 6.1. Determine the rate of water flow through a 12-inch-diameter riveted-steel pipe (e = 0.01 ft) when the head loss due to friction is 10 ft in 1000 ft of pipe length. Assume the water temperature is 60°F.

For water at 60°F., $\nu = 1.2 \times 10^{-5}$ ft²/sec. (Table 6.1). The relative roughness of the pipe e/d = 0.01/1.0 = 0.01. From the Moody diagram (Fig. 6.1) the curve giving the relationship between f and N_R is nearly constant over the greater part of its range, with f = 0.039. Assume a value of 0.39 for f, substitute this into the Darcy-Weisbach equation, and solve for the velocity V. From Eq. 6.5,

$$h_f = fL/d(V^2/2g)$$

or

$$10 = 0.039(1000)/(1.0)/(V^2/64.4)$$

Solving for V gives

$$V = 4.06 \text{ ft/sec.}$$

For this velocity, the Reynolds number is

$$N_R = Vd/\nu = 4.06(1.0)/1.2 \times 10^{-5} = 3.4 \times 10^5$$

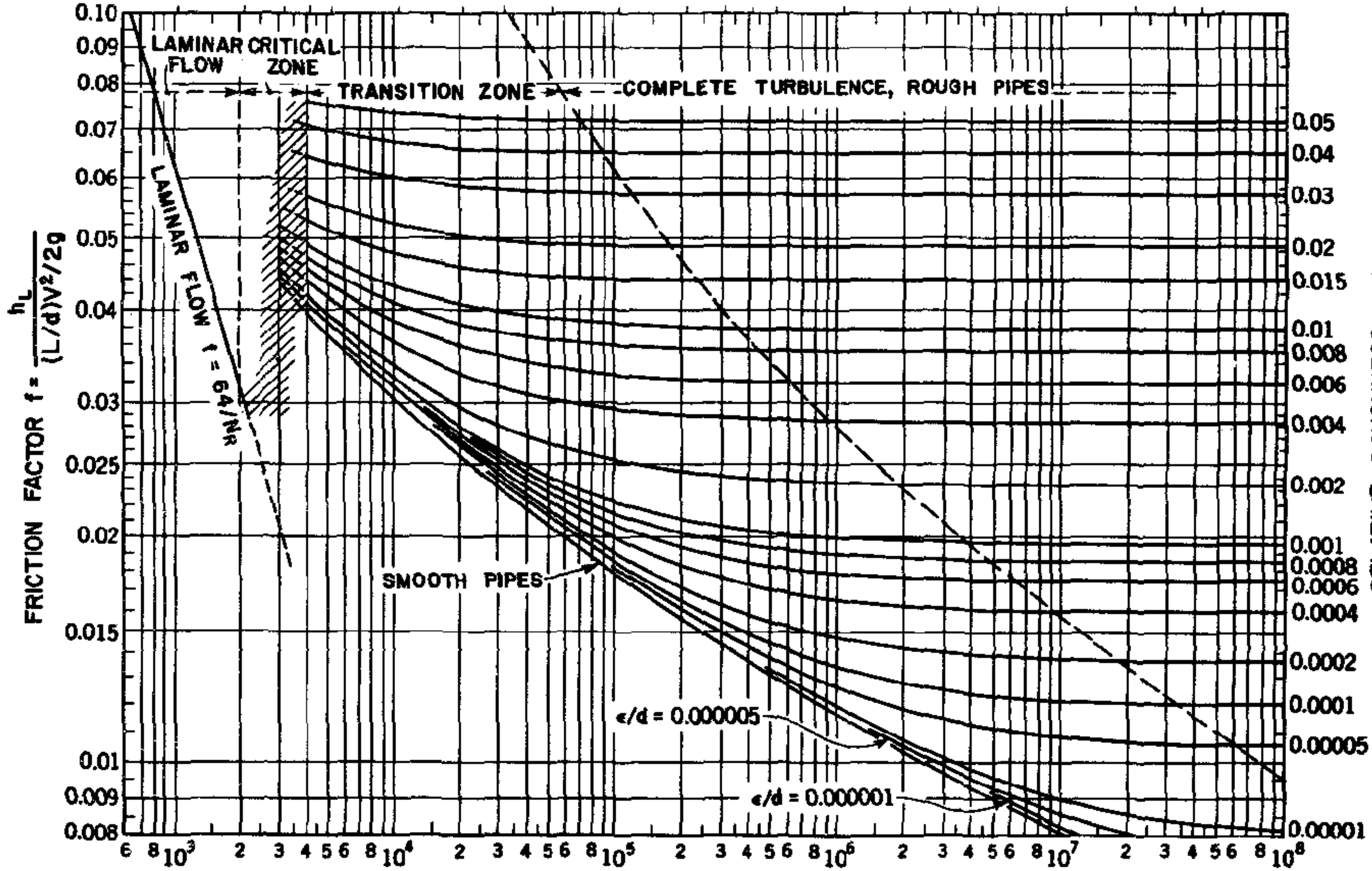

Fig. 6.1. Moody diagram. REYNOLDS NUMBER $N_R = Vd/\nu$ (d, ft; V, fps; ν, ft^2/sec)

From the Moody diagram, f = 0.038.

A second trial can now be made with f = 0.038. From Eq. 6.5

$$V = \left\{[10(1.0)\ (64.4)] \ /\ [0.038(1000)]\right\}^{1/2} = 4.12 \text{ ft/sec}$$

For this velocity,

$$N_R = 3.4 \times 10^5$$

From the Moody diagram, f is again equal to 0.038, and therefore, the rate of flow through the pipe is

$$Q = AV = (\pi/4)(1.0)^2(4.12) = 3.23 \text{ cfs.}$$

6.4.2. Empirical Formulas

In spite of their accuracy in determining headlosses in pipe flow, the Darcy-Weisbach equation and friction factor are not commonly used by engineers for practical flow problems. Various empirical formulas are widely used, and as long as these formulas are applied within the range of conditions for which they have been developed, they give accurate solutions to pipe flow problems.

Hazen-Williams Formula.— One of the most popular of the empirical equations is the Hazen-Williams formula. In the English system of units the formula is written as

$$Q = 1.318\ C_{HW} R^{0.63} A S^{0.54} \qquad (6.6)$$

where

C_{HW} = the Hazen-Williams roughness coefficient
R = hydraulic radius, R = A/P (ft)
A = cross sectional area (ft)
P = *wetted perimeter (ft)*
S = slope of the energy gradeline

The energy gradeline slope is the energy loss h_f divided by the pipe length ($S = h_f/L$).

For circular pipe

$$R = A/P = (\pi d^2/4)/(\pi d) = d/4 \qquad (6.7)$$

In contrast to the Darcy-Weisbach coefficient, the Hazen-Williams roughness coefficient is not given as a function of the Reynolds number, but is taken merely as a function of pipe material. Some typical values are given in Table 6.3. The tabulated values apply to pipes that are larger than 2 inches in diameter and which have velocities less than 8 to 10 ft/sec. The highest values of C_{HW} are 140 to 150 and are for very smooth pipe. In many types of pipes (such as cast iron or steel) there is a large reduction in the coefficient with pipe age. For the design of pipe systems a conservatively low value of C_{HW} is chosen to prevent the system from being under-designed as the pipes deteriorate with age.

It should be kept in mind that Eq. 6.6 is applicable only when English units are being used. If SI units (Systeme International d'Unites) are used, then the equation is written as

$$Q = 0.849\ C_{HW}R^{0.63}AS^{0.54} \tag{6.8}$$

It is often necessary to compute the headloss in a pipe for a given flow. The Hazen-Williams formula can be rearranged using the substitution $S = h_f/L$ to permit direct determination of h_f. In English units the equation is

$$h_f = \frac{4.73\quad LQ^{1.85}}{C_{HW}^{1.85}\quad d^{4.87}} \tag{6.9}$$

The equation is now in a form that is similar to the Darcy-Weisbach equation. Note that when this formulation is used, the head loss is proportional to the discharge to the 1.85 power rather than to the discharge squared.

TABLE 6.3. HAZEN-WILLIAMS COEFFICIENTS FOR VARIOUS TYPES OF PIPE MATERIALS

Type of Pipe	C_{HW}
Very smooth pipe	140-150
PVC(plastic) pipe	140-150
New Cast Iron or Steel	130
Concrete	120
Clay	110
Riveted Steel	110
Brick Sewer	100
Old cast iron	80-100

Example 6.2.: For the data of Example 6.1, compute the discharge by the Hazen-Williams equation.

$$S = h_f/L = 10/1000 = 0.01$$

$$R = d/4 = 1.0/4 = 0.25 \text{ ft.}$$

$$A = (\pi/4)d^2 = (\pi/4)(1.0)^2 = 0.785 \text{ ft}^2$$

From Table 6.3, C_{HW} for riveted steel pipe is 110. Therefore,

$$Q = 0.849(110)^{0.63}(0.25)(0.785)(0.01)^{0.54} = 2.55 \text{ cfs.}$$

Manning's Equation.— Civil and agricultural engineers frequently use the Manning equation for pipe flow calculations, especially for culverts and irrigation systems. Although this empirical equation was originally developed for analysis of open channel flows, if the range of flow conditions for which the roughness coefficients were determined is not exceeded, good results can be obtained. Manning's equation in English units is

$$Q = \frac{1.486}{n} R^{2/3} S^{1/2} A \qquad (6.10)$$

where n = Manning's roughness coefficient, and the other symbols are as defined above. The equation was originally derived with metric units, so for applications using SI units, the constant 1.486 should be replaced with unity. Values of Manning's n for water flow in pipes are listed in Table 6.4.

TABLE 6.4. MANNING'S ROUGHNESS COEFFICIENTS, n, FOR VARIOUS TYPES OF PIPES

Type of pipe	n
Very smooth pipe	0.008–0.010
PVC (Plastic) pipe	0.009–0.011
New cast iron or steel	0.012–0.014
Concrete	0.012–0.015
Clay	0.013–0.017
Riveted steel	0.013–0.017
Brick sewer	0.014–0.022
Old cast iron	0.020–0.030
Corrugated metal	0.020–0.025

Manning's equation can also be written in terms of the friction loss h_f in a pipe of given length L. For English units the equation is

$$h_f = 4.64\ n^2\ Q^2\ L/D^{16/3} \qquad (6.11)$$

The head loss due to friction in this particular expression is proportional to the discharge squared.

Example 6.3.: Using the data provided in the previous two examples, compute the discharge using Manning's equation with n = 0.015 as the average value for riveted steel (Table 6.4).

$$Q = (1.486/n) R^{2/3} S^{1/2} A = (1.486/0.015)(0.25)^{2/3}(0.01)^{1/2}(0.785)$$

$$= 3.09 \text{ cfs.}$$

Note that for this particular example this agrees reasonably closely with the value computed by the Darcy-Weisbach friction factor. The Hazen-Williams equation (Example 6.2) gives a discharge that is about the same as would be given by a Manning's n of 0.018 for this size pipe.

6.5. Minor Losses

In addition to energy losses resulting from pipe boundary friction, losses also occur as a result of changes in the pipe flow patterns caused by changes in the conduit area or direction of flow. These changes create additional turbulence which in turn produces additional energy loss as the turbulence is dissipated. These losses are referred to as minor losses, because in long pipelines friction losses are usually much larger than other types of losses.

In many cases, however, these losses may be the largest losses in the system. For example, a partially closed valve can produce much larger losses than pipe friction in a given system. Typically, minor losses include those due to:

1. Pipe entrance from a larger water body.
2. Pipe exit.
3. Expansions (abrupt or gradual).
4. Contractions (abrupt or gradual).
5. Pipe fittings such as bends, elbows, tees, etc.
6. Valves (even when fully opened).

Because of the complex nature of flow in the above situations the losses must be determined from experimental data. The head loss is usually computed as a function of the pipe velocity head by means of a head loss coefficient K,

$$h_L = KV^2/2g \tag{6.12}$$

Tables of loss coefficients for various types of pipe fittings are available. These data are frequently provided by manufacturers of specific fittings. Some representative values are given in Table 6.5.

TABLE 6.5. HEAD LOSS COEFFICIENTS FOR VARIOUS PIPE FITTINGS

Fitting	K
Fully open check valve (Ball type)	70
(Lift type)	12
(Swing type)	2.5
Fully open foot valve	15
Fully open globe valve	10
Fully open angle valve	5
Gate valve - fully open	0.2
Gate valve - 3/4 open	1.0
Gate valve - 1/2 open	5.6
Close return bend	2.2
Standard short radius elbow	0.9
Medium sweep elbow	0.8
Long sweep elbow	0.6
45^o elbow	0.4

The minor losses in a pipe system are related to $V^2/2g$, and for a pipe with constant diameter the losses can be summed into a single total loss term h_T (including friction also) as:

$$h_T = K_{system} V^2/2g \tag{6.13}$$

The losses can also be expressed as a function of the discharge squared where the magnitude of K'_{system} is dependent on the areas of the various pipes in the system as well as the individual loss coefficients

$$h_T = K'_{system} Q^2 \tag{6.14}$$

where Q is the system discharge.

6.5.1. Bend Losses

Bend losses are usually computed as an increase in head loss over the loss that would occur in a straight length of pipe. The loss is related to the bend angle and the ratio of bend radius to pipe diameter. Fig.6.2 gives loss coefficient data of this type. In some cases the change in pipe direction is accomplished by means of a mitred joint. Fig. 6.2 can also be used to obtain the loss coefficient for mitred bends.

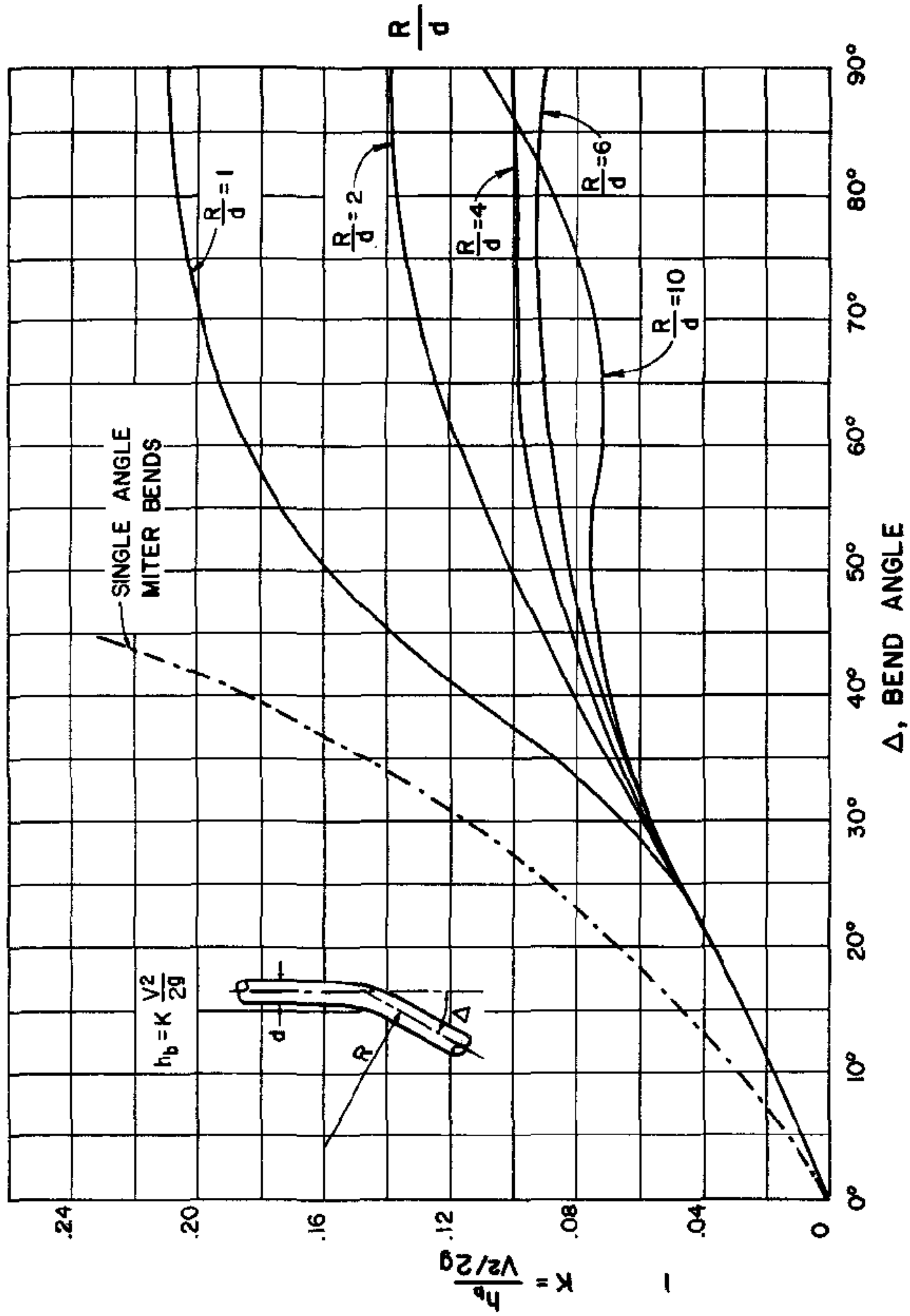

Fig. 6.2. Head loss coefficients for pipe bends (Bend loss).

6.5.2. Entrance and Exit Loss Coefficients

When flow enters a pipe from a reservoir separation effects may be present unless the inlet is rounded to prevent this. Fig. 6.3 shows loss coefficients for various types of inlets from reservoirs. Note that even a moderate degree of rounding of the inlet can reduce the head loss significantly.

Little can be done to reduce the loss at the exit into a reservoir. There is usually no way to change the flow kinetic energy (which is represented by the velocity head) into potential energy. The kinetic energy is usually entirely dissipated as turbulence, making $K_{outlet} = 1.0$ for all types of outlets.

6.5.3. Expansion and Contractions

Abrupt changes in the diameter of the pipe are termed sudden expansions and contractions. For sudden expansions, the theoretical loss coefficient (which agrees well with experimental values) is given by

$$K = \left(1 - \frac{d_1^2}{d_2^2} \right)^2 \qquad (6.15)$$

The loss coefficient is applied to the velocity head for the smaller pipe, with diameter d_1. When the flow is into a large reservoir $d_1/d_2 = 0$, and the loss coefficient approaches 1.0.

The loss coefficient for a sudden contraction can be determined from the empirical formula

$$K = 0.42 \left(1 - \frac{d_2}{d_1} \right)^2 \qquad (6.16)$$

where d_2 is the small pipe diameter.

Gradual enlargements (also known as diffusers) have head loss coefficients which depend on the expansion angle of the enlargement. The experimental data of Gibson (1930) are shown in Fig. 6.4. With small central angles the loss depends primarily on surface friction. With larger angles separation occurs, and the loss is a maximum for angles of about 40^o to 60^o. The loss coefficient for this range of angles is actually greater than for the sudden enlargement. The optimum enlargement angle is about 7^o, because for smaller angles the friction losses become large.

6.6. Pipeflow Calculations - Single Pipe

In pipeflow analysis it is useful to make sketches of the pipeline with the energy grade line and hydraulic gradelines plotted to scale. These sketches clearly illustrate changes in velocity, pressure, and energy along the length of the pipeline.

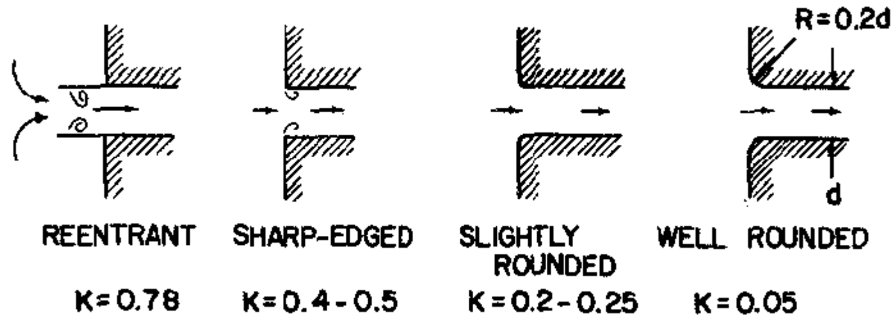

Fig. 6.3. Head loss coefficients for inlets to pipes from a reservoir.

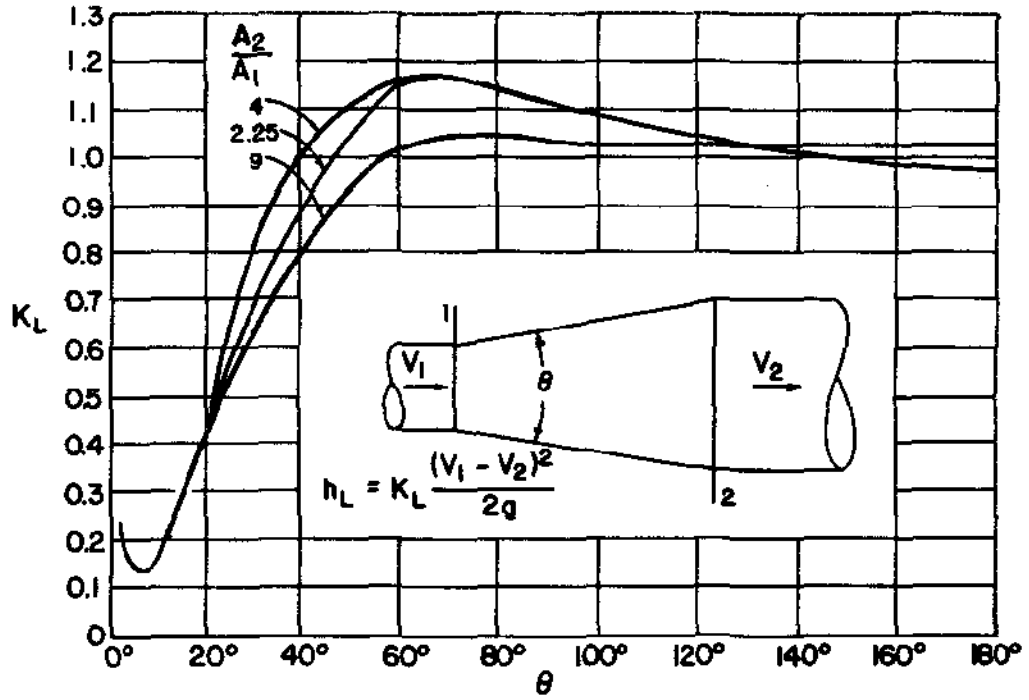

Fig. 6.4. Head loss coefficients for conical enlargements (Gibson).

Steady flow pipe hydraulic problems can be solved using the principles of energy and mass conservation. Engineering pipe flow problems are of three general types:

1. Calculation of <u>head loss</u> and variation in head from discharge and pipe system characteristics,
2. Calculation of <u>discharge</u> from given head and pipe system characteristics, and
3. Calculation of the required <u>pipe diameter</u> to provide a given discharge for a given head (or head difference).

The first type of problem can be solved directly by the Darcy-Weisbach equation. The second and third types cannot be solved directly when the Darcy-Weisbach friction factor is used, because in the general case, the friction factor f and loss coefficient K depend on the Reynolds number which involves discharge and pipe diameter (the unknown quantities in these types of problems). Usually a trial-and-error procedure is required to permit a solution.

Consider the following simple example: A pipe of constant diameter carries flow between two reservoirs. The difference in reservoir water levels is H_o. The pipe system is shown in Fig. 6.5. As indicated by the energy grade line changes, (as expressed by the energy equation):

$$H_o = h_e + h_f + h_o \qquad (6.17)$$

where h_e = head loss at the entrance of the pipe
h_f = head loss due to pipe friction, and
h_o = head loss at the outlet to the downstream reservoir.

If $K_e = 0.5$, $K_o = 1.0$, and if the Darcy-Weisbach equation is used to compute h_f, then Eq. 6.17 can be written in terms of the pipe velocity head $V^2/2g$ as

$$H = (0.5 + fL/d + 1.0)\ V^2/2g \qquad (6.18)$$

The value of f is a function of pipe roughness (e/d and the Reynolds number N_R. The relative importance of the head loss terms is strongly related to the length of the pipeline. For large values of L/d, the other loss terms are insignificant and can be neglected. This is illustrated by the following examples:

Example 6.4: The reservoirs in Fig. 6.5 have a difference in level of 20 ft. For a steel pipe with e = 0.0002, a pipe diameter of 2.0 ft and L of 2000 ft, what is the pipe discharge?

From Eq. 6.18

$$20 = [(0.5 + f\ (2000/2.0) + 1.0]\ V^2/2g$$

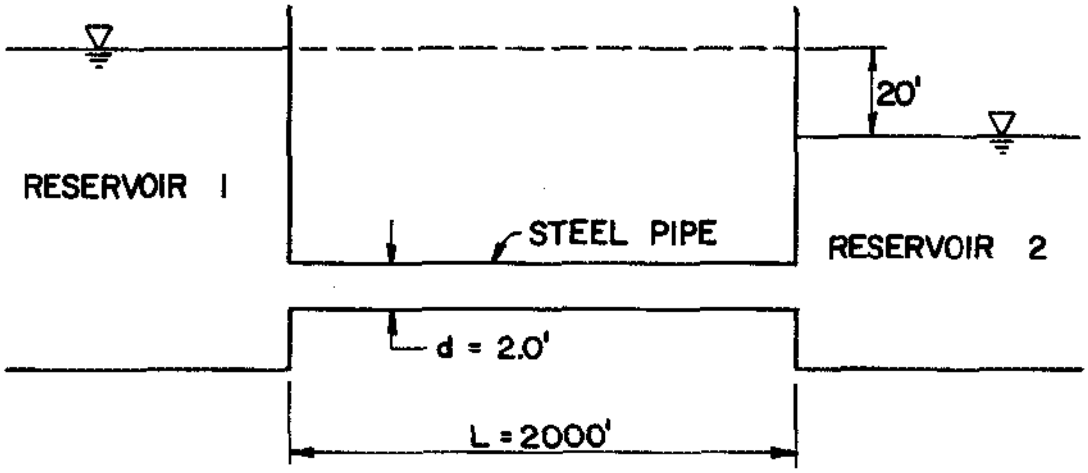

Fig. 6.5. *Reservoirs connected by a single pipe (Example 6.4.).*

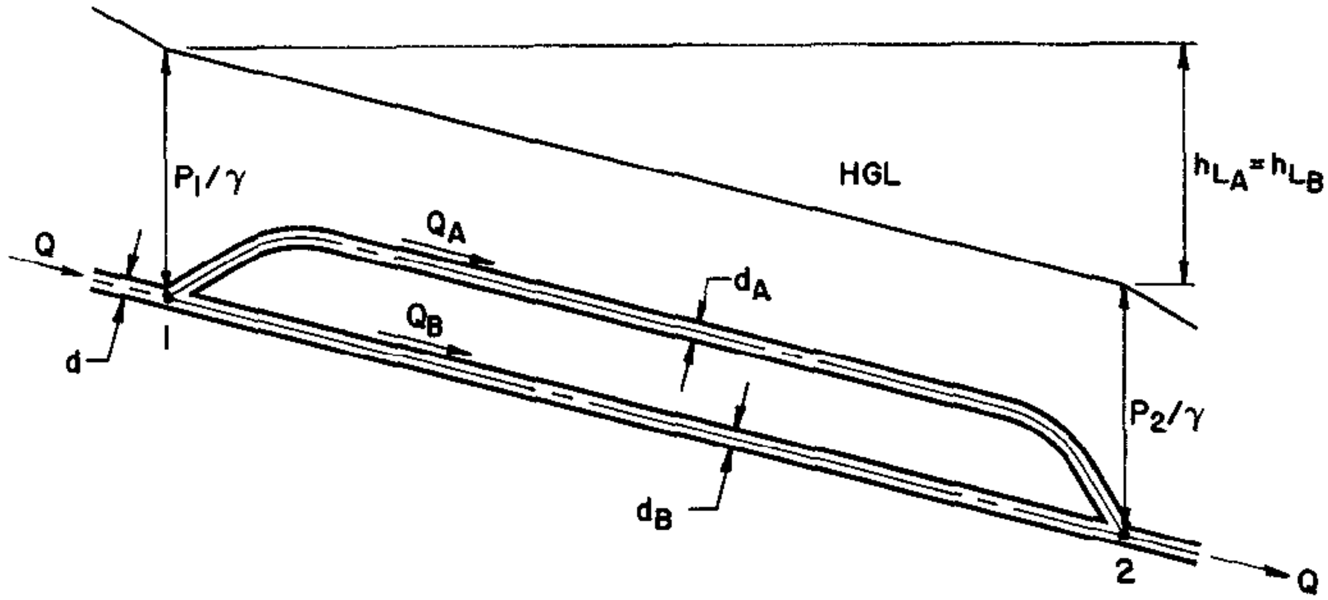

Fig. 6.6. *Parallel pipe system.*

Two unknowns are present in this problem: f and V (or Q). We can assume a value of f and solve by trial in this case. Try f = 0.02. Then

$$40 = (0.5 + 0.02\ (2000/2) + 1.0)\ V^2/2g$$

and

$$V^2/2g = 40/21.5 = 0.930,$$

thus

$$V = 7.74 \text{ft/sec}.$$

Assume $\nu = 1.2 \times 10^{-5}$ ft^2/sec. Then $N_R = Vd/\nu = 7.74\ (2)/1.2 \times 10^{-5} = 1.29 \times 10^6$. For e/d = 0.0002/2 = 0.0001, from the Moody diagram (Fig. 6.1), f = 0.013, which is significantly lower than the assumed value of 0.02.

Taking f = 0.013 for the next trial:

$$20 = [0.5 + 0.013\ (2000/2) + 1.0]\ V^2/2$$

$$V^2/2g = 20/14.5 = 1.379$$

and

$$V = 9.42 \text{ ft/sec}$$

For this V, $N_R = 9.42\ (2.0)/1.2 \times 10^{-5} = 1.57 \times 10^6$. From the Moody diagram f = 0.012 for this value of N_R. The third trial, therefore, is based on f = 0.012.

$$20 = [0.5 + 0.012\ (2000\ /2) + 1.0]\ V^2/2g$$

$$V^2/2g = 20/13.5 = 1.481$$

$$V = 9.77 \text{ ft/sec}$$

$$N_R = 9.77(2)/1.2 \times 10^{-5} = 1.6 \times 10^6$$

and f = 0.012. Thus V = 9.77 ft/sec is the correct value. The pipe discharge is

$$Q = AV = (\pi/4)(2.0)^2(9.77) = 30.7 \text{ cfs}.$$

6.7. Flow in Noncircular Conduits

The Hydraulic Diameter— In a circular pipe with diameter d, the hydraulic radius R is

$$R = d/4 \text{ or } d = 4R \qquad (6.19)$$

where $R = A/P$, and P is the wetted perimeter. The Darcy–Weisbach equation (6.5) can be written in terms of the hydraulic radius instead of the pipe diameter d

$$h_f = f \frac{L}{4R} \frac{V^2}{2g} \tag{6.20}$$

where d is replaced by 4R. It is customary to define the hydraulic diameter D_h as

$$D_h = 4A/P = 4R \tag{6.21}$$

The Moody diagram can be used to relate the friction factor to the Reynolds number (given in this case by $N_R = VD_h/\nu$) and a relative roughness (e/D_h). In the turbulent flow range, the relationship holds quite closely.

Example 6.5: What is the friction loss in 100 ft of a 2-ft-square concrete conduit when the discharge is 20 cfs?

$R = A/P = (2)(2)/(4)(2) = 0.50$ ft

$D_n = 4R = 4(0.50) = 2.0$ ft

$N_R = VD_h/\nu = (20/4)(2.0)/1.2 \times 10^{-5} = 8.3 \times 10^5$

From Table 6.2, choose $e = 0.002$ ft.

$e/D_h = 0.002/2.0 = 0.001$

From the Moody diagram (Fig. 6.1), $f = 0.02$. Therefore,

$$h_f = 0.02 \frac{(100)}{4(0.5)} \frac{(20/4)^2}{64.4} = 0.39 \text{ ft}$$

6.8. Multiple Pipes

Multiple pipe systems can become quite complex. As an introduction to systems of multiple pipes consider the parallel pipe system shown in Figure 6.6. In engineering practice the capacity of a pipeline is frequently increased by laying a second pipeline parallel to it. The pipes, labeled A and B in the figure, are connected at points 1 and 2. At the points where the pipelines are connected the energy grade line elevations must be the same in the two pipes. Because the head loss between points 1 and 2 is the difference in the EGL elevations,

$$h_{L_A} = h_{L_B} \tag{6.22}$$

A second equation (conservation of mass) can also be written

$$Q = Q_A + Q_B \tag{6.23}$$

In general terms the head loss can be related to the discharge by writing it as

$$h_L = KQ^n$$

Equation (6.22) can then be written as

$$K_A Q_A{}^n = K_B Q_B{}^n \tag{6.24}$$

Example 6.6: An 18-inch diameter, 4000 ft long pipeline connects two reservoirs which have an elevation difference of 50 ft. The maximum discharge through this line is 6.0 cfs. It is desired to increase the discharge between the reservoirs by laying a 2500 ft long pipe of the same material and size parallel to the first 2500 feet of the original line. If this is done, what maximum discharge between the reservoirs can be attained?

Solution: Assume that the Hazen-Williams formula is the appropriate equation for determining the friction loss. In this case n = 1.85, and for the initial pipe

$$50 = K(6.00)^{1.85}$$

hence

$$K = 1.811$$

K is directly proportional to pipe length, so K for each 2500 ft pipe section is (2500/4000) (1.811) = 1.132. For the common 1500 ft long section K = (1500/4000) (1.811) = 0.679.

The head loss through the common pipe plus branch A is

$$50 = 0.679\ Q^{1.85} + 1.132\ Q_A{}^{1.85}$$

Similarly for branch B

$$50 = 0.679\ Q^{1.85} + 1.132\ Q_B{}^{1.85}$$

Simultaneous solution of these two equations gives $Q_A = Q_B$, and since $Q = Q_A + Q_B$, $Q_A = Q_B = Q/2$.

$$50 = 0.679\ Q^{1.85} + 1.132(Q/2)^{1.85}$$

Solving gives Q = 8.30 cfs, thus the increase in discharge is 2.30 cfs.

6.9. Three Reservoir Analysis

The three reservoir problem provides another example of the use of the principle of mass and energy conservation to solve a flow problem. In this problem, as illustrated in Fig. 6.7, pipes connected to three reservoirs meet at a common point, 0. Three possible flow conditions can exist:

1. Flow may be out of reservoir A into B and C
2. Flow may be from reservoirs A and B into C
3. If the head at the junction is just equal to the reservoir elevation at B, there can be flow from A to C without flow into or out of reservoir B.

If the head losses are computed using $h_L = KQ^n$, then for Case 1:

$$Q_A - Q_B - Q_C = 0$$
$$Z_A - K_A Q_A^n = Z_C + K_C Q_C^n \qquad (6.26)$$
$$Z_A - K_A Q_A^n = Z_B + K_B Q_B^n$$

For Case 2:

$$Q_A + Q_B - Q_C = 0$$
$$Z_A - K_A Q_A^n = Z_C + K_C Q_C^n \qquad (6.27)$$
$$Z_B - K_B Q_B^n = Z_B - K_B Q_B^n$$

For Case 3: Since $Q_B = 0$, either of the two sets of equations can provide a solution.

One can determine which of these sets of equations (i.e., Case 1 or Case 2) should be used by making a trial calculation assuming $Q_B = 0$, using the two head equations for either case and computing Q_A and Q_B. If $Q_A > Q_C$, then the Case 1 equations should be used. If $Q_A < Q_C$, then Case 2 applies.

The solution is made by assuming a value for the head at the junction, h_j, solving for the corresponding discharges in each pipe, and then making a correction to the assumed head. For example, for Case 1 let $\Delta Q = Q_A - Q_B - Q_C$. When ΔQ reaches some appropriately small value, the problem is solved.

$$K_A Q_A^n = Z_A - h_j = h_{L_A}$$
$$K_B Q_B^n = h_j - Z_B = h_{L_B} \qquad (6.28)$$
$$K_C Q_C^n = h_j - Z_B = h_{L_C}$$

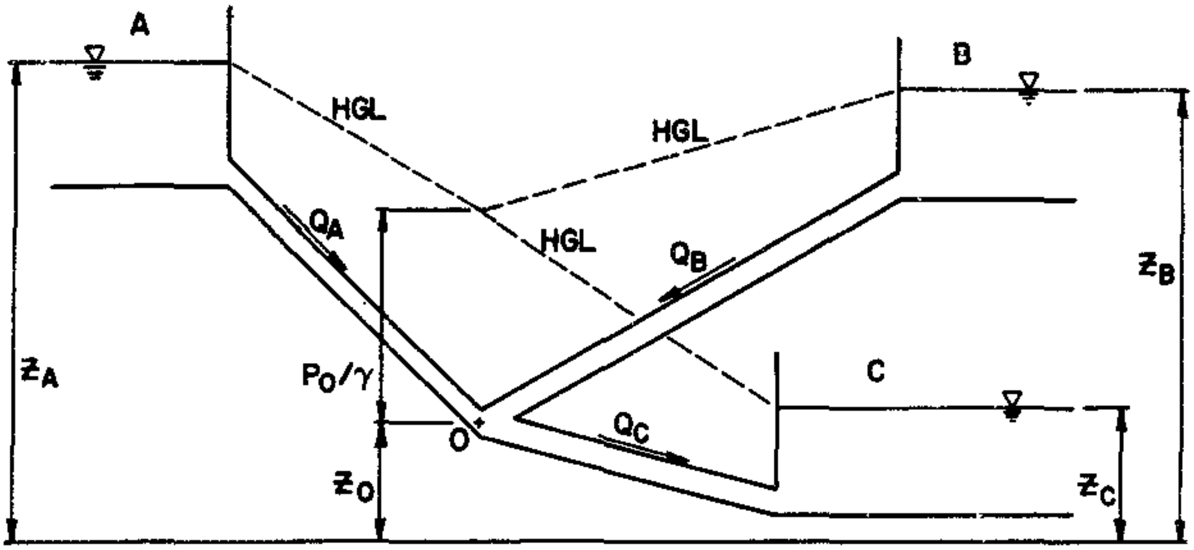

Fig. 6.7. *Three reservoir problem.*

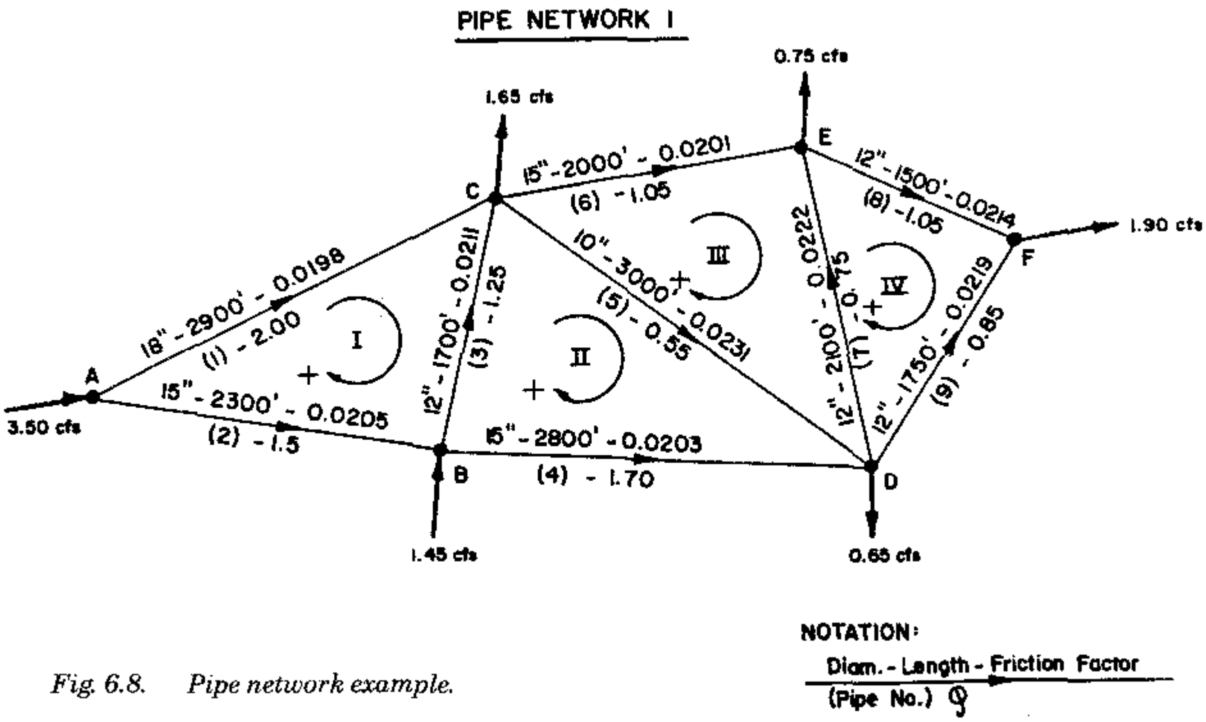

Fig. 6.8. *Pipe network example.*

The correction to the assumed head at the junction is given by

$$h_{j\ new} = h_j + \Delta h \tag{6.29}$$

The computational process is repeated until ΔQ is sufficiently small.

6.10. Pipe Networks

Pipe systems consisting of many interconnected pipes can become very complex. An example of a pipe network system is the water distribution system for a city. In these networks there may be many pipe connections and many points where flow leaves the system, and there are frequently several points where flow is supplied to the system. The pipes are of various sizes and have differing hydraulic characteristics. Pumps, valves and fittings and reservoirs may also have to be taken into account.

A simple pipe network is shown schematically in Fig. 6.8. In this network the pipe junctions (nodes) are labeled using capital letters, the individual pipes are labled by numbers, the pipe 'loops' (closed circuit through which flow can pass) are labeled using Roman numerals. Flow in the clockwise direction is assumed to be positive.

Conservation of mass (continuity) and conservation of energy must be satisfied throughout the network. Because continuity must be satisfied at each junction the following must hold:

$$\sum_{j=1}^{M} Q_j = 0 \tag{6.30}$$

where j = the number of the element connected to the junction
Q_j = discharge in each pipe (or inlet or outlet) at the junction
M = number of elements connected to the junction

At any junction there can be only one position of the energy grade line. This means that the net head loss around any single loop in the network must be zero. Thus, around each loop

$$\sum_{i=1}^{N} h_{L_i} = 0 \tag{6.31}$$

where i = the pipe number
h_{L_i} = the head loss in pipe i (if the flow in the pipe is counterclockwise, the value of h_L is negative)
N = number of pipes in the loop

As discussed earlier, the head loss can be written in the form

$$h_{L_i} = K_i Q_i^n \tag{6.32}$$

The solution is made by assuming initial values Q_i for all flows in the system. If these initial values are reasonably close to the actual flows, then the actual flows Q_i can be expressed as being equal to the initial estimate plus a correction ΔQ.

Therefore,

$$Q_{Ai} = Q_i + \Delta Q_i \tag{6.33}$$

Equation 6.31 for each loop can now be written (using Eq. 6.32 and 6.33 as

$$\sum_{i=1}^{N} K_i \left(Q_i + \Delta Q\right)^n = 0 \tag{6.34}$$

where ΔQ is a flow correction applied to the entire loop. Once ΔQ is known the problem is solved.

6.11. Hardy-Cross Method

In the general case, the exponent n in Eq. (6.34) is not an integer. To determine ΔQ we can expand Eq. (6.34) by the binomial theorem. If ΔQ is small, all terms after the first two terms in the binomial expansion can be dropped. Solving for ΔQ gives

$$\Delta Q = \frac{\sum_{i=1}^{N} K_i Q_i^n}{- n \sum_{i=1}^{N} K_i Q_i^{n-1}} \tag{6.35}$$

The correction ΔQ can be either positive or negative. In order to retain the correct sign for ΔQ, the absolute value of the discharge is used in the calculations of the summation in the denominator of Eq. (6.35), while the terms in the summation in the numerator retain the appropriate sign for the discharge in the individual pipe.

In the Hardy-Cross method for network analysis corrections are made to each loop computation. Pipes which are common to more than one loop have multiple ΔQ corrections. The correct flows cannot be determined, therefore, by a single pass through the network. An iterative procedure must be used, and the corrections must be made to each loop until the value of the correction becomes so small that further corrections will have only very small effects on the solution.

In some cases the solution procedure will not converge. This problem can be frequently overcome by assuming a new set of initial values which are closer to the actual values and repeating the calculations. The results obtained from the first set of calculations are often useful in providing a better estimate.

The Hardy-Cross Method was developed as a hand calculation procedure. More efficient procedures exist for computer applications, but this procedure will give answers that are just as accurate as other methods because the same set of equations are solved in all of the currently used network analysis methods. A simple FORTRAN program is given below to illustrate the application of the Hardy-Cross procedure. In order to properly prepare the input for the program, it is suggested the following procedure be followed:

1. Draw the network and label loops, pipes, and junctions.

2. Tabulate (or show on the drawing) pipe diameters, length, and roughness.

3. Show on the drawing all flows into or out of the network (magnitude and direction).

4. Estimate and show on the drawing the discharges in each loop. Continuity must be satisfied at each junction. Use an arrow to indicate flow direction.

5. Tabulate the data for each pipe in numerical sequence, listing: pipe number, diameter, length, roughness value, initial flow estimate, its primary loop number, and its secondary loop number. The sign of the discharge is positive if the flow is in the clockwise direction, negative if the flow is in the opposite direction. An example run, (with program data,) for the example shown in Fig. 6.8. is shown in Fig. 6.9.

6.12. Hardy-Cross Pipe Network Computer Program

This computer program can be used to determine the steady-state flow in each pipe in a pipe network. The program arrays are dimensioned to allow up to 200 pipes and 100 nodes to be used. If needed, additional system elements can be added merely by changing the dimension statements at the beginning of the program.

Program Structure— This program is simple enough to permit all computations to be done using a single program unit without subroutines. The initial section of the program deals with data input. The data are read from a file, the name of which is supplied by the user from the keyboard. The name of the file to which the output data are written is also specified at the beginning of the run. Once the data are read, the program goes

```
================================================================================
HARDY CROSS PIPE NETWORK ANALYSIS:
================================================================================

NETWORK HAS   9 PIPES AND   4 LOOPS

FLOW RATES ARE COMPUTED TO NEAREST 0.01 CFS
================================================================================
```

NETWORK ELEMENTS:

PIPE NO.	DIAM.(IN)	LENGTH(FT)	DARCY F	Q(CFS)	PRIMARY LOOP	SECONDARY LOOP
1	18.0	2900.0	0.0198	2.00	1	0
2	15.0	2300.0	0.0205	-1.50	1	0
3	12.0	1700.0	0.0211	-1.25	1	2
4	15.0	2800.0	0.0203	-1.70	2	0
5	10.0	3000.0	0.0231	0.55	2	3
6	15.0	2000.0	0.0201	1.85	3	0
7	12.0	2100.0	0.0222	-0.75	3	4
8	12.0	1750.0	0.0219	-0.85	4	0
9	12.0	1500.0	0.0214	1.05	4	0

```
================================================================================
RESULTS OF NETWORK ANALYSIS:
================================================================================

NO. OF ITERATIONS =  10        FLOW RATES ARE COMPUTED TO NEAREST 0.01 CFS
```

PIPE NO.	CONVEYANCE	Q (CFS)	HEADLOSS (FT)
1	0.190344	2.38	1.08
2	0.388923	-1.12	0.49
3	0.902950	-0.82	0.60
4	0.468853	-1.76	1.44
5	4.340821	0.45	0.87
6	0.331595	1.90	1.19
7	1.173558	-0.54	0.35
8	0.964749	-1.01	0.98
9	0.808048	0.89	0.64

Fig. 6.9. Data and results for Hardy-Cross analysis of pipe network shown in Fig. 6.8.

through the iterative process to determine the flows throughout the network. At the completion of the calculations the results are written to the output file. A message is printed in the output if the solution does not converge after 20 iterations.

Example Runs— Two examples are given to illustrate how the input data are coded from the physical description of the pipe network and how the printed program results look. The first example is the network shown in Figure 6.8, and consists of a network with 9 pipes making 4 loops.

Note that an initial estimate of the flow in each pipe must be supplied as program input. These initial flows must satisify the condition that the sum of the flows leaving the node must equal the sum of the flows entering. The sign for the numerical value for the flow in each pipe must be positive if the flow is in the clockwise direction for the primary loop of the pipe (and negative, of course, if in the negative direction).

For example, in Loop I, the initial flow in Pipe 1 is assumed to be 2.00 cfs (positive), and therefore, the flow in Pipe 2 must be -1.50 cfs to satisfy continuity at the first node. At the node at the intersection of Pipes 2, 3, and 4 the flow into the node is 1.50 + 1.45 = 2.95 cs. The flow out of the node into Pipes 3 and 4 must be assumed to be -1.25 cfs (negative, because it is in the counterclockwise direction for Loop I), and, therefore, the flow in Pipe 4 must be -1.70 cfs.

The input and output data for this example are shown in Fig. 6.9. Note that 10 iterations are required to determine if the network flows to within 0.01 cfs. The convergence would be more rapid if a larger value of DELTAQ were used in the input. If convergence does not occur, a second trial should be made using the output from the previous run to estimate the pipe flow for the next trial.

The second example (Fig. 6.10 and 6.11) is provided to illustrate a more complex example. In this case there are 21 pipes and 10 loops. Procedures exist which are somewhat more efficient than the Hardy Cross method for the analysis of very large pipe networks. The reader is referred to the texts by Jeppson (1983) and Watters (1979) for these more advanced procedures.

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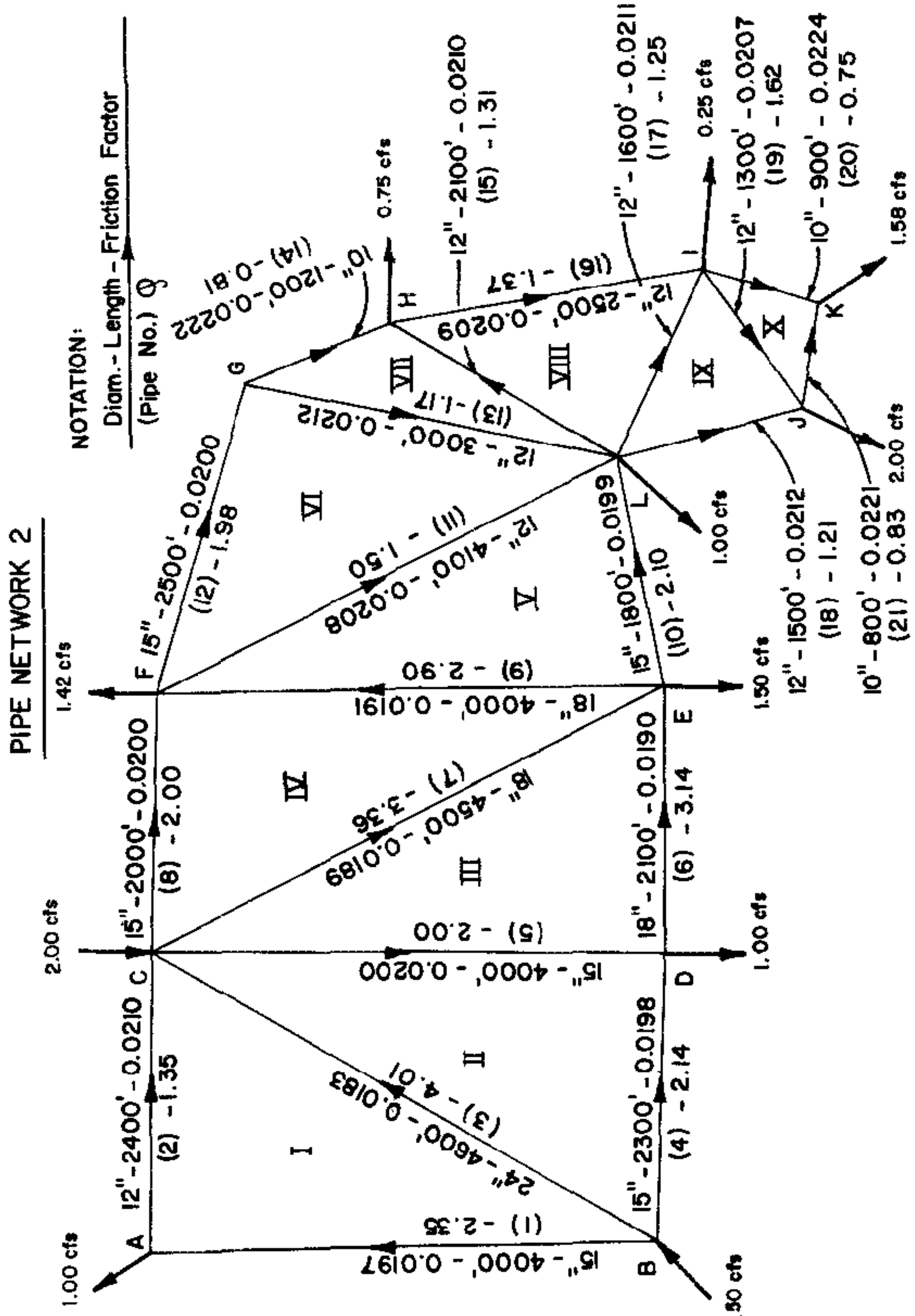

Fig. 6.10. *Pipe network with 21 pipes and 10 loops.*

HARDY CROSS PIPE NETWORK ANALYSIS:

NETWORK HAS 21 PIPES AND 10 LOOPS

FLOW RATES ARE COMPUTED TO NEAREST 0.01 CFS

NETWORK ELEMENTS:

PIPE NO.	DIAM.(IN)	LENGTH(FT)	DARCY F	Q(CFS)	PRIMARY LOOP	SECONDARY LOOP
1	15.0	4000.0	0.0197	2.35	1	0
2	12.0	2400.0	0.0210	1.35	1	0
3	24.0	4600.0	0.0183	-4.01	1	2
4	15.0	2300.0	0.0198	-2.14	2	0
5	15.0	4000.0	0.0200	2.00	2	3
6	18.0	2100.0	0.0190	-3.14	3	0
7	18.0	4500.0	0.0189	3.36	3	4
8	15.0	2000.0	0.0200	2.00	4	0
9	18.0	4000.0	0.0191	-2.90	4	5
10	15.0	1800.0	0.0199	-2.10	5	0
11	12.0	4100.0	0.0208	1.50	5	6
12	15.0	2500.0	0.0200	1.98	6	0
13	12.0	3000.0	0.0212	1.17	6	7
14	10.0	1200.0	0.0222	0.81	7	0
15	12.0	2100.0	0.0210	-1.31	7	8
16	12.0	2500.0	0.0209	1.37	8	0
17	12.0	1600.0	0.0211	-1.25	8	9
18	12.0	1500.0	0.0212	-1.21	9	0
19	12.0	1300.0	0.0207	1.62	9	10
20	10.0	900.0	0.0224	0.75	10	0
21	10.0	800.0	0.0221	-0.83	10	0

RESULTS OF NETWORK ANALYSIS:

NO. OF ITERATIONS = 11 FLOW RATES ARE COMPUTED TO NEAREST 0.01 CFS

PIPE NO.	CONVEYANCE	Q (CFS)	HEADLOSS (FT)
1	0.649993	1.36	1.21
2	1.268711	0.36	0.17
3	0.066220	-4.56	1.38
4	0.375643	-2.57	2.49
5	0.659891	1.30	1.12
6	0.132266	-2.88	1.09
7	0.231936	2.80	2.21
8	0.329946	2.82	2.63
9	0.253262	-1.27	0.41
10	0.295466	-2.90	2.49
11	2.146740	0.98	2.07
12	0.412432	1.69	1.18
13	1.600993	0.76	0.93
14	1.668679	0.93	1.45
15	1.110123	-0.68	0.52
16	1.315281	0.86	0.98
17	0.849835	-1.32	1.49
18	0.800497	-1.64	2.16
19	0.677401	0.99	0.67
20	1.262784	0.94	1.12
21	1.107442	-0.64	0.45

Fig. 6.11. *Hardy-Cross network analysis for pipe system of Fig. 6.10.*

PROGRAM 6.1.

```
C*--------------------------------------------------------------------*
C*                    **** PROGRAM HARDY ****                         *
C*--------------------------------------------------------------------*
C*     HARDY CROSS PIPE NETWORK ANALYSIS                              *
C*     ENERGY LOSSES ARE CALCULATED BY THE DARCY-WEISBACH EQUATION    *
C*--------------------------------------------------------------------*
C*                                                                    *
C*     LIST OF VARIABLES:                                             *
C*     ------------------                                             *
C*                                                                    *
C*           NPIPES  = NUMBER OF PIPES                                *
C*           NLOOPS  = NUMBER OF LOOPS                                *
C*           DELTAQ  = ACCURACY OF FLOW CALCULATIONS IN CFS.          *
C*           NO(I)   = PIPE IDENTIFICATION NUMBER                     *
C*           D(I)    = DIAMETER OF PIPE 'I' IN INCHES                 *
C*           L(I)    = LENGTH OF PIPE 'I' IN FEET                     *
C*           F(I)    = DARCY-WEISBACH FRICTION FACTOR FOR PIPE 'I'    *
C*           INPUT   = NAME OF INPUT FILE  ('FILENAME.DAT')           *
C*           OUTPUT  = NAME OF OUTPUT FILE ('FILENAME.PRT')           *
C*--------------------------------------------------------------------*
C
      REAL L(200)
      CHARACTER *20  INPUT, OUTPUT
      DIMENSION SN(100), SD(100), Q(200), CONVEY(200), F(200), D(200),
     1 NP(200), NS(200), NO(200), DEL(100), HL(200)
      WRITE (4,*) 'NAME THE DATA FILE'
      READ (4,*) INPUT
      WRITE (4,*) 'NAME THE OUTPUT FILE'
      READ (4,*) OUTPUT
      OPEN (UNIT=5,FILE=INPUT,STATUS='OLD')
      OPEN (UNIT=6,FILE=OUTPUT,STATUS='NEW')
      READ (5,10) NPIPES, NLOOPS, DELTAQ
 10   FORMAT (2I10,F10.2)
      WRITE (6,20) NPIPES, NLOOPS, DELTAQ
 20   FORMAT (3X,80('=')/3X,'HARDY CROSS PIPE NETWORK ANALYSIS:'/
     1 3X,80('=')//3X,'NETWORK HAS ',I3,' PIPES AND ',I3,' LOOPS'//
     2 3X,'FLOW RATES ARE COMPUTED TO NEAREST ',F4.2,' CFS'/3X,80('='),
     3 //)
C
C  READ IN DATA FOR EACH PIPE AND WRITE OUT INPUT DATA
C  -----------------------------------------------
C
      READ (5,110) (NO(I),D(I),L(I),F(I),Q(I),NP(I),NS(I), I=1,NPIPES)
 110  FORMAT (I10,2F10.1,F10.4,F10.2,2I10)
      WRITE (6,120)
 120  FORMAT (3X,'NETWORK ELEMENTS:'/3X,17('-')//3X,80('-')/3X,
     1 'PIPE NO.',2X,'DIAM.(IN)',2X,'LENGTH(FT)',2X,'DARCY F',2X,
     2 'Q(CFS)',2X,'PRIMARY LOOP',2X,'SECONDARY LOOP'/3X,80('-'))
      WRITE (6,130) (NO(I),D(I),L(I),F(I),Q(I),NP(I),NS(I),
     1 I = 1, NPIPES)
 130  FORMAT (I8,F12.1,F13.1,F10.4,F8.2,I10,I14)
      WRITE (6,140)
 140  FORMAT (3X,80('-'))
C
C  INITIAL CALCULATIONS
C  --------------------
C
      IT = 0
```

```
      G  = 32.2
      DO 210 I = 1, NPIPES
             D(I) = D(I)/12.0
210   CONVEY(I) = F(I) * L(I)/(2.*G*0.7854*0.7854*(D(I)**5.0))
220   DO 230 I = 1, NLOOPS
             SN(I) = 0.0
230          SD(I) = 0.0
      DO 240 I = 1, NPIPES
             QA    = ABS(Q(I))
             ANUM  = -CONVEY(I)*Q(I)*QA
             DENOM = 2.*CONVEY(I)*QA
             J     = NP(I)
             K     = NS(I)
             SN(J) = SN(J) + ANUM
             SD(J) = SD(J) + DENOM
             IF (K .LE. 0) GO TO 240
             SN(K) = SN(K) - ANUM
             SD(K) = SD(K) + DENOM
240   CONTINUE
      DO 250 I = 1, NLOOPS
             IF (SD(I) .EQ. 0) GO TO 250
             DEL(I) = SN(I)/SD(I)
250   CONTINUE
      DO 270 I = 1, NPIPES
             J = NP(I)
             K = NS(I)
             IF (K .LE. 0) GO TO 260
             Q(I) = Q(I) + DEL(J) - DEL(K)
             GO TO 270
260          Q(I) = Q(I) + DEL(J)
270   CONTINUE
      IT = IT + 1
      IF ((IT-20) .GT.0) GO TO 380
280   DO 290 I = 1, NLOOPS
             IF ((ABS(DEL(I))-DELTAQ) .GT. 0) GO TO 220
290   CONTINUE
      DO 300 I = 1, NPIPES
             HL(I) = CONVEY(I)*Q(I)*Q(I)
300   CONTINUE
C
C  WRITE THE RESULTS
C  -----------------
C
310   WRITE (6,320) IT,DELTAQ
320   FORMAT (////3X,80('=')/3X,'RESULTS OF NETWORK ANALYSIS: '/
    1 3X,80('=')//3X,'NO. OF ITERATIONS = ',I3,8X,
    2 'FLOW RATES ARE COMPUTED TO NEAREST ',F4.2,' CFS '/3X,80('-'))
330   WRITE (6,340)
340   FORMAT (3X,'PIPE NO.   CONVEYANCE   Q (CFS)   HEADLOSS (FT)'/
    1 3X,80('-'))
350   WRITE (6,360) (NO(I),CONVEY(I),Q(I),HL(I), I = 1, NPIPES)
360   FORMAT (I8,F15.6,F8.2,F12.2)
      WRITE (6,370)
370   FORMAT (3X,80('-'))
      GO TO 999
380   WRITE (6,390)
390   FORMAT (//3X,'RUN TERMINATED AFTER 20 ITERATIONS',
    1 /3X,'SOLUTION DID NOT CONVERGE')
999   CONTINUE
      STOP
      END
```

CHAPTER 7

STORM DRAIN SYSTEM ANALYSIS

7.1. Introduction

In this chapter, computer software will be presented for the analsis of pipeline storm drain systems. For design pupposes, a computer-aided-design-interaction (CADI) program is developed which allows the engineer to prepare the initial design of the pipeline. This system of programs is based upon the assumption that a cost effective pipeline system is one that is under (or close to) pressure flow conditions. In this fashion, the storm drain system is initially sized using the pressure flow CADI model. Open channel flow effects within the storm drain pipeline are evaluated using the computer software appropriate for junctions and gradually varied flow analysis (for example, Hromadka et. al., 1984).

In this chapter, suggested screen text pages are provided which present an architecture for designing a user-friendly CADI environment for the program user. All data entry prompts are accompanied by a range of allowable values. Additionally, program commands which should be made available to the program user are also included. Software to incorporate these user-friendly features is machine dependent and therefore is not provided in this book. (Software for developing the example user-friendly features is contained in Hromadka et. al., 1984).

7.2. Pressure Flow CADI Model

A common approach for the development of an urban watershed flood control system is the design of a pipeflow storm drain network. A strategy for the design of the storm drain and the selection of pipe sizes is to attempt to achieve pressure flow conditions whenever possible. This strategy is especially useful in regions where the land topography has a mild gradient (approximately 0.0010 to 0.0040 ft/ft). The resulting system would be somewhat optimized in that the design consideration of the hydraulic grade line or HGL would closely conform to the maximum allowable value while providing a reasonable design for flood control protection purposes.

In the following, computer code for the development of a CADI software system is presented in subroutine form. Each subroutine represents a separate pressure pipeflow analysis procedure which computes the change in the energy grade line or EGL due to the specific flow process or processes being considered. For example, the friction loss subroutine computes an estimate of the change in the EGL as a function of the user entered data of pipe length, friction factor, and upstream and downstream elevations. Storing the EGL and HGL which is identified at the downstream point of the study reach, the HGL and EGL can be estimated at the upstream point of the study reach by adding the friction loss to the downstream EGL value. As in all the model processes, the HGL for the upstream point of the study reach is computed by subtracting the pressure flow condition velocity head Hv (where $Hv = V^2/2g$, and $V = Q/A$, where A is the pipe cross section area) from the EGL, i.e. HGL = EGL - Hv. In this fashion, the engineer can proceed in the upstream direction along the pipeline, calling the appropriate subroutine analysis procedure to adjust the HGL and EGL as the study progresses.

The CADI approach would involve linking all the subroutines together by a Main Menu program which allows the engineer to branch to the desired subroutine analysis procedures. Because the entire hydraulic analysis direction is in the upstream direction, the total memory of the various analysis steps is summarized by the position values of the HGL and the EGL. Consequently, the computer program operates only upon these two quantities as the study proceeds in the upstream direction. At each point of the storm drain, the engineer decides whether the computed data is acceptable for design purposes. If acceptable, the engineer ACCEPTS the most recent data values entered during the current subroutine data prompt sequence and the program automatically stores the data and returns to the Main Menu. If the engineer REJECTS the computed results, the program automatically erases the recently entered data values and returns to the Main Menu. Depending on the engineer's decision, the HGL and EGL are adjusted accordingly. This interface between the software and the engineer allows for the efficient design of the storm drain system on the first pass of data entry. The computer program does not replace the engineering design process, but amplifies the engineer's capability by removing the tedious calculation phase of the study while incorporating the design capacity of the engineer.

After the analysis of the main line storm drain is completed, the connecting storm drain laterals which merge with the main line drain at junction points can also be analyzed as separate problems. Using the main line HGL as the hydraulic control for the lateral drain, an analysis is developed in the upstream direction along the lateral analogous to the study process along the main line.

Because the CADI software determines changes in the EGL due to pressure pipeflow energy losses, the procedures are inadequate for non-pressure flow conditions. However, at locations where the storm drain unseals (i.e., the flow transitions from pressure flow conditions to non-pressure flow conditions), the program assumes soffit control (where the HGL is specified to occur at the soffit of the pipe), and the EGL is recomputed according to the new HGL and for a pressure flow condition. The engineer is informed as to this HGL adjustment, and the assumed change in the HGL is also displayed. Generally, in this condition the assumed pipesize may have been too large, in which case the engineer may reject the last computed results and attempt another design approach.

The STORM DRAIN PRESSURE FLOW CADI MODEL is composed of a Main Menu program and nine subroutine analysis procedures. The model is developed by linking the Main Menu selection program to each subroutine in order to enable the engineer to branch to the desired analysis procedure when optioned. The various programs are listed in Table 7.1.

TABLE 7.1. STORM DRAIN PRESSURE FLOW CADI MODEL PROGRAMS

PROGRAM	Description	Name
7.1	Main Menu	MAIN
7.2	friction losses	HFA
7.3	manhole losses	HMA
7.4	bend losses	HBA
7.5	sudden enlargement losses	HEA
7.6	junction losses	HJA
7.7	angle point losses	HAPA
7.8	sudden contraction losses	HCA
7.9	catch basin losses	HCBA
7.10	transition losses	TRANSA

In the following, descriptions for each subroutine used in the CADI model are presented which also include the relationships used to compute the various energy losses. These relationships were obtained from design manuals which are referenced in the Chapter Bibliography. Also included in the discussions are general design considerations which may be useful in the development of a pressure flow storm drain system. It is noted, however, that design criteria varies between agencies and that the programmed energy loss computations may be inappropriate. Generally, the necessary changes to the programs required to accommodate the local design criteria can be accommodated by simply changing the various program coefficients used to compute the energy losses. The engineer should review the local agency design criteria in order to verify the basic relationships used in the supplied programs.

PROGRAM 7.1. CADI Main Menu (Pressure Flow)

Before initiating the CADI program analysis, the engineer must prepare the necessary information about the storm drain system in order to enter the data values prompted by the program. The data preparation process is simply the procedure of assigning nodal points along the storm drain system. Although the actual node numbers have no impact upon the program, it is useful to number the nodes sequentially along the pipeline in a single direction. Should additional node numbers be needed at a later time, decimal points can be added to the neighboring integer node numbers (for example, 7.1, 7.2, and so forth). The storm drain system should provide the information of estimated pipeline flowline elevations, estimated pipe sizes (which may be changed during the study process), friction factors (Manning's equation), ground surface elevations at critical points or node numbers, pipeflow quantities (cubic feet per second or cfs), lengths of pipe reaches, bend and angle point angles, and the identification of a suitable hydraulic control at the most downstream point of the system (otherwise soffit control will be assumed by the program).

After the data preparation phase is completed, the CADI study phase can be initiated. The first data entry process is the definition of the hydraulic control HGL elevation at the most downstream node number of the system. The assumed pipe diameter, pipeflow, pipe flowline elevation, and node number is requested. Based on the entered data, the program establishes the control HGL and EGL.

The program continues by advancing to the Main Menu. The engineer proceeds with the analysis in the upstream direction along the pipe system. The downstream and upstream node numbers are entered as a reference to the position of the study. The engineer then selects the analysis procedure from the provided nine subroutines which determines the necessary energy losses in the pipeline reach under study. Should several analysis procedures be required in a single reach, the appropriate subroutines are called sequentially so that the sum of the individual energy losses equals the total energy loss of the system.

After the selection of a subroutine, the program branches to that analysis procedure. Once the required subroutine data is entered, the appropriate energy loss is estimated and the upstream node HGL and EGL are displayed. Also displayed is the prompt for the engineer to ACCEPT or REJECT the recent analysis results.

Should the engineer ACCEPT the results, the recently entered data is stored in a data file (for future use and editing). Otherwise, the REJECT option results in the elimination of the most recent analysis. In both cases, the program returns to the Main Menu for the selection of the next analysis procedure.

PROGRAM 7.2. Friction Losses

Friction losses for pressure pipeflow conditions are computed from Manning's equation for steady flow

$$Q = 1.486\ A\ R^{.667}\ S_f^{\ .5} / n \tag{7.1}$$

where

Q = flow rate, in cfs
n = friction factor (see Table 7.2)
A = cross section area of pipe, in ft^2
R = hydraulic radius (ratio of area to pipe circumference)
S_f = friction slope

For storm drain design purposes, the friction factor is usually assumed to be a constant (regardless of the flow rate). Typical values for the friction factor are given in Table 7.2.

TABLE 7.2. TYPICAL MANNING'S FRICTION FACTORS

Conduit Description	n
reinforced concrete pipe (RCP)	0.013
asbestos cement pipe (ACP)	0.012
corrugated metal pipe (CMP)	0.024
asphalt lined CMP	0.015

The increase in the EGL due to friction losses is given by the energy head loss, H_f, which is estimated by

$$H_f = S_f L \tag{7.2}$$

where L is the length of pipe (ft) in the reach under study.

PROGRAM 7.2 computes the pressure pipeflow friction loss H_f by using the above equations. The engineer enters the downstream and upstream nodal point elevations, the length of pipe between the nodal points, a friction factor, and the flowrate. This data is then used to compute the EGL at the upstream node by

$$\text{EGL(upstream node)} = \text{EGL(downstream node)} + H_f \tag{7.3}$$

PROGRAM 7.3. Manhole Losses

Manhole structures are generally constructed along the storm drain line in order to provide an adequate maintenance access to the pipeline. The loss in energy due to the passage of flow through the manhole, H_m, can be estimated as a function of the flow velocity head by

$$H_m = K_m\ Hv \tag{7.4}$$

where K_m is a coefficient which is experimentally determined. The computer program computes H_m by assuming K_m is the constant value, $K_m = 0.05$. The EGL at the upstream node is computed as

$$\text{EGL(upstream node)} = \text{EGL(downstream node)} + H_m \tag{7.5}$$

The spacing of manhole locations is generally a function of pipe size. For example, for pipe diameters less than 30 inches, manholes may be spaced at 200 to 300 foot intervals. Spacings of about 400 feet may be suitable for pipe sizes between a 30 and 45 inch diameter. For pipe sizes larger than a 45 inch diameter, a spacing of 500 feet may be adequate.

PROGRAM 7.4. Bend Losses

Similar to the energy losses due to flow through manholes, the energy losses due to flow through pipeline bends, H_b, is assumed to be a function of the flow velocity head by

$$H_b = K_b\ Hv \tag{7.6}$$

where the coefficient K_b is experimentally determined. In PROGRAM 7.4, the value of K_b is assumed to be independent of the flowrate and a function of the bend angle by

$$K_b = 0.25(\text{angle}/90)^{.5} \tag{7.7}$$

where 'angle' is the central angle (less than 90 degrees) of the bend, (see Fig. 7.1).

Because the analysis proceeds from node to node, PROGRAM 7.4 includes the effects of both the bend and friction in the EGL computation for the upstream node. Thus, the EGL at the upstream node is estimated by

$$\text{EGL(upstream node)} = \text{EGL(downstream node)} + H_f + H_b \tag{7.8}$$

PROGRAM 7.5. Sudden Enlargement Losses

As the CADI study proceeds in the upstream direction, the reduction in pipesize (in the upstream direction) without a transition structure is equivalent to a 'sudden enlargement' of pipesize in the downstream direction. The energy head loss due to a sudden enlargement of pipesize, H_e, is given by the relationship

$$H_e = (V1-V2)^2/2g \tag{7.9}$$

where V1 is the upstream flow velocity and V2 is the downstream flow velocity. Due to the sudden enlargement occurring for the flow direction proceeding downstream, V1 is greater than V2. The EGL for the upstream node is computed as

$$\text{EGL(upstream node)} = \text{EGL(downstream node)} + H_e \tag{7.10}$$

It must be noted that although the study is proceeding in the upstream direction, the sudden enlargement analysis procedure accounts for the increase in pipesize occurring in the downstream direction, (see Fig. 7.2).

PROGRAM 7.6. Junction Losses

The most complex analysis procedure included in the STORM DRAIN PRESSURE FLOW CADI MODEL is the pressure flow junction analysis. This program estimates the change in EGL at a junction of pressure pipeflows where up to two laterals may confluence with the mainline flow, and the junction may also serve as an inlet for flows entering from the top of the junction structure. Both the mainline and laterals may have various angles of confluence. Because the program accomodates all energy losses between nodal points, friction losses through the structure are included. Additionally, entrance losses are included should there be inlet flows.

Friction losses are computed using Eq. 7.2 to estimate the friction slope for both the upstream and downstream reaches. Using the average of the two friction slopes, the friction loss, H_f, is computed based on the length of the junction structure. Should flows enter the junction structure through an inlet constructed at the top of the structure, an additional entrance loss may be included. For example, the entrance loss may be estimated as a loss associated to a catch basin inlet, H_{cb} (see PROGRAM 7.9), where

$$H_{cb} = 0.20 \quad Hv \tag{7.11}$$

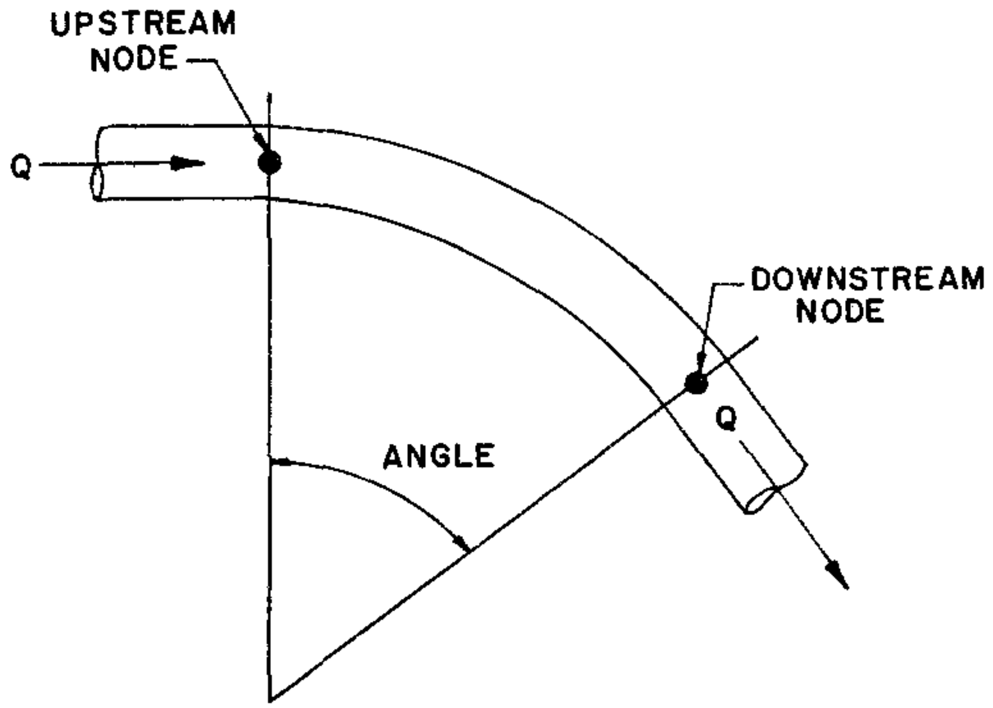

Fig. 7.1. *Bend loss model geometry.*

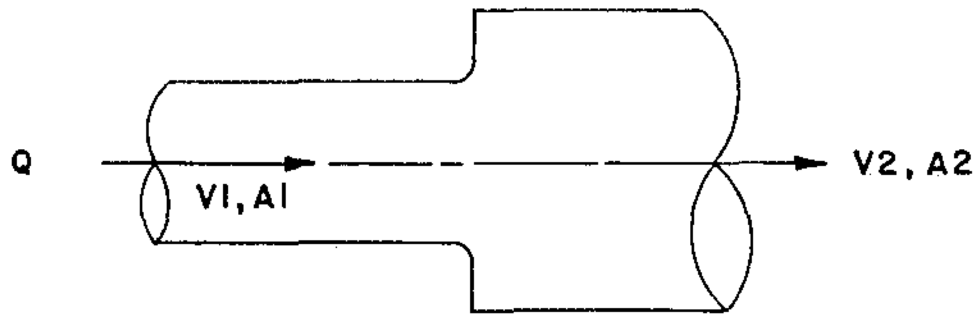

Fig. 7.2. *Sudden enlargement model geometry.*

The junction losses due to the merging of flows of a mainline flow with one or two lateral pipeline flows may be estimated by a pressure plus momentum analysis. For example, the City of Los Angeles' Thompson equation relates the energy loss, H_j, to the change in the HGL (with friction loss H_f) by

$$H_j = dHGL + Hv1 - Hv2 + H_{cb} + H_f \qquad (7.12)$$

where dHGL is the change in the HGL estimated by

$$dHGL = 2(Q2V2 - Q1V1\cos(ang1) - Q3V3\cos(ang3) - Q4V4\cos(ang4))/g(A1+A2) \qquad (7.13)$$

where

Q1,V1,A1 = upstream flow rate, flow velocity, and pipe area
Q2,V2,A2 = downstream flow rate, flow velocity, and pipe area
Q3,V3 = lateral flow rate, and flow velocity
Q4,V4 = lateral flow rate, and flow velocity
ang1 = angle of confluence between upstream and downstream pipes
ang3,ang4 = angles of confluence between laterals and downstream pipes

In Eq. 7.12, Hv1 and Hv2 refer to the velocity heads for the upstream and downstream pressure pipeflows, respectively, and H_{cb} follows from Eq. 7.11. The EGL for the upstream node is computed by summing the various losses which are assumed to occur through the junction structure,

$$EGL(upstream\ node) = EGL(downstream\ node) + H_j \qquad (7.14)$$

It is noted that the junction loss relations do not include losses resulting from a transition structure. This type of analysis is addressed in PROGRAM 7.10. Another consideration is that Eq. 7.13 may result in the estimated junction loss, H_j, being negative. To avoid this difficulty, the computer program compares H_j to the manhole loss H_m given in PROGRAM 7.3. Should H_m exceed H_j, then Hm is used for H_j and a warning message is printed with the computer results.

PROGRAM 7.7. Angle Point Losses

Energy losses due to an angle point, H_{ap}, are computed as a function of the pressure flow velocity head by

$$H_{ap} = K_{ap}\ Hv \qquad (7.15)$$

where K_{ap} is a coefficient which is experimentally determined. In PROGRAM 7.7, the coefficient K_{ap} is assumed to be a function of the central angle (Fig. 7.4) as shown in Table 7.3.

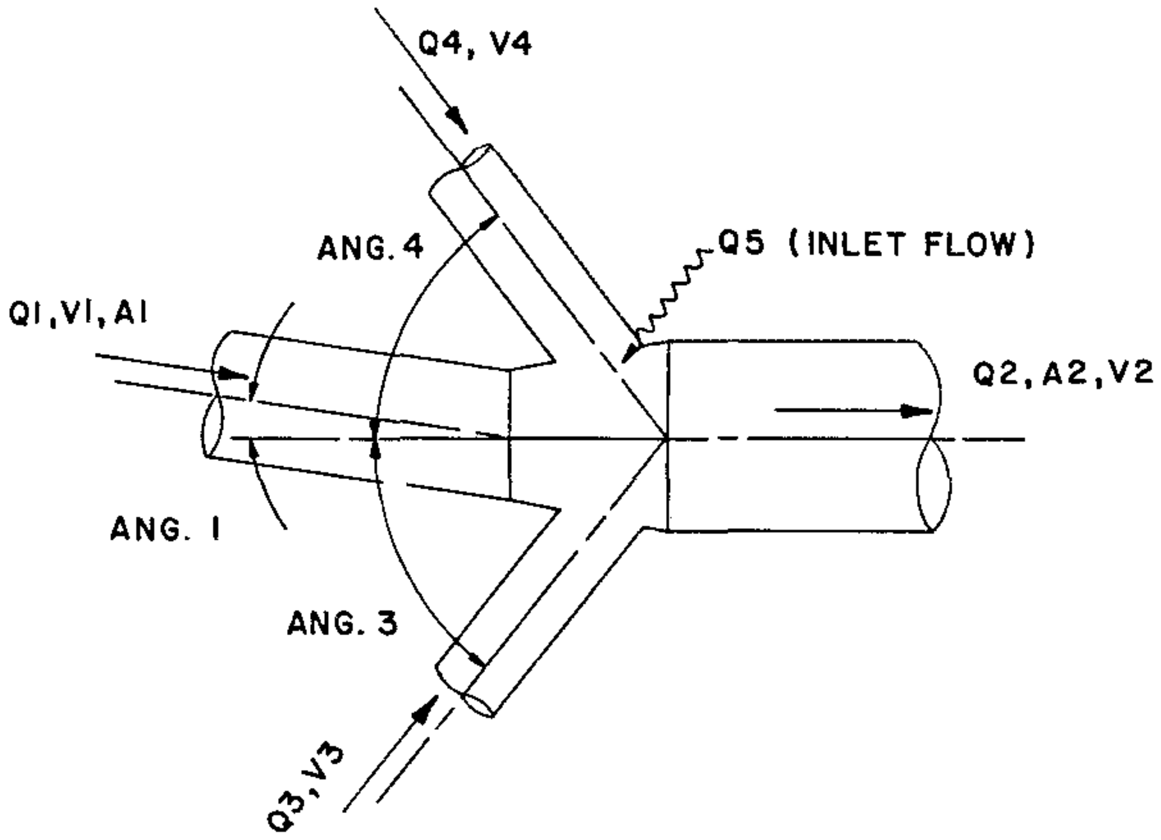

Fig. 7.3. Junction loss model geometry.

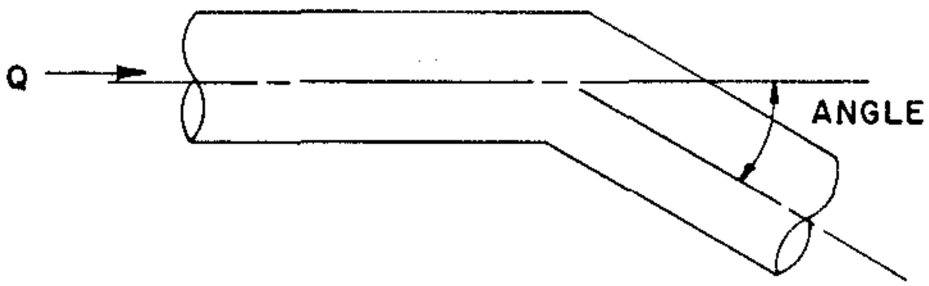

Fig. 7.4. Angle point loss model geometry.

TABLE 7.3. TYPICAL VALUES OF K_{ap}

Angle (degrees)	K_{ap}	Angle (degrees)	K_{ap}
1	.005	10	.030
2	.008	12	.037
3	.011	15	.047
4	.014	20	.067
5	.017	25	.090
6	.020	30	.115
7	.022	35	.146
8	.024	40	.184
9	.027	45	.234

The PROGRAM linearly interpolates between point values of K_{ap} in order to provide a smooth value of the coefficient as a function of the angle point. The EGL for the upstream side of the angle point is computed as

$$\text{EGL(upstream node)} = \text{EGL(downstream node)} + H_{ap} \tag{7.16}$$

The tabulated values are estimates of K_{ap} only, and replacement values can be substituted by a recoding of the linear interpolation functions used in PROGRAM 7.7. Although the tabulated values provide for K_{ap} coefficients for a range of angles up to 45 degrees, many design criteria manuals suggest avoiding angle points greater than 6 degrees when designing storm drain pipelines.

PROGRAM 7.8. Sudden Contraction Losses

Similar to PROGRAM 7.5 (Sudden Enlargement Losses), this program computes the energy losses due to a sudden reduction in pipesize in the flow direction. Because the CADI study proceeds in the upstream direction, PROGRAM 7.8 would be used when the pipe sizes suddenly increase as the study proceeds in the upstream direction. A convenient procedure for estimating the sudden contraction losses, H_c, is to assume the energy head loss to be a function of the downstream velocity head, Hv2, by (Fig. 7.5)

$$H_c = K_c \text{Hv2} \tag{7.17}$$

where K_c is a coefficient related to the ratio of upstream and downstream pressure pipe flow areas A2/A1 given by Table 7.4.

TABLE 7.4. TYPICAL VALUES OF K_c

A2/A1	K_c
.10	.46
.20	.41
.30	.36
.40	.30
.50	.24
.60	.18
.70	.12
.80	.06
.90	.02
1.00	.00

In Table 7.4, A2 is the cross section area of the downstream pipe and A1 is the cross section area of the upstream pipe. Other values for K_c may be substituted into PROGRAM 7.8 by revising the linear interpolation formulas used to relate K_c as a function of the pipeflow area ratio.

As with the other models, the upstream node EGL is computed by

$$\text{EGL(upstream node)} = \text{EGL(downstream node)} + H_c \quad (7.18)$$

PROGRAM 7.9. Catch Basin Losses

Inlets into the storm drain system (or catch basins) are often designed in anticipation that both the HGL and EGL coincide with the ponded water surface within the inlet. Consequently, the kinetic energy of flow (or velocity head) and any losses due to the entrance of the flow into the pipeline must be accounted for in the computation of the EGL within the basin. The sum of the entrance loss and the velocity head Hv is called the catch basin loss, H_{cb}, and is assumed to be a function of the flow velocity head by

$$H_{cb} = K_{cb}\ Hv \quad (7.19)$$

where K_{cb} is a coefficient which is experimentally determined. In PROGRAM 7.9, K_{cb} is assumed to be 1.2 where 0.2 is attributed to the entrance loss. The subroutine also defines the HGL to equal the EGL in order to model a ponded water surface condition. Consequently, this subroutine may be considered to model the pipeline where the storm drain connects to the catch basin, at locations where all kinetic energy is lost due to a ponded water condition, or where angle points are greater than about 60 degrees. The upstream node EGL and HGL are computed as

$$\text{EGL(upstream node)} = \text{EGL(downstream node)} + H_{cb} - Hv \quad (7.20)$$
$$\text{HGL(upstream node)} = \text{EGL(upstream node)}$$

PROGRAM 7.10. Transition Losses

Abrupt changes in pipesize are accompanied by high energy losses. In order to reduce the losses due to a sudden expansion or contraction, structures may be designed which provide for a smooth transition for the change in pipe size. The approach used in PROGRAM 10 for the estimation of the transition losses, H_t, is to model the energy losses as a function of the change in the velocity head due to the change in pipe size by

$$H_t = K_t(Hv2 - Hv1) \text{ . . . for Hv2 greater than Hv1} \quad (7.21a)$$

$$H_t = K_t(Hv1 - Hv2) \text{. . . for Hv1 greater than Hv2} \quad (7.21b)$$

Eq. 7.21 is assumed to apply when the transition structure wall of convergence or divergence is less than 5.75 degrees. PROGRAM 7.10 uses $K_t = 0.1$ in Eq. 7.21a, and $K_t = 0.2$ in Eq. 7.21b. For angles of convergence or divergence greater than 5.75 degrees, transition losses are computed by the emperical relationship

$$H_t = 3.5((\text{Tan}(\text{delta} \times .00872665))^{1.22}) \quad (7.22)$$

where delta is the total transition angle which is equal to twice the angle of convergence or divergence.

Because the transition structure also includes losses due to friction, Eq. 7.2 is used to compute the appropriate H_f value. Consequently, the EGL at the upstream node is computed by

$$\text{EGL(upstream node)} = \text{EGL(downstream node)} + Ht + H_f \quad (7.23)$$

Chapter Bibliography

"Design Manual: Channel Hydraulics and Structures," Orange County Flood Control District, OCEMA, Ca., July, 1972.

"Design Manual: Hydraulic," Los Angeles County Flood Control District, Los Angeles, Ca., 1970.

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Hromadka, T.V., J.M. Clements, H. Saluja, Computer Methods in Urban Watershed Hydraulics, Lighthouse Publications, Mission Viejo, CA, 1984.

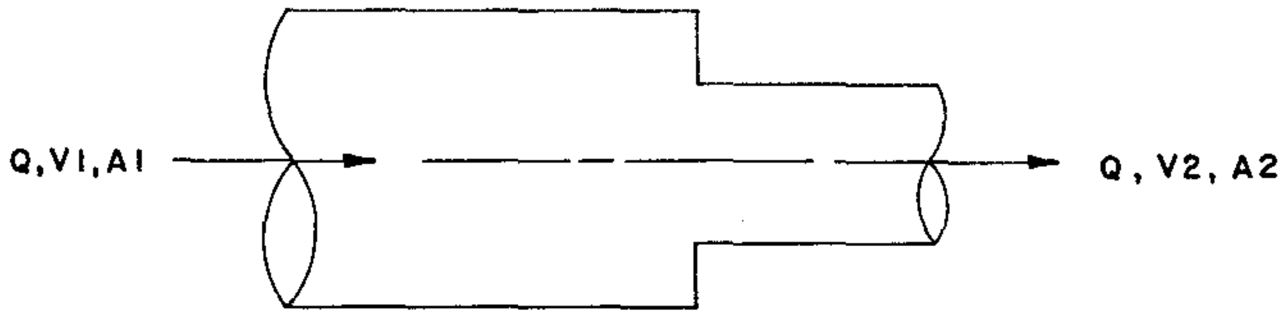

Fig. 7.5. *Sudden contraction model geometry.*

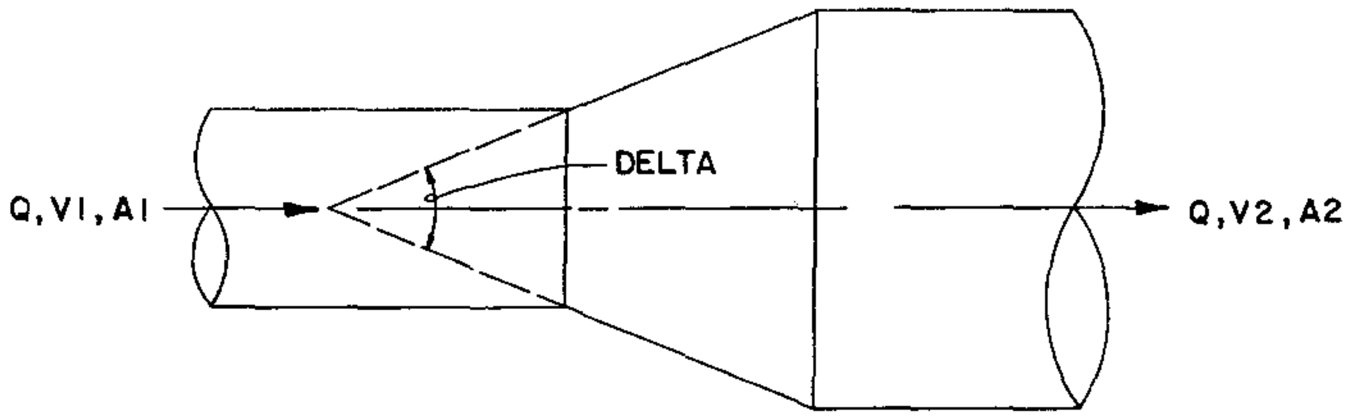

Fig. 7.6. *Transition loss model geometry.*

PROGRAM 7.1. DATA ENTRY

```
Enter node number where pressure pipe flow hydraulic
control is specified................................. ===>    "ZN1"
:ALLOWABLE VALUES ARE [0.00  ] TO [9999.99   ]

Enter pipe flowline elevation of nodal point........ ===>    "ELE"
:ALLOWABLE VALUES ARE [-99999.99 ] TO [+99999.99 ]

Enter pressure flow pipe diameter(INCHES)........... ===>    "D"
:ALLOWABLE VALUES ARE [3] TO [240   ]

Enter pressure pipe flow(CFS)....................... ===>    "Q"
:ALLOWABLE VALUES ARE [0] TO [100000]

Enter assumed hydraulic grade line(HGL) at nodal
point .............................................. ===>    "HGL"
(NOTE: EGL IS ENERGY GRADE LINE)
:ALLOWABLE VALUES ARE [-99999.99 ] TO [+99999.99 ]

------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
```

```
Enter downstream node number........................ ===>    "ZN1"
:ALLOWABLE VALUES ARE [0.00  ] TO [9999.99   ]

Enter upstream node number.......................... ===>    "ZN2"
:ALLOWABLE VALUES ARE [0.00  ] TO [9999.99   ]

PRESSURE PIPE FLOW PROCESSES:
    1= Friction Losses
    2= Manhole Losses
    3= Pipe-bend Losses
    4= Sudden Pipe-enlargement
    5= Junction Losses
    6= Angle-point Losses
    7= Sudden Pipe Reduction
    8= Catch Basin Entrance Losses
    9= Transition Losses
                  Select pressure pipe flow process ===>    "KODE"

Enter pipe flowline elevation of node number........ ===>    "ELE"
:ALLOWABLE VALUES ARE [-99999.998] TO [+99999.998]

------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
      MAIN to go to main menu
```

PROGRAM 7.1.

```
C ----------------------------------------------------------------------
        PROGRAM PRESSURE
C ----------------------------------------------------------------------
C
C  EXECUTIVE DRIVER FOR PRESSURE BATCH SYSTEM.
C
        COMMON/NUT/NUT
C
C
        NUT=6
        HGL=0.
        EGL=0.
C
C  OPEN FILES
        OPEN 5,"PRESS.DAT"
        OPEN 6,"PRESS.ANS"
C
C  PROCESS DATA FILE
C
C  PROBLEM CONTROLS FIRST
        CALL INITP(HGL,EGL,D,Q)
C
C  SEGMENTS
100     READ FREE(5,END=1000)ZN1,ZN2,KODE,ELE
        WRITE(NUT,601)
        WRITE(NUT,600)ZN1,ZN2,KODE,ELE
        WRITE(NUT,601)
C  SEGMENTS
        IERR=0
        IF(KODE.EQ.1)CALL HFA(TEST,HGL,EGL,ERROR)
        IF(KODE.EQ.2)CALL HMA(TEST,HGL,EGL,ERROR)
        IF(KODE.EQ.3)CALL HBA(TEST,HGL,EGL,ERROR)
        IF(KODE.EQ.4)CALL HEA(TEST,HGL,EGL,ERROR)
        IF(KODE.EQ.5)CALL HJA(TEST,HGL,EGL,ERROR)
        IF(KODE.EQ.6)CALL HAPA(TEST,HGL,EGL,ERROR)
        IF(KODE.EQ.7)CALL HCA(TEST,HGL,EGL,ERROR)
        IF(KODE.EQ.8)CALL HCBA(TEST,HGL,EGL,ERROR)
        IF(KODE.EQ.9)CALL TRANSA(TEST,HGL,EGL,ERROR)
        IF(IERR.NE.0)GO TO 50
        GO TO 100
C
C  ERROR PROCESSING
50      WRITE(NUT,602)
1000    CONTINUE
C  FORMATS
600     FORMAT(3X,'NODE ',F8.2,' : HGL= <',F9.3,'>;EGL= <',F9.3,'>;',
        'FLOWLINE= <',F9.3,'>')
601     FORMAT(1X,76('*'))
602     FORMAT(1X,'***FATAL ERROR - CHECK DATA INPUT***')
        RETURN
        END
```

```
C  -----------------------------------------------------------------
        SUBROUTINE INITP(HGL,EGL,D,Q)
C  -----------------------------------------------------------------
C
C  PROBLEM CONTROLS
C
        COMMON /NUT/NUT
C
C
C..READ DATA INPUT
        READ FREE(5)ZN1,ELE,D,HGL,Q
        ZN2=ZN1
C
C
        WRITE(NUT,6000)
        WRITE(NUT,5018)
        WRITE(NUT,6000)
C
        WRITE(NUT,5020) ZN1,ELE,D,Q
        WRITE(NUT,5001) HGL
        WRITE(NUT,7000)
        A = .7854*D*D/144.
        V=Q/A
        HV=V*V/64.4
        TEST = ELE+D/12.
        XHGL=HGL
        IF(HGL .GE. TEST) GO TO 30
        WRITE(NUT,6500)
        XHGL=TEST
30      EGL=XHGL+HV
        WRITE(NUT,1000) ZN1,XHGL,EGL,ELE
C
C  LOAD HGL WITH XHGL TO CONTINUE CASCADING LOGIC (INPUT HAS BEEN
C  PRESERVED)
        HGL=XHGL
C
C   FORMATS
C
1000    FORMAT(3X,'NODE ',F8.2,' : HGL= <',F9.3,'>;EGL= <',F9.3,'>;',
     .  'FLOWLINE= <',F9.3,'>')
5020    FORMAT(3X,'NOTE: STEADY FLOW HYDRAULIC HEAD-LOSS COMPUTATIONS',
     .  ' BASED ON THE MOST',/,3X,'CONSERVATIVE FORMULAE FROM THE ',
     .  'CURRENT LACRD,LACFCD, AND OCEMA',/,3X,'DESIGN MANUALS.',
     .  //,3X,'DOWNSTREAM PRESSURE PIPE FLOW CONTROL DATA:',/,
     .  3X,'NODE NUMBER = ',F8.2,11X,'FLOWLINE ELEVATION = ',F8.2,/,
     .  3X,'PIPE DIAMETER(INCH) = ',F6.2,5X,'PIPE FLOW(CFS) = ',F10.2)
5001    FORMAT(3X,'ASSUMED DOWNSTREAM CONTROL HGL = ',F9.3)
5018    FORMAT(/,11X,'PRESSURE PIPE-FLOW HYDRAULICS COMPUTER PROGRAM',
     .  ' PACKAGE',/,11X, '(Reference: LACFD,LACRD,& OCEMA HYDRAULICS',
     .  ' CRITERION)',/)
6500    FORMAT(3X,'SOFFIT CONTROL ASSUMED AT BEGINNING OF PIPE',
     .  ' SYSTEM')
7000    FORMAT(4('===================='))
6000    FORMAT(4('********************'))
C
        RETURN
        END
```

PROGRAM 7.2. DATA ENTRY

```
---DATA ENTRY FOR FRICTION LOSSES---

Enter pressure pipe flow(CFS)........................ ===>    "Q"
:ALLOWABLE VALUES ARE [0] TO [1000000   ]

Enter pipe diameter(INCHES).......................... ===>    "D"
:ALLOWABLE VALUES ARE [3] TO [240   ]

Enter length of pipe(FEET)........................... ===>    "XL"
:ALLOWABLE VALUES ARE [0] TO [1000000   ]

Enter mannings friction factor....................... ===>    "XN"
:ALLOWABLE VALUES ARE [.002  ] TO [.5    ]



---------------------------------------------------------------------------
   TYPE: EXIT to leave program ; TOP  to go to top of page
```

PROGRAM 7.3. DATA ENTRY

```
---DATA ENTRY FOR MANHOLE LOSSES---

Enter pressure pipe flow(CFS)........................ ===>    "Q"
:ALLOWABLE VALUES ARE [0] TO [1000000   ]

Enter pipe diameter(INCHES).......................... ===>    "D"
:ALLOWABLE VALUES ARE [3] TO [240   ]



---------------------------------------------------------------------------
   TYPE: EXIT to leave program ; TOP  to go to top of page
```

PROGRAM 7.2.

```
C  ---------------------------------------------------------------------
         SUBROUTINE HFA(TEST,HGL,EGL,ERROR)
C  ---------------------------------------------------------------------
C
C  FRICTION LOSSES
C
         COMMON/NUT/NUT
C
C
C..READ DATA INPUT
         READ FREE (5)Q,D,XL,XN
C
C
         TEST=D/12
         DD=D/12.
         XK=35.628*DD**2.66666667
         XK=XK*.013/XN
         SF=Q/XK*Q/XK
         XHF=XL*SF
         TEGL=EGL+XHF
         A = .7854*DD*DD
         HV = Q*Q/A/A/64.4
         THGL = TEGL-HV
         WRITE(NUT,605)
         WRITE(NUT,50)Q,D,XL,XN,Q,XK,SF,XL,SF,XHF
50       FORMAT(3X,'PIPE FLOW = ',F10.2,' CFS',5X,'PIPE ',
     C   'DIAMETER = ',F6.2,' INCHES',/,3X,'PIPE LENGTH = ',
     C   F10.2,' FEET',5X,'MANNINGS N = ',F7.5,/,3X,
     C   'SF=(Q/K)**2 = ((',F10.2,')/(',F10.3,'))**2 =',F10.7,/,
     C   3X,'HF=L*SF = (',F10.2,')*(',F10.7,') =',F10.3)
         ERROR=0.
C
         HGL=THGL
         EGL=TEGL
C
C   FORMATS
C
605      FORMAT(3X,'CALCULATE PRESSURE FLOW FRICTION LOSSES(LACFCD):')
         RETURN
         END
```

PROGRAM 7.3.

```
C  ---------------------------------------------------------------
       SUBROUTINE HMA(TEST,HGL,EGL,ERROR)
C  ---------------------------------------------------------------
C
C  MANHOLE LOSSES
C
       COMMON /NUT/NUT
C
C
C...READ OFF DATA FILE
       READ FREE(5)Q,D
C
C
       TEST=D/12
       A=.7854*D*D/144.
       V=Q/A
       HV=V*V/64.4
       XHM=.05*HV
C
       WRITE(NUT,605)
       WRITE(NUT,30)Q,D,A,V,HV,HV,XHM
30     FORMAT(3X,'PIPE FLOW = ',F10.2,' CFS',5X,'PIPE ',
     C 'DIAMETER = ',F6.2,' INCHES',/,3X,'PRESSURE FLOW AREA = ',
     C F7.3,' SQUARE FEET',/,3X,'FLOW VELOCITY = ',
     C F6.2,' FEET PER SECOND',/,3X,'VELOCITY HEAD = ',F6.3,/,
     C 3X,'HMN = .05*(VELOCITY HEAD) = .05*(',F6.3,') = ',
     C F6.3)
       ERROR=0.
       TEGL=EGL+XHM
       THGL = TEGL-HV
C
       HGL=THGL
       EGL=TEGL
C
C   FORMATS
C
605    FORMAT(3X,'CALCULATE PRESSURE FLOW MANHOLE LOSSES(LACFCD):')
C
       RETURN
       END
```

PROGRAM 7.4. DATA ENTRY

```
---DATA ENTRY FOR PIPE-BEND LOSSES---

Enter pressure pipe flow(CFS)......................... ===>    "Q"
:ALLOWABLE VALUES ARE [0] TO [1000000   ]

Enter pipe diameter(INCHES)........................... ===>    "D"
:ALLOWABLE VALUES ARE [3] TO [240   ]

Enter pipe bend angle(DEGREES)........................ ===>    "DELTA"
:ALLOWABLE VALUES ARE [0] TO [90    ]

Enter length of pipe(FEET)............................ ===>    "XL"
:ALLOWABLE VALUES ARE [0] TO [1000000   ]

Enter mannings friction factor........................ ===>    "XN"
:ALLOWABLE VALUES ARE [.008  ] TO [.500  ]

---------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
```

PROGRAM 7.5. DATA ENTRY

```
---DATA ENTRY FOR SUDDEN PIPE ENLARGEMENT---

Enter pressure pipe flow(CFS)......................... ===>    "Q"
:ALLOWABLE VALUES ARE [0] TO [1000000   ]

Enter downstream pipe diameter(INCHES)................ ===>    "D2"
:ALLOWABLE VALUES ARE [3      ] TO [240   ]

Enter upstream pipe diameter(INCHES).................. ===>    "D1"
:ALLOWABLE VALUES ARE [3      ] TO [240   ]

---------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
```

PROGRAM 7.4.

```
C  -----------------------------------------------------------------
         SUBROUTINE HBA(TEST,HGL,EGL,ERROR)
C  -----------------------------------------------------------------
C
C   PIPE-BEND LOSSES
C
         COMMON /NUT/NUT
C
C
C..READ DATA INPUT
         READ FREE (5)Q,D,DELTA,XL,XN
C
C
         TEST=D/12
         XKB=.25*(DELTA/90.)**.5
         A=.7854*D*D/144.
         V=Q/A
         HV=V*V/64.4
         XHB=XKB*HV
         XK=35.628*(D/12.)**2.66666667
         XK=XK*.013/XN
         SF=Q/XK*Q/XK
         XHF=XL*SF
C
         WRITE(NUT,605)
         WRITE(NUT,100)Q,D,DELTA,XL,XN
100      FORMAT(3X,'PIPE FLOW = ',F10.2,' CFS',5X,'PIPE DIAMETER = ',
     C   F6.2,' INCHES',/,3X,'CENTRAL ANGLE = ',F6.3,' DEGREES',/,3X,
     C   'PIPE LENGTH = ',F10.2,' FEET',5X,'MANNINGS N = ',F7.5)
         WRITE(NUT,110)A,V,HV,XKB,XKB,HV,XHB
110      FORMAT(3X,'PRESSURE FLOW AREA = ',F7.3,' SQUARE FEET',/,
     C   3X,'FLOW VELOCITY = ',F6.2,' FEET PER SECOND',/,3X,
     C   'VELOCITY HEAD = ',F6.3,5X,'BEND COEFFICIENT(KB) = ',
     C   F6.4,/,3X,'HB=KB*(VELOCITY HEAD) = (',F6.3,')*(',F6.3,
     C   ') = ',F6.3)
         WRITE(NUT,120)XK,SF,XL,SF,XHF
120      FORMAT(3X,'PIPE CONVEYANCE FACTOR = ',F10.3,
     C   5X,'FRICTION SLOPE(SF) = ',F10.7,/,
     C   3X,'FRICTION LOSSES = L*SF = (',F10.2,')*(',F10.7,') = ',F6.3)
         ERROR = 0.
         TEGL=EGL+XHF+XHB
         THGL = TEGL-HV
C
         HGL=THGL
         EGL=TEGL
C
C   FORMATS
C
605      FORMAT(3X,'CALCULATE PRESSURE FLOW PIPE-BEND LOSSES(OCEMA):')
         RETURN
         END
```

PROGRAM 7.5.

```
C  ----------------------------------------------------------------
         SUBROUTINE HEA(TEST,HGL,EGL,ERROR)
C  ----------------------------------------------------------------
C
C  SUDDEN PIPE ENLARGEMENT
C
         COMMON /NUT/NUT
C
C
C..READ DATA INPUT
         READ FREE(5)Q,D2,D1
C
C
         TEST=D1/12
         A1=.7854*D1*D1/144.
         A2=.7854*D2*D2/144.
         V1=Q/A1
         V2=Q/A2
         HV1=V1*V1/64.4
         XHE=((V1-V2)**2.)/64.4
C
         WRITE(NUT,605)
         WRITE(NUT,100)Q,D2,D1
100      FORMAT(3X,'PIPE FLOW = ',F10.2,' CFS',/,
     C   3X,'DOWNSTREAM PIPE DIAMETER = ',F6.2,
     C   ' INCHES',/,3X,'UPSTREAM PIPE DIAMETER = ',F6.2,
     C   ' INCHES')
         WRITE(NUT,110)V1,V2
110      FORMAT(3X,'UPSTREAM PRESSURE FLOW VELOCITY = ',F6.2,' FEET/SEC.'
     .   ,/,3X,'DOWNSTREAM PRESSURE FLOW VELOCITY = ',F6.2,' FEET/SEC.',
     .   /,3X,'SUDDEN PIPE-FLOW ENLARGEMENT LOSSES = ')
         WRITE(NUT,120)V1,V2,XHE
120      FORMAT(3X,'((V1-V2)**2)/64.4 = ((',F7.3,'-',F7.3,')**2/64.4 = '
     .   ,F6.3)
         TEGL = EGL +XHE
         THGL = TEGL -HV1
         ERROR=0.
C
         HGL=THGL
         EGL=TEGL
C
C   FORMATS
C
605      FORMAT(3X,'CALCULATE PRESSURE FLOW SUDDEN PIPE ENLARGEMENT',
     .   ' LOSSES(LACRD):')
C
         RETURN
         END
```

PROGRAM 7.6. DATA ENTRY

```
---DATA ENTRY FOR JUNCTION LOSSES---PAGE 1

Enter downstream pipe flow(CFS).................... ===>    "Q1"
:ALLOWABLE VALUES ARE [0] TO [9999.99   ]

Enter upstream pipe flow(CFS)...................... ===>    "Q2"
:ALLOWABLE VALUES ARE [0] TO [9999.99   ]

Enter first lateral pressure pipe flow(CFS)........ ===>    "Q3"
:ALLOWABLE VALUES ARE [0] TO [9999.99   ]

Enter second lateral pressure pipe flow(CFS)....... ===>    "Q4"
:ALLOWABLE VALUES ARE [0] TO [9999.99   ]

Catch basin flow(CFS) into junction structure...... ===>    "Q5"



--------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
```

```
---DATA ENTRY FOR JUNCTION LOSSES---PAGE 2

Enter downstream pipe diameter(INCHES)............. ===>    "D1"
:ALLOWABLE VALUES ARE [3] TO [240   ]

Enter upstream pipe diameter(INCHES)............... ===>    "D2"
:ALLOWABLE VALUES ARE [3] TO [240   ]

Enter first lateral pipe diameter(INCHES).......... ===>    "D3"
(NOTE: IF LATERAL ABSENT, ENTER 0)
:ALLOWABLE VALUES ARE [0] TO [240   ]

Enter second lateral pipe diameter(INCHES)......... ===>    "D4"
(NOTE: IF LATERAL ABSENT, ENTER 0)
:ALLOWABLE VALUES ARE [0] TO [240   ]



--------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
                            ; BACK to go back one page
```

```
---DATA ENTRY FOR JUNCTION LOSSES---PAGE 3

Enter upstream pipe angle with respect to
 downstream pipe(DEGREES)........................... ===>    "DELTA1"
:ALLOWABLE VALUES ARE [0] TO [90    ]

Enter first lateral pipe angle with respect to
 downstream pipe(DEGREES)........................... ===>    "DELTA3"
:ALLOWABLE VALUES ARE [0] TO [90    ]

Enter second lateral pipe angle with respect to
 downstream pipe(DEGREES)........................... ===>    "DELTA4"
:ALLOWABLE VALUES ARE [0] TO [90    ]

Enter junction structure length(FEET)............... ===>    "XL"
:ALLOWABLE VALUES ARE [1] TO [100   ]

------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
                            ; BACK to go back one page
```

```
---DATA ENTRY FOR JUNCTION LOSSES---PAGE 4

Enter downstream pipe mannings friction factor...... ===>    "XN2"
:ALLOWABLE VALUES ARE [.008  ] TO [.500  ]

Enter upstream pipe mannings friction factor........ ===>    "XN1"
:ALLOWABLE VALUES ARE [.008  ] TO [.500  ]

------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
                            ; BACK to go back one page
```

PROGRAM 7.6.

```
C ----------------------------------------------------------------
      SUBROUTINE HJA(TEST,HGL,EGL,ERROR)
C ----------------------------------------------------------------
C
C  JUNCTION LOSSES
C
      COMMON /NUT/NUT
C
C
C..READ DATA INPUT
      READ FREE(5)Q1,Q2,Q3,Q4,Q5,D1,D2,D3,D4,DELTA1,DELTA3,
     . DELTA4,XL,XN1,XN2
C
C
      D1=D1/12.
      D2=D2/12.
      D3=D3/12.
      D4=D4/12.
      PI=.7854
      A1=PI*D1*D1
      A2=PI*D2*D2
      A3=PI*D3*D3
      A4=PI*D4*D4
      V1=Q1/A1
      V2=Q2/A2
      V3=0.
      V4=0.
      IF(A3.NE.0.)V3=Q3/A3
      IF(A4.NE.0.)V4=Q4/A4
      HV2=V2*V2/64.4
      HV1=V1*V1/64.4
      X3=COS(DELTA3*PI/45.)
      X4=COS(DELTA4*PI/45.)
      X5=COS(DELTA1*PI/45.)
      TLOSS=0.
      DELY=(Q2*V2-Q1*V1*X5-Q3*V3*X3-Q4*V4*X4)/16.1/(A1+A2)
      HE=.2*HV2
      HM=.05*HV2
      SF1=(Q1/(35.628*.013/XN1*D1**2.666667))**2.
      SF2=(Q2/(35.628*.013/XN2*D2**2.666667))**2.
      SFM=.5*(SF1+SF2)
      HF=XL*SFM
C
      WRITE(NUT,605)
      WRITE(NUT,200)Q1,D11,A1,V1,DELTA1,HV1,Q2,D21,A2,V2,HV2,Q3,
     C D31,A3,V3,DELTA3,Q4,D41,A4,V4,DELTA4,Q5
200   FORMAT(4X,'NO. DISCHARGE  DIAMETER    AREA     VELOCITY',
     C '   DELTA        HV'
     C ,/,5X,'1',3X,F7.1,3X,F6.2,4X,F7.3,3X,F6.3,4X,F6.3,
     C 4X,F6.3,
     C /,5X,'2',3X,F7.1,3X,F6.2,4X,F7.3,3X,F6.3,6X,'--',6X,F6.3,/,5X,
     C '3',3X,F7.1,3X,F6.2,4X,F7.3,3X,F6.3,4X,F6.3,7X,'-',/,5X,
     C '4',3X,F7.1,3X,F6.2,4X,F7.3,3X,F6.3,4X,F6.3,7X,'-',/,5X,
     C '5',3X,F7.1,'===Q5 EQUALS BASIN INPUT===')
      WRITE(NUT,210)
210   FORMAT(/,3X,'LACFCD AND OCEMA PRESSURE FLOW JUNCTION FORMULAE',
     C ' USED:')
      WRITE(NUT,211)
211   FORMAT(3X,'DY=(Q2*V2-Q1*V1*COS(DELTA1)-Q3*V3*COS(DELTA3)-',
```

```
     C  /,7X,'Q4*V4*COS(DELTA4))/((A1+A2)*16.1)',/)
C
        WRITE(NUT,220)XN1,XN2,SF1,SF2,SFM,XL,HF
220     FORMAT(3X,'UPSTREAM MANNINGS N = ',F7.5,/,3X,
     .  'DOWNSTREAM MANNINGS N = ',F7.5,/,
     .  3X,'UPSTREAM FRICTION SLOPE = ',F7.5,/,
     C  3X,'DOWNSTREAM FRICTION SLOPE = ',F7.5,/,
     C  3X,'AVERAGED FRICTION SLOPE IN JUNCTION ASSUMED AS ',F7.5,/,
     C  3X,'JUNCTION LENGTH(FEET) = ',F6.2,5X,'FRICTION LOSS = ',F6.3)
        IF(Q5.EQ.0.)HE=0.
        WRITE(NUT,225)HE
225     FORMAT(3X,'ENTRANCE LOSSES = ',F6.3)
        TLOSS=TLOSS+HE
230     XHJ=DELY+HV1-HV2
        IF(XHJ.GE.HM)GOTO 240
        WRITE(NUT,235)XHJ,HM
235     FORMAT(3X,'MANHOLE LOSSES GREATER THAN THOMPSON MOMENTUM LOSSES'
     C  ,/,3X,'MOMENTUM LOSSES = ',F6.3,5X,'MANHOLE LOSSES = ',F6.3)
        XHJ=HM
240     TLOSS=TLOSS+HF+XHJ
        TEGL=EGL+TLOSS
        THGL=TEGL-HV1
        WRITE(NUT,250)DELY,HV1,HV2,HF,HE,TLOSS
250     FORMAT(3X,'JUNCTION LOSSES = DY+HV1-HV2+(FRICTION LOSS)+',
     C  '(ENTRANCE LOSSES)',/,3X,'JUNCTION LOSSES = ',
     C  F6.3,'+',F6.3,'-',F6.3,'+(',F6.3,')+(',F6.3,') = ',F6.3)
        ERROR=0.
C
        HGL=THGL
        EGL=TEGL
C
C    FORMATS
C
605     FORMAT(3X,'CALCULATE PRESSURE FLOW JUNCTION LOSSES:')
C
        RETURN
        END
```

PROGRAM 7.7. DATA ENTRY

```
---DATA ENTRY FOR ANGLE-POINT LOSSES---

Enter pressure pipe flow(CFS)........................ ===>    "Q"
:ALLOWABLE VALUES ARE [0] TO [1000000   ]

Enter pipe diameter(INCHES).......................... ===>    "D"
:ALLOWABLE VALUES ARE [3] TO [240   ]

Enter pressure flow angle-point angle(DEGREES)...... ===>    "DELTA"
:ALLOWABLE VALUES ARE [0] TO [45    ]

TYPE: EXIT to leave program ; TOP  to go to top of page
```

PROGRAM 7.8. DATA ENTRY

```
---DATA ENTRY FOR SUDDEN PIPE REDUCTION---

Enter pressure pipe flow(CFS)........................ ===>    "Q"
:ALLOWABLE VALUES ARE [0] TO [1000000   ]

Enter downstream pipe diameter(INCHES)............... ===>    "D2"
:ALLOWABLE VALUES ARE [3     ] TO [240   ]

Enter upstream pipe diameter(INCHES)................. ===>    "D1"
:ALLOWABLE VALUES ARE [3     ] TO [240   ]

TYPE: EXIT to leave program ; TOP  to go to top of page
```

PROGRAM 7.7.

```
C ----------------------------------------------------------------------
      SUBROUTINE HAPA(TEST,HGL,EGL,ERROR)
C ----------------------------------------------------------------------
C
C  ANGLE-POINT LOSSES
C
      COMMON /NUT/NUT
C
C
C..READ DATA INPUT
      READ FREE(5)Q,D,DELTA
C
C
      TEST=D/12
      A=.7854*D*D/144.
      V=Q/A
      HV=V*V/64.4
      IF(DELTA.LE.1.)XKA=.005-(1.-DELTA)*.005
      IF(DELTA.GT.1..AND.DELTA.LE.6.)XKA=.005+(DELTA-1.)*.003
      IF(DELTA.GT.6..AND.DELTA.LE.8.)XKA=.02+(DELTA-6.)*.002
      IF(DELTA.GT.8..AND.DELTA.LE.10.)XKA=.024+(DELTA-8.)*.003
      IF(DELTA.GT.10..AND.DELTA.LE.12.)XKA=.03+(DELTA-10.)*.0035
      IF(DELTA.GT.12..AND.DELTA.LE.15.)XKA=.037+(DELTA-12.)*.00333333
      IF(DELTA.GT.15..AND.DELTA.LE.20.)XKA=.047+(DELTA-15.)*.004
      IF(DELTA.GT.20..AND.DELTA.LE.25.)XKA=.067+(DELTA-20.)*.0046
      IF(DELTA.GT.25..AND.DELTA.LE.30.)XKA=.09+(DELTA-25.)*.005
      IF(DELTA.GT.30..AND.DELTA.LE.35.)XKA=.115+(DELTA-30.)*.0062
      IF(DELTA.GT.35..AND.DELTA.LE.40.)XKA=.146+(DELTA-35.)*.0076
      IF(DELTA.GT.40..AND.DELTA.LE.45.)XKA=.184+(DELTA-40.)*.01
      IF(XKA.LE.0.)XKA=0.
      HAP=HV*XKA
C
      WRITE(NUT,605)
      WRITE(NUT,100)Q,D,DELTA
100   FORMAT(3X,'PIPE FLOW = ',F10.2,' CFS',5X,'PIPE DIAMETER = ',
     C F6.2,' INCHES',/,3X,'PIPE ANGLE POINT DELTA = ',F6.2,' DEGREES')
      WRITE(NUT,103)XKA
103   FORMAT(3X,'PRESSURE FLOW ANGLE-POINT COEFFICIENT KA = ',
     C F6.4)
      WRITE(NUT,110)V,HV,XKA,HV,HAP
110   FORMAT(3X,'PRESSURE FLOW VELOCITY = ',F6.2,' FEET/SEC.',/,3X,
     C 'VELOCITY HEAD = ',F6.3,/,3X,'HAPT=KA*(VELOCITY HEAD) = ',
     C '(',F6.4,')*(',F6.3,') = ',F6.3)
      ERROR=0.
      TEGL=EGL+HAP
      THGL = TEGL-HV
C
      HGL=THGL
      EGL=TEGL
C
C   FORMATS
C
605   FORMAT(3X,'CALCULATE PRESSURE FLOW ANGLE-POINT LOSSES(LACRD):')
C
      RETURN
      END
```

PROGRAM 7.8.

```
C ----------------------------------------------------------------------
      SUBROUTINE BCA(TEST,HGL,EGL,ERROR)
C ----------------------------------------------------------------------
C
C  SUDDEN PIPE REDUCTION
C
      COMMON /NUT/NUT
C
C
C..READ DATA INPUT
      READ FREE(5)Q,D1,D2
C
C
      TEST=D1/12.
      X=D2/D1*D2/D1
      IF(X.GE..1)GOTO 50
      WRITE(WT,655)
      GO TO 3020
C
50    CONTINUE
C
      IF(X.GE..1.AND.X.LT..3)XK=.46-.5*(X-.1)
      IF(X.GE..3.AND.X.LT..8)XK=.36-.6*(X-.3)
      IF(X.GE..8.AND.X.LT..9)XK=.06-.4*(X-.8)
      IF(X.GE..9.AND.X.LE.1.)XK=.02-.2*(X-.9)
      A2=.7854*D2*D2/144.
      A1=.7854*D1*D1/144.
      V1=Q/A1
      HV1=V1*V1/64.4
      V2=Q/A2
      HV2 = V2*V2/64.4
      V=V1
      HV = HV1
      IF(HV2 .GT. HV1) HV=HV2
      XHC = XK*HV
      IF(HV2 .GT. HV1) V=V2
C
      WRITE(NUT,605)
      WRITE(NUT,100)Q,D1,D2
100   FORMAT(3X,'PIPE FLOW = ',F10.2,' CFS',/,
     C 3X,'UPSTREAM PIPE DIAMETER = ',F6.2,
     C ' INCHES',/,3X,'DOWNSTREAM PIPE DIAMETER = ',F6.2,
     C ' INCHES')
      WRITE(NUT,110)V,HV,XK,XK,HV,XHC
110   FORMAT(3X,'PRESSURE FLOW VELOCITY = ',F6.2,' FEET/SEC.',
     C /,3X,'VELOCITY HEAD = ',F10.2,/,3X,
     C 'PIPE REDUCTION LOSS COEFFICIENT KC = ',F6.3,/,
     C 3X,'HC(PER LACFCD)=KC*(VELOCITY HEAD) = ',
     C '(',F6.3,')*(',F10.2,') = ',F6.3)
      ERROR=0.
      TEGL=EGL+XHC
      THGL=TEGL-HV1
C
      HGL=THGL
      EGL=TEGL
3020  CONTINUE
C
C   FORMATS
C
605   FORMAT(3X,'CALCULATE PRESSURE FLOW SUDDEN PIPE REDUCTION'
     . ,' LOSSES(LACRD):')
655   FORMAT(3X,'*ERROR-SUDDEN PIPE SIZE REDUCTION TOO SEVERE')
C
      RETURN
      END
```

PROGRAM 7.9. DATA ENTRY

```
---DATA ENTRY FOR CATCH BASIN ENTRANCE LOSSES---


Enter pressure pipe flow(CFS) from catch basin....... ===>    "Q"
:ALLOWABLE VALUES ARE [0] TO [1000000   ]

Enter pipe diameter(INCHES) from catch basin......... ===>    "D"
:ALLOWABLE VALUES ARE [3] TO [240   ]










---------------------------------------------------------------------
 TYPE: EXIT to leave program ; TOP  to go to top of page
```

PROGRAM 7.9.

```
C  ------------------------------------------------------------------------
      SUBROUTINE HCBA(TEST,HGL,EGL,ERROR)
C  ------------------------------------------------------------------------
C
C  CATCH BASIN ENTRANCE LOSSES
C
      COMMON /NUT/NUT
C
C
C..READ DATA INPUT
      READ FREE(5)Q,D
C
C
      TEST=D/12
      A=.7854*D*D/144.
      V=Q/A
      HV=V*V/64.4
      HE=.2*HV
C
      WRITE(NUT,605)
      WRITE(NUT,100)Q,D
100   FORMAT(3X,'PIPE FLOW(CFS) = ',F10.2,5X,'PIPE DIAMETER(INCH) = ',
     C F6.2)
      WRITE(NUT,105)HV,HV,HE
105   FORMAT(3X,'PRESSURE FLOW VELOCITY HEAD = ',F6.3,/,
     C 3X,'CATCH BASIN ENERGY LOSS = .2*(VELOCITY HEAD) = .2*(',
     C F6.3,') =',F6.3)
      ERROR = 0.
      TEGL=EGL+HE
      THGL=TEGL
C
      HGL=THGL
      EGL=TEGL
C
C  FORMATS
C
605   FORMAT(3X,'CALCULATE PRESSURE FLOW CATCH BASIN ENTRANCE'
     . ,' LOSSES(LACFCD):')
C
      RETURN
      END
```

PROGRAM 7.10. DATA ENTRY

```
---DATA ENTRY FOR TRANSITION LOSSES---PAGE 1

Enter pressure pipe flow(CFS)........................ ===>    "Q"
:ALLOWABLE VALUES ARE [0] TO [1000000   ]

Enter downstream pipe diameter(INCHES)............... ===>    "D2"
:ALLOWABLE VALUES ARE [3] TO [240   ]

Enter upstream pipe diameter(INCHES)................. ===>    "D1"
:ALLOWABLE VALUES ARE [3] TO [240   ]

Enter length of transition(FEET)..................... ===>    "XL"
:ALLOWABLE VALUES ARE [0] TO [1000   ]

--------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
```

```
---DATA ENTRY FOR TRANSITION LOSSES---PAGE 2

Enter downstream pipe mannings friction factor...... ===>    "XN2"
:ALLOWABLE VALUES ARE [.008   ] TO [.500   ]

Enter upstream pipe mannings friction factor........ ===>    "XN1"
:ALLOWABLE VALUES ARE [.008   ] TO [.500   ]

Enter total-angle-of-transition(DEGREES)............ ===>    "DELTA"
(NOTE: SEE LACFCD DESIGN MANUAL CHART No. B-11
FOR DESCRIPTION.)
:ALLOWABLE VALUES ARE [0] TO [22.666666 ]

--------------------------------------------------------------------------
TYPE: EXIT to leave program ; TOP  to go to top of page
                            ; BACK to go back one page
```

PROGRAM 7.10.

```
C ---------------------------------------------------------------------
        SUBROUTINE TRANSA(TEST,HGL,EGL,ERROR)
C ---------------------------------------------------------------------
C
C  TRANSITION LOSSES
C
        COMMON /NUT/NUT
C
C
C..READ DATA INPUT
        READ FREE(5)Q,D1,D2,XL,XN1,XN2,DELTA
C
C
        TEST=D1/12.
        A1=.7854*D1*D1/144.
        A2=.7854*D2*D2/144.
        V1=Q/A1
        V2=Q/A2
        HV1=V1*V1/64.4
        HV2=V2*V2/64.4
        XK1=35.628*.013/XN1*(D1/12.)**2.6666667
        XK2=35.628*.013/XN2*(D2/12.)**2.6666667
        SF1=(Q/XK1)**2.
        SF2=(Q/XK2)**2.
        SFM=.5*(SF1+SF2)
        HF=XL*SFM
C
        WRITE(NUT,605)
        WRITE(NUT,200)Q,D1,D2,XL,XN1,XN2,DELTA
200     FORMAT(3X,'PRESSURE PIPE FLOW(CFS) = ',F10.2,/,
     C  3X,'UPSTREAM DIAMETER(INCH) = ',F6.2,5X,
     C  'DOWNSTREAM DIAMETER(INCH) = ',F6.2,/,
     C  3X,'TRANSITION STRUCTURE LENGTH(FEET) = ',F6.2,/,
     C  3X,'UPSTREAM MANNINGS N = ',F6.4,5X,
     C  'DOWNSTREAM MANNINGS N = ',F6.4,/,3X,'TOTAL-ANGLE-OF-',
     C  'TRANSITION(DEGREES) = ',F6.3)
        WRITE(NUT,202)V1,HV1,V2,HV2
202     FORMAT(3X,'UPSTREAM VELOCITY(FPS) = ',F7.3,3X,
     C  'UPSTREAM VELOCITY HEAD = ',F6.3,/,3X,
     C  'DOWNSTREAM VELOCITY(FPS) = ',F7.3,3X,
     C  'DOWNSTREAM VELOCITY HEAD = ',F6.3)
        IF(DELTA.LE.11.5)GOTO 1000
        WRITE(NUT,201)
        IF(V1.GT.V2)WRITE(NUT,300)
        IF(V2.GT.V1)WRITE(NUT,301)
201     FORMAT(3X,'DELTA EXCEEDS 11.5 DEGREES, USE LACFCD "GIBSON"',
     C  ' NOMOGRAPH FOR KE VALUE')
        XKE=3.5*((TAN(DELTA*.00872665))**1.22)
300     FORMAT(3X,'VELOCITY DECREASE IN TRANSITION',/,3X,
     C  'HT = KO*(V1-V2)*(V1-V2)/64.4')
301     FORMAT(3X,'VELOCITY INCREASE IN TRANSITION',/,3X,
     C  'HT = KI*(V2-V1)*(V2-V1)/64.4')
        IF(V2.GE.V1)XKE=XKE/2.
        XHT=(V2-V1)*(V2-V1)*XKE/64.4
        GOTO 2000
1000    XKE=.2
        IF(V2.GE.V1)XKE=.1
        IF(V2.GE.V1)WRITE(NUT,305)
        IF(V1.GT.V2)WRITE(NUT,306)
```

```
305      FORMAT(3X,'HT = .1*(V2*V2-V1*V1)/64.4')
306      FORMAT(3X,'HT = .2*(V1*V1-V2*V2)/64.4')
         XHT=(ABS(V2*V2-V1*V1))*XKE/64.4
2000     WRITE(NUT,2010)XKE,XHT
2010     FORMAT(3X,'TRANSITION LOSS COEFFICIENT = ',F6.4,/,
     C   3X,'TRANSITION LOSS = ',F6.3)
         WRITE(NUT,2020)SF1,SF2,SFM,HF
2020     FORMAT(3X,'UPSTREAM FRICTION SLOPE = ',F7.5,/,3X,
     C   'DOWNSTREAM FRICTION SLOPE = ',F7.5,/,3X,
     C   'AVERAGE TRANSITION STRUCTURE FRICTION SLOPE ASSUMED = ',F7.5,/,
     C   3X,'HF = L*SF = ',F6.3)
         WRITE(NUT,2030)XHT,HF
2030     FORMAT(3X,'TOTAL LOSSES = (HT)+(HF) = (',F6.3,')+(',F6.3,')')
         TEGL=EGL+XHT+HF
         THGL=TEGL-HV1
         ERROR=0.
C
         HGL=THGL
         EGL=TEGL
C
C    FORMATS
C
605      FORMAT(3X,'CALCULATE PRESSURE FLOW TRANSITION LOSSES(LACFCD):')
C
         RETURN
         END
```

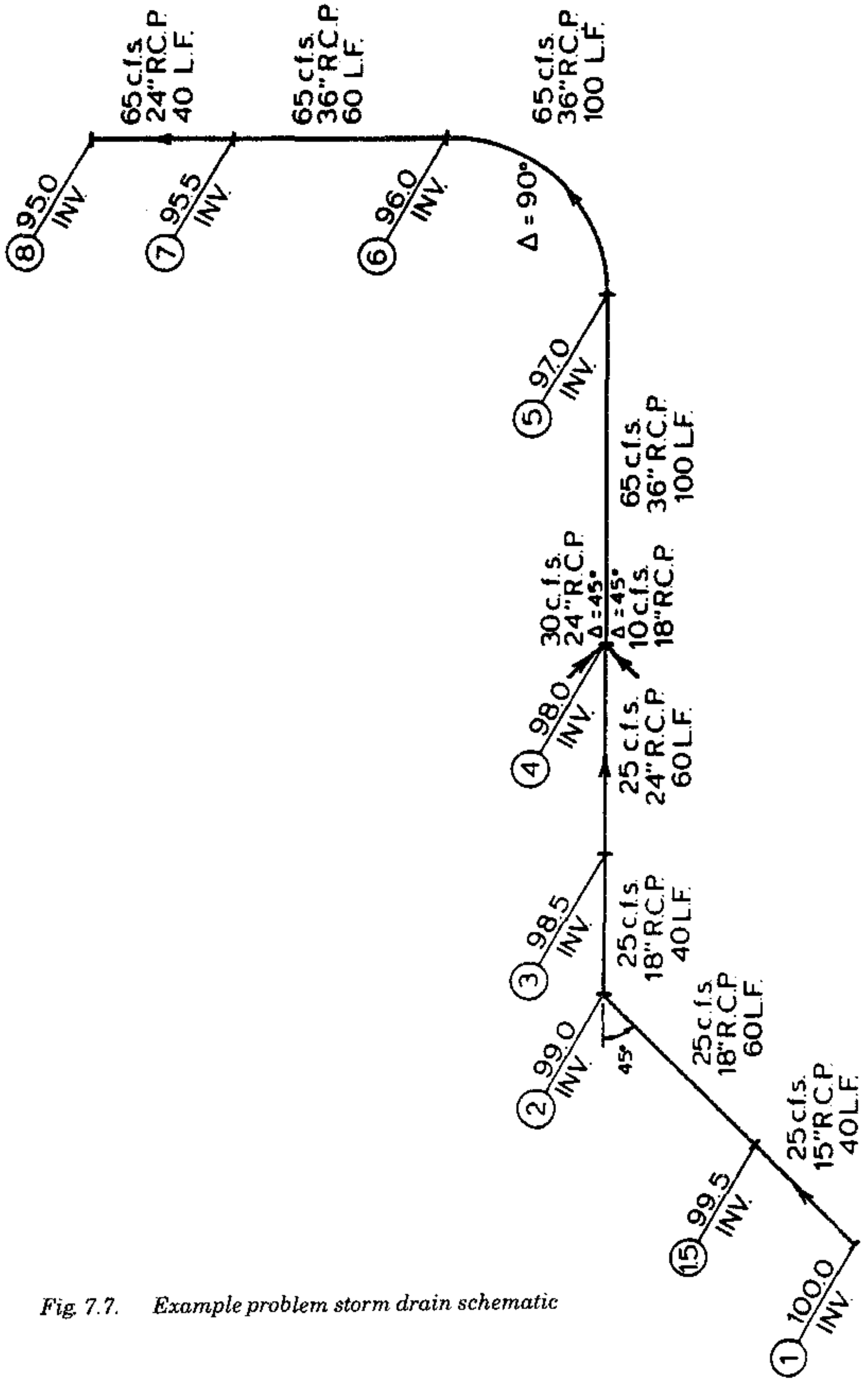

Fig. 7.7. Example problem storm drain schematic

```
*******DESCRIPTION OF RESULTS**************************************************
*                                                                             *
*                             EXAMPLE PROBLEM                                 *
*                                                                             *
*******************************************************************************

*******************************************************************************
  NOTE: STEADY FLOW HYDRAULIC HEAD-LOSS COMPUTATIONS BASED ON THE MOST
  CONSERVATIVE FORMULAE FROM THE CURRENT LACRD,LACFCD, AND OCEMA
  DESIGN MANUALS.

  DOWNSTREAM PRESSURE PIPE FLOW CONTROL DATA:
  NODE NUMBER =      8.00            FLOWLINE ELEVATION =     95.00
  PIPE DIAMETER(INCH) =  24.00       PIPE FLOW(CPS) =        65.00
  ASSUMED DOWNSTREAM CONTROL HGL =     97.000
===============================================================================
  NODE      8.00 : HGL= <   97.000>;EGL= <  103.647>;FLOWLINE= <    95.000>


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                 Advanced Engineering Software [AES]
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                 REV. 2.0      RELEASE DATE: 7/30/82

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===============================================================================
  PRESSURE FLOW PROCESS FROM NODE      8.00 TO NODE      7.00 IS CODE =  1
  UPSTREAM NODE      7.00      ELEVATION =     95.50
-------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW FRICTION LOSSES(LACFCD):
  PIPE FLOW =        65.00 CFS       PIPE DIAMETER =  24.00 INCHES
  PIPE LENGTH =        40.00 FEET       MANNINGS N =  .01300
  SF=(Q/K)**2 = ((      65.00)/(    226.224))**2 =  .0825563
  HF=L*SF = (       40.00)*(  .0825563) =          3.302
  NODE      7.00 : HGL= <  100.302>;EGL= <  106.949>;FLOWLINE= <    95.500>


===============================================================================
  PRESSURE FLOW PROCESS FROM NODE      7.00 TO NODE      7.00 IS CODE =  7
  UPSTREAM NODE      7.00      ELEVATION =     95.50
-------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW SUDDEN PIPE REDUCTION LOSSES(LACRD):
  PIPE FLOW =        65.00 CFS
  UPSTREAM PIPE DIAMETER =  36.00 INCHES
  DOWNSTREAM PIPE DIAMETER =  24.00 INCHES
  PRESSURE FLOW VELOCITY =  20.69 FEET/SEC.
  VELOCITY HEAD =          6.65
  PIPE REDUCTION LOSS COEFFICIENT KC =     .273
  HC(PER LACPCD)=KC*(VELOCITY HEAD) = (   .273)*(        6.65) =   1.817
  NODE      7.00 : HGL= <  107.453>;EGL= <  108.766>;FLOWLINE= <    95.500>

===============================================================================
  PRESSURE FLOW PROCESS FROM NODE      7.00 TO NODE      6.00 IS CODE =  1
  UPSTREAM NODE      6.00      ELEVATION =     96.00
-------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW FRICTION LOSSES(LACFCD):
  PIPE FLOW =        65.00 CFS       PIPE DIAMETER =  36.00 INCHES
  PIPE LENGTH =        60.00 FEET       MANNINGS N =  .01300
  SF=(Q/K)**2 = ((      65.00)/(    666.983))**2 =  .0094972
  HF=L*SF = (       60.00)*(  .0094972) =           .570
  NODE      6.00 : HGL= <  108.023>;EGL= <  109.336>;FLOWLINE= <    96.000>

===============================================================================
  PRESSURE FLOW PROCESS FROM NODE      6.00 TO NODE      5.00 IS CODE =  3
  UPSTREAM NODE      5.00      ELEVATION =     97.00
-------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW PIPE-BEND LOSSES(OCEMA):
  PIPE FLOW =        65.00 CFS       PIPE DIAMETER =  36.00 INCHES
  CENTRAL ANGLE = 90.000 DEGREES
  PIPE LENGTH =        100.00 FEET       MANNINGS N =  .01300
  PRESSURE FLOW AREA =    7.069 SQUARE FEET
  FLOW VELOCITY =     9.20 FEET PER SECOND
  VELOCITY HEAD =  1.313        BEND COEFFICIENT(KB) =  .2500
  HB=KB*(VELOCITY HEAD) = (   .250)*( 1.313) =     .328
  PIPE CONVEYANCE FACTOR =     666.983      FRICTION SLOPE(SF) =    .0094972
  FRICTION LOSSES = L*SF = (      100.00)*(  .0094972) =     .950
  NODE      5.00 : HGL= <  109.301>;EGL= <  110.614>;FLOWLINE= <    97.000>

===============================================================================
  PRESSURE FLOW PROCESS FROM NODE      5.00 TO NODE      5.00 IS CODE =  2
  UPSTREAM NODE      5.00      ELEVATION =     97.00
-------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW MANHOLE LOSSES(LACFCD):
  PIPE FLOW =        65.00 CFS       PIPE DIAMETER =  36.00 INCHES
  PRESSURE FLOW AREA =    7.069 SQUARE FEET
  FLOW VELOCITY =     9.20 FEET PER SECOND
  VELOCITY HEAD =  1.313
  HMN = .05*(VELOCITY HEAD) = .05*( 1.313) =      .066
  NODE      5.00 : HGL= <  109.367>;EGL= <  110.680>;FLOWLINE= <    97.000>

===============================================================================
  PRESSURE FLOW PROCESS FROM NODE      5.00 TO NODE      4.00 IS CODE =  1
  UPSTREAM NODE      4.00      ELEVATION =     98.00
-------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW FRICTION LOSSES(LACFCD):
  PIPE FLOW =        65.00 CFS       PIPE DIAMETER =  36.00 INCHES
  PIPE LENGTH =       100.00 FEET       MANNINGS N =  .01300
  SF=(Q/K)**2 = ((      65.00)/(    666.983))**2 =  .0094972
  HF=L*SF = (      100.00)*(  .0094972) =           .950
  NODE      4.00 : HGL= <  110.317>;EGL= <  111.630>;FLOWLINE= <    98.000>

===============================================================================
  PRESSURE FLOW PROCESS FROM NODE      4.00 TO NODE      4.00 IS CODE =  5
  UPSTREAM NODE      4.00      ELEVATION =     98.00
-------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW JUNCTION LOSSES:
   NO. DISCHARGE  DIAMETER     AREA    VELOCITY     DELTA        HV
    1      25.0     24.00     3.142     7.958        .000       .983
    2      65.0     36.00     7.069     9.196         ---      1.313
    3      10.0     18.00     1.767     5.659      45.000        -
    4      30.0     24.00     3.142     9.549      45.000        -
    5         .0===Q5 EQUALS BASIN INPUT===
```

Fig. 7.8. *Example problem program results*

```
LACFCD AND OCEMA PRESSURE FLOW JUNCTION FORMULAE USED:
DY=(Q2*V2-Q1*V1*COS(DELTA1)-Q3*V3*COS(DELTA3)-
    Q4*V4*COS(DELTA4))/((A1+A2)*16.1)

UPSTREAM MANNINGS N =  .01300
DOWNSTREAM MANNINGS N =  .01300
UPSTREAM FRICTION SLOPE =  .01221
DOWNSTREAM FRICTION SLOPE =  .00950
AVERAGED FRICTION SLOPE IN JUNCTION ASSUMED AS  .01085
JUNCTION LENGTH(FEET) =    5.00      FRICTION LOSS =    .054
ENTRANCE LOSSES =   .000
JUNCTION LOSSES = DY+HV1-HV2+(FRICTION LOSS)+(ENTRANCE LOSSES)
JUNCTION LOSSES =    .950+  .983- 1.313+(  .054)+(  .000) =   .675
NODE       4.00 : HGL= <  111.321>;EGL= <  112.304>;FLOWLINE= <   98.000>

******************************************************************************
  PRESSURE FLOW PROCESS FROM NODE      4.00 TO NODE      3.00 IS CODE =  1
  UPSTREAM NODE      3.00      ELEVATION =     98.50
------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW FRICTION LOSSES(LACFCD):
  PIPE FLOW =       25.00 CFS       PIPE DIAMETER =  24.00 INCHES
  PIPE LENGTH =        60.00 FEET       MANNINGS N =  .01300
  SF=(Q/K)**2 = ((      25.00)/(    226.224))**2 =  .0122125
  HF=L*SF = (       60.00)*(  .0122125) =        .733
  NODE       3.00 : HGL= <  112.054>;EGL= <  113.037>;FLOWLINE= <   98.500>

******************************************************************************
  PRESSURE FLOW PROCESS FROM NODE      3.00 TO NODE      3.00 IS CODE =  4
  UPSTREAM NODE      3.00      ELEVATION =     98.50
------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW SUDDEN PIPE ENLARGEMENT LOSSES(LACRD):
  PIPE FLOW =       25.00 CFS
  DOWNSTREAM PIPE DIAMETER =  24.00 INCHES
  UPSTREAM PIPE DIAMETER =  18.00 INCHES
  UPSTREAM PRESSURE FLOW VELOCITY =  14.15 FEET/SEC.
  DOWNSTREAM PRESSURE FLOW VELOCITY =   7.96 FEET/SEC.
  SUDDEN PIPE-FLOW ENLARGEMENT LOSSES =
  ((V1-V2)**2)/64.4 = (( 14.147-  7.958)**2/64.4 =    .595
  NODE       3.00 : HGL= <  110.524>;EGL= <  113.632>;FLOWLINE= <   98.500>

******************************************************************************
  PRESSURE FLOW PROCESS FROM NODE      3.00 TO NODE      2.00 IS CODE =  1
  UPSTREAM NODE      2.00      ELEVATION =     99.00
------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW FRICTION LOSSES(LACFCD):
  PIPE FLOW =       25.00 CFS       PIPE DIAMETER =  18.00 INCHES
  PIPE LENGTH =        40.00 FEET       MANNINGS N =  .01300
  SF=(Q/K)**2 = ((      25.00)/(    105.043))**2 =  .0566427
  HF=L*SF = (       40.00)*(  .0566427) =       2.266
  NODE       2.00 : HGL= <  112.790>;EGL= <  115.898>;FLOWLINE= <   99.000>

******************************************************************************
  PRESSURE FLOW PROCESS FROM NODE      2.00 TO NODE      2.00 IS CODE =  6
  UPSTREAM NODE      2.00      ELEVATION =     99.00
------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW ANGLE-POINT LOSSES(LACRD):
  PIPE FLOW =       25.00 CFS       PIPE DIAMETER =  18.00 INCHES
  PIPE ANGLE POINT DELTA =  45.00 DEGREES
  PRESSURE FLOW ANGLE-POINT COEFFICIENT KA =  .2340
  PRESSURE FLOW VELOCITY =  14.15 FEET/SEC.
  VELOCITY HEAD =  3.108
  HAPT=KA*(VELOCITY HEAD) = ( .2340)*( 3.108) =    .727
  NODE       2.00 : HGL= <  113.517>;EGL= <  116.625>;FLOWLINE= <   99.000>

******************************************************************************
  PRESSURE FLOW PROCESS FROM NODE      2.00 TO NODE      1.50 IS CODE =  1
  UPSTREAM NODE      1.50      ELEVATION =     99.50
------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW FRICTION LOSSES(LACFCD):
  PIPE FLOW =       25.00 CFS       PIPE DIAMETER =  18.00 INCHES
  PIPE LENGTH =        60.00 FEET       MANNINGS N =  .01300
  SF=(Q/K)**2 = ((      25.00)/(    105.043))**2 =  .0566427
  HF=L*SF = (       60.00)*(  .0566427) =       3.399
  NODE       1.50 : HGL= <  116.916>;EGL= <  120.023>;FLOWLINE= <   99.500>

******************************************************************************
  PRESSURE FLOW PROCESS FROM NODE      1.50 TO NODE      1.50 IS CODE =  9
  UPSTREAM NODE      1.50      ELEVATION =     99.50
------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW TRANSITION LOSSES(LACFCD):
  PRESSURE PIPE FLOW(CFS) =        25.00
  UPSTREAM DIAMETER(INCH) =  15.00      DOWNSTREAM DIAMETER(INCH) =  18.00
  TRANSITION STRUCTURE LENGTH(FEET) =    5.00
  UPSTREAM MANNINGS N =  .0130      DOWNSTREAM MANNINGS N =  .0130
  TOTAL-ANGLE-OF-TRANSITION(DEGREES) =  1.430
  UPSTREAM VELOCITY(FPS) =  20.372    UPSTREAM VELOCITY HEAD =  6.444
  DOWNSTREAM VELOCITY(FPS) =  14.147    DOWNSTREAM VELOCITY HEAD =  3.108
  HT = .2*(V1*V1-V2*V2)/64.4
  TRANSITION LOSS COEFFICIENT =  .2000
  TRANSITION LOSS =    .667
  UPSTREAM FRICTION SLOPE =  .14978
  DOWNSTREAM FRICTION SLOPE =  .05664
  AVERAGE TRANSITION STRUCTURE FRICTION SLOPE ASSUMED =  .10321
  HF = L*SF =    .516
  TOTAL LOSSES = (HT)+(HF) = (  .667)+(  .516)
  NODE       1.50 : HGL= <  114.762>;EGL= <  121.207>;FLOWLINE= <   99.500>

******************************************************************************
  PRESSURE FLOW PROCESS FROM NODE      1.50 TO NODE      1.00 IS CODE =  1
  UPSTREAM NODE      1.00      ELEVATION =    100.00
------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW FRICTION LOSSES(LACFCD):
  PIPE FLOW =       25.00 CFS       PIPE DIAMETER =  15.00 INCHES
  PIPE LENGTH =        40.00 FEET       MANNINGS N =  .01300
  SF=(Q/K)**2 = ((      25.00)/(     64.598))**2 =  .1497765
  HF=L*SF = (       40.00)*(  .1497765) =       5.991
  NODE       1.00 : HGL= <  120.753>;EGL= <  127.198>;FLOWLINE= <  100.000>

******************************************************************************
  PRESSURE FLOW PROCESS FROM NODE      1.00 TO NODE      1.00 IS CODE =  8
  UPSTREAM NODE      1.00      ELEVATION =    100.00
------------------------------------------------------------------------------
  CALCULATE PRESSURE FLOW CATCH BASIN ENTRANCE LOSSES(LACFCD):
  PIPE FLOW(CFS) =        25.00      PIPE DIAMETER(INCH) =  15.00
  PRESSURE FLOW VELOCITY HEAD =  6.444
  CATCH BASIN ENERGY LOSS = .2*(VELOCITY HEAD) = .2*( 6.444) = 1.289
  NODE       1.00 : HGL= <  128.486>;EGL= <  128.486>;FLOWLINE= <  100.000>
******************************************************************************
  END OF PRESSURE FLOW HYDRAULICS PIPE SYSTEM
```

Appendix A - Lag Relationships

		DRAINAGE AREA SQ MI	L MILES	L_{CA} MILES	S FT./MI.	LAG HOURS
1.	SAN GABRIEL RIVER AT SAN GABRIEL DAM NO. 1*	162	23.2	11.6	350	3.3
2.	WEST FORK SAN GABRIEL RIVER AT SAN GABRIEL DAM NO. 2	40.4	9.3	4.2	450	1.6
3.	SANTA ANITA CREEK AT SANTA ANITA DAM	10.8	5.8	2.5	880	1.1
4.	SAN DIMAS CREEK AT SAN DIMAS DAM	16.2	8.6	4.8	440	1.5
5.	EATON WASH AT EATON DAM	9.5	7.3	4.4	800	1.3
6.	MURRIETA CREEK AT TEMECULA	220	27.2	10.3	95	4.0
7.	SANTA CLARA RIVER NEAR SAUGUS	355	38.0	15.8	140	5.4
8.	TEMECULA CREEK AT PAUBA CANYON**	168	26.0	11.3	150	3.7
9.	SANTA MARGARITA RIVER NEAR FALLBROOK	645	46.0	22.0	105	7.3
10.	EAST FULLERTON CREEK AT FULLERTON DAM	3.1	3.2	1.7	140	0.8
11.	TUJUNGA CREEK AT BIG TUJUNGA DAM NO. 1	81.4	15.1	7.3	280	2.5
12.	SANTA MARGARITA RIVER AT YSIDORA	740	81.2	34.3	85	9.3

* EXCLUDES AREA ABOVE SAN GABRIEL DAM NO. 2

** PALOMAR MOUNTAIN PORTION. ENTIRE AREA IS 319 SQUARE MILES, OF WHICH 151 SQUARE MILES DID NOT CONTRIBUTE APPRECIABLE FLOOD FLOWS DURING THE 1937 AND 1940 OCCURRENCES

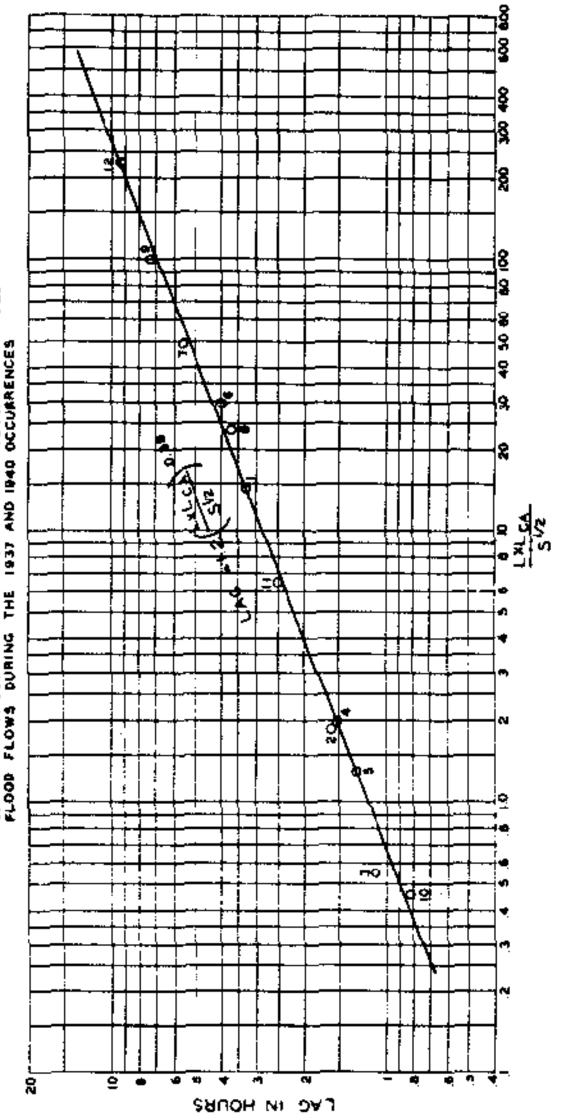
TERMINOLOGY

L = LENGTH OF LONGEST WATERCOURSE.

L_{CA} = LENGTH OF LONGEST WATERCOURSE, MEASURED UPSTREAM, TO POINT OPPOSITE CENTER OF AREA.

S = OVER-ALL SLOPE OF DRAINAGE AREA BETWEEN HEADWATERS AND COLLECTION POINT.

LAG = ELAPSED TIME FROM BEGINNING OF UNIT RAINFALL TO INSTANT THAT SUMMATION HYDROGRAPH REACHES 50% OF ULTIMATE DISCHARGE.



LAG RELATIONSHIP
FOR WATERSHEDS
SOUTHERN CALIFORNIA